VIBRATION OF CYLINDRICAL STRUCTURES INDUCED BY AXIAL FLOW

by

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SYNOPSIS

This paper discusses the effect of axial flow on cylindrical structures. It is shown that axial flow may cause *hydroelastic instabilities* at sufficiently high flow velocities. However, for the range of flow velocities and other parameters pertaining to industrial systems, the effect of purely axial, uniform, steady flow is to damp free motions. Nevertheless, departures from such ideal flow conditions induce small amplitude vibration, termed *sub-critical vibration*. The underlying mechanism of this vibration is examined, and the various means available for predicting its amplitude are discussed. These latter are either empirical or analytical (generally semi-empirical); the analytical methods are further classified into three categories accordingly as they postulate the vibration to be *forced*, *parametric* or *self-excited*. The measure of success achieved in predicting sub-critical vibration amplitude is discussed, and possible reasons to account for its being generally poor. At present, the amplitude may be predicted typically to within one order of magnitude.

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1. *INTRODUCTION*

Unlike the case of vibrations of cylindrical structures induced by *transverse* flow, the study of vibration of such structures induced by *axial* flow is a relatively new phenomenon, beginning *circa* 1958. The reasons for this are easily understood upon considering that, whereas in the former case, large amplitudes of transverse vibration may result in structures of practical interest subjected to moderate flow velocities, in the latter case the resultant transverse vibrations are generally of very small amplitude, typically 10^{-3} cm.

Accordingly, such vibrations would normally be of no concern, were it not for the peculiar geometry in certain applications rendering vibration of even such small amplitude worrisome; namely (i) in the case of nuclear reactor fuel-element bundles(for systems where the mean flow is nominally axial) and (ii) in the case of heat exchanger tubes, in regions where the flow is mainly axial. In both these cases, the problem arises from close spacing, either between the cylindrical elements and 'spacers' on adjacent elements, or between the cylindrical elements and intermediate imperfect supports; thus, even small amplitude vibration results in impact within the cylindrical structures and, hence, fretting and wear - which,in time, might cause the rupture of the (tubular) cylindrical elements, with serious consequences.

Studies of the subject under consideration were conducted independently in the U.S.A. by Burgreen, Byrnes & Benforado

(1958), Shields (1960), Quinn (1962,1965) and Pavlica & Marshall (1966); in France at SOGREAH (1962); in Sweden by Roström & Andersson (1964a,b,c); and in Canada by Paidoussis (1965,1966a). These were the first studies to appear in the literature and they had one or more of the following aims: (i) measurement of the amplitude of vibration of particular cylindrical structure configurations, modelling nuclear reactor components and flow conditions; (ii) understanding the causes of vibration; (iii) development of means for predicting the vibration amplitudes. Although from the designer's point of view, item (iii) was the most important, it became evident that the lack of understanding of the underlying mechanism causing the vibration rendered its prediction, at least with any degree of confidence, unsatisfactory. Thus the need for more fundamental and systematic as opposed to *ad hoc* studies became evident.

The free motions of a cylinder in axial flow were studied by Paidoussis (1966a,b,c) both analytically and experimentally and the equation of motion was derived for the first time. It was shown that the effect of axial flow was to damp free oscillations at small and moderate flow velocities and to reduce the natural frequencies of oscillation. However, at sufficiently high flow velocities, cylinders in axial flow become subject to hydroelastic instabilities, namely to *buckling* (divergence) and to *oscillatory instabilities* (flutter). These hydroelastic instabilities were shown to be of no practical concern (Paidoussis 1966a), as the *critical* flow velocities necessary for their inception — for cylindrical structures of normal engineering application — are much too high (typically > 100 m/s).

Thus, the vibrations of practical interest occur with flow velocities smaller than the critical; we term these *sub-critical vibrations*, to differentiate them from those associated with oscillatory instabilities. The sub-critical vibrations are random in character, with the dominant frequency component corresponding to the first-mode of the cylinders; as mentioned earlier, the amplitude rarely exceeds 10^{-3} cm.

Having obtained the equation of motion, at least some of the parameters influencing the vibrations of a cylinder in axial flow could be identified, thus enabling the formulation of a semi-empirical expression for the prediction of amplitude of sub-critical vibration (Paidoussis 1966a,1969) which proved to be perhaps the most successful, at least until very recently (Den Hartog 1970). This semi-empirical theory for sub-critical vibration, as well as those of Burgreen *et al.* and of Quinn's, suffered from a common weakness; namely that, in the course of their formulation, the forces causing sub-critical vibration were either incompletely specified, or not at all; accordingly, some of the forces and parameters influencing vibration could not have been incorporated properly in the expressions obtained for the amplitude. Moreover, the process being random, the predicted 'maximum' amplitudes were of questionable interpretation.

Accordingly, it became clear that the mechanism of sub-critical vibration excitation would have to be clarified. This proved to be a difficult task and one that is still pre-occupying researchers in this field. First, the question arises whether sub-critical vibration is forced, in the classical sense or self-excited. In the former case, vibration may arise from departures from steady axial flow, near-field and far-field noise, vibration transmittted through piping and supports, etc; however, the characterization of some of these forces is quite difficult and their prediction questionable. In the latter case, if the vibration is self-excited, the feedback mechanism involved would have to be identified.

Reavis(1967,1969) postulated that the exciting force arose from boundary-layer pressure fluctuations around the cylindrical structures (i.e., from near-field noise). Using available correlations of boundary-layer pressure fluctuations in pipe-flow, he obtained the first stochastic vibration model for the problem at hand and a theoretical prediction of the vibration amplitude; alas, the predicted amplitude was, in some cases, one or two orders of magnitude smaller than the measured values. Later, this particular excitation model was refined and the theory extended by Gorman (1969,1971) — particularly with regard to two-phase flow —— and by S.S. Chen & Wambsganss (1970,1972). These theories will be discussed in § 5.3.

At about the same time, Y.N. Chen (1970a,b) postulated another mechanism of excitation, where it is supposed that the cylindrical structure is excited parametrically, in the manner of a column subjected to a compressive load with a periodic component. In the case here under consideration, the excitation parameter is the axial flow velocity, which is postulated to contain periodic perturbations. This theory is further discussed in § 5.4.

In recent years, the volume of work in the general area of flow-induced vibration and in the particular topic discussed here has increased sharply, as evidenced by the number of special conferences in this area. In spite of this, the problem at hand cannot be considered to have been solved, not by a long chalk. Indeed, it is questionable whether the amplitude of vibration of cylindrical structures in axial flow may be predicted today better than within an order of magnitude with any of the existing theories.

This paper attempts to present a comprehensive review of the published literature, emphasizing the work that has not yet been superseded, and to give the *state of the art* for the benefit of the designer as well as the researcher.

2. THE EQUATION OF SMALL LATERAL MOTIONS

2.1 *Derivation of the equation of motions and of the boundary conditions*

Consider the flexible slender cylinder shown in figure 1(a) immersed in a steady, axial flow, parallel to the x-axis. The x- and y- axes lie in a horizontal plane wherein all motions are supposed to be confined. At its two ends, the cylinder is tapered over a sufficiently short length, compared with its overall length, so that in dealing with its dynamics it may be considered essentially of uniform cross-section; yet the tapered sections are presumed to be sufficiently long so as to admit no discontinuities in the flow past the cylinder. We denote the cylinder diameter by D, its cross-sectional area by S, its flexural rigidity by EI, and its length by L, its mass per unit length by m, the fluid density by ρ and the axial flow velocity by U.

Consider now a small element δx of the cylinder undergoing small free lateral motions y(x,t). The cylinder is subjected to a lateral force due to inviscid flow around it, $F_A \delta x$, and to viscous forces $F_N \delta x$ and $F_L \delta x$, in the lateral and longitudinal directions respectively. We also assume that it is subjected to a tension T(x). We consider the cylinder as an Euler-Bernoulli beam subject to lateral shear forces Q(x) and to bending moments $M(x)$, as shown in figure 2. Now, taking force balances in the x- and y- directions and a moment balance, we obtain,

$$\frac{\partial T}{\partial x} + F_L + (F_N + F_A)\frac{\partial y}{\partial x} = 0\ , \tag{1}$$

$$\frac{\partial Q}{\partial x} - F_N - F_A + F_L\frac{\partial y}{\partial x} + \frac{\partial}{\partial x}\left(T\frac{\partial y}{\partial x}\right) - m\frac{\partial^2 y}{\partial t^2} = 0\ , \tag{2}$$

$$Q = -\frac{\partial M}{\partial x}\ . \tag{3}$$

Using slender-body, inviscid flow theory, and a number of other assumptions given in detail elsewhere (Paidoussis 1966b), it is found that

$$F_A = M[(\partial/\partial t) + U(\partial/\partial x)]^2\ y \tag{4}$$

as shown by Lighthill (1960), where M is the virtual mass of the fluid per unit length, which is equal to ρS for a circular cylinder oscillating in an infinite fluid medium, provided that the wavelength of motion is large in comparison with the diameter of the cylinder.

The viscous forces acting on long inclined cylinders have been discussed by Taylor (1952). For rough cylinders and turbulent boundary layers, Taylor proposed the following expressions:

$$F_N = \tfrac{1}{2}\rho D U^2 (C_{Dp}\ \sin^2 i + C_f\ \sin i),$$

$$F_L = \tfrac{1}{2}\rho D U^2 C_f \cos\ i,$$

where C_{Dp} and C_f are the coefficients associated with form and friction drag for a cylinder in cross-flow. Taking i as the angle of incidence of the moving cylinder, we may write

$$i = \tan^{-1}(\partial y/\partial x) + \tan^{-1}[(\partial y/\partial t)/U];$$

moreover, linearizing the above expressions for small $\partial y/\partial x$ and $(\partial y/\partial t)/U$, we obtain

$$F_N = \tfrac{1}{2}c_N(M/D)U[(\partial/\partial t) + U(\partial/\partial x)]y + \tfrac{1}{2}c_D(M/D)(\partial y/\partial t),$$
$$F_L = \tfrac{1}{2}c_T(M/D)U^2, \qquad (5)$$

where c_D represents the viscous damping force at zero flow velocity.

From (4) and (5), it is clear that the third term in equation (1) is of second order small and hence, may be neglected. Accordingly, the axial tension may be found by substituting F_L from (5) into (1) and integrating from x to L, yielding

$$T(x) = T(L) + \tfrac{1}{2}c_T MU^2(L-x)/D .$$

A non-zero value of T(L) can only arise if the downstream end is free, or at least free to move axially, in which case it is associated with form drag and may be taken to be $T(L) = \tfrac{1}{2}c_T' MU^2$, where c_T' is the form drag coefficient. Accordingly, in such cases, we have

$$T(x) = \tfrac{1}{2}c_T MU^2(L-x)/D + \tfrac{1}{2}c_T' MU^2 .$$

Where both ends are supported and the distance between supports is fixed, the tension is given by

$$T(x) = T_o + \tfrac{1}{2}c_T MU^2(\tfrac{1}{2}L-x)/D ,$$

where T_o is a tension imposed by external means. Combining the two cases, we may write generally

$$T(x) = \delta T_o + \tfrac{1}{2}c_T\frac{MU^2}{D}\{(1-\tfrac{1}{2}\delta)L-x\} + \tfrac{1}{2}(1-\delta)c_T' MU^2, \qquad (6)$$

where $\delta = 0$ corresponds to the case where the downstream end is free to move axially and $\delta = 1$ where it is not.

Finally, assuming internal damping within the cylinder to be of the Kelvin type, we may write

$$M = EI(\partial^2 y/\partial x^2) + \mu I(\partial^3 y/\partial t \partial x^2) . \qquad (7)$$

Now, substituting equations (4) to (7) into equations (2) and (3), and making use of equation (1), we obtain the equation of small lateral motions

$$EI \frac{\partial^4 y}{\partial x^4} + \mu I \frac{\partial^5 y}{\partial t \partial x^4} + M\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 y$$

$$- \frac{1}{2} c_T \frac{MU^2}{D} \{(1-\tfrac{1}{2}\delta)L-x\} \frac{\partial^2 y}{\partial x^2} - \{ \delta T_o + \tfrac{1}{2}(1-\delta) c_T' MU^2\} \frac{\partial^2 y}{\partial x^2}$$

$$+ \frac{1}{2} c_N \frac{MU}{D} \left(\frac{\partial y}{\partial t} + U\frac{\partial y}{\partial x}\right) + \frac{1}{2} c_D \frac{M}{D} \frac{\partial y}{\partial t} + m \frac{\partial^2 y}{\partial t^2} = 0 . \qquad (8)$$

This equation differs somewhat from that originally derived (Paidoussis 1966b) in the following respects: (a) internal dissipation is taken into account, as well as flow-independent viscous damping; (b) the term corresponding to $(\partial T/\partial x)(\partial y/\partial x)$ is, quite properly, missing from the above equation having been cancelled by $F_L(\partial y/\partial x)$. In the original derivation of the equation of motion, this was not realized because of the inconsistent manner in which the frictional forces were resolved (Paidoussis 1971). Unfortunately, this error remained undetected by all workers who based their work on the author's original derivation. Fortunately, however, the error is not very important; the numerical values given by Paidoussis (1966b,c) are only slightly affected, and the general conclusions regarding stability are not affected at all.

We next consider the boundary conditions. We use the following generalized conditions (cf. Chen & Wambsganss 1972)

$$EI\frac{\partial^3 y}{\partial x^3} + k_o y = EI\frac{\partial^2 y}{\partial x^2} - c_o\frac{\partial y}{\partial x} = 0 \quad \text{at } x = 0$$

and (9)

$$EI\frac{\partial^3 y}{\partial x^3} - k_L y = EI\frac{\partial^2 y}{\partial x^2} + c_L\frac{\partial y}{\partial x} = 0 \quad \text{at } x = L,$$

from which all the standard boundary conditions may be obtained accordingly as k_o, k_L, c_o and c_L are either zero or infinite. If the downstream end is free, terminating in a tapering end, the cross-sectional area of which varies smoothly from S to zero in a distance ℓ ($<<L$), the following boundary conditions derived previously (Paidoussis 1966b) should be used

$$EI\frac{\partial^3 y}{\partial x^3} + fMU\left(\frac{\partial y}{\partial t} + U\frac{\partial y}{\partial x}\right) - (m+fM)x_e\frac{\partial^2 y}{\partial t^2} = 0$$

(10)

and $$\frac{\partial^2 y}{\partial x^2} = 0$$

at $x=L$, where $x_e = \frac{1}{S}\int_{L-\ell}^{L} S(x)\,dx$. The parameter f is a measure of departures from ideal slender-body, inviscid flow theory arising from (i) the lateral flow not being truly two-dimensional and (ii) boundary layer effects. Thus $f=1$ represents the ideally slender case, while normally $0<f<1$. It is seen that equations (10) simplify to the particular form of equations (9) corresponding to a free downstream end ($k_L=c_L=0$), as $f\to0$ and $x_e\to0$, i.e., for a blunt free end.

2.2 *The equation of motion in dimensionless form*

In order to render the equation of motion and the boundary conditions dimensionless, we define the following dimensionless terms:

$$\xi = x/L, \quad \eta = y/L, \quad \tau = \{EI/(m+M)\}^{\frac{1}{2}}\, t/L^2,$$

$$\beta = M/(M+m), \quad \Gamma = T_o L^2/EI, \; \varepsilon = L/D, \quad u = (M/EI)^{\frac{1}{2}}\, UL, \tag{11}$$

$$\alpha = \{I/[E(M+m)]\}^{\frac{1}{2}} \; \mu/L^2, \quad c = c_D L (M/EI)^{\frac{1}{2}},$$

$$\kappa_o = k_o L^3/EI, \quad \kappa_1 = k_L L^3/EI, \quad \kappa_o' = c_o L^2/EI, \quad \kappa_1' = c_L L^2/EI,$$

$$\chi = x_e/L .$$

Substituting into equations (8), (9) and (10) we obtain the dimensionless equation of motion

$$\frac{\partial^4\eta}{\partial\xi^4} + \alpha\frac{\partial^5\eta}{\partial\xi^4\partial\tau} + \{u^2[1-\tfrac{1}{2}\varepsilon c_T(1-\tfrac{1}{2}\delta-\xi) - \tfrac{1}{2}(1-\delta)c_T'] - \delta\Gamma\}\frac{\partial^2\eta}{\partial\xi^2}$$

$$+ 2\beta^{\frac{1}{2}}\, u\frac{\partial^2\eta}{\partial\xi\partial\tau} + \tfrac{1}{2}\varepsilon c_N u^2\frac{\partial\eta}{\partial\xi} + \tfrac{1}{2}\beta^{\frac{1}{2}}\,(\varepsilon c_N u + \varepsilon c)\frac{\partial\eta}{\partial\tau} + \frac{\partial^2\eta}{\partial\tau^2} = 0 \tag{12}$$

and the standard boundary conditions

$$\frac{\partial^3\eta}{\partial\xi^3} + \kappa_o\eta = \frac{\partial^2\eta}{\partial\xi^2} - \kappa_o'\frac{\partial\eta}{\partial\xi} = 0 \qquad \text{at } \xi = 0,$$

$$\frac{\partial^3\eta}{\partial\xi^3} - \kappa_1\eta = \frac{\partial^2\eta}{\partial\xi^2} + \kappa_1'\frac{\partial\eta}{\partial\xi} = 0 \qquad \text{at } \xi = 1, \tag{13}$$

with the special boundary conditions for a tapered free end (ξ=1)

$$\frac{\partial^3\eta}{\partial\xi^3} + fu^2\frac{\partial\eta}{\partial\xi} + f\beta^{\frac{1}{2}}\, u\frac{\partial\eta}{\partial\tau} - \{1+(f-1)\beta\}\,\chi\frac{\partial^2\eta}{\partial\tau^2} = \frac{\partial^2\eta}{\partial\xi^2} = 0 . \tag{14}$$

2.3 *Equation of forced motions*

There are several ways in which the cylinder may be excited and the equation of motion would have to be modified to take the excitation forces into account. Thus, if the tension Γ varied harmonically, we may have to introduce $\Gamma=\Gamma_o(1+\mu\cos\omega\tau)$ in equation (12). On the other hand, if the cylinder is subjected to a lateral force distributed along its length, $Q(\xi,\tau)$ then equation (12) takes the form

$$\mathcal{L}[\eta(\xi,\tau)] = Q(\xi,\tau) \tag{15}$$

where $\mathcal{L}[\eta(\xi,\tau)]$ stands for the left-hand-side of equation (12).

Some forms of these excitation mechanisms will be discussed in §4 and §5.

3. THE EFFECT OF AXIAL FLOW ON VIBRATION

Here, we shall discuss the effect of the mean axial flow on vibration of cylindrical structures, assuming the flow to be purely axial and steady. Moreover, we shall not concern ourselves for now, with the forces exciting sub-critical vibration(e.g. near-field noise). Succinctly, we shall be considering *how free motions of cylindrical structures in axial flow differ from those in vacuo.*

To this end, we consider solutions of equation (12) of the form

$$\eta(\xi,\tau) = Y(\xi)e^{i\omega\tau}$$

subject to the appropriate boundary conditions. Here ω is a dimensionless frequency related to the circular frequency of motion, Ω, by

$$\omega = \{(M+m)/EI\}^{\frac{1}{2}}\Omega L^2 . \qquad (16)$$

3.1 *Behaviour at zero flow velocity*

The equation of motion of a cylinder *in vacuo* reduces to

$$\frac{\partial^4\eta}{\partial\xi^4} + \alpha\frac{\partial^5\eta}{\partial\xi^4\partial\tau} + \frac{\partial^2\eta}{\partial\tau^2} = 0 \qquad (17a)$$

where, in the dimensionless terms involved, we understand that M=0. Now, in the absence of internal dissipation, this equation reduces to the dimensionless form of the Euler-Bernoulli beam equation, the solution of which yields wholly real eigenfrequencies, ω_r, which are simply the squares of the corresponding dimensionless eigenvalues(e.g. for a pinned-pinned beam $\omega_r = \pi^2$,

$4\pi^2$, $9\pi^2$ ....for r = 1, 2, 3 ....). The presence of the dissipation term (i) renders the frequencies complex (for α small), with a positive real part (i.e., $\mathrm{Im}(\omega_r) > 0$, and the oscillation is damped), and (ii) decreases the values of the frequency from the aforementioned values.

Now, if the cylinder is immersed in a *stationary* fluid, the equation of motion becomes

$$\frac{\partial^4\eta}{\partial\xi^4} + \alpha\frac{\partial^5\eta}{\partial\xi^4\partial\tau} + c\,\frac{\partial\eta}{\partial\tau} + \frac{\partial^2\eta}{\partial\tau^2} = 0\,. \tag{17b}$$

In this case, the presence of the viscous dissipation term will increase the damping and further reduce the real parts of the ω_r, $\mathrm{Re}(\omega_r)$. Moreover, the *dimensional* frequencies Ω_r will be further reduced, as the effective mass of the beam is now m+M.

This is a convenient place to say a few more words about the virtual mass, M. It was mentioned before that $M=\rho S$, provided that the wavelength of motion is long (Niordson 1953) and provided that the cylinder is immersed in an infinite fluid. Recently, Chen & Wambsganss (1972) investigated the case where the fluid is not infinite, but is confined by an outer cylinder of diameter d, concentric with the vibrating cylinder. Both the vibrating (inner) cylinder and the outer cylinder are assumed to be infinitely long and rigid. Writing $M=C_M\rho S$, they found that for $d/D \simeq 10$ essentially $C_M = 1$; however, as the fluid becomes more confined, C_M takes higher values, as shown in figure 3. (In that figure, $a = \Omega D/2c$, where Ω is the circular frequency of oscillation and c is the velocity of sound; C_M is found to be insensitive to a.) It is evident that taking $M=\rho S$ is an oversimplification, in general. In a densely spaced array of cylinders, it is clear that $M>\rho S$, although the exact value of M is difficult to assess in such a geometry.

3.2 *Behaviour in a flowing fluid*

The effect of flow must be assessed by obtaining solutions to equation (12) in its full form. Some solutions, obtained by methods detailed elsewhere (Paidoussis 1966b), in the case where $\alpha=c=0$, will now be discussed.

Figure 4 shows the frequencies (plotted in an Argand diagram) associated with the lowest three modes of a pinned-pinned cylinder ($\delta= 1$), as functions of the flow velocity u. This is a typical case illustrating the behaviour of such a system. Since we have taken $\alpha=c=0$, the frequencies at u = 0 are wholly real and they correspond to $Re(\omega) =\pi^2, 4\pi^2, 9\pi^2$. As the flow velocity increases, the effect of flow is to diminish $Re(\omega)$ and to produce $Im(\omega)>0$; i.e., the effect of flow is to damp free motions of the cylinder. This action of the flow is associated partly with the damping term $\frac{1}{2}\varepsilon c_N \beta^{\frac{1}{2}} u(\partial\eta/\partial\tau)$ and partly with the Coriolis-type term $2\beta^{\frac{1}{2}} u(\partial^2\eta/\partial\xi\partial\tau)$.

As the flow velocity increases further, the situation changes. The first mode ceases being oscillatory at about u = 3.14, and at slightly higher flow one branch of this mode becomes associated with a *negative* $Im(\omega)$; thus, the system becomes unstable by *buckling* at that point (u = 3.145). The destabilizing action is associated with the term $u^2(\partial^2\eta/\partial\xi^2)$; recognizing that $\partial^2\eta/\partial\xi^2$ is proportional to the inverse of the radius of curvature, we see that this term represents a 'centrifugal' force. Thus, the buckling instability in this case is not unlike that of a flat panel subjected to axial flow [cf. Miller (1960), Rosenberg & Youngdahl (1962), for instance].

At higher flow velocities, the damping influence of the flow begins diminishing in the higher modes also, and eventually the system becomes subject to *oscillatory instability* in its second mode, at $u = 5.7$, and in its third mode at $u = 8.3$, approximately. The destabilizing action is associated with the frictional terms in this case; in all cases where both ends of the cylinder are supported, in the absence of frictional terms, the cylinder has been shown (Paidoussis 1966b) to be exclusively subject to buckling instabilities - unless of course, the cylinder is tubular with very thin walls, in which case flutter in the *shell modes* may occur.

The behaviour with increasing flow of a typical cantilevered cylinder having a streamlined, tapered downstream end ($f=1$) is illustrated in figure 5. It is quite similar to that of a cylinder supported at both ends. However, the destabilizing action for both buckling and oscillatory instabilities in this case is associated mainly with the $u^2(\partial^2\eta/\partial\xi^2)$ term (Paidoussis 1966b, 1971). It is noteworthly that the stability of cantilevered cylinders is strongly influenced by the parameter f. Cylinders with sufficiently small f, i.e., with a blunt downstream end, are not subject to hydroelastic instabilities.

The effect of flow on the free vibration characteristics of cylinders at low u was recently tested by Chen & Wambsganns(1972) and found to agree with theory remarkably well. Some of their results are shown in figure 6.

4. SUB-CRITICAL VIBRATIONS

4.1 *General Character of Sub-critical Vibrations*

Let us consider a typical tubular cylinder which might be used as a nuclear-reactor fuel element. Let us take the diameter to be 15 mm and the wall-thickness 0.4 mm, the length 60 cm and Young's modulus $E = 7 \times 10^5$ kg/cm^2. If the cylinder is immersed in axial water flow with $U = 10$m/s, the dimensionless flow velocity is $u = 0.43$, approximately. Indeed for most realistic applications, $u < 1$; this, incidentally, was the reason for using rubber cylinders in the experimental work on stability of such cylinders in axial flow (Paidoussis 1966c), as the value of E for rubber is quite small.

Therefore, it is obvious that for industrial applications, of the current type at least, we need not worry about hydroelastic instabilities, and we are only concerned with sub-critical vibrations. Now, as discussed in §3.2 (figures 4 and 5), the effect of the steady axial flow for $u < 1$ is (i) to damp the motions of the cylinder, and (ii) to decrease slightly its natural frequencies of oscillation, at least when both ends of the cylinder are supported. Recently Chen & Wambsganss (1972) have measured this damping effect of axial flow (figure 6).

Typically, sub-critical vibration amplitudes are of the order of 10^{-4} to 10^{-2} cm. They are random, with a fairly narrow frequency spectrum, the predominant frequency corresponding to that of the first mode. The process has been found to be ergodic and the amplitude distribution approximately Gaussian (Wambsganss & Chen 1971).

Measurement of the vibrations has been performed by various means, e.g. by internally mounted accelerometers, proximity gauges, inductive transducers involving internally mounted magnets, optical methods, etc. In this paper we are not concerned with experimental techniques. It should be pointed out, however, that the interpretation of vibration measurements is no easy task. This is because the vibration amplitudes are of the same order of magnitude as might be excited by giving the test-section a hammer-blow or a healthy kick; accordingly, the separation of the 'signal' from 'background noise' is problematical.

The predominant frequency of vibration being known*, the designer is mainly interested in being able to predict its amplitude; specifically, in being able to predict its r.m.s. value, and perhaps the standard deviation and other statistics that would enable him to predict what is the probability of a certain amplitude of motion not being exceeded.

Certain features of sub-critical vibration, which could almost have been supposed *a priori*, are that the amplitude increases with the length of the cylinders and with the flow velocity, but decreases with the flexural rigidity, etc. Such observations form the basis for the empirical expressions for predicting vibration amplitude, which will be discussed in §5.

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* Some cases have been reported (Paidoussis & Sharp 1967) where the dominant frequency of vibration was quite different than the first-mode natural frequency.

4.2 *Mechanism of Excitation of Sub-critical Vibrations*

Of course real flows are not ideally uniform, steady and purely axial; accordingly, the damping effect on vibrations (for small u) is but one aspect of the effect of axial flow on cylindrical structures subjected to it. Thus, axial flow may be regarded as something of a Trojan horse; although apparently beneficent, damping free motions, it contains within it the very forces which excite sub-critical vibration. Substantial departures from ideally uniform, steady and axial flow conditions, or even small perturbations in it, could well cause vibration. Remember that the vibration is of small amplitude so that the forces involved need not be large.

For instance, unsteady separated flow upstream of the cylindrical structure may give rise to unsteady lateral forces on the cylinders. Boiling, or changes in flow régime in two-phase flows, unless they occur in an axially symmetric fashion about each cylinder, may equally produce lateral forces acting on the cylinders. Swirl, as well as quasi-steady transverse flow components, as may be generated at the entrance of a flow channel, could also give rise to lateral forces.

Small-scale turbulence within the boundary layer around the cylinders generates random pressure perturbations around the cylinders; the pressure perturbations are not spatially uniform at any given instant, and when integrated around the circumference of a cylinder give rise to a random lateral load.

All the above may be considered to be sources of energy in the flow which could excite and sustain sub-critical vibration as a classical *forced vibration*. Another possibility is that the observed motions are a *parametric oscillation*. Parametric oscillations are defined as those which are dependent for their excitation upon the time-dependence of one or more of the parameters of the oscillatory system. For instance, if the flow velocity is not quite steady it may be written in the form $u = u_o \ (1 + \sum_n a_n \cos\omega_n\tau)$, where the a_n are small. It may be shown that this gives rise to a time-dependent 'compressive' force, the term $u^2(\partial^2\eta/\partial\xi^2)$ in equation (12) being equivalent to a force $-T(\partial^2\eta/\partial\xi^2)$. By analogy to a column subjected to a harmonically time-dependent compressive force, the cylinders may be subject to parametric oscillations when the ratio of ω_n to one of the natural frequencies of the cylinders, ω_r, is $\omega_n/\omega_r = 2, 1, 2/3$ ...... (Bolotin 1964).

Yet another possibility is that the vibration* is *self-excited*. The characteristic feature of self-excited vibrations is that there is a source of energy available upon which the system may draw in synchronism to its own natural vibrations (Magnus 1965). The energy source, in this case, is of course the flowing fluid. Suppose, for instance, that the axial flow exaggerates a natural bow in the cylinder; then, if the increased bowing causes an increased pressure drop in the flow and a reduction in the flow velocity, the bow would diminish under the action of flexural restoring forces; this

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* the term *sub-critical vibration* will henceforth be simplified to *vibration*.

completes the feedback and a self-excited vibration would be possible.

Thus the observed vibration may be forced, parametric or self-excited. In all the above, the sources of excitation lie within the fluid flow. In addition we have vibrations transmitted mechanically through the apparatus and via the supports of the cylindrical structures.

One interesting aspect of the problem that becomes clear from the above discussion is that, whatever the source of excitation, there is very little one can do to control the excitation mechanism. As Den Hartog (1970) put it, rather succinctly - he usually does - "The only recourse we have is to limit the flow velocity or to make the tube so stiff that the stresses caused by the random turbulent forces are within tolerable limits".

5. *PREDICTION OF SUB-CRITICAL VIBRATION AMPLITUDE*

We have seen that the observed vibration may be forced, parametric, or self-excited - or a combination of these. A number of analytical models have been proposed based on one or the other of these excitation mechanisms. All these models have certain common features which should be mentioned at the outset. Firstly, vibration transmitted through supports, piping, etc., i.e. mechanically transmitted vibration is not taken into account. Secondly, far-field noise, such as disturbances in the flow originating from pumps, valves and such, are not taken into account analytically but only empirically, or not at all. It is virtually impossible to do otherwise; these are *'system' characteristics*, where by 'system' we understand the whole flow system of which the cylindrical structures are only a small part. At present, there is no way of characterizing such disturbances *a priori*.

In this section we shall discuss the various means available forpredicting the vibration characteristics, first by empirical means and then with the aid of analytical models as classified above. The dominant frequency of vibration is taken to be equal to essentially that of the first-mode frequency of the cylinders in stationary fluid; accordingly, we shall concentrate on describing the means available for predicting vibration amplitude.

5.1 *Empirical Expressions*

Most of the means for predicting vibration amplitude are to a greater or lesser extent semi-empirical. However, in this classification we consider only Burgreen's *et al.* (1958) and Paidoussis' (1969) expressions to fall into this category. Quinn's semi-empirical expression is based on an analytical self-excitation model and will be dealt with separately.

The basis for using empirical expressions for predicting the vibration amplitude is no different, in principle, than the analogous case in heat transfer where, for instance, knowing that the Nusselt number may depend on the Reynolds and Prandtl numbers we may construct an empirical expression from which the heat transfer coefficient may be calculated. In the case of vibration induced by axial flow, knowing that the amplitude depends on u, β, ε, and the frictional coefficients (and, hence, the Reynolds number), etc., one may proceed in the same way.

Here we give the more recent and apparently more successful of the empirical expressions, namely that of Paidoussis' which is the following:

$$\frac{\delta}{D} = \alpha_1^{-4} \left[\frac{u^{1.6}\varepsilon^{1.8}N_R^{0.25}}{1 + u^2} \right] \left[\left(\frac{D_h}{D}\right)^{0.4} \right] \left[\frac{\beta^{2/3}}{1+4\beta} \right] \left[5\times10^{-4}K \right] \tag{16}$$

where,

- δ is the 'maximum' amplitude,
- α_1 is the dimensionless first-mode eigenvalue of the cylinder

N_R is the Reynolds number based on the hydraulic diameter

D_h is the hydraulic diameter,

K is a parameter

u, ε, β and D were defined in §2.

Here K represents a measure of departures from axial, steady and uniform flow conditions and of mechanically transmitted vibration. K = 1 corresponds to conditions that may exist if care has been exercised in producing disturbance-free flow conditions upstream of the cylindrical structure and low mechanically-transmitted vibration level. On the other hand, for realistic, industrial environments, K = 5.

As mentioned, earlier, the 'forcing function' was not specified in deriving this empirical expression, and this cannot but have a deleterious effect on its success.

The empirical expression is compared with the experimental data of Paidoussis and Sharp (1967), Burgreen *et al.* (1958), Quinn (1962), SoGREAH (1962) and Roström & Andersson (1964a,b,c). As shown in figure 7 the agreement is reasonable, but leaves a great deal to be desired. Every set of experimental results shows a large discrepancy at low flow velocities. This may be explained as being due to mechanically transmitted vibration and to other 'system' characteristics, which are overshadowed at higher flow velocities. If this 'background' noise were subtracted, then agreement between the empirical expression and the experimental points would improve greatly; however, this device, used by some authors in the field represents inadmissible cosmetic surgery from the designer's point of view.

This last point is important in this respect: it forces us to examine what we mean by flow-induced vibration. Is it the total vibration which arises when a cylinder is placed in circulating flow system, or is it that component of vibration which is due to flow over the cylinders *per se*? In the above expression the former view was adopted with the aim of producing something immediately usable by the designer; others take the latter view, but do not always make it clear that they do.

In this empirical expression the question arises as to what does the 'maximum' amplitude of vibration represent, recalling that the vibration is random. An answer to this was given by Reavis (1969) who said that it is "..... the maximum displacement from equilibrium to be expected if one were to scan through an oscillograph record of rod vibration about five feet in length. This length of record corresponds to the author's experience that a data sample is about two arms' lengths long. The author conjectures that the experimenters discussed in this paper*, too, scanned vibration-oscillograph records of comparable length." Although from the theoretical point of view this is an unsatisfactory answer, it is a reasonable one from the practical point of view.

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* Burgreen *et al.*, Quinn, SoGREAH and Pavlica & Marshall.

5.2 *Quinn's Self-excited Vibration Model*

Quinn (1962) states his fundamental postulate of self-excited vibration as follows: "If the rod is not in motion, the action of the flow on the rod in the transverse direction is centrifugal+, owing to what curvature the rod may have. If the rod is in motion, the centrifugal force will vary with rod velocity, because the radial flow distribution is a function of rod position. Of course, the curvature is very small and therefore the centrifugal force is also small. However, fluid turbulence is three dimensional and the resulting quadratic damping acts in a weak manner for small transverse rod velocities ....."

Quinn proposed a 'tentative' analytical model based on this postulate, involving the following two equations: an equation of motion of the cylinder

$$EI(\partial^4 y(\partial x^4) + MU^2(\partial^2 y/\partial x^2)^* + MU^2(d^2 y_o/dx^2)^*$$
$$+ \tfrac{1}{2}\rho D C_D |\partial y/\partial t|(\partial y/\partial t) + (m+M)(\partial^2 y/\partial t^2) = 0 \qquad (17)$$

and a momentum equation for one-dimensional axial flow

$$\rho(dU/dt) + \tfrac{1}{2}\rho f U^2\{D_h[1+\beta_e(e+y_o+y)]\}^{-1} = -\partial P/\partial x, \qquad (18)$$

+ Cf. term $u^2(\partial^2\eta/\partial\xi^2)$ in equation (12).

* Quinn, unaccountably, used a negative sign for these terms.

where y_o represents the permanent rod bow, f the friction coefficient of channel flow, D_h is the hydraulic diameter of the rod, β_e is a coefficient of eccentricity, e is the hydraulic eccentricity of rod supports and P is the pressure.

Since the damping term is retained in nonlinear form, it is clear that amplitude information may be obtained by approximate solution of these equations. Quinn reduces the system to an equivalent one of one-degree-of-freedom, assuming a first-mode deflection, and obtains a criterion for existence of self-excited vibration depending entirely on the eccentricity of the supports, the permanent bow and the time-dependent bow. He similarly obtains an expression for the amplitude of vibration which depends on the same parameters. In fact, the amplitude obtained is linearly proportional to the permanent bow of the cylinder. Herein lies the main difficulty in using Quinn's theory for estimating vibration amplitude, since the permanent bow in the cylinders is not known ahead of time.

Quinn tested his model mainly by comparing trends in experimental data with trends indicated by his approximate solutions; on this basis the model was found to be in agreement with experiments. He also attempted a comparison in absolute terms between theory and experiment and he states that "this comparison may be said to not refute the model".

It may be concluded that Quinn's model is difficult to apply. Work along similar lines was not pursued until very recently, by Avanzini (1971); the author, however, has not yet had time to evaluate this last model. Whether there is sufficient hard evidence to support the postulate of self-excited vibration is difficult to assess. Quinn's contention that there is a 'critical' flow velocity for the onset of self-excited oscillation may be explained, equally plausibly, by the argument in the last paragraph but two of §5.1. Also, if vibration is indeed random, this would contradict the possibility of self-excited vibration which implies some sort of quasi-stable limit-cycle motion.

5.3 *Forced-vibration Models*

A complete forced-vibration model would take into consideration all the forces which could directly excite lateral motion of the cylinder. However, since most of these forces cannot be characterized easily, and hence cannot be known *a priori* for a given system, the forced-vibration models invariably take into account only those forces which *are* known. This amounts to taking into account only the forces arising from pressure perturbations in the turbulent boundary layer surrounding the cylindrical structures. Statistical correlations of wall-pressure fluctuations of turbulent boundary layers are available, and these have been utilized to obtain estimates of vibration amplitude.

Theories of this type have been developed by several workers, e.g. Reavis (1967, 1969), Gorman (1969, 1971), Chen & Wambsganss (1970, 1972), and Kanazawa & Boresi (1970). The basis of all these methods is the following. It is assumed that motion in any one (x,y)-plane is representative of motion as a whole. Accordingly, the random pressure field, $p(x,\theta,t)$, on the surface of a cylinder is translated to an equivalent lateral force field, $f(x,t)$, by integrating around cross-sections of the cylinder, as shown in figure 8(a); thus

$$f(x,t) = \int_0^{2\pi} p(x,\theta,t)\cos\theta \frac{D}{2} d\theta \qquad (19)$$

The cylinder is now subject to a random distributed lateral force $f(x,t)$, shown in figure 8(b), and the motion of the cylinder may be analyzed by standard means.

Some of the assumptions usually made in these theories are as follows:

a) all far-field pressure fluctuation components are neglected, as well as pressure components arising from turbulence induced by supports, grids, etc., and from boiling and flow-régime changes;

b) no correlation is assumed between the pressure fields on adjacent cylinders (in the case of multi-cylinder structures);

c) the motion of a cylinder is assumed to have no effect on the pressure field, nor on the motion of adjacent cylinders;

d) the process is assumed to be ergodic and the pressure field homogeneous;

e) the cylinder is lightly damped, and response in the first-mode of the cylinder is considered to be dominant.

The first study of this type was by Reavis (1967, 1969). At the time there was no available information on turbulent wall-pressure fluctuations on a cylinder, and Reavis used Bakewell's (1962) measurements for turbulent wall-pressure fluctuations in a pipe. Gorman (1969, 1971) extended Reavis' work, concentrating mainly on two-phase flows; he made his own measurements of wall-pressure fluctuations, once again on the flow tube. Chen & Wambsganss (1970), on the other hand, used more recent measurements by Bakewell (1968) on wall-pressure fluctuations on a body of revolution, as well as measurements by Clinch (1969) and others.

Here we shall outline the theory, essentially as proposed by Chen & Wambsganss (1970, 1972), which may be considered to be the most up-to-date, and then discuss the differences between this and Reavis' work; subsequently, we shall discuss Gorman's work as it pertains to two-phase flows.

The equation of motion used is essentially equation (15); note, however, that Chen & Wambsganss used the uncorrected form of this equation (§2.1). The displacement $\eta(\xi,\tau)$ is expanded in terms of eigenfunctions $\phi_n(\xi)$ as follows:

$$\eta(\xi,\tau) = \sum_n \phi_n(\xi)\, q_n(\tau) \tag{20}$$

Since classical normal modes do not exist in this case, the eigenfunctions of the following system are used:

$$\phi^{iv} + \{u^2[1-\tfrac{1}{2}\varepsilon c_T(1-\tfrac{1}{2}\delta) - \tfrac{1}{2}(1-\delta)c_T'] - \delta\Gamma\}\phi'' - \lambda\phi = 0,$$

$$\text{with} \qquad \phi''' + \kappa_0\phi = \phi'' - \kappa_0'\phi' = 0 \qquad \text{at} \qquad \xi = 0, \tag{21}$$

$$\text{and} \qquad \phi''' - \kappa_1\phi = \phi'' + \kappa_1'\phi' = 0 \qquad \text{at} \qquad \xi = 1,$$

where λ is the eigenvalue to be determined, and the primes denote differentiation with respect to ξ. In general this eigenvalue problem is non-self-adjoint. Accordingly, to be able to decouple the equations of motion we must also define the adjoint eigenvalue problem, which of course will yield the same eigenvalues but a different set of eigenfunctions $\psi_n(\xi)$. The $\phi_n(\xi)$ and $\psi_n(\xi)$ form a bi-orthogonal set; i.e. each of the eigenfunctions ϕ_n is orthogonal to all eigenfunctions ψ_n, except where they both correspond to the same eigenvalue. (It should be mentioned here that for some of the 'standard' boundary conditions, e.g. clamped-clamped, pinned-pinned, the eigenvalue problem is indeed self-adjoint, and the eigenfunctions ϕ_n are orthogonal; in such cases the analysis is slightly simplified).

Substituting now equation (20) into (15) while making use of (21), multiplying by ψ_m and integrating from $\xi=0$ to 1, we obtain:

$$\ddot{q}_m + \alpha\sum_n a_{nm}\dot{q}_n + 2\beta^{\frac{1}{2}}u\sum_n b_{nm}\dot{q}_n + \tfrac{1}{2}\beta^{\frac{1}{2}}(\varepsilon c_N u+\varepsilon c)\dot{q}_m$$

$$+ \tfrac{1}{2}\varepsilon c_N u^2\sum_n b_{nm}q_n + \tfrac{1}{2}\varepsilon c_T u^2\sum_n d_{nm}q_n + \lambda_m q_m = Q_m\,, \tag{22}$$

$m = 1, 2, 3, \ldots;$ where

$$a_{nm} = \int_0^1 \phi_n^{iv}\psi_m d\xi \Big/ \int_0^1 \phi_m\psi_m d\xi,$$

$$b_{nm} = \int_0^1 \phi_n'\psi_m d\xi \Big/ \int_0^1 \phi_m\psi_m d\xi,$$

$$d_{nm} = \int_0^1 \xi\phi_n''\psi_m d\xi \Big/ \int_0^1 \phi_m\psi_m d\xi,$$

$$Q_m(\tau) = \int_0^1 Q(\xi,\tau)\psi_m d\xi / \int_0^1 \phi_m\psi_m d\xi.$$

As may be seen, equation (22) is not decoupled. Making the usual assumptions for small damping, the off-diagonal terms associated with internal and Coriolis-type damping are neglected; similarly, for the fifth and sixth terms in the equation. Accordingly, we obtain the following decoupled set of equations:

$$\ddot{q}_m + [\alpha a_{mm} + 2\beta^{\frac{1}{2}} u b_{mm} + \tfrac{1}{2}\beta^{\frac{1}{2}}(\varepsilon c_N u + \varepsilon c)]\dot{q}_m$$

$$+ [\lambda_m + \tfrac{1}{2}\varepsilon c_N u^2 b_{mm} + \tfrac{1}{2}\varepsilon c_T u^2 d_{mm}] q_m = Q_m \ , \qquad (23)$$

which may be re-written into the standard form:

$$\ddot{q}_m + 2\zeta_m\omega_m\dot{q}_m + \omega_m^2 q_m = Q_m. \qquad (24)$$

We define the correlation function of the generalized force field by:

$$R_{Q_nQ_m}(\tau_o) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} Q_n(\tau)Q_m(\tau+\tau_o)d\tau$$

$$= \frac{1}{M_nM_m} \int_0^1 \int_0^1 \psi_n(\xi)\psi_m(\xi+\xi_o)R_{QQ}(\xi,\xi+\xi_o,\tau_o)d\xi d(\xi+\xi_o), \qquad (25)$$

where $$R_{QQ}(\xi,\xi+\xi_o,\tau_o) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} Q(\xi,\tau)Q(\xi+\xi_o,\tau+\tau_o)d\tau$$

and $$M_n = \int_0^1 \phi_n \psi_n d\xi \, .$$

Note that $Q(\xi,\tau)$ is the force field and $Q_m(\tau)$ is the generalized force field, as defined in equation (22). Now, the cross-correlation spectral density is the Fourier transform of the correlation function. Hence,

$$\Phi_{QQ}(\xi,\xi+\xi_o,\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} R_{QQ}(\xi,\xi+\xi_o,\tau_o)e^{i\omega\tau_o}d\tau_o, \tag{26}$$

and $$\Phi_{Q_nQ_m}(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} R_{Q_nQ_m}(\tau_o)e^{i\omega\tau_o}d\tau_o;$$

and the Fourier transform of equation (25) may be written in the form

$$\Phi_{Q_nQ_m}(\omega) = \frac{1}{M_nM_m}\int_0^1\int_0^1 \psi_n(\xi)\psi_m(\xi+\xi_o) \times \Phi_{QQ}(\xi,\xi+\xi_o,\omega)d\xi d(\xi+\xi_o). \tag{27}$$

We may similarly write down expressions for the cross-correlation function and cross-correlation spectral density of the displacement from equation (20), namely

$$R_{\eta\eta}(\xi,\xi+\xi_o,\tau_o) = \sum_n \sum_m R_{q_nq_m}(\tau_o)\phi_n(\xi)\phi_m(\xi+\xi_o),$$

$$\Phi_{\eta\eta}(\xi,\xi+\xi_o,\omega) = \sum_n \sum_m \Phi_{q_n q_m}(\omega)\phi_n(\xi)\phi_m(\xi+\xi_o), \tag{28}$$

where $R_{q_n q_m}(\tau_o) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} q_n(\tau)q_m(\tau+\tau_o)d\tau,$

and $\Phi_{q_n q_m}(\omega)$ is the Fourier transform of $R_{q_n q_m}(\tau_o)$.

But from linear random vibration theory we know that

$$\Phi_{q_n q_m}(\omega) = |H_n(\omega)H_m(\omega)|\Phi_{Q_n Q_m}(\omega), \tag{29}$$

where $H_n(\omega) = [(\omega_n^2 - \omega^2) + 2\zeta_n\omega_n\omega i]^{-1}$. Hence, substituting equation (29) into (28) and using equation (27) we obtain the cross-correlation spectral density of the displacement

$$\Phi_{\eta\eta}(\xi,\xi+\xi_o,\omega) = \sum_n \sum_m |H_n(\omega)H_m(\omega)| \frac{\phi_n(\xi)\phi_m(\xi+\xi_o)}{M_n M_m}$$

$$\times \int_0^1 \int_0^1 \Phi_{QQ}(\xi,\xi+\xi_o,\omega)\psi_n(\xi)\psi_m(\xi+\xi_o)d\xi d(\xi+\xi_o).$$

Now we presume that we can write

$$\Phi_{QQ}(\xi,\xi+\xi_o,\omega) = A_{QQ}(\omega)\Psi_{QQ}(\xi,\xi+\xi_o,\omega)$$

which will be justified *a posteriori* by equation (33). $A_{QQ}(\omega)$ is the power spectral density of the force field and Ψ_{QQ} is a spatial correlation function. Substituting this into the equation above and letting $\xi_o=0$ we obtain the power spectral density of the displacement

$$\Phi_{\eta\eta}(\xi,\xi,\omega) = A_{QQ}(\omega) \sum_n \sum_m |H_n(\omega) H_m(\omega)|$$

$$\times \frac{\phi_n(\xi)\phi_m(\xi)}{M_n M_m} \int_0^1 \int_0^1 \Psi_{QQ}(\xi,\xi+\xi_o,\omega)$$

$$\times \psi_n(\xi)\psi_m(\xi+\xi_o) d\xi d(\xi+\xi_o). \quad (30)$$

Finally, the mean-square displacement is given by

$$\overline{\eta^2(\xi)} = \int_0^\infty \Phi_{\eta\eta}(\xi,\xi,\omega) d\omega. \quad (31)$$

The solution, as given by equations (30) and (31) may be simplified by assuming that damping is sufficiently small for coupling between the modes to be negligible; the double summation in equation (30) may then be reduced to a single summation. Moreover, if we assume that the response is predominantly in the first mode, we can consider only the leading term in the series. Hence

$$\Phi_{\eta\eta}(\xi,\xi,\omega) \simeq A_{QQ}(\omega) H_1^2(\omega) [\phi_1^2(\xi)/M_1^2] \times \int_0^1 \int_0^1 \Psi_{QQ}(\xi,\xi+\xi_o,\omega)$$

$$\times \psi_n(\xi)\psi_m(\xi+\xi_o) d\xi d(\xi+\xi_o). \quad (32)$$

This equation is quite similar to that first proposed by Reavis (1967).

Now we have to correlate the pressure field to the force field. Since the pressure field is assumed to be homogeneous, pressure correlations will depend on the separation between measurement points rather than absolute positions. Thus the cross-correlation spectral density will in general be $\Phi_{pp}(\xi_o,\theta_o,\omega)$ where $\xi_o=|\xi_1-\xi_2|$ and $\theta_o = |\theta_1-\theta_2|$. Using Corcos' (1963) model, one may write

$$\Phi_{pp}(\xi_o,\theta_o,\omega) = A_{pp}(\omega)\Psi_1(\omega\xi_o/U_c)\Psi_2(\omega\theta_o/U_c), \tag{33}$$

where $A_{pp}(\omega)$ is the power spectral density of the wall-pressure fluctuations, Ψ_1 and Ψ_2 are spatial correlation functions in the axial and circumferential directions, respectively, and U_c is the convection velocity. These functions will not be reproduced here for the sake of brevity. The translation from the pressure field to the force field is accomplished by using equation (19), in dimensionless form.

Wambsganss & Chen (1971) developed a simplified expression for the amplitude, based on their theory, *applicable to systems within a limited range of parameters;* this is given below

$$[\overline{y^2}]^{\frac{1}{2}} = \frac{0.018KD^{1.5}D_h^{1.5}U^2\phi_1(x)}{L^{0.5}f_o^{1.5}(M+m)\zeta^{0.5}\left[1 - \dfrac{\beta_1 MU^2L^2}{EI+\beta_1 TL^2}\right]^{0.75}} \tag{34}$$

where $K = 2.56 \times 10^{-3} lb \times sec^{2.5}/ft^{5.5}$;

$\beta_1 = 0.101$ for pinned-pinned cylinder and $\phi_1(x) = \sqrt{2} \sin(\pi x/L)$; $\beta_1 = 0.0246$ for a clamped-clamped cylinder, with the corresponding form for the eigenfunction $\phi_1(x)$;

$\zeta = \zeta_o + a_1 U + a_2 U^2$, with ζ_o being the damping factor in stationary fluid, $a_1 = 2.4 \times 10^{-4}$ sec/ft, and $a_2 = 3.4 \times 10^{-6}$ sec^2/ft^2;

f_o = fundamental frequency in stationary fluid, in Hz; the other quantities being as elsewhere in this paper.

As mentioned earlier, Reavis (1967, 1969) developed a closely similar theory. By the time the various simplications, necessary to render Chen & Wambsganss' theory tractable, are introduced, these differences become less important, in practice.

Reavis (1967, 1969) obtained a simplified expression for the amplitude, as follows:

$$[\overline{y^2(t)}]^{\frac{1}{2}} = C\eta_d \eta_D \eta_L \frac{DLN^{0.5}}{mf^{1.5}\zeta^{0.5}} U\rho\nu^{0.5} \tag{35}$$

where C is as given in figure 11, η_d, η_D, η_L are functions of the Strouhal numbers based on the diameter, hydraulic diameter and the length, respectively, N is the number of cylinders, f the first-mode natural frequency in Hz and ζ the corresponding damping factor, and ν is the kinematic viscosity; other symbols are as elsewhere in this paper.

So far Chen & Wambsganss' theory has been applied to only one or two cases, and agreement between theory and experiment as shown here in figure 9 is fairly good. The experimental points are their own, obtained in a well-designed circulation system with minimum 'system' disturbances. On the other hand, when Reavis (1969) applied his theory, which is closely similar to Chen & Wambsganss', to some of the available data, he obtained very large discrepancies as shown in figure 10. After introducing a correction factor which is a function of D_h/L - presuming the r.m.s. pressure fluctuations to decrease with decreasing D_h/L - as shown in figure 11, agreement is vastly improved (figure 12); but then, so it should, as figure 11, although qualitatively sound, quantitatively is no less than a procrustean couch.

Now the main differences between Chen & Wambsganss' theory and Reavis' are as follows. In the latter theory (i) wall-pressure correlations *on the pipe*, rather than on the cylinder, are used; (ii) effects arising from the mean axial flow, such as flow-induced damping, are neglected; (iii) only pinned-pinned cylinders are considered. Point (i) is not as serious as might be supposed as will become evident in the discussion of Gorman's work below. Point (ii) represents of course a serious omission, but for small u the main effect of the mean axial flow is to damp motions; this, if taken into account, would clearly make agreement between theory and experiment even worse. Point (iii), probably, has also a minor effect, as the *actual* first-mode natural frequency of the cylinder is used in Reavis' equation and the error (for cylinders that are not pinned-pinned) arises from departures of the actual eigenfunctions from the half-sinusoidal shape pertaining to pinned-pinned end conditions; once again use of the correct eigenfunctions would probably result in lower theoretical values. Accordingly, at the time of writing, there is no reason to suppose that Chen & Wambsganss's theory, albeit more elegant and rigorous, would give substantially different agreement with experiment than Reavis' in its pristine form - i.e. similar to figure 10.

We next consider Gorman's work (1967, 1969). Gorman's analysis follows Reavis' fairly closely and as such requires no further elaboration here. His work differs from that of Reavis' and Chen & Wambsganss' in that he obtained his own pressure correlations experimentally, first in water (Gorman 1967) and then in 'two-phase' flows simulated by air-water mixtures. In all cases he measured the near-field pressure correlations on the wall of the flow channel, reasoning that the pressure field must be closely similar to that which would be acting on a cylindrical structure within. His correlation measurements in water suffered from lack of resolution because of the large size of his pressure transducers; in any case some may be in doubt, according to Chen & Wambsganss (1970), as "the lateral cross-correlation of pressure does not agree with that given in other reports".

Gorman's (1969) measurements with air-water mixtures were much more refined. Some results of vibration amplitudes are reproduced as figure 13, where agreement between theory and experiment is truly very good. (For the first time we see a *linear scale* for plotting vibration amplitude.) The experimental points in figure 13 are from Gorman's own measurements with air-water mixtures. In figure 14 we see some measurements by Pettigrew in steam-water mixtures, in another circulation system, which exhibit a similar amplitude peak at about 12% quality as in Gorman's results of figure 13. It should be stressed that no correction factors are introduced by Gorman in his theoretical expression for predicting amplitude. This expression is given below

$$[\overline{y^2(t)}]^{\frac{1}{2}} = \frac{LD\psi_D\psi_L[\overline{p_\omega^{\,2}(t)}]^{\frac{1}{2}}(\pi f_1)^{\frac{1}{2}}}{m\Omega_1^{\,2}(4\zeta)^{\frac{1}{2}}} \qquad (36)$$

where ψ_D and ψ_L are equivalent to Reavis' η_d and η_L, $\overline{p_\omega^{\,2}(t)}$ is power-spectral density of the pressure field in the vicinity of the first-mode natural frequency of the cylinder, and $\Omega_1 = 2\pi f_1$ is the first-mode circular frequency of vibration of the cylinder.

Similarly good agreement between theory and experiment was recently reported by Cedolin *et al.* (1971), using Gorman's theory but their own measured pressure correlation functions; the fluid used was a nitrogen-water mixture. Theory and experiment are compared in figure 15.

The question arises as to why such good agreement was obtained by Gorman and by Cedolin *et al.* in two-phase flows, and by Gorman even in water-flows (figure 16), while Reavis' work showed large discrepancies between his own theory and others' experiments. The answer is fairly clear. Two-phase flows have notoriously short memory; i.e. the internal damping in two-phase flows is quite high. Accordingly, pressure fluctuations arising upstream of the cylindrical structure simply do not survive to be felt there - except (plane) pressure waves which, in any case, were electronically suppressed in Gorman's measurements of pressure correlations, as is normal for near-field pressure correlation measurements. Accordingly, it is the locally generated pressure disturbances, i.e. those generated in the immediate vicinity, that matter. In other words, in two-phase flows, the near-field pressure field essentially represents the total pressure field so far as vibration of cylinders is concerned.

What remains to be explained is the good agreement between theory and experiment obtained by Gorman in the case of single-phase (water) flow. Here we may invoke the "peculiarity" of his measured correlations to suggest that they did not correspond solely to boundary-layer pressure fluctuations, but also contained components of 'system' distrubances - other than those which are axially symmetric. Of course, this may be true, to a certain extent, for Gorman's two-phase flow pressure-correlation measurements, but we have nothing to compare them against. (Unfortunately, Gorman did not apply his theory to others' experimental results, presumably because they are all associated with steam-water flows rather than air-water flows).

Now, presuming that 'system' peculiarities are inherent in Gorman's pressure correlations, and accepting the validity of the forced-vibration excitation mechanism, then (in the context of linear vibration theory) agreement between theory and experiment simply becomes a test of the validity of Gorman's analytical model; alternatively, accepting the various approximations, simplifications, etc., inherent in Gorman's analytical model, the observed agreement supports the validity of the forced-vibration excitation model. Nevertheless, *from the point of view of vibration amplitude prediction*, we are no further ahead - at least in single-phase flows - for, if one has to measure the pressure field characteristics of his flow system before one can predict vibration amplitude of cylinders in it, one might as well measure the vibration amplitude directly *in situ*.

5.4 *Y.N. Chen's Parametric Vibration Model*

Y.N. Chen (1970a,b) used a simplified form of equation (8) by considering (i) the Coriolis acceleration term to be small, (ii) $U(\partial y/\partial x)$ to be small compared to $(\partial y/\partial t)$, and (iii) the axial drag force to be acting at the mid-point of the cylinder. With these and other minor simplifications, he obtained the following approximate equation of motion:

$$EI(\partial^4 y/\partial x^4) + (MU^2+\tfrac{1}{4}c_L\rho U^2 LD)(\partial^2 y/\partial x^2)$$

$$+ \tfrac{1}{2}c_N\rho D(\partial y/\partial t) + (m+M)(\partial^2 y/\partial t^2) = 0 . \qquad (37)$$

Assuming pinned boundary conditions, he considered solutions of the form

$$y(x,t) = \sum_n \sin(n\pi x/L)\, q_n(t) \qquad (38)$$

which substituted into equation (37) yield

$$(m+M)\ddot{q}_n+\tfrac{1}{2}c_N\rho D\dot{q}_n+[(\tfrac{n\pi}{L})^4 EI-(\tfrac{n\pi}{L})^2(\tfrac{1}{4}c_L\rho DL+M)U^2]q_n = 0. \qquad (39)$$

Next, Chen argues as follows: "The natural frequency of the fuel rods usually used is very low, so they can only be excited to vibration in resonance by the large eddies, the size of which is comparable with the path width of the flow along the rod assembly. The diameter of the rod will be small compared with this eddy size, so that the phase shift of the dynamic eddy force along the periphery of the circular cross section of the rod will be small too. No significant resultant dynamic force perpendicular to the rod axis can therefore be exerted by the eddy. This force will be neglected in further considerations."

Elsewhere, Chen states the following: "No significant resultant dynamic force perpendicular to the rod axis can therefore be exerted. According to the calculation of Reavis this dynamic force is about 3.5 to 240 times less than would be necessary to maintain the intensities of the vibrations which actually occured on a number of fuel rods. The results obtained by several authors, yield such as 3.5 to 20 times less by Quinn, 15 by Pavlica, 50 by SoGREAH and 50 to 240 by Burgreen. However, this small excitation may possess a special effect on the initiation of the instability postulated by Chen. This will be shown later."

It is postulated next that the observed vibration is parametric and arises through the effect of velocity fluctuations which in turn affect the effective 'centrifugal', or compressive, force represented by the second term in equation (37). Accordingly, we set

$$U = \bar{U} + u' = \bar{U} + u'_o \cos\Omega t, \qquad (40)$$

and changing variables by introducing $\tau = \Omega t$, equation (39) may be re-written in the form

$$\frac{d^2q_n}{d\tau^2} + 2\,\zeta\frac{dq_n}{d\tau} + [\delta'_n - \varepsilon_n \cos\tau]q_n = 0 \tag{41}$$

where

$$2\zeta = \tfrac{1}{2}c_N\rho D/[(M+m)\Omega];$$

$$\delta'_n = (\frac{\Omega_n}{\Omega})^2[1 - \frac{(\frac{1}{4}c_L\rho DL + M)\overline{U}^2}{(n\pi/L)^2 EI}],$$

Ω_n being the nth natural frequency (42)

$$\Omega_n^2 = (n\pi)^4[EI/(m+M)L^4];$$

and $$\varepsilon_n = (\frac{\Omega_n}{\Omega})^2\ [\frac{2(\frac{1}{4}c_L\rho DL + M)}{(n\pi/L)^2 EI}]\ \overline{U}u'_o.$$

In these expressions $(n\pi/L)^2EI$ is the nth Euler buckling load, while $(\frac{1}{4}c_L\rho DL + M)\overline{U}^2$ represents the effective steady-state 'centrifugal' or compressive load, and $2(\frac{1}{4}c_L\rho DL+M)\ \overline{U}u'_o$ represents the effective fluctuating component of that load. Clearly, if the steady-state compression equals the Euler buckling load, the cylinder would buckle; this defines a critical buckling velocity

$$\overline{U}_c^2 = [(\frac{n\pi}{L})^2 EI]/[\tfrac{1}{4}c_L\rho DL+M]. \tag{43}$$

With this notation the expressions for ε_n and δ'_n may be considerably simplified to

$$\delta_n' = \left(\frac{\Omega_n}{\Omega}\right)^2 \left[1 - \left(\frac{\bar{U}}{\bar{U}_c}\right)^2\right] \tag{44}$$

and $$\varepsilon_n = 2\left(\frac{\Omega_n}{\Omega}\right)^2 \bar{U}u_o'/\bar{U}_c^2 \ .$$

Of course u_o' is unknown generally; based on some measurements by Motzfeld and by himself, Chen proposes

$$u_o' = \alpha\xi^2 S\bar{U} \ ,$$

where $S = fD_h/\bar{U}$ is the Strouhal number, ξ is the initial turbulence level at the entrance of the cylindrical structure, and α is a proportionality factor. Substituting this expression for u_o' into equation (44) we obtain

$$\varepsilon_n = 2\left(\frac{\Omega_n}{\Omega}\right)^2 \left(\frac{\xi\bar{U}}{\bar{U}_c}\right)^2 \alpha S \ . \tag{45}$$

With these definitions equation (41) may be written in a more easily recognizable from by letting

$$q_n(\tau) = e^{-\zeta\tau} B_n(\tau) \text{ and } \delta_n = \delta_n' - \zeta^2 \ , \tag{46}$$

which yields

$$\frac{d^2 B_n}{d\tau^2} + (\delta_n - \varepsilon_n \cos\tau) B_n = 0 \ . \tag{47}$$

This is a Mathieu-Hill type equation. It admits solutions which are stable or unstable according to the combination of values of δ_n and ε_n (Bolotin 1964). Now for small damping $\delta_n \simeq \delta'_n$ and hence δ_n is the square of the ratio of the natural frequency in flow to the frequency Ω, i.e.

$$\delta_n = [\Omega_{nf}/\Omega]^2 , \tag{48}$$

where $\Omega_{nf}^2 = [1 - (\bar{U}/\bar{U}_c)^2]\Omega_n^2$, in the absence of damping. It is known from the solution of equations such as (47) that the regions of parametric oscillations (instabilities) occur, in the case of $\varepsilon_n \to 0$, at $\Omega/\Omega_{nf} = 2, 1, 2/3, \ldots$. For $\varepsilon_n > 0$ there are alternate regions of stability and instability depending on the values of δ_n; the unstable regions are wider, the larger the value of ε_n. Normally the unstable region associated with $\Omega/\Omega_{nf} = 2$ is the most important (primary region). The secondary instability region, corresponding to $\Omega/\Omega_{nf} = 1$ is normally truly secondary, especially if damping is present. However, here Chen reasons as follows: "The turbulence force component, which acts perpendicularly to the rod axis has been neglected in the derivation of the differential equation due to its small strength. But this force will cause a resonance of the rod, however weak it might be ..... The excitation of the perpendicular force will therefore reinforce the trend of the (parametric) instability." Accordingly, it is presumed that the secondary instability region is the 'primary' one in this case, and $\Omega = \Omega_{nf}$.

Furthermore, Chen reasons that the amplitude of vibration would be proportional to ε_n and, fairly arbitrarily, sets

$$fy/\overline{U} = k\varepsilon_n \, , \tag{49}$$

where y here represents the amplitude of vibration and k is a proportionality constant. Using the definition of ε_n in equation (45) we have

$$fy/\overline{U} = 2k\left(\frac{\Omega_n}{\Omega}\right)^2\left(\frac{\xi\overline{U}}{\overline{U}_c}\right)^2\alpha S.$$

But since $\Omega_{nf}=\Omega$, the definition of Ω_{nf} in equation (48) implies that $(\Omega_n/\Omega)^2 = (\Omega_{nf}/\Omega)^2[1 - (\overline{U}/\overline{U}_c)^2]^{-1}$. Also, $S = fD_h/\overline{U}$. Combining α and k into a new constant $K = 2k\alpha$ the above equation may now be written as

$$\frac{y}{D_h} = K\left[1 - \left(\frac{\overline{U}}{\overline{U}_c}\right)^2\right]^{-1}\left(\frac{\xi\overline{U}}{\overline{U}_c}\right)^2. \tag{50}$$

Chen subsequently lets $\xi = \beta\xi_o$ where ξ_o is the turbulence level at the entrance to the cylindrical structure for a normal technical case, such that $\beta = \frac{1}{2}$ represents ideally quiet flow conditions (e.g. the experiments at SoGREAH), $\beta = 1$ represents the normal, and $\beta = 2$ for particularly poor flow conditions. Then $\xi_o{}^2K$ is evaluated empirically, finally yielding the following expression for predicting vibration amplitude.

$$\frac{y}{D_h} = \left[1 - \left(\frac{\overline{U}}{\overline{U}_c}\right)^2\right]^{-1}\left(\frac{\beta\overline{U}}{\overline{U}_c}\right)^2. \tag{51}$$

Chen tested this theory by comparing the predicted amplitudes, given by equation (51), to most available experimental amplitudes of vibrations (obtained by Burgreen *et al.*, Pavlica & Marshall, Quinn, Roström & Andersson, SoGREAH, Basile *et al.* and Reavis). Agreement is quite good, being always better than within an order of magnitude, as shown in figure 17.

It must be stressed that, by the time the various assumptions are made [particularly that of equation (49)] this becomes a semi-empirical theory.

6. *CONCLUSION*

In this paper, we have considered a number of aspects of vibration of cylindrical structures in axial flow.

We have shown that cylinders immersed in axial flow are subject to hydroelastic instabilities. However, these instabilities occur at sufficiently high flow velocities to be of no concern for industrial applications. The mean axial flow velocity has mainly two effects on the free motions of industrially feasible cylindrical structures: (i) it slightly lowers their natural frequencies (in the case of cylinders with both ends supported) and (ii) it introduces additional damping to their motions (flow-induced damping).

However, axial flow - not being purely axial, uniform and steady - has an additional effect on cylindrical structures; it induces random vibration of very small amplitude ('sub-critical' vibration) which *is* of concern in certain industrial applications. The amplitude of these vibrations is generally so small that instrumented experimental assemblies for its study ".... could act as a fairly sensitive seismograph", as suggested by Gorman. The frequency of vibration is essentially the first-mode natural frequency of the cylinders.

Means of predicting sub-critical vibration amplitude

Originally, it was attempted to predict vibration amplitude by the use of empirical expressions. This proved to have only limited success and discrepancies between predicted and measured amplitudes of one order of magnitude were not too uncommon. Accordingly, it was sought to understand the underlying mechanism of vibration so as to achieve better success. Three types of mechanism were proposed corresponding to whether the vibration was postulated to be self-excited, forced or parametric.

Quinn's *self-excited vibration model* is difficult to use for predicting vibration amplitude and has not been adequately tested. A more recent and not entirely analytical model by Avanzini was not reviewed here; preliminary review indicated discrepancies between prediction and measurement no smaller than those attainable by the empirical expressions.

Forced vibration models were proposed by Reavis, Gorman, Chen & Wambsganss and others. Here it is postulated that vibration is caused by random pressure fluctuations on the surface of the cylinders which, integrated around the circumference,give rise to random transverse forces. Since the only pressure fluctuations which have been characterized, i.e., which can be predicted independently of the particularities of the system,are the wall-pressure fluctuations in turbulent boundary layers, these theories perforce take only these (*near-field*) pressure fluctuations into account. Reavis applied this theoretical model to others' available experimental data and found the theory to underestimate vibration amplitude by as much as two and one-half orders of magnitude. Chen & Wambsganss' theory, which is more elegant and refined, but quite similar to Reavis' has not been adequately tested but should not be significantly more successful. Reavis empirically modified his theoretical model and obtained much improved prediction.

Gorman, using a slightly modified model in the case of flows of air-water mixtures, achieved remarkable success. Agreement between theory and experiment of the order of 30% or better was attained. In two-phase flows, far-field pressure fluctuations (excepting plane pressure waves) are quickly damped, and hence, the near-field pressure fluctuations represent the total excitation field. This explains the success of the theory in two-phase flows and the lack of it in single-phase flows. It also lends strong support to the forced-vibration model. Unfortunately, Gorman's theory has not been applied to any experiments performed in a circulation system other than that in which the pressure correlations were measured. As his measured correlations may contain perculiarities of his own system, it is possible that such good agreement may not be attained when applied to another system. Moreover, the above experiments of Gorman's were all with a single cylinder.

Finally, a *parametric excitation model* by Y.N. Chen was discussed. It supposes that turbulence-induced changes in the axial flow velocity excite flexural vibration, in the same way that periodic changes in the compressive load on a column may cause parametric instabilities. This theory is semi-empirical and gives agreement with experiment within one order of magnitude.

We may conclude the following:

(i) prediction of vibration amplitude in single-phase and two-phase flows is possible by Paidoussis', Reavis' and Y.N. Chen's semi-empirical expressions (all other theories being considered inadequately tested to be able to say), within one order of magnitude.

(ii) it may be possible to achieve better agreement in two-phase flows,e.g., by Gorman's theory, if pressure correlations for two-phase flows (as opposed to flows of air-water mixtures) become available.

Concluding Remarks

The sad aspect of the situation may be viewed as follows. If one excludes Paidoussis' own experimental data for bundles of cylinders (which have not been used by any other workers to test their theories), then it may be said that his empirical expression achieves prediction to within one order of magnitude. Reavis' semi-empirical expression improves on this slightly and Y.N. Chen's hardly at all (although it must be said that a great deal of newer data are used to test the latter). This point is being made solely to indicate that, in terms of ability to predict vibration amplitude, we have not progressed very far, despite the considerable analytical effort expended.

It *must* be admitted that without empirical correction, all purely analytical theories have failed in their ability of predicting vibration amplitude in single-phase flows. (This phraseology is here used advisedly, to remind the reader of the difference between the *truly flow-induced* components of vibration and the *total vibration in axial flow*, including mechanically transmitted vibration arising from pumps, etc. The designer would clearly like to know the latter). The situation in two-phase flows may be considerably better, but we shall not know definitely until Gorman's theory, for instance, is applied to experimental results obtained in an experimental rig other than his own and to bundles of cylinders.

Nevertheless Gorman's success with flows of air-water mixtures suggests that the forced vibration model is sound. Let us presume that his theory, when applied to other experimental data, proves equally successful. Then we would conclude that, if only we could characterize adequately the pressure field in single-phase flows, we might be more successful in this case also. What is needed is such information as the power spectral densities of pressure fluctuations induced by valves, supports, bends, etc., and their decay characteristics with distance; we need wall-pressure correlations on closely spaced bundles and how they vary with spacing, etc. This, of course, implies a long and tedious research program. Even so, all this, assuming that it eventually becomes available, may not be enough, as it will still be difficult to know *a priori* what mechanically transmitted vibration may be present. It may well be that prediction of vibration to better than 100% or 200% is unattainable and that it will remain so.

In contrast to the pessimistic note struck above, our basic qualitative understanding of the effects of axial flow on cylindrical structures has improved greatly. Starting with Reavis' original proposal, the remarkable work by Chen & Wambsganss and Y.N. Chen's interesting parametric excitation postulate have added enormously to the understanding of this interesting subject.

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8. REFERENCES

AVANZINI, P.G. 1971 "Self-sustained vibrations of rods in parallel flow". Preprint E3/5. 1st Int'l Conf. *Structural Mechanics in Reactor Technology;* Berlin.

BAKEWELL, H.P.,Jr., CAREY, G.F., LIBUHA, J.J., SCHLOEMER, H.H. & VON WINKLE, W.A. 1962 "Wall pressure fluctuations in turbulent pipe flow". *U.S. Navy Underwater Sound Lab.* Report No. 559.

BAKEWELL, H.P.,Jr. 1968 "Turbulent wall-pressure fluctuations on a body of revolution". *J. Acoust. Soc. Amer.*, 43, 1358-1363.

BASILE, D., FAURE, J. and OHLMER, E. 1968 "Experimental study on the vibrations of various fuel rod models in parallel flow". *Nucl. Eng. Design,* 7, 517-534.

BOLOTIN, V.V. 1964 *The Dynamic Stability of Elastic Systems.* San Francisco: Holden-Day Inc.

BURGREEN, D., BYRNES, J.J. and BENFORADO, D.M. 1958 "Vibration of rods induced by water in parallel flow". Trans. ASME, 80, 991-1003.

CEDOLIN, L., HASSID, A., ROSSINI, T., SOLIERI, R. 1971 "Vibrations induced by the two-phase (gas + liquid) coolant flow in the power channels of a pressure tube type nuclear reactor". Preprint E4/5, 1st Int'l Conf. *Structural Mechanics in Reactor Technology;* Berlin.

CHEN, S.S. and WAMBSGANSS, M.W.,Jr. 1970 "Response of a flexible rod to near-field flow noise". *Proc. Conf. on Flow-Induced Vibrations in Reactor System Components;* Argonne, Illinois. Argonne National Lab. Report ANL-7685, 5-31.

CHEN, S.S. and WAMBSGANSS, M.W. 1972 "Parallel-flow-induced vibration of fuel rods". *Nucl. Eng. Design*, 18, 253-278.

CHEN, Y.N. 1970a "Turbulence-induced instability of fuel rods in parallel flow". *Sulzer Technical Review*, Research Number.

CHEN, Y.N. 1970b "Flow-induced vibrations in tube bundle heat exchangers with cross and parallel flow. Part 1: Parallel flow". *Flow-Induced Vibration in Heat Exchangers*; New York: ASME, 57-66.

CLINCH, J.M. 1969 "Measurements of the wall pressure field at the surface of a smooth-walled pipe containing turbulent water flow". *J. Sound Vib.*, 9, 398-419.

CORCOS, G.M. 1963 "Resolution of pressure in turbulence". *J. Acoust. Soc. Amer.*, 35, 192-199.

DEN HARTOG, J.P. 1970 "The mechanism of tube vibrations". *Flow-Induced Vibration in Heat Exchangers*; New York: ASME, 78.

GORMAN, D.J. 1969 *The role of turbulence in the vibration of reactor fuel elements in liquid flow.* Atomic Energy of Canada, Report AECL-3371.

GORMAN, D.J. 1971 "An analytical and experimental investigation of the vibration of cylindrical reactor fuel elements in two-phase parallel flow". *Nucl.Sci. Eng.*, 44, 277-290.

KANAZAWA, R.M. and BORESI, A.P. 1970 "Calculation of the response of rods to boundary-layer pressure fluctuations". *Proc. Conf. on Flow-Induced Vibrations in Reactor System Components*; Argonne, Illinois. Argonne National Lab. Report ANL-7685, 47-63.

LIGHTHILL, M.J. 1960 "Note on the swimming of slender fish". *J. Fluid Mech.*, 9, 305-317.

MAGNUS, K. 1965 *Vibrations* London: Blackie & Son Ltd.

MILLER, D.R. 1960 "Critical flow velocities for collapse of reactor parallel-plate fuel assemblies". *J. Eng. Power*, 82, 83-95.

NIORDSON, F.I. 1953 "Vibrations of a cylindrical tube containing flowing fluid . *Kungl, Tek. Högsk. Handl.*,No. 73.

PAÏDOUSSIS, M.P. 1965 *The amplitude of fluid-induced vibration of cylinders in axial flow*". Atomic Energy of Canada Report AECL-2225.

PAÏDOUSSIS, M.P. 1966a "Vibration of flexible cylinders with supported ends, induced by axial flow". *Proc. Instn Mech. Engrs* , 180, 268-278.

PAÏDOUSSIS, M.P. 1966b "Dynamics of flexible slender cylinders in axial flow. Part I: Theory". *J. Fluid Mech.*, 26, 717-736.

PAÏDOUSSIS, M.P. 1966c "Dynamics of flexible slender cylinders in axial flow. Part II: Experiments". *J. Fluid Mech.*, 26, 737-751.

PAÏDOUSSIS, M.P. 1969 "An experimental study of vibration of flexible cylinders induced by nominally axial flow". *Nuclear Science Eng.*, 35, 127-138.

PAÏDOUSSIS, M.P. 1970 "Dynamics of submerged towed cylinders". *Proc. 8th Symp. on Naval Hydrodynamics*, Pasadena, Calif. To be published by the Office of Naval Research,(U.S.) Department of the Navy.

PAÏDOUSSIS, M.P. and SHARP, F.L. 1967 *An experimental study of the vibration of flexible cylinders induced by nominally axial flow.* Atomic Energy of Canada Report CRNL-76.

PAVLICA, R.T. and MARSHALL, R.C. 1966 "An experimental study of fuel assembly vibrations induced by coolant flow". *Nuclear Eng. Design*, 4, 54-60.

QUINN, E.P. 1962 *Vibration of fuel rods in parallel flow.* U.S. Atomic Energy Commission Report GEAP-4059.

QUINN, E.P. 1965 *Vibration of SEFOR fuel rods in parallel flow.* U.S. Atomic Energy Commission Report GEAP-4966.

REAVIS, J.R. 1967 "WVI - Westinghouse vibration correlation for maximum fuel-element displacement in parallel turbulent flow". *Trans. Am. Nucl. Soc.*, 10, 369-371.

REAVIS, J.R. 1969 "Vibration correlation for maximum fuel-element displacement in parallel turbulent flow". *Nucl. Sci. Eng.*, 38, 63-69.

ROSENBERG, G.S. and YOUNGDAHL, C.K. 1962 "A simplified dynamic model for the vibration frequencies and critical coolant flow velocities for reactor parallel plate fuel assemblies". *Nucl. Sci. Eng.*, 13, 91-102.

ROSTRÖM, K.G. and ANDERSSON,N. 1964a *Boiler element for Marviken, vibration test with one rod.* Ab. Atomenergi, (Stockholm), Arbetsrapport RPL-724.

ROSTRÖM, K.G. and ANDERSSON, N. 1964b *Superheater element for Marviken, vibration test with one rod.* Ab. Atomenergi, Arbetsrapport RPL-725.

ROSTRÖM, K.G. 1964c *Seven-rod fuel element Vibration tests.* Ab. Atomenergi, Arbetsrapport RPL-726.

SHIELDS, C.M. 1960 *N.S. Savannah fuel design and development program - Fuel rod vibration.* U.S. Atomic Energy Commission Report GEAP-3583.

SOGREAH 1962 *Study of vibrations and load losses in tubular clusters.* Société Grenobloise d'Études et Applications Hydrauliques. Special Report No. 3, EURAEC-288.

TAYLOR, G.I. 1952 "Analysis of the swimming of long and narrow animals". *Proc. Roy. Soc. (London)*, (A), 214, 158-183.

WAMBSGANSS, M.W. and CHEN, S.S. 1971 *Tentative Design Guide for Calculating the Vibration Response of Flexible Cylindrical Elements in Axial Flow.* Argonne National Lab. Report ANL-ETD-71-07.

WAMBSGANSS, M.W. and ZALESKI 1970 "Measurement, interpretation and characterization of near-field flow noise". *Proc. Conf. on Flow-Induced Vibration in Reactor System Components.* Argonne, Illinois. Argonne National Lab. Report ANL-7685, 112-140

9. FIGURES

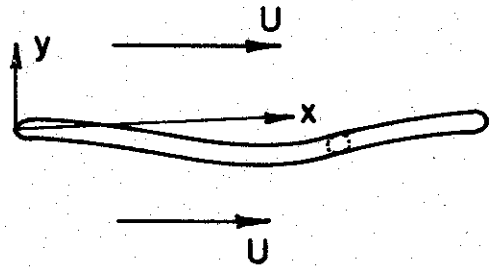


FIG.1 CYLINDER IN PARALLEL FLOW

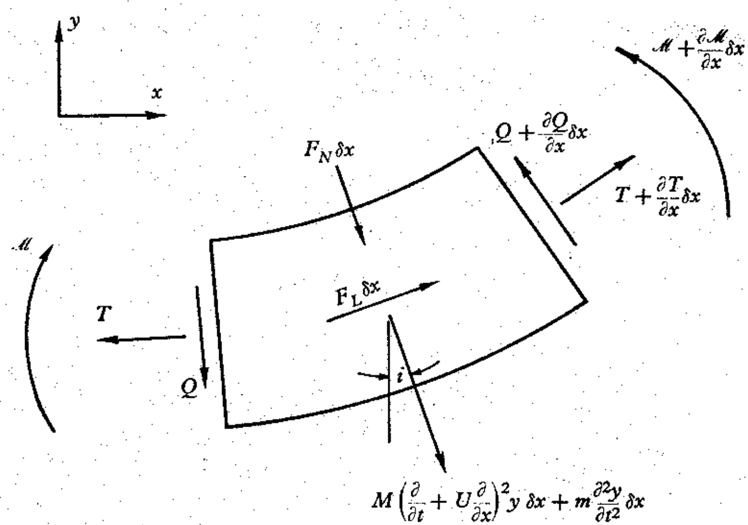


FIG.2 ELEMENT OF CYLINDER SHOWING FORCES ACTING ON IT

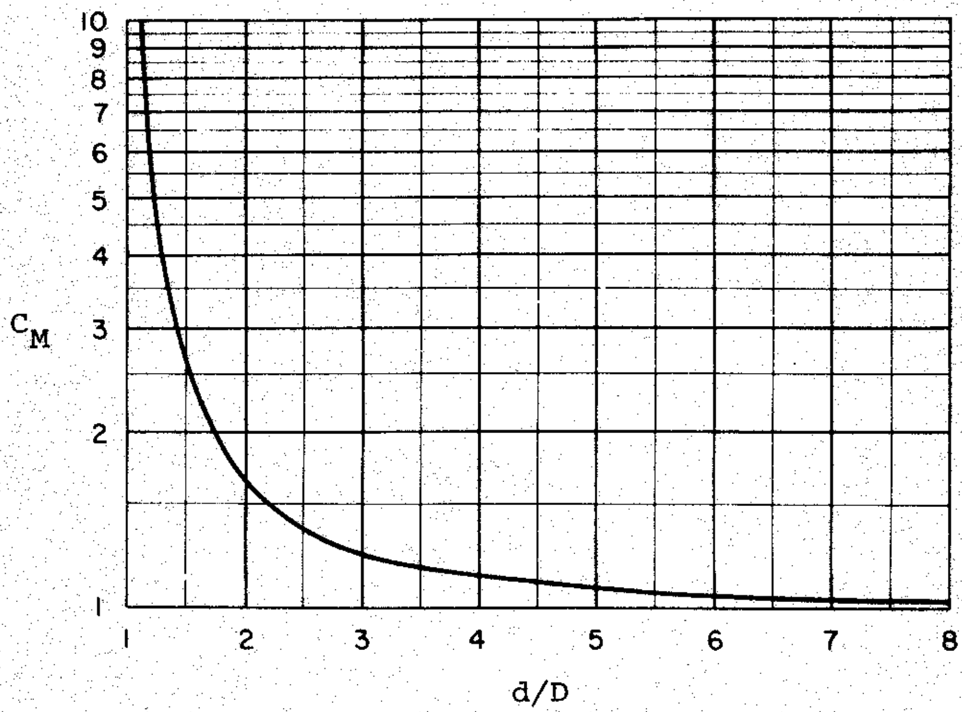


FIG. 3 Virtual mass coefficient as a function of d/D, i.e., (annulus diameter)/(cylinder diameter); [from Wambsganss and Chen (1971)].

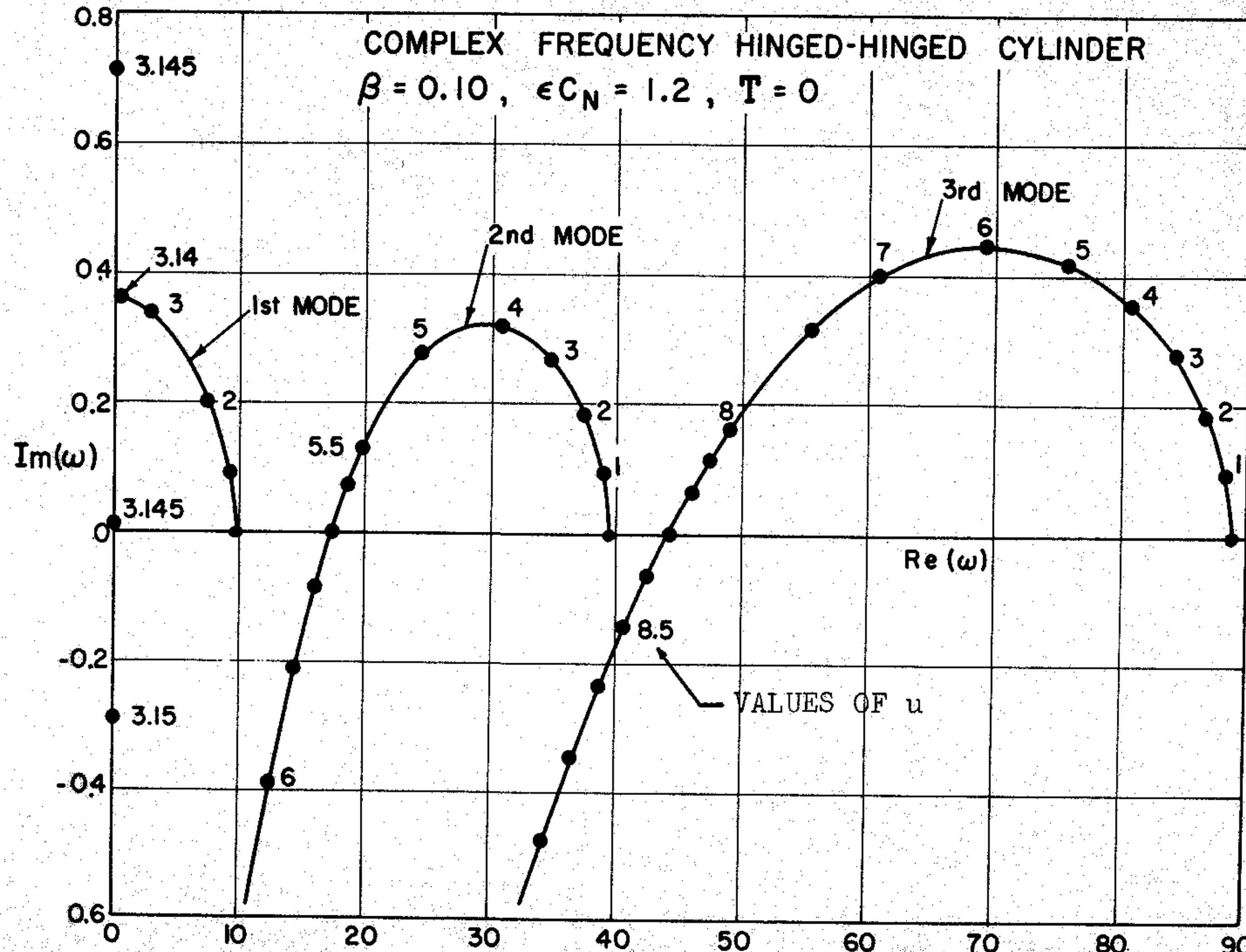


FIG. 4 The dimensionless complex frequency of the three lowest modes of a typical cylinder pinned at both ends, as a function of the flow velocity.

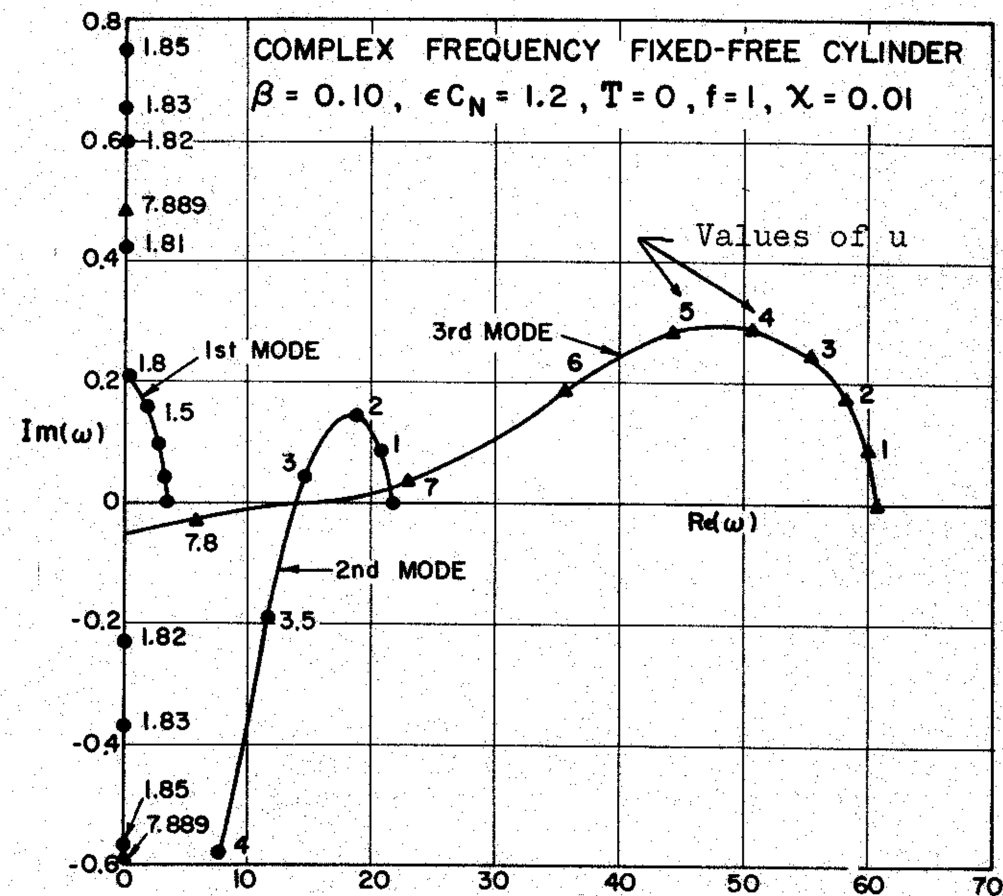


FIG. 5 The dimensionless complex frequency of the three lowest modes of a typical cantilevered cylinder with a tapered free end, as a function of the flow velocity.

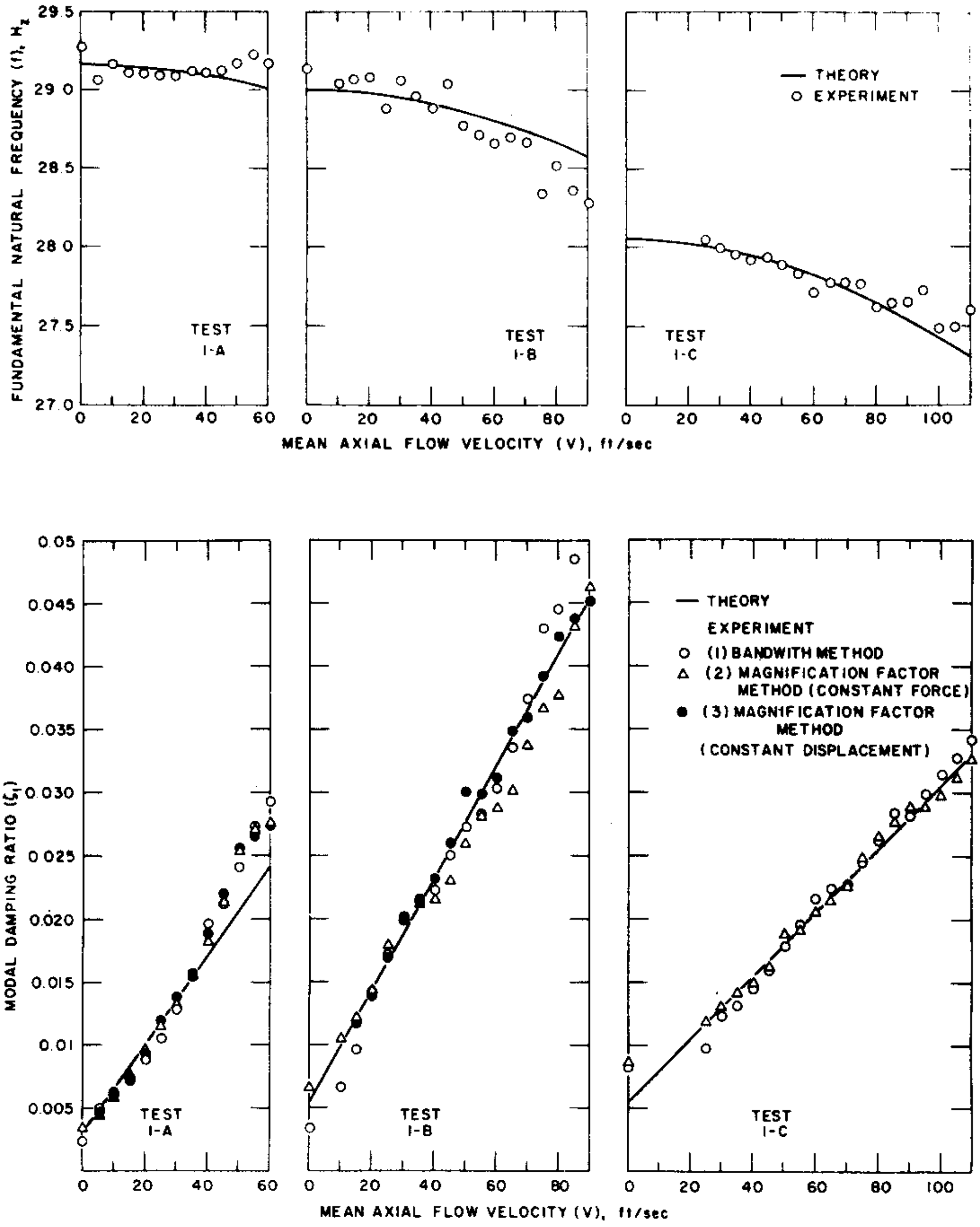


FIG. 6 The effect of the mean flow, for small values of u, on free vibration characteristics of a cylinder with clamped ends [from Chen and Wambsganss (1972)].

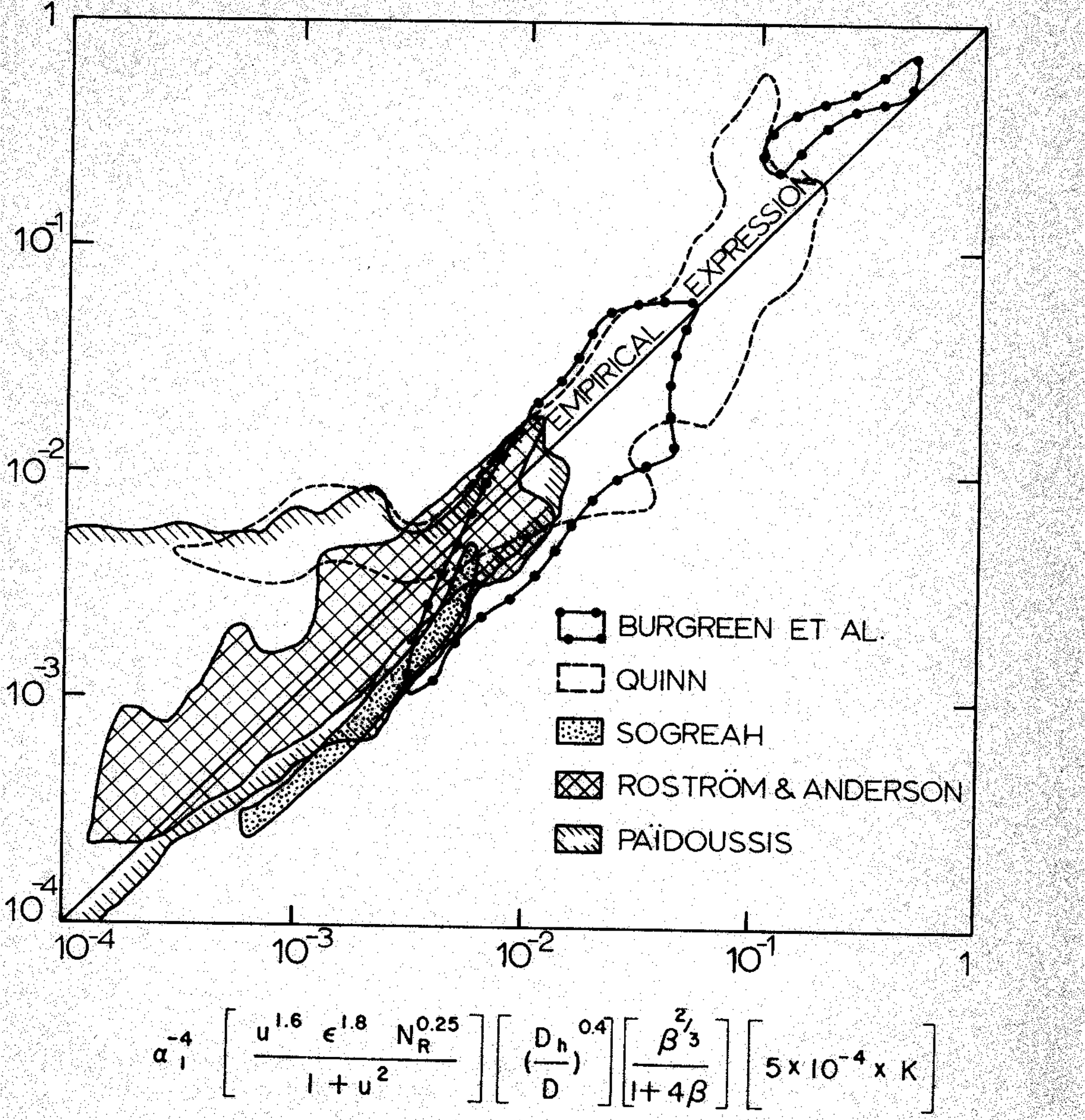


FIG. 7 Agreement between measured and predicted amplitudes of vibration according to Paidoussis' (1969) empirical expression.

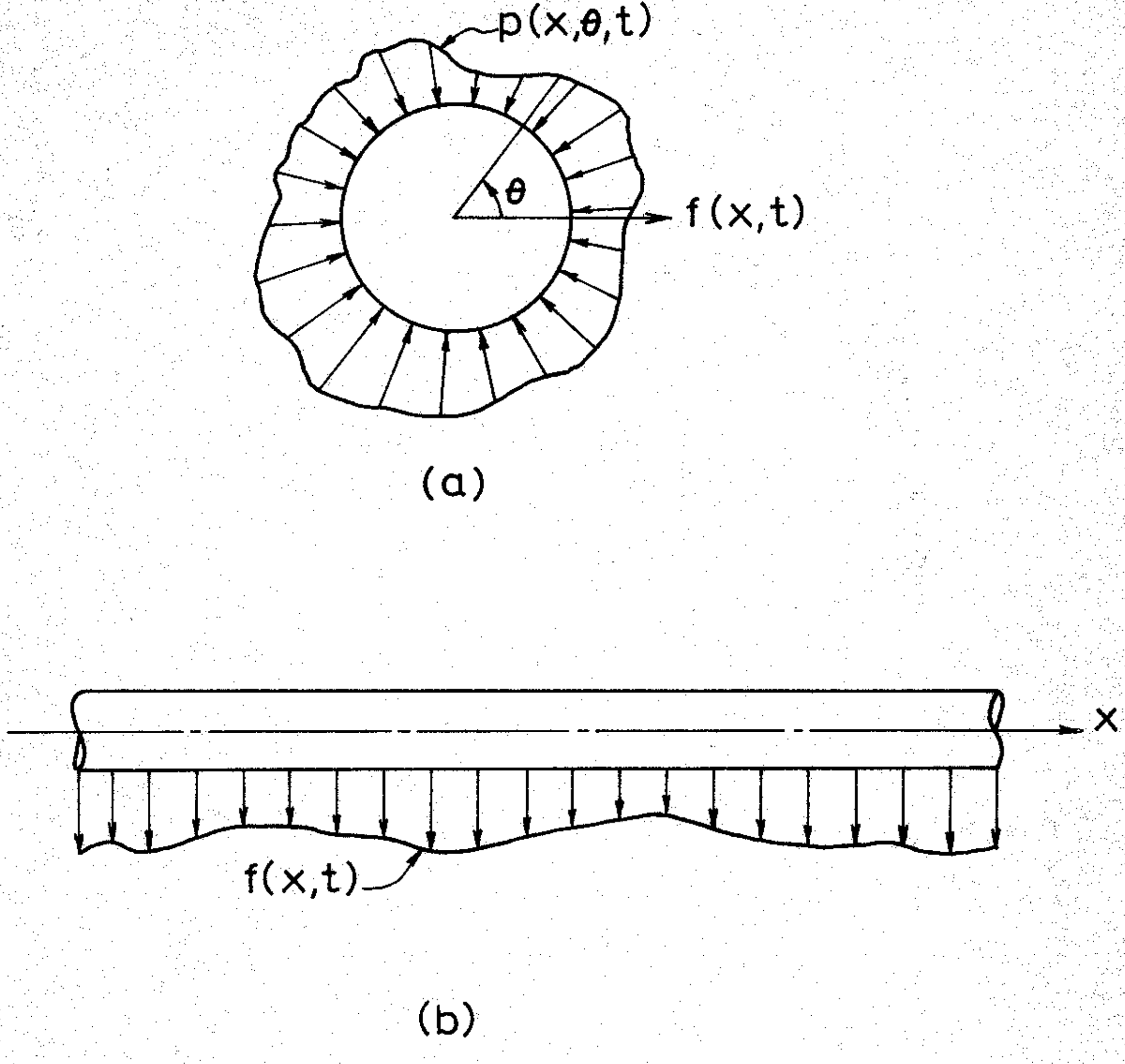
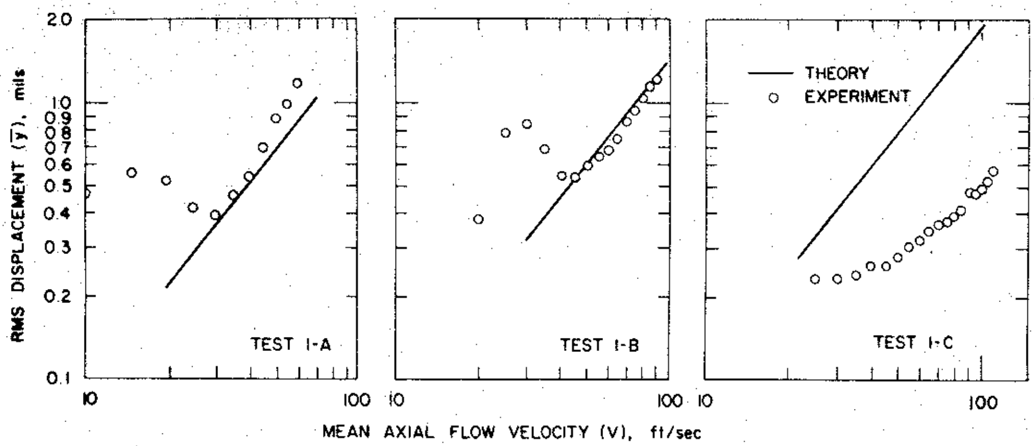
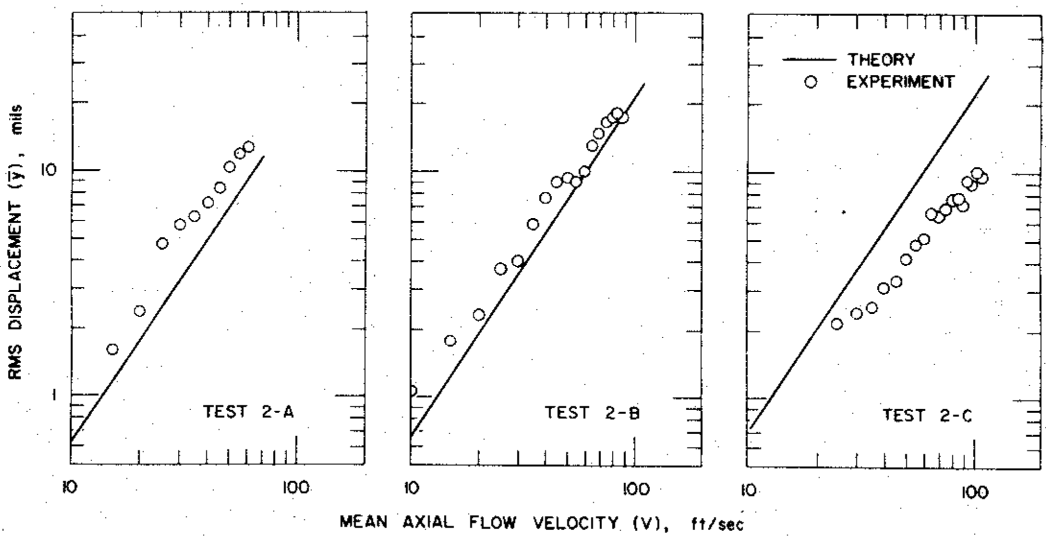


FIG. 8 Translation of random pressure field into a random force field on a cylinder.

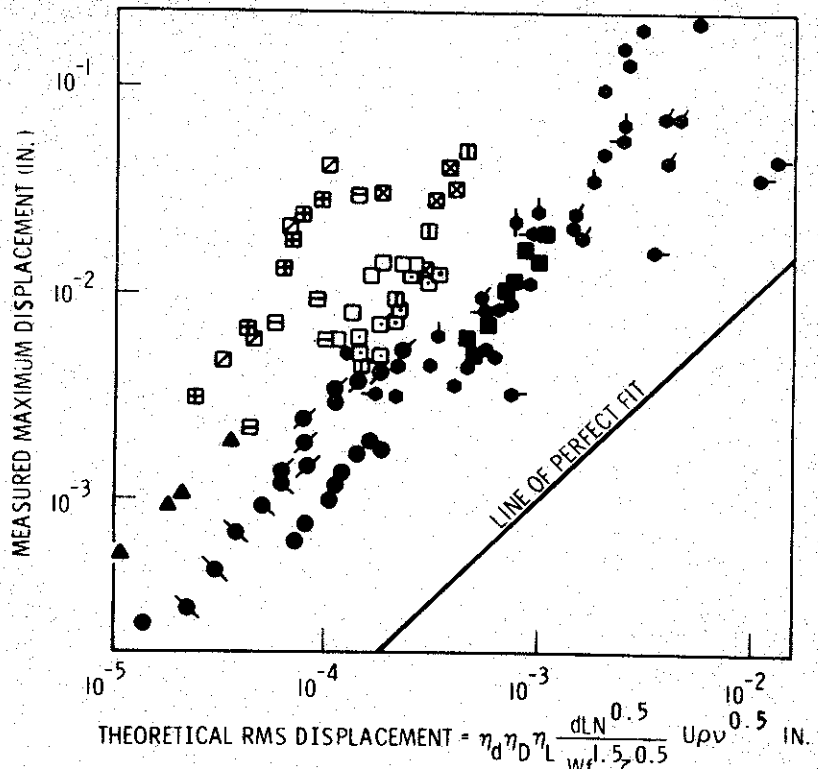


RMS displacement of fixed-fixed rods at $x = \frac{1}{2}l$.



RMS displacement of cantilevered rods at x = 2 ft.

FIG. 9 Agreement between Chen and Wambsganss' theoretical and experimental values of the amplitude of subcritical vibration [from Chen and Wambsganss(1972)].



B - BURGREEN; Q - QUINN S - SOGREAH P - PAVLICA

SYMBOL	▲	◫	⊞	⊟	⊠	⍁	⊡	□	■	●	●	●	●	●	●	●	●
INVESTIGATOR	S	B	B	B	B	B	B	B	B	P	P	P	Q	Q	Q	Q	Q
WATER TEMP., °F	70	70	70	70	70	70	70	70	70	70	70	70	500	200	93	80	80
NO. RODS IN BEAM	1	1	1	1	1	1	1	1	1	16	16	16	1	1	1	1	1
BEAM WT., LB	1.0	.25	1.5	1.5	.92	4.5	1.5	4.5	.25	57	57	57	3.3	3.3	3.3	3.3	3.3
BEAM NAT. FREQ., HZ	41	20	15	15	12	10	15	10	20	11	6	10	3.1	3.1	3.0	3.1	3.1
ROD DIA., IN.	.47	.62	.62	.62	.50	.62	.62	.62	.62	.44	.44	.44	.39	.39	.39	.39	.39
HYDRAULIC DIA. IN.	5.8	2.4	5.6	2.4	2.4	2.4	.85	.85	.85	.58	.58	.58	.86	.86	.24	1.2	.86
DAMPING RATIO X 10^3	10	12	12	12	12	12	12	12	12	18	18	18	13	13	13	13	13
BEAM LENGTH, IN.	45	48	48	48	48	48	48	48	48	72	72	72	74	74	74	74	74

FIG. 10 Agreement between Reavis' theory and others' experiments for the amplitude of sub-critical vibration [from Reavis (1969)]

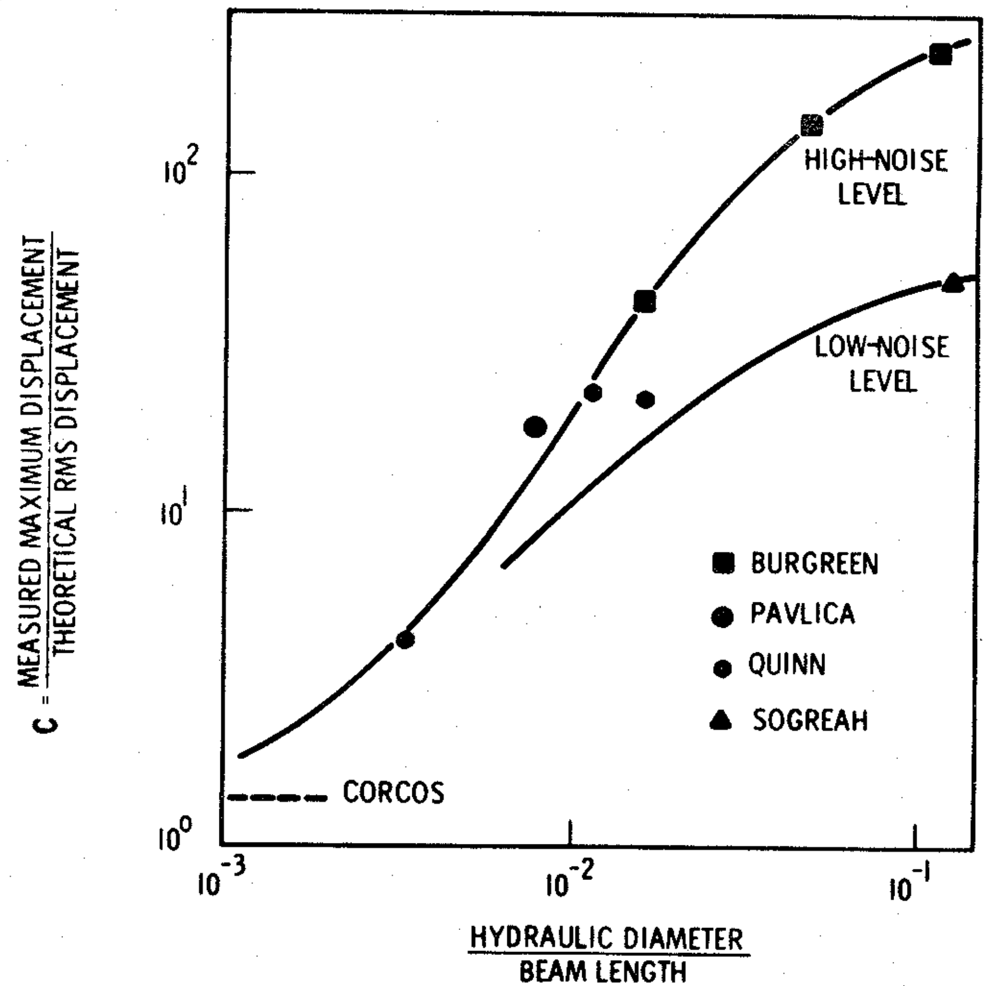
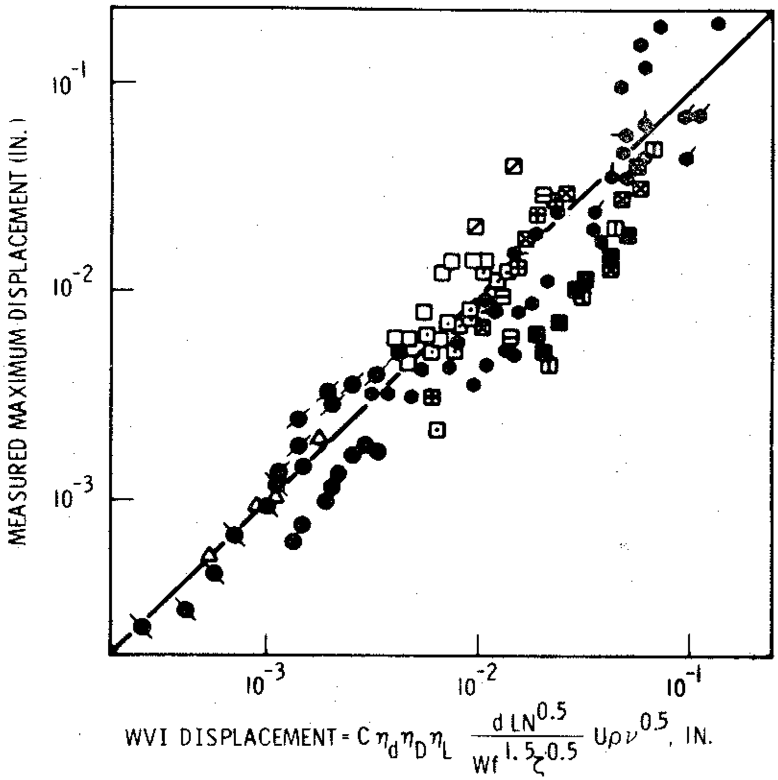


FIG. 11 Average disparity between theoretical and experimental amplitude of vibration defining the constant C [from Reavis (1969)].



B - BURGREEN; Q - QUINN S - SOGREAH P - PAVLICA

SYMBOL	△	◫	⊞	⊟	⊠	⍁	⊡	□	■	●	●	●	●	●	●	●	●
INVESTIGATOR	S	B	B	B	B	B	B	B	B	P	P	P	Q	Q	Q	Q	Q
WATER TEMP., °F	70	70	70	70	70	70	70	70	70	70	70	70	500	200	93	80	80
NO. RODS IN BEAM	1	1	1	1	1	1	1	1	1	16	16	16	1	1	1	1	1
BEAM WT., LB	1.0	.25	1.5	1.5	.92	4.5	1.5	4.5	.25	57	57	57	3.3	3.3	3.3	3.3	3.3
BEAM NAT. FREQ., HZ	41	20	15	15	12	10	15	10	20	11	6	10	3.1	3.1	3.0	3.1	3.1
ROD DIA., IN.	.47	.62	.62	.62	.50	.62	.62	.62	.62	.44	.44	.44	.39	.39	.39	.39	.39
HYDRAULIC DIA. IN.	5.8	2.4	5.6	2.4	2.4	2.4	.85	.85	.85	.58	.58	.58	.86	.86	.24	1.2	.86
DAMPING RATIO X 10^3	10	12	12	12	12	12	12	12	12	18	18	18	13	13	13	13	13
BEAM LENGTH, IN.	45	48	48	48	48	48	48	48	48	72	72	72	74	74	74	74	74

FIG. 12 Agreement between Reavis' semi-empirical expression (incorporating the constant C) and others' experiments [from Reavis (1969)].

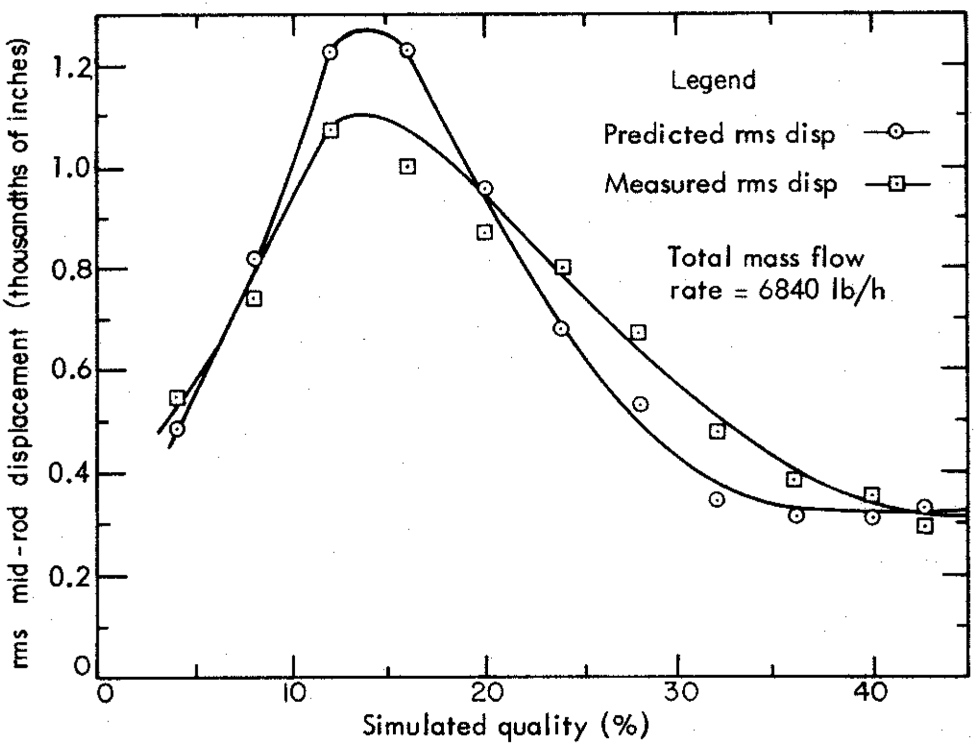


FIG. 13 Agreement between Gorman's theoretical and experimental values of the amplitude of vibration for flows of air-water mixtures [from Gorman (1969)].

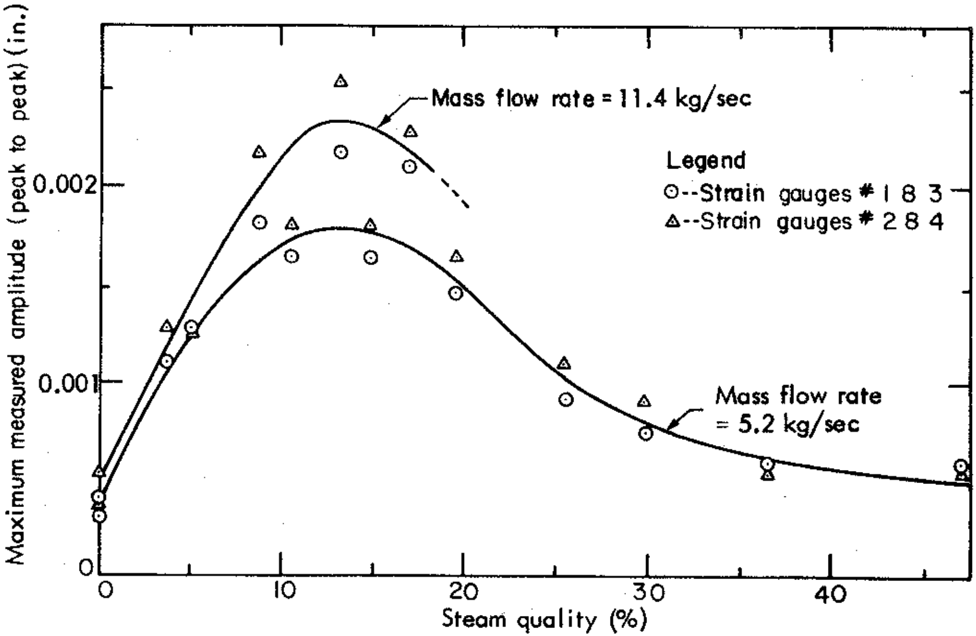


FIG. 14 Pettigrew's experimental values of amplitude of vibration for two-phase (steam-water) flow [from Gorman (1969]

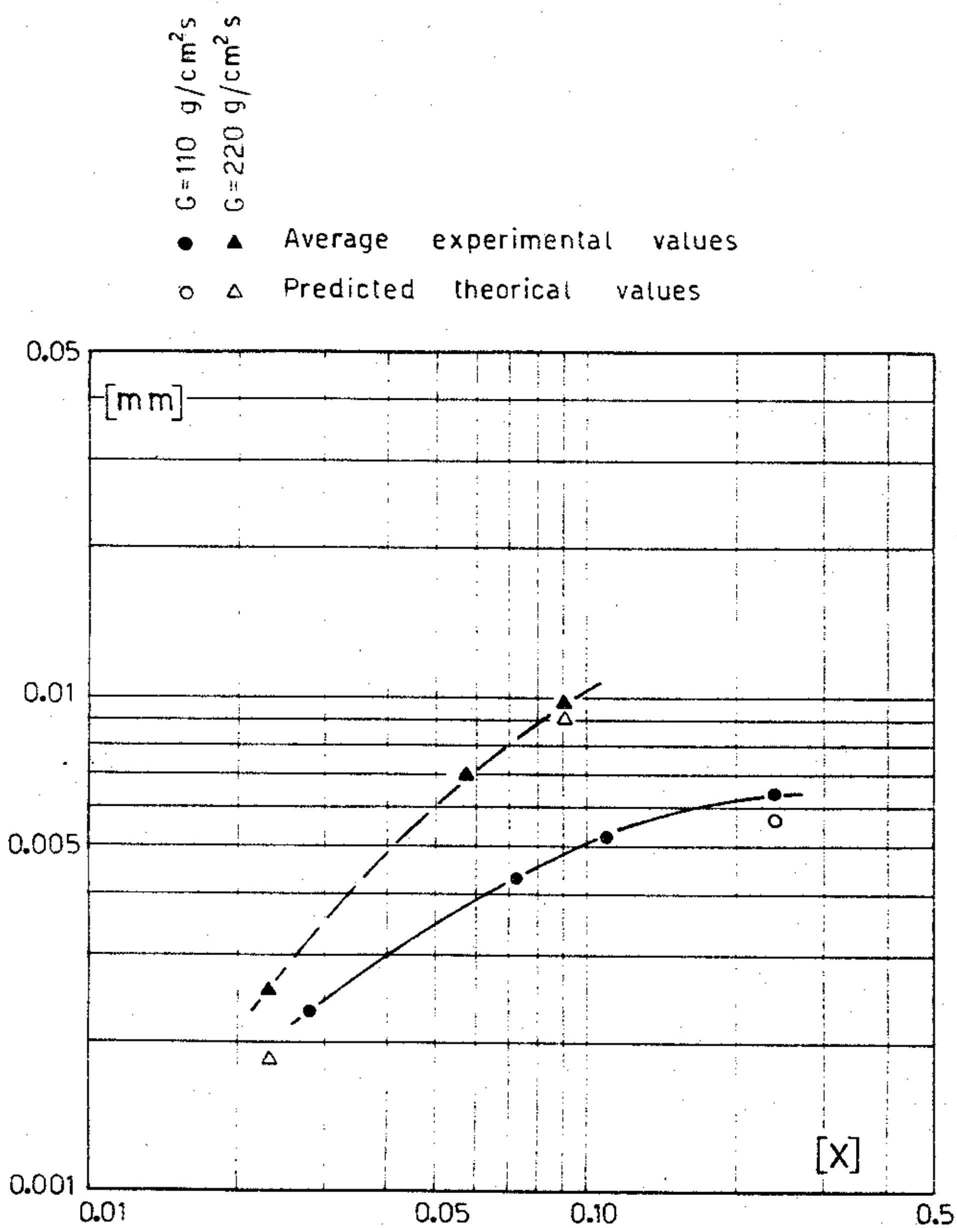


FIG. 15 Agreement between Cedolin *et al.* theoretical and experimental values of the amplitude of vibration for flows of nitrogen-water mixtures [from Cedolin *et al.*,(1971)]

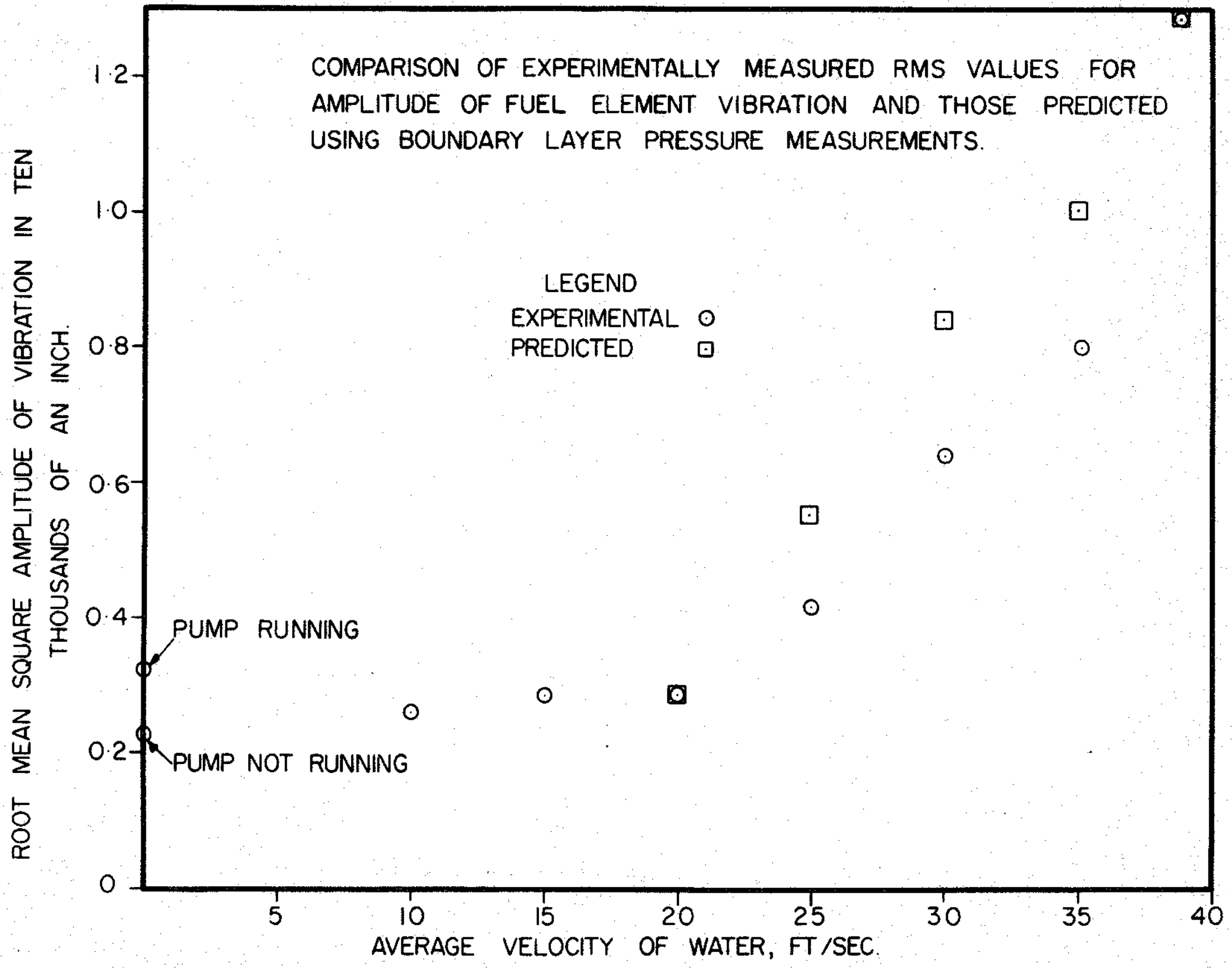


FIG. 16 Agreement between Gorman's theoretical and experimental amplitudes of vibration for water flow [from Gorman (1967)].

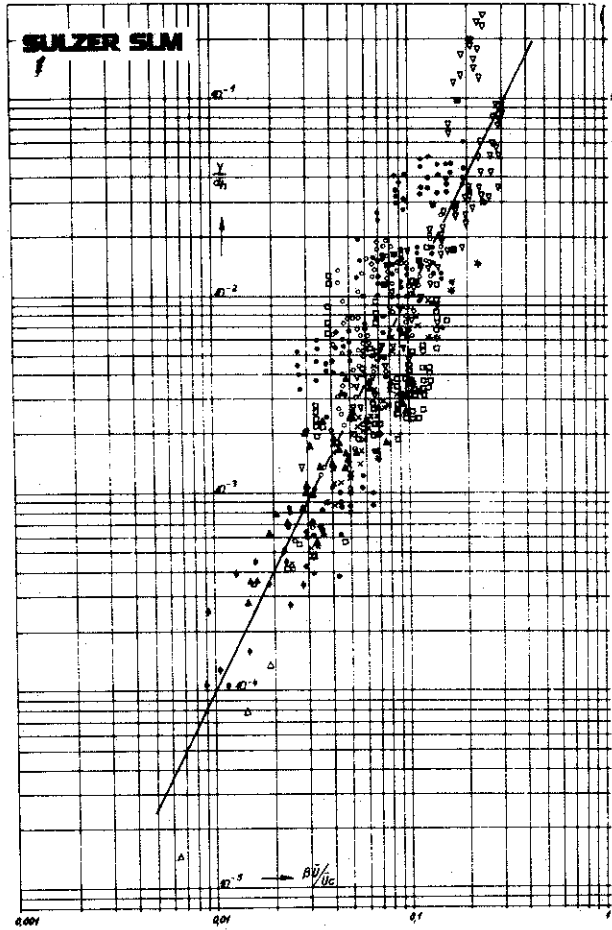


FIG. 17 Agreement between measured and predicted amplitudes of vibration according to Y.N. Chen's (1970b) model.