RENAISSANCE INTERVAL-SUCCESSION THEORY: TREATISES AND ANALYSIS

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To my mother, Giselle Morgan, for her
unwavering love, support, and encouragement.

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Abstract

Interval-succession treatises convey idiomatic polyphony by explaining what vertical intervals between two voices could follow one another in improvisation and counterpoint, as connected by specified melodic motions. They were the primary means of the transmission of contrapuntal teaching in treatises for centuries. This dissertation is the first to take a comprehensive, computer-assisted approach to the study of these treatises' numerous examples. I focus on the interval-succession lists in book I of Johannes Tinctoris's *Liber de arte contrapuncti* (1477), and in book II of Pietro Pontio's *Ragionamento di musica* (1588). By comparing Tinctoris's list of 768 interval successions to the theoretical list of all the successions that follow his five explicitly-stated voice-leading principles, I refute the commonly-held misconception that Tinctoris's list is exhaustive. In my examination of the 167 interval successions that respect all of Tinctoris's explicit constraints, yet that he omits from his list, I uncover nine tacit voice-leading principles that all of Tinctoris's successions respect despite the fact that he does not explicitly state them.

This study is also the first in-depth examination of the contrapuntal teaching found in Pontio's list of 123 interval successions. I draw attention to the multitude of contextual enrichments that he brings to the theoretical genre, the most significant of which is the inclusion of dissonance directly into interval-succession theory. I expose the tacit theoretical assumptions and analytical methods that are apparent in his interval-succession examples when considered together as a coherent whole. Well-known for the numerous repertoire references that Pontio makes to his own and other composers' works, I show that his approach to interval-succession theory is more akin to teaching through the use of commonplaces than through the memorization of tables of numerous examples. My examination of Tinctoris and Pontio's interval-succession treatises culminates in a comparison of their theoretical and pedagogic approaches. An appendix provides a transcription of all the examples from book II of Pontio's treatise.

I draw on these newly-uncovered facets of Tinctoris and Pontio's intervallic thinking and some important concepts from Nicola Vicentino and Joachim Burmeister's treatises to problematize the discernment of structural tones in a contrapuntal passage. These authors highlight important points to consider, but do not offer a viable approach to counterpoint reduction. I develop Ruth DeFord's (2015) theoretical discussions of her terms "compositional

tactus" and "contrapuntal rhythm" into a historically-informed method of determining the structural tones of Renaissance counterpoint. An automated implementation of this approach, which I call the dynamic-offset method, can be used to reduce a contrapuntal passage, or to more generally characterize its rhythmic profile. It is made freely available online as part of the VIS Framework (Vertical Interval Successions) for music analysis of scores in symbolic notation. This music-analysis framework is a tool open to music researchers looking to make empirical observations about a piece or corpus of music. By taking my research on Renaissance interval-succession treatises into account in the computational tools I have created, my research findings and new analysis methods are made available to other researchers both in written form in my dissertation, and as computational tools in the VIS Framework.

Résumé

Les traités de suites d'intervalles enseignent les idiomes de la polyphonie en expliquant les intervalles pouvant s'enchaîner dans l'improvisation et dans la composition, telles que connectées par des mouvements mélodiques précis. Ces traités ont été, pendant des siècles, le principal moyen de transmettre l'art du contrepoint. Cette thèse est la première à adopter une approche systématique et assistée par l'ordinateur pour l'analyse des nombreux exemples de ces traités. Mon étude porte spécifiquement sur les listes de suites d'intervalles incluses dans le premier livre du *Liber de arte contrapuncti* (1477) de Johannes Tinctoris, et dans le deuxième livre de *Ragionamento di musica* (1588) de Pietro Pontio. En comparant la liste des 768 suites d'intervalles de Tinctoris à la liste théorique de toutes les suites qui respectent ses cinq principes de maniement de voix explicitement développés dans son traité, je réfute l'idée fausse mais courante que son inventaire est exhaustif. Dans mon analyse des 167 enchaînements d'intervalles qui se conforment à toutes les contraintes explicites de Tinctoris mais qu'il a omis de sa liste, j'ai découvert dix principes tacites de maniement des voix que toutes ses suites d'intervalles respectent, bien qu'il ne les mentionne pas explicitement.

Cette étude constitue le premier examen en profondeur de l'enseignement pédagogique fait à partir de la liste des 123 suites d'intervalles identifiées par Pontio. Je souligne la multitude d'enrichissements contextuels qu'il apporte au genre théorique, le plus important étant l'inclusion de la dissonance directement dans sa théorie des suites d'intervalles. J'expose les suppositions tacites de son approche et de ses méthodes d'analyse qui sont apparentes dans ses exemples de suites intervalliques lorsque considérées comme un tout cohérent. Bien connu pour les nombreuses références à ses propres œuvres et à celles d'autres compositeurs, je montre que l'approche de Pontio concernant la théorie des suites d'intervalles est plus semblable à l'enseignement avec des lieux communs qu'à celui fondé dans la mémorisation de tables de nombreux exemples. Mon examen des traités de suites d'intervalles de Tinctoris et Pontio aboutit ainsi à une comparaison de leurs approches théoriques et pédagogiques. Une annexe fournit une transcription de tous les exemples du deuxième livre du traité de Pontio.

Je puise dans ces facettes nouvellement découvertes de la pensée intervallique de Tinctoris et Pontio et dans les traités de Nicola Vicentino et Joachim Burmeister afin de problématiser le discernement des notes structurales dans un passage de contrepoint. Ces auteurs

soulignent beaucoup d'éléments importants à considérer, sans toutefois offrir une approche viable pour la réduction contrapuntique. Je développe la discussion théorique de Ruth DeFord (2015) à propos de ses termes « *tactus* de composition » et « rythme contrapuntique » en utilisant une méthode historiquement informée qui détermine les notes structurales dans le contrepoint de la Renaissance. Une mise en œuvre automatisée de cette approche, que j'appelle le « dynamic-offset method », peut être employée pour réduire un passage contrapuntique, ou pour caractériser son profil rythmique de façon plus générale. Cet outil est librement disponible en tant que partie du VIS Framework pour l'analyse musicale d'œuvres en notation symbolique. Ce logiciel d'analyse est mis à la disposition des chercheurs en musique qui souhaitent faire des observations empiriques à propos d'un morceau ou d'un corpus. En prenant en compte ma recherche sur les traités de suites d'intervalles de la Renaissance dans la création de ces outils d'analyse, mes conclusions et nouvelles méthodes d'analyses sont disponibles aux autres chercheurs dans cette thèse, et aussi comme outils numériques dans le VIS Framework.

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My time spent in Montreal has been an overwhelmingly positive experience, both academically and personally. From day one I have been fortunate to be surrounded by many outstanding people, some of whom I would like to thank at present. To begin, my advisors Peter Schubert and Julie Cumming have had a transformative influence on me as a music scholar. Peter's frank and honest approach has helped me be more critical of my own ideas and more open to those of others. Julie's steadfast work ethic and unparalleled facility in clearly expressing complex ideas have encouraged me to strive to reach my potential. So fortunate am I that Peter and Julie are not the first exceptional music teachers who had a profound impact on me as a student. I must also thank Severine Neff, Luc Marty, Bernard Maurin, and Catherine Losada, for helping me grow into the person I am today.

Many other students at McGill also had a strongly positive impact on my studies. Julie and Peter run a "musicology lab" that meets regularly to discuss a wide range of topics and critique one another's run-throughs of presentations. Professor Ichiro Fujinaga has similar lab meetings, and I was very fortunate to be able to be invited to these as well, despite not formally being Ichiro's student. Between these two groups, I regularly engaged with other scholars from music theory, musicology, music technology, and computer science. Learning to follow and share ideas with others both within and outside of my domain has been one of the most important conceptual lessons I learned during my degree.

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As my dissertation makes use of the VIS-Framework, I would like to thank some of the main programmers on the project, namely Christopher Antila, Jamie Klassen, Marina Borsodi,

and Ryan Bannon. Michael Cuthbert's software contributions via music21 also greatly facilitated our progress.

Before even starting my coursework at McGill, I had the good fortune to work side-by-side with Christopher Antila on the ELVIS project. I quickly found myself in a role for which I had no experience, as a computer programmer. In a show of saintly patience, Christopher took me under his wing and beyond just teaching me how to code, he taught me to think computationally. In fact, he is the one who showed me how to write the script that generates the list of all the interval successions that follow Tinctoris's explicitly stated principles, and this was the starting point of what eventually developed into this dissertation.

Alessandra Ignesti helped me work through a number of challenging Italian and Latin sources. In addition to this invaluable linguistic help, she was also a great reference concerning medieval and Renaissance treatises so I have much to thank her for. Similarly, Marie-Ève Piché has helped me with my French on more than one occasion, and her diligence has been inspiring.

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Chapter 1

Introduction and Literature Review

"… And to tell you the truth, the difficulty of improvisation and composition, and their beauty, consists solely in knowing how best to accommodate the aforementioned consonances and dissonances in their proper place."[1] This is a preliminary bit of advice the master Don Paolo gives to the student Don Hettore in Pietro Pontio's treatise set as a dialogue, *Ragionamento di musica* (*RM*, 1588), before embarking on the interval-succession list that is the focus of book II. By the end of the 16th century, interval-succession lists were a long-established means of conveying idiomatic counterpoint and voice leading. Replete with theoretical premises as well as pedagogical advice, interval-succession treatises offer a penetrating window into the musical thinking and improvisational and compositional craft of Renaissance musicians. I examine Tinctoris's interval-succession list in book I of the *Liber de arte contrapuncti* (*LAC*, 1477) and Pontio's in book II of *RM* with the explicit intention of demonstrating that interval succession theory continued well into the Renaissance and that it stayed current by constantly adapting to new developments in composition and improvisation. I chose these two treatises because they were both important for the theoretical genre, and considerably different in their approaches. I look at the interval-succession lists of Tinctoris (chapter 2) and Pontio (chapter 3), first alone to see what each theorist's set of examples reveals about his intervallic thinking by querying those examples as a coherent corpus. Then I compare the theoretical and pedagogical approaches of the two authors in a side-by-side comparison which is also in chapter 3. Surprisingly little literature exists on interval-succession theory after Tinctoris, which is one of the reasons I find it important to examine Pontio's treatise. Drawing on these and other primary sources, in chapter 4 I problematize the analytical process of identifying the structural tones of Renaissance counterpoint, showing that while the primary sources highlight important points that must be considered, they offer unviable approaches to reduction. For this reason I propose a new analytical method which can be used to

[1] "& per dirvi il vero, la difficultà del contrapunto, & della compositione, & la sua bellezza solo consiste in saper bene, & con bel modo accommodare nel suo proprio luogo le dette consonantie, & dissonantie." Pietro Pontio, *Ragionamento di musica* (1588), 24, accessed from: Christophe Dupraz, *Traités Musicaux Romans* (www.tremir.fr), 2013, specifically: http://www.ums3323.paris-sorbonne.fr/TREMIR/TReMiR_Pontio/R0_start.htm.

reduce Renaissance counterpoint to its structural tones. I demonstrate how this is crucial to achieving more accurate depictions of contrapuntal passages in n-gram analyses in both small and large scopes. I also show how this method can be used to describe the rhythmic profile of the structural tones of a passage of counterpoint, thereby adding a new dimension to contrapuntal analysis. In this introduction, I will explain special terminology and refer to the secondary literature as necessary.

Primary Sources

Albert Seay's modern Latin edition and English translation of the *LAC* have undoubtedly contributed to the abundance of research on it by making this important reference more accessible.[2] More recently Jeffrey Dean has been working on a revised authoritative edition of the *LAC* available online. Dean's version allows for the side-by-side comparison of the manuscripts as well as an edited Latin version and an English translation.[3] This edition is a well-developed facet of the larger Early Music Theory project.[4] At present, only book I of the *LAC* is available on this website. Therefore, my citations of book I (the vast majority of my references to the *LAC*) refer to Jeffrey Dean's online versions of the text and any citations of books II and III refer to Seay's editions.

Pontio's *RM* is also a central text for this dissertation and I consult two versions of the treatise. The first is the facsimile compiled by Suxanne Clercx.[5] For ease of reading and study, I also refer to the electronic version available on Christophe Durpaz's TRéMiR website which is of great help in locating the compositions cited in *RM* and also makes the text easier to read (though it is still in Italian), especially for a non-native reader of Italian such as myself.[6] While I consulted both versions, unless otherwise noted, all of my references are to the online version. When confronted with antiquated Italian words with which I am not familiar and which are not found in modern dictionaries, I searched for them in a primary-source Italian-to-English dictionary from

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[2] Johannes Tinctoris, *Liber de arte contrapuncti*, in *Opera Theoretica*, ed. Albert Seay (np: American Institute of Musicology, 1975), vol. 2; Johannes Tinctoris, *Liber de arte contrapuncti*, trans. Albert Seay (Rome: American Institute of Musicology, 1961).
[3] Johannes Tinctoris, prologue to *Liber de arte contrapuncti*, trans. Jeffrey Dean, http://earlymusictheory.org/Tinctoris/texts/deartecontrapuncti/.
[4] See: http://earlymusictheory.org/.
[5] Pietro Pontio, *Ragionamento di musica*, facs. comp. Suzanne Clercx (1588; Kassel: Bärenreiter, 1959).
[6] Pietro Pontio, *Ragionamento di musica*, available on: Christophe Dupraz, *Traités Musicaux Romans* (www.tremir.fr, 2013), http://www.ums3323.paris-sorbonne.fr/TREMIR/TReMiR_Pontio/R0_start.htm.

1611.[7] When these sources were not enough for me to understand the Italian text I was very fortunate to receive continuing help from my colleague at McGill Alessandra Ignesti, for which I am extremely grateful.

Interval-Succession Theory

This study takes as its point of departure Sarah Fuller's chapter on the theoretical traditions known as *organum*, *gradi*, *intervalschritt lehre*, *discantus*, and *contrapunctus*.[8] Insofar as they all describe ways of improvising and/or composing idiomatic polyphony, they are all part of a greater theoretical tradition which I refer to as interval-succession theory. While the treatises considered here would generally be labelled as members of the *contrapunctus* branch, the distinctions between the branches of interval-succession theory have more to do with the repertoire to which any given treatise pertains than with the fundamental theoretical approach of the author. Klaus-Jürgen Sachs proposed an evolution of mostly 14th-century treatises belonging to the discant branch giving way to mostly 15th-century *contrapunctus* treatises. However, Felix Diergarten has recently argued that Sachs was only able to depict this neat teleological development by citing the treatises in a chronological order which has since been seriously revised such that Sachs's assertion has been undermined.[9] This suggests that any distinctions between these branches of interval-succession theory are subtle and that these branches' lineages are intertwined. Therefore in referring to interval-succession theory rather than more specifically to *contrapunctus* theory, I deliberately imply that many of the observations I make about Renaissance treatises may well also apply to treatises that came before and after this period, though an in-depth discussion of all interval-succession treatises is beyond the scope of this study.[10] Peter Schubert has shown that many

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[7] John Florio, *Queen Anna's New World of Words, or Dictionarie of the Italian and English Tongues*, comp. online Greg Lindahl (London: Melch Bradwood, 1611), available at: http://www.pbm.com/~lindahl/florio/.

[8] Sarah Fuller, "Organum – discantus – contrapunctus in the Middle Ages," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 477–502.

[9] Klaus-Jürgen Sachs, *Der Contrapunctus im 14. und 15. Jahrhundert: Untersuchungen zum Terminus, zur Lehre und zu den Quellen*, (Wiesbaden: Steiner, 1974); and Felix Diergarten, "Beyond Contrapunctus. On a Hypothesis by Hugo Riemann and Klaus-Jürgen Sachs," (paper presented at the Medieval and Renaissance Music Conference [MedRen], Brussels, Belgium, July 6–9, 2015.) Available on academia.edu at: https://www.academia.edu/14084984/Beyond_contrapunctus._On_a_hypothesis_by_Hugo_Riemann_and_Klaus-J%C3%BCrgen_Sachs (accessed November 8, 2015).

[10] *Tractatus* is an example of an interval-succession treatise that comes after 1600. Christoph Bernhard, *The Tractatus* (late 1650's), trans. Walter Hilse in *The Music Forum*, ed. William Mitchell and Felix Salzer (New York: Columbia University Press, 1973), vol. III, 35–61.

Renaissance treatises use the word "counterpoint" to refer not to a musical texture or abstract concept, but rather to a practice modern musicians would generally label as improvisation.[11] In this dissertation, unless otherwise noted, I use the term "counterpoint" in the modern sense and so I translate primary-source uses of the word counterpoint as improvisation.

Fuller's main point in her aforementioned chapter was to stress the fact that as even as new approaches to polyphony were described and taught in treatises, older methods (explanations of which were typically not included in later treatises) continued to be used in practice. This is a valuable observation concerning the reality of polyphonic practice and experience for medieval and Renaissance musicians. Tinctoris's famous quote "Nor, which I cannot wonder at enough, does there exist any composition except this side of forty years that is considered worth hearing by the learned," seems to agree with Fuller's point in that Tinctoris was aware of polyphony from previous generations from first-hand experience, however he regrets that these older styles of music persist.[12] Similarly to what I do in chapter 4, in a different study Fuller has compared different interval-succession treatises in order to extract their theoretical stances on dissonance treatment.[13] Fuller's study, however, examined 14th-century treatises whereas the primary sources I work with are from the 15th and 16th centuries or the very beginning of the 17th century.

As Fuller's work mentioned above examines treatises and polyphonic practice spanning some 600 years we can hardly critique her for not having considered any after Tinctoris. Bonnie Blackburn's chapter on music theory after 1450 considers the *LAC* and treatises written after it, however, her focus is on the rapidly changing face of music theory in the decades around 1500. She puts the work and ideas of the most important theorists of this time in context of how they were interacting and responding to one another, as well as to the other arts. Blackburn's broader examination of all aspects of music theory in surviving manuscripts informs the present study with its more limited focus on interval-succession theory; her summaries of Tinctoris's stances on

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[11] Peter Schubert, "Counterpoint Pedagogy in the Renaissance," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen, (Cambridge, UK: Cambridge University Press, 2002), 503-33.
[12] Tinctoris, prologue to *Contrapuncti*, http://earlymusictheory.org/Tinctoris/texts/deartecontrapuncti/.
[13] Sarah Fuller, "Contrapunctus Theory, Dissonance Regulation, and French Polyphony of the Fourteenth Century," in *Medieval Music in Practice: Studies in Honor of Richard Crocker*, ed. Judith Peraino (Middleton, Wisconsin: American Institute of Musicology, 2013), 113–52.

proportions and speculative theory are particularly relevant to chapter 4 where I draw on this knowledge in the creation of a new analysis method.[14]

Secondary Literature on Tinctoris

The *summa* that Tinctoris's combined theoretical writings represent has received considerable scholarly attention. Alexander Blachly explains that modern scholars revere him to a fault, commenting that he is "…probably the most highly esteemed theorist of the fifteenth century… but so great is his authority that modern misapprehensions concerning Tinctoris's passing observations on mensuration and tempo have led to a distorted view of fifteenth-century mensural relationships generally."[15] The wealth of scholarly attention given to Tinctoris's writings with respect to mensuration, mode, notation, pedagogy, or the general atmosphere of 15th-century music and music theory notwithstanding, less attention has been devoted to the theoretical implications of Tinctoris's examples and approach to interval-succession theory.[16] Margaret Bent's research is a notable exception. Much of her scholarship deals directly with the theoretical and practical (especially concerning accidentals) implications of Tinctoris's treatises, most notably the *LAC*.[17] Bent's focus has often been on providing a more nuanced interpretation of Tinctoris's explicit, though sometimes inscrutable or even contradictory, statements. My examination of book

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[14] Bonnie Blackburn, "Music Theory and Musical Thinking after 1450," in *Music as Concept and Practice in the Late Middle Ages*, ed. Reinhard Strohm and Bonnie Blackburn (New York: Oxford University Press, 2001), 325-33.

[15] Alexander Blachly, "Reading Tinctoris for Guidance on Tempo," in *Antoine Busnoys: Method, Meaning, and Context in Late Medieval Music* (Oxford: Clarendon Press, 1999), 399–427.

[16] For studies on mensuration see especially: Anna Maria Busse Berger, *Mensuration and Proportion Signs* (Oxford: Clarendon Press, 1993); Blachly, "Guidance on Tempo;" Rob Wegman, "What is 'Acceleratio mensurae'?", *Music and Letters*, 73 (1992), 515–24; and Eunice Schroeder, "Dissonance Placement and Stylistic Change in the Fifteenth Century: Tinctoris's Rules and Dufay's Practice," *The Journal of Musicology*, 7/3 (Summer, 1989), 366–89. On mode see: Jeffrey Dean, "Okeghem's Attitude towards Modality: Three-Mode and Eight-Mode Typologies," in *Modality in the Music of the Fourteenth and Fifteenth Centuries*, ed. Ursula Günther, Ludwig Finscher, and Jeffrey Dean (Neuhausen-Stuttgart: American Institute of Musicology: Hänssler-Verlag, 1996), 203–46. On notation see: Ron Woodley, "Minor Coloration Revisited: Okeghem's *Ma bouche rit* and Beyond," in *Théorie et analyse musicales (1450-1650) Proceedings of the International Conference, Louvain-la-Neuve*, ed. A. E. Ceulemans and Bonnie Blackburn (2001), 39–63; and Christian Goursaud, "The Neapolitan Presentation Manuscripts of Tinctoris's Music Theory: Valencia 835 and Bologna 2573," (PhD diss., Birmingham City University, 2015).

[17] See especially: Margaret Bent, "*Musica Recta* and *Musica Ficta*," "Renaissance Counterpoint and *Musica Ficta*," "Diatonic *Ficta*," and "*Resfacta* and *Cantare Super Librum*," all in *Counterpoint, Composition, and* Musica Ficta (New York: Routledge, 2002) 61–94, 105–14, 115–60, and 301–319; and "On False Concords in Late Fifteenth-Century Music: Yet Another Look at Tinctoris," in *Théorie et analyse musicales (1450-1650) Proceedings of the International Conference, Louvain-la-Neuve*, ed. A. E. Ceulemans and Bonnie Blackburn (2001), 65–118.

I of the *LAC* adds to Bent's scholarship on Tinctoris by interpreting the tacit tenets of his intervallic thinking that exert their influence on his numerous examples.

Similar to my research, Adam Whitaker has focused on the examples found in treatises to discern how they serve the expressions of their author's ideas.[18] While Whittaker does take an example-driven approach, as I do, he generally only considers one or a very small number of examples at a time, and so stops short of querying Tinctoris's set of examples as a corpus communicating certain facets of Tinctoris's intervallic thinking as a whole. The systematic approach I take to studying the consonant interval successions in book I of the *LAC* is my greatest and most novel contribution to scholarship on Tinctoris and on interval-succession theory in general. This approach demonstrates how an interval-succession treatise can be interpreted as an author's comprehensive stance on valid contrapuntal successions.

An important concept in interval-succession theory is that of the referential voice. In an interval succession, the referential voice is that against which the vertical intervals are measured. Tinctoris adopts the standard of measuring intervals against the pre-existing tenor. Pontio takes a different approach and generally measures them against the lowest-sounding voice. This means that Pontio never has to deal with situations (as Tinctoris routinely does) where his referential voice is actually the higher voice.

Related to the concept of the referential voice is the term fundamental counterpoint. I use this term to refer to the structural notes in a passage which I deduce by removing non-structural dissonances and consonant ornaments via a method I explain in detail in chapter 4. There are two situations in which the concept of fundamental counterpoint is pertinent in this dissertation. The first is with respect to octave equivalency and the position of the referential voice. For the purposes of this study, I consider successions that are octave-equivalent, such as those in Figure 1, to be different instances of the same fundamental counterpoint.

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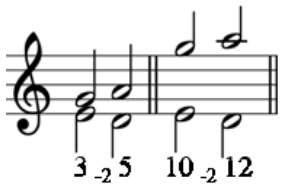
[18] Most notably: Adam Whittaker, "Thoughts on the integration of musical examples in Johannes Tinctoris's *Expositio manus* and *Liber de arte contrapuncti*," (paper presented at the Medieval and Renaissance Music Conference, Birmingham, England, July 3–6, 2014); and "Exemplifying Imperfection and Alteration in Fifteenth-Century Theory: A Comparison of the Approaches of Johannes Tinctoris and Franchino Gaforus," (paper presented at the Medieval and Renaissance Music Conference, Sheffield, England, July 5–8, 2016).

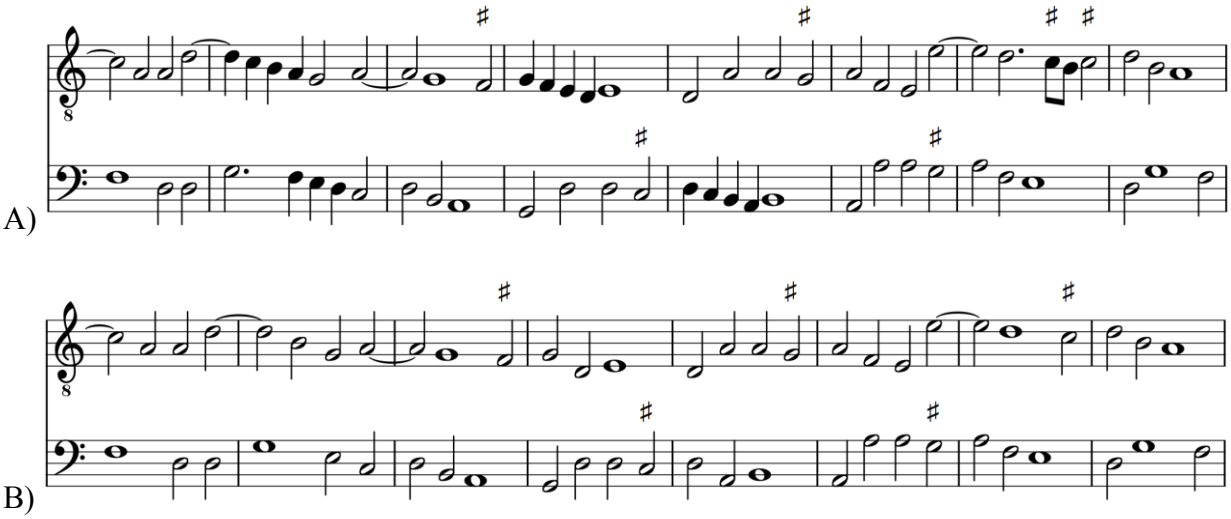
Figure 1: Octave-equivalent interval successions. The numerical notation below the example will be explained on page 17.



Simplified to the octave, the 10 -2 12 interval succession in Figure 1 would be the same as the first, 3 -2 5. Note that I do not extend this relation to successions related by invertible counterpoint at the octave. If we change the names of the two voices (i.e. the tenor can be the lower or the higher voice), but none of the actual melodic lines change, I also consider the fundamental counterpoint to be the same.

The second situation where the term fundamental counterpoint comes into play is in the reduction of a passage. Reduction is important because it is essential to many types of repertoire corpus studies. This reductive process relies on the application of highly objective dissonance types identified in a two-voice contrapuntal model. Figure 2-A has a number of consonant and dissonant ornaments and when we reduce them out, as in Figure 2-B, we are left with the fundamental counterpoint of the passage.[19]

Figure 2: Josquin, Crucifixus, NJE [13.12], mm. 6-13 in A and a reduction to its fundamental counterpoint in B.



[19] Josquin, Crucifixus, NJE [13.12], http://josquin.stanford.edu/work/?id=Jos1312.

This fundamental counterpoint can still have dissonances because my reductive method assesses suspensions as structural, so they are not reduced away. I borrow this approach from Renaissance counterpoint treatises; in *RM*, for example, thirty-six of the 123 interval-succession examples include dissonant intervals that arise from suspensions so suspensions are clearly more structural than the several semiminim passing tones that get completely ignored in Pontio's text descriptions. The reduction above samples the counterpoint at regular minims, except for the accented semiminim dissonances in bars 9 and 10 where I reduce the passage to the notes on the weak semiminims. If instead we sample the counterpoint of Figure 2 at every new note, our resultant analysis would not correspond to the musical syntax because ornaments would be represented as being on the same hierarchical level as the structural tones in the fundamental counterpoint. The reduction in Figure 2-B avoids such errors.

These two levels of abstraction with respect to fundamental counterpoint are motivated by interval-succession treatises. In Pontio's treatise in particular it is apparent that he expects the reader to be able to recognize the basic form of his successions irrespective of any octave doublings, voice crossings, or ornamentation with consonant or dissonant non-structural tones. In Tinctoris's interval-succession list, he uses vertical intervals within a maximum range of three octaves instead of simplifying them to within one and introduces each succession twice to allow for the tenor to be the lower voice or the upper voice. Because of these decisions, he routinely gives six different instances of what I consider to be the same fundamental counterpoint. By examining the discussion or exclusion of the six instances of the same fundamental counterpoint, I reveal nine tacit voice-leading principles that Tinctoris systematically followed despite not writing them out in prose.

With a definition of fundamental counterpoint in place, we can now consider two repertoire studies related to Tinctoris's compositions and theoretical writings. William Melin's dissertation provides a modern edition of Tinctoris's compositions and insightful commentary and observations on the mensurations he uses, his general musical style, and his dissonance treatment.[20] Melin provides the results of some basic queries about selected "representative" movements or parts of movements. The main issues with Melin's study are that he only counts

[20] William Melin, "The Music of Johannes Tinctoris (ca. 1435 –1511): A Comparative Study of Theory and Practice" (PhD diss., Ohio State University, 1973).

direct melodic motions (i.e. no vertical intervals or contrapuntal patterns are counted), and he queries the surface layer of the music instead of analyzing the fundamental-counterpoint reduction.

Lee Rothfarb contributes a case study wherein he identifies instances of dissonance types as they were described by Tinctoris in book II of the *LAC*.[21] While Rothfarb concentrated on Tinctoris's descriptions of dissonance treatment, chapter 2 of this dissertation focuses on his interval successions which are all consonant. Whereas Rothfarb sought to reconcile discrepancies between Tinctoris's prose descriptions of dissonance treatment, and observed dissonance treatment in the work of Tinctoris and his contemporaries, I work with Tinctoris's treatise examples by themselves, without seeking to justify or refute any interval successions based on a repertoire study. Such a corpus study comparing Tinctoris's consonant interval-succession descriptions to their observed use and frequency in the repertoire would be a valuable way of building on Rothfarb and my research. In this respect, Rothfarb's study wisely focused on dissonance, because Tinctoris is more explicit about the rhythmic and durational aspects of their use. In fact, in this volume I had originally planned to include a study counting the occurrences of Tinctoris's consonant interval successions in the repertoire, but after making some progress I found that it was first necessary to ascertain at what metric level Tinctoris's consonant interval successions occur. In order to progress towards this eventual goal of being able to reliably identify fundamental-counterpoint progressions, I propose an analytical method that can be used for reduction in chapter 4. I will return to a more involved discussion of this method shortly.

While Tinctoris's treatises are among the most frequently studied theoretical writings on music from the Renaissance, most scholars that reference his interval-succession list in book I of the *LAC* limit themselves to the recognition of how thorough it is, and to a citation of it as the height of the interval-succession theory tradition. It has been called "an exhaustive inventory" (Fuller) of "*all* possible interval progressions" (Busse Berger, emphasis hers), but in chapter 2 I show that this common assertion is incorrect.[22] Tinctoris is very clear about five voice-leading

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[21] Lee Rothfarb, "Tinctoris vs. Tinctoris: Theory and Practice of Dissonance in Counterpoint," *In Theory Only*, 9/2 (Ann Arbor: Michigan Music Thoery Society, 1986), 3–32.

[22] Fuller, "Organum," 498; Anna Maria Busse Berger, *Medieval Music and the Art of Memory* (Los Angeles: University of California Press, 2005), 141; and more recently: Anna Maria Busse Berger, "Oral Composition in Fifteenth-Century Music," in *The Cambridge History of Fifteenth-Century Music*, ed. Anna Maria Busse Berger and Jesse Rodin (Cambridge: Cambridge University Press, 2015), 140. See also Knud Jeppesen, *Counterpoint: The Polyphonic Vocal Style of the Sixteenth Century*, trans. Glen Haydon (New York: Dover Publications, 1992), 10.

principles that guide the creation of his list of 768 of his interval successions. But if we generate the set of all interval successions that respect these principles we get a list of 935 successions. This corrects the common misconception that his list is exhaustive as an additional 167 were logically expected, but omitted. As mentioned above, I compare these omitted successions by considering them to be different manifestations of the same fundamental counterpoint if they are: 1) octave-equivalent interval successions; and/or 2) successions with the same voice leading, but whose parts have swapped their "tenor" and "contrapunctus" labels. Given that Tinctoris names interval successions in a range of three octaves, and with two possible positions for the tenor, there are six instances of the same fundamental counterpoint that Tinctoris can incorporate in his list. When he exhibits strong consistency across all six of these successions (either in their discussion, or their exclusion from the list) in a way that goes beyond his five explicitly-stated principles structuring his list, I deduce tacit principles that he systematically respects. By means of an examination of Tinctoris's examples, my study uncovers nine tacit (i.e. unwritten) principles all of his interval successions follow, despite the fact that he does not discuss them in prose.

Rules vs. Principles

Short lists of general principles were common in Renaissance counterpoint treatises. Tinctoris's eight general principles for composition which make up book III of the *LAC* are representative exemplars. An example of a general principle is the common restriction on parallel perfect intervals. A general principle does not apply to a single specific contrapuntal situation, but rather to a group of related situations. The relationship between this principle and the specific interval successions to which it applies is one-to-many; there is one general principle discouraging the use of parallel perfect intervals, but there are many possible interval successions that constitute such forbidden parallels.

In contrast to general principles, I borrow Anna Maria Busse Berger's term "rule" to refer to a precise interval succession.[23] We are used to thinking of the word "rule" as a synonym for a general principle, but Busse Berger likens them to the medieval traditions of memorization of formulaic tables of mathematical equations or grammatical declensions.[24] When explaining how

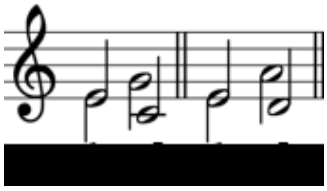
[23] Busse Berger, *Art of Memory*, 118–9.
[24] Mary Carruthers, *The Book of Memory: A study in Medieval Culture* (Cambridge: Cambridge University Press, 1990).

to recognize if a counterpoint treatise is in this memorization tradition, Busse Berger explains that: "If the same or similar material is presented again and again with a multitude of rules… we have an organization familiar to us from the grammar and abacus texts and can assume that the text and the music examples were learned by heart."[25] An apt example can be found in Ugolino's description of how one can go from a unison to a fifth. Given its position in Ugolino's list of successions, the reader already knows that the first vertical interval is a unison and the pre-existing tenor will ascend from there:

Si tertia vel quarta tendit,	If [the tenor] goes up a third or a fourth,
infra diapente tenebit.	take the fifth below.[26]

This rhymed couplet actually conveys two "rules", possible realizations of which are provided in Figure 3.

Figure 3: Possible pitch-specific realizations of Ugolino's abstract rules stated above. The tenor's notes all have downward-pointing stems.



The idea is that, having memorized these rules ahead of time, the student will see the tenor moving from the unison up a third from E4 to G4 (as in the first succession in Figure 3) and sound a fifth below this G4 by moving down a third from E4 to C4.

As Peter Schubert and Busse Berger have pointed out, Ugolino's interval successions are particularly good examples of rules because the fact that they are presented as rhymed couplets supports the idea that they were meant for memorization.[27] The point of a rule, in this sense, is to provide a clear and precise written or improvised contrapuntal reaction to a stimulus. That stimulus usually takes the form of given starting and ending vertical intervals and a given melodic motion in the pre-existing voice. A musician who has appropriately memorized enough rules is supposed

[25] Busse Berger, *Art of Memory*, 118–9.
[26] Ugolino di Orvieto, *Declaratio musicae disciplinae*, in *Corpus scriptorium de musica*, comp. Albert Seay (Rome: American Institute of Musicology, 1959), no. 7, vol. 2, bk. II chap. 26, 32.
[27] Schubert, "Counterpoint Pedagogy," 506; Busse Berger, *Art of Memory*, 139.

to be able to recognize these contrapuntal stimuli, and recall and execute (i.e. sing or write) the response in real time. Principles can also be memorized but the task is quite different from that of memorizing rules. Since a rule is in a one-to-one relationship to a specific desired voice leading, one memorizes it to recall it as a reflex. Principles, however, are general and slightly more abstract and thereby demand more reflection to be properly applied. While rules as specific examples often travel in large packs, each principle covers multiple contrapuntal scenarios so they are often presented in a short lists.

Busse Berger gives three characteristics of interval-succession treatises that suggest alignment with the rule-based tradition of memorization: 1) a multitude of similar rules instead of general principles; 2) versification (as with Ugolino above); 3) the tabular and well organized graphic layout of information. While I agree that this the first and third of these characteristics aptly describe Tinctoris's list, none of them are pertinent to Pontio's which is detailed, not methodical, and accompanied by comparatively verbose text descriptions.

Secondary Literature on Pontio

The dearth of scholarly attention to Pontio is not confined to his take on interval-succession theory. In speaking of Pontio's treatises and music, Russell Murray noted that "Little has been written about either."[28] The most significant study on Pontio is Russell Murray's dissertation which is devoted to a presentation and comparison of his compositions and two treatises, *RM* and the *Dialogo* (1595). In it, Murray provides a valuable appendix of Pontio's works, which is the only readily-available source for these scores.

What literature there is on Pontio's treatises tends to focus on *RM* and specifically on his advice to composers; Klaus-Jürgen Sachs, Harold Powers, and James Armstrong have studied *RM* from this angle.[29] Similarly, Peter Schubert has also examined *RM* in order to ascertain the improvisatory techniques one can learn from it, and to clarify the distinction that Pontio makes

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[28] Russell Murray, "The Voice of the Composer: Theory and Practice in the Works of Pietro Pontio" (PhD diss., University of North Texas, 1989), 155.

[29] Klaus-Jürgen Sachs, "Musikalische 'Struktur' im Spiegel der Kompositionslehre von Pietro Pontios *Ragionamento di musica* (1588)," *Zeichen und Struktur in der Musik der Renaissance* (New York: Bärenreiter, 1989), 141–57; James Armstrong, "How to compose a Psalm: Ponzio and Cerone compared," *Studi musicali* 7 (1978), 103–39. Powers's study in particular focused on Pontio's modal analysis: Harold Powers, "From Psalmody to Tonality," in *Tonal Structures in Early Music*, ed. Cristle Collins Judd (New York: Garland Publishing, 1998), 281–301.

between improvisation and composition.[30] In a separate study Sachs compared theory and practice though he focused on Pontio's second treatise, the *Dialogo*.[31] All of these studies, in addition to Murray's dissertation, share the common theme of comparing Pontio's theory and practice. This is understandable given the size of his compositional output and the clarity of presentation in his theoretical writings.

My study contributes to the literature on Pontio's underexplored theoretical writings by offering the first in-depth examination of his interval-succession examples. These examples are extremely rich in musical context and details including considerations for duration, metric position, affect, genre, texture, etc. His list of 123 examples is founded on different structuring principles than those of Tinctoris, such that the same analytical techniques I used to examine the latter's list cannot be applied to that of the former. Instead, in studying Pontio's examples I focus on defining the theoretical and pedagogical assumptions they rely on. In comparing Pontio's text descriptions to his two-voice examples, I confirm that he measures vertical and melodic intervals at on-beat, non-syncopated minims and reduces away any shorter intervening notes. Pontio's views on adding accidentals are also gathered by considering together the relevant successions that are scattered throughout his list.

The one scholar to have briefly touched on this topic is Russell Murray. He portrays the aforementioned abundance of musical context and details in Pontio's interval-succession examples as "superfluous."[32] The present study, therefore offers an alternative reading to Pontio's approach to interval-succession theory by demonstrating the utility of all the context and details that he adds. My valorization of these added elements is based in large part on my interpretation of Pontio's pedagogical approach, the subject to which I will now turn.

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[30] Peter Schubert, "From Improvisation to Composition: Three 16th-century Case Studies," in *Improvising Early Music* ed. by Dirk Moelants (Leuven: Leuven University Press, 2014) 93–130.
[31] Klaus-Jürgen Sachs, "'*Theorica e Prattica di Musica*' in Pietro Pontios *Dialogo (Parma: 1595)*," *Musiktheorie* vol. 4 (Laaber: Laaber-Verlag, 1989), 127–41.
[32] Russell Murray, "Zacconi as Teacher: A Pedagogical Style in Words and Deeds," in *Music Education in the Middle Ages and the Renaissance*, ed. Russell Murray, Susan Forscher Weiss, and Cynthia Cyrus (Indianapolis: Indiana University Press, 2010), 308.

Renaissance Pedagogical Models

I contrast the pedagogical approaches of Tinctoris and Pontio by unpacking the way they present interval-succession examples. In chapter 3, I characterize Tinctoris's list as corresponding more closely to what Busse Berger portrays as a tradition founded on a rule-based model of memorization.[33] While she suggests that memorization was the cornerstone of interval-succession pedagogy, she points out that Leo Treitler and Steven Immel came to opposing conclusions about the use of memorization in conjunction with the Vatican Organum Treatise.[34] Whatever the role of memorization was in interval-succession theory, I agree that the *LAC* corresponds to the first and third of Busse Berger's useful criteria to determine if a treatise is in the rule-based memorization tradition.

I liken Pontio's approach, by contrast, to the practice of learning with commonplaces, which are contextualized fragments traditionally collected from literary works and pertaining to a given heading (under which they are stored) that could be recalled and reused.[35] There is some uncertainty about the applicability of the commonplace approach to music learning. While Cristle Collins Judd found that, despite certain similarities, one cannot apply the traditionally literary commonplace approach to music, Peter Schubert has pointed out that two Renaissance treatises explicitly identify themselves in this way: Francisco de Montanos's *Arte de musica teorica y pratica* (1592), and Pedro Cerone's *El melopeo y maestro* (1613).[36] This latter treatise is particularly relevant to the connection I draw between *RM* and the commonplace approach because, as Russell Murray notes, Cerone was particularly indebted to the ideas of Pontio as *El melopeo y maestro* incorporates a paraphrase of almost all of *RM*.[37] I add to this conversation about

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[33] Busse Berger, *Art of Memory*; and Busse Berger, "Oral Composition."
[34] Busse Berger, *Art of Memory*, 121–6; Leo Treitler, "Der Vatikanische Organumtraktat und das Organum von Notre Dame de Paris. Perspektiven der Entwicklung einer schriftlichen Musikkultur in Europa," *Basler Jahrbuch für historische Musikpraxis* 7 (1983), 29–30; Steven Immel, "The Vatican Organum Treatise Re-examined," *Early Music History* 34 (2001): 164–6.
[35] Ann Moss, *Printed Commonplace-Books and the Structuring of Renaissance Thought* (New York: Oxford University Press, 1996).
[36] Cristle Collins Judd, *Reading Renaissance Music Theory: Hearing with the Eyes* (Cambridge: Cambridge University Press, 2000), 175; Peter Schubert, "Musical Commonplaces in the Renaissance," in *Music Education in the Middle Ages and the Renaissance*, ed. Russell Murray, Susan Forscher Weiss, and Cynthia Cyrus (Indianapolis: Indiana University Press, 2010), 162; Francisco de Montanos, *Arte de musica theorica y pratica* (Valladolid: 1592); Pietro Cerone, *El melopeo y maestro* (Naples, 1613; repr. Bologna, 1969).
[37] Russell Murray "Pontio, Pietro," *Grove Music Online* (Oxford University Press, accessed July 21, 2016), http://www.oxfordmusiconline.com/subscriber/article/grove/music/22096.

early-music pedagogy by contrasting the markedly different approaches of two authors writing interval-succession theory treatises. Significantly, this reading of *RM* breaks with the hitherto direct association of interval-succession theory treatises with the rule-based memorization model that Fuller and Busse Berger describe.

Musica Speculativa versus *Musica Practica*

Lawrence Gushee promotes a three-pronged scheme for classifying medieval works of music theory that considers each treatise to be a response to the expectations of an institution or audience, a particular type of music, and intellectual style.[38] While Gushee works to unpack issues of lineage, influence, and borrowing in the medieval period, my study focuses on theoretical works in the Renaissance but benefits from Gushee's suggested classification criteria. Gushee proposed a system of genre classification of theoretical treatises based on three variable: 1) intellectual style, 2) institutional or social function, and 3) the kind of music discussed. With respect to his approach, the present study pertains to questions surrounding the authors' pedagogic approaches and the *LAC* and *RM*'s audiences and their intended manner of use. In this way, my research engages directly with Gushee's first two classification variables.

Complementing Gushee's research, Herlinger argues that a theoretical work should be considered as a member of the *musica practica* or *musica speculativa* traditions based not on its content alone, but rather in the manner of presentation of its information.[39] Bonnie Blackburn has also contributed to this discussion, most notably with respect to the theoretical works of Tinctoris. She broadly states that: "Tinctoris has very little interest in speculative music."[40] This bold statement is largely based on the fact that Tinctoris's treatises collectively address almost all aspects of practical music-making that were commonly studied in his time, from beginners' lessons to more complex subjects. The work of these three scholars demonstrates that the way modern scholars classify and conceive of theoretical works from the past is heavily dependent on the way

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[38] Lawrence Gushee, "Questions of Genre in Medieval Treatises on Music," in *Gattungen der Musik Einzeldarstellungen: Gedenkschrift Leo Schrade*, ed. Wulf Arlt, Ernst Lichtenhahn, and Hans Oesch (Munich: Francke, 1973), 366–7.

[39] Jan Herlinger, "Music Theory of the Fourteenth and Early Fifteenth Centuries," in *Music as Concept and Practice in the Late Middle Ages*, ed. Reinhard Strohm and Bonnie Blackburn (New York: Oxford University Press, 2001), 244–300.

[40] Bonnie Blackburn, "Theory after 1450," 327.

we think those treatises were used. I contribute to this dialogue by re-examining the question of who the intended readers of the *LAC* and *RM* are. By exploring who used these treatises and how, my findings inform our assessment of the theoretical genres to which they belong.

Also on the topic of *musica speculativa*, the lineage of the conceptual status of the unison in the history of Western music theory is a point David Cohen has explored extensively.[41] My study corroborates Cohen's findings by showing that the interval-succession lists of the *LAC* and *RM* afford, in different ways, special treatment to the successions that include a unison, reflective of its special conceptual category.

Origins of this Study

The topic of this dissertation grew naturally out of my work with the ELVIS (Electronic Locator of Vertical Interval Successions) research project at McGill University funded by the Digging into Data Challenge program, 2012-2014; Julie Cumming was the principal investigator.[42] ELVIS continues as the search and analysis axis of SIMSSA (Single Interface for Musical Score Searching and Analysis). SIMSSA is a music-research project funded by the Social Sciences and Humanities Council of Canada at the Schulich School of Music, McGill University and Ichiro Fujinaga is the principal investigator.[43] SIMSSA is composed of two axes, the content axis led by Ichiro Fujinaga, and the search and analysis axis led by Julie Cumming. This dissertation is supported by and directly affiliated with SIMSSA's search and analysis axis. Members of this axis develop and use software tools to analyze music with the goal of identifying and quantifying components of musical style change in scores in symbolic notation. A score in symbolic notation is a computer file that contains the notes, rests, and other elements of a digital encoding of a score. Common file types for scores in symbolic notation include .mid, .mei, .xml, and .krn.

The VIS Framework is the suite of music analysis tools developed by the ELVIS research team.[44] It is built on top of `music21` which is an established and reliable Python library for music

[41] David Cohen, "Metaphysics, Ideology, Discipline: Consonance, Dissonance, and the Foundations of Western Polyphony," *Theoria* 7 (1993): 1–86.
[42] See: http://elvisproject.ca/.
[43] See: https://simssa.ca/.
[44] The VIS Framework is available on github: https://github.com/ELVIS-Project/vis-framework.

analysis developed by Michael Cuthbert at MIT.[45] The VIS Framework allows the user to combine simple analytical steps (such as identifying notes, intervals, and durations) to arrive at more complex queries (such as n-gram tabulation, dissonance classification, and reduction) on a single piece or a corpus of pieces in symbolic notation. VIS stands for Vertical Interval Successions.

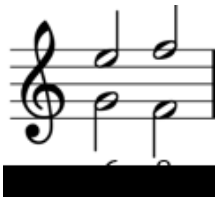
While the repertoire we work with is diverse, many of our tools are specially designed for the analysis of Renaissance music. Tinctoris's interval-succession list was an important model for Cumming and Peter Schubert when they developed a way we identify and label units of counterpoint for the ELVIS project. Christopher Antila and Cumming used these software tools to show how three consecutive generations of Renaissance composers shared certain contrapuntal patterns, which the authors refer to as n-grams, but that there was some variation in the most common patterns from one generation to the next.[46] Improving automated n-gram analysis is one of the main motivations behind the analytical method I present in chapter 4, so now I will define n-grams in more detail.

Interval-Succession N-Grams

An n-gram is a sequence of consecutive analysis events and their use in polyphonic music research has been documented by Shyamala Doraisamy.[47] In the ELVIS research team, we often describe counterpoint with n-grams in two voices that are modeled after interval successions. These interval-succession n-grams describe the counterpoint between two voices by presenting two vertical intervals connected by the melodic motion of the lower voice. The typical voice-leading scenario wherein a sixth expands to an octave by having the lower voice descend and the upper voice ascend by step is encoded as 6 $_{-2}$ 8 (see Figure 4).

[45] Michael Cuthbert, `music21`, [toolkit for computer-aided musicology], http://web.mit.edu/music21/.
[46] Christopher Antila and Julie Cumming, "The VIS Framework: Analyzing Counterpoint in Large Datasets," in *Proceedings of the International Society for Music Information Retrieval*, 2014, 71–76.
[47] Shyamala Doraisamy, "Polyphonic Music Retrieval: The n-gram approach," (PhD diss., University of London, 2004).

Figure 4: A concrete example of the abstract 2-gram 6 $_{-2}$ 8.



We call this representation a 2-gram because it contains a sequence of two vertical intervals.[48] The first and third numbers in the n-gram (6 and 8) correspond to the vertical intervals in the order of their appearance, and the second number in subscript ($_{-2}$) is the melodic motion of the lower voice. The melodic motion of the upper voice is typically not provided because, given the two vertical intervals and the melodic interval of lower voice, we can infer that the upper voice's melodic interval between the two vertical intervals is up a step. More precision can be added to this label if we include interval quality. This is done by prepending the Arabic numbers (which indicate interval quantity) with the standard interval-quality abbreviations of P, M, m, d, and A for perfect, major, minor, diminished, and augmented respectively. With interval quality, the succession in Figure 4 would be M6 $_{-M2}$ P8, though I usually omit quality in my labels because Tinctoris does in his interval successions, and Pontio is somewhat inconsistent with his use of quality. Note that even with the inclusion of interval quality, the label still does not specify particular pitches; it specifies intervallic relations. In this dissertation, I will usually refer to interval successions rather than n-grams in order to more closely match scholarship on the theoretical tradition examined here, interval-succession theory. I make use of the ELVIS team's n-gram annotations wherever I label interval successions. Schubert and Cumming have likened the interval-succession n-gram to the word in literary analysis because it is the smallest unit that conveys syntactical meaning.[49] So can these interval-succession n-grams be equated to the interval successions of Renaissance treatises? I maintain that they can be if a piece is first properly reduced to its fundamental counterpoint.

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[48] Some might object to the use of the term "2-gram" for interval successions because more than two pieces of information are conveyed. Furthermore, the four pieces of encoded information are in two different dimensions, one "vertical" (harmonic) and the other "horizontal" (time/melody). Often in text analysis, a 2-gram is two words that appear one after another in a text. However this is also two-dimensional in that diction is mapped out over time so I find the term to be appropriate for contrapuntal analysis, or at the very least, no more inappropriate than it is for text. For a more in-depth discussion of musical n-grams see: Peter Schubert and Julie Cumming, "Another Lesson from Lassus: using Computers to Analyze Counterpoint," *Early Music* 43.4 (November, 2015): 577-86.

[49] Schubert and Cumming, "Another Lesson," 577–86.

Before problematizing this reduction, I must first introduce my new analytical method, the dynamic-offset method.

Dynamic-Offset Method

Three terms are necessary to the understanding of the dynamic-offset method, the first of which is compositional *tactus*. DeFord differentiates between six uses of the word *tactus*, and in chapter 4 I often make reference to one of these, her compositional *tactus*. She describes it as: "The time unit that serves as a standard of reference for various aspects of rhythm, such as the rate of contrapuntal motion, dissonance treatment, and syncopation, in a composition."[50] Put in terms of fundamental counterpoint, we can think of compositional *tactus* as the first mensural level at which the individual intervals of the fundamental counterpoint group into progressions. Klaus-Jürgen Sachs has equated *tactus* understood this way to Tinctoris's *mensurae directio*, or measuring note.[51] Pontio uses the terms *misura* and *battuta* to refer to the compositional *tactus*.

Contrapuntal rhythm is another term borrowed from DeFord, and it is directly related to her compositional *tactus*. She defines contrapuntal rhythm as "the rhythm of the structural contrapuntal progressions on which a piece is based."[52] It is expressed as a duration, typically the minim, and I abbreviate it "CR". Since the analytical method I present in chapter 4 is currently only suitable for pieces with duple divisions of the compositional *tactus*, the CR in all my analyses is equal to half of the compositional *tactus*. For example, in Figure 2 the compositional *tactus* is the semibreve and the contrapuntal rhythm is the minim.

As is the convention in the VIS Framework, I use the term offset in the same way `music21` does. An offset is a specific time point in a piece in symbolic notation. Each note or rest in a piece is found at a specific offset which is determined by how far that note or rest is set off from the beginning of the piece. The offset is expressed as a number; and the scale evaluates notated quarter notes as 1's, half notes as 2's, eighth notes as .5's, etc. So a note that is ten half notes into a piece is found at offset 20.

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[50] Ruth DeFord, Tactus*, Mensuration, and Rhythm in Renaissance Music* (Cambridge: Cambridge University Press, 2015), 51.
[51] Klaus-Jürgen Sachs, "Counterpoint," *Grove Music Online* (Oxford University Press, accessed July 11, 2016), http://www.oxfordmusiconline.com/subscriber/article/grove/music/06690.
[52] DeFord, Tactus, 84.

The dynamic-offset method is the name of the automated implementation of the analysis method I propose in chapter 4. It examines a score in symbolic notation and pinpoints the offsets at which the fundamental counterpoint is found. This information can be used to reduce a piece to its fundamental counterpoint, or to more generally characterize its contrapuntal character, by which I mean the relative steadiness or variability of its CR. The method is "dynamic" because the offsets it designates as time points at which to sample the counterpoint, are not necessarily regular. The time interval between offsets is determined by an automated analysis of dissonance treatment and attack density.

Problematizing Reduction

I begin chapter 4 by using the theoretical writings of Tinctoris, Nicola Vicentino, Joachim Burmeister, and Pontio to problematize the sampling of counterpoint. Do any of these authors offer a viable method for reducing a passage to its fundamental counterpoint? We need to know when to make analytical observations in a piece in order to find n-grams that we actually consider meaningful. Instead of resolving this issue, I show that a close reading of the four authors above offers no precise working method for intelligently reducing a passage to its fundamental counterpoint. Furthermore, these primary sources demonstrate that neither of the two most common existing automated approaches to sampling counterpoint is completely satisfactory for the analysis of Renaissance polyphony.

Tinctoris and Pontio take opposing and equally unhelpful approaches to reduction. Tinctoris offers no precise information about what mensural level his consonant interval successions correspond to, so one is left wondering what specific durational values to realize them in. Pontio, by contrast, is specific but overly rigid. In all of his interval-succession examples his text descriptions make it clear that he reduces to regular minims. For Pontio the minim is the CR, regardless of the specific musical context. This causes him to analyze passing-tone dissonances a minim in duration in same way he does consonant intervals. This approach makes these passing tones appear structural.

While not offering a concrete solution, Vicentino and Burmeister offer similar examples that demonstrate that the same cadential suspension figures can occur in durationally augmented or diminished forms. Julie Cumming has portrayed the contrapuntal cadence of the Renaissance

as the original contrapuntal module, that is, a contrapuntal pattern that gets repeated.[53] If we want to be able to recognize all the instances of repetition of cadential patterns as well as of other contrapuntal patterns, we need an analytical method that can adjust to counterpoint that advances at different rates. I offer an automated approach called the dynamic-offset method that discerns the CR of a piece based primarily on its dissonance treatment and attack density.

Repertoire Studies

One of the main goals of the dynamic-offset method is to improve the accuracy of contrapuntal sampling in repertoire studies. In this sense my method is basic to repertoire studies, and is therefore only influenced by existing studies to a limited extent.

Michael Cuthbert used n-gram analysis to show that a corpus of 1,000 14th-century monophonic pieces exhibited high amounts of melodic borrowing from one another.[54] As notation of the pieces in Cuthbert's corpus conveyed pitch but not rhythm, the question of how often to sample music was very simple; he looked at every pitch. For studies on polyphonic Renaissance music, however, this method will not work. Cuthbert's study therefore demonstrates that the most appropriate method of analysis depends on one's particular repertoire and analysis question.

Joshua Albrecht and David Huron's 2015 study wherein modes are derived from a pitch count of the first ten and last ten measures of a piece, is well-designed with respect to the statistical methodology employed.[55] However there are three important issues with this Albrecht and Huron's study: 1) no consideration for the harmonic or melodic aspects of counterpoint are taken into consideration; 2) pieces examined are excessively truncated as only their first ten and last ten measures are considered; 3) Renaissance mode is anachronistically extrapolated from a process optimized for identifying the key in early 18th-century music. This study demonstrates the need for a historically informed approach to analysis.

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[53] Julie Cumming, "From Two-Part Framework to Movable Module," in *Medieval Music in Practice: Studies in Honor of Richard Crocker* (Middleton, Wisconsin: American Institute of Musicology, 2013) 177–215.
[54] Michael Scott Cuthbert, "Hidden in our Publications: Uncovering Concordances, Citations, and Influence in Medieval Music through Databases and Programming," (public lecture, All-Souls College, Oxford, October 22, 2015).
[55] Joshua Albrecht and David Huron, "A Statistical Approach to Tracing the Historical Development of Major and Minor Pitch Distributions, 1400–1750," *Music Perception: An Interdisciplinary Journal*, 31/3 (December 2012), 223–43.

The specific analytical tools and methodology used in this dissertation borrow extensively from two studies in particular. First, Schubert and Cumming's corpus study on Lassus's twelve duets is a concise demonstration of how n-gram analysis can be a revealing component of the contrapuntal examination of a piece or corpus of pieces.[56] The corpus study at the end of chapter 4 revisits these twelve duets, along with another twelve by Morley, and ten two-part pieces or movements by Josquin. The second corpus study that had a profound influence on this dissertation is Christopher Antila and Julie Cumming's study on n-gram expression in three repertoire corpora from chronologically adjacent generations of Renaissance composers.[57] Antila and Cumming show the effectiveness of n-grams in capturing musically-salient shifts and tendencies in compositional style. While these authors have already produced important findings, their studies sampled counterpoint at regular minims and always on the strong part of those minims. In light of the variety of contrapuntal levels theorized by Vicentino and Burmeister, and that I demonstrate in chapter 4, their analyses would benefit from a more nuanced and historically-informed approach to fundamental-counterpoint reduction. This would allow their queries to more faithfully pinpoint the fundamental counterpoint for analysis in each piece. Instead of analyzing the expression of vertical intervals, melodic intervals, repeating contrapuntal patterns, or modes in the repertoire as the studies mentioned above did, in chapter 4 I present a new automated method that discerns the CR of Renaissance music along with a few demonstrations of its applicability to the analysis of single pieces or passages, and of corpora. Ruth DeFord has taken a decisive step in this same direction by dealing with many of the most important theoretical issues.[58] Her approach, however, does not specify a systematic procedure to follow in order to identify the CR of a passage. The present study makes this important contribution.

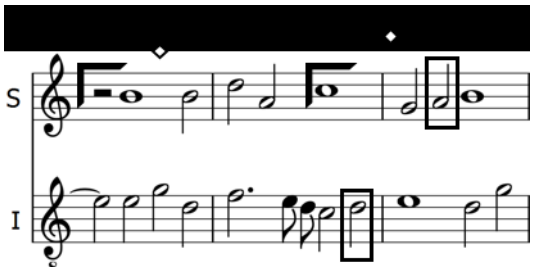
Contrapuntal Character

As mentioned above, the contrapuntal character of a passage is based on its steadiness or variability with respect to CR. The contrapuntal character of Figure 2 is regular, because its fundamental counterpoint advances at regular minims. By contrast the passage in Figure 5 begins with a minim CR, but decelerates to the semibreve when passing tones a minim in duration get

[56] Schubert and Cumming, "Another Lesson," 577–86.
[57] Antila and Cumming, "The VIS Framework," 71–76.
[58] DeFord, Tactus.

introduced.[59] This is because the analytical method I offer in chapter 4 interprets passing and neighbor tones as being active one metric level faster than the CR (i.e. a minim passing tone projects a semibreve CR); suspensions, by contrast, are active on the same metric level as that of the CR.

Figure 5: Josquin, Agnus Dei, NJE [13.11], mm. 13–15.



By applying the CR analysis to describe the contrapuntal character of a passage, I put my dynamic-offset method (which determines the CR) in dialogue with the extensive body of modern contrapuntal research. Building on Jesse Anne Owens's concept of the module, or repeating contrapuntal pattern, John Milsom has described imitation after one time unit as stretto fuga.[60] Peter Schubert and Julie Cumming have both jointly and each individually developed a classification method for points of imitation which Schubert calls presentation types.[61] Schubert and Cumming make note of the presence of stretto fuga when its presence dominates a point of imitation. In chapter 4, I demonstrate that my dynamic-offset method can add further precision to the analysis of contrapuntal passages including points of imitation and stretto fuga, and can therefore potentially further refine the classification of presentation types by distinguishing between passages with different contrapuntal characters. I show the usefulness of considering

[59] Josquin des Pres, *Agnus Dei*, NJE [13.11], http://josquin.stanford.edu/work/?id=Jos1311.
[60] Jessie Ann Owens, *Composers at Work: The Craft of Musical Composition 1450–1600* (New York and Oxford: Oxford University Press, 1997); John Milsom, "'Imitatio,' 'intertextuality', and early music," in *Citation and authority in Medieval and Renaissance musical culture: Learning from the learned*, ed. Suzannah Clark and Elizabeth Eva Leach (Woodbridge: Boydell & Brewer, 2005), 141–51.
[61] Peter Schubert, "Hidden Forms in Palestrina's First Book of Four-Voice Motets," *Journal of the American Musicological Society* 60 (2007), 483–556. For a corrected version of the appendix see: http://www.music.mcgill.ca/~schubert/finaltable.pdf; Julie Cumming and Peter Schubert, "The Origins of Pervasive Imitation," in *The Cambridge History of Fifteenth-Century Music*, ed. Anna Maria Busse Berger and Jesse Rodin (New York: Cambridge University Press, 2015), 200–28; Julie Cumming, "Text-Setting and Imitative Technique," in *The Motet around 1500: On the Relationship of Imitation and Text Treatment*, ed. Thomas Schmidt-Beste (Turnhout: Brepols, 2012).

contrapuntal character in analysis in chapter 4 by revisiting some of Schubert and Cumming's analyses of points of imitation involving stretto fuga.[62]

Chapter Outline

This dissertation is the first study to take a comprehensive approach to examining interval-succession treatises in order to better understand the authors' contrapuntal thinking. The interval successions in the *LAC* and *RM* are taken as coherent sets representing the author of each treatise's stance on a variety of musical issues. Each interval-succession list becomes a corpus of examples. Chapter 2 is devoted to this subject with respect to Tinctoris's *LAC*, and chapter 3 with respect to Pontio's *RM*. Approaching the primary documents in this way opens the door to a variety of means of comparing the examples which reveal finer points of the authors' voice-leading practice clearly demonstrated in the examples, albeit not explicitly stated in prose.

On the basis of the important distinctions I uncover concerning Tinctoris and Pontio's theoretical and pedagogical approaches in the *LAC* and *RM* considered individually, I conclude chapter 3 by comparing the two authors. The latter occasionally deviates from the ordering principles that are otherwise clearly present in his list unlike the former who is pointedly systematic. In addition to this, Pontio relies on the reader's ability to calculate octave-equivalent successions from those he provides which stay mostly within the octave, and he often describes general categories of successions, rather than specific individual ones as Tinctoris does. Often, a single example in Pontio's treatise will represent a group of similar but distinct interval successions, and it is the student's task to negotiate this important level of abstraction. I demonstrate that these differences are indicative of the two theorists' disparate pedagogical approaches. The numerous interval successions in the *LAC* are terse and formulaic, and correspond to what Anna Maria Busse Berger has described as indicators of a tradition grounded in memorization. Pontio's list, by contrast, is rich in musical context, not very formulaic in its presentation, demands a fair amount of sophistication from its reader, all of which I interpret as being indicative of a pedagogical approach of commonplaces. This reading is corroborated by the numerous repertoire references, true commonplaces, that Pontio makes throughout *RM*.

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[62] Cumming and Schubert, "Origins".

In the final chapter of this dissertation, I look to the Renaissance treatises of Tinctoris, Vicentino, Pontio, and Burmeister to problematize issues surrounding the reduction of Renaissance counterpoint. The primary sources I consult usefully highlight central issues, but I show that the wholesale adoption of Tinctoris or Pontio's approach to analysis is unviable. This is because Tinctoris gives no indication of the mensural level to which his successions apply. Conversely, Pontio forgoes any analytical flexibility by always applying his successions in regular minims. For this reason I propose an analytical method based primarily on dissonance treatment and attack density. This method can be used to make a reduction of a piece of Renaissance music, or more generally to describe the contrapuntal character of a piece or corpus of pieces. Alternative means of sampling counterpoint already exist but they are along the lines of Tinctoris and Pontio's problematic approaches mentioned above. My historically informed approach to this analysis problem serves to improve the degree to which detected n-grams represent the fundamental-counterpoint progressions in n-gram analyses of Renaissance music. An automated implementation of this tool called the dynamic-offset method is made available along with this dissertation.[63] The automation of my approach makes it easier for others to review and refine, and also opens up the possibility of doing corpus studies with my new analytical tool. I conclude with three analyses making use of this automated analytical tool ranging in scope from the analysis of a single piece, to a medium-sized corpus study.

[63] The entire VIS Framework is available on github at: https://github.com/ELVIS-Project/vis-framework
The dynamic-offset is accessible by using the "dynamic" setting of the offset indexer.

Chapter 2

The Tacit Principles of Tinctoris's Interval Successions

Medieval and Renaissance counterpoint treatises offer a practical guide to composing and improvising polyphony that respects stylistic norms. These counterpoint treatises are also replete with theoretical implications and can serve as a penetrating window into Renaissance musical thought, training, and style. Sarah Fuller has described Tinctoris's *Liber de arte contrapuncti* (hereafter *LAC*) as a "culmination of *contrapunctus* theory from the preceding two centuries," underlining its importance to our understanding of the period.[1] In this chapter I re-examine the *LAC* in order to rectify the common misconception that its interval-succession list is exhaustive, and to reveal nine tacit contrapuntal principles that Tinctoris systematically followed in the examples in the treatise. In addition to providing insight into Tinctoris's intervallic thinking, my methodology also serves as a general model for taking a corpus-study approach in researching the examples in counterpoint treatises in order to enrich our understanding of them. While I do not discuss other treatises in great detail in this chapter, chapter 3 will take a similar approach to Pontio's interval succession treatise, and conclude with a comparison of the two authors' theoretical and pedagogical approaches.

Background

The present investigation of early counterpoint treatises takes Fuller's chapter on the theoretical traditions known as *organum*, *gradi*, *Intervalschritt Lehre*, *discantus*, and *contrapunctus* as its point of departure.[2] Much has been made of the distinction between *discantus* and *contrapunctus* treatises in particular; as explained in the introduction, insofar as all five of these terms describe ways of improvising or composing idiomatic polyphony, they are all part of a greater theoretical tradition which I refer to as interval-succession theory.[3] Jan Herlinger also

[1] Sarah Fuller, "Organum – discantus – contrapunctus in the Middle Ages," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 498.
[2] Fuller, "Organum," 477–502.
[3] For more on the distinction between *discantus* and *contrapunctus* see: Klaus-Jürgen Sachs, *Der Contrapunctus im 14. und 15. Jahrhundert: Untersuchungen zum Terminus, zur Lehre und zu den Quellen*, (Wiesbaden: Steiner, 1974); and Felix Diergarten, "Beyond Contrapunctus. On a Hypothesis by Hugo Riemann and Klaus-Jürgen Sachs,"

provides illuminating commentary on this tradition; his examination, however, ends at the year 1450 and so lacks an in-depth discussion of the *LAC*.[4] In the same volume, Bonnie Blackburn examines Tinctoris's theoretical works in some detail, but her focus is on contextualizing a web of mutually influencing authors, not on the tacit voice-leading guidelines conveyed by the *LAC* as mine is in this chapter.[5]

The *LAC* is organized in three books. In all three Tinctoris generally refers to melodic intervals by giving the number of steps they encompass. For example the melodic motion of four steps corresponds to the interval of a fifth. The largest section of the *LAC* is the first book, which is mainly a list of acceptable consonant interval successions. Specifically it cites successions that are two vertical intervals long. These successions begin and end with a consonant interval between the unison and triple octave inclusive. Therefore this list is roughly three times as long as it would be if he had restricted himself to successions beginning on a vertical interval within the octave. Tinctoris orders the successions in this long list by increasing interval size of: 1) the first vertical interval in the succession; 2) the second vertical interval; and 3) the melodic interval of the tenor, first ascending, then descending. Tinctoris presents each set of intervals twice, first with the tenor as the lower voice, then as the upper voice effectively doubling the length of the list. I often refer to "fundamental counterpoint" and by this I mean the relationship of the voices irrespective of their names or functions, octave doublings, and any ornamentations. For example, when Tinctoris presents the same interval succession twice but calls the lower voice the tenor the first time, then the upper voice, these two examples convey the same fundamental counterpoint. All of Tinctoris's interval-succession examples are note-against-note counterpoint so there is never any issue with reducing out ornamentations. In all, Tinctoris provides the reader with 768 interval succession in the first book. This chapter unpacks some of Tinctoris's theoretical assumptions that can be observed through a detailed examination of these examples taken as a coherent corpus. A

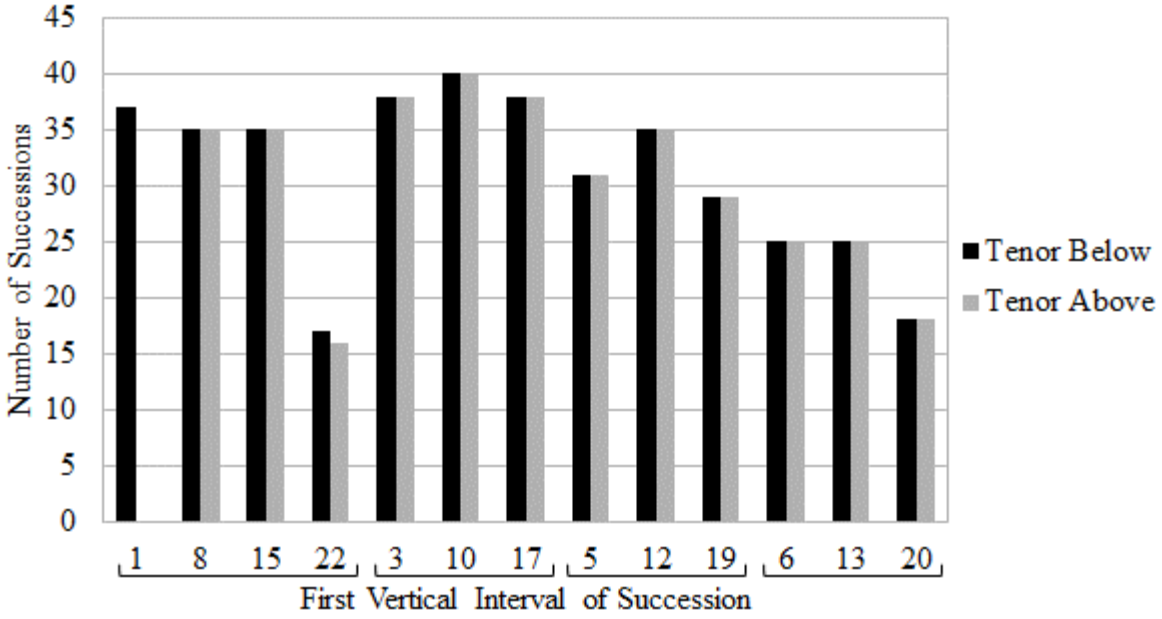
(paper presented at the Medieval and Renaissance Music Conference [MedRen], Brussels, Belgium, July 6–9, 2015.) Available on academia.edu at: https://www.academia.edu/14084984/Beyond_contrapunctus._On_a_hypothesis_by_Hugo_Riemann_and_Klaus-J%C3%BCrgen_Sachs (accessed November 8, 2015).

[4] Jan Herlinger, "Music Theory of the Fourteenth and Early Fifteenth Centuries," in *Music as Concept and Practice in the Late Middle Ages*, ed. Reinhard Strohm and Bonnie Blackburn (New York: Oxford University Press, 2001), 244–300.

[5] Bonnie Blackburn, "Music Theory and Musical Thinking after 1450," in *Music as Concept and Practice in the Late Middle Ages*, ed. Reinhard Strohm and Bonnie Blackburn (New York: Oxford University Press, 2001), 301–345.

breakdown according to the vertical interval of the succession of this list is given in Figure 1.[6] Right away we can see that thirds, tenths, and seventeenths are the most commonly used starting intervals, in contrast to sixths, thirteenths, and twentieths which are the least commonly used.

Figure 1: The number of interval successions given in the LAC *grouped according to the first vertical interval.*



The second book of the *LAC* provides the most thorough explanation of dissonance treatment ever committed to paper at that time.[7] Book III presents eight general principles to be observed in composition.

The massive interval-succession list in book I has generally been taken as "an exhaustive inventory" (Fuller) of "*all* possible interval progressions" (Busse Berger, emphasis hers), however it is far from comprehensive.[8] Tinctoris structures his list by imposing five voice-leading principles summarized in Figure 2.

[6] Note that Tinctoris does not provide two positions for the tenor for interval successions that start on the unison. For this reason all of these successions have been put in the "Tenor Below" column.
[7] Blackburn, "Music after 1450," 330.
[8] Fuller, "Organum," 498. Anna Maria Busse Berger, *Medieval Music and the Art of Memory* (Los Angeles: University of California Press, 2005), 141. Busse Berger has more recently renewed this claim stating again that Tinctoris lists "*all* available interval progressions…" Anna Maria Busse Berger, "Oral Composition in Fifteenth-Century Music," in *The Cambridge History of Fifteenth-Century Music*, ed. Anna Maria Busse Berger and Jesse Rodin (Cambridge: Cambridge University Press, 2015), 140. See also Knud Jeppesen, *Counterpoint: The Polyphonic Vocal Style of the Sixteenth Century*, trans. Glen Haydon (New York: Dover Publications, 1992), 10.

Figure 2: Tinctoris's explicitly stated voice-leading principles.

1. No dissonances are included: (Prologue) "Now, therefore, among other things, I have decided expressly to write down those few things that I have perceived by sleepless study concerning the art of counterpoint, which is produced from consonances." [9]
2. No melodic intervals greater than a fifth are used in either voice: (Bk. I, chap. 19) "Besides, it must be understood that rarely in composed music, and hardly ever or never in plainchant, the tenor ascends or descends beyond the fourth step, wherefore we have not ordered the concords according to the excess of that fourth step [i.e. interval of a fifth]."[10]
3. A maximum range of three octaves (a twenty-second) should be respected at all times: (Bk. I, chap. 2) "But since I cannot attain infinity and wish to reject unnecessary things, I have restricted myself to those twenty-two concords [which span three octaves]."[11]
4. No parallel perfect intervals may be used: (Bk. III, chap. 2) "We should ascend and descend with the tenor by imperfect concords but not, however, by perfect ones of the same type."[12]
5. A sixth can go to a fifth (or octave-equivalent succession) only when the tenor remains in place: (Bk. I, chap. 7) "this sixth can be taken up above the tenor if that tenor, remaining in the same position, has a fifth, likewise above it."[13]

He does not present all five principles together and only one comes from the eight principles stated in book III. However all five of these principles affecting two-voice interval successions are explicitly stated and every interval succession given in the *LAC* obeys them. Adhering to these five principles generates a list of 935 interval successions; so what about the 167 that are seemingly missing from Tinctoris's list of 768?[14] I will show that the *LAC* exhibits a high degree of uniformity

[9] Note that at the time of writing this chapter, only bk. I of Jeffrey Dean's translation of the *LAC* is available online so all references to bk. I are to his addition. The few references to bks. II and III are to Seay's edition. Johannes Tinctoris, prologue to *Liber de arte contrapuncti*, trans. Jeffrey Dean, http://earlymusictheory.org/Tinctoris/texts/deartecontrapuncti/.
[10] Tinctoris, *Arte contrapuncti*, bk. I chap. 19.
[11] Tinctoris, *Arte contrapuncti*, bk. I chap. 2.
[12] This is Book III's second rule. That same rule allows parallel perfect intervals as an extreme tolerance in three or more voices when a third voice moves in contrary motion to the parallels. Johannes Tinctoris, *Liber de arte contrapuncti*, trans. Albert Seay (Rome: American Institute of Musicology, 1961), bk. III chap. 2, 133.
[13] Note that this quote and the others like it in the chapters on the thirteenth and twentieth provide an exception to the standard concatenation options for a sixth which are another sixth, an octave, or a tenth. This exception he makes for the resolution of a sixth to a fifth is given in the context of an interval succession three intervals long wherein a fifth or an octave move to a sixth and then to a fifth with the tenor staying in place the whole time. Tinctoris, *Arte contrapuncti*, bk. I chap. 7.
[14] Many thanks to Christopher Antila for helping me write the script that generates the possible interval successions that follow Tinctoris's explicit rules.

in the successions it omits, and that we can use these omissions to uncover tacit contrapuntal constraints that Tinctoris respected when compiling this list. I will refrain from conjecture as to whether or not the patterns I find in Tinctoris's selection and discussion of interval successions were intentional.

Methodology

A distinct advantage of integrating computational techniques into my analysis of the *LAC* is that it facilitates non-linear examination and comparison of the interval successions. By organizing the examples in a variety of ways we can reveal tacit principles that structure the author's intervallic thinking. Given a set of voice-leading principles as well codified as Tinctoris's, it is easy to compare the interval successions that are specifically named to the larger list of all those that satisfy the explicitly stated principles. I take any succession that follows all of his explicit principles yet is omitted from Tinctoris's list to be a potential window into additional tacit principles governing acceptable voice leading. In uncovering various similarities, gaps, and patterns in the interval successions included and the language used to describe them, we can deduce the voice-leading principles whose influence on the list is evident, despite the fact that they are not written out explicitly.

Octave Equivalence

Comparing octave-equivalent interval successions is an extremely fruitful way of revealing the tacit principles in the *LAC*. There are three slight caveats about octave-equivalent successions summarized in Figure 3.

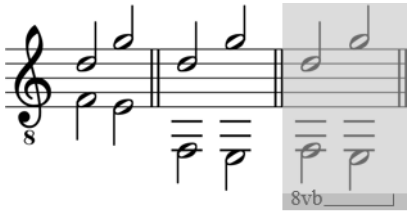
Figure 3: Limitations on octave equivalence.

1. A maximum range of three octaves
2. The unison has a special status and its treatment is therefore unlike that of the octave or any other interval
3. There is a unique disposition of consonances around the unison

First and most simply, Tinctoris's maximum range of three octaves does not allow, for example, a twentieth to go to a twenty-fourth in the same way that a sixth is shown going to a

tenth, because intervals larger than a twenty-second (the triple octave) are not considered as shown in Figure 4.[15]

Figure 4: Demonstration of the LAC*'s three-octave maximum range. The grayed-out succession is not found in the* LAC.

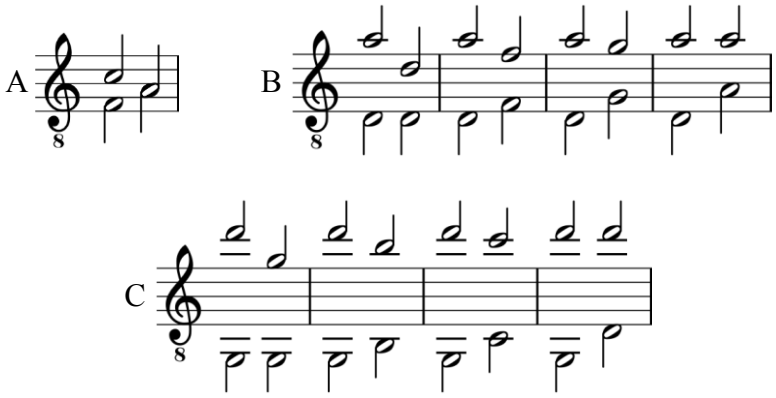


Second, the unison is not equivalent to any other interval. This is in keeping with a long tradition of according a unique status to the unison of which Tinctoris seems to have been aware in light of, among other things, his comment that "[because it produces one and the same sound] some declare that the unison is not a concord."[16] To summarize Tinctoris's stance on the unison, he considers it to be a consonance, but does not treat it in exactly the same way as the octave. For example there is only one interval succession of a fifth going to a unison (5 $_3$ 1, Figure 5-A), however there are four beginning on a twelfth and going to an octave (Figure 5-B) and octave-equivalent versions of the same four interval successions are also named beginning on the nineteenth and going to the fifteenth (Figure 5-C). The succession of a 12th (or 19th) contracting to an 8^{ve} (or 15th) a step above the lower note which may appear to be missing from Figure 5-B and Figure 5-C will be discussed later in this chapter.

[15] This and all further notated examples taken from the *LAC* all use downward-stemmed notes for the voice that is designated as the tenor, and examples that do not actually occur in the *LAC* are grayed out.

[16] David Cohen has traced the historical and philosophical origins of this special status of the unison in "Metaphysics, Ideology, Discipline: Consonance, Dissonance, and the Foundations of Western Polyphony," *Theoria* 7 (1993): 1–86. It should be noted, however, that not all Medieval and Renaissance writers are in agreement. For example, the author of the *grado* treatise *Dico che 'l contrapuncto* states that: "I affirm that the octave is a consonance insofar as it originates from the unison," which is the same language he uses to relate the third to the tenth, the fifth to the twelfth, etc. as in "The tenth is a dissonance [imperfect consonance] because it originates from the third" and "The twelfth is a consonance because it originates from the fifth." Many thanks to Alessandra Ignesti for providing me with complete English translations of both Scattolin's article and the *Dico che'l contrapuncto* treatise (included in Scattolin's article). See *Dico che 'l contrapuncto* ch. 3.28–3.33 in Pier Paolo Scattolin, "La regola del 'grado' nella teoria medievale del contrappunto," *Rivista Italiana di Musicologia* 14 (1979): 11–74.

Figure 5: Successions listed in the LAC *of a fifth, twelfth, or nineteenth going to a unison, octave, or fifteenth respectively.*

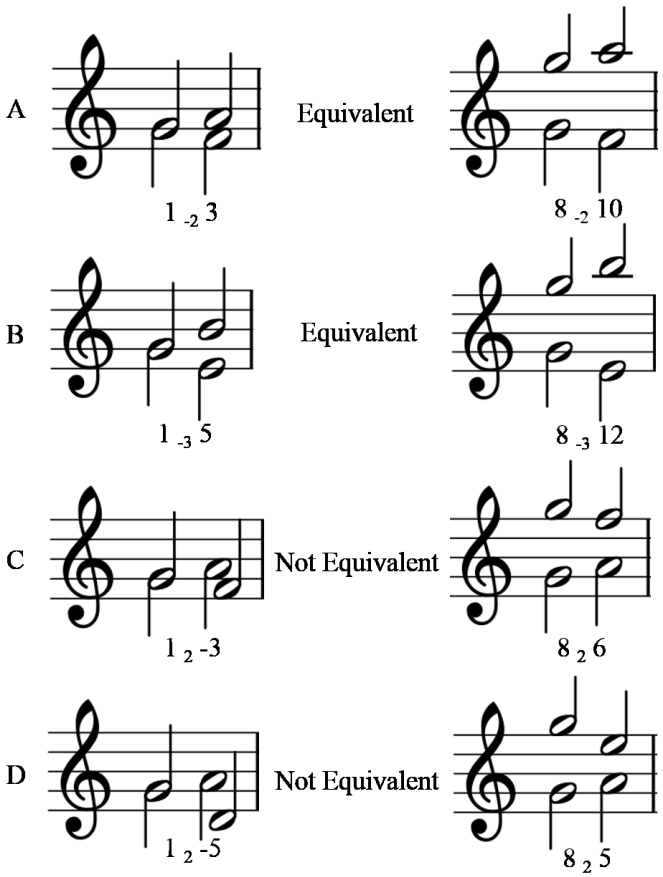


The fact that the listed options for arriving at a unison or octave are so different suggests that the unison is demands special treatment with respect to the ways it can be approached. By contrast, almost all other intervals an octave apart (such as thirds and tenths) receive nearly identical treatment both as originating and destination intervals. In short, not all successions beginning or ending on the unison have octave-equivalent versions in the *LAC* beginning or ending with the octave. Some modern musicians may consider the unison and the octave equivalent but they generally were not held to be so in Tinctoris's time and therefore there is no reason to expect them to have the same interval-succession possibilities.

Third, many successions that begin with an interval of a fifth or smaller and involve a voice crossing cannot have exact counterparts at the octave because of the special disposition of consonant intervals around the unison as compared with those around the octave. While consonant intervals lie two and four steps greater than both the octave and the unison (i.e. a tenth and a twelfth for the octave and a third and fifth for the unison, see Figure 6-A and Figure 6-B), on the other side consonant intervals lie two and three steps smaller than the octave (i.e. a sixth and a fifth) yet two and *four* steps "smaller" than the unison (i.e. a third and fifth arrived at by voice crossing, see Figure 6-C and Figure 6-D).[17] In short, the pitch space allows for a necessarily different set of successions when the threshold of the unison is traversed. Other than the times that one of these three points is a factor, the interval-succession content in each of Tinctoris's three octaves is identical save for one isolated case I will discuss shortly.

[17] See the introduction for a more detailed explanation of interval successions that contain a voice crossing and their annotation.

Figure 6: Consonances above and below a unison and their closest octave-equivalent counterparts. The tenor is always the voice with downward stems; negative numbers for harmonic intervals indicate that the tenor is above the other voice. Pair C's successions are not equivalent because although each voice preserves its melodic motion, the second vertical intervals are not octave equivalent. Pair D's successions are not equivalent because, while both successions end on a perfect fifth, the melodic motion of the contrapuntal voice changes.



The Two Positions of the Tenor

The comparisons between interval successions I have drawn so far have been based on octave equivalence. Another useful way of reordering Tinctoris's list is to put side-by-side the successions that have the same voice leading but different positions for the tenor (i.e. the tenor is the lower voice in one and the upper voice in another). We can combine these two means of examining the voice leading conveyed in the treatise (i.e. octave equivalence and swapping the position of the tenor) to come up with as many as six versions of any given voice-leading scenario. A thorough example of this is provided later in Figure 15. When the *LAC* demonstrates strong consistency through similar treatment or repeated exclusion of all six theoretically possible versions of a generic two-voice succession, I interpret this to be indicative of well-defined

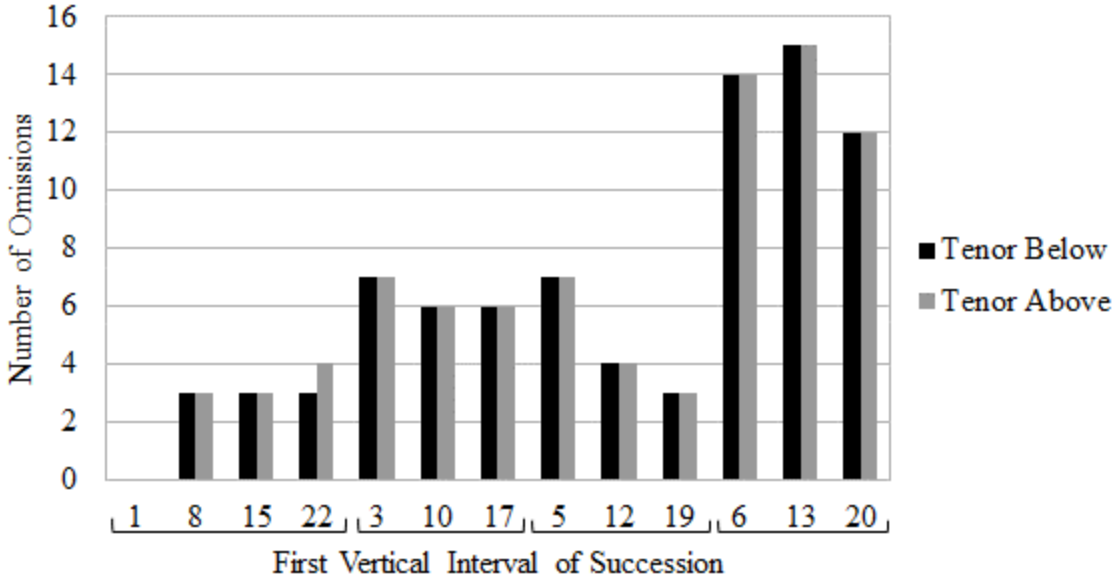
contrapuntal principles. When these contrapuntal principles are not provided in prose, I take these to be tacit principles uncovered by treating Tinctoris's examples as a coherent whole.

Interval-Succession List as Corpus

As mentioned earlier, integrating computational methods in the study of the *LAC* is very helpful because it allows for non-linear examination of Tinctoris's corpus of interval-succession examples. A breakdown of the interval successions included in the *LAC* was already provided in Figure 1 and in this section the same technique is applied to the interval successions that follow Tinctoris's explicitly stated principles (summarized in Figure 2) yet get omitted from the list. In so doing, the tacit principles of Tinctoris's contrapuntal thinking are revealed through inductive reasoning.

The fact that Tinctoris names interval successions when the tenor is the upper or lower voice may seem to suggest that different fundamental counterpoint is allowed in the two situations. This assumption is not borne out by the *LAC*'s successions. In all cases save one, exactly the same successions are presented in both tenor positions. Figure 1 demonstrates this by showing that there are just as many successions starting on any given interval when the tenor is above as when it is below, with the minor exception of the successions starting on the twenty-second where the two counts differ only by one. The same conclusion can be drawn from Figure 7 though instead of tallying the successions that Tinctoris provided, it counts the successions that he omitted from his list, despite the fact that they follow his five explicitly stated principles which were given back in Figure 2.

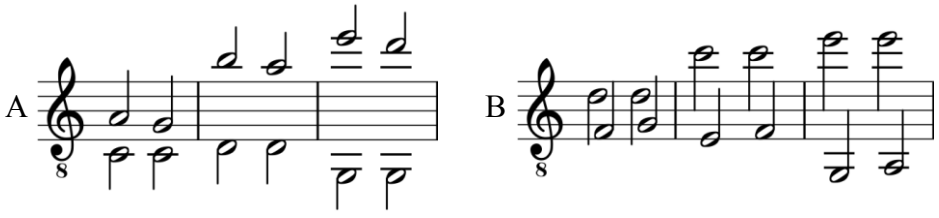
Figure 7: The number of omitted interval successions grouped by their first vertical interval.



The graph shows that the omissions are equal in number among octave-equivalent intervals, and (though it is not shown on the graph) they are also contrapuntally equivalent successions save one category of successions which I will address now.

Of the twenty-four possible successions going from a sixth to a fifth (or octave-equivalent successions) Tinctoris only includes one per octave: 6 $_1$ 5; 13 $_1$ 12; and 20 $_1$ 19 (Figure 8-A). When the tenor is the upper voice, Tinctoris still only includes one succession of a sixth to a fifth (or octave-equivalent successions) but they are not the same fundamental counterpoint because the upper voice is now the stationary voice: -6 $_1$ -5; -13 $_1$ -12; and -20 $_1$ -19 (Figure 8-B).[18]

Figure 8: Motions of sixths going to fifths provided in the LAC *with the tenor below (A) and the tenor above (B). Note that the tenor is always shown as the voice with downward-stemmed notes.*



[18] Note that the medial number shown in subscript conveys the melodic motion of the tenor even when the tenor is the upper voice. This has been done to most closely match Tinctoris's examples and text descriptions.

So, in keeping with the fifth explicitly-stated guideline given earlier in Figure 2, Tinctoris only shows the sixths going to fifths when the tenor does not move, regardless of whether it is the upper voice or the lower voice. This is the only instance in the *LAC* that systematically conveys different fundamental counterpoint for when the tenor is the upper or lower voice. It may seem that this is in keeping with the "closest-approach rule" which would only allow a minor sixth to move to a perfect fifth in order to preserve semitonal motion in one voice, however the notes in Tinctoris's examples generally show a major sixth going to a perfect fifth; the beginning of the chapter on the twentieth even specifies a B-flat below a G to show a major twentieth going to a perfect nineteenth.[19] With the exception of the sixths going to fifths, the fact that the successions are treated identically regardless of the position of the tenor confirms that the tenor's position is not an important concern for the fundamental counterpoint. One possibility for why Tinctoris included it anyway is that having separate successions for each tenor position may facilitate the already conceptually demanding task of composing or improvising against a pre-existing tenor for music students. This possibility will be considered in more detail in chapter 3 when I compare Tinctoris's approach to interval-succession theory to that of Pontio.

Keeping in mind that Tinctoris's list is ordered by the first vertical interval in the succession, then by the second vertical interval, and finally by the melodic interval of the tenor, we can focus on the first step of this three-pronged ordering and count how many interval successions starting with each interval get omitted despite following all of Tinctoris's explicit principles. Figure 7 shows that no potential interval successions beginning with a unison (shown as a "1") get omitted, so Tinctoris is true to his word when he states that the unison is "the source and origin of all concords".[20] By contrast the potential interval successions that begin with a sixth or one of its octave doublings get omitted the most. This restriction on the use of the sixth is wholly consistent with Tinctoris's prose descriptions of sixths as having "more of harshness than of sweetness" when used in only two voices.[21] Occasionally Tinctoris explains that a given interval succession should not be used in note-against-note counterpoint in two voices. These restrictive

[19] For more on the "closest-approach" principle, see: Peter Schubert, "Counterpoint Pedagogy in the Renaissance," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 506; and Karol Berger, *Musica Ficta* (New York: Cambridge University Press, 1987), 122; and Jan Herlinger. "Marchetto da Padova." *Grove Music Online*. *Oxford Music Online*. Oxford University Press, accessed August 14, 2016,http://www.oxfordmusiconline.com/subscriber/article/grove/music/17738.
[20] Tinctoris, *Arte contrapuncti*, bk. I chap. 3.
[21] Tinctoris, *Arte contrapuncti*, bk. I chap. 7.

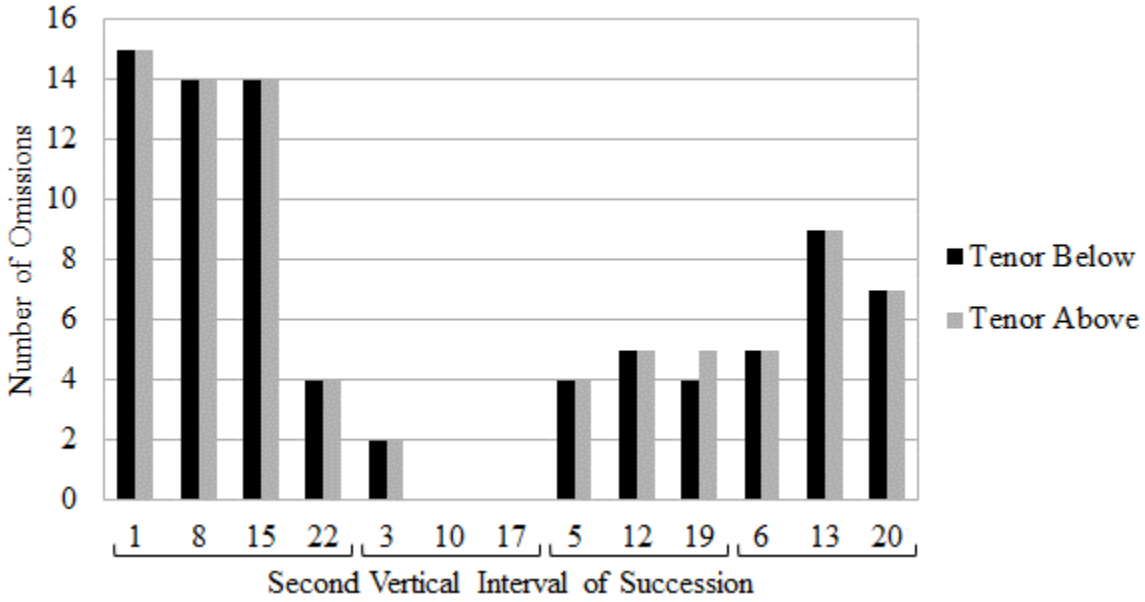
comments are made exclusively on interval successions that start on a sixth, a thirteenth, or a twentieth. The 136 interval successions starting on a sixth, thirteenth, or twentieth correspond exactly to one another through octave equivalence. Of these, 68 are not recommended in simple two-voice counterpoint, and the same exact ones are flagged with this restriction in all three octaves.[22] More broadly, the fact that Tinctoris repeatedly felt the need to specify that certain interval successions are not appropriate for two-voice counterpoint implies that the remainder of his successions do apply to passages in more than two voices. There are many ways to adopt a "pairwise model" of music analysis, the predominant ones being examining all pairs that include some referential voice, or examining all two-voice combinations.[23] The fundamental point of pairwise analysis is that, as much as possible, only one pair of voices in a composition is examined at a time. With respect to the types of pairwise analysis presented in the introduction, Tinctoris's approach relates a main reference voice, the tenor, to each other voice in the piece one at a time. This produces one fewer pair than there are voices in a given piece. Since Tinctoris's approach applies equally to two-voice pieces and pieces in more than two voices, he must specify if a succession cannot be used when the threshold of three voices is not attained.

The fact that Tinctoris omitted so many successions that begin on a sixth begs the question: does Tinctoris similarly omit a disproportionately high number of successions that end on a sixth? We can group the successions that Tinctoris omitted by their second vertical interval in order to shed light on the varying acceptableness of the different consonances as a destination interval. Figure 9 shows this grouping and illustrates that Tinctoris advocates going to thirds, tenths, and seventeenths in more ways than to any other interval group.

[22] Neither the number 136 nor 68 is divisible by three. Despite the fact that the successions beginning on a sixth, thirteenth, and twentieth correspond to one another exactly, Tinctoris's explicitly stated three-octave limit for intervals causes fourteen successions that begin on a twentieth to be omitted from the *LAC*; ten of these fourteen logically would have carried the note-against-note 2-voice counterpoint restriction.

[23] For more on the pairwise model of music analysis, see: Andie Sigler and Jon Wild, "Schematizing the Treatment of Dissonance in 16th-century Counterpoint," in *Proceedings of the International Society for Music Information Retrieval*, 2015, 645–650.

Figure 9: The number of omitted interval successions grouped by their second vertical interval.

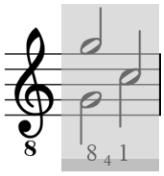


The four successions ending on a third that Tinctoris omits all come from a sixth and involve a voice crossing that necessitates large crossing leaps in both voices; perhaps this is why Tinctoris omits just these four.[24] While the sixth and its octave-equivalent intervals are the second vertical interval of a higher-than-average number of successions, the successions ending with the unison or simple or double octaves clearly stand out as being omitted the most. The triple octave would belong to more omitted successions if Tinctoris had allowed intervals larger than the triple octave as the first vertical interval. It may come as a surprise to see that the number of omitted successions ending on a fifth, twelfth, or nineteenth was not higher. One must keep in mind, however, that only those successions that follow all five of Tinctoris's explicitly stated principles (see Figure 2) but were still omitted from the list are counted. Since Tinctoris's last two of these principles (stating that parallel fifths should not be used and that a sixth can go to a fifth only when the tenor remains in place) directly rule out 45 successions ending on a fifth that would have otherwise been acceptable, it should come as no surprise that the remaining successions are subject to relatively few omissions.

[24] In his treatise from 1555, Vicentino is extremely sensitive to large melodic leaps, and often discourages the use of interval successions that include them in one and especially in both voices. Nicola Vicentino, *Ancient music adapted to modern practice* (*L'antica musica ridotta alla moderna prattica*, 1555; New Haven, Yale University Press 1996), 131–134.

The highly consistent nature of such fine-grained distinctions about Tinctoris's interval-succession content in all three octaves demonstrates just how closely he followed several voice-leading principles despite the fact that they were not written out explicitly. Every chapter in book I of the *LAC* begins with a general discussion of the first interval in the succession including a list of these equivalencies, such as this one on the eighteenth (i.e. a fourth plus two octaves): "Moreover, just like the diatessaron, to which it corresponds at the double diapason, and the diatessaron over diapason, to which at the diapason…".[25] To take an example that begins on an octave, no instances of contracting consecutive octaves by contrary motion can be found in the interval-succession list.[26] Following Tinctoris's principle that neither voice should move by more than a fifth melodically, there are only two basic ways one can move from an octave to a unison: 1) the lower voice can move up a fourth and the upper voice down a fifth (shown in Figure 10); or 2) the lower voice moves up a fifth and the upper voice down a fourth.

Figure 10: Hypothetical example (not included in the LAC*) of consecutive contracting octaves.*



These two interval successions are found in Tinctoris's list neither when the tenor is the lower voice nor when it is the upper voice; while this allows for the existence of the tacit principle that contracting octaves by contrary motion are forbidden, it is hardly convincing proof. But continuing on, we note that these two interval successions doubled at the octave (i.e. a fifteenth contracting to an octave) and at the triple octave (i.e. a twenty-second contracting to a fifteenth) appear neither when the tenor is the lower voice, nor when it is the upper. In light of these twelve systematic omissions, I infer that Tinctoris had a tacit principle that contracting octaves by contrary motion are not allowed. [27] All twelve possible instances of the conceptually similar case of consecutive expanding octaves by contrary motion are found in Tinctoris's list, however their use is restricted by his language; ten of these are qualified as only "*raro*" or "*rarissime*" so one could say that they

[25] Tinctoris, *Arte contrapuncti*, bk. I chap. 15.
[26] Note that throughout this paper when I refer to "consecutive octaves" these "octaves" include the unison with the octave, double octave, and triple octave.
[27] For clarity, there are twelve possibilities in this case because there are two interval successions in each of three octaves with the tenor as the lower or the upper voice, yielding 2 x 3 x 2 = 12.

are at best tolerated.[28] The allowance of consecutive expanding octaves despite the exclusion of consecutive contracting octaves may stem from the aforementioned allowance for the unison to go to any suitably proximate interval. This is because a unison can expand to an octave, but it cannot contract to any smaller interval; projecting this type of succession to other intervals via octave equivalence allows for consecutive expanding octaves but not consecutive contracting octaves. If this is indeed the reasoning behind the concatenation options for the simple, double, and triple octave, it suggests that the implicit principles Tinctoris followed are hierarchically organized at least to some extent.

With a restriction on contracting octaves by contrary motion, one might imagine that consecutive fifths (simple or compound) by contrary motion would be similarly marginalized. To the contrary, however, Tinctoris not only included all such possible instances, he even praised them stating that the first fifth "very appropriately," "very elegantly," "very sweetly," or "very well," calls for the second fifth.[29] Perhaps he felt obliged to allow consecutive fifths by contrary motion, such as those from the *LAC* shown in Figure 11, in light of their idiomatic use in the "leaping-contratenor cadence" which can be found in numerous examples in the treatise such as the possibilities for improvisation shown in book I chapters 10 and 15, and the excerpt reproduced in Figure 12.[30]

[28] The other two have the qualifier "*aliquando*" [sometimes] which although also limiting, is used much more liberally in the treatise than *raro* or *rarissime* and is not as insistent.
[29] Tinctoris, *Arte contrapuncti*, bk. I chap. 6, 11, and 16.
[30] Note that Tinctoris does not explicitly allow for the interval succession that the tenor and the contratenor execute at the moment of the leaping-contratenor cadence because it involves a leap of an octave in the contratenor, which goes against his principle that no voice should move by more than a fifth melodically. However, just after the interval-succession list proper Tinctoris does concede that voices do move by leaps of a sixth, seventh or octave on occasion, and that in these cases the other voice should move to the closest consonance; the voice leading of the leaping contratenor cadence conforms to this principle. Tinctoris, *Arte contrapuncti*, bk. I chap. 19. For more on the leaping-contratenor cadence see: Julie Cumming, *Motet in the Age of Du Fay* (Cambridge: Cambridge University Press, 1999), 76–78. Tinctoris, *Arte contrapuncti*, bk. II chap. 24, 116.

Figure 11: Interval successions from the LAC *of consecutive fifths by contrary motion which are all described in a flattering manner in the text.*

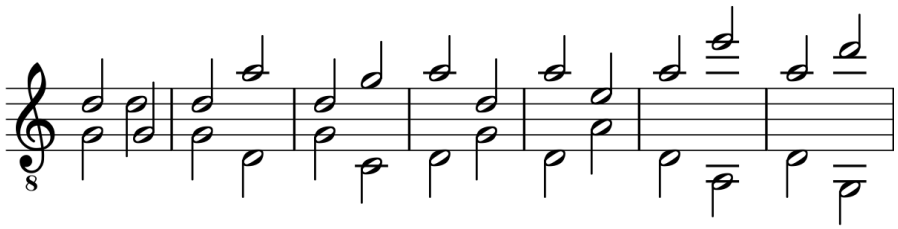
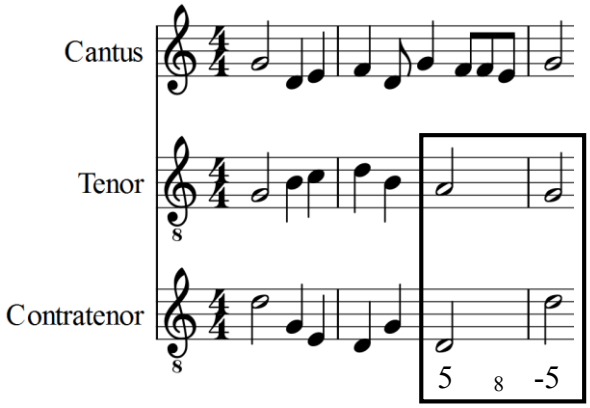


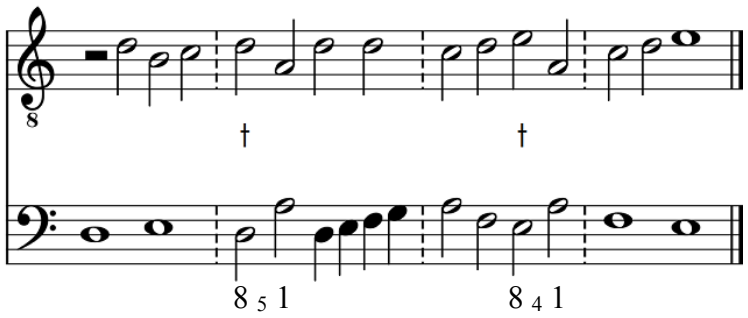
Figure 12: Example from the LAC *that employs an interval succession between the tenor and contratenor that is not included in the interval-succession list.*



While it is easy to group together instances of consecutive octaves and consecutive fifths by contrary motion into the broader category of consecutive perfect intervals by contrary motion, observing these consistencies in Tinctoris's intervallic thinking reveals just how different these two types of successions were for him. Pontio's interval-succession treatise (bk. II of *Ragionamento di musica*) does allow for contracting consecutive octaves by contrary motion as shown in Figure 13, but only when this scenario obtains between the two basses of an eight-voice polychoral work, a texture that was not in use when Tinctoris wrote his treatise.[31]

[31] Pietro Pontio, *Ragionamento di musica* (1588), comp. Suzanne Clercx (New York: Bärenreiter, 1959), 66. Note that I have shifted the placement of Pontio's cross symbol ahead by one note to indicate the first pitch in the upper voice that participates in the interval succession being discussed. Pontio is somewhat inconsistent with the placement of these symbols with respect to the successions he discusses.

Figure 13: Pontio's example allowing for contracting consecutive octaves between the two basses of a polychoral texture.



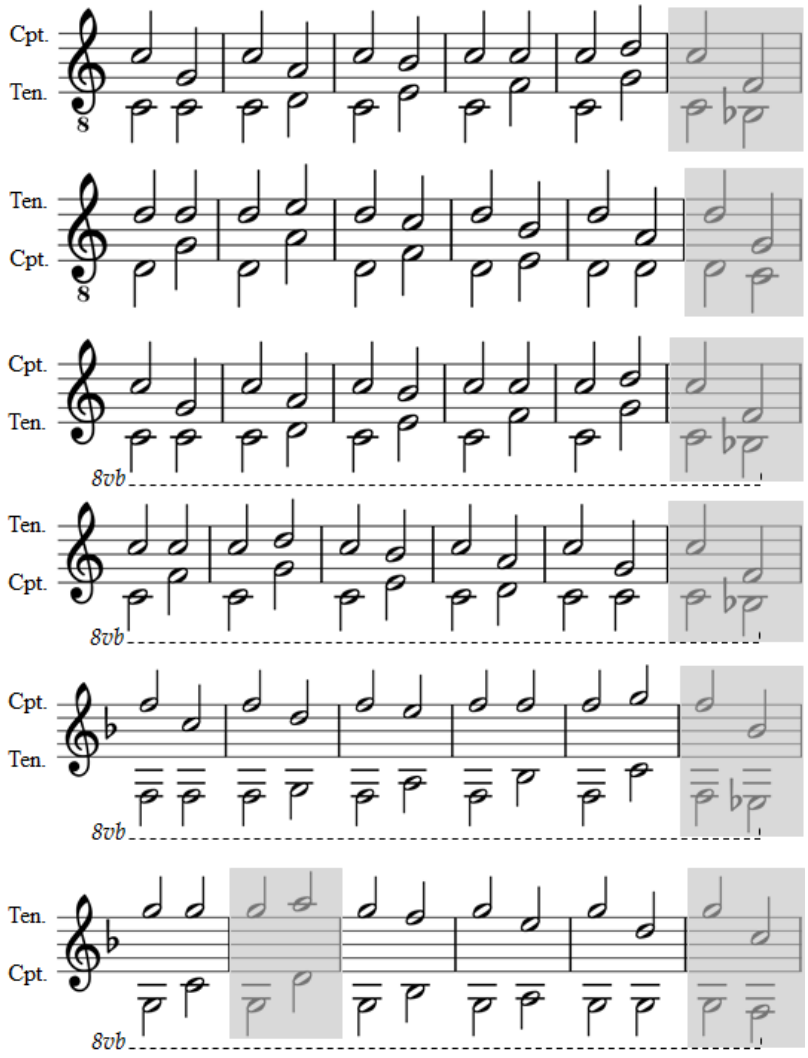
This precision vividly demonstrates how the details and circumstances surrounding which interval successions are permitted by theorists reflect the questions about and practical approaches to evolving stylistic concerns faced by theorists, composers, and performers alike.

In examining the ways a twelfth or a nineteenth can contract to an octave or a fifteenth respectively, there is an apparent gap in the successions that Tinctoris provides. A twelfth is shown being able to go to an octave when the tenor stays on the same note, or rises by a third, fourth or fifth (i.e. 12 $_1$ 8, 12 3 8, 12 $_4$ 8, 12 $_5$ 8 shown in Figure 5-B). The "missing" interval succession would have a twelfth going to an octave when the tenor moves up by a second, 12 $_2$ 8, but this succession and others like it are systematically excluded from the treatise. The successions 5 $_2$ 1, 12 $_2$ 8, and 19 $_2$ 15 (shown in Figure 14) are all absent and seemingly skipped over but it is not clear what logic is behind this gap, if any.

Figure 14: Octave-equivalent successions omitted from the LAC*; the second and third in particular appear to be gaps if one follows the logical structure of Tinctoris's list.*

Of the thirty-six possible ways an octave can move to a fifth (or octave-equivalent succession) that follow Tinctoris's explicit principles, one subtype with six members is systematically missing, namely those that would have the lower voice move down by step and the upper voice move down by fifth. All thirty-six are shown in Figure 15.

Figure 15: All interval successions of an octave going to a fifth that follow Tinctoris's explicitly stated principles. Those that do not appear in the LAC *are shown grayed out.*



The voice-leading of twenty-nine of the remaining successions from this category that are present in the *LAC* is described almost identically in all three octaves irrespective of whether the tenor is the upper or lower voice. A seventh succession of the thirty-six in this category is also omitted, one that would begin on a triple octave. It is not clear if the extreme register of this succession played a role in its omission (the extremes of the succession would have encompassed three octaves and a second, though this is not without precedent in the treatise), if there is some other reason, or if this is just an inexplicable discrepancy.[32] To call this discrepancy rare is an

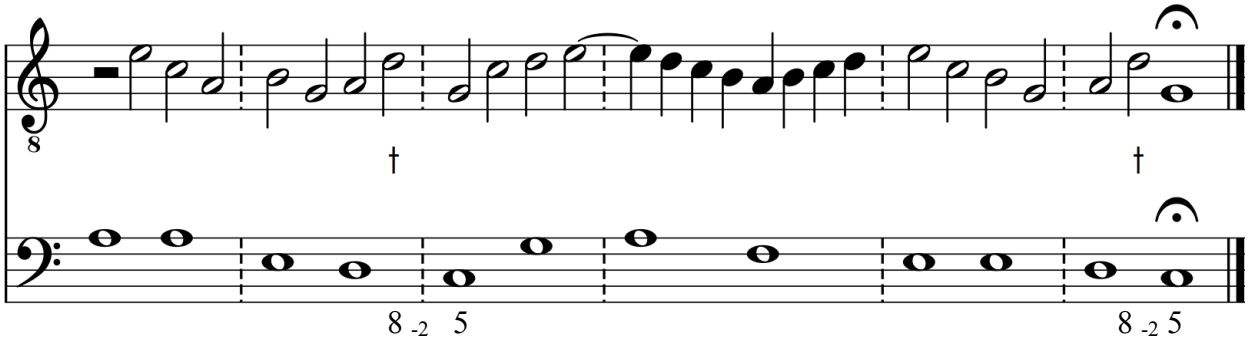
[32] While most of the omitted successions produce, and are therefore accounted for by, the list of nine tacit rules compiled in Figure 18, I was not able to identify a logical explaination behind a handful of them.

understatement; it is the only difference between the successions starting on the octave, fifteenth, and twenty-second that is not due to register constraints and the only otherwise inexplicable discrepancy in the fundamental counterpoint of the successions with the tenor above or below. The fact that Tinctoris was following several tacit principles, whether consciously or subconsciously, is still abundantly clear.

Tinctoris's silent exclusion of the sub-category of six that are missing (the right-most successions in Figure 15) is emphatically voiced by later theorists. Pontio expresses essentially the same voice-leading content with an opposite approach; instead of detailing which ways one could correctly proceed from an octave to a fifth, he inclusively states that (see Figure 16):

> …In all ways that they will be done, be they in two- or three-voice improvisations, & compositions; & these on the strong part of the beat as on the weak, in all ways I say they will have a good effect. but there is a passage from the octave to the fifth, that is not sensible in two-voice improvisation; & this is, when both of the parts descend, one by step, & the other by leap, as here.[33]

Figure 16: Pontio's counterexample illustrating the one way that going from an octave to a fifth is undesireable.



So instead of naming all the interval successions in this category that work, Pontio says that all reasonable options will work save one, the very same that Tinctoris omitted. This corroborates my interpretation that when Tinctoris omits an interval succession it is tantamount to prohibiting its

[33] This and all other translations of Pontio are my own. Note that the peculiar punctuation here is implemented to follow the original as closely as possible: "…in tutti i modi, che saranno fatti, cioè nei contrapunti, Terzetti, & compositioni; & questo così in principio di misura, come in fine, in tutti i modi dico faranno buono effetto. si truova però un passaggio dall'Ottava alla Quinta, che nel contrapunto di due voci, non hà del giudicioso; & questo sarà, quando ambedue le parti discenderanno, una con movimento congiunto, & l'altra con movimento separato, come quì." Pontio, *Ragionamento*, 63.

use.[34] Going further, it underlines how some interval-succession treatises clearly tried to convey general principles rather than an abundance of specific "rules" (i.e. individual interval successions). The significance of this last point will be discussed in more detail in chapter 3.

Conclusion

Earlier I pointed out that the *LAC*'s interval-succession list respects five principles that Tinctoris explicitly states, and these are reproduced in Figure 17 for the reader's convenience.

Figure 17: Tinctoris's explicitly stated voice-leading principles.

1. No dissonances are included: (Prologue) "Now, therefore, among other things, I have decided expressly to write down those few things that I have perceived by sleepless study concerning the art of counterpoint, which is produced from consonances." [35]
2. No melodic intervals greater than a fifth are used in either voice: (Bk. I, chap. 19) "Besides, it must be understood that rarely in composed music, and hardly ever or never in plainchant, the tenor ascends or descends beyond the fourth step, wherefore we have not ordered the concords according to the excess of that fourth step [i.e. interval of a fifth]."[36]
3. A maximum range of three octaves (a twenty-second) should be respected at all times: (Bk. I, chap. 2) "But since I cannot attain infinity and wish to reject unnecessary things, I have restricted myself to those twenty-two concords [which span three octaves]."[37]
4. No parallel perfect intervals may be used: (Bk. III, chap. 2) "We should ascend and descend with the tenor by imperfect concords but not, however, by perfect ones of the same type."[38]
5. A sixth can go to a fifth (or octave-equivalent succession) only when the tenor remains in place: (Bk. I, chap. 7) "this sixth can be taken up above the tenor if that tenor, remaining in the same position, has a fifth, likewise above it."[39]

[34] Thomas Morley discouraged the use of a very similar interval progression by claiming it amounted to "hitting the eight [i.e. the octave] on the face," however his reasoning is that the added voice should not leave an octave by leap melodically. Thomas Morley, *A plaine and easie introduction to practicall musicke* (London: Peter Short, 1597; Amsterdam: Theatrum Orbis Terrarum, 1969), 82.
[35] Tinctoris, prologue *Arte contrapuncti*.
[36] Tinctoris, *Arte contrapuncti*, bk. I chap. 19.
[37] Tinctoris, *Arte contrapuncti*, bk. I chap. 2.
[38] Tinctoris, *Arte contrapuncti*, bk. III chap. 2, 133.
[39] Tinctoris, *Arte contrapuncti*, bk. I chap. 7.

In addition to these explicit principles, I showed that the list also respects at least nine tacit principles which can be extrapolated from the *LAC* when it is studied as a coherent corpus of examples. These nine tacit principles are listed together in Figure 18.

Figure 18: Tinctoris's tacit voice-leading principles.

1. The unison has no restrictions on its movement; it can go to any other interval
2. After the unison, the third, tenth, and seventeenth are the intervals with the greatest mobility as Tinctoris cites the greatest number of interval succession that begin with them
3. Movement from sixths, thirteenths, and twentieths is subject to the most restrictions especially in note-against-note counterpoint in two voices
4. Movement to perfect unisons and octaves is disproportionately restricted given that many interval successions that follow all of Tinctoris's rules and end on one of these intervals still get omitted
5. Contracting consecutive octaves by contrary motion are not permitted
6. Expanding consecutive octaves by contrary motion are tolerated but discouraged
7. Consecutive fifths by contrary motion are well received as Tinctoris's text descriptions of these interval successions are very positive
8. The succession of an octave moving to a fifth with the lower voice moving down a step (i.e. 8 $_{-2}$ 5 or octave-equivalent succession) is not permitted
9. While melodic motions of a fifth are permitted in either voice, *both* voices should not move in opposite directions by a fifth[40]

Going back to Fuller and Busse Berger's assertions that Tinctoris's interval-succession list is exhaustive, in fact it nearly is so, however, only when we consider the added voice-leading constraints of the nine tacit principles in Figure 18. In light of such consistency between theoretically equivalent versions of the same fundamental counterpoint, one may wonder why Tinctoris included all three octaves and situations with the tenor as the bottom voice and then as the top voice. This raises important questions about the nature and purpose of the *LAC*. Why go to all the trouble of writing out reams of interval-succession examples six subtly different ways when

[40] The only exception to this principle is the situation wherein two voices sound a fifth and then sound the same fifth but with the voices parts in swapped positions. The remaining possibilities that are systematically omitted would involve parallel sixths where both voices move by a fifth in the same direction, a fifth expanding to a thirteenth (or a twelfth to a twentieth) or the reverse, i.e. a thirteenth contracting to a fifth (or a twentieth to a twelfth).

one octave would generally do? Especially given the considerable time, effort, and expense that writing or copying a book represented in the fifteenth century, why not simply let Tinctoris's ubiquitous explanations that "the tenth is like the third," "the twelfth is like the fifth," etc., suffice to convey appropriate voice leading to the reader? Plausible explanations for this are: to facilitate memorization; to facilitate the use of the treatise by students who did not yet fully grasp octave equivalence; to produce a more theoretically complete work; and because the ratios that represent octave equivalent ratios (such as 3/2 and 3/1 for a fifth and a twelfth respectively) can seem to underline their differences. Ultimately we cannot know for sure why Tinctoris realized his treatise in the manner that he did.

Renaissance musical thinking was intervallic in nature. For this reason interval successions are an appropriate lens through which to conceive of and analyze Renaissance musical syntax. This explains why interval-succession lists were so common in counterpoint treatises from the period. As this chapter works directly with Tinctoris's interval successions, the study is essentially *emic*.[41] By assessing the treatise in this historically informed way, I demonstrated that, in addition to his five explicitly stated constraints on interval successions, Tinctoris also systematically respected at least nine further tacit principles when compiling his list. Chapter 3 will take a similar corpus-study approach to the examples in Pontio's interval-succession treatise.

[41] For a discussion of the *emic* and *etic* dichotomy, see: Harold Powers, "Tonal Types and Modal Categories in Renaissance Polyphony," *Journal of the American Musicological Society* 34, no. 3 (1981): 428–470.

Chapter 3

Pietro Pontio's Approach to Interval-Succession Theory

Introduction

Ragionamento di musica (*RM*, 1588) is the first of Pietro Pontio's two treatises and Russell Murray described it as the more practical one.[1] While he moved multiple times between Parma, Bergamo, Pavia, and Milan, both of his treatises were written in his final years spent in Parma. By this stage in his life, he was an established and prolific composer of sacred works, some of which had multiple editions.[2] The evidence suggests that Pontio enjoyed greater popularity in his own time than he does today. The numerous and varied references he makes to contemporary and recent compositions bear witness to his musical erudition.

RM is set as a dialog between student and master, and broken down into four books the subjects of which are given in Figure 1.

Figure 1: The subject matter of Ragionamento di musica*, by book.*

Book	Subject Matter
I	Broad definitions and categories of music; theory of musical proportions
II	Interval-succession list
III	Characteristics of improvisation and composition; mode
IV	Mensuration; genre

This chapter examines the interval-succession examples in florid two-voice counterpoint provided in book II of *RM* together as a coherent group to infer various aspects of Pontio's intervallic thinking and general theoretical assumptions.

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[1] Except for references to Suzanne Clercx's *postface* from her facsimile edition of *Ragionamento di musica*, all references to the treatise are taken from Christophe Dupraz's online edition: Pietro Pontio, *Ragionamento di musica* (1588), accessed from: Christophe Dupraz, *Traités Musicaux Romans* (www.tremir.fr), 2013, specifically: http://www.ums3323.paris-sorbonne.fr/TREMIR/TReMiR_Pontio/R0_start.htm.
Russell Murray, "Pontio, Pietro," *Grove Music Online* (Oxford University Press, accessed July 21, 2016), http://www.oxfordmusiconline.com/subscriber/article/grove/music/22096.

[2] With the exception of a single madrigal, all of Pontio's known works are sacred. Murray, "Pontio, Pietro."

Similar to his compositions, Pontio's treatises have received only limited attention from modern researchers. Klaus-Jürgen Sachs, Harold Powers, and James Armstrong have examined *RM* focusing on his compositional advice.[3] In a separate study Sachs compared theory and practice though he focused on Pontio's second treatise, the *Dialogo*.[4] Peter Schubert has also studied *RM*, though with the intent of deciphering what improvisatory techniques could be learned from it, and to unpack the relationship between improvisation and composition according to Pontio.[5] To the best of my knowledge, this study is the first thorough examination of Pontio's interval-succession list.

This study will show that Pontio integrated numerous contextual enrichments into interval-succession theory compared to his predecessors, especially Tinctoris. Most notably, these include consideration of: interval quality, dissonance treatment, duration, metric placement, accidentals the number of active voices, genre, texture, timbre, text setting, and affect. By grouping and cross-referencing his examples according to different criteria, I will show that a thorough investigation of Pontio's body of interval-succession examples reveals insights into his analytical thinking concerning dissonances as well as his approach to accidentals both labelled and unlabeled. I will conclude with a comparison of Tinctoris and Pontio's interval-succession lists as well as of their theoretical and pedagogical approaches.

Transcriptions and Translations

One minor impediment to research on this interval-succession treatise is that there was previously no modern edition of the examples. Throughout this dissertation I present several of Pontio's examples transcribed in modern notation, and a transcription of all of the examples from book II of *RM* can be found in the appendix. All transcriptions in this dissertation use original note values (minims in the mensural notation are half notes in modern notation). With respect to Pontio's interval-succession examples in particular, each one concludes with longs in both

[3] Klaus-Jürgen Sachs, "Musikalische 'Struktur' im Spiegel der Kompositionslehre von Pietro Pontios *Ragionamento di musica* (1588)," *Zeichen und Struktur in der Musik der Renaissance* (New York: Bärenreiter, 1989), 141–57; James Armstrong, "How to compose a Psalm: Ponzio and Cerone compared," *Studi musicali* 7 (1978), 103–39. Powers's study in particular focused on Pontio's modal analysis: Harold Powers, "From Psalmody to Tonality," in *Tonal Structures in Early Music*, ed. Cristle Collins Judd (New York: Garland Publishing, 1998), 281–301.

[4] Klaus-Jürgen Sachs, "'*Theorica e Prattica di Musica*' in Pietro Pontios *Dialogo (Parma 1595)*," *Musiktheorie* vol. 4 (Laaber: Laaber-Verlag, 1989), 127–41.

[5] Peter Schubert, "From Improvisation to Composition: Three 16th-century Case Studies," in *Improvising Early Music* ed. by Dirk Moelants (Leuven: Leuven University Press, 2014) 93–130.

voices. These terminal longs have been transcribed as either whole notes or double whole notes depending on which one will complete the last measure of 4/2 time. 4/2 is meant to reflect 𝄵, the likely implied time signature, and has been adopted as a convenient convention.[6] As the bar lines are another editorial addition, they are shown as dotted lines. All translations of Pontio are my own, and I am very grateful to Alessandra Ignesti who helped me on numerous occasions with this task.

Interval-succession Examples

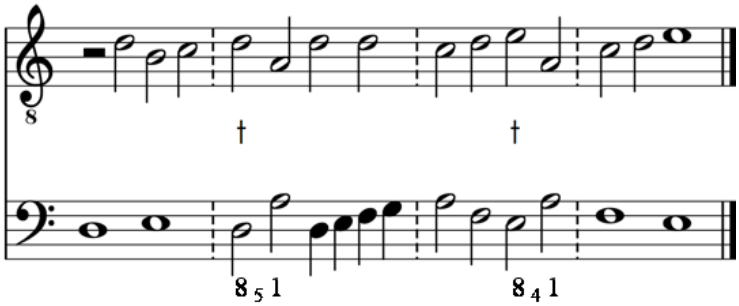
The wealth of musical context with which Pontio augments his examples is an important contribution to interval-succession theory. Figure 2 is a good example that conveys almost all of this contextual information.[7]

Figure 2: RM example showing two exceptional ways an octave can go to a unison.

Il secondo poi sarà, quando, fatta l'Ottava, anderete all'Unisono in principio di battuta, over' in elevatione della battuta, movendosi le parti con movimento separato, una ascendente, & l'altra discendente, come quì.	The second [way to go from an octave to a unison] will be, when, having made an octave, you will go to a unison on the strong part of the beat, or on the weak part of the beat, each part moving by leap, one rising, and the other descending, as here.

[6] Suzanne Clercx has noted that Pontio combined the parts in score in his second treatise: Pontio, *Ragionamento*, in *Postface* by Suzanne Clercx; Pietro Pontio, *Dialogo Del R. M. Don Pietro Pontio Parmigiano, Ove Si Tratta Della Theorica, è Prattica di Musica, & anco si Mostra la Diversità de' Contraponti & Canoni*, (Parma: Viotti, 1595). For more on the process of scoring up parts in the 16th century, see: Suzanne Clercx, "D'une ardoise aux partitions du XVIe siècle," in *Mélanges d'histoire et d'esthétique musicales offerts à Paul-Marie Masson professeur honoraire en Sorbonne* (Paris: Richard-Masse, 1955), 157–170.

[7] Pontio, *Ragionamento*, 65–6.



Questo movimento dico nelli contrapunti, di due, & tre voci esser di niun valore, & non esser permesso anco nelle compositioni di quattro, ò cinque voci. Vero è, ch'è permesso, anzi è suo proprio, il darlo alle compositioni d'otto voci à Choro separato; à fine ch'ogni Choro habbia il suo fondamento; perche un sol Basso non potrebbe supplire per la lontananza delli Chori; & cosi si permette, ch'essi Bassi passino dall'Ottava all'Unisono; & per contrario dall'Unisono all'Ottava, come anco vi dissi ragionando dell'Unisono; il che fece Don Pietro Pontio nel Magnificat Quarti Toni, il qual si truova nelli suoi Salmi ; & parimente nel Motetto ad otto voci *Iste homo*, ch'è nel primo libro de' suoi Motetti à cinque."

I say this movement in two or three-voice improvisations is worthless and is not permitted in four- or five-voice compositions either. It is true that it is allowed, and indeed finds its proper use, in eight-voice compositions for divided choir so that each choir may have its own fundamental, because a single bass could not compensate for the distance of the choirs. And in this way the basses are allowed to go from the octave to the unison and conversely from the unison to the octave, as I told you today in discussing the unison. Don Pietro Pontio does this in the *Magnificat Quarti Toni*, which is found in his psalms. And likewise in the eight-voice motet *Iste homo*, which is in the first book of his five-voice motets.

This example is very representative of Pontio's list because he provides a text description, an idiomatic two-voice example that features the succession twice, the metric position of the notes,

and an explanation of how many voices the succession is allowed in, both for improvisation and composition. In many of the examples he also provides details about interval quality, duration (especially for the successions containing a dissonance), texture, the other interval successions which are like the one in question, timbre, and genre. To accompany these examples he cites no fewer than 78 specific interval successions in repertoire from 21 different masters in book II.[8]

Example Annotations

With respect to the annotations of interval successions, Pontio generally uses a dagger, †, in the faster-moving part (usually the upper part) to direct the reader to the location of the interval succession being discussed, though sometimes this is absent, and in some cases it is used to designate other salient musical phenomena, such as points of imitation. As each succession usually appears twice in its example, Pontio often labels both occurrences. He is somewhat inconsistent with where these labels go, so I regularize the position of his daggers in my examples so that they are all found at the location of the first vertical interval in the succession being discussed. I indicate this editorial shifting where it occurs by putting the dagger in square brackets, [†]. In some cases a dagger appears at an interval succession that is similar to, but distinct from that being discussed. Where this appears to have been done in error, I convey this with a dagger and a question mark in square brackets, [†?]. In Figure 3 this is used to show that Pontio's second dagger appears to be a mistake, given that it designates the succession 5 $_{-3}$ 8 rather than 5 $_{1}$ 8 and its metric placement differs from that discussed in the text. While mistakes of this kind may provide insight into the copying and editorial process Pontio used, as well as any help he may have received in the setting of the examples, questions of this nature are beyond the scope of this study. Finally, a dagger in parentheses, (†), is used to indicate interval successions that correspond to that being discussed, but were not designated with a dagger. Figure 3 demonstrates how I render one of Pontio's examples.[9]

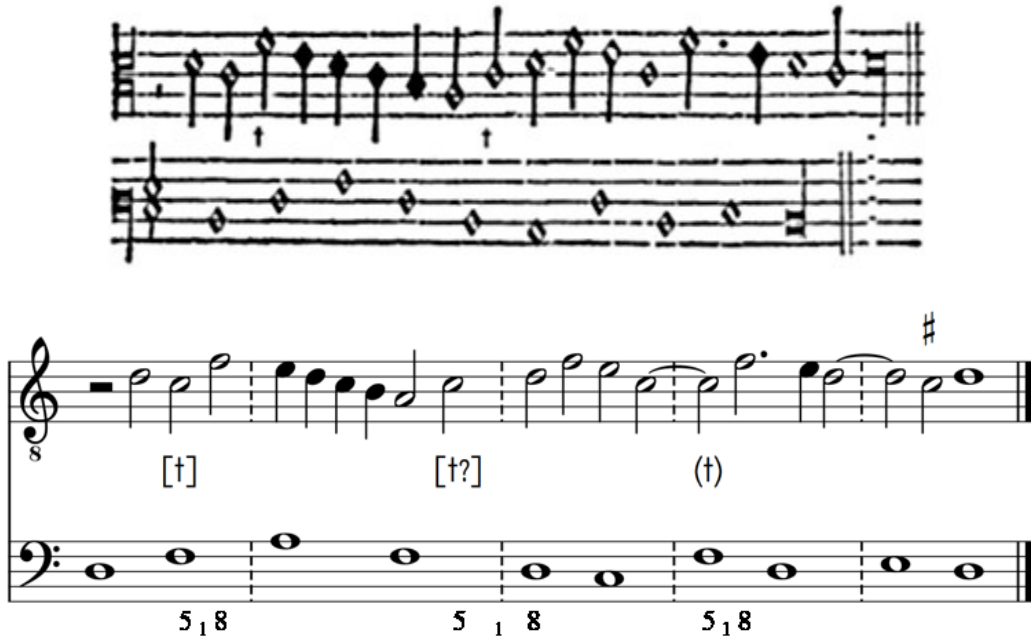
Figure 3: A comparison of the original image taken from the online edition and my transcription of Pontio's example for the 5 $_{1}$ 8 succession.

[8] Suzanne Clercx breaks these references down by composer for all of *RM* in her *Postface* and Christophe Dupraz provides a modernization of each composer's name. Pontio, *Ragionamento*, in *Postface* by Suzanne Clercx; Christophe Dupraz, *Traités Musicaux Romans* (www.tremir.fr), 2013, specifically: http://www.ums3323.paris-sorbonne.fr/TREMIR/TReMiR_Pontio/R0_start.htm .

[9] Pontio, *Ragionamento*, 45.

Il terzo è parimente di passare dalla Quinta all'Ottava; & questo si fà, stando ferma una parte, & l'altra andando alla Ottava in elevatione della misura, come quì.

The third [way] is likewise to go from the fifth to the octave, and this is done [with] one part staying still, and the other going to the octave on the weak part of the beat, as here.



Et questo è il più bello, & più gentile di tutti questi tre passaggi; & di questo si può servire in ogni sorte di contrapunti, Terzetti, & compositioni.

And this is the most beautiful and finest of all these three passages, and you can make use of this in every sort of improvisation, *terzetti*, and composition."

Terzetti

Peter Schubert has shown that in Renaissance counterpoint treatises, the term *contrapunto* generally referred to improvisation so I have translated the term accordingly.[10] In addition to improvisation and composition, Pontio mentions *terzetti* thirty-nine times in book II of *RM*, one of which is shown in Figure 3. I have been unable to find any mention of *terzetti* in any other primary source or modern literature. While it is clearly a piece of music in three

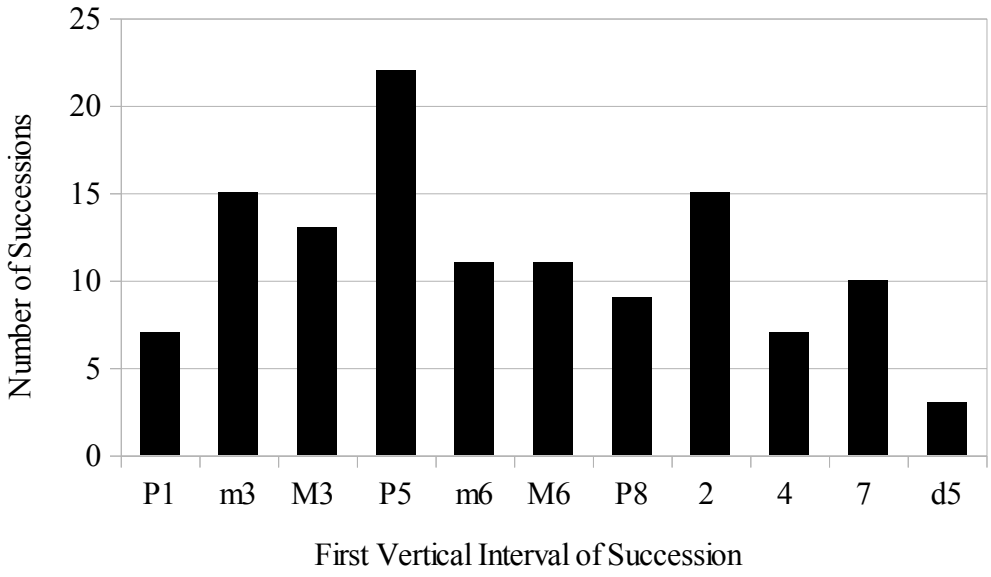
[10] Schubert, "From Improvisation to Composition," 122–6.

voices, it is not clear if it is improvised or composed as Pontio also makes references to improvisations and compositions in three voices. Though it is not certain, *terzetti* appear to be three-voice improvisations. The word *terzetti* appears unchanged in my translations.

The Structure of Pontio's List

Book II of *RM* contains Pontio's interval-succession list, with 130 musical examples in all. The list proper begins after five introductory examples serving to define simple or note-against-note counterpoint, florid or diminished counterpoint, perfect versus imperfect intervals, and stepwise motion versus motion by leap. There are two further examples in the middle of the interval-succession series that are of a general character and therefore do not convey any specific interval successions. This adds up to a total 123 interval-succession examples eighty-one of which convey fully consonant successions, and forty-two that include at least one dissonant interval in the succession. Figure 4 serves as an overview of these 123 examples, grouping them by the first vertical interval in the succession described in the order the intervals appear as chapter headings in *RM*. The logic behind the ordering of these chapters will be discussed shortly.

Figure 4: Bar chart of Pontio's interval-succession examples, as grouped by their first vertical interval.



No interval quality (major, minor, etc.) is assigned to the seconds, fourths, and sevenths because Pontio does not specify their quality in his chapter headings. I count repeated interval successions separately because generally each new instance adds something to the discourse. The

one instance of a consonant fourth is grouped with the "4" column which otherwise contains successions including dissonant fourths. These dissonant fourths are dissonant because they are over the bass (as almost all of Pontio's successions are), not because of an augmented or diminished interval quality. The consonant fourth is consonant because it is not against the bass and this succession, 4 $_4$ 1, is shown in Figure 13 and will be discussed shortly. In Figure 4 we can see that Pontio only includes intervals within the octave in his chapter titles and this is because he relies on octave equivalence to make his list applicable to simple and compound intervals rather than including several octaves. I will return to the pedagogical significance of this decision to rely on octave equivalence in describing interval successions.

After dividing his list into two main groups of successions (starting with a consonance or a dissonance), Pontio further distinguishes between successions according to the size of the first vertical interval in each succession. The list is not structured by the second vertical interval, nor by the connecting melodic motion of either of the two voices like Tinctoris's list is, as we saw in the previous chapter. Instead, the third and final means of categorization of Pontio's list structure is based on preferred successions. Pontio begins each chapter with the succession or successions that he considers to be the first vertical interval's most idiomatic continuation. Since these preferred successions are deemed as such based neither on the quantity nor quality of their successions' constituent intervals (as is the main organization of the list), their placement at the beginning of each chapter results in what may at first seem to be a haphazard ordering of successions within each chapter, but this is pedagogically-motivated and can be understood as a convenient convention for the reader; for any given starting vertical interval, the best choice of interval to follow it with appears first. For instance, the labels of all the successions in the chapter on the minor sixth are shown in Figure 5 in the order in which they appear.

Figure 5: Pontio's interval-succession examples in the chapter on the minor sixth, in order of appearance.

1. m6 $_{P1}$ P5
2. m6 $_{P1}$ P5
3. m6 $_{P1}$ m3
4. m6 $_{m2}$ M3
5. m6 $_{P1}$ m3
6. m6 $_{P5}$ m3

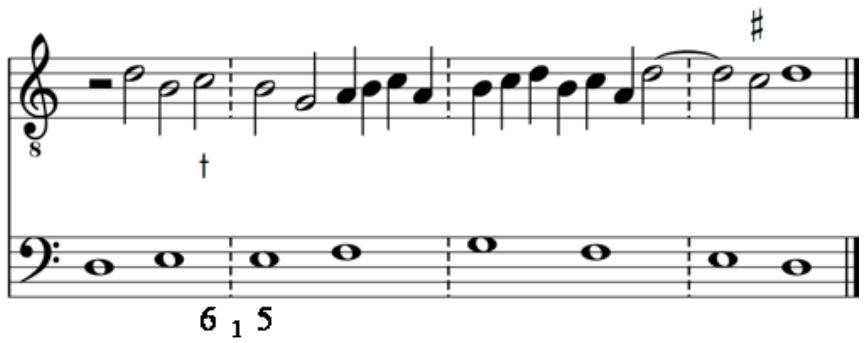
7. m6 $_{-M3}$ P8
8. m6 $_{m2}$ P1
9. m6 $_{P4}$ P1
10. m6 $_{-M2}$ m10
11. M6 $_{P5}$ M2
12. m6 $_{m3}$ P4

The first two of these, both m6 $_{P1}$ P5, respect the closest-approach rule in going from a sixth to a fifth. The reason that the same label appears twice is that Pontio finds it important enough to provide two examples for, the first when the minor sixth begins on a weak minim, the second a strong minim. The text description of these two examples, reproduced in Figure 6, maintains that this motion is something of a *passe-partout* in improvisation and composition.[11]

Figure 6: Pontios two preferred concatenations on the minor sixth, both of which are 6 $_{I}$ 5.

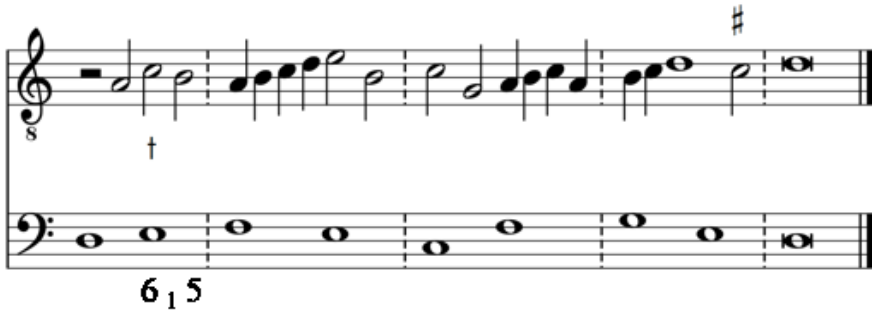
Et quanto al primo, che sarà, quando fatta la Sesta passerete alla Quinta, l'havrete da fare in dui modi.	And concerning the first, which will be when, having sounded the [minor] sixth, you go on to the fifth. You will have two ways of doing this.
Il primo, quando fatta la Sesta in elevatione della misura, verrete alla Quinta, rimanendo ferma una parte, come quì.	The first, when the sixth is on the weak part of the beat, you will come to the fifth [with] one part staying in place.

[11] Pontio, *Ragionamento*, 53–4.



L'altro, quando ambedue le parti ascenderanno, over discenderanno; una d'esse con movimento separato, l'altra con movimento congiunto, & oltra di ciò in principio di misura ambedue le parti si troveranno nella Sesta, & poi le medesime parti in elevatione della misura se truoveranno in Quinta, rimanendo ferma una parte, come quì.

The other, when both the parts will ascend or descend, one of these by leap, the other by step, and after this both the parts will be a sixth apart, and afterwards the aforementioned parts will be a fifth apart on the weak part of the beat, with one part staying still, as here.



E sappiate anco, che de questi due passaggi in ogni sorte di contrapunti, & compositioni vi potrete servire senza dubbio alcuno.

And know that you can make use these two passages in every sort of improvisation and composition without any hesitation whatsoever.

In summary, Pontio structures the interval-succession examples in his list on the basis of three hierarchic criteria with only minor exceptions (such as the penultimate member of the list in Figure 5). These three criteria are shown in order of importance in Figure 7 and will be compared to those of Tinctoris later in this chapter.

Figure 7: Hierarchy of criteria Pontio uses to structure his interval-succession list.

1. Consonance/dissonance of first vertical interval
2. Interval quantity and quality of first vertical interval, in ascending order (except d5)
3. Preferred successions

I will show that Pontio emphasizes the transmission of a variety of musical lessons rather than the strict organization of his examples. These lessons include how to differentiate between primary notes and their ornamentations, how to apply accidentals, and what justifies counterpoint that would normally be considered bad. One outcome of the lack of strictness in Pontio's approach is that his resultant list does not lend itself to the same analysis techniques applied to Tinctoris's in the last chapter. Nonetheless, conclusions about Pontio's musical thinking can be drawn by considering certain subsets of Pontio's examples together. In what follows, I will present these observations and by way of conclusion, I will compare Pontio's interval-succession list to Tinctoris's especially with respect to their theoretical and pedagogical approaches.

Bass Orientation

Pontio explicitly states that his interval-succession examples should be understood not against the tenor, but rather against the lowest-sounding voice, effectively cutting the number of successions he has to address in half since his referential voice is never the higher of the two. He expresses this conceptual point in no uncertain terms at the end of book II: [12]

> Non lascio anco d'avertirvi, che de tutt' i passaggi, de quali hò ragionato, così de buoni, come de non buoni, io intendo, che debbano esser fatti nella parte più Bassa ;
>
> I will not leave you either without mentioning that all these successions which I have reasoned about, for better or worse, I intend, should be done against the lowest

[12] Pontio, *Ragionamento*, 87.

percioche nelle parti di mezo questo ordine non può servarsi.

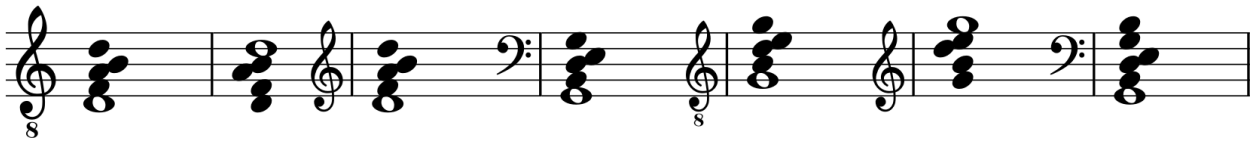
part; because none of these rules can apply to the internal voices.

Pontio is clearly aware of what a departure this bass orientation is from the standard tenor-centric model, given that he adopts an apologetic tone as he asserts the point. That being stated, this new orientation does not come out of nowhere; a fledging transition to lowest-sounding-voice orientation was already nascent in Tinctoris's *LAC*. It is true that in all 768 of his interval-succession examples Tinctoris refers to the pre-existing voice as the tenor, and that he articulates the "tenor principle," which states that in a polyphonic piece, the mode is decided by the tenor.[13] Yet despite this well-defined tenor orientation, Tinctoris explains that when several musicians are singing *super librum* (i.e. improvising over a given chant) they should avoid singing a sixth above the lowest voice at stopping points irrespective of the tenor's position in the texture.[14] Tinctoris demonstrates this with the example reproduced in Figure 8 wherein, regardless of the position of the tenor (given in open note heads), the other singers should avoid a sixth above the lowest-sounding voice because it would clash with other voices singing a fifth above that lowest voice.[15]

Figure 8: Tinctoris's example advising improvisers against singing a sixth above the lowest-sounding voice on the penultimate sonority before a stopping point. This is independent of the tenor's position, shown with open note heads.

> In addition, some, with whom I agree, say that it is not a bad thing if, with many singing *super librum*, some of them stop on an imperfect concord. However, I believe it must be understood that, where there are many more singing than [there are] perfect concords brought out by their voices, they should, therefore, also avoid the sixth, thirteenth and twentieth above the lower note; for none of these agree with a perfection because of their hardness, and particularly so with the fifth, twelfth and nineteenth, as here:

[13] Bernhard Meier, *The Modes of Classical Vocal Polyphony*, (New York: Broude Bros., 1988).
[14] Though not specifically defined by Tinctoris, these stopping points are presumably the goal points of internal or final cadences.
[15] Translation Seay's; example re-notated from: Johannes Tinctoris, *Johannes Tinctoris Opera Theoretica*, comp. Albert Seay. (Rome: American Institute of Musicology, 1961), bk III chap. 1, 146.



In Tinctoris's instructions above, the "imperfect concord" he allows at stopping points when improvising in many voices should be understood as a third (or octave-equivalent interval) above the lowest voice. Since a sixth above the lowest voice is not allowed, this amounts to the imperative that the arrival sonority of a cadence be a chord that, once note doublings are simplified, is composed of either a root and its fifth, or a root, third, and fifth.

Like Tinctoris, in spite of the general tenor orientation in his approach to music theory, in the interval-succession section of *Le institioni harmoniche* Zarlino accorded a special status to the bass:[16]

> As the earth is the foundation of the other elements, the bass has the function of sustaining and stabilizing, fortifying and giving growth to the other parts. It is the foundation of the harmony and for this reason is called bass, as if to say the base and sustenance of the other parts.

So the transition to a bass orientation was a long time in the making.[17] Practically speaking, it greatly simplifies interval-succession theory as a general means of understanding voice leading in polyphonic music because it places the referential voice in a pre-defined location, the bottom of the voice pair.

Interval Quality

Pontio mostly ordered his interval successions by ascending size of the first vertical interval, first with the consonances, then the dissonances. He deviates from this order at the very

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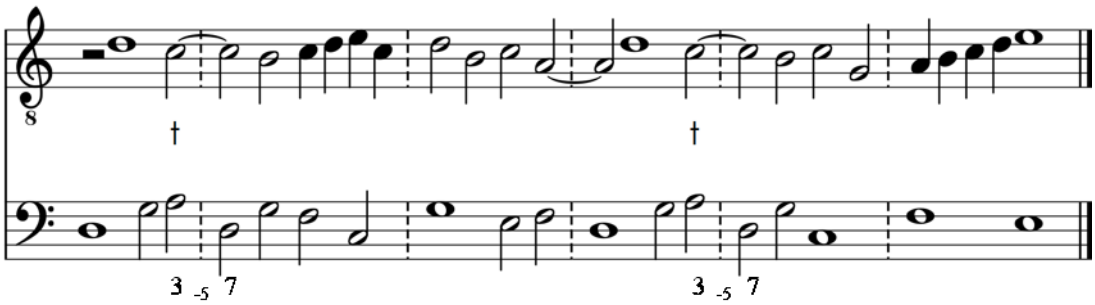
[16] Zarlino, *Art of Counterpoint*, 179–80.

[17] Note that while Pontio did not cite Tinctoris directly, he did cite Gaffurius who was heavily influenced by Tinctoris, and cited Zarlino twice, as noted in: Pontio, *Ragionamento*, in *Postface* by Suzanne Clercx. For more on the gradual transition from a tenor to a bass orientation in the Renaissance, especially with respect to Zarlino, see: Richard Crocker, "Discant, Counterpoint, and Harmony," in *Journal of the American Musicological Society* 15/1 (Spring, 1962) 1–21. Concerning developments in compositional process, see: Bonnie Blackburn, "On Compositional Process in the Fifteenth Century," *Journal of the American Musicological Society*, 40/2 (University of California Press, Summer, 1987), 210–284. Kevin Moll traces this issue in the Germanic literature in his editor's introduction to: *Counterpoint and Compositional Process in the Time of Dufay: Perspectives from German Musicology*, ed. and trans. Kevin Moll (New York: Garland Publishing Inc., 1997), 35–48.

end of the list, as a discussion of diminished fifths appears after that on sevenths. It is not clear if this means that Pontio considers diminished fifths to be a different, presumably more pronounced, category of dissonance, if they appear out of the established order because they represent the only interval which is dissonant only by virtue of its quality, or if this slight aberration has no explanation or is an error.

Pontio has separate chapters in book II of *RM* for interval successions starting on minor or major thirds and minor or major sixths but he is not overly rigid with his interval-quality organization. In his chapters on dissonance he does not distinguish between minor and major seconds and sevenths, or diminished, perfect, or augmented fourths in any systematic way, though any specific example is of course a manifestation of one or another specific interval quality. Because of this, four interval-succession examples are given out of order with respect to interval quality, such as Figure 9 that describes an interval succession beginning on a minor third despite being found in the *Della Terza Maggiore* chapter.[18]

Figure 9: Some of Pontio's interval-succession examples appear out of order with respect to interval quality such as this one which begins on a minor third but is found in the major third chapter.



It is not evident why some examples are found out of order in this way, but this does demonstrate Pontio's general lack of emphasis on maintaining a rigid organization throughout his interval-succession list.

Theoretical Consistency

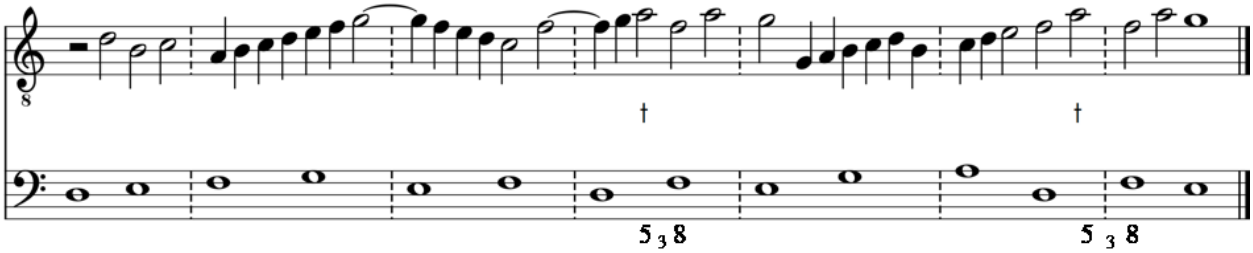
In this chapter I have been referring to Pontio's two-voice excerpts as examples; this is because they do not always have a one-to-one correspondence with what I refer to throughout this dissertation as rules, that is, specific individual interval successions. A "rule" in this sense is

[18] Pontio, *Ragionamento*, 42.

meant to work just like a mathematical equation: there is one correct answer and it should be memorized so that it can be recalled immediately, like 2 x 3 = 6. Pontio's interval-succession examples do not work like this. Rather, many of Pontio's examples resemble general principles that apply to a group of specific interval successions so there is not a one-to-one correspondence between one of Pontio's examples, and a specific interval-succession result.

One reason for this lack of one-to-one correspondence is that Pontio is content to rely on octave equivalence to address interval successions that begin on intervals greater than an octave rather than including extensive repetition across three octaves like Tinctoris. The clearest manifestation of Pontio's application of octave equivalence can be found in his example of a twelfth contracting to an octave reproduced in Figure 10, which is found in the chapter describing successions beginning on a perfect fifth, *Della Quinta*.[19]

Figure 10: Pontio's example of "fifth" going to an octave 5 3 8, though it is actually 12 3 8.



Octave equivalence is the most basic reason that each one of Pontio's examples does not correspond to a single interval succession. Pontio expects the reader to be able to simplify intervals or make simple intervals compound as necessary.

Another reason his examples are more like general principles is the fact that in many cases a single example corresponds to a group of similar interval successions, such as that shown in Figure 2. The two places in the example that Pontio designates with a dagger are actually slightly different successions, specifically 8 4 1 and 8 5 1. On top of this, in the text Pontio explains that you can do this in reverse as well, which effectively adds two more successions, 1 4 8 and 1 5 8. So this one example by Pontio is one-to-many because it corresponds to at least four distinct interval successions which all have subtly different fundamental counterpoint.

[19] Pontio, *Ragionamento*, 46.

While Pontio's presentation of interval successions eliminates most forms of theoretical redundancy, there are a few places in his list that amount to a repetition of an interval succession already discussed. While there are never six concrete manifestations of the same voice-leading idea as we saw was the norm in Tinctoris's list where successions were repeated across three octaves as well as with the tenor as the lower or upper voice, the few redundancies in Pontio's list allow for some checking to see how consistent he is. The purpose of these verifications is to get a sense of what Pontio prioritized while making his list. Strong consistency across his entire list, such as that we saw with Tinctoris's, would suggest that the organization of his list was very important. I will show that there are some inconsistencies in the list, and these are due to Pontio's prioritization of practical musical concerns over systematic organization. This suggests that the tenets of Pontio's interval-succession theory are like what Peter Schubert has termed "soft rules," that is, concepts to apply whenever possible but which can be overridden by a variety of musical concerns.[20]

Bi-directional List

Of note is the special organization of the first chapter of his list proper, *Dell'Unisono*. It begins with successions that go from some *other* interval to a unison, and then continues with those that begin on a unison and go to another interval. Pontio was not the first to group interval successions based on their second vertical interval; Vicentino and Zarlino did the same some thirty years prior.[21] This dual-ordered organization greatly increases the applicability of such a list because it allows a musician to also work backwards from a given interval, and see how he or she wishes to get there. The fact that the reverse order was not incorporated in the treatise for other intervals may suggest one of two things. Pontio may have originally planned an interval-succession list of twice the scope, but chose to simplify once he realized how involved his list already was with all of his other innovations; or the reverse order could have only been included in this one case because it is particularly appropriate for the unison given that one may easily identify a note as a cadential goal and plan to finish at a unison on it, and then backtrack to figure out how to get there. If this second hypothesis were the case, one would expect a similar dual

[20] Peter Schubert, *Modal Counterpoint: Renaissance Style*, 2nd ed. (New York: Oxford University Press, 2008), xi.
[21] Nicola Vicentino, *Ancient music adapted to modern practice* (*L'antica musica ridotta alla moderna prattica*), trans. Maria Rika Maniates, ed. Claude Palisca, (1555; New Haven, Yale University Press 1996), 87–93; and Zarlino, *Art of Counterpoint*, 77–8.

ordering in the chapter *Della Ottava*. Even more importantly, one would expect to find the standard closest approach interval succession m3 $_{m2}$ P1; this is included in the chapter on the minor third, but not in the reverse-ordered section of the unison chapter.[22] Vicentino's dual-ordering is also only given for intervals including a unison. This dual ordering in the unison chapter engenders some redundancy in Pontio's list as several of the reverse-ordered successions treated in this chapter go on to be repeated in subsequent chapters in the chapters corresponding to their first harmonic intervals. I will now turn to an examination of these "redundant" examples.

The first part of the unison chapter comprises eight interval successions shown with seven examples. Figure 11 provides these in the order of their appearance.

Figure 11: Pontio's examples from the Dell'Unisono *chapter which go to a unison.*

1. m3 $_{M2}$ P1
2. m3 $_{P1}$ P1
3. m3 $_{P1}$ P1
4. P5 $_{M2}$ P1
5. P5 $_{M3}$ P1
6. P4 $_{P4}$ P1
7. m3 $_{m3}$ P1
8. P5 $_{M3}$ P1

The second and third of these appear to be the same, but Pontio shows them as two separate examples because in the second, the note of the unison is attacked in both voices, and Pontio generally disapproves of "striking" (*percotendosi*) the unison in this way.[23] The fifth and eighth of the successions in Figure 11 also have the same label. The second of the two, shown in Figure 12, was added because that example contains a great deal of motivic repetition which Pontio often cites as a justification for what would otherwise be an undesirable interval succession.[24]

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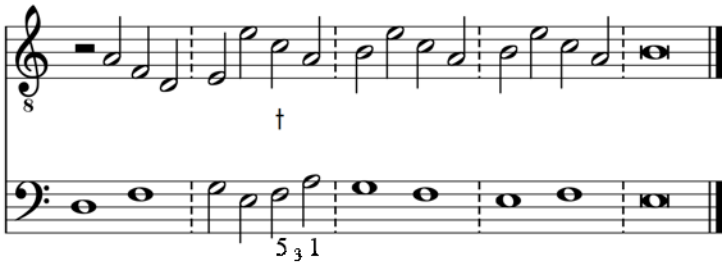
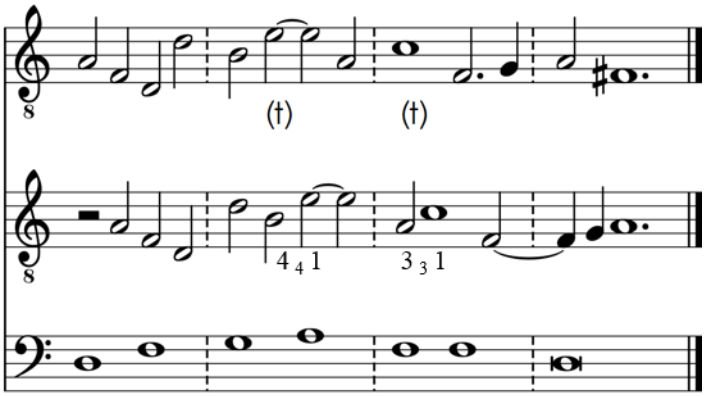
[22] The phrygian version of this succession, m3 $_{M2}$ P1, however, is found in the unison chapter.
[23] Judging an interval succession by the number of attacks it makes on a given note is effectively a timbral consideration.
[24] Pontio, *Ragionamento*, 28.

Figure 12: Pontio's second example for the P5 M3 P1 interval succession found in the chapter on the unison.



The sixth and seventh items in the list in Figure 11 are the two that share the same notated example. In this rather exceptional example in three voices, Pontio describes counterpoint occurring between the two upper voices. This is because the example in question, shown in Figure 13, consists of a canon in two upper voices harmonized with a supporting bass.[25]

Figure 13: Pontio's only interval-succession example in three voices, which he uses to convey the successions 4 4 1, and 3 3 1.



While this does technically contradict book II's concluding remarks cited above in the Bass Orientation section of this chapter, it does agree with Pontio's frequent tolerances of otherwise undesirable counterpoint for the sake of imitation, text painting, or other musical reasons.

Not all of the successions ending on the unison from subsequent chapters are found in this initial section, so the successions given in Figure 11 can in no way be taken as an exhaustive list of ways to get to a unison that Pontio approves of. Similarly three of the eight successions listed in Figure 11 do not appear in their normal place in the subsequent chapters. Of the five that do reappear in later chapters, two of these make reference to different musical-context details

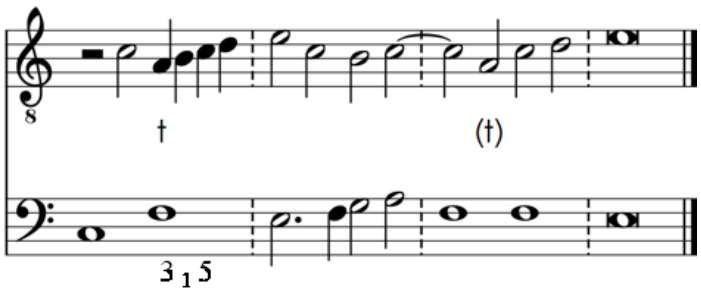
[25] Pontio, *Ragionamento*, 28.

than they did in their unison-chapter versions. These examples, indeed the very first interval-succession examples in *RM*, should give pause to any reader expecting to find the kind of unwavering theoretical consistency that, as we saw in chapter 2, is so characteristic of Tinctoris's list. Specifically, it is not feasible to identify interval successions that seem to be missing from Pontio's list because he does not provide enough clearly stated general voice-leading restrictions concerning which successions are permissible. Instead, my focus will be on determining the unstated analytical assumptions operant in Pontio's examples.

Analysis at the Minim

Upon examining all of his interval-succession examples it is clear that Pontio analyzes the vertical and melodic intervals of his examples at the regular time interval of the minim. This is apparent in Pontio's description of the example shown in Figure 14 wherein a third moves to a fifth with the lower voice staying still and the upper voice rising in semiminims.[26]

Figure 14: 3 ı 5 interval succession showing that Pontio analyzes at the minim.



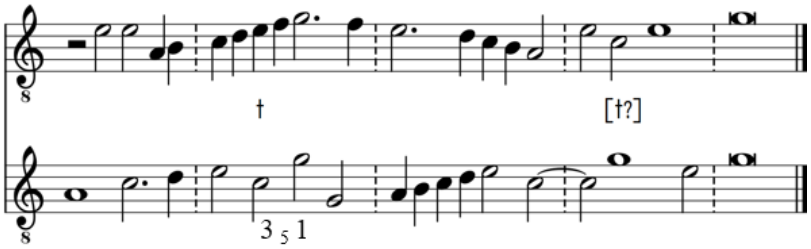
In the text description, Pontio explains that the third can go to the fifth in this way, either on the strong part of the semibreve or on the weak. Both metric placements are included in this example though Pontio only indicated the first one with a dagger, the parenthetical dagger is an editorial addition. Given that, as mentioned above, Pontio usually tried to insert the interval succession he was discussing into the example twice, I take the second instance as an intentional inclusion even though Pontio does not label it like he does the first. It is striking that the instance that is filled in with a weak semiminim passing tone has the same text description as the second instance of the interval succession where the upper voice is in minims. The example reproduced in Figure 15 goes even a step further because the text describes both parts as moving by leap, however at the

[26] Pontio, *Ragionamento*, 37.

first dagger (the second dagger appears to be a mistake) the upper voice can only be understood to be moving by leap if we reduce away the F semiminim passing tone which falls between the two intervals named in Pontio's text description.[27]

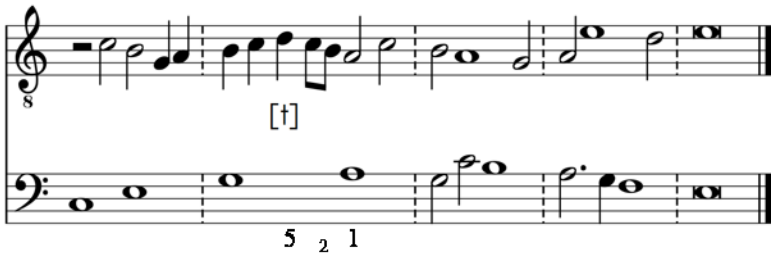
Figure 15: Example showing both parts moving "by leap" at the first dagger, confirming that Pontio analyzes at the minim.

Il quarto modo sarà, quando ambedue le parti per movimento separato, fatta che sarà la Terza, si troveranno in Unisono ascendendo, come quì.	The fourth way [to go from a major third to a perfect unison] will be, when both of the parts [move] by leap, having made a Third, they find themselves at a unison, rising as here.

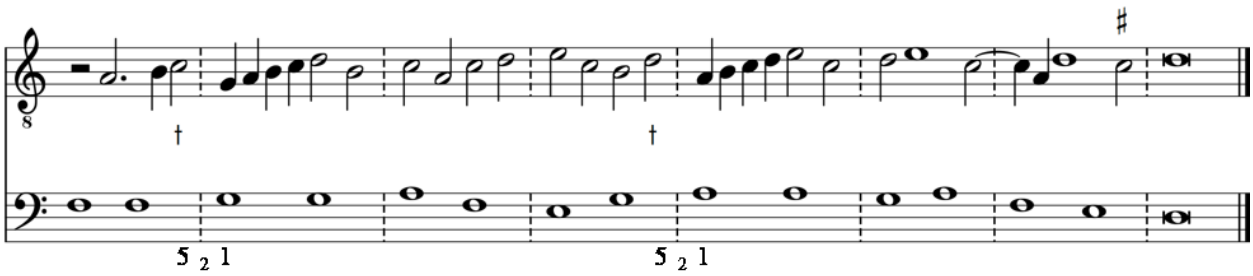


A final piece of evidence that Pontio analyzed his passages at regular minim intervals, can be found in the comparison of two of the times the P5 $_{M2}$ P1 succession is discussed. It appears ornamented with passing tones the first time, but unornamented the later, as can be seen in Figure 16.[28]

Figure 16: Two instances of P5 $_{M2}$ P1 found in RM. The first succession is ornamented though the second is not.



[27] Pontio's wording is "quando ambedue le parti per movimento separato…"; Pontio, *Ragionamento*, 39–40.
[28] Pontio, *Ragionamento*, 27 and 50.



In analyzing Palestrina, Peter Schubert has written that "the basic unit of consonance is the semibreve."[29] This may appear to disagree with the idea that the minim is the default durational value at which fundamental counterpoint goes by in Renaissance music, though Schubert's general analytical model in this examination of Palestrina clearly makes intervallic observations at regular minims, and this analysis decision is reaffirmed in his later analysis of duets by Lassus done with Julie Cumming wherein the authors explicitly define the regular minim spacing of their analysis.[30]

Consonant vs. Dissonant Interval Successions

Pontio divides his interval-succession list into two main sections, the first providing the interval successions that start on a consonance, and the second those that begin with a dissonance. Those beginning on a consonance can also go to a dissonance and this occurs in eight of the 89 successions that begin with a consonance. This means that 81 of his successions are fully consonant while 42 include at least one dissonance. In one of these 42, a dissonance resolves to another dissonance but this is a negative example and will be discussed later in this chapter.[31] The fact that there are about twice as many fully consonant examples in Pontio's list may seem to suggest that he is more concerned with describing consonant counterpoint. This, however, is not a fair conclusion given that the motion of at least one voice in a dissonant succession is highly constrained if the dissonance is to be prepared and resolved correctly, so there is naturally a more limited number of idiomatic successions that include dissonance. As a

[29] Peter Schubert, "Hidden Forms in Palestrina's First Book of Four-Voice Motets," *Journal of the American Musicological Society* 60 (2007), 498. For a corrected version of the appendix see: http://www.music.mcgill.ca/~schubert/finaltable.pdf.

[30] Peter Schubert and Julie Cumming, "Another Lesson from Lassus: Using Computers to Analyze Counterpoint," *Early Music* 43.4 (November 2015): 577–86. See also Peter Schubert, "A Lesson from Lassus: Form in the Duos of 1577," *Music Theory Spectrum* 17/1 (1995): 1–26.

[31] The interval succession is M2 -m2 d5. Pontio, *Ragionamento*, 71.

diminished fifth contains two tendency tones and therefore has highly constrained resolution for both voices, the influence of this concept can be noted in Pontio's chapter on this interval; it contains only three examples, the fewest of any chapter in book II, one of which is a negative example.

The inclusion of dissonances in an interval-succession list proper was not a first, as Vicentino had already included them in his 1555 treatise.[32] Vicentino's treatise was arguably much less accessible given that he bombards the reader with an array of complex tuning ideas and new notational symbols. Palisca has characterized Vicentino's approach as avant garde since he sought to push current music in new directions. [33] In contrast to this Pontio's treatise is much more traditional, easier to read and comprehend, and seeks to describe musical practices of past and present masters rather than change them. Whatever their readership and theoretical motives, Vicentino and Pontio were indeed modernizers of the interval-succession theory tradition which had dealt exclusively in consonances for centuries prior. Pontio was demonstrably well-read with respect to theoretical treatises, however, despite the numerous apparent influences Vicentino's treatise had on *RM*, Pontio never cites Vicentino in *RM* and I could establish no connection between the two.[34]

An example of a more conservative approach to interval-succession theory, Prosdocimo's stance on purely consonant interval successions was more normative:[35]

> The first rule is this: that the discords named above… are not used in counterpoint in any way, because, on account of their dissonance, they are deeply hostile to harmony and nature, which seem to be the end of this art.

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[32] Vicentino even provides a typology of suspensions based on their durations. Vicentino, *Ancient music*, 93–5. Also note that, while not exactly discussed as interval successions, Zarlino also provides similar examples in: Gioseffo Zarlino, *Art of Counterpoint*, trans. Guy Marco and Claude Palisca, ed. Claude Palisca (1558; New Haven and London: Yale University Press, 1968) 92–102.

[33] Concerning Vicentino's avant garde approach and readership, see: Claude Palisca, "Foreward by the Series Editor" to Nicola Vicentino, *Ancient music adapted to modern practice* (*L'antica musica ridotta alla moderna prattica*, 1555; New Haven, Yale University Press 1996), vii–viii.

[34] Russell Murray, "The Voice of the Composer: Theory and Practice in the Works of Pietro Pontio," (PhD diss., University of North Texas, 1989) vol. 1, 166 and 192.

[35] "Prima ergo regula est hec, quod discordantie superius nominate... nullo modo in contrapuncto usitande sunt, eo quod propter ipsarum dissonantiam cordialiter armonie et nature inicantur, que finis huius artis existere videtur"; Prosdocimo de' Beldomandi, *Contrapunctus*, trans. Jan Herlinger (Lincoln: University of Nebraska Press, 1984), 58–9.

Instead, Pontio addresses the use of seconds, fourths, sevenths, and diminished fifths in his interval-succession list and stated that:[36]

> Prima dunque dovete sapere, che due varietà di contrapunto si truovano; uno è detto contrapunto semplice, overo uniforme; l'altro florido, ò per dir meglio deminuito; & questo secondo è quello, che conviene à coloro, che di Musica vogliono far professione: perche con questo contrapunto ve ne passate alla compositione.
>
> The first thing you should know, is that there are two types of counterpoint. One is called simple or uniform counterpoint, the other florid, or to put it better diminished. And this second [type] is the one that is suitable for those who want to make a profession out of music because with this [type of] counterpoint you will go on to composition.

So Pontio's inclusion of dissonance is better understood as the theoretical application of the interval-succession approach to florid counterpoint.[37] This shift from note-against-note to florid counterpoint is a significant change for interval-succession theory. Concerning note-against-note versus florid counterpoint, Prosdocimo wrote that: "Counterpoint construed in the proper or strict sense is the placement of one single note against some other single note in a melody..."[38] Here the shift in musical thought surrounding dissonances is considerable. For Prosdocimo, florid counterpoint was not even truly counterpoint properly speaking; for Pontio, florid counterpoint is the counterpoint of professional musicians, whereas only beginner students or amateurs would use the note-against-note variety.[39] This marked contrast between Prosdocimo and Pontio is a

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[36] Pontio, *Ragionamento*, 21.

[37] For more on diminished counterpoint, see: Klaus-Jürgen Sachs, *Der Contrapunctus im 14. und 15. Jahrhundert: Untersuchungen zum Terminus, zur Lehre und zu den Quellen*, (Wiesbaden: Steiner, 1974); Ross Duffin, "*Contrapunctus Simplex et Diminutus*: Polyphonic Improvisation for Voices in the Fifteenth Century," *Basler Jahrbuch für historische Musikpraxis* (2007): 76–77; and Daniel Leech-Wilkinson "Petrus frater dictus Palma ociosa," *Grove Music Online* (Oxford University Press, accessed July 21, 2016), http://www.oxfordmusiconline.com/subscriber/article/grove/music/21495.

[38] Prosdocimo, *Contrapunctus*, 29.

[39] Note that for Prosdocimo the term *contrapunctus* did not mean improvisation as Peter Schubert has shown it generally did in the Renaissance. Instead, for Prosdocimo, unless otherwise qualified, this referred to the modern conception of note-against-note counterpoint which could pertain to improvisation or composition. Peter Schubert,

testament to how much interval-succession theory evolved in the Renaissance and by extension underscores the need for research on late-Renaissance interval-succession treatises.

Metric Position

Metrically speaking, in all of Pontio's examples we can distinguish between strong and weak minims, and by extension interval successions that start on strong or weak minims. In some cases the text description stipulates that either of the two metric positions is acceptable, and in others the succession in question appears twice in the example, in both metric positions. This information is particularly indispensable for the dissonant examples because in these cases the dissonance must be prepared, sounded, and resolved properly and metric position is a large part of this treatment, as can be seen in the succession shown in Figure 17 which reproduces one of Pontio's dissonant successions, namely 7 $_2$ 5. Note the abundance of durational and metric-position information he provides.[40]

Figure 17: Pontio's example for the interval successions 7 $_2$ 5.

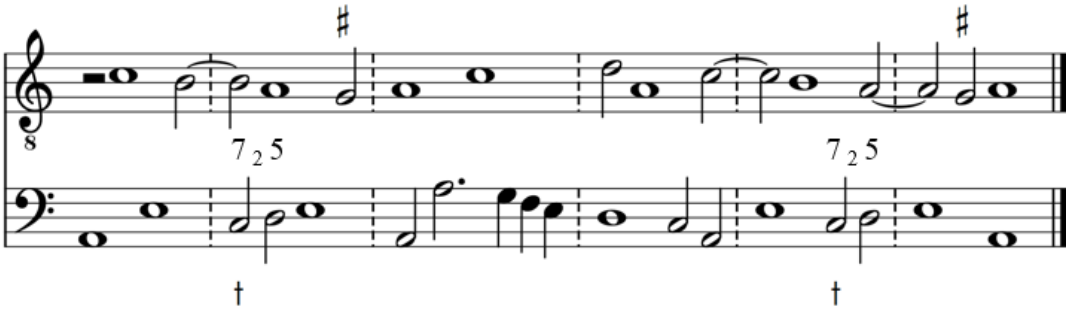
Primo, quando la parte Bassa passerà con movimento disgiunto con due figure di Minime all'incontro d'una figura di Semibreve posta nella parte superiore, & in elevatione della battuta, e quella prima figura della parte Bassa farà la Quinta in elevatione della battuta ; la Seconda poi farà la Settima in principio di misura, & se moverà per grado disgiunto ; la qual poi, fatta la Set[t]ima, ascenderà per movimento congiunto, & la superiore con	The first [of these successions from the seventh happens], when the Bass leaps [a third downward] with two minims against a semibreve in the upper part, and on the weak part of the beat. And on this first note in the bass a fifth will be sounded on the weak part of the beat. The second [note] then will make a seventh on the strong part of the beat, and will go by leap. After this, having sounded a Seventh, it will rise by step, and the upper part will descend by step; and the parts will

"Counterpoint Pedagogy in the Renaissance," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 503–533.

[40] Pontio, *Ragionamento*, 78.

movimento congiunto discenderà ; & le parti si troveranno lontano in Quinta. Del qual passaggio voi vi potete servire ne' contrapunti fatti sopra il canto figurato ; & anco ne' Terzetti, & di più anco nelle compositioni di più voci, come apertamente potete quì vedere.

be a Fifth apart. You can make use of this passage in improvisations made over a figurated chant; and also in *Terzetti*, and in compositions in more voices as well, as you can plainly see here.



Pontio does not use the metric information as an organizational criterion in his list. Given the abundance of details Pontio gives for each example, it would be tedious and infeasible for him to meticulously categorize the entire list on account of all these details. Perhaps for this reason, the list is only ordered according to a subset of these details, namely: consonance/dissonance, interval quantity and quality, and preferred usages. The remaining details provided, including metric position, therefore reduce the impression that the list is systematically conceived, as they do not participate in the organizational structuring of the list yet are constantly being discussed. The purely local scope of many of these tenets of musical context reduces the methodical quality some have closely associated with interval-succession theory.[41] Instead Pontio discusses the metric position of each of his examples individually in a way that does not seek to taxonomically order them according to this criterion.

[41] See especially: Sarah Fuller, "Organum – discantus – contrapunctus in the Middle Ages," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 477–502; and Anna Maria Busse Berger, *Medieval Music and the Art of Memory* (Los Angeles: University of California Press, 2005).

Repertoire References

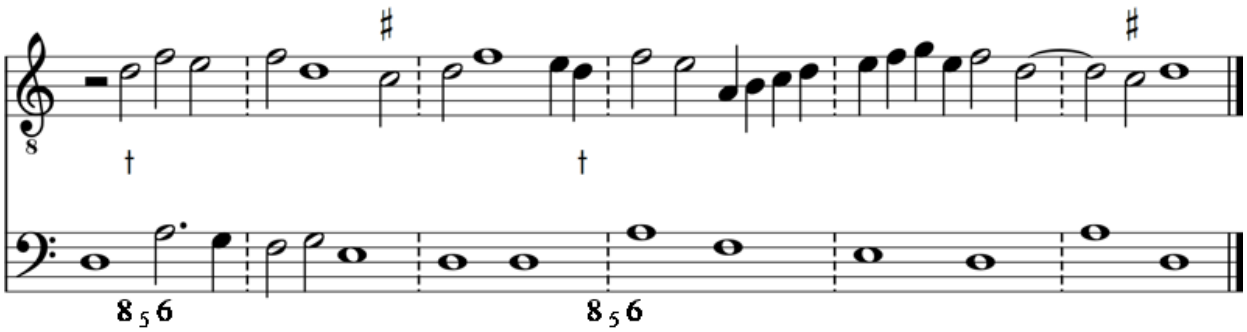
A further observation that only becomes apparent when one considers all of Pontio's examples together is the considerably greater concentration of repertoire references for examples that begin with a dissonance.[42] The eighty-one fully-consonant and the forty-two that contain dissonant intervals are accompanied by sixteen and sixty-two repertoire citations respectively. This amounts to an average of .2 repertoire citations per consonant interval succession and 1.48 per dissonant interval succession. This is significant because the dissonant examples contain over seven times as many repertoire citations as their consonant counterparts which suggests that Pontio sought to justify his inclusion of dissonance. This corroborates my previous assertion that Pontio was well aware of just how progressive the inclusion of dissonance in interval-succession theory was, and may have sought to preemptively quiet critics by directly linking it to actual musical practice. While idiomatic use of dissonance had already been a feature of early music for centuries, in-depth discussion of it in treatises only began with Tinctoris in 1477, and even more recently in the context of interval successions with Vicentino in 1555.[43] Pontio combined Tinctoris's new strand of music theory, the detailed discussion of dissonance, with the time-honored interval-succession approach.

Accented Dissonances

Occasionally, certain idiomatic uses of dissonance can displace a note in an interval succession from the strong to the weak part of the minim. Pontio conveys this point with the example of the succession 8 $_5$ 6, reproduced in Figure 18, which includes a dissonant third quarter.[44]

Figure 18: 8 $_5$ 6 interval-succession example demonstrating that a dissonant third quarter can displace the regular minim intervallic rhythm by one semiminim.

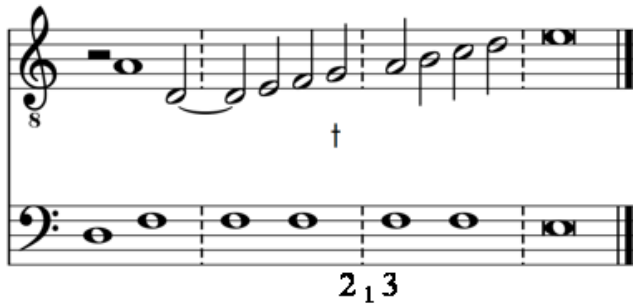
[42] For a comprehensive list of the repertoire references Pontio makes including a modern normalization of the names, see Christophe Dupraz, *Traités Musicaux Romans* (www.tremir.fr), 2013, specifically: http://www.ums3323.paris-sorbonne.fr/TREMIR/TReMiR_Pontio/R0_start.htm .
[43] Tinctoris, *Art of Counterpoint*; Vicentino, *Ancient music*.
[44] A dissonant third quarter is a special type of accented passing tone described in: Schubert, *Modal Counterpoint*, 87; Pontio, *Ragionamento*, 64.



The text description makes no reference to the E in the upper voice (in the second-to-last semiminim of the third editorial measure) and the interval succession can only be an octave to a minor sixth if one measures the D in the upper part against the D in the lower part. This vividly demonstrates that Pontio took musical syntax, especially idiomatic uses of dissonance, into consideration along with metric factors in order to determine which notes correspond to the fundamental counterpoint of a passage.

Despite the example shown in Figure 18 one cannot conclude that Pontio simply disregarded dissonances when describing interval successions because we already established that he allows dissonances to serve as the first or second interval in the successions he lists. So what determines whether or not Pontio will discuss a dissonance? An important example to consider concerning this question is one of the resolutions of a second Pontio offers, shown in Figure 19.[45]

Figure 19: One of Pontio's interval-succession examples including a passing tone on a weak minim.

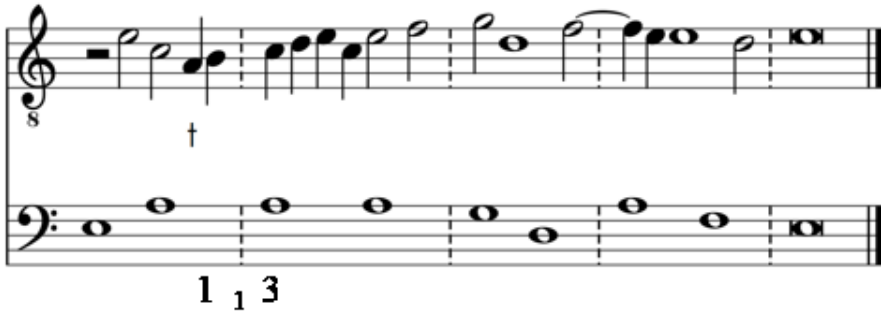


Here a clear instance of a passing tone on a weak minim gets its own interval-succession explanation. The main difference between this passing tone and those from Figure 14 and Figure

[45] Pontio, *Ragionamento*, 68.

15 that were omitted from discussion is that the one in Figure 19 is set with a longer duration, namely a minim. Since Pontio takes the minim as the default level at which he makes analytical observations, this dissonant second gets discussed on an equal footing with the consonant third that follows it. With this analysis Pontio makes the implicit assertion that a weak passing tone can occur at the same metric level as that on which the fundamental counterpoint is proceeding. Exposing this point is a critical observation about Pontio's reductive method to which I will return in the next chapter. For the moment we can simply observe that, had Pontio analyzed this passage at the intervallic rhythm of the semibreve (whole note), this passing tone would have been reduced out in the same way as those from Figure 14 and Figure 15. After applying a semibreve intervallic rhythm analysis, the resulting interval succession would be that of a unison going to a major third with the lower voice staying in place, P1 $_{P1}$ M3. Pontio allows for precisely this interval succession, though in shorter values, in his example reproduced in Figure 20.[46]

Figure 20: Pontio example of a unison moving to a third, P1 $_{P1}$ m3.



The "leap" of a third here during the interval succession is filled in with a passing tone making it even more similar to that shown in Figure 19. In all, the dissonance in five of Pontio's dissonant successions is a passing tone on a weak minim, and if analyzed at the semibreve instead, the succession of four of these five can be found among Pontio's consonant examples.

Considered together, these examples confirm that in his interval-succession treatise, Pontio generally analyzed at regular minim time intervals. This analysis decision seems to be fixed in Pontio's thinking. The only nuance he adds to this reductive method is to disregard

[46] While the third in this interval succession is actually minor, Pontio's description specifies that the progression is equally good when the third is major.

accented dissonances other than suspensions. While not stated explicitly, this analytical methodology is of great import to anyone wishing to apply Pontio's treatise to improvisation or composition because, in order to make use of his interval-succession examples one must know to what mensural level they correspond. Since there is no mensural variety in his analytical method (he always samples the counterpoint at regular minims), it is unclear how his approach applies to passages employing shorter or faster values, or different mensurations.

Dissonance Types

Examining Pontio's forty-two successions that contain dissonances, we can note that thirty-six of the dissonances are suspensions while only five are weak passing tones.[47] There is one accented passing tone (though not a dissonant third quarter) but this one, in the succession d5 $_{-M3}$ M6, is a negative example. This breakdown may seem strange since passing tones are generally the most common type of dissonance found in Renaissance music, and given that there are several other dissonance types idiomatic to this period (such as neighbor tones, nota cambiatas, escape tones, etc.) which are entirely absent in the successions.[48] This is not a reflection of the frequency of these various dissonance types in repertoire or even in Pontio's examples, but rather of the metric level on which they are operative. Since Pontio's discussion of interval successions analyzes at the minim, this greatly favors suspensions as this is the metric level where they are most commonly found. By contrast passing tones, neighbors, and other dissonance types are most commonly found on weak semiminims or fusae. As shown in Figure 14, Figure 15, and Figure 18, these shorter and metrically weaker dissonances get ignored by Pontio's reductive method.[49] Like the one from Pontio's example shown in Figure 19, occasionally passing tones do occur on weak minims, and in chapter 4 I will revisit the analytical importance of these longer dissonances. No diminished or augmented suspensions are shown in book II of *RM*.

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[47] Note that Pontio does not use the terms "suspension" and "passing tone."

[48] For more on the nature and types of dissonances found in Renaissance music, see: Andie Sigler and Jon Wild, "Schematizing the Treatment of Dissonance in 16th-century Counterpoint," in *Proceedings of the International Society for Music Information Retrieval*, 2015, 645–650.

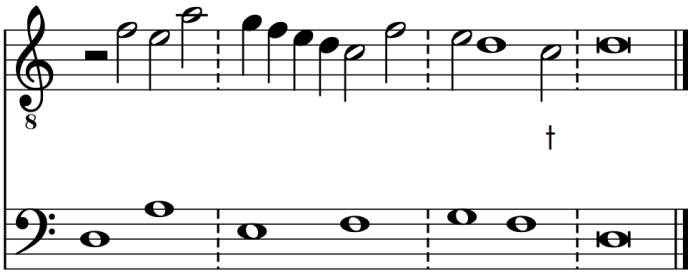
[49] For sample repertoire queries which analyze at regular minims, see: Schubert and Cumming, "Another Lesson," 577-86; and Christopher Antila and Julie Cumming, "The VIS Framework: Analyzing Counterpoint in Large Datasets," in *Proceedings of the International Society for Music Information Retrieval*, 2014, 71–76.

Accidentals

By grouping Pontio's examples that include and/or discuss accidentals, we can shed light on where and why he would make use of them in improvising or composing. As accidentals were often not notated in Renaissance music, it is especially interesting to examine treatises for information on the subject, as they are potentially sources where the use and discussion of accidentals is more explicit in order to train musicians to correctly add them where appropriate when no signs are provided. In a few places in book II of *RM* Pontio explicitly addresses the use of accidentals and explains what types of interval successions most often require them. The only other place in *RM* that has a considerable treatment of questions surrounding accidentals is in book IV, though the actual focus of that book is on mensuration and genre.[50]

With the examples and text description given in Figure 21 he explains that adding a sharp should be avoided if it will create an augmented fifth.[51]

Figure 21: Examples showing that some suspension resolutions should not be raised in order to avoid an augmented fifths.



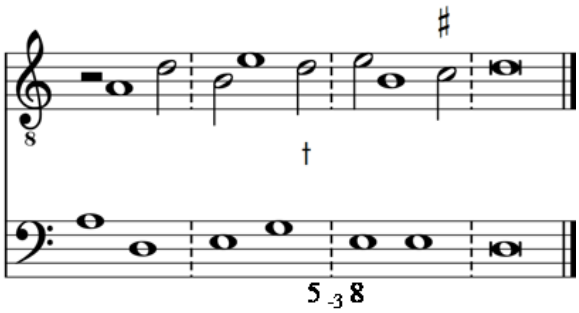
La cagione è questa ; perche quel sol, fa, sol, è posto in modo di cadenza ; & per che ordinariamente tutte le cadenze, che da se non fanno il semitono naturale, come, Mi, fa ; accidentalmente lo fanno, ancora che non siano segnate queste virgolette, ♯ per questa cagione il pratico cantore, vedendo tal

The reason [that such passages are to be avoided] is that this sol, fa, sol is set in the manner of a cadence. And since ordinarily all cadences which do not have the natural semitone, mi-fa, include it via accidentals, even when they are not marked with this sharp, #. For this reason, the practice of singers,

[50] Pontio, *Ragionamento*, 150–158.
[51] Pontio, *Ragionamento*, 46–7.

movimento accrescerà uno semitono quel fa ; talche sarà una Quinta falsa. La onde si deve lasciarlo. Se sarà poi naturale come quì, alquanto sarà men cattivo.

when seeing such motion, is to raise this fa. So then it will be an augmented fifth. Hence, this should be avoided. Like this it will be more natural, or rather less bad.



The fact that Pontio includes this example suggests that, at least at first, he assumes musicians will add accidentals on the basis of their line alone, not based on the vertical intervals their part makes with another part.[52] In both examples the suspensions are consonant therefore not cadential. Despite this, he says the D-C-D movement in the upper voice of the first example in Figure 21 (referenced as "sol-fa-sol") is "in the manner of a cadence" (*in modo di cadenza*). This is because, considered alone, the upper voice could potentially cadence to D at this point.

Less than 20 years after Pontio's *RM*, Burmeister's *Musica Poetica* distinguishes two definitions of a cadence:[53]

> There are two types of cadence: *tou meleos*, or that pertaining to melody, and *tes harmonias*, pertaining to harmony. A melodic cadence is the cadence of one voice extracted from the combined structure of a harmonic cadence. It consists of a beginning, middle, and end and is aimed at terminating the melodic period and the melody itself.

[52] Peter Urquhart, "Cross-Relations by Franco-Flemish Composers after Josquin," *Koninklijke Vereniging voor Nederlandse Muziekgeschiedenis*, vol. 43 (Koninklijke Vereniging voor Nederlanse Muziekgeschiedenis: 1993), 3–41.

[53] Joachim Burmeister, *Musical Poetics*, trans. Benito Rivera (1606; New Haven: Yale University Press, 1993) 106–109.

> A melodic cadence takes its origin from the combined structure of a harmonic cadence in the following way. The *triphonic* harmonic cadence (concerning which later) is constructed out of a combination of four primary voices. It has four combined melodies, the first suitable for the discant, the second for the alto, the third for the tenor, the fourth for the bass. The melody of any one of these primary voices, taken alone and apart from its bond with the other voices, is a melodic cadence.

Burmeister's harmonic cadence is the two-or-more-part cadence that is defined by its vertical intervals and includes a dissonant suspension; among modern scholars, this is the common understanding of the term cadence in Renaissance music.[54] His melodic cadence is any one-voice subset of the harmonic cadence, and this is what Pontio means when he says that a voice by itself is "in modo di cadenza" though Pontio only uses this phrase to refer to what Burmeister calls the discant cadence.[55] Burmeister uses the discant cadence to explicitly show how a melodic cadence typically associated with a particular voice part can be found in any other voice.[56]

Figure 22: Burmeister's example of the transferal of a melodic cadence from one voice to another.

> When the cadence of one melody is transferred to another voice, there is a change of register but not of name.

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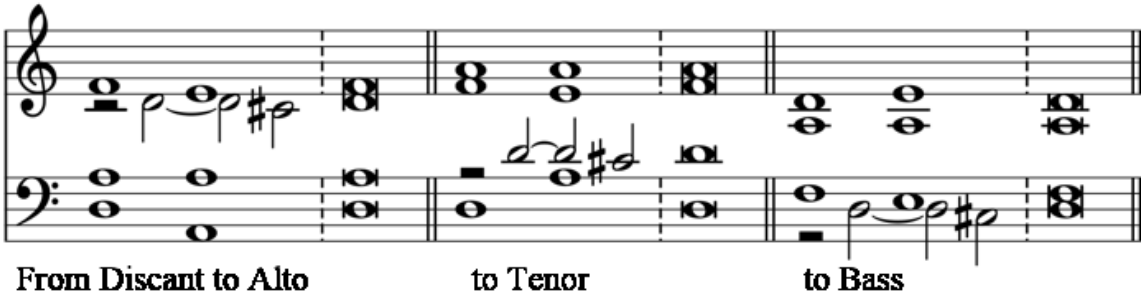
[54] Concerning cadential voice functions, see: Bernhard Meier, *The Modes of Classical Vocal Polyphony*, (New York: Broude Bros., 1988), 90–101; Karol Berger provides a more ample discussion of Renaissance treatise on the topic of accidentals at cadences, though he does not discuss Pontio and the theorists he does discuss justify accidentals based on the vertical intervals involved rather than on cadential patterns in individual melodies as Pontio does. Karol Berger, *Musica Ficta* (Cambridge: Cambridge University Press, 1987), 122–154. A comparison of Meier and Berger's approaches can be found in: Michèle Fromson, "Cadential Structure in the Mid-Sixteenth Century: The Analytical Approaches of Bernhard Meier and Karol Berger Compared," *Theory and Practice*, vol. 16 (Music Theory Society of New York State, 1991): 179–213. See also: Julie Cumming, "From Two-Part Framework to Movable Module," in *Medieval Music in Practice: Studies in Honor of Richard Crocker* (Middleton, Wisconsin: American Institute of Musicology, 2013) 181–2.

[55] Burmeister, *Musical Poetics*, 108–109.

[56] To this quoted translation, Benito Rivera adds the footnote: "That is to say, a discant cadence remains a discant cadence even when it is found in the alto part." Burmeister, *Musical Poetics*, 120–121.

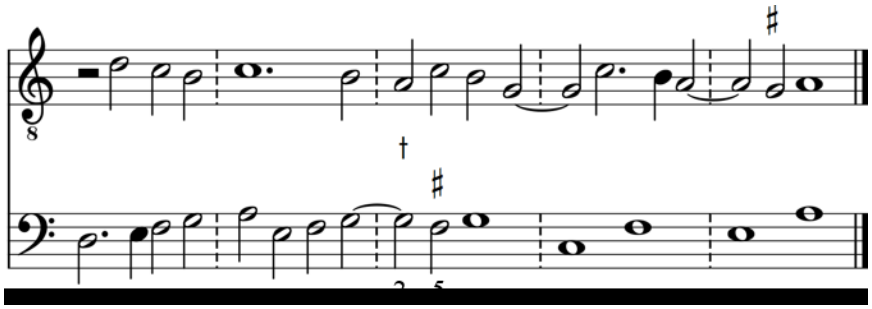


The default approach Pontio refers to, of adding accidentals primarily based on one's own line, is not so surprising given that people performed from parts, not scores. Nonetheless Pontio cites the example in Figure 21 precisely to show that the vertical intervals that are created are ultimately the most important criterion for when to add accidentals, even if they are not the first considered.

In light of the fact that Pontio generally assumes that musicians will add accidentals as necessary to cadence correctly, he must discuss interval successions where this approach runs into problems. His discussion of the interval succession M2 -m2 d5 shown in

Figure 23 is a good demonstration of the fact that Pontio assumes musicians will add a sharp to the note of resolution of suspensions.

Figure 23: Pontio's example which includes two consecutive dissonances if the F# is included.

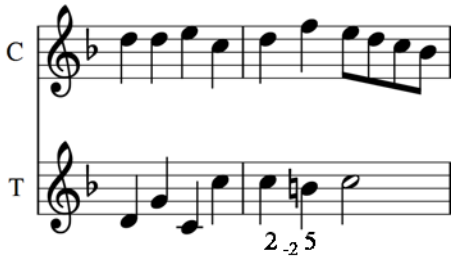


The F# is needed to make a cadential arrival on G. It does not appear in Pontio's musical example, however the accidental is discussed explicitly in the text. He does not need to include the F# because the bass is fulfilling the cantizans cadential function here (Pontio again says it is *in modo di cadenza*) so he expects the reader to know the F would usually be raised.[57] We can speak of a cantizans cadential melody when a voice sounds a suspension, resolves that down by

[57] Pontio, *Ragionamento*, 46.

step, then goes back up a step to the first pitch of the figure. Pontio refers to this example twice, the first time as a negative example when the F# is included, and the second as a regular example of good counterpoint when the F# is omitted.[58] Pontio admits that his disapproval of this passage is not uniformly shared by his contemporaries. He cites Kerle's *Magnificat Terzi Toni* as evidence of a master doing precisely this succession with the F#. This is, however, Pontio's only citation of Kerle, so one might conclude that this is a subtle critique of Kerle's craft rather than a more even-handed presentation of two differing views on the subject. Recently Sabine Feinen pointed to Pontio's frequent citation of Morales in *RM* as an indicator of an amelioration of the latter's status in the eyes of his peers, so by the same logic one might infer that only one reference to a composer with respect to what is called a negative example could be perceived as a slight critique.[59] Whatever the case, very similar counterpoint can be found in one of Morley's two-voice canzonets, shown in Figure 24, provided that we analyze at the level of the semiminim.[60]

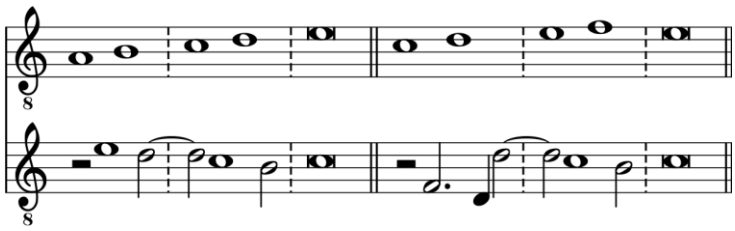
Figure 24: Mm. 5-6 from Morley's Goe yee my canzonets wherein the same M2 -m2 d5 interval succession that Pontio discouraged can be observed if we analyze at the level of the semiminim.



Pontio would also seem to differ with Zarlino, as the latter presents a similar resolution of a diminished to a major third at the end of one of his diminished counterpoint examples, reproduced in Figure 25.[61]

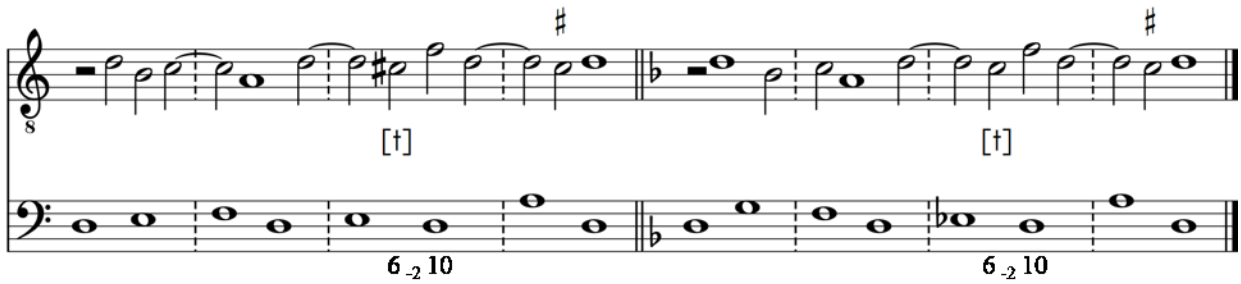
[58] Pontio, *Ragionamento*, 71.
[59] Sabine Feinen, "Cristóbal de Morales, the Light of Spanish Music: Cristóbal de Morales' Magnificats in Renaissance Music Theory" (paper presented at the Medieval and Renaissance Music Conference [MedRen], Sheffield, England, July 5–8, 2016).
[60] Thomas Morley, *First Book of Canzonets to Two Voyces*, ed. Bernard Thomas (1595; London: London Pro Musica, 2000), 2.
[61] Zarlino, *Art of Counterpoint*, 99.

Figure 25: Zarlino, diminished counterpoint example of a diminished fifth resolving to a major third.



In discussing accidentals, Pontio also addresses melodic concerns with the example shown in Figure 26.[62] Here the C should not be raised because it would create a melodic diminished fourth between C# and F in the upper voice. Instead a flat can be added to the E in the lower part in order to set up a cadence to D (a cadence that gets evaded) with a seventh resolving to a major sixth.

Figure 26: Pontio notates a passage twice with different accidentals to show that setting up a cadence with a flat instead of a sharp can avoid a dissonant melodic leap.

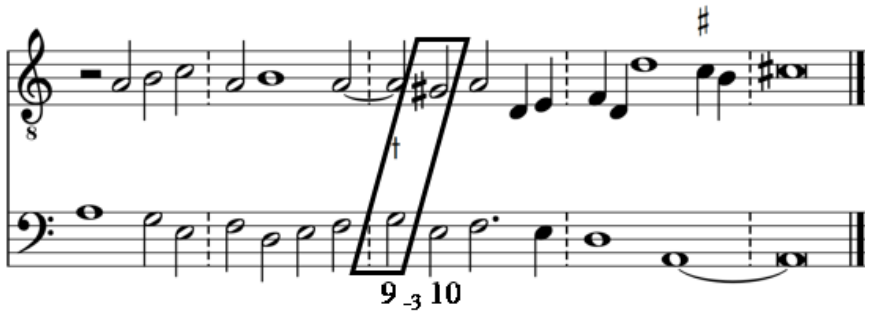


It is noteworthy that the recommended solution adds a flat to the lower part rather than a sharp to the upper part. Though Pontio does not comment on the applicability of this example to improvisation or composition, it is presumably included for composers only, as an improviser would not have control over both parts.

As a final example of Pontio's comments pertaining to accidentals, he uses Figure 27 to warn the reader against cross relations between the two voices.[63]

[62] Pontio, *Ragionamento*, 61.
[63] Pontio, *Ragionamento*, 69.

Figure 27: Pontio discourages cross relations between two voices in this example (between the G and the G#).



The examples reproduced in the last three Figures all pertain to the application of accidentals, but they are spread out across three non-adjacent chapters. They are representative of Pontio's interval-succession teaching in that they are replete with invaluable information, however, since that information does not participate in the organizational structure of the list it is found scattered throughout.

That examples relating to similar points are not necessarily found together combined with the fact that Pontio densely packs so much and so many types of musical context make this interval-succession treatise a poor candidate for memorization. Pontio's list is essentially the antithesis of the interval-succession lists that Busse Berger has generally characterized as systematic, repetitive, and easy to memorize. Instead, Busse Berger explains that theoretical accounts of florid counterpoint were generally not memorized, and Pontio's list seems to fall in this category, though she did not address any treatises that discuss florid counterpoint with an interval-succession-theory approach.[64] In lieu of the memorization of rules, *RM* invites a close reading and demands a great deal of reflection from the student. Their greater complexity makes his examples more akin to commonplaces, a point to which I will return shortly.

Tolerances

Pontio often tolerates interval successions of which he is not particularly fond, if there is a musical or expressive justification. These tolerances are usually for the sake of imitation or affect. A fascinating example, reproduced in Figure 28, of a tolerance that Pontio grants

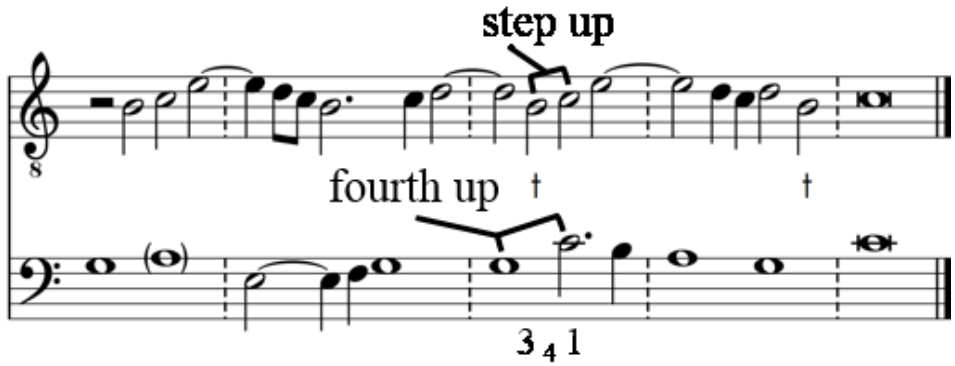
[64] Busse Berger, *Art of Memory*, 111–158.

begrudgingly for the sake of imitation or canon, starts as a laudable example, but this changes when the interval succession is subjected to mirror inversion.[65] The mirror inversion involves a step up in the upper voice becoming a step down in the lower voice, and a fourth up in the lower voice becoming a fourth down in upper voice.

Figure 28: A negative example that Pontio tolerates for the sake of imitation or canon.

Il terzo modo di far doppò la terza l'Unisono sarà, che tutte le parti ascendino, una per movimento congiunto, & questa sarà la parte superiore, & l'altra con movimento separato, come quì.

The third way to make a unison after a third will be, that all parts rise, one by step, and this will be the upper part, and the other by leap, as here.



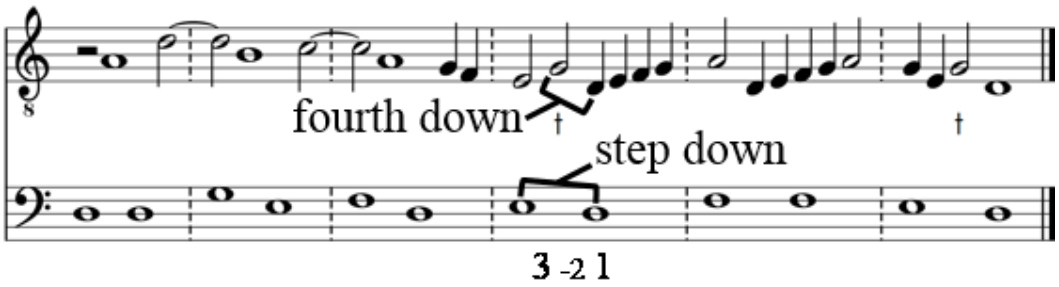
Il qual passaggio è perfettissimo in ogni sorte di contrapunto, & compositione, ne si può in ciò errare. Avertendo però, che detto passaggio con gli medesimi movimenti fatto, ma per modo contrario (ilche sarà, quando la parte Bassa anderà per movimento congiunto discendendo, & la

This passage is most perfect in every sort of improvisation and composition; one cannot go wrong with it. I am warning though, that this passage with the aforementioned movements done, but in the opposite way (which will be, when the Bass part will go down by step, and the upper [part]

[65] Among other things, this excerpt is notable for being, to the best of my knowledge, the earliest reference to diatonic mirror inversion of an interval succession. Pontio, *Ragionamento*, 38–9.

superiore per movimento separato, pur discendendo) senza dubbio sarà di niuno valore, & questo aviene per il modo del cantare, che discendendo ambedue le parti fanno appresso gli ascoltanti giuditiosi non buono effetto ; Et per tal rispetto in ogni sorte di compositioni, & contrapunti non è da farsi, come potete veder quì.

descending by leap) without a doubt will be worthless, and this comes about because of the way the singers, in making both parts descend, will make a bitter impression on judicious listeners. And for this concern, in every sort of composition and improvisation it is not to be done, as you can see here.



Non già perche sia consonantia falsa, ma è ben passaggio senza alcuna gratia, & per dirlo più chiaro è goffo : Et questo ancora sia detto della Terza minore, avenga che da alcuni scrittori sia concesso. Ma dirò con buona pace loro, ch'à me pare, non habbiano ragione alcuna ; perche se questo fosse tenuto buono passaggio, come sono gli altri, da i Musici sarebbe stato fatto ; ilche non si vede esser usato da alcuno, se non è stato astretto da qualche sua imitatione, come fece Cipriano nel Madrigale. *Era il bel viso suo,*

Not really because it would be a diminished interval, but it is indeed a passage without any grace, and is clumsy to put it more clearly. And even coming from the minor third, although some writers would allow it. But I will say *pace* them, that it seems to me that it would make no sense whatsoever. Because if this were held as a good passage, as are the others, it would be done by musicians; [yet] this is not seen in use by anyone unless he is constrained by some imitation, as [Rore] is in the madrigal *Era il bel*

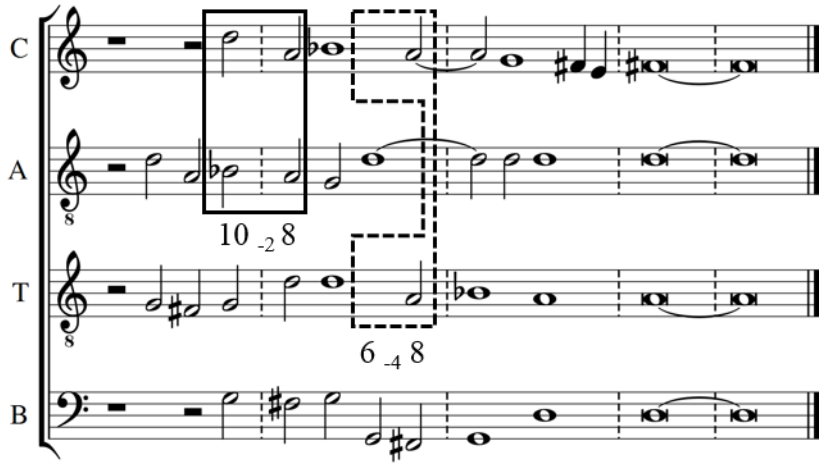
qual esser suole, nel secondo libro de suoi Madrigali à quattro, nel fine della seconda parte di esso Madrigale, & lo fece per la imitatione. sì che ò per imitatione, ò per canoni il compositore sarà escusato, altrimente io per me non laudo, che si faccia in modo alcuno.

viso suo, qual esser suole in the second book of his four-voice madrigals [where] he does it by imitation. So, for the sake of either imitation or canon, the composer will be forgiven. Otherwise for me, I do not praise its use in any way.

It seems unlikely that this is a veiled attack on Rore's compositional craft. While there is no surviving proof that Pontio was Rore's student, the master did recommend a young Pontio for his first job in Bergamo.[66] Furthermore, with nine repertoire references Rore is the third-most cited composer in book II of *Ragionamento* of the 21 to which Pontio refers (after only Pontio himself and Jacquet of Mantua), so he is clearly held in high esteem.

After close examination of Rore's madrigal *Era il bel viso suo*, I was unable to find the exact interval succession Pontio cited. The closest match is the succession M10 -m2 P8 found between the cantus and the altus at the end of the *seconda parte*, reproduced in Figure 29.[67]

Figure 29: Mm. 97–100 from Rore's Era il bel viso *which include the passage Pontio seems to be referring to.*



[66] Russell Murray "Pontio, Pietro," *Grove Music Online* (Oxford University Press, accessed July 21, 2016), http://www.oxfordmusiconline.com/subscriber/article/grove/music/22096.

[67] Recopied with a minim-equals-half-note transcription from: Cipriano de Rore, *Cipriani Rore opera omnia*, ed. Bernhard Meier (Rome: American Institue of Musicology, 1959), 3–7.

In addition to appearing in an octave-equivalent format between the altus and cantus (designated with a box), the succession also appears inverted at the octave between the tenor and the cantus (designated with a dashed box).[68] Having familiarized ourselves with the example, we should re-consider our reading of Pontio's text description. There are two key observations to be made. At first it seemed that he meant that this interval succession entailed a minor third contracting to a unison, but in light of the fact that Rore's third (in fact a tenth) is major, it is apparent that Pontio's quality description "And even, coming from the minor Third…" was meant to convey that even if the third is minor, the succession is still undesirable. Even more significant, the succession is found in a voice pair that does not include the bass, seemingly contradicting Pontio's assertion, cited in the Bass Orientation section of this chapter, that one should measure interval successions against the lowest-sounding voice. We already saw another exception to this bass-orientation principle in the Bi-directional List sub-heading above, and this was for the sake of canon. This suggests that Pontio felt the need to have an analytical approach that responded to the contrapuntal texture of the music he was examining, namely whether it was imitative or not.

Similar to those exceptions made for imitative textures, Pontio includes some tolerances in his list for the expression of affect, as shown in Figure 30.[69]

Figure 30: An interval succession that Pontio allows for the sake of expressing affect.

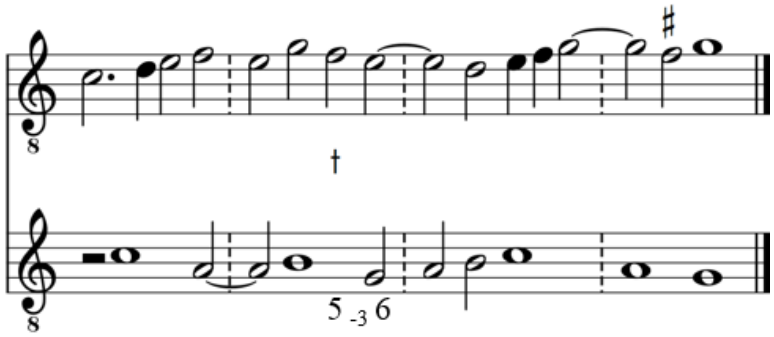
Il secondo passaggio dunque sarà, quando fatta la Quinta, passerete alla Sesta ; & questo in tal modo si fà, cioè una parte anderà per movimento congiunto, & l'altra per movimento separato. Ma sarà molto migliore, quando la Quinta sarà in elevatione, se ben' anco si può fare	The second passage therefore will be when, having made a fifth, you go on to the sixth, and this in such a way that one part will go by step, and the other by leap. But it will be much better, when the fifth will be on the weak part of the beat, though it can also be done on the strong

[68] Note that this inversion at the octave is not the same type of mirror inversion that Pontio discussed in his text quoted above.

[69] Concerning the period perception of interval affect, see: Timothy McKinney, *Adrian Willaert and the Theory of Interval Affect: the 'Musica nova' Madrigals and the Novel Theories of Zarlino and Vicentino* (Franham Surrey, England: Ashgate, 2010); Pontio, *Ragionamento*, 86.

in principio di misura, come quì.

part of the beat, as here.



Il qual passaggio vi servirà in qualche vostro soggetto ne' contrapunti, & nelle compositioni. Ma vi dico bene, che non vi essendo occasione, io per me non laudo cotal movimento ; percioche facendolo si rende duro, & aspro il contrapunto, & la compositione, & per questo rispetto laudo il lasciarlo da banda. Pur quando ve ne vogliate servire, potete farlo per imitare qualche effetto duro, & aspro, & così con giuditio, & à proposito sarà fatto.

This passage will serve you in some of your motives in improvisations and in compositions. But I tell you, without there being special circumstances, I personally do not applaud such movement because doing it renders the improvisation or composition hard and harsh. And for this concern I recommend leaving it by the wayside. So when you want to make use of it, you can do it in order to imitate some hard, and harsh effect, and in this way judiciously, and in this context, it will be done.

We already saw that this accented passing tone at the minim level is the only one discussed in Pontio's entire treatise, and now we see that this is because it is tolerated precisely to convey a harsh affect, the diminished fifth coming as it does on a strong minim. Otherwise, Pontio explicitly states that this is better on a weak minim. The fact that Pontio allows for regular musical syntax to be changed in the name of affect puts him more in line with his contemporaries

of the nascent *secunda pratica* and suggests that Clercx's portrayal of him as decidedly conventional was perhaps slightly overstated.[70]

Comparing Tinctoris and Pontio

Having discussed the basic structure of Pontio's interval-succession list as well as the aspects of his intervallic thinking that can be gleaned from it, I will now examine the extent to which his list can be compared to Tinctoris's. Both theorists are important authors in discourse on interval-succession theory, and I will explain the most significant differences in their theoretical and pedagogic approaches.

The main structuring principles of Tinctoris's interval-succession list were given in chapter 1 and those of Pontio earlier in this chapter. They are reproduced next to each other for easier comparison in Figure 31.

Figure 31: Ordering principles in Tinctoris and Pontio's interval-succession treatises.

Structural Level	Tinctoris - *LAC*	Pontio - *RM*
First	1st vertical interval, asc. quantity	1st vertical interval consonant/dissonant
Second	2nd vertical interval, asc. quantity	1st vertical interval, asc. quantity and quality
Third	Melodic motion of tenor, asc. quantity	Preferred successions

Pontio's discussion of interval successions in general groups and with both regular examples and negative examples make his list less systematic than Tinctoris's. This is part of why Pontio's list is considerably shorter than Tinctoris's. This means that it is not possible to extrapolate tacit guidelines from consistencies in Pontio's list to the same extent and with the same techniques as I did in the previous chapter with Tinctoris's.

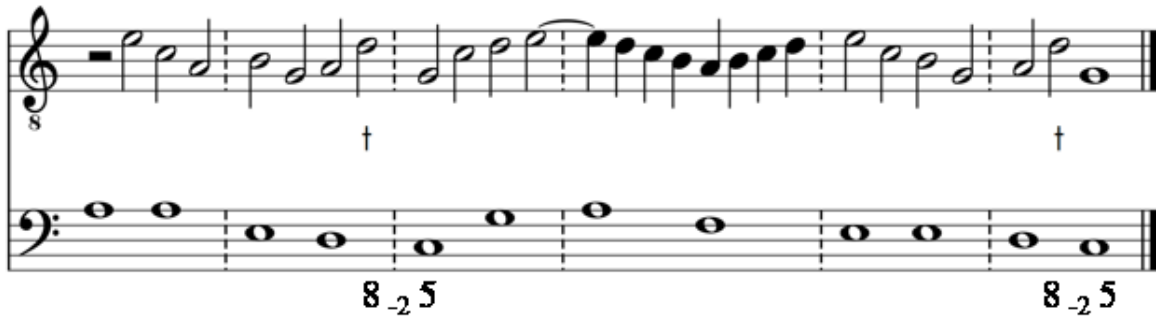
Tinctoris lists all the interval successions he finds acceptable, whereas Pontio is occasionally content to generalize that all ways of going from a given interval to another are

[70] Pontio, *Ragionamento*, in *Postface* by Suzanne Clercx.

excellent. For example, Pontio's makes the broad generalization that all successions of an octave to a fifth are good, save one which he presents as a negative example shown in Figure 32.[71]

Figure 32: Pontio's negative example illustrating the one way that going from an octave to a fifth is undesireable.

…in tutti i modi, che saranno fatti, cioè nei contrapunti, Terzetti, & compositioni ; & questo così in principio di misura, come in fine, in tutti i modi dico faranno buono effetto. si truova però un passaggio dall'Ottava alla Quinta, che nel contrapunto di due voci, non hà del giudicioso ; & questo sarà, quando ambedue le parti discenderanno, una con movimento congiunto, & l'altra con movimento separato, come quì.	…In all ways that they will be done, be they in two- or three-voice improvisations and compositions, and these on the strong part of the beat as on the weak, in all ways I say they will have a good effect. But there is a passage from the octave to the fifth that is not sensible in two-voice improvisation and this is, when both of the parts descend, one by step, and the other by leap, as here.



This brief text explanation accompanied by one negative example succinctly conveys an amount of voice-leading information that took Tinctoris thirty interval successions to articulate. With all these differences in mind, Tinctoris and Pontio's interval-succession lists have relatively little ground for comprehensive comparison.

Beyond being a good general sample of Pontio's examples, the specific successions in Figure 2 of octaves contracting to a unison by leaps of fourths or fifths in both voices are

[71] Pontio, *Ragionamento*, 63.

evidence of the fact that interval-succession theory was in need of an update for 16th-century musical practices. This voice-leading scenario is one that Tinctoris systematically excluded from his treatise in its contracting versions (i.e. 8 $_4$ 1 and 8 $_5$ 1) and systematically discouraged with his word choice in its expanding versions (i.e. 1 $_{-4}$ 8 and 1 $_{-5}$ 8). In chapter 1 I established that when Tinctoris omitted an interval succession that followed all of his explicit general principles, it amounted to him discouraging its use. Tinctoris could simply omit discussion of these interval successions because the one texture in which Pontio permits this succession, polychoral works with multiple bass parts, was not in use in the 15th century. This confirms that interval-succession theory was still relevant and evolving in the late 16th century as it was manifestly responding to the thicker textures that became relatively common in the early 16th century. This also demonstrates the usefulness and specificity of even just one component of the wealth of musical context (in this case the texture) that Pontio applied in his interval-succession theory.

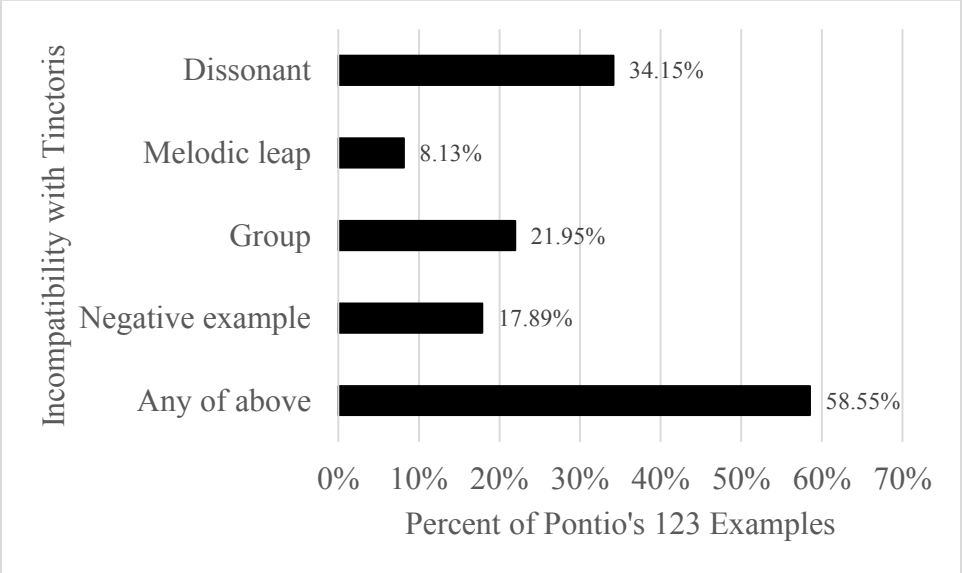
Tinctoris's 768 interval successions may at first seem to dwarf Pontio's 123, but when we keep in mind that Pontio does not include octave repetition and does not describe the same succession twice when the tenor switches from the lower to the upper voice, we see that the two lists are of comparable length. If we only consider one of each group of six successions which convey the same fundamental counterpoint, Tinctoris's list contracts to approximately 140.[72] Furthermore, as Pontio's examples often stand for a group of similar but distinct successions, we could easily consider Pontio's list to be more than 123 successions long. It is intriguing that the two theorists treat roughly the same number of fundamental counterpoint scenarios in their treatises despite their considerably disparate approaches. Therefore Tinctoris's afore-mentioned reputation as the most thorough interval-succession treatise author is somewhat unfounded given that his record number of examples has more to do with the theoretical assumptions behind the structuring of his list than with its contrapuntal completeness.

So Tinctoris and Pontio present about the same number of fundamental-counterpoint scenarios, but are they the same ones? There is overlap but the two theorists are too conceptually far apart to have identical lists. The main difference between the two lists is the fact that Pontio allows either the first or the second harmonic interval in his successions to be dissonances. Forty-

[72] This is *approximately* 140 because there is some minor variation in Tinctoris's list in light of the special circumstances discussed in chapter 1.

two of Pontio's 123 interval successions fall in this category. Also in contradiction with Tinctoris's explicitly stated voice-leading principles, ten of Pontio's successions employ a melodic interval in one of the voices which Tinctoris did not allow for in his list. Twenty-seven of Pontio's examples correspond to *groups* of interval successions (such as the group containing 8 $_{4}$ 1, 8 $_{5}$ 1, 1 $_{-4}$ 8, and 1 $_{-5}$ 8) rather than a single succession, making them impossible to compare to Tinctoris's successions. As a final point pertaining to the incompatibility of the two lists, Pontio's includes twenty-two negative examples, whereas Tinctoris simply omits undesirable interval successions from his list, as I demonstrated in chapter 1. At least one of the incompatibilities mentioned above is operative in seventy-two of Pontio's 123 examples. These features of Pontio's examples that are incompatible with Tinctoris's interval-succession list are illustrated as percentages of the total 123 examples in Figure 33.

Figure 33: Incompatible features of Pontio's examples with Tinctoris's interval-succession list.



While their interval-successions lists allow for only nominal comparison, we can compare the principles behind the two author's theoretical and pedagogical approaches.

By virtue of the length of his list, Tinctoris is very thorough, methodical, and explicit about what he thinks of numerous interval successions, but he does not comment on key theoretical assumptions and issues, such as how the musical surface should be reduced to arrive at the interval successions he provides. So he is very specific with respect to the intervallic aspect of music (both vertical and melodic) whereas he is temporally general in that his

successions do not communicate any durational or metric information. Pontio is the complete opposite. He relies more on the reader's common sense to ensure that interval successions are idiomatic. We saw he writes that all ways of going from an octave to a fifth are excellent, save one which he cites as a negative example. Does Pontio really mean *all* other ways? With respect to intervallic information, Pontio is often more general by virtue of his groups which contain slightly different interval successions. Conversely Pontio is much more precise with respect to both duration and meter. His examples are much more ecologically correct than the abstract ones of Tinctoris, as Pontio's are shown in convincing two-voice passages.[73]

Russell Murray has offered a differing interpretation of Pontio's interval-succession examples. In discussing Pontio's first succession coming from a minor sixth (the first shown in Figure 6) he maintains that "…we see that there is really only one crucial thing to learn—the passage marked with asterisks [daggers]. The remainder is superfluous, and it is hard to tell whether the examples are intended to be sung or are merely there for study."[74] Unlike Murray, I interpret the wealth of context in Pontio's examples as a useful and natural outgrowth of his pedagogical approach which is very hands-on and expects much from the student.

The difference between the approaches of Tinctoris and Pontio may be due to their intended purposes. Pontio seems to be targeting more mature and more advanced students. Murray has pointed out that *RM*'s intended audience was presumably students who had mastered basic musicianship skills because no simple elements are discussed in the treatise.[75] While none are discussed in the *LAC* either, Tinctoris could skip these lessons because he had already covered them in his other treatises, most notably the exposition on the Guidonian hand.[76] Don Hettore, the student in the dialog of *RM*, even cites the theorist Luigi Dentice so he cannot be seen as a beginner student. Tinctoris, however, seems to be expecting complete beginners given the concluding remarks to the *LAC*:

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[73] Russell Murray has described Pontio's list as more carefully-organized than that of Tinctoris, but does not justify this claim and I cannot see how one can come to this conclusion. Murray, "The Voice," vol. 1, 176.
[74] Russell Murray, "Zacconi as Teacher: A Pedagogical Style in Words and Deeds," in *Music Education in the Middle Ages and the Renaissance*, ed. Russell Murray, Susan Forscher Weiss, and Cynthia Cyrus (Indianapolis: Indiana University Press, 2010), 308.
[75] Murray, "The Voice," 162.
[76] Johannes Tinctoris, *Expositio manus*, in *Opera Theoretica*, ed. Albert Seay (Place of Publication Not Specified: American Institute of Musicology, 1975), vol. 1.

> …so, in our time, I have known not even one man who has achieved eminent or noble rank among musicians, if he began to compose or sing *super librum* [i.e., to improvise] at or above his twentieth year of age.

The age limit itself is not so important, but rather that he is speaking about students that are just beginning to improvise or compose. It may seem strange that a book written in Latin would be destined for young choir boys. It is more likely that the *LAC* is intended as a workbook for teachers instructing beginner students, or that it was written without a specific student in mind, a thorough-going list being a conceivable goal for its own sake.

If Tinctoris's audience is indeed beginners, this may explain why Tinctoris's successions provide no rhythmic information; it would not be needed if the students were only improvising in first-species counterpoint. Perhaps by the end of book III of the *LAC*, Tinctoris expected the student to be more advanced. But Pontio starts right away with florid counterpoint examples as he considers first-species counterpoint below the serious music student.

Anna Maria Busse Berger takes Tinctoris's interval-succession list as an exemplary model of a treatise full of terse, straight-forward, and systematically-presented rules meant for memorization.[77] We have seen that this characterization does not match Pontio's list in *RM*. Instead, Pontio's pedagogy is more akin to the tradition of commonplace books in Renaissance education. Though mainly used in literary education, Peter Schubert has discussed their explicit use in two musical treatises written just after *RM*: *Arte de musica teorica y pratica* by Francisco de Montanos, and *El melopeo y maestro* by Pedro Cerone (1613).[78] Schubert contrasts Erasmus's definitions of *exempla*, which are mainly factual, with commonplaces, which "are more elaborate propositions or arguments."[79] Rich in context, Pontio's two-voice examples are convincing specimens of real music and thus are appropriate to the slightly more advanced student hoping to use them as commonplaces. Pontio's abundance of repertoire reference, true commonplaces, corroborates this interpretation of his pedagogical approach.

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[77] Busse Berger, *Art of Memory*, 141–44.

[78] Peter Schubert, "Musical Commonplaces in the Renaissance," in *Music Education in the Middle Ages and the Renaissance*, ed. Russell Murray, Susan Forscher Weiss, and Cynthia Cyrus (Indianapolis: Indiana University Press, 2010), 162–92; Francisco de Montanos, *Arte de musica theorica y pratica* (Valladolid, 1592); Pietro Cerone, *El melopeo y maestro* (Naples, 1613; repr. Bologna, 1969).

[79] Note that Schubert goes on to detail more definitions of *exempla* and commonplaces, and each one ends up being able to serve as a conceptual container of the other. Schubert, "Commonplaces," 165.

Having compared the theoretical and pedagogical approaches of the two theorists, I will now examine an important voice-leading situation about which the authors differ considerably.

Parallel Sixths

I have shown that Tinctoris and Pontio's interval-succession lists were conceived of with such different structural principles in mind that they are generally incompatible. But surely there is a way to compare their discussions of at least a few interval successions. I will now examine how the two theorists dealt with interval successions that involve motion by parallel sixths in the two voices.[80] I chose this group of interval successions because parallel sixths are a crucial part of Renaissance polyphony.[81] Examining the case of parallel sixths will serve to identify some important methodological issues with contrapuntal corpus studies of Renaissance music.

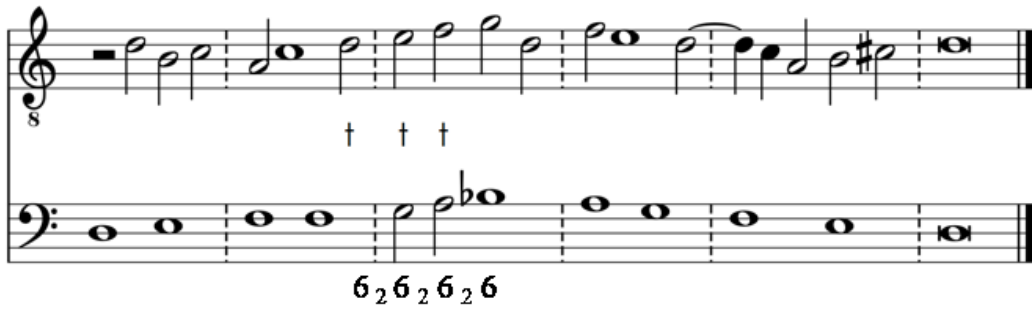
In the *LAC*, Tinctoris shows a sixth going to another sixth with both voices standing still, or ascending or descending a second, third, or fourth. Though parallel sixths that leap a fifth technically follow his explicitly stated principles, he omits these. These seven interval successions are given with the two positions of the tenor in three octaves, meaning that Tinctoris presents a total of 42 interval successions which go from one sixth to another. By contrast Pontio only allows for parallel sixths *by step* up or down. He even cites two voices moving by a leap of a third in parallel sixths as a negative example, as shown in Figure 34.[82]

Figure 34: Pontio's examples approving of parallel sixths by step, but not by leap.

Hora mi soviene, c'havendovi detto, che delle Seste ne potete fare due, & tre (si come è vero, che si può) non v'avertì però di questo, c'hora gli aggiongo, che siano tutte due le parti per movimento congiunto,	Now I recall having told you that you can use two or three sixths [in a row] (as it is true that you can). I did not, however, warn you of this, that now I am adding, that both parts [should] be by step, as here.

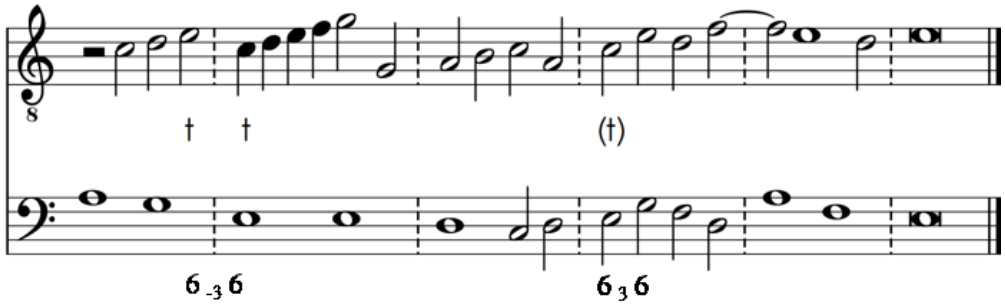
[80] Strictly speaking this group of successions may more accurately be referred to as "consecutive sixths" because Tinctoris allows for one sixth to follow another when neither voice moves and the same exact interval is repeated, though typically we think of the parallel sixth model involving motion. I will continue to use the term parallel sixths to be more in line with modern terminology.
[81] Peter Schubert, *Modal Counterpoint, Renaissance Style*, 2nd ed. (New York: Oxford University Press, 2008), 190–2.
[82] Pontio, *Ragionamento*, 62.

come quì.



Et questo à fine, che facciano grato udire à gli ascoltanti ; perche facendosi altrimente, cioè, essendo le parti ascendenti, ò discendenti con movimento separato, come quì.

And this so that they would welcome hearing of the listeners. Because otherwise, the parts ascend or descend by leap, as here.



Alla purgata orecchia non farian grato udire ; ma nasceria durezza, & sconciatura ne i vostri contrapunti, & nelle vostre compositioni ; & tanto più se fussero le Seste maggiori. Vero è, che di tal passaggio vi potrete servire in qualche compositione, come sogliono usare i diligenti, & pratici compositori, quando le parole trattano di cosa dura ; ò di cadere, ò d'altra cosa goffa ; ch'in tal caso

All clear ears will not welcome hearing [them]. Instead harshness and annoyance will be born in your improvisations and in your compositions and all the more so if they are major sixths. It is true that you can make use of such passages in some compositions, as even diligent and pragmatic composers are wont to do, when the words deal with something hard, or falling, or some other clumsy thing.

giovano à far ben qualche passaggio strano, & goffo ; & cosi proportionatamente accompagnando il canto alle parole, sarete giudicato di sano intelletto.	In this case the young are right to make some strange, and clumsy passage, and in this way, proportionately matching the singing to the words, you will be judged of sound intellect.

The interval successions these two theorists allow for are summarized in Figure 35 with just one tenor position and octave shown for those of Tinctoris.

Figure 35: Interval successions involving consecutive sixths allowed by Tinctoris and Pontio.

Succession	6 $_{1}$ 6	6 $_{2}$ 6	6 $_{3}$ 6	6 $_{4}$ 6	6 $_{5}$ 6	6 $_{-2}$ 6	6 $_{-3}$ 6	6 $_{-4}$ 6	6 $_{-5}$ 6
Tinctoris	✓	✓	✓	✓	✗	✓	✓	✓	✗
Pontio	✗	✓	✗	✗	✗	✓	✗	✗	✗

We can conclude that Tinctoris and Pontio give different advice about interval successions involving parallel sixths, but why is this so? Two primary explanations must be considered. Do they disagree about the way parallel sixths can or should be used, or has music practice changed between their two treatises, and both theorists accurately reflect their generation's style? This is precisely the sort of question a corpus study could address. Margaret Bent has similarly called for "the testing of [her] findings on the [*LAC*] on a wide spectrum of Renaissance music."[83] But devising a specific query to convincingly answer this research question quickly becomes a complicated matter. The main issue relates to the theoretical approach of both theorists, but in different ways for each one.

For Tinctoris, it is unclear on what level of counterpoint his interval successions are active. It cannot be the literal musical surface, because then his successions would have had to include dissonances. Instead book II of the *LAC* addresses the use of dissonance which is presumably to ornament the fully consonant successions listed in book I. So then by what

[83] Margaret Bent, "On False Concords in Late Fifteenth-Century Music: Yet Another Look at Tinctoris," in *Music Theory and Analysis 1450–1650*, dir. Anne-Emmanuelle Ceulemans and Bonnie Blackburn (Louvain-la-Neuve: Publications d'histoire de l'art et d'archéologie de l'université catholique de Louvain - C, 2001), 118.

reductive method should one arrive at the fundamental counterpoint where his successions can be understood to apply? The greater variety of mensurations in the 15th century makes this issue all the more important for Tinctoris's treatise.

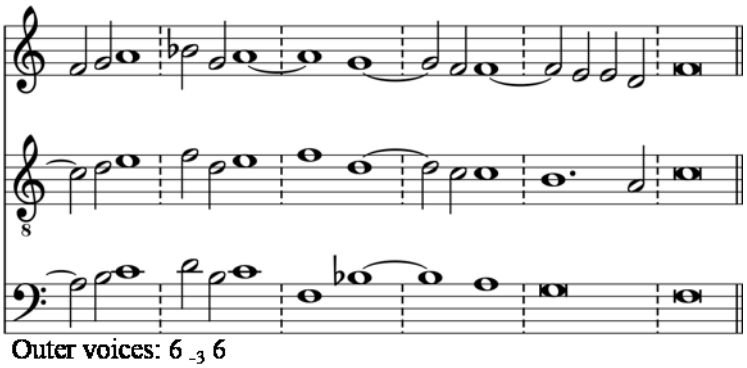
By contrast, Pontio clearly analyzes all of his interval-succession examples by looking at what is happening at regular minims. The only exception is for brief accented dissonances such as dissonant third quarters, as shown in Figure 18. But does this analytical assertion hold true for all pieces and in all mensurations? Within any given piece and without a change in mensuration, can the regular duration at which fundamental counterpoint progresses shift from the minim to the semibreve or from the minim to the semiminim?

These are all questions that must be resolved before we can decisively compare the counterpoint of different pieces, genres, and generations of Renaissance composers, as well as the theoretical treatises that are meant to directly relate to this repertoire. One may be tempted to simply do the same corpus study multiple times changing the metric level at which the music is analyzed each successive time. The issue with that approach is that analysis at any metric level will produce *some* result. Without first working out the theoretical foundations of how we want to parse the counterpoint of a passage, we can draw no substantive conclusions about the patterns we find. The main issue, therefore, is not a computational one but is inherently theoretical. We can see this issue more clearly with some examples from the repertoire.

One thing is certain, 15th-century music frequently has pairs of voices moving in parallel sixths. This is extremely common by step, and it is also found by leap, such as at the end of the Christe of Du Fay's *Missa Se la face ay pale*, shown in Figure 36.[84]

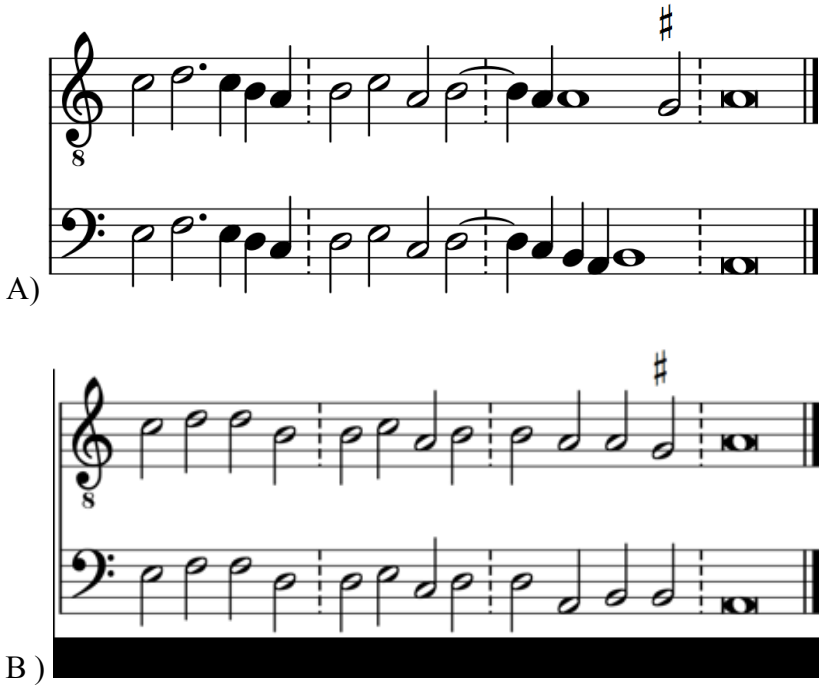
[84] Du Fay, *Missa Se la face ay pale*, Christe. Renotated from: http://josquin.stanford.edu/work/?id=Duf1005.

Figure 36: The end of the Christe from Du Fay's Missa Se la face ay pale *which includes pair of voices leaping together in parallel sixths.*



But we saw that Pontio explicitly discouraged even small leaps in parallel sixths, so does this sort of passage not happen in 16th-century music? A passage in two voices by Jacquet of Mantua reproduced in Figure 37 shows that, as one may have suspected, parallel sixths happen by leap in the 16th century as well.[85]

Figure 37: Mm. 137-140 of Jacquet of Mantua's Sanctus from his Missa In illo tempore *in A and its reduction in regular minims in B.*



[85] Recopied with a minim-equals-half-note transcription from: Jacquet of Mantua, "Missa In illo tempore," *Collected Works*, ed. Philip Jackson and George Nugent (np: American Institute of Musicology Hänssler-Verlag, 1986), vol. VI, 151.

Jacquet of Mantua is the second-most cited composer in book II of *RM* (after only Pontio himself) and, though not shown, the text being set in this passage is "*in nomine Domini*" so there is no reason to think this is what Pontio would have considered a tolerance of poor counterpoint to express something harsh mentioned in the text. If we analyze at regular minims and disregard any notes between these observation points, we get two instances of parallel sixths by leap of a third as shown in the reduction in Figure 37-B. But would Pontio have really objected to such a passage? Would he really analyze the two instance of 6 $_{-3}$ 6 the same way even though the first is filled in with stepwise semiminims? If we apply his analytical approach wholesale, the answer to these questions is yes.

In the preceding examples I highlighted some repertoire examples that do not follow some of Pontio's strongest admonishments. Before we can assess these discrepancies in an empirical way, we will need to define the most appropriate way to sample the counterpoint of a passage. The following chapter begins to address this analytical issue by offering a dynamic and context-dependent means of reducing a musical passage to its fundamental counterpoint. Beyond serving to facilitate future corpus studies on Renaissance counterpoint, the theoretical insights this approach will offer will be important findings in and of themselves.

Conclusions

In this chapter I detailed the organizational structure of Pontio's interval-succession list, and explored what musical lessons book II of *RM* communicates with its successions. We saw that while he explicitly takes the bass as the reference voice against which he measures interval successions, this is not without certain exceptions. Further exceptions were found in the application of Pontio's structuring principle of interval quality, and in the interval successions that appeared more than once but sometimes with slightly different details given. While not obscuring his points, these minor inconsistencies suggest that Pontio's focus was really on all of the contextual details he provided about each of his examples. Pontio turned the established medium of the interval-succession treatise into a vehicle by which to convey his stance on a number of contrapuntal issues.

In considering together several of Pontio's interval-succession examples that are spread throughout his list but deal with the same topics, we were able to discern his theoretical stance on

four important issues. The first of these is that Pontio generally analyzes at regular minims. The second point adds nuance to this analysis at the minim, by accounting for idiomatic uses of dissonance, namely brief accented passing dissonances that displace the observation point from the strong to the weak part of the minim when they occur. These two observations will be instrumental to the analytical method proposed in the next chapter. The third point we uncovered about Pontio's theoretical approach was that he first considers single parts when deciding where to apply accidentals, and only afterwards does he verify that his decisions form the correct vertical intervals at cadence points. The last point we observed about Pontio's approach, is that for the sake of imitation, canon, or expression of the text, he explicitly allows certain interval successions to which he would normally object. The abundance of detail and context in Pontio's examples in conjunction with the numerous repertoire references he makes, led us to characterize Pontio's take on interval-succession theory as book of musical commonplaces.

Pontio's interval-succession list serves as an up-to-date account of 16th-century musical practice and cites an unprecedented number (in the context of an interval-succession treatise) of specific moments in masterworks and makes special considerations for the thicker textures more endemic to the 16th century.[86] In reference to *RM*'s relevance to the nascent *seconda practica*, Suzanne Clercx has asserted that "Pontio seemed to be yet unaware of [the novelties of the *stile nuovo*] when he published his first treatise [*RM*]; one would have to wait until the year 1600 for Artusi, whose conceptions of music do not differ so much from those of Pontio, to express concern over the "imperfections of modern music."[87] This seems unjustified, however, given that on multiple occasions Pontio cites expressing the text as a valid justification of otherwise undesirable counterpoint. In light of all these points, book II of *RM* represents a mature

[86] For a discussion of repertoire citations in a number of Renaissance theoretical writings see: Cristle Collins Judd, *Reading Renaissance Music Theory: Hearing with the Eyes*, (Cambridge: Cambridge University Press, 2000); and Burmeister, *Musical Poetics*, xxxviii–xlv.

[87] Truncated and translated from French: "Cependant, en cette fin du XVIe siècle, apparaissaient les signes avant-coureurs du «stile nuovo». Mais ces nouveautés, Pontio ne semblait pas encore les connaître lorsqu'en 1588, il publia son premier traité; il faudra attendre l'an 1600 pour que l'Artusi, dont les conceptions musicales ne diffèrent pas tellement de celles de Pontio, s'inquiète des «imperfections de la musique moderne.» Pontio, *Ragionamento*, in *Postface* by Suzanne Clercx.

manifestation of interval-succession theory, though it was far from the last as Christoph Bernhard contributed a treatise to this tradition in the late 1650's.[88]

Compared side by side, we saw that the differences between Tinctoris and Pontio's lists are considerable enough to preclude a comprehensive comparison of the interval-successions they contain. Tinctoris's focus seems to have been the systematic approach he takes wherein almost all interval successions he approves of are included. Instead, Pontio focuses on adding a plethora of musical context. Gone are all traces of a *musica speculativa* impulse to classify and to pursue theoretical completeness for its own sake. Russell Murray sums up the tone of Pontio's two treatises well: "His treatises exhibit a direct approach to meeting the needs of these [church] musicians within the realm of *musica practica*. The emphasis is always on the practical, with little regard for the more ethereal arguments of the humanist theorists, and as little concern for the traditional matters of the venerable science of *musica speculativa.*"[89]

My analysis of the two pillars of interval-succession theory that the *LAC* and *RM* represent in this and the previous chapter is based on the treatises alone. In the interest of moving closer to the laudable goal of comparing these treatises to real contrapuntal practice as observed in the repertoire, in the next chapter I address the main issues surrounding the best way of sampling counterpoint for analysis from both theoretical and computational standpoints.

[88] Christoph Bernhard, *The Tractatus* (late 1650's), trans. Walter Hilse in *The Music Forum*, ed. William Mitchell and Felix Salzer (New York: Columbia University Press, 1973), vol. III, 35–61.
[89] Russell Murray, "The Voice," vol. 1, 154.

Chapter 4

Contrapuntal Rhythm in Theory and Analysis

In this chapter I present an analytical method by which to determine the contrapuntal rhythm (CR) of a passage or piece of music. I will define the key terms involved later in this chapter, but at this stage a simple definition of CR will suffice. As mentioned in the introduction, Ruth DeFord defines contrapuntal rhythm as "the rhythm of the structural contrapuntal progressions on which a piece is based."[1] Before going into any detail about this analytical method, I will examine some primary sources that address issues pertinent to CR. The compositions I cite will be shown without the accompanying text in order to leave space for analytical annotations.

Primary Sources on Contrapuntal Rhythm

As CR is a new term proposed by DeFord, it is not something that is directly referred to in any primary sources. Nonetheless it is a salient feature of Renaissance music and so it is only natural that a wide range of topics that were covered by period authors directly pertain to the subject. Commentary on the following subjects is especially noteworthy: the dimensions of a piece, musical pacing, dissonance treatment, and conventions in musical reductions and analysis. While a great many authors could be cited on these topics, the current examination limits itself to those of Tinctoris, Vicentino, and Pontio.

Johannes Tinctoris

Tinctoris is a good theorist to begin with, because his writings in the latter half of the 15th century came at a watershed moment in theoretical discourse. Most notably, the extensive attention he gave to dissonance prompted almost all of his successors to follow suit and thereby triggered rapid development and refinement of pedagogical advice concerning dissonance found in treatises. Klaus-Jurgen Sachs has written that Tinctoris's examples of dissonance treatment in book II of the *LAC* "open a new phase in the history of counterpoint."[2] More than his discussion

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[1] Ruth DeFord, Tactus*, Mensuration, and Rhythm in Renaissance Music* (Cambridge: Cambridge University Press, 2015), 84.

[2] Klaus-Jürgen Sachs, *Der Contrapunctus im 14. und 15. Jahrhundert: Untersuchungen zum Terminus, zur Lehre und zu den Quellen*, (Wiesbaden: Steiner, 1974), 154. Translation and citation from Eunice Schroeder, "Dissonance

of dissonance treatment itself, Tinctoris's greatest contribution to the modern concept of CR is the amount of attention that he rightfully accorded to the subject. Ruth DeFord has noted this, writing that "Tinctoris was the first theorist to recognize and describe in detail the relation between mensuration and dissonance treatment in composition."[3]

Beyond his discussion of consonance and dissonance, some of the terms that Tinctoris used are relevant to the pacing of Renaissance music. The first of these is "quantity". In his dictionary of musical terms Tinctoris defined this as "that according to which it is understood how large a piece is." [4] By quantity, Tinctoris was referring to mensuration. The four quantities correspond to the four mensural levels of major and minor mode, tempus, and prolation.[5] The most important point here is that Tinctoris speaks of using quantity to *understand* a piece of music. This is relevant to the modern concept of CR because it is an early expression of a regular durational value used to comprehend and parse a piece of music.

Also related to pacing is Tinctoris's *mensurae directio*, or measuring note. Klaus-Jürgen Sachs has equated Tinctoris's measuring note to the *tactus* and *battuta* of later writers.[6] Eunice Schroeder has reiterated Sachs's explanation of Tinctoris's dissonance rules, and proposes that a change in compositional practice with respect to the duration and metric placement of dissonances occurring in the 1430's and 1440's was a significant stylistic shift and an important motivation for Tinctoris's often cited comment:[7]

> Nor, which I cannot wonder at enough, does there exist any composition except this side of forty years that is considered worth hearing by the learned. But at the

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Placement and Stylistic Change in the Fifteenth Century: Tinctoris's Rules and Dufay's Practice," *The Journal of Musicology*, 7/3 (Summer, 1989), 367.

[3] Ruth DeFord, Tactus, 18.

[4] Translation from: Bonnie Blackburn, "Music Theory and Musical Thinking after 1450," in *Music as Concept and Practice in the Late Middle Ages*, ed. Reinhard Strohm and Bonnie Blackburn (New York: Oxford University Press, 2001), 329.

[5] Johannes Tinctoris, *Dictionary of Musical Terms (Terminorum Musicae Diffinitorium, c. 1475)*, trans. Carl Parrish (London: The Free Press of Glencoe, 1963), 52–3 and 91; and Johannis Tinctoris, *Terminorum Musicae Diffinitorium (c. 1475) Lexique de la Musique*, trans. Armand Machabey (Paris: Richard-Masse Editeurs, 1951), 45–45[A].

[6] Note that Pontio occasionally uses the term *battuta* when discussing the metric position of a note: Pontio, *Ragionamento*; Klaus-Jürgen Sachs, "Counterpoint," *Grove Music Online* (Oxford University Press, accessed July 11, 2016), http://www.oxfordmusiconline.com/subscriber/article/grove/music/06690.

[7] Schroeder, "Dissonance Placement," 366–89; Johannes Tinctoris, prologue to *Liber de arte contrapuncti*, trans. Jeffrey Dean, http://earlymusictheory.org/Tinctoris/texts/deartecontrapuncti/.

> present time, if I may put aside innumerable improvisers who perform very beautifully, I know not whether by the strength of some heavenly inspiration or by the force of hard practice, countless composers flourish, such as Johannes Okeghem, Johannes Regis, Antoine Busnoys, Frémin le Caron, Guillaume Faugues, who pride themselves on having as their teachers in this divine art the recently deceased John Dunstaple, Gilles Binchois, Guillaume du Faÿ. Nearly all the works of them all breathe such sweetness that (at least in my opinion) they should be considered worthy not only of men and heroes but even of the immortal gods.

Since the CR of a piece is directly related to compositional *tactus* which is equated to Tinctoris's measuring note, Schroeder's study effectively identifies CR as an important stylistic marker.[8] Schroeder takes Tinctoris's generalizations about the measuring note active in different mensurations and quantifies the number of dissonances of various durations used by Du Fay in compositions with these mensurations.[9] She then compares these dissonance tallies to Tinctoris's suggestions. An issue with Schroeder's study, however, is that she is not specific enough about which dissonance types are found at different durations. Later in this chapter we will see that, while all dissonances suggest a specific CR value, different dissonance types do this in different ways.

Another important lesson that Tinctoris imparts is that an interval in the fundamental counterpoint of a pair of voices can repeat no movement in either of the two voices. This type of immobile counterpoint is easy to overlook and indeed Pontio did not include these sorts of successions in his list. We will see, however, that allowing for the counterpoint in any given pair of voices to repeat is essential to analyzing CR, especially in more than two voices. This is because not all voices in a texture articulate a new note at the same time. So when we are driven to sample the counterpoint (that is, observe what intervals are sounding) at a given point because

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[8] Schroeder, "Dissonance Placement," 366–89.

[9] Alexander Blachly disagrees with Feldmann, Schroeder, and Busse Berger's interpretations of Tinctoris's stance on mensuration and tempo. Nonetheless, this in this chapter I cite primary sources primarily in order to identify what aspects of music were salient and important to them. Tinctoris's precise understanding of mensuration, measuring note, tempo, etc. does not directly impact any of the specific components of the analytical method proposed here. See: Alexander Blachly, "Reading Tinctoris for Guidance on Tempo," in *Antoine Busnoys: Method, Meaning, and Context in Late Medieval Music* (Oxford: Clarendon Press, 1999), 399–427; and Rob Wegman, "What is 'Acceleratio mensurae'?", *Music and Letters*, 73 (1992), 515–24.

of activity in some of the voices, it is important to be able to say that the intervals of the voices that have not moved continue, even if they are not reattacked.

Tinctoris's eighth and final rule given in book III of the *LAC* is an extended exhortation promoting variety above all in composition and improvisation. Tinctoris states:

> Also, any composer or improviser of the greatest genius may achieve this diversity if he either composes or improvises now by one quantity, then by another, now by one perfection, then by another, now by one proportion, then by another, now by one conjunction, then by another, now with syncopations, then without syncopations, now with fugae, then without fugae, now with pauses, now without pauses, now diminished, now as written… Every composed work, therefore, must be diverse in its quality and quantity…[10]

Here, several of the contributors to variety that Tinctoris is recommending will have a direct impact on the CR, namely quantity, perfection, proportion, syncopations, fuga, rests, and diminished counterpoint. These all have to do with the dimensions of a composition, contrapuntal activity, and the hierarchical or metric relationship between different notes or note values. In so far as Tinctoris is championing variety in these characteristics of music, we can assume that they are often manipulated in improvisations and compositions. By extension, this underscores the necessity for the constant re-evaluation of the CR of a piece, because it could easily change if one or more pertinent features of the music changes and so an analytical model must be able to adapt to these changes. The fact that Tinctoris cites these musical characteristics as important features of a piece to vary for the sake of variety, allows us to conclude that changes in CR correspond to audible and salient changes in the music.[11]

In comparing Tinctoris's descriptions of dissonance in book II of the *LAC* to his compositions, Lee Rothfarb has further informed the discussion of pacing with respect to dissonance treatment.[12] He points out that while Tinctoris engaged in some uses of dissonance in

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[10] Tinctoris, *Contrapuncti*, bk. III, ch. 8, http://earlymusictheory.org/Tinctoris/texts/deartecontrapuncti/.
[11] See also Luko, "Tinctoris on *Varietas*," *Early Music History*, 27 (2008) 99–136.
[12] Lee Rothfarb, "Tinctoris vs. Tinctoris: Theory and Practice of Dissonance in Counterpoint," *In Theory Only*, 9/2 (Ann Arbor: Michigan Music Thoery Society, 1986), 3–32.

his compositions that he condemns in his theoretical writings, instances of passing dissonances of excessive duration in a given mensuration are rare.

Nicola Vicentino and Joachim Burmeister

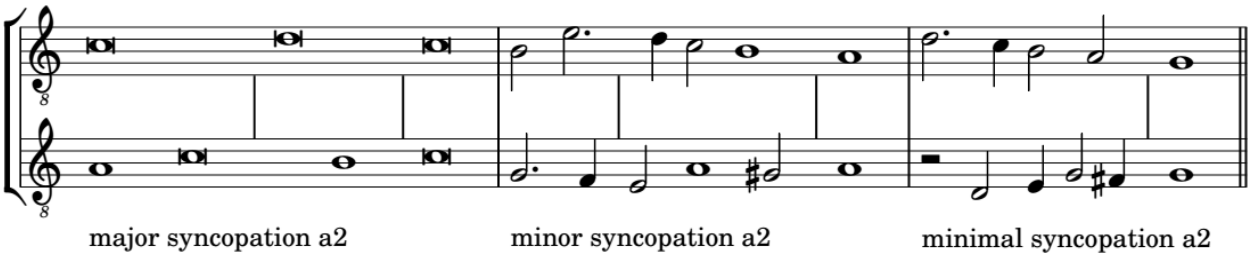
Nicola Vicentino goes a step further than Tinctoris by stressing the importance of "motion" in composition in general, and also of utilizing a rate of motion that is appropriate to the text:[13]

> Motion is extremely important in compositions, for it is so potent that it transforms the nature of steps, consonances, words, and instruments. A composition lacking the pace appropriate to the subject of the words or the design of other ideas will not gratify listeners, for it will seem to have been made without care and judgment.

Vicentino was speaking of precise durational values here rather than of organizational levels as Tinctoris was, however, as I will show later in this chapter, rhythmic density is a key factor in the assessment of CR, and the durations used in a piece heavily influence its rhythmic density.

A second point where Vicentino provides important insight into CR is in his discussion of suspensions at different durational levels. He shows an example of "major syncopation," "minor syncopation," and "minimal syncopation" where the dissonant suspensions last a semibreve, a minim, and a semiminim respectively, as shown in his example reproduced in Figure 1.[14]

Figure 1: Vicentino's three levels of dissonant syncopation.



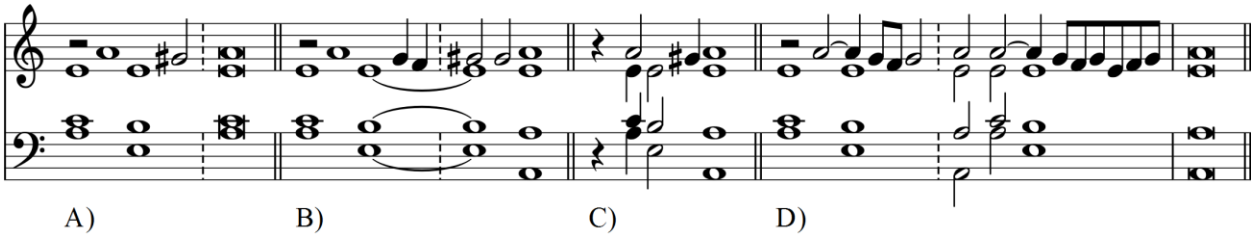
This example draws attention to the fact that suspensions occur at different rhythmic levels. Dissonance treatment, especially involving suspensions, is an important determinant of CR. This

[13] Nicola Vicentino, *Ancient Music Adapted to Modern Practice*, trans. Maria Rika Maniates, ed. Claude Palisca (*L'antica musica ridotta alla moderna prattica*, 1555; New Haven, Yale University Press 1996), 135.
[14] Special thanks to Christopher Antila for renotating this example from: Nicola Vicentino, *Ancient music*, 93–5.

example reinforces the idea that dissonances of different durations will point to different CR values.

Joachim Burmeister reiterates these same ideas with four-voice examples in his treatise from 1606. After several examples of basic cadences without ornamentation and with CR values at the minim (such as that in Figure 1-A), he shows one augmented example (Figure 2-B), one diminished example (Figure 2-C), and two ornamented examples (Figure 2-D).[15]

Figure 2: Burmeister's examples of cadences in different values and with different ornamentations.



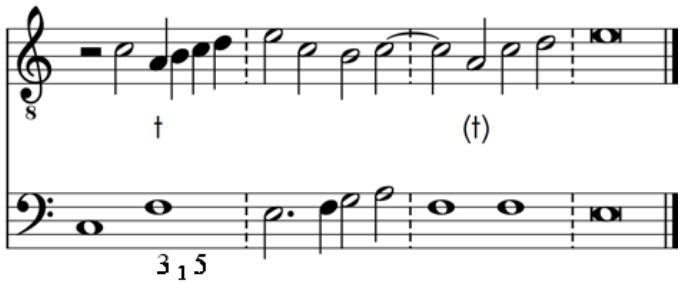
These examples demonstrate very similar counterpoint occurring in different durational values. Each of these examples clearly points to a specific rate at which the counterpoint advances, so if we do not have an analytical approach that can adapt to these differences than our analysis will label them differently, and obscure the fact that these passages all exhibit exceedingly similar counterpoint.

Pietro Pontio

In chapter 2 we saw a number of Pontio's examples that convey various aspects of his reductive approach to analysis. Now I will reconsider many of these examples with a focus on their implications for CR analysis. We saw that Pontio generally analyzes music with a steady minim CR. This eliminates the briefest, metrically weak dissonances from his analytical reductions, such as the passing semiminim B in the upper voice which gets ignored in Pontio's discussion of the 3 $_1$ 5 interval succession, Pontio's example of which is shown in Figure 3.[16]

[15] Benito Rivera has translated example B as "re-acticulation", and while the G#4 (Burmeister's "middle" of the cadence) is re-articulated, in this case augmentation seems like a more appropriate translation. Joachim Burmeister, *Musical Poetics* (1606), trans. Benito Rivera (New Haven: Yale University Press, 1993), 114–5 and 118–9.
[16] Pontio, *Ragionamento*, 37.

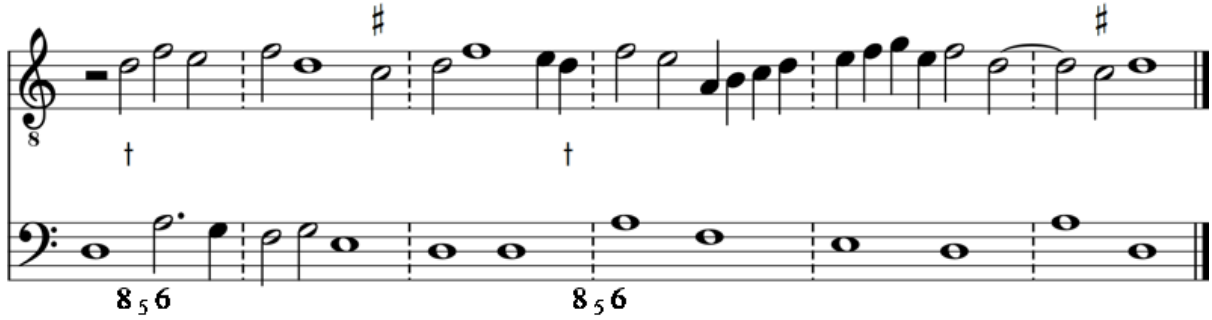
Figure 3: 3 1 5 interval succession showing that Pontio analyzes at the minim.



While she does not discuss *RM* directly, DeFord maintains that this approach Pontio took was the norm in the Renaissance: "Theorists take the imperfect semibreve as the principal model for the note corresponding to a binary compositional *tactus*, though they sometimes illustrate other possibilities as well."[17]

We also saw that Pontio's text discussions of interval successions even reduce away metrically strong semiminims that are identifiable as dissonant third quarters, given that the following is the example of an octave (marked with a dagger) going to a sixth: [18]

Figure 4: 8 5 6 interval-succession example demonstrating that a dissonant third quarter can displace the regular minim contrapuntal rhythm by one semiminim.



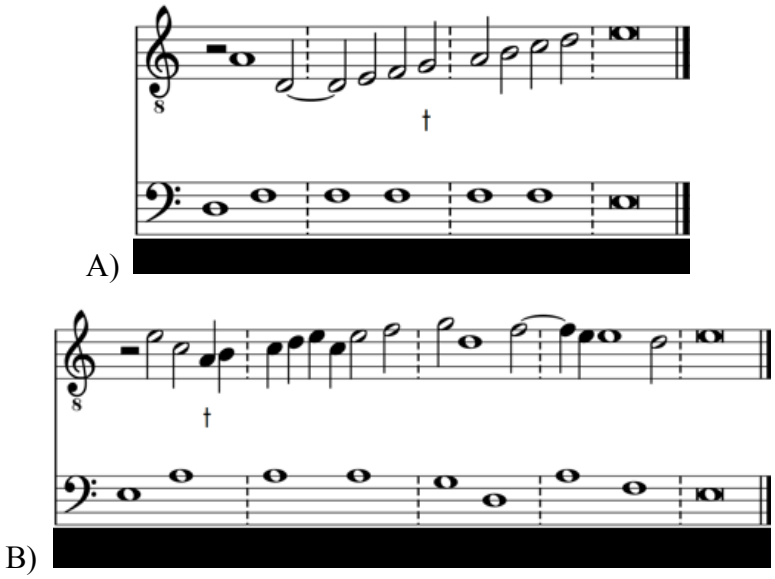
This demonstrates that taking analytical observations at the beginning of each unit equal to the CR (for Pontio this is always the minim) is a simplistic method of reduction and would be problematic here because it would suggest that this ninth is structural. Instead, Pontio considers musical syntax by accounting for idiomatic uses of dissonance, both metrically strong and weak.

[17] DeFord, Tactus, 82.
[18] A dissonant third quarter is a special type of accented passing tone described in: Schubert, *Modal Counterpoint*, 87; Pontio, *Ragionamento*, 64.

In *RM* we saw that in thirty-six of Pontio's forty-two dissonant interval-succession examples, the dissonance in question is a suspension. Given that suspensions are less common than passing tones, this led to the conclusion that the imbalance was probably because suspensions are dissonant at a larger metric level than are passing tones.[19] In this chapter I will use this observation to guide the assignment of a projected CR value for each dissonance type with a given duration and metric position.

Finally, Pontio's analysis of passing tones a minim in duration highlights the fact that he allows passing tones to occur on the level of the CR. This may seem like a small point but it is quite important. Allowing for passing tones to occur on the level of the CR in effect allows for the possibility that music notated in longer values functions fundamentally differently from music notated such that the minim is the CR. We saw this point in the previous chapter by comparing two of Pontio's interval-succession examples reproduced in Figure 5.[20]

Figure 5: The interval successions marked in the two examples are the same if we analyzed the two with CRs at the semibreve and minim respectfully. Both would be analyzed as 1 ı 3.



[19] For the distribution of different dissonance types, see Andie Sigler and Jon Wild, "Schematizing the Treatment of Dissonance in 16th-century Counterpoint," in *Proceedings of the International Society for Music Information Retrieval*, 2015, 645–650.
[20] Pontio, *Ragionamento*, 68 and 29.

If we apply Pontio's regular-minim reductive method to both examples, we would get the intervallic analysis shown below the score. These are quite different because in the second the analytical method allows notes that sound like passing dissonances to attain to an equal status as the consonant notes they ornament. If instead we analyze Figure 5-A at regular semibreves, the succession in the box would be annotated as 1 $_1$ 3 in the manner of Figure 5-B analyzed with a minim CR. This makes the syntax of Figure 5-A much more similar to that of Figure 5-B. The most valuable lesson to be learned from Pontio here is not his reductive method itself, but rather that one's theoretical assumptions and methodology should be as clear and transparent as possible in order to facilitate scholarly critique of one's approach.

As Pontio was a prolific composer and given that we have distilled relatively specific details about his analytical approach, one may easily wonder how well the tenets of this approach mesh with Pontio's oeuvre. The discussion of Pontio in this chapter and throughout this dissertation concentrates solely on his theoretical writings, and specifically just from *RM*. The only edition of any of his works that I am aware of can be found in the third appendix of Russell Murray's doctoral dissertation.[21] The pieces he provides, however, are not made available as symbolic notation files so I cannot apply the automated-analysis method I present later in this chapter to his works at present. [22] That being stated, examination of the scores in Murray's valuable appendix reveals their operative CR to usually be the minim, with occasional expansions to the semibreve, much like DeFord's description of the masses of Pontio's contemporary Palestrina. DeFord writes that, in the masses of Palestrina, "The compositional *tactus* is principally the semibreve, but it may shift occasionally to the breve."[23]

Summary of Primary-Source Findings

While none of these authors directly addressed the topic of CR, consideration for this theoretical concept is palpable in their writing. By way of summary, in the interval-succession treatises of Tinctoris, Vicentino, and Pontio, we gleaned the following lessons pertinent to CR analysis:

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[21] Russell Murray, "The Voice of the Composer: Theory and Practice in the Works of Pietro Pontio" (PhD diss., University of North Texas, 1989).
[22] While optical music recognition software is available to turn score images into symbolic notation files, this software is not yet reliable enough to be used as a source for conducting contrapuntal queries.
[23] DeFord, Tactus, 375.

1. Dissonance is a crucial part of Renaissance musical syntax
2. Pacing is a salient characteristic of a piece and was discussed by multiple Renaissance authors often in terms of proportions or durational values
3. A vertical interval in the fundamental counterpoint between two voices can repeat with no movement in either voice
4. Variety was sought in many musical domains that directly influence CR analysis
5. At least three different durational values exist for suspensions
6. Most weak and even some strong brief dissonances such as dissonant third quarters are disregarded in some period analysis
7. A suspension points to a shorter CR value than a weak passing tone of the same duration
8. Given that Pontio's analyses asserted a fixed CR value, the minim, we can at least conclude that the CR is generally stable
9. In some cases, if we analyze two different passages with different CRs, we can recognize the recurrence of the interval succession at different rhythmic values (see Figure 5)
10. It is important to make theoretical assumptions and analytical methodology clear

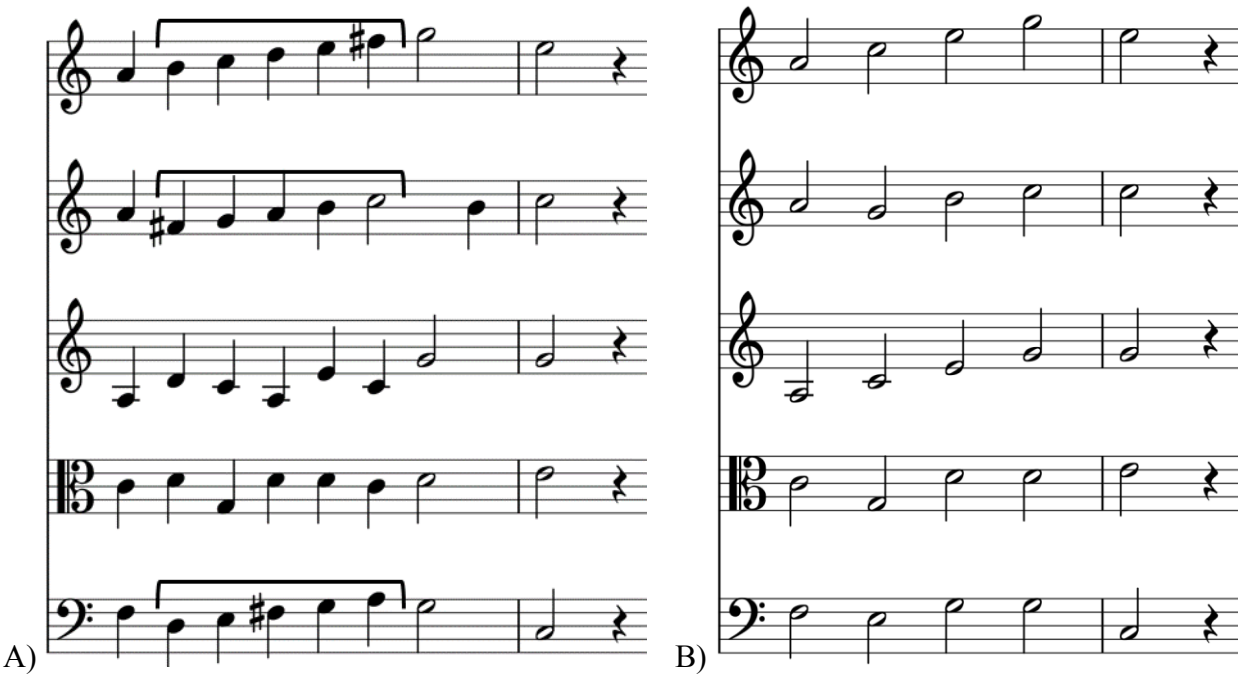
The insight into reduction provided by period theorists is invaluable, but also incomplete. Ruth DeFord has put it concisely: "Rhythmic styles and notational practices in real music are much more diverse than those described by theorists."[24] While we do not need to adopt all of these ideas wholesale, they will inform and guide the creation of a systematic approach to CR analysis.

While one can learn a great deal about counterpoint from the *LAC* and *RM*, Tinctoris and Pontio's writings on the subject leave us with two important and related problems. Tinctoris's interval successions are totally imprecise with respect to duration and rhythm, so it is unclear if they correspond to the musical surface, or some reduction of that surface. Pontio is just the opposite; he is overly specific about rhythmic level to which his successions correspond. Given that Pontio always samples the counterpoint of his examples at regular minims, one is left to wonder if these same progressions can occur in expanded or contracted form. The following

[24] DeFord, Tactus, 1.

excerpt from a canzon by Giovanni Gabrieli is a good example of a scenario in which Pontio's overly rigid approach is problematic.[25]

Figure 6: Giovanni Gabrieli, Canzon I (C195), mm. 45-46 in A, and its reduction at regular minims in B.

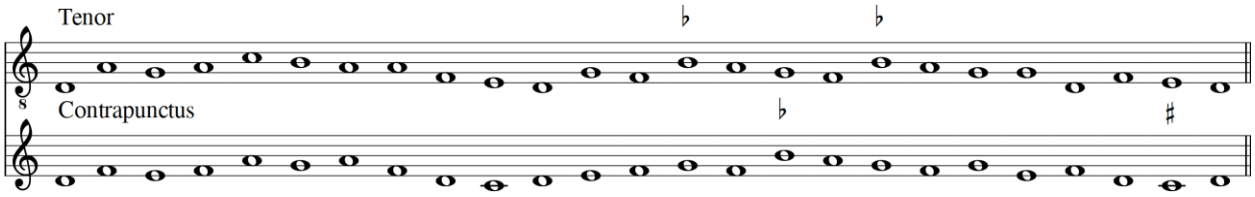


From the second to the sixth semiminims in Figure 6-A (designated with braces), the bass and the two upper voices sound a new 6/3 sonority on each semiminim, rising by step each time. This fauxbourdon texture is elaborated with the two other internal voices which do not participate in the stepwise pattern, but do interact with its harmonies. The reduction at regular minims in Figure 6-B demonstrates how inaccurately a rigid reductive method, like the one Pontio uses, can represent the counterpoint of a passage. This reduction raises a small conflict with Pontio's point that parallel sixths should only happen by step. When we reduce at regular minims, we see that the outer voices of Figure 6-B move in parallel sixths by leap from the second to the third minim. The middle voice and the highest voice get reduced such that they are in parallel octaves for all of bar 45, though this is also a relatively minor issue. The main problem is that the proper of a

[25] Renotated from: Giovanni Gabrieli, *Opera omnia*, ed. Richard Charteris (Rome: American Institute of Musicology, Hänssler, 1998), vol. 11.

passage of parallel sixths is to eventually resolve out to an octave by stepwise contrary motion. Tinctoris summarized this voice-leading convention well:[26]

> … the upper sixth can never be melodiously taken up unless one or more other sixths follow it, tending finally to an upper octave or tenth (without the interposition of a concord of another kind), or unless the tenor descends after it only one step or three, against the notes of which steps either an upper octave or tenth is placed.



In the Gabrieli excerpt, this sixth-to-octave resolution does happen from the A3-F#5 in the outer voices on the last semiminim in the braces in Figure 6-A, to the G's which immediately follow them. But since the A3-F#5 happens on a weak semiminim, this point in the music gets reduced away in Figure 6-B and the resolution of the fauxbourdon passage no longer appears idiomatic. Although the primary CR of this canzon is the minim, clearly this rate has sped up to the semiminim in this passage, and any analytical model used to reduce tonal music must be able to adapt to these sorts of shifts.

So while the primary source have helped direct us to the main issues surrounding CR analysis, if we unquestioningly adopt their analytical models and methods we end up with untenable analyses for certain passages. The remainder of this chapter is devoted to the presentation of a new method for CR analysis, and its application to some repertoire studies.

Contrapuntal Rhythm

In this section I address the central questions about CR, namely how it relates to the *tactus*, why it is important, what repertoire CR analysis as detailed in this chapter pertains to,

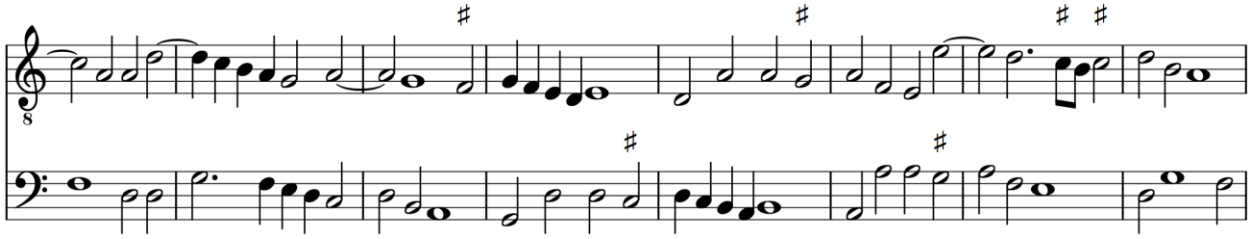
[26] Tinctoris, *Contrapuncti*, bk. I, ch. 7, http://earlymusictheory.org/Tinctoris/texts/deartecontrapuncti/.

what the benefits of automated analysis are, and how my new analytical method, the dynamic-offset method, is an automated implementation of CR analysis.

Basics of Contrapuntal Rhythm

Adding to DeFord's definition stated above (see p. 102), by way of comparison we can say that CR is to early music, as harmonic rhythm is to tonal music; the difference is in the intervallic as opposed to chordal basis of analysis. Put another way, we can think of the CR of an excerpt of music as the speed at which the fundamental counterpoint progresses, independent of most dissonant ornamentation.[27] It is expressed as a durational value such as a minim. By extension, CR analysis consists of the discernment of the CR in music. For example, CR analysis of the excerpt reproduced in Figure 7 reveals the CR to be at the minim.[28] I will address the specifics of how I come to that conclusion shortly, but for now we can say that every minim is attacked with either a consonance or the idiomatic use of a metrically accented dissonance type.[29] Note that the harmonic rhythm of this passage is primarily at the semibreve, so while harmonic rhythm is similar in principle to CR, they are completely independent.

Figure 7: Josquin, Crucifixus, NJE [13.12], mm. 6-13. The CR is equal to the minim in the entire excerpt.



The example above is taken from the Crucifixus of a mass attributed to Josquin; I will refer to it repeatedly in my description of CR because it is a lucid and concise example of the main concepts at hand. When we say that the CR is at the minim, it is understood that this is on-beat, non-syncopated minims.

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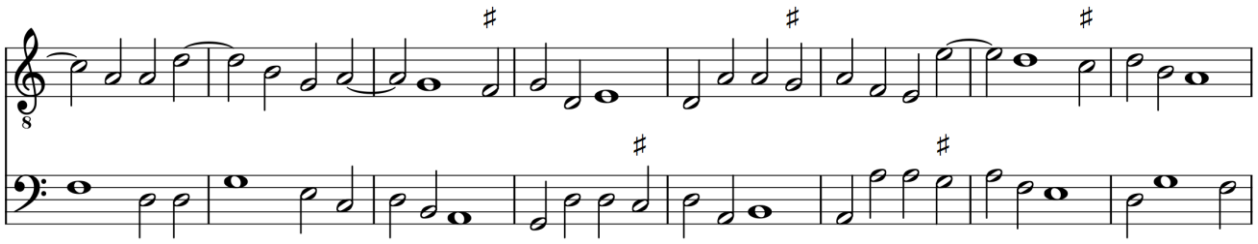
[27] For more on the regularity of contrapuntal rhythm, see: Ruth DeFord, Tactus*, Mensuration, and Rhythm in Renaissance Music* (Cambridge: Cambridge University Press, 2015), 84.
[28] Josquin, Crucifixus, NJE [13.12], http://josquin.stanford.edu/work/?id=Jos1312.
[29] As Peter Schubert notes, the "agent" and "patient" terminology for the two voices involved in a suspension figure was first introduced in music by Artusi. The agent is the voice that moves into the dissonance by step or by leap, and the patient is the voice that sustains the note that becomes dissonant when the agent moves, before resolving down a step. Schubert, *Modal Counterpoint*, 76–7; Giovanni Maria Artusi, *L'Artusi overo delle imperfettioni della moderna musica* (1600; Bologna: Forni, 1968).

In this dissertation I use this term "fundamental counterpoint" to refer to the first level of reduction of a passage of music. Fundamental counterpoint and CR are closely related but distinct terms. The former corresponds to the actual notes of a reduction, whereas the latter is the rate at which new events (vertical intervals) occur. Figure 8 shows the fundamental counterpoint reduction of Figure 7.

Figure 8: Fundamental counterpoint reduction of Josquin, Crucifixus, NJE [13.12], mm. 6-13 (Figure 7).

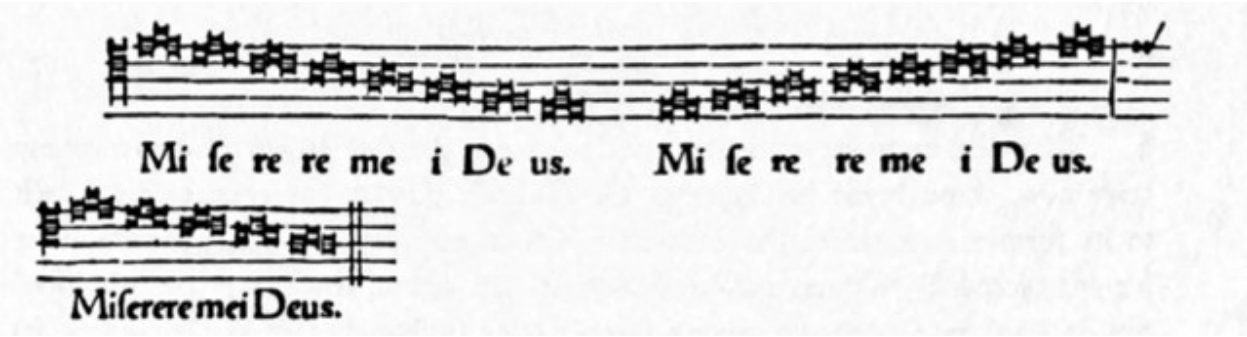


Here we see that the fundamental counterpoint of a passage corresponds to specific notes. By contrast, the CR is the *rate* at which this fundamental counterpoint progresses. As mentioned earlier, the CR is uniformly at the minim in this passage. Comparing the previous two figures, one can observe that the attacks of the musical surface (Figure 7) are more densely concentrated than those of the fundamental-counterpoint reduction (Figure 8).

Deeper levels of structure beyond that of fundamental counterpoint can and do exist in Renaissance music and this was apparent to period musicians as well. Figure 9 reproduces Glarean's reduction from the *Dodecachordon* of the cantus-firmus tenor from Josquin's five-voice motet *Miserere mei, Deus*.[30]

[30] Many thanks to Ian Lorenz for directing me to both this motet and Glarean's analysis of it. Henricus Glareanus, *Dodecachordon* (1547), trans. Clement Miller, (Place of publication not identified: American Institute of Musicology, 1965) 260. For the score of Josquin's motet *Miserere mei, Deus* see: http://josquin.stanford.edu/work/?id=Jos1803 (accessed July 24, 2016).

Figure 9: Glarean's reduction of the first tenor from Josquin's five-voice motet Miserere mei, Deus.



This piece comprises 422 bars in modern notation so this concise reduction of the first tenor vividly illustrates that Renaissance musicians were thinking about musical structure, at least to a certain extent, in a hierarchical way. On some level, anything in between these notes of Glarean's reduction is ornamentation. But this kind of extremely high-level architectural perspective has little to do with the actual musical syntax and counterpoint of the work. CR sits much closer to the musical surface.

Contrapuntal Rhythm and Compositional *Tactus*

In the context of Renaissance music, *tactus* can have several meanings all relating to the measurement of musical time.[31] Ruth DeFord has defined no fewer than six definitions of *tactus*, but for the purposes of this study, I will focus on what she calls "compositional *tactus*." [32] DeFord defines this term as "the time unit that serves as a standard of reference for various aspects of rhythm, such as the rate of contrapuntal motion, dissonance treatment, and syncopation, in a composition." The compositional *tactus* is in a well-defined relationship to CR; the latter is the duration that is the first metric subdivision of the former. As the method for determining CR I outline here only applies to music with fully duple divisions, in the pieces I will consider the CR is equal to half the value of DeFord's compositional *tactus*. Since my method for determining the CR does not need an analysis for the value of the *tactus*, I do not

[31] For a basic introduction to *tactus*, see: Antoine Auda, *Théorie et Pratique du Tactus: Transcription et Exécution de la Musique antérieure aux environs de 1650*, (Brussels: Oeuvres de Don Bosco, 1965), 1–36. For an in depth exploration of the relationship between *tactus* and mensuration proportions signs, see: Anna Maria Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution* (Oxford: Clarendon Press, 1993).

[32] Antoine Auda has differentiated three *tactus*, however, all three are subcategories of what DeFord calls compositional *tactus*. Antoine Auda, *Théorie et Pratique du TactusM Transcription et Exécution de la Musique antérieure aux environs de 1650*, (Brussels: Oeuvres de Don Bosco, 1965), 19–25; Ruth DeFord, Tactus*, Mensuration, and Rhythm in Renaissance Music* (Cambridge: Cambridge University Press, 2015), 51.

have the problem of first having to define the level of the *tactus*. Despite the fact that I take the term CR from DeFord, when I reference her work I will more often refer to her use of the term compositional *tactus* because this is the term she uses most often in her analyses. Zarlino wrote that each half of a *tactus* is salient and J. A. Bank has corroborated this statement. [33] In the following quote from Zarlino, the *tactus* is the semibreve:[34]

> Thus over every semibreve of the subject we may place two minims… but with this consideration, that… each of them be consonant, because these two parts of the semibreve are recognized strongly by the sense with respect to the *tactus*, which is considered in two ways, namely, in the downstroke and in the upstroke.

As an example of the relationship between the two, the passages that DeFord analyzes as having a semibreve compositional *tactus* almost all get assigned a minim CR using the method presented in this chapter. In light of this point, we can understand DeFord's assessments of compositional *tactus* as also pertaining to the CR of the passage in question.

Why Study Contrapuntal Rhythm?

There are three main reasons to study CR. The first is that assessing the CR of a passage is essential to determining its fundamental counterpoint. Reduction is a common analytical procedure, and I argue that assessing the CR is an essential step to reducing early music that cannot be skipped.[35]

The second reason is that it quantifies the pacing of a composition by which I mean the rate at which fundamental counterpoint progresses. This is particularly important when that pacing changes in the middle of a piece. These changes do not occur so often, but that only serves to make them all the more notable when they do. Towards the end of this chapter I will show some examples of pieces which contain changes in CR, both by augmentation and diminution. Changes in the CR are understood as occurring in tandem with changes in the *tactus*.

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[33] J. A. Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century* (Amsterdam: Annie Bank, 1972), 7.
[34] Zarlino, *Le institutioni harmoniche*, bk. 3, ch. 42, 195. The translation is DeFord's, cited from: Deford, Tactus, 88.
[35] For an example of reductive analysis in the context of early music, see: Sarah Fuller, "Exploring Tonal Structure in French Polyphonic Song of the Fourteenth Century," in *Tonal Structures in Early Music*, ed. Cristle Collins Judd (New York: Garland Publishing, 1998), 61–86.

The third reason to study CR flows naturally from the first two. One of the most advantageous aspects of studying CR is that, when evaluated systematically, it comes to the fore as a very audible, but often overlooked stylistic characteristic of early music. As with any other stylistic characteristic, once it is better codified and understood we can potentially use it to distinguish between different composers, time periods, styles, or genres. If a composer's use of CR is particularly idiosyncratic it would be a good criterion by which to associate pieces of uncertain attribution to a specific composer or conversely by which to refute such claims.

Scope of Study

CR analysis is particularly appropriate for early music, as it was intervallically conceived. That being stated, this study only concerns itself with the CR of Renaissance music from roughly 1450 to 1600. This constraint is largely due to the fact that CR analysis is heavily dependent on dissonance treatment. Dissonance treatment across the period studied is fairly well-defined and consistently applied.

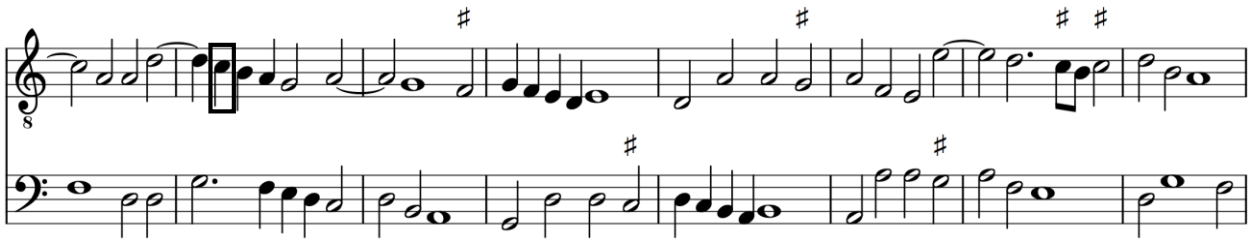
The second limitation of scope in this study is that my method only applies to pieces that are duple on the mensural level of the CR, typically that of prolation which defines the number of minims in the semibreve. This is again because of dissonance treatment. There is simply no comprehensive assessment of dissonance in sections with triple divisions, neither in primary sources, nor in modern ones. Though DeFord does address ternary compositional *tactus*, applying this to the level of CR poses unique problems that she did not have to deal with on the level of the compositional *tactus.*[36] One example of an issue with pieces with a ternary CR is that it is not clear how suspensions function since they normally appear on strong beat and are resolved on the following weak beat. With a ternary CR, beats two and three are both weak, which would suggest that suspensions can only occur on beat one, however one finds scenarios where they occur on beat two suggesting that either beat two is stronger than beat three, or that our definition of a suspension should be adjusted accordingly. The accommodation of ternary divisions in this model and more generally of dissonance in triple meter is an important avenue for future research.

[36] DeFord, Tactus, 90–3.

Benefits of Automated Analysis

There are several benefits to automating CR analysis and ironically the most important one has nothing to do with computers. The main benefit is that automation necessitates a systematic approach. A systematic approach is valuable because it is reproducible, and therefore easier for other scholars to critique and improve upon. While formalization can also be more rigid than analysis "by hand," this is not necessarily a bad thing as the specific parameters being analyzed, dissonance types and attack density, are relatively objective and so do not suffer greatly from the lack of nuance that often accompanies automated analysis. Formalization also makes key analysis questions, decisions, and assumptions explicit. For example, in Figure 5 we saw that Pontio allows for passing tones to occur at the level of the contrapuntal rhythm when those passing tones are a minim in duration. Later we will see how this is problematic in the analysis of some pieces, even if it is the norm for much of the repertoire. In my method, each dissonance points to a specific CR value at that moment in the piece, based on its dissonance type, metric placement, and duration. For example, in Figure 10 the C4 passing tone in the upper voice in the second bar shown (designated with a square) is on the weak part of the first half-note beat.[37]

Figure 10: Josquin, Crucifixus, NJE [13.12], mm. 6-13.



This suggests that the operative CR at this point in the piece is the half note. But had all the values been doubled, my method would adjust accordingly and analyze the CR at the semibreve. The limpid nature of a systematic approach brings issues like this to the fore, rather than allowing them to be glossed over. This in turn accelerates the improvement of the tenets of the approach in question.

[37] Josquin, Crucifixus, NJE [13.12], http://josquin.stanford.edu/work/?id=Jos1312.

Another significant benefit of automation is that it opens up the possibility of applying an analysis to large corpus studies. Corpus studies are not inherently better than individual examinations of a single piece (for which the automation of CR analysis would not be a necessity, though is still helpful), but they do address analysis questions of a different nature. For example, earlier in this chapter I mentioned that CR analysis could be used to inform cases of uncertain attribution. Comparing a single piece to the norms defined by the musical language of a set of pieces is precisely the sort of inquiry that is greatly facilitated with automated analysis.

Lastly, automation also facilitates score annotation by associating a specific offset, or time point in the piece, with every analytical observation, making them readily accessible for annotation. This makes results easier to share with other scholars, and even more importantly it greatly facilitates the refinement of a new analytical method. Given that explicit CR analysis is a new take on the centuries-old analytical procedure of reduction, I experimented with many new ideas some of which I ultimately rejected before arriving at my final analytical method. A fast and accurate means of visualizing the impact of analysis decisions (usually in the form of annotating a score) is an invaluable tool in the development of new a method.

Dynamic-Offset Method

The dynamic-offset method is the name of my automated approach to CR analysis. This method is part of the VIS Framework for music analysis wherein it applies a CR analysis of a Renaissance score to reduce it to its fundamental counterpoint.[38]

How exactly is CR analysis "dynamic?" Dynamic in this context means that the CR is continually re-evaluated for every new moment in a given piece. This is what allows it to adapt to changes in the middle of a piece, and change values accordingly. Such a change does not occur in every piece, and it tends to happen relatively few times in the pieces where it does change. The key point here is that it *can* change because it is contextually defined.

Related to this term is the concept that CR analysis is "sticky." This means that, all else being equal, as the CR is dynamically defined throughout a piece, at each moment there is a certain preference to continue to measure the CR at the same regular rate as used in the

[38] The VIS Framework is software in development at McGill University that analyzes music in symbolic notation. Its code is available on github: https://github.com/ELVIS-Project/vis-framework.

immediately preceding moment. So while CR is dynamic and can change, the fact that it is sticky makes it resist fleeting and unsustained changes in the rhythm.

The term "offset" has many meanings in the context of music. I use this term in the same way it is used in music21, which is as a specific time point in a piece. As my method works with scores in symbolic notation, this time point is not expressed in seconds as a method working with audio files might, but rather in musical durations. Any point in a score can be expressed as the number of quarter notes that it is set off from the beginning of the piece. This is that point's offset. So an offset is simply a specific moment in a score. Therefore, the dynamic-offset method takes its name from the fact that it makes a reduction of a piece by sampling the counterpoint at time points in a piece that can be at variable distances from each other.

Having reviewed the basics about CR, the scope of this study, the benefits of automating CR analysis, and the dynamic-offset method, I will now break down my analytical method, step by step.

Analytical Method

While Ruth DeFord has written extensively on the theoretical history of CR and highlighted a number of the musical factors that must be considered for this type of analysis, she did not detail a precise method one can use to ascertain CR. That task is the subject of the present section.

DeFord cites contrapuntal structure, surface rhythm, cadences, and text setting as the most important rhythmic events that differentiate levels of mensuration, of which her compositional *tactus* always corresponds to one at any given moment.[39] All of these, save text setting, are incorporated in my dynamic-offset method in some fashion. There are two main musical components that weigh in on my method's assessment of CR: attack density, and dissonance treatment. Surface rhythm is addressed by attack density, and contrapuntal structure and cadences by dissonance analysis. I will discuss each one independently and then explain how eventual conflicts between the two analytical methods are reconciled. After that I will explain how this method is applied to every new event by sliding an analysis window across a piece of

[39] DeFord, Tactus, 82.

music. The main contribution of my method is not the components of my analysis, but rather the explicit nature of my methodology.

While text setting is a valid analytical concern in music analysis it is omitted from consideration here because it is often problematic to quantify especially given that it is not always explicit in Renaissance music.[40] The number of active voices in a passage plays an indirect role in how it is analyzed. Currently this information is only operative in that some steps in the process rely on pairwise analysis (explained below), so if the texture ever drops to a single voice, the analysis is suspended until two or more voices are present.

Pairwise Model

The pairwise model of analysis does not necessarily consider only one pair of voices in a texture with several parts. Rather, it analyzes one pair of voices *at a time*, completely independently from all the other voices in the texture, though as many as all the pairs may be considered in succession.[41] Renaissance descriptions of dissonance treatment are generally pairwise. It is hard to say whether this is due to the legacy of the successive model of composition where one part is added at a time to a pre-existing melody, or whether it is because contrapuntal theory at that time was essentially dyadic. Andie Siegler and Jon Wild analyzed dissonance treatment in Palestrina and Victoria in a pairwise fashion to the extent that it was possible, but it was necessary to consider more than two voices at a time in order to, among other things, evaluate fourths as consonant or dissonant.[42] The dissonance analysis, as well as all other automated music analysis done here, is done with VIS. As dissonance detection in VIS is mostly pairwise, the detection of CR begins in a pairwise fashion. But VIS considers all the parts together to calculate the attack-density so ultimately CR analysis is not entirely pairwise. This in turn makes the key analytical assumption that a musical passage projects a single primary CR

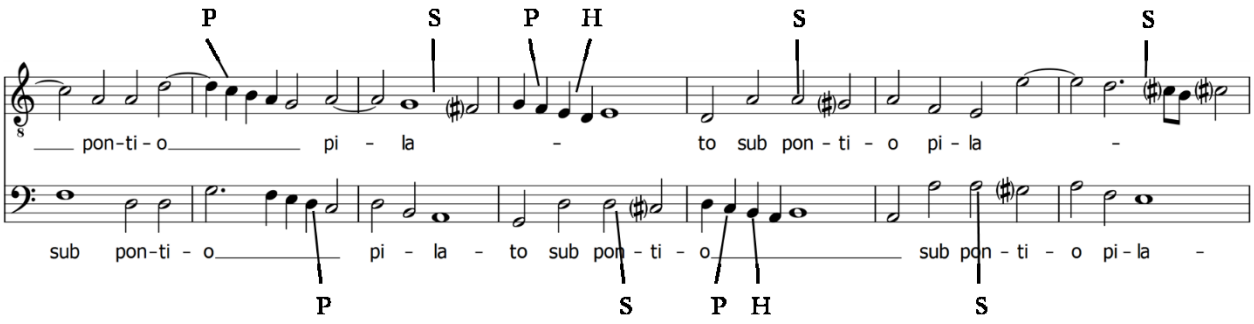
[40] DeFord, Tactus, 105–6.
[41] The number of pairs analyzed in the pairwise model depends on one's specific approach. Tinctoris, for example, considered all pairs that include the tenor resulting in 1 pair less than the number of voices in the piece. Pontio analyzed all pairs that include the bass, also resulting in 1 pair less than the number of voices in the piece. Another possibility is to consider all pairs without the necessity of the inclusion of a referential voice.
[42] The program Siegler and Wild used to detect and categorize dissonances was entirely separate from VIS, though the classification of dissonance in both programs is based on the same pseudo-code dissonance-type descriptions which Jon Wild distilled from Peter Schubert's modal counterpoint textbook. Schubert, *Modal Counterpoint*.

despite the fact that we may find that, to some extent, contrapuntal progressions occur on multiple levels as DeFord has asserted.[43]

Dissonance Treatment

Now we will see how each dissonance also projects an operative CR at its location in a piece, based on its dissonance type, metric position, and duration. The reduction of the musical surface that was made to calculate the attack density is not used in this step; instead the original musical surface is. The result of dissonance classification is shown for the same excerpt shown earlier, now reproduced in Figure 11 with its dissonances labelled.[44]

Figure 11: Josquin, Crucifixus, NJE [13.12], mm. 6-13, with dissonance annotations.



This excerpt only contains three types of dissonances: passing tones, chanson idioms, and suspensions labelled as P, H, and S respectively. A chanson idiom is a type of accented passing tone that precedes the agent of a suspension.[45] Each dissonance is understood to suggest a specific CR at its point in the piece. When we find a passing tone on the weak part of some longer containing duration, this points to that longer containing duration as the contrapuntal rhythm. For example, as mentioned earlier, the C4 semiminim passing tone in the upper voice in the second bar of Figure 11 projects a contrapuntal rhythm at the half note. All four of the passing tones in this excerpt are on weak semiminims, so they all point to a CR at the minim. Chanson idioms also project a CR of the note value that is twice their duration. All those found in the example above are a semiminim in duration, so they also point to the minim as the CR of the passage. Suspensions suggest the CR of the metric level on which their resolutions are weak.

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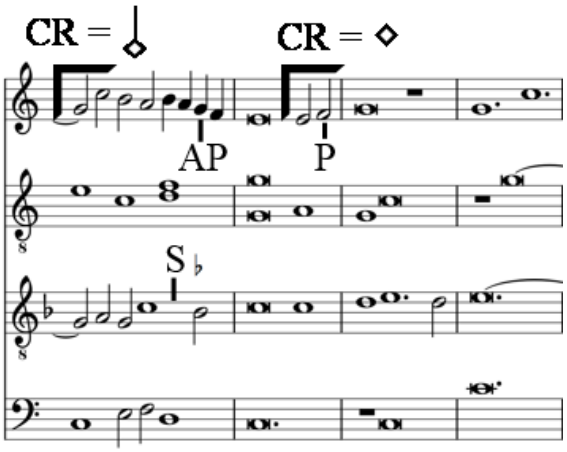
[43] DeFord, Tactus, 82.
[44] Josquin, Crucifixus, NJE [13.12], http://josquin.stanford.edu/work/?id=Jos1312.
[45] Schubert, *Modal Counterpoint*, 87.

The five suspensions in Figure 11 occur on strong minims and resolve on weak minims, and thereby all project a CR at the minim. Indeed all instances of the three dissonance types found in this passage agree in that they all project a minim CR.

It is possible to have some disagreement amongst the dissonances within a given time span of a piece. In these cases some dissonances will suggest one CR, while others will suggest another. These tend to be found in transitional passages such as that in Figure 12, wherein the CR shifts from the minim to the semibreve.[46]

Figure 12: Du Fay, Missa Ave regina celorum*, Kyrie I, mm. 7–10.*



Here the suspended C4 in m. 7 of the tenor as well as the accented passing tone G4 (labelled "AP") in the cantus both project a CR at the minim, and the rhythmic density up to the cadence on the downbeat of bar 8 agrees with this analysis. While there is no change in the number of sounding voices (with the exception of the abandonment of the brief divisi in the contratenor), this cadence does mark an important cleavage in the contrapuntal texture of the piece. Starting at bar 8 the piece begins to project a semibreve CR with a considerably sparse attack density. The question of when the listener perceives such CR shifts is beyond the scope of this dissertation.[47] The new value is confirmed by the F4 passing tone in bar 8 of the cantus as a passing tone on a weak minim points to a semibreve CR.

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[46] Du Fay, *Missa Ave regina celorum*, Kyrie I, http://josquin.stanford.edu/work/?id=Duf1002.

[47] While my analysis method does not work reliably in all scenarios where the CR is in triple groupings as it is beginning in m. 8 of Figure 12, the analysis of this particular except can be analyzed because it does not include suspensions, the locus of my method's problem in triple meter.

While there is no theoretical limit to the number of levels suggested by dissonance analysis, in practice I have not encountered more than two levels being projected in close proximity to one another and DeFord seems to agree with this observation.[48] In situations where different dissonances in the same time span of a piece point to CR values, the value supported by the greatest number of dissonances is taken. The fact that temporally proximate dissonances can point to different contrapuntal levels should not be interpreted as evidence that dissonance analysis is an inaccurate indicator of the primary contrapuntal level. Rather, this corroborates DeFord's idea that multiple contrapuntal levels can be simultaneously projected.[49] Insofar as multiple contrapuntal levels occasionally do co-exist in a prominent fashion, choosing the most prominent one as *the* CR is an expedient simplification of the musical reality at any given point in a piece. For this reason, future implementations of the dynamic-offset method may seek to convey the extent to which all present contrapuntal levels are projected. I will now explain the way that attack density independently suggests a second, possibly conflicting, CR value.

Attack Density

With the term attack density, I refer to the average time interval between note attacks (i.e. note onsets) in any voice of a piece of music. To calculate this average, notes are represented as numbers according to the standard used by Michael Cuthbert in `music21` wherein a notated quarter note is assigned to 1, an eighth note .5, a half note 2, etc.[50] These values are then averaged together and rounded to the nearest note value which is a plausible operative CR. In music with duple divisions, DeFord's corpus studies of a wide range of Renaissance music found compositional *tactus* values at the minim, semibreve, or breve.[51] These point to CR values at the semiminim, minim, or semibreve respectively. To this group of three values, I also add the possibility of a fusa (eighth note) duration for the CR. This is primarily to accommodate the analysis of scores in symbolic notation that have been transcribed with the minim equal to the modern quarter note. Unless otherwise stated, all transcriptions in this volume are in original

[48] DeFord, Tactus, 82.
[49] DeFord, Tactus, 82.
[50] Michael Cuthbert, `music21`, [toolkit for computer-aided musicology], http://web.mit.edu/music21/.
[51] DeFord, Tactus, 88. See also: Busse Berger, *Signs*, 6.

note values, however, when creating tools suitable for corpus studies, it is important for one's method to be able to adapt to a range of type-setting practices.[52]

To which notes should my basic method of calculating attack density be applied? On some level, any attack at any metric position stresses that point to a certain extent, but the attacks of the musical surface are almost always more densely concentrated than those of the fundamental counterpoint by virtue of the fact that the latter is extracted from the musical surface by means of reduction. After reducing out most types of ornamentation, attack density is calculated by taking the average of the remaining notes. This average keeps CR readings from becoming too volatile and shifting because of one or two longer or shorter values. A rejection of such volatility is in line with most contrapuntal research where the CR is generally understood to be regular.[53]

So how do we reduce out most types of ornamentation? This reduction is achieved by accounting for two factors: dissonance treatment, and agogic accent. Concerning dissonances, I classify them into two broad categories: structural dissonances and non-structural dissonances. Only suspensions are considered structural dissonances because they are an integral part of Renaissance contrapuntal syntax, especially at cadences.[54] As they are structural, suspensions do not get reduced away. All other dissonance types are non-structural, so they get reduced out before the average attack density is calculated. The precise manner this is done depends on the metric placement of a non-structural dissonance. For a non-structural dissonance on a weak beat, its duration gets added to the preceding note before it is reduced away, as demonstrated in Figure 13 which shows a passage by Morley alongside its reduction using this method.[55]

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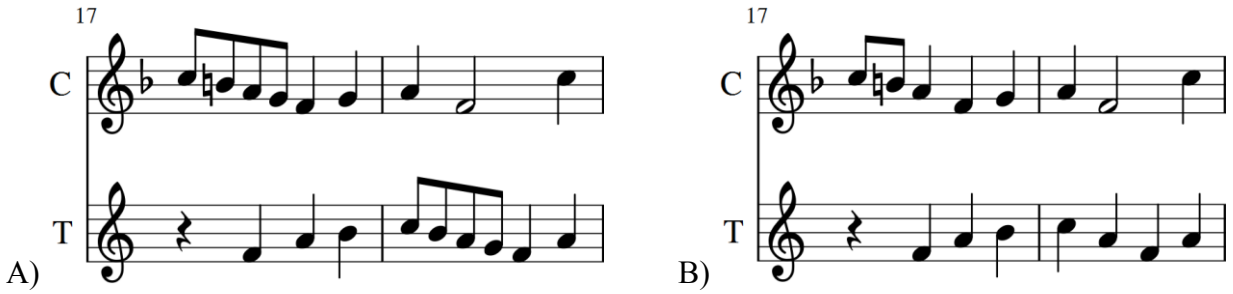
[52] Going further, this process could be abstracted to allow for any duration, including shorter and longer ones, such as sixteenth notes and breves. This abstraction was not deemed musically necessary and the implementation with four possible values was preferred for its simplicity.

[53] As in, for example: Peter Schubert and Julie Cumming, "Another Lesson from Lassus: Using Computers to Analyze Counterpoint," *Early Music* 43.4 (November 2015): 577–86.

[54] On contrapuntal syntax at cadences, see: DeFord, Tactus, 101–5; and Margaret Bent, "Ciconia, Prosdocimus, and the Workings of Musical Grammar as Exemplified in *O felix templum* and *O Padua*," in *Johannes Ciconia, musicien de la transition*, ed. Philippe Vendrix (Turnhout: Brépols, 2003), 65–106.

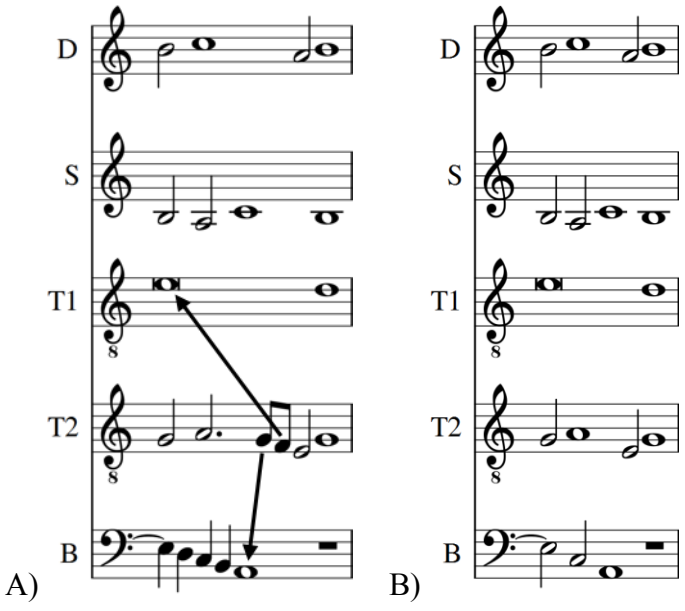
[55] Thomas Morley, *First Book of Canzonets to Two Voyces*, ed. Bernard Thomas (London: Thomas Este, 1595; New York: Performers' Facsimiles, 1988).

Figure 13: Morley, Sweet Nimphe*, mm. 17–8 in A, its basic reduction in B.*



This procedure applies to any type of weak non-structural dissonance including passing tones, neighbor tones, and échappées. In Figure 13-A the semiminim G4 in each voice gets reduced away, along with the semiminim B4 in the second measure shown of the tenor. One may think of the semiminim B4 in the cantus as a passing tone, but since there is no other sounding voice for it to be dissonant against, the pairwise dissonance detection implemented in VIS does not label this as a dissonance. In the cases where there are two consecutive passing tones, their combined durational value is added to the note preceding the first passing tone before the dissonances are discarded. An example of this non-structural dissonance reduction is given in Figure 14.[56]

Figure 14: Ockeghem, Missa Fors seulement*, Credo, m. 19 in A, its basic reduction in B.*

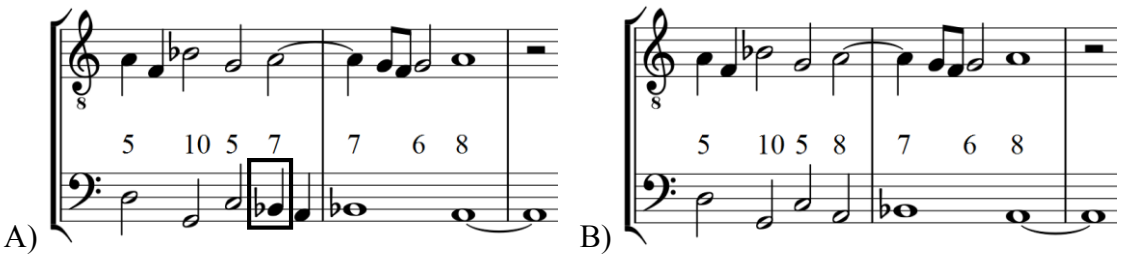


[56] Johannes Ockeghem, *Masses and Mass Sections*, ed. J. van Benthem (Utrecht: 1994), vol. 2/4. Renotated from: http://josquin.stanford.edu/cgi-bin/jrp?a=notationEditText&f=Ock1007c.

Since two consecutive passing-tone are usually dissonant against different notes, they are typically only found in thicker textures. The semiminim G3 in the second tenor is dissonant against the *bassus*, whereas the semiminim F3 is dissonant against the first tenor.

Metrically strong non-structural dissonances are handled in a similar fashion to their weak counterparts. The difference is that instead of folding their durations into the end of the preceding note, they are prefixed to the duration of the following note as is the B♭2 designated with a box in Figure 15-A.[57] This is done because the metrically strong non-structural dissonance can be understood as displacing the note that immediately follows it. Figure 15-B shows the result of this step in the reductive process.

Figure 15: Lassus, Sancti Mei*, mm. 16–8 in A, its basic reduction in B.*



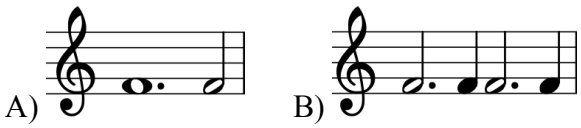
Though Figure 15-B only represents an intermediary step in the process and we have not yet calculated the CR, it is worth noting that the CR of this passage is the minim.[58] When we compare the vertical intervals (shown between the staves) at regular minims of Figure 15-A to Figure 15-B, we see that the intervallic syntax of the latter makes more sense. The problem with Figure 15-A is that if we analyze at regular minims, a dissonant seventh is prepared by another dissonant seventh. In Figure 15-B the analysis conveys the hearing that the operative vertical interval on the last minim of m. 16 is an octave, which suitably prepares the dissonant seventh that arrives on the following minim. In the previous chapter we saw that Pontio reduced metrically strong non-structural dissonances in this same way.

Agogic accent is based on the relative durations of notes. It has the effect of stressing longer and metrically stronger notes. In my method of reduction, agogic accent is only

[57] Orlando di Lasso, *Magnum opus musicum*, typeset Carl Proske, ed. Franz Xaver Haberl (New York: Broude Brothers Limited, 1973), vol. I, 1–7.
[58] For a more in-depth analysis of Lassus's twelve duets, see: Schubert and Cumming, "Another Lesson."

considered to a limited extent. DeFord has described five different ways note-value combinations can project or obscure various metric levels of a mensural structure.[59] Her two examples, reproduced in Figure 16-A and Figure 16-B, underscore the breve and the semibreve levels respectively.[60]

Figure 16: DeFord's example of agogic accent projecting the breve and semibreve levels respectively.



This is because the notes are in a durational relationship of 3:1 and the longer first note begins at the regular metric level of the note whose duration would be the sum of the two in both examples, a breve and a semibreve respectively. My model accounts for agogic accent by taking what DeFord has described as the clearest ways of projecting a given level by means of this type of accentuation. In my reductive method, if a note is repeated in the same voice, and the duration and beat strength of the first note are greater than those of the second, then the two notes are replaced by the sum of their durations at the attack point of the first note. Figure 17 shows the result of applying this reductive step to the examples in Figure 16.

Figure 17: Reduction of Figure 16 based on the agogic accent of the notes.

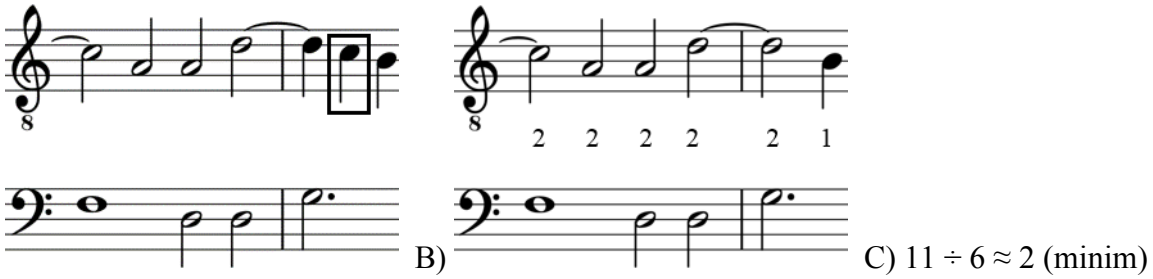
These comparisons are kept relative in order to more easily adapt to different levels of transcription and different mensurations (though, as mentioned earlier my method only currently works with duple divisions). VIS uses `music21` to calculate the beat strength of a note, and `music21` assigns an equal weight to the downbeats of all measures. Therefore this reductive step will not affect consecutive downbeats, even if one may consider one measure to be stronger than the other. In modern transcriptions of mensural notation, the measure does not necessarily correspond to the same mensural level in all mensurations. A more refined approach to beat strength calculation for mensural music could be a fruitful subject for future research. Practically

[59] DeFord, Tactus, 94–5.
[60] DeFord, 95.

speaking, the approach taken here only allows for the reduction of notes based on their agogic accent when the note that gets reduced out is relatively short. In practice, this step reduces out notes in almost every piece I have examined, however, it has little appreciable impact on the final attack density of the piece because the attack point of the note that gets reduced away is often marked by some other voice.

In summary, after reducing the musical surface by eliminating non-structural dissonances and accounting for agogic accentuation, the average attack density of successive overlapping time spans of this reduction is calculated. When we round this value for each time span to the nearest undotted note value, this gives the CR value for that moment in the piece, based solely on the attack density.[61] I demonstrate this calculation in Figure 18 for a short stretch of the Josquin Crucifixus seen earlier.[62]

Figure 18: Attack density analysis of Josquin, Crucifixus, NJE [13.12], mm. 6-7.



A) B) C) 11 ÷ 6 ≈ 2 (minim)

In Figure 18-A non-structural dissonances are identified and reduced out. Figure 18-B calculates the duration of each event slice. Finally, these durations are averaged and rounded to the nearest reasonable CR value, in this case 2, the minim. I have described how my method makes use of dissonance analysis and attack density, so now I will describe how they are windowed across an entire piece.

Windowing

Windowing, also known as w-shingling, is an analytical technique wherein a set number of consecutive observations are considered together for some type of analysis, and then the same number is used for observations one unit later…. The number of consecutive observations

[61] Dotted note values are not considered here because this method currently only applies to music with entirely duple divisions.
[62] Josquin, Crucifixus, NJE [13.12], http://josquin.stanford.edu/work/?id=Jos1312.

considered together is called the window size. Windowing is often used for datasets that have a time dimension, such as stock prices, where the window size can be expressed easily as an amount of time, such as three days. If we had data for the closing price of a stock for ten days, we could figure out what the six-day average starting on the sixth day, because the first five days do not have enough observations before them to satisfy the window size. This windowing process of the ten day period with a window of size six is illustrated in Figure 19. Here we see that the first valid window ends on the sixth day, and a new and mostly overlapping window appears every day after that. So how does this apply to music analysis?

Figure 19: Abstract illustration of windowing process.

Window 1	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Window 2	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Window 3	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Window 4	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Window 5	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10

The two key analytics of the dynamic-offset method are attack density and dissonance treatment. We have already discussed how to calculate each one, so now we just need to apply those calculations to a given window size. We can quantify the passage of musical time in at least two ways that are pertinent to windowing: either we can set off windows of a certain durational value, such as a dotted breve (six minims given duple divisions) which will have a variable number of events in it, or we take the last *n* events which will occupy a variable durational value of musical time, depending on how long each of those events last.

The dynamic-offset method blends these two approaches, starting by applying the attack-density analysis over a six-event window. The reasoning behind the choice of the size will be explained shortly. Instead of looking at the events on the musical surface, the events in the reduction from the attack density analysis are windowed. This helps normalize the durational values of the windows, and also makes the technique perform more uniformly when dealing with

both highly ornamented and sparsely ornamented or unornamented textures. It should be noted that this approach operates on the assumption that the degree and nature of ornamentation in a piece of music is secondary to its fundamental counterpoint.

But this reduction has most of its dissonances abstracted away! So to calculate intermediate CR values based on the dissonance analysis, we take the beginning and ending points of the attack-density analysis windows, and consider the dissonances that occur between these time points in the original musical surface. In this way we get two potential CR values, one based on attack density, the other on dissonance analysis, for almost every time-point in the intermediary reduction from the attack-density analysis. This is not every point because the first five points do not satisfy the window size of six. This initial lack of a reading is intentional. Without a full window the CR analysis is considered to be inconclusive. Furthermore, every time the texture drops to one or no voices, the windowing process starts anew. This kind of change in texture will often happen immediately after a cadence. Starting the windowing again after a cadence that is followed by a severe reduction in texture is also intentional. A cadence of this sort usually convincingly ends the previous musical passage allowing for the next one to begin in any way the composer desires. This could be with a new passage with the same CR, or it could be different. Often important internal cadences will be followed by an ensuing passage that has a longer CR value such as that we saw in Figure 12. These brief periods that are initially considered inconclusive with respect to the CR until a minimum amount of metric and syntactic entrainment to the new beginning has taken place.

Window Size

One may wonder why a window size of six was chosen. In some sense this is admittedly a "magic number" in that it is a constant value that is not context-dependent. I tried out a few different window sizes and I found that values smaller than six rendered the CR too volatile because it took too few events from different metric levels to heavily influence a window's average.[63] On the other hand, window sizes larger than six were occasionally too resistant to change, and missed brief passages with a different CR value, or simply took too long to change.

[63] The end of the passage in Figure 20 is a good example of a typical idiom that is problematic with window sizes that are smaller than six events as very short but consonant notes would overly skew the attack-density reading of a smaller window.

A more refined windowing approach would perhaps take proximity to cadence points into account by making the window size smaller around cadence to facilitate the detection of shifts in the CR. DeFord has stated that changes in contrapuntal rhythm may be brief, coming often only at the end of a section: "Like Du Fay, Ockeghem often speeds up the note values and the contrapuntal rhythm to create a sense of climax at the end of a section."[64]

Determining the Contrapuntal Rhythm

Having described how the two analytics are applied via a sliding six-event window, I will now explain how the readings from the attack density and dissonance analyses are reconciled into one final CR associated with the time point at the end of each window in the piece. The process is very simple: if the two readings agree, that value is assigned as the CR of that point in the piece. If they are different, then the tie is broken by choosing the one that agrees with the CR value for the previous moment in the piece. This is what makes CR analysis sticky, thereby resisting change to a limited extent. Though simple, this method of reconciling disagreement between the two components of the analytical method with the preceding reading means that for the CR value to change, the new value has to be projected by both the metric and dissonance profiles of a piece. While perceptual observations are beyond the scope of this dissertation, the stickiness of this method can be likened to a listener's need for a minimum amount of entrainment to new musical stimuli.

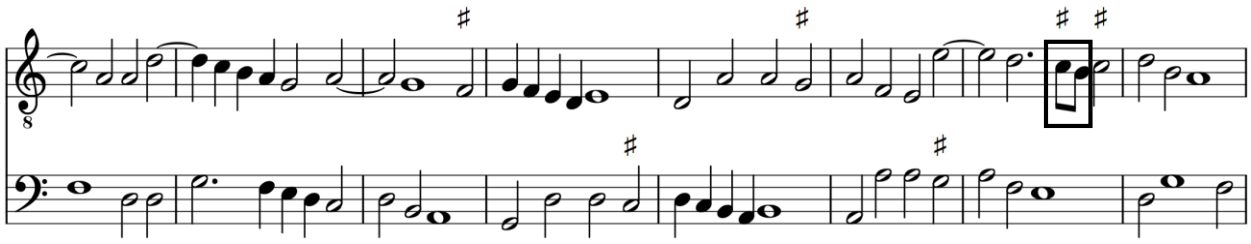
This is particularly important at beginnings of passages wherein voices often enter one or two at a time. In many cases, the attack density of this momentarily reduced setting will not clearly express the CR to the same extent that it would when all voices are active. In these cases the dissonance analysis is particularly helpful because any single dissonance points to a specific CR whereas clearly projecting a CR via attack density takes several notes.

On the other hand dissonance analysis has its issues as well. Given that Renaissance dissonances are defined intervallically between two voices, notes that would likely be flagged as dissonances in thicker textures may not be accompanied by a note that fulfills the harmonic requirements of their potential dissonance types in passages with only two or three voices. This

[64] DeFord, Tactus, 265.

is because there are fewer other notes for any given note to form dissonant intervals against. The end of the Josquin passage we examined earlier is a good example, reproduced in Figure 20.[65]

Figure 20: Josquin, Crucifixus, NJE [13.12], mm. 6-13.



We may think of the fusa C#4 as a dissonant anticipation of the minim C#4, and that the fusa B3 serves as a dissonant lower neighbor to the two C#4's, however the C#4 and B3 fusae are not actually dissonant against anything. The only other pitch present in this context is an E3 and the fusae form a major sixth and a perfect fifth above it. In this particular case, the attack density reading still returns the correct CR since it works with a windowed average. The minim-value CR can be said to be correct because the eight-note C#4 and B3 form a typical cadential ornament despite their being consonant, and so these short values should not derail the regular minim attack-density readings. In fact it is precisely for cases like this that an average must be taken as opposed to the duration of each individual slice. One can imagine adding a voice that could fulfill the bassizans role at this cadence moving from A2 to D3 against the E3 and D3 of the lower voice in the duo. In this case, the B3 fusa would be detected as a dissonance against the A2. This demonstrates the fact that the more voices there are present in a texture, the greater likelihood there is for diminished counterpoint in one voice to be dissonant against another voice in the texture.

On the other hand, musical passages with no dissonances whatsoever are encountered from time to time. In these situations the music denies the listener one of the primary means of metric orientation. Passages of this nature, such as that shown in Figure 21, are a common feature in Morley's duets.[66]

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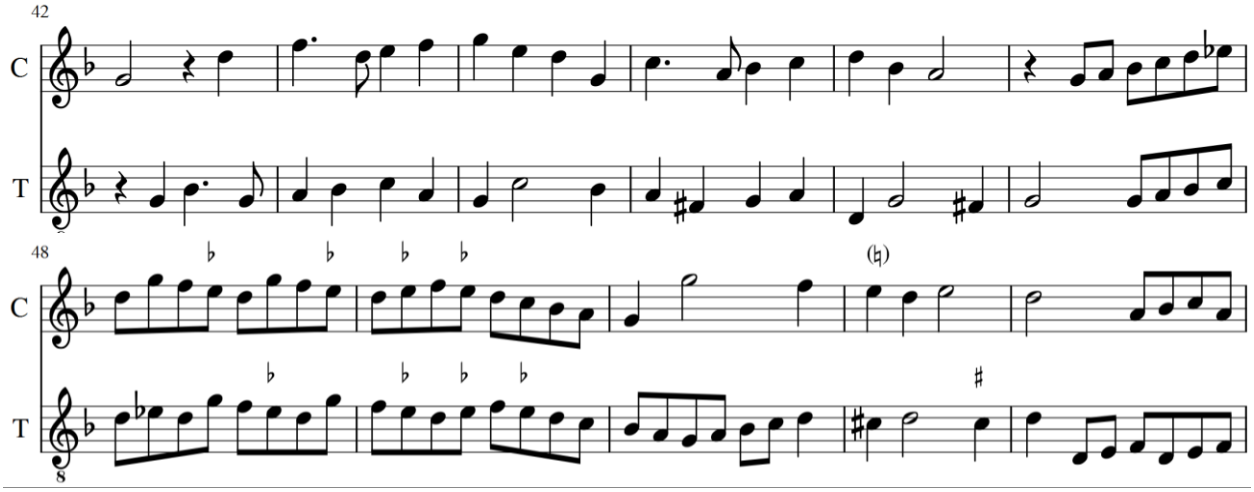
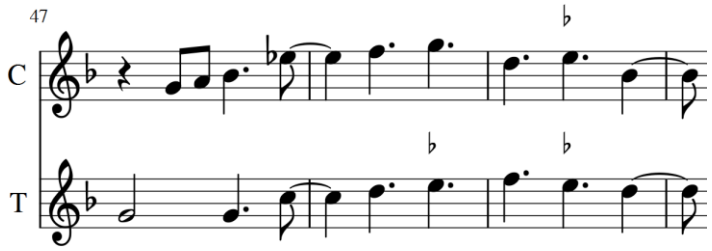
[65] Josquin, Crucifixus, NJE [13.12], http://josquin.stanford.edu/work/?id=Jos1312.
[66] Renotated from Bernard Thomas's edition, though the editorial accidentals are my own. Thomas Morley, "Fantasie: La Girandola," in *First Book of Canzonets to Two Voyces* (1595; London: Pro Musica, 2000), 12–3.

Figure 21: Morley, Fantasie: La Girandola*, mm. 42–52.*



From halfway through bar 47 to the downbeat of bar 50 both voices are in a constant stream of consonant eighth notes. In exclusively consonant passages such as this, any sampling rate will generally produce passable counterpoint, even non-isochronous patterns or absurd values such as, for the passage above, the dotted semiminim. To illustrate this point, a reduction that samples the counterpoint after every dotted semiminim starting from halfway through bar 47 is provided in Figure 22.

Figure 22: Improbable reduction of Morley, Fantasie: La Girandola*, mm. 47-50.*



In lieu of the clarity of metric expression the idiomatic use of dissonances provides, listeners can orient themselves according to other musical phenomena. This is the sort of scenario that the dynamic-offset method of contrapuntal sampling would normally analyze at the semiminim by virtue of the fact that this was the last unambiguously expressed CR. John Milsom's term, stretto fuga, is pertinent here; a stretto fuga is imitation at a given pitch interval after one unit of time.[67]

[67] John Milsom, "'Imitatio,' 'intertextuality', and early music," in *Citation and authority in Medieval and Renaissance musical culture: Learning from the learned*, ed. Suzannah Clark and Elizabeth Eva Leach (Woodbridge: Boydell & Brewer, 2005), 141–51.

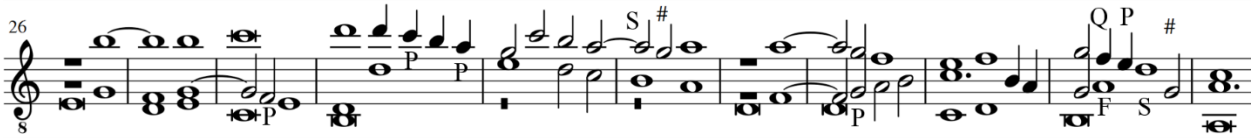
In Morley's duet, this section constitutes a stretto fuga at the semiminim which can be understood as corroborating the CR = semiminim analysis, though the time interval of imitation in stretto fugas are not always equal to the CR.[68] The consideration of imitative models in CR analysis is a promising avenue for future research.

An important point here is that the components of CR analysis, attack density and dissonance treatment, complement one another well. One must keep in mind that occasional instances of ambiguity and indeterminacy are not weaknesses of the dynamic-offset method. To the contrary, since this analysis is contextually based, uncertainty in the results reflects an unclear projection of CR in the music, which may well have been explicitly desired and sought after by the composer.

By way of summary, I list the steps of the dynamic-offset method in Figure 23 as applied to the three-voice chanson attributed to Josquin *Ce povre mendiant/Pauper sum ego*.[69] The discantus, tenor, and bassus parts are all shown on the same staff.

Figure 23: Analysis steps of the dynamic-offset method, all of which are fully automated.

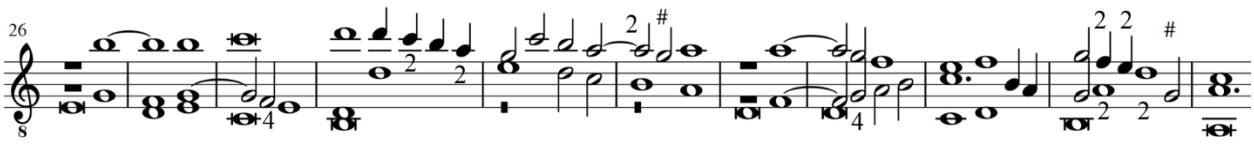
1. **Dissonance classification:** The process begins with the classification of dissonance types. The labels mean the following: P = passing tone, Q = dissonant third quarter (a type of accented passing tone), S = suspension, and F = fake suspension (a dissonant preparation to a suspension).



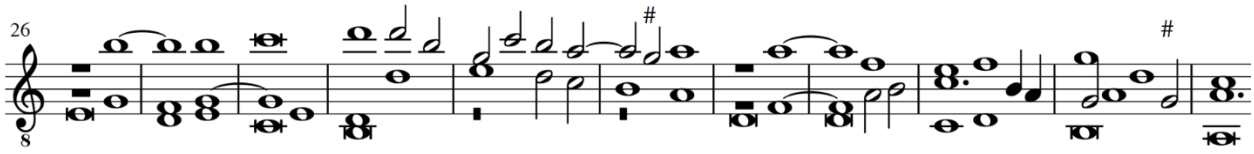
2. **Dissonance-based assessment of CR:** Each dissonance projects a CR value by virtue of its dissonance type, duration, and metric placement. These values are expressed with positive numbers which represent the number of quarter notes in modern notation (all transcriptions set the minim equal to the half note). Passing and neighbor tones project a

[68] Peter Schubert, "From Improvisation to Composition: Three 16th-century Case Studies," in *Improvising Early Music* ed. by Dirk Moelants (Leuven: Leuven University Press, 2014) 108–117.
[69] Josquin, *Ce povre mendiant/Pauper sum ego*, NJE 27.5, http://josquin.stanford.edu/work/?id=Jos2705.

CR at the metric level above the dissonance so in the example below, the passing tone in measure 28 is a minim in duration and therefore projects a semibreve CR (equal to 4 when measured in quarter notes). Suspensions project the CR of the level of their resolution so the suspension in measure 31 below that resolves on a weak minim projects a minim CR (equal to 2 when measured in quarter notes).

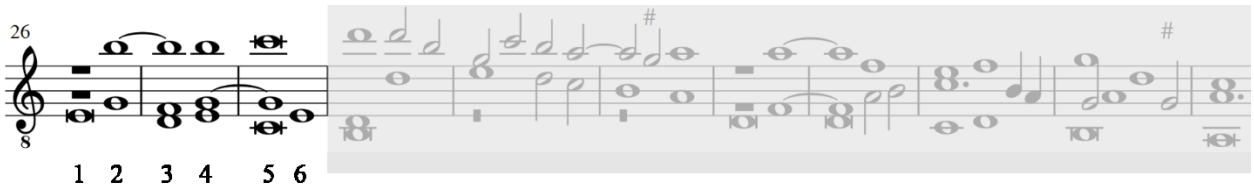


3. **Reduction:** All dissonances except for suspensions (including fake) are reduced out.



4. **Attack-density assessment of CR and comparison to dissonance-based assessment:** In this model an "event" is a time point in a piece that has at least one note onset in any voice. In a window of six consecutive events from the reduction of the piece from step 3, the attack density is calculated. In this case the six events span three breves or the equivalent of 24 quarter notes. 24 ÷ 6 = 4, so here the attack density suggests a CR at the semibreve. This reading is compared to the primary CR projected by the dissonances (see step 2); if they agree, this value is assigned as the CR of the end of this six-event window; if they disagree, the previous confirmed value is used. In this case both readings point to a CR at the semibreve.

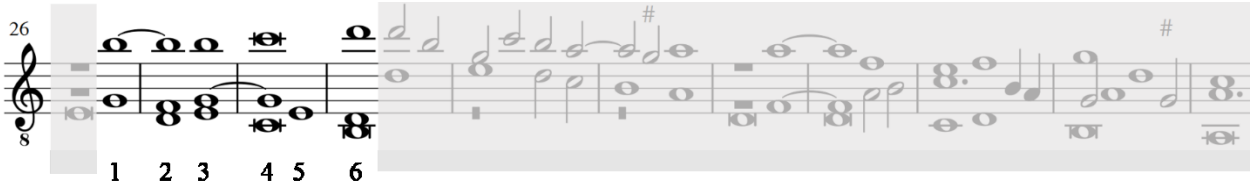
Window 1: The attack density is 24 ÷ 6 = 4 (semibreve)



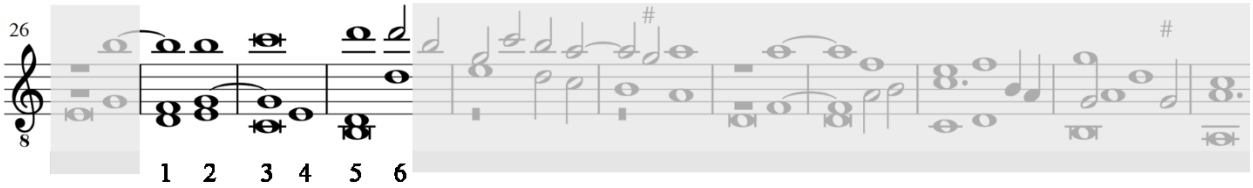
5. **Windowing:** Step 4 is repeated by sliding the six-event window over every event in the intermediary reduction from step 3. Average note values are rounded up or down to

reasonable duple-division note durations. In this example, all the attack densities are between 2 and 4 before being rounded to 2 or 4 (a minim and a semibreve respectively). The rounding threshold between two values in this model is 25% more than the shorter value, in this case 2.5. This means that any values below or equal to 2.5 get rounded down to 2, and values between 2.5 and 4 get rounded up to 4. In window seven of the example below, tighter attack density drops below the threshold of 2.5 and along with the weak semiminim passing tones in the highest voice which confirm this minim reading (see step 2) this causes the CR reading to shift to the minim.

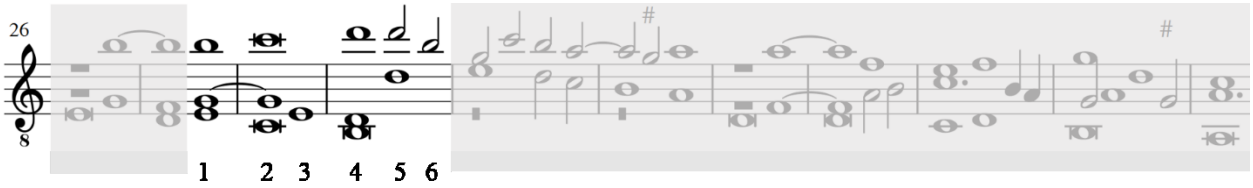
Window 2: The attack density is 24 ÷ 6 = 4 (semibreve)



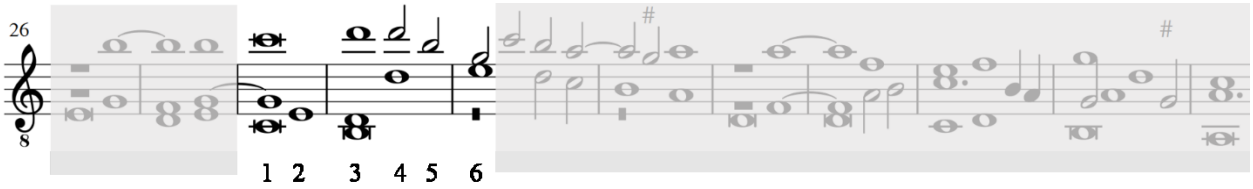
Window 3: The attack density is 22 ÷ 6 = 3.67 which gets rounded to 4 (semibreve)



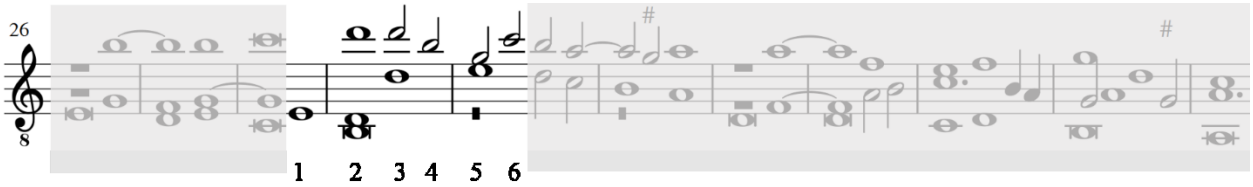
Window 4: The attack density is 20 ÷ 6 = 3.33 which gets rounded to 4 (semibreve)



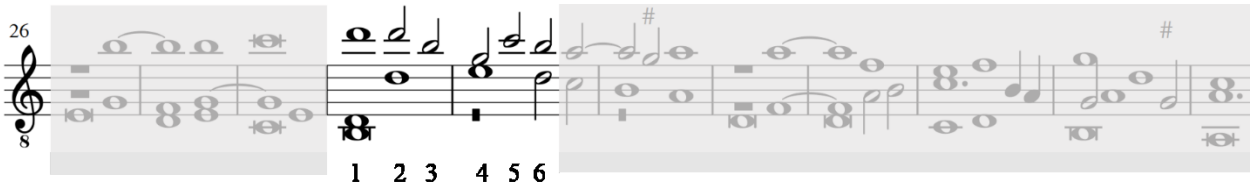
Window 5: The attack density is 18 ÷ 6 = 3 which gets rounded to 4 (semibreve)



Window 6: The attack density is 16 ÷ 6 = 2.67 which gets rounded to 4 (semibreve)



Window 7: The attack density is 14 ÷ 6 = 2.33 which gets rounded to 2 (minim)

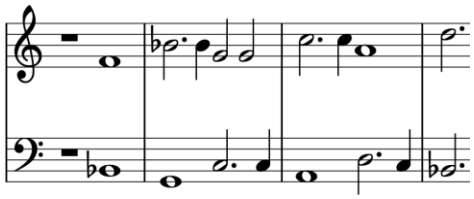


Etc…

Contrapuntal Levels

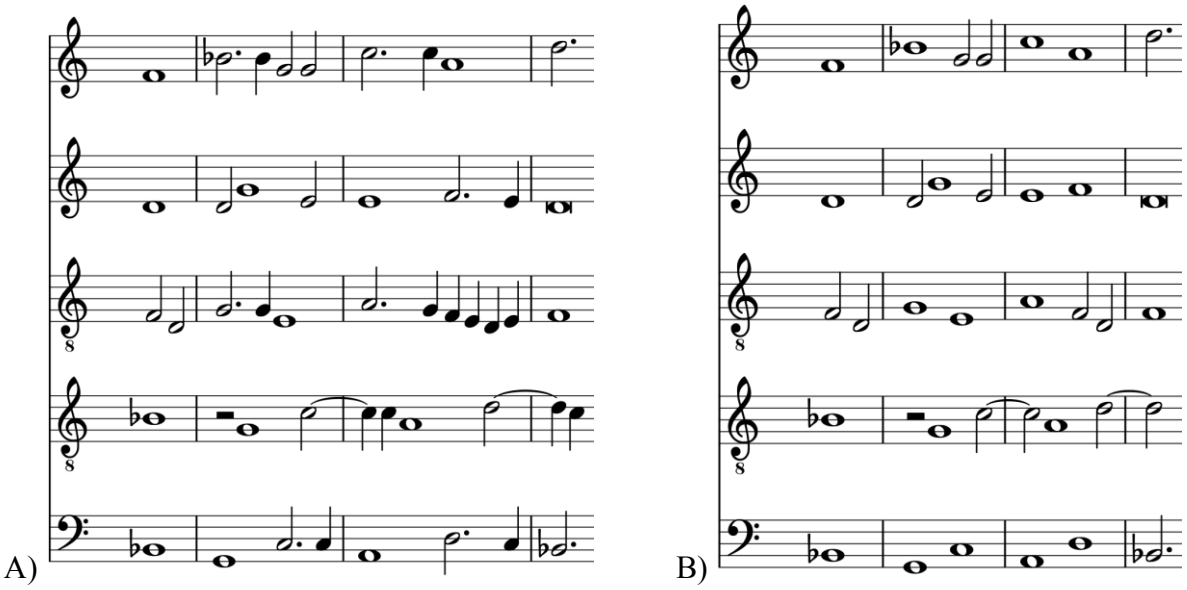
Fundamental counterpoint at the level of the CR groups into interval successions at the compositional *tactus* level. It is important to address cases where certain musical factors point to different mensural levels as the level of the CR. The two-voice excerpt in Figure 24 seems to project a semibreve CR.

Figure 24: Two-voice excerpt with an attack density that points to a semibreve CR.



Intervals mostly change at regular semibreves, and most of the values that are shorter than the semibreve still project this level by virtue of their agogic accent on consonant repeated notes. The two parts are even in stretto fuga at the seventh below after the time interval of a semibreve. Yet the excerpt above is not a duo but the two outer voices of a passage in five voices. The same passage in all five voices is provided in Figure 25-A, and in a reduced version in Figure 25-B.

Figure 25: Palestrina Missa Pro defunctis*, mm. 165–8.*



When all five voices are considered, the case for a minim CR becomes much stronger. The minim is the shortest note value that is consistently articulated throughout this passage. Attack-density analysis points to a minim CR because new intervals in the fundamental counterpoint are articulated at every minim. The passing tones on weak semiminims in the third measure shown corroborate this hearing. The rhythmic figure of a dotted minim followed by a semiminim seemed to stress the semibreve level by virtue of the placement of its agogic accent when we were just considering the outer voices. When all five are considered we see that this same figure appears twice beginning on weak minims in an internal voice. This is because the stretto fuga that we originally noted between the outer voices at the time interval of a semibreve has been intensified by the second tenor who follows the bass in a stretto fuga at the octave after just a minim. The result of the intermediary reduction used in the attack-density step is shown in Figure 25-B.

Considered together, Figure 24 and Figure 25 allow us to observe that the CR rhythm projected by any given pair of voices does not necessarily correspond to that of the entire piece. This is the reason that CR analysis cannot be pairwise in nature, but instead must be calculated for a piece as a whole. The fact that the three inner voices cause this shift of our assessment of the CR may help us better appreciate the rhythmic and contrapuntal interest they add to this passage. Figure 25 is also a particularly good example of the relationship between contrapuntal

tactus and CR, here the semibreve and minim respectively. The former generally contains many strong contrapuntal progressions (wherein both voices in any given pair move, often by contrary motion) and corresponds to the harmonic rhythm of the passage, whereas the latter contains many weak contrapuntal progressions (where only one or no voices move in any given pair) and corresponds to the time interval of imitation in the stretto fuga between the bass and second tenor.[70]

Alternative Automated Solutions

There are currently two primary methods by which to sample the counterpoint of a passage. These will be referred to as the "salami-slicing" method and the regular-offset method. I will now describe both methods, explaining what their strengths and weaknesses are, and how they differ from the dynamic-offset method I propose.

Salami Slicing

The term "salami slicing" is taken from Ian Quinn.[71] This approach consists of taking a new analytical reading every time there is a new event in the musical texture. A "new event" consists of a new note onset, or a new note or rest onset, depending on the particular implementation of the technique. It is called salami slicing because, with respect to duration, the different "slices" (events) of one's analysis will often be of varying length. Only a piece with constant and evenly spaced perpetual motion would have salami slices of equal duration.

This method is completely unsuitable for fundamental-counterpoint analysis because it does not reduce away anything from the musical surface. This means that it makes no distinction between fundamental counterpoint and ornamentation. The inappropriateness of this method for our purposes here does not mean that it is useless. Ian Quinn and Christopher White successfully used this method to reveal a hidden Markov model in the voice leading of Bach's chorales.[72] Michael Cuthbert also made good use of this method to demonstrate the extent to which pieces in

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[70] DeFord, Tactus, 83–7.
[71] Ian Quinn, "What's 'Key for Key': A Theoretically Naïve Key-Finding Model for Bach Chorales," *Zeitschrift der Gesellschaft für Musiktheorie*, 7.
[72] Ian Quinn and Christopher White, "Expanding Notions of Harmonic Function Through a Corpus Analysis of the Bach Chorales," (paper presented at the annual meeting for the Society for Music Theory, Charlotte, North Carolina, November, 2013).

a corpus of 14th-century monophonic Italian music made melodic borrowings from one another.[73] This method may even be of some use for our purposes here in conjunction with the dynamic-offset approach proposed. For example, comparing a dynamic-offset reduction of a piece to a salami-sliced representation of the same piece could help categorize the different ways the same fundamental counterpoint gets ornamented which could in turn point to tendencies in ornamentation.

Regular Offset

The regular-offset method is essentially what Pontio used in his analysis of his own interval successions. It samples a piece of music at a regular time interval. Generally this does not distinguish between notes that are attacked at the point they are sampled, and those that are still sounding from a previous attack. It takes advantage of the fact that, even in the pieces where it changes, the CR in Renaissance music is generally stable. The regular-offset method performs relatively well in pieces where the CR does not change, provided of course, that the researcher running the query has chosen the most appropriate duration at which to make regular analytical observations. Peter Schubert and Julie Cumming have made effective use of the regular-offset method in analysis of twelve duets by Lassus from 1577 without reducing away short accented passing tones.[74] As we will see later in the corpus study, these duets are quite consistent in their projection of a minim CR.

There are two main issues with this approach: 1) it is fully dependent on the quality of the researcher's decision at which regular offset to sample the counterpoint of a passage. If the researcher chooses poorly the results will be an inaccurate reflection of the fundamental counterpoint of the piece; and 2) if the CR changes in the course of the piece, the method will sample that entire passage incorrectly. By extension, whatever contrapuntal patterns, chord progressions, etc. the researcher is observing will not be an accurate representation of the piece.

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[73] Michael Scott Cuthbert, "Hidden in our Publications: Uncovering Concordances, Citations, and Influence in Medieval Music through Databases and Programming," (public lecture, All-Souls College, Oxford, October 22, 2015).

[74] Schubert and Cumming, "Another Lesson." For similar contrapuntal studies via interval-succession analysis, see: Christopher Antila and Julie Cumming, "The VIS Framework: Analyzing Counterpoint in Large Datasets," in *Proceedings of the International Society for Music Information Retrieval*, 2014, 71–76; and Darrell Conklin and Mathieu Bergeron, "Discovery of Contrapuntal Patterns," in *Proceedings of the International Society for Music Information Retrieval*, 2010, 201–206.

Even if we assume that a researcher is able to accurately and reliably identify the main CR of a piece, having to do so for each individual file in a large corpus can quickly become a time-consuming and error-prone task. Concerning the second point above, some may maintain that the CR of a piece of Renaissance music does not change. Mid-piece changes in mensuration make this an untenable argument, and even in the pieces which do not change mensuration, this stance oversimplifies the music. Ruth DeFord's study on compositional *tactus*, mensuration, and rhythm, includes detailed analyses of seven large corpora, from the songs of Du Fay to the masses of Palestrina. In each of these corpora the *tactus* was found to be mainly at a specific level, though it occasionally shifts to a different one. Her depiction of compositional *tactus* in the five- and six-voice motets of Josquin is representative: "The compositional *tactus* corresponds to the semibreve most of the time, but the breve takes on that function intermittently."[75] The only repertoire she found where a second level of *tactus* was particularly rare is in villanescas and villanellas, though even in these contexts she explains that they are found.[76] Furthermore, she explains that staying at a minim compositional *tactus*, which DeFord considers to be the lowest possible value, contributes to the "low-style character" of these pieces.[77]

When using the regular-offset method, one can choose to reduce out short accented passing tones as did Pontio in *RM*.[78] This momentarily shifts the analytical observation point off of the grid of absolutely regular durations, so it technically adds a measure of irregularity into this otherwise regular approach. As accented dissonances other than suspensions are more common in the 15th than in the 16th century, omitting this step is less of an issue in later Renaissance repertoire.

Measuring Success

Given that there are at least three automated ways of sampling the counterpoint of a piece in symbolic notation, how can we measure the success of the different methods? The best way is to see how the different approaches work in different musical situations. We can compare the results of the three methods to one another, and also to "ground-truth" analysis. In this case we

[75] DeFord, Tactus, 301.
[76] DeFord, Tactus, 448–54.
[77] DeFord, Tactus, 448.
[78] Pontio, *Ragionamento, 37*.

can take as ground truth the analysis of a piece done without a computer. Specifically, in so far as the goal here is to discern the fundamental counterpoint from the musical surface, the most successful method will be that which most closely resembles our analysis done by hand. The finer points of a reduction will of course vary from analyst to analyst, however, with well-chosen test cases, we will be able compare the results of the different approaches in important musical situations.

Ideally, the dynamic-offset method would produce the same exact results as a human analyst. But this is not necessarily an attainable goal and given that our ground truth is itself somewhat problematically subjective, a more modest goal will be more appropriate. If the dynamic-offset method identifies the fundamental counterpoint of a passage better than the two primary existing methods, the new analysis tool will be a useful contribution to the field.

Analyses and Applications

There are two basic ways CR can be used in music analysis: 1) as an observation about the counterpoint of a specific passage of music, and 2) as a way of generalizing about the contrapuntal character of a piece, corpus, or composer's style. By contrapuntal character I mean an assessment of how many times the piece or corpus changes CR, as well as what specific CR values it uses. In what follows, I will demonstrate specific applications of this tool. I begin by using it to analyze excerpts of a piece, and to compare two pieces. Then I show how CR analysis can inform the analysis of points of imitation. I close this section with a corpus study of duets. In addition to the findings these analyses present, my goal is to demonstrate the utility and breadth of this tool in the analysis of Renaissance music.

CR in the Analysis of Individual Pieces

CR analysis can inform our understanding of individual passages or pieces of music and I will show both of these types of results for the Gloria of Josquin's *Missa De beata virgine*. We have already seen some of the finer points about how it discerns the fundamental counterpoint of a passage, so now I will show how it can aptly describe rate at which counterpoint progresses in transitions between sections, and also characterize the counterpoint of an entire piece.

The counterpoint of the first section of Josquin's aforementioned Gloria shifts the mensural level on which it is active. My reductive method identifies 5 shifts between minim and

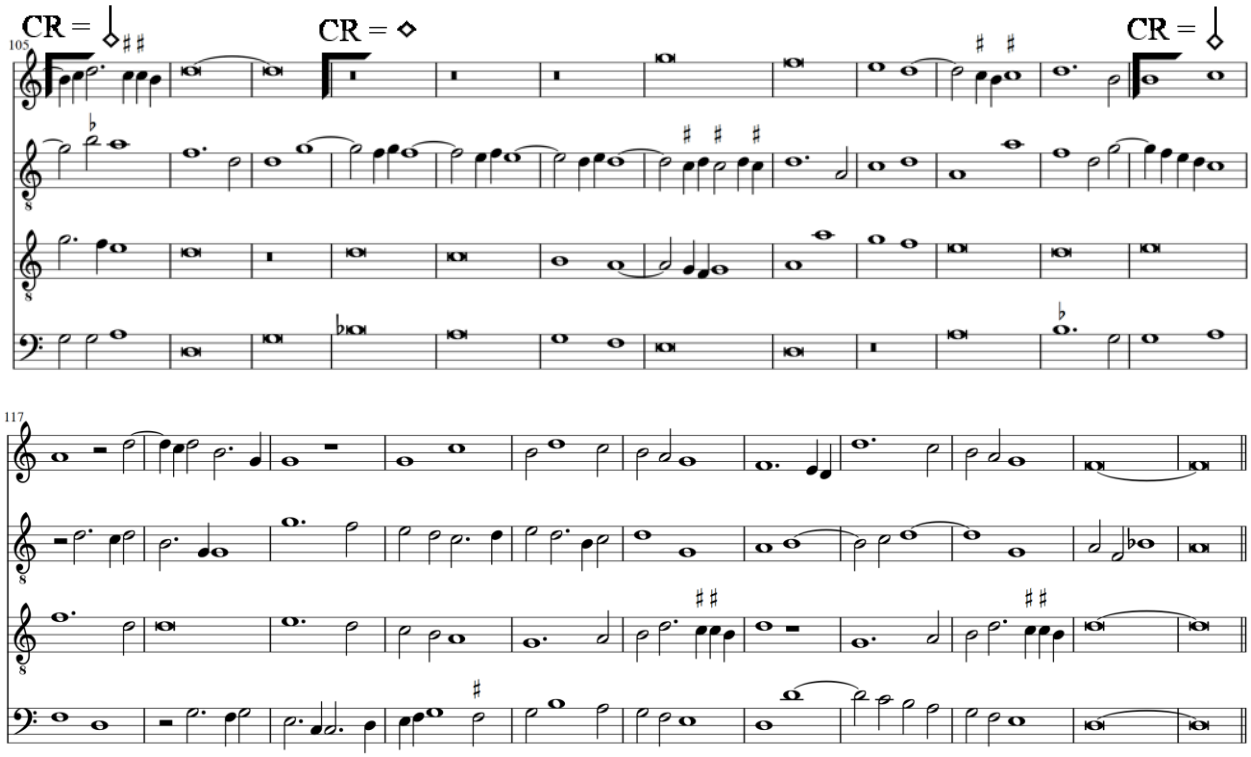
semibreve values in the first part of the Gloria. At first these points are identified by their offsets, or time points in the score. The offsets where the CR changes, as well as the offset of these changes are given in Figure 26.[79] These offsets are not very human-reader friendly, so I have also provided the measure numbers. The CR values of 2 and 4 correspond to the minim and semibreve respectively.

Figure 26: CR changes and their locations in Josquin's Missa De beata virgine*, Gloria, first part.*

CR Value	4	2	4	2	4	2
Offset	0.0	28.0	764.0	780.0	856.0	920.0
Measure	1	4.5	96.5	98.5	108	116

Most deviations from the primary CR value of the minim are brief. Figure 27 contains the longest shift away from the primary minim CR in favor of a semibreve CR, and the return to the minim.

Figure 27: Josquin, Missa De beata virgine*, Gloria, mm. 105–27.*



[79] Josquin, *Missa De beata virgine*, Gloria, NJE 3.3, http://josquin.stanford.edu/work/?id=Jos0303.

The passage begins with a minim CR, most clearly communicated with the cadence to D ending in bar 106. It employs a suspension which, though ornamented, resolves to C#5 on the last minim of bar 105 (the C#5 that immediately precedes this last minim is a consonant anticipation). This cadence in bar 106 marks the beginning of a new passage with a considerably more sparse attack density. There is no dissonance to confirm a shift to a semibreve CR until the next cadence which starts in bar 111. The suspension here features a slightly less common ornament, though the concept of a suspension a semibreve in duration is audible. The cadence with a deceptive bassizans resolution in bar 115 more clearly conveys the semibreve CR as it is ornamented in a more common fashion. After this evaded cadence, the music projects a minim CR with more and more clarity. Almost every minim is attacked from bar 116 to the end of the section, and all the cadential suspensions (in bars 120, 122, and 125) confirm the transition to a minim CR value, as do all the non-structural dissonances in bars 116–27.

The primary CR of this Gloria is the minim. Several briefer passages with a semibreve CR, such as the one shown in Figure 27, add considerable variety to the pacing of the piece despite not constituting changes in mensuration. It is crucial to note that passages with a longer durational value for their CR do not just employ longer notes, they adapt their counterpoint and, notably, their dissonances to match this new level of mensural orientation. By acknowledging the fact that the value of the CR of a piece can change, we can recognize similar contrapuntal patterns across shifts in the CR. We can only consider the cadences ending in bars 112 and 123 as inherently similar if we accept that fundamental counterpoint progresses at variable note values. And these two cadences are inarguably similar; the tenor and altus just switch cadential roles, and the bassus and superius sing the same notes, though the superius's are down an octave the second time. Julie Cumming has shown that the movement of cadential roles from one voice to another became increasingly common over the course of the Renaissance.[80] Indeed the main difference between these two cadences is their CR. Observations of this nature cannot be made with either the salami-slicing or the regular-offset methods for sampling counterpoint.

By way of contrast, we can compare this Gloria by Josquin to a Kyrie by Palestrina; the two movements are characterized by markedly different CR profiles. The Kyrie of his *Missa Pro*

[80] Julie Cumming, "From Two-Part Framework to Movable Module," in *Medieval Music in Practice: Studies in Honor of Richard Crocker* (Middleton, Wisconsin: American Institute of Musicology, 2013) 177–215.

Defunctis, part of which was shown in Figure 25, never strays from its minim CR. Even in music that never alters the value of its CR, the dynamic-offset method provides a more informed analysis than the regular-offset method because one can be sure that the regularity of contrapuntal measurements corresponds to the syntax of the music rather than simply being imposed. It remains to be determined along what lines, if any, variation in the CR profile of Renaissance music is found. Pieces that have a constant as opposed to variable durational CR value may be more common among certain composers, genres, ensemble sizes, periods, etc.

CR in Presentation-Type Analysis

Peter Schubert has theorized different compositional approaches to Renaissance points of imitation which he calls presentation types.[81] Focusing on Schubert's periodic-entry and imitative-duo presentation types, Julie Cumming and Schubert did an extensive study on how the use of imitation changed in 15th-century music.[82] They found a number of differences between mid and late 15th-century uses of imitation.[83] CR analysis reveals an aspect of points of imitation that could help further differentiate between presentation types and thereby potentially refine the use of this established theoretical model. I will demonstrate this by revisiting three of Cumming and Schubert's examples.

We can classify imitation points into two broad categories according to their CR profiles, those that maintain a steady CR, and those that accelerate it. I will revisit three of Cumming and Schubert's examples to demonstrate these two types. It is nearly impossible for a point of imitation to project a slowdown in the CR because whatever lines were projecting the initial CR get subsequently repeated. Any given melody or passage can get reinterpreted at a different CR when set in a new musical context, however, this change can generally only occur through acceleration because new added voices can only increase the pre-existing passage's attack

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[81] Peter Schubert, "Hidden Forms in Palestrina's First Book of Four-Voice Motets," *Journal of the American Musicological Society* 60 (2007), 483–556. For a corrected version of the appendix see: http://www.music.mcgill.ca/~schubert/finaltable.pdf.

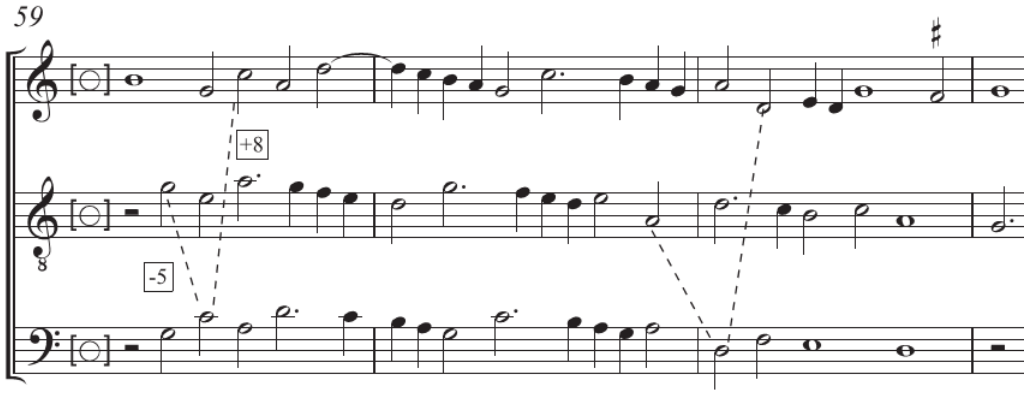
[82] Julie Cumming and Peter Schubert, "The Origins of Pervasive Imitation," in *The Cambridge History of Fifteenth-Century Music*, ed. Anna Maria Busse Berger and Jesse Rodin (New York: Cambridge University Press, 2015), 200–28.

[83] The authors also added a couple of new imitation types to Schubert's list. Julie Cumming discusses these additional presentation types in: Julie Cumming, "Text-Setting and Imitative Technique," in *The Motet around 1500: On the Relationship of Imitation and Text Treatment*, ed. Thomas Schmidt-Beste (Turnhout: Brepols, 2012).

density. Therefore the gradual building of texture inherent to points of imitation is antithetical to a decrease in CR.

The passage of Henricus Isaac's *O decus ecclesie* cited by the authors and reproduced in Figure 28 is a good example of a point of imitation that projects a steady CR at the minim.[84]

Figure 28: Henricus Isaac, O decus ecclesie, mm. 59–62 reproduced from Cumming and Schubert 2015.

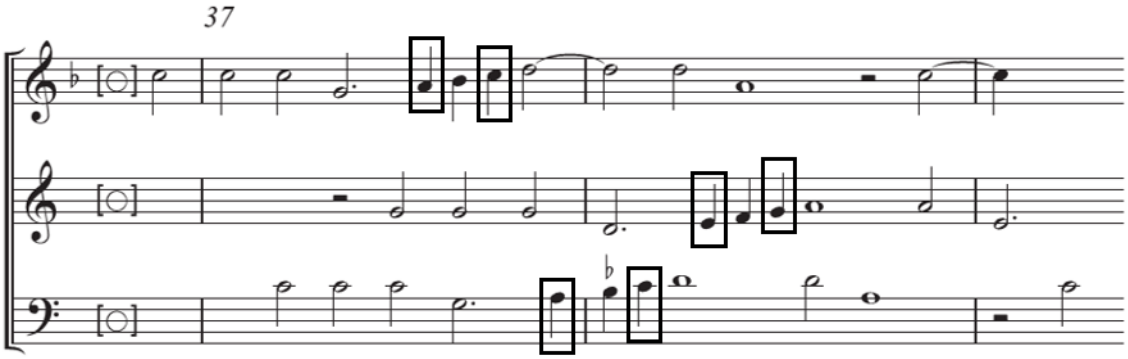


While the semiminims that begin with the fourth note of the *soggetto* do increase the surface rhythmic activity, the weak passing notes they add to the texture serve to confirm the steady minim CR. The suspension on a strong minim in the cadence that concludes this point of imitation is the final affirmation of its steady minim CR.

By contrast we can consider the passage in Figure 29, also reproduced from Cumming and Schubert, in which the CR is initially ambiguous, but then projects a minim CR.[85]

[84] Note that this example and the next one are in perfect tempus. My analytical model can be applied to these passages despite my previous stipulation that all divisions must be binary because there is no hemiola or other metric devices that exert an influence on the regular binary minims of the minor prolation used in both examples. Each perfect breve contains three semibreves, but as the CR never expands to the semibreve level these examples are not problematic for my method. Cumming and Schubert, "Origins," 212.
[85] Cumming and Schubert, "Origins," 213.

Figure 29: Henricus Isaac, O decus ecclesie, mm. 37–39 reproduced from Cumming and Schubert 2015.

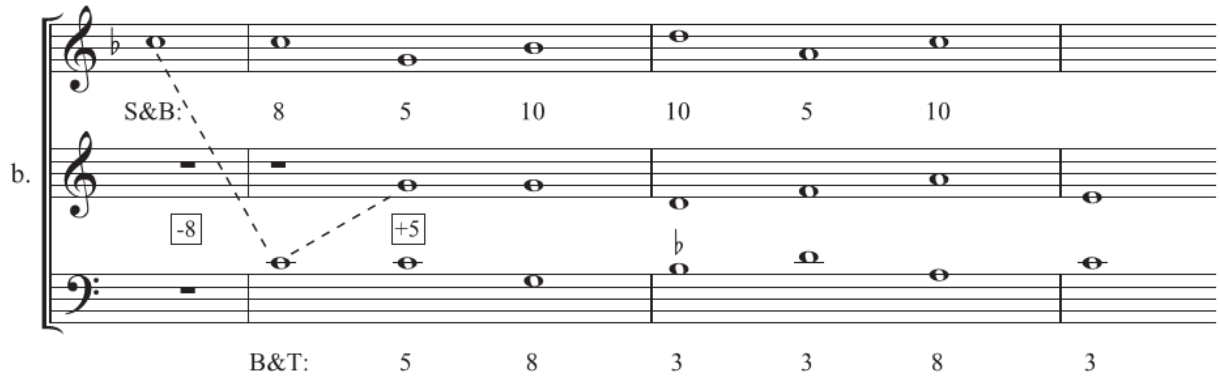


Despite articulating every minim from the beginning of the point of imitation, the passage begins ambiguously with respect to CR because the semibreve CR of the preceding passage (not shown) is not conclusively overridden in favor of the minim until the arrival of the semiminim passing tones on the fifth note of the *soggetto*. There are only three of these semiminims in the *soggetto* but two of them are ascending passing tones and the staggered entries cause six weak semiminims in a row to be articulated as dissonances all pointing to a minim CR. These six semiminims are designated with a box in Figure 29.

On top of accelerating the preceding CR, this passage is a good example of a stretto fuga that projects a CR that is shorter than the time interval of imitation. This is unlike the excerpt taken from the same piece shown in Figure 28 in which the CR and the time interval of imitation are both at the minim.[86] Cumming and Schubert also made a reduction of the passage in Figure 29 which is shown in Figure 30.[87]

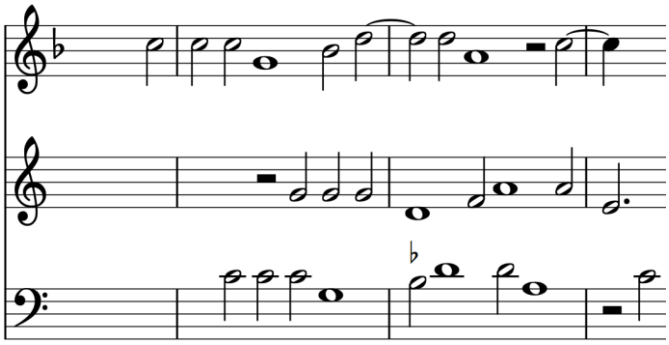
[86] Though it should be noted that the top voice shown can be thought of as following the middle voice after a semibreve.
[87] Cumming and Schubert, "Origins," 213.

Figure 30: Reduction of Henricus Isaac, O decus ecclesie, mm. 37–39 reproduced from Cumming and Schubert 2015.



This reduction in regular semibreves corresponds to the level that DeFord would call the compositional *tactus*. If instead we use the CR analysis to make a reduction of the excerpt Cumming and Schubert provided, shown in Figure 31, this ends up being at the minim.

Figure 31: Reduction of Figure 29 using the dynamic-offset method.

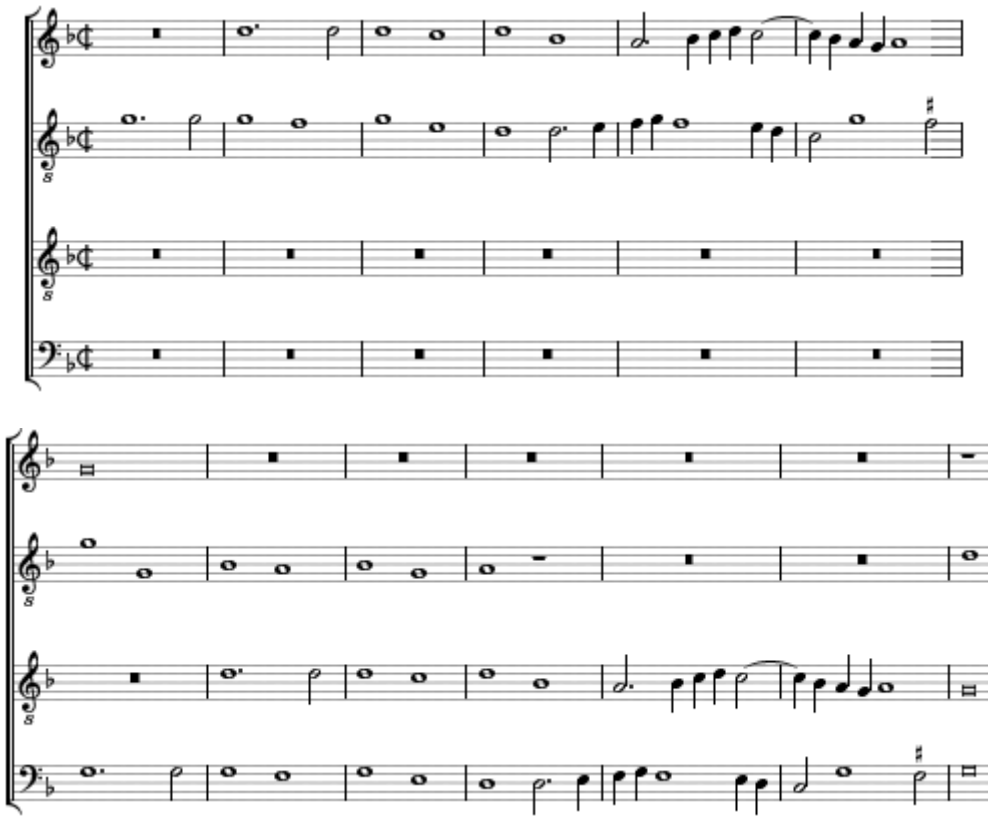


One reduction is not necessarily better or worse than the other, they are simply at different levels of reduction. This reduction at the minim (Figure 31) is heavily influenced by the passing-tone semiminims that go reduced away which underscores the orientational and analytical import that surface level dissonance can have on some theoretical models.

As a final demonstration of the influence CR analysis can have on presentation-type analysis, I will re-examine the incipit of Josquin's *Vultum tuum deprecabuntur*, shown in Figure 32.[88]

[88] Josquin, Crucifixus, NJE 25.14, http://josquin.stanford.edu/work/?id=Jos2514.

Figure 32: Josquin Vultum tuum deprecabuntur, tertia pars*, NJE 25.14, mm. 1–13.*



As Cumming and Schubert noted, the time interval of imitation begins at the breve, but then contracts to the semibreve.[89] The attack density mirrors this contraction of the time interval of imitation by accelerating from the semibreve to the minim. In light of this very audible shift in the attack density of the point of imitation, we may want to consider CR analysis to further distinguish between presentation types. The CR is generally stable in Renaissance music, but with the imitative duo repeating in the lower voices, one may hear the CR value shifting much more often than is normal in the space of just 24 semibreves: semibreve - minim - semibreve - minim.[90] CR variation of this sort is somewhat endemic, but not essential to the imitative duo presentation type. This is because the four-voice "periodic entries" tend to normalize the CR by

[89] Cumming and Schubert, "Origins," 223–4.
[90] The stretches that could be heard as being at a CR of the semibreve are not confirmed by dissonances, so they are somewhat ambiguous.

virtue of the fact that the same melody is sounding in at least one voice throughout the point of imitation.[91]

CR in a Corpus Study

Automating an analysis facilitates its application in corpus studies and CR analysis has a number of potential applications in this regard. One can imagine doing a large study of ornamentation by comparing the musical surface to a fundamental counterpoint reduction based on the CR. Another possibility would be to use the CR profiles of a composer's securely attributed pieces in order to inform situations of doubtful attribution though this would depend on the extent to which the composer's CR profile is idiosyncratic and/or uniform. It is my hope that other researchers will use my CR analysis tools for these and other studies. In what follows I will describe the results of a study of a corpus of duets by Josquin, Lassus, and Morley to compare and contrast their compositional styles based on the CR profiles of their compositions.

The corpus consists of twelve duets each by Lassus and Morley, and nine mass movements entirely in two voices and one two-voice motet by Josquin for a total of 34 pieces or movements. The attribution of the pieces by Lassus and Morley is secure, but less so for some of the Josquin pieces. The full list of pieces is given at the end of this section in Figure 37. These pieces were chosen because they are of comparable length, are entirely in two voices, and maintain minor prolation throughout. Two further two-voice mass movements by Josquin were found, but one was excluded because it is too short, and the other because it makes use of major prolation. The Josquin pieces were all taken from the Josquin Research Project database, and the Lassus from the ELVIS database.[92] The Morley files were transcribed from a facsimile of the 1595 print.[93] Given that, as we saw, several components of the dynamic-offset method are sensitive to the number of active voices in a texture, it was important to have pieces with all the same number of voices throughout. In this way any issues concerning the application of the dynamic-offset method in two voices apply to all pieces in the corpus equally.

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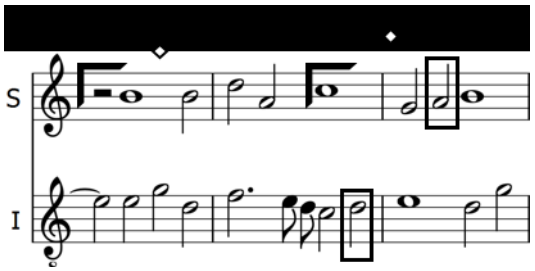
[91] The "imitative duo" and "periodic entries" terminology is taken from: Schubert, "Hidden Forms."
[92] Josquin des Pres, Josquin Research Project led by Jesse Rodin: http://josquin.stanford.edu/; Orlando di Lasso, *Magnum opus musicum*, typeset Carl Proske, ed. Franz Xaver Haberl (New York: Broude Brothers Limited, 1973), vol. I, 1–7; the files in symbolic notation of Lassus's duets Morley's canzonets were taken from the database of the ELVIS team, led by Julie Cumming: http://database.elvisproject.ca/.
[93] Thomas Morley, *First Book of Canzonets to Two Voyces*, ed. Bernard Thomas (London: Thomas Este, 1595; New York: Performers' Facsimiles, 1988).

Josquin and Lassus's pieces exhibited a similar amount of variety in their CR values, whereas Morley's pieces never changed in this regard. For Josquin and Lassus the primary CR value is the minim. The two times that this differs in Josquin's pieces, it is in favor of a semibreve CR. By contrast, the three times Lassus's duets express a CR change, it is by acceleration to the semiminim. A sample passage from Josquin that decelerates from the minim to the semibreve is shown in Figure 33.[94]

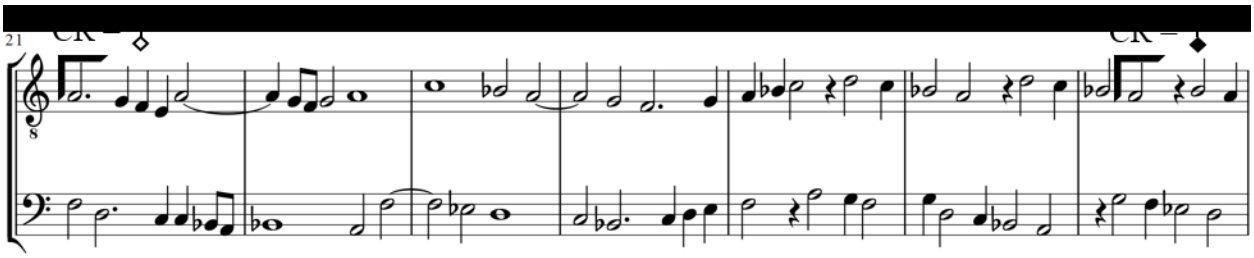
Figure 33: Josquin, Agnus Dei, NJE [13.11]*, mm. 13–15.*



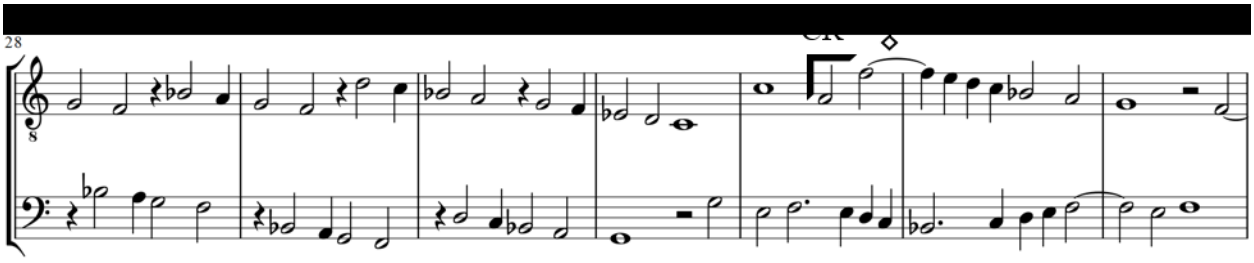
Crucial here are the passing tones on weak minims which are designated with boxes. In conjunction with the attack-density analysis, they express the CR's expansion to the semibreve. This semibreve CR value is not sustained for very long. The suspended G in m. 16 already begins to reorient the passage to a minim CR.

In the interest of examining a piece that conveys a departure from the primary CR by means of acceleration, we can consider the excerpt from Lassus shown in Figure 34.

Figure 34: Lassus, Qui vult*, mm. 21–34.*



[94] Josquin, Agnus Dei, NJE [13.11], http://josquin.stanford.edu/work/?id=Jos1311.



This excerpt begins by clearly projecting a minim CR. Then, from mm. 25–30 Lassus includes syncopated minims. While the attack density suggests a semiminim CR soon after their arrival, this shift is not immediately present in the dissonance analysis because, at first, these syncopated minims are purely consonant. This is why the syncopated minim G3 in the lower voice in bar 27 (designated with a box) is so important. It is a dissonant suspension on a strong semiminim and finally corroborates the semiminim-CR analysis. This hearing is further fortified by the syncopated B♭4 in bar 28 of the upper voice (also designated with a box) which is a dissonant suspension for the second half of its duration. This passage comes to a close on C3 with the concluding gesture in bar 31. The ensuing passage unambiguously returns to a minim CR.

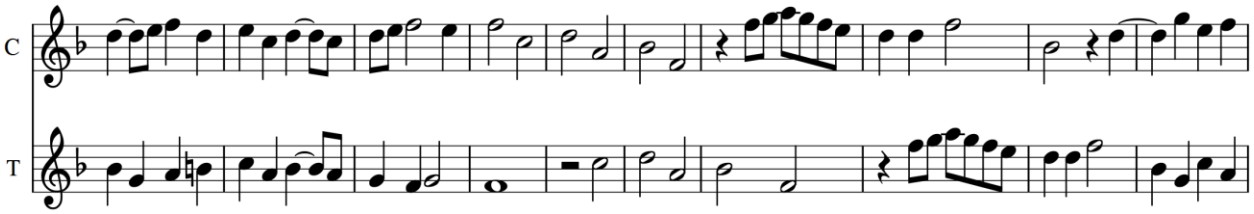
Comparing Josquin and Lassus with respect to CR, we can say that they employ a similar amount of variety in this facet of their composition but in opposite ways; Josquin's changes of the CR are by deceleration from the minim to the semibreve, whereas Lassus's are by acceleration from the minim to the semiminim. We will look at quantifying these composers' use of CR in more precise terms shortly.

By contrast Morley's pieces all returned CR readings uniformly at the semiminim. This would point to a minim compositional *tactus* using DeFord's method. In her analysis of villanescas and villanellas she connected their lack of variety in this regard with low-style composition, especially given that this minim level at which they mainly function is what DeFord deemed to be the shortest possible compositional *tactus*.[95] The fact that his primary (and only) CR value is shorter than those of Josquin and Lassus is more of a notational difference than a true compositional one. The lack of observed CR variety, however, constitutes a significant musical difference.

[95] DeFord, Tactus, 448.

The lack of diversity in the CR of Morley's duets is not due to their attack density readings. On the contrary, there are some passages in these twelve duets whose attack density suggests a minim CR, such as that shown in Figure 35.[96]

Figure 35: Morley Goe yee my canzonets*, mm. 22–31.*

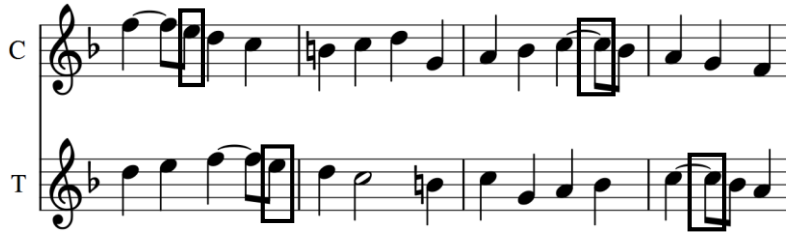


Here the attack density is uniformly at the minim for six minims from mm. 25–8. However the lack of dissonance to confirm the minim level in this section is what keeps the CR from shifting from the semiminim to the minim. From bar 25 to the downbeat of bar 28 in the excerpt above we have one of the extremely rare situations in which the salami-slicing method can potentially return a CR analysis closer to what a human analyst would want than the dynamic-offset method currently does.

Although passages such as that in Figure 35 do not get supported by dissonances projecting a minim CR, Morley's twelve canzonets do exhibit some variety in the CR values projected by their dissonances. Notably, while the dissonant portion of most suspensions is a semiminim in duration, some last for only a fusa. As noted earlier, a dissonant suspension that resolves on a weak fusa points to a fusa CR value. In terms of DeFord's compositional *tactus*, this would be a semiminim, though she maintains that the minim is the shortest possible value.[97] This is another case in which the two components of my analysis disagree. Similarly to what we saw with the sections potentially analyzed at the minim in Figure 35, these fusa CR values projected by the dissonances are not corroborated by the attack-density readings. A good example of this is shown in Figure 36.

[96] Thomas Morley, *First Book of Canzonets to Two Voyces*, ed. Bernard Thomas (London: Thomas Este, 1595; New York: Performers' Facsimiles, 1988).
[97] DeFord, Tactus, 448.

Figure 36: Morley, When Loe by Break of Morning*, mm. 7–10.*



In this excerpt there are four suspensions (shown in boxes) where the dissonant portion lasts only a fusa. At no point in this passage, however, does the attack density project a fusa CR. Rather, the attack density conforms more closely to the suspension lasting a semiminim in the second bar of this excerpt. If we used a smaller window size in this corpus study, the CR in Morley would demonstrate some variety, such as in sections like Figure 36 where it would accelerate briefly to the fusa. But if we apply the same window-size reduction to all the pieces in the corpus, more CR variation would also be found in the pieces by Josquin and Lassus. So while some of the finer points of CR analysis as implemented in the dynamic-offset method may require further revision to more precisely assess *exact* amounts CR variation, it already provides informative results concerning the *relative* amounts of CR variation in the works different composers. It is also important to note that there is a fifth suspension in the second measure shown in Figure 36. The second half of the minim C5 is a dissonant suspension. This suspension points to a semiminim CR and is cadential, unlike the four designated with boxes. Distinguishing between cadential and non-cadential suspensions may be a useful avenue of future refinement to the dynamic-offset method.

In light of this closer examination of Morley's writing, we see that his canzonets do contain some rhythmic variety despite their overall uniform CR values. Considering my two criteria separately, we can note similar diversity in the attack density in the pieces of the three composers, but not in their dissonance treatment. Since my model requires agreement between the two readings in order for the CR analysis to change, the dissonance treatment seems to be the main locus of difference between Morley on the one hand, and Lassus and Josquin on the other, at least with respect to CR.

Figure 37: Summary of CR findings in duets. SM, M, and SB stand for semiminim, minim, and semibreve respectively.

Composer	Piece	CR	
		Primary	Secondary
Josquin	Agnus Dei	M 96.9%	SB 3.1%
	Crucifixus	M 100%	NA
	Et incarnatus est	M 100%	NA
	Missa Ad fugam, Sanctus - Pleni	M 100%	NA
	Missa Ad fugam, Sanctus - Qui venit	M 100%	NA
	Missa Allez regretz, Sanctus - Pleni	M 100%	NA
	Missa Ave maris stella, Sanctus - Benedictus	M 100%	NA
	Missa Ave maris stella, Sanctus - Qui venit	M 100%	NA
	Missa Pange lingua, Sanctus - Benedictus	M 100%	NA
	Qui edunt me	M 62.6%	SB 37.4%
Lassus	Beatus vir	M 100%	NA
	Beatus homo	M 100%	NA
	Oculus	M 93%	SM 7%
	Justus	M 100%	NA
	Expectatio	M 100%	NA
	Qui sequitur me	M 100%	NA
	Justi	M 100%	NA
	Sancti mei	M 100%	NA
	Qui vult	M 82.1%	SM 17.9%
	Serve bone	M 100%	NA
	Fulgebunt justi	M 100%	NA
	Sicut rosa	M 91.6%	SM 8.4%
Morley	Goe yee my canzonets	SM 100%	NA
	When loe by break of morning	SM 100%	NA
	Sweet nimphe	SM 100%	NA
	I goe before my darling	SM 100%	NA
	La Girandola	SM 100%	NA
	Miraculous loves wounding	SM 100%	NA
	Lo heere another love	SM 100%	NA
	Fyre and Lightning	SM 100%	NA
	Flora wilt thou torment mee	SM 100%	NA
	In nets of golen wyers	SM 100%	NA
	O thou that art so cruell	SM 100%	NA
	I should for griefe and anguish	SM 100%	NA

A summary of the CR analysis for each piece in this corpus is given in Figure 37. There are at least two ways to quantify the large-scale use of CR by a composer. We can count the

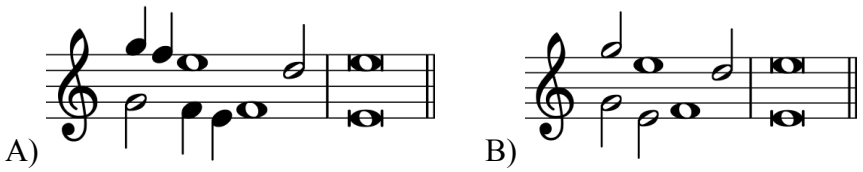
number of pieces in a corpus that exhibit shifts in CR. For the corpora shown in Figure 37 this would 20% of Josquin's duets, 25% of Lassus's, and 0% of Morley's. Another means of comparison would be to see, on average, how much time is spent at a CR other than the primary CR. This results in 4% of the time for Josquin, 3.3% for Lassus, and 0% for Morley. This returns lower percentages than the first method, but keeps the composers in similar proportions of CR variety. Josquin's reading using this second method is slightly higher than that of Lassus, which suggests that when Josquin does use more than one CR level in a piece, he uses it much more extensively.

Known Issues

The main issue with the dynamic-offset method is that it is currently limited to pieces with duple divisions. Similarly, it is currently incapable of intelligently handling passages containing hemiola. This problem is due to a general lack of theorization of suspensions in triple meter by both period and modern authors.

It is theoretically possible for the reduction produced by the dynamic-offset method to include parallel octaves or unisons where there was a particular realization of a chanson idiom. A passage such as Figure 38-A would get simplified to Figure 38-B using this method.

Figure 38: Problematic reduction of a hypothetical chanson idiom.

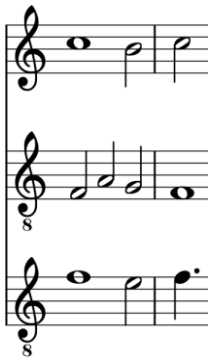


While this is a potential issue, this is arguably an accurate representation of the fundamental counterpoint of this passage. (I have not actually encountered a realization of a chanson idiom such as that in Figure 38-A in the repertoire. Peter Schubert's description of chanson idioms in his modal counterpoint textbook provides six abstract examples of the dissonance, none of which correspond exactly to Figure 38-A above.)[98] While parallel perfect intervals were generally considered anathema to good counterpoint, tolerance of them is not entirely unprecedented. Tinctoris's second general principle about counterpoint in book III of the *LAC* allows for parallel

[98] Schubert, *Modal Counterpoint*, 147–8.

perfect intervals in three or more voices when a third voice moves in contrary motion. The example he provides is reproduced in Figure 39 with the parallel fifths in the outer voices. The two places this happens are indicated with an asterisk, and the first one is particularly clear.[99]

Figure 39: Tinctoris's example of tolerated parallel fifths renotated from Seay's translation.



Peter Schubert has pointed out that Pedro Cerone's suggested two-voice interval successions in first species semibreves "often contain voice-leading errors that will be corrected when one line is diminished.[100] Similarly, Geoffrey Chew has found many parallels in reductions of Monteverdi.[101] Therefore when removing diminution for the sake of a reduction, one must accept that some voice-leading errors may creep into the reduction.

I point out this case of apparent parallel octaves in a hypothetical chanson-idiom reduction because any time reductions are made, one must entertain the idea that they will reveal "bad" counterpoint at deeper levels of the hierarchy. A great many contrapuntal mistakes of this sort would likely suggest that the level of reduction being considered is deeper than that of any syntactical contrapuntal progressions. The dynamic-offset method does not venture into such deep levels of reduction.

Another issue is that when no dissonance is used, the method has trouble adapting to apparent changes in CR as we saw in the Morley excerpt in Figure 21 and Figure 22. It remains to be determined if the best way to deal with such passages is to continue with the last secure

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[99] Johannes Tinctoris, *Liber de arte contrapuncti*, trans. Albert Seay (Rome: American Institute of Musicology, 1961), bk. III chap. 2, 133.
[100] Peter Schubert, "Counterpoint Pedagogy in the Renaissance," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 510; Pedro Cerone, *El melopeo y maestro* (Naples, 1613; repr. Bologna, 1969), bk IX, chap. 24, 587.
[101] Geoffrey Chew, "The Perfections of Modern Music: Consecutive Fifths and Tonal Coherence in Monteverdi," *Music Analysis*, 8/3 (October, 1989): 247–73.

reading, label the passage as indeterminate, or just rely on the attack-density reading. The current implementation takes the first of these options.

Future implementations of this automated approach to CR analysis may make more sophisticated use of the number of active voices by detecting large and abrupt changes in texture around which the CR may be more likely to change, and also would distinguish between when a voice drops out of the texture for an extended period of time and when it leaves only momentarily, for example to participate in a hoquet.

A final issue I will raise is that no distinction is currently made between strong and weak contrapuntal progressions. As defined by DeFord, strong contrapuntal progressions include motion in both voices when analyzing with a pairwise approach, and these are often by contrary motion. Weak contrapuntal progressions, on the other hand involve motion in only one voice in any given pair.[102] DeFord has associated strong contrapuntal progressions with the articulation of the compositional *tactus*. It remains to be seen what impact a consideration of different types of contrapuntal progressions could have on CR analysis.

Conclusions

This study of CR adds precision and detail to the broad exploration of CR and compositional *tactus* that DeFord's recent book offers.[103] After reviewing some theoretical points put forward by Tinctoris, Vicentino, and Pontio, I described a step-by-step process by which one can discern the CR of a passage. This process is based primarily on two components, attack density and dissonance analysis. I then compared the dynamic-offset method to alternative automated approaches of sampling counterpoint. In almost all cases, the dynamic-offset method outperformed or at least matched the results of the alternative methods.

Having grounded my approach in period sources and described its constituent steps, I demonstrated several analytical uses of the dynamic-offset method. My first analysis case showed how Josquin added to the variety of the Gloria from *Missa De beata virgine* by repeatedly altering the CR. This was contrasted to the Kyrie from Palestrina's *Missa Pro defunctis* which includes no change of CR. I then demonstrated how considering CR could help

[102] DeFord, Tactus, 83.
[103] DeFord, Tactus, especially 82–113.

further refine a categorization of presentation types. This was achieved by differentiating between points of imitation that continued the preceding CR, or accelerated it. As a final demonstration of this tool in analysis, I applied it to a corpus study of thirty-four duets by Josquin, Lassus, and Morley. While the CR values for Josquin and Lassus's pieces were different, they contained a similar amount of variety. By contrast, Morley's duets consistently projected a semiminim CR and therefore displayed no variety with respect to this aspect of composition. This lack of CR variety was similar to that seen in Palestrina's *Missa Pro defunctis*, Kyrie. In addition to the specific findings they contribute, these studies testify to the strength and versatility of CR as an analysis tool.

The dynamic-offset method amounts to the first systematic approach to CR analysis in the context of analysis and reduction of modal counterpoint. Revisions to the method will be necessary. I have identified a few known issues with my implementation of this method, but the main reason for this is that the biggest theoretical issues surrounding contrapuntal analysis and reduction in Renaissance music are yet to be definitively resolved. The unprecedented level of precision that this study provides on the topic is an important contribution to the field because it promotes scholarly discussion of CR and proposes an analytical methodology for future studies to build on. I also foster analytical accountability by automating analysis. Among other benefits, automation rapidly exposes the analytical repercussions of new theoretical assumptions and decisions thereby making much more rapid development of analytical models possible. The fact that CR analysis can be used to make a reduction of a piece greatly broadens its potential applications as reduction is such a common analytical procedure. The goal of the computational approach I offer here is therefore to facilitate and accelerate the theoretical aspects of CR.

Conclusion

In this dissertation I began by uncovering a number of new observations about Tinctoris and Pontio's interval-succession treatises. Beyond the implications these findings have on research on the *LAC* and *RM*, they more broadly promote the integration of computational techniques in the analysis of interval-succession treatises. Examining each of the treatises individually led naturally to a comparison of the theoretical and pedagogical approaches of the two authors' takes on interval-succession theory. With this comparison, I revealed important distinctions in some basic tenets of the authors' approaches which show that while Tinctoris's list is more in line with treatises associated with a tradition of rule-based memorization, Pontio's list can be understood as participating in the commonplace tradition. Building on these observations, I evaluated the applicability of the two authors' analytical models for relating their successions to real music. I found that Tinctoris's abstract successions give no indication of the mensural level to which they pertain, and conversely that Pontio was rigidly strict, always analyzing at regular minims.[1] Neither of these two authors, therefore, provides an acceptable means of assessing the fundamental counterpoint of a passage. Furthermore the writings of Vicentino and Burmeister clearly demonstrate that the same cadential passages can be found at three mensural levels, so I concluded that a degree of flexibility in any reductive model is a necessity. In offering the dynamic-offset method in this dissertation, I aim to satisfy this need in the field, or at least begin to do so. In what follows I will summarize my main findings in this dissertation in slightly more detail.

Interval-Succession Treatise Examples as Corpora

Liber de arte contrapuncti

Tinctoris was an exceptionally thorough music theorist, and he stayed true to form when writing the *LAC*. Well known for its impressive scope, his interval-succession list has often been characterized as exhaustive. By comparing the list of interval successions that follow his five

[1] The one exception to this in Pontio occurred when a semiminim dissonant third quarter (an accented passing tone) displaced his analysis by one semiminim. Pietro Pontio, *Ragionamento di musica* (1588), 64, accessed from: Christophe Dupraz, *Traités Musicaux Romans* (www.tremir.fr), 2013, specifically: http://www.ums3323.paris-sorbonne.fr/TREMIR/TReMiR_Pontio/R0_start.htm.

explicitly stated voice-leading principles (935 successions) to the list of successions that he actually provides (768 successions), I correct this misunderstanding as my analysis shows that there are 167 successions that are unaccounted for. More importantly, by accounting for octave-equivalency (his successions are given in a maximum range of three octaves) and situations where two successions are the same, but the tenor has gone from being the lower voice to the upper voice, I show that Tinctoris exhibits strong consistency in the six different successions that convey the same fundamental counterpoint. By identifying categories of interval successions that he routinely omits from his treatise, I reveal nine tacit voice-leading principles that all of his given successions respect, but that Tinctoris does not write out in prose.

Ragionamento di musica

While still part of the greater tradition of interval-succession theory, Pontio's list of 123 examples breaks with some strongly established norms in the theoretical genre. The most significant among these is the fact that he allows dissonances to be included in the vertical intervals of his successions. Related to this, Pontio's successions are found in ecologically correct examples in two-voice diminished counterpoint again breaking with the standard which was in abstract first-species counterpoint. Pontio significantly enriches interval-succession theory by adding a plethora of musical context. All his examples include comments about interval quality, metric placement, and the suitability in a specified number of voices in both improvisation and composition. Some also include further comments about: accidentals, duration, texture, genre, imitation, affect, and timbre.

In examining Pontio's body of interval succession examples I showed that on more than one occasion he cites a tolerance for an interval succession when set in an 8-voice polychoral texture. This is significant not only because it is a very specific type of musical contextualization, but also because polychoral settings did not come into use until the 16th century. This demonstrates that Pontio's interval-succession list provided some much needed updating to the theoretical genre.

The considerable attention to detail that Pontio affords each of his examples through his involved contextualization contrasts sharply with the more relaxed overall structure of his list. I showed that the general structure of Pontio's interval-succession list is clearly structured on three factors:

Figure 1: Hierarchy of criteria Pontio uses to structure his interval-succession list.

1. Consonance/dissonance of first vertical interval
2. Interval quantity and quality of first vertical interval, in ascending order (except d5)
3. Preferred successions

The third of these criteria is particularly unspecific as he gives no set definition for what he generally considers to be a preferred succession. More important than these structuring principles themselves, however, is the fact that Pontio is somewhat inconsistent in his application of them. Some successions are found in the chapters on the minor third and minor sixth that actually start on a major third or major sixth, and vice versa. Furthermore, at the end of book II Pontio asserts that he takes the lowest-sounding voice as the referential voice in his successions. Despite this assertion, in his only example in three voices he measures intervals against the middle voice. Similarly, the interval succession in one of the pieces he references by Rore is found between the upper two voices of a four-voice piece.

While these minor deviations from his main overall structure and general theoretical approach are not confusing for the reader, they do diminish the impression that Pontio is being systematic. These deviations suggest that Pontio is appropriating interval succession theory as a means to communicate lessons about the topics to which he continually returns, including accidentals, imitation, affect, and genre.

The fact that Pontio is not as systematic in the building of his list as, for example, Tinctoris was, means that in studying his list it is not possible to apply the same methodology that I used to discover tacit principles operative in Tinctoris's list. However, other types of observations can be made about Pontio's list when we study his examples as a coherent corpus. For example, by considering all of the examples that include passing dissonances within the succession being discussed, I show that Pontio samples counterpoint at regular minims throughout his list. This means that any metrically weak passing dissonance between on-beat minims do not even get mentioned in the text descriptions of his examples. Though somewhat simple, this approach to reduction is a telling aspect of Pontio's intervallic thinking. It quickly gets him into trouble. Of his forty-two interval-succession examples that include dissonance, thirty-six of the dissonances in question are suspensions. Five of the six remaining dissonances are passing tones on weak minims. The fact that he makes new analytical observations at regular minims (as opposed to switching to

some other mensural level) means that these passing tones a minim in duration get treated on equal status with the consonances in his successions. These passing tones are therefore problematically depicted as structural.

Beyond what this dissertation shows about the *LAC* and *RM*, my methodology is an important contribution to this field in and of itself. The techniques I used and further ones inspired by them would be very appropriate to the study of other interval-succession treatises.

Theoretical and Pedagogical Approaches

After examining the *LAC* and *RM* each individually, I went on compare their theoretical and pedagogical approaches. The basic structuring principles of their lists are different, as I pointed out with the table in Figure 2.

Figure 2: Ordering principles in Tinctoris and Pontio's interval-succession treatises.

Structural Level	Tinctoris - *LAC*	Pontio - *RM*
First	1st vertical interval, asc. quantity	1st vertical interval consonant/dissonant
Second	2nd vertical interval, asc. quantity	1st vertical interval, asc. quantity and quality
Third	Melodic motion of tenor, asc. quantity	Preferred successions

In comparing the two, I noted that while Tinctoris only lists the successions that he recommends or at least tolerates, Pontio occasionally includes negative examples. In my queries on Tinctoris's list by itself, I noted that he systematically excludes the succession 8 $_{-2}$ 5 from his list in all octave redoublings and when the tenor is the lower or the upper voice. When I later showed that Pontio states an octave to go to a fifth in all ways *except* 8 $_{-2}$ 5 which he explicitly gives as a negative example, this corroborated my conclusion that Tinctoris's systematic exclusion of a succession is tantamount to him prohibiting its use.

Owing to their different structuring principles, I exposed the general incompatibility of these two interval-successions lists, however, the authors' theoretical and pedagogical approaches remain open to comparison. I showed that Tinctoris is precise and painstakingly thorough; even when he omits an interval succession he is methodical about the omission. This precision is expressed mainly in his specificity about vertical and melodic intervals. With respect to duration and meter, however, Tinctoris is completely abstract. Pontio is the exact opposite as his examples

are less specific about their intervallic content as many of them are broad generalizations, but very specific about other musical details, especially duration and meter. I interpreted these important differences as pointing to different intended readers. As Tinctoris spells out many octave doublings and swaps of the position of the referential voice, he makes many relatively simple musical abstractions explicit, suggesting a younger and less sophisticated reader. By contrast Pontio relies on the reader's ability to work with his examples that are often specific instances of general principles to be respected. The student in the dialogue of *RM* even cites a theorist at one point reinforcing our impression of his sophistication. Given the sophistication Pontio demands of his reader, the abundance of context in his examples, as well as his numerous repertoire references, I interpret Pontio's approach to interval-succession theory as more akin to a pedagogy based on commonplaces. This interpretation is supported by the fact that Pedro Cerone based entire sections of his *El melepeo y maestro* on *RM* and also explicitly associated his treatise with the commonplace tradition.[2] The *LAC*, by contrast, is a more traditional exemplar of interval-succession theory, as its examples are more formulaic and repetitive.[3] The marked differences between the structure and content of the *LAC* and *RM* in addition to those between the theoretical and pedagogical approaches of the authors were the primary reasons for my selection of these two treatises for examination in this dissertation. The range of forms these treatises can take, attests to the breadth of interval-succession theory.

I concluded my comparison of the *LAC* and *RM* by contrasting the successions involving parallel sixths each author allows. I illustrated these differences with the following table.

Figure 3: Interval successions involving consecutive sixths allowed by Tinctoris and Pontio.

Succession	6 $_{1}$ 6	6 $_{2}$ 6	6 $_{3}$ 6	6 $_{4}$ 6	6 $_{5}$ 6	6 $_{-2}$ 6	6 $_{-3}$ 6	6 $_{-4}$ 6	6 $_{-5}$ 6
Tinctoris	✓	✓	✓	✓	✗	✓	✓	✓	✗
Pontio	✗	✓	✗	✗	✗	✓	✗	✗	✗

[2] Peter Schubert, "Musical Commonplaces in the Renaissance," in *Music Education in the Middle Ages and the Renaissance*, ed. Russell Murray, Susan Forscher Weiss, and Cynthia Cyrus (Indianapolis: Indiana University Press, 2010), 161–92; Pedro Cerone, *El melopeo y maestro* (Naples, 1613; repr. Bologna, 1969); and Russell Murray, "The Voice of the Composer: Theory and Practice in the Works of Pietro Pontio" (PhD diss., University of North Texas, 1989), 382.

[3] Anna Maria Busse Berger, *Medieval Music and the Art of Memory* (Los Angeles: University of California Press, 2005), 111–58, especially 141–44.

Pontio only allowed for parallel sixths up or down by step, whereas for Tinctoris they could stay in place, or move in tandem up or down a second, third, or fourth. Given that parallel sixths are an important part of Renaissance intervallic syntax, this point invited repertoire queries to determine if a change in musical practice accounted for the divergence of the two authors. Before that repertoire query could be realized, however, I determined that a precise and historically informed means of reducing Renaissance counterpoint would be needed first. The problematizing of reduction and the presentation of that analytical method was the subject of my last chapter.

Dynamic-Offset Method

Problematization

I found the main impetus for the creation of my analytical method in the *LAC* and *RM*. The interval successions they describe seemed like an ideal theoretical basis for corpus studies. Indeed Peter Schubert and Julie Cumming described the interval succession as the musical equivalent of the literary word, as it is the smallest unit that holds syntactical meaning.[4] The issue was knowing where to look for those successions, or more specifically, at what mensural level of a piece. In my comparison of Tinctoris and Pontio's interval-succession treatises, I noted that Tinctoris did not specify at which mensural level his interval succession should be used and that Pontio always reduced passages to the minim. As Tinctoris is too vague, and Pontio too rigid, I looked to other treatises for concrete, but flexible approaches to the discernment of fundamental counterpoint. Vicentino and Burmeister both present cadences side-by-side that have different durational values as their operative CRs. Similar to Pontio, they seem to think that the minim was the primary or default value, but they significantly demonstrate that this was subject to variation.

Analytical Approach

With these primary-source bearings in place, I devised a new method for determining the fundamental counterpoint of a passage called the dynamic-offset method. Drawing heavily on the ideas and terminology of Ruth DeFord, this method is based primarily on the analysis of

[4] Peter Schubert and Julie Cumming, "Another Lesson from Lassus: Using Computers to Analyze Counterpoint," *Early Music* 43.4 (November 2015): 577–86.

dissonance treatment and attack density.[5] These two components ensure that the analysis takes contrapuntal syntax into account, and they allow it to adjust dynamically to a shift in the CR at any moment in a piece. The main purpose of this tool is to improve the accuracy of n-gram analysis in the context of queries on Renaissance polyphony. This function of the dynamic-offset method puts my work in dialogue with existing Renaissance repertoire studies such as those of Peter Schubert and Julie Cumming, Christopher Antila and Julie Cumming, Darrell Conklin, William Melin, and Lee Rothfarb.[6]

Another application of the dynamic-offset method consists of the description of the contrapuntal character of a piece or corpus of music. The contrapuntal character of a piece is a quantification of the relative stability or variability of its CR. I demonstrated this application with three analyses ranging in scope from a single piece, to a medium-sized corpus. The second of these demonstrations focused on the analysis of points of imitation. In revisiting some examples from an article by Julie Cumming and Peter Schubert, I showed that it is possible to distinguish between points of imitation on the basis of their contrapuntal character.[7] Namely, the CR of a point of imitation was found to either be stable, or to accelerate. In my final demonstration of the dynamic-offset method, I quantified the contrapuntal character of a corpus of thirty-four duos by Josquin, Lassus, and Morley. I found the duos by Josquin and Lassus to be similarly varied in this regard, but that Morley's demonstrated little to no changes in their CR.

In light of the commonplaceness of reduction as an analytical process, the applications of contrapuntal-character analysis that I exhibited with my queries, and the availability of my tools online as part of the VIS Framework, it is my hope that other researchers will also make use of the

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[5] Ruth DeFord, Tactus*, Mensuration, and Rhythm in Renaissance Music* (Cambridge: Cambridge University Press, 2015).

[6] Schubert and Cumming, "Another Lesson;" Christopher Antila and Julie Cumming, "The VIS Framework: Analyzing Counterpoint in Large Datasets," in *Proceedings of the International Society for Music Information Retrieval*, 2014, 71–76; Darrell Conklin and Mathieu Bergeron, "Discovery of Contrapuntal Patterns," in *Proceedings of the International Society for Music Information Retrieval*, 2010, 201–206; William Melin, "The Music of Johannes Tinctoris (ca. 1435 –1511): A Comparative Study of Theory and Practice" (PhD diss., Ohio State University, 1973); Lee Rothfarb, "Tinctoris vs. Tinctoris: Theory and Practice of Dissonance in Counterpoint," *In Theory Only*, 9/2 (Ann Arbor: Michigan Music Thoery Society, 1986), 3–32.

[7] Julie Cumming and Peter Schubert, "The Origins of Pervasive Imitation," in *The Cambridge History of Fifteenth-Century Music*, ed. Anna Maria Busse Berger and Jesse Rodin (New York: Cambridge University Press, 2015), 200–28.

dynamic-offset method.[8] My study builds on DeFord's theorization of CR by laying bare the steps of a method for its discernment in Renaissance music. With the openness of my approach, I enthusiastically invite critiques and corrections to my method from other scholars. Faithful identification of the fundamental counterpoint of Renaissance music is an ambitious, and perhaps unattainable pursuit. My aim with the present study, therefore, is to encourage the field to continue in this direction so that with each improvement we better our understanding of the language of Renaissance counterpoint.

[8] Concerning the code of the VIS Framework, see: https://github.com/ELVIS-Project/vis-framework. For more on the ELVIS research group, see: http://elvisproject.ca/.

Appendix

Notes to the Reader

This appendix contains all of Pontio's notated examples from book II of *Ragionamento di musica*. As many of these were already presented in chapter 3, I will now repeat the notes I gave there about this edition of the examples.

Each of Pontio's interval-succession examples concludes with longs in both voices. These terminal longs have been transcribed as either whole notes or double whole notes depending on which one will complete the last measure of 4/2 time. 4/2 is meant to reflect 𝄵, the likely implied time signature, and has been adopted as a convenient convention. As the bar lines are another editorial addition, they are shown as dotted lines.

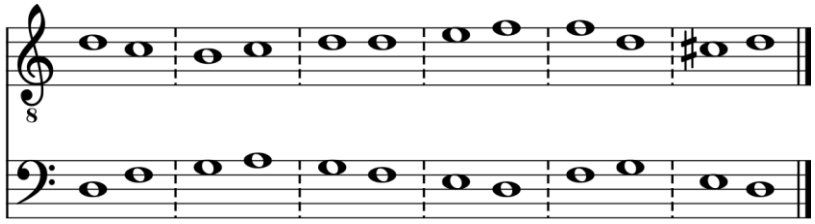
With respect to the annotations of interval successions, Pontio generally uses a dagger, †, in the faster-moving part (usually the upper part) to direct the reader to the location of the interval succession being discussed, though sometimes this is absent, and in some cases it is used to designate other salient musical phenomena, such as points of imitation. As each succession usually appears twice in its example, Pontio often labels both occurrences. He is somewhat inconsistent with where these labels go, so I regularize the position of his daggers in my examples so that they are all found at the location of the first vertical interval in the succession being discussed. I indicate this editorial shifting where it occurs by putting the dagger in square brackets, [†]. In some cases a dagger appears at an interval succession that is similar to, but distinct from that being discussed. Where this appears to have been done in error, I convey this with a dagger and a question mark in square brackets, [†?]. Finally, a dagger in parentheses, (†), is used to indicate interval successions that correspond to that being discussed, but were not designated with a dagger.

While Pontio generally includes interval quality in his discussion of examples, he is somewhat inconsistent with this and so quality has been omitted from my labels. Concerning my labelling conventions, I borrow adopt the approach of the ELVIS research team. Interval successions two vertical intervals long are labelled with three numbers. The first and the third correspond to the two vertical intervals in the succession and the second number, which I put in

subscript, depicts the melodic motion of the voice on the lower voice. So we see that Figure 4 is labelled 3 $_2$ 1 because the first vertical interval is a third, the second vertical is a unison, and the lower voice moves up a second between the two vertical intervals. If the voices cross in the middle of the succession, I show the second vertical interval (the third number in the label) as a negative number, such as in Figure 93.

Del Contrapunto Semplice

Figure 1: Example of simple counterpoint, p. 22.



Del contrapunto Florido

Figure 2: Example of how to put two, four, or more notes against one in florid counterpoint, p. 22.

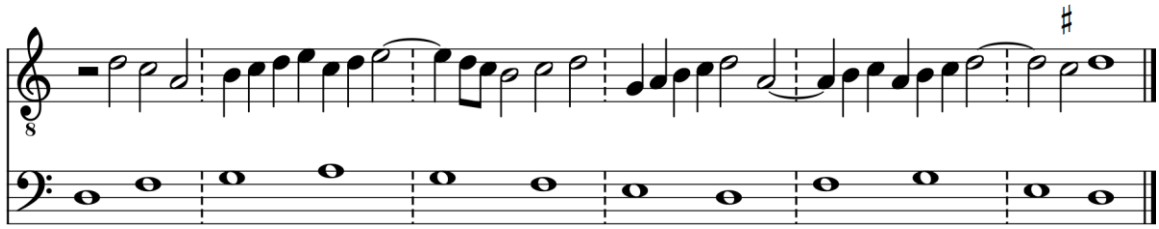
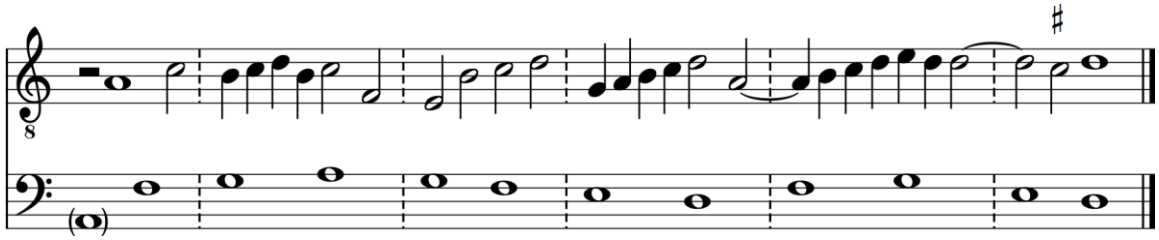


Figure 3: Example of how to mix perfect and imperfect consonances, p. 24.



Dell'Unisono

Interval Successions Ending on the Unison

Figure 4: 3 2 1, p. 26.

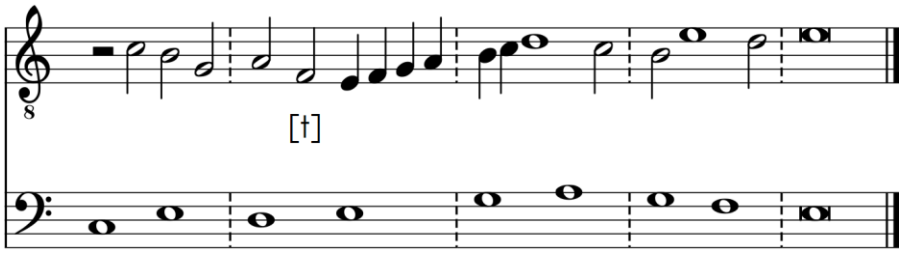


Figure 5: 3 1 1, p. 26.

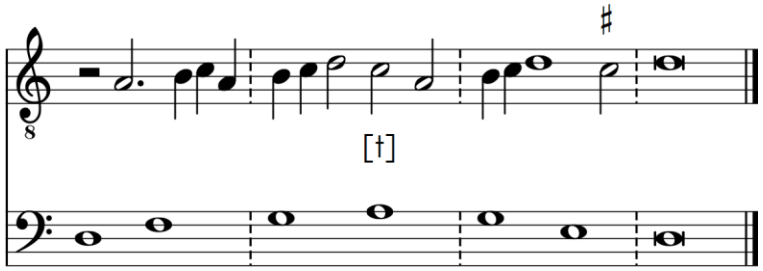


Figure 6: 3 1 1, p. 26.

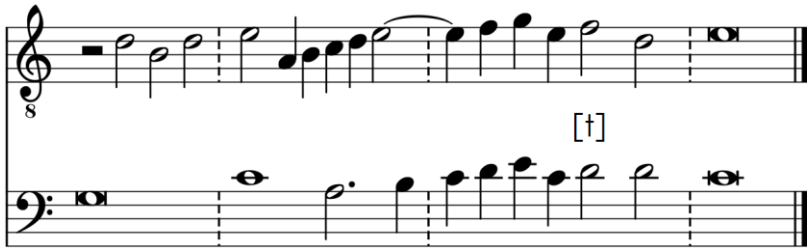


Figure 7: 5 2 1, p. 27.

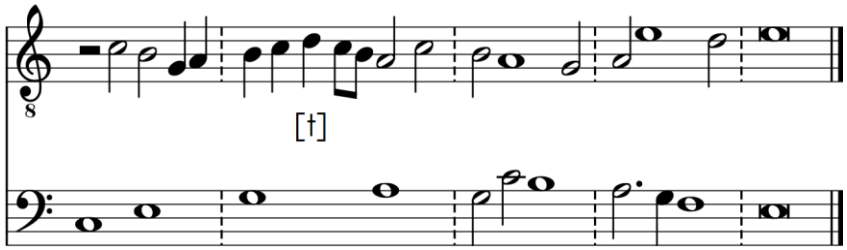


Figure 8: 5 3 1, p. 27.

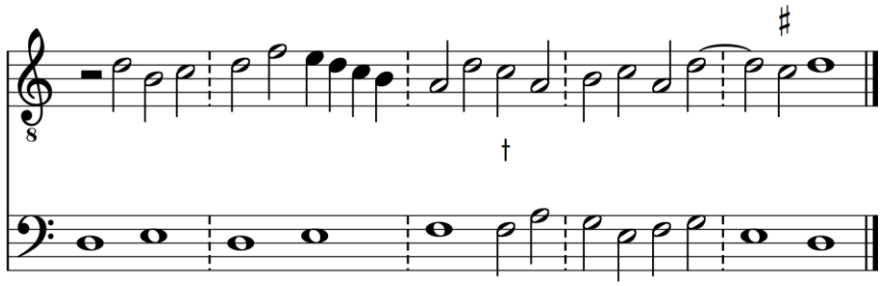


Figure 9: 4 4 1 and 3 3 1, p. 28.

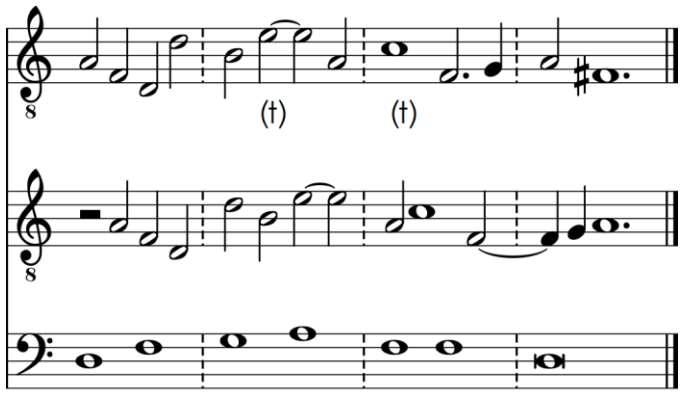
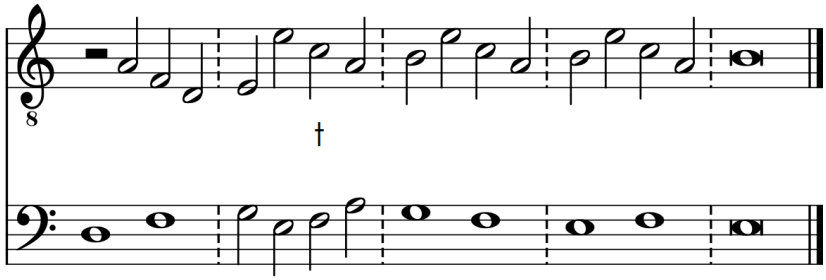


Figure 10: 5 3 1, p. 28.



Interval Successions Starting on the Unison

Figure 11: 1 1 3, p. 29.

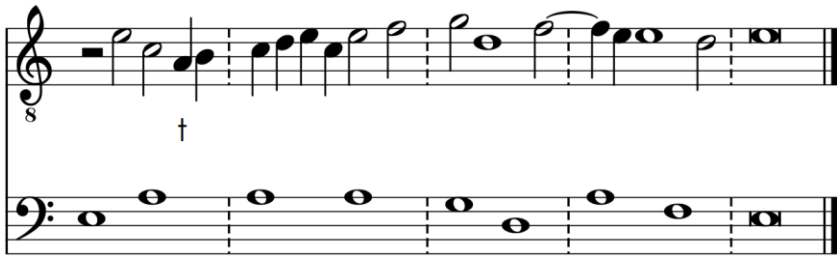


Figure 12: 1 $_{1}$ 5, p. 29.

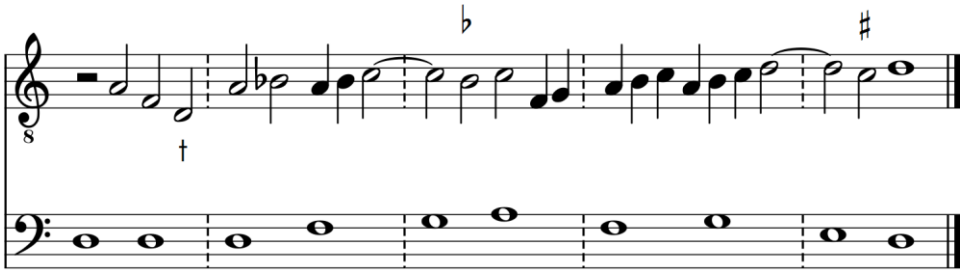


Figure 13: 1 $_{4}$ 5, p. 29.

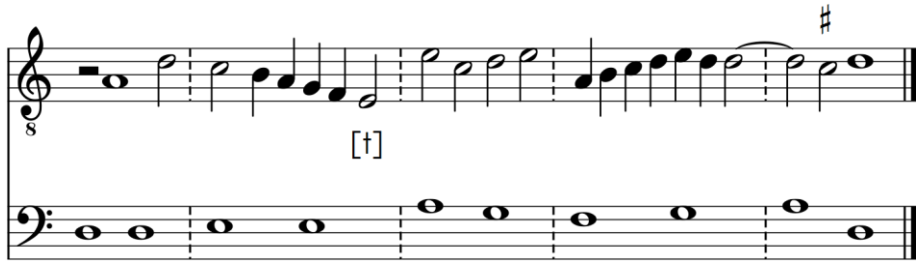


Figure 14: 1 $_{1}$ 6, p. 30.

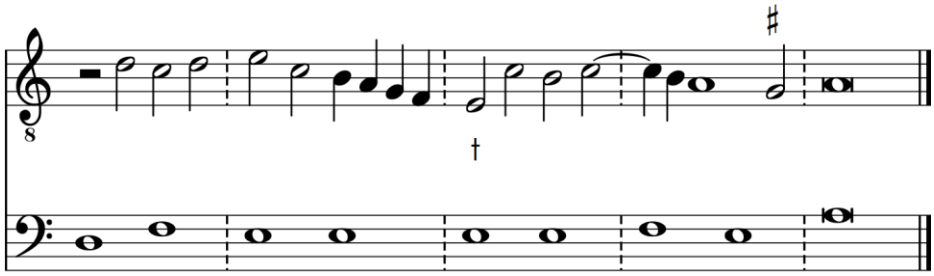


Figure 15: 1 $_{-3}$ 6, p. 30.

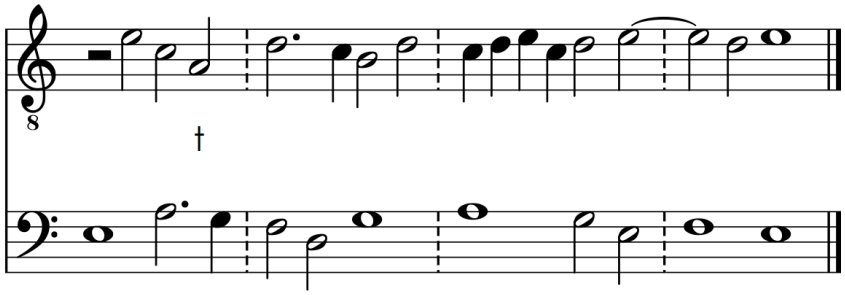


Figure 16: 1 $_{1}$ 8, p. 30.

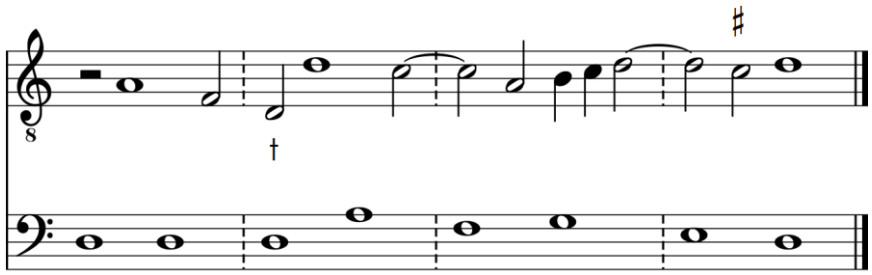


Figure 17: 1 $_{-4}$ 8, p. 31.

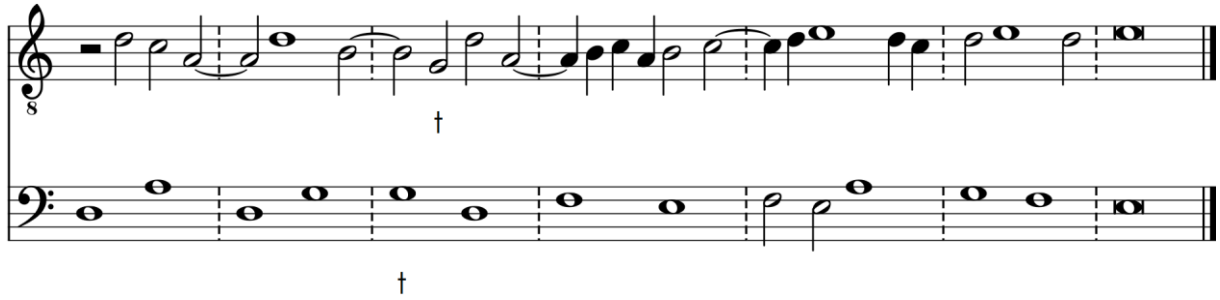


Figure 18: General example, p. 31.

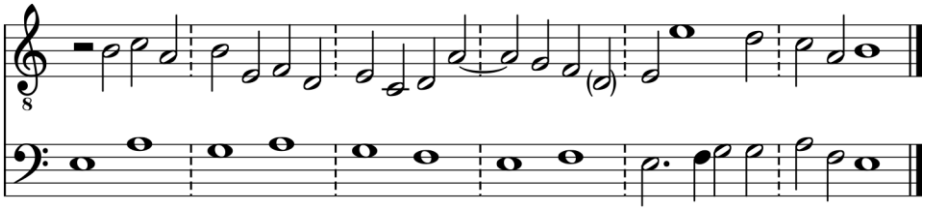


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Figure 19: 3 $_{2}$ 1, p. 32.

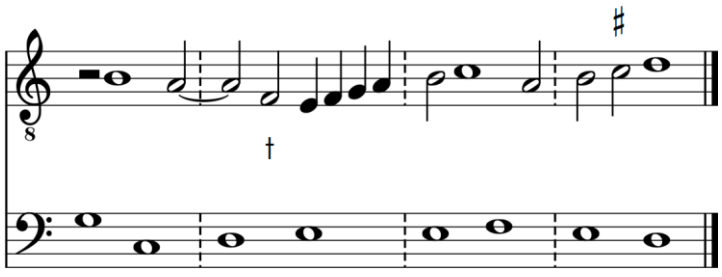

Figure 20: 3 1 1, p. 33.

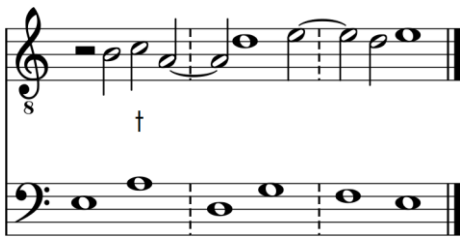

Figure 21: 3 5 1, p. 33.

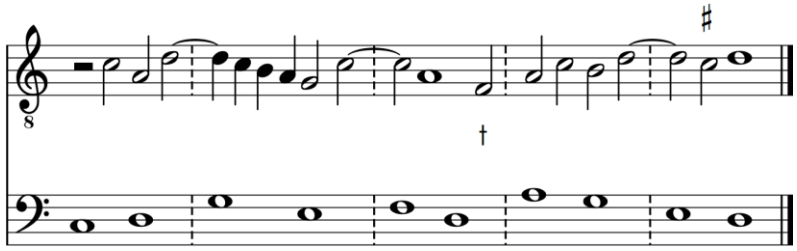

Figure 22: 3 1 5, p. 33.

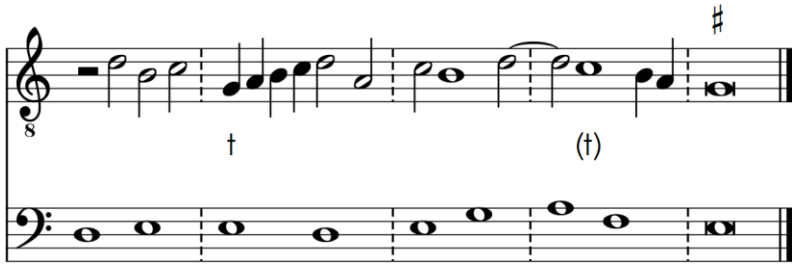

Figure 23: 3 1 6, p. 34.

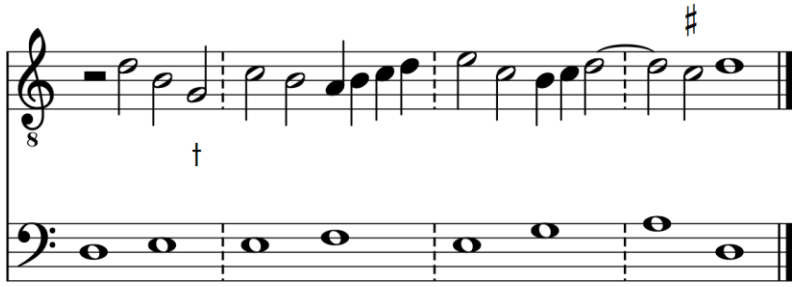

Figure 24: 3 -2 6, p. 34.

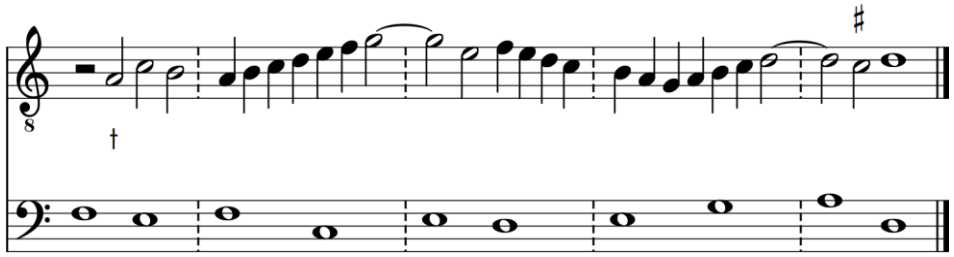

Figure 25: 3 $_{2}$ 6, p. 35.

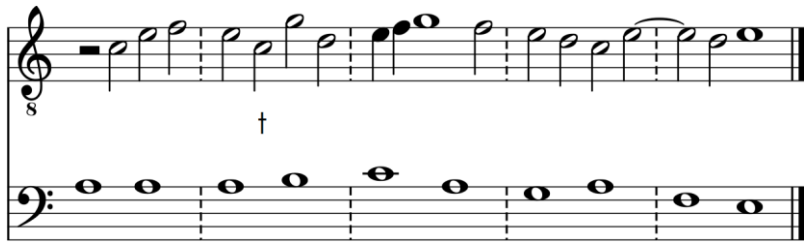

Figure 26: 3 $_{-2}$ 8, p. 35.

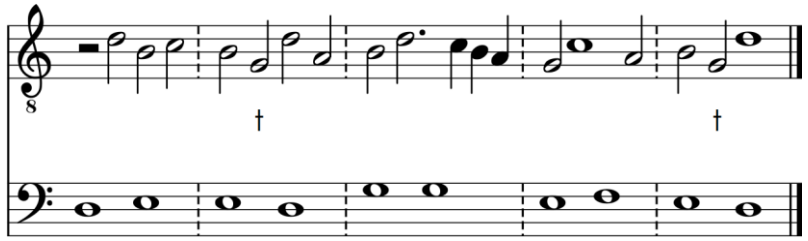

Figure 27: 3 $_{-3}$ 8, p. 36.

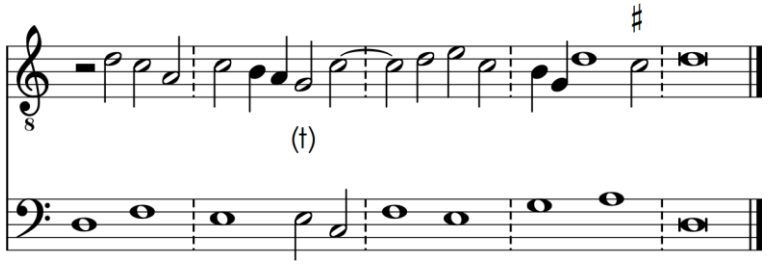

Figure 28: 3 $_{1}$ 2, p. 36.

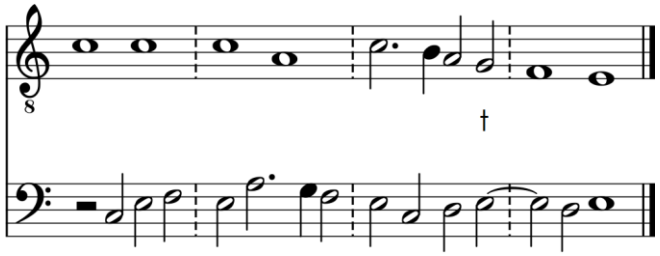

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Figure 29: 3 $_{1}$ 5, p. 37.

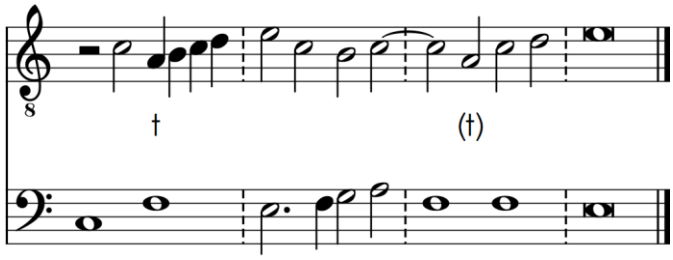

Figure 30: 3 $_{-2}$ 6, p. 37. Note that the succession Pontio appears to be discussing begins on a weak semiminim.

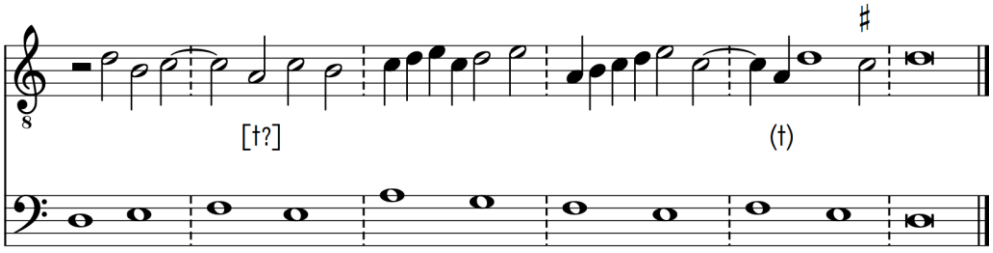

Figure 31: 3 $_{-5}$ 6, p. 37.

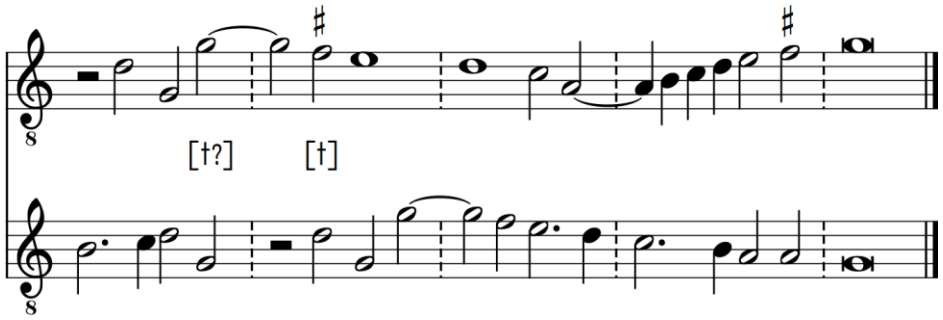

Figure 32: 3 $_{1}$ 1, p. 38.

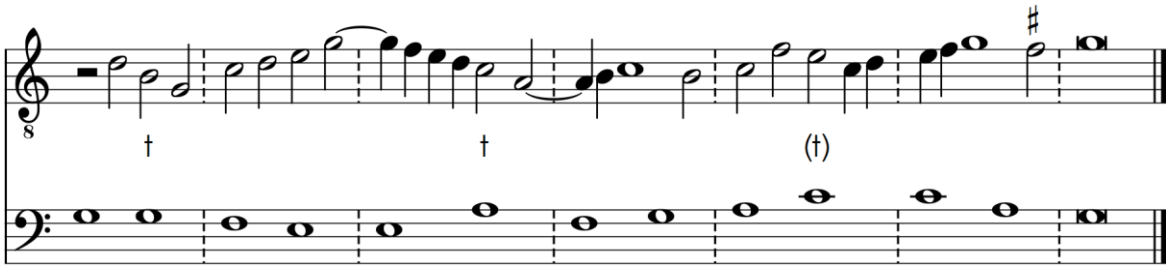

Figure 33: 3 $_{2}$ 1, p. 38.

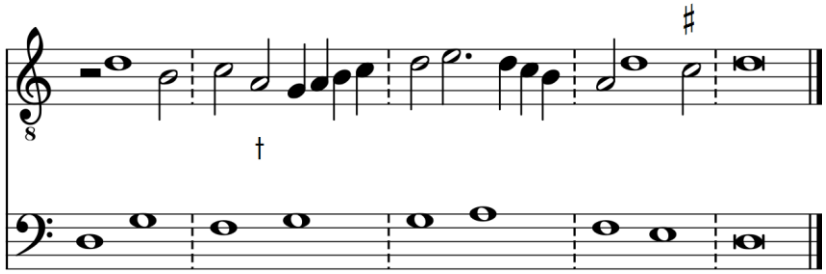

Figure 34: 3 $_{4}$ 1, p. 38.

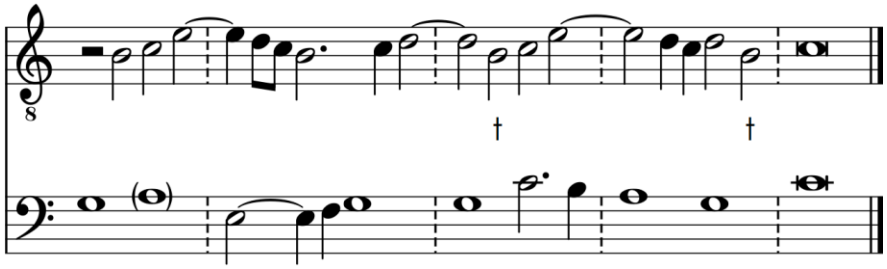

Figure 35: 3 $_{-2}$ 1, p. 39.

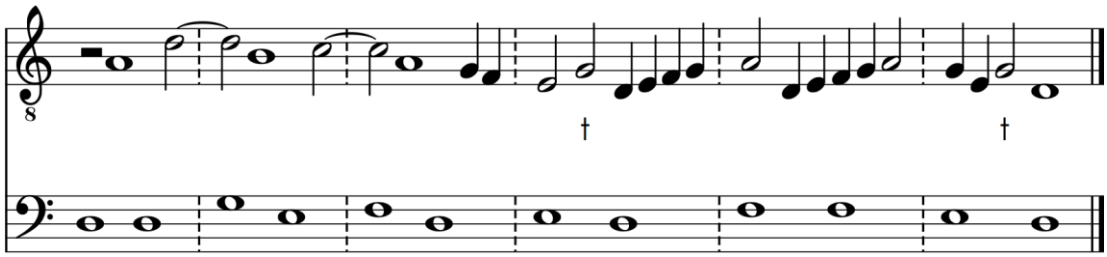

Figure 36: 3 $_{5}$ 1, p. 40.

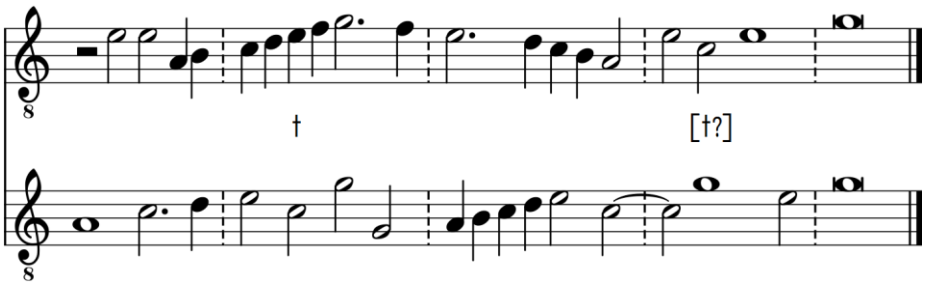

Figure 37: 3 $_{1}$ 8, p. 40.

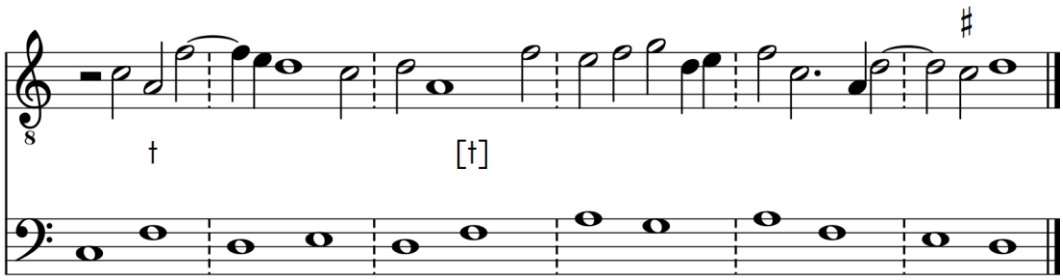

Figure 38: 3 $_{-5}$ 8, p. 41.

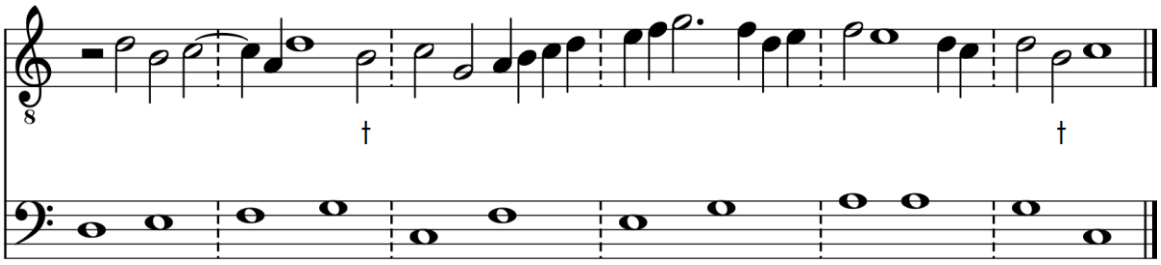

Figure 39: 3 $_{-2}$ 8, p. 41.

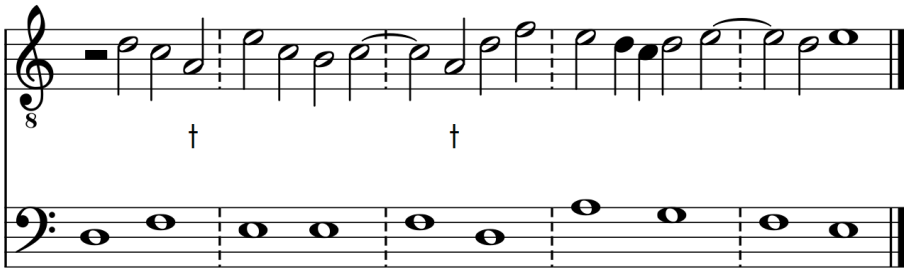

Figure 40: 3 $_{-5}$ 7, p. 42.

Figure 41: 3 2 2, p. 42.

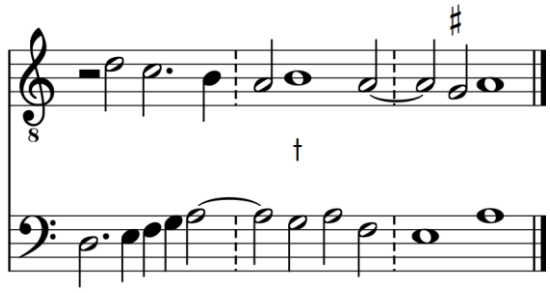

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Figure 42: 5 1 6, p. 43.

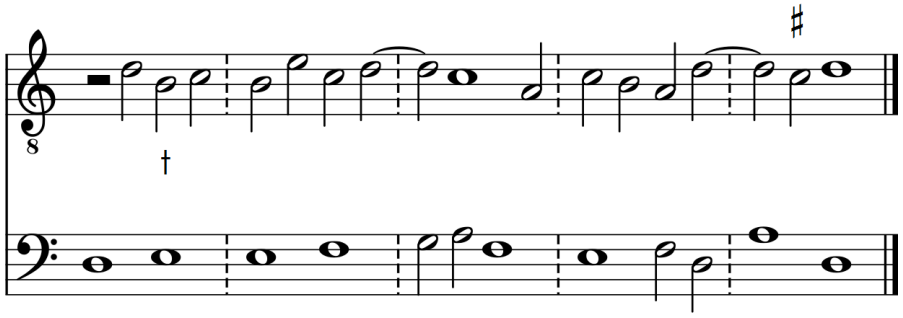

Figure 43: 5 -4 6, p. 44.

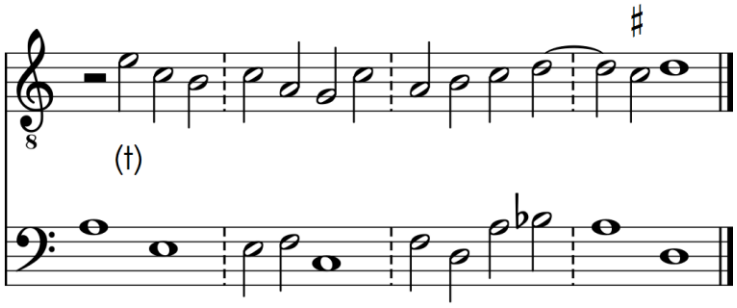

Figure 44: 5 -2 8, p. 44.

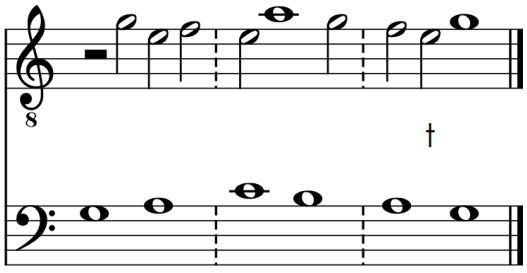

Figure 45: 5 $_{2}$ 8, p. 44.

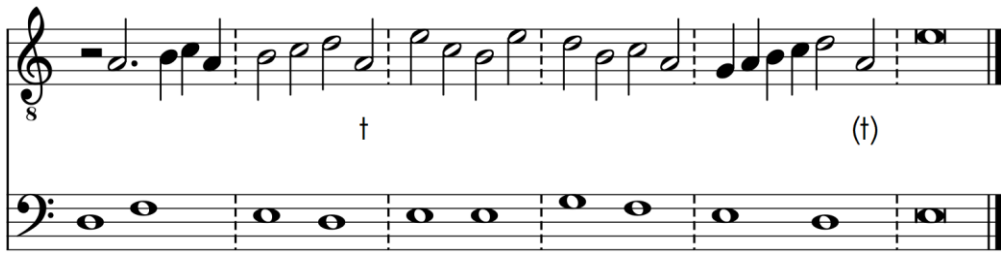

Figure 46: 5 $_{1}$ 8, p. 45.

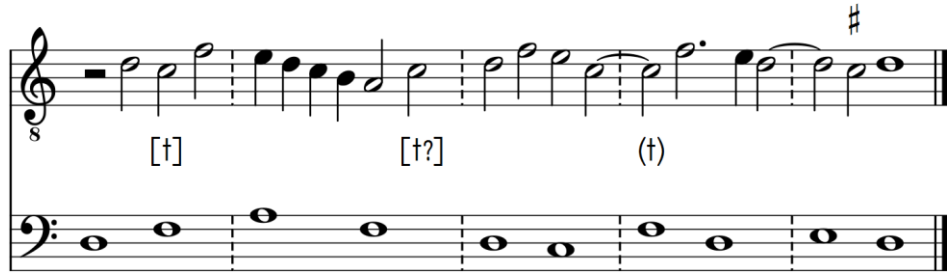

Figure 47: 5 $_{3}$ 8, p. 45.

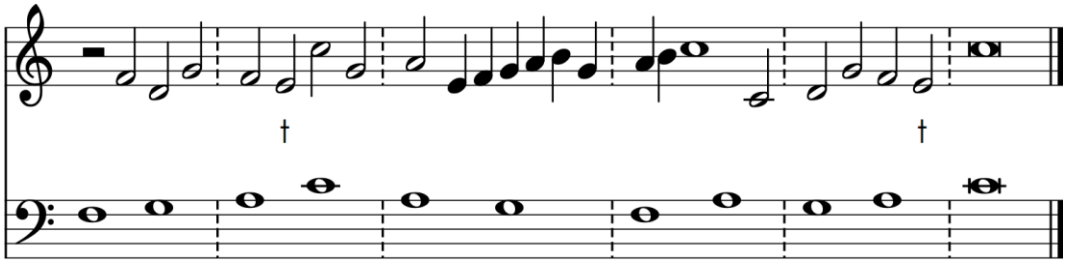

Figure 48: 5 $_{3}$ 8 (12 $_{3}$ 8), p. 46.

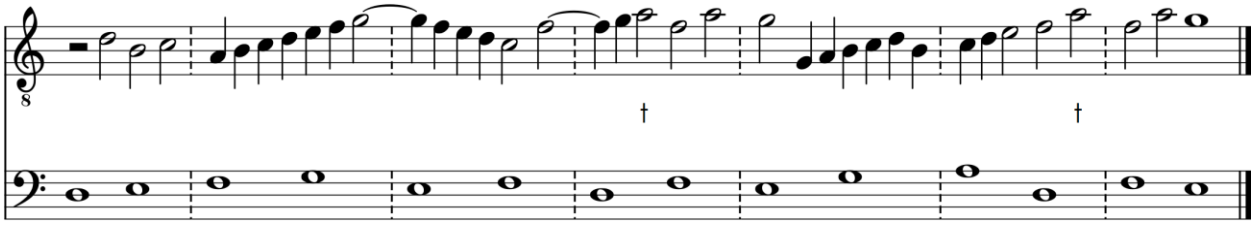

Figure 49: 5 $_{-3}$ 8, p. 46.

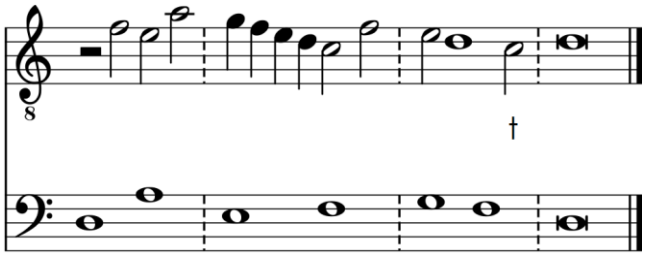

Figure 50: 5 -3 8, p. 47.

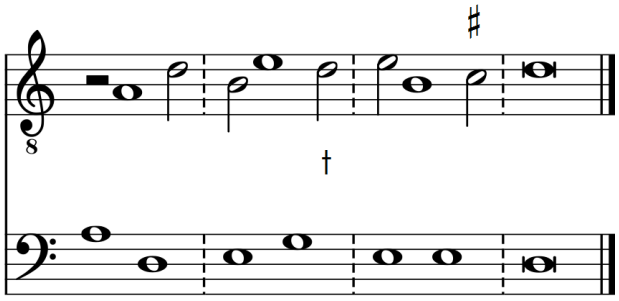

Figure 51: 5 -2 8, p. 47.

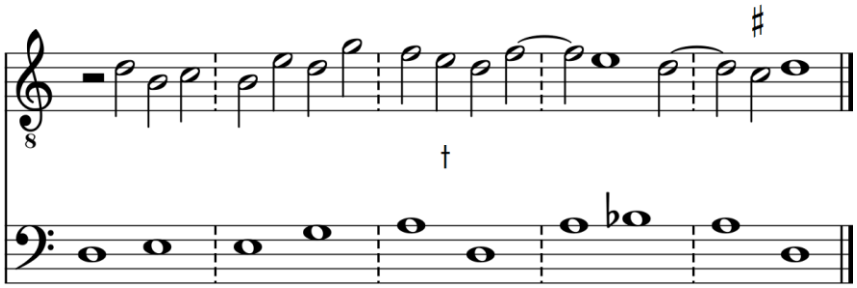

Figure 52: 5 2 3, p. 48.

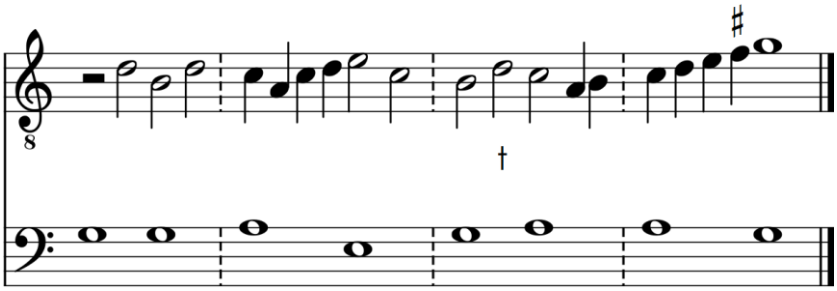

Figure 53: 5 1 1, p. 48.

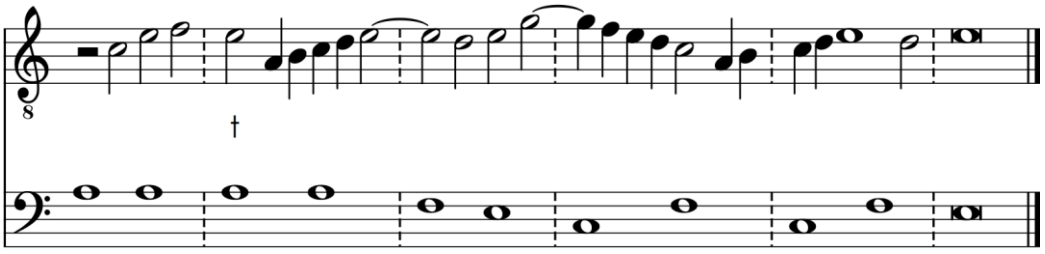

Figure 54: 5 4 1, p. 48.

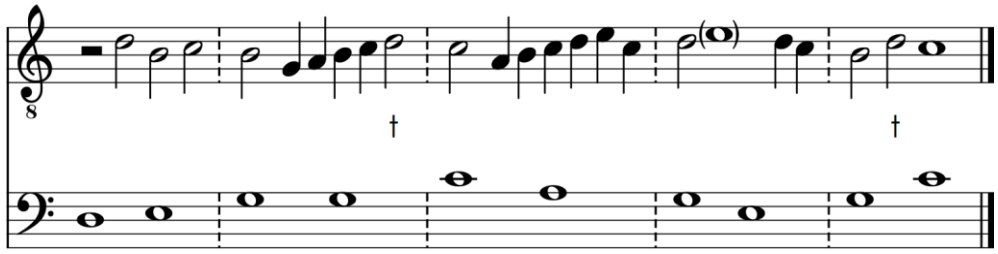

Figure 55: 5 3 1, p. 49.

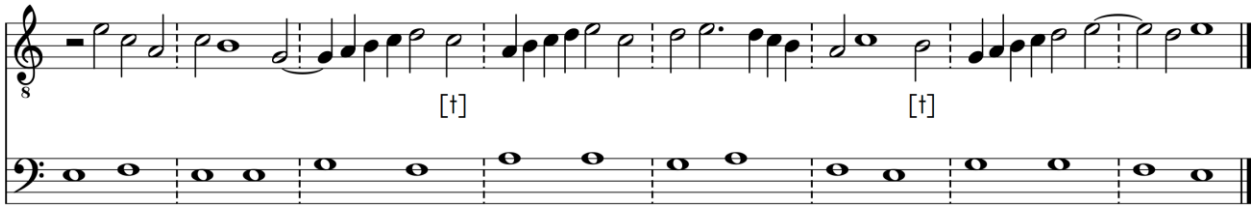

Figure 56: 5 6 1, p. 49.

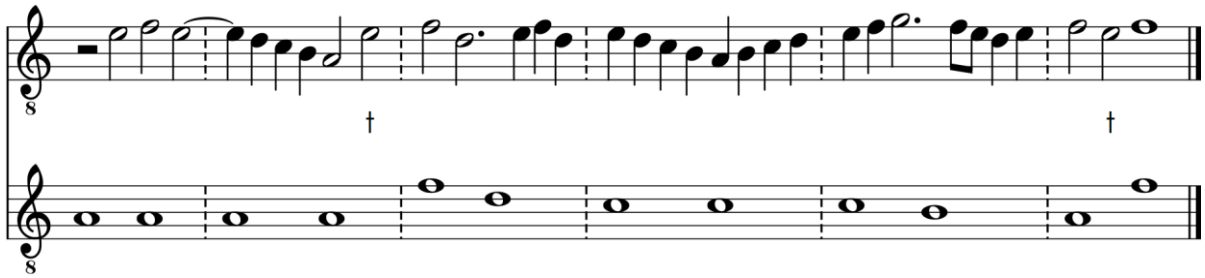

Figure 57: 5 2 1, p. 50.

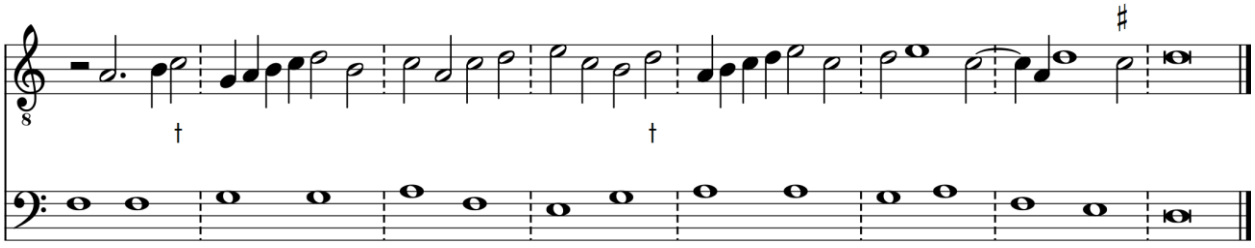

Figure 58: 5 -3 7, p. 50.

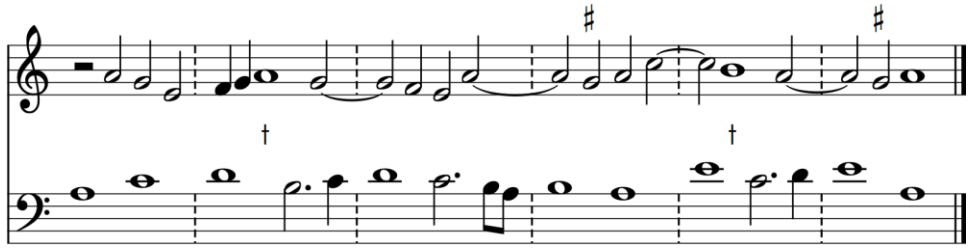

Figure 59: 5 -3 7, p. 51.

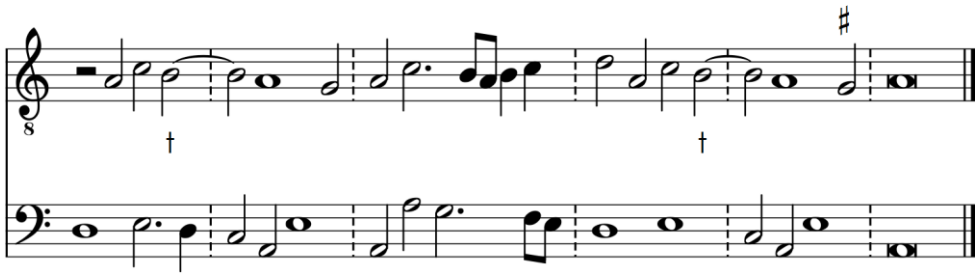

Figure 60: 5 4 2, p. 51.

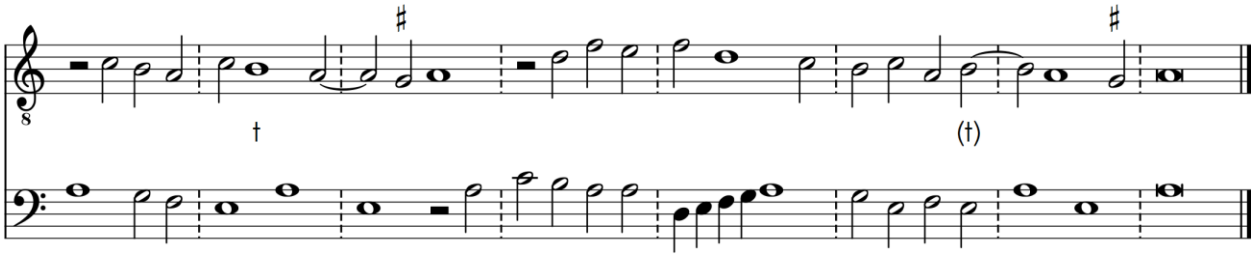

Figure 61: Example concerning motivic repetition beginning on a fifth, p. 52.

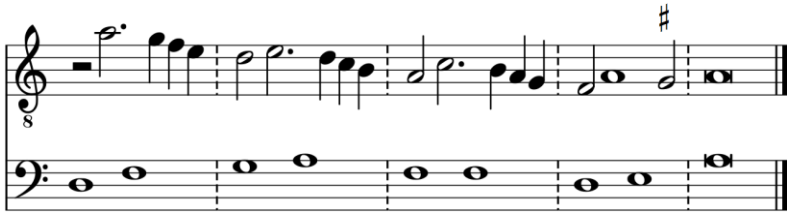

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Figure 62: 6 1 5, p. 53.

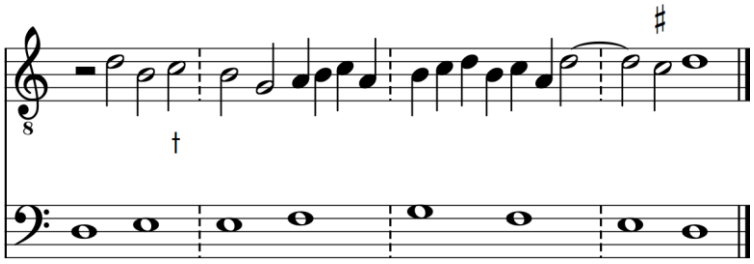


Figure 63: 6 1 5, p. 54.

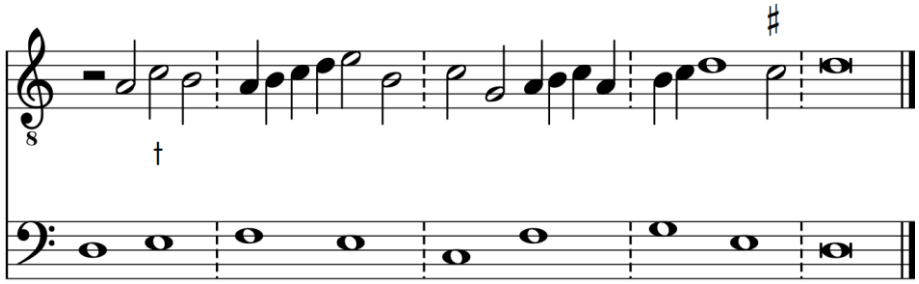


Figure 64: 6 1 3, p. 54.

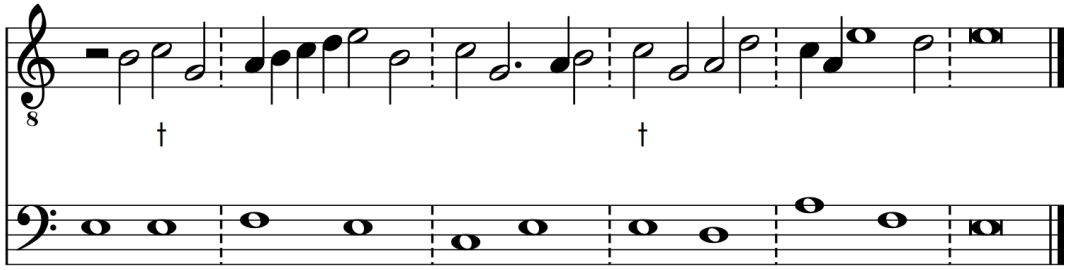


Figure 65: 6 2 3, p. 54.

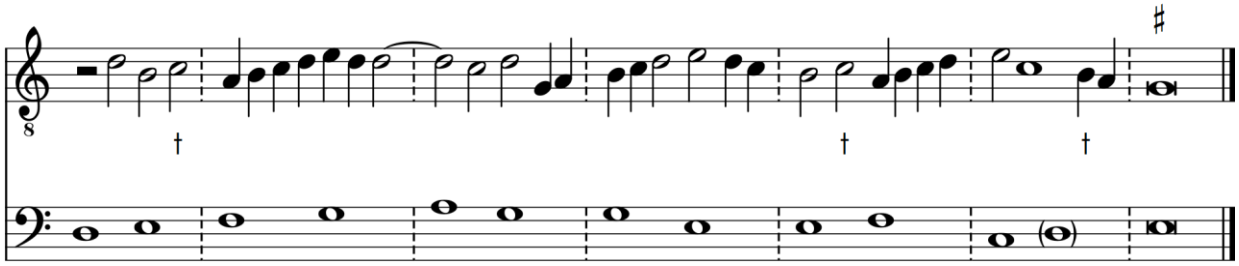


Figure 66: 6 1 3, p. 55.

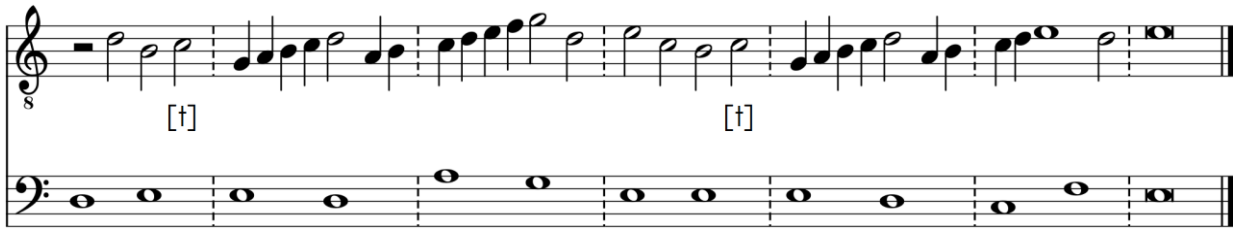


Figure 67: 6 5 3, p. 55. The first annotation, not quite Pontio's normal dagger, appears to be a mistake.

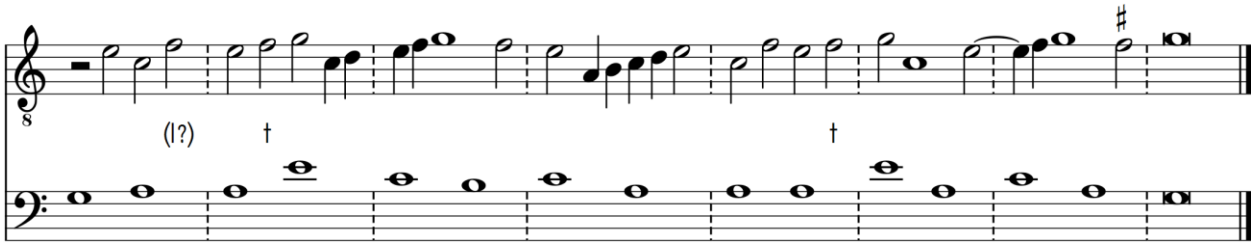


Figure 68: 6 -3 8, p. 55.

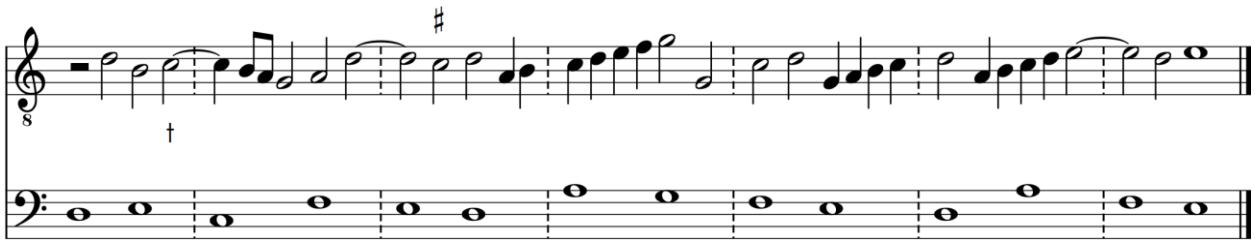


Figure 69: 6 2 1, p. 56.

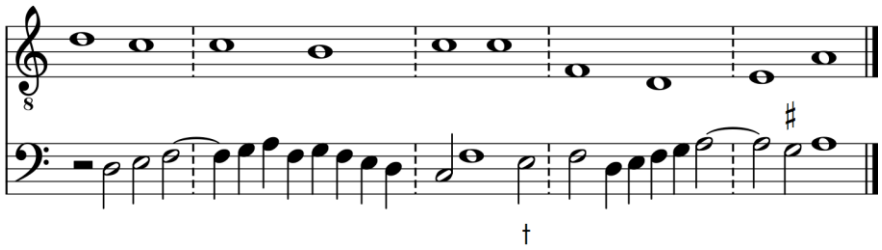


Figure 70: 6 4 1, p. 56.

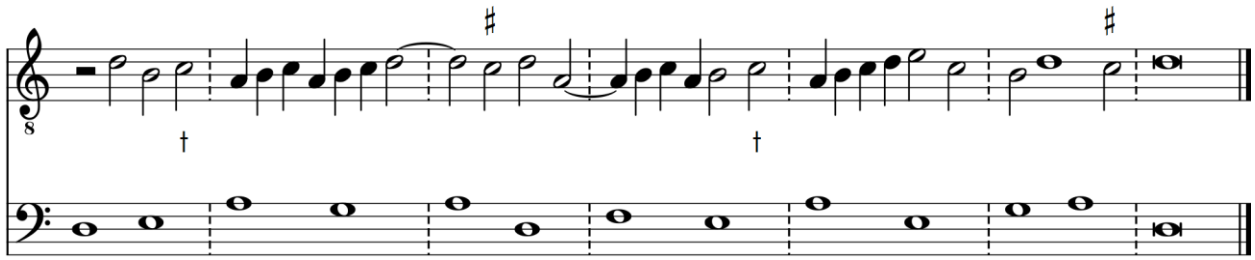


Figure 71: 6 -2 10, p. 57.

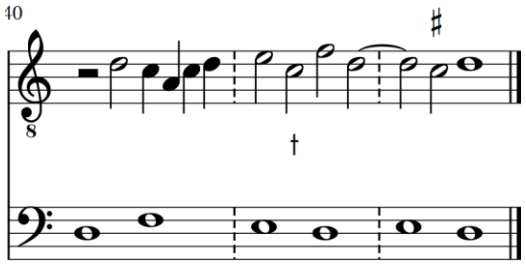


Figure 72: 6 5 2, p. 57.

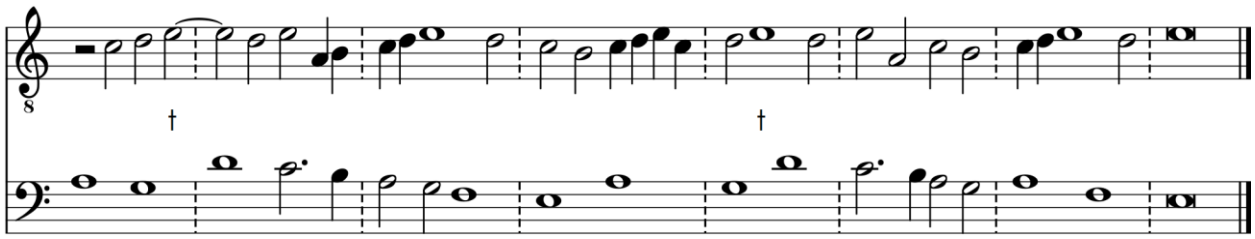
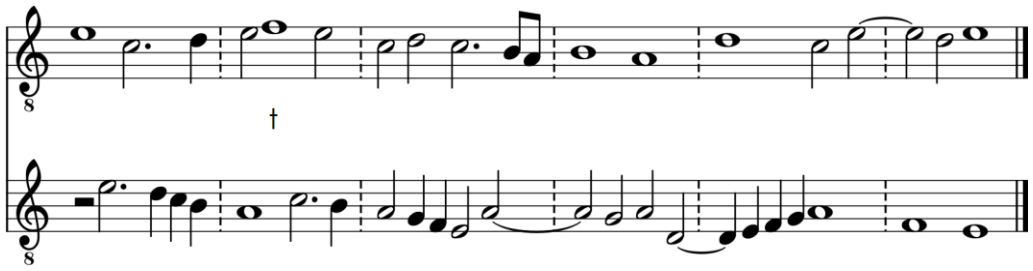


Figure 73: 6 $_{3}$ 4, p. 58.



Della Sesta Maggiore

Figure 74: 6 $_{-3}$ 8, p. 58.

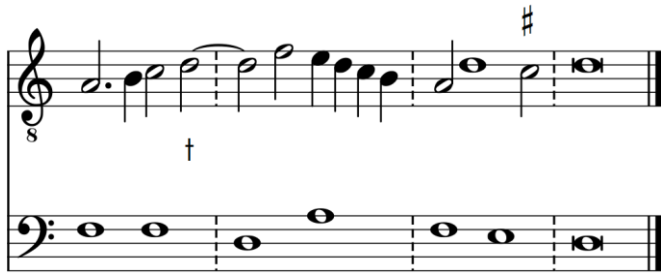


Figure 75: 6 $_{-2}$ 8, p. 59.

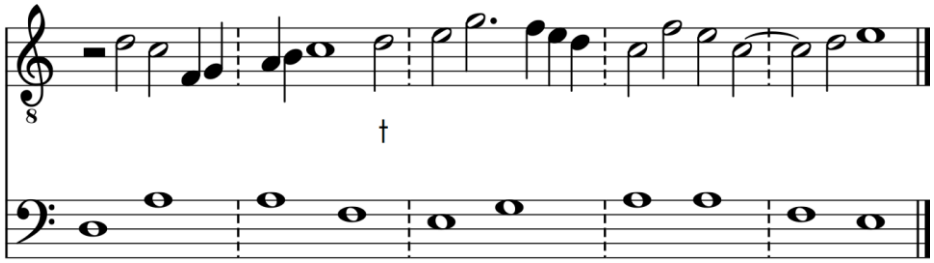


Figure 76: 6 $_{-5}$ 8, p. 59.

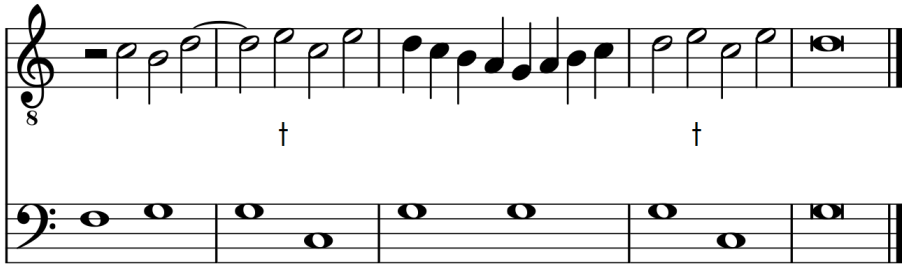


Figure 77: 6 $_{1}$ 8, p. 60.

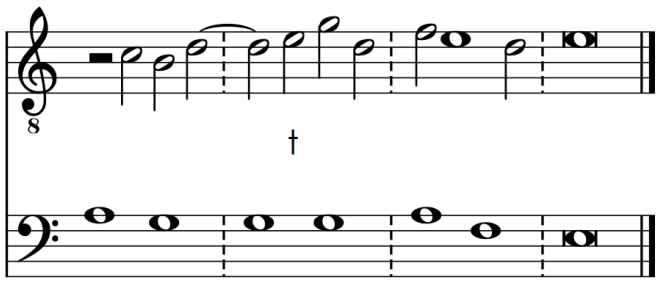


Figure 78: 6 $_{-2}$ 5, p. 60.

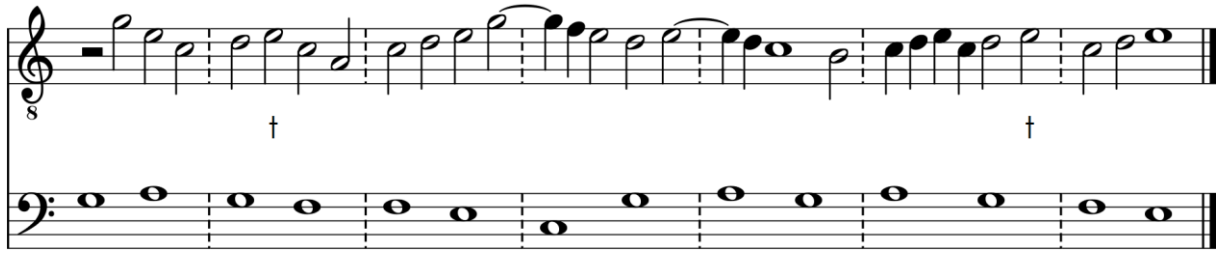


Figure 79: 6 $_{-2}$ 10, p. 61.

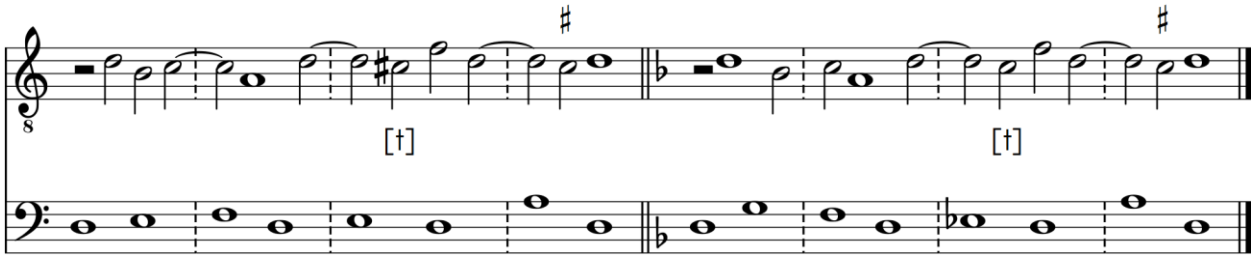


Figure 80: 6 $_{2}$ 6, p. 62.

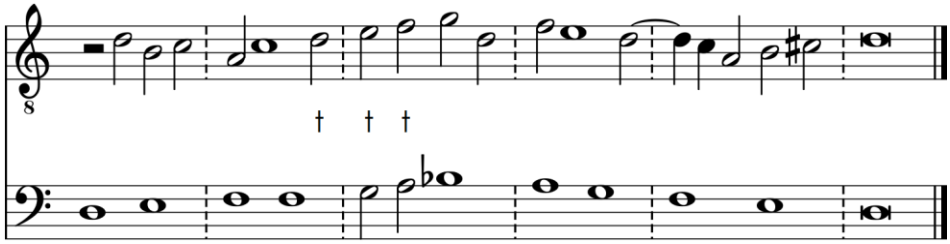


Figure 81: 6 -3 6, p. 62.

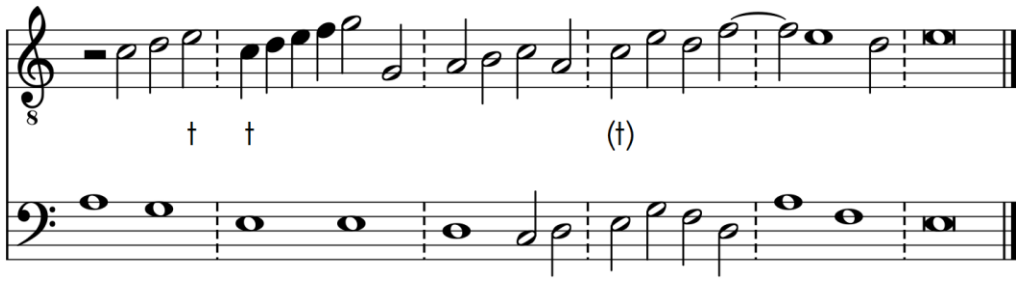
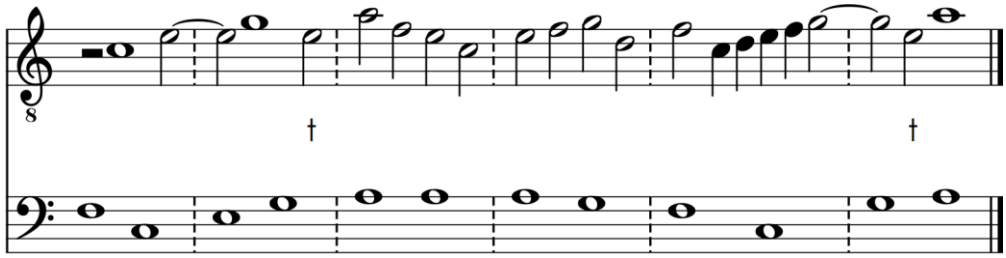


Figure 82: 6 2 8, p. 63.



Della Ottava

Figure 83: 8 -2 5, p. 63.

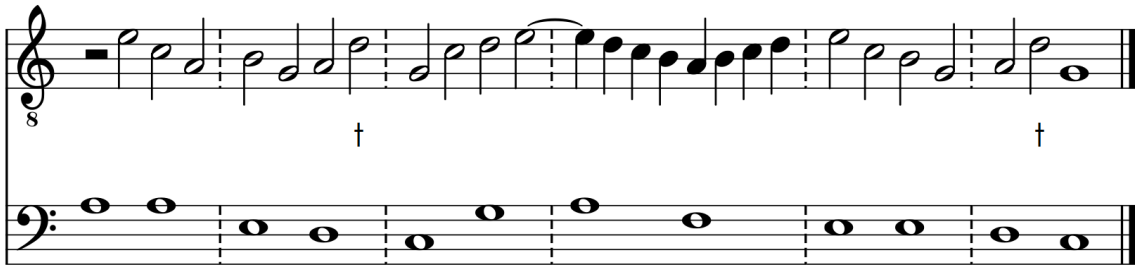


Figure 84: 8 1 6, p. 64.

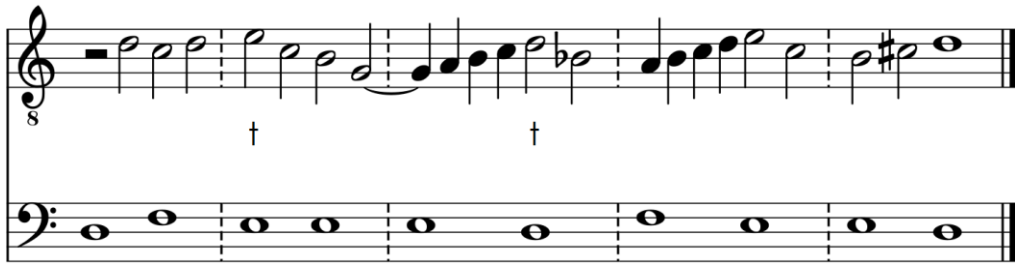


Figure 85: 8 4 6, p. 64.

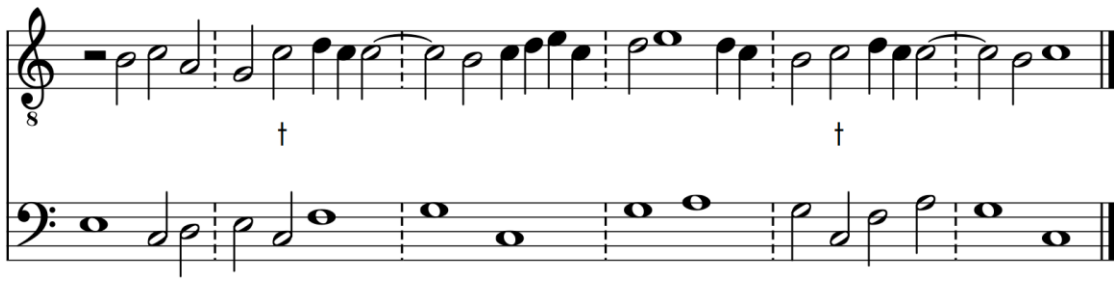


Figure 86: 8 5 6, p. 64.

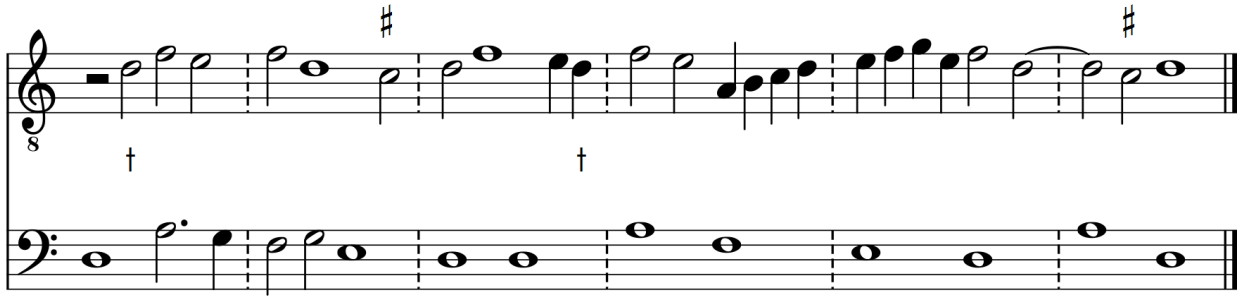


Figure 87: 8 1 3 (8 1 10), p. 65.

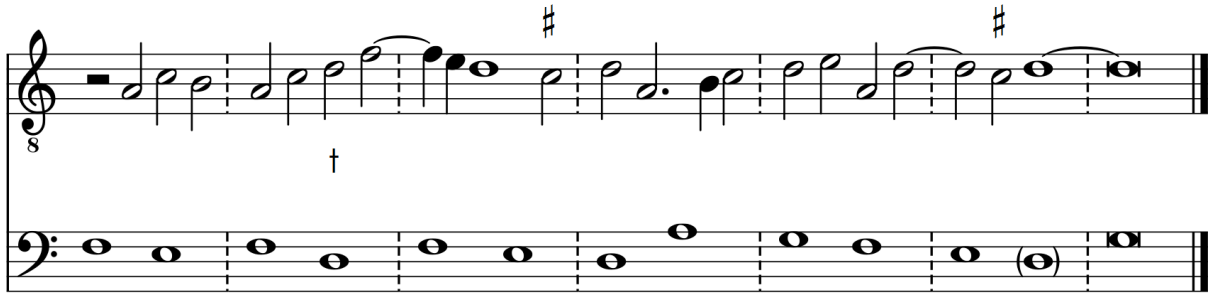


Figure 88: 8 1 1, p. 65.

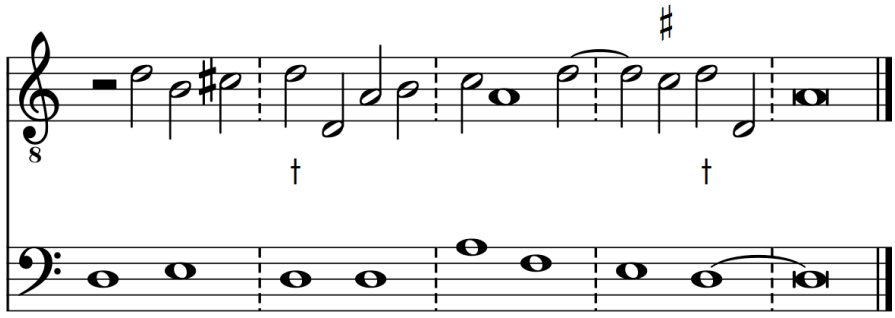
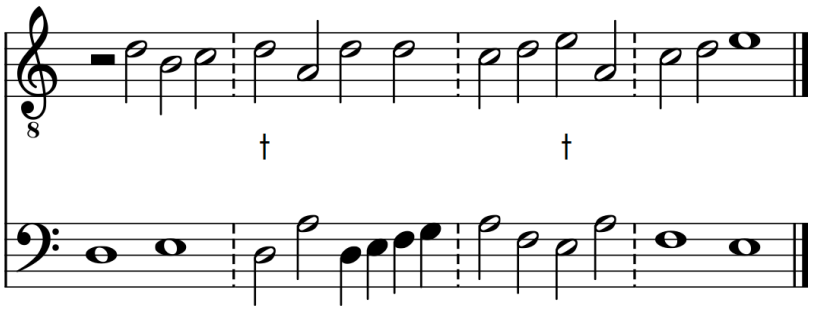


Figure 89: 8 5 1 and 8 4 1, p. 66.



Della Seconda

Figure 90: 2 1 1, p. 67.

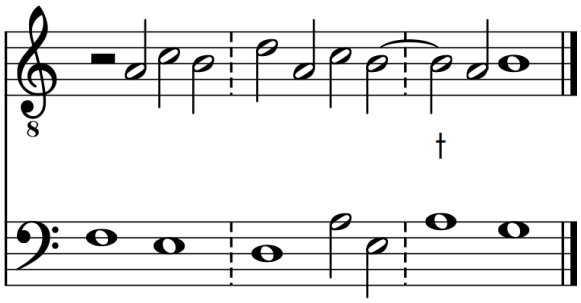


Figure 91: 2 1 1, p. 67.

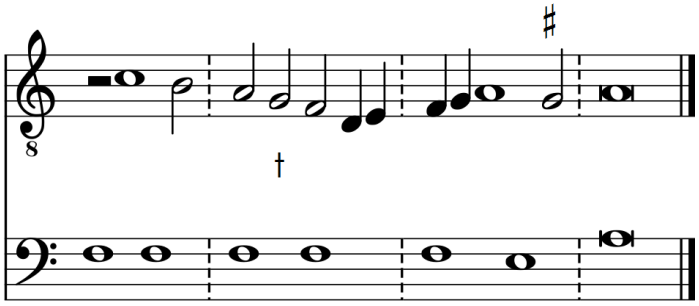


Figure 92: 2 1 3, p. 68.

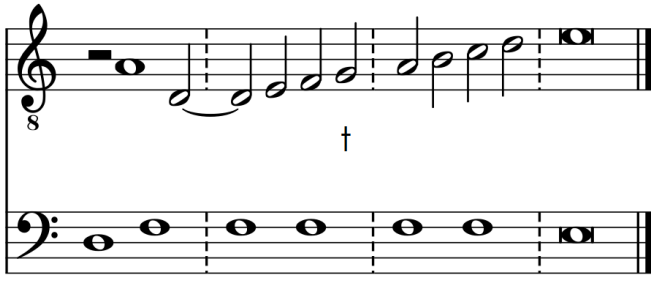


Figure 93: 2 $_{3}$ -3, p. 68.

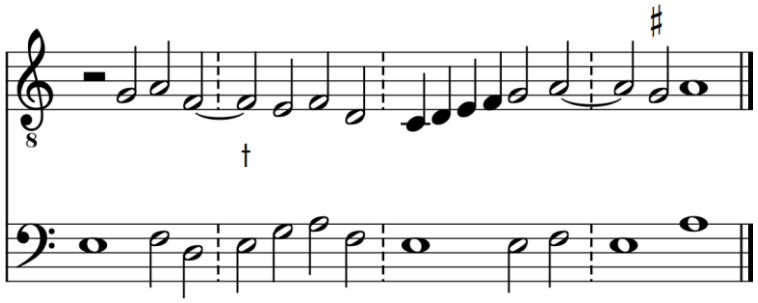


Figure 94: 2 $_{-3}$ 3, p. 69.

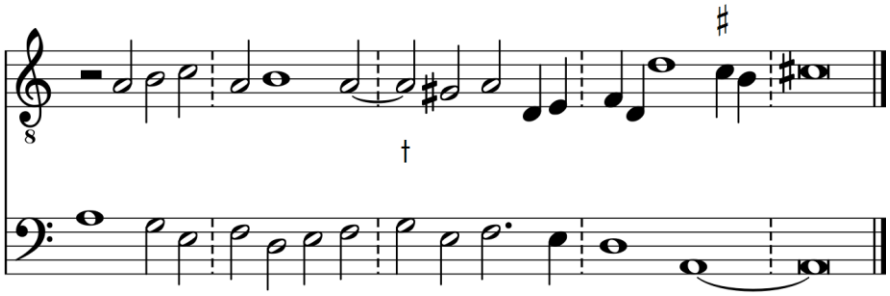


Figure 95: 2 $_{-2}$ -3, p. 69.

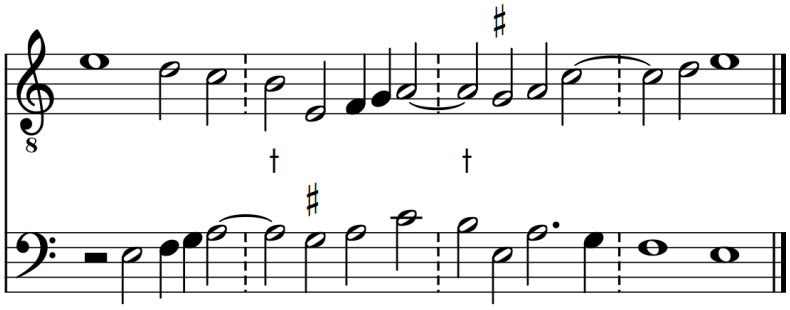


Figure 96: 2 $_{-3}$ 3, p. 70.

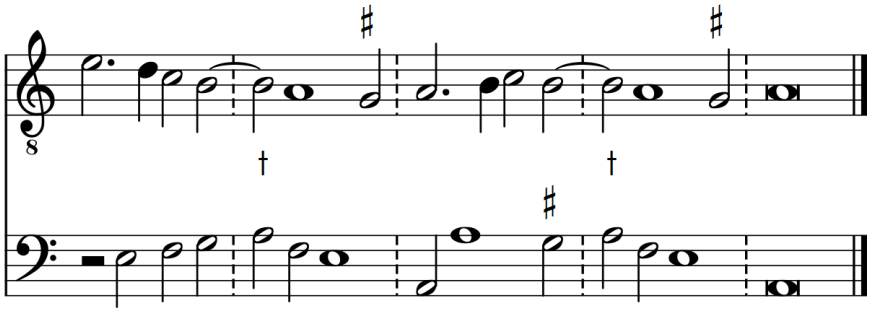


Figure 97: -2 $_{1}$ -3, p. 70. Note that the voice in the upper staff is the lowest-sounding voice at the succession.

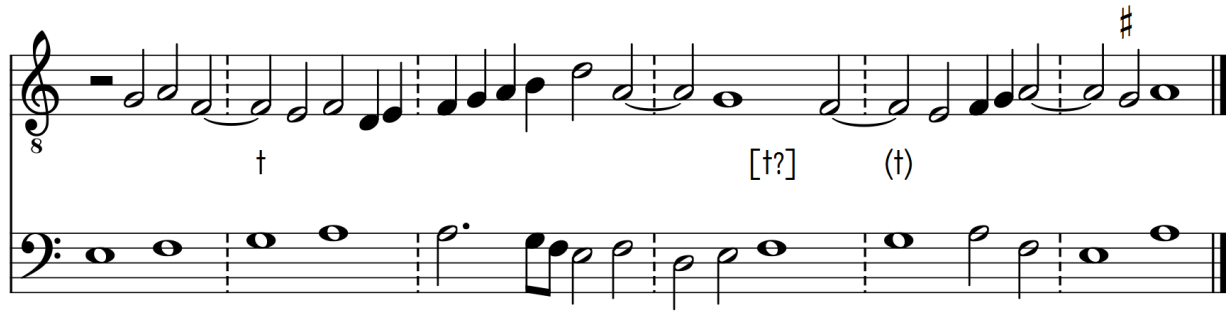


Figure 98: 2 $_{-3}$ 5, p. 70.

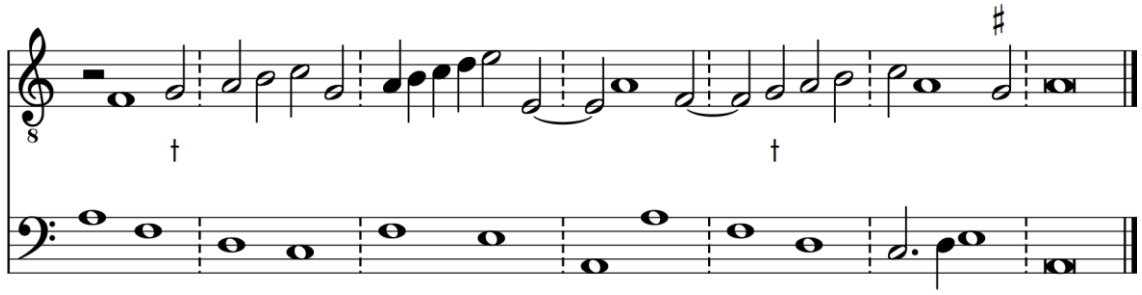


Figure 99: 2 $_{-2}$ 5, p. 71.

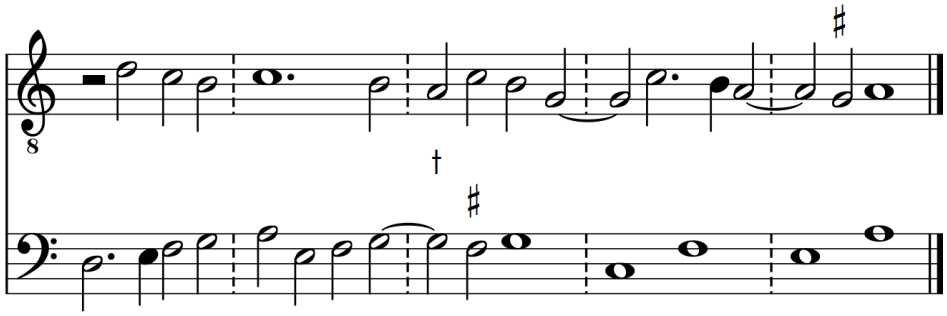


Figure 100: 2 $_{-5}$ 5, p. 71.

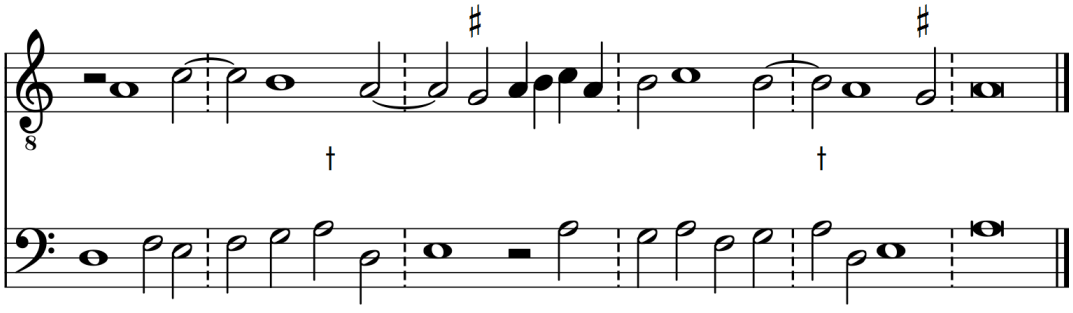


Figure 101: 2 -4 5, p. 72.

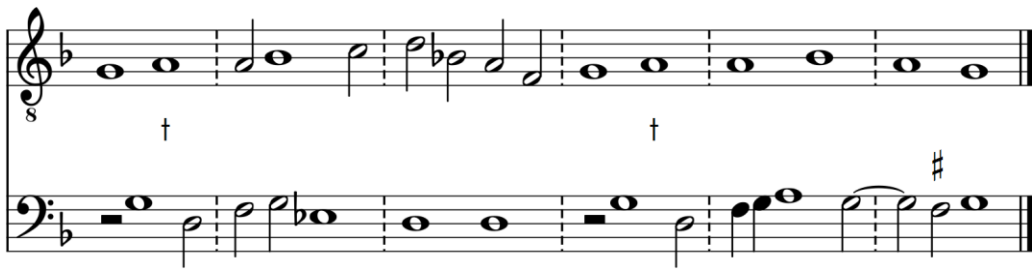


Figure 102: 2 -2 6, p. 72.

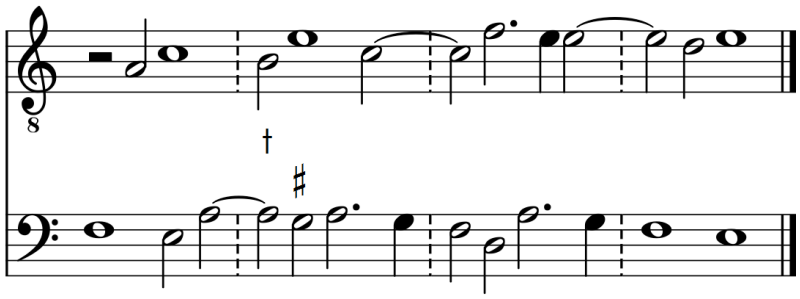


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Figure 103: 4 1 3, p. 74.

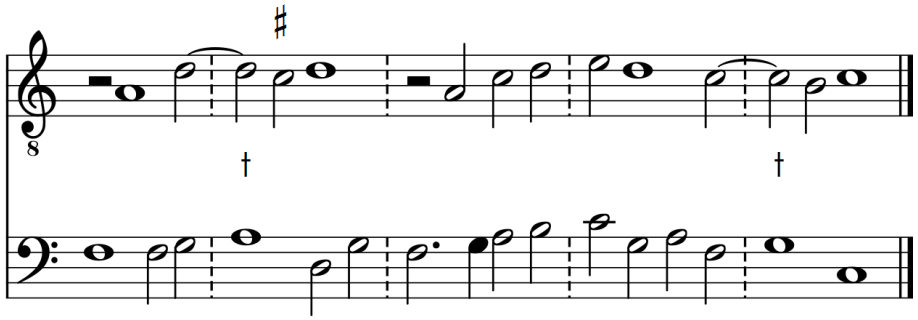


Figure 104: 4 5 -3, p. 74.

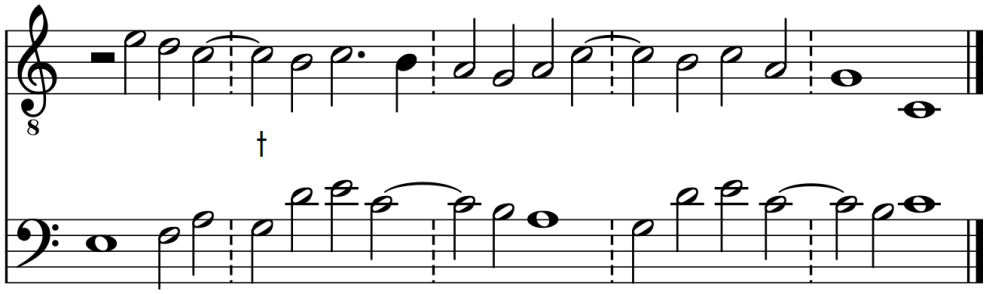


Figure 105: 4 -2 5, p. 75.

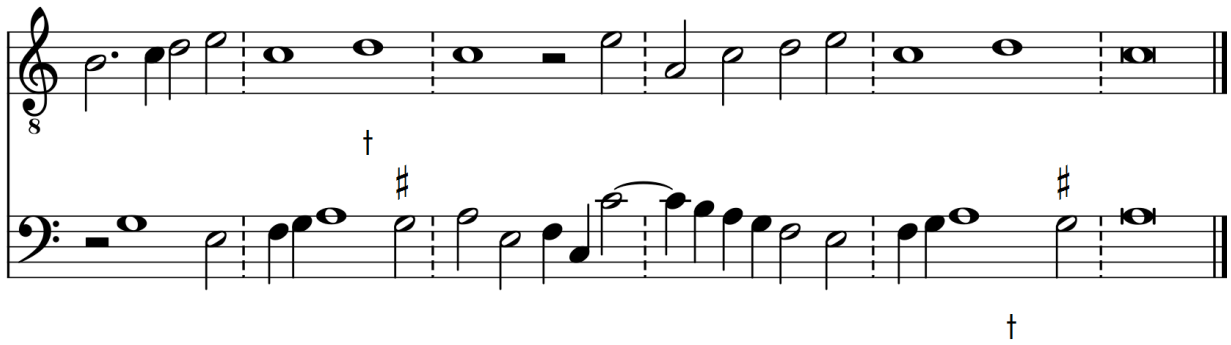


Figure 106: 4 -3 5, p. 75.

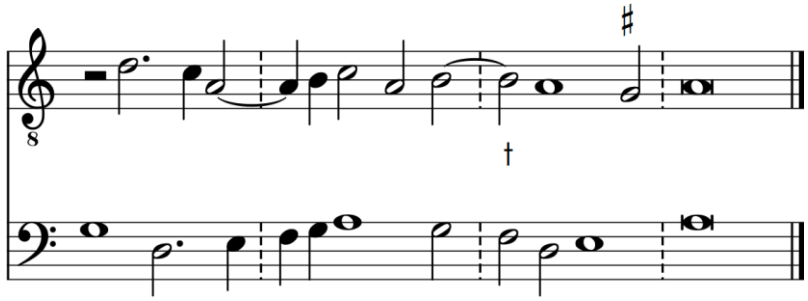


Figure 107: 4 -2 6, p. 76.

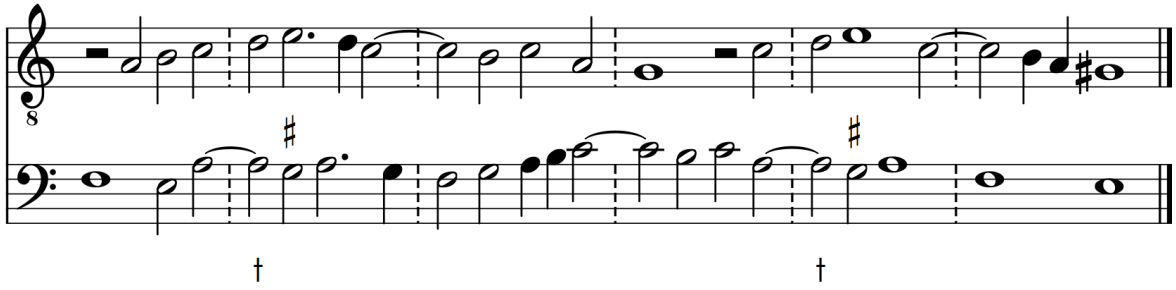
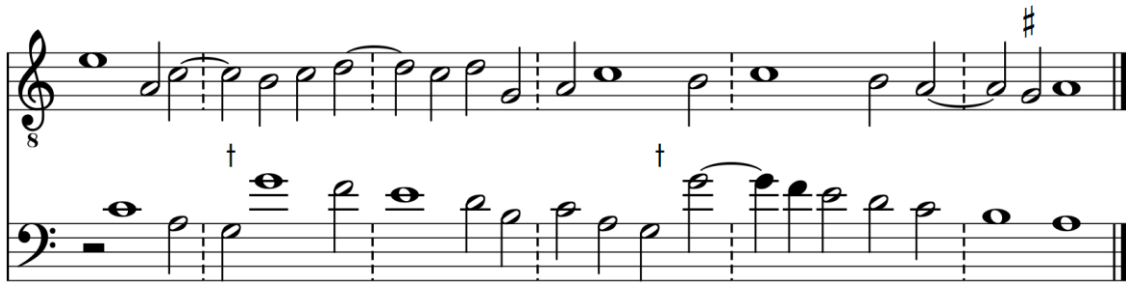


Figure 108: 4 8 -6, p. 77.



Della Settima

Figure 109: 7 1 6, p. 78.

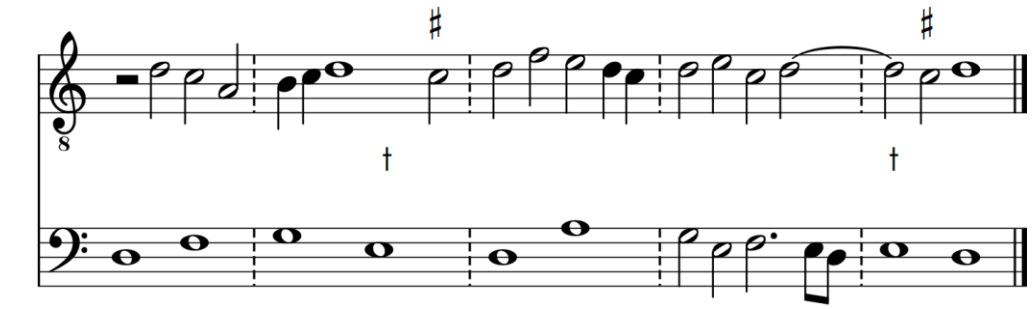


Figure 110: 7 2 5, p. 78.

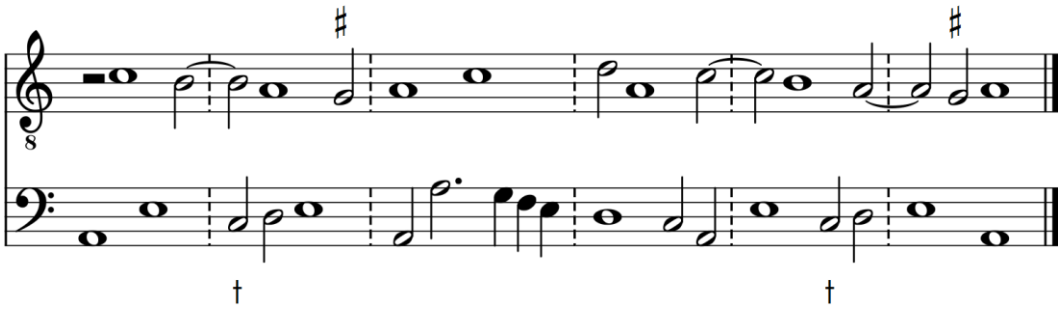


Figure 111: 7 2 5, p. 79.

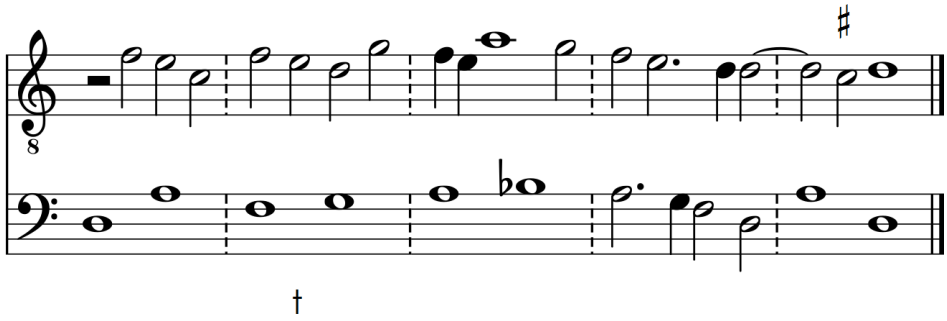


Figure 112: 7 2 5, p. 80.

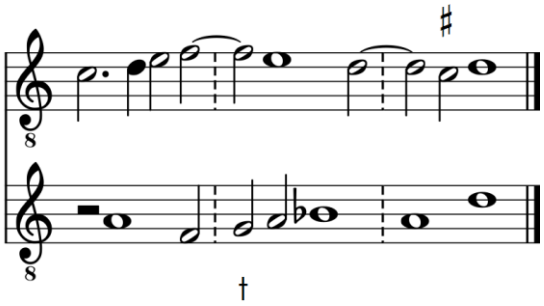


Figure 113: 7 1 5, p. 80.

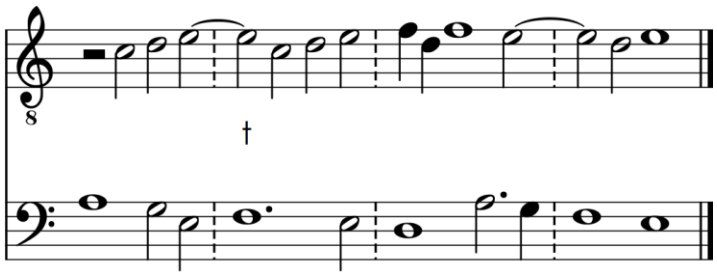


Figure 114: 7 4 3, p. 81.

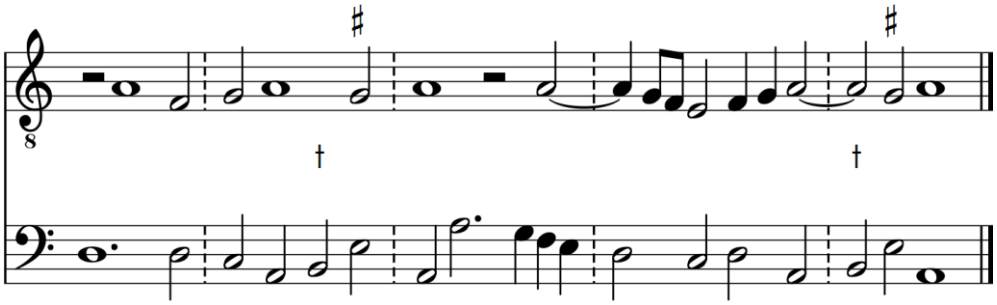


Figure 115: 7 1 3, p. 82.

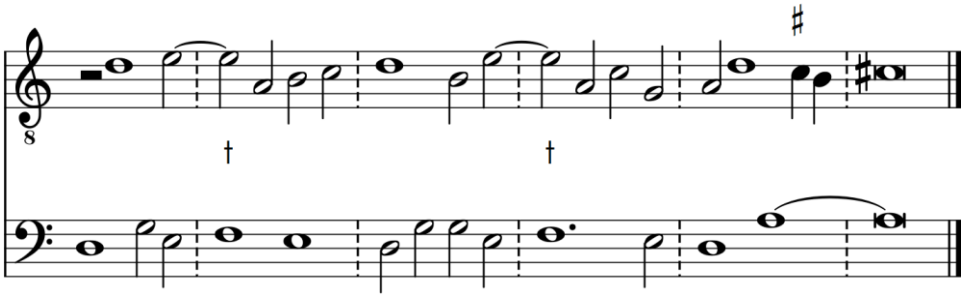


Figure 116: -7 8 3, p. 83.

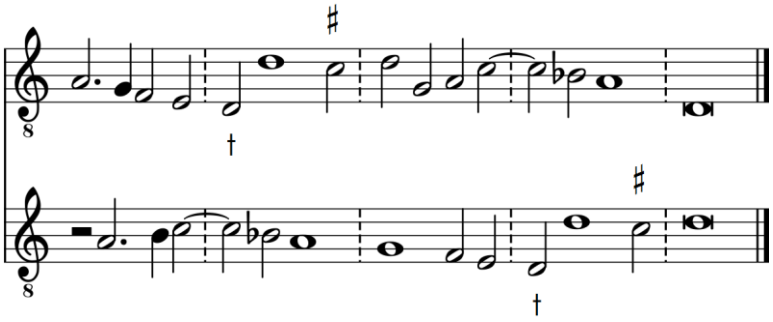


Figure 117: 7 -3 8, p. 84.

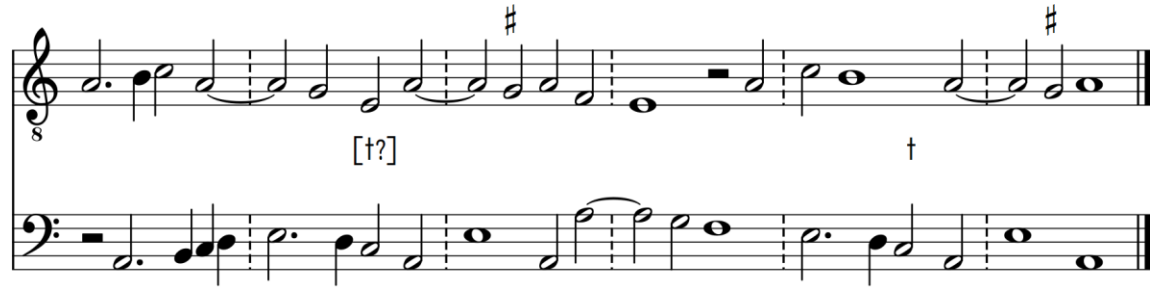


Figure 118: 7 -5 3 (7 -5 10), p. 84.

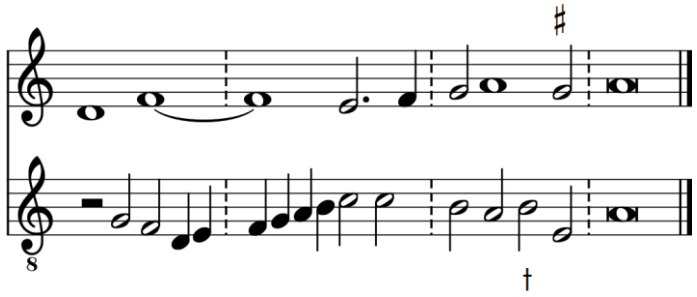


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Figure 119: 5 2 3, p. 85.

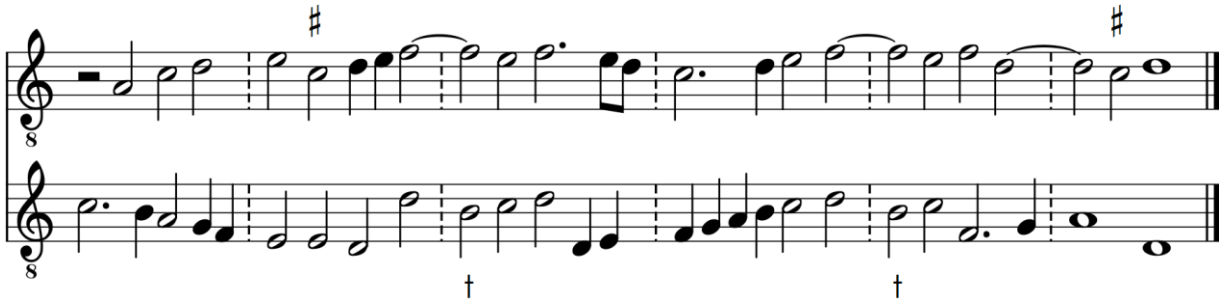


Figure 120: 5 2 3, p. 86.

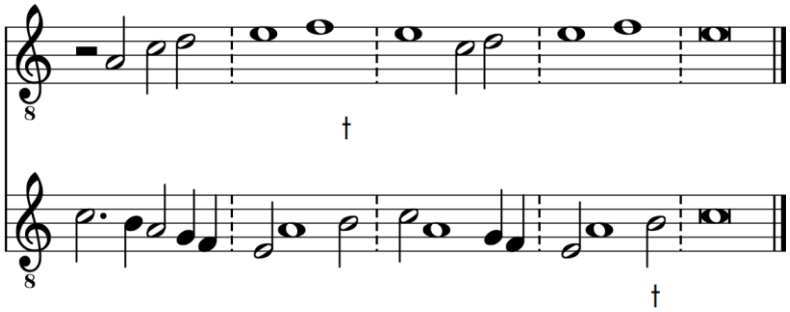
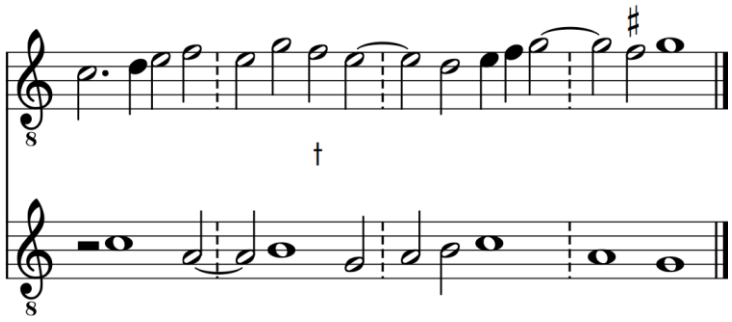


Figure 121: 5 -3 6, p. 86.



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