DERIVATION AND APPLICATIONS OF OPTIMUM BUS INCREMENTAL COSTS

by

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ABSTRACT

The thesis is mainly concerned with the derivation and applications of optimum bus load incremental costs at all system buses. For a completely lossless system it is shown that at least one active transmission constraint is sufficient to bring about a large disparity between the optimum bus incremental costs and the system incremental cost. Furthermore, on including system losses into the study it is shown that even in the absence of active transmission limits, the optimum bus incremental costs could differ from each other and the system incremental cost. Hence economy interchanges (i.e., the transactions of a utility among its customers and neighbouring utilities) based on the system incremental cost are no longer optimum, as this will cause some customers to subsidize others, and could even bring about large financial losses (or gains as the case may be) to utility companies.

Applications of bus incremental costs in areas such as ascertaining the profits and losses incurred by utilities (or customers) through such economy interchanges, determining the best location in the system for cogeneration customers of the electrical load following type, and in the expansion planning of power systems are demonstrated through tests carried out on the 'IEEE 24-Bus Reliability Test System', and a smaller 10-bus experimental test system.

RESUME

Cette étude se concentre principalement sur le calcul des coûts incrémentaux des charges individuelles aux barres d'un réseau, et les applications qui en découlent. On démontre, pour un modèle de réseau sans perte, que la présence d'une contrainte fonctionnelle active (sur une ligne de transport, par exemple) engendre de grandes disparités entre les coûts incrémentaux du système et celles des charges individuelles. Lorsque les pertes sont incorporées au modèle, ces disparités existent même en absence des contraintes fonctionnelles. Ainsi, dans les échanges d'énergie entre producteurs ou entre le producteur et les clients, dont les tarifs sont basés sur le coût incrémental du système, il se peut qu'une partie subventionne l'autre.

Quelques applications sont presentées, touchant les sujets suivants: le calcul du profit ou de la perte associé aux échanges d'énergie, l'établissement des meilleures emplacements pour la cogénération, et l'étude de l'expansion du réseau. Des essais utilisant des modèles de 10 et de 24 barres démontrent la fiabilité de la méthode.

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NOMENCLATURE

$\underline{A}$	A given matrix in the LLS problem.
a_i, b_i, c_i	Generation cost coefficients of unit i.
$\underline{B} \triangleq$	diag $[b_1, b_2, \ldots \ldots, b_m]$
B_{i-j}	Susceptance of line $i-j$.
$\underset{\sim}{b}$	An n-dimensional vector of negative admittances connected to the slack bus, but not coincident with the slack bus.
$\underline{c} \triangleq$	$[c_1, c_2, \ldots\ldots, c_m]^t$
$C(\underline{P})$	System operating cost in \$/H.
$C_i(P_i)$	Hourly operating cost of unit i.
$C_i'(P_i) \triangleq$	$\frac{d}{dP_i}(C_i(P_i))$
C_{tot}	Daily operating cost of the system in \$.
C_i^{op}	Daily cost of fuel for the operation of unit i.
C_i^{bus}	Cost incurred by the utility per day in supplying the energy requirements of bus i.
$\underline{d}$	A vector of dimension p, forming the lower bound of the functional inequalities.
$\underline{e}_{i-j}$	An n-dimensional vector, having 1 in position i and -1 in position j.

$\underline{e}_j$	A general vector of dimension j, whose elements are all unity.
$\underline{E}$	An elementary Hermitian matrix (Householder matrix).
E	Mathematical expectation.
$\hat{\underline{G}}$	A matrix of active functional inequalities.
$\underline{H}$	Hessian matrix defined in equation (B.4).
$\underline{J}$	The nxn non-singular DC load flow Jacobian.
$\underline{K}$	Vector of incremental transmission losses, at all generation buses.
$\underline{L}$	A vector defining the bus loads at all the system buses except the slack bus.
$\underline{P}$	An m dimensional vector of controllable generations.
p	Number of lines in the system.
$\bar{\underline{P}}$	Same as $\underline{P}$, but defined for the equivalent lossless system.
$\underline{P}^*$	A vector of optimum generations in the ED problem with losses.
P_i	Real generation of unit i.
P_i^{max}	An upper bound for the generation of unit i.

P_i^j		Value of P_i at the jth break point of the IC curve.
P_L		Total system losses in a 'δ' neighbourhood of the optimum $\underline{P}^*$.
PF_i	$\triangleq$	$\frac{\partial P_L}{\partial P_i}$, i.e., the penalty factor at bus i .
P_{ci}		Real power output of a cogeneration unit i .
$\underline{Q}$		An orthogonal matrix.
$\underline{R}$		An upper trapezoidal matrix.
$\underline{S}$		A matrix defined by equation (2.17) .
T		Time period under study.
T_n		Normalized time.
$\underline{T}$		Bus load system demand correlation vector.
$\underline{X}$		A vector of unknowns in the LLS problem.
$\underline{Y}$		A matrix defined in equation (2.15.c) .
$\underline{Y}_{i-j}$	$\triangleq$	$B_{i-j}\ \underline{e}_{ij}$.
$\underline{Z}$		Vector of real bus injections.
$\underline{u}_1$		Vector of Lagrange multipliers corresponding to the active generator constraints excluding the slack bus.

$\underline{\alpha}_2$ — Vector of LM's corresponding to the active functional inequality constraints (including the slack bus) .

α_{m+1} — Component of $\underline{\alpha}_2$ corresponding to the slack bus.

γ_{i-j} — Power flow sensitivity vector of line i-j due to injection $\underline{Z}$.

ΔP_L — Incremental change in the system losses due to an incremental shift $\Delta \underline{P}^*$ from the optimum $\underline{P}^*$.

$\underline{\theta}$ — An n dimensional state vector.

$\underline{\tilde{\theta}}$ — Same as $\underline{\theta}$ but applicable to the equivalent lossless model.

$\underline{\Lambda} \triangleq [\lambda_1, \lambda_2, \ldots, \lambda_m, \lambda_{m+2}, \ldots \lambda_{m+1}]^t$

λ_{m+1} — Lagrange multiplier corresponding to the load flow equality constraint of the slack bus.

$\underline{\tilde{\Lambda}}$ — Same as $\underline{\Lambda}$ but applicable to the equivalent lossless model.

$\underline{\nu}$ — Cogeneration - system demand correlation vector.

$\underline{\Gamma}^G$ — Generation constrained participation factors.

$\phi(P_D)$ — Distribution function of system demand.

$\phi(P_D)$	Normalized load duration curve of the system.
$Q_i(P_D) \triangleq$	$C_i'(P_i(P_D))\,\phi(P_D)$.
$[\cdot]^t$	Transpose of a matrix (vector) .
$[\cdot]^{-1}$	Inverse of a matrix.
$\lVert\cdot\rVert_2$	2-norm of a matrix (vector) .

ABBREVIATIONS

BIC	Bus Incremental Cost
ED	Economic Dispatch
ELM	Equivalent Lossless Model
EICC	Equal Incremental Cost Criterion
IC	Incremental Cost
ITL	Incremental Transmission Loss
LLS	Linear Least Squares
LM	Lagrange Multiplier
LDC	Load Duration Curve
PF	Penalty Factor
PDF	Probability Density Function
PU	Per Unit
SIC	System Incremental Cost
s.t.	subject to
TSLF	Taylor Series Loss Formula
w.r.t.	with respect to

CHAPTER I

INTRODUCTION

1.0 A Brief Preview

The concept of pricing electricity has always been a controversial issue. To date no method has been developed which forms a fair pricing strategy. Utility Companies usually carry out their transactions based on the System Incremental Cost, abbreviated SIC . However, the outcome of the present study shows that this is optimum only under special system conditions, and cannot be applied in general. Furthermore the optimum Bus Incremental Cost, abbreviated BIC , of the various system buses are shown to provide better information, than the SIC in economics related to system operation.

The SIC is the cost of supplying the next MWH increment in the system demand, while the BIC is the cost of supplying the next MWH increment in the corresponding bus demand. The unit measure of both these quantities are in $/MWH .

1.1 Time Varying Nature of Electricity Prices

The hourly load curve of any power system is stochastic in nature. Variations take place with time of day, day of week, and week of year [IEEE Reliability Test System 1979, Elgerd 1982, Sullivan 1977, Singh 1977] . Since the demand for electricity must always be satisfied, i.e., the generation must always equal the load, it becomes necessary for the utility companies to vary their generation scheduling with time of day. However, this scheduling must be carried out in an optimum manner. The solution to the problem of optimum scheduling comes under the topic of Economic Dispatch [Happ 1977] .

The exact cost incurred by the utility in supplying the next MWH increase in the system demand, is reflected in the SIC . The SIC will be low at times of low demand and high at times of high demand. The reason being that at low demands relatively inexpensive units are in operation and at high demands more expensive units like combustion turbine generator units have to be started up to meet the growing system demand. However, this is not always the case, as the SIC can be high even at times of relatively low demand. Such a situation arises when a transmission line to a load center reaches an operating limit, and in order to meet the demand at this bus it may be necessary to start up more expensive generating units. A similar situation may result when a relatively cheap source of electrical energy, such as a nuclear plant reaches an operating limit, or is in outage.

The daily operating cost of an electric utility cannot be forecast exactly, the reasons being :

(1) The uncertain nature of the availability of power system components [Billinton 1976] ,

(2) The uncertainty in the daily demand pattern,

(3) Uncertainty in maintenance scheduling.

For these reasons estimating the cost of meeting the energy needs of various system buses becomes a formidable task.

The time varying nature of electricity production brings about the need for a time varying price structure. This has led to many versions of "time of day pricing" [Raynolds and Creighton 1980] . In this form of pricing, the price charged to customers changes according to a predetermined schedule. Such pricing policies have been used in Europe for more than two decades, and are coming into use in the U.S. and other countries [Bohn 1980] . However, the major drawback in such a pricing scheme is that since prices are established as much as one year in advance, the actual price charged for a given period does not reflect the exact cost of generation at that time. Further, such a scheme does not reflect the variations in the price of electricity at different locations in the system, generally caused by outages, and transmission losses.

A second approach, is to sell electricity at a price which is determined at the time the sale is made, rather than in advance. This approach is used between utilities for purchasing electricity. Such transactions are called economy interchanges. The selling price of electricity is based on the SIC . Often interconnected systems are operated at the same SIC . Sales being usually carried out when a difference exists between these SIC's . Usually the system with the lower SIC is the seller to the one with the higher SIC , which is the buyer [Vojdani 1982, Bohn 1980] .

Energy transfer between utilities takes place through one or more tie lines linking the respective buses belonging to either system. Hence fixing the selling price on the SIC could bring about large monetary discrepancies, if the corresponding bus and system incremental costs differ significantly, i.e., depending on the BIC of the selling bus being higher or lower than the SIC , the buyer stands to either gain or lose by this transaction. Transactions based on optimum BIC's serve to remove this anomaly and bring about an equity in such transactions.

The advantages of carrying out transactions based on time varying optimum BIC's are summarized below.

(a) Improves system load factor, and reduces the average cost of a KWH generated by encouraging lowering demand at times of high cost. This voluntary action is far superior to load management where the customer's supply is interrupted by the utility without prior warning ['Impacts of Several Major Load Management Projects 1982'] .

(b) Reduces the demand for spinning reserve by providing a means of rapidly reducing the demand, when facing a system emergency, such as a line or generator outage.

(c) Gives more opportunity to customers, to reduce their electricity bills, by scheduling their pattern of electricity use appropriately.

(d) Establishes a good basis to compute revenues payable to customers with cogeneration facilities.

Many versions of time varying prices can be found in the literature. These are known by different names, namely spot price, responsive price, load adaptive price, real time price, flexible price [Bohn 1980, Schweppe 1978] . However most of these prices require complex computation, and makes one to doubt whether they can be applied to large systems in real time. On the contrary, it is shown in subsequent chapters that very little numerical computations are needed to determine the optimum BIC's . This makes determining BIC's in real time quite feasible, and hence its real time implementation in economy interchanges should prove very attractive to utility companies.

1.2 Types of Customers and their Responses to a Time Varying Price

Customers in a system come under two different classes, namely storage and non-storage customers [Bohn 1980] .

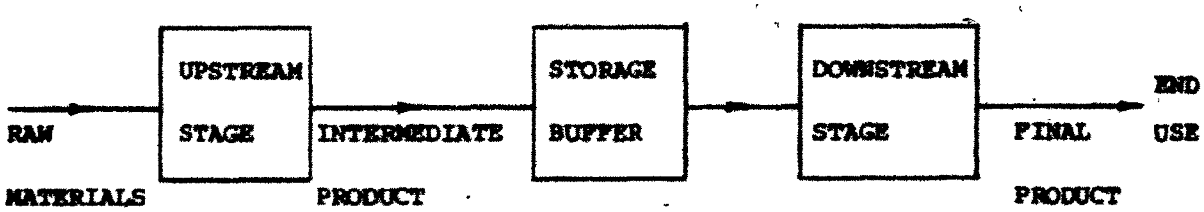

Figure 1.1 Storage Process Model.

Figure 1.1 shows a storage customer. The basic processes involved here are an upstream stage, an intermediate stage and a downstream stage. The downstream stage is in operation at all times, which means that when the storage is empty the upstream stage must be turned on. It is assumed that the storage buffer has a fixed size, negligible storage costs compared to the cost of electricity, and is 100 % efficient. Electricity is consumed only in the upstream stage. The production of the intermediate product is proportional to the electricity consumed. Electricity costs are the only variable costs, other costs such as labour, supplies, etc., are assumed to be uncorrelated to the electricity price.

In this model the customer tries to minimize the cost of electricity while maintaining the downstream stage in full production. Hence, when the BIC's are relatively low, the customer will operate the upstream stage at full capacity building up an optimum level of intermediate product, subject to storage requirements, to maintain the downstream stage in full operation at times of high cost.

An example of such a process is an air based electrical space heating system. The downstream storage stage is the area being heated, and the upstream stage is the electrically operated heater. The storage buffers being air, surrounding walls and other thermal mass of the conditioned area. Storage takes the form of heating to above the thermostat set points and letting it cool off during periods of high cost.

Other examples include continuous process chemical plants, chilled water systems, air conditioning systems, gas operation plants, heat storage pads, and other similar facilities.

There are many different classes of non-storage customers, and their responses to time varying electricity prices could be any one, or a combination of the following, i.e.,

1. Turning off unessential lighting, heating, or air-conditioning equipment.

2. Slowing down to stopping a chemical or physical process using electricity.

3. Slowing down or stopping an assembly line.

4. Starting up customer's own generating equipment, when the marginal cost of customer's electricity production is below the corresponding optimum BIC .

5. Increasing the output of a cogeneration unit.

1.3 Forecasting the BIC's

In a lossless system, the BIC's are piecewise linear in the system demand P_D [Vojdani 1982] , i.e.,

$$BIC_i \triangleq \lambda_i = f(P_D) \qquad (1.1)$$

This implies that the variance of the BIC , equals the variance in system demand, i.e.,

$$Var(\lambda_i) = Var(P_D) \qquad (1.2)$$

Hence the BIC's can be predicted to the same level of statistical precision as the system demand. However if customers do respond to time varying price of electricity, then the demand will be price dependent. This makes the overall problem of forecasting BIC's , quite a formidable task. However, mathematical models can be built to reflect optimum customer behaviour. These models can be used as shown in the schematic diagram of Figure 1.2 in order to forecast price dependent loads.

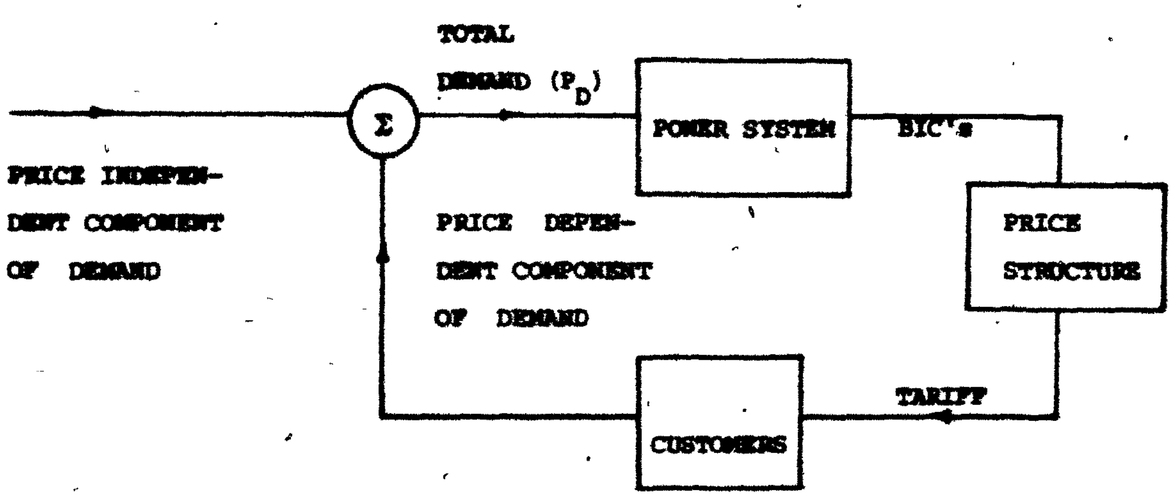

Figure 1.2 Schematic Diagram for Forecasting Price Dependent Demand.

A good forecast of the BIC's are very essential, to increase the social benefits of customers participating in a time varying price structure. This will cause more customers to participate, thereby causing the national benefit to increase due to time varying electricity prices [Bohn 1980, Caramanis et al., 1980, Roger et al., 1980] .

1.4 An Overview of Cogeneration

Cogeneration is the simultaneous production of electrical (or mechanical energy), and useful thermal energy, both from the same fuel source. This is different to the conventional system that produces either electrical or thermal (steam) energy. A cogeneration system produces both, and requires 10 to 30 percent less fuel than conventional systems [De Renzo 1983] . One of the many advantages of cogeneration is that it saves fuel. Cogeneration if developed and operated optimally will provide substantial amounts of energy saving potential to the nation, as well as provide dollar savings for a cogeneration customer.

Although the name cogeneration is new, the practice is nearly a century old. In the early 1900's, most industrial plants generated their own electricity, and many practised cogeneration by using the waste steam for process heating. As the centrally generated electricity became cheaper, widely available and more reliable, industrial electricity generation declined. Currently as industries face higher electricity bills, due to spiralling fuel costs, renewed interest is shown in cogeneration, as more customers see the potential savings involved.

1.5 Types of Cogeneration Systems

Technically there are two fundamental types of cogeneration systems, namely topping and bottoming cycle plants [De Renzo 1983, Harkins 1981] . There are also combined cycle plants which embody both topping and bottoming cycle principles.

In a topping cycle plant electricity (or mechanical energy) is produced first, and the thermal energy from the exhaust is captured for further use. Figure 1.3 shows a topping cycle plant. In a bottoming cycle plant, thermal energy is captured from a waste stream, and then is used to produce electricity, by driving a turbine generator unit. A combined cycle plant is shown in Figure 1.4 .

1.6 Current Technology for Cogeneration Systems

A variety of concepts and technologies are available for both topping and bottoming cycle plants. For example in a steam turbine topping cycle, high pressure steam produced by the combustion of a wide variety of fuels is used to drive a turbine. The turbine exhaust

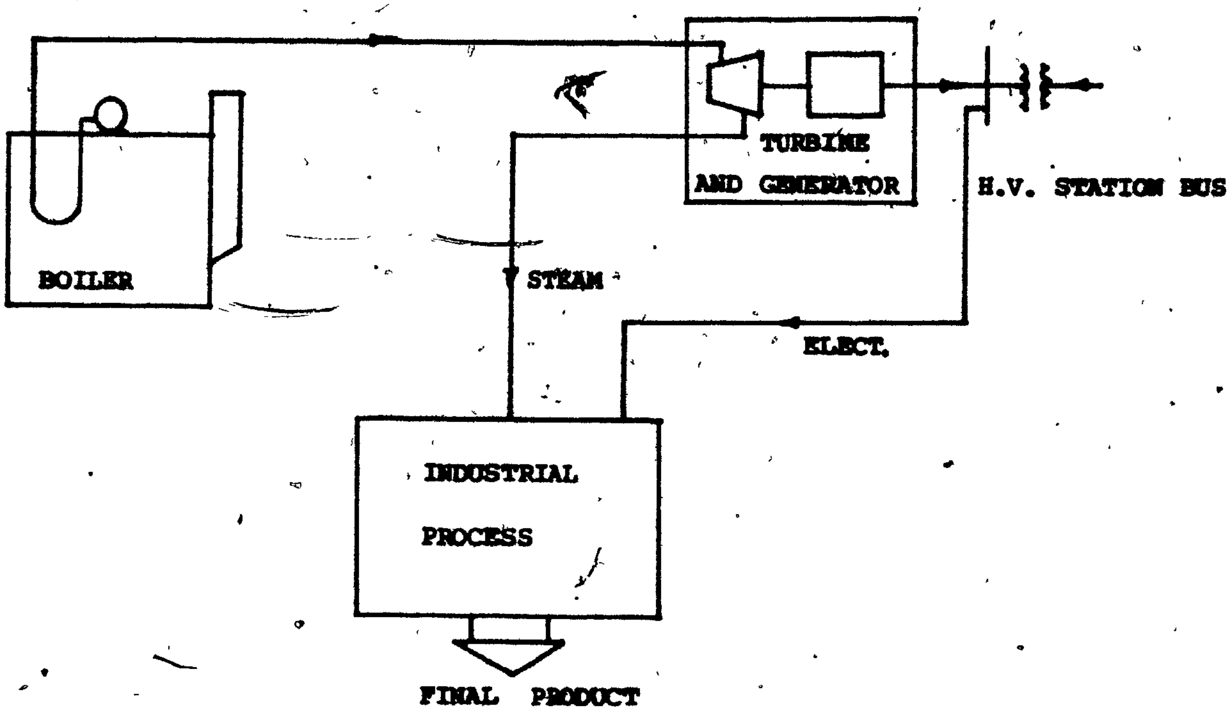

Figure 1.3 Topping Cycle Plant.

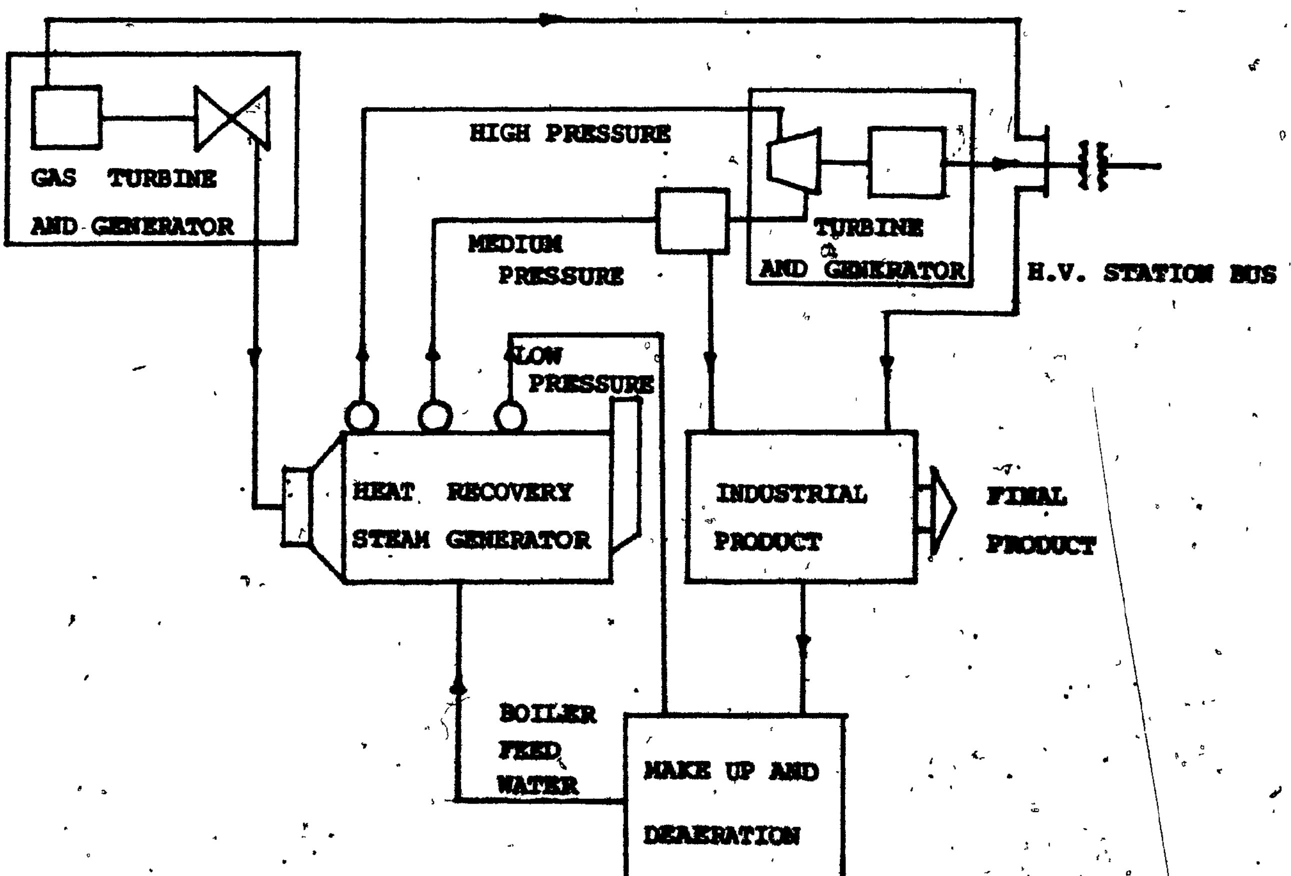

Figure 1.4 Combined Cycle Plant.

is used for industrial processes, and the mechanical energy is converted to electricity in an alternator. Other topping cycle options include gas turbines, diesels and fuel cells.

Bottoming cycle plants can operate with steam or an organic compound as the prime working fluid. Heat pumps may be used with cogeneration prime movers to upgrade the low temperature process heat [De Renzo 1983] .

1.7 Operation of Cogeneration Units

A cogeneration unit can be operated either to track its own electrical demand, or its thermal demand. If the unit tracks its electrical demand, the process is called electrical load following, and if it tracks its thermal demand the process is called thermal load following. The present study assumes that the cogeneration units involved, are of the electrical load following type. This mode of operation will be beneficial, only if the marginal operating cost of the cogeneration unit, does not exceed the BIC of the corresponding bus,

1.8 Future Prospects for Cogeneration

Industries utilizing process steam, can benefit by the cogeneration technology of today, by investing in such facilities. Many cogeneration systems currently prove to be economical, providing attractive returns on investments. Cogeneration is now encouraged by utilities as an alternative to power system expansion [Cougan and Williams 1983] .

The prices of oil and gas will continue to rise in future, causing the price of bulk electricity to rise. Thus any cogeneration systems that conserve these fuels, are going to receive wide acceptance. This will force cogeneration systems of the future to use coal as the primary source of fuel [Merril 1983] . Further, as industries, institutions and utilities become more aware of the technologies, and the economics of cogeneration, free market forces could work to expand the role of cogeneration in the industry [Schweppe et al., 1980, Benette et al., 1980] .

1.9 Objections to the Implementation of a Price Structure Based on BIC's

The participants of a pricing scheme are utilities, customers, and regulators. Given below is a list of effects, which could form the basis for objections to a time varying price structure based on BIC's, i.e.,

(1) Revenues (profits) to utilities may become uncertain.

(2) Approving uncertain prices in advance may prove difficult to regulatory authorities.

(3) The controls needed on the part of the customer, to modify its load according to the present and forecast price, may prove to be too complicated, and could require expensive hardware and software equipment.

(4) The mismatch between the forecasted BIC , and the exact BIC at a given instant need not always be small due to various uncertainties in the system parameters. This calls for secondary equipment, needed to communicate the exact BIC at a given instant in real time.

1.10 The Need for a Radical Change in the Philosophy of Supply of Electricity

From the time commercial electricity production and distribution became viable, the philosophy behind its successful operation has been of a "supply follows demand" type rather than "demand follows supply" . Commercial electricity companies had to supply any demand called for by the customers. In recent times there is evidence to show that this mode of operation is receiving less acceptance among the utility circles, the reasons being summarized below.

1. Utilities cannot handle contingencies efficiently, as handling of such contingencies must be shared by both the utility and the customer, in an interactive manner.

2. The failure of customers to see the exact cost of electricity they consume, causes some customers to subsidize others.

3. Lack of customer cooperation under normal and abnormal system conditions brings about large fluctuations in the daily system demand. This calls for large reserve generation and transmission capacities. The large investments that go along with these reserves brings about large financial burdens on the utility companies. Recovering the capital invested may take long, as these reserves come into operation only at times of peak demand.

For these reasons, the philosophy of current utility operation must undergo a radical change to the philosophy of "demand follows supply" . The system would now be in a better state of equilibrium, in which the customers play an active role in the control of the power system by dropping demand at times of high cost and increasing demand at times of low cost. Assuming a positive response from customers, the system could come very close to deregulation [Schweppe et al., 1980, Golub et al., 1981, Schweppe 1978] .

1.11 Objectives of the Thesis

This Thesis is mainly concerned with the derivation of optimum (minimum) BIC's and ascertaining the relationships amongst the various BIC's and the SIC . It is shown that in the absence of constrained transmission lines, all BIC's are in parity with each other and the SIC in a lossless system. However at least one active transmission constraint is sufficient to bring about a large disparity among these incremental costs. The 'IEEE Reliability Test System 1979', is used extensively in the exemplification of certain applications of these BIC's .

Further applications of BIC's in various categories of planning and operation of a power system are considered. Specific examples being :

(a) Estimating the cost of daily energy requirements at different buses.

(b) Analyzing the economic impact on (i) the customer, (ii) the utility due to a change in the pricing philosophy.

(c) Locating the economically best bus in the system for economy interchanges (i.e., transactions of a utility among its customers and other neighbouring utilities) .

(d) Choosing the best alternative amongst many system expansion alternatives, which would minimize the daily operating cost of the system.

1.12 Conclusions

The cost of electricity varies with time. This makes the buying (from other utilities, or from customers with cogeneration facilities) and selling (to customers or a neighbouring utility) of electricity dependent on the time at which transaction is made. Hence it becomes necessary to compute exactly the cost of delivering the electrical energy (usually produced at various locations in the network) to the various customers. The present study shows that the BIC's , which reflect the exact cost of the next MWH increment in bus demand, can be used successfully to compute these costs.

These BIC's , if implemented by the utilities may prove to change the electricity supply philosophy from one of supply follows demand to demand follows supply. From fundamental principles in economics it is known that this latter form will bring about an equilibrium between the supplier (utility) and the consumer (customers) . Many advantages result from such an equilibrium. Other applications of BIC's are in the fields of power system planning and operation. Chapter IV of this thesis exemplifies these applications.

CHAPTER II

DERIVATION OF OPTIMUM BUS INCREMENTAL COSTS

2.0 Introduction to the Problem of Economic Dispatch

Economic dispatch (ED) as the name implies is the most economical mode of dispatching electrical energy to meet the system demand and losses, giving due regard to generation and transmission limits. Interest in this complex problem was shown as early as 1930 [Happ 1977] , and the method of equal incremental cost became widely accepted as the solution to the lossless ED problem, of a system comprising of only thermal units. A few years later transmission losses were included into the basic problem, and the solution was established. A good background of the necessary theory in this regard is given in the work by Kirchmayer [Kirchmayer 1958] .

The problem of ED can be formulated as a mathematical nonlinear optimization problem comprising of an objective function (the generation costs) which must be minimized subject to certain constraints. The constraints vary depending upon the power system model used for the study. Since this study utilizes the DC load flow model, the corresponding constraints become the real power flow limits in the generators and transmission lines.

In subsequent sections detailed compact expressions for the objective function and constraints are derived and the problem of

ED is formulated. The necessary conditions to be satisfied at the optimum, give the required optimum bus incremental costs.

2.1 Operating Cost of Thermal Units

The input to a thermal generating unit is measured in MBtu/H while the output is measured in MW. As auxiliary power requirements of a thermal unit must be accounted for, the given input - output data must be converted to input versus net plant send out. Unfortunately these characteristics are non linear. In practice these non linear characteristics are approximated by a quadratic function, through a linear least squares (abbreviated LLS) fit. The necessary theory is given in Appendix C.

These quadratic cost functions enable the derivation of very elegant analytical solutions to the problem of ED. The work of Vojdani [Vojdani 1979] highlights these points. Many practical applications relating to this theory can be found in the work by Al-Jishi [Al-Jishi 1980].

The operating cost of each unit in \$/hr can be approximated by the following function, i.e.,

$$C_i(P_i) = c_i + a_i P_i + \frac{1}{2} b_i P_i^2 \quad ; \quad \forall\, i = 1, \ldots, m+1 \qquad (2.1)$$

where $m+1$ is the total number of dispatchable units.

Thus the objective function can be written as follows, i.e.,

$$C(\underline{P}) = \sum_{i=1}^{m+1} C_i(P_i) \tag{2.2}$$

By separating the operating cost of the slack generator, for reasons which will become clear in the subsequent reading, equation (2.2) can be written as :

$$C(P) = \underline{e}_m^t \underline{C} + \underline{a}^t \underline{P} + \frac{1}{2} \underline{P}^t \underline{B} \underline{P} + c_{m+1} + a_{m+1} P_{m+1} + \frac{1}{2} b_{m+1} P_{m+1}^2 \tag{2.3}$$

where the following are defined, i.e.,

$$\underline{C} \triangleq [c_1, c_2, \ldots\ldots\ldots, c_m]^t \tag{2.4.a}$$

$$\underline{B} \triangleq \text{diag } [b_1, b_2, \ldots\ldots, b_m] \tag{2.4.b}$$

$$\underline{a} \triangleq [a_1, a_2, \ldots\ldots, a_m]^t \tag{2.4.c}$$

$$\underline{e}_m \triangleq [1, 1, \ldots\ldots\ldots 1]_{mx1}^t \tag{2.4.d}$$

$$\underline{P} \triangleq [P_1, P_2, \ldots\ldots\ldots P_m]^t \tag{2.4.e}$$

Now that the objective function to the ED problem is established, the next task in formulating the ED problem is to establish the related constraints. This will be handled in the next section.

2.2 Formulating the Constraints to the ED Problem

The constraints related to the ED problem are two fold; namely, (i) equality constraints, (ii) inequality constraints. The next two subsections are devoted to formulating the necessary equations related to these constraints.

2.2.1 Equality Constraints

The line resistances are assumed to be very small, compared to the line reactances, i.e., for all lines i-j

$$\frac{R_{i-j}}{X_{i-j}} << 1 \tag{2.5}$$

Further disregarding the flow of reactive power by holding the bus voltages fixed at 1.0 pu and considering only the flow of real power the following set of n+1 load flow equations (the details of which are given in Appendix A) results, i.e.,

$$\hat{Z} = \hat{J}\,\hat{\theta} \tag{2.6}$$

where :

n+1 is the total number of system buses.

$\hat{Z}$ vector of all net bus injections.

$\hat{\underline{J}}$ is the $(n+1) \times (n+1)$ singular DC load flow Jacobian.

$\hat{\underline{\theta}}$ is the vector of all bus voltage angles.

Matrix $\hat{J}$ is singular, as the sum of its columns are zero. This problem is overcome by choosing a slack bus and deleting the row and column corresponding to this bus. If bus number $(m+1)$ is chosen as the slack bus, equation (2.6) becomes :

$$\underline{Z} = \underline{J}\,\underline{\theta} \tag{2.7}$$

By definition :

$$\underline{Z} \triangleq \underline{P} - \underline{L} \tag{2.8.a}$$

$$\underline{L} \triangleq [L_1, L_2, \ldots L_m, L_{m+2} \ldots\ldots L_{n+1}]^t \tag{2.8.b}$$

$\underline{Z}$ is the vector of n bus injections.

$\underline{J}$ is the $(n \times n)$ non singular DC load flow Jacobian.

$\underline{\theta}$ is the state vector of n bus voltage angles.

The load flow equation corresponding to the slack bus can be written as follows, i.e.,

$$P_{m+1} - L_{m+1} = \hat{\underline{b}}^t\,\underline{\theta} \tag{2.9}$$

Where $\tilde{\underline{b}}$ is an n dimensional vector whose elements are

$$\tilde{b}_i \triangleq \begin{cases} -B_{(m+1)-i} \\ \text{or} & \forall\, i \in (m+1) \\ 0 \end{cases} \tag{2.10}$$

$B_{(m+1)-i}$ is the susceptance of the line joining buses $(m+1)$ and i. The symbol $i \in (m+1)$ implies that bus number i is connected to bus number $(m+1)$ but is not coincident with $(m+1)$.

2.2.2. Inequality Constraints

Two sets of inequality constraints exist. The first set corresponds to generation limits and the second set corresponds to transmission limits.

2.2.2.1 Limits on Real Generations

The bounds on the controllable generators can be written as follows, i.e.,

$$\underline{P}^{min} \leq \underline{P} \leq \underline{P}^{max} \tag{2.11}$$

Real power limit on the slack generator can be written as follows, i.e.,

$$P_{m+1}^{min} \leq P_{m+1} \leq P_{m+1}^{max} \tag{2.12}$$

From equations (2.9) and (2.12), the following results, i.e.,

$$P_{m+1}^{min} - L_{m+1} < \underline{\tilde{b}}^{t} \, \underline{\theta} < P_{m+1}^{max} - L_{m+1} \quad (2.13)$$

2.2.2.2 <u>Limits on Real Line Flows</u>

For all lines (i-j), the real power flow limits can be written as follows, i.e.,

$$P_{i-j}^{min} < P_{i-j} < P_{i-j}^{max} \quad (2.14)$$

From equations (A.21) and (2.14), the following result is obtained, i.e.,

$$- P_{i-j}^{max} < \underline{Y}_{i-j}^{t} \, \underline{\theta} < \underline{P}_{i-j}^{max} \quad (2.15.a)$$

In a compact form, for all lines i-j equation (2.15.a) becomes :

$$- \underline{P}_{line}^{max} < \underline{Y} \, \underline{\theta} < \underline{P}_{line}^{max} \quad (2.15.b)$$

By definition :

$$\underline{Y} \triangleq \begin{bmatrix} \vdots \\ \underline{Y}_{i-j}^{t} \\ \vdots \end{bmatrix}_{p \times l} \leftarrow \text{corresponds to line } i-j \quad (2.15.c)$$

$$\underline{P}_{line}^{max} \triangleq \begin{bmatrix} \vdots \\ P_{i-j}^{max} \\ \vdots \end{bmatrix}_{px1} \leftarrow \text{corresponds to line } i-j \qquad (2.15.d)$$

Where p is the number of transmission lines in the system.

The following compact form for all functional inequalities is obtained by combining equations (2.13) and (2.15.b), i.e.,

$$\underline{S}\,\underline{\theta} > \underline{d} \qquad (2.16)$$

where by definition :

$$\underline{S} \triangleq \begin{bmatrix} \underline{Y} \\ -\underline{Y} \\ \underline{\tilde{b}}^t \\ -\underline{\tilde{b}}^t \end{bmatrix} \qquad (2.17.a)$$

$$\underline{d} \triangleq \begin{bmatrix} -P_{line}^{max} \\ \\ P_{m+1}^{min} - L_{m+1} \quad \text{or} \quad -P_{m+1}^{max} + L_{m+1} \end{bmatrix} \qquad (2.17.b)$$

Now that the objective function and the corresponding inequality constraints are well established, the following section combines all previous derivations to formulate the ED problem in a compact form.

2.3 Formulation of the Transmission Constrained ED Problem

The ED problem, formulated as a non-linear optimization problem would read as follows, i.e.,

$$\underset{\text{w.r.t. } \underline{P}}{\text{Minimize}} \quad \underline{e}_m^t \underline{C} + \underline{a}^t \underline{P} + \frac{1}{2} \underline{P}^t \underline{B} \underline{P} + c_{m+1} + a_{m+1} P_{m+1} + \frac{1}{2} b_{m+1} P_{m+1}^2 \tag{2.18.a}$$

$$\text{s.t.} \quad \underline{Z} = \underline{J}\, \underline{\theta} \tag{2.18.b}$$

$$P_{m+1} - L_{m+1} = \underline{\hat{b}}^t \underline{\theta} \tag{2.18.c}$$

$$\underline{P}^{min} < \underline{P} < \underline{P}^{max} \tag{2.18.d}$$

$$\underline{S}\, \underline{\theta} > \underline{d} \tag{2.18.e}$$

It will be assumed for the purpose of generality, that the optimum generation schedule corresponding to the ED problem above is defined by the following equation, i.e.,

$$\underline{P} \triangleq \begin{bmatrix} \underline{P}_1 \\ \hline \underline{P}_2 \end{bmatrix}_{m \times 1} \begin{matrix} (\ell+u) \times 1 \\ \\ \end{matrix} \tag{2.18.f}$$

where : ℓ and u are the number of generators operating at lower and upper limits respectively.

$\underline{P}_1$ is the set of bound generators.

$\underline{P}_2$ is the set of free generators.

The optimum BIC's to be derived next, will be based upon the ED model defined in this section.

2.4 Solution of the ED Problem for Optimum Bus Incremental Costs

2.4.1 Derivation of Necessary Conditions - Slack Generator is Free

Assuming a total of q active constraints, at the transmission constrained optimum, which are characterized as follows, i.e.,

1. ℓ generators operate at their lower limit,
2. u generators operate at their upper limit,
3. $q-\ell-u$ functional inequality constraints, of equation (2.16) are active. Since the slack generator is assumed free, these constraints correspond to the bound lines only.

By definition :

$$\underline{g} \triangleq \begin{bmatrix} \underline{g}_b \\ \hline \underline{g}_f \end{bmatrix} \begin{matrix} (q-\ell-u)\times 1 \\ (p\times 1) \end{matrix} \qquad (2.19)$$

$$\underline{d} \triangleq \begin{bmatrix} \underline{d}_b \\ \hline \underline{d}_f \end{bmatrix} \begin{matrix} (q-\ell-u)\times 1 \\ (p\times 1) \end{matrix} \qquad (2.20)$$

Where subscripts b and f are for the bound and free constraints respectively. The Lagrangian to be minimized becomes :

$$\mathcal{L} = \underline{e}_m^t \underline{C} + \underline{a}^t \underline{P} + \frac{1}{2} \underline{P}^t \underline{B} \underline{P} + c_{m+1} + a_{m+1} P_{m+1} + \frac{1}{2} b_{m+1} P_{m+1}^2$$

$$+ \underline{\Lambda}^t(\underline{J}\,\underline{\theta} + \underline{L} - \underline{P}) + \lambda_{m+1}(\underline{\tilde{b}}^t \underline{\theta} + L_{m+1} - P_{m+1}) + \underline{\alpha}_1^t(\underline{P}_1^{limit} - \underline{P}_1)$$

$$+ \underline{\alpha}_2^t(\underline{d}_b - \underline{S}_b \underline{\theta}) \tag{2.21}$$

The following necessary conditions must hold at the optimum, i.e.,

$$\frac{\partial \mathcal{L}}{\partial \underline{P}_1} = \underline{a}_1 + \underline{B}_{11} \underline{P}_1 - \underline{\Lambda}_1 - \underline{\alpha}_1 = 0 \tag{2.22.a}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{P}_2} = \underline{a}_2 + \underline{B}_{22} \underline{P}_2 - \underline{\Lambda}_2 = 0 \tag{2.22.b}$$

$$\frac{\partial \mathcal{L}}{\partial P_{m+1}} = a_{m+1} + b_{m+1} P_{m+1} - \lambda_{m+1} = 0 \tag{2.22.c}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\theta}} = \underline{J}^t \underline{\Lambda} + \underline{\tilde{b}} \lambda_{m+1} - \underline{S}_b^t \underline{\alpha}_2 = 0 \tag{2.22.d}$$

The following equations are a consequence of equations (2.22.d) and (2.22.c), i.e.,

$$\underline{J}^t \underline{\Lambda} = [\underline{S}_b^t \underline{\alpha}_2 - \underline{\tilde{b}} \lambda_{m+1}] \tag{2.23.a}$$

$$\lambda_{m+1} = a_{m+1} + b_{m+1} P_{m+1} \tag{2.23.b}$$

It is shown below that the elements of $\underline{\Lambda}$ are the optimum BIC's at all buses except the slack bus. The BIC of the slack bus is obtained from equation (2.23.b) .

At the optimum, the Lagrangian $\mathcal{L}$ of equation (2.21) is equal to the optimum system operating cost $C(\underline{P})$. If one considers an infinitesimal change in only the bus loads L_i's at all buses, by using the necessary conditions given by equations (2.22.a-d) and a Taylor series expansion about the optimum $\underline{P}$, along with some algebraic manipulations one can arrive at the following equation, i.e.,

$$d\,C(\underline{P}) = d\mathcal{L} = d\,\underline{L}^t\,\underline{\Lambda} + dL_{m+1}\,\lambda_{m+1} \qquad (2.24)$$

If the change in bus loads is confined only to bus i , that is, if only L_i is allowed to vary by an infinitesimal amount while other bus loads are held fixed, one could obtain the following equation from equation (2.24) , i.e.,

$$\frac{dC}{dL_i} = \lambda_i \qquad \forall\ i = 1, 2, \ldots\ldots, n+1 \qquad (2.25)$$

Equation (2.25) suggests that a MWH increase in energy demand at bus i will cost the system a minimum of λ_i \$. Hence elements of $\underline{\Lambda}$ give the optimum BIC's of all buses except the slack bus. The optimum BIC of the slack bus is given by equation (2.23.b) . One sees from this equation that, whenever the slack generator is unbounded its IC is equal to the optimum BIC . The next section derives the necessary modifications to this rule when the slack generator is bound.

2.4.2 Derivation of Necessary Conditions - Slack Generator is Bound

The problem in this case is identical to that in Section 2.4.1 . The only difference being that the slack generator now operates at a limit (could be an upper or lower limit) .

Recalling that the slack generator belongs to the functional inequality constraints of equation (2.18.e) one can rewrite the last term of the Lagrangian of equation (2.21) as follows, i.e.,

$$\underline{\alpha}_2^t(\underline{d}_b - \underline{S}_b\,\underline{\theta}) = [\tilde{\underline{\alpha}}_2^t \;\vdots\; \alpha_{m+1}] \left[\begin{array}{c} \tilde{\underline{d}}_b - \tilde{\underline{S}}_b\,\underline{\theta} \\ \hline s \end{array}\right] \tag{2.26}$$

Depending on whether the slack generator is at an upper limit or lower limit, s in equation (2.26) can take either one of the two values shown below, i.e.,

$$s = -P_{m+1}^{max} + P_{m+1} \tag{2.27}$$

$$s = P_{m+1}^{min} - P_{m+1} \tag{2.28}$$

Thus the Lagrangian to be minimized becomes (assuming that the slack generator is at an upper limit, without loss of generality) :

$$\begin{aligned}\mathcal{L} = {} & \underline{e}_m^t\,\underline{C} + \underline{a}^t\,\underline{P} + \frac{1}{2}\underline{P}^t\,\underline{B}\,\underline{P} + c_{m+1} + a_{m+1}\,P_{m+1} + \frac{1}{2}\,b_{m+1}\,P_{m+1}^2 \\ & + \underline{\lambda}^t(\underline{J}\,\underline{\theta} + \underline{L} - \underline{P}) + \lambda_{m+1}(\tilde{\underline{b}}^t\,\underline{\theta} + L_{m+1} - P_{m+1}) + \underline{\alpha}_1^t(\underline{P}_1^{limit} - \underline{P}_1) \\ & + \alpha_2^t(\tilde{\underline{d}}_b - \tilde{\underline{S}}_b\,\underline{\theta}) + \alpha_{m+1}(-P_{m+1}^{max} + P_{m+1})\end{aligned} \tag{2.29}$$

The necessary conditions are the same as those of Section 2.4.1 except for equation (2.22.c) which becomes either one of the two equations given below depending on whether the slack generator is at an upper or lower limit respectively.

$$\frac{\partial \mathcal{L}}{\partial P_{m+1}} = a_{m+1} + b_{m+1} P_{m+1} - \lambda_{m+1} + \alpha_{m+1} = 0 \qquad (2.30.a)$$

$$\frac{\partial \mathcal{L}}{\partial P_{m+1}} = a_{m+1} + b_{m+1} P_{m+1} - \lambda_{m+1} - \alpha_{m+1} = 0 \qquad (2.30.b)$$

In the next section it will be shown that these results are quite general and hold true for any bus provided it is a generation bus. This means that any generation bus can be chosen as the slack bus for the purpose of computing the BIC's and the results should be consistent. This has been found to be true in numerical tests as well.

2.4.3 Independence of BIC's of the Choice of a Slack Bus

The elements of vector $\underline{\alpha}_1$ in equation (2.22.a) which corresponds to the bound generators $\underline{P}_1$ can be negative or positive depending on whether they correspond to generators operating at upper or lower limits. Hence the ith component of this equation would be either equation (2.31.a) or equation (2.31.b) depending on whether generator i is at an upper limit or lower limit, i.e.,

$$a_i + b_i P_i + |\alpha_i| = \lambda_i \quad (2.31.a)$$

$$a_i + b_i P_i - |\alpha_i| = \lambda_i \quad (2.31.b)$$

The notations are assumed to be understood.

The results of equations (2.31.a, b) suggest that the results of equations (2.30.a, b) can be extended to any bus provided it carries a generator operating at a limit.

The ith component of equation (2.22.b) can be written as follows, i.e.,

$$a_i + b_i P_i = \lambda_i \quad (2.32)$$

Once again the notations are assumed to be understood.

From equations (2.23.b) and (2.32) one sees that the results of equation (2.23.b) can be generalized to any bus provided it carries a generator which is free.

The results of this section can be stated in the form of the following Lemma.

<u>Lemma</u> : In a lossless power system comprising of only thermal generating units having quadratic cost functions the optimum bus incremental cost of a generation bus will be equal to the incremental cost of the corresponding

generator if and only if it is free - else it is either equal to the IC of the corresponding generator plus the corresponding LM or minus the corresponding LM depending on whether that generator is at an upper or lower limit.

This Lemma serves to check the validity of numerical results for optimum BIC's obtained by using equation (2.23.a) and proves to give consistent results. Hence one can conclude that the model used for determining optimum BIC's in this study are independent of the choice of the slack bus. Numerical studies carried out further confirms this fact.

The next section describes an algorithm for computing optimum BIC's continuously from the minimum loadability limit to the maximum loadability limit. As can be seen from equations (2.23.a, b) one needs to know the LM's* of the constrained lines, i.e., $\underline{\alpha}_2$ and the value of the generation corresponding to the generator chosen to be the slack. This information is provided through a simulation package working on the theory of Appendix B . This theory uses the energy balance equation as an equality constraint, while the model developed in this chapter utilizes the (n+1) load flow equations as the equality constraints. It can be shown through simple algebra that the LM's involved in both models are in fact the same, i.e.,

$$\underline{\lambda}_1 = \underline{\alpha}_1 \qquad (2.33.a)$$

$$\underline{\lambda}_2 = \underline{\alpha}_2 \qquad (2.33.b)$$

---

*Lagrange Multipliers.

where vectors $\underline{\lambda}_1$ and $\underline{\lambda}_2$ are defined in Appendix B . This in fact simplifies the interaction between the routine developed in this study and the simulation package used to supply optimum LM's [Vojdani 1982] . The next section describes the algorithm and highlights some computational advantages.

2.4.4 Description of the Algorithm to Compute Optimum BIC's

In the first phase the algorithm reads in the constrained generators at a change point*, and determines whether the slack generator chosen for the study is bound or not. Depending on the state of the slack generator, the BIC of the slack bus is computed, and correspondingly that portion of the matrix $\underline{S}_b$ is constructed and stored.

In the next phase the LM's corresponding to the active transmission lines are read in and correspondingly the remaining portions of matrix $\underline{S}_b$ are constructed and stored. It is worthy of mention that the final form of matrix $\underline{S}_b$ at a change point has only a few rows, i.e., the number of active transmission constraints plus one, a number which is normally never greater than five.

The DC load flow Jacobian need not be changed every time the slack generator reaches an operating limit. This was demonstrated in

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*A change point is a value of system demand P_D^*, at which an inequality constraint given by equations (2.18.d, e) becomes active or inactive.

Sections 2.4.2 and 2.4.3 . Hence the Jacobian need be factorized only once at the first change point, either through sparse Cholesky factors or otherwise, and stored appropriately to be recalled at every change point in order to solve equation (2.23.a) for the optimum BIC's . This results in considerable savings in computer time.

As the BIC's are piecewise linear within a pair of load intervals, the routine for computing BIC's need be called only twice per interval, once at the change point at the beginning of the interval and next at the change point at the end of the interval. The BIC's at points in between these change points being obtained through linear interpolation. The analytical expression for the BIC's completely eliminates the need for iterations and makes the solution very fast, thus making it attractive for real time implementation in economy interchanges. The operation of the subroutine is shown in the form of a flow chart in Figure 2.1 .

The next section derives a relationship between the SIC and the corresponding BIC's of all system buses and demonstrates how the former may be computed from the latter, discussing the implications of constrained lines in these results.

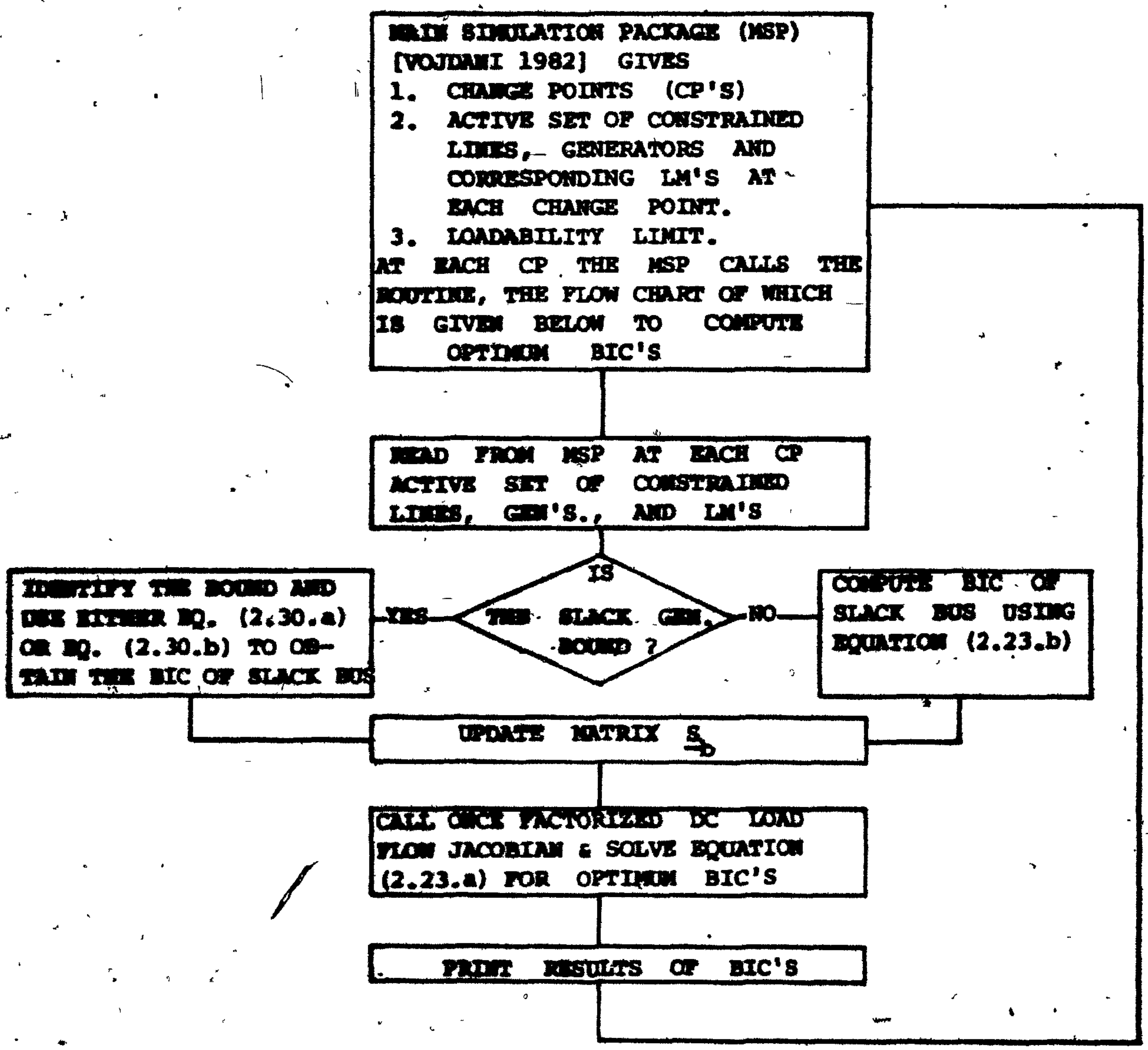


Figure 2.1 Flow Chart indicating the manner in which optimum BIC'S are computed through the loadability range.

2.4.5 The System Incremental Cost

In the present study it is assumed that the bus loads are linearly correlated to the system demand P_D [IEEE Reliability Test System 1979] .

$$L_i = t_i P_D \qquad \forall i = 1, \ldots, n+1 \tag{2.34}$$

Separating the bus load at the slack bus, equation (2.34) becomes :

$$\underline{L} = \underline{T} P_D \tag{2.35.a}$$

$$L_{m+1} = t_{m+1} P_D \tag{2.35.b}$$

where

$$\underline{T} \triangleq [t_1, \ldots, t_m, t_{m+2}, \ldots, t_{n+1}]^t \tag{2.35.c}$$

is the bus load system load correlation vector.

Thus substituting for $\underline{L}$ in either equation (2.21) or (2.29) and taking the derivative with respect to system demand P_D results in :

$$\frac{d\mathcal{L}}{dP_D} = \underline{\Lambda}^t \underline{T} + \lambda_{m+1} t_{m+1} \tag{2.36.a}$$

$$= \sum_{i=1}^{m+1} t_i \lambda_i \tag{2.36.b}$$

However, in the absence of transmission line constraints, equation (2.23.a) becomes :

$$\underline{J}^t \underline{\Lambda} = -\tilde{\underline{b}}\, \lambda_{m+1} \qquad (2.37)$$

the solution of which is trivial, due to the structure of the DC load flow Jacobian $\underline{J}$, and is given by :

$$\lambda_1 = \lambda_2 = \dots\dots\dots\dots = \lambda_{m+1} \qquad (2.38)$$

where λ_i is the ith element of $\underline{\Lambda}$.

From equations (2.36) and (2.38) , the following equation results, i.e.,

$$\frac{d\mathcal{L}}{dP_D} = \frac{dC}{dP_D} = \lambda_{m+1} \qquad (2.39)$$

i.e., the system λ , defined by $\frac{dC}{dP_D}$ is equal to the BIC of the slack bus.

2.4.6 Summary of Results

In the absence of constrained transmission lines, all BIC's are in parity with each other and the SIC . However, one active transmission constraint is sufficient to bring about a large disparity among BIC's and the SIC . The difference are much more pronounced close to the loadability limit of the power system. This latter fact is demonstrated through tests carried out in Chapter IV .

The transmission losses have been ignored up to now in this study. The next section demonstrates how these losses can be incorporated into the study, and it is shown that even in the absence of constrained transmission lines there could be disparity among the BIC's due to transmission losses.

2.5 Effect of Transmission Losses on Optimum BIC's

2.5.0 A Brief Overview of Transmission Loss Approximation Formulae

Classically, transmission losses have been incorporated into the ED problem through B-Coefficients [Kirchmayer 1958] . However, other more accurate methods exist such as the DC - Loss Formula (DCLF) and Taylor Series Loss Formula (TSLF) . Numerical studies carried out to ascertain the accuracy of these models demonstrate the TSLF to give the best results [Galiana and Banakar 1979, Yiu Cho Man 1979] . In this study losses are approximated by a linear expression to the TSLF . It will be seen in the subsequent reading that this approximation simplifies the computations relating to optimum BIC's in a lossy system. However, in order to apply this approximation, an expression must be derived for system losses. This task will be undertaken in the next section.

2.5.1 The Dependence of Losses on Controllable Generations and Loads

In the analysis to follow, the bus voltage magnitudes will be assumed fixed, i.e., they will be treated as parameters to comply with the

DC - Load Flow model. Thus all buses except the slack bus will be modelled as voltage controlled buses.

The n load flow equations can therefore be written from equation (A.12) as follows, i.e.,

$$\underline{Z} = \underline{Z}(\theta) \tag{2.40}$$

The ith element of equation (2.40) is given by [Elgerd 1982, El-Abiad 1968] the following, i.e.,

$$Z_i = \sum_{k \in i} |Y_{i-k} V_i V_k| \cos(\theta_i - \theta_k - \alpha_{ik})$$

$$i = 1, \ldots, m, m+2, \ldots, n+1 \tag{2.41}$$

By definition :

$$Y_{i-k} \triangleq |Y_{i-k}| e^{j\alpha_{ik}} \tag{2.42}$$

where $k \in i$ implies that k is connected to i , but is not coincident with i .

Thus knowing the n-bus injections the n-load flow equations (2.41) can be solved to give a unique solution of the state vector $\underline{\theta}$ [Tinney and Hart 1967] . Hence the state vector $\underline{\theta}$ can be written as follows, i.e.,

$$\underline{\theta} = \underline{\theta}(\underline{P}, \underline{L}) \tag{2.43.a}$$

The transmission system loss is a dependent load flow variable, and hence can be expressed in terms of the state vector $\underline{\theta}$, i.e.,

$$P_L = P_L(\underline{\theta}) \qquad (2.43.b)$$

From equations (2.43.a) and (2.43.b) the following result is obtained, i.e.,

$$P_L = P_L(\underline{P}, \underline{L}) \qquad (2.43.c)$$

Before proceeding to the derivation of the Equivalent Lossless Model (ELM), the ED model with losses will be formulated and solved for the optimum BIC's . Thenceforth the theory proceeds to show how these BIC's can be obtained from the ELM . It will be shown that the solution to this latter problem is trivial for the case to be investigated, and hence would simplify the determination of optimum BIC's in the ED model with losses.

2.5.2 Derivation of Optimum BIC's with Transmission Losses

In the presence of transmission losses, the ED problem can be formulated as follows, i.e.,

$$\text{minimize} \sum_{i=1}^{m+1} C_i(P_i) \qquad (2.44.a)$$

w.r.t. P_i , for $i = 1, 2, \ldots , m+1$

$$\text{s.t.} \qquad \underline{P} - \underline{L} = \underline{P}(\underline{\theta}) \tag{2.44.b}$$

$$P_{m+1} - L_{m+1} = P_{m+1}(\underline{\theta}) \tag{2.44.c}$$

By definition

$$\hat{\underline{P}} = [P_1, P_2, \ldots\ldots, P_{m+1}]^t . \tag{2.44.d}$$

Hence the Lagrangian to be minimized becomes :

$$\mathcal{L} = \sum_{i=1}^{m+1} C_i(P_i) + \underline{\Lambda}^t(\underline{P}(\underline{\theta}) + \underline{L} - \underline{P}) + \lambda_{m+1}(P_{m+1}(\theta) + L_{m+1} - P_{m+1}) \tag{2.45}$$

The first order necessary conditions are

$$\frac{\partial \mathcal{L}}{\partial P_i} = C_i'(P_i) - \lambda_i = 0 \quad , \quad \forall\, i = 1, \ldots\ldots, m \tag{2.46.a}$$

$$\frac{\partial \mathcal{L}}{\partial P_{m+1}} = C_{m+1}'(P_{m+1}) - \lambda_{m+1} = 0 \tag{2.46.b}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\theta}} = \frac{\partial P^t(\underline{\theta})}{\partial \underline{\theta}} \underline{\Lambda} + \lambda_{m+1} \frac{\partial P_{m+1}(\underline{\theta})}{\partial \underline{\theta}} = 0 \tag{2.46.c}$$

The ith component of equation (2.46.c) can be written as follows, i.e.,

$$\sum_{\substack{j=1 \\ j \neq m+1}}^{n} \lambda_j \frac{\partial P_j(\underline{\theta})}{\partial \theta_i} = -\lambda_{m+1} \frac{\partial P_{m+1}(\underline{\theta})}{\partial \theta_i} \tag{2.47.a}$$

From equation (2.47.a) the following equation can be written :

$$\lambda_i = -\lambda_{m+1} \frac{\partial P_{m+1}(\underline{\theta})}{\partial P_i(\underline{\theta})} \tag{2.47.b}$$

From equations (2.44.b, c) and (2.43.c) the following result can be obtained, i.e.,

$$P_L(\underline{P}, \underline{L}) = P_{m+1}(\underline{\theta}) + e_n^t \underline{P}(\underline{\theta}) \tag{2.48}$$

By definition :

$$e_n \triangleq [1, 1, \ldots\ldots, 1]^t_{1xn} \tag{2.49}$$

From equation (2.48) the following result can be obtained, i.e.,

$$\frac{\partial P_{m+1}(\underline{\theta})}{\partial P_i(\underline{\theta})} = \frac{\partial P_L}{\partial P_i} \frac{\partial P_i}{\partial P_i(\underline{\theta})} - 1 \tag{2.50}$$

From the ith component of equation (2.44.b) the following result is obtained, i.e.,

$$\frac{\partial P_i(\underline{\theta})}{\partial P_i} = 1 \tag{2.51}$$

From equations (2.50) and (2.51) the following result is obtained, i.e.,

$$\frac{\partial P_{m+1}(\underline{\theta})}{\partial P_i(\underline{\theta})} = \frac{\partial P_L}{\partial P_i} - 1 \tag{2.52}$$

The Incremental Transmission Loss (ITL) at bus i is defined by the following [Kirchmayer 1958], i.e.,

$$\frac{\partial P_L}{\partial P_i} \triangleq k_i \quad , \qquad \forall\, i = 1, 2, \ldots , m, m+2, \ldots\ldots , n+1 \qquad (2.53.a)$$

$$\frac{\partial P_L}{\partial P_{m+1}} \triangleq k_{m+1} \triangleq 0 \qquad (2.53.b)$$

Thus from equations (2.47), (2.52) and (2.53.a, b) the following result is obtained, i.e.,

$$\lambda_i = \lambda_{m+1} (1 - k_i) \qquad (2.54)$$

The Penalty Factor at bus i (PF_i) is defined by the following equation, i.e.,

$$PF_i \triangleq \frac{1}{(1 - k_i)} \qquad (2.55)$$

From equations (2.54) and (2.55) the following result is obtained, i.e.,

$$PF_i\, \lambda_i = \lambda_{m+1} \qquad \forall\, i = 1, \ldots\ldots , n \qquad (2.56)$$

Following an approach similar to that given in Section 2.5.1 one can show that λ_{m+1} and the elements of $\underline{\Lambda}$ are the BIC's of the respective buses. Furthermore the optimum operating strategy in the presence of transmission losses implies that the penalized BIC's of all system buses must be equal to the BIC of the slack bus.

In summary one could say that when transmission lines are lossy, the optimum operation of the system corresponds to the penalized IC of all generators being equal. Components of equation (2.56) are the modified coordination equations [Galiana and Vojdani 1979]. The following summary is made from equations (2.56), (2.46.a, b, and c), i.e.,

$$\frac{dC}{dL_i} \triangleq \lambda_i = C_i'(P_i) = (1 - k_i)\, C_{m+1}'(P_{m+1}) \qquad \forall\, i = 1, \ldots, m \tag{2.57.a}$$

$$\frac{dC}{dL_{m+1}} \triangleq \lambda_{m+1} = C'_{m+1}(P_{m+1}) \tag{2.57.b}$$

$$\frac{dC}{dL_j} \triangleq \lambda_j = (1 - k_j)\, C_{m+1}'(P_{m+1}) \qquad \forall\, j = m+2, \ldots, n+1 \tag{2.57.c}$$

In practice the modified coordination equations are solved iteratively, with an outer iteration loop which computes both the total system loss P_L, and the incremental transmission losses k_i from a load flow study or loss approximation formula [Vojdani 1979].

The approximate equivalent lossless model will be derived next from the lossy ED problem, and it will be shown how one could compute the BIC's defined by equations (2.57.a,b, and c) trivially from the ELM.

2.5.3 The Equivalent Lossless Problem

From equations (2.53.a, b) the following results are obtained, i.e.,

$$\Delta P_L = \underline{K}^t \Delta \underline{P} \qquad (2.58.a)$$

This can be written in the form shown below, i.e.,

$$P_L = P_{LO} + \underline{K}^t (\hat{\underline{P}} - \underline{P}^*) \qquad (2.58.b)$$

Where P_{LO} is the total system losses at the optimum defined by $\underline{P}^*$.

By definition :

$$\underline{K} \triangleq [k_1, k_2, \ldots\ldots\ldots, k_{m+1}]^t \qquad (2.59.a)$$

$$\underline{P}^* \triangleq [P_1^*, P_2^*, \ldots\ldots\ldots, P_{m+1}^*]^t \qquad (2.59.b)$$

$$\underline{P}^* + \Delta \underline{P}^* \triangleq \hat{\underline{P}} = [P_1, P_2, \ldots., P_{m+1}]^t \qquad (2.59.c)$$

Equation (2.58.b) defines the total system losses in a 'δ' neighbourhood of the base case optimum, defined by $\underline{P}^*$. This is the linearized form of Taylor series loss formula.

Rewriting the ED problem of Section 2.5.2 with the energy balance equation results in the following set of equations, i.e.,

$$\text{Minimize} \quad \sum_{i=1}^{m+1} C_i(P_i) \tag{2.60.a}$$

$$\text{w.r.t.} \quad P_i \quad \text{for} \quad i = 1, 2, \ldots, m+1$$

$$\text{s.t.} \quad \sum_{i=1}^{m+1} P_i = P_D + P_L \tag{2.60.b}$$

Eliminating P_L in equation (2.60.b) by using equation (2.58.b) results in the following, i.e.,

$$\underline{e}_{m+1}^t \, \hat{\underline{P}} = P_D + P_{LO} + \underline{K}^t (\hat{\underline{P}} - \underline{P}^*) \tag{2.61.a}$$

By definition :

$$e_{m+1} \triangleq [1, \ldots\ldots\ldots\ldots, 1]^t \tag{2.61.b}$$

On simplifying equation (2.61.a) results in the following, i.e.,

$$(\underline{e}_{m+1} - \underline{K})^t \, \hat{\underline{P}} = P_D + P_{LO} - K^t P^* \tag{2.61.c}$$

Hence, equation (2.61.c) gets simplified to the following, i.e.,

$$(\underline{e}_{m+1} - \underline{K})^t \, \hat{\underline{P}} \simeq P_D \tag{2.62.a}$$

$$\text{i.e.,} \quad \sum_{i=1}^{m+1} \bar{P}_i \simeq P_D \tag{2.62.b}$$

Equation (2.62.b) is similar to the energy balance equation of a lossless system.

By definition :

$$\bar{P}_i \triangleq (1 - k_i)P_i \qquad \forall\, i = 1, \ldots\ldots, m+1 \qquad (2.62.c)$$

Thus equation (2.60.b) in model (2.60) becomes equation (2.62.b) , thus mathematically transforming ED model with losses, i.e., equation (2.60) to the following equivalent lossless model, i.e.,

$$\text{Minimize} \sum_{i=1}^{m+1} \{C_i(P_i) = \bar{C}_i(\bar{P}_i) \triangleq c_i + \bar{a}_i P_i + \frac{1}{2}\bar{b}_i \bar{P}_i^2\} \qquad (2.63.a)$$

$$\text{w.r.t.} \quad P_i , \quad \text{for } i = 1, 2, \ldots\ldots, m+1$$

$$\text{s.t.} \quad \sum_{i=1}^{m+1} \bar{P}_i \cong P_D \qquad (2.63.b)$$

Where by definition :

$$\bar{a}_i \triangleq \frac{a_i}{1 - k_i} \qquad (2.64.a)$$

$$\bar{b}_i \triangleq \frac{b_i}{(1 - k_i)^2} \qquad (2.64.b)$$

$\bar{P}_i$ in this model will be the modified generations, and $\bar{a}_i$, $\bar{b}_i$ the modified cost coefficients.

Due to the insignificant component of system losses that are usually involved in each load flow equation, the energy balance equation (2.63.b) can be written as the following approximate load flow equations, i.e.,

$$\underline{\bar{P}} - \underline{L} = \underline{J}\,\underline{\bar{\theta}} \qquad (2.65.a)$$

$$\bar{P}_{m+1} - L_{m+1} = \tilde{b}^{t}\,\underline{\bar{\theta}} \qquad (2.65.b)$$

Where by definition :

$$\bar{P} \triangleq [\bar{P}_1, \bar{P}_2, \ldots\ldots\ldots, \bar{P}_m]^t \qquad (2.65.c)$$

$$\underline{\bar{\theta}} = [\bar{\theta}_1, \bar{\theta}_2, \ldots, \bar{\theta}_m, \bar{\theta}_{m+2}, \ldots\ldots \bar{\theta}_{m+1}]^t \qquad (2.65.d)$$

The justification of equations (2.65.a) and (2.65.b) lies in the fact that they are mathematically correct, i.e., addition of these equations does give the energy balance equation (2.63.b) . Stated differently it means that the component of total system loss inherent in each of the load flow equations is small enough to be neglected. However the addition of these insignificant losses (to give the total system losses P_L) cannot be considered to be negligible, and hence are taken into account in the energy balance equation (2.60.b) . Thus considering equation (2.63.a) as the objective function and equations (2.65.a, b) as equality constraints, the Lagrangian to be minimized becomes :

$$\mathcal{L} = \sum_{i=1}^{m+1} C_i\,(\bar{P}_i) + \underline{\bar{\lambda}}^{t}\,(\underline{J}\,\underline{\bar{\theta}} + \underline{L} - \underline{\bar{P}}) + \bar{\lambda}_{m+1}\,(\tilde{\underline{b}}^{t}\,\underline{\bar{\theta}} + L_{m+1} - \bar{P}_{m+1})$$

(2.66.a)

The first order necessary conditions are :

$$\frac{\partial \mathcal{L}}{\partial \bar{P}_i} = \bar{C}_i'(\bar{P}_i) - \bar{\lambda}_i = 0 , \quad \forall\, i = 1, \ldots\ldots , m+1 \qquad (2.66.b)$$

$$\frac{\partial \mathcal{L}}{\partial \bar{P}_{m+1}} = \bar{C}'_{m+1} (\bar{P}_{m+1}) - \bar{\lambda}_{m+1} = 0 \qquad (2.66.c)$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\bar{\theta}}} = \underline{J}^t \, \underline{\bar{\Lambda}}^t + \bar{\lambda}_{m+1} \, \underline{\tilde{b}} = 0 \qquad (2.66.d)$$

As before it can be shown that the elements of $\underline{\Lambda}$ and λ_{m+1} are the BIC's of the respective buses.

From equations (2.66.c, d) the following result is obtained, i.e.,

$$\underline{\bar{\Lambda}} = \underline{a}_n \, \bar{C}'_{m+1} (P_{m+1}) = \underline{a}_n \, C'_{m+1} (P_{m+1}) \qquad (2.67)$$

From equation (2.67) one sees that the BIC's of all buses in the HLM are equal to the IC of the slack generator No. (m+1) . This solution can be obtained easily through the subroutine described in Section 2.4.4 , the only additional information needed being the incremental transmission losses, which must be obtained from a different computer program. As these ITL's do not vary greatly over the load interval, they need be computed only once at the beginning of each load interval [Al-Jishi 1982] .

On replacing $C'_{m+1}(P_{m+1})$ in equations (2.57.a, b, c) with the corresponding values in equation (2.67), results in the following equations relating the BIC's in the ELM to those of the lossy ED model and are summarized below, i.e.,

$$\frac{dC}{dL_i} \triangleq \lambda_i = C'_i(P_i) = (1 - k_i)\,\bar{\lambda}_i \quad \forall\, i = 1, \ldots\ldots, m \tag{2.68.a}$$

$$\frac{dC}{dL_{m+1}} \triangleq \lambda_{m+1} = C'_{m+1}(P_{m+1}) \tag{2.68.b}$$

$$\frac{dC}{dL_j} \triangleq \lambda_j = (1 - K_j)\,\bar{\lambda}_j \quad \forall\, j = m+2, \ldots\ldots (m+1) \tag{2.68.c}$$

It can be observed that the BIC of the slack bus, i.e., bus number (m+1), remains invariant in both models. Thus, in solving for the BIC's of the ED model with losses, one could first solve the corresponding ELM by utilizing the algorithm of Section 2.4.4, and by suitably modifying the generation cost coefficients of all generators. Once the solution to this problem is obtained, the BIC of the ED model with losses can be obtained through the transformations defined by equations (2.68.a, b, and c).

CHAPTER III

OPTIMUM OPERATING COST OF A POWER SYSTEM

3.0 Introduction

The optimum operating cost of a power system plays an important role in the planning stages of a power system, as the power system planner would always try to select those alternative expansion strategies which would minimize this operating cost. The term operating cost, implies the total cost of fuel used by the system generators over a specified period of time, preferably a day. The operating cost of a system is dependent on the loading procedure, availability of units, and the system demand. The present study views the problem of operation as a short term deterministic problem where forced outages are ignored and a specified network structure is assumed. However, the theory developed can be used to simulate forced generator outages by suitably modifying the load duration curve [Vardi and Avi-Itzhak 1981] .

The load duration curve will serve as the basis for computing the optimum operating cost of the system. Hence, a discussion of this will be considered in the next section.

3.1 The Load Duration Curve

The load duration curve (LDC) is defined as a graph showing the fraction of time in a given time interval for which any given system load level is exceeded. The load duration curve can be used for the following computations, namely the operating cost of a power system, the energy delivered by each generating unit, and any associated cogeneration units, and the minimum cost incurred by a utility in supplying the energy requirements of a given bus (either based on the BIC of the corresponding bus or the SIC) . In its conventional form the abscissa of the LDC specifies the number of hours in a given period (assumed to be a day in this study) during which the customers demand for power equals or exceeds the associated demand level specified on the ordinate (usually as a percentage of the annual peak demand) . In practice the role of the two coordinate axes are interchanged and the time variable is normalized, the base chosen for the normalization being the corresponding period (24 H in this study) . Thus, the ordinate of the normalized LDC can be interpreted as the probability that a given demand may be equalled or exceeded.

If $\psi(P_D)$ defines the normalized LDC , the cumulative density (or distribution function) is defined by the following equation, i.e.,

$$1 - \psi(P_D) = \phi(P_D) \tag{3.1}$$

The derivative of equation (3.1) with respect to the demand P_D gives the corresponding density function of the demand, i.e.,

$$PDF(P_D) \triangleq \phi'(P_D) \tag{3.2}$$

In the above equation $PDF(P_D)$ is the probability density function of the system demand P_D. Figure 3.1* gives a graphical illustration. The $PDF(P_D)$ can be used to compute the expected values (weighted average values) of the bus and system IC's. This gives an indication of the relative variation. The process of determining the expected values are discussed in the next section.

3.2 Computing the Expected Values of Bus and System IC's

The BIC's are piecewise linear functions in the system demand. However, the theory to follow does not explicitly assume this, and would be valid even otherwise. The expected value of the BIC of bus i, i.e., λ_i, is computed from the following equation, i.e.,

$$E(\lambda_i) = \int_{-\infty}^{\infty} BIC_i\,(P_D)\;\phi'(P_D)\;dP_D \qquad (3.3.a)$$

where E stands for the mathematical expectation.

The SIC is also piecewise linear in the system demand P_D. The expected value of which is given by

$$E(SIC) = \int_{-\infty}^{\infty} SIC(P_D)\;\phi'(P_D)\;dP_D \qquad (3.3.b)$$

---

*All figures are given at the end of this chapter.

In the absence of transmission line constraints, it can be shown from equations (2.23.a, b) and (3.3.a, b) that the following is true, i.e.,

$$E(SIC) = E(\lambda_i) \qquad \forall\, i = 1, \ldots\ldots, n+1 \qquad (3.3.c)$$

The computation of the expected values would help observe relative variations in BIC's from the SIC due to saturating transmission lines.

The method for computing the optimum operating costs of a power system, and those of individual generators will be discussed in the next section. The computer package developed for these computations approximates the LDC by a piecewise linear function. This latter approximation being used to simplify computations.

3.3 Theory for Computing the Optimum Operating Cost of a Power System

For demonstrating the theory, a power system with only two generating units will be considered. However the results derived are quite general, and can be applied to a system with any number of generating units.

Figure 3.2 shows a geometrical procedure for constructing the SIC curve, load duration curves of generating units, provided the IC curves of the individual generating units, the LDC of the system are known. Presently it is assumed that no lines are saturated, hence the equal incremental cost criterion (EICC) can be used to compute the SIC curve of Figure 3.2.c . Furthermore, it can be seen from Figures 3.2.d, e, f, that the ordinates of the normalized LDC of generating units are equal to the ordinates of the normalized LDC of the system, within the range of operation of each individual unit, i.e.,

$$\psi_i(P_i(P_D)) = \psi(P_D) \qquad \text{for} \quad P_i^{min} < P_i < P_i^{\ell t}$$
$$\forall\, i = 1, \ldots, m+1 \qquad (3.4.a)$$

where $\psi_i(P_i(P_D))$ is the normalized LDC of unit i and

$P_i^{\ell t}$ is the limiting generation of unit i. Generally $P_i^{\ell t} < P_i^{max}$ for all units.

In the event that transmission lines reach operating limits, the EICC may no longer be employed to compute the SIC curve. Assuming that the computer package provides the piecewise linear variation in the SIC curve, and the piecewise linear variation in the generation of unit i with demand P_D, Figure 3.3 gives a geometrical procedure for obtaining the normalized LDC of generating units in this case. To be general it is assumed here that the unit picks up load at a point beyond P_D^{min} . Once again it will be seen that within the working range of each unit equation (3.4.a) is valid.

The minimum and maximum system demands are assumed to lie in the range given by the following, i.e.,

$$\sum_{i=1}^{m+1} P_i^{min} < P_D^{min} \; , \; P_D^{max} < \sum_{i=1}^{m+1} P_i^{max} \qquad (3.4.b)$$

The optimum total operating cost of the system is given by (in \$) :

$$C_{tot} = C_{ABCD} + \int_{\sum_{i=1}^{m+1} P_i^{min}}^{P_D^{max}} C'(P_D)\, \phi(P_D)\, dP_D \qquad (3.4.c)$$

From Figures 3.2.d, e, f the shaded areas A_1, A_2, and A_3 are related by the following equation, i.e.,

$$A_1 + A_2 = A_3 \qquad (3.4.d)$$

The cost incurred by the system (in \$) , to supply the energy represented by area A_3 is given by the following, i.e.,

$$C_{ABCD} = \{C_1(P_1^{min}) + C_2(P_2^{min})\} \times T \qquad (3.4.e)$$

where T is the time duration under study taken to be 24 H in this work. The next section shows how these very same ideas can be used to compute the optimum operating cost of generating units. First the theory is developed for a unit with one IC segment and subsequently it is extended to cover multiple IC segments.

3.4 Theory for Computing the Optimum Operating Cost of a Single Generating Unit with only One IC Segment

The optimum operating cost of unit i (in \$) is given by the following equation, i.e.,

$$C_i^{op} = C_i(P_i^{min}) \times T + \int_{P_i^{min}}^{P_i^{\ell t}} C_i'(P_i)\, \phi_i(P_i)\, dP_i \tag{3.5.a}$$

On simplifying equation (3.5.a) results in the following equation, i.e.,

$$C_i^{op} = C_i(P_i^{min}) \times T + \int_{\sum_{i=1}^{m+1} P_i^{min}}^{P_D^{max}} C_i'(P_i)\, \phi(P_D)\, m_i(P_D)\, dP_D \qquad \forall\, i = 1, \ldots, m+1 \tag{3.5.b}$$

where by definition :

$$m_i(P_D) \triangleq \frac{dP_i}{dP_D} \tag{3.5.c}$$

The IC of unit i, i.e., $C_i'(P_i)$ can be parameterized in P_D. This is possible as under lossless ED, the generation of unit i is piecewise linear in the demand P_D, i.e.,

$$P_i = P_i(P_D) \tag{3.5.d}$$

Thus equation (3.5.b) becomes :

$$C_i^{op} = C_i(P_i^{min}) \times T + \int_{\sum_{i=1}^{m+1} P_i^{min}}^{P_D^{max}} \Omega_i(P_D)\, m_i(P_D)\, dP_D \quad \forall\, i = 1, \ldots\ldots, m+1 \qquad (3.5.e)$$

where by definition :

$$\Omega_i(P_D) \triangleq C_i'(P_i(P_D))\, \phi(P_D) \qquad (3.5.f)$$

This theory will now be extended to the case where the generating units have multiple IC segments.

3.5 Theory for Computing the Optimum Operating Cost of a Single Generating Unit with Multiple IC Segments

Multiple IC segments are typical, when a generating plant comprises of many units which are not identical. The single equivalent unit which results through the application of the EICC will have many IC segments as shown in Figure 3.4 . If the single equivalent unit, denoted as i , has n IC segments, equation (3.5.b) gets modified to the following equation, i.e.,

$$C_i^{op} = C_i(P_i^{min}) \times T + \int_{\sum_{i=1}^{m+1} P_i^{min}}^{P_D^1} \Omega_i(P_D)\, m_i^1(P_D)\, dP_D$$

$$+ \int_{P_D^1}^{P_D^2} \Omega_i(P_D)\, m_i^2(P_D)\, dP_D$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$+ \int_{P_D^{n-1}}^{P_D^n} \Omega_i(P_D)\, m_i^n(P_D)\, dP_D \, ,$$

$$\forall\, i = 1, \ldots\ldots, m+1 \qquad (3.6)$$

In the next two sections it is shown how the ideas of cost computations may be extended to compute the energy delivered by each generating unit and the energy requirements of the different system buses.

3.6 Total Energy Delivered by Each Generating Unit

The total energy delivered by each generating unit i, i.e., W_i (in MWH) over a period T is given by the following equation, i.e.,

$$W_i = P_i^{min} \times T + \int_{P_i^{min}}^{P_i^{lt}} \psi_i(P_i)\, dP_i \qquad (3.7)$$

This could be simplified to give the following, i.e.,

$$W_i = P_i^{min} \times T + \int_{\sum_{i=1}^{m+1} P_i^{min}}^{P_D^{max}} \psi(P_D)\, m_i(P_D)\, dP_D \qquad (3.8)$$

$$\forall\ i = 1, \ldots\ldots, m+1$$

Parameterizing in system demand P_D is particularly useful as it helps to work with only the system LDC . The similarity between equations (3.5.e) and (3.8) implies that the algorithm for computing the operating cost of a unit can be used to compute the total energy delivered by each unit. The next section discusses the optimum cost incurred by the utility to supply electricity to different system buses.

3.7 Optimum Cost of Supplying Electricity to a Given Bus

The cost incurred by the utility in supplying the energy requirements of a given bus based on the BIC of that bus is given by the following equation, i.e.,

$$C_i^{bus} = t_i \left[C_{ABCD} + \int_{\sum_{i=1}^{m+1} P_i}^{P_D^{max}} \lambda_i(P_D)\, \psi(P_D)\, dP_D\right] \qquad (3.9)$$

where λ_i is the BIC of bus i , and t_i is the component of the correlation vector $\underline{T}$ corresponding to bus i . By replacing $\lambda_i(P_D)$ in the integrand of equation (3.9) by the SIC one could compute the cost of supplying electricity to bus i based on the SIC . This would aid computations relating to economy interchanges. The similar structure of equations (3.4.c) and (3.9) implies that one could use the algorithm which computes optimum operating cost of a system to compute the cost incurred by a utility to supply electricity to a given bus.

3.8 Modelling Cogeneration

The real power output from a cogeneration unit could have any general shape and need not resemble the load curve of the system in any way. However, in order to simplify computations the theory to be developed assumes a constant correlation between the real generation of a cogeneration unit and the system load curve. In the more general case this can be extended to a piecewise linear correlation function in the system demand P_D , the theory being still valid and being applied to a segment at a time.

The load duration curve of the system is known a priori . The BIC parameterised in P_D will be provided by the computer package developed in this study. By assuming a constant correlation ν_i to exist between the ith cogeneration unit and system demand P_D , Figure

3.5 gives the geometrical procedure for constructing the LDC of the cogeneration unit and the corresponding BIC as seen by this unit. The construction is assumed to be self-explanatory.

The area A_c in Figure 3.5 is the contribution by the cogeneration unit to the base load of the system. Hence, from Figures 3.2 and 3.5 one sees that the utility saves an amount (in \$) given by the following, i.e.,

$$R_i = R_i^{base} + \int_{P_{ci}^{min}}^{P_{ci}^{max}} \psi_{ci}(P_{ci})\ \lambda_{ci}(P_{ci})\ dP_{ci} \tag{3.10.a}$$

$$R_i^{base} \triangleq \frac{A_c}{A_3} \times C_{ABCD} \tag{3.10.b}$$

$\psi_{ci}(P_{ci})$ is the normalized LDC of the cogeneration unit and λ_{ci} is the BIC of bus i parameterized in P_{ci}. The computations involved in computing R_i can be greatly simplified by parameterizing ψ_i and λ_{ci} in P_D. Hence, equation (3.10.a) gets simplified to the following, i.e.,

$$R_i = R_i^{base} + \nu_i \int_{\sum_{i=1}^{m+1} P_i^{min}}^{P_D^{max}} \psi_{ci}(P_{ci}(P_D))\ \lambda_{ci}(P_{ci}(P_D))\ dP_D \tag{3.11}$$

The similar nature of equations (3.4.c) and (3.11) enables one to use the same algorithm in the computations of both the total operating cost of the system, i.e., c_{tot} and utility savings R_i.

Having a cogeneration unit in the system will affect the net LDC, and the bus load - system demand correlation vector $\underline{T}$. The next section investigates these changes.

3.9 Effect of A Cogeneration Unit on the Correlation Vector

From equation (2.34) the system demand would be written in the following form, i.e.,

$$P_D = \sum_{j=1}^{n+1} t_j P_D \qquad (3.12)$$

On introducing a cogeneration customer to bus i, having a fixed correlation ν_i to the system demand P_D, equation (3.12) gets modified to the following, i.e.,

$$(1 - \nu_i)P_D = \sum_{j=1}^{i-1} t_j P_D + (t_i - \nu_i)P_D + \sum_{j=i+1}^{n+1} C_j P_D \qquad (3.13)$$

Equation (3.13) can be simplified to the following form, i.e.,

$$P_D^{mod} = \sum_{j=1}^{n+1} t_j^{mod} P_D \qquad (3.14)$$

where by definition :

$$P_D^{mod} \triangleq (1 - \nu_i)P_D \qquad (3.15.a)$$

$$t_j^{mod} \triangleq \frac{t_j}{(1 - \nu_i)} \qquad \forall\, j = 1, \ldots, n+1 \quad j \neq i \tag{3.15.b}$$

and

$$t_j^{mod} \triangleq \frac{t_j - \nu_i}{(1 - \nu_i)} \qquad j = i \tag{3.15.c}$$

Hence every time a cogeneration unit is added to the system, the components of the correlation vector gets modified according to equations (3.15.b) and (3.15.c) respectively and hence do not require major modificiations to the algorithms for computing the costs discussed in the previous sections.

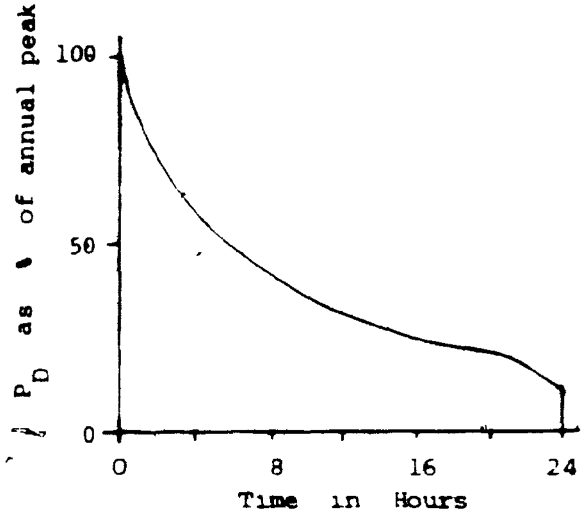


Figure 3.1.a Daily system load duration curve.

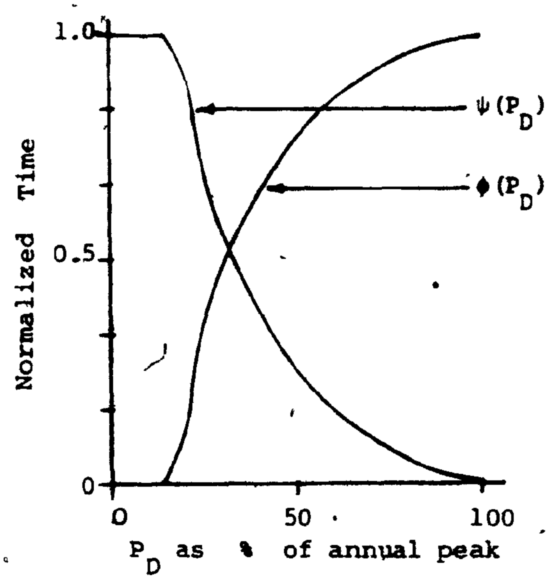


Figure 3.1.b Normalized LDC $\psi(P_D)$, and the load distribution function $\phi(P_D)$.

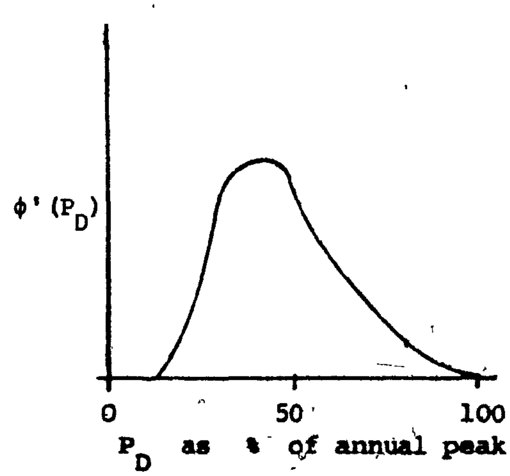


Figure 3.1.c The load density function.

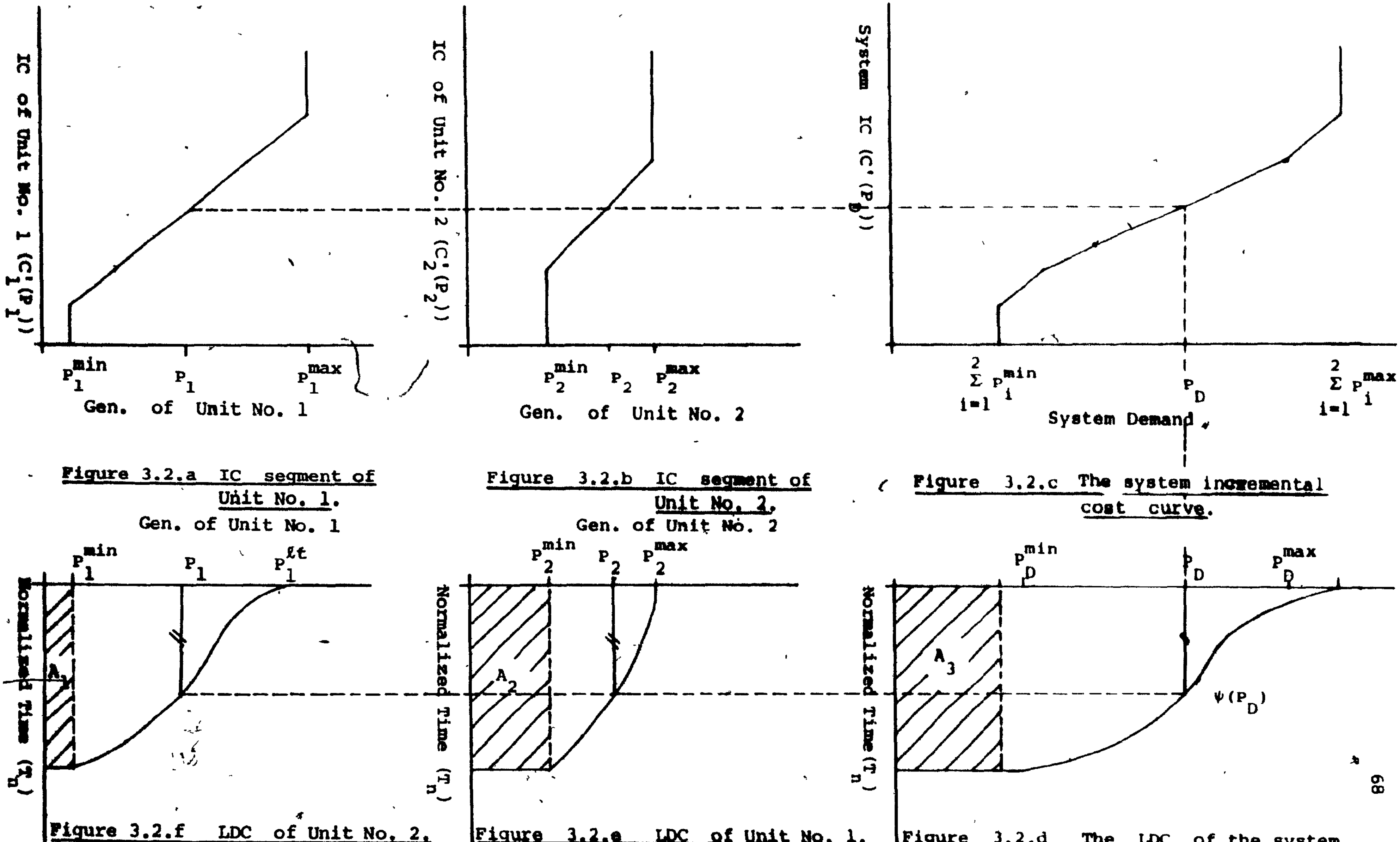


Figure 3.2.a IC segment of Unit No. 1.

Figure 3.2.b IC segment of Unit No. 2.

Figure 3.2.c The system incremental cost curve.

Figure 3.2.f LDC of Unit No. 2.

Figure 3.2.e LDC of Unit No. 1.

Figure 3.2.d The LDC of the system.

Figure 3.2 Constructing the LDC of units when transmission constraints are inactive.

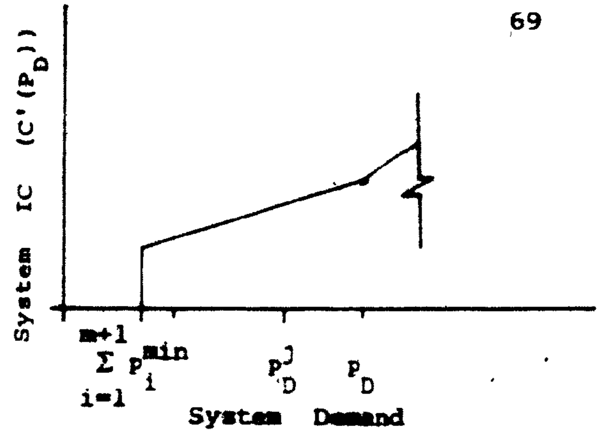


Figure 3.3.a One segment of the SIC curve.

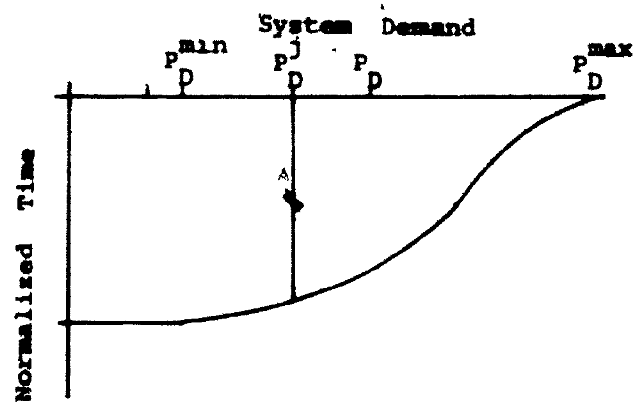


Figure 3.3.b The system LDC .

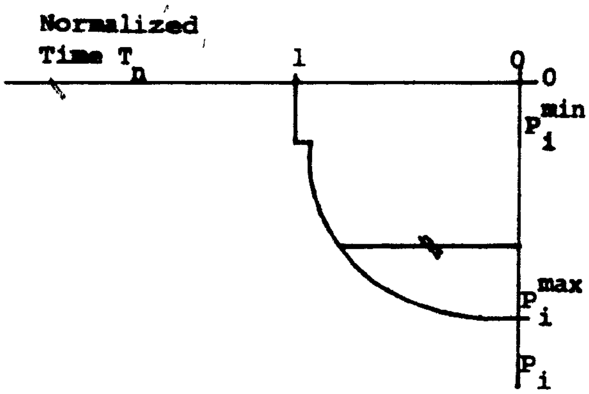


Figure 3.3.d LDC of Unit i .

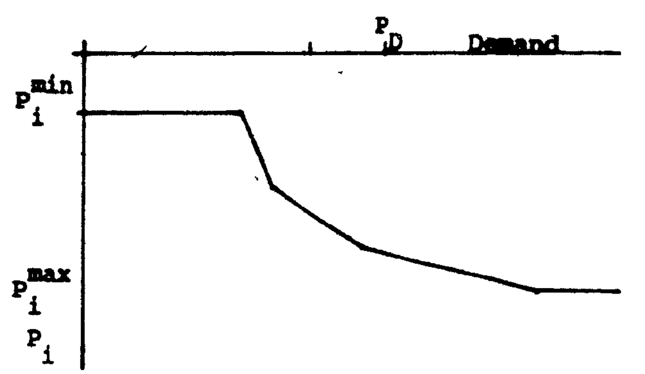


Figure 3.3.c Variation of P_i with P_D .

Figure 3.3 Constructing the normalized LDC of Unit i when transmission constraints are active.

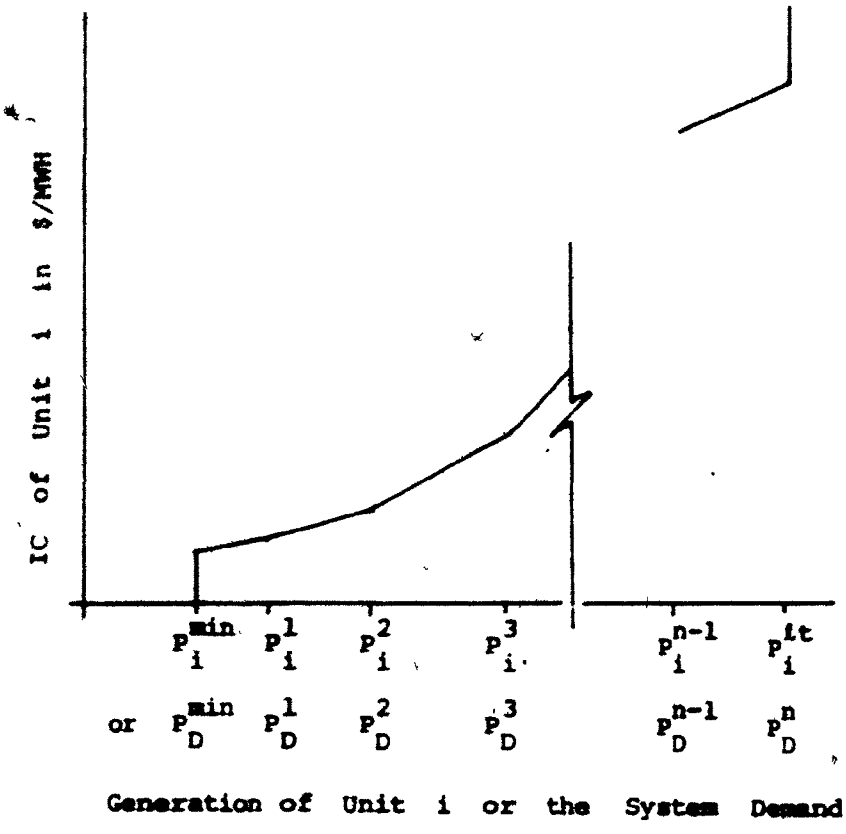
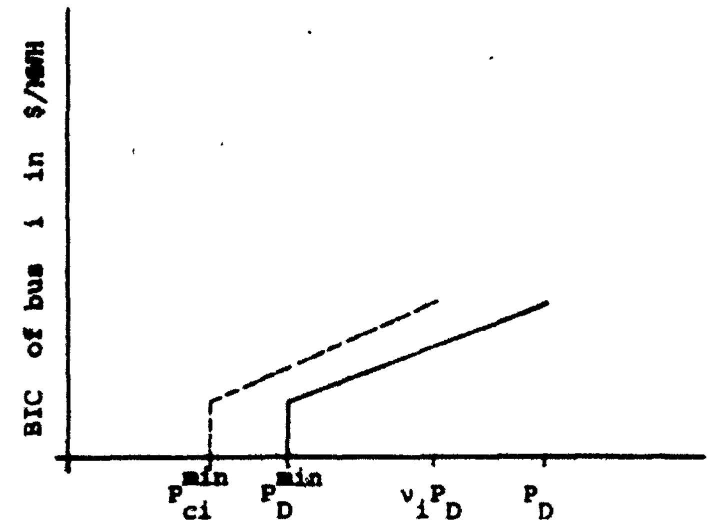


Figure 3.4 IC segments for Unit i .

NOTE : 1. The abscissa has two scales namely (i) the generation of unit i (P_i) , (ii) the system demand P_D .

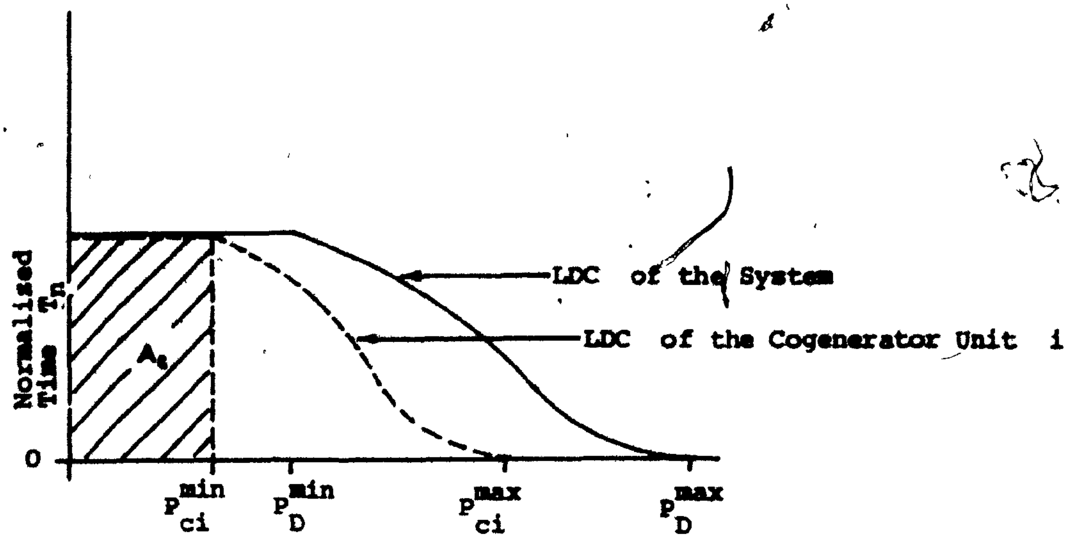
2. P_D^j is the value of P_D corresponding to P_i , for $j = 1, \ldots, n$.

3. $P_D^n < P_D^{max}$.



Output of Cogeneration Unit or Demand

Figure 3.5.a One segment of the BIC curve.



Output of Cogeneration Unit or Demand

Figure 3.5.b Load duration curves

Figure 3.5 Constructing the LDC of Cogeneration Unit i, and the BIC seen by this unit.

CHAPTER IV

ANALYSIS OF SIMULATION RESULTS

4.0 Description of Objectives

The numerical results discussed in this chapter are a consequence of two simulations carried out on the IEEE 24-bus test system [IEEE Reliability Test System 1979] , and a simulation on a 10-bus experimental test system [Vojdani 1982] . A major portion of this chapter is devoted to demonstrating the theory developed in previous chapters through these simulations. Although the theory developed is quite general within the framework of the necessary assumptions stated in their derivation, modifications were needed to the data cited in the first reference in order to overcome certain programming limitations inherent in the main simulation package. These modifications were needed only in the heat rate data of some generators and the configurations of generators. The modified data corresponding to these changes are given in Appendix D . Appendix E gives the data used for simulations carried out on the 10-bus system, with modifications made only to the network parameters to demonstrate the much pronounced variations in BIC's from the SIC close to the loadability limit with three constrained lines existing at the loadability limit.

The first two simulations cited above, to be carried out on the 24-bus system are meant to highlight specific theoretical aspects, and hence needed few changes to the network structure. These changes and the points emphasised by the simulation results will be the topic of Section 4.2 .

The data given for the IEEE 24-bus system in the corresponding reference specifies that certain hydro units are available for dispatch along with the remaining all thermal units. As one cannot define a fuel cost for hydro units, it cannot be incorporated into the ED model, along with other thermal generating units. Hence an optimum method for utilizing this hydro energy must be sought. Such a method is developed in the next section.

4.1 Optimum Utilisation of Available Hydro Energy

The data applicable to the 24-bus system specifies 769.23 MWH as the total amount of hydro energy available for dispatch on the first day of week 18 , which has been chosen as a typical day to demonstrate the theory of previous chapters. Further, this amount of energy can be dispatched at 100 % capacity, i.e., 3 PU MW , as specified in the data. However, if this energy is dispatched at 100 % capacity it would last only for 2.56 H . For better dispersion of this energy (3.85 H) it will be assumed that the dispatch is carried out at a fixed capacity of 66.7 % (i.e., at 2 PU MW) until all the available energy is expended. In order to carry out this dispatch optimally it becomes necessary to determine the time of day τ at which the hydro generator must be started. By varying τ from 0 to 24 H one could determine the value of τ which corresponds to the minimum system operating cost. This problem can be stated as the following optimization problem, i.e.,

$$\begin{aligned} &\text{minimize} \quad C(\underline{P}, H) \\ &\text{w.r.t.} \quad \tau \end{aligned} \tag{4.1}$$

where :

τ is the time of day at which hydro units are started.

H is the daily availability of hydro energy in MWH .

$C(\underline{P},H)$ The minimum daily system operating cost.

The solution to this problem is carried out numerically by incrementing τ in discrete time steps and computing the overall system operating cost. The results are given in Table 4.1*. From these results it becomes evident that 15:00 H is the best time to start the hydro units for this particular day as this brings about the lowest daily system operating cost of \$ 362.0×10^3 (cf. \$ 372.19×10^3 in the absence of hydro units) . This is found to be physically acceptable as well, as the system peak for this day occurs close to 15:00 H .

On dispatching the hydro energy in this manner the system load duration curve gets affected in the time interval 15:00 H to 18:51 H , by 2 PU MW . Hence the load duration curve is modified to

*All relevant tables and figures are given at the end of this chapter.

give the net load duration curve, the data of which normalized to 24 H is given in Table 4.2 . The area of the net load duration curve now becomes the net energy requirement of the system. Subsequent simulations and discussions to follow will assume that the dispatch of hydro units are fixed at the above level, and will utilize the net LDC of Table 4.2 for computations.

Table 4.3 gives the energy delivered by each generator and the corresponding cost of this energy. It can be seen from this table that generator 21 (nuclear) picks up the largest portion of system energy demand. The low IC of 9.7 \$/MWH at minimum generation of 200 MW at this plant and the relatively small gradient (0.0078 \$/MW^2-H) of the IC curve for this plant are the prime cause of this effect. On the contrary, the generating plant at bus # 15 picks up the least system energy demand, the reasons being a high IC of 22.11 \$/MWH at a minimum generation of 12 MW and the high slope of the IC curve of 0.2986 \$/MW^2-H . For these reasons, this plant remains static at its minimum generation level of 12 MW through the entire range of system loads. In carrying out these computations double lines 18 - 21 and one line 15 - 21 are assumed to be in outage in Figure 4.1 . These network modifications being necessary to demonstrate the disparity arising among the BIC's due to a single line, namely line 15 - 21 reaching an operating limit at a system demand of 14.436 PU MW . To demonstrate this and other theoretical points of interest on the 24-bus system, it was necessary to make certain other modifications to the network structure. This will be the subject of the next section.

4.2 Objectives and Assumptions Relating to Simulations

(a) Simulation # 1 : This is meant to demonstrate that the BIC's vary from the SIC when at least one transmission line reaches an operating limit. By removing the double lines 18 - 21 and one line 15 - 21 of Figure 4.1 , it is demonstrated that the single line remaining between buses 15 - 21 reaches an operating limit at a system demand of 14.436 PU MW (on 100 MVA base) . The resulting BIC's and the SIC are tabulated in Table 4.4 . Furthermore, necessary computations are carried out to demonstrate that if electricity is priced based on the SIC , it could cause some customers connected to certain buses to subsidize other customers connected to the remaining network buses. It is also demonstrated that by a change in the pricing philosophy, the utility neither gains nor loses so long as no purchases are made either from a neighbouring utility or from cogeneration customers. However if such purchases are based on the SIC , it is shown that either the buyer or the seller stands to lose or gain unfavourably.

(b) Simulation # 2 : The intention here being to investigate the changes in the results of simulation # 1 brought about by a second transmission line getting constrained. Once again any line could have been chosen to represent this second line, provided it permits the system to supply its peak demand. For purposes of this study the power flow limit on line 8 - 9 is is reduced from 175 to 75 MW , keeping the remaining data as in simulation # 1 . This causes line 8 - 9 to reach an operating limit at a system demand of 19.558 PU MW . This occurs in addition to line 15 - 21 getting constrained at 14.436 PU MW as before. The results are tabulated in Table 4.7 , from which one can see the much pronounced variations in some of the BIC's with respect to the SIC . A graphical plot of these variations are given in Figures 4.2.1 to 4.2.24 .

The pertinent aspects of simulations 1 and 2 will be discussed in the next section.

4.3 A Discussion of Results

Table 4.4 gives the variation in BIC's through the range of system loads, pertaining to the first day of week 18 as given in the data for the 24-bus system. This data is modified appropriately to consider the optimum dispatch of hydro energy as a negative load. The normalized LDC corresponding to this modified system demand is given in Table 4.2 , which will serve as the basis for cost computations to follow in Section 4.3 .

As the first transmission line, i.e., line 15 - 21 gets constrained at a system demand of 14.436 PU MW , the BIC's of all system buses begin to differ from each other and the SIC in intervals arising beyond this point. These results correspond to bus # 23 being chosen as the slack bus. However, on trying other buses in the system as the slack bus consistent results were obtained, confirming the theory of Chapter II . These results are computed using the general equation (2.23.a and b) . Further the BIC's of generation buses were recomputed through using the Lemma cited in Chapter II . Good agreement were found to exist between corresponding results. The SIC's given in this table were provided by the main simulation package. However on recomputing these SIC's from the BIC's and correlation data, using equation (2.36.b) good agreements were also obtained. These additional tests and the consistency of results are adequate justification that the results are accurate and are void of programming errors.

The low BIC of bus # 21 compared to the SIC , even at load levels close to maximum system demand, while the high BIC of bus # 15 can be explained through the Lemma cited in Chapter II ; i.e., the nuclear plant at bus # 21 which has the lowest IC at minimum demand, and a relatively low gradient to the IC curve, keeps the BIC of bus # 21 , which must be equal to the IC of generator # 21 , low . An argument to the contrary explains why the BIC of bus # 15 is high. The structure of the network is the prime reason for the BIC's of buses 17 and 18 being equal and those of 7 and 8 being equal - i.e., one could note that the structure of the network pertaining to the network modifications of simulation # 1 , isolates bus # 18 from the rest of the system except for bus # 17 . Hence so long as line 17 - 18 does not reach an operating limit, buses 17 and 18 will have the same BIC's . A similar situation prevails at buses 7 and 8 .

The cost of delivering the daily energy requirements of the various load buses are computed based on the SIC and the corresponding BIC . These results are tabulated in Table 4.5 . The net customer savings* due to a change in the pricing policy as given in Table 4.5 indicates that customers on buses 14, 16, 18, 19, and 20 will subsidize the electricity use of customers in the remaining buses if cost computations are based on the SIC . The high net savings at bus # 18 is due to (i) its proximity to bus # 21 which carries a cheap source of electrical energy and (ii) no generation facilities being available at this bus - for should a generator be present here by the Lemma of Chapter II , the BIC would be equal to the IC of the generator. The large negative savings at bus # 15 are mainly due to the expensive generating plant at this bus, which continues to remain constrained at its lower limit through the entire range of system loads. Hence, the BIC at this bus by the Lemma of Chapter II becomes the IC of the plant less its LM . Since the IC is high this makes the BIC more expensive. The simplest method of reducing this large financial loss at bus # 15 would be to completely shut down the plant at this bus, and restarting the units when the BIC becomes equal to the IC at minimum generation. By adopting such a control strategy, especially when

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*Net customer savings is defined as the difference between electricity prices based on the SIC and the corresponding BIC .

dealing with expensive units such as gas turbines, the net benefits of a cheap source of electrical energy such as that from a nuclear plant could be spread over a wider margin of customers. One can observe in Table 4.5 that the sum of electricity costs remain equal to \$ 362.0 x 10^3, which in turn is equal to the total cost of operating the system generators given in Table 4.3 .

It will now be demonstrated how a utility could gain or lose unfavourably when economy interchanges are carried out between two or more neighbouring utilities. If 85 % of the bus demand at bus # 18 corresponds to sales to a neighbouring utility, the daily cost of energy based on the SIC as computed by the seller is \$ 36.0 x 10^3, while if the same cost were computed based on the BIC it would be \$ 35.42 x 10^3 . Hence by maintaining electricity sales based on SIC , brings about an unwarranted financial benefit of \$ 580.0 per day. If sales similar to the one mentioned above are carried out at bus # 15 , would result in a loss of \$ 306.0 per day to the seller or an unwarranted financial gain to the buyer. A similar situation can prevail if electricity purchases are made from a neighbouring utility.

The theory developed in Chapter II assumes a fixed correlation to exist between the real generation of a cogeneration customer and the system demand. This assumption is used mainly for simplifying the numerical computations. However more generally the above specific correlation can be extended to a more general one by assuming a piecewise linear correlation as a function of system demand. Through this latter assumption practically any complex generation function, to represent the real injection of a cogeneration unit may be constructed, in which case the general theory developed remains valid.

The net savings to the utility, due to a cogeneration unit, capable of generating 1.5 % of system demand at all times is investigated, by applying the unit to all 17 load buses. The savings computed on the SIC and the corresponding BIC is given in Table 4.6 . For purposes of studying the effect of such cogeneration units on the constrained line 15 - 21 , the maximum value of the LM of this line (which corresponds to a system demand of 22.185 pu MW) is also given in this table. It is found that in the absence of the cogeneration unit, the maximum value of the LM of line 15 - 21 is given by 8.689 $/MWH . On comparing this value with those reported in Table 4.6 , one sees that the constraint on this line is relieved partially the greatest amount being when the cogeneration customer is situated at bus # 15 , and is constrained to the greatest level when the cogeneration unit is at bus # 18 . The data of Table 4.6 can be used in expansion studies to identify the best locations for siting cogeneration units, and to identifying those cogeneration units directly responsible for relieving constrained lines. Studies of the latter form can be used for working out suitable incentive schemes to encourage cogeneration units in strategic locations to generate more at times of contingencies, to release line constraints.

Next the study will focus attention on the effect of a second line getting constrained on the BIC's . The results of simulation # 2 demonstrates these effects, and are given in Table 4.7 . A graphical plot of these results are given in Figures 4.2.1 to 4.2.24 . The pronounced variations between the BIC's and the SIC can be seen in buses 7, 8, 17, 18, 21 and 22 , the greatest difference occurring at the maximum system demand of 22.185 PU MW . On comparing Tables 4.4 and 4.7 one observes that the results are identical up to a value of system demand equal to 15.336 PU MW and start differing beyond this value of system demand. This is due to line 8 - 9 reaching an operating limit at this value of system demand. More pronounced variations in the BIC's and the SIC beyond this point can be observed in the results corresponding to buses 17, 18, 21 and 22 . It is demonstrated through simulations carried out on a 10-bus system that these variations could be spread throughout all system buses if many lines get constrained, a common feature close to the loadability limit. Results of these simulations carried out to the system loadability limit is shown graphically in Figures 4.4.1 to 4.4.10 . Some features not found in previous simulations are highlighted here, namely the case where BIC's could be significantly higher than the SIC are observed in figures corresponding to buses 3, 4 and 5 . Furthermore lines reaching operating limits are clearly demarcated in these latter figures which will now be discussed briefly.

Transmission line 1 - 2 reaches an operating limit at a system demand of 4.091 PU MW . This is clearly demarcated in all Figures 4.4.1 to 4.4.10 . At 6.192 PU MW a second line 5 - 6 reaches an operating limit. This change is well pronounced in Figure 4.4.5 where the gradient of the BIC segment is seen to increase and in Figure 4.4.6 where the gradient is seen to decrease. The reduction is due to a free generator at this bus, capable of supplying the bus load at its IC (Lemma Chapter II) . At 6.466 PU MW a third line 2 - 3 reaches an operating limit. This causes the generation of unit # 3 to rise from 20 MW to 102.55 MW . As this generator now operates close to its upper limit its IC will be high. This is reflected as a sharp rise in the BIC of bus # 3 seen in Figure 4.4.3 . At a system demand of 9.567 PU MW a fourth line 9 - 10 reaches an operating limit. This causes the next segment of the BIC of bus # 9 to take a higher gradient and that at bus # 10 to take on a lower gradient. This is self-explanatory as bus # 10 carries a free generator (also through Lemma Chapter II) . At 10.290 PU MW line 1 - 2 gets unconstrained, and at 10.429 PU MW generator at bus # 10 reaches an operating limit. This is shown as a near step rise in the SIC curve and some of the BIC curves. The limit of system loadability is reached at 10.646 PU MW . One notes that approaching loadability limit is not reflected in the cost. Hence artificial means of keeping customers away from the loadability limit must be sought possibly through the incorporation of suitable penalty functions into the BIC's, and charging the customers based on these penalised BIC's .

Although none of the three simulations investigated show discontinuities in the BIC's, the next section discusses situations where such discontinuities may occur.

4.4 Discontinuities in the Bus Incremental Costs

Discontinuities in BIC's are due to multiple constraints becoming active or inactive (in addition to those of the previous load interval) at a change point. However numerical demonstrations of this feature were not possible due to the main simulation package limiting such changes in the active set to one. However, a qualitative explanation of this phenomenon will be discussed.

If the generator of bus # 7 in Figure 4.1 has a very high IC (higher than that of any other system unit), it will continue to operate at a lower limit until line 7 - 8 reaches an operating limit. At this point this generator must be released from its lower bound to meet at least the bus demand. This means that the LM corresponding to this unit must instantaneously become zero. By Lemma of Chapter II, this causes the BIC of bus # 7 to increase instantaneously to the corresponding IC value of this generator. A phenomenon to the contrary occurring if the line 7 - 8 is released from the active set in subsequent loading of the system.

The discussion to follow subsequently is to outline other areas in power system planning where information provided by BIC's can be successfully exploited.

4.5 Application of BIC's in the Expansion Planning of Power Systems

Very often the power system planner is faced with the problem of choosing the best alternative among many expansion alternatives, in order to determine the best strategy which would minimize daily operating cost of the system. The application of BIC's in this field of study will be explained through the following example. The data corresponding to simulation # 1 being used in this study.

The following expansion alternatives are assumed to be available, i.e.,

(i) Commissioning a cogeneration unit capable of generating 1.5 % P_D MW at all times at bus # 15 .

(ii) Building a new transmission line either between buses 18 and 21 or between 15 and 21 , assuming that these lines are not available at present.

(iii) Expanding the generation facilities at bus # 15 through the addition of three new 12 MW units of the type already in existence at this bus.

The daily operating cost of the system were found to change from $ 362.0 x 10^3 to the following values in each of the above alternatives. In the first alternative the daily operating cost becomes $ 354.0 x 10^3 . This however excludes the amount paid by the utility to the cogenerator concerned, which could be computed either based on the SIC or BIC . However, it is more appropriate to assume that the utility pays this cogeneration customer at least the expense the utility would have incurred had it supplied this energy, which amounts to $ 5.36 x 10^3 (computed based on the BIC of bus # 15) . Hence the total operating cost for alternative # 1 becomes $ 359.36 . It is found that the addition of a new transmission line either between buses 18 and 21 , or between buses 15 and 21 is capable of removing the constraint on line 15 - 21 and result in a daily operating cost of $ 359.7 x 10^3 . The third alternative is found to be by far the most expensive operationally, as the operating cost now becomes $ 373.6 x 10^3 . The high IC at the minimum generation level and the steep slope of the IC curve, precludes this unit from picking up the system demand, and hence continues to operate at the lower generation limit which is operationally more expensive. Hence operationally the first alternative will prove to be the best.

Furthermore if electricity sales to neighbouring utilities are being contemplated, information provided by the BIC's can be used successfully in theoretical studies for estimating changes in revenue brought about due to network uncertainties, and break down of tielines.

4.6 Conclusions

Discussions and numerical examples in this chapter have shown that the BIC's of a system give better information relating to electricity prices than those provided by the SIC . Furthermore the reasons for the variations in BIC's from the SIC are discussed so that if one were to base cost computations on the SIC one would know where precautionary measures must be taken (like having additional transmission facilities in system locations where lines are likely to reach operating limits) in the power system to alleviate unwarranted subsidy and financial losses (or gains) to both a utility and its customers.

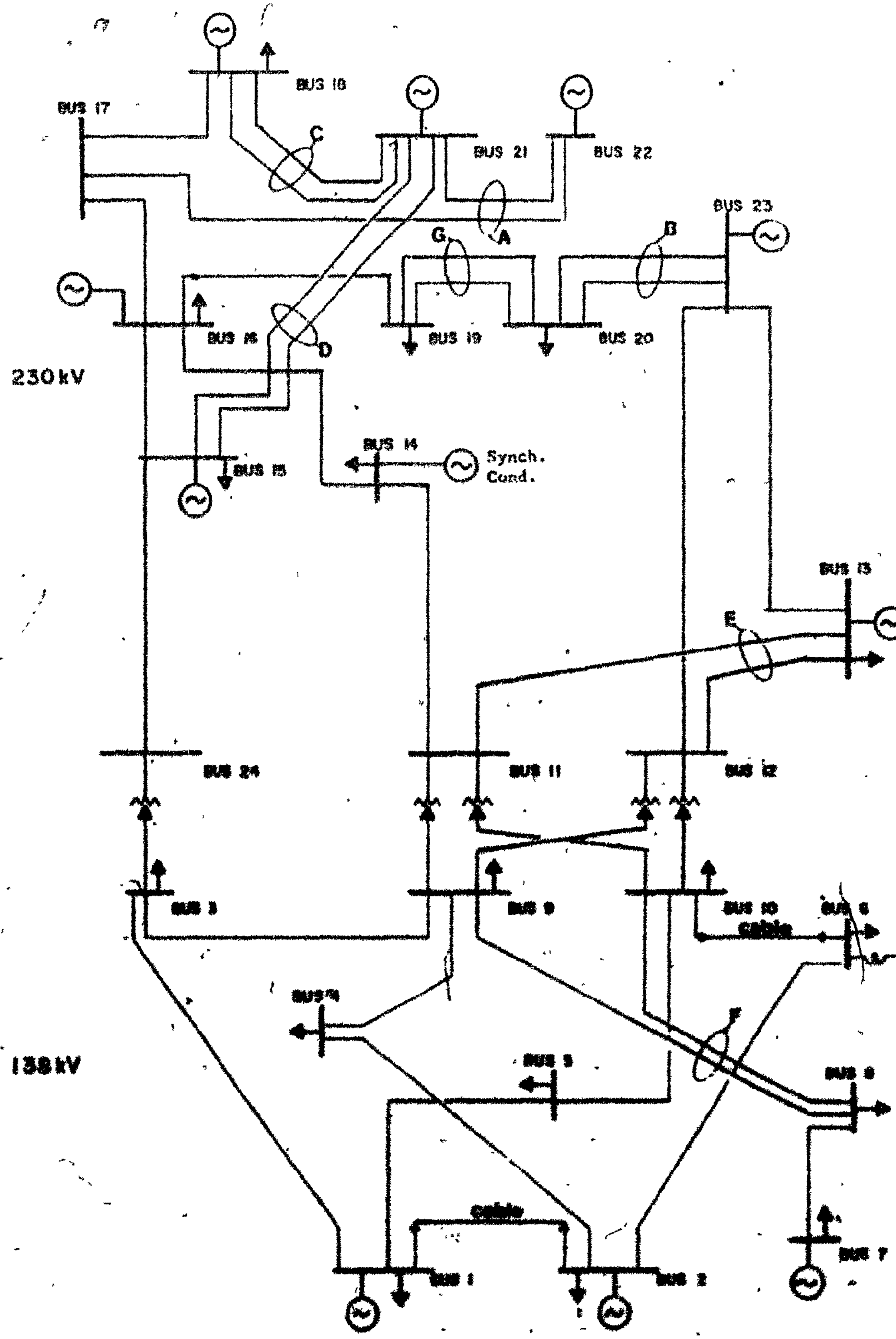

Figure 4.1 IEEE 24-bus test system.

TABLE 4.1

OPTIMUM OPERATING COST FOR VARYING HYDRO DISPATCH

T (H)	COST ($/DAY)
0:00	368.13×10^3
3:00	368.34×10^3
7:00	362.99×10^3
11:00	362.03×10^3
15:00	362.00×10^3
19:00	362.86×10^3

TABLE 4.2

NORMALIZED LDC WITH HYDRO UNITS

DEMAND PU MW (100 MVA BASE)	NORMALIZED TIME (24H BASE)
0.000	1.000
12.423	1.000
12.867	0.917
13.311	0.854
14.196	0.792
15.973	0.729
16.860	0.688
18.635	0.646
19.301	0.542
19.522	0.438
20.410	0.375
20.632	0.313
21.075	0.271
21.963	0.208
22.185	0.000

TABLE 4.3

ENERGY PRODUCED AND THE MINIMUM OPERATING COST OF THERMAL UNITS PER DAY

GEN. BUS #	ENERGY (PU MWH/DAY)	OPERATING COST ($/DAY)
1	0.485	11.87×10^3
2	0.830	25.26×10^3
7	0.752	37.90×10^3
13	2.069	29.81×10^3
15	0.120	5.72×10^3
16	2.297	59.64×10^3
21	6.386	90.09×10^3
23	5.503	101.71×10^3
TOTAL	18.44	362.0×10^3

TABLE 4.4

VARIATION OF SYSTEM & BUS INCREMENTAL COSTS

LOAD(PU MW)	9.631	12.921	14.436	15.336	19.946	19.953	20.882	20.993	21.995	22.185	E(BIC & SIC)
SIC ($/MWH)	4.745	6.422	7.107	10.711	12.345	12.376	12.836	13.281	13.762	13.844	11.225
BIC (1)	4.745	6.422	7.106	10.776	12.444	12.471	12.939	13.391	13.882	13.965	11.302
BIC (2)	4.745	6.422	7.106	10.774	12.440	12.468	12.935	13.388	13.878	13.961	11.299
BIC (3)	4.745	6.422	7.106	10.844	12.543	12.579	13.047	13.508	14.007	14.092	11.382
BIC (4)	4.745	6.422	7.106	10.768	12.431	12.459	12.926	13.377	13.866	13.950	11.292
BIC (5)	4.745	6.422	7.106	10.762	12.423	12.451	12.916	13.367	13.856	13.939	11.285
BIC (6)	4.745	6.422	7.106	10.754	12.411	12.438	12.903	13.353	13.840	13.923	11.275
BIC (7)	4.745	6.422	7.106	10.755	12.413	12.440	12.905	13.355	13.843	13.926	11.277
BIC (8)	4.745	6.422	7.106	10.755	12.413	12.440	12.905	13.355	13.843	13.926	11.277
BIC (9)	4.745	6.422	7.106	10.763	12.424	12.451	12.918	13.369	13.857	13.940	11.286
BIC (10)	4.745	6.422	7.106	10.747	12.402	12.429	12.893	13.342	13.829	13.912	11.268
BIC (11)	4.745	6.422	7.106	10.731	12.376	12.404	12.867	13.314	13.798	13.881	11.248
BIC (12)	4.745	6.422	7.106	10.737	12.387	12.414	12.877	13.325	13.810	13.892	11.256
BIC (13)	4.745	6.422	7.106	10.729	12.375	12.402	12.864	13.311	13.795	13.878	11.247
BIC (14)	4.745	6.422	7.106	10.708	12.346	12.371	12.831	13.275	13.756	13.838	11.222
BIC (15)	4.745	6.422	7.106	10.981	12.741	12.770	13.264	13.742	14.259	14.347	11.543
BIC (16)	4.745	6.422	7.106	10.687	12.314	12.340	12.797	13.239	13.717	13.799	11.197
BIC (17)	4.745	6.422	7.106	10.221	11.638	11.659	12.057	12.441	12.858	12.929	10.648
BIC (18)	4.745	6.422	7.106	10.221	11.638	11.659	12.057	12.441	12.858	12.929	10.648
BIC (19)	4.745	6.422	7.106	10.698	12.329	12.356	12.814	13.257	13.737	13.819	11.209
BIC (20)	4.745	6.422	7.107	10.707	12.343	12.369	12.829	13.273	13.754	13.836	11.220
BIC (21)	4.745	6.422	7.106	7.103	7.102	7.102	7.109	7.109	7.116	7.117	6.981
BIC (22)	4.745	6.422	7.106	8.324	8.878	8.887	9.047	9.197	9.365	9.394	8.417
BIC (23)	4.745	6.422	7.107	10.712	12.350	12.377	12.837	13.281	13.763	13.845	11.226
BIC (24)	4.745	6.422	7.106	10.929	12.665	12.694	13.181	13.652	14.163	14.250	11.481

E(BIC & SIC): EXPECTED VALUE OF BUS AND SYSTEM IC.

TABLE 4.5

COST OF ENERGY REQUIREMENTS AT DIFFERENT BUSES PER DAY

BUS #	BUS LOAD - SYSTEM LOAD CORRELATION %	COST OF ELECTRICITY IN $/DAY BASED ON SIC	BIC	NET CUSTOMER SAVINGS IN $ / DAY
1	3.8	13.75×10^3	13.79×10^3	- 40.0
2	3.4	12.31×10^3	12.33×10^3	- 20.0
3	6.3	22.81×10^3	22.91×10^3	-100.0
4	2.6	9.41×10^3	9.43×10^3	- 20.0
5	2.5	9.05×10^3	9.07×10^3	- 20.0
6	4.8	17.38×10^3	17.40×10^3	- 20.0
7	4.4	15.93×10^3	15.95×10^3	- 20.0
8	6.0	21.72×10^3	21.75×10^3	- 30.0
9	6.1	22.08×10^3	22.12×10^3	- 40.0
10	6.8	24.62×10^3	24.64×10^3	- 20.0
13	9.3	33.66×10^3	33.68×10^3	- 20.0
14	6.8	24.62×10^3	24.61×10^3	10.0
15	11.1	40.18×10^3	40.54×10^3	-360.0
16	3.5	12.67×10^3	12.66×10^3	10.0
18	11.7	42.35×10^3	41.67×10^3	680.0
19	6.4	23.17×10^3	23.16×10^3	10.0
20	4.5	16.29×10^3	16.29×10^3	0.0
TOTAL COST ($/DAY)		362.0×10^3	362.0×10^3	0.0 (NET TOTAL SAVINGS)

TABLE 4.6

NET DAILY SAVING TO A UTILITY DUE TO COGENERATION

CO-GEN. BUS #	NET UTILITY SAVINGS IN $/DAY BASED ON SIC	NET UTILITY SAVINGS IN $/DAY BASED ON BIC	MAXIMUM VALUE OF THE LM OF LINE 15 - 21
1	5.35×10^3	5.36×10^3	8.672
2	5.35×10^3	5.36×10^3	8.672
3	5.35×10^3	5.36×10^3	8.663
4	5.35×10^3	5.36×10^3	8.672
5	5.35×10^3	5.36×10^3	8.673
6	5.35×10^3	5.36×10^3	8.674
7	5.35×10^3	5.36×10^3	8.669
8	5.35×10^3	5.36×10^3	8.674
9	5.35×10^3	5.36×10^3	8.673
10	5.35×10^3	5.36×10^3	8.674
13	5.35×10^3	5.36×10^3	8.677
14	5.35×10^3	5.36×10^3	8.680
15	5.35×10^3	5.36×10^3	8.647
16	5.35×10^3	5.36×10^3	8.682
18	5.36×10^3	5.24×10^3	8.738
19	5.35×10^3	5.36×10^3	8.680
20	5.35×10^3	5.36×10^3	8.680

TABLE 4.7

VARIATION OF SYSTEM & BUS INCREMENTAL COSTS

LOAD(PU MW)	9.631	12.921	14.436	15.336	19.588	19.528	19.864	20.073	20.992	21.124	22.166	E(BIC & SIC)
SIC ($/MWH)	4.745	6.422	7.107	10.711	12.211	12.367	12.490	12.536	12.546	13.367	13.887	11.274
BIC (1)	4.745	6.422	7.106	10.776	12.303	12.366	12.471	12.514	12.939	13.396	13.871	11.236
BIC (2)	4.745	6.422	7.106	10.774	12.300	12.388	12.462	12.504	12.930	13.386	13.863	11.268
BIC (3)	4.745	6.422	7.166	10.844	12.400	12.222	12.415	12.454	12.897	13.401	13.853	11.268
BIC (4)	4.745	6.422	7.106	10.766	12.291	12.202	12.292	12.330	12.766	13.262	13.745	11.203
BIC (5)	4.745	6.422	7.106	10.762	12.263	12.437	12.549	12.598	13.012	13.443	13.911	11.288
BIC (6)	4.745	6.422	7.106	10.754	12.271	12.474	12.591	12.638	13.051	13.467	13.921	11.297
BIC (7)	4.745	6.422	7.106	10.756	12.273	13.761	13.998	14.085	14.406	14.476	14.861	11.798
BIC (8)	4.745	6.422	7.106	10.755	12.273	13.761	13.995	14.086	14.407	14.476	14.861	11.798
BIC (9)	4.745	6.422	7.106	10.763	12.284	12.074	12.154	12.186	12.630	13.159	13.649	11.188
BIC (10)	4.745	6.422	7.106	10.747	12.262	12.511	12.631	12.686	13.085	13.492	13.982	11.307
BIC (11)	4.745	6.422	7.106	10.731	12.239	12.248	12.346	12.387	12.810	13.276	13.749	11.166
BIC (12)	4.745	6.422	7.106	10.737	12.246	12.259	12.358	12.399	12.823	13.285	13.762	11.208
BIC (13)	4.745	6.422	7.106	10.729	12.237	12.245	12.343	12.384	12.807	13.273	13.745	11.194
BIC (14)	4.745	6.422	7.106	10.708	12.207	12.206	12.303	12.343	12.764	13.230	13.700	11.166
BIC (15)	4.745	6.422	7.106	10.981	12.593	12.575	12.677	12.720	13.174	13.600	14.166	11.476
BIC (16)	4.745	6.422	7.106	10.687	12.177	12.167	12.262	12.302	12.721	13.187	13.666	11.138
BIC (17)	4.745	6.422	7.106	10.221	11.516	11.507	11.550	11.625	11.591	12.396	12.864	10.597
BIC (18)	4.745	6.422	7.106	10.221	11.516	11.507	11.550	11.625	11.591	12.396	12.864	10.597
BIC (19)	4.745	6.422	7.106	10.698	12.192	12.186	12.282	12.322	12.743	13.209	13.678	11.188
BIC (20)	4.745	6.422	7.107	10.707	12.205	12.203	12.300	12.340	12.762	13.227	13.657	11.166
BIC (21)	4.745	6.422	7.106	7.103	7.102	7.101	7.102	7.103	7.110	7.110	7.147	6.581
BIC (22)	4.745	6.422	7.106	8.324	8.831	8.827	8.860	8.874	9.022	9.180	9.348	8.397
BIC (23)	4.745	6.422	7.107	10.712	12.212	12.213	12.310	12.350	12.772	13.238	13.766	11.171
BIC (24)	4.745	6.422	7.106	10.929	12.516	12.478	12.577	12.618	13.068	13.673	14.074	11.401

E(BIC & SIC): EXPECTED VALUE OF BUS AND SYSTEM IC.

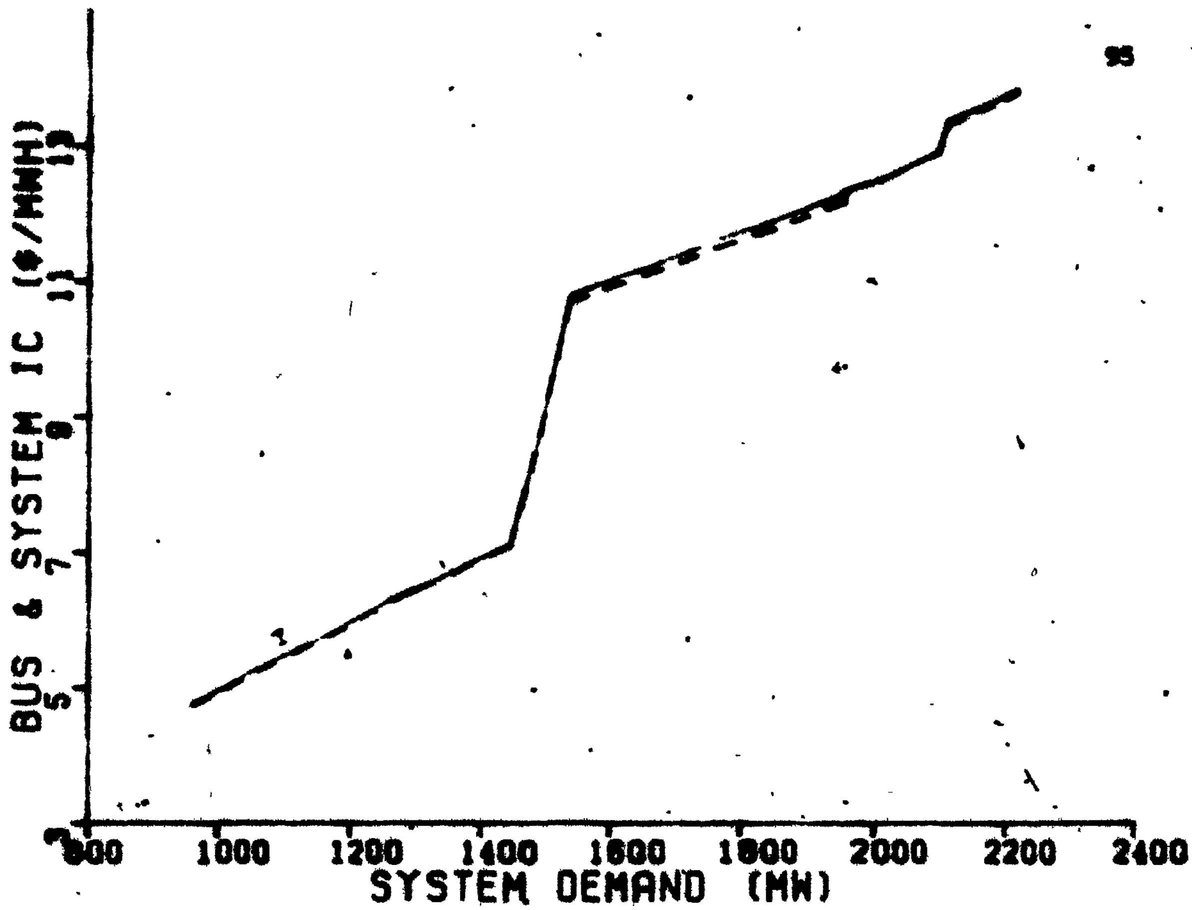

Figure 4.2.1 BIC of bus # 1 and SIC versus demand.

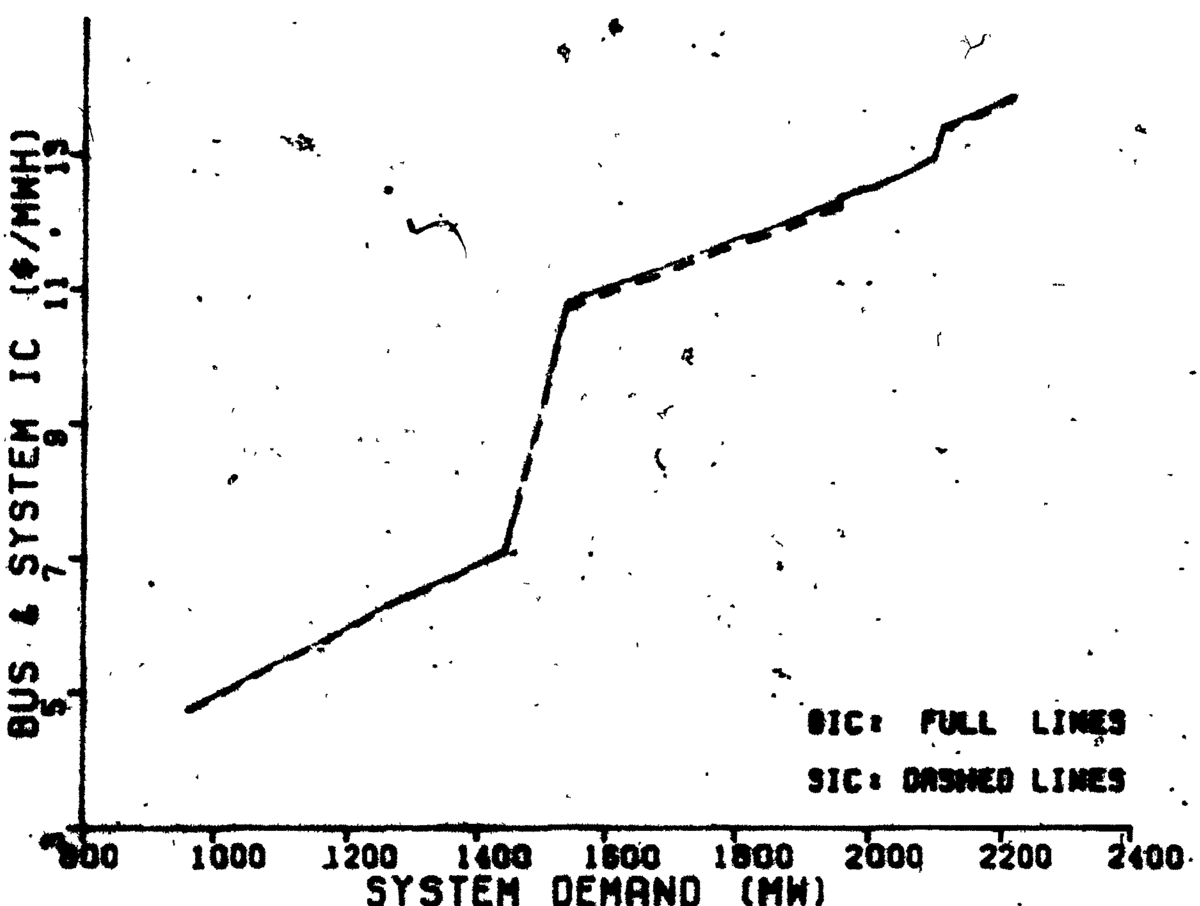

Figure 4.2.2 BIC of bus # 2 and SIC versus demand.

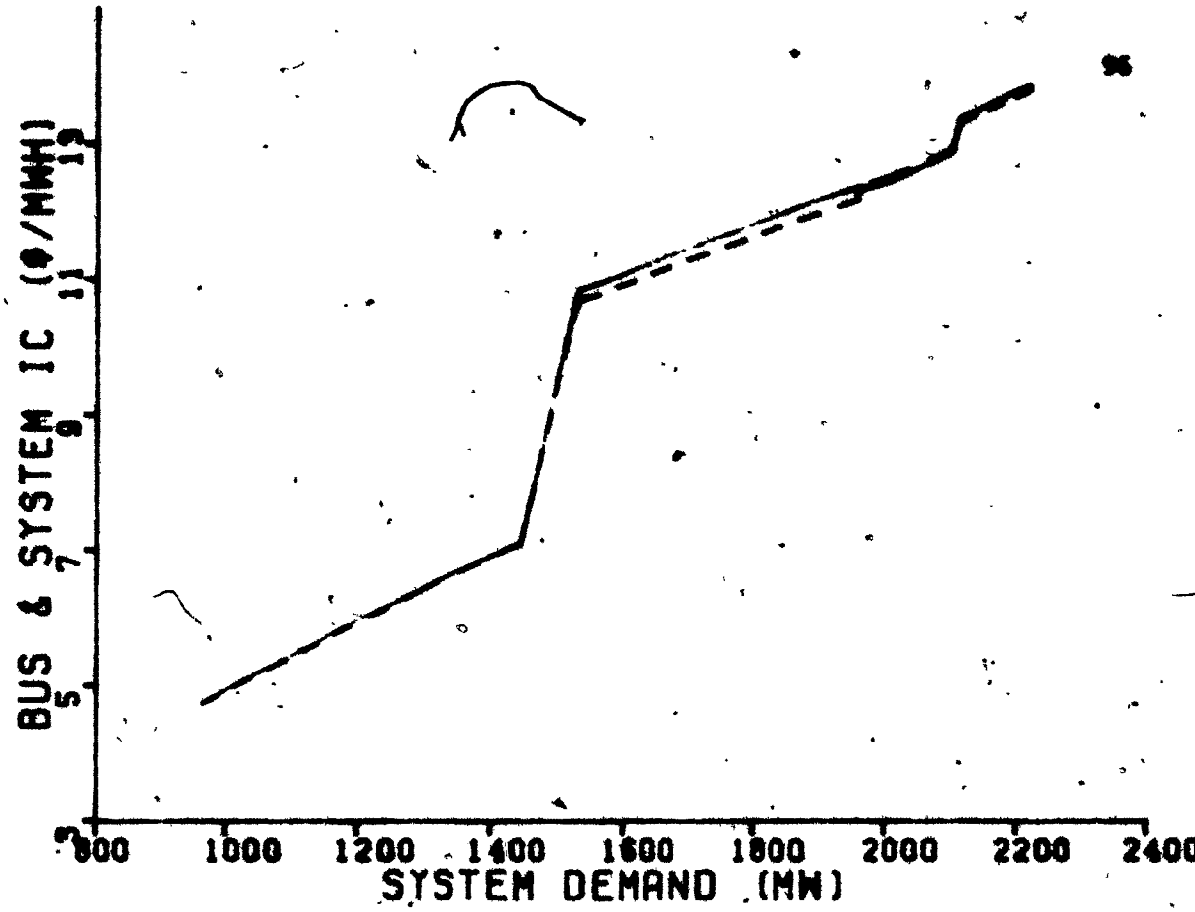

Figure 4.2.3 BIC at bus # 3 and SIC versus demand.

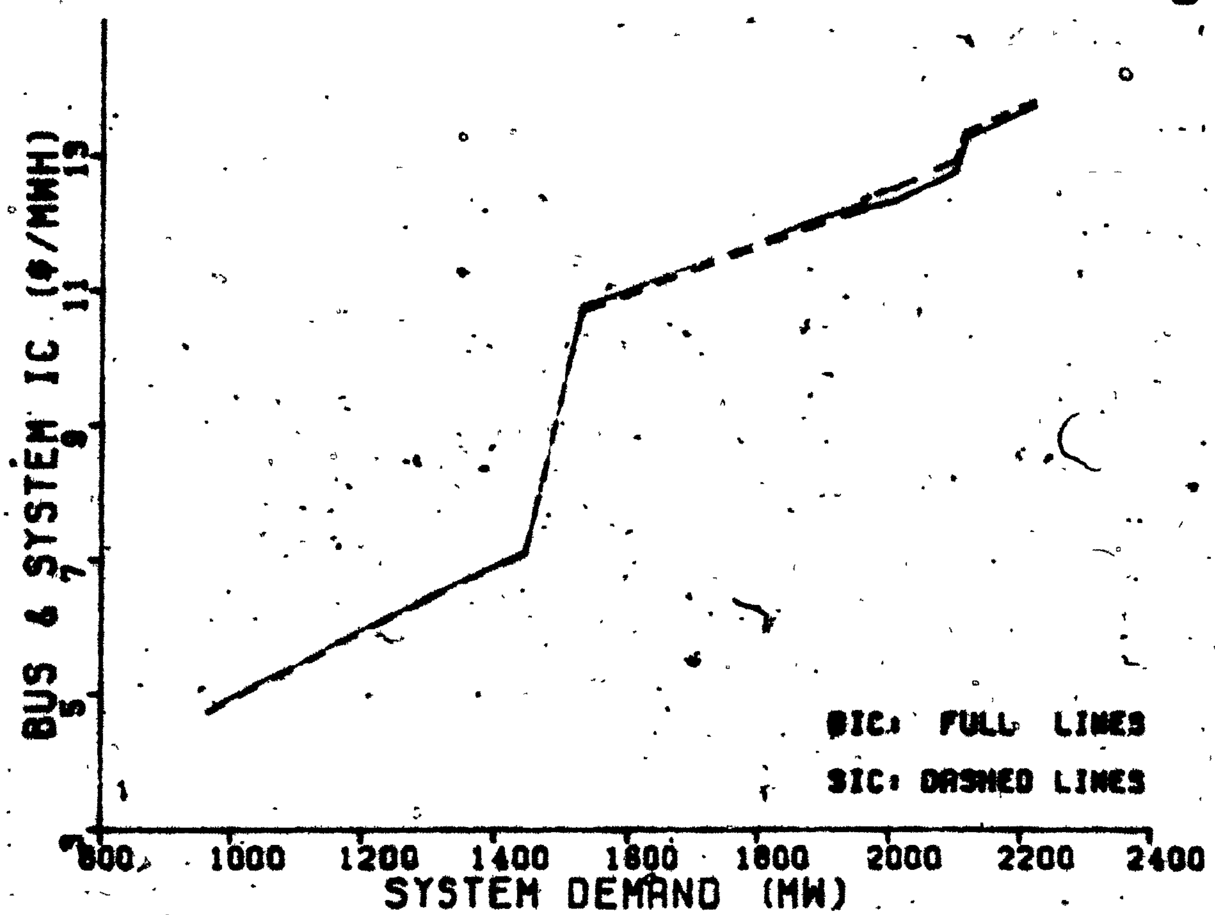

Figure 4.2.4 BIC at bus # 4 and SIC versus demand.

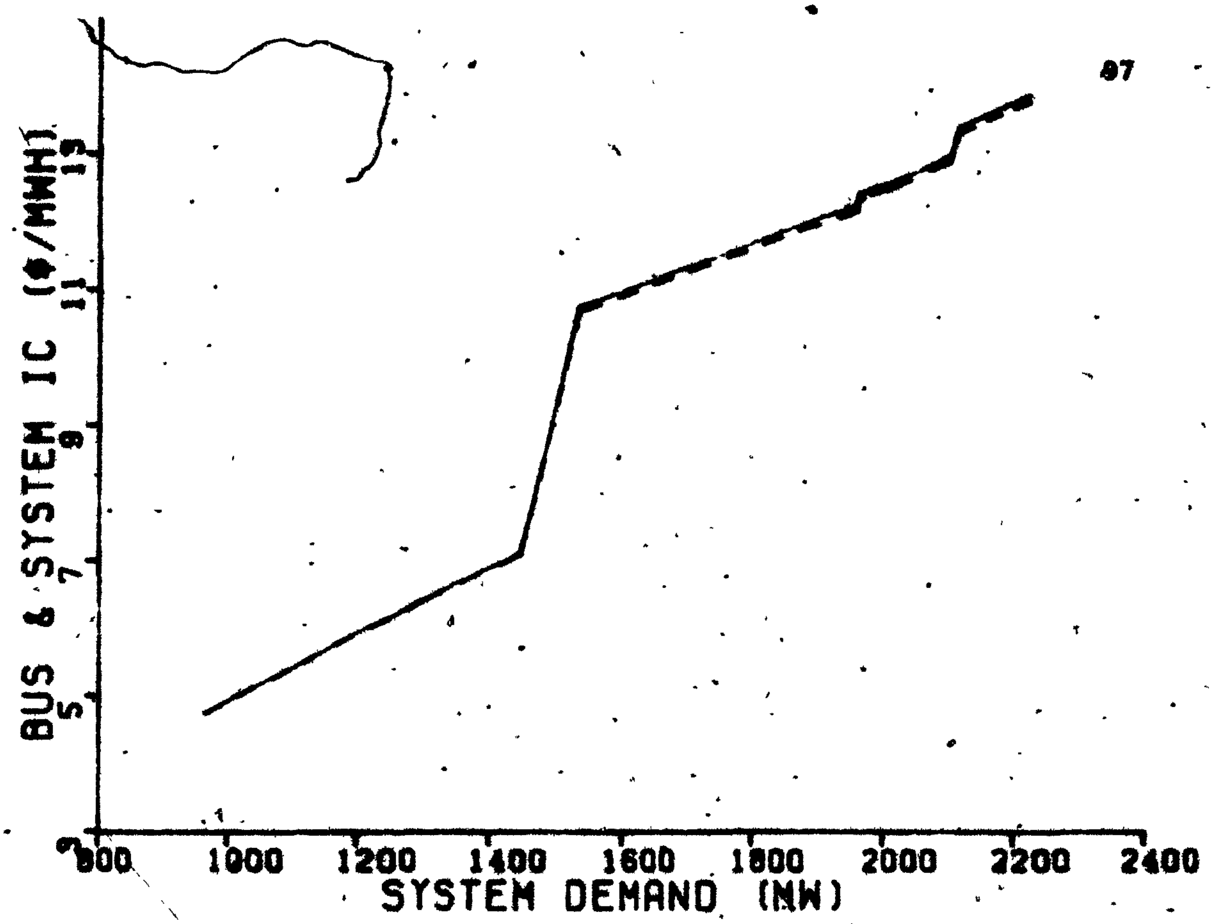

Figure 4.2.5 BIC at bus # 5 and SIC versus demand.

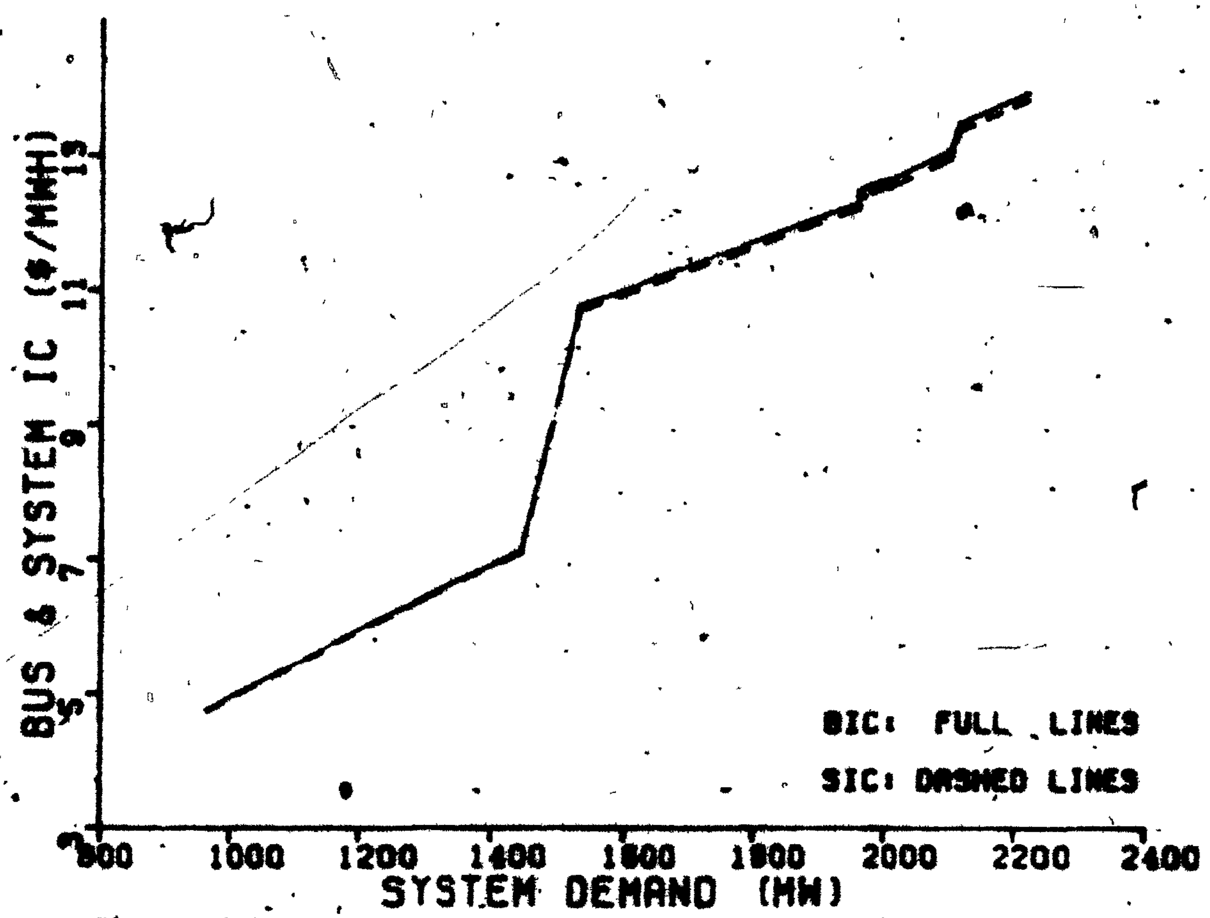

Figure 4.2.6 BIC at bus # 6 and SIC versus demand.

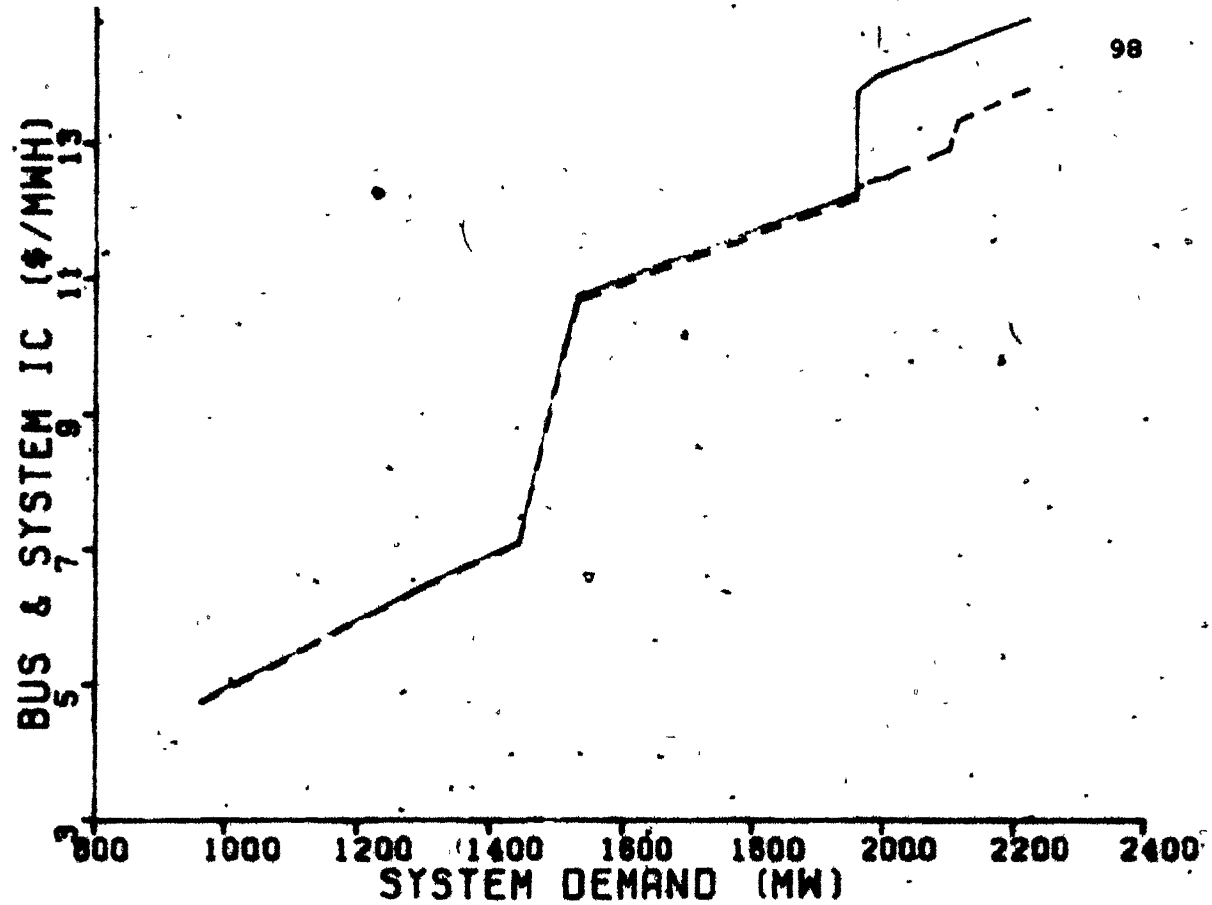

Figure 4.2.7 BIC at bus # 7 and SIC versus demand.

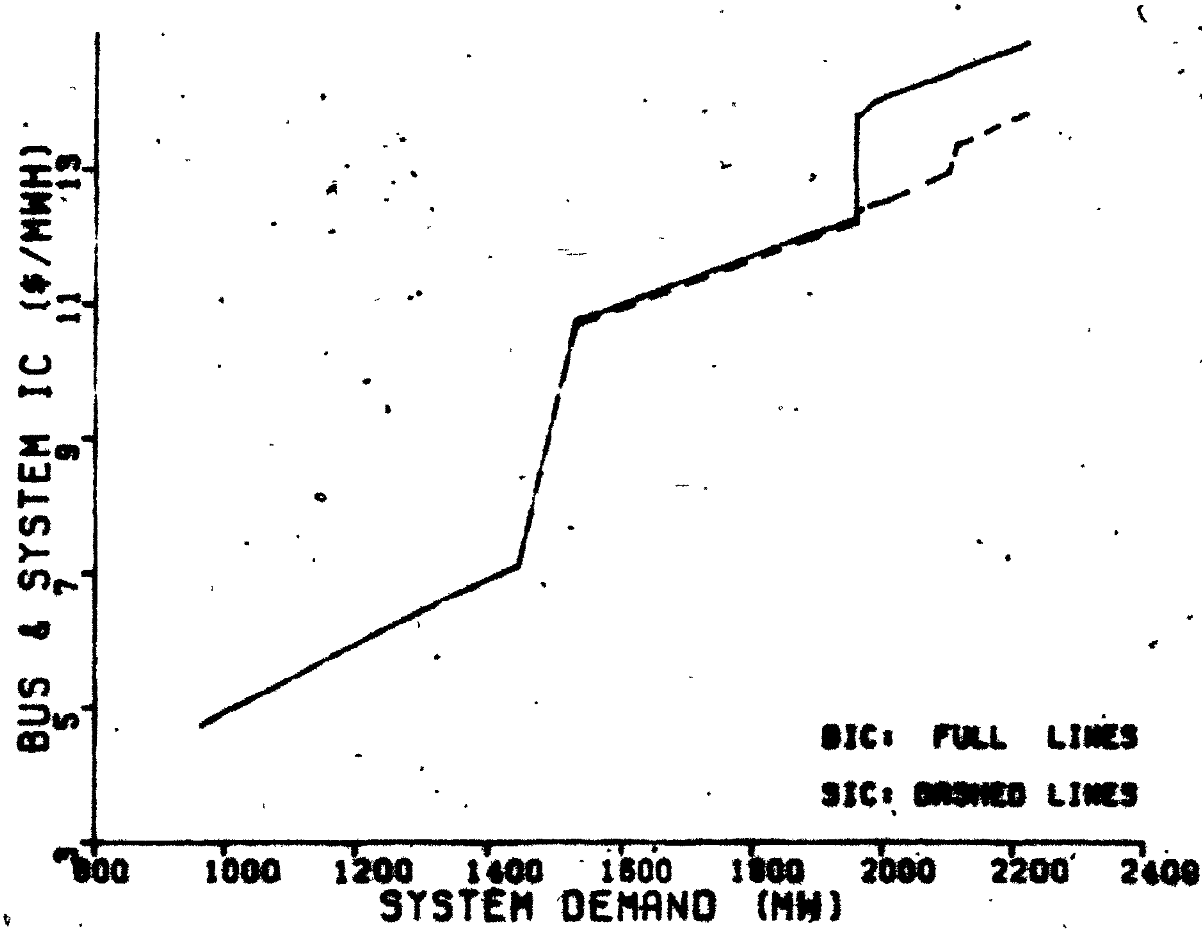

Figure 4.2.8 BIC at bus # 8 and SIC versus demand.

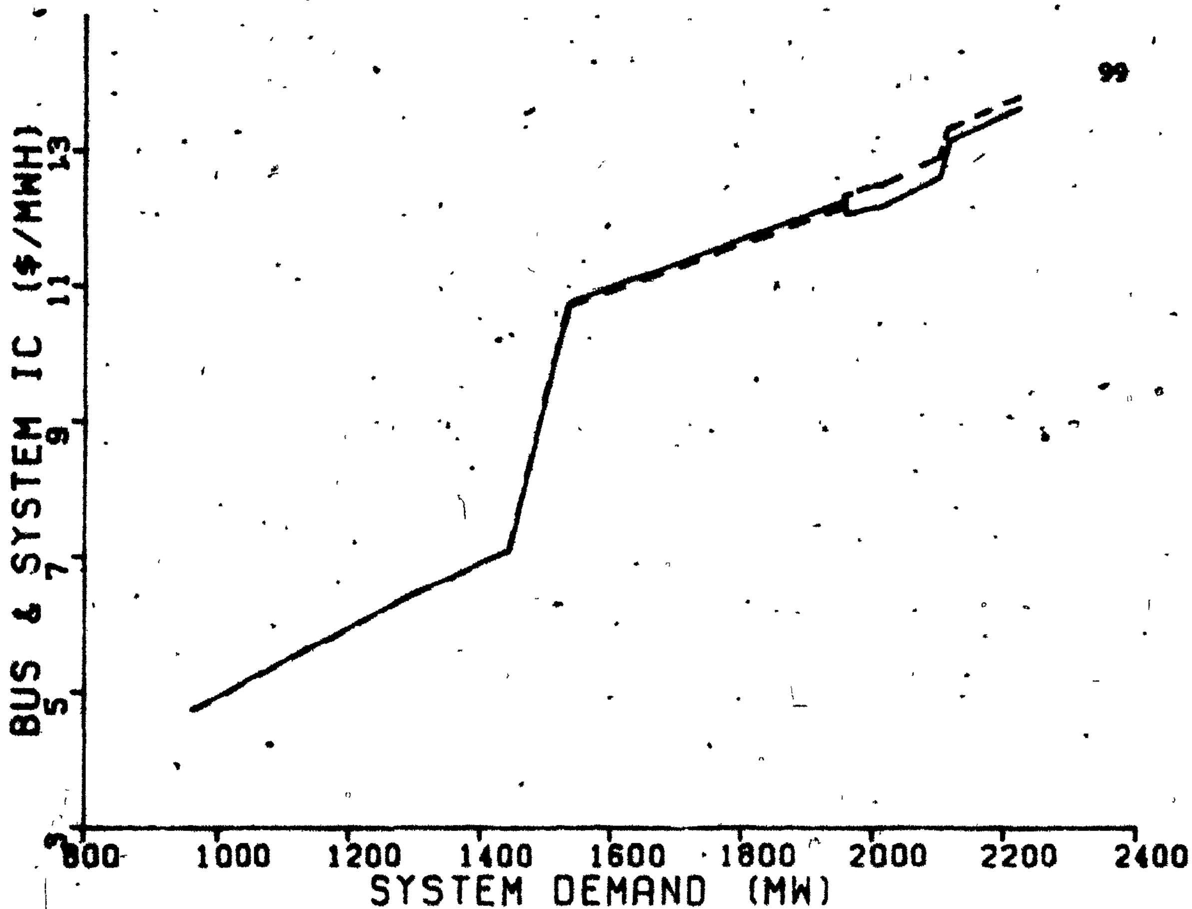

Figure 4.2.9 BIC at bus # 9 and SIC versus demand.

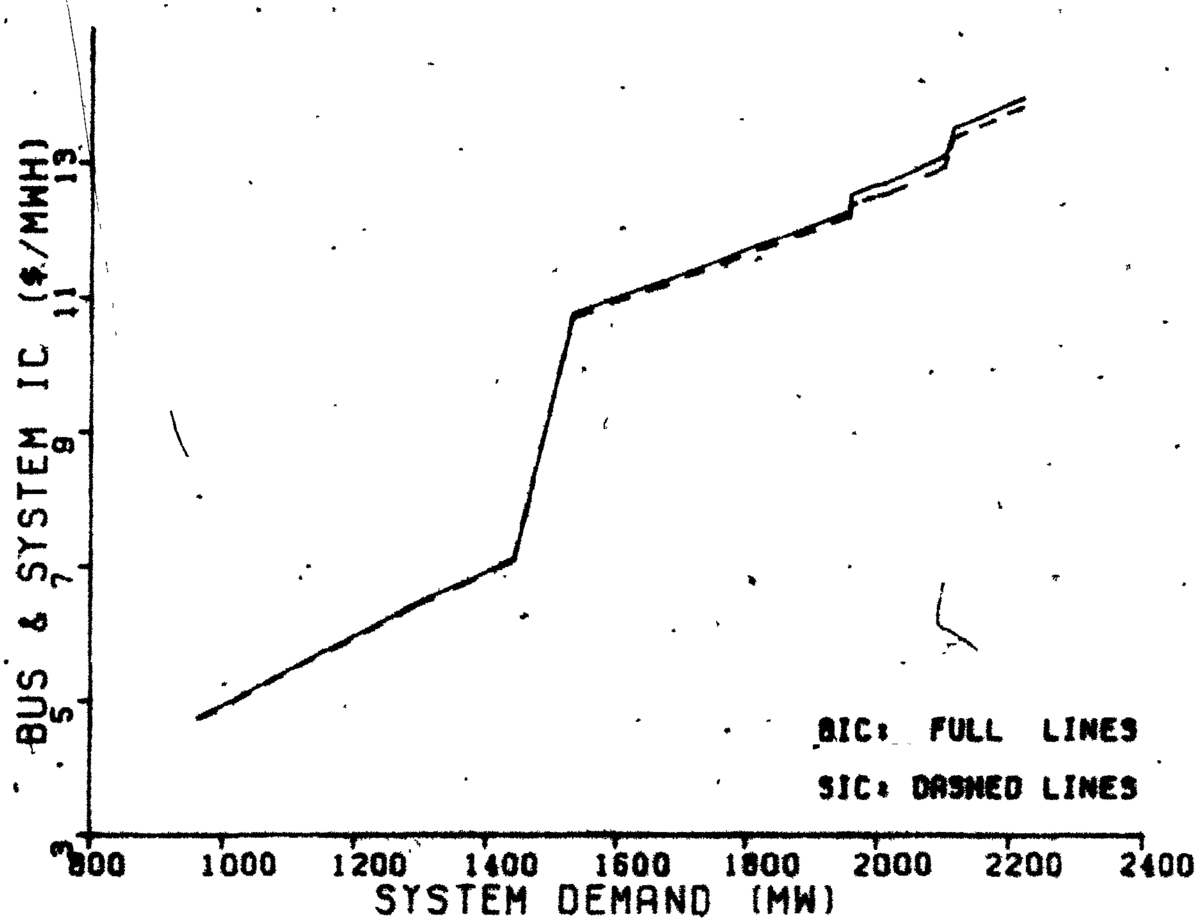

Figure 4.2.10 BIC at bus # 10 and SIC versus demand.

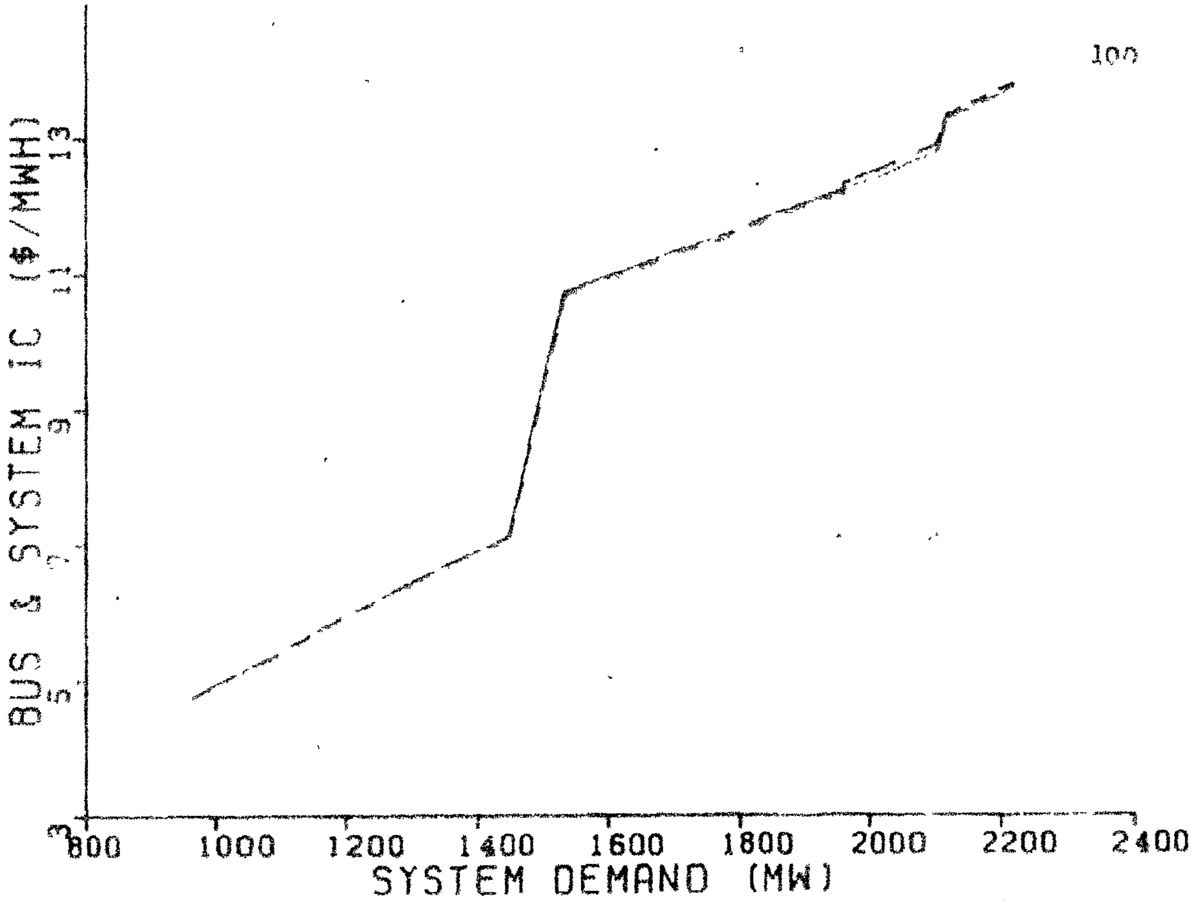

Figure 4.2.11 BIC at bus # 11 and SIC versus demand.

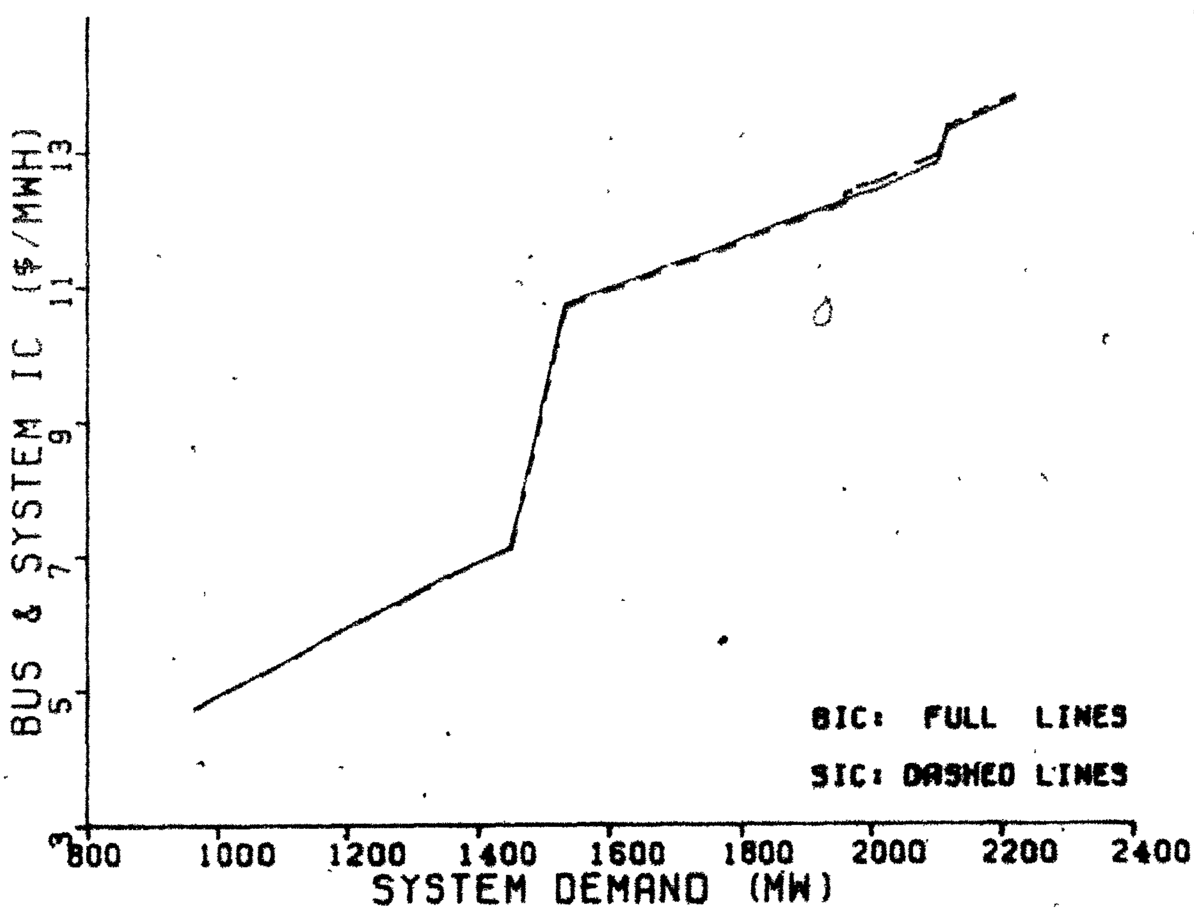

Figure 4.2.12 BIC at bus # 12 and SIC versus demand.

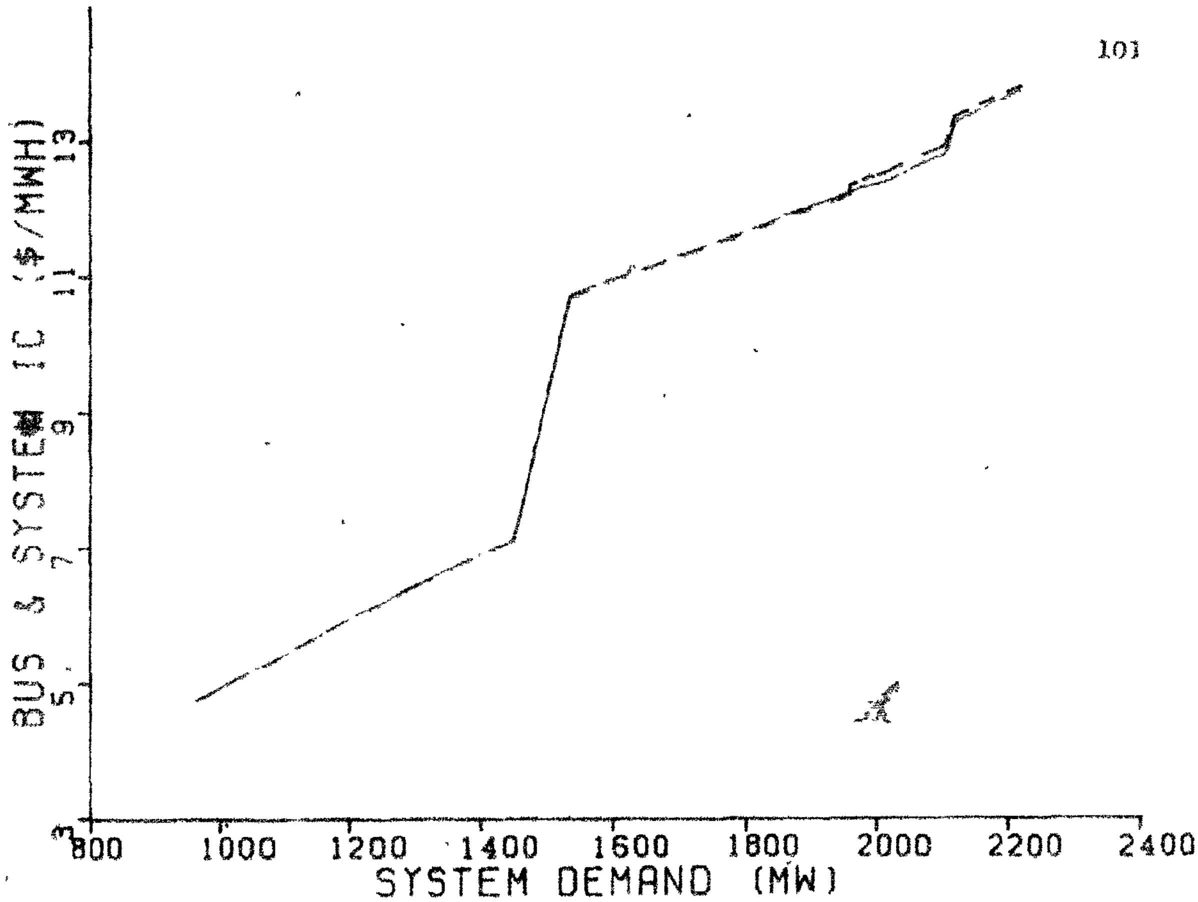

Figure 4.2.13 BIC of bus # 13 and SIC versus demand.

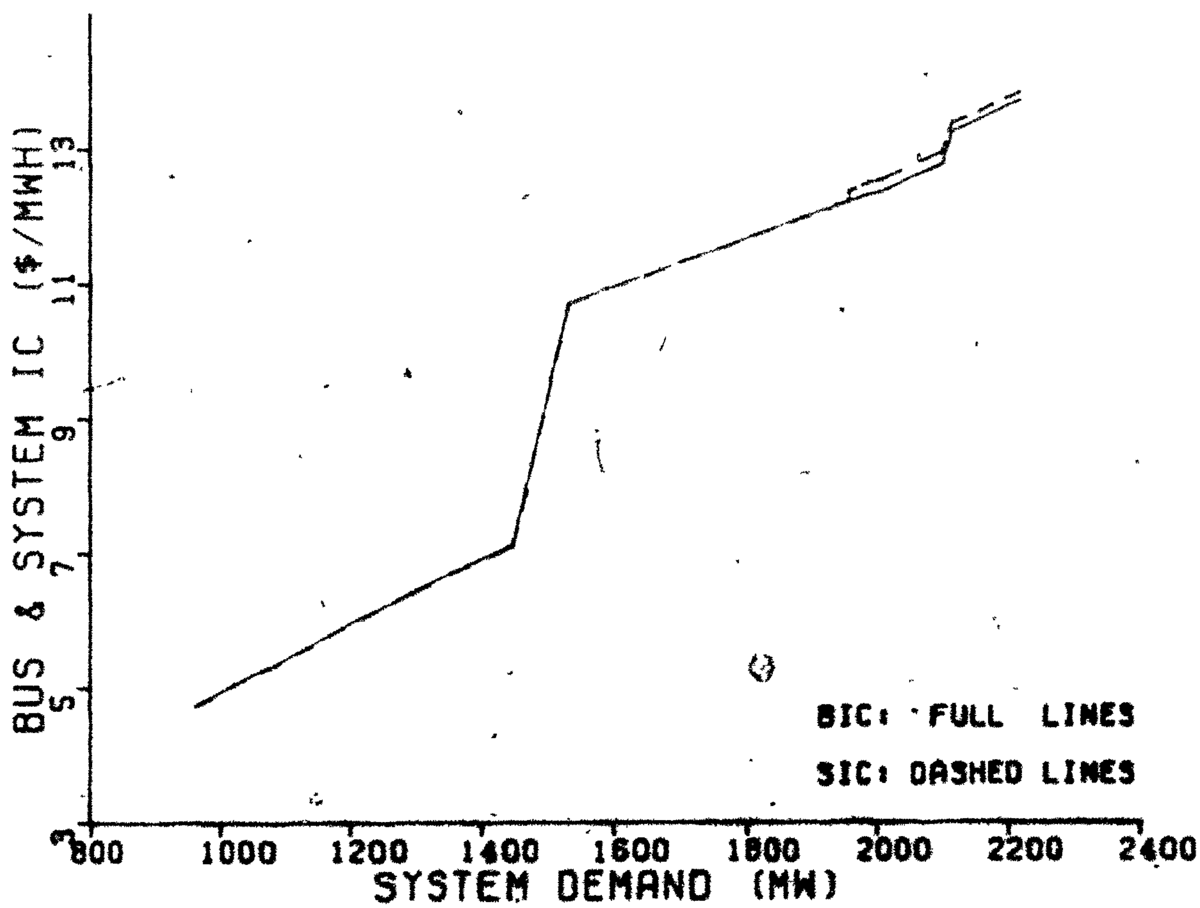

Figure 4.2.14 BIC of bus # 14 and SIC versus demand.

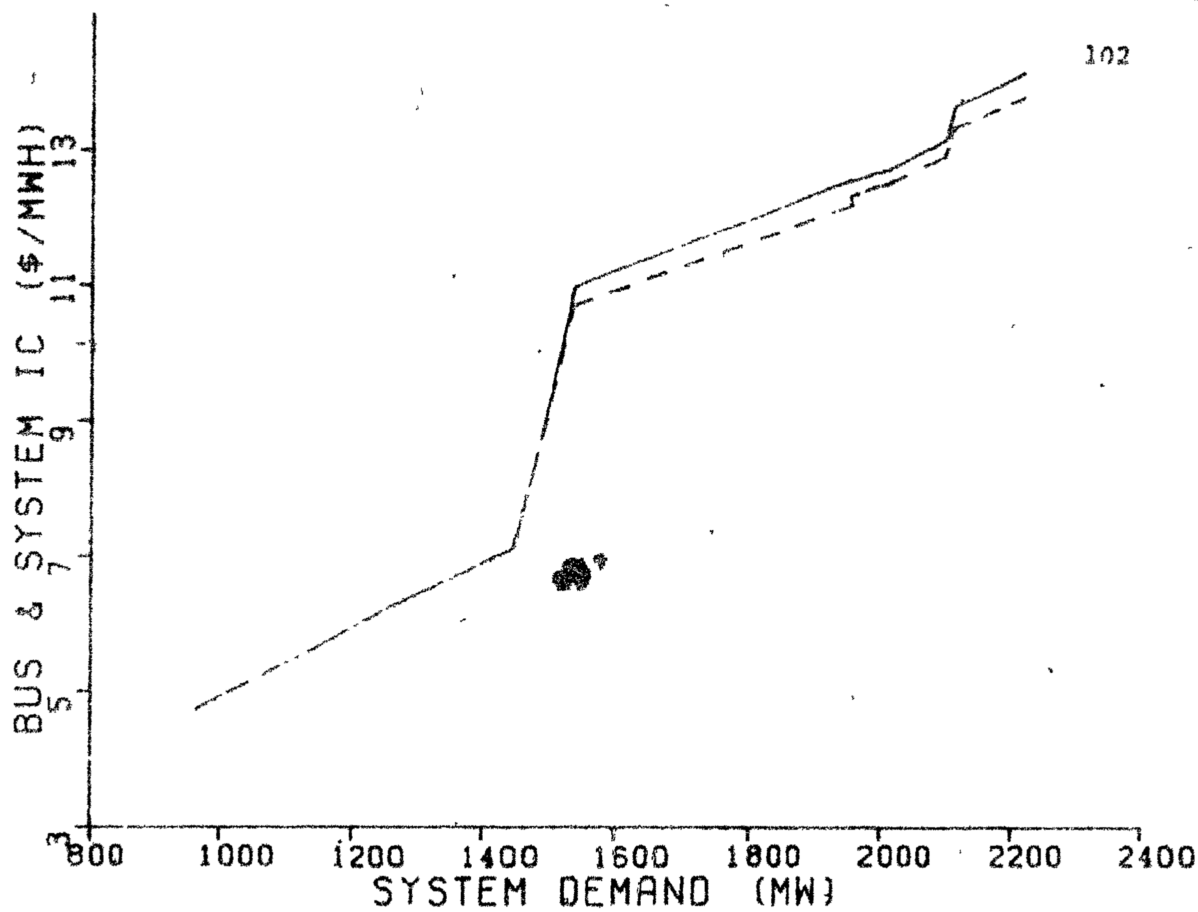

Figure 4.2.15 BIC at bus # 15 and SIC versus demand.

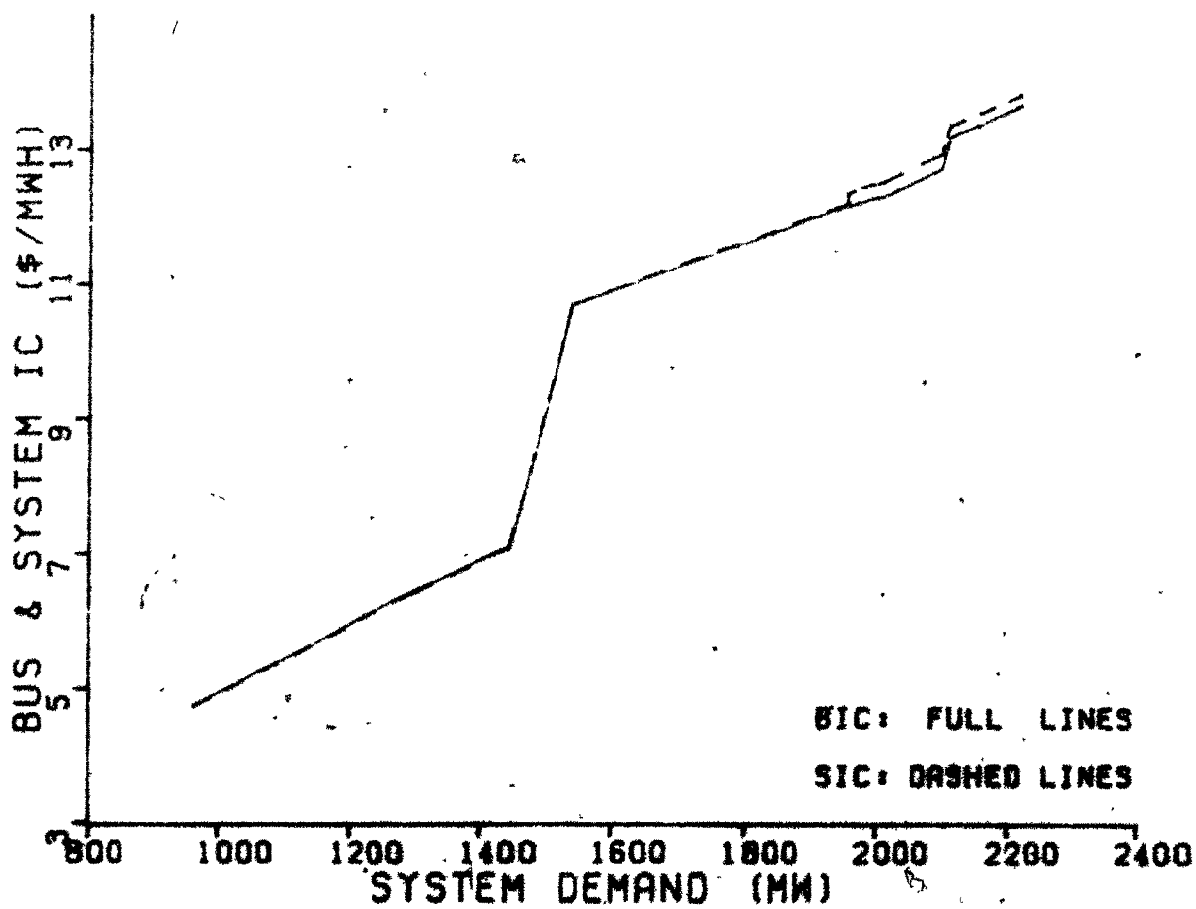

Figure 4.2.16 BIC of bus # 16 and, SIC versus demand.

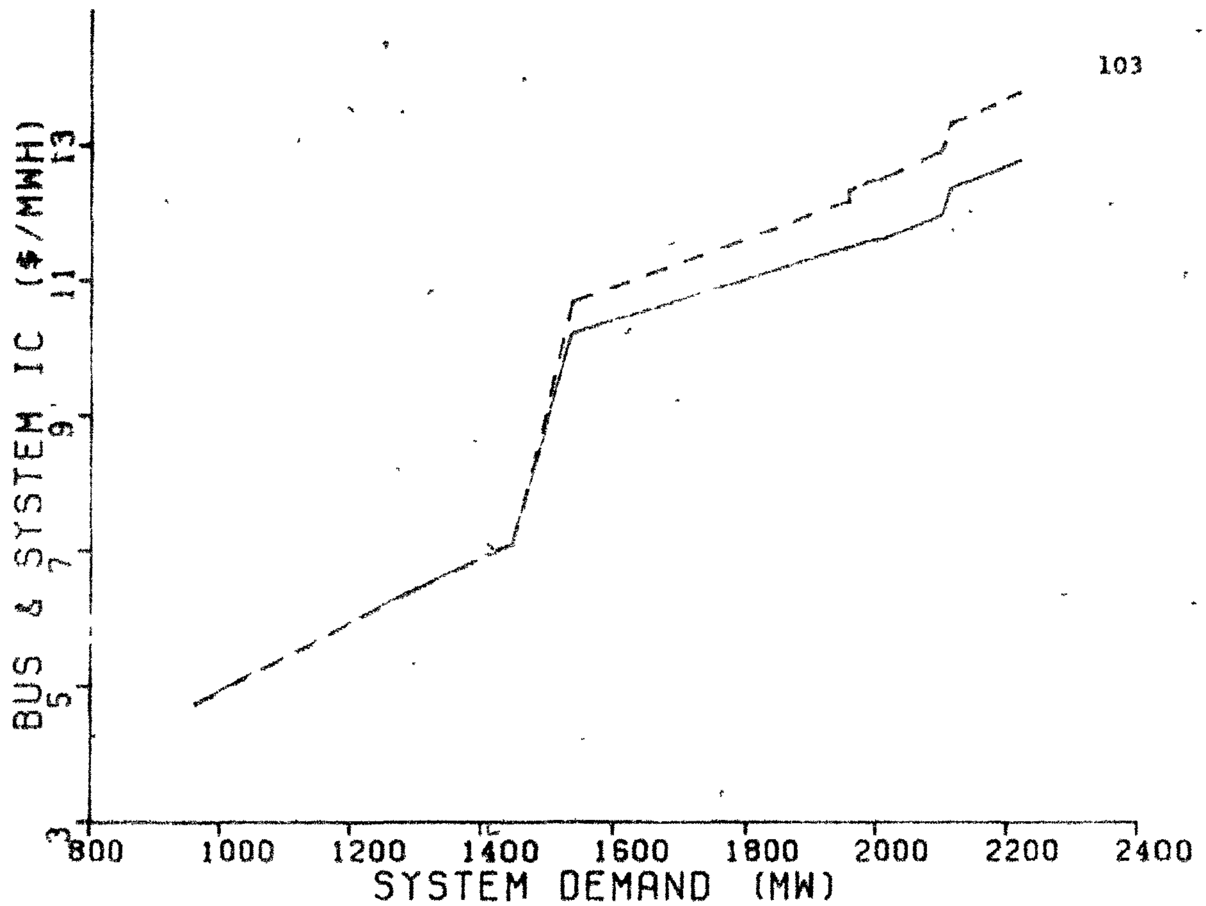

Figure 4.2.17 BIC at bus # 17 and SIC versus demand.

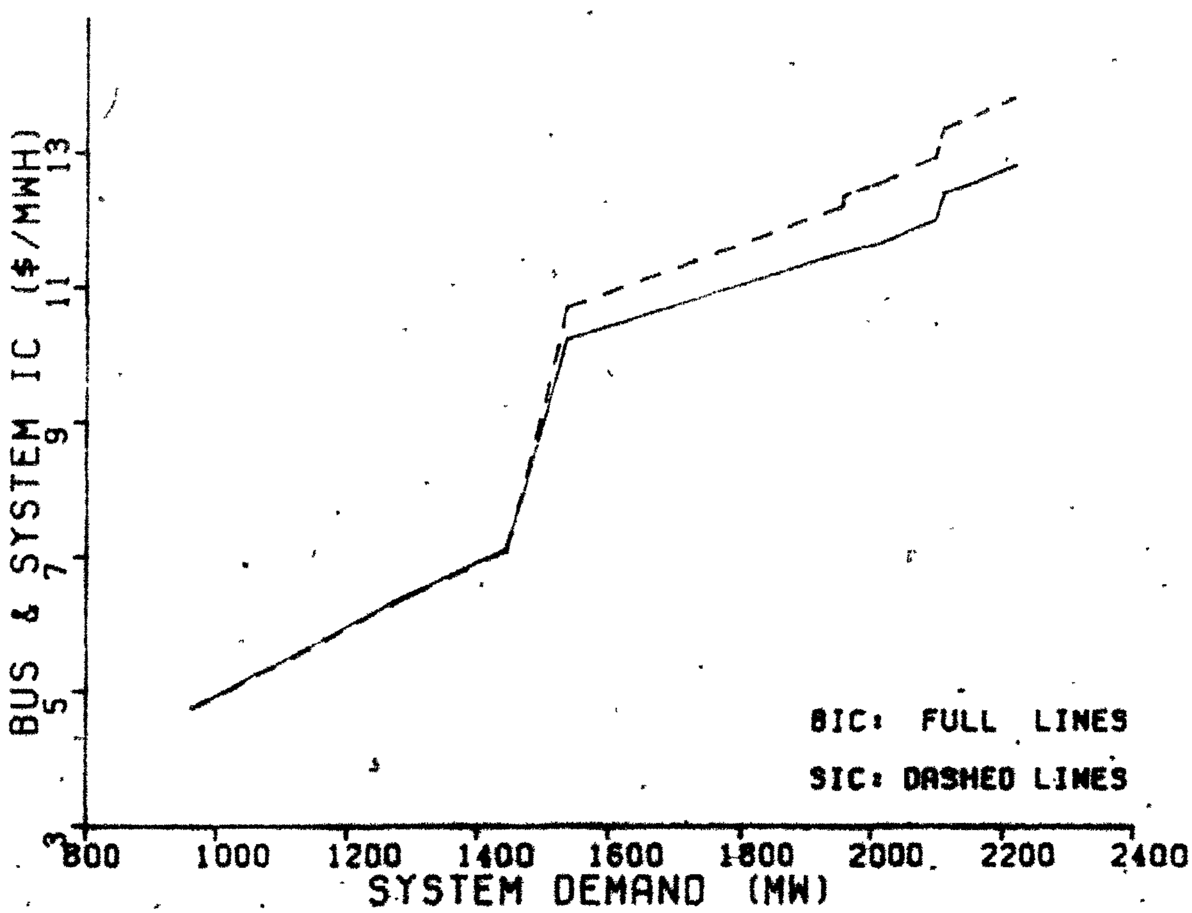

Figure 4.2.18 BIC at bus # 18 and SIC versus demand.

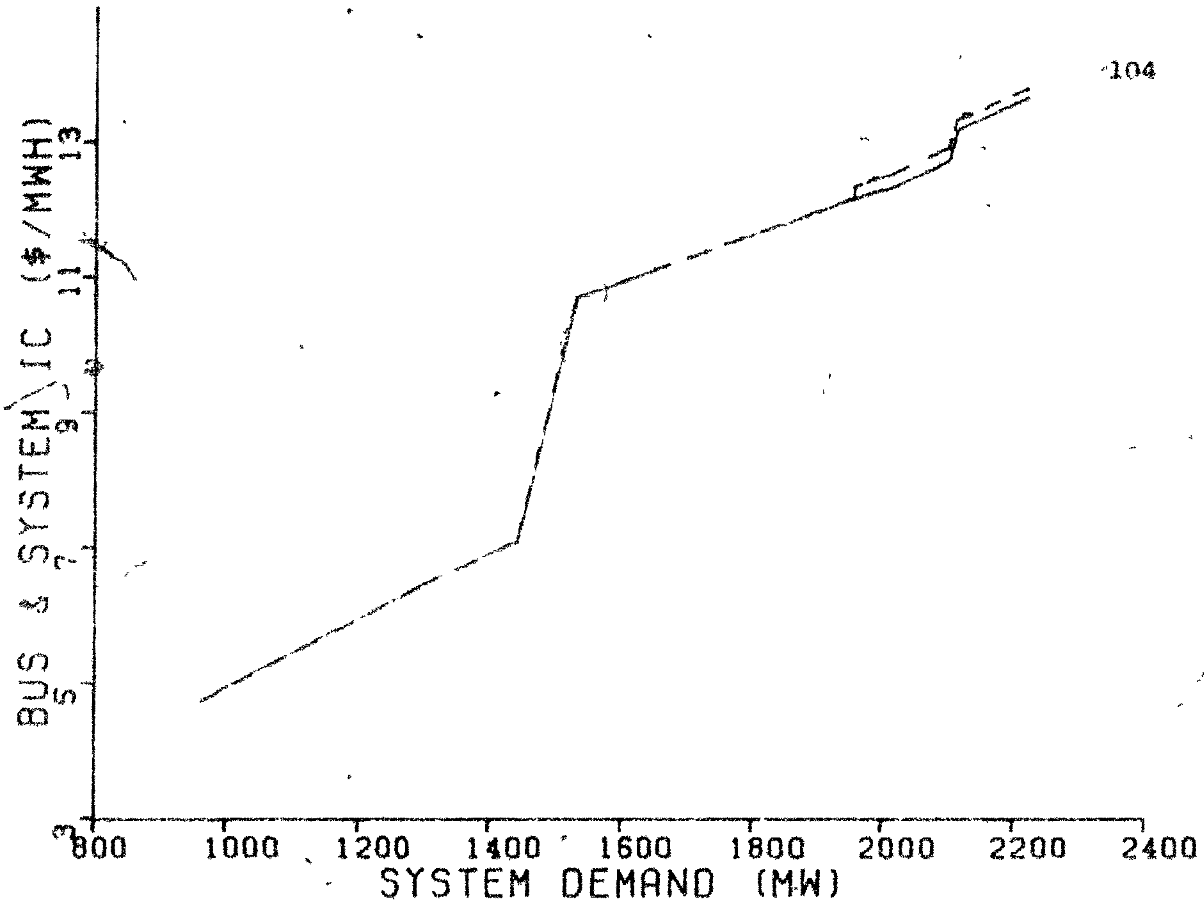

Figure 4.2.19 BIC of bus # 19 and SIC versus demand.

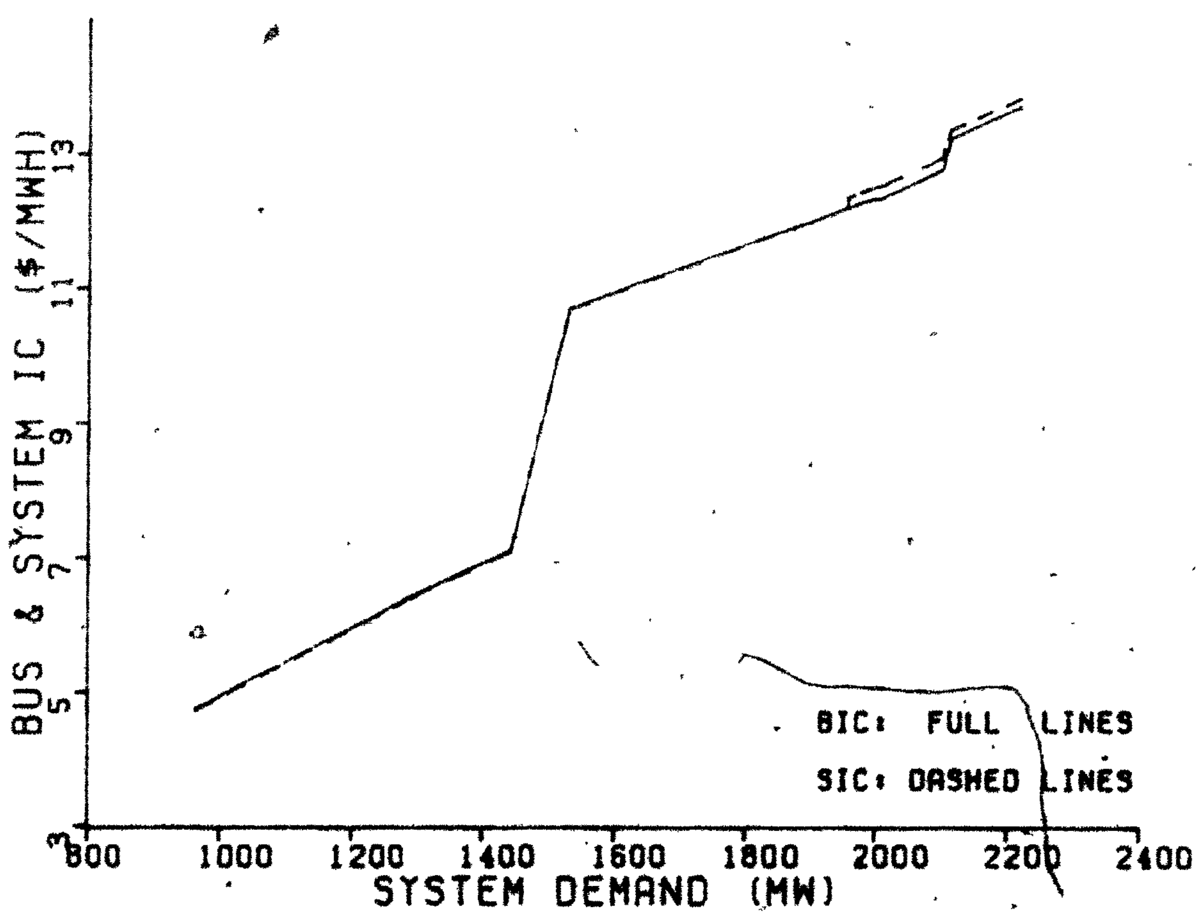

Figure 4.2.20 BIC of bus # 20 and SIC versus demand.

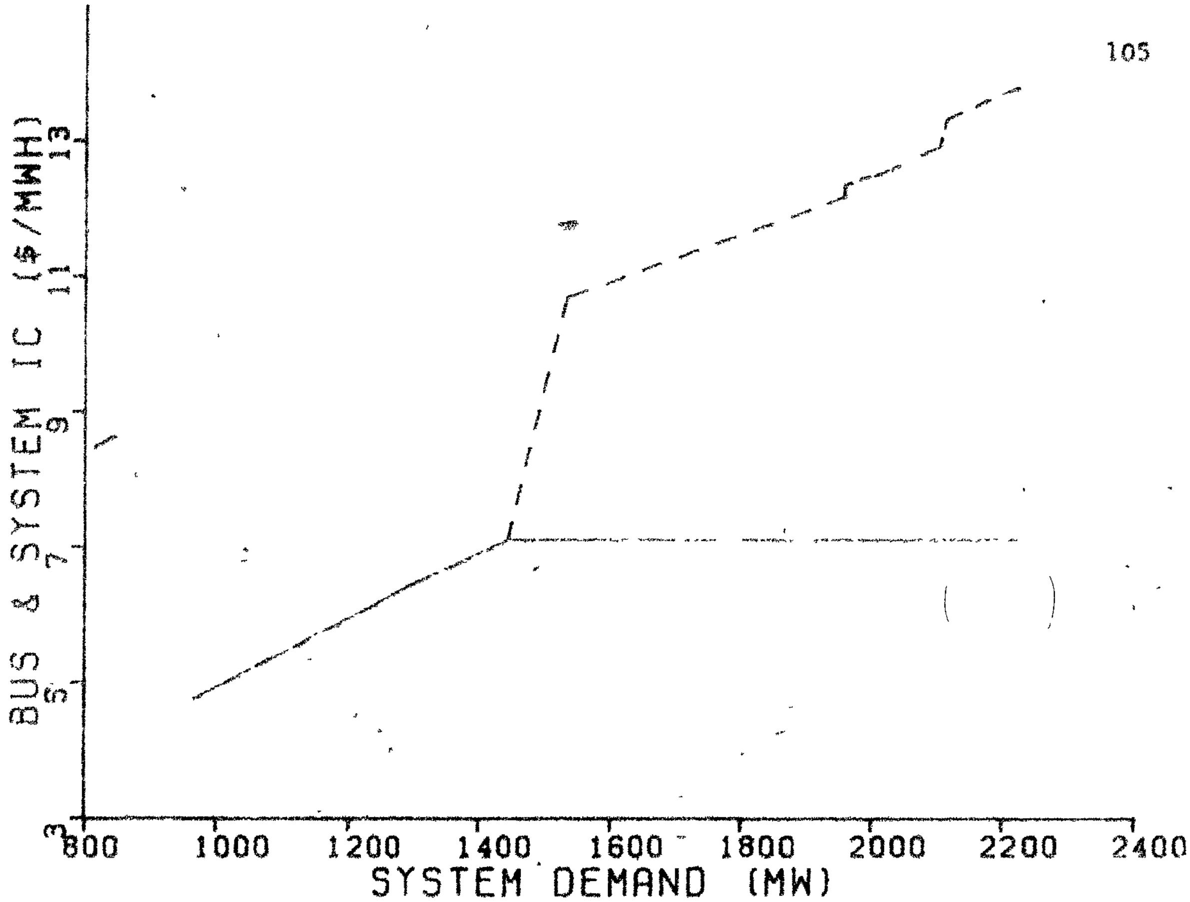

Figure 4.2.21 BIC of bus # 21 and SIC versus demand.

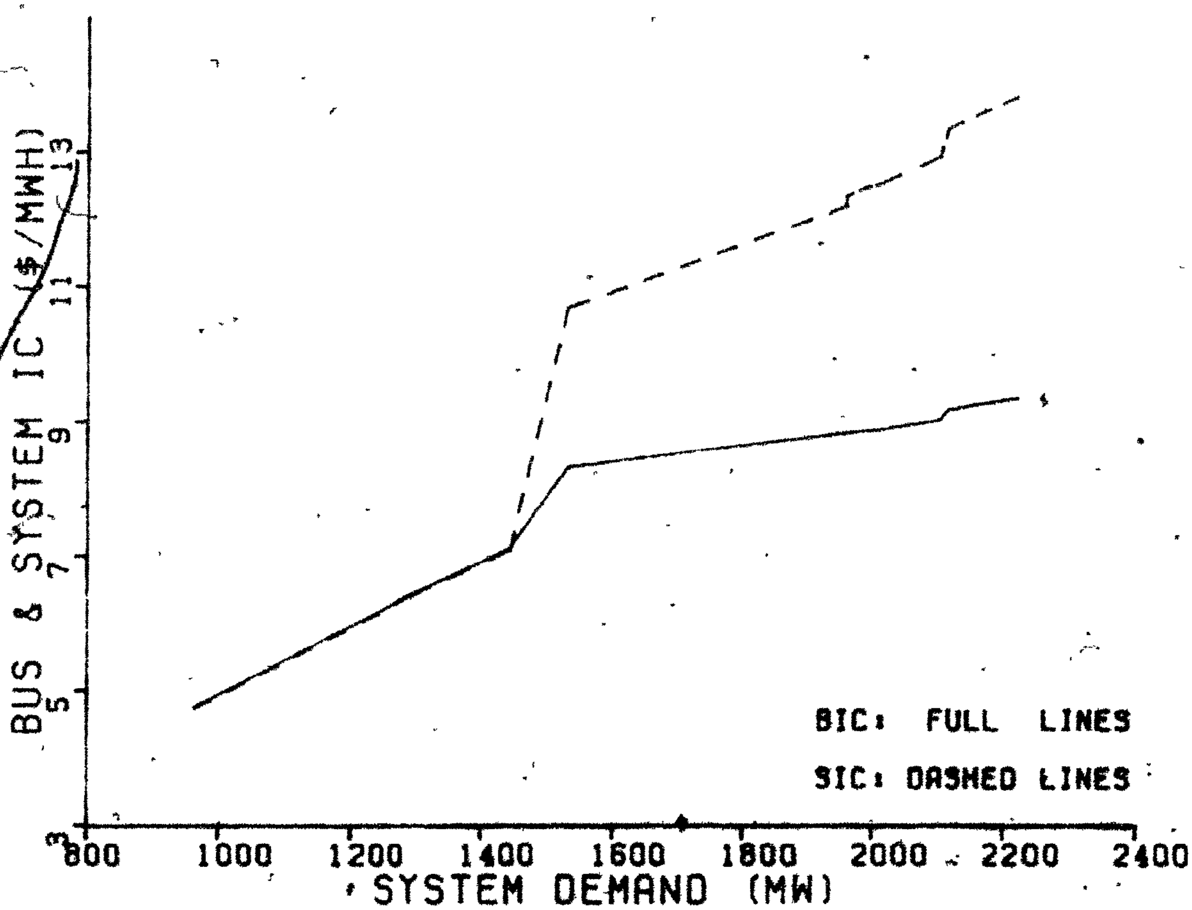

Figure 4.2.22 . BIC of bus # 22 and SIC versus demand.

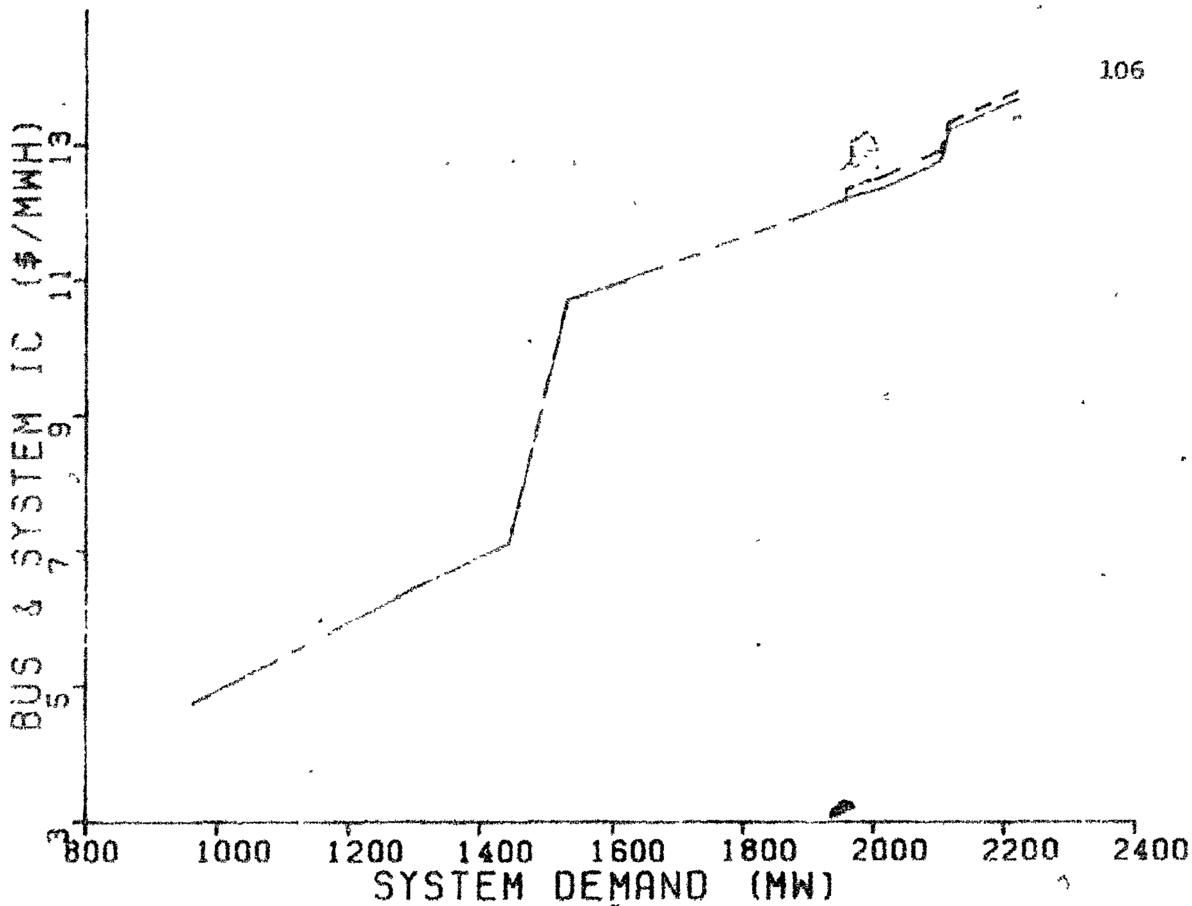

Figure 4.2.23 BIC of bus # 23 and SIC versus demand.

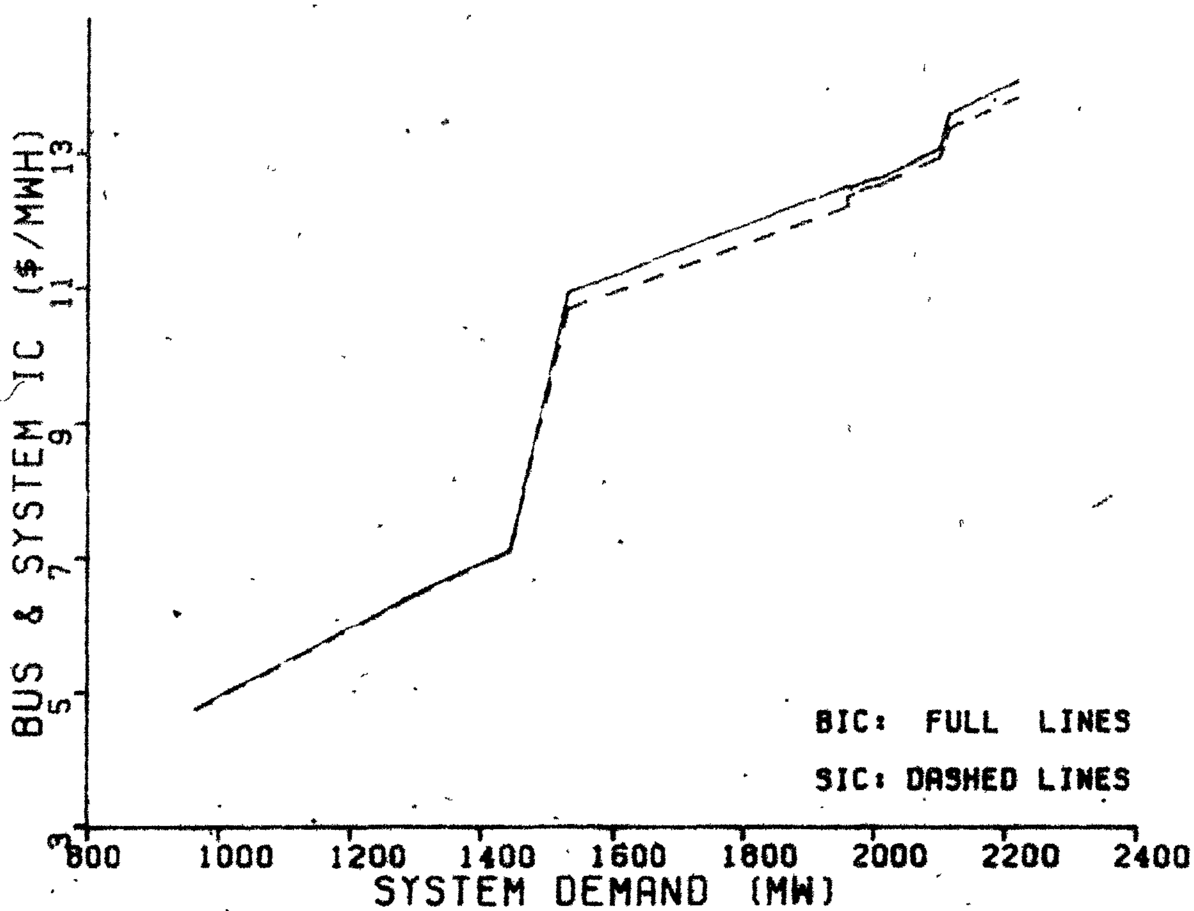

Figure 4.2.24 BIC of bus # 24 and SIC versus demand.

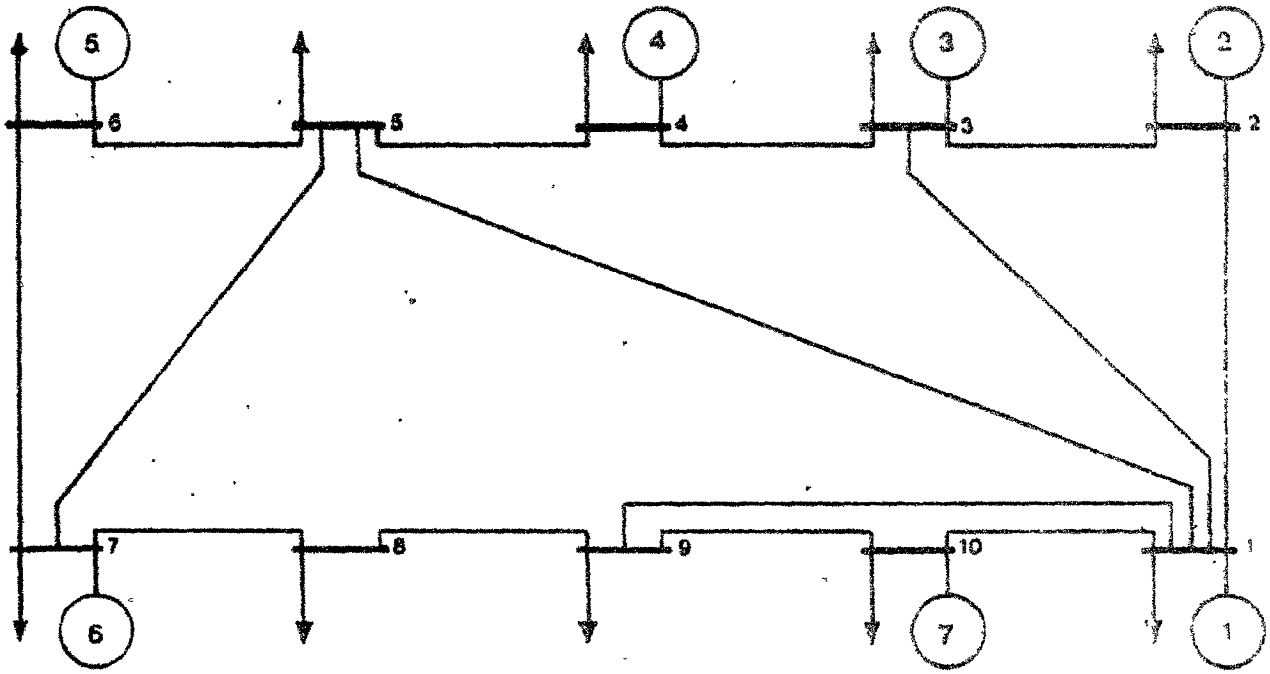

Figure 4.3 10-Bus experimental test system.

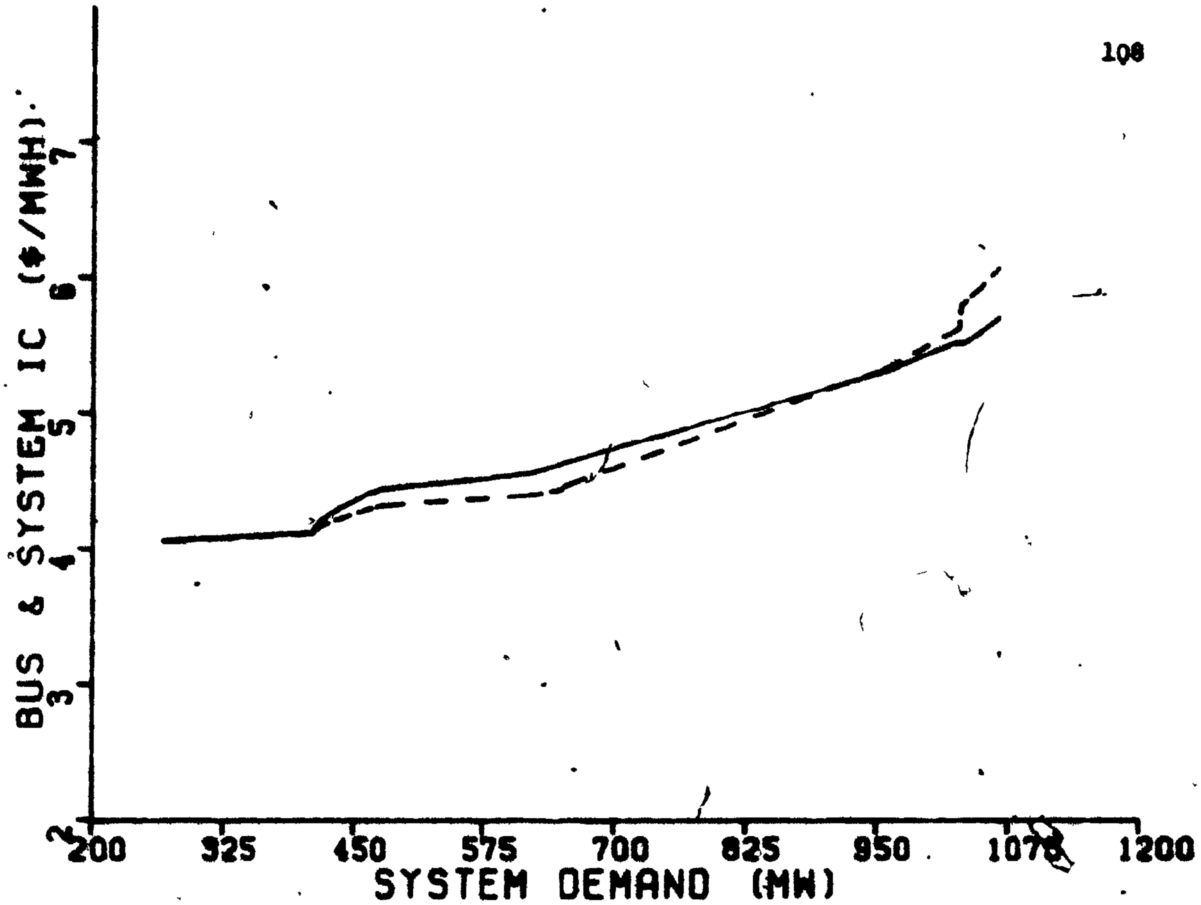

Figure 4.4.1 BIC of bus # 1 and SIC versus demand.

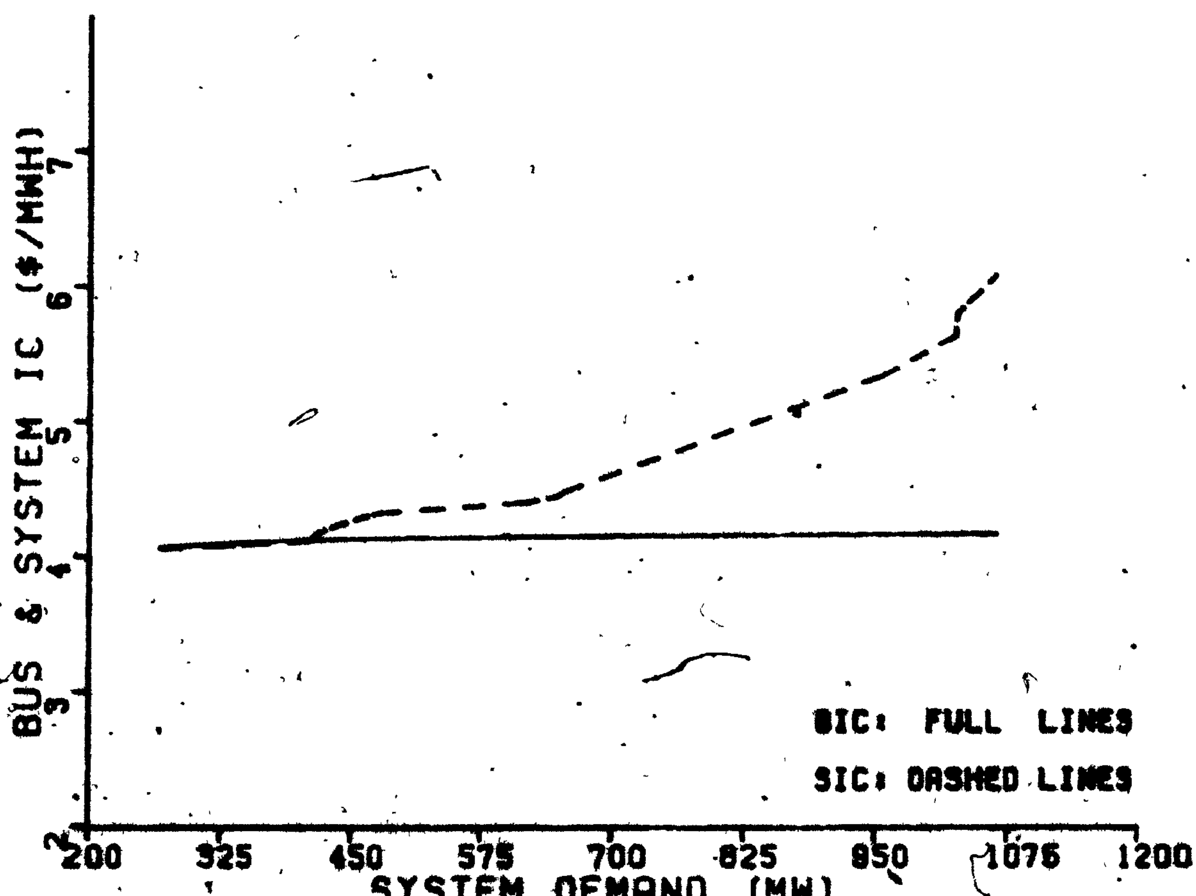

Figure 4.4.2 BIC of bus # 2 and SIC versus demand.

Figure 4.4 Variations in bus and system IC's in the 10-bus system.

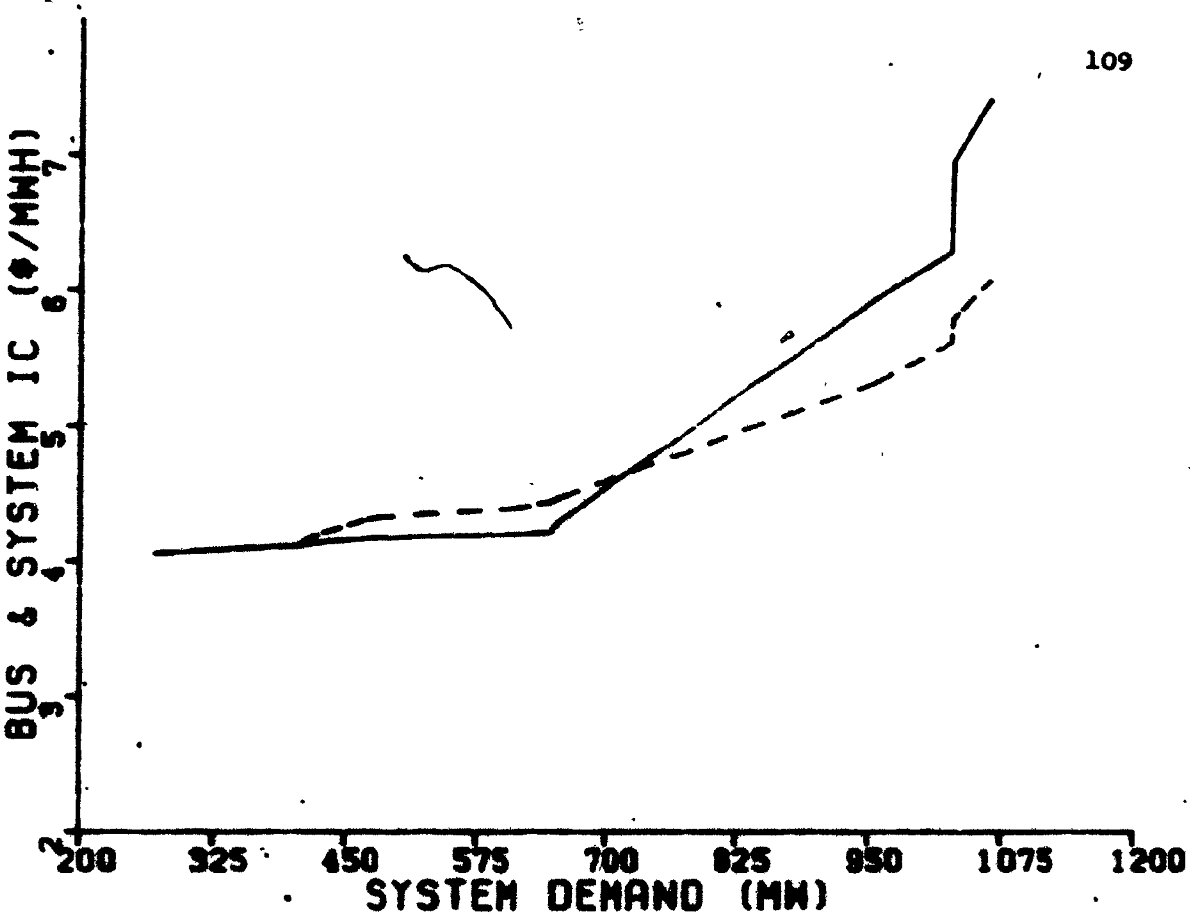

Figure 4.4.3 BIC of bus #3 and SIC versus demand.

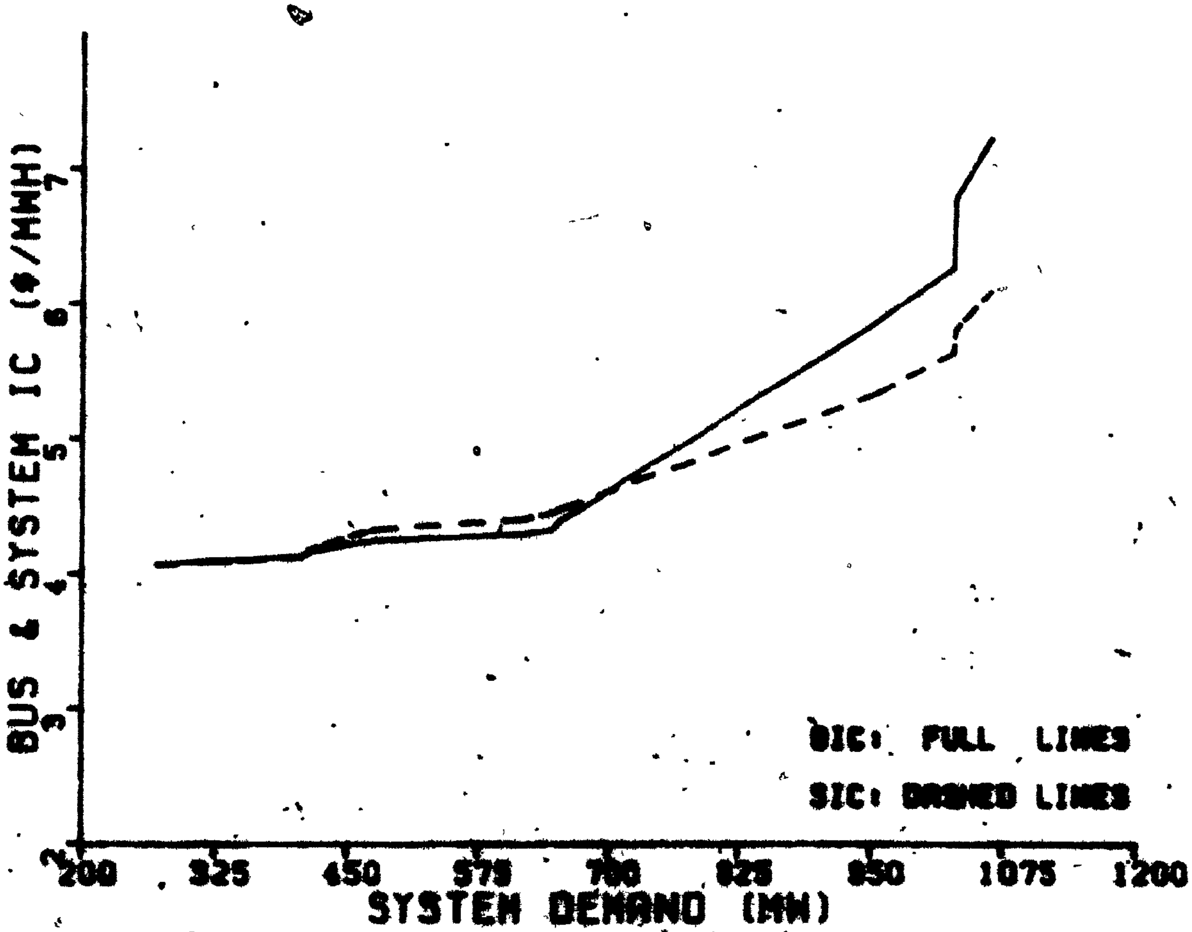

Figure 4.4.4 BIC of bus # 4 and SIC versus demand.

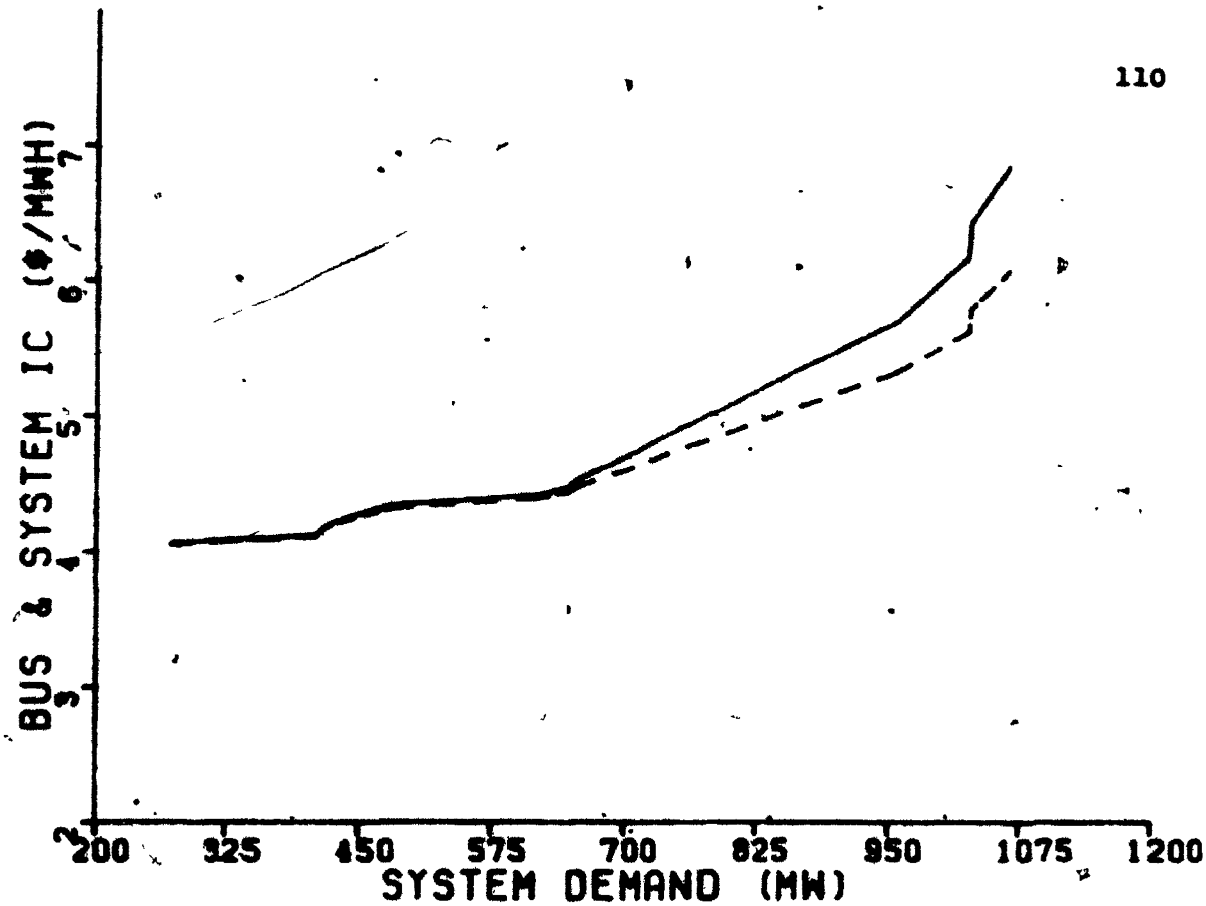

Figure 4.4.5 BIC of bus # 5 and SIC versus demand.

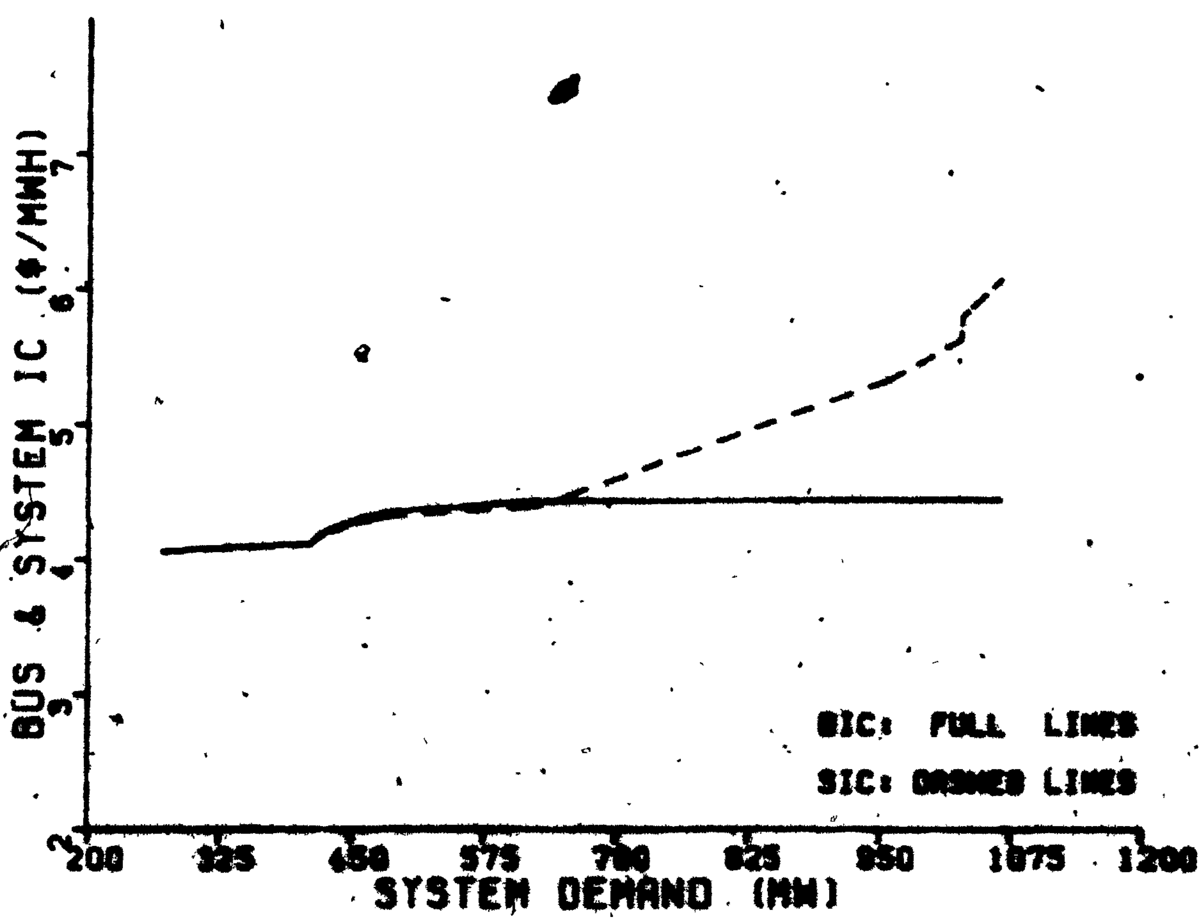

Figure 4.4.6 BIC of bus # 6 and SIC versus demand.

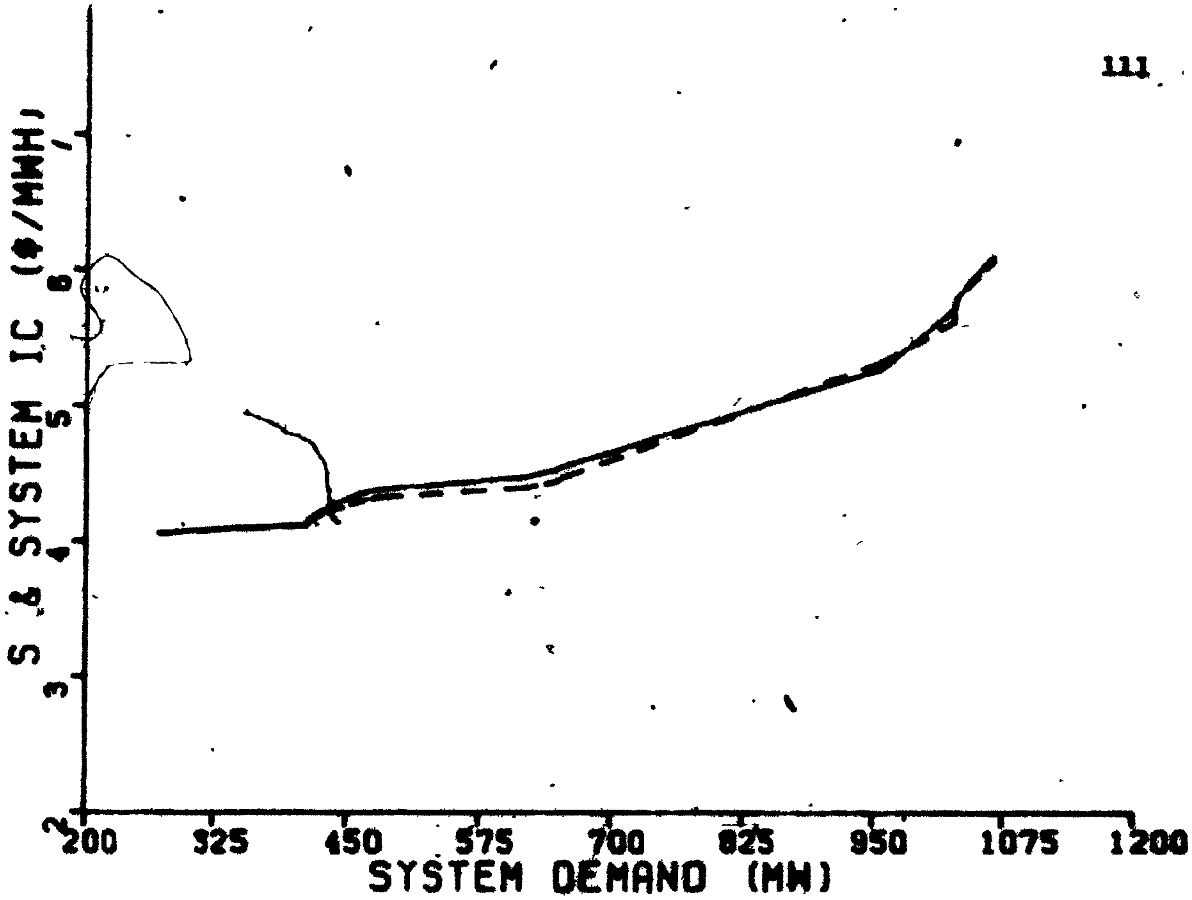

Figure 4.4.7 BIC of bus # 7 and SIC versus demand.

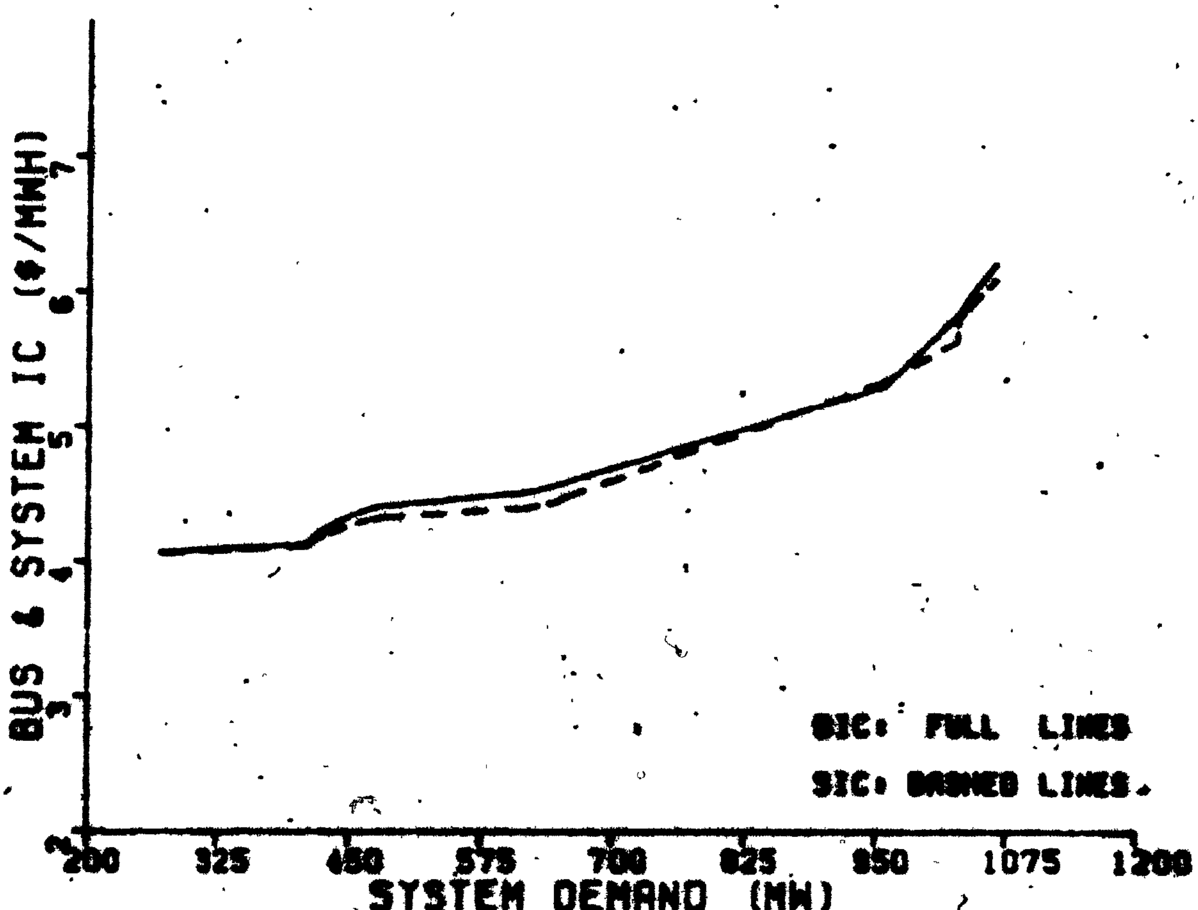

Figure 4.4.8 BIC of bus # 8 and SIC versus demand.

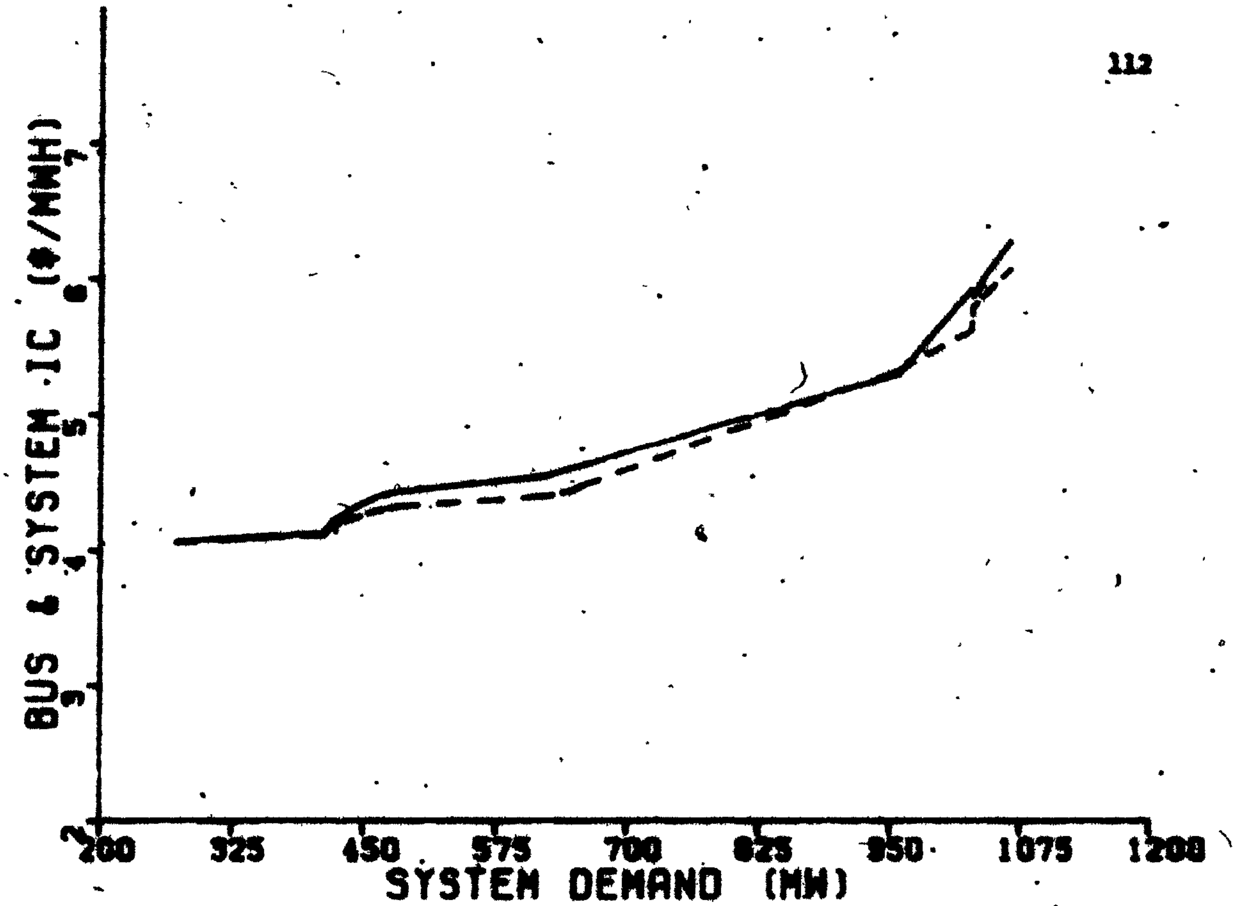

Figure 4.4.9 BIC of bus # 5 and SIC versus demand.

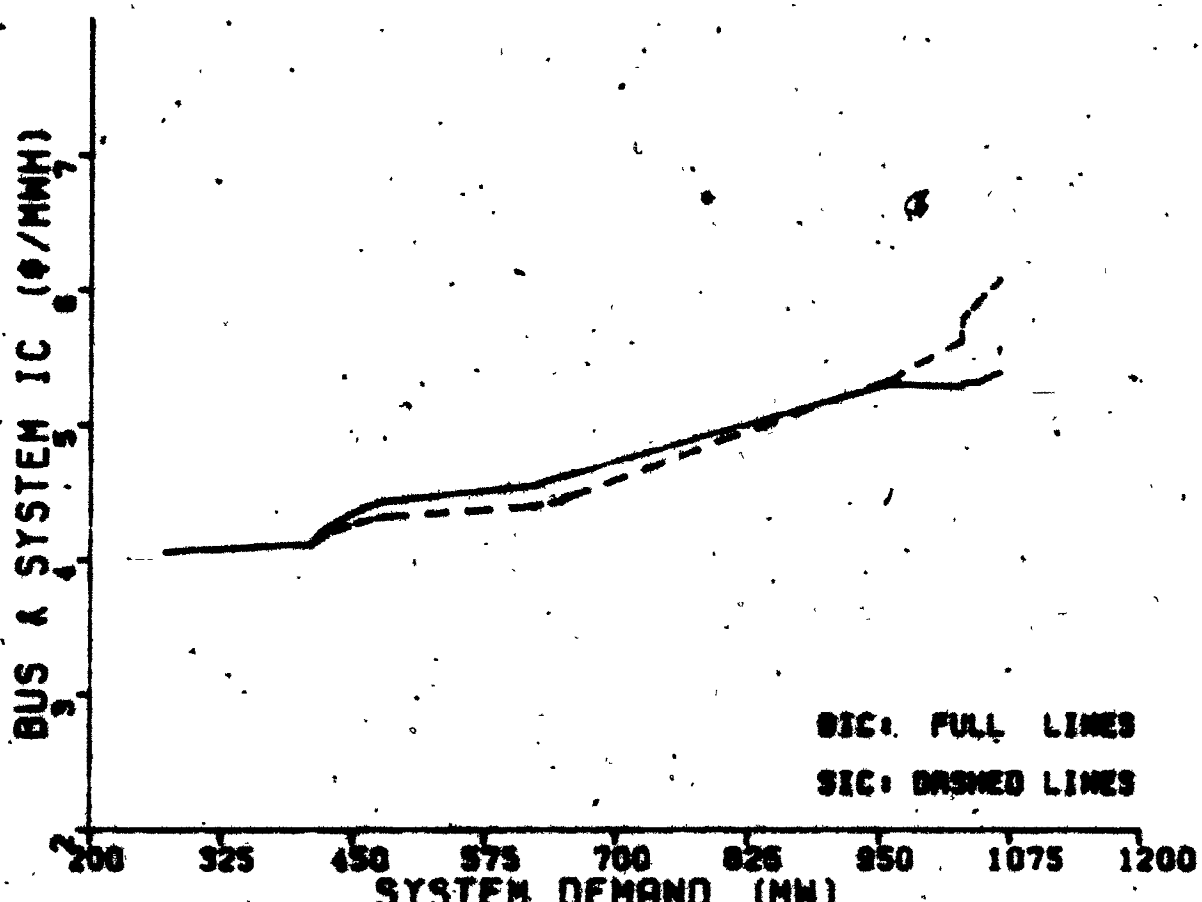

Figure 4.4.10 BIC of bus # 10 and SIC versus demand.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

5.0 Conclusions

This study has investigated the fact that the cost incurred by the utility in supplying a fixed amount of energy to different system localities need not always be the same in a lossless system. It is demonstrated through studies on a 24-bus system that these costs could have wide differences once transmission lines reach operating limits. One saturating line is sufficient to bring about a large disparity. Continuing to carry out economy interchanges based on the BIC will then cause some customers to subsidise others, and utilities to incur losses (or gains) in an unfavourable manner. It is also shown that in a lossless system so long as electricity sales by the utility are confined among its customers, the utility neither gains nor loses by a change in the pricing philosophy.

It is envisioned that a pricing strategy based on BIC's, could bring about a radical change in the current philosophy of dispatching electricity. It is conjectured that under a pricing policy based on BIC's, customers will adjust their demand according to the incremental cost of electricity supply, not demanding excessive amounts of energy at times of limited supply. A better mode of cooperation may result as the controlling force is now the load varying price of electricity. However, since an approaching system loadability limit is not reflected in the

BIC's, it may be necessary to price customers based on penalized BIC's rather than the true optimum BIC's, designed to discourage load increments close to system loadability limit.

Finally, the study has also shown how the optimum BIC's can be utilized in power system expansion studies (both generation and transmission expansion), where one could consider encouraging cogeneration at certain strategic locations as an alternative to building new plants or transmission lines. In the operation aspect of a power system, suitably modified optimum BIC's can be used to control different bus loads, encouraging or discouraging electricity consumption depending on prevailing circumstances.

5.1 Recommendations for Future Research

(a) The results reported in this work assumes a lossless system. Chapter II gives the necessary theory to include transmission losses into the model. Hence a program developed to combine this theory along with the program used in the present study will provide more realistic values for the BIC's.

(b) It is found that no indication of an approaching loadability limit is reflected in the BIC's . Hence, the pricing philosophy based on BIC's must be modified to keep customers away from this loadability limit. This can be accomplished by multiplying the optimum BIC's by suitable penalty functions, and pricing the customers based on these penalized BIC's . Further research is needed to determine such penalty functions which not only keep customers away from the loadability limit, but also provide a means of reflecting the large capital and other operation related costs incurred by a utility. In Figure 5.1* one type of penalty function capable of meeting these requirements is shown.

(c) The present study assumes all units in the system to be available at all times. However, some units may be out of commission due to forced outages or for maintenance. Hence for more realistic results a means of incorporating unit commitment into the present study has to be formulated.

(d) Customer response to time varying electricity prices is not clearly understood. Hence before utilities can make a complete change in their pricing philosophy, it would be most appropriate to study the problem mathematically, through building suitable models to simulate possible customer responses to load varying prices. The experience gained from

*This figure is given at the end of this chapter.

from such studies will provide valuable guidelines to the actual implementation issue of a new pricing philosophy based on BIC's.

(e) The present study assumes a constant correlation between the bus load and system demand in computing optimum BIC's. It will prove useful if one can track these optimum BIC's for different bus loads without having to rerun the simulation program. It is possible to parameterize the optimum BIC's in terms of a homotopy and to track the optimum BIC's by varying this homotopy between 0 and 1. However this process could involve activating or deactivating multiple constraints of the active set making the problem more complex. More investigation is needed in this area. Appending a program successfully developed to implement this issue to the existing package will provide a range of BIC's valid for varying bus loads not necessarily having a fixed correlation to the system demand.

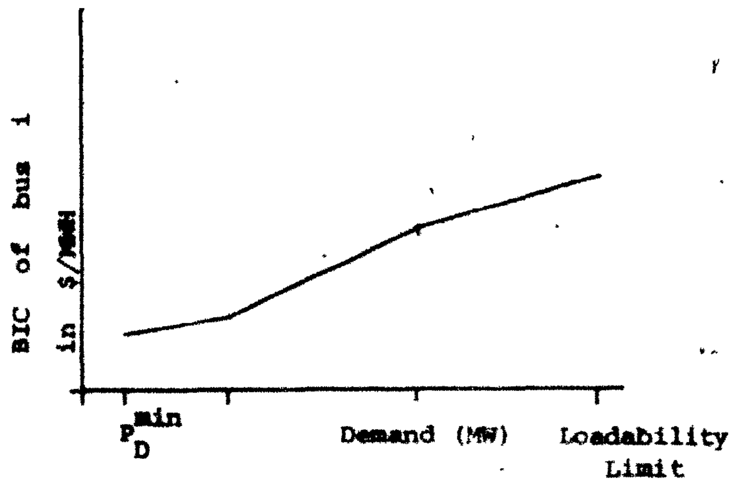

Figure 5.1.a BIC of bus i .

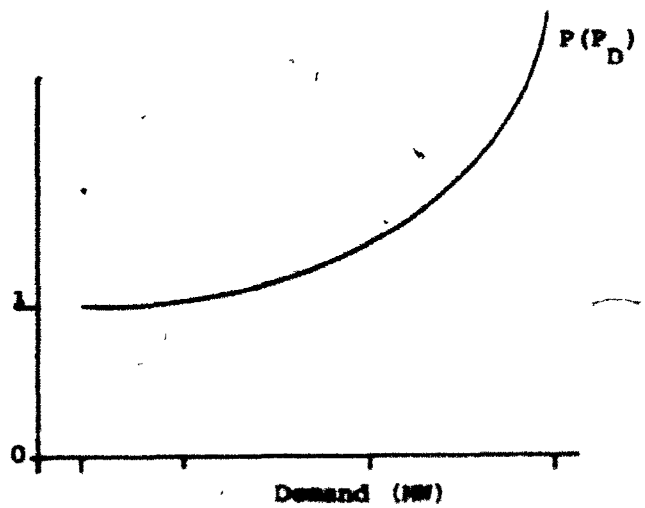

Figure 5.1.b A proposed penalty function.

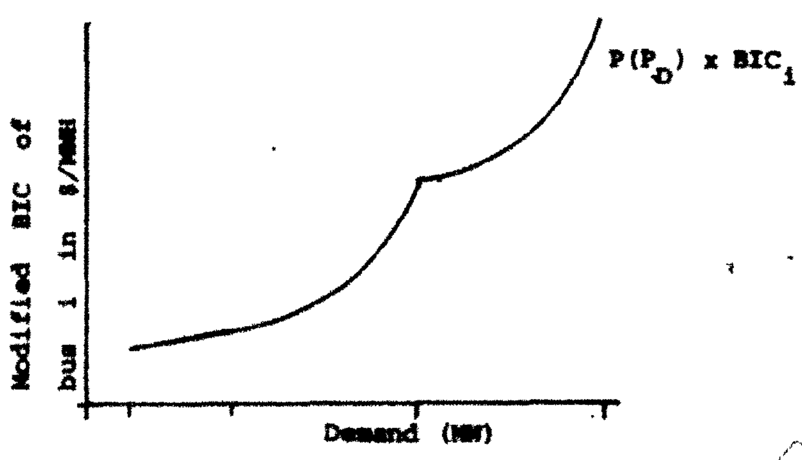

Figure 5.1.c Penalised BIC of bus i .

Figure 5.1 Modifying BIC's through suitable penalty functions.

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APPENDIX A

A DC LOAD FLOW MODEL

This appendix covers the necessary background on the DC - load flow.

A.1 Basic Assumptions and Derivations

The following lists the basic assumptions under which this model operates:

1. All line resistances are assumed to be negligibly small [see also equation (2.5)] .

2. All reactive power flow is ignored and only the real power flow is considered.

3. The difference in the voltage angles $(\theta_i - \theta_j)$ are assumed to be sufficiently small such that the following assumption holds; i.e.,

$$\sin(\theta_i - \theta_j) \simeq (\theta_i - \theta_j) \qquad \text{(A.1)}$$

where θ_i , θ_j are measured in radians.

4. All bus voltages are equal to 1.0 pu .

The power flow in line i-j is given by [Elgerd 1982, Sullivan 1977] :

$$P_{i-j} = B_{i-j} V_i V_j \sin(\theta_i - \theta_j) \qquad \text{(A.2)}$$

where B_{i-j} is the susceptance of the line joining, buses i and j ,

By definition :

$$\text{Real generation at bus } i \triangleq P_i \qquad \text{(A.3.a)}$$

$$\text{Real load at bus } i \triangleq L_i \qquad \text{(A.3.b)}$$

$$\text{Real injection at bus } i,\ Z_i \triangleq P_i - L_i \qquad \text{(A.3.c)}$$

The real injection at bus i can be written as,

$$Z_i = P_i - L_i = \sum_{j \in i} B_{ij} V_i V_j \sin(\theta_i - \theta_j), \ \forall i = 1, \ldots, n+1 \qquad \text{(A.4)}$$

where $j \in i$ implies that bus j is connected to bus i , but is not coincident with bus i .

By assumptions 3 and 4 , equation (A.4) becomes

$$Z_i = \sum_{j \in i} B_{ij}(\theta_i - \theta_j) \ , \quad \forall i = 1, \ldots\ldots, n+1 \qquad \text{(A.5)}$$

Let $\hat{\underline{Z}}$, be the vector of (n+1) bus injections, given by :

$$\hat{\underline{z}} \triangleq [P_1 - L_1, \ldots\ldots, P_m - L_m, P_{m+1} - L_{m+1}, -L_{m+2}, \ldots\ldots\ldots, -L_{n+1}]^t_{1xn+1} \quad (A.6)$$

Let $\hat{\underline{\theta}}$, be the vector of bus voltage angles, given by :

$$\hat{\underline{\theta}} \triangleq [\theta_1, \ldots\ldots, \theta_m, \theta_{m+1}, \theta_{m+2}, \ldots\ldots\ldots \theta_{n+1}]^t_{1xn+1} \quad (A.7)$$

Thus the (n+1) load flow equations (A.4) becomes,

$$\hat{\underline{z}} = \hat{\underline{J}}\,\hat{\underline{\theta}} \quad (A.8)$$

where $\hat{J}$ is the (n+1) x (n+1) symmetric DC load flow Jacobian whose elements are

$$\hat{J}_{ij} = -B_{i-j} \quad (A.9)$$

$$\hat{J}_{ii} = \sum_{j \neq i} B_{i-j} \quad (A.10)$$

The (n+1) x (n+1) DC load flow Jacobian is singular, as :

$$\sum_{i=1}^{n+1} \hat{\underline{j}}_i = 0 \quad (A.11)$$

where $\hat{\underline{j}}_i$ is the ith column of $\hat{\underline{J}}$.

To overcome this problem a new Jacobian $\underline{J}$ is defined by deleting the row and column corresponding to the slack bus, which is shown as bus (m+1) .

Thus the non-singular set of n load flow equations become :

$$\underline{Z} = \underline{J}\,\underline{\theta} \tag{A.12}$$

By definition :

$$\underline{Z} \triangleq [P_1 - L_1, \ldots\ldots, P_m - L_m, -L_{m+2}, \ldots\ldots\ldots, -L_{n+1}]^t_{1xn} \tag{A.13}$$

$$\underline{\theta} \triangleq [\theta_1, \ldots\ldots, \theta_m, \theta_{m+2}, \ldots\ldots\ldots, \theta_{n+1}]^t_{1xn} \tag{A.14.a}$$

$$\theta_{m+1} \triangleq 0 \tag{A.14.b}$$

The D.C. load flow model used in this study is given by equation (A.12) .

A.2 Sensitivities of Line Flows to Independent Bus Injections

The line flow sensitivity in line i-j is given by :

$$\underline{Y}_{i-j} = \frac{\partial P_{i-j}}{\partial \underline{Z}} \tag{A.15}$$

From equation (A.2) , and assumptions 3, 4 the following result , i.e.,

$$P_{i-j} = B_{i-j}\,\underline{\delta}^t_{ij}\,\underline{\theta} \tag{A.16}$$

By definition :

$$\underline{e}_{ij} \triangleq [0, \ldots\ldots 1 \ldots\ldots -1 \ldots\ldots 0]^{t}_{1xn} \qquad \text{(A.17)}$$

$$\begin{array}{cc} \uparrow & \uparrow \\ i & j \end{array}$$

From equations (A.16) and (A.12) substituting for the state vector $\underline{\theta}$, we get

$$\underline{\theta} = \underline{J}^{-1} \underline{Z} \qquad \text{(A.18)}$$

$$P_{i-j} = B_{i-j} \underline{e}^{t}_{ij} [\underline{J}^{-1} \underline{Z}] \qquad \text{(A.19)}$$

$$P_{i-j} = \underline{Y}^{t}_{i-j} \underline{J}^{-1} \underline{Z} \qquad \text{(A.20)}$$

where

$$\underline{Y}_{i-j} \triangleq B_{i-j} \underline{e}_{ij} \qquad \text{(A.21)}$$

Thus from equations (A.15) and (A.19), the sensitivity of line flows become :

$$\underline{Y}_{i-j} = \underline{J}^{-1} \underline{Y}_{ij} \qquad \text{(A.22)}$$

From equations (A.19) and (A.21), the line flow on line (i-j) becomes

$$P_{i-j} = \underline{Y}^{t}_{ij} \underline{Z} \qquad \text{(A.23)}$$

APPENDIX B

SOLUTION OF THE CONSTRAINED ED PROBLEM BY THE CONTINUATION METHOD [Vojdani 1982]

B.1 The Objective Function

Under the assumptions of quadratic costs of generation given by equation (2.1), the objective function can be written as follows:

$$C(\underline{P}) = c_0 + \tfrac{1}{2}(\underline{P} - \underline{P}^0)^t \, \underline{H}(\underline{P} - \underline{P}^0) \qquad (B.1)$$

where :

$\underline{P}^0$ is the generation schedule of the unconstrained problem, and is given by the following equation [Vojdani 1979], i.e.,

$$\underline{P}^0 = [\underline{B} + b_{m+1}\,\underline{e}_m\,\underline{e}_m^t]^{-1}\,[(a_{m+1} + b_{m+1}\,P_D)\underline{e}_m - \underline{a}] \qquad (B.2)$$

By definition

$$\underline{H} \triangleq \frac{\partial^2 C(\underline{P})}{\partial \underline{P}\,\partial \underline{P}^t} = \underline{B} + b_{m+1}\,\underline{e}_m\,\underline{e}_m^t \qquad (B.3)$$

The Hessian matrix $\underline{H}$ will be positive definite, provided the following relation holds, i.e.,

$$b_i > 0 \qquad \forall\, i = 1, \ldots\ldots, m \qquad (B.4)$$

C_0 is the optimum cost of the unconstrained problem in \$/hr .

B.2 Inequality Constraints

B.2.1 Generation Bounds

The bound on all controllable generators $\underline{P}$, and the slack generator is written as

$$\underline{P}^{min} < \underline{P} < \underline{P}^{max} \qquad (B.5.a)$$

$$P_D - P^{max}_{m+1} < \underline{e}^t_m \underline{P} < P_D - P^{min}_{m+1} \qquad (B.5.b)$$

B.2.2 Transmission Bounds

The line flow sensitivity vector $\underline{Y}_{i-j}$ defined by equation (A.22) can be partitioned as follows :

$$\underline{Y}_{i-j} = \begin{bmatrix} Y^1_{i-j} \\ \hline Y^2_{i-j} \end{bmatrix} \begin{matrix} m \times 1 \\ n \times 1 \end{matrix} \qquad (B.6)$$

From equations (2.8), (A.13) and (A.23) , the following results ; i.e.,

$$(\underline{Y}_{i-j})^t \underline{L} - F^{max}_{i-j} < (\underline{Y}^1_{i-j})^t \underline{P} < (\underline{Y}_{i-j})^t \underline{L} + F^{max}_{i-j} \qquad (B.7)$$

By combining equations (B.5.6) and (B.7), the following functional inequality results, i.e.,

$$\underline{G}\,\underline{P} > \underline{K} \qquad \text{(B.8)}$$

B.3 Solution to the Constrained ED Problem

The ED problem viewed as a nonlinear optimization problem would be as follows, i.e.,

$$\text{Minimize} \quad \frac{1}{2}\,(\underline{P} - \underline{P}^{0})^{t}\,\underline{H}\,(\underline{P} - \underline{P}^{0}) \qquad \text{(B.9.a)}$$

w.r.t. $\underline{P}$

$$\text{s.t.} \quad \underline{P}^{min} < \underline{P} < \underline{P}^{max} \qquad \text{(B.9.b)}$$

$$\underline{G}\,\underline{P} > \underline{K} \qquad \text{(B.9.c)}$$

Let the optimum be characterized by q active inequality constraints, the details of which are as follows, i.e.,

$$\underline{P}_i = \underline{P}_i^{min} \qquad i = 1, 2, \ldots\ldots, \ell \qquad \text{(B.10.a)}$$

$$\underline{P}_j = \underline{P}_j^{max} \qquad j = \ell+1, \ldots\ldots, \ell+u \qquad \text{(B.10.b)}$$

$$\underline{P}_f^{min} < \underline{P}_f < \underline{P}_f^{max} \qquad f = \ell+u+1, \ldots\ldots, m \qquad \text{(B.10.c)}$$

$$\hat{\underline{G}}\,\underline{P} = \hat{\underline{K}} \qquad \text{a total of } q - \ell - u \text{ equations} \qquad \text{(B.10.d)}$$

The sizes of matrices $\underline{\hat{G}}$ and $\underline{\hat{K}}$ are assumed understood.

By definition :

The unconstrained solution, i.e., solution to equation (B.9.a) only is given by :

$$\underline{P}^0 \triangleq \left[\begin{array}{c} \underline{P}_1^0 \\ \hline \underline{P}_2^0 \end{array}\right] \begin{array}{l} (\ell+u)\mathrm{x}1 \\ m\mathrm{x}1 \end{array} \qquad \text{(B.11.a)}$$

The generation constrained solution, i.e., solution to (B.9.a) and (B.9.b) only is given by the following, assuming the slack generator to be free without loss of generality, i.e.,

$$\underline{P}^G \triangleq \left[\begin{array}{c} \underline{P}^{limit} \\ \hline P_2^G \end{array}\right] \begin{array}{l} (\ell+1)\mathrm{x}1 \\ m\mathrm{x}1 \end{array} \qquad \text{(B.11.b)}$$

The security constrained solution, i.e., solution to equations (B.9.a - c) is given by :

$$\underline{P}^S \triangleq \left[\begin{array}{c} \underline{P}_1^{limit} \\ \hline \underline{P}_2^S \end{array}\right] = \left[\begin{array}{c} \underline{P}_1 \\ \hline \underline{P}_2 \end{array}\right] \qquad \text{(B.11.c)}$$

Partitioned Hessian :

$$\underline{H} \triangleq \left[\begin{array}{c|c} \underline{H}_{11} & \underline{H}_{12} \\ \hline \underline{H}_{12}^T & \underline{H}_{22} \end{array}\right]_{m \times m} \qquad \text{(B.11.d)}$$

$\underline{H}_{11}$ has dimension $(\ell+u) \times (\ell+u)$

Partitioned matrix of active functional inequalities :

$$\hat{G} \triangleq \Big[\underbrace{\hat{\underline{G}}_1}_{(\ell+u)} \;\Big|\; \hat{\underline{G}}_2\Big]_{(q-\ell-u) \times m} \qquad \text{(B.11.e)}$$

Vector of Lagrange multipliers :

$$\lambda \triangleq \left[\begin{array}{c} \underline{\lambda}_1 \\ \hline \underline{\lambda}_2 \end{array}\right] \begin{array}{l} (\ell+u) \times 1 \\ (q \times 1) \end{array} \qquad \text{(B.11.f)}$$

The Lagrangian to be minimized reads :

$$\mathcal{L} = C_0 + \frac{1}{2} (\underline{P} - \underline{P}^0)^T \underline{H} (\underline{P} - \underline{P}^0) + \underline{\lambda}_1^t (\underline{P}_1^{limit} - \underline{P}_1) + \underline{\lambda}_2^t (\hat{\underline{g}} - \hat{\underline{G}}\,\underline{P}) \qquad \text{(B.12.a)}$$

The first order necessary conditions are :

$$\frac{\partial \mathcal{L}}{\partial \underline{P}_1} = \underline{H}_{11}(\underline{P}_1 - \underline{P}_1^0) + \underline{H}_{12}(\underline{P}_2 - \underline{P}_2^0) - \underline{\lambda}_1 - \hat{\underline{G}}_1^T \underline{\lambda}_2 = 0 \qquad \text{(B.12.b)}$$

$$\frac{\partial f}{\partial \underline{P}_2} = \underline{H}_{22}(\underline{P}_2 - \underline{P}_2^0) + \underline{H}_{21}(\underline{P}_1 - \underline{P}_1^0) - \underline{G}_2^{*t}\,\underline{\lambda}_2 = 0 \qquad \text{(B.12.c)}$$

For the case of generations constrained ED problem $\underline{\lambda}_2 = 0$, thereby giving the following result from equation (B.12.c)

$$\underline{P}_2^G = \underline{P}_2^0 - \underline{H}_{22}^{-1}\,\underline{H}_{21}\,(\underline{P}_1 - \underline{P}_1^0) \qquad \text{(B.13.a)}$$

With some algebraic manipulations, (B.13.a) can be shown to be equal to :

$$\underline{P}_2^G = \underline{\Pi}^G P_D + \underline{\eta}^G \qquad \text{(B.13.b)}$$

Where $\underline{\Pi}^G$ are the generation constrained participation factors defined by

$$\Pi_f^G \triangleq \frac{1}{b_f\, S_1^G} \; , \; f = \ell+u+1 \; , \; \ldots\ldots\ldots \; , \; m \qquad \text{(B.13.c)}$$

$$S_1^G \triangleq \sum_{f=\ell+u+1}^{m+1} \frac{1}{b_f} \qquad \text{(B.13.d)}$$

$$\underline{\eta}^G \triangleq \underline{\Pi}^G\Big(S_2^G - \sum_{i=1}^{\ell+u} P_i^{limit}\Big) - \frac{a_f}{b_f} \; , \; f = \ell+u+1 \; , \; \ldots\ldots \; m \qquad \text{(B.13.e)}$$

$$S_2^G = \sum_{f=\ell+u}^{m+1} \frac{a_f}{b_f} \qquad \text{(B.13.f)}$$

From equations (B.12.c) and (B.13.a) the following result is obtained, i.e.,

$$\underline{P} = \underline{P}_2^G + \underline{H}_{22}^{-1} \; \underline{G}_2^t \; \underline{\lambda}_2 \qquad \text{(B.13.g)}$$

From equations (B.10.d), (B.11.c), and (B.13.g) results in the following equation for $\underline{\lambda}_2$, i.e.,

$$\underline{\lambda}_2 = [\hat{\underline{G}}_2 \; \underline{H}_{22}^{-1} \; \hat{\underline{G}}_2^t]^{-1} \; [\hat{\underline{K}} - \hat{\underline{G}} \; \underline{P}^G] \qquad \text{(B.13.h)}$$

From equation (B.12.b)

$$\underline{\lambda}_1 = [\underline{H}_{11} \; \vdots \; \underline{H}_{12}] [\underline{P} - \underline{P}^0] - \underline{G}_1^t \; \underline{\lambda}_2 \qquad \text{(B.13.i)}$$

The u components of $\underline{\lambda}_1$ associated with generators at upper limits, will be negative, while the remaining components of $\underline{\lambda}_1$ and $\underline{\lambda}_2$ will be positive. For purposes of consistency the u components of $\underline{\lambda}_1$ are multiplied by -1 in order to have all LM's positive.

B.4 Parameterising in the Domain of System Demand

B.4.1 Parameterising the Lagrange Multipliers in P_D

The bus loads are linearly correlated to the system demand P_D, as given by the following equation :

$$L_i = t_i \, P_D \qquad \forall \; i = 1, \ldots\ldots, (m+1) \qquad \text{(B.14.a)}$$

$$\sum_{i=1}^{m+1} t_i = 1 \qquad \text{(B.14.b)}$$

From equations (B.14.a-b) and (B.13.c-d) the LM's can be parameterized in P_D as follows :

$$\underline{\lambda}_1 = \underline{\beta}_1 P_D + \underline{\beta}_1' \quad \text{(B.15.a)}$$

$$\underline{\lambda}_2 = \underline{\beta}_2 P_D + \underline{\beta}_2' \quad \text{(B.15.b)}$$

B.4.2 Parameterizing the Set of Free Generators in P_D

From equations (B.13.b), (B.13.q) and (B.15.b), the following results, i.e.,

$$\underline{P}_2 = \underline{\Pi}^s P_D + \underline{\Pi}^s \quad \text{(B.16.c)}$$

where $\underline{\Pi}^s$ is the security constrained participation factors.

B.4.3 Parameterizing the Line Flows in P_D

From equations (A.13), (B.14.a), (B.16.c) and (A.12) the state vector $\underline{\theta}$ can be written as :

$$\underline{\theta}(P_D) = \underline{\tau} P_D + \underline{\tau}' \quad \text{(B.17.a)}$$

From equations (B.17.a) and (2.16)

$$\underline{\phi}(P_D) + \underline{\phi}' \geq \underline{d} \qquad \text{(B.17.b)}$$

B.5 The Continuous Simulation Algorithm

1. Identify the optimum generation schedule at the minimum loadability limit, i.e.,

$$P_D^{min} = \sum_{i=1}^{m+1} P_i \qquad \text{(B.18.a)}$$

2. Determine the generation constraint to be released from the active set.

   This corresponds to generator i where

$$C_i'(P_i) < C_j'(P_j) \qquad \forall \; j = 1, \ldots\ldots, m \quad j \neq i \qquad \text{(B.18.b)}$$

3. Determine the minimum positive value of P_D which either brings about an equality of equation (B.17.b) or makes a Lagrange multiplier in equations (B.15.a) or (B.15.b) zero.

   This will mark the beginning of another load interval.

4. Repeat step 3 until limit of system loadability is reached, i.e.,

$$P_D = P_D^{max}$$

APPENDIX C

SOLUTION OF THE LLS PROBLEM

BY THE METHOD OF HOUSEHOLDER ORTHOGONALIZATION

C.1 Householder Matrices

Householder matrices are Unitary Elementary Hermitian matrices which have the general form :

$$\underline{H} = \underline{I} - \frac{2\,\underline{u}\,\underline{u}^t}{\underline{u}^t\,\underline{u}} \tag{C.1}$$

By definition

$$\underline{x} \triangleq [x_1, \ldots\ldots\ldots, x_k]^t_{1xk} \tag{C.2.a}$$

$$\tilde{\underline{x}} \triangleq [\rho, 0, \ldots\ldots\ldots, 0\,]^t_{1xk} \tag{C.2.b}$$

It can be shown that a transformation of the following nature is possible [Stewart, 1973] .

$$\underline{H}\,\underline{x} = \tilde{\underline{x}} \tag{C.3.a}$$

Provided the following equation is satisfied, i.e.,

$$\underline{u} = x \pm \rho e' \tag{C.3.b}$$

The sign chosen must be such that no cancellation results.

This is a necessary condition, for purposes of precision in computing.

By definition :

$$\underline{e}' \triangleq [1, 0, \ldots\ldots, 0]^t_{1\times k} \qquad \text{(C.4.a)}$$

$$\rho \triangleq \| x \|_2 \qquad \text{(C.4.b)}$$

In the LLS problem, Householder matrices will be used to convert a rectangular matrix into an upper trapezoidal matrix.

C.2 Solution of the LLS Problem

The LLS problem can be stated as follows

$$\text{Minimise } \|\underline{\tilde{b}} - \underline{A}\, x\|_2^2 \qquad \text{(C.5.b)}$$

w.r.t. $\underline{x}$

where the following matrices are given :

$$\underline{\tilde{b}} = [\tilde{b}_1 \ldots\ldots\ldots \tilde{b}_m]^t_{1\times m} \qquad \text{(C.6.a)}$$

$$\underline{A} = \begin{bmatrix} a_{11} & \ldots\ldots & a_{1n} \\ & & \\ a_{m1} & \ldots\ldots & a_{mn} \end{bmatrix}, \quad m > n \qquad \text{(C.6.b)}$$

The $\| \cdot \|_2$ is invariant under orthogonal multiplication.

Thus equation (C.5) can be written as

$$\text{Minimize} \quad \| \underline{Q}(\underline{b} - \underline{A}\,\underline{x}) \|_2^2 \qquad \text{(C.7.a)}$$

w.r.t. $\underline{x}$

$$\text{where} \quad \underline{Q} = \underline{E}_m \, \underline{E}_{m-1} \cdot \cdot \cdot \cdot \cdot \underline{E}_1 \qquad \text{(C.7.b)}$$

Where $\underline{E}_1, \underline{E}_2, \ldots \ldots, \underline{E}_m$ are Householder matrices designed on the columns of $\underline{A}$ to bring it into an upper trapezoidal form.

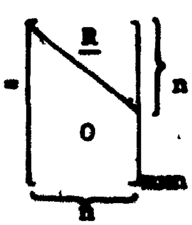
Thus the objective function (C.7.a) becomes

$$\text{Minimize} \quad \| \bar{\underline{b}}_1 - \underline{R}\,\underline{x} \|_2^2 + \| \bar{\underline{b}}_2 \|_2^2 \qquad \text{(C.8.a)}$$

w.r.t. $\underline{x}$

By definition:

$$\underline{Q}\,\tilde{\underline{b}} \triangleq \begin{bmatrix} \bar{\underline{b}}_1 \\ \bar{\underline{b}}_2 \end{bmatrix}_{m \times 1}^{n \times 1} \qquad \text{(C.8.b)}$$

$\underline{Q}\,\underline{A} =$  (C.8.c)

The solution to equation (C.8.a) is given by

$$\underline{R}\,\hat{\underline{x}} = \bar{\underline{b}}_1 \qquad \text{(C.9)}$$

The solution of equation (C.9) is trivial, as R is now upper-triangular and square, and the solution $\hat{x}$ is the best LLS estimate of $\underline{x}$.

Algorithms to solve the LLS Problem, based on this theory are proven to be numerically stable. Householder matrices are proven to exist, for any given matrix [Stewart 1973].

Note : The optimum generation schedule, and the optimum operating costs are very sensitive to the generation cost coefficients [Vojdani 1979]. This calls for a very accurate method for their determination. In the present study, an algorithm based on the above theory is used to compute these cost coefficients.

APPENDIX D

MODIFIED DATA FOR GENERATION UNITS

IN THE 24-BUS SYSTEM

Table D.1 , gives the heat rate data, used in this thesis. The sources for the data are :

(1) IEEE Reliability Test System 1979 .

(2) El Hawary and Christensen 1979 .

TABLE . D.1

GENERATING UNIT OPERATING COST DATA

SIZE (MW)	TYPE	FUEL	OUTPUT %	HEAT RATE BTU/KWh	O & M COST FIXED $/KW/YR.	VARIABLE $/MWH
12	Fossil Steam	#6 Oil	20	15,500	10.0	1.6
			50	12,150		
			80	11,460		
			100	11,310		
50	Fossil Coal	Coal	20	14,400	1.75	0.9
			50	11,680		
			80	11,110		
			100	10,970		
76	Fossil Coal	Coal	20	15,600	10.0	0.9
			50	12,900		
			80	11,900		
			100	12,000		

TABLE D.1 (cont'd)

SIZE (MW)	TYPE	FUEL	OUTPUT %	HEAT RATE BTU/KWh	O & M COST FIXED $/KW/YR.	O & M COST VARIABLE $/MWH
100	Fossil Steam	#6 Oil	25	12,400	8.5	0.80
			55	9,200		
			80	8,870		
			100	8,980		
155	Fossil Steam	Coal	35	11,200	7.0	0.80
			60	9,000		
			80	8,840		
			100	9,120		
197	Fossil Steam	#6 Oil	35	10,750	5.0	0.70
			60	9,850		
			80	9,840		
			100	9,600		
350	Fossil Steam	Coal	40	10,200	4.5	0.70
			65	9,600		
			80	9,500		
			100	9,500		
400	Nuclear Steam	LWR.	25	12,550	5.0	0.30
			50	10,400		
			80	10,323		
			100	10,625		

Table D.2 gives the generation cost coefficients. The technique employed in determining these coefficients is described in Appendix C .

TABLE D.2

COST COEFFICIENTS OF GENERATING UNITS

UNIT SIZE	COST COEFFICIENTS		
(MW)	c ($/H)	a ($/MWH)	b ($/MW^2-H)
12	33.9109	21.3943	0.2986
50	66.8300	12.3535	0.0117
76	100.4397	12.1449	0.0226
100	428.3764	9.7177	0.1330
155	539.7767	1.3365	0.0790
197	301.2237	20.0227	0.0060
350	388.2519	8.9196	0.0078
400	358.7356	3.4247	0.0102

The locations of the generating units are given in Table D.3 .

TABLE D.3

GENERATING UNIT LOCATIONS

BUS NO.	UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5
1	50	50			
2	76	76	76	76	
7	100	100	100		
13	197	197	197		
15	12	12	12	12	12
16	155	155			
21	400	400			
23	350	350			

APPENDIX E

GENERATION AND MODIFIED TRANSMISSION DATA FOR THE 10-BUS SYSTEM [VOJDANI 1982]

TABLE E.1

IMPEDANCE AND RATING DATA FOR LINES

FROM BUS	TO BUS	X (PU ON 100 MVA)	RATING (PU MVA ON 100 MVA)
1	2	0.230	0.75
1	3	0.300	0.50
1	5	0.730	0.50
1	9	0.150	0.75
1	10	0.073	1.00
2	3	0.040	1.50
3	4	0.200	1.00
4	5	0.350	0.75
5	6	0.280	0.50
5	7	0.370	0.50
6	7	0.670	0.50
7	8	0.150	0.50
8	9	0.170	0.50
9	10	0.170	0.50

TABLE E.2

COST COEFFICIENTS OF GENERATING UNITS

UNIT TYPE	MIN. GEN. (MW)	MAX. GEN. (MW)	COST COEFFICIENTS a ($/MWH)	b ($/MW^2-H)
A	10	60	3.80	0.040
B	20	100	4.00	0.003
C	5	60	3.90	0.040
D	10	65	3.90	0.040
E	20	50	4.05	0.002
F	10	60	4.33	0.002
G	5	30	3.90	0.074
H	10	60	4.00	0.022

TABLE E.3

GENERATING UNIT LOCATIONS

BUS NO.	UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5	UNIT 6
1	A	A	A	A	A	A
2	B	B	B	B	B	B
3	C	D				
4	E					
6	F	F				
7	G	G	G			
10	H	H				

TABLE E.4

BUS LOAD DATA

BUS NO.	BUS LOAD % OF SYSTEM DEMAND
1	20
2	10
3	15
4	10
5	10
6	5
7	5
8	5
9	10
10	10