A RADIO INVESTIGATION OF AURORA

by

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ABSTRACT

Radar equipment operating at a frequency of 106.5 Mc/s was used to investigate polar aurora. Radio reflections were observed and their characteristics determined by means of directive aerial arrays, one of which was rotatable about a vertical axis. The reflections were found to arise from many small reflecting centres situated within the auroral structures, and located about 100 km above the surface of the Earth. A free electron density of at least 10^8 cm^{-3}, within the reflecting centres, was deduced. Auroral echoes were not observed at altitude angles greater than 7.5^o (altitude angles between 7.5^o and 13^o were not investigated). An explanation of this fact is suggested. The auroral echoes apparently arose only during the operation of the primary ionizing mechanism. The experimental evidence concerning the occurrence of auroral echoes during the daylight hours is inconclusive. Some of these results are incompatible with the views held by earlier workers.

FOREWORD AND ACKNOWLEDGMENTS

An important consideration in experimental investigations of polar aurora is the choice of a suitable location. The author was particularly fortunate in being permitted to do the experimental work at Saskatoon where the excellent facilities of the University of Saskatchewan were available. Dr. B.W. Currie of that University supervised the experimental work at Saskatoon during the summer months, and Professor G.A. Woonton supervised the work at McGill University, which consisted principally of the analysis and interpretation of the accumulated data.

The research progressed in three stages corresponding to experimental work done during the summers of 1949, 1950, and 1951, however, this chronological division is largely ignored in the organization of the thesis. Instead, the material is divided, somewhat arbitrarily, into three parts each concerned with one aspect of the reflection problem. These three parts correspond to Chapters III, IV and V. Chapter I contains a general introduction to the problem as well as a brief description of the research. Chapter II gives a theoretical discussion of various reflection mechanisms including the one which Herlofson considers to be responsible for the reflection of radio waves by auroral structures. Evidence supporting the auroral origin of the observed radio echoes is presented in Chapter III. The reflection process is examined from the experimental point of view in Chapter IV, where it is shown that the observed radio echoes are not explained adequately by Herlofson's theory. In Chapter V further experimental evidence is presented and the influence of atmospheric absorption on the reception of radio echoes is discussed. Also in Chapter V a qualitative picture of the distribution of ionization in an auroral structure is described and is shown to be consistent with the experimental observations.

In Chapter VI the results are summarized and their significance is indicated. Some suggestions concerning further work in the field are also made in Chapter VI.

The first Appendix is devoted to an elementary derivation of the various expressions which enter into the theoretical discussions. Some technical details of the equipment are given in the second Appendix.

The author wishes to acknowledge his indebtedness to those who contributed to the research. At McGill University, he benefited from the guidance provided by Professor G.A. Woonton and from association with various students working in the Eaton Laboratory. In particular, discussions with Mr. J.H. Chapman were helpful in clarifying the author's ideas concerning the upper atmosphere.

While at the University of Saskatchewan the author was fortunate to receive guidance from Dr. B.W. Currie, who gave generously of his extensive knowledge of auroral processes. Dr. Currie first suggested the research and obtained the equipment necessary for its prosecution. Dr. W. Petrie made many useful suggestions derived from his spectroscopic studies of aurora.

Mr. F.E. Vawter assisted with all the observations and undertook much of the routine maintenance of the equipment. Without this assistance it would have been necessary to confine the research to a much more modest program.

During the period of continuous operation mentioned in section 5.06, Messrs. G.R. Small and R.E. Jensen carried out the observations made during the daylight hours. Mr. C.H. Costain assisted with the observations during May and June, 1951.

The aircraft flights mentioned in section 4.04 were arranged through the cooperation of Wing Commander J. Baillie, Commanding Officer, 406 Squadron, R.C.A.F. (Reserve). Wing Commander Baillie took a personal interest in the research and piloted one of the aircraft.

The project was sponsored by the Defence Research Board of Canada. Valuable radar equipment (initially the property of the United States Air Force) was loaned to the Defence Research Board for the project, through the cooperation of the Geophysical Research Directorate, A.F. Cambridge Research Laboratories.

The author received financial assistance from the Defence Research Board during the summer months. The work done during the winter months was assisted by two National Research Council Scholarships, the author receiving a Studentship in 1949 and a Fellowship in 1950.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

CHAPTER I

Introduction

1.01 Early and contemporary experiments

Much of the present knowledge concerning auroral processes was derived from observation and analysis of the light emitted by the various auroral forms. Such investigations have yielded information about the height and frequency of occurrence of aurora as well as a large body of spectroscopic data, but it is still not known to what extent the upper atmosphere is ionized during auroral displays. The problem of auroral ionization is currently the subject of considerable interest, both because of its influence on high latitude communications and because it represents important evidence in any general theory of aurora. As yet spectroscopic studies of this problem are inconclusive. Radio observations of the polar ionosphere have shown that radio waves are highly absorbed in passing through the lower ionosphere during auroral periods (64). These observations are interpreted as indicating an increase of ionization at such times. The radio observations of Appleton, Naismith and Ingram (5) may include instances involving reflection of the radio waves from auroral structures but the first specific investigation of this effect seems to be that of Harang and Stoffregan which was started at Tromso in 1938 (48). They used equipment operating at a frequency of 40 Mc/s and observed echoes of small amplitude which they associated with auroral displays.

In 1947 radio echoes associated with aurora were observed by Pierce who used ionospheric equipment operating at a frequency of 3.5 Mc/s (73), and by Lovell, Clegg and Ellyett who used meteor detection equipment operating at a frequency of 46 Mc/s (55). The latter authors interpreted their measurements as indicating that the free electron density within the auroral structure was about 3×10^7 cm^{-3}; however, Herlofson pointed out that their observations could have resulted from a much lower electron density (49). In 1950 Aspinall and Hawkins used Herlofson's method to calculate the electron density from data provided by Lovell, Clegg and Ellyett and also from their own results which they obtained using equipment operating at a frequency of 72 Mc/s (6). They found that the electron density within an auroral structure was approximately 7×10^5 cm^{-3}, or slightly higher than that which exists in the normal E-region during the day. Radio reflections associated with aurora were also observed by McKinley and Millman in the course of meteor studies (56). Their equipment operates at a frequency of 32.7 Mc/s.

Gerson has analysed the observations made on the night of November 19, 1949 by a number of radio amateurs and concluded that the signals received at a frequency of 50 Mc/s were reflected by auroral structures (42). Observations similar to those of Pierce were made by Davidson, who used equipment operating on frequencies of 3.4 Mc/s and 17.3 Mc/s (31).

The highest frequency at which auroral reflections have been reported is 106.5 Mc/s. Observations at this frequency were first made by the author in 1949 (35).

1.02 Nature of the problem

The present research is an outgrowth of an earlier radio investigation of aurora in which the author participated. This earlier work was started in 1948 and made use of equipment operating at a frequency of 3,000 Mc/s. No reflections were observed which could be associated with aurora but short bursts of electromagnetic energy at this frequency were received. The energy was emitted apparently by particular auroral structures, notably by active forms exhibiting much structure (36). Covington has measured the radiation received from the sky by a receiver tuned to 3,000 Mc/s. He recently reported an increase of such radiation during auroral periods (27). His results would seem to offer a partial confirmation of the earlier observations. The phenomenon suggests the presence of small volumes of intense ionization (as high as 10^{11} free electrons per cubic centimetre) within the auroral structures (37). Some support for this interpretation is found in the calculations of Barbier (7), but there remains a large discrepancy between the experimental results obtained in Great Britain and those obtained in Canada.

1.03 Object of the research

This thesis is concerned with a systematic investigation of radio reflections associated with aurora. Several new techniques were employed in an attempt to gain information concerning the reflection mechanism and thereby concerning auroral electron densities. With this object in view a number of separate experiments were conducted, each designed to provide some additional information about the reflection process. The choice of a frequency above 100 Mc/s had a twofold purpose: firstly, the observation of radio reflections at so high a frequency had not been reported previously; secondly, directive aerial systems were considered desirable, and these are easier to build for the higher frequencies.

1.04 Subsidiary results

In the course of the experiments it became apparent that the techniques employed were well suited to the general study of polar aurora. However, this demands that the individual experiments be continued over long periods of time. No effort was made to accumulate sufficient data for statistical purposes. The capabilities of various techniques are demonstrated by examples drawn from the data collected by the author. He wishes to make it clear that measurements of the occurrence, duration and movement of auroral structures, contained in this thesis, are only intended to illustrate the capabilities of the radio method. No claim is made that the results are statistically reliable. The only justification for their inclusion is the evident advantages offered by the radio techniques over those previously employed.

1.05 Description of the research

The research made use of radar equipment operating at a frequency of 106.5 Mc/s. A short pulse of electromagnetic energy was sent out from the transmitter, and the time taken for the energy to reach the auroral display and return to the receiver was taken as a measure of the range to the reflecting surface. The use of directive aerial systems with equipment permitted the further localization of the reflecting surface.

Photographic methods were used to record the amplitude, range, direction and duration of the radio echoes. Direct photography of the visible aurora was also used to provide a record of the location and type of aurora present. One of the aerial systems was rotatable about a vertical axis so that records could be made of the echoes returned from different parts of an auroral display or, alternatively, continuous records could be made of the echoes returned from a given direction.

The extensive use of photographic recording was dictated by the nature of the experiments. The occurrence and duration of an auroral display are not readily predicted. In addition, some of the experiments could only be performed when the sky was clear and the aurora in a particular location. For these reasons, it was necessary to observe almost continuously during the dark hours. However, only a small fraction of the time spent in observation (less than

1/500th for most of the experiments) was spent in the actual taking of records. It was expedient, therefore, to record photographically as much information as possible during the auroral displays, leaving the analysis and interpretation of the data to a later time.

The conclusions drawn in the thesis are based upon a considerably larger volume of data than has hitherto been available. The versatility of the equipment and the geographical location of the project have been important factors in determining the amount of data collected.

1.06 Location of the project

The experimental part of the research project was carried out at Saskatoon (52.1° N; 106.6° W). Aside from the excellent facilities for auroral studies available at the University of Saskatchewan, the geographical location was found singularly appropriate. The auroral zone (the zone of maximum frequency of auroral occurrence) lies some 800 km north of Saskatoon, the nearest point lying in a direction somewhat east of north. While overhead auroral displays occur frequently at Saskatoon, by far the greatest number of visible displays occur near the northern horizon at a range of from 600 to 1000 km. These displays were well suited to radio investigations of the type described in this thesis.

1.07 Auroral designations

In describing the various auroral forms different authors have resorted to different classifications. In the following pages the designations of auroral forms (e.g. arc, band, ray arc, ray band, isolated ray, etc.) correspond to those used in the Photographic Atlas of Auroral Forms, International Geodetic and Geophysical Union, 1930. In addition, it sometimes will be necessary to describe the aurora as "active" or "exhibiting structure". The latter description will include all those forms, such as ray arcs, ray bands, draperies, etc., which exhibit vertical discontinuities as observed visually. The term "active" will include all forms in which rapid movement is visually observable.

CHAPTER II

Reflection from an Ionized Medium

2.01 Introduction

When a radio wave passes from free space into a region containing free electrons part or all of the energy is reflected or scattered. The amount of energy which is returned along the path of the incident wave is determined by the following factors:

(i) The power density in the incident wave;

(ii) The frequency of the incident wave;

(iii) The density of free electrons;

(iv) The electron density gradient at the boundary of the ionized region;

(v) The shape of the ionized region;

(vi) The size of the ionized region;

(vii) The absorption loss, i.e., the amount of power lost within the ionized region as a result of collisions between the free electrons and heavier particles.

The purpose of this chapter is to review the influence of these parameters. Four separate processes are discussed, each corresponding to a different range of the parameters. In Chapter IV the experimental evidence will be examined with a view to deciding which of the processes best describes the production of auroral radio echoes. The influence of (iv), (v), (vi) and (vii) will be discussed again in more detail in Chapters IV and V.

While these four processes are discussed separately in the following sections, it should be borne in mind that more than one of them may operate simultaneously in the same part or different parts of the ionized region.

The discussion makes use of two reflection coefficients. The first, R_a, is the "optical" reflection coefficient which represents the ratio of the amplitude of the reflected wave to that of the incident wave. The second, R_p, is the power reflection coefficient which represents the corresponding ratio of power densities in the reflected and incident waves.

2.02 "Complete" reflection

It is shown in Appendix I that for a given wave frequency, f, there is a critical free electron density, N_o, given by, (Equation VI, Appendix I),

$$N_o = 1.24 \times 10^{-8} f^2.$$

If the electron density, N (cm^{-3}), rises above this critical value, the wave can not penetrate the ionized region. If the absorption loss is small (see section 5.02), the apparent reflection coefficient of such a region is unity and the process is independent of the electron density gradient between the ionized and non-ionized regions.

2.03 Partial reflection at a boundary

Let us now consider an ionized region in which the electron density is considerably less than N_o. The index of refraction of such a region is given by, (Equation VII, Appendix I),

$$n = 1 - \frac{Ne^2}{2\pi mf^2}\left(\frac{\omega^2}{\omega^2 + f_c{}^2}\right)$$

where the expression in brackets represents the effect of electron collisions.

We must now consider the influence of the electron density gradient at the boundary. First, let us assume that the transition region between the ionized and non-ionized regions is many wavelengths thick and that the electron density gradient is nowhere a rapidly varying function. Then, as the wave penetrates the region the change of index of refraction which it experiences per wavelength of travel is small compared with the total change. This situation is analogous to a tapered junction between two transmission lines of different characteristic impedances. There is no appreciable reflection of energy and the reflection coefficient is effectively zero. Now, let us gradually reduce the thickness of the transition region. Again, by analogy to the case of transmission lines, appreciable reflection of energy begins to occur when the thickness is reduced to the order of a wavelength. As the thickness is further reduced to values small compared with the wavelength, the reflected energy reaches a limiting value. This situation is similar to that encountered in optics when reflection occurs at the interface between two media of differing indices of refraction. The appropriate optical reflection coefficient has been derived from electromagnetic theory by Slater and Frank (79). It is,

$$R_a = \frac{1 - n}{1 + n} .$$

Neglecting the absorption loss and remembering that we assumed N small compared with N_o, i.e., n approximately equal to unity, then we may write,

$$R_a \approx \frac{Ne^2}{4\pi mf^2} .$$

The power reflection coefficient, R_p, is just

$$R_p = R_a^{\,2}.$$

Equivalent expressions have been developed by Herlofson (49).

2.04 Electron scattering from a small cluster of electrons

When the electron density in the ionized region is so low that the index of refraction differs very little from unity then each electron may scatter energy independently. This problem has been treated by Lovell and Clegg in connection with radio reflections from meteor trails (54).

The scattering cross-section of a single free electron may be deduced from classical scattering theory (see, for example, Slater and Frank (80)). Since the frequency of the incident radiation is much higher than the natural frequency of oscillation of the scatterer (in this case the natural frequency is zero since the electron is free), the appropriate cross-section is the Thomson scattering cross-section, namely,

$$\frac{8\pi}{3} \left(\frac{e^2}{mc^2}\right)^2 \text{cm}^2.$$

Lovell and Clegg have shown that this cross-section may be modified to include the effect of electron collisions by multiplying it by the factor,

$$\frac{\omega^2}{\omega^2 + f_c^{\ 2}} \cdot$$

We may define a power scattering coefficient, S_p, such that it represents the ratio of the total power scattered by the electrons encountered by one cm^2 of the wave front to that in the original wave. For a small cluster of electrons where the dimension of the cluster in the direction of wave travel is small compared with the wave-length, all the electrons move in unison. Since we are interested only in radiation returned along the incident path we may treat the cluster of electrons as a coherent source and add the amplitudes of the waves scattered by individual electrons. Thus the power scattering coefficient becomes

$$S_p = \frac{8\pi}{3} \left(\frac{e^2}{mc^2}\right)^2 N_t^{\ 2} ,$$

where N_t is the total number of electrons encountered by one cm^2 of the wavefront.

Lovell and Clegg have also pointed out that the electrons do not scatter uniformly in all directions but rather as Hertzian dipoles so that the power scattered back along the incident path will be 1.5 times as great as that calculated for a uniform scatterer.

2.05 Electron scattering from a large cluster of electrons

The treatment of scattering from a large cluster of electrons is similar to that given in the preceding section except for the following consideration. If the dimension of the cluster in the direction of wave travel is large compared with the wavelength, the electrons do not vibrate in unison and they cannot be considered as coherent scatterers. In this instance, it is necessary to add the intensities of the scattered waves and the power scattering coefficient becomes

$$S_p = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 Nt \,.$$

2.06 Reflection from a plane surface

In the preceding sections we have been concerned with the evaluation of reflection and scattering coefficients. We now consider the utilization of these coefficients in the calculation of the amount of power returned to a receiver (coincident with the transmitter) by reflection from an ionized region.

The auroral echo received by Lovell, Clegg and Ellyett (55) was interpreted by them as being due to the process outlined in section 2.02. This process normally operates in the ionosphere and permits the measurement of critical reflection frequencies and, hence, the calculation of electron densities. The above-mentioned authors assumed that reflection took place at a large plane surface at vertical incidence in the same manner as ionospheric reflections. The critical electron density, N_0, is 2.6×10^7 cm^{-3} for the frequency which they used. The calculation of the returned power in such a situation is comparatively simple. The only condition under which appreciable power will be reflected to the receiver is that of normal incidence on the plane (specular reflection). For this condition the received power is calculated simply from the inverse square law. If P_T is the transmitted power, the power density at any distance, r, is given by

$$\frac{P_T G_T}{4\pi r^2} \quad \text{watt cm}^{-2},$$

where G_T is the aerial gain which is defined as the ratio of the power radiated in the direction concerned to that which would be radiated in the same direction by an isotropic radiator. If the effective area of the receiving aerial is A_R, the power received is,

$$P_R = \frac{P_T G_T A_R}{4\pi(2d)^2},$$

where d is the range of the reflecting surface. Friis and Lewis have shown from reciprocity considerations that the following simple relation, between gain and effective area, holds for any aerial (39);

$$G = \frac{4\pi A}{\lambda^2},$$

where λ is the wavelength.

Herlofson (49) made these calculations for the echo received by Lovell, Clegg and Ellyett and found that the ratio between the received power and that given by the equation derived above was 5×10^{-7}. He therefore assumed that this figure represented the reflection coefficient. By further assuming that the boundary of the ionized region was sharp he was able to calculate the electron density, N, which he found to be 4×10^4 cm^{-3}. This value is considerably lower than that found in the normal E-region during the day. However, he pointed out that diffuseness in the boundary of the order of a wavelength would cause the electron density to be underestimated.

2.07 Reflection from a cylindrical surface

In order to interpret the echoes which they observed, Aspinall and Hawkins (6) classified them as "diffuse" or "discrete". The latter type were associated with auroral rays and these were used in the calculation of electron densities. They assumed that reflections occurred at the surface of an ionized region in the form of a homogeneous cylinder with sharp boundaries. From the visual appearance of rays, they estimated the radius to be 1 km.

The power reflected from a large cylinder may be calculated from ray theory in the following simple manner. Since no appreciable power will be reflected from the cylinder unless the condition for specular reflection is fulfilled, i.e., the direction of incidence perpendicular to the axis of the cylinder, let us assume a system of rectangular coordinates with the transmitting-receiving aerial located at the origin and the cylinder located a distance, d, along the y axis with its axis parallel to the z axis. We may imagine a square of unit cross-section parallel to the x-z plane and situated a unit distance from the origin along the y axis. The power density on this square will be $P_T/4\pi$. If we project rays passing through this square back to the x-z plane after reflection by the cylinder, we find a rectangle with x dimension

$$\frac{2d(a + d)}{a}$$

and y dimension 2d, where a is the radius of the cylinder. The power density at the receiver is found directly from the ratio of the area of the square to that of the projected rectangle. Using the symbols of the previous section, the power received is given by,

$$P_R = \frac{R_p P_T G_T A_R}{4\pi(2d)^2} \left(\frac{a}{(a + d)}\right),$$

where the reflection coefficient, R_p, has been included. Aspinall and Hawkins, using an equivalent expression given by Smith (81) to calculate the reflection coefficient, found an electron density in the aurora of 6×10^5 cm^{-3}. Upon recalculating the data of Lovell, Clegg and Ellyett, according to these assumptions, they found a value of 9×10^5 cm^{-3}. These values are slightly in excess of the electron density in the normal daytime E-region, and approximately equal to that in the normal night-time F-region.

2.08 Reflection from a small target

In this section we consider the power returned from a small, roughly spherical reflector. If this power is to be detectable, and if the size of the sphere is to be no larger than reasonably may be associated with an auroral structure (a diameter of less than, say, a kilometer), the reflection coefficient at the surface of the sphere must be of the order of unity. Such a sphere may be assumed to scatter the incident power uniformly in all directions (cf. Burrows and Attwood (13)). This problem is treated by first finding the power density at the target. The reflected power density at the receiver may be found by assuming that the power intercepted by the target is scattered uniformly in all directions. This results in the "radar equation", one form of which is as follows:

$$P_R = \frac{P_T G_T A_R K A_S}{16\pi^2 d^4},$$

where A_S is the projected area of the target and K is a constant, often called the "target gain", introduced to allow for deviation from the ideal isotropic scatterer. (K = 1 for an isotropic scatterer).

If many such scatterers are present, we must add intensities of the scattered waves. Thus, the apparent target area will be just the sum of the individual target areas.

2.09 Electron scattering in an extended region

The power received from a small element of an extended ionized region by the process of section 2.04 may be calculated by the method of the preceding section. This power may be converted into the corresponding wave amplitude or receiver voltage amplitude by the usual methods. The contributions of all the elements must then be combined by the use of the appropriate form of Fresnel's integral. This has been done for the particular case of a long thin cylinder by Lovell and Clegg (54). Since their development is concerned specifically with meteor echoes, no attempt will be made to summarize it here; however, it is perhaps necessary to point out that reflections arising in this manner are extremely sensitive to the angle of incidence. The condition for specular reflection must be fulfilled, if echoes of appreciable amplitude are to be observed.

The calculation of power scattered by the process of section 2.05 involves the evaluation of the total power scattering cross-section for the ionized region. The expression given in section 2.08 may then be used to calculate the power received. However, when radar techniques are employed to study such a region, only a finite volume of the region may contribute to the scattered power at any instant in time. In order that the energies scattered from different parts of the region be additive they must arrive at the receiver simultaneously. If the energy is transmitted for a time, t, then the waves scattered by two electrons situated at distances r_1 and r_2 may combine only if

$$|r_1 - r_2| < \frac{ct}{2} .$$

Since radar transmitters are designed to transmit energy for only a few microseconds at a time, very sensitive equipment would be necessary to detect the small amount of power scattered by this process.

CHAPTER III

The Auroral Origin of the Radio Echoes

3.01 Introduction

Radio echoes occurring simultaneously with aurora were reported by a number of workers (see section 1.01); however, a close association between the two phenomena has not been demonstrated previously. The lack of positive evidence linking the radio reflections to the visible aurora has caused some confusion regarding auroral ionization. Many casual observations made by radio amateurs seem to indicate radio reflections from visible aurora but it is probable that many of the instances reported represent reflection from clouds of "Sporadic E" ionization (cf. reference (85)). The following sections of this chapter are devoted to an examination of the experimental evidence with a view to determining whether or not the radio echoes have their origin in the visible aurora.

3.02 Characteristics of the radar equipment

The principal characteristics of the radar equipment used for this experiment may be summarized as follows:

Transmitter:

Frequency....................106.5 Mc/s (λ = 2.8 metres)

Peak power...................variable, usually about 10 kw

Pulse length.................30 microseconds

Pulse recurrence frequency..125 per second

Aerial array:

Height......................2λ

Width.......................4λ

Height of centre............3λ above surrounding terrain

Radiation pattern:

Horizontal width..........$\sim 15^{\circ}$

Elevation of primary lobe...maximum at 4.8° above horizontal

Field of view...............clear view of horizon except for the sector between 080° and 240° (true bearings)

Most of the energy radiated was contained in the primary lobe, so that radar observation was effectively limited to an angle of less than 10° above the horizon. The region in space illuminated by the radiated energy is shown schematically in Figure 3.01. The centre of the radar beam intersected the auroral region between 600 and 1000 km from the radar station, so that true auroral echoes were expected to occur predominantly between these ranges. The aerial array rotated about a vertical axis and could be caused to sweep continuously between predetermined azimuth bearings.

3.03 The radar displays

The radar echoes were observed simultaneously on two oscilloscopes. In each case the time base represented a total range of 400 km but the start of the time base could be delayed by various fixed intervals after the occurrence

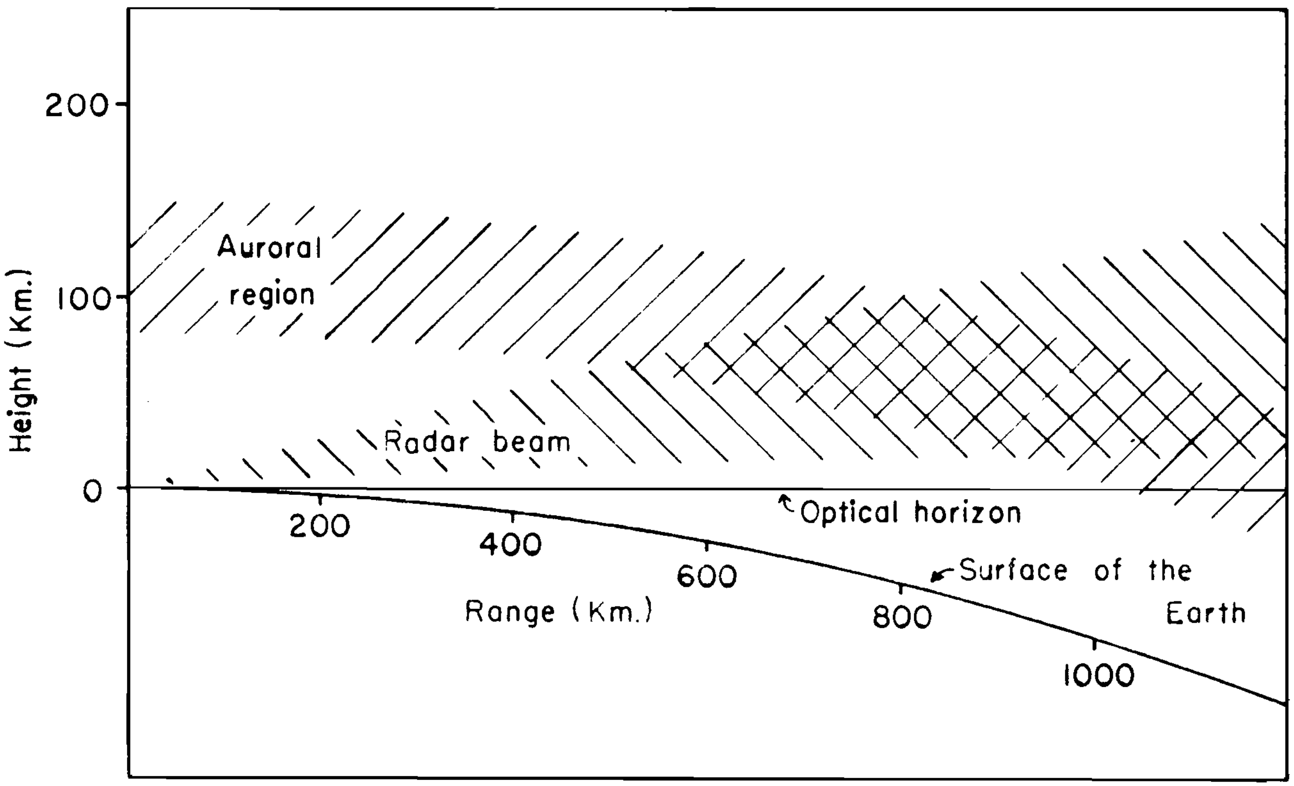


Figure 3.01 - Schematic diagram showing vertical section through the auroral region and the radar beam.

of the transmitter pulse. A switching arrangement provided delay times corresponding to ranges of 0, 300, 600 and 900 kilometres. In practice, auroral echoes were observed only on the last two range scales (600 - 1000 km and 900 - 1300 km). The succeeding transmitter pulse appeared on the last time base at an apparent range of 1200 km so that there was an ambiguity in range between 0 to 100 km and 1200 to 1300 km, but no echoes were observed at these ranges.

One of the displays was operated in the form of a conventional range presentation. This type of presentation corresponds to a graph drawn with reference to a rectangular set of coordinates. The base line consists of a horizontal line and distance along this line is proportional to range. The echoes are presented as vertical deflections from the base line, the amplitude of deflection being proportional to the amplitude of the received signal.

The second display was a modification of the conventional Plan Position Indicator (P.P.I.). In this presentation the trace is a faint radial line originating at the centre of the face of the cathode ray tube. Radial distance from the centre corresponds to range. The trace rotates about the centre of the tube face in synchronism with the aerial array and is intensity modulated, i.e., an echo produces a bright spot; its position on the face of the cathode ray tube corresponding to the range and bearing of the reflecting object. The conventional form of

P.P.I. display presents a map or plan view of the reflecting objects within range of the radar equipment, but when it is used with a delayed time base, (as in the present instance), the configuration of extended echoes is badly distorted. Each of the presentations was displayed on a 12-inch cathode ray tube. Technical details of the circuits employed to produce these displays are given in Appendix II.

3.04 Auroral photography

In order to provide a record of the aurora, the sky was photographed by a remotely controlled camera. This camera had a lens of aperture f/1.25 and by using the faster commercial films good photographs of the brighter auroral features could be obtained in a few seconds. However, exposure times as long as 36 seconds were often necessary to record the distant aurora because of a low-lying atmospheric haze. The camera was mounted on a remotely controlled turn-table and fixed at a small elevation angle so that the region covered by the radar beam would fall near the centre of its field of view. From the radar operating position the auroral camera could be rotated to any azimuth position, the shutter opened for a predetermined interval and the film advanced. Further details of the camera are given in Appendix II.

3.05 The recording procedure

The radar records were taken in the form of photographs of each of the radar displays. The procedure may be summarized as follows: The radar aerial array was caused to sweep continuously through a chosen angle, usually about 120° centred on magnetic north. When echoes were observed the P.P.I. camera was exposed for one sweep of the aerial array. At the same time the auroral camera was rotated to an appropriate azimuth bearing and exposed. As the rotating P.P.I. trace swept through the echoes it was momentarily brightened by a manually operated control and at the same time a photograph was taken of the range display. The resultant bright line on the P.P.I. photograph indicated the bearing corresponding to the range display photograph. This procedure resulted in three photographs, two of the radar displays and one of the aurora, all taken as nearly as possible at the same time. Sets of photographs were taken as often as possible while the echoes persisted. This usually resulted in one set of photographs every two minutes but occasionally rates as high as one set every minute were obtained for short periods.

This experiment was one of the least productive of records. Even on apparently clear nights it was found that the presence of a slight low-lying haze interfered with auroral photography. In addition many echoes were observed which only lasted for a fraction of a minute and these echoes could not be recorded using the procedure outlined above. The latter limitation is due to the nature of the experiment and does not represent a true limitation of the radar method for auroral investigation. Transient echoes may be recorded and, in fact, were recorded in the course of other experiments (see Chapter V). Nevertheless, some 70 complete sets of photographs were secured and these proved adequate for the intended purpose. A typical set of photographs is reproduced in Figure 3.02.

3.06 Analysis of the records

In order to establish a positional correspondence between the visible aurora and the point of origin of the radar echoes it was assumed that the lower edge of each display was located between 80 and 130 km above the surface of the Earth and further that at any one time this height was nearly constant along the length of the display. Both these assumptions are in accord with the known characteristics of auroral displays. It was thus possible to ascertain the approximate range to any part of the auroral display by measuring the angle between the horizon and the corresponding part of the lower edge. Even though the uncertainty as to the precise height which should have been used in each case led to a relatively large uncertainty in range, it was possible to make use of the second assumption (constant height) to determine the configuration of the auroral display as projected on the Earth's surface. This configuration could then be compared with that obtained from the radar echoes.

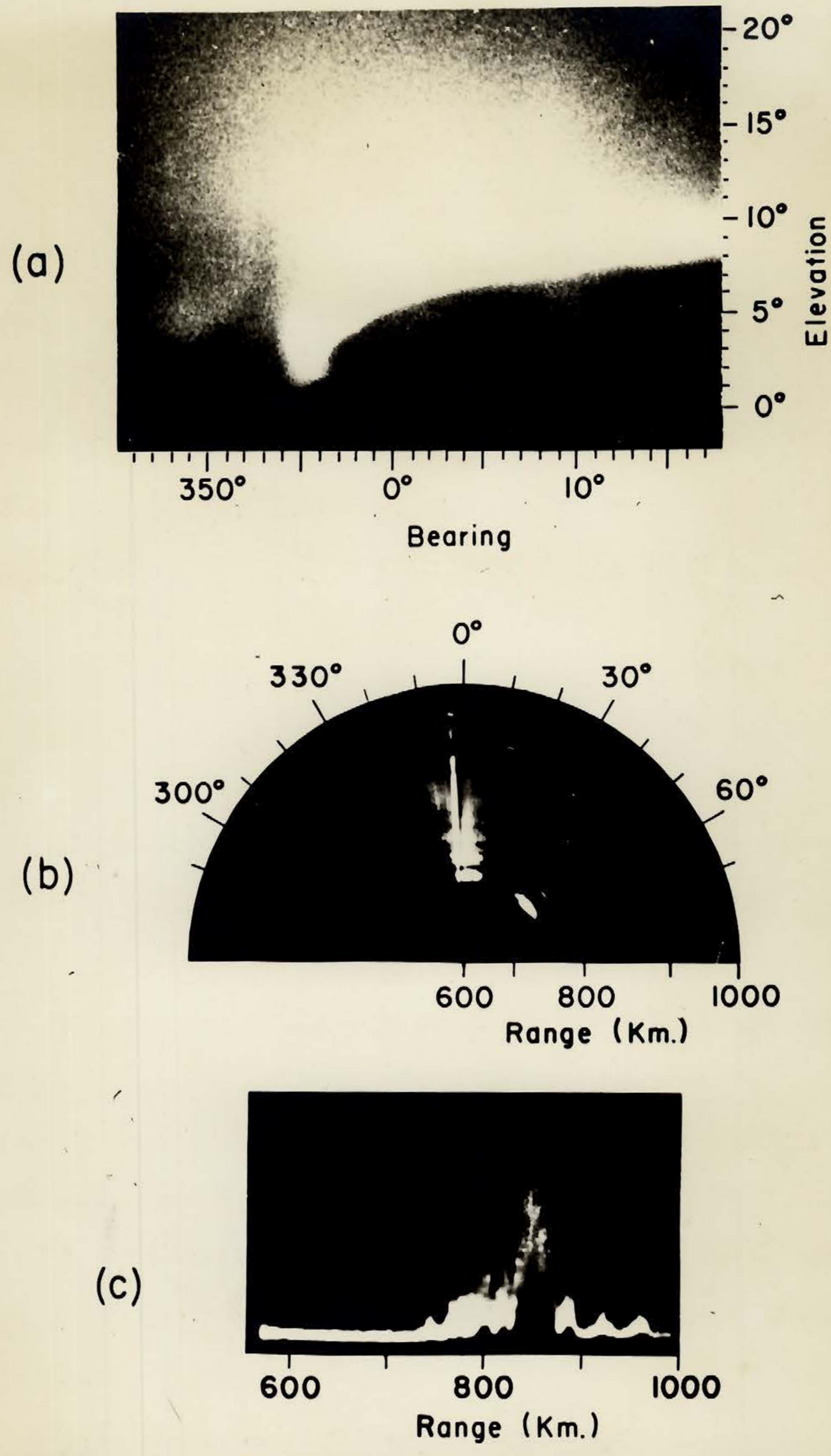


Figure 3.02 - (a) Auroral photograph, (b) P.P.I. photograph, (c) Range display photograph, all taken at 22 hrs, 26 min on the night of August 15, 1949.

Altitude angles corresponding to various parts of the auroral display were measured directly from the enlarged auroral photographs. For this purpose the auroral camera was calibrated by photographing various stars while they were in known positions. The auroral range was calculated for each degree in azimuth for assumed auroral heights of 80, 90, 100 and 120 km. These ranges were then plotted for comparison with the radar echoes. Except for the initial determination of the altitude angles the method used is essentially that used by Currie and Jones to find the directional characteristics of auroral displays (29). The simplified method used to obtain the altitude angles in the present instance is only valid for small angles.

The range which was plotted was actually the true or slant range. The relation between the slant range and ground range (range along the Earth's surface to a point under the display) for points situated 100 km above the Earth's surface is shown in Figure 3.03. The slant range exceeds the ground range by approximately 10 km for ranges between 600 and 1000 km.

The measurement of relative bearing from the auroral photographs was much more accurate than the deduced range, whereas for the radar photographs the converse was true. For this reason the positional correspondence could only be taken as significant when the radar echoes were sufficiently extensive, so that they presented a distinctive

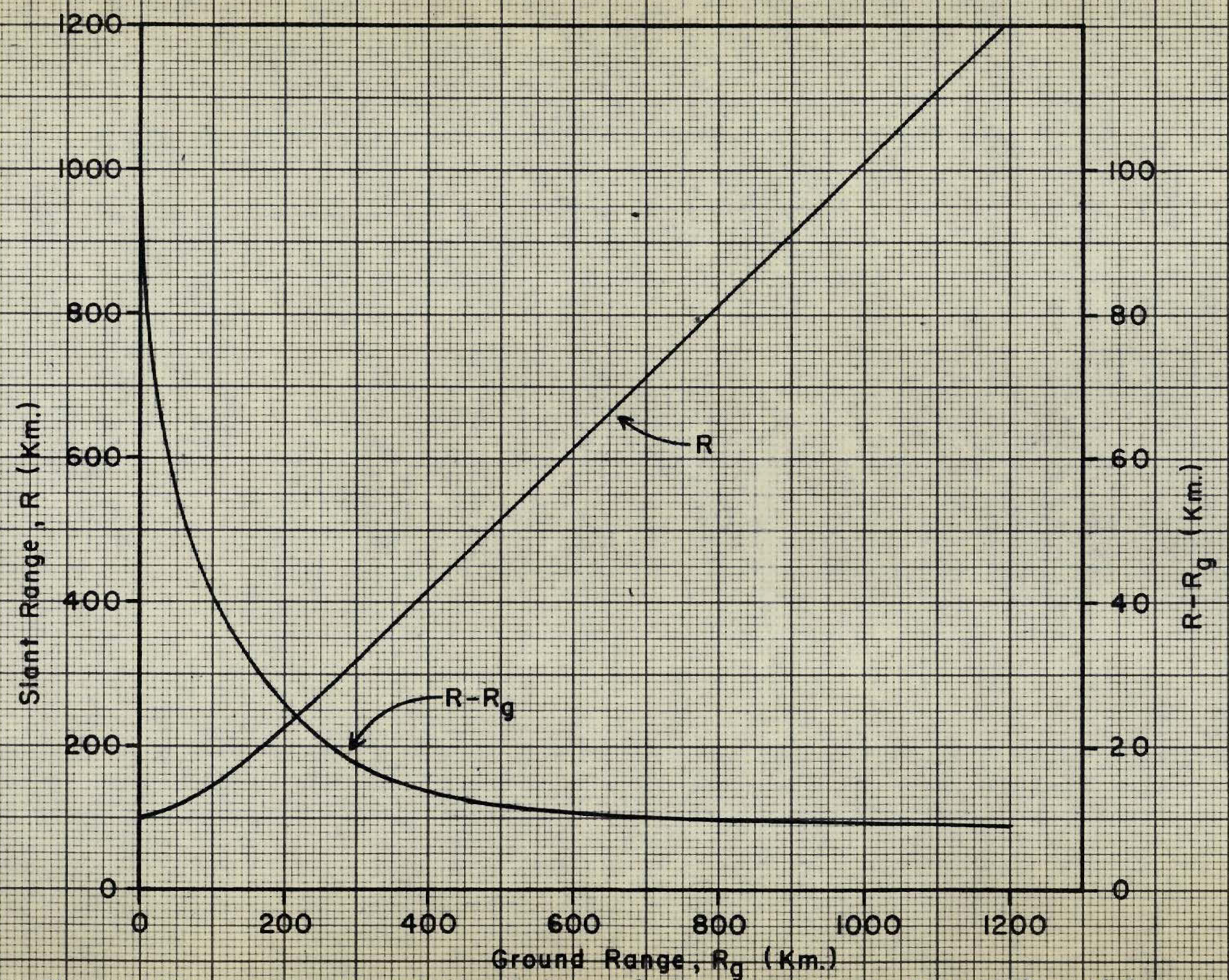


Figure 3.03 - Showing the relation between slant range and ground range for points 100 km above the Earth's surface.

configuration. Such echoes were recorded on three occasions when nine complete sets of photographs were secured. The results found by analysis of these photographs are shown in Figures 3.04, 3.05 and 3.06. In each case 0° represents true north. The local magnetic declination is about 20° East. In Figures 3.05 and 3.06(a), the auroral positions corresponding to two different assumed heights are included and serve to indicate the sensitivity of the method to auroral height. In the remainder only the auroral curve which falls closest to the echoes is plotted. In Figures 3.05(a) and 3.06(a), (b) and (d), it was necessary to apply a bearing correction of about $+7^{\circ}$ to the radar echoes in order to obtain the agreement shown. This was found to correspond to an error which sometimes occurred in the bearing transmission system between the aerial array and the P.P.I. cathode ray tube.

The aurora plotted in Figure 3.04 was in the form of an arc with some structure. That in Figures 3.05 and 3.06 consisted of active bands. The aurora of Figure 3.06 exhibited much movement.

3.07 Determination of the apparent height of aurora

The records which exhibited only single or small groups of echoes were used in an alternative test of the echoes for auroral origin. The radar ranges and the auroral altitude angles were determined from the records as before. These were then combined to determine the

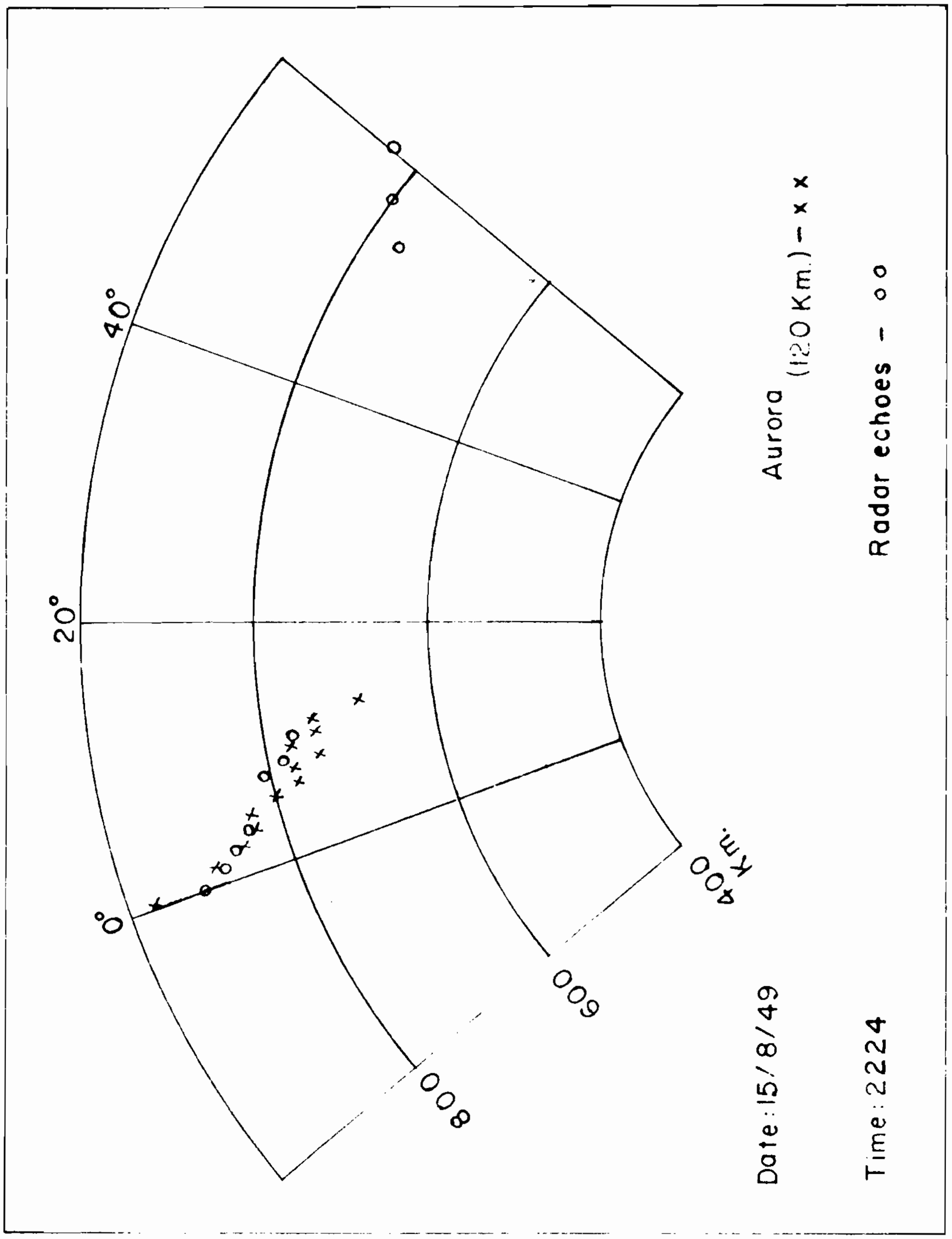

0°
20°
40°
400 Km.
600
800
Aurora (120 Km.) — x x
Radar echoes — o o
Date: 15/8/49
Time: 2224

Figure 3.04 (a)

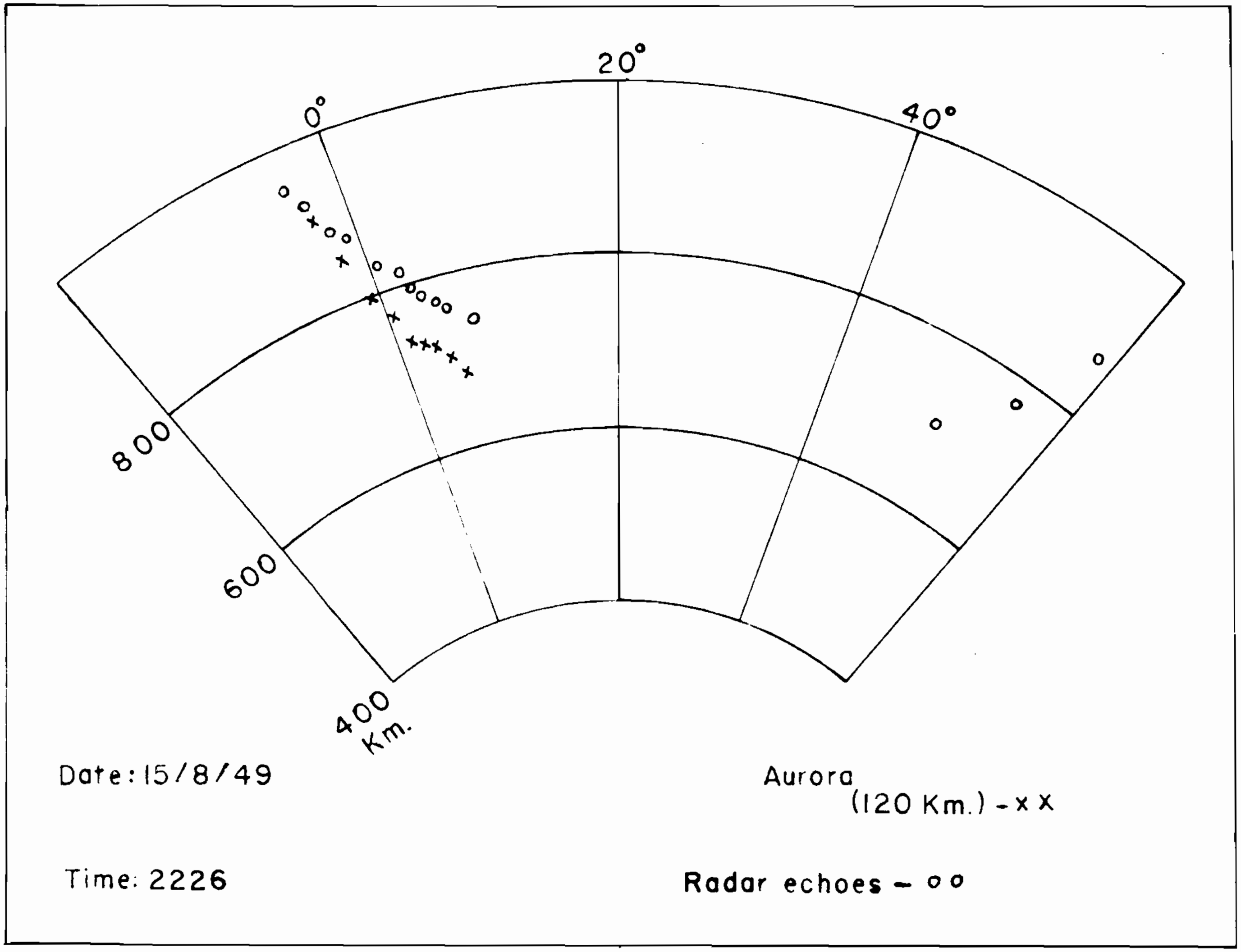
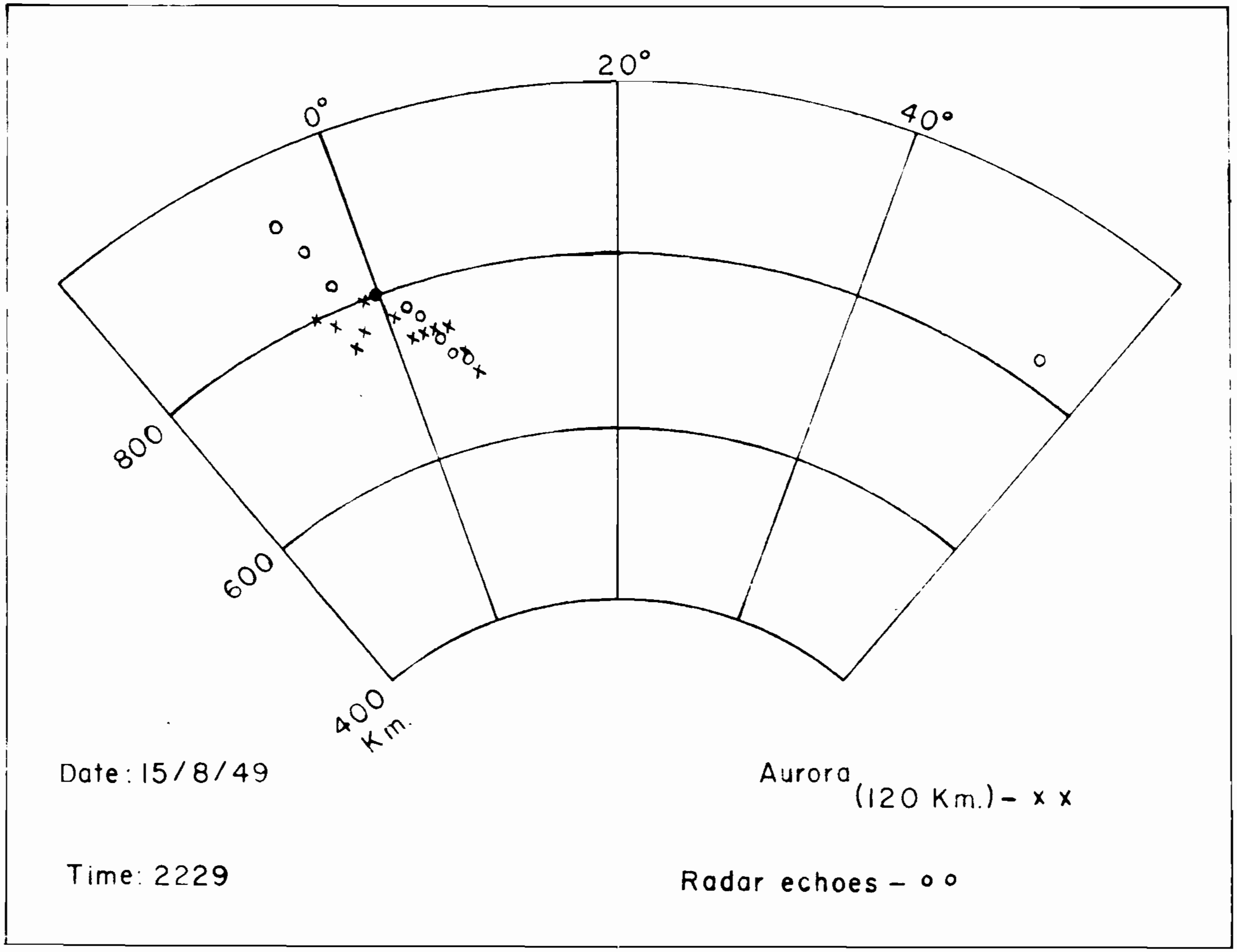
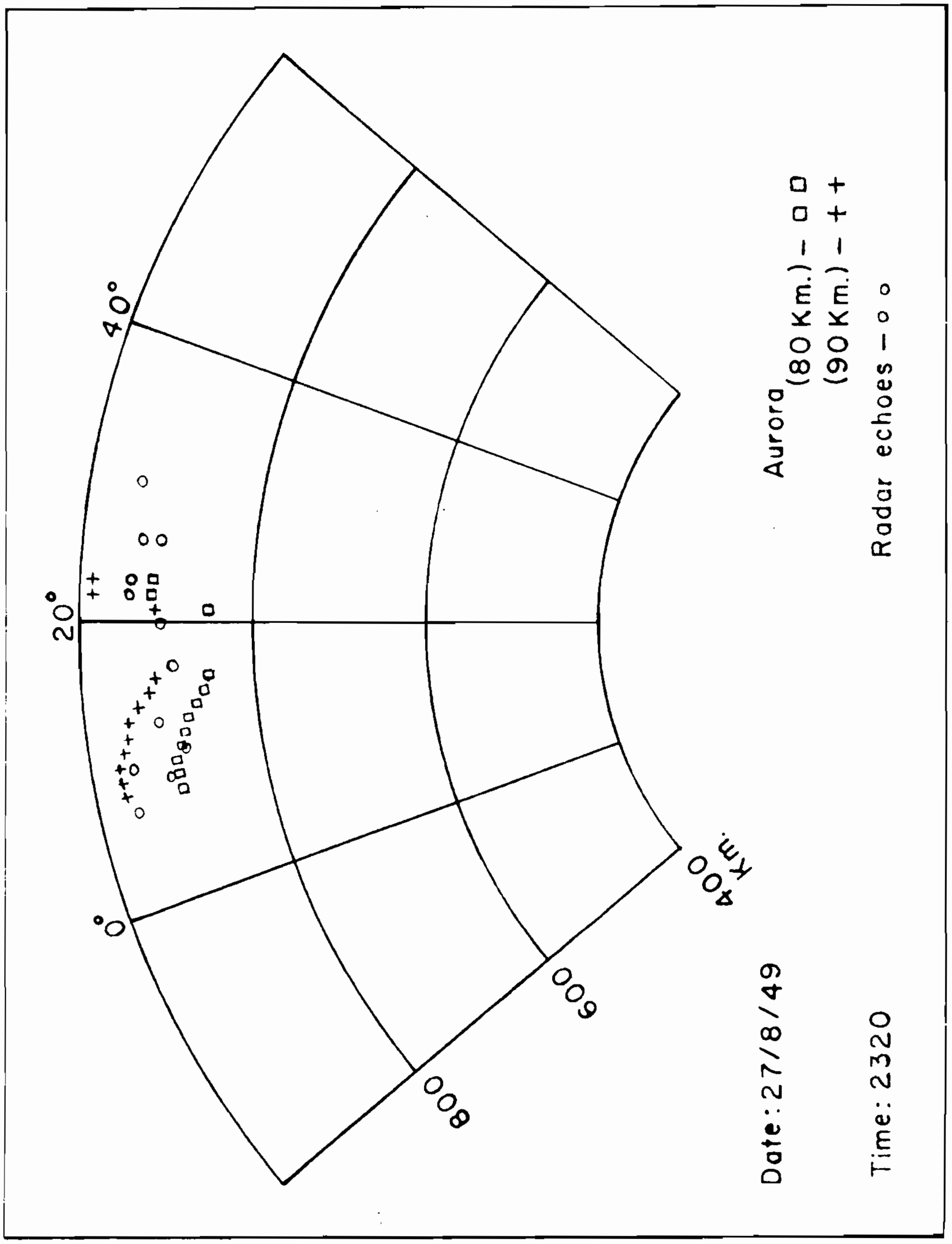

20°
0°
40°
800
600
400
Km.
Date: 15/8/49
Time: 2226
Aurora (120 Km.) - x x
Radar echoes — o o

Figure 3.04 (b)



20°
0°
40°
800
600
400 Km.
Date: 15/8/49
Time: 2229
Aurora (120 Km.) – x x
Radar echoes – o o

Figure 3.04 (c)



0°
20°
40°
400 Km.
600
800
Date: 27/8/49
Time: 2320
Aurora (80 Km.) – □□
(90 Km.) – ++
Radar echoes – ○○

Figure 3.05 (a)

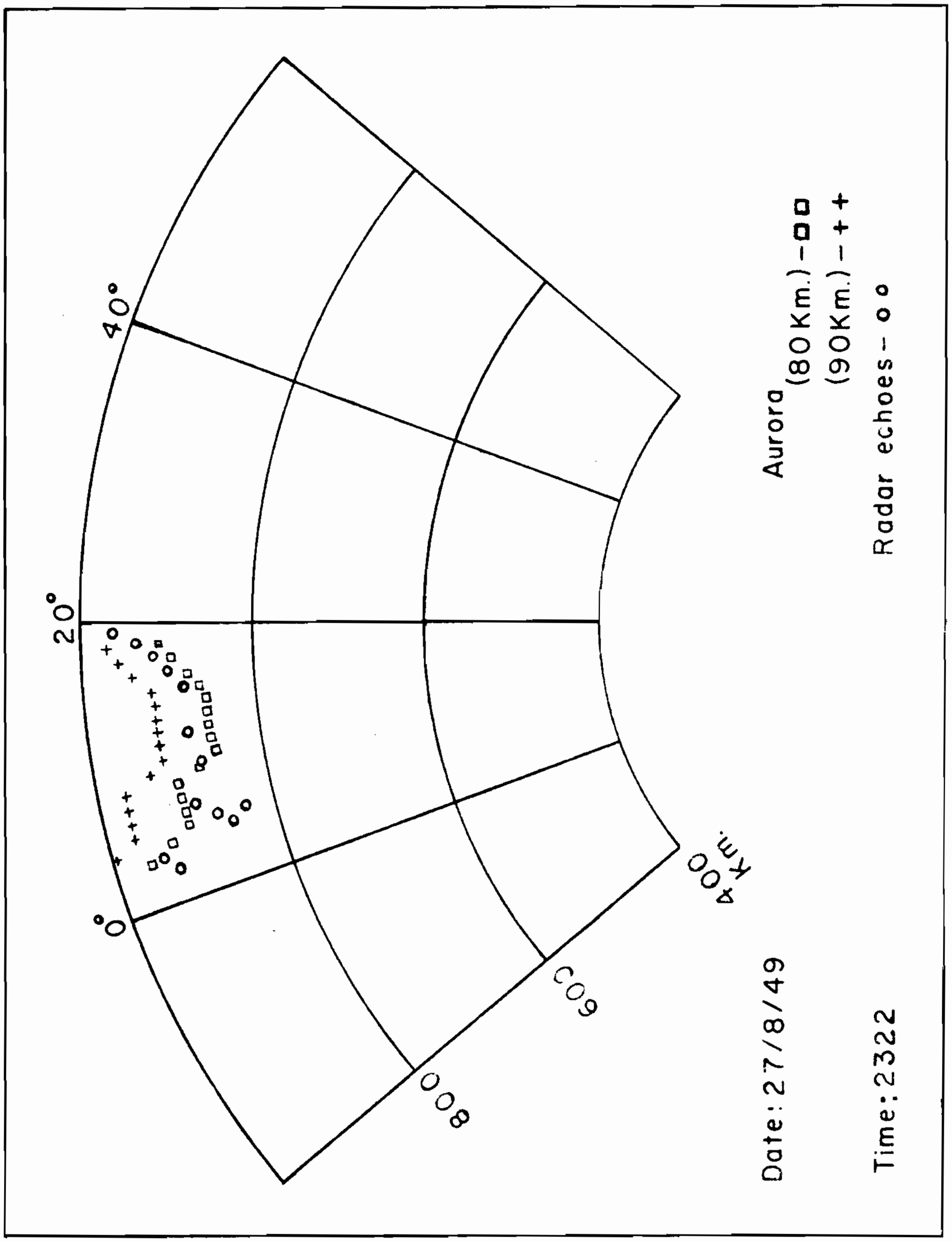

0°
20°
40°
400 Km.
600
800
Aurora (80 Km.) - □□
(90 Km.) - + +
Radar echoes - o o
Date: 27/8/49
Time: 2322

Figure 3.05 (b)

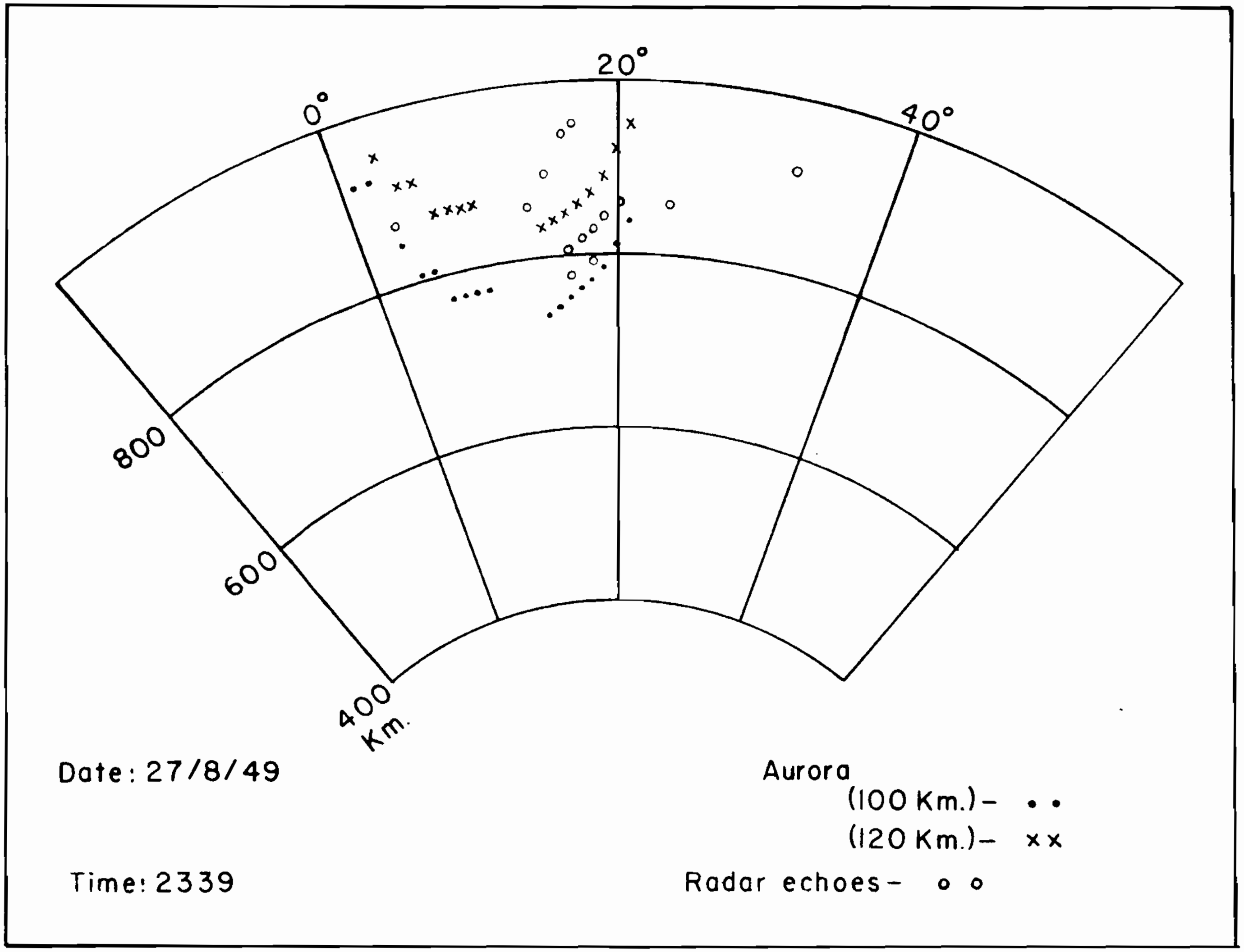

20°
0°
40°
800
600
400
Km.
Date: 27/8/49
Time: 2339
Aurora
(100 Km.)- • •
(120 Km.)- x x
Radar echoes- o o

Figure 3.06 (a)

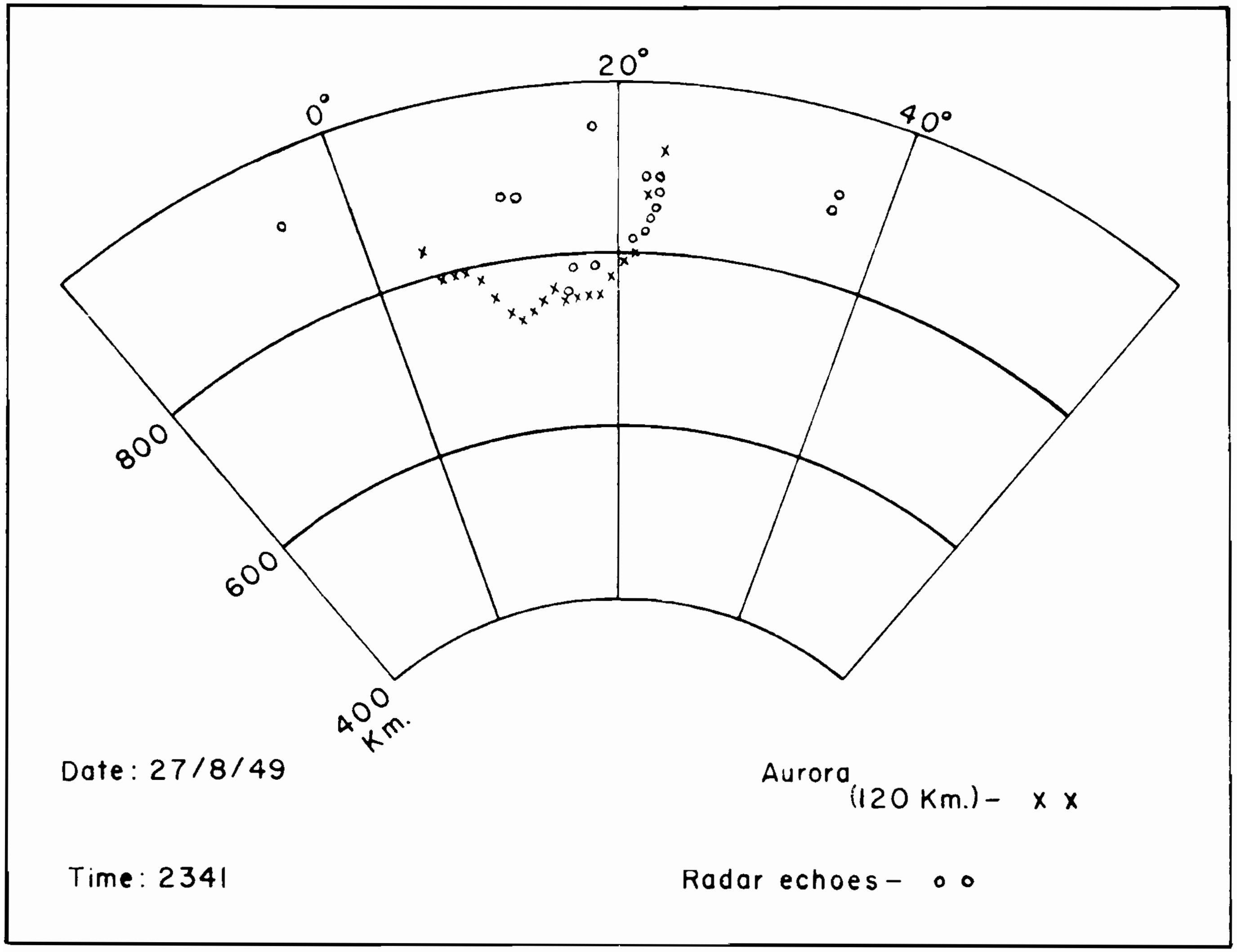

20°
0°
40°
800
600
400 Km.
Date: 27/8/49
Time: 2341
Aurora (120 Km.)- x x
Radar echoes- o o

Figure 3.06 (b)

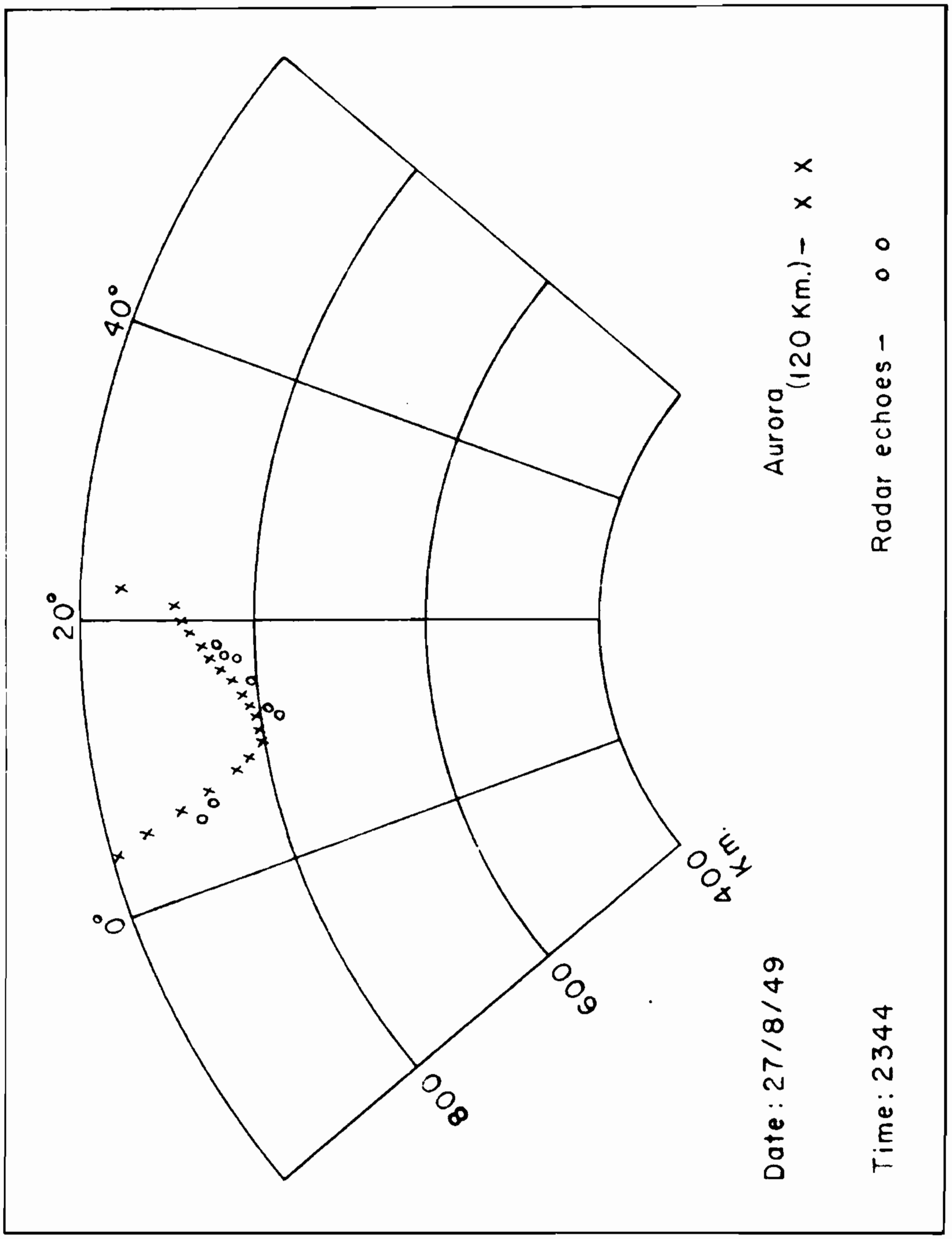

20°
40°
0°
400 Km.
600
800
Aurora (120 Km.) – x x
Radar echoes – o o
Date: 27/8/49
Time: 2344

Figure 3.06 (c)

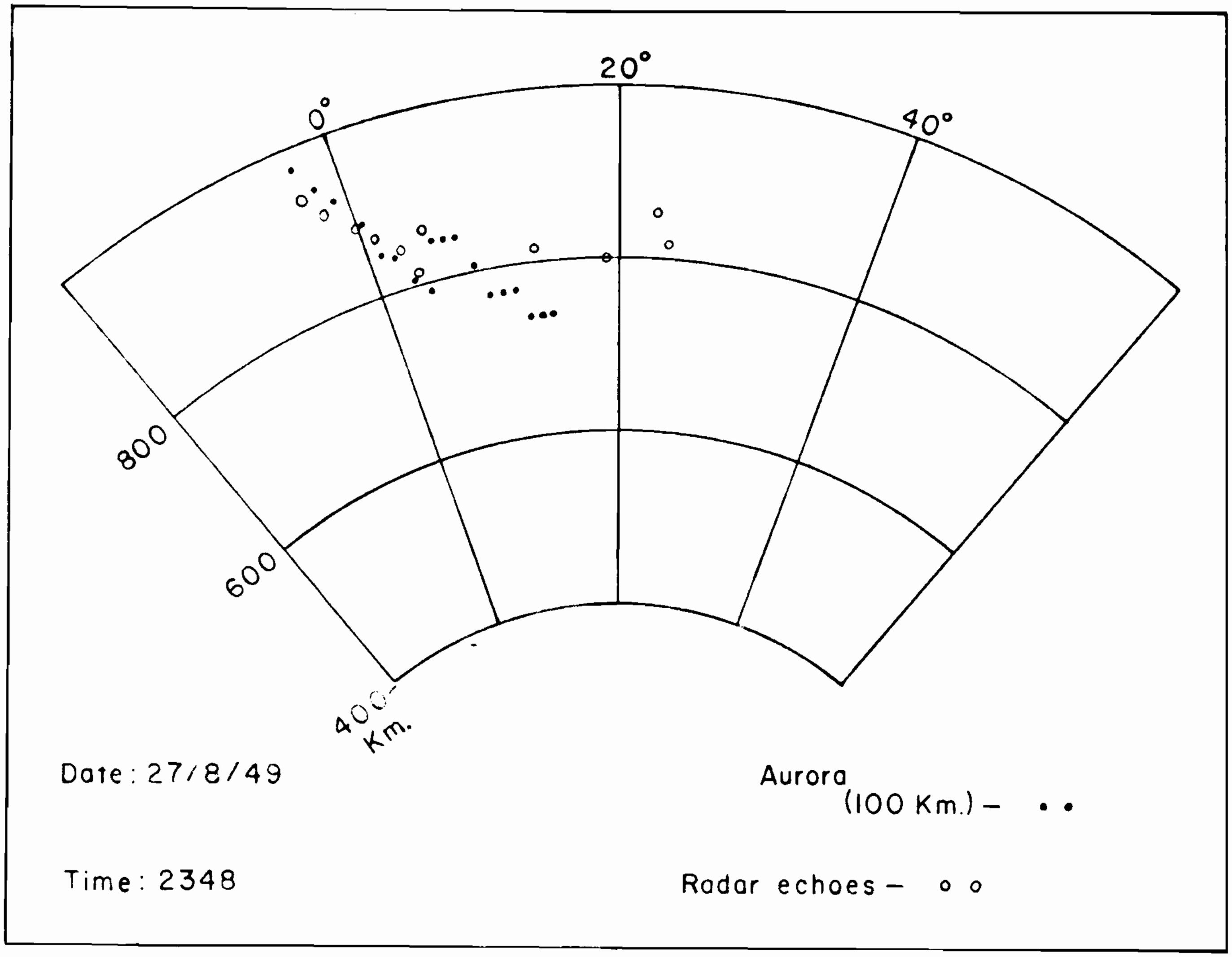

0°
20°
40°
800
600
400 Km.
Date: 27/8/49
Time: 2348
Aurora (100 Km.) —
Radar echoes —

Figure 3.06 (d)

apparent height of the lower edge of the aurora. If the reflecting centres were located within the visible aurora, then the heights found in this manner should agree with the heights which have been found by other workers from double station photographs.

The uncertainty in the true bearing of a single radar echo as determined from the P.P.I. photographs was of the order of $\pm 7^{\circ}$ so that it was necessary to take into account the total variation of the auroral altitude angle within these limits. The accuracy of the height found was thus a function of the rate of change of the auroral altitude angle in the neighbourhood of the reflecting centre.

The results of this analysis are shown in Figure 3.07 where the determined heights are plotted against range. The length of each vertical line represents the inaccuracy mentioned in the preceding paragraph. The most precise determinations are represented by lines having a length corresponding to 5 km. The inherent accuracy of the method is well within this value. Each small circle represents a more accurate determination made by the method of the preceding section, while the horizontal line associated with the circle indicates the total extent of the echoes in range. The lack of any consistent trend in determined height with increasing range is taken to indicate the absence of any systematic error in the determination of either range or auroral altitude angle.

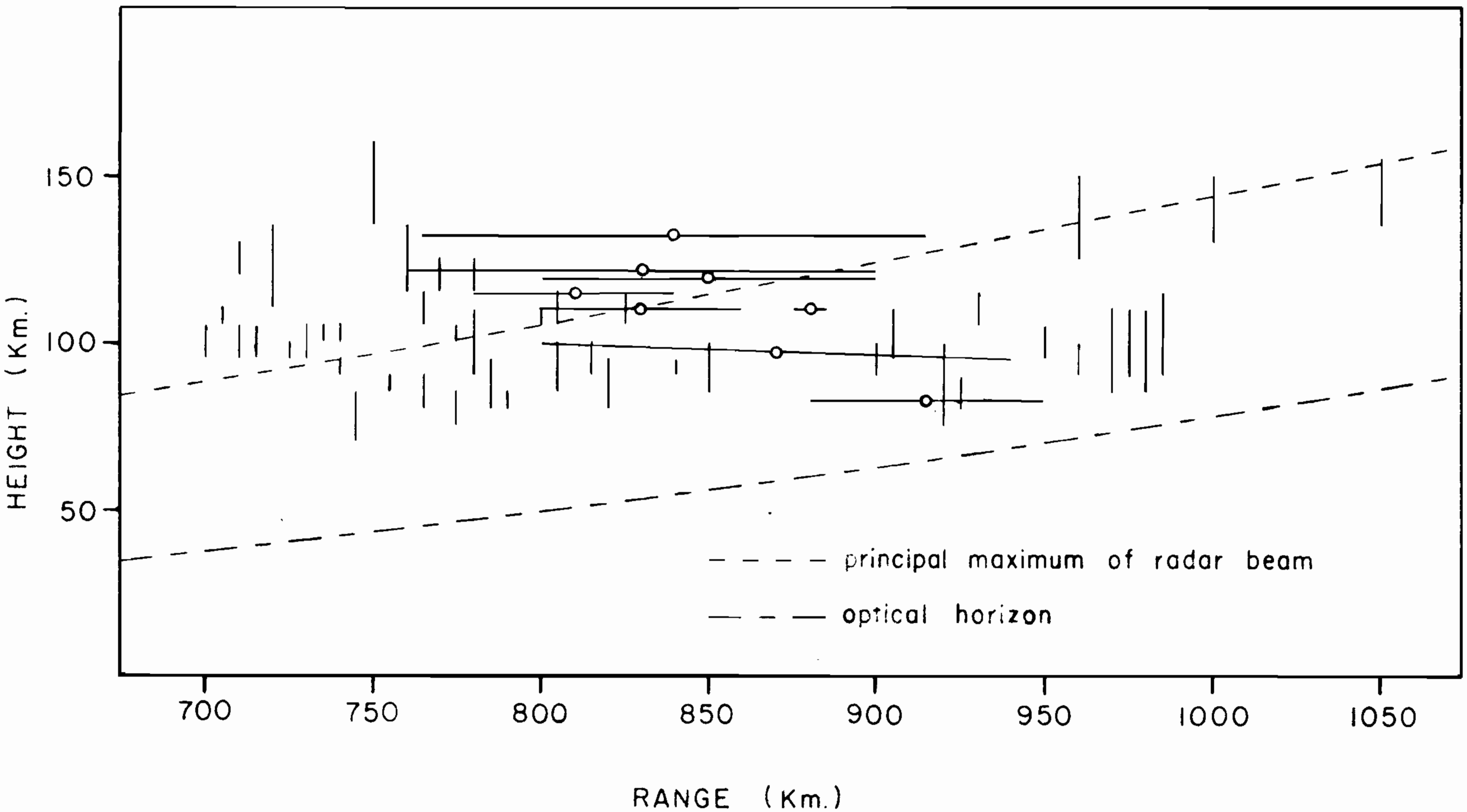


Figure 3.07 - Distribution of apparent auroral heights with range.

The number of records involved hardly could represent an adequate sample of aurora at Saskatoon; however, the distribution of frequency of occurrence with height shows an excellent correlation with that found by other methods. Alty and Wilson used a double station photographic method to find the height of aurora at Saskatoon. They found a similar distribution for a much larger number of displays which occurred during the International Polar Year of 1932-33 (1). Both these distributions are plotted in Figure 3.08. In plotting the frequency distribution for the radar method each determination was given the same weight, and a height increment of 5 km was used.

It should be noted that the height found by this method can be only the height of the lower edge of the visible aurora and need not bear any relation to the height of the reflecting region. It was assumed implicitly that the range to the reflecting region and that to the lower edge of the aurora differed by a negligible amount.

3.08 Conclusions

The evidence presented in the preceding sections indicates that the echoes were closely associated with the visible aurora. The direct evidence refers only to positional correspondence in a two-dimensional sense. The determination of the height of reflection will be discussed in Chapter IV since it is closely related to

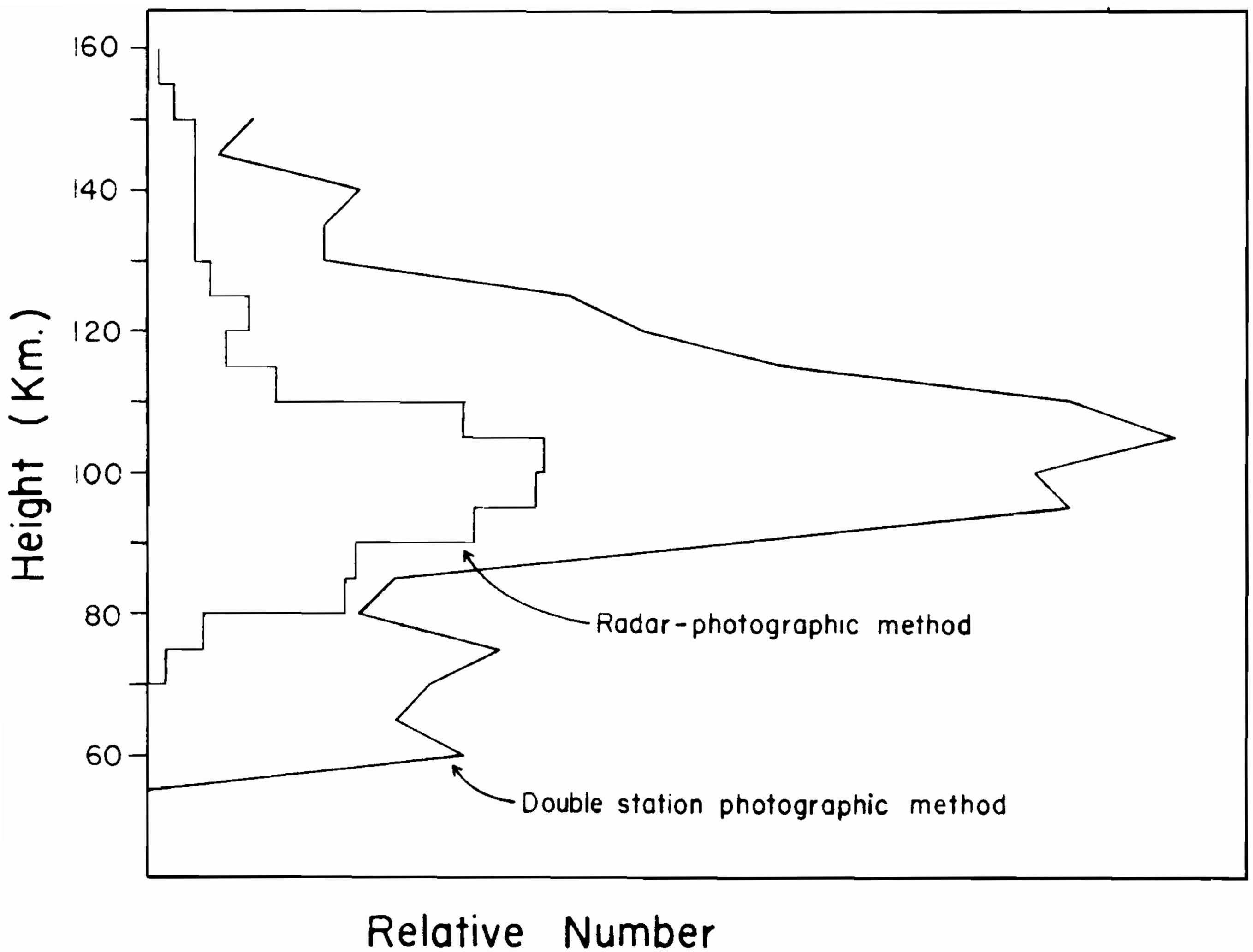


Figure 3.08 - Distribution of frequency of occurrence with height as found by Alty and Wilson (double station photographic method) and as found by the radar-photographic method.

the problem which is treated there. In the meantime, however, it is worth noting that the approximate height of reflection may be inferred from Figure 3.07. The echoes were distributed in range about the point where the principal maximum of the radar beam intersected the 100 km level. The extensive echoes were concentrated near this point. This distribution indicates that the reflecting region was located about 100 km above the surface of the earth. It is not possible to infer the thickness of the reflecting region from results of this experiment but it is evident that the reflecting centres must have been located within or just below the visible aurora.

In order to measure range the radar equipment measures the time elapsing between the transmission of the electromagnetic energy and the reception of that portion of it which is reflected by the distant target. The range is derived by assuming that the velocity of propagation of the energy is constant and equal to its free space value (3×10^{10} cm sec^{-1}). The fact that the radar ranges measured in this experiment appear to have been the true ranges to the auroral structures indicates that there was no appreciable retardation of the wave during its passage to and from the aurora.

3.09 Subsidiary results

Radio reflections are well suited to the study of the occurrence and motion of active auroral displays. In some respects they offer a distinct advantage over photographic methods. Rapidly moving aurora is difficult to record photographically but may be studied by the radar method. Double station photographs of distant aurora yield accurate results only when the two stations are separated by a great distance. The same information may be obtained by combining radar observations with single station visual or photographic records.

The movement in range of auroral structures is evident in Figures 3.04, 3.05 and 3.06. While the form of the records was not the most suitable, it was possible also to ascertain the apparent vertical motion of particular structures over significant periods of time. Three such instances are shown in Figure 3.09. The auroral heights were derived by the method of section 3.07. In each case, the records involve the same structure over the periods shown, although in Figure 3.09(b) the band faded for a short interval about the middle of the period.

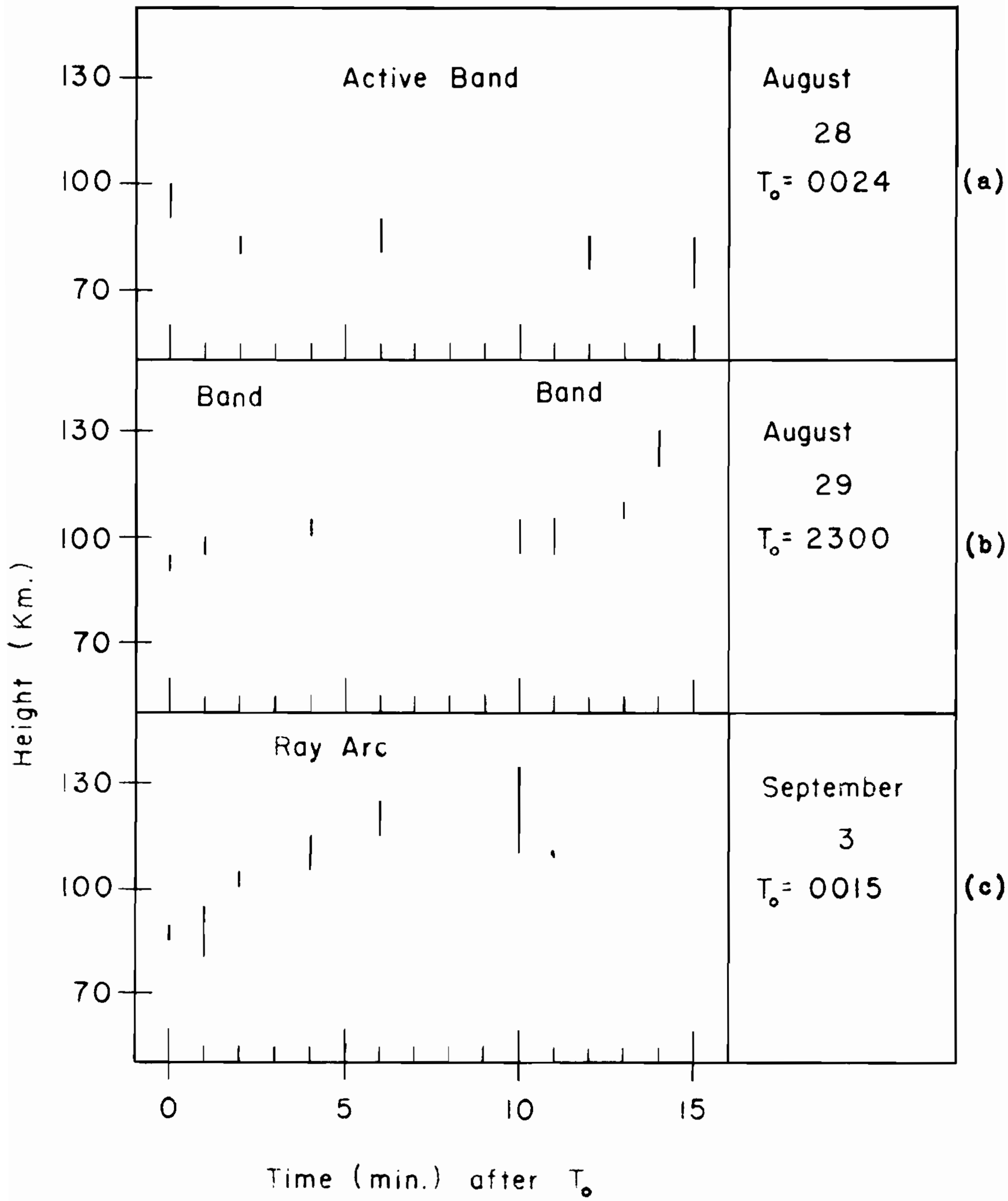


Figure 3.09 - The apparent vertical motion of particular auroral structures.

CHAPTER IV

The Reflection Process

4.01 Characteristics of auroral echoes

Typical range display photographs of auroral echoes are reproduced in Figure 4.01. The echoes show a multiple fine structure which suggests that they are composite echoes arising from a large number of individual reflecting centres. The individual components vary rapidly in amplitude, as might be expected if the reflecting centres were continuously being formed and dissipated. The surfaces of the reflecting centres would thus be in motion relative to one another and the waves reflected from different centres would interfere in a complicated manner giving rise to the amplitude fluctuation. The echoes are also characteristically triangular in shape. The triangle has a narrow base when the radar beam is directed at right angles to the length of the auroral display and a wide base when the beam makes a small angle with the display. This is apparently due to the fact that in the first case all the reflecting centres are at approximately the same range whereas in the second the radar beam illuminates reflecting centres widely distributed in range. The horizontal radiation pattern of the radar beam has a central maximum and the power falls off uniformly on either side. Thus, when the beam makes a small angle with the length of the display, the weak extremities of the echo may be associated with the weak edges of the beam

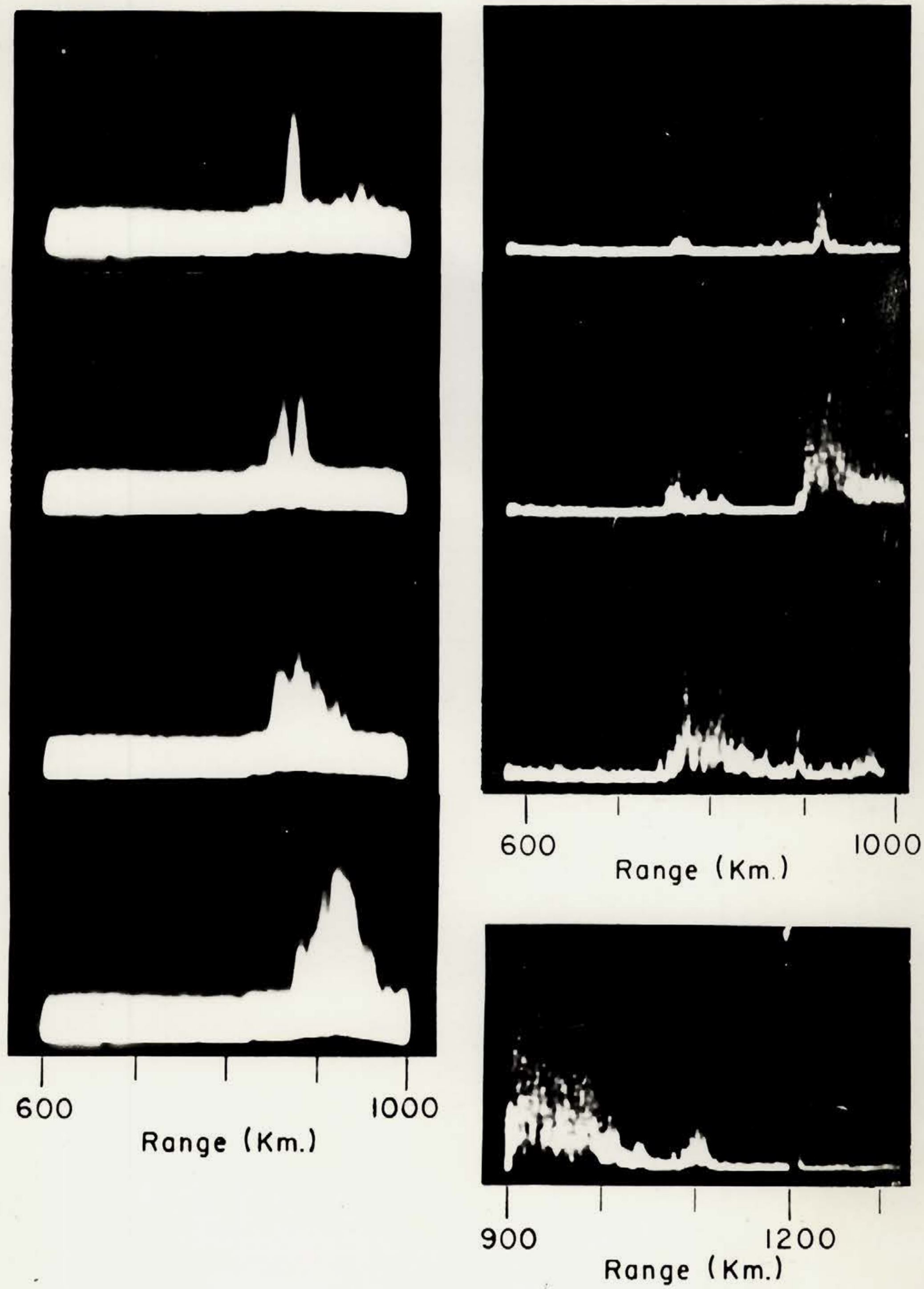


Figure 4.01 - Typical photographs of the range display showing auroral echoes. Those in the left hand column were taken with long exposures to show the outline of the echo, whereas the others were taken with short exposures to show the fine structure.

and the central maximum of the echo with that of the beam.

Radar echoes were not observed at this frequency from all auroral displays. Echoes were usually observed when the aurora exhibited some structure and was moderately bright, but some echoes were observed that apparently originated in weak aurora. The presence of structure seemed to be more important than the visible brightness of the display. The duration of the echoes was variable between wide limits and rarely coincided with that of the display itself. The echoes usually appeared and disappeared many times at different bearings during the course of the display. Persistent echoes were obtained from stable auroral arcs and sporadic echoes from the more active forms. The duration of various auroral echoes may be noted from the continuous records reproduced in Chapter V.

4.02 Echoes arising from aurora in the west

A few echoes arising from aurora in the west were observed. These echoes were not recorded but, because of their importance in the discussion to follow, they are described here. The observations were made during two nights when the auroral displays were particularly intense and various auroral structures were frequently overhead. During the observations the radar operator was in telephonic communication with an observer situated so that he had a clear view of both the sky and the radar aerial array.

In particular, the observations concern occasions when a ray arc passed nearly overhead and extended to the western horizon in an unbroken curve. Usually no echoes were received from the extremity of such an arc, but short echoes were observed when the arc developed a sharp bend near the horizon. These echoes were observed at azimuth bearings between 280° and 290° and at ranges between 700 and 800 km. Similar observations could not be made on the eastern ends of the displays because various buildings limited the view of the radar equipment and of the visual observer.

4.03 Atmospheric refraction

We digress for a moment to discuss the effect of atmospheric refraction on the radar measurements. The subject of refraction of radio waves in the lower atmosphere has been discussed thoroughly by Burrows and Attwood (14, 15, 16, 17,). The index of refraction, n, of the atmosphere is given approximately by,

$$(n - 1)10^6 = \frac{Ap}{T} - \frac{De}{T} + \frac{Be}{T^2}$$

where,

p = barometric pressure (millibars);

e = water vapor pressure (millibars);

T = temperature (°K);

A = 79;

B = 3.8×10^5;

D = 11.

It is customary to discuss the modified index of refraction, M, given by

$$M = \left((n - 1) + \frac{h}{a}\right)10^6 ,$$

where h = height of point considered;

a = radius of the Earth.

For the "standard" atmosphere, i.e., the average atmosphere for the middle latitudes, M decreases linearly with height and, to a good approximation, is given by

$$(M - M_0)10^{-6} = \frac{h}{ka},$$

where M_0 = value of M at the surface of the Earth;

$k = 4/3$.

Rays leaving the transmitter at low angles are actually curved but for the purpose of calculation may be replaced by straight rays if the radius of the Earth, a, is replaced by a modified radius, ka. This method of calculation may be used with confidence for the region of the atmosphere extending up to about 10 km above the Earth's surface. For the part of the atmosphere between 10 km and 100 km there is a paucity of reliable data from which to calculate the index of refraction; however, at least for the upper region it is certain that the density is so low and the index of refraction so near unity that refraction is very slight.

For the purpose of this thesis, calculations dependent upon refractive effects were made twice -- once assuming that the standard atmosphere existed up to the 100 km level and again neglecting all atmospheric refraction. It is assumed that the correct result lies somewhere between the two. In this respect it may be noted that for a ray leaving the Earth tangentially and reaching the 100 km level almost 1/3 of its path length lies below the 10 km level. Thus, if there is no anomalous refraction near the 80 km level, (which is unique in so many ways), a mean between the two calculated values may well be a close approximation to the true value.

4.04 Height of reflection

The approximate height of reflection was mentioned in section 3.08. That determination was dependent upon a knowledge of the angular position of the primary lobe maximum in the radiation pattern. The positions of the maxima of several of the lower lobes were calculated by simple theory in order to find their approximate positions. Arrangements were then made to have an aircraft fly at a constant height (20,000 ft) and constant bearing. The amplitude of the echo returned from the aircraft was plotted against range giving a graph which exhibited maxima corresponding to the maxima in the radiation pattern. The angular positions of these maxima were then established from the geometry of the situation. Since the echo arose in the lower atmosphere refraction was taken into account in the determination.

Ideally, each lobe of the radiation pattern should exhibit a single maximum and the power should fall uniformly to zero above and below the maximum. However, when a large aerial array is situated close to the ground, the finite size of the array and local irregularities in the terrain give rise to interference effects which may cause irregularities in the outline of the lowest lobe. It was evident that if a distinctive irregularity could be found in the radiation pattern it could be used to determine the height of reflection.

The frequency of occurrence of echoes was plotted against range for various azimuth sectors. Since auroral photographs were not needed, many more records were available for this purpose then for the analysis of the previous chapter. The graph for echoes obtained in the sector 350° - 010° is given in Figure 4.02(a) and shows a distinctive minimum at about 875 km range. An aircraft flight was arranged to measure the radiation pattern in this sector. The radar cross-section of an aircraft is very sensitive to aspect so that it was essential that the aircraft maintain a constant course as well as a constant altitude during the flight. The success of the experiment was due largely to the skill of the aircrew. Two flights were made, one at an altitude of 10,300 feet and azimuth bearing of 003°, the second at an altitude of 9,300 feet, the course

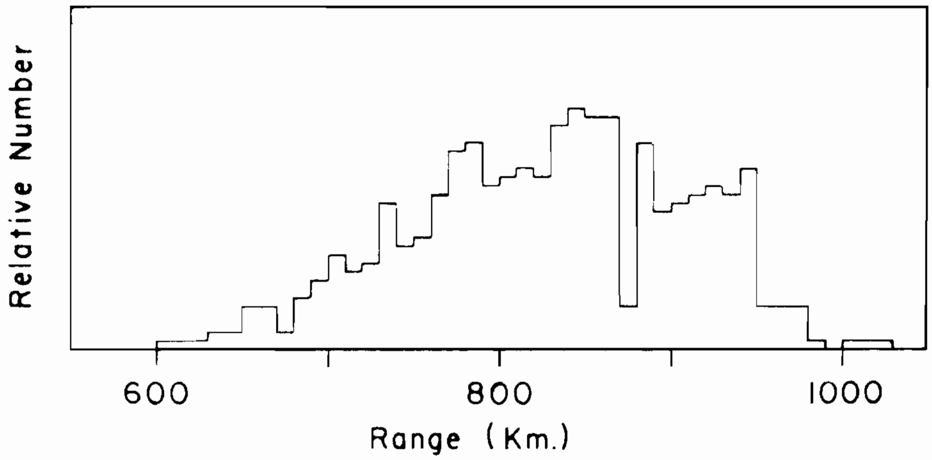


Figure 4.02 (a) - Frequency of occurrence of auroral echoes for azimuth sector 350° - 010°.

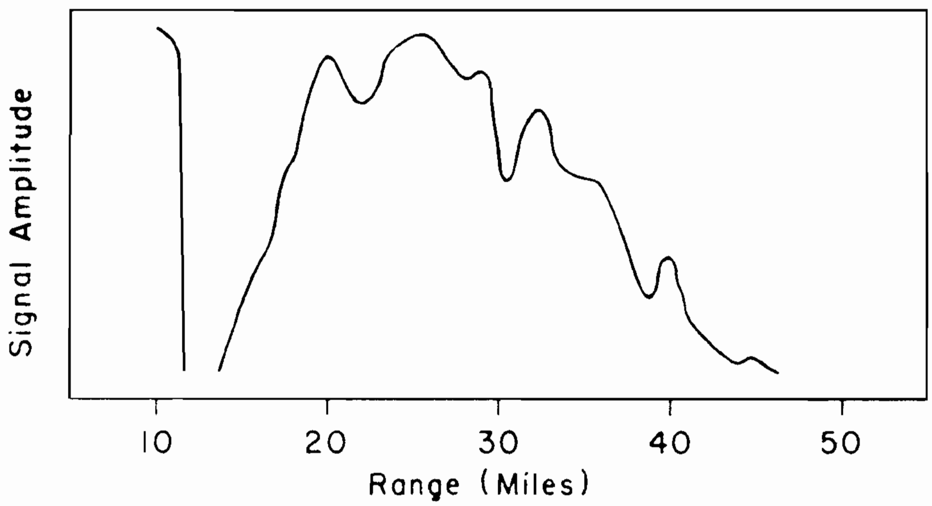


Figure 4.02 (b) - Amplitude of echo from aircraft at an altitude of 10,300 feet.

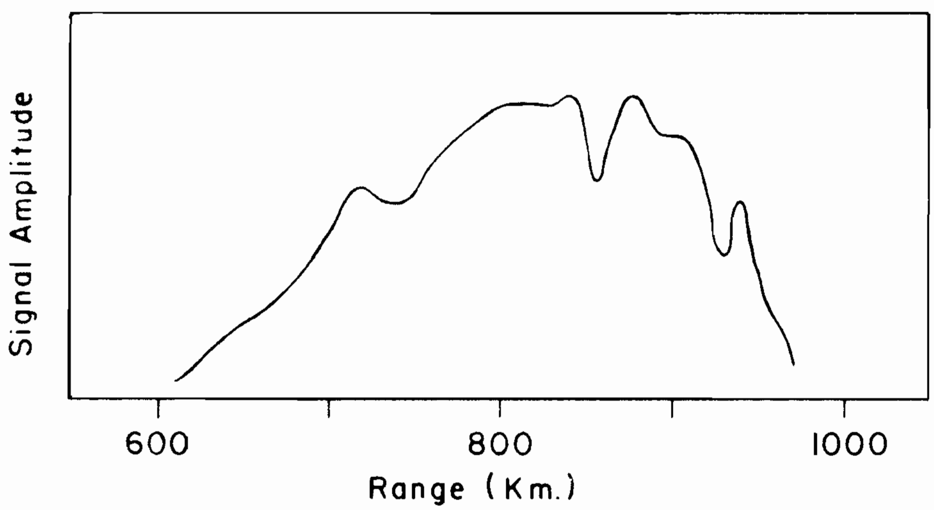


Figure 4.02 (c) - Data of (b) projected to 100 km level.

being the reciprocal of the first. The data from the second flight was projected geometrically to the same height as the first and the two sets of data were then averaged. The resulting graph of echo amplitude versus range is given in Figure 4.02(b). Figure 4.02(c) shows the same data projected to an altitude of 100 km, refraction being neglected. The signal amplitude varies as the inverse square of the range (the intensity varies as the inverse fourth power). This variation was included in the calculations involved in projecting the data to the higher level. There is qualitative agreement between this graph and that of Figure 4.02(a), but since small errors in close range measurements in Figure 4.02(b) are greatly accentuated in Figure 4.02(c), no attempt was made to obtain a positional correspondence between all the features of the two graphs. Instead, the minimum at about 855 km range in the latter graph was identified with that at 875 km range in the former. This minimum was found to correspond to an angle of 3.6° above the horizontal (we refer here to the ray which leaves the radar station at this angle, its inclination may be changed by refraction). For a standard atmosphere extending up to the reflecting region the height of reflection was found to be 98 km above the surface of the Earth. When refraction was neglected the corresponding height was 114 km. The inaccuracy of the measurement is probably somewhat less than the difference between these two values.

The height of reflection, as found in this section, is in good agreement with that found in section 3.08 but the greatest value of the determination is to show that there was a discrete level from which the auroral echoes arose. If the echoes had arisen from all levels within the auroral structures no such determination would have been possible. Evidently the height was constant for most of the displays which were observed.

The heights of reflection found by other workers are:

Lovell, Clegg and Ellyett (55)	480 km;
Herlofson (49)	200 km;
Aspinall and Hawkins (6)	100 - 320 km.

4.05 A distinguishable characteristic of the reflection mechanism.

The various processes by which auroral echoes might arise were summarized in Chapter II. Of these it was pointed out in section 2.09 that electron scattering was a most improbable choice. There remains, then, partial reflection from a plane surface, partial reflection from an extended cylindrical surface and scattering by a small body. It is not possible to differentiate between these processes by measuring the power returned to the receiver, but echoes of detectable amplitude will result from the first two processes only when the condition for specular reflection is fulfilled. If reflection takes place at an extended plane surface, echoes will be detectable in only one direction, namely, along the normal to the surface. Echoes which arise from an extended cylindrical surface will only be detectable in the plane at right angles to the axis of the cylinder. A small scattering centre will give rise to echoes when viewed from any direction. A directive rotatable aerial array is an adequate tool for distinguishing between these processes.

Since echoes were received at widely differing azimuth bearings from the same auroral display, reflection from a plane surface may be discarded without further consideration. Before discussing the second process in detail we must give some thought to the direction in space along which the cylinders may be assumed to lie. If the reflecting cylinders are to be closely associated with the auroral structures, they must have the same orientation as the auroral rays. It is well established that these rays follow very closely the lines of force of the Earth's magnetic field. The problem has been investigated by various workers during the last hundred years. The results obtained have been summarized by Vegard (86). A series of measurements made by Vegard and Krogness (88, 89) during the early part of this century established that auroral rays usually point to a position on the celestial sphere lying about 1° below the magnetic zenith, i.e., the angle between the auroral ray and the corresponding undisturbed magnetic line is about 1°, both the ray and the magnetic line lying in the plane of the magnetic meridian. Examination of the angle between the direction of wave travel and the magnetic field, for the regions from which echoes were observed, should reveal the condition for specular reflection, if the reflecting surfaces were actually cylinders.

4.06 The magnetic field near Saskatoon

In this section we are interested in finding the angle, θ, between a ray which leaves the radar station at a specified angle of elevation and the magnetic line which it intersects at a specified location. If the ray leaves the radar station at an angle of elevation, V, and an azimuth bearing, A, then from the geometry of the situation we obtain

$$\cos\theta = \cos I \cos(D - A) \cos(R + V) - \sin I \sin(R + V),$$

where

I = angle of inclination of the magnetic field;

D = angle of declination of the magnetic field;

R = range angle, i.e., the angle subtended at the centre of the Earth by the arc of the great circle joining the radar station to the point under investigation.

This expression may be modified to include the effect of refraction in a standard atmosphere by substituting R' for R where

$$R' = \frac{3R}{4}.$$

The data concerning the magnetic field of the Earth were taken from the detailed maps given by Vestine (91) for the epoch 1945.5. The calculations were carried out in two different ways. In the first instance the elevation angle of the radar ray was fixed at 4.8^{o} (to coincide with the first maximum of the radiation pattern) and the angle, θ, was calculated for a representative number of points in the area from which the echoes were observed. These results were then used to draw a map showing lines of constant θ. This map is reproduced in Figure 4.03. The calculations included the effect of refraction since this yielded results more favourable to the assumption that the reflectors were cylinders with their axes parallel to the magnetic field. The effect of neglecting refraction would have been to increase the value of θ for all points on the map. It may be seen that the condition for specular reflection ($\theta = 90^{o}$) is not fulfilled for any point at a range greater than 600 km, the minimum range of the observed auroral echoes. In fact, power was radiated at elevation angles less than 4.8^{o}. From aircraft data we may set the lowest angle at which appreciable energy was radiated at about 2.5^{o} below the maximum. The condition for specular reflection could then be fulfilled for all

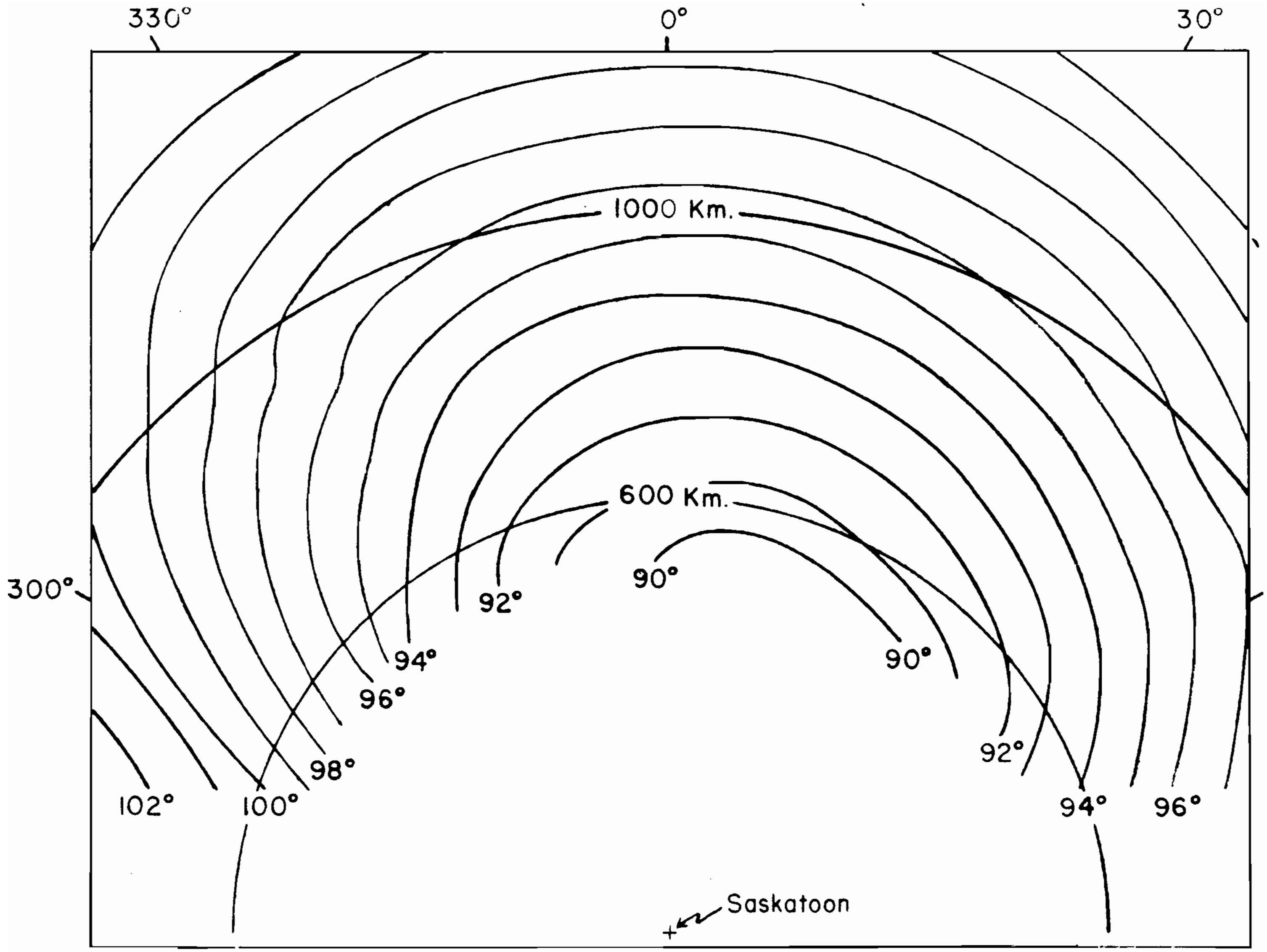


Figure 4.03 - Map showing lines of constant θ for the ray leaving the radar station at an elevation angle of 4.8°.

points having θ less than 92.5°; however, this would place the height of reflection considerably below 100 kilometres.

The region from which echoes were obtained is shown on a map drawn to the same scale in Fig. 4.04. Each line on this map indicates the position and extent of the echoes present on a single record, although the echoes were not necessarily continuous along the whole length of the line.

The second method of calculating the angle, θ, made use of the knowledge that the reflecting centres were close to the 100 km level. The height of reflection was fixed at 100 km and the elevation angle of the ray from the radar station was allowed to vary from point to point in accordance with this condition. The map showing lines of constant θ calculated in this manner is given in Figure 4.05. In order to interpret this map it is not necessary to use our knowledge of the radiation pattern. The effect of refraction in a standard atmosphere is again included in Figure 4.05. The approximate effect of disregarding refraction, as well as that of altering the height of reflection, may be inferred from Figure 4.06. In this figure R + V is the angle of inclination (with respect to the horizontal) of the ray from the radar station to the reflection level, measured at the reflection point.

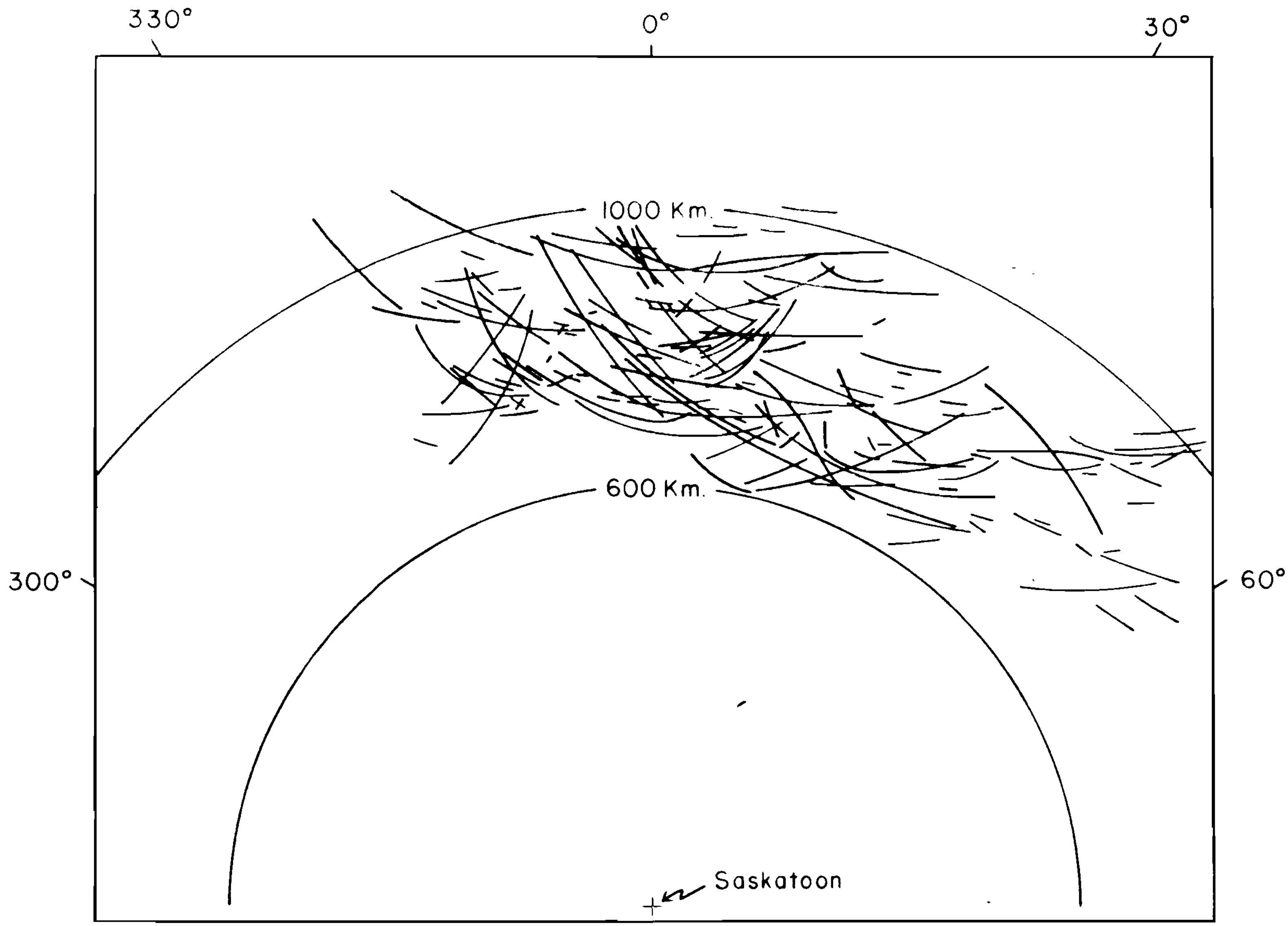


Figure 4.04 - Map showing the distribution of recorded auroral echoes.

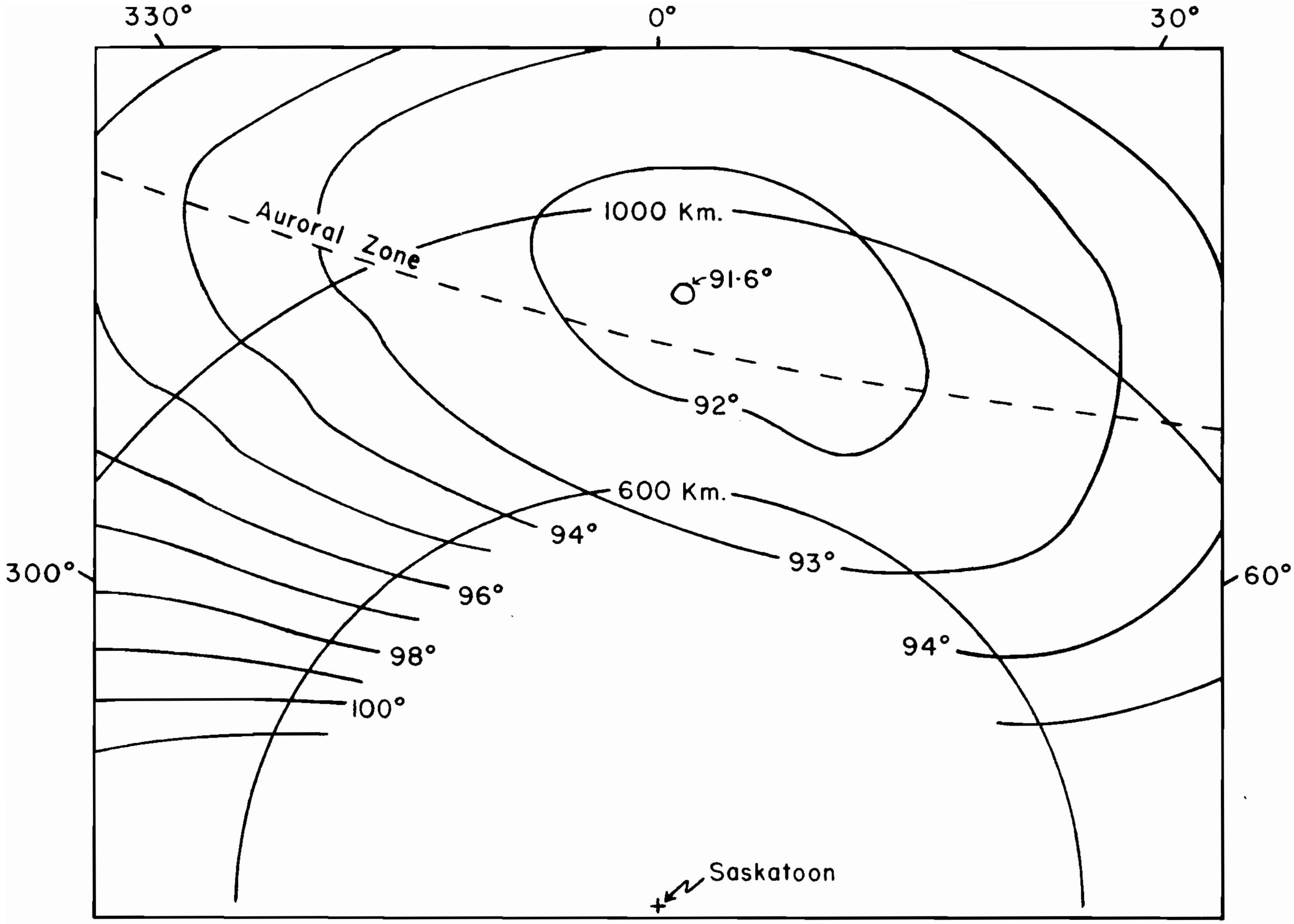


Figure 4.05 - Map showing lines of constant θ for the rays which intersect the level 100 km above the Earth's surface.

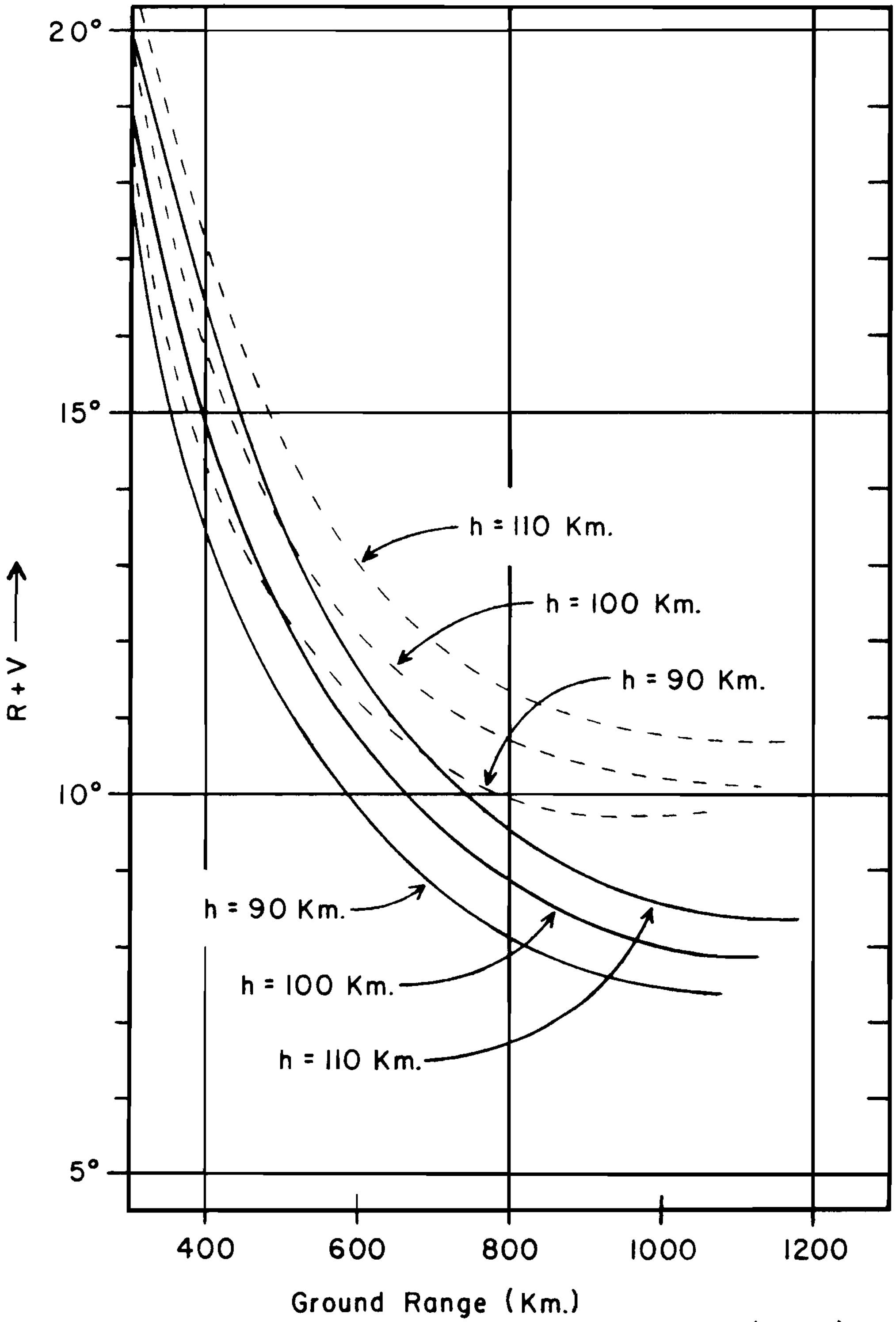


Figure 4.06 - Showing the angle of inclination, (R + V), of rays at the reflection level. Solid lines refer to propagation in the standard atmosphere, broken lines to a non-refracting atmosphere.

This angle is plotted against range for three different reflection levels for a standard atmosphere (solid lines) and again disregarding refraction (broken lines).

In Figure 4.05 the position of the auroral zone, after Vestine (90), is also plotted. The lowering of the magnetic zenith during auroral displays would tend to reduce the value of θ. particularly in the direction of the magnetic meridian (about 20° azimuth bearing) but even when this effect is taken into account, the condition for specular reflection apparently is not fulfilled for any part of the region and certainly is not fulfilled over the whole region from which echoes were obtained.

4.07 The angular limits of the reflected energy

In section 4.05 we saw that the electromagnetic energy was scattered uniformly in all directions in the horizontal plane. An important clue to the nature of the reflecting centres is given by the echoes from aurora in the west (section 4.02). These echoes must have arisen in the region where the angle between the ray from the radar station and the direction of the magnetic field was at least 100° and probably somewhat greater. This means that the angle between the incident and specularly reflected rays was at least 20°. The scattered radiation probably was distributed symmetrically about the specularly reflected ray, i.e., the scattered power was distributed fairly uniformly through a vertical angle of at least 40°. Obviously, no great error is involved in assuming that the echoes arose from scattering centres which behaved as isotropic scatterers. At this point in the argument it is not necessary to decide whether the scattering centres acted in this manner because of their shape (large approximately spherical volumes) or their size (small volumes of indefinite shape).

<u>4.08</u> The total scattering cross-section

The scattering cross-section represented by the observed echoes may be calculated from the expression given in section 2.07. The approximate values of the necessary constants are,

Aerial gain - 100;

Transmitter peak power - 10 kw;

Minimum detectable signal - 2×10^{-14} watts.

The minimum cross-section which is detectable at 800 km range is found to be about 1.6×10^{4} metre^{2}. The cross-section corresponding to the largest observed echoes (which saturated the receiver) must have been at least 100 times as great as this value. Since all the echoes show the characteristic fine structure mentioned in section 4.01 the calculated cross-section represents only the total cross-section of all the scattering centres which contribute to the echo.

It is interesting to note that if a power reflection coefficient is calculated for the observed echoes by the method used by Aspinall and Hawkins, many values are obtained which are ten times as great as the one which they reported. If the echoes arose in the manner which they suggest, the reflection coefficient should be inversely proportional to the fourth power of frequency, i.e., the reflection coefficient which they found should be four times as great as our value for comparable auroral structures.

4.09 The free electron density

From the discussion presented in the preceding sections we see that no volume of ionization which reasonably may be associated with auroral structures could scatter the incident power over large angles and still return the observed amount of power to the receiver, unless the reflection coefficient for the volume were of the order of unity. The minimum value of the free electron densities corresponding to the observed echoes may then be calculated from Equation VI, Appendix I. The density is found to be 10^8 free electrons per cubic centimetre. This value is considerably in excess of the densities which occur in the undisturbed ionosphere. The maximum ionization of the daytime F-region is about 10^6 cm^{-3}.

4.10 Amplitude records and pulse length experiments

Several continuous records of the amplitude of individual echoes were obtained. For this experiment the transmitter pulse length was changed rapidly from 20 microseconds to 40 microseconds and back again by a motor-driven switch which operated 30 times a second. The echo amplitudes corresponding to each of the transmitter pulse lengths were displayed separately on a cathode ray tube and recorded on continuously moving film.

The experiment was performed in the hope that if only a few scattering centres were simultaneously contributing to the echo, the amplitude records corresponding to the two pulse lengths would show quite different variations with time. In fact, the records show a random fluctuation in amplitude at a rate too high to be measured with a switching frequency of 30 cycles/sec.

The pulse length was also varied manually between the limits mentioned above, while the echoes were observed visually. Even small isolated echoes seemed to show a uniform increase of amplitude with increasing pulse length. Since the bandwidth of the receiver was 2 Mc/s, the attenuation suffered by the shorter pulses in passing through the receiver would not be appreciable. The observations are interpreted, therefore, as indicating that each echo was due to a large number of scattering centres. The effect described in the latter part of section 2.09 would account for the observed variation of echo amplitude with pulse length, if the scattering centres were distributed in range over more than say 5 km.

Single, isolated volumes of ionization capable of reflecting 100 megacycle waves undoubtedly exist during auroral displays but since they were not detected we must conclude that they do not reach a size sufficient to be detected by the apparatus which was used. Only when a number of these volumes were situated close together did they give a detectable effect. Increasing the sensitivity of the apparatus would decrease the number of reflecting centres necessary to produce a detectable echo until, finally, a simple echo arising from a single reflecting centre could be observed. Such an observation would be of great value in determining the size of the individual reflecting centres.

4.11 Considerations concerning the size of the reflecting centres

The smallest observed echoes correspond to a total cross-section of about 2×10^4 metre^2 (section 4.08), but apparently this cross-section corresponds to several individual scattering centres. It is not unreasonable to assume that each scattering centre consists of a volume of ionization only a few metres across. The large echoes may arise either by an increase in the number of reflecting volumes or by an increase in their size. In any case it would seem that they represent considerably smaller volumes than that occupied by the visible auroral structures. Gartlein has examined auroral structures with the aid of field glasses (40). In particular, he noticed that the individual rays of ray arcs were made up of a multitude of very fine rays which were in continual motion, and continuously appeared and disappeared. It is logical to associate the reflecting volumes with the individual components of this fine structure, rather than with the gross structure visible to the unaided eye.

CHAPTER V

Details of the Reflection Process

5.01 High-angle observations

The aerial array used in the experiments already described radiated most of the energy at low altitude angles. In order to extend the observations to higher angles a new aerial array was built. The aerial consisted of fifteen colinear, half-wave elements mounted at a distance of one-quarter wavelength above a reflecting screen, and produced a fan-shaped beam passing through the zenith. The angular dimensions of the beam were about 7° in the east-west plane and about 140° in the north-south plane. The major axis of the beam lay along the magnetic meridian so that it encountered the auroral displays at right angles to their length. Two views of this aerial array are reproduced in Figure 5.01.

It was not possible to use an aircraft to measure the radiation pattern accurately but the calculated position of the beam in space was confirmed by the simultaneous observation of meteor trails and their radar echoes during the Perseid meteor shower of 1950.

The fixed aerial array was operated during a number of auroral displays occurring at various altitude angles between 20° and 90° but no auroral echoes were observed. In view of this result, an attempt was made to observe auroral echoes from the second lobe of the original aerial

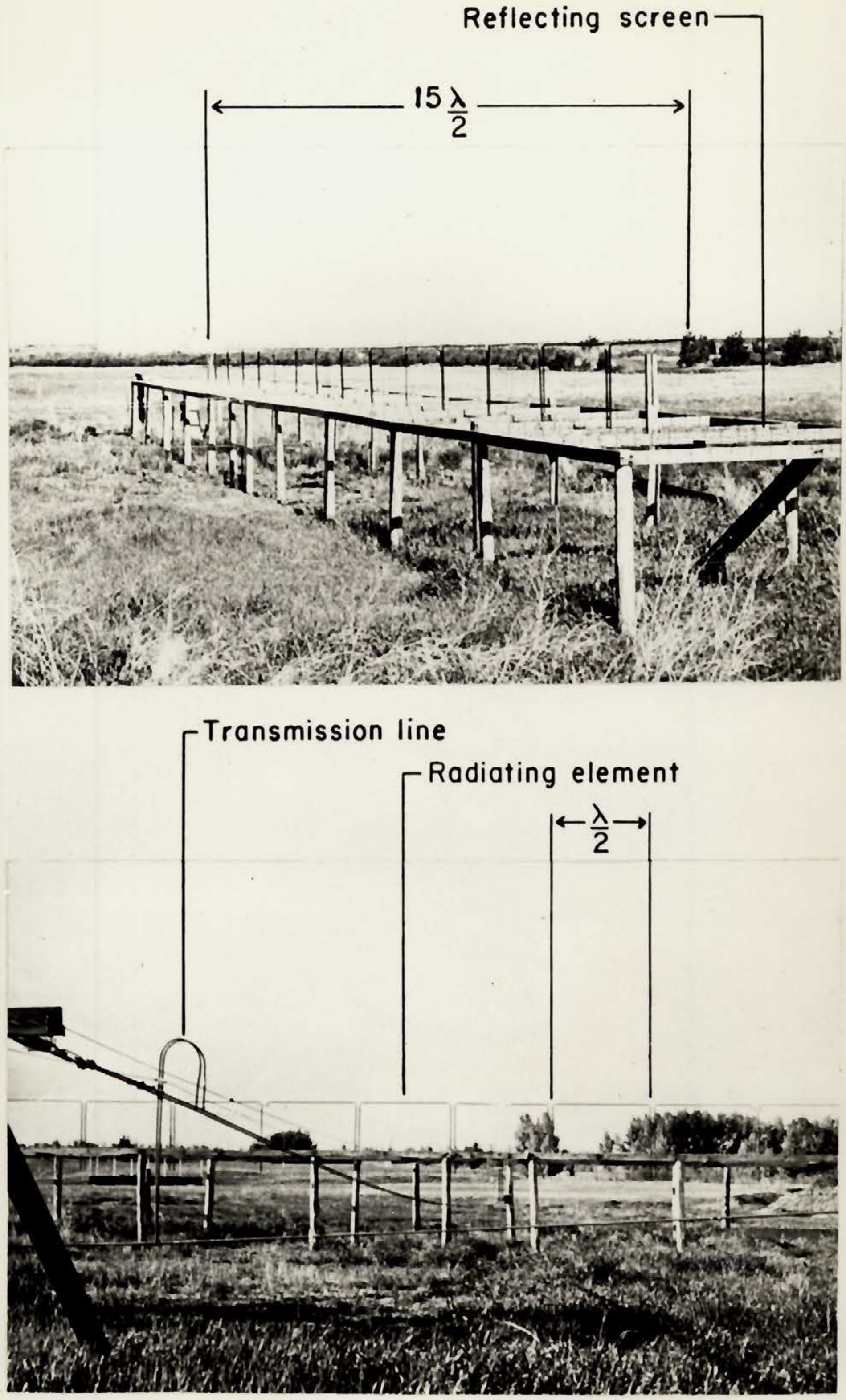


Figure 5.01 - The aerial array used for high-angle observations.

array. The maximum of this lobe is inclined at about 14.5° above the horizontal. The aerial array was tilted back 10° from the vertical so that approximately the same power was radiated into each of the first two lobes. With this arrangement auroral echoes were still observed from the first lobe (4.8° elevation) but none from the second. This left a region of the sky between 7.5° and about 13° which could not be investigated.

At first sight the lack of auroral echoes from the higher altitude angles would seem to indicate that the vertical angle through which the power was scattered by the auroral reflecting centres was very limited. However, calculations similar to those of section 4.06 show that if the direction of the magnetic field were the critical parameter the lack of echoes from the north at 13° altitude would not be consistent with the presence of echoes from the west at 5° altitude. We therefore must look for an effect which is dependent primarily on the altitude angle in order to explain the observations.

5.02 Atmospheric absorption

In this section we will discuss the effect of atmospheric absorption on the reception of radio echoes from aurora. The power absorption coefficient, K, is given by (Equation VIII, Appendix I).

$$K = \frac{4\pi}{c} \frac{Ne^2}{m} \left(\frac{f_c}{(\omega^2 + f_c^2)} \right)$$

where f_c is the frequency of collision between an electron and atmospheric molecules.

In order to evaluate the power lost it is necessary to know f_c and N for all points on the wave path. The collision frequency is given by

$$f_c = anv ,$$

where,

a = effective collision cross-section (cm^2);

n = numerical molecular density (cm^{-3});

v = mean electronic velocity (cm sec^{-1}).

The appropriate collision cross-section is the one which includes only processes which remove energy from the electron, i.e., ionization and excitation. The inelastic cross-section of nitrogen molecules for collision with low velocity electrons was discussed by Francis and Jenkins (38). From their discussion it appears that an appropriate value of the cross-section for the calculations is about 3×10^{-16} cm^2. The molecular densities at various heights were taken from the extensive tables published by Grimminger (45).

There is some doubt as to the appropriate value for v; however, the collision frequencies as a function of height were calculated for three different values of v and the resulting curves are plotted in Figure 5.02. The concept of temperature as applied to the ionosphere has been discussed by Mitra (67), who concludes that the

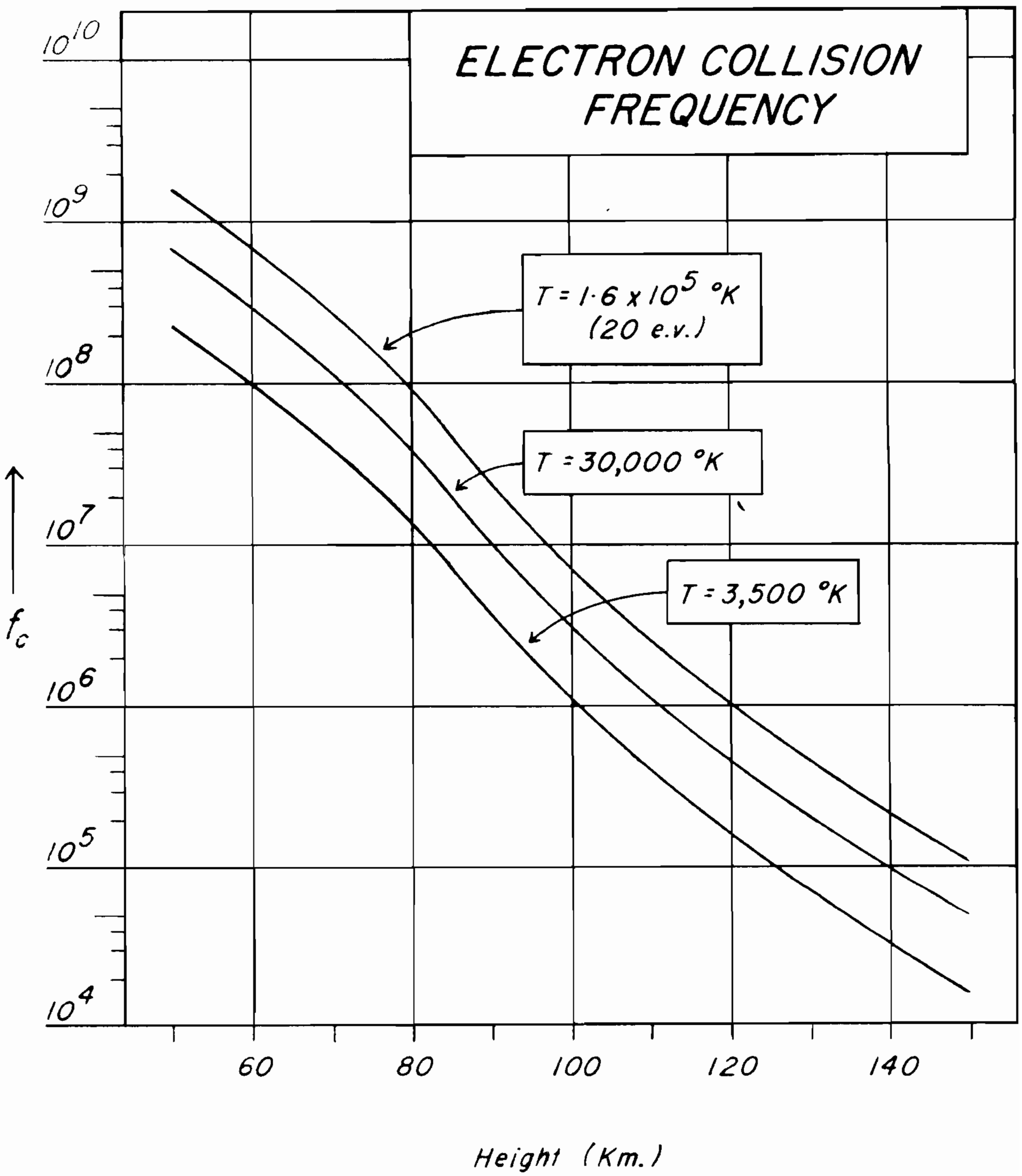

ELECTRON COLLISION FREQUENCY
T = 1·6 x 10^5 °K (20 e.v.)
T = 30,000 °K
T = 3,500 °K
10^10
10^9
10^8
10^7
10^6
10^5
10^4
fc
60
80
100
120
140
Height (Km.)

Figure 5.02

normal ionosphere is in thermal equilibrium. Scott has indicated his agreement with this conclusion (78). On the other hand, the experimental results of Riefman and Dow indicate that the electrons in the E-region are not in thermal equilibrium with the other atmospheric constituents (76). Petrie has discussed the concept of temperature as derived from spectroscopic observation of aurora and pointed out that since thermal equilibrium does not exist in the aurora the distribution of velocities is not Maxwellian (71). Lacking a knowledge of the distribution of electronic velocities we may define a temperature for our present purpose by the kinetic energy relation,

$$\frac{mv^2}{2} = \frac{3}{2} kT ,$$

where,

k = Boltzmann's constant;

T = electron temperature (^{o}K);

v = effective velocity of the electron;

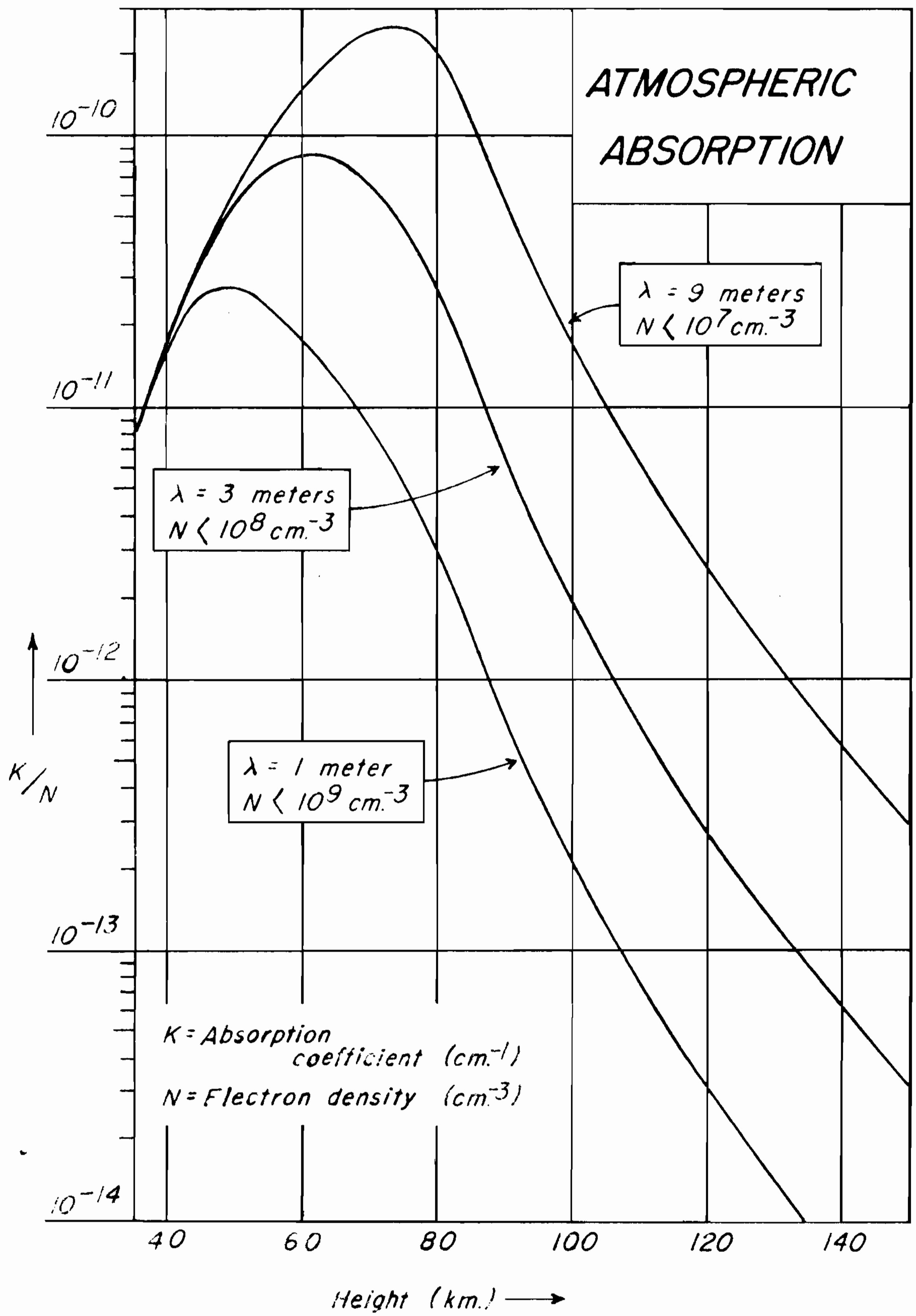
m = mass of the electron.

Barbier has deduced a value of approximately 30,000 ^{o}K for the electron temperature in the aurora (7). The highest value that this temperature is likely to have may be deduced in the following manner, after a suggestion made to the author by Dr. Petrie. If the ionizing particles are protons (see Chapter VI) they will lose about 36 electron volts energy per ionizing collision (43). The ionization potentials of atmospheric constituents are about 15 volts so that the maximum energy of the ejected electron will be about 20 ev corresponding to a temperature of 1.6×10^5 °K. A lower limit of about 3,500 °K may be inferred from the rocket measurements of the undisturbed E-region made by Riefman and Dow (76). The curves of Figure 5.02 correspond to these three values of electron temperature.

Typical curves showing the variation of the power absorption coefficient, K, with height and wavelength are reproduced in Figure 5.03. The quantity plotted is K/N or the absorption coefficient per unit electron density. The curves correspond to an electron energy of 20 ev. The region to the left of each peak is characterized by the condition $f_c > \omega$. Booker has pointed out that appreciable reflection does not occur in such a region (11). Obviously, if appreciable ionization exists below the height of maximum absorption, a vertically directed wave will suffer large attenuation both before and after reflection. This consideration undoubtedly explains the "fade-outs" which occur during auroral displays for the frequencies at which ionospheric equipment normally operates.

The total attenuation suffered by a wave may be expressed in the form,

$$A = \frac{10}{2.3} \ln \frac{P_o}{P} = \frac{10}{2.3} \frac{4\pi}{c} \int_0^x \frac{Ne^2}{m} \left(\frac{f_c}{(\omega^2 + f_c^2)} \right) dx ,$$



ATMOSPHERIC ABSORPTION
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
K/N
λ = 9 meters
N < 10^7 cm.$^{-3}$
λ = 3 meters
N < 10^8 cm.$^{-3}$
λ = 1 meter
N < 10^9 cm.$^{-3}$
K = Absorption coefficient (cm.$^{-1}$)
N = Electron density (cm.$^{-3}$)
40
60
80
100
120
140
Height (km.)

Figure 5.03

where A is the attenuation expressed in decibels and x is the distance measured along the ray path. Since there is no appreciable wave retardation during the reflection process (see section 3.08) the ionization must be confined to the region close to the auroral display. In this region, for the particular case of nearly horizontal rays, f_c, is almost constant along the ray path. We may, therefore, take all quantities except Ndx outside the integration sign and the attenuation is dependent only upon the total number of electrons encountered by each square centimetre of the wave front.

We may consider each reflecting centre to be surrounded by a sheath of electrons and we are concerned with the attenuation suffered by the wave in traversing this sheath before and after encountering the reflecting surface. Diffusion in the normal ionosphere was discussed by Ferraro (33, 34), but his conclusions are of little value in the present instance. While the precise form of the electron distribution assumed for the sheath makes little difference to the argument, we may assume that the electron density as a function of distance from the reflecting surface is given by,

$$N = N_0 \, e^{-x/D} \, ,$$

where D is a constant which we shall call the "sheath thickness". N_0 is, of course, the critical electron density given by Equation VI, Appendix I. Substituting this value into the expression for attenuation, performing the integration and remembering that the wave makes two transits of the sheath, we obtain,

$$A = \frac{20D\, f_c}{2.3c} \left(\frac{\omega^2}{(\omega^2 + f_c^2)} \right),$$

which may be written in the form,

$$\frac{D}{A} = \frac{2.3c}{20\, f_c} \left(1 + \frac{f_c^2}{\omega^2} \right).$$

Since the expression is only of interest when $\omega > f_c$, the second term in the brackets may be neglected. This expression is plotted in Figure 5.04 for the three values of electron temperature. The scale on the right of the figure indicates the lowest point on each curve which is valid for any particular frequency, i.e., the point at which $f_c = \omega$. The figure shows the sensitivity of attenuation to height and sheath thickness. By way of illustration, assuming the middle curve is appropriate to auroral conditions, echoes may not be expected from a height less than 100 km at a frequency of 500 kilocycles. At higher frequencies the value of D/A at this height is 10^3, so that, for example, the attenuation would be 1 db if the sheath thickness were 10 metres or 10 db if the sheath thickness were 100 metres. It is evident that echoes may only be

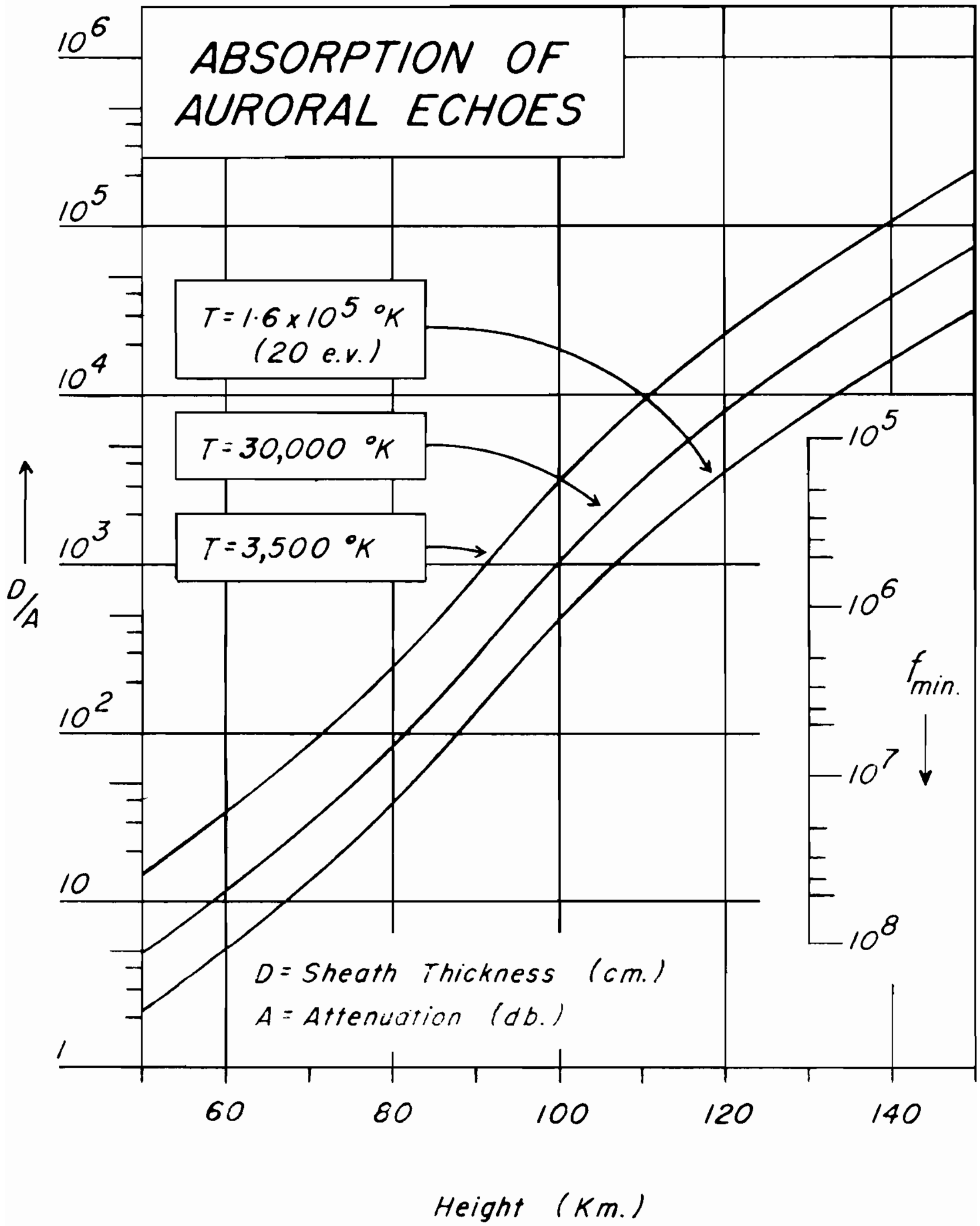

ABSORPTION OF AURORAL ECHOES
T = 1·6 x 10^5 °K (20 e.v.)
T = 30,000 °K
T = 3,500 °K
D = Sheath Thickness (cm.)
A = Attenuation (db.)
D/A
f min.
Height (Km.)
10^6
10^5
10^4
10^3
10^2
10
1
60
80
100
120
140
10^5
10^6
10^7
10^8

Figure 5.04

expected from the lower portion of an auroral display when the sheath thickness is small, i.e., the electron density gradient is high.

5.03 Auroral ionization

The variation of the light emitted by the aurora as a function of height has been discussed extensively by Harang (46, 47). He showed that there were three factors which favour a maximum emission of light near the lower edge of the displays:

(a) The atmospheric density decreases almost exponentially with increasing height so that incoming particles would suffer more collisions at the lower edge of the display than at the top.

(b) The collision cross-section of most charged particles is known to increase with decreasing velocity (see, for example, the observations of Geiger and Rutherford (77)).

(c) The incoming particles spiral around the magnetic lines of force and the convergence of the magnetic lines will eventually cause the particles to execute a great many revolutions at nearly constant height and then return to outer space.

Vegard has suggested that the spiral nature of the orbits is very important in determining the luminosity-height curve of the aurora (87, 88). While (b) and (c) can not, in general, operate simultaneously all these effects tend to cause a very rapid increase of auroral luminosity with decreasing height. Vegard and Harang have each suggested that the ionization distribution and the luminosity distribution are closely similar. In any case, we should expect the electron density within the display to increase rapidly with decreasing height.

There are two additional factors which should be considered with respect to auroral ionization: Firstly, studies of radio echoes from meteors made by McKinley and Millman (57) and of the fading of radio echoes from the ionosphere (see references 12, 57, 68, 69, 70, 74) show that the upper atmosphere has a discrete structure. It is not known whether this fine structure comes about by reason of density variations or by some parts of the ionosphere being more easily ionized than others, but it is reasonable to assume a similar structure for the auroral ionization. Secondly, the lateral diffusion of electrons will be inhibited by the Earth's magnetic field. Such an effect has been mentioned by Johnson (52) with respect to meteoric ionization and Ferraro has pointed out that this effect will only be present for regions where the collision frequency is less than the gyro frequency of the electrons in the magnetic field (33).

We may visualize the auroral ionization as consisting of many columns, each column lying along a magnetic line. The electron density increases rapidly toward the bottom but nevertheless shows a discrete structure. The lateral electron density gradient of the column is high because of the magnetic field. At a certain height, h_c, near the bottom of the column the collision frequency is of the same order as the gyro frequency (about 1.6 Mc/s) and below this height the electrons are free to diffuse rapidly in the lateral direction. This results in a region of low electron density gradient, and high collision frequency, which is, according to the preceding section, an absorbing region for radio waves. A schematic diagram of a vertical section through such a column is reproduced in Figure 5.05. We may further identify one or more reflecting centres with the region just above h_c and thus obtain a simple explanation for the apparent constancy of the height of reflection.

We may find an approximate value of h_c from Figure 5.02, although the electron collision frequencies shown in this Figure should be increased slightly to include the effect of elastic collisions. The curve for T = 30,000 ^{o}K gives a height, h_c, equal to about 104 km. The upper and lower curves give heights about 10 km above and below this value. The agreement between h_c found in this manner and the height of reflection as found in Chapter V is excellent.

It is not possible to estimate the dimensions of the

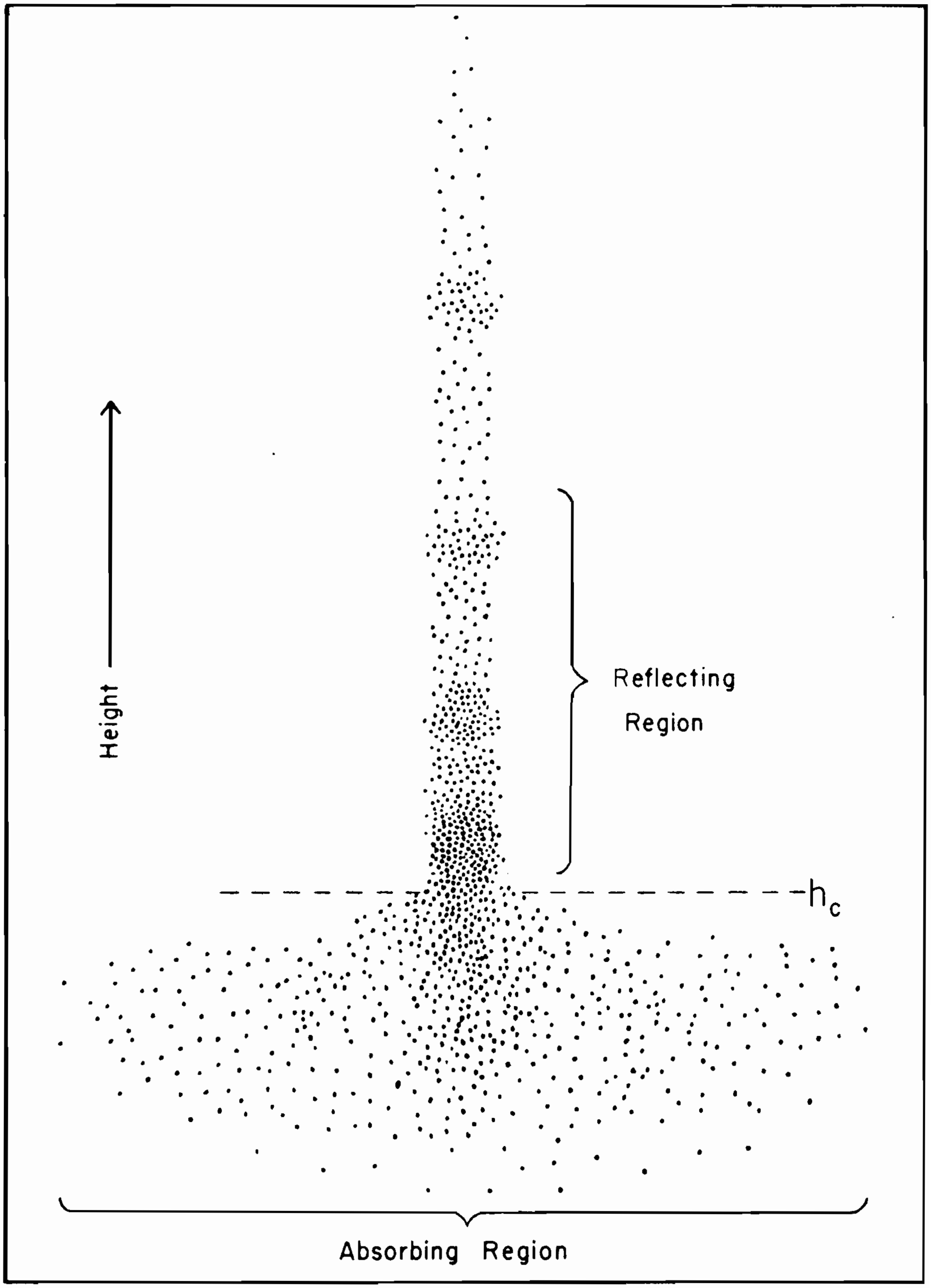


Figure 5.05 - A schematic diagram of the suggested electron distribution in a reflecting centre.

reflecting and absorbing regions with any degree of accuracy but again we may note here that the lateral dimension of the reflecting region need be only a few metres in order to satisfy the experimental observations.

5.04 A suggested explanation of high angle observations

While the configuration of auroral ionization is probably much more complicated than we suggested in the preceding section, we may use that picture to explain the high angle observations. If the reflecting region is confined to the region of highest electron density (just above h_c) then rays approaching at angles well above the horizontal will be prevented from reaching it by the lateral extension of the absorbing region. The geometry of the situation is represented in Figure 5.06(a). In this figure,

z = vertical extension of the screened region above h_c;

b = lateral extension of the absorbing region;

V = altitude angle of the ray for the radar station;

h = height of reflection, very nearly h_c;

r = radius of the Earth;

R = range angle of the aurora.

Evidently, $$S = R + V$$

and $$z = b \tan S.$$

In Figure 5.06(b) the variation of the screening function, z/b, with range is represented. While the effect of refraction is not great it has been included in this

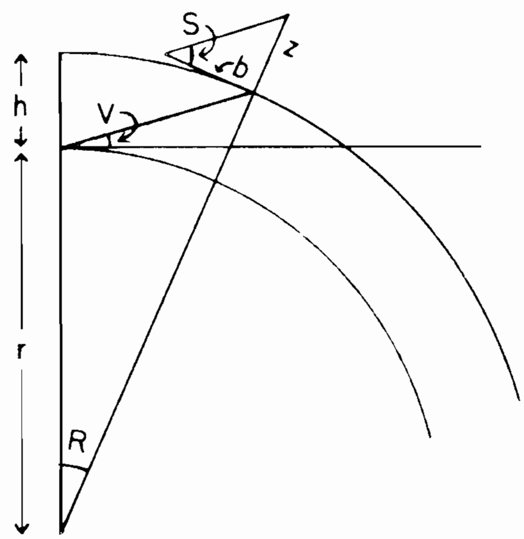


Figure 5.06 (a) - The geometry of high-angle observations.

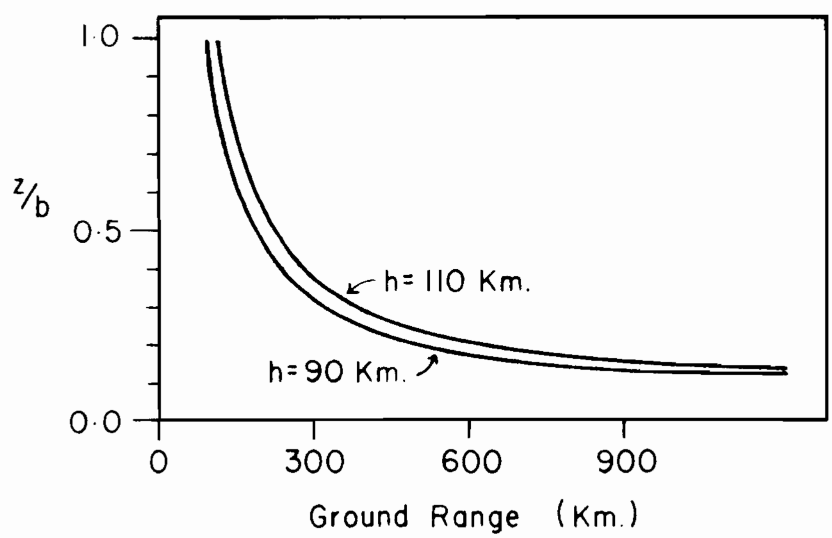


Figure 5.06 (b) - Form of function which may control high-angle observation.

Figure. The portion of the sky which was investigated without receiving auroral echoes corresponds to ground ranges between zero and about 270 km. Echoes were obtained at ranges greater than 600 km. The portion of the sky corresponding to the intervening ranges was not investigated.

We may also use the suggested electron density configuration to explain the occurrence of echoes from aurora in the west. When the arc was in the form of a smooth curve extending to the horizon in the west the radar beam encountered the display in a direction parallel to its length. For this situation we may visualize each reflecting centre as being screened from the radar beam by the absorbing regions associated with neighbouring reflecting centres at closer ranges. Evidently no echoes would result. On the other hand, when the display exhibited a sharp bend away from the length of the display, the reflecting centres in the bend would be exposed to the radar beam and give rise to the observed echoes.

5.05 Range-time records

A number of records were obtained of the variation of auroral echoes with time. Some of the records were made by causing the radial time base of the P.P.I. oscilloscope to rotate at a constant velocity while the aerial remained stationary. The camera was exposed for the duration of one cycle of the time base (6.25 minutes). Other records were taken with a separate oscilloscope using a vertical, intensity modulated trace. The time base in this case corresponded to a range of 150 km and could be delayed by various intervals so that echoes occurring at any range could be displayed. The oscilloscope was photographed by a camera in which the film was drawn horizontally at a constant rate. Portions of several records taken in this manner are reproduced in Figure 5.07, in which the records were chosen to show variety rather than because they were typical.

Several interesting characteristics are evident from the records. The duration of the echoes varied from a few seconds to about 30 minutes. The echoes showed a marked tendency to remain at a constant range, often disappearing and reappearing at the same range. Even for those records which show a range gradient, simultaneous observation of the range display showed that this apparent change of range came about by the growth and decay of separate echoes on the extremities of the primary echo.

There is ample evidence from ionospheric observations (see references 12, 58, 68, 69, 70, 74) that winds with velocities in excess of 50 metres/second exist in the ionosphere. Similar results have been reported by Greenhow who measured the rate of fading of meteor echoes (44).

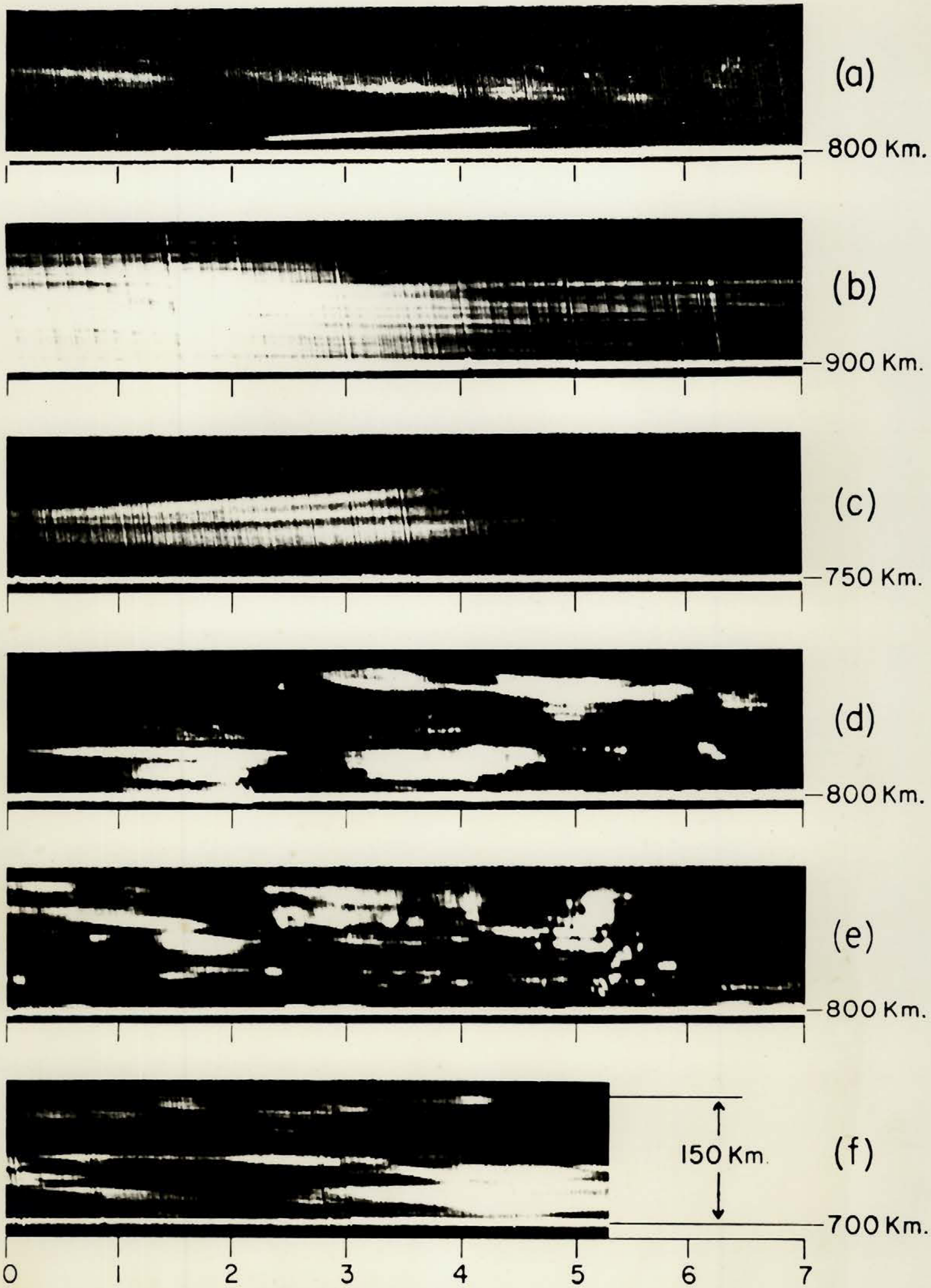


Figure 5.07 - Range-time records of auroral echoes.

Observations made by J.H. Chapman at McGill University seem to indicate that the winds have higher velocities during times of magnetic disturbance. No uniform drift of individual auroral echoes has been recorded which could be interpreted as being due to ionospheric winds. It appears that electron densities above the critical reflection value for 100 Mc/s do not persist for any appreciable time after the cessation of the primary ionizing process. The apparent drift of auroral echoes which has been noted and measured by Aspinall and Hawkins, and by McKinley (56) is undoubtedly due to the motion of the ionizing agent. Presumably this motion is the apparent motion of the point of precipitation of the incoming particles.

The record shown in Figure 5.07(a) displays an echo of unknown origin. This echo which appears at about the middle of the record and lasts for a little more than two minutes is quite unlike an auroral echo in appearance. While the auroral echoes at greater range show a progressive decrease in range this echo shows a uniform increase in range at an apparent radial velocity of about 100 metres/second. Meteor echoes observed at this frequency do not normally persist for more than a few seconds; however, it is possible that a large meteor passing through the residual ionization of the nearby auroral display caused the electron density to rise to a sufficiently high value to produce this enduring echo. In any case, the echo must have arisen from ionization formed in some such anomalous manner and its uniform velocity suggests the influence of an ionospheric wind.

A few records were obtained which show even greater range resolution. For these records the length of the time base corresponded to 40 km. One-half of the film was devoted to an intensity modulated range-time record similar to those discussed above. At ten second intervals a range-amplitude presentation was photographed on the remaining half of the film. The resultant record provided a continuous record of the range and extent of the echoes as well as an intermittent record of echo amplitude. The technical details involved in obtaining this type of record are given in Appendix II. A typical record is reproduced in Figure 5.08(a). The vertical dark lines in the continuous record indicate the times corresponding to the amplitude photographs. The continuous record is also broken at 30 second intervals to show range calibration marks which are 8 km apart. Figure 5.08(b) shows an enlarged section of the same record.

5.06 Echoes arising in sunlit aurora

One of the important questions which arise in any discussion of a radio investigation of aurora is

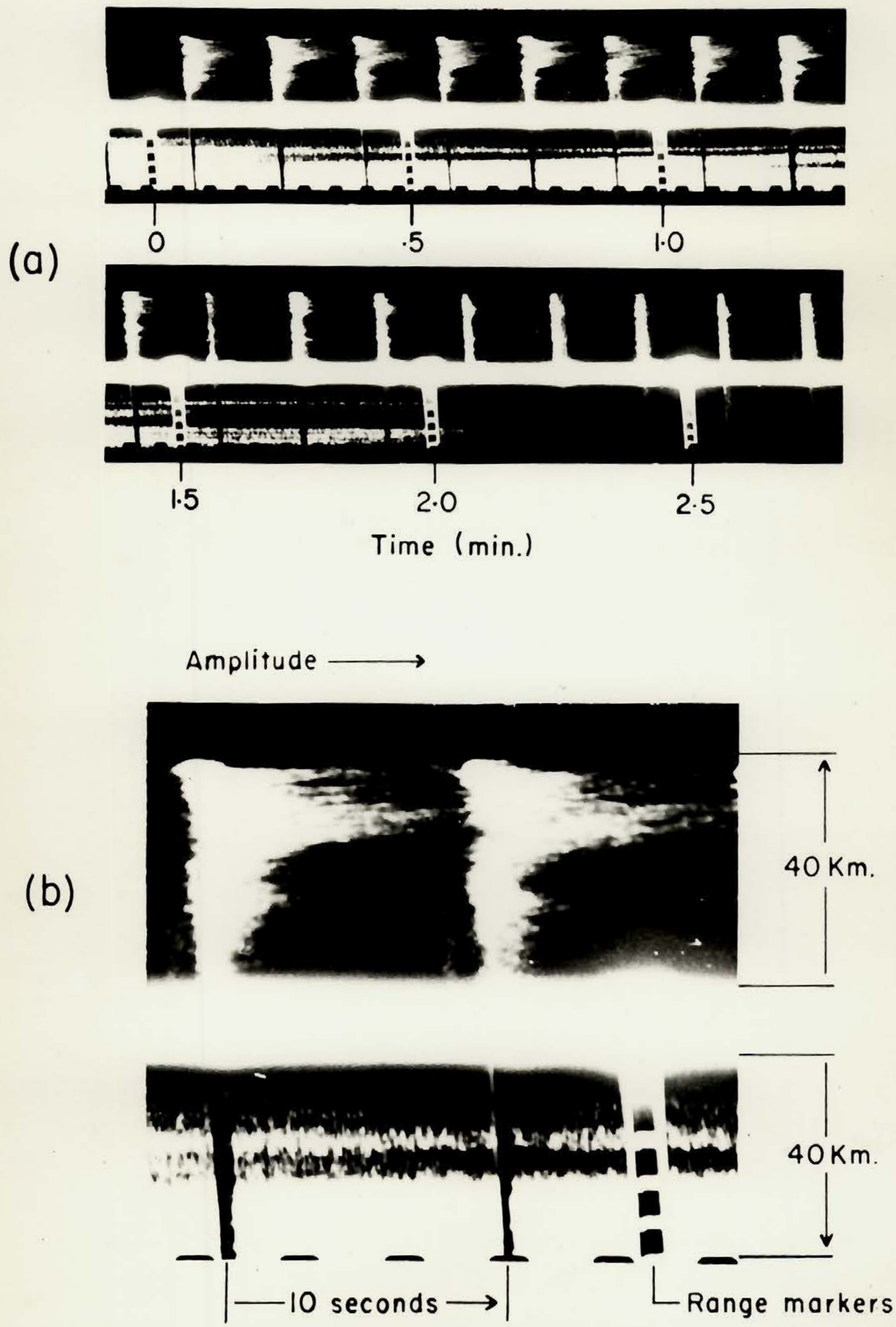


Figure 5.08 - Continuous record showing the variation in amplitude and range of auroral echoes.

whether or not auroral echoes may be observed during the daylight hours. The evidence pertaining to this question is not conclusive. Casual observation during many daylight periodsof operation has revealed no auroral echoes. The radar equipment was operated continuously for 56 hours during the auroral period of August 18-21, 1950. Auroral echoes were observed during the nights of August 18/19 and 19/20 but not on the night of August 20/21. At 07 hrs, 15 min of the 19th small echoes were observed for a few seconds at a range of 970 km. Sunrise occurred at about 04 hrs, 15 min. Since this is the only occasion upon which echoes of possible auroral origin were observed during daylight hours, the author hesitates to attach any great significance tc the occurrence. Unfortunately, the observation was made by a radar operator who was not well acquainted with the appearance of auroral echoes.

On the other hand, auroral echoes were often received from sunlit aurora, i.e., from aurora which was sunlit although the surface of the Earth immediately below the display was in darkness. Such aurora is often observed visually. Table 5.01 lists the relevant data for a number of such echoes observed after sunset on the 15th and 16th of May 1951. The second and third columns give the times (Mountain Standard) of appearance and disappearance of the echoes. The fourth and fifth columns give the latitude

TABLE 5.01

Echoes from Sunlit Aurora

Date	Time of Appear-ance	Time of Disappear-ance	Lat.	Long.	Depression of the Sun	
	(MST)	(MST)	(°N)	(°E)	°	'
15/5/51	2102		60	249	1	30
		2107				
	2109		60	249	2	07
		2121			3	08
	2123		59	251	4	43
		2128				
	2130		60	251	4	30
		2136				
	2137		58.5	251	6	13
		2139			6	24
16/5/51	2110		58.5	259	7	00
		2117				
	2117		58	259	7	21
		2118			7	24

and longitude of the point immediately below the reflecting region and the last column gives the angular depression of the Sun below the horizon for an observer situated at this point. The calculations involved in finding these depression angles were made by Mr. C.H. Costain, to whom the author is indebted. The level 100 km above the surface of the Earth remains sunlit until the Sun is 10° below the horizon.

CHAPTER VI

Concluding Remarks

6.01 Summary of the experimental evidence

In Chapter III we saw that the radio echoes observed during auroral displays actually arose from reflecting centres within the visible aurora. In Chapter IV we examined the evidence pertaining to the nature of the reflecting centres and found that they were small volumes of ionization in which the electron density was greater than 10^8 electrons per cubic centimetre. The reflecting centres were found to occur only in a thin layer situated about 100 km above the surface of the Earth. In Chapter V high angle radio observations of aurora were described. Auroral echoes were not observed at altitude angles above 7.5^o, although altitude angles between 7.5^o and 13^o were not investigated because of the limitations of the equipment. Based upon several reasonable assumptions concerning the aurora, an explanation of this fact was presented, and experimental evidence supporing the explanation was cited. We also saw that auroral echoes arose only during the operation of the ionizing mechanism. The evidence concerning auroral echoes during the daylight hours is inconclusive but indicates that such echoes either do not exist or are very rarely observed at a frequency of 100 Mc/s. We will discuss the significance of these results in the succeeding sections.

6.02 Height of reflection

The history of experiments and theories concerning the polar aurora was summarized in 1937 by Hewson (50) and more recently by Currie (28) and by Chapman (19). Chapman has reviewed the overwhelming evidence in support of the view that aurora is caused by solar corpuscles (20), but pointed out that the famous calculations of Stormer (82) could not apply to the motion of the particles between the Sun and Earth, although they might describe adequately the trajectories of the particles close to the Earth. The entry of protons into the Earth's atmosphere during auroral displays has been shown by observations of the Doppler shift exhibited by the Hα line in the auroral spectrum. Such observations have been made within the past year by Meinel (62) and Gartlein (41). Martyn recently has proposed a theory (59) which contains many of the features of an earlier theory by Chapman and Ferraro (see references 21 to 26). According to this theory charged particles are accelerated into the Earth's atmosphere. The current represented by the particles flows down the lines of force of the magnetic field (since this is the direction of highest conductivity), until it reaches a particular level in the atmosphere where it flows horizontally across the auroral zone, eventually returning to outer space. The height at which the horizontal conductivity of the atmosphere becomes appreciable is just the height of our absorbing regions according to section 5.03. We have also suggested that this height is very nearly the height at which reflections of radio waves occur. Martyn has suggested that the horizontal current flows at about the 90 km level. According to the viewpoint presented here, our observations show that if such a current exists it will flow somewhat closer to the 100 km level.

The apparent height of the visible aurora was determined in section 3.07 by combining radar and photographic observations. Four of the determinations yielded heights about 20 km above the height of reflection. Vegard has suggested that all such extreme values of the determined height are due to errors arising from diffuseness of the photographic image, and that the true height of all draperies and ray arcs is close to 100 km (88). While this statement is open to doubt, it is certainly true that the height of the lower edge of most of these forms is close to 100 km. It is apparent that this level is unique in some way and it is possible that the unique characteristic is just the one which, according to section 5.03, fixes the height of reflection.

6.03 The electron density in auroral displays

The density of 10^8 electrons per cubic centimetre which was derived from our observations is higher than that found by other workers (see section 1.01). It has been suggested by Bates (8) and Petrie (72) that such a high electron density is inconsistent with spectroscopic data. The principal difficulty is that an electron density of 10^8 cm^{-3} would lead to a rate of emission of light quanta higher than is observed. We note, however, that according to our observations the electron density of 10^8 cm^{-3} applies only to volumes much smaller than that occupied by the whole auroral display. The average electron density throughout the display is certainly much smaller. It is this average density (which can not be derived from our observations) that should be consistent with the spectroscopic measurements.

6.04 Persistence of the reflecting centres

Petrie has also pointed out that if the electron density is as high as 10^8 cm^{-3} near the 100 km level, the electrons will be removed rapidly by recombination (72). Our observations show that electron densities of this magnitude do not persist for any perceptible interval after the cessation of the ionizing process. We may also note that the ionizing agent is only strong enough to produce this electron density for short periods of time compared with the total duration of the auroral displays.

6.05 Daylight aurora

The fact that auroral echoes have not been observed frequently during daylight hours should not be construed as indicating the absence of daylight aurora. The presence of the E-region at about the 100 km level during the day may be expected to exert a marked influence on any further ionization which occurs. The configuration of auroral ionization suggested in Chapter V would certainly be altered considerably by the presence of the E-region ionization. It is quite possible that aurora could exist during the daylight hours and still not be detectable by the use of 100 Mc/s radio waves.

The author can offer no suitable explanation as to why auroral echoes should be observed just after the Sun has set for an observer under the display and not before this time. It does not seem likely that any marked change can occur at the 100 km level at the instant that the Earth's surface ceases to be illuminated.

6.06 The results of other workers

It is not wise to attempt a detailed comparison of results obtained by various workers at different locations and using different radio frequencies. However, some aspects of the observations reasonably may be compared. The radio echoes discussed in the preceding chapters indicate a higher apparent reflection coefficient for the aurora than those reported by other workers using lower radio frequencies. Aspinall and Hawkins used a directive aerial array from which the first maximum of the radiation pattern was directed at 8.5° above the horizontal. The other workers used simple aerials from which most of the power was directed vertically. Since it is not reasonable to assume that the reflection coefficient of an auroral display increases with increasing frequency, we must conclude that the echoes increase in amplitude with decreasing angle of elevation of the radar beam. This is in agreement with our high-angle observations. If one assumes that all the radio echoes arose from reflecting centres near the 100 km level, examination of the minimum ranges reported indicates that the maximum elevation angle at which echoes may be detected increases slightly with decreasing frequency. This also is in accord with the suggested configuration of auroral ionization.

The observations reported by Gerson (42), which were made at a frequency of 50 Mc/s, support several of the conclusions drawn from the present data. The report concerns a specific occasion when a number of widely separated amateur radio stations were able to communicate with each other by directing their aerials toward an auroral display in the north. Although Gerson suggests that reflections took place at an auroral "surface", the fact that the radio stations were separated by as much as several hundred kilometres in both the north-south and east-west directions indicates that the energy was scattered widely in both the vertical and horizontal planes. This is in agreement with the conclusions drawn in Chapter IV. The reflected signals were also modulated in such a manner as to make speech unintelligible. This effect probably comes about in the same manner as the rapid fluctuation of the individual components of the observed radar echoes. In section 4.10 we pointed out that lengthening the transmitted pulse would increase the apparent reflection coefficient of the aurora, if the reflections actually arose from a large number of reflecting centres. The equipment used by amateurs is relatively insensitive but operates by means of continuous wave transmission (infinite pulse length). The fact that auroral reflections could be detected by such equipment is ample confirmation of the presence of many reflecting centres.

6.07 Suggestions regarding further research

A considerably larger volume of data than hitherto has been available, was accumulated in the course of the present research. However, much remains to be done before the potentialities of the method are exhausted. A few aspects of the problem which appear to the author to be worthy of further research are listed below:

(a) Observations should be made to determine the actual upper limit in frequency at which radio reflections occur. This would indicate the upper limit of auroral electron densities.

(b) Simultaneous observations on two or more frequencies would provide a much clearer picture of the auroral processes involved in the formation of the reflecting centres.

(c) The maximum elevation angle at which auroral radio reflections are observable should be determined, and the variation of this angle with frequency should be investigated.

(d) A comprehensive program of observations should be undertaken during the daylight hours in order to settle the question as whether or not echoes from daylight aurora actually occur.

(e) Radar observations should be combined with ionospheric records taken at the point under the display to test the validity of the conclusions drawn in Chapter V.

(f) Radar observations should be combined with spectroscopic and photometric observations in order to determine the relations existing between the electronic density and the various ionic densities.

(g) The radar method has already proven its usefulness as a tool in the study of the occurrence, duration, movement and configuration of auroral displays. Observations similar to those described in the previous chapters could be carried on over longer periods of time in order to provide much needed data on the general characteristics of aurora.

APPENDIX I

Propagation in an Ionized Medium

The propagation of radio waves in an ionized medium has been discussed extensively in the literature (see, for example, references 2, 11, 30, 51, 63, 64, 65, 83, 84); however, these treatments are, in general, far more complex than necessary for the present purpose. The complications arise from the presence of the Earth's magnetic field which renders the medium anisotropic to the frequencies employed for ionospheric studies. The now famous Appleton-Hartrie equation applies to propagation at these frequencies. The frequency used in the present study is far removed from the gyro frequency of electrons in the Earth's magnetic field (about 1.6 Mc/s) and, in consequence, the effect of the Earth's field is so slight that it may be neglected. On the other hand, the effect of electron collisions is often neglected in ionospheric studies, but in view of the high electron temperatures which undoubtedly exist during auroral displays this effect must be taken into account in the present work. The following elementary treatment, while containing nothing which is not inherent in the more extensive treatments cited above, serves to indicate the origin of the expressions used in the body of the thesis and their regions of validity.

We consider that the ionized region is rendered conducting by the presence of free electrons (neglecting the effect of the heavy ions) and that the dielectric constant and magnetic permeability of the medium are each unity. In common with much of the literature c.g.s. (Gaussian) units are used throughout. The following notation is used:

e = electronic charge;

c = velocity of light;

f_c = frequency of collision between an electron and atmospheric molecules;

f = wave frequency;

$\omega = 2\pi f$;

x = space coordinate in direction of propagation;

t = time coordinate;

γ = propagation constant;

n = index of refraction (real);

σ = conductivity of the medium;

k = optical absorption coefficient;

N = numerical free electron density;

J = current density;

E = electric field vector of the wave;

H = magnetic field vector of the wave.

The appropriate forms of Maxwell's equations are,

$$\nabla \times H = \frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi\sigma E}{c}$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t}$$

$$\nabla \cdot E = \nabla \cdot H = 0,$$

which results in a wave equation of the form,

$$\nabla^2 E = \frac{4\pi\sigma}{c^2}\frac{\partial E}{\partial t} + \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2},$$

and we may assume, as a solution, the propagation equation,

$$E = E_0\, e^{j(\gamma x - \omega t)} \qquad \text{I.}$$

If γ is complex we may separate the real and imaginary parts and write the propagation equation in the form,

$$E = E_0\, e^{j\omega(\frac{nx}{c} - t)}\, e^{-kx} \qquad \text{II.}$$

Substituting I into the wave equation we find,

$$\gamma = \frac{\omega}{c}\left(1 + \frac{j4\pi\sigma}{\omega}\right)^{1/2} \qquad \text{III.}$$

It is now necessary to introduce the equation of motion of the electrons and from it deduce a value of σ. Appleton and Chapman (3), following Lorentz (53), have shown that the rate of loss of momentum by a single electron, due to collisions, is $m\dot{x}f_c$. The equation of motion is then,

$$m\ddot{x} + mf_c\dot{x} = eE.$$

There has been much discussion as to whether or not another term, the so-called "Lorentz polarization term", should be added to this equation. Darwin has examined the question theoretically (30). Mitra (66) has examined the experimental evidence of Booker and Berkner (9, 10), Martyn and Munro (60, 61) and Appleton, Farmer and Ratcliffe (4, 32, 75). Both Darwin and Mitra conclude that it is incorrect to include the polarization term. In any case the inclusion of the term only reduces the deduced value of N_o by one-fourth and has a negligible effect upon the absorption coefficient for auroral conditions.

The solution of the equation of motion is,

$$x = \frac{jeE}{\omega m\ (f_c - j\omega)},$$

and

$$\dot{x} = \frac{e}{m} \frac{E}{(f_c - j\omega)},$$

but

$$J = Ne\dot{x} = \frac{Ne^2E}{m(f_c - j\omega)} = \sigma E,$$

therefore,

$$\sigma = \frac{Ne^2}{m} \frac{(f_c + j\omega)}{(f_c{}^2 + \omega^2)},$$

and if we define,

$$G = \frac{4\pi Ne^2}{m(f_c{}^2 + \omega^2)}, \qquad \text{IV.}$$

then

$$\sigma = \frac{G}{4\pi}(f_c + j\omega).$$

Substituting this value of σ into III we obtain,

$$\gamma = \frac{\omega}{c}\left(1 + jG\frac{f_c}{\omega} - G\right)^{1/2} \qquad \text{V.}$$

The real part of the expression is never zero for finite values of f_c, however, let us first assume $f_c = 0$. For this particular case the expression is real and comparison of I and II shows us that,

$$k = 0,$$

and

$$n = (1 - G)^{1/2},$$

and for $G = 1$, $n = 0$.

At the point in the medium where this condition is fulfilled, the phase velocity becomes infinite, the group velocity is zero and total reflection takes place. The value of N for which total reflection occurs at a given frequency is called the critical density, N_o. With the aid of IV we find,

$$N_o = \frac{\omega^2 m}{4\pi e^2} = 1.24 \times 10^{-8} f^2 \,. \qquad \text{VI.}$$

In reality f_c is finite but, for $f_c < \omega$ both the real part of V, and the index of refraction have a minimum near $N = N_o$, resulting in partial reflection. The reflection coefficient, however, is of the order of unity for all values of f_c appreciably below ω (cf. Booker (11)). For this range of f_c and for $N < N_o$, it is permissible to expand V with the aid of the binomial theorem. Neglecting powers of G higher than the first and once again comparing I and II we obtain,

$$n = 1 - \frac{G}{2} = 1 - \frac{Ne^2}{2\pi m f^2}\left(\frac{\omega^2}{(\omega^2 + f_c^{\,2})}\right), \qquad \text{VII.}$$

and

$$k = \frac{G f_c}{2c} \,.$$

The power absorption coefficient, K, is just twice this value. On substitution from IV,

$$K = \frac{4\pi}{c}\,\frac{Ne^2}{m}\left(\frac{f_c}{(\omega^2 + f_c^{\,2})}\right) . \qquad \text{VIII.}$$

APPENDIX II

Technical details of the equipment.

Introduction

The radar equipment used for the experiments was a standard long-range warning set designed for military application. In the course of the experiments the display and modulation systems were modified in order to make the equipment more suitable for auroral investigation. The particular form of the modifications was determined largely by the existing circuits so that the end result was often more complicated than would be necessary if new equipment were being designed for the purpose. For this reason a detailed discussion of all the electronic circuits used for each experiment would be of little value. Instead a typical arrangement of the apparatus is discussed and those circuits which may be of particular interest are reproduced.

The author wishes to acknowledge the contributions made by Mr. F.E. Vawter to the development of the equipment. The electronic circuits used in modifying the equipment were designed initially by the author; however, many of the units which were designed and built in some haste during the summer months were rebuilt in an improved form during the winter months by Mr. Vawter.

The remotely operated auroral camera

The remotely operated camera was actually designed by the author for use in an earlier investigation (36), but it is mentioned here because it has not been described previously. The camera consists of three units. The first unit is similar to an ordinary 35 mm still camera except that in place of the shutter a dark slide is fitted and the framing wheel drives a cam which operates a switch. The second unit, into which the first fits, consists of a weather-tight box and contains the solenoid-operated shutter and the film-winding motor. The motor is connected to the film take-up spool by a flexible shaft. The third unit, located at the operating position, contains the camera control circuit. A schematic diagram of the control circuit is given in Figure A2.01 from which the operating cycle is apparent. The cycle is initiated by closing the "reset-expose" switch. The shutter first opens for a period of time determined by the settings of the "exposure" and "multiplier" switches, then closes and the motor winds the film on a distance corresponding to one frame. The motor is controlled by two relays. This is necessary in order to cause the motor to run continuously, once energized, until the framing switch opens after closing once.

A second remotely controlled camera has been built recently by Mr. Vawter in which the "reset-exposure" switch is replaced by a relay operated by a push button. In using this circuit it is not necessary to reset the circuit manually between exposures.

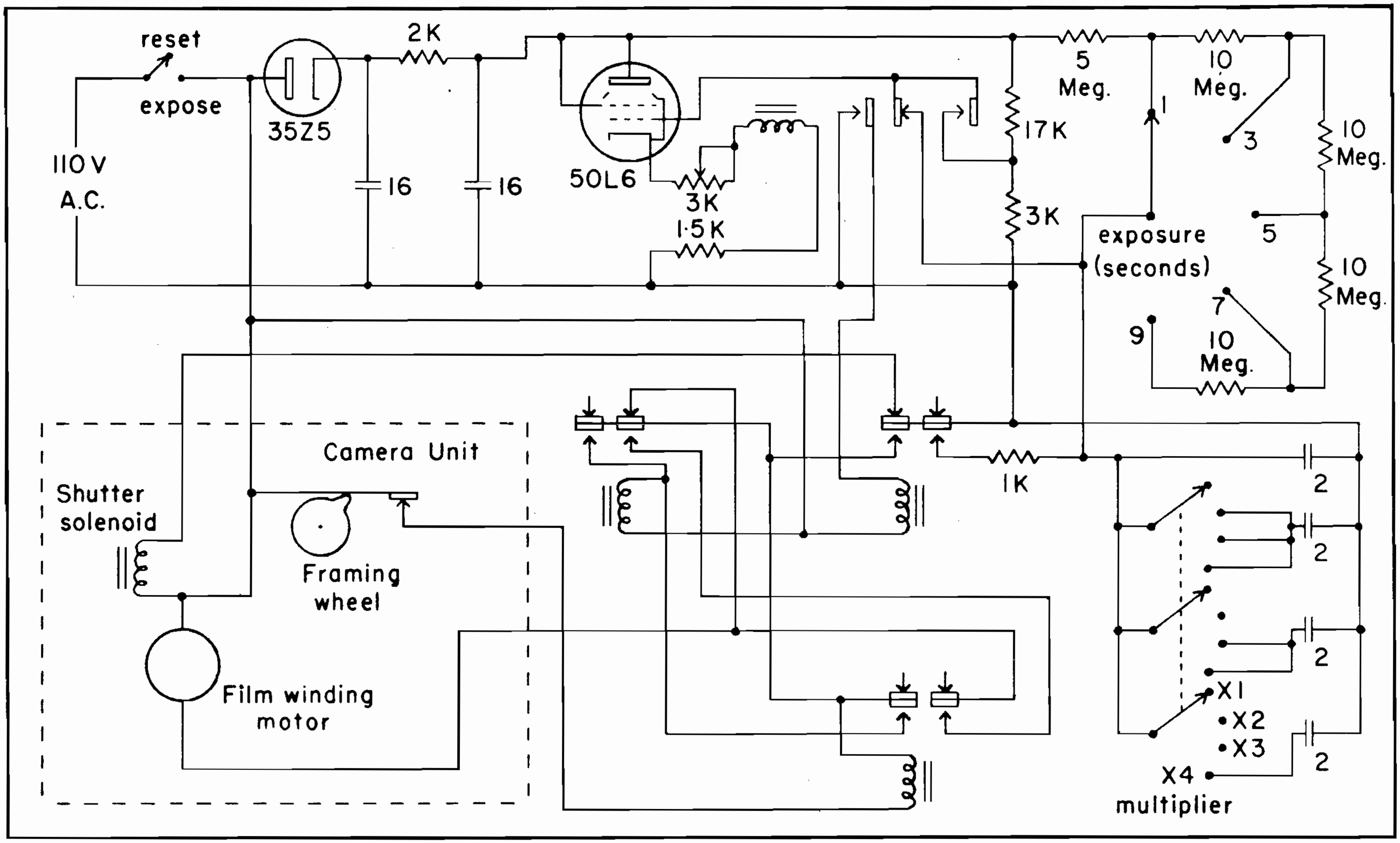


Figure A2.01 - Schematic diagram of the remotely controlled auroral camera.

The camera was mounted on a remotely controlled turn-table and the four connections necessary for its operation were made through slip-rings in the base of the turn-table.

Display and modulation circuits

The types of displays used were described in section 3.03. A schematic diagram of the circuits associated with these displays is given in Figure A2.02. In this figure the part to the left of the broken line includes the units which were present in the original equipment. The only alterations necessary in these units were to reduce the oscillator frequency to 250 cycles/sec and to increase the length of the time base to 400 km. Every second trace was removed from the cathode ray tubes by the application of a suitable waveform to the control grids. It was then necessary to trigger the transmitter at any one of four times relative to the start of the visible trace. These times differed by 1/500th of a second or one-half cycle of the master oscillator. The sine wave voltage from the master oscillator was passed through the phasing unit to the paraphase circuit which provided two, 250-cycle sine waves, 180° out of phase with each other. By choosing between these two voltages, the transmitter pulse could be placed at either of two positions on the cathode ray tube trace, separated by 300 km in range. This sine wave was applied to the square wave generator (an overdriven amplifier)

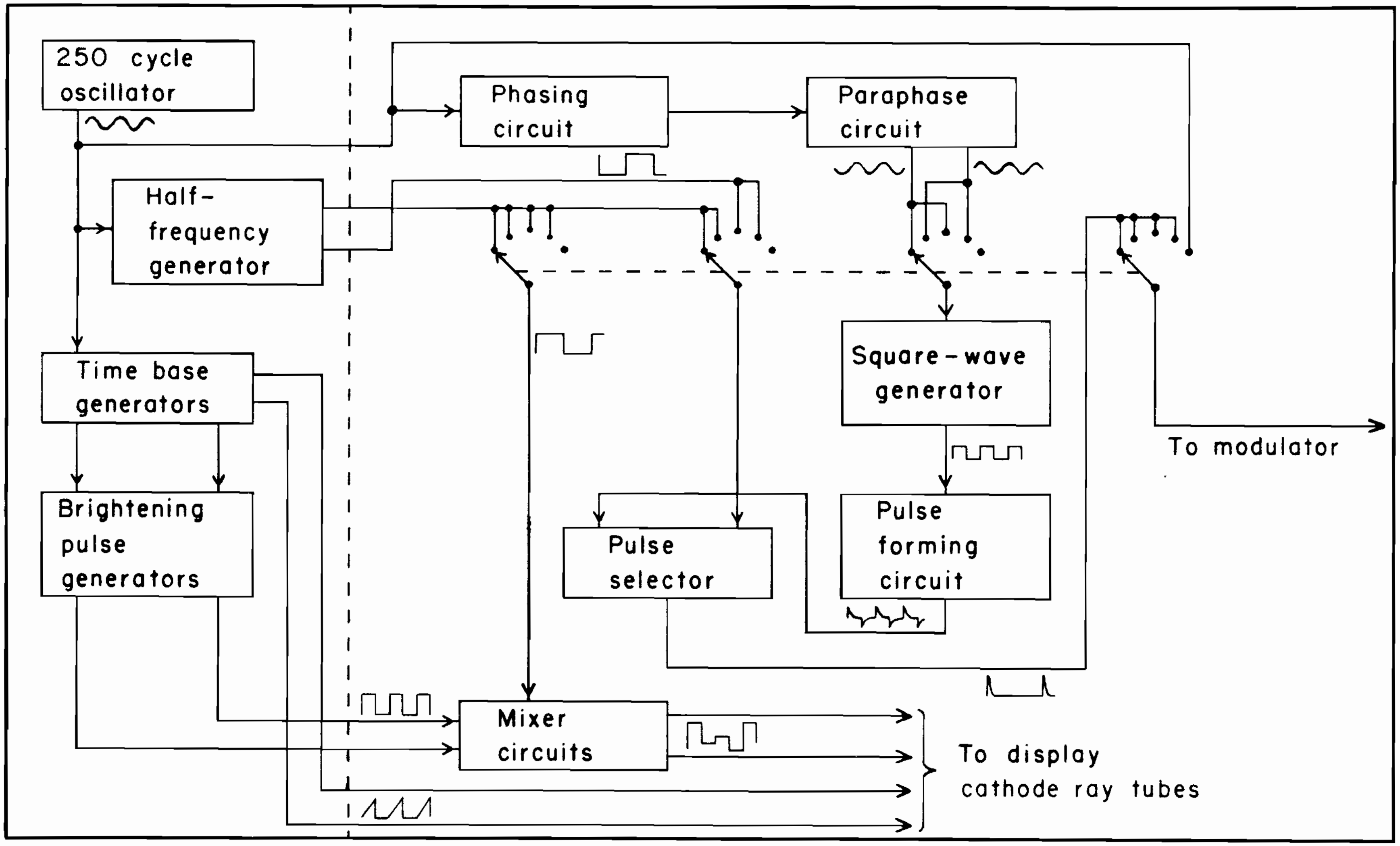


Figure A2.02 - Block diagram of the display and modulation systems.

and then differentiated in the pulse forming circuit. Every second pulse applied to the pulse selector circuit was allowed to pass on to the modulator to trigger the transmitter. A 125-cycle square wave was applied to the mixer circuits and to the pulse selector. The pulse selector allowed only those pulses, which occurred during one-half cycle of the square wave, to pass through it. If the same phase of the square wave were applied to both the mixer circuits and the pulse selector the transmitter pulse appeared on the visible trace, otherwise it was not visible on the cathode ray tube. The purpose of the mixer circuits was to add the 125-cycle square wave to the normal brightening pulse in order to render every second trace invisible. The four modes of operation were selected by the first four positions of the switch shown in the diagram. The fifth position provided normal operation of the radar equipment at a pulse recurrence frequency of 250 per second.

The form of most of the circuits is evident from the names given them in Figure A2.02. A circuit diagram of the pulse selector is reproduced in Figure A2.03. This circuit operates on the principle of diode clamping. The square wave is applied to the first stage and paraphase voltages appear at the anodes of the first 6SN7. These anodes are connected through two series diodes so that they conduct during alternate half cycles of the square wave. The input

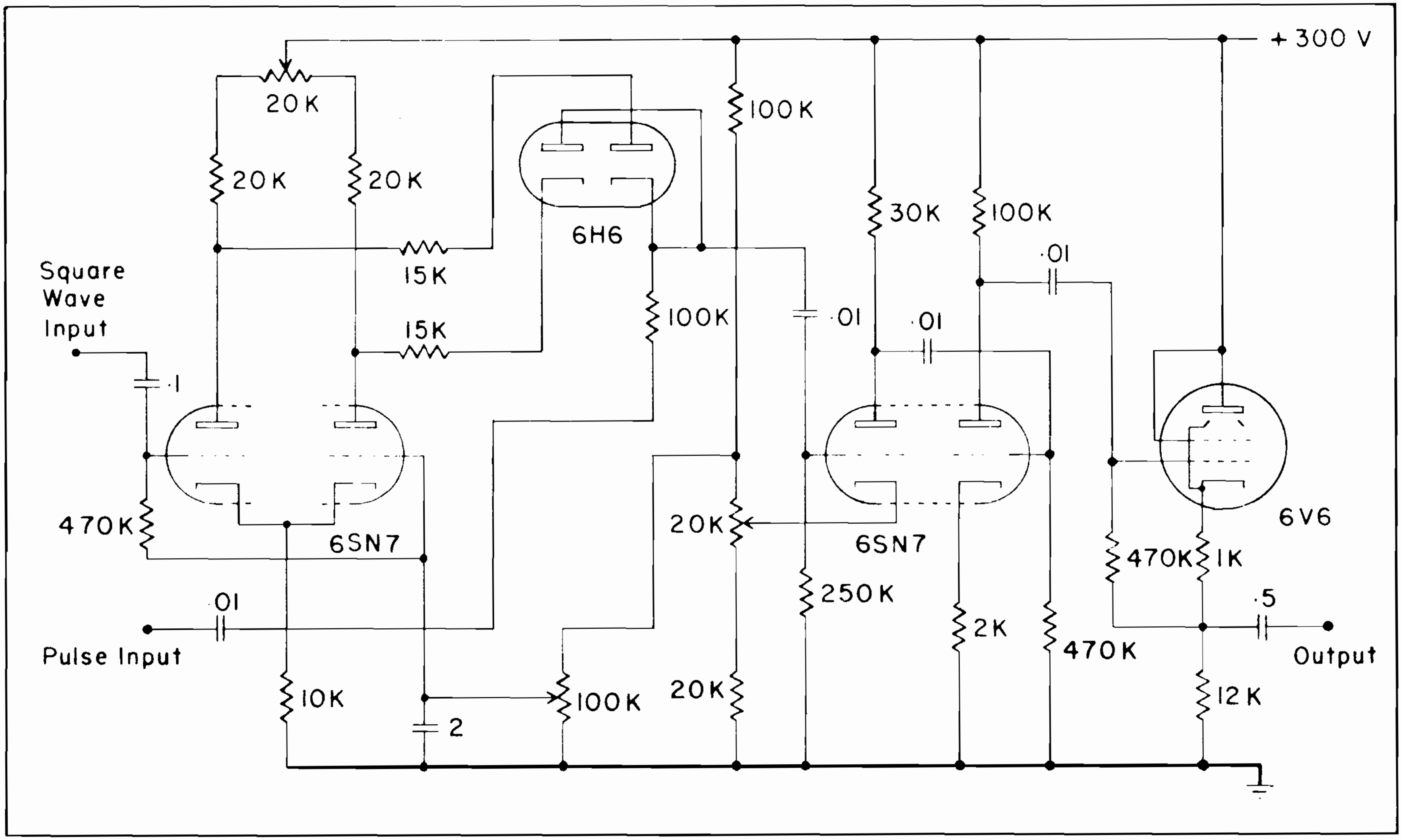


Figure A2.03 - Circuit diagram of the pulse selector.

pulses are applied to the point between the two diodes so that those pulses which occur while the diodes are conducting, are much reduced in amplitude. The remainder of the circuit is designed to select only those pulses of greatest amplitude and amplify them. The cathode follower output stage is necessary since the pulses are carried to the modulator through a long cable.

<u>Apparatus for continuous recording</u>

The arrangement of the apparatus for recording the auroral echoes on continuously moving film is shown in Figure A2.04. The part of this figure to the right of the dotted line represents circuits originally associated with the oscilloscope. The output from the variable delay circuit consisted of pulses which were delayed by a variable interval after the occurrence of the transmitter pulse. These pulses were used to trigger the oscilloscope time base. The operation of the apparatus is evident from the diagram. When the switch was in the position shown, an intensity modulated trace was presented on the upper half of the face of the cathode ray tube. When the switch was in the other position a range-amplitude trace was presented on the lower half of the face of the cathode ray tube. In fact, the contacts shown in the diagram were operated by a relay which in turn was energized once every ten seconds by a mechanical timer driven by a synchronous motor. In order to record only the intensity modulated trace the relay was disconnected

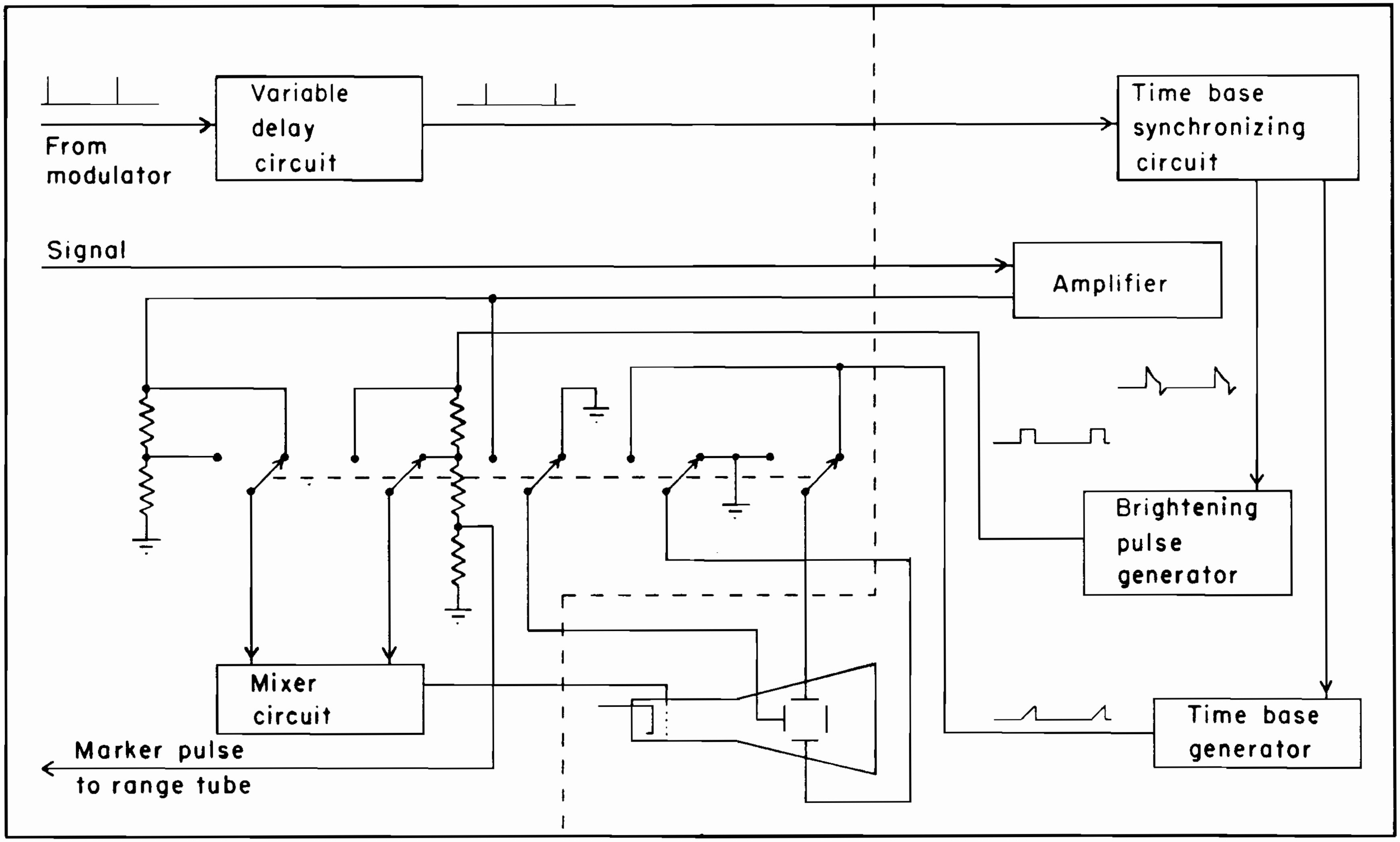


Figure A2.04 - Schematic diagram of the equipment used for continuous recording of auroral echoes.

from the timer and the time base amplitude was increased to a convenient size.

A small part of the brightening pulse was applied to the vertical deflection plates of the range display tube. This pulse, appearing on the range trace, indicated the portion of the trace being displayed on the recording oscilloscope and facilitated the setting of the variable delay control.

A circuit diagram of the variable delay circuit is given in Figure A2.05. The circuit values shown have been found to provide particularly stable operation over long periods of time. The principle of operation is that of the so-called "sanatron" circuit which is described in the Radiation Laboratory Series (18). The delay time is linearly related to the potential on the slider of the delay control, which makes the circuit well suited to remote control.

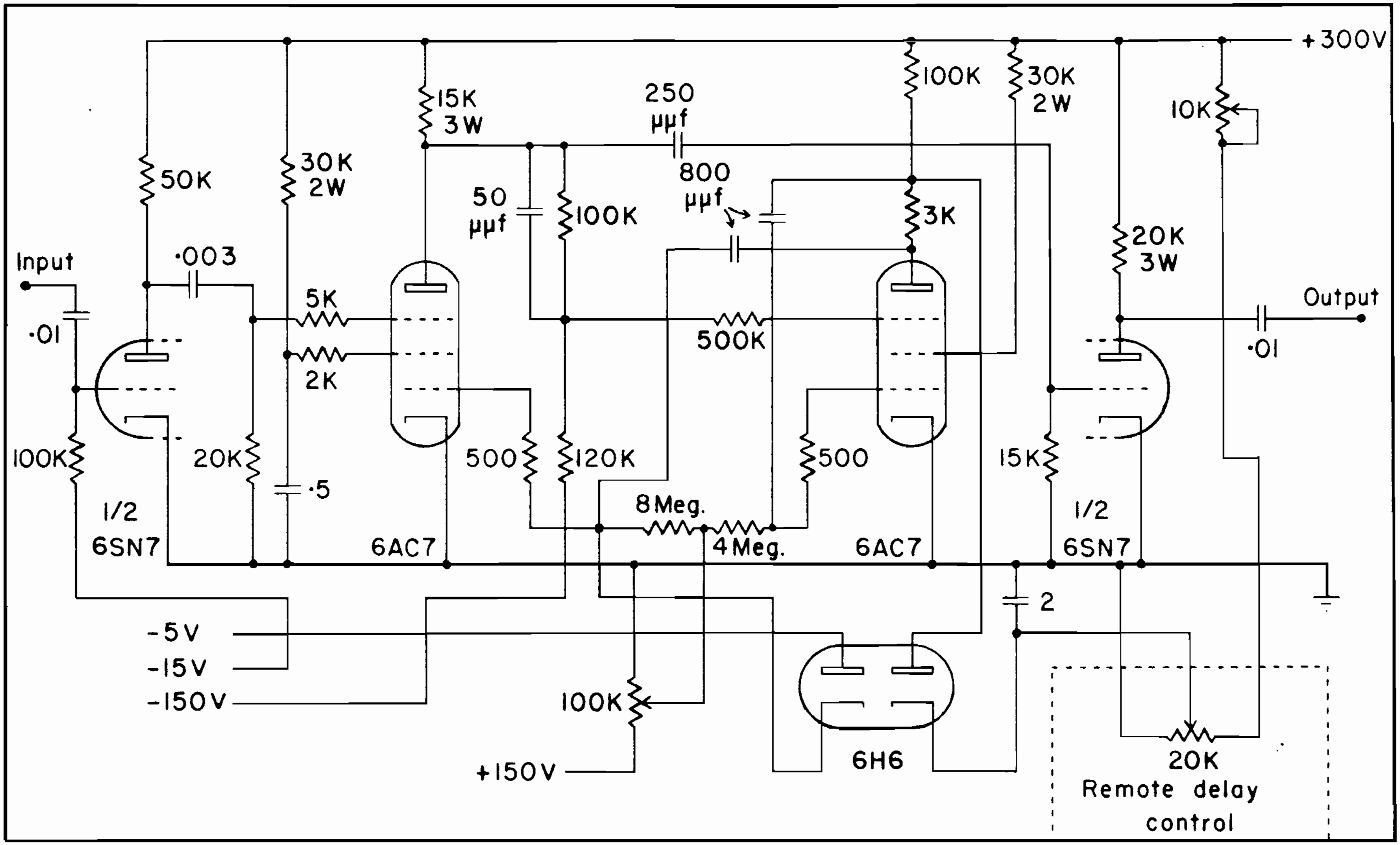


Figure A2.05 - Circuit diagram of the variable delay unit.

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