A New Realization of the Ekpyrotic Scenario

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Abstract

The standard Hot Big Bang with Inflation is the current standard paradigm of the early universe. However, there exist different unsolved theoretical issues for the inflation scenario. The thesis explores a new version of an alternative scenario named Ekpyrosis. It suggests that by adding an S-brane located after the Ekpyrotic contraction phase, all the puzzles of the standard Hot Big Bang cosmology and the theoretical inconsistencies of inflation can be resolved in this new model. Besides, using the methodology of cosmological perturbation theory connecting between the primordial universe and late-time cosmological observations, it is shown that the predictions of the model are consistent with the current constraints. Furthermore, to bridge the phase Ekpyrotic contraction in the very early universe and Standard Hot Big Bang, a natural process of reheating is studied for S-brane Ekpyrosis. The result shows that the S-brane efficiently decays into radiation so that the emergent scenario can be smoothly connected to Hot Big Bang cosmology. Finally, by studying the entropy of cosmological perturbations, we derive a lower bound on the reheating temperature after Ekpyrosis.

Abrégé

Le Hot Big Bang standard avec inflation est le paradigme standard actuel de l'univers primitif. Cependant, il existe différents problèmes théoriques non résolus pour le scénario d'inflation. La thèse explore un scénario alternatif nommé Ekpyrosis. Il suggère qu'en ajoutant une S-brane située après la phase de contraction ekpyrotique, toutes les énigmes de la cosmologie standard du Hot Big Bang et les incohérences théoriques de l'inflation peuvent être résolues dans ce nouveau modèle. De plus, en utilisant la méthodologie de la théorie des perturbations cosmologiques reliant l'univers primordial et les observations cosmologiques tardives, les prédictions du modèle sont bien cohérentes avec les contraintes actuelles. De plus, pour combler la contraction ekpyrotique dans le tout premier univers et le Big Bang chaud standard, un processus naturel de réchauffement est bien étudié pour l'ekpyrosis S-brane. Le résultat montre que la S-brane se désintègre efficacement en rayonnement afin que le scénario émergent puisse être connecté en douceur à la cosmologie du Hot Big Bang. Enfin, en étudiant l'entropie des perturbations cosmologiques, nous dérivons une borne inférieure sur la température de réchauffage après Ekpyrosis.

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Last but not least, I will express my appreciation to my families for their support and love over the years.

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Preface

Contributions of the Author.

This manuscript thesis contains four peer-reviewed and published articles that are original and consist of distinct contributions to knowledge. These are presented in their original form, due to copyright, in Chapters 5, 6, 7 and 8. The authors are listed in alphabetical order.

In the paper listed in chapter 5, the main idea is inspired through the discussion between Prof.Brandenberger and me. We equally shared the calculation of section 5.3 and of the appendix and I checked all the calculations. The paper writing was mostly done by Prof.Brandenberger based on our calculation note.

In the paper listed in chapter 6, Prof.Brandenberger raised the idea and led the project. I contributed most parts of calculation in section 6.4 and the appendix and I checked all the calculations. The paper was written mostly by Prof.Brandenberger based on our calculation note.

In the paper listed in chapter 7, Prof.Brandenberger, Prof.Dasgupta and I raised the idea and and I led the project. I contribute all parts of the calculation and all three authors participated in the discussions. The paper was written by Prof.Brandenberger and me. I participated in 70% of the writing.

In the paper listed in chapter 8, Prof.Brandenberger, Dr.Brahma and I raised the idea. I contributed 80% of calculation in section 8.2,8.3,8.4 and half of 8.5 and all three authors participated in the discussions. The paper was written by Prof.Brandenberger and Dr. Brahma and me. I participated in 35% of the writing.

Publications not included in this thesis during my Ph.D. study

- Yuichi Nakai, Ryo Namba, Ziwei Wang, Light Dark Photon Dark Matter from Inflation, arXiv:2004.10743

- Ziwei Wang, Robert Brandenberger, Lavinia Heisenberg, Eternal Inflation, Entropy Bounds and the Swampland, Eur.Phys.J.C80 (2020) 9 arXiv:1907.08943

- Yi-Fu Cai, Xingang Chen, Mohammad Hossein Namjoo, Misao Sasaki, Dong-Gang Wang, Ziwei Wang, Revisiting non-Gaussianity from non-attractor inflation models, JCAP1805, 012(2018) arXiv:1712.09998

- Aditya VarnaIyer,Shi Pi,Yi Wang,Ziwei Wang,Siyi Zhou , Strongly Coupled Quasi-Single Field Inflation,JCAP1801, 041(2018), arXiv:1710.03054

Chapter 1

Progress on Observational Cosmology

1.1 Cosmic microwave background and primordial universe

The Cosmic Microwave Background (CMB) as a smoking gun for modern observational cosmology bridges theories of primordial cosmology with data. According to the Big Bang model the early universe expanded and cooled down as time proceeded. While the universe was hot and opaque, radiation was scattered from one particle to another. It was still too hot for electrons to be associated with a particular nucleus; such free electrons are effective at scattering photons, thus ensuring that no radiation ever got very far in the early universe without having its path changed. When the temperature had dropped to about 3000 K, electrons and nuclei managed to combine to form stable atoms of hydrogen and helium. With no free electrons to scatter photons, the universe became transparent for the first time in cosmic history. From this point on, matter and radiation interacted much less frequently; we say that they decoupled from each other and evolved separately. Suddenly, electromagnetic radiation could really travel, and it has been traveling through the universe ever since.

First accurate measurements of CMB anisotropies were made with a satellite orbiting Earth. Named the Cosmic Background Explorer (COBE), it was launched by NASA in November 1989 and then followed by Wilkinson Microwave Anisotropy Probe (WMAP) from June 2001 with more accuracy. The data it received quickly showed that the CMB closely matches that expected from a blackbody with a temperature of 2.73 K. This is exactly the result expected if the CMB was indeed redshifted radiation emitted by a hot gas that filled all of space shortly after the universe began [5, 6].

The latest CMB measurement from space was performed by the Planck satellite [1, 7, 8]. The average temperature of the CMB is measured to be $T_{\rm cmb} = 2.7255$ K, and the fluctuations in temperature of the microwave radiation about the average, with the order of $\frac{\delta T}{T} \sim 10^{-4}$ [9], is displayed in Fig. 1.1.

The observed fluctuations of the CMB represent a picture of the universe at the time of recombination. Before that occured, the universe went through a radiation dominated phase during which Big Bang Nucleosynthesis (BBN) happened. After BBN, the universe is dominated by a plasma of electrons, atomic nucleis and photons. Both statistical mechanics and gravity played significant roles in how the fluctuations evolved (see, e.g Refs. [10] for

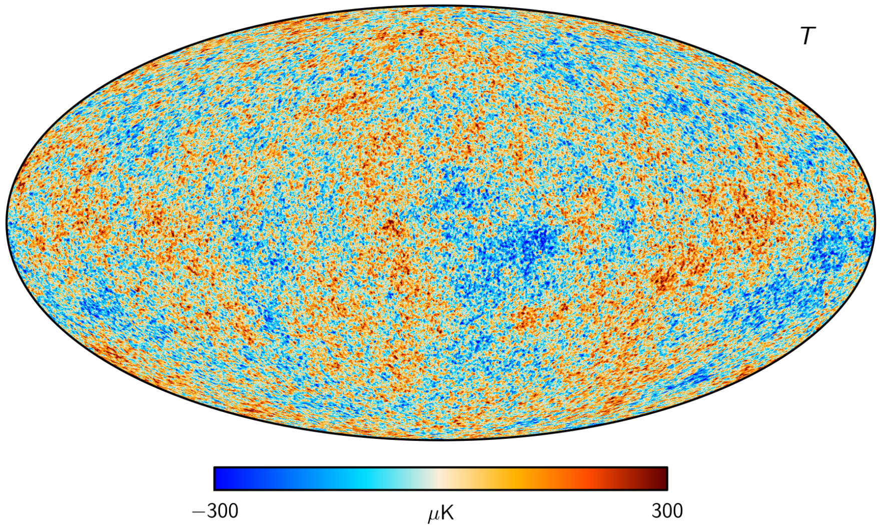

Figure 1.1: Pillars of anisotropy of CMB temperature fluctuation [1].

reviews). In summary, with well understood physics during the phase of hot Big Bang cosmology, physicists can perform a reverse process revealing the primordial fluctuations which yield the origin of currently observed temperature fluctuations of the CMB [11].

The initial conditions for the hot Big Bang may have been created by quantum fluctuations during a period of inflationary expansion or a period of Ekpyrotic contraction. These mechanisms predict the statistics of the initial conditions, for instance, fluctuations in different directions in the sky, rather than the specific value of the temperature fluctuation in a specific direction. For Gaussian initial conditions, these correlations are completely specified by the two-point correlation function of the curvature perturbation $\mathcal{R}$

$$\langle \mathcal{R}(x)\mathcal{R}(x') \rangle = \frac{2\pi^2}{k^3} P_{\mathcal{R}}(k)\delta^3(k-k'), \tag{1.1}$$

where $P_{\mathcal{R}}(k)$ is the primordial curvature perturbation power spectrum and k is the wavenumber of the fluctuations. Observations show that the power spectrum is slightly red tilted, which is parameterized as follows

$$P_{\mathcal{R}}(k) = A_s(\frac{k}{k_*})^{\mathrm{n_s}-1}, \tag{1.2}$$

where A_s and n_s are the amplitude and the tilt of the power spectrum at pivot scale k_*

Also primordial gravitational waves may be generated during the early universe, which can contribute to the polarization of the CMB photons. Mathematically the polarization can be decomposed into two types : the curl-free E-mode component and the divergence-free B-mode component. Both scalar and tensor perturbations create CMB E-mode but only tensor perturbations create B-modes. Measuring of the B-B correlation function leads to a probe of the primordial gravitational waves [12, 13].

Like the scalar perturbation spectrum, the primordial tensor power spectrum is usually parameterized as follows,

$$P_t(k) = A_t(\frac{k}{k_*})^{\mathrm{n_t}} \tag{1.3}$$

where A_t is the amplitude and n_t is the tilt of the tensor spectrum. Since there is no direct detection of primordial gravitational wave, only upper bounds on the scalar amplitude are known. A convenient quantity is the tensor-to-scalar ration which is defined as

$$r \equiv \frac{P_t}{P_\zeta} \tag{1.4}$$

where the scalar to tensor ratio r is constrained to be $r < 0.069$ based on the current CMB polarization measurements [14]. An early universe model, predicting a tensor-to-scalar ratio in excess of the observational bound is ruled out.

1.2 Notation

Through the thesis, we try to be consistent in our notation and conventions. Greek indices indicate spacetime coordinates, $\mu, \nu, \cdots \in \{0, 1, 2, 3\}$, while Latin indices indate spatial coordinates, $i, j, \ldots, \in \{1, 2, 3\}$. We use the metric signature, $(-, +, +, +)$. We define the reduced Planck mass by $\mathrm{m_{pl}} \equiv \sqrt{8\pi \mathrm{G_N}}$, where G_N is Newton's gravitational constant. The spped of light and reduced Planck constant are set to unity: $c = \hbar = 1$. Derivative with respect to cosmic time is denoted by an overdot, $\dot{} \equiv d/dt$. And a derivative with respect to conformal time is denoted by a prime, $' \equiv d/d\tau$.

Chapter 2

Review of primordial cosmology

In the last chapter we introduced a set of cosmological observables that encode information about the early universe. In this chapter, we will go through the dynamics of the early universe model : standard Hot Big bang cosmology model as foundation of the rest of the thesis. This chapter is based on many textbooks, lecture notes, and review articles; we list Refs. [10, 15–18] for recommendations, but many more exist.

2.1 Homogeneous and isotropic universe

2.1.1 Background

Standard Big Bang cosmology assumes homogeneity and isotropy of space on large scales, which leads to the Friedmann-Robertson-Walker (FRW) metric for the spacetime of the universe:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{2.1}$$

Here, the scale factor $a(t)$ characterizes the relative size of a spacelike hypersurfaces Σ at different times. The curvature parameter k is $+1$ for positively curved Σ, 0 for flat Σ, and -1 for negatively curved Σ.

The kinetics of cosmology is determined by General Relativity

$$G_{\mu\nu} = R_{\mu\nu} + \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2.2}$$

where $R_{\mu\nu}$ and R are Ricci tensor and scalar which are derived from the metric $g_{\mu\nu}$,

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{\mu\nu} - \Gamma^{\alpha}_{\beta\nu}\Gamma^{\beta}_{\mu\alpha}, \quad R \equiv g^{\mu\nu}R_{\mu\nu} \tag{2.3}$$

$$\Gamma^{\mu}{}_{\alpha\beta} \equiv \frac{g^{\mu\nu}}{2}\left[g_{\alpha\nu,\beta} + g_{\beta\nu,\alpha} - g_{\alpha\beta,\nu}\right] . \tag{2.4}$$

For a perfect fluid, the energy momentum tensor on the right hand side of (2.2) is organized as follows

$$T^{\mu}_{\ \nu} = g^{\mu\alpha}T_{\alpha\nu} = (\rho + p)\, u^{\mu}u_{\nu} - p\,\delta^{\mu}_{\nu} \tag{2.5}$$

where ρ is the energy density of the fluid, p its pressure and u^{μ} its 4-velocity. Note that ρ and p are defined in the rest frame of the fluid and they are functions only of time.

Inserting the geometry of an FRW universe (2.1) (2.3) (2.4) and the energy momentum tensor of perfect fluid (2.5) without dark energy into the Einstein equation (2.2), the Friedmann equations are derived

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho - \frac{k}{a^2} \tag{2.6}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) \tag{2.7}$$

where the Hubble parameter $H \equiv \frac{\dot{a}}{a}$ is defined. In an expanding universe $H > 0$ with matter consistent with the Strong Energy Condition $\rho + 3p \geq 0$ impling $\ddot{a} < 0$. This indicates the existence of a singularity $a(t = t_{begin}) = 0$ in the finite past. The conclusion relies on General Relativity, namely that the Friedmann Equations are applicable, and assumes no matter violating the Strong Energy Condition We will return to this topic in a later section.

Besides, the continuity equation for perfect fluid from the covariant conservation equation $\nabla_{\mu} T^{\mu}_{\nu} = 0$, becomes

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{2.8}$$

where the two Friedmann equations (2.6)(2.7) and the continuity equation (2.8) are not independent with each other. To solve the kinetics of the universe from the equations above, more information about the perfect fluid should be introduced. We define the equation of state parameter $\omega \equiv \frac{p}{\rho}$ and for most components the equation of state is constant in time.

For a single component universe, the continuity equation (2.8) is integrated as

$$\rho \propto a^{-3(1+w)}, \tag{2.9}$$

and together with friedmann equations (2.6)(2.7) the time evolution of the scale factor is obtained as a characterization of early universe kinetics.

$$a(t) \propto \begin{cases} t^{2/3(1+w)} & w \neq -1 \\ e^{Ht} & w = -1 \end{cases} \tag{2.10}$$

For non-relativistic matter ($w = 0$), radiation ($w = \frac{1}{3}$) and a cosmological constant ($w = -1$), the scale factor grows as $a(t) \propto t^{2/3}, a(t) \propto t^{1/2}, a(t) \propto e^{Ht}$.

For components with different equations of state, the ρ and p are defined as the sum of the density and pressure contributions from ρ_i and p_i for each component.

$$\rho \equiv \sum_i \rho_i\,, \qquad p \equiv \sum_i p_i\,. \tag{2.11}$$

We define the present ratio of the energy density for each component relative to the critical energy density $\rho_{\rm crit} \equiv 3H_0^2$

$$\Omega_i \equiv \frac{\rho_{i0}}{\rho_{\rm crit}}\,, \tag{2.12}$$

and the corresponding equations of state $w_i \equiv \frac{p_i}{\rho_i}$ Here the subscript '0' denotes evaluation of a quantity at the present time t_0. Under normalization of the scale factor such that $a_0 = a(t_0) \equiv 1$, the Friedmann equations(2.6) (2.7) yield

$$\left(\frac{H}{H_0}\right)^2 = \sum_i \Omega_i a^{-3(1+w_i)} + \Omega_k a^{-2}, \tag{2.13}$$

with $\Omega_k \equiv -k/a_0^2 H_0^2$ parameterizing the curvature. Consistently the parameterizing curvature and all components contributing to the whole energy density yields

$$\Omega \equiv \sum_i \Omega_i = 1 - \Omega_k\,. \tag{2.14}$$

2.1.2 Puzzles for standard hot big-bang cosmology

According to (2.6) (2.7), we may find evolution of parameterized curvature which describes the relative flatness of the universe.

$$-\Omega \equiv \frac{\Omega - 1}{\Omega} = \frac{k}{3H_0^2} a^{1+3w} \tag{2.15}$$

or, as a differential equation

$$\frac{d|\Omega - 1|}{dt} = (1 + 3w)\Omega|\Omega - 1|H \tag{2.16}$$

From (2.16) it is concluded that an exactly flat $\Omega = 1$ universe remains flat as time evolves. However, for an expanding universe $H > 0$ with matter components satisfing $w > -\frac{1}{3}$, we find $\frac{d|\Omega-1|}{dlna} > 0$ which indicates any small deviation from exact flatness will grow.

According to current observational constraints, deviations from flatness at Big Bang Nucleosynthesis satisfy the following condition

$$|\Omega(a_{BBN}) - 1| < \mathcal{O}(10^{-16}), \tag{2.17}$$

and for the universe during earlier periods the deviation should be even smaller. Therefore, the standard hot big bang cosmology inevitable suffers from a fine-tuning problem of the flatness of the universe.

Another problem concerns the large scale homogeneity in temperature of the CMB. For thermal equilibrium photons within a volume of at least the size of the observed CMB patch should be in causal contact in the early universe. The maximum comoving distance light can propagate between an initial time t_i and some later time t is

$$\Delta\tau = \tau - \tau_i = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_0^a da \frac{1}{Ha^2} \tag{2.18}$$

And we may define the comoving particle horizon to be this maximum distance of causality. For a universe consisting of a single component fluid with equation of state ω, we find that the comoving particle horizon behaves as follows

$$\tau \propto a^{\frac{1}{2}(1+3w)} \tag{2.19}$$

For a matter and radiation dominated universe with $w = 0,\ \frac{1}{3}$, the Hubble radius grows linearly with time, which indicates that large scales inside the horizon currently were outside the horizon during last scattering. However, the near homogeneity of the CMB indicates homogeneity of universe in the past. Unless we accept a fine-tuned condition, the framework of hot big bang cosmology could not explain the homogeneity of the universe. The inflation scenario is one proposal for resolving puzzles from Hot Big Bang Cosmology

2.2 Inflationary cosmology

In order to solve the flatness and homogeneity problems listed above, we consider a universe with an early phase during which the scale factor grows exponentially, namely an Inflationary universe or Inflation.

$$a(t) \propto e^{Ht} \tag{2.20}$$

From (2.15) and (2.16), we may use the comoving Hubble radius to express the effective apparent curvature

$$|1-\Omega(a)| = \frac{1}{(aH)^2} \tag{2.21}$$

For inflation, $|1-\Omega(a)|$ decreases exponentially which means driving the universe towards flatness even with a non-flat initial condition. If inflation lasts enough time, the universe at the end of inflation is exponentially flat which explains the flatness problem of Hot Big Bang cosmology.

The comoving particle horizon for inflation is calculated from (2.18) and (2.20)

$$\tau = -\frac{1}{aH} \tag{2.22}$$

A decreasing Hubble radius also means that large scales were inside the Hubble radius before inflation. Causality before inflation may explain spatial homogeneity.

2.2.1 Slor-roll inflation

The simplest models of inflation involve a single scalar field ϕ, the inflaton. The dynamics of a scalar field minimally coupled to gravity is governed by the action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]. \tag{2.23}$$

The energy-momentum tensor for the scalar field is

$$T_{\mu\nu}^{(\phi)} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi + V(\phi)\right). \tag{2.24}$$

where $V_{,\phi} = \frac{dV}{d\phi}$. Assuming the FRW metric (2.1) for $g_{\mu\nu}$ and a homogeneous field $\phi(t, \mathbf{x}) \equiv \phi(t)$, the scalar energy-momentum tensor takes the form of a perfect fluid with

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{2.25}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,, \tag{2.26}$$

$$\tag{2.27}$$

The equation of state of the single field

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{2.28}$$

could lead to phases with accelerated expansion when the potential energy $V(\phi)$ dominates over the kinetic energy $\frac{1}{2}\dot{\phi}^2$.

The field equation of motion is

$$\frac{\delta S_\phi}{\delta\phi} = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu\phi) + V_{,\phi} = 0\,, \tag{2.29}$$

and in a homogenous FRW universe, the field equation of motion and the Friedmann equation becomes

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \quad H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{2.30}$$

We define ϵ as first order slow-roll parameter as follows

$$\epsilon \equiv -\frac{\dot{H}}{H^2}. \tag{2.31}$$

Accelerated expansion occurs if $\epsilon < 1$. The de Sitter limit, $p_\phi \to -\rho_\phi$, corresponds to $\epsilon \to 0$. In this case, the potential energy dominates over the kinetic energy yields

$$\dot{\phi}^2 \ll V(\phi). \tag{2.32}$$

Accelerated expansion will only be sustained for a sufficiently long period of time if the time derivative of the first slow-roll parameter ϵ is also small. Rigorously speaking $|\eta| < 1$ ensures that the fractional change of ε per e-fold is small.

$$\eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \ll 1 \tag{2.33}$$

which requires smallness of the second time derivative of field ϕ

$$|\ddot{\phi}| \ll |3H\dot{\phi}|\,,\ |V_{,\phi}|\,. \tag{2.34}$$

The slow-roll conditions are directly related to the derivatives of the potential during slow-roll inflation.

$$\epsilon \approx \epsilon_V\,, \quad \eta \approx 2\eta_V - 2\epsilon_V\,, \tag{2.35}$$

where

$$\epsilon_{\mathrm{v}}(\phi) \equiv \frac{M_{\mathrm{pl}}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2, \quad \eta_{\mathrm{v}}(\phi) \equiv M_{\mathrm{pl}}^2 \frac{V_{,\phi\phi}}{V}\,. \tag{2.36}$$

In spite of its successes at providing a solution of some problems of standard Big Bang cosmology and mechanism for the origin of structure, inflation suffers from a number of conceptual problems. In the following we discuss one issue which has recently been rasied.

2.2.2 Problems of inflation

While inflation makes several successful predictions, it suffers from a few conceptual issues.

It is believed that most effective field theories containing scalar fields are inconsistent with a quantum theory of gravity. It has been argued [19] that an effective field theory can only hold for a finite range of expectation values of ϕ, the reason being that there is an infinite tower of states whose mass scales as $m \sim m_{pl} e^{-\alpha \Delta\phi}$. Hence, if ϕ evolves for more than m_{pl}, these states become low mass and must be included in the effective field theory analysis. Thus, the range of validity of a fixed effective field theory is constrained by

$$|\frac{\Delta\phi}{m_{pl}}| \leq \mathcal{O}(1) \tag{2.37}$$

Meanwhile it appears to be difficult to construct any de Sitter vacuum with string constructions of scalar potentials [19]. It is proposed that the potential for scalar fields in string theory satisfies either,

$$|\frac{\nabla V}{V}| \geq \frac{c}{m_{pl}}\,, \tag{2.38}$$

or

$$\min(\frac{\nabla_i \nabla_j V}{V}) \leq -\frac{c'}{m_{pl}}\,, \tag{2.39}$$

for some constant $c, c' > 0$, which are of the order 1. The conjecture can be derived by combing entropy considerations in de Sitter space and the distance conjecture of string theory [20, 21].

We may assign the entropy of de Sitter spacetime to the logarithm of the dimension of the Hilbert space associated to the microstates. At large distances in field space towers of states become light thereby increasing the number of states and the entropy of de Sitter spacetime. The exponential nature of the tower of states in the distance conjecture corresponds to the property of the potential that its derivative should be proportional to itself, which gives the first condition (2.38). The second condition (2.39) arises including an instability which implies that the entropy-based argument for the first condition (2.38) breaks down.

Almost all inflation models obeying the observational bound are in modest tension with the distance conjecture (2.37). More seriously, models with power-law or plateau potential in which $\nabla V/V \to 0$ for some values of ϕ, are all inconsistent with the de Sitter conjecture (2.38) [22].

Another problem concerns the origin of fluctuation modes in the modes origins in sub-Planckian regime. The perturbations that account for the CMB fluctuations at late times start their evolution deep in the sub-Hubble regime. Specifically, for modes currently inside the Hubble $\lambda(t_0) < H_0^{-1}$, if inflation lasts sufficient long, these modes will be in the sub-Planckian regime $\lambda(t_i) < l_{\text{lp}}$ at the beginning of inflation. Modes satisfying both conditions above are initally inside Trans-Planckian regime, where the semi-classical quantum field theory in curved spacetime breaks down. The results of predictions from inflation based on semi-classical quantum field theory can not be trusted for modes in the Trans-Planckian regime [23].

The trans-Planckian Censorship Conjecture (TCC) states that in models consistent with quantunm gravity the situation can never arise that an initally trans-Planckian wavelength of a fluctuation mode grows to be super-Hubble [24, 25].

$$\frac{a(t)}{a(t_i)} l_{\text{lp}} \leq H^{-1}(t)\,, \tag{2.40}$$

where the condition must hold for any $t > t_i$. For inflation, we take the t as time of reheating t_R and t_i as the beginning of inflation, and thus we have the following TCC condition

$$\frac{a(t_R)}{a(t_i)} l_{\rm lp} \leq H^{-1}(t_R)\,. \tag{2.41}$$

From the condition one can obtain a strict constraint on the energy scale of inflation after given some any knowledge or assumption about reheating. For instant reheating,

$$V^{\frac{3}{4}} < \sqrt{3} M_{\rm pl}^2 (T_0 T_{\rm eq})^{\frac{1}{2}} \tag{2.42}$$

where T_0 and $T_{\rm eq}$ are temperatures at present and at the time of matter and radiation equilibrium. Only inflation models with energy scale $V^{\frac{1}{4}} < 10^9$GeV are consistent with the TCC conjecture. However, inflation under such low level energy scale predicts an extremely tiny tensor fluctuation spectrum.

$$P_h(k) = \frac{2H^2}{\pi^2 M_{\rm pl}} < 10^{-40}, \quad r < 10^{-30} \tag{2.43}$$

Although at the present time we do not have any direct detaction of tensor perturbations, any discovery of primordial gravitational waves with finite tensor-to scalar ratio $r > \mathcal{O}(10^{-30})$ would rule out inflation given the TCC.

Chapter 3

Alternative Scenarios for the Early Universe

3.1 General Criterion for Alternative Scenario

In previous chapters, it has been shown the inflation scenario can solve the horizon and flatness problems in a natural way for Hot Big Bang cosmology. Primordial Fluctuations seed the cosmological perturbation which successfully yield the initial conditions for the large scale structure. On the other hand, inconsistencies of effective theory modeling inflation and string theory pushes physicists to search for new possibilities in the extreme early universe, i.e, see Ref [26, 27]. for reviews and details.

Before diving into different examples of alternative scenarios for the early universe, I will emphasize general criteria for a successful alternative scenario.

Firstly, scales which we observe today must have started out early at sub-Hubble lengths. Secondly, the comoving particle horizon during the very early universe should shrink rapidly. Concerning the second point, sufficient contacts of causality before modes exiting the particle horizon ensures the homogeneity of the universe on large scale at the time of recombination, which explains the horizon puzzles. Concerning the first point, in order to have a causal mechanism for the origin of fluctuations, they must have started from sub-Hubble scales. Thus, the modes of fluctuations inside the observational windows should exit the Hubble radius during the early universe and finally re-enter the Hubble in the late time universe.

New scenarios are expected to contain at least one mechanism generating fluctuations for the initial condition of cosmological perturbations. A nearly scale invariant power spectrum of the scalar part of perturbations should be generated during the scenario in order to be consistent with observational constraints for primordial fluctuations. In Fig.3.1, the space-time sketch of inflationary cosmology are displaced.

Finally, the homogeneous and isotropic solution for the early phase should be an attractor in initial condition space. In particular, any initial deviation from spatial flatness should be decreasing in time.[1]

[1] As discussed earlier, these criteria are obeyed in the inflationary scenario

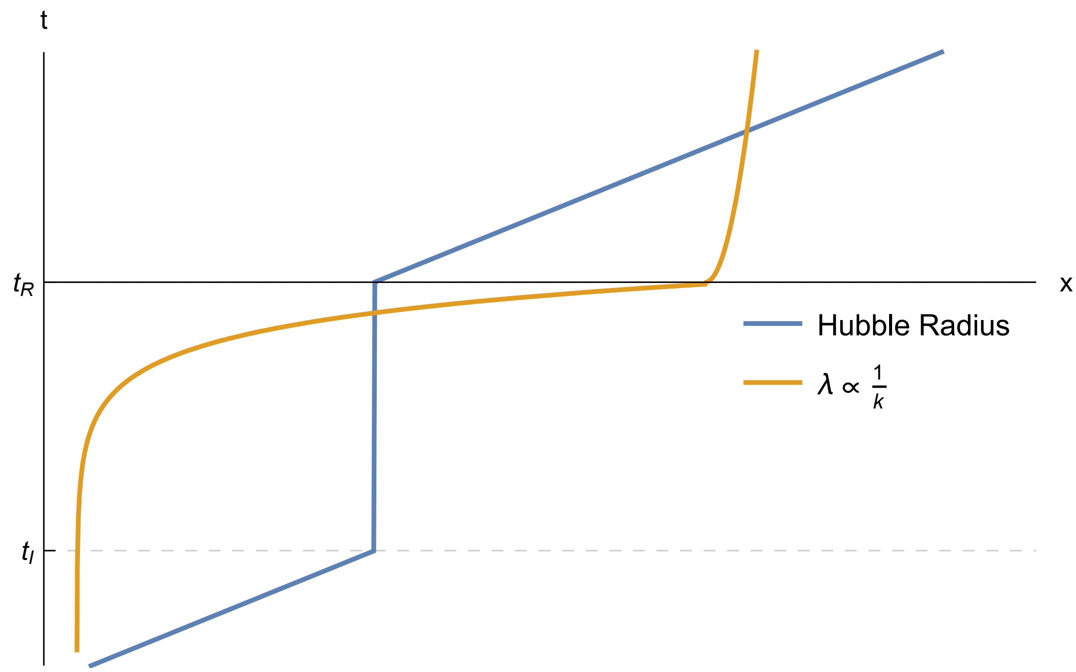

Figure 3.1: Space-time sketch of inflationary cosmology. The vertical axis is time, the horizontal axis corresponds to physical distance. The solid line labelled k is the physical length of a fixed comoving fluctuation scale.

3.2 Examples

3.2.1 String gas cosmology

String gas cosmology is a model of the very early universe based on coupling a gas of closed string matter to a background space-time geometry [28–30]. Assuming that all spatial sections are compact toroidal manifolds, with R denoting the radius of the torus, the degrees of freedom of closed strings include, in addition to the momentum modes whose energies are quantized in units of $1/R$, and string winding modes whose energies are quantized in units of R. and oscillatory modes whose energies are independent of R. There is a maximal temperature of a gas of strings, the “Hagedorn temperature” T_H. The presence of string winding modes leads to a symmetry of the string mass spectrum under the transformation $R \rightarrow 1/R$. Under this transition, momentum and winding modes are exchanged. This is part of the T-duality symmetry. In Fig.3.2, the relation between temperature T and torus radius R are displayed.

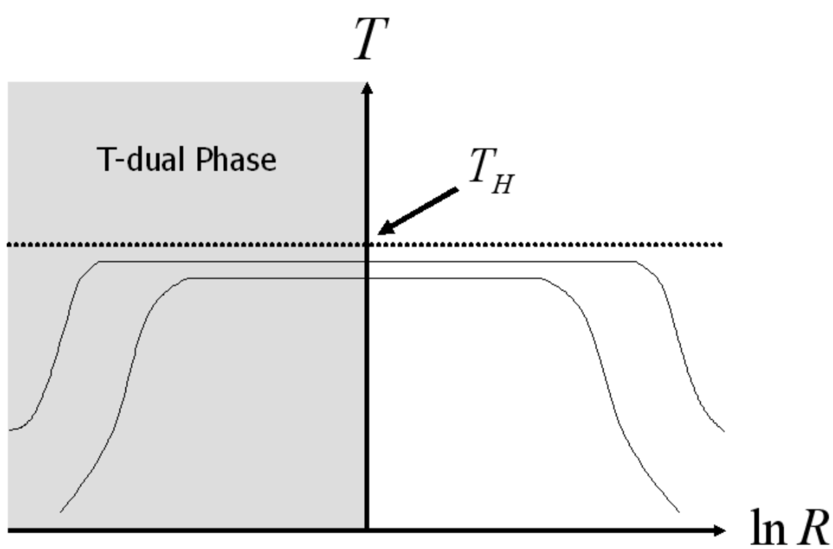

Figure 3.2: The temperature (vertical axis) as a function of radius (horizontal axis) of a gas of closed strings in thermal equilibrium. Note the absence of a temperature singularity. The range of values of R for which the temperature is close to the Hagedorn temperature TH depends on the total entropy of the universe. The upper of the two curves corresponds to a universe with larger entropy [2]

In string gas cosmology, the universe begins close to the Hagedorn temperature T_H. This configuration is a metastable fixed point. A sketch is given in Fig.3.3.

Fixed co-moving scales which are subject of observational cosmology today originate

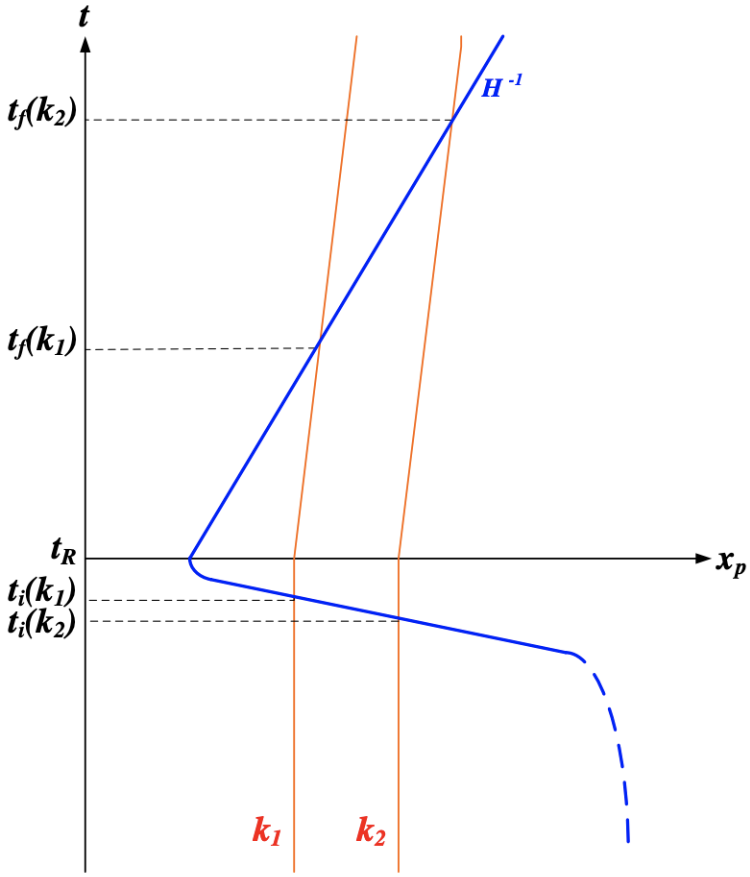

Figure 3.3: Space-time diagram (sketch) showing the evolution of fixed co-moving scales in string gas cosmology. The vertical axis is time, the horizontal axis is physical distance. The solid curve represents the Einstein frame Hubble radius H^{-1} which shrinks abruptly to a micro-physical scale at t_R and then increases linearly in time for $t > tR$. Fixed co-moving scales (the dotted lines labeled by k_1 and k_2) which are currently probed in cosmological observations have wavelengths which are smaller than the Hubble radius before t_R. They exit the Hubble radius at times $ti_(k)$ just prior to t_R, and propagate with a wavelength larger than the Hubble radius until they reenter the Hubble radius at times $t_f(k)$ [2].

inside the Hubble radius. Unlike in inflationary cosmology, where the fluctuations are of quantum vacuum nature because any classical matter is being red-shifted during the pahse of accelerated expansion, in string gas cosmology the dominant fluctuations are thermal fluctuations of the dominant matter, namely thermal string fluctuations.From this figure it is clear how the criteria of a successful early universe cosmology are satisfied: the horizon is infinite since time runs to $-\infty$, and is hence unrelated to the Hubble radius. Scales of cosmological interest originate inside the Hubble radius in the Hagedorn phase, allowing for a causal structure formation scenario. The power spectrum of cosmological fluctuations is

$$P_\Phi(k) = \frac{T}{l_s^2}\frac{1}{1 - T/T_H} \tag{3.1}$$

where l_s is the string length, and T is the temperature when the mode k exits the Hubble radius at the end of the Hagedorn phase. The values of the spectral tilt and the running are determined by the details of the transition between the Hagedorn phase and the radiation phase. Since the temperature is approximately constant during the Hagedorn phase, this power spectrum is scale-invariant. Taking into account the fact that larger scales exit the Hubble radius at slightly higher temperatures, one obtains a slight red tilt of the spectrum.

There are still open questions on the topics of String gas cosmology, of which the most important one is absence of a consistent dynamical description of the Hagedorn phase. The scenario, however, does not yet naturally explain the origin of spatial flatness [2].

3.2.2 Matter Bounce

The idea is that instead of originating from a Big Bang singularity, the universe has emerged from a cosmological bounce with a matter-dominated phase of contraction. Bouncing cosmology as an alternative paradigm of early universe cosmology not only addresses some of the conceptual problems of Standard Big Bang cosmology such as the horizon, flatness and entropy problems, but also can provide a mechanism for generating the scale invariant power spectrum of primordial fluctuations [3, 31, 32].

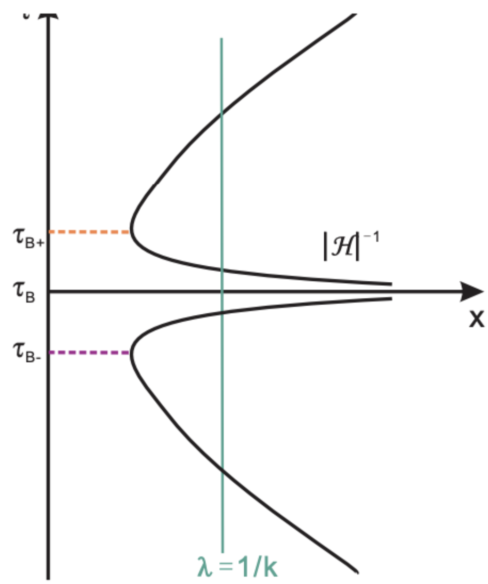

Figure 3.4: Space-time sketch of a non-singular Matter Bounce. The vertical axis is conformal time, the horizontal axis corresponds to comoving spatial coordinates. The vertical line indicates the wavelength for some fixed perturbation mode [3].

The space-time background cosmology which we have in mind has time t running from

$-\infty$ to ∞. The bounce point can be taken to be 0. For negative times the universe is contracting. In figure (Fig), fixed comoving scales start out with a wavelength smaller than the Hubble radius. Only vacuum fluctuations which exit the Hubble radius in a phase of matter domination will end up with a scale-invariant spectrum, because for the dominant mode of the curvature fluctuation there is a duality between a phase of exponential expansion and a phase of matter-dominated contraction [33]. The power spectrum is given by

$$P_R(k,\tau) = \frac{1}{12\pi}k^3|v_k(\tau)|^2a^2(\tau) = \frac{1}{12\pi}k^3(\frac{\tau_H(k)}{\tau(k)})^2|v_k(\tau_h)|^2 \tag{3.2}$$

where $\tau_H(k)$ represents the time that modes with comoving wave number k exit the Hubble during the contracting phase. In a matter dominated contracting phase, substituting the condition $\tau_H \propto k^{-1}$, from (3.2) the matter bounce generates a nearly scale invariant power spectrum of curvature perturbations.

The bouncing point where the Null Energy Condition is violated needs physics beyond the Standard Model and matter satisfying the usual energy conditions. Since General Relativity is a non-renormalizable theory, in any approach to quantum gravity there will be terms in the action which contain higher derivatives. There are existence proofs of higher derivative actions which have non-singular cosmological solutions. In addition Horava-Lifshitz gravity [34], can lead to a non-trivial bounce solution [35].

The main problem of the matter Bounce scenario is the isotropy problem. The energy density in anisotropies blows us faster than the energy density in cold matter and radiation, hence destroying the homogeneity of the bounce. The Ekpyrotic scenario which will be discussed next nicely avoids this problem.

3.2.3 Ekpyrotic

The Ekpyrotic phase is the central insight that shows how a contracting phase preceding the big bang can solve the standard cosmological puzzles [4, 36–38]. This is a surprising statement since one would naively think that a contracting, gravitating system would lead to large curvatures near a singularity. And we know from the near-flatness of our current universe that shortly after the big bang the universe must have been extremely flat.

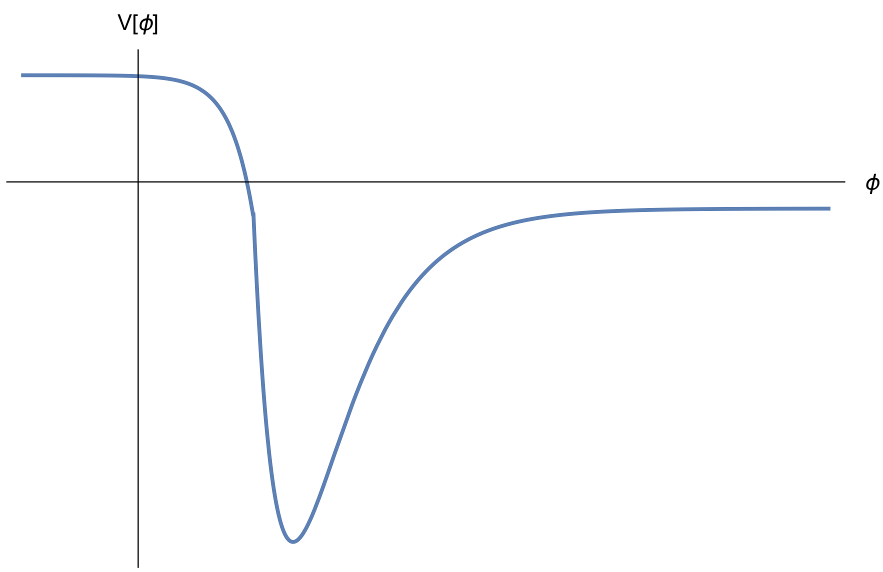

Figure 3.5: The potential during ekpyrosis is negative and steeply falling. The contracting phase involves the motion of ϕ down the negative exponential form $V \propto -e^{-c\phi}$ of potential, and the part of the potential to the left of the minimum leads to bounce [4]

Consider the Friedmann equation relating the Hubble parameter to the total energy density in the universe, which is the sum of kinetic and potential energy. Now suppose that the scalar ϕ has a negative exponential potential, as shown in Fig. 3.5,

$$V(\phi) = -V_0 e^{-c\phi}, \tag{3.3}$$

where V_0 and c are constants. This automatically implies an equation of state $\omega > 1$,. In a contracting universe, the contribution of anisotropies to the effective energy density proportional to a^{-6} comes to dominate the cosmic evolution. However, if there is a matter component with $w > 1$, then the component will scale with an even larger negative power of a, and hence will come to dominate over the anisotropy term in a contracting universe.

In fact it is straightforward to generalize the treatment to having many scalars ϕ_i with potentials $V_i(\phi_i)$[2]. Then, in a flat FRW background, the equations of motion become

$$\ddot{\phi}_i + 3H\dot{\phi}_i + V_{i,\phi_i} = 0 \tag{3.4}$$

and

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\sum_i \dot{\phi}_i{}^2 + \sum_i V_i(\phi_i)\right], \tag{3.5}$$

where $V_{i,\phi_i} = (\partial V_i/\partial\phi_i)$ with no summation implied.

If all the fields have negative exponential potentials $V_i(\phi_i) = -V_i\, e^{-c_i\phi_i}$ and if $c_i \gg 1$ for

[2]Note that in our work we only use a single scalar field

all i, then the Einstein-scalar equations admit the scaling solution

$$a = (-t)^p, \qquad \phi_i = \frac{2}{c_i} \ln(-\sqrt{c_i^2 V_i/2}t), \qquad p = \sum_i \frac{2}{c_i^2}. \tag{3.6}$$

Thus, we have a very slowly contracting universe with (constant) equation of state

$$w = \frac{2}{3p} - 1 \gg 1. \tag{3.7}$$

The time coordinate is negative during the ekpyrotic phase. The steeply falling scalar fields act as a very stiff fluid with the condition $w \gg 1$ to be the defining feature of Ekpyrosis. The main consequence is that the extra term in the Friedmann equation (3.5) with $w \gg 1$ comes to dominate the cosmic evolution, and once more, the fractional energy densities $\Omega_\kappa \propto a^{-2}H^{-2}$ and $\Omega_\sigma \propto a^{-6}H^{-2}$ quickly decay. Thus, neglecting quantum effects, the universe is left exponentially flat and isotropic as it approaches the big crunch. The inclusion of quantum effects superposes small fluctuations on this classical background.

From the higher-dimensional point of view, one of the scalar fields ϕ_i determines the distance between two branes of the size of a compact extra dimension. The potential then represents a conjectured attractive force between the end-of-the-world branes. The Ekpyrotic phase causes the branes to become very flat and parallel over large patches by diluting inhomogeneities in the brane curvature. Thus the homogeneity puzzle is also solved by the ekpyrotic phase.

On large scales, the power spectrum for $\mathcal{R}$ is given by

$$P_R(k) = \frac{k^3}{2\pi^2}\frac{|v|^2}{z^2} \propto k^{3-|\frac{\epsilon-3}{\epsilon-1}|}, \tag{3.8}$$

which means that the spectral index for $\mathcal{R}$ is

$$n_R - 1 = 3 - |\frac{\epsilon-3}{\epsilon-1}|. \tag{3.9}$$

In ekpyrotic models, ϵ is large, and thus the spectrum of the curvature perturbation $\mathcal{R}$ is extremely blue, meaning that there is much more power on smaller scales, in disagreement with observations.

3.3 New Ekpyrotic

A few works [39, 40] have been done to resolve the problem. One approach [39] is to introduce multiple fields. Curvature perturbations are converted from scale-invariant entropy perturbations that may be generated by scalar fields in a contracting universe. As a simplest example, two fields with negative exponential potential give as a phenomenology model describing two boundary branes with an attractive force

$$V = -V_1 e^{-\int c_1(\phi_1)d\phi_1} - V_2 e^{-\int c_2(\phi_2)d\phi_2} \tag{3.10}$$

where c_1, c_2, V_1, V_2 are positive. The slow roll parameter and trajectory angle $\dot{\theta}$ of the two fields model is defined as follows

$$\epsilon \equiv \frac{3}{2}(1+\omega) = \frac{\dot{\phi}_1^2 + \dot{\phi}_2}{2H^2} \tag{3.11}$$

$$\dot{\theta} = \frac{\dot{\phi}_2 V_{,\phi_1} - \dot{\phi}_1 V_{,\phi_2}}{\dot{\phi}_1^2 + \dot{\phi}_2^2} \tag{3.12}$$

For simplicity, we consider negative exponent potentials and tune the trajectory during contraction as a straight line.

$$V = -Ve^{-c\frac{\phi_1}{Mpl}} - \gamma^2 V e^{-\frac{c\phi_2}{Mpl\gamma}} \tag{3.13}$$

In a contracting universe, a growing mode is given by the entropy perturbation $\delta\sigma$, namely the relative fluctuation in the two fields ϕ_1 and ϕ_2, which is at linear order defined as follows

$$\delta\sigma \equiv \frac{\dot{\phi}_1 \delta\phi_2 - \dot{\phi}_2 \delta\phi_1}{\sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}} \tag{3.14}$$

The gauge-invariant entropy perturbation represents the perturbation orthogonal to the background scalar field trajectory. On large scales, at linear order the equation of motion of entropy field s is derived as

$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + V_{ss}\delta\sigma = 0\,, \tag{3.15}$$

where V_{ss} satisfies

$$V_{ss} = \frac{1}{\sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}}\left(\dot{\phi}_1^2 V_{,\phi_2\phi_2} + \dot{\phi}_2^2 V_{,\phi_1\phi_1} - 2\dot{\phi}_1\dot{\phi}_2 V_{,\phi_1\phi_2}\right)\,. \tag{3.16}$$

The power spectrum of the σ field which follows from (3.15) is nearly scale invariant.

$$n_\sigma - 1 = \frac{2}{\epsilon} - \frac{d\epsilon}{\epsilon^2 dN} = \frac{4(1+\gamma^2)}{c^2} \tag{3.17}$$

The mixing between curvature and entropy perturbations relies on a turning trajectory of the background fields. The converting happens after the Ekpyrotic contraction. At linear order, a non-zero entropy perturbation combined with a bending $\dot{\theta}$ can source the curvature perturbation on large scales

$$\dot{\zeta} \approx -\frac{1}{\sqrt{2\epsilon}}\delta s \tag{3.18}$$

For a sharp turning point, the large scale curvature perturbation is dramatically lifted and its spectrum obtains the features of the entropy perturbation, and a correct tilt is obtained.

In the main part of this thesis, we propose another Ekpyrosis model originating from key aspects of string theory to generate a nearly scale invariant power spectrum with an S-brane. However, before we introduce the model, the theory of cosmological perturbation is reviewed briefly in the next chapter.

Chapter 4

Theory of Cosmological Perturbation

The theory of cosmological perturbation plays a significant role in exploring early universe physics. In the context of General of Relativity and quantum field theory in curved spacetime, physicists calculate primordial fluctuations during the early universe and predict late-time observables. A lot of textbooks, lecture notes, and review articles have addressed the topic well and we take Refs. [17, 41–44] as recommendations.

4.1 Relativistic cosmological perturbation theory

4.1.1 Transformation of different gauges

We will work in the context of General Relativity as the theory of space-time and consider as matter source a scalar field. In this case we define perturbations around the homogeneous background solutions for the scalar field $\bar{\phi}(t)$ and the general first order perturbation in a spatial flat FRW metric $\bar{g}_{\mu\nu}(t)$,

$$\phi(t, \mathbf{x}) = \bar{\phi}(t) + \delta\phi(t, \mathbf{x}) \,, \qquad g_{\mu\nu}(t, \mathbf{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t, \mathbf{x}) \,, \tag{4.1}$$

where

$$\begin{aligned} ds^2 &= g_{\mu\nu}\, dx^\mu dx^\nu \\ &= -(1+2\Phi)dt^2 + 2aB_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j \,. \end{aligned} \tag{4.2}$$

In real space, the scalar-vector tensor (SVT) decomposition of the metric perturbations (4.2) is

$$B_i \equiv \partial_i B - S_i \,, \qquad \text{where} \quad \partial^i S_i = 0 \,, \tag{4.3}$$

and

$$E_{ij} \equiv 2\partial_{ij} E + 2\partial_{(i} F_{j)} + h_{ij} \,, \qquad \text{where} \quad \partial^i F_i = 0 \,, \quad h^i_i = \partial^i h_{ij} = 0 \,. \tag{4.4}$$

The SVT decomposition in real space corresponds to the distinct transformation properties of scalars, vectors and tensors on spatial hypersurfaces. The vector perturbations S_i and F_i decay with the expansion of the universe. For this reason we ignore vector perturbations here. Our focus will be on scalar and tensor fluctuations which are observed as density fluctuations and gravitational waves in the late universe.

The association between points in the background spacetime and the perturbed spacetime is via the coordinate system, which means there are many possible coordinate systems in the perturbed spacetime, all close to each other. A gauge transformation in General Relativity perturbation theory describes a coordinate transformation between such coordinate systems in the perturbed spacetime

Tensor fluctuations are gauge-invariant, which will be discussed in the following sections, but scalar fluctuations change under a change of coordinates. Consider the gauge transformation

$$t \rightarrow t + \alpha \tag{4.5}$$

$$x^i \rightarrow x^i + \delta^{ij}\beta_{,j}\,. \tag{4.6}$$

where α and β_j are small quantities.

Under these coordinate transformations the scalar metric perturbations transform as

$$\Phi \rightarrow \Phi - \dot{\alpha} \tag{4.7}$$

$$B \rightarrow B + a^{-1}\alpha - a\dot{\beta} \tag{4.8}$$

$$E \rightarrow E - \beta \tag{4.9}$$

$$\Psi \rightarrow \Psi + H\alpha\,. \tag{4.10}$$

We assume that in the early universe the energy density of the sclar field matter is the dominant contribution to the stress-energy of the universe, so that the scalar field perturbations $\delta\phi$ backreact on the spacetime geometry. This coupling between matter perturbations and metric perturbations is described by the Einstein Equations .

After the end of the early universe phase, the perturbations to the total stress-energy tensor of the universe can be modelled as that of a fluid

$$
\begin{aligned}
T^0_0 &= -(\bar{\rho} + \delta\rho) &(4.11)\\
T^0_i &= (\bar{\rho} + \bar{p})\, a v_i &(4.12)\\
T^i_0 &= -(\bar{\rho} + \bar{p})(v^i - B^i)/a &(4.13)\\
T^i_j &= \delta^i_j(\bar{p} + \delta p) + \Sigma^i_j\,. &(4.14)
\end{aligned}
$$

where ρ and p are energy density and pressure while v^i is the peculiar velocity and Σ^j_i is the anisotropic stress. The anisotropic stress Σ^i_j is gauge-invariant while the density, pressure and momentum density $((\delta q)_{,i} \equiv (\bar{\rho} + \bar{p})v_i)$ transform as follows

$$
\begin{aligned}
\delta\rho &\to \delta\rho - \dot{\bar{\rho}}\,\alpha &(4.15)\\
\delta p &\to \delta p - \dot{\bar{p}}\,\alpha &(4.16)\\
\delta q &\to \delta q + (\bar{\rho} + \bar{p})\,\alpha\,. &(4.17)
\end{aligned}
$$

4.1.2 Gauge-Invariant Variables

In Gauge-Invariant formalism, the gauge artifacts cancel within the gauge-invariant quantities. We can choose combinations of metric and matter perturbations [45] to construct gauge invariant variables. An important gauge-invariant scalar quantity is the curvature perturbation on uniform-density hypersurfaces

$$-\zeta \equiv \Psi + \frac{H}{\dot{\bar{\rho}}}\delta\rho\,. \tag{4.18}$$

where ζ is the spatial curvature of constant-density hypersurfaces. The variable ζ remains constant outside the Hubble radius for adiabatic matter perturbations that satisfy

$$\delta p - \frac{\dot{\bar{p}}}{\dot{\bar{\rho}}}\delta\rho = 0 \tag{4.19}$$

In the early universe models with single field, the condition (4.19) is always satisfied, so the perturbation $\zeta_{\mathbf{k}}$ does not evolve outside the horizon, $k \ll aH$.

Another gauge-invariant scalar is the comoving curvature perturbation

$$\mathcal{R} \equiv \Psi - \frac{H}{\bar{\rho} + \bar{p}}\delta q\,, \tag{4.20}$$

where δq is the scalar part of the 3-momentum density $T^0_i = \partial_i \delta q$. In the case of scalar field, during inflation $T^0_i = -\dot{\bar{\phi}}\,\partial_i\delta\phi$ and hence

$$\mathcal{R} = \Psi + \frac{H}{\dot{\bar{\phi}}}\delta\phi\,. \tag{4.21}$$

Geometrically, $\mathcal{R}$ measures the spatial curvature of comoving (or constant-ϕ) hypersurfaces.

The linearized Einstein equations relate ζ and $\mathcal{R}$ in the following

$$-\zeta = \mathcal{R} + \frac{k^2}{(aH)^2}\frac{2\bar{\rho}}{3(\bar{\rho}+\bar{p})}\Psi_{\rm B}\,, \tag{4.22}$$

where

$$\Psi_{\rm B} \equiv \psi + a^2 H(\dot{E} - B/a)\,, \tag{4.23}$$

is one of the Bardeen potentials [45]. ζ and $\mathcal{R}$ are therefore equal on super-Hubble scales. The correlation functions of ζ and $\mathcal{R}$ are therefore equal at Hubble radius crossing and both ζ and $\mathcal{R}$ are conserved on super-Hubble scales to linear order and in the absence of entropy fluctuations.

4.1.3 Examples of different gauges

Another method to treat gauge degrees of freedom is simply fixing the gauge. Once the gauge is fixed, pure gauge artifacts cannot appear. In the following section, we list different gauges used in cosmology as follows.

Newtonian Gauge

The Newtonian gauge reduces to Newtonian gravity in the small-scale limit with weak gravity field. It is defined by

$$B = E = 0\,, \tag{4.24}$$

and thus

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Psi)\delta_{ij}dx^i dx^j\,. \tag{4.25}$$

The linearized Einstein Equations are

$$\begin{aligned}
3H(\dot{\Psi} + H\Phi) + \frac{k^2}{a^2}\Psi &= -4\pi G\,\delta\rho & (4.26)\\
\dot{\Psi} + H\Phi &= -4\pi G\,\delta q & (4.27)\\
\ddot{\Psi} + 3H\dot{\Psi} + H\dot{\Phi} + (3H^2 + 2\dot{H})\Phi &= 4\pi G\left(\delta p - \frac{2}{3}k^2\delta\Sigma\right) & (4.28)\\
\frac{\Psi - \Phi}{a^2} &= 8\pi G\,\delta\Sigma\,. & (4.29)
\end{aligned}$$

and the continuity equations are

$$\begin{aligned}
\dot{\delta\rho} + 3H(\delta\rho + \delta p) &= \frac{k^2}{a^2}\delta q + 3(\bar{\rho} + \bar{p})\dot{\Psi}\,, & (4.30)\\
\dot{\delta q} + 3H\delta q &= -\delta p + \frac{2}{3}k^2\delta\Sigma - (\bar{\rho} + \bar{p})\Phi\,. & (4.31)
\end{aligned}$$

Comoving gauge

By setting the scalar momentum density as zero, the comoving gauge is defined by

$$\delta q = 0\,, \qquad E = 0\,. \tag{4.32}$$

It is also conventional to define curvature perturbation $\mathcal{R} \equiv -\Psi$ in this gauge. Unlike Newtonian gauge, we prefer describing the perturbation in the large scale. The linearized Einstein Equations are

$$\begin{aligned}
3H(-\dot{\mathcal{R}} + H\Phi) + \frac{k^2}{a^2}\left[-\mathcal{R} - aHB\right] &= -4\pi G\,\delta\rho & (4.33)\\
-\dot{\mathcal{R}} + H\Phi &= 0 & (4.34)\\
-\ddot{\mathcal{R}} - 3H\dot{\mathcal{R}} + H\dot{\Phi} + (3H^2 + 2\dot{H})\Phi &= 4\pi G\left(\delta p - \frac{2}{3}k^2\delta\Sigma\right) & (4.35)\\
(\partial_t + 3H)B/a + \frac{\mathcal{R} + \Phi}{a^2} &= -8\pi G\,\delta\Sigma\,. & (4.36)
\end{aligned}$$

The continuity equations are

$$\begin{aligned}
\dot{\delta\rho} + 3H(\delta\rho + \delta p) &= (\bar{\rho} + \bar{p})[-3\dot{\mathcal{R}} + k^2 B/a]\,. & (4.37)\\
0 &= -\delta p + \frac{2}{3}k^2\delta\Sigma - (\bar{\rho} + \bar{p})\Phi\,. & (4.38)
\end{aligned}$$

Equations (4.38) and (4.34) may be combined into

$$\Phi = \frac{-\delta p + \frac{2}{3}\Sigma}{\bar{\rho} + \bar{p}}\,, \qquad kB = \frac{4\pi G a^2 \delta\rho - k^2 \mathcal{R}}{aH}\,. \tag{4.39}$$

This comoving gauge is convenient to discuss the super-Hubble scale physics. Gravity effects accumulate while for adiabatic matter perturbation only, ζ is a conserved quantity on super-Hubble scale. This gauge provide a connection between the early universe physics and current observations.

Spatially-flat gauge

Another convenient gauge for computing cosmological perturbation is spatially-flat gauge

$$\Psi = E = 0\,. \tag{4.40}$$

The Einstein Equations are

$$\begin{aligned}
3H^2\Phi + \frac{k^2}{a^2}\left[-aHB)\right] &= -4\pi G\,\delta\rho && (4.41)\\
H\Phi &= -4\pi G\,\delta q && (4.42)\\
H\dot{\Phi} + (3H^2 + 2\dot{H})\Phi &= 4\pi G\left(\delta p - \frac{2}{3}k^2\delta\Sigma\right) && (4.43)\\
(\partial_t + 3H)B/a + \frac{\Phi}{a^2} &= -8\pi G\,\delta\Sigma\,. && (4.44)
\end{aligned}$$

The continuity equations are

$$\dot{\delta\rho} + 3H(\delta\rho + \delta p) = \frac{k^2}{a^2}\delta q + (\bar{\rho} + \bar{p})[k^2 B/a]\,, \tag{4.45}$$

$$\dot{\delta q} + 3H\delta q = -\delta p + \frac{2}{3}k^2\delta\Sigma - (\bar{\rho} + \bar{p})\Phi\,. \tag{4.46}$$

During the early universe phase driven by a scalar field ϕ, all scalar parts of perturbations are described by $\delta\phi$. Because on sub-Hubble scale but not as small as Planck scale, gravity is weak. Thus it is intuitive to think about gravity as decoupled.

In early universe, in most cases scalar fields contribute to the matter. In then next section, we will briefly go through the calculation of the curvature perturbation of a scalar field in quadratic order as an example.

4.2 Calculating the action of curvature perturbation of a scalar field in quadratic order

We consider models in which matter is described by a canonical scalar field ϕ minimally coupled to gravity

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,\left[M_{\rm pl}^2 R - (\nabla\phi)^2 - 2V(\phi)\right]\,, \tag{4.47}$$

in units where $M_{\rm pl}^{-2} \equiv 8\pi G$. We will study perturbations of this action due to fluctuations in the scalar field $\delta\phi(t, x^i) \equiv \phi(t, x^i) - \bar{\phi}(t)$ and the metric. We will treat metric fluctuations in the Arnowitt-Deser-Misner (ADM) formalism [18].

We consider a flat background metric

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j = a^2(\tau)(-d\tau^2 + \delta_{ij}dx^i dx^j)\,, \tag{4.48}$$

with scale factor $a(t)$ and Hubble parameter $H(t) \equiv \partial_t \ln a$ satisfying the Friedmann Equations

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \qquad \dot{H} = -\frac{1}{2}\dot{\phi}^2\,. \tag{4.49}$$

The scalar field satisfies the Klein-Gordon Equation

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0\,. \tag{4.50}$$

The standard slow-roll parameters are

$$\epsilon_{\rm v} = \frac{1}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \approx \frac{1}{2}\frac{\dot{\phi}^2}{M_{\rm pl}^2 H^2}\,, \qquad \eta_{\rm v} = \frac{V_{,\phi\phi}}{V} \approx -\frac{\ddot{\phi}}{H\dot{\phi}} + \frac{1}{2}\frac{\dot{\phi}^2}{M_{\rm pl}^2 H^2}\,. \tag{4.51}$$

We treat fluctuations in the ADM formalism [46] where spacetime is sliced into three-dimensional hypersurfaces

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\,. \tag{4.52}$$

where g_{ij} is the three-dimensional metric on slices of constant t. The lapse function $N(\mathbf{x})$ and the shift function $N_i(\mathbf{x})$ are non-dynamical Lagrange multipliers in the action. We will solve them as constrains. The action (4.47) becomes

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\Big[NR^{(3)} - 2NV + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot{\phi} - N^i\partial_i\phi)^2 - Ng^{ij}\partial_i\phi\partial_j\phi - 2V\Big]\,, \tag{4.53}$$

where

$$E_{ij} \equiv \frac{1}{2}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)\,, \qquad E = E^i_i\,. \tag{4.54}$$

E_{ij} is related to the extrinsic curvature of the three-dimensional spatial slices $K_{ij} = N^{-1}E_{ij}$.

We choose the following gauge for the dynamical fields g_{ij} and ϕ

$$\delta\phi = 0\,, \qquad g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]\,, \qquad \partial_i h_{ij} = h^i_i = 0\,. \tag{4.55}$$

In this gauge the matter field is unperturbed and all scalar degrees of freedom are parameterized by the metric fluctuation $\mathcal{R}(t,\mathbf{x})$.

4.2.1 Constraint Equations

The ADM action (4.53) implies the following constraint equations for the Lagrange multipliers N and N^i

$$\nabla_i[N^{-1}(E^i_j - \delta^i_j E)] = 0\,, \tag{4.56}$$

$$R^{(3)} - 2V - N^{-2}(E_{ij}E^{ij} - E^2) - N^{-2}\dot{\phi}^2 = 0\,. \tag{4.57}$$

To solve the constraints, we split the shift vector N_i into scalar and vector parts

$$N_i \equiv \psi_{,i} + \tilde{N}_i\,, \quad \text{where} \quad \tilde{N}_{i,i} = 0\,, \tag{4.58}$$

and define the lapse perturbation as

$$N \equiv 1 + \alpha\,. \tag{4.59}$$

The quantities α, ψ and $\tilde{N}_i$ then admit expansions in powers of $\mathcal{R}$,

$$\begin{aligned}\alpha =& \alpha_1 + \alpha_2 + \dots\,,\\ \psi =& \psi_1 + \psi_2 + \dots\,,\\ \tilde{N}_i =& \tilde{N}_i^{(1)} + \tilde{N}_i^{(2)} + \dots\,,\end{aligned} \tag{4.60}$$

where small parameters satisfy the relation $\alpha_n = \mathcal{O}(\mathcal{R}^n)$. The constraint equations may then be set to zero order-by-order.

We solve the constrains by orders. At first order (4.57) implies

$$\alpha_1 = \frac{\dot{\mathcal{R}}}{H}\,\partial^2 \tilde{N}_i^{(1)} = 0\,. \tag{4.61}$$

With an appropriate choice of boundary conditions one may set $\tilde{N}_i^{(1)} \equiv 0$. At first order Eqn. (4.56) implies

$$\psi_1 = -\frac{\mathcal{R}}{H} - \epsilon_{\mathrm{v}}\frac{a^2}{H}\,\partial^{-2}\dot{\mathcal{R}}\,, \tag{4.62}$$

where ∂^{-2} is defined via $\partial^{-2}(\partial^2\phi) = \phi$.

Substituting the first-order solutions for N and N_i back into the action, one finds the following second-order action

$$S_2 = \frac{1}{2}\int d^4x\, a^3\,\frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right] \tag{4.63}$$

We introduce the canonical variable v by $v \equiv z\mathcal{R}$ where $z \equiv \sqrt{2\epsilon}a$. The action then becomes

$$S_2 = \frac{1}{2}\int d\tau d^3x\left[v'^2 + (\partial_i v)^2 + \frac{z''}{z}v^2\right]. \tag{4.64}$$

Then we will use the mode functions from solving the equation of motion derived from (4.64) to quantize the perturbation of early universe.

4.3 Quantization

We promote the field v and its conjugate momentum v' to quantum operators

$$v \quad \rightarrow \quad \hat{v} = \int \frac{d\mathbf{k}^3}{(2\pi)^3} \left[v_k(\tau)\hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + v_k^*(\tau)\hat{a}_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}} \right] . \tag{4.65}$$

Alternatively, the Fourier components $v_{\mathbf{k}}$ are promoted to operators and expressed via the following decomposition

$$v_{\mathbf{k}} \quad \rightarrow \quad \hat{v}_{\mathbf{k}} = v_k(\tau)\hat{a}_{\mathbf{k}} + v_{-k}^*(\tau)\hat{a}_{-\mathbf{k}}^\dagger , \tag{4.66}$$

where the creation and annihilation operators $\hat{a}_{-\mathbf{k}}^\dagger$ and $\hat{a}_{\mathbf{k}}$ satisfy the canonical commutation relation

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] \ , \quad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \, , \tag{4.67}$$

To manifest the canonical commutation condition for the field operator $\hat{v}$, the mode functions are normalized as follows

$$v_k^{*\prime} v_k - v_k^* v_k' = i \, , \tag{4.68}$$

which provides one of the boundary conditions on the solutions. The second boundary conditions that fixes the mode functions completely comes from vacuum selection.

4.3.1 Boundary Conditions and Bunch-Davies Vacuum

The vacuum state for the fluctuations is defined as

$$\hat{a}_{\mathbf{k}}|0\rangle = 0\,, \tag{4.69}$$

which corresponds to specifying an additional boundary conditions for v_k. The standard choice is the Minkowski vacuum of a comoving observer in the far past, $\tau \to -\infty$ or $|k\tau| \gg 1$ or $k \gg aH$. In this limit the mode equation (4.64) becomes

$$v_k'' + k^2 v_k = 0\,. \tag{4.70}$$

For this case a unique solution exists if we require the vacuum to be the minimum energy state. Hence we impose the initial condition

$$\lim_{\tau \to -\infty} v_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\,. \tag{4.71}$$

The boundary conditions (4.68) and (4.71) completely fix the mode functions on all scales.

4.3.2 Solution in de Sitter Space

In a de Sitter limit $\varepsilon \to 0$, $\frac{z''}{z} = \frac{a''}{a} = \frac{2}{\tau^2}$, the mode equation is solved

$$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right) v_k = 0\,. \tag{4.72}$$

By linear combination of two independent solutions, we give solution to Eqn. (4.72) is

$$v_k = \alpha\, \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) + \beta\, \frac{e^{ik\tau}}{\sqrt{2k}} \left(1 + \frac{i}{k\tau}\right). \tag{4.73}$$

We may fix the free parameters α and β by considering the quantization condition (4.68) together with the choice of Bunch-Davies vacuum (4.71). This gives $\alpha = 1$, $\beta = 0$ and leads to the unique Bunch-Davies mode functions

$$v_k = \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right)\,. \tag{4.74}$$

4.3.3 Power Spectrum in Quasi-de Sitter

The de Sitter result for v allows us to compute the power spectrum of $\mathcal{R}$ at Hubble radius crossing, $a(t_\star)H(t_\star) = k$,

$$\langle \mathcal{R}_{\mathbf{k}}(t) \mathcal{R}_{\mathbf{k}'}(t) \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \frac{H_\star^2}{2k^3} \frac{H_\star^2}{\dot{\phi}_\star^2}\,. \tag{4.75}$$

Here, $(...)_\star$ indicates that a quantity is to be evaluated at Hubble radius. We define power spectrum $P^2_{\mathcal{R}}(k)$ by

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} P_{\mathcal{R}}(k) \,, \tag{4.76}$$

This gives

$$P_{\mathcal{R}}(k) = \frac{H^2_\star}{(2\pi)^2} \frac{H^2_\star}{\dot{\phi}^2_\star} = \frac{H^2}{8\pi^2 \epsilon_{\mathrm{v}} M_{\mathrm{pl}}} \,. \tag{4.77}$$

Since $\mathcal{R}$ is a constant on super-Hubble scales, the spectrum at Hubble radius crossing determines the future spectrum until a given fluctuation mode re-enters the Hubble radius.

The power spectrum does not depend on k at the lowest order. Thus the inflationary perturbations are nearly scale invariant. However, different modes exit the Hubble as slightly different times when $a_\star H_\star$ has a different value. This fact gives the correct result for the power spectrum during slow-roll inflation. It is important to consider the small non-scale-invariant correction to the power spectrum. This correction is called the tilt of the spectrum, or the spectral index n_s defined as

$$n_s - 1 \equiv \frac{d\ln \mathrm{P}_{\mathcal{R}}}{d\ln \mathrm{k}} = -6\epsilon_V + 2\eta_V \tag{4.78}$$

4.3.4 Tensor Mode

By expansion of the Einstein-Hilbert action one may obtain the second-order action for tensor fluctuations

$$S_h = \frac{M_{\rm pl}^2}{8}\int d\tau dx^3 a^2 \left[(h_{ij}')^2 - (\partial_l h_{ij})^2\right] . \tag{4.79}$$

where h_{ij} is a dimensionless tensor field.

We define the following Fourier expansion

$$h_{ij} = \int \frac{d^3k}{(2\pi)^3}\sum_{s=+,\times}\epsilon^s_{ij}(k)h^s_{\mathbf{k}}(\tau)e^{i\mathbf{k}\cdot\mathbf{x}}\,, \tag{4.80}$$

where $\epsilon_{ii} = k^i\epsilon_{ij} = 0$ and $\epsilon^s_{ij}(k)\epsilon^{s'}_{ij}(k) = 2\delta_{ss'}$. The tensor action (4.79) becomes

$$S_{(2)} = \sum_s \int d\tau d\mathbf{k}\frac{a^2}{4}M_{\rm pl}^2\left[h^s_{\mathbf{k}}{}'h^s_{\mathbf{k}}{}' - k^2 h^s_{\mathbf{k}}h^s_{\mathbf{k}}\right] . \tag{4.81}$$

After fields are canonically normalized as follows

$$v^s_{\mathbf{k}} \equiv \frac{a}{2}M_{\rm pl}h^s_{\mathbf{k}}\,, \tag{4.82}$$

the action of tensor perturbation in quadratic order is derived as

$$S_h = \sum_s \frac{1}{2}\int d\tau d^3\mathbf{k}\left[(v^s_{\mathbf{k}}{}')^2 - \left(k^2 - \frac{a''}{a}\right)(v^s_{\mathbf{k}})^2\right] , \tag{4.83}$$

where

$$\frac{a''}{a} = \frac{2}{\tau^2} \tag{4.84}$$

holds in de Sitter space.

Similarly we define the power spectrum of the tensor perturbation just like that of the scalars. Each polarization of the gravitational wave is therefore just a renormalized massless field in de Sitter space , the power spectrum for tensor perturbations with two polarizations,

$$P_{\mathrm{h}}^2 = \frac{2}{\pi^2}\frac{H_\star^2}{M_{\mathrm{pl}}^2}\,. \tag{4.85}$$

To compare with the scalar power spectrum, a tensor to scalar ratio is defined and calculated as

$$r = \frac{P_h}{P_{\mathcal{R}}} = \frac{8}{M_{\mathrm{pl}}^2}\left(\frac{d\phi}{dN}\right)^2 \approx 16\epsilon\,. \tag{4.86}$$

It is also interesting to note that, the tensor to scalar ratio is related to the distance that the inflaton rolled in field space during inflation. The inflaton's rolling distance per e-fold isThe total field evolution between the time when CMB fluctuations exited the Hubble radius. at N_{cmb} and the end of inflation at N_{end} can therefore be written as the following integral

$$\frac{\Delta\phi}{M_{\mathrm{pl}}} = \int_{N_{\mathrm{end}}}^{N_{\mathrm{cmb}}} dN\,\sqrt{\frac{r}{8}}\,. \tag{4.87}$$

During slow-roll evolution, $r(N)$ is approximately a constant, which may obtain the following approximate relation [47]

$$\frac{\Delta\phi}{M_{\rm pl}} \approx \mathcal{O}(1) \times \left(\frac{r}{0.01}\right)^{1/2} , \tag{4.88}$$

where $r(N_{\rm cmb})$ is the tensor-to-scalar ratio on CMB scales. In short, large tensor to scalar ratio r indicates a longer distance of inflaton's motion. This relation is known as the Lyth bound

In the following chapters we will apply the theory of cosmological fluctuations to our new proposed version of the Ekpyrotic Scenario.

Chapter 5

Nonsingular Ekpyrotic Cosmology with a Nearly Scale-Invariant Spectrum of Cosmological Perturbations and Gravitational Waves

We propose a mechanism borrowed from string theory which yields a non-singular transition from a phase of Ekpyrotic contraction to the expanding phase of Standard Big Bang cosmology. The same mechanism converts the initial vacuum spectrum of cosmological fluctuations before the bounce into a scale-invariant one, and also changes the spectrum of gravitational waves into an almost scale-invariant one. The scalar and tensor tilts are predicted to be the same, in contrast to the predictions from the "String Gas Cosmology" scenario. The amplitude of the gravitational wave spectrum depends on the ratio of the string scale to the Planck scale and may be in reach of upcoming experiments.

5.1 Introduction

The Inflationary Universe scenario [48–51] has become the standard paradigm of early universe cosmology. It is based on the assumption that there was a period of almost exponential expansion during a time period in the very early universe. Inflation provides a solution of the horizon and flatness problems of Standard Big Bang cosmology, and provides a causal mechanism for producing cosmological perturbations and microwave background anisotropies based on the assumption that all fluctuation modes start our in their vacuum state inside the Hubble radius at early times [52]. The spectrum of curvature perturbations is predicted to be almost scale-invariant, with a slight red tilt. Inflation also produces [53] an approximately scale-invariant spectrum of gravitational waves, again with a slight red tilt.

Inflation is usually obtained by working in the context of Einstein gravity and assuming that there is a new scalar field, the *inflaton* field φ, whose stress-energy tensor has an equation of state

$$w \equiv \frac{p}{\rho} \simeq -1 \tag{5.1}$$

(where p and ρ are pressure and energy density, respectively) which leads to accelerated expansion.

Recently, however, inflationary cosmology has come under some pressure. First of all, recall that in order for a scalar field φ to serve as an inflaton, its potential energy $V(\varphi)$ has to be very flat in order that the potential energy dominates over the kinetic energy [1]. However, general arguments from string theory lead to the *swampland constraint* [55] (see [21] for reviews)

$$\frac{V'}{V} > \frac{c}{m_{pl}} \tag{5.2}$$

for slowly rolling scalar fields whose energy density dominates the universe, where c is a constant of the order 1, m_{pl} is the four space-time dimensional Planck mass, and a prime indicates the derivative with respect to the field φ. Effective field theory models which violate this constraint are said to lie in the *swampland* and are not consistent with string theory [2]. Single scalar field models of slow-roll inflation are thus [22] in the swampland [3].

A second constraint on inflationary cosmology comes from the recently proposed *Trans-Planckian Censorship* conjecture (TCC) [24] which states that during cosmological evolution no scales whose wavelengths were smaller than the Planck length ever exit the

[1]Warm inflation [54] provides an avenue of relaxing this constraint.
[2]These considerations also have implications for scalar field models of Dark Energy [56]
[3]Once again, models of warm inflation avoid this constraint [57]

Hubble horizon. This conjecture can be viewed as analogous to Penrose's *Cosmic Censorship* hypothesis [58] which states that timelike singularities must be hidden by horizons. If the TCC is satisfied, then trans-Planckian modes are hidden from the classical region [4]. The TCC also shields the classical region of cosmology from non-unitarities associated with setting up quantum field theory in an expanding background [60]. Since during inflation the physical wavelength of fluctuation modes increases almost exponentially, while the Hubble radius remains almost constant, the TCC clearly provides severe constraints on inflationary cosmology. In [25] it was shown that, assuming that the post-inflationary cosmology is like in Standard Big Bang cosmology, and that the potential energy during inflation is approximately constant, the potential energy is constrained to obey the upper bound

$$V^{1/4} \, < \, 3 \times 10^9 \mathrm{Gev}\,, \tag{5.3}$$

which leads to an upper bound on the tensor-to-scalar ratio r of

$$r \, < \, 10^{-30}\,. \tag{5.4}$$

Even if these constraints are enforced, an initial condition problem remains [25].

In light of these constraints on inflationary cosmology it is interesting to re-consider some alternative early universe scenarios. Any viable alternative to inflation should produce

[4]Fluctuation modes oscillate on sub-Hubble scales, but become squeezed states and classicalize on super-Hubble scales [59] (see [42] for reviews of the theory of cosmological perturbations).

an approximately scale-invariant spectrum of almost adiabatic cosmological fluctuations on scales which are being observed today. In this case, as shown in the pioneering papers [61,62], acoustic oscillations in the angular power spectrum of the Cosmic Microwave Background (CMB) and baryon acoustic oscillations in the matter power spectrum will be generated. Two promising classes of alternative scenario (see e.g. [27] for a review and comparison of these alternatives) are *bouncing* and *emergent* cosmologies. In bouncing cosmologies (see e.g. [63, 64] for reviews) it is assumed that the universe begins in a contracting phase, and new physics produces a bounce which leads to the current expanding phase of Big Bang cosmology. Fluctuations are taken to be in their vacuum state in the far past. In the *emergent* scenario, it is assumed that the current phase of cosmological expansion starts after a phase transition from a novel state of space-time-matter. One example is *String Gas Cosmology* [28] where it is assumed that the early phase is a hot gas of fundamental superstrings near the critical temperature of string theory, and thermal fluctuations with holographic scaling in this hot gas lead to an almost scale-invariant spectrum of curvature fluctuations [30] with a slight red tilt and an almost scale-invariant spectrum of gravitational waves [65] with a slight blue tilt. Provided that the energy scale of the bounce or of the emergent phase is lower than the Planck scale, there are no constraints on the scenarios resulting from the TCC. Note that in bouncing and emergent scenarios the horizon problem is trivially solved - the causal horizon is infinite, and there is hence in principle no problem in explaining the near isotropy of the CMB.

Among bouncing scenarios, the *Ekpyrotic Scenario* [4, 66] (see also [37, 38, 67] for a cyclic version) has a number of attractive features. The Ekpyrotic scenario assumes that the contracting phase has an equation of state parameter

$$w \gg 1 \,. \tag{5.5}$$

This can be realized if matter is given by a scalar field φ with negative exponential potential

$$V(\varphi) \;=\; -V_0 e^{-\sqrt{2/p}\varphi/m_{pl}} \tag{5.6}$$

with $V_0 > 0$ and $0 < p \ll 1$, and assuming that φ begins at positive values with positive total energy density. In this case, the scale factor evolves as

$$a(t) \;\sim (-t)^p \tag{5.7}$$

(note that t is negative in the contracting phase) and

$$w \;\simeq\; \frac{4}{3p}\,. \tag{5.8}$$

Note that negative exponential potentials arise rather generically in string compactifications (see e.g. [68] for a review). The initial Ekpyrotic model was in fact based on heterotic

M-theory [69, 70] (see also [71]) [5].

A nice feature of the Ekpyrotic scenario is that anisotropies are diluted in the phase of contraction [36], unlike what happens in a symmetric bounce where the anisotropies blow up [74]. A further nice feature is that the homogeneous contracting trajectory is a local attractor in initial condition space. This feature is shared by models of large field inflation [75–79], but not models of small field inflation [80]. As in inflationary cosmology, spatial curvature is diluted. Hence, the Ekpyrotic scenario also solves the flatness problem of Big Bang cosmology [6].

The Ekpyrotic scenario faces two main challenges. The first is how to obtain a non-singular bounce from the early contracting phase to the late time expanding phase of Standard Big Bang cosmology. The second is how to obtain a roughly scale-invariant spectrum of curvature fluctuations. It can be shown that the adiabatic curvature fluctuations in a phase of Ekpyrotic contraction retain a nearly vacuum spectrum [82, 83] in spite of the fact that the spectrum of fluctuations of the scalar field φ obtains a scale-invariant spectrum [84, 85]. It is possible to obtain a scale-invariant spectrum of curvature fluctuations making use of en entropy field which acquires a scale-invariant spectrum [33, 39, 86–91], and converting the entropy fluctuations to curvature perturbations [7]. It has also been shown in [93, 94] that non-trivial matching conditions of fluctuations on

[5] For newer versions of the Ekpyrotic scenario see e.g. [72, 73]. Our discussion, however, will be based on the original scenario,

[6] Note that some of these features are shared by the Pre-Big-Bang scenario [81] which is based on a phase of contraction with $w = 1$.

[7] An almost scale-invariant spectrum can also be obtained [40, 92] by making use of a rapidly changing

the space-like surface separating the contracting phase from the expanding one can convert the scale-invariant spectrum of φ to that of curvature perturbations [8]. The spectrum of gravitational waves, on the other hand, remains close to vacuum, and hence a negligible amplitude of such waves on cosmological scales is predicted.

In this paper we suggest a way of simultaneously obtaining a cosmological bounce and obtaining a scale-invariant spectrum of both curvature fluctuations and gravitational waves [9]. Our mechanism is based on the fact that (in the context of string theory), at the string scale, enhanced symmetries in the low energy effective action are expected to appear (see e.g. [101, 102]): a tower of string states which has string scale mass in a Minkowski space-time background becomes massless and has to be included in the low energy effective action [10]. In the low energy effective action, there is thus an extra term which appears at the time $t = t_B$ when the density reaches the string scale density. It is a delta function term localized on the space-like hypersurface $t = t_B$ (as will be discussed in Section 3 we are working in the uniform density gauge), and we hence call this term an *S brane*. In the same way that a D-brane has negative pressure (equals positive tension) in the spatial directions along the brane and vanishing pressure in the normal direction, an S-brane has vanishing energy density and negative pressure. This is discussed in detail in [103, 104] in the context

equation of state before the phase of Ekpyrotic contraction.

[8]In the case of a singular bounce between an Ekpyrotic contracting phase and an expanding phase the transfer of fluctuations was studied in [95] (see also [96, 97] for a study of the transfer in a holographic cosmology setup.)

[9]See [98–100] for attempts at obtaining a nonsingular Ekpyrotic bounce using a cubic Galileon Lagrangian.

[10]This is the same physics discussed under the name *distance conjecture* in the recent superstring literature [55]

of a specific string model with thermal duality in the Euclidean temporal direction. The S-brane hence yields a contribution to the effective energy-momentum tensor which violates the Null Energy Condition and hence, as shown in [103, 104] can meditate a transition from contraction to expansion. As we show here, the effect of the S-brane on the cosmological perturbations and gravitational waves converts initial vacuum fluctuations before the bounce to scale-invariant ones after the bounce.

In the following section we discuss the origin of the S-brane and how this object mediates the transition between contraction and expansion. Then, in Section 3 we study the coupling of cosmological perturbations and gravitational waves to the S-brane and show that the slope of the power spectrum of curvature perturbations changes by a factor of k^{-2}, k being comoving momentum. Thus, the vacuum power spectrum with a small red tilt δn produced during the Ekpyrotic phase of contraction is converted into a scale-invariant one with the same small red tilt δn. The spectrum of gravitational is enhanced by the same mechanism when passing through the bounce. Thus, unlike in the other approaches to Ekpyrotic cosmology [33, 39, 86–91], we obtain a roughly scale-invariant spectrum of gravitational waves [11].

Our discussion is at the level of an effective field theory, but we have in mind a setting coming from string theory. We will work in units where the speed of light and the Planck and Boltzmann constants are set to 1. Space-time indices are denoted by Greek symbols,

[11] Note that the anamophic scenario of [72] also produces an approximately scale-invariant spectrum of gravitational waves.

spatial indices by lower case Latin ones. The cosmological scale factor is denoted by $a(t)$, and $H(t)$ is the Hubble expansion rate. G is Newton's gravitational constant, related to the reduced Planck mass m_{pl} via $8\pi G = m_{pl}^{-2}$. Since spatial curvature is not important in the early universe we will set it to zero.

5.2 S-Brane and Nonsingular Bounce

We are working with a four-dimensional effective action S of the form

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\Big[R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\Big] \\ &\quad -\int d^4x\kappa\delta(\tau-\tau_B)\sqrt{\gamma}\,, \end{aligned} \tag{5.9}$$

where R is the Ricci scalar of the four-dimensional space-time metric $g_{\mu\nu}$ with determinant g, φ is the scalar field with negative exponential potential $V(\varphi)$ (7.1), γ_{ij} is the induced metric on the hypersurface $t = t_B$ with determiant γ, and κ is the tension of the S-brane. One of the conditions on our coordinates (this will be important when studying fluctuations in the next section) is that the constant t surfaces correspond to constant density. The time t_B is the time when the density reaches the critical value when the extra tower of string states becomes massless and when hence the S-brane appears.

As discussed in detail in [103, 104], the S-brane induces a localized stress-energy tensor with energy density ρ_B and pressure p_B given by

$$\rho_B = 0\,, \tag{5.10}$$

$$p_B = -\kappa\delta(t-t_B)\,. \tag{5.11}$$

Integrating the Friedmann equations across the bounce time t_B yields the following change of the Hubble constant:

$$\begin{aligned}\delta H &\equiv \lim_{\epsilon\to 0} H(t_B+\epsilon) - H(t_B-\epsilon) \\ &= 4\pi G\kappa\,.\end{aligned} \tag{5.12}$$

Hence, provided that the energy density just before the bounce obeys the constraint

$$\rho(t_B)^{1/2} < \frac{\sqrt{3}}{2} m_{pl}^{-1}\kappa\,, \tag{5.13}$$

then the S-brane will induce a cosmological bounce. We expect the S-brane to appear at the string energy scale η_s, and hence $\rho(t_B) \sim \eta_s^4$. We expect κ to be given by

$$\kappa \sim N\eta_s^3\,, \tag{5.14}$$

where N is an integer given by the number of string states which become massless at the enhanced symmetry point (where the brane appears). Hence, provided that the $N\eta_s$ is larger than the Planck mass, the S-brane will induce a cosmological bounce.

We will assume that small-scale interactions between the enhanced symmetry states and the degrees of freedom of the Standard Model lead to a bath of radiation after the bounce. This is the analog of the reheating process at the end of inflation. With this assumption, the bounce will induce a transition between an Ekpyrotic phase of contraction and a radiation phase of expansion after the bounce.

5.3 Fluctuations Passing Through the S-Brane Bounce

Let us now consider how metric fluctuations couple to the S-brane. In this section it is convenient to use conformal time τ in terms of which the homogeneous and isotropic background metric takes the form

$$ds^2 \,=\, a^2(\tau)\Big(d\tau^2 - d\mathbf{x}^2\Big)\,. \tag{5.15}$$

We will consider scalar metric fluctuations and gravitational waves separately [12]. The metric for scalar fluctuations takes the form (see e.g. [42, 105] for reviews of the theory of

[12]We postpone the discussion of vector modes to a later study.

cosmological perturbations)

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & -B_{,i} \\ -B_{,i} & (1+2\Psi)\delta_{ij}+E_{,ij} \end{pmatrix}, \tag{5.16}$$

where the fluctuation variables Φ, Ψ, B and E are functions of space and time. The scalar field is

$$\varphi(\tau,\mathbf{x}) = \varphi_0(\tau)+\delta\varphi(\mathbf{x},\tau)\,, \tag{5.17}$$

where φ_0 is the background scalar field and $\delta\varphi$ is the field fluctuation.

Not all of the fluctuation variables are independent. We will work in comoving gauge $\delta\varphi = 0$ in which the scalar field energy density is constant on constant time surfaces (on large scales where the spatial gradient energy is negligible). We can impose a second gauge condition and choose it to be $E = 0$ for computational ease.

We wish to obtain the effects of the S-brane on the equation of motion for the fluctuations. To this end, we insert the above ansatz for the perturbated metric and perturbed matter into the full action (7.3) and expand to second order in the fluctuation variables. Note that the terms linear in the fluctuations vanish if the background satisfies the background equations of motion. The contribution of the bulk term in the action to the second order action for scalar fluctuations is

$$S^{(2)} = \frac{1}{2}\int d^4x\Big[v'^2 - v_{,i}v_{,i}+\frac{z''}{z}v^2\Big]\,, \tag{5.18}$$

where v is the Mukhanov-Sasaki variable [106, 107] which is given by

$$v = z\zeta \,, \tag{5.19}$$

where ζ is the curvature fluctuation in comoving gauge, and

$$z(\tau) \propto a(\tau) \tag{5.20}$$

if the equation of state of the background is time-independent. A prime in the above equations denotes a derivative with respect to conformal time.

During the phase of Ekpyrotic contraction we have (see (5.7))

$$\tau(t) = -\frac{1}{1-p}(-t)^{1-p} \,, \tag{5.21}$$

and hence

$$z(\tau) \sim \tau^{p/(1-p)} \,, \tag{5.22}$$

from which it follows that

$$\frac{z''}{z} = \frac{p(2p-1)}{(1-p)^2}\tau^{-2} \,, \tag{5.23}$$

which implies that an initial vacuum spectrum remains almost vaccum on super-Hubble scales, acquiring only a small red tilt proportional to p. After the bounce in the radiation phase of expansion we have

$$\frac{z''}{z} \;=\; 0\,, \tag{5.24}$$

as is well known.

In the gauge we are using the induced metric γ_{ij} is

$$\gamma_{ij} \;=\; a^2(1+2\Psi)\delta_{ij}\,, \tag{5.25}$$

and hence the contribution $S_B^{(2)}$ of the brane term in the action to quadratic order in Ψ is

$$S_B^{(2)} \;=\; \int d^4x\kappa a^3(1+3\Psi+\frac{3}{2}\Psi^2)\delta(\tau - t\tau_B)\,. \tag{5.26}$$

Note that the sum of the terms linear in the fluctuations cancel since we are expanding about a solution of the background equations.

In the gauge we are using Ψ is proportional to the Sasaki-Mukhanov variable v:

$$\Psi \;=\; z^{-1}v\,, \tag{5.27}$$

where z is a function of the background cosmology which depends both on the geometry and on the matter (its form will be discussed below), and hence the brane contribution to the second order action can be re-written as

$$S_B^{(2)} = \frac{1}{2}\int d^4x \frac{a^3}{z^2} 3\kappa\delta(\tau-\tau_B)v^2 \,, \tag{5.28}$$

and thus the full second order action for v is

$$S^{(2)} = \frac{1}{2}\int d^4x \Big[v'^2 - v_{,i}v_{,i} + \Big(\frac{z''}{z} + 3\frac{a^3}{z^2}\kappa\delta(\tau-\tau_B)\Big)v^2\Big] \,, \tag{5.29}$$

and the resulting equation of motion for v is

$$v'' + \Big[k^2 - \frac{z''}{z} - 3\frac{a^3}{z^2}\kappa\delta(\tau-\tau_B)\Big]v = 0 \,. \tag{5.30}$$

Since $z'' = 0$ after the bounce and z''/z is proportional to p and hence very small before the bounce, we can to first approximation neglect this term.

As shown in the Appendix, the δ function contribution to the mass in the equation of motion (6.15) leads to an enhancement of the mode functions by a factor

$$\beta_k = \frac{m}{k} \,, \tag{5.31}$$

where m is the coefficient of the delta function. From (6.15) it follows that

$$\beta_k = 3\kappa a\Big(\frac{a}{z}\Big)^2(\tau_B)k^{-1} \,. \tag{5.32}$$

This factor leads to a conversion of the vacuum power spectrum of v before the bounce to a scale-invariant one after the bounce. This is the main result of our analysis. If the spectrum before the bounce is a vacuum spectrum modulated by a slight red tilt (as it is in the case of Ekpyrotic contraction), the power spectrum after the bounce will be a scale-invariant one modulated by the same red tilt.

Let us now study the coupling of gravitational waves to the S-brane. If we consider a gravitational wave travelling in the x direction, then the induced metric γ_{ij} is

$$\gamma_{ij} = a^2(\tau) \begin{pmatrix} 1 & 0 \\ 0 & 1 + h\epsilon_{ab} \end{pmatrix} , \tag{5.33}$$

where ϵ_{ab} is the polarization tensor of gravitational waves in the y/z plane. Hence, to leading order in the amplitude h^2

$$\sqrt{\gamma} = a^3 \Big(1 - \frac{1}{2} h^2\Big) . \tag{5.34}$$

The canonical variable for gravitational waves is

$$u \equiv ahm_{pl} , \tag{5.35}$$

where the factor of m_{pl} is important to have the right dimensions. The bulk action for u is

$$S^{(2)} = \frac{1}{2} \int d^4x \Big[u'^2 - u_{,i} u_{,i} + \frac{a''}{a} u^2 \Big] , \tag{5.36}$$

and the brane contribution is

$$S_B^{(2)} = \frac{1}{2}\int d^4x\kappa a\delta(\tau-\tau_B)u^2 m_{pl}^{-2}\,, \tag{5.37}$$

Hence, the equation of motion for u_k becomes

$$u'' + \Big[k^2 - \frac{a''}{a} - \kappa a m_{pl}^{-2}\delta(\tau-\tau_B)\Big]u = 0\,. \tag{5.38}$$

This equation is analogous to the equation for scalar modes, except that the coefficient of the delta function term is different. In analogy with what was shown above for the scalar modes, we find that the tensor modes are enhanced by the factor

$$\beta_k = \kappa a m_{pl}^{-2}k^{-1}\,. \tag{5.39}$$

Our results (5.32) and (5.39) show that the passage through the S-brane leads to the conversion of an initial vacuum spectrum for the scalars and tensors to a scale-invariant one for both scalars and tensors. This is very different from what is obtained in previous realizations of Ekpyrotic cosmology where the tensors retain their vacuum form.

The tensor to scalar ratio r can be read off of the amplitude ratio of the Bogoliubov coefficients β_k. To evaluate this ratio, recall [42, 105] that

$$z(t) \;=\; a(t)\frac{\dot{\varphi}_0}{H}\,, \tag{5.40}$$

where $\varphi_0(t)$ is the background scalar field. For the Ekpyrotic potential of (7.1), this background is given by

$$\varphi_0(t) \;=\; \sqrt{2p}m_{pl}\log(-\frac{\sqrt{V_0}}{m_{pl}\sqrt{p(1-p)}}t)\,, \tag{5.41}$$

and hence

$$\frac{\dot{\varphi}_0}{H} \;=\sqrt{2/p}m_{pl}\,. \tag{5.42}$$

Hence, comparing (5.39) and (5.32) and making use of (5.42) we find that the tensor to scalar ratio r becomes

$$r \;= 16\pi\Big(\frac{72}{p^3}\Big) \;\sim\; \frac{10^3}{p^3}\,. \tag{5.43}$$

The factor of 16π stems from the different normalizations of the scalar and tensor power spectrum, the other term is the square of the ratio of the Bogoliubov coefficients (5.39) to (5.32).

We find that the amplitude of the tensor spectrum is larger than the amplitude of the scalar spectrum, a result which is obviously inconsistent with observations. This is related to the fact that the analog of the inflationary slow-roll parameter ϵ is proportional to $1/p \gg 1$

in the case of Ekpyrotic contraction. Hence, in our model we need to invoke a separate mechanism to boost the scalar modes, as considered in previous work on the Ekpyrotic scenario [33, 39, 86–91, 93, 94]. This will be explored in an upcoming paper.

Let us now consider the corrections to the spectrum due to the z''/z term in the mode equation. Since

$$\frac{z''}{z} = -\frac{p(1-2p)}{(1-p)^2}\frac{1}{\tau^2} \equiv -\alpha(p)\frac{1}{\tau^2}\,, \tag{5.44}$$

the dominant mode in for the v and u modes on super-Hubble wavelengths scales as

$$v(\tau) \sim \tau^{\alpha} \tag{5.45}$$

and similarly for u. Hence we find for the Fourier modes of v and u on super-Hubble lengths

$$v_k(\tau) \simeq v_{k,0}\Big(\frac{\tau}{\tau_H(k)}\Big)^{\alpha}\,, \tag{5.46}$$

where $\tau_H(k)$ is the time of Hubble radius crossing of the k'th mode and is given by

$$\tau_H(k) = \frac{p}{(1-p)}k^{-1}\,. \tag{5.47}$$

If we use vacuum initial conditions then

$$v_{k,0} = \frac{1}{\sqrt{2k}}\,. \tag{5.48}$$

The dimensionless power spectrum is given by

$$P_v(k) \;=\; k^3|v_k|^2\,. \tag{5.49}$$

According to the definition of the scalar tilt n_s we have

$$P(k) \;\sim\; k^{n_s-1}\,. \tag{5.50}$$

Combining (5.46, 5.47, 5.48, 5.49) and (5.50) we find that the tilt of the scalar modes before the bounce is given by

$$n_s \;=\; 3+2\alpha \;\simeq\; 3+2p\,. \tag{5.51}$$

Since the passage through the S-brane changes the spectral tilt by -2, the tilt after the bounce is

$$n_s-1 \;=\; 2\alpha \;\simeq\; 2p\,. \tag{5.52}$$

Thus, we predict a blue tilt of the scalar spectrum with the deviation of the tilt from that of scale-invariance whose magnitude is $2p$. The tensor spectrum has the same tilt. Note that the tensor index n_t is defined via

$$P(k) \;\sim\; k^{n_t}\,. \tag{5.53}$$

Thus, the spectrum aquires a slight blue tilt.

Our new Ekpyrotic scenario hence predicts roughly scale invariant scalar and tensor spectra with tilts which obey the consistency relation

$$n_t \;=\; n_s - 1\,. \tag{5.54}$$

The amplitude of the spectrum of gravitational waves after the bounce can be found from (5.39). The power spectrum of the canonical variable u is

$$P_u(k,\tau) \;\simeq\; a^2\kappa^2 m_{pl}^{-4}\Big(k\tau_B\Big)^{-p/2}\,, \tag{5.55}$$

and hence the power spectrum of gravitational waves becomes

$$P_h(k,\tau) \;\simeq\; \kappa^2 m_{pl}^{-6}\Big(k\tau_B\Big)^{-p/2}\,, \tag{5.56}$$

Recall from Section 2 that the tension κ is expected to be given by the string scale η_s, and thus the amplitude $\mathcal{A}$ of the power spectrum (dropping the last factor above which represents the small red tilt) will be

$$\mathcal{A} \;\simeq\; N^2\Big(\frac{\eta_s}{m_{pl}}\Big)^6\,. \tag{5.57}$$

If the string scale is the one preferred by particle physics considerations in the early textbook

[108, 109] on string theory, namely $\eta_s \sim 10^{17}$GeV, then the amplitude is

$$\mathcal{A} \sim 10^{-12}\,, \tag{5.58}$$

which, using the observed amplitude of the scalar spectrum, corresponds to

$$r \sim 10^{-3}\,. \tag{5.59}$$

Note, however, that this valuie depends sensitively on the ratio between the string scale and the Planck scale.

5.4 Conclusions and Discussion

It is generally expected that at string energy density scales a new tower of string states becomes massless. If this is the case, this tower of states has to be included in the low energy effective action for cosmological evolution. It will appear as a term in the effective action localized at a particular time, i.e. as an S-brane. This object has zero energy density and negative pressure, and can hence induce the transition from contraction to expansion.

We have used this S-brane construction to provide a new realization of the Ekpyrotic scenario. We have shown that the coupling of the S-brane to cosmological fluctuations and in particular to gravitational waves leads to a change in the spectral index of super-Hubble cosmological perturbations of $\delta n_s = -2$. This converts a vacuum spectrum into a scale-invariant one. Our minimal construction leads to a larger amplitude for the tensor modes than the scalar modes, and hence has to be supplemented by a separate source of cosmological perturbations. A natural way to obtain this will be presented in a followup paper.

The amplitude of the power spectrum of gravitational waves depends on the ratio of the string scale η_s and Planck scale m_{pl} (see (5.57)). The tilt of the gravitational wave spectrum is predicted to be the same as the tilt of the scalar spectrum. Both tilts are blue.

It is interesting to compare these predictions with those obtained in *String Gas Cosmology* [28], an emergent universe scenario motivated by string theory. String Gas Cosmology (SGC) also yields a roughly scale-invariant spectrum of scalar and tensor modes, and the amplitude of the power spectrum of cosmological perturbations is given by a combination of the string scale and the Planck scale, but in this case the fourth power and not the sixth power as here (see e.g. [29, 110] for detailed discussions). SGC yields two consistency relations between the four basic cosmological observables. In particular, the tensor tilt is $n_t = 1 - n_s$. The predicted tensor tilt is blue, as it is predicted to be in the scenario studied here. The scalar tilt, however, is red.

In this paper we have assumed that the state after the bounce is that of radiation, in a similar way that the state after reheating in inflation is that of radiation. The explicit model of ultraviolet physics that yields the production of radiation across the bounce remains to be explored. In future work we plan to study the amplitude of non-Gaussianities generated in our scenario.

Appendix

Let us consider the equation of motion

$$X''_k(\tau) + \Big[k^2 + m\delta(\tau - \tau_B)\Big] X_k(\tau) \;=\; 0\,. \tag{5.60}$$

The solutions are plane waves for $\tau < \tau_B$ and for $\tau > \tau_B$. If we denote the positive frequency solutions as f_k and the negative frequency ones as f_k^*, where the $*$ indicates complex conjugation, then the solution which is pure positive frequency before τ_B can be written for $\tau > \tau_B$ as

$$X_k \;=\; \alpha_k f_k + \beta_k f_k^*\,, \tag{5.61}$$

where α_k and β_k are the Bogoliubov mode matching coefficients which obey the relationship

$$|\alpha_k|^2 - |\beta_k|^2 \;=\; 1\,. \tag{5.62}$$

By integrating the equation (5.60) over time τ against a test function (a smooth function which decays exponentially at $\tau \to \pm\infty$) $f(\tau)$ it can be easily shown that

$$\beta_k \;=\; \frac{m}{k}\,. \tag{5.63}$$

This result implies that the power spectrum of X_k is boosted by a factor of $(m/k)^2$. This turns a vacuum spectrum into a scale-invariant one.

Chapter 6

Ekpyrotic Cosmology with a Zero-Shear S-Brane

In a recent paper [111] we proposed a mechanism for a continuous transition between a contracting Ekpyrotic phase and the Standard Big Bang phase of expansion: the bounce is generated by an S-brane which represents the effects of higher mass string states in the low energy effective field theory. We showed that gravitational waves on cosmological scales obtain a nearly scale-invariant spectrum. Here, we study the cosmological fluctuations in this setup, assuming that the S-brane has zero shear. We find a nearly scale-invariant spectrum of cosmological perturbations with a slight red tilt. The scenario yields two consistency relations for cosmological observations, the first one relating the tensor to scalar ratio with the scalar spectral tilt, the second relating the tensor tilt to the scalar tilt. The predicted tensor to scalar ratio is within the reach of upcoming CMB observations. The tensor tilt is blue.

6.1 Introduction

In light of the recent challenges for inflationary cosmology (both the inability to embed canonical single field slow roll inflation into string theory as a consequence of the *swampland criteria* (see [19, 22, 55] for original articles and [21, 112] for reviews) and the *Trans-Planckian Censorship Conjecture* [24, 25]) it is of great interest to explore possible alternatives to inflation and further develop their predictions for cosmological observations.

The Ekpyrotic scenario [4, 66] is a promising alternative to cosmological inflation for explaining the isotropy of the microwave background and the spatial flatness of today's universe. The scenario is based on the assumption that the universe begins in a phase of slow contraction. In the context of Einstein gravity such a slow contracting phase can be generated by a scalar matter field φ with a negative exponential potential with the exponent chosen such that the homogeneous scalar field trajectory has an equation of state paramter $w \gg 1$, where w is the ratio between the pressure and energy density, and the scale factor evolves as

$$a(t) \sim (-t)^p \quad (t < 0) \tag{6.1}$$

with [1]

$$p \ll 1\,. \tag{6.2}$$

In terms of conformal time τ the scale factor evolves as

$$a(\tau) \sim (-\tau)^q \quad (\tau < 0) \tag{6.3}$$

with

$$q = \frac{p}{1-p}\,. \tag{6.4}$$

A nice feature of the Ekpyrotic scenario is the fact that anisotropies and initial inhomogeneities are diluted in the contracting phase. Thus, the homogeneous and isotropic contracting phase space trajectory is an attractor in initial condition space [36].

The main challenge for the Ekpyrotic scenario has been to obtain a well-controlled transition from the contracting Ekpyrotic phase to the expanding phase which is postulated to evolve as in Standard Big Bang cosmology, i.e. beginning with a radiation-dominated phase after the transition [2] . A related challenge has been to obtain a robust computation of the spectrum of cosmological perturbations and gravitational waves, assuming that these fluctuations begin early in the contracting phase in their vacuum state.

[1] The Pre-Big-Bang scenario of [81] is the case $p = 1$ and can be obtained in the context of dilaton gravity.

[2] In the original paper [4, 66] it was argued that the transition would be singular from the point of view of an effective field theory coupled to Einstein gravity. More recently, however, there have been a lot of attempts to realize the Ekpyrotic scenario in the context of a non-singular effective field theory [39, 88, 89]. Note that violations of the Null Energy Condition are required in order to obtain a non-singular bounce.

In a recent paper, we [111] have proposed a mechanism which yields a nonsingular transition between contraction and expansion. We argued that, in the context of string theory, at the string density new stringy degrees of freedom must be included in the effective action for the background and fluctuations. These terms yield an *S-brane*, a space-like hypersurface which is reached when the background density hits the string density. Since such an S-brane has vanishing energy density and negative pressure, it yields the violation of the energy conditions required to obtain a transition between contraction and expansion.

The second main result of [111] was the demonstration that, on super-Hubble scales, gravitational modes passing through the S-brane experience an enhancement of their amplitude proportional to k^{-1}, where k is the comoving wavenumber. This amplification is precisely the correct one required to turn an initial vacuum spectrum into a scale-invariant one. The same mechanism also changes the spectrum of the canonical curvature fluctuation variable v, but the amplitude of the enhancement was found to be smaller than the amplitude of the enhancement of the gravitational waves. It this were the only amplification mechanism of the scalar spectrum, an unacceptably large tensor to scalar ratio would result.

However, the evolution of the scalar metric fluctuations is more complicated than that of the tensor modes. The precise specification of the location of the S-brane is crucial. Here, we demontrate that, if the S-brane has no shear, then the growing mode of the scalar field fluctuation $\delta\varphi$ seeds an approximately scale-invariant spectrum of curvature fluctuations in the expanding phase. Combined with the results of [111] for the gravitational wave spectrum we obtain two consistency relations for current observables, the first relating the tensor to scalar ratio r with the scalar tilt n_s, the second relating the tensor tilt n_t to the scalar tilt. The first relation is

$$r = \mathcal{B}(1 - n_s)^2 \,, \tag{6.5}$$

where $\mathcal{B}$ is a constant of the order one, and the second relation is

$$n_t = (1 - n_s) \,. \tag{6.6}$$

Since the tilt of the scalar spectrum is red, (6.6) represents a blue tilt of the tensor spectrum, like [28, 65] in the case of String Gas Cosmology [113, 114].

In the folliowing section we discuss the computation of scalar metric fluctuations in Ekpyrotic cosmology, reviewing known results and emphasizing the role of the specific location of the matching surface between the contracting and expanding phases. In Section 3 we review the S-brane scenario introduced in [111]. Section 4 provides a summary of our calculations, and in Section 5 we discuss some implications of our results. We assume a spatially flat homogeneous and isotropic background space-time with scale factor $a(t)$, where t is physical time. It is often convenient to use conformal time τ determined by $dt = a(t)d\tau$. The Hubble expansion rate is $H \equiv \dot{a}/a$, and its inverse is the Hubble radius, the length scale which plays a key role in the evolution of cosmological fluctuations. We use units in which the speed of light and Planck's constant are set to 1. Comoving spatial coordinates are denoted by $\mathbf{x}$, and the corresponding comoving momentum vector is $\mathbf{k}$ (its magnitude is written as k). The reduced Planck mass is denoted by m_{pl} and it is related to Newton's gravitational constant G via $m_{pl}^{-2} = 8\pi G$.

6.2 Curvature Fluctuations in Ekpyrotic Cosmology

During a phase of Ekpyrotic contraction ($\tau < 0$), fluctuations of the scalar field φ which start as vacuum perturbations in the far past acquire [4, 66] a nearly scale-invariant power spectrum $\mathcal{P}_{\delta\varphi}(k)$

$$\mathcal{P}_{\delta\varphi}(k) \equiv \frac{1}{2\pi^2} k^3 |\delta\varphi_k|^2 \sim k^{n_s - 1} \tag{6.7}$$

(where $\delta\varphi_k$ is the Fourier mode of $\delta\varphi$) with a red tilt given by the spectral index

$$n_s - 1 = -2q \,, \tag{6.8}$$

to leading order in q, while the spectrum of gravitational waves (with the same initial conditions) retains its vacuum form, modulo a small blue tilt given by

$$\delta n_t = 2q \,, \tag{6.9}$$

again to leading order in q. Note that the tensor tilt of the canonically normalized variable

$$\tilde{h} \equiv ah\,, \tag{6.10}$$

(where h is the amplitude of a particular gravitational wave polarization mode) is defined via (we are using the conventions from [68])

$$\mathcal{P}_{\tilde{h}}(k) \equiv \frac{1}{2\pi^2}|\tilde{h}_k|^2 \sim k^{n_t} \tag{6.11}$$

($\tilde{h}_k$ again denoting the Fourier mode of $\tilde{h}$). The transformation of the spectrum of $\delta\varphi$ is a consequence of the fact that $\delta\varphi_k$ has a mode which grows on super-Hubble scales, whereas $\tilde{h}_k$ does not have such a growing mode.

On the other hand, as pointed out in [82,83], the canonical fluctuation variable v [106,107] which describes the amplitude of curvature fluctuations in comoving coordinates, does not grow on super-Hubble scales. In fact, the dominant mode of v_k slightly decreases, like the leading mode of $\tilde{h}_k$. This implies that the spectrum of fluctuations of the comoving gauge curvature at the end of the contracting phase remains close to vacuum. In fact, it acquires the same slight blue tilt which the spectrum of gravitational waves does.

The fact that the spectra of $\delta\varphi$ and v have a very different spectral index in the contracting phase of the Ekpyrotic scenario (they have the same index in the case of single field slow-roll inflation) implies that great care must be taken to correctly evolve the fluctuations from the end of the contracting phase to the beginning of the expanding phase. In the following we will review this issue. Note that this ambiguity does not effect the evolution of the gravitational wave spectrum since gravitational waves are gauge-independent.

Following the notation of [42] (see also [105] for an overview) we will work in longitudinal gauge, a coordinate system in which the metric (including scalar fluctuations) is diagonal and given by the line element

$$ds^2 \;=\; a(\tau)^2\Big[(1+2\Phi)d\tau^2 - (1-2\Phi)d\mathbf{x}^2\Big]\,, \tag{6.12}$$

where $\Phi(\mathbf{x},\tau)$ is the scalar fluctuations variable, the relativistic generalization of the Newtonian gravitational potential, and we have assumed the absence of anisotropic matter stress, an assumption which is satisfied at linear order in the case of a scalar matter field. The scalar matter field φ can be decomposed into its background value $\varphi_0(\tau)$ and the fluctuation $\delta\varphi(\mathbf{x},\tau)$:

$$\varphi(\mathbf{x},\tau) \;=\; \varphi_0(\tau) + \delta\varphi(\mathbf{x},\tau)\,. \tag{6.13}$$

The amplitude of the late time anisotropies in the microwave background and of the inhomogeneities which lead to the observed large-scale structure of the universe are determined by the spectrum of the curvature fluctuation variable $\mathcal{R}$, the curvature perturbation in comoving coordinates. This quantity is closely related to the Sasaki-Mukhanov variable v [106, 107], the variable in terms of which the action for linear cosmological perturbations has a canonical kinetic term. This variable is

$$v \; = \; a\Big(\delta\varphi + \frac{\dot{\varphi}_0}{H}\Phi\Big)\,, \tag{6.14}$$

and its Fourier mode obeys the equation

$$v_k'' + \Big(k^2 - \frac{z''}{z}\Big)v_k \; = \; 0\,, \tag{6.15}$$

where

$$z \; \equiv \; a\frac{\varphi_0'}{\mathcal{H}}\,, \tag{6.16}$$

with a prime denoting a derivative with respect to conformal time and $\mathcal{H} \equiv a'/a$.

In the phase of Ekpyrotic contraction, z and a differ only by a constant, and hence (on super-Hubble scales where the k^2 term is negligible) (6.15) becomes

$$v_k'' + q(1-q)v_k \; = \; 0\,, \tag{6.17}$$

with a dominant solution

$$v_k \; \propto \; (-\tau)^{q(1-q)} \; \sim \; (-\tau)^q\,, \tag{6.18}$$

a solution which is in fact slightly decaying as a function of time. As a consequence, an initial vacuum spectrum for fluctuations of v does not get transformed into a scale-invariant one.

Rather, the spectrum acquires a slight blue tilt $\Delta n_s > 0$ compared to a vacuum spectrum:

$$\Delta n_s \,=\, 2q\,, \tag{6.19}$$

to leading order in the small parameter q. To derive this relation, note that v_k oscillates with constant amplitude on sub-Hubble scales, and then scales as $(-\tau)^q$ on super-Hubble scales. Hence, the total change in amplitude of the mode v_k compared to its vacuum value is

$$\frac{v_k(\tau)}{v_k(\tau_H(k))} \,\sim\, (-\tau_H(k))^{-q} \,\sim\, k^q\,, \tag{6.20}$$

where $\tau_H(k)$ is the time when the mode k exits the Hubble radius (see Figure 1). Note that the canonical variable $\tilde{h}$ for gravitational waves obeys the same equation (6.15).

A different view of the evolution of scalar metric fluctuations in the phase of Ekpyrotic contraction is obtained by following the relativistic potential Φ, or equivalently the rescaled variable

$$u \,\equiv\, \frac{m_{pl}}{\mathcal{H}} a\Phi\,. \tag{6.21}$$

It obeys the mode equation [84, 93, 94]

$$u_k'' + \Big(k^2 - \frac{2}{a}\mathcal{H}^2 - \frac{a''}{a}\Big)u_k \,=\, 0\,, \tag{6.22}$$

which on super-Hubble scales (and in the case of Ekpyrotic contraction) becomes

$$u_k'' - q(-\tau)^{-2} u_k \,=\, 0\,, \tag{6.23}$$

which leads to a growing mode with

$$u_k \,\sim\, (-\tau)^{-q} \,\sim\, a(\tau)^{-1}\,, \tag{6.24}$$

and a decaying mode with

$$u_k \,\sim\, (-\tau)^{1+q}\,, \tag{6.25}$$

always to leading order in q. This implies that in the infrared limit there are two modes for Φ, a growing mode with

$$\Phi_k(\tau) \,=\, A(k)\frac{\mathcal{H}}{a}(\tau)\frac{a(\tau_H(k))}{a(\tau)}\,, \tag{6.26}$$

and a constant mode

$$\Phi_k(\tau) \,=\, B(k)\,, \tag{6.27}$$

where the coefficients $A(k)$ and $B(k)$ are determined by the vacuum initial conditions which are [3]

$$\lim_{\tau\to-\infty} u_k \,=\, \frac{1}{k^{3/2}} e^{-ik\tau}\,. \tag{6.28}$$

[3]Note that the factor $k^{-3/2}$ comes from the fact that u is related to the canonical fluctuation variable by a factor which in the ultraviolet scales as k^{-1}, and the canonical variable having vacuum initial conditions $\sim k^{-1/2}$.

It is the factor of $k^{-3/2}$ in the vacuum normalization of u which leads to the scale-invariance of the power spectrum of u. In fact, the slight growth of u on super-Hubble scales yields a red tilt of the spectrum with

$$n_s - 1 \, = \, -2q \, , \tag{6.29}$$

which should be compared to the tensor tilt of

$$n_t \, = \, 2q \, . \tag{6.30}$$

Since at the end of the contracting phase the dominant modes of Φ and v have very different scalings, the key question is which of the variables is continuous when passing through the S-brane. In the case of single field slow-roll inflation the canonical variable v is continous while Φ jumps if we treat reheating as instantaneous (see e.g. [115]). In the case of bouncing cosmologies with a smooth transition from contracting to expansion (which always requires new physics input), one can often (see e.g. [3, 32, 116–118] for some specific examples, and [64] for a review) show that it is again the variable v which determines the spectrum of the dominant mode at late times (however, an example where this is not true was studied in [119, 120]). In our case, the transition is not smooth and happens on a space-like hypersurface, the S-brane. The transfer of fluctuations across a space-like matching surface was studied in detail in [93, 94] (see also [121]), generalizing the Israel matching conditions [122]. It was found that the location of the surface in space-time is crucial in order to determine whether Φ or v determine the spectrum of late time fluctuations. It was found that if the hypersurface is at a constant density surface (as assumed in [111]), then the dominant mode of Φ in the contracting phase does not couple to the dominant mode in the expanding phase, while v is continuous, and it is hence the spectrum of v which determines the amplitude of the late time curvature fluctuations. On the other hand, for any other matching surface, in particular in the case of a shear-free surface which we will be using in this paper, then the growing mode of Φ in the contracting phase couples to the dominant mode of the late time curvature fluctuation variable.

6.3 S-Brane and Nonsingular Bounce

We now briefly review the scenario introduced in [111]. It is based on the assumption that our Ekpyrotic scenario is embedded in superstring theory. In this context, the scalar field φ is the lightest modulus field of the particular string theory background being considered (e.g. the radius of the large extra dimension in Horava-Witten theory [69, 70], the original setting of the Ekpyrotic scenario [4, 66]). However, as the background energy density increases, the mass of a large tower of string states no longer becomes negligible. At the string scale, this tower of states has to be included. The suggestion of [111] (see also [103, 104, 123] for a similar proposal in a slightly different setting) is that these degrees of freedom must be included as an *S-brane*, a relativistic space-like hypersurface which is located where the density hits the string scale η_s. Specifically, the low energy action of the theory is assumed to take the form

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\Big[R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\Big] \\ &\quad - \int d^4x\kappa\delta(\tau-\tau_B)\sqrt{\gamma}\,, \end{aligned} \tag{6.31}$$

where τ_B is the time when the string density is reached, R is the Ricci scalar of the four-dimensional space-time metric $g_{\mu\nu}$ with determinant g, φ is our Ekpyrotic scalar field, γ_{ij} is the induced metric on the hypersurface with determiant γ, and κ is the tension of the S-brane. The tension of the brane is set by the string scale η_s

$$\kappa \equiv N\eta_s^3\,, \tag{6.32}$$

where N is a large integer given by the number of new states which become low mass.

Since the S-brane is a space-like relativistic object, its energy density vanishes and it has negative pressure. This can be understood in analogy with the energy-momentum tensor of a domain wall which has vanishing pressure in orthogonal direction and negative pressure (i.e. positive tension) in the parallel directions. Hence, the S-brane leads to a violation of the null energy condition, such a violation being required to obtain a transition from contraction to expansion. If we denote the background time at the position of the S-brane by t_B, then we can integrate the Friedmann equation across the brane and obtain a change

$$\delta H \equiv \lim_{\epsilon \to 0} H(t_B + \epsilon) - H(t_B - \epsilon) = 4\pi G \kappa \,. \tag{6.33}$$

We see that the integer N has to be sufficiently large in order to allow for a transition from contraction to expansion. We will assume that the bounce is symmetric in the sense

$$\delta H = 2|H_-| \,, \tag{6.34}$$

where H_- is the value of H at the end of the contracting phase when the energy density reaches the string scale. Making use of the Friedmann equation

$$\delta H = \frac{2}{\sqrt{3}} \eta_s^2 m_{pl}^{-1} \,, \tag{6.35}$$

comparing (6.35) with (6.33) and inserting (6.32) leads to the condition

$$N = \frac{4}{\sqrt{3}} \frac{m_{pl}}{\eta_s} \,. \tag{6.36}$$

Our model has two free parameters, firstly the ratio of the string scale to the Planck mass, and secondly the index q. As we will see in the next section, the amplitude of the scalar spectrum is set by the ratio of the string scale to the Planck mass, and also depends on the value of q. The amplitude of the tensor modes depends on N, as shown in [111]. In this way, two consistency relations for observables result, the first being the relation (6.6) between the tensor and scalar tilts, and the second a relation between the tensor to scalar ratio and the scalar tilt which will be derived below.

6.4 Curvature Fluctuations with a Zero Shear S-Brane

In our previous paper we took the S-brane to be located at the surface of constant density. In this case, as shown in [93, 94], the growing mode of Φ in the contracting phase does not couple to the dominant mode in the expanding phase. From the point of view of the dynamics of the S-brane, it is, however, natural to assume that the brane has no shear. In this case, the brane is located at the surface of constant time in the longitudinal gauge which we are using.

The matching conditions of both the background and the cosmological fluctuations were studied in detail in [93,94], and we will make use of the results derived there. The matching conditions state that firstly the induced metric computed from both sides of the brane is identical, and the second condition states that the extrinsic curvature jumps by an amount which is set by the surface tension. For the background, it is this second condition which is equivalent to the condition (6.33) which we have used above.

Scales we are interested in are in the far infrared compared to the Hubble radius. Hence, we can focus on the infrared limit of the solutions of the equation of motion for u. In this limit, and following the notation of [93, 94], we write the solution of Φ in the contracting phase as

$$\Phi_-(k,\tau) \;=\; A_-(k)\frac{\mathcal{H}}{a^2} + B_-(k)\,, \tag{6.37}$$

where we have absorbed the factor of $a(\tau_H(k))$ from (6.26) into the coefficient A_-. Analogously, in the expanding phase the solution can be written as

$$\Phi_+(k,\tau) \;=\; A_+(k)\frac{\mathcal{H}}{a^2} + B_+(k)\,. \tag{6.38}$$

For a zero shear brane and neglecting fluctuations in the S-brane tension, matching the induced metric on both sides of the brane, and having the jump in the extrinsic curvature being given by the surface tension, leads to the following relations between the A and B coefficients (see [93, 94])

$$\begin{aligned} A_+ \;&=\; \frac{\mathcal{H}_-}{\mathcal{H}_+}A_- + \frac{a_B^2}{\mathcal{H}_+}(B_- - B_+) \\ B_+ \;&=\; \Big(\frac{\mathcal{H}_+(\mathcal{H}_-{}'/\mathcal{H}_- - \mathcal{H}_-) - \mathcal{H}_+{}' + \mathcal{H}_+{}^2}{2\mathcal{H}_+{}^2 - \mathcal{H}_+{}'}\Big)\frac{\mathcal{H}_-}{a_B^2}A_- \\ &\quad + \Big(1 + \frac{\mathcal{H}_-\mathcal{H}_+ - \mathcal{H}_+{}^2}{2\mathcal{H}_+{}^2 - \mathcal{H}_+{}'}\Big)B_-\,, \end{aligned} \tag{6.39}$$

where a_B is the value of a at the transition surface [4]. As is evident from (6.39), the growing mode of Φ in the contracting phase couples to the dominant mode in the expanding phase.

[4] We assume that the perturbations in the surface tension are negligible in the infrared limit.

The effect of the subdominant mode in the contracting phase on B_+ is negligible. Making use of the scaling of $\mathcal{H}$ in the contracting phase we obtain

$$B_+(k) \simeq -\frac{\mathcal{H}_+}{a_B^2}\frac{1}{3q}A_-(k)\,. \tag{6.40}$$

The amplitude $A_-(k)$ is obtained by matching to the vacuum initial conditions of (6.28) and yields [93, 94]

$$A_-(k) \simeq 2^\mu\Gamma(\mu)m_{pl}^{-1}k^{-3/2}(k\tau_B)^{-q}\,, \tag{6.41}$$

where τ_B is the conformal time when the S-brane arises, Γ stands for the gamma function, and $\mu = q + 1/2$. Making use of (6.40) we find the following result for the late time power spectrum of Φ:

$$\mathcal{P}_\Phi(k) \simeq \frac{1}{2\pi^2}(k\tau_B)^{-2q}\Big(\frac{\mathcal{H}_+}{a_B^2 m_{pl}}\Big)^2\frac{1}{9q^2}2^{2\mu}\Gamma(\mu)^2\,. \tag{6.42}$$

Making use of the Friedmann equation to solve for $\mathcal{H}_+$ we obtain

$$\mathcal{P}_\Phi(k) \simeq \frac{1}{2\pi^2}(k\tau_B)^{-2q}\Big(\frac{\eta_s}{m_{pl}}\Big)^4\frac{1}{27q^2}2^{2\mu}\Gamma(\mu)^2\,. \tag{6.43}$$

We see that the spectrum is approximately scale-invariant with a small red tilt of magnitude $2q$.

If we demand that the power spectrum of Φ has the observed valuie 10^{-9} [14] at the pivot scale k_C (the scale of the CMB quadrupole), we find

$$\left(\frac{\eta_s}{m_{pl}}\right)^4 \simeq (2\pi^2)27q^2 2^{-2\mu}\Gamma(\mu)^{-2}(k_C\tau_B)^{2q}10^{-9} . \tag{6.44}$$

The amplitude of the power spectrum of gravitational waves obtained from [111] is

$$\mathcal{P}_h(k) \simeq \frac{1}{2\pi^2}\kappa^2 m_{pl}^{-6}(k\tau_B)^{2q} . \tag{6.45}$$

Hence, the predicted tensor to scalar ratio r (evaluated at the pivot scale k_C) is

$$\begin{aligned} r &\equiv \frac{\mathcal{P}_h(k)}{\mathcal{P}_\Phi(k)} \\ &\simeq 144(k_C\tau_B)^{4q}2^{-2\mu}\Gamma(\mu)^{-2}q^2 \end{aligned} \tag{6.46}$$

which is obtained by combining (6.45) and (6.43) and making use of the expression (6.36) for N.

Since the value of q is given by the scalar tilt $q = (1 - n_s)/2$, (6.46) yields a consistency relation between the tensor to scalar ratio and the scalar tilt. Its approximate form is

$$r \sim 36(k_C\tau_B)^{4q}(1 - n_s)^2 . \tag{6.47}$$

6.5 Conclusions and Discussion

We have extended our work [111] studying the spectrum of cosmological fluctuations in an Ekpyrotic model in which the transition from contraction to expansion is mediated by an S-brane which represents the effects of higher energy string states which enter the effective action at the string scale. In a previous paper we discovered that the coupling of the gravitational fluctuations to the S-brane leads to the conversion of an initial vacuum spectrum to a scale-invariant one. This conversion occurs both for gravitational waves and also for the Sasaki-Mukhanov variable v representing the cosmological perturbations. However, this mechanism gives a greater boost to the gravitational waves then to the variable v. As emphasized in [84, 93, 94], in the case of a transition from contraction to expansion the variable v does not necessarily carry the full information about the scalar metric fluctuations. The Newtonian gravitational potential Φ grows in the contracting phase and obtains a roughly scale-invariant spectrum. Here, we have shown that if the S-brane has zero shear (instead of being located at the constant density surface), then the dominant fluctuation mode at late time is strongly coupled to the growing mode in the contracting phase. Hence, the spectrum of cosmological perturbations at late times is approximately scale-invariant and obtains an amplitude which is larger than that of the gravitational wave spectrum.

Our model has two free parameters: the ratio between the string scale and the Planck scale, and the index q which describes the rate of Ekpyrotic contraction. If we fix these two parameters in terms of the observed amplitude and tilt of the spectrum of cosmological perturbations, we obtain predictions for the tensor to scale ratio r, and for the tensor tilt:

$$\begin{aligned} n_t &= 1 - n_s \\ r &= \mathcal{B}(1 - n_s)^2 , \end{aligned} \tag{6.48}$$

where $\mathcal{B}$ is a number of order one. Note that these consistency relations are different from those of canonical single field inflation (which always predicts a red tilt of the tensor spectrum). The relation between the tilts n_t and n_s is the same as that obtained in String Gas Cosmology, but the predicted value of r has a different dependence on the parameters than what is obtained in String Gas Cosmology [30, 110].

An important open problem for our scenario is to study the generation of radiation during the transition through the S-brane. Work on this topic is in progress. It would also be interesting to study the amplitude and shape of the non-Gaussianities. In inflationary cosmology the amplitude of the non-Gaussianities is suppressed by the slow-roll parameter ϵ. In the Ekpyrotic scenario the analog of the inflationary slow-roll parameter is a large number, and hence one suppression mechanism of non-Gaussianities disappears. In the *New Ekpyrotic Scenario* [33, 39, 86–89], a two field model in which scale-invariant fluctuations of a second scalar field χ source scale-invariant curvature fluctuations via the induced entropy fluctuations, it has in fact been shown that the induced non-Gaussianities are larger than the current upper bounds [39, 124, 125]. However, since our mechanism of generating scale-invariant fluctuations is different, the conclusions of [39, 124, 125] will not apply.

Appendix

In this Appendix we demonstrate that the S-brane does not effect the equation of motion for the scalar fluctuation variable u discussed in this paper. This contrasts to the large effect which the S-brane has on the evolution of the v variable and which we discussed in our previous paper [111].

The starting point of our analysis is the total action (7.3). We insert our ansatz for metric and matter fluctuations (6.12) and (6.13) into this action and expand to second order in our metric potential variable Φ. The calculation is explained in detail in [42], and we only quote the crucial points. We need to focus on the relative normalizations with which Φ enters the bulk and the brane part of the action. The term in the bulk action quadratic in Φ' is (see Eq. (10.35) in [42])

$$S^{(2)} = \frac{3}{8\pi G}\int d^4xa^2\Big[-\Phi'^2 - 4\mathcal{H}\Phi\Phi' - 6\mathcal{H}^2\Phi^2 - (\partial_i\Phi)^2\Big]. \tag{6.49}$$

On the other hand, the term quadratic iin the brane action is

$$S_B^{(2)} = \frac{1}{2}\int d^4x\kappa a^3 3\Phi^2\delta(\tau - \tau_B). \tag{6.50}$$

The resulting equation of motion for Φ is (in Fourier space)

$$\Phi'' + 2\mathcal{H}\Phi' + \Big[k^2 + 2\mathcal{H}^2 - 2\mathcal{H}' - 4\pi G a\kappa\delta(\tau-\tau_B)\Big]\Phi \,=\, 0\,. \tag{6.51}$$

We see that the brane contribution is suppressed compared to the bulk contribution by a factor $\mathcal{F}$ which is

$$\mathcal{F} \,\sim\, a\kappa m_{pl}^{-2}\,. \tag{6.52}$$

Note that $\mathcal{F}$ has mass dimension one since the brane contribution is multiplied by $\delta(\tau-\tau_B)$. The brane thus yields a δ function contribution to the effective mass in the equation of motion (6.22) for u which becomes

$$u_k'' + \Big[k^2 - \frac{2}{a}\mathcal{H}^2 - \frac{a''}{a} + \mathcal{F}\delta(\tau-\tau_B)\Big]u_k \,=\, 0\,. \tag{6.53}$$

We see that, unlike in the case of the equation of motion for the canonical variable v studied in [111], the contribution of the brane inside the equation of motion for u is suppressed by a and therefore negligible. Hence, the u_k modes do not acquire the infrared enhancement which the v modes experience.

Chapter 7

Reheating after S-Brane Ekpyrosis

In recent work, two of us proposed a nonsingular Ekpyrotic cosmology making use of an S-brane which forms at the end of the phase of Ekpyrotic contraction. This S-Brane mediates a transition between contraction and expansion. Graviitational waves passing through the S-Brane acquire a roughly scale-invariant spectrum, and if the S-Brane has zero shear, then a roughly scale-invariant spectrum of cosmological perturbatiions results. Here, we study the production of gauge field fluctuations driven by the decay of the S-Brane, and we show that the reheating process via gauge field production will be efficient, leading to a radiation-dominated expanding phase.

7.1 Introduction

Although the inflationary scenario [48–51] has become the standard paradigm of early universe cosmology, it has recently been challenged from considerations based on fundamental physics. On one hand, inflation (at least canonical single field slow-roll inflation) seems hard to realize in string theory based on the *swampland constraints* [19, 22, 55, 126] (see e.g. [21, 112] for a review). On the other hand, the effective field theory description of inflation is subject to the *Trans-Planckian Censorship* constraint (TCC) [24] which forces the energy scale of inflation to be many orders of magnitude smaller than the scale of particle physics Grand Unification (the scale used in the canonical single field slow-roll models of inflation), leading to a negligible amplitude of primorial gravitational waves [25] (see, however, [127] for an opposite point of view on this issue).

Inflation is not the only early universe scenario which can explain the current observational data (see e.g. [27, 128] for a review of some alternative scenarios). Bouncing cosmologies (see e.g. [64] for a recent review) and emergent scenarios such as *String Gas Cosmology* [28, 30] are promising alternatives. However, at the moment none of the alternatives have been developed to the level of self-consistency that the inflationary scenario has. However, in light of the conceptual challenges to inflation, there is a great need to work on improvements of alternative scenarios.

Among bouncing cosmologies, the Ekpyrotic scenario [4, 66] has a preferred status. The Ekpyrotic scenario is based on the hypothesis that the contracting phase was one of very slow contraction. Such a phase can be realized in the context of Einstein gravity by assuming that matter is dominated by a scalar field φ with a negative exponential potential (but positive total energy density) such that the equation of state is $w \gg 1$, where $w = p/\rho$ is the ratio betwen pressure and energy density. As a consequence of this equation of state, all initial cold matter, radiation, spatial curvature and anisotropy is suppressed relative to the energy density in φ. Thus, the Ekpyrotic scenario solves the flatness problem of Standard Big Bang cosmology, and the homogeneous and isotropic contracting trajectory is an attractor in initial condition space, as first discussed in [36]. In the words of the recent paper [129], the Ekpyrotic scenario is a *super-smoother* [1].

In order to yield a successful early universe cosmology, there must be a well-controlled transition from the contracting Ekpyrotic phase to the radiation phase of Standard Big Bang Cosmology. In the past, the transition was either assumed to be singular (breakdown of the effective field theory description [4,66]) or else obtained by invoking matter fields which violate the Null Energy Condition [33,39,86–89] (see e.g [130] for a review). In a recent work, two of us have suggested [111] that the transition arises as a consequence of an S-brane which appears once the background energy density reaches the string scale [2]. From the point of view of string theory, the S-brane represents string degrees of freedom which decouple at low energy densities but become comparable in energy to the energy scale of the background and

[1] Note that the homogeneous and isotropic expanding phase in large field inflation is also a local attractor in initial condition space [75], as recently reviewed in [76]. Large field inflation is, however, in tension with the swampland criteria and the TCC [25]

[2] See [103, 104, 123] for previous work on obtaining a non-singular bounce making use of an S-brane in a string theoretical setting.

hence have to be included in the low energy effective field theory as a space-filling object which is localized at a fixed time. Such a brane has vanishing energy energy density, and negative pressure (positive tension). As such, it can mediate the transition between the contracting phase and an expanding universe. In [111] it was shown that gravitational waves passing through the S-brane acquire a scale-invariant spectrum (with a slight blue tilt) if they begin as vacuum fluctuations early in the contracting phase. In [131] it was then shown that, assuming that the S-brane has zero shear, the curvature fluctuations similarly obtain a scale-invariant spectrum (this time with a slight red tilt), and two consistency relations which related the amplitudes and tilts of the scalar and tensor spectra were derived.

In the works [111, 131] it was assumed that the universe is radiation dominated after the transition. Here we study the production of radiation during the decay of the S-brane and show that, indeed, the post S-brane state is dominated by radiation. In analogy to the reheating which takes place at the end of inflation, where particle production is driven by the dynamics of the inflaton field [132, 133] (see [134, 135] for reviews), we can speak of the process of *reheating after Ekpyrosis* which we analyze here.

In the following section, we review the S-brane bounce mechanism and discuss the string theoretic setting of the S-brane. In Section 3 we then discuss the coupling of the S-brane to the Standard Model radiation field and then (in Section 4) estimate the efficiency of radiation production during the decay of the S-brane. We conclude with a summary and discussion of our results.

We assume a spatially flat homogeneous and isotropic background space-time with scale factor $a(t)$, where t is physical time. It is often convenient to use conformal time τ determined by $dt = a(t)d\tau$. The Hubble expansion rate is $H \equiv \dot{a}/a$, and its inverse is the Hubble radius, the length scale which plays a key role in the evolution of cosmological fluctuations. We use units in which the speed of light and Planck's constant are set to 1. Comoving spatial coordinates are denoted by $\mathbf{x}$, and the corresponding comoving momentum vector is $\mathbf{k}$ (its magnitude is written as k). The reduced Planck mass is denoted by m_{pl} and it is related to Newton's gravitational constant G via $m_{pl}^{-2} = 8\pi G$.

7.2 S-Brane Bounce

The Ekpyrotic scenario is based on an effective field theory involving a canonically normalized scalar field φ with negative exponential potential

$$V(\varphi) \;=\; -V_0 e^{-\sqrt{2/p}\varphi/m_{pl}} \tag{7.1}$$

with $V_0 > 0$ and $0 < p \ll 1$ coupled to Einstein gravity. The universe is assumed to begin in a contracting phase at large values of φ and positive total energy. The equation of state of the scalar field matter is

$$w \;\equiv\; \frac{p}{\rho} \;=\; \frac{4}{3p} \;\gg\; 1\,. \tag{7.2}$$

Hence, in the contracting phase, φ comes to dominate over all other forms of energy which might have been present at the initial time.

Negative exponential potentials are ubiquitous in string theory. In fact, the first realization of the Ekpyrotic scenario [4, 66] was based on the potential of a bulk brane moving in a Horava-Witten background [69, 70]. In the context of string theory, we know that as the energy density of the background approaches the string scale, new stringy degrees of freedom become low mass and have to be included in the low energy effective action. In [111], we proposed to do this by adding an S-brane [136] to the action to yield

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\left[R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right] \\
&\quad - \int d^4x\kappa\delta(t-t_B)\sqrt{\gamma}\,. \qquad (7.3)
\end{aligned}
$$

The second term is the S-brane. It is localized at the time t_B when the background density reaches the string scale. In the above, R is the Ricci scalar and g is the determinant of the four-dimensional space-time metric $g_{\mu\nu}$, γ is the determinant of the induced metric on the S-brane world volume, and κ is the tension of the S-brane, given by the string scale η_s.

As shown in [111], the S-brane mediates a non-singular transition between Ekpyrotic contraction and expansion. Since the homogeneous and isotropic contracting solution is an attractor in initial condition space and any pre-existing classical fluctuations at the beginning of the phase of Ekpyrotic contraction get diluted relative to the contribution of φ, it is reasonable to assume that both scalar and tensor fluctuations are in their vacuum state on sub-Hubble scales. It was shown [111] the gravitational waves passing through the S-brane acquire a scale-invariant spectrum, and in [131] it was shown that, provided that the S-brane has zero shear, a scale-invariant spectrum of curvature fluctuations is generated. The scenario leads in fact to two consistency relations between the four basic cosmological observables [131], namely the amplitudes and tilts of the scalar and tensor spectra.

In [111, 131] it was simply assumed that the state after the cosmological bounce would be the radiation phase of Standard Big Bang cosmology, analogously to how the Standard Model radiation phase follows after inflation. In order for this to be the case, there has to be efficient energy transfer from the S-brane to radiation. In this paper, we show that this reheating process exists and is indeed very efficient.

In order to be able to study reheating after the S-brane bounce, we need to better understand the S-brane microphysics. Let us consider, to be specific, Type IIB superstring theory. In this theory, the gauge fields of the Standard Model of particle physics, including the photon field, live on the world volume of D-branes. In the context of Type IIB string theory, our four-dimensional space-time can be considered to be the world volume of a D3 brane (three spatial and one time dimension) which is located at a particular point along the compactified spatial dimensions.

Similar to what was done in the original Ekpyrotic proposal [4, 66], we assume that φ is a Kähler modulus related to the size of one of the compact dimensions of space. Negative exponential potentials for such moduli fields are ubiquitous in string theory (see e.g. [68] for a review). In line with the setup of the Ekpyrotic scenario, the evolution begins in a phase of contraction with positive total energy density, the kinetic energy density being slightly larger than the potential energy density. As the universe contracts, the energy density increases until it reaches the string scale, which happens at a value of φ which we denote by φ_s. At that point, the kinetic energy density of φ is large enough to excite an S-brane. Similar to what was done in [136], we will model the S-brane by a tachyon condensate. The tachyon configuration $T(x,t)$ can be viewed as an unstable four brane, the extra spatial dimension being, for example, the dimension corresponding to φ. Since anisotropies and matter inhomogeneities are smoothed out in the contracting phase, the tachyon configuration will be homogeneous $T(x,t) = T(t)$ on the three brane of our space-time volume.

Note that the potential of φ at φ_s is negative. Its value can be viewed as the magnitude of the (negative) cosmological constant in the anti-de-Sitter ground state of the string theory. The tachyon configuration can be described by a positive potential energy density $V(T)$ on top of the negative value of the background. We take the potential to have the form

$$V(T) = \begin{cases} -\frac{1}{2}\lambda\eta^2 T^2 + \frac{1}{2}\lambda\eta^4, & -\eta < T < \eta \\ \\ 0, \quad |T| > \eta\,, \end{cases} \tag{7.4}$$

where we expect η to be given by the string scale. A sketch of this potential is given in Fig. 1. In order to represent an S-brane, the total energy density $V_{\rm grav}$ (tachyon energy plus contribution from the cosmological constant) must vanish at $T = 0$. This forces η to be of the order of the string energy scale. It is the quantity $V_{\rm grav}$ which determines the evolution of the gravitational background.

The action of a Standard Model gauge field (e.g. the photon field) on the D3 brane has the Born-Infeld form (see e.g. [137])

$$S_{BI} \,=\, \int dt d^3x V_{\rm total}(T)\sqrt{\det(\eta_{\mu\nu} + F_{\mu\nu})}\,, \tag{7.5}$$

where τ is the tension of the D-brane, $V_{\rm total}(T)$ is the total potential energy of the tachyon, and $F_{\mu\nu}$ is the field strength of the gauge field A_μ (in string units). The total potential energy gets a constant contribution from the tension τ of the D3-brane plus the contribution from the tachyon field:

$$V_{\rm total}(T) \,=\, \tau + V(T) + f(\partial_0 T, \partial_0^2 T, ...)\,. \tag{7.6}$$

where $f(\partial_0 T, \partial_0^2 T, ...)$ can be ignored for our case [138, 139], and the constant term coming from the brane tension does not have any effect on the particle production calculation which we discuss below.

We are interested in the growth of fluctuations of A_μ which start out in their vacuum state. Hence, we can expand the square root and keep only the leading term in $F_{\mu\nu}F^{\mu\nu}$

$$\sqrt{\det(\eta_{\mu\nu} + F_{\mu\nu})} \simeq 1 + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \,. \tag{7.7}$$

In particular, we then recover the usual gauge theory action for $|T| > \eta$ where $V_{\text{total}}(T)$ is a non-vanishing constant.

If we choose the gauge $A_0 = 0$, then for a homogeneous tachyon kink $K(t)$ the action for the gauge fields becomes [137]

$$S = - \int dt d^3x \dot{K}^2(t) \left(\frac{1}{4} F_{ab} F^{ab} - \frac{1}{2} \dot{A}_a \dot{A}^a \right) , \tag{7.8}$$

where the indices a, b are spatial ones, and we have extracted the terms with time derivatives. We have also used the on-shell condition in (7.4) to replace V_{total} by $\dot{K}(t)$, ignoring constant factors. Note that, during passage through the S-brane, the coefficient $\dot{K}$ depends on time, and hence gauge field particle production is possible. This is what we study in the following two sections.

7.3 Coupling of Radiation to the S-Brane

We have seen that the coupling of the S-brane to the gauge field A_μ is given by [137]

$$S = \int dt d^3x \dot{K}^2(t) \left(\frac{1}{4} F_{ab} F^{ab} - \frac{1}{2} \dot{A}_a \dot{A}^a \right) , \tag{7.9}$$

where the indices a, b run only over space, and we have used temporal gauge. This action can be put into canonical form by defining a rescaled gauge field

$$B_a \equiv \dot{K} A_a \tag{7.10}$$

with associated field strength tensor $\tilde{F}_{ab}$. In terms of this rescaled gauge field, the action is given by [137]

$$S = \int dt d^3x \left[\frac{1}{4} \tilde{F}_{ab} \tilde{F}^{ab} + \frac{1}{2} B_a \left(-\frac{\partial^2}{\partial^2 t} + \frac{K_{,ttt}}{K_{,t}} \right) B^a \right] , \tag{7.11}$$

where $K_{,ttt}$ stands for the third time derivative of K.

The equation of motion for T in the range $-\eta < T < \eta$ is

$$\ddot{T} - \lambda \eta^2 T = 0 , \tag{7.12}$$

and its solution is

$$T(t) = \begin{cases} v\sqrt{\frac{\eta^4\lambda}{v^2}+1}(t+P)-\eta, & t<-P \\ \frac{v\sinh(\eta\sqrt{\lambda}t)}{\eta\sqrt{\lambda}}, & -P<t<P \\ v\sqrt{\frac{\eta^4\lambda}{v^2}+1}(t-P)+\eta, & t>P\,, \end{cases} \tag{7.13}$$

where we have adjusted the time axis such that T is at the top of the potential at time $t=0$, and v is the velocity of the field at the top. The time interval P is the duration of the bounce, and is determined by when $T(P)=\eta$.

The scenario we have in mind is now the following. During the phase of Ekpyrotic contraction, φ is decreasing from a very large initial positive value. Once the energy density of φ approaches the string density the S-brane forms. The formation of the S-brane is the process where T increases from $T=-\eta$ to $T=0$. The decay of the S-brane corresponds to the motion of T from $T=0$ to $T=\eta$. During the phase of Ekpyrotic contraction, the total energy density in the φ field is slightly positive. This implies that the magnitude of the kinetic energy is close to (but larger) than the absolute value of the total potential energy V_{total}. Hence we expect

$$v \simeq \lambda^{1/2}\eta^2\,, \tag{7.14}$$

where v is defined in (7.13). This implies that

$$P \simeq \lambda^{-1/2}\eta^{-1}\,. \tag{7.15}$$

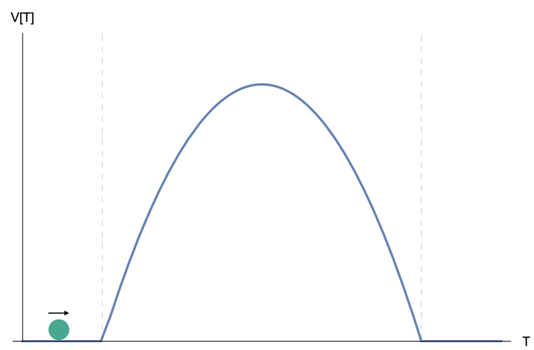

Figure 7.1: The sketch of tachyon potential $V(T)$ during the bounce phase.

To make contact with the discussion in [137]), we note that in our case the profile function $K(t)$ is given by the soliton solution of the equation of motion for $T(t)$:

$$K(t) = T(t)\,, \tag{7.16}$$

which implies that

$$\frac{K_{,ttt}}{K_{,t}} = \lambda\eta^2\,. \tag{7.17}$$

7.4 Reheating from S-Brane Decay

As discussed in the previous section, the coupling of the rescaled gauge field B_μ to the S-brane profile function $K(t)$ is given by (7.11). The resulting equation of motion for the gauge field B_i is

$$\ddot{B}_i - \partial_a\partial^a B_i - \lambda\eta^2 B_i = 0\,, \tag{7.18}$$

which is a wave equation with tachyonic mass term. The solution for the Fourier modes is given by

$$B_i(k,t) = c_1 e^{t\sqrt{\eta^2\lambda - k^2}} + c_2 e^{-t\sqrt{\eta^2\lambda - k^2}}, \quad k < k_c \tag{7.19}$$

$$B_i(k,t) = c_1 e^{it\sqrt{\eta^2\lambda - k^2}} + c_2 e^{-it\sqrt{\eta^2\lambda - k^2}}, \quad k > k_c\,,$$

with $k_c \equiv \lambda^{1/2}\eta$. From (7.20) it is not hard to see that the growing mode $k \to 0$ for $t > 0$ asymptotically scales as $e^{\eta\sqrt{\lambda}t}$. The corresponding modes of A_i field, however, is a constant mode. Therefore there is no growing mode for A_i after $t = 0$.

In terms of the canonical gauge field, the equation of motion becomes (inserting the expression (7.16) for $K(t)$)

$$\ddot{A}_i + 2\eta\sqrt{\lambda}\tanh\left(\eta\sqrt{\lambda}t\right)\dot{A}_i + k^2 A_i \;=\; 0\,. \tag{7.20}$$

The solution of this equation can be written as follows

$$A_i(t,k) = \mathrm{sech}\left(\eta\sqrt{\lambda}t\right) \times \begin{cases} \tilde{c}_1 e^{-t\sqrt{\eta^2\lambda - k^2}} + \tilde{c}_2 e^{t\sqrt{\eta^2\lambda - k^2}}, & k < k_c = \eta\sqrt{\lambda} \\ \tilde{c}_1 e^{-it\sqrt{-\eta^2\lambda + k^2}} + \tilde{c}_2 e^{it\sqrt{-\eta^2\lambda + k^2}}, & k > k_c = \eta\sqrt{\lambda} \end{cases} \tag{7.21}$$

In the asymptotic regions $t \to \pm\infty$, the equation (7.20) has solutions

$$A_i(k, t \to \infty) \sim e^{-\eta\sqrt{\lambda}t} \times \begin{cases} e^{\pm\sqrt{\eta^2\lambda - k^2}t}, & k < k_c = \eta\sqrt{\lambda} \\ e^{\pm i(-\sqrt{\eta^2\lambda + k^2})t}, & k > k_c = \eta\sqrt{\lambda} \end{cases} \tag{7.22}$$

and

$$A_i(k, t \to -\infty) \sim e^{+\eta\sqrt{\lambda}t} \times \begin{cases} e^{\pm\sqrt{\eta^2\lambda - k^2}t}, & k < k_c = \eta\sqrt{\lambda} \\ e^{\pm i(-\sqrt{\eta^2\lambda + k^2})t}, & k > k_c = \eta\sqrt{\lambda}\,. \end{cases} \tag{7.23}$$

During the phase of Ekpyrotic contraction, any pre-existing gauge field fluctuations are diluted compared to the energy density in φ. Hence, the gauge field fluctuations will be in their ground state when φ hits the S-brane. Hence, focusing on modes with $k < k_c$, we can write the solution for A_i in the form

$$A_i = \begin{cases} A_i^{(1)} = \frac{e^{-ikt}}{\sqrt{2}\sqrt{k}} & t < -P \\ A_i^{(2)} = \operatorname{sech}\left(\eta\sqrt{\lambda}t\right)\left(c_1 e^{-t\sqrt{\eta^2\lambda - k^2}} + c_2 e^{t\sqrt{\eta^2\lambda - k^2}}\right) & -P < t < P \\ A_i^{(3)} = \alpha\frac{e^{-ikt}}{\sqrt{2}\sqrt{k}} + \beta\frac{e^{+ikt}}{\sqrt{2}\sqrt{k}} & t > P\,, \end{cases} \tag{7.24}$$

where the constants c_1, c_2, α and β (which are all functions of k) are determined by matching A_i and $\dot{A}_i$ at $t = -P$ and $t = P$, i.e. using the junction conditions

$$\begin{cases} A_i^{(1)}(-P) = A_i^{(2)}(-P), \quad \dot{A}_i^{(1)}(-P) = \dot{A}_i^{(2)}(-P) \\ A_i^{(2)}(P) = A_i^{(3)}(P), \quad \dot{A}_i^{(2)}(P) = \dot{A}_i^{(3)}(P)\,. \end{cases} \tag{7.25}$$

Using (7.24) and (7.25) we can solve for the Bogoliubov mode mixing coefficients β. Notice that $|\beta|^2 - |\alpha|^2 = 1$ by the requirement of unitarity. The result is

$$\begin{aligned} |\beta|^2 &= \frac{\mu^2 \mathrm{sech}^4(\mu P)}{16k^2(1-\frac{k^2}{\mu^2})} \\ &\quad \left[\left(\sqrt{1-\frac{k^2}{\mu^2}}+1\right)\sinh\left(2\mu P\left(1-\sqrt{1-\frac{k^2}{\mu^2}}\right)\right)\right. \\ &+ \left.\left(\sqrt{1-\frac{k^2}{\mu^2}}-1\right)\sinh\left(2\mu P\left(\sqrt{1-\frac{k^2}{\mu^2}}+1\right)\right)\right]^2 \end{aligned} \tag{7.26}$$

where we introduced the constant $\mu \equiv \sqrt{\lambda}\eta$ to simplify the expression. In the infrared limit $k \ll k_c$ we obtain

$$|\beta|^2 \approx \frac{k^2 \mathrm{sech}^4(\mu P)\Big(4P\mu + \sinh(4P\mu)\Big)^2}{64\mu^2\left(1-\frac{k^2}{\mu^2}\right)} \sim k^2\,, \tag{7.27}$$

while for $k \sim k_c$ we get

$$|\beta|^2 \approx -\frac{\mu^2 \mathrm{sech}^4(\mu P)(\sinh(2\mu P) - 2\mu P\cosh(2\mu P))^2}{4k^2} \tag{7.28}$$

Note that for $k > k_c$ the Bogoliubov coefficients are suppressed.

Since the particle number density is given by the Bogoliubov mode mixing coefficient, i.e. $n(k) \sim |\beta|^2$, we can use the above results to give an order of magnitude estimate of the energy density in gauge fields produced during S-brane decay. The energy density in the gauge fields can be obtained by integrating over the contribution of all modes with $k < k_c$, making use of the initial vacuum value

$$A_k(0) = \frac{1}{\sqrt{2k}} \,. \tag{7.29}$$

Making use of the solution (7.24) we obtain

$$\rho_A(P) \sim \int_0^{k_c} dk \frac{k^2}{2\pi^2} \frac{1}{2k} |\beta_k|^2 k^2 \sim \frac{1}{32\pi} \lambda^2 \eta^4 \,. \tag{7.30}$$

Here, the first factor of k^2 comes from the phase space measure, and the second factor of k^2 comes from the gradients (recall that the energy is gradient energy). The integral is dominated by the upper limit, and, inserting the value for k_c and the result (7.28) for the Bogoliubov coefficients, we obtain

$$\frac{\rho_A(\tau)}{V_0} \sim \frac{3}{16\pi} \lambda \,. \tag{7.31}$$

This is the main result of our analysis. We expect $\lambda \sim 1$, and hence we see that the S-brane can very efficiently decay into radiation.

7.5 Conclusions and Discussion

We have studied the embedding of the S-brane bounce scenario proposed in [111] into string theory. Considering Type IIB superstring theory, our space-time can be viewed as a D3-brane on which the usual Standard Model fields (including the gauge field of electromagnetism) live. At the end point of a phase of Ekpyrotic contraction, an unstable D4-brane is excited and acts gravitationally as an S-brane. This tachyonic brane fills our three spatial dimensions and is extended in one further direction (e.g. the direction which corresponds to the Ekpyrotic scalar field φ). The tachyon configuration couples to the gauge fields via a Born-Infeld action. Hence, during the formation and decay of the D4-brane, gauge field production can occur. We have shown that the process of gauge field production is very effective and can transfer a fraction of order one of the S-brane energy into radiation. Hence, there is no obstruction to efficient reheating after the S-brane bounce. The S-brane will automatically mediate not only the transition between an initial contracting phase and an expanding phase, but in fact to the expanding radiation phase of Standard Big Bang cosmology.

Chapter 8

Entanglement entropy of cosmological perturbations for S-brane Ekpyrotic scenario

We calculate the entanglement entropy of scalar perturbations due to gravitational non-linearities present in any model of canonically-coupled, single-field ekpyrosis. Specifically, we focus on a recent model of improved ekpyrosis which is able to generate a scale-invariant power spectrum of curvature perturbations and gravitational waves as well as have a non-singular bounce due to an S-brane at the end of ekpyrotic contraction. By requiring that the entanglement entropy remians subdominant to the thermal entropy produced during reheating, we get an upper bound on the energy scale of the bounce.

8.1 Introduction

In a recent paper [140] we studied the momentum space entanglement entropy between sub- and super-Hubble modes generated by the intrinsic gravitational nonlinearities. We applied the formalism to inflationary cosmology and found that the entanglement entropy is a growing function of time since the phase space of super-Hubble modes is increasing. By demanding that the entanglement entropy remain smaller than the thermal entropy after reheating, we found an upper bound on the duration of inflation which is consistent with the constraint [25] coming from the *trans-Planckian censorship conjecture* [24]. In this paper we will apply the formalism developed in [140] to the new version of the Ekpyrotic scenario recently proposed by two of us [111]. Once again, the phase space of super-Hubble modes is an increasing function of time, and hence the entanglement entropy will be increasing. By demanding that the entanglement entropy at the end of the phase of Ekpyrotic contraction be smaller than the thermal entropy after the bounce, we find an upper bound on the energy scale of the bounce.

The Ekpyrotic scenario [4, 66] is a promising alternative to cosmological inflation. The scenario is based on the assumption that the cosmological scale factor $a(t)$ is decreasing very slowly

$$a(t) \sim t^p \quad \text{with} \quad 0 < p \ll 1 \tag{8.1}$$

as a function of physical time t, where $t < 0$ in the contracting period. In the context of

Einstein gravity, this time dependence of the scale factor can be obtained if matter is dominated by a scalar field with a negative exponential potential. Such potentials are ubiquitous in string theory compactifications (see e.g. [68]). A contracting universe with $w > 1$ leads to a dilution of spatial curvature and of anisotropies [36]. It is also an attractor in initial space [129]. These are significant advantages compared to other cosmological models with a contracting phase (see e.g. [64] for a review of bounce scenarios).

Recently, two of us [111] proposed a new version of the Ekpyrotic scenario in which an S-brane (see e.g. [136]) arising at string densities mediates the non-singular transition between contraction and expansion. If the S-brane has zero shear, then roughly scale-invariant spectra of both cosmological perturbations and gravitational waves are generated [131], with two consistency relations between the four basic cosmological observables (the amplitudes and tilts of the scalar and tensor spectra). The decay of the S-brane after the bounce naturally generates [141] the radiation of the post-bounce universe. In this paper we will consider entropy constraints on this new scenario.

In the following section we briefly review the equations which describe the phase of Ekpyrotic contraction. In Section III we summarize the framework for computing momentum space entanglement between sub- and super-Hubble modes developed in [140]. We then turn to the computation of the entanglement entropy at the end of the phase of Ekpyrotic contraction, as a function of the Hubble parameter at that time, and derive the resulting upper bound on the energy scale of the bounce by demanding consistency with the second law of thermodynamics for the full time evolution of the model.

We close this section with some general remarks. We already introduced the cosmological scale factor $a(t)$ which yields the Hubble expansion rate

$$H(t) = \frac{\dot{a}}{a}, \tag{8.2}$$

where an overdot represents a derivative with respect to time. The inverse of H is the Hubble radius. The Hubble radius plays a crucial role concerning cosmological perturbations (see e.g. [42, 105] for reviews): on length scales smaller than the Hubble radius fluctuations oscillate. On super-Hubble scales the oscillations freeze out, the fluctuations can be squeezed and classicalize [59]. Both during an expanding inflationary cosmology and during Ekpyrotic scenario the Hubble radius decreases in comoving coordinates, and in both scenarios a natural assumption is that fluctuations start as quantum vacuum perturbations on sub-Hubble scales. The modes we have access to are the ones which have been able to classicalize. Hence, it is natural to divide the total Hilbert space of fluctuation modes into sub- and super-Hubble modes, and to study the entanglement entropy between these two sets of modes. Since the phase space of super-Hubble modes is increasing as a function of time, the resulting entanglement entropy will also be increasing. In order to be consistent with the entropy in the expanding radiation phase, the entanglement entropy at the end of the contracting phase cannot be too large. It is the resulting constraint on the model which we will here explore.

We will be using natural units in which the speed of light, Planck's constant and Boltzmann's constant are set to 1. We denote the energy density by ρ and the pressure by P, keeping the symbol p for the exponent appearing in the expression for the time evolution of the scale factor (to be consistent with the usual notation in papers on the Ekpyrotic scenario).

8.2 Description of S-brane ekpyrosis

In the usual realizations of the Ekpyrotic scenario, the slow contraction phase is driven by a scalar field with negative exponential potential [4, 66]

$$S = \int dt d^3x \Big(-\frac{1}{2}\partial^\mu \varphi \partial_\mu \varphi + V_0 e^{-\sqrt{\frac{2}{p}}\frac{\varphi}{M_{\rm Pl}}} \Big) , \tag{8.3}$$

where $M_{\rm Pl}$ is the Planck mass. Since the potential is steep, the field is not slowly rolling and we can find an approximate solution by ignoring the Hubble parameter H. This approximation is further justified since during Ekpyrosis the equation of state parameter $w \equiv \frac{P}{\rho} \gg 1$ and hence the contraction of the universe is very slow. Using this approximation, we find that the scalar field evolves according to

$$\varphi(t) = \sqrt{2p}\, M_{\rm Pl}\, \log \left(-\sqrt{\frac{V_0}{p}} \frac{t}{M_{\rm Pl}} \right) . \tag{8.4}$$

Further, using the relation $\dot{\varphi}^2 = -2\dot{H}M_{\rm Pl}^2$, we find that

$$\dot{H} = -\frac{p}{t^2}, \qquad H = -\frac{p}{-t}, \qquad a = a_0(-t)^p \,. \tag{8.5}$$

One can check the self-consistency of our approximation by calculating the equation of state

$$\begin{aligned} w &= \frac{P}{\rho} = \frac{-\rho + \dot{\varphi}^2}{\rho} \\ &= \frac{-3H^2\mathrm{m}_{\rm pl}^2 - 2\dot{H}\mathrm{m}_{\rm pl}^2}{3H^2\mathrm{m}_{\rm pl}^2} = -1 + \frac{2}{3p} \gg 1 \,. \end{aligned} \tag{8.6}$$

From the above, the "slow-roll" parameters can be calculated in a straightforward manner. Note that only the first order slow-roll parameter is non-zero and is given by

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{p}, \qquad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon} = 0 \,. \tag{8.7}$$

It will be beneficial for us later on to rewrite the quantities above in terms of conformal time

$$\tau(t) \;=\; \int \frac{dt}{a(t)} = -\frac{1}{1-p}(-t)^{1-p} \,. \tag{8.8}$$

The background quantities are given by

$$
\begin{aligned}
a(\tau) &= \tilde{a}_0(-\tau)^{\frac{p}{1-p}}, \\
H(\tau) &= p(1-p)^{-\frac{1}{1-p}}(-\tau)^{-\frac{1}{1-p}} \\
\mathcal{H}(\tau) &\equiv \frac{da}{ad\tau} = -\left(\frac{p}{1-p}\right)\frac{1}{\tau}.
\end{aligned}
\tag{8.9}
$$

In [111] it was suggested that an S-brane (a space-like brane) will be generated once the background energy density reaches the string scale. The S-brane is an object with vanishing energy density and negative pressure (since it has positive tension). Hence, it is an object which violates the NEC (null energy condition) and allows for the nonsingular transition between the Ekpyrotic contracting phase and a radiation dominated phase of expansion.

8.3 States for super-Hubble modes

In order to study the entanglement entropy due to mode-coupling between cosmological perturbations, we study linear fluctuations about a spatially flat FLRW metric. In longitudinal gauge, the metric can be written as [42, 105]

$$
ds^2 = -a^2(\tau)\Big[\mathrm{d}\tau^2(1+2\phi) - (1+2\psi)\mathrm{d}\mathbf{x}^2\Big], \tag{8.10}
$$

where ϕ and ψ are the metric perturbation variables. A particularly useful variable is the curvature fluctuation ζ in uniform density gauge which is given by

$$\zeta \;=\; -\psi + \frac{H}{\dot{\rho}}\delta\rho\,, \tag{8.11}$$

where ρ is the background energy density and $\delta\rho$ is the density perturbation. If matter is a single field minimally coupled to Einstein gravity (as is the case in the Ekpyrotic scenario), the variable in terms of which the action for fluctuations has canonical form is

$$v(\boldsymbol{x},\tau) \;\equiv\; z(\tau)\zeta(\boldsymbol{x},\tau)\,, \tag{8.12}$$

where

$$z(\tau) \;=\; \sqrt{2\epsilon}\; a\; M_{\rm Pl}c_s^{-1}\,, \tag{8.13}$$

where c_s^2 is the speed of sound squared of the matter source. We can expand this field in Fourier modes and canonically quantize it, introducing mode creation and annihilation operators $a_k^\dagger$ and a_k, respectively.

The Hamiltonian derived from the quadratic action in momentum space is organized as follows (see e.g. [142])

$$\begin{aligned} H_2 &= \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[c_s k(a_{\boldsymbol{k}}a^\dagger_{\boldsymbol{k}} + a_{-\boldsymbol{k}}a^\dagger_{-\boldsymbol{k}})\right] \\ &-\frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[i\left(\frac{z'}{z}\right)(a_{\boldsymbol{k}}a_{-\boldsymbol{k}} - a^\dagger_{\boldsymbol{k}}a^\dagger_{-\boldsymbol{k}})\right], \end{aligned} \tag{8.14}$$

where a prime denotes a derivative with respect to conformal time. The second term is the source of the two-mode squeezing term while the first one is the usual free part of the quadratic Hamiltonian, as in Minkowski space.

In the Heisenberg picture, the equation for $a_{\boldsymbol{k}}$ is written

$$\frac{da_{\boldsymbol{k}}}{d\tau} = \frac{z'}{z}a^\dagger_{\boldsymbol{k}} - ic_s k a_{\boldsymbol{k}}\,. \tag{8.15}$$

In the case of inflation $\frac{z'}{z} = -\frac{1}{\tau}$. Hence, on super-Hubble scales (for which the momentum term on the right hand side of (8.15) is negligible), the z term results in a highly squeezed vacuum. In contrast, during a phase of slow Ekpyrotic contraction $\frac{z'}{z} = -\frac{p}{(1-p)\tau}$ for super-Hubble modes. Since $p \ll 1$ the amount of squeezing is very small. In the limit $p \to 0$, the equation for the annihilation operators becomes the same as in flat spacetime quantum field theory.

Since the squeezing term is sub-dominant for Ekpyrosis, we can safely ignore the squeezing entropy which plays a crucial role for inflation, and focus on the entanglement entropy generated by the leading gravitational nonlinearities, i.e. by the cubic interaction term in the Hamitonian. This will be the topic of the following sections.

8.4 Entanglement entropy for scalar perturbations due to gravitational non-linearities

In this section we turn to the computation of the momentum space entanglement entropy between super- and sub-Hubble modes generated by the gravitational nonlinearities. In applications to black hole physics and in the context of the AdS/CFT correspondence, entanglement entropy is usually considered in terms of position space entanglement [143–147]. However, when considering the entropy of cosmological perturbations, it is more natural to consider momentum space entanglement. The reason is that the basic variables are the momentum modes of the fluctuations. The modes that classicalize and become accessible to cosmological experiments are the super-Hubble modes, with the sub-Hubble modes acting as a sea which are not directly accessible. Hence, it is natural to consider the entanglement between super- and sub-Hubble modes. In a previous paper [140], momentum space entanglement entropy was studied in the context of inflation, generalizing the formalism developed in [148] (see also [149–153] for earlier work on the entropy of cosmological perturbations).

The entanglement entropy due to the mode coupling by the leading order cubic nonlinear terms can be calculated as follows. First, we break up the Hilbert space of scalar perturbations into sub-Hubble and super-Hubble modes, *i.e.*

$$\mathcal{H} \;=\; \mathcal{H}_{\rm sub} \otimes \mathcal{H}_{\rm super}\,. \tag{8.16}$$

The super-Hubble modes shall be treated as our system which is coupled to the bath of sub-Hubble modes. We shall carry out this calculation in Fourier space. Since our analysis is in the framework of an effective field theory, we need to apply $M_{\rm Pl}$ as a physical cutoff for our sub-Hubble modes. The total Hamiltonian can be described as

$$\mathbf{H} \;=\; \mathbf{H}_{\rm sub} \otimes \mathbb{I} + \mathbb{I} \otimes \mathbf{H}_{\rm super} + \mathbf{H}_{\rm int}\,, \tag{8.17}$$

where $\mathbf{H}_{\rm sub}$ and $\mathbf{H}_{\rm super}$ refer to the quadratic Hamiltonian for the scalar modes with momenta $k < aH$ and $k > aH$, respectively. $\mathbf{H}_{\rm int}$ refers to the interaction Hamiltonian due to our cubic non-Gaussian term.

As discussed in [140] (based on [154] and [43]), in terms of the variable ζ, the integrated interaction Lagrangian is given by

$$\begin{aligned} S_3 \;=\; M_{\rm Pl}^2 \int dt\, \mathrm{d}^3x\, \Big[& a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 \\ & -\frac{\mathrm{d}}{dt}\left(a^3\epsilon^2\zeta^2\dot{\zeta}\right)\Big] \end{aligned} \tag{8.18}$$

where we neglected nonlocal terms and used the fact that ϵ is constant. Since in the case of the Ekpyrotic scenario the dominant mode of ζ is constant, the leading interaction term in the action is

$$S_3 = M_{\rm Pl}^2 \int dt\, \mathrm{d}^3 x a \epsilon^2 \zeta (\partial\zeta)^2 \,. \tag{8.19}$$

Using conformal time, and in terms of the canonical variable v, this interaction term takes the form

$$S_3 = \frac{\sqrt{\epsilon}}{2\sqrt{2}\, a\, M_{\rm Pl}} \int \mathrm{d}\tau\, \mathrm{d}^3\mathbf{x}\, v\, (\partial v)^2 \tag{8.20}$$

To make contact with the formalism to compute the entanglement entropy developed in [148] and applied in [140] to the case of inflationary cosmology, we note that the effective coupling λ is

$$\lambda = \frac{\sqrt{\epsilon}}{2\sqrt{2} a M_{\rm Pl}} \,, \tag{8.21}$$

from which we can define a dimensionless interaction parameter, given by

$$\tilde{\lambda} = \frac{\sqrt{\epsilon}\Lambda}{2\sqrt{2} a M_{\rm Pl}} \,, \tag{8.22}$$

where Λ is a renormalization scale which we expect to be the Planck scale.

8.5 Entanglement Entropy of scalar perturbations

As a consequence of the nonlinearities, an initial vacuum state of both system and bath modes gets excited. At time t the state $|\Omega\rangle$ becomes

$$\begin{aligned} |\Omega\rangle(t) \;=\; |0,0\rangle \;\; &+ \;\; \sum_{n\neq 0} A_n(t)|n,0\rangle + \sum_{n\neq 0} B_N(t)|0,N\rangle \\ &+ \;\; \sum_{n,N\neq 0} C_{n,N}(t)|n,N\rangle \,, \end{aligned} \tag{8.23}$$

where $|n\rangle$ denotes an n-particle state of the system (in fact, a product state over all super-Hubble k modes), and $|N\rangle$ is the corresponding state for the bath. At the initial time, the coefficients A_n, B_N and $C_{n,N}$ all vanish, and they build up gradually over time due to the gravitational interactions.

As shown in [148] and generalized in [140] in the case of a time-dependent background, the induced entanglement entropy between the super- and sub-Hubble modes is induced by the interaction coefficients $C_{n,N}$ in the following way

$$S_{\rm ent} \;=\; -\lambda^2 \log\left(\tilde{\lambda}^2\right) \sum_{n,N\neq 0} |\tilde{C}_{n,N}|^2 \,. \tag{8.24}$$

For an infinite set of modes labelled by a continuous index k, the summation becomes a momentum space integral. We use the dimensionless effective coupling as the argument of the logarithm to make this term well-defined. However, in the end, this term will not be too important for our purposes as we shall ignore the logarithm term to focus only on the rest of the factors to get an order of magnitude estimate for the bound on the energy scale of the bounce.

We follow the analyses of [140] in order to calculate the entanglement entropy in this case. For the contracting phase in Ekpyrosis, the squeezing of the state of fluctuations on super-Hubble scale is negligible and we can set $r_k(\eta) \sim 0$ for the calculation. Hence, the general formula (62) of [140] simplifies to yield the following expression for the induced entanglement entropy density per comoving volume (please see the Appendix for details)

$$
\begin{aligned}
s \; &= (2\pi)^3 \lambda^2 \log\left(\tilde{\lambda}^2\right) \int_{k_I}^{aH} \mathrm{d}^3 p_3 \int_{aH}^{aM_{\mathrm{Pl}}} \mathrm{d}^3 p_2 \int_{aH}^{aM_{\mathrm{Pl}}} \mathrm{d}^3 p_1 \, \delta^3(p_1 + p_2 + p_3) \, \frac{1}{(p_1+p_2+p_3)^2} \left(\frac{p_1 p_2}{p_3}\right) \\
&= -\, \lambda^2 \log\left(\tilde{\lambda}^2\right) \int_{k_I}^{aH} \frac{\mathrm{d}^3 p_3}{(2\pi)^3} \int_{aH}^{aM_{pl}} \frac{\mathrm{d}^3 p_2}{(2\pi)^3} \left(\frac{p_2 \sqrt{p_2^2 + p_3^2 + 2 p_2 p_3 \cos\theta}}{p_3}\right) \\
&\quad \frac{1}{(\sqrt{p_2^2 + p_3^2 + 2 p_2 p_3 \cos\theta} + p_2 + p_3)^2}
\end{aligned}
\tag{8.25}
$$

Notice that the super-Hubble mode p_3 gets an infrared cut-off by k_I while the sub-Hubble mode p_2 has a natural, physical UV-cutoff given by aM_{pl}. The meaning of the IR cut-off can be thought of as follows: If there is a phase prior to Ekpyrosis, then we shall only consider super-Hubble modes generated during Ekpyrosis for our calculations. However, note that unlike in the case of inflation, we are free to take the limit $k_I \to 0$ for our integral without encountering any divergences. Generally we take $p_3 \ll p_2$ when (8.25) is evaluated. Thus, we obtain

$$
\begin{aligned}
s &\approx \frac{\lambda^2 \log\left(\tilde{\lambda}^2\right)}{4\pi^4} \int_{k_I}^{aH} dp_3\, p_3^2 \int_{aH}^{aM_{\rm Pl}} dp_2\, p_2^2 \frac{1}{4p_3} \\
&\approx \frac{\lambda^2 \log\left(\tilde{\lambda}^2\right)}{48\pi^4} \left[(aH)^2 - k_I^2\right] \cdot \left[(aM_{\rm Pl})^3 - (aH)^3\right] \\
&\sim \frac{1}{192\pi^4 p} \log\left(\frac{1}{2\sqrt{2}pa}\right) a^3 M_{\rm Pl} H^2\,.
\end{aligned}
\tag{8.26}
$$

In the last line, we have replaced the slow-roll parameter ϵ by the small number p and set $\Lambda = M_{\rm Pl}$. Dividing by the factor a^3 we obtain the the entropy density per physical volume element. We see that the entanglement entropy (per unit physical volume) grows logarithmically as a function of time in the contracting phase, ignoring the time-dependence of H.

If we interpret entanglement entropy as a contribution to the entropy which should obey the second law of thermodynamics, then we can obtain an upper bound on the energy scale of the bounce (or equivalently on the value of H just before the bounce) by demanding that the entanglement entropy (8.26) be smaller than the thermal entropy density after the bounce which is calculated as follows

$$
\frac{s}{V_0} = \frac{4}{3}\Big(\frac{30}{\pi^2 g}\Big)^{1/4} \rho^{1/4} = \frac{4}{3}\Big(\frac{90}{\pi^2 g}\Big)^{1/4} H^{3/2} M_{pl}^{3/2} \tag{8.27}
$$

where g is the number of effective spin degrees of freedom in the thermal bath after the bounce. From (8.27) and (8.26) we obtain the condition

$$\left(\frac{H}{M_{\rm Pl}}\right)^{1/2} < \, 256\pi^{7/2}\left(\frac{90}{g}\right)^{1/4} p \, . \tag{8.28}$$

This sets an upper bound on the string scale, the energy scale where the S-brane will occur. It is not a very stringent bound. Assuming that the S-brane arises at string energy density, then the above equation yields the corresponding bound on the string energy scale in Planck units.

8.6 Conclusions

We have computed the entanglement entropy between sub- and super-Hubble modes during a phase of Ekpyrotic contraction. We found that this entropy grows logarithmically with decreasing scale factor. Demanding that the entanglement entropy at the end of the phase of contraction does not exceed the thermal entropy after the bounce, we obtain an upper bound on the energy scale of the bounce.

Appendix: Calculation of entanglement entropy

In order to explicitly calculate the entanglement entropy, we first need to evaluate the matrix elements, given by

$$C_{n,N} := -i \int_{\tau_0}^{\tau} \mathrm{d}\tau' \, e^{i(p_1+p_2+p_3)\tau'} \, \langle n, N| \, H_{\rm int}(\tau') \, |0,0\rangle \, . \tag{8.29}$$

Note that the interaction Hamiltonian can be derived from (8.20) and is given by

$$\begin{aligned}\lambda(\tau) H_{\rm int} \quad &= \quad \lambda(\tau) \int_\Delta \left[\sqrt{\frac{k_2 k_3}{k_1}} \left(c^\dagger_{-\mathbf{k}_1} c^\dagger_{-\mathbf{k}_2} c^\dagger_{-\mathbf{k}_3} + c_{\mathbf{k}_1} c^\dagger_{-\mathbf{k}_2} c^\dagger_{-\mathbf{k}_3} + \dots \right) + \sqrt{\frac{k_2 k_1}{k_3}} \left(c^\dagger_{-\mathbf{k}_1} c^\dagger_{-\mathbf{k}_2} c^\dagger_{-\mathbf{k}_3} + \dots \right) \right. \\ &\left. + \sqrt{\frac{k_1 k_3}{k_2}} \left(c^\dagger_{-\mathbf{k}_1} c^\dagger_{-\mathbf{k}_2} c^\dagger_{-\mathbf{k}_3} + \dots \right) \right] \, ,\end{aligned} \tag{8.30}$$

where λ is given by (8.21) and we have defined $\int_\Delta := \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} \frac{\mathrm{d}^3 k_2}{(2\pi)^3} \frac{\mathrm{d}^3 k_3}{(2\pi)^3} \, (2\pi)^3 \, \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$ and the 'dots' are permutations.

Since the squeezing is negligible for Ekpyrosis, both the sub- and super-Hubble modes are in the Minkowski vacuum. This makes the evaluation of $\langle n, N| \, H_{\rm int}(\tau') \, |0,0\rangle$ relatively simpler compared to the case for inflation, and we need to only consider terms in $H_{\rm int}$ of the form $c^\dagger_{-\mathbf{k}} c^\dagger_{-\mathbf{k}} c^\dagger_{-\mathbf{k}}$, where two of the modes can be sub-Hubble and one super-Hubble or the other way around. Once again, since the vacuum state is the same for both sub- and super-Hubble modes, the dominant contribution is going to be from the case where there are two super Hubble modes. We find:

$$\langle n, N| \, H_{\rm int}(\tau') \, |0,0\rangle \sim \sqrt{\frac{p_1 p_2}{p_3}}\,, \tag{8.31}$$

where p_1, p_2 are sub-Hubble and p_3 is the super-Hubble mode. (The other terms proportional to $\sqrt{p_1 p_3/p_2}$ and $\sqrt{p_2 p_3/p_1}$ are sub-dominant in this case.) Plugging this into the expression for the matrix element (8.29), we find

$$C_{n,N} = -i \int_{\tau_0}^{\eta} \mathrm{d}\tau' \, e^{i(p_1+p_2+p_3)\tau'} \sqrt{\frac{p_1 p_2}{p_3}}\,. \tag{8.32}$$

This is where the negligible squeezing for Ekpyrosis plays a crucial role so that we can evaluate the matrix elements as if for *time-independent* perturbation theory. Using this, one can calculate the entanglement entropy as

$$\begin{aligned} S_{\rm ent} \;\; \sim \;\; (2\pi)^3 \;\; & \int_H^{aH} \frac{\mathrm{d}^3 p_3}{(2\pi)^3} \int_{aH}^{aM_{\rm Pl}} \frac{\mathrm{d}^3 p_2}{(2\pi)^3} \int_{aH}^{aM_{\rm Pl}} \frac{\mathrm{d}^3 p_1}{(2\pi)^3} \, \delta^3(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3) \left(\frac{p_1 p_2}{p_3}\right) \times \\ & \left| \int_{\tau_0}^{\tau} \mathrm{d}\tau' \; e^{i(p_1+p_2+p_3)\eta'} \right|^2 \lambda^2(\tau') \log\left(\tilde{\lambda}^2(\tau')\right) \;\; . \end{aligned} \tag{8.33}$$

The time-dependence of the coupling parameter $\lambda(\tau)$ is extremely weak in Ekpyrosis since it is only due to the presence of the scale factor $a(\tau)$, which itself changes very slowly in conformal time. Therefore, one can pull the $\lambda^2 \log\left(\tilde{\lambda}^2\right)$ term outside the integral and then one arrives at the result given in (8.24), with $\tilde{C}_{n,N}$ being the time-independent matrix elements. As shown in this Appendix, in the case of Ekpyrosis, one can easily work with the time-independent matrix elements (due to the weak dependence of the interaction parameter on conformal time and having an usual flat vacuum for the super-Hubble modes) and we recover the results discussed in the main body of the text.

Chapter 9

Conclusion and Future work

In this thesis, I propose a new mechanism generating Ekpyrosis as an alternative scenario to inflation, which can successfully solve the puzzles of the Hot Big Bang Universe and avoid other theoretical inconsistencies of inflation.

In chapter 5, we propose an Ekpyrotic model with an S-brane located at the bouncing point and show that this results in a nearly scale-invariant tensor power spectrum which is different from results of other Ekpyrotic models, which generally predict a highly blue tilted vacuum spectrum. In chapter 6, the matching conditions at the matching surface are discussed in detail. By carefully choosing the matching condition on a zero-shear surface, a slightly red tilted scalar power spectrum is obtained with the correct amplitude. All of the results from chapters 5 and 6 are either consistent with current cosmological observation or waiting for future tests.

In chapter 7, a reheating mechanism after the Ekpyrotic contraction phase is studied. We illustrate that the decaying process of the S-brane into radiation fields is efficient enough to generate matter to support Hot Big Bang Cosmology. In chapter 8, the entropy of cosmological fluctuations during the contracting phase is calculated. The result provides a theoretical lower bound on the reheating temperature after the bounce.

Current cosmological observations measures non-gaussianities at higher precision in the past, which can be a powerful tool to distinguish or falsify different early universe models. By studying the non-gaussianities of cosmological fluctuations from S-brane Ekpyrosis, the parameters space of the model couble be constrained more. Besides, as mentioned in chapter 4, the ekpyrotic contraction is an attractor towards spatial flatness. It is promising to explore whether the existence of the S-brane will break the solution. These are topics for further research.

Bibliography

[1] Planck collaboration, N. Aghanim et al., *Planck 2018 results. I. Overview and the cosmological legacy of Planck*, *Astron. Astrophys.* **641** (2020) A1, [1807.06205].

[2] R. H. Brandenberger, *String Gas Cosmology: Progress and Problems*, *Class. Quant. Grav.* **28** (2011) 204005, [1105.3247].

[3] Y.-F. Cai, T. Qiu, Y.-S. Piao, M. Li and X. Zhang, *Bouncing universe with quintom matter*, *JHEP* **10** (2007) 071, [0704.1090].

[4] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, *The Ekpyrotic universe: Colliding branes and the origin of the hot big bang*, *Phys. Rev. D* **64** (2001) 123522, [hep-th/0103239].

[5] C. L. Bennett, A. Banday, K. M. Gorski, G. Hinshaw, P. Jackson, P. Keegstra et al., *Four year COBE DMR cosmic microwave background observations: Maps and basic results*, *Astrophys. J. Lett.* **464** (1996) L1–L4, [astro-ph/9601067].

[6] WMAP collaboration, C. L. Bennett et al., *First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Preliminary maps and basic results*, *Astrophys. J. Suppl.* **148** (2003) 1–27, [astro-ph/0302207].

[7] Planck collaboration, P. A. R. Ade et al., *Planck 2013 results. I. Overview of products and scientific results*, *Astron. Astrophys.* **571** (2014) A1, [1303.5062].

[8] Planck collaboration, R. Adam et al., *Planck 2015 results. I. Overview of products and scientific results*, *Astron. Astrophys.* **594** (2016) A1, [1502.01582].

[9] D. J. Fixsen, *The Temperature of the Cosmic Microwave Background*, *Astrophys. J.* **707** (2009) 916–920, [0911.1955].

[10] S. Dodelson, *Modern Cosmology*. Academic Press, Amsterdam, 2003.

[11] A. Lewis, A. Challinor and A. Lasenby, *Efficient computation of CMB anisotropies in closed FRW models*, *Astrophys. J.* **538** (2000) 473–476, [astro-ph/9911177].

[12] U. Seljak and M. Zaldarriaga, *Signature of gravity waves in polarization of the microwave background*, *Phys. Rev. Lett.* **78** (1997) 2054–2057, [astro-ph/9609169].

[13] M. Kamionkowski, A. Kosowsky and A. Stebbins, *Statistics of cosmic microwave background polarization*, *Phys. Rev. D* **55** (1997) 7368–7388, [astro-ph/9611125].

[14] Planck collaboration, N. Aghanim et al., *Planck 2018 results. VI. Cosmological parameters*, *Astron. Astrophys.* **641** (2020) A6, [1807.06209].

[15] V. Mukhanov, *Physical Foundations of Cosmology.* Cambridge University Press, Oxford, 2005.

[16] G. F. R. Ellis, *Relativistic cosmology*, *Proc. Int. Sch. Phys. Fermi* **47** (1971) 104–182.

[17] P. Peter and J.-P. Uzan, *Primordial Cosmology.* Oxford Graduate Texts. Oxford University Press, 2, 2013.

[18] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation.* W. H. Freeman, San Francisco, 1973.

[19] G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, *De Sitter Space and the Swampland*, 1806.08362.

[20] H. Ooguri, E. Palti, G. Shiu and C. Vafa, *Distance and de Sitter Conjectures on the Swampland*, *Phys. Lett. B* **788** (2019) 180–184, [1810.05506].

[21] E. Palti, *The Swampland: Introduction and Review*, *Fortsch. Phys.* **67** (2019) 1900037, [1903.06239].

[22] P. Agrawal, G. Obied, P. J. Steinhardt and C. Vafa, *On the Cosmological Implications of the String Swampland*, *Phys. Lett. B* **784** (2018) 271–276, [1806.09718].

[23] J. Martin and R. H. Brandenberger, *The TransPlanckian problem of inflationary cosmology*, *Phys. Rev. D* **63** (2001) 123501, [hep-th/0005209].

[24] A. Bedroya and C. Vafa, *Trans-Planckian Censorship and the Swampland*, *JHEP* **09** (2020) 123, [1909.11063].

[25] A. Bedroya, R. Brandenberger, M. Loverde and C. Vafa, *Trans-Planckian Censorship and Inflationary Cosmology*, *Phys. Rev. D* **101** (2020) 103502, [1909.11106].

[26] R. H. Brandenberger, *Introduction to Early Universe Cosmology*, *PoS* **ICFI2010** (2010) 001, [1103.2271].

[27] R. H. Brandenberger, *Alternatives to the inflationary paradigm of structure formation*, *Int. J. Mod. Phys. Conf. Ser.* **01** (2011) 67–79, [0902.4731].

[28] R. H. Brandenberger and C. Vafa, *Superstrings in the Early Universe*, *Nucl. Phys. B* **316** (1989) 391–410.

[29] R. H. Brandenberger, A. Nayeri, S. P. Patil and C. Vafa, *String gas cosmology and structure formation*, *Int. J. Mod. Phys. A* **22** (2007) 3621–3642, [hep-th/0608121].

[30] A. Nayeri, R. H. Brandenberger and C. Vafa, *Producing a scale-invariant spectrum of perturbations in a Hagedorn phase of string cosmology*, *Phys. Rev. Lett.* **97** (2006) 021302, [hep-th/0511140].

[31] Y.-F. Cai, S.-H. Chen, J. B. Dent, S. Dutta and E. N. Saridakis, *Matter Bounce Cosmology with the f(T) Gravity*, *Class. Quant. Grav.* **28** (2011) 215011, [1104.4349].

[32] Y.-F. Cai, D. A. Easson and R. Brandenberger, *Towards a Nonsingular Bouncing Cosmology*, *JCAP* **08** (2012) 020, [1206.2382].

[33] F. Di Marco, F. Finelli and R. Brandenberger, *Adiabatic and isocurvature perturbations for multifield generalized Einstein models*, *Phys. Rev. D* **67** (2003) 063512, [astro-ph/0211276].

[34] P. Horava, *Quantum Gravity at a Lifshitz Point*, *Phys. Rev. D* **79** (2009) 084008, [0901.3775].

[35] R. Brandenberger, *Matter Bounce in Horava-Lifshitz Cosmology*, *Phys. Rev. D* **80** (2009) 043516, [0904.2835].

[36] J. K. Erickson, D. H. Wesley, P. J. Steinhardt and N. Turok, *Kasner and mixmaster behavior in universes with equation of state w >= 1*, *Phys. Rev. D* **69** (2004) 063514, [hep-th/0312009].

[37] P. J. Steinhardt and N. Turok, *Cosmic evolution in a cyclic universe*, *Phys. Rev. D* **65** (2002) 126003, [hep-th/0111098].

[38] P. J. Steinhardt, N. Turok and N. Turok, *A Cyclic model of the universe*, *Science* **296** (2002) 1436–1439, [hep-th/0111030].

[39] E. I. Buchbinder, J. Khoury and B. A. Ovrut, *New Ekpyrotic cosmology*, *Phys. Rev. D* **76** (2007) 123503, [hep-th/0702154].

[40] J. Khoury and P. J. Steinhardt, *Adiabatic Ekpyrosis: Scale-Invariant Curvature Perturbations from a Single Scalar Field in a Contracting Universe*, *Phys. Rev. Lett.* **104** (2010) 091301, [0910.2230].

[41] D. Baumann, *Inflation*, in *Theoretical Advanced Study Institute in Elementary Particle Physics: Physics of the Large and the Small*, pp. 523–686, 2011. 0907.5424. DOI.

[42] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, *Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions*, *Phys. Rept.* **215** (1992) 203–333.

[43] J. M. Maldacena, *Non-Gaussian features of primordial fluctuations in single field inflationary models*, *JHEP* **05** (2003) 013, [astro-ph/0210603].

[44] A. R. Liddle and D. H. Lyth, *Cosmological inflation and large scale structure.* 2000.

[45] J. M. Bardeen, *Gauge Invariant Cosmological Perturbations*, *Phys. Rev. D* **22** (1980) 1882–1905.

[46] R. Arnowitt, S. Deser and C. W. Misner, *Dynamical structure and definition of energy in general relativity*, *Phys. Rev.* **116** (Dec, 1959) 1322–1330.

[47] D. H. Lyth, *What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?*, *Phys. Rev. Lett.* **78** (1997) 1861–1863, [hep-ph/9606387].

[48] A. H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, *Phys. Rev. D* **23** (1981) 347–356.

[49] R. Brout, F. Englert and E. Gunzig, *The Creation of the Universe as a Quantum Phenomenon*, *Annals Phys.* **115** (1978) 78.

[50] A. A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, *Phys. Lett. B* **91** (1980) 99–102.

[51] K. Sato, *First Order Phase Transition of a Vacuum and Expansion of the Universe*, *Mon. Not. Roy. Astron. Soc.* **195** (1981) 467–479.

[52] V. F. Mukhanov and G. V. Chibisov, *Quantum Fluctuations and a Nonsingular Universe*, *JETP Lett.* **33** (1981) 532–535.

[53] A. A. Starobinsky, *Spectrum of relict gravitational radiation and the early state of the universe*, *JETP Lett.* **30** (1979) 682–685.

[54] A. Berera, *Warm inflation*, *Phys. Rev. Lett.* **75** (1995) 3218–3221, [astro-ph/9509049].

[55] H. Ooguri and C. Vafa, *On the Geometry of the String Landscape and the Swampland*, *Nucl. Phys. B* **766** (2007) 21–33, [hep-th/0605264].

[56] L. Heisenberg, M. Bartelmann, R. Brandenberger and A. Refregier, *Dark Energy in the Swampland*, *Phys. Rev. D* **98** (2018) 123502, [1808.02877].

[57] S. Das, *Warm Inflation in the light of Swampland Criteria*, *Phys. Rev. D* **99** (2019) 063514, [1810.05038].

[58] R. Penrose, *Gravitational collapse: The role of general relativity*, *Riv. Nuovo Cim.* **1** (1969) 252–276.

[59] C. Kiefer, D. Polarski and A. A. Starobinsky, *Quantum to classical transition for fluctuations in the early universe*, *Int. J. Mod. Phys. D* **7** (1998) 455–462, [gr-qc/9802003].

[60] N. Weiss, *Constraints on Hamiltonian Lattice Formulations of Field Theories in an Expanding Universe*, *Phys. Rev. D* **32** (1985) 3228.

[61] R. A. Sunyaev and Y. B. Zeldovich, *Small scale fluctuations of relic radiation*, *Astrophys. Space Sci.* **7** (1970) 3–19.

[62] P. J. E. Peebles and J. T. Yu, *Primeval adiabatic perturbation in an expanding universe*, *Astrophys. J.* **162** (1970) 815–836.

[63] M. Novello and S. E. P. Bergliaffa, *Bouncing Cosmologies*, *Phys. Rept.* **463** (2008) 127–213, [0802.1634].

[64] R. Brandenberger and P. Peter, *Bouncing Cosmologies: Progress and Problems*, *Found. Phys.* **47** (2017) 797–850, [1603.05834].

[65] R. H. Brandenberger, A. Nayeri, S. P. Patil and C. Vafa, *Tensor Modes from a Primordial Hagedorn Phase of String Cosmology*, *Phys. Rev. Lett.* **98** (2007) 231302, [hep-th/0604126].

[66] J. Khoury, B. A. Ovrut, N. Seiberg, P. J. Steinhardt and N. Turok, *From big crunch to big bang*, *Phys. Rev. D* **65** (2002) 086007, [hep-th/0108187].

[67] J. Khoury, P. J. Steinhardt and N. Turok, *Designing cyclic universe models*, *Phys. Rev. Lett.* **92** (2004) 031302, [hep-th/0307132].

[68] D. Baumann and L. McAllister, *Inflation and String Theory*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 5, 2015, 10.1017/CBO9781316105733.

[69] P. Horava and E. Witten, *Eleven-dimensional supergravity on a manifold with boundary*, *Nucl. Phys. B* **475** (1996) 94–114, [hep-th/9603142].

[70] P. Horava and E. Witten, *Heterotic and type I string dynamics from eleven-dimensions*, *Nucl. Phys. B* **460** (1996) 506–524, [hep-th/9510209].

[71] N. Turok, M. Perry and P. J. Steinhardt, *M theory model of a big crunch / big bang transition*, *Phys. Rev. D* **70** (2004) 106004, [hep-th/0408083].

[72] A. Ijjas and P. J. Steinhardt, *The anamorphic universe*, *JCAP* **10** (2015) 001, [1507.03875].

[73] A. Ijjas and P. J. Steinhardt, *A new kind of cyclic universe*, *Phys. Lett. B* **795** (2019) 666–672, [1904.08022].

[74] Y.-F. Cai, R. Brandenberger and P. Peter, *Anisotropy in a Nonsingular Bounce*, *Class. Quant. Grav.* **30** (2013) 075019, [1301.4703].

[75] R. H. Brandenberger and J. H. Kung, *Chaotic Inflation as an Attractor in Initial Condition Space*, *Phys. Rev. D* **42** (1990) 1008–1015.

[76] H. A. Feldman and R. H. Brandenberger, *Chaotic Inflation With Metric and Matter Perturbations*, *Phys. Lett. B* **227** (1989) 359–366.

[77] W. E. East, M. Kleban, A. Linde and L. Senatore, *Beginning inflation in an inhomogeneous universe*, *JCAP* **09** (2016) 010, [1511.05143].

[78] K. Clough, E. A. Lim, B. S. DiNunno, W. Fischler, R. Flauger and S. Paban, *Robustness of Inflation to Inhomogeneous Initial Conditions*, *JCAP* **09** (2017) 025, [1608.04408].

[79] R. Brandenberger, *Initial conditions for inflation — A short review*, *Int. J. Mod. Phys. D* **26** (2016) 1740002, [1601.01918].

[80] D. S. Goldwirth and T. Piran, *Initial conditions for inflation*, *Phys. Rept.* **214** (1992) 223–291.

[81] M. Gasperini and G. Veneziano, *Pre - big bang in string cosmology*, *Astropart. Phys.* **1** (1993) 317–339, [hep-th/9211021].

[82] R. Brandenberger and F. Finelli, *On the spectrum of fluctuations in an effective field theory of the Ekpyrotic universe*, *JHEP* **11** (2001) 056, [hep-th/0109004].

[83] D. H. Lyth, *The Primordial curvature perturbation in the ekpyrotic universe*, *Phys. Lett. B* **524** (2002) 1–4, [hep-ph/0106153].

[84] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, *Density perturbations in the ekpyrotic scenario*, *Phys. Rev. D* **66** (2002) 046005, [hep-th/0109050].

[85] S. Gratton, J. Khoury, P. J. Steinhardt and N. Turok, *Conditions for generating scale-invariant density perturbations*, *Phys. Rev. D* **69** (2004) 103505, [astro-ph/0301395].

[86] A. Notari and A. Riotto, *Isocurvature perturbations in the ekpyrotic universe*, *Nucl. Phys. B* **644** (2002) 371–382, [hep-th/0205019].

[87] F. Finelli, *Assisted contraction*, *Phys. Lett. B* **545** (2002) 1–7, [hep-th/0206112].

[88] J.-L. Lehners, P. McFadden, N. Turok and P. J. Steinhardt, *Generating ekpyrotic curvature perturbations before the big bang*, *Phys. Rev. D* **76** (2007) 103501, [hep-th/0702153].

[89] P. Creminelli and L. Senatore, *A Smooth bouncing cosmology with scale invariant spectrum*, *JCAP* **11** (2007) 010, [hep-th/0702165].

[90] A. Ijjas, J.-L. Lehners and P. J. Steinhardt, *General mechanism for producing scale-invariant perturbations and small non-Gaussianity in ekpyrotic models*, *Phys. Rev. D* **89** (2014) 123520, [1404.1265].

[91] A. Fertig, J.-L. Lehners, E. Mallwitz and E. Wilson-Ewing, *Converting entropy to curvature perturbations after a cosmic bounce*, *JCAP* **10** (2016) 005, [1607.05663].

[92] J. Khoury and P. J. Steinhardt, *Generating Scale-Invariant Perturbations from Rapidly-Evolving Equation of State*, *Phys. Rev. D* **83** (2011) 123502, [1101.3548].

[93] R. Durrer and F. Vernizzi, *Adiabatic perturbations in pre - big bang models: Matching conditions and scale invariance*, *Phys. Rev. D* **66** (2002) 083503, [hep-ph/0203275].

[94] C. Cartier, R. Durrer and E. J. Copeland, *Cosmological perturbations and the transition from contraction to expansion*, *Phys. Rev. D* **67** (2003) 103517, [hep-th/0301198].

[95] A. J. Tolley, N. Turok and P. J. Steinhardt, *Cosmological perturbations in a big crunch / big bang space-time*, *Phys. Rev. D* **69** (2004) 106005, [hep-th/0306109].

[96] R. H. Brandenberger, E. G. M. Ferreira, I. A. Morrison, Y.-F. Cai, S. R. Das and Y. Wang, *Fluctuations in a cosmology with a spacelike singularity and their gauge theory dual description*, *Phys. Rev. D* **94** (2016) 083508, [1601.00231].

[97] E. G. M. Ferreira and R. Brandenberger, *Holographic Curvature Perturbations in a Cosmology with a Space-Like Singularity*, *JCAP* **07** (2016) 030, [1602.08152].

[98] D. A. Easson, I. Sawicki and A. Vikman, *G-bounce*, *Journal of Cosmology and Astroparticle Physics* **2011** (Nov, 2011) 021–021.

[99] A. Ijjas and P. J. Steinhardt, *Fully stable cosmological solutions with a non-singular classical bounce*, *Phys. Lett. B* **764** (2017) 289–294, [1609.01253].

[100] A. Ijjas and P. J. Steinhardt, *Classically stable nonsingular cosmological bounces*, *Phys. Rev. Lett.* **117** (2016) 121304, [1606.08880].

[101] S. Watson, *Moduli stabilization with the string Higgs effect*, *Phys. Rev. D* **70** (2004) 066005, [hep-th/0404177].

[102] L. Kofman, A. D. Linde, X. Liu, A. Maloney, L. McAllister and E. Silverstein, *Beauty is attractive: Moduli trapping at enhanced symmetry points*, *JHEP* **05** (2004) 030, [hep-th/0403001].

[103] R. H. Brandenberger, C. Kounnas, H. Partouche, S. P. Patil and N. Toumbas, *Cosmological Perturbations Across an S-brane*, *JCAP* **03** (2014) 015, [1312.2524].

[104] C. Kounnas, H. Partouche and N. Toumbas, *S-brane to thermal non-singular string cosmology*, *Class. Quant. Grav.* **29** (2012) 095014, [1111.5816].

[105] R. H. Brandenberger, *Lectures on the theory of cosmological perturbations*, *Lect. Notes Phys.* **646** (2004) 127–167, [hep-th/0306071].

[106] M. Sasaki, *Large Scale Quantum Fluctuations in the Inflationary Universe*, *Prog. Theor. Phys.* **76** (1986) 1036.

[107] V. F. Mukhanov, *Quantum Theory of Gauge Invariant Cosmological Perturbations*, *Sov. Phys. JETP* **67** (1988) 1297–1302.

[108] M. B. Green, J. H. Schwarz and E. Witten, *SUPERSTRING THEORY. VOL. 1: INTRODUCTION*. Cambridge Monographs on Mathematical Physics. 7, 1988.

[109] M. B. Green, J. H. Schwarz and E. Witten, *SUPERSTRING THEORY. VOL. 2: LOOP AMPLITUDES, ANOMALIES AND PHENOMENOLOGY*. 7, 1988.

[110] R. H. Brandenberger, A. Nayeri and S. P. Patil, *Closed String Thermodynamics and a Blue Tensor Spectrum*, *Phys. Rev. D* **90** (2014) 067301, [1403.4927].

[111] R. Brandenberger and Z. Wang, *Nonsingular Ekpyrotic Cosmology with a Nearly Scale-Invariant Spectrum of Cosmological Perturbations and Gravitational Waves*, *Phys. Rev. D* **101** (2020) 063522, [2001.00638].

[112] T. D. Brennan, F. Carta and C. Vafa, *The String Landscape, the Swampland, and the Missing Corner*, *PoS* **TASI2017** (2017) 015, [1711.00864].

[113] R. H. Brandenberger, *String Gas Cosmology*, 8, 2008. 0808.0746.

[114] T. Battefeld and S. Watson, *String gas cosmology*, *Rev. Mod. Phys.* **78** (2006) 435–454, [hep-th/0510022].

[115] R. H. Brandenberger and R. Kahn, *COSMOLOGICAL PERTURBATIONS IN INFLATIONARY UNIVERSE MODELS*, *Phys. Rev. D* **29** (1984) 2172.

[116] R. Brandenberger, H. Firouzjahi and O. Saremi, *Cosmological Perturbations on a Bouncing Brane*, *JCAP* **11** (2007) 028, [0707.4181].

[117] S. Alexander, T. Biswas and R. H. Brandenberger, *On the Transfer of Adiabatic Fluctuations through a Nonsingular Cosmological Bounce*, 0707.4679.

[118] Y.-F. Cai, T. Qiu, R. Brandenberger, Y.-S. Piao and X. Zhang, *On Perturbations of Quintom Bounce*, *JCAP* **03** (2008) 013, [0711.2187].

[119] B. Xue and P. J. Steinhardt, *Unstable growth of curvature perturbation in non-singular bouncing cosmologies*, *Phys. Rev. Lett.* **105** (2010) 261301, [1007.2875].

[120] B. Xue and P. J. Steinhardt, *Evolution of curvature and anisotropy near a nonsingular bounce*, *Phys. Rev. D* **84** (2011) 083520, [1106.1416].

[121] N. Deruelle and V. F. Mukhanov, *On matching conditions for cosmological perturbations*, *Phys. Rev. D* **52** (1995) 5549–5555, [gr-qc/9503050].

[122] W. Israel, *Singular hypersurfaces and thin shells in general relativity*, *Nuovo Cim. B* **44S10** (1966) 1.

[123] C. Kounnas, H. Partouche and N. Toumbas, *Thermal duality and non-singular cosmology in d-dimensional superstrings*, *Nucl. Phys. B* **855** (2012) 280–307, [1106.0946].

[124] J.-L. Lehners and P. J. Steinhardt, *Intuitive understanding of non-gaussianity in ekpyrotic and cyclic models*, *Phys. Rev. D* **78** (2008) 023506, [0804.1293].

[125] J.-L. Lehners and P. J. Steinhardt, *Non-Gaussian density fluctuations from entropically generated curvature perturbations in Ekpyrotic models*, *Phys. Rev. D* **77** (2008) 063533, [0712.3779].

[126] S. K. Garg and C. Krishnan, *Bounds on Slow Roll and the de Sitter Swampland*, *JHEP* **11** (2019) 075, [1807.05193].

[127] G. Dvali, A. Kehagias and A. Riotto, *Inflation and Decoupling*, 2005.05146.

[128] R. H. Brandenberger, *Cosmology of the Very Early Universe*, *AIP Conf. Proc.* **1268** (2010) 3–70, [1003.1745].

[129] W. G. Cook, I. A. Glushchenko, A. Ijjas, F. Pretorius and P. J. Steinhardt, *Supersmoothing through Slow Contraction*, *Phys. Lett. B* **808** (2020) 135690, [2006.01172].

[130] J.-L. Lehners, *Ekpyrotic and Cyclic Cosmology*, *Phys. Rept.* **465** (2008) 223–263, [0806.1245].

[131] R. Brandenberger and Z. Wang, *Ekpyrotic cosmology with a zero-shear S-brane*, *Phys. Rev. D* **102** (2020) 023516, [2004.06437].

[132] A. D. Dolgov and D. P. Kirilova, *ON PARTICLE CREATION BY A TIME DEPENDENT SCALAR FIELD*, *Sov. J. Nucl. Phys.* **51** (1990) 172–177.

[133] J. H. Traschen and R. H. Brandenberger, *Particle Production During Out-of-equilibrium Phase Transitions*, *Phys. Rev. D* **42** (1990) 2491–2504.

[134] R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine and A. Mazumdar, *Reheating in Inflationary Cosmology: Theory and Applications*, *Ann. Rev. Nucl. Part. Sci.* **60** (2010) 27–51, [1001.2600].

[135] M. A. Amin, M. P. Hertzberg, D. I. Kaiser and J. Karouby, *Nonperturbative Dynamics Of Reheating After Inflation: A Review*, *Int. J. Mod. Phys. D* **24** (2014) 1530003, [1410.3808].

[136] M. Gutperle and A. Strominger, *Space - like branes*, *JHEP* **04** (2002) 018, [hep-th/0202210].

[137] J. A. Minahan and B. Zwiebach, *Gauge fields and fermions in tachyon effective field theories*, *JHEP* **02** (2001) 034, [hep-th/0011226].

[138] A. Sen, *Supersymmetric world volume action for nonBPS D-branes*, *JHEP* **10** (1999) 008, [hep-th/9909062].

[139] A. Sen, *Non-BPS D-branes in string theory*, *Class. Quant. Grav.* **17** (2000) 1251–1256.

[140] S. Brahma, O. Alaryani and R. Brandenberger, *Entanglement entropy of cosmological perturbations*, *Phys. Rev. D* **102** (2020) 043529, [2005.09688].

[141] R. Brandenberger, K. Dasgupta and Z. Wang, *Reheating after S-brane ekpyrosis*, *Phys. Rev. D* **102** (2020) 063514, [2007.01203].

[142] E. Nelson, *Quantum Decoherence During Inflation from Gravitational Nonlinearities*, *JCAP* **03** (2016) 022, [1601.03734].

[143] P. Calabrese and J. L. Cardy, *Entanglement entropy and quantum field theory*, *J. Stat. Mech.* **0406** (2004) P06002, [hep-th/0405152].

[144] S. Ryu and T. Takayanagi, *Holographic derivation of entanglement entropy from AdS/CFT*, *Phys. Rev. Lett.* **96** (2006) 181602, [hep-th/0603001].

[145] S. Ryu and T. Takayanagi, *Aspects of Holographic Entanglement Entropy*, *JHEP* **08** (2006) 045, [hep-th/0605073].

[146] J. Eisert, M. Cramer and M. B. Plenio, *Area laws for the entanglement entropy - a review*, *Rev. Mod. Phys.* **82** (2010) 277–306, [0808.3773].

[147] T. Nishioka, S. Ryu and T. Takayanagi, *Holographic Entanglement Entropy: An Overview*, *J. Phys. A* **42** (2009) 504008, [0905.0932].

[148] V. Balasubramanian, M. B. McDermott and M. Van Raamsdonk, *Momentum-space entanglement and renormalization in quantum field theory*, *Phys. Rev. D* **86** (2012) 045014, [1108.3568].

[149] S. S. Kumar and S. Shankaranarayanan, *Role of spatial higher order derivatives in momentum space entanglement*, *Phys. Rev. D* **95** (2017) 065023, [1702.08655].

[150] R. H. Brandenberger, T. Prokopec and V. F. Mukhanov, *The Entropy of the gravitational field*, *Phys. Rev. D* **48** (1993) 2443–2455, [gr-qc/9208009].

[151] R. H. Brandenberger, V. F. Mukhanov and T. Prokopec, *Entropy of a classical stochastic field and cosmological perturbations*, *Phys. Rev. Lett.* **69** (1992) 3606–3609, [astro-ph/9206005].

[152] M. Gasperini and M. Giovannini, *Entropy production in the cosmological amplification of the vacuum fluctuations*, *Phys. Lett. B* **301** (1993) 334–338, [gr-qc/9301010].

[153] M. Gasperini and M. Giovannini, *Quantum squeezing and cosmological entropy production*, *Class. Quant. Grav.* **10** (1993) L133–L136, [gr-qc/9307024].

[154] P. Adshead, W. Hu, C. Dvorkin and H. V. Peiris, *Fast Computation of Bispectrum Features with Generalized Slow Roll*, *Phys. Rev. D* **84** (2011) 043519, [1102.3435].