# Analysis and Modeling of Metal Commodity Price Cycles Through Band-Pass Filters and Elliott Wave Principle

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# Abstract

The great interest of the main metal and mineral industry stakeholders for understanding the behavior of metal prices and their dynamics with the rest of the human activity has resulted in the development and application of a wide variety of models. Nevertheless, the most popular methods for price modeling and forecasting are not capable of fully capture the empirical evidence about cyclical behavior (especially in the long-term), the potential connection with long economic cycles, and factors of increasing interest such as mass psychology.

Given this scenario, two techniques, little explored by the mineral economics, are studied and applied. The main objective is to evaluate whether they can address some of the drawbacks of classical methods and improve the understanding and modeling of the metallic commodities price behavior.

The first technique is the Band-Pass Filter, which proved to be useful in extracting cyclical components of different frequencies and expanding the means for the study of long-term price, co-movement, substitution effect, and the dynamic with explanatory variables in different cyclical components. The second is the Elliott Wave Principle, a technique vastly used by technical analysts and built on the belief that market prices are a reflection of participants' mass psychology, which did not demonstrate to be a strong approach to model metal commodity prices.

# Résumé

Le grand intérêt manifesté par les principaux acteurs de l'industrie des métaux et des minéraux pour comprendre le comportement des prix des métaux et leur dynamique avec le reste de l'activité humaine a conduit à la mise au point et à l'application d'une grande variété de modèles. Néanmoins, les méthodes les plus populaires de modélisation et de prévision des prix ne permettent pas de saisir pleinement les preuves empiriques concernant le comportement cyclique (en particulier à long terme), le lien potentiel avec de longs cycles économiques et les facteurs d'intérêt croissant tels que la psychologie de masse.

Dans ce scénario, deux techniques peu explorées par l'économie des minéraux sont étudiées et appliquées. L'objectif principal est d'évaluer s'ils peuvent résoudre certains des inconvénients des méthodes classiques et améliorer la compréhension et la modélisation du comportement des prix des produits métalliques.

La première technique est le filtre passe-bande, qui s'est avéré utile pour extraire des composantes cycliques de fréquences différentes et pour étendre les moyens d'étude du prix à long terme, du mouvement simultané, de l'effet de substitution et de la dynamique avec variables explicatives dans différentes composantes cycliques. Le second est le Elliott Wave Principle, une technique largement utilisée par les analystes techniques et fondée sur la conviction que les prix du marché reflètent la psychologie de masse des participants, ce qui n'a pas démontré une approche solide pour modéliser les prix des produits métalliques.

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## Contributions of Authors

The author of this thesis is Matias Marañon. Dr. Mustafa Kumral was the supervisor of the author's Master of Engineering Degree. As a result of this research, Mr. Marañon and Dr. Kumral co-authored three papers. The first, titled "Exploring the Elliott Wave Principle to interpret metal commodity price cycles", and the second, titled "Kondratiev Long Cycles in Metal Commodity Prices", have been published in the Policy Resources Journal. The third, titled "Dynamics behind cycles and co-movements in metal prices: An empirical study using band-pass filters", is under review.

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## Abbreviations

| | |
|---|---|
| **ACF** | Asymmetrical Christiano-Fitzgerald Band-Pass Filter |
| **ADF** | Augmented Dickey-Fuller Test |
| **AR** | Autoregressive model |
| **ARCH** | Autoregressive Conditional Heteroscedastic |
| **ARIMA** | Autoregressive Integrated Moving Average |
| **ARMA** | Autoregressive Moving Average |
| **BK** | Baxter-King Band-Pass Filter |
| **BMII** | Base Metals & Iron ore Index |
| **BPF** | Band-Pass Filter |
| **CT** | Chaos Theory |
| **EMH** | Efficient Market Hypothesis |
| **EWP** | Elliott Wave Principle |
| **FA** | Fibonacci Analysis |
| **FR** | Fibonacci Ratios |
| **FS** | Fibonacci Series |
| **GARCH** | Generalized Autoregressive Conditional Heteroscedastic |
| **GBM** | Geometric Brownian Motion model |
| **HP** | Hodrick-Prescott High-Pass Filter |
| **LC** | Long-term cyclical component |
| **LPF** | Low-Pass Filter |
| **MA** | Moving Average |
| **MC** | Medium-term cyclical component |
| **ML** | Machine Learning |
| **MR** | Mean Reversion model |

| | |
|---|---|
| **MUS$** | Millions of American Dollars |
| **PCA** | Principal Component Analysis |
| **PDF** | Probability Distribution Function |
| **RWH** | Random Walk Hypothesis |
| **SBM** | Stochastic Brownian Motion model |
| **SC** | Short-term cyclical component |
| **SVAR** | Structural Vector Auto Regression |
| **T** | Trend component |
| **US** | United States |
| **USGS** | United States Geological Survey |
| **VAR** | Vector Auto Regression |
| **VECM** | Vector Error Correction Model |
| **VRT** | Variance Ratio Test |

# 1. Introduction

## 1.1. Problem Statement

The understanding of the cyclical behavior of metal commodities prices has historically attracted much interest from a wide range of economic agents linked to the mineral and metal industries. The reason for this is the important economic implications that price fluctuations have on the mining business, local economies and other stakeholders such as governments.

Given the long-term characteristics of this business, most of the economic agents involved in the metal and mineral industries are more interested in long-term price outlooks (years), rather than in the near future. However, there are important shortcomings in the current models typically used for the study of the behavior of the price of metals and their prediction. These are especially related to a lack of recognition of the empirical cyclicality in metal commodity prices. On the other hand, human and mass psychology is a factor that is receiving increasing interest as a causal factor, as is considered to affect several components of the supply and demand dynamics.

In this context, two techniques little studied by the mineral economics are known for addressing some of the shortcomings identified in the models commonly used, in the sense that they allow for the study of cyclical components of longer-term and incorporates human psychology as a fundamental factor to explain the fluctuations in prices. These are the Band-Pass Filter (BPF) and the Elliott Wave Principle (EWP).

## 1.2. Research Objectives

The main purpose of this research is to study and apply two techniques, scarcely used by the mineral economics, to identify those aspects that may help in the modeling and forecasting of metal commodity prices, as well as expand the current understanding of their cyclical behavior. These techniques are the Band-Pass Filter (BPF) and the Elliott Wave Principle (EWP).

The BPF is a tool developed and popularized in recent decades by econometricians interested in studying the cyclical components of macroeconomic time series. One of the benefits of the BPF is that allows for isolating cyclical components of particular interest, for instance, short-, medium- and long-term cycles. However, for its rational implementation, a crucial input is the identification of the cyclical components to be studied a priori. Therefore, it is fundamentally a deep understanding of the cyclicity in different frequencies observed in the prices of metals and the economy in general.

The EWP is one of the most popular tools used by technical analysts to define changes in trends, description of cycles and predict the prices of all kind of instrument traded under the scheme of a competitive market. The method is based on the belief that the fluctuations in markets follow recognizable and repetitive patterns caused by changes in mass psychology.

These techniques are popular for the study of cyclical components observed empirically in the prices of financial assets and macroeconomic time series, while EWP is also vastly used as forecasting tool by technical analysts. Furthermore, they address shortcomings identified in the method commonly used for the modeling of metal and mineral prices.

## 1.3. Originality and Success

The study of these tools for the mineral economy has been very limited, and according to the author's knowledge, there is no consensus on whether the mineral economics can benefit from these tools to improve its understanding of the cyclicality of real metals prices and forecasts of short-, medium- and long-term behaviors.

Regarding the BPF, its use to investigate the cyclical components of metal prices has recently taken more attention in the academic circle. This increasing interest was caused by the greater interest in understanding the long-term cyclicality during the recent period of high metal commodity prices so-called the "super-cycle". Although recent research using BPF are indicative of the benefits of this tool for the study of cycles in metal prices, there is still no consensus regarding the cyclical components of interest and their potential link with economic activity cycles. In this research, the cyclical components are examined, incorporating the most recent knowledge of the theory of long economic cycles, resulting in an unprecedented cyclical decomposition in the literature.

In the case of EWP, despite its long history and vast implementation by technical analysts in trading, including base metals and precious metals, the formal academic study of EWP is very scarce. While the mineral economics, as well as other fields, maintains a skeptical view of the Technical Analysis, there is no much literature identifying these weaknesses and/or whether it can help in understanding the cyclical characteristic of metal prices. Therefore, this research makes an important contribution to the formal evaluation of this method.

Regarding the BPF evaluation, the following findings are highlighted:

- Using an unpublished cyclical decomposition, it was possible to extract cyclic components of short-, medium- and long-term correlated with the recent findings in metal prices cyclicity.
- A long-term cyclical component of 48-60 years was found, not negligible and highly correlated with those of the long economic cycle theory.
- The tool allowed to corroborate and complement some classical hypothesis of the mineral economics with respect to the trend and co-movement of metal prices.
- The BFP, and particularly the Asymmetrical Christiano-Fitzgerald Band-Pass Filter, is a technique that can improve the understanding of the cyclical components. Further research is required to incorporate its benefits into forecasting.

Regarding the EWP evaluation, the following findings are highlighted:

- The technique has a weak conceptual framework for its applicability in the metal commodities markets.
- The subjectivity of technical analysts cannot be fully removed.

- The fluctuations and cycles observed in the prices of metals commodities would not be subject to the dynamics suggested by EWP.

## 1.4. Social Impact and Economic Benefits

It is striking that despite the cyclical characteristics of the mining business, backed by a large body of empirical support, there is no convincing evidence of a countercyclical behavior of the business agents as a whole. A clear example of the above is the fact that the techniques commonly used for price modeling and forecasting, whether to be used for mine planning processes, project evaluation or something else, do not properly integrate this behavior.

This research shows that, on one hand, the BPF is a tool that can substantially improve the understanding of price cyclicality. Its correct implementation can provide support in investment timing, studying the dynamics between metals (co-movement and substitution effect) and dynamics with macroeconomic variables. Moreover, there is potential to be used as a complementary input for the improvement of price forecasts. On the other hand, there is evidence to suggest that EWP alone is a limited tool for metals prices cycles modeling and forecasting.

## 1.5. Thesis Organization

The thesis is divided into six chapters as follows:

**Chapter 1** introduces the problem and the research objectives, and briefly describes the main findings and benefits.

**Chapter 2** provides a detailed literature review about the understanding of the cyclical behavior in metal commodity prices, the recent findings of their long-term behavior, and the most common techniques used for the cycles modeling and forecasting of metals prices.

**Chapter 3** describes the methodology applied in the research. Given that the two techniques in evaluation belong to different conceptual frameworks, the methodologies for the application of these tools differ and are carefully explained in **Section 4.3** for the BPF and **Section 5.3** for the EWP.

**Chapter 4** is devoted to the study and application of the BPF.

**Chapter 5** is devoted to the study and application of the EWP.

**Chapter 6** concludes the thesis by indicating the benefits, or weaknesses, of each tool and how they can improve the mineral economics understanding of the metal price cycles. Furthermore, it provides guidance for further research in this field.

# 2. Literature Review

## 2.1. The Importance of Understanding the Cycles in Metal Commodity Prices

In mineral economics, the hypothesis of cycles[1] in commodity prices has been extensively documented, with a large body of empirical evidence suggesting that metal commodity prices behave cyclically. This behavior has been commonly linked to demand shocks, global economic activity, and the time lag between supply and demand decisions (Heap, 2005; Radetzki, 2006; Roberts, 2009; Humphreys, 2011; Rossen, 2015; Tilton et al., 2011; Tilton and Guzmán, 2016; Marañon and Kumral, 2018). In other words, although there is no one unified hypothesis that explains cycles in metal and mineral prices, they can be understood as the reflection of a constant process of disturbance and restoration of the equilibrium in the supply and demand.

The reason why research groups, business analyst and the academia has been long devoted to the understanding of this behavior is the large economic impact on the mining business. In this endeavor, the long-term behaviors of metals prices are of particular interest, given the distinctive features of the mineral and metal industry:

- It is a business that has very long-term lead times to provide value creation: 12-15 years from discoveries to production, 3-7 years from project approval to production, 20-40 years of operation, and 15-50 years for total closure (Gandhi and Sarkar, 2016; Haldar, 2018).

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[1] It is important to mention that for mineral economist and macroeconomist, the concept of cycle is far from its precise definition (i.e., a series of events that are repeated regularly in the same order). Instead, they are understood as recurrent patterns, which their time and amplitudes could largely differ.

- It requires massive investments while having a long period of negative cash flows. For instance, in the current copper project portfolio, an average copper project (about 150 kt Cu/year) requires about MUS$ 2,500 as initial investment (Plusmining, 2018).
- The timing of the major decisions, such as the approval of a project, can have a major impact on the value of the business (Auger and Guzmán, 2010).
- Its financial performance is closely related to the behavior of prices.
- In addition to the evident high financial risk, it is also subject to a wide range of operational uncertainties: geology (grade distribution, grade continuities, hardness, etc.), geomechanics (rock burst, failures, etc.), metallurgy (recovery, processing times, etc.), assets management (equipment availability and productivity, infrastructure failures, availability of inputs, etc.), organizational (strikes). Therefore, it is a sector keen to quantified the uncertainties and treat them.

The vast economic implications also extend to most of the sector's stakeholders. One of major concern is the impact on governments finance. While some mineral-rich economies have developed means to cope with the variation in their income derived from the changes in metal and mineral prices, some others have largely struggled in this regard, leading to problems such as social unrest, resources nationalism, corruption, and Dutch diseases. Furthermore, and from a local economic point of view, the sector generates jobs, develops infrastructure, support the development of the tertiary economic sector, promote the technological developments, and so on and so forth (Ayres and van den Bergh, 2005; Gordon and Tilton, 2008; Darling, 2011).

## 2.2. Recent Findings of Long Cycles in Metals Prices

In the last decade, the unfolding of a quite unusual cycle resulted in a fruitful period for the study of long cycles.

Among the most recent findings, Cuddington and Jerrett (2008) and Jerrett and Cuddington (2008) concluded that there would be evidence in favor of long metal price cycles of 20-70 years. Roberts (2009) introduced the idea that cycles are not symmetrical, in the sense that a contraction period is typically shorter than the expansions. Subsequently, the work of Jack (2013) delves into this line establishing the existence of long-run trends, medium-run cycles, and short-run boom-bust episodes. The work of Erten and Ocampo (2012) allowed them to identify long cycles of 30-40 years in the real prices of commodities for the period 1865-2009. Finally, Rossen (2015) provided evidence that corroborates cycles of approximately 28-30 years between 1910 and 1996.

Although recent evidence suggests long cycles of up to 70 years, a less addressed aspect is the eventual connection of these long cycles with the business cycles. In economics, there are several lines of research that study the cyclicity of businesses for different horizons, their dynamics, and driving forces. For instance, the classical business cycles (2-8 years), Kitchin cycles (3-5 years), Juglar cycles (7-11 years), the Kuznets swings (15-25 years), the "medium-term" cycles lastly popularized of 30 to 40 years, and long economic cycles or Kondratiev waves (45-60 years). In **Chapter 4**, it is connected the long cycles observed in metal prices with those of the long economic cycle theory.

## 2.3. Techniques for the Study of Metals Prices Dynamics

**Section 2.1** summarized the reasons that explain the interest in understanding the price behavior in metal and mineral industries. With this hope, an important amount of theories and techniques have been used, with different degrees of success and acceptance. Some of them more focus on modeling cyclical components and longer-term trends, while others were mostly interested in short-term forecasting.

**Section 2.3** introduces a brief review of the techniques that are commonly used in this task, as well as some other methods that are slowly gaining more attention. Depending on the theoretical framework, these techniques can be classified in the following categories: fundamental analysis, technical analysis, econometric models, time series analysis, stochastic models, and specific algorithms. As it is seen from **Sections 2.3.1-6**, all these techniques are not exempt from criticism, being particularly interesting the fact that cyclical components are not commonly incorporated.

### 2.3.1. Fundamental Analysis

This analysis is rooted in the classical economic view that metal and mineral commodities prices are generally defined by the supply and demand. It turns out that supply and demand are just the reflections of a vast amount of information and variables.

For instance, demand is not only a function of the economic activities of relevant consumers but also considers factors such as the prices of substitutes and complements materials, technological changes, government activity, changes in consumption patterns and the activity of investors and speculators.

On the other hand, supply is a larger function of factors such as production costs, productivity, inputs prices, government decision, availability of financial and geological resources, market structure, disruptions, as well as the activity of investors (Darling, 2011; Humphreys, 2011; Tilton and Guzmán, 2016; Kumral, 2018). Note that authors such, as Simon (1959), have long ago alleged that human psychology must be a factor of consideration since may have major implications in systems subject to equilibrium dynamics through time.

It is important to mention that while some changes in this factor may perturb the short-term equilibrium, e.g. disruptions, others are changes that slowly unfold, e.g. technological improvements. Hence, the relevance of these factors varies according to the time horizon of interest.

Therefore, although the objective of the fundamental analysis is to understand how the said factors weight in the future supply-demand balance dynamic, this turns out to be a quite complex task considering the massive amount of information that it is needed to be incorporated. Moreover, it requires a wide range of expertise and professionals. This approach, which pays more attention to long-term perspectives, in practice is performed by entities who have the economic means such as mineral economics research groups, banks' departments, as well as government institutions.

### 2.3.2. Technical Analysis

As metals and minerals are becoming more used as financial instruments by investors and speculators, some of the techniques intensively used for them have expanded the means for the study of the metal prices. The Technical analysis is one of them. This field groups a wide variety of techniques derived from different theories and with different purposes. EWP, Fibonacci analysis (FA), Arc analysis, oscillators, Aroon indicator, and on-balance volume are some of the most popular. In spite of a large number of techniques and tools in this field, most of them are based on the belief that prices follow identifiable patterns. Thus, an extremely rigorous analysis of the charts allows analysists to make a prediction on future prices, cycles, and trend.

Despite their wide used, especially in trading activities, several academic fields have shown strong skepticism regarding their usefulness, alleging that they are based on the subjectivity of the analysts and does not integrate factors that have proven to have causality power (Mandelbrot and Hudson, 2004; Humphreys, 2011). On top of that, the assumed short-term power of these tools (days, weeks and few months) make them of less importance for the mineral economics (Humphreys, 2011).

### 2.3.3. Econometric Models

Given the tight relationship between metal prices and global economic dynamics, econometric models have been vastly used for the understanding of prices (Watkins, 2010; Erten and Ocampo, 2012; Gargano and Timmermann, 2014; Tapia-Cortez et al., 2018). These are model aiming to mathematical capture past dynamics (correlation, co-integration, causality, etc.) in a large number of variables. Co-integration tests, Causality test, Structural Vector Auto Regression (SVAR), Vector Auto Regression (VAR) and Vector Error Correction (VEC) are typical econometric techniques used to understand the dynamics of complex systems.

However, a typical critic of these techniques emerges from the idea that past information (or part of it) is not useful since reality never repeats with the same characteristics. Nevertheless, much research using these techniques have demonstrated empirical close cyclical and temporal relationships between metal prices and some economic variables, such as the exchange rate of metal producers (Ciudad, 2005; Wu, 2013; Chipili, 2015; Haque et al., 2015). Other lines of critics relate to the deep technical knowledge needed, the large database required, their condition of static (Tapia-Cortez, 2018), and their little power in estimating long-term trends.

### 2.3.4. Time Series Analysis

Time series techniques can be understood as a convenient means to study prices behaviors in the sense that is substantially based only on the information of the time series and does not require larger models. Nevertheless, their simplicity is partially explained by the scope of interest, which is mainly forecasting, rather than the study of the dynamics or causalities. Models such as Autoregressive (AR), Moving Average (MA), Autoregressive Moving Average (ARMA), Autoregressive Integrated Moving Average (ARIMA), Autoregressive Conditional Heteroscedastic (ARCH) and its generalized version (GARCH) are typically used in metals prices forecasting, especially in the financial sector.

One of the critics against this model is the lack of representativeness of the empirical behavior of metal commodity prices. Lastly, wavelet-ARIMA model has been introduced to represent better the true behavior of some variables that present cyclical components, such as the metal prices or others (Kriechbaumer et al., 2014; Hasan et al., 2015).

As these models are mostly based on the intertemporal relationship of a sequence of data, it also faces similar shortcomings than econometric models, i.e., the past may not be useful for understanding the future. Furthermore, as their interest is on the forecast with the more “parsimonious” model possible, they may lack real world representativity as they do not capture dynamics empirically proven (Dooley and Lenihan, 2005).

### 2.3.5. Stochastic Models

As a consequence of the efficient markets hypothesis (EMH), the price of financial instruments, such as the stocks, should have a memoryless condition, thus expected to have random behavior (random walk hypothesis or RWH). This was the root for the development of stochastic models for the representation of such behavior, based on Monte Carlo simulations (Lee et al.; 2006). In practice, discrete models are typically addressed by Geometric Brownian Motion (GBM) and Stochastic Brownian Motion (SBM) models. The Mean Reversion (MR) model is an alternative which has been introduced to capture the empirical evidence of metal prices reverting to a long-term price. However, this is not more than the continuous-time version of the discrete autoregressive model of order one or AR(1).

From the mineral economics point of view, the introduction of this perception has changed how it is understood the dynamic in metals commodity prices, and led to a long debate on whether metal prices have or not unit root (i.e., whether they are random or not). This debate is far from its end and there is an important body of literature supporting the RWH. It turns out that the tests developed, mostly be econometricians, for the study of the RWH often results in favor of this behavior in metal commodity prices traded in metal exchanges, feeding the debate (Wang and William, 2007; Andersson, 2007; Oglend and Asche, 2016).

The most important critics to these methods relate to the belief that these models result in a lack of rationality since they depict behavior that is contradicting with the empirical evidence in favor of the supply-demand equilibrium dynamics.

### 2.3.6. Algorithm Specifics

As data science and artificial intelligence develop, there more and more methods that are catching the attention as means for the study of the dynamics of metal prices. As shown by Tapia-Cortez et al. (2018), two of them are the Chaos theory (CT) and machine learning (ML) techniques. These models have demonstrated some advances in representing the temporal relationships of variables in the metal markets. For instance, while CT could be applied in determining a system dynamic, ML tools have been implemented with forecasting purposes thanks to its capacity to find hidden patterns in a large database.

Nevertheless, although they have been implemented with relative success, it turns out that these techniques can provide little clues regarding the dynamics behind. Indeed, Tapia-Cortez et al. (2018) concluded that at the current level of development and assessment of these techniques, neither of them are capable of properly represent the real metal commodity markets and identify the main drivers of these systems, becoming a major problem for the study of longer-term prices.

# 3. Methodology

Two additional methods will be assessed as techniques for the study and modeling of metal prices and cycles. One could be categorized as a time series technique, while the other comes from the technical analysis. These are the BPF, evaluated in **Chapter 4**, and the EWP, evaluated in **Chapter 5**.

The reasons for their evaluation lay on what was shown in **Section 2.3**, which can be summarized as follows:

- The assessment of metal commodity prices is highly important for metal industry stakeholders and no mean has proved to be fully satisfactory.
- This assessment is especially important for the long-term, rather than the near future and the methods under evaluation may help in this regard.
- There are recent improvements in the comprehension of long cycles in metal prices, but there is a lack of understanding of their connection with long business activity dynamics and other variables such as psychology.
- There is a lack of incorporation of more realistic cyclical components into forecasting.

Therefore, both methods under evaluation may, in principle, help to address some of the shortcomings of the classical methods, especially with respect to cyclicality, long-term behaviors and dynamics with related variables, as well as take into account human psychology.

The overall methodology follows a straightforward process illustrated in **Figure 3.1**.

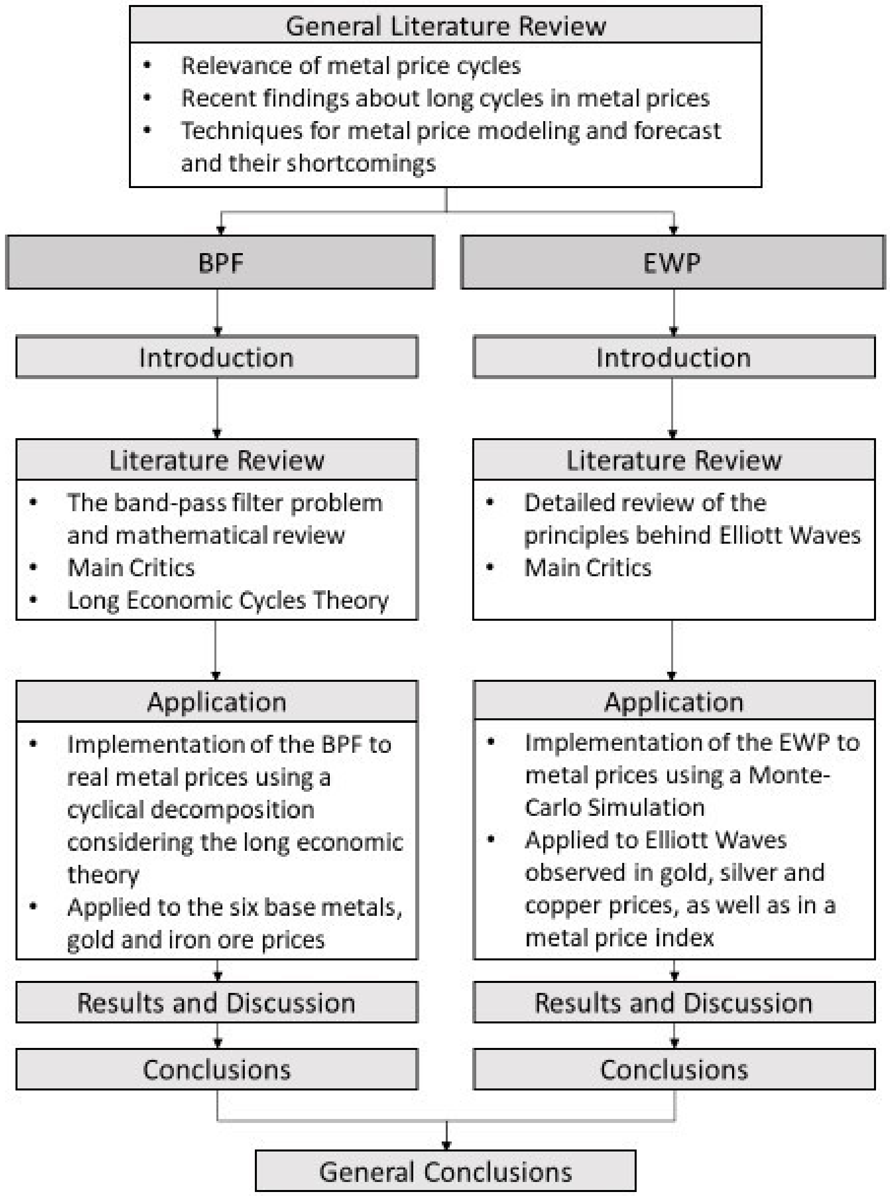



**Figure 3.1**: Overall Methodology for the Assessment of the BPF and EWP

Given that the two techniques in evaluation belong to different conceptual frameworks, the methodologies for the application of these tools differ and are described in detail in their respective chapters.

# 4. Evaluation of the Band-Pass Filters

## 4.1. Introduction

The study of cycles in macroeconomic data has been a task that has gathered economist and econometrician several decades ago, aiming to characterized primarily the business cycles. As stated by Baxter and King (1999), this must start with the business cycle measurement. In this endeavor, a problem that researchers have faced is how to decompose the time series into the cyclical components and the trend. The techniques developed and widely used during most of the twentieth century has been largely discredited using two main arguments: i) they are arbitrary methods with the sole purpose of obtaining stationary cyclical component; ii) they do not allow researchers to take into account the empirical features of these cycles (such as the length). These drawbacks led business-cycle researchers to developed the band-pass filter within the time series analysis domain.

In time series analysis, the spectral analysis theoretically establishes that a covariance-stationary time series can be approximated by a finite linear combination of trigonometric functions evaluated in increasing frequency components (Cramér and Leadbetter, 1967). This is known as the Fourier representation of a sequence. For this research, these frequency components can be understood as cycles of different frequencies and they will be called cyclical components.

Using this theoretical framework, one could extract the desired frequency components through the ideal band-pass filter. For instance, if a macroeconomist is studying the Kuznets swings of extensions between 15 to 25 years, then the ideal band-pass filter would allow the researcher to decompose the time series under analysis in the "Kuznets cycle" components (cyclical component with periodicity of 15 to 25 years), the slow secular trend (trend component with, if any, periodicity of 25 or more years) and the irregular component (those with periodicity lower than 15 years) (see **Equation 4.1**). This is the reason for its name: it allows the research to pass through the time series fluctuations within a band of desired periodicities/frequencies.

| $Y_t = IC_{2-15} + CC_{15-25} + TC_{25\ or\ more}$ | **Eq. 4.1** |
|---|---|

Nevertheless, the implementation of the ideal band pass-filter faces a major problem. For the ideal band-pass filter to be computed, it is required either infinite data, or the process under analysis to be a covariance-stationary process; hence, limiting its usefulness in empirical research. As a result of the above, in the last decades, econometricians have developed approximations to the ideal band-pass filter.

## 4.2. Literature Review: The Band-Pass Filters and Long Cycles

### 4.2.1. The Band-Pass Filter Problem

#### 4.2.1.1. Mathematical Review

For the understanding of the band-pass filter, it is better to start by its basic units, the low-pass filter (LPF), since a band-pass filter is technically obtained as a combination of LPFs. Their application on a time series can be interpreted as the extraction of its trend component in the context of this research. Frequency filters are typically designed in the frequency-domain for simplicity, and the following review will be made using both time- and frequency-domains[2]. The Fourier transform and the inverse Fourier transform operators allow for mapping time-domain functions into their frequency-domain equivalent, and vice versa.

The previous point can be summarized by **Equation 4.2**, which shows the trend ($T_t$) component of a time series $y_t$, and **Equation 4.3**, which shows that $y_t$ is the sum of its trend and not trend ($NT_t$) components. Note that *F(L)* in these equations denote the time-domain ideal low-pass filter.

| $T_t = F(L) * y_t$ | **Eq. 4.2** |
|---|---|

| $y_t = T_t + NT_t$ | **Eq. 4.3** |
|---|---|

[2] For more details, see Chapter 6 of Hamilton, 1994

In the frequency domain, the ideal LPF, i.e., a filter that passes only frequencies lower than $w_L$, has a frequency-domain function (the frequency-domain equivalent to *F(L)*) given by **Equation 4.4**. This filter only captures frequencies lower than $w_L$, while leaving unaffected the components with frequencies equal or higher than $w_L$. Note that the lowercase *f* refers to the filter in the frequency-domain, while the uppercase *F* refers to the time-domain filter. This notation will be used through the mathematical review.

| | |
|---|---|
| $$f(w) = \begin{cases} 0\ if\ w_L < \lvert w\rvert \leq \pi \\ 1\ if\ \lvert w\rvert \leq w_L \end{cases}$$ $$Note\ that\ F\ refers\ to\ (w) = \begin{cases} 0\ if\ w_L < \lvert w\rvert \leq \pi \\ 1 \qquad if\ \lvert w\rvert \leq w \end{cases}$$ | **Eq. 4.4** |

It turns out that the time-domain ideal LPF could be computed by **Equation 4.5**.

| | |
|---|---|
| $$F(L) = \sum_{h=-\infty}^{\infty} F_h * L^h$$ $$with\ h\ representing\ the\ lag/lead$$ $$and\ L\ the\ lag/lead\ operator\ (L^h = e^{iwh}, i = \sqrt{-1})$$ | **Eq. 4.5** |

Using the Fourier transform theorem, the weights (*Fh*) are obtained by the Fourier integral of the frequency-domain function as shown in **Equation 4.6**.

| | |
|---|---|
| $$F_h = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(w) * e^{(ieh)} dw$$ | **Eq. 4.6** |

It follows that as *f(w)=1* when $\lvert w\rvert \leq w_L$ and 0 when $\lvert w\rvert > w_L$, the solution to **Equation 4.6** are the weights $F_h$, shown in **Equations 4.7-8**.

| | |
|---|---|
| $$F_0 = \frac{w_L}{\pi}$$ | **Eq. 4.7** |

| | |
|---|---|
| $$F_h = \frac{\sin(hw)}{h\pi}\ for\ h = 1, 2, \ldots, \infty$$ | **Eq. 4.8** |

Note that $F(L)$ requires an infinite time series to be solved. This set a major limitation since the interest in the research is to obtain a decomposition of the time series in the time-domain, a task that becomes impossible unless an infinite sequence of data is had. Nevertheless, there is a straightforward path for estimating $T_t$ directly. For this, it is required to obtain the Fourier transform of $y_t$, i.e., $y^*_t$, and calculate the inverse Fourier transform of the product $f(w)\ y^*_t$. The major drawback of this path is that $y_t$ is required to be covariance-stationary, assumption normally violated in macroeconomic and financial variables (Christiano and Fitzgerald, 2003).

Therefore, econometric researchers have come up with optimal approximations to solve the problem of obtaining time-domain filters that allow for extracting the cyclical components within a range of frequencies and are not subject to the constraint of an infinitive sequence. Furthermore, the alternative proposed for an optimal approximation to the band-pass filter must achieve certain desired conditions for the proper study of cyclical components. In other words, the band-pass filter must:

- Extract the specific range of frequencies without affecting the properties of the extracted components (for instance, not modifying the amplitude).
- Not introduce or minimize phase shifts (i.e., the filter does not induce phase shift among the underlying periodic series).
- Obtain a stationary time series even when applied to trending data. In other words, the cyclical component extracted must be stationary, otherwise, the filter is not removing trend components. This is particularly important considering the large body of literature suggesting the presence of stochastic trends (commonly, integrated of order one or two) in several macroeconomic and financial series.
- Obtain cyclical components which are unrelated to the extension of the known time series. To put it simply, this means that cyclical components must be intrinsic to the time series observed and not be dependent on the extension.

Hodrick and Prescott (1980) developed a time-domain high-pass filter (HP filter) that, as pointed out by Baxter and King (1999), provides a reasonable approximation to the ideal high-pass filter, but in quarterly data. On the other hand, one of the major critics is that the cyclical components obtained depends on the extension of the data. Moreover, by construction, this filter is limited only for the extraction of a cyclical component over certain frequency and not a band of frequencies.

Bearing this drawback in mind, Baxter and King (1999) develop a band-pass filter (BK), demonstrating that it was superior to the HP filter under certain circumstance (such as annual data). This solution was obtained as a symmetrical moving average with lags and leads defined by the researcher[3]. The weights used are obtained as the solution of the minimization problem shown in **Equation 4.9**[4] subject to **Equations 4.10-11**. The first restriction deals with their objective to obtain a symmetrical filter in the time-domain, while the second with obtaining a stationary filtered series[5].

$$Min \int_{-\pi}^{\pi} |f(w) - bk(w)|^2 dw \qquad \textbf{Eq. 4.9}$$

$$BK(L) = BK_0 + \sum_{h=1}^{K} BK_h(L^h + L^{-h}) \qquad \textbf{Eq. 4.10}$$

$$BK(1) = BK_0 + 2\sum_{h=1}^{K} BK_h \qquad \textbf{Eq. 4.11}$$

It turns out that Christiano and Fitzgerald (2003), demonstrated that their filter (the asymmetric band-pass filter or ACF) dominates the BK, especially when interested in extracting low-frequency components. In other words, the optimal approximation to the ideal band-pass filter obtained by

[3] In their paper, they also demonstrate that the extension of number of lags and leads (i.e. K value) affects the performance of the filter and reduce the data available. Therefore, a tradeoff must be considered when defining the K value. The authors conclude their work by suggesting to take K=3 in annual data for standard macroeconomic data.
[4] Minimization of the equal-weighted average square modulus of the difference between the ideal and the approximation, both in their frequency-domain representations.
[5] Exploiting the fact that when a filter using symmetric moving averages which sum to zero, the filter has trend elimination properties. For more details see Appendix A in Baxter and King, 1999.

Christiano and Fitzgerald it is more efficient (reduces leakage, exacerbation and compression problems), especially when extracting very low cyclical components (see **Figure 4.1** for a representation of the above). On the other hand, this filter asymptotically approaches the ideal filter as the sample size growth (Estrella, 2007).

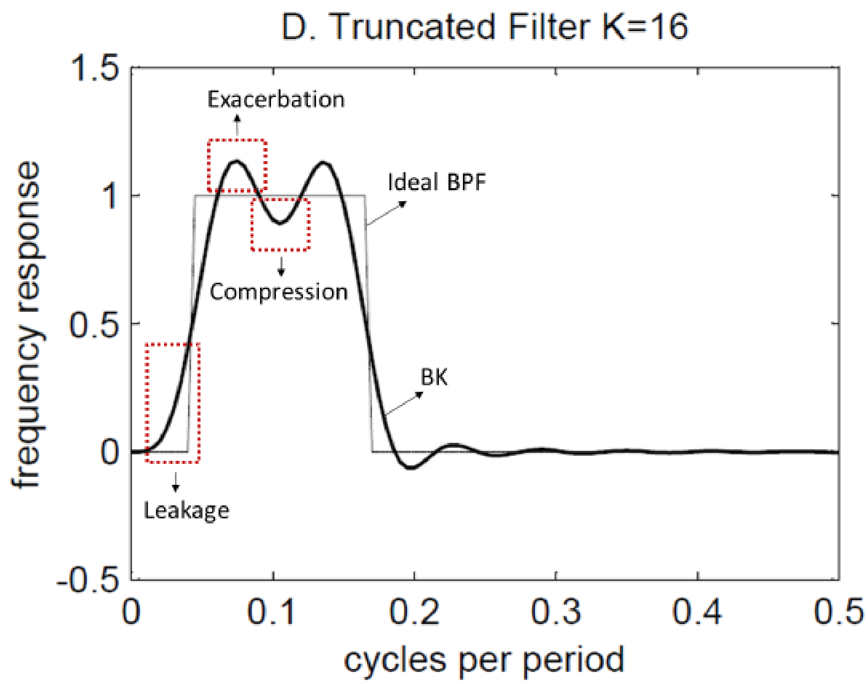



**Figure 4.1:** Leakage, exacerbation and compression problems in BK in frequency-domain.
**Source:** Modification from Baxter and King, 1999.

Christiano and Fitzgerald derived their band-pass filters as follow. Consider the orthogonal decomposition of **Equation 4.12** of the stochastic process $y_t$. Here $C_t$ captures the cyclical components with a certain periodicity, let say $2 \leq p_L < p_U < \infty$[6]. $NC_t$ captures the non-cyclical time series components, usually interpreted as the trend and short-term irregularities.

| | |
|---|---|
| $y_t = C_t + NC_t$ | **Eq. 4.12** |

[6] Note that periodicity ($p$) and frequency ($w$) are related as $w=2\pi/p$.

Here, the band-pass problem aims to obtain a filter capable of obtaining the cyclical component as a result of applying this filter on the time series under study (i.e., $y_t$), as shown in **Equation 4.13**.

| $$C_t = B(L) * y_t$$ | **Eq. 4.13** |
|---|---|

Note that *B(L)* represents the ideal band-pass filter, previously shown in **Equation 4.14.**

| $$B(L) = \sum_{h=-\infty}^{\infty} B_h * L^j$$ | **Eq. 4.14** |
|---|---|

Its Fourier transform is shown in **Equation 4.15**, which shows the frequency-domain filter.

| $$b(w) = \begin{cases} 1\ if\ \lvert w \rvert \in [a, b] \\ 0\ otherwisw \end{cases} \\ with -\pi \leq w \leq \pi$$ | **Eq. 4.15** |
|---|---|

Note that, in the frequency domain, this filter takes values of 1 when within $a=2\pi/p_U$ and $b=2\pi/p_L$, and *0* otherwise. As shown before, the calculation of this filter is infeasible. Thus, let assume a hypothetical optimal approximation in **Equation 4.16**.

| $$\hat{C}_t = \hat{B}(L)y_t = \tilde{B}_{t-1}y_1 + B_{t-2}y_2 + \cdots + B_1 y_{t-1} + B_0 y_t + B_1 y_{t+1} + \cdots + B_{T-1-t}y_{T-1} + \tilde{B}_{T-t}y_T \\ for\ t = 2, \ldots, T-1$$ | **Eq. 4.16** |
|---|---|

Christiano and Fitzgerald treated the problem of estimating the $B_h$ as a projection one. Indeed, the estimation of $C = [C_1, \ldots, C_T]$ is $\hat{C}$, the projection of $C$ onto the data available. **Equation 4.17** indicates the set of respective projection problems.

| $$\widehat{C_t} = P[C_t \vert y], t = 1, \ldots, T$$ | **Eq. 4.18** |
|---|---|

Therefore, the general solution to each projection problem is the linear function in **Equation 4.19**.

| | |
|---|---|
| $$\widehat{C_t} = \sum_{h=-j}^{k} \widehat{B}_h^{k,j} * y_{t-h}$$ $$j = T - t \ and \ k = t - 1$$ | **Eq. 4.19** |

Christiano and Fitzgerald defined the $\widehat{B}_h^{k,j}$ as the weights that solve the minimization problem in the time-domain in **Equation 4.20**. Note that the solution of this problem depends on *t*, and therefore the solution is one filter for each date. Moreover, for each $C_t$, its filter weights past and future values of $y_t$ asymmetrically. This explains why this filter is known as Christiano-Fitzgerald asymmetrical band-pass filter.

| | |
|---|---|
| $$Min_{\widehat{B}_h^{k,j}, h=-j,\ldots,k} \ E\left[\left(C_t - \widehat{C_t}\right)^2 \middle| y\right]$$ | **Eq. 4.20** |

For simplicity, this problem is solved in the frequency domain, which representation is shown in **Equation 4.21**.

| | |
|---|---|
| $$Min_{\widehat{B}_h^{k,j}, h=-j,\ldots,k} \int_{-\pi}^{\pi} \left|b(w) - \hat{b}^{k,j}(w)\right|^2 s_y(w) \, dw$$ | **Eq. 4.21** |

Note that this minimization problem is slightly different to the one solved by Baxter and King since here the squared deviations between the approximate filter $\hat{b}^{k,j}(w)$ and the ideal filter $b(w)$ are weighted by the spectral density function of $y_t$.

Thus, in order to solve **Equation 4.21**, it is required to know the spectral density function[7]. Christiano and Fitzgerald provide a simple solution for processes with representations as shown in **Equation 4.22**.

| | |
|---|---|
| $$y_t = y_{t-1} + \theta(L)\varepsilon_t \, , \theta(L) \ a \ polinomy \ of \ m \ lags$$ | **Eq. 4.22** |

[7] The spectral density ($s_y(w)$) and autocovariance-generating ($g_y(h)$) functions are related by $s_y(w) = \frac{1}{2\pi} \sum_{h=-\infty}^{\infty} \gamma_h \, e^{-iwh}$, with $g_y(h) = \sum_{h=-\infty}^{\infty} \gamma_h z^h$ (z a complex number).

The interest in this thesis is on random walk processes, i.e., when *m=0*, since there is a great body of literature indicating that prices of commodities are not covariance-stationary. For the purpose of confirming the latter, **Section 4.3** will test the random walk hypothesis for the series of prices under evaluation. In addition, the method requires the elimination of drift[8].

It turns out that if $y_t$ follows a random walk process, then the weights that solve **Equation 4.22** are presented in **Equation 4.23-26**.

| | |
|---|---|
| $B_h = \dfrac{\sin\left(\frac{2\pi h}{p_L}\right) - \sin\left(\frac{2\pi h}{p_U}\right)}{\pi h}, h > 1$ | **Eq. 4.23** |

| | |
|---|---|
| $B_0 = 2(1/p_L - 1/p_U)$ | **Eq. 4.24** |

| | |
|---|---|
| $\tilde{B}_{T-t} = -\frac{1}{2}B_0 - \sum_{h=1}^{T-t-1} B_h$ | **Eq. 4.25** |

| | |
|---|---|
| $\tilde{B}_{t-1} = -B_0 - B_1 - \cdots - B_{T-1-t} - \tilde{B}_{T-t} - B_1 - \cdots - B_{t-2}$ | **Eq. 4.26** |

Since the introduction of the BPF, the main focus has been associated with the study of business cycles with a certain periodicity. Nevertheless, its use has slowly expanded to the study of the dynamic among macroeconomic variables in different cyclical components. For instance, in the relationship between money growth and inflation.

The authors Cuddington and Jerret incorporated, in 2008, the use of the ACF in the mineral economics field. However, they deeply studied only two cyclical components (2 to 20 years and 20 to 70 years). These tools can continue to be exploited for a deeper study of the cyclical dynamics of the metal commodity prices. Indeed, Cuddington and Jerret (2008) demonstrated the existence of cycles of up

[8] The drift adjust in this thesis was performed following the guideline of Christiano and Fitzgerald (2003), for more details see footnote 13.

to 70 years; it would be hard to argue that cycles of 20 or 70 years are consequences of the same phenomenon.

Although the long cycles have caught the attention in the last decade (in the context of the "super-cycle"), there is still much to understand about them, for instance, its connection with long economic cycle theories. Therefore, **Section 4.2.2** reviews the literature focuses on the long economic cycle theory in order to enhance the cyclical decomposition to be applied in **Section 4.3**. Furthermore, it has been identified other areas of interest that could also be explored by this tool. Specifically, testing some of the classical hypothesis regarding the long-term behavior in metals prices, as well as the co-movement in different cyclical components. To the author knowledge, these aspects have been vaguely explored through the band-pass filter and are addressed in the application of the ACF in **Section 4.3**.

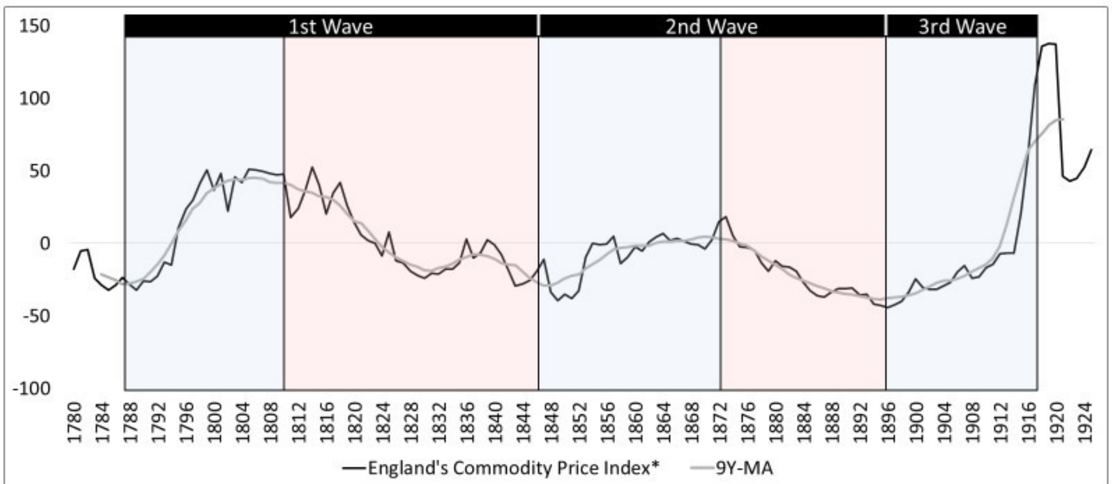
#### 4.2.1.3. Main Critics

The main critic to this method is the underlying assumption that a time series is the result of a finite summation of cyclical components of increasing frequencies. Nevertheless, there is a wide consensus that being applied with the required discipline, they can be quite successful in extracting cyclical components of time series (Pollock, 2014).

On the other hand, the specialized literature attempting to answer which band-pass filter is better has concluded that there is no filter as a unique best solution (Baxter and King, 1999; Christiano and Fitzgerald, 1999, 2003; Estrella, 2007, Pollock, 2014). Instead, depending on whether the interest is on lower or higher frequency components, as well as assumptions on the data-generating process, there are guidelines about which ideal band-pass filter approximation the researcher should use. In this context, filters, such as the Hodrick-Prescott (HP), Trigonometric Regression Filters (Hamilton, 1994), the Frequency Domain Method (FD), Baxter-King (BK) and Christiano-Fitzgerald (CF), are commonly used depending on the research's purposes.

## 4.2.2. Long Economic Cycle Theory

Regarding the long economic cycles, it was the Russian economist N. Kondratiev who formally proposed[9] in the 1920s that it is very likely that the economic dynamic[10] is subject to cycles of a duration of 48 to 55 years (Kondratiev, 1984). This theory was the result of Kondratiev's extensive empirical study on several macroeconomic indicators[11] over 1780-1925, including the commodity price indices (as shown in **Figure 4.2**), for the major economies at that time (i.e. United States, England, and France).




* For England the data used by Kondratiev was the Silberling's index for 1780-1846, and from 1846 on it was used the Sauerbeck's index (sources: Silberling's book, Annuaire statistique and Journal of the Royal Statistical Society. For USA, 1791-1801 the H.V. Roelse's Index, 1801-1825 the A.H. Hansen's index, 1825-1839 the C.H. Jurgens's index, 1840-1890 the Falkner's Index, and since 1890 on the Bureau of Labor Statistic (source: Annuaire statistique de la France 1922 and 1924). For France, the data was taken from the Annuaire statistique de la France 1922 and 1924.

**Figure 4.2**: Long cycles identified by Kondratiev in the England's commodity price index 1780-1924. The time series is shown as the deviations from the linear trend and long cyclical component was calculated as the 9-years centered moving average.
**Source**: Modified from Kondratiev, 1984.

After Kondratiev, the theory was consolidated in the academic circles through the contributions of economists such as Schumpeter (1939), Mandel (1980), Dickson (1983), Goldstein (1988), Ayres

[9] Before Kondratiev, the long cycles were a topic first discussed by several economists such as Gelderen (1914), Moore (1914), Layton (1922), Trotsky (1923) and S. de Wolff (1924). However, none of them systematized their research and findings.
[10] Back then, Kondratiev was referring specifically to the capitalist economic system.
[11] Commodity prices, production and consumption, interest rate, labor wages, foreign trade and savings in banks.

(2006), Dator (2006), Korotayev and Tsirel, 2010, Korotayev et. al. (2011), Grinin et al. (2012) and Grinin et al. (2016). Thus, Kondratiev's work has been extended to the twentieth and twenty-first centuries, proposing cycles of about 45-60 years identified in **Table 4.1**.

| | Upward | | Downward | | Length |
|---|---|---|---|---|---|
| | Start | End | Start | End | [years] |
| 1st Long Wave | 1790 | 1810/17 | 1810/17 | 1844/51 | 54-61 |
| 2nd Long Wave | 1844/51 | 1870/75 | 1870/75 | 1890/96 | 46-52 |
| 3rd Long Wave | 1890/96 | 1914/20 | 1914/20 | 1939/50 | 60 |
| 4th Long Wave | 1939/50 | 1968/74 | 1968/74 | 1984/91 | 52 |
| 5th Long Wave | 1984/91 | 2008/10 | 2008/10 | - | - |

**Table 4.1**: Kondratiev long cycles proposed for the twentieth and twenty-first centuries.

Regarding the economic dynamics behind these cycles, Kondratiev reflected on the idea that during the inflection between the bearish and bullish trend, there was an important technological transformation in production techniques. However, he focused on the investment and capital dynamics as the main explanation; indirectly linked to innovation. Nevertheless, later contributions have placed in the technological innovation at the center of the explanation (Schumpeter, 1939; Mensch, 1979; Haustein and Neuwirth, 1982; Singer, 1998; Korotayev et al., 2011), the reason why the Kondratiev waves are associated with a set of disruptive technologies. To put it simply, the economic long cyclicality is currently explained by the life stages of a new set of technologies capable of redefining the global productive capacity. These stages are: i) disruptive innovation; ii) assimilation; iii) standardization; iv) obsolescence and conditions for a new technology revolution.

Once the technological revolution is placed as the main driver, an interesting discussion is on whether these cycles should be shorter with the technological progress. The perception that innovation and technological advance is accelerated has been actually demonstrated by several researchers (Kurzweil, 2000; Buchanan, 2008). Nevertheless, this faster advance does not necessarily mean shorter economic cycles. Indeed, the vast majority of Kondratiev followers, and for whom technology is the main driver of the long cycles, the waves are likely to remain in the range observed (i.e. 45-60 years). Indeed, Devezas and Corredine (2002) postulated that the length of the long cycles (about 60 years) is also linked to the social diffusion of new revolutionary innovations, which in turns is dominated by two "biological control parameters", i.e. human cognition and transfer of knowledge between generations. In contrast, Šmihula (2009) has proposed an alternative interpretation for the long waves, which indeed reflects an acceleration of these cycles; however, it is quite decoupled of Kondratiev long cycle mainstream.

The potential links between the fluctuations of these long economic cycles and commodity prices could lay in two ideas: i) changes in the intensity of use of inputs are driven by technological innovations, where metals have historically played a relevant role; ii) innovations that allow the metals and minerals industries to increase their reserves. Indeed, a quick visual inspection[12], as shown in **Figure 4.3**, extends the Kondratiev's findings of a high synchronization degree between a metal commodities prices index[13] and the Kondratiev waves. **Appendix 1** shows the same visual analysis for the prices of the six base metals, gold, and iron ore.

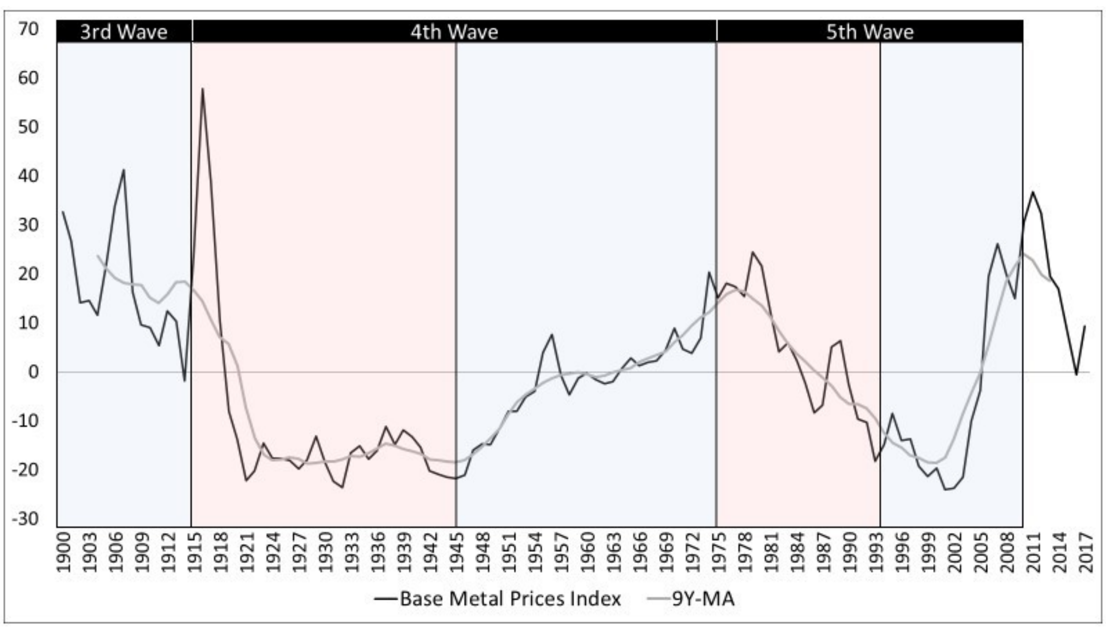



**Figure 4.3**: Extension of Kondratiev analysis for the base metals price index[14] using the same methodology applied in Kondratiev's original work (Kondratiev, 1984).

[12] Applying the same method than Kondratiev to extract the long cyclical components, i.e., the centered moving average on the deviation of the linear trend.
[13] Index built using the copper, nickel, zinc, lead, tin, aluminum and iron ore prices and the weights used by the World Bank (2018) in the construction of its Metal and Mineral Prices Index.
[14] Built using the 2018 World Bank's weights (1900=100).

Although there is extensive literature supporting the long economic cycles theory, it is considered a controversial theory and has received important criticism. Some of the most important relate to the ideas that long waves would be the result of outlier events non-related to innovation (Metz, 2006) or that long trends observed are the results of stochastic changes (Rosenberg and Frishtak, 1983). It is also pointed out that the detrending techniques could derive in misinterpreting the data, leading to the emergence of long cycles (Charles and Kang, 1981). Other lines of criticism are those initiated by Solomou (1986), who refuse the idea of a connection between cluster-of-innovation and long waves, and Freeman et al. (1987), who believes that technological innovation is only a part of a more integral explanation for long cycles.

Regarding mineral economics, the latest findings would validate the presence of long cycles in real metals prices. However, there seems to be no consensus on the extensions, and aspects such as their linkage to the global economic dynamics or factors behind long-term changes in the industry are even less explored.

## 4.3. Application of Band-Pass Filters for the Study of Metal Commodity Cycles

### 4.3.1. Specific Area of Interest

The main objective of this section is to apply the ACF to the metal commodity prices, but incorporating the long economic cycles provided by Kondratiev’s theory for a novel cyclical decomposition. This is made to deepen the understanding of cycles in metals prices, focus especially on the long cycles and their possible link with innovation and technology, through the Kondratiev’s hypothesis of long economic cycles. This is especially suitable in the current scenario where, on one hand, the world is facing technological revolutions (expected to importantly shape the future supply and demand for several metals) and high macroeconomic uncertainty derived from a global trade conflict (partially linked to technological domination). On the other hand, the mining industry has steady suffer from cost inflation in the last decade and portfolios for some base metals are abnormally depressed (Mikitchook et al., 2018). Furthermore, the results of the study are expected to contribute to the understanding of higher frequency cycles, the co-movement in different cyclical components and price behavior in the very long term (trend). The above in the five points below:

i. **Long cycles and Kondratiev waves:** formal study the presence of long cycles in the real prices of metals and their connection with Kondratiev waves. For this purpose, it will study the cyclical components of the time series ranging from 45-60 years as suggested by the long economic cycle theory.

ii. **Higher frequency cycles:** identification of short- and medium-term cycles (short and medium cycles), and link them to the last findings in macroeconomics and the mineral economics. In practice, this translates into business cycles (periods of no more than 10 years) and medium-term cycles, documented by the mineral economics and ranging from 10 to 40 years.

iii. **Co-movement in different cyclical components:** the co-movement has received significant attention in recent years due to its implications for investors seeking diversification within metals markets. However, a less studied aspect is the persistence of this synchronized movement in different cyclical frequencies. This study will allow the evaluation of co-movement for metals short, medium and long cycles components, as well as in their trends.

iv. **Trend Analysis:** The dominant view in the mineral economics is that, in the very long term, the supply curve is rather flat since it can react to increases in demand through discoveries of competitive resources (in the context of the predominant technologies) and/or technological improvements that reduce production costs and/or enhance the deposit discovery process. As a consequence of the above, the prices of metals in the very long term should be rather trendless, being unaffected by changes in demand. The present study adds antecedents in favor of this vision and also establish a measure of what it is meant by "very long term".

v. **Dynamics of the long cycles**: Finally, a working hypothesis on the explanatory dynamics for long-term cyclicality observed in the real prices of metals is proposed and aspect for further research are suggested.

### 4.3.2. The ACF

In order to answer the previous questions, it will be used the Christiano-Fitzgerald's optimal "linear" approximation assuming processes *I(1)* (drift adjusted). As it was previously acknowledged, the debate on whether commodity prices behave under the RWH remains open and there is an important body of literature validating this (Wang and William, 2007; Andersson, 2007; and Oglend and Asche, 2016).

Although this research does not attempt to deeply study whether the real metal prices are unit root process, a quick study is implemented based on two of the most common econometric tests for this purpose: The Augmented Dickey-Fuller[15] test (ADF) and the variance ratio test (VRT). The results are shown in **Table 4.2**. Considering a joint interpretation of the ADF and VRT, it can be stated that there is statistical evidence to consider the series as integrated of order 1, i.e., they are unit root processes.

[15] The implementation of an ADF without considering structural breaks leads to a bias towards the non-rejection of the null hypothesis (Perron, 1989). Therefore, the analysis of the RWH is be complemented with the variance ratio test (VRT). The VRT has gained popularity in recent years as a complement to test the RWH (Charles, 2009), although it is not without weaknesses as Kim and Kim (2010) point out.

| Test | ADF Test | | VRT* | | | Conclusion |
|---|---|---|---|---|---|---|
| Null | It is a random walk | | It is a random walk | | p-value | |
| Cu | Observations | 216 | Max \|z\| (at period 16) | | 0.270 | I(1) |
| | p-value | 0.005 | Wald (Chi-Square) | | 0.052 | |
| Ni | Observations | 177 | Max \|z\| (at period 16) | | 0.396 | I(1) |
| | p-value | 0.345 | Wald (Chi-Square) | | 0.275 | |
| Zn | Observations | 141 | Max \|z\| (at period 16) | | 0.133 | I(1) |
| | p-value | 0.0001 | Wald (Chi-Square) | | 0.088 | |
| Pb | Observations | 117 | Max \|z\| (at period 16) | | 0.436 | I(1) |
| | p-value | 0.060 | Wald (Chi-Square) | | 0.184 | |
| Sn | Observations | 136 | Max \|z\| (at period 16) | | 0.388 | I(1) |
| | p-value | 0.029 | Wald (Chi-Square) | | 0.200 | |
| Al | Observations | 121 | Max \|z\| (at period 16) | | 0.281 | I(1) |
| | p-value | 0.002 | Wald (Chi-Square) | | 0.107 | |
| Au | Observations | 116 | Max \|z\| (at period 16) | | 0.044 | I(1) |
| | p-value | 0.212 | Wald (Chi-Square) | | 0.072 | |
| Fe | Observations | 114 | Max \|z\| (at period 16) | | 0.091 | I(1) |
| | p-value | 0.069 | Wald (Chi-Square) | | 0.152 | |
| * Standard error estimates assume no heteroskedasticity, user-specified lags: 2 4 8 16, test probabilities computed using wild bootstrap: dist=twopoint. | | | | | | |

**Table 4.2**: Augmented-Dickey Fuller and Variance Ratio Tests.

Returning to the ACF, the reasons of its usage are briefly described below:

- Allows for decomposing the time series in components that fluctuate within a frequency range. For this study, high frequencies (short cycles), medium frequencies (medium cycles), low frequencies (long cycles) and trend.
- Allows for considering the full data in the estimation of the filter coefficients, unlike the fixed length symmetric options, such as BK filter.
- Has a greater adjustment to the ideal band-pass filter at low frequencies; the focus of interest in this research.

- Fulfills the desired requirements in a band-pass filter: minimizes the difference with the ideal filter, extracts the desired frequencies without compromising the remnant, minimizes the introduction of phase shift[16] and eliminates stochastic trends in the filtered series.

The decomposition of the series of real prices in their cyclical components is as shown in **Equation 4.27**:

| | |
|---|---|
| $A\_Price_t = A\_SC_t + A\_MC_t + A\_LC_t + A\_T_t$ | **Eq. 4.27** |

*A_Price* is the natural logarithm of the price *A* in *t*. *A_SC* is the short-term cyclical (or high frequency) component (2-10 years) of *A_Price* in *t*. *A_MC* is the medium-term cyclical (or medium frequency) component (10-45 years) of *A_Price* in *t*. *A_LC* is the long-term cyclical (or low frequency) component (45-60 years) of *A_Price* in *t*. Finally, *A_T* is the trend of *A_Price* in *t*. **Figure 4.4** illustrates how the ACF band-pass filter was applied to extract each component step by step.

[16] Although the asymmetric method does not ensure that there is no phase change, under conditions such as random walk, its impact is minimal and with significant gains in relation to minimizing the difference with the ideal band-pass filter.

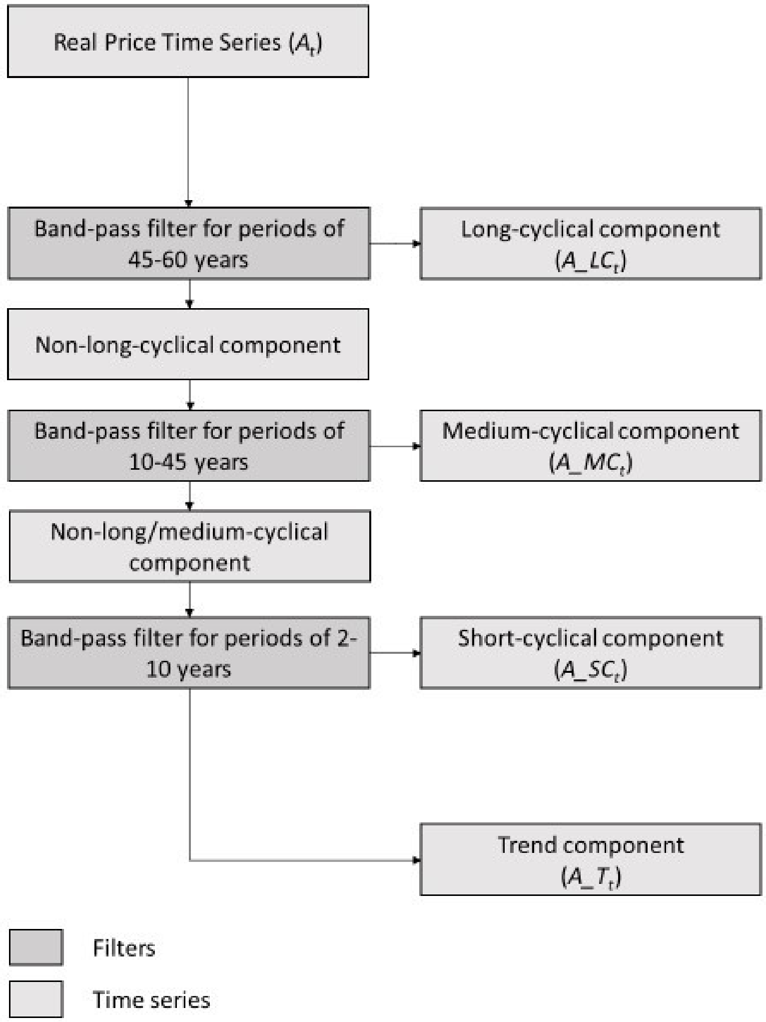



**Figure 4.4**: Representation of the band-pass filtering process for the extraction of the short-, medium-, and long-term cyclical components, as well as the trend.

For the co-movement analysis, the Pearson correlation coefficient (considering all available data pairs) and the principal component analysis (PCA, considering a balanced sample) will be studied[17], complementing the band-pass filter analysis.

### 4.3.3. The Data

Given the low frequency of the cycles of main interest, the extension of the series of real prices[18] is fundamental. Unfortunately, this task is not trivial considering the limited availability of reliable data

[17] Similar than Cuddington and Jerret (2008).
[18] The US's CPI is used as the deflator. For further details of this choice, see Appendix III: Choice of Deflators: CPI vs. PPI (Cuddington and Jerrett, 2008).

with more than 100-150 years. As a consequence, the large body of studies related to cycles in metals prices is based on information from the beginning of the twentieth century, or at best since the mid-nineteenth century. However, this thesis considers data -in real terms- that go as far as the year 1800, covering all the long-term cycles identified in Kondratiev's theory. **Table 4.3** describes the series used. All the series were transformed to natural logarithm since this allows for evaluating the deviations of the cycles with respect to the trend as percentage approximations.

| Variable | Code | Unit | Period | Source | Obs. |
|---|---|---|---|---|---|
| Copper Price | CU | Ln(US$2017/t) | 1800-2017 | USGS, Makridakis et al. (1997) | 218 |
| Nickel Price | NI | Ln(US$2017/t) | 1840-2017 | USGS | 178 |
| Zinc Price | ZN | Ln(US$2017/t) | 1875-2017 | USGS | 143 |
| Lead Price | PB | Ln(US$2017/t) | 1880-2017 | USGS | 118 |
| Tin Price | SN | Ln(US$2017/t) | 1900-2017 | USGS | 138 |
| Aluminum Price | AL | Ln(US$2017/t) | 1895-2017 | USGS | 123 |
| Gold Price[19] | AU | Ln(US$2017/t) | 1900-2017 | USGS | 118 |
| Iron Ore Price | FE | Ln(US$2017/t) | 1900-2017 | USGS | 118 |

**Table 4.3**: Metal and mineral prices.

Note that the price of copper begins in 1800, which means a great opportunity in this research considering that it has historically proven to be strongly correlated with macroeconomic trends (Heap, 2005; Tapia-Cortez et al., 2018), as a consequence of its broad global use in key economic sectors for economic growth and development, such as infrastructure and energy.

## 4.4. Results and Discussions

### 4.4.1. Long cycles and Kondratiev waves

For each of the 8 metals, the short, medium, and long cyclical components, as well as the trend and the real price are shown in **Figures 4.5-4.12.** For most of the figures displayed in **Section 4.4**, the

[19] Given its characteristic as a financial asset, especially safe haven, a negative correlation would be expected with the rest of the elements.

results for copper are presented for the reasons explained in **Section 4.3**. Moreover, the abbreviations following the commodity name, i.e. SC, MC, LC, and T, refer to the short-, medium-, long cyclical, and trend components of the time series.

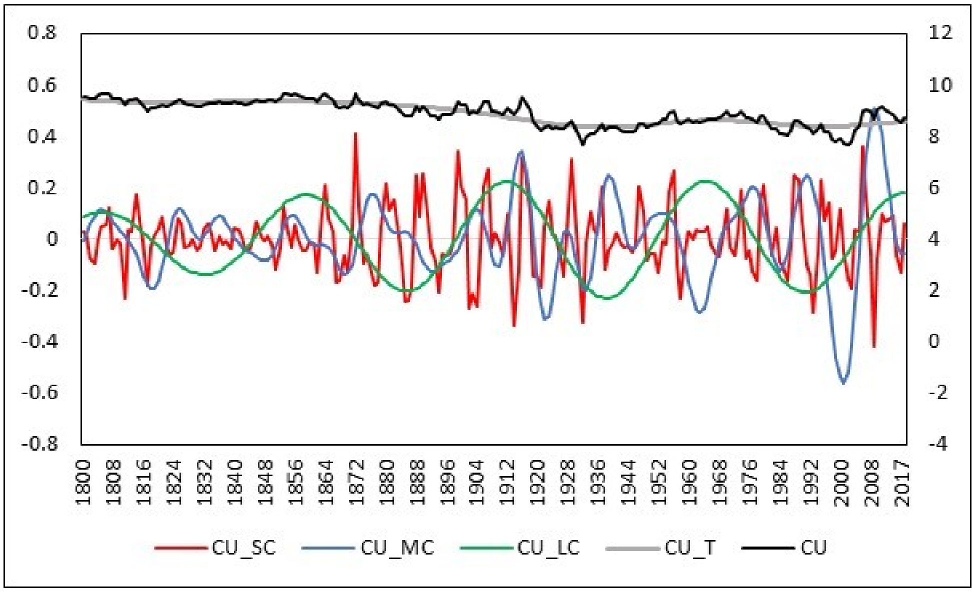



**Figure 4.5:** Copper real price and its cyclical components and trend (log scale).

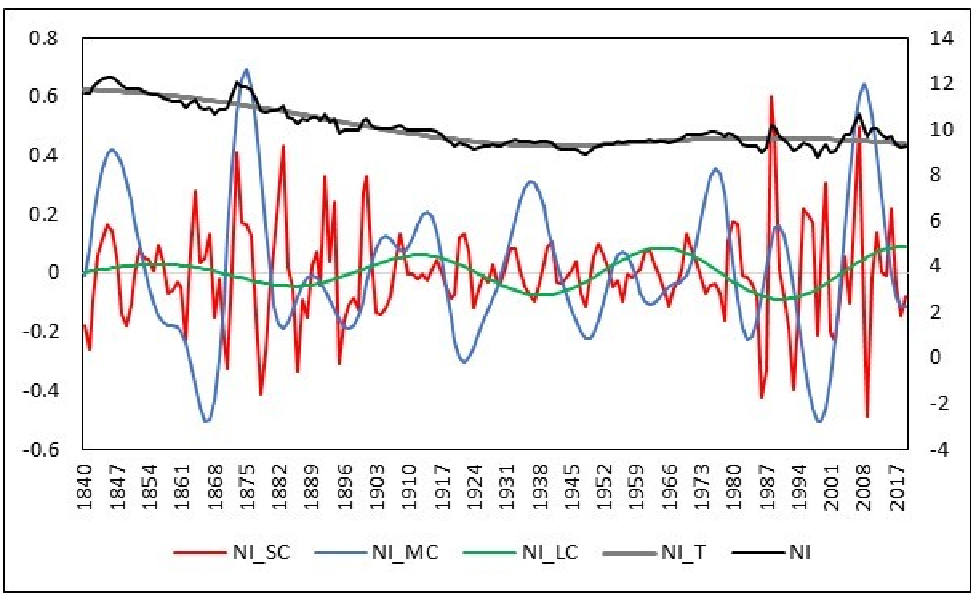



**Figure 4.6:** Nickel real price and its cyclical components and trend (log scale).

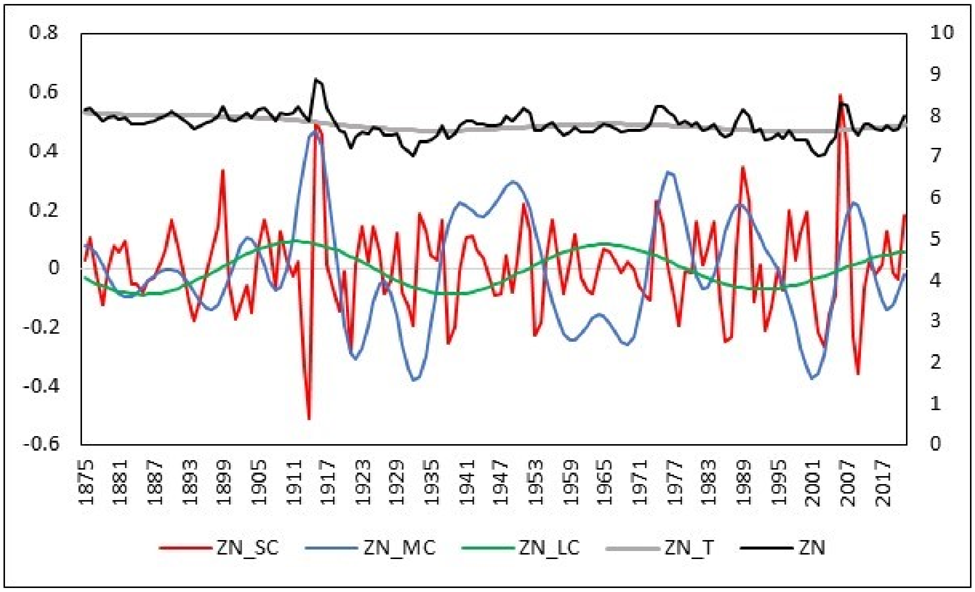



**Figure 4.7:** Zinc real price and its cyclical components and trend (log scale).

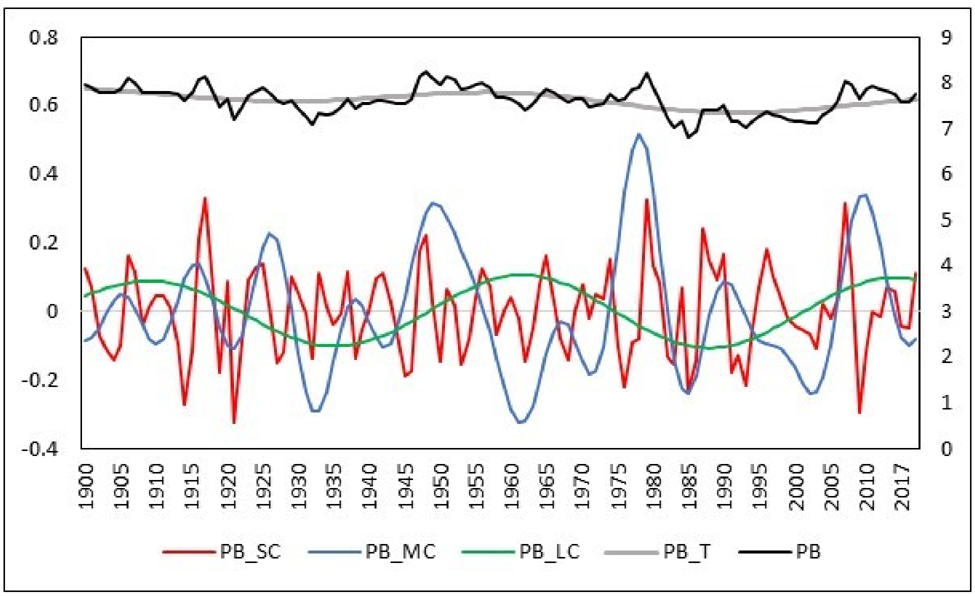



**Figure 4.8:** Lead real price and its cyclical components and trend (log scale).

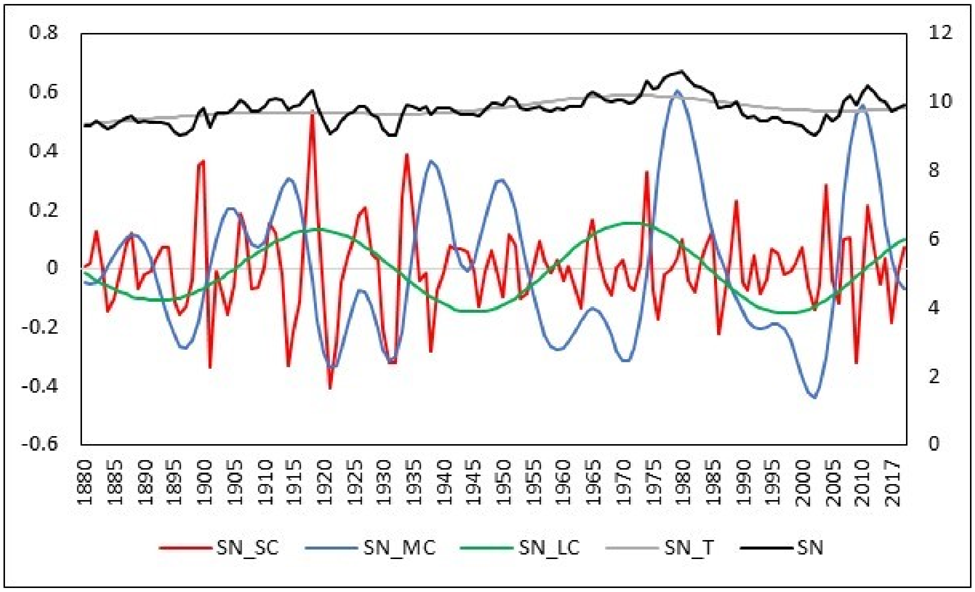



**Figure 4.9:** Tin real price and its cyclical components and trend (log scale).

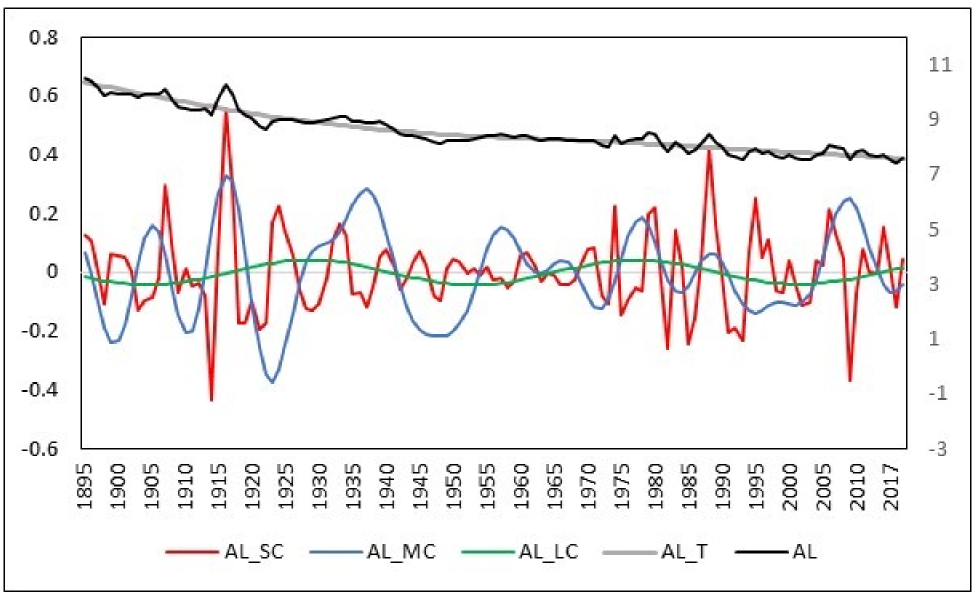



**Figure 4.10:** Aluminum real price and its cyclical components and trend (log scale).

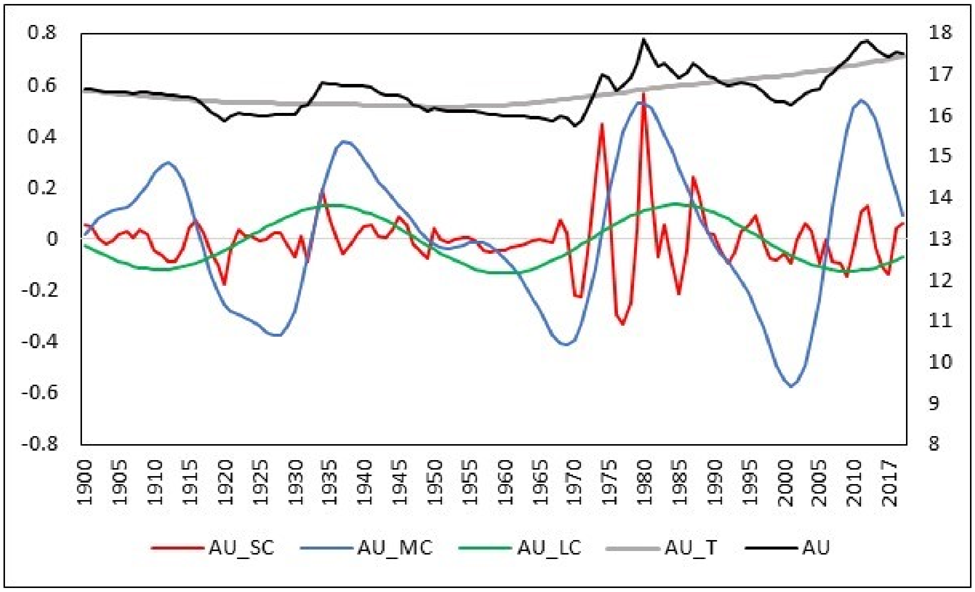



**Figure 4.11:** Gold real price and its cyclical components and trend (log scale).

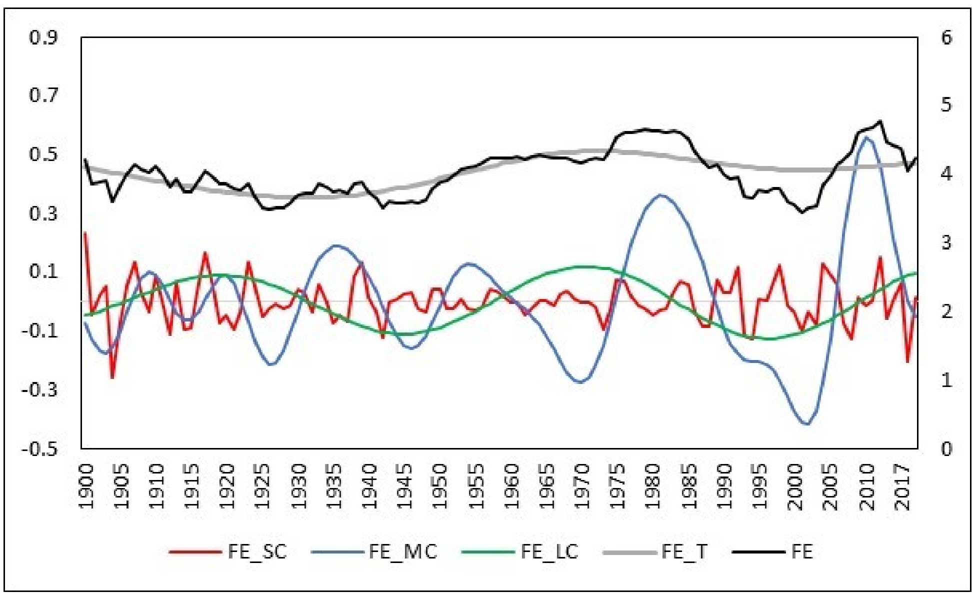



**Figure 4.12:** Iron ore real price and its cyclical components and trend (log scale).

**Figure 4.13** and **Figure 4.14** show the long cycles for base metals and gold and iron ore, respectively. It can be observed that in all cases there are clear cycles of about 45-60 years. Analyzing the results as a whole, the long cyclical components have typically a length of 49 to 60 years, with an average of 53 years for the raw material (base metals and iron ore) and 50 years for gold. A larger correlation is seen in a subset of elements. As shown in **Figure 4.15**, the long cycles of copper, nickel, zinc, lead, and gold have a high correlation, while lower between this subgroup and aluminum, iron ore and tin.

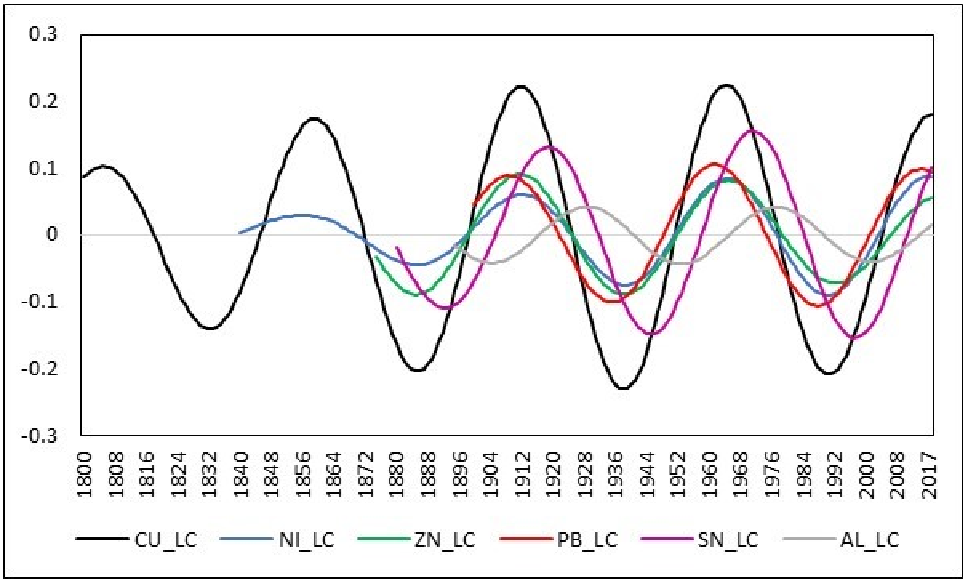



**Figure 4.13:** Long cycles components for base metals (log scale).

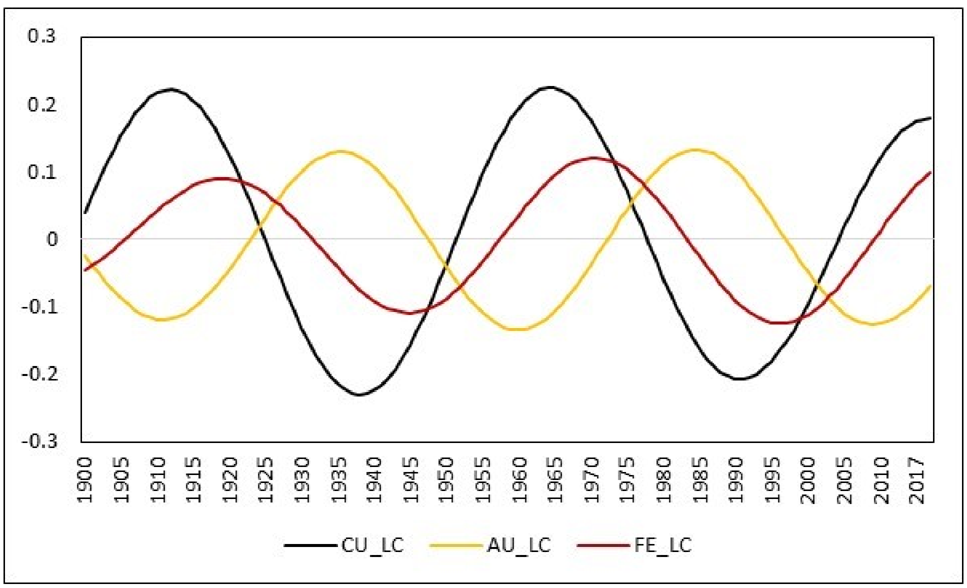



**Figure 4.14:** Long cycles components for copper, iron ore and gold (log scale).

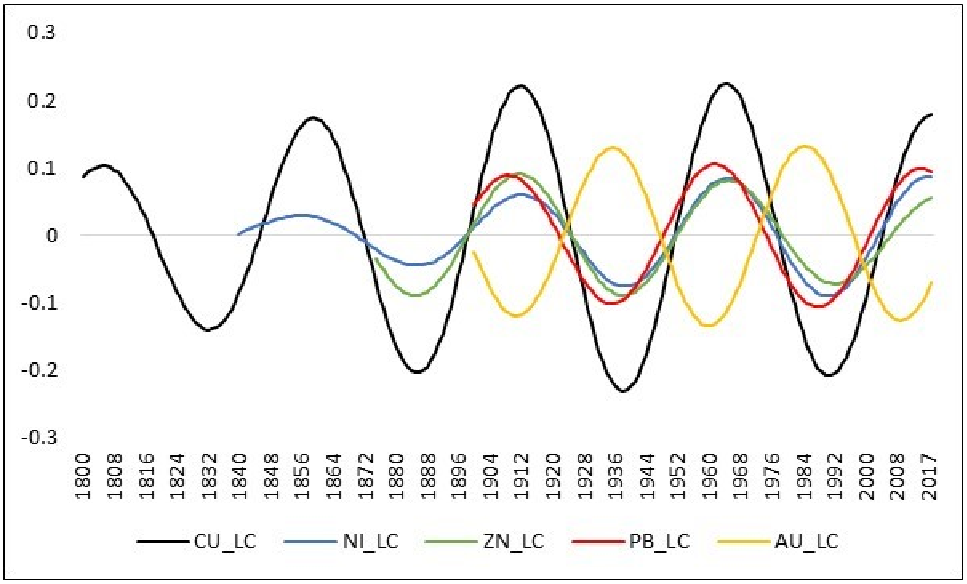



**Figure 4.15:** Highly correlated long cycles components (log scale).

By contrasting the Kondratiev cycles of **Table 4.1** with the cycles of **Figure 4.15**, **Figure 4.16** is obtained. This image is very illustrative to clearly confirm the close correlation between Kondratiev waves and long cycles in real metals prices. As known, the correlation does not mean causality, but the high level of synchronization is clearly a matter of interest. This topic will be further discussed.

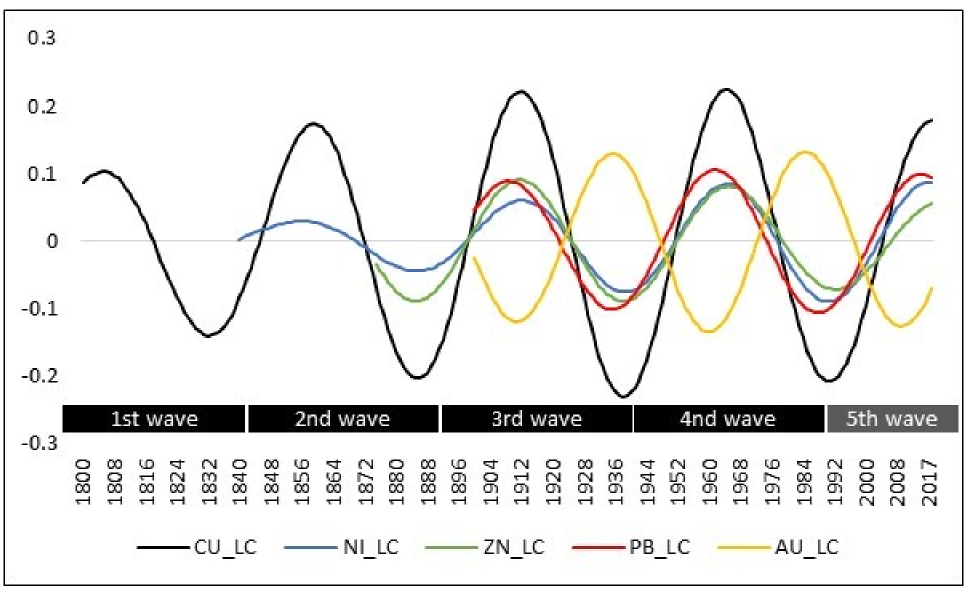



**Figure 4.16:** Contrast between highly correlated long cycles and the Kondratiev waves (log scale). (Grey color in the 5th wave means on development)

It is worth mentioning that the safe haven characteristic of gold would be validated for the long-term through the high negative correlation with most of the base metals (**Figure 4.15**). On the other hand, as shown in **Figure 4.17,** the decoupling between the price of aluminum and copper is striking. One hypothesis is the substitute dynamic between them, which in practice can be more valid for longer periods than in short cycles; being reflected in this phase shift in their long cyclical components.

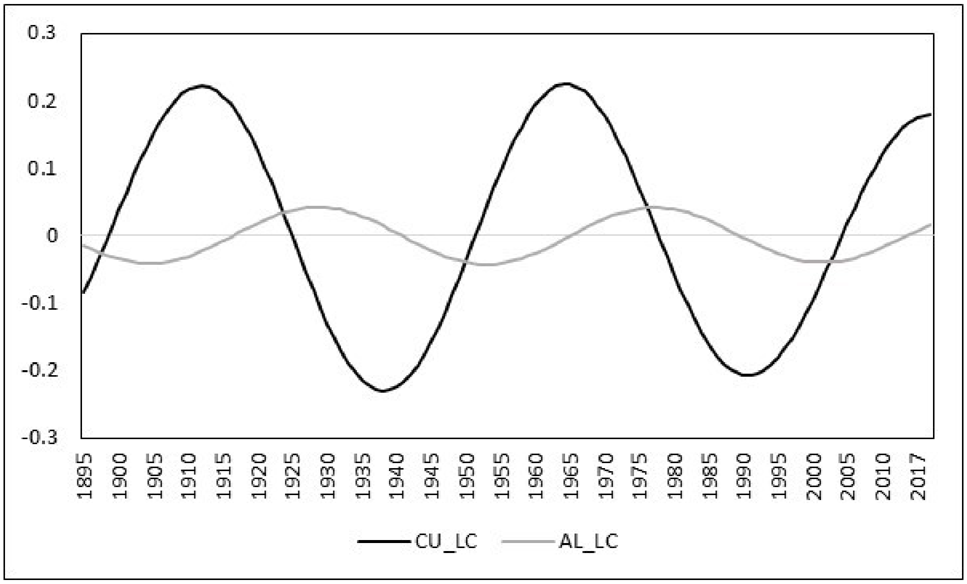



**Figure 4.17:** Cyclic phase shift between copper and aluminum (log scale).

### 4.4.2. Higher frequency cycles

**Figure 4.18** and **Figure 4.19** show medium cycles components for base metals and gold and iron ore, respectively. The results confirm medium-term cycles with periods of approximately no more than 25 years. Indeed, for the base metals and iron ore, the medium cyclical components have typically a length of 11 to 25 years, with an average of 15 years. In contrast, the gold case presents duration much larger of typically 23 to 31 years, with an average of 28 years. **Figure 4.20**[20] contrasts the cycles identified in the work of Erten and Ocampo (2012) and Rossen (2015) with the medium cyclical components. Although the medium cyclical component for gold matches well the cycles identified

[20] The Figure 8 presents the results just for copper, nickel, zinc and gold to favor the visualization. However, a conclusion from these figures is made taking into account the full sample.

by those researchers, the same level of consistency is not observed for the rest. Furthermore, it is important to mention that although co-movement is seen, the correlation apparently is less strong than in the long cyclical components.

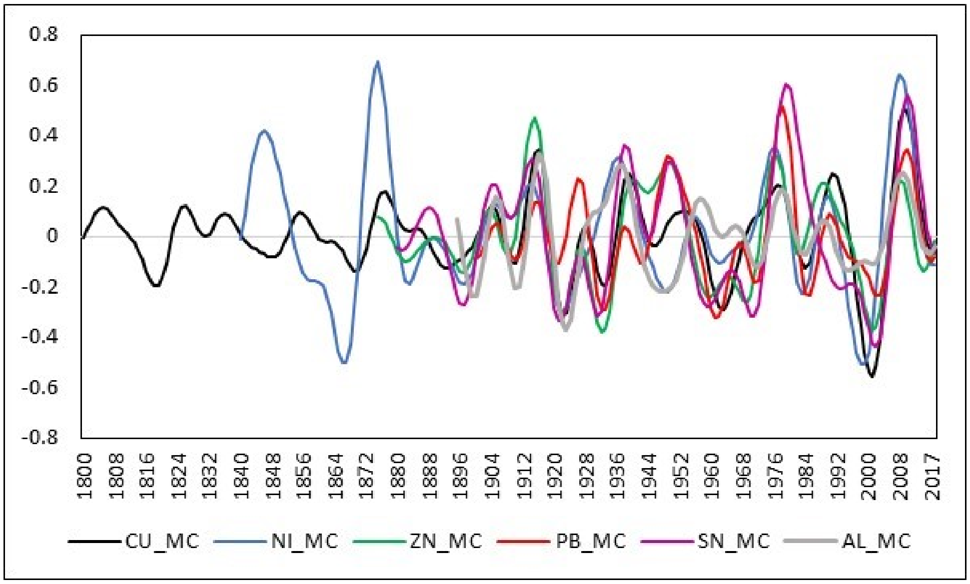



**Figure 4.18:** Medium cycles components for base metals (log scale).

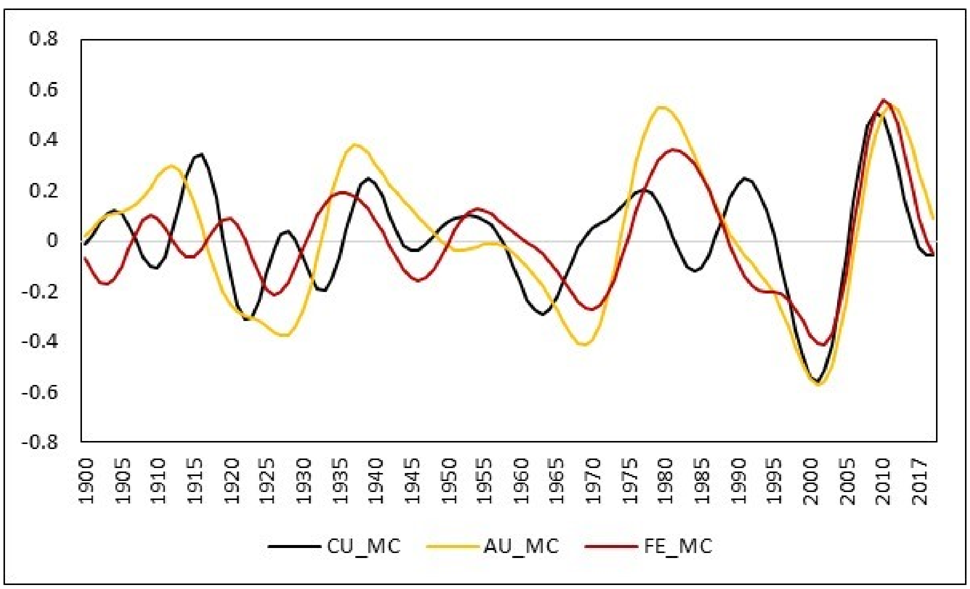



**Figure 4.19:** Medium cycles components for gold and iron ore (log scale).

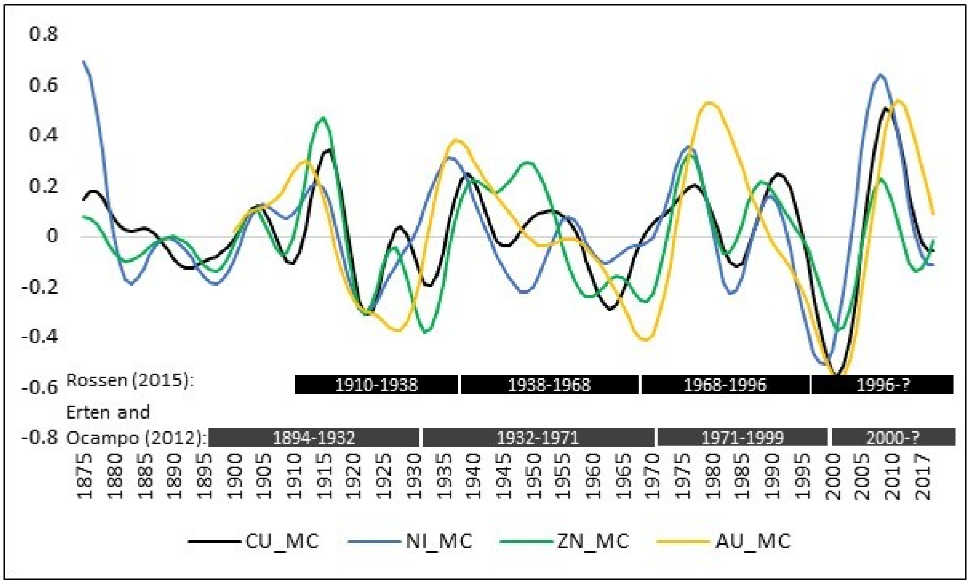



**Figure 4.20:** Contrast between proposed cycles and medium cycles component (log scale). (Black labels refer to cycles proposed in Rossen (2015), while grey by Erten and Ocampo (2012))

**Figure 4.21** and **Figure 4.22** show the short-term cyclical components for base metals and gold and iron ore, respectively. Analyzing the results as a whole, the short cyclical components have typically a length of 2 to 9 years, with a slightly different average between raw material prices (base metals and iron ore) and gold of 4 and 5 years, respectively. Note that this cyclical component could be aligned with the business cycles of modern occidental and the Chinese economies; the main source of raw material consumption in the last 200 years. Although there is still a correlation in specific periods, this relationship begins to be much more elusive than the previous cases. **Figure 4.23** shows the short cycles for the three most-traded metals, along with gold. Although there are several highly correlated peaks, the co-movement is apparently weaker than in the low-frequency components. Therefore, the lower degree of co-movement could indicate that the particularities of each market weigh importantly in the price formation.

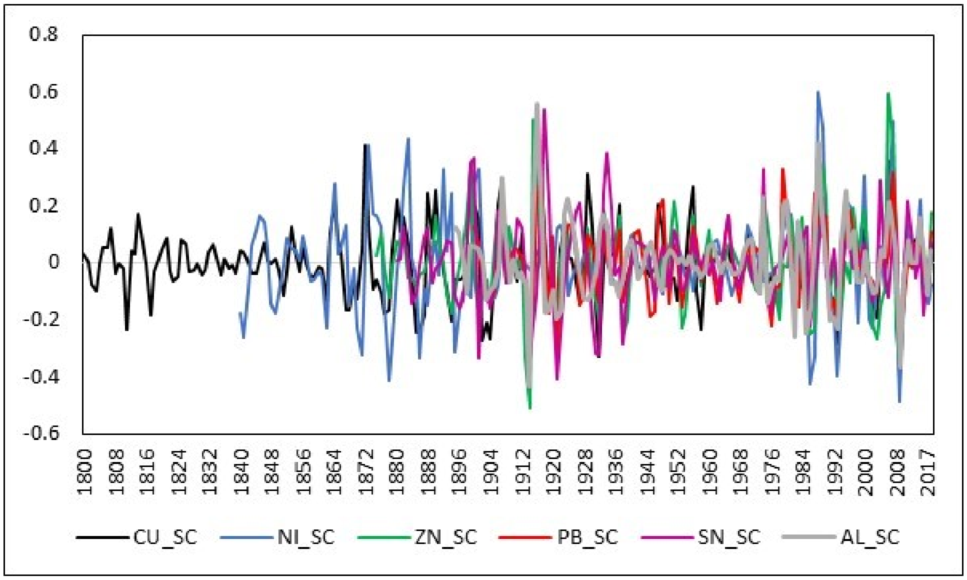



**Figure 4.21**: Short cycles components for base metals (log scale).

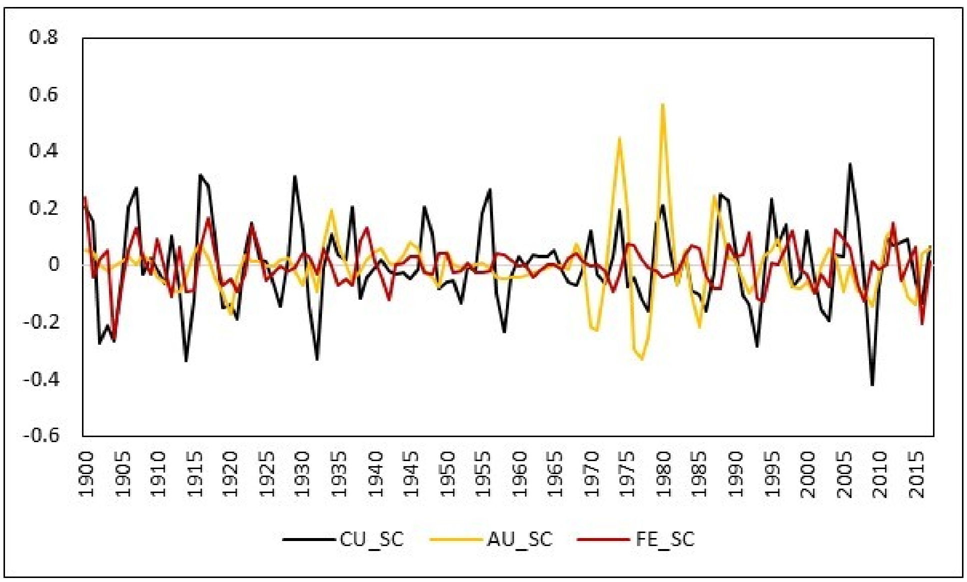



**Figure 4.22:** Short cycles components for gold and iron ore (log scale).

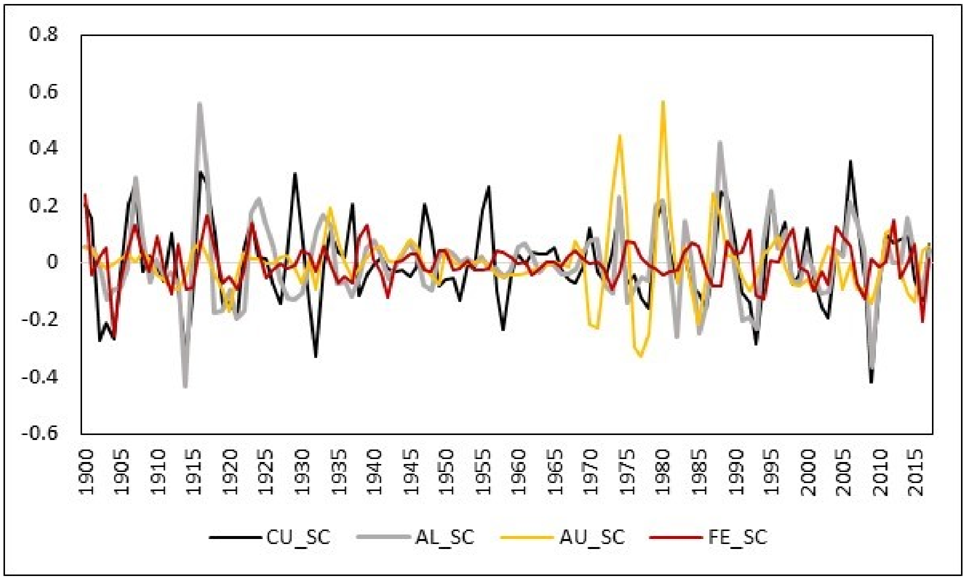



**Figure 4.23:** Weaker correlation in short cycles component of Cu, Al, Au and Iron Ore (log scale).

The statistical characteristics of the cyclical components are summarized in **Table 4.4**.

| Metal | Statistic | Prices Cyclical Components | | |
|---|---|---|---|---|
| | | LC | MC | SC |
| CU | No.[21] | 4 | 13 | 45 |
| | Ave. Length[22] | 53 | 16 | 5 |
| | SD[23] | 1 | 4 | 2 |
| NI | No. | 3 | 11 | 33 |
| | Ave. Length | 55 | 16 | 5 |
| | SD | 5 | 4 | 2 |
| ZN | No. | 2 | 10 | 34 |
| | Ave. Length | 54 | 13 | 4 |
| | SD | 1 | 3 | 2 |
| AL | No. | 2 | 7 | 30 |
| | Ave. Length | 49 | 15 | 4 |
| | SD | 1 | 5 | 2 |
| PB | No. | 2 | 9 | 25 |
| | Ave. Length | 53 | 13 | 4 |
| | SD | 1 | 3 | 2 |
| SN | No. | 2 | 9 | 30 |
| | Ave. Length | 52 | 14 | 5 |
| | SD | 0 | 2 | 2 |
| AU | No. | 2 | 9 | 30 |
| | Ave. Length | 50 | 28 | 5 |
| | SD | 1 | 3 | 2 |
| FE | No. | 2 | 9 | 30 |
| | Ave. Length | 52 | 17 | 4 |
| | SD | 0 | 6 | 2 |
| Raw Material[24] | Ave. Length | 53 | 15 | 4 |
| Gold | Ave. Length | 50 | 28 | 5 |

**Table 4.4:** Characterization of the cyclical components.

To answer the question about how relevant each of these components is in the price variation, it is studied the approximated range of variation of each cyclical component with respect to the trend, shown in **Table 4.5**.

[21] Number of cycles identified over the sample.
[22] Average cycles length in years.
[23] Standard deviation in years.
[24] Base Metals plus Iron Ore.

| Commodity | Deviation Sign | LC | MC | SC |
|---|---|---|---|---|
| CU | Negative | 23% | 56% | 42% |
| | Positive | 22% | 51% | 41% |
| NI | Negative | 9% | 51% | 49% |
| | Positive | 9% | 69% | 60% |
| ZN | Negative | 9% | 38% | 51% |
| | Positive | 9% | 47% | 59% |
| PB | Negative | 11% | 32% | 32% |
| | Positive | 11% | 52% | 33% |
| SN | Negative | 15% | 44% | 41% |
| | Positive | 16% | 61% | 54% |
| AL | Negative | 4% | 37% | 44% |
| | Positive | 4% | 33% | 56% |
| AU | Negative | 13% | 57% | 33% |
| | Positive | 13% | 54% | 57% |
| FE | Negative | 12% | 41% | 26% |
| | Positive | 12% | 56% | 24% |

**Table 4.5:** Deviations of the Cyclical Components from the Trends.

The short and medium cycles are the most important in explaining the deviations of the price with respect to the trend, with ranges typically of about -40% to +50% in the cycles trough/crest. However, the long cycles are far from being negligible, explaining about 5% to 20% (depending on the case) of the variation with respect to the trend. Note that the long cycles component is especially important for copper, tin, gold, and iron ore, while less relevant in aluminum, nickel, and zinc. Therefore, although prices deviations from their trend are considerably affected by their long cycles components, the impact of the higher frequency components are of more relevancy as expected.

### 4.4.3. Co-movement in different cyclical components

The Pearson correlation coefficient (considering all available data pairs) and the principal component analysis (PCA, considering a balanced sample) results are shown in **Tables 4.6-8** and **Table 4.9**, respectively.

| | |
|---|---|
| | Medium to high positive correlation (>0.4) |
| | Medium to high negative correlation (<-0.4) |

| | CU_LC | NI_LC | ZN_LC | PB_LC | SN_LC | AL_LC | AU_LC | FE_LC |
|---|---|---|---|---|---|---|---|---|
| CU_LC | 1.00 | | | | | | | |
| NI_LC | 0.95 | 1.00 | | | | | | |
| ZN_LC | 0.99 | 0.93 | 1.00 | | | | | |
| PB_LC | 0.93 | 0.95 | 0.88 | 1.00 | | | | |
| SN_LC | 0.70 | 0.66 | 0.73 | 0.39 | 1.00 | | | |
| AL_LC | -0.18 | -0.16 | -0.13 | -0.47 | 0.58 | 1.00 | | |
| AU_LC | -0.84 | -0.85 | -0.78 | -0.95 | -0.21 | 0.62 | 1.00 | |
| FE_LC | 0.73 | 0.72 | 0.76 | 0.47 | 0.99 | 0.53 | -0.29 | 1.00 |

**Table 4.6:** Correlation across the long cyclical components.

| | CU_MC | NI_MC | ZN_MC | PB_MC | SN_MC | AL_MC | AU_MC | FE_MC |
|---|---|---|---|---|---|---|---|---|
| CU_MC | 1.00 | | | | | | | |
| NI_MC | 0.57 | 1.00 | | | | | | |
| ZN_MC | 0.73 | 0.41 | 1.00 | | | | | |
| PB_MC | 0.64 | 0.33 | 0.65 | 1.00 | | | | |
| SN_MC | 0.66 | 0.52 | 0.70 | 0.71 | 1.00 | | | |
| AL_MC | 0.51 | 0.68 | 0.22 | 0.16 | 0.39 | 1.00 | | |
| AU_MC | 0.60 | 0.56 | 0.55 | 0.41 | 0.86 | 0.40 | 1.00 | |
| FE_MC | 0.49 | 0.52 | 0.25 | 0.35 | 0.70 | 0.46 | 0.82 | 1.00 |

**Table 4.7:** Correlation across the medium cyclical components.

| | CU_SC | NI_SC | ZN_SC | PB_SC | SN_SC | AL_SC | AU_SC | FE_SC |
|---|---|---|---|---|---|---|---|---|
| CU_SC | 1.00 | | | | | | | |
| NI_SC | 0.40 | 1.00 | | | | | | |
| ZN_SC | 0.59 | 0.33 | 1.00 | | | | | |
| PB_SC | 0.69 | 0.31 | 0.49 | 1.00 | | | | |
| SN_SC | 0.42 | 0.16 | 0.34 | 0.45 | 1.00 | | | |
| AL_SC | 0.59 | 0.49 | 0.49 | 0.57 | 0.27 | 1.00 | | |
| AU_SC | 0.32 | 0.19 | 0.18 | 0.27 | 0.22 | 0.35 | 1.00 | |
| FE_SC | 0.24 | 0.14 | 0.12 | 0.11 | 0.20 | 0.24 | -0.07 | 1.00 |

**Table 4.8:** Correlation across the short cyclical components.

| Cycle | Group | Number | Value | Difference | Proportion | Cumulative Value | Cumulative Proportion |
|---|---|---|---|---|---|---|---|
| LC | 8 metals | 1 | 5.58 | 3.29 | 0.70 | 5.58 | 70% |
| | Base Metals | 1 | 4.32 | 2.73 | 0.72 | 4.32 | 72% |
| | Cu & Au | 1 | 1.84 | 1.68 | 0.92 | 1.84 | 92% |
| MC | 8 metals | 1 | 4.82 | 3.57 | 0.60 | 4.82 | 60% |
| | Base Metals | 1 | 3.72 | 2.54 | 0.62 | 3.72 | 62% |
| | Cu & Au | 1 | 1.60 | 1.21 | 0.80 | 1.60 | 80% |
| SC | 8 metals | 1 | 3.55 | 2.48 | 0.44 | 3.55 | 44% |
| | Base Metals | 1 | 3.33 | 2.41 | 0.55 | 3.33 | 55% |
| | Cu & Au | 1 | 1.32 | 0.65 | 0.66 | 1.32 | 66% |
| Principal Components Analysis, extracting 8/6/2 of 8/6/2 possible components, respectively, and showing only the first principal component, balanced sample (listwise missing value deletion), included observations: 118 after adjustments. | | | | | | | |

**Table 4.9:** Principal Component Analysis for long, medium and short cyclical components.

In the case of long cycles, there is good synchronization observed in the high correlation coefficients. In addition, the negative relationship with gold and the shift phase between aluminum and the rest is validated. The idea of a long-term substitution effect between copper and aluminum may have more empirical support when observing the small and negative correlation between them. Regarding the PCA, if the main component is considered as cyclicity factor (as indicated by Cuddington and Jerret, 2008), this would explain 70% of the joint co-variance for the 8 metals, while 72% only considering the base metals. This reaffirms the strong correlation in long-term cyclicality and allows for thinking of a common factor behind its causality. Similar results are found in Cuddington and Jerret, 2008, and Cuddington et al., 2015; although with a different decomposition for long cycle components.

In the results for the medium and short cycles, it is seen how correlation begins to decrease as the frequency increases. Note that, while gold was negatively correlated with the rest of commodities for long cycles components, this relationship turns to be positive for the medium cycles components and almost null for the short cycles components. Thus, one could say that for metal commodities investors, gold could become relevant hedging mean in the long term (given the opposite movements), while its hedging characteristics vanish as frequency increases, but still providing diversification potential. The gold different behavior in comparison with the other metals is related to the fact that its price is much more susceptible to macroeconomic and supply conditions. This is even more evident from the beginning of the 1970's; period related with the agony and eventually the end of the Gold standard in the US, and the rest of the superpowers. On the other hand, the phase shift between aluminum and the rest is no longer observed.

The latter point suggests that as the frequency of cyclicity increases, the co-movement is less strong. This finding is relatively counterintuitive in the sense that one would expect a high correlation in the short term (Fernandez, 2015). To illustrate this point, **Table 4.10** shows the correlations in the extreme case of base metals prices in daily-basis for 2017, corroborating the above for the case of copper, nickel, zinc, lead, and aluminum. Hence, the co-movement could be considered as stronger in the more extreme cases.

| | CU | NI | ZN | PB | SN | AL |
|---|---|---|---|---|---|---|
| CU | 1.00 | | | | | |
| NI | 0.89 | 1.00 | | | | |
| ZN | 0.94 | 0.86 | 1.00 | | | |
| PB | 0.83 | 0.74 | 0.91 | 1.00 | | |
| SN | 0.01 | -0.03 | 0.04 | -0.08 | 1.00 | |
| AL | 0.87 | 0.71 | 0.84 | 0.71 | 0.06 | 1.00 |

| | |
|---|---|
| (dark grey) | Medium to high positive correlation (>0.4) |
| (light grey) | Medium to high negative correlation (<-0.4) |

**Table 4.10**: Correlation of the London Metal Exchange daily prices for Base Metals.

### 4.4.4. Trend Analysis

Once cyclical components are removed from metals prices, one gets the trends. **Figure 4.24** and **Figure 4.25** show the resulting trends for base metals and gold and iron ore, respectively. The classical view of the mineral economy suggests that, in the very long term, real prices should follow a constant trend, given that supply is capable of adapting. This adaptation is probably through the discovery of competitive resources and/or technological advances that compensate the expected cost increase caused by the depletion of the "non-renewable" resources. As seen in **Figure 4.24** and **Figure 4.25**, until the beginning of the twentieth century, the trends had a negative slope (in most of the cases). That is, either by technology or by the startup of new competitive reserves, the industry was capable of more than compensate the losses coming from depletion. Nevertheless, the trends stabilize from around the 1920/30s for most of the metals evaluated. This observation is quite aligned with Cuddington and Jerrett (2008) and Deverell and Turner (2017), who detected the flattering from the 1920s for several base metals.

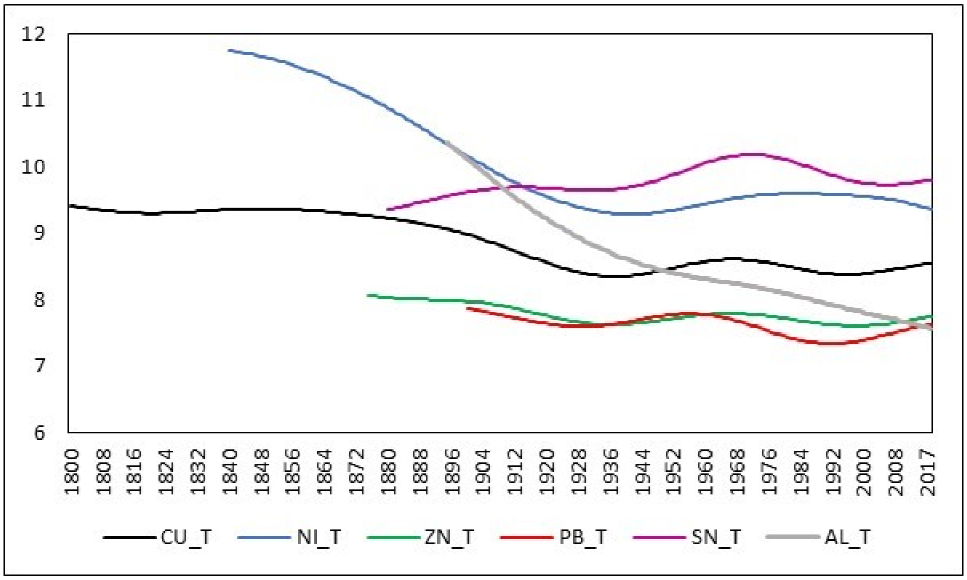



**Figure 4.24:** Trend components for the base metals (log scale).

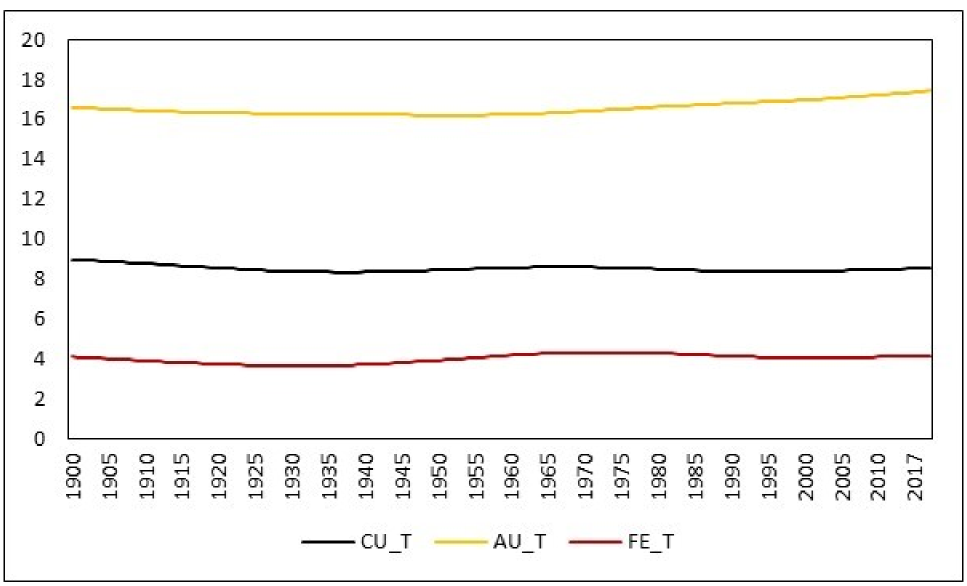



**Figure 4.25:** Trend components for copper, gold and iron ore (log scale).

**Figures 4.26-28** show the trends, in real price, for all cases between 1930-2017[25]. As it can be observed, in the case of base metals, the hypothesis of trendless prices is more or less fulfilled[26], except in the case of aluminum, where the trend has been strongly downward; probably due to a combination of abundant new resources, the increase in recycling, reduction of electricity cost and/or improvements in casting techniques[27].

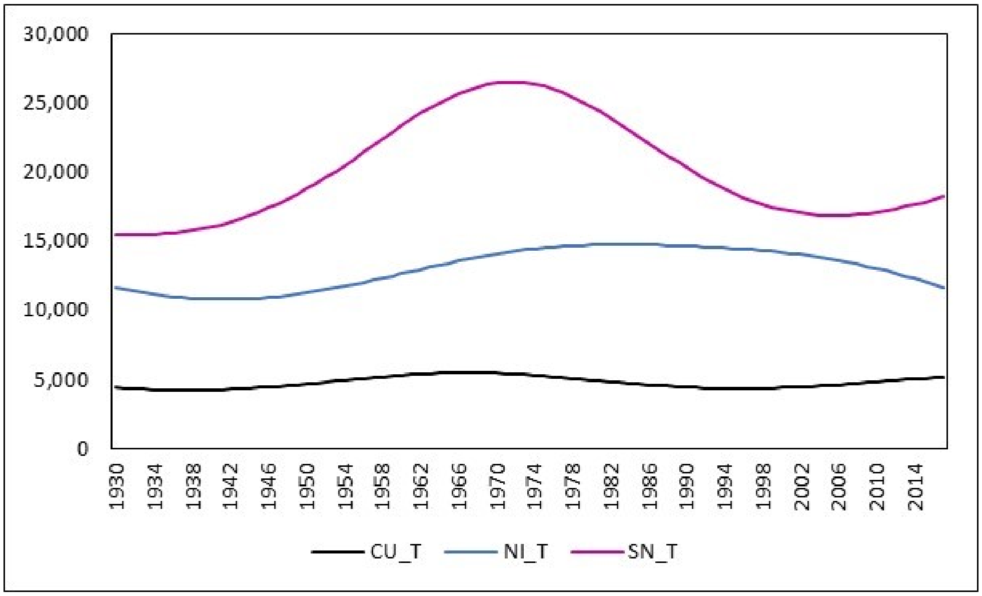



**Figure 4.26:** Copper, nickel and tin trends in real prices (US$2017/t).

[25] The trends are shown in real prices (US$2017/t) for the six base metals, gold and iron ore. For purpose of scale and visualization, the metals prices have been split in 3 figures.
[26] As a note of caution, it is valuable to comment on the discussion about the impact of the deflator in the identification of trends in real prices. For more details see Svedberg and Tilton (2006) and Fernandez (2012).
[27] Considering that world's consumption for aluminum has accelerated in the last 2 decades.

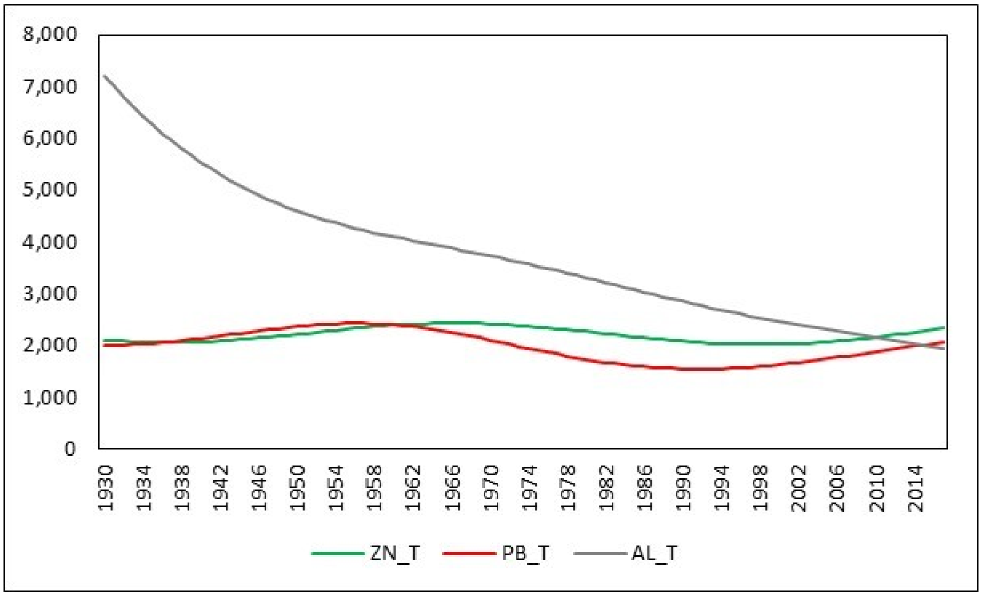



**Figure 4.27:** Zinc, lead and aluminum trends in real prices (US$2017/t).

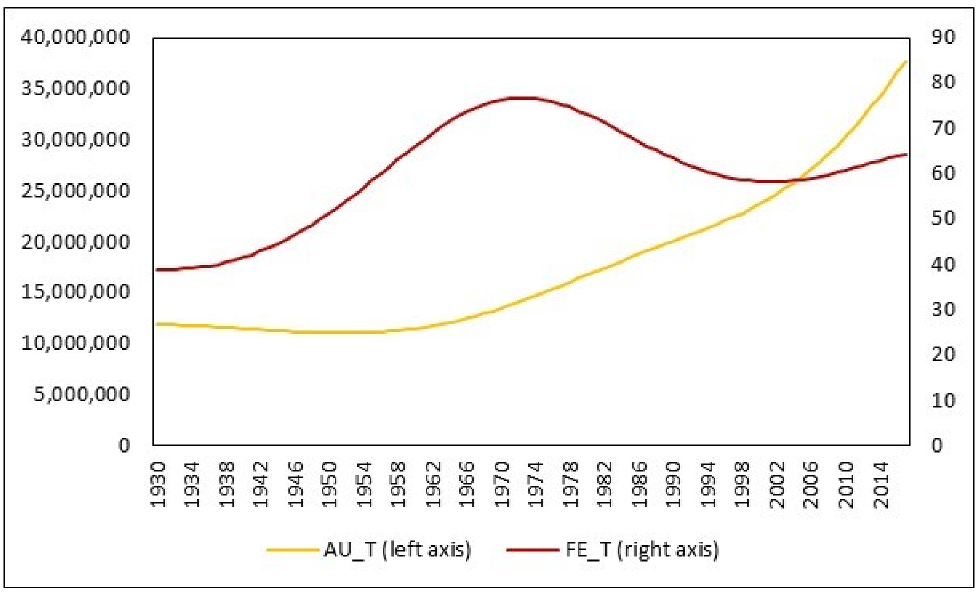



**Figure 4.28:** Gold and iron ore trends in real prices (US$2017/t).

In the particular case of gold, this follows an increasing trend (and accelerating), indicative of the harder conditions to obtain it. This is what partially explains that more than 50% of the world's exploration expenditures are on gold campaigns (S&P Global Market Intelligence, 2018); and the number is even higher if it is measured by the number of prospects. In the case of iron ore[28], the flattening of the trend occurs rather in the 1960s.

Therefore, the results suggest that, first, what is understood as a very long-term trend could be defined by horizons of over 60-70 years. Second, the presented evidence is in favor of the mineral economics classical view, at least for copper, nickel, zinc, lead, and tin, starting in 1930, and for iron ore, from 1960. Thus, long cycles should be more associated with changes in supply, and therefore, understanding the dynamics of these changes is key. Note that in most long cyclical components analyzed are highly correlated, suggesting that for metals with trendless prices, the changes in the supply might occur rather in a synchronized manner.

### 4.4.5. Long Cycles Drivers: Technology, Resources Availability or Both?

Up to this point, the evaluation allowed to identify highly correlated cyclical components of 48-60 years for most of the assessed real metals prices. Moreover, these long cycles components demonstrated to be well synchronized with the Kondratiev's long economic cycles theory. Therefore, the next step is to make sense of what might be behind these long cycles and whether this economic theory can help for a better understanding of the long cycles in metal commodities.

First, it is necessary to recognize that a slow unfolding global phenomenon is necessary to trigger these long, steady and synchronized up-and-down episodes in metals prices. Moreover, if it is assumed that real prices are trendless in the very long-term as a consequence of the supply's adaptation to long-term demand, then one should expect that long cyclical components are supply-side related; reflecting the restoration of the long-term equilibrium.

In this adaptative long-term process, the penetration of technological innovation in the production and commercialization processes is one of the possible answers of how the mining industry could be

[28] Highly correlated with the trend of tin.

able to bring the depletion-related long-term costs increases down[29]. This path could be more or less in line with the "cluster innovation" hypothesis. A problem with this is that if this is the case, then Kondratiev waves and long cycles components in metals should show a shift phase: the rise of a new innovative revolution triggers a new long economic cycle, and at the same time the incorporation of these innovations in the mining industry drives cost and prices down. This is not what is observed in the results of **Section 4.4.1** (recall **Figure 4.16**). Although, in defense of the synchronization seen one might argue that this could be related with the well-known fact that the mining business (especially the purely extractive activity) has fewer incentives for innovation, and therefore is a late adopter when compared to other industries (Andai, 2017).

Even if the latter is true, it is unlikely that this factor alone may trigger such a long period of decreasing costs. Otherwise, how it could be explained, for instance, that industry such as gold and copper have such diverging price trends, considering the overall similar extractive technologies.

Thus, these technological improvements must be complemented with the entry into production of an important amount of "highly" competitive projects, which, giving the high correlation of long cyclical components in four of the six base metals, may occur in a "synchronized" manner. Nevertheless, for this scenario to occur, it is required the existence of such a competitive portfolio of projects[30]. At this point is when the cyclical behavior of the exploration activity can be an important factor. Knowing the cyclical behavior of prices, it would be expected countercyclical levels of exploration activity. Nevertheless, exploration expenditure has proven to be quite correlated with the metals prices (S&P Global Market Intelligence, 2018), reflecting an investors' risk aversion approach toward this activity.

Hence, when the industry faces a long period of increasing prices, exploration activity is not only boosted (Jacks, 2013), but it would be successful in identifying new resources given the previous

[29] It also applies to other productive value chain cost increases, such as transport costs.
[30] This aspect is often missed when reviewing the literature on long cycles in metals prices, but it may be a quite differentiating factor among metals and minerals markets.

results. However, from discovery to production is a long process that can typically take up to 12-15 years (Gandhi and Sarkar, 2016), and therefore it is likely that in an extended upward trend in prices, this "cluster discoveries" are building up the future project portfolio. Eventually, the economic conditions are given for these competitive deposits become operations (favored for a portfolio in good shape and technological innovations) more than correcting the deviation from the long term. While this situation could be the long-term dynamic in most of the base metals, it may not be the case for gold, were the price follows a clear upward trend, or for aluminum, which presents the opposite situation.

Of course, this explanation is very simplistic and has only focused on long-term cycles. However, the purpose is to strengthen the hypothesis that very long cyclical components are defined by changes in supply, rather than in the demand. Following this logic, medium cycles are expected to be more responsive to changes in both supply and demand, and short cycles to changes mostly in demand.

An interrogative that is still unsolved is what could explain the high level of synchronization between most of the base metals and gold with the theoretical Kondratiev wave. Note that although Kondratiev waves are more associated with innovation cycles, this, in turn, could be related to major and sequential macroeconomic changes: the industrial revolutions in England and Western Europe (1760-1840), the post-civil war reconstruction and industrialization in the United States (1860-1900), Progressive Era in the United States and World Wars (1900-1945), the post Second World War "re-urbanization and re-industrialization" of Europe and Japan and the Cold War (1945-1990), and the take-off of the Asian economic development, led by China (1985-present).

The episodes mentioned took decades to unfold and are typically characterized by sequential stages of urbanization and industrialization[31], trade and logistics expansion, and continuous innovation and technological development. All these stages are intensive in raw material consumption, especially of

[31] In some cases, the most accurate description is one of re-urbanization and re-industrialization.

metals and minerals widely used in the modern economy, which might explain the high level of synchronization across most of the base metals long cycles and with the Kondratiev waves. On the other hand, over this period of analysis, technological innovation has been a key factor in the "battles" between an emerging and dominant economy[32].

As it has been shown, there is a likely tight relationship between the long economic cycle and the long cyclical component observed in the metal prices. Using the same logic, one may establish a connection between metal prices short and medium cyclical components and economic cycles that must be further study. **Table 4.11** summarized the potential link between the cyclical components of the metal prices and the economic cycles, their drivers, and how they could shape the metal commodity markets. Note that the table does not imply that these economic cycles are the only reason behind the fluctuation in metal prices, but rather the potential connection and impact to the metal price formation process.

| Cyclical Component | Economic Cycle Potentially Influencer | Economic Cycles Drivers | Influence on the Metal Markets |
|---|---|---|---|
| SC | Business cycles (2-8 years), Kitchin cycles (3-5 years), Juglar cycles (7-11 years) | Changes in fiscal and monetary policies, as well as fixed investment in major economies | Mostly changes in demand, with a limited response from the supply side. |
| MC | Kuznets swings (15-25 years) | Urbanization, infrastructure, and demographic transformation in large economies | Changes in the demand cause by major economic transformation and adaptation of the supply |
| LC | Kondratiev waves (45-60 years) | Innovation and disruptive technology cycles | Changes in the supply, potentially linked to changes on technology changes and/or mineral resources endowment |

**Table 4.11:** Potential link between metal prices cyclical components and economic cycles.

[32] Such as England vs. US (industrialization of US), the US vs. Eastern Europe/Japan (post Second World War), and US vs. China (currently). For more details, revise Allison (2017).

## 4.5. Conclusions

Applying the ACF on the real metals prices (base metals, iron ore, and gold) and using a novel cyclical decomposition (which considers the main findings of one of the most documented theories of long economic cycles), it was identified long cycles of 45-60 years, highly correlated in most cases. Although both the short- and medium-term cyclical components are more relevant in explaining the deviations of real prices from trends, the long-term cyclical component is not negligible.

Furthermore, long cycles in the prices of metals present a high level of coincidence with Kondratiev's theoretical waves; which are typically explained by the theory of innovation. However, the link between both cycles is still not evident. Considering that slow and steady changes in the supply side, perhaps connected to technological improvements, are more likely to explain the long cycles observed in metals prices, it is suggested to study how innovations flow into the metal industries and reshape the supply curve.

Through the study of the correlation and the PCA for the short, medium and long cyclical components, it was identified that the degree of co-movement varies with the cycles frequencies. Particularly, it can be highlighted a high degree of co-movement for most of the metals analyzed in long cycles, while as the frequency increases, the co-movement is less intense; although still high. One possible explanation is that short and medium cycles can be more susceptible to particularities of their industries and markets, while long cycles due to a more generalized phenomenon. At least two particular cases are important to highlight. First, the view of gold as a strong safe haven would be validated only for the long cycles. Second, the substitution dynamic between copper and aluminum it is supported for long cycles, but not for cyclical components of short and medium term.

The evaluation of the trend allows for validating the classic view of the mineral economics about trendless prices in the very long term. This result would be valid for copper, nickel, zinc, lead and tin from about the 1920/30s, and iron ore from the 1960s. The confirmation of this behavior in the very long term would suggest that long-term cyclicity are likely to be connected to changes in supply. Therefore, future efforts in long cycles in metals prices should focus on understanding the potential cyclical dynamic of the changes in the supply-side factors, such as cyclical behavior in exploration activity, the evolution of the project portfolios and how technological innovation reshape the long-term supply curve.

# 5. Evaluation of the Elliott Wave Principle

## 5.1. Introduction

The Wave Principle, known as the Elliott Wave Principle (EWP), is a theory widely used in technical analysis. It was developed by Ralph Nelson Elliott in the 1930s and reached great popularity during the 1930s and 1940s thanks to successful predictions of the United States (US) market. However, with Elliott's death in 1948, the theory remained unknown for several decades, until the 1980s, when regained great popularity again.

The theory claims that fluctuations in markets follow recognizable and repetitive patterns (called waves), caused by changes in mass psychology of market participants. Hence, one of the most important principles in Elliott's theory is that markets behave like waves that repeatedly unfold as a result of recurrent changes in the mass psychology of the markets' participants.

On the other hand, the effect of mass physiology has been pointed out long ago as an important factor to consider in any system exhibiting dynamic equilibrium through time (Smith, 1776; Skar, 2004). For the specific case of metal commodity markets, authors such as Simon (1959) and Tapia-Cortez et al. (2018) have clearly indicated that mass psychology is a factor that must be considered in metal prices modeling. Empirically, the business mood has shaped the decision-making process for investments in mining, either in projects or exploration. A clear example of the latter is the high correlation observed between the exploration budget, which should be driven by long-term drivers, and the metal prices (**Figure 5.1**). Moreover, given the increasing participation of investors and speculators in the last decades on the metal commodity markets, mass psychology could be even more important for the current metal commodity markets.

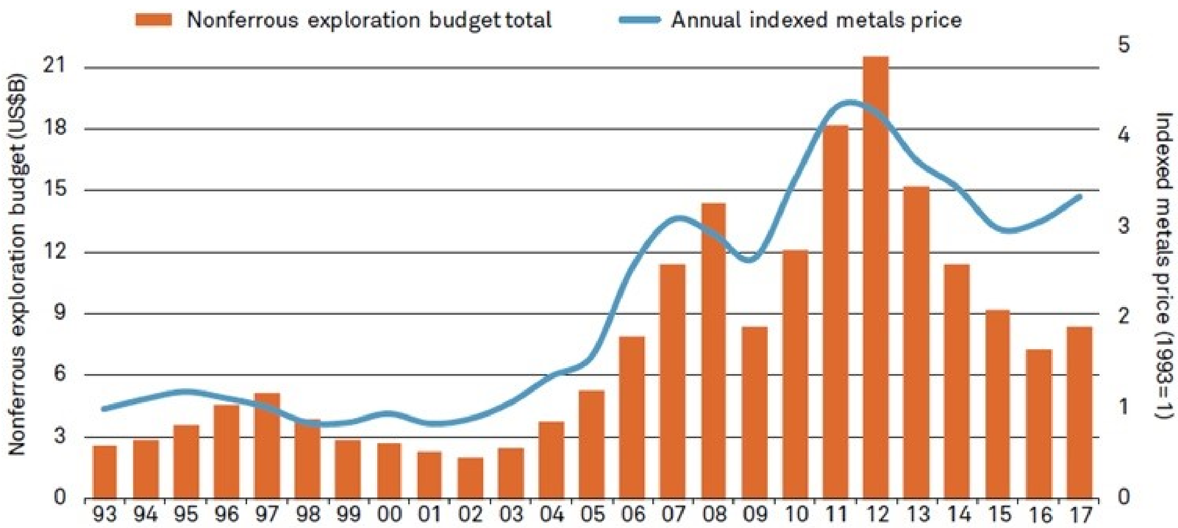


**Figure 5.1:** Cyclicality of prices and exploration budget.
**Source:** S&P Global Market Intelligence, 2018.

Therefore, the main objective of **Chapter 5** is to explore the EWP to evaluate whether it is a suitable approach for price modeling and there are aspects of the theory that could complement the understanding of the cyclical behavior of metal commodity markets.

## 5.2. Literature Review: The Elliott Wave Principles and Mass Psychology Affecting Metal Commodity Prices

In 1939 Elliott published "*The Wave Principle*", where he introduced the principles[33] that govern the markets, the patterns in which the market unfold, and the mass psychology associated with the market fluctuation phenomenon (Bolton, 1994). In 1946, Elliott added his wave principles to a more extensive work called "*Nature's Law: The Secret of the Universe*". In this work, he connected his wave theory with the use of the numbers and ratios of the Fibonacci series (FS) for the prediction of market trends (Prechter and Frost, 2017), related to what is known today as Fibonacci Analysis (FA).

Although Elliott successfully used his theory to predict the bullish trend of the Dow Jones Industrial Average (DJIA) in the 1940s, his theory gained real interest from the financial sector at the end of the

[33] Note that some of the EWP where vaguely introduced by Charles H. Dow in his Dow Theory.

1970s, thanks to very accurate predictions made by the most renowned modern EWP practitioner, Robert Prechter[34]. This was followed by his publication, along with A.J. Frost, of the "Elliott Wave Principle". This book, which is considered today the definitive guide of the theory, is a compilation of the work of Elliott, where the authors also introduced some adjustments to the original theory. Part of the popularity gained faded in the 1990s, due to poorly estimations of the bull trend in 1995 for the United States financial market. According to Blackman and Green (2017), these estimations turned out to be quite wrong for both the index value and the timing.

Today, this theory is still very popular and widely used by practitioners of technical analysis, and three are the main principles behind the theory: the impulsive-corrective cycle, mass psychology as explanatory variable and the Fibonacci series ruling the Elliott's waves.

### 5.2.1. Impulsive-Corrective Cycle

The foundation of Elliott theory is on the belief that markets are governed by the mass psychology, which is expressed repetitively in the form of waves. The way in which markets reflect the social psyche, therefore, is through cycles made up of the union of two patterns identified by Elliott as impulsive and corrective waves (or patterns).

The impulse wave reflects an advance (increase) in the price, as shown in **Figure 5.1**, and is composed of 5 waves called waves 1, 2, 3, 4 and 5. The waves 1, 3 and 5 are responsible for defining the wave trend. Waves 2 and 4 are corrective movements to the trend. In contrast, the corrective wave or pattern introduces partial retracement[35] to the impulse wave and is composed of three waves named A, B, and C; as shown in **Figure 5.1**. From the union of an impulse wave (1, 2, 3, 4, 5) with a corrective

[34] Robert Prechter is considered one of the most important practitioners of the EWP. He has a bachelor in psychology from Yale University, worked in Merrill-Lynch as a market technician, and founded the Elliott Wave International (an investment adviser firm). Today he is also known for his Socionomics theory.

[35] The theory also introduced other corrective patterns that do not follow exactly this behavior (i.e. triangular waves).

wave (A, B, C) an 8-waves cycle is obtained. From the union of several of this minimum unit, cycles of higher degree are built.

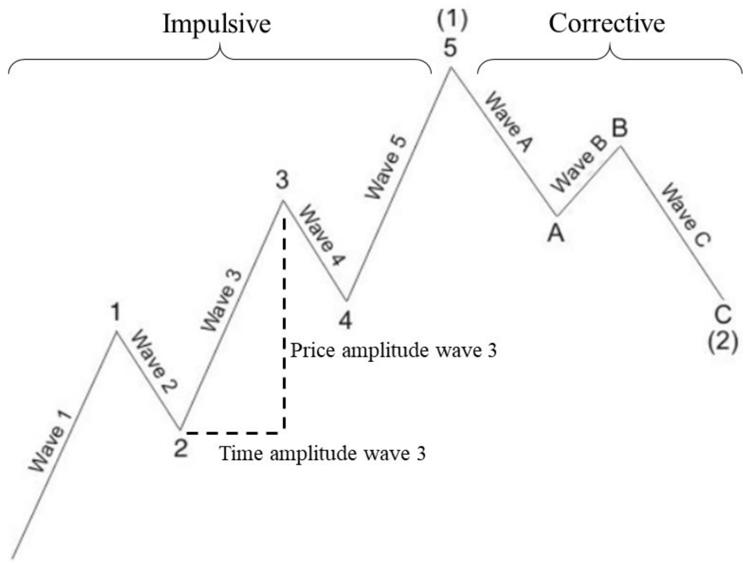



**Figure 5.1:** Impulsive and Corrective patterns forming one impulsive-corrective cycle.
**Source:** Modified from Prechter and Frost (2017).

Elliott affirmed that he could observe these patterns regardless of the time scale considered. This allowed him to extrapolate the impulsive-corrective cycle to all time scales, providing the fractal[36] characteristic of his theory (idea illustrated in **Figure 5.2**). This was also supported by his belief that human nature does not change, is repetitive, and its activity is highly predictive. Consequently, fractals allow the theory to be valid regardless of the time scale used to analyze the market[37].

[36] Considering that the pattern is the same but it differs in its price and time amplitudes, the fractal obtained is called a statistical self-similarity fractal.
[37] Note that the idea that markets and business cycles could be modeled by fractals have been also study by B. Mandelbrot et al. (1997) through his Multifractal Model of Asset Returns (MMAR). The use of fractal allowed him to capture two of the empirically erroneous assumptions of the EMH: non-normality of the logarithm of returns and their autocorrelation.

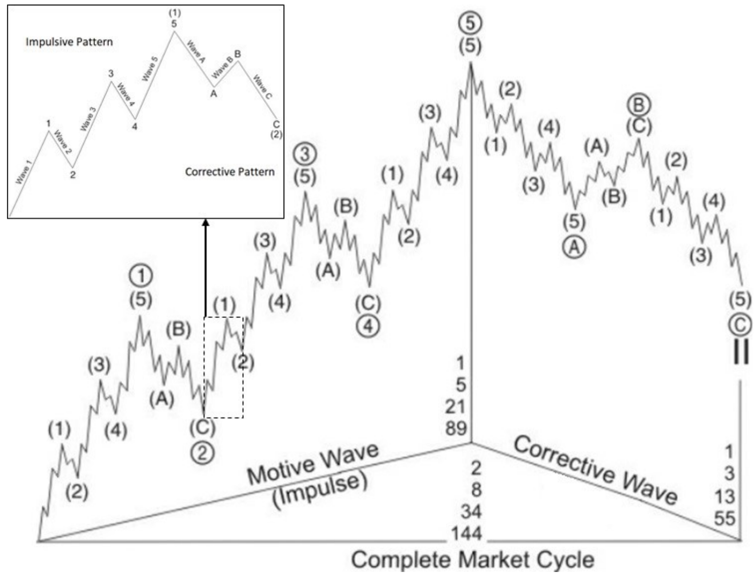



**Figure 5.2:** Elliott's fractal market cycle.
**Source:** Modified from Prechter and Frost (2017).

Elliott provided a system that classifies the waves according to the degree of the cycle (**Table 5.1**), giving a notation system to avoid interpretation problems with nested waves and indicating time references. For instance, a Grand Supercycle can typically comprise 100 years or more, and each sub-cycle considers progressively shorter periods of time, up to the Subminuette Cycle, which represents a few minutes (Droke, 2000). This is also illustrated in **Figure 5.2**.

| Wave Degree/Cycle | Impulsive Pattern (5 waves) | Corrective Pattern (3 waves) |
|---|---|---|
| Supermillennium | [1] [2] [3] [4] [5] | [A] [B] [C] |
| Millennium | (1) (2) (3) (4) (5) | (A) (B) (C) |
| Submillennium | 1 2 3 4 5 | A B C |
| Grand Supercycle | [I] [II] [III] [IV] [V] | [a] [b] [c] |
| Supercycle | (I) (II) (III) (IV) (V) | (a) (b) (c) |
| Cycle | I II III IV V | a b c |
| Primary | [1] [2] [3] [4] [5] | [A] [B] [C] |
| Intermediate | (1) (2) (3) (4) (5) | (A) (B) (C) |
| Minor | 1 2 3 4 5 | A B C |
| Minute | [i] [ii] [iii] [iv] [v] | [a] [b] [c] |
| Minuette | (i) (ii) (iii) (iv) (v) | (a) (b) (c) |
| Subminuette | i ii iii iv v | a b c |

**Table 5.1:** Elliott wave degrees and the counting nomenclature.
**Source:** Droke (2000), Prechter and Frost (2017).

### 5.2.2. Mass Psychology as Explanatory Variable

For EWP, mass psychology is the main driver of the market's behaviors. Thus, each of the 8 waves of a cycle (whether it is a Grand Supercycle of 100 years or Subminuette of a few minutes) reflects the different status of the market agent mood. Nevertheless, he and his follower are keen to stress the idea that mass psychology is more trackable through markets average indices, such as the DJIA or S&P500. This because it is in averages where independent views cancel out, allowing the emergence of the overall participants' mood.

Elliott defined the waves psychology as follow:

- **Wave 1:** These are hard to identify due to the rationality behind their formation since they are generated in an environment of conflicting expectations. This trade-off occurs mainly between a predominantly negative view, due to the important *momentum* of the previous decline and the non-encouraging news and indicators of the economy, versus a growing questioning of this pessimism and its foundations.
- **Wave 2:** Initially generated by the massive realization of the gains obtained during Wave 1. However, this selling process recalls the negative expectations of the near past, which tend to be temperamental due to a feeling of confirming previous pessimism. Due to this, a significant retracement of the market is observed.
- **Wave 3:** The so-called powerful wave starts shyly with a moderate pessimism because of the previous correction. However, increasing good news and recovery of the fundamentals strengthen a generalized optimism, which leads to better earnings estimates. The consolidation of the recovery encourages investors to enter the market massively, causing a large volume of acquisition, even in assets that were previously unquoted (Calvo and Jiménez, 2001). Although most EWP practitioners agree that the wave 3 of a stock market tends to be the most intense, some indicate that in commodities this occurs in wave 5 (Kotyrba, 2013).

- **Wave 4:** After a period of optimism -perhaps irrational- the questions about the foundations of this recovery begin. It is a period where there is a discrepancy between market participants; some of them wanting to benefit and others still betting on a greater recovery. However, a lack of a market worsening confirmation creates the basis for the next rise (Prechter and Frost, 2017). As a consequence, this corrective wave tends to be extensive in time, complex in structure and with a discrete price retrace due to divergent views.
- **Wave 5:** After the observed deterioration, many fears are dissipated, and a rising pressure begins again, but of less dynamism than the wave 3. Partially because the optimism shown during the complete uptrend begins to show signs of weakening, which is expressed in a slowdown toward the new peak. Additionally, this period is influenced by an important irrationality derived from the late entry into the market of a significant number of participants, wanting to take advantage of what they could not in the previous trends.
- **Wave A:** This wave begins the change of trend. There is a remnant optimism of the extensive upward trend experienced, so the fall is associated with a profit taking. However, the first signs of a more fundamental deterioration emerge.
- **Wave B:** As the authors Prechter and Frost describe, B waves are: "false, silly games, bull traps, speculators' paradise, and/or expressions of silly institutional complacency.". The above is expresses through a strong and unsupported dynamism in the market derived from a mistaken perception that the market still remains in an uptrend.
- **Wave C:** After a strange period of expectations outside the fundamentals, there is a total decline, where panic spreads and an unsuccessful race begins for haven assets. Ultimately, the market collapses when accepting that a bear market has been consolidated.

### 5.2.3. Fibonacci Series Ruling Elliott Waves

In Elliott's first work, he did not introduce a rational system that allowed to make estimations of the waves time and price amplitudes (recall **Figure 5.1** for the illustration of the wave's time and price amplitude). However, as the author Bolton (1994) indicated, Elliott developed the fundamentals for the impulse-correction shape through the use of the FS in his second work: "*Nature's Law: The Secret of the Universe*". With this, Elliott established the basis for what is known today as FA.

The FS is attributed[38] to the work of the Italian mathematician Leonardo Fibonacci da Pisa, who derived such a series -in the thirteenth century- as a result of the study of the growth of rabbit population. The first two terms of the series are 1 and 1, and afterward, the series is described by **Equation 5.1**, obtaining: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ..., ∞. Some of the most relevant properties of the FS are the values obtained asymptotically from the division between consecutive numbers of the series, i.e., 0.618 and 1.618[39]; the latter being the golden number represented by the Greek letter *phi* (φ).

| | |
|---|---|
| $s_{n=1} = 1; s_{n=2} = 1; s_n = s_{n-1} + s_{n-2}, \forall n \in N^+ \; with \; n > 2$ | **Eq. 5.1** |

Elliott began to study the FS and its ratios (Fibonacci Ratios or FR), theorizing that there is a universal structural force that governs natural phenomena and being the humankind -and everything derived from it- the result of nature, then it is subject to the same forces.

Initially, Elliott noticed that the counting of his waves fitted to the series numbers, and therefore he declared that the FS governs the number of waves observed in the markets (see the waves counting in **Figure 5.2**). Then, Elliott applied the Fibonacci series to the duration of bull and bearish markets (e.g. observing the DJIA, the cycle between the lows of 1921 and 1942 lasted 21 years, between the high of 1928 and the low of 1949 lasted 21 years, the bull cycles of 8 years between 1921 and 1929 and 1949 and 1957, etc.). However, Bolton (1994) said that Elliott's findings did not provide useful foundations for the prediction of cycles extensions.

[38] There is evidence that the series was formalized by Indian mathematicians called Pingala and Virahanka between 200 and 700 DC.

[39] Note that these ratios (0.618 and 1.618), as well as other ratios obtained from the division of more distant values in the FS, called Fibonacci ratios (FR), are not exclusive to the FS, and are obtained from any series where the next term is the sum of the two previous terms.

Finally, Elliott introduced the notion that waves' time and price amplitude were connected to each other by the golden number, as well as others FR. In his work, Elliott found several cases where these ratios fitted to time and price amplitudes, becoming a pioneer in what is known today in technical analysis as FA. These findings led Elliott and its followers to establish that Elliott's waves are governed by the FS and FR.

The FA results are crucial for Elliott's theory since the results provide a base to model cycles and perform forecasts. More specifically, consecutive waves are related through the FR. Therefore, once the correct wave has been identified (typically the wave 1), the forecasting of next Elliott's waves is based on the concatenation of waves through the FR[40]. At this point, it is very important to highlight that the FA relies on the correct counting of waves, which in practice has been proven to be open to the interpretation of the technical analysts (Droke, 2000; Aronson, 2007). Furthermore, modern technical analyst state that to obtain reliable predictions with EWP, it is crucial to empirically define the probabilities associated with the advance and retracement of a wave toward the FR (Teseo, 2001).

There is some consensus among EWP practitioners about the most useful FR, which are listed in **Table 5.2** (Teseo, 2001; Rinehart, 2004; Brown, 2012; Prechter and Frost, 2017). Moreover, there are three ratios widely used by technical analysts in spite of the fact that they do not belong -at least directly- to the FR (Balan, 1989; Rinehart, 2004). These are 0.5000, 0.7862, 1.7862. The reason for the first (0.5000) is that is the result of the division between the third and second term of the Fibonacci ratios. The second (0.7862) is the result of the square root of the inverse of the golden number[41]. The third (1.7862) is the second plus 1.

[40] The forecast requires the correct identification of time and price amplitudes associated with wave 1, since next waves will be related to wave 1 through the Fibonacci ratios.
[41] $(1/\varphi)^{0.5} = 0.7862$, where $\varphi = s_n / s_{n+1}$, with $s_n$ and $s_{n+1}$ elements of the FS.

| Series Element Involved | Ratio | Value when n→+∞ |
|---|---|---|
| $s_n, s_{n+3}$ | $s_n / s_{n+3}$ | 0.2361 |
| $s_n, s_{n+2}$ | $s_n / s_{n+2}$ | 0.3820 |
| $s_2, s_3$ | $s_2/s_3$ | 0.5000 |
| $s_n, s_{n+1}$ | $s_n / s_{n+1}$ | 0.6180 |
| $s_n, s_{n+1}$ | $(s_n / s_{n+1})^{0.5}$ | 0.7862 |
| $s_n$ | $s_n / s_n$ | 1.0000 |
| $s_n, s_{n+1}$ | $s_{n+1}/ s_n$ | 1.6180 |
| $s_n, s_{n+1}$ | $1+(s_n / s_{n+1})^{0.5}$ | 1.7862 |
| $s_n, s_{n+2}$ | $s_{n+2}/ s_n$ | 2.6180 |
| $s_n$, s $s_{n+3}$ | $s_{n+3}/s_n$ | 4.2361 |

**Table 5.2**: FR commonly used by EWP and FA practitioners.
**Source**: Teseo (2001), Rinehart (2004).

For instance**, Figure 5.4** shows the formation of three different impulsive-corrective cycles using different combinations of Fibonacci ratios for the time and price amplitude for the internal waves 2 to 5 in the impulsive pattern and the full corrective pattern[42].

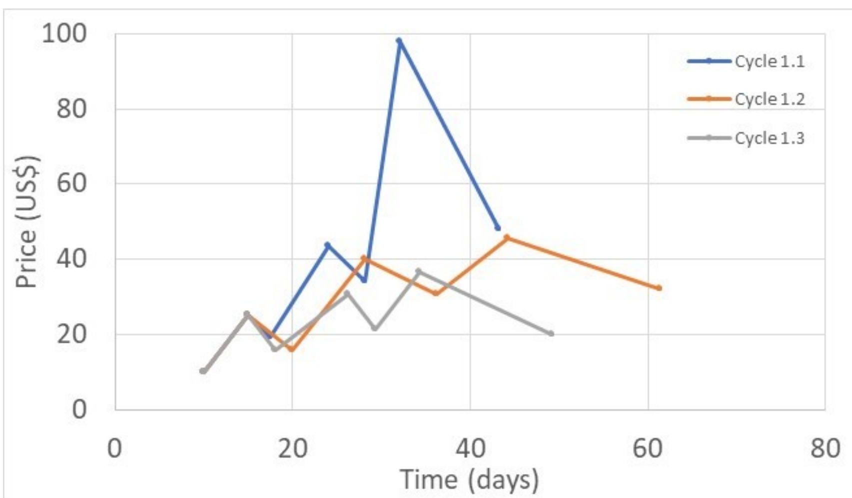



**Figure 5.4**: Examples of the impulsive-corrective wave (one cycle) using EWP and FA.

## 5.2.4. Main Criticism

Despite the popularity among technical analysts, EWP and its applicability have been subject to criticism by the academic circles, indicating at least three core weaknesses: i) mass psychology as the

[42] Note that all of them start from the same wave 1, which is the starting point for modeling the rest of the cycle.

sole explanatory variable; ii) use of FA for the pattern formation; and iii) Subjectivity and high flexibility.

### 5.3.1 Mass psychology as unique explanatory variable

EWP considered that the only explanatory variable behind markets cycles is mass psychology, an assumption that does not have scientific support so far. Although mass psychology might influence markets and business cycles (field currently study by the behavioral economics), there are several variables and phenomenon that have been empirically assessed as explicative of these economic cycles. Some of them are related to the theory of the countries' economic development in stages, changes in countries' factor endowment, international trading context, and technological shift.

In response to this line of criticism, Prechter -one of the most famous EWP practitioner- developed the Socionomics Theory (**Table 5.3** summarizes the contrast between the traditional economic model and the Socionomics). In this theory, Prechter posits that mass psychology is not the result of world events, but in the other way around. In other words, the social mood, which happens to be measurable by the markets' behavior, is endogenous, patterned and responsible for all phenomena in the world (Szala and Holder, 2004).

| Economic Model | Socionomic Model |
|---|---|
| 1. Objective, conscious, rational decisions to maximize utility determine financial values | 1. Subjective, unconscious, pre-rational impulses to herd determine financial values |
| 2. Financial markets are random | 2. Financial markets are patterned |
| 3. Financial markets are unpredictable | 3. Financial markets are probabilistically predictable |
| 4. Financial markets "trend toward equilibrium" and "revert to the mean" | 4. Financial markets are dynamic and do no revert to anything |
| 5. Investors' decisions are based on knowledge and certainty | 5. Investors' decisions are fraught with ignorance and uncertainty and they use the information to rationalize emotional imperatives |
| 6. Changing occasions prognostic changes in the values of related financial instruments | 6. Changing values of financial instruments presage changes in associated events |
| 7. Economic principles govern finance | 7. Socionomics principles govern finance |

**Table 5.3**: Contrast of the market behavior assumptions for classic economic & socionomic.
**Source**: Szala and Holter, 2004.

### 5.3.2. Use of FA for the pattern formation

The second critic largely discussed relates to the claims that the FS and FR govern the geometry of the impulsive-corrective pattern. Once again, these assumptions have not been scientifically proven, beyond some researchers who have found rather low correlations between FR and market movements (Bhattacharya and Kumar; 2006). Furthermore, opponents stated that considering the inability of Elliott's original theory to predict, the usage of FA was included.

### 5.3.3. Subjectivity and high flexibility

Regarding the high flexibility, the critics point out that the large set of rules, the fractal nature, and the FA provide to the method with a high level of flexibility. As a consequence, contrarians to the theory argue that practitioners are capable of interpreting any market moves on the basis of EWP. Mandelbrot and Hudson (2004) stated that the judgment of wave analysts is more important than the objectivity of numbers, reflecting that the flexibility is vast enough so that subjectivity becomes crucial in its application. In addition, Aronson (2007) remarked that EWP capability to fit any market history segment was based on the method's large degree of freedom. However, the capacity to fit past history has nothing to deal with the capacity to forecast[43].

## 5.3. Application of EWP for the Study of Metal Commodity Cycles through a Monte Carlo Simulation

The next step is to apply the EWP for the modeling of metal commodity cycles. It is very important to mention that as the method does not establish clear rules, but rather principles, it is virtually impossible to fully remove the subjectivity. Nevertheless, capturing the empirical knowledge from EWP practitioners allows for narrowing down the principles to a set of rules that can be simulated.

[43] Aronson used the analogy of a large-order polynomial, which is capable of fitting well to any historical data chart, however it is likely useless in forecasting.

Therefore, a Monte Carlo simulation was implemented for the modeling of cycles based on EWP practitioners' knowledge. The methodology is summarized in **Figure 5.5** and is described in the next paragraphs.

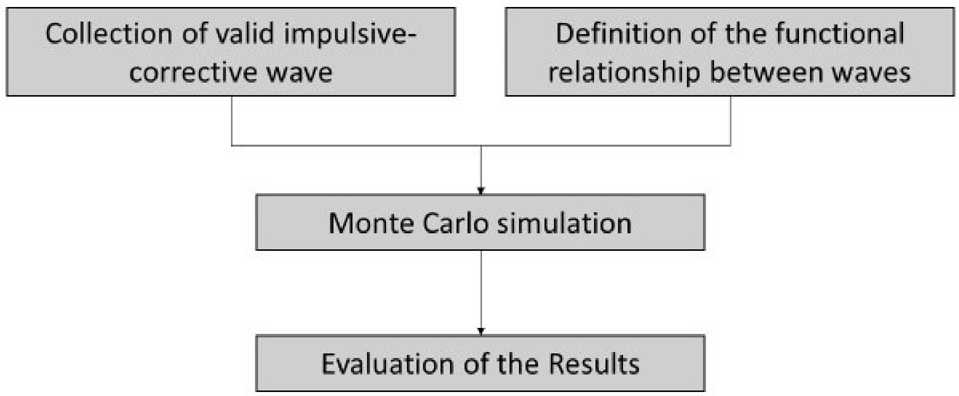



**Figure 5.5**: Methodology for the forecasting of Elliott waves.

### 5.3.1. Collection of valid impulsive-corrective wave

Given that the forecasting of Elliott waves is based on the concatenation of waves through FR, starting from a correctively identified wave 1[44], the objective of this step is to select valid impulsive-corrective cycles observed in the metal commodity prices. In practice, these were the results of an investigation that gathered Elliott patterns from well-known practitioners. The wave 1 of the selected impulsive-corrective waves will be the initial value upon which the impulsive-corrective cycle is modeled. Moreover, the modeled cycles will be contrasted with the actual values of the impulsive-corrective cycle.

The valid impulsive-corrective waves and their sources are presented in **Figure 5.6** (the charts with the original wave are shown in **Appendix 2**) and **Table 5.4**, respectively. The objective was to select a group of metals historically used as investment means (since investors' mass psychology could be

[44] The correct identification of a wave 1 is subject to the judgment of practitioners.

more presented in their price formation process) and representative of the economic status, as well as a metal price index, which summarized the overall behavior of base metals.

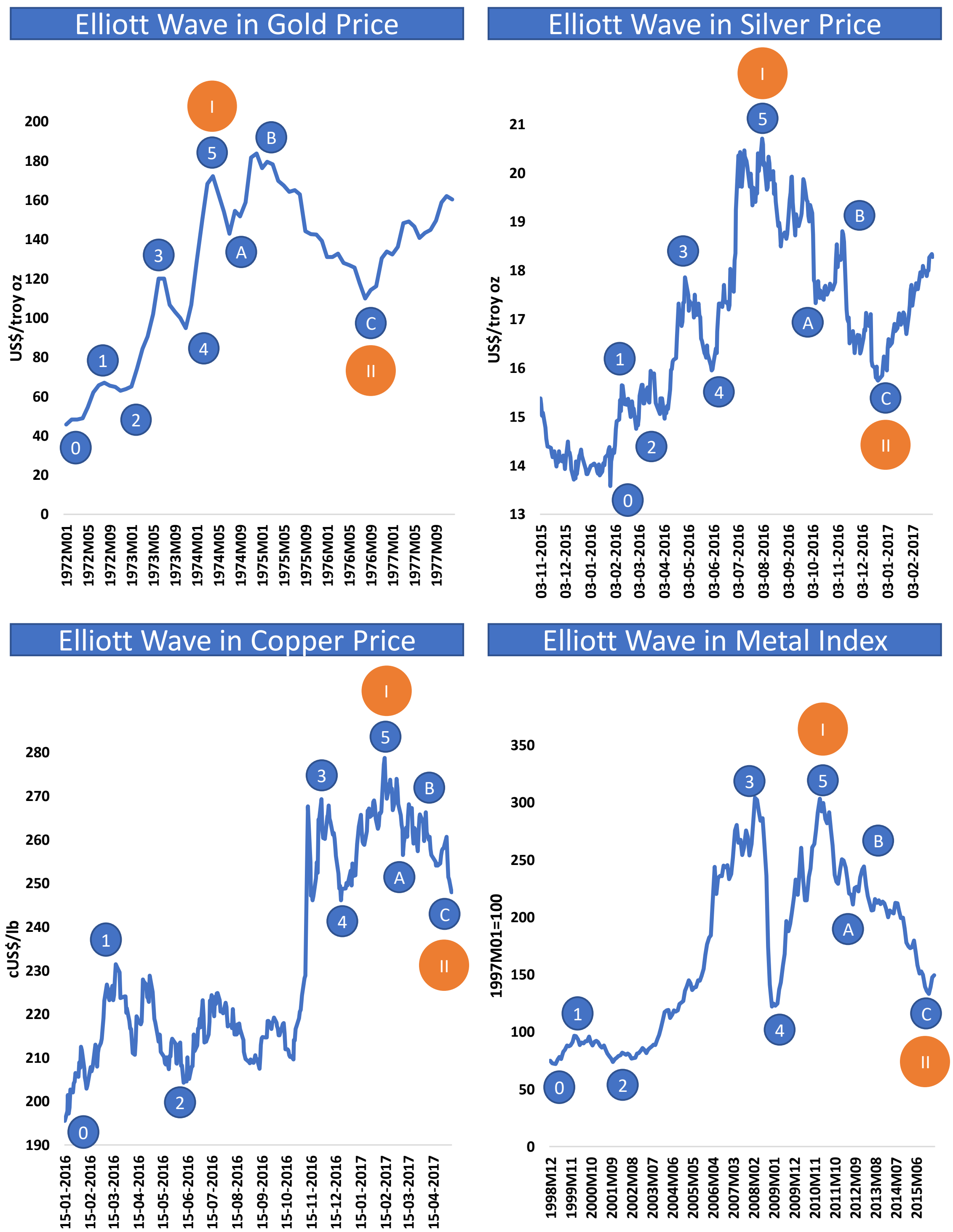



**Figure 5.6**: Elliott wave detected by practitioners in the gold, silver and copper prices and the base metal and iron ore index.

| Metal/Index | Period | Frequency | Source of Elliot Wave | Source of Data |
|---|---|---|---|---|
| Gold | 01/1972-09/1976 | Monthly | Prechter and Frost (2017) | Cochilco |
| Silver | 01/2016-12/2016 | Daily | elliottwave5.com (2017) | Cochilco |
| Copper | 01/2016-05/2017 | Daily | Ramki Ramakrishnan (2017), wavetimes.com | Cochilco |
| Base Metals & Iron ore Index (BMII) | 03/1999-01/2016 | Monthly | Isaac (2017), www.elliottwave.com | World Bank |

**Table 5.4**: Details of the valid impulsive-corrective waves.

Regarding the Elliott wave observed in the metal price index, no publicly available detection of an Elliott wave over a well-known metal price index was found. However, the work of Elliott Wave International (2017) provides the detection of an impulsive-corrective cycle over the discontinued Continuous Commodity Index (CCI)[45] between 1998-2017[46]. Over this period, the CCI shows a similar pattern than the World Bank's base metal and iron ore prices index, which is corroborated by a Pearson correlation coefficient of 92%. Thus, the "equivalent" Elliot wave observed on the base metal and iron ore prices index (BMII) was used (**Appendix 2** shows the comparison between the CCI and the base metal and iron ore prices index).

### 5.3.2. Definition of the functional relationship among waves

The goal of the second stage is to define the functional relationship among the internal waves[47] of the impulsive-corrective cycle. This is done in a two-step process. The first is the definition of the FR towards which a wave trend in time and price amplitude (called the "sections") and their probabilities. Once defined the section towards which a wave is trending, the next step is to select a random number

[45] The Continuous Commodity Index, today known as Thomson Reuters Equal Weight Commodity Index, is an index currently provided by Thomson Reuters about the futures prices of 17 commodities evenly weighted: crude oil, heating oil, natural gas, gold, silver, platinum, copper, cocoa, coffee, corn, soybeans, sugar, orange juice, wheat, cotton, live cattle and live hogs.

[46] Interesting period of analysis as it contains the super cycle of commodities.

[47] Precisely, waves 2, 2, 3 ,4, 5 and II.

from a probability distribution function (PDF) able to model the wave behavior of "trending" towards certain FR.

For instance, let say that one wants to simulate the wave 2 (W2). Let assume that W2 is bonded to wave 1 (W1). Its price amplitude trends towards sections defined by the Fibonacci ratios $FR_1$, with 60% of probability, and $FR_2$, with 40%. Moreover, its time amplitude trends towards sections defined by the Fibonacci ratios $FR_3$, with 70% of probability, and $FR_4$, with 30%. The selection of the sections of trending in price and time amplitude is simulated by the respective discrete PDFs. Once the sections have been selected (let assumed sections defined by $FR_2$ and $FR_4$), the definitive price and time values for W2 are selected from PDFs defined over these sections. The method explained is shown in **Figure 5.7**. This modeling is able not only to capture the empirical knowledge observe in the literature but also the essence of the principle which is the idea of prices trending "towards" Fibonacci ratios.

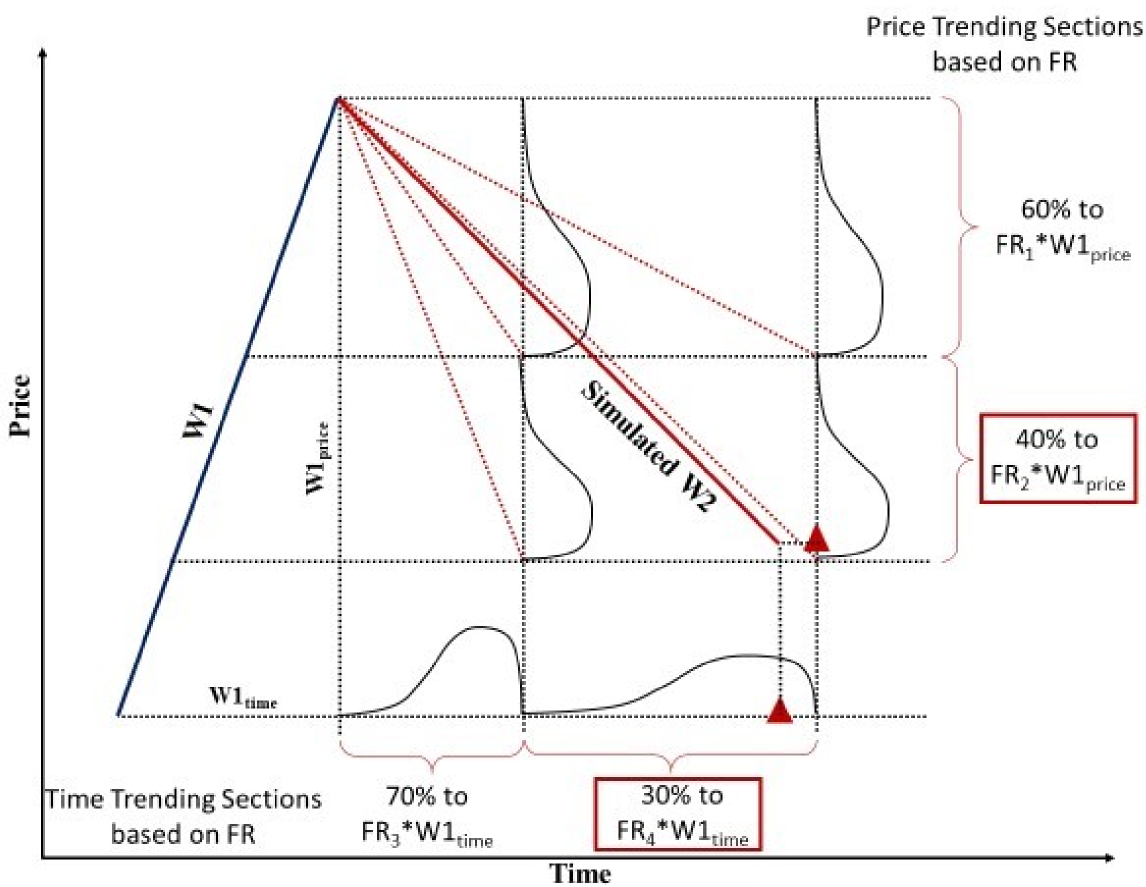



**Figure 5.7**: Hypotheticals modelling of wave 2.

The concatenation of waves through FR, the definition of the discrete PDFs for the sections of trending, and the PDFs for the selection of the ratios over the section are based intensively on the work of Teseo (2001), Rinehart (2004) and Prechter and Frost (2017).

Particularly, for the linkage among waves through FR and the probability of advance or retracement towards these FR, the work of Teseo (2001) and Rinehart (2004) was used. They defined the FR typically observed in markets and their probabilities[48]. No literature was found on the probability of the time amplitude realization, hence it was assumed they are equiprobable.

For the shape of the PDF within sections, no extensive bibliography was found. Nevertheless, the contribution of Swannell (2003A, 2003B) is one to be highlighted; although no enough for the purpose of the simulation of a complete impulsive-corrective cycle. Therefore, the triangular distribution was assumed for the moves within a section, since its parameters (i.e., a=min, b=max, and c=mode) are easily defined considering the limits of the sections. For example, possible price retracement of Wave 4, which are subject to the price amplitude of Wave 3, are simulated drawing random numbers from a triangular distribution with parameters (0.23, 0.38, 0.38) with 15% of chances, a triangular distribution with parameters (0.38, 0.50, 0.50) with 60% of chances, and so on. The same applied for the time amplitude. The information used for the stochastic model is summarized in **Tables 5.5-6**.

[48] For the estimation of time amplitude, the work of Teseo (2001) provides the FR that relate waves. However, he did not provide the probability of this FR, and therefore, unlike the case of the price amplitude, it was assumed that FR for time amplitud were equiprobable

| Wave | Connected to Price of… | Move | Price Amplitude Section | Section Probability | PDF Within Section |
|---|---|---|---|---|---|
| W1 | Not Applicable | Not Applicable | Not Applicable | Not Applicable | Not Applicable |
| W2 | W1 | Retracement | 0.23-0.38 | 12% | Triangle(0.23,0.38,0.38) |
| | W1 | Retracement | 0.50-0.61 | 73% | Triangle(0.50,0.61,0.61) |
| | W1 | Retracement | 0.61-0.78 | 15% | Triangle(0.61,0.78,0.78) |
| W3 | W1 | Advance | 0.78-1.00 | 2% | Triangle(0.78,1.00,1.00) |
| | W1 | Advance | 1.00-1.61 | 15% | Triangle(1.00,1.61,1.61) |
| | W1 | Advance | 1.61-1.78 | 45% | Triangle(1.61,1.78, 1.78) |
| | W1 | Advance | 1.78-2.61 | 30% | Triangle(1.78,2.61,2.61) |
| | W1 | Advance | 2.61-4.23 | 8% | Triangle(2.61,4.23,4.23) |
| W4 | W3 | Retracement | 0.23-0.38 | 15% | Triangle(0.23,0.38,0.38) |
| | W3 | Retracement | 0.38-0.50 | 60% | Triangle(0.38,0.50, 0.50) |
| | W3 | Retracement | 0.50-0.61 | 15% | Triangle(0.50,0.61,0.61) |
| | W3 | Retracement | 0.61-0.78 | 10% | Triangle(0.61,0.78,0.78) |
| W5, if W3>1.61W1 | W1 | Advance | 0.78-1.00 | 2% | Triangle(0.78,1.00,1.00) |
| | W1 | Advance | 1.00-1.61 | 16% | Triangle(1.00,1.61,1.61) |
| | W1 | Advance | 1.61-2.61 | 82% | Triangle(1.61,2.61,2.61) |
| W5, if W3<1.61W1 | W1-W2+W3 | Advance | 0.50-0.61 | 2% | Triangle(0.50,0.61,0.61) |
| | W1-W2+W3 | Advance | 0.61-1.00 | 16% | Triangle(0.61,1.00,1.00) |
| | W1-W2+W3 | Advance | 1.00-1.61 | 82% | Triangle(1.00,1.61,1.61) |

**Table 5.5**: Price amplitude waves link through FR and probability of scenarios.

| Wave | Connected to Time of… | Move | Time Amplitude Section | Section Probability | PDF Within Section |
|---|---|---|---|---|---|
| W1 | Not Applicable | Not Aplicable | Not Applicable | Not Applicable | Not Applicable |
| W2 | W1 | Advance | 0.00-0.50 | 33.3% | Triangle(0.00,0.50, 0.50) |
| | W1 | Advance | 0.50-0.61 | 33.3% | Triangle(0.50,0.61,0.61) |
| | W1 | Advance | 0.61-0.78 | 33.3% | Triangle(0.61,0.78,0.78) |
| W3 | W1 | Advance | 0.00-1.61 | 50% | Triangle(0.00,1.61,1.61) |
| | W2 | Advance | 0.00-2.61 | 50% | Triangle(0.00,2.61,2.61) |
| W4 | W3 | Advance | 0.00-0.50 | 16.7% | Triangle(0.00,0.50, 0.50) |
| | W3 | Advance | 0.50-0.61 | 16.7% | Triangle(0.50,0.61,0.61) |
| | W2 | Advance | 0.00-0.61 | 16.7% | Triangle(0.00,0.61,0.61) |
| | W2 | Advance | 0.61-1.00 | 16.7% | Triangle(0.61,1.00,1.00) |
| | W1-W2+W3 | Advance | 0.00-0.38 | 16.7% | Triangle(0.00,0.38, 0.38) |
| | W1-W2+W3 | Advance | 0.38-0.61 | 16.7% | Triangle(0.38,0.61,0.61) |
| W5 | W2 | Advance | 0.00-0.61 | 12.5% | Triangle(0.00,0.61,0.61) |
| | W2 | Advance | 0.61-1.00 | 12.5% | Triangle(0.61,1.00,1.00) |
| | W3 | Advance | 0.00-0.61 | 12.5% | Triangle(0.00,0.61,0.61) |
| | W3 | Advance | 0.61-1.00 | 12.5% | Triangle(0.61,1.00,1.00) |
| | W4 | Advance | 0.00-0.61 | 12.5% | Triangle(0.00,0.61,0.61) |
| | W4 | Advance | 0.61-1.00 | 12.5% | Triangle(0.61,1.00,1.00) |
| | W1-W2+W3 | Advance | 0.00-0.61 | 12.5% | Triangle(0.00,0.61,0.61) |
| | W1-W2+W3 | Advance | 0.61-1.00 | 12.5% | Triangle(0.61,1.00,1.00) |

**Table 5.6**: Time amplitude waves link through FR and probability of scenarios.

**Figure 5.8** illustrates graphically the modeling, showing an example of the potential materialization of wave 3 based on the information of **Tables 5.5-6**.

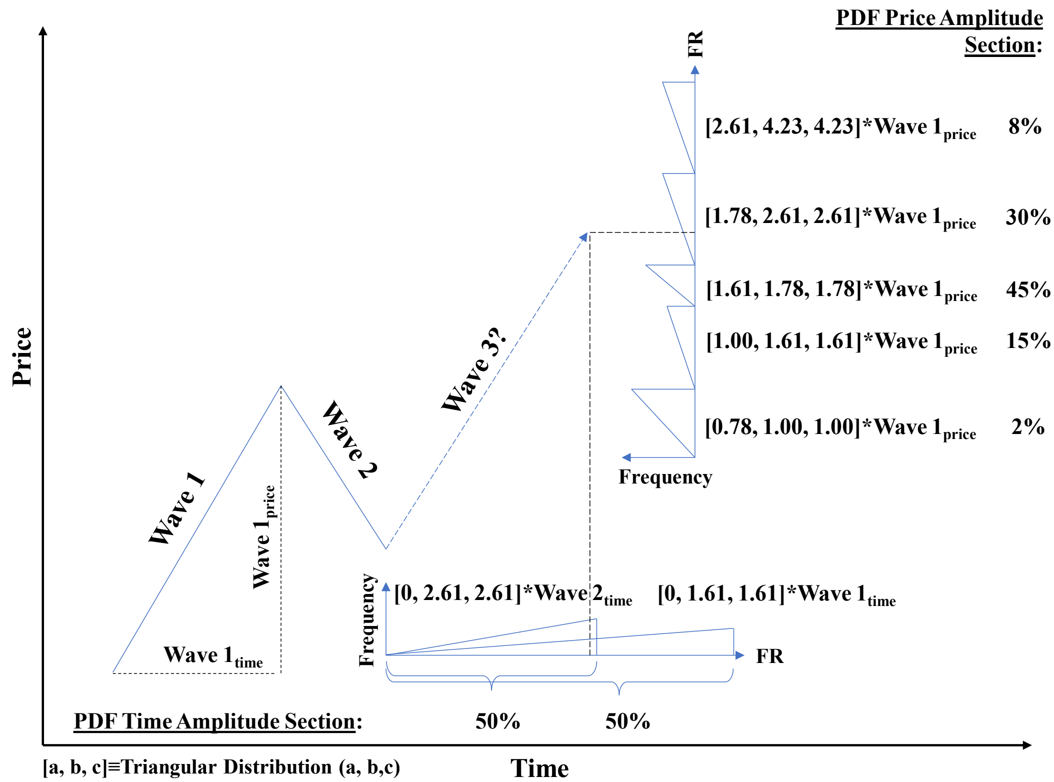



**Figure 5.8**: Idealization of the linkage between the wave 3 and 1 based on the work of Prechter and Frost (2017), Teseo (2001) and Rinehart (2004).

### 5.3.3. Monte Carlo Simulation

Once the empirical knowledge is assembled into a model to forecast Elliott waves, the Monte Carlo simulation is performed with 10.000 iterations. Due to the low complexity of the model, random-number generation processes and the simulation were programmed in VBA Excel 2016. It is important to mention that the random number generator of Microsoft Excel 2007 and previous versions (based on the Wichmann–Hill 1982 RNG method) have been vastly discredited (McCullough and Heiser, 2008). Nevertheless, from 2010 versions, Microsoft has improved the random number generator and today this version generates random numbers that pass standard tests of randomness (Mélard, 2014).

**Figure 5.9** shows one simulation for the gold case presented in **Figure 5.6** where: i) the original price series is the orange line; ii) the actual Elliott pattern is the black line; iii) the simulated Elliott wave is the blue line.

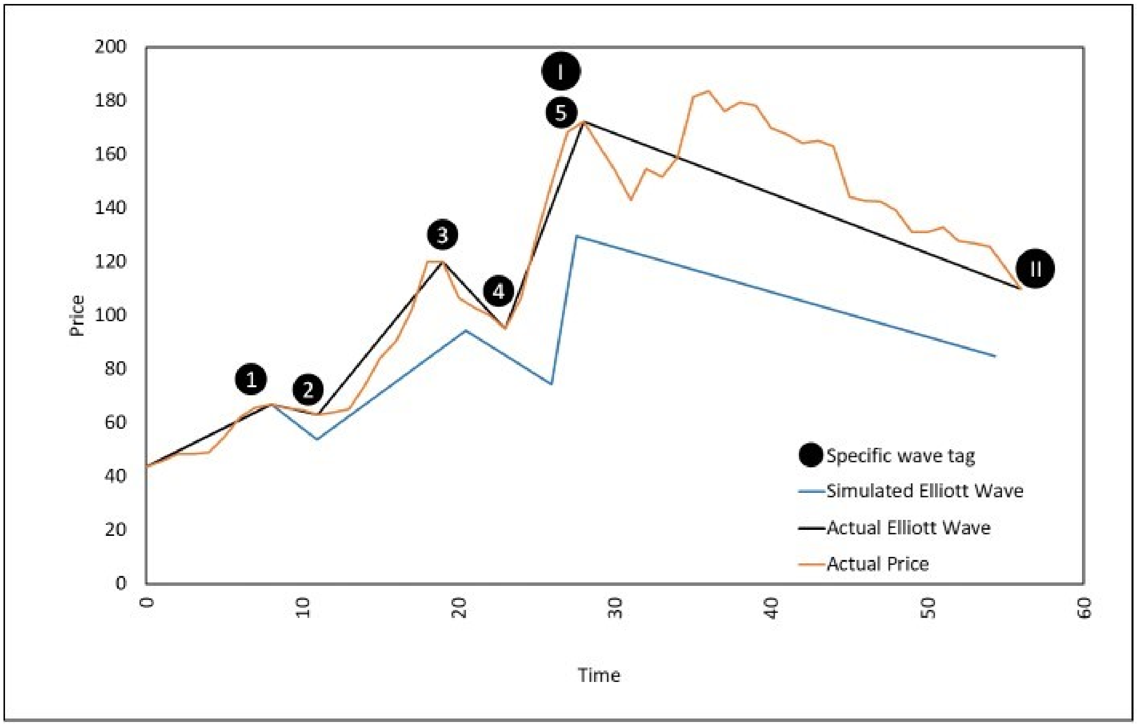



**Figure 5.9**: Impulsive-corrective cycle as result of one simulation.

## 5.4. Results and Discussion

**Figures 5.10-13** show the simulated results[49] of wave I (composed by waves 1, 2, 3, 4, 5) and wave II obtained for the gold, silver, copper and BMII cases, respectively. The actual values of the waves are identified with the black circle tag, while the simulated value for each wave is identified with different colors.

The results of the simulation are conclusive in indicating that this methodology systematically fails in representing the cyclical behavior observed in the metal prices under evaluation through the impulsive-corrective cycles suggested by the EWP practitioners. Indeed, when visually comparing

[49] Just 1,000 sample were displayed to favor the visualization of the results.

the actual value of each internal wave whit their corresponding simulated values (called the simulated domain), generally they are not contained or, at the very best, they are at the edge of the respective domain. For instance, when observing the actual wave 2 of the gold case (**Figure 5.10**), this is not contained in the simulated domain observed in the grey dots. Although waves 3 and 4 are within their domain (yellow and brown dots), they are at the edge. Wave 5 is also out of the domain (green dots). Finally, wave II is also at the edge of the simulated domain (blue dots). The pattern previously described becomes even more dramatic for the cases evaluated in silver and BMII.

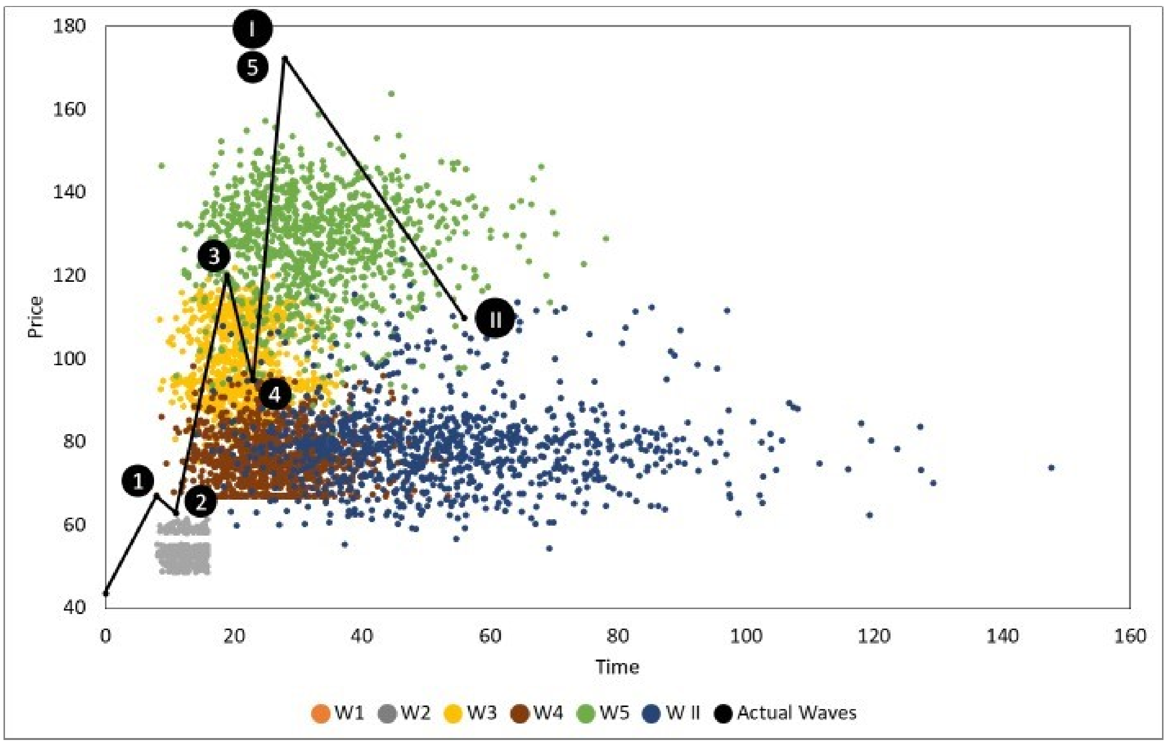



**Figure 5.10**: Price-time simulations for internal waves of the impulsive-corrective cycle in the gold price.

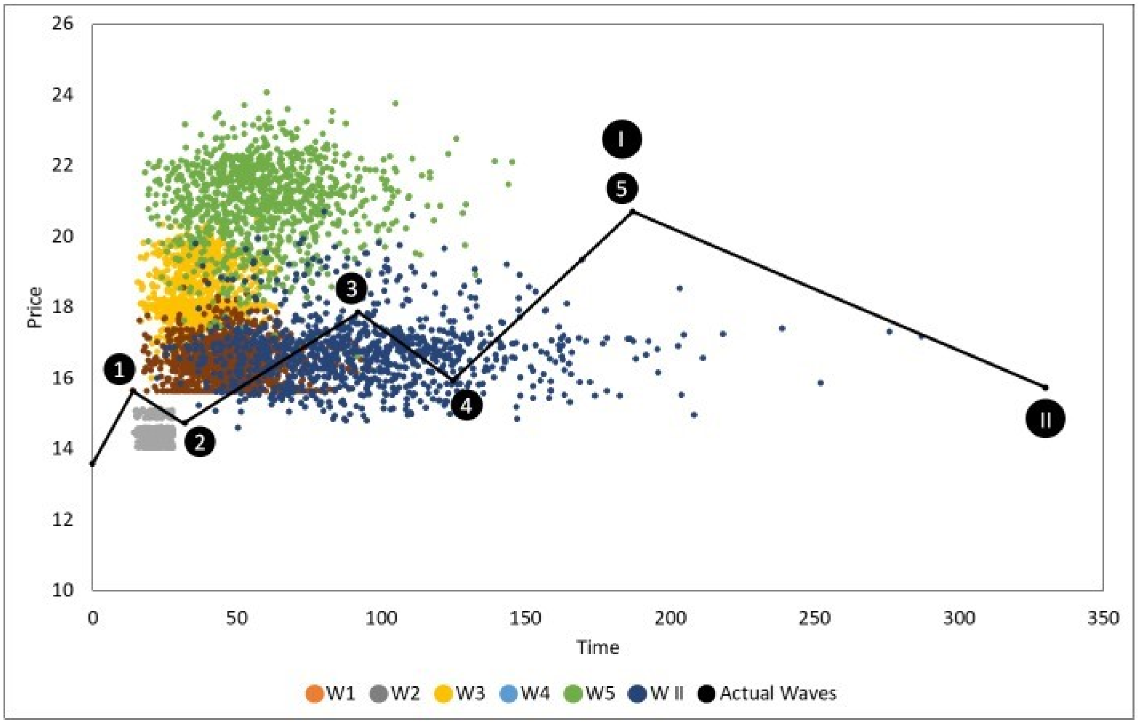



**Figure 5.11**: Price-time simulations for internal waves of the impulsive-corrective cycle in the silver price.

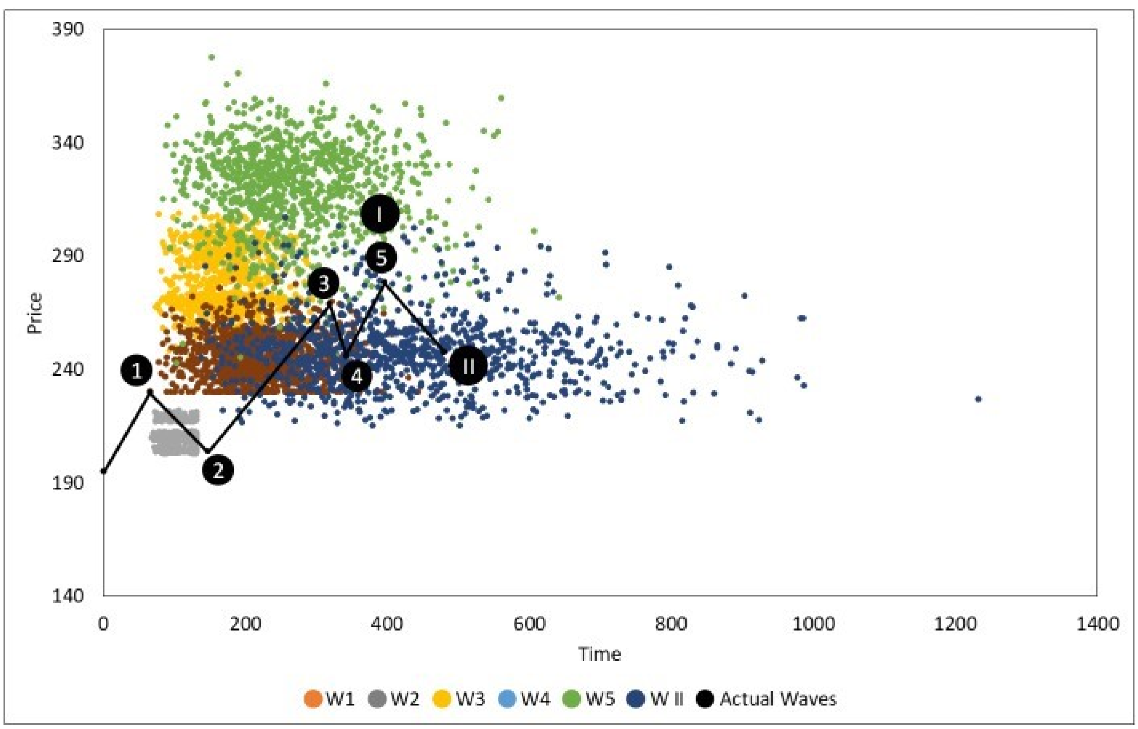



**Figure 5.12**: Price-time simulations for internal waves of the impulsive-corrective cycle in the copper price.

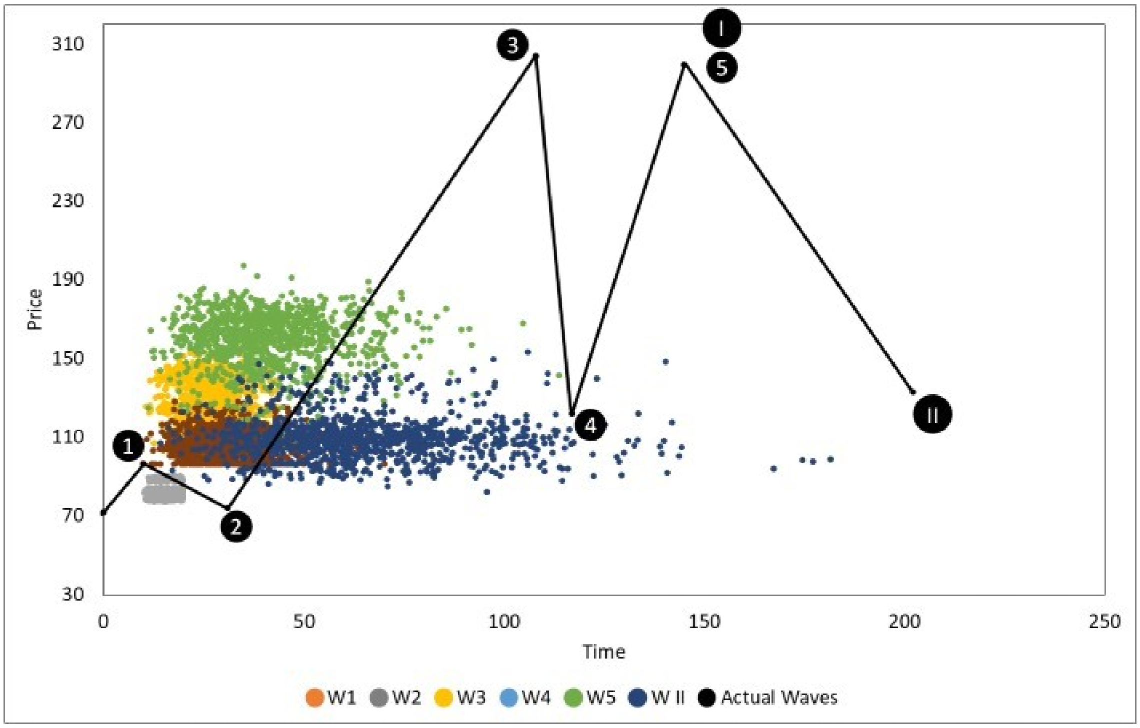


**Figure 5.13**: Price-time simulations for internal waves of the impulsive-corrective cycle in the BMII.

To corroborate the previous perception, **Figures 5.14-18** display the simulated domain for each of the simulated waves in the silver case, with their respective percentiles 5 and 95 in both price and time, and the actual waves values. The results for the gold, copper, and BMII are presented in **Appendix 3**. As can be observed, the actual values of the waves are consistently out of the most likely price-time region (i.e., the region defined within the limits of the percentiles 5 and 95).

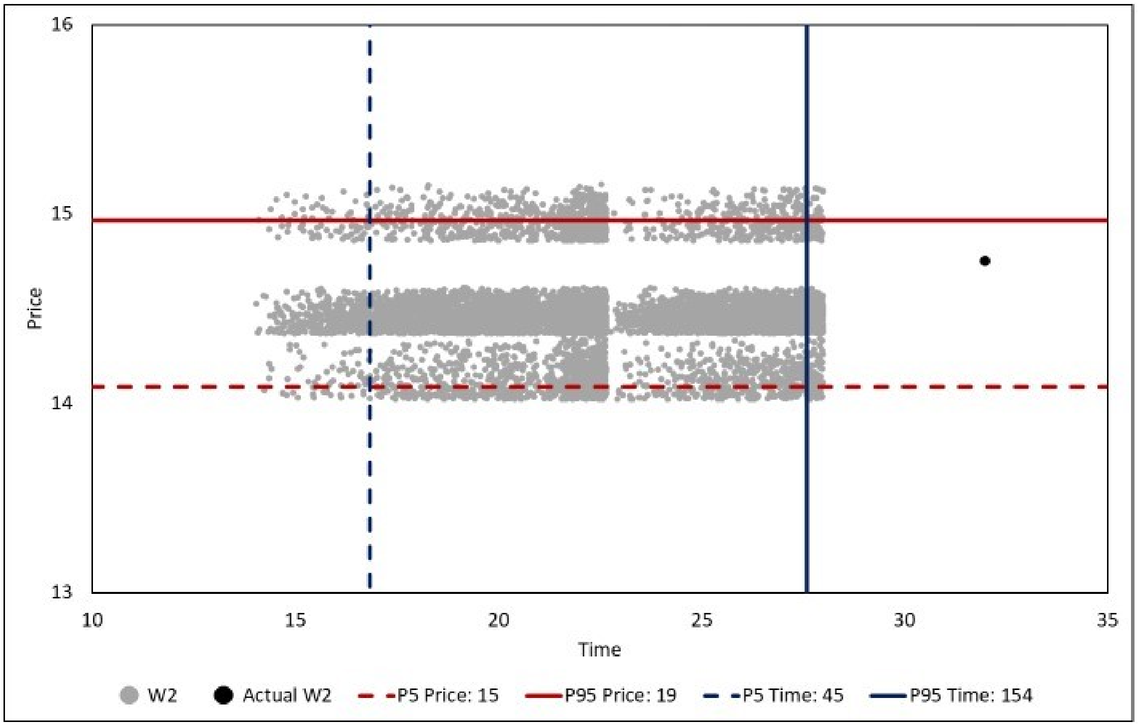



**Figure 5.14**: Simulated values of W2 in the silver price.

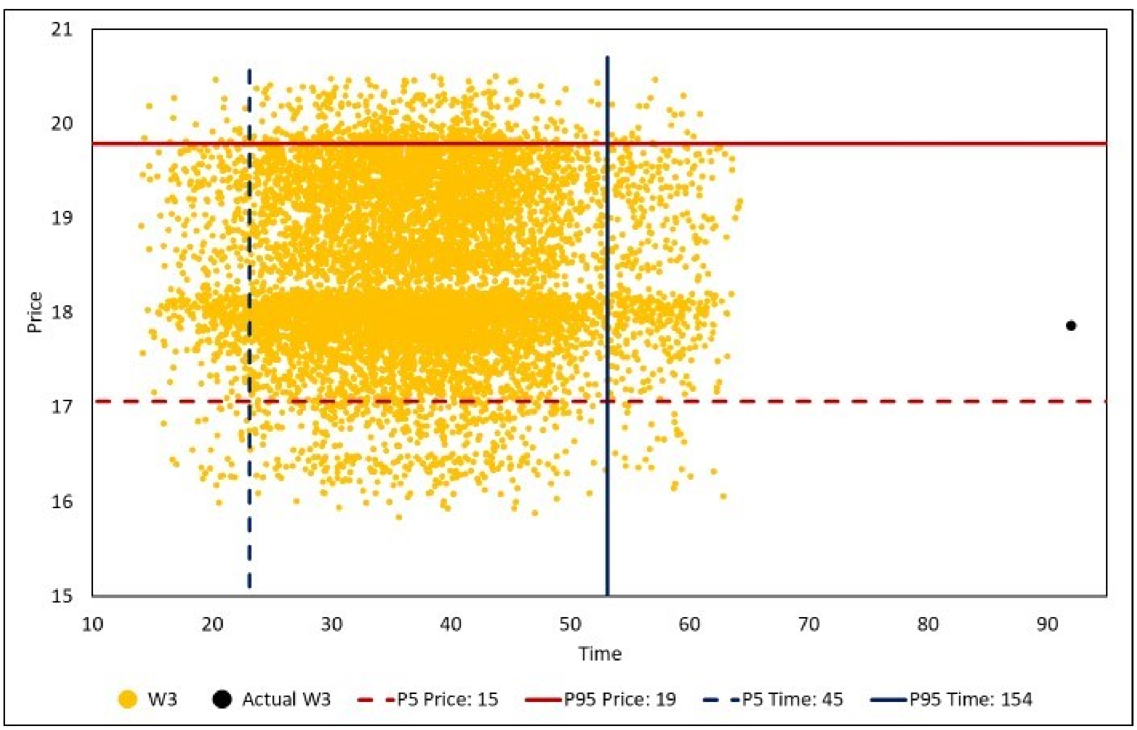



**Figure 5.15**: Simulated values of W3 in the silver price.

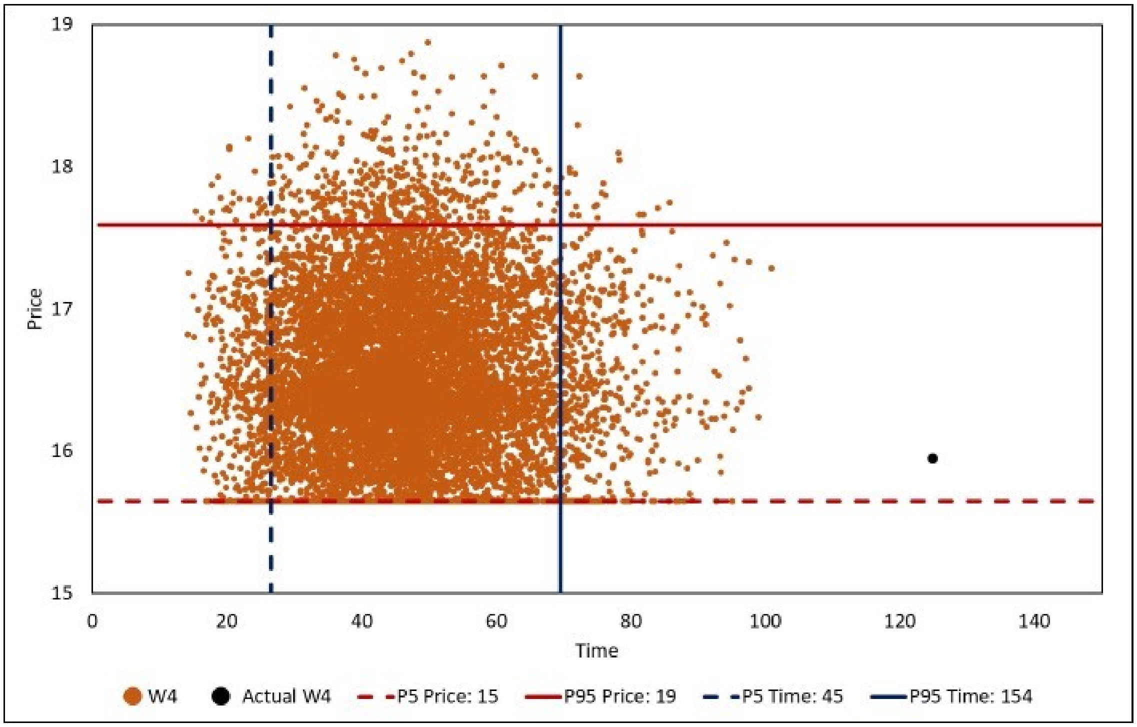



**Figure 5.16**: Simulated values of W4 in the silver price.

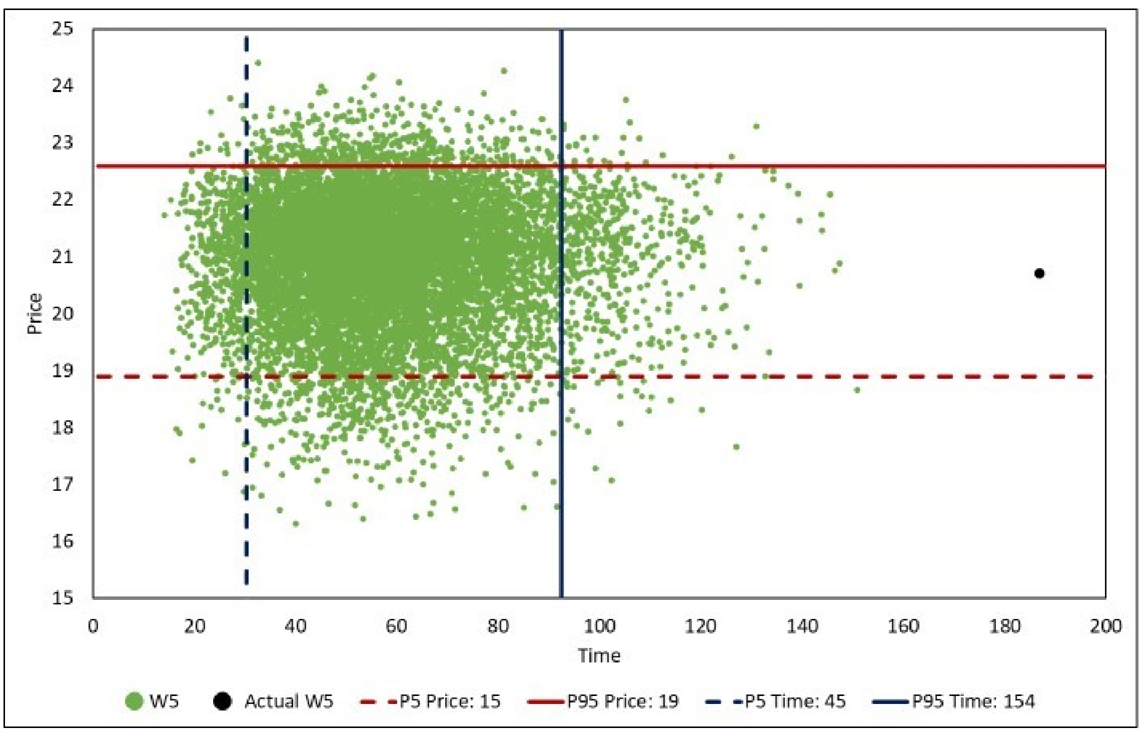



**Figure 5.17**: Simulated values of W5 in the silver price.

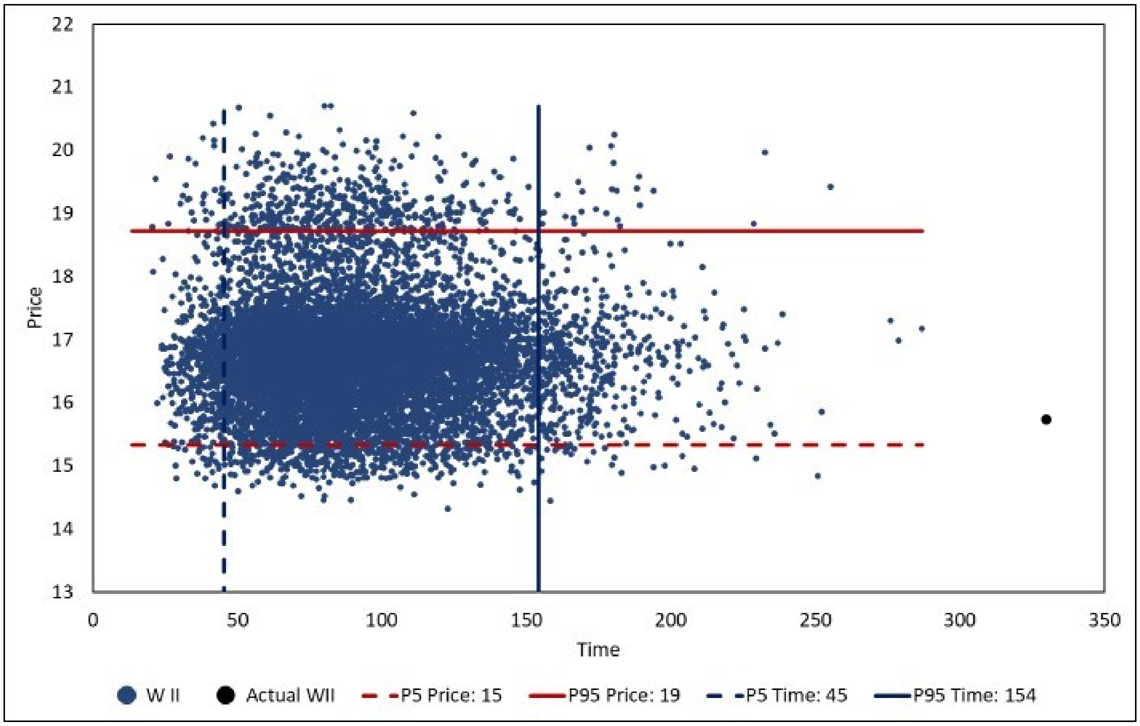



**Figure 5.18**: Simulated values of WII in the silver price.

Furthermore, it is important to highlight the large dispersion of the price-time bivariate distribution, being importantly volatile in time. Thus, the method provides high uncertainty on the length of the cycle. For instance, the impulsive-corrective cycle in the copper case could have last between 210 to 720 days with 90% confidence. This is quite aligned with one of the main critics about high flexibility.

## 5.5. Conclusions

The fact that the method is based on principles rather than on a set of scientifically verifiable rules makes it impossible to completely eliminate a subjective component. Although a thorough bibliographic review of several of the most important EWP practitioners allowed for generating stochastically modellable rules, the subjectivity in the model remains present, at least through the assumption of the correct identification of wave 1. Thus, the applicability is bounded to the expertise of EWP experts.

Over the three cases analyzed, only the copper case was partially well modeled. However, the wide dispersion in the price-time domain validates the critic that the EWP provides enough flexibility to fit Elliott waves to a wide range of results[50].

Although it has been indicating by EWP practitioners that the wave principle is not suitable for individual assets. Nevertheless, EWP is suggested for the study of metal commodity markets where investor involvement is substantial, such as gold and silver[51]. The results provide evidence to suggest that the trends observed in metal commodity prices, including gold and silver, do not target FR, which is in line with the literature review.

Assuming that the EWP practitioners provided accurate waves counting, the results of the simulations suggest that the method is weak in properly model cycle in the metal commodity analyzed. This conclusion is independent of whether the evaluation is over days, months, and years.

Finally, to the question of whether investigating mining companies' stocks instead of the metal commodity prices, one could recall the idea that is in averages where independent views cancel out, allowing the emergence of the overall mass psychology. Using this argument, one could say that this idea could bring little benefit. Indeed, knowing the high correlation between the mining companies' stocks prices and the metal commodity prices and given the results of the simulation, one could also argue that this may not bring benefit.

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[50] Note that this flexibility could be further extended by adding more FR into the model.

[51] Indeed, investors' consumption explain 36% (World Gold Council, 2017) and 20% (The Silver Institute, 2017) of current world consumption for gold and silver, respectively.

# 6. Conclusions and Recommendations

An exhaustive bibliographic review allowed a successful formal evaluation of the BPF and EWP for the study of the cyclicity in commodity prices. The evaluations have allowed expanding the knowledge of the use of these tools in mineral economics, in addition to complementing the current understanding of the behavior of metal prices. In particular, the BFP was applied under an unpublished decomposition, supported by empirical evidence. On the other hand, the vast empirical knowledge gathered from important EWP practitioners allowed to reduce the subjectivity of the method through a simulation of Monte Carlos, permitting a formal evaluation.

Regarding the BPF technique, it can be said that using a novel cyclical decomposition, it was possible to extract cyclic components of short-, medium- and long-term correlated with the recent findings in metal prices cyclicity. Moreover, a long-term cyclical component of 45-60 years was found, not negligible and highly correlated with those of the long economic cycle theory. Furthermore, the tool allowed to corroborate and complement some classical hypothesis of the mineral economics with respect to the trend and co-movement of metal prices. Finally, The BFP, and particularly the Asymmetrical Christiano-Fitzgerald Band-Pass Filter, is a technique that can improve the understanding of the cyclical components. Further research is required to incorporate its benefits in cycles modeling into forecasting.

Regarding the EWP evaluation, the technique has a weak conceptual framework for its applicability in the metal commodities markets. Indeed, the subjectivity of technical analyst cannot be fully removed. The fluctuations and cycles observed in the prices of metals commodities would not be subject to the dynamics suggested by EWP.

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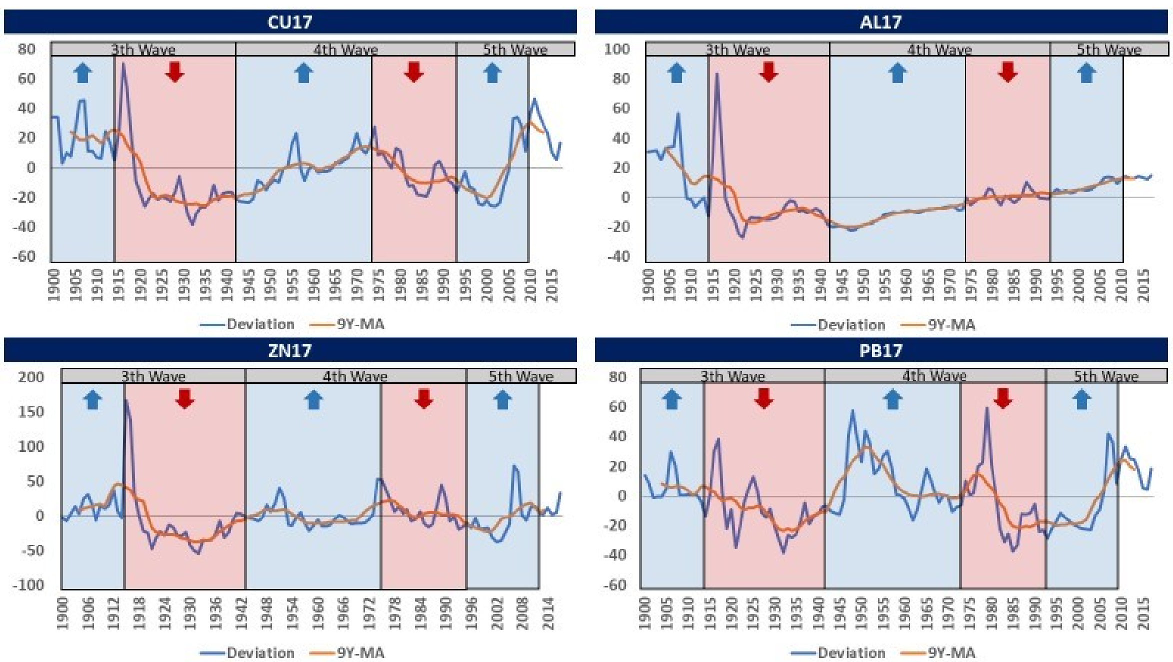
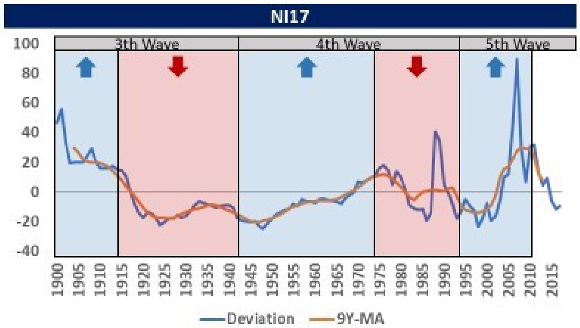
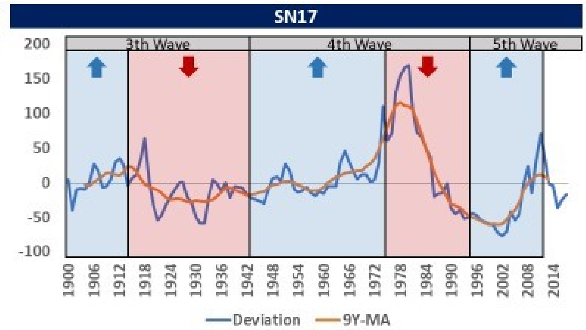
# Appendices

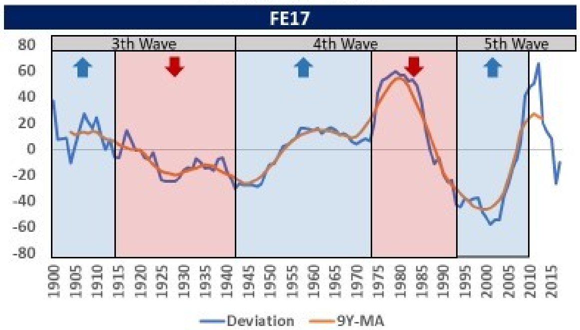
## Appendix 1: Visual Inspection of Kondratiev Waves in Metal Commodity Prices

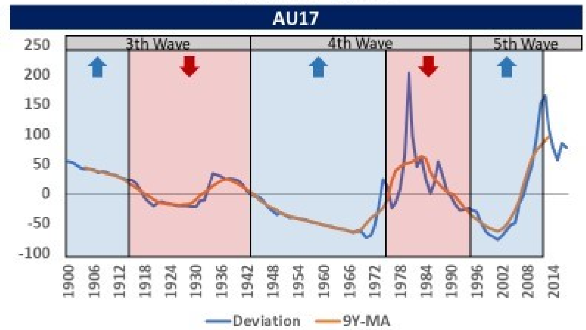
The method for extracting the long cyclical component is the same applied by Kondratiev in its original work and has the sole purpose to provide a first approach to the observation of long cycles in metal prices highly synchronized with the long economic cycles of Kondratiev theory. Prior application of the centered 9-years moving average to the deviations from the trend, the prices were inflation-adjusted and indexed to 1900=100.




NI17
3th Wave
4th Wave
5th Wave
Deviation
9Y-MA

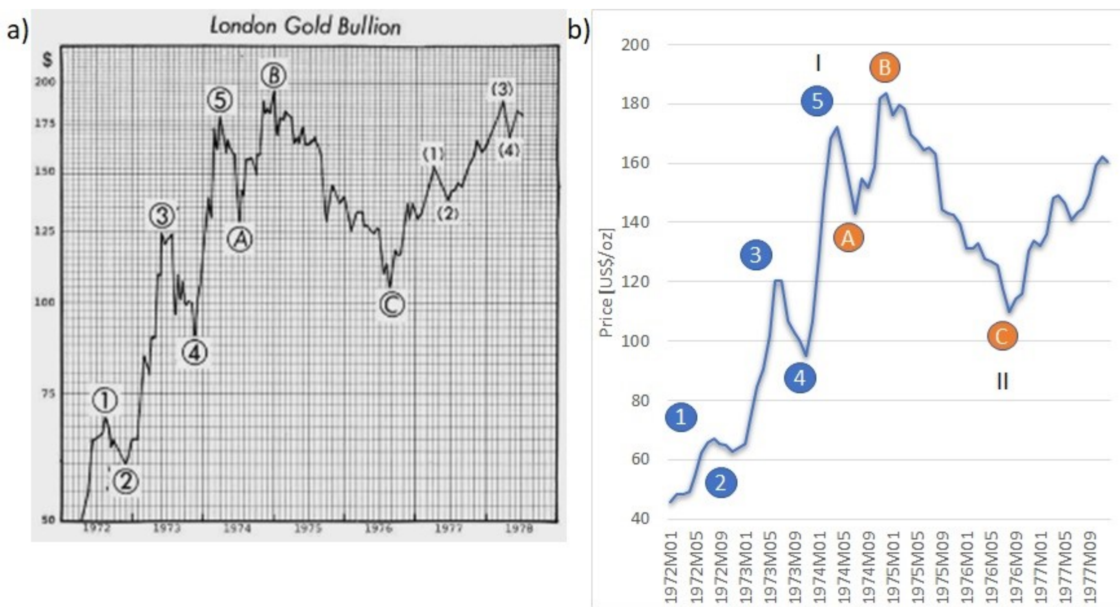

SN17
3th Wave
4th Wave
5th Wave
Deviation
9Y-MA


FE17
3th Wave
4th Wave
5th Wave
Deviation
9Y-MA


AU17
3th Wave
4th Wave
5th Wave
Deviation
9Y-MA

## Appendix 2: Original Elliott wave pattern in gold, silver, and copper prices and the CCI and base metal and iron ore index

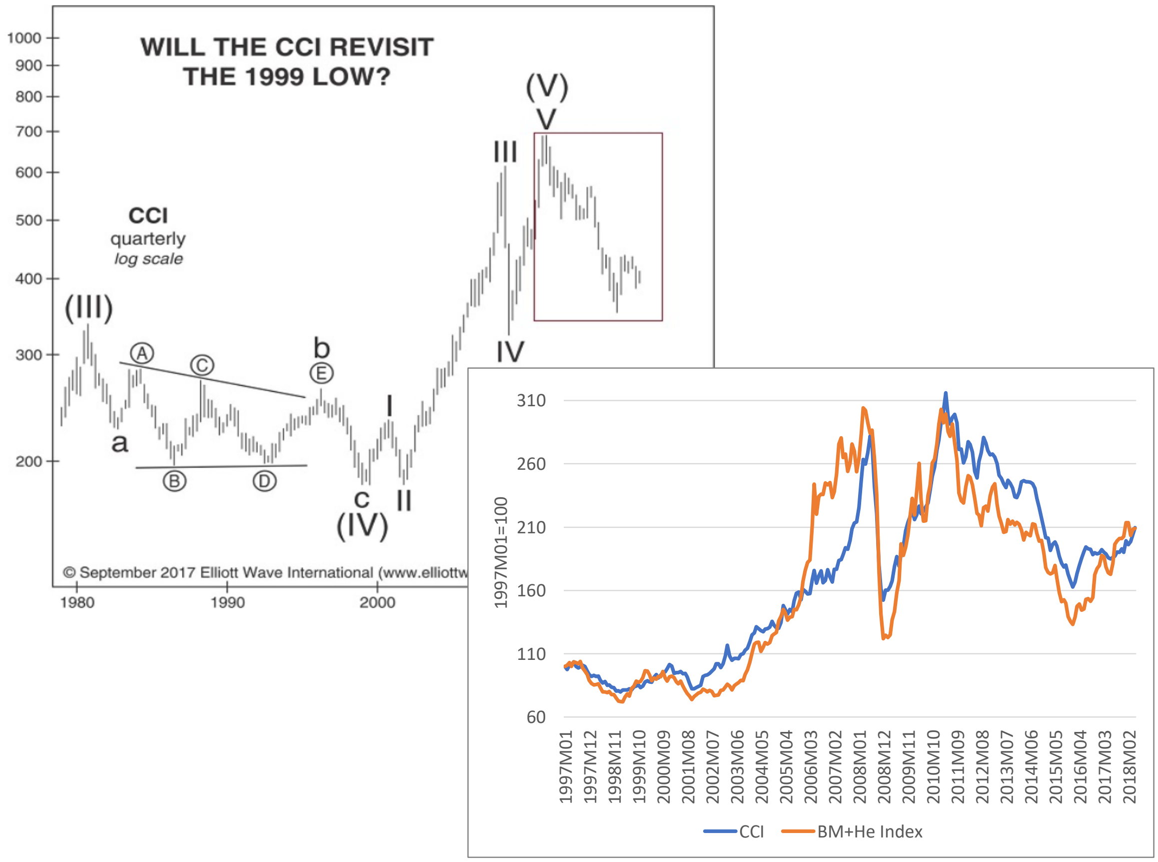
**Gold Case:** Elliott impulsive-corrective cycle detected in the gold price by Prechter and Frost (2017). between 1972 and 1974 on monthly basis. This Elliott wave was used as the base for the simulation on **Section 5.4**.



**Silver Case:** Elliott impulsive-corrective cycle detected in the silver price by ElliottWave5.0. between January 2015 and March 2017 on a daily basis. This Elliott wave was used as the base for the simulation on **Section 5.4**.

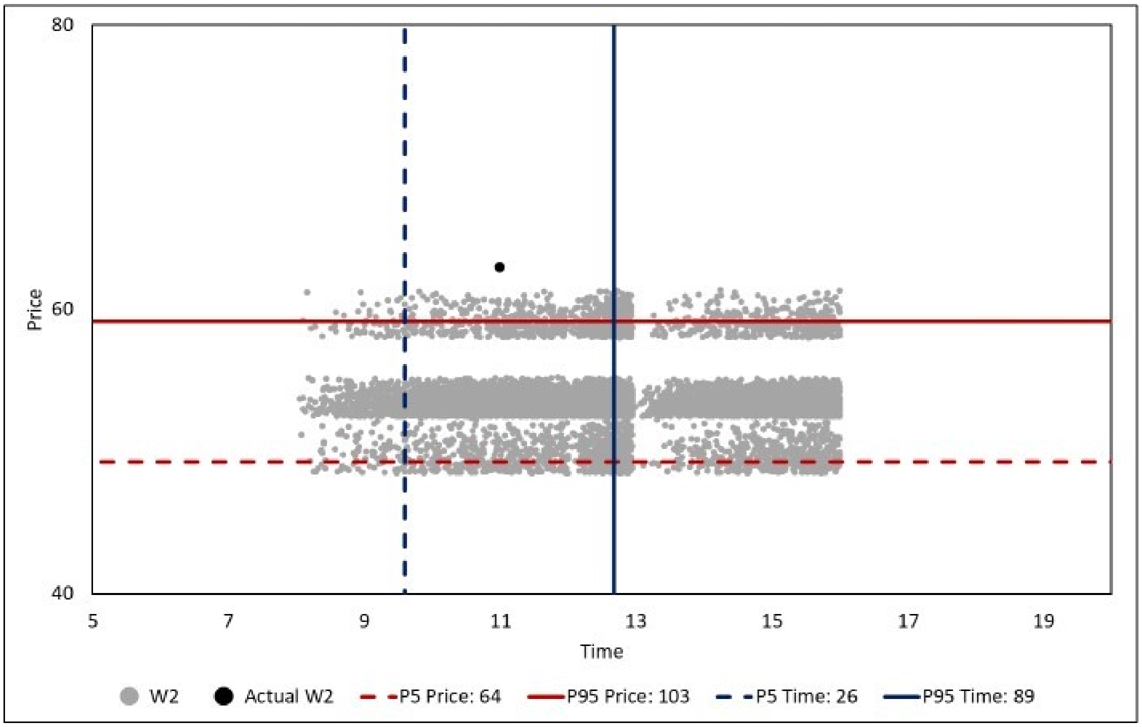

**Copper Case:** Elliott impulsive-corrective cycle detected in the copper price by Ramki Ramakrishnan (WaveTimes.com) between August 2015 and July 2017 on a daily basis. This Elliott wave was used as the base for the simulation on **Section 5.4**.

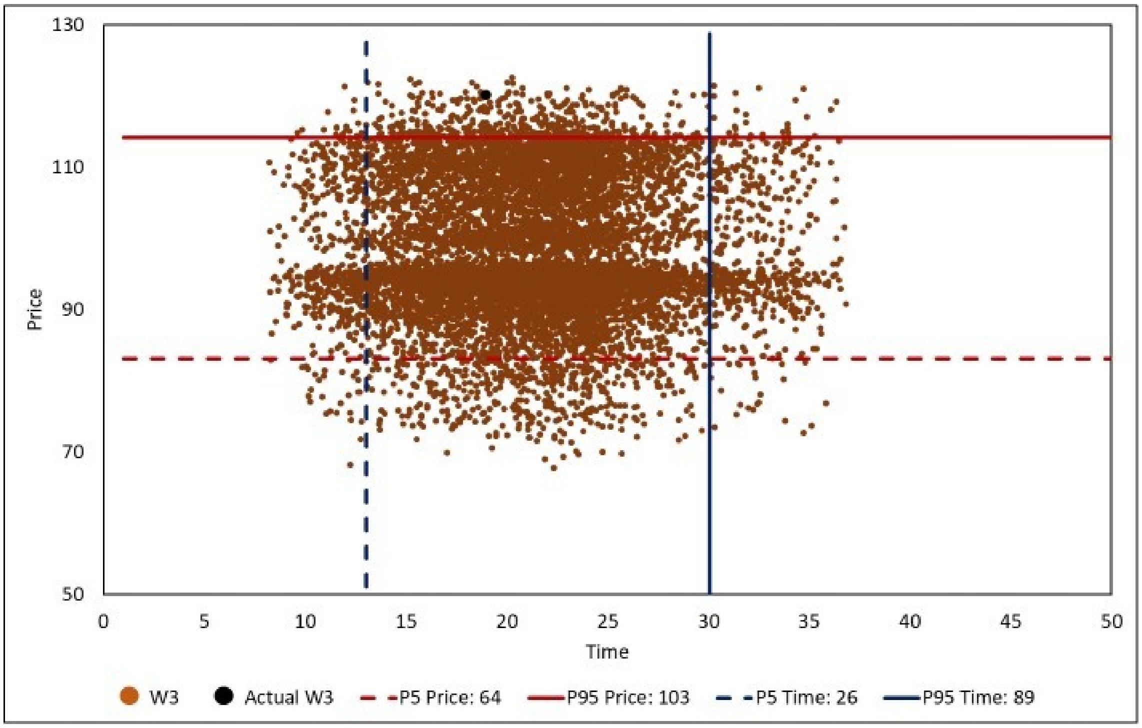
**CCI & Base metal and iron ore index**: Elliott impulsive-corrective cycle detected in the Continuous Commodity Index (CCI) by J. Kennedy and N. Isaac (ElliottWaveInternational.com) between 1980 and 2017 on an annual basis. This Elliott wave was used as the base for the simulation on **Section 5.4**.

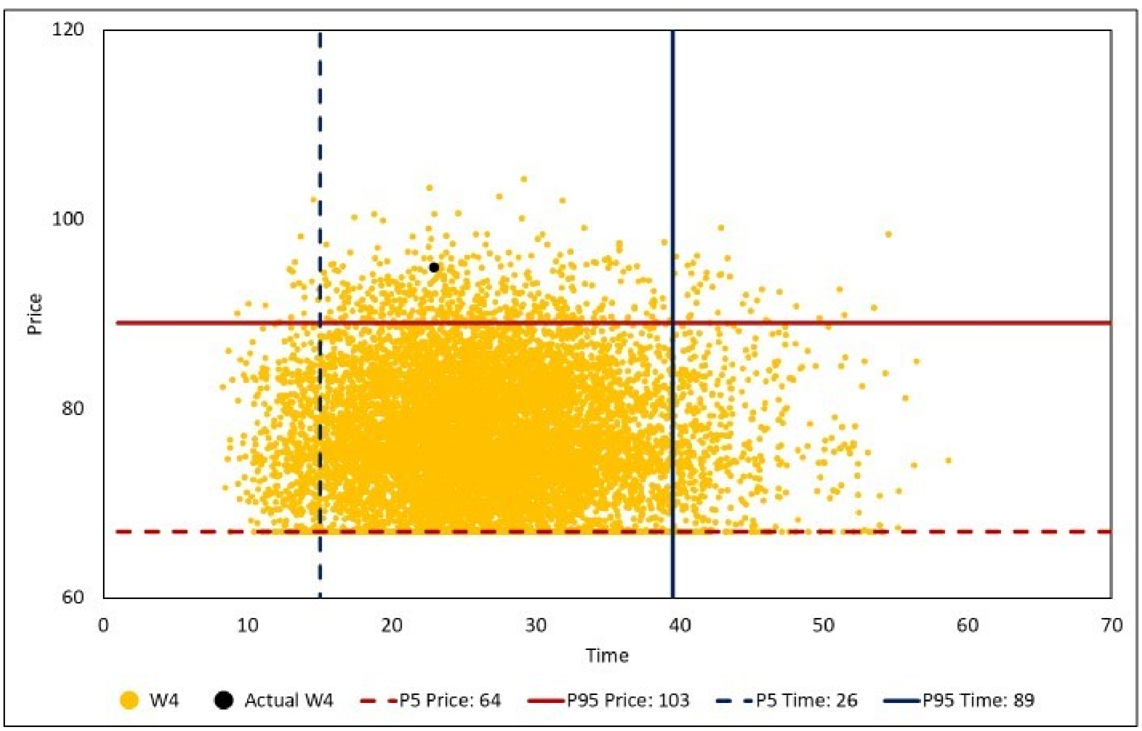


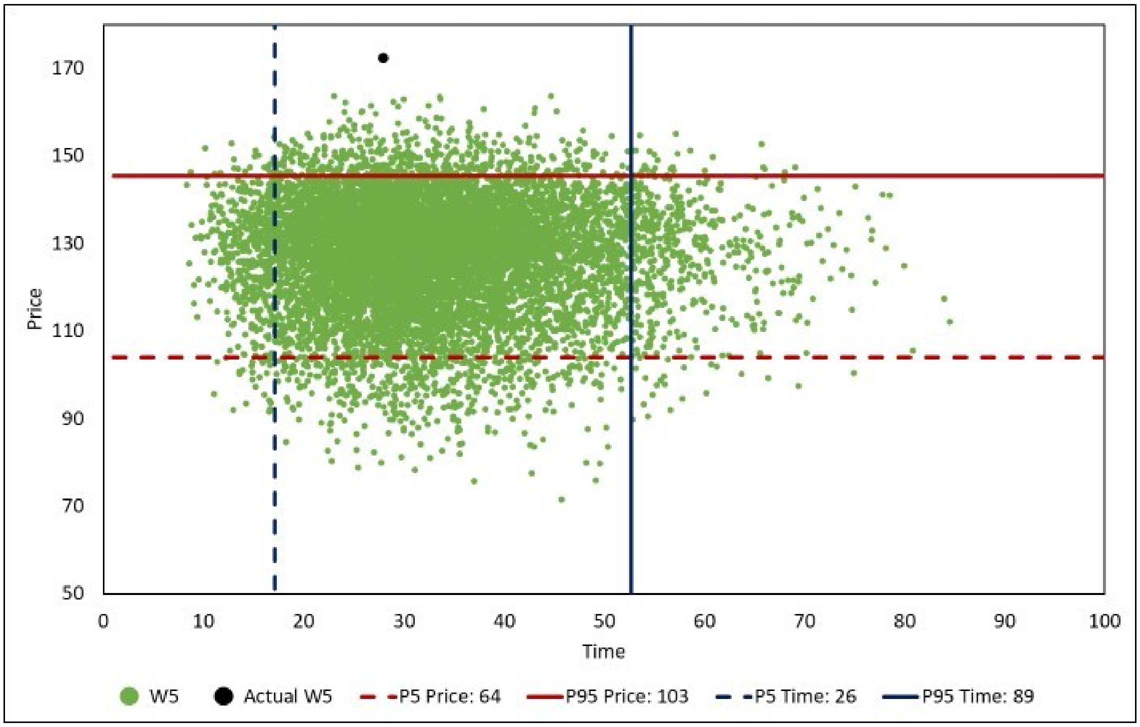
## Appendix 3: Simulated values of W2 to W5, and WII for gold, copper and BMII cases

**Gold Case:** Simulated values of W2, W3, W4, W5, and WII, along with their percentiles 5 and 95 for price and time amplitudes, for the gold case.

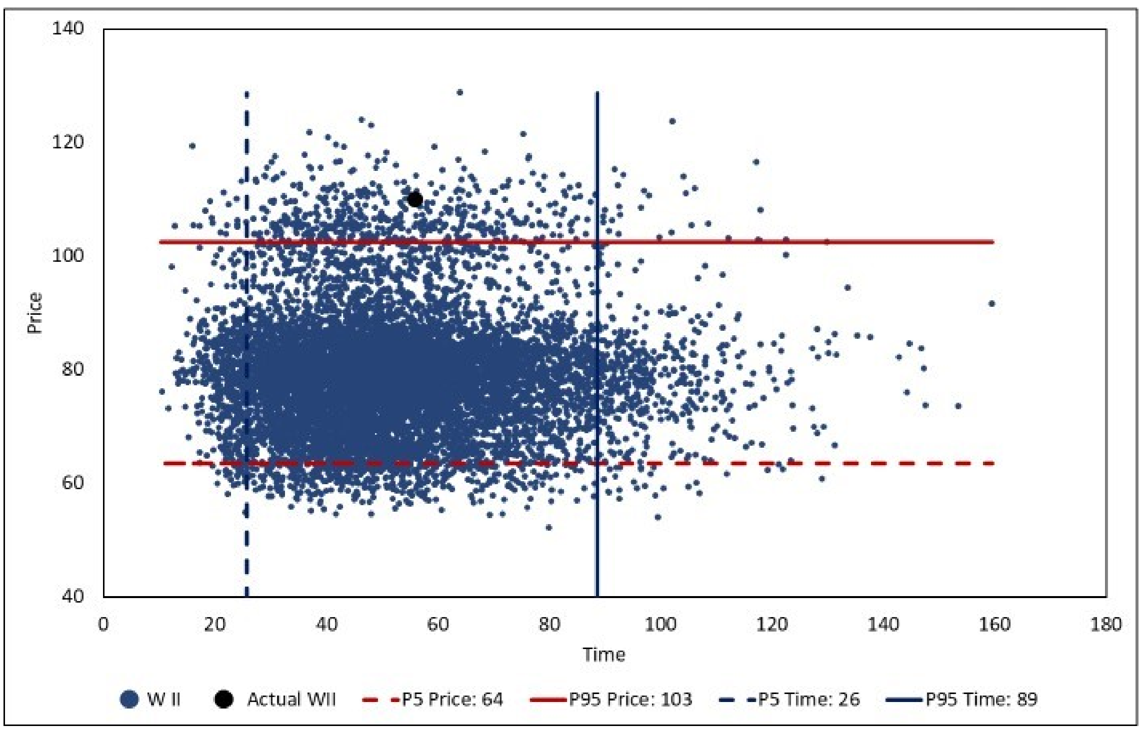



130
110
90
70
50
Price
0
5
10
15
20
25
30
35
40
45
50
Time
W3
Actual W3
P5 Price: 64
P95 Price: 103
P5 Time: 26
P95 Time: 89


120
100
80
60
Price
0
10
20
30
40
50
60
70
Time
W4
Actual W4
P5 Price: 64
P95 Price: 103
P5 Time: 26
P95 Time: 89

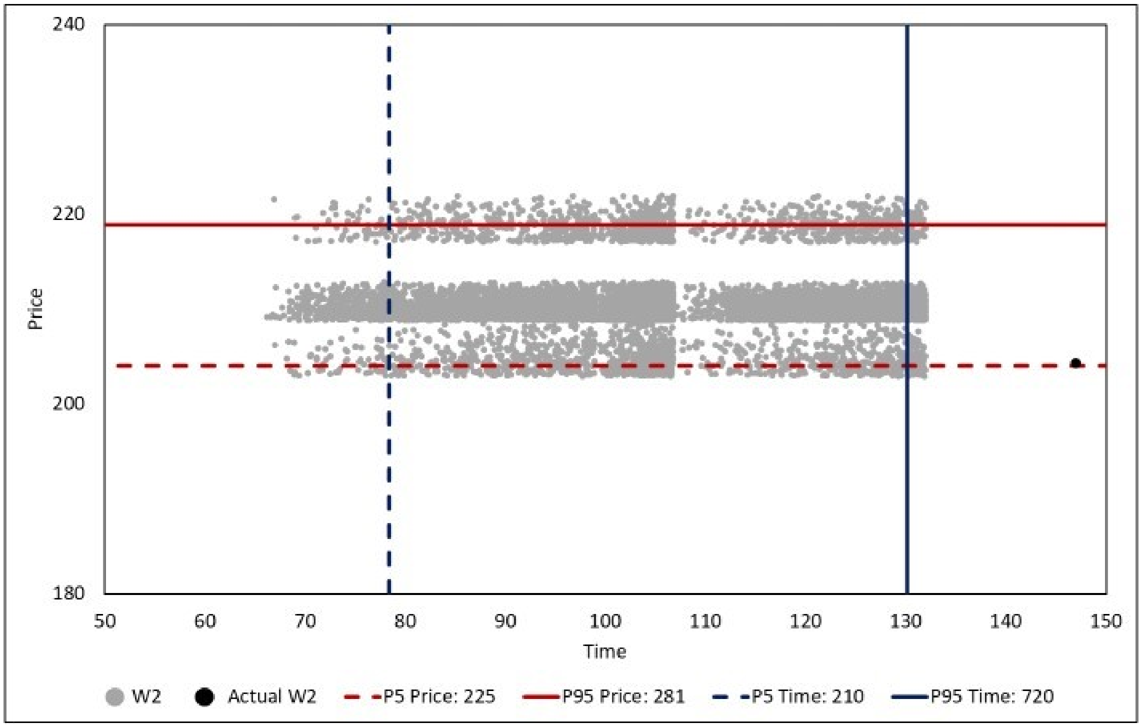


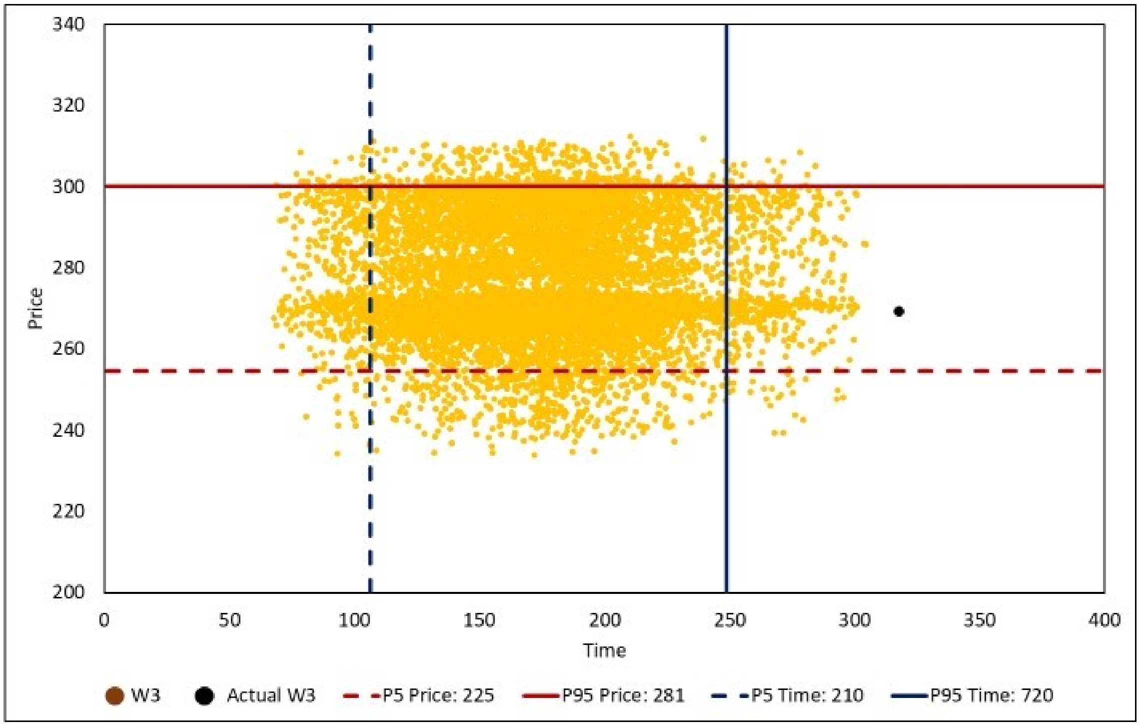
170
150
130
110
90
70
50
Price
0
10
20
30
40
50
60
70
80
90
100
Time
W5
Actual W5
P5 Price: 64
P95 Price: 103
P5 Time: 26
P95 Time: 89

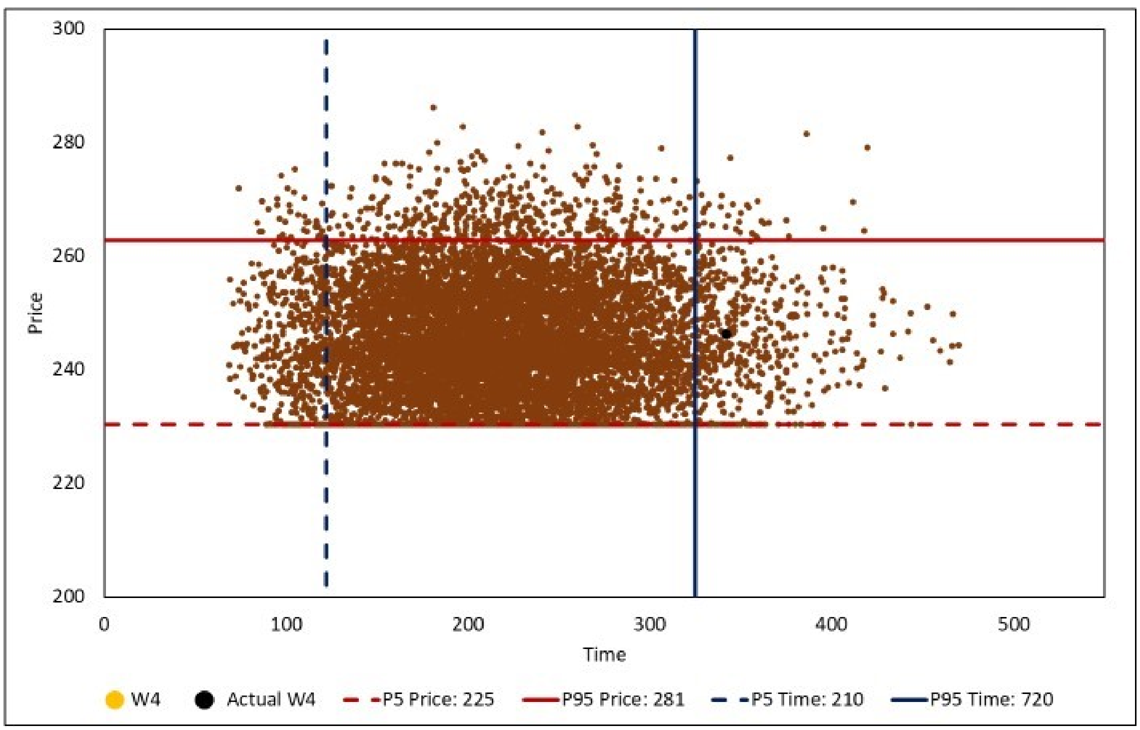



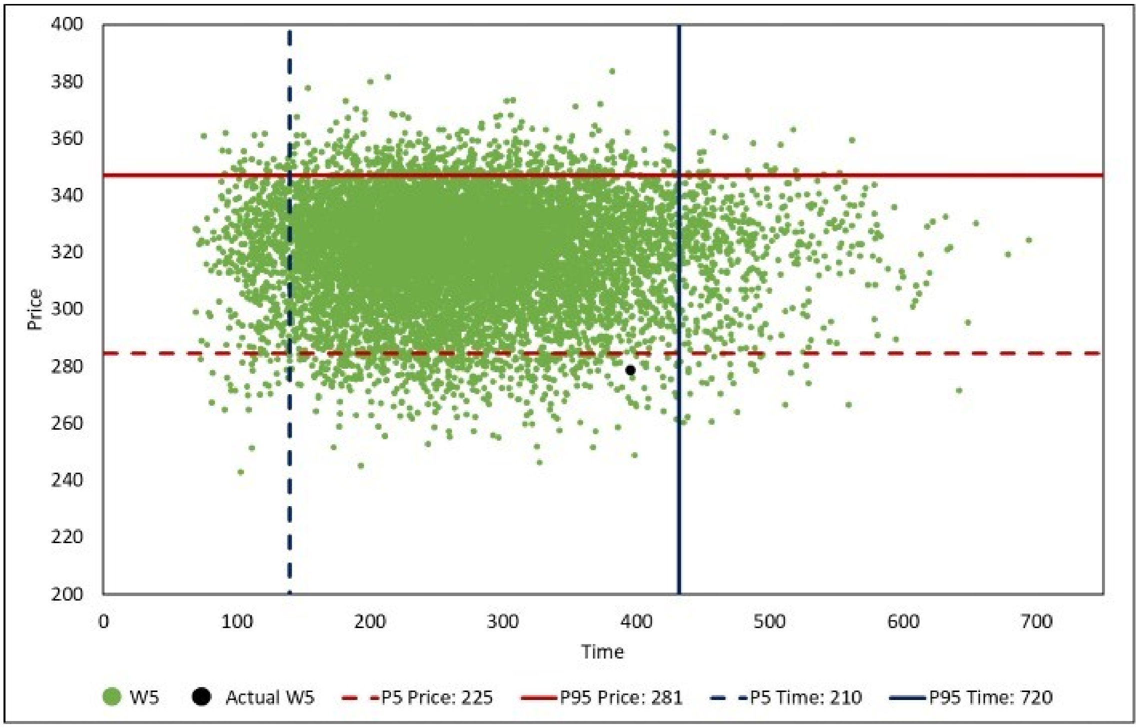
140
120
100
80
60
40
Price
0
20
40
60
80
100
120
140
160
180
Time
W II
Actual WII
P5 Price: 64
P95 Price: 103
P5 Time: 26
P95 Time: 89

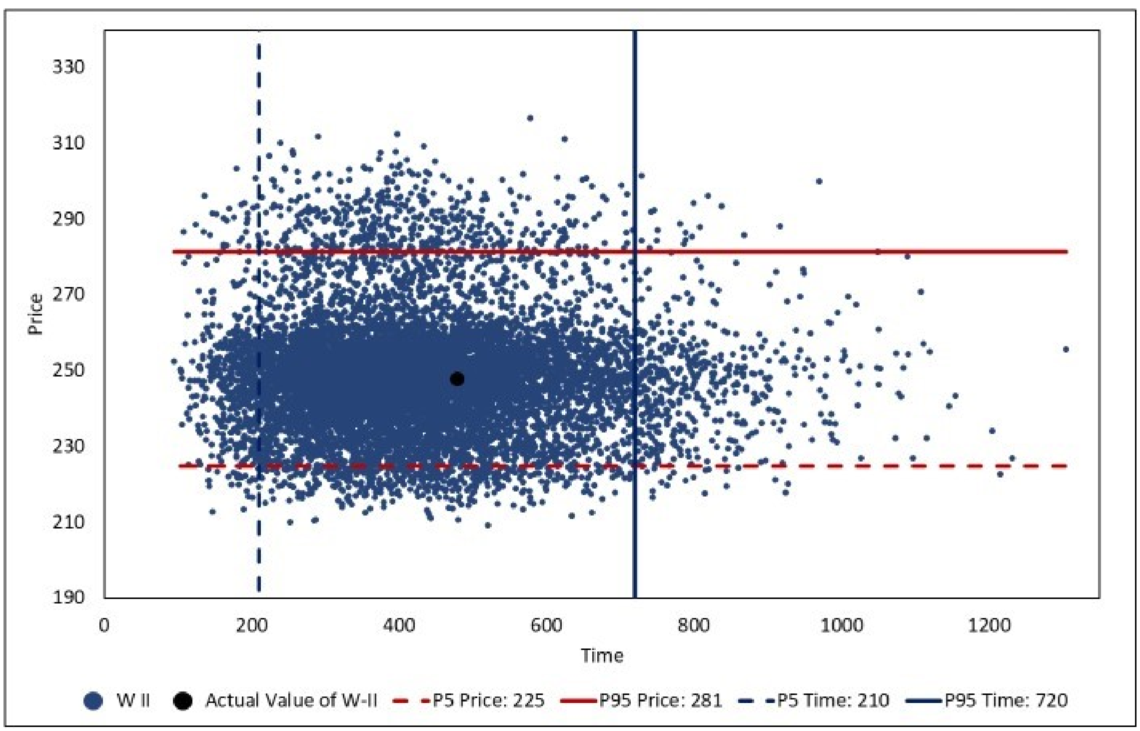
**Copper Case:** Simulated values of W2, W3, W4, W5, and WII, along with their percentiles 5 and 95 for price and time amplitudes, for the copper case.



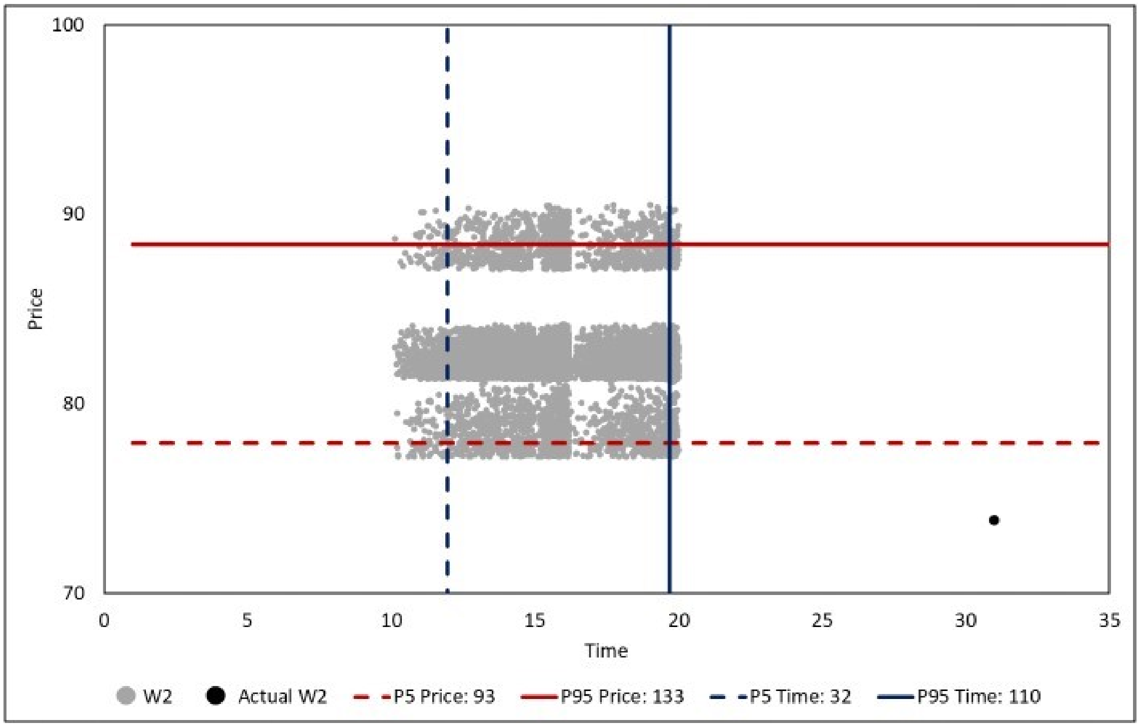

340
320
300
280
260
240
220
200
Price
0
50
100
150
200
250
300
350
400
Time
W3
Actual W3
P5 Price: 225
P95 Price: 281
P5 Time: 210
P95 Time: 720

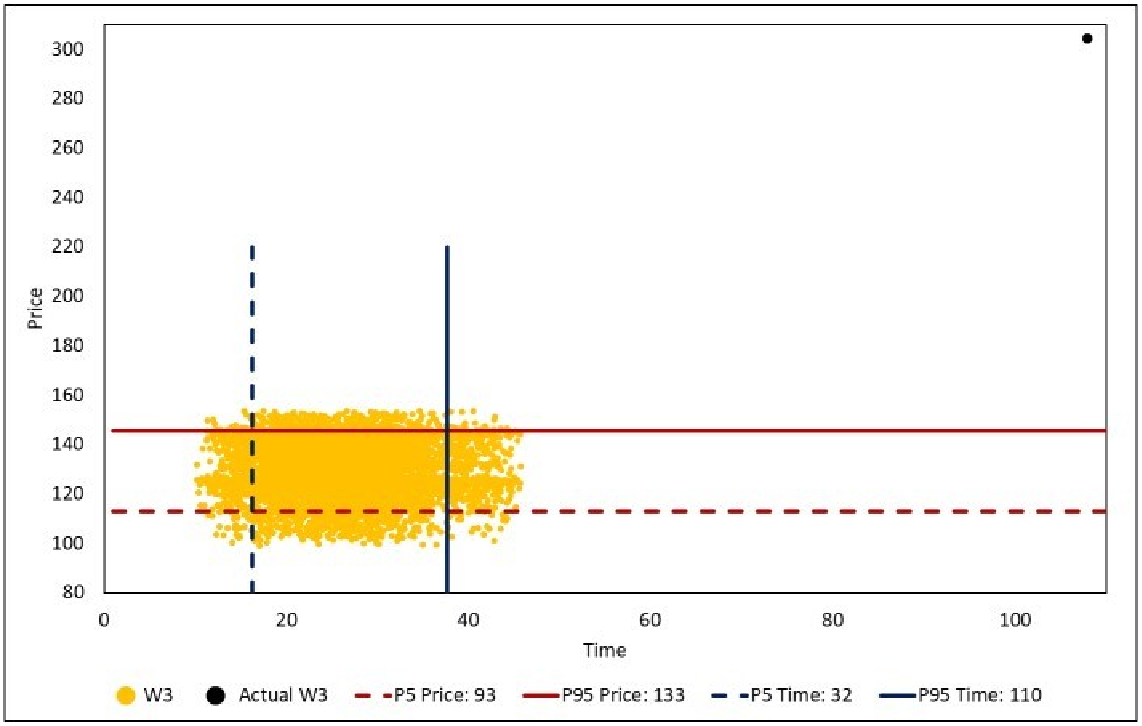

300
280
260
240
220
200
Price
0
100
200
300
400
500
Time
W4
Actual W4
P5 Price: 225
P95 Price: 281
P5 Time: 210
P95 Time: 720


Price
Time
W5
Actual W5
P5 Price: 225
P95 Price: 281
P5 Time: 210
P95 Time: 720


Price
Time
W II
Actual Value of W-II
P5 Price: 225
P95 Price: 281
P5 Time: 210
P95 Time: 720

**BMII Case:** Simulated values of W2, W3, W4, W5, and WII, along with their percentiles 5 and 95 for price and time amplitudes, for the BMII case.




300
280
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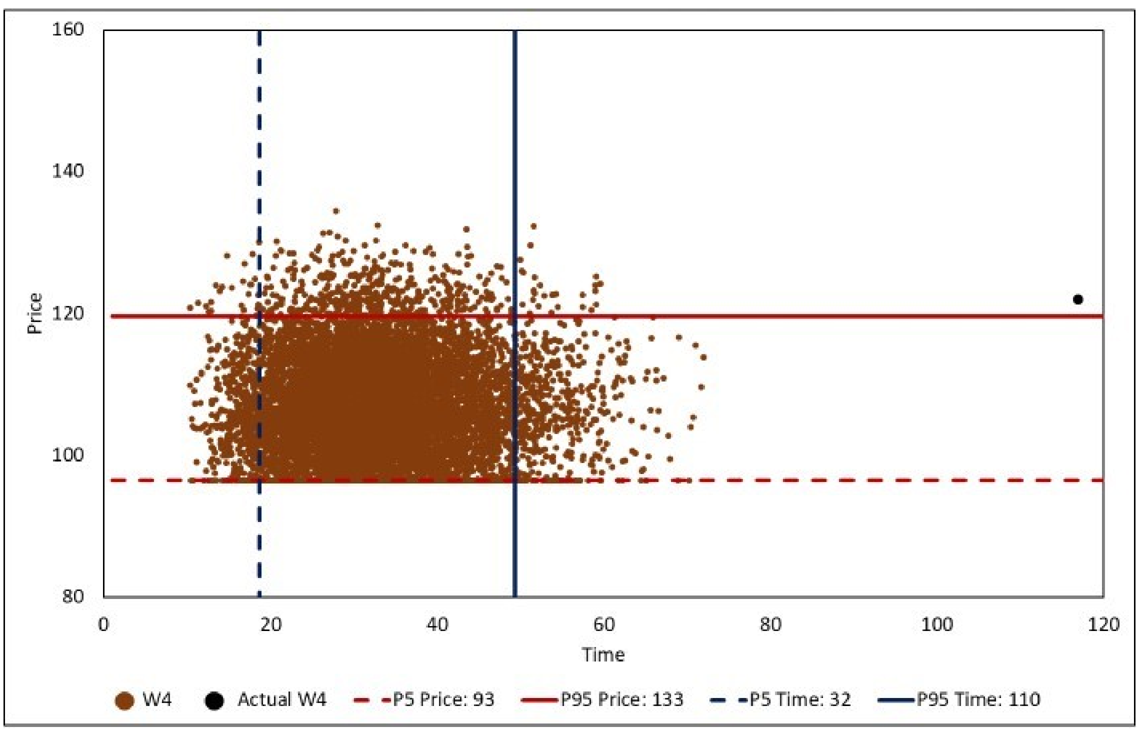
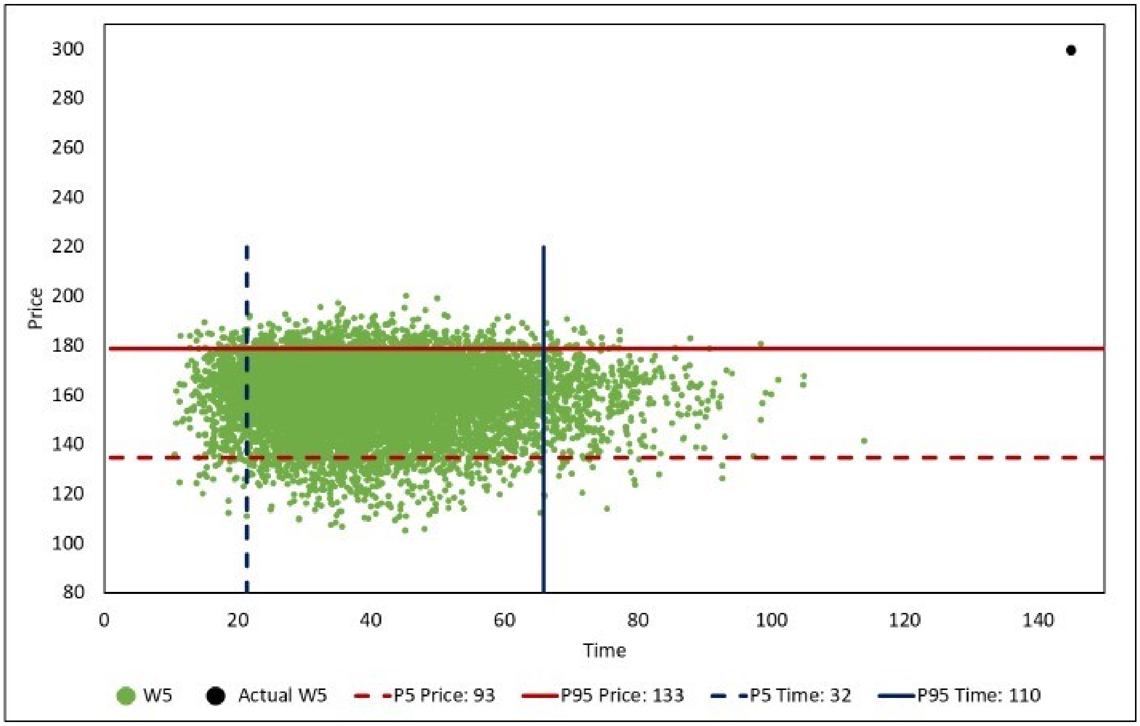
220
200
180
160
140
120
100
80
Price
0
20
40
60
80
100
Time
W3
Actual W3
P5 Price: 93
P95 Price: 133
P5 Time: 32
P95 Time: 110


160
140
120
100
80
Price
0
20
40
60
80
100
120
Time
W4
Actual W4
P5 Price: 93
P95 Price: 133
P5 Time: 32
P95 Time: 110



Price
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120
100
80
0
20
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Time
W5
Actual W5
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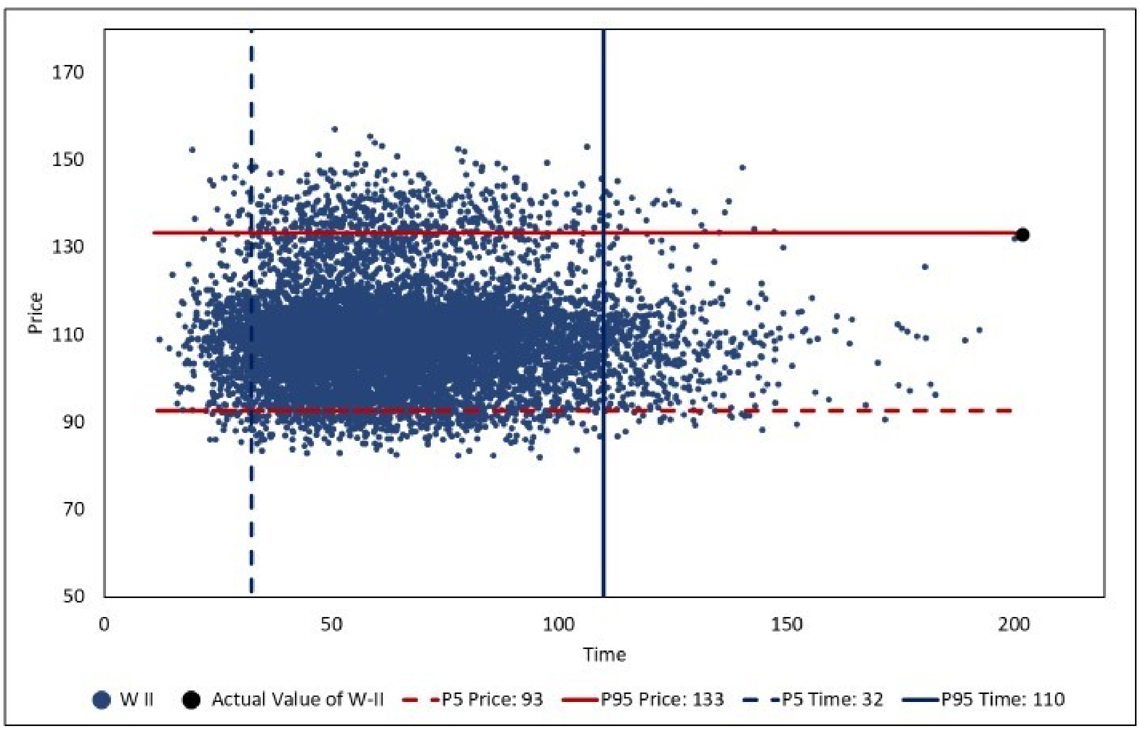
P95 Price: 133
P5 Time: 32
P95 Time: 110

Price
170
150
130
110
90
70
50
0
50
100
150
200
Time
W II
Actual Value of W-II
P5 Price: 93
P95 Price: 133
P5 Time: 32
P95 Time: 110