

EXAMINING SPATIAL VISUALIZATION IN RELATION TO GEOMETRY

Examining Spatial Visualization in Relation to Geometry

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Abstract

In this thesis, I present a study of how elementary students used spatial visualization while engaging in geometric tasks. In particular, I examined: (a) students' visualization practices when engaging in spatial visualization tasks in geometry, (b) the difficulties students encountered when participating in such tasks, (c) the differences between early, middle and late elementary students in how they used spatial visualization when solving geometric tasks and (d) the embodied resources the students drew upon to engage in spatial visualization when solving the geometric tasks. This research used case study methodology. I interviewed six elementary students and video recorded the interviews. Within the interviews, each student completed six different geometric tasks, with multiple subtasks within. Drawing upon existing research on spatial visualization and embodied theory, I characterized moments when students used spatial visualization and how they used it, paying attention to both their talk and gestures. I identified common problems students had solving each task, and using the previous two steps of analysis, I compared themes across grade levels while considering the characteristics of each case. When analyzing the importance of gestures in my study, I looked at how and when students used gestures. Additionally, I looked at what types of visualization practices were used in conjunction with gestures. From that point, I did an in-depth analysis of two students' gestures and what that meant in terms of whether they were successful or not when completing the geometric tasks. My analysis revealed 17 visualization practices that students used, five of which were used in multiple tasks. Students showed difficulties in tasks where they had to imagine a composition. Additionally, students who used visualization practices that were based on perception had difficulty answering tasks correctly. There were three main differences between early, middle

and late elementary: (a) the youngest student had the lowest success rate, and the oldest student had the highest success rate when completing tasks; (b) older students were able to use a variety of visualization practices, some of which were more advanced than the lower grades; and (c) students with higher success rates tended to use visualization practices that encompassed some form of imagining. The analysis of gestures showed three important aspects: (a) the role of manipulation of tools in shaping the visual plane, (b) the importance of gesture in helping students focus on isolated mathematical properties and their motion, and (c) the importance of gestures that match the motion being visualized. The results of my study suggest possible areas for future research and may allow teachers to create a more comprehensive pedagogy for spatial visualization that is grounded in student thinking.

Résumé

Dans cette thèse, je présente une étude démontrant la façon dont les élèves du primaire ont utilisé la visualisation spatiale tout en s'engageant dans des tâches géométriques. J'ai examiné plus particulièrement: (a) les pratiques de visualisation des élèves lorsqu'ils s'engagent dans des tâches de visualisation spatiale en géométrie, (b) les difficultés rencontrées par les élèves lorsqu'ils participent à de telles tâches, (c) les différences entre les élèves au début du primaire, au milieu et à la fin du primaire, comment ils ont utilisé la visualisation spatiale lors de la résolution de tâches géométriques et (d) les ressources incorporées sur lesquelles les élèves se sont appuyés pour s'engager dans la visualisation spatiale lors de la résolution des tâches géométriques. Cette recherche a utilisé la méthodologie des études de cas. J'ai passé l'entrevue à six élèves du primaire et chacune des entrevues a été enregistrée par une vidéo. Chacun des six élèves était leur propre cas de visualisation spatiale. Au cours des entrevues chaque élève a effectué six tâches géométriques différentes, avec plusieurs sous-tâches à l'intérieur. Je me suis inspirée des recherches existantes sur la visualisation spatiale et sur la théorie incarnée pour m'aider à créer mes questions de recherche, à concevoir mes tâches et à analyser mes résultats. Les données d'entrevues ont servi de données pour l'analyse. Pour examiner les données, j'ai caractérisé les moments où les élèves ont utilisé la visualisation spatiale et comment ils l'ont utilisée (j'ai prêté une attention particulière à leurs discours et à leurs gestes). J'ai identifié les problèmes courants rencontrés par les élèves pour résoudre chaque tâche, et en utilisant les deux étapes précédentes de l'analyse, j'ai comparé les thèmes à tous les niveaux tout en tenant compte des caractéristiques de chaque cas. Des contrastes ont été faits entre les niveaux scolaires et les expériences individuelles afin d'éclairer les nuances dans la progression de la visualisation spatiale de l'élève. Lors de l'analyse de

l'importance des gestes dans mon étude, j'ai examiné comment et quand les élèves utilisaient les gestes. De plus, j'ai regardé quels types de pratiques de visualisation étaient utilisées en conjonction avec les gestes. À partir de là, j'ai fait une analyse approfondie des gestes de deux élèves et de ce que cela signifiait en matière de réussite ou d'échec lors de l'exécution des tâches géométriques. Mon analyse a révélé dix-sept pratiques de visualisation que les élèves ont utilisées, dont cinq ont été utilisées dans plusieurs tâches. Les élèves ont montré des difficultés dans les tâches où ils devaient imaginer une composition. De plus, les élèves qui ont utilisé des pratiques de visualisation basées sur la perception ont eu du mal à répondre correctement aux tâches. Il y avait trois différences principales entre le début du primaire, le milieu et la fin du primaire: (a) le plus jeune élève avait le taux de réussite le plus faible, et l'élève le plus âgé avait le taux de réussite le plus élevé à la fin des tâches; (b) les élèves plus âgés pouvaient utiliser une variété de pratiques de visualisation, dont certaines étaient plus avancées que les classes inférieures; et (c) les élèves ayant des taux de réussite plus élevés avaient tendance à utiliser des pratiques de visualisation qui englobait une certaine forme d'imagination. L'analyse des gestes a montré trois aspects importants: (a) le rôle de la manipulation des outils dans la formation du plan visuel, (b) l'importance du geste pour aider les élèves à se concentrer sur les propriétés mathématiques isolées et leur mouvement, et (c) l'importance de gestes qui correspondent au mouvement visualisé. Les résultats de mon étude suggèrent des domaines possibles pour de futures recherches et pourraient permettre aux enseignants de créer une pédagogie plus complète, fondée sur la réflexion des élèves.

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Contributions of Authors

I am the sole author of this thesis. My supervisor provided guidance and feedback. Below I describe each of our roles in more detail.

Before the study

To start my work with this thesis, I created a thesis outline and obtained approval from my thesis supervisor. I then went through the ethics review process where I obtained approval from McGill to conduct my research. I additionally obtained the approval from the school board where I conducted my research (Lester B Pearson School Board), as well as the principal from the school where the student interviews took place. I created the teacher interviewer booklet (protocol), the student booklet and teacher questionnaire. I obtained written consent from all the guardians/parents of the participants as well as verbal consent from the participants.

Chapter 1

Along with assistance from my supervisor, I created four research questions that would guide my research. I read various literature associated with my topic (Spatial Visualization), and then wrote about its importance and why I decided to study that particular topic. The writing for Chapter 1 was done by me and was edited with suggestions by my supervisor. The search for relevant literature was done by myself and was supplemented with various readings by my supervisor.

Chapter 2

In Chapter 2, my supervisor and I looked into various literature that could be used. My supervisor gave me suggestions on articles that I might find useful to help me substantiate my claims. I was the sole author of Chapter 2 but received constant feedback from my supervisor.

Chapter 3

In Chapter 3, I continued to support my claims with literature that I had read. Additionally, in Chapter 3, I speak to the data collection and data analysis process. I collected all the data on my own. I analyzed the data on my own using various methods and software. I wrote Chapter 3 on my own but received guidance and feedback from my supervisor.

Chapter 4

In Chapter 4, I reported all the results and wrote about the importance of embodied resources. My thesis supervisor read my work and provided comments and feedback on areas that needed more elaboration or improvement.

Chapter 5

In the last Chapter of my thesis, I provided a summary of the results from my research, highlighted the contributions to current research, provided implications for teachers and spoke about limitations and future directions for research. I wrote this Chapter and obtained feedback from my supervisor.

Chapter 1: Introduction

Since I was a child, I have fluctuated between excelling in math and failing miserably, specifically in geometry. I was never good at visualizing the shapes and rotations inside my head. It was through my own failures and insecurities that I was drawn towards conducting research that explored what visualization was, and why it perplexed me. As I explored my insecurities, I realized that the visualization that I struggled with in geometry was actually called spatial visualization.

Spatial visualization is the ability to see objects inside of one's head, mentally rotate those objects, and imagine the outcome of a problem before it is solved (Mulligan, 2015). For example, being able to read a map and then orient yourself in a way to get to your destination is an example of spatial visualization in action. When people read a map, they need to pinpoint where they are currently on the map and then locate where they would like to get to. They have to see the spatial relationship between where they are on the map and where they are in the real world. Once they have oriented themselves within the map, they then need to see their current surroundings and decide to turn right or left, walk straight, etc. Without spatial visualization, people would be unable to navigate their surroundings.

As an elementary school teacher, I see the need for direct support when it comes to building students' spatial visualization, for both geometry learning as well as learning for everyday life. Gravemeijer et al. (2016) believe that geometry, along with the notion of measurement, "build a bridge between everyday reality and mathematics" (p. 8). The researchers speculate that geometry provides the groundwork for understanding spatial aspects of reality.

This means that spatial visualization is not just useful in an abstract manner, but rather, it has real world, explicit uses.

I have observed my own students struggling with various concepts within geometry, such as rotation, translation and transformation. All of these concepts require spatial visualization in order to understand them. Moreover, spatial visualization and reasoning are also necessary components when learning about composition and decomposition of objects. Yet, Quebec's curriculum does not provide guidance to teachers, such as myself, on how to teach spatial visualization or even if we should teach spatial visualization at all. In particular, we have few resources which detail how students tend to engage in spatial visualization. Such resources could provide valuable insight for teachers to design and implement lessons targeting spatial visualization.

Despite the lack of resources for teachers in Quebec, research has shown that spatial visualization skills are not innate but can be taught and developed from an early age (Casey et al., 2008). Block building and puzzle play in children as young as two to four years old, has shown to be effective in increasing spatial visualization (Verdine, Golinkoff, Hirsh-Pasek, Newcombe et al., 2014). Furthermore, a study conducted last year took 327 grade five and six students from ten different elementary schools in rural Australia (Lowrie et al., 2019). The students were randomly selected to go into two groups: an intervention condition group (where the students received six hours of a spatial visualization program) or the control group (where they received no formal spatial visualization instruction). The results showed that students who received six hours of the spatial visualization program not only had higher scores on spatial visualization tasks than their peers in the control group, but their overall performance in general

mathematics increased and even bypassed their control group peers. Intervention programs like the one stated above would be particularly beneficial for students since children's minds are thought to be more malleable than adults, thus resulting in greater spatial visualization skills throughout their entire lives (Uttal et al., 2013).

Since research has shown that spatial visualization skills are not only malleable but can be taught at a young age, these skills need to be added to the curriculum in hopes to increase students' understanding and future success in mathematics and beyond. However, such changes require better understanding of how children engage in spatial visualization in the elementary grades. This has motivated me to conduct research to look further at spatial visualization and how students use it in school. In this thesis, I present a study of elementary students' spatial visualization in geometry. In this introductory chapter, I begin by describing the importance of spatial visualization and why we should care about it. I then speak to the relationship between spatial visualization and geometry. Finally, I will touch upon spatial visualization in schools, or the lack thereof.

Importance of Spatial Visualization

Spatial visualization is a skill that is necessary for many parts of life, but it is remarkably important for success in school mathematics (Ontario Ministry of Education. (n.d.); Battista, 1990; Mix & Cheng, 2012). One study indicated that spatial thinking was a better predictor of later mathematics success than either verbal or mathematical skills (Ontario Ministry of Education. (n.d.)). That means that students who have strong skills in mathematical areas such as arithmetic, word problems, measurement, geometry, algebra and calculus, but do not have strong spatial skills, would not necessarily be successful in mathematics in the future.

Moreover, students' spatial ability can impact their *current* mathematical performance (Cheng & Mix, 2014; Verdine, Golinkoff, Hirsh-Pasek, Newcombe et al., 2014). For example, various studies have found that children who performed better on spatial tasks also performed better on counting tasks, number line estimation and nonverbal problem solving (Cheng & Mix, 2014; Kyttälä et al., 2003; Geary et al., 2007, Rasmussen & Bisanz, 2005). In 1990, Battista conducted a study with 145 high school students. These students were administered an exam that tested spatial visualization, logical reasoning, knowledge of geometry and geometric problem solving. These four areas were tested using various tests/tasks specific to each area. The results were compounded, and Battista found that mathematical achievement was directly related to spatial visualization. Battista's study and others' research collectively point to a strong relation between mathematics achievement and spatial visualization. Mix & Cheng (2012) captured this well: The link between spatial ability and mathematics is so strongly entrenched that there is no longer any point to ask whether or not the two are related.

Spatial visualization is also important in predicting future career success. Increasing evidence supports the idea that spatial visualization and reasoning contributes not only to success in school mathematics but also in subsequent careers related to science, technology, engineering and mathematics (STEM) (Buckley et al., 2019; Linn & Peterson, 1985; Mulligan, 2015; Mulligan et al., 2018; Verdine, Golinkoff, Hirsh-Pasek & Newcombe, 2014; Tosto et al., 2014; Kell et al., 2013; Young et al., 2018). For example, one particular study, conducted with 400,000 students, spanning over 11 years, found that the students who had higher spatial visualization skills in high school later reported being employed in careers within the fields of physical sciences, math, computer science and engineering (Wai et al., 2009). Additionally, in a separate

study, people who were employed in careers within geosciences and geography reported an increased ability to navigate their environments, compared to people working in other occupations (Newcombe, 2013).

In addition to spatial visualization being salient for mathematics achievement as well as future career success, thinking spatially allows people to navigate within our three-dimensional physical world (Ontario Ministry of Education. (n.d.)). For example, navigating within our world could mean using a mall directory to physically find a store. Similarly, it could mean figuring out how to cut a piece of pie so that everyone gets an even piece. Equally, navigation might mean being able to follow pictorial directions on how to assemble furniture from Ikea. On a daily basis, numerous demands are placed upon us to visualize something before we know the outcome (i.e., how we are going to pack all the clothes needed into a suitcase), and to mentally problem solve. It is spatial skills that allow us to be able to meet these requirements. With no formal instruction surrounding spatial skills, students will leave school unable to adequately respond to these demands.

Further to the reasons above, educational policy efforts outside of Quebec have begun emphasizing the importance of focusing on spatial visualization. Most notably, in 2006, the National Research Council in Ontario placed a call for action so that spatial thinking would be recognized for its importance. They postulated that spatial thinking was not only important in mathematics but also across subject areas. Added stress was put on researchers in education, as well as educational leaders, to better develop their understanding of spatial thinking and create supports to foster spatial literacy in students (Ontario Ministry of Education. (n.d.)).

Relationship between Geometry and Spatial Visualization

In ancient Greek times, geometry was the main focus of mathematics (Sinclair & Bruce, 2015). However, in today's society, geometry is much more than a strand of mathematics. As Gravemeijer et al. (2016) describe, geometry is all around us, right from the time we are born. The researchers believe that geometry is present when babies are given a ball to play with or a cone-shaped glass to drink from. The baby learns about spaces, both abstract and physical (their confined space of a crib and then eventually larger spaces like a room, the house, the neighborhood, the city, etc.). Geometry is used when building homes and creating objects. Geometry plays a large role in our lives, not just in the mathematical sense.

Beyond our everyday experiences, geometry is recognized as an important part of primary and secondary school education (Boakes, 2009). Geometry includes notions and concepts about shapes (different forms), space (in the sense of the area surrounding our environment and the physical and abstract space between objects), composition (how shapes can be combined to create other shapes), decomposition (how shapes can be broken down into other shapes), nets (two-dimensional shapes that can be folded to create a three-dimensional shape), rotations (turning objects) and perspectives (different points of view), to name a few (these will be explored in more detail in Chapter 3). Within elementary and high school education, geometry allows students to: (a) visualize, draw and construct figures; (b) study the spatial aspects of the physical world; (c) represent nonvisual mathematical concepts and relationships; and (d) understand mathematics as a system (Sinclair & Bruce, 2015).

A key aspect of engaging in geometry in both school and everyday mathematics is visualization. In fact, research has shown links between students' spatial visualization and their

ability to perform geometric tasks (Battista, 1990; Boakes, 2009; Pittalis & Christou, 2010). Pittalis & Christou, for example, conducted a study with 269 students from elementary and secondary schools in Cyprus. In this study, the students were given two tests. The first test measured students' three-dimensional geometry thinking and the second test measured students' spatial abilities – defined as being able to observe, construct and mentally deal with three-dimensional objects. The results showed that students who had strong spatial abilities performed better when reasoning about geometry compared to their peers with low spatial abilities (Pittalis & Christou, 2010). Moreover, Boakes (2009) similarly found in his study with 56 seventh-grade students that spatial visualization skills were necessary to be successful in geometry. This was concluded after conducting a pretest with all the participants, then having the students in the test group engage in daily 20-minute lessons focusing on spatial visualization skills. Afterwards, both the test group who had the extra lessons and the control group, who had no further instruction, took a final test to gauge the students' spatial visualization skills. The results show that the students who had extra training did better on the tests and allowed Boakes to conclude that spatial visualization was an essential part of geometric thought (Boakes, 2009).

A large part of using spatial visualization in geometry is being able to visualize, draw and construct objects. However, at times it may be difficult for students to complete such tasks. Evidence suggests that gestures play a large role in spatial understanding (Ehrlich et al., 2006) and that by paying close attention to the gestures that students use when explaining their answers, we can gain greater insight into their spatial skills. For example, one study showed that the more students gestured when they were asked to explain how they solved a spatial transformation task, the better they performed (Ehrlich et al., 2006). In geometry in particular,

Alibali and colleagues postulated that it is difficult for students to describe an irregular shape when they are just speaking about it, but it becomes easier to describe the shape when using gestures (Alibali et al., 1999). Thus, it is important to consider the role of gestures when understanding how students engage in spatial visualization in geometry.

Spatial Visualization in Schools

Over numerous centuries, students have struggled with the many complex notions of mathematics. With technological advances and increased knowledge and resources, one would postulate that math achievement would increase. However, over the past 15 years, almost all Canadian provinces showed significant declines in international math scores (Stokke, 2015). In 2015, Canadian students scored 48% in mathematical reasoning, compared to students in Singapore who scored 90% in the same category (Mullis et al., 2016). These statistics represent an urgent need for increased attention and research in mathematics. An area of mathematics that requires additional research is in geometry, specifically with regards to spatial visualization.

Seeing as spatial visualization is such an important part of learning and knowledge, especially for STEM subjects, it is surprising that such little emphasis is given in schools. For example, results from a recent study shows that in over 50 years of examining spatial reasoning, there was very little implementation of spatial visualization within the curriculum, and even less instruction of spatial visualization in educational settings, even in STEM areas, where spatial visualization is considered to be extremely relevant (Van den Heuvel-Panhuizen et al., 2015; Wai et al., 2009).

This is particularly true in our own country, Canada. After inspection of the different curricula across the country, only Ontario and Prince Edward Island (P.E.I), place a great

importance on spatial visualization and its need to be fostered in the school setting. In provinces like Alberta, British Columbia, Saskatchewan, Manitoba, New Brunswick, Nova Scotia, Newfoundland, Yukon and the Northwest Territories (NWT), which all use the same mathematical framework, visualization is only mentioned in reference to the importance of seeing two-dimensional and three-dimensional objects (The Common Curriculum Framework, 2006). They do, however, speak to the importance of spatial sense. Quebec is the only province where there is no explicit emphasis on spatial visualization whatsoever.

Among the provinces that do include spatial visualization, they approach the topic differently. For example, the Alberta curriculum (along with the other provinces who share the same framework) sees spatial sense as a students' ability to make predictions and communicate in an effective manner concerning shapes, objects and their own representations (Alberta Ministry of Education, 2016). Alberta also believes that spatial sense can be developed through various experiences and interactions within the school environment and that it would lead to a better understanding and appreciation of two-dimensional and three-dimensional shapes in addition to being able to interpret and reflect on one's surrounding physical environment (Alberta Ministry of Education, 2016).

In comparison, the province of Prince Edward Island (P.E.I) believes that visualization is an important component of the mathematics curriculum from the beginning of elementary through to graduation. They believe that visualization is where the students are able to think "in pictures and images and [have] the ability to perceive, transform, and recreate different aspects of the visual-spatial world" (Prince Edward Island Ministry of Education, 2015, p. 7). The P.E.I curriculum writers postulate that the use of visualization within the study of mathematics

provides different opportunities for students to understand and make connections to different mathematical concepts and that the use of visual images and visual reasoning are important aspects of “number, spatial, and measurement sense” (p. 7). This province explicitly speaks to spatial visualization and its importance when describing relationships between objects and space (Prince Edward Island Ministry of Education, 2015).

In contrast to P.E.I, curriculum writers in the province of Ontario are so concerned about spatial visualization, that they produced a 28-page support document to be used alongside their curriculum. The province looks at spatial visualization as not only placing oneself in a place and time and being able to mentally and physically move spatially throughout that specific environment, but rather it sees spatial visualization as a complex process made up of considerable amounts of concepts and tools that are necessary to living in our world (Ontario Ministry of Education. (n.d.).) In this support document, Ontario stresses that spatial visualization is not just used in geometry (e.g., when we physically or mentally combine shapes to make new shapes or take shapes apart, or when we scale objects to make them proportionally bigger or smaller) but rather it is involved in perspective-taking, creating and designing all kinds of objects, shifting dimensions (two-dimensional and three-dimensional), diagramming (flow charts and renderings of drawings), locating objects, using non-verbal reasoning, orienting, navigating and much more (Ontario Ministry of Education. (n.d.)). Ontario acknowledges that spatial thinking plays an important role from kindergarten to high school graduation, and not just in mathematics: Regardless of subject matter, spatial skills and abilities are important. For example, in high school when students are learning about molecules they need to think about the spatial structure of those molecules. In physical education class, the students need to use

proprioception (an awareness of where their body is in space and relation to other objects).

Within the arts, whether that be drama, music or visual arts, spatial skills are required to manipulate forms, and create musical notes.

Research Gaps and Objectives

The vast differences of the importance placed upon spatial visualization, just within our country, is cause for concern. As noted earlier, literature suggests that students' poor understanding of geometry and limited spatial ability poses a threat not only for fluency and mastery of geometric problems, but for other topics within the domain of mathematics, in addition to other subjects as well (Clements & Sarama, 2011). This existing research on spatial reasoning has mostly focused on the relationship between spatial reasoning and visualization as well as math achievement and future career success (Mulligan et al., 2018). However, there is a lack of research on the nature of students' spatial visualization, including how they engage in spatial visualization and how this engagement varies across grade levels. In addition, there is limited research on how gestures provide insightful information surrounding students' spatial thinking. Looking at how students use spatial visualization when completing geometric tasks can provide insight into students' difficulties/ weaknesses. By understanding the ways that students engage with spatial visualization, we can provide teachers with guidance for development of curricula and teaching materials. The limited research on students' use of spatial visualization, its progression, as well as the accompanying gestures when completing spatial tasks, has led me to the following research questions.

This study aims to understand how elementary students engage with spatial visualization when completing geometric tasks. Specifically, my research will explore the following

questions: (RQ1) What difficulties do elementary students encounter when undertaking spatial visualization tasks in geometry? (RQ2) What is the nature of students' spatial visualization when solving geometric tasks? In particular, I was interested in understanding the types of *visualization practices* that students engaged in when completing such tasks. Additionally, in order to illuminate the progression of students' spatial visualization, I will ask: (RQ3) What are the differences between early, middle and late elementary students, in how they use spatial visualization when solving geometric tasks? Lastly, I will ask: (RQ4) What embodied resources do students draw upon to engage in spatial visualization when solving geometric tasks?

Overview of Thesis

I present this thesis over five chapters. Here in Chapter 1, in order to motivate my research, I briefly outlined the importance of spatial visualization, the link it has to geometry and how it is approached in Canadian curricula. Additionally, I introduced the research objectives that allowed this research to take place. In Chapter 2, I will provide a review of the literature in spatial visualization in order to outline how I will conceptualize it in my research. I will also discuss the theoretical perspective guiding this thesis, namely Embodied Cognition and why it is important in my research. In Chapter 3, I will highlight my methodological framework, in this case, Case Study Methodology, and describe my setting, participants, and data collection and analysis methods. Chapter 4 encompasses the results of the study and Chapter 5 looks at the various contributions my research has made, implications for the teaching of spatial visualization, and limitations and further directions for research.

Chapter 2: Conceptual Framework

In this chapter, I begin by providing insight into how spatial visualization has been characterized. Furthermore, I will highlight various research that has been conducted regarding children's spatial visualization and end with embodied theory (the theoretical perspective that has helped guide me in this thesis).

Characterizing Spatial Visualization

In this section, I will speak to the different terms used when describing spatial visualization. Additionally, I will delve deeper into what spatial visualization means and how I have conceptualized it for my study.

Spatial Visualization as a Field

Within the field of spatial visualization, many different terms arise, terms such as spatial ability, spatial thinking, spatial skills, and spatial reasoning. All of these terms are aspects of spatial visualization. Mulligan and colleagues (2018) found that these terms are intertwined and are often used interchangeably. Some researchers might add a component that differs slightly from another researcher or perhaps place more emphasis on one component, but in general there is very little distinction between the terms. Across the different terms, the aspects that are constant are: (a) the ability to mentally manipulate objects, (b) the spatial relationship between objects and (c) abstract thought (Bruce & Hawes, 2015; Hawes et al., 2015; Mulligan, 2015; Mulligan et al., 2018; Newcombe, 2013; Sack & Vazquez, 2016; Shumway, 2013; Uttal et al., 2013; Verdine, Golinkoff, Hirsh-Pasek & Newcombe, 2014; Verdine, Golinkoff, Hirsh-Pasek, Newcombe et al., 2014; Wai et al., 2009). The ability to mentally manipulate objects means that

people are able to see a shape/form/object inside of their mind and then manipulate it in some way (e.g., make it bigger or smaller and/or moving it to another location) (Fennema & Tarte, 1985; Mulligan et al., 2018). Spatial relationships between objects refers to the relationship between the space that encompasses and separates different objects and how that relationship is formed (Mulligan, 2015). In this study, abstract thought refers to the ability to think about shapes, forms and ideas that are not physically present. These terms will be elaborated on more in the next section.

Although the terms are viewed as interchangeable, some research has shown slight differences when defining them (Whiteley et al., 2015). The most notable difference pertains to the term spatial reasoning. Indeed, spatial reasoning is similar to the aforementioned terms, in such that it conceptualizes spatialization as involving the same components described above: manipulations of mental representations, spatial relations, and abstract thought (NCTM, 2000; Bruce & Hawes, 2015; Arcavi, 2013; Wai et al., 2009; Khine, 2017; Mulligan, 2015; Verdine, Golinkoff, Hirsh-Pasek, Newcombe et al., 2014; Sack & Vazquez, 2016). However, spatial reasoning places a greater emphasis on the “reasoning” aspect. That is, it is not just about mental manipulation but also about the *thinking processes* people undergo to understand the relationship between oneself, and the world around them (Whiteley et al., 2015). Spatial reasoning is the *process* of: (a) *making sense of* what can be mentally manipulated and (b) *creating connections between* the visual representations and abstract thought. For example, when a child puts a puzzle together, they must engage in thinking processes to make sense of how the pieces can be mentally manipulated. They need to think about how the pieces can be rotated or rearranged to fit together. In doing so, they must make sense of which pieces can be moved or turned and how.

An example of creating connections between visual representations and abstract thought is when a child tries to figure out which path is the fastest to take when getting from home to school. The child must imagine the different ways (visual representation) that can be taken to get to their destination. The abstract thought comes into play when thinking about how much time each path takes. In their thinking process, the child must make connections between the visual representation (the different paths) and how they know which path is faster or slower.

Spatial ability, spatial skills, spatial thinking, and spatial reasoning are all part of spatial visualization. Within this thesis, I am using the term spatial visualization because it encompasses the ideas of spatialization from the other terms, but also highlights how people “see”. Since mental manipulation is part of all of the terms, I chose spatial visualization because I wanted to see how students were able to visualize the objects and their corresponding rotations and transformations. I wanted to see if students were able to imagine outcomes before they knew the answers. Montello and colleagues (2014) believed that spatial visualization helps “make visible the kinds of processes and relationships that are normally unobservable to the naked eye” (p. 99). This is what I wanted to focus on in my research.

Aspects of Spatial Visualization

Spatial visualization has been defined by many researchers in various ways but the definition that I chose to work with comes from Mulligan’s (2015) idea that spatial visualization is the ability to see objects inside of one’s head, mentally manipulate those objects, and imagine the outcome of a problem before it was solved. Additionally, I am working with the notion that spatial relationships between objects is essential to spatial visualization (Bruce & Hawes, 2015; Mulligan, 2015). If a person engages in any one of these aspects, they are engaging in spatial

visualization. Although these aspects are inter-related and often occur simultaneously, in the paragraphs to follow, I will speak more in-depth to each of these different components of spatial visualization (seeing objects inside one's head, mental manipulation of objects, imagining the outcome of a problem before it is solved, and spatial relationships).

Seeing Objects Inside of One's Mind. One of the main components of spatial visualization is being able to *mentally envision objects* (Wai et al., 2009). This means that when an object or shape is presented to someone, they are able to close their eyes and see that image clearly in their mind. When a person is engaged in mentally envisioning objects that they had been previously shown, they are utilizing their spatial reasoning process of making connections between visual representations and abstract thought. This is done when a person uses the memory of the object (abstract thought) to create the visual representation in their mind (which is the image that they see).

Mental Manipulation of Objects. The second component of spatial visualization is being able to *mentally manipulate envisioned objects or shapes* (Hawes et al., 2015). Manipulations of an object or shape can take on many different meanings. To manipulate something means the object/shape can be rotated (turned), it can be transformed (made bigger, smaller), it can include a composition (it can be added to another object) or a decomposition (it can be taken away from an object). When mentally manipulating an object, a person is engaged in the spatial reasoning process of making sense of what can be mentally manipulated and making connections between visual representations and abstract thought. An example of a mental manipulation is when some is putting together furniture. That person manipulates the various parts inside their mind before trying to piece them together physically. The pieces of

furniture are not physically present inside one's mind, therefore abstract thought is needed in creating those images (the visual representations), and then manipulating them.

Imagining an Outcome. Another component of spatial visualization that I will draw upon in my thesis is *imagining an outcome*. This is where a person is able to imagine the outcome of a problem (mathematical or not) before it is solved. When engaging in predicting an outcome, people must engage the spatial reasoning process of making sense of what might be mentally manipulated. For example, imaging an outcome occurs when someone first imagines how to pack all of their clothes in their suitcase for vacation before placing them in the suitcase. Alternatively, a student imagines an outcome when they imagine what solid a two-dimensional net would make in geometry when it is folded in three-dimensional space. This example utilizes abstract thought to help the person navigate the shape in space.

Spatial Relationships. The last component of spatial visualization is the spatial relationships between objects (Verdine, Golinkoff, Hirsh-Pasek & Newcombe, 2014). Spatial relationships speak to how objects are connected or separated in space and what that space means. When people think about the spatial relationships between objects, they are partaking in the process of making connections between visual representations and abstract thought. The space between objects can be physical (a person can see two objects on a table and they can physically measure the distance between the two objects), or it can be abstract (a person is imagining two objects in their head but they are separated by an immeasurable distance).

All of these aspects of spatial visualization strengthened my resolve that it was a necessary component when undertaking not only geometric tasks, but tasks involved in any subject area where you needed to think abstractly about something (art, chemistry, biology, etc.).

In my thesis, I am especially interested in the *visualization practices* that students use when undertaking geometric tasks. Visualization practices refer to the different ways that students use spatial visualization when completing geometric tasks. This is evidenced by speech, gestures and the processes that students use. These practices involve engaging in the common aspects that we have seen previously (mental manipulations, spatial relationships, abstract thought) and the spatial reasoning processes that one undertakes when completing geometric tasks.

Research on Children's Spatial Visualization

Spatial reasoning and thinking (components of spatial visualization) have been studied extensively within the domain of psychology; however, there is little research that has been conducted surrounding spatial thinking and how students use it (Hawes et al., 2015; Wai et al., 2009). The literature on children's spatial visualization mostly focuses on the connection between significant mathematical achievement and high spatial ability, as well as how spatial skills are malleable and can be taught (Casey et al., 2008; Verdine, Golinkoff, Hirsh-Pasek & Newcombe, 2014; Uttal et al., 2013, Lowrie et al., 2019). Although there is little research on how students' engage in spatial visualization, some research has characterized types of spatial thinking and skills used in spatial visualization. This section of my thesis will explore various ways researchers have characterized students' spatial visualization. As I elaborate on in the following sections, these include: (a) perceptual versus property, and (b) intrinsic/extrinsic to static/dynamic skills

Perceptual vs Property Focused

One way that researchers have characterized spatial visualization is by whether students focus on visual perceptions versus mathematical properties (Lehrer et al., 1998). Children who

focus on visual perceptions tend to focus solely on what they see holistically and exclude mathematical characteristics (Lehrer et al., 1998; Uttal et al., 2013). In doing so, they often draw upon analogies based on their own experiences. For example, if a child who is focused on visual perceptions saw a sphere, they might describe it as a ball, something they get to play with. The child does not yet attend to the fact that the sphere is a three-dimensional representation of the two-dimensional circle and that it holds specific properties, such as the diameter is twice the size of the radius.

Alternatively, children who focus on mathematical properties tend to see and are able to describe characteristics that are key to defining an object. For example, a rhombus and a square look very similar. When presented with a rhombus, the child whose focus is visual perception might say that the rhombus is a square. The child whose focus is mathematical properties would instead say that it could not be a square because squares have four right angles with equal measures.

The basis of perceptual and property-based skills comes from the Van Hiele theory (Alibous, 2016). Van Hiele found that children were first drawn to the overall appearance of shapes and forms. Based on their theory, children tend to focus on what something looked like and ignore the properties that were associated with it. Only later with proper instruction do children gradually understand that shapes have specific properties (like the number of sides or the measurements of its angle(s)) (Lehrer et al., 1998). I drew upon these distinctions in analyzing my data, as described in more detail in the Methodology chapter.

Intrinsic/Extrinsic to Static/Dynamic Skills

Another way that researchers have characterized students' spatial visualization is through students' static and dynamic spatial skills (Uttal et al., 2013). These two ways of thinking are considered essential dimensions of spatial reasoning (Okamoto et al., 2015; Uttal et al., 2013). For this thesis, I defined static to mean that the object or frame of reference that a student is dealing with remains motionless (Okamoto et al., 2015). For example, a student is looking at a pattern block on a desk (the pattern block is not moving, nor is the desk). On the contrary, dynamic would mean that the object or perspective that a student is working with is transformed or has been moved. Dynamic spatial skills allow people to mentally and physically transform objects through rotation, folding and bending (Okamoto et al., 2015). Dynamic skills tend to be studied more compared to static skills, because dynamic skills are often associated with STEM careers (Okamoto et al., 2015; Froese et al., 2013). For example, a child may be shown a figurine and then told to move to the other side of the room and asked if the figurine looks the same. Here, the child was first looking at the figurine from the front but then was viewing it from the side or the back, changing the perspective.

Moreover, when thinking about static and dynamic skills, *intrinsic* and *extrinsic* dimensions can be applied. The word intrinsic is used when one's visualization focuses on defining or describing an object (Okamoto et al., 2015). For example, the intrinsic dimension for a ball would be to describe its size or color (specifications of the ball are given). Conversely, extrinsic means focusing on "the relation[s] among objects or [the] relation of [the] object to a [specific] frame of reference" (Okamoto et al., 2015, p. 16). For example, visualizing the

extrinsic dimension when using that same ball would occur when it is in relation to another object or when it is in a given space (is it on a table, on the floor, in a corner).

Within the category of static spatial skills there are two distinctions: *Intrinsic-static* and *Extrinsic-static*. Intrinsic-static means that people are able to distinguish objects within a distracting but immovable situation/frame of reference (Okamoto et al., 2015). Intrinsic-static skills are commonly associated with interior designers, painters, and scientists (Kozhevnikov et al., 2005; Mix & Cheng, 2012). This is because these professions tend to be able to recognize, describe and classify spatial qualities of an object and the relation of parts within that object (Okamoto et al., 2015). For example, an interior designer is able to look at the physical space in a room and then imagine what types of objects could be placed where. For children, intrinsic-static skills mean they can identify two-dimensional and three-dimensional geometric shapes within other shapes or on their own but with distractors present (Okamoto et al., 2015). For example, Clements and colleagues (1999) conducted a study with 97 preschool and elementary students in which they asked the students to look at a paper that had different geometric shapes and to classify them. The results found that all students were able to identify circles even with distractors present because they relied on visual cues to help them – illustrating their use of intrinsic-static skills.

Extrinsic-static spatial skills are present when one can see the relationship between objects or a frame of reference. For example, such skills are used when someone is looking at a map and planning which path to take to get to their destination (Okamoto et al., 2015). The map is not moving, but the person looking at the map has to see the relationship between where they are starting and where they want to end up. A larger amount of research has been placed on

extrinsic-static skills because they are believed to be the easiest to teach in an educational setting (Uttal et al., 2013). Often, extrinsic-static spatial skills are assessed in children using maps, models and tasks that alter images from straight to vertical. Studies suggest that extrinsic-static skills improve with age, and a deeper understanding of one's place in relation to their surroundings (Okamoto et al., 2015).

Similar to static skills, dynamic skills are also divided into intrinsic and extrinsic skills. *Intrinsic-dynamic skills* allow people to alter or mentally transform objects (Okamoto et al., 2015). For example, an intrinsic-dynamic task would be to show an image of a small person to a child and then provide three choices of what it would look like if it were rotated (Okamoto et al., 2015). In this situation, the child would be able to see the original person in their mind and rotate them. Children's intrinsic-dynamic skills are often studied through block design and paper folding tasks (Okamoto et al., 2015). These skills are thought to be the most teachable to children, and therefore studied the most (Uttal et al., 2013).

Extrinsic-dynamic skills allow people to see the changes in spatial relations between two or more objects as well as between one's own body and objects in their environment (Okamoto et al., 2015). Researchers characterizing extrinsic-dynamic skills have broken that term down to encompass two additional types of skills, *object-to-object navigation* and *self-to-object navigation*. Object-to-object navigation is where a student needs to update the spatial information they had because one or more objects are now in or were in motion (Okamoto et al., 2015). Object-to-object navigation is a multi-step process. For example, if a child's toy was hidden in one place but then has been moved and is now hidden somewhere else, the child needs to understand that the object has moved and update their spatial information in order to find the

object in its new hiding place. Self-to-object navigation occurs when the child is able to understand their position in relation to a specific object or place. For example, a child might use this type of navigation when they know that there are different routes to take to get to their friend's house based on whether their point of departure is home, their grandparents house or school. These additions allow for researchers to study how children situate themselves in space in addition to perspective. For example, Moll et al. (2013) found that children as young as three years old could identify other people's visual perspectives, not just their own.

The four distinctions between spatial skills – intrinsic-static, extrinsic-static, intrinsic-dynamic, extrinsic-dynamic – helped me to select the tasks that I used when interviewing students. These four distinctions allowed me to adapt/create tasks that targeted different spatial skills.

Embodied Cognition

In this section, I am going to speak to the theoretical perspective that I chose to guide my research, which is embodied theory. I will outline what embodied cognition/theory is and why it is important for understanding spatial visualization.

Embodied cognition or embodied theory is the idea that the mind and the body are not two separate beings but rather one essence that works in tandem to learn and reason about phenomena (Abrahamson & Bakker, 2016). When using embodied theory to examine students' thinking, researchers look at how students' physical actions represent their mental reasoning processes (Abrahamson, 2017). Within the constructs of this study, students' physical actions are the gestures that they make in addition to their speech when they are explaining how they solved

a task. I am using the word gesture to refer to hand movements that are directly linked to speech (Goldin-Meadow, 2003; Sweetser, 2004)).

Alibali and colleagues (1999) found that gestures can reveal strategies used when problem solving that are not notably expressed in speech. This was also speculated by Goldin-Meadow (2003) when she suggested that the gestures that are used when someone is speaking are elemental to both speech and a person's thought process. This means that by examining the gestures that students use, in conjunction with their speech, it can provide a greater understanding of students' mental reasoning. The same study done by Alibali et al. (1999) found that people will often omit information that they find difficult to verbalize (such as information pertaining to images). This is especially useful in my own research since I am asking students to look at a variety of visual arrangements and then explain how they came up with their answers. As Alibali et al. summarized, "speech and gesture together provided a more complete preview of solution strategies than speech alone" (p. 331). These notions will help me to answer my fourth research question (RQ4): What embodied resources do students draw upon to engage in spatial visualization when solving geometric tasks?

As evidence has shown, speech and gestures, when taken together, give a more accurate explanation of what someone is trying to express (Ehrlich et al., 2006; Hostetter & Alibali, 2008). According to Hostetter & Alibali (2008), mental imagery (which is an important aspect of spatial visualization) is an embodied process. As the researchers explain, "successful verbal recall is strongly associated with how easy a concept is to visualize... [and] visual mental images are tied to simulated movement" (p. 499). This means that if students are able to imagine something mentally, it will be easier for them to speak and gesture about it.

Furthermore, research in mathematics education demonstrates that students' actions (in this case their gestures) can influence how they think (Tran, Smith & Buschkuehl, 2017; Moeller et al., 2012; Fischer & Brugger., 2011). According to Trans, Smith & Buschkuehl (2017), mathematics automatically connects students' perceptions and their actions, thus establishing embodied theory in the study of mathematics as "especially helpful in understanding its abstract, complex nature" (p. 1). For example, when young children learn to count and solve arithmetic problems, it is common to see them use their fingers (also known as embodied numerosity (Moeller et al., 2012)). Research has shown that gestures and other bodily movement has been used by children to reason about mathematical ideas in topics ranging from geometry (Hostetter & Alibali, 2008; Ehrlich et al, 2006), arithmetic (Moeller et al., 2012), and measurement (Gravemeijer et al.,2016).

Moreover, research indicates that there is a strong link between gesture and spatial visualization (Hostetter & Alibali, 2008; Ehrlich et al., 2006). For example, one study analyzed the test results of spatial tasks given to 80 five-year-old students. The results found that children who performed better on the spatial transformation task often referred to movement in their gestures and not in their speech (Ehrlich et al., 2006).

As seen throughout the studies, gestures can be a very important component when trying to understand what someone is explaining, and especially what someone is visualizing. This is why I decided to use embodied cognition in my own research. This theory is important to my work because it will provide a deeper understanding of students' visual thinking, rather than relying solely on their verbal contributions.

Summary of Chapter 2

In this chapter, I highlighted various ways that spatial visualization, in conjunction with other spatial terms, have been defined by researchers. I used this to define my own conceptualization of spatial visualization as consisting of four aspects: (a) seeing objects inside one's head, (b) mental manipulation of objects, (c) imagining the outcome of a problem before it is solved, and (d) spatial relationships. Additionally, I spoke to what research has revealed regarding children and spatial visualization. This research provided me insight into two conceptions that, as I describe in the next chapter, I drew upon in my analysis: perceptual versus property-based visualization. In addition, I highlighted four types of spatial skills: intrinsic-static, extrinsic-static, intrinsic-dynamic, and extrinsic-dynamic. As I elaborate on next, I used to select interview tasks and interpret my results. I ended the chapter by speaking to my theoretical perspective for this thesis, embodied cognition, and its importance in my research.

Chapter 3: Methodological Framework and Methods

In chapter three, I will be discussing the methodology that I chose to work with in this thesis: case study methodology. Afterwards, I will describe the setting and participants of my study. Furthermore, I will speak to the data collection process (student interviews, teacher questionnaires, and the tasks associated with the student interviews). I will end the chapter with a review of the five phases of data analysis that I conducted for this study (transcribing and organizing data, episode creation, coding, comparison and analyzing embodied resources).

Case Study Methodology

For my thesis I decided to use case study methodology. In the paragraphs that follow, I will describe what case study methodology is, why I decided to use it to help guide my research and its importance in my work.

Case study methodology is a research approach where the researcher follows the philosophy “of analyzing an existing, real-life situation in all its complexity, [and] exploring it as close to the people concerned as possible” (Kyburz-Graber, 2004, p. 3). Case study methodology is especially useful in situations where researchers ask “how” questions and have limited control over behavioural outcomes (e.g. how a student responds to a prompt) (Yin, 2014).

There are four different types of designs for case studies: holistic single case, embedded single case, holistic multiple case and embedded multiple case (Yin, 2014). The first and second type of case studies focus on a single case. A *holistic single case study* means that the researcher is examining just one case and that there is only a single unit of analysis. For example, a researcher might decide to use a holistic single case study when that particular case is extreme or

rare (Yin, 2014). An *embedded single case study* means that the researcher is still examining just one case, but they have multiple units of analysis. The first two types of case studies did not apply to my research. Despite that fact that each of the students who participated were the subject of an individual case study, my study as a whole looked at multiple students and then compared them.

The last two types of case studies are: *holistic multiple case study* and an *embedded multiple case study*. As with the previous two types of case studies mentioned above, holistic and embedded mean the same thing. The difference here is, just like the name suggests, this type of case study focuses on multiple cases. For my research, I chose to work with an embedded multiple case study. Each student made up their own case, but I examined multiple cases. Additionally, I chose to use embedded rather than holistic because there were different types of spatial tasks students engaged in that presented different aspects of each case's visualization. Also, my units of analysis were the different episodes that I created based on the interview questions that I asked the students (I will speak more to this later on in the data analysis section).

I found this methodology to be very much in line with what I wanted to do with my own research. I did not set up an experiment where I had a control and a test group. I did not know the outcome of what the students would say when I asked my questions. I wanted to use this research approach because I wished to view each of my participants as unique individuals whose distinct approaches to problem solving, original answers and gestures could be viewed as separate cases of spatial visualization.

Moreover, case study methodology aims to explore real-life situations and identify factors that promote or inhibit various phenomena (Kyburz & Graber, 2004). Kyburz-Graber's

(2004) highlighted that case study methodology can assist in distinguishing various elements that can either aid or hinder a situation. I found this notion helpful when thinking about my own research. In particular, throughout my study, I aimed to characterize the nature of students' spatial visualization when undertaking geometric tasks. These instances of spatial visualization allowed me to see how students engaged in spatial visualization (the visualization practices they used) and if these ways helped them to successfully complete a geometric task or not. If students were unable to successfully complete a task, I could then consider what spatial visualization practices were present, if any, and what was lacking. These notions helped answer my research questions, but ultimately will also help teachers and curriculum designers approach spatial visualization in a comprehensive teachable manner (See Chapter 5 - Discussion for more on this).

This aspect of case study methodology also assisted me when making tasks for the student interviews. I designed the various tasks to be something that could be used in real-life situations. The tasks that I created for my participants were similar to questions that they would encounter throughout their educational journey.

Furthermore, case study methodology was important to my thesis because it allowed me to follow a research approach where I could be flexible and adapt to what students said, where I could add or remove questions depending on how students responded and where my focus during the interview was very much tailored to that specific student. Case study methodology allowed me to be able to look at each interview as a separate case of spatial visualization. Additionally, I could compare and contrast each case which would allow me to answer my first, third, and fourth research questions: (RQ1) What difficulties do elementary students encounter when undertaking

spatial visualization tasks in geometry? (RQ3) What are the differences between early, middle and late elementary students, in how they use spatial visualization when solving geometric tasks? (RQ4) What embodied resources do students draw upon to engage in spatial visualization when solving geometric tasks?

Setting and Participants

For this study, I recruited students from a rural school in Western Quebec. The students were chosen based on their participation in an afterschool program (daycare). This was to ensure that the interviews did not interfere with their normal class schedules. The students who were participants of the study attended the school that I taught at. I did not interview any of my own students as not to skew results or put added pressure on the students to feel the need to participate. Participation in the study was entirely voluntary. Students could withdraw from the study at any time or decide to only answer a few questions if they chose. To enlist the students, I spoke briefly about what I was planning on doing and provided a consent form for their parent/legal guardian to sign. The students were deemed ready to participate if their parents/legal guardians signed the form.

There were six students who partook in my research (see Table 1 for an overview): one from grade one (ST1), one from grade three (ST3), two from grade four (ST4-A, ST4-B), one from grade five (ST5) and one from grade six (ST6). According to the students' teachers (based on a questionnaire I provided them), ST3, ST4-A, ST4-B, and ST5 all had at least two to three weeks of formal instruction on geometric concepts. ST1's and ST6's teachers did not complete the questionnaire, so I cannot speak to the amount of geometry taught. However, according to the

Quebec curriculum, it is likely that they spent the same two to three weeks learning about geometry as their peers.

Table 1

Overview of Student Participants

Pseudonym	Age	Grade	Geometry concepts covered	Spatial visualization taught by the teacher and if so, how?
ST1	Between 6-7 years old	1	N/A	Questionnaire not completed
ST3	Between 8-9 years old	3	-2D shapes -3D solids -Types of triangles -Cartesian plane -Angles and lines	No.
ST4-A	Between 9-10 years old	4	-Symmetric Figures -Reflection -Tessellation -Cartesian plane -Polygons and quadrilaterals -Classification of solids -Nets of solids	No.
ST4-B	Between 9-10 years old	4	-Symmetric Figures -Reflection -Tessellation -Cartesian plane -Polygons and quadrilaterals	No.

			-Classification of solids	
			-Nets of solids	
ST5	Between 10-11 years old	5	- Describing and classifying prisms and pyramids -Using faces, vertices and edges, -Solids of nets -Triangles	Yes. By using “3D images to show how they can’t always see all the parts.”
ST6	Between 11-12 years old	6	N/A	Questionnaire not completed

Note. In this table, the 4th column explains which geometry concepts were covered according to the teacher questionnaire. The 5th column explains if the particular teacher taught any spatial visualization and if so, how the teacher went about teaching it.

Data Collection

Data collection consisted of student interviews and teacher questionnaires. Below I describe the student interviews, as well as the teacher questionnaires.

Student Interviews

Interview Procedure. The interviews consisted of six visualization tasks, with two to four subtasks within each of the six main tasks pertaining to geometry (described in more detail below). The students were taken after school to my classroom where I conducted semi-structured task-based interviews (see Appendix A for Interviewer Booklet). The interviews lasted between 20 and 60 minutes depending on the individual student’s responses. Students answered questions related to each task in a student booklet (see Appendix B). While the students worked with the booklet, depending on the task, they were provided additional materials such as blocks and three-

dimensional figures. As I describe in more detail below, the tasks that the students participated in assessed their use of spatial visualization by asking them to imagine different combinations and to think about what something would look like in the future (Ontario Ministry of Education, n.d).

The interviews were video recorded. Given my interest in students' embodied resources (RQ4: What embodied resources do students draw upon to engage in spatial visualization when solving geometric tasks?), the video camera focused only on the students' hands, the student booklet, and the materials they were working on.

The procedure for the interviews included three steps which were repeated for every task. First, I read and explained the task with the students and made sure they understood the problem. Second, I provided students with time to solve the task, while asking them to speak aloud about their solving process. Last, I asked questions to probe on how they saw something, what techniques they used and why they thought in a specific way. Examples of questions were: "How did you imagine that inside your head?" "Show me what you saw." "How did you decide to complete that task?" "Why did you complete it that way?"

Interview Tasks. For the interviews, I had the students perform six main tasks, each consisting of two to four subtasks. Each task was broken down according to different ideas and concepts within geometry (see Table 2). I selected these tasks to range in geometric concepts, aspects of spatial visualization (described in Chapter 2) and types of spatial skills. As I describe in my limitations in Chapter 5, due to time limitations when working with young children, I was not able to include a task that targeted extrinsic-static skills.

Table 2*Overview of Interview Tasks*

Name of Main Task	Geometric Concepts	Targeted Components of Spatial Visualization	Types of Spatial Skills Targeted	Source
Composition & Decomposition	Shapes, space, composition, decomposition	-Seeing Objects -Mental Manipulation -Imagining an Outcome - Spatial Relationships	Intrinsic-Dynamic Intrinsic-Static	Ontario Ministry of Education. (n.d.). Page 9
Nets	Nets	-Mental Manipulation -Imagining an Outcome -Spatial Relationships -Seeing Objects	Intrinsic-Dynamic	Created my own after looking at examples from Ontario Ministry of Education. (n.d.).
Rotation	Rotation, transformation	-Seeing Objects -Mental Manipulation -Imagining an Outcome	Intrinsic-Dynamic	Created from looking at different ideas in the Ontario Ministry of Education (n.d.) document & Okamoto et al., 2015, p.16
Comparison of 3D objects	Composing/ decomposing 3D shapes	-Seeing Objects -Mental Manipulation -Imagining an Outcome - Spatial Relationships	Intrinsic- Static	Hawes, Tepylo & Moss 2015, p. 43
Flips	Transformations, fractions, angles.	-Seeing Objects -Mental Manipulation -Imagining an Outcome	Intrinsic-Dynamic	Hawes, Tepylo & Moss 2015, p. 43

- Spatial Relationships				
Perspective	Self-to-object navigation, perspective	-Spatial Relationships - Seeing Objects	Extrinsic-Dynamic	Okamoto et al., 2015, p. 25

Note. In table 2 provides an overview of the different tasks that I had students complete, the geometry and spatial visualization concepts that were targeted and where I drew my inspiration from to create the tasks.

Task 1: Composition and Decomposition. The first task encompassed the idea of composition and decomposition (see Figure 1). The students were asked to look at a series of pattern blocks and determine: (a) the smallest number of pattern blocks needed to fill the empty figure shown and (b) the greatest number of pattern blocks needed to fill the same empty figure. To solve question (a) correctly students would have said that the least number of blocks needed were two – the hexagon and the trapezoid. In order to solve question (b) correctly, the students would have said they needed nine blocks – all green triangles.

Figure 1

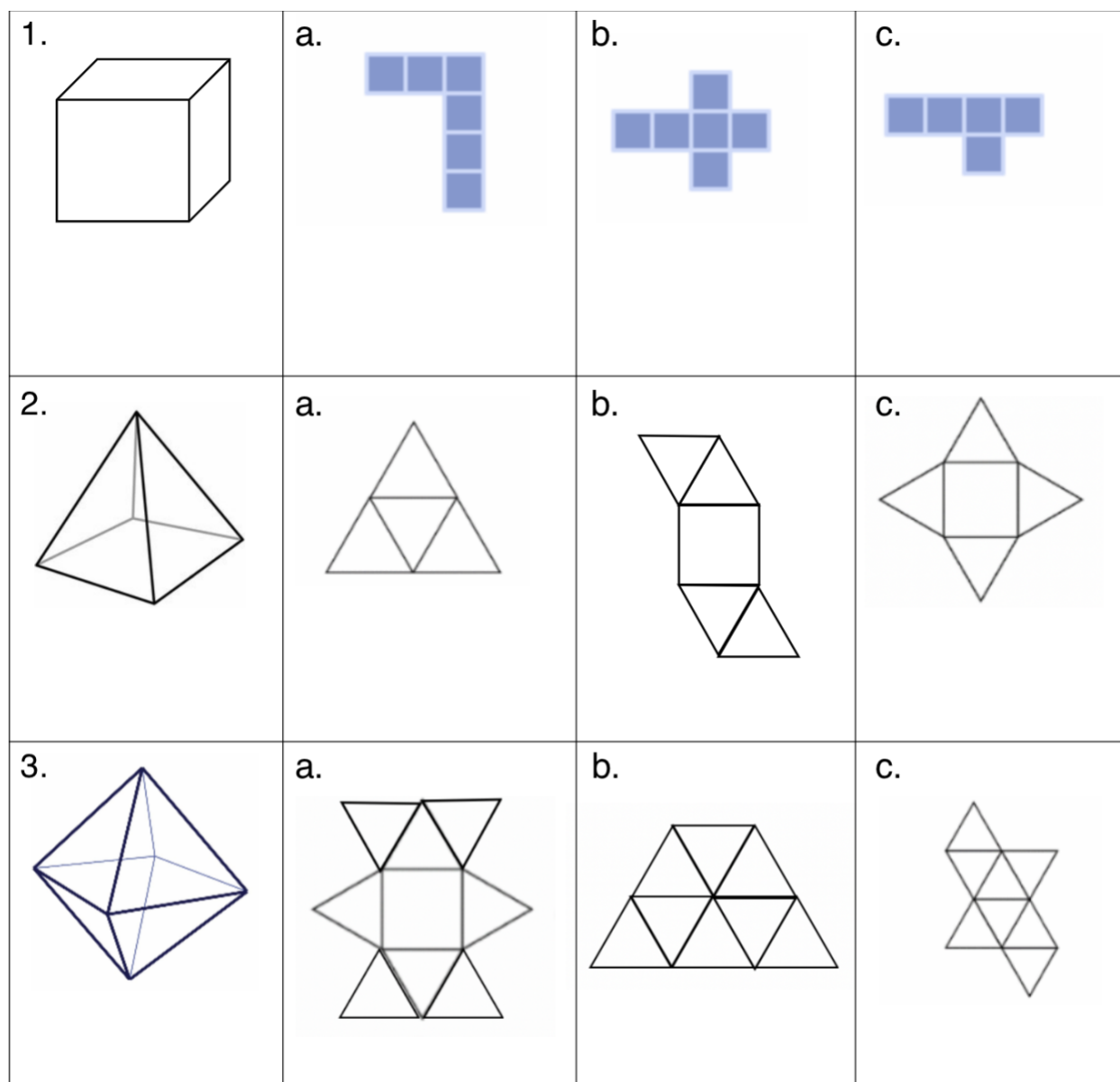
Task 1: Composition and Decomposition Task.



Additionally, the students were given 21 pattern blocks and were asked to show as many ways possible to make a square with the blocks provided. Of the 21 blocks provided, there were:

three hexagons, three trapezoids, six equilateral triangles, four rhombi, four squares and one diamond. For the students to have answered this question correctly, they would have had to use the four squares. Since the triangles given were equilateral, they were not able to use them to create a square.

Task 2: Nets. The second task assessed students' visualization of nets. The student booklet (see Appendix B) showed three different three-dimensional figures along with three subsequent nets (see Figure 2). The letters and numbers in Figure 2 were added to provide clarification for this thesis. The students were asked to look at the three-dimensional figures and then imagine unfolding them. They were then asked to choose a net that they thought corresponded with the three-dimensional figure. After the students had given their answers, the actual physical representations of the figures were brought out for the student to examine. The students were not allowed to unfold them but were asked to feel the shape in their hands and look at it to see if they wanted to stay with their original answer or change it. In order for the students to solve the cube net correctly, they would have chosen 1b, the 2nd choice that looked like the letter "t". To solve the prism net, they could have chosen either 2b or 2c. For the octahedron, the students would have had to choose the last option, 3c.

Figure 2*Task 2: Nets*

Note: The letters and numbers in Figure 2 were added to provide clarification for this thesis (adapted from Ontario Ministry of Education. (n.d.)).

Task 3: Rotation. The third task focused on rotation (see Figure 3). Two different triangles were shown in the booklet. One triangle was an equilateral and the other an isosceles. The student had to imagine that they turned the triangle in their mind and then drew what they thought it would look like. In order to solve this task correctly, the students would have had to draw the exact same triangle for the first question since it was an equilateral triangle. The second triangle was an isosceles triangle, so that top of the triangle would have to be pointing to the right in order for the students' answer to be correct.

After responding about the two triangles, the students were shown a picture of a little person standing upright, and then three images of the little person in various positions. The students were asked to imagine turning the first person in their head and to choose which of one of the three little people they thought it would be. For the answer to be correct the students would have had to choose the first option.

Figure 3

Task 3: Images for Rotation (Okamoto et al., 2015, p. 16)

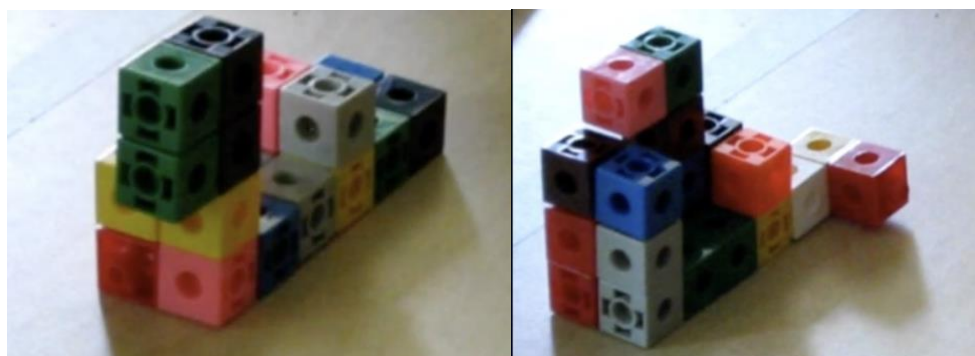


Task 4: Composition of 3D objects. In the fourth task, students were asked to recreate two different objects (see Figure 4). The first object was shown to the student for 10 seconds and then hidden. The students were asked to recreate it from memory. Once they were done, the object was put back in sight and the similarities and differences between the original and their

rendition were discussed. The second step was to leave a more complex shape out for them to see and ask if they could recreate it, but without having to memorize what it looked like. For the task to be deemed successful, the students' recreated objects had to match the original object that was shown for both instances.

Figure 4

Task 4: First Object for Recreation on the Left and Second Object for Recreation on the Right



Task 5: Flips. The fifth task had the students folding a piece of paper into quarters (they were asked to fold the paper in half, and then in half again). Once that was done, they punched a hole in the upper corner (see Figure 5). In the student booklet (see Appendix B), there were five images of a paper with holes in them for the students to choose from (see Figure 6). Note that the letters under the paper options were added for clarification in this thesis. The students were asked to imagine unfolding the paper, and then to choose which image they thought it would look like. To solve this task correctly, students would have had to choose the last option, “e”. The only exception was for ST1. Because they punched the hole in the wrong corner, when unfolded, the correct answer was the 3rd option, “c”. Once the students guessed, they were able

to open the paper and discuss if there were any differences between what they saw and what their answer was.

Figure 5

Task 5: Paper Folding Process

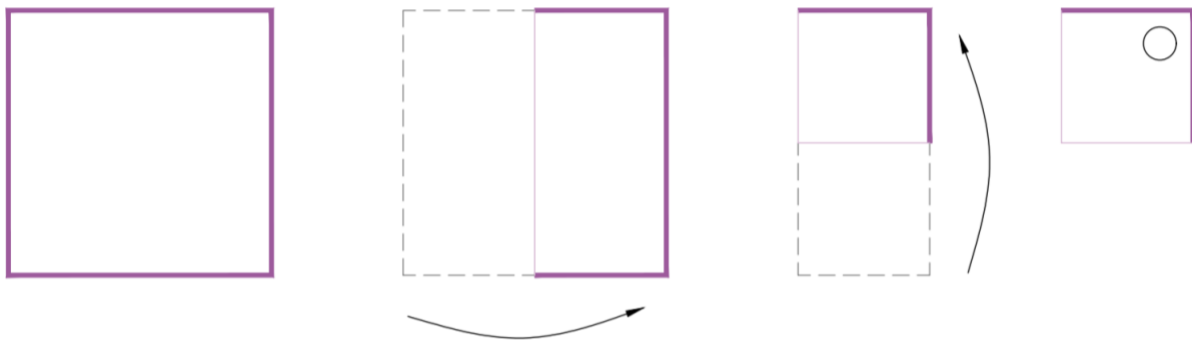
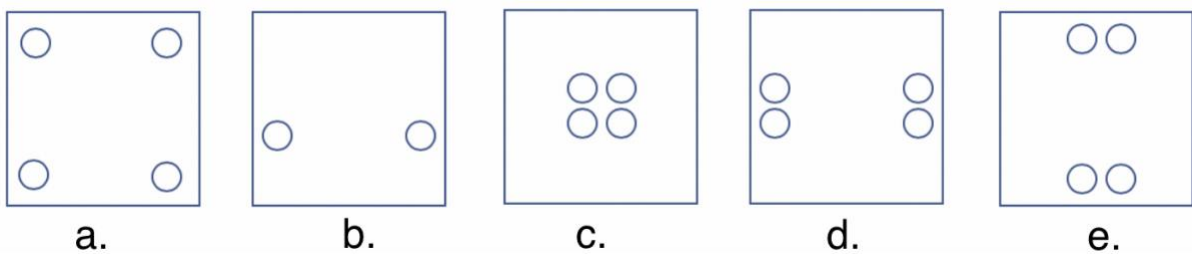


Figure 6

Task 5: Paper Options



Note: The letters were added to help clarify for the thesis (adapted from Hawes, Tepylo & Moss., 2015, p. 40).

Task 6: Perspective. The sixth and final task assessed students' visualization of perspective. The students were shown an image with three dinosaurs (see Figure 7), and then

they were asked to circle the image of what the main dinosaur saw (see Figure 8). Note that the letters in the figure were added to help clarify for the thesis. To answer this question correctly the students had to choose the middle image, with the lone dinosaur, option “b”.

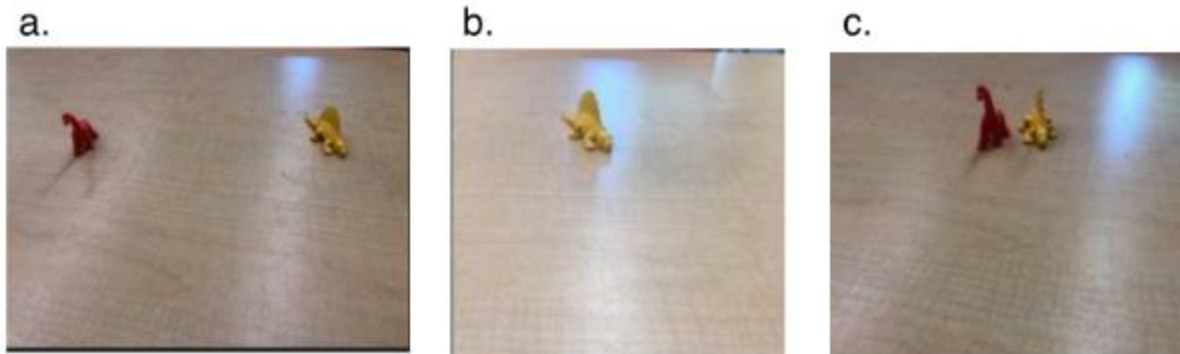
Figure 7

Task 6: Initial View of the Dinosaurs (adapted from Okamoto et al., 2015, p. 25)



Figure 8

Task 6: The Three Possible Choices



Note: The letters in the figure were added to help clarify for the thesis (adapted from Okamoto et al., 2015, p. 25).

Teacher Questionnaire

In addition to the video recorded interviews, I distributed teacher questionnaires (see Appendix C) to the teachers of the students who partook in my study. The aim of the teacher questionnaire was to provide context for the students' responses. The questionnaire asked the teachers to describe what they had taught in mathematics during that year, what they taught about geometry, and what they taught about spatial visualization. These responses allowed me to understand what supports for spatial visualization the child had during the year in order to situate each of the cases.

Data Analysis

In order to analyze the interview data, I decided to organize the analysis into five different phases. I describe each in what follows.

Phase 1: Transcribing and Organizing the Data

In the first phase, I watched all the videos of the interviews and created a data log. Within the data log, I compiled a list of the students who participated (denoted by pseudonym only), what grade they were in, the number of tasks they completed, the length of their specific interviews and if a teacher questionnaire had been completed by their teacher or not.

Afterwards, I created tables where I wrote the student's answers or inserted an image of their work. For example, in the first task I asked students to tell me what the largest number of blocks needed to fill a shape were. Answers ranged from four to twelve blocks. I decided to create this table so that I could easily compare answers between students. This was particularly useful in answering my first and third research questions: (RQ1) What difficulties do elementary

students encounter when undertaking spatial visualization tasks in geometry? (RQ3) What are the differences between early, middle and late elementary students, in how they use spatial visualization when solving geometric tasks?

Once I had completed the table with the answers, in order to help me answer all of my research questions, I transcribed all of the interview data and then created another table where I placed the transcriptions of students' answers and their gestures in rows based on each answer. For example, one particular answer with gestures looked like this:

Well because this is an octagon [*traces hexagon with pencil*] and if I just cut this in half right here, [*draws a line on the empty figure*] its where the octagon would be, I'd put it there [*uses pencil to point to hexagon and back to empty figure*]. Then I realized that this [*points to trapezoid with pencil*] was this [*uses pencil to point to empty figure*] but in a bigger form.

Documenting the gestures that students used when completing the tasks was not only important for RQ2 and RQ3, in order to provide elaborations to students' verbal explanations, but was also particularly important for answering my fourth research question (RQ4): What embodied resources do students draw upon to engage in spatial visualization when solving geometric tasks?

Phase 2: Episode Creation

In the next phase of analysis, with assistance from my thesis supervisor, I established rules in order to create episodes of my interviews. These episodes were created using a software called MaxQDA. MaxQDA is a qualitative analysis software that allows researchers to code and

compare multiple video data at the same time. The purpose of creating episodes was to help distinguish between tasks and get a better idea of the different aspects of spatial visualization that were being displayed. These episodes were based on the questions probing into what the student did during the different tasks. Episodes started and finished when a new question was asked, with the exceptions of non-leading probes, clarifications and the actual tasks. Non-leading probes were questions that I asked that did not delve deeper into the students' thinking. For example, during one interview, a student seemed to be struggling with the task, so I asked, "Do you want to look at toys to help you out?" Clarifications refer to questions asked when a student either (a) did not seem to understand what was being asked so I asked it in another way or (b) they asked a question to make sure they understood what needed to be done. For example, during one interview, ST1 asked, "Do they have to stay the exact same size?" and I answered, "No, the size no." The moments when I introduced the tasks to the students were not included as episodes because I was looking at how the student responded to the questions following their engagement in the task. Their responses provided insight into what they saw. Table 3 provides an overview of the number of episodes for each student's interview and duration of those episodes.

Table 3*Episode Information*

Student Interviewed	Number of Episodes during the interview	Range of length of episodes
ST1	24	2 seconds to 14 seconds
ST3	21	8 seconds to 32 seconds

ST4-A	26	4 seconds to 2.5 minutes
ST4-B	22	6 seconds to 1 minute
ST5	22	2 seconds to 30 seconds
ST6	22	7 seconds to 2.5 minutes

Phase 3: Coding

Once the episodes were created, I started the process of developing my coding scheme to characterize the visualization practices that students used when solving the tasks (RQ2). Since each task targeted a different aspect of geometric visualization, I created a coding scheme for each task. To develop each of the six coding schemes, I inductively looked at the data I collected, and I deductively drew upon constructs in existing literature. To determine whether something constituted a spatial visualization practice, I considered whether the actions they took, how they spoke and their gestures related to one of the four aspects of spatial visualization that I described in Chapter 2: seeing an object in one's mind, mental manipulation, imagining an outcome, and spatial relationships. In doing so, I considered a visualization *practice* to be evidenced by the speech, actions and gesture students used to describe their process of how they completed the tasks. Additionally, to help me see nuances in students' visualization practice, I drew upon the distinctions of perceptual versus property-focused thinking when making sense of how the students were engaging with the tasks. I also attended to both verbal and nonverbal evidence. To assist in operationalizing my codes, I listened for keywords students used when explaining their thinking (for example when a student said they “imagined”, “rotated” or “flipped” something). I

also described the hand movements and drawings students made. Each coding scheme was related to different aspects of spatial visualization that the students displayed while solving each geometric task.

I created four drafts of my coding scheme before finally settling on the fifth version with the help of my thesis supervisor (see Tables 4 to 9 in the results section). In order to create each draft, I sampled my data and tested each new scheme. Whenever there were gaps or rules did not work well, I made revisions and tried again.

Once I had finalized my coding scheme, I then used MaxQDA to assign codes to each episode I had created previously. I assigned more than one code if necessary. I used this software because it allowed me to code the video directly, which allowed me to attend to the verbal and non-verbal cues in how students visualized.

Phase 4: Comparison

Once all the episodes were coded, I then compared codes to attend to my third research question (RQ3): What are the differences between early, middle and late elementary students, in how they use spatial visualization when solving geometric tasks? To determine what counted as early, middle and late elementary students, I used the Quebec education progression of learning cycle system (Quebec Education Program, 2009). When looking at early elementary students, I used the Quebec cycle one system, which refers to grades one and two. In this study I only had a grade one student, ST1, so they made up my “early” elementary student section. For my middle elementary students, they fell into cycle two, which refers to grades three and four. In my study, that meant that ST3, ST4-A and ST4-B were in this section. For late elementary students, this

was Quebec's cycle three, which consisted of grade 5 and grade 6. ST5 and ST6 fell into this category.

When comparing across cycles, I looked at the differences between the number and types of codes the students had in each cycle. I also looked at the success rates between cycles. Additionally, I used the tables I had created in Phase 1 to look at each student's answers.

Phase 5: Analyzing Embodied Resources

The fifth phase of my analysis helped me answer my fourth research question (RQ4): What embodied resources do students draw upon to engage in spatial visualization when solving geometric tasks? I conducted two types of analysis. First, in order to identify *whether* gestures supported spatial visualization, I created tables per each subtask that documented gestures, and if the students gave the correct or incorrect answer. In doing so, I was looking for relations between correctness and gesture and incorrectness and no gesture.

Second, in order to understand how *types* of gesture might play a role in spatial visualization, I conducted a fine-grained analysis of two students (ST1 and ST6). I decided to use ST1 and ST6 because they had the largest difference between success rate and the types of visualization practices used, as well as the type and amount of gestures they used. Additionally, ST1 was in the lowest grade level for this study and ST6 was in the highest-grade level. To compare the two students, I chose three tasks to focus on (task 1c, 3b and 5). I chose these specific tasks by looking at the interview videos and the transcripts to see when the students engaged in certain visualization practices and the types of gestures that accompanied those practices. The three tasks mentioned above showed the greatest differences in visualization practice, correctness and gestures. When contrasting ST1 and ST6, I compared the verbal and

non-verbal discourse of each transcript, documenting the kinds of gestures that the student used in relation to what they said. I also looked at which types of visualization practices were used and what gestures accompanied them.

Issues of Validity and Trustworthiness within my Research

When I first started thinking about my research, I knew that I had to spend a considerable amount of time anticipating what my study would entail, who my participants would be and how I was going to hold myself accountable to my participants and the results of my study. In the paragraphs that follow, I will speak to my biases, expectations and values as a researcher, my reactivity, the lens that I used when conducting my research and provide a brief overview of the different ways I tried to make my study valid and trustworthy.

Bias, Expectations, Values and Reactivity

Bias and reactivity are considered to be the two main threats to validity for a qualitative study such as my own (Maxwell, 2013). Bias refers to the theories, beliefs and perceptual lens that a researcher brings with them when conducting research (Maxwell, 2013). Before I partook in any interviews with the participants, I had to understand my own values and expectations that I brought to my study and how those would influence how I conducted the study and the types of conclusions I made from conducting it. I knew that I would most likely be interviewing students at the school that I worked at, so I had to make sure that none of the students that I interviewed were current students of mine. I did not want current students because I knew that some students would volunteer because they either wanted to make me happy or they thought by not doing so, it would upset me. Also, if I were to interview my own students it would be harder for me to take what they were saying at face value. Since I know what I have taught and I was aware of the

capabilities of my students, if the student got an answer wrong that I thought they should have gotten correct, it would be hard for me to not probe on why they got it wrong, or perhaps ask them questions to try and get them to arrive at the correct answer.

Another bias I had to acknowledge was that I believed students in the higher grades would fare better than students in the lower grades. I had to take that bias and make sure that when conducting the interviews, I did not try to coach or provide extra guidance to the younger grades. I also had to make sure that my questions were as similar as possible to each other regardless of grade level. I had a protocol to follow for the tasks but asked additional questions depending on what the student said or did. Additionally, I had to make sure that this bias did not interfere with my analysis. I did this by conducting different kinds of analysis. I did not solely focus on visualization practices but also looked at the correctness and incorrectness of each student (this was a much more objective analysis procedure). The measures that I used to deem if something was correct or incorrect were based on the literature (since the tasks I used were adapted from previous research). Therefore, I did not determine the answers, thus making sure that I was unable to influence that process.

In addition to my biases, values and expectations, I also had to take into account reactivity. Reactivity refers to the effect that a researcher has on their participants (Maxwell, 2013). According to Maxwell, the researcher cannot eradicate the influence they have over their participants or the setting. Since my data came from the interviews with the students, I was physically part of the environment with the participants. What I said and how I conducted myself irrefutably influenced the participants' actions and answers, and therefore affects the validity of the inferences that I can draw from the interviews. To take this into account, during my

analysis I went back and looked at the videos as well as the transcripts to make sure that there were no leading questions. My interpretation of the data was based on the coding scheme I created through an iterative process and therefore was more objective (age of the student or if I had a preference did not play any role). When looking at how students engaged in a task, if a visualization practice was used, they were coded with that practice. If no practice was used, that was noted as well.

Lens

Researchers have speculated that procedures of validity are governed by two perspectives: (a) the lens in which the researcher chooses to validate their study (that lens being either their own, the participants in the study, or people independent of the study), and (b) the researchers' paradigm assumptions (Creswell & Miller, 2000). For my research I used my own lens when recruiting participants, creating tasks and protocols and analyzing the data. I decided to use a researcher's lens because I was not looking at students' beliefs or perspectives, I was trying to understand their cognitive processes. Using my own lens allowed me to see things that I would not have been able to if I had taken the lens of the participant. For example, I was looking to see the types of visualization practices that students used when engaging with geometric tasks. If I were to ask the students which practices, they used, they would either not know the types of practices and would not be able to answer, or if I explained it all to them, then perhaps it would have changed the way they answered the question.

When using the researchers' lens, I did so with the paradigm assumption of a critical influence. Critical influence refers to the assumption that the researcher situates themselves historically, and based on "social, political, cultural, economic, ethnic and gender antecedents"

(Creswell & Miller, 2000, p. 126). I knew that my role as a teacher in the school held more weight with the students than someone who was either (a) not a teacher or (b) not from the school and unfamiliar with the students. Therefore, when interviewing the students, I told them that at any time if they did not want to answer a question, they did not have to or if they wanted to stop the interview at any time, they could. I also reassured them that I was not looking for right or wrong answers, but that I just wanted to see how they completed geometric tasks. Additionally, before the interview started, I explained to them about the study and asked them if they consented. Although legal consent is not legally required for under aged children (just signed consent from parent/guardian), asking their consent allows young participants to understand what is happening and that they have a choice (Butler-Kisber, 2010).

Validation and Trustworthiness

Since I was limited with the time I had with my participants (I will speak more about this limitation in Chapter 5), I was unable to use a vast variety of validity procedures. Specific procedures like prolonged time in the field and multiple sources of data provide great opportunities to validate studies; however, I was unable to participate in those types of procedures (Butler-Kisber, 2010; Creswell & Miller, 2000; Lincoln & Guba, 1986; Maxwell, 2013). Despite this, there were some procedures that I did practice in order to add more credibility to my research. The first procedure that I used was called *respondent validation*. This is where I would regularly check in with the participants to make sure that I was not distorting the meaning of what they were saying or doing (Maxwell, 2013). Another way I made sure to add credibility to my study was being careful with the generalizations that I made from my results (see Chapters 4 and 5). According to Maxwell (2013), there are two types of

generalizability in qualitative research: internal and external generalization. Internal generalization refers to generalizing the results within the group of participants (Maxwell, 2013). External generalization refers to generalizing the results to a wider group of individuals outside of the study (Maxwell, 2013). I did not make any external generalizations and additionally, I was hesitant to make internal generalizations because of the sample size of my study. In the cases when I did make internal generalizations, I used the teacher survey to help situate the cases to make sense of the results in relation to the kinds of learning experiences the participants had had. Although not all teachers filled out the survey for the students, based on my knowledge of the Quebec curriculum, and the mandate of teachers to teach that curriculum, I knew that all students had some experience with geometry. The last way that I tried to make my study more valid and trustworthy was by providing a comprehensive report of my research process in this thesis.

Summary of Chapter 3

In this chapter I presented my methodological framework, case study methodology, and its significance within my thesis. I spoke about the setting and the participants that I recruited for my study. Additionally, I described the process of data collection and data analysis that I used when tabulating my results. To conclude chapter 3, I spoke to the issues of validity and reliability within my study.

Chapter 4: Results

In chapter 4, I will share my findings. In order to properly portray my results, I have categorized them according to my research questions. I will speak to each research question, the interview task that was applied, which students were able to complete the task successfully or not and the visualization practices that were utilized.

Student Difficulties with, and the Nature of their Spatial Visualization

This section of my thesis focuses on answering my first and second research questions: (RQ1) What difficulties do elementary students encounter when undertaking spatial visualization tasks in geometry? and (RQ2) What is the nature of students' spatial visualization when solving geometric tasks? As mentioned earlier, there were six main tasks that I had students complete during our interviews. Within each task, there were another two to three sub tasks. For all of the six main tasks, I assigned visualization practices (see Tables 4 to 9) to each student's response. The visualization practices that students used allowed me to characterize the nature of their spatial visualization. Some of the visualization practices were distinct to one main task, whereas some of them overlapped with other main tasks. Furthermore, since students' reason with spatial visualization in various and multiple ways, more than one visualization practice could be assigned to a student per task. In the sections that follow, I will describe the various visualization practices and the difficulties the students encountered for each task.

Task 1: Composition and Decomposition

Visualization Practices for Task 1. In tasks 1a and 1b, students were shown a picture of an empty shape and smaller shapes beside it. The students were asked to determine first, what the

smallest number of blocks needed to fill the shape was and second, what the largest number of blocks needed to fill the shape was. Task 1c asked students to create a square with blocks provided. For all of task one, (including the sub tasks) I identified four visualization practices that students used: *seeing sameness*, *imagining transformation*, *imagining decomposition* and *imagining composition*. Table 4 outlines the name of the visualization practice, a description of what that practice means and an example of the practice in action.

Table 4*Visualization Practices for Composition & Decomposition Tasks*

Visualization Practice	Description of Practice	Example of Practice
Seeing Sameness	When students describe replications of shapes embedded within another shape(s).	ST1: “Because it (the empty figure) looked like [points to empty figure] like this form here [points to hexagon] and and this one [points to trapezoid] here [points to empty figure where the trapezoid would go].”
Imagining transformation	When students describe seeing a transformation (in particular, rotation and translation).	ST5: “...this one [points to trapezoid] you just need to flip it [makes a turning gesture with hand] and it looks like that.”
Imagining decomposition	When students describe being able to imagine an object getting broken down into a different object.	ST6: “Well because this is an octagon [traces hexagon with pencil] and if i just cut this in half right here, [draws a line on the empty figure] its where the octagon would be...”
Imagining composition	When students describe being able to imagine putting objects together to form another object.	ST4-A: “....those 2 together would make this [points to empty figure]...”

The first visualization practice was *seeing sameness*. This encompasses the idea that the student was able to solve the task by looking at shapes and seeing replications within other shapes (e.g., when a student pointed out that the bottom of the empty shape resembled a trapezoid) – a process that requires attending to spatial relationships. This was evidenced by the student verbally expressing the similarities. *Seeing sameness* lends itself to the idea that students tend to rely on perception rather than property-based reasoning.

The second practice some students used to help solve task 1 was *imagining transformation*. Students who used this practice were able to perform a mental manipulation of a rotation or translation (e.g., a student took the trapezoid shape and mentally turned it so that it could fit inside the empty figure). This was evidenced when the students described the transformation they had done. This practice highlights mental manipulation, one of the defining aspects of spatial visualization.

The third visualization practice was *imagining decomposition*. This visualization practice meant that students were able to see a shape and then break it down into either different shapes or smaller versions of the same shape. Here students used the third defining aspect of spatial visualization: imagining the outcome. For example, ST6 talked about cutting the hexagon “in half” for task 1a.

The last visualization practice that certain students used to solve task 1 was *imagining composition*. This practice is the opposite of decomposition. Here, the students were able to imagine a different outcome: one of putting together different shapes/objects to make another one (e.g., when ST4-A pointed to the trapezoid and the hexagon and said, “those two together would make this,” “this” being the empty shape). For all four of the visualization practices that

could be employed for task 1, with the exception of *seeing sameness*, the aspects of spatial visualization, such as seeing objects in one's mind and mentally manipulating those envisioned objects were present. The students had to use those two aspects of spatial visualization in order to solve the task correctly.

Trends in Difficulties and Visualization Practices for Task 1. For task 1a, students had to decide the smallest number of blocks needed to fill the shape (see Figure 2 in Chapter 3). Within task 1a, all of the students were able to answer the question successfully. Out of the six students, five of them (ST1, ST3, ST4-B, ST5 and ST6) used the visualization practice of *seeing sameness*. The students were able to acknowledge that the empty figure that they had to place blocks into resembled a hexagon and a trapezoid. Moreover, two of the students (ST3 and ST4-B) who had used the visualization practice of *seeing sameness* also used the practice of *imagining composition*. ST6 was the only student to additionally use the practice *imagining decomposition*.

In task 1b, students were asked to determine what the largest number of blocks needed to fill the shape was (see Figure 2 in Chapter 3). Every student answered this question incorrectly. One of the six students (ST1) did not use any of the four visualization practices. ST1 chose the same two blocks they chose in task 1a. When I noted that those were the smallest number of blocks and now, we were looking for the largest, ST1 still stated those two blocks and could not explain why they chose them. Four of the six students (ST3, ST-A, ST4-B and ST6) used the visualization practice of *seeing sameness*. ST3 drew three rhombi and one trapezoid in the empty figure. ST4-A also used the practice of *seeing sameness* by drawing shapes within the empty figure, but additionally used the practice of *imagining composition*. This was evidenced while

ST4-A described why they chose which blocks to add to the shape: “those two together would make this.” ST4-B drew lines from the shapes on the paper into the empty figure and ST6 drew shapes within the empty figure. ST5 used the practice of *imaging transformation* and drew six triangles and two rhombi.

Task 1a and 1b both asked the students to use the same types of blocks to fill the same empty figure. In task 1a, the task appeared to be easier because 83% of the students were able to use the practice of *seeing sameness*, which allowed them to choose the trapezoid and hexagon easily. In 1b, however, only 33% of the students were able to see sameness between the empty space and the shapes provided, making it difficult to fill the empty figure. Task 1b allowed me to see that even though some students were trying to use spatial visualization to help them solve this task, when it came to visualizing many shapes and how they would fit perfectly within another shape, it proved to be too difficult. The students needed to coordinate more shapes to imagine the composition. Imagining the outcome (one of the defining aspects of spatial visualization) requires more than just seeing sameness. That may be why the students struggled with this task.

In task 1c, the last sub task for composition and decomposition, the students were given 21 pattern blocks (three hexagons, three trapezoids, six triangles, four rhombi, and four squares). They were then asked to recreate a square with the blocks given. All students with the exception of ST1 were able to create another square, using the four smaller squares. ST1 made different figures such as a sun but did not focus on the task at hand even with additional support¹.

¹ Additional support refers to asking the question again and trying to get the student to stay on task.

However, none of the students were able to create a square with any of the other blocks provided. ST5 was the only student who was able to create a rhombus (very similar to a square). 50% of the students used the visualization practice of seeing *sameness* (ST1, ST4-A and ST5). The remaining 50% of the students (ST3, ST4-B and ST6) used the practice *imagining composition*.

Task 2: Nets

Visualization Practices for Task 2. In task 2, I characterized the nature of students' spatial visualization with six visualization practices: *imagining flipping*, *seeing sameness*, *imagining the final product*, *counting the number of sides*, *mapping* and *instinct*. Table 5 outlines the name of each visualization practice and provides a description of what the practice means with an example of the practice in action.

Table 5

Visualization Practices for Nets

Visualization Practice	Description of the Practice	Example of Practice
Imagining Flipping *	When the student describes being able to see the net flipping in their mind or evidenced through pointing to parts of a net.	ST6: "So if you take this [<i>uses pencil to point to top of net and makes flipping up motion</i>] and you flip it up...
Seeing sameness*	When the student describes seeing similarities between the net they are looking at and other shapes.	ST1: "Because because it looks like a almost like half of a square [<i>uses pencil to trace the first net</i>]."

Imagining the final product *	When the student is able to describe how they imagined what an unfolded net would look like when it was put together.	ST6: “...it would be the roof of it”.
Counting # of sides	When the student counts the number of sides on a net.	ST5: “This one, [points to the 3rd net] um because well there’s 4 sides , there’s 4 sides here [uses pencil to point to the 4 sides on the 3rd net]
Mapping	When a student maps out/ points to different parts on the net and to the corresponding point on the 3D object.	ST6: “That one, this would be right here, that one would be there, that one would be there and this one would be over here”.
Instinct	When a student cannot use mathematical reasoning to justify their answer but rather uses their instincts (meaning the student describes an innate feeling toward a particular answer, or claims they have seen it before, or it seems familiar).	ST5: “It just looks like it would be this one”.

*These practices were used in more than one main task.

The first visualization practice I observed was *imagining flipping*. This practice occurred when a student described being able to visualize in their mind the different parts of the net flipping up (e.g., when ST5 was explaining why they chose that particular net, they said “you just need to flip it”). This first practice attends to three of the four aspects of spatial visualization- seeing an object in one’s mind, mental manipulation and imagining an outcome.

The next practice students used in task 2 was *seeing sameness* – the perception-based practice that also occurred in task 1. When using this practice in task 2, students were able to

make out geometric shapes within the nets. For example, when ST3 was explaining their thinking for choosing one net they said, “Because there’s lots of triangles and I know it’s not this one because there are no squares in it.”

The third practice for task 2 was *imagining the final product*. Students were characterized as using the practice of *imagining the final product* when they described what they thought the final product would look like, often drawing connections to objects from their lives (e.g., “it would be the roof of it”). This third practice draws upon the third aspect of spatial visualization, imagining an outcome.

The fourth practice for task 2 was *counting the number of sides*. This is when a student counted the number of sides a net had (e.g., “there’s four sides...”). This practice helped students determine which net matched. They counted the number of sides the net had and counted the number of sides the three-dimensional figure had. If they were the same, it helped the student make their decision (to help with the decision making, the students had to use their knowledge of spatial relationships, the fourth aspect of spatial visualization).

The fifth visualization practice was *mapping*. This occurred when a student pointed to a specific part of the net and then pointed to where that part would be on the three-dimensional object (e.g., ST3 pointed to the bottom of the square pyramid and then the square shape on the net and said “it has the same bottom”). Although this visualization practice is similar to *seeing sameness*, it does differ. *Seeing sameness* means the child sees similarities between shapes/forms. *Mapping* is the practice where the student physically points to specific parts. Although they are seeing what is similar, in this practice, what is important is the actual act of pointing to parts of an incomplete form (net) and then to the corresponding parts on the

completed three-dimensional object. In this task, the visualization practice of *mapping* corresponds to the fourth defining aspect of spatial visualization, spatial relationships. The child is able to see the relationship between the net and the three-dimensional object.

The last visualization practice for the second task was *instinct*. This practice occurred when a student was unable to use mathematical reasoning to explain their thinking but rather relied on a “gut feeling” or explained having seen the same answer before somewhere else. For example, when asking ST5 why they chose their net, they responded, “It just looks like it would be this one.” ST5’s imagined outcome was based on perception of the object rather than the mathematical properties.

Trends in Difficulties and Visualization Practices for Task 2. For task 2a, the students were asked to look at an image of a cube and then choose one possible net solution out of the three provided (see Figure 3 in Chapter 3). Four students were able to answer this task correctly, specifically: ST3, ST4-A, ST5 and ST6. For the four students who answered the task correctly, all of them used the visualization practice of *imagining flipping*. The two students who answered incorrectly (ST1 and ST4-B), did so differently. ST4-B used the practice of *instinct* to defend their answer. When I probed further on why ST4-B chose those particular nets (ST4-B chose two different nets), they stated, “I’ve seen a cube unfolded many times like that.” That student did not try to imagine flipping it or imagine the final product; rather, they used their past experience to guide their answer. In contrast, ST1 stated that they chose their answer because it looked “like almost half of a square.” I characterized ST1’s thinking with the visualization practice of *seeing sameness*. This corresponds with the Van Hiele theory mentioned early in Chapter 2, that younger or inexperienced students tend to focus on the physical appearance before they focus on

the mathematical properties. ST1, being the youngest of the participants, said they had chosen that net because they thought it looked similar to the cube. This particular task allowed me to see that students who successfully completed this task were only able to do so because they were able to mentally flip the image (this is the mental manipulation aspect of spatial visualization).

In task 2b, I used the same procedure of asking students to look at a three-dimensional shape and select its corresponding net. The three-dimensional shape for 2b was a square pyramid (see Figure 3 in Chapter 3). 100% of the students were able to complete 2b successfully. When looking at how the students answered the question, none of the students used all of the same visualization practices. All of the students used multiple visualization practices to complete task 2b, with the exception of ST4-B who just used one practice, *seeing sameness*. When I pressed ST4-B on why they chose that net, their answer focused solely on the geometric shapes (e.g.: “Well, there’s a square in it and that was the only one with a square”). ST1, ST3 and ST6 used the practice of *imagining flipping*. ST1, ST4-A and ST5 all *counted the number of sides* that the net had. ST3, ST4-A, ST4-B and ST5 all *saw sameness* between the net they chose, and other geometric shapes, some of which were present in the three-dimensional pyramid. In addition, ST3 and ST4-A used *mapping* to help explain their thinking for why they solved task 2b in the way that they did. ST4-A and ST6 both used the practice of *imagining the final product*. Task 2b highlighted the fact that multiple visualization practices were used in order to solve task 2b correctly.

Task 2c, which was an octahedron (see Figure 3 in Chapter 3), proved to be the most difficult for the students. 50% of the students were unable to complete this task successfully. From the three students who answered correctly (ST3, ST4-A and ST5), there was no

visualization practice that was used unanimously. ST3 and ST4-A both tried to *imagine the final product*, whereas ST5 relied on their *instinct* to defend their answer. Among the three students who answered incorrectly (ST1, ST4-B and ST6), ST1 kept repeating that they did not know which one it was, and when they chose one, stated they did not know why they chose it. Both ST4-B and ST6 used the practice of *seeing sameness*. When asked why they chose the first net option, they both replied that it was the only one with a square in it so it could not be any other option. This task highlighted the fact that sometimes students could get stuck because they focused on perceptual practices and that could impact their answer, as well as their reasoning process.

Task 3: Rotation

Visualization Practices for Task 3. For the third task, students were asked to perform three activities, all of which included rotation of the whole object. In task 3, I characterized the nature of students' spatial visualization with six practices: *imagining rotation*, *imagining the final product*, *justification based on appearance*, *equality*, *imagining rotation of a point* and *mapping*. Table 6 outlines the name of the visualization practice and provides a description of what that practice means with an example of the practice in action.

Table 6*Visualization Practices for Rotation Tasks*

Visualization Practice	Description of Practice	Example of Practice
Imagining rotation	When a student describes how they imagined a rotation.	ST4-A: “so if I turned it like this [<i>same hand movement as before</i>] his head would be here”
Imagining the final product *	When the student is able to describe how they imagined what the triangle or the little person would look like.	ST6: “Cause this guy, [<i>points to original image</i>] if you flip him over [<i>uses pencil to make a flipping motion</i>] this arm should be the other way, [<i>points to original image’s arm out in front</i>] like that one, [<i>points to the arm in front on the image ST1 chose</i>] and this one should be opposite like that [<i>points to the other arm on the original drawing and makes a motion to show how the arm would look</i>] and then his legs.”
Justification based on appearance *	When a student doesn’t use mathematical reasoning but gives a reason to their answer based on their perception of what the object should look like.	ST5: “Well, because these sides [<i>points to left and right side of first triangle image</i>] they’re not perfectly straight they go upwards [<i>traces finger along triangle line upwards</i>] so they wouldn’t be like touching the bottom here [<i>points to the bottom of the drawing</i>] so they would go more upwards.”
Equality	When a student is able to express that they see equal sides.	ST6: “Cause they all have equal sides here, [<i>uses pencil to point to sides of triangle</i>] so if you flip it over [<i>uses pencil to make a flipping motion</i>] it will still have the same sides.”
Imagining rotation of a point	When a student imagines rotating a single point.	ST6: gesture: <i>points to the tip of the triangle and rotates it.</i>
Mapping*	When a student focuses on specific pieces of the drawing, and points to where they should	ST6: “this arm would be here...”

be on the original.

Note: *These visualization practices were used in more than one main task.

The first practice, *imagining rotation*, occurred when students described how they mentally saw a rotation inside of their head (an important defining aspect of spatial visualization). When I say rotation, I am speaking about how the students were able to turn the object inside of their head. For example, when I asked ST6 how they knew the triangle would look like it did, they stated, “you just rotate it like that.”

The second visualization practice observed was *imagining the final product* – a practice that was also used by students in task 2. In task 3, *imagining the final product* occurred when the student was able to see what something (the outcome) would look like before it was finished.

The third visualization practice that was used for task 3 was *justification based on appearance* (note this practice was also used in tasks 4, 5 and 6. More information to follow.). Similar to what the name suggests, students were characterized as using this practice when they were unable to use mathematical reasoning and instead focused on their own perception of what an object should look like. For example, ST5 answered the question of rotating the equilateral triangle correctly, but when asked how they knew that the triangle would look like that, they said, “these sides... they go upwards so they wouldn’t be like touching.” Although this practice entailed students imagining an outcome, the way they did so was purely based on holistic perception.

The visualization practice of *equality* was used when students expressed that they saw equal sides (e.g., “they all have equal sides”). The idea of equality helped students to draw the

triangle, because they knew they just had to replicate the same image since all the sides were the same. This process drew upon three aspects of spatial visualization- seeing an object in one's mind and imagining an outcome. This was particularly relevant to task 3a because the triangle was equilateral. This visualization practice highlighted which students were able to acknowledge the mathematical property that an equilateral triangle has all equal sides, and which students instead focused on perception.

Imagining rotation of a point occurred when a student imagined turning a single point of an object/shape and not the object or shape in its entirety (e.g., ST6 pointed towards the tip of the triangle and said, "Well cause you take it like this and you just rotate it like that"). This practice relates to spatial visualization aspects of seeing an object in one's mind, mental manipulation and imagining an outcome.

The last practice for task 3 was *mapping* – a practice that students also used in task 2. This practice occurred when a student pointed to a specific part of an image and then pointed back to where that part would be on the original image. For example, when asking ST3 why they chose the specific image of the little person that they did (see Figure 4 in Chapter 3), they pointed to a leg on the incorrect answer and pointed to the leg on the original image and said, "This leg would be up."

Trends in Difficulties and Visualization Practices for Task 3. In task 3a, students were asked to look at an equilateral triangle, imagine rotating it in their head and then draw what they thought it would look like once it was turned (see Figure 4 in Chapter 3). 100% of the students were able to answer this task correctly. However, only 50% of the students used the same visualization practices. ST4-A, ST4-B and ST6 used the practices of *equality* and *imagining*

rotation. Those three students knew that the triangle would look the same after the rotation because it was an equilateral triangle, meaning all three sides were equal or the same. Additionally, even though the students said they knew it would look the same because of its mathematical property, they also explained how they imagined rotating the triangle mentally. Among the other three students, ST1 used the practice of *imagining rotation* but did not seem to understand that the triangle would look the same because of equality. ST3 used the practice of *equality* and said that they did not need to imagine the actual rotation because they knew that it would look the same since all the sides were equal. ST5 used the practice *justification based on appearance*. ST5 went into detail of how the triangle would look slightly different because they created it by hand.

For task 3b, students were asked to look at an isosceles triangle (see Figure 4 in Chapter 3), and then repeat the same steps as in task 3a. For this task every student except for ST1 was able to answer correctly. ST1 used the visualization practice of *justification based on appearance*. They drew an identical triangle beside the original isosceles triangle and just stated they would look the same. When I asked why, ST1 said they just would. For the students who answered 3b correctly, three of them used the practice of *imagining rotation* (ST3, ST4-A and ST4-B). ST6 imagined a rotation but used the practice of *imagining rotation of a point*. ST6 specifically used the top of the isosceles triangle to rotate and help them answer this task. Interestingly, ST5 gave the correct answer however, when I probed into why they drew the isosceles triangle in that manner, they used the practice of *justification based on appearance*. It appeared that the student was focusing too literally on their drawing. They stated, “well, because these sides they’re not perfectly straight.” ST5 drew the triangle in the correct way but seemed to

focus more on how well they drew the triangle rather than the fact that they had to rotate the triangle and draw it with the tip pointing to the right.

For the last task in the rotation category, the students were asked to look at a picture with a little person (see Figure 4 in Chapter 3) and then imagine that they rotated that person inside their mind. Afterwards, they needed to choose one of the three images displayed for what they thought the rotated person would look like. 100% of the students were able to answer this task correctly. All six students used at the minimum the visualization practice of *imagining rotation*. All of the students, with the exception of ST4-B, also used the practice of *mapping*. The students would often use different body parts of the person to help them orient where the person should be facing. ST3 and ST4-A additionally used the practice of *imagining the final product*. Both ST3 and ST4-A pointed to the other two possible answers and spoke to how they did not match the original person, so they could not be considered for the final product.

Task 4: Comparison of 3D objects

Visualization Practices for Task 4. In task 4 students were asked to recreate two different figures made of blocks. For this task, I characterized the nature of students' spatial visualization with three practices: *justification based on appearance*, *focusing on groups*, and *focusing on single blocks*. Table 7 outlines the name of the visualization practice, a description of what that practice means and an example of the practice in action.

Table 7

Visualization Practices for Comparison of 3D Objects Tasks

Visualization Practices	Description of Practice	Example of Practice
Justification based on appearance *	When the student's focus is on non-mathematical features like the color.	ST4-B: "They look the same".
Focusing on groups	When a student looks at groups of blocks together rather than focusing on individual blocks.	ST3: "There's 6 of these [<i>runs finger along the base of the model and the reconstructed figure</i>]. And there's 2 of these [<i>places finger on top of the 2 blocks that come up a bit higher on the base</i>]."
Focusing on single blocks	When the student focuses their attention block by block, rather than on a group of blocks.	ST3: "There's one down here [<i>points to the empty space on the reconstructed figure where a block is missing</i>]"

Note: *This visualization practice was used in more than one main task.

For task 4, students used three different practices. The first practice *justification based on appearance* was also used in task 3 and task 5. This practice occurred when students focused on appearance (e.g., "they look the same").

The second practice was *focusing on groups*. This occurred when the student focused on a group of blocks when explaining their reasoning. For example, when asking ST4-A why they thought there were differences between their recreated figure and the original, they stated that they tried to focus on groups since they only had 10 seconds to look at it, "I just tried to count like the differences so four... and I thought this was only two...". For this second practice, the spatial visualization aspect of seeing an object in one's mind was essential. If the student was not able to see the object in their mind, it would be difficult for them to recreate it afterwards once the object was hidden.

The last practice for task 4, *focusing on single blocks*, occurred when students focused on individual blocks when explaining their reasoning (e.g., “this one is like that”). At times, students would compare the single blocks when explaining their thinking. This showed the defining aspect of spatial relationships since students were relating individual blocks to one another.

Trends in Difficulties and Visualization Practices for Task 4. In task 4a, students were shown a three-dimensional figure (see Figure 5 in Chapter 3) for 10 seconds. After the 10 seconds, the figure was hidden, and the students were asked to try and recreate it from memory. For task 4a none of the students were able to complete it successfully. All of the students, with the exception of ST1, used the visualization practice of *focusing on groups*. ST4-B and ST6 also used the visualization practice of *justification based on appearance*. ST6 along with ST5 used *focusing on single blocks*. When asking the students what the differences were between the original and their recreation, all of the students except for ST1 spoke about specific groups of blocks that were not matched correctly. Additionally, these same students stated that they tried to focus on groups within the 10 seconds they were shown the object to help them remember what it looked like. ST1 did not use any of the four practices of visualization and just responded that everything was different. When asking ST1 how they tried to solve the task, they said they did not know.

In task 4b, the students were again asked to look at a figure (see Figure 5 in Chapter 3), but this time a bit more complex, and try to recreate it. This time however the figure was not hidden, the students could look at the figure and recreate it at the same time. 100% of the students were able to successfully complete task 4b. When asking the students how they solved

this task, ST1, ST4-B, ST5 and ST6 said they all used the practice of *justification based on appearance*. When speaking about the similarities between their own models and the original figure, these students spoke about colors or how they “just looked the same.” ST4-A stated that they completed the task by *focusing on the groups* and also used this practice when describing the similarities between the original and their recreation. ST3 used the practice of *focusing on single blocks* to recreate their figure and pointed to individual blocks when stating the similarities between the two objects. Task 4b proved to be too easy for the students (this will be discussed in more detail in Chapter 5).

Task 5: Flips

Visualization Practices for Task 5. In task 5 students were asked to take a piece of 8 ½ x 11-inch paper and fold it in half. They then were instructed to fold the paper in half again and to punch a hole in the top right corner (see Figure 6 in Chapter 3). Afterwards students were asked to choose one answer from the possible five options (see Figure 7 in Chapter 3). For task 5, I characterized students' spatial visualization through two practices: *imagining flipping* and *justification based on appearance* (see Table 8).

Table 8

Visualization Practices for Flipping Task

Visualization Practice	Description of Practice	Example of Practice
Imagining flipping *	When the student describes being able to see the paper flipping in their mind or evidenced through pointing to parts of the paper.	ST4 “Because it’s folded in half so there would be 2 dots like this and if we unfold it again there would be 2 dots on the other side.”

Justification based on appearance *	Students focus on what they think the image will look like rather than the concept of the flips.	ST3 “Because the dots[points to holes on hole punched paper] on this one [points to first image option] are there [points to corner of hole punched paper] and then these ones the dots are there [points to the last image option and to the holes on the hole punched paper] and I thought they would be there [points to edge of hole punched paper]”
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Note: *These practices were used in more than one main task.

Both of the visualization practices used in task 5 overlap with other main tasks. The practice of *imaging flipping* was used in task 2. However, in task 5, it referred to when a student was able to mentally see the paper unfolding. This was evidenced by the students’ hand gestures (folding and unfolding hand movements) and verbal descriptions. For example, when asking ST6 how they knew which option to choose they said, “If I were to unfold it that way there would be two like this, and then I would unfold it that way and there’d be another two on this side.” Additionally, ST6 used their hands to make gestures to imitate as if they were unfolding the paper.

For the practice of *justification based on appearance*, this occurred when a student focused on what they thought the object would look like based on general appearance of the paper (e.g., focusing on the holes in the corners), without describing how it got to the final state. For example, when asking ST4-A why they chose their answer they said the holes were “...in the corner and because it’s four, there would be four, because it’s folded in four.” This practice can be similar to counting sides as seen in previous tasks; however, unlike in counting, where the student mapped from one object to another, here the student focused on overall appearance, which could be signified by the number of holes.

Trends in Difficulties and Visualization Practices for Task 5. In task 5, only two students were able to complete this task successfully (ST3 and ST6). Both of the students who answered correctly did so by using the visualization practice of *imagining flipping*. ST3 and ST6 explained to me that because the paper was folded in half twice that meant that there would be four holes and that since it was folded in half, that meant that what was done on one side had to be done on another. For example, when asking ST3 how they knew which option to choose they said, “... it’s folded in half there’s two dots here so there is going to be two dots on the other side if you unfold it again”. The four students who had the incorrect answer (ST1, ST4-A, ST-B, and ST5) all used the practice of *justification based on appearance*. When questioning the students on why they chose the option that they did, every student understood that there would be four holes, since it was folded, but they said that they punched a hole in the corner, therefore once unfolded the holes would appear in the corners. For example, when I asked ST4-A why they chose the answer that they did, ST4-A replied “...none of these have gone into the corners at all, and this one would be in the corner.” Here, ST4-A was talking about the other options. Since only one of the five options had holes in the corner, ST4-A reasoned it could not be any of the other four options. Similar to task 2c, task 5 shows how students often focus on perception which could in turn make it hard for students to visualize in their mind an alternative answer.

Task 6: Perspective

Visualization Practices for Task 6. For the final task, students were asked to look at a picture of one dinosaur in front of two other dinosaurs (see Figure 8 in Chapter 3). The students were then asked to choose from three images provided in order to decide what the first dinosaur could see (see Figure 8 in Chapter 3). In order to characterize the students’ spatial visualization

for the final task, I identified three visualization practices: *direction the object is facing*, *imagining the extent of peripheral view* and *justification based on appearance*. Table 9 details the name of the visualization practice, a small description of that practice and an example of the practice in action.

Table 9*Visualization Practices for Perspective Task*

Visualization Practice	Description of Practice	Example of Practice
Direction the object is facing	Student justifies the answer with the direction the object is facing.	ST5: “Because his head [<i>pointing to Jerry’s head</i>] is facing towards that one [<i>takes finger and drags it from Jerry to the other dinosaur that is in Jerry’s line of sight</i>].”
Imagining extent of peripheral view	The student imagines what the dinosaur might be able to see and what they might not be able to see based on their understanding of peripheral vision/view.	ST6: “...well I’m guessing his view from this side [<i>points to the side of the dinosaur and again traces in the air a line of sight</i>] would only go about to here, so it wouldn’t go all the way to see that one [<i>points to the dinosaur that can’t be seen</i>].”
Justification based on appearance *	Student does not take into account the point of view of the dinosaur. They acknowledge solely what they see.	ST1: “Because this one [<i>circles the dinosaur on the left in the first image option</i>] looks like this one [<i>points to the dinosaur on the left in the original image</i>] and this one [<i>circles the dinosaur on the right in the first image option</i>] looks like this one [<i>points to the dinosaur on the right in the original image</i>].”

Note: *This visualization practice was used in more than one main task.

For task 6, I identified three different visualization practices. The first practice was the *direction the object was facing*. In this practice, students justified their answer by looking at the direction in which the main object was facing (e.g., “because his head was pointing towards the

dinosaur...”). This visualization practice aligns with seeing spatial relationships (an aspect of spatial visualization). In this first practice, students need to take into account the spatial relationship between where the main dinosaur is located, where the other two dinosaurs are located, the location between the three dinosaurs, as well as the direction that the main dinosaur is looking. Ignoring the spatial relationship between the dinosaurs would result in an incorrect answer.

The second practice, *imagining the extent of peripheral view*, occurs when students see from another perspective. In this particular practice, students imagined what someone else (in this case a dinosaur) might or might not be able to see based on where the dinosaur was located, not where the student was located (e.g., “I know his peripheral view wouldn’t go like that”). This visualization practice uses three of the four aspects of spatial visualization: seeing objects inside of one’s mind, imagining an outcome and spatial relationships. The students needed to be able to see the dinosaur inside their mind, decide the extent to which the dinosaur could see (spatial relationship) and then imagine what the dinosaur would see (imagine the outcome).

The final practice for task 6 is *justification based on appearance*. This practice was used in tasks 3, 4 and 5, but in task 6 this meant that students did not take into account the point of view of others; they only acknowledged what they saw, focusing on the holistic appearance of the arrangement of the dinosaurs. For example, when I asked ST3 how they knew that the main dinosaur saw that, they pointed to the two dinosaurs in front of the main dinosaur (see Figure 8 in Chapter 3) and said, “There is one dinosaur here and one dinosaur laying down here.” ST3 did not acknowledge that since the main dinosaur's head was facing only one dinosaur, they would not be able to see both.

Trends in Difficulties and Visualization Practices for Task 6. In task 6, 67% of the students answered correctly. For the students who answered correctly (ST4-A, ST4-B, ST5 and ST6), all of them used the visualization practice where they assessed the *direction the object was facing*. The four students understood that since the main dinosaur was pointed to the right, they would only see one dinosaur in front of them, not both. Additionally, ST4-A and ST6 used the practice *imagining the extent of peripheral view*. Both of the students stated that because the main dinosaur was pointed extremely to the right, the dinosaur on the left would not be in view because it was too far away. The two students who answered incorrectly (ST1 and ST3) did so in the same manner. Both of the students used the visualization practice of *justification based on appearance*. Neither ST1 nor ST3 paid attention to which direction the main dinosaur was facing. When asking them to explain their answers, both students made reference to the dinosaurs in the original picture (see Figure 8 in Chapter 3) and then pointed them out in the picture option that they chose (see Figure 9 in Chapter 3).

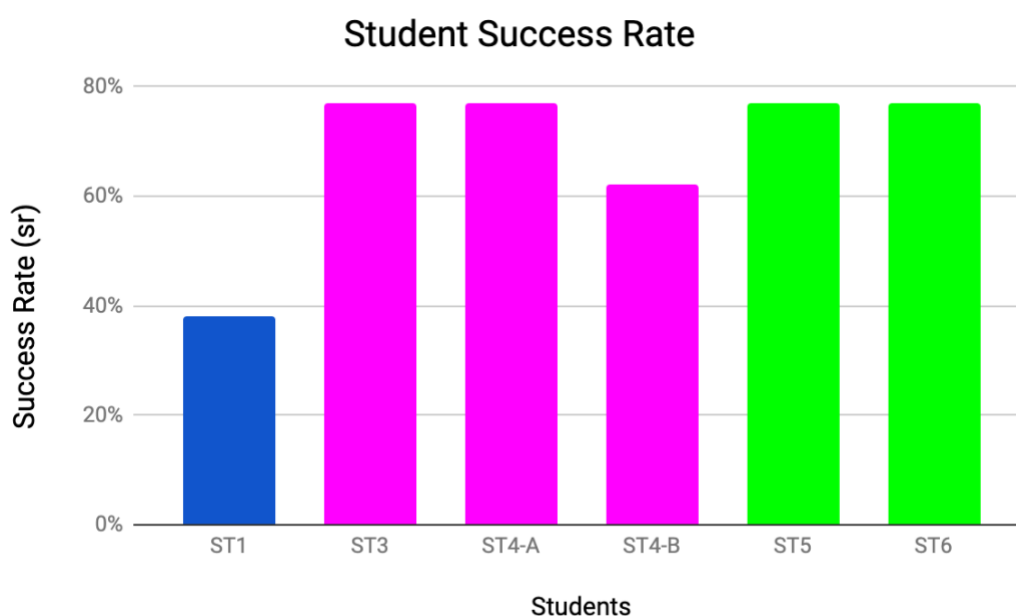
Differences in use of Spatial Visualization Based on Cycles

This section of my thesis focuses on answering my third research question (RQ3): What are the differences between early, middle and late elementary students, in how they use spatial visualization when solving geometric tasks? As discussed earlier, in Quebec, elementary education is separated by cycles. Cycle 1 encompassing grades one and two, cycle 2 being made up of grades three and four, and cycle 3 being the last part of elementary with grades five and six. For my study I only had one student in cycle 1, three in cycle 2 and two in cycle 3 (I will speak more to this as a limitation of my study in Chapter 5).

To help me analyze the differences between early, middle and late elementary students, I looked at their success rate (sr). Figure 9 showcases each student's success rate and compares it to the result of the other students. The same color columns show students in the same cycle.

Figure 9

Student Success Rates



As we look across the three cycles, we see that the lowest success rate was in early elementary (cycle 1) and the highest average success rate was in late elementary (cycle 3). However, what is interesting to note is that ST3 and ST4-A (two students from cycle 2) and ST5 and ST6 (the two students from cycle 3) all had the same success rate. However, even with the same success rate, the students used a variety of different visualization practices and their level of explaining was diverse. From my results, there seemed to be a relationship with higher success rates in task completion and more visualization practices used.

Additionally, a higher success rate was shown when the students used visualization practices that encompassed some sort of imagining (*imagining composition or decomposition, imagining transformation, imagining flipping, imagining the final product, imagining rotation, imagining extent of peripheral view*). For example, in task 5, the two students who were successful (ST3 and ST6), used the visualization practice of *imagining flipping*. The other four students answered incorrectly and did not use a visualization practice that included imagining. An important thing to note is that all 17 visualization practices include imagining an outcome, which is one of the aspects of spatial visualization. Additionally, with the exception of the visualization practices: *imagining the final product* and *imagining extent of peripherals view*, the other 15 practices consisted of the aspect of spatial visualization of mental manipulation.

Cycle 1

For the one student in cycle 1, they used 12 visualization practices to help them answer the tasks that were asked during the interview. Most of the visualization practices that were used relied on perception, such as *seeing sameness* and *justification based on appearance*. These two practices were used 58% of the time. As noted earlier in chapter 2, this aligns with the Van Hiele theory that with less experience, younger students tend to focus on what they see rather than the mathematical properties. This could be in part because they have not learned the properties prior to this interview or because they do not know how to attend to them as of yet. Additionally, when I asked ST1 to explain their thinking, they often answered with, “I don’t know”.

Cycle 2

For cycle 2, there were three participants. One in grade three (ST3) and two in grade four (ST4-A and ST4-B). ST3 and ST4-A had the same success rate and between them used an

average of 22.5 visualization practices to complete the tasks. ST4-B had a lower success rate and used 15 visualization practices while solving the interview tasks. Students in cycle 2 had a greater success rate than the student in cycle 1 and used more visualization practices. In addition to the use of more practices, the type of practices used varied more. Students in cycle 2 were able to use mathematical properties when solving the tasks. This was evidenced by the use of visualization practices such as *equality*, *imagining the extent of peripheral view*, *imagining composition* and *imagining decomposition* (in contrast, ST1 used 0% of these practices when solving the different tasks). When asking students in cycle 2 to explain their thinking, they would often point to specific parts of an object and explain how they saw something. For example, when asking ST4-A how they knew what the smallest number of blocks needed to fill the empty space (see Figure 2 in Chapter 3) was, they responded, “So I just add them, because I see this [*points to shape*] going in here.” In contrast, ST1 responded to the same question by saying, “because it looked like like this form here.”

Cycle 3

The students in cycle three (ST5 and ST6) had the highest common success rate. Although ST5 and ST6 had the same success rate as ST3 and ST4-A (two students from cycle 2), overall when comparing the two cycles, cycle 3 was the most consistent. ST5 and ST6 each used 21 practices of visualization when solving the geometric tasks. Their visualization practices extended a little further than cycle 2 students’ practices. For example, in task 2a (the nets task for the cube, see Figure 3 in Chapter 3), both students from cycle 3 were able to use the visualization practice of *imagining the final product*. None of the other students from cycle 1 or cycle 2 used that practice and instead mostly relied on *instinct* or *seeing sameness*. When asking students in

cycle 3 to explain their thinking, they were able to not only point to different aspects of shapes but were also able to articulate their thinking, spatial reasoning processes and answers in a more comprehensive manner. For example, when ST6 was explaining why they chose their answer to task 5 (see Figure 7 in Chapter 3), they stated, “Cause I folded it this way, so if I were to unfold it that way there would be two like this, and then I would unfold it that way and there’d be another two on this side.” Comparatively, ST1 (student in cycle 1) answered the same question with “Cause... I don’t know”.

When comparing students from early, middle and late elementary school, it is clear that the students in late elementary school were able to complete more tasks successfully than the students in early and middle elementary school. Additionally, the results showed that not only did students in higher grades complete tasks with a higher success rate, they also used a larger repertoire of spatial visualization practices. The higher number of visualization practices corresponded with more correct answers. This was evidenced by looking at the two students with the lowest success rates (ST1 and ST4-B) and the number of visualization practices used (12 and 15 respectively), compared to the students with the higher success rates (ST3, ST4-A, ST5 and ST6), who all used over 20 visualization practices when completing the geometric tasks.

Embodied Resources and Spatial Visualization

For this portion of the results, I will share findings related to my fourth research question (RQ4): What embodied resources do students draw upon to engage in spatial visualization when solving geometric tasks? I will present the trends between correct/ incorrect answers and the accompanying gestures used. Afterwards, I will present an in-depth comparison between ST1 and ST6 and focus on the nuances in correct answers, and the use of gestures, or lack thereof.

Trends between Answers and Gestures

When analyzing the results of my study, there was no clear relation between occurrence of gestures and right answers. In some tasks, gestures were used, and the students answered correctly. For example, in task 2b all of the students were able to complete the task successfully and they all used gestures to do so. However, in other tasks, such as task 4a, all of the students answered incorrectly and 83% of them used gestures.

On the other hand, students' gestures were tied intricately to their explanations. Over multiple tasks, students would point to certain parts of a net or a shape and use that to help them explain. Had the students not pointed and only described with words, a lot of their thinking would be unclear. For example, during task 2a (see Figure 3 in Chapter 3) the students had to choose a net for the corresponding three-dimensional cube. Four of the six students (ST3, ST4-A, ST5 and ST6) used the visualization practice of *imagining flipping*. However, when they were explaining what they did, they also spoke about *flipping* the various parts of the net. Without their pointing and gestures, it would have been impossible to understand. For example, when ST6 explained their thinking they said, "So if you take this and you flip it up, and that one flip it up, and that one flip it up, and flip it up, it would look like that from the top." Without the accompanying gestures, we would not be able to tell which parts ST6 was flipping up and what they meant by "it would look like that from the top."

When looking at the explanations that students gave to their solutions, the students who used gestures tended to elaborate more and provide justifications. For the students who did not use gestures, a lot of their answers were "I don't know" or they used their instinct. For example,

ST4-B used no gestures when solving 1c. When I asked about their thinking, they answered, “I don’t see any other way.”

Comparison of ST1 and ST6

For this next part of my results, I will compare ST1 and ST6 in order to highlight the types of gestures that appeared supportive of visualization. As described in Chapter 3, I chose these two particular students because of the differences in the type and number of visualization practices used as well as the usage of gestures, or lack thereof. The results of the comparison between ST1 and ST6 show that gesture is important in (a) the role of manipulation of tools in shaping the visual plane, (b) the importance of gesture in helping students focus on isolated mathematical properties and their motion, and (c) the importance of gestures that match the motion being visualized. In the paragraphs to follow, I will examine three different tasks and how each student used gestures when completing the task.

Task 1c. In task 1c, students were given pattern blocks in order to recreate a square (see Figure 2 in Chapter 3). ST1 took three triangles, placed them on the table and then stopped and said, “Mm, how like how?” ST1 did not rotate any blocks or compare them to other blocks. They took one rhombus and four triangles but when they saw that that did not work, they went on to say, “I can make a sun” and then created a sun with the blocks provided. ST1 did not try to manipulate the blocks. Rather, after they tried an idea, they clasped their hands in front of them. ST1’s lack of gesture limited their visual plane, which may have led to fewer opportunities for ST1 to see different configurations.

Conversely, ST6 took the pattern blocks and rotated them in different directions and said, “This one there, that goes there, wait that would be more like a rectangle. No wait, if I do this...

That. That'd be a rectangle too. These two... Hmm that wouldn't work." Additionally, ST6 tried using the same shapes together as well as mixing them with other types of shapes. Since ST6 was continuously moving the blocks, this provided more opportunities to imagine an outcome and mentally manipulate the objects (two components of spatial visualization).

As the task continued, I asked both students if they felt that there were more ways to make a square, but they were not able to do it because of the blocks they were given. ST1 did not answer the question whereas ST6 stated, "[you would] need like... an isosceles triangle that would be like cut off like here." Additionally, ST6 then moved various shapes together and said that if they had an isosceles triangle then, "this would be like straight here (ST6 had pointed to the top of the shape to indicate that it wouldn't cave in but rather it would be straight allowing for a straight line to appear) instead of it going like that." The continuous use of the student's hands allowed them to construct a multitude of various shapes and expanded their field of vision.

Task 3b. For task 3b, students were asked to look at an isosceles triangle (see Figure 4 in Chapter 3), imagine rotating it in their head and then draw what they thought the rotated version would look like. ST1 drew the same image of the triangle beside it and stated that it would look the same. They did not use any visualization practices or gestures and answered the question incorrectly. When questioned further, ST1 did not answer.

In contrast, ST6 drew the triangle with the tip pointed towards the right and answered the question correctly. When questioned on how they knew it would look like that, ST6 took their pencil and pointed to the tip of the triangle. They then made a line from the tip of the triangle downwards and said, "Well cause you take it like this, and you just rotate it like that, this part here would be there...". ST6 was able to express in their speech the rotation that occurred but

also through pointing with their pencil and making that downward motion. The gesture of pointing with their pencil allowed the student to isolate a part of the object and see how it would move. The comparison of ST1 and ST6 in this task showed that gestures were important in guiding the visual field towards focusing on the mathematical properties and imagining the isolated motion of those properties.

Task 5. Finally, when both students made gestures, there were differences in whether their gestures were aligned or mismatched (Breckinridge Church & Goldin-Meadow, 1986). For example, in task 5, I asked ST1 why they thought the unfolded paper would look like the option they chose (see Figure 7 in Chapter 3) and they said, “Because it’s like the corners” and moved their hands around in a circular motion in front of them. ST1 believed the holes would be in all four corners, yet the gesture did not mimic what they said; it was mismatched.

ST6, however, used their hands to show an unfolding motion and then pointed to two holes on the image they chose and said, “Cause I folded it this way, so if I were to unfold it that way there would be two like this.” ST6 then repeated the unfolding motion with their hands and pointed to the other two holes on the image of their choice and said, “...and then I would unfold it that way and there’d be another two [holes] on this side.” ST6’s gestures were directly related to their thinking and helped explain their reasoning process. This alignment may have assisted ST6 in seeing the process of unfolding, leading to the correct answer (or at least confirm this process).

Summary of Chapter 4

In chapter 4, I presented the different results of my study based on my four guiding research questions. I spoke to the trends in and the difficulties with spatial visualization that

students had when undertaking geometric tasks. Additionally, I outlined the differences between how students in early, middle and late elementary school used spatial visualization practices when completing the geometric tasks during the student interviews. Lastly, I spoke to my last research question, which looked at embodied theory and whether and how gestures, along with speech served as a resource for students' visualization when undertaking geometric tasks.

Chapter 5: Discussion

In chapter 5, I will speak to the purpose of my research and the four research questions that guided this study. I will then briefly summarize the results from Chapter 4. Additionally, in this chapter I will discuss the different contributions my research has made as related to the pre-existing literature on spatial visualization and how students' reason about it. Additionally, this chapter provides a discussion of the limitations of the study and areas for future research. This chapter concludes with implications for how my results may be valuable for use by teachers and curriculum designers.

Summary of Research Findings

The purpose of this study was to understand how elementary students engaged with spatial visualization when completing geometric tasks. This was done through answering my four research questions:

(RQ1) What difficulties do elementary students encounter when undertaking spatial visualization tasks in geometry?

(RQ2) What is the nature of students' spatial visualization when solving geometric tasks? In particular, I was interested in understanding the types of visualization practices that students engaged in when completing such tasks.

(RQ3) What are the differences between early, middle and late elementary students, in how they use spatial visualization when solving geometric tasks?

(RQ4) What embodied resources do students draw upon to engage in spatial visualization when solving geometric tasks?

Summary for RQ1 and RQ2

To help me answer my first and second research questions, I created different categories of visualization practices that students could use in each task (as discussed in detail in Chapter 4). The amount and type of visualization practices varied per task. In task 1, students used four different visualization practices (*seeing sameness, imagining transformation, imagining decomposition and imagining composition*). In task 2, students used six different visualization practices (*imagining flipping, seeing sameness, imagining the final product, counting the number of sides, mapping and instinct*). In task 3, students again used six practices but these differed to some extent from those used in task 2 (*imagining rotation, imagining the final product, justification based on appearance, equality, imaging rotation of a point and mapping*). For task 4, students used three practices (*justification based on appearance, focusing on groups, and focusing on single blocks*). Task 5 included two visualization practices (*imagining flipping and justification based on appearance*). For the final task, students used three visualization practices (*direction the object is facing, imagining the extent of peripheral view and justification based on appearance*). From all the different visualization practices, there were five that were common throughout different tasks (*imagining flipping, seeing sameness, imagining final product, justification based on appearance, and mapping*).

In addition to the visualization practices changing between tasks, they were also classified as either perceptual or property-based practices. The perceptual based practices were ones where students focused on what something looked like. These included: *seeing sameness, instinct, mapping, focusing on groups, focusing on single blocks, imagining the final product and justification based on appearance*. The property-based practices were those through which

students focused on the mathematical properties. These included: *counting the number of sides*, *equality*, *direction the object is facing*, *imagining flipping*, *imagining rotation* (since they need to zero in on something rotated to communicate that), *imagining rotation of a point*, *imagining decomposition and composition*, *imagining transformation* and the *extent of the peripheral view*.

In this thesis, I answered research questions one and two together because the nature of students' spatial visualization and the difficulties associated with them were connected. Throughout the study, it became clear that there were a few areas in particular that students struggled with when undertaking spatial visualization tasks in geometry. First, students struggled with the visualization practice of being able to *imagine composition*. In task 1c (see Figure 2 in Chapter 3), a task that was designed for students to perform a composition, only 50% of the students were able to use the visualization practice of *imagining composition*. Students were not able to mentally take an object and manipulate it in order to create another object.

The second difficulty that students encountered was when they relied on practices based on perception, most notably the visualization practice of *justification based on appearance*. The study shows that, at least with this sample, correctness of answers was related to the number and type of visualization practices used. Students who used practices based on perception, such as *justification based on appearance* or *seeing sameness*, tended to answer incorrectly. This appeared to be partly due to the fact that they were unable to use mathematical properties to reason but also because they were using visualization practices which focused on visual perceptions instead. In particular, for tasks 2c, 5 and 6, the visualization practices that were based on perception appeared to hinder the students in answering correctly. In task 2c (see Figure 3 in Chapter 3), the students had a choice of three nets, one of which corresponded to the

octahedron. Out of the three students who answered this question incorrectly (ST1, ST4-B and ST6), two of the students (ST4-B and ST6) did so because they used the visualization practice of *seeing sameness*. ST1 did not use a visualization practice. For example, ST4-B and ST6 felt that the three-dimensional octahedron had a square within it and used their visual perception to choose and then defend their answer. In task 5 (see Figure 7 in Chapter 3), students had to fold a paper in half twice and hole punch the top corner and then decide where the holes would appear on the paper once unfolded. Only two students were able to answer this question correctly. The other four students answered incorrectly because they were too concerned with what they thought it should look like instead of visualizing the process of unfolding, illustrated through their use of the visualization practice of *justification based on appearance*. In task 6 (see Figure 8 in Chapter 3), the students had to choose which image they thought the main dinosaur could see (see Figure 8 in Chapter 3). The two students who answered incorrectly, ST1 and ST3 did so because they used the visualization practice of *justification based on appearance*. The students looked at the first image of the main dinosaur looking across the land (see Figure 8 in Chapter 3), and they saw two dinosaurs in front of it. The students then looked for the image choice that showed the two dinosaurs (see Figure 8 in Chapter 3) in about the same position and chose that. They did not take into account that the main dinosaur's head was turned and therefore part of its vision was obscured. *Justification based on appearance* occurred in multiple tasks which suggests that students had difficulty visualizing aspects of the problems because they fixated on a holistic image or visual representation instead of attending to mathematical properties.

The two difficulties that I identified correspond with both types of intrinsic skills: intrinsic-dynamic and intrinsic-static skills. The tasks when students had a hard time imagining

composition required the use of intrinsic-dynamic skills. Moreover, when students struggled with tasks because they focused on perception rather than properties, they were using their intrinsic-dynamic and intrinsic-static skills.

Summary for RQ3

When examining the results of the study, the student in early elementary answered fewer tasks correctly, used fewer visualization practices when partaking in the geometric tasks and often focused on perception. Although four students had the same success rate (two from middle elementary and two from late elementary), the students in late elementary were more consistent with completing tasks successfully and with their visualization practices. Students in late elementary were better able to articulate their thinking whereas students in the lower elementary levels had difficulty explaining their thinking process. Additionally, the types of practices used differed greatly between early, middle and late elementary. As compared to students in late elementary, students in middle elementary school tended to use practices that were a bit more focused on perceptual rather than property-based practices.

As mentioned in Chapter 3, I used the teacher questionnaires to situate the cases to make sense of the results in relation to the kinds of learning experiences the participants had. ST1's teacher did not complete a questionnaire but having taught that same grade level before myself, I knew that the type of geometry taught pertained more to identifying shapes based on number of sides. This contrasted to the older students whose geometry learning was more advanced, as they learnt not only about shapes in relation to their sides but many other mathematical properties as well (e.g. angles and equality). Since ST1 did not have the same level of geometry learning, it may explain why the older students had more ease with certain tasks.

Summary for RQ4

My last research question focused on the embodied resources that students used to complete geometric tasks. When analyzing the results, it appeared that the use of gestures and correct answers were not related for most of the tasks.

As noted in Chapter 4, I did a fine-grained analysis of two students (ST1 and ST6), comparing their visualization practices and the gestures that they used for three different tasks (task 1c, 3b and 5). When I did an in-depth analysis of ST1 and ST6, I found three differences.

First, gesture seemed to play a role in the manipulation of tools in shaping the visual plane. This was evidenced in task 1c when the students had to use pattern blocks to recreate a square. ST1 did not try to continuously use the blocks to make different arrangements and therefore their visual plane was limited. In comparison, ST6 continuously manipulated the pattern blocks in various ways, thus creating a larger visual plane which allowed for different arrangements of the pattern blocks.

Second, gestures also seemed important in helping students focus on isolated mathematical properties and their motion. In task 3b, the triangle that the students had to mentally envision and then rotate was an isosceles triangle (different from 3a where the sides were all equal, here they were not). When completing this task, ST1 said that the rotated triangle would look the same as the original triangle. They did not understand that the sides were not equal and therefore the image would look different. Conversely, ST6 knew that the triangle would look different. They pointed to the tip of the triangle with their pencil and then drew a line downwards while explaining how they rotated the triangle.

Finally, I found the importance of gestures that match the motion being visualized. ST1's gestures were mismatched to what they were saying. The word mismatched refers to the idea that the information that is being communicated in the gesture does not match the information that is being conveyed with the accompanying speech (Breckinridge Church & Goldin-Meadow, 1986). Research has shown that mismatches between gesture and speech indicate that the child does not fully understand the specific concept they are talking about (Breckinridge Church & Goldin-Meadow, 1986). This appeared to be the case with ST1. For example, in task 5 (see Figure 7) ST1's gestures did not match their words when trying to explain why they thought the holes would appear in the corner. Instead, ST1 moved their hands around in a circular motion in front of them but said, "Because it's like the corners." Once ST1 unfolded the paper and saw that their answer was incorrect I asked them to try and make sense of the correct answer. I asked them "Why do you think it looks like that?", indicating to the unfolded paper. Rather than trying to recreate the gestures, ST1 said, "cause". In contrast, ST6's gestures mimicked what they were saying.

Contributions of the Research

My research has four distinct contributions: (a) the novelty of my framework of visualization practices; (b) continuing to establish a link between spatial skills and mathematical performance; (c) finding difficulties elementary students have when using spatial visualization to partake in geometric tasks; and (d) furthering existing understanding of the relation between gesture and cognition.

Novelty of Framework

The most significant contribution of my study is the framework that I have constructed. The majority of the research that characterizes students' spatial visualization does so in a broad manner (e.g., perceptual vs property based, intrinsic-static/dynamic, extrinsic-static/dynamic) (Lehrer et al., 1998; Okamoto et al., 2015; Uttal et al., 2013). The framework that I have created builds from these previous ideas but does so in a way that provides finer distinctions. My framework allows teachers to anticipate what types of visualization practices students could use and the difficulties they might face.

Continuation of Establishing a Link Between Spatial Skills and Mathematical Performance

As seen in previous research (Battista, 1990; Tosto et al, 2014; Uttal et al., 2013; Verdine, Golinkoff, Hirsh-Pasek, Newcombe et al., 2014; Wai et al., 2009) there is a link between spatial and mathematical skills. My results substantiate this finding. However, unlike previous studies, I identified this relation by relating the visualization practices used to their success on the mathematical tasks they completed during our interviews. I found that students who were able to use a higher number and a larger variety of spatial visualization practices tended to perform better on geometric tasks than their peers who used fewer visualization practices. This was evidenced by the success rate of the students and the number and type of visualization practices used. The number of spatial visualization practices used indicated students were able to use and reason with a multitude of spatial visualization practices. This led to a greater number of tasks completed successfully, and more thorough explanations of students' reasoning.

Difficulties Elementary Students Have When Using Spatial Visualization in Geometric Tasks

Unlike previous research where most of the focus was the malleability of spatial skills (Casey et al., 2008; Uttal et al., 2013; Lowrie et al., 2019), my research has focused on the difficulties that students face with spatial visualization when undertaking geometric tasks. Trends in my study show that imagining a composition is a spatial ability that many students have difficulty with. Similarly to the Van Hiele theory and other researchers (Okamoto et al., 2015; Van den Heuvel-Panhuizen & Robitzsch, 2015), my study substantiates the claims that students who focus more on the physical appearance of objects rather than on mathematical properties tend to: (a) be students who are younger and have less experience with mathematical concepts, and (b) often get the answer incorrect.

Furthering Existing Understanding of the Relation between Gesture and Cognition

Throughout my study, gestures were an important aspect for me. Although my findings did not lead to any evidence to suggest the gestures directly correlated to correct answers, my research did substantiate previous research that gestures help students articulate their thought process, especially their spatial visualization (Ehrlich et al., 2006). Additionally, my research helped to substantiate research on gesture mismatch (Breckinridge Church & Goldin-Meadow, 1986). My research corroborates two decades of research that postulates that the gestures used during speech are essential to both understanding what is being said and the thought processes that are being used (Alibali et al., 1999; Ehrlich et al., 2006; Goldin-Meadow, 2003; Hostetter & Alibali, 2008).

Moreover, my results show a difference in how certain gestures afforded different visual planes and potentially played a role in narrowing the visual field to focus on the motion of

isolated mathematical properties. In all three tasks (1c, 3b and 5), ST6 used gestures to engage in spatial visualization when completing the geometric tasks. Task 1c highlighted that gestures were important for visual planes. In task 1c where the students had to recreate a square with pattern blocks, ST6's hands were in constant movement, choosing different pattern blocks and putting them in various arrangements which allowed for a greater visual plane. In task 3b (see Figure 4 in Chapter 3), where the students had to look at an isosceles triangle, imagine rotating it in their mind then draw what they thought it would look like, ST6 pointed to the tip of the triangle with their pencil and then traced a line with the pencil to show the imagined trajectory of the tip of the triangle. Additionally, ST6 used their pencil to point to the part of the drawn triangle and where they would be on the original triangle (part of the visualization practice of *mapping*). The gesture of pointing to the tip of the triangle and drawing the trajectory allowed the student to zero in on that mathematical property and its motion. In task 5 (see Figure 7 in Chapter 3), where the students had to fold a piece of paper in half twice, hole punch the top right corner and then guess where the holes would be, ST6 made unfolding gestures with their hands and pointed to the holes on the image they chose to indicate how they knew to choose that image. By identifying these gestures, these can be helpful for teachers to know what to look for when students are solving a task. Moreover, teachers could encourage students to use such gestures when thinking about certain geometric concepts.

Although more research is needed to establish clear links between these types of gesture and spatial visualization, my research acts as a starting point. In order to illuminate concrete links, future researchers might look more systematically at how types of gestures support spatial visualization. Using methods such as microgenetic studies, researchers can try to understand

change as it occurs (Siegler & Crowley, 1991). For example, researchers might ask students to first perform a task and then repeat the task, but by enacting particular gestures. By using microgenetic methods, researchers would be able to analyse the gestures in the two scenarios and look for relations between the performance of the task with or without the gesture, thus allowing for more conclusive data (Siegler & Crowley, 1991).

Limitations and Future Directions for Research

One of the limitations to my study was the number and composition of participants. When creating my study, originally, I had wanted to have a larger sample size. However, since students were chosen based on multiple criteria (e.g., if they were part of an after school program and they stayed for more than 45 minutes, if their parents/guardians gave consent, if they were not my current student, etc....), it made it difficult to recruit a large number of students. Since my sample size only included six students, it would be impossible to generalize the trends that I have discovered in my study. A larger sample size would be needed in order to see if similar trends arise. Additionally, only one student was categorized in the early elementary section. Therefore, that one student would not be representative of the whole early elementary population. Moreover, research has shown that gender differences play a role in spatial visualization skills (Battista, 1990; Fennema & Tartre, 1985; Terlecki et al., 2008), with the favourability being towards males. The one student in the early elementary category (ST1) was female, which further points to the need for a larger number of participants. Furthermore, all the students who I interviewed were from a school that was part of an affluent neighborhood. Research suggests that there is a link between higher spatial ability and higher socio-economic status (SES) (Bruce & Hawes, 2015; Verdine, Golinkoff, Hirsh-Pasek, Newcombe et al., 2014).

Therefore, further research would be needed to encompass a larger sample size, with a variety of genders in neighborhoods with varying SESs.

In addition to the limitations due to my participants, there were two specific limitations related to tasks that were too easy to reveal a range of ways in which students visualized. The first limitation came from task 1c. Task 1c had students using various pattern blocks to try and recreate a square. This task was limited due to the number of pattern blocks that were supplied to the participants. A greater number of blocks as well as diversity within the pattern blocks themselves may have provided an opportunity to view a greater range in students' spatial visualization. The second limitation came from task 4b. In task 4b, students were asked to look at a figure (see Figure 5 in Chapter 3) that was visible to them the whole time and recreate it. This task, unlike task 4a, did not provide a good opportunity to see how the students used the three visualization practices: *justification based on appearance*, *focusing on groups*, and *focusing on single blocks*. In task 4a students only had 10 seconds to look at the object before it was hidden, and they were asked to recreate it. By hiding the figure, the students had to use all of the aspects of spatial visualization (seeing objects inside one's mind (with the help of abstract thought), mentally manipulating the envisioned object, imaging the outcome and spatial relationships) in order to recreate the image successfully.

Another limitation is that the kinds of visualization practices that I observed may have been tied to the nature of the tasks. When designing my interview questions, I did so in a manner to try to have students engage in various tasks that would have students using different practices of visualization and that would test various spatial skills (intrinsic static/dynamic and extrinsic static/dynamic). The tasks that I created drew on three out of the four categories of spatial skills.

Specifically, my tasks had students working with skills that focused on intrinsic-dynamic, intrinsic-static, and extrinsic-dynamic skills. My study did not have any tasks that used extrinsic-static skills. Thus, future research might look to further expand my framework using other tasks to encompass all four categories.

In developing my framework further, a potential future research design could use eye cameras. Eye cameras are currently being used in research involving teacher professional vision and noticing (Sherin et al., 2008). Although the research is looking at the teacher view, the same can be applied for the student. The typical videotaping of an interview focuses on the manipulatives and the students' hands; this provides a skewed view of what the student is actually seeing (Sherin et al., 2008). By placing an eye camera on the student, future studies would have a greater understanding of what the student focuses on when solving spatial visualisation problems since the researchers would be able to see from the students' perspective, thus allowing for a richer interpretation.

Implications for Teachers

I believe my study has produced a few implications for teachers. First, I have started a framework on visualization practices that, when further developed, could be used as a tool for professional development for teachers. Research has shown that teachers often do not feel comfortable when teaching geometric tasks and will spend less time on geometry than other mathematical concepts (Swafford et al., 1997). This is in part due to their own insecurities about the concepts and also because of their own lack of spatial abilities (Swafford et al., 1997). Professional development is thus needed to help teachers become more versed in spatial visualization. My framework can be used in two ways during professional development. First, it

can be used as a tool to support teachers' development of spatial visualization in geometry. In addition, the framework can be used to help teachers anticipate how students will engage in visualization tasks (Uttal et al., 2013). This framework encompasses different visualization practices and how they are commonly used when completing geometric tasks. The importance of this framework is that it is grounded in student thinking. This allows teachers to leverage past experiences to help them design instruction that is rooted in student thinking. For example, if a teacher can anticipate what students will say or what they will likely struggle with when learning about a specific concept, they can then use that knowledge to help students see concepts and ideas in a different way.

Second, the types of gestures that I have seen from students throughout my study show that the use of gestures while using more complex spatial visualization practices can lead to a greater articulation of their thought processes. Understanding the types of gestures that students might use or knowing the types of gestures that are useful in understanding abstract concepts can be used as knowledge for teachers to help support students in visualizing. When students are gesturing, teachers can look out for mismatch, limitation to visual field and how gestures can help students zero in on mathematical properties and their motion. Some of the gestures that ST6 used (pointing to the top of the isosceles triangle with their pencil and drawing a line downwards to show the trajectory of the rotated triangle in task 3b, or making unfolding gestures in task 5, are some gestures that students could be encouraged to do.

Conclusion

The aim of my study was to understand how elementary students engage with spatial visualization when completing geometric tasks. In order to answer my research questions, I conducted task-based interviews with six students. I analyzed the types of visualization practices they used and the accompanying gestures (if any) when solving the geometric tasks presented in the interviews. The results showed higher consistent success rates in late elementary, but these results cannot be generalized due to the various limitations of the study, specifically sample size of participants. My research brought forth a new framework of visualization practices that is grounded in student thinking and continued to substantiate the idea that spatial visualization and mathematical performance are linked. Additionally, I identified various difficulties that students had with visualization practices while doing geometric tasks. The new framework as well as the student difficulties identified can serve as important knowledge that can be disseminated through professional development courses for teachers. My hope is that this research will inform future research in spatial visualization, as much more is needed to ensure that students develop the skills they need to be successful in their everyday, school and careers.

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Appendix A: Interviewer Booklet

Examining Spatial Visualization in Relation to Geometry

Researcher: Ashley Cameron

Interviewer Booklet

spatial visualization

The puzzle consists of a 3x3 grid of squares. The patterns in the grid are as follows:

- Row 1: Top-left triangle yellow, bottom-right triangle purple; Top-left triangle purple, bottom-right triangle yellow; Top-left triangle yellow, bottom-right triangle purple.
- Row 2: Top-left triangle purple, bottom-right triangle yellow; Top-left triangle yellow, bottom-right triangle purple; Top-left triangle purple, bottom-right triangle yellow.
- Row 3: Top-left triangle purple, bottom-right triangle yellow; Top-left triangle yellow, bottom-right triangle purple; Empty square.

Below the grid are five options labeled A through E:

- A: Top-left triangle yellow, bottom-right triangle purple; Top-left triangle purple, bottom-right triangle yellow; Top-left triangle yellow, bottom-right triangle purple.
- B: Top-left triangle purple, bottom-right triangle yellow; Top-left triangle yellow, bottom-right triangle purple; Top-left triangle purple, bottom-right triangle yellow.
- C: Top-left triangle yellow, bottom-right triangle purple; Top-left triangle purple, bottom-right triangle yellow; Top-left triangle yellow, bottom-right triangle purple.
- D: Top-left triangle purple, bottom-right triangle yellow; Top-left triangle yellow, bottom-right triangle purple; Top-left triangle purple, bottom-right triangle yellow.
- E: Top-left triangle yellow, bottom-right triangle purple; Top-left triangle purple, bottom-right triangle yellow; Top-left triangle yellow, bottom-right triangle purple.

Naglieri Nonverbal Ability Test (NNAT[®] 2Test)

TestingMom.com

Interviewer: Before we start today, I just want to remind you that if ever you feel uncomfortable or want to stop at any time you can ok? There are no right or wrong answers, I just want to see how you are thinking about things. While you work through each math problem, I would like you to speak out loud whatever it is that you are thinking about math and how to solve the problem. For example, if the math problem in front of me said “what is 2 plus 2?” I would say something like “ok so I see two different groups of two, I’m going to draw them to make it easier for me to see. If I take the first group of two and add it to the second group, then I will have 4 in total.” Does that make sense to you? What if the math problem said “3 plus 1?” what would you say out loud when solving it? [Listen to response, provide suggestions or probe for more information.]

Interviewer: Do you have any questions before we start? [Listen to response.]

Interviewer: Today we are going to be looking at some geometry problems. When I say the word geometry do you know what I mean by that? [wait for student’s response, depending on their response either agree with their description of geometry or clarify].

Interviewer: I brought a few tools with me to help you solve some of the geometry problems we are going to do today. Do you know what any of these are? What they are used for? [Let student look at all the tools that were brought. Ask child to describe or name each item if they can. If not help to clarify.]

Task # 1- Composing & Decomposing Tasks



- a) Using any combination of the pattern blocks above, determine the fewest number of blocks needed to fill the empty figure.

Can you draw it in the first figure below? **Point to figure.**

How did you decide that those would be the fewest blocks needed to fill the shape?

- b) What is the greatest number of blocks needed to fill the figure?

Can you draw it in the 2nd figure below? ** Point to figure.**

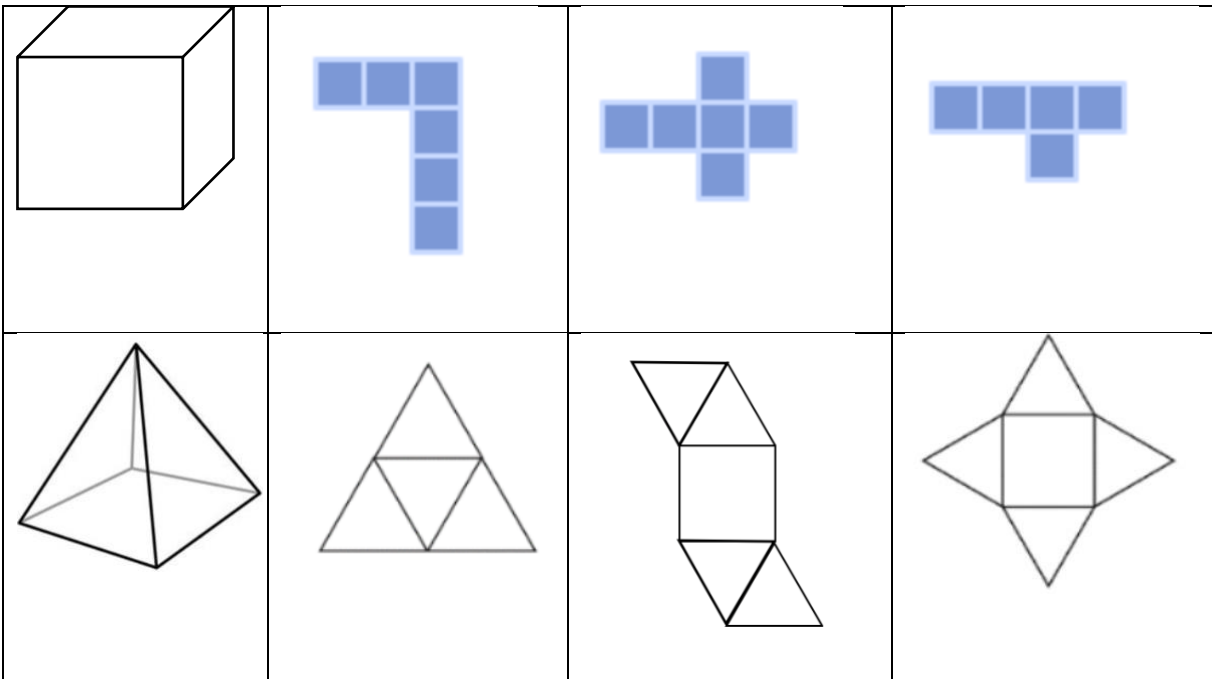
How did you decide that those would be the most blocks needed to fill the shape?

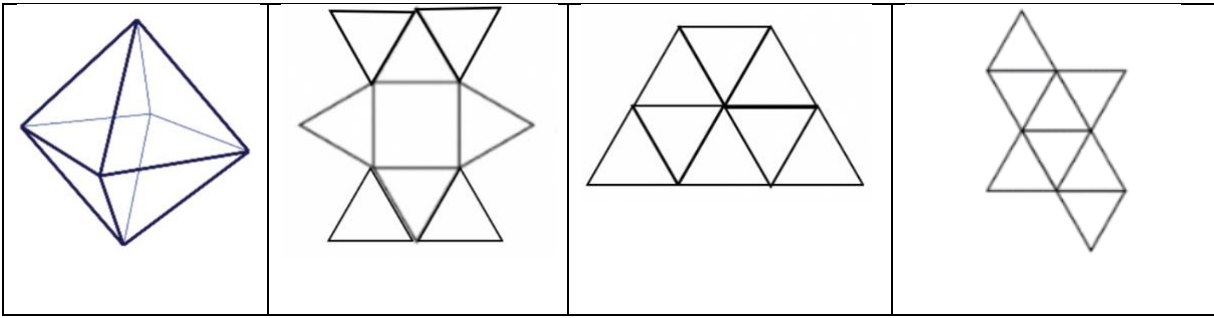
I'm going to give you some blocks and I would like you to show me all the different ways to make a square. Before we do that, can you draw a square for me on your paper?

- c) Using the blocks provided, can you show me all the different ways to make a square?

Did you feel like there were more ways to make a square, but you couldn't do it because of the blocks that were given?

Task # 2- Nets





Each of these are showing a 3-D image. Do you know what a 3-D image is?

So, imagine you were going to unfold this object **Point to cube** If you unfolded it, which one of these **points to nets** do you think it would look like?

How did you know which one to choose? Repeat with all 3 objects.

Once the students have completed this paper, *show them the physical object.*

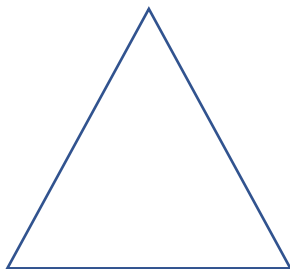
Now looking at the physical form do you want to change which image you chose? Do not let them decompose the object.

If they changed their mind ☐ **why did you change your mind? What influenced your decision?**

If they didn't change their mind-> **why did you decide to keep the same image?**

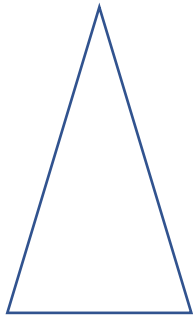
Task # 3- Rotation

- a) Look at the triangle below. Imagine that you turn it **make turning gesture with hand** and then it lands on this side ** point to side** Do not physically move the triangle. Student cannot move it either. Can you draw what it would look like if it was rotated?



Why do you think it would look like that?

Is what you drew, the same as what you imagined in your head?

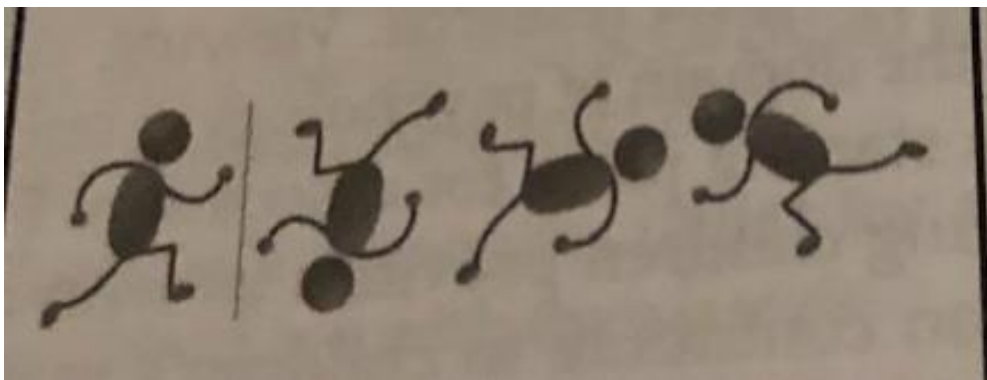


Now imagine that you turned this triangle and it landed on this side. Draw what it would look like.

Why do you think it would look like that?

Is what you drew, the same as what you imagined in your head?

- b) Look at the picture below. Imagine you were going to turn the little person in the picture. Once you have turned him in your mind, which one would it look like? **point to the 3 different people**



Why did you pick that image?

How did you imagine turning it in your head?

Task # 4- Can you build this?

I'm going to show you a shape now. I want you to pay close attention to it because I am going to show it to you for 10 seconds and then I want you to recreate it using the blocks in front of you. What does it mean to recreate something? [Wait for student response, if correct show shape, if not, clarify].

[Show student a shape made of linked blocks for 10 seconds. Allow time for student to try to recreate it].

[Once student has recreated it, bring out original and compare.]

You did a great job! **Do you see any similarities? Do you see any differences?** [Wait for student response].

[If differences] **Why do you think there were differences? Do you think if the object was placed in front of you the whole time you were have made it exactly the same?**

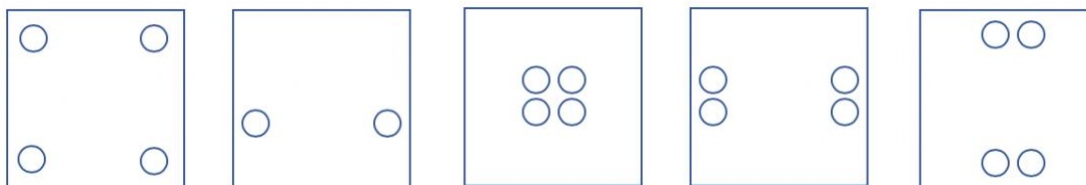
Let's try another one where the object is placed in front of you and you don't have a time limit [place another more complex object in front of student and wait for them to recreate it. Talk about any differences that come up and why they think that is].

Do you see any differences?

Why do you think there were differences?

Task # 5- Paper Folding

[Take a piece of paper.] Here we have a piece of paper. Can you fold it in half? Can you fold it in half again? Now I'm going to whole punch the paper here **whole punch paper** Imagine we were going to unfold the paper, which one of these **point to the 5 options below** do you think it would look like?



Why do you think it will look like that?

If time-> unfold paper and compare actual to what the student chose.

Is this **point to unfolded paper the same as what you chose?**

If different-> **Why do you think it's different?**

If the same-> **How did you know it would look like that?**

Now that you see the paper, why do you think it's this one **point to right picture instead of the one you chose originally?**

Task # 6- Perspective

- a) Jerry the dinosaur was looking across the land. Circle the image that represents what he sees.



How did you know that Jerry saw it like that?

If time-> Take out dinos and set up a situation and ask the same questions.

Appendix B: Student Booklet

Examining Spatial Visualization in Relation to Geometry

Researcher: Ashley Cameron

Student Booklet

spatial visualization

A B C D E

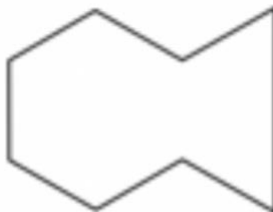
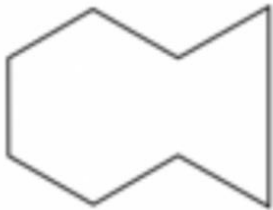
Naglieri Nonverbal Ability Test (NNAT[®] 2Test)

TestingMom.com

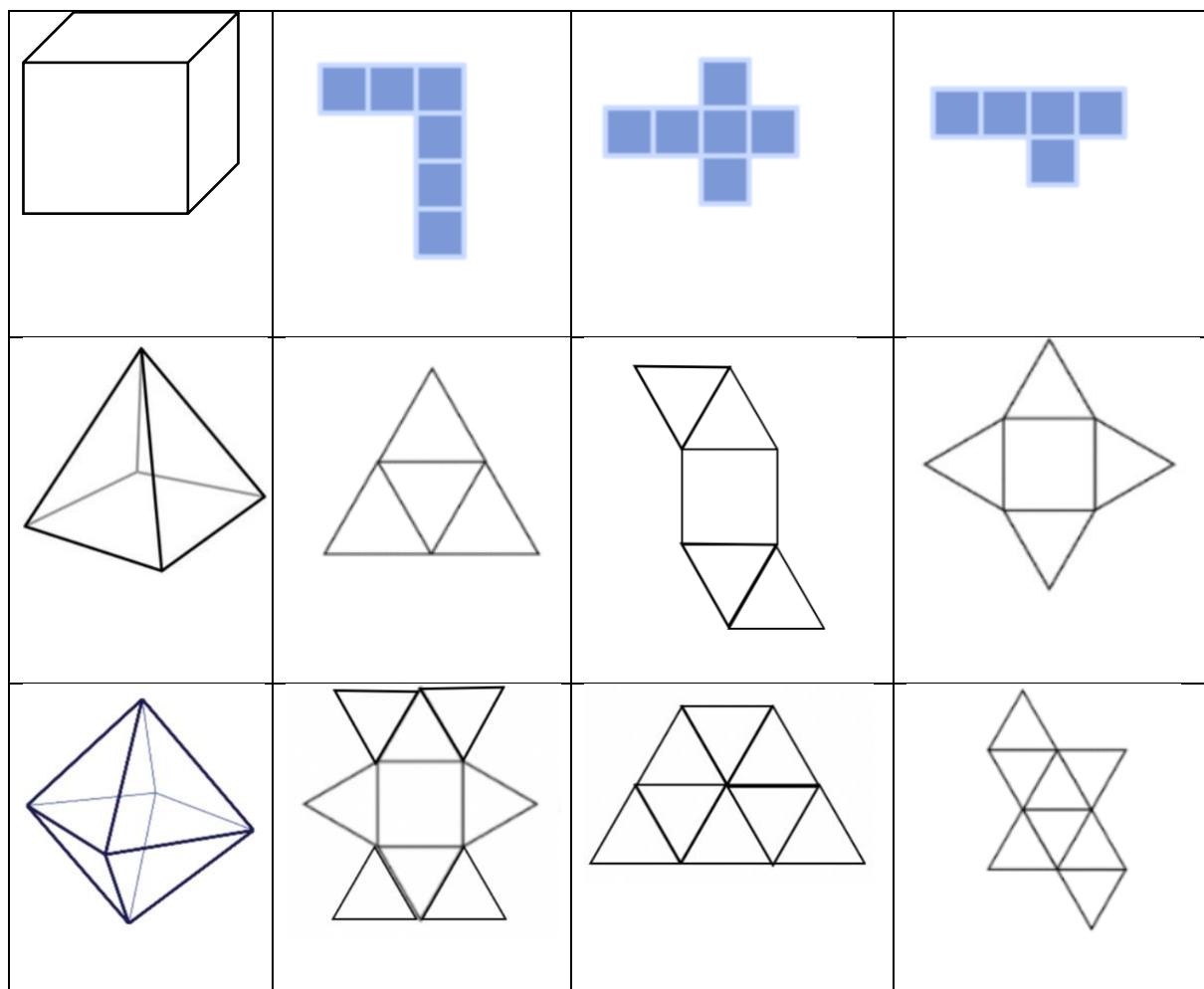
Student Name: _____

Date: _____

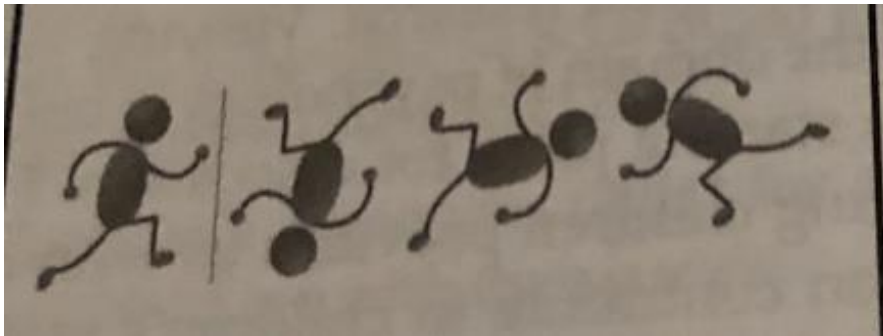
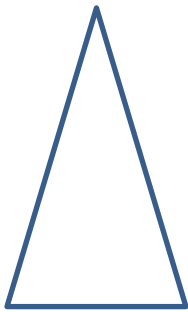
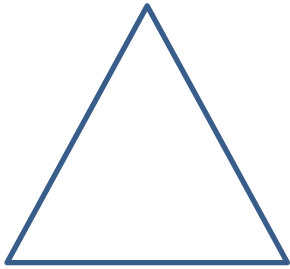
Task # 1- Composing & Decomposing Tasks



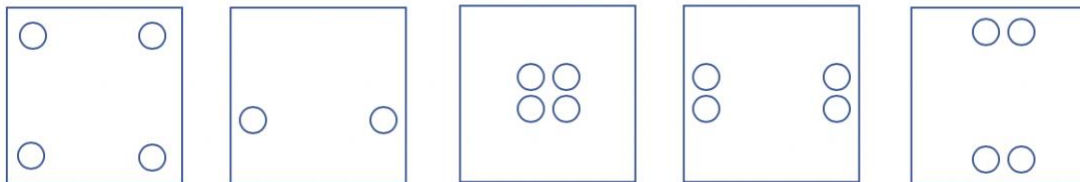
Task # 2- Nets



Task # 3- Rotation



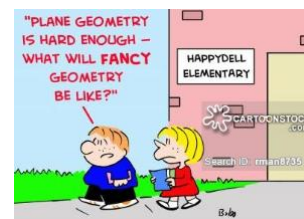
Task # 5- Paper Folding



Task # 6- Perspective



Appendix C: Teacher Questionnaire



Name: _____ School: _____

Please answer the following 6 questions to the best of your ability and return this form through internal school mail or in person in room 102. If at any point you do not feel comfortable answering a question, you are not obliged to do so.

- 1) How many days or weeks of the year do you teach geometry?

- 2) What geometry content have you covered thus far this school year?

- 3) In what manner did you cover the geometry content? (e.g.: followed the math text book, created activities, hands on projects, etc.) Provide as much detail as possible (e.g., text book name).

- 4) What geometry content do you still plan to cover this school year?

- 5) How would you describe spatial visualization?

- 6) Have you ever taught spatial visualization? If yes, in what way?
