A resolution of unnatural inflation in string theory? D-terms on the resolved conifold

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August 2008

A thesis submitted to McGill University in partial fulfilment of the requirements of the degree of Master of Science

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ACKNOWLEDGEMENTS

I would like to thank first and foremost my supervisor, Keshav Dasgupta, for his guidance and commitment to myself and his other students.

I also would like to thank my collaborators on the work on which this thesis is based: Keshav Dasgupta, Paul Franche, and Anke Knauf. This thesis would not have been possible without their work and scholarship. I and my collaborators are also grateful to the many people who gave helpful discussion and advice during our research. These include Cliff Burgess, Jim Cline, Hassan Firouzjahi, Andrew Frey, Hans Jockers, Shamit Kachru, Peter Ouyang, Gary Shiu, Radu Tatar and Angel Uranga.

I am grateful for the help and guidance of many other professors in the McGill department of physics who have played such an important role in my education.

I must equally acknowledge the crucial support of my fellow students. In particular, I thank Francis-Yan Cyr-Racine, Paul Franche, Rhiannon Gwyn, Jean Lachapelle, Nima Lashkari, Andrew Stewart, and Aaron Vincent.

Lastly, I would like to acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC) during the writing of this thesis and throughout the course of my degree.

ABSTRACT

We give an overview of flux compactification and a brief introduction to the present challenges in embedding inflation into string theory. With these challenges in mind, we suggest a novel string theory setup in which to study brane inflation. We construct a deformation of the warped resolved conifold background with supersymmetry breaking ISD (1,2) fluxes by adding D7-branes. We find spontaneous supersymmetry breaking without generating a bulk cosmological constant. In the compactified form, our background will no longer be a Calabi–Yau manifold. In the presence of D7-branes, the (1,2) fluxes can give rise to non-trivial D-terms. We study the Ouyang embedding of D7-branes and find that in this case the D-terms are indeed non-zero. In the limit when we approach the singular conifold, the D-terms vanish for Ouyang's embedding, although supersymmetry is broken. We discuss the puzzle of determining the correct 4D supergravity superpotential and suggest an approach to study the inflationary dynamics in this background.

ABRÉGÉ

Nous donnons un survol de la compactification des flux dans la théorie des supercordes, ainsi qu'une brève introduction aux défis présents dans la réalisation de l'inflation par le biais de la théorie des cordes. Nous proposons ainsi une concrétisation originelle de l'inflation due aux branes. En ajoutant des branes D7 à la théorie de type IIB, nous construisons une modification d'un fond de variété conique déformée à l'aide de flux auto-duaux imaginaires ISD (1,2) brisant la supersymmétrie, sans toutefois générer de constante cosmologique dans le volume. Dans sa forme compactifiée notre fond ne sera plus une variété Calabi-Yau, puisqu'une classe de Chern non-nulle sera permise. En présence des branes D7, les flux (1,2) peuvent de plus générer des termes D non-triviaux. Nous étudions le plongement Ouyang de branes D7 et trouvons que dans ces cas, les termes D sont effectivement non-nuls. Dans la limite de la variété conique singulière, les termes D deviennent zéro pour le plongement de Ouyang, alors que la supersymmétrie paraît être brisée. Nous discutons du problème de la détermination du superpotentiel effectif en 4 dimensions, et suggérons une approche pour l'étude de la dynamique de l'inflation dans ce contexte.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS					
ABS	TRAC	Τ	iii		
ABR	RÉGÉ		iv		
1	Introd	luction	1		
2	A brie	f review of string compactification	6		
	 2.1 2.2 2.3 2.4 2.5 	Type IIB string theory at a glance	7 10 13 13 14 16 20 20 22 26		
3	Inflati	on in string theory	29		
	$\begin{array}{c} 3.1 \\ 3.2 \\ 3.3 \\ 3.4 \\ 3.5 \\ 3.6 \\ 3.7 \end{array}$	The inflationary paradigm $\dots \dots \dots$	29 31 32 33 36 38 39		
4	D7–br	anes on the resolved conifold	42		
	4.1	The background	42		

	4.2	The warped resolved conifold with fluxes	45			
	4.3	The scalar potential and supersymmetry	48			
	4.4	Ouyang embedding of D7–branes on the resolved conifold	53			
	4.5	D-terms from non–primitive background flux on D7–branes	61			
5	Future	e Directions and Applications to Cosmology	70			
	5.1	Compactification and non-Kählerity	70			
		5.1.1 Inflationary dynamics	72			
		5.1.2 Supersymmetry restoration	72			
	5.2	AISD Fluxes and Anti-Branes	73			
	5.3	Conclusions	74			
А	The ge	eometry of the resolved conifold	75			
В	Ouyar	g embedding of D7–branes on the resolved conifold	80			
Refe	References					

CHAPTER 1 Introduction

There should be little wonder that the inflationary model continues to be the dominant paradigm of early-universe cosmology with its robustly successfull observational predictions [1]. Nevertheless, many of its proponents are dissatisfied with standard effective field theoretic approaches to inflation and remain in search of a fundamental theory in which to make sense of the model's conjectured characteristics and requirements. Although effective field theories have given much valuable insight, understanding the naturality and stability of a specific model requires a more thorough understanding of the UV physics. Without a clear picture of how inflation is embedded into a fundamental physical theory, inflation will continue to be a model of questionable motivation.

It is accordingly valuable to see if it is possible, first, to embed the inflationary paradigm into a realistic string theory background and, second, to determine how natural such an embedding is within string theory. While the first challenge as been answered affirmatively to most investigator's satisfaction [2, 3, 4, 5, 6, 7], finding examples that satisfy the somewhat nebulous requirement of naturalness has been problematic. In this manuscript we will construct a background that shows promise for generating inflation without excessive fine-tuning.

The requirements of an inflationary model can be fit into string theory in a vast number of ways. We make the choice to construct a background for a particular subset of these models known in the literature under 'brane-inflation'. Current brane inflation models (e.g. [2, 3, 4, 5]) are usually situated in a particular type IIB string theory setup known as the "warped throat". In this setup, one considers supergravity solutions with non-vanishing background fluxes which generate a strongly warped Calabi–Yau geometry via their backreaction on the metric. The warping introduces a hierarchy in the 4D Planck scale dependent on the position in the background manifold. Almost uniformly in the literature, the Calabi-Yau background is taken to be the deformed conifold [8], although many of the calculations are actually done using the metric of a singular conifold for ease of calculation. The small compact end of the conifold is referred to as the infrared tip of the throat while the non-compact end is referred to as the UV, in accordance with the hierarchy generated by the warping.

Typically in brane inflation models an anti–D3-brane is placed at the infrared tip of the throat and a D3-brane at some distance from it towards the UV. The $D3/\overline{D3}$ pairing breaks supersymmetry, uplifts the vacua [9], and generates a force on the branes [10, 11]. The long-range attractive forces between the branes result specifically from a gravitational attraction between the branes and from the Ramond-Ramond background fields under which the branes are charged. The potential corresponding to these forces is flattened by the warping of the metric between the brane and anti-brane [2]. Due to this flatness, the field in the 4D theory corresponding to the brane/anti-brane distance is an ideal candidate for the inflaton field.

Nevertheless, such a setup is fraught with problems; generating a proper inflationary model under this program is exceedingly difficult [2]. When one takes into account the full spectum of moduli fields, it has not yet been possible to isolate a slow-rolling inflaton and simultaneously stabilize the remaining fields without appeals to fine-tuning [2, 3, 4]

In this paper, we consider a variant approach that compactifies on a warped resolved conifold [12] instead of the warped deformed conifold considered in other work to date. The salient difference between these two backgrounds is the blow-up of a 2-cycle at the tip in the resolved conifold as opposed to a 3-cycle for the deformed. The broken \mathbb{Z}_2 symmetry of the blown up 2-cycle should capture some effects of more general non-Calabi-Yau conifold backgrounds that are closely related to the resolved and deformed conifolds.

This setup obviates the need for anti-branes as the background fluxes break supersymmetry spontaneously. Despite this fact, we will show conclusively that the fluxes nevertheless obey all of the equations of motion up to linear order in a suitable expansion parameter. We will solve the field equations for the background by following a method first used by Ouyang in [13] according to a general ansatz for warped backgrounds established by [14, 15, 16]. Though the fluxes we find are consistent on the non-compact Calabi-Yau manifold considered, they cannot exist on a compact Calabi-Yau. We will argue that the presented background should be thought of as a CY coordinate patch on some more general compact background. Further, we will show that the SUSY-breaking background fluxes generate D– terms that uplift the potential¹ if D7-branes present in the background are wrapped on particular cycles (and possibly no D–terms for other choices of embedding). D– terms from D-branes in string theory have been well-studied by a variety of authors [20, 21, 22, 23, 24], who have established methods to calculate this uplifting. As D– terms necessitate the presence of F–terms [25], we will consider how SUSY breaking can be seen in the F–term potential. We will show that there will be no F-term uplifting and some aspects of this breaking will remain open questions for further study. Namely, there will remain a puzzle as to how to establish the correct 4D superpotential for the generalized compactification into which our setup must be embedded.

The addition of D7-brane to this setup should not be seen as precarious. D7branes are a standard ingredient in F-theory compactifications of type IIB string theory and play a key role in the stabilization of Kähler moduli through nonperturbative corrections to the superpotential in most IIB string compactifications [9]. Moreover, the inclusion of the D7-brane is key to our proposed slow-roll inflationary model. As has been show in D3/D7 inflationary models, the pull-back of SUSY-breaking fluxes onto a D7 brane generates an attractive potential between D3and D7-branes [6]. On the other hand, the SUSY-breaking fluxes in our background will also attract a D3-brane towards the tip of the throat. In such a manner, one can

¹ This idea was put forward in [17], but needed some corrections [18, 19]

create opposing forces on a D3-brane controlled by the same SUSY-breaking parameter. The conjecture is that with these two forces controlled by the same parameter, a fine balance will emerge naturally and result in a stable slow-roll potential.

This manuscript is outlined as follows: In Chapter 2, we will give an overview of the basic ideas behind flux compactifications in string theory. We will discuss a specific class of supergravity solutions known as warped Calabi-Yau compactifications. We will also discuss the conditions for preserving $\mathcal{N} = 1$ supersymmetry in the 4D effective theory and how to compute the corresponding 4D SUSY potentials. In Chapter 3, we will outline the basic ideas of brane-inflation, the current dilemmas, and then motivate a choice of new background. In Chapter 4, we will explicitly construct the string theory setup and compute the relevant background fields and potentials.

Contribution of authors

The original research contained in this thesis is based on work done in collaboration with K. Dasgupta, P. Franche, and A. Knauf [26]. Specifically, chapters 4 and 5, to which this author was a primary contributor, are largely taken from this work. Sections of [26] discussing the F-theory lift of this work, to which this author was not a primary contributor, have not been included in this thesis.

CHAPTER 2

A brief review of string compactification

String theory, for mathematical consistency, is formulated as a theory in 10 spacetime dimensions. This naturally must be reconciled with the existence of a universe of only four dimensions that we observe and measure. If a 10D theory is to be taken seriously, it must also provide a mechanism whereby modes that propagate in the extra dimensions are decoupled from low-energy observed physics. One such method is to compactify six spatial dimensions on some manifold of small volume. Kaluza-Klein excitations on the internal compact manifold gain masses of order the inverse compactification length scale (M_c) , and are not visible in the low-energy 4D effective theory.

There are also strong reasons to believe that some supersymmetry may be manifest in our universe at relatively low energies (well below the scale M_c). This, and the technical simplifications gained in supersymmetric theories, motivate us to search for compactifications of string theory that result in supersymmetric effective theories. Nevertheless, the full 10D string theory may retain too large a number of unbroken supercharges when reduced to 4D or, conversely, none at all. Theories with $\mathcal{N} > 2$ in 4D are heavily constrained and are not viable examples in which to embed the standard model. One thus must carefully select the compactification conditions to give a suitable $\mathcal{N} = 1, 2$ low-energy theory.

We will begin by discussing the most naive trivial compactifications of string theory as a starting point to examine more general warped compactifications. We will then examine the 4D effective theory that results from warped compactifications on Calabi-Yau manifolds and discuss the stabilization of moduli fields.

Because the history of and motivations for string compactifications are expansive, we will attempt to take the most direct path to the relevant results while hopefully laying down an intelligible background. In doing so, we will inevitably pay insufficient attention to the phenomenology that motivated these string theory constructions and to some key results. We will similarly only concern ourselves with discussing type IIB string theory, which, as will become clear, is a profitable limit in which to study the 4D effective theory of the compactifications. Similar work—some of it taking historical precedence over the results described here—has been carried out in M-theory and heterotic string theory. This work will be described either in its IIB variation or, unfortunately, not all.

2.1 Type IIB string theory at a glance

We now will summarily establish a large amount of notation used in this paper by detailing the field content and action of the low-energy supergravity limit of type IIB string theory. We will only detail the bosonic degrees of freedom in the theory, although one should not forget that appropriate fermionic superpartners exist for the mentioned fields. Derivations of these results are well-documented in standard string theory texts, eg. [27].

Type IIB string theory is a theory of closed strings, with open strings stretching between Dp-branes of odd dimension. The Dp-branes are charged under (p+1)-form R-R fields and act as sources for background flux. Thus, in the R-R sector, there are fields C_0 , C_2 , and C_4 with respective field strengths F_1 , F_3 , and F_5 . In the NS-NS sector there is the metric g, the dilaton Φ , and a two-form field B_2 with field strength H_3 .

Type IIB string theory has a global $SL(2,\mathbb{Z})$ symmetry and it will be useful to redefine the above fields into a form that makes the symmetry of the theory explicit. We define

$$\tau = C_0 + ie^{-\Phi} \tag{2.1}$$

$$G_3 = F_3 - \tau H_3 \tag{2.2}$$

$$g = e^{-\Phi/2}g.$$
 (2.3)

The above metric transformation changes from the action from the String Frame to the Einstein Frame. We also will define a five-form field strength

$$\tilde{F}_5 = F_5 + \frac{1}{2}B_2 \wedge H_3 - \frac{1}{2}C_2 \wedge F_3 \tag{2.4}$$

and impose the constraint that it is self-dual

$$\star \tilde{F}_5 = \tilde{F}_5 \,, \tag{2.5}$$

as an equation of motion (to match the string spectrum) because it cannot be derived from a covariant 10D supergravity action. Under an $SL(2,\mathbb{Z})$ transformation

,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad a, b, c, d \in \mathbb{Z}$$
(2.6)

these fields transform as

$$\tau \to \frac{a\tau + b}{c\tau + d} \quad G_3 \to \frac{G_3}{c\tau + d}$$
 (2.7)

$$g \to g \qquad \tilde{F}_5 \to \tilde{F}_5 \,.$$
 (2.8)

In these variables, the bulk bosonic action for the low-energy supergravity limit of type IIB string theory, in the Einstein frame where it is $SL(2,\mathbb{Z})$ invariant, is given by

$$S_{Bulk}^{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}X \sqrt{-g} \left(R - \frac{\partial_M \tau \partial^M \bar{\tau}}{2(Im\tau)^2} - \frac{G_3 \cdot \bar{G}_3}{12Im\tau} - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right) \,. \tag{2.9}$$

It will also be useful to recall the action for D-branes in the IIB theory. For a Dp-brane, the Dirac-Born-Infeld (DBI) action is given by

$$S_{DBI} = -T_p \int d^{p+1}\sigma \, e^{-\Phi} \sqrt{-\det(\hat{g} + \hat{B} + 2\pi\alpha' F)} \,, \tag{2.10}$$

where σ are coordinates on the D-brane, \hat{g} and \hat{B} are the pullbacks of the respective fields onto the worldvolume of the brane, and F is the gauge field living on the brane. The Chern-Simons (CS) action for a Dp-brane is given by

$$S_{CS} = T_p \int d^{p+1} \sigma C \, e^{B + 2\pi \alpha' F} \,, \qquad (2.11)$$

where we are implicitly only keeping the (p+1)-forms in the action and have defined the quantity C by

$$C = \sum_{n=0}^{4} C_{2n} , \qquad (2.12)$$

a sum of RR gauge fields. Note that for \overline{Dp} -branes, the DBI action stays the same, but the CS action changes sign.

2.2 Trivially factorized compactifications

The simplest (and naive) assumption for a compactification scenario is that spacetime trivially factorizes into a product space

$$\mathcal{M} = \mathcal{M}_4 \times \mathcal{M}_6 \,, \tag{2.13}$$

where \mathcal{M}_4 is some maximally symmetric Poincaré invariant 4-manifold and \mathcal{M}_6 is the internal 6D Riemannian compact manifold. For now we make the further simplifying assumptions that the dilaton is constant along the internal manifold and that all background fluxes vanish.

We would like to preserve the minimal fraction of the full supersymmetry that could possibly descend to the resulting 4D effective theory. This is equivalent to finding an internal manifold that supports the existence of exactly one covariantly conserved spinor. One can then show that this condition implies it is possible to endow the internal manifold with a complex structure and Kähler metric that is Ricci flat:

$$R_{mn} = 0.$$
 (2.14)

Such manifolds are called Calabi-Yau (CY). The definition of a compact CY manifold typically also includes the condition that it be simply connected. We use this definition here.¹

In the case of type IIB string theory on a CY, the $\mathcal{N} = 2$ supersymmetries in 10D (one each for the left-moving and right-moving states) reduce to $\mathcal{N} = 2$ in 4D.

CY moduli fields in the 4D theory

A Calabi-Yau manifold is connected to other Calabi-Yau manifolds through continuous deformations of the metric and complex structure that preserve the condition of Ricci-flatness. Thus, a CY manifold is a point in a geometric space of CY manifolds on which the coordinates are called moduli. For CY manifolds, this moduli space is a finite dimensional product space [28],

$$M = M^{1,1} \times M^{2,1}, \qquad (2.15)$$

where $M^{1,1}$ corresponds to the space of Kähler deformations and $M^{2,1}$ corresponds to the space of complex structure deformations. Kähler deformations of the metric leave the complex structure unchanged and are locally parameterized by a basis of $h^{1,1}$ real harmonic (1, 1)-forms on a CY manifold at a point in moduli space:

$$\delta \tilde{g}_{a\bar{b}} = a^i (\psi_i)_{a\bar{b}} \,, \tag{2.16}$$

¹ A more complete treatment of CY manifolds and the relation to SUSY can be found in any number of string theory reviews or textbooks, such as [27].

where ψ_i a basis of $H^{1,1}$. Conversely, complex structure deformations of the metric require a corresponding deformation of the complex structure to keep the metric Kähler. These deformations are locally parameterized by a basis of $h^{2,1}$ complex harmonic (2, 1) forms on a CY manifold at a point in moduli space:

$$\delta \tilde{g}_{\bar{a}\bar{b}} = b^j \frac{1}{\|\Omega\|^2} \bar{\Omega}_{\bar{a}}^{cd}(\chi_j)_{cd\bar{b}} = b^j \chi'_j , \qquad (2.17)$$

where χ_j are a basis of $H^{2,1}$ and Ω the unique holomorphic 3-form on a CY. Thus we can parameterize deformations of the metric locally in moduli space as

$$\delta \tilde{g} = a^i \psi_i + b^j \chi'_j \,. \tag{2.18}$$

If we allow these parameters to vary with the position in non-compact space, it is then clear that these moduli parameters $a_i(x^{\mu}), b_j(x^{\nu})$ will enter the 4D effective theory as scalar fields. One finds kinetic terms for these fields from dimensional reduction of the Ricci scalar

$$\int_{\mathcal{M}_6} d^6 y \sqrt{g} R + \dots \qquad (2.19)$$

As the moduli controlling the background geometry of the compactification are actually fields of the 4D theory, we immediately conclude that they must be stabilized by a large mass to explain the absence of scalar fields observed in our universe. This problem is commonly known as moduli stabilization. Without other contributions to the action, we will only find kinetic terms for these moduli fields in the trivial compactification without background fluxes. Enticingly, we can heuristically see from the fact that the G-flux (G_3 field strength) is built from the same basis of (2,1) forms that they may have the possibility to generate potentials for some of the moduli. We will examine this in the section 2.4.

2.3 Flux compactification

2.3.1 Warped compactifications

A simple generalization of the trivially factorized background is a warped compactification with metric

$$g_{MN}dX^{N}dX^{M} = e^{2A(y)}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + e^{-2A(y)}\tilde{g}_{mn}dy^{m}dy^{n}, \qquad (2.20)$$

where X_M, X_N are coordinates on the full 10D metric, x_μ, x_ν are coordinates on the non-compact space, and y_m, y_n are coordinates on the internal compact space. A(y)is the warp factor and is independent of the non-compact coordinates. Thus, this background retains 4D Poincaré invariance. For now, we will not demand that the unwarped internal manifold $(\mathcal{M}_6, \tilde{g}_{mn})$ is Calabi-Yau, although this will be a useful specialization to consider later on.

Warped compactifications are of interest to us because of the prominent role they play in brane inflation scenarios. The warping of the metric allows one to greatly flatten the inflaton potential, allowing for slow-roll inflation to take place. This will be described in Section 3.2. Warped compactifications are also of general interest because the warping of the metric allows one to generate an exponential hierarchy between the Planck scale and weak scale physics [16]. Warped compactifications were first realized in string theory in [14] in the context of M-theory. There, the authors derived a consistent compactification of the low-energy supergravity limit of M-theory on a warped Calabi-Yau 4-fold, resulting in an $\mathcal{N} = 2$ D = 3 effective theory. They did so by turning on non-vanishing 4form flux and finding constraints which relate it to the warp factor of the geometry. This work was extended in [29] to D = 4 effective theories in type IIB by lifting the M-theory background to F-theory, and then was elaborated on by [16], again in type IIB. In describing the relevant results here, we will follow the derivation provided in [16].

2.3.2 A general warped ansatz

Consider the general warped metric given in (2.20). We will construct an ansatz for a consistent string background on this metric. This entails finding consistent solutions for G_3 , the self-dual five-form flux \tilde{F}_5 , and the axio-dilaton τ (recall the field content of type IIB string theory as described earlier in section 2.1) that obey all constraints and equations of motion.

The five-form flux is self-dual, and thus cannot be constrained to live only in internal dimensions. We thus must be careful to choose a form that is Poincaré invariant. We can do so by choosing the legs in external space to be proportional to the 4D volume form and then adding the self-duality explicitly:

$$\tilde{F}_5 = d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 + \star d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3, \qquad (2.21)$$

for some function $\alpha(y)$ as yet to be determined and where we use \star to denote the Hodge star operator. Further, suppose that the axio-dilaton is non-constant, but

only depends on the internal coordinates $(\tau(y))$ and the G_3 -flux is non-vanishing with legs only along the internal manifold (again so as to satisfy 4D Poincaré invariance). Because we are allowing non-vanishing background fluxes on the compactification manifold, we call such a background a (warped) flux compactification.

Field equations and constraints

Given this ansatz, let us list the entire set of constraints and field equations from the IIB action in 2.1

$$S = S_{bulk} + S_{DBI} + S_{CS} \tag{2.22}$$

that must be satisfied for a consistent supergravity solution. First, one must solve Einstein's equation, which can be rewritten as an explicit equation for the Ricci tensor as

$$R_{MN} = \kappa_{10}^2 \left(T_{MN} - \frac{1}{8} g_{MN} T \right) \,, \tag{2.23}$$

where T_{MN} is the stress energy tensor. For our particular ansatz this field equation becomes, for the non-compact components,

$$R_{\mu\nu} = -g_{\mu\nu} \left(\frac{G_3 \cdot \bar{G}_3}{48Im\tau} + \frac{e^{-8A}}{4} \partial_M \alpha \partial^M \alpha \right) + \kappa_{10}^2 \left(T_{\mu\nu}^{loc} - \frac{1}{8} g_{\mu\nu} T^{loc} \right) \,. \tag{2.24}$$

Here we have separated out the contribution from the fluxes, leaving the stress-energy from localized sources (D-branes) $T^{loc}_{\mu\nu}$. For the compact internal components, we similarly find

$$\tilde{R}_{mn} = \kappa_{10}^2 \, \frac{\partial_m \tau \partial_n \bar{\tau} + \partial_n \tau \partial_m \bar{\tau}}{4(Im\tau)^2} + \kappa_{10}^2 \left(\tilde{T}_{mn}^{loc} - \frac{1}{8} \tilde{g}_{mn} \tilde{T}^{loc} \right) \tag{2.25}$$

(There is a tilde on the stress-energy tensor here because we are varying the action with respect to the unwarped internal metric \tilde{g}_{mn}).

We also must solve the field equations for the flux, G_3 , and for the axio-dilaton, τ :

$$d\left(e^{4A}\star_{6}G_{3}-i\alpha G_{3}\right)+\frac{i}{Im\tau}d\tau\wedge\Re\left(e^{4A}\star_{6}G_{3}-i\alpha G_{3}\right)=0$$
(2.26)

$$\tilde{\nabla}^2 \tau = \frac{\tilde{\nabla} \tau \cdot \tilde{\nabla} \tau}{iIm\tau} - \frac{4\kappa_{10}^2 (Im\tau)^2}{\sqrt{-g}} \frac{\delta \tilde{S}^{loc}}{\delta \bar{\tau}}, \qquad (2.27)$$

where \star_6 is the Hodge star operator on the internal manifold. In addition, we must make sure that all the fluxes obey the appropriate Bianchi identities:

$$dF_3 = 0 \tag{2.28}$$

$$dH_3 = 0$$
 (2.29)

$$d\tilde{F}_5 = H_3 \wedge F_3 + 2\kappa_{10}^2 T_3 \rho_3^{loc} \,. \tag{2.30}$$

where ρ_3^{loc} is the localized D3 charge density. Lastly, we must impose the self-duality of the five-form flux, although this has already been taken care of by our choice of ansatz.

2.3.3 Solving the equations of motion and the Bianchi identities

We now outline the computation of a series of constraints arising from the above field equations for the metric and Bianchi identities for the five-form flux. The constraints can be satisfied by a simple refinement of the initial ansatz and a restriction on the types of allowed localized sources (branes). With this refined ansatz, it is then possible to show that most of the remaining field equations take on a simplified form or are trivially satisfied. First, we note that the external components of the Ricci tensor can be computed explicitly from the warped metric ansatz:

$$R_{\mu\nu} = -\eta_{\mu\nu} e^{4A} \tilde{\nabla}^2 A \,. \tag{2.31}$$

Next, it is possible to rewrite the Bianchi identity from (2.30), using our ansatz, as

$$\tilde{\nabla}^2 \alpha = i e^{2A} \frac{G_3 \cdot \star_6 \bar{G}_3}{12 I m \tau} + 2 e^{-6A} \partial_m \alpha \partial^m e^{4A} + 2 \kappa_{10}^2 e^{2A} T_3 \rho_3^{loc} \,. \tag{2.32}$$

Combining these two conditions with the expression for the Ricci tensor in terms of the background fields (2.24) gives the constraint equation

$$\tilde{\nabla}^{2}(e^{4A} - \alpha) = \frac{e^{2A}}{6Im\tau} |iG_{3} - \star_{6}G_{3}|^{2} + e^{-6A} |\partial(e^{4A} - \alpha)|^{2} + (2.33)$$

$$2\kappa_{10}^{2} e^{2A} \left[\frac{1}{4} \left(T_{m}^{loc\,m} - T_{\mu}^{loc\,\mu}\right) - T_{3}\rho_{3}^{loc}\right].$$

We can identify an immediate consistent solution as

$$\alpha = e^{4A} \tag{2.34}$$

$$\star_6 G_3 = iG_3 \tag{2.35}$$

assuming we only include localized sources that satisfy

$$T_m^{loc\,m} - T_\mu^{loc\,\mu} = T_3 \rho_3^{loc} \,. \tag{2.36}$$

It is worth noting that, although this identification is always a consistent choice locally, the conclusion can be strengthened in a compact setting. In this case, the LHS of (2.34) integrates to zero. Under the weakened assumption only that

$$T_m^{loc\,m} - T_\mu^{loc\,\mu} \ge T_3 \rho_3^{loc},$$
 (2.37)

the positivity of the individual terms on the RHS implies that (2.34) and (2.36) must hold. Thus, given our initial ansatz, our solution is sufficient in non-compact backgrounds and necessary in compact ones.

The assumption (2.36) limits the types of branes that can be consistently embedded in our background. Namely, it is valid for D3 branes, fractional D3 branes (D5 branes wrapped on homologically trivial cycles), and D7 branes, but not anti-D3 branes or D5 branes [16].

Solving the remaining constraints

What remains to be checked is that this solution obeys the remaining Bianchi identities for F_3 and H_3 (2.28, 2.29) and equations of motion for G_3 and τ (2.26, 2.27), and satisfies Einstein's equations for the internal compactification manifold \tilde{g}_{mn} . While the Bianchi identities must be imposed by hand on our choice of background flux, the equation of motion for G_3 is satisfied automatically by this ansatz because the individual terms of (2.26) vanish exactly:

$$e^{4A} \star_6 G_3 - i\alpha G_3 = 0. (2.38)$$

Solving the dilaton equation of motion (2.27) and ensuring that the internal manifold is consistent with the dilaton (2.25) remains non-trivial. In the absence of D7 branes, these equations are satisfied by a Calabi-Yau manifold with constant dilaton. In the presence of D7 branes, a non-trivial dilaton will be necessary to solve these field equations. From the F-theory perspective, the choice of dilaton is determined uniquely by the monodromy of the D7 branes [30]. If one can find a globally well-defined lift of the IIB background to F-theory such that it is a fibration over a flat CY base, then these equations must be satisfied. Such a lift is discussed in the paper on which this work is based [26], but is not within the scope of this manuscript.

Imaginary self-dual fluxes

We call fluxes that satisfy the second condition in (2.35),

$$\star_6 G_3 = iG_3 \,, \tag{2.39}$$

imaginary self-dual (ISD). Conversely, we call fluxes anti-imaginary self dual (AISD) if they satisfy

$$\star_6 G_3 = -iG_3 \,. \tag{2.40}$$

On a compact Calabi-Yau, there is a limited cohomology which allows us to be able to make precise statements about the space of fluxes that can satisfy this constraint. The complete cohomology of harmonic three-forms on a compact Calabi-Yau is determined by the basis of (2,1) forms $\{\chi_{\alpha}\}$ (whose number is determined by the Hodge number $h^{2,1}$), the unique (3,0) form Ω , and the conjugates of these forms. One can show that on a compact Calabi-Yau $\{\chi_{\alpha}, \overline{\Omega}\}$ are always ISD forms and hence their conjugates are AISD forms (See, for example, [27]).

These assumptions must be relaxed for non-compact Calabi-Yau manifolds, or compact non-Calabi-Yau manifolds. Such manifolds may support a non-trivial (1,0) cohomology. Thus we can construct a three-form from the wedge product of a (1,0)or (0,1)- form with an element of the (1,1)-cohomology.

2.4 The 4D effective theory

2.4.1 Preserving $\mathcal{N} = 1$ supersymmetry

Recall that we would like our background to retain $\mathcal{N} = 1$ supersymmetry up to some parametrically controlled spontaneous breaking. However, we do not generically expect that the background fluxes in solutions of the type discussed here will preserve all (or any) of the original supersymmetry.

In [31, 32, 33], necessary and sufficient conditions were found to preserve $\mathcal{N} = 1$ supersymmetry in warped type IIB compactifications with a non-constant dilaton of the particular variety described in the previous section (such backgrounds are commonly denoted 'B-type' solutions). The authors of [31, 32, 33] found first that the internal unwarped manifold, $(\mathcal{M}_6, \tilde{g})$, must be a complex Kähler manifold. Thus it must be a complex manifold with closed Kähler form J. Second, they found that G_3 must be ISD, that it must be primitive:

$$J \wedge G_3 = 0; \tag{2.41}$$

and, lastly, that it must be of Hodge type (2,1).

We can see the supersymmetry breaking that arises when we have non-ISD fluxes on a background by examining the scalar potential of the 4D theory that arises from the G-flux. We find this potential by direct dimensional reduction of the 10D theory. In particular, it is induced by the flux kinetic term

$$S_G = -\frac{1}{4\kappa_{10}^2} \int \frac{G_3 \wedge \star_6 \overline{G}_3}{Im \,\tau} \,, \qquad (2.42)$$

where the Hodge star is on the cohomology of the internal manifold and we are integrating over the internal manifold. In this kinetic term we have left out the warp factor as well as the kinetic term for \tilde{F}_5 . Though not strictly complete, this simplified calculation gives the correct features of the calculation as all fields couple to the warp factor and the five-form flux in the same ratio [16]. One can think of this as a large radius limit in which the warp factor is constant and hence the five-form flux similarly vanishes as it depends on $[d\alpha] = 0.^2$ The unwarped kinetic term can be rewritten as a potential plus a topological term if we split G_3 into its constituent ISD and anti-ISD part:

$$G_3 = G^{\text{ISD}} + G^{\text{AISD}}, \qquad G^{(\text{A})\text{ISD}} \equiv \frac{1}{2} (G_3 \pm i * G_3)$$

$$\star_6 G^{\text{ISD}} = i G^{\text{ISD}}, \qquad \star_6 G^{\text{AISD}} = -i G^{\text{AISD}}. \qquad (2.43)$$

Then the flux kinetic term in the action becomes

$$S_G = -\frac{1}{2\kappa_{10}^2} \int \frac{G^{\text{AISD}} \wedge \star_6 \overline{G}^{\text{AISD}}}{Im\tau} + \frac{i}{4\kappa_{10}^2} \int \frac{G_3 \wedge \overline{G}_3}{Im\tau}$$
$$= -V - N_{\text{flux}}. \qquad (2.44)$$

² For a more precise treatment that also includes warping, the Einstein term and the F_5 flux term see [34]. The qualitative result remains unchanged and it was shown that the GVW superpotential is not influenced by warping.

The second term is topological and independent of the moduli. In a compact setup it will be cancelled by the localized charges, if we use the tadpole cancellation condition $\int H_3 \wedge F_3 = -2\kappa_{10}^2 T_3 Q_3^{\text{loc}}$ (D7-branes also carry an effective D3-charge given by $-\chi(X)/24$, the Euler character of the corresponding F-theory 4-fold). The potential for the moduli is given by the anti-ISD fluxes only

$$V = \frac{1}{2\kappa_{10}^2} \int \frac{G^{\text{AISD}} \wedge \star_6 \overline{G}^{\text{AISD}}}{Im\tau} \,. \tag{2.45}$$

This means that the potential vanishes identically for ISD flux and gives rise to the ensuing condition $\star_6 G_3 = iG_3$. Note, however, that this potential remains vanishing for fluxes that are non-primitive or that are not of Hodge type (2,1), but still ISD. It requires a more careful analysis, which we will do in the next section, to see how these break supersymmetry.

It is worth stressing again that the conditions and results outlined above are independent of the type of manifold $(\mathcal{M}_6, \tilde{g})$ chosen in the warped compactification.

2.4.2 SUSY for the compact warped Calabi–Yau

For a compact Calabi-Yau, the limited cohomology supported on the manifold allows us to make precise statements about SUSY preserving fluxes. In fact, one can show that any (2,1) flux on a compact Calabi-Yau must be both ISD and primitive. Using the Lefschetz decomposition, we can decompose any harmonic (2,1) flux on a Kähler (and hence also on a CY) manifold into components

$$\chi_{\alpha} = \chi_{\alpha'} + v \wedge J \,, \tag{2.46}$$

where $\chi_{\alpha'}$ and v are primitive forms.³ However, a CY manifold does not support any globally defined one-forms. Thus all 3-forms on a CY manifold must be primitive. One can show that these primitive forms must be ISD by explicit calculation. It follows by simple conjugation of these forms that all (1,2) forms are AISD.

It is clear that any flux that solves the equations of motion (ie. the flux is ISD) on a compact CY will preserve SUSY.

The Kähler potential and superpotential

In order to understand the potential for moduli fields, it will be useful to understand the effective theory in terms of the Kähler potential (\mathcal{K}) and superpotential (W) of an $\mathcal{N} = 1$ 4D effective theory. In terms of these variables, we can write the F-term potential in its canonical supergravity form

$$V_F = e^{\mathcal{K}} \left(\sum_{\alpha} |D_{\alpha}W|^2 - 3|W|^2 \right) , \qquad (2.47)$$

with $D_{\alpha}W = \partial_{\alpha}W + W\partial_{\alpha}\mathcal{K}$ and α running over all Kähler moduli k_a , complex structure moduli z_i and the dilaton Φ .

The Kähler potential can be found through explicit dimensional reduction of the 10D action, comparing the resulting non-canonical kinetic terms with the generic form for the kinetic term of an $\mathcal{N} = 1$ scalar:

$$S = \frac{1}{2\kappa_4^2} \int d^4 \sqrt{-g} x \partial_\alpha \partial_\beta \mathcal{K} \partial_\mu \phi^\alpha \partial^\mu \phi^\beta \,. \tag{2.48}$$

³ Note that primitivity for fluxes that are not 3-forms has a more general meaning than given earlier. Namely, an n-form α on a complex 3-fold is primitive if $J^{4-n} \wedge \alpha = 0$. Thus v is annihilated by $J \wedge J \wedge J$

One finds that the Kähler potential for the complex structure moduli and dilaton is given by [28]

$$\mathcal{K} = -\ln(-i(\tau - \bar{\tau})) - \ln\left(-i\int_{\mathcal{M}_6} \Omega \wedge \bar{\Omega}\right).$$
(2.49)

It is also necessary to find the Kähler potential for the Kähler moduli.

In [35], it was conjectured (and later supported by direct computation [16]), that the correct form of the superpotential is

$$W = \int G_3 \wedge \Omega \,, \tag{2.50}$$

where Ω is the unique (3,0) form on the CY manifold. It is easy to see that this is the natural choice to enforce the previously established supersymmetry constraints. The supersymmetry condition

$$W = \int G_3 \wedge \Omega = 0 \tag{2.51}$$

enforces that W must not be of type (0,3). Note next that

$$\frac{\partial\Omega}{\partial z_j} = -\partial_j \mathcal{K}(z,\bar{z})\Omega^{(3,0)} + \chi_j^{(2,1)}, \qquad (2.52)$$

for z_j complex structure moduli and $\chi_j^{(2,1)}$ the corresponding primitive (2,1) basis element of $H^{(2,1)}$. (see e.g. [28]). Thus

$$D_{j}W = -\partial_{j}\mathcal{K}\int G_{3}\wedge\Omega + \int G_{3}\wedge\chi_{j} + (\partial_{j}\mathcal{K})\int G_{3}\wedge\Omega = \int G_{3}\wedge\chi_{j} \qquad (2.53)$$

and the vanishing of this equation requires that G_3 have no (1,2) components. Lastly, we find that

$$D_{\tau}W = \frac{-1}{\tau - \bar{\tau}} \int \bar{G}_3 \wedge \Omega \,. \tag{2.54}$$

This enforces the last constraint that G_3 not have a (3,0) component.

This perspective also allows us to see more clearly how background G-flux leads to the stabilization of the complex structure moduli. Under changes of the complex structure, the Hodge type of the G_3 will naturally change as Hodge type is determind by the complex structure of the manifold. The G_3 -flux will generically pick up components that are of type (1, 2) and this will break supersymmetry, generating terms $D_jW \neq 0$. A similar argument suggests that G_3 will also generically change its Hodge type under shifts of the dilaton. Because G_3 is determined by the addition of two three forms (recall $G_3 = F_3 - \tau H_3$), changing τ will upset the balance that leads to pure (2,1)-flux and generically introduce (3,0) components. Thus, as above, $D_{\tau}W \neq 0$ and we see that there is a potential for the dilaton.

Although the matching of constraints from the 10D and 4D perspectives is compelling evidence that the Gukov-Witten-Vafa superpotential is correct, one can make a stronger statement. Expanding the scalar potential derived in (2.45) in terms of a basis of 3-form fluxes on a CY, one finds that the potential can be written in the form (2.47) with the above defined superpotential [16].

We also have similar contributions from the Kähler moduli, although these do not enforce any additional supersymmetry constraints. For example, for the volume modulus ρ , one finds

$$D_{\rho}W = \frac{-3W}{\rho - \bar{\rho}} \,. \tag{2.55}$$

One might naively expect that any generic supersymmetry breaking would generate a non-zero potential. However, (2.45) tells us that this is not the case as only supersymmetry breaking with AISD fluxes will generate an uplifted potential. This can be seen similarly from the standpoint of (2.47). Imaginary self-dual SUSYbreaking fluxes correspond to $W \neq 0$, but this does not automatically imply that V is non-zero. Notably, there is a corresponding term

$$G^{\rho\bar{\rho}}D_{\rho}W\overline{D_{\rho}W},\qquad(2.56)$$

which can often exactly cancel this contribution. These so-called no-scale models have broken supersymmetry, but have vanishing potential and cannot stabilize the volume modulus. The form of solution discussed herein is exactly of this type. This can be seen easily because the constraint equations that define our solution are independent of a rescaling of the metric (or, more generally, of perturbations to the Kähler moduli of the metric). Nevertheless, this is merely the leading-order perturbative potential. It does not account for non-perturbative corrections to W, nor to α' corrections to the Kähler potential. Non-perturbative corrections to the superpotential that stabilize the volume modulus (or Kähler moduli) [9] will be an important element in studying inflationary dynamics in chapter 3.

2.5 Non-compact warped solutions

Although we have demonstrated that it is possible to create consistent warped Calabi-Yau compactifications that preserve $\mathcal{N} = 1$ SUSY, this is not a sufficient accomplishment to facillitate the study of inflationary dynamics in this background. We need further to be able to write an explicit metric for the CY manifold. However, this has not been possible for compact CY 3-folds as no explicit metrics are known. One can, instead, consider warped solutions where the internal manifold is Calabi-Yau, but non-compact. In this case, explicit metrics are known and can be simply written. One must be able to show that such non-compact solutions can be consistently embedded into a compact solution as local geometric data.

A particularly interesting variety of solutions are non-compact warped conifolds [8, 36, 12]. Conical singularities naturally arise in the moduli space of compact Calabi-Yau manifolds [28] and hence these non-compact solutions can, in general, be consistently embedded into compact CY backgrounds [16].

The simplest such conifold solution is the singular conifold [37]. It is a hypersurface in \mathbb{C}^4 defined by

$$yz - uv = 0 \tag{2.57}$$

for x, y, u, v complex coordinates. Topologically, the singular conifold can be thought of as cone over the base $S^2 \times S^3$ and has a Ricci flat metric given explicitly by

$$ds^{2} = d\rho^{2} + \frac{1}{9}\rho^{2} \left(d\psi + \cos\theta_{1} d\phi_{1} + \cos\theta_{2} d\phi_{2} \right)^{2} + \frac{\rho^{2}}{6} \left(d\theta_{1}^{2} + \sin^{2}\theta_{1} d\phi_{1}^{2} \right) + \frac{\rho^{2}}{6} \left(d\theta_{2}^{2} + \sin^{2}\theta_{2} d\phi_{2}^{2} \right).$$
(2.58)

At $\rho = 0$, this metric has a true singularity where the curvature becomes infinite.

In [36], Klebanov and Tseytlin constructed warped backgrounds with the singular conifold as the unwarped internal manifold, commonly called the Klebanov-Tseytlin (KT) warped throat. It was shown that placing D3 branes at the tip of the conifold sources F_3 flux, while wrapping D5 branes on a 2-cycle of the conifold would source H_3 flux. Making an exactly analogous ansatz to section 2.3.2, they constructed a consistent supergravity solution with background fluxes turned on. This flux 'compactification' is exactly of the form we described. Recall that this form of solution is sufficient to satisfy all the constraints outlined in a non-compact background and further it is necessary that this form holds locally in compact backgrounds (assuming that all the localized objects obey the relation between the stress-energy tensor and tension outlined in (2.36)).

One can find similar 'compactifications' on other types of conifold backgrounds that are non-singular. These will be described in more detail in chapter 4.

CHAPTER 3 Inflation in string theory

In this chapter we will give a (briefer still) introduction to inflation and its embedding into explicit string theory models. We will focus on brane/anti-brane models of inflation in IIB string theory. The complete avoidance of other interesting models of stringy inflation does not reflect an evaluation of their promise.

Following this overview, we will use the described characteristics of brane/antibrane inflation and some of its remaining challenges to motivate a proposal for a variant inflationary scenario. This scenario will, in turn, be the motivation for the background that we construct and study in this manuscript.

3.1 The inflationary paradigm

Inflation, as a phenomenological paradigm of early-universe cosmology, has been fantastically successful. It provides a simple explanation for the observed flatness and homogeneity of the universe and has been remarkably accurate in predicting and explaining the perturbations observed in the cosmic microwave background [1].

In coarsest of terms, the inflationary model posits a scalar field, rolling down to the minimum of a potential well and, in doing so, generating an exponential expansion of space in the early universe. For inflation to generate sufficient e-foldings, the inflaton field must roll slowly down a potential. If the inflaton field has a canonical kinetic term, then slow-roll requires that the parameters

$$\epsilon = \frac{M_{pl}^2}{2} \left(\frac{V'}{V}\right)^2 \tag{3.1}$$

$$\eta = M_{pl}^2 \left(\frac{V''}{V}\right) \tag{3.2}$$

be much less than unity. For the inflationary models we will be discussing, it will be the η -parameter whose smallness is difficult to control.

While there is much to be understood by considering inflation from the perspective of an effective field theory, it is important to embed the paradigm into a UV complete model such as string theory. If string theory really is the correct UV complete model of our universe, then it must be able to contain the low-energy dynamics of inflation, assuming we are convinced that inflation is, in fact, the correct model of the early universe. More concretely, we also do not expect that the full parameter space of an EFT will be accessible by its realization in a UV complete theory. Finding explicit embeddings into string theory allows one to better measure how realistic and generic the parameters of an EFT really are in that setting.

Perhaps most importantly, from the perspective of an EFT it is not possible to construct models that have $\eta \ll 1$ without reaching to poorly motivated finetuning. This is absolutely generic to all EFTs of slow-roll inflation and can be seen quite simply: We naturally expect the renormalization group flow to generate a higher-dimensional operator

$$\frac{V(\phi)}{M_{pl}^2}\phi\phi^{\dagger}, \qquad (3.3)$$
as this term automatically respects all the symmetries of theory. The EFT thus predicts a contribution to η of order 1 that must be finely tuned away. In a UV complete theory, on the other hand, we might hope that such contributions are forbidden by uniquely stringy mechanisms.

3.2 Brane inflation and warped compactifications

Brane inflation, first proposed in [10] was given a true stringy realization in [11]. In their scenario, they imagined a gas of branes and anti-branes extending across non-compact space and moving in the compact directions. The long-range attraction between oppositely-charged branes pulls them together. The field in the 4D effective theory corresponding to the distance between the branes acts as the inflaton.

We will make the embedding of this model more explicit and consider a D3brane placed in a CY background and extending along all 4 non-compact directions. Further, we will assume that the brane has been fixed by some potential such that it is only free to move towards an attractive anti D3-brane some distance r away.

A simple calculation, first put forward in [11], can show that a trivially factorized Calabi-Yau compactification is too naive a setting for slow-roll brane inflation. In such a background, the 4D Planck mass is determined simply by $M_{4,Pl}^2 = M_{10,Pl}^8 L^6$, where L is the length scale of the CY manifold. Meanwhile, the potential will have a contribution independent of r from the brane tension and a contribution from the exchange of massless bulk modes. All of these bulk modes must have a long range behaviour (by Gauss's Law) that generates a potential with scaling $1/r^4$. We thus have a potential

$$V = A - \frac{B}{r^4} \tag{3.4}$$

and it is then immediate that (3.2) becomes

$$\eta \sim k \left(\frac{L}{r}\right)^6 \,, \tag{3.5}$$

for k some constant. Through a more careful analysis determining A and B and by canonically normalizing the kinetic term for r [38], it can be shown that this constant is of order 1. We thus cannot make η small without having the branes at distances larger than the compactification scale, an impossibility in our approach. While this can seemingly be avoided by introducing extra dimensions of anisotropic scales, it has been shown that this will not render a solution without introducing tachyonic directions in the potential [2].

3.3 Solving the First η -problem with Warped Backgrounds

In [2] (commonly known in the literature as KKLMMT), it was shown that sufficiently small η can be found by generalizing trivially factorized backgrounds to warped compactifications. Consider a warped solution as set forth in the section 2.3.2 with some warp factor $\alpha(r)$ that only depends on an internal coordinate r, a constant dilaton, and a compact CY compactification manifold. Adding a D3-brane at some radius r_1 , one can calculate a perturbation to the warp factor by solving

$$\nabla_6^2 \Delta \alpha = C \delta^6 (r - r_1) \tag{3.6}$$

to find a new warp factor $\alpha(r, r_1) = \alpha(r) + \Delta \alpha(r, r_1)$. An anti-D3-brane, situated at some point r_0 with $r_1 >> r_0$, can then be used as a probe brane to calculate the potential between the brane/anti-brane pair. This can be extracted simply from the DBI and Chern-Simons actions for these branes, given by

$$S_{\overline{D3}} = -T_3 \int d^4x \sqrt{-g} \sqrt{\alpha(r_0, r_1)^2 - \alpha(r_0, r_1)\partial_\mu r_0 \partial^\mu r_0} - T_3 \int d^4x \, \alpha(r_0, r_1) (3.7)$$

$$\simeq \int d^4x \left(T_3 \partial_\mu r_0 \partial^\mu r_0 - 2T_3 \alpha(r_0, r_1) \right).$$
(3.8)

The resulting potential is

$$V(r_0) = 2T_3\alpha(r_0, r_1) \tag{3.9}$$

and the canonical field can be seen from the kinetic term to be $\phi = \sqrt{T_3}r_0$. In KKLMMT, they considered such a setup for a simple background with warpfactor

$$\alpha(r) = \frac{R^4}{r^4} , \quad R = 4\pi a g_s N \alpha'^2 , \quad (3.10)$$

where the parameter a depended on the precise choice of compactification manifold and N is a free parameter determining the number of units of flux wrapping the compact manifold. The resulting perturbed potential was then found to be

$$V = 2T_3 \frac{r_0^4}{R^4} \left(1 - \frac{1}{N} \frac{r_0^4}{r_1^4} \right) \,. \tag{3.11}$$

It is then easy to calculate η again and see that, when $r_0 \ll R$, the factor of r_0^8/R^4 is enough to flatten the potential and generate a small mass term.

3.4 Moduli stabilization and the second η -problem

While at first glance it may seem like [2] solved our η -problem, they also point out that we have crucially neglected the moduli fields discussed in section 2.4.2. If the moduli fields do not have a potential, then it is not consistent to consider our background to be stable and to consider brane dynamics while the moduli are static. Even if the moduli fields have some potential, the slow roll of the field associated to the brane/anti-brane pair is irrelevant if one of the moduli fields rolls quickly.

There are two immediate approaches one may take to this problem. One could consider one or a combination of these moduli fields to be the relevant inflaton field instead of a brane/anti-brane pair. Though there is a vast and productive literature on this approach (see [39], for example, as a review), we will not take this angle here. Rather, we will attempt to find a mechanism that generates a potential to stabilize all of these fields and assume that the moduli fields are already at the minima of their potential.

For warped flux compactifications on compact Calabi-Yau backgrounds, we have shown in section 2.4.2 that the complex structure moduli fields and dilaton will all be stabilized by the potential generated by the *G*-flux wrapping the internal manifold. It remains then only to stabilize the Kähler moduli. We will only consider the Kähler moduli associated to the overall volume of the compactification ρ that is generic to all CY solutions. It will be seen that stabilizing this volume modulus will generically induce a new η -problem that is not nearly as easy to overcome as the one solved by warped backgrounds.

Following [2], we consider a generic potential that stabilizes the volume superfield ρ . Such a potential may arise from non-perturbative corrections to the superpotential from gauge condensates on D7 branes or Euclidean D3-brane instantons [9]

$$W = W_0 + W(\rho), \qquad (3.12)$$

although we will keep the exact form of the potential generic. Further, we note that all potentials arising from brane tensions or fluxes vanish with some power of the physical scale L^{-a} , for a some order one number. This gives a potential of the form

$$V = \frac{X(\rho)}{L^a},\tag{3.13}$$

neglecting explicit dependencies on other moduli. However, the complex volume superfield is related to the physical length modulus of the compactification by [2]

$$2L = \rho + \bar{\rho} - k(\phi, \bar{\phi}), \qquad (3.14)$$

for position fields ϕ_i of the mobile D3. The stabilization of the volume field will thus introduce a dependence on the brane position into the potential. The potential (3.13), when rewritten in terms of complex fields, will have the form

$$V = \frac{X(\rho)}{(\rho + \bar{\rho} - k(\phi, \bar{\phi}))^a}.$$
 (3.15)

We can further assume that there is a minimum in moduli space at $\rho = \rho_0$ and $\phi = 0$, at which point we have that $k(\phi, \bar{\phi}) = \phi \bar{\phi}$. Expanding the potential about this minimum, we find that

$$V = V(\rho_0, 0) \left(1 + a \frac{\phi \bar{\phi}}{\rho_0 + \bar{\rho}_0} \right) .$$
 (3.16)

When the inflaton field ϕ is canonically normalized and renamed φ ¹, this becomes

$$V = V(\rho_0, 0) \left(1 + 3a\varphi\bar{\varphi}\right) \tag{3.17}$$

with a mass term of order one. This argument is also made more explicitly for particular backgrounds of brane inflation in [2].

We see that a generic attempt to stabilize the volume modulus leads to mass terms for the inflation field that give an order one contribution to η . In order to eliminate this effect, one has to introduce further dependence on the brane position into the scalar potential potential. That this further contribution should have the correct sign and cancel the existing term to high precision is not naively expected and would necessitate a finely-tuned potential. The hope, then, is that one can find such a tuned contribution that arises naturally.

3.5 Finely tuned remedies

A particularly successful explicit scenario for examining the η -problem in D3/D3 inflation was put forward in [3]. They consider a warped deformed conifold background² with a stack of D7 branes placed on a specific cycle determined by the Kuperstein Embedding [40].

¹ The normalization that gives the canonical form of the inflaton kinetic term is $\varphi = \phi/\sqrt{3(\rho + \bar{\rho})}$. That this is correct can easily by seen by examining the relevant term in the Kahler potential: $K \ni -3\ln(\rho + \bar{\rho} - k(\phi, \bar{\phi}))$. Again, we are taking a quadratic approximation to $k(\phi, \bar{\phi})$.

 $^{^2}$ A more detailed introduction to this background will be given at the beginning of chapter 4.

In previous work, the non-perturbative corrections to the superpotential due to gauge condensates on the D7 brane were calculated [41]. It was found that not only did the condensate contribution depend on the volume superfield, as suggested first in [9], but also on the position of a D3-brane. This further effect is due to the D3-brane's backreaction on the cycle on which the D7 is wrapped. For a stack of nD7-branes wrapped on a cycle determined by the holomorphic equation

$$f(z_i) = 0, (3.18)$$

the non-perturbative correction is given by

$$W(\rho, z_i) = W_0 + A_0 \left(\frac{f(z_i)}{f(0)}\right)^{1/n} e^{-a\rho}, \qquad (3.19)$$

for some constants A_0 and a that will not be of particular importance to us here.

The explicit dependence of the superpotential on the brane position contributes additional terms to the F-term potential. These new terms correct the potential for the mobile D3-brane only at 3rd order; they found no direct cancellation of the large mass term for the inflaton. More concretely, they found that η takes the form

$$\eta = \frac{2}{3} - \eta_{-1/2} \left(\frac{\phi}{\phi_{\mu}}\right) \,, \tag{3.20}$$

for some parameters ϕ_{μ} , $\eta_{-1/2}$ that are determined by the Kuperstein embedding itself and by the precise embedding of the entire deformed conifold scenario into a complete compact CY background. It is clear that the η -problem will only disappear near inflection points where these two terms cancel. Such a set-up remains finelytuned and does not present itself as a natural solution.

3.6 D3/D7 inflation

A variant model of brane inflation considers instead a D3/D7-brane system, where again the inter-brane distance serves as the inflaton field [6]. In this case, the branes are attracted by a potential generated from gaugino condensates on the D7-brane and SUSY-breaking fluxes on the D7 world volume. The F-term and Dterm potentials appearing from the gaugino condensate and SUSY-breaking fluxes conspired to give a consistent resolution of the anomalies associated with the FI terms.

D3/D7 inflation has primarily been studied in toroidal manifolds of the form T^n/Γ , for Γ some discrete symmetry group, the most common background of which is $K3 \times T^2/\mathbb{Z}_2$ studied in [6, 42]. It has not been studied in warped conifold backgrounds of the type discussed here for $D3/\overline{D3}$ inflation.

The D3/D7 inflationary model did not have an initial η -problem in [6]. As its inflationary potential is generated by D-terms rather than F-terms, it may inherit an η -problem from non-perturbative effects needed to stabilize the volume modulus. Although mass terms from the non-perturbative F-term potential are prohibited by a shift symmetry at the classical level, there is a strong indication that this will not be preserved in quantum corrections [7, 42].

3.7 A possible scenario for natural inflation

Let us now sketch a possible model of inflation using the resolved conifold background with D7 branes and additional D3 branes. This scenario will combine the basic premise of $D3/\overline{D3}$ in the 'warped throat' with D3/D7 models³.

We want to balance a D3-brane that is attracted towards the D7-brane (because of the non-primitive flux on the D7 worldvolume) with another force that drives the D3 toward the tip. This can be achieved by using a background in which the addition of a D3 explicitly breaks supersymmetry, such as the resolved warped deformed conifold [15]. The motion of D3-branes towards the tip in the latter background is a consequence of the running dilaton. One could supplement the attraction due to the running dilaton by adding additional anti-D3-branes at the bottom of the throat.

While either of these potentials alone are still too steep for slow–roll inflation, combining both forces we might hope to slow down the motion of the D3 in either the one or the other direction. There are two possible scenarios, depending on which force dominates:

- The D-term potential created by the non-primitive flux dominates and attracts the D3-brane towards the wrapped D7 brane. Inflation ends when the D3 dissolves into the D7 as non-commutative instantons and supersymmetry is restored.
- The attraction towards the anti–D3 brane at the bottom of the throat (or by a running dilaton in a more general background) dominates. Inflation ends as

³ A Similar idea has been proposed independently by Cliff Burgess.

all or some D3 branes getting annihilated by the anti–D3 brane(s) at the tip of the throat.

The naive hope is that the motion in either direction will be slow because the D3 branes are pulled in both directions. Achieving such slow roll will not necessarily be another appeal to fine-tuning. A proper arrangement of D3/D7-branes in the singular conifold will preserve supersymmetry; there will be no forces on the branes and the configuration will be static. The appearance of a running dilaton and of supersymmetry-breaking fluxes on the D7-branes should be controlled by the small deformation of the background away from the singular geometry. If both forces are controlled by the same parameter, there is hope that they will be of the same magnitude.

If we want to combine the D-term and F-term potentials we are faced with an issue pointed out by [25]: for a supersymmetric background it is impossible to have a *D*-term potential that could be used to pull the D3 brane towards the D7 branes. Thus if we want to switch on non-primitive fluxes on the wrapped D7 branes we have to embed the D7 branes in a non-supersymmetric background. Our problem becomes threefold:

- Construct a supergravity background with embedded D7 branes that breaks supersymmetry spontaneously;
- Allow for a possible D-term uplifting by avoiding the no-go theorem of [25], as pointed out by [19]. Note that the D7 worldvolume theory will not only contribute the D but also possible F-terms, such that the issue of [25] might be resolved;

• Balance the D3 brane using the two forces: one from the D-term potential and the other from the attractive force at the tip of the deformed conifold in the setup of [2].

In this manuscript we will address the first two problems by constructing a nonsupersymmetric background with D-terms on the D7 branes given by the pullback of a non-primitive flux. To analyze the last problem, we might have to go to a more generic background like the resolved warped deformed conifold [15], whereas most of the literature deals with the limit where the manifold looks like a singular conifold.

CHAPTER 4 D7–branes on the resolved conifold

4.1 The background

The simplest "throat" studied so far is the singular conifold, a warped flux background known as the Klebanov–Tseytlin (KT) solution [36], which we introduced in section 2.5. The singularity at the tip of the conifold can be smoothed out in two different ways: by blowing up a 3–sphere (the deformed conifold) or by blowing up a 2–sphere (the resolved conifold). Both of these manifolds are still Calabi–Yau. These particular backgrounds, with added fluxes, have been studied by Klebanov–Strassler (KS) [8] and Pando Zayas–Tseytlin (PT) [12] respectively.

On the other hand, one could imagine a less simple, but more general, background that allows for both blown-up 2– and 3–cycles. The "resolved warped deformed conifold" can be interpreted as such a manifold. It was suggested [43] (see also [44]) as an interpolating solution between the KS and Maldacena–Nunez (MN) solutions. It turns out [45, 15] that the correct interpolating solution does not have a blown-up 2–cycle but a broken \mathbb{Z}_2 symmetry in comparison to the singular or deformed conifold metric. It is not a CY anymore, but an SU(3) structure manifold. Since the resolved conifold is the simplest manifold with such a broken \mathbb{Z}_2 (corresponding to the exchange of the two 2-cycles in (4.1)) we will use it as a toy model here, keeping in mind that our motivation stems from the "resolved warped deformed conifold". Apart from the broken \mathbb{Z}_2 symmetry, there is another interesting feature: the background exhibits a running dilaton, in contrast to the KT, KS or PT solutions on warped CY's with constant dilaton. Placing a D3-brane in this background will result in a force due to this running dilaton. This does not mean that the resolved warped deformed conifold breaks supersymmetry, but rather that the D3 oriented along Minkowski space does not preserve the same subset of supercharges. There is another source of a running dilaton that will be of interest to us: D7-branes. Their behavior will be determined by the particular embedding we choose for the D7.

The most general "throat" background, taken as one that allows for blown–up 2– and 3–cycles, has the metric

$$ds^{2} = F_{3} dr^{2} + F_{4} (d\psi + \cos\theta_{1} d\phi_{1} + \cos\theta_{2} d\phi_{2})^{2} + F_{1} (d\theta_{1}^{2} + \sin^{2}\theta_{1} d\phi_{1}^{2}) + F_{2} (d\theta_{2}^{2} + \sin^{2}\theta_{2} d\phi_{2}^{2})$$
(4.1)
+ $2b \bigg[\cos\psi (d\theta_{1} d\theta_{2} + \sin\theta_{1} \sin\theta_{2} d\phi_{1} d\phi_{2}) - \sin\psi (\sin\theta_{2} d\phi_{2} d\theta_{1} - \sin\theta_{1} d\phi_{1} d\theta_{2}) \bigg],$

where the coefficients F_i , b are functions of the radial coordinate r, (θ_i, ϕ_i) parameterize two 2-spheres, and $\psi = 0 \dots 4\pi$ is a U(1) fibration over those spheres. The commonly known backgrounds are found in the limits:

singular conifold: F₁ = F₂ and b = 0, i.e. both 2-spheres have equal radii (and shrink to zero size as r → 0), the cross-terms in the third line in (4.1) are absent;

- deformed conifold: F₁ = F₂ and b ≠ 0, i.e. both 2-spheres have equal radii, but the U(1) shift symmetry is broken due to the more complicated fibration in the third line;
- resolved conifold: F₁ ≠ F₂ and b = 0, i.e. the 2-spheres have unequal size (this corresponds to the breaking of a discrete Z₂ exchanging both) and the third line in (4.1) is absent.

For a complete definition of the functions F_i we refer the reader to [15, 37, 46]. They are of course more restricted than outlined above in order to guarantee an SU(3) holonomy or SU(3) structure. In [5], the limit

$$F_1 \approx F_2 = \frac{r^2}{6}, \quad b \to 0, \quad F_3 = 1, \quad F_4 = \frac{r^2}{9}$$
 (4.2)

was employed. In this limit the background becomes a (non-compact) singular conifold, and one can add D7 branes using the technique discussed in [13]. This is the simplest choice and works well in the situation when we are far from the tip of the throat and the resolution parameter (the size of the 2–sphere that remains finite) is very small. Here, we intend to go beyond this simplification. However, the resolved warped deformed conifold is difficult to study, mostly because it is not a CY. We therefore choose the simplest approximation that captures the essential feature of the broken \mathbb{Z}_2 : we choose to restrict ourselves to the resolved conifold.

We will turn on fluxes (or rather borrow them from the PT solution [12]) that break supersymmetry because they are not only of cohomology type (2,1), but also (1,2). This is not possible on a compact CY. (1,2) flux can only be ISD if it is of the form $J^{1,1} \wedge \bar{m}^{0,1}$, for some antiholomorphic 1–form \bar{m} (J is the Kähler form). This would require a nontrivial one-cycle, so the first Chern class cannot be zero anymore. This argument breaks down for non-compact manifolds, as Poincaré duality fails. In a consistent compactification, one therefore has to change the background as to not be conformally CY, or to glue it onto a compact bulk in such that the entire compactification manifold is no longer CY. This would lead us beyond the case of conformal CY with flux compactifications examined in [29] or GKP [16], and is beyond the scope of this work. In section 4.2 we will review the PT background and explain why it already breaks supersymmetry. It will be shown, however, that this does not lead to uplifting as the cosmological constant remains zero (this is explained in section 4.3). Only after we embed D7-branes in this background (see section 4.4) we can observe the D-terms and uplift our potential. This calculation is performed in section 4.5.

4.2 The warped resolved conifold with fluxes

Similar to the Klebanov–Strassler model, a warped geometry can be created by fluxes in the *resolved* conifold background (see appendix A for a thorough discussion of this geometry and definition of coordinates). The full supergravity solution for the resolved conifold was derived by Pando–Zayas and Tseytlin [12] (PT) and includes non–trivial RR and NS flux with constant dilaton. It can be understood as placing a stack of fractional D3–branes (i.e. D5–branes that wrap a 2–cycle) in this background. The ten–dimensional metric is found to be

$$ds_{10}^2 = h^{-1/2}(\rho) \eta_{\mu\nu} dx^{\mu} dx^{\nu} + h^{1/2}(\rho) ds_6^2, \qquad (4.3)$$

where ds_6^2 refers to the resolved conifold metric given by

$$ds_{6}^{2} = \kappa(\rho)^{-1} d\rho^{2} + \frac{\kappa(\rho)}{9} \rho^{2} \left(d\psi + \cos\theta_{1} d\phi_{1} + \cos\theta_{2} d\phi_{2} \right)^{2} + \frac{\rho^{2}}{6} \left(d\theta_{1}^{2} + \sin^{2}\theta_{1} d\phi_{1}^{2} \right) + \frac{\rho^{2} + 6a^{2}}{6} \left(d\theta_{2}^{2} + \sin^{2}\theta_{2} d\phi_{2}^{2} \right).$$
(4.4)

Note that as $\rho \to 0$, the (θ_2, ϕ_2) sphere remains finite, whereas for the singular conifold both (θ_i, ϕ_i) spheres scale with $\rho^2/6$. The parameter *a* is called the resolution parameter because it determines the size of the resolved 2–sphere. This asymmetry in the geometry also determines an asymmetry in the flux on the 2–cycles and is the source of supersymmetry breaking. The 3–form fluxes in this background are¹

$$H_3 = d\rho \wedge [f_1'(\rho) d\theta_1 \wedge \sin \theta_1 d\phi_1 + f_2'(\rho) d\theta_2 \wedge \sin \theta_2 d\phi_2]$$
(4.5)

$$F_3 = Pe_{\psi} \wedge (d\theta_1 \wedge \sin \theta_1 \, d\phi_1 - d\theta_2 \wedge \sin \theta_2 \, d\phi_2) \tag{4.6}$$

and the self-dual 5-form flux is given by

$$F_5 = \mathcal{F} + *\mathcal{F} , \qquad \mathcal{F} = K(\rho) e_{\psi} \wedge d\theta_1 \wedge \sin \theta_1 \, d\phi_1 \wedge d\theta_2 \wedge \sin \theta_2 \, d\phi_2 , \qquad (4.7)$$

where

$$f_{1}(\rho) = \frac{3}{2}g_{s}P\ln(\rho^{2} + 9a^{2})$$

$$f_{2}(\rho) = \frac{1}{6}g_{s}P\left(\frac{36a^{2}}{\rho^{2}} - \ln[\rho^{16}(\rho^{2} + 9a^{2})]\right)$$

$$K(\rho) = Q - \frac{1}{3}g_{s}P^{2}\left(\frac{18a^{2}}{\rho^{2}} - \ln[\rho^{8}(\rho^{2} + 9a^{2})^{5}]\right)$$
(4.8)

¹ There is a typo in eq. (4.3) in [12], concerning the sign of F_3 .

and where P is proportional to the number of fractional D3-branes and Q proportional to the number of regular D3-branes, and both are proportional to α' . This solution is of the exact form of our ansatz in the non-compact CY setting discussed in chapter 2.

It was pointed out in [47] and confirmed in [46] that this solution breaks supersymmetry. The reason lies in the fact that the 3-form flux has not only a (2,1), but also a (1,2) part (see Section 2.4.1). It is, nevertheless, a consistent supergravity solution because the 3-form flux $G_3 = F_3 - iH_3$ obeys the imaginary self-duality condition $*_6G_3 = iG_3$ as was derived earlier in Section 2.3.3.

It is sufficiently important to show explicitly that this statement is correct. Using (A.15) we can rewrite the 3-form flux in terms of vielbeins

$$G_3 = -\frac{18P}{\rho^3 \sqrt{\kappa}} \left(e_2 \wedge e_3 \wedge e_4 + i \, e_1 \wedge e_5 \wedge e_6 \right) + \frac{18P \left(e_2 \wedge e_5 \wedge e_6 + i \, e_1 \wedge e_3 \wedge e_4 \right)}{\rho \sqrt{\rho^2 + 6a^2} \sqrt{\rho^2 + 9a^2}} \,. \tag{4.9}$$

The vielbein notation is extremely convenient to see that this flux is indeed imaginary self-dual. The Hodge dual is simply found by

$$*_6(e_{i_1} \wedge e_{i_2} \wedge \ldots \wedge e_{i_k}) = \epsilon_{i_1 i_2 \dots i_k}^{i_{k+1} \dots i_6} e_{i_{k+1}} \wedge \ldots \wedge e_{i_6}$$

and does not involve any factors of \sqrt{g} . We use the convention that $\epsilon_{123456} = \epsilon_{123}^{456} =$ 1. With the complex structure (A.17) the PT flux becomes

$$G_3 = \frac{-9P}{\rho^3 \sqrt{\rho^2 + 6a^2} \sqrt{\rho^2 + 9a^2}} \left[\left(\rho^2 + 3a^2\right) \left(E_1 \wedge E_2 \wedge \overline{E}_2 - E_1 \wedge E_3 \wedge \overline{E}_3\right) + 3a^2 \left(E_2 \wedge \overline{E}_1 \wedge \overline{E}_2 + E_3 \wedge \overline{E}_1 \wedge \overline{E}_3\right) \right]. (4.10)$$

We make several observations: This flux is neither primitive² nor is it of type (2,1). We can see that it has a (1,2) and a (2,1) part, which cannot be avoided by a different choice of complex structure. Consequently, this flux does indeed break supersymmetry.

We also observe that, in the limit $a \rightarrow 0$, the (1,2) part vanishes, the flux becomes primitive, and we recover the singular conifold solution. This indicates that the resolution forbids a supersymmetric supergravity solution, i.e. the blow-up of a nontrivial 2-cycle in a conifold geometry can lead to supersymmetry breaking. We will exploit this fact to our advantage.

4.3 The scalar potential and supersymmetry

We have just argued that the non-primitive (1,2) flux breaks supersymmetry. One might therefore wonder if it can be used to uplift our potential to a positive vacuum. The answer is a resolute no because the scalar potential always remains zero when the flux is ISD, regardless of whether or not the vacuum breaks supersymmetry. This follows immediately from (2.45), but it is not clear why this is true from the supergravity perspective. Keeping in mind that the supergravity form of the potential derived earlier (2.47) is only valid for a compact CY manifold, can we nevertheless understand the vanishing potential from this point of view? Clearly, there is no F-term associated to derivatives w.r.t. the Kähler parameter or the dilaton, as the superpotential (2.50) does not depend on them. But what about an F-term $D_{z_j}W$?

 $[\]overline{\sum_{i=1}^{2} \text{Since } J = \frac{i}{2} \sum_{i} (E_i \wedge \overline{E_i})} \text{ it follows immediately that } J \wedge G_3 \text{ has a nonvanishing } E_2 \wedge E_3 \wedge \overline{E_1} \wedge \overline{E_2} \wedge \overline{E_3} \text{ part that is proportional to } a^2.$

Let us for a moment assume we are still talking about a CY, although (1,2) ISD flux cannot exist on a compact CY. So we still assume our moduli space to be parameterized by Ω and χ_i the holomorphic three-form and primitive basis of (2,1) forms. Let us furthermore assume the superpotential is still given by (2.50). Then it is easy to see that there could be a non-vanishing derivative of W w.r.t. a complex structure parameter. Using (2.52) one finds

$$\partial_{z_i} W = k_i(z, \bar{z}) W + \int G_3 \wedge \chi_i^{(2,1)},$$
(4.11)

which could be nonvanishing for G_3 of type (1,2). But (1,2) flux can only be ISD if it is proportional to the Kähler form, $G^{(1,2)} = J^{(1,1)} \wedge \overline{m}^{(0,1)}$, so this becomes

$$\partial_{z_i} W = \int J^{(1,1)} \wedge \bar{m}^{(0,1)} \wedge \chi_i^{(2,1)} = 0$$
(4.12)

when we use the fact that χ_i is primitive, $J^{(1,1)} \wedge \chi_i^{(2,1)} = 0$. If there is no (0,3) part present, W vanishes identically and

$$D_{z_i}W = \partial_{z_i}W + W \partial_{z_i}\mathcal{K} = 0, \qquad (4.13)$$

so all F-terms vanish in our setup. Note that in the non-compact scenario the term $-3|W|^2$ is absent (we neglected M_P in above formulae). However, our argument does not depend on the no-scale structure of the model. W is identically zero, because we don't have any (0,3) flux turned on, and all F-terms vanish individually.

This discussion has two immediate weak points: First of all, we can no longer assume our moduli space is only parameterized by Ω and χ_i if we allow for a (1,2) flux. Once we compactify, there has to be a basis for the one-form $m^{(1,0)}$ as well (for simplicity of the argument let us assume there is only one such 1-form in the following). This would modify the derivative of Ω , the natural guess respecting the (3,0)+(2,1) structure³ being

$$\frac{\partial\Omega}{\partial_{z_j}} = k_j(z,\bar{z})\Omega^{(3,0)} + \chi_j^{(2,1)} + \nu_j J^{(1,1)} \wedge m^{(1,0)} \,. \tag{4.14}$$

If we keep using the GVW superpotential, we get an additional term

$$\partial_{z_j} W = \int G_3 \wedge (\nu_j J^{(1,1)} \wedge m^{(1,0)}) = \int J^{(1,1)} \wedge \bar{m}^{(0,1)} \wedge \nu_j J^{(1,1)} \wedge m^{(1,0)}, \quad (4.15)$$

which will in general be non-zero for the type of G_3 flux we have turned on. However, the superpotential will also change since we have to expand G_3 in this new basis as well:

$$G_3^{\text{AISD}} = g_1 \Omega + g_2^i \, \bar{\chi}_i + g_3 \, J \wedge \bar{m} \,. \tag{4.16}$$

Plugging this into the scalar potential (2.45) does not give (2.47) in terms of the superpotential defined for CY manifolds. To bring this into the form of the standard F-term potential we would need to know the metric on the new moduli space, which does not correspond to a CY anymore. Finding the relevant moduli space would allow one to see how W changes. It is likely that it will contain terms with J, and thus will introduce a dependence on Kähler structure moduli. This breaks the no-scale structure and we have to re-examine the cancellation between $D_{k_a}W$ and W.

 $[\]overline{}^{3}$ In the case of a complex manifold, the original derivation [28] holds and (4.14) would not acquire an extra term.

Regardless, we know that the combination $\sum_{\alpha} |D_{\alpha}W|^2 - 3|W|^2$ has to vanish, as (2.45) remains valid. ISD flux cannot give a non-zero potential.

In addition, it is worth noting that we may have to modify the superpotential as to include a term enforcing primitivity. In the compact CY setting this is already taken care of, because an ISD (2,1) form is always primitive. The ISD (1,2) form we are concerned with, on the other hand, is not. If we allow for this type of flux, we should introduce a term that reproduces the primitivity condition as a SUSY condition DW = 0. This was already considered in an M/F-theory context [35], where it was conjectured that

$$\widetilde{W} = \int J \wedge J \wedge G_4 \,. \tag{4.17}$$

Given this definition, $D_J \widetilde{W} = 0$ leads to the primitivity condition $J \wedge G_4 = 0$ for the 4-form flux on the 8-manifold. It is not obvious how this term reduces to type IIB. It will not give rise to a superpotential, but rather to a D-term, as it depends on the Kähler moduli and not the complex structure moduli. For a $K3 \times K3$ orientifold, the dimensional reduction of \widetilde{W} has been carried out [21] and the result agrees with that obtained in type IIB from a D7-worldvolume analysis [22]. Also in the F-theory setup, only the non-primitive fluxes on the D7-branes create a D-term in the effective four-dimensional theory. We can therefore safely conclude that the supersymmetry breaking due to the (1,2) flux will not be visible in the scalar potential that appears from the reduction of the IIB bulk action.

There is also an enlightening discussion in [48] where it was illustrated that, from an F-theory point of view, a flux of type (0,4), (4,0), or proportional to $J \wedge J$ can break supersymmetry without generating a cosmological constant. It is the latter case that corresponds to non-primitive ISD flux in IIB. We do not have an explicit map between these two types of fluxes, but we do present some arguments in [26]. It should be clear that ISD flux lifts to self-dual flux in F-theory and that the non-primitivity property is preserved in this lift.

To summarize, the supersymmetry breaking associated to non-primitive (1,2) fluxes will not give rise to an F-term uplift, as the scalar potential generated by the flux in the IIB bulk action remains zero, so does the superpotential if we rely on the CY property of the resolved conifold. We can, however, in the spirit of KKLMMT allow a non-vanishing W_0 that is created by fluxes in the compact bulk that is glued to the throat. It does not appear in the scalar potential because of the no-scale structure of these models (but it will, once the no-scale structure is broken by non-perturbative effects or because the superpotential is not simply the one from GVW [35] anymore).

The (1,2) flux gives rise to an "auxiliary D-term" [33], which is absent in the 4d scalar potential but can be understood as an FI-term from an anomalous U(1) on the D7 worldvolume (the pullback of the B-field on the D7 worldvolume enters into the DBI action). Let us therefore turn to the question how to embed a D7 in the resolved conifold background; we will then turn to the computation of the D-terms in section 4.5.

4.4 Ouyang embedding of D7–branes on the resolved conifold

We consider now the addition of D7-branes to the PT background. In [13] a holomorphic embedding of D7-branes into the singular conifold background was presented. Such an embedding is necessary to preserve supersymmetry on the submanifold, although not alone sufficient (complete BPS conditions are found in [49, 50]). In particular, supersymmetry requires that, in the absence of gauge flux on the D7, the pullback of the flux is (1,1) and primitive on the cycle wrapped by the D7.

The holomorphic embedding chosen in [13] is described by

$$z = \mu^2, \qquad (4.18)$$

where z is one of the holomorphic coordinates defined in (A.8). It is commonly known as the Ouyang embedding. Although we already know that the PT background breaks supersymmetry, we will use precisely the same embedding (we consider only $\mu = 0$ for simplicity). It is worth emphasizing that this embedding, first considered on the singular conifold, remains holomorphic on the resolved conifold (details are found in Appendix B). As a consistency check we should always be able to recover the original singular solution in the limit $a \to 0$.

The singular solution from [13] is actually not supersymmetric, though one might have expected otherwise. The primitivity of the pulled-back flux is not met by the singular Ouyang embedding in [13]. It is possible to restore supersymmetry by turning on appropriate gauge flux such that the combination of the gauge flux and background flux is primitive [51]. Nevertheless, as we will demonstrate in section 4.5, this susy breaking in [13] does not manifest itself in a D-term. The D7-brane induces a non-trivial axion-dilaton

$$\tau = \frac{i}{g_s} + \frac{N}{2\pi i} \log z , \qquad (4.19)$$

where N is the number of embedded D7-branes and z is now a holomorphic coordinate on the resolved conifold, (see (A.8) for its definition). As pointed out in [5], there is an additional running of the dilaton in the "resolved warped deformed conifold" background, but in does not show up in our toy model because we use a conformal CY. As we focus on the limit where the geometry looks like the resolved conifold (i.e. $b \rightarrow 0$ in (4.1)), we recover the PT supergravity solution, which has a constant dilaton. We will therefore concentrate on the running of the dilaton (4.19) as generated by the D7-brane embedding⁴. This running dilaton was not taken into account by [3], where the D7 is embedded in the singular conifold and a D3-brane is attracted towards an anti-D3 at the bottom of the throat. The given reasoning is that the dilaton contribution should be exactly cancelled by a change in geometry when approaching the supersymmetric limit (if the D7-brane embedding is supersymmetric and the D3-brane preserves the same supersymmetry, the scenario has to be stable when the SUSY-breaking anti-D3 is removed). Our setup, on the other

⁴ Note that the choice of our axio-dilaton is the minimal one that guarantees that under a z monodromy we will recover the correct charge of a single D7 brane. However there is a much deeper reason for choosing this. As we will discuss later in section 3.3, our background appears from a F-theory compactification on a fourfold with Euler number 19728. For such a fourfold the local D7 brane charge is exactly given by the above choice of the axio-dilaton with N being the effective number of coincident local and non-local seven branes.

hand, is non-supersymmetric from the start and therefore we are not led to conclude that the running of the dilaton should vanish from a similar line of argument. It will, however, be suppressed by the susy breaking scale. For a viable inflationary scenario one should rather use the resolved warped deformed conifold; its running dilaton will be the primary reason for a D3 to move towards the tip⁵. In this section we simply want to study the backreaction of the dilaton onto the background.

We determine the change the dilaton induces in the other fluxes and the warp factor at linear order $g_s N$, (see appendix B for details of the calculation). We neglect any backreaction on the geometry beyond a change in the warp factor, i.e. we will assume the manifold remains a conformal resolved conifold. This ansatz turns out to be justified because we are able to solve the eom's with the internal manifold still being the resolved conifold. A distortion of the conifold with Ouyang embedding has been studied in [52], but the D7-branes are smeared over the angular directions, such that the dilaton does not exhibit the behavior (4.19) and runs as log ρ only. Instead of choosing this approximation, we will prove the consistency of our approach by solving all equations of motion explicitly.

⁵ Such a scenario has been studied in [5], where the running dilaton due to a blown–up 2–cycle was parameterized by $\delta N(a) \log z$, where a is a small resolution. This analysis was based on the original Ouyang embedding [13], which we will now reconsider for the resolved conifold.

Consider first the Bianchi identity, which in leading order becomes

$$d\hat{G}_3 = d\hat{F}_3 - d\tau \wedge \hat{H}_3 - \tau \wedge d\hat{H}_3 = -d\tau \wedge H_3 + \mathcal{O}((g_s N)^2)$$

$$= -\left(\frac{N}{2\pi i}\frac{dz}{z}\right) \wedge \left(df_1(\rho) \wedge d\theta_1 \wedge \sin\theta_1 \, d\phi_1 + df_2(\rho) \wedge d\theta_2 \wedge \sin\theta_2 \, d\phi_2\right) + \mathcal{O}((g_s N)^2),$$
(4.20)

where H_3 indicates the unmodified NS flux from (4.5), whereas the hat indicates the corrected flux at leading order. In order to find a 3–form flux that obeys this Bianchi identity, we make an ansatz

$$\hat{G}_3 = \sum \alpha_i \,\eta_i \,, \tag{4.21}$$

where $\{\eta_i\}$ is a basis of imaginary self-dual (ISD) 3-forms on the resolved conifold. In accordance with the observations about the cohomology of G_3 , we do not restrict ourselves to (2,1) forms, but allow for η_i of (1,2) cohomology as well. With the convention (A.17) we define

$$\eta_{1} = E_{1} \wedge E_{2} \wedge \overline{E}_{2} - E_{1} \wedge E_{3} \wedge \overline{E}_{3}$$

$$\eta_{2} = E_{1} \wedge E_{2} \wedge \overline{E}_{3} - E_{1} \wedge E_{3} \wedge \overline{E}_{2}$$

$$\eta_{3} = E_{1} \wedge E_{2} \wedge \overline{E}_{1} + E_{2} \wedge E_{3} \wedge \overline{E}_{3}$$

$$\eta_{4} = E_{1} \wedge E_{3} \wedge \overline{E}_{1} - E_{2} \wedge E_{3} \wedge \overline{E}_{2}$$

$$\eta_{5} = E_{2} \wedge E_{3} \wedge \overline{E}_{1}$$

$$\eta_{6} = E_{1} \wedge \overline{E}_{1} \wedge \overline{E}_{3} + E_{2} \wedge \overline{E}_{2} \wedge \overline{E}_{3}$$

$$\eta_{7} = E_{1} \wedge \overline{E}_{1} \wedge \overline{E}_{2} - E_{3} \wedge \overline{E}_{2} \wedge \overline{E}_{3}$$

$$\eta_{8} = E_{2} \wedge \overline{E}_{1} \wedge \overline{E}_{2} + E_{3} \wedge \overline{E}_{1} \wedge \overline{E}_{3}$$

$$(4.22)$$

•

Note that there are five (2,1) ISD forms, but only three (1,2) ISD forms. This is due to the fact that a form of type (1,2) can only be ISD if it is proportional to J.

Not surprisingly, there is no solution to the Bianchi identity involving only the (2,1) forms. We find a particular solution in terms of only four of above eight 3-forms

$$P_3 = \alpha_1(\rho) \eta_1 + e^{-i\psi/2} \alpha_3(\rho, \theta_1) \eta_3 + e^{-i\psi/2} \alpha_4(\rho, \theta_2) \eta_4 + \alpha_8(\rho) \eta_8, \quad (4.23)$$

with

$$\alpha_{1} = \frac{3g_{s}NP}{8\pi\rho^{3}} \frac{\left[18a^{2} - 36(\rho^{2} + 3a^{2})\log\left(\frac{\rho}{a}\right) + (10\rho^{2} + 72a^{2})\log\left(\frac{\rho^{2}}{\rho^{2} + 9a^{2}}\right)\right]}{\sqrt{\rho^{2} + 6a^{2}}\sqrt{\rho^{2} + 9a^{2}}}$$

$$\alpha_{3} = -3\sqrt{6}g_{s}NP \frac{72a^{4} - 3\rho^{4} + a^{2}\rho^{2}(\log(\rho^{2} + 9a^{2}) - 56\log\rho)}{8\pi\rho^{3}(\rho^{2} + 6a^{2})^{2}} \cot\frac{\theta_{1}}{2}$$

$$\alpha_{4} = -9\sqrt{6}g_{s}NP \frac{\rho^{2} - 9a^{2}\log(\rho^{2} + 9a^{2})}{8\pi\rho^{4}\sqrt{\rho^{2} + 6a^{2}}} \cot\frac{\theta_{2}}{2}$$

$$(4.24)$$

$$3a^{2} \left[a_{2} + NP \frac{-9(\rho^{2} + 4a^{2}) + 28\rho^{2}\log\rho + (81a^{2} + 13\rho^{2})\log(\rho^{2} + 9a^{2})}{8\pi\rho^{4}\sqrt{\rho^{2} + 6a^{2}}}\right]$$

$$\alpha_8 = \frac{3a^2}{\rho^2 + 3a^2} \left[3g_s NP \frac{-9(\rho^2 + 4a^2) + 28\rho^2 \log \rho + (81a^2 + 13\rho^2) \log(\rho^2 + 9a^2)}{8\pi\rho^3 \sqrt{\rho^2 + 6a^2} \sqrt{\rho^2 + 9a^2}} + \alpha_1 \right].$$

Note that a_8 is implicitly given by α_1 . Furthermore, we find a homogeneous solution

$$G_{3}^{hom} = \beta_{1}(z,\rho) \eta_{1} + e^{-i\psi/2} \beta_{3}(\rho,\theta_{1}) \eta_{3} + e^{-i\psi/2} \beta_{4}(\rho,\theta_{2}) \eta_{4} \qquad (4.25)$$
$$+ e^{-i\psi} \beta_{5}(\rho,\theta_{1},\theta_{2}) \eta_{5} + \beta_{8}(z,\rho) \eta_{8},$$

with β_i given in (B.10). This solution has the right singularity structure at z = 0 and $\rho = 0$, but it does not transform correctly under $SL(2,\mathbb{Z})$. When $\psi \to \psi + 4\pi$, the axion–dilaton transforms as $\tau \to \tau + N$. This would imply that G_3 has to be invariant under this shift, which is true for the particular solution, but not the homogeneous one. We therefore conclude that the correction to the 3–form flux, which is in general

a linear combination of P_3 and G_3^{hom} , is given by (4.23) only

$$\hat{G}_3 = G_3 + P_3. \tag{4.26}$$

Note that in terms of η_i the original 3–form flux was given by

$$G_3 = -9P \frac{(\rho^2 + 3a^2)\eta_1 + 3a^2\eta_8}{\rho^3\sqrt{\rho^2 + 6a^2}\sqrt{\rho^2 + 9a^2}}.$$
(4.27)

We can now determine the change in the remaining fluxes and the warp factor, at least to linear order in $(g_s N)$. We find the corrected RR and NS flux from the real and imaginary part of \hat{G}_3 , respectively

$$\hat{H}_3 = \frac{\hat{G}_3 - \hat{G}_3}{\tau - \bar{\tau}}$$
 and $\tilde{F}_3 = \frac{\hat{G}_3 + \hat{G}_3}{2}$. (4.28)

This results in the closed NS-NS 3–form

$$\hat{H}_{3} = d\rho \wedge e_{\psi} \wedge (c_{1} d\theta_{1} + c_{2} d\theta_{2}) + d\rho \wedge (c_{3} \sin \theta_{1} d\theta_{1} \wedge d\phi_{1} - c_{4} \sin \theta_{2} d\theta_{2} \wedge d\phi_{2}) + \left(\frac{\rho^{2} + 6a^{2}}{2\rho} c_{1} \sin \theta_{2} d\phi_{2} - \frac{\rho}{2} c_{2} \sin \theta_{1} d\phi_{1}\right) \wedge d\theta_{1} \wedge d\theta_{2}$$

$$(4.29)$$

and the non–closed RR 3–form (note that $\tilde{F}_3 = \hat{F}_3 - C_0 \hat{H}_3$, where \hat{F}_3 is closed)

$$\widetilde{F}_{3} = -\frac{1}{g_{s}} d\rho \wedge e_{\psi} \wedge (c_{1} \sin \theta_{1} d\phi_{1} + c_{2} \sin \theta_{2} d\phi_{2}) + \frac{1}{g_{s}} e_{\psi} \wedge (c_{5} \sin \theta_{1} d\theta_{1} \wedge d\phi_{1} - c_{6} \sin \theta_{2} d\theta_{2} \wedge d\phi_{2}) - \frac{1}{g_{s}} \sin \theta_{1} \sin \theta_{2} \left(\frac{\rho}{2} c_{2} d\theta_{1} - \frac{\rho^{2} + 6a^{2}}{2\rho} c_{1} d\theta_{2}\right) \wedge d\phi_{1} \wedge d\phi_{2}, \quad (4.30)$$

(see (B.15) for the coefficients c_i). This allows us to write the NS 2–form potential $(dB_2 = \hat{H}_3)$

$$B_{2} = \left(b_{1}(\rho)\cot\frac{\theta_{1}}{2}d\theta_{1} + b_{2}(r)\cot\frac{\theta_{2}}{2}d\theta_{2}\right) \wedge e_{\psi}$$

$$+ \left[\frac{3g_{s}^{2}NP}{4\pi}\left(1 + \log(\rho^{2} + 9a^{2})\right)\log\left(\sin\frac{\theta_{1}}{2}\sin\frac{\theta_{2}}{2}\right) + b_{3}(\rho)\right]\sin\theta_{1}d\theta_{1} \wedge d\phi_{1}$$

$$- \left[\frac{g_{s}^{2}NP}{12\pi\rho^{2}}\left(-36a^{2} + 9\rho^{2} + 16\rho^{2}\log\rho + \rho^{2}\log(\rho^{2} + 9a^{2})\right)\log\left(\sin\frac{\theta_{1}}{2}\sin\frac{\theta_{2}}{2}\right) + b_{4}(\rho)\right]$$

$$\times \sin\theta_{2}d\theta_{2} \wedge d\phi_{2},$$

$$(4.31)$$

where the coefficients are given in (B.17). This mirrors closely the result for the singular conifold [13] and we can indeed show that we produce this result in the $a \to 0$ limit. Away from the singular limit, we find an asymmetry between the (θ_1, ϕ_1) and (θ_2, ϕ_2) spheres, which was to be expected since our manifold (the resolved conifold or its more complicated cousin, the resolved warped deformed conifold) does not have the \mathbb{Z}_2 symmetry that exchanges the two 2–spheres in the singular conifold geometry. The lesser degree of symmetry is naturally also expressed in the fluxes.

The five-form flux is as usual given by $(\tilde{*}_{10} \text{ indicates the Hodge star on the full}$ 10-dimensional *warped* space)

$$\hat{F}_5 = (1 + \tilde{*}_{10})(d\hat{h}^{-1} \wedge d^4 x), \qquad (4.32)$$

which requires knowledge of the warp factor $\hat{h}(\rho)$ that is consistent with these new fluxes. In order to solve the supergravity equations of motion one requires

$$\hat{h}^2 \Delta \hat{h}^{-1} - 2\hat{h}^3 \partial_m \hat{h}^{-1} \partial_n \hat{h}^{-1} g^{mn} = -\Delta \hat{h} = *_6 \left(\frac{\hat{G}_3 \wedge \overline{\hat{G}}_3}{6(\overline{\tau} - \tau)} \right) = \frac{1}{6} *_6 d\hat{F}_5, \quad (4.33)$$

where Δ is the Laplacian on the unwarped resolved conifold and all indices are raised and lowered with the unwarped metric. After some simplifications the Laplacian on the resolved conifold takes the form

$$\Delta \hat{h} = \kappa \partial_{\rho}^{2} \hat{h} + \frac{5\rho^{2} + 27a^{2}}{\rho(\rho^{2} + 6a^{2})} \partial_{\rho} \hat{h} + \frac{6}{\rho^{2}} \left(\partial_{\theta_{1}}^{2} \hat{h} + \cot \theta_{1} \partial_{\theta_{1}} \hat{h} \right) + \frac{6}{\rho^{2} + 6a^{2}} \left(\partial_{\theta_{2}}^{2} \hat{h} + \cot \theta_{2} \partial_{\theta_{2}} \hat{h} \right)$$

This should be evaluated in linear order in N, since we solved the SuGra eom for the fluxes also in linear order. As the the right hand side of

$$\frac{1}{6} *_{6} d\hat{F}_{5} = \frac{54g_{s}P}{\pi\rho^{6}(\rho^{2}+6a^{2})(\rho^{2}+9a^{2})} \Biggl\{ 12\pi\rho^{4}+9a^{2}\rho^{2}(8\pi-g_{s}N)+54a^{4}(4\pi+g_{s}N) + g_{s}N \Biggl[(25\rho^{4}+66a^{2}\rho^{2}-54a^{2})\log\rho + (10\rho^{4}+102a^{2}\rho^{2}+189a^{4})\log(\rho^{2}+9a^{2}) + 6(\rho^{4}+6a^{2}\rho^{2}+18a^{4})\log\left(\sin\frac{\theta_{1}}{2}\sin\frac{\theta_{2}}{2}\right) \Biggr] \Biggr\}$$

$$(4.34)$$

appears sufficiently complicated, we need to employ some simplification. The obvious choice is to consider $\rho \gg a$, i.e. we only trust our solution sufficiently far from the tip. As in the limit $a \rightarrow 0$ we recover the singular conifold setup, we know our solution takes the form [13]

$$\hat{h}(\rho,\theta_1,\theta_2) = 1 + \frac{L^4}{r^4} \left\{ 1 + \frac{24g_s P^2}{\pi \alpha' Q} \log \rho \left[1 + \frac{3g_s N}{2\pi \alpha'} \left(\log \rho + \frac{1}{2} \right) + \frac{g_s N}{2\pi \alpha'} \log \left(\sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2} \right) \right] \right\} + \mathcal{O}\left(\frac{a^2}{\rho^2}\right)$$

$$(4.35)$$

with $L^4 = 27\pi g_s \alpha' Q/4$. Apart from the a^2/ρ^2 -correction, this is the same result as for the singular conifold [13]. We have not been able to find an analytic solution at higher order, but considering that most models work with even cruder approximations of the warp factor (i.e. $h(r) \sim \log r/r^4$), we believe this should suffice.

4.5 D-terms from non–primitive background flux on D7–branes

Soft supersymmetry breaking via D-terms on D7-branes has been considered in [20] and was later applied to more realistic type IIB orientifolds [22, 23] or their F-theory lift [21, 24] (see also [53] for a IIA scenario); the most general study for generalized CYs has appeared in [54]. The established consensus is that non-primitive flux on the D7–worldvolume gives rise to D-terms in the effective 4–dimensional theory, which can only under certain conditions remain non-zero in the vacuum. One way to phrase the necessary condition is to require that the 4-cycle wrapped by the D7-branes admits non-trivial 2-forms that become trivial in the ambient Calabi-Yau, i.e. the H^2 -cohomology on the four-cycle is bigger than just the pullback of $H^2(CY)$. (Equivalently [22] states that the 4-cycle needs to intersect its orientifold image over a 2-cycle that supports non-trivial flux. The same is true in the case of two stacks [23] intersecting over a 2-cycle.) This condition can be satisfied for the Ouyang embedding in the $\mu \neq 0$ case: The resolved conifold admits only one non-trivial 2-cycle, the sphere that remains finite at the tip. The 4-cycle that the D7 wraps, on the other hand, can also have a non-trivial cycle spanned by (θ_1, ϕ_1) , if the D7 in the Ouyang embedding do not reach all the way to the bottom of the throat. On the D7, this cycle will never shrink completely. Nevertheless, we are mostly concerned with the case $\mu = 0$ here. In contrast to [22, 23] we consider the pullback of a background field with non-vanishing fieldstrength, not the zero mode fluctuations, i.e. we do not expand the worldvolume flux in a basis of H^2 . This gives rise to a D-term that depends on the overall volume of the manifold and the resolution parameter a. Though an orientifold will be necessary to consistently compactify our background, we will not specify any orientifold action here, as we do not know a specific compactification for our background.

Following the derivation in [23, 54], we extract the D-terms from the DBI action. Suppose our stack of N D7-branes wraps a 4-cycle Σ , as specified by the Ouyang embedding in section 4.4. The full DBI action for the 8-dimensional worldvolume (in string frame) reads

$$S_{D7} = -\mu_7 \int_{\Sigma \times \mathcal{M}_4} d^8 \xi \, e^{-\Phi} \sqrt{|\check{g} + \check{B} - 2\pi \alpha' F|} \,, \tag{4.36}$$

where the symbol $\check{}$ indicates the pullback of the metric and the NS field onto the D7 and where F is the worldvolume gauge flux. With the warped product ansatz for the spacetime this expression becomes

$$S_{D7} = -\mu_7 \int d^4 x \, e^{-\Phi} \sqrt{|\check{g}_4|} \, \sqrt{\left|1 + 2\pi\alpha' \check{g}_4^{-1} F_4\right|} \, \Gamma \,, \tag{4.37}$$

where g_4 and F_4 indicate the 4-dimensional part of the metric and gauge flux and one defines

$$\Gamma = \int_{\Sigma} d^4 \xi \sqrt{|\check{g}_{\Sigma} + \mathcal{F}|}, \qquad (4.38)$$

where we have introduced $\mathcal{F} = \check{B} - 2\pi \alpha' F$. In the following, the pullback is always understood as onto the 4-cycle Σ . We do not consider any gauge fields along the external space \mathcal{M}_4 . The quantity (4.38) is the main parameter for the D-terms. Expanding the full action (4.36) at low energies yields the potential contribution

$$V_{D7} = \mu_7 e^{3\Phi} \mathcal{V}^{-2} \Gamma \,, \tag{4.39}$$

where the volume \mathcal{V} of the resolved conifold is defined as

$$\mathcal{V} = \frac{1}{6} \int_{Y} J \wedge J \wedge J = \frac{(4\pi)^2}{108} \int_{0}^{R} \rho^3(\rho^2 + 6a^2) \, d\rho = \frac{8\pi^3}{81} \, R^4(R^2 + 9a^2) \,. \tag{4.40}$$

This integral has to be regularized by an explicit cut-off, as we study the noncompact case. Simply cutting off the radial direction does probably destroy the holomorphicity condition, but we will ignore this subtlety here.

One can write [23] $\Gamma = \tilde{\Gamma} e^{-i\zeta} = |\tilde{\Gamma}| e^{i(\tilde{\zeta} - \zeta)}$, where ζ is determined from the BPS calibration condition and

$$\tilde{\Gamma} = \frac{1}{2} \int_{\Sigma} \left(\check{J} \wedge \check{J} - \mathcal{F} \wedge \mathcal{F} \right) + i \int_{\Sigma} \check{J} \wedge \mathcal{F} \,. \tag{4.41}$$

Then the condition for the D7 to preserve the same supersymmetry as the O7 corresponds to $\zeta = \tilde{\zeta} = 0$, or equivalently $Im\tilde{\Gamma} = 0$. Allowing for a small supersymmetry breaking, one expands the D7-potential (4.39) in $Im\tilde{\Gamma} \ll \Re\tilde{\Gamma}$ and finds

$$V_{D7} = \mu_7 e^{3\Phi} \mathcal{V}^{-2} \Gamma = \mu_7 e^{3\Phi} \mathcal{V}^{-2} \sqrt{(\Re \tilde{\Gamma})^2 + (Im \tilde{\Gamma})^2} = \mu_7 e^{3\Phi} \mathcal{V}^{-2} \Re \tilde{\Gamma} + \frac{1}{2} \mu_7 e^{3\Phi} \mathcal{V}^{-2} \frac{(Im \tilde{\Gamma})^2}{\Re \tilde{\Gamma}}.$$
(4.42)

The first term in this expansion will be cancelled by the tadpole cancellation condition in a consistent compactification. The second term is interpreted as the SUSY– breaking D–term. The real and imaginary part of $\tilde{\Gamma}$ are easily read off from (4.38) (the integrals are real) and can be calculated for our explicit case at hand. All we need to know is the pullback of the Kähler form onto the 4–cycle and the worldvolume flux \mathcal{F} . We would like to consider the simple case such that

$$\dot{B} \neq 0, \qquad F = 0, \qquad (4.43)$$

as we have an explicit solution of this form. There could be gauge flux on the D7-brane to could restore supersymmetry in the $a \rightarrow 0$ limit. It is noted again that to preserve supersymmetry, holomorphicity is not enough. One also needs the worldvolume flux to be of pure (1,1) type and primitive [49]. The reason that it is so difficult to achieve non-trivial D-terms with closed \check{B} is that F could always cancel the non-primitive part of \check{B} [22], unless some non-trivial topological conditions are met.

In calculating the D–terms, we must treat the D7 as a probe. Thus the B–field that is pulled back is not the one we calculated in (4.31), but the original PT solution

$$B = f_1(\rho) \sin \theta_1 \, d\theta_1 \wedge d\phi_1 + f_2(\rho) \sin \theta_2 \, d\theta_2 \wedge d\phi_2 \,, \tag{4.44}$$

where f_1 and f_2 were defined in (4.8). The embedding z = 0 we use has actually 2 branches, since

$$z = 0 = \left(9a^2\rho^4 + \rho^6\right)^{1/4} \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$
(4.45)

can be satisfied by either $\theta_1 = 0$ or $\theta_2 = 0$. This implies that also ϕ_1 =fixed or ϕ_2 =fixed, as θ_i being zero refers to the pole of one of the 2–spheres where the circle described by ϕ_i collapses. The full holomorphic cycle is then a sum over these 2 branches.

Consider the 2 four-cycles $\Sigma_1 = (\rho, \psi, \phi_1, \theta_1)$ and $\Sigma_2 = (\rho, \psi, \phi_2, \theta_2)$ that correspond to the branches $\theta_2 = 0$ and $\theta_1 = 0$, respectively. The complex structure induced on them is actually a trivial pullback of the complex structure on the resolved conifold. Using the complex vielbeins (A.17), we see that

$$\Sigma_1 = (E_1|_{\theta_2=0}, E_2), \qquad \Sigma_2 = (E_1|_{\theta_1=0}, E_3), \qquad (4.46)$$

where in $E_1|_{\theta_2=0}$ and $E_1|_{\theta_1=0}$ the imaginary part is truncated to

$$ImE_1|_{\theta_2=0} = \frac{\rho\sqrt{\kappa}}{3} \left(d\psi + \cos\theta_1 \, d\phi_1 \right) \quad \text{and} \quad ImE_1|_{\theta_1=0} = \frac{\rho\sqrt{\kappa}}{3} \left(d\psi + \cos\theta_2 \, d\phi_2 \right),$$

respectively. It is easy to show that the induced complex structure on the four-cycle still allows for a closed Kähler form. With this observation we find the pullback of B onto both branches

$$\check{B}|_{\Sigma_1} = \frac{-3i}{\rho^2} f_1 E_2 \wedge \bar{E}_2, \qquad \check{B}|_{\Sigma_2} = \frac{-3i}{\rho^2 + 6a^2} f_2 E_3 \wedge \bar{E}_3, \qquad (4.47)$$

which turn out to be of type (1,1). But that does not mean they are primitive. In fact, as we will see shortly, the pullback of B is not primitive on each individual branch, but in the limit $a \rightarrow 0$ the D-term generated by them vanishes when summing over both branches. So it appears that the Ouyang embedding in the singular conifold [13] breaks supersymmetry due to this non-primitivity, but generates neither an F-term nor a D-term. Supersymmetry could possibly be restored by choosing appropriate gauge flux, but we solved the equations of motion only for the case F = 0, so we will keep working with this assumption. In general, F would mix with the metric in the e.o.m., changing our original setup. If we consider the B-field (4.31) that reflects the D7-backreaction, we find its pullback onto Σ_1 (the case of Σ_2 is completely analogous)

$$\check{B}_{2}|_{\Sigma_{1}} = b_{1}(\rho) \cot \frac{\theta_{1}}{2} d\theta_{1} \wedge (d\psi + \cos \theta_{1} d\phi_{1})$$

$$+ \left[\frac{3g_{s}^{2}NP}{4\pi} \left(1 + \log(\rho^{2} + 9a^{2}) \right) \log \left(\sin \frac{\theta_{1}}{2} \cdot 0 \right) + b_{3}(\rho) \right] \sin \theta_{1} d\theta_{1} \wedge d\phi_{1}.$$
(4.48)

We encounter the usual problem that B contains terms with $\log z$, so naturally we find a log-divergent term if we pull back onto a cycle that is described by z = 0. However, this is not our concern here. We highlight the fact that this B-field is not of pure (1,1) type anymore, but rather contains (2,0) and (0,2) terms as well:

$$\check{B}_{2}|_{\Sigma_{1}} = \frac{3\sqrt{3}i \, b_{1}(\rho)}{2\rho^{2}\sqrt{2\kappa(\rho)}} \cot \frac{\theta_{1}}{2} \left[e^{i\psi/2} (E_{1} \wedge \bar{E}_{2} - \bar{E}_{1} \wedge \bar{E}_{2}) + e^{-i\psi/2} (E_{1} \wedge E_{2} + E_{2} \wedge \bar{E}_{1}) \right]
- \frac{3i}{\rho^{2}} \left[\frac{3g_{s}^{2}NP}{4\pi} \left(1 + \log(\rho^{2} + 9a^{2}) \right) \log \left(\sin \frac{\theta_{1}}{2} \cdot 0 \right) + b_{3}(\rho) \right] E_{2} \wedge \bar{E}_{2}. \quad (4.49)$$

For our considerations the probe approximation shall suffice. We could not obtain any sensible result with the B-field (4.48) anyway, as we would have to integrate over the divergent points $\theta_i = 0$. Naturally, this self-interaction is divergent.

Let us now turn to the calculation of the D-terms for the embedding $\mu = 0$. The crucial integral for the D-term coming from (4.41) is given by the pullbacks of J and B. We still need to give the pullback of J onto both branches:

$$\check{J}|_{\Sigma_1} = \frac{\rho}{3} d\rho \wedge (d\psi + \cos\theta_1 \, d\phi_1) + \frac{\rho^2}{6} \sin\theta_1 \, d\phi_1 \wedge d\theta_1
\check{J}|_{\Sigma_2} = \frac{\rho}{3} d\rho \wedge (d\psi + \cos\theta_2 \, d\phi_2) + \frac{\rho^2 + 6a^2}{6} \sin\theta_2 \, d\phi_2 \wedge d\theta_2.$$
(4.50)
And we repeat the pull-back of B in terms of real coordinates:

$$\check{B}|_{\Sigma_1} = f_1(\rho) \sin \theta_1 \, d\theta_1 \wedge d\phi_1 \,, \qquad \check{B}|_{\Sigma_2} = f_2(\rho) \sin \theta_2 \, d\theta_2 \wedge d\phi_2 \,. \tag{4.51}$$

The D-term is now obtained from $Im\tilde{\Gamma}$ in (4.41)

$$D = \int_{\Sigma_1} \check{J}|_{\Sigma_1} \wedge \check{B}|_{\Sigma_1} + \int_{\Sigma_2} \check{J}|_{\Sigma_2} \wedge \check{B}|_{\Sigma_2}$$

=
$$\int_{\Sigma_1} \frac{\rho}{3} f_1 \sin \theta_1 \, d\rho \wedge d\psi \wedge d\theta_1 \wedge \phi_1 + \int_{\Sigma_2} \frac{\rho}{3} f_2 \sin \theta_2 \, d\rho \wedge d\psi \wedge d\theta_2 \wedge \phi_2 4.52)$$

We see immediately that for the case $f_1 = -f_2$, i.e. the singular $a \to 0$ limit of the KT solution, the D-term vanishes after summing both cycles, even though the pullback of B is non-primitive in this case. For the case $a \neq 0$ we can perform the integrals, again introducing a cut-off R for the radial direction. We find

$$D = \frac{32\pi^2 g_s P}{9} \left[9a^2 \log(9+a^2) - (9a^2 - 2R^2) \log R - (9a^2 + R^2) \log(9a^2 + R^2) \right].$$
(4.53)

To obtain the full D-term potential, we also need $\Re \tilde{\Gamma}$ from (4.41). Looking at the pullbacks of the B-fields (4.47) we see that $\check{B} \wedge \check{B}$ vanishes for both branches, so

$$\Re \tilde{\Gamma} = \frac{1}{2} \int_{\Sigma_1} \check{J}|_{\Sigma_1} \wedge \check{J}|_{\Sigma_1} + \frac{1}{2} \int_{\Sigma_2} \check{J}|_{\Sigma_2} \wedge \check{J}|_{\Sigma_2}$$

$$= \frac{4\pi^2}{9} R^2 (R^2 + 6a^2). \qquad (4.54)$$

The total D-term potential then reads

$$V_{D7} = \frac{1}{2} \mu_7 e^{3\Phi} \mathcal{V}^{-2} \frac{(Im\tilde{\Gamma})^2}{\Re\tilde{\Gamma}} = \frac{59049 \,\mu_7 \, e^{3\Phi}}{512\pi^8} \frac{D^2}{R^{10} (R^2 + 6a^2) (R^2 + 9a^2)^2}, \qquad (4.55)$$

with the D-term D from (4.53). In the probe approximation, Φ is just the constant background dilaton and can be set to zero. This is one of the main results of our paper. We find a non-zero D-term created by non-primitive (1,2) flux when pulled back to non-primitive flux on D7-branes. Their magnitude is highly suppressed in a large volume compactification. It would be most desireable to find a consistent compactification for our setup, in which we do not have to introduce a cut-off by hand that spoils holomorphicity. Let us stress again that these (1,2) fluxes did not lead to the creation of a bulk cosmological constant because they are ISD. We would expect, however, a modification of the superpotential, i.e. in general D-terms on D7-branes also create F-terms [21, 22, 23].

We have so far neglected any zero modes. Once we study D3/D7 inflation, there will also be degrees of freedom that become light when the two branes approach each other. The D- and F-terms in this case have to be re-evaluated. As already outlined in the beginning of this section, we believe that the conditions to have nonzero D-terms in the vacuum (i.e. intersection over a two-cycle with non-trivial flux or a cohomology $H^2(\Sigma)$ of the 4-cycle that is greater than the pullback of the CY cohomology $H^2(CY)$) can be met when $\mu \neq 0$. For $\mu = 0$, it appears rather the opposite: there is only one non-trivial 2-cycle in the resolved conifold, the blownup (ϕ_2 , θ_2)-sphere. With $\mu = 0$, the cycle Σ_1 is topologically trivial (it contains the shrinking 2-sphere), the cycle Σ_2 is not. However, once we compactify, we will introduce another cycle on which the (0,1) form is supported. This should be in (ρ , ψ) direction, as $G^{(1,2)} \sim J \wedge \overline{E}_1$, and E_1 extends along ρ and ψ . However, from (4.51), we see that this 2-cycle does not support any flux. We believe this puzzle might be clarified once the original Ouyang embedding in the singular conifold background is made supersymmetric with appropriate gauge fluxes. Note however, that there is an essential difference between the singular KT and the resolved PT backgrounds: the B-field in the bulk is primitive, i.e. $J \wedge J \wedge B =$ 0, for the first case but not for the latter.

The next step would be to consider the embedding $\mu \neq 0$. The integrals becomes much more complicated and cannot be solved analytically. Only for the case a = 0have we been able to show by numerical integration that D = 0. For $a \neq 0$ the integrand's strong oscillatory behavior has prevented us from finding a solution so far. Note that the pullback of J and B is much more involved. We have to use the embedding equations

$$(\rho^6 + 9a^2\rho^4) = \left(\frac{|\mu^2|}{\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}}\right)^4, \quad \psi = \phi_1 + \phi_2 + \text{const}.$$
(4.56)

It is then tedious but straightforward to calculate the pullback

$$\dot{J}_{\alpha\beta} = \partial_{\alpha} y^m \partial_{\beta} y^n J_{mn} , \qquad (4.57)$$

where $m, n = \rho, \psi, \theta_1, \theta_2, \phi_1, \phi_2$ run over the whole 3-fold, whereas $\alpha, \beta = \theta_1, \theta_2, \phi_1, \phi_2$ parameterize the 4-cycle. A similar formula gives the pullback of the NS field \check{B} . Note, however, that the pullback will contain terms with $(\sin \theta_i)^{-1}$, which diverge at the integration boundaries $\theta_i = 0$. For the case a = 0, this seems to be under control; for the resolved case we cannot make any definite statement.

CHAPTER 5

Future Directions and Applications to Cosmology

5.1 Compactification and non-Kählerity

One remaining issues that has not been given sufficient attention thus far is the nature of a possible compactification of our background. It will certainly not be a Calabi–Yau, nor even Kähler. We argued in chapter 4 that the six–dimensional base cannot be a Calabi–Yau manifold as it has a non-vanishing first Chern class. One can go further, as was discussed in [26], and argue that by reducing our background's M-theory lift to IIA, the T–dual IIB background will indeed be non–Kähler. This construction follows the ones laid out in [55, 56]. Specifically, the three-form flux that we get in type IIA will dissolve in the metric once we T-dualize to type IIB theory, making the background non-Kähler¹. Once the background is non-Kähler there would be *extra* sources of fluxes, namely "geometric flux". One can replace the

¹ In M-theory once $d\mathcal{J} \neq 0$ the four-form flux $\mathcal{J} \wedge \mathcal{J}$ is not closed.

type IIB three-form NS flux by

$$\tilde{H}_3 \equiv \hat{H}_3 + id(e^{-\phi}J).$$
(5.1)

This complexification of the three-form flux is not new and has been observed earlier in heterotic compactifications [57, 58, 59, 60], which in turn gave rise to a new superpotential in the heterotic theory [61, 62]. An interesting observation here is that the type IIB background itself becomes non-Kähler now as compared to the heterotic background where the type IIB background was conformally Kähler.

We also remarked on possible generalizations of the IIB superpotential in section 4.3. It seems clear that the GVW superpotential will get corrected if the moduli space is enlarged by non-trivial one-forms. For the case of a background that is mirror to a Calabi–Yau with NS flux (so it acquires a non-trivial T^3 fibration when the mirror symmetry is interpreted as three T-dualities — the NS B-field becomes part of the metric in the mirror manifold [55]), a superpotential has been proposed [24]. Whether or not this is suggestive for our case requires further study. Thus far, we have no reason to believe that our IIB manifold (globally) admits an SU(3) structure. The space of generalized Calabi–Yau manifolds is much larger, though some work on superpotentials in this case appeared in [63, 64, 65, 66]. If we could infer that our IIB background admits an SU(3) structure, then it would be guaranteed to be complex [67, 68, 69] if it preserved supersymmetry. However, in the presence of SUSY– breaking flux we cannot infer the structure of the manifold. A complex manifold would have the advantage of giving control over the space of complex structure deformations.

5.1.1 Inflationary dynamics

The warped resolved conifold may be a good model of inflation with D-term uplifting, despite the difficulties in computing non-zero D-terms in this work. We would have to extend our analysis beyond the case $\mu = 0$ (in this case the D7 extend all the way down the throat, which would not allow us to place a D3 *between* the D7 and the tip) and to other embeddings, such as the Kuperstein embedding [40]. Our preliminary analysis indicates that the value of the D-terms should depend on the choice of embedding.

Taking a resolved warped deformed conifold creates non-trivial dilaton profile from two sources now:

- From the D7 branes, and
- From the broken \mathbb{Z}_2 symmetry in the metric.

The running of dilaton from the first case can already be seen at a supersymmetric level in the Ouyang background [13], which was originally analyzed for a noncompact singular conifold background. Once we generalize the metric to one with broken \mathbb{Z}_2 and switch on fluxes, the second case mentioned above becomes relevant, and we must discuss the combined effects to get the full background geometry. This makes the problem much harder to solve.

5.1.2 Supersymmetry restoration

If the D3-brane falls into the D7-branes at the end of inflation we expect supersymmetry to be restored. Such a SUSY restoration was first described in [6]. For our case, the situation is more involved. From the F-theory point of view (more detail is given in the original paper [26]), one can argue that in the presence of F flux on the D7–brane we can in fact demand:

$$\mathcal{J} \wedge G_{\text{total}} = 0 \tag{5.2}$$

and therefore restore supersymmetry with (2,2) fluxes.

The F flux used to restore supersymmetry in the above paragraph could be interpreted in two ways: switching on the second Chern class or switching on first Chern class. The former, which leads to instantons, is the end point of the D3 brane dissolving on the D7 branes. The latter, however, gives rise to a bound state of a D5 brane with the D7 branes. Such a technique of restoring supersymmetry has already been discussed in [70, 71] and could probably be used to restore supersymmetry in the limit where the resolution parameter a goes to zero. This would then be one simple way of restoring supersymmetry in the original Ouyang construction [13].

5.2 AISD Fluxes and Anti-Branes

Related to our flux choice is another issue that deserves mentioning. The (1, 2) flux that we have described in this paper is ISD and solves the equations of motion. One may also choose AISD flux if one changes the ansatz for the background geometry, i.e. if one ventures beyond conformal Calabi–Yau compactifications or allows objects that do not respect the BPS-like bound assumed in our initial ansatz (see [72]). Typically, one can show that a compact conformally Calabi-Yau background only allows ISD fluxes that are also primitive. As we saw above, non-primitive ISD fluxes are allowed on a compact non-Kähler background or on a non-compact Calabi-Yau background. However, AISD fluxes are generically part of the solution to the

equations of motion on non-Kähler backgrounds. Some recent papers dealing with this are [73, 74, 72].

5.3 Conclusions

Motivated by the possibility of a naturally balanced background that would generate slow-roll inflation, we have applied the methods of [13] to the warped resolved conifold background of Pando–Zayas and Tseytlin [12]. We found a supergravity background that breaks supersymmetry spontaneously due to fluxes of type (1,2) without generating a bulk cosmological constant. The pullback of the NS B–field onto the D7–worldvolume gives rise to D-terms, which vanish in the limit of vanishing resolution parameter $a \rightarrow 0$, i.e. when we approach the original singular background of [13]. In the case we studied, the D7 gauge fluxes were zero and the D-terms were entirely due to the non–primitive NS B–field. In general, we would also expect F–terms from the D7 worldvolume theory.

To continue to study inflationary dynamics in this background, it will be necessary to study other D7 embeddings to find non-zero, analytically computable D– terms. It will also be necessary to properly understand the F–term potential by establishing to correct superpotential in the 4D theory. This, in turn, will require a more complete understanding of the compactification of the warped resolved conifold into more general non-CY backgrounds.

APPENDIX A

The geometry of the resolved conifold

The resolved conifold is a manifold which looks asymptotically like the singular conifold, but is non-singular at the tip. Its geometry can be derived by starting with the singular version, a non-compact Calabi–Yau 3-fold, that can be embedded in \mathbb{C}^4 as [37]

$$\sum_{i=1}^{4} z_i^2 = 0.$$
 (A.1)

This describes a cone over $S^2 \times S^3$, which becomes singular at the origin. By a change of coordinates this can also be written as

$$yz - uv = 0, (A.2)$$

which is equivalent to non-trivial solutions to the equation

$$\begin{pmatrix} z & u \\ v & y \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = 0, \qquad (A.3)$$

in which ξ_1, ξ_2 are homogeneous coordinates on $\mathbb{CP}^1 \sim S^2$. For $(u, v, y, z) \neq 0$ (away from the tip), they describe again a conifold. But at (u, v, w, z) = 0 this is solved by any pair (ξ_1, ξ_2) . Due to the overall scaling freedom $(\xi_1, \xi_2) \sim (\lambda \xi_1, \lambda \xi_2)$ we can mod out by this equivalence class and (ξ_1, ξ_2) actually describe a $\mathbb{CP}^1 \sim S^2$ at the tip of the cone. The resolved conifold can be covered by two complex coordinate patches $(H_+ \text{ and } H_-)$, given by

$$H_{+} = \{\xi_{1} \neq 0\} = \{(u, y; \lambda) | u, y, \lambda \in \mathbb{C}\}, \ \lambda = \frac{\xi_{2}}{\xi_{1}}$$
(A.4)

$$H_{-} = \{\xi_2 \neq 0\} = \{(v, z; \mu) | v, z, \mu \in \mathbb{C}\}, \ \mu = \frac{\xi_1}{\xi_2}.$$
 (A.5)

On H_+ we have that

$$z = -u\lambda , v = -y\lambda , \qquad (A.6)$$

on H_-

$$y = -v\mu , u = -z\mu , \qquad (A.7)$$

and on the intersection of these two patches, the coordinates are related by

$$(v, z; \mu) = (-y\lambda, -u\lambda; 1/\lambda).$$

The holomorphic coordinates are conveniently parameterized by

$$z = (9a^{2}\rho^{4} + \rho^{6})^{1/4} e^{i/2(\psi - \phi_{1} - \phi_{2})} \sin(\theta_{1}/2) \sin(\theta_{2}/2)$$

$$y = (9a^{2}\rho^{4} + \rho^{6})^{1/4} e^{i/2(\psi + \phi_{1} + \phi_{2})} \cos(\theta_{1}/2) \cos(\theta_{2}/2)$$

$$u = (9a^{2}\rho^{4} + \rho^{6})^{1/4} e^{i/2(\psi + \phi_{1} - \phi_{2})} \cos(\theta_{1}/2) \sin(\theta_{2}/2)$$

$$v = (9a^{2}\rho^{4} + \rho^{6})^{1/4} e^{i/2(\psi - \phi_{1} + \phi_{2})} \sin(\theta_{1}/2) \cos(\theta_{2}/2).$$
(A.8)

Here, $\theta_i = 0 \dots \pi$, $\phi_i = 0 \dots 2\pi$ are the usual Euler angles on S^2 , $\psi = 0 \dots 4\pi$ describes a U(1) fiber over the two 2-spheres and $\rho = 0 \dots \infty$ is the radial coordinate. Note that our radial coordinate ρ is related to the commonly used r via $\rho^2 = 3/(2r^2)F'(r^2)$, where $F(r^2)$ appears in the Kähler potential K of the resolved conifold

$$K(r) = F(r^{2}) + 4a^{2}\log(1+|\lambda|^{2}).$$
(A.9)

Note that the Kähler potential is not a globally defined quantity, since λ is only defined on the patch H_+ that excludes $\xi_1 = 0$. For completeness let us also quote [37, 12]

$$F'(r^2) = \frac{\partial F(r^2)}{\partial r^2} = \frac{1}{r^2} \left(-2a^2 + 4a^2 N^{-1/3}(r) + N^{1/3}(r) \right) \quad \text{with} \quad (A.10)$$

$$N(r) = \frac{1}{2} \left(r^4 - 16a^2 + \sqrt{r^8 - 32a^6r^4} \right).$$
 (A.11)

The inverse relation between ρ and r is found to be

$$r = \left(\frac{2}{3}\right)^{3/4} \left(9a^2\rho^4 + \rho^6\right)^{1/4}.$$
 (A.12)

In terms of these real coordinates the Ricci–flat Kähler metric on the resolved conifold reads

$$ds_{\rm res}^2 = \kappa(\rho)^{-1} d\rho^2 + \frac{\kappa(\rho)}{9} \rho^2 \left(d\psi + \cos\theta_1 \, d\phi_1 + \cos\theta_2 \, d\phi_2 \right)^2 + \frac{\rho^2}{6} \left(d\theta_1^2 + \sin^2\theta_1 \, d\phi_1^2 \right) + \frac{\rho^2 + 6a^2}{6} \left(d\theta_2^2 + \sin^2\theta_2 \, d\phi_2^2 \right), \quad (A.13)$$

with $\kappa(\rho) = (\rho^2 + 9a^2)/(\rho^2 + 6a^2)$. In the limit $a \to 0$ one recovers the singular conifold metric; therefore, *a* is called "resolution" parameter and gives the radius of

the blown–up 2–sphere at the tip.

It will be useful later on to have a set of vielbeins that describes this metric, i.e.

$$ds^{2} = \sum_{i=1}^{6} (e_{i})^{2}.$$
 (A.14)

Following [46] we choose

$$e_{1} = \kappa^{-1/2} d\rho$$

$$e_{2} = \frac{\rho\sqrt{\kappa}}{3} (d\psi + \cos\theta_{1} d\phi_{1} + \cos\theta_{2} d\phi_{2}) = \frac{\rho\sqrt{\kappa}}{3} e_{\psi}$$

$$e_{3} = \frac{\rho}{\sqrt{6}} (\sin\psi/2 \sin\theta_{1} d\phi_{1} + \cos\psi/2 d\theta_{1})$$

$$e_{4} = \frac{\rho}{\sqrt{6}} (-\cos\psi/2 \sin\theta_{1} d\phi_{1} + \sin\psi/2 d\theta_{1}) \qquad (A.15)$$

$$e_{5} = \frac{\sqrt{\rho^{2} + 6a^{2}}}{\sqrt{6}} (\sin\psi/2 \sin\theta_{2} d\phi_{2} + \cos\psi/2 d\theta_{2})$$

$$e_{6} = \frac{\sqrt{\rho^{2} + 6a^{2}}}{\sqrt{6}} (-\cos\psi/2 \sin\theta_{2} d\phi_{2} + \sin\psi/2 d\theta_{2}),$$

as they lead to a closed Kähler form J as well as a closed holomorphic 3–form Ω with a simple complex structure induced by

$$J^{(1,1)} = e_1 \wedge e_2 + e_3 \wedge e_4 + e_5 \wedge e_6, \qquad \Omega^{(3,0)} = (e_1 + ie_2) \wedge (e_3 + ie_4) \wedge (e_5 + ie_6).$$
(A.16)

We define our complex vielbeins to be

$$E_1 = e_1 + i e_2, \qquad E_2 = e_3 + i e_4, \qquad E_3 = e_5 + i e_6.$$
 (A.17)

This results in a coordinate expression for J as

$$J = \frac{\rho}{3} d\rho \wedge (d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2) + \frac{\rho^2}{6} \sin\theta_1 d\phi_1 \wedge d\theta_1 + \frac{\rho^2 + 6a^2}{6} \sin\theta_2 d\phi_2 \wedge d\theta_2.$$
(A.18)

APPENDIX B

Ouyang embedding of D7–branes on the resolved conifold

In this appendix we describe how D7–branes can be embedded in the PT background. We use the Ouyang [13] embedding

$$z = \mu^2, \tag{B.1}$$

where z is one of the holomorphic coordinates defined in (A.8). While this choice was originally made for the singular conifold, it continues to give a consistent holomorphic embedding on both patches. From (A.8), it is clear that selecting $z = \mu^2$ on $H_$ implies that $-u\lambda = \mu^2$ on the intersection with H_+ , which consistently gives $z = \mu^2$ on all of H_+ .

While the case $\mu \neq 0$, where the D7-brane does not extend to the tip of the throat, is of primary interest for inflationary models, we set $\mu = 0$ for simplicity of calculation. As a consistency check, we should always be able to recover a supersymmetric solution in the limit $a \rightarrow 0$. The D7-brane induces a non-trivial axion-dilaton

$$\tau = \frac{i}{g_s} + \frac{N}{2\pi i} \log z , \qquad (B.2)$$

where N is the number of embedded D7-branes. Our goal is to determine the change the dilaton induces in the other fluxes and the warp factor. We will closely follow the method laid out in [13] and solve the SuGra equation of motion only in linear order $g_s N$. That said, we neglect any backreaction onto the geometry beyond a change in the warp factor, i.e. we will assume the manifold remains a conformal resolved conifold.

Consider first the Bianchi identity, which in leading order becomes (H_3 indicates the unmodified NS flux from (4.5), whereas the hat indicates the corrected flux at leading order)

$$d\hat{G}_3 = d\hat{F}_3 - d\tau \wedge \hat{H}_3 - \tau \wedge d\hat{H}_3 = -d\tau \wedge H_3 + \mathcal{O}((g_s N)^2)$$

$$= -\left(\frac{N}{2\pi i}\frac{dz}{z}\right) \wedge \left(df_1(\rho) \wedge d\theta_1 \wedge \sin\theta_1 \, d\phi_1 + df_2(\rho) \wedge d\theta_2 \wedge \sin\theta_2 \, d\phi_2\right) + \mathcal{O}((g_s N)^2) \,.$$
(B.3)

In order to find a 3-form flux that obeys this Bianchi identity, we make an ansatz

$$\hat{G}_3 = \sum \alpha_i \eta_i \,, \tag{B.4}$$

where $\{\eta_i\}$ is a basis of imaginary self-dual (ISD) 3-forms on the resolved conifold given in (4.22). We find a particular solution in terms of only four of above eight 3-forms

$$P_3 = \alpha_1(\rho) \eta_1 + e^{-i\psi/2} \alpha_3(\rho, \theta_1) \eta_3 + e^{-i\psi/2} \alpha_4(\rho, \theta_2) \eta_4 + \alpha_8(\rho) \eta_8, \quad (B.5)$$

with

$$\alpha_{3} = -3\sqrt{6}g_{s}NP \frac{72a^{4} - 3\rho^{4} + a^{2}\rho^{2}(\log(\rho^{2} + 9a^{2}) - 56\log\rho)}{8\pi\rho^{3}(\rho^{2} + 6a^{2})^{2}} \cot\frac{\theta_{1}}{2}$$

$$\alpha_{4} = -9\sqrt{6}g_{s}NP \frac{\rho^{2} - 9a^{2}\log(\rho^{2} + 9a^{2})}{8\pi\rho^{4}\sqrt{\rho^{2} + 6a^{2}}} \cot\frac{\theta_{2}}{2}$$
(B.6)

$$\alpha_8 = \frac{3a^2}{\rho^2 + 3a^2} \left[3g_s NP \frac{-9(\rho^2 + 4a^2) + 28\rho^2 \log \rho + (81a^2 + 13\rho^2) \log(\rho^2 + 9a^2)}{8\pi\rho^3 \sqrt{\rho^2 + 6a^2} \sqrt{\rho^2 + 9a^2}} + \alpha_1(\rho) \right].$$

Note that a_8 is implicitly given by α_1 , which in turn is determined via the first order ODE

$$\alpha_{1}'(\rho) = \frac{-3}{\rho(\rho^{2} + 3a^{2})(\rho^{2} + 9a^{2})\sqrt{\rho^{2} + 6a^{2}}} \left[\frac{(162a^{6} + 78a^{4}\rho^{2} + 15a^{2}\rho^{4} + \rho^{6})}{\sqrt{\rho^{2} + 6a^{2}}} \alpha_{1}(\rho) + 3g_{s}NP \frac{-162a^{6} + 99a^{4}\rho^{2} + 63a^{2}\rho^{4} + 6\rho^{6} + 14a^{2}\rho^{2}(\rho^{2} + 9a^{2})\log\frac{\rho^{2}}{\rho^{2} + 9a^{2}}}{4\pi\rho^{3}\sqrt{\rho^{2} + 9a^{2}}} \right]$$

Letting $a \to 0$ in above equations, we do indeed recover the singular conifold solution of [13]. Keeping the resolution parameter a finite instead, we can solve for $\alpha_1(\rho)$

$$\alpha_1(\rho) = \frac{3g_s NP}{8\pi\rho^3} \frac{\left[18a^2 - 36(\rho^2 + 3a^2)\log\left(\frac{\rho}{a}\right) + (10\rho^2 + 72a^2)\log\left(\frac{\rho^2}{\rho^2 + 9a^2}\right)\right]}{\sqrt{\rho^2 + 6a^2}\sqrt{\rho^2 + 9a^2}} \,.$$
(B.8)

Furthermore, we find a homogeneous solution

$$G_{3}^{hom} = \beta_{1}(z,\rho) \eta_{1} + e^{-i\psi/2} \beta_{3}(\rho,\theta_{1}) \eta_{3} + e^{-i\psi/2} \beta_{4}(\rho,\theta_{2}) \eta_{4}$$
(B.9)
+ $e^{-i\psi} \beta_{5}(\rho,\theta_{1},\theta_{2}) \eta_{5} + \beta_{8}(z,\rho) \eta_{8},$

with

$$\beta_{1} = \frac{-3i}{8\rho^{3}\sqrt{\rho^{2} + 6a^{2}}\sqrt{\rho^{2} + 9a^{2}}} \left[12(\rho^{2} + 3a^{2})\log z + 18a^{2} + 10(\rho^{2} - 9a^{2})\log \rho + (13\rho^{2} + 99a^{2})\log(\rho^{2} + 9a^{2})\right]$$

$$\beta_{3} = 3i\sqrt{6} \left(\frac{-36a^{4} + 3\rho^{4} + 2a^{2}\rho^{2}(20\log \rho - \log(\rho^{2} + 9a^{2}))}{4\rho^{3}(\rho^{2} + 6a^{2})^{2}}\right) \cot \frac{\theta_{1}}{2}$$

$$\beta_{4} = -9i\sqrt{6} \left(\frac{\rho^{2} - 6a^{2}\log(\rho^{2} + 9a^{2})}{4\rho^{4}\sqrt{\rho^{2} + 6a^{2}}}\right) \cot \frac{\theta_{2}}{2}$$

$$\beta_{5} = \frac{-9i(\cot \frac{\theta_{1}}{2}\cos \theta_{2} + \cot \theta_{1})}{2\rho^{2}\sqrt{\rho^{2} + 9a^{2}}\sin \theta_{2}}$$

$$\beta_{8} = \frac{-27ia^{2}}{8\rho^{3}\sqrt{\rho^{2} + 6a^{2}}\sqrt{\rho^{2} + 9a^{2}}} \left[4\log z + 6 - 10\log \rho - \log(\rho^{2} + 9a^{2})\right].$$

This solution has the right singularity structure at z = 0 and $\rho = 0$, but it does not transform correctly under $SL(2,\mathbb{Z})$; only the particular solution does. We therefore conclude that the correction to the 3-form flux, which is in general a linear combination of P_3 and G_3^{hom} , is given by (B.5) only

$$\hat{G}_3 = G_3 + P_3 \,. \tag{B.11}$$

We can now determine the change in the remaining fluxes and the warp factor, at least to linear order in $(g_s N)$. We find the corrected fluxes from the equations

$$\hat{H}_3 = \frac{\overline{\hat{G}}_3 - \hat{G}_3}{\tau - \overline{\tau}}$$
 and $\widetilde{F}_3 = \frac{\hat{G}_3 + \overline{\hat{G}}_3}{2}$, (B.12)

which evaluates to

$$\hat{H}_{3} = d\rho \wedge e_{\psi} \wedge (c_{1} d\theta_{1} + c_{2} d\theta_{2}) + d\rho \wedge (c_{3} \sin \theta_{1} d\theta_{1} \wedge d\phi_{1} - c_{4} \sin \theta_{2} d\theta_{2} \wedge d\phi_{2}) \\
+ \left(\frac{\rho^{2} + 6a^{2}}{2\rho} c_{1} \sin \theta_{2} d\phi_{2} - \frac{\rho}{2} c_{2} \sin \theta_{1} d\phi_{1}\right) \wedge d\theta_{1} \wedge d\theta_{2}, \quad (B.13)$$

$$\widetilde{F}_{3} = -\frac{1}{g_{s}} d\rho \wedge e_{\psi} \wedge (c_{1} \sin \theta_{1} d\phi_{1} + c_{2} \sin \theta_{2} d\phi_{2}) \\
+ \frac{1}{g_{s}} e_{\psi} \wedge (c_{5} \sin \theta_{1} d\theta_{1} \wedge d\phi_{1} - c_{6} \sin \theta_{2} d\theta_{2} \wedge d\phi_{2}) \\
- \frac{1}{g_{s}} \sin \theta_{1} \sin \theta_{2} \left(\frac{\rho}{2} c_{2} d\theta_{1} - \frac{\rho^{2} + 6a^{2}}{2\rho} c_{1} d\theta_{2}\right) \wedge d\phi_{1} \wedge d\phi_{2}. \quad (B.14)$$

We have introduced the coefficients

$$c_{1} = \frac{g_{s}^{2}PN}{4\pi\rho(\rho^{2}+6a^{2})^{2}} \left(72a^{4}-3\rho^{4}-56a^{2}\rho^{2}\log\rho+a^{2}\rho^{2}\log(\rho^{2}+9a^{2})\right) \cot\frac{\theta_{1}}{2}$$

$$c_{2} = \frac{3g_{s}^{2}PN}{4\pi\rho^{3}} \left(\rho^{2}-9a^{2}\log(\rho^{2}+9a^{2})\right) \cot\frac{\theta_{2}}{2}$$
(B.15)
$$c_{3} = \frac{3g_{s}P\rho}{\rho^{2}+9a^{2}} + \frac{g_{s}^{2}PN}{8\pi\rho(\rho^{2}+9a^{2})} \left[-36a^{2}-36\rho^{2}\log a+34\rho^{2}\log \rho + (10\rho^{2}+81a^{2})\log(\rho^{2}+9a^{2})+12\rho^{2}\log\left(\sin\frac{\theta_{1}}{2}\sin\frac{\theta_{2}}{2}\right)\right]$$

$$c_{4} = \frac{3g_{s}P(\rho^{2}+6a^{2})}{\kappa\rho^{3}} + \frac{g_{s}^{2}NP}{8\pi\kappa\rho^{3}} \left[18a^{2}-36(\rho^{2}+6a^{2})\log a+(34\rho^{2}+36a^{2})\log \rho + (10\rho^{2}+63a^{2})\log(\rho^{2}+9a^{2})+(12\rho^{2}+72a^{2})\log\left(\sin\frac{\theta_{1}}{2}\sin\frac{\theta_{2}}{2}\right)\right]$$

$$c_{5} = g_{s}P + \frac{g_{s}^{2}PN}{24\pi(\rho^{2} + 6a^{2})} \left[18a^{2} - 36(\rho^{2} + 6a^{2})\log a + 8(2\rho^{2} - 9a^{2})\log \rho + (10\rho^{2} + 63a^{2})\log(\rho^{2} + 9a^{2}) \right]$$

$$c_{6} = g_{s}P + \frac{g_{s}^{2}PN}{24\pi\rho^{2}} \left[-36a^{2} - 36\rho^{2}\log a + 16\rho^{2}\log \rho + (10\rho^{2} + 81a^{2})\log(\rho^{2} + 9a^{2}) \right].$$

This allows us to write the NS 2–form potential

$$B_{2} = \left(b_{1}(\rho)\cot\frac{\theta_{1}}{2}d\theta_{1} + b_{2}(\rho)\cot\frac{\theta_{2}}{2}d\theta_{2}\right) \wedge e_{\psi}$$

$$+ \left[\frac{3g_{s}^{2}NP}{4\pi}\left(1 + \log(\rho^{2} + 9a^{2})\right)\log\left(\sin\frac{\theta_{1}}{2}\sin\frac{\theta_{2}}{2}\right) + b_{3}(\rho)\right]\sin\theta_{1}d\theta_{1} \wedge d\phi_{1}$$

$$- \left[\frac{g_{s}^{2}NP}{12\pi\rho^{2}}\left(-36a^{2} + 9\rho^{2} + 16\rho^{2}\log\rho + \rho^{2}\log(\rho^{2} + 9a^{2})\right)\log\left(\sin\frac{\theta_{1}}{2}\sin\frac{\theta_{2}}{2}\right) + b_{4}(\rho)\right]$$

$$\times \sin\theta_{2}d\theta_{2} \wedge d\phi_{2},$$
(B.16)

with the $\rho\text{-dependent}$ functions

$$b_{1}(\rho) = \frac{g_{S}^{2}NP}{24\pi(\rho^{2}+6a^{2})} (18a^{2} + (16\rho^{2}-72a^{2})\log\rho + (\rho^{2}+9a^{2})\log(\rho^{2}+9a^{2}))$$

$$b_{2}(\rho) = -\frac{3g_{s}^{2}NP}{8\pi\rho^{2}} (\rho^{2}+9a^{2})\log(\rho^{2}+9a^{2})$$
(B.17)

and $b_3(\rho)$ and $b_a(\rho)$ are given by the first order differential equations

$$b'_{3}(\rho) = \frac{3g_{s}P\rho}{\rho^{2} + 9a^{2}} + \frac{g_{s}^{2}NP}{8\pi\rho(\rho^{2} + 9a^{2})} \Big[-36a^{2} - 36a^{2}\log a + 34\rho^{2}\log \rho + (10\rho^{2} + 81a^{2})\log(\rho^{2} + 9a^{2}) \Big] b'_{4}(\rho) = -\frac{3g_{s}P(\rho^{2} + 6a^{2})}{\kappa\rho^{3}} - \frac{g_{s}^{2}NP}{8\pi\kappa\rho^{3}} \Big[18a^{2} - 36(\rho^{2} + 6a^{2})\log a + (34\rho^{2} + 36a^{2})\log\rho + (10\rho^{2} + 63a^{2})\log(\rho^{2} + 9a^{2}) \Big] .$$
(B.18)

The five–form flux is as usual given by

$$\hat{F}_5 = (1 + \tilde{*}_{10})(d\hat{h}^{-1} \wedge d^4 x).$$
(B.19)

In order to solve the supergravity equations of motion, the warp factor has to fulfill

$$\hat{h}^2 \Delta \hat{h}^{-1} - 2\hat{h}^3 \partial_m \hat{h}^{-1} \partial_n \hat{h}^{-1} g^{mn} = -\Delta \hat{h} = *_6 \left(\frac{\hat{G}_3 \wedge \overline{\hat{G}}_3}{6(\overline{\tau} - \tau)} \right) = \frac{1}{6} *_6 d\hat{F}_5, \quad (B.20)$$

where Δ is the Laplacian on the unwarped resolved conifold and all indices are raised and lowered with the unwarped metric. This should be evaluated in linear order in N, since we solved the SuGra eom for the fluxes also in linear order. However, we were unable to find an analytic solution to this problem, so we need to employ some simplification. We can take the limit $\rho \gg a$, i.e. we restrict ourselves to be far from the tip. As in the limit $a \rightarrow 0$ we recover the singular conifold setup [13], we know our solution takes the form

$$\hat{h}(\rho,\theta_1,\theta_2) = 1 + \frac{L^4}{r^4} \left\{ 1 + \frac{24g_s P^2}{\pi \alpha' Q} \log \rho \left[1 + \frac{3g_s N}{2\pi \alpha'} \left(\log \rho + \frac{1}{2} \right) \right] + \frac{g_s N}{2\pi \alpha'} \log \left(\sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2} \right) \right\} + \mathcal{O}\left(\frac{a^2}{\rho^2} \right),$$
(B.21)

with $L^4 = 27\pi g_s \alpha' Q/4$. Unfortunately, we cannot give an explicit expression for the a^2/ρ^2 corrections. However, the above result is already an improvement over using the simple Klebanov–Tseytlin warp factor (which is strictly only valid for the singular solution, but is often employed in the deformed KS geometry).

References

- WMAP Collaboration, E. Komatsu *et al.*, "Five-Year Wilkinson Microwave Anisotropy Probe (WMAP 1) Observations: Cosmological Interpretation," arXiv:0803.0547 [astro-ph].
- [2] S. Kachru *et al.*, "Towards inflation in string theory," *JCAP* 0310 (2003) 013, hep-th/0308055.
- [3] D. Baumann, A. Dymarsky, I. R. Klebanov, and L. McAllister, "Towards an explicit model of d-brane inflation," arXiv:0706.0360 [hep-th].
- [4] A. Krause and E. Pajer, "Chasing brane inflation in string-theory," arXiv:0705.4682 [hep-th].
- [5] C. P. Burgess, J. M. Cline, K. Dasgupta, and H. Firouzjahi, "Uplifting and inflation with d3 branes," *JHEP* 03 (2007) 027, hep-th/0610320.
- [6] K. Dasgupta, C. Herdeiro, S. Hirano, and R. Kallosh, "D3/d7 inflationary model and m-theory," *Phys. Rev.* D65 (2002) 126002, hep-th/0203019.
- [7] J. P. Hsu, R. Kallosh, and S. Prokushkin, "On brane inflation with volume stabilization," JCAP 0312 (2003) 009, arXiv:hep-th/0311077.
- [8] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and chisb-resolution of naked singularities," *JHEP* 08 (2000) 052, hep-th/0007191.
- [9] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, "De sitter vacua in string theory," Phys. Rev. D68 (2003) 046005, hep-th/0301240.
- [10] G. R. Dvali and S. H. H. Tye, "Brane inflation," *Phys. Lett.* B450 (1999) 72-82, arXiv:hep-ph/9812483.
- [11] C. P. Burgess *et al.*, "The inflationary brane-antibrane universe," JHEP 07 (2001) 047, arXiv:hep-th/0105204.

- [12] L. A. Pando Zayas and A. A. Tseytlin, "3-branes on resolved conifold," JHEP 11 (2000) 028, hep-th/0010088.
- P. Ouyang, "Holomorphic d7-branes and flavored n = 1 gauge theories," Nucl. Phys. B699 (2004) 207-225, hep-th/0311084.
- [14] K. Becker and M. Becker, "M-theory on eight-manifolds," Nucl. Phys. B477 (1996) 155–167, hep-th/9605053.
- [15] A. Dymarsky, I. R. Klebanov, and N. Seiberg, "On the moduli space of the cascading su(m+p) x su(p) gauge theory," *JHEP* 01 (2006) 155, hep-th/0511254.
- [16] S. B. Giddings, S. Kachru, and J. Polchinski, "Hierarchies from fluxes in string compactifications," *Phys. Rev.* D66 (2002) 106006, hep-th/0105097.
- [17] C. P. Burgess, R. Kallosh, and F. Quevedo, "de sitter string vacua from supersymmetric d-terms," *JHEP* 10 (2003) 056, hep-th/0309187.
- [18] A. Achucarro, B. de Carlos, J. A. Casas, and L. Doplicher, "de sitter vacua from uplifting d-terms in effective supergravities from realistic strings," *JHEP* 06 (2006) 014, hep-th/0601190.
- [19] D. Cremades, M. P. Garcia del Moral, F. Quevedo, and K. Suruliz, "Moduli stabilisation and de sitter string vacua from magnetised d7 branes," *JHEP* 05 (2007) 100, hep-th/0701154.
- [20] P. G. Camara, L. E. Ibanez, and A. M. Uranga, "Flux-induced susy-breaking soft terms on d7-d3 brane systems," *Nucl. Phys.* B708 (2005) 268–316, hep-th/0408036.
- [21] D. Lust, P. Mayr, S. Reffert, and S. Stieberger, "F-theory flux, destabilization of orientifolds and soft terms on d7-branes," *Nucl. Phys.* B732 (2006) 243–290, hep-th/0501139.
- [22] H. Jockers and J. Louis, "D-terms and f-terms from d7-brane fluxes," Nucl. Phys. B718 (2005) 203–246, hep-th/0502059.
- [23] M. Haack, D. Krefl, D. Lust, A. Van Proeyen, and M. Zagermann, "Gaugino condensates and d-terms from d7-branes," *JHEP* 01 (2007) 078, hep-th/0609211.

- [24] P. Berglund and P. Mayr, "Non-perturbative superpotentials in f-theory and string duality," hep-th/0504058.
- [25] K. Choi, A. Falkowski, H. P. Nilles, and M. Olechowski, "Soft supersymmetry breaking in kklt flux compactification," *Nucl. Phys.* B718 (2005) 113–133, hep-th/0503216.
- [26] K. Dasgupta, P. Franche, A. Knauf, and J. Sully, "D-terms on the resolved conifold," arXiv:0802.0202 [hep-th].
- [27] K. Becker, M. Becker, and J. H. Schwarz, "String theory and M-theory: A modern introduction,". Cambridge, UK: Cambridge Univ. Pr. (2007) 739 p.
- [28] P. Candelas and X. de la Ossa, "Moduli space of calabi-yau manifolds," Nucl. Phys. B355 (1991) 455–481.
- [29] K. Dasgupta, G. Rajesh, and S. Sethi, "M theory, orientifolds and G-flux," JHEP 08 (1999) 023, hep-th/9908088.
- [30] C. Vafa, "Evidence for f-theory," Nucl. Phys. B469 (1996) 403–418, hep-th/9602022.
- [31] M. Grana and J. Polchinski, "Gauge / gravity duals with holomorphic dilaton," *Phys. Rev.* D65 (2002) 126005, arXiv:hep-th/0106014.
- [32] M. Grana and J. Polchinski, "Supersymmetric three-form flux perturbations on ads(5)," Phys. Rev. D63 (2001) 026001, hep-th/0009211.
- [33] S. Kachru, M. B. Schulz, and S. Trivedi, "Moduli stabilization from fluxes in a simple iib orientifold," JHEP 10 (2003) 007, hep-th/0201028.
- [34] O. DeWolfe and S. B. Giddings, "Scales and hierarchies in warped compactifications and brane worlds," *Phys. Rev.* D67 (2003) 066008, hep-th/0208123.
- [35] S. Gukov, C. Vafa, and E. Witten, "Cft's from calabi-yau four-folds," Nucl. Phys. B584 (2000) 69–108, hep-th/9906070.
- [36] I. R. Klebanov and A. A. Tseytlin, "Gravity duals of supersymmetric su(n) x su(n+m) gauge theories," Nucl. Phys. B578 (2000) 123–138, hep-th/0002159.
- [37] P. Candelas and X. C. de la Ossa, "Comments on conifolds," Nucl. Phys. B342 (1990) 246–268.

- [38] F. Quevedo, "Lectures on string/brane cosmology," Class. Quant. Grav. 19 (2002) 5721-5779, arXiv:hep-th/0210292.
- [39] J. M. Cline, "Inflation from string theory," arXiv:hep-th/0501179.
- [40] S. Kuperstein, "Meson spectroscopy from holomorphic probes on the warped deformed conifold," JHEP 03 (2005) 014, hep-th/0411097.
- [41] D. Baumann et al., "On D3-brane potentials in compactifications with fluxes and wrapped D-branes," JHEP 11 (2006) 031, arXiv:hep-th/0607050.
- [42] M. Haack *et al.*, "Update of D3/D7-Brane Inflation on $K3 \times T^2/Z_2$," arXiv:0804.3961 [hep-th].
- [43] G. Papadopoulos and A. A. Tseytlin, "Complex geometry of conifolds and 5-brane wrapped on 2- sphere," *Class. Quant. Grav.* 18 (2001) 1333–1354, hep-th/0012034.
- [44] S. S. Gubser, C. P. Herzog, and I. R. Klebanov, "Variations on the warped deformed conifold," *Comptes Rendus Physique* 5 (2004) 1031–1038, arXiv:hep-th/0409186.
- [45] A. Butti, M. Grana, R. Minasian, M. Petrini, and A. Zaffaroni, "The baryonic branch of klebanov-strassler solution: A supersymmetric family of su(3) structure backgrounds," *JHEP* 03 (2005) 069, hep-th/0412187.
- [46] R. Gwyn and A. Knauf, "The geometric transition revisited," hep-th/0703289.
- [47] M. Cvetic, G. W. Gibbons, H. Lu, and C. N. Pope, "Ricci-flat metrics, harmonic forms and brane resolutions," *Commun. Math. Phys.* 232 (2003) 457–500, hep-th/0012011.
- [48] K. Becker, M. Becker, M. Haack, and J. Louis, "Supersymmetry breaking and alpha'-corrections to flux induced potentials," *JHEP* 06 (2002) 060, hep-th/0204254.
- [49] J. Gomis, F. Marchesano, and D. Mateos, "An open string landscape," JHEP 11 (2005) 021, hep-th/0506179.

- [50] M. Marino, R. Minasian, G. W. Moore, and A. Strominger, "Nonlinear instantons from supersymmetric p-branes," *JHEP* 01 (2000) 005, hep-th/9911206.
- [51] H.-Y. Chen, P. Ouyang, and G. Shiu, "On Supersymmetric D7-branes in the Warped Deformed Conifold," arXiv:0807.2428 [hep-th].
- [52] F. Benini, F. Canoura, S. Cremonesi, C. Nunez, and A. V. Ramallo, "Unquenched flavors in the klebanov-witten model," *JHEP* 02 (2007) 090, hep-th/0612118.
- [53] G. Villadoro and F. Zwirner, "D terms from D-branes, gauge invariance and moduli stabilization in flux compactifications," *JHEP* 03 (2006) 087, hep-th/0602120.
- [54] L. Martucci, "D-branes on general N = 1 backgrounds: Superpotentials and D-terms," JHEP 06 (2006) 033, hep-th/0602129.
- [55] S. Gurrieri, J. Louis, A. Micu, and D. Waldram, "Mirror symmetry in generalized calabi-yau compactifications," *Nucl. Phys.* B654 (2003) 61–113, hep-th/0211102.
- [56] M. Becker, K. Dasgupta, A. Knauf, and R. Tatar, "Geometric transitions, flops and non-kaehler manifolds. i," *Nucl. Phys.* B702 (2004) 207–268, hep-th/0403288.
- [57] K. Becker and K. Dasgupta, "Heterotic strings with torsion," JHEP 11 (2002) 006, hep-th/0209077.
- [58] K. Becker, M. Becker, K. Dasgupta, and P. S. Green, "Compactifications of heterotic theory on non-kaehler complex manifolds. i," *JHEP* 04 (2003) 007, hep-th/0301161.
- [59] K. Becker, M. Becker, P. S. Green, K. Dasgupta, and E. Sharpe, "Compactifications of heterotic strings on non-kaehler complex manifolds. ii," *Nucl. Phys.* B678 (2004) 19–100, hep-th/0310058.
- [60] M. Becker and K. Dasgupta, "Kaehler versus non-kaehler compactifications," hep-th/0312221.

- [61] K. Becker, M. Becker, K. Dasgupta, and S. Prokushkin, "Properties of heterotic vacua from superpotentials," *Nucl. Phys.* B666 (2003) 144–174, hep-th/0304001.
- [62] G. Lopes Cardoso, G. Curio, G. Dall'Agata, and D. Lust, "Bps action and superpotential for heterotic string compactifications with fluxes," *JHEP* 10 (2003) 004, hep-th/0306088.
- [63] M. Grana, J. Louis, and D. Waldram, "Hitchin functionals in n = 2 supergravity," JHEP 01 (2006) 008, hep-th/0505264.
- [64] I. Benmachiche and T. W. Grimm, "Generalized n = 1 orientifold compactifications and the hitchin functionals," Nucl. Phys. B748 (2006) 200-252, hep-th/0602241.
- [65] M. Grana, J. Louis, and D. Waldram, "Su(3) x su(3) compactification and mirror duals of magnetic fluxes," JHEP 04 (2007) 101, hep-th/0612237.
- [66] P. Koerber and L. Martucci, "From ten to four and back again: how to generalize the geometry," JHEP 08 (2007) 059, arXiv:0707.1038 [hep-th].
- [67] A. R. Frey, "Notes on SU(3) structures in type IIB supergravity," JHEP 06 (2004) 027, hep-th/0404107.
- [68] M. Grana, R. Minasian, M. Petrini, and A. Tomasiello, "Supersymmetric backgrounds from generalized Calabi-Yau manifolds," *JHEP* 08 (2004) 046, hep-th/0406137.
- [69] G. Dall'Agata, "On supersymmetric solutions of type IIB supergravity with general fluxes," Nucl. Phys. B695 (2004) 243-266, hep-th/0403220.
- [70] K. Dasgupta, J. Guffin, R. Gwyn, and S. H. Katz, "Dipole-deformed bound states and heterotic kodaira surfaces," *Nucl. Phys.* B769 (2007) 1–30, hep-th/0610001.
- [71] K. Dasgupta *et al.*, "Gauge gravity dualities, dipoles and new non-kaehler manifolds," *Nucl. Phys.* B755 (2006) 21–78, hep-th/0605201.
- [72] O. DeWolfe, S. Kachru, and M. Mulligan, "A gravity dual of metastable dynamical supersymmetry breaking," arXiv:0801.1520 [hep-th].

- [73] P. G. Camara, L. E. Ibanez, and A. M. Uranga, "Flux-induced susy-breaking soft terms," Nucl. Phys. B689 (2004) 195–242, hep-th/0311241.
- [74] P. G. Camara, L. E. Ibanez, and A. M. Uranga, "Flux-induced susy-breaking soft terms on d7-d3 brane systems," *Nucl. Phys.* B708 (2005) 268–316, hep-th/0408036.