GEOMETRICAL PHYSICS: MATHEMATICS IN THE NATURAL PHILOSOPHY OF THOMAS HOBBES

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ABSTRACT

My thesis examines Thomas Hobbes's attempt to develop a mathematical account of nature. I argue that Hobbes's conception of how we should think quantitatively about the world was deeply indebted to the ideas of his ancient and medieval predecessors. These ideas were often amenable to Hobbes's vision of a demonstrative, geometrically-based science. However, he was forced to adapt the ancient and medieval models to the demands of his own thoroughgoing materialism. This hybrid resulted in a distinctive, if only partially successful, approach to the problems of the new mechanical philosophy.

ABRÉGÉ

Notre thèse consiste en une étude du projet de Thomas Hobbes de produire un compte rendu mathématique de la Nature. Nous affirmons que l'idée hobbesienne de la façon dont nous devons concevoir le monde, en termes quantitatifs, est profondément influencée par les vues de ses prédécesseurs antiques et médiévaux. Ces vues se sont souvent avérées compatibles avec l'idée hobbesienne d'une science démonstrative et fondée sur la géométrie. Cependant, Hobbes a dû adapter les modèles antiques et médiévaux aux exigences de son matérialisme intégral. Le résultat de ce croisement est une approche distinctive, bien que non entièrement satisfaisante, des problèmes de la nouvelle philosophie mécanique.

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ABBREVIATIONS

Works by Hobbes

CB — *Elements of Philosophy, the First Section, Concerning Body* (Hobbes 1656). References are to part, chapter, and section number, followed by a volume and page number from *EW* after a semicolon.

DCp — *Elementorum philosophiae sectio prima de corpore* (Hobbes 1655). References are to part, chapter, and section number, followed by a volume and page number from *OL* after a semicolon.

DP — *Dialogus Physicus* (Hobbes 1661). References are to the whole work, followed by a volume and page number from *OL* after a semicolon, followed by a page number from Hobbes (1985) after a second semicolon.

EL — *The Elements of Law Natural and Politic* (Hobbes 1994b). References are to part, chapter, and section number, followed by a page number from Hobbes (1994b) after a semicolon.

Ex — *Examinatio et Emendatio Mathematicae Hordiernae* (Hobbes 1660). References are to dialogue number, followed by a volume and page number from *OL* after a semicolon.

EW — *The English Works of Thomas Hobbes of Malmesbury, now First Collected and Edited by Sir William Molesworth* (Hobbes 1839-45b).

Lev — *Leviathan* (Hobbes 1651). References are to part and chapter number, followed by a page number from Hobbes (1991b) after a comma.

OL — *Thomae Hobbes Malmesburiensis Opera Philosophica Quae Latinae Scripsit Omnia in Unum Corpus Nunc Primum Collecta* (Hobbes 1829-45a).

PRG — *De Principiis et Ratiocinatione Geometrarum* (Hobbes 1666). References are to chapter number, followed by a volume and page number from *OL* after a semicolon.

SL — *Six Lessons to the Professors of the Mathematiques* (appendix to Hobbes 1656). References are to lesson number, followed by a volume and page number from *OL* after a semicolon.

TO — *Tractatus Opticus* (Hobbes 1644). References are to hypothesis or proposition number, followed by a volume and page number from *OL* after a semicolon.

TO II — *Tractatus Opticus II* (Hobbes 1963). References are to chapter and section number, followed by a page number from Hobbes (1963).

Works by Other Authors

CSM — *The Philosophical Writings of Descartes* (Descartes 1985).

Elements — *The Thirteen Books of Euclid's "Elements"* (Euclid [1925] 1956). References are to book number and proposition, definition, or axiom number, followed by a volume and page number after a semicolon.

M — *Principles of Philosophy* (Descartes 1983).

Op — *Optics*. References are to discourse number, followed by a volume and page number from *CSM* after a semicolon.

Pr — *Principles of Philosophy*. References are to part and section number, followed by a page number from *M* after a semicolon.

Rules — *Rules for the Direction of the Mind*. References are to rule number, followed by a volume and page number from *CSM* after a semicolon.

Other Abbreviations

ED — Epistle Dedicatory.

AL — Epistle Ad Lectorem.

INTRODUCTION

Thomas Hobbes is widely thought of first and foremost as a political philosopher. This emphasis on his political work tends to overshadow his contributions to natural philosophy. By the late 1640s, Hobbes had a significant reputation within the scientific community, and he continued to write on scientific matters throughout his career. Hobbes discussed optics with Marin Mersenne and René Descartes, attended dissections with William Harvey, and debated the status of the new experimental science with Robert Boyle.

In this dissertation I will examine one aspect of Hobbes's scientific work: his attempt to develop a mathematical science of the physical world. The so-called "mathematisation" of natural philosophy was one of the most significant changes to occur during the scientific revolution. Prior to the seventeenth century, the dominant way of distinguishing between mathematics and physics was based on the Aristotelian model. Physics was a qualitative science which sought explanations for the properties of natural bodies, such bodies being characterized as those things which contain in themselves the principles of rest and motion. Their behaviour was explained in terms of their natures: for example, the behaviour of falling bodies could be explained in terms of their natural motion towards the centre of the earth.

By contrast, mathematics was a quantitative science. It did not discuss the natures or properties of bodies. Instead, according to Aristotle, mathematics investigates abstract concepts that are immovable and separate from matter.

The mathematical sciences, such as optics, harmonics, and astronomy do not fit neatly into either of these categories, as they appear to apply mathematical methods to non-mathematical objects. They were considered to be subordinate branches of mathematics: for example, optics was subordinated to geometry, harmonics to arithmetic. The subject matter of the mathematical sciences was thus distinct from that of physics.

The seventeenth century saw the gradual collapse of the Aristotelian distinction between physics and mathematics, and a corresponding acceptance both of a mathematical physics, and of a physicization of formerly mathematical disciplines, such as optics. The mathematisation of nature took many different forms in the work of its different proponents, and this dissertation will discuss how Hobbes fit into this movement: how he thought that mathematics should be applied to the study of nature, and how his views both resembled and differed from those of other practitioners of the new mathematical sciences.

The most prominent feature of Hobbes's mathematical physics is its overwhelmingly geometrical nature. As legend has it, Hobbes had an epiphany upon his first exposure to Euclid's *Elements*, and he was forever after a fierce defender of geometry against such interlopers as the new symbolic algebra. Hobbes not only promoted geometry as the fundamental mathematical science, but also held that it was the means by which mathematics and physics could be brought together. Galileo famously said that the book of the universe "is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures." This is also a fair statement of Hobbes's view.

However, Hobbes's approach was, in an important sense, more ambitious than Galileo's. Some seventeenth-century scientists, such as Galileo, were most interested in applying mathematics to particular problems. They were less concerned with provided an ontological or metaphysical justification for the application of mathematics to natural philosophy. Hobbes, on the other hand, was less interested in solving particular problems, aiming instead to build a complete philosophical system. He believed that a comprehensive geometrical physics, demonstrative in character and grounded in the principles of the new mechanical philosophy, would provide a solid foundation for his scientific system. As I will show, in developing his geometrical physics Hobbes looked to ancient and medieval ways of thinking quantitatively about the world, as these models were often more compatible with his goals than the approaches taken by his contemporaries.

Hobbes was, however, forced to adapt the ancient and medieval ways of thinking in light of his own thoroughgoing materialism. Hobbes held that matter is all that exists and, consequently, that geometry must be redescribed as a science of body. He carried out this redescription by arguing that geometrical objects are the products of bodies in motion (so, for example, he claims that a line is the path of a moving body). Hobbes then claims, and this is the part of his mathematics that I will be most interested in, that since mathematical objects just are the products of matter in motion, we can, by considering those objects, learn important things about the motions that caused them. In this way motion, the source of all change in the world, becomes subject to mathematical analysis. As we will see, this approach to natural philosophy was, while ingenious, only partially successful.

In his approach to the mathematisation of nature Hobbes was often (though not always) swimming against the current of scientific change. This raises the question of why we should bother to study Hobbes's natural philosophy. As I suggest above, Hobbes's account of body was intended to provide a foundation for his whole philosophical system. Considering Hobbes's natural philosophy will therefore improve our understanding of his broader project. Furthermore, although Hobbes was not always on the winning side of scientific debates, I hope to show that he was a significant participant in these discussions. As such, looking closely at his views will contribute to a richer and more accurate understanding of the massive changes that occurred in seventeenth-century natural philosophy.

My dissertation begins in chapter 1 with an overview of Hobbes's scientific system. After these preliminaries, I discuss Hobbes's use of mathematics in the various divisions of his natural philosophy. My primary source will be Hobbes's *De Corpore*. This text, published in 1655, was intended to be the first instalment of Hobbes's three-part system of the elements of philosophy: *De Corpore*, *De Homine*, and *De Cive*. It contains Hobbes's most extensive account of his natural philosophy.

Chapter 2 examines Hobbes's scientific method. I begin by discussing Hobbes's attempt to adapt the mathematical method of analysis and synthesis to the study of nature in general. Chapter 2 also considers Hobbes's views on the nature and status of hypotheses. For Hobbes physical hypotheses, an essential part of natural philosophy, must be formulated in terms of mathematical principles.

Hobbes's first philosophy contains his fundamental scientific principles. In chapter 3 I discuss the roles that some of these principles play in Hobbes's mathematisation of nature. I consider his accounts of body, time, motion, and (most significantly) quantity. This last concept represents the basis for his geometrical physics: Hobbes argues that all quantities of bodies must be conceived as lines, surfaces, and solids, and can hence be made the subject of geometry.

Chapters 3 and 4 discuss Hobbes's mathematical account of motion. Since, for Hobbes, all natural phenomena must be explained in terms of the motions of bodies, his mathematical mechanics is the core of his geometrical account of nature. In chapter 4 I consider Hobbes's kinematics, which I argue was modelled on medieval efforts to analyse the spatio-temporal effects of motion geometrically. Although Hobbes's use of this model allowed him to develop a fairly coherent kinematics, the medieval techniques failed to provide him with the tools he needed to develop a materialist dynamics.

Hobbes's dynamics is the subject of chapter 5. In this chapter I discuss the reasons why Hobbes was unable to develop a coherent geometrical treatment of impact. This inability accounts for his fragmented concept of force, as well as the puzzling lack of quantitative analysis in his dynamics. Furthermore, I will argue that Hobbes's strict materialism and vision of mechanics as a demonstrative science contributed significantly to his failure to provide a mathematical account of circular motion.

In the final chapter I consider some of the extensive work on the nature of light that Hobbes did at the beginning of his career. I begin by arguing that Hobbes, by treating optics as a mathematical science of matter in motion, was able to develop a significant and influential account of refraction. I then show that some of the most important aspects of the geometrical mechanics in *De Corpore* can also be found in Hobbes's early optical work. Hobbes's early successes in his work on light may have encouraged him to apply the same methods to the science of motion in general. This would account for Hobbes's optimism about the possibility of developing a geometrical mechanics. despite the implausibility of many aspects of this project.

CHAPTER 1

THE SCIENTIFIC SYSTEM

De Corpore is the first part of Hobbes's elements of philosophy, which was intended to present a systematic account of the foundations of all scientific thought. In this chapter I will present an overview of Hobbes's scientific system. This will prepare the way for more detailed discussions, in the following chapters, of the particular sciences that are included in *De Corpore*.

In the first section I will discuss some general features of Hobbes's system: the criteria by which Hobbes distinguishes science from non-science, the ways in which he differentiates between the particular sciences, and the order in which he thinks that those special sciences should be studied and presented.

In the second section I will present an introduction to the subject matter of each of the parts of *De Corpore*: the first section on logic, and the accounts of the particular sciences of first philosophy, mathematics, and physics. Hobbes has some unusual ideas (from a seventeenth century and a contemporary point of view) about both the content of these areas of inquiry, and how they should be assembled to form his grand system. It will therefore be useful to have a sketch of these ideas before embarking on discussions of the various parts of his system.

In short, this chapter will provide a kind of aerial survey of Hobbes's system. With this map in hand, when we are on the ground exploring particular aspects of the Hobbesian landscape, we will nonetheless know where we are relative to the whole countryside.

1.1 General Features of Hobbes's System

Hobbes's system is supposed to include all the truly scientific disciplines, and he presents definite criteria for distinguishing science from non-science. He defines philosophy as the knowledge that we acquire, by true reasoning, of effects from their causes, or alternatively of possible causes from their effects (*DCp* I.1.2; *OL* I, 2). As we will discuss in further detail below, Hobbes makes an important distinction between the certain knowledge that arises when we reason from cause to effect and the hypotheses that are generated when we describe the possible causes of natural effects. The subject matter of philosophy or science (Hobbes uses these terms interchangeably) is everything to which either form of causal analysis can be applied.

The idea that scientific or philosophical knowledge should be causal was not, in itself, an unusual one. Aristotle states in the *Posterior Analytics* that scientific knowledge must be causal,[1] and this conception of scientific knowledge was still prominent in Hobbes's time.[2] There are, however, significant differences between the concepts of cause espoused by Hobbes and by Aristotle and his followers. There are four Aristotelian causes — formal, material, efficient, and final — and a scientific explanation could appeal to any of them. Hobbes, on the other hand, claims that the only true causes are bodies in motion.

In *De Corpore*, Hobbes uses his account of the subject matter of philosophy to exclude a number of topics: false doctrines, as well as those that are not well-founded (such as astrology), are excluded, since neither can have been the result of right reasoning.

Theology, which Hobbes seems to be taking to be the study of God's nature, is denied scientific status because God is uncaused, and hence cannot be the subject of causal analysis (*DCp* I.1.8; *OL* I, 9). This is not to say that causal reasoning has no part to play in religion. In *Leviathan*, Hobbes claims that our belief in God likely arose from our curiosity about the causes of natural bodies:

> For he that from any effect hee seeth come to passe, should reason to the next and immediate cause thereof, and from thence to the cause of that cause, and plonge himselfe profoundly into the pursuit of causes; shall at last come to this, that there must be (as even the Heathen Philosophers confessed) one First Mover; that is, a First, and an Eternal cause of all things; which is that which men mean by the name of God. (*Lev* I.12, 77)

By reasoning repeatedly from effect to cause we come to the idea that there must have been some first cause, and we refer to this first cause as God. Causal reasoning can thus lead us to the knowledge that God must exist. However, we can have no knowledge of God's properties beyond this, since we have no knowledge of his causes. As I have noted, for Hobbes this has the consequence that theology cannot be considered a scientific discipline.

The exclusion of theology from science does not mean that there can be no religious knowledge. Hobbes does, for example, refer to the knowledge that can be acquired by divine inspiration and revelation (*DCp* I.1.8; *OL* I, 9). However, this knowledge is not philosophical, because it is acquired by supernatural means, rather than by reason.

Things which are thought to be immaterial (such as angels) do not fall within the subject matter of philosophy. Hobbes offers little argument for this claim, stating only that with regard to these things there is no opportunity for resolution or composition, and hence for reasoning. As we will see, Hobbes thinks that reason involves the resolution of our conceptions into their constituent parts, and the subsequent composition of those parts. However, according to Hobbes, all conceptions must originate in sense-impressions, and sense-impressions can only be caused by the impinging of external bodies.[3] Each of our conceptions is a representation of the external body or object which caused the idea in question.[4] Hobbes therefore seems to be arguing that we cannot resolve or compose our conceptions of immaterial things, since no such conceptions exist.

Hobbes also denies scientific status to history. Although historical knowledge can be very useful to philosophers, it is not philosophical knowledge, since it is acquired by experience rather than reason. Hobbes's denial that history is a philosophical discipline is indicative of his attitude towards experience in general: although he freely acknowledges that experience is a valuable thing to have, he denies that it constitutes scientific knowledge. This is the basis for the important distinction that Hobbes makes between prudence and philosophy. He defines "prudence" as the ability, based on past experience, to make successful conjectures about what will happen in the future. Experience increases prudence, for "by how much one man has more experience of things past, than another: by so much also he is more Prudent, and his expectations the seldomer fail him" (*Lev* I.3, 22). Although prudence can be helpful in guiding our actions, it is not science or philosophy in the strict sense — it does not provide us with wisdom, or the ability to demonstrate (not merely conjecture) that certain effects will follow from certain causes.

Hobbes hoped that making a firm distinction between science and non-science would prevent people from believing those who falsely claim the name of philosophy for non-scientific subjects. In particular, he was concerned with the use of pseudo-philosophy to encourage political dissent and create religious conflict. In the Epistle Dedicatory to *De Corpore*, for example, he claims that when the ancient Greeks sent their children to be instructed in what they thought was philosophy, the children were in fact taught "nothing...but to dispute, and, neglecting the laws, to settle every question each according to his own wishes" (*DCp* ED; *OL* I, unpaginated).[5] Similarly, some of what has been called philosophy, but is in fact the result of the Church fathers drawing on certain false doctrines of Plato and Aristotle for the purposes of defending Christianity, could be called "pernicious; for it stirred up innumerable controversies in the Christian world concerning religion, and from controversies wars" (*DCp* ED; *OL* I, unpaginated).[6]

Hobbes thus distinguishes science from non-science on the basis of his definition of philosophy, and this definition states that all subjects of scientific inquiry must have certain characteristics. All the sciences that make up Hobbes's system will therefore attempt to explain their subjects in importantly similar terms since all objects of scientific study are, to a degree, the same kind of thing. Most obviously, all philosophical explanations, on Hobbes's account, should appeal to the causes or possible causes of the thing being examined. As I have mentioned, Hobbes draws important distinctions among the kinds of causal analyses that are appropriate in different situations. Nonetheless, one of the more interesting aspects of Hobbes's philosophy is his attempts to fit all scientific subjects into his explanatory framework.

A further aspect of Hobbes's systematic approach is his belief that we can differentiate and catalogue all possible areas of scientific inquiry. In his chapter on method, Hobbes claims that those who seek "knowledge of the causes of all things, to such an extent as it can be acquired"[7] must follow a plan of study consisting of five divisions: first philosophy, geometry, physics, moral philosophy, and civil philosophy (*DCp* I.6.4-7; *OL* I, 61-6). Similarly, in the dedication to *De Cive*, Hobbes claims that there are a certain number of subjects that human reason is capable of understanding, and a branch of philosophy suited to the study of each kind of thing:

> Now look, how many sorts of things there are, which properly fall within the cognizance of human reason; into so many branches does the tree of philosophy divide itself. And from the diversity of matter about which they are conversant, there hath been given to those branches a diversity of names too. For treating of figures, it is called *geometry*; of motion, *physic*; of natural right, *morals*; put altogether, and they make up *philosophy*. Just as the British, the Atlantic, and the Indian seas, being diversely christened from the diversity of their shores, do notwithstanding all together make up *the ocean*.[8]

As is clear from the differing accounts cited above, Hobbes is not always consistent when enumerating the different divisions of philosophy. Nor is he consistent in describing how we should distinguish between them: for example, in *De Corpore* he distinguishes between geometry and physics by claiming that they study different kinds of motion (*DCp* I.6.6; *OL* I, 63-4). However, only physics is described as a science of motion in the passage above. As we will see, for Hobbes the cause of everything is motion, and hence motion becomes the subject of every science. In the above passage from *De Cive*, Hobbes may be taking it for granted that the subject matter of the various special sciences will be various forms of motion — that in geometry, for example, figures will be explained in terms of the motions of points and lines. Nonetheless, this does not account for why

Hobbes states that physics is the science of motion *per se*, rather than the discipline that explains the motions that cause natural phenomena, as he does elsewhere. Despite these tensions, Hobbes clearly thought that there was a number of types of objects suitable for human contemplation, and that these types could in some way be enumerated.

Finally, Hobbes maintained that the various sciences should be both taught and investigated in a particular order. He describes this order in two places in *De Corpore* (*DCp* I.6.6, I.6.17; *OL* I, 62-5, 77-8). The briefer of these accounts appears in a discussion of what "is proper to methodical demonstration." This discussion also describes the order in which the sciences should be investigated, since Hobbes thinks that we should, insofar as is possible, demonstrate or teach things in the same order in which they were originally discovered. He begins by claiming that we must reason according to the rules of syllogism, and that syllogisms must start with definitions. Then,

> that after definitions, he who teaches should proceed by the same method by which he discovered something; namely that first those things be demonstrated which are immediate to the most universal definitions (in which is contained that part of philosophy which is called first philosophy), then those things which can be demonstrated by means of motion simply (in which geometry consists), after geometry, those things which can be taught through manifest action, that is, through pushing and pulling. Next he must descend to the motion of invisible parts, or mutation, and to the doctrine of the sense and imagination, and to the internal passions of animals, but especially of man, in which are contained the first foundations of duties or civil doctrine, which holds the final place. Now that that which I described ought to be the order of universal doctrine, can be known from this: that which we said must be taught in the latter place, cannot be demonstrated, unless by those ideas which are proposed to be taught in the first place. (*DCp* I.6.17; *OL* I, 77)[9]

The sciences must therefore be investigated and taught in the following order: first philosophy, geometry, physics, moral philosophy, and, finally, civil philosophy. A part of *De Corpore* is dedicated to each of the first three sciences, and the final two represent the subject matter of the other two books of Hobbes's elements of philosophy: *De Homine* and *De Cive*.

Hobbes is not claiming that the content of each science can be derived from the content of those that precede it.[10] It sometimes seems that Hobbes should make the claim that all the sciences can be deduced from the definitions of first philosophy, given that he at different times claims that all demonstrations should be cast in syllogistic form, that the first principles of all demonstrations are definitions, and that all definitions should be considered the subject matter of first philosophy. However, Hobbes does not, in practice, follow the third precept. Definitions are introduced in each part of *De Corpore* which cannot be inferred from the parts that precede, and these definitions are used as first principles within each of the sciences. Hobbes is, in this respect, following the example of Euclid, who introduced new definitions in each book of the *Elements*.

We should study the sciences in the order that Hobbes has suggested because each of the latter sciences incorporates principles which have been established in the previous chapters. For example, Hobbes attempts to deduce the principles of motion from the definitions of first philosophy and some new definitions introduced in part III of *De Corpore*.

A somewhat different relationship exists between physics and those sciences that must be mastered before it is undertaken. As I have mentioned, Hobbes thinks that physics is a hypothetical science. An extended discussion of the place of hypotheses in Hobbes's scientific system will be presented in chapter 2. Briefly, the hypothetical nature of physics depends on two factors: first, there are multiple possible causes for any given natural effect. Furthermore, Hobbes, like many other mechanical philosophers, favoured explanations in terms of minute corpuscles of matter. These corpuscles being so small as to be unobservable, we can never be sure that a given corpuscular explanation is, in fact, the correct one. For these reasons, the correct explanation of any natural phenomenon cannot be deduced from the principles of the previous sciences, or from any other information available to us. However, Hobbes nonetheless claims that those who investigate natural philosophy must begin with geometry (*DCp* I.6.6; *OL* I, 65). The principles of geometry place limits on the kinds of explanations that can be presented in physics: every natural phenomenon must be explained in terms of the motions by which it could have been generated. As we will see, geometry describes the different kinds of motion that are possible, and how they can be produced, and hence delineates the various kinds of motion which can be appealed to in accounting for natural phenomena.

1.2 The Parts of De Corpore

1.2.1 Logic or Computation

In the first section of *De Corpore*, titled "Logic and Computation," Hobbes claims that a proper method is needed to bring philosophy out of its primitive state. Philosophy, he argues, is in the same condition that wine and corn were in ancient times — although corn and vines always existed, men lived on acorns for want of the ability to plant and sow them. Similarly, every man is born with natural reason, and uses it continuously to some degree. However, this natural reason is insufficient when we are faced with complicated problems requiring lengthy consideration. Method allows us to improve our reason, in the same way that farming techniques allow us to improve the natural yield of plants and trees (*DCp* I.1.1; *OL* 1-2). The first part of *De Corpore* is thus supposed to provide the method that will allow the sciences to be properly developed.

Hobbes's ideas regarding the role of method were not unusual — he thought that scientific method should present both a means to the discovery of new knowledge, and a way of presenting and teaching that knowledge in the clearest way possible. The method that Hobbes presents is thus supposed to facilitate both the discovery and the demonstration of causal knowledge. As such, it is applicable (at least in theory) to all the areas of inquiry that make up Hobbes's philosophical system. However, as we will see, Hobbes also makes it clear that the method should, in some respects, be applied differently to different subjects.

1.2.2 First Philosophy

In his message to the reader at the beginning of *De Corpore* Hobbes claims that in the second part of the work, which is titled "First Philosophy" (*Philosophia Prima*), he will "mutually distinguish by accurate definition the ideas of the most common notions for the purpose of eliminating ambiguity and obscurity" (*DCp* AL; *OL* I, unpaginated).[11] His first philosophy is intended to provide proper definitions of our most fundamental concepts. Hobbes's discussions of the content of first philosophy often contain partial lists of the concepts to be defined. Included in one or more of the lists are body, space, time, place, matter, form, essence, subject, accident, power, act, finite, infinite, quantity, quality, motion, action, passion, cause, effect, unity, and number.

These definitions are intended to provide the foundational concepts for all the other sciences, and are thus not supposed to be specific to any special science. This is made clear in the second of the *Six Lessons to the Professors of the Mathematiques* (1656), where Hobbes is responding to an objection from John Wallis.[12] Wallis had criticized Hobbes's definition of "magnitude," claiming that it brought the subject matter of natural philosophy into a mathematical definition. Hobbes responds that Wallis is ignorant of the roles of the different parts of philosophy:

> It seems by this, that all this while you think it is a piece of the geometry of Euclid, no less to make the definitions he useth, than to infer from them the theorems he demonstrateth. Which is not true. For he that telleth you in what sense you are to take the appellations of those things which he nameth in his discourse, teacheth you but his language, that afterwards he may teach you his art. But teaching of language is not mathematic, nor logic, nor physic, nor any other science; and therefore to call a definition, as you do, mathematical, or physical, is a mark of ignorance, in a professor inexcusable. (*SL* 2; *EW* VII, 225)

The formulation of philosophical terminology that occurs in first philosophy is not a part of any of the special sciences, but is a necessary precursor of them all. Since science assumes the existence of a language, both for the formulation and the communication of demonstrations, the defining of terms must precede any particular science.

Despite this initial degree of clarity regarding the purpose and content of first philosophy, it quickly becomes difficult to discern what distinguishes it from the other parts of philosophy. It is immediately apparent that Hobbes does not, as one would expect from the above rebuke to Wallis, present all of his scientific definitions in his first philosophy. Furthermore, it is unclear what, exactly, distinguishes the definitions that appear in the second part of *De Corpore* from those that appear elsewhere in Hobbes's work. Hobbes makes a number of claims about what makes the concepts defined in his first philosophy more fundamental, and, correspondingly, how the definitions included in this part of *De Corpore* can be distinguished from those found elsewhere in the work. In his discussions of the organization of the sciences, Hobbes claims that the subject matter of first philosophy is universals, universal things, or universal definitions. However, there are at least two ways that the term "universal" can be understood: first, universal things are described as "those accidents which are common to all bodies, that is to all matter, rather than singular, that is the accidents by which one thing can be distinguished from another" (*DCp* I.6.4; *OL* I, 61).[13] One possibility is thus that the definitions of first philosophy are of those things that are common to all matter. This would be consistent with Hobbes's claim in his message to the reader of *De Corpore* that first philosophy deals with "ideas of the most common things," and with his similar claim in his critique of Thomas White's *De Mundo* that it is "the science where theorems concerning the attributes of being at large are demonstrated."[14]

In order to evaluate this suggestion, we first need to clarify what, exactly, Hobbes means by "universal" and "singular" or "particular" things. Hobbes provides some helpful examples in his account of the different kinds of definitions. He states that definitions are either of names of things of which we can conceive some cause, or of names of things of which we can conceive no causes at all:

> Of the first sort are body or matter, quantity or extension, motion simply, in short that which is in all matter. Of the second sort are such a body, such and so great a motion, so great a magnitude, such a figure, and all others by which one body can be distinguished from another. (*DCp* I.6.13; *OL* I, 71)[15]

This passage makes the distinction between universal and singular things seem fairly clear-cut: the first are general properties that are possessed by each and every body, the latter are those properties that are possessed by only some bodies. So, for example, "magnitude" is a universal thing (all bodies must possess some magnitude) while "2 feet long" is a particular thing (only some bodies have this particular magnitude). Similarly, all bodies must have some figure, but only some are circular. Hobbes thinks that we can offer causal definitions of the latter sort of thing, but not of the former.

Unfortunately, this initial distinction falls apart in practice. The list of things common to all matter that Hobbes presents in the above passage would pretty much exhaust the concepts of first philosophy on this account, and part II of *De Corpore* clearly includes more than five definitions. Hobbes's scientific method also demands a more extensive list of basic concepts. According to Hobbes, in order to develop a general account of the world, we must generate a catalogue of the most universal components of our concepts. We acquire knowledge of these universal things by resolving our particular concepts into their most general parts:

> For example, let there be proposed any concept or idea of a singular thing, suppose a square. Therefore the square will be resolved into *plane, terminated with a certain number of equal lines and right angles*. Therefore we have these universal things, or things agreeable to all matter, *line, plane,* (in which is contained surface) *terminated, angle, straightness, equality,* of which if anyone should find out the causes or generations, they may be composed into the cause of the square. Again, if someone should propose to himself the concept of gold, then by resolving should come to the ideas of *solid, visible, heavy,* (that is tending towards the centre of the earth or motion downwards) and many others more universal than gold itself, which in turn can be resolved, until they come to the most universal. (*DCp* I.6.4; *OL* I, 61)[16]

The method of resolution will be discussed at length in chapter 2. For our present purposes it is important to note the examples of universal things that Hobbes presents in this passage. The idea of a square is initially resolved into the ideas of particular lines and angles. The process of resolution concludes when we reach the universal things line, plane, terminated, angle, straightness, and equality. In accordance with Hobbes's previous statements, one would expect these to be properties possessed by all matter. This seems to be the case for some of the properties that Hobbes mentions: all of our conceptions (at least for Hobbes) must be terminated. To say that something is terminated is hence to distinguish it from no other thing.

However, it hard to see how some of these properties could be said to be common to all matter. Not all of our ideas have the property of "straightness," for example — we can distinguish something straight from something curved. Hobbes seems to have even more difficulties resolving the concept of "gold" into its most universal parts. He suggests that there will be some way to resolve "solid," "gold," and "visible" into more universal ideas, but he does not carry through with the resolution. It is likely that he simply could not figure out what those more universal ideas would be.

There is not only confusion regarding what sorts of concepts should be defined in Hobbes's first philosophy, but also a corresponding confusion regarding what kinds of definitions are appropriate for the concepts of first philosophy. As we have seen, at some points in *De Corpore* Hobbes distinguishes between definitions of things which have no conceivable cause and definitions of things of which we can conceive a cause. The former type of definition is associated with those things which are common to all matter (or universal things), while the latter is associated with things by which we can distinguish one body from another (or particular things). At another point, however, he states that first philosophy consists of the knowledge of universals *and their causes* (*DCp* I.6.6; *OL* 70). Again, this confusion in theory is reflected in practice: there are, in fact, both causal and non-causal definitions in part II of *De Corpore*.

One could remove some of these difficulties by hypothesizing that, despite its title, the whole of part II of *De Corpore* is not intended to be an exposition of first philosophy. This is suggested by a passage in the Author's Epistle of *Six Lessons*, where Hobbes states that "from the seventh chapter of my book *De Corpore*, to the thirteenth, I have rectified and explained the principles of the science" (*SL* ED, *EW* VII, 185). If we assume that "the science" that Hobbes is referring to in this passage is first philosophy, the last chapter, or chapter 14, of *De Corpore*'s second part is not included in Hobbes's first philosophy.[17] Eliminating chapter fourteen, "Of Strait and Crooked, Angle and

Figure" would be of some help in making Hobbes's account consistent, as this chapter includes many causal definitions of seemingly "singular" things.

However, it is clear that the science that Hobbes is talking about in this passage is geometry, not first philosophy. Immediately following this passage, Hobbes states "*id est*, I have done the work for which Dr. Wallis receives the wages" — in other words, he has done the work of a professor of geometry. Furthermore, he goes on to claim that "in the seventh [chapter], I have exhibited and demonstrated the proportion of the parabola and parabolasters to the parallelograms of the same height and base" (*SL* ED; *EW* VII, 185). The reference to the "seventh" chapter is clearly a typo here, since Hobbes presents his theory of parabolasters (or curves of higher degree than the simple parabola)[18] in the *seventeenth* chapter of *De Corpore*.

It appears impossible to save Hobbes's account of first philosophy from some degree of inconsistency. On a final survey, it seems that he intended his first philosophy to be an account of the most general or universal concepts, in the sense that the concepts which are defined in the first philosophy tend to be those which are most widely appealed to in the chapters which follow. For example, the concept of circular motion, which rests on the definition of a circle presented in part II, plays a prominent role in Hobbes's dynamics and his explanations of natural phenomena. The bulk of these definitions will be non-causal. However, Hobbes cannot sustain the thesis that they should all be, as some of his most fundamental concepts seem to require causal definitions.

1.2.3 Mathematics

For Hobbes, mathematics is essentially equivalent to geometry. As we will see, he claims that arithmetic is merely a special branch of geometry, and restricts his own mathematical work to geometrical proofs. In order to justify this conception of mathematics, Hobbes reworks the traditional division of the subject into arithmetic and geometry. Pure mathematics was customarily divided into geometry, which examined continuous quantity (lines, surfaces, and solids), and arithmetic, which examined discrete quantity (numbers). According to the traditional view, therefore, geometry and arithmetic are separate disciplines with different classes of objects.

Hobbes makes something like this traditional distinction in *De Corpore*. He differentiates between discrete and continuous quantity by claiming that we acquire our ideas of these different kinds of quantity in different ways. We obtain our ideas of continuous quantity by considering bodies in motion, while our idea of number arises from the consideration of points or numeral names (*DCp* II.12.3-5; *OL* I, 124-6).

In the *Examinatio et Emendatio Mathematicae Hordiernae* (1660), Hobbes is much more forceful concerning the dependence of arithmetic on geometry. Having agreed that geometry concerns itself with continuous magnitudes, one of the participants in this dialogue adds that "because any given continuous magnitude can be divided into as many aliquot parts as one wishes, having its ratio to any other thing unchanged, it is clear that arithmetic is contained in geometry" (*Ex* I; *OL* IV, 28).[19] Hobbes therefore endorses the traditional claim that the subject of geometry is continuous magnitude. However, he also uses this classification to claim that arithmetic is merely a part of geometry, by arguing that the units of discrete quantity that are the subject of arithmetic can be generated by the division of geometrical objects.

This argument is not a particularly strong one, since it seems that discrete units and our ideas of them could also be generated by other means. We might, for example, develop an idea of discrete quantity by considering individual objects (so, for example, our concept of the number "three" could have been generated by the consideration of a group of three apples). As we saw above, in *De Corpore* Hobbes himself allows that we can acquire our ideas of discrete quantity from our sense impressions of discrete items, such as points.

Hobbes may have increased his insistence on the dependent status of arithmetic because of his increased involvement in debates with John Wallis.[20] The classification of the mathematical sciences was in flux in the late sixteenth and seventeenth centuries, during which time new divisions were being proposed, and debates were carried out over which discipline should have priority.[21] This debate was prompted by the rise of algebraic techniques in mathematics. Algebra was understood by many as a generalization of arithmetic techniques. At the same time, it was often assumed that algebraic symbols could represent different kinds of quantity. These sorts of considerations led some mathematicians, including Wallis, to argue that arithmetic must be the most fundamental mathematical discipline.[22] Given the increasingly vituperative nature of Hobbes's debate with Wallis, Hobbes may have progressively strengthened his opposition to all of Wallis's positions, including that regarding the relative priority of arithmetic and geometry.

It is not difficult to see why Hobbes would want to make geometry the fundamental mathematical science. Hobbes was, of course, a strict materialist, holding that only matter exists and that all things must be explained in terms of the motion and impact of bodies. In order to place mathematics on a firm foundation, he hence had to redescribe its subject matter in terms of bodies in motion, and the objects of geometry are more amenable to such a redescription. There were even precedents for considering mathematical objects as the products of motion. In *De Anima* Aristotle states that "since they say a moving line generates a surface and a moving point a line, the movements of the psychic units must be lines."[23] As Hobbes was fond of noting, Euclid defines a "sphere" as the figure which is created when "the diameter of a semicircle remaining fixed, the semicircle is carried round and restored again to the same position from which it began to be moved."[24]

Hobbes describes the subject matter of geometry in two ways. Although these accounts are not incompatible, it is worth looking at each in turn. One way that Hobbes understands geometry is in terms of ratio or comparison — as, for example, the title of part III of *De Corpore* is "Ratios of Motions and Magnitudes".[25] Similarly, in the *Six Lessons* "geometry" is defined as "the science of determining the quantity of anything, not measured, by comparing it with some other quantity or quantities measured" (*SL* 1; *EW* VII, 191).

It was not unusual, in Hobbes's day, to refer to the theory of ratios and proportions as the "essence" of geometry. This is not surprising, given the importance of ratios and proportions for Greek geometry. Within Greek mathematics, measurement was carried out by appeal to ratios, rather than by direct measurement using standard units.[26] For Euclid, for example, determining the area of a figure meant constructing a square with an equal area. The *Elements* thus contains such problems as "[t]o construct a square equal to a given rectilinear figure."[27] The doctrine of ratios and proportions is particularly prominent in the latter books of the *Elements*, which are largely devoted to figuring out the ratios and proportions between different kinds of figures.[28]

It may have been common to think of the theory of ratios and proportions as an essential part of mathematics, but there was a significant amount of controversy about what that theory should look like. Hobbes defines "ratio" in the chapter "Of Identity and Difference," where the relation of two bodies is described as their likeness or unlikeness, equality or inequality. The first body is called the "antecedent," and the second the "consequent," while the

> *[r]elation* [...] of the antecedent to the consequent according to magnitude, namely its equality, or excess, or defect, is called the *ratio* or *proportion* of the antecedent to the consequent; so that *ratio* is nothing other than the equality or inequality of the antecedent compared to the consequent according to magnitude. (*DCp* II.11.3; *OL* I, 119)[29]

Hobbes distinguishes between two kinds of ratio: first, we can compare a magnitude to another by saying that it is greater or less than another by some fixed amount. For example, we can say that five exceeds two by three, or that seven is less than nine by two. Alternatively, we can say that a magnitude is "greater or less than another, by so much of its part or parts, as 7 is less than 10, by three tenths of its ten parts" (*DCp* II.13.1; *OL* I, 129).[30] The former case is of an arithmetic ratio, which gives only the numerical difference between the two quantities. In providing a geometrical ratio, on the other hand, we explain what part one magnitude is of another.

It is notable, in the context of contemporary debates over the nature of ratios, that Hobbes chooses to define a ratio as a "relation". In the seventeenth century there was conflict between two ways of understanding ratios. I will give a brief sketch of this debate, in order to situate Hobbes's views.[31] One camp conceived of ratios as relations. This view finds its roots in book 5 of the *Elements*, which presents an account of ratio and proportion applicable to all geometric magnitudes. In the third definition of this book, Euclid states that "[a] **ratio** is a sort of relation is respect of size between two magnitudes of the same kind."[32] Euclid's account therefore treats a ratio as a relation between terms, rather than as a quotient.

Furthermore, as is indicated by the above definition, only magnitudes of the same kind can have a ratio to one another. Magnitudes were thought to fall into heterogeneous kinds (such as number, point, line, and surface) which are kept separate by the Euclidean doctrine of ratios. This segregation is also entailed by definition 5 of book 5, which states that "[m]agnitudes are said to **have a ratio** to one another which are capable, when multiplied, of exceeding one another." A point cannot have a ratio to a line because no number of points will ever be able to exceed the magnitude of a line. Definition 5 also precludes ratios between infinitesimal and finite magnitudes since, again, no multiple of the former will exceed the latter. Euclid's theory does allow for ratios between incommensurable magnitudes: for example, the side and hypotenuse of a right-angled triangle can be compared, since they are both line segements. A ratio between two magnitudes of the same kind can also be compared to a ratio between two magnitudes of a different kind, and Euclid hence states (in definition 6):"[l]et magnitudes which have the same ratio be called **proportional**."

The second theory of proportion (which is sometimes called the "numerical" rather than the "relational" theory)[33] assigns each ratio a "size" or "denomination".[34] It thus views ratios as quantities, rather than relations, and ratios are said to be equal when they have the same size. In the seventeenth century, the magnitude of a ratio was often equated with the quotient arising from the division of the consequent into the antecedent.

The numerical theory tended to homogenize those kinds of magnitudes which the relational theory kept separate. It encouraged, for example, the assumption that A:B :: C:D because AxD = BxC, and this implies that A can be multiplied into D, and B into C, despite the fact that this might involve the multiplication of kinds of magnitude that the relational theory would keep separate. The numerical theory also demanded a new way of thinking about ratios between commensurable and incommensurable magnitudes: viewing ratios as quotients seems to require an account of how many times the consequent can be divided into the antecedent. This is obviously problematic if, for example, the antecedent and consequent are the side and diagonal of a square.

Many proponents of the numerical theory responded to such difficulties by proposing an expanded concept of number. Wallis, for example, asserts that when magnitudes are compared in a ratio, they are transferred into the genus of number.[35] This new concept of number includes all kinds of magnitude, as well as irrational numbers, whereas the traditional concept of number included only positive integers.[36]

Hobbes's account of proportion clearly falls into the first tradition, although he disagrees with Euclid and other proponents of the relational view on some key points, including (as we will see below) the definition of "sameness of ratios".[37] On the other hand, his nemesis Wallis was, as has been mentioned, a strong supporter of the numerical theory.

The second way that Hobbes characterizes the subject matter of geometry is as the study of what can be produced or demonstrated simply from motion (*ex motu simpliciter*) (*DCp* I.6.6; *OL* I, 63, 65; *DCp* I.6.17; *OL* I, 77). The most extensive description along these lines is presented in the sixth section of *De Corpore*'s first part, where Hobbes, in an overview of the various parts of his philosophical system, states that we begin by considering

> ...a moved body if nothing else is considered in it besides the motion which it will produce; now it is immediately clear that a line or length is produced; next what a long body will make if it is moved, and it will be ascertained that it makes a surface, and in this manner we see what can be made simply from motion; next, in a similar way, it ought to be contemplated what effects, what sorts of figures, and what sorts of properties will proceed from the addition, multiplication, subtraction, and division of these sorts of motions; and arising from this contemplation is that part of philosophy which is called geometry. (*DCp* I.6.6; *OL* I, 63)[38]

As I have discussed, for Hobbes geometry is the study of ratios and proportions of magnitudes. However, Hobbes also holds that the concept of magnitude is essentially kinematic, since, according to his doctrine, we must conceive of magnitude as the product of bodies in motion. This is suggested by the above passage, where Hobbes defines various geometrical objects in kinematic terms. Like all other objects of scientific inquiry, if we are going to understand magnitude, we must understand the means by which it generated. The study of motion is thus an essential aspect of Hobbes's mathematics. In the above passage Hobbes seems even to suggest that geometry is roughly equivalent to what we would call kinematics, in that he appears to describe geometry as the study of the spatial and temporal effects of motion without reference to particular forces or bodies acting to produce or alter it.[39]

However, the account of motion in Hobbes's geometry goes beyond kinematics. Immediately following the discussion quoted above, Hobbes states:

> After the consideration of those things which are made simply from motion, the consideration follows of those things, which the motion of one body produces in another body, and because there can be motion in the individual parts of a body, in such a way that the whole nevertheless does not yield its place [*suo loco non decedat*], it must be inquired in the first place which motion produces which motion in the whole; that is, by some body running into another body which rests, or which is already moved by the same motion, what way and with what velocity it will be moved after that collision, and in turn which motion that second motion will generate in a third, and so on, from which contemplation arises that part of philosophy which is about motion. (*DCp* I.6.6; *OL* I, 63)[40]

In order to properly understand motion, we must also consider the effects that the motion of one body can have on the motion of another. We must therefore venture into what we would call dynamics, wherein the forces that bring about the production or modification of motion in bodies are examined. For Hobbes, this is equivalent to examining the effects of bodies colliding with other bodies, since he holds that bodies can only act on each other through contact. It is difficult, from the above passage, to tell if Hobbes's dynamics is supposed to be a part of geometry or of physics. However, as we will see in chapter 5, part III of *De Corpore* does include an extended discussion of dynamics.

As I mentioned at the beginning of this section, there is no conflict between Hobbes's two characterizations of geometry (in terms of proportion and motion). It is clear that for Hobbes a comparison of motions and magnitudes must include a study of the motions by which the various magnitudes are generated. The link between these two conceptions of geometry is Hobbes's account of sameness of ratios. Euclid had said that

> Magnitudes are said to **be in the same ratio**, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.[41]

Hobbes rejected the complexities of the Euclidean definition, stating instead that "one geometrical ratio is the same as another geometrical ratio, when some cause can be assigned which, producing equal effects in equal times, determines both ratios" (*DCp* II.13.6; *OL* I, 132).[42] In geometry we compare ratios, but in order to do so we must consider the causes of the magnitudes that make up ratios. These causes will invariably be motions.

1.2.4 Physics

Hobbes holds that, once we understand the effects of whole bodies on other whole bodies, we should examine the invisible changes which occur in the parts of bodies. The latter task is saved for the final part of Hobbes's natural philosophy:

> In the third place we reach the investigation of those things which are made by the motion of parts, as that in which the same thing continues, yet to the senses it does not seem the same but changed; therefore sensible qualities are investigated, such as *light, colour, transparency, opacity, sound, odour, taste, heat, cold* and so on, which because they cannot be understood without knowledge of the causes of the senses themselves,

> consideration of the causes of *sight, hearing, smell, taste*, and *touch*, will constitute the third part; but those qualities mentioned earlier, and all changes must be deferred to the fourth part, which two considerations comprise that part of philosophy which is called physics. (*DCp* I.6.6; *OL* I. 64)[43]

The primary task of physics is therefore the study of the motions of the imperceptible parts of bodies, these motions being the cause of the perceptible qualities.[44] In order to understand the causes of changes in sensible qualities, we must first understand how the senses that perceive those changes work.

One of the most notable things about the transition from geometry to physics is the difference that Hobbes posits between the levels of certainty that can be attained in each of the two sciences. This distinction is made clearly in the following well-known passage at the beginning of the *Six Lessons*:

> Of arts, some are demonstrable, others indemonstrable; and demonstrable are those the construction of the subject whereof is in the power of the artist himself, who, in his demonstration, does no more but deduce the consequences of his own operation. The reason whereof is this, that the science of every subject is derived from a precognition of the causes, generation, and construction of the same; and consequently where the causes are known, there is place for demonstration, but not where the causes are to seek for. Geometry therefore is demonstrable, for the lines and figures from which we reason are drawn and described by ourselves; and civil philosophy is demonstrable because we make the commonwealth ourselves. But because of natural bodies we know not the construction, but seek it from the effects, there lies no demonstration of what the causes be we seek for, but only of what they may be. (*SL* ED; *EW* VII, unpaginated)

Hobbes claims that we can only reason from cause to effect, and hence have true demonstration, when we know for certain what the causes of a given effect are, and he states that this only occurs when the cause is within our control, i.e., when it is something that we ourselves construct or generate. This is what happens in geometry and civil philosophy, when we generate the objects from which demonstrations proceed. However, in the case of physics, we begin from a phenomenon which is given, and attempt to provide an explanation for it. Because God can create any phenomenon in a number of ways, we can never be sure that the causal explanation that we arrive at describes the way that the effect was actually brought about. In physics we must therefore be satisfied with supplying possible causes for the effects with which we are presented.

Based on this way of drawing the distinction, it does not seem that Hobbes can maintain that there is a difference in principle between the demonstrable and indemonstrable sciences. The distinction is based on the claim that we cannot know for sure how any natural phenomenon was generated. However, this could also be the case in our dealings with geometrical objects. For example, one could be presented with a line without being told if it had been generated progressively by the motion of a point (as, for example, if it had been traced out by the tip of a pencil) or by a single, discrete action (as if one had dipped the straight edge of a ruler in ink and pressed it onto a piece of paper). On Hobbes's account, one could not have demonstrable knowledge of the line's properties under these circumstances.

Wallis brings up, in a different context, the issue of the various ways that geometrical objects can be constructed. He questions whether Hobbes's definition of a line in terms of the motion of a point is really a definition, since it is not reciprocal, i.e. the motion of a point will necessarily trace out a line, but a line need not be generated by the motion of a point. Hobbes replies that "it is reciprocal. For not only the way of a body whose quantity is not considered is a line, but also every line is, or may be conceived to be, the way of a body so moved" (*SL* 2; *EW* VII, 214). He notes that Euclid defines several geometrical objects in terms of their generation (including, for example, his definition of a sphere in terms of the circumduction of a semicircle):

> Euclid saw that what proper passion soever should be derived from these his definitions would be true of any other cylinder, sphere, or cone, though it were otherwise generated; and the description of the generation of any one being by the imagination applicable to all, which is equivalent to convertible, he did not believe that any rational man could be misled by learning logic to be offended to it. (*SL* 2; *EW* VII, 214)

Hobbes justifies his definition by claiming that if a given object could have been constructed in the manner used to define it, all of the properties which follow from the definition will also apply to the object, regardless of how it was actually generated. However, there seems to be no good reason why this reasoning could not also be applied to natural effects: if a phenomenon can be imagined to have been generated in a certain way, and we define it accordingly, why can't its properties be demonstrated from that definition?

Hobbes presents a more robust version of the distinction between geometry and physics in *De Homine*. He begins by reiterating the distinction between sciences whose objects are within our power to create, and whose demonstrations can therefore be described as *a priori*, and sciences which are *a posteriori*, because they study things whose causes are outside of our sphere of influence. He goes on to state:

> And since one cannot proceed in reasoning about natural things that are brought about by motion from the effects to the causes without a knowledge of those things that follow from that kind of motion; and since one cannot proceed to the consequences of motions without a knowledge of quantity, which is geometry; nothing can be demonstrated by physics without something also being demonstrated *a priori*. Therefore physics (I mean true physics), that depends on geometry, is usually numbered among the mixed mathematics. For those sciences are usually called mathematical that are learned not from use and experience, but from teachers and rules. Therefore those mathematics are pure which (like geometry and arithmetic) revolve around quantities in the abstract so that work in the subject requires no knowledge of fact; those mathematics are mixed, in turn, which in their reasoning also consider any quality of the subject, as is the case with astronomy, music, physics, and the parts of physics that can vary on account of the variety of species and the parts of universe.[45]

In this passage Hobbes uses traditional language to describe the distinction between physics and mathematics: the former is *a posteriori* and the latter is *a priori*. Furthermore, the former is learned from experience, while the latter involves the study of abstract quantities. For Hobbes, of course, all knowledge begins from sense experience to some degree. However, the practising of physics depends on having knowledge of fact — experience of particular, historically specific events. For example, in doing astronomy one might need experience of the path of a particular comet. However, knowledge of quantity can be abstracted from experience of any matter in motion.

The distinction that is drawn in *De Corpore* is thus a distinctly Hobbesian version of a common way of differentiating between geometry and physics. Traditionally, it was argued that the objects of mathematics are abstractions, whose properties remain the same regardless of how the world happens to be. Its theorems thus have a certainty which is lacking in those sciences which concern themselves with the contingencies of the physical world. Hence, according to the traditional view, the mathematical and physical sciences are differentiated according to their objects. Mathematics studied abstractions, while natural philosophy examined material objects.

For Hobbes, however, body is the subject of both sciences. He must therefore draw the distinction in terms of the different kinds of properties that matter can have. On the one hand, we can consider the most general features of matter and motion. As we will see, the science of these most general properties is mathematics, or the study of quantity. Hobbes holds that any possible universe would be made of matter, and that all matter would have the same essential properties. Mathematics would therefore be true even if the world were arranged in a different way. Natural philosophy, on the other hand, explores the qualitative properties of bodies. For Hobbes, qualities are sense impressions or phantasms, rather than properties that actually inhere in the bodies that we perceive. As such, they could easily vary if the world, and especially the perceivers in it, were different.

Not everyone was enamoured of Hobbes's way of distinguishing between mathematics and natural philosophy. Wallis ascribed to the traditional view whereby mathematics is an *a priori* science because its objects are abstract and their natures independent of the structure of the physical world. Wallis repeatedly criticized Hobbes for introducing matter and motion into his mathematics, arguing that these notions belong to natural philosophy, and rob mathematics of its clarity and certainty.[46] So for example, Wallis objects to Hobbes's notion of a line, stating that the nature of a line can be understood without introducing principles of body and motion. Such principles are "plainly accidental, nor do they pertain to their essences, so it is strange to find motion in the definition of a point or line."[47]

For Hobbes, as we have seen, the principles of matter and motion are not "accidental" to the nature of mathematical objects. He holds that mathematics, like any other discipline, must be redescribed in materialist terms if it is to be a true science.[48] Furthermore, as I have argued, Hobbes thinks that there are certain general properties of matter and motion, and it is these properties that Hobbes refers to as mathematical. For Hobbes, mathematical quantity is not abstract because it is independent of the structure of the physical world, but precisely because it represents that world's essential features.

ENDNOTES TO CHAPTER 1

1. Aristotle states that "we suppose ourselves to possess unqualified scientific knowledge of a thing, as opposed to knowing it in the accidental way in which the sophist knows, when we think that we know the cause on which the fact depends, as the cause of that fact and of no other, and, further, that the fact could not be other than it is" (*Posterior Analytics* 1.2, 71b 8-11).

2. In the sixteenth century, for example, this conception of science was the basis for a debate over whether mathematics and mathematical demonstrations meet Aristotelian standards for scientific knowledge. This debate began with the 1547 publication of Alessandro Piccolomini's *Commentarium de certitudine mathematicarum disciplinarum*, which claimed, among other things, that mathematics is not a causal science. Mancosu argues that the ensuing debate reached into the seventeenth century, and had an influence on Hobbes's work. On this debate, see Mancosu (1992) and (1996, ch. 1), Jardine (1988, 693-94), Wallace (1984, 136-48), and Dear (1995, 35-42).

3. In the first chapter of *Leviathan*, for example, Hobbes states that the "Originall of [all thoughts], is that which we call SENSE; (For there is no conception in a mans mind, which hath not at first, totally, or by parts been begotten upon the organs of Sense.)" (*Lev* I.1, 13). Furthermore, "[t]he cause of Sense, is the Externall Body, or Object, which presseth the organ proper to each Sense" (*Lev* I.1, 13).

4. As Hobbes states in *Leviathan*, "*[s]ingly*, [the thoughts of man] are every one a *Representation* or *Apparence*, of some quality, or other Accident of a body without us; which is commonly called an *Object*. Which Object worketh on the Eyes, Eares, and other parts of mans body; and by diversity of working, produceth diversity of Apparences" (*Lev* I.1, 13)

5. ...nihil...praeterquam disputare, neglectisque legibus de omni quaestione suo quemque arbitrio constituere.

All translations of Hobbes's Latin works are my own, unless otherwise specified. Throughout I include the original Latin of translated passages in the endnotes.

6. ...perniciosam; innumerabiles enim illa in orbe Christiano de religione controversias, et ex controversiis bella excitavit.

7. ...cognitione causarum quantum fieri potest omnium rerum.

8. Hobbes (1991a, 91).

9. Ut procedatur post definitiones eadem methodo qua qui docet, ipsa quaeque invenerat; nempe ut primo demonstrentur ea quae sunt definitionibus maxime universalibus proxima (in quo continetur pars philosophiae illa quae philosophia prima dicitur) deinde ea quae demonstrari possunt per motum simpliciter (in quo consistit geometria) post geometriam, ea quae doceri possunt per actionem manifestam, id est, per impulsionem et tractionem. Inde ad motum partium invisibilium, sive mutationem, et ad doctrinam sensuum imaginationisque descendendum est, et ad animalium passiones internas, praesertim vero hominis, in quibus continentur fundamenta prima officiorum sive doctrinae civilis quae locum tenet ultimum. Quod autem doctrinae universae ordo is quem dixi esse debeat, ex eo cognosci potest; quod quae posteriore loco docenda esse dicimus, nisi iis cognitis quae priore loco tractanda proponuntur, demonstrari non possunt.

10. Malcolm attributes a version of this thesis ("that Hobbes envisaged a single continuous chain of derivation leading from physics, via psychology, to politics") to Alan Ryan (Malcolm 1990, 145-6). Malcolm argues against this interpretation, as does Sorell (1986).

11. ...rerum communissimarum ideas ad sublationem ambigui et obscuri, definitionibus accuratis inter se distinguo.

For other accounts in Hobbes of the content and purpose of first philosophy, see Hobbes (1991a, 103); *Lev* IV.46, 463; Hobbes (1976, 23); and *SL* 2; *EW* VII, 226.

12. As is well known, Hobbes carried out a long and extremely vituperative dispute with Wallis, a prominent mathematician and Presbyterian theologian. They debated about a wide range of topics, including mathematics, politics, and Latin grammar. The *Six Lessons* is one of many texts that Hobbes produced in response to attacks from Wallis. For an account of this dispute, see Jesseph (1999).

13. ...accidentium eorum quae sunt omnibus corporibus, hoc est omni materiae communes, quam singularium, hoc est accidentium quibus una res ab alia distinguitur.

14. Hobbes (1976, 23).

15. Prioris generis sunt corpus sive materia, quantitas sive extensio, motus simpliciter, denique quae omni materiae insunt. Secundi generis sunt corpus tale, motus talis et tantus, magnitudo tanta, talis figura, aliaque omnia quibus unum corpus ab alio distingui potest.

16. Exempli gratia, proposito quolibet conceptu sive idea rei singularis, puta quadrati. Quadratum ergo resolvetur in *planum, terminatum lineis,* et *angulis rectis, certo numero,* et *aequalibus*. Itaque habemus universalia haec, sive materiae omni convenientia, *lineam,*

planum, (in quo continetur superficies) *terminatum, angulum, rectitudinem, aequalitatem*, quorum causas sive generationes si quis invenerit, in causam quadrati eas componet. Rursus, si proponat sibi conceptum auri, venient inde resolvendo ideae *solidi, visibilis, gravis*, (id est conantis ad centrum terrae sive motus deorsum) aliaque multa magis universalia quam est ipsum aurum, quae rursus resolvi possunt, donec perveniatur ad universalissima.

17. Sorell (1986, 60) reads this passage as eliminating chapter 14 from first philosophy.

18. Jesseph (1999, 111n).

19. Et quoniam magnitudo continua quaelibet data dividi potest in partes quotlibet aliquotas, ratione eius ad quamlibet aliam non mutata, manifestum est arithmeticam in geometria contineri.

The other character in the dialogue concurs, stating: "Itaque qui de quantitate loquens continua, geometra est, idem de eadem loquens quantitate ut divisa in partes aliquotas, est arithmeticus".

In the *Six Lessons*, Hobbes similarly says that "So also is number quantity; but in no other sense than as a line is quantity divided into equal parts" (*SL* 1; *EW* VII, 194).

20. This was suggested to me by Douglas Jesseph.

21. See Mancosu (1996, 86). In the third of his *Mathematical Lectures*, "Of the Identity of Arithmetic and Geometry," Barrow defends the position that geometry is the more fundamental science. The lecture also presents a useful summary of some of the important arguments of those who defended the opposite position.

22. On the debate over the status of geometry and arithmetic, and John Wallis's role in it, see Jesseph (1999, 37-40).

23. *De Anima* I.4, 409a 4-5.

24. *Elements* XI, defn. 14; II, 261. Hobbes cites this definition in the *Six Lessons*, in response to an objection from Wallis to the use of motion in geometrical definitions (*SL* I; *EW* VII, 215).

25. De Rationibus Motuum, et Magnitudum.

26. Murdoch and Sylla (1978, 230). Murdoch (1963, 261-65).

27. *Elements* II, prop.14; 1, 409.

28. There were some dissenters: Barrow, for example, while acknowledging that proportion constitutes an extremely important part of mathematics, objects to Hobbes's definition, claiming that it doesn't account for many of the problems that geometricians address:

> *Geometry* (says [Hobbes]) *is the Science of determining the Magnitude of any thing not measured, by its Comparison with another measured Magnitude or Magnitudes*. But I ask those Definers of our Science; when the Geometrician *bisects a Right Line, or a Rectilineal Angle;* when he *erects* or *lets fall a right-lined Perpendicular* from a *given Point*; when he *draws* a *Parallel* to a *given Right Line* through a *given Point*; or when he *draws* a *right-lined Tangent* to a *given Curve*; when he *constitutes* and *equilateral Triangle*, or *Square*, upon a *given Right Line*; when he *describes* a *Circle* though *three given Points*, or *circumscribes* a *Circle* about a *given Triangle*; when he *investigates* the *Center* of a given *Circumference*, or the *Focus* of a given *Conic Section*; I say, when he does many such Things, and *resolves Problems* respecting only the *Position* of *Magnitudes*: whether does he perform the Office of a Geometrician as he ought, and when he compares Magnitudes together this Way, as to their Quantity, what Relation have they to any *Measure*? Why, none at all; he only determines the *Situation* of *Lines*, and enquires after the *Position* of *certain Points*. (Barrow 1970, 246-7)

29. *Relatio*...antecedentis ad consequens secundem magnitudinem, nimirum aequalitas, vel excessus, vel defectus ejus, *ratio* et *proportio* antecedentis ad consequens dicitur: ut *ratio* nihil aliud sit quam aequalitas vel inaequalitas antecedentis comparati ad consequens secundum magnitudinem.

30. ...major vel minor alia, tanta ejus parte vel partibus, ut 7 minor est 10, tribus ipsius denarii partibus decimis.

31. On the two theories of ratio and proportion, and their place in seventeenth-century debates, see Jesseph (1999, 85-94), Grosholtz (1987, 209-12), Sylla (1984), Sasaki (1985), and Barrow (1970, XVII-XXIII, 312-440).

32. *Elements* V, defn.3; II, 114.

33. Sylla (1984), Jesseph (1999, 86).

34. Sylla (1984, 22-3) traces this tradition back to Theon of Alexandria's commentary on Ptolemy's *Almagest*, and finds it communicated in the Middle Ages in the work of Jordanus Nemorarius, Campanus, and Roger Bacon.

35. Jesseph (1999, 88).

36. The development of this new concept of number is explored in Klein (1968). Klein sees Wallis's theory as the culmination of this process (Klein 1968, 211-224).

37. In another example, Hobbes disagreed with Barrow, an important defender of the relational theory, when he argued that ratios (at least of excess and defect) are quantities (Jesseph 1999, 90-92).

38. ...primo enim videndum, corpus motum, si nihil aliud consideretur in eo praeter motum quid efficiat; apparet autem statim effici lineam sive longitudinem; deinde quid faciat corpus longum si moveatur, constabitque fieri superficiem, atque ita porro quid fiat ex motu simpliciter; deinde simili modo, ex huiusmodi motibus additis, multiplicatis, subtractis, divisisque, qui effectus, quales figurae, et quales earum existent proprietates, contemplandum est; atque ex hac contemplatione orta est philosophiae pars ea quae appellatur geometria.

39. As we will see, one should always be careful in applying such terminology to Hobbes, since the meanings which he gives to such terms as "force" are significantly different from modern ones.

40. Post considerationem eorum quae fiunt ex motu simpliciter, sequitur consideratio eorum, quae motus unius corporis efficit in corpus aliud, et quoniam motus esse potest in partibus corporis singulis, ita tamen ut totum suo loco non decedat, inquirendum est primo loco quis motus quem motum efficit in toto; hoc est, incurrente aliquo corpore in aliud corpus quod quiescit, vel quod motu aliquo jam movetur, qua via et qua velocitate movebitur illud post incursum, et rursus quem motum motus ille secundus generabit in tertio, et sic deinceps, ex qua contemplatione existet philosophia pars illa quae de motu est.

41. *Elements* V, defn. 3; II, 114.

42. Ratio geometrica rationi geometricae eadem est, quando causa aliqua aequalibus temporibus aequalia faciens, utramque rationem determinans eadem assignari potest.

43. Tertio loco ad eorum inquisitionem devenietur quae fiunt ex motu partium, ut in quo consistit quod eadem res, sensui tamen eaedem non videantur sed mutatae; itaque investigantur hoc loco, qualitates sensibiles, quales sunt, *lux, color, diaphaneitas, opacitas, sonus, odor, sapor, calor, frigus,* et similia, quae quia sine cognitione causae

ipsius sensionis cognosci non possunt, consideratio causarum *visionis, auditus, olfactus, gustus,* et *tactus*, tertum locum obtinebit, qualitates autem illae praedictae, mutationesque omnes in locum quartum differendae sunt, quae duae considerationes eam partem philosophiae continent quae vocatur physica.

44. Charleton makes a similar distinction when differentiating between the local motion with which he is concerned in the *Physiologia Epicuro-Gassendo-Charltoniana* (1654), and mutation:

> But, our subject is *Motion as proper to a body Concrete, which sensibly changes the Place of its whole, or some sensible part.* For, herein motion plainly distinguisheth it self from mutation, that in *motion* the whole Body, V.G. of a man, or some sensible part thereof, as his hand or foot is translated from one place to another: but in *Mutation* only the insensible particles of a body, or any part thereof, change their positions and places, though the whole or sensible parts thereof, remain quiet. (Charleton [1654] 1966, 438)

45. Hobbes (1991a, 42).

46. For an overview of the debate between Hobbes and Wallis on the proper relationship between natural philosophy and mathematics, see Jesseph (1999, 132-142).

47. Quoted in Jesseph (1999, 134).

48. *PRG* 12; *OL* IV, 421.

CHAPTER 2

THE METHOD OF NATURAL PHILOSOPHY

As was mentioned in the previous chapter, Hobbes defines "philosophy" as "*knowledge, acquired by right reasoning, of effects or phenomena from conceptions of their causes or generations, and in turn [knowledge] of generations that can be from conceptions of effects*" (*DCp* I.1.2; *OL* I, 2).[1] Philosophical knowledge is hence causal knowledge.

Hobbes believed that causal knowledge would be a powerful tool, as it would allow us, insofar as is physically possible, to generate any effect that we might conceive and desire. This, in turn, would greatly improve the conditions of human life.[2] Hobbes thus conceives of philosophy as a practical enterprise, claiming that "all speculation is undertaken for the sake of some action or work" (*DCp* I.1.6: *OL* I, 6).[3]

In order to maximize our causal knowledge, Hobbes thinks that we need to develop an effective scientific method. "Method," not surprisingly, is defined as "*the briefest [means of] investigation of effects by their known causes, and causes by their known effects*" (*DCp* I.6.1; *OL* I, 58-9).[4] The first section of this chapter will examine Hobbes's version of the method of analysis and synthesis. My discussion will focus on the tension between two aspects of this method: Hobbes's notion of conceptual analysis, and his claim that the method of analysis will produce causal knowledge. This subject is particularly relevant to the topic of this dissertation, since it concerns an attempt on Hobbes's part, albeit a less than successful one, to adapt a mathematical method to the study of nature in general. I do not pretend to be addressing all significant aspects of

Hobbes's account of analysis and synthesis: most notably, I will not be discussing Hobbes's rejection of analytic or algebraic methods in mathematics. This aspect of Hobbes's thought has recently received an extensive treatment by Jesseph.[5]

The second section of the chapter will discuss Hobbes's views on the status of hypotheses. As was discussed in the previous chapter, Hobbes thinks that hypotheses play an important role in the non-demonstrative sciences. These hypotheses must, however, be grounded in the principles of previously established, demonstrative sciences. As we will see, Hobbes's mathematical account of motion is in part shaped by the fact that it will serve to ground the hypotheses of physics.

2.1 The Method of Analysis and Synthesis

In the seventeenth century it was common to emphasize the importance of scientific method. In his *Rules for the Direction of the Mind*, for example, Descartes states that "it is far better never to contemplate investigating the truth about any matter than to do so without a method."[6] It was also standard to include analysis and synthesis as an important part of this method. Descartes thus continues:

> So useful is this method that without it the pursuit of learning would, I think, be more harmful than profitable. Hence I can readily believe that the great minds of the past were to some extent aware of it, guided to it even by nature alone. For the human mind has within it a sort of spark of the divine, in which the first seeds of useful ways of thinking are sown, seeds which, however neglected and stifled by studies which impede them, often bear fruit of their own accord. This is our experience in the simplest of sciences, arithmetic and geometry: we are well aware that the geometers of antiquity employed a sort of analysis which they went on to apply to the solution of every problem, though they begrudged revealing it to posterity.[7]

Descartes thought that seventeenth-century algebra had reconstructed some of the method that had provided the ancients with so much success.

Descartes was not alone in referring to the classical origins of the method of analysis.[8] The most complete description of the method in ancient sources is from Pappus of Alexandria's *Mathematical Collections*. This text was translated into Latin by Federico Commandino and published in 1589.[9] It is worth quoting Pappus's description of the method of analysis at length:

> Analysis, then, takes that which is sought as if it were admitted and passes from it through its successive consequences to something which is admitted as the result of synthesis: for in analysis we assume that which is sought as if it were already done, and we inquire what it is from which this results, and again what is the antecedent cause of the latter, and so on, until by so retracing our steps we come upon something already known or belonging to the class of first principles, and such a method we call analysis as being solution backwards. But in synthesis, reversing the process, we take as already done that which was last arrived at in the analysis and, by arranging in their natural order as consequences what before were antecedents, and successively connecting them one with another, we arrive finally at the construction of what was sought; and this we call synthesis [...] Now analysis is of two kinds, the one directed to searching for the truth and called 'theoretical' the other for finding what we are told to find and called 'problematical'. In the theoretical kind we assume what is sought as if it were existent and true, after which we pass through its successive consequences, as if they too were true and established by virtue of our hypothesis, to something admitted: then if that something admitted is true, that which is sought will also be true and the truth will correspond in the reverse order to the analysis, but if we come upon something admittedly false, that which is sought will also be false. In the problematical kind we assume that which is propounded as if it were known, after which we pass through its successive consequences taking them as true, up to something admitted: if then that is admitted as possible and obtainable, that is, what mathematicians call given, what was originally proposed will also be possible, and the proof will again correspond in the reverse order to the analysis, but if we come upon something admittedly impossible the problem will be impossible.[10]

According to Pappus's description, there are two kinds of analysis: theoretical analysis, which is used to produce proofs of theorems, and problematical analysis, which is used to solve problems.[11] In both cases, the method involves taking what is to be proven or constructed as if it were true or given, then reasoning back through successive principles from which that thing can be derived. At some point we either come to something already known to be true, in which case we reverse the analysis in order to produce a demonstration, or we come to a falsehood, in which case the analysis serves as a *reductio ad absurdum* for the result originally sought.

In part II of *De Corpore* (which, to recall, contains Hobbes's geometry) Hobbes gives an account of the analysis which is sometimes similar to Pappus's, but also differs in significant ways. Hobbes states that

> [a]nalysis is continuous reasoning from the definitions of the terms of some statement which we suppose true, and in turn from the definitions of the terms of those definitions, until we come to something known, the composition of which is the demonstration of the truth or falsity of the statement supposed. And that very composition or demonstration is that which is called synthesis. *Analytica* is therefore the art of reasoning from something supposed to principles, that is, to first propositions or those demonstrated from first propositions, as many as suffice for the demonstration of the truth or falsity of the thing supposed. But synthesis is the art itself of demonstrating. Therefore *synthesis* and *analysis* do not differ otherwise than as forwards and backwards. (*DCp* III.20.6; *OL* I, 252)[12]

In line with Pappus's description, Hobbes's procedure involves reasoning backwards from some proposition that is taken as if true. Hobbes goes on to emphasize that each step in the analysis must be convertible: not only must the consequent follow from the antecedent, but the antecedent must also follow from the consequent.

Despite the similarities with Pappus, Hobbes has adopted the method of analysis to suit his own philosophical presuppositions. The process of analysis must stop at something we know to be true. To recall, for Hobbes all first principles are definitions. Accordingly, in the above passage analysis is described as a process wherein we reason from the definitions of the terms in the admitted proposition, to the definitions of the terms in those definitions, and so on. This makes the form of Hobbes's analysis different from Pappus's: at each step of a Hobbesian analysis, a single proposition is simultaneously analysed into many propositions. In a traditional mathematical analysis, however, each step involves reasoning successively from one proposition to another. As we will see, this aspect of Hobbes's method will prove problematic when it comes to putting the method into practice.

Hobbes goes on to explain how analysis will generate causal knowledge (which is, of course, the goal of all scientific inquiry). In a mathematical analysis one seeks the proportions of two quantities, and the solution involves the construction of a figure of a given quantity. The problem is thus solved when one reaches the cause of the construction sought (or detects the impossibility of such a construction). Since analysis ends when one reaches prime propositions (or definitions), the definitions must contain the causes of the construction. Hobbes therefore offers a second account of analysis and synthesis, stating that "*analysis is reasoning from the supposed construction or thing made to the efficient or many coefficient causes of the construction or thing made.* And *synthesis is reasoning from the first causes of the construction continued through the middle causes to the thing itself made*" (*DCp* III.20.6; *OL* I. 254).[13] Reasoning from effect to cause and term to definition is essentially the same thing, since all definitions must include the causes of the thing being defined.

Hobbes draws a similar connection between causal and linguistic or conceptual analysis in *De Corpore*'s chapter "Of Method," in which he provides a very general account of scientific method. He there reiterates that philosophy is knowledge that we acquire by right reasoning, either of effects from their causes or causes from their effects. Reason, however, consists in

> composition and division or resolution. Therefore every method through which we investigate the causes of things, is either compositive, or resolutive, or partly compositive, and partly resolutive. And the resolutive is commonly called *analytic*, the compositive *synthetic*. (*DCp* I.6.1; *OL* I. 59)[14]

Hobbes elaborates that we will be able to obtain knowledge of causes by some combination of breaking our conceptions into their parts, and adding those parts together. Hobbes introduces the terms "composition" and "synthesis" to describe the latter part of the process, "resolution" and "analysis" to describe the former.[15] He is careful to specify that by parts he means parts of the nature of the thing, rather than parts of the thing itself — so, for example, the concept of a man would not be resolved into his head, shoulders, legs, and other body parts, but into his figure, motion, quantity, sense, reason, and so on. These accidents "being compounded or put together, constitute the whole nature of man, but not the man himself" (*DCp* I.6.2; *OL* I, 60).[16] In other words, it appears that we divide our conception into those properties which would be both necessary and sufficient for our applying the name of that conception to a given object. These properties would, of course, be those that would appear in a definition of that object.

Such resolution leads to an understanding of causes because "the cause of the whole is composed from the causes of the parts, but it is necessary to know the things to be compounded before [we can know] the compound" (DCp I.6.2; *OL* I, 59-60).[17] Resolving a compound object into its parts is a necessary precursor of finding the causes of those parts, and of compounding those causes into the cause of the whole.

In both discussions of his method Hobbes claims that linguistic or conceptual analysis will lead to causal knowledge. However, this claim is problematic, for reasons related to the structure of a Hobbesian analysis. As I mentioned earlier, at each stage of a traditional mathematical analysis a single consequence is drawn from the previous proposition. Because each consequence was generated in succession, the order of the corresponding synthesis will be clear: one simply reverses the successive steps of the analysis. On the other hand, at each stage of a Hobbesian analysis a notion is divided into multiple conceptual parts. The process establishes no specific order as to how the parts should be compounded when it comes time to perform the synthesis.

This becomes problematic when we consider that in compounding the parts of something we are also supposed to be compounding the causes of those parts, so as to ascertain how to generate the whole. It seems that I could know the parts of thing, and how to generate those parts, without knowing how to generate the whole — we need a procedure that tells us how to assemble the parts that we are able to construct.

To illustrate, we can look at the example of a square. According to Hobbes's method, in order to figure out how to generate a square, we would begin with an object assumed to be such a figure. We would then analyse the supposed square into its conceptual parts, which Hobbes states are four sides, equality of sides, and right angles (*DCp* I.1.3; *OL* I, 4). However, this process does not tell us how, exactly, to reassemble those parts in such a way that they form a square. Hobbes's proposal can be compared with the following part of the solution that Euclid presents in the *Elements*[18] to the problem of describing a square on a given straight line:

> Let AB [in figure 2.1]be the given straight line; thus it is required to describe a square on the straight line AB.
>
> Let AC be drawn at right angles to the straight line AB from the point A on it [I.11], and let AD be made equal to AB; through the point D let DE be drawn parallel to AB, and through the point B let BE be drawn parallel to AD.

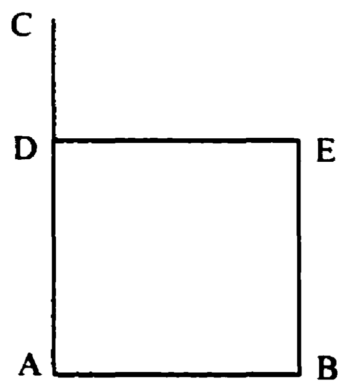


Figure 2.1

Euclid's procedure involves drawing equal straight lines at right angles, and he hence presupposes that we know how to construct these parts of a square (for example, he refers to Book I, proposition 11, which describes how to draw a straight line at right angles to a given straight line from a given point on it). However, it also specifies the order and configuration in which these parts should be constructed. Hobbes's method does not tell us how to generate such a procedure.

This difficulty can be further illustrated by looking at a famous passage from the Author's preface to *De Cive*, in which Hobbes explains his methodology with the example of a watch:

> Concerning my method, I thought it not sufficient to use a plain and evident style in what I have to deliver, except I took my beginning from the very matter of civil government, and thence proceeded to its generation and form, and the first beginnings of justice. For everything is best understood by its constitutive causes. For as in a watch, or some such small engine, the matter, figure, and motion of the wheels cannot well be known, except it be taken insunder and viewed in parts.[19]

It is true that taking a watch apart can provide insight into how it functions. However, in taking the watch apart we would not just be interested in identifying the different parts and figuring out how they might have been made. A crucial aspect of the appeal of taking the watch apart is that it would allow us to see how the parts were put together, and hence how we could develop a procedure for reassembling the watch and constructing others like it.[20] If the process of resolution left us with nothing but a jumble of parts, it would be of little use.

It is interesting, in light of these problems, that Hobbes often equates reasoning with computation, which is "*to collect the sum of many things added together, or to know what remains when one thing is taken away from another*" (*DCp* I.1.2; *OL* I, 3).[21] He allows that reasoning can also include multiplication and division, stipulating that multiplication is just the addition of equals to one another, and division a subtraction of equals. We can compute not only with numbers, but "magnitude can also added to and taken away from magnitude" and, in similar fashion, we can compute with body, motion, time, degrees of quality, action, conception, proportions, speech, and names (*DCp* I.1.3; *OL* I, 4-5). Reasoning about, or adding together, the various parts of our conceptions is supposed to generate causal knowledge. However, addition is a commutative operation — reason, as modelled on this operation, can tell us that we should compound a set of causes, but cannot tell us the order in which they should be compounded. Nor does it appear able to provide information about where parts should be generated in relation to each other.

Hobbes might reply that in describing reasoning as a process of "addition" and "subtraction" he was speaking metaphorically. Indeed, this sometimes seems to be the case — in *Leviathan*, for example, Hobbes (having defined "reason" as nothing but addition and subtraction) provides some examples of non-numerical reasoning:

> Logicians teach the same [to add and subtract] in *Consequences of words*; adding together *two Names*; to make an *Affirmation*; and *two Affirmations*, to make a *Syllogisme*; and *many Syllogismes* to make a *Demonstration*; and from the *summe*, or the *Conclusion* of a *Syllogisme*, they substract one *Proposition*, to finde the other. Writers of Politiques, adde together *Pactions*, to find mens *duties*; and Lawyers, *Laws*, and *facts*, to find what is *right* and *wrong* in the actions of the private men. (*Lev.* I.5, 32)

The order in which we, for example, "sum" up names will often make a significant difference to the resulting affirmation. Since Hobbes clearly intends such considerations to be part of the reasoning process, his arithmetic imagery should not, perhaps, be taken too seriously.

However, elsewhere in *Leviathan* and *De Corpore* Hobbes provides relatively detailed accounts of how affirmations and syllogisms should work. Similarly, his work includes extensive discussions of the relationship between facts and laws. In the above passage Hobbes uses the terms "addition" and "subtraction" metaphorically, but he backs up this metaphorical terminology with detailed accounts of how the relevant reasoning processes actually work.

Unfortunately, this is not the case when it comes to Hobbes's method of analysis and synthesis. As I have discussed, Hobbes does not explain how, exactly, resolution will preserve the configuration of the parts of the thing being resolved, thus allowing the causes of those parts to be recomposed into the cause of the whole. This is not an insignificant omission on Hobbes's part. The general philosophical method that Hobbes presents in *De Corpore* is supposed to provide a foolproof guide that anyone can use to reason from cause to effect or effect to cause. Unless Hobbes's method tells us exactly how to disassemble and reassemble the parts of the thing being subject to analysis, it has not done what it was intended to do. A method that leaves us with a pile of watch parts will not help us become watchmakers.

In this regard it is useful to look again at Hobbes's use of the method of analysis in his mathematics. Hobbes thinks that there are three different kinds of geometrical analysis,[22] but

> in none of these ways can a certain rule be established, in somewhat complicated questions, from the supposition of which unknown the analysis should begin, nor from the variety of equations which disclose themselves at the beginning, which should be chosen, but success will be assigned according to ingenuity, science previously acquired, and partly by fortune. (*DCp* III.20.6; *OL* I, 255)[23]

In all three types of mathematical analysis it is impossible to develop definite rules by which the method can be applied to more complicated situations. This is the very reason why Hobbes chooses not to discuss geometrical analysis in the context of his more general discussion of the method. At the end of the account of method in *De Corpore*'s second part, Hobbes states that the first part of geometrical analysis is the equation of known and unknown things, "and this equation cannot be discovered except by those who have at hand the nature, properties, and transpositions of proportions, the addition, subtraction, multiplication, and division of lines and surfaces, and the extraction of roots, that which is already of no mediocre geometer" (*DCp* I.6.19; *OL* I, 79-80).[24] The mathematical method of analysis is an art which cannot be practised without extensive and specialized training, and it cannot therefore be detached from geometry itself. Although the geometrical method may have inspired Hobbes's enthusiasm for analysis, it cannot, in the absence of significant changes, supply a methodological model which can be applied to all of the sciences. Furthermore, success at the geometrical method of analysis does not depend on method alone: "in the discovering of equations there is no method, but each succeeds so much as he exhibits natural wit" (*DCp* I.6.19; *OL* I, 80).[25]

Hobbes is looking for a method that will allow us to move from a given effect to its causes and back again, but he also requires that the method, if followed properly, will provide accurate results to those with varying levels of natural ability and specialized knowledge. Hobbes thought that linguistic, and hence conceptual, resolution supplied such a method. For example, despite Hobbes's belief that civil philosophy is based on the principles of geometry and physics, he nonetheless claims that those who do not understand mathematics and natural philosophy can understand the principles of civil philosophy by means of a process of resolution:

> For if a question be propounded, as, *whether such an action be just or unjust*: if that *unjust* be resolved into *fact against law*, and that notion *law* into the *command* of him or them that have *coercive power*; and that *power* be derived from the *wills* of men that constitute such power, to the end they may live in peace, they may at last come to this, that the appetites of men and the passions of their minds are such, that, unless they be restrained by some power, they will always be making war upon one another: which may be known to be so by any man's experience, that will but examine his own mind. (*DCp* I.6.7; *OL* I, 64)[26]

If one understands the meanings of the terms in a proposition (a necessary precursor to doing any kind of philosophy), breaking them down into their parts is a mechanical process, and hence one that requires no special ability. Nor does this procedure require any specialized knowledge, as Hobbes indicates that it could potentially be carried out by any person. However, for reasons we have discussed, this form of linguistic resolution, while potentially easy to carry out, does not produce the kind of causal knowledge that Hobbes's goal.

Hobbes had political reasons for aspiring to develop a foolproof method. At the beginning of *De Corpore*, Hobbes claims that civil conflict arises because people do not know the causes of war and peace, or the rules of civil life. He then asks "[w]hy have they not learnt this, unless because it has thus far been taught by no one with a clear and proper method?" (*DCp* I.1.7; *OL* I, 7).[27] He goes on to compare the consensus generated by geometrical texts with the controversy generated by volumes of ethics, suggesting that the latter results from a lack of clear demonstration. A proper method is needed so that people will be able to determine the causes of war and peace, and adjust their behaviour accordingly. If such a method were at all difficult to use, it would be unlikely to eliminate controversy and civil conflict.

Hobbes's confidence that his method of analysis would be both accessible and productive of scientific knowledge may have been due to the influence of Bacon. There are strong similarities between Hobbes's account and some features of the method presented by Bacon in *The New Organon*. Furthermore, there is evidence of an association between Hobbes and Bacon. Aubrey reports that Hobbes acted as a secretary to Bacon, saying that Hobbes

> was beloved by his lordship, who was wont to have him walk with him in his delicate groves when he did meditate; and when a notion darted into his head, Mr Hobbes was presently to write it down, and his lordship was wont to say that he did it better than any one else about him; for that many times when he read their notes, he scarce understood what they writ, because they understood it not clearly themselves.[28]

The programmes of Hobbes and Bacon differ, of course, in a number of important ways, not least of all in their vastly different attitudes towards experiment. However, given their relationship, it would not be surprising if Hobbes were influenced by some aspects of the Baconian programme.

Bacon and Hobbes shared the view that the goal of scientific knowledge should be power over nature, for the sake of bettering the human condition.[29] Bacon held the aim of this power to be the ability "on a given body, to generate and superinduce a new nature or new natures."[30] These natures are the properties that bodies can have. Bodies can be regarded "as a troop or collection of simple natures", i.e., a given body just is the collection of its essential properties. Gold, for example, can seen as the union of the properties of being yellow, heavy, malleable, and so on. Hence if we know how to superinduce all of these properties on a body, we know how to generate gold.[31]

In order to produce natures, Bacon claims that we need to know their forms. The form, according to Bacon, is the reality underlying the nature's appearance to us.[32] Since the form is present when the nature is present, absent when it is absent, and increases and decreases in presence with the presence of the nature,[33] someone who knows how to generate the form will also know how to generate the nature. In order to generate a thing we therefore need to be able to bring about the forms which underlie its essential properties.

Even this brief account of Bacon's method suggests some similarities with Hobbes's account of analysis and synthesis. Both suggest that we consider a body as a composition of its essential properties: for Bacon, these are its "simple natures"; for Hobbes, the "parts of its nature." They hence propose that if we can discover the conditions under which these properties come about, we can compound or superinduce them collectively onto a body, and hence generate the object that we desire.

Furthermore, both thinkers claim that a complete knowledge of nature would result from a catalogue of the simple natures. Bacon claims that a knowledge of simple natures "gives entrance to all the secrets of nature's workshop," just as a knowledge of the letters of the alphabet provides the basis for all discourse.[34] Similarly, Hobbes suggests that in order to obtain knowledge of the causes of all things, we must first resolve our ideas down to their most simple and universal parts. These simple or universal things can then be compounded to generate ideas of other, more complex objects.[35]

Finally, it was noted that Hobbes rejects the geometrical method of analysis as a methodological model on the grounds that it can only be used by those who have some natural wit. Bacon similarly claims that his "way of discovering sciences goes far to level men's wit and leaves but little to individual excellence, because it performs everything by the surest rules and demonstrations." As one needs no special talent to draw a circle with the aid of a compass, so those with little ability should be able to reason scientifically with the help of Bacon's method.[36]

Commentators have made other suggestions regarding the origins of Hobbes method. Hobbes's talk of resolving things into their parts has led some to contend that Hobbes was influenced by the methodology of the School of Padua. Watkins, for example, claims that "the intuitive idea which informs this methodological tradition was this: the way to understand something is to take it apart, in deed or in thought, ascertain the nature of its parts, and then reassemble it — resolve it and recompose it."[37] If this description is accurate, it would indeed provide an explanation for some of Hobbes's views on method.[38]

There are some similarities between Hobbes's work on method and that of the Paduans. For example, the logician Jacopo Zabarella (1533-89), one of the main proponents of the Paduan methodology, shares with Hobbes the view that all scientific knowledge is causal, and is gained by reasoning either from cause to effect or effect to cause. The former process can be called resolution, the latter composition. Hobbes and Zabarella also share the view that the latter is the superior form of reasoning, although the understanding of effects will ultimately involve both.

These shared ideas do not establish a particularly Paduan influence on Hobbes, since they were standard views within the Aristotelian tradition. Furthermore, there is a significant difference between the ways in which Hobbes and Zabarella use the terms "resolution" and "composition." Zabarella distinguishes between two types of resolution:[39] first, he describes the *a posteriori* proof, which allows us to discover causes that are not immediately perceived by the senses.[40] This is distinguished from demonstrative induction, which reveals those less hidden causes which are immediately perceivable to the senses.[41] While both kinds of resolution involve reasoning from effect to cause, there is no indication that Zabarella thought that either process should entail a

Hobbesian division of the given effect into its constituent conceptual parts.[42] Similarly, for Zabarella composition is just a reversal of these processes, wherein we reason from cause to effect. Again, nothing resembling the addition of Hobbesian parts is involved.

This becomes clear if we look at an example of a Zabarellan resolution.[43] In the following passage Zabarella discusses how we come to know the existence of prime matter:

> Let us consider the demonstration in Book One of the *Physics* by which Aristotle infers from the generation of substances the existence of prime matter: from a known effect an unknown cause. For generation is known by our senses, but the material substrate is in the highest degree unknown. Having considered the proper subject, that is, a perishable natural body in which generation occurs, he shows that there inheres in it a cause on account of which this effect inheres; and it is *demonstratio quia* [demonstration from effect to cause] which is thus formed: 'wherever generation occurs there is a material substrate; but in a natural body there is generation; so in a natural body there is prime matter'. In this demonstration the minor premiss is known to us confusedly (*confuse*), because we do in fact observe that natural bodies are generated and perish, but we do not know the cause. The major premiss, although not known by the senses, is easily made known by applying some mental consideration.[44]

In this example, the existence of prime matter is proven through the existence of bodies that are generated and perish, and the demonstration reveals something that was not knowable to the senses. The process involves reasoning that the sensible effect of generation and corruption entails the cause of prime matter, and hence involves the direction of causal reasoning the Hobbes associates with resolution. However, Zabarella does not begin, as Hobbes proposes, by analysing the concept of body into all of its parts. Furthermore, Zabarella reasons successively from effect to prior cause, thus incorporating order into his resolution.

Some commentators have made the further suggestion that Hobbes's method was influenced by that of Galileo.[45] Leaving aside the question of whether Galileo was himself influenced by the School of Padua, and hence was the link between Hobbes and the

Paduans,[46] there are similarities between their methods. In particular, some point to Galileo's account of the motion of projectiles in the Fourth Day[47] of *Dialogues Concerning Two New Sciences* (1638). Galileo posits that the motion of a projectile is derived from the composition of a uniform horizontal motion and a naturally accelerated downward motion, and derives some of the properties of projectile motion by considering it in this way.[48]

The procedure used by Galileo in this and other examples does bear some resemblance to Hobbes's method of analysis and synthesis: first, Galileo actually resolves the motion into its constituent parts in order to determine its properties. That this method worked so well in explaining the properties of motion would have appealed to Hobbes, given that his materialism commits him to describing all phenomena in terms of matter in motion. Furthermore, there is no doubt that Hobbes read and was influenced by Galileo,[49] as we will be discussing in future chapters.

There appears to be no textual evidence which would exclude Galileo as the source of Hobbes's method. However, the parallels described above do not provide conclusive evidence in favour of Galileo's influence. Most importantly, his work on motion does little to account for the peculiarities of Hobbes's method of analysis and notion conceptual or linguistic resolution.

In sum, it seems that Hobbes, like many others in the seventeenth century, was inspired by the method of analysis, in all likelihood as that method was depicted in mathematical sources. He adapted the method in accordance with two of his most important philosophical presuppositions: the importance of causal knowledge, and the axiomatic status of definitions. These two aspects of Hobbesian analysis sit uneasily together, however, since the linguistic or conceptual analysis that Hobbes proposes will not tell us how to generate the thing being analysed.

2.2 The Nature and Status of Hypotheses

As we have seen, Hobbes's method includes both synthesis, or reasoning from cause to effect, and analysis, or reasoning from effect to cause. However, these aspects of

Hobbes's method do not have equal status. As he states in *De Homine*, "[b]oth of these methods of proof are usually called demonstrations; the former kind is, however, preferable to the latter; and rightly so; for it is better to know how we can best use present causes than to know the irrevocable past, whatsoever its nature."[50] Although Hobbes's assertion of the superiority of synthesis is standard, his justification of that superiority is not: Hobbes's assertion depends on the assumption that a cause must temporally precede its effect, which an Aristotelian would, of course, reject.

As we saw in chapter 1, geometry and politics are demonstrative or *a priori* sciences because the generation of their subjects is within our power. Physics, on the other hand, is *a posteriori* because it involves reasoning from natural effects to their possible causes. Because we can never be sure what the causes are of a given natural effect, explanations in physics will always be hypothetical. As a result, in natural philosophy we can only aspire "to have such opinions as no certayne experience can confute, and from which can be deduced by lawfull argumentation, no absurdity."[51]

Hobbes does not, however, think that there are no restrictions on possible physical hypotheses. Legitimate hypotheses must have two properties: "of which the first is, that it be conceivable, that is, not absurd; the other, that, by conceding it, the necessity of the phaenomenon may be inferred" (*DP*; *OL* IV, 254; 362). Hobbes places significant restrictions on what counts as a conceivable hypothesis. Many of these arise from Hobbes's fundamental presuppositions: all hypotheses, for example, must explain the given effect in terms of the motion and impact of bodies.[52] He thus berates those scientists who "arouse in very learned men, not only of our country but also abroad, the expectation of advancing physics, when [they] have not yet established the doctrine of universal and abstract motion (which was easy and mathematical)" (*DP*; *OL* IV, 273; 379). Since all hypotheses must be formulated in terms of matter in motion, we need to understand the doctrine of motion before we will be able to frame adequate hypotheses.

Hobbes dismisses various Aristotelian notions, including the doctrines of immaterial substances and essences, as inconceivable, and hence having no place in the hypotheses of natural philosophy.[53] So, for example, Hobbes states in *Dialogus Physicus*:

> In physics books, many things present themselves which cannot be grasped, such as those things said of rarefaction and condensation, of immaterial substances, of essences and many other things: which if you try to explain with their words, it is useless, and if with your own, you will say nothing. (*DP*; *OL* IV, 238; 349)

However, as we can see from the above passage, Hobbes could, in the same breath, criticize Aristoteleanism and claim that versions of the mechanical philosophy other than his own are absurd. Hobbes was a plenist, arguing that a space not filled with body is impossible. Some other seventeenth-century scientists, including Robert Boyle, attempted to explain natural phenomena in terms of material particles in a vacuum. The vacuists claimed that bodies could become rarer or more dense as their constituent particles occupied more or less space. Hobbes renounces this doctrine for being not only incorrect, but inconceivable:

> What is that *Condensed*, and *Rarefied*? Condensed, is when there is in the very same Matter, lesse Quantity than before; and Rarefied, when more. As if there could be Matter, that had not some determined Quantity: when Quantity is nothing else but the Determination of matter; that is of Body, by which we say one Body is greater, or lesser than another, by thus, or thus much. Or as if a Body were made without any Quantity at all, and that afterwards more, or else were put into it, according as it is intended the Body should be more, or lesse Dense. (*Lev* III.46, 468)

We will discuss Hobbes's account of quantity in the next chapter. What is significant for our present purposes is that Hobbes thinks that the notions of "rarefaction" and "condensation" are precluded by the very nature of body and quantity, if body and quantity are understood properly.

Despite there being such significant constraints on possible hypotheses, Hobbes holds that it is nonetheless possible that more than one equally plausible causal account will exist for a given effect. Experiments and experience can help to eliminate potential explanations or render them more probable,[54] but may not be able to narrow the field to a single hypothesis.

Controversies over the status of hypotheses were widespread in the sixteenth and seventeenth centuries, particularly in the context of debates over that nature and goals of astronomy. Some argued that the hypotheses of astronomy should aid in the calculation of heavenly motions, and not include speculations about the true nature of the universe. An example of this type of view is the famous preface that Andreas Osiander, concerned about the theological implications of Copernicus's work, inserted into Copernicus's *De Revolutionibus orbium coelestium* (1543). Osiander's preface states, in part:

> It is the task of an astronomer to compose a history of the celestial motions through careful and skilful observation. Then, since he cannot by any means apprehend the true causes, he must conceive and devise causes or hypotheses of such a kind that when assumed they enable those motions to be calculated correctly from the principles of geometry, for the future as well as for the past. [...] Nor is it necessary for those hypotheses to be true or even probable; provided that they yield a reckoning consistent with the observations, that alone is sufficient [...] For it is quite evident that the causes of the apparent motions are completely and absolutely unknown to this art. And if using his imagination he thinks up any causes, and he will certainly think up as many as possible, he on no account does so to persuade anyone that that is how things are, but merely to establish a correct method of calculation.[55]

According to Jardine, Osiander was only one of a substantial number of sixteenth-century authors who denied or doubted the capacity of astronomers' models to represent the dispositions and motions of the heavenly bodies, and consequently insisted on a strict distinction between mathematical astronomy and natural philosophy.[56] Jardine argues that the most significant motivation for this division was a desire to avoid conflict between the planetary models of the astronomers and Aristotelian cosmology, and thus refers to it as the "pragmatic compromise."

This demarcation between astronomy and natural philosophy was widely challenged around the end of the sixteenth century.[57] Kepler was one significant figure who argued for a close affiliation between natural philosophy and mathematical astronomy. The following passages from Kepler's *Epitome astronomiae Copernicae* (1618) describe his views on the nature of astronomy and how it is related to other sciences:

> *What is astronomy?* It is a science setting out the causes of those things which appear to us on earth as we attend to the heavens and the stars, and which the vicissitudes of time bring forth: and when we have perceived these causes, we are able to predict the future face of the heavens, that is, the celestial appearances, and to assign particular times to things in the past [...] *What is the relation between this science and others?* 1. It is a part of physics, because it seeks the causes of things and natural occurrences, because the motion of the heavenly bodies is amongst its subjects, and because one of its purposes is to inquire into the form of the structure of the universe and its parts [...] *Concerning the causes of hypotheses*. What, then, is the third part of the task of an astronomer? The third part, physics, is popularly deemed unnecessary for the astronomer, but truly it is in the highest degree relevant to the purpose of this branch of philosophy, and cannot, indeed, be dispensed with by the astronomer. For astronomers should not have absolute freedom to think up anything they please without reason; on the contrary, you should be able to give *causas probabiles* for your hypotheses which you propose as the true causes of the appearances, and thus establish in advance the principles of your astronomy in a higher science, namely physics or metaphysics — yet you are not prevented from using those geometrical, physical or metaphysical considerations about matters pertaining to these higher disciplines that are supplied to you by the very exposition of the specific discipline, provided you do not introduce any begging of the question. This being granted, it comes about that the astronomer (master of what he has set out to do insofar as he has devised causes of the motions which are in accord with reason and fit to give rise to everything that the history of observations contains) now draws together in a single form those things which he had previously determined one at a time.[58]

There are some striking similarities between Kepler's account of astronomy and Hobbes's views on natural philosophy. Both emphasize the importance of causal knowledge, with Kepler claiming that the subject matter of astronomy is the causes of heavenly motions.

As opposed to astronomers such as Osiander, Kepler argues that hypotheses are not mere calculating devices, but should describe the nature and causes of celestial appearances. Like Hobbes, he holds that there are constraints on the kinds of hypotheses that astronomers can offer: hypotheses must be "in accord with reason" and observation, and supported by principles from the "higher" sciences of physics and metaphysics.[59] Furthermore, Kepler's physics and metaphysics are decidedly quantitative in nature.[60]

Given these similarities, it is not unlikely that Hobbes's views on the nature and status of hypotheses were influenced by Kepler. In the Epistle Dedicatory to *De Corpore* Hobbes praises Kepler, along with Gassendi and Mersenne, for advancing astronomy and physics.[61]

Hobbes and Kepler differed, however, as to the epistemological status of hypotheses. As I discussed, for Hobbes there was an essential indeterminacy with regard to physical hypotheses: because of the fact that God can create any given effect in numerous ways, and the imperceptibility of the material particles that bring about physical effects, we can never be sure that a given explanation is, in fact, the correct one.

Kepler, on the other hand, held that we could come to know the true causes of the celestial motions. Although two hypotheses might seem to yield the same results, Kepler denies that this can, in fact, be the case. If there are two demonstrations, from different hypotheses, of the same conclusion, Kepler asks us to consider whether these hypotheses in fact fall under the same genus. For example, some have argued that whether we assume that the earth is moved within the heavens, or that the heavens are turned around the earth, "the same emergences of the signs of the zodiac follow, the same days, the same risings and settings of the stars, the same features of the night."[62] Kepler replies:

> For the occurrences listed above, and a thousand others, happen neither because of the motions of the heavens, nor because of the motion of the earth, insofar as it is a motion of the heaven or of the earth. Rather, they happen insofar as there occurs a degree of separation between the earth and the heaven along a path which is regularly curved with respect to the path of the sun, by whichever of the two bodies separation is brought about.[63]

When the same conclusion seems to be demonstrated from two different hypotheses, the hypotheses are often, with regard to that demonstration, actually one and the same.

It is possible, on rare occasions, that a false hypothesis will, by accident, end up yielding a true conclusion. These false hypotheses can easily be weeded out, however, if we consider their other consequences. Although a falsehood may yield the truth once by chance, if that falsehood is incorporated into other demonstrations it will eventually betray itself, as a liar, though sometimes convincing, will eventually be caught in his own contradictions.[64] Accordingly, Kepler doubts that if someone should consider both physical and mathematical consequences, "he will come across any hypothesis, whether simple or complex, which will not turn out to have a conclusion peculiar to it and separate and different from all the others."[65]

Since Kepler has a greater confidence than Hobbes in our ability to discern the true causes of physical phenomena, he also holds our hypotheses to a higher standard. Astronomers should not be satisfied until they can demonstrate their conclusions in syllogisms from true premises.[66] Hobbes, on the other hand, believing that the true causes of natural phenomena are ultimately beyond our ability to discern, holds that physics must be a non-demonstrative science.

Hobbes therefore occupies a position in between those of Kepler and the promoters of the "pragmatic compromise." He agrees with Kepler that hypotheses must include a causal account of the phenomena being explained, and that hypotheses must be grounded in previously established, quantitative sciences. On the other hand, Hobbes is more sceptical about the possibility of establishing true hypotheses.

This is not unlike the position espoused by Descartes on the status of hypotheses. Descartes agreed with Hobbes that physics must be founded on a prior science (in this case Cartesian metaphysics), and that the content of that prior science would place constraints on possible physical hypotheses. However, these constraints do not eliminate an essential indeterminacy regarding physical hypotheses. Hence Descartes states in the *Principles*:

> We noticed earlier that it is certain that all the bodies which compose the universe are formed of one [sort of] matter, which is divisible into all sorts of parts and already divided into many which are moved diversely and the motions of which are in some way circular, and that there is always an equal quantity of these motions in the universe: but we have not been able to determine in a similar way the size of the parts into which this matter is divided, nor at what speed they move, nor what circles they describe. For, seeing that these parts could have been regulated by God in an infinity of diverse ways; experience alone should teach us which of these ways He chose. That is why we are now at liberty to assume anything we please, provided that everything we shall deduce from it is {entirely} in conformity with experience.[67]

Descartes, like Hobbes, holds that there will always be multiple possible hypotheses for any given effect, both because we cannot perceive which particular configuration of material particles brought about that effect, and because of God's ability to bring about the same effect in different ways. However, both Cartesian and Hobbesian hypotheses must be based on pre-established principles and in line with experience.

Descartes also holds that adequate physical hypotheses can nonetheless be false. When discussing his denial of the earth's motion, Descartes states that he does not intend his account "to be accepted as entirely in conformity with the truth, but only as an hypothesis {or supposition which may be false}."[68] Furthermore, before presenting a hypothetical discussion of how everything in the visible universe could have been generated by the motion of small particles, Descartes acknowledges that his story will necessarily be false, since it contradicts the Christian account of creation.

However, Descartes does hold that, although we can never achieve absolute certainty in our hypotheses, we can sometimes come very close to this. At the end of part IV of the *Principles*, Descartes states that,

> it must be considered that there are things which are held to be morally certain, that is, [certain] to a degree which suffices for the needs of everyday life; although if compared to the absolute power of God, they are uncertain. Thus, for example, if someone wishes to read a message written in Latin letters, to which however their true meaning has not been given

> and if, upon conjecturing that wherever there is an A in the message, a B must be read, and a C wherever there is a B, and that for each letter, the following one must be substituted; he finds that by this means certain Latin words are formed by these letters: he will not doubt that the true meaning of that message is contained in these words, even if he knows this solely by conjecture, and even though it may perhaps be the case that the person who wrote the message did not put the immediately following letters but some others in the place of the true ones, and thus concealed a different meaning in the message. It would however be so difficult for this to happen, {especially if the message contains many words}, that it does not seem credible. But those who notice how many things concerning the magnet, fire, and the fabric of the entire World have been deduced here from so few principles (even though they may suppose that I adopted these principles only by chance and without reason), will perhaps still know that it could scarcely have occurred that so many things should be consistent with one another, if they were false.[69]

Although there is always a chance that his hypotheses will be false, Descartes argues that the fact that his principles can explain a wide range of phenomena makes this chance very small. Descartes is suggesting that we can greatly limit the collection of potential hypotheses by ensuring that they are consistent with any phenomenon of nature that we should choose to examine.

Hobbes does not make similarly bold claims about the "moral" certainty of his hypotheses. He does, however, seem to acknowledge that hypotheses are rendered more probable if they can account for a greater number of phenomena. As I have noted, in the *Dialogus Physicus* Hobbes claims that the results Boyle's experiments, which Hobbes thinks he can explain by means of his own principles, only serve to render his own hyptheses more probable (*DP*; *OL* IV, 273; 379).

Ultimately, both Hobbes and Descartes hold that even if their hypotheses are false, they will nonetheless be of great benefit. Descartes thinks that his hypotheses regarding the creation of the world, though necessarily false, will nonetheless produce true and useful conclusions. He offers a number of justifications for this claim, but the most interesting for our purposes is that his "hypothesis will be as useful to life as if it were true, {because we will be able to use it in the same way to dispose natural causes to produce the effects which we desire}."[70]

Hobbes can offer a similar justification for his own use of potentially false hypotheses. Although Hobbes's physical hypotheses are only possible explanations, they do outline a chain of causes that would necessarily produce the effect being explained. Hobbes's interest in the practical applications of physical hypotheses is in line with his claim that the end of philosophy is the ability to reproduce previous effects for the benefit of humanity (*DCp* I.1.6; *OL* I,6). For both Hobbes and Descartes the attaining of absolute truth may not be possible in natural philosophy, but there is nonetheless an instrumental value to be had in explaining how effects could have been brought about, and hence could be generated again in the future.

ENDNOTES TO CHAPTER 2

1. *Philosophia est Effectuum sive Phaenomen[o]n ex conceptis eorum Causis seu Generationibus, et rursus Generationum quae esse possunt, ex cognitis effectibus per rectam ratiocationem acquisita cognitio.*

2. As Hobbes states, "[f]inis autem seu scopus philosophiae est, ut praevisis effectibus uti possimus ad commoda nostra, vel ut effectibus animo conceptis per corporum ad corpora applicationem, effectus similes, quatenus humana vis et rerum materia patietur, ad vitae humanae usus industria hominum producantur" (*DCp* I.1.6; *OL* I, 6).

3. ...omnis denique speculatio, actionis vel operis alicujus gratia instituta est.

4. ...*effectuum per causas cognitas, vel causarum per cognitos effectus brevissima investigatio.*

5. Jesseph (1999, 224-246).

6. *Rules* IV: *CSM* I, 16.

7. *Rules* IV: *CSM* I, 17.

8. For example, François Viéte's *In artem analyticam isagoge* (1591) begins with the statement that

> there is a certain way of searching for the truth in mathematics that Plato is said first to have discovered; Theon called it analysis, and he defined it as assuming that which is sought as if it were admitted [and working] through the consequences [of that assumption] to what is admittedly true, as opposed to synthesis, which is assuming what is [already] admitted [and working] through the consequences [of that assumption] to arrive at and to understand that which is sought. (Viéte 1983, 11)

9. Hanson (1990, 602).

10. Quoted in and translated by Heath ([1921] 1981, vol.2, 400-1).

11. Hintikka and Remes (1974, 1).

12. Analysis ergo est, ex terminorum alicujus dicti, quod pro vero supponimus, definitionibus, et rursus ex terminorum illarum definitionum definitionibus ratiocinatio perpetua, donec ad nota aliqua ventum sit, quorum compositio est veritatis vel falsitatis

dicti suppositi demonstratio. Atque ea ipsa compositio sive demonstratio id ipsum est, quod appellatur synthesis. *Analytica* itaque est, ars ratiocinandi a supposito ad principia, id est, ad propositiones primas vel ex primis demonstratas, quot sufficiunt ad suppositi veritatem vel falsitatem demonstrandam: synthetica autem, ars ipsa demonstrandi.

Synthesis ergo et *analysis* aliter quam ut prorsum et retrorsum non differunt.

13. *...analysis est ratiocinatio a supposito constructo vel facto ad facti sive constructi causam efficientem vel multas coefficientes.* Ut et *synthesis ratiocinatio est a causis primis constructionis per media ad ipsum factum perpetua.*

14. ...in compositione et divisione sive resolutione. Itaque omnis methodus per quam causas rerum investigamus, vel compositiva est, vel resolutiva, vel partim compositiva, partim resolutiva. Et resolutiva quidem *analytica*; compositiva autem *synthetica* appellari solet.

15. "Resolutio" and "compositio" are the Latin terms that Commandino used to translate the Greek *analysis* and *synthesis* in his 1588 translation of Pappus's *Mathematical Collections* (Gilbert 1990, 82-3; Jardine 1976, 306-7).

16. ...quae sunt accidentia quae composita simul constituunt totam hominis, non molem, sed naturam.

17. Componitur enim causa totius ex causis partium, componenda autem prius cognosci necesse est quam compositum.

18. *Elements* I, prop.46; I, 347.

19. Hobbes (1991a, 98-9).

20. Given Hobbes's distinction between resolving a thing into its parts and into the parts of its nature, Hobbes is clearly using the watch example as a metaphor. However, given Hobbes's distinction, it is interesting to consider how he would actually go about figuring out how to generate a watch — according to his stated method he would have to begin by doing a conceptual analysis into personal device, keeps time, and so on.

21. Computare vero est *plurium rerum simul additarum summam colligere, vel una re ab alia detracta, cognoscere residuum.*

22. These correspond to the three ways that one can determine the equality or inequality of geometrical objects:

> For from motion and time the equality or inequality of any quantities can be argued, no less than by congruence: and some motion can be found so

> that two quantities, whether lines or surfaces, although one is straight and the other curved, are by extension congruent and coincide; which method Archimedes used in his treatise on spirals [...] Moreover equality and inequality are often found by the section of two quantities into parts which are considered as indivisibles, as Cavalieri Bonaventura has done in our time, and Archimedes in many places. Finally the same can be considered by the powers of lines or the roots of powers, by multiplication, division, addition, subtraction and the extraction of roots from powers, or by finding where right lines terminate in the same ratio. (*DCp* III.20.6; *OL* I, 254)

> Nam ex motu et tempore argui potest aequalitas et inaequalitas quarundam quantitatum, non minus quam per congruentiam: et potest aliquo motu fieri, ut duae quantitates sive lineae sive superficies, etsi altera recta altera curva sit, per extensionem congruant et coincidant; qua methodo usus est Archimedes in spiralibus [...] Praeterea aequalitas et inaequalitas invenitur saepenumero ex sectione utriusque quantitas in partes quas considerant ut indivisibiles, ut fecit nostris temporibus Cavalerius Bonaventura, et idem Archimedes in multis locis. Idem denique fit considerando linearum potestates vel potestatum latera, per multiplicationem, divisionem, additionem, subtractionem, et laterum e potestatibus extractionem, vel inveniendo ubi terminantur rectae ejusdem rationis.

23. ...in nullo horum modorum certa statui regula potest, in quaestione aliquanto complicatiore, a quo potissimum ex ignotis supposito ordienda sit analysis; neque ex variis aequationibus quae ab initio sese produnt, quaenam potissimum sit eligenda: sed ingenio, scientae prius acquisitae, et partim etiam fortunae successus tribuendus est.

24. ...aequatio autem illa inveniri non potest nisi ab iis qui proportionis naturam proprietates et transpositiones, linearum et superficierum additionem, substractionem, multiplicationem, divisionem, radicumque extractionem in promptu habent, id quod jam geometrae non mediocris est.

25. ...in aequationibus inveniendis nulla est methodus, sed tantum quisque valet quantum solertia praestat naturali.

26. Nam proposita quaestione qualibet, *ut, an actio talis justa an injusta sit,* resolvendo illud *injustum* in *factum* et *contra leges*, et notionem illam *legis*, in *mandatum* ejus qui coercere *potest*, et *potentiam* illam in *voluntatem* hominum pacis causa talem potentiam constituentium, pervenietur tandem ad hoc quod tales sunt hominum appetitus et motus animorum ut nisi a potentia aliqua coerciti, bello se invicem persecuturi sint, id quod per uniuscujusque proprium animum examinantis experientiam, cognosci potest.

27. Quare autem eam non didecerunt, nisi quod a nemine clara et recta methodo hactenus tradita sit?

28. Aubrey (1898, vol.1, 331). For more on the association between Hobbes and Bacon, see Hobbes (1994a, vol.2, 624, 628, 628-9 n 13), Martinich (1999, 65-9).

29. "Human knowledge and human power meet in one; for where the cause is not known the effect cannot be produced" (Bacon 1960, Bk. I, aph. III, 121). Cp. *DCp* I.1.6; *OL* I, 7.

30. Bacon (1960, Bk. II, aph. I, 121).

31. "The first [rule or axiom for the transformation of bodies] regards a body as a troop or collection of simple natures. In gold, for example, the following properties meet. It is yellow in color, heavy up to a certain weight, malleable or ductile to a certain degree of extension; it is not volatile and loses none of its substance by the action of fire; it turns into a liquid with a certain degree of fluidity; it is separated and dissolved by particular means; and so on for the other natures which meet in gold. This kind of axiom, therefore, deduces the thing from the forms of simple natures. For he who knows the forms of yellow, weight, ductility, fixity, fluidity, solution, and so on, and the methods for superinducing them and their gradations and modes, will make it his care to have them joined together in some body, whence may follow the transformation of that body into gold" (Bacon 1960, Bk. II, aph. V, 124).

32. "...the form of a thing is the very thing itself, and the thing differs from the form no otherwise than as the apparent differs from the real, or the external from the internal, or the thing in reference to man from the thing in reference to the universe..." (Bacon 1960, Bk. II, aph. XIII, 142).

33. "For the form of a nature is such, that given the form, the nature infallibly follows. Therefore it is always present when the nature is present, and universally implies it, and is constantly inherent in it. Again, the form is such that if it be taken away the nature infallibly vanishes. Therefore it is always absent when the nature is absent, and implies its absence, and inheres in nothing else" (Bacon 1960, Bk. II, aph. IV, 123).

"...no nature can be taken as the true form, unless it always decrease when the nature in question decreases, and in like manner always increase when the nature in question increases" (Bacon 1960, Bk. II, aph. XIII, 144).

34. Bacon (1960, Bk. I, aph. CXXI, 110).

35. *DCp* I.6.4; *OL* I, 61-2. The distinction that Hobbes attempts to draw in this section between universal and singular things will prove difficult to maintain, but it is clear that he intends it to be a distinction between simple or irreducible accidents, and those things

which are generated by compounding them.

36. Bacon (1960, Bk. I, aph. CXXII, 112).

37. Watkins (1965, 52).

38. On the methodology of Zabarella and the Paduan school, see Gilbert (1960, 167-73) and Jardine (1988, 689-93). On the possible influence of this methodology on Hobbes, see Jesseph (1996, 96-6), Shapin and Schaffer (1985, 147-48), and Watkins (1965, 54-9). Watkins also argues that Hobbes was influenced by the Paduan School via Galileo. Prins (1990) argues that Hobbes has a different and incompatible conception of science from that of the Paduans, and hence could not have been influenced by them.

39. See Zabarella ([1597] 1966) Book III, Chapter XIX: "de speciebus methodi resolutivae, & earum differentiis". Prins (1990, 38 n 51) provides an account of this distinction.

40. For example, this kind of resolution would allow us to demonstrate the existence of prime matter.

41. Zabarella ([1597] 1966, Bk. III, ch. XIX, column 269A).

42. Prins (1990, 40-1) makes this point in his discussion of the differences between the uses of the term "resolution" by Hobbes and Zabarella.

43. See Zabarella ([1597] 1966) Book III, Chapter XIX: "de speciebus methodi resolutiuae, & earum differentis".

44. Quoted and translated by Jardine (1976, 301). The quotation is from Zabarella's treatise *De regressu*, but the example is cited in chapter XIX of *De methodis* as an example of the first kind of resolution.

45. See Watkins (1965, 55-63), Macpherson (1968, 25-7).

46. Jardine (1976) argues against such an influence.

47. The *Dialogues Concerning Two New Sciences* is divided into four "days" of discussion, each addressing a different topic.

48. Galileo (1954, 244-94).

49. For example, in the Epistle Dedicatory of *De Corpore* Hobbes famously describes Galileo as "the first that opened to us the gate of natural philosophy universal, which is the knowledge of the nature of *motion*. So that neither can the age of natural philosophy

be reckoned higher than to him" (*DCp* ED; *OL* I, unpaginated).

50. Hobbes (1991a, 41).

51. Hobbes (1994a, vol.1, 33-4).

52. In the *Dialogus Physicus*'s Epistle to the Reader, Hobbes states that "[n]ature does all things by the conflict of bodies pressing each other mutually with their motions. So, in the conflict of two bodies, whether fluid or hard, if you understand how much motion performs in each body, that is by what path and quantity, as a not unsuitable reader you will come to physics and you will find the very probable causes of motion rightly calculated." He contrasts this way of doing physics with resting "content with the worthless statements of others" (*DP*; *OL* IV, 238; 348-9).

53. Hobbes's dismissal of these Aristotelian concepts is particularly emphatic in chapter 46 of *Leviathan*, "Of Darkness from Vain Philosophy, and Fabulous Traditions" (*Lev.* IV.46, 462-466).

54. In the *Dialogus Physicus*, for example, Hobbes replies to an account of Boyle's air-pump by stating:

> So you admit there to be nothing yet from your colleagues for the advancement of the science of natural causes, except that one of them has found a machine that can excite the motion of the air so much that parts of the sphere simultaneously tend from everywhere towards the centre, and so that the hypotheses of Hobbes, which indeed were probable enough beforehand, may by this be rendered more probable. (*DP*; *OL* IV, 273; 379)

55. Quoted in Jardine (1984, 43) .

56. Jardine (1984, 237).

57. Jardine (1984, 244-47) discusses some possible reasons for this change in attitude.

58. Quoted in Jardine (1984, 250).

59. Jardine (1984, 251) argues that "[w]hen Kepler urges the astronomer to seek *causas probabiles* for his hypotheses, he means, I suggest, that he is to seek causes that are acceptable because adequately warranted."

60. Jardine (1984, 253-55).

61. Hobbes states that "[a]t post hos astronomiam et physicam quidem universalem Johannes Keplerus, Petrus Gassendus, Marinus Mersennus, physicam vero humani corporis specialem ingenia et industria medicorum, id est, vero physicorum, praesertim vero nostrorum e Collegio Londinensi doctissimorum hominum, pro tam exiguo tempore egregie promoverunt" (*DCp* ED; *OL* I, unpaginated).

62. Jardine (1984, 142).

63. Jardine (1984, 142).

64. Jardine (1984, 140).

65. Jardine (1984, 141).

66. Jardine (1984, 139).

67. *Pr* III.46; *M* 106.

68. *Pr* III.19; *M* 91.

69. *Pr* IV.204; *M* 286-7.

70. *Pr* III.44; *M* 105.

CHAPTER 3

FIRST PHILOSOPHY AND

THE FOUNDATIONS OF A MATHEMATICAL ACCOUNT OF NATURE

This chapter will discuss some of the definitions that appear in part II of *De Corpore*, "Philosophia Prima." As was discussed in the first chapter of this dissertation. the definitions contained in this part of *De Corpore* are supposed to represent the fundamental principles of Hobbes's scientific system. They are not a part of any of the particular sciences. but will be appealed to by all of them.

We have already encountered some of Hobbes's first philosophy definitions (such as those of "ratio" and "proportion") and others will be discussed in the chapters to come. The definitions that I will discuss in this chapter merit special attention because of the important roles that they play in Hobbes's account of nature. The purpose of my discussion is two-fold: the first is simply to acquaint us with some of the most significant of Hobbes's basic concepts. In addition. I will discuss the role that each of these concepts plays in Hobbes's mathematisation of nature.

3.1 Body

In the seventeenth century Hobbes was far from alone in his belief that the sciences should form a system. In the Preface to the French edition of the *Principles of Philosophy*. Descartes famously states that "Philosophy as a whole is like a tree; of which the roots are Metaphysics. the trunk is Physics. and the branches emerging from the trunk are all the other branches of knowledge."[1] As we have seen. Hobbes preferred the term "first philosophy" to "metaphysics," arguing that the latter name suggested the study of something supernatural.[2] He also, of course, disagreed with Descartes regarding the content of metaphysics, rejecting, among other things, Descartes's doctrine of the immaterial nature of the soul. However, both Hobbes and Descartes agreed that the study of nature should be based upon a foundation of first principles. Furthermore, both thinkers included amongst those principles the basis for a mathematical treatment of nature.

The metaphysical basis for Descartes's mathematisation of nature is his conception of body. He argues that extension, or the geometrical properties of length, breadth, and depth, is the essence of body. In the *Principles*, he claims that each of the two substances of mind and body has a principal attribute or "property which constitutes its nature and essence, and to which all the other properties are related."[3] Extension in length, breadth, and depth constitutes the essence of corporeal substance, while thought constitutes the essence of thinking substance.

On Descartes's account this means that all other properties which can be attributed to body presuppose extension. For example, we cannot think of size or shape without thinking of them as mode of some extended thing, while we can understand extension independently of the properties of size and shape.[4] As Descartes states in his Replies to Hobbes's Objections to the *Meditations*:

> Now there are certain acts that we call 'corporeal', such as size, shape, motion and all others that cannot be thought of apart from local extension; and we use the term 'body' to refer to the substance in which they inhere. It cannot be supposed that one substance is the subject of shape, and another substance is the subject of local motion etc., since all these acts fall under the common concept of extension.[5]

There is thus no property of body that is not understood through the attribute of extension. As Garber has stated, "[i]n this way Cartesian bodies are just the objects of geometry made real, purely geometrical objects that exists outside of the minds that conceive them."[6] The properties of willing, understanding, imagining, and sensing cannot be clearly and distinctly perceived to be the properties of an extended thing, and hence must fall under the attribute of thought.

Hobbes's account of body is similar in some ways to Descartes's, but there are important differences in the ways in which the two conceive of the connection between body and extension. Hobbes defines "body" as "*anything which, not depending on our thought, coincides or is coextended with some part of space*" (*DCp* II.8.1; *OL* I. 91).[7] This account might seem very close to Descartes's, especially given Hobbes's comment that we call something "body" on account of its having extension.[8] However, Hobbes's claims about the centrality of extension to our conception of body are not as strong as Descartes's. At a later point in his chapter on body and accident, Hobbes presents the following definition of "essence":

> Now the accident on account of which we impose a certain name on some body, or the accident which denominates its subject, is usually called the *essence*, as rationality is called the *essence* of man, whiteness, the *essence* of a white thing, and extension the *essence* of body. (*DCp* II.8.23; *OL* I. 104)[9]

When Hobbes says that body is something extended, he just means that we apply the name "body" to those things, and only those things, that have the property of extension. Unlike Descartes's notion of essence, this leaves open the possibility that bodies could have accidents that could be understood without reference to extension and the geometrical properties of length, breadth, and depth.

The basis in Hobbes's own first philosophy of his mathematisation of nature is suggested by his discussion of accidents and how they relate to bodies. Hobbes defines an "accident" as "*the faculty of a body by which it impresses in us a conception of itself*" (*DCp* II.8.2; *OL* I. 91).[10] We have ideas of a whole range of accidental properties, and these accidents are alike in being faculties by which bodies produce those ideas in us. Hobbes goes on to state, when explaining how an accident is in a body, that "as magnitude, or rest, or motion is in that which is great, which rests, or which is moved (which, how it ought to be understood everyone understands) so every other accident ought to be understood to be in its subject" (*DCp* II.8.3: *OL* I. 92).[11] All accidents are thus in their subjects in the same way.

Hobbes defers a detailed explanation of how accidents are in their subjects, claiming that it is more properly a part of natural philosophy:

> Now because it can seem to some, that not all accidents are in their bodies in the same manner as *extension*, motion, rest, and figures are; for example, that colour, heat, order, virtue, vice and the like are in them in another manner and (as they say) inhere; I propose that they suspend for the present their judgement concerning this matter, and wait for a while, until it is investigated by reason, whether these very accidents are not also certain motions, either of the imagining mind, or of the bodies themselves which are perceived with the senses; for to investigate this, is the greatest part of natural Philosophy. (*DCp* I.8.3; *OL* I, 93)[12]

In these passages Hobbes is arguing against the Aristotelian doctrine that accidents (rather than essences) "inhere in" individual substances but do not constitute them.[13] He is claiming, on the contrary, that all accidents, be they essential or not, are in bodies in the same way — as he will show, they are all actually motions of the mind or of external bodies.

As we will see, it is this doctrine that forms the foundation for Hobbes's mathematisation of nature. By showing that the motions by which our ideas are generated can be represented and analysed mathematically, Hobbes hopes to be able to present a quantitative account of natural phenomena.

Hobbes and Descartes share a commitment to mechanism and a belief that all natural phenomena should be explained in terms of matter in motion. However, they differ as to which is of these fundamental entities is the metaphysical basis for the mathematisation of nature: for Descartes, it is body, but for Hobbes, motion.

3.2 Time

For Hobbes, the ideas of time and motion are closely connected. He claims that a phantasm of a body moving continuously from space to space is the same as an idea of time, and hence defines "time" as "the phantasm of motion, insofar as we imagine in the motion before and after, or succession" (*DCp* II.7.3; *OL* I, 84).[14] He claims that this definition is close to common opinion, and agrees with Aristotle's definition.[15]

Hobbes presents two significant arguments for this definition: first, that no one considers time, or any unit of time, to be an accident or state of an external body. If time is not in external bodies, he maintains that it must be in the mind.

Having argued that time is a phantasm, Hobbes claims that it must be a phantasm of motion,

> for when we wish to know, by what moments time slips away, we employ some motion, as for example of the sun, or an automaton, or a water clock [*clepsydrae*], or we mark out a line, over which we will imagine something to be borne; but by no other method does time appear. (*DCp* II.7.3; *OL* I, 84)[16]

We can only perceive the passage of time through the perception of motion, so the two phantasms must be the same.

Of course, not all seventeenth century thinkers concurred with this account of time. In particular, the above argument connecting time and motion would be unsatisfactory for anyone who denied the original claim that time must be a phantasm. Isaac Barrow, for example, writes that "[t]ime does not imply motion, as far as its absolute and intrinsic nature is concerned; not any more than it implies rest; whether things move or are still, whether we sleep or wake, Time pursues the even tenor of its way."[17] Barrow thinks that time would elapse whether or not things were in motion, although he acknowledges that without motion we would be unable to perceive or measure its passage.

This account of time represents Hobbes's first and most frequently cited example of an entity being reduced to a perception of matter in motion. As we will see, these sorts of reductions play an important part in Hobbes's mathematisation of nature. Hobbes may have hoped that this definition of time would clear the way for other such accounts, as it is among the more plausible of his reductions, and the one for which he could most credibly claim historical precedent.

3.3 Motion

Hobbes's first philosophy also contains some basic principles regarding the nature of motion. These principles will feature prominently in the natural philosophy that we will discuss in subsequent chapters. Most of Hobbes's definitions are standard in their form, although they often become problematic when interpreted in light of other aspects of his philosophy. "Motion" is defined as "*the continual forsaking of one place and acquisition of another*" (*DCp* II.8.10; *OL* I, 97), the place that is forsaken being called the *terminus a quo*, and the place that is acquired the *terminus ad quem*. With this definition Hobbes is, of course, reducing all motion to local motion. This was a standard move for mechanists. For Aristotle, motion was a general concept meaning change from one state to another. In addition to local motion (change from one place to another) it included alteration (qualitative change), augmentation and diminution (quantitative change) and sometimes[18] generation and corruption.[19] For the practitioners of the new mechanical philosophy, however, all other kinds of change were to be explained in terms of the local motion of bodies. The explanatory primacy of local motion led many mechanists to simply identify it with motion. Hobbes is presenting a standard definition of local motion: as is suggested by the above, it was accepted by Aristotle and the Scholastics.[20] It was also the preferred definition of many seventeenth-century theorists.[21]

As he states in his definition, for Hobbes motion is continuous. He elaborates on what he means by this by stating that

> however small a body is, it cannot leave at once its whole former place, so that a part of it is not in a part which is common to each place, namely to the relinquished and the acquired places. For example, [in figure 3.1] let any body be in the place ABCD, that body cannot arrive at the place BDEF, but it must first be in GHIK, of which the part GHBD is common to both the place ABCD, and the place GHIK, and the part BDIK is common to both the place GHIK and the place BDEK (*DCp* II.8.10; *OL* I. 97).[22]

In other words, a moving body does not jump from place to place, but must pass through a succession of intermediate places. Each of these intermediate places will have a part in common with the initial place, as well as a part in common with the final place.

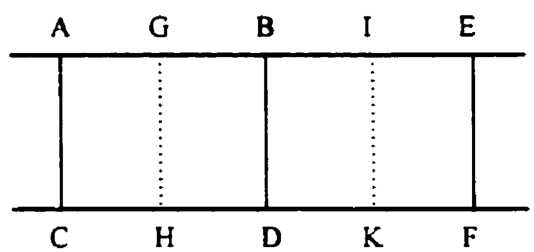


Figure 3.1

Hobbes's discussion of the continuity of motion foreshadows some tensions in his account of the continuum. Although Hobbes does not make this explicit, it appears that the moving body discussed above will have to move through an infinite number of intermediate spaces, with the part common to the intermediate place and the initial place becoming progressively smaller. This presumes that space is infinitely divisible: otherwise, there would be some smallest part of the moving body that would jump from being in ABCD to being in BDEF, without any intermediate steps. However, this contradicts Hobbes's statements elsewhere, particularly in his mathematics, that there are "least parts" to the continuum. I will examine these tensions in Hobbes's account at length in chapter 4.

Hobbes's first philosophy also contains a set of definitions regarding velocity, none of which are without precedent. Motion, "insofar as by it, a certain length, in a certain time, can be traversed, is called *velocity*" (*DCp* II.8.15; *OL* I, 100).[23] Similarly, equally swift motions are those by which equal spaces are traversed in equal times, while the velocity is greater when a greater length is made in equal time, or an equal length in less time. Uniform velocity is that "by which in equal parts of time equal lengths are passed over" (*DCp* II.8.17; *OL* I, 101).[24] Of non-uniform motions, "those, which in equal parts of time are accelerated, or retarded in increments or decrements always equal, are said to be uniformly accelerated, or uniformly retarded" (*DCp* II.8.17; *OL* I, 101).[25]

Hobbes does present two immediately conspicuous definitions in his first philosophy account of motion: first, he claims that a motion being greater than, lesser than, or equal to another motion is not only a factor of the motions' velocities, but also "of the velocity applied to each part of the magnitude." He illustrates this by claiming that while the velocity of two horses abreast is equal, the motion of the two together is double that of each considered alone. Hence motions are equal

> *when the velocity of one computed through its whole magnitude is equal to the velocity of the other, likewise computed through the whole of its magnitude.* But *a motion is greater than another motion when its velocity so computed, as was said, is greater than this other similarly computed. Less,* in fact, *when less.* (*DCp* II.8.18; *OL* I, 101-2)[26]

At an earlier point in the chapter, Hobbes had promised that he would show that velocity being applied to all the parts of a solid makes a magnitude of motion, as "the goodness of gold computed in the several parts of it make its price" (*DCp* II.8.12; *OL* I, 99).[27] In both these cases Hobbes is claiming that we can generate a quantitative measure of a quality by considering the intensity of that quality throughout the dimensions of a body.

In this passage we see the first signs of the influence of medieval theories of motion on Hobbes. I will discuss this influence in much greater detail in chapter 4. Very briefly, some medieval theorists distinguished between the intensity of a quality and its extension in a subject.[28] One could, for example, distinguish between the intensity of heat, or its temperature, and the quantity of heat, or the temperature considered throughout a body's extension. During the early to mid fourteenth century a group of scholars working at Oxford's Merton College — most notably Thomas Bradwardine, William Heytesbury, Richard Swineshead, and John Dumbleton — began to extend this way of thinking about qualities to their treatments of motion. In this case, the intensity of a motion was taken to be its velocity. Velocity could be considered either through the extension of the moving body, or through its extension in time, i.e., the duration of the motion.[29] The Mertonians could thus distinguish between the quality of a motion, or its swiftness or slowness, and the quantity of a motion, or that swiftness considered throughout the motion's duration or the extension of the moving body. As we will see, Hobbes adopts this way of thinking about motion in his discussion of kinematics.

Hobbes was not the only one in the seventeenth century to speak in similar ways about the "quantity of motion." In the *Principles*, for example, Descartes states that "if one part of matter moves twice as fast as another which is twice as large, we must consider that there is the same quantity of motion in each part."[30] Descartes, like Hobbes, claims that quantity of motion is a factor of the velocity and the magnitude of a body. However, as I will argue in chapter 4, there are other affinities between Hobbes's account of motion and that of the medievals which make Descartes an unlikely source for Hobbes's views.

Another notable aspect of Hobbes's first philosophy account of motion is a pair of quasi-inertial principles:

> What rests, is understood to always rest, unless some other body besides itself, having gotten into its place, makes it the case that the first body can no longer rest [...] Similarly, what is moved, is understood to always be moved, unless there is another thing outside of itself on account of which it rests. (*DCp* II.8.19; *OL* I, 102-3)[31]

Hobbes argues for these principles by claiming that there is no intrinsic reason why a resting body would move one way or another, nor is there anything in a moving body which would give it reason to rest. Therefore, both the causes of a resting body beginning to move in a particular direction, and of a moving body coming to rest, must be external to the bodies in question.

Two things should be noted about Hobbes's quasi-inertial principles: first, Hobbes does not limit their application to rectilinear motion, as Descartes did in his own inertial principle. Brandt argues that this is because Hobbes was following Galileo's notion of circular inertia. In chapter 5 I will argue that a preferable explanation can be given in terms of the overwhelming nature of Hobbes's desire to explain phenomena in terms of the impact of moving bodies. This aspect of Hobbes's project will also account for the fact that, as Brandt has noted,[32] Hobbes makes little use of his quasi-inertial principles in his natural philosophy .

3.4 Quantity

In the first section of this chapter, I argued that Hobbes's account of body does not represent the basis of his mathematisation of nature. Instead, this foundation can be found in his concept of quantity, as it is this concept that allows Hobbes to argue that all motions, and hence the causes of all natural phenomena, can be represented by geometrical objects. The argument for this conclusion occurs in three stages: first, Hobbes claims that the geometrical objects of line, surface, and solid and the three dimensions of body are the products of the same motions, considered in different ways. Second, he defines "quantity" as dimension determined, and claims that all quantities can be represented by lines, surfaces and solids. Finally, as I will discuss in the next chapter, he claims that all qualities can also be represented in this way by geometrical objects, and can thus also be subject to mathematical analysis.

The first stage of Hobbes's argument occurs in chapter 8, where he states that

> If the magnitude of a body which is moved is not considered (although it is always something), the path through which it passes is called a *line*, or a *single* and *simple dimension*, but the space it passes through [transit] is called *length*, and the body itself a *point*; in that sense in which the earth is a point, and its annual path is usually called the ecliptic line. But if a body, which is moved, is considered now as *long*, and is also supposed to be so moved, that its separate parts are understood to make separate lines, the way of every single part of that body is called *breadth*, the space which it makes is called *surface*, consisting of the two-fold dimension *length* and *breadth*, of which the whole of one is applied to the separate parts of the other.
>
> In turn, if a body is now considered as having a *surface*, and is understood to be moved, so that its separate parts make separate lines, the way of each part of that body is *thickness* or *depth*, the space which is made is called *solid*, composed from three dimensions, of which all of any two are applied to the individual parts of the third. (*DCp* II.8.12; *OL* 98-9)[33]

This passage is notable in that it presents Hobbes's genetic and materialist definitions of the mathematical objects of line, surface, and solid. These definitions are extremely important for Hobbes's effort to turn mathematics into a science of body, and their decidedly non-traditional nature would get him into endless trouble with adversaries such as Wallis.[34]

However, these definitions also establish a relationship of identity between the three dimensions and the geometrical objects of line, surface, and solid. For Hobbes, things must be defined according to their causes. Since, as he supposes, our ideas of line and a single dimension are generated by considering the same motions, these ideas must coincide. Similarly, if a line moves it simultaneously sweeps out a surface and two dimensions, while both a solid and the three dimensions are generated by the motion of a surface.

Hobbes proceeds to define quantity in terms of these three dimensions. He begins his chapter "Of Quantity" by stating:

> What dimension is, and how manifold, it was said before in chapter 8, that without doubt it is three, line (or length), surface and solid. Each one of

> these, if it is determined, that is, if its limits or termini are made known, is usually called *quantity*. (*DCp* II.12.1; *OL* I, 123)[35]

Quantity is any suitable answer to the question "how much," and this question is always answered in terms of one of the three dimensions determined. As this definition confirms, the dimensions are identified with line, surface, and solid. Hobbes's definition, which makes all ideas of quantity geometrical, immediately raises the question of the status of number in Hobbes's theory, a matter which I will discuss at some length below.

A quantity can be determined, or its limits set out, in two ways: first, by sense, as when a line, surface or solid of a given measure is marked out and observed. Hobbes refers to this way of determining as "exposition," and the quantity so determined is called "exposed."[36] If this method were used to answer the question "how much?" the answer provided would be of the form "as much as you see (or sense by some other means) exposed." Alternatively, we can determine the quantity of a thing by memory, or comparing it to some exposed quantity (for example, if I say that a road is a thousand feet long, I am comparing it with the quantity of a foot, which I know by exposition).

Hobbes was not alone in asserting the primacy of our sense impressions of quantity. For example, Barrow, in his *Mathematical Lectures*, asserts that we can know quantity in four different ways: "In the *first* Place, one Kind of *Knowledge* is *radical, absolute* and *primary*, whereby the Quantity of a Thing is exposed to the Senses, and immediately discerned and estimated by them, being as it were seen by a Kind of Intuition without further Comparison with other Quantities."[37] This is the way that all "primitive" measures, i.e., those measures to which measures of the same kind are referred, are known. Their quantity "can be scarce any Way explained but by pointing the Finger at them and answering him that enquires about their Quantity. *It is as much as you see or perceive by your Sense*."[38] The second way that we can have knowledge of quantity is by comparison with a measure which has been exposed to the senses in this way. Barrow refers to quantities known in this way as "mediate" or "secondary" measures.[39]

Hobbes's doctrine of exposition is obviously tied to his empiricism, as it entails that at least our primitive ideas of quantity must have originated in sense experience. However, given Hobbes's identification of the three dimensions with geometrical objects, this notion of how quantities are determined also serves the function of ensuring that all quantities must be exposed by lines, surfaces, and solids, and hence that all of our ideas of quantity will be geometrical.

It is thus not surprising that Hobbes begins his account of how various quantities can be exposed by discussing geometrical objects. Lines, surfaces, and solids can be exposed in three ways: first, by motion, when they are generated in such a way that the marks of their motions are permanent (as occurs, for example, when a line is drawn on paper). Second, "by *apposition*, as when a line is added to a line, that is, a length to a length, a breadth to a breadth, a thickness to a thickness" (*DCp* II.12.3; *OL* 125).[40] Thus a new line can be exposed by laying two other lines end to end, or a new solid can be exposed by placing two solids side by side and adjacent to each other. Finally, lines and surfaces can be exposed by sections, as when a line is generated by cutting a surface, or a surface by cutting a solid.

Hobbes then explains how all further quantities are themselves exposed by lines, surfaces, and solids (the three dimensions). So, for example, time is said to be exposed by a line over which a body is moved uniformly — this provides a sensible representation of our idea of time, which is, as Hobbes has claimed, the idea of before and after in motion.

Hobbes also thinks that motions and their properties can be exposed. For the exposition of velocity,

> (which, by definition, is the motion by which a certain space is traversed in a certain time,) it is required both that time be exposed, and also that that space be exposed, which is to be traversed by the mobile whose velocity we wish to determine, and that the mobile is understood to be moved on. Therefore two lines must be exposed, the one over which uniform motion should be understood to be made, so that time is determined; the other over which velocity should be estimated: so that if we wish to expose the velocity of the mobile A [see figure 3.2], we will draw two lines, AB, and CD, and we will also place a mobile on C; then we will say that the

> velocity of the mobile is so much, that it traverses the line AB in the same time in which the mobile C traverses the line CD with uniform motion. (*DCp* II.12.6; *OL* 126)[41]

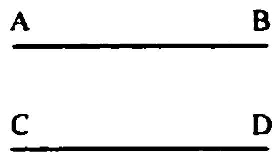


Figure 3.2

Hobbes had said that we get the idea of velocity when we consider motion, but insofar as a certain length can be traversed in certain time by that motion. Therefore, in order to expose velocity we need to expose the motion in question, and hence some length traversed by it, as well as the time in which it passes over that distance. Velocity is thus exposed by two lines: one corresponding to the motion and the other to the time in which it occurs.

At this point it is worth taking a brief detour to discuss one of the apparent problems with Hobbes's account of quantity: it seems to leave no room for discrete quantity, since it is difficult to explain how number can be exposed by means of the continuous magnitudes of line, surface, and solid. Hobbes's account of how this occurs is brief and somewhat cryptic. He suggests two ways in which number can be exposed: either "by the exposition of points, or of the names of number, *one*, *two*, *three*, *&c*." (*DCp* II.12.5; *OL* I. 125).[42] If the former procedure is used, we must be able to discern one point from another (this is why number is called discrete, rather than continuous, quantity). If we are going to expose number by its names,

> they must be recited in order and from memory, as one, two, three, &c. For even if someone should say thus, one, one, one, &c. nevertheless he does not know the number, unless perhaps of two or three, which it is possible to remember, but as figures of a certain kind, not as numbers. (*DCp* II. 12.5; *OL* I, 126)[43]

As I discussed in chapter 1, this passage is puzzling in that it is only partially consistent with Hobbes's strong statements elsewhere about the dependence of arithmetic on geometry. He does claim that we can expose numbers by points, which is in line with his programme of establishing that all of our ideas of quantity must be geometrical. However, he does not suggest that discrete quantity could be exposed by dividing a line into parts, as he does in the *Examinatio*. As I mentioned, this may be because Hobbes had yet to embark on his most heated debates with Wallis.

This passage also proposes that we can generate ideas of discrete quantity by hearing the names of numbers recited. Pycior has suggested that this means that Hobbes was moving away from viewing arithmetic as dependent on geometry (a view that he would return to in the *Examinatio*) and towards the idea that it is an independent science of names.[44] Pycior finds support for this view in Hobbes's discussions of negative numbers. At the time at which Hobbes was writing, there was considerable controversy about the status of negative and irrational numbers. These numbers seemed impossible to define or conceive of clearly. They were, however, becoming increasingly useful in mathematical practice. There were therefore many attempts by mathematicians in the seventeenth century to understand the nature of these "impossible" numbers.[45] In "Of Names," an early chapter of *De Corpore*, Hobbes presents the following comment which relates to these difficulties:

> It is not in fact necessary that every name be the name of some thing [...] this word *nothing* is a name, yet it cannot be the name of a thing. For if (for example) taking away two and three from five, we do not perceive anything remaining, if we should wish to remember that taking away, this speech *nothing is remaining*, and in that speech the name *nothing*, is not unuseful. Also on account of the same reason *less than nothing* is correctly spoken of the remainder when a greater is taken away from a lesser. For the mind imagines to itself a remainder of this kind for the sake of teaching, and desires, as many times as it needs, to recall it in memory. (*DCp* I.2.6; *OL* I, 15-16)[46]

To recall, for Hobbes quantity is determined by being exposed to the senses, or by being compared to some quantity already so exposed. So, for example, positive numbers may be exposed by points. Pycior claims that in the above passage we find we find evidence that Hobbes, finding no corresponding way to set negative numbers before the senses, is moving towards the "view of numbers as names."[47] It appears that she is claiming that, on Hobbes's view, the names of numbers are not signs for our conceptions of the numbers — they are the numbers themselves. Unless Pycior is making this stronger assertion, the claim that arithmetic is a science of names would be a trivial one in the context of Hobbes's view that all of science involves a manipulation of meaningful names.[48] If this were the case, Pycior's claim would not involve the kind of novelty that she suggests is at stake.

This strong thesis is not supported by the text of *De Corpore*. The context of the above passage makes it clear that when Hobbes states that it is not "necessary that every name should be the name of something," he is not claiming that it is unnecessary for every name to be associated with a conception that gives it meaning. Rather, he is claiming that the conception which gives a word meaning need not be of something which we conceive as being actual and existent.[49]

In addition, Hobbes claims that words must always be used in conjunction with the conceptions which give them meaning.[50] It is failing to abide by this rule which results in the dangerous errors of the vain philosophy of the schools. Given his emphatic opposition to the vain philosophy, it is unlikely that Hobbes would have recommended that arithmetic be considered as a science of names alone.

However, this is not to say that Hobbes's account of discrete quantity is without problems. His solution to the problem of negative numbers is not to claim that these numbers are just names, but rather to suggest that the conceptions that give meaning to these names are not of actually existing things. His account of what these ideas would be like is extremely vague, leaving open the question of how, exactly, we can conceive of negative numbers. There do seem to be the resources within Hobbes's system for such an account. He could, for example, use direction to expose the negative numbers to the senses — a number line could be generated with successive points in one direction representing positive numbers, in the other direction negative numbers.

However, Hobbes presents no such explicit account of our conceptions of negative numbers. It appears, given his overwhelming interest in geometry and the study of motion, that he did not pay enough attention to working out the details of his number theory. As we will see, it is continuous, rather than discrete, quantity that does most of the work in his account of nature.

ENDNOTES TO CHAPTER 3

1. *Pr*; *M* xxiv.

2. Hobbes (1976. 16).

3. *Pr* I.53; *M* 23.

4. *Pr* I.53; *M* 23-4 .

5. *CSM* I. 124.

6. Garber (1992b. 295).

7. ...*corpus est quicquid non dependens a nostra cognitatione cum spatii parte aliqua coincidit vel coextenditur.*

8. ...est quod appellari solet. propter extensionem quidem. *corpus.*

9. Accidens autem propter quod corpori alicui certum nomen imponimus. sive accidens. quod subjectum suum denominat. *essentia* dici solet. ut rationalitas. hominis: albedo. albi; extensio. corporis dicitur *essentia.*

10. ...*accidens esse facultatem corporis qua sui conceptum nobis imprimit.*

11. ...sed sicut magnitudo. vel quies, vel motus est en eo quod magnum est. quod quiescit. vel quod movetur (quod quo modo intelligendum est unusquisque intelligit) ita etiam omne aliud accidens inesse subjecto suo intelligi debet.

12. Quod autem alicui videri possit. non omnia accidentia suis corporibus ita inesse. sicut inest *extensio*. motus. quies, aut figura; exempli causa colorem. calorem. odorem. virtutem. vitium. et similia. aliter inesse et (ut dicunt) inhaerere; velim eum in praesentia judicium suum de ea re suspendere. et parumper expectare, donec ratiocinatione investigatum est. an haec ipsa accidentia non sint etiam motus quidam. aut animi imaginantis. aut corporum ipsorum quae sentiuntur; nam illud explorare. magna pars est Philosophiae naturalis.

13. Bolton (1998. 178-9).

14. ...tempus est phantasma motus, quatenus in motu imaginamur prius et posterius.

15. Aristotle states in the *Physics*: "It is clear, then, that time is 'number of movement in respect of the before and after'" (*Physics* IV.11, 220a25).

16. ...cum enim, quibus momentis tempus labatur, congoscere volumus, adhibemus motum aliquem, ut solis, vel automati, aut clepsydrae, vel lineam signamus, super quam aliquid ferri imaginabimur; alio autem modo tempus nullum apparet.

17. Barrow (1916, 35).

18. For Aristotle, generation and corruption are, strictly speaking, "mutations" rather than "motions" (*Physics* V.1, 224a21-226b15). However, he does treat generation and corruption in his physics which is, as we will see, closely aligned with the study of motion.

19. *Physics* III.1, 201a10.

20. *Physics* IV.4, 211a12-16.

21. Gabbey (1998, 650) notes that some seventeenth-century theorists were content with the common "change-of-place" definition, while some others "had new and important things to say about motion *qua* motion, believing that the common definition harboured difficulties that had to be resolved before solid achievements in natural philosophy could be assured."

22. ...corpus quantulumcumque sit, non potest totum simul a toto loco priore ita excedere, ut pars ejus non sit in parte quae sit utrique loco, nimirum relicto et acquisito, communis. Exempli causa, sit corpus quodcunque in loco ABCD, non potest illud pervenire ad locum BDEF, quin prius sit in GHIK, cujus pars GHBD sit communis utrique loco ABCD, et GHIK, et pars BDIK communis utrique loco GHIK et BDEF.

23. Motus, quatenus eo, longitudo certa, tempore certo, transmitti potest, appellatur *velocitas*.

24. It is somewhat surprising that Hobbes does not note that equal distances should be traversed in *any* equal intervals of time, given that this proviso appears in Galileo's definition of uniform motion, as well as some medieval definitions of the same.

25. ...illi, qui aequalibus temporis partibus accelerantur, vel retardantur aequalibus semper incrementis vel decrementis, dicuntur uniformiter accelerati, vel uniformiter retardati.

26. *...cum unius velocitas per omnem ejus magnitudinem computata,aequalis est alterius velocitati, per omnem item magnitudinem ejus computatae. Major* autem *motus motu est quando velocitas illius sic, ut dictum est, computata, hujus velocitate similiter computata, major est.*

27. ...bonitas auri in singulis partibus computata facit pretium ejus.

28. Clagett (1959, 212).

29. See Clagett (1959, 213-4).

30. *CSM* I. 240.

31. Quod quiescit, semper quiescere intelligitur, nisi sit aliud aliquod corpus praeter ipsum, quo supposito, quiescere amplius non possit...Similiter quod movetur, semper moveri intelligitur, nisi aliud sit extra ipsum propter quo quiescit.

32. Brandt (1928, 328).

33. Si corporis quod movetur magnitudo (etsi semper aliqua sit) nulla consideretur. via per quam transit. *linea*. sive *dimensio una* et *simplex*. dicitur. spatium autem quod transit *longitudo*. ipsumque corpus *punctum* appellatur; eo sensu quo terra *punctum*. et via ejus annua linea eccliptica vocari solet. Quod si corpus, quod movetur, consideretur jam ut *longum*. atque ita moveri supponatur, ut singulae ejus partes singulas lineas conficere intelligantur. via uniuscujusque partis ejus corporis *latitudo*, spatium quod conficitur *superficies* vocatur. constans ex duplici dimensione *latitudine* et *longitudine*. quarum altera tota ad alterius partes singulas sit applicata.

Rursus si corpus consideretur ut habens jam *superficiem*, et ita intelligatur moveri. ut singulae ejus partes singulas conficiant lineas, uniuscujusque que partes via corporis illius *crassities* seu *profunditas*, spatium quod conficitur *solidum* vocatur, conflatum ex dimensionibus tribus, quarum quaelibet duae totae applicantur ad singulas partes tertiae.

34. On the place of the these definitions in the mathematical tradition, and the various critical responses to them. see Jesseph (1999, 76-83).

35. De dimensione quid sit. et quotuplex. dictum supra est capite 8. nimirum tres esse, lineam (sive longitudinem) superficiem, et solidum. Harum unaquaeque. si sit determinata. id est. si fines seu termini ejus cogniti fiant. *quantitas* appellari solet.

36. In using the term "exposition" for this process. Hobbes may be adopting the terminology traditionally used to describe the divisions of a geometrical proposition. The "setting-out" [expono] is the part of a proposition which "marks off what is given. by itself. and adapts it beforehand for use in the investigation" (*Elements* commentary, I.129).

37. Barrow ([1734] 1970. 272).

38. Barrow ([1734] 1970, 273).

39. Barrow ([1734] 1970, 275) mentions two other ways of knowing quantity: first, when we know the ratio, expressed in numbers, of the quantity to some other quantity previously estimated. Finally, "Every Quantity is said to be after a Sort known and determined (as we have said above) whose general Nature we comprehend, though we are ignorant of its particular Quantity, or do not consider it. Thus we know how a Radius is affected in a Circle, or a Side in a Square, though we are ignorant, or neglect the Quantity of this or that particular Radius of a Circle, or Side of a Square." Barrow is particularly concerned in this case about quantities that are set out and used in the generation of others, and can thus be used as a sort of measure of the final product (so if a square is generated by the drawing its side into itself, the quantity of the square will depend on that of the side, and hence the side can be called a kind of primitive measure thereof).

40. ...per *appositionem*, ut quando linea lineae, id est, longitudo longitudini, latitudo latitudini, crassities crassitiei adjungitur.

41. Ad expositionem autem velocitatis, (quae, per definitionem, est motus quo certum spatium certo tempore percurritur,) requiritur tum ut tempus exponatur, tum etiam ut illud spatium, quod a mobili, cujus velocitatem determinare volumus, transmittendum est expositum sit, et in eo mobile moveri intelligatur. Duae itaque lineae exponendae sunt, altera super quam intelligatur fieri motus uniformis, ut tempus certum sit; altera super quam velocitas aestimetur: ut si velocitatem velimus exponere mobilis A, ducemus duas lineas AB, et CD, et mobile in C quoque statuemus; tum vero dicemus velocitatem mobilis A tantam esse, ut percurrat lineam AB eodem tempore quo mobile C percurret lineam CD motu uniformi.

42. ...per expositionem punctorum, vel etiam nominum numeralium, *unum, duo, tria, &c.*

43. ...ordine et memoriter recitari debent, ut unum, duo, tria, &c. nam etsi quis dicat sic, unum, unum, unum, &c. numerum tamen nescit, nisi forte binarium aut ternarium, cujus meminisse quidem potest, sed ut figurae cujusdam, non ut numeri.

44. Pycior (1987, 272-3) and (1997, 141-3).

45. Pycior (1987, 267-8) and (1997).

46. Neque vero ut omne nomen, alicujus rei nomen sit, necessarium est...vox haec *nihil* nomen est, rei tamen nomen esse non potest. Nam si (exempli gratia) subducentes binarium et ternarium ex quinario, non videmus ullum residuum, si illius subductionis meminisse velimus, oratio haec *nihil residuum est*, et in illa nomen *nihil* inutile non est. Propter eandem rationem etiam *minus quam nihil* dicetur recte residuo, ubi majus detrahitur a minore. Hujusmodi enim residua doctrinae causa fingit sibi animus, cupitque, quoties opus est, in memoriam revocare.

47. Pycior (1997, 142).

48. See, for example, *Lev* I.V, 31-7.

49. This is made clear by some of the other examples, besides that of negative numbers, that Hobbes appeals to in the passage which Pycior cites:

> It is not in fact necessary that every name be the name of some thing. For as the words *man, tree, stone*, are the names of the things themselves, so too the images of a man, tree, stone, which occur to people sleeping, have for themselves their own names, however they are not things, but just figments of things and phantasms for given that we remember these things, and therefore it is necessary that they be designated and signified by names no less than the things themselves. Also this word *future* is a name, but any future thing does not exist, nor do we know what we call the future, or whether the future ever exists, but yet because by imagination, we are used to fastening past to present things, we signify with the name *future* such a fastening together. (*DCp* I.2.6; *OL* I, 15)
>
> Neque vero ut omne nomen, alicujus rei nomen sit, necessarium est. Sicut enim voces *homo, arbor, lapis*, ipsarum rerum nomina sunt, ita quoque imagines hominis, arboris, lapidis, quae occurrunt somiantibus, sua sibi habent nomina, quamvis res non sint, sed rerum figmenta tantum et phantasmata. Datur enim ipsarum meminisse, ideoque nominibus eas non minus quam res ipsas notari et significari oportet. Etiam vox haec *futurum* nomen est, sed res futura nondum ulla est, neque scimus quod futurum vocamus, an futurum umquam sit; attamen quia cogitatione, praeterita praesentibus subnectere soliti sumus, nomine *futuri* talem subnexionem significamus.

When we use the word "man" to signify the image of a man in a dream, we are not using the word to signify a thing which exists in the world. Similarly, "future" does not signify an actual thing, because the future does not yet exist. However, the words "man" and "future" do refer do certain conceptions — the phantasm of the man in a dream and our idea of the things which have yet to exist. In the same way, there is no such thing as a negative number, but the words "less than nothing" derive their meaning from our conception of what remains when more is subtracted from less.

50. See, for example, *EL* I.VI.4; 41.

CHAPTER 4

MATHEMATICAL KINEMATICS

For Aristotle, the study of physics and the study of motion are closely aligned, since the subject matter of physics is natural body in general, and nature is defined as "a source or cause of being moved and of being at rest in that to which it belongs primarily."[1] The motion of natural bodies falls under the domain of physics, and is hence, according to the Aristotelian classification of the sciences, not subject to mathematical analysis. As we saw in chapter 3, Aristotle's conception of motion was much broader than our own: to recall, Aristotelian motions included not only local motions, but also alteration, augmentation and diminution, and (at least sometimes) generation and corruption.

The Aristotelian understanding of motion and how it should be studied was widely challenged in the seventeenth century. The very idea of motion became reduced to that of local motion, which the mechanical philosophers used to explain all other forms of change. Local motion itself was subjected to a variety of mathematical analyses. The next two chapters will place Hobbes's account of motion in the context of these changes. They will show that Hobbes's materialist mathematics allowed him to adopt a distinctive approach to the mathematisation of motion, and discuss the ways in which this approach was developed in part III of *De Corpore*. The chapters will also compare Hobbes's account of motion with those of other seventeenth-century theorists who thought that motion could be treated mathematically, most notably with work of Galileo, as well as with the work of some of Hobbes's significant medieval predecessors.

This chapter will examine Hobbes's kinematics, while chapter 5 will look at his dynamics. This distinction must, of course, be used with caution, since the terminology is not Hobbes's own. However, as was discussed in chapter 1, he did distinguish between the study of *motus simpliciter* and the study of the effects of the motion of one body on another, a distinction which can be fairly described as one between kinematics and dynamics. Furthermore, this will represent a significant division in Hobbes's treatment of motion.

As Brandt has noted, part III of *De Corpore* is both geometrical mechanics and mechanical geometry.[2] It should be noted at the outset that I will be discussing Hobbes's use of mathematics to explore problems in mechanics. I will not consider Hobbes's use of the principles of matter and motion to address traditionally mathematical problems, except when it is relevant to the subject at hand.

4.1 The Quantitative Analysis of Qualities

One of the primary tasks of this chapter will be to compare Hobbes's account of motion with the one presented by Galileo in the *Two New Sciences*. This comparison will be particularly helpful since, as has been widely noted, Hobbes was a great admirer of Galileo's work, referring to him in *De Corpore* as the person who "first opened to us the principal gate of universal physics, natural motion" (*DCp* ED; *OL* I, unpaginated).[3] Hobbes was no doubt impressed by Galileo's achievements in describing motions in mathematical terms. In his work, Galileo endeavoured to replace vague descriptions of the properties of motion with precise, mathematical ones. As he states at the beginning of the Third Day, Galileo's purpose is to present "a very new science dealing with a very ancient subject."[4] Although many books had been written on the subject of kinematics, Galileo believed that he had discovered and demonstrated some previously unobserved properties of motion. For example, the "superficial" observation had been made that a freely falling object accelerates continuously, "but to just what extent this acceleration occurs has not yet been announced; for so far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity."[5] Similarly, although the path of a projectile had been described as curved, no one had noted that its path is parabolic.

As we will see, the *Two New Sciences* was clearly an influence on part III of *De Corpore*. However, Hobbes also turns away from Galileo at a number of significant points in his account. Examining these differences will allow us to see how Hobbes's approach to mechanics led him to reject some important aspects of Galileo's much more successful account.

I will also be comparing Hobbes's work with that of Nicole Oresme, one of the foremost practitioners of medieval kinematics. I will argue that Hobbes's account of motion represents, in some significant senses, a return to the medieval perspective. In making this comparison, I do not mean to suggest that Oresme is the only medieval thinker who could have influenced Hobbes. As I mentioned briefly in chapter 3, and will discuss again below, Hobbes's work also has a great deal in common with that of a group of philosophers working out of Merton College in the early to mid fourteenth century. However, Oresme shared many of the Mertonians's doctrines, and was, in addition, the first to develop a systematic account of how motions could be represented geometrically. I will therefore treat Oresme's work as representative of a medieval approach to the study of motion that clearly had an influence, through some channel, on Hobbes.

Hobbes is, in many ways, more of a medieval than a Galilean. To begin, for both Hobbes and Oresme kinematics was part of a larger effort to provide a quantitative analysis of qualities. As I mentioned in chapter 3, this was a project which they shared with the Mertonians. The Mertonians's interest in quantitative kinematics was prompted by their consideration of the philosophical problem of how qualities vary in intensity. There were thought to be two ways that qualitative variation could be explained: either the quality itself varies, or the subject participates to a greater or lesser degree in an unchanging quality or form.[6] The Mertonians held the former view, and argued that increases and decreases in qualitative intensity should be analysed in terms of the addition and subtraction of degrees of intensity.[7] This analysis naturally led to the mathematical treatment of such changes. The same quantitative techniques were then applied by analogy to changes in motion, thereby allowing the Mertonians to do significant work in kinematics.[8]

Oresmes contributed to this area of study by giving a clear account of how these variations in intensity could be represented by geometrical figures.[9] According to the method that Oresme lays out in his *De configurationibus qualitatum et motuum* (c.1350), a line representing the extension of a quality in its subject is taken as the base of a figure.

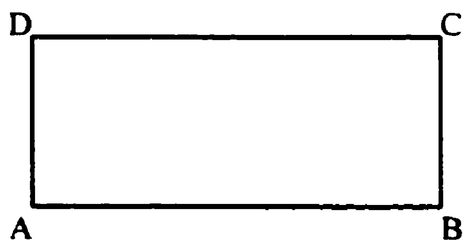


Figure 4.1

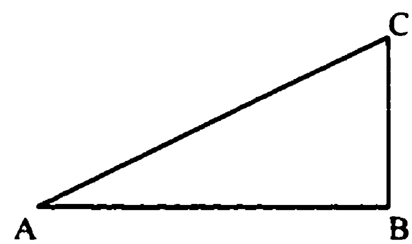


Figure 4.2

Lines are then erected perpendicular to each point on the line, each representing the intensity of the quality at some point in that extension. A rectangle, such as ABCD (in figure 4.1), would represent a uniform quality, since the lines representing the quality's intensity are equal at every point. However, a right-angled triangle represents a quality which increases or decreases uniformly (what Oresme and other medievals called a "uniformly difform" quality). As we can see in triangle ABC (in figure 4.2), the lines erected on the base AB increase uniformly from the point A to the line BC. Using the same techniques, figures could be constructed to represent any of the infinite number of ways that a quality could alter non-uniformly.

In the second and third parts of the *De configurationibus*, Oresme shows how his geometrical analysis of qualities can also be applied to the analysis of local motions. In the most important case for our purposes, a subject's velocity can be represented by a figure with the baseline representing the duration of the motion,[10] and lines erected perpendicular to the base representing the instantaneous velocities at various points of time. A rectangle would therefore represent a uniform motion, while a right-angled triangle would represent a motion whose velocity increases or decreases uniformly. The area of the figure represents the quantity of the subject's "total velocity."[11]

Hobbes, like Oresme, is interested in the quantitative representation of qualities. However, Hobbes's mechanistic physics demands that he take a somewhat different approach than Oresme's to the problem. For Hobbes, local motion is the fundamental form of change, since it is that by which all other changes must be explained. Accordingly, he does not merely treat kinematics as a special case of the mathematical analysis of qualitative variations. Instead, Hobbes begins with the geometrical representation of local motion: as we saw in chapter 3, local motions and their velocities must be exposed to the senses by means of geometrical figures. Hobbes then states that qualities can be represented by the quantities of the velocities by which they were generated:

> Also concerning heat, light, and divers other qualities, which have degrees, there lieth a question of *how much*, to be answered by a *so much*, and consequently they have their quantities, though the same with the quantity of swiftness: because the intensions and remissions of the swiftness of that motion by which the agent produceth such a quality. And as quantity may be considered in all the operations of nature, so also doth geometry run quite through the whole body of natural philosophy. (*SL* 1; *EW* VII, 196)

Oresme argued that the same geometrical techniques could be used to analyse qualitative changes and variations in local motion. Hobbes's mechanism leads him to claim that the very same mathematical object can simultaneously represent both kinds of change.

Although, as we will see, Galileo applies similar techniques to the investigation of motion, he shows no interest in using them to study qualitative change. He is content to provide a quantitative account of motion's effects, without placing this project in the context of a broader account of continuous change.

4.2. Endeavour And Impetus

Before continuing to compare his work with that of Galileo and Oresme, we need to discuss some of the basic principles of Hobbes's account of motion. The bulk of Hobbes's kinematics is presented in chapters 15 and 16 of *De Corpore*'s third part. Chapter 15 begins with a review of Hobbes's first philosophy discussion of motion, then presents some new principles. First and foremost among these new concepts are "endeavour" (*conatus*)[12] and "impetus." These notions play an important role in Hobbes's kinematics and in his dynamics, and are probably the principles in his account of motion which have been most extensively discussed by others.[13]

"Endeavour" is defined as "*motion through less space and time than any given, that is, determined, or marked out by exposition or number,* that is, *through a point*" (*DCp* III.15.2; *OL* I, 177).[14] To recall, in his first philosophy Hobbes defines a point as a body whose magnitude is not brought into computations, in which sense the earth itself can be regarded as a point. Hobbes is referring to the claim, made by some astronomers, that the earth can, for the sake of our observations and calculations, be considered as a point. For example, Ptolemy states in his *Almagest* (c.150 A.D) that "the earth has, to the senses, the ratio of a point to the distance of the so-called fixed stars."[15] He offers as evidence of this claim the lack of a parallax: there is no measurable difference in the apparent sizes and distances of the stars from any point on earth.[16] Aristarchus of Samos (c.310-230 B.C.), in his *On the Sizes and Distances of the Sun and Moon*, claims that the earth "is in the relation of a point and centre" not to the sphere of the fixed stars, but "to the sphere in which the moon moves."[17]

Ptolemy and Aristarchus are not claiming that there is no ratio between the earth and the spheres of the stars or moon (both believed the world to be finite) — they are merely saying that that ratio is so great that the size of the earth makes no measurable difference to our calculations and observations. Hobbes is hence suggesting that the ideas of these astronomers represent a precedent for his own view that the ratio between a point and a line is so great that the size of the point can be disregarded. He thus reiterates, following his definition of "endeavour," that a point is not something "which has no quantity, or can by no means be divided (for there is nothing of this sort in the nature of things); but that whose quantity is not considered, that is, neither its quantity nor any part thereof is computed in demonstration; so that a point is not taken for indivisible, but for undivided. As also an instant is to be taken for an undivided time, not for an indivisible time" (*DCp* III.15.2; *OL* I, 177-8).[18]

A motion through a point would thus be a motion through a space and in a time too small to be considered. As Hobbes states, endeavour is motion, "but so that neither the quantity of time in which is made, nor of the line through which it is made, can be compared in a demonstration with the quantity of time or line of which it is a part" (*DCp* III.15.2; *OL* I, 178).[19]

Although points cannot be so compared with lines, the magnitude of one point can be greater or lesser than the magnitude of another point (*DCp* III.15.2; *OL* I, 178). Although the magnitude of every point can be disregarded in comparison with the magnitude of a line, there may nevertheless be a considerable ratio between two such disregardable magnitudes. So, for example, Ptolemy would probably agree that there is a considerable ratio between the magnitudes of the earth and moon, while maintaining that neither magnitude is comparable with the distance to the stars.

Hobbes elaborates that in the same way that there are greater and lesser points, there are also greater and lesser endeavours. He purports to explain how this can be the case in the following puzzling passage:

> In the same manner if there are two motions both beginning and ending together, their endeavours will be equal or unequal in the proportion of the velocities [of the two motions]; as we see that a ball of lead descends with a greater endeavour than a ball of wool. (*DCp* III.15.2; *OL* I,178)[20]

The most obvious difficulty with this passage is Hobbes's apparent claim that objects with different weights will accelerate at different rates in free fall. This suggests that Hobbes completely missed the point of Galileo's argument to the contrary, despite his familiarity with Galileo's work.[21]

Brandt, finding it hard to believe that Hobbes could misunderstand Galileo in this way, offers an alternative interpretation of this passage. He suggests that Hobbes's example introduces a dynamic aspect to the concept of endeavour which, as we saw above, was defined in a purely kinematic way. On Brandt's interpretation, when Hobbes claims that the two motions "begin and end together" he means to convey the Galilean point that when a ball of lead and a ball of wood fall simultaneously from a certain height, they will reach the ground simultaneously. However, there is "a considerable difference between the two cases, viz. with respect to the dynamic. The effect of the bullet of lead is considerably greater than that of the ball of wool. This difference is the difference in endeavour."[22] Brandt acknowledges that this interpretation does not fit well with the text, since Hobbes specifies that the difference between the endeavours of the two motions will be proportional to their velocities and, on Brandt's reading, the velocities of the falling balls will be the same. However, Brandt claims that he cannot bring himself to attribute a significant misreading of Galileo to Hobbes, and awkwardly accommodates the reference to velocity by surmising that Hobbes was trying to express two different lines of thought in the passage.[23]

As we will see, Hobbes does make use of his endeavour concept in his dynamics. However, it is questionable to claim, on the basis of this passage, that the concept has an intrinsically dynamic aspect. Hobbes does not explicitly mention the magnitude or weight of the bodies as a factor to be considered when comparing their endeavours (something which he certainly does raise when he actually does get around to defining force). Nor does he mention the notions of effect or force of impact. Finally, there is the acknowledged problem that the passage posits a correlation between the velocities of the bodies and their endeavours.

A less convoluted reading of this passage is available — one which does not involve attributing a misreading of Galileo to Hobbes. Brandt assumes that the balls of lead and wool are meant to be imagined as falling under ideal conditions. However, if we instead suppose that Hobbes meant to take into account the effects of air resistance, this would explain why he thought that the balls of wool and lead would acquire different velocities after falling for the same amount of time. Galileo himself, in the *Dialogue Concerning the Two Chief World Systems* (1632), adduces against the claim that the speeds of naturally falling bodies will be proportional to their weight the fact that a ball of lead will fall only twice as fast as a ball of cork, although it weighs many times more.[24] Attributing to Hobbes the claim that, under normal circumstances, naturally falling balls of lead and wool will accelerate at different rates does not mean that he misunderstood Galileo.

If this interpretation is correct, when Hobbes states, in his account of greater and lesser endeavours, that the motions of the lead and wool "begin and end together," he means that the two balls begin and end their falls at the same points in time. The ball with the greater endeavour is that which traverses a greater space in that period of time, i.e., it is that with the greater velocity. Similarly, as Hobbes thinks that points can be of different sizes, so a motion which traverses a point with a greater magnitude in the same instant of time will have a greater velocity, and hence endeavour.

Hobbes goes on to define the closely related concept of impetus, which is "*the velocity itself, but considered in any point of time in which a transition is made [fit transitus].* So that the impetus is nothing other than the quantity or velocity of the endeavour itself." (*DCp* III.16.15; *OL* I, 178)[25] A body's impetus, then, is just its instantaneous velocity.

4.3 Impetus, Total Velocity, And The Nature of The Continuum

Hobbes begins chapter 16, "Of Uniform and Accelerated Motion and of Motion by Concourse," by relating impetuses, or instantaneous velocities, to the velocities of motion through extended periods of time. However, before examining this aspect of Hobbes's theory, it will be helpful to look at his notions of the infinite and the nature of the continuum.

The ancient world bequeathed two primary ways of understanding the nature of continuous magnitude: first, one could hold that any such magnitude is infinitely divisible. Aristotle held this position — although he denies the existence of actual infinities, he argues that there are potential infinities by division, claiming in the *Physics* that "it is plain that everything continuous is divisible into divisibles that are infinitely divisible."[26] Accordingly, Aristotle denies that a line can be composed of points, a line being continuous and points indivisible.[27] On the other hand, the seventeenth century saw the revival of the ancient atomism of Democritus and Epicurus. The atomists held that a magnitude cannot be divided indefinitely, but only until some indivisible or atomic magnitude is reached, beyond which further division is impossible.[28]

The debate over the nature of the continuum, which extended through the medieval and early modern periods, was too complex to discuss in detail here.[29] Many variations of both the divisibilist and indivisibilist positions were developed, with proponents of each side attempting to answer the other's arguments, and some thinkers attempting to find a compromise between the two positions.

Hobbes's account of the continuum is one of those that draws on both the Aristotelian and the atomist positions. Hobbes, like Aristotle, rejects the notion that we can have an idea of an actual infinity: we get all of our ideas through the senses, and the senses cannot provide us with ideas either of the infinitely large or the infinitely small. Hence our idea of the infinite is just the idea of something whose limits or bounds we cannot conceive:

> Whatsoever we imagine, is *Finite*. Therefore there is no Idea, or conception of any thing we call *Infinite*. No man can have in his mind an Image of infinite magnitude; nor conceive infinite swiftness, or infinite force, or infinite power. When we say any thing is infinite, we signifie onely, that we are not able to conceive the ends, and bounds of the thing named; having no Conception of the thing, but of our own inability. (*Lev* I.3, 23)

There are some differences between Hobbes's views regarding the infinitely large and the infinitely small. Hobbes does not hold that the infinitely large is impossible, only that it is impossible for us to conceive of it. He does not, for example, deny the possibility that the world might be infinite in space or time, just that it is beyond our rational capacities to know whether this is in fact the case. He thus thinks that philosophers should abandon their wranglings over the magnitude and duration of the world, leaving these questions to those who are authorized to determine the nature of religious observances (*DCp* IV.26.1; *OL* I, 335-6).

Hobbes does, however, deny the possible existence of the infinitely small. He maintains that anything that has quantity, and hence everything in nature, is infinitely divisible, holding that no matter how small a given quantity is, it can always be divided into yet smaller parts (*DCp* II.7.13, *OL* I, 89; *DCp* III.15.2; *OL* I, 177-8) He presents little justification for this assertion. In his first philosophy he offers a standard argument for the infinite divisibility of space and time, which could be applied to other kinds of quantity: take a part of time or space, assumed to be of that magnitude at which divisibility becomes impossible. Assume this part to be contiguous on either side with two equal parts, then divide this whole space or time (which, being greater than the least divisible, must itself be divisible) into two, in the process dividing the middle part into two equal parts. The indivisible has thus been divided (*DCp* II.7.13; *OL* I, 89).

There are, however, places in *De Corpore* where Hobbes says things that seem to contradict his statements about the infinite divisibility of matter: first, when attempting to solve various mathematical problems, including the squaring of the circle, he sometimes speaks of a quantity being infinitely divided or the "least parts" of a magnitude being found.[30] Second, as we have seen, Hobbes defines "endeavour" and "impetus" in terms of motions through points. However, motions through points must somehow make up motions through a line. As I mentioned above, this is a position usually avoided by those who believe that matter is infinitely divisible. I will look at each issue in turn.

In order to understand the first set of difficulties, we need to look at Hobbes's comments in the context of contemporary debates over the use of indivisibles in mathematics. The emergence of the method of indivisibles was one of the most significant developments in early modern mathematics. The method was first presented by Bonaventura Cavalieri in his *Geometria indivisibilibus continuorum nova quadam ratione promota* (1635).[31] Cavalieri's method was based on the idea that the ratio of the areas of two plane figures or two solids is the same as the ratio of what he calls "all the lines" of the figures or "all the planes" of the solids. He also referred to these lines or planes as "indivisibles." To focus on the first case, "all the lines" of a figure can be generated by inserting the figure between two parallel tangents, then moving one of the parallels (called the "regula") through the figure until the two meet. For example, if (in figure 4.3) we take the figure ABC between the parallel tangents WX and YZ, then move WX towards YZ until the two parallels meet, WX will have passed through all of the indivisibles or all of the lines in ABC.

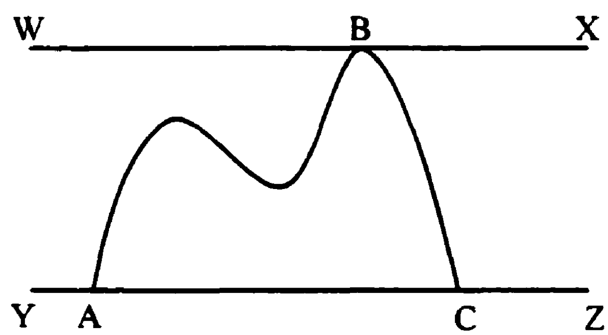


Figure 4.3

Intuitively, the regula will pass through a different line during each instant of its motion. The aggregate of these is called "all the lines" of the figure.

Cavalieri holds that we can determine the ratio between two figures by determining the ratio between their respective collections of lines. He treats these collections as a new form of magnitude which can be accommodated by the Euclidean theory of proportion. The second theorem of the second book of his *Geometria* thus states that "[a]ll the lines of rectilinear transit of arbitrary plane figures, and all the planes of arbitrary solids, are magnitudes which have a ratio among each other."[32]

By appealing to motion, whose continuity he took to be relatively unproblematic, Cavalieri avoided the claim that the figure is composed of all the lines, or, more generally, that any continuum is the sum of its indivisibles. He also avoided the question of whether the indivisibles are infinitesimal, and whether there is an infinite number of them, believing that his theory would hold up regardless of how questions of the infinite were settled.

As Jesseph has shown, Hobbes was deeply influenced by Cavalieri, including some of Cavalieri's proofs, with very little modification, in chapter 17 of *De Corpore*.[33] Hobbes saw similarities between Cavalieri's method and his own philosophy of mathematics. In the *Examinatio*, for example, Hobbes states:

> Those things that when multiplied can exceed one another are homogeneous, and measurable by the same kind of measure, as lengths are measurable by lengths, surfaces are measurable by surfaces, solids are measurable by solids. However, those things which are heterogeneous, are measured by different kinds of measures. But if lines are considered as the most minute parallelograms, as they are considered by those who use that method of demonstration that Bonaventura Cavalieri uses in his doctrine of *Indivisibles*, there will also be a ratio between *straight lines* and *plane surfaces*. For such lines multiplied will be able to exceed any given finite plane surface. (*Ex* 2; *OL* 4, 74-5)[34]

Hobbes attributes to Cavalieri the view that lines have breadth, a view that is clearly close to Hobbes's own. Furthermore, he interprets Cavalieri's doctrine to entail the homogeneity of indivisibles and continuous magnitudes.

Some of Cavalieri's own statements may have led Hobbes to this view. In his *Exercitationes Geometricae Sex* (1647) Cavalieri states that "it is manifest that we can conceive of plane figures in the from of cloth woven out of parallel threads, and solids in the form of books, which are built up out of parallel pages."[35] This does suggest that lines, like threads or pages, have breadth. However, Cavalieri quickly clarifies that, while the threads of cloth and pages of a book are finite, the "lines in plane figures (or planes in solids) are to be supposed, without any thickness." It is difficult to say why Hobbes saw the parallels that he did between his own mathematics and Cavalieri's. As Jesseph notes, "[s]ince there is no deep doctrinal affinity between these two thinkers, the latter's reticence on foundational issues is responsible for Hobbes's approval (and reinterpretation) of Cavalieri's doctrines."[36]

Unlike Cavalieri, Wallis was not at all hesitant in his use of infinitesimal methods. He held that lines are composed of an infinite number of indivisible points, and figures of an infinite number of lines. In doing so, he simply ignored the classical distinction between discrete and continuous magnitude. As I have discussed, Wallis also held that arithmetic is the foundation of mathematics. In works including his *Arithmetica Infinitorum* (1656) he argued that geometrical problems could be solved arithmetically by considering the area of a figure to be made up of an infinite number of infinitesimals, then calculating the area of that figure by calculating the infinite sum of those infinitesimals.[37]

Hobbes, of course, disagreed vehemently with almost every aspect of Wallis's programme.[38] Most notably for our purposes, he criticizes Wallis's account of the nature of indivisibles. So, in the *Six Lessons*, Hobbes declares to Wallis that

> you think it will pass for current, without proof, that a point is nothing. Which if it do, geometry also shall pass for nothing, as having no ground nor beginning but in nothing. But I have already in a former lesson sufficiently showed you the consequence of that opinion. To which I may add, that it destroys the method of *indivisibles*, invented by Bonaventura; and upon which, not well understood, you have grounded all your scurvy book of *Arithmetica Infinitorum*. (*SL* V; *EW* VII, 300-1)

Wallis often criticized Hobbes's notion of a point as a body with magnitude. Hobbes retorts that considering points as "nothing" would undermine the method of indivisibles, because an infinite sum of nothings is nothing.

Wallis does not actually argue that indivisibles are "nothing," but that they are infinitely small. He hence says in his *Conic Sections* that the indivisibles that make up a figure can be supposed to be parallelograms, but that the altitude of each parallelogram "is supposed to be infinitely small, that is, no altitude, for a quantity infinitely small is not quantity, scarcely differing from a line." Hobbes replies:

> How do you determine this word *scarce*? The least altitude, is somewhat or nothing. If somewhat, then the first character of your arithmetical progression must not be a cypher; and consequently the first eighteen propositions of this your *Arithmetica Infinitorum* are all nought. If nothing, then your whole figure is without altitude, and consequently your understanding nought. (*SL* V; *EW* VII, 308)

Hobbes is charging that Wallis's indivisibles must either be nothing or something. If nothing, then the figure that they make up will also be nothing. If, on the other hand, the indivisibles do have some altitude, they will not be "cyphers," as Wallis puts it, but Hobbesian indivisibles, complete with magnitudes.

Hobbes thought that the method of indivisibles could be useful, but only if indivisibles were properly conceived as extended points or lines with breadth. Since he claims in the *Examinatio* that lines conceived in this way, i.e., as minute parallelograms, can be multiplied so as to exceed a plane surface, his position seems to be that continuous magnitude is composed of some finite, though indefinitely large, number of indivisibles of the kind that he attributes to Cavalieri. It is interesting, however, that Hobbes avoids treating a figure as the "sum" of its constituent lines, and maintains Cavalieri's practice of describing indivisibles by means of the motion of a line through a figure. In doing so he avoids any association with Wallis's arithmetic approach to the method of indivisibles.

It remains to be considered whether Hobbes's version of the method of indivisibles can be reconciled with his commitment to the infinite divisibility of continuous magnitude. In order to render these aspects of his theory consistent Hobbes could appeal to some of the same notions that he did in his definitions of "point," "line," and "surface." To recall, Hobbes had, in defining these geometrical objects, stated that they have magnitudes that are (in various respects) too small to be considered in demonstration. Hobbes could use the same resources to distinguish indivisibles from continuous magnitudes: continuous magnitudes are infinitely divisible, but at some point in the process of division the resulting quantities become too small to be considered in demonstration. At this point we regard them as indivisibles. The threshold between considerable and inconsiderable quantities would, on this account, be a pragmatic one: as we have seen, Hobbes describes the length of a point as less space than can be determined or assigned by exposition. Magnitude becomes inconsiderable when it cannot be exposed, i.e., set before the senses. Hobbes's empiricism dictates which quantities need to be taken into account in demonstrations.

The primary difficulty with this proposal is that it would conflate the means of distinguishing points, lines, and surfaces with the means of distinguishing indivisibles from continuous magnitudes. In other words, a mathematical line would be just the same thing as the "least part" of a surface, or a line considered as a minute parallelogram. But Hobbes needs to maintain the distinction between these two classes of things: indivisibles need to be homogeneous with continuous magnitude, so that the former multiplied will be able to exceed the latter. On the other hand, if the heterogeneity of different kinds of magnitude is to be maintained, points, lines, and surfaces cannot be thought to measure each other. It is clear from Hobbes's comments in the *Examinatio* that he sees the need to maintain the independent existence of both types of object. His mathematics demands the use of the former. However, he must also maintain the heterogeneity of the various types of geometrical magnitude, lest he fall into something like the numerical theory of ratios, which, to recall, tends to encourage the homogenization of the kinds of magnitude. However, it does not seem that Hobbes has the resources to develop distinct ways of differentiating indivisibles from continuous magnitudes and the various kinds of geometric magnitudes from each other.

Hobbes does not specify whether the points he refers to in his definitions of "endeavour" and "impetus" are Hobbesian indivisibles or geometrical points. However, either option involves difficulties for Hobbes. If the points in question are geometrical, Hobbes is left in the uncomfortable position of having to explain how motion through one kind of magnitude can be made up of motions through magnitudes of a heterogeneous kind. On the other hand, it would not be helpful for Hobbes to appeal to his method of indivisibles at this point. If he claims that a motion through a line is the "sum" of motions through points (considered as minute lines) he again runs the risk of seeming to support an analysis like Wallis's. On the other hand, Cavalieri's way of avoiding the issue of the composition of the continuum is not open to Hobbes at this point. Cavalieri appealed to motion as a relatively unproblematic example of the continuous, and used it to account for more complicated cases. Hobbes followed him in using the motion of a line through a figure to understand continuous magnitudes. However, Hobbes's analysis of motion in terms of endeavours problematizes motion itself. It would, of course, be less than helpful for Hobbes to appeal to the continuous nature of motion to sidestep this problem.

It is with these difficulties in mind that we should look at Hobbes's attempt to relate impetuses to the velocity of a perceptible motion. Hobbes states that "[t]he velocity of any body moved during some time is as great as that which is made from the impetus (which it has in a point of time) multiplied into the time of its motion" (*DCp* III.16.1; *OL* I, 184).[39] In this passage, Hobbes avoids saying that a body's overall velocity is the sum of its impetuses. Instead, the velocity is taken to be proportional to the motion's impetus (by which Hobbes means its mean impetus)[40] "calculated into" the time of its motion. As we will see in the next section, Hobbes is referring to the idea that a body's velocity can be represented by a figure of which one side represents the body's mean impetus, the other the time of its motion. The issue is no longer one of the composition of a continuous motion; rather, Hobbes is making a claim about a proportion between a given velocity and a mathematical representation thereof.

Hobbes's approach here is in some senses the opposite of Cavalieri's (and Hobbes's own elsewhere): while Cavalieri appeals to motion to sidestep questions regarding the nature of geometrical continuity, Hobbes here takes geometrical magnitude to be the unproblematic case, since he appeals to geometrical figures to explain the relationship between impetuses and perceptible motions.

The principle quoted above, and the treatments of motion which follow, owe much to medieval kinematics. To recall, Oresme claimed that a subject's "total velocity" can be represented by a figure wherein the baseline represents the duration of the motion, and lines erected perpendicular to the base represent the instantaneous velocities at various points of time. There is an obvious similarity between Oresme's doctrine and the statements that Hobbes makes in the first section of chapter 16. Hobbes claims that since an impetus is the velocity at a single point of time, if we take all the impetuses in all the points of time that make up a finite motion, they will be equal to "the [mean] impetus calculated into the total time, or with the velocity of the total motion" (*DCp* III.16.1; *OL* I, 185).[41] Hobbes's use of the language of "total" motion and velocity is of course similar to Oresme's. The following is presented as a corollary to this principle:

> If the impetus is everywhere the same and any straight line taken for the measure of time, and the impetuses applied [ordinately][42] to that straight line, they will describe a parallelogram that will represent the velocity of the total motion. If however the impetus beginning from rest should increase uniformly, that is, always in the same proportion with the time passed, the total velocity of the motion will be represented by a triangle, of which one side is the total time, the other the greatest impetus acquired in that time. (*DCp* III.16.1; *OL* I, 185)[43]

Hobbes goes on to claim that uniformly accelerated motion can also be represented by parallelograms equal in area to the triangle described above. Hobbes and Oresme therefore used geometrical figures in similar ways to represent various kinds of motions.

Hobbes was not the only one to use these techniques in the seventeenth century. There is a strong resemblance between Oresme's proof of what was called the mean speed theorem and Galileo's presentation of the same theorem in the *Two New Sciences*. The mean speed theorem, the discovery of which was one of the most significant achievements of medieval kinematics, states that in a given time the same space will be traversed by a body moving with uniformly accelerated motion and the same body moving with a uniform speed equal to the mean between the starting and final speeds of the first motion.

Oresme's proof (which is originally presented in terms of qualities, but is later said to apply also to velocities) is as follows: in figure 4.4, let there be a uniformly difform motion represented by the triangle ABC, and let D be its middle instant of time. The velocity at this point in time is thus represented by the line DE, and a velocity uniformly of the degree DE throughout the time AB would be represented by the rectangle AFGB. But EFC and EGB are equal. BAC and AFGD are therefore equal, and hence the velocities designated by these figures must also be equal.

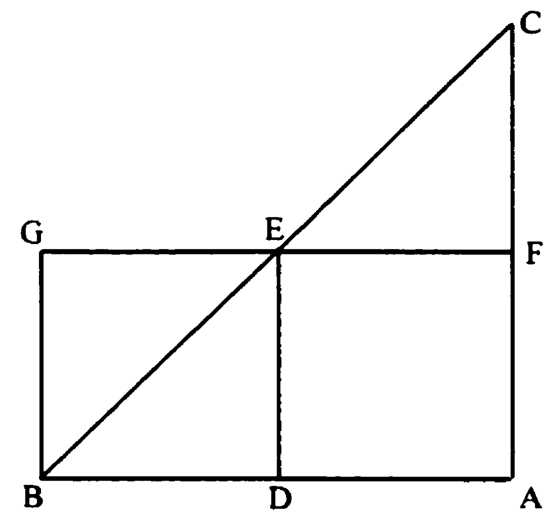


Figure 4.4

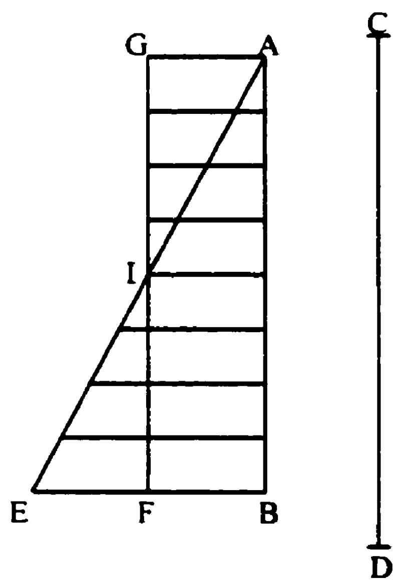


Figure 4.5

Galileo's proof of the mean speed theorem uses a figure which is essentially the same as Oresme's, although Galileo turns it onto its end and introduces a line CD which represents the space traversed by a body moving with uniform acceleration during the time AB (see figure 4.5). All the lines drawn perpendicular to AB represent values of speed — those contained in the triangle AEB represent values which increase uniformly, while those contained in the rectangle AGFB represent values which are uniformly of the degree which the uniformly accelerated motion reaches at its midpoint. Since the parallels contained in the triangle AGI are equal to those contained in the triangle IEF, while those in the trapezium AIFB are common, the sum of all the parallels in AEB is equal to the sum of all those in AGFB. Furthermore,

> Since each and every instant of time in the time-interval AB has its corresponding point on the line AB, from which points parallels drawn in and limited by the triangle AEB represent the increasing values of the growing velocity, and since parallels contained within the rectangle represent the values of a speed which is not increasing, but constant, it appears, in like manner, that the momenta [*momenta*] assumed by the moving body may also be represented, in the case of the accelerated motion, by the increasing parallels of the triangle AED, and in the case of the uniform motion, by the parallels of the rectangle GB.[44]

Since the ratio between the triangle AEB and the rectangle AGFB is that same as that between the spaces traversed by the two bodies, the mean speed theorem has been demonstrated.

It should be noted that Galileo proves a very similar theorem in a comparable manner in the *Dialogue Concerning Two Chief World Systems*.[45] As Clagett notes, this proof uses the vocabulary of medieval kinematics, describing a figure's surface as representing "the mass and sum of the whole velocity" (*"la massa e la summa di tutta la velocita"*).[46]

The use of these techniques in the *Two New Sciences* and *Dialogue Concerning Two Chief World Systems* might suggest that Hobbes encountered them in Galileo's work, then adapted them to suit his own purposes. There is, however, some evidence that

Hobbes was directly influenced by medieval sources. As we saw in chapter 2, Hobbes shared with the medievals the project of quantitatively representing qualitative change. However, it is impossible to say for sure what Hobbes's sources were, since he nowhere states that he was aware of the medieval work in kinematics. As will be discussed in subsequent sections, at the very least the theoretical perspective that Hobbes brought to the study of motion was much more in line with that of the medievals than with Galileo's. Even if it was Galileo's work that suggested the geometrical representation of motions to Hobbes, the framework into which he fit these techniques is one that would look more familiar to Oresme.

An example of this tendency is Hobbes's failure to address questions surrounding the nature of continuous motion. Hobbes's use of figures to represent motions does not entail an answer to the kinds of difficulties that were discussed above. *De Corpore* makes no effort to explain how impetuses (represented by the straight lines erected perpendicular to the baseline) relate to the motion's total velocity (represented by the area of the figure). Nor is this subject discussed in Oresme's work. On the other hand, as we saw in the above proof, Galileo talks about the "sums" of the parallel lines in different figures. He is able to do so because he had a more worked-out view on the nature of continuous magnitude. We will be looking at his arguments at a later point in the chapter, but, briefly, Galileo claims that the continuum is made up of an infinite number of infinitely small indivisibles.

4.4 The Proofs

This section will look at two of Hobbes's proofs from chapter 16: his proof that the distances traversed in motion uniformly accelerated from rest are as the odd numbers beginning from one, and his demonstration that a body borne by two movements, one uniform and one uniformly accelerated, will trace the path of a semiparabola. We will also look at Galileo's proofs of similar propositions in the *Two New Sciences*, which are, to recall, those that Galileo mentions at the beginning of the Third Day as illustrations of his novel contributions to the study of motion. Having these proofs at hand will facilitate a comparison of Hobbes's methods with Galileo's. Subsequent sections of this chapter will discuss the ways in which Hobbes differed from Galileo in the scope of the other proofs that Hobbes presents in chapter 16.

4.4.1 Distances Traversed in Uniformly Accelerated Motion

In the Third Day of the *Two New Sciences*, Galileo sets out to prove that "the spaces described by a body falling from rest with a uniformly accelerated motion are to each other as the squares of the time-intervals employed in traversing these distances."[47]

In figure 4.6, let AB represent a time beginning at A, in which the intervals AD and AE are taken. Let HI be the distance traversed by a body falling from rest at H with uniform acceleration, and let HL be the space traversed in the time AD, and HM the space traversed in AE. Therefore Galileo wants to prove that HM is to HL as AE^2 is to AD^2, i.e., that $HM:HL :: AE^2:AD^2$.

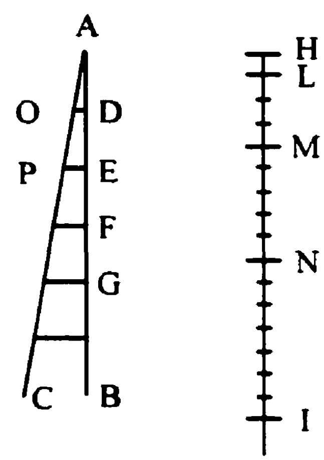


Figure 4.6

Using the mean speed theorem, which he has just demonstrated, Galileo begins by showing that distances traversed with uniformly accelerated motion can be reduced to distances traversed with uniform motion. Let the straight line AC be drawn at any angle with AB, and let two parallel lines DO and EP be drawn, with DO representing the greatest velocity attained during AD and EP the greatest velocity attained during AE. According to the mean speed theorem, the distances HM and HL are equal to those that would be traversed during the times AD and AE by bodies with velocities which are half of DO and EP, respectively.

Galileo now appeals to his previous results concerning uniform motion. At an earlier point in the Third Day he had demonstrated that the spaces traversed by two particles in uniform motion bear to one another a ratio which is equal to the product of the ratio of the velocities by the ratio of the times. Therefore, using modern notation, (HM / HL) = (AE / AD) x (½ED / ½DO). But in this case the ratio of the velocities is equal to the ratio of the time intervals (since the motions in question are uniformly accelerated). Thus (AE / AD) = (ED / DO) = (½ED / ½DO), and (HM / HL) = (AE^2 / AD^2), which was to be proved.

An almost identical theorem appears as a corollary to one of Hobbes's proofs (although Hobbes does not specify that the result applies to the motion of a falling body). The second proof of chapter 16 demonstrates that

> [i]n motion uniformly accelerated from rest (that is, where the impetus increases continually according to the proportion of the times) the length traversed in one time to the length traversed in another time will also be as the product of the impetus into the time to the product of the impetus into the time. (*DCp* III.16.3; *OL* I, 186-7)[48]

In figure 4.7, let AB be a time. At the beginning of this time, the body's impetus is as the point A, i.e. the body starts with zero impetus. Let the impetus increase uniformly until, in the last point of the time AB, namely B, it is BI. Take another time AF, at the beginning of which the body also has zero impetus, and let the impetus increase uniformly until the instant F, at which point let the impetus acquired be FK. Finally, let the length traversed in time AB be DE. Hobbes wishes to claim that the length DE is to the length traversed in time AF, as the time AB multiplied into the impetus increasing continuously to BI is to the time AF multiplied into the impetus increasing continuously to FK.

He begins by equating the area of the triangle ABI with the total velocity of the body moved in time AB, and area of the triangle AFK with the total velocity of the body moved in time AF. Appealing to the fact that the areas of these figures also represent the distances traversed in AB and AF respectively, he also asserts that DE is to the length traversed in AF as triangle ABI to the triangle AFK, i.e., as the duplicate proportion of the time AB to the time AF.

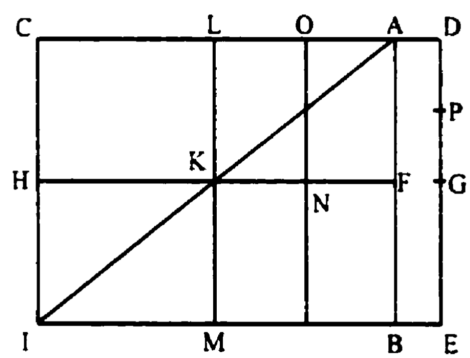


Figure 4.7

These claims can be summarized as DE:distance travelled in AF :: ABI:AFK :: AB^2:AF^2.[49]

Let DE be to DP as ABI is to AFK, so that the length traversed in the time AB will be to the length traversed in the time AF as the triangle ABI is to the triangle AFK. But the triangle ABI is made by the multiplication of the time AB into the impetus increasing continuously to BI, and the triangle AFK is made by the multiplication of the time AF into the impetus increasing continuously to FK, so the proposition has been demonstrated.

Hobbes presents three corollaries to this proof: first, that in motion uniformly accelerated the lengths traversed are in duplicate proportion of the time. For DE:DP :: ABI:AFK. But ABI:AFK :: AB^2:AF^2; therefore DE:DP :: AB^2:AF^2.

Second, that in motion uniformly accelerated the lengths traversed in equal times successively from the beginning of motion are as the differences of the square numbers beginning from unity (as 3, 5, 7 etc.). (This follows from the first corollary.)

Third, in a uniformly accelerated motion beginning from zero, the length traversed is to another length traversed in the same time, but uniformly and with an impetus equal to that acquired in the final point of time of the other motion, as a triangle is to a parallelogram, whose height and base are the same. This result is related to the mean speed theorem, since it entails that a body moving with uniform acceleration will, in the same time, traverse a length half that that the same body would traverse moving uniformly with the final speed of the former motion (since the parallelogram that Hobbes describes has twice the area of the triangle).

4.4.2 Paths of Bodies Moving with Compounded Motion

As was mentioned in chapter 2, in the *Two New Sciences* Galileo demonstrates that "[a] projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola."[50] Again, Galileo's theorem pertains to a natural motion, in this case that of a projectile.

We are asked to imagine an object moving with equable motion along the plane AB (in figure 4.8). At B it loses the support of the plane, and the horizontal motion is hence compounded with a naturally accelerated motion along BN. The intervals BC, CD, and DE represent equal times. Straight lines parallel to BN are dropped from the points C, D, and E. On the first we take a part CI, on the next its quadruple DF, and on the next its nontuple EH. If the projectile gains the amount of vertical motion represented by the line CI in the interval of time BC, it will be at point F after the interval BD, and at the point H

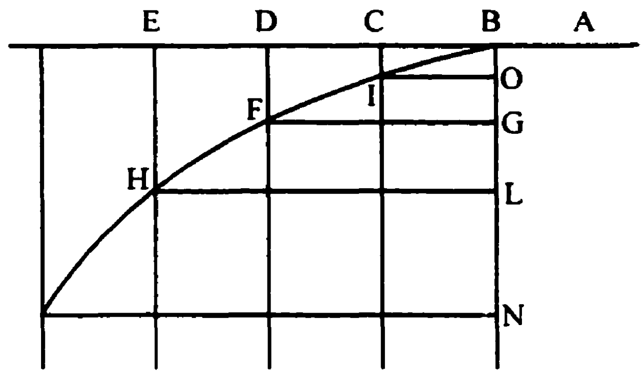


Figure 4.8

after the interval BE, given that Galileo has established that a naturally accelerated body will move through distances proportional to the square of the times. Galileo goes on to show, by comparing the properties of the curve generated by joining these points and those of a parabola that the two are identical.

In a later part of chapter 16, Hobbes offers a number of proofs which demonstrate the characteristics of the paths traced by bodies with movements formed from the concourse of two motions. In the second of these, Hobbes sets out to demonstrate that

> [i]f a mobile is borne by two movements together, meeting in any given angle, of which the one is moved uniformly, the other with a motion accelerated uniformly from rest (that is, that the impetuses are in the ratio of the times; that is, that the ratio of the lengths is the duplicate of the ratio of the times) until it acquires by acceleration an impetus equal to the impetus of the uniform motion, the line in which the mobile is borne will be the curved line of the semiparabola, of which the base is the impetus ultimately acquired. (*DCp* III.16.9; *OL* I, 196)[51]

In figure 4.9, let AB be a straight line, which is moved with uniform motion to CD, during which time the straight line AC also moves, but with a uniformly accelerated motion to BD (until the impetus acquired be BD equal to the straight line AC). Hobbes then asks that the semiparabola AGDB be described. He does not explain how this should be done, apparently taking the possibility and the means of the construction, as well as the properties of the semiparabola, for granted. Hobbes's claim is that "by the concourse of both movements together, it will happen that the mobile traverses the semiparabolic curve AGD" (*DCp* III.16.9; *OL* I,196).[52] Hobbes will prove this by arguing first about the properties of the points at which the two moving lines AB and AC intersect (and hence about the path of a body whose motion is compounded from the motions of each of the lines). He will then show that these properties match those of the separately defined semiparabola.

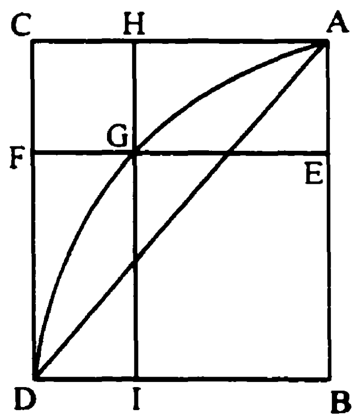


Figure 4.9

Let the parallelogram ABDC be completed, then any point E in the straight line AB be taken. From the point E let EF be drawn parallel to AC and cutting the semiparabola at G. Through the point G let HI be drawn parallel to the line AB and CD. According to our suppositions about the motions of the lines AC and AB, the distance that AB will traverse in some given time will vary as the square of the distance traversed by AC. However, a parabola is a line with the property that the distance along the vertical y-axis (or, in this case, along the line AB) varies as the square of the distance along the horizontal y-axis (or the line AC). Therefore, when AC is in EF, AB will be in HI, and the moving body will be at the point G in the parabola. Since this will occur no matter where the point E is taken between A and B, the body will always be found in the parabola AGD.

There is no question that there are strong similarities between the proofs presented by Galileo and Hobbes, suggesting that Galileo was an important influence on Hobbes's work. This is not surprising, given that, as I have noted, Hobbes was a great admirer of Galileo's work.[53]

However, Hobbes's proofs are not identical to Galileo's. Most notably, in the proof just discussed Hobbes describes the path of the body moving with compounded motion by means of the intersection of the two moving lines AB (moving with uniform motion) and AC (moving with uniformly accelerated motion). In doing so, he avoids appealing, as Galileo does in his proof, to the kinds of motions possessed by actual projectiles. As we will see, it is significant that, in his mathematics, Hobbes makes no reference to naturally occurring motions.

4.5 The Nature of Geometrical Representation

It is clear from the previous sections that geometrical representations are a prominent feature of the work of Hobbes, Oresme, and Galileo. This section will compare the roles that geometrical figures play in the kinematics presented by these theorists, as well as the rationales that Hobbes, Oresme, and Galileo offer for the use of such figures.

For both Hobbes and Oresme, determining the ratios of any measurable things entails examining geometrical entities that in some way correspond to the things in question. In particular, there are correlations between certain geometrical objects and certain motions, and by using mathematics to examine the objects we can also determine the characteristics of the corresponding motions. Both Oresme and Hobbes justify these claims by appealing to the manner in which we conceive of quantity. However, there are important differences between the correlations that Oresme and Hobbes posit and the ways in which they justify these correlations.

In the first section of *De configurationibus*, Oresme claims that "[e]very measurable thing except numbers is imagined in the manner of continuous quantity. Therefore, for the mensuration of such a thing, it is necessary that points, lines, and surfaces, or their properties, be imagined."[54] Oresme presents a number of different reasons for maintaining this claim: first, he states that we initially find measure or ratio in geometrical entities, "while in other things it is recognized by similarity as they are being referred by the intellect to them."

Second, there are significant similarities between geometrical objects and the qualities or intensities that Oresme is interested in measuring (Oresme later claims that these similarities also hold between geometrical objects and motions):

> For whatever ratio is found to exist between intensity and intensity, in relating intensities of the same kind, a similar ratio is found to exist between line and line, and vice versa. For just as one line is commensurable to another line and incommensurable to still another, so similarly in regard to intensities certain ones are mutually commensurable and others incommensurable in any way because of their [property of] continuity. Therefore, the measure of intensities can be fittingly imagined as the measure of lines, since an intensity could be imagined as being infinitely decreased or infinitely increased in the same way as a line.[55]

Oresme also thinks that intensities and geometrical objects can be infinitely divided in the same ways. For all of these reasons, he holds that lines and intensities can be similarly manipulated. Performing demonstrations on the relevant lines can therefore teach us about the ratios of the corresponding intensities.

Furthermore, Oresme thinks that it will be more fruitful to examine the lines than the qualities themselves, as "the quantity or ratio of lines is better known and is more readily conceived by us — nay the line is in the first species of continua, therefore such intensity ought to be imagined by lines."[56] Since we first meet with measurement in geometrical objects, this is where we most easily understand ratios. This is borne out by the relative ease with which people can understand propositions illustrated by geometrical examples: "something is quickly and perfectly understood when it is explained by a visible example. Thus it seems quite difficult for certain people to understand the nature of a quality that is uniformly difform. But what is easier to understand than that the altitude of a right triangle is uniformly difform?"[57]

Descartes offers an account of the use of mathematical representations that is similar in some significant ways. In the *Rules for the Direction of the Mind* he suggests that if we are considering a problem it "should be re-expressed in terms of the real extension of bodies and should be pictured in our imagination entirely by means of bare figures. Thus it will be perceived much more distinctly by our intellect."[58] Properly abstracted from superfluous considerations, all problems have to do with comparisons of magnitudes. It will thus be helpful to transfer these problems to the species of magnitude which is most easy for us to conceive: "the real extension of a body considered in abstraction from everything else about it save its having a shape." Nothing else

> displays more distinctly all the various differences in proportions. One thing can of course be said to be more or less white than another, one sound more or less sharp than another, and so on; but we cannot determine exactly whether the greater exceeds the lesser by a ratio of 2 to 1 or 3 to 1 unless we have recourse to a certain analogy with the extension of a body that has shape.[59]

Descartes therefore thinks that picturing extension and shape in the imagination can help us solve problems by allowing us to perceive proportions distinctly.

Both Oresme and Descartes begin by establishing a correlation between physical phenomena (such as motions and qualities) and geometrical objects. They posit that there are important similarities between mathematical and physical entities that allow the former to represent the latter. They then argue that the geometrical representations allow for a much easier and more distinct perception of the proportions which are the objects of their interest.

In line with his materialist philosophy of mathematics, Hobbes attempts to establish an even closer relationship between bodies and mathematical representations. To recall, in his part II account of quantity, Hobbes claims that geometrical objects just are the results of considering bodies in motion in a particular way. Furthermore, quantities must be exposed to the senses by lines, surfaces, or solids. For Hobbes, we must measure things by conceiving of them in terms of geometrical objects because such objects are the *only* means through which we can conceive of quantity, and hence its comparison.

Hobbes's justification for this position lies, of course, in his empiricist conception of how we can come to know quantity: a quantity can only be determined by having its limits set out to our senses. We can only perceive the three dimensions of body, which are identified with line, surface, and solid. If this is the only way of perceiving quantity, there is no need to establish the kinds of similarities between geometrical figures and other magnitudes that feature in the accounts of Oresme and Descartes.

Hobbes's strong sense of mathematical representation leads him to make some controversial claims, most notably about the existence of ratios between magnitudes which were traditionally assumed to be of different kinds. As was discussed in chapter 1, the classical account of proportion denied that there could be ratios between different kinds of magnitudes. So, to recall, Oresme claims a ratio between line and line can always be found which is similar to any given ratio between intensity and intensity, so long as we are "relating intensities of the same kind."

Hobbes appears to challenge this assumption in the first proof of chapter 16, which considers the lengths traversed in uniform motion. In a corollary to this proof, he states that

> Since it was shown that the lengths traversed in uniform motion are as the parallelograms made from the impetuses calculated into the time, that is (on account of the equal impetuses) as the times themselves, it will also be, by exchanging, as time to length so time to length, and in general all the properties and transformations of proportions which we demonstrated and are enumerated in chapter 13 are applicable here. (*DCp* III.16.2; *OL* I, 186)[60]

Hobbes is claiming (among other things) that, given two lengths L1 and L2 and two times T1 and T2, if L1:L2 :: T1:T2, then L1:T1 :: L2:T2. This, of course, involves the formation of ratios comparing time and length directly.

In the *Six Lessons* Hobbes explains why he thinks that this kind of comparison is acceptable. He defines homogeneous quantities as "those which may be compared by [...] application of their measures to one another; so that solids and superficies are heterogeneous quantities, because there is no coincidence or application of those two dimensions" (*SL* I, *EW* VII, 198). Hobbes thus maintains the traditional claim that there cannot be ratios between magnitudes of different dimensions , i.e., between a solid and a surface, a solid and a line, or a line and a solid. However, he goes on to make the more surprising claim that "[h]omogeneous also are line, and the quantity of time; because the quantity of time is measured by the application of a line to a line; for though time be no line, yet the quantity of time is a line, and the length of two times is compared by the length of two lines" (*SL* I, *EW* VII, 198). Homogeneous quantities are those whose measures can be directly compared. However, Hobbes claims that there are only three ways of measuring things: according to the three dimensions of body, or the geometrical objects of line, surface, solid.[61] All quantity falls into these three broad categories, and all magnitudes within each category are homogeneous.

This doctrine creates difficulties within Hobbes's own system. It seems, for example, to generate a circularity in his account of velocity. To recall, Hobbes had stated that equally swift motions are those by which equal spaces are traversed in equal times. If, however, time itself is measured by a line traversed with uniform velocity, this is hardly an adequate definition.

Hobbes was also roundly criticized by others for holding this view. Wallis, not surprisingly, took Hobbes to task on his notion of quantity and measurement, arguing that time and length cannot be directly compared. Hobbes fails to really respond to Wallis's protests, as it clear from Hobbes's account of their exchange: "And to your question, what is the proportion of an *hour* to an *ell*? I answer, it is the same proportion that *two* *hours* have to *two ells*. You see that your question is not so subtle as you thought it" (*SL* I; *EW* VII, 273).

Barrow, who, unlike Wallis, defended a number of Hobbes's positions in his *Mathematical Lectures*, nonetheless criticizes Hobbes's account of homogeneous quantities. As was discussed in chapter 3, Barrow's concept of "primitive" measure bears some resemblance to Hobbes's doctrine of exposition. However, Barrow thinks that there are a number of additional meanings that the term "measure" can have, and that Hobbes's "absurd" view stems from an equivocation between two of these: first, one magnitude can be a measure of another by being a part of it. In this sense, time can be measured by minutes, distance by miles, and so on. On the other hand, "measure" can refer to "*any Thing, which may either conveniently represent or any Way notifie another*."[62] In this sense, a thermometer can be calibrated to measure temperature.[63] Barrow's point is that any quantity can be made to measure another in the second sense, through an appropriate process of calibration. Only homogeneous quantities can be directly compared, and hence measure each other in the first sense. Hobbes's error is to mistake the second kind of measure, by which a line can be used to measure time, for the first.

Hobbes is clearly trying to find a middle ground between the strong classical theory of homogeneous magnitudes, and the completely arbitrary sense of measure that Barrow accuses him of espousing. His attempt relies on the connections which he posits between motions and the geometrical magnitudes that they are said to generate. To recall, Hobbes defines a line (or a single dimension) as the path that a body makes when its magnitude is disregarded. In the case where we observe a particular path being made by the motion of a particular body, a form of natural calibration occurs. There is a non-arbitrary connection between (for example) the amount of time in which the body is moved and the quantity of length which is generated, which might seem to justify a direct comparison between time and length.

Barrow anticipates this kind of argument when he claims that "*heterogeneous Quantities* are sometimes as the *Measures* of others, because they administer a Kind of Knowledge of *homogeneous Measures*."[64] Barrow notes that a certain arc of the equator (determined by the position of the sun and the horizon) is sometimes said to measure the time of day, because the ratio of it to the circumference of the earth is in the same proportion as the time to the entire day.[65] The example is meant to illustrate that the natural relationship between some heterogeneous magnitudes sometimes provides us with knowledge of the proportions that hold between homogeneous magnitudes (as between the time and the whole day). In the same sense, length could be said to be the measure of time, since we can determine the ratio of two times by observing the lengths that a body moving uniformly traverses in that time. However, according to Barrow, this is another improper and equivocal use of the term "measure."

Furthermore, Hobbes's own use of lines in proofs undercuts the possibility of appealing to such a process of natural calibration as a general justification for his doctrine of homogeneous quantities. As one can see in the proofs described above, Hobbes takes arbitrary lines to be representative of the times and velocities of various motions. In doing so, he eliminates any natural connection which might exist between a particular line and a given motion. The sense in which the lines in Hobbes's proofs can be said to measure time is a completely arbitrary one.

Galileo, like Hobbes and Oresme, appeals to mathematical representations and their various aspects in his kinematic proofs. In addition, like these other two thinkers, Galileo takes the proportions between parts of these figures to correspond to proportions between certain features of the motions that they represent.

However, unlike Hobbes and Oresme, Galileo provides no ontological or metaphysical justification for his use of geometrical representations. As we will see in the next section, he sometimes appeals to experience to convince his interlocutors that particular results which have been demonstrated mathematically also apply to motions in the physical world. However, he does not offer any overarching arguments to justify the assumed correlations between figures and motions. This is in line with his overall approach, which is not to give theoretical justifications of his uses of mathematics, but rather to show how well they work in practice.

4.6 The Role of Experience

In this chapter I have been emphasizing the fact that, despite the obvious and acknowledged influence that Galileo had on Hobbes, there are also significant differences in their approaches to the mathematical study of motion. One such disparity is in the role that experience plays in their accounts. It is easy to find Hobbes wanting when he is compared to Galileo in this regard — Brandt, for example, implies that the differences between their approaches show that Hobbes failed the recognize the most significant aspects of Galileo's enterprise. This section will compare the uses to which Hobbes, Galileo, and Oresme put experience in their treatments of motion, and offer an explanation for the approach that Hobbes adopted.

There are two ways in which Hobbes's use of experience can be compared to Galileo's: first, there is a significant difference in the range of motions that they chose to address. At the beginning of his account of naturally accelerated motion Galileo states that he is most interested in finding a definition of accelerated motion that best fits with the behaviour of actual falling bodies:

> For anyone may invent an arbitrary type of motion and discuss its properties; thus, for instance, some have imagined helices and conchoids as described by certain motions which are not met with in nature, and have very commendably established the properties which these curves possess in virtue of their definitions; but we have decided to consider the phenomena of bodies falling with an acceleration such as actually occurs in nature and to make this definition of accelerated motion exhibit the essential features of observed accelerated motion.[66]

Galileo's account concentrates on the kinds of motions that we actually experience in nature.

In marked contrast, Hobbes examines the characteristics of a wide variety of motions, including many which do not occur naturally. For example, in chapter 16 Hobbes not only considers uniformly accelerated motion, but also motion where the velocity increases in triplicate proportion to the time. He concludes his treatment by stating that his method could easily be used to compute the lengths that would be traversed by bodies with velocities in proportions quadruplicate, quintuplicate, and so on to the times of their motions. Similarly, Hobbes extends his analysis of compounded motions to include those of bodies carried by one uniform motion and one accelerated at any of the rates just mentioned. Furthermore, Hobbes does not indicate the special status of any of the motions that he discusses (for example, that uniformly accelerated motion is that which belongs to bodies in free fall, or that the motion of a projectile is compounded of one uniform and one uniformly accelerated motion), treating them all as equally significant. Considering these aspects of Hobbes's kinematics to be at odds with his statement that his mathematics will include only that which is conducive to natural philosophy (*DCp* III.15.1; *OL* I, 176), Brandt claims that "Hobbes both directly and indirectly shows a conspicuous lack of interest in mathematics as applied to the motions occurring in experience [...] This is mathematically warrantable but it seems strange in a philosopher who is driving at a mathematico-mechanical explanation of nature."[67]

There are a number of replies that one can make to Brandt's concern: first, one can question whether the whole of part III of *De Corpore* was really intended as a contribution to natural philosophy. Jesseph points to Hobbes's desire to present a mathematical programme which could be applied to the most general of problems. As such, his materialist mathematics would have to include elements with no immediate physical application.[68] Hobbes's interest in purely mathematical problems clearly represents one reason for the wide range of motions that Hobbes discusses in part III.

However, Hobbes's very general kinematics can also be seen as playing a part of his overall programme of accounting for physical phenomena, as Hobbes promised in the statement quoted above. To recall, Hobbesian physics is a hypothetical science. Any physical phenomenon that we experience may have been caused by any number of invisible motions. Since we can never be sure of the actual cause, the best that we can do is provide a plausible explanatory hypothesis. This attitude towards our ability to explain particular phenomena implies a general agnosticism towards the possibility of making reliable claims about what classes of motions are and are not involved in the production of natural phenomena. Mathematics, since it involves the study of all simple motions, plays the role of providing a sort of menu of the different kinds of motions which could be drawn on in the formation of hypotheses.

Furthermore, there is a similarity between Hobbes's approach and that taken by the practitioners of medieval kinematics. The medieval study of the intension and remission of forms was largely disconnected from the study of nature. Oresme, for example, considers configurations representing a wide variety of qualities and motions, without concern for whether they correspond to any motions found in nature. His interest is purely with discovering the range of configurations that are possible, and examining their characteristics. In a particularly telling example of this medieval tendency, although, as has been noted, the Mertonian practitioners of kinematics demonstrated the mean speed theorem, none of them considered applying it to the motion of naturally falling bodies.[69]

Oresme does, at some points in *De configurationibus*, posit that the configurations of qualities and motions might offer explanations for various physical phenomena. Exhibiting another similarity with Hobbes, however, these explanations are presented as mere hypotheses. For example, after attempting to explain the differing rates at which various substances heat up, Oresme states that "if in similar cases someone wished to assign another cause or causes in addition to this one, I shall not argue about it. It suffices for me that this could sometimes have a place [among the causes]."[70]

A second issue regarding Hobbes and Galileo's use of experience is the role of experience in their proofs. Galileo, claims Brandt, "never loses sight of experience. Experience is his point of departure, and to experience he returns, armed with mathematics, in order to verify his deductions. Without this verification the matter does not interest him."[71] As an example, Brandt notes that Galileo uses experiments at the beginning of the Third Day to verify that his definition of uniformly accelerated motion corresponds to what actually occurs in nature, and to verify deductions that were drawn from the definition. In *De Corpore*, however, "there is no mention whatever of experiments or experience"[72] in Hobbes's accounts of accelerated and compounded motions.

Brandt is right to observe that Hobbes makes no reference to experience or experiment in his kinematics. However, it is worthwhile to present a more detailed account of why Hobbes thought that such references were unnecessary — one which does not just claim, as Brandt's does, that Hobbes was more interested in the mathematical than the physical. I will discuss the uses to which Galileo put experience, focussing on his account of uniformly accelerated motion, then explain how Hobbes's programme attempts to achieve these ends in other ways.

There are two primary uses of experience in Galileo's account of uniformly accelerated motion: first, he employs it to overcome scepticism that his abstractly formulated definition of this kind of motion can be applied to what actually occurs in the physical realm. At the beginning of the Third Day discussion of naturally accelerated motion, Salviati (Galileo's spokesperson) posits that a "motion is said to be uniformly accelerated, when starting from rest, it acquires, during equal time-intervals, equal increments of speed."[73] Salviati initially supports this definition by appealing to the criterion of simplicity: the simplest kind of addition is that which always repeats itself in the same manner, and so this is the kind of increment that we should posit when trying to explain the relationship between time and motion. Although one could question whether the criterion of simplicity really recommends Galileo's over other potential definitions, it purports to provide an abstract reason for accepting Galileo's formulation.

Sagredo responds that he cannot object to the definition itself, since all definitions are by nature arbitrary. However, he claims that he should "nevertheless without offense be allowed to doubt whether such a definition as the above, established in an abstract manner, corresponds to and describes that kind of accelerated motion which we meet in nature in the case of freely falling bodies."[74] The difficulty that he raises is as follows:

> When I think of a heavy body falling from rest, that is, starting with zero speed and gaining speed in proportion to the time from the beginning of the motion: such a motion as would, for instance, in eight beats of the

> pulse acquire eight degrees of speed; having at the end of the fourth beat acquired four degrees; at the end of the second, two; at the end of the first, one: and since time is divisible without limit, it follows from all these considerations that if the earlier speed of a body is less than its present speed in a constant ratio, then there is no degree of speed however small (or, one may say, no degree of slowness however great) with which we may not find this body travelling after starting from infinite slowness, i.e. from rest.[75]

However. Sagredo finds it difficult to imagine that a falling body could experience such extremely slow speeds, given that our senses perceive that it acquires great speed in a very short period of time.

Sagredo is having difficulty seeing how the evidence of sense can be reconciled with abstract reasoning. There seems to be a contradiction between what is mathematically and physically possible, and Galileo's task is thus to show that a given motion *could* exist in nature. He does so by appealing to experience: he claims, through the voice of Salviati, that the same experiment which troubles Sagredo can also be used to show how the initial motions of a falling body must be very slow. He notes that if a heavy body is placed on some kind of yielding material, it will have only a small effect, i.e., it will leave only a small depression. However, the body will exert a greater pressure if it is dropped from a height, due to the greater velocity that it will have when it come into contact with the substance. The effect will grow greater as the height from which the body is dropped, and hence the velocity that it has when it reaches the material, increases. Since the effect becomes greater as the velocity does, and the effect is minimal when the body is dropped from a small height, it seems reasonable to assume that the velocity is also very small at that point.

An important feature of this discussion is Galileo's claim that the experiment can seem to support either Galileo's definition or Sagredo's objection, depending on how it is interpreted. The experiment cannot thus be said to simply confirm or disconfirm the proposed definition. Interpreted with the help of reason, it helps the mind become accustomed to seeing the possibility of certain mathematical properties existing in natural motions: it changes our intuitions about what sorts of mathematical descriptions of nature are possible.

Such a use of experience would not be necessary in Hobbes's mathematical programme. First of all, by taking mathematics to be a science of matter and motion, Hobbes equates the mathematically and the physically possible. If a line is just the path of a moving body, when one describes the generation of a line, one is at the same time describing the possible motion of a body.

Hobbes's confidence in his method may also have led him to think that the kind of conceptual aids that Galileo uses were unnecessary in his own account. One thing that attracted Hobbes to the deductive method, as exemplified by the method of geometry, was its ability to demonstrate the truth of initially implausible conclusions. This is exemplified by Aubrey's famous description of Hobbes's discovery of Euclid:

> He was 40 years old before he looked on Geometry; which happened accidentally. Being in a Gentleman's Library, Euclid's Elements lay open, and 'twas the *47 E. libri I.* He read the Proposition. *By G—*, sayd he (he would now and then sweare an emphaticall Oath by way of emphasis) *this is impossible!* So he reads the Demonstration of it, which referred him back to such a Proposition; which proposition he read. That referred him back to another, which he also read. *Et sic deinceps* that at last he was demonstratively convinced of that trueth. This made him in love with Geometry.[76]

According to this tale, Hobbes became enamoured of geometry because it showed him how the Pythagorean theorem, a theorem he had initially found impossible, could be demonstrated. Although there is reason to doubt the absolute truth of Aubrey's version of events,[77] it captures an important aspect of Hobbes's admiration for the geometric method: Hobbes did think that people could be persuaded of the most surprising results if those results were properly demonstrated. This may explain why Hobbes was confident that the careful reader would have little difficulty accepting the results of his mechanical-mathematical demonstrations. Demonstration alone would show the possibility, and indeed the truth, of his conclusions, even if those conclusions seemed implausible at the outset.

It should be noted that, although Hobbes does not make use of experience in his account of kinematics, he does so elsewhere in his mathematics. So, as we will see, in his discussions of dynamics, Hobbes will often support proposed explanations by showing how well they account for some perceived phenomenon. However, as we will also discuss in the following chapter, Hobbes's account of dynamics is remarkable for its non-demonstrative character. Hobbes was not above appealing to experience to make his results seem more plausible, but only when he was unable to provide an adequate demonstration.

The second use to which Galileo puts experience is a more familiar one. Having established that the definition of uniformly accelerated motion could correspond to motions found in nature, he then has to show that it in fact does. In order to achieve this end, Galileo uses experience to confirm the hypothesis that actual falling bodies possess continuously accelerated motion, as he has defined it. So, in a subsequent part of the Third Day, Simplicio says that he is still doubtful as to whether Galileo's definition corresponds to the motion of falling bodies, and suggests that this might be "the proper moment to introduce one of the those experiments — and there are many of them, I understand — which illustrate in several ways the conclusions reached."[78] In response Galileo presents his famous experiment involving the measurement of the descent of balls down an inclined plane. This experiment, he claims, assured him that falling balls actually experience uniformly accelerated motion. This is a straightforward case of verifying a hypothesis by seeing whether the results predicted by it are those that actually occur in nature.

As has been discussed, discovering if the mathematically demonstrated properties of possible motions actually occur anywhere in nature was not something that Hobbes undertook to do in his kinematics. Given that he made no such hypotheses, it is not surprising that he did not use experiments to confirm them.

4.7 The Medieval And The Galilean in Hobbes's Kinematics

Throughout this chapter, and in parts of the last one, I have tried to point out similarities between the work that Hobbes presents in part III of *De Corpore* and Oresme's kinematics. I have also noted that the medieval aspects of Hobbes's programme are much stronger and wide-ranging than those in Galileo's work. Thus, even if Hobbes adopted elements of his geometrical kinematics via Galileo, he incorporates them into a framework which includes many of the features of medieval kinematics that Galileo had left behind.

This being said, the question arises of how to reconcile the medieval aspects of Hobbes's work with his obvious regard for Galileo. Hobbes clearly admired the results that Galileo was able to achieve by means of his mathematical treatment of motion. Why, then, did not Hobbes adopt more of Galileo's method? Why did he instead embed some of Galileo's most prominent results in his own theoretical framework?

Despite Galileo's successes, Oresme's work had more of the features that Hobbes demanded of a mathematical mechanics. As I have noted, for Oresme, like Hobbes, the mathematisation of motion is part of a broader project involving the quantitative treatment of qualities. This was a project which held no interest for Galileo.

Second, Oresme's work provided a model of an *a priori* kinematics, in that it is not based on or confirmed by our experiences of motion. As I discussed in chapter 1, Hobbes that mathematics, as opposed to physics, is an *a priori* science. For Hobbes, this meant that mathematics should consider the most general properties of matter and motion, without reference to particular facts or experiences. As we have seen, Galileo's kinematics makes extensive use of experience and experiment.

In a related point, both Hobbes and Oresme provide very general accounts of motion, including many motions that we do not perceive in nature. As I have argued, this abstract kinematics allowed Hobbes, among other things, to develop an account of the various kinds of motion that one could appeal to in physical hypotheses. Galileo, on the other hand, focuses his attention on those motions of which we have direct experience.

Finally, it is useful to imagine what the structure of the *Two New Sciences* must have looked like to Hobbes. Hobbes believed that demonstrative sciences should be rigorous: his synthetic method was intended to provide syllogistic demonstrations from self-evident first principles. Although he frequently failed to meet his own standards of clarity and demonstration, he nonetheless held up the geometrical method's embodiment of these properties as an ideal. Galileo paid homage to the deductive ideal,[79] and many of his findings are set out in terms of theorems, propositions, and corollaries. However, his proofs are interspersed throughout with detours, observations, discussions of experiments, and speculations about secondary topics. Galileo's willingness to use all of these tools and approaches was, of course, one of the reasons why his work was so successful. For Hobbes, however, a unified, systematic approach like the one that we find in Oresme's work must have held out more promise as the means to the rigorous mathematics of motion that he was determined to build.

ENDNOTES TO CHAPTER 4

1. *Physics* II.1, 192b22.

2. Brandt (1928, 293).

3. ...primus aperuit nobis physicae universae portam primam, naturam motus.

4. Galileo (1954, 153).

5. Galileo (1954, 153).

6. These are the two possibilites suggested by Aristotle in chapter 8 of the *Categories*. On this debate and the Mertonian contribution to it, see Lewis (1980, 19-32), Grant (1996, 99-100), Clagett (1959, 106), Sylla (1971).

7. Because of this approach to the problem, they came to be called "calculators".

8. On the Mertonians, see Clagett (1959, ch. 4), Lewis (1980).

9. Clagett (1959, 332) claims that Oresme's application of two-dimensional geometry to kinematics was preceded by the work of an Italian Franciscan named Giovanni di Casali. Di Casali did not, however, develop the suggestion of a geometrical mechanics systematically.

10. As was discussed in the previous chapter, the baseline could also be taken to represent the extension of the motion's subject, according to which the motion is said to be "great" or "small" rather than "long" or "short".

11. On Oresme's geometrical kinematics, see Clagett (1959, ch. 6), Oresme (1968).

12. I will be using the term "endeavour" as a translation of the Latin "conatus" (deriving from the verb "conor", meaning to try or attempt). "Endeavour" is the English equivalent which is used in Moleworth's translation of *De Corpore*. Although many commentators have chosen to retain the Latin "conatus", I have found that "endeavour" allows for less awkward translations of the various other derivations of "conor" that Hobbes uses in his dynamics.

13. For an extensive discussion of these concepts see Brandt (1928, 294-316). Bernstein (1980) discusses these concepts and their possible influence on Leibniz.

14. *...conatum esse motum per spatium et tempus minus quam quod datur, id est, determinatur, sive expositione vel numero assignatur,* id est, *per punctum.*

15. Ptolemy (1984, 43).

16. The apparent lack of such a parallax was also a popular objection against the Copernican theory of the universe. If the earth revolved around the sun one would have even greater reason to expect a shift in one's view of the stars, given accepted views of the size of the universe. In response, the Copernicans had to argue that the universe is, if not infinite, at least immensely larger than suggested by the ancients.

17. Heath (1932, 100).

18. ...quod quantitatem nullam habet, sive quod nulla ratione potest dividi (nihil enim est ejusmodi in rerum natura); sed id cujus quantitas non consideratur, hoc est, cujus neque quantitas neque pars ulla inter demonstrandum computatur; ita ut punctum non habeatur pro indivisibili, sed pro indiviso. Sicut etiam *instans* sumendum est pro tempore indiviso, non pro indivisibili.

19. ...sed ita ut neque temporis in quo fit, neque lineae per quam fit quantitas, ullam comparationem habeat in demonstratione cum quantitate temporis vel lineae cujus ipsa est pars.

20. Eodem modo si sint duo motus simul incipientes et simul desinentes, conatus eorum erunt aequales vel inaequales in ratione velocitatum; quemadmodum videmus majore conatu descendere pilam plumbeam, quam laneam.

21. Bernstein (1980, 31n22), for example, states that "[i]t is well-known that Hobbes professed himself a disciple of Galileo's [...] The evidence that he studied *Two new Sciences*, furthermore, is overwhelming, *e.g. De Corpore*, Chapter 16, *Decameron Physiologicum*, EW, VIII, p. 148. Whether Hobbes understood what he read is quite another matter. In the aforementioned chapter of *De Corpore* Hobbes discusses the distances traversed by bodies which acquire equal increments of velocity in equal time, and asserts that in uniformly accelerated motion distances are acquired in proportion to the square of the times. Since one is inclined to think that he followed the kinematics well enough, and also read Galileo closely, the only alternative left is that Hobbes failed to connect the mathematical picture with physical reality."

22. Brandt (1928, 297-8).

23. Brandt states that "if we put the best construction on the passage, we can make the infelicitous "velocities" fit in the following way: Hobbes, when he wrote the passage in question, has confused two different lines of thought. First he has thought of two bodies

of the same mass (weight). When these move in a certain time (the motion of descent is not specially thought of), the effects of the two bodies, their conatus, will depend on their velocity; the body having the greatest velocity will have the greatest effect, and consequently, the greatest conatus. After this his thoughts have taken another direction. He has remembered that the effect of a body moved also depends on its weight, and in accordance herewith he gives the example of the bullet of lead and the ball of wool. Briefly put: it is the concept of momentum, the only concept of "force" Hobbes has, which he has had in mind, but which, by a lapse, has not been presented clearly" (Brandt 1928, 299).

24. Galileo (1967, 202).

25. *...ipsam velocitatem, sed consideratam in puncto quolibet temporis in quo fit transitus.* Adeo ut impetus nihil aliud sit quam quantitas sive velocitas ipsius conatus.

26. *Physics* VI.1, 231b15-16.

27. *Physics* VI.1, 231a 25.

28. On Aristotelian reactions to atomism, see Miller (1982).

29. Kretzmann (1982) is a collection of articles discussing treatments of infinity and continuity in antiquity and the Middle Ages. For an overview of the debate in the 13th and 14th centuries, see de Wodeham (1998), especially Rega Wood's introduction. As Wood notes, the literature at this time assumed that indivisibles were simple, unextended entities. The indivisibilists included those who held that the continuum is composed of a finite number of indivisibles and those who held that the continuum is composed of an infinite number of indivisibles, either mediately or immediately conjoined. Garber et al (1998) provides an overview of various early modern atomist responses to the Aristotelian doctrine of infinite divisibility.

30. Jesseph (1999, 184) points this out.

31. On Cavalieri's method, see Mancosu (1996, 34-64), Jesseph (1989, 225-31) and (1999, 40-42), and Anderson (1985).

32. Quoted in Mancosu (1996, 41).

33. Jesseph (1993, 181-9), (1999, 112-117).

34. Quae enim multiplicata se mutuo possunt superare, homogenea sunt; eodemque genere mensurae mensurabilia; ut longitudines longitudinibus, superficies superficiebus, solida solidis. Quae vero heterogenea sunt, diverso genere mensurae mensurantur. Sin

lineae pro minutissimis parallelogrammis considerentur, ut ab iis considerantur qui methodo demonstrandi utuntur ea, qua Bonaventura Cavalerius in doctrina *Indivisibilium* usus est, habebunt inter se rationem etiam *lineae rectae* et *superficies planae*: poterunt enim tales lineae multiplicatae quamlibet finitam superficiem planam superare.

35. Quoted in Jesseph (1999, 40n56).

36. Jesseph (1999, 179-80).

37. On Wallis's theory of indivisibles, see Jesseph (1999, 42-4, 174-8) and Mancosu (1996, 145-7).

38. On Hobbes's criticisms of Wallis, see Jesseph (1999, 177-88).

39. Velocitas cujuscunque corporis per aliquod tempus moti tanta est, quantum est quod fit ex impetu (quem habet in puncto temporis) ducto in tempus ipsius motus.

40. An English edition of *De Corpore* (Hobbes 1656a) was published a year after the Latin edition. There is controversy regarding whether Hobbes himself is the author of the translation. At the very least, Hobbes seems to have made some changes in the English edition in response to objections from Wallis. The English edition is careful to specify that it is the mean impetus that Hobbes is referring to here, no doubt in response to Wallis's objection that the Latin version does not specify which impetus should be multiplied into the time (for Wallis's objection and Hobbes's response, see *SL* 4: *EW* VII, 269-70).

41. ...cum impetu in totum tempus ducto, sive cum ipsa totius motus velocitate.

42. Again, an adjustment has been made in the English edition, apparently in response to an objection to the Latin. The English reads: "and the quicknesses or impetus applied ordinately to any strait line making an angle with it, and representing the way of the body's motion" (*CB* III.16.1; *EW* I, 219).

43. Si impetus ubique idem sit et sumatur recta quaelibet pro mensura temporis, impetus ad illam rectam ordinatim applicati, designabunt parallelogrammum, quod representabit velocitatem totius motus. Sin impetus a quiete incipiens crescat uniformiter, id est, in eadem semper ratione cum temporibus consumptis, tota velocitas motus erit representata per triangulum, cujus unum latus est totum tempus, alterum impetus maximus eo tempore acquisitus.

44. Galileo (1954, 173-4).

45. Galileo (1967, 227-9).

46. Clagett (1959, 416).

47. Galileo (1954, 174).

48. In motu uniformiter a quiete accelerato (hoc est, ubi impetus continuo crescunt in ratione temporum) est quoque longitudo percursa uno tempore ad longitudinem percursam alio tempore, ut factum ex impetu in tempus ad factum ex impetu in tempus.

49. In the English edition, Hobbes adds the proviso that this only holds if the triangles are similar (or, as he puts it, if the triangles are "like"). If the triangles are "unlike," the proportion of DE to DG and ABI to AFK is compounded of the proportions of ABI to AFK and of BI to FK. In modern notation, DE/DG = ABI/AFK = (AB/AF x AB/FK) (*CB* III.16.3; *EW* I, 222).

50. Galileo (1954, 245).

51. Si mobile feratur a duobus simul moventibus, in dato quolibet angulo concurrentibus, quorum alterum movetur uniformiter, alterum motu a quiete uniformiter accelerato (hoc est, ut impetus sint in ratione temporum; id est, ut ratio longitudinum sit rationis temporum duplicata) donec impetum acquisierit acceleratione impetui motus uniformis aequalem, linea in qua fertur mobile erit linea curva semiparabolae, cujus basis est impetus ultimo acquisitus.

52. Dico per concursum amborum simul moventium fieri, ut mobile percurrat curvam semiparabolicam AD.

53. As Jesseph (1999, 117-25) has shown, in addition to the acknowledged influence of Galileo, Hobbes's account of the composition of motion may have been shaped by the work of Roberval.

54. Oresme (1968, 160-1).

55. Oresme (1968, 167).

56. Oresme (1968, 167).

57. Oresme (1968, 175).

58. *Rules* 14; *CSM* I, 56.

59. *Rules* 14; *CSM* I, 58.

60. Quoniam ostensum est in motu uniformi longitudines percursas esse ut parallellogramma ex impetu ducto in tempora, id est (propter aequales impetus) ut ipsa

tempora, erit quoque, permutando, ut tempus ad longitudinem ita tempus ad longitudinem, et in universum habent hic locum omnes analogismorum proprietates et metamorphoses quas capite decimo tertio enumeratas demonstravimus.

61. And first there is the quantity of bodies, and that of three kinds: length, which is by one way of measuring; superficies, made of the complication of two lengths, or the measure taken two ways; and solid, which is the complication of three lengths, or of the measure taken three ways, for breadth or thickness are but other lengths. (*SL* I; *EW* VII, 193)

62. Barrow (1970, Lecture XVI, 306).

63. This example is Jesseph's (1999, 146).

64. Barrow (1970, Lecture XV, 269).

65. Barrow states that "[a] proposed Arch of the Equator intercepted between the Horizon and Center of the Sun in the Equator is therefore called the *Measure* of the elapsed Time of the Day, because the Knowledge of its Proportion to the entire Circumference of the Equator argues the Proportion of that Time to the entire Day" (Barrow 1970, Lecture XV, 269). He is referring to the fact that when the sun is on the celestial equator, the time of day can be measured by the arc of the celestial equator between the point where the sun rises and the sun's current position. This arc, as a fraction of the whole equator, is equal to the time since sunrise, as a fraction of 24 hours. If the sun is not on the equator, the time of day can be measured (somewhat imprecisely) by measuring the arc between the point where the sun rises and the intersection of the equator and the circle running through the celestial pole and the centre of the sun.

66. Galileo (1954, 160).

67. Brandt (1928, 320).

68. Jesseph (1999, 111).

69. Grant (1996, 151). Grant claims that this did not occur until the middle of the sixteenth century.

70. Oresme (1968, 233).

71. Brandt (1928, 317).

72. Brandt (1928, 318).

73. Galileo (1954, 162).

74. Galileo (1954, 162).

75. Galileo (1954, 162).

76. Aubrey (1898, I: 332).

77. Jesseph (1999, 5n7), Grant (1996, 111).

78. Galileo (1954, 178).

79. Galileo (1954, 6).

CHAPTER 5

DYNAMICS AND THE LIMITS OF HOBBESIAN GEOMETRY

In the previous chapter, we began our discussion of Hobbes's mathematical mechanics. In part III of *De Corpore* Hobbes brings together the two dominant elements of his natural philosophy: mathematics and motion. In his discussion of the spatio-temporal effects of motion, these two components come together fairly well. As we have seen, there are many difficulties with his account. Chapter 16 does, however, present a coherent, quantitative analysis of certain types of motion.

In his descriptions of the subject matter of mathematics, Hobbes had also promised to provide an account of the effects of moving bodies on other bodies. Given his insistence that all changes in the world must be explained by means of such interactions, this account is an essential part of his system. Unfortunately, in his dynamics Hobbes fails to unite mathematics and mechanics. His discussions of this topic are rarely quantitative and frequently riddled with inconsistencies.

In this chapter I will offer at least a partial explanation for this failure of the Hobbesian project. In the first section, I will discuss the many functions that Hobbes's endeavour concept plays in his descriptions of the interactions amongst bodies. In the second, I will examine Hobbes's attempt to provide a quantitative account of these interactions. I will argue that the limitations of Hobbesian geometry make such an account impossible. In the final section, I will look at Hobbes's treatment of circular motion. His account will be compared with those of Galileo and Descartes, particularly with regard to their various uses of quasi-inertial principles.

5.1 The Many Uses of Endeavour

As we have seen, Hobbes's ontology is spare in the extreme. His world is completely filled with matter. In and of themselves, bodies are all the same — they can be distinguished only by the variety of their internal motions. (*DCp* III.21.5; *OL* I, 263-4). Correspondingly, all change in the world must be brought about by the motions of bodies and their parts.

In this system, the concepts of dynamics, like all others, must be defined in terms of bodies and motion. This is evident in the first three dynamic principles of part III, which Hobbes introduces immediately after the definitions of "endeavour" and "impetus." To recall, an endeavour is a point motion, and impetus the magnitude or instantaneous velocity of an endeavour. "Resistance" is then defined as "*upon the contact of two mobiles, an endeavour contrary to an endeavour; whether wholly or in part*" (*DCp* III.15.2; *OL* I, 178).[1] Whether the endeavours are wholly or only partly contrary to each other depends on the angle at which they meet. If one of the bodies in contact succeeds in displacing either the whole or some part of the other, the former is said to press the latter: "*of two mobiles we say that the one presses the other, when by means of its own endeavour one of them brings it about that the other or part of it yields its place*" (*DCp* III.15.2; *OL* I, 178-9).[2] Finally, "*we say that a body pressed and not moved away restores itself, when, the pressing body have been removed, on account of the internal constitution of the body itself its moved parts return each to its own place*" (*DCp* III.15.2; *OL* I, 179).[3] These definitions will be discussed in more detail below. What I would like to note here is that they account for all the ways, within Hobbes's framework, that one body can have an effect on another: a body can come into contact with another body, the latter body can resist that contact, and, the contact having ceased, the pressed body can restore itself. Furthermore, all three means of interaction are described in terms of actual, if very small, motions.

As is suggested by the above definitions, the concept of endeavour is the most prominent device in Hobbes's dynamics. It is this concept, rather than that of impetus, or the magnitude of endeavour, which appears with by far the most frequency in the chapters of *De Corpore* that deal with dynamics. This has led some commentators to suggest that the concept of endeavour has an inherently dynamic aspect which is lacking in the notion of impetus.[4] In the previous chapter, I argued that the chapter 15 discussions of endeavour and impetus offer no evidence for this claim. I will also be disputing this view in the following two sections: first, I will describe the various forms of motion that Hobbes refers to as "endeavours." As we will see, he applies this term to motions which are, at least some of the time, imperceptible. Such motions play an important role in *De Corpore*, since they allow Hobbes to explain in mechanical terms phenomena in which no apparent motions are involved. The endeavour concept is also used to describe the tendencies that bodies have to certain kinds of motions, and sometimes includes an element of directionality.

In the second section, I will discuss Hobbes's idea of force, which is an attempt to quantify, using the concept of impetus, the effects of moving bodies. This attempt will, however, fail: first of all, because it does not allow for a method of geometrical representation that would encompass all the various forms of endeavour that Hobbes posits. In addition, the representations of force that he eventually settles on are incapable of capturing the notion of directionality. The prominence of the endeavour concept is not due to its possessing an inherently dynamic aspect, but to difficulties in developing an analysis in which impetus, its quantitative counterpart, could have a place.

In this discussion that follows, I delineate and describe five uses of the endeavour concept. This is not meant to imply that these aspects of the concept always appear in isolation (for example, the term "endeavour" is sometimes used to refer to an imperceptible motion in a particular direction). However, they can and should be distinguished, since they serve different purposes in Hobbes's system.

The first use of endeavour is to describe the propagation of motions through media. Such propagation of motion occurs when "any body, endeavouring in opposition to [another] body, moves it, and this moved body moves likewise a third, and so on" (*DCp* III.22.3; *OL* I, 272).[5] This kind of motion is not possessed by a single body, but is transferred from body to body through a medium. There is no reason why this definition could not apply to a perceptible motion (it would seem, for example, to be applicable to waves moving through a body of water). However, Hobbes mostly uses the concept to refer to imperceptible motions.

Propagated imperceptible motions are a prominent feature of Hobbes's physics, appearing in the account of sense at the beginning of *De Corpore*'s fourth part. Sensation, like all other phenomena, must be explained in terms of motions — in this case, those of the organs of sense. When such an organ is touched and pressed, the resulting motion is instantaneously propagated to the organ's innermost part. The organ's reaction or resistance (which is caused by its own "internal natural motion") is the resulting phantasm or idea, which we perceive as external because it is an endeavour outwards. Sense, therefore, "*is a phantasm remaining for some time made by the reaction from an outwards endeavour of the organ of sense, which is generated from the inwards endeavour of the object*" (*DCp* IV.25.2; *OL* I, 319).[6]

Second, the concept of endeavour is used to characterize what Hobbes calls the "beginning of motion". This use is particularly prominent in Hobbes's account of appetite and aversion. Pleasure and pain arise from the helping and hindering (respectively) of vital motion, i.e., the motion of the blood that originates with the heart. The beginnings of those movements that we make in order to increase pleasure and avoid pain are called appetite and aversion: "appetite and aversion or avoidance of the spirit [*animi*] are the first endeavours of animal motion" (*DCp* IV.25.12; *OL* I, 332).[7] In this case, the endeavour is the very small initial part of a movement towards some pleasant or away from some unpleasant thing.[8] This was one of the first uses to which Hobbes put the endeavour concept: in the *Elements of Law* appetite and aversion are also defined as endeavours or the beginnings of certain internal motions.[9]

Thirdly, the term "endeavour" sometimes refers to the imperceptible motions possessed by bodies that appear to be at rest. For Hobbes, the state of rest has no efficacy: "*rest is inactive [inertem] and devoid of all efficacy [efficaciae]; motion alone is that which both gives motion to resting things and takes it away from moving things*" (*DCp* III.15.3; *OL* 180).[10] A body that was truly at rest would have no power to change the motion of a body with which it came into contact.[11] Since apparently resting bodies clearly do have such effects, Hobbes posits that such bodies actually possess endeavours (and can hence be said to resist and press according the definitions cited above). For example, Hobbes defines weight (*pondus*) as "the aggregate of all the endeavours, by which the individual points of a body, which presses the beam [of a scale], in straight lines mutually parallel to each other; the pressing body itself is called the ponderans" (*DCp* III.23.1; *OL* I, 287).[12] Even when the scale is balanced, and the body appears to be at rest, its imperceptible endeavours continue to exert an influence. In another chapter of part III, Hobbes describes the phenomena of a crossbow which, having being bent for a long period of time, can only be returned to a straight posture by a great deal of force. His explanation is that the endeavours possessed by the crossbow have, over time, become accustomed to a new kind of motion. In this case, the imperceptible endeavours resist the force of someone trying to straighten out the crossbow (*DCp* III.22.20; *OL* I, 284-85).

In addition to those that will be discussed in future sections, there is an immediate problem with these uses of the endeavour concept: it is not at all clear how a number of the endeavours that Hobbes posits perpetuate themselves. To return to our example of the crossbow which appears to be at rest, if the endeavours by which it resists external forces are always present, how are they continually renewed? If, on the other hand, they are somehow caused by contact with the body being resisted, how does this occur? One might expect Hobbes to account for the perpetuity of such endeavours by making the imperceptible motions circular. However, it would be difficult to explain how such a motion could be in continual opposition to all external forces. Furthermore, some of Hobbes's uses of the endeavour concept would preclude such an explanation. For example, appetite is described as an endeavour outwards. Since many appetites persist over time, their constitutive endeavours must have a constant outward direction over the same period of time. The same argument could be made with regard to the outwardly-directed endeavours that Hobbes identifies with our phantasms.

It is likely that Hobbes intended such explanations to be the task of physics: mathematics identifies when an imperceptible motion must be at work, while part IV of

De Corpore would describe possible causes for those motions. Unfortunately, if this was his intention, Hobbes does not always follow through in part IV with the appropriate explanations. In some cases he does: part IV does, for example, contain a mechanistic explanation of heaviness. Briefly, Hobbes hypothesizes that when heavy bodies begin to descend towards the earth air rushes in behind them to prevent the formation of a vacuum. The force of this air thrusts the heavy bodies towards the earth (*DCp* IV.30.2; *OL* I, 415-17). However, the difficult case of the resistance of an apparently resting body is not addressed in Hobbes's physics.

There are two further aspects to Hobbes's dynamical applications of the endeavour concept: first, an endeavour is frequently used to account for a body's tendency or apparent effort to move in a particular way. These tendencies are sometimes manifested in perceivable motions. For example, people can, by jumping, acquire an endeavour upwards. This allows them to acquire a temporary motion in this direction, although it is quickly extinguished by the effects of gravity (*DCp* IV.30.13; *OL* I, 424). On the other hand, as in the case of a heavy body balanced on a scale, the tendency (in this case an endeavour to move downwards) is often imperceptible.

Hobbes presents no explicit argument for equating tendencies to motion with actual motions. It does, however, follow from his foundational belief that all effects must be explained by reference to bodies in motion. As we will see, Hobbes's analysis of tendencies in terms of endeavours put him at odds with Descartes. Their differences on this subject become particularly evident in a comparison of their optical theories, and will therefore be taken up in chapter 6.

Finally, Hobbes's notion of endeavour often includes an element of directionality. As we have seen, when discussing heavy bodies Hobbes often refers to their downwards endeavours (although they can acquire temporary endeavours in other directions). The directional aspect of endeavour is also apparent in the following principle from chapter 15: "[a]nd if while a mobile is borne in any line by a motion which is made from the concourse of two movents, at that point, when it is first abandoned by the force of one of the movents, its endeavour changes into an endeavour along the line of the other movent" (*DCp* III.15.5; *OL* I, 182).[13]

Again, *De Corpore* includes no argument for attributing direction to endeavour. It is not surprising, however, that Hobbes would make this assumption, since endeavours are motions. Every motion is the transfer of a body from one place to another. and hence has direction. Hobbes makes such an argument in his correspondence, where he debated with Descartes about whether determination or directionality should be treated as something separate from motion. As I will discuss in chapter 6, in his optics Descartes claims that the force of a motion can be distinguished from the motion's determination. Hobbes argues. on the other hand, that as every man is an individual, despite the fact that we use the common name "man,"

> in the same way. therefore, every motion is either this. or that motion. in other words. a motion *determined* by the limits of its start and finish. So just as Socrates and man are not two men, nor two things, but one man described by two names (since it is the same thing which is named 'Socrates' and named 'man'), in the same way 'motion' and 'determined motion' are one motion, and the same thing under two names.[14]

Although we use the term "motion" to refer to various motions. we should not lose sight of the fact that each of those motions has a particular determination, and hence direction.

We have thus seen that Hobbes's dynamics makes wide use of his concept of endeavour. It is this notion, rather than the quantitative impetus. which appears most frequently in his discussions of dynamics. In the next section. we will describe Hobbes's attempt to mathematise his dynamics, and discuss the attendant difficulties. These difficulties forced him to fall back on non-quantitative concepts.

5.2 Force: Mathematising the Effects of Motion

As we have seen, In part III of *De Corpore* Hobbes describes numerous interactions between bodies. However, his vague discussions do not explain how such interactions can be quantitatively analysed. An attempt at such an analysis is suggested by his concept of force (*vis*), which is first introduced in *De Corpore*'s second part. To recall, Hobbes there defined greater, lesser, and equal motions in terms of the velocities of the bodies being compared as multiplied into their respective magnitudes. Following this definition, Hobbes goes on to say that "the magnitude of motion which we just said was computed in this way, is precisely that which we generally call *force*" (*DCp* II.8.18: *OL* I, 102).[15]

Hobbes picks up on this suggestion when, in this course of introducing his new principles at the beginning of part III, he defines force as "*the impetus multiplied either into itself, or into the magnitude of the moving body, by which the moving body acts more or less upon the body which resists*" (*DCp* III.15.2; *OL* I, 179).[16] This definition represents the dynamic aspect of Hobbes's programme for the mathematisation of motion. Force, for Hobbes is an essentially quantitative concept: it tells us how to quantify the ability of one body to either move, or resist the motion of, another. This quantity is identified with the body's magnitude times its velocity.

Hobbes's notion of force, or the magnitude of motion, should not be confused with that of momentum, or the product of mass and velocity. When Hobbes refers to a body's magnitude, he simply means its extension.[17] Furthermore, Hobbes's definition of force does not include an element of directionality. Again, I will be arguing that this aspect of his endeavour concept was not transferred to the idea of force because it would be impossible to represent mathematically within the Hobbesian framework.

It is difficult to see what is meant by the first disjunct in Hobbes's definition of "force." Hobbes seems to be suggesting that force can vary as the square of the velocity. However, this is a view that is consistently contradicted by what Hobbes says elsewhere. It may be that Hobbes, was trying to account for the force of an individual point. The magnitude of a single point is so small that it cannot enter into computations, including that of force. As we will see, Hobbes claims that the force of a single point can be exposed by a line representing the velocity of that point over some period of time. However, that velocity must, in some sense, be made up of numerous impetuses. Perhaps in the above passage Hobbes is suggesting that the force of a point can be estimated by considering that point's multiple impetuses.

The latter part of Hobbes's definition reflects an idea of force that was common in the seventeenth century.[18] Most notably, it corresponds to Descartes's use of the idea of a body's quantity of motion, which functions, in Gabbey's words, as "the criterion of its dynamical supremacy relative to other bodies with which it interacts."[19] As is evident from the rules of impact that Descartes presents in the *Principles of Philosophy*, calculating quantity of motion is, at least in most situations, the means by which we measure the force to move or resist movement that a body possesses. This quantity is equal to size times speed:

> For although motion is only a mode of the matter which is moved, nevertheless there is a fixed and determined quantity of it; which, as we can easily understand, can be always the same in the universe though there may be more or less motion in certain of its individual parts. That is why we must think that when one part of matter moves twice as fast as another twice as large, there is as much motion in the smaller as in the larger; and that whenever the movement of one part decreases, that of another increases exactly in proportion.[20]

One of the fundamental principles of Descartes's physics is that God created the universe containing a certain amount of motion, and God's immutability assures us that that amount must remain constant. Descartes took this principle to entail the conservation of motion in particular interactions amongst bodies. As is suggested by the above passage, the quantity of motion that is preserved is measured by size times speed.

Despite these similarities, the idea of magnitude or quantity of motion had some distinct features in Hobbes's system. Most notably, on Hobbes's account the magnitude of a body's motion is equal to the aggregate of the motions possessed by each of the points that make up its magnitude. This conception of force, which I will refer to as an "additive" conception, is reflected in two of the new theorems that Hobbes introduces in chapter 15: first, he claims that "*a resting point, to which another point with an impetus* *however small is brought into contact, will be moved by that impetus*" (*DCp* III.15.3; *OL* I, 179).[21] Hobbes's reasoning is that if the resting point is not moved by the given impetus, neither will the point be moved by any multiple of that impetus — since any multiple of nothing is nothing. If this were the case, it would be impossible that the body at rest would ever be moved.

Furthermore, Hobbes argues that if a point with however small an impetus should come into contact with a body at rest, the resting body must yield to some degree, regardless of how hard it is. He argues similarly that if the resting body did not yield to the given impetus, nor would it yield to the action of any number of points with equal impetuses:

> for since all these points act equally, if one of them should have no effect, likewise the whole aggregate together will have so many times no effect, as there are accumulated points, that is, none. And by consequence there would be some bodies so hard that they could be broken by no force, that is, a finite hardness or a finite force that would not yield to an infinite one, which is absurd. (*DCp* III.15.3; *OL* I, 180)[22]

Both of these theorems assert that even the smallest impetus must have an effect, since any larger impetus must be considered a multiple of that initial effect.

When discussing the propagation of motion, Hobbes presents other arguments that support this conclusion. He notes that a very small object, such as a grain of sand, can be placed at a sufficiently great distance that it will not be visible. Because all endeavours are propagated to an infinite distance,[23] it must nonetheless have an effect on the organs of sense. Furthermore, if a sufficient number of grains were added to that one, at some point the aggregate would become visible. This would be impossible, Hobbes thinks, if each part of that aggregate did not act on the organs of sight (*DCp* III.22.9; *OL* I, 278-9).

These arguments have interesting repercussions for Hobbes's theory of perception. Before presenting these arguments, he states that "[n]ow although an endeavour of this sort, perpetually propagated, does not always appear to the senses as if it is some motion; nevertheless it appears as an action, or the efficient cause of some change" (*DCp* III.22.9; *OL* I, 278-9).[24] We do not directly perceive all of the motions that work on our organs of sense. We can, however, by considering the effects of which they are a partial cause, ascertain that they must exist. This explains how Hobbes can refer to certain motions which act on the sense organs as "imperceptible," despite the fact that they are partial causes of our sense perceptions.

Descartes was committed neither to the additive analysis of force, nor to the idea that any moving bit of matter, however small, will have an effect upon impact with another body. In fact, he challenges this principle in a letter to Hobbes via Mersenne:

> Further, his assumption that 'that which does not yield to the slightest force cannot be moved by any force at all' has no semblance of truth. For who can believe, for example, that a weight of 100 pounds on a pair of scales yields ever so slightly to a weight of one pound placed on the other arm of the scales, because it does yield to a weight of 200 pounds?[25]

Hobbes responded with an argument similar to the one in *De Corpore*: "if the slightest force does not cause the thing struck by it to yield, at least by a tiny amount, then twice that force will not suffice to do so; for twice nothing is nothing, and will remain nothing however many times you multiply the force."[26] Descartes's error is to assume that, on Hobbes's account, the whole body which is struck must yield. Although the one pound weight will not cause the whole 100 pound weight to move, it will lower slightly that part of the scale's arm with which it is in contact.

Leibniz also comments, though more favourably, on this aspect of Hobbes's doctrine. In a letter to Hobbes, he comments that many of Hobbes's principles have been misused because of ignorance as to how they should be applied:

> Take, for example, the general principles of motion: 'nothing can begin to move, unless it is moved by another thing; a body at rest, however large, can be made to move by the slightest motion of another body, however small'. If anyone applied those principles inappropriately to the physical objects we perceive, without preparing the minds of his audience by

> showing that many things which seem to be at rest are imperceptibly moved, the common people would pour scorn on him.[27]

That both Descartes and Leibniz commented on this principle suggests that it was one of the more controversial amongst Hobbes's teachings on mechanics.

Hobbes may have been influenced by the atomists in developing his additive analysis of force. The atomists held that all atoms possess their own natural and perpetual motion. As Charleton[28] claims, in his Christian version of the doctrine,

> *at their Creation, God invigorated or impregnated [atoms] with an Internal Energy, or Faculty Motive, which may be conceived the First Cause of all Natural Actions, or Motions,* (for they are indistinguishable) *performed in the World...their internal Motive Virtue necessitates their perpetual Commotion among themselves, from the moment of its infusion, to the expiration of Natures lease.*[29]

Despite the fact that our senses tell us that many larger bodies are at rest, these natural motions continue when atoms assemble to form "concretions." Furthermore, the perpetual agitation of atoms is the source of the motions of these compounded bodies.[30]

There are some obvious parallels between this theory and the one that Hobbes presents in *De Corpore*. Hobbes, like Charleton, holds that the motions of bodies are derived from the motions of their constitutive parts (be they points or atoms). Furthermore, both think that the motions of these tiny parts are frequently imperceptible to the senses. Hobbes does not say whether or not every point of matter is perpetually in motion. However, this is a likely consequence of his belief in the infinite propagation of even the smallest motion. The most significant difference between these doctrines is, of course, that Hobbes's principles demand that such motions be explained by means of external, mechanical causes. As I have noted, *De Corpore* fails to provide many of these explanations. Hobbes may have adopted the atomists' ideas about motion without fully working out an account of their causes.

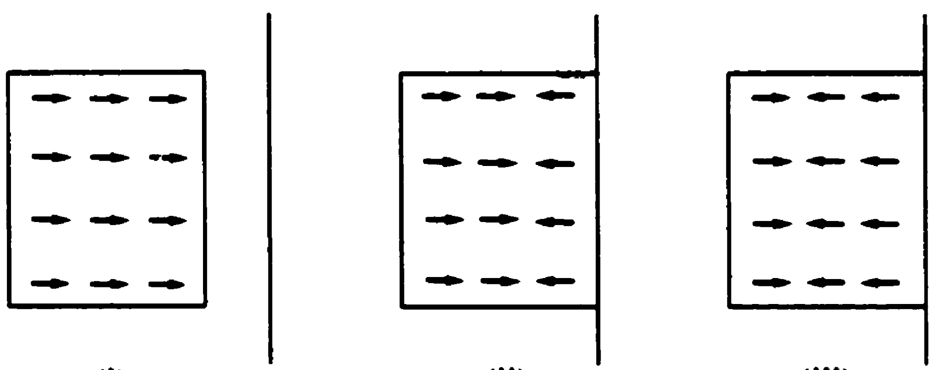
As we have seen, although quantity or magnitude of motion was not a new idea, Hobbes presents his own variation on the concept. He also adapts to fit his own system a common seventeenth-century understanding of how we should analyse interactions between bodies. Hobbes subscribes to what Gabbey calls "the contest view" of dynamic interaction.[31] According to this account, interactions between bodies are contests between opposing forces, in which the body with the greater force will be the winner and the body with the lesser force the loser. Hence, when listing the various ways that we can reason about motions, Hobbes says that "[s]ometimes motion is considered in relation to the effect alone which the moving body has upon the mobile, and then it is usually called *momentum*. But *momentum* is the excess motion of the moving body over the motion or endeavour of the resisting body" (*DCp* III.15.4; *OL* I, 181).[32] This suggests that the effect that a moving body will have varies with the amount of motion that it has over and above that possessed by the moved body. Before this vague suggestion could be useful, however, Hobbes would clearly have to specify how a number of factors besides magnitude of motion can have an effect on the final result: is motion, or something else, conserved during the interaction? does it matter if the bodies involved are heavy or light, hard or soft? what if the bodies are moving in the same, or different, directions?

This apparently common understanding of force is also, however, influenced by Hobbes's additive notion of force. In the following passage, for example, he attempts to account for why differences in magnitude and speed influence the ability to bring about change in other bodies:

> Upon a body, which resists motion, the force of the movent (the magnitude being equal) of that which is moved more swiftly is greater than [the force of] that which is moved more slowly: likewise the force of the greater body (the velocity being equal) is greater than that of the lesser. For to the extent that (the magnitude being equal) the movent presses upon the mobile with a greater velocity, it impresses a greater motion on it. And to the extent that (the velocity being equal) the movent presses with a greater bulk [*mole*] upon the same point, or the same part of the mobile, it loses less of its velocity; for the very reason that the resisting body acts on only that part of the movent which it touches: therefore it weakens the impetus

> of that part alone, while meanwhile the parts not touched proceed and preserve their whole force, until those parts should come into contact, at which point their force has some effect of its own. Therefore, for example, for battering, the longer piece of wood works more upon a wall than the shorter with the same thickness and velocity, and the thicker works more than the thinner with the same length and velocity. (*DCp* III.15.8; *OL* I, 183)[33]

A larger body will be more effective than a smaller one because a lesser proportion of its parts will come into contact with the resisting body at the first moment of impact. The force of those points that do not experience direct contact will continue for some further, though undoubtedly very small, amount of time. The underlying assumption is that each



(i) (ii) (iii)

Figure 5.1

point has its own motion, and hence each must press upon another body if its motion is to change. Consequently a body loses its force by a kind of domino effect: first the parts that are directly touched are slowed or stopped. These in turn reduce the endeavours of the points immediately adjacent to them, and so on (see figure 5.1).

This analysis has interesting consequences for the influence that the figures of bodies would have on their interaction. Take (in figure 5.2) two bodies of equal magnitude and velocity, A and B, each in turn coming into contact with the surface of another body C (that surface being larger than the contact surfaces of either A or B). Both will exert the same force, since they possess the same magnitude of motion. However, if

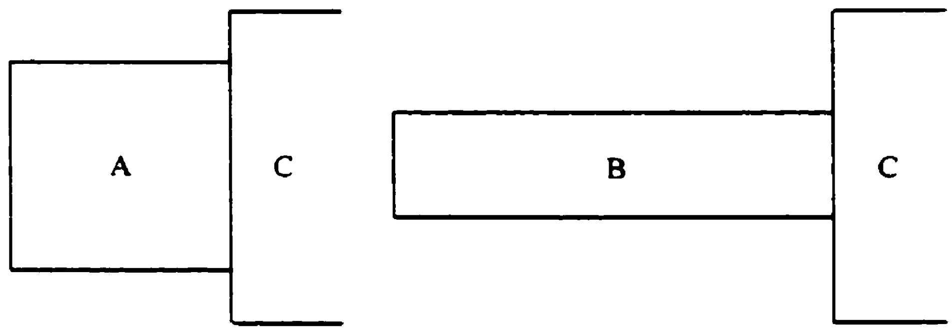


Figure 5.2

the surface of A that comes into contact with C has a greater area than the corresponding surface of B, it will presumably take longer (if only by an imperceptibly small time) for B to bring about the same effect as A. A will exert a larger portion of its force immediately upon contact with C, while in the case of B the influence of a larger proportion of its points will not be felt until they have themselves pressed points whose motion has already been abated.

Hobbes's idea of force is also notable for its compatibility with another conception of quantity of motion. As was mentioned in the previous two chapters, the practitioners of medieval kinematics shared with Hobbes the idea that motions could be described as "great" or "small" in accordance with the extension of their subject. To recall, in addition to representing the total velocities of motions by means of geometrical figures, the medievals also suggested that a body's motion could be represented by the area of a figure with a base representing the body's magnitude, and lines erected perpendicular to each point in the base representing the velocity of a point of the body's magnitude. The quantity of motion, as represented by the area of the figure, is compounded from the velocities of all the body's constitutive points, as represented by the perpendicular lines. On the face of it, this would seem like an entirely appropriate way of representing geometrically Hobbes's additive account of force.

There are further reasons why one would expect this mode of representation to be attractive to Hobbes. It would be a natural extension of the system of representation which he adopted with such enthusiasm in his kinematics. Furthermore, he uses a similar vocabulary when defining quantity of motion and total velocity, defining the latter (to recall) as the impetus *multiplied into* the time of the body's motion.

However, Hobbes's own principles would have prevented him from adopting this form of geometrical representation in his dynamics. In Hobbes's kinematics, total velocity is represented by a figure with a baseline representing the duration of the motion, and lines erected perpendicular to the baseline representing the instantaneous velocity at various points in time. In the representation of total velocity, two quantities (time and velocity) which are both exposed by lines, and are hence homogeneous, are multiplied into each other. If, however, a similar figure were constructed to represent quantity of motion, the baseline would represent the magnitude of the moving body, and the lines erected perpendicular to the baseline would represent the instantaneous velocity of each of the body's parts. This would involve both velocity and three-dimensional magnitude being represented by lines. To recall, Hobbes claims in part III of *De Corpore* that each kind of quantity must be exposed by means of one (and only one) of the three types of geometrical object: line, surface, or solid. Since magnitude is a three-dimensional quantity, it must be exposed by a solid. Its quantity is thus heterogeneous with that of a line, by which it would need to be represented if the medieval mode of representation were to be adopted.

Hobbes's attitude towards the principle that heterogeneous magnitudes should not be compared was sometimes ambivalent. As I discussed in the previous chapter, Hobbes sometimes describes a figure as being made up of indivisibles, a position that seems to involve the direction comparison of points and lines. Furthermore, Hobbes claims that magnitudes of any kind in a ratio can be represented by a ratio between lines. In his account of the composition of ratios, Hobbes sets out to demonstrate that "[i]f there should be any three magnitudes, or any three things having some ratio between themselves, as three numbers, three times, three degrees, etc., the ratios of the first to the second, and the second to the third, taken together, are equal to the ratio of the first to the third" (*DCp* II.13.13: *OL* I, 140).[34] He demonstrates this proposition by considering lines, stating that "any ratio can be reduced to a ratio between lines" (*DCp* II.13.13; *OL* I, 140).[35]

In both these cases, however, there are ways for Hobbes to avoid, or at least temper, the accusation that he is allowing the direct comparison of heterogeneous magnitudes. As we have seen, Hobbes claims that his indivisibles must be thought of as very thin parallelograms, and hence homogeneous with continuous magnitude. In the above passage from his account of ratios, Hobbes does not say that the ratio between magnitudes of different kinds can be exposed by lines. He seems to be claiming that for any ratio there is an equivalent ratio between two lines, and that we can learn about the former by manipulating the latter. This would not be dissimilar from the justification that, as we saw in the previous chapter, Oresme presented for the use of geometrical representations. Hobbes could argue that such a representation does not involve the comparison of heterogeneous magnitudes, but merely the construction of a ratio equal to the ratio between the two homogeneous magnitudes being considered. This is not say that there is no tension between Hobbes's work on indivisibles and ratios and the principles of the incommensurability of heterogeneous magnitudes. However, Hobbes apparently did not find the tension so significant that it could not be overlooked.

He could not, however, have avoided the difficulties that would have been entailed by the representation of force or quantity of motion in the manner described above. Accordingly, in later parts of *De Corpore* Hobbes assumes that the force of a moving body will be exposed by means of both a solid and a line, with the solid exposing the body's magnitude and the line its velocity. However, this form of representation brings its own problems. In particular, it has problematic consequences for the analysis of dynamic interactions between bodies perceived to be at rest and those perceived to be in motion. To recall, on Hobbes's account a resting body has no efficacy — it can neither produce motion in nor remove it from other bodies. Hobbes thus explains the effects of apparently resting bodies by positing that they possess imperceptible endeavours. According to Hobbes's initial definition, the force of such a body would be measured by multiplying its magnitude into its imperceptibly small velocity. In order to evaluate the momentum that some moving body possessed in an interaction with a resting body, and hence the effect that it would have on the resting body, one would have to compare the magnitude of its motion with the force of the resting body.

Apart from the practical difficulties which seem inherent in this proposal, for Hobbes it also turns out to be mathematically impossible. This becomes clear in a discussion of the differences between thrusting (*trusio*) and percussion (*percussio*). In chapter 15 Hobbes discusses the various features of motions. One such feature is the position of the movent with respect to the mobile: the motion is called pushing (*pulsio*) when the movent precedes the mobile, pulling (*tractio*) when it causes the mobile to follow it. A further distinction is drawn between two kinds of pushing: when the motions of the movent and the mobile begin together, it is called thrusting (*trusio*). If the movent begins its motion before that of the mobile, it is called percussion. (*DCp* III.15.4: *OL* I, 181)

At a later point in *De Corpore*, Hobbes argues that despite there being only this one difference between thrusting and percussion, their effects are nonetheless so different "that it does not seem possible to compare their forces with one another":

> I say that by any given effect of percussion, for example, by the stroke of a beetle[36] of any weight, by which a stake is driven with a given power into earth of a given tenacity, to determine by how much weight, without the

> stroke, and in what time the same stake would be driven just as far into the same earth, seems to me to be if not impossible, yet very difficult. Now the cause of the difficulty is, that the velocity of the percutient (*percutientis*) seems to be compared with the magnitude of the ponderant (*ponderantis*). But velocity, which is estimated from the length of space, must be regarded according to one dimension; but weight, which we judge according to the dimension of the whole body, is as a solid. But there is no comparison of a solid and a length, that is, a line. (*DCp* III.22.16; *OL* I, 282-3)[37]

The problem that Hobbes is addressing here is relatively clear:[38] in dynamic interactions, the effects of a striking body cannot be compared with those of a static weight. This problem is one of comparing the relative effects of a static and a moving force on some third body. However, the problems that Hobbes encounters in trying to describe this situation would also apply to describing a direct interaction between a moving body and one at rest.

Hobbes's rationale for thinking that this comparison is difficult obviously stems from the incommensurability of the striking body's velocity with the magnitude of the thing being struck. To recall, Hobbes defines weight as the aggregate of the downward endeavours possessed by a body pressing down on the beam of a scale (presumably the body would have the same weight if it were pressing down on something else).[39] If we take this definition seriously, it seems that the force of the ponderant and the percutient should be directly comparable, since they could both be measured in terms of magnitude times velocity (although, in the case of the ponderant the velocity would, of course, be imperceptible).

However, in the above passage, Hobbes suggests that the force of the ponderant is due to its magnitude alone. One way to make sense of this would be to assume that the ponderant's motions are too small to be exposed, and thus the only way that its force can be represented is by an exposition of its magnitude. On the other hand, since the velocity of the percutient is perceptible, its exposition will be included in the exposition of the percutient's total force. A comparison of these two forces is therefore impossible, because of Hobbes's views regarding the incommensurability of different kinds of geometrical objects: magnitude is exposed by a solid, velocity by a line. This difficulty would obviously extend to any interaction between a body which moves perceptibly and another whose motion is imperceptible. It would thus rule out quantitative analyses of many interactions involving the kinds of endeavours that were discussed in the first section of this chapter.

Such difficulties do not, however, cause Hobbes to abandon the idea that the forces of different kinds of bodies should be exposed in various ways. He clarifies and expands his position in the *Six Lessons*, where he states explicitly that there are different categories of force, each demanding a proper form of geometrical representation. When describing how various types of quantity can be determined, i.e., exposed by means of one of the three dimensions of body, Hobbes states that "[i]f the force consist in swiftness, the determination is the same with that of swiftness, namely, by a line; if in swiftness and quantity of body jointly, then by a line and a solid; or if in quantity of body only, as weight, by a solid only" (*SL* I; *EW* VII. 195). These determinations line up at least roughly with the uses of endeavour that we have discussed. The quantity of some forces varies only with the body's velocity. This suggests that these are the forces of bodies with perceptible motions but imperceptible sizes. The motions of the points or atoms that are added together to generate the forces of composite bodies would be exposed in this way. The quantity of motion possessed by other bodies depends on both their velocity and their magnitude. This suggests the forces of perceptible bodies with perceptible motions, such as the percussive bodies discussed above. Still other forces depend on magnitude alone. As has been discussed, these correspond to perceptible bodies which seem to be at rest.

By fragmenting the concept of force, Hobbes manages to fit the various kinds of motions that he needs to explain physical phenomena within his system of geometrical representation. However, he pays a price, since in many cases these representations cannot be compared. Hobbes has claimed that we evaluate the effect that one body can have on another by calculating and comparing the magnitudes of their respective motions. If the tenets of Hobbesian geometry preclude such comparisons, they also preclude, on

Hobbes's own terms, quantitative analyses of many of the interactions in the physical world.[40]

Furthermore, given Hobbes's views on exposition and imperceptible motions, there would be no distinction between the geometrical representations of magnitude and what he calls "weight." Both are exposed by a solid, since the imperceptible motions that account for differences in weight cannot be exposed by means of lines. Hobbes thus has no means of bringing the influence of weight into his quantitative dynamics. This ambiguity is suggested by the fact that in the above passage Hobbes treats weight as a determining factor with regard to the effects of impact. As we have seen, he elsewhere describes momentum and force in terms of magnitude alone.

The above passages suggest that the various types of endeavour can be represented geometrically, albeit in incommensurable forms. To recall, however, for Hobbes, endeavours, like all forms of motion, include the further element of directionality. Directionality is never mentioned in connection with impetus. There is simply no room for directionality within Hobbes's system of mathematical representation. Most of his representations do not include abstracted versions of the path of a given motion (the exceptions being the representations of projectile motion). They represent in various ways changes in a body's speed, but not in its direction.

In the initial sections of this chapter, I have shown that there are two competing tendencies in Hobbes's dynamics: first, Hobbes treats the concept of endeavour as an all-purpose explanatory tool, using it to describe a wide variety of motions accounting for diverse physical phenomena. On the other hand, his account of dynamic interaction demands that the magnitudes of these motions be directly compared. Hobbes's extensive use of the term "endeavour," rather than "impetus," in his dynamics is an indication that we was unable to bring these tendencies together, and hence could not develop a truly quantitative dynamics.

5.3 Circular Motion

Before leaving Hobbes's attempt at a mathematical mechanics, it is worth looking at his account of circular motion. Providing a mathematical analysis of this type of motion was one of the great puzzles of seventeenth-century mechanics. Hobbes makes little progress towards this solution. However, as we will see, the reasons for his failure are not those commonly supposed.

Chapter 21, "Of Circular Motion," deals almost exclusively with a particular variety of the same, which Hobbes calls "simple circular motion." Hobbes's procedure here is thus significantly different from the one that he followed in his discussions of uniform and accelerated motion. To recall, he there described a wide range of motions, without regard for whether or not they actually appear in nature. In chapter 21, however, Hobbes's account is very much focussed on one kind of motion. Furthermore, as I will discuss, Hobbes's interest is due to the role that it plays in his causal explanations of a number of physical phenomena.

He begins by presenting a kinematic description of simple circular motion. In chapter 15, Hobbes had defined simple motion to be that whereby the several parts of a moving body describe several equal lines (*DCp* III.15.4; *OL* I. 181). In simple circular motion, the lines that each part of the body trace are circular. Any straight line in a body

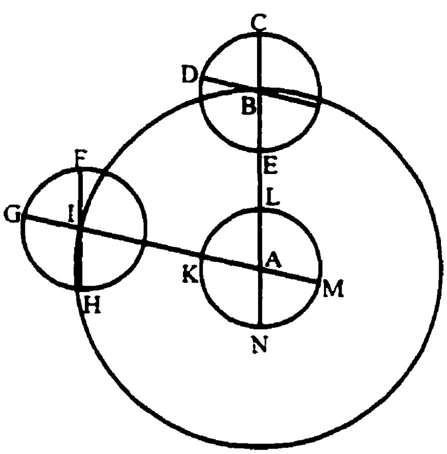


Figure 5.3

so moved will always be carried parallel to itself (*Dcp* III.21.1; *OL* I, 258-9).

Hobbes's initial kinematic account also describes a particular instance of simple circular motion. In figure 5.3, let there be a circle with the centre A and the radius AB on which, at point B, there is an epicycle CDE with a radius BC. Let AB be moved around A until it is coincident with AI, and the epicycle CDE with it until it is coincident with FGH. But suppose that during this time CDE has a uniform contrary rotation about B from E by D towards C, in such a way that the angle formed by movement of the radius BE is the same as that made by BA. In these circumstances, the axis CE of the epicycle will always be carried parallel to itself, i.e., when AB is in AI, CE will be in FH. (*DCp* III.21.2; *OL* I, 260-1)

This is the kind of motion possessed by the earth in its orbit. Hence Hobbes says that "those two motions which Copernicus ascribes to the earth, both annual, are reduced to this one simple circular motion, by which all the points of the moved body are carried always with equal velocity, that is, in equal times they complete equal revolutions uniformly" (*DCp* III.21.2; *OL* I, 261).[41] This is an allusion to the discussion of the same subject in Galileo's *Dialogue Concerning the Two Chief World Systems*, where he argues that a motion like that which Hobbes calls simple circular is all that is needed to account for the motion of the earth.[42]

This discussion is the first indication that Hobbes's interest in simple circular motion is motivated by the presence of this type of motion in the physical world. He goes on to say that simple circular motion is the most frequent of all circular motions, it being "of such a kind as they use, who turn something with their arms, as those who grind or sift" (*DCp* III.21.21; *OL* I, 261).[43] As we will see, this sort of motion will turn out to be Hobbes's favourite hypothesis when explaining physical phenomena. In the final chapters of his mathematical mechanics, Hobbes abandons the abstract stance of his chapters on uniform and accelerated motion.

Following these kinematic descriptions, Hobbes presents several principles characterizing the dynamic effects of simple circular motion. There is no need to discuss all of the principles that Hobbes presents in chapter 21. However, it will be useful to go over one in order to get a sense of the tenor of Hobbes's discussion. The first property of simple motion described by Hobbes is that "if a body is borne with simple motion in a fluid and full medium, it changes the position of every part of the ambient fluid opposing its motion, even the smallest, so that into every single place new particles of fluid are continually replacing each other" (*DCp* III.21.3; *OL* I, 261).[44]

Hobbes argues for this claim by asking us to consider a body moving with a simple circular motion of any quantity. He asserts that, during this motion, each point of the body will eventually be carried in every possible direction. When the body moves in any direction, it will move the bodies next to it. However, for reasons that have been discussed, this motion will be perpetually propagated through the medium. Since the body will eventually move in all directions, every part of the ambient will be moved by the resulting propagated motion.

The first thing to note about Hobbes's analysis is its lack of mathematical precision. He often describes the dynamic effects of simple motion in terms of geometrical objects and their paths, but these principles are not of the sort that would allow us to make quantitative predictions. This is not something that we should hold against Hobbes: at the time *De Corpore* was written no one else had developed a successful mathematical analysis of circular motion, let alone an account of fluid mechanics. However, as we will see, there may have been particular reasons for Hobbes's failure to produce quantitative treatments of these phenomena.

The second thing to note is that Hobbes is again concerned with motions that will feature in his explanations of natural phenomena. In his natural philosophy, Hobbes often refers back to the principles from chapter 21 when justifying physical hypotheses. So, for example, in chapter 21 Hobbes presents the principle: "[i]f a spherical body should be moved in a fluid medium with simple circular motion, and in the same medium another sphere made from a matter not liquid should be floating, this sphere will also be moved with simple circular motion" (*DCp* III.21.10; *OL* I, 268-9).[45] Hobbes's arguments for this proposition are particularly vague and ad hoc.[46] However, having established the proposition to his satisfaction, in part IV of *De Corpore* Hobbes states that "[w]e have demonstrated, in chapter 21, article 10, that from the supposed simple circular motion of the sun, that the earth will be so moved around the sun, that its axis will always be held parallel to itself" (*DCp* IV.26.6; *OL* I, 349).[47] Hobbes seems to be positing that the sun is perpetually moving with a small simple circular motion, apparently imperceptible to observers on earth, and that this motion is transmitted through the celestial medium, resulting in the motion of the earth. Continual simple circular motion on the part of the sun and planets is one of the six suppositions that Hobbes presents for the saving of celestial phenomena (*DCp* IV.26.5; *OL* I, 348).

By placing the doctrine of simple circular motion front and centre in his physics, Hobbes appears to be contrasting his own principal explanatory device with Descartes's. Descartes uses vortices to account for the celestial motions, arguing that each star is surrounded by a huge vortex of fluid matter, in which planets can be carried around. Planets can have their own, smaller vortices, which account for the rotation of moons around those planets. In claiming that he can explain all the various motions of the heavens, Hobbes is attempting demonstrate the superiority of the device of simple circular motion over Cartesian vortices.

Descartes had also appealed to vortices to explain a range of terrestrial phenomena, including magnetism and the behaviour of the tides. Hobbes follows him by showing off the explanatory power of simple circular motion. To present just one example, Hobbes claims that if a body with simple circular motion is placed in a medium, it will cause bodies floating in that medium to congregate if they are homogeneous, and disperse if they are heterogeneous (*DCp* III.21.5; *OL* I, 263). As we have seen, Hobbesian bodies differ only insofar as they have different internal motions:

> But bodies which so differ, experience a common external motion differently. Wherefore they will not be borne together, that is, they will be dispersed. But being dispersed they will at some time or other come upon bodies similar to themselves, and will be moved similarly and together with those, and these too coming upon similar bodies will unite and make greater bodies. Whereby homogeneous beings in a medium, where they

> float, are congregated by simple circular motion; heterogeneous bodies however are dispersed. (*DCp* III.21.5; *OL* I, 264)[48]

This process is identified with what is commonly called fermentation, which Hobbes notes is a process that we find in natural phenomena such as new wine. Hobbes is attempting to show that fermentation, a notoriously difficult phenomenon to explain,[49] can be explained by means of simple circular motion. In subsequent parts of *De Corpore*, Hobbes appeals to fermentation in explaining phenomena as diverse as cloud formation (*DCp* IV.28.15; *OL* I, 392-3) and the motion of the blood (*DH* I.1; *OL* 2, 4).

Brandt expresses dissatisfaction with Hobbes's account of simple circular motion, stating that it includes "not the faintest indication of a discussion of the mechanical possibility of the motion itself."[50] This is, in general, a legitimate criticism of Hobbes, since Hobbes does not provide an adequate physical account to go with his kinematic description of simple circular motion. Brandt, however, has a particular difficulty with Hobbes's doctrine: he is disappointed that Hobbes shows no concern with reconciling the quasi-inertial principle that is presented in part II with simple circular motion, asking "how can any one in one place maintain the principle of inertia and in another the simple circular motion and not make the least mention of how these two plainly contrary principles can be reconciled?".[51]

Brandt attributes Hobbes's disinterest to a particular understanding of inertial motion. As we saw in chapter 3, Hobbes, unlike Descartes, does not restrict his principle to rectilinear motion. If Hobbes had believed that inertial motion was rectilinear, Brandt thinks that he would have felt compelled to offer a mechanical explanation of simple circular motion. He therefore asks why Hobbes failed to follow Descartes's lead on this point.

Brandt argues that Hobbes was instead following Galileo. Galileo assigned a special status to circular motion.[52] On the first day of the *Two Chief World Systems*, Salviati professes that, although he differs with Aristotle on many other points, he agrees with the claim that the world is perfect and "most orderly, having its parts disposed in the highest and most perfect order among themselves."[53] In such a world, however, no body could be naturally suited to straight motion. If something moves in a straight line, it is continually moving away from every place in which it has been. This could not occur in a perfectly ordered world, in which everything is in its proper place. One might hypothesize that in the world's original disordered state, straight motions were used to transport bodies to their original places. Once, however, they reached these places, the bodies must have been set in circular motion.[54]

Furthermore, since straight lines are infinite and indeterminate, straight motion is also by nature infinite. Galileo held that the universe is finite. A rectilinear principle of motion is therefore impossible, since nature would not undertake to move a body towards an impossible end.[55]

Circular motions can be part of a well-ordered world: if a body moves around its own centre, it always keeps the same place. Alternatively, if a body moves about the circumference of a circle with a fixed centre, its motion is finite and disturbs nothing outside of that circumference. Furthermore, circular motion is the only uniform motion. Acceleration occurs when a body approaches the place to which it tends, and retardation when it recedes from that place. In circular motion about its proper place a moving body "is continually going away from and approaching its natural terminus," and hence "the inclinations are always of equal strength (*forze*) in it."[56] All of these considerations lead Galileo to conclude that

> only circular motion can naturally suit bodies which are integral parts of the universe as constituted in the best arrangement, and that the most which can be said for straight motion is that it is assigned by nature to its bodies (and their parts) whenever these are to be found outside their proper places, arranged badly, and are therefore in need of being restored to their natural state by the shortest path. From which it seems to me one may reasonably conclude that for the maintenance of perfect order among the parts of the universe it is necessary to say that movable bodies are movable only circularly; if there are any that do not move circularly, these are necessarily immovable, nothing but rest and circular motion being suitable to the preservation of order.[57]

Again, since Galileo believes that the world is perfectly ordered, he also holds that all the motions in the universe must be circular.[58]

At a later point in the *Dialogue*, Galileo elaborates on how this could be the case. In the Second Day, he formulates his own precursor to the principle of inertia. Salviati asks Simplicio to imagine a smooth plane surface made of some hard material. It is agreed that a hard, spherical ball placed on such a surface would roll down it spontaneously and with a constant acceleration. On the other hand, a ball thrust up the same surface would be retarded at a constant rate as its impulse lessened. When asked what would happen if the ball were to be placed on a plane with no upward or downward slope, Simplicio concludes that "[t]here being no downward slope, there can be no natural tendency toward motion; and there being no upward slope, there can be no resistance to being moved, so there would be an indifference between the propensity and the resistance to motion."[59] The ball would therefore remain stable if placed down firmly. When pressed, Simplicio continues that if the ball were given an impulse in some direction, there would likewise be no cause for acceleration or deceleration, and hence the ball would continue to move as far as the surface extended — perpetually, if the surface were boundless.[60]

In keeping with his statements in the First Day, Galileo restricts his notion of inertia to circular motion. Since the acceleration of the ball on the downward slope and its deceleration on the upward slope are due to the tendency of heavy bodies to move towards the centre of the earth, a surface whose parts are all equidistant from the earth's centre will produce neither. When asked if any such surfaces exist in the world, Simplicio replies that there are "[p]lenty of them; such would be the surface of our terrestrial globe if it were smooth, and not rough and mountainous as it is. But there is that of the water, when it is placid and tranquil."[61] Motions around the earth's circumference are therefore inertial, and would continue perpetually if they did not meet with resistance.

This principle is introduced to counter an argument against one of the main objections to the idea of a moving earth. Simplicio had claimed that a stone dropped from the mast of a moving ship would land as far from that mast as the ship had advanced during the stone's fall. Analogously, an object thrown upwards on a moving earth would land far from where it was thrown. The fact that this does not occur is, by this reasoning, evidence that the earth is not in motion.

Armed with his notion of circular inertia, Galileo argues that the stone would actually fall at the foot of the mast. As had been argued, the ship, if moving on a calm sea, moves with a perpetual circular motion. The stone, since it is moving with the ship, shares in this "ineradicable" motion. Simplicio is therefore forced to conclude that "the stone, moving with an indelibly impressed motion, is not going to leave the ship, but will follow it, and finally will fall at the same place where it fell when the ship remained motionless."[62]

According to Brandt, Hobbes was following Galileo's belief that all motions are "by nature" circular, and he therefore failed to recognize the import of Descartes's version of the inertial principle. There are some significant similarities and connections between the accounts that Hobbes and Galileo give of circular motion: first, circular motion obviously plays a central role in both of their accounts of nature. Furthermore, the reference in *De Corpore* to Galileo's discussion of Copernicus suggests that Hobbes adopted his notion of simple circular motion from Galileo's account of the same in the *Two Chief World Systems*. Finally, both refer to circular motion as the "natural" motion of terrestrial bodies. Hobbes's use of this terminology is particularly evident in the *Dialogus Physicus*, where he uses the hypothesis of simple circular motion to explain the spring of the air. The cause of the spring is there attributed to tiny particles of air, which "effect that [simple circular] motion of restitution, returning into themselves, with their own natural motion of which there is no beginning" (*DP*; *OL* IV, 249; 358). Similarly, he posits that there is a "*simple circular* motion of the earth, congenital to its nature" (*DP*, *OL* IV, 252; 361).

However, there are also important differences between the views expressed in the *Two Chief World Systems* and *De Corpore*: first, Hobbes offers no metaphysical reasons for the primacy of circular motion. He does not claim that motion in a circle is more perfect than that in a straight line, nor, given his agnosticism regarding the finitude or infinitude of the universe, would he object to the possibility of an infinite motion. Hobbes justifies the positing of circular rather than straight motion on a case by case basis, on the grounds that the former provides a better explanation of the phenomenon being discussed. In the *Dialogus Physicus*, for example, Hobbes's interlocutor explains why the restitution of a crossbow's steel plate must involve circular motions:

> That motion cannot be straight, since, if it were straight, the whole body (so to speak) would be carried away by the motion of the crossbow itself, in the way that a missile is usually carried off. Therefore it is necessary that the endeavour be circular, such that every point in a body restoring itself may perform a circle. (*DP*; *OL* IV, 248; 357)

Secondly, Galileo's inertial circular motion applies to bodies moving in or around their natural places, including the motion of the earth around the sun, the earth around its centre, and bodies around the earth's circumference. Hobbes has no idea of natural place, and would clearly object to the teleological aspects of Galileo's discussion.

Thirdly, since it is simple circular motion that Hobbes often describes as "natural", he would not be able to adopt Galileo's arguments for circular inertia, even if he were so inclined. Hobbes could not, for example, use Galileo's argument that motions around the circumference of the earth would continue perpetually in the absence of impediments – a ship moving with simple circular motion around the earth would be floating upside down when it reached the antipodes. Furthermore, there is no indication that Galileo would have described other circular motions, such as those that Hobbes often attributes to the tiny particles of earth and air in the atmosphere, as inertial. If Hobbes were claiming that these motions are inertial, it would be an unjustified extension of Galileo's views.

In fact, there is no reason to suppose that this was the way that Hobbes thought of these tiny motions (or, for that matter, the motion of the earth itself). He does not claim that these motions would continue perpetually in the absence of resistance. Instead, Hobbes's discussion is focussed on providing mechanical explanations for how these motions are produced and maintained in a plenum. For example, as we have seen, one of Hobbes's dynamic principles is "[i]f a spherical body should be moved in a liquid medium with simple circular motion, and in the same medium should be floating another sphere made from a material not liquid, that too will be moved with simple circular motion" (*DCp* III.21.10: *OL* I, 268-69).[63] In part IV, he refers back to this section, saying that this is where he demonstrated the simple circular motion of the earth from that of the sun (*DCp* IV.25.6; *OL* I, 349). Like all physical phenomena, the motion of the earth requires a causal, mechanical hypothesis.

Corresponding to these differences between Hobbes and Galileo's accounts is a difference in the reasons why they refer to the motion of the earth and terrestrial particles as "eternal" and "natural." Hobbes thinks that the simple circular motion of the sun causes that of the earth, which in turn causes a similar motion in the air and any particles suspended in it. Since Hobbes appears to assume that the sun has always moved with simple circular motion, the resulting motions of the earth and its parts would be eternal. Similarly, descriptions of the earth's "natural" motion may simply refer to it being the motion that the earth has in the natural course of things. In both cases Hobbes's terminology is not due to a belief in circular motion as a reflection of nature's perfection, nor a conviction that this motion would continue in the absence of resistance. The earth and many of its particles have always experienced, and in all likelihood will continue to experience, simple circular motion. This perpetual motion can be explained, however, in terms of nothing but their mechanical causes.

In a sense, Brandt's criticism is a valid one. Hobbes does not seem to have recognized that the principle of inertia applies only to rectilinear motion, and he makes almost no use of the principle in his mechanics. However, these omissions are not due to a tacit acceptance of circular inertia. Instead, I suspect that they are due to Hobbes's ambition to offer mechanical explanations of how various effects can occur in a plenum, and his failure to see the extent to which principles describing what would happen to a body in the absence of any resistance could help with this project.

Brandt's objections are thus largely off the mark. It is not entirely fair to accuse Hobbes of saying nothing about the "mechanical possibility" of simple circular motion. He provided, using the conceptual resources of his system, causal, mechanical explanations (albeit vague and sometimes implausible ones) for the various circular motions that he posited. For Hobbes, these qualified as complete explanations of the phenomena in question.

Hobbes's approach is also evident when he describes what it is for a body to have a tendency to motion. The peculiarities of his account are particularly evident when compared with Descartes's account of the same. In the *Principles*, Descartes summarizes his second law of nature by stating that

> each part of matter, considered individually, tends to continue its movement only along straight lines, and never along curved ones; even though many of these parts are frequently forced to move aside because they encounter others in their path, and even though, as stated before, in any movement, a circle of matter which moves together is always in some way formed.[64]

A body's tendency to move in a straight line is not an actual motion — in fact, given that the world is a plenum, this tendency will never be realized. Because every body is always encountering others, which resist its motion with their own, all matter is forced to move in circular paths.[65]

Descartes supports this principle by analysing the circular motion of a stone in a sling:

> For example, when the stone A is rotated in the sling EA and describes the circle ABF; at the instant at which it is at point A, it is inclined to move along the tangent of the circle toward C. We cannot conceive that it is inclined to any circular movement: for although it will have previously come from L to A along a curved line, none of this circular movement can be understood to remain in it when it is at point A. Moreover, this is confirmed by experience, because if the stone then leaves the sling, it will continue to move, not toward B, but toward C.[66]

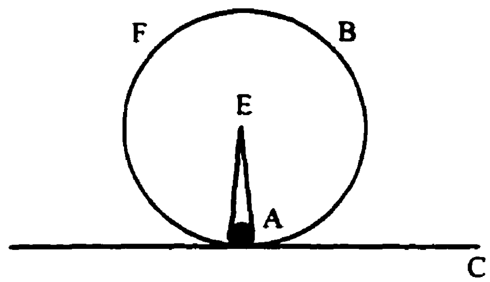


Figure 5.4

Leaving aside the mechanical difficulties with this description of circular motion, Descartes thinks that a body's inclination at every instant to move in a straight line accounts for the fact that the stone will move along the tangent if released from the sling.

As we have seen, Hobbes describes a body's tendency to move as an endeavour in some direction. As such, it is always an actual, though sometimes imperceptibly small, motion. In our full world, tendencies must therefore be explained as the effects of other moving bodies. Accordingly, Hobbes offers a different description of the circular motion of a body around a centre. It appears as an illustration of one of the principles of motion described in chapter 15: "And if while a mobile is borne in any line with a motion which is made from the concourse of two movents, in that point, where it is first abandoned by the force of one of the movents, its endeavour will be changed into an endeavour along the line of the other movent" (*DCp* III.15.6; *OL* I, 182).[67] The first example that Hobbes produces in conjunction with this principle is that of a body carried by the concourse of two winds: in which case, if one of the wind stops, the body will continue to move in the direction of that which remains. Furthermore,

> in a circle, where a motion is determined by a movent along the tangent and by a radius retaining the mobile at a certain distance from the centre,

> its endeavour which had previously been in the circumference in the circle, if the retention of the radius should be taken away, it will afterwards be in the tangent alone, that is, in a straight line. (*DCp* III.15.6; *OL* I, 182)[68]

There are some obvious similarities between this passage and the parallel example in Descartes: both describe motion around a circumference as composed of an inclination or endeavour to move along the tangent and a retaining influence along the radius. In Descartes's case, the former is attributed to an inertial tendency. Hobbes, on the other hand, uses this as an example of a body being carried by two movents: both the tendency to move along the tangent and the retaining force are assumed to be the effects of other moving bodies. He does not appeal to inertia, circular or rectilinear, to explain this kind of circular motion — he appeals only to movements and their effects.

What Hobbes goes on to say in this passage is, however, puzzling. Immediately after the above quotation, he continues:

> For since endeavour is estimated in a part of the circumference less than that which can be given, that is, in a point, the way of the mobile along a circumference will be composed from infinitely many straight lines, of which each one is less than can be indicated, and which on account of that fact are called points. Therefore the mobile proceeds, after it was freed from the retention of the radius, along the same straight line, that is along the tangent.(*DCp* III.15.6; *OL* I. 182)[69]

This seems almost identical to the argument that Descartes presents for the rectilinear nature of inertial motion. However, Descartes and Hobbes have different reasons for supposing that at each point of its circular motion a body will tend in a straight line. Descartes claims that his second law of nature, like his first,

> results from the immutability and simplicity of the operation by which God maintains movement in matter; for He only maintains it precisely as it is at the very moment at which He is maintaining it, and not as it may perhaps have been at some earlier time. Of course, no movement is accomplished in an instant; yet it is obvious that every moving body, at

> any given moment in the course of its movement, is inclined to continue that movement in some direction in a straight line, and never in a curved one.[70]

Descartes's account, like that of Hobbes, depends on the claim that a body moving through a circle has a tendency to rectilinear motion at every point. For Descartes, however, a rotating body has a tendency at any moment to move in a straight line because only straight, linear motion can be constant, and hence maintained by God's immutable operation. Descartes thus argues from metaphysical considerations to the rectilinearity of the tendency to motion.

On the other hand, Hobbes begins by claiming that instantaneous motion must be rectilinear, because of the very nature of a point (Hobbes seems to be relying here on the notion of point that he developed in the context of his theory of indivisibles). Hobbes's reasoning is not based on quasi-inertial principles, but on the purported character of an an actual, instantaneous motion.

Nevertheless, Hobbes's statements about point motion, along with things that he says elsewhere, should lead him to something like Descartes's second law. He claims that motion through a point, and hence motion in an instant, must be in a straight line. Together with his statement that a moving body, unless impeded by another, will continue with the same speed and *along the same way* (*per eandem viam*) (*DCp* III.15.1; *OL* I. 177)[71], this implies that were any motion to continue unimpeded, it would necessarily be rectilinear.

However, these are not conclusions that Hobbes himself draws, and they play no part in his argument regarding the direction in which a body released from circular motion will travel. As I have noted, Hobbes explains the circular motion of a body around a centre by means of two movents, or the actions of two moving bodies other than that being carried, rather than by the influence of a force restraining an inertial tendency. Hobbes either did not see the consequences of his statements about point motion, or did not think that they were relevant in this case.

There is reason to think that Hobbes failed to think through his position on the nature of point motion, and the consequences that that position might have. As we saw in our discussion of his arithmetic, this was not an unusual tendency for Hobbes when it came to subjects that he considered secondary to his main project. For example, in the section immediately preceding that from which the above passages were drawn, Hobbes says that:

> Every endeavour tends towards that place, that is, along that way, which the motion of the movent determines, if there is one movent, or (if there are more movents) which the motion determines, which is made from the concourse of those movents. For example, if a mobile is borne by a straight motion, its first endeavour will be in a straight line; if it is borne by a circular motion, its first endeavour will likewise be in the circumference of a circle. (*DCp* III.15.5; *OL* I, 182)[72]

Contrary to his argument about the rectilinear nature of point motion, Hobbes here suggests that endeavours can be either straight or circular. Brandt cites this passage as evidence that "according to Hobbes, there is a curved inert [*sic*] motion".[73] Again, this is not the case — Hobbes is just saying that if multiple movents are affecting a body in such a way that its movement is circular, its endeavour will also be circular. It could just as easily be straight if the movent(s) influencing it were different. Once again, Hobbes's confusion cannot be explaining to attributing to him a belief in circular inertia. Instead, his difficulties stem from a failure to give the topic of inertial motion much thought at all.

These considerations help to illuminate a particularly odd passage in chapter 21. As has been noted, the bulk of chapter 21 is devoted to an account of simple circular motion and its dynamic effects. In the midst of describing these effects, however, Hobbes turns his attention to compounded circular motion, wherein the various parts of the moving body describe greater or larger perimeters according to their distance from a common centre. He claims that moving bodies of this sort carry around other solid bodies that adhere to them, but after the contact is broken off, the same solid bodies will be cast off along the tangent of the point where the breaking off occurred (*eadem autem a* *contactu abrupta per tangentem puncti abruptionis projicit*). Hobbes's argument for this principle is as follows:

> For let there be a circle [in figure 5.5] whose radius is AB, and some body situated in the circumference at B, which if it should be fixed at B, would be carried around together with the circle, as is manifest enough by itself. But while moving, let that adhesion be supposed to be however removed, just when it is in point B. I say that the mobile will advance from B along the tangent BC. Let it be understood that the radius AB and the body B consist of hard material. And let us suppose the radius AB to be struck at point B by a body falling along the tangent DB. Therefore the motion originated from the concourse of two things, of which the one is the endeavour along DB produced towards C (for the body would advance from B, along BC itself, unless it were restrained towards the radius AB), the other is the retention itself. But that retention gives no endeavour towards the centre to the body at B. Therefore the retention having been removed, that which is done at the breaking off, only one endeavour remains in B upon the breaking off, and that is along the tangent BC. Therefore the broken off point B will be moved along the tangent BC: which was to be demonstrated. (*DCp* III.21.9: *OL* I. 268)[74]

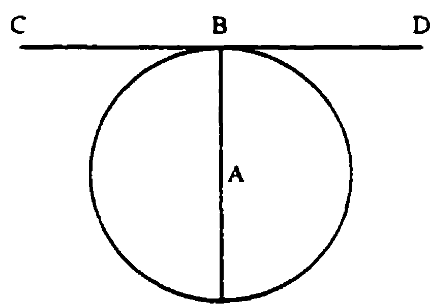


Figure 5.5

This passage is puzzling for a number of reasons, but most notably because of Hobbes's supposition that the body at B is struck by another hard body moving along the tangent DB. It seems obvious that the body at B will move towards C under such circumstances. From our point of view, the interesting case to consider is why a rotating circular body casts off objects that are not struck in such a fashion.

This passage reflects that same beliefs about how to explain circular motion that were evident in Hobbes's earlier account of the tendency of a rotating body to move along the tangent in the absence of a retaining force. Again, the motion of a body attached to a circumference with compounded circular motion is composed of a tendency to move along the tangent plus the retention along the radius. However, as I have discussed, for Hobbes both elements of this motion are supposed to be caused by external forces, i.e., by the motions of external bodies. Given this framework, the simplest explanation of the tendency to move along the tangent would be the impact of a body moving along DB. If the retention is removed, the body moving along DB will naturally cause that at B to be cast off. Hobbes is thus supposing that compounded circular motion is the result, in part, of the rotating body being struck at each point of its circumference by bodies moving along the tangent to each point. If bodies along the circumference are not restrained, the impact of these external bodies will force them to be cast off along the tangent.

Our discussion of Hobbes on circular motion has, in a sense, taken use away from the subject of mathematics. However, it may offer one explanation for why Hobbes was unable to provide a quantitative account of such motions. When Christian Huygens eventually provided a successful treatment of what Hobbes calls compounded circular motion, it was by measuring the effect that centrifugal force had on the rotating body: Huygens measured the distance between a given point on the circumference and the point on the tangent that the body would have reached had the centrifugal force not acted on it. In other words, he reasoned about the force by considering its effects. Given his view of science as a causal enterprise, this approach would not have been open to Hobbes. In his mathematics he had to reason from cause to effect, not the other way around.

Endnotes to Chapter 5

1. *...resistentiam esse, in contactu duorum mobilium, conatum conatui, vel omnino vel ex aliqua parte, contrarium.*

2. Quarto, ut definiamus quid sit *premere; duorum mobilium alterum alterum premere dicimus, quando conatu suo unum eorum facit ut alterum vel pars ejus loco cedat.*

3. Quinto, *restituere se corpus pressum nec dimotum dicimus, quando, sublato premente, partes ejus motae propter ipsam corporis internam constitutionem in suum quaeque locum redeunt.*

4. See, for example, Bernstein (1980, 29-33), Brandt (1928, 296-300), and Westfall (1971b, 110).

5. Quando ergo corpus aliquod, corpus contra conans, illud movet, et hoc motum movet itidem tertium, et sic deinceps, illam actionem motus propagationem appellabimus.

6. *...sensio est ab organi sensorii conatu ad extra, qui generatur a conatu ab objecto versus interna, eoque aliquandiu manente per reactionem factum phantasma.*

7. Sunt ergo appetitus et fuga sive animi aversio motus animalis conatus primi.

8. Barnouw (1992) argues that the concept of endeavour used by Hobbes in his account of appetite and aversion ("le *conatus* psychique") is fundamentally different from the concept of endeavour used in Hobbes's physics ("le *conatus* physique"). Barnouw claims that these types of endeavour share a number of features: both are, for example, small motions which can result in perceptible actions, enter into compounded motions, and be suppressed by contrary endeavours (119). He also claims, however, that the internal motions involved in generation of appetites and aversions "donnent à ces corps [animés] ce qui manque à ces corps ce que manque à tout autre corps: l'orientation vers une fin" (118). In other words, the endeavours possessed by animate bodies are goal-oriented in a way that the endeavours of inanimate objects are not. I see no reason to attribute this distinction to Hobbes. As I will discuss, Hobbes often uses the term "endeavour" to describe the apparent striving of inanimate bodies towards some end. In using the same term to refer to appetites and aversions of human beings, he seems only to be emphasizing that those impulses are no more than the products of matter in motion.

9. In the *Elements of Law*, Hobbes states that "[t]his motion, in which consisteth pleasure or pain, is also a solicitation or provocation either to draw near to the thing that pleaseth, or to retire from the thing that displeaseth. And this solicitation is the endeavour or internal beginning of animal motion, which when the object delighteth, is called

APPETITE; when it displeaseth, it is called AVERSION, in respect of the displeasure present; but in respect of the displeasure expected, FEAR" (*EL* I.VII.2; 40).

10. Manifestum ergo est, *quietem inertem atque efficaciae omnis expertem esse; motum autem solum esse qui motum et quiescentibus dat et motis adimit.*

11. Hobbes presumably imagines that a moving body, however small, would be able to move an actually resting body, however large, without the first body losing any of its own motion.

12. *Pondus* est aggregatum omnium conatuum, quibus singula puncta corporis, quod radium premit, in rectis sibi invicem parallelis conantur; ipsum autem corpus premens ponderans nominatur.

13. Et si quidem dum fertur mobile in linea qualibet motu qui fit a concursu duorum moventium, in eo puncto, ubi primum destituitur a vi unius moventis, mutabitur conatus ejus in conatum per lineam moventis alterius.

14. Hobbes (1994a, vol. 1, 108).

15. ...motus magnitudo eo quo jam diximus modo computata, id ipsum est quod appellamus vulgo *vim*.

16. Sexto, *vim* definiemus *esse impetum multiplicatum sive in se, sive in magnitudinem moventis, qua movens plus vel minus agit in corpus quod resistit.*

17. As Hobbes states, "[e]xtensio corporis idem est quod magnitudo ejus, sive id quod aliqui vocant *spatium reale*" (*DCp* II.8.4; *OL* I.93). As I discussed in chapter 2, this was one of the more controversial doctrines in *De Corpore*, leading to extensive debates with Wallis over the nature of quantity and the possibility of rarefaction and condensation.

18. Gabbey (1971, 20), (1973, 283).

19. Gabbey (1971, 20).

20. *Pr* II.36; *M* 58.

21. ...*quod punctum quiescens, cui aliud punctum quantulocunque impetu usque ad contactum admovetur, ab eo impetu movebitur.*

22. ...nam cum omnia illa puncta aequaliter agant, unum autem eorum nullum habeat effectum, etiam aggregatum omnium simul habebit toties nullum effectum, quot sunt accumulata puncta, id est, nullum. Et per consequens essent aliqua corpora ita dura ut

nulla vi frangi possent, id est, durities finita, id est, vis finita infinitae non cederet; quod est absurdum.

23. *DCp* III.15.7; *OL* I, 182-3.

24. Quanquam autem hujusmodi conatus, perpetuo propagatus, non semper ita appareat sensibus tanquam esset motus aliquis; apparet tamen ut actio, sive mutationis alicujus efficiens causa.

25. Hobbes (1994a, vol.1, 58).

26. Hobbes (1994a, vol.1, 75).

27. Hobbes (1994a, vol.2, 717).

28. There is evidence in Hobbes's correspondence that he had access to Charleton's work. In a 1655 letter, Adam du Prat wrote to Hobbes: "Please send me, via a friend, two copies of the 'Epicuro-Gassendo-Charletonian Philosophy'. M. Gassendi asked me to write to you about it" (Hobbes 1994a, vol.1, 214). Du Prat later thanks Hobbes for sending it to him (Hobbes 1994a, vol.1, 246).

29. Charleton ([1654] 1966, 126).

30. Charleton states that "[t]he Third Propriety of the Universal Matter, Atoms, is *Mobility*, or *Gravity*: and from that fountain is it that all Concretions derive their *Virtue Motive*. For, though our deceptable *sense* inform us, that the minute Particles of Bodies are fixt in the act of their Coadunation, wedged up together, and as it were fast bound to the peace by reciprocal concatenation and revinction: yet, from the Dissolution of all Compound natures, in process of time, caused by the intestine Commotions of their Elementary Principles, without the hostility of any External Contraries, may our more judicious *Reason* well inferr, that Atoms are never totally deprived of that their essential Faculty, Mobility; but are uncessantly agitated thereby even in the centrals of Concretions, the most solid and compact; some tending one, others another, in a perpetual attempt of Eruption, and when the Major part of them chance to affect one and the same way of emancipation, then is their united force determined to one part of the Concretion, and motion likewise determined to one region, respecting that Part." (Charleton [1654] 1966, 269)

31. Gabbey (1971, 27). Gabbey includes the following caveat regarding Hobbes (and others): "Now in the special case of a body at rest, for some (Hobbes, the young Leibniz, Malebranche and others) such a body had no force to resist motion, so the contest notion did not apply, the total available force being redistributed among the bodies according to the conservation principle." Although it is true that, on Hobbes's account, a body which is

truly at rest will have no force to resist, it should be emphasized that (as I have argued) the bodies that we perceive to be at rest actually possess imperceptible endeavours, and hence do possess a force to resist.

32. Consideratur motus aliquando in solo effectu quem habet movens in mobile, et tunc vocari *momentum* solet. Est autem *momentum*, excessus motus corporis moventis super motum vel conatum corporis resistentis.

33. In corpus, quod motui resistit, major est moventis vis (pari magnitudine) ejus quod velocius quam ejus quod tardius movetur: item moventis majoris (pari velocitate) quam minoris. Nam quod (pari magnitudine) majore velocitate impingit in mobile, majorem ipsi imprimit motum. Et quod (pari velocitate) majore mole impingit in idem punctum, vel eandem partem mobilis, minus deperdit velocitatis; propterea quod corpus resistens agit in eam partem moventis solam quam contingit: ejus ergo partis solius impetum retundit, cum interea partes non tactae procedant et vires suas integras conservent, quoad et illae ad contactum veniant, ubi vires earum effectum suum obtinent aliquem. Itaque, exempli causa, arietando, lignum longius quam brevius eadem crassitudine et velocitate, et crassius quam exilius eadem longitudine et velocitate plus operatur in parietem.

34. Si fuerint tres magnitudes quaecunque, vel tria quaecunque habentia inter se rationem aliquam, ut tres numeri, tria tempora, tres gradus, etc. rationes primi ad secundum, et secundi ad tertium simul sumptae, sunt aequales rationi primi ad tertium.

35. ...ratio quaevis ad rationem linearum reduci potest.

36. A "beetle" is a heavy instrument, usually with a wooden head, used for ramming paving stones, driving wedges, and so on.

37. Dato, inquam, percussionis effectu aliquo, exempli causa, ictu fistucae dati ponderis, quo palus in terram datae tenacitatis, data mensura infigitur, definire quanto pondere, sine ictu, et quanto tempore idem palus in eandem terram tantundem infigatur, mihi quidem si non impossibile, tamen difficillimum esse videtur. Difficultatis autem causa est, quod velocitas percutientis cum ponderantis magnitudine comparanda esse videatur. Velocitas autem, quae ex longitudine spatiorum aestimatur, pro unica dimensione habenda est; pondus autem, quod dimensione totius corporis metimur, est instar solidi. Solidi autem et longitudinis, id est, lineae, comparatio nulla est.

38. This passage is, however, extremely confusing in the translation reprinted in the *English Works*, which reads: "I say, any effect of percussion being propounded, as for example, the stroke of a beetle of any weight assigned, by which a pile of any given length is to be driven into earth of any tenacity given, it seems to me very hard, if not impossible, to define with what weight, *or with what stroke*, and in what time, the same

pile may be driven to a depth assigned into the same earth" (*CB* III.22.16; *EW* I, 347) (italics mine).

39. It should be noted that, although Hobbes uses the concept of weight here, he doesn't get around to defining it until chapter 23.

40. There were potential ways out of Hobbes's difficulties that he apparently did not recognize. He might, for example, have developed an account of impact incorporating relativity of motion principles. Huygens adopted such an approach in his mature collision theory, arguing that the data of a collision within one reference frame could be transformed into the data of another collision by setting the first reference frame into uniform motion with regard to a second reference frame. If one of the collision problems could be solved, so could the others into which it could be transformed (Gabbey 1998, 66). If Hobbes had adopted a similar approach, it seems that he could have transformed the problematic collisions between apparently resting bodies and bodies in perceptible motion into collisions between two perceptibly moving bodies, then applied solutions to the latter to the former. There are reasons to suspect that, even if he had considered it, Hobbes would not have accepted such an analysis. As I have discussed, for Hobbes a body truly at rest would offer no resistance to the impact of another moving body, while a body with even imperceptible motion would have some force of resistance. There is, for Hobbes, a real difference between rest and motion, with significant implications. He was thus committed to a view whereby collision problems could only be solved by considering whether a body is, in fact, at rest or in motion.

41. Constat hinc, duos illos motus, quos ascribit telluri Nicolaus Copernicus, annuos recidere ambos ad hunc unum motum circularem simplicem, nimirum, per quem fit ut puncta moti aequali semper ferantur velocitate; id est, ut aequalibus temporibus aequales absolvant circulos uniformiter.

42. Galileo (1967, 398-99).

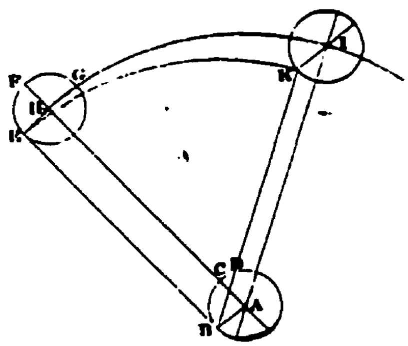
43. ...quali utuntur, qui aliquid brachiis circumagunt, veluti qui molunt vel cribant.

44. Primo, corpus si feratur motu simplice in medio fluido et pleno, mutat situm partium omnium ambientis fluidi motui suo obstantium, etiam minimarum, ita ut in unumquemque locum novae continuo subintrent fluidi particulae.

45. Si corpus spaericum moveatur in medio liquido motu circulari simplice; sitque in eodem medio natans alia sphaera ex materia non liquida, ea quoque movebitur motu circulari simplice.

46. Hobbes asks us to let BCD to be a circle with the centre A, around the perimeter of which a sphere is said to move with simple circular motion. Let EFG be another sphere

made of consistent matter, whose semidiameter is EH, and centre H, and let the circle HI with the radius AH be described. Hobbes claims that the sphere EFG will, by the motion of the body in BCD, be moved in the circumference HI with simple motion (*DCp* III.21.10; *OL* I, 269).



Hobbes begins by claiming that since the motion of BCD makes it the case that every point in the fluid medium will describe equal circular lines in the same time, the points E, F, and G will in the same time describe equal circles with equal radii.

He justifies this claim be referring us back to the fourth section of chapter 21, which states that

> supposing this simple motion to be in air, water, or another liquid, the parts of that liquid, which adhere nearest to the moving body, will be carried by the same motion and with the same velocity, so that in the time in which any point of the moving body completes its circle, in the same time any part of the liquid adhering near the moving body will describe a part of its circle equal to the whole circle of the moving body. I say that it will describe part of the circle, not the whole circle, because all of those parts have their motion from the moving body in an interior concentric motion, and of concentric circles the exterior are always greater than the interior, nor can any motion impressed by some moving body be greater than the motion of the impressing body. (*DCp* III.21.4; *OL* I, 262)

> supposito motu hoc simplice in aere, aqua, aliove liquido, partes ejus liquidi, quae corpori moventi proxime adhaerescunt, circumferentur eodem motu eademque velocitate; ita ut quo tempore punctum quodlibet moventis suum absolverit circulum, eodem tempore quaelibet pars liquidi proxime moventi adhaerens circuli sui describet partem circulo moventis integro aequalem. Describet, inquam, partem circuli, non circulum integrum; propterea quod omnes illae partes motum suum habent a movente in circulo interiore concentrico, et sunt circulorum concentricorum exteriores interioribus semper majores; nec potest esse

motus impressus a movente ullo, velocior quam est motus imprimentis.

Hence, according to Hobbes, the parts of the medium that are nearest to the moving body will be carried around in circles, but the parts of the medium will take more time to complete their circles than the moving body will. The parts of the medium that adhere to those parts nearest the moving body will also describe circles, but, again, in times which increase with their distance from the moving body.

To mention just a few difficulties with this passage, Hobbes does not describe the nature of the "adherence" between the parts of the medium and the moving body, or explain why those parts should be carried around by the body's motion. He assumes without justification that the moving body cannot impart a velocity on some part of the medium greater than that possessed by the body itself, and does not clarify what circles described by the medium's parts will look like.

Hobbes's lack of precision on this last point comes in handy when he attempts to demonstrate that a sphere moving with simple circle motion in a liquid medium will cause other bodies in that medium to also move with simple circular motion. To refer again to the above figure, let EB be drawn equal and parallel to AH, and AB connected, which will therefore be equal and parallel to EH, and let on the centre B and with the radius BE the arch EK be drawn equal to the arch HI, and the straight lines AI, BK, and IK be drawn. AI and BK will be equal and parallel, as will AB and KI, since the arches EK and HI, and hence the angles KBE and IAH are equal.

Claiming, as above, that (because of the motion in BCD) E and G will "in the same time describe equal circles with equal radii," Hobbes infers that E and H will be moved in the same time to K and I, and IK will be parallel to the line EH from which it began. Hobbes is extending, without argument, his claims about the effects of the simple circular motion of a body in a fluid medium on the parts of that medium to the effects of a similarly moving body on another body floating in the ambient fluid. Furthermore, he is interpreting the vague talk of "equal circles" from proposition 4 in such a way that any straight line in the sphere EFG will always (by the motion in BCD) be moved parallel to itself. Based on these dubious claims, Hobbes confidently concludes that the sphere will thereby be moved with simple circular motion.

47. Nos autem ex supposito motu solis circulari simplice, fore demonstravimus, cap. XXI, art. 10, ut terra ita moveatur circa solem, ut axis ejus semper sibi teneatur parallelus.

48. Quae autem sic differunt, motum ab externo communem dissimiliter patiuntur. Quapropter non ferentur una, hoc est, dissipabuntur. Dissipata autem incident aliquando in corpora sibi similia, unaque cum ipsis et similiter movebuntur, et haec quoque in alia similia incidentia unientur et fient majora. Quare homogenea quidem in medio, ubi naturaliter fluctuant, a motu simplice congregantur; heterogenea vero dissipantur.

49. See Newton ([1931] 1952, 401).

50. Brandt (1928, 324).

51. Brandt (1928, 325).

52. It should be noted that there is controversy regarding the interpretation of Galileo's statements on circular motion. Many commentators, like Brandt, attribute to Galileo a principle of circular inertia. However, Stillman Drake (1970, ch.13) argues that this is a serious misreading of Galileo's view. According to Drake, Galileo consistently held that observable terrestrial objects, to which an impulse was inparted by a push or release from a rotating sling, conserved the received impetus in the form of uniform rectilinear motion. On the other hand, Drake claims, Galileo also attributed essential circularity to various "natural" terrestrial motions. There is no need to settle this debate here, since, as I will argue, Hobbes does not hold the position that Brandt is describing, regardless of Galileo's position on the subject.

53. Galileo (1967, 19).

54. Galileo (1967, 19-20).

55. Galileo (1967, 19).

56. Galileo (1967, 31-2).

57. Galileo (1967, 32).

58. Drake (1970, 276-7) holds that when Galileo referred to "circular motion" in this context, he must have meant "circulation," i.e., nothing more than recurrent motion over a closed path. Drake argues that Galileo could not possibly have believed the planetary paths to be perfectly circular, a position held by "no competent astronomer since Aristotle."

59. Galileo (1967, 147).

60. Galileo (1967, 147).

61. Galileo (1967, 148).

62. Galileo (1967, 148).

63. Si corpus spaericum moveatur in medio liquido motu circulari simplice; sitque in eodem medio natans alia sphaera ex materia non liquida, ea quoque movebitur motu circulari simplice.

64. *Pr* II.39; *M* 60.

65. As he explains in the *Principles*, this type of motion means that a body will move "in such a way that it drives another body out of the place which it enters, and that other takes the place of still another, and so on until the last, which enters the place left by the first one at the moment at which the first one leaves it" (*Pr* II.33; *M* 56).

66. *Pr* II.39, *M* 61.

67. Et si quidem dum fertur mobile in linea qualibet motu qui fit a concursu duorum moventium, in eo puncto, ubi primum destituitur a vi unius moventis, mutabitur conatus ejus in conatum per lineam moventis alterius.

68. Et in circulo, ubi motus determinatur a movente per tangentem et a radio retinente mobile in certa a centro distantia, conatus ejus qui prius erat in circuli circumferentia, si auferatur retentio radii, erit postea in tangente sola, id est, in linea recta.

69. Cum enim conatus aestimatur in parte circumferentiae minore quam quae dari possit, id est, in puncto, erit via mobilis per circumferentiam composita ex lineis rectis, quarum una quaeque minor est quam quae dici possit, numero infinitis, et quae ob eam rem appellantur puncta. Procedet itaque mobile, postquam a retentione radii liberatum est, secundum eandem rectam, id est, secundum tangentem.

70. *Pr* II.39; *M* 60-1.

71. *...quidquid movetur, eadem celeritate et per eandem viam semper progressurum esse, nisi a corpore moto et contiguo impediatur.*

72. Conatus autem omnis tendit eo versum, id est, per eam viam, quam determinat motus moventis, si movens unum sit, vel (si plura sint moventia) quam motus determinat, qui fit ex eorum moventium concursu. Exempli causa; si mobile motu feratur recto, primus conatus ejus erit in linea recta; si feratur motu circulari, etiam conatus ejus primus erit in circumferentia circuli.

73. Brandt (1928, 328).

74. Sit enim circulus (in fig. 4) cujus radius AB, et corpus aliquod positum in circumferentia ad B; quod quidem si fixum sit in B, una circumferetur, ut satis per se manifestum est. Inter movendum autem, adhaesio illa supponatur quomodocunque tolli, tunc cum est in puncto B. Dico fore ut mobile a B procedat per tangentem BC. Intelligatur tam radius AB quam ipsum corpus B, consistere ex materia dura. Et supponamus radium AB percussum esse in puncto B a corpore incidente secundum DB tangentem. Orietur ergo motus circularis ex concursu duarum rerum, quarum altera est conatus per DB productam versus C (nam procederet corpus a B, per ipsum BC, nisi esset retentum ad AB radium), altera est retentio ipsa. Sed retentio illa nullum dat corpori in B conatum

versus centrum. Sublata igitur retentione, id quod fit in abruptione, restat unicus in B abrupto conatus, et is per tangentem BC. Ergo per tangentem BC movebitur punctum B abruptum: quod erat demonstrandum.

CHAPTER 6

MATHEMATICS IN HOBBES'S THEORY OF LIGHT

Previous chapters have examined the uses of mathematics in the foundational divisions of Hobbes's natural philosophy. In this chapter I will discuss Hobbes's attempt to mathematise one of the special sciences: optics, and, in particular, the study of light. Like many of his contemporaries, Hobbes was fascinated with optics. He wrote widely on the subject, and his work in this area represents the bulk of his early scientific efforts. Hobbes's theories of light and vision raise many issues, of which this chapter will address only a few. In keeping with the subject matter of previous chapters, I will focus on Hobbes's attempts to treat optics as a geometrical science of motion.

The primary source for my discussion will be a treatise by Hobbes, written around 1640,[1] which Mersenne published in 1644 as part of his *Cogitata Physico-Mathematica* (it represented chapter 7 of the "Optics" contained therein). This treatise, which contains an extensive discussion of the nature of light and refraction, received the title *Tractatus Opticus* when it was reprinted as part of Molesworth's *Opera Latina*. I will adopt this title in the discussion that follows. The *Tractatus Opticus* represents the most complete account of Hobbes's theory of light to be published in his lifetime, and contains the version of his views to which many of his contemporaries reacted. I will consider both the treatise itself and some significant responses to it. Most notably, I will draw on a correspondence that Hobbes had with Descartes about optics. These letters, written between February and April of 1641, were exchanged through Mersenne. In them, Hobbes offers various criticisms of Descartes's *Optics*, and Descartes comments (with varying degrees of nastiness) on what appears to be an early draft of the *Tractatus Opticus*.

Hobbes clarifies and expands on some of the views expressed in the *Tractatus Opticus* in a longer Latin manuscript which exists in the British Library.[2] This manuscript remained unpublished in Hobbes's lifetime. Ferdinand Tönnies published excerpts from the treatise, which he gave the unfortunate name of *Tractatus Opticus*, in his edition of *The Elements of Law, Natural and Politic*.[3] In order to avoid confusion, I will refer to this treatise as *Tractatus Opticus II*.[4]

Finally, I will consider the discussions of light and refraction that are included in *De Corpore*. As we will see, Hobbes's account of light in *De Corpore* is significantly different from the treatments in his earlier optical works. In the last section of this chapter, I will consider why Hobbes changed his views on the nature of refraction.

This chapter is divided into five sections: first, in order to establish the context of Hobbes's work, I will discuss some earlier accounts of light and vision. The second section will consider Hobbes's physical explanation of light. I will then examine Hobbes's notion of a ray, which represents the basis for his account of light, and show how he uses this concept to provide a demonstration of the sine law of refraction. In the fourth section of the chapter, I will look at the mathematical techniques that Hobbes uses in his theory of light, and argue that these techniques anticipate some important aspects of the geometrical mechanics that Hobbes presents in *De Corpore*. Finally, I will consider why Hobbes adopted aspects of his approach from the *Tractatus Opticus* in his mechanics, but, at the same time, abandoned the *Tractatus* account of refraction in *De Corpore*.

6.1 Theories of Light And Vision Before Hobbes

In order to understand what is distinctive about Hobbes's theory of light, we must begin by looking at the work of some of his predecessors.[5] The history of optics is, of course, a vast topic, and the limits of this chapter will allow only a brief discussion of those theories most relevant for our understanding of Hobbes's work. Given our interests in this chapter, my discussion will focus on early theories of light. I will pay most attention to early theories of rays and refraction, as well as to considerations of the extent to which thinkers before Hobbes integrated the mathematical and physical aspects of their optical theories. I will also, however, consider pertinent aspects of some ancient accounts of vision. The ancients were, on the whole, more interested in the nature of vision than that of light (although, of course, these topics can never be completely separated). However, aspects of their theories of vision, most notably their various notions of the visual ray, clearly influenced the independent accounts of light developed by some medieval and early modern theorists.

Among the first theories of light was that of the atomists, who held that bodies emit material replicas (called *eidola* or *simulcra*) in all directions. Lucretius illustrates this idea by comparing the replicas to the skin shed by a serpent.[6] Vision occurs when one of these replicas enters the eye of a perceiver.[7] Light is not involved in this process, except insofar as it burns off the dark, heavy air that was thought to block the passage of the *eidola*.

Light plays a more central role in Aristotle's theory of vision. Aristotle argued that light could not be an emission from some kind of body,[8] but must be a state of the medium between the perceiver and the perceived. He begins by defining "transparent" as "what is visible, and yet not visible in itself, but rather owing its visibility to the colour *of something else*: of this character are air, water, and many solid bodies."[9] The transparent is thus that through which we perceive the colours of objects that are visible in themselves. Aristotle goes on to state that light is

> the activity of what is transparent so far forth as it has in it the determinate power of becoming transparent; where this power is present, there is also the potentiality of the contrary, viz darkness. Light is as it were the proper colour of what is transparent and exists whenever the potentially transparent is excited to actuality by the influence of fire or something resembling 'the uppermost body'.[10]

Light is the state of the transparent in which it is actual, i.e., the state in which bodies are visible through the transparent medium. Light is not something that travels or is propagated through the medium,[11] but something that the medium can acquire all at once, as water may conceivably become frozen simultaneously throughout.

The accounts presented by Aristotle and the atomists included almost no mathematics. This was certainly not the case for all ancient theories of light and vision: in Euclid's *Optica*, for example, we find a theory of vision that is almost entirely geometrical.[12] The *Optica* takes the form of a geometrical treatise, beginning with seven postulates. In the first three of these, Euclid asks us to assume:

> 1. That the rectilinear rays proceeding from the eye diverge indefinitely;
> 2. That the figure contained by a set of visual rays is a cone of which the vertex is at the eye and the base at the surface of the objects seen;
> 3. That those things are seen upon which visual rays fall and those things are not seen upon which visual rays do not fall.[13]

Euclid held that vision is brought about by discrete rectilinear rays that emerge from the eye, forming a cone. As he states in his third postulate, in order to be perceived, an object has to intercept one of these rays. Visual rays were treated as mathematical lines, thus allowing for a geometrical treatment of vision.

Euclid's theory solved some of the problems presented earlier accounts. For example, as Galen pointed out,[14] we would not be able to judge the size of an object based on an emitted *eidola*, since that *eidola* would have to shrink, often significantly, in order to fit into the eye. Euclid posits that "things seen under a larger angle appear larger, those under a smaller angle appear smaller, and those under equal angles appear equal." By treating optics as a geometrical science, Euclid can explain how we perceive objects at varying distances. However, Euclid offered little explanation of the physical nature of visual rays[15] and how they relate to light.

Ptolemy followed Euclid in explaining vision in terms of a visual cone emerging from the observer's eye.[16] However, in his own *Optica*, Ptolemy significantly altered and expanded on Euclid's theory: first, Ptolemy offered a more robust account of the physical nature of rays. He seems to have thought of the visual cone as being physically real, and in the same genus as the light from an external luminous source.[17]

Furthermore, for Ptolemy the visual cone is not made up of a collection of discrete rays. Since we perceive objects as continuous wholes, he argued, the visual cone itself must be continuous. Treating rays as mathematical lines does not reflect the physical reality of vision. Ptolemy thus referred to the continuous cone itself as a "ray."

Ptolemy's work is also notable for the serious attention he pays to the phenomenon of refraction.[18] Ptolemy did experimental work on refraction,[19] and knew that there was a connection between the change of direction that occurs when light passes from one medium to another and the density of those media, such that the light will turn towards the normal when it passes from a rare to a dense medium, and will turn away from the normal if the media are reversed. He also held that the extent of the change of direction will be related to the degree of the difference in density between the media, and attempted to measure the angles of refraction of light rays entering water. Ptolemy's work on refraction was, however, based on observation and experiment, and not on his account of the physical nature of light or an understanding of the causes of refraction.[20]

The next significant advances in optics occurred eight hundred years after Ptolemy. In the ninth century A.D. many Greek philosophical and scientific texts, including many of the optical texts, were translated into Arabic. The ideas in these texts were then discussed and criticized by a number of Islamic scholars. Euclid and Ptolemy's extramission[21] theory of vision was, for example, developed and promoted by al-Kindi (d. ca. 866) in his treatise *De aspectibus*.[22]

Like Ptolemy, and against Euclid, al-Kindi held that the cone of visual radiation must be continuous. He argued, first of all, that if visual rays were mathematical lines, they would terminate in dimensionless points. However, visual rays can only perceive that part of an object with which they have contact, and hence, on the Euclidean account, visual rays would be unable to perceive three-dimensional bodies. The visual rays must therefore be three-dimensional themselves. Furthermore, they must form a continuous cone, for if there were gaps between the visual rays, there would be corresponding blank spaces in our visual field.[23]

Al-Kindi did make some claims about the physical nature of rays — he held that visual rays are not corporeal entities emitted from the eye, but an instantaneous transformation of the ambient air, so as to allow the air to transmit the properties of the perceived object to the eye.[24] In the tradition of Euclid, however, al-Kindi's *De aspectibus* is primarily geometrical.

Some of the most significant work in this history of optics was done by another Islamic scholar, Ibn al-Haytham (also know by the Latin Alhazen) (d. ca. 1039). Ibn al-Haytham made a revolutionary contribution to optics by arguing that vision is the result of light or colour radiating in all directions from each point on the surface of a luminous or illuminated body.[25] Most importantly for our purposes, Ibn al-Haytham also developed an extremely influential account of refraction. As I noted above, Ptolemy provided a basic description of the behaviour of refracted rays, and Ibn al-Haytham's *Optics* expands on Ptolemy's account. Ibn al-Haytham presented a causal account of the refraction of light, claiming that light travels with a great (though finite) speed in transparent bodies, and that its speed will be greater in rare bodies than in dense bodies. This is due to the fact that the denser medium offers more resistance to the motion of light. When light moves from a rarer to a denser medium, its speed is thus altered, and this is the cause of refraction.

Ibn al-Haytham also held that rays falling along the perpendicular to a surface are stronger than the rays that strike the surface obliquely.[26] He supports this principle by appealing to two mechanical analogies:[27]

> If one takes a thin board and fastens it over a wide opening, and if he stands opposite the board and throws an iron ball at it forcefully and observes that the ball moves along the perpendicular to the surface of the board, the board will yield to the ball; or if the board is thin and the force moving the ball is powerful, the board will be broken [by the ball]. And if he then stands in a position oblique with respect to the board and at the

> same distance as before and throws the ball at the same board, the ball will be deflected by the board (unless the latter should be unduly delicate) and will no longer be moved in its original direction, but will deviate toward some other direction.
>
> Similarly, if one takes a sword and places a rod before him and strikes the rod with the sword in such a way that the sword is perpendicular to the surface of the rod, the rod will be cut considerably; and if the sword is oblique and strikes the rod obliquely, the rod will not be cut completely, but perhaps partially, or perhaps the sword will be deflected. And the more oblique the [motion of the] sword, the less forceful it acts on the rod. And there are many other similar things, from which it is evident that motion along the perpendicular is stronger and easier and that the oblique motion which approaches the perpendicular is [stronger and] easier than that which is more remote from the perpendicular.[28]

Light, like the sword and ball, will act more forcefully it strikes a surface perpendicularly.[29] Thus when light falls upon a surface along the perpendicular it will continue to move in a straight line. On the other hand, if a ray enters the denser medium obliquely, it will turn towards the direction in which its traversal of the medium will be easiest, i.e., towards the perpendicular.

In his discussion of refraction, Ibn al-Haytham argues that the motion of a ray can be divided into components parallel and perpendicular to the refracting surface.[30] He uses this analysis to explain why, when a ray moves from a rarer to a denser medium, it does not, in the denser medium, follow the path perpendicular to the surface (which Ibn al-Haytham has said would be the easiest course). Ibn al-Haytham claims that the parallel component of the motion, although it will be weakened by the denser medium, will not be destroyed, and the ray will thus traverse a path between the original direction and the normal to the surface.

Ibn al-Haytham also appeals to this understanding of refraction to account for why a ray moving from a denser to a rarer medium will turn away from the perpendicular. According to Ibn al-Haytham, a medium's resistance will only act on a motion's parallel component. Hence, when the motion enters the rarer medium, the decreased resistance will result only in the increase of that parallel component, and the ray will turn away from the normal.

Ibn al-Haytham's account of refraction was extremely influential on medieval and early modern optical theory. As Sabra has noted, "[p]ractically all subsequent explanations of refraction, up until the publication of Descartes's *Dioptric*, were almost entirely dependent upon Ibn al-Haytham."[31] Descartes himself seems to have been influenced by Ibn al-Haytham's account, although the influence may have been an indirect one, through the versions of Ibn al-Haytham's approach that appear in the work of Roger Bacon, Witelo, and Kepler.[32]

Descartes's theory of light is without doubt the most significant for our understanding of Hobbes's optics. In the mid-1630s Hobbes already had an interest in optics. This is clear from the letters that Hobbes wrote to his patron, the earl of Newcastle, during this time, in which optics are a frequent topic.[33] Hobbes had already developed some of the features of his mechanist theory of light, including the idea that light is a phantasm caused by motion in a medium. He thus states in a letter of October 15, 1636: "But whereas I vse the phrases, the light passes, or the coulor passes or diffuseth it selfe, my meaning is that the motion is onely in y^e medium, and light and coulor are but the effects of that motion in y^e brayne."[34]

Descartes published his *Discourse on the Method* and the accompanying *Essays*, including the *Optics*, in 1637. Hobbes soon received a copy from his friend Kenelm Digby. In 1640 Hobbes sent a manuscript commenting on the *Optics* to Descartes via Mersenne, sparking, as was mentioned at the beginning of this chapter, a sometimes vituperative exchange on the nature of light and vision. During the course of this debate, Hobbes wrote the *Tractatus Opticus*. As we will see, many aspects of this text seem to have been developed as a response to Descartes's doctrine.

Hobbes would have been impressed by two aspects of Descartes's theory: first, Descartes was committed to presenting an account of light in terms of matter in motion. As such, his work represents a new physicalist approach to optics. Second, Descartes was the first to demonstrate the sine law of refraction.[35] Hobbes was no doubt excited by the new mathematical precision that Descartes brought to optics. However, as we will see, Descartes's theory was not without its difficulties. Most notably, it is often difficult to reconcile Descartes's physical account of light with his mathematical treatment of refraction. As even my brief survey above shows, Descartes was not the first theorist to encounter this problem. As I will discuss, Hobbes attempted to provide more unified physico-mathematical account of light in his own work.

Descartes describes light as an "action" or "tendency to move."[36] He often explains this idea by claiming that the action of light is like that of a stick with which we can "perceive" objects around us.[37] He appeals to this analogy in a letter to Hobbes, where he challenges Hobbes's fundamental assumption that all action is local motion, stating that

> in his first hypothesis he makes a false assumption when he says that 'all action is local motion'. For when I press, for example, with a stick against the ground, the action of my hand is communicated to the whole of that stick, and is transmitted as far as the ground, even though we do not suppose in the slightest that the stick is moved — not even indiscernibly, as he goes on to assume.[38]

Although Descartes did not think that there could be such a thing as instantaneous motion, he did hold that tendencies to motion are transmitted instantaneously. Although his account is mechanical, Descartes is in agreement with the Aristotelians with regard to the instantaneous transmission of light.

Although Descartes conceived of light as a tendency to motion, he explains refraction in terms of the motion of a ball or stone. In his account of refraction, Descartes discusses the motion of such a projectile as it passes through a thin linen sheet:

> We come now to refraction. First let us suppose that a ball impelled from A towards B encounters at point B not the surface of the earth, [as was supposed in Descartes's account of reflection] but a linen sheet CBE which is so thin and finely woven that the ball has enough force to puncture it and pass right through, losing only some of its speed (say, a

half) in doing so. Now given this, in order to know what path it must follow, let us consider again that its motion is entirely different from its determination to move in one direction rather than another — from which it follows that the quantity of these two factors must be examined separately. And let us also consider that, of the two parts of which we can imagine this determination to be composed, only the one which was making the ball tend in a downward direction can be changed in any way through its colliding with the sheet, while the one which was making the ball tend to the right must always remain the same as it was, because the sheet offers no opposition at all to the determination in his direction. Then, having described the circle AFD with its centre at B [figure 6.1], and having drawn at right angles to CBE the three straight lines AC, HB, FE so that the distance between FE and HB is twice that between HB and AC, we shall see that the ball must tend towards the point I. For, since the ball loses half its speed in passing though the sheet CBE, it must take twice as much time to descend from B to some point on the circumference of the circle AFD as it took to go from A to B above the sheet. And since it loses none of its former determination to advance to the right, in twice the time it took to pass from the line AC to HB it must cover twice the distance in the same direction, and consequently it must arrive at some point on the straight line FE simultaneously with its reaching some point on the circumference of the circle AFD. This would be impossible if it did not go towards I, as this is the only point below the sheet CBE where the circle AFD and the straight line FE intersect.[39]

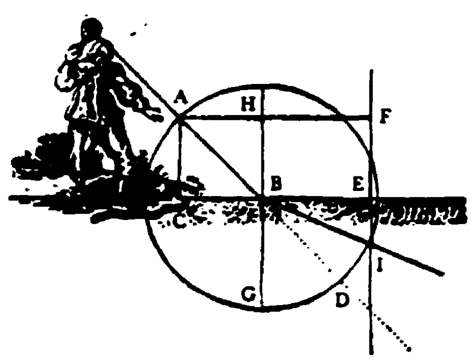


Figure 6.1

Descartes, like Ibn al-Haytham, divides the motion of the light (or the projectile representing it) into components parallel and perpendicular to the refracting surface (or, in this case, the linen sheet). Descartes assumes that the velocity of the ball decreases when it passes through the sheet. However, unlike Ibn al-Haytham, Descartes holds that this change in velocity is due only to a change in the component of the motion perpendicular to the surface.

Descartes proof of the sine law relies on two assumptions: first, he supposes that the velocity of light depends only on its medium. Accordingly, if we let v_i be the velocity of incidence and v_r the velocity of refraction, the ratio of v_i to v_r will be a constant (let it be μ):

$$v_i/v_r = \mu$$

Descartes also assumes that the component of the velocity parallel to the refracting surface will remain constant. Thus, if i is the angle of incidence and r is the angle of refraction:

$$v_i \sin i = v_r \sin r$$

Combining these two assumptions, we have:

$$\sin i = \mu \sin r$$

This is, of course, the sine law of refraction.

This is a perfectly good mathematical proof of the sine law. However, as I mentioned above, difficulties arise when it comes to interpreting the proof in terms of Descartes's physical account of light: first, although Descartes holds that light is a tendency to motion, his treatment of refraction considers actual motions. Descartes defends this approach by claiming that tendencies to motion behave in the same ways as true motions, stating that "it is very easy to believe that the action or tendency to move (which, I have said, should be taken for light) must in this respect obey the same laws as motion itself".[40] Many of Descartes's contemporaries were not persuaded, however, arguing that there is no reason to suppose that light, which is transmitted instantaneously, will obey the same laws as a body moving with successive motion.[41] Hobbes was among those who objected to Descartes's use of the ball analogy in his treatment of refraction. Descartes makes reference to Hobbes's criticism in a letter of January 1641, where he states that Hobbes

> is no more felicitous on the subject of refraction, when he distinguishes between the refraction which takes place when the moved body itself passes through media, and the refraction which takes place when it does not; for in both cases, if the bodies are of the same kind, they will be refracted in the same direction.[42]

Although Descartes acknowledges Hobbes's objection, he does little to address it, merely reasserting that actual motions and tendencies to motion behave in the same way.

A related difficulty involves how we should understand Descartes's talk of "velocity" in his account of refraction. As I discussed above, Descartes holds that light is transmitted instantaneously. However, his proof seems to depend upon there being a variation in the velocity of the light in the two media, and such a variation seems impossible to account for unless we make reference to time.

The idea that the velocity of light varies in different media was not a new one (as I mentioned above, Ibn al-Haytham made use of this notion in his account of refraction). Descartes does, however, make the somewhat unusual claim that the velocity of light increases when it enters a denser medium. This becomes clear in a passage from the *Optics*, where he is discussing the apparently "amazing" fact that while a ball moving from air into water will turn away from the normal, light will be refracted towards the normal when it passes from air into water. Given Descartes's account of refraction, this implies that the component of the light's velocity perpendicular to the refracting surface actually increases when the light enters the denser medium. Descartes explains this phenomenon by again appealing to the ball analogy:

> You will no longer find this strange, however, if you recall the nature that I ascribed to light, when I said it is nothing but a certain movement or an action received in a very subtle matter which fills the pores of other bodies. And you should consider too that, just as a ball loses more of its motion in striking a soft body than a hard one and rolls less easily on a carpet than on a completely bare table, so the action of this subtle matter can be impeded much more by the parts of the air (which, being as it were soft and badly joined, do not offer it much resistance) than by those of water, which offer it more resistance.[43]

Accordingly, if the minute parts of a transparent body are "harder and firmer," they allow the light to pass more easily, given that the light "does not have to drive any of them out of their places, as a ball must expel the parts of water in order to find a passage through them."

6.2 Hobbes on The Physical Nature of Light

The *Tractatus Opticus* is presented in the form of a geometrical treatise, with five hypotheses and fourteen propositions. In the hypotheses Hobbes presents some basic principles, observational truths, and definitions. The propositions sometimes include several parts, and vary in their content, including explanations of phenomena, mathematical laws regarding the motion of light, and further definitions.

That Hobbes, like Descartes, intends to present a materialist account of light is in evidence from the beginning of the *Tractatus Opticus*. In the first hypothesis, Hobbes begins by stating that "[a]ll action is local motion in the agent, as all passion is local motion in the patient. By the name *agent* is understood a *body*, by whose motion an effect is produced in another body; by *patient*, some body in which motion is generated by another body" (*TO* hyp.1; *OL* V, 217).[44] Even at this early stage in his scientific career, Hobbes was dedicated to the principle that all effects must be explained in terms of bodies in motion. He describes a hammer striking a nail as an example of the action of one body bringing about motion in another, but goes on to state:

> Likewise, while a glowing coal heats a man, although neither the coal nor the man departs from their place, and neither is therefore moved, nevertheless there is some matter or subtle body in the coal, which is moved, and it produces motion in the medium all the way to the man; and there is in the man, who has remained immobile, some motion generated therefrom in his internal parts. Now this motion in the internal parts of the man is heat; and to be so moved, and heated, this is to undergo [*pati*]; and that motion which is in the parts of the glowing coal, is its action or heat, and so to be moved, is to heat. (*TO* hyp.1; *OL* V, 217)[45]

Hobbes is thus precluding action at a distance as an explanatory resource: if one apparently immobile body has an effect on another, the former must be causing an insensible motion to be propagated to the latter.[46] This passage also suggests that Hobbes will present an account of light in terms of motion propagated through a medium.

In the second hypothesis, Hobbes defines vision in terms of action and passion, stating that "[v]ision is the passion produced in the seeing [thing] by the action of a luminous or illuminated object" (*TO* hyp.2; *OL* V, 217).[47] For Hobbes, we can only perceive by means of our vision those bodies that are illuminated or produce their own light.

Hobbes goes on to eliminate a competing explanation of vision by claiming that "[i]n vision, neither the object, nor any part of it passes its place to the eye" (*TO* hyp.3: *OL* V, 217).[48] Hobbes is clearly attacking the atomists' theory of vision. The atomists, to recall, explained vision by claiming that visible bodies emit replicas of themselves in every direction. Hobbes gives little argument for rejecting this account of vision, except to claim that it is unnecessary to posit a body actually moving from the luminous body to the eye, since small motions can easily be propagated to any distance (*TO* hyp.3; *OL* V, 217-8).[49] Furthermore, Hobbes, in his first proposition, claims that were perceivable objects to constantly give off parts of themselves in all directions, as an explanation in terms of the constant emission of *eidola* requires, those objects would soon disintegrate.

Up to this point, Descartes and Hobbes are largely in agreement on the subject of light. Descartes, as we have seen, shared Hobbes's commitment to explaining all phenomena, including light, in terms of matter and motion. Furthermore, both thinkers held that light is caused by an action propagated through a medium, rather than a body transmitted from the perceived object to the eye (or vice versa). There are, however, significant differences in the particular mechanical theories that Descartes and Hobbes offer. As is suggested by his first hypothesis, Hobbes argues that light consists in actual, if insensibly small, local motions.

The first three propositions of the *Tractatus Opticus* set out the particulars of Hobbes's explanation of light: first, he claims that "*[e]very luminous thing dilates itself,* *and swells into a greater bulk [molem], and contracts itself again, having continual systole and diastole*" (*TO* prop.1; *OL* V, 218).[50] Hobbes is comparing the action of a body with the continual *systole* and *diastole* of a heart.[51] Light is the action or motion of a luminous object, but it must also be seen from every direction at once. Since Hobbes thinks that it would be impossible for a bright object to be constantly dispersing itself in every direction, "it remains that the parts of the luminous body which were shown to be moved towards every direction at the same time, must withdraw themselves again" (*TO* prop.1; *OL* V, 218).[52]

In the third proposition, "*[t]o consider how light is made, and what it is*" (*TO* prop.1; *OL* V, 219)[53], Hobbes proposes that we consider (in figure 6.2) a luminous body, such as the sun, with a centre A and a radius B. Let the body be circumscribed by concentric orbs[54], each containing an equal quantity of matter, and hence having the

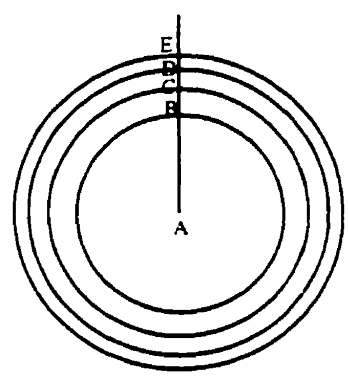


Figure 6.2

decreasing thicknesses BC, CD, DE. If the sun swells, such that its radius becomes AC, it will push that part of the surrounding medium that was in BC into CD. At the same time, that part of the medium that was in CD will be pushed into DE, and so on. If an eye should be located at any distance from the sun, as soon as the process of dilation begins, the motion will be propagated to the retina. From thence it will be propagated through the optical nerve to the brain, which will react, sending an opposing motion back along the nerve to the retina and back along the same lines to the sun (*TO* prop.3; *OL* V, 219-20). All of this occurs in the same instant.

According to Hobbes, this account explains why the light from a luminous object weakens with distance: the distance from BC is greater than CD, which is greater than DE, and so on. However, the motion is propagated along all of the distances in the same instant. Thus, the motion will be propagated more swiftly, and hence with greater strength, in BC than in CD, and in CD than in DE (*TO* prop.3; *OL* V, 220). It is worth noting that, at this early stage in his scientific career, Hobbes had developed the idea that motions occurring in an instant can have greater or lesser velocities.

Hobbes goes on to clarify that we do not call the motion from the luminous object light until it is propagated back from the brain towards the luminous object (*TO* prop.3: *OL* V, 220-21). Light is the phantasm produced by the dilation of a luminous object, not the motion itself. This is confirmed by the fact that we can experience the phantasm that we call light in the absence of a dilating object, as, for example, when the optical nerve is disturbed by a vigorous shaking of the head. To recall, in Hobbesian physics we begin with sensible qualities, which are all nothing but phantasms in sensing beings,[55] then reason backwards to their possible causes. Light, though in fact the phantasm of a lucid body, is thus a legitimate subject of physical inquiry.

It should be noted that in *De Corpore* Hobbes abandons the idea that light is caused by the *systole* and *diastole* of a luminous body. He instead posits that it is the result of simple circular motion on the part of a luminous body. So, for example, the light of the sun is caused by its simple circular motion, whereby the sun pushes away the surrounding matter, sometimes in one direction, sometimes in another, generating motion which is propagated to the eyes of sentient beings. (*DCp* IV.27.2; *OL* I, 364-5). Shapiro argues that Hobbes abandoned his theory of expansion and contraction because, by the time he wrote *De Corpore*, he had become convinced of the impossibility of a vacuum.[56] The impossibility of a vacuum would certainly pose a difficulty for the contraction and expansion model. Hobbes could, as he does in other apparent cases of contraction and dilation, posit that some fine substance rushes to fill in the spaces that must be created when a body seems to expand. This would make it difficult, however, to explain why the expansion would generate a propagated motion, since the displaced medium would be supposed to rush inward, rather than outward.

There are other physical difficulties that follow if we assume that the motion of the luminous body is like that of the heart. Most notably, on this model, rays are emitted from the centre of the radiant body, and not in every direction from each point on the surface of the luminous body. As a consequence, only part of the body will be visible from any given direction.[57] This problem is, however, hardly avoided by the simple circular motion hypothesis, given that simple circular motion produces propagated motions successively in various directions.

In the end, it is likely that Hobbes modified his hypothesis in order to fit his account of light into the general explanatory framework of *De Corpore* which, as we have seen, placed a great deal of emphasis on simple circular motion. As I noted in chapter 5, Hobbes often seems to be comparing his own explanatory device with Descartes's vortex theory. It is therefore interesting that for Descartes light consists in the pressure exerted due to centrifugal force by which aetherial particles strive away from the centres of their vortices.[58] In the *De Corpore* discussion of light Hobbes is again contrasting Descartes's use of vortices with his own use of simple circular motion.

6.3 Rays and Refraction

As I discussed above, there are a number of significant inconsistencies between Descartes's account of the physical nature of light and his mathematical treatment of refraction. In this section I will describe Hobbes's attempts to provide a more unified account of light, based on his notion of a ray. As I will discuss, Hobbes's account is not only more coherent than Descartes's, it was also a significant precursor to the wave theory of light.

At the end of the third proposition, Hobbes introduces his definition of a "ray":

> I call a *ray*, the path through which motion from a luminous body is propagated through a medium. For example, let there be a luminous body

> AB, by whose motion towards CD the part of the medium which is interposed between AB and CD, is pushed forward to EF: and from that part of the medium which was between CD and EF, moved forward further to GH, propelled that part which was between EF and GH, forward to IK, and so on, either in a straight line or not, suppose towards LM. Now the space which is contained between the lines AIOL, and BKM, is that which I call a ray, or the path through which motion from a luminous body is propagated. (*TO* prop.3; *OL* V, 221-2)[59]

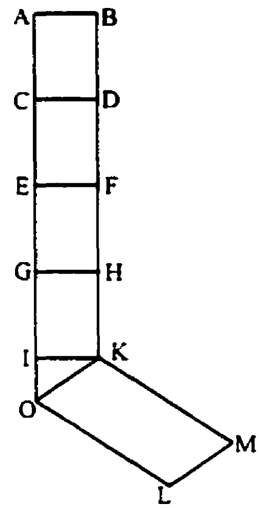


Figure 6.3

As I discussed in the previous section, the concept of a "ray" was a contested notion in optics. In light of this, there are several notable features of Hobbes's definition: first, for Hobbes rays are propagated from the luminous or illuminated body to the eye. Hobbes's rays are rays of light, rather than visual rays.

Second, as he states in his fourth proposition, for Hobbes a ray is solid space, since "a ray is the path through which motion is projected from a luminous body, it can only be the motion of a body: it follows that the ray is the place of a body, and consequently has three dimensions. A ray is therefore solid space" (*TO* prop.4; *OL* V, 222).[60] As we have seen, this was not an uncommon view, having been held by Ptolemy, al-Kindi, and Ibn al-Haytham, as well as some medieval thinkers.[61]

However, there are other respects in which Hobbes's notion of a ray represents a break with tradition: first, a Hobbesian ray is the path of a motion, and is hence generated successively. As we have seen, for those in the Aristotelian tradition, a ray represents a single qualitative change which occurs all at once. Furthermore, Descartes held that light, as a form of static pressure, is propagated instantaneously. For Hobbes, by contrast, the generation of a ray takes some finite period of time.

The related notion of a "line of light" (*linea lucis*) is especially important for Hobbes's account of refraction. The line of light is that line from which the sides of a ray begin (for example, AB in figure 6.3). In addition, those lines "which are derived from the line of light by a continual protrusion [*protrusione*], such as CD, EF, etc." (*TO* prop.4; *OL* V, 222)[62] are also called "lines of light." Hence lines of light represent each successive outer boundary of the propagated motion, and these lines are always normal to the sides of the ray. As such, they bear a strong resemblance to the concept of a wave front.[63]

Hobbesian rays can be either "straight" or "refracted":

> A ray is *straight* which, cut by a plane through its axis, produces in the cut plane the figure of a parallelogram, as AK. A ray is *refracted*, which is composed from straight rays making an angle, together with an intermediate part: as the ray AM is called *refracted*, because it is composed from the straight rays AK and KL, together with the part IKO.(*TO* prop.4; *OL* V, 222)[64]

If a cross-section of a ray, taken through that ray's axis, is a paralleogram, that ray is straight. This seems at odds with Hobbes's understanding of the luminous body, since his notion that rays emerge from the centre of a sphere suggests that those rays will be conical. As I will discuss in section 4, Hobbes will argue that the width of a ray is so small that we can treat its sides as if they were parallel. A refracted ray is formed from two straight rays with a intermediate part. As we will see, although the above figure suggests that the intermediate part will be triangular, Hobbes thinks that it must, in reality, be a portion of a sector.

With these definitions in hand, Hobbes offers his explanation of the physical causes of refraction, appealing to the varying velocities with which propagated motions travel in different media (*TO* prop.4; *OL* V, 223-4). His explanation relies on his fifth hypothesis, which states that a rarer medium is that which is less unyielding to motion, a denser medium that which is more unyielding (*TO* hyp.5; *OL* V, 218).[65] This is, of course, the opposite of Descartes's claim that light penetrates a denser medium more easily. Hobbes offers no argument for this claim, but the implication of his hypothesis seems simply to be that a denser medium will offer more resistance to, i.e., be less yielding to, motion than a rarer medium.

Hobbes's account of refraction includes both a physical explanation of the phenomenon and a pair of comparisons illustrating this explanation. He begins by noting that if each part of the line of light AB moves towards CD with equal swiftness, it will describe a parallelogram. It will thus be as if AB were a cylinder being rolled from AB towards CD. AB will behave in this way if it is moving in a uniform medium, since all of its parts will be moving with the same velocity. If, on the other hand, AB should enter a different medium obliquely, part of AB will move with a different velocity from the rest of the line of light. Its path will the same as that traced by a rolling cone with the bases AE and BF, i.e., its path will trace the figure AHRB (see figure 6.4).

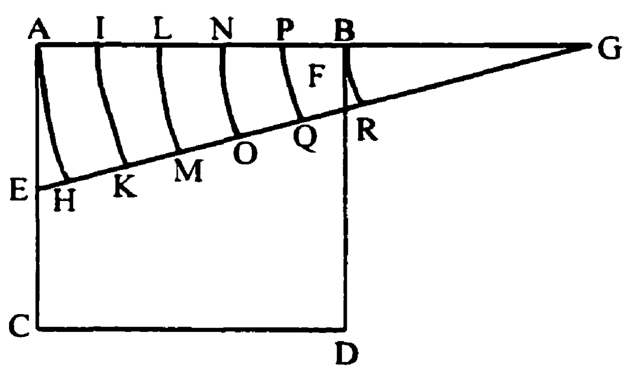


Figure 6.4

Hobbes's explanation of refraction is thus based on the idea that if a ray enters a different medium obliquely, its ends will move with different speeds. If the ray enters a denser medium, the end of the ray that enters first will be slowed down, and the ray will be refracted towards the perpendicular. If, on the other hand, the ray enters a rarer medium, the end of the ray that enters first will speed up, and the ray will be refracted away from the perpendicular. This is an intuitively plausible account of refraction, and, as we will see, it has a significant influence on other seventeenth-century theorists.

The cone model is problematic, however, because it suggests that the various points on the line of light do not slow down because they enter the dense medium, but that some points are always slower than other because of their spatial relationship with the other points on the line. Furthermore, the analogy contradicts the fact that one would expect the line of light to bend and change width as each of its parts enters the second medium. These difficulties with the cone analogy are indicative of problems with two of Hobbes's assumptions about the nature of the line of light, namely, that must it be continuously straight and of constant width. Hobbes offers no argument for these significant assumptions.

Hobbes's treatment of rays and refraction, having been published in Mersenne's *Optica*, found a receptive audience in Robert Hooke. Hooke discusses the nature of light in his *Micrographia* (1665), in the context of an account of the colours of thin, transparent bodies. Hooke is often cited as an important precursor to Huygens's wave theory. Sabra, for example, states that "it was Hooke's merit to have introduced the concept of a wave-front; and, by considering what the wave-front undergoes in passing from one medium into another, he has replaced Descartes' comparisons with a clear mechanical picture that was later more successfully used by Huygens."[66] Hooke was, however, clearly influenced by Hobbes's earlier proto-wave theory.[67]

Hooke, like Hobbes and Descartes, thought that light must be propagated through a medium. In the *Micrographia* he presents five remarks explaining how this must occur. The fourth and fifth remarks are most important for our purposes: the fourth states that "the motion is propagated every way through an *Homogenous medium* by *direct* or *straight* lines extended every way like Rays from the centre of a Sphere."[68] The fifth remark asserts that

> in an *Homogenous medium* this motion is propagated every way with *equal velocity*, whence necessarily every *pulse* or *vibration* of the luminous body will generate a Sphere, which will continually increase, and grow bigger, just after the same manner (though indefinitely swifter) as the waves or rings on the surface of the water do swell into bigger and bigger circles about a point of it, where by the sinking of a Stone the motion was begun, whence it necessarily follows, that all the parts of these spheres undulated through an *Homogenous medium* cut the Rays at right angles.[69]

Hooke, like Hobbes, understands light to be a series of pulses generated by the swelling motion of a spherical body and propagated through a medium. The waves are, at each point, perpendicular to the rays or direction of propagation.

Hooke uses his notion of a "pulse" to explain refraction:

> But because all transparent *mediums* are not *Homogeneous* to one another, therefore we will next examine how this pulse or motion will be propagated through differingly transparent *mediums*. And here, according to the most acute and excellent Philosopher *Des Cartes*, I suppose the sign [sine] of the angle of inclination in the first *medium* to be to the sign of refraction in the second, As the density of the first, to the density of the second. By density, I mean not the density in respect of gravity (with which the refractions or transparency or *mediums* hold no proportion) but in respect onely to the *trajection* of the Rays of light, in which respect they only differ in this; that the one propagates the pulse more easily and weakly, the other more slowly, but more strongly. But as for the pulses themselves, they will by refraction acquire another propriety, which we shall now endeavour to explicate.
>
> We will suppose therefore in the first Figure [figure 6.5] ACFD to be a physical Ray, or ABC and DEF to be two Mathematical Rays, *trajected* from a very remote point of a luminous body through an *Homogenous* transparent *medium* LLL, and DA, EB, FC, to be small portions of the orbicular impulses which must therefore cut the Rays at right angles; these Rays meeting with the plain surface NO of a *medium* that yields an easier *transitus* to the propagation of light, and falling *obliquely* on it, they will the medium MMM be refracted towards the

perpendicular of the surface. And because this *medium* is more easily *trajected* then the former by a third, therefore the point C of the orbicular pulse FC will be mov'd to H four spaces in the same time that F the other end of it is mov'd to G three spaces, therefore the whole refracted pulse GH shall be *oblique* to the refracted Rays CHK and GI.[70]

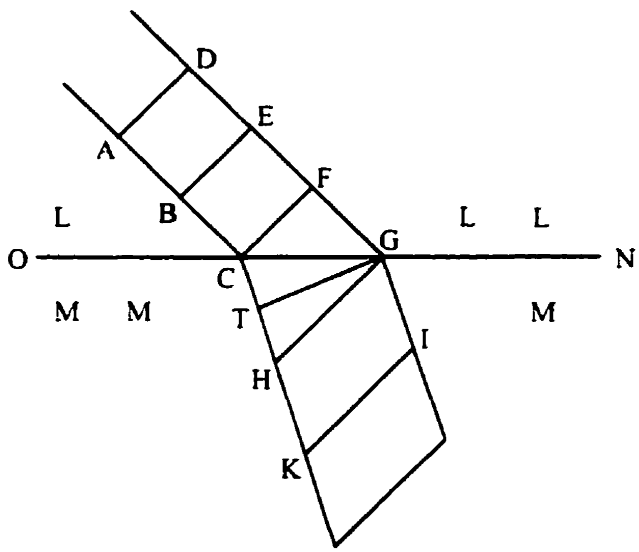


Figure 6.5

Like Hobbes, Hooke, when discussing refraction, considers a ray with parallel sides, despite the fact that that ray is a "small portion of the orbicular impulses." He argues that refraction occurs because, when a pulse enters a different medium obliquely, parts of that pulse will, for a time, be travelling with different velocities.

Hooke does depart from Hobbes on some important points. Like Descartes, he holds that light moves faster in a denser medium.[71] As we can see from the above passage, Hooke also thought that after being refracted, the ray front would be at an oblique angle to the direction of propagation. Hooke accounted for colours by positing that white light is the result of uniform pulse at right angles to the direction of propagation, while colours result when pulses are disturbed and "obliquated" by refraction.[72] He criticized the Hobbesian notion that a ray's pulse must always be perpendicular to the direction of propagation, arguing that this makes it impossible to explain the perception of colours.[73]

In the above passage, Hooke simply assumes Descartes's sine law. Hobbes, however, sets out to demonstrate the law based on his own account of light and its relative speed in rarer and denser media. Hobbes sets the stage for his proof in proposition 11, which states:

> *If there are any two inclinations of rays from the same rare medium to the same dense medium, or vice versa, and the common surface of the media is a plane: the progress of the light in the first medium to the progress of the light made in the same time in second, will have the same ratio in the one inclination as in the other.* (*TO* prop.11; *OL* V, 236)[74]

This proposition states that the ratio of the progress of light in the first medium to the progress of light in the second medium is a constant, independent of inclination. The progress of light is simply the distance traversed by a line of light. It can be measured in two ways: either by considering the actual distance traversed, or by considering a straight line, perpendicular to the refracting surface, and drawn from the initial position of the ray to that surface.

Let (in figure 6.6) there be a ray AC, with the line of light AB, forming the angle

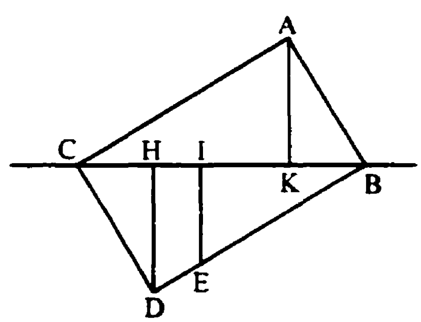


Figure 6.6

ACB with the plane CB. Let the parallelogram ABCD be completed. Let the plane CB be assumed to separate two different media, and let the rarer medium be that above CB, the denser below. The progress of light in the denser medium is represented by AC, or AK if the progress is measured perpendicularly. Similarly, let the progress of light in the second medium (in the same time) be BE, or IE if measured perpendicularly. Hobbes intends to demonstrate that the ratio of AC to BE is the same as the ratio of AK to IE, and that that ratio is a constant.

Hobbes assumes that the velocity of light in a medium is wholly determined by the density of that medium. Accordingly, the progress of light in a given medium will always vary with the time of the motion. But Hobbes has stipulated that the time in which the light travels from A to C is the same time in which it travels from B to E. Hence the ratio of AC to BE is a constant. From the equality of AC and BD, and the similarity of the triangles BDH and BEI:

$$(AC/BE) = (BD/BE) = (HD/IE)$$

But AK is equal to HD, and therefore:

$$(AC/BE) = (AK/IE)$$

Thus the ratios of the distances travelled by a line of light in two media will be the same, regardless of how that distance is measured, and that ratio will be the same for any inclination of the ray. This proof can easily be adjusted to show that the same will hold if the line of light is moving from a denser to a rarer medium.

The sine law itself is the subject of proposition 12:

> *If there are two inclinations of rays from the same rare medium to the same dense medium, or vice versa, and the common surface of the media is a plane, it will be as the sine of the angle of inclination to the sine of the angle of refraction in one inclination, so the sine of the angle of inclination to the sine of the angle of refraction in the other inclination.* (*TO* prop.12; *OL* V, 230)[75]

Let CB (in figure 6.7) be a plane surface dividing two media, with the denser below, and the rarer above. Let AC be a ray with line of light AB, incident to the plane with an angle of incidence ACO. Let CK be the refracted ray from AC, with an angle of refraction KCL. Hobbes intends to demonstrate that the ratio of the sine of ACO to the sine of KCL is a constant, regardless of inclination. Let AB be drawn perpendicular to AC, cutting the

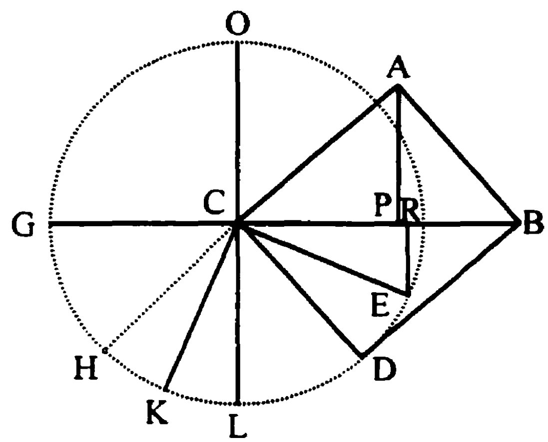


Figure 6.7

plane at B, and let the parallelogram ABCD be completed. Let CE be drawn perpendicular to CK at point C, and let CE be equal to CD or AB. Therefore the circle drawn with the centre C and radius CE will go through D.

Because CE is perpendicular to the refracted ray CK, and equal to the line of light AB, AP drawn perpendicular to the plane CB will be the progress of the light in the rare medium, ER the progress of the light in the dense medium. But according to proposition 11, the ratio of AP to ER is a constant.

Now Hobbes has to demonstrate that AP is the sine of the angle of inclination ACO, and ER is the sine of the angle of refraction KCL. The angles OCA and ACB together make a right angle, as do ABC and ACB. Hence ABC is equal to the angle of inclination OCA. Since the circle was constructed with the radius BA, which can be stipulated as unity, AP is the sine of the angle ABC, and hence of the angle of inclination. By a similar procedure it can be shown that ER is the sine of angle of refraction.[76]

Hobbes's proof is less than elegant. However, as Shapiro has shown. Isaac Barrow and Emanuel Maignan developed simpler and clearer proofs of the sine law based on Hobbes's approach to refraction.[77] Although Hobbes's priority has not always been acknowledged, either by early modern theorists or contemporary scholars, his work on light had a significant influence on seventeenth-century optics.

6.4 Mathematical Techniques in the Account of Refraction

As I have discussed, Hobbes achieved some success with his theory of light: his theory represent a relatively coherent mechanistic alternative to Descartes's account of light and refraction. Furthermore. Hobbes's approach was a notable precursor to the wave theory of light.

In developing this approach, Hobbes relied heavily a way of thinking of about bodies in motion which allows him to treat physical rays as the objects of mathematics. This aspect of Hobbes's optics holds particular interest in light of the topics discussed in previous chapters, as it resembles the approach that he took in *De Corpore*'s geometrical account of motion.

As we have seen, for Hobbes a ray is the path of a propagated motion, and, as such, has breadth. However, Hobbes also holds that, in some circumstances, a ray can be considered as a mathematical line. A ray is so considered when we disregard its breadth (although it must have some) for the sake of demonstration. For example, the fifth proposition of the *Tractatus Opticus* states that "*a ray falling perpendicularly upon a plane surface, can be considered as a mathematical line: but falling upon the same surface obliquely, it must be considered as having breadth*" (*TO* prop.5; *OL* V, 225).[78] Hobbes asks us to consider the ray ABCD (in figure 6.8)[79] falling upon the plane BD. AB and CD are equal and both perpendicular to BD, and they are separated by the equal lines AC and BD. When the ray falls upon BD, all the points on the line of light will slow down at the same time and to the same degree. We need not consider the ray's breadth,

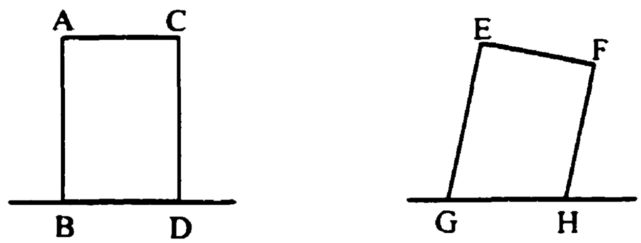


Figure 6.8

because the same thing will happen at each point of that breadth. In other words, we can consider the ray as a mathematical line. On the other hand, if the ray EFGH falls obliquely on the plane surface GH, we must consider separately what happens to the sides FH and EG. The ray is refracted because its sides enter the denser medium at different times and hence, for some interval, have different velocities. If EFGH were considered as a mathematical line, it could not be considered as having width, and thus the differences in velocities between the two sides could not be considered. Although Hobbes thinks that rays cannot always be treated as mathematical lines, he already has the concept of a line that he would appeal to in *De Corpore*.

Hobbes also appeals to this notion of a line in the *Tractatus Opticus II*. Furthermore, in discussing the ways in which we can consider a luminous body he refers to the familiar idea of a point as a body whose magnitude is not considered. When Hobbes, in the *Tractatus Opticus II*, introduces his ray concept, he differentiates it from previous understandings of what a ray is (in the *Tractatus Opticus II* Hobbes decides to use the term "radiation" to distinguish his own concept from those of his predecessors). Having argued that a ray cannot be characterized as a mathematical line, he concludes:

> Therefore a ray is not length without breadth, but a solid, whose length is terminated by the surface of the luminous or radiating body; although it is sometimes possible for that luminous body to be considered not as a surface, but as a point, namely when by ratiocination, the magnitude of the

> object or luminous body is not considered; something is not called a mathematical point, or line, or surface because it is without dimensions, but because they are not assumed in argument. (*TO II* II.1; 160)[80]

Again, although rays and luminous bodies have magnitude, that magnitude can (in certain situations) be disregarded for the sake of mathematical demonstration.

In Hobbes's early optical work we also see rays being characterized, as lines are in *De Corpore*, as things whose breadth is "less than any given magnitude." In the *Tractatus Opticus II*, Hobbes describes the relationship between actual rays (or radiation) and the "wide lines" that he considers in his demonstrations, stating that

> although I here consider that radiation which is made by any small and imperceptible part of a luminous body, nevertheless because both dimensions [length and breadth] must sometimes be so contemplated, I give to every irradiation a conspicuous width, as much as is sufficient for the addition in writing of marks, or letters, by which it is possible to name and distinguish every dimension easily, and then, after the demonstration has been completed, each person can, in their imagination, reduce this width to the thinness of lines. (*TO II* II.2; 160)[81]

The figures that we use to represent rays or radiation must have perceptible width, in order to allow for the labelling and proper consideration of their various dimensions. However, we can, by means of our imagination, disregard these visible dimensions, in order to apply our results to a more accurate conception of rays or radiation as imperceptibly thin lines.

This understanding of a ray proved useful in Hobbes's optics. Many of Hobbes's demonstration rely on the assumption that the sides of a given light ray are parallel. This cannot be the case since, according to Hobbes's contraction and expansion model, the rays must be conical. Hobbes admits as much in the *Tractatus Opticus II*, but argues that the differences in width in a ray are insensible, and hence the sides of the ray can be considered parallel (*TO II* II.2; 160).[82]

Although Hobbes thinks of rays in much the same way as he thinks of mathematical lines in *De Corpore*, in his optics he is more likely to exploit the fact that these rays do have breadth, which we can choose to consider or not, depending on the situation. This allows Hobbes to avoid some difficulties that he runs into as a result of his conception of a physical ray. In proposition 7 of the *Tractatus Opticus*, for example, Hobbes demonstrates that a ray falling obliquely on the plane surface of a rarer medium will be refracted away from the perpendicular (*TO* prop.7; *OL* V, 228-30). Having argued that a ray from the line of light AB (in figure 6.9) in a dense medium, having fallen upon the plane surface ED of a rarer medium, would form the refracted ray ABEFNO, he states:

> Now if for AB, the line of light, a magnitude is supposed less than every magnitude proposed, what is demonstrated of the wide line ABEFNO is demonstrated of the [line] drawn AEN. Whereby the ray is refracted from E into N, i.e., in the direction opposed to the perpendicular. And for that reason the ray from the dense medium etc. Which was to be proven. (*TO* prop.7; *OL* V, 230)[83]

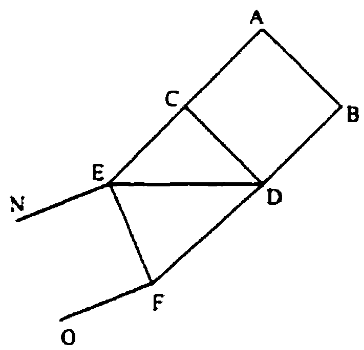


Figure 6.9

Proposition 6, a demonstration that a ray falling from a rare into a dense medium is refracted towards the perpendicular, is concluded in a similar way.[84] As was discussed above, Hobbes holds that, in order to understand the nature of refraction, we must consider rays as entities possessing breadth. However, having demonstrated the sixth and seventh propositions by assuming such breadth, Hobbes asks us to assume that such demonstrations would also hold concerning a ray with a breadth "less than any magnitude proposed." In essence, he proposes that a demonstration concerning a ray with perceptible breadth will also apply to a ray whose breadth is less than any we can imagine or need consider in a demonstration.

This was an important and necessary claim for Hobbes to make. The final proposition of the *Tractatus Opticus* states that "*[t]he refraction of a ray falling obliquely upon a different medium, whose surface is curved, is the same as if it had fallen upon the plane surface touching the curve itself*" (*TO* prop.14; *OL* V, 245).[85] If the breadth of rays always had to be considered, Hobbes would have difficulty accounting for the refraction of rays at curved surfaces, since each point of a ray's breadth would have a different angle of incidence.[86] Descartes objects to Hobbes's manoeuvre in this proposition, stating in a letter that Hobbes's

> main error lies in his explanation of the physical cause of the refraction of rays: it is completely illusory, and contrary to the principles of mechanics. Illusory, because it is based on the breadth which he gratuitously attributes to rays (and which, in his fourteenth proposition, he takes away from them, while saying nevertheless that they are refracted in the same way).[87]

Descartes objects both to Hobbes's claim that a ray must have breadth, and to Hobbes's habit of abstracting away that breadth in some proofs.

As the passages that I have discussed in this section show, in his optics Hobbes thinks of a light ray as the path of a motion through a medium. This path must have breadth, but that breadth can be disregarded for the sake of demonstration. This is, of course, very close to the notion of a mathematical line that Hobbes appeals to in *De Corpore*. In Hobbes's optics we therefore find aspects of the understanding of geometrical objects that would prove so central to his mathematical mechanics. It is impossible to say for sure whether Hobbes already had his notion of a mathematical line prior to his work on light, or if he developed this concept in the context of his theorizing about optics. At the very least, I suspect that Hobbes was greatly encouraged by the success of his mathematical account of light. Since the study of light is, for Hobbes, just the study certain types of matter in motion, Hobbes's sucesses in mathematizing this science may have emboldened him in his attempts to apply geometry to mechanics in general.

6.5 Refraction in *De Corpore*

I have been suggesting that a significant aspect of Hobbes's geometrical mechanics is anticipated by his work in the optics, and that his relatively successful account of light and refraction may have encouraged Hobbes to think that his approach in his optics could be applied to the study of motion in general. These claims are rendered problematic by the fact that Hobbes, in *De Corpore*, presents a much different account of refraction than the one which appears in the *Tractatus Opticus*. Why, if Hobbes was so pleased with his approach in the *Tractatus Opticus*, would he change it in *De Corpore*, where (as I claim) he seems to apply aspects of the *Tractatus* approach to other subjects?

The *De Corpore* account of refraction occurs in the final chapter of *De Corpore*'s third part (which, to recall, contains Hobbes's mathematics). The chapter, which is titled "Of Refraction and Reflection," begins with a set of definitions, from which it is immediately clear that Hobbes's approach has changed from his earlier optical works. Refraction is defined as "the breaking [*fractio*] of a line, along which a moved body or its action would proceed in one and the same medium, into two straight lines according to the different nature of the two media" (*DCp* I.24.1; *OL* I, 305).[88] There is no mention of a ray — instead, Hobbes refers to refraction as something that occurs to the *line* along which something moves. Similarly, in later definitions he refers to the "line of incidence" and the "refracted line." As we have seen, in the *Tractatus Opticus* Hobbes claims that a light ray can sometimes be treated as a mathematical line. In *De Corpore*, on the other hand, it seems that the refracted path of a moving body or propagated motion will always be described as a line.

This change is indicative of a shift in *De Corpore* towards a more general account of refraction. He is concerned not just with the refraction of light, but with the refraction of all motions and moved bodies. Hence Hobbes begins chapter 24 by discussing the behaviour of moving bodies, such as stones (*DCp* I.24.2-3; *OL* I, 306-308). His treatment is similar to Descartes's, which, to recall, Hobbes had earlier criticized as being irrelevant to the study of light.

Similarly, in section 4 of the chapter Hobbes presents a demonstration of a version of the sine law. However, the *De Corpore* version of law applies to any propagated motion passing from one medium to another:

> If in any medium it should be supposed that from some one point an endeavour is propagated at the same time in every direction and into all the parts of the medium; and to that endeavour there should be obliquely opposed a medium of a different nature, that is either rarer or denser; that endeavour will be so refracted, that the sine of the angle of refraction will be to the sine of the angle of inclination, as the density of the first medium is to the density of the second, taken reciprocally. (*DCp* I.24.4; *OL* I, 308)[89]

Since light is one example of propagated endeavour, this law will apply to light, but it will also apply to other forms of propagated motion. Again, it is clear that Hobbes is trying to develop a more general account of refraction than the one that he presented in the *Tractatus Opticus*.

In his proof, Hobbes considers (in figure 6.10) an endeavour propagated from A to B, which then falls upon the surface DH of a denser medium, and is refracted along BI. Hobbes adopts an approach similar to that of Ibn al-Haytham, Kepler, and Descartes, and divides AB and BI into components perpendicular and parallel to the refracting surface: in the case of AB, BF and AF, respectively, and in the case of BI, BH and BK. While Descartes assumed that the parallel component remains constant during refraction, Hobbes, perhaps following Ibn al-Haytham or Kepler, assumes that it is the perpendicular component of the endeavour is that is constant. Since Hobbes assumes that light moves

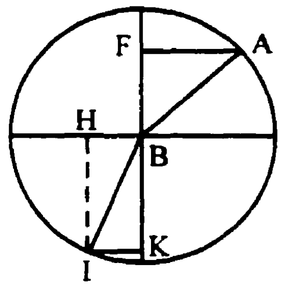


Figure 6.10

more slowly in the denser medium, the endeavour will be propagated through AF in the rarer medium, but only through IK in the denser medium. Let d_i and d_r represent the densities of the upper and lower media. Hobbes assumes that the ratio of the distances traversed by the light in each of the media will be inversely proportional to the ratio of the media's densities. Hence,

$$(d_i/d_r) = (BH/AF) = (IK/AF)$$

Let i be the angle of incidence and r the angle of refraction. Since AF = sin i and IK = sin r,

$$(IK/AF) = (\sin r/\sin i)$$

We therefore have,

$$(d_i/d_r) = (\sin r/\sin i)$$

Since d_i/d_r is a constant, this is the sine law.

There are a number of difficulties with Hobbes's proof. As Shapiro points out, the motions from A to B and B to I are supposed to occur in the equal times, and hence their distances are proportional to their velocities. However, since AB and BI are equal, their velocities must also be the same, which is contrary to Hobbes's assumption about the velocities of endeavours in rarer and denser media. Furthermore, it is a consequence of Hobbes's approach that the perpendicular component of the motion BI is increased while its parallel component is decreased.[90]

It is difficult to say why Hobbes abandoned his earlier approach to refraction. I suspect that he was trying to fit his treatment of refraction into the scientific system which he describes at the outset of *De Corpore*. As is clear by the very general account discussed above, when he wrote *De Corpore* Hobbes had apparently become convinced that refraction is not something that light alone undergoes: any moving body or propagated motion will change direction when it moves from one medium to another of a different density. On this understanding of refraction, it is a subject for Hobbesian geometry, or the study of the most general properties of motion. Hobbes does offer separate accounts of the refraction of moving bodies and propagated motions. He is still left, however, to provide a proof of the sine law for all propagated motions. Such motions can, of course, be of a number of kinds — most importantly for our purposes, they can begin with the motion of either a single point or a line. In the *Tractatus Opticus* account of refraction, Hobbes assumes the latter case, and can hence appeal to the breadth of the propagated motion in his proof. In *De Corpore*, however, he is attempting to provide a more general account, suitable for his mathematics. He thus presents a proof of the sine law in terms of point motions. As we have seen, for Hobbes the motion of a line, surface, or solid is in some sense made up of the motion of its constituent points, so he presumably thought that his account of refraction could be generalized to account for other types of propagated motion.

The changes in Hobbes's account of refraction from the *Tractatus Opticus* to *De Corpore* illustrate the nature of his scientific project. As I noted at the beginning of this dissertation, Hobbes, unlike Galileo, was less interested in applying mathematics to particular problems than in building a comprehensive system based on a mathematical account of the nature of body and motion. Hobbes was determined enough to build such a system that he was willing to abandon one of his most notable scientific results. With the first volume of his elements of philosophy, Hobbes had hoped to become the Euclid of the new mechanical philosophy. Unfortunately, the world cannot be drawn with a straight-edge and compass.

ENDNOTES TO CHAPTER 6

1. Brandt (1927, 87-99)

2. British Museum, Harlean 6796, ff. 19-266.

3. Hobbes ([1889] 1969).

4. There is no complete edition of the *Tractatus Opticus* currently available. I will refer to the version published by Franco Alessio in the *Rivista critica di storia della filosofia* (Alessio 1963. 147-228). This version is, however, of limited use, since it does not include the manuscript's figures.

5. My account in this section is indebted to Lindberg (1976) and Sabra (1981).

6. Lindberg (1976. 2-3).

7. On the atomists' theory of vision. see Lindberg (1976. 2-3).

8. Aristotle states that "light is neither fire nor any kind whatsoever of body nor an efflux from any kind of body" (*De anima* II.7. 418b14-15).

9. *De anima* II.7. 418b5-6.

10. *De anima* II.7 418b8-13.

11. Aristotle thus criticizes Empedocles. stating that he "was wrong in speaking of light as 'travelling' or being at a given moment between the earth and its envelope. its movement being unobservable by us; that view is contrary both to the clear evidence of argument and to the observed facts" (*De anima* II.7, 418b20-4).

12. On Euclid's theory of vision. see Lindberg (1976. 12-4), Ronchi (1970, 15-23), and Park (1997, 55-8).

13. Cohen and Drabkin (1958, 257-8).

14. Lindberg (1976. 10).

15. Although. as Lindberg points out (Lindberg 1976. 13-4). some of the *Optica*'s postulates do have implications regarding the nature of visual rays: for example. from his statements that visual rays proceed from the eye. it is clear that Euclid holds an extramission theory of vision.

16. On Ptolemy's theory of vision, see Lindberg (1976, 15-7), Park (1997, 63-8).

17. Lindberg (1976, 15). Lindberg is drawing on the work of Albert Lejeune (see Lejeune 1948 and Ptolemy 1956).

18. On this aspect of Ptolemy's work, see Sabra (1981, 93).

19. On Ptolemy's refraction experiments, see Park (1997, 66-8).

20. Lindberg states that "Ptolemy express no explicit interest in the cause of refraction — one could argue that Book V of his *Optica* is related to the study of refraction as his *Almagest* is related to the science of astrophysics" (1969, 24).

21. An extramission theory of vision is one which posits that vision occurs because the eye sends forth a power or ray to the object. On the other hand, intromission theories hold that the object of vision sends its image or ray through the medium to the eye.

22. On al-Kindi's theory of vision, see Lindberg (1976, 18-32).

23. Lindberg (1976, 24-6).

24. On al-Kindi's account of the nature of visual rays, see Lindberg (1976, 30-1) and Park (1997, 74).

25. On Ibn al-Haytham's theory of vision, see Lindberg (1976, ch.4) and (1967, 322-30).

26. This aspect of Ibn al-Haytham's account of refraction played an important role in his theory of vision. As I noted above, Ibn al-Haytham held that light or colour radiates from every point on the surface of a luminous or illuminated body. This proposal raises immediate problems, however, since every point on the eye will receive numerous rays at any point — Ibn al-Haytham has to explain how this could result in any kind of coherent perception. He thus noted that although many rays will fall upon any point on the surface of the eye, only one of these rays will be incident upon it perpendicularly, and hence proceed into the eye unrefracted. All of the unrefracted rays will together form a cone with the visual field as a base and the apex at the centre of the eye. The rays, being rectilinear, will maintain their order, thus allowing a coherent image of the perceived bodies to form in the eye.

27. Again, Ibn al-Haytham seems to be building on Ptolemy's account, since Ptolemy had also compared the behaviour of light moving one medium to another with the motion of a projectile.

28. Quoted in Lindberg (1976, 75).

29. Lindberg notes some difficulties with these analogies, most notably that "they have nothing to do with the proposition they are meant to demonstrate. Both concern *impact on* a body, not *transmission through* a medium" (1968, 27).

30. Ibn al-Haytham also took this approach in his treatment of reflection (Sabra 1981, 75-6).

31. Sabra (1981, 98).

32. It should be noted that although Kepler drew on Ibn al-Haytham's account of refraction, he greatly improved Ibn al-Haytham's intromission account of vision, and developed the theory of the retinal image. On Kepler's theory of vision, see Lindberg (1976, ch.9).

33. See, for example, Hobbes (1994a, vol.1, 28-9, 33-4, 37-8).

34. Hobbes (1994a, vol.1,34, 38).

35. There was, and continues to be, debate over whether it is Descartes or Snell that first discovered this law (they seem to have detected the law almost simulataneously). For a discussion of this debate, see Sabra (1981, 99-105).

36. *Op* II; *CSM* I, 155.

37. In the first discourse of his *Optics*, for example, Descartes states:

> No doubt you have had the experience of walking at night over rough ground without a light, and finding it necessary to use a stick in order to guide yourself. You may then have been able to notice that by means of this stick you could feel the various objects situated around you, and that you could even tell whether they were trees or stones or sand or water or grass of mud or any other such thing. It is true that this kind of sensation is somewhat confused and obscure in those who do not have long practice with it. But consider it in those born blind, who have made use of it all their lives: with them, you will find, it is so perfect and so exact that one might almost say that they see with their hands, or that their stick is the organ of some sixth sense given to them in place of sight. In order to draw a comparison from this, I would have you consider the light in bodies we call 'luminous' to be nothing other than a certain movement, or very rapid and lively action, which passes to our eyes through the medium of the air and other transparent bodies, just as the movement or resistance of the bodies encountered by a blind man passes to his hand by means of his stick. (*Op* II; *CSM* I, 153)

38. Hobbes (1994a, vol.1, 91).

39. *Op* II; *CSM* I, 158.

40. *Op* II; *CSM* I, 155.

41. Pierre de Fermat, for example, wrote in a letter to Mersenne that "it seems that there is a particular disproportionality in that the motion of a ball is more or less violent, according as it is pushed with different forces, whereas light penetrates the diaphanous bodies in an instant and seems to have nothing successive in it" (Quoted in Sabra 1967, 112).

42. Hobbes (1994a, vol.1, 58). Hobbes reiterates his concerns in a subsequent letter (1994a, vol.1, 77), where he objects that a body moving from medium to medium is refracted in a different direction then when a motion is propagated through a medium.

43. *Op* II; *CSM* I, 162-3.

44. Omnis actio est motus localis in agente, sicut et omnis passio est motus localis in patiente. *Agentis* nomine intellego *corpus*, cujus motu producitur effectus in alio corpore: *patientis*, in quo motus aliquis ab alio corpore generatur.

45. Item dum carbo ignitus calefacit hominem, etsi neque carbo neque homo suo loco exeat, neque ideo moveatur, est tamen aliquid materiae sive corporis subtilis in carbone, quod movetur, et motum ciet in medio usque ad hominem; et est et in homine stante immoto, motus aliquis in partibus internis inde generatus. Motus autem hic in partibus hominis internis est calor; et sic moveri, calefieri, hoc est pati; et motus ille qui est in partibus carbonis igniti, est actio ejus, sive calefactio; et sic moveri, calefacere.

46. Some theorists had appealed to action at a distance to explain how we perceive light. This explanation occurs, for example, in Ockham's account of vision (Lindberg 1976, 142).

47. Vision est passio producta in vidente per actionem objecti lucidi vel illuminati.

48. In visione, neque objectum, neque pars ejus quaecunque transit a loco suo ad oculum.

49. Ut motus possit motum generare ad quamlibet distantium, non est necessarium ut corpus illud a quo motus generatur, transeat per totum illud spatium quod motus propagatur: sufficit enim ut parum, imo insensibiliter motum, protudat id quod proxime adstat; nam id quod adstat, pulsum suo loco, pellit quoque quod est proximum sibi, atque eo modo motus propagabitur quantum libueris.

50. *Omne lucidum dilatat se, tumescitque in molem majorem, iterumque contrahit se, perpetuam habens systolem et diastolem.*

51. This analogy is made explicitly in the Harlean manuscript: "Supponendum ergo ulterius est Lucidum omne non modo se dilatare sed etiam contrahere, nimirum alternis vicibus; quem ad modum Cor humanum alterna illa contractione et dilatatione quae vocatur *systole* et *diastole*, pellit continuo et protrudit sanguinem per arterias" (*TO II* 1.8; 150).

52. ...restat ut partes lucidi quas ostensum est moveri versus omnes partes simul, se iterum recipiant.

53. *Considerare quomodo fiat lumen, et quid sit.*

54. An orb (*orbis*) being defined as a solid contained between two concentric spherical surfaces.

55. In *De Corpore* Hobbes thus states that "light and *colour*, and *heat*, and *sound*, and the other qualities, which are usually called sensible, and not objects, but phantasms of the sentient being" (lux enim et *color*, et *calor*, et *sonus*, et caeterae qualitates, quae sensibiles vocari solent, objecta non sunt, sed sentientium phantasmata) (*D.Cp.* IV.25.3; *O.L.* I, 319).

56. Shapiro (1973, 461).

57. It is difficult to see why Hobbes did not see this problem with his model, given that Kepler had already established that light must be emitted in every direction from every point of a luminous body. Shapiro (1970, 149) posits that this difficulty may have been the result of Hobbes's basing his account of light on a theory of sound: "This is exactly the sort of confusion which would arise if a theory of light were modeled too closely in an analogy with sound, as Hobbes's theory appears to be. All the parts of an acoustic source vibrated together, while in an optical source all the points must vibrate independently."

58. *Pr* III 54-64; *M* 111-18.

59. *Radium* appello, viam per quam motus a lucido per medium propagatur. Exempli gratia: sit lucidum AB, a quo moto ad CD pars medii quae interjacet inter AB et CD, protrudatur ad EF: et a parte medii quae erat inter CD et EF, promota ulterius ad GH, propellatur pars illa quae erat inter EF et GH, ulterius ad IK, et sic deinceps, sive directe sive non, puta versus LM. Spatium jam quod continetur inter lineas AIOL, et BKM, et id quod voco radium, sive viam per quam motus a lucido per medium propagatur.

60. Quoniam enim radius est via per quam motus projicitur a lucido, neque potest esse motus nisi corporis: sequitur radium locum esse corporis, et proinde habere tres dimensiones. Est ergo radius spatium solidum.

61. For example, Roger Bacon holds that rays must be three-dimensional, citing the views of Ibn al-Haytham and al-Kindi as support:

> It is not to be understood that the lines along which multiplication occurs do not consist of length alone, extended between two points, but all of them have width and depth [as well], as the authors of books on optics determine. Alhazen demonstrates in his fourth book that every ray coming rom a part of a body necessarily has width and depth, as well as length. Similarly, Jacob Alkindi says that an impression is similar to that which produces it; now, the impressing body has three dimensions, and therefore the ray has [this same] corporeal property. And he adds that rays do not consist of straight lines between which are intervals, but that multiplication is continuous, and therefore it does not lack width. And, in the third place, he says that whatever lacks width, depth, and length is not perceived by sight; therefore a ray [if it were to lack width and depth] would be unseen, which it is not. And we know that a ray must pass through some part of the medium; but every part of the medium has three dimensions. (Bacon 1983, 95)

62. ...quae a linea lucis continua protrusione derivantur, quales sunt CD, EF etc.....

63. Shapiro (1973, 151) makes the point.

64. Radius *directus* est, qui sectus plano per axim, exhibet in plano secante figuram parallelogrammam, ut AK. Radius *refractus* est, qui componitur ex directis angulum facientibus, una cum parte intermedia: ut radius AM *refractus* dicitur, quia componitur ex directis AK et KL, una cum parte IKO.

65. Medium rarius voco quod minus contumax est adversus motum recipiendum: densius quod magis.

66. Sabra (1981, 195). As Sabra notes, this is also the position of Whittaker, who states that "Hooke introduces, moreover, the idea of the *wave-front*, or locus at any instant of a disturbance generated originally at a point, and affirms that it is a sphere, whose centre is the point in question, and whose radii are the rays of light issuing from the point" (Whittaker [1951, 1953], 15).

67. For an extended discussion of Hooke and Hobbes's theory of light, see Shapiro (1970, 189-207).

68. Hooke ([1665] 1961, 56-7).

69. Hooke ([1665] 1961, 57).

70. Hooke ([1665] 1961, 57).

71. On the Cartesian influence on Hooke, see Sabra (1981 186-95).

72. Whittaker ([1951, 1953] 1989, 16).

73. The following comment, from the *Micrographia*, is clearly directed at the Hobbesian account of light, although Hooke does not seem to be aware that the theory is Hobbes's:

> ...that *Hypothesis* which the industrious *Moreanus* [Mersenne] has publish'd about the slower motion of the end of a Ray in a denser *medium*, then in a more rare and thin, seems altogether unsufficient to solve abundance of *Phenomena*, of which this is not the least considerable, that it is impossible from that supposition, that any colours should be generated from the refraction of the Rays; for since by that *Hypothesis* the *undulating pulse* is always carried perpendicular, or at right angles with the Ray or Line of direction, it follows, that the stroke of the pulse of light, after it has been once or twice refracted (through a Prisme, for example) must affect the eye with the same kind of stroke as if it had not been refracted at all. Nor will it be enough for a Defendant of that *Hypothesis*, to say, that perhaps it is because the refractions have made the Rays more weak, for if so, then two refractions in the two parallel sides of a *Quadrangular Prisme* would produce colours, but we have no such *Phaenomena* produc'd. (Hooke [1665] 1961, 100)

74. *Sit sint duae quaelibet inclinationes radiorum ab eodem medio raro ad idem medium densum, vel contra, superficies autem mediorum communis sit plana: progressus lucis in primo medium ad progressum lucis simul factum in secundo, habebit eandem rationem in una inclinatione quam in altera.*

75. *Si sint duae quaelibet inclinationes radiorum ab eodem medio raro ad idem medium densum, vel contra, sitque superficies mediorum communis plana, erit ut sinus anguli inclinationis ad sinum anguli refracti in una inclinatione, ita sinus anguli inclinationis ad sinum anguli refracti in altera inclinatione.*

76. As Shapiro (1973, 258-9) has noted, in his proof of the sine law Hobbes seem to abandon his physical model based on the curved path of a refracted ray. Hobbes does demonstrate that the curved path model is compatible with the approach based on the "progress of light," but this demonstration occurs in proposition 5 of the *Tractatus Opticus*, six propositions earlier than the proof of the sine law.

77. Although, as Shapiro notes, Barrow and Maignan adapt Hobbes's approach to suit their own emission theories of light.

78. *Radius incidens perpendiculariter in superficiem planam, considerari potest tanquam linea mathematica: sed incidens in eandem oblique, considerandus est ut habens latitudinem.*

79. This figure is my own (Hobbes does not provide an illustration for proposition 5).

80. Non est ergo radius longitudo sine latitudine, sed solidum, cuius longitudo terminatur superficie corporis lucidi sive radiantis; quanquam possit interdum illa considerari non ut superficies, sed ut punctum, nimirum cum ratiocinatione, obiecti sive lucidi magnitudo non consideratur; neque dicitur aliquid punctum vel linea, vel superficies mathematica propterea quod dimensionibus careat, sed quia in argumentum non assumuntur.

81. ...quamquam radiationem hic considero eam quae fit a qualibet minima et imperceptibili parte lucidi, tamen quia et sic ambae dimensiones aliquando contemplandae sunt, dabo omni irradiationi latitudinem conspicuam, quantum sufficit adscriptioni notarum, sive literarum, quibis commodius omnis dimensio distingui et nominari possit, quam latitudinem, finita demonstratione, ad exilitatem linearem revocare imaginatione sua unusquisque potest...

82. ...quia cum tota AC linea lucis intelligenda sit ut insensibilis, quamadmodum etiam caeterae lineae lucis ab ea propagate (et latitudo qua pinguntur ad id solum inserviat, ut detur spatium adscribendis notis) differentia latitudinum, quamquam revera aliqua sit (quia omnis irradiatio est conica) insensibilis erit, et proinde latera radiationis pro parallelis haberi possunt.

83. Jam si pro AB, linea lucis, sumatur magnitudo omni magnitudine proposita minor, quod demonstratur de linea lata ABEFNO demonstrabitur de ducta AEN. Quare radius refringitur ab E in N, in partes scilicet aversas a perpindiculo. Et propterea radius e medio denso etc. Quod erat probandum.

84. Jam si pro latitudine AB sumamus latitudinem radii minorem quavis magnitudine data, demonstratio haec eadem existens applicabitur lineae EN, ut EN sit ipse radius refractus versus perpendicularem ED. Quare radius e medio raro, etc. Quod erat probandum.

85. *Radii incidentis oblique in medium diversum, cujus superficies est curva, refractio eadem est ac si incidisset in contactum planae superficiei ipsam curvam contingentis.*

86. Shapiro (1970, 160).

87. Hobbes (1994a, vol.1, 91).

88. REFRACTIO est lineae, secundum quam procederet corpus motum vel actio ejus in uno et eodem medio, in duas lineas rectas propter duorum mediorum naturam diversam fractio.

89. Si in medio quolibet supponatur ab uno aliquo puncto conatum in omnes simul partes quaqua versum propagari; oppositumque oblique ei conatui sit medium naturae diversae, id est, rarius vel densius; ita refringetur conatus ille, ut sinus anguli refracti sit ad sinum anguli inclinationis, ut densitas primi medii ad densitatem secundi reciproce sumptam.

90. Shapiro (1973, 171).

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