A DISCOUNT CASH FLOW AND REAL OPTION VALUATION APPROACH for IRON ORE MINING COMPANY

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ABSTRACT

Discounted Cash Flow (DCF) analysis is simply straightforward analysis with the assumption that mining investment can proceed if the Net Present Value (NPV) of all its future cash flows discounted to the present is positive. In most circumstances the use of a relatively simple static DCF valuation, along with a small number of key sensitivities such as prices, operating cost, discount rate, interest rate, and discrete scenarios, is sufficient to guide mining investment decisions. Some of the major drawbacks of DCF analysis lie with its static nature as it does not consider managerial flexibility. DCF analysis does not consider the cyclical nature of the mining industry especially market and margin uncertainty, nor does it consider the long-term impact of sovereign risk. It does not consider grade and structural uncertainty.

In some projects, the use of quantitative methods to obtain additional insight into project risks is encouraged. These tools can assist project teams to prioritise identified risks, evaluate the overall effect of identified risks on predicted project outcomes, set realistic project targets, validate project schedules and cost estimates, and help quantify appropriate levels of project contingency.

Uncertainty and competition within the market affects timing of investment for iron ore mining companies. The concept of real options has shed light on managerial

flexibility of irreversible investment under uncertainty in contrast to the DCF models.

Real option analysis does not necessarily replace DCF analysis but rather complements it. As pointed out before, and as will be evident throughout this thesis, the application of real option theory builds on DCF and the underlying concepts integrating them into a new valuation paradigm. Real option analysis considers various uncertainties such as prices, discount rate, and correlation associated with mining investments.

In this research, a case study is carried out on an iron ore deposit. The main contributions to this thesis are: iron ore price modelling; stochastic correlation using Jacobi process between prices and risk discount factor; incorporating Real Option Valuation (ROV) (managerial flexibility) into strategic mine planning; and comparison between Modern Asset Price (MAP) and DCF analysis.

The case study reveals that iron ore price does revert to the long term mean equilibrium and that prices and risk discount factor do correlate. It is shown that managerial flexibility adds value to resource development, and, with the inclusion of risk adjusted prices and risk discount factor to DCF analysis, it does add value to the project. That simply defines MAP as a complement to DCF.

Keywords:

Discounted Cash Flow (DCF), Real Option Valuation (ROV), Geometric Brownian Motion (GBM), Mean Reversion, Correlation, Modern Asset Pricing (MAP)

RÉSUMÉ

L’analyse de flux de trésorerie actualisé (DCF) est une analyse assez simple selon laquelle l'investissement minier peut s’effectuer si la valeur actuelle nette (NPV) de l'ensemble des flux de trésorerie futurs actualisés selon la valeur présente est positif. Dans la plupart des cas, l'utilisation d'une évaluation DCF statique relativement simple, avec un petit nombre de sensibilités clés tels que les prix, les coûts d'exploitation, le taux d'actualisation, le taux d'intérêt et des scénarios discrets sont suffisants pour guider les décisions d'investissement minier. Quelques-uns des inconvénients majeurs de l'analyse DCF viennent de sa nature statique, car elle ne tient pas compte de la souplesse en matière de gestion. L’analyse DCF ne tient pas compte de la nature cyclique de l'industrie minière, plus particulièrement le marché et la marge d'incertitude, et elle n’examine pas l'impact à long terme du risque souverain. Elle ne tient pas compte de la teneur et de l'incertitude structurelle.

Dans certains projets, l'utilisation de méthodes quantitatives pour obtenir des informations supplémentaires sur les risques du projet est encouragée. Ces outils peuvent aider les équipes de projet à hiérarchiser les risques identifiés, évaluer l'effet global des risques identifiés sur les résultats prévus, fixer des objectifs réalistes pour leurs projets, valider les calendriers des projets et les estimations

de coûts, et aider à quantifier les niveaux de contingence appropriés pour les projets.

L'incertitude et la concurrence au sein du marché affectent le calendrier des investissements pour les entreprises d'exploitation de minerai de fer. Le concept des options réelles a mis en lumière la souplesse en matière de gestion de l'investissement irréversible en cas d'incertitude contrairement aux modèles DCF.

L'analyse des options réelles ne remplace pas nécessairement l'analyse DCF, elle vient plutôt en complément. Tel que souligné précédemment, et comme cela sera évident tout au long de cette thèse, l'application de la théorie des options réelles se construit plutôt sur celle de la DCF et ses concepts sous-jacents en les intégrant dans un nouveau paradigme d'évaluation. L'analyse des options réelles tient compte des diverses incertitudes tels que les prix, le taux d'actualisation et la corrélation en lien avec les investissements miniers.

Dans le cadre de cette recherche, une étude de cas a été réalisée sur un gisement de minerai de fer. Les principaux apports de cette thèse sont: une modélisation de prix du minerai de fer; une corrélation stochastique utilisant le procédé Jacobi entre les prix et le facteur de réduction des risques; l'intégration de la valorisation d'option réelle (ROV) (souplesse en matière de gestion) dans la planification

stratégique de la mine; et la comparaison entre l'évaluation des actifs modernes (MAP) et l'analyse DCF.

L'étude de cas révèle que le prix du minerai de fer revient effectivement à son équilibre moyen à long terme et qu'il y a une corrélation entre les prix et le facteur de réduction des risques. On démontre que la souplesse en matière de gestion apporte une valeur ajoutée au développement des ressources et avec l'inclusion des prix ajustés au facteur de risque et le taux de risque actualisé à l'analyse DCF, on ajoute de la valeur au projet. Cela définit simplement MAP comme un complément à la DCF.

Mots-clés: Flux de Trésorerie Actualisés (DCF), Valorisation D'options Réelles (ROV), Mouvement Brownien Géométrique (GBM), Rendement Moyen, Corrélation, Évaluation des Actifs Financier Modernes (MAP).

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DCF	Discounted Cash Flow
DR	Discount Rate
IRR	Internal Rate of Return
NI	National Instrument
JORC	Joint Ore Reserves Committee
MAP	Modern Asset Pricing
NPV	Net Present Value
PB	Payback Period
RADR	Risk Adjusted Discounted Rate
ROV	Real Option Valuation
RDF	Risk Discount Factor
CFS	Concentrate for Sale
GIOC	Gardner Iron Ore Company

CHAPTER 1: INTRODUCTION

Mineral project valuation plays a key role as companies operate, expand, merge, and acquire other properties. Mineral project evaluation involves understanding the key project drivers, as well as quantifying the level of risk to determine project viability.

The mining industry requires large capital investment to start a project. Most project study stages are used to evaluate mineral projects and develop a deposit. The trend of the studies starts from scoping studies/order of magnitude studies (±50% accuracy), prefeasibility studies (±35% accuracy), feasibility studies (±25% accuracy), and bankable feasibility (±15% accuracy).

These project study stages are required to be undertaken in line with international codes, such as Joint Ore Reserves Committee (JORC) or National Instrument (NI) 43-101 if they are publicly listed companies, or funded by a bank.

In any mineral project the important areas of evaluation include licensing, exploration, orebody knowledge, mining, processing, infrastructure, hydrogeology, environment, capital and operating costs, and economic analysis and risk evaluation. Economic analysis and risk assessment are very significant as the former defines the final economic value and the latter the risk for both the company and investors.

Traditional economic analyses of mineral projects are commonly based on a traditional Discounted Cash Flow (DCF) methodology. A cash flow is designed to capture all cash inflows and outflows over the life of a project and avoid inclusion of non-cash accruals. The cash flow model must recognize the time-value of money by discounting at a suitable discount rate to obtain their Net Present Value (NPV).

1.1 Problem Definition

Traditional DCF analysis is the most widely utilized investment valuation tool, it has several shortcomings which make it a less than ideal choice for analyzing development investments (Dixit & Pindyck, 1994). Dixit and Pindyck (1994) documented three drawbacks that make valuation of a potential investment via the DCF approach less accurate: (1) when the capital cost of the investment is not irreversible, (2) when there is uncertainty in the expected future expected cash flows, (3) when no managerial flexibility is accounted for in the timing of the investment.

The difference between a project's NPV and its actual expected NPV grows broader as each of the above drawbacks increase in magnitude. DCF analysis assumes that investment expenditures can be completely recovered if the potential of an investment cannot be achieved, or if the investment is assumed to be non-recoverable, then it becomes a now or never decision. When irreversibility, uncertainty, and timing is taken into account in mining investment decisions, it gives options to decision makers thereby changing the investment rules of the "traditional DCF investment model." (Dixit & Pindyck, 1994).

The opportunity cost of dismissing an option, by taking the investment decision and not updating with current information, must be included in the NPV rule: "when the NPV of a mining project breaks even or NPV is greater than zero then you can invest." The current value of future cash flows must be greater than the current value of investment cost as well as the present value of keeping the investment option viable (Dixit & Pindyck, 1994).

The motivation of the topic and the following problem statement originates from my interest in mineral economics and iron ore mining industry. The common literature on DCF and Real Option Value (ROV) has always been associated with commodities such as gold and copper. The recent increase in demand for iron ore has increased the importance of this matter and the need for sufficient decision making tools. Due to the insufficient decision making tools for iron ore investment, I have focused my research on market and margin uncertainty, correlation between prices and risk discount factor, resource and technical uncertainty, managerial flexibility, modern asset pricing in relation to iron ore investment.

1.2 Scope and Objectives

a. To show the merits of real options in mine valuation.

b. To model multiple variables in through mean reversion process (i.e., price and discount rate).

c. To treat correlation between price and discount rate stochastically through Jacobi process.

d. To introduce real options into iron ore investment analysis to increase profit and operation efficiencies.

e. To reproduce iron ore price cycles.

f. To advance in finding the most proper values of parameters (i.e., volatility, drift, reversion speed etc.) used in stochastic process.

g. To incorporate managerial flexibility into strategic mine planning rather than using binomial tree analysis.

h. To compare Modern Asset Pricing (MAP) with DCF evaluation.

i. To prepare companies for future requirements of NI43-101

The case chosen is a real world project in the Canadian iron ore mining industry. Due to a confidentiality agreement with the company, the name of the company is modified. Microsoft Excel, @Risk, and COMET Strategic Mine Planning Tool are the software used to complete the work on the case studies. Though five components of ROV were mentioned, only three were considered in the research.

Sovereign Risk and Resource and Technical Uncertainty were not considered due to the limited data available for this research.

1.3 Originality and Success

This thesis concentrates on developing new mine planning practices using multiple stochastic processes and real option valuations.

- Price, discount rate and correlation were uncertain parameters. In this research price and discount rate were modelled by mean reversion. Correlation between price and discount rate were modelled by Jacobi processes. In other words these three uncertain parameters were treated by two different stochastic processes. This is the more realistic approach to reproduce actual case.
- Incorporating these three models with managerial flexibility into real option valuation. Two options were considered if (i) new deposit should be mined or not, (ii) mineral processing facility capacity should be expanded or not.
- Using price, discount rate, and correlation models into modern asset pricing model to compare with traditional discounted cash flow model.

The developed models and real option approaches were tested through real case studies to observe the performance of the proposed techniques. It was concluded that;

- the proposed stochastic models showed that iron ore prices revert to long-term mean

- managerial flexibility added value such that profit increased
- modern asset pricing assisted to understand the risk associated decision making on investment

1.4 Social Impact and Economic Benefits

Since the traditional approaches undervalue the projects, the findings will help companies to understand the real value of their assets. Hence, companies will have new means to assess their projects. Mining operations over the years have been one of the biggest contributes to employment opportunities to local people. Under current low prices, proposed approach can allow the companies to stay in business. The proposed approach will help junior mining companies to maximize project value under market uncertainty.

1.5 Thesis Outline

This thesis consists of nine chapters organized according to the following description:

- Chapter 1 contains an introduction about mineral project evaluation, the problem definition, scope, and objectives of the thesis.
- Chapter 2 presents a literature review on the dynamics of valuation in the iron ore mining industry and a description of the common risk and uncertainty associated with this valuation.

- Chapter 3 presents a literature review on traditional discount cash flow analysis and a description of the common risk and uncertainty associated with this valuation.
- Chapter 4 presents a literature review on real option valuations and a description of the five components of ROV.
- Chapter 5 presents the concept of modelling iron ore prices under uncertainty. This chapter is key input for the two case studies captured in the thesis report.
- Chapter 6 presents an overview of resource and technical uncertainty. This chapter is more of a literature review because of the lack of data available for research purposes.
- Chapter 7 presents the concept of managerial flexibility in a case study. This chapter shifts the paradigm of managerial flexibility techniques used in past research papers.
- Chapter 8 presents a comparison of MAP with DCF in a case study. This chapter uses output from Chapter 5 as an input for the MAP analysis. Both MAP and DCF undergo risk analysis to quantify the evaluation results.
- Chapter 9 presents the conclusions of the thesis and recommendations for future research. After this section, the list of the references and important

data used to conduct this research is included in the bibliography and appendix sections.

CHAPTER 2: DYNAMICS OF VALUATION IN THE IRON ORE INDUSTRY

2.1 Iron Ore Prices

The most important factor affecting the iron ore industry is the price of iron ore. It is impossible to foresee the level of future iron ore prices. The iron ore price is determined by demand and supply. BHP, Rio Tinto, and Vale are the bigger players of the iron ore market and together contribute about 66% of the total iron ore production in the world as shown in Figure 2-1.

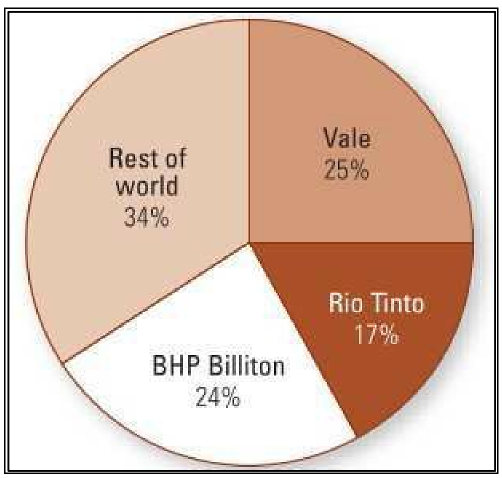

Figure 2-1: Seaborne Iron-Ore Total Trade Market by Supplier in 2009

(Source: ATKEARNEY. “Steel’s Challenge – Living with Higher and More Volatile Iron-Ore Prices”)

Contracts of iron ore trade are controlled by the big three (BHP, Rio Tinto, and Vale). Most junior iron ore companies use iron ore swap platforms to trade iron ore which are mostly bid and ask trades. The swap platform allows iron ore producers (mining companies) to ask for a selling price of their products based on quality and at the consumers (steel makers) bidding until an agreement is reached. The agreed price becomes the spot price for the day. Various institutions such as steel makers, governments of iron ore importing countries, and firms, all have the potential to influence iron ore price levels as well as the global economy, expectations, and unexpected events such as natural disasters and weather conditions.

2.2 Supply and Demand

Supply and demand are one of main determinants of the price of a good. The other explanation for demand and supply is by looking at how producers and consumers interact. The supply and demand model depends on a high level of competition, meaning that there are enough producers and consumers in the market for bidding to take place. Consumers bid against each other and thereby raise the price, while producers bid against each other and thereby lower the price. The law of demand states that when the prices of goods rise, and everything else remains stable, the magnitude of demand will fall.

Supply is not just the amount of something there, but the willingness and ability of potential producers to produce and sell it. The law of supply states that “when the price of a good increases, and everything else remains the same, the amount of the good supplied will also surge.”

China has been emerged as the world’s largest steel producer. The reason behind this was its fast track urbanization. Approximately 20% of steel usage is for residential buildings and over 50% was accounted for by the construction sector (Robert and Anthony

2012). Most of the China's domestic iron ore is much lower than a standard 63.5% grade. To meet the basic requirement of 66% grade, most high grade is imported from overseas making China dependent on overseas mining companies (Ma et al., 2013).

China consumes 70% of the world's iron ore due to its economic boom. The high demand for iron ore from 2008 to 2011 brought iron prices to a historical high level. The pace and growth of the global economy affects the demand for iron ore. Due to the current overproduction of iron ore, this has created a bubble in China and for that reason prices have decreased significantly. Both Europe and North America's demand for iron ore has decreased over the years due to the current economic crisis. Companies like Tata Steel and AcerlorMittal supplies their own iron ore from their mining sites to their steel factories.

2.3 Technology

As demand for iron ore increases, most exploration will discover extractable iron ore reserves in new and old areas. Existing and future technology makes it possible to undertake projects in unknown areas with more difficult conditions. Examples include companies such as Labrador Iron Mines reopening the abandoned Iron Ore of Canada Directing Shipping Ore (DSO) mines at Schefferville, Quebec, due to improvements in processing technology in the mining industry. Rio Tinto iron mines at Pilbara, Western Australia, are currently using autonomous trucks that are controlled from Perth. This helps to bridge the shortage of truck drivers, thereby improving and increasing productivity to meet market demand. As technology improves, resources are degraded. This requires more research and also becomes expensive. History has documented it well, latest technologies are always expensive. Technology represents an uncertain factor in pioneer projects where new technology and new concepts play important roles. Investments in technology and the possibility of failure and delay make technology an uncertain factor.

2.4 Costs and Inflation

The level of capital investment (CAPEX) and operating costs (OPEX) are factors representing uncertainty in the evaluation of iron ore projects. When iron ore demand increases, ore price also increases. Thus, most companies expand or explore new resources to meet the demand. The demand for high salary by highly skilled labour and the increase in cost of equipment due high iron ore price, increases cost significantly. As commodity prices increases the cost of labour and equipment increases. The level of costs influences the profitability of projects.

According to Emhjellen and Osmundsen (2002), one might expect that cost overruns have the same probability as completing projects below cost estimates. However, the authors argued that observations clearly indicated an over representation of cost overruns. This may be a consequence of two selection biases: (1) Project selection, it is typically the projects with the most positive internal cost estimates that are being trailed by the investing firm; and (2) Tender selection, opposition sees to it that tenders with negative and representative cost estimates are ruled out. The authors discussed the importance of accurate cost estimates and the implication of these estimates on investment decisions.

Uncertainty regarding inflation over longer life of mine creates difficulties in estimating future levels of revenues and costs (Kvaleg, 2009).

2.5 Mineable Iron Ore Reserves

In the iron ore industry the exact amount of mineable iron ore reserves is one of the main uncertainty factors. Mineable iron ore reserves are forecasted by converting resources to reserves based on the level of confidence of drill hole information as well as economics. The conversion of resource to reserve is guide by international stand codes such as JORC, NI 43-101 etc. Figure 2-2 is an example of mineral resource to reserve conversion by NI 43-101 code standards.

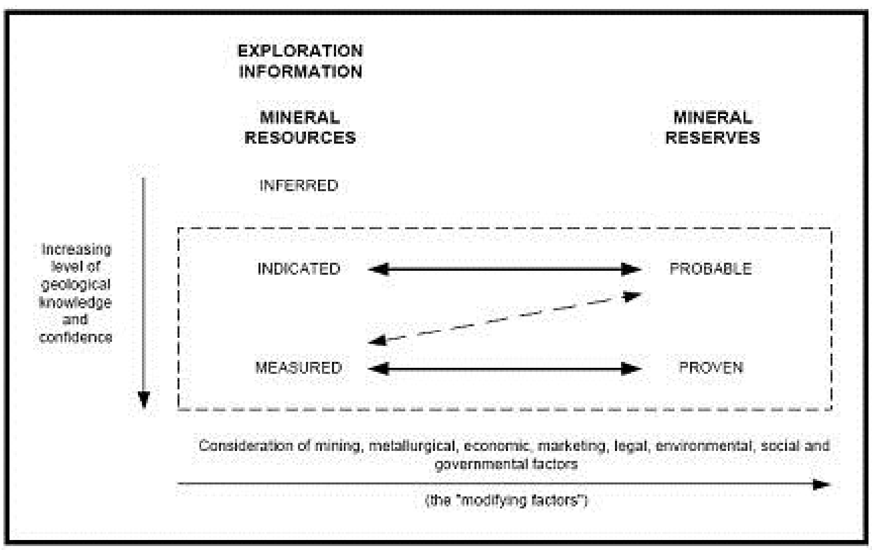

Figure 2-2: Conversion of Mineral Resources to Mineral Reserves

(Source: http://www.cim.org/committees/cimdefstds_dec11_05.pdf)

Human errors during core logging, sample preparation errors and grade uncertainties may be present in the classification of the resources. Due to these errors and uncertainties, the actual amount of extractable reserves may turn out to be less than predicted. Predictions about production rates are subject to the same uncertainty as predictions about the amount of iron ore reserves. The level of confidence based on drill-hole distance as well as the knowledge of the ore body. Predictions and calculations about the total amount of extractable reserves and production rates are important inputs in valuations of mining developments. Wrong inputs may lead to bad investment decisions and unprofitable projects. The conversion of a resource to reserve takes into consideration price of commodity, cost of mining and processing, and cut-off grade.

2.6 The Issue of Time

Iron Ore mining project developments are often characterized by long time horizons commonly known in the mining industry as life of mine (LOM). The mines of most companies have been actively mined for over 30 years. BHP has been mining iron ore at the Mount Whaleback Operations, Pilbara, Western Australia since 1968 and is still expanding as well as increasing annual production rates. Rio Tinto has also been mining at their Mount Tom Price operation at Pilbara since 1966.

The challenging factor for evaluating projects with huge reserves and mineral inventory is discount rate. Though many analysts have come up with different rules of thumb for evaluating such projects, there is still no sufficiently certain method. Discount rate varies over the life of mine but traditionally most companies have a treated it as a single static parameter for valuating mineral projects. This has hurt most projects before it begins. Discount rate is an uncertain parameter and must be treated as such.

2.7 Prospects and Licenses

Geographic areas with potential mineral resources must be approved by the federal and provincial government before any mining activity can find place. When specific areas are approved, the government issues a licensing round where interested mining companies can apply for the areas.

Licenses contain both rights and obligations. It gives holders an exclusive right for exploration and exploration drilling. After companies have enough drilling information to progress to mining, they have to attain a mining licence. Companies have to convert from mining licenses to leases within a timeframe depending on the provincial laws. Each license also contains a specified work obligation to be met by the holders within a specified period of time. These obligations may include surveys and exploration drilling. If all the licensees agree, the license can be given back to the government after work obligations have been met.

Considerations should be made regarding beliefs and investments needed to be taken regarding geophysics data acquisitions and surveys and/or exploration drilling in order to meet specified obligations in the license. By applying for a lease, companies agree to make investments that could potentially uncover profitable mining reserves. This phase of any mineral development is uncertain as it can be treated as an opportunity cost.

Companies can lose lot of money as in trying to convert from licence to lease. There is no guarantee of capital investment from investors either if a company converts a licence to lease.

2.8 Resources under Evaluation

The aim of exploration drilling is to obtain answers about the presence of iron ore resources, the size of the resource, and the quality of the grades. These indicators determine whether a project should be taken to the next stage, which is development. Holders of licenses must make decisions about further exploration drilling, waiting, abandonment, or development. A final decision about development should incorporate economic, technological and environmental considerations. Decisions about iron ore developments confiscate both financial resources and labour for a long time horizon, and should be given great attention.

2.9 Resources under Development

A decision about developing an iron ore project means that investments in production facilities, necessary equipment, and infrastructure must be made. If the mine is located close to existing mining companies, it may be possible to take advantage of existing infrastructure and facilities such as roads, air strips, and power, etc. One of the biggest challenges for most iron ore companies is transportation paths either to port or to consumers due to the commonly remote locations of mines. The process of developing, engineering, building, and installing technological and practical solutions may be time demanding. This results in a period of high investments and negative cash flows. This period is followed by possible positive cash flows as production gets going.

2.10 Resources in Production

The common mining production profile involves ramping to a steady production and then a decline in production towards the end of mine life, unless new resources are discovered during the intervening steady stage. Different development concepts and production rates can give different production profiles. As the mine progresses, different techniques are used to maintain as steady a production as possible. This often results in increasing marginal costs and decreasing profits per tonne of ore mined. Decisions about the shutting down and abandonment of a mine should be made on the basis of the total amount of extractable reserves left, production rates, iron ore price, costs of bankruptcy, and alternative investment and resource distribution opportunities. When the cost of production is higher than the price of iron ore, production should be shut down and the company should abandon the mine for a time, possibly forever. An example is Cliff Resource Iron Ore Mine at Wabush, Labrador and Newfoundland, Canada.

2.11 Closure Cost

Holders of mining licenses are responsible for shutting down production and the bankruptcy of developed facilities. Closure costs and accomplishment of bankruptcy should be taken into account and estimated already at the time of valuation.

2.12 Phases of Iron Ore Mining Developments

There are four main phases of all mining development, including iron ore mining development: exploration studies, engineering, and construction and production.

Mining companies normally start with an exploration program that includes drilling to define resources and obtain geological structural information. Exploration drilling can go on for years and, based on the information obtained, helps companies to move to the next phase of mining development.

Mining studies are the next phase after exploration development. Most companies normally contract mining companies to complete studies ranging from conceptual studies to bankable feasibility studies. Companies can choose to continue to the next phase of the project depending on the outcome of the studies.

Detailed engineering is the last phase before construction. It involves detailed designing and commissioning. Construction and production is the last phase of mining development.

2.13 Summary

In this Chapter, the dynamics of valuation led to uncertainty and risk in iron ore industry were reviewed. These risks and uncertainties are key when evaluating iron ore projects.

CHAPTER 3: LITERATURE REVIEW ON DISCOUNT CASH FLOW ANALYSIS

3.1 Introduction

DCF is the most popular and applied methodology used by analyst for mineral project valuation. The first to derive present value calculations as an ordinary economic outcome in wealth calculation, and to justify maximization of present value as the goal of production was Irving Fisher. The determinants of interest rates to calculate present value was also derived by Fisher (Fisher 1930).

DCF analysis can be broken into two parts, (1) the net present value (NPV), and (2) the internal rate of return. Although these two methods have similarities, there are also some important differences. The following sections explains the components of DCF analysis as well the differences between NPV and IRR.

3.2 Components of DCF Analysis

Traditionally, DCF analysis has been subjected to forecasting future cash flows and discounting the cash flow back to the its present value over time. With this process, a discount rate is applied and this represents uncertainty and risk. The sole purpose of this analysis is to help investors understand the expected present value of future income and cost in comparison with investment cost. The variance between the present value of future income and the project's investment costs is the projects expected NPV. The standard expression for NPV calculation is show in Equation (3.1).

$$NPV = C_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r)^t} \qquad (3.1)$$

3.2.1 Capital Investment - C_0

Capital investment have to be made in order to develop any mineral project. The common investment for mineral projects involves exploration, order of magnitude/scoping studies, pre-feasibility studies, feasibility studies, detailed engineering, facilities for production, processing plants, equipment's, and camps. Some of these investments are normally classified as sunk cost and are not used in the DCF analysis. The bigger as well as remote the project is the higher the investment required.

3.2.2 Cash Flows – C_t

Future cash flow must be forecasted in order to calculate NPV. There are multiple factors that contribute to forecasting future cash flows but these can be easily broken into two categories namely revenue and cost. Revenue from mining projects can be defines as the total production multiplied by price of the commodity. Due to price uncertainty, it is sometimes difficult to realistically forecast revenue.

To forecast future cash flows, cost is subtracted from revenue. Cost can be commonly broken into two categories in DCF analysis. The two common cost categories are operational and maintenance cost. Operational costs normally involve haulage cost, processing cost, labour cost, fuel cost, electrical cost, transportation cost, explosives cost, and rental cost. Maintenance costs involve the cost for maintain and repairing equipment and other facilities connected to the mining operation.

Taxes is another important factor that affects the outcome of expected cash flow. Taxes varies from country to country. Investment is subject to depreciation when calculating taxable income for both ordinary and special taxes.

3.2.3 Discount Rate - r

Discount rate is the most influential variable in the present value function. It determines expected present value of future expected cash flows. Discount rate influences every cash flow, which constitutes costs and revenue. Alternative terms for the discount rate are required rate of return, capital cost and alternative cost. Risky projects, all other things equal, are less valuable than safe projects. As a consequence, investors or companies demand higher rates of return from risky projects. When cash flows are uncertain, like in mining developments, they are normally represented by their expected values and the rate of return is increased on the basis of the Capital Asset Pricing Model (CAPM) in order to outweigh the possibilities for undesirable outcomes.

Defining the appropriate discount rate for mineral project has never been easy. Weitzman (2001) states: “The utmost critical single problem with discounting future benefits and costs is that no agreement now exists about what actual interest to use. Therefore we should be operating from within a framework that incorporates the complex uncertainty about interest rates directly into our benefit-cost methodology”.

Most company’s management define the way discount rate is calculated and this usually from a model such as WACC, CAPM, and Modern Asset Pricing Model (Mun, 2006).

3.2.4 Project life - n

Project life span varies from project to project and this subject to the amount mineable reserves, production rate and other economic factors. Discount rate increases linearly with the length of time of the project. This creates additional challenges in discount rate estimation. A minimal change in discount rate can make a huge difference in expected NPV of the project. The longer the project life the more uncertain the NPV becomes. This due to the fact that prices change and cost increases due to inflation. Longer life of project does make the project more risky with the assumption that traditional DCF assumes price and cost to be constant as well as discount rate.

3.3 Different Traditional Discounted Cash Flow Techniques

3.3.1 Net Present Value

NPV is the present value of an investment’s projected cash inflows minus the costs of attaining the investment. NPV is the most common technique used in DCF. Most decision makers use the single figure output to define the worth of the project in terms of money. However, the NPV should always be evaluated in terms of the financial size of the project.

Companies should be careful about investing in projects with high CAPEX and high OPEX and only marginally positive NPV.

3.3.2 Internal Rate of Return

Internal rate of return is well-defined as the discount rate often used in capital costing that makes the NPV of all cash flows from a specific project equivalent to zero. The IRR guidelines states that all companies must accept investment prospects offering rates of return in excess of their opportunity costs of capital (Kvalevåg, 2009).

Kvalevåg, T. (2009) stated that finding the IRR of a project lasting T years is solved for IRR in equation (3.2). This calculation usually involves trial and error. The expression can also be solved graphically.

$$NPV = C_0 + \frac{C_1}{1+IRR} + \frac{C_2}{(1+IRR)^2} + \cdots + \frac{C_T}{(1+IRR)^T} = 0 \qquad (3.2)$$

Brealey, Myers, and Allen (2006) report four pitfalls of the IRR method;

1. The determination of whether cash flows represent equity or loan. If the result of a project offers positive cash flows followed by negative cash flows, NPV can rise as the discount rate is increased.

2. Projects with cash flows that fluctuates more than one time. If this is the case, the project may have several IRRs or no IRR at all. In the case of mining developments, where capital (CAPEX) and operating (OPEX) costs or expenses and production targets cause cash flows to fluctuate more than one time throughout the life of mine.

3. IRR does not have the ability to differentiate project of different sizes as well as not differentiate projects with different patterns of cash flows over time.

4. The IRR does require comparison of projects' IRR with the opportunity cost of discount rate. Most of the time, discount rate varies over time, and there may be no simple path for evaluating IRRs of projects. There is always the possibility of discount rate for current cash flows being different from discount rate for future cash flows.

3.3.3 Payback Period

Payback period is well-defined as the period required to recuperate the initial investment in a project from operations. The payback period method of financial evaluation is used to evaluate mining projects and to calculate the cash flow per year from the start of the production until the accrued cash flow are equivalent to the cost of the investment at which period the investment is said to have been paid back. The time taken to achieve this payback is referred to as the payback period.

Management of mining companies normally define the limit of payback period and expect the calculated payback period to be less than their limit if possible. Most management with concern of possible liquidity tries to minimise risk by recovering the initial investment quickly. This concern is taken into account when management defines the payback period limit. Payback periods often used for small disbursements that have obvious paybacks for which the use of more sophisticated capital costing methods is not required or justified (Cooper, Morgan et al., 2001).

Certain projects are not considered viable based on the outcome of the payback period analysis as they take longer to recover. What is normally not considered by the analyst and management is the fact that benefits will accrue sometime in the future and well beyond the normal payback period. However, such projects may actually be vital for the long-term success of the business. It is therefore important to use the payback method more as a measure of project liquidity rather than project profitability. Payback period analysis is commonly used for evaluation of mining project investments by companies despite its shortcomings. In the investment world, investors use payback period to attract of investors for capital investments. Payback period as a single measure for the viability of a project has decreased overtime but has rather increased as a secondary measure (Segelod, 1995).

3.4 Accounting for Uncertainty and Risk

The biggest challenge for most analysts is the ability to incorporate the factors of uncertainty into a valuation model. Uncertainty impacts upon the valuation process are in two ways as stated by French and Gabrielli (2005): (1) cash flows from investment are to fluctuating grades of uncertain, and (2) the resulting valuation figure is therefore open to uncertainty.

There are several methods that are used for accounting for uncertainty in DCF. The two commonly used approaches are: (1) adjustment of discount rate, and (2) sensitivity of variables to adjust forecasted cash flow. When discount rate is increased, future and uncertain cash flows are valued lower. Secondly, forecasted cash flows can be adjusted. Variables such as price and discount rate used in determining cash flows can shift in different directions. By forecasting subjective or statistical movements of price and discount rate companies can account for uncertainty. The risk adjusted NPV of a project is only as accurate as the correctness of forecasted cash flows and the correctness of the discount rate.

3.4.1 Sensitivity Analysis

A sensitivity analysis is an iterative process of the base analysis by substituting alternative decisions or ranges of values for decisions to determine the profitability of projects.The common factors used in sensitivity analysis for mineral project valuation are price, total amount of mineable reserves, production rates, OPEX, and CAPEX. As the project valuation progresses, other unidentified variables may be identified. Optimistic and pessimistic estimates should be given for the value of these identified factors as stated by Kvalevåg, T. (2009). The author also stated that by changing key factors one at a time according to the optimistic and pessimistic estimates, it is possible to see how profitability is affected by changes.

Sensitivity analysis enables investors and analyst to get a better understanding of key drivers of their mining project, and helps with investment decisions. Investors and analyst are able to identify uncertainty factors of highest importance, and to define areas where the company should focus in order to get additional information before final decisions are made.

Sensitivity analysis has some drawbacks such as ambiguity of results. The analyst might understand the results but to the investor or shareholder is sometimes not clear what the results represent. Whenever mining commodity price levels increases there is lot of activities in the mining industry, creating higher demand and probably higher costs. Estimation of future correlation between different key factors such as price and discount rate represents an important challenge in valuation of mining projects (Brealey, Myers, and Allen, 2006). Jacobi process has been identified as an option for determining the correlation between prices and discount rate.

3.4.2 Scenario Analysis

With the assumption that key factors such price and discount rate may be correlated, a possible approach is to analyse various possible scenarios. With regards to mining project development, scenario analysis can be used to anticipate different commodity price levels with combinations of different values of other key factors. This allows the analyst to look at different, but consistent, combinations of key factors. This quantitative view of scenario analysis may be challenged by tacticians, who have traditionally viewed scenario analysis as a qualitative application, the primary benefit of which is to broaden the thinking of decision makers.

In scenario analysis we estimate expected cash flows and asset value under various circumstances with the intent of getting a better sense of the consequence of risk on value. Different NPVs are calculated using different key factors such as price, CAPEX, OPEX, discount rate and serve as a help in the decision making process (Brealey, Myers, and Allen, 2006).

3.4.3 Monte Carlo Simulation

Monte Carlo simulation is a method used to appreciate the impact of risk and uncertainty of a project. The method is used by specialists in such broadly disparate fields as finance, project management, energy, engineering, manufacturing, mining, research and development, insurance, oil & gas, transportation, and the environment.

Computer software such as @Risk, Crystal Ball, and Risk Simulator are used to simulate all possible outcomes for the project. The simulation can be done as many times as possible. By scrutinizing the results it is possible to plot a frequency distribution of the results and to calculate expected values, upper bounds, and lower bounds.

Various components of financial model such as price, cost, and discount rate can be transformed into probability distributions. The probability distributions can be incorporated into a Monte Carlo simulation. Different analyst can work individually, but at the same time incorporate all their input into one model and get one output (Mccray, 1975).

The key drawbacks of Monte Carlo simulation include the resource and time required to build an accurate model for a specific project. Correlations between variables such as commodity price and discount rate is difficult to estimate. If the input variables for the model are wrong, the results of the simulations will be wrong. The outcome of the simulation is as good as the input parameters and model used (Brealey, Myers, and Allen, 2006).

3.5 Advantages and Disadvantages of DCF

An important part of understanding and improving DCF analysis involves analyzing its advantages and disadvantages. The following two sections give an overview of some of the advantages and disadvantages of DCF analysis.

3.5.1 Advantages of Using the DCF

- Clear, consistent decision criteria for all projects;

- Same results regardless of the risk preferences of investors;
- Quantitative, decent level of precision, and economically rational;
- Not as vulnerable to accounting conventions (depreciation, inventory valuation);
- Factors in the time value of money and basic risk structures;
- Relatively simple, widely taught, and widely accepted; and
- Simple to explain to management: “If benefits outweigh the costs, do it!”

3.5.2 Disadvantages of using the DCF

- DCF ignores randomness in cash flow variables.
- DCF analysis may use sensitivity analysis that varies a single variable at a time by a set percentage to gain insight into how NPV changes with random outcomes for that variable.
- DCF ignores the effects of contingent cash flows and flexibility.
- DCF risk adjustments do not recognize the dynamic variation of cash flow risk through time.
- DCF uses of a single discount rate implies that project cash flow uncertainty increases through time in a regular manner.

3.6 Summary

In this chapter, traditional discounted cash flow or DCF valuation is explained. The components of DCF analysis were clearly explained. Different valuation techniques for DCF were explained as well accounting for uncertainty and risk for this technique. The difference between DCF assumptions and realities as well as disadvantages of the techniques were emphasized.

CHAPTER 4: REAL OPTIONS VALUATIONS

4.1 Introduction

Real options are basically application of the idea behind financial options to production industries. It is a combined solution using financial principles, economic analysis, management flexibility, statistics, and econometric modelling. Real options applies options theory in valuing real physical assets (e.g. mining, oil and gas, agriculture and real estates), on the contrary to financial assets such as stocks, in an environment where managerial flexibility can be incorporated in investment decision-making, valuing opportunity cost of investments, and options of when project capital expenditures should be spent (Mun, 2006).

A real options valuation (ROV) model can be further supplemented with a residual project risk discount rate applied to the risk-adjusted cash flow to account for aspects of technical and commercial project uncertainty that have not been explicitly recognized under the ROV methodology (Smith and McCardle, 1999).

4.2 Components of Real Options

ROV has five enhancements in comparison to a standard DCF valuation model; see Figure 4-1. These enhancements depend on a set of additional input assumptions that must be developed jointly with relevant group subject matter experts. As these assumptions are realistic, ROV will be more applicable.

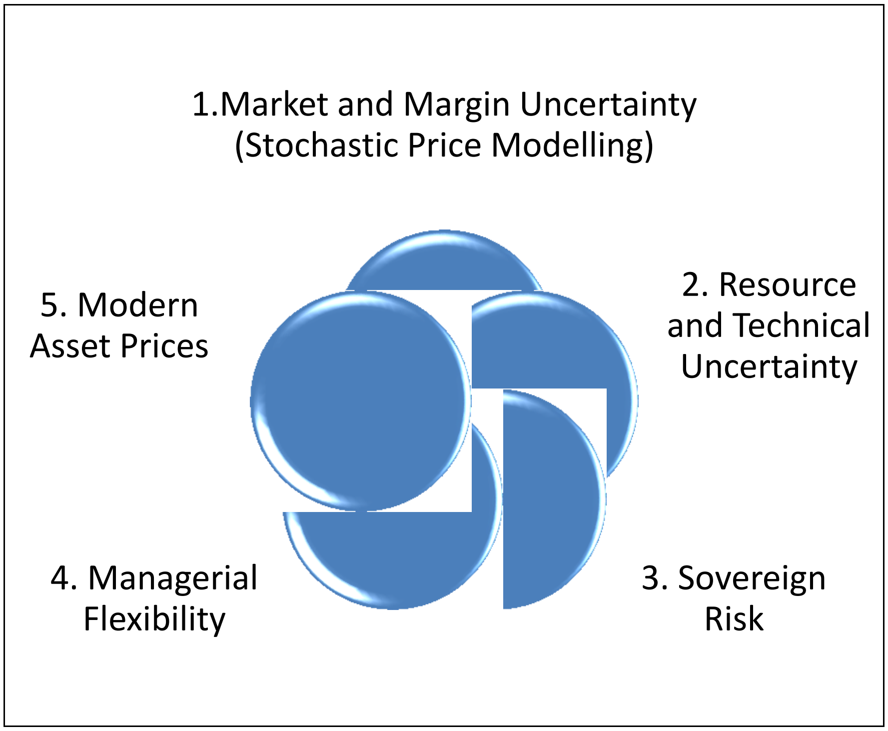

Figure 4-1: Five Enhancement of ROV models

4.2.1 Price and margin uncertainty

Rather than using a single price series, or running distinct alternative scenarios, ROVs are based on a stochastic price model. As well as allowing for price shocks resulting from short-term business cycles, the price models used include the potential for structural breaks in long-term price trends.

This approach generates a distribution or range of project values under-price uncertainty. This allows for the project valuation to capture the impact of any non-linear effects resulting from factors such as a resource super-tax or price participation agreements with floors and ceilings. It also provides the basis of applying other ROV modules, in particular managerial flexibility.

Where the scale of production of a particular project could affect market prices this should be included in the price simulation. This may require a deeper level of engagement with economics and specific supply and demand models.

4.2.2 Resource and technical uncertainty

Standard valuations are based on a single set of resource and technical assumptions – often "most likely" rather than expected values. These assumptions include:

- size and quality of resources;
- impact of technical factors (such as recovery rates and innovative technology) on production; and
- availability of key inputs, such as access to water and land infrastructure.

Resource estimates can be highly uncertain, and the ROV approach replaces these single point estimates with a set of probability weighted scenarios. Multiple simulation of orebody can be considered. These simulations are equally probable and provide an opportunity for risk analysis and uncertainty management.

Different mine plan and cash flow projections are required for each scenario. For this to remain a manageable exercise, scenarios should address only those uncertainties that have a material impact on the valuation outcome, and mine plans will only be developed at a level that allows for a fact based approximation of expected investment and operating cost.

4.2.3 Sovereign risk

In the standard evaluation methodology a set of “country risk premia” are added to the underlying group cost of capital used to discount cash flows. These premia are calculated for individual countries as the equivalent of applying a set of risk events to the cash flows of a “standard” operating mine, weighted by a set of likelihoods. These risk events include the possibility of the mine being expropriated, changes in taxation, and a set of operating risks caused by disruptions to local markets.

In the ROV approach the initial step is to apply the same set of risk events and their assumed likelihoods to the specific cash flows of the project under review. This allows for a more correct calculation of the value adjustment than using the generalised discount rate premia.

The ROV approach also allows for a more refined assessment of country risk based on the circumstances of the particular project. This alternative assessment could incorporate:

- specific risks facing an individual project;

- adjustment of country risk assessments based on in-house knowledge; and
- reduction in likelihood or impact of risk events from mitigation activities;

Preparing this assessment is a difficult task that depends on what can be highly subjective opinions. As such this assessment should be normalised against risk estimates for other group operations and incorporate input from internal experts including external affairs, tax, and group risk functions. At all times a comparison to third party opinions and external indicators such as sovereign risk premiums is advised.

4.2.4 Management flexibility

Mine and plant operations evolve over time, most frequently in response to changes in variables described above. For example, a step change in expected long run prices, or the discovery of additional resource, may trigger a capacity expansion.

The ROV process simulates these management choices through a decision tree within the valuation model. Choices within the decision tree – for example, to expand or not – are made by comparing 'trigger prices' with simulated price forecasts at appropriate times. The precise form of the decision tree and the timing at which "triggers" are tested are key inputs, and are best established through an initial workshop process.

4.2.5 Modern Asset Pricing (MAP)

The standard approach to valuation is to discount net project cash flows by a single, time-constant, discount rate (Guj and Garzon, 2007). ROV incorporates the modern asset pricing (MAP) approach. MAP first adjusts individual cash flow lines for their particular risk profile (to derive certainty equivalents), and then discounts the sum of these risk adjusted cash flow lines by a compounding risk free rate, plus a “residual risk premium” capturing remaining systemic project risks (Espinoza and Morris, 2013; Espinoza, 2014).

Despite more difficult to parameterise than the standard NPV discounting approach, modern asset pricing is theoretically more correct, and does result in lower discount rates and so higher values—for projects that common sense would suggest are lower risk. For example, modern asset pricing tends to raise the value of longer life and higher margin projects, and reduce the value of high cost and short life operations. Therefore, MAP tends to generate a premium for assets in line with our strategy, and penalize assets that do not fulfil all of our strategic criteria.

MAP also allows superior valuations of deal structures. For example, it will also attribute more value to a steady stream of payments, like a fixed royalty, than a stream of payments that are more risky, such as price participation or profit shares. One important difference in the approach is that since the degree of price uncertainty tends to saturate over time (e.g., beyond a certain timeframe the incremental extent to which prices may diverge from their long run equilibrium level given a certain probability interval begins to reduce) a key element of project uncertainty—price risk—will level out at some point.

Application of modern asset pricing requires three discount factors:

1. A price risk adjustment, which is specific to each commodity product depending on its particular level of uncertainty;
2. A risk free rate; and
3. A residual risk premium.

This risk premium is calibrated so that, on average, the valuation of company's assets would be equivalent when discounted at the company's cost of capital.

In terms of the application of these rates:

- revenue, price linked costs such as conversion costs and royalties should be fully price-risk adjusted;
- non-contracted CAPEX, closure costs and operating expenses (including freight) should be assumed to be 40% correlated with price and 40% of the price-risk adjustment is initially applied;
- tax and working capital are calculated using risk-adjusted cash flows;
- net cash flows are then discounted by the risk-free rate and the residual risk premium; and
- risk-adjusted closure costs are discounted separately at risk-free rate plus a 1% premium.

4.3 Motivation for Real Option Methodology

A group of petroleum industry managers and consultants came together in 2003 to evaluate various approaches of estimating asset value. During the workshop, two key approaches of estimation evaluated were DCF and ROV methods. There was also confusion over the concept of the two approaches to value estimation and the relationships among them as well as the implications of this for standards of best practice.

Based on the outcome of the workshop, a taxonomy of valuation methods came to being known as the "the Banff taxonomy". Figure 4.2 is similar to "the Banff taxonomy".

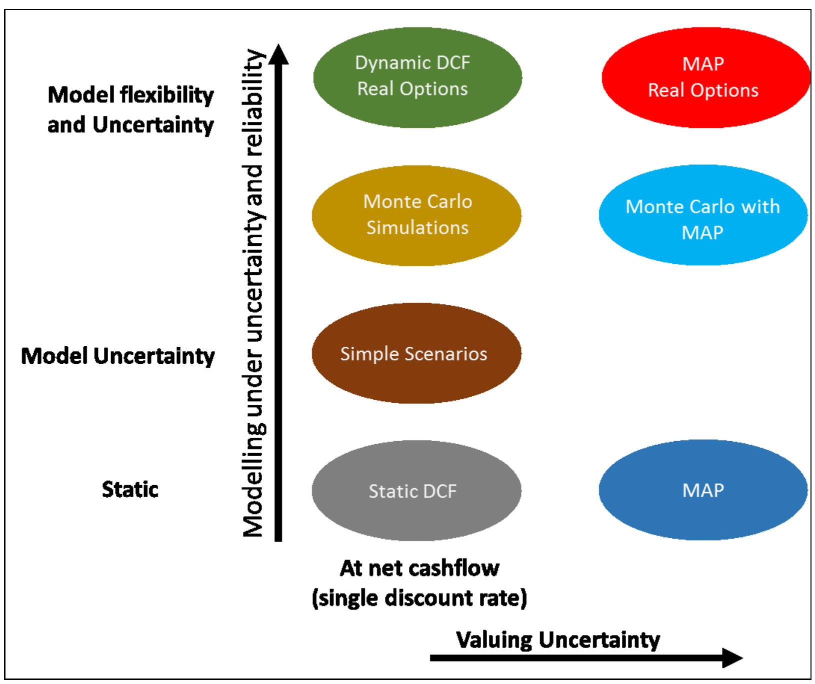

Figure 4-2: Motivation for Real Options Simplified ("the Banff taxonomy")

Laughton (2007) stated based on the structure of the taxonomy, there are two characteristics of cash flow that help to determine financial market prices; (1) timing and,(2)uncertainty. The author emphasised that timing is relatively simple, and the effects of time on value are determined in roughly the same way by most asset methods, using prices determined from claims to risk-free cash flows, frequently in government debt markets. Uncertainty on other hand, uncertainty is complex and multidimensional.

Real options valuation as simplified by “the Banff taxonomy” focuses on model uncertainty and the effect of valuing the uncertainty of the asset value. Model uncertainty can be either qualitative or quantitative. The quantitative models are normally presumed as either static or dynamic.

Laughton (2007) also stated that the effect of uncertainty on asset value is determined at the level of either:

- The asset of cash flows themselves; or
- The sources of uncertainty in the asset cash flows (such as the term structure of oil or gas prices or geological uncertainties like oil-in-place)

The asset of cash flow is on the left hand side of Figure 4.2 and at source is on the right. If an asset cash flow (net cash flow, single discount rate) is valuated, a risk premium is incorporated as well as risk free interest rate into the discount rate which is used to discount the cash flow.

On the other hand if valuation of uncertainty is done at source, the uncertain parameters are risk adjusted and used to predict a risk adjusted cash flow. The uncertain parameters are discounted for time using risk free interest rates to estimate asset value. This method normally applies to market prices.

4.4 Summary

In this chapter, real options valuation (ROV) is introduced with a historical review of how it evolved. The components of real options valuation are broken into five parts and briefly explained. The motivation for real option methodology is summarized in a Figure 4.2.

CHAPTER 5: MARKET AND MARGIN UNCERTAINTY

5.1 Introduction

Pricing of commodities is of great importance to mineral project evaluation. Through the use of various pricing techniques, some experts in the industry have predicted that precious metal forecasts move with spot prices while base metal forecasts exhibit some form of price reversion. Both characteristics have important value effects when projecting evaluation.

Mean reverting process integrates the tendency of base metal prices to gravitate towards a "normal" equilibrium price level that is usually overseen by the cost of production, and in the case of iron ore, level of demand. This process incorporates implied volatility to predict future prices.

Volatility is a key part in finance with significant role in investment, security assessment, risk management, and financial policy making. Thus, most of the activity in the study area of a financial institution is dedicated to modeling and forecasting of asset volatility (Wang and Xu, 2015).

5.2 Geometric Brownian Motion

Geometric Brownian Motion (GBM) is one the most popular stochastic process model used in financial economics theory as well as practice Dias (2009). In several cases this is not the better model, even being a reasonable mapping of probabilities with the time.

Assuming a project value V or the value of the developed reserve that follows a Geometric Brownian Motion, the stochastic equation for its disparity with the time t is:

$$dV = \alpha V dt + \sigma V dz \qquad (5.1)$$

Where

dz = Wiener increment = $\varepsilon\, dt^{1/2}$ (where ε is the standard normal distribution);

α is the drift; and

σ is the volatility of V.

In Equation (5.1), $\alpha V dt$ is the expectation (trend) term and $\sigma V dz$ is the variation term (deviation from the tendency or term of uncertainty). This specification from Wiener process leads to a "jumpy" changes or spikes in the stochastic variable V. The simple reason behind the theses changes are:

- For a small time interval Δt, the standard deviation movement will be much higher than the mean of price movement. This is because, for small Δt, $(\Delta t)^{1/2}$ is much larger than Δt, and this will define the behavior of sample paths of a Wiener process.
- For similar motives, a Wiener process has no time derivative in a orthodox sense: $\Delta z/\Delta t = \varepsilon\, (\Delta t)^{-1/2}$, becomes infinite as Δt approaches zero.

In real options valuation, there is a dividend like income stream δ for the holder of the asset. The dividend yield is associated with the cash flows generated by the resources in place.

For commodities prices this is called convenience yield or rate of return of shortfall. In all circumstances, the equilibrium requires that the total expected return μ to be the summation of expected capital gain and the expected dividend, so that:

$$\mu = \alpha + \delta \qquad (5.3)$$

So, the stochastic equation can be written:

$$dV = (\mu - \delta)\, V\, dt + \sigma\, V\, dz \qquad (5.4)$$

Popular models such as the Paddock & Siegel & Smith model, uses this stochastic process.

GBM has the great advantage of the simplicity. GBM has been compared with Mean-Reverting process using cumulative investment examples. It was realised mean-reverting process considers supply and demand with regards to increasing market price of certain commodity prices. It was concluded that cumulative investment is unaffected by the use of mean reversion rather than GBM (Metcalf & Hasset, 1995). This is a defense against more sophisticated models. However, sometimes more specific models are required, mainly for economical business, where even a not huge difference is important.

5.3 Mean-Reverting Process

The modelling of commodity price uncertainty has attracted a great deal of attention in the mathematical finance literature. In this research, we will use a model where the commodity spot price is assumed to follow the stochastic process (Schwartz, 1997):

$$dp = \eta(\ln \bar{p} - \ln p)p dt + \sigma dz \tag{5.5}$$

Let $y = \ln p$, applying Ito's Lemma allows characterization of the log price by an Ornstein-Uhlenbeck stochastic mean reverting process:

$$dy = \eta(\bar{y} - y)dt + \sigma dz \tag{5.6}$$

With

$$\bar{y} = \ln(\bar{p}) - \frac{\sigma^2}{2\eta} \tag{5.7}$$

Where

$\bar{p}$ is the long-run equilibrium commodity price;

dz is an increment to a standard Brownian motion;

η measures the speed of mean reversion to the long run mean log price; $\bar{y}$

σ is price volatility rate.

Using the properties of the lognormal distribution, the expectation of the forward price given the current spot price p_o is given

$$E_t(p_o) = \exp\left\{\ln(p_o)e^{-\eta t\Delta t} + \left[\ln(\bar{p}) - \frac{\sigma^2}{2\eta}\right]\left(1 - e^{-\eta t\Delta t}\right) + \sigma^2(1 - e^{-2\eta t\Delta t})/4\eta\right\} \qquad (5.8)$$

Where

P_t is the spot price at time t;

Δt is the fixed time interval from t to t+1;

The correct discrete time format for the continuous time evolution of mean reversion is the static first order autoregressive process (Dixit & Pindyck, 1994).The sample path simulation equation for y_t is achieved by using the exact discrete time expression:

$$y_t = y_{t-1}e^{-\eta\Delta t} + \bar{y}\left(1 - e^{-\eta\Delta t}\right) + N(0,1)\sigma\sqrt{(1 - e^{-2\eta\Delta t})}/2\eta \qquad (5.9)$$

Where

N (0, 1) is the normally distributed random variable.

By substituting Equation 5.9 to $p=e^y$, we have the exact discrete-time equation for p_t given by

$$p_t = \exp\left\{\ln(p_{t-1})e^{-\eta\Delta t} + \left[\ln(\bar{p}) - \frac{\sigma^2}{2\eta}\right]\left(1 - e^{-\eta\Delta t}\right) + N(0,1)\sigma\sqrt{(1 - e^{-2\eta\Delta t})}/2\eta\right\} \qquad (5.10)$$

Let P = {p_t, t=0,…,T} denote a price scenario with spot prices p_t is determined by Equation 10. Let p_o = {Et (p_o), t=0,…,T} denote the expected price scenario given the current spot price p_o, where Et (p_o) is determined by Equation 5.10. Figure 5-1 presents a sample path of copper price simulated using the above price model combined with a path of the anticipated price scenario p_o as well as the actual historic price path from which the sample price path is developed.

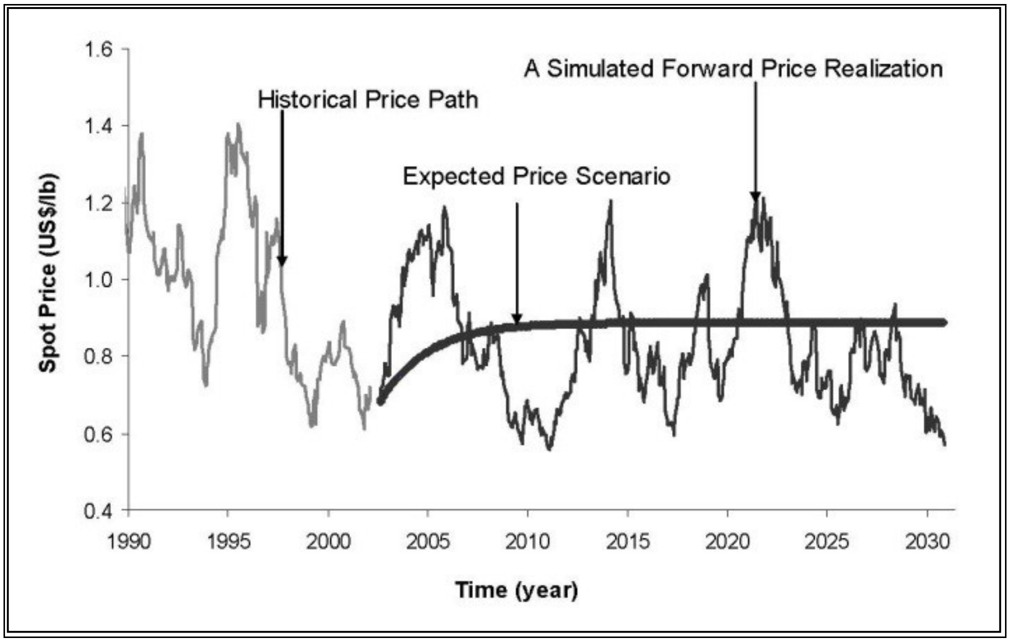

Figure 5-1: Simulation of the copper price using a Lognormal Mean Reverting Price Model

(Source: Schwartz, 1997)

The Mean Reversion Process is a lognormal diffusion process, but with the variance increasing not uniformly to the time interval. The variance reverts upwards in the beginning and after sometime reverts downwards to the long-term mean. Figure *5-2*-3 illustrates this, for "low prices" case.

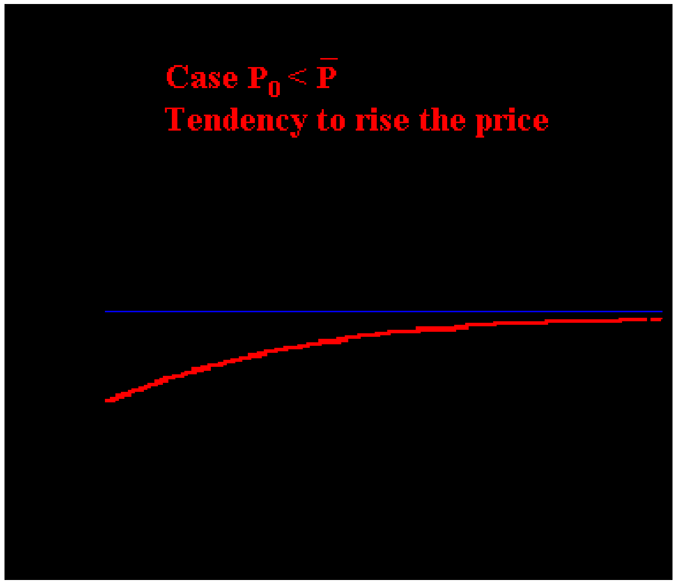

Figure 5-2: Low prices (Lognormal diffusion process)

(Source: http://marcoagd.usuarios.rdc.puc-rio.br/revers.html#mean-rev)

Figure *5-3* illustrates the mean-reversion for the “high prices” case.

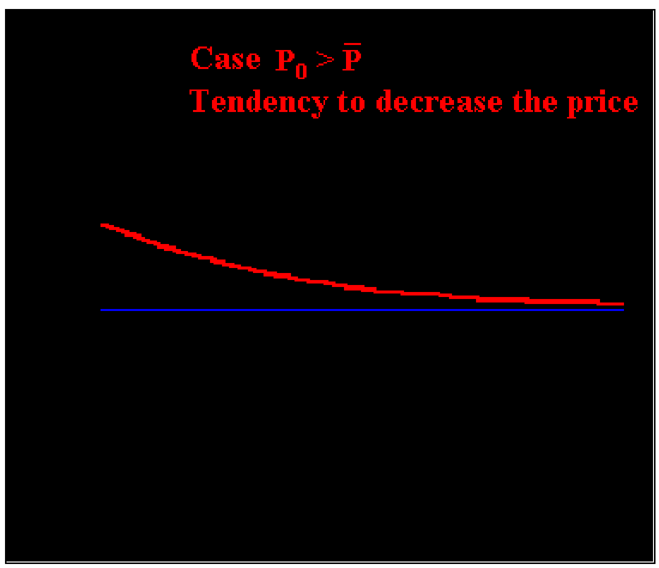

Figure 5-3: High price (Lognormal diffusion process)

(Source: http://marcoagd.usuarios.rdc.puc-rio.br/revers.html#mean-rev)

Mean reversion may only reveal itself over very long time frames. Usually, a decision to model a quantity with a mean reverting stochastic process is established both on empirical observation of that quantity over time, as well as some hypothetical argument as to why it should be mean reverting. Models that the financial brokers use for mean reverting modelling is the GARCH MODEL and RISK METRIC MODEL is used for non-mean reverting models.

The mean reversion process has been considered the normal choice for commodities. Microeconomics theory has proven that, in the long term, the price of a commodity ought to be tied to its long-term marginal production cost or, "in case of a commodity like iron ore, the long-term profit capitalize on price sought by shareholders" (Laughton & Jacoby, 1995).

As futures prices decreases (toward the long-term mean, in backward manner) if the spot prices are "high," and are increasing if prices are "low," the prices are random walk, the volatility in the futures prices are presumed equal to volatility of the spot price, but the study have proven that spot prices are much more volatile than futures prices (Baker et al, 1998). In both cases, the mean-reverting model is far more consistent with the futures prices data than random walk model.

Over the years, the other biggest component that have not been considered with mean reverting process for commodity price modelling is using bound stochastic correlation process such as Jacobi process. In this thesis Jacobi process is consider in modelling the stochastic correlation between price returns.

Generally, Jacobi diffusion and its generalizations are ideal diffusions to model stochastic correlation. It is presumed that stochastic correlation is mean reverting to a long term mean and is driven by a Brownian motion whose oscillation can be improved by using a higher volatility factor for the stochastic correlation process.

5.4 Jacobi Process

One of the most common models used in modeling stochastic correlation is the Vasicek 1977 model (Ulhenbeck and Orienstien, 1930). The formula for this model is;

$$d_\rho = a\left(m_\rho - \rho_t\right)dt + \sigma_\rho \varepsilon_t \sqrt{dt} \qquad (5.11)$$

Where

a is the mean reversion speed (i.e. the degree at which the correlation at time t, ρ_t, is reverted to it long-term mean. "a" can take values $0 \le a \le 1$.

m_ρ is the long-term mean of the correlation ρ

The limitation to Vasicek 1977 model with respect to stochastically modeling correlation is that the model is not bounded. The results from this stochastic model was either bigger than one or smaller than one. These are likely to occur when mean reversion *"a"* is lower and volatility σ_ρ is high.

Based on the limitation of Vasicek 1977 model with regards to no bounds, a bounded Jacobi process is introduced. Applying Jacobi process to Equation (5.11);

$$d_\rho = a(m_\rho - \rho_t)dt + \sigma_\rho\sqrt{(u-\rho_t)(\rho_t - l)}\varepsilon_t\sqrt{dt} \quad (5.12)$$

Where

u is the upper boundary level,

l is the lower boundary level (i.e. $u \geq \rho \geq l$)

Considering Pearsons framework for the correlation modeling with the boundaries $u = +1$ and $l = -1$. Based on this consideration, Equation (5.12) reduces to;

$$d_\rho = a(m_\rho - \rho_t)dt + \sigma_\rho\sqrt{(1-\rho_t^2)}\varepsilon_t\sqrt{dt} \quad (5.13)$$

5.5 Motivation for Mean- Reverting Prices

- Most stochastic process research on real options is geometric Brownian motion.
- This is a natural model for exponential growth under uncertainty.
- Assume a real option where the key variable is the price of some input or output.
- Without a doubt, price may be subject to random shocks, but that in the long term, competitive pressures will ensure that it tends to revert to long-term mean.
- GBM uses random walk used to model prices with the assumption that price changes are independent of one another. It is also presumed that the historical path the price followed to achieve its current price is irrelevant for forecasting the future

price path. Mean reversion can be measured as a modification of the random walk, when price changes are correlated but not completely autonomous of one another.

5.6 Modelling Input Parameters

The most collective ROV model was developed by Paddock, Siegel and Smith (1983 and 1988). It models the underlying's price as a Geometric Brownian Motion.

Microeconomics theory has proven that, in the long term, the price of a commodity ought to be tied to its long-term marginal production cost or, “in case of a commodity like iron ore, the long-term profit maximizing price sought by shareholders”, which differs mostly across the countries mainly because of the geologic features.

Generally, marginal interaction between production and demand, between depletion and new reserve discoveries leads to smooth price changes. In the case of iron ore, as it is a cartelized commodity, the situation is a little bit more complex. Indeed, the iron ore price is also tied to the long-run profit-maximizing price sought by the big three (Vale, Rio Tinto and BHP) and Chinese Steel makers. The role of emerging countries such as China furthermore remains very important in the production game of the steel industry as most of the iron ore sales are purchased by them. For instance, the large rise in iron ore prices in the year 2008 (Figure 5-4 and Figure 5-5) is mainly due to the demand from China. In Figures 5-4 and 5-5, one can see the evolution of iron prices over the last century.

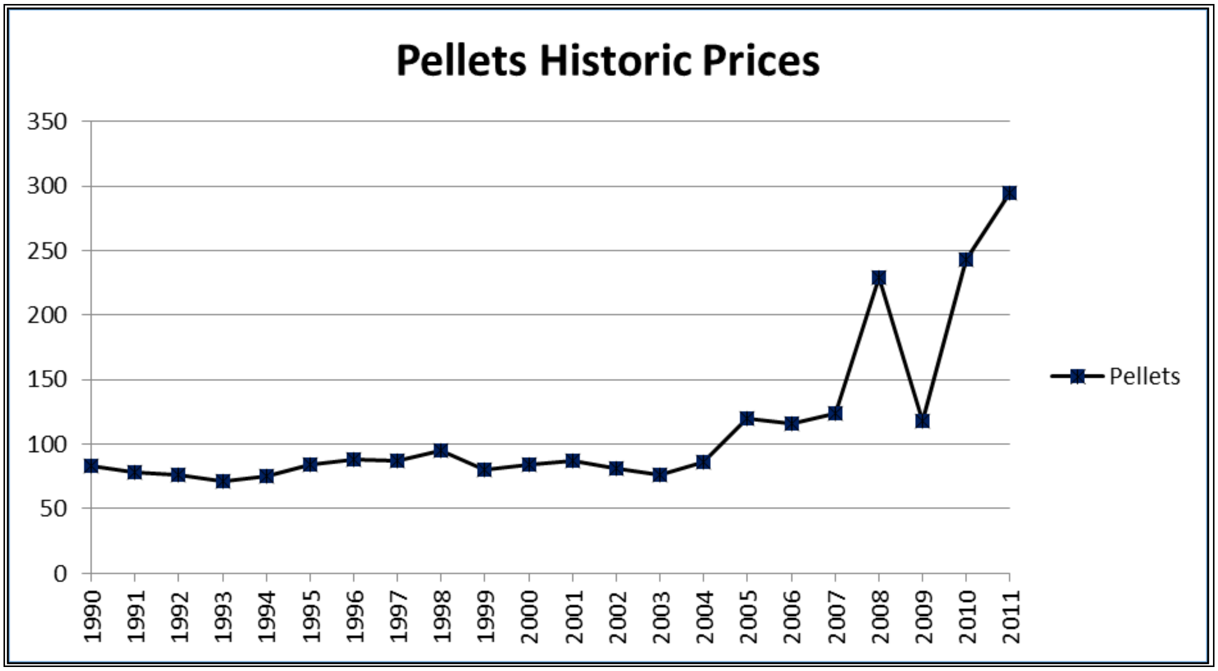

Figure 5-4: Pellets historic prices

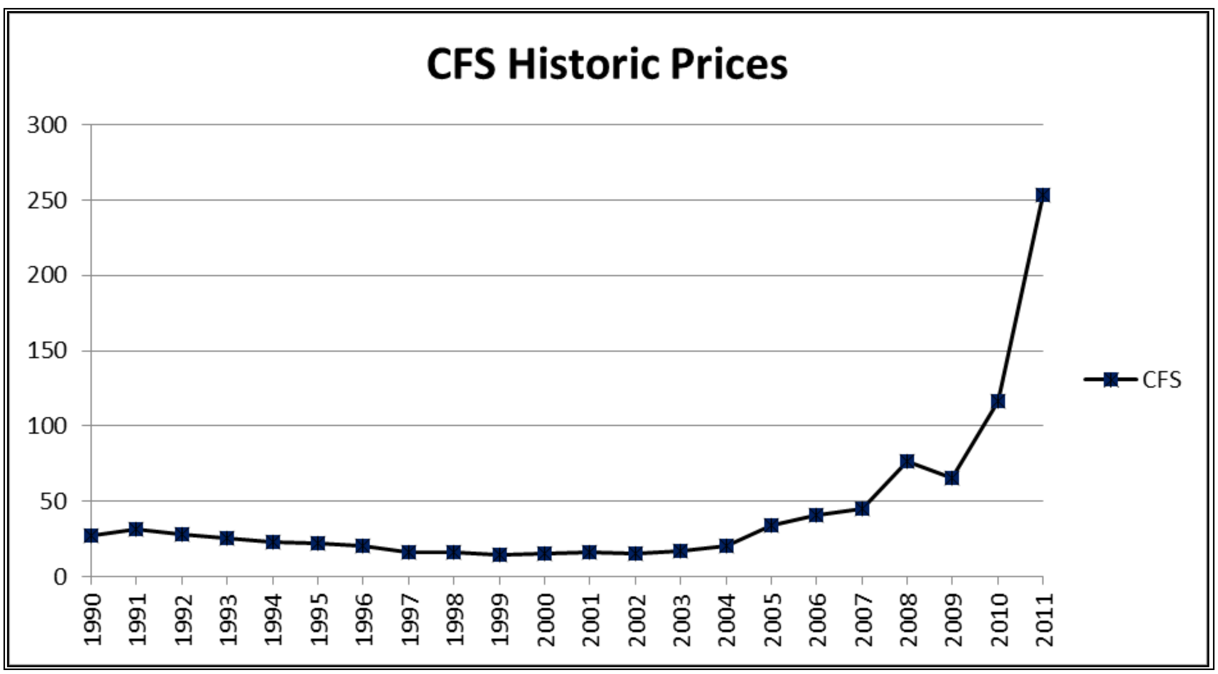

Figure 5-5: CFS historic prices

In this thesis, square root of time rule is used in modelling the input parameters for the price modelling. The Square root of time assumes independent price moves and constant volatility:

- If there is substantial mean reversion, time scaling will overestimate volatility (mean reversion has a statistical tendency to revert to a long-term mean).
- If there is considerable trending, time scaling will underestimate volatility (trending has a statistical inclination to keep moving in single path).
- If volatility fluctuates over time, or there are jumps, time scaling will be incorrect.

5.6.1 How Mean Reversion Rates, Mean Reversion Levels and Volatilities are Practically Calculated in Excel using the "square root of time rule"

Square root of time rule is generally used in risk management to transform certain Value at Risk (VaR) measures between different holding periods as well as impact of mean reverting process on forecasting volatility. If prices follow a mean reverting path, the square root of time rule would give us very huge volatility estimates. Below is the practical steps taken in using the square root of time to calculate mean reversion speed, volatility and long run mean. These steps are in reference to both Table 2 and 3.

- Column B (rows 2–23) contains historical annual iron ore prices for both Pellets and CFS over 22 years period in both Table 2 and 3.

Determine the standard deviation assuming that returns are independent. Column C (rows 3–23) shows the annual yields. Using Excel's built-in STDEV (sample standard deviation) function, the annual volatility was calculated. Using the "square root of time rule", we annualise to obtain STDEV* 365days assuming that time is restrained in working days. In this case, the annualised volatility for Pellets is equal to 583.2%, (24.7% x √365). And the annualised volatility for CFS is equal to 524.83 %,(27.5% x √365).

Calculating the absolute price changes, Column D (rows 3–23) shows the annual changes.

I estimated the Mean Reversion rate in a moderately simple and vigorous manner by regressing absolute price changes (Column D) on the previous price levels (Column E).

I used the Excel functions SLOPE, INTERCEPT and STEYX (residual standard deviation) to calculate the parameters from the regression. The mean reversion speed is defined as the negative of the slope and the long run mean as the intercept approximation of that regression divided by the mean reversion speed.

The volatility of dollar price changes is specified by the residual standard deviation calculated with STYDX. To attain percentage volatility, I divided by the long run mean. Refer to Table 1 and 3

Table 1: Mean Reversion Rates, Mean Reversion Levels and Volatilities for Pellets

A	B	C	D	E	F
1	Year	Pellets Prices	$ Price Change	$Previous Price	Price Change %
2	1990	83.7			
3	1991	78.5	-5.2	83.7	-6.4%
4	1992	75.9	-2.6	78.5	-3.4%
5	1993	71.3	-4.6	75.9	-6.3%
6	1994	75	3.7	71.3	5.1%
7	1995	83.9	8.9	75	11.2%
8	1996	88.2	4.3	83.9	5.0%
9	1997	87.2	-1	88.2	-1.1%
10	1998	95.3	8.1	87.2	8.9%
11	1999	80.7	-14.6	95.3	-16.6%
12	2000	84.3	3.6	80.7	4.4%
13	2001	87.3	3	84.3	3.5%
14	2002	81.1	-6.2	87.3	-7.4%
15	2003	76.6	-4.5	81.1	-5.7%
16	2004	86.1	9.5	76.6	11.7%
17	2005	120	33.9	86.1	33.2%
18	2006	116	-4	120	-3.4%
19	2007	124	8	116	6.7%
20	2008	229	105	124	61.3%
21	2009	118	-111	229	-66.3%
22	2010	243	125	118	72.2%
23	2011	295	52	243	19.4%
			Y values	X values	

Table 2: Mean Reversion Rates and Volatilities Output for Pellets

Standard Deviation		*Regression Parameters*	
STDEV(u)	27.4%	SLOPE	-8%
SQRT	4.5825757	INTERCEPT	9.3
		STEYX	47.16
Speed	8%		
Long Run Mean	115.9		
Volatility	40.7%		

Table 3: Mean Reversion Rates, Mean Reversion Levels and Volatilities for CFS

A	B	C	D	E	F
1	Year	CFS Prices	$ Price Change	$Previous Price	Price Change %
2	1990	27			
3	1991	31	4	28	13.8%
4	1992	28	-3	31	-10.2%
5	1993	25	-3	29	-11.3%
6	1994	23	-2	26	-8.3%
7	1995	22	-1	22	-4.4%
8	1996	20	-2	24	-9.5%
9	1997	16	-4	25	-23.1%
10	1998	16	0	16	2.8%
11	1999	15	-2	16	-11.7%
12	2000	15	1	15	4.3%
13	2001	16	1	15	4.2%
14	2002	15	0	16	-2.4%
15	2003	17	1	15	8.6%
16	2004	20	3	17	17.1%
17	2005	34	14	20	53.9%
18	2006	41	7	34	17.4%
19	2007	45	4	41	9.1%
20	2008	76	32	45	53.6%
21	2009	65	-11	76	-15.5%
22	2010	116	51	65	57.8%
23	2011	253	137	116	77.6%
			Y values	X values	

Table 4: Mean Reversion Rates and Volatilities Output for CFS

Standard Deviation		*Regression Parameters*	
STDEV(u)	27.5%	SLOPE	100.9%
SQRT	4.5825757	INTERCEPT	-22.5
		STEYX	21.51
Speed	-100.9%		
Long Run Mean	22.3		
Volatility	96.5%		

Correlation and volatility between iron ore prices is a very important factor to consider when developing risk management and mean reversion models for iron ore prices. The efficiency of hedging strategies for instance, depends on the existence of robust and steady correlation between spot and futures iron ore prices; the lack of correlation on the other hand, or even unexpected changes in the level of correlations may have negative consequences not only for hedging and risk management but also in determining the efficiency of a company's financial policies.

In general, demand and supply of iron ore may be attributed to common macroeconomic shocks on world markets, and the substitutability in the production or consumption of related commodities like steel. It is also an established fact that although iron ore prices could be correlated, correlation fluctuations over time and, in particular, correlation fluctuations have become more irregular over the last seven years.

Recent research by Buyuksahin et al. (2010) and Silvennoinen and Thorp (2010) has established that returns correlation between various commodities has increased substantially during the 2008 financial crisis. Tang and Xiong (2011) also indicated that the surge in the correlations between the returns of various commodity futures started long before the 2008 financial crisis.

The biggest challenge for this project was the minimal amount of historical data and the spike in prices in the last three years. The correlations between historical prices for Pellets were well correlated until 2007 till date and it was same instance for Concentrate prices. The variations in prices made it quite difficult to calculate correlation and this shown in Tables 5 and 6.

It was noted between the years 2009 to 2011, there was a big spike in prices with both pellets and concentrate which made it difficult to correlate the historical prices. This was due to the demand of iron ore by China. The correlation variation for both pellets and concentrate prices are shown in Figures 5-6 and 5-7.

Table 5: Pellets Corrected Correlation Used

Year	Pellets Prices	$ Price Change	$ Previous Price	Price Change %	Auto Correlation
1990	83.7				0.084
1991	78.5	-5.2	83.7	-0.064	
1992	75.9	-2.6	78.5	-0.03	
1993	71.3	-4.6	75.9	-0.06	
1994	75	3.7	71.3	0.05	
1995	83.9	8.9	75	0.11	
1996	88.2	4.3	83.9	0.05	
1997	87.2	-1	88.2	-0.01	
1998	95.3	8.1	87.2	0.09	
1999	80.7	-14.6	95.3	-0.17	
2000	84.3	3.6	80.7	0.04	
2001	87.3	3	84.3	0.03	
2002	81.1	-6.2	87.3	-0.07	
2003	76.6	-4.5	81.1	-0.06	
2004	86.1	9.5	76.6	0.12	
2005	120	33.9	86.1	0.33	
2006	116	-4	120	-0.03	
2007	124	8	116	0.07	
2008	229	105	124	0.61	
2009	118	-111	229	-0.66	
2010	243	125	118	0.72	
2011	295	52	243	0.19	
		Y values	X values		

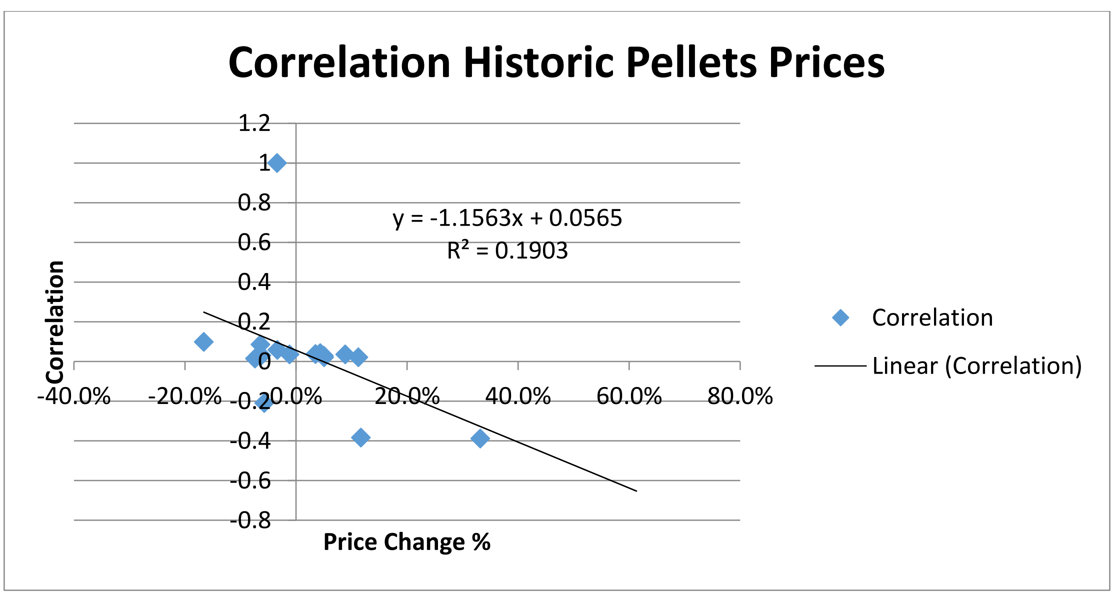

Figure 5-6: Pellets price correlation graph

Table 6: CFS Corrected Correlation Used

Year	CFS Prices	$ Price Change	$ Previous Price	Price Change %	Auto Correlation
1990	27.00				0.33
1991	31.00	4.00	28.00	0.14	
1992	28.00	-3.00	31.00	-0.10	
1993	25.00	-3.00	29.00	-0.11	
1994	23.00	-2.00	26.00	-0.08	
1995	22.00	-1.00	22.00	-0.04	
1996	20.00	-2.00	24.00	-0.10	
1997	15.87	-4.13	25.00	-0.23	
1998	16.31	0.45	15.87	0.03	
1999	14.52	-1.80	16.31	-0.12	
2000	15.15	0.64	14.52	0.04	
2001	15.80	0.65	15.15	0.04	
2002	15.43	-0.38	15.80	-0.02	
2003	16.82	1.39	15.43	0.09	
2004	19.95	3.13	16.82	0.17	
2005	34.21	14.26	19.95	0.54	
2006	40.71	6.50	34.21	0.17	
2007	44.58	3.87	40.71	0.09	
2008	76.23	31.65	44.58	0.54	
2009	65.27	-10.96	76.23	-0.16	
2010	116.39	51.12	65.27	0.58	
2011	253.00	136.61	116.39	0.78	
		Y values	X values		

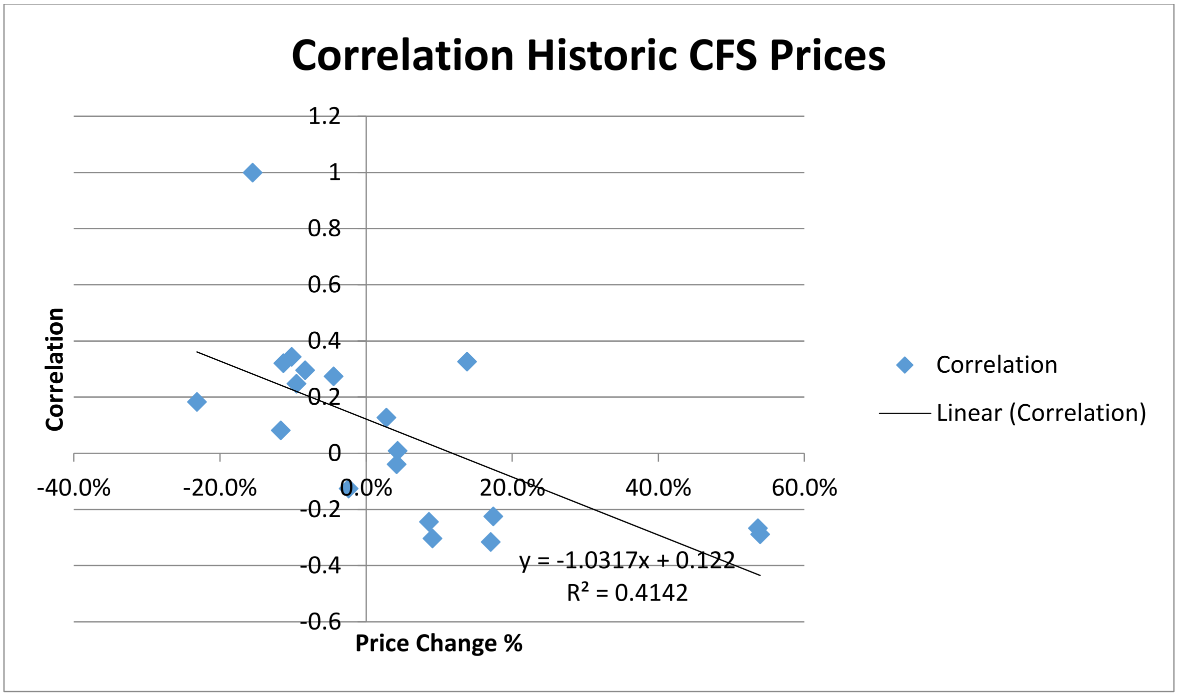

Figure 5-7: CFS price correlation graph

5.7 Iron Ore Prices Modelling

The modelling of the mean reverting model was broken into two parts. The Stochastic model was developed on the Mean Reversion process and the simulated path model was developed on the mean reversion process with the wieners process associated with it. This model was initially built by Ernst and Young for training purposes and it has been modified for the purpose of this project. The model and results are included in Appendix A.

One key feature of the models is the stepping time. The step time was built in to capture the volatility of the prices and for this project specifically the annual prices took into consideration the possible changes in prices with respect to volatility and market risk throughout the year. These drifts were captured on a weekly base. As of 2011, when this research started, the spot prices were \$250 per tonne and \$180 per tonne for both PELLETS and CFS respectively. Risk discounted factor was modelled stochastically to risk adjust all the expected prices.

All calculations were built from the mean reversion with a modification to the random walk assumption as shown below.

$$\underbrace{S_{t+1} - S_t}_{\substack{Expected\ change\ in \\ price\ at\ t+1\ and\ t}} = \underbrace{\alpha(S^* - S_t)}_{\substack{Mean\ Reversion \\ Component}} + \underbrace{\sigma\varepsilon_t}_{\substack{Random \\ Component}} \tag{5.14}$$

Where:

S^* mean reversion level or long run equilibrium price

S_t spot price

α mean reversion rate

σ volatility

ε random shock to price from *t* to *t+1*

The results from the modelling are shown in Appendix A. Below are the input parameters for both pellets and concentrate prices as shown in Table 7 and 8. For this project only 100 iterations were considered for running the Monte Carlo simulation process. After the simulation the spreadsheet was 64 megabytes in size and this due to the fact that Risk simulator stores all of its information in Microsoft Excel. This was quite a challenge when saving and editing the spreadsheet.

Table 7: Pellets Input Parameters

<u>PELLETS</u>		
Economic assumptions		
Riskfree interest rate (unescalated; %):	2.0%	
Market price of risk:	3.00	
DCF Risk Adjusted Discounted Rate (RADR) (%):	8.0%	
Mineral price models		
Current mineral price:	250	$/unit
Current long-term mineral median:	180	$/unit
Mineral median trend (%):	0	
Short-term price uncertainty (%):	40.7%	
Price reversion half-life:	2.5	years
Reversion factor:	**0.0800**	
Price correlation:	0.09	
Commodity price of risk:	**0.26**	
Monte Carlo parameters		
Simulation time step	0.0192	years

Table 8: CFS Input Parameters

<u>CFS</u>		
Economic assumptions		
Riskfree interest rate (unescalated; %):	2.0%	
Market price of risk:	3.00	
DCF Risk Adjusted Discounted Rate (RADR) (%):	8.0%	
Mineral price models		
Current mineral price:	180	$/unit
Current long-term mineral median:	100	$/unit
Mineral median trend (%):	0	
Short-term price uncertainty (%):	3.7%	
Price reversion half-life:	2.5	years
Reversion factor:	**0.5**	
Price correlation:	0.485	
Commodity price of risk:	**1.32**	
Monte Carlo parameters		
Simulation time step	0.0192	years

5.8 Comparison between Stochastic and Simulated Paths Models

Both stochastic model and simulated path model were built on the same basis. The key difference between the two models is the wieners process which used in the simulated path model and this populated from a Monte Carlo normal distribution. The prices in both models were risked adjusted by discounting with risk adjustment factor to get the Risk Adjusted expected price.

The key observation in both models was to keep track of the price reverting to the long term median price at a point and this is shown in Figures 5-8 and 5-10. The black line is the simulated median prices for the 10 year period taking into consideration the simulation steps of 0.0192 years. As can be seen in the Figure 5-8 and Figure 5-10, at one stage in the year the price reverts to the long term median price. The blue, pink, lime and yellow lines are the median price from the stochastic model and the boundaries are the 90th and 10th percentile boundaries. Both models complement each other but depending on the simulation tool used the simulated path price prediction can become a challenge. Figures 5-9 and 5-11 shows a plot of the stochastic modelled risk discount factor applied over the 10 years price model for both Pellets and CFS.

Table 9: Pellets Risk Adjusted Prices

Pellets Mineral price path simulation											
Year 0 expected mineral price model											
Project time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Forecast time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Static stochastic price model											
Long-term mineral median	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
Associated price variance (%)	0.0000	0.1531	0.2835	0.3947	0.4894	0.5701	0.6389	0.6975	0.7475	0.7900	0.8263
Median price	250.00	243.76	238.15	233.08	228.49	224.34	220.57	217.15	214.04	211.21	208.63
Expected price	250.000	263.155	274.417	283.924	291.838	298.335	303.590	307.771	311.034	313.520	315.357
90th percentile price	250.000	402.537	471.307	521.527	560.230	590.603	614.583	633.517	648.416	660.062	669.079
10th percentile price	250.000	147.617	120.334	104.165	93.191	85.215	79.164	74.434	70.656	67.585	65.054
Risk discount factor	1.0000	0.9051	0.8255	0.7582	0.7010	0.6520	0.6098	0.5733	0.5416	0.5138	0.4895
Risk-adjusted expected price	250.00	238.17	226.52	215.27	204.57	194.52	185.14	176.46	168.45	161.10	154.36
Year 2 expected mineral price model											
Project time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Forecast time			0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
Static stochastic price model											
Long-term mineral median	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
Associated price variance (%)			0.0000	0.1531	0.2835	0.3947	0.4894	0.5701	0.6389	0.6975	0.7475
Median price			377.40	356.51	338.27	322.25	308.14	295.66	284.59	274.75	265.96
Expected price			377.395	384.872	389.783	392.550	393.567	393.183	391.706	389.396	386.471
90th percentile price			377.395	588.723	669.446	721.059	755.515	778.372	792.965	801.536	805.681
10th percentile price			377.395	215.895	170.922	144.017	125.676	112.307	102.141	94.175	87.792
Risk discount factor			1.0000	0.9051	0.8255	0.7582	0.7010	0.6520	0.6098	0.5733	0.5416
Risk-adjusted expected price			377.40	348.34	321.75	297.63	275.88	256.36	238.88	223.26	209.31
Year 4 expected mineral price model											
Project time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Forecast time					0.0	1.0	2.0	3.0	4.0	5.0	6.0
Static stochastic price model											
Long-term mineral median	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
Associated price variance (%)					0.0000	0.1531	0.2835	0.3947	0.4894	0.5701	0.6389
Median price					542.27	498.19	460.68	428.57	400.92	376.98	356.15
Expected price					542.272	537.816	530.845	522.067	512.069	501.322	490.196
90th percentile price					542.272	822.675	911.718	958.963	983.000	992.451	992.346
10th percentile price					542.272	301.689	232.779	191.534	163.516	143.195	127.823
Risk discount factor					1.0000	0.9051	0.8255	0.7582	0.7010	0.6520	0.6098
Risk-adjusted expected price					542.27	486.76	438.19	395.83	358.95	326.87	298.94
Year 6 expected mineral price model											
Project time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Forecast time							0.0	1.0	2.0	3.0	4.0
Static stochastic price model											
Long-term mineral median	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
Associated price variance (%)							0.0000	0.1531	0.2835	0.3947	0.4894
Median price							147.05	149.35	151.51	153.53	155.42
Expected price							147.045	161.229	174.583	187.022	198.506
90th percentile price							147.045	246.625	299.844	343.533	381.063
10th percentile price							147.045	90.442	76.556	68.614	63.388
Risk discount factor							1.0000	0.9051	0.8255	0.7582	0.7010
Risk-adjusted expected price							147.05	145.92	144.11	141.80	139.15
Year 8 expected mineral price model											
Project time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Forecast time									0.0	1.0	2.0
Static stochastic price model											
Long-term mineral median	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
Associated price variance (%)									0.0000	0.1531	0.2835
Median price									241.98	236.54	231.62
Expected price									241.979	255.351	266.896
90th percentile price									241.979	390.599	458.390
10th percentile price									241.979	143.239	117.036
Risk discount factor									1.0000	0.9051	0.8255
Risk-adjusted expected price									241.98	231.11	220.31

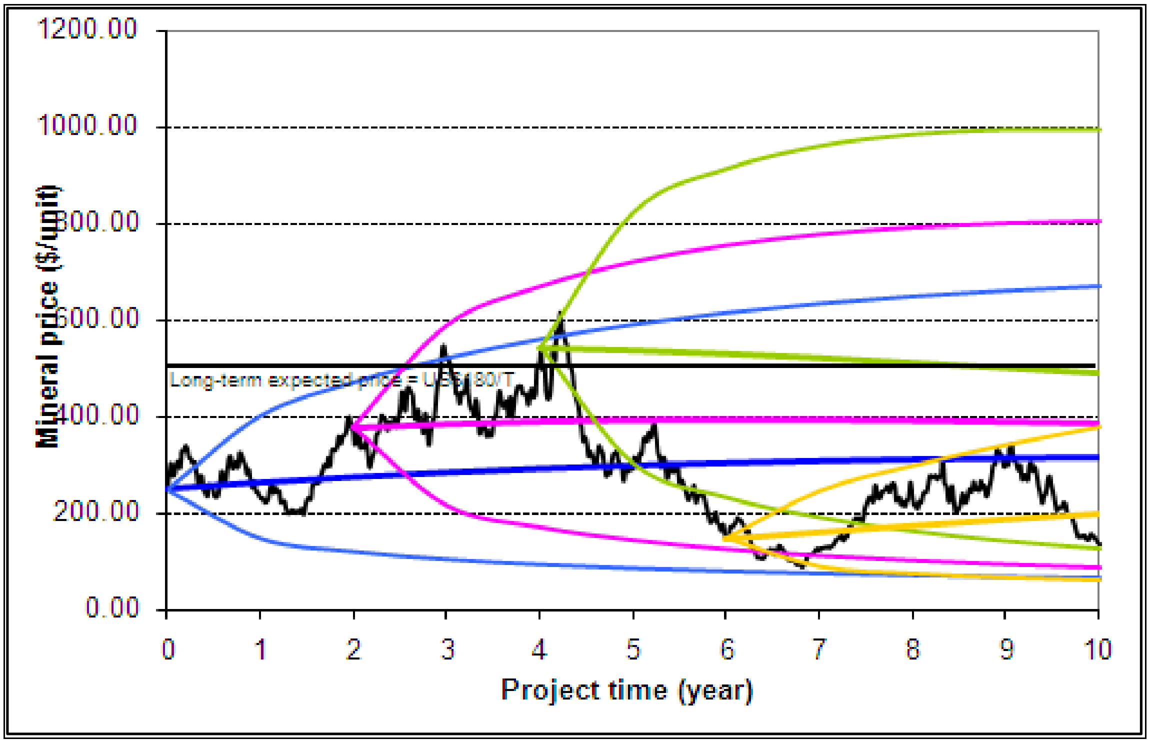

Figure 5-8: Pellets graph

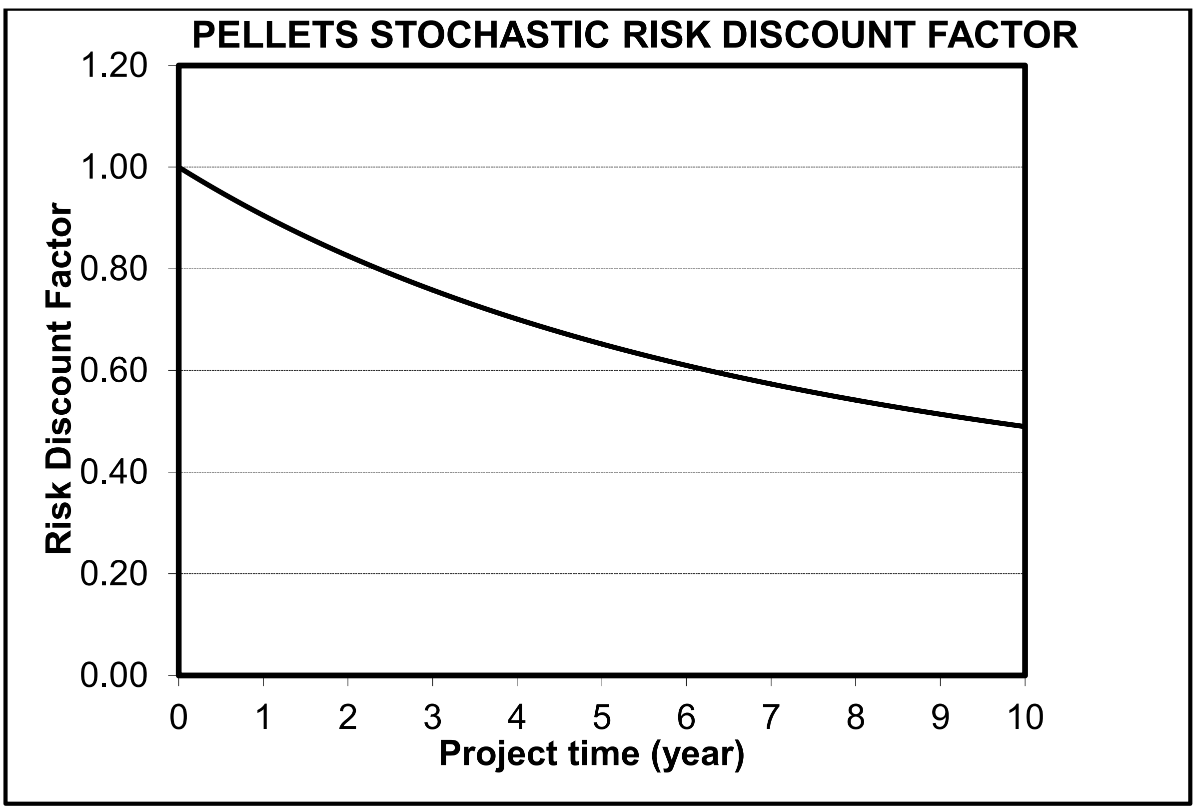

Figure 5-9: Pellets stochastic risk discount factor graph

Table 10: CFS Risk Adjusted Prices

CFS Mineral price path simulation											
Year 0 expected mineral price model											
Project time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Forecast time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Static stochastic price model											
Long-term mineral median	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Associated price variance (%)	0.0000	0.0009	0.0012	0.0013	0.0013	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014
Median price	180.00	142.83	124.14	114.01	108.28	104.94	102.97	101.79	101.08	100.66	100.40
Expected price	180.000	142.895	124.213	114.088	108.353	105.015	103.040	101.860	101.152	100.724	100.466
90th percentile price	180.000	148.323	129.737	119.410	113.490	110.023	107.965	106.733	105.992	105.544	105.274
10th percentile price	180.000	137.547	118.783	108.862	103.309	100.098	98.205	97.077	96.400	95.992	95.746
Risk discount factor	1.0000	0.9585	0.9342	0.9198	0.9111	0.9059	0.9028	0.9008	0.8997	0.8990	0.8986
Risk-adjusted expected price	180.00	136.97	116.04	104.93	98.72	95.13	93.02	91.76	91.01	90.55	90.28
Year 2 expected mineral price model											
Project time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Forecast time			0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
Static stochastic price model											
Long-term mineral median	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Associated price variance (%)			0.0000	0.0009	0.0012	0.0013	0.0013	0.0014	0.0014	0.0014	0.0014
Median price			125.15	114.57	108.60	105.13	103.08	101.86	101.12	100.68	100.41
Expected price			125.148	114.625	108.667	105.201	103.152	101.928	101.192	100.749	100.480
90th percentile price			125.148	118.978	113.500	110.108	108.043	106.789	106.029	105.568	105.289
10th percentile price			125.148	110.334	103.916	100.382	98.350	97.155	96.444	96.018	95.761
Risk discount factor			1.0000	0.9585	0.9342	0.9198	0.9111	0.9059	0.9028	0.9008	0.8997
Risk-adjusted expected price			125.15	109.87	101.52	96.76	93.98	92.34	91.35	90.76	90.40
Year 4 expected mineral price model											
Project time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Forecast time					0.0	1.0	2.0	3.0	4.0	5.0	6.0
Static stochastic price model											
Long-term mineral median	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Associated price variance (%)					0.0000	0.0009	0.0012	0.0013	0.0013	0.0014	0.0014
Median price					107.30	104.37	102.63	101.58	100.96	100.58	100.35
Expected price					107.301	104.412	102.687	101.651	101.026	100.649	100.420
90th percentile price					107.301	108.378	107.254	106.392	105.816	105.449	105.220
10th percentile price					107.301	100.504	98.198	96.995	96.323	95.936	95.708
Risk discount factor					1.0000	0.9585	0.9342	0.9198	0.9111	0.9059	0.9028
Risk-adjusted expected price					107.30	100.08	95.93	93.49	92.05	91.18	90.65
Year 6 expected mineral price model											
Project time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Forecast time							0.0	1.0	2.0	3.0	4.0
Static stochastic price model											
Long-term mineral median	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Associated price variance (%)							0.0000	0.0009	0.0012	0.0013	0.0013
Median price							97.14	98.26	98.94	99.36	99.61
Expected price							97.144	98.300	98.998	99.420	99.676
90th percentile price							97.144	102.034	103.401	104.057	104.402
10th percentile price							97.144	94.621	94.670	94.866	95.036
Risk discount factor							1.0000	0.9585	0.9342	0.9198	0.9111
Risk-adjusted expected price							97.14	94.22	92.48	91.44	90.81
Year 8 expected mineral price model											
Project time	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Forecast time									0.0	1.0	2.0
Static stochastic price model											
Long-term mineral median	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Associated price variance (%)									0.0000	0.0009	0.0012
Median price									92.49	95.38	97.17
Expected price									92.492	95.418	97.227
90th percentile price									92.492	99.042	101.551
10th percentile price									92.492	91.846	92.977
Risk discount factor									1.0000	0.9585	0.9342
Risk-adjusted expected price									92.49	91.46	90.83

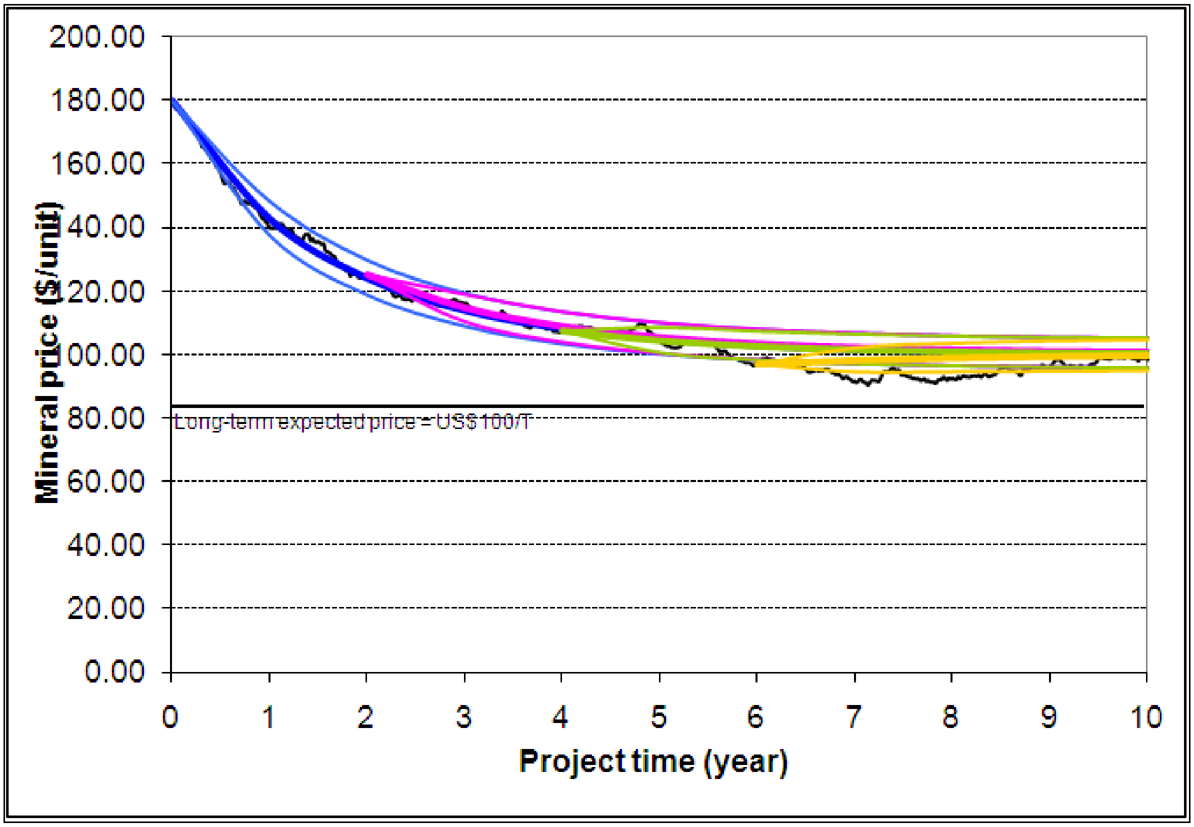

Figure 5-10: CFS graph

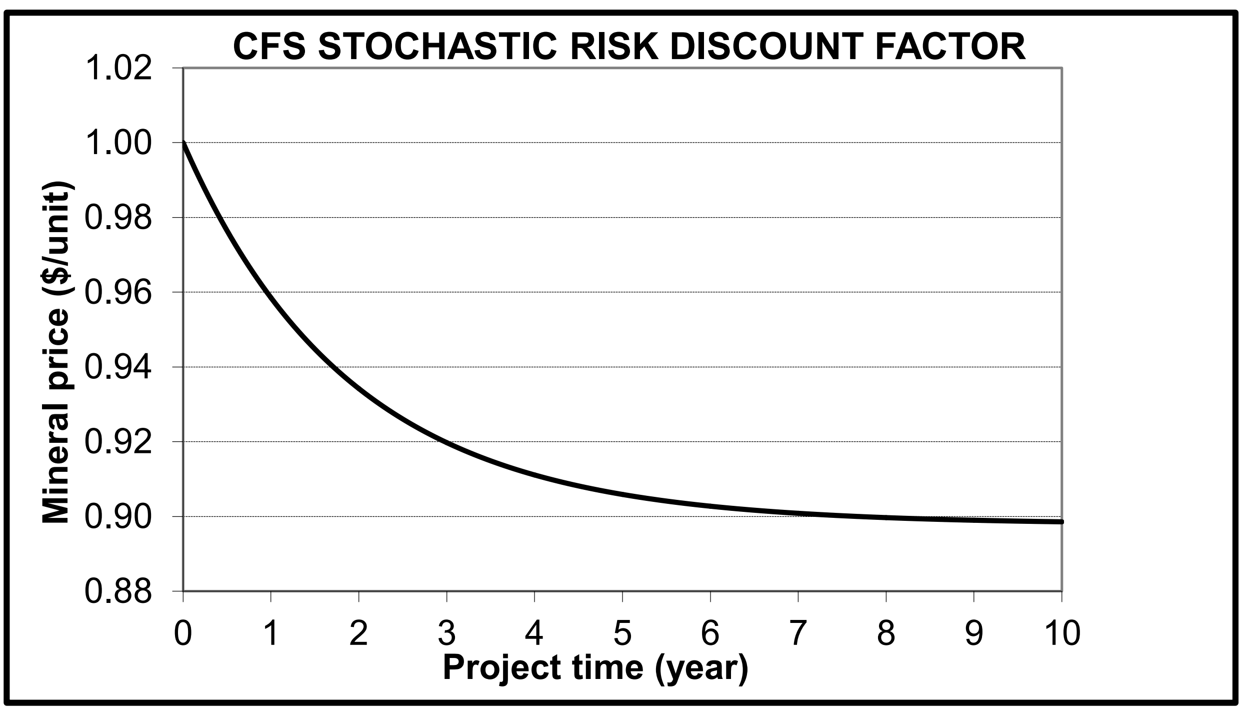

Figure 5-11: CFS risk discount factor graph

5.8.1 Mean Reversion Component or Drift

Mean reversion component or “drift” term is bounded by the distance between the current price, the mean reversion level, and the mean reversion rate. The mean reversion component can be positive resulting in an upward influence on spot price. This influence is due to the fact that the spot price is below the mean reversion level.

On the other hand, the mean reversion component can be negative resulting in a downward influence on spot price because the spot price is above the mean reversion level. This normally results in a price path that drifts towards the mean reversion level over time, at a speed determined by the mean reversion rate (Blanco and Soronow, 2001). This shown in Figures 5-12 and 5-14.

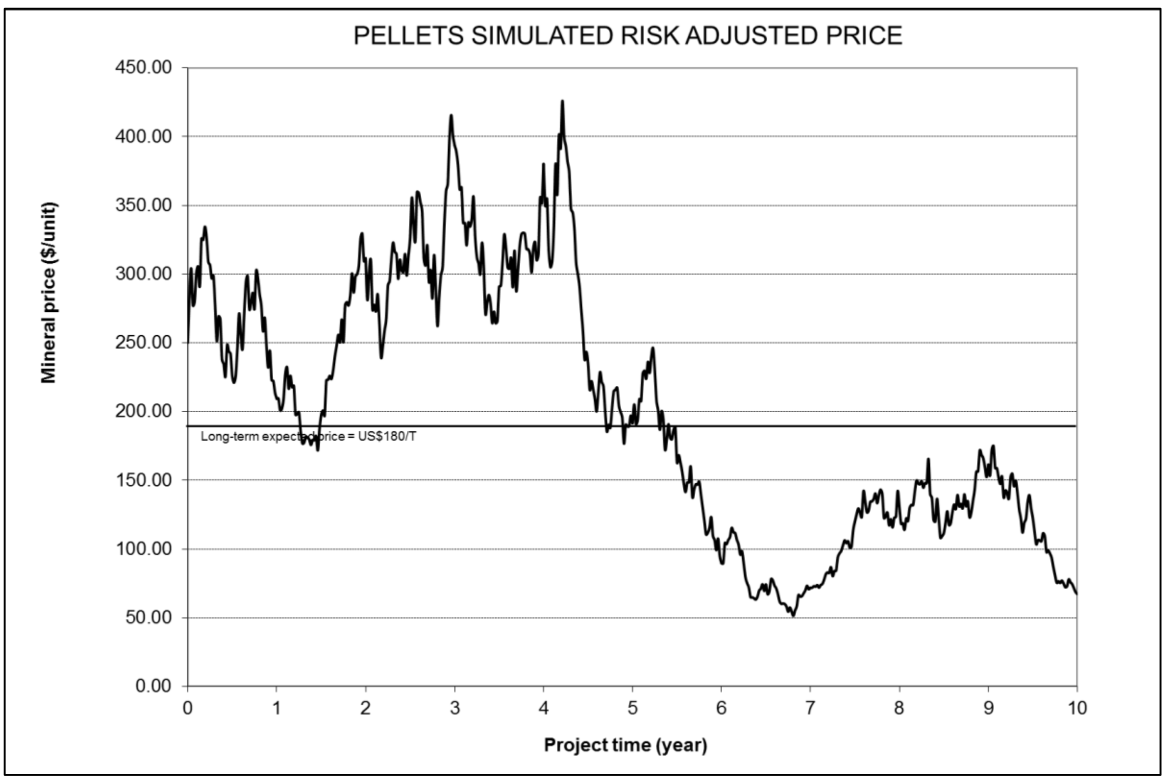

Figure 5-12: Pellets simulated risk adjusted price

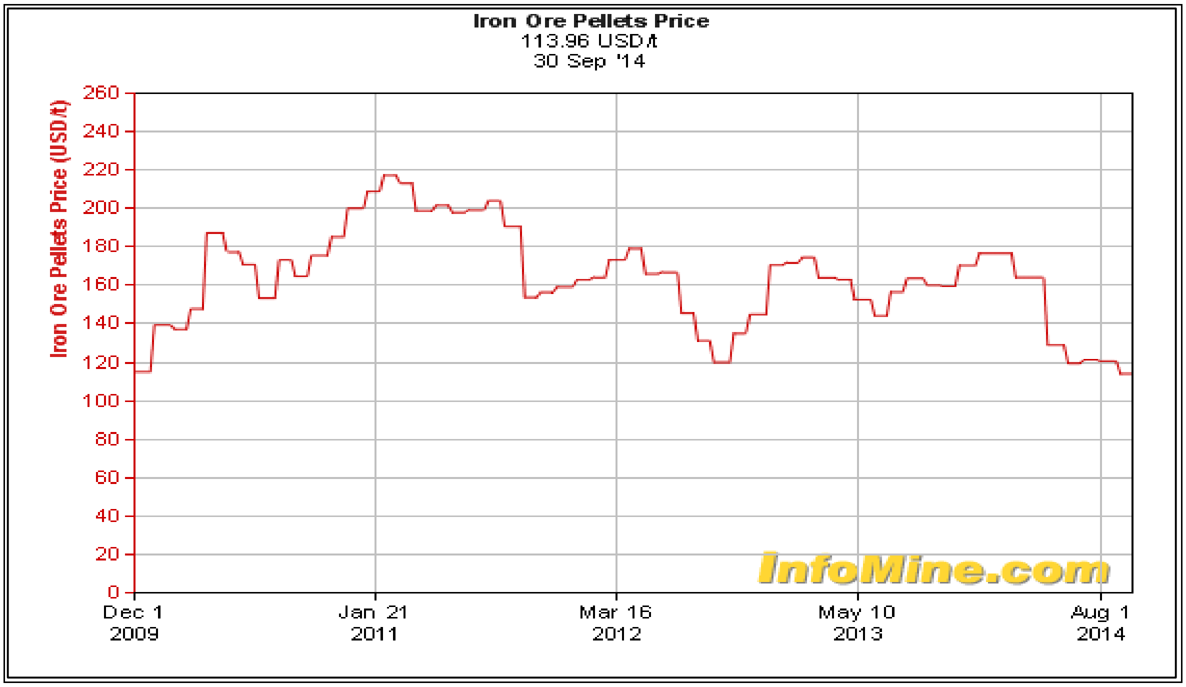

Figure 5-13: Iron ore pellet price

(Source: http://www.infomine.com/investment/metal-prices/iron-ore-pellets/all/))

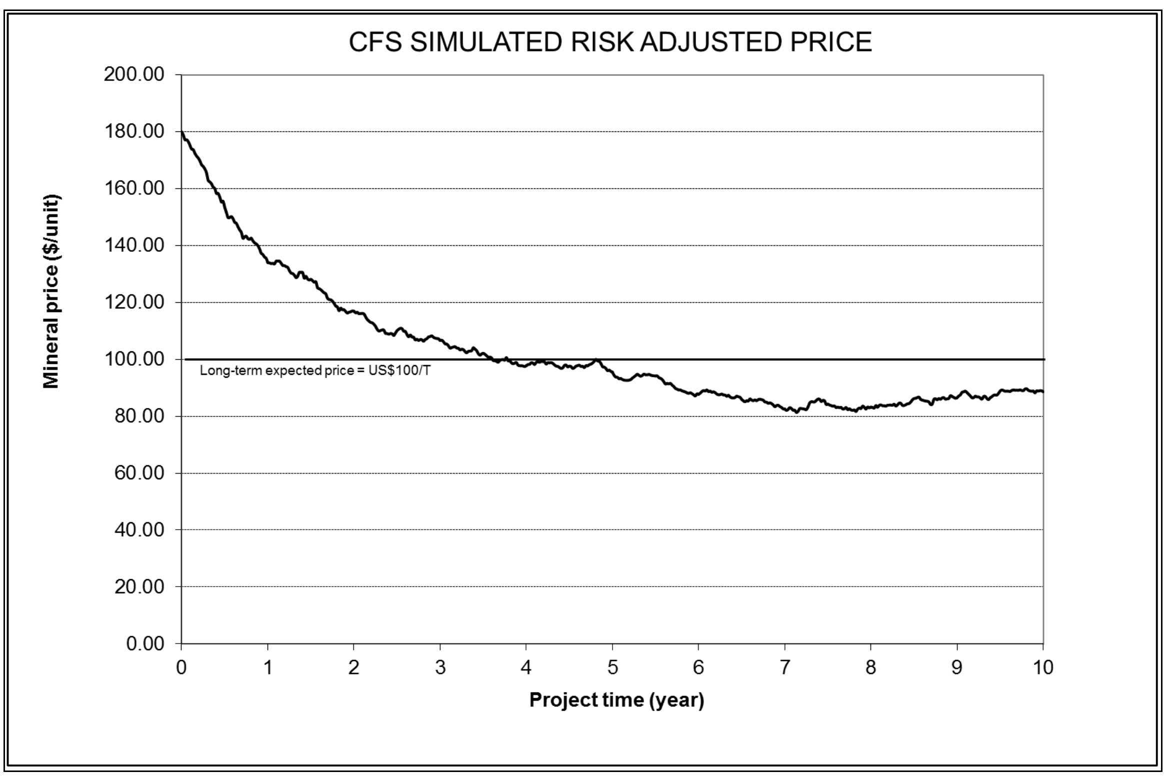

Figure 5-14: CFS simulated risk adjusted price

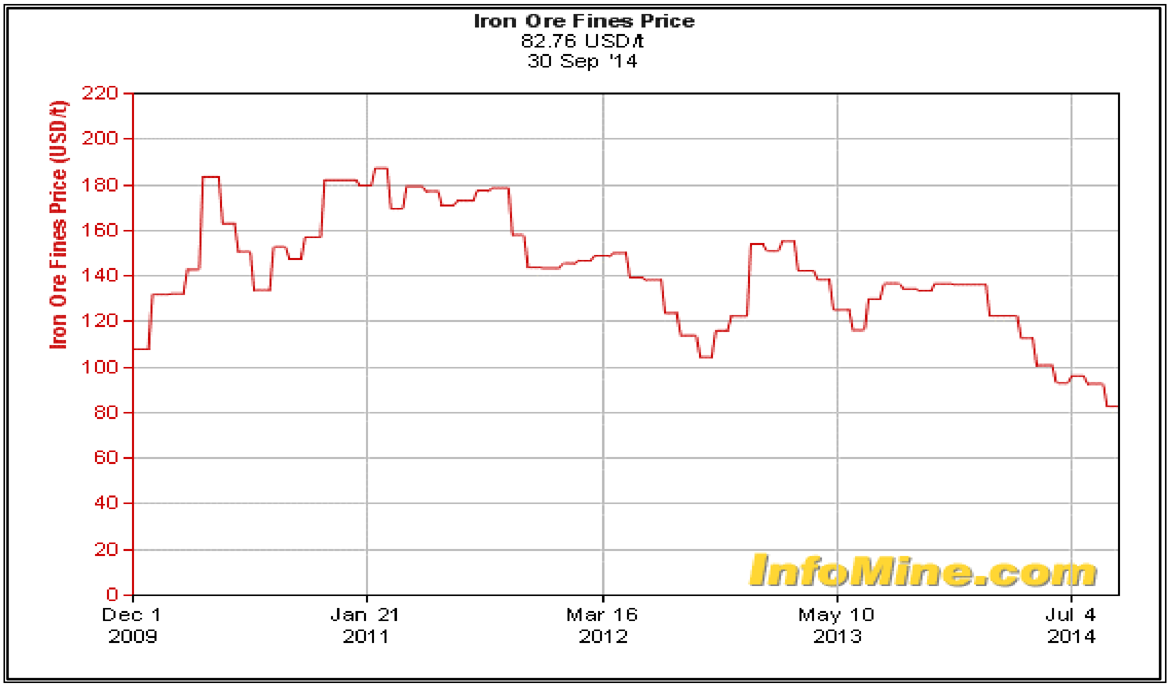

Figure 5-15: Iron ore CFS price

(Source: http://www.infomine.com/investment/metal-prices/iron-ore-fines/all/)

Comparatively, Pellets simulated risk adjusted prices drift wiggly towards the long term equilibrium whilst CFS simulated risk adjusted prices drift smoothly towards the long term equilibrium. This due to the fact that Mean Reversion considers historical prices to be related to predicting future prices. Mean Reversion assumes the price changes are related based on random walk.

Comparing Figures 5-14 and 5-16 (Year 0 (2011)-Year 10(2021)) to Figures 5-15 and 5-17, the simulated adjusted prices are similar to the actual historical prices. This shows that Mean Reversion Process is the suitable method of modelling future iron ore prices.

Mean Reversion process for predicting commodities like iron ore is still at its early stages. Since iron ore prices are customer and client biased it makes it difficult to gather information to model the market risk. The other most important part is the input parameter is linked to the amount of historical data.

Overall the path for the model has been defined and since there are iron ore swaps now it will be easy to get more information from the stock market on iron ore trades with relation to the steel indices. From all indications, if more historical information is gathered, the better it will be to model the input parameters to predict better output of prices. To say the least iron prices does mean revert and both stochastic/simulated path model can be used in future predict risk-adjusted expected prices.

CHAPTER 6: MANAGERIAL FLEXIBILITY

6.1 Introduction

Most projects often offer multiple options for management to choose from. Each of these options derives its value from its estimated NPV and from any opportunity cost associated with future project. Management and Analyst normally selects the project with the highest NPV. This ability is called managerial flexibility and it impacts project value because it helps management to resolve uncertainty in current and future mining projects.

Over the years, evaluation methods that considers managerial flexibility or ROV method are generally believed to be more precise and realistic than static evaluation techniques, i.e., DCF (Bengtsson 2001; Samis et al. 2006). When management of a mining company understands possible trends of the commodity market, they can achieve additional expected profit by selecting a risk adjusted optimal price threshold. ROV can assist the owner of the mine to;

(1) Evaluate the value of a mine by considering the uncertainties and managerial flexibility incorporated in the project;

(2) Making managerial decisions such as starting a mine, abandonment, expansion, and reduction of production rates accordingly to changes in market condition and;

(3) Nonflexible techniques may undervalue a project.

In any mining project, uncertainty increases its risk, and as a consequence measure, the mining company manager or management requires flexibility to manage risks in the project (Tang, 2007).

ROV techniques which was derived from financial options are one of the most important corporate finance decision-making tools to have been introduced in the past 30 or 40 years. ROV methodology captures the present value of managerial flexibility decisions at a future date in response to the arrival of new data. Traditional NPV methodology implicitly assumes prices and discount rate as static, i.e. no flexibility (Hall and Nicholls, 2007).

Kazakidis and Scoble (2003) state that managerial flexibility needs to be built into the project to help reduce risk, but also to enable managers to anticipate opportunities that may develop during the life phase of the operation.

Dimitrakopoulos and Sabour (2007) and Sabour and Poulin (2010) mentioned that the main advantage of the ROV was that it allowed management to incorporate flexibility and to have the ability to revise decisions over time, based on new information from the likes of drill hole data, commodity prices and cost.

Managerial flexibility grants a transparency standard for analyzing the timing of strategic and managerial decisions, as it deals with all sort of uncertainty individually, accounting for all possible scenarios of future outcomes.

6.2 Strategic Mine Planning

Strategic mine planning aims at maximizing the value to be realized from a strategic resource development. Strategic mine planners use various mine planning software's with the help of economic input such as prices, capital and operating costs, scheduling scenarios, orebody knowledge, mining method, pit optimization, pit design, and phase designs.

Uncertainty abounds even with all the information available and is a core component to both the forecasting and valuation of a project. Some of the uncertainties that have been determined to have significant impact on strategic mine planning include technical uncertainty, geological uncertainty, and market uncertainty. The focus of this work is on market uncertainty, which is mostly affected by commodity prices, discount rate and in the case of iron ore is quite difficult to predict. With the help of trading platforms and the stochastic price modelling, market uncertainty could be incorporated into long-term resource development plans.

Long-term planning can also be labelled as "Strategic Mine Planning" depending on the individual mine planner. The development of strategic mine plans using software such as NPV Scheduler, MineMax planner, Whittle Optimiser, COMET, and SimSched has become very common (Stone et al., 2004). These tools allow strategic mine planners to schedule using objective functions such as maximizing and minimizing NPV and cost respectively. An apparent challenge with this methodology is that the value objective is highly sensitive to the assumed deterministic forward commodity price and in most cases, historical data proves to be quite volatile.

This has raised two key questions for the strategic mine planner;

1. Whether this valuation approach can be used to accurately rank different investment opportunities.

2. Whether the strategic mine plan which is considered optimal for the assumed static commodity price will still be optimal, or more reasonably near-optimal, when price uncertainty is accounted for (Zhang et al., 2007).

6.2.1 Comet Strategic Mine Planning Tool

COMET is a strategic mine planning tool that is currently being used by many companies. COMET is based on dynamic programming. In COMET the NPV is calculated from a series of cash flows using a constant discount rate, δ, over the life of the operation:

$$\mathrm{NPV} = \sum_{\mathrm{year}=1}^{\mathrm{life}} \mathrm{Cash\ flow}_{\mathrm{year}} * (1+\delta)^{-\mathrm{year}} \tag{6.1}$$

As can be seen from Equation 6.1, there are only two parameters that can impact the NPV of a deposit, the discount rate and the annual cash flows. The cash flow component can be a function of a large number of properties including:

- Price of commodity;
- Quantity of rock processed for ore and sent to waste;

- Recovered proportion of processed products;
- Operating cost;
- Sustaining capital;
- Rehabilitation cost; and
- Royalties and taxes.

This bulleted list is not intended to be comprehensive, it is clear there are many decisions that can affect the cash flow, and hence the NPV.

The ultimate objective is to maximise the NPV which is manipulated through parameters such as production rate, cut-off grade, and price of commodity. These parameters, as stated by King (2000), are divided into two categories:

Uncontrollable: A few of the parameters that impact the annual cash flow are not under the control of the mine. These may be structural uncertainty, geological uncertainty, and commodity prices.

Controllable: These parameters may include the production rates, cut-off grade, and, to some degree, product recoveries.

King (1999, 2000) has shown how the following operating policies can be optimized using dynamic programming techniques.

- Cut-off grade policy;

- Comminution (including blasting and grind/recovery/throughput) policies;
- Dilution/loss policy; and
- Phase sequencing policies;

All the above policies can be simultaneously optimized.The basis of optimization algorithm has not changed since Ken Lane's work in 1964 (The Economic Definition of Ore – Cut-off Grades in Theory and Practice, by Ken Lane, 1988 (and revised in 1997)):

$$V(R,T) = \underset{\substack{0 \le r \le R \\ \text{all } \omega}}{\text{Max}} \left\{ c(t,r,\omega) + \frac{V(R-r,T+t)}{(1+\delta)^t} \right\} \tag{6.2}$$

In Equation 6.2 the maximum NPV of a resource (R) at a time (T) is calculated by finding the best strategy (ω) that maximizes the sum of: the cash flow (c) for an increment resource (r) and the maximum NPV of the remaining reserve. The remaining resource cash flows are discounted (by δ) and start after the increment of resource is mined (at time t).

King (1999) indicated that there were two limitations to the above algorithm which restrict its application in practical mining problems. First, calculation of the cash flow for the resource can be dependent on the time it is mined. Aging facilities, processing expansions, and commodity price changes are some of the reasons the cash flow is dependent on the time of mining. Without knowing the time that each resource increment is mined, cash flow is unknown and therefore the best operating policy and remaining value are also unknown.

The second problem with this algorithm is that it must start with the last increment of resource. The algorithm breaks down if the operation has multiple resources that could be the last block. Operations with stockpiles are a particular problem since the quantity and qualities of stockpiled material are unknown at the end of the mine life.

King's solution to both these problems is to start at the beginning rather than at the end of the mine life. This is intuitively much more logical as this is how the deposit is actually mined. The status of all pushbacks, stopes, and stockpiles are known at the beginning of the operation.

The main problem with starting at the beginning that King discovered was that we do not know the maximum remaining value to use in Equation 6.2. This problem is overcome if an estimate of the remaining maximum value is used as shown in Equation 6.3 below:

$$V'(R,T) = \underset{\substack{0 \le r \le R \\ \text{all } \omega}}{\text{Max}} \{c(t,r,\omega) + \frac{V(R-r,T+t)}{(1+\delta)^t}\} \quad (6.3)$$

Equation 6.2 has the same general form as Equation 6.1 except that the maximum value(V) of the entire reserve and remaining reserve is replaced by an estimate of this maximum value (V'). An estimate of the remaining value can be obtained from initially scheduling the resource using default properties, referred to as the "seed" schedule in COMET.

The seed schedule needs not to be very close to the optimum though it should be good enough to produce a positive NPV for the project.

6.3 Managerial Flexibility Case Study

6.3.1 Location and Geology

Gardner iron ore mining operations and exploration program are centred on Lower Proterozoic iron formations of the Knob Lake Group. This formations lies within the lithotectonic Gagnon Terrane, in the Grenville Province of Western Labrador.

The Knob Lake Group falls into the continental-margin metasedimentary categorisation, comprising of pelitic schist, iron formations, and native mafic volcanics. It unconformably overlies the Archean Ashuanipi Metamorphic Complex, which is predominantly comprises granitic and granodioritic gneisses.

6.3.2 Assumptions and Results

Gardner Iron Ore Mining Company (GIOC) has five reserves pits and two resource pits. Out of the five reserve pits, three are active. They are Williams, Mehta, and Blechynden pit. The other two dormant reserve pits are Szwedska and Koufos. The two resource pits are Caroline and Perez pits.

As part of the ongoing GIOC expansion program, resource development groups were asked to put in place a strategic plan with respect to all operative pits and resource pits to guide the business decision making. The expansion program is broken into expansion 1 and 2.

Expansion 1 comprises additional spiral lines for ore processing and possibly bringing the Caroline pit into production. Expansion 2 comprises an additional ore delivery system and bringing the Perez pit into production. COMET was the strategic mine planning tool chosen for analysing the strategic plan for GIOC. The planning process is shown in Figure 6-1.

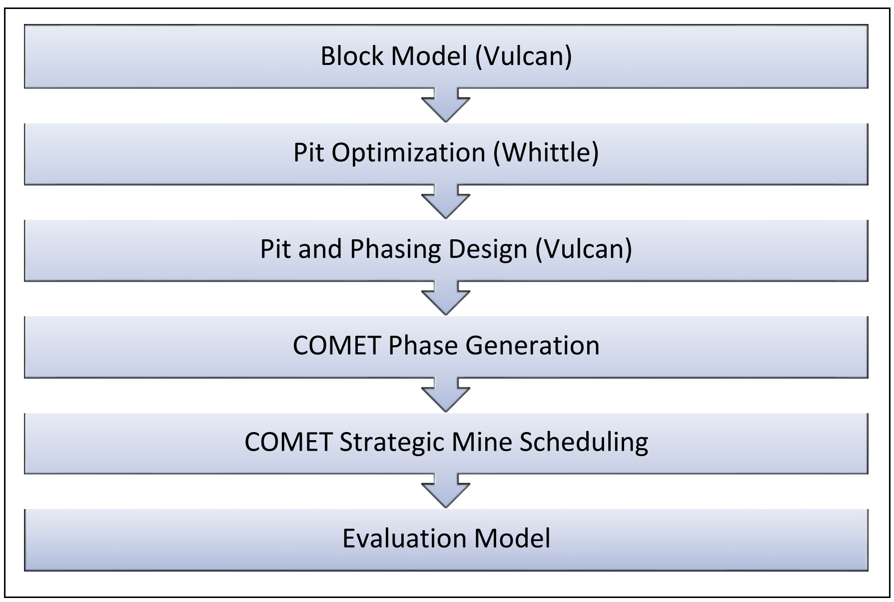

Figure 6-1: Planning flow sheet for GIOC

The most important factor affecting the studies is the market uncertainty of iron ore prices and discount rate. Relatively, the price to cost ratio is a challenge for most operating and upcoming mining companies. In the case of GIOC, business analysts group have always used a static long term price and single discount rate with static cost associated with it.

This makes it quite difficult to build managerial flexibility into the LOM plans and in this case, the strategic mine plans.

The goal for this exercise is to maximize NPV at the same time coordinating the various expansion scenarios to make sense out of it. Table 11 is the case matrix used to measure the impact of our schedule with respect to managerial flexibility as well as accounting for the market uncertainty concerning the expansion phase.

Table 11: Scenario Matrix for Case Study (F=Future prices, S=Static Prices, RDF=Risk Discount Factor, and DR= Discount rate (8%))

Scenarios	*Price*	*Discount Rate/Risk Discount Factor*	*Rev Pits*	*Res Pits*	*ODS Exp*	*Extra Spiral Lines*
1a	F	RDF	Yes	-	-	-
1b	S	DR	Yes	-	-	-
2a	F	RDF	Yes	Yes	-	-
2b	S	DR	Yes	Yes	-	-
3a	F	RDF	Yes	Yes	Yes	-
3b	S	DR	Yes	Yes	Yes	-
4a	F	RDF	Yes	Yes	-	Yes
4b	S	DR	Yes	Yes	-	Yes
5a	F	RDF	Yes	Yes	Yes	Yes
5b	S	DR	Yes	Yes	Yes	Yes
6a	F	RDF	Yes	-	-	Yes
6b	S	DR	Yes	-	-	Yes
7a	F	RDF	Yes	-	Yes	-
7b	S	DR	Yes	-	Yes	-

Using the scenario matrix above, all scenarios were evaluated with COMET. Each scenario process went through five iterations, which took an average of 800 seconds to complete the process. The results of the ore mined with respect to pits were plotted and used in an analytical process. Figures 6-2 and 6-3 are ore chat plots from the scenario 5a and 5b.

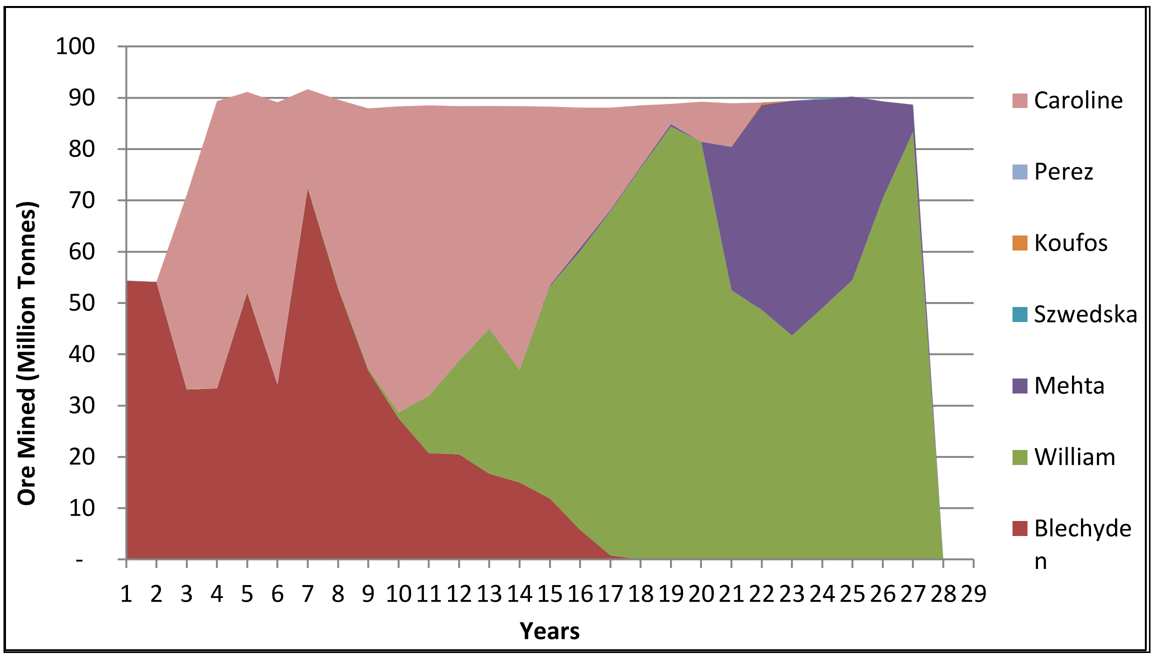

Figure 6-2: Scenario 5a

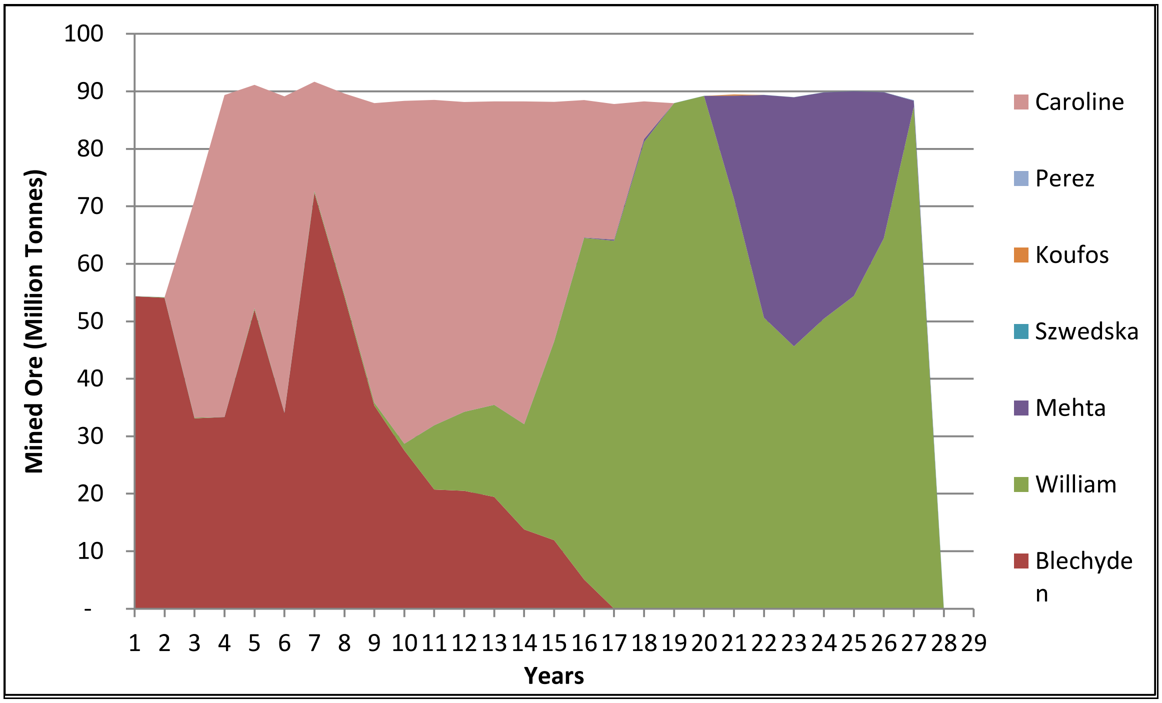

Figure 6-3: Scenario 5b

The modelled future risk discount factor and prices for pellet are shown in Figures 6-4 and 6-5.

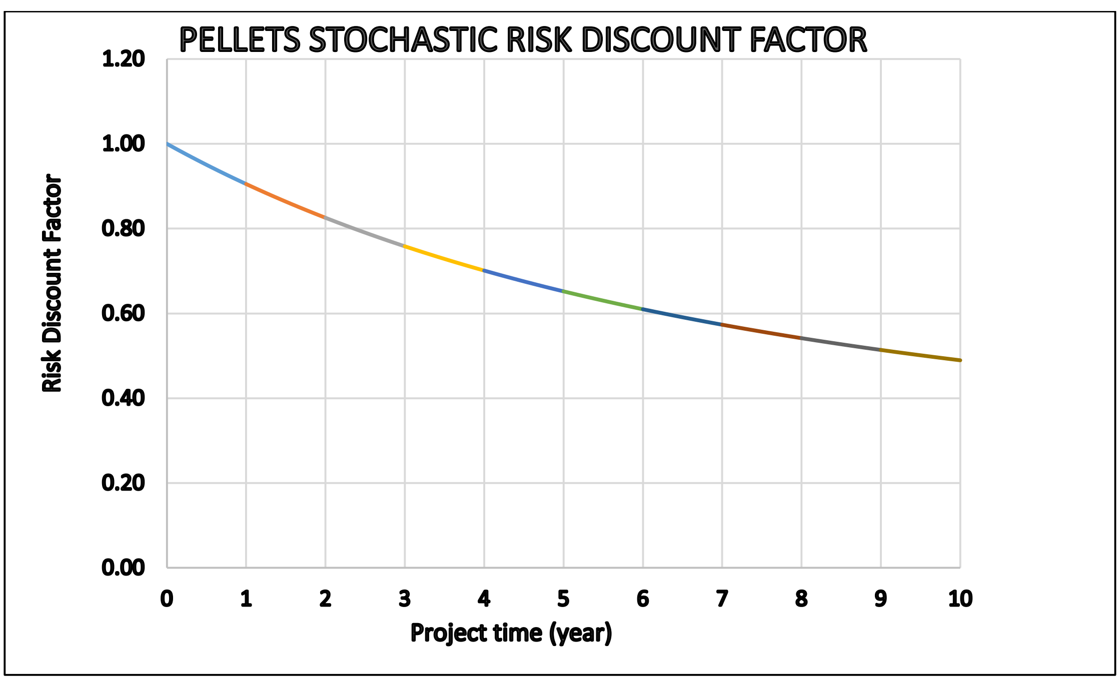

Figure 6-4: Pellet modelled risk discount factor

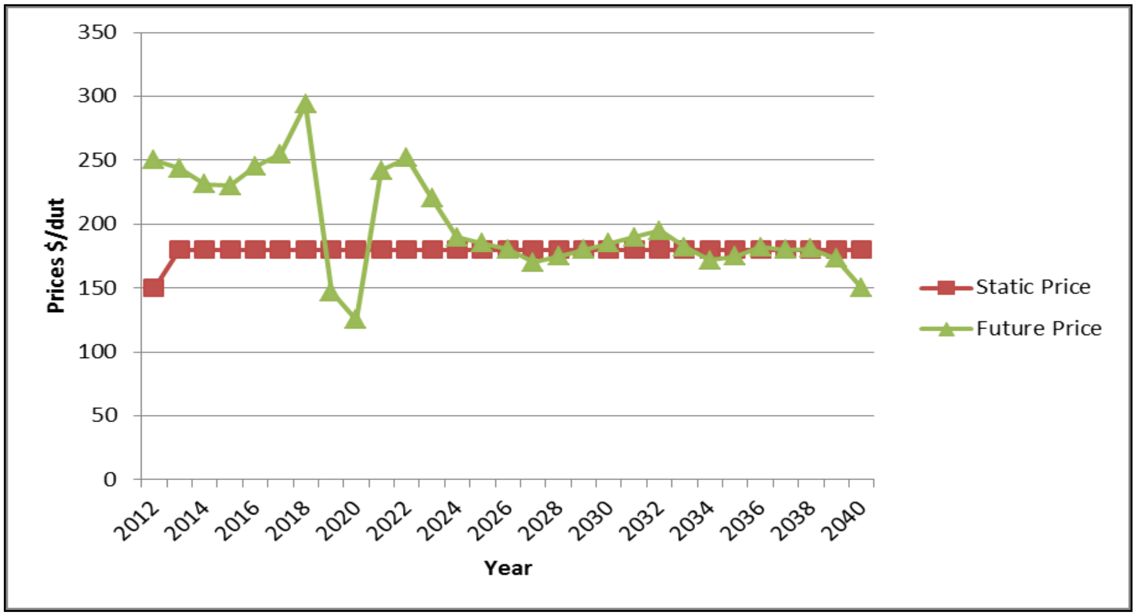

Figure 6-5: Static and future pellet prices

The modelled future risk discount factor and prices for concentrate are shown in Figures 6-6 and 6-7.

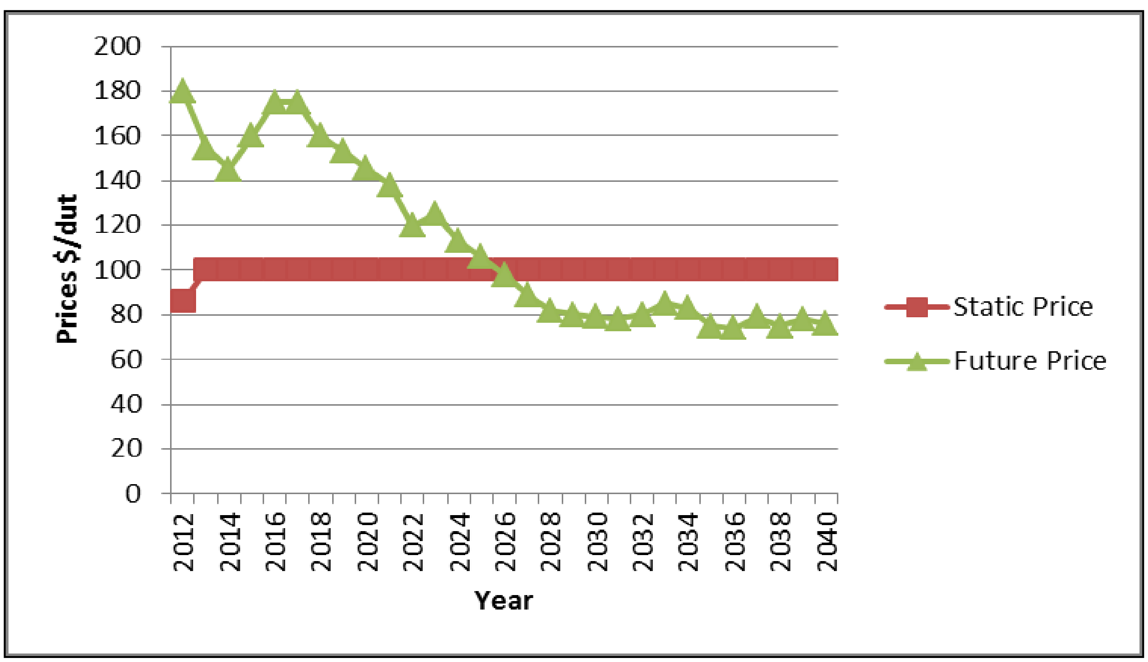

Figure 6-6: Static and future concentrate prices

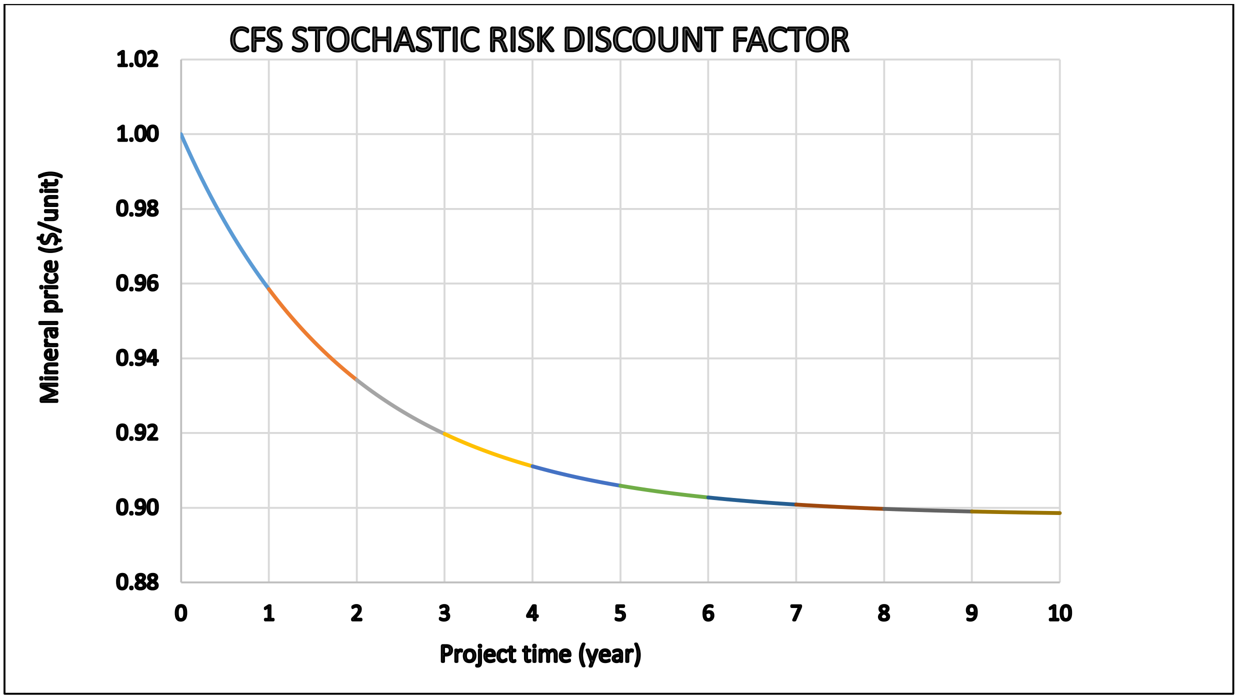

Figure 6-7: Concentrate modelled risk discount factor

Based on these price assumptions as well as discount rate and risk discount factor, all the scenarios were run with the same static cost. Taking a close look at the results, NPV from all scenarios with future prices and risk discount factor were higher (Figure 6-8). The main factors that had impact on outcome of the NPV are prices and discount rate/risk discount factor. The simulated prices and risk discounted factor gave higher NPV comparable to the static price and static discount rate (8%). More noticeably, ore mined was mostly driven by geology, prices and discount rate. In the real world when prices of a commodity go up the costs go up and when prices go down the costs still go up.

Processing capacity also had an impact on the schedule because of the clipping curve for both the base case and the expansion case.

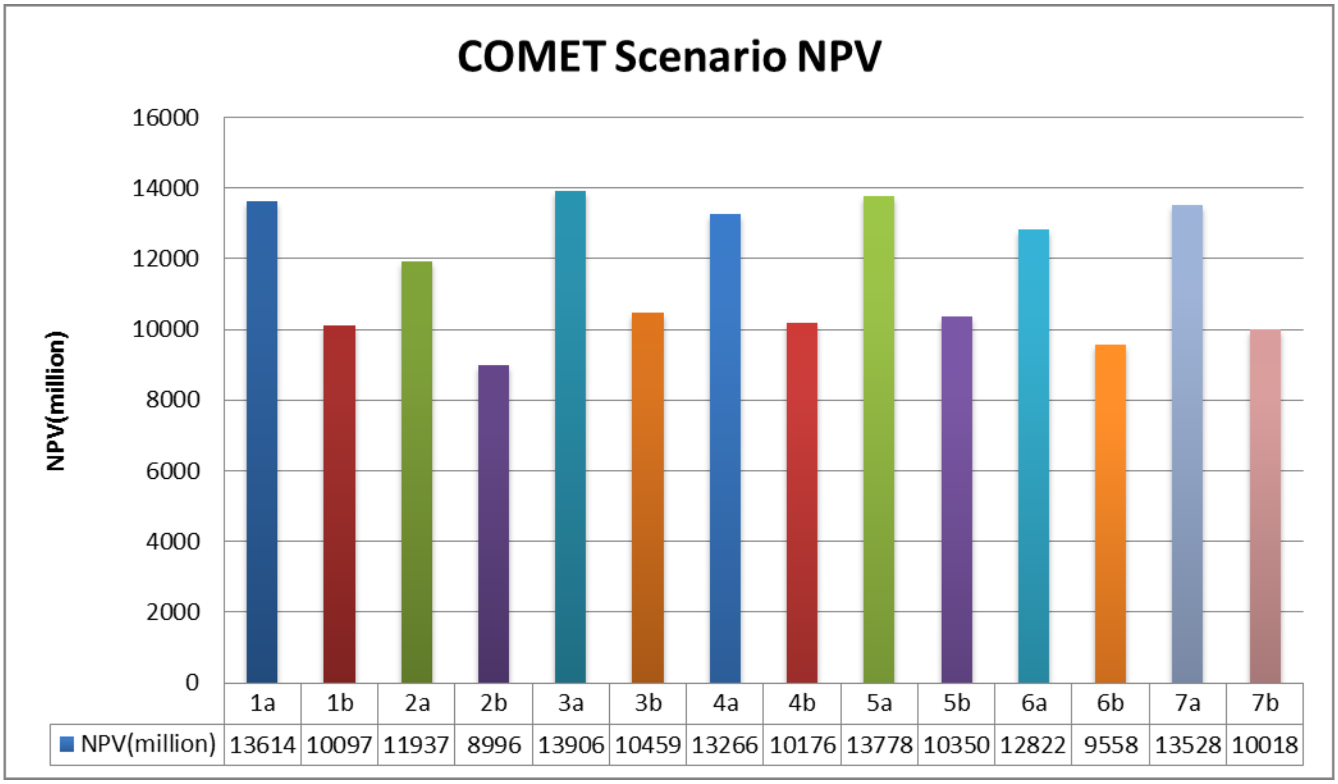

Figure 6-8: NPV (millions) plots from all the 14 scenarios

At GIOC, MIF is classified as ore. Limonite content in the ore is mostly used as the cut off and this has little impact on ore mined. Out of the reserve and resource pits, Blechynden and Caroline had the lowest stripping ratio. This made the two pits a better choice for COMET to select when both reserve and resources pits were turned on with the scenario runs. Nevertheless, not to lose sight of the expansion program for GIOC, Scenario 5a/b both gave similar schedules but the NPV difference was about \$2.9 billion in favour of future prices.

COMET strategic mine planning tool emulates a mine planning strategy which reacts to the commodity price and discount rate changes and accordingly updates the mine plan. Therefore in the strategic mine plan under future prices, the expansion program can start in 2014 since price starts to rise up. With all the expansion capital required, a higher NPV was achieved with future prices and risk discount factor.

CHAPTER 7: MODERN ASSET PRICING

7.1 Introduction

Modern Asset Pricing (MAP) is used in minimising risk associated with mineral project evaluation and it is a compliment to DCF. Over the years most projects NPV has been over stated due the tendency of mining companies using a static price as well as directly discounting project cash flow.

Over the years MAP has incorporated other risk adjusted components besides prices such as discount rate, cost and taxes. There are couple of advantages of MAP over DCF and this stated as follows;

1. MAP also allows analysts to include the value of future flexibility in their evaluations. It removes the bias induced by the inappropriate use of a static rate for projects with different patterns of risk.

2. MAP inspires analyst to evaluate a wider variety of project options for consideration, plus those where future management flexibility is an issue.

3. Discounting individual project determinants, as MAP does, involves many fewer considerations than directly discounting project cash-flows. It allows project analysts to discount future commodity prices, while DCF methods demand that they discount the cash flows directly.

4. MAP methods integrate the risk analysis by determining project value as a property of all potential scenarios considered as a group, as opposed to determining a sensitivities for each variable , and using the outcome of the sensitivities in the evaluation process.

This evaluation technique is an elementary form of ROV (Samis, 2001; Salahor, 1998; Laughton, 1998), which is known as Modern Asset Pricing. MAP is mostly easy to apply and gives dependable, conservative estimates of the value of a mining project under the assumption that the majority of its risk is related to the volatility of the price of the commodity produced. More sophisticated forms of ROV entail neutralization of the volatility of the overall cash flows of a project, including all other bases of risk besides that of the commodity prices.

Although there are a numerous papers published over the years about MAP evaluation methodology, they may seem to the unexperienced reader hard to apply it in reality. The main purpose of this chapter is to demystify this apparent complexity by developing in the following sections a realistic mining related evaluation model in a step by step manner.

7.2 A Practical Example of Modern Asset Pricing Modelling of an Iron Ore Mine

7.2.1 Modelling Steps and Assumptions

Most practical numerical examples are normally the most effective way to fully understand a theory, this section presents the evaluation of a simplified iron ore-mining project in a step-by-step fashion. Both the DCF/NPV and MAP/NPV of the project are derived. The steps involved involve the development of the project cash flow model as follows:

For the DCF valuation:

- A stochastic model is used in projecting the risky expected spot prices over the life of a project;
- The risky expected spot prices is used as an input in the revenue function of the DCF model; and
- The project's net after tax cash flows are discounted to the present using risk and time adjusted rate of discount to obtain the DCF/NPV.

For the MAP valuation:

- Obtaining available iron ore swaps quotes from the Singapore and Chinese Iron Ore Swap Markets;
- The iron ore price market risk was determined from market statistics;
- The futures prices are forecasted beyond the longest market estimate to the end of the project life using both stochastic and simulated model as well the assessed iron ore price risk; and
- The projected future prices are used as input to help calculate MAP model revenue and use the risk free rate of interest to discount the net after-tax cash flows to obtain its MAP/NPV.

Once the model is created the DCF and MAP outcomes can be compared. The example assumes that it is 13 May 2012 and that an operating iron ore mine with 10 years of mining life is offered for outright purchase. The mine produces and sells 16 million tonnes of iron ore in concentrate per annum to a smelter and pellets as well. To progress with negotiations it will become imperative to estimate a realistic expected order of magnitude of the value of the project as well as an idea of its minimum value.

Table 12: Summary of Economic Input for DCF and MAP Valuation

Life of Mine	*10 Years*
Annual Production (tonnes per annum)	$16,000,000
Operating Cost ($/t)	$80.4
Depreciation(straight line)	30%
Tax	30%
Rehab cost (last year-$18,000,000 and Rehab year-$30,000,000)	$48,000,000
Capital Cost (1st year-$25,000,000, 2nd year-$70,000,000, 3rd year-$135,000,000 CAD)	$230,000,000
Working Capital ($CAD)	$13,500,000
RADR	8%

7.2.2 MAP versus DCF Modelling

To facilitate the study, both the MAP and DCF models are developed using Canadian dollars as the currency and millions of tonnes of iron contained in concentrates/pellets as the units of production. Please note that the assumed data in Table 12 was used for all the models (Pellet MAP, Pellet DCF, CSF MAP, and CFS DCF).

Appendix A illustrates how in the MAP model (Pellets/CFS) of the above project:

- Future expected price is used in computing the project revenue; and
- The net after-tax cash flows were discounted by the risk-adjusted discount rate (8%) and a risk-free rate of interest (2.0%) is considered. The future expected prices were not considered in the model again.

The MAP/NPV for pellets is CAD$8.62 billion and CAD$1.192 billion for CFS represents a minimum risk-adjusted value for this project.

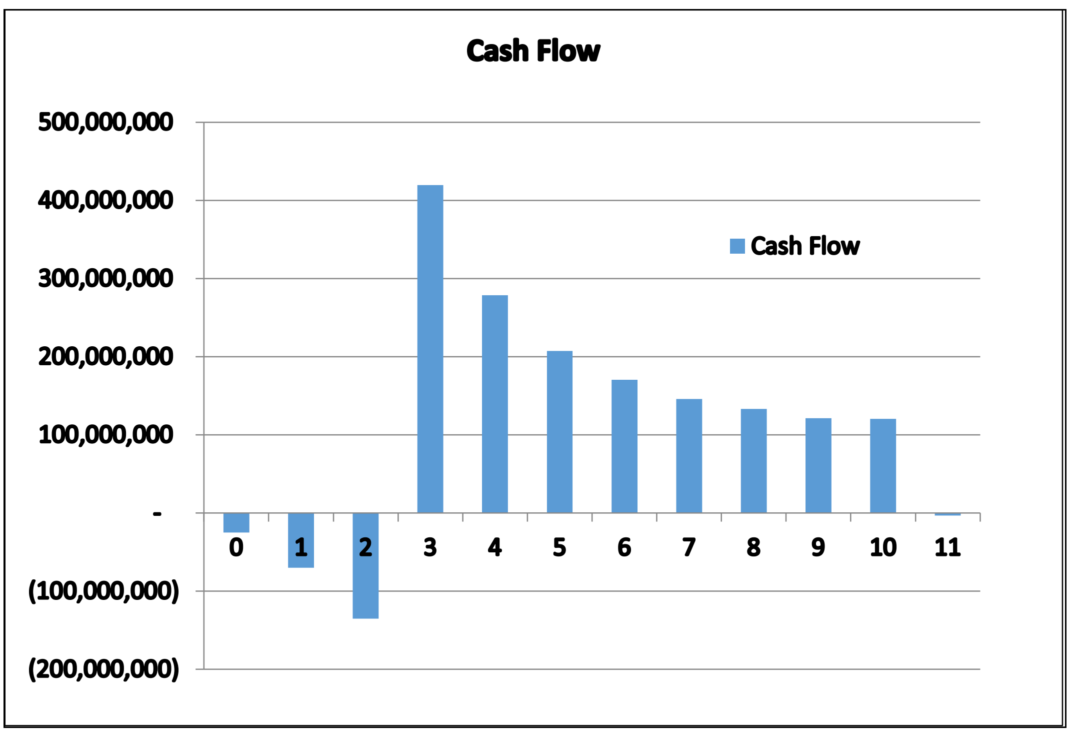

Figure 7-1: CFS MAP

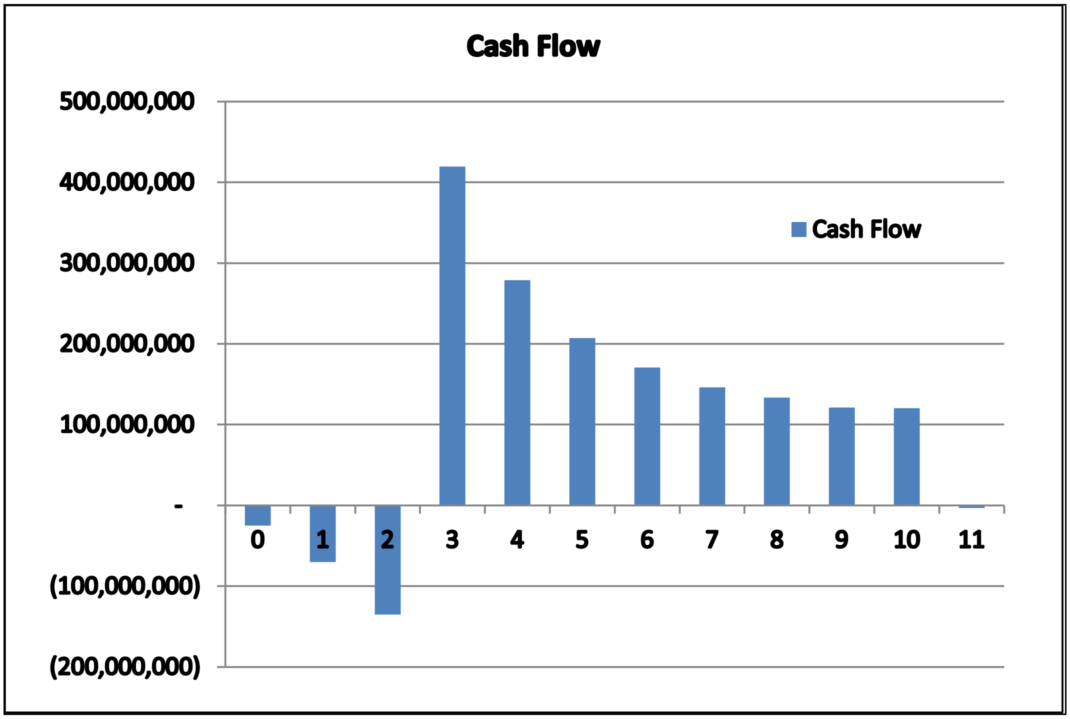

Figure 7-2: Pellet MAP

Appendix B shows the corresponding conventional DCF model using the estimated expected spot prices (instead of the futures expected prices) and the corporate rate of discount of 8%. The relevant expected DCF/NPV for Pellets is CAD$9.012 billion and for CFS is CAD$5.220 billion. The difference of CAD$394 million between DCF/NPV and MAP/NPV (Pellets) represents the economic value of the price risk discount. Similarly, the difference of CAD$4.028 billion between DCF/NPV and MAP/NPV (CFS) represents the monetary value of the price risk discount. The above DCF/NPV is “expected,” i.e., it is the mean of all potential NPVs weighted by their respective probability of occurrence. Thus the DCF/NPV is uncertain as either higher or lower NPVs than expected are probable depending on whether the actual prices of iron ore during the life of the mine are higher or lower than estimate.

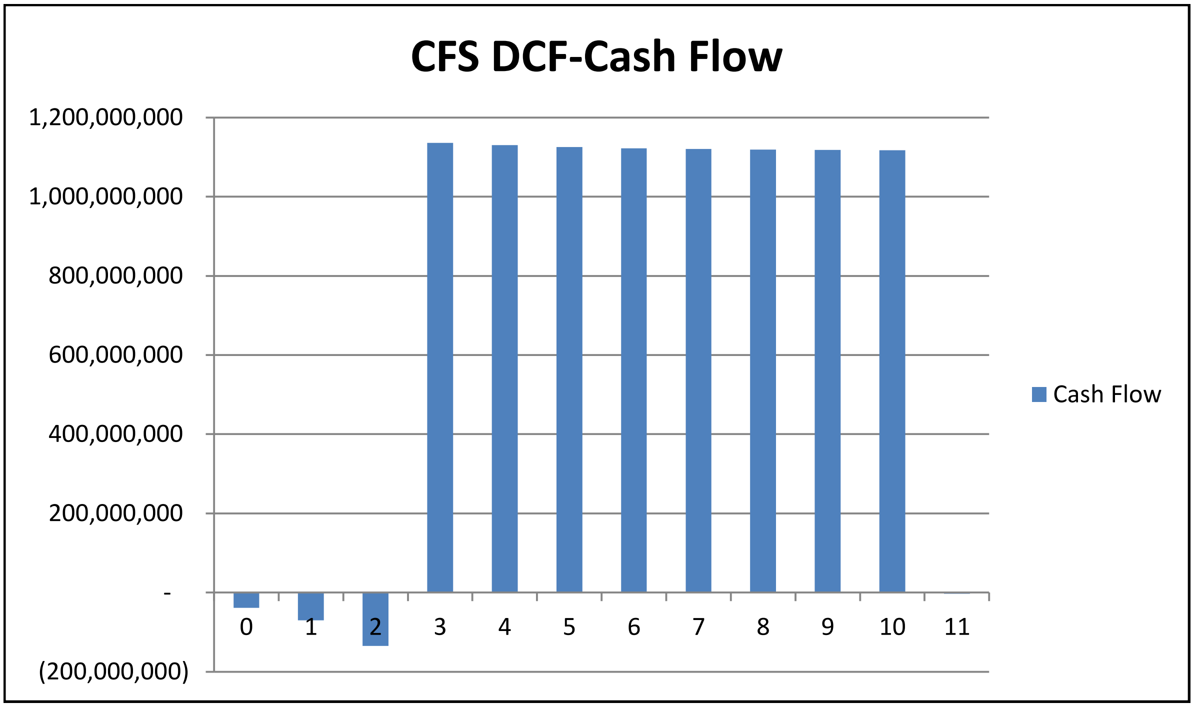

Figure 7-3: CFS DCF

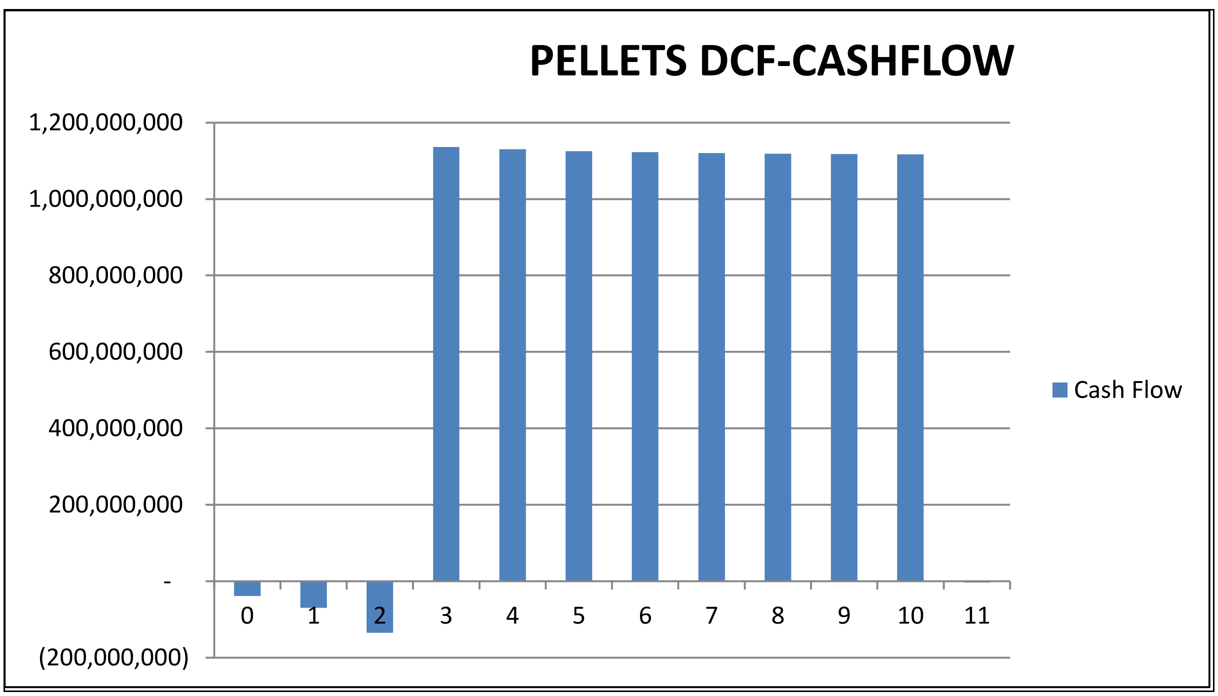

Figure 7-4: Pellets DCF

Table 13: NPV Results for Pellets and CFS (DCF/MAP)

NPV for Pellet DCF	9,012,078,824
NPV for CFS DCF	5,220,291,798
NPV for Pellet MAP	8,617,504,014
NPV for CFS MAP	1,192,285,961

7.2.3 Risk Analysis

Risk is the amount of uncertainty in the outcome of a result. There are traditional attitudes to risk. Risk analysis does give valuable additional value in contrast to a more usual one number evaluation answer.

Risk analysis was conducted on all four models using @RISK modelling software. Monte Carlo simulation was the technique used and prices were the decision variables using a normal distribution for all four models (PELLET-DCF, PELLETS-MAP, CFS – DCF, CFS–MAP). NPV was the output variable for all four models. The outcome of the simulation is shown in Figures 7-5, 7-6, 7-7, and 7-8. Table 15 is the summary of the simulation results.

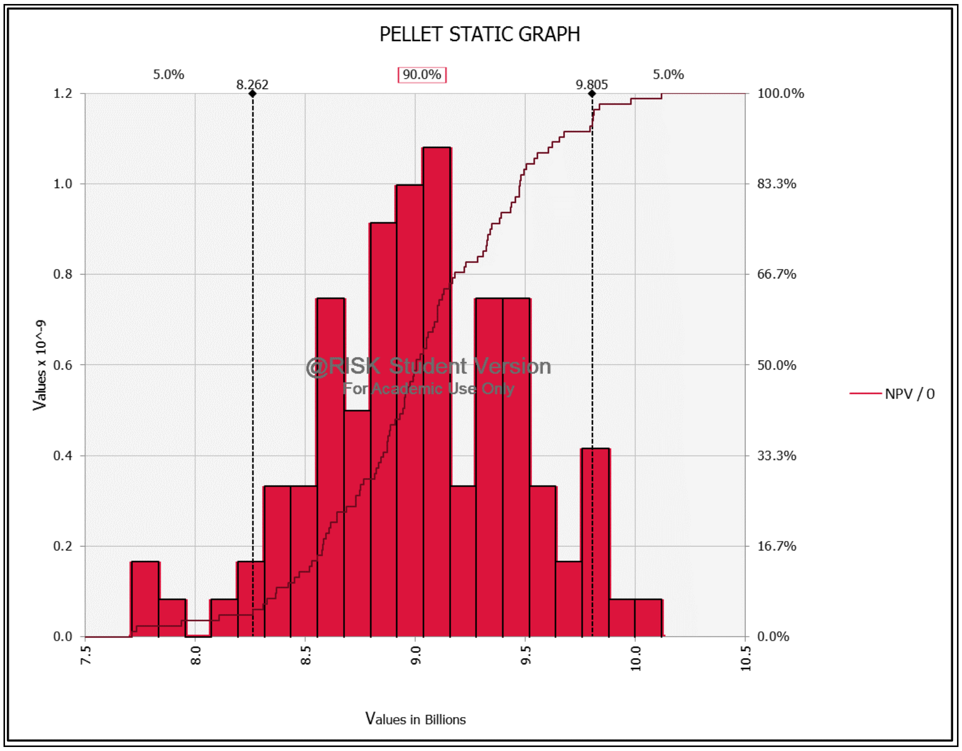

Figure 7-5: Simulation Pellet DCF

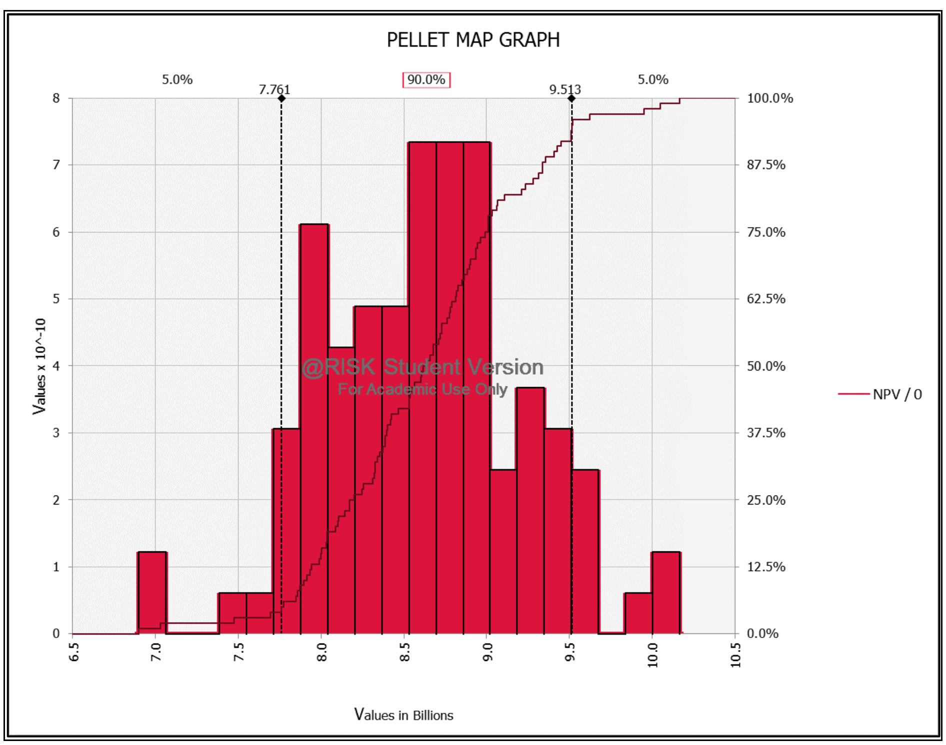

Figure 7-6: Simulations Pellets MAP

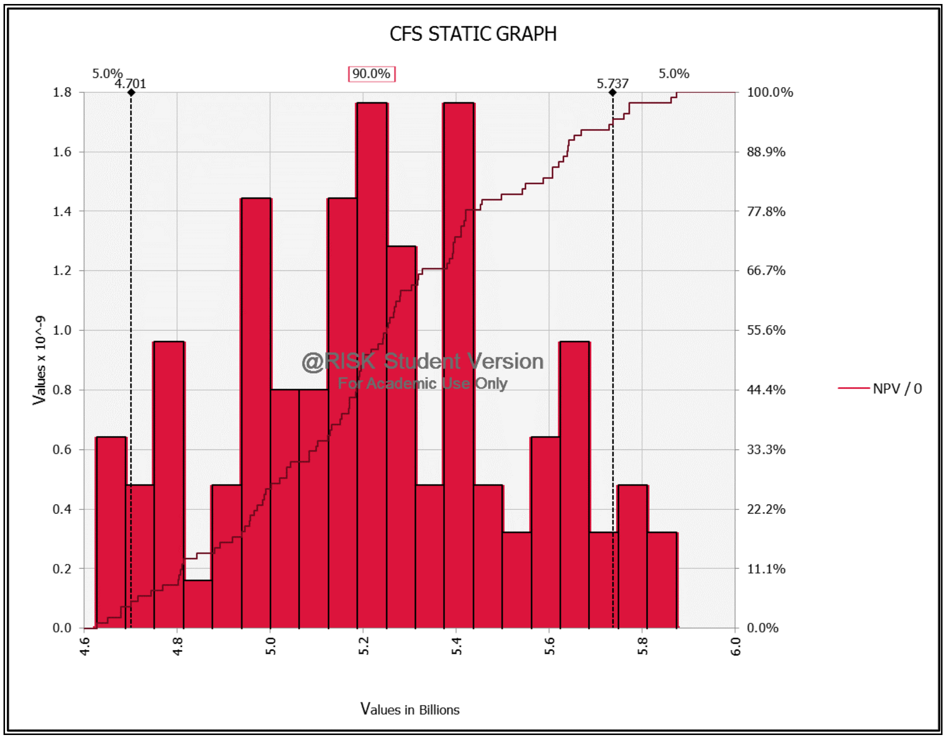

Figure 7-7: Simulations CFS DCF

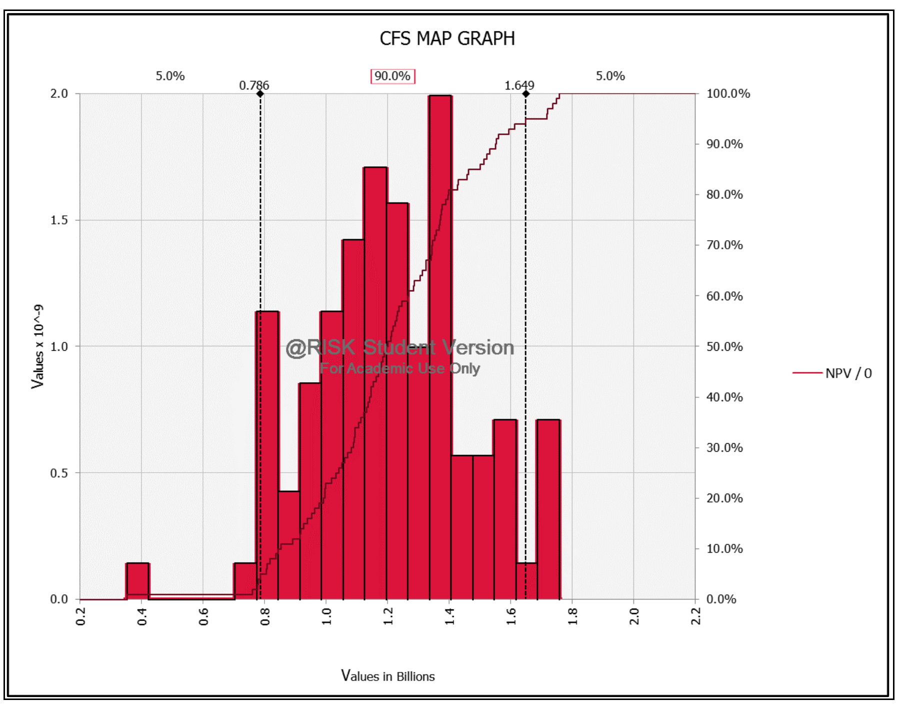

Figure 7-8: Simulation CFS MAP

Table 14: Risk Analysis NPV Results for Pellets and CFS (DCF/MAP)

	Minimum	*Mean*	*Maximum*
PELLETS – DCF	7,713,574,964	9,016,449,859	10,120,963,004
PELLETS – MAP	6,896,451,250	8,620,377,272	10,165,655,221
CFS – DCF	4,352,286,707	5,219,019,692	6,034,788,581
CFS MAP	353,035,967	1,206,884,690	1,758,280,938

CHAPTER 8: CONCLUSIONS AND FUTURE WORK

In this research, a new real option framework is proposed to valuate iron ore projects. For the prediction of iron ore prices, square root of time rule is used to define parameters such as mean reversion rates, mean reversion levels and volatilities. Jacobi process is used to help define the correlation boundary between price and risk discount factor. A strategic mine plan is developed using input from the expected future risk adjusted price and static prices with consideration of managerial flexibility. A comparison between DCF and MAP is presented using both the expected future risk adjusted price and static price. Risk Analysis is conducted on both the MAP and DCF using @Risk software.

Mean reversion process for predicting commodities such as iron ore is still at its early stages. Since iron ore market is not very well structured, it is difficult to gather information to model the market risk. With respect to this project, the data was limited in both cases of pellets and concentrates. This made it difficult to model the volatility, reversion rate and correlation.

Another challenge is to simultaneously simulate multiple variables (e.g. price, discount rate, and correlation). To overcome this @Risk Simulator and Risk Simulator Software were both used in the Monte Carlo simulation. To say the least, iron price does mean revert and both stochastic/simulated path models can be used to predict risk-adjusted expected prices in future.

Binomial tree valuation is widely used in real option valuation researches and applications. However this method is not able to consider multiple uncertainties at the same time. This thesis is presenting an alternate life-of-mine evaluation method under-price uncertainty and managerial flexibility. This approach calculates the expected NPV based on mine plans that are updated according to both static and future risk adjusted price model. Comparing with the most commonly used evaluation method which is based on a static price (current spot price), the future risk adjusted price has led to a higher project valuation which is close to the absolute upper bound on the achievable expected NPV of the mining project in the case study given in this thesis. This higher valuation arises from the capability of future price modelling to incorporate managerial flexibility and options in mining project evaluation.

In the case study, it is observed that there is no substantial difference between the mine plans generated by using options such as resource development and mill processing capacity. This is due to the fact that there is less variation in geology with respect to grade quality in all the deposits.

As a conclusion, MAP represents a powerful complement to DCF analysis in providing a minimum risk-adjusted value for mining projects. As all the risk-related inputs are sourced from actual market data and subject to limited interpretation, they are fairly objective. MAP is also a valid quantitative approach to generate realistic assumptions.

Comparison between the MAP and the expected DCF provides a quantitative monetary value for the risk premium implicit in values derived from conventional DCF analysis. This, in turn, helps to assess the realism of the corporate risk-and-time adjusted discount rate used.

Decisions based on maximising expected DCF imply a risk-neutral attitude to investment, while decisions based on MAP are consistent with a high level of risk aversion. DCF clearly produces higher project values. As a consequence, it will not be the methodology of choice for project proponents, but may provide a valuable insight for the provider of the project finance. In addition, MAP may prove helpful in better understanding the financial characteristics of a project and in choosing between alternative developments with different possible combinations of capital and operating costs.

In future work, as new information becomes available, this research can be revisited. The new information could be obtained through iron ore swap market in future. Iron ore swap is fairly new and will take time to gather historic data. The proposed methodology can be also used a base approach for future research.

Another possible future work will be incorporating other options such as the impact of cost volatility against future prices into strategic mine planning as well as the MAP/DCF analysis. Given that geology uncertainty has significant influence on project value, this uncertainty can be considered. Finally, as iron ore market becomes more global, sovereignty risk will be important source of risk. Therefore this risk can be a concern of real option valuation for iron ore.

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APPENDIX A

MAP CFS		
Riskless interest rate (%):		**2.0%**
Current spot price ($/unit):		**$180.00**
Current long-term price median ($/unit)		**$100.00**
Price of market risk		**0.5000**
Correlation of mineral an	0.5	**0.0900**
Price of mineral risk		**0.0500**
Price median growth rate (%/year)		**0.0%**
Short-term price volatility (%/year)		**0.0%**
Reversion half-life (years)		**2.50**
Reversion factor		**0.080**
Period length (years)		**1**

Units-Million $	
Tax	30%
Depreciation	30%

Year	0	1	2	3	4	5	6	7	8	9	10	11
Price				116	104	98	95	93	92	91	91	
Production Rate	0.00	0.00	0.00	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	0.00
Revenue	0.00	0.00	0.00	1,856,000,000	1,664,000,000	1,568,000,000	1,520,000,000	1,488,000,000	1,472,000,000	1,456,000,000	1,456,000,000	0.00
Operating Cost	0.00	0.00	0.00	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	0.00
Rehab Cost	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18,000,000	30,000,000
Operating profit	0.00	0.00	0.00	569,600,000	377,600,000	281,600,000	233,600,000	201,600,000	185,600,000	169,600,000	169,600,000	-30000000.00
Depreciation Pool												
Start balance	-	25,000,000	95,000,000	230,000,000	161,000,000	112,700,000	78,890,000	55,223,000	38,656,100	27,059,270	18,941,489	13,259,042
Addition	25,000,000	70,000,000	135,000,000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Maximum Allowance	7,500,000	28,500,000	69,000,000	69,000,000	48,300,000	33,810,000	23,667,000	16,566,900	11,596,830	8,117,781	5,682,447	3,977,713
Allowance Taken	-	-	-	69,000,000	48,300,000	33,810,000	23,667,000	16,566,900	11,596,830	8,117,781	5,682,447	(30,000,000)
Ending Balance	25,000,000	95,000,000	230,000,000	161,000,000	112,700,000	78,890,000	55,223,000	38,656,100	27,059,270	18,941,489	13,259,042	43,259,042
Income After Depreciation	0.00	0.00	0.00	500,600,000	329,300,000	247,790,000	209,933,000	185,033,100	174,003,170	161,482,219	163,917,553	-
Taxable Income	0.00	0.00	0.00	500,600,000	329,300,000	247,790,000	209,933,000	185,033,100	174,003,170	161,482,219	163,917,553	-
Taxes@30%	0.00	0.00	0.00	150,180,000	98,790,000	74,337,000	62,979,900	55,509,930	52,200,951	48,444,666	49,175,266	-
Capital cost	25,000,000	70,000,000	135,000,000									
Working capital	13,500,000											(13,500,000)
Salvage Value												13,259,042
Rehab cost												
Cash Flow	(38,500,000)	(70,000,000)	(135,000,000)	419,420,000	278,810,000	207,263,000	170,620,100	146,090,070	133,399,049	121,155,334	120,424,734	(3,240,958)
Risk Adjusted DCF	(38,500,000)	(68,613,907)	(129,706,574)	394,994,881	257,374,069	187,539,318	151,326,454	127,004,605	113,675,171	101,197,442	98,595,433	(2,600,929)

Payback Period	
CAPEX	243,500,000
Amount to pay back(CF)	1,222,203,170
Payback Period yrs	1.00

Dicounted Payback Period	
CAPEX	236,820,481
Amount to pay back(DCF)	1,333,111,939
DPP yrs	1.24
NPV	1,192,285,961
SUM OF DCF	1,192,285,961
PVR	-5.03
IRR	76%

ERR												
Time	0	1	2	3	4	5	6	7	8	9	10	11
Positive CF	0.00	0.00	0.00	419,420,000	278,810,000	207,263,000	170,620,100	146,090,070	133,399,049	121,155,334	120,424,734	0.00
Negative CF	(38,500,000)	(70,000,000)	(135,000,000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	(3,240,958)
PV, Negative CF	214,842,547											
FV,Postive CF	2,754,860,026											
ERR	29.06%											

Appendix A1 Output MAP CFS Analysis

MAP PELLETS

Riskless interest rate (%):		**2.0%**
Current spot price ($/unit):		**$250.00**
Current long-term price median ($/unit)		**$180.00**
Price of market risk		**0.5000**
Correlation of mineral an	0.5	**0.0900**
Price of mineral risk		**0.0500**
Price median growth rate (%/year)		**0.0%**
Short-term price volatility (%/year)		**0.0%**
Reversion half-life (years)		**2.50**
Reversion factor		**0.080**
Period length (years)		**1**

Units-Million $

Tax 30%

Depreciation 30% Operating Cost $ 80.40

Year	0	1	2	3	4	5	6	7	8	9	10	11
Price				226	215	204	194	185	176	168	161	0
Production Rate	0.00	0.00	0.00	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	0.00
Revenue	0.00	0.00	0.00	3,616,000,000	3,440,000,000	3,264,000,000	3,104,000,000	2,960,000,000	2,816,000,000	2,688,000,000	2,576,000,000	0.00
Operating Cost	0.00	0.00	0.00	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	0.00
Rehab Cost	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18,000,000	30,000,000
Operating profit	0.00	0.00	0.00	2,329,600,000	2,153,600,000	1,977,600,000	1,817,600,000	1,673,600,000	1,529,600,000	1,401,600,000	1,289,600,000	(30,000,000)
Depreciation Pool												
Start balance	-	25,000,000	95,000,000	230,000,000	161,000,000	112,700,000	78,890,000	55,223,000	38,656,100	27,059,270	18,941,489	13,259,042
Addition	25,000,000	70,000,000	135,000,000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Maximum Allowance	7,500,000	28,500,000	69,000,000	69,000,000	48,300,000	33,810,000	23,667,000	16,566,900	11,596,830	8,117,781	5,682,447	3,977,713
Allowance Taken	-	-	-	69,000,000	48,300,000	33,810,000	23,667,000	16,566,900	11,596,830	8,117,781	5,682,447	(30,000,000)
Ending Balance	25,000,000	95,000,000	230,000,000	161,000,000	112,700,000	78,890,000	55,223,000	38,656,100	27,059,270	18,941,489	13,259,042	43,259,042
Income After Depreciation	0.00	0.00	0.00	2,260,600,000	2,105,300,000	1,943,790,000	1,793,933,000	1,657,033,100	1,518,003,170	1,393,482,219	1,283,917,553	-
Taxable Income	0.00	0.00	0.00	2,260,600,000	2,105,300,000	1,943,790,000	1,793,933,000	1,657,033,100	1,518,003,170	1,393,482,219	1,283,917,553	-
Taxes@30%	0.00	0.00	0.00	678,180,000	631,590,000	583,137,000	538,179,900	497,109,930	455,400,951	418,044,666	385,175,266	-
Capital cost	25,000,000	70,000,000	135,000,000									
Working capital	13,500,000											(13,500,000)
Salvage Value												13,259,042
Rehab cost												
Cash Flow	(38,500,000)	(70,000,000)	(135,000,000)	1,651,420,000	1,522,010,000	1,394,463,000	1,279,420,100	1,176,490,070	1,074,199,049	983,555,334	904,424,734	(3,240,958)
Risk Adjusted DCF	(38,500,000)	(68,613,907)	(129,706,574)	1,555,248,786	1,404,992,310	1,261,762,300	1,134,743,834	1,022,791,331	915,372,048	821,534,472	740,480,344	(2,600,929)

Payback Period	
CAPEX	243,500,000
Amount to pay back(CF)	7,023,803,170
Payback Period yrs	0.17

Dicounted Payback Period	
CAPEX	236,820,481
Amount to pay back(DCF)	8,116,445,082
DPP yrs	0.20
NPV	8,617,504,014
SUM OF DCF	8,617,504,014
PVR	-36.39
IRR	209%

ERR												
Time	0	1	2	3	4	5	6	7	8	9	10	11
Positive CF	0.00	0.00	0.00	1,651,420,000	1,522,010,000	1,394,463,000	1,279,420,100	1,176,490,070	1,074,199,049	983,555,334	904,424,734	0.00
Negative CF	(38,500,000)	(70,000,000)	(135,000,000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	(3,240,958)
PV, Negative CF	214,842,547											
FV,Postive CF	16,374,054,686											
ERR	54.24%											

Appendix A2 Output MAP Pellet Analysis

APPENDIX B

DCF CFS		
Riskless interest rate (%):		2.0%
Current spot price ($/unit):		$180.00
Current long-term price median ($/unit)		$100.00
Price of market risk		0.5000
Correlation of mineral an	0.5	0.0900
Price of mineral risk		0.0500
Price median growth rate (%/year)		0.0%
Short-term price volatility (%/year)		0.0%
Reversion half-life (years)		2.50
Reversion factor		0.080
RADR		8%
Period length (years)		1

Units-Million $

Tax	30%											
Depreciation	30%											
Year	0	1	2	3	4	5	6	7	8	9	10	11
Price				180	180	180	180	180	180	180	180	
Production Rate	0.00	0.00	0.00	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	0.00
Revenue	0.00	0.00	0.00	2,880,000,000	2,880,000,000	2,880,000,000	2,880,000,000	2,880,000,000	2,880,000,000	2,880,000,000	2,880,000,000	0.00
Operating Cost	0.00	0.00	0.00	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	0.00
Rehab Cost	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18,000,000	30,000,000
Operating profit	0.00	0.00	0.00	1,593,600,000	1,593,600,000	1,593,600,000	1,593,600,000	1,593,600,000	1,593,600,000	1,593,600,000	1,593,600,000	-30000000.00
Depreciation Pool												
Start balance	-	25,000,000	95,000,000	230,000,000	161,000,000	112,700,000	78,890,000	55,223,000	38,656,100	27,059,270	18,941,489	13,259,042
Addition	25,000,000	70,000,000	135,000,000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Maximum Allowance	7,500,000	28,500,000	69,000,000	69,000,000	48,300,000	33,810,000	23,667,000	16,566,900	11,596,830	8,117,781	5,682,447	3,977,713
Allowance Taken	-	-	-	69,000,000	48,300,000	33,810,000	23,667,000	16,566,900	11,596,830	8,117,781	5,682,447	(30,000,000)
Ending Balance	25,000,000	95,000,000	230,000,000	161,000,000	112,700,000	78,890,000	55,223,000	38,656,100	27,059,270	18,941,489	13,259,042	43,259,042
Income After Depreciation	0.00	0.00	0.00	1,524,600,000	1,545,300,000	1,559,790,000	1,569,933,000	1,577,033,100	1,582,003,170	1,585,482,219	1,587,917,553	-
Taxable Income	0.00	0.00	0.00	1,524,600,000	1,545,300,000	1,559,790,000	1,569,933,000	1,577,033,100	1,582,003,170	1,585,482,219	1,587,917,553	-
Taxes@30%	0.00	0.00	0.00	457,380,000	463,590,000	467,937,000	470,979,900	473,109,930	474,600,951	475,644,666	476,375,266	-
Capital cost	25,000,000	70,000,000	135,000,000									
Working capital	13,500,000											(13,500,000)
Salvage Value												13,259,042
Rehab cost												
Cash Flow	(38,500,000)	(70,000,000)	(135,000,000)	1,136,220,000	1,130,010,000	1,125,663,000	1,122,620,100	1,120,490,070	1,118,999,049	1,117,955,334	1,117,224,734	(3,240,958)
Risk Adjusted DCF	(38,500,000)	(64,618,144)	(115,039,412)	893,782,308	820,555,673	754,554,474	694,658,673	640,034,084	590,039,721	544,167,281	502,001,432	(1,344,294)

Payback Period	
CAPEX	243,500,000
Amount to pay back(CF)	5,635,003,170
Payback Period yrs	0.22

Dicounted Payback Period	
CAPEX	218,157,556
Amount to pay back(DCF)	4,937,792,215
DPP yrs	0.31
NPV	5,220,291,798
SUM OF DCF	5,220,291,798
PVR	-23.93
IRR	158%

ERR												
Time	0	1	2	3	4	5	6	7	8	9	10	11
Positive CF	0.00	0.00	0.00	1,136,220,000	1,130,010,000	1,125,663,000	1,122,620,100	1,120,490,070	1,118,999,049	1,117,955,334	1,117,224,734	0.00
Negative CF	(38,500,000)	(70,000,000)	(135,000,000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	(3,240,958)
PV, Negative CF	214,842,547											
FV,Postive CF	14,151,400,263											
ERR	52.01%											

Appendix B1 Output CFS DCF Analysis

DCF PELLETS		
Riskless interest rate (%):		2.0%
Current spot price ($/unit):		$250.00
Current long-term price median ($/unit)		$180.00
Price of market risk		0.5000
Correlation of mineral an	0.5	0.0900
Price of mineral risk		0.0500
Price median growth rate (%/year)		0.0%
Short-term price volatility (%/year)		0.0%
Reversion half-life (years)		2.50
Reversion factor		0.080
RADR		8%
Period length (years)		1

Units-Million $

Tax 30%

Depreciation 30% Operating Cost $ 80.40

Year	0	1	2	3	4	5	6	7	8	9	10	11
Price				250	250	250	250	250	250	250	250	
Production Rate	0.00	0.00	0.00	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	16,000,000	0.00
Revenue	0.00	0.00	0.00	4,000,000,000	4,000,000,000	4,000,000,000	4,000,000,000	4,000,000,000	4,000,000,000	4,000,000,000	4,000,000,000	0.00
Operating Cost	0.00	0.00	0.00	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	1,286,400,000	0.00
Rehab Cost	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18,000,000	30,000,000
Operating profit	0.00	0.00	0.00	2,713,600,000	2,713,600,000	2,713,600,000	2,713,600,000	2,713,600,000	2,713,600,000	2,713,600,000	2,713,600,000	(30,000,000)
Depreciation Pool												
Start balance	-	25,000,000	95,000,000	230,000,000	161,000,000	112,700,000	78,890,000	55,223,000	38,656,100	27,059,270	18,941,489	13,259,042
Addition	25,000,000	70,000,000	135,000,000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Maximum Allowance	7,500,000	28,500,000	69,000,000	69,000,000	48,300,000	33,810,000	23,667,000	16,566,900	11,596,830	8,117,781	5,682,447	3,977,713
Allowance Taken	-	-	-	69,000,000	48,300,000	33,810,000	23,667,000	16,566,900	11,596,830	8,117,781	5,682,447	(30,000,000)
Ending Balance	25,000,000	95,000,000	230,000,000	161,000,000	112,700,000	78,890,000	55,223,000	38,656,100	27,059,270	18,941,489	13,259,042	43,259,042
Income After Depreciation	0.00	0.00	0.00	2,644,600,000	2,665,300,000	2,679,790,000	2,689,933,000	2,697,033,100	2,702,003,170	2,705,482,219	2,707,917,553	-
Taxable Income	0.00	0.00	0.00	2,644,600,000	2,665,300,000	2,679,790,000	2,689,933,000	2,697,033,100	2,702,003,170	2,705,482,219	2,707,917,553	-
Taxes@30%	0.00	0.00	0.00	793,380,000	799,590,000	803,937,000	806,979,900	809,109,930	810,600,951	811,644,666	812,375,266	-
Capital cost	25,000,000	70,000,000	135,000,000									
Working capital	13,500,000											(13,500,000)
Salvage Value												13,259,042
Rehab cost												
Cash Flow	(38,500,000)	(70,000,000)	(135,000,000)	1,920,220,000	1,914,010,000	1,909,663,000	1,906,620,100	1,904,490,070	1,902,999,049	1,901,955,334	1,901,224,734	(3,240,958)
Risk Adjusted DCF	(38,500,000)	(64,618,144)	(115,039,412)	1,510,498,551	1,389,856,518	1,280,085,390	1,179,784,852	1,087,861,990	1,003,436,981	925,781,050	854,275,340	(1,344,294)

Payback Period	
CAPEX	243,500,000
Amount to pay back(CF)	9,555,003,170
Payback Period yrs	0.13

Dicounted Payback Period	
CAPEX	218,157,556
Amount to pay back(DCF)	8,377,305,333
DPP yrs	0.18
NPV	9,012,078,824
SUM OF DCF	9,012,078,824
PVR	-41.31
IRR	211%

ERR												
Time	0	1	2	3	4	5	6	7	8	9	10	11
Positive CF	0.00	0.00	0.00	1,920,220,000	1,914,010,000	1,909,663,000	1,906,620,100	1,904,490,070	1,902,999,049	1,901,955,334	1,901,224,734	0.00
Negative CF	(38,500,000)	(70,000,000)	(135,000,000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	(3,240,958)
PV, Negative CF	214,842,547											
FV,Postive CF	24,013,710,160											
ERR	60.26%											

Appendix B2 Output DCF Pellet Analysis