Essays on Macro-Development and Inequality

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Abstract

This dissertation consists of three essays that study development and inequality at the macroeconomic level. In the first chapter, I examine the impact of policy reforms on promoting wage employment in low-income countries. My empirical findings indicate that the wage-employed and the self-employed occupy distinct roles and offer different goods in these economies. I extend an occupational choice model that reflects the true nature of these employment sectors. With the calibrated model, I investigate policy reforms aiming at increasing the relative wage to promote wage employment in the formal sector. Quantitative analyses reveal that due to the low elasticity of substitution between goods produced by two employment sectors, any attempt to promote wage employment will be weakened by the accompanying increase in the relative price of goods the self-employed provide due to decreased supply. What's more, the elasticity of substitution matters for the success of policy reforms: a higher elasticity results in a lower relative price adjustment, thus giving the self-employed more incentives to switch sectors.

The second chapter evaluates the effects of closing the gender gap in the labor market on fiscal policies and economic outcomes in Japan. Using a neoclassical growth model that incorporates gender considerations, counterfactual analysis indicates that equalizing the labor force participation rate in Japan would lead to several benefits. These include a substantial decrease in the consumption tax rate, increased economic activity, reduced working hours, and a slight enhancement in debt sustainability.

The third chapter examines the disparities in wealth accumulation channels between homeowners and renters, focusing on asset allocation, rate of return, and saving rates. Empirical findings indicate that, on average, homeowners maintain a more diversified portfolio, leading to higher overall returns. However, renters also achieve competitive

returns as homeowners in terms of individual assets (excluding housing). Combining the analysis of overall and individual returns, it suggests housing provides a significant return that drives up the overall return rate. Additionally, homeowners demonstrate a higher saving rate than renters, mainly due to the mortgage acting as a commitment device.

Abrégé

Cette thèse est composée de trois essais qui étudient le développement et les inégalités au niveau macroéconomique. Dans le premier chapitre, j'examine l'impact des politiques sur la promotion de l'emploi salarié dans les pays à faible revenu. Mes résultats empiriques indiquent que les salariés et les travailleurs indépendants occupent des rôles distincts et offrent différents biens dans ces économies. J'étends un modèle de choix professionnel qui reflète la véritable nature de ces secteurs d'emploi. Avec le modèle calibré, j'étudie les politiques visant à augmenter le salaire relatif pour promouvoir l'emploi salarié dans le secteur formel. Les analyses quantitatives révèlent qu'en raison de la faible élasticité de substitution entre les biens produits par deux secteurs d'emploi, toute tentative de promotion de l'emploi salarié sera affaiblie par l'augmentation concomitante du prix relatif des biens fournis par les travailleurs indépendants en raison de la diminution de l'offre. De plus, l'élasticité de substitution est importante pour le succès des réformes politiques : une élasticité plus élevée entraîne un ajustement des prix relatifs plus faible, ce qui incite davantage les travailleurs indépendants à changer de secteur.

Le deuxième chapitre évalue les effets de la réduction de l'écart entre les hommes et les femmes dans la participation sur le marché de l'emploi, sur les politiques budgétaires et la performance économiques du Japon. En utilisant un modèle de croissance néoclassique qui intègre des considérations de genre, une analyse contrefactuelle indique que l'égalisation du taux d'activité au Japon entraînerait plusieurs avantages. Il s'agit notamment d'une baisse substantielle du taux de taxe à la consommation, d'une activité économique accrue, d'une réduction des heures de travail et d'une légère amélioration de la viabilité de la dette.

Le troisième chapitre examine les disparités dans les canaux d'accumulation de richesse entre propriétaires et locataires, en se concentrant sur l'allocation d'actifs, le taux de rendement et les taux d'épargne. Les résultats empiriques indiquent qu'en moyenne, les propriétaires conservent un portefeuille plus diversifié, ce qui entraîne des rendements globaux plus élevés. Cependant, les locataires obtiennent également des rendements compétitifs en tant que propriétaires en termes de biens individuels (hors logement). En combinant l'analyse des rendements globaux et individuels, il suggère que le logement offre un rendement significatif qui fait augmenter le taux de rendement global. De plus, les propriétaires affichent un taux d'épargne plus élevé que les locataires, principalement en raison du fait que l'hypothèque agit comme un instrument d'engagement.

Acknowledgements

My PhD journey at McGill University in Montréal, Canada has been incredibly fortunate. This experience would not have been possible without the unwavering support of my advisors, professors, colleagues, friends, and family. I am deeply grateful to all who have contributed to my growth during this remarkable chapter of my academic career.

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in guiding me through this pivotal stage of my career. As an expert on debt issues, he has provided invaluable guidance for my research project, greatly enhancing both my understanding of and approach to the subject. We are so lucky to have him at McGill.

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My journey in economics began as a master's student at Duke University, where I first developed a passion for economic research. I am deeply grateful to Prof. Attila Ambrus and Prof. Kent Kimbrough for igniting this interest with their inspiring teaching and insightful guidance in my research project. Their consistent encouragement and support throughout the years have been invaluable, providing a strong foundation for my academic and research pursuits.

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Contribution to Original Knowledge

This thesis contributes original knowledge in several key areas. The first chapter evaluates the impact of policy reforms aimed at promoting wage employment in low-income countries, within a realistic framework that acknowledges the distinct nature of wage employment versus self-employment. Previous studies have typically analyzed occupational choice assuming that wage-employed and self-employed individuals produce identical goods. My contributions are twofold.

Empirically, I document heterogeneity in occupational distribution across two employment sectors in low-income countries, suggesting that wage-employed and self-employed individuals are involved in producing different types of goods. Quantitatively, I extend an occupational choice model by incorporating empirical evidence that distinguishes between goods produced by wage-employed and self-employed individuals. I calibrate this model to Tanzania's economy and subsequently analyze the implications of two sets of policy reforms aimed at increasing the relative wage of working in the self-employment sector: one is to reduce the corporate tax rate, and the other is to expand the regulation of the informal sector, where the self-employed typically work in developing economies. I find that the elasticity of substitution between goods produced by two employment sectors is an important parameter that influences the success of policy reforms, as it governs the adjustment of relative price, thus playing an essential role in the agent's occupational choice.

The second chapter contributes to the literature by linking fiscal policies with gender gaps in the labor market. Given Japan's high debt-to-GDP ratio due to population aging, this chapter studies how to ease the current situation through a new angle: increasing the female labor force participation rate. It examines the impact of labor market equalization on Japan's fiscal policies and economic outcomes. We extend a neoclassical growth model to include the gender dimension and calibrate the steady state to reflect Japan's economy. Subsequently, we perform a counterfactual analysis on closing the gender gap in the labor force participation rate. The findings indicate that increasing female labor participation has significant benefits in the new steady state, as the government can reduce the consumption tax rate to keep the primary balance at the previous level, the economy experiences expanded activity, and the household enjoys lower working hours. It, therefore, provides a new incentive for the government to close the gender gap in the labor market.

The third chapter's contribution lies in decomposing the mechanisms of wealth accumulation. Previous literature has demonstrated that homeowners accumulate wealth at a higher rate than renters. However, they overlook the channels that lead to this result. This chapter fills in the gap by analyzing the difference in wealth accumulation channels between homeowners and renters. Using detailed wealth data from the PSID covering the years 1999 to 2017, the empirical analysis reveals that homeowners achieve higher wealth accumulation due to a higher savings rate, which housing provides as a commitment device, and a higher return rate on housing assets.

Contribution of Authors

This thesis is composed of three chapters. I am the sole author of the first and third chapters. The second chapter is based on my summer internship project at the IMF and is coauthored with Diego Gomes and Jiajia Gu. Diego Gomes and Jiajia Gu came up with the research idea. Together, we contributed equally to building the model, collecting data, calibrating the model, conducting numerical analysis, and writing the manuscript.

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Introduction

The differences in living standards are vast and persistent, not only between countries but also among different groups within a country. Globally, there is a stark contrast among countries at various stages of development, with differences in production technology, employment status, and income levels. Within a country, inequality manifests in gender gaps in the labor market and income disparities between homeowners and renters.

This thesis explores issues related to development and inequality. Chapter 1 investigates the effects of policy reforms aimed at promoting wage employment in low-income countries. Chapter 2 examines the impact of closing the gender gap in the labor market on Japan's fiscal policies and economic outcomes. Chapter 3 analyzes the differences in wealth accumulation channels that lead to wealth inequality between homeowners and renters in the US.

In the first chapter, titled "Policy Reforms and Self-Employment in Developing Countries: A Multi-Good Approach," I examine how heterogeneous agents' occupational choices respond to government policies. Low-income countries have a disproportionately higher number of people working in the self-employment sector compared to high-income countries. Policymakers aim to reduce the self-employment rate in developing countries because the self-employment sector is characterized by lower productivity and higher informality rates than the wage-employment sector. My empirical evidence shows that the self-employed in low-income countries tend to concentrate on occupations related to providing home production substitutes, whereas fewer wage-employed individuals engage in these types of occupations.

I then extend a baseline occupational choice model to incorporate this fact. In the model, heterogeneous agents with varying earning abilities and preferences for self-employment choose between wage employment in producing a generic manufacturing good and self-employment in providing a home production substitute. I calibrate the model to Tanzania's economy.

In an economy with two types of goods, their relative price plays a crucial role in occupational choices, alongside the equilibrium wage rate in the corporate sector. A quantitative analysis reveals that when the elasticity of substitution between goods is low, policy reforms aimed at promoting wage employment, such as decreasing the corporate tax rate and increasing the enforcement of the informal sector, are often unsuccessful. This outcome aligns with findings in some empirical literature. When some self-employed individuals switch to wage employment, the relative price of home production substitute goods increases, making it less appealing for others to make the same transition.

One policy recommendation to promote wage employment in developing countries is to encourage the development of a substantial corporate sector that supplies home production substitutes. To do this, the government could subsidize the corporate taxation tied to those sectors and improve the productivity in these sectors through innovation. In these ways, these formal firms can effectively compete with the self-employment sector, thereby driving down the self-employment rate.

In the second chapter, titled "Quantifying the Effect of Closing the Gender Gap in the Labor Market on Japan's Economy," we examine the impact of increasing female labor force participation on Japan's fiscal policies and economic outcomes. Japan currently has the highest debt-to-GDP ratio in the world, making its debt the least sustainable among developed nations. At the same time, the gap between male and female labor force participation rates is significantly wider than in other developed countries. This study investigates whether raising the female labor force participation rate to match that of males would improve Japan's fiscal conditions and serve as a new engine of economic growth.

Extending a neoclassical growth model to include a gender dimension, we assume an infinitely-lived representative household consisting of a male and a female. Both individuals choose their working hours, while the household makes joint decisions on consumption and investment. To address the primary balance deficit, the government issues public debt, which the household can purchase. We calibrate the model's initial steady state to Japan's economy in 2022.

Using the calibrated model, we conduct a counterfactual analysis to study the effects of equalizing the female labor force participation rate with the male. As a first step, we assume that government expenditure remains constant throughout the years. This change in female labor force participation could result from a change in social norms that support gender equality or flexible work arrangements and stronger parental leave policies. In the new steady state, the government can significantly reduce the consumption tax, lowering it from 13.33% to 9.65%, while maintaining a fixed primary balance. The higher household income from increased female labor market participation and the reduced consumption tax rate also lead to expanded economic activities.

Then, we assume that government expenditures are raised proportionately to the GDP level to account for the higher government expenditures to stimulate higher female labor force participation. For example, the governments can expand access to affordable childcare and eldercare services. These measures aim to alleviate barriers to female employment and support a balance between work and family responsibilities. These policies will promote a higher level of female labor force participation, which is also a fiscal burden for the government. In this case, the above results remain consistent in the new steady state, though at a smaller magnitude.

With Japan's population aging rapidly, the government's fiscal capacity is under significant pressure due to increased spending on public health and pension payments. In this context, promoting higher female labor force participation can be a crucial strategy for boosting fiscal revenue and serving as a new engine of economic growth. By implementing strategies to facilitate women's entry into the labor market, Japan can enhance economic growth and increase fiscal revenue amidst an aging population.

In the third chapter, titled "Housing Investment and Wealth Accumulation," I investigate the mechanisms through which homeowners accumulate higher levels of wealth. Wealth holdings vary significantly across households, and researchers have been exploring factors that could explain this polarized wealth distribution. Previous studies have shown that homeowners tend to accumulate more wealth. However, the specific channels through which housing investment contributes to this increased wealth accumulation have not been thoroughly studied.

The US data from the Panel Study of Income Dynamics (PSID), the longest-running longitudinal household survey in the world, provides comprehensive information on wealth and income. This allows for direct measurement of a household's savings, portfolio allocations, and returns from assets. From analyzing this dataset, I have identified three key empirical findings. Firstly, the allocation of non-housing wealth is similar between homeowners and renters within the same wealth distribution. Secondly, homeowners generally experience a higher overall rate of return compared to renters. However, when looking at individual non-housing assets, there is no statistically significant difference in returns between the two groups. This indicates that the substantial return on housing itself is a major contributor to the higher overall rate of return seen among homeowners. Additionally, homeowners demonstrate a higher saving rate than renters, even after considering demographic differences. My study suggests that homeownership serves as a commitment device, encouraging better saving behavior among homeowners. In summary, homeowners accumulate more wealth due to a combination of high returns on housing and higher saving rates.

1

Policy Reforms and Self-Employment in Developing Countries: A Multi-Good Approach

1.1 Introduction

This scene might look familiar to anyone who has ever been to a developing country: bumpy yet busy streets are constantly flooded with street vendors shouting out to sell, contrasting with the silent boulevards in the developed world. In low-income countries, disproportionally, more people set up small informal businesses and become self-employed. In high-income countries, becoming an employee for a formal sector firm is the norm. International Labor Organization (ILO) statistics show that, on average, self-employment takes up 61% of the non-agricultural labor force in low-income countries, while barely 12% in high-income countries. Despite numerous attempts to crack down on informal self-employment and promote employment in more efficient firms, high self-employment seems like a deep-rooted phenomenon in many low-income countries. Why is it so hard to encourage wage employment in developing countries? This paper aims to answer this question by evaluating policy experiments in a quantitative model featuring a realistic characterization of self-employment and wage employment.

Admittedly, the ubiquity of the self-employed might provide some convenience to daily life, but it also poses challenges to developing economies. First, most self-employed work in the informal sector and don't contribute to tax revenue, and their economic activities are notoriously hard to monitor. The substantial presence of tax-avoiding self-employment dramatically reduces the fiscal capacity of a nation, causing severe budgetary strain for governments in developing countries. Second, the literature has shown that self-employment activities are less productive than the corporate sector. Having so many labor forces centering on the low-productivity sector and with scanty earnings to spare may be a source of misallocation, thus worrying the policymakers.

Encouraging wage employment is generally a goal for policymaking in low-income countries. According to the United Nations' Sustainable Development Goals, promoting decent work, fostering industrialization, and reducing inequality is paramount. Hence, it is essential to have a framework to understand the high rate of self-employment in developing countries and how it reacts to policies aimed at reducing it.

A strand of literature studies whether policy reforms are effective at promoting wage employment. Quantitative analysis (Ordonez, 2014; Gollin, 2006; Ihrig and Moe, 2004) shows that tax reforms have a large effect on reducing the informal self-employment rate and increasing the share of employment in formal firms. However, natural experiments in Vietnam (Pham, 2020), China (Li et al., 2021), India (Hasan et al., 2021), and Brazil (Rocha et al., 2018) suggest the opposite. This paper attempts to reconcile the discrepancy between the quantitative and the empirical literature's findings. My main insight is that the quantitative models miss a crucial feature of self-employment: it supplies goods and services very different from the wage sector.

My paper makes several main contributions. Empirically, I establish two stylized facts. First, a remarkable difference exists between the self-employed and the wage-employed occupations in low-income countries. The self-employed tend to concentrate on jobs that provide marketable home production goods or services. Therefore, it is unsurprising that you see many street food vendors, hairdressers, cleaners, and helpers in developing countries. Since the self-employed and the wage-employed take on different professions, it implies the goods or services they bring to the economy will be different. Second, the reduction in home-production-related self-employment accounts for around 70% of the decline in self-employment rate with economic development. High-income countries have a much lower self-employment rate than low-income countries, primarily because of a sharp decrease in home-production-related self-employment with income level.

Theoretically, my innovation is to incorporate the empirical evidence in an occupational choice model by assuming the wage-employed and the self-employed produce different goods, in contrast to the existing literature. In a simple setting, heterogeneous agents choose occupations based on their idiosyncratic earning ability and taste for self-employment. The wage-employed work in a representative firm, which produces manufacturing goods and is subject to corporate income tax. At the same time, the self-employed provide home production substitute goods and escape the tax burden. Assuming all wage-employed produce manufacturing goods and all self-employed make home production substitute goods is designed to capture the fact that two employment sectors bring distinct goods and services to the market. Since there are two goods in the economy, the relative price of home production substitute goods directly affects self-employment income. Thus, when making an occupational choice, not only does the wage rate matter but so does the relative price. Moreover, as there are home-production substitute goods in the economy, I also model households' home production time so that they can make home-production goods themselves, like cooking at home instead of buying lunch from food trucks. It's a general equilibrium model where agents solve the occupational choice problem by choosing the optimal occupation, time use, and consumption bundle.

I calibrate the model to Tanzania's economy. Tanzania is a sub-Saharan country that fits my model setting well. Around 60% of the self-employed are doing home-production-related professions for a living. The high informality rate is a pressing issue as the former agricultural workers move to urban areas at the initial stage of structural transformation. The country also has a good data source that facilitates calibration. The Tanzania Integrated Labour Force Survey (ILFS) covers a wide range of topics that provide a comprehensive view of Tanzania's economy. More importantly, ILFS contains the Time Use Survey (TUS) data, which is uncommon for low-income countries. TUS data allows me to target moments in the household's time use. ILFS thus provides a useful set of calibration targets.

Using the calibrated model, I study the effect of corporate tax cuts on promoting wage employment. In a setting where the self-employed and the wage-employed produce different, not too highly substitutable goods, wage employment is essentially inelastic to the tax reform, consistent with the natural experiment evidence. According to the model, with lower corporate taxes, the corporate sector demands more labor, and the equilibrium wage rate increases, making wage employment more appealing. The existing literature also captures this effect. However, while the wage incentive moves the self-employed to work in the formal sector firm, my model predicts that fewer people provide home-production-substitute goods; thus, the relative price of these goods will increase, making self-employment still a relatively attractive choice. The policy experiment implies that the elasticity of substitution between the goods produced by two employment sectors is a crucial parameter of the effectiveness of tax reforms. When the two goods are good substitutes, corporate tax cuts are three times as powerful in promoting wage employment as the benchmark scenario because the demand for home-production-substitute goods is subdued. Thus, the relative price effect that makes self-employment equally favorable becomes less influential.

Related Literature. Besides contributing to the literature that theoretically and empirically studies how tax policies affect wage employment, as mentioned above, this paper also contributes to the following strands of literature.

First, the literature on self-employment and development (Poschke, 2023; Gindling and Newhouse, 2014; Gollin, 2008) shows that the self-employment rate negatively correlates with the national income level. I contribute by investigating the occupations of the self-employed in low-income countries and finding out that the self-employed and wage-employed take different professions. Moreover, I identify that the self-employed that provide home-production-substitute goods account for around 70% of the decline in the self-employment rate with economic development.

Second, the model is linked to the literature on occupational choice and entrepreneurship (Feng and Ren, 2023; Bento et al., 2023; Gu, 2021; Buera, 2009) and more closely to "necessity entrepreneur" (Herreno and Ocampo, 2023; Fairlie and Fossen, 2018; Poschke, 2013b), who are more likely to have low-skills, be own-account workers, and take jobs mainly for subsistence needs. I contribute by looking deeper into the nature of these necessity entrepreneurs and explicitly modeling them as providing home production substitute goods, an imperfect substitute of goods made by the wage-employed.

Third, this paper is closely related to the literature studying the informal sector and its regulation (Abras et al., 2018; Ulyssea, 2018; Ordonez, 2014; Almeida and Carneiro, 2012). Most self-employed work in the informal sector in developing countries, so the quantitative exercise provides a rigorous foundation to explain why formalization efforts were unsuccessful in some experiments (De Mel et al., 2013; Kaplan et al., 2011).

Fourth, this paper broadly falls in the literature on structural transformation and home production (Gottlieb et al., 2023; Ngai et al., 2022; Dinkelman and Ngai, 2021; Ngai and Petrongolo, 2017; Ngai and Pissarides, 2007). I contribute by studying a setting that fits into the initial stage of structural change, where labor moves out of the agricultural sector. More uniquely, agents must decide between being wage-employed in the manufacturing sector or self-employed in the service sector. This paper provides some insights on occupational choice along the structural transformation process.

The remainder of the paper follows. Section 1.2 presents two stylized facts regarding self-employment and wage employment across countries. Section 1.3 introduces a simple occupational choice model. Section 1.4 details the calibration where Tanzania is a case study. Section 1.5 shows the policy experiments. Finally, Section 1.6 concludes.

1.2 Stylized Facts

This section presents two stylized facts on wage employment and self-employment. The first fact regards whether wage-employed and self-employed are engaged in different professions in developing contexts. The second fact probes into what accounts for the major decline in the self-employment rate with economic development.

1.2.1 Occupational heterogeneity

First, I empirically test if a difference exists between jobs taken by the wage-employed and the self-employed. The quantitative literature, which assumes two employment sectors produce the same goods, implies that wage-employed and self-employed have similar jobs. Hence, I examine the empirical occupational distribution to check if this assumption holds.

The International Labour Organization (ILO)'s dataset on employment by status in employment and occupation sheds light on this point. The dataset compiles labor force survey data from 136 countries from 2000 to 2022. The country coverage ranges from low-income countries whose GDP per capita (in 2023 US dollar) is lower than $500, such as Burundi, Cambodia, Congo, Ethiopia, and Somalia, to high-income countries whose GDP per capita is higher than $100,000, such as Luxembourg and Norway. Therefore, the dataset provides a comprehensive view of how occupational choices differ by status in employment for countries with different income levels.

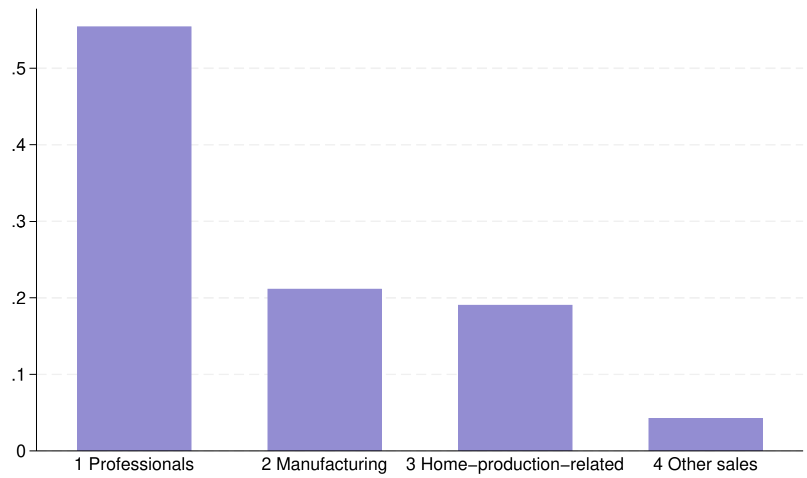
Classifications of employment status and occupations used in the ILO dataset follow international standards, which helps to harmonize labor force surveys in different countries. By status in employment, an employed person could be wage-employed or self-employed, where the self-employed category includes employers, own-account workers, members of producers' cooperatives, and contributing family members. The ILO follows the International Standard Classification of Occupation, 2008 (ISCO-08) and divides all occupations into ten major groups. I exclude agricultural and armed forces occupations because I focus on occupational choice in an urban setting, thus leaving me with eight major occupation groups.

I consolidate the eight occupation groups in the ILO into four major groups. The consolidated groups include (1) managers, professionals, technicians, associate professionals, and clerical support workers; (2) manufacturing occupations; (3) home-production-related occupations; (4) other sales. Details of the consolidation can be found in the Appendix 1.7.1.

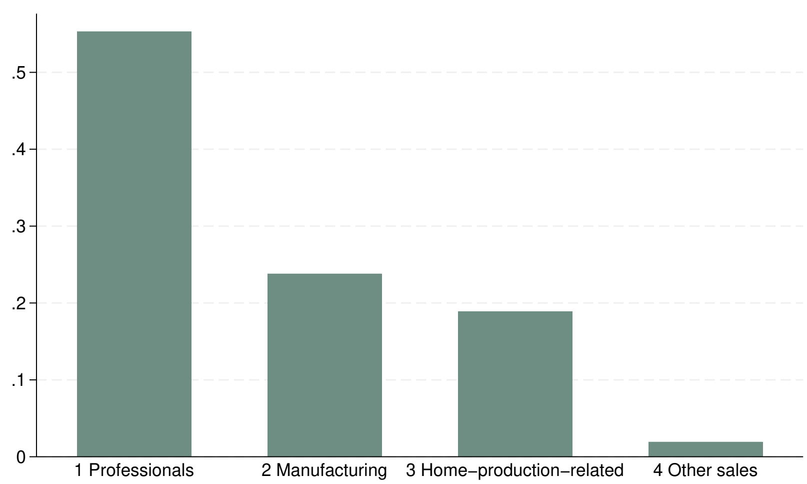
Occupational distribution by employment status is quite diverse for countries at different stages of economic development. Therefore, it is essential to separate the occupation distribution for countries at different income levels. Following the standard of the World Bank, I classify countries into four groups based on GDP per capita in 2019: low income, lower-middle income, higher-middle income, and high income. This section focuses on presenting the results in low-income and high-income countries.

I use each country's most recent survey data to calculate the occupational distribution in four major groups for wage-employed and self-employed separately. Within each employment status, the probability in four occupation groups sum up to 1. Then, I take the average for each occupation group across all countries in the same income-level category.

Figure 1.1 presents the average occupational distribution by employment status in high-income countries.The wage-employed (Panel a) and the self-employed (Panel b) exhibit very similar distributions across four major occupational groups. In contrast, Figure 1.2 reveals a markedly different pattern in low-income countries, where the occupational distributions between the wage-employed and the self-employed are distinct. In these countries, around half of the self-employed are engaged in home-production-related occupations, while most of the wage-employed are concentrated in professional and manufacturing occupations.



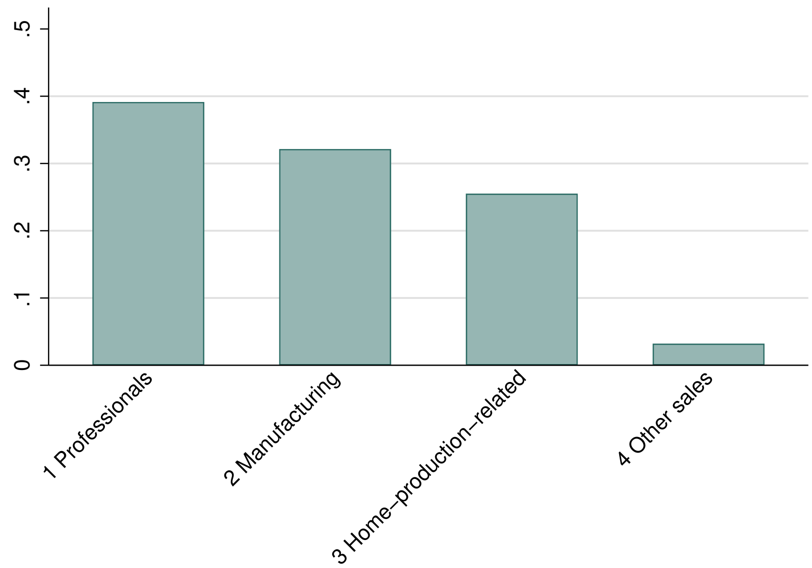
(a) Wage-employed



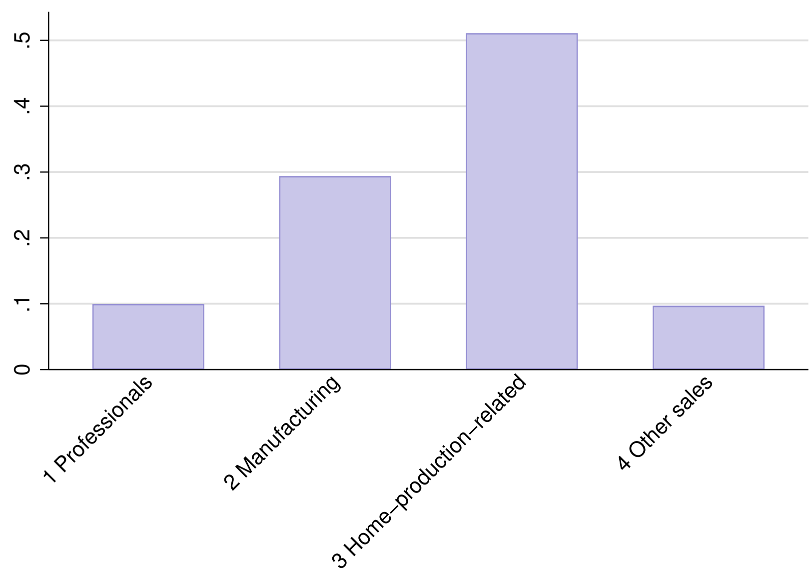
(b) Self-employed

Figure 1.1: Consolidated occupational distribution by employment status in HICs

Notes. This figure presents the average occupational distribution of the wage-employed (Panel (a)) and the self-employed (Panel (b)), respectively, in high-income countries. Data source: International Labour Organization (ILO) dataset on employment by status in employment and occupations.



(a) Wage-employed



(b) Self-employed

Figure 1.2: Consolidated occupational distribution by employment status in LICs

Notes. This figure presents the average occupational distribution of the wage-employed (Panel (a)) and the self-employed (Panel (b)), respectively, in low-income countries. Data source: International Labour Organization (ILO) dataset on employment by status in employment and occupations.

In developing contexts, the self-employed primarily focus on home-production-related occupations, whereas most wage-employed individuals work in professions that require higher skill levels, such as managers, professionals, and technicians. Consequently, it is inappropriate to assume that both employment sectors produce the same types of goods. In the model section, I will address the differences in the nature of work between these two employment sectors based on this observed pattern.

1.2.2 The decline of the self-employment with development

The literature indicates that self-employment decreases as countries develop. Empirical findings in the previous subsection suggest that in poorer countries, many self-employed individuals work in home-production-related occupations. This raises a natural question: how are these phenomena related? Specifically, does self-employment in home-production-related occupations decline with development, and how significant is this decline in contributing to the overall decrease in self-employment as development progresses?

ILO's data on employment by status in employment and occupation answers this question. I focus on non-agricultural occupations and divide them into home-production-related or non-home-production-related occupations. Then, I calculate the proportion of the self-employed in these two occupational groups out of all employed for each country using its latest survey data. A country's income level is measured by the log GDP per capita in 2019 (in 2023 US dollars).

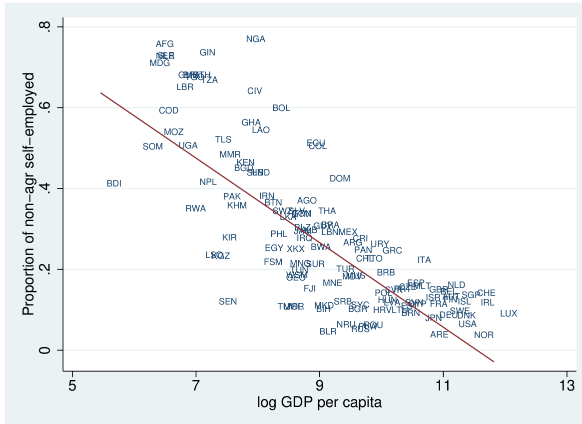
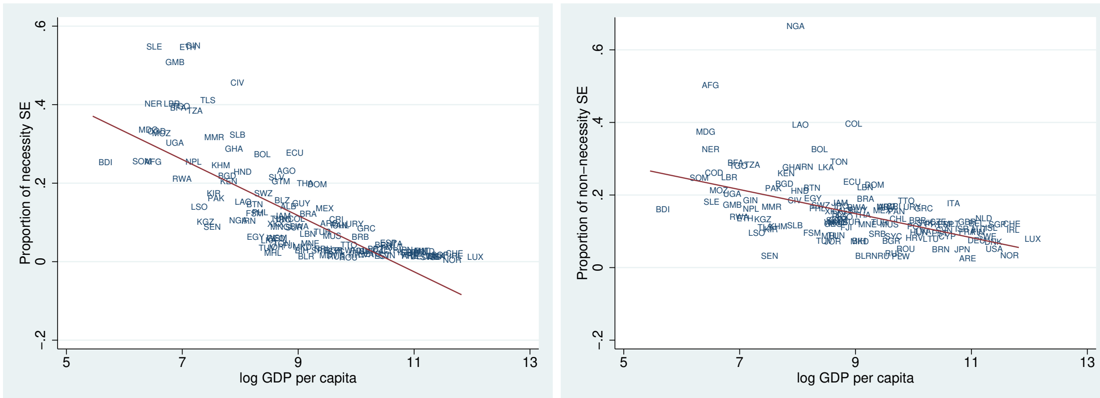


Figure 1.3: Proportion of (non-agr.) self-employed out of all (non-agr.) employed

Notes. This figure shows the relationship between the proportion of non-agricultural self-employed out of all employed with log GDP per capita. Data source: International Labour Organization (ILO) dataset on employment by status in employment and occupations.

Figure 1.3 replicates the empirical findings in (Poschke, 2023; Gollin, 2008) that the self-employment rate has a negative correlation with economic development. Self-employment is the dominant mode of employment in low-income countries. On average, 61% are self-employed in low-income countries where log GDP per capita is less than 7, compared to merely 12% of the self-employment rate in high-income countries whose log GDP per capita is more than 10.

Figure 1.4 indicates that the decline of self-employment in home-production-related occupations is the primary driver of the decrease in the self-employment rate with economic development. The average home-production-related self-employment rate plunges from 36% in low-income countries to 2% in high-income countries, while the average non-home-production-related self-employment rate decreases from 24% to 10% from low-income to high-income group. A back-of-envelope calculation shows that the sharp decrease in self-employment in home-production-related occupations accounts for around 70% of the decline in self-employment with GDP per capita. More rigorously, the slope coefficient for regressing the proportion of non-agricultural self-employment on log GDP per capita, as shown in Figure 1.3, is -0.1046; while the slope coefficients of the fitted lines in the two panels of Figure 1.4 are -0.0715 and -0.0331, respectively. The regression coefficients indicate that the plunge in the self-employment rate is largely attributed to home-production-related self-employment.



(a) Home-production SE

(b) Non-home-production SE

Figure 1.4: Prop. of home-production and non-home-production SE out of non-agr. employed

Notes. This figure shows the relationship between the proportion of home-production-related self-employed (Panel a) and non-home-production-related self-employed (Panel b) out of all employed with log GDP per capita. Data source: International Labour Organization (ILO).

As the decrease in home-production-related self-employment rate is the key to the decline in self-employment as a country develops, it is essential to understand how policies affect the size of this specific group of people.

1.3 A Model of Occupational Choice

Since self-employment is the dominant form of employment in low-income countries, I need a model to quantitatively evaluate if policies can effectively lessen the self-employment rate and encourage wage employment in more productive firms. This section presents a standard occupational choice model (Gollin, 2008, 2006), incorporating the empirical evidence that the self-employed and the wage-employed produce different goods. My innovation is that I assume the self-employed produce home production substitute goods, while the wage-employed produce other distinguishable goods, which I will call *manufacturing goods* for simplification. Since there are marketable home production substitute goods in the economy, I also explicitly model home production goods and services the households make that are not tradeable, such as homemade meals, care for children, cleaning the houses, etc.

1.3.1 The model setup

Heterogeneous agents. A continuum of agents of measure 1 populates the economy. Agents are heterogeneous in two dimensions: earning ability as a worker, ν, and taste for self-employment, ι. I assume that ν follows log-normal distribution, ι follows normal distribution, and both distributions are independent, i.e., $\ln\nu \sim \mathbf{N}(\mu_\nu, \sigma_\nu^2)$, $\iota \sim \mathbf{N}(\mu_\iota, \sigma_\iota^2)$, and $\nu \perp \iota$. The probability distribution functions of ν and ι are $f(\nu)$ and $g(\iota)$, respectively.

Time use. Each agent has $\bar{T}$ amount of time endowment, which they can allocate in three activities: market work, n, home production, h, and leisure, l.

Preference. Agents value composite consumption goods, c_{com}, and leisure time, l. The utility function has the CRRA form:

$$u = \theta\frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma}. \tag{1.1}$$

The composite consumption good c_{com} is a CES aggregate of manufacturing goods, c_m, and home goods, c_h, with elasticity of substitution being ϵ:

$$c_{com} = \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{1.2}$$

The corporate sector produces manufacturing goods, c_m. Home goods, c_h, is another CES aggregate of home production goods, c_{sh}, and home production substitute goods purchased on the market, c_{ph}, with elasticity of substitution being ζ:

$$c_h = \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)\, c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}. \tag{1.3}$$

The corporate sector. A representative firm produces manufacturing goods, c_m, which is the numeraire in the economy. The wage-employed work in this firm. The corporate sector's production technology is $Y_m = zN^{1-\alpha}$, where z is the TFP term and N is the sum of the efficient amount of labor by all workers. For a worker with earning ability, ν, and spends n amount of hour on market work, the efficient labor she supplies is νn. The firm is subject to the corporate tax at the rate τ. I model the corporate tax as a simple rate levied on corporate output rather than on profit. This approach aims to capture the various distortions that the formal sector experiences compared to the informal sector, which can arise not only from corporate income taxes but also from government regulations, entry barriers, failure to protect property rights, and other contributing factors.

The corporate sector hires the optimal amount of labor by maximizing its profit:

$$\max_{N} \quad \Pi = (1-\tau)zN^{1-\alpha} - wN \tag{1.4}$$

From the optimization, the corporate sector's labor demand is: $N^d = \frac{(1-\tau)(1-\alpha)Y_m}{w}$, and it earns positive profit:

$$\Pi^* = (1-\tau)\alpha Y_m. \tag{1.5}$$

Home production technology. Two types of home goods exist in the economy. The first is home production goods, c_{sh}, that everyone makes and then consumes, which are not tradable on the market. The second is home production substitute goods, c_{ph}, that the self-employed make and are marketable. The two types of home goods are not identical. Due to customization and sophistication, comparing the productivity of the production of these two goods ex-ante is not apparent.

Home production goods. Agents do home production using their home production hours with a linear production technology: $y_{sh} = \rho h$. ρ measures the home production productivity. The inventions of more efficient home appliances, like washing machines and vacuum robots, greatly reduce the time required to perform a certain amount of home production, which can be captured by an increase in ρ. Home production goods are not tradeable, and agents consume all the home production goods they produce: $c_{sh} = y_{sh}$.

Home production substitute goods. Agents can purchase home production substitute goods on the market, c_{ph}. For example, you can buy your lunch from a food truck instead of cooking at home. The self-employed in the economy produce home production substitute goods using their market work time: $y_{ph} = \xi n$. ξ measures the productivity of the self-employed in their market work. The home production substitute goods are tradable at the equilibrium price p.

The government sector. The government collects taxes in the unit of manufacturing goods, and then spends all the tax revenue. I assume the agents do not value government spending for simplification, which is also a standard assumption in the literature.

Occupational choice. There are two occupational choices available in the economy. An agent could either be a wage worker, working in the corporate sector. Alternatively, an agent could be self-employed and produce home production substitute goods.

For the wage-employed, earning ability ν determines the efficient amount of labor a worker can supply for each working hour. Given equilibrium wage rate w, an agent's earning ability ν, and working hours n, the income for a worker is $w\nu n$.

For the self-employed, by assumption, they produce home production substitute goods, like restaurant meals, cleaning services, personal care services, etc. Home-production-related occupations take up a significant proportion of the self-employed in developing countries. Therefore, assuming all the self-employed are providing home-production substitute goods is a simplification to distinguish the nature of the self-employed is different from the wage-employed in developing countries. Given the equilibrium price of home production substitute goods p, the self-employed productivity ξ, and working hours n, the income for a self-employed is $p\xi n$.

Agents choose an occupation based on higher utility. Besides utility from composite consumption goods, c_{com}, and leisure time, l, agents receive an additional amount of relative utility, ι, from self-employment. ι follows normal distribution $\iota \sim N(\mu_\iota, \sigma_\iota^2)$; but for a given individual, his/her ι is fixed instead of random. The relative taste for self-employment differs across agents. Some agents might have a higher ι since they value the flexibility from being self-employed; others may have a lower ι if they think providing home production substitute goods is less prestigious. The utility from each occupation comes from each agent's optimization problem by choosing the consumption bundle and time allocation. $u^*_{we}(c_m, c_h, l; \nu)$ is the optimized utility an agent can get from being wage-employed by solving the problem (1.7), which depends on the state variable, ν; while $u^*_{se}(c_m, c_h, l; \nu, \iota)$ is the optimized utility an agent can obtain by being self-employed from the problem (1.9), which depends on two state variables, ν and ι. By comparing optimized utility from two occupations, an agent chooses either wage-employed, $o(\nu, \iota) = we$, or self-employed, $o(\nu, \iota) = se$.

$$\max_{o \in \{we, se\}} \{u^*_{we}(c_m, c_h, l; \nu), u^*_{se}(c_m, c_h, l; \nu, \iota)\} \tag{1.6}$$

1. wage-employed:

$$\max_{h,l,c_m,c_{ph}} u_{we}(c_m, c_h, l; \nu) = \theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma} \tag{1.7}$$

$$s.t. \quad c_m + pc_{ph} = w\nu(\bar{T} - h - l) \tag{1.8}$$

2. self-employed with taste as ι:

$$\max_{h,l,c_m,c_{ph}} u_{se}(c_m, c_h, l; \nu, \iota) = \iota + \left[\theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma}\right] \tag{1.9}$$

$$s.t. \quad c_m + pc_{ph} = p\xi(\bar{T} - h - l) \tag{1.10}$$

where c_{com} is a function of manufacturing goods, c_m, and home goods, c_h, as defined in equation (1.2); c_h is a function of home production goods, c_{sh}, and home production substitute goods, c_{ph}, as defined in equation (1.3); c_{sh} is a function of home production time h since $c_{sh} = \rho h$.

General equilibrium. The equilibrium consists of the wage rate w, relative price of home production substitute goods p, agent's career decision $o(\nu, \iota) \in \{we, se\}$, decision on time allocation in market work, home production, and leisure, $\{n, h, l\}$, and decision on consumption in c_m, c_{ph}, such that given prices, idiosyncratic earning ability ν, and preference for self-employment ι, agents are maximizing their utility and the following markets clear:

1. Manufacturing goods market clears. The manufacturing goods the representative

firm produces, Y_m, equals the total demand of goods $c_m(\nu, \iota)$ from everyone in the economy plus the government expenditure and the corporate sector's profit.

$$Y_m = \int \int c_m(\nu, \iota) f(\nu) g(\iota) d\nu d\iota + \tau Y_m + \Pi^* \tag{1.11}$$

2. Home production substitute goods market clears. The home production substitute goods supplied by the agents who decide to become self-employed equals the total demand from the economy. Both the wage-employed and the self-employed can demand home production substitute goods. The idea is that there are different kinds of home production substitute goods owing to specialization. For example, street food vendors can also pay someone to babysit their children when necessary.

$$\int \int_{o=se} \xi n(\nu, \iota) f(\nu) g(\iota) d\nu d\iota = \int \int c_{ph}(\nu, \iota) f(\nu) g(\iota) d\nu d\iota \tag{1.12}$$

3. The labor market clears. The efficient amount of labor supplied by the agents who opt to become wage-employed equals the labor demand from the corporate sector. Workers' decision on how much time to devote to market work, n_w, depends only on their idiosyncratic earning ability ν.

$$\int \int_{o=we} \nu n(\nu) f(\nu) g(\iota) d\nu d\iota = \frac{(1-\tau)(1-\alpha) Y_m}{w} \tag{1.13}$$

1.3.2 Equilibrium occupational choice

Using calibrated parameter values (details in Section 1.4), Figure 1.5 shows the equilibrium occupational choice in the $\ln \nu - \iota$ space. This figure is an illustrative simulation of $N = 1000$ agents drawn randomly from calibrated distributions. The blue dots represent the wage-employed, while the yellow dots represent the self-employed. Agents with higher earning ability ν provide more efficient labor in a given working hour as a worker, thus receiving higher income, are more likely to be wage-employed. Agents with higher ι have a stronger preference for self-employment, and are thus more likely to be self-employed.

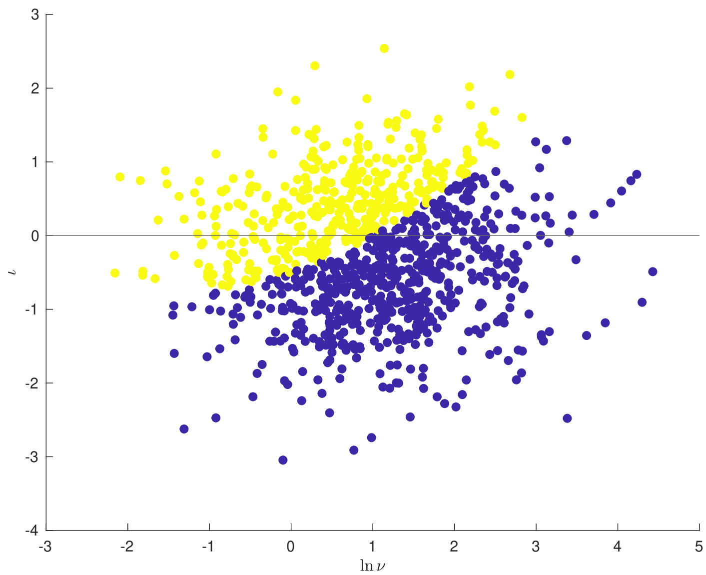
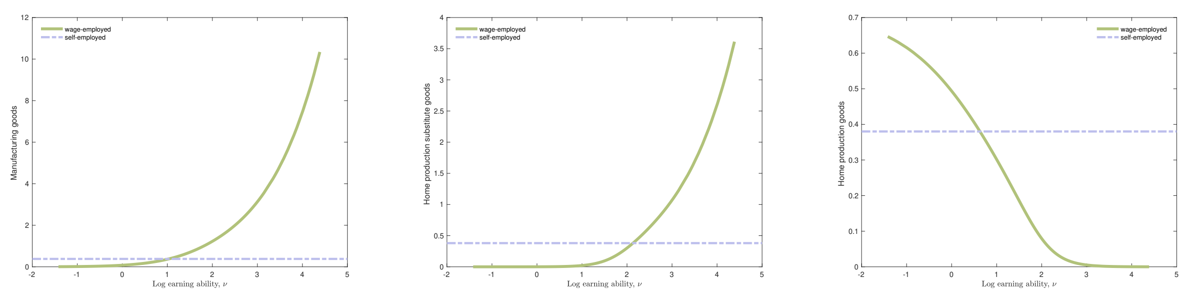


Figure 1.5: Occupational choice in ln ν - ι space

Notes. This figure illustrates the equilibrium occupational choices for each agent, selected randomly from calibrated distributions. Each agent has a unique earning ability and taste for self-employment. Blue dots represent agents choosing wage employment, while yellow dots represent self-employment.

1.3.3 Consumption choice



(a) c_m (b) c_{ph} (c) c_{sh}

Figure 1.6: Optimal consumption choice

Notes. This figure presents the optimal consumption choices for each agent in equilibrium, which include manufacturing goods (panel a), home production substitute goods (panel b), and home production goods (panel c). The green line shows the consumption choice for the wage-employed with different earning abilities, while the purple dotted line is for the self-employed.

Figure 1.6 displays the consumption choices by agents using calibrated parameters detailed in Section 1.4. Agents who decide to be self-employed have the same productivity in both market work and home production work. Therefore, all self-employed make the same consumption choices. Agents with higher earning ability, ν, have higher income and thus can afford more manufacturing goods, c_m, and home production substitute goods, c_{ph}. Meanwhile, higher-ability workers produce fewer home production goods due to less time devoted to it (see Figure 1.7).

Income allocation

Each agent spends their income between buying manufacturing goods and home production substitute goods. The optimal consumption bundle between these two goods, c_m and c_{ph}, is derived explicitly in the Appendix 1.7.4 but presented here:

(a) For the wage-employed:

$$\frac{c_m}{c_{ph}} = \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \cdot \left[\psi + (1-\psi)\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{1-\zeta}\right]^{\frac{\zeta}{\zeta-1}} \tag{1.14}$$

(b) For the self-employed:

$$\frac{c_m}{c_{ph}} = \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \cdot \left[\psi + (1-\psi)\left(\frac{\psi\xi}{\rho(1-\psi)}\right)^{1-\zeta}\right]^{\frac{\zeta}{\zeta-1}} \tag{1.15}$$

Holding other things constant, the $\frac{c_m}{c_{ph}}$ ratio is negatively correlated with p. The intuition is simple: when home production substitute goods become relatively more expensive, agents will respond by consuming more manufacturing goods compared to home production substitute goods. The $\frac{c_m}{c_{ph}}$ ratio is also positively correlated with ϵ, the elasticity of substitution between manufacturing goods and home goods. When two types of goods are easier to substitute, an increase in the price of c_{ph} will lead the agents to adjust consumption bundles by maintaining a higher $\frac{c_m}{c_{ph}}$ ratio.

Home goods allocation

Agents also have an optimal consumption bundle between home production goods, c_{sh}, and home production substitute goods, c_{ph}. The optimal ratio between two types of home goods:

(a) For the wage-employed:

$$\frac{c_{ph}}{c_{sh}} = \left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta} \tag{1.16}$$

(b) For the self-employed:

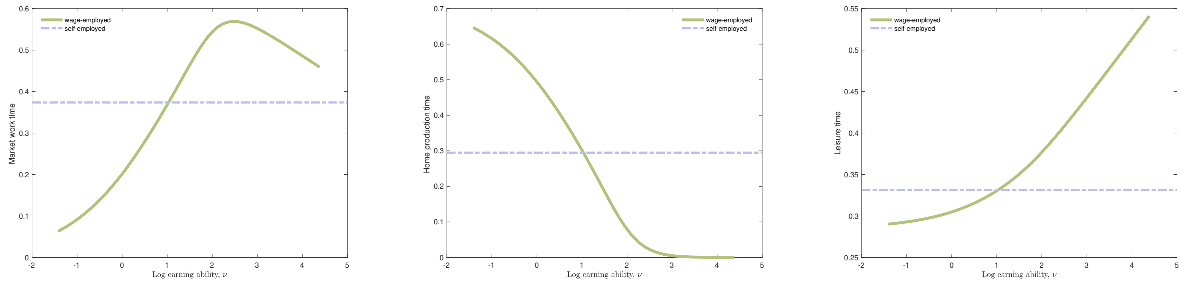
$$\frac{c_{ph}}{c_{sh}} = \left(\frac{\psi\xi}{(1-\psi)\rho}\right)^{\zeta} \tag{1.17}$$

On the one hand, for the wage-employed, $\frac{c_{ph}}{c_{sh}}$ is negatively correlated with the relative price, p. Workers will demand less of c_{ph} and increase their home production time to produce more c_{sh} when home production substitute goods become more expensive. On the other hand, the self-employed people's $\frac{c_{ph}}{c_{sh}}$ is independent of the relative price p. When the price of goods they produce, p, increases, their incomes increase at the same rate, thus allowing them to maintain the same consumption bundle between c_{ph} and c_{sh}.

1.3.4 Time use choice

An agent allocates time between market work, home production, and leisure. The optimal time allocation for the three activities can be derived analytically, which you can find in the Appendix 1.7.4. Figure 1.7 presented the optimal time use for agents with different earning abilities and occupations using calibrated parameter values detailed in Section 1.4.

Agents who opt to be self-employed have the same productivity and preference, therefore, they have the same time use choice. Wage-employed with higher earning abilities spend more time on leisure and less time on work (market work plus home production), due to a stronger income effect. Among work, market work time has an inverse U-shape with workers' abilities, while home production time has a negative relationship with earning ability.



(a) Market work (b) Home production (c) Leisure

Figure 1.7: Optimal time allocation

Notes. This figure presents the optimal time use choices for each agent in equilibrium, which include market work time (panel a), home production time (panel b), and leisure time (panel c). The green line shows the consumption choice for the wage-employed with different earning abilities, while the purple dotted line is for the self-employed.

Market hour-to-home production hour ratio. In general, higher-ability workers maintain a higher market hour-to-home production hour ratio. Since wage-employed are heterogeneous in the productivity of market work while everyone has the same productivity in home production, high-ability workers will work more in the market, earn more income, and buy more home production substitute goods. Equation (1.18) confirms that $\frac{n}{h}$ has a positive correlation with earning ability ν.

$$\frac{n}{h} = \rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1}+(1-\psi)\right)^{\frac{\zeta-\epsilon}{\zeta-1}}+\left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1} \tag{1.18}$$

Meanwhile, an increase in the price of home production substitute goods p will bring down the $\frac{n}{h}$ ratio. The wage-employed will readjust their time allocation by spending more time on home production to make home goods. For example, when it becomes more expensive to eat outside, a rational worker will spend more time cooking at home and bringing her food.

1.4 Calibration: The Case of Tanzania

I calibrate the model to fit Tanzania's economy in 2020. Tanzania is a low-income country where around 60% of the self-employed are doing home-production-related occupations.

The country's Development Vision 2050 highlights industrialization as one of the pillars to foster economic growth. Raising wage employment and reducing self-employment in a low-income country is generally a development goal of policymakers. As outlined in the United Nations' Millennium Development Goals, promoting inclusive and sustainable economic growth, employment, and decent work for all is paramount. Tanzania had some tax reforms to accommodate its development goal. For example, in 2018/2019, the government reduced a few industries' corporate income tax rates to encourage investment and increase employment opportunities for five years.

Tanzania also has excellent survey data that facilitates the calibration. The Tanzania Integrated Labour Force Survey 2020/2021 covers a wide range of topics and incorporates the Time Use Survey (TUS). It is very rare for a low-income country to have time-use data, which is critical for the calibration exercise. With TUS, I can target moments on household time allocation and gain better insights into how households spend time among market work, home production, and leisure. I set some parameters to common values from the literature and calibrate the rest internally.

1.4.1 Predetermined parameter values

To calibrate the model, I first predetermine some parameters using standard values in the literature or through normalization. α is $\frac{1}{3}$ so that $1-\alpha$, the labor income share, is $\frac{2}{3}$. I take σ, the relative risk aversion, from the estimated value in Fang and Zhu (2017). I normalize all the productivity parameters to 1. The benchmark corporate income tax rate, $\tau = 30\%$, is the tax rate in Tanzania before the tax reform.

ϵ, the elasticity of substitution between c_m and c_h, is an important parameter. When assuming the wage-employed and the self-employed produce homogeneous goods as in the previous literature (Gollin, 2008; Ihrig and Moe, 2004), it is equivalent to assume that the elasticity of substitution between manufacturing goods and home goods is infinity. Aguiar et al. (2012) survey the literature that estimates the elasticity of substitution between market and home goods. The estimated parameter ϵ range from slightly less than 2 to 2.3, and I choose $\epsilon = 2$ as in Gottlieb et al. (2023). In the quantitative exercise, I also consider policy implications with a higher ϵ.

I set ζ, the elasticity of substitution between c_{ph} and c_{sh}, to be 4 because it is easier to substitute within home goods than between home goods and manufacturing goods ($\zeta > \epsilon$). Moro et al. (2017) estimate this parameter value in their online Appendix B. Their estimates range from 0.267 to 6.850, depending on different specifications. Therefore, setting $\zeta = 4$ is around the average of the estimates.

Table 1.1: Predetermined parameter values

Parameter	Value
α	$\frac{1}{3}$
σ, relative risk aversion	1.4
ϵ, elasticity of substitution between c_m and c_h	2
ζ, elasticity of substitution between c_{ph} and c_{sh}	4
z, TFP term	1
ρ, home production productivity	1
ξ, NE productivity	1
$\bar{T}$, total time endowment	1
μ_ν, mean of $\ln \nu$	1
τ, corporate tax rate	0.3

1.4.2 Targeted moments

I use the following six moments to calibrate the six remaining unknown parameters: (1) standard deviation of log earning ability, σ_ν; (2) weight on consumption in the utility function, θ; (3) weight on c_m in the c_{com} composite, ϕ; (4) weight on c_{ph} in the c_h composite, ψ; (5) mean of taste for self-employment distribution, μ_ι; (6) standard deviation of taste for self-employment distribution, σ_ι.

Proportion of the self-employed. The data is from the ILO data on employment by status in employment and occupation for Tanzania in 2020. I exclude occupations in agriculture and armed forces and the self-employed who are not doing home-production-related occupations. Then, the remaining sample includes the wage-employed and the self-employed doing home-production-related occupations. The self-employment rate is 38%.

Average time use. The Time Use Survey within Tanzania's Integrated Labour Force Survey 2020/2021 provides a detailed 24-hour diary for each interviewee. I categorize each activity group into either market work, home production, or leisure. The market work includes time spent on employment and related activities and the production of goods for its own final use. Home production includes time spent on unpaid domestic services for household and family members, unpaid caregiving services for household and family members, unpaid volunteer, trainee, and other unpaid work. Leisure includes time spent on learning, socializing, community participation, and culture leisure mass-media and sports practices. I assume everyone spends 12 hours daily on self-care and maintenance; therefore, everybody allocates the remaining 12 hours daily to market work, home production, and leisure.

The working age population, those between 15 and 65 years old, spend, on average, 4.67 hours and 3.10 hours per day on market work and home production, respectively. For those who do multitasking, I consider only the primary activity and allocate all the time to it. Therefore, among the 12 hours of discretionary time, an average person allocates 39% to market work and 26% to home production, which become two targeted moments.

Standard deviation of log wage. The imputed hourly wage of the employees follows a log-normal distribution, whose standard deviation is one of the targets. I compute the hourly wage by dividing reported last week's total paid income by last week's working hours for the working-age employees. A normal distribution fits the imputed log hourly wage as seen in Figure 1.14 in the Appendix. The standard deviation of the log wage is 0.95.

Average income ratio of the wage-employed and the self-employed. The wage-employed have a higher average income than the self-employed, as revealed in the ILFS. I consider total paid income as income for the wage-employed and total self income as income for the self-employed doing home-production-related occupations. I exclude reported self income that is negative. On average, the income of the wage-employed is 2.77 that of the self-employed.

Non-home-production-related goods expenditure share. Tanzania's National Account sheds light on household expenditure between marketable home-production and non-home-production-related goods. I separate each non-agricultural activity into either group, which you can find detailed classification in the Appendix 1.7.2. In 2020, around 71% of the non-agricultural GDP occurred in the non-home-production-related sectors, while the remaining 29% was in the home-production-related sector.

Table 1.2: Targeted moments

	Model	Data
Prop. of self-employment	0.38	0.38
Avg market work time	0.39	0.39
Avg household work time	0.27	0.26
Std log wage	0.95	0.95
Consumption goods expenditure share	0.71	0.71
Average income ratio: worker over NE	2.77	2.77

1.4.3 Calibrated parameter values

I calibrate the 6 unknown parameters jointly by minimizing the sum of squared distances between each moment in the data and that of the model. Table 1.3 presents the calibrated parameter values.

Some moments are more informative for calibrating specific parameters. Total working hours, market plus home production, help to pin down the weight of consumption θ. The weight of the manufacturing goods in the c_{com} composite, ϕ, determines household expenditure share. The weight of home production substitute goods in the total home goods composite, ψ, plays a role in how households allocate working time between market work and home production. The standard deviation of log ability, σ_ν, directly governs the workers' standard deviation of log hourly wage.

Both ability and taste for self-employment determine an agent's occupational choice, the distribution of taste for self-employment is essential to understand how the occupation choice is different from the one solely governed by the ability. The mean taste for self-employment, μ_ι, sheds light on the proportion of the self-employed in the economy. As you can see, on average, agents don't prefer the self-employment over wage-employment. A more dispersed taste distribution is more likely to lead to high-ability agent choose self-employment sector due to extremely higher preference, thus reducing the average income ratio between the wage-employed and the self-employed.

Table 1.3: Calibrated parameter values

	Parameter value
σ_ν, std of $\ln \nu$	1.0175
θ, weight on consumption in the utility function	0.6471
ϕ, weight on c_m in the c_{com} composite	0.4348
ψ, weight on c_{ph} in the c_h composite	0.3462
μ_ι, mean of ι	-0.2568
σ_ι, std of ι	0.8238

1.5 Policy Experiments

Based on the calibrated model that fits Tanzania's economy in 2020, I conduct policy experiments to study the effectiveness of tax reforms on promoting wage employment. I consider two sets of policy reform: the first is to reduce the corporate tax rate, and the second is to increase regulation against informal self-employment.

1.5.1 Corporate tax cut

The rationale for using corporate tax cuts to promote wage employment is this: a lower corporate tax rate increases the firm's labor demand, thus elevating the equilibrium wage rate. As the corporate sector offers more competitive incomes, it persuades some self-employed to switch to being wage-employed. In 2018/2019, Tanzanian government reduced the corporate income tax rate for the pharmaceutical and leather industries, intending to promote employment in these two industries. No post-reform data is available yet to empirically examine this tax reform's effectiveness. Meanwhile, I will evaluate it quantitatively using the calibrated model.

Benchmark scenario. In this section, I assess quantitatively the effect of corporate income tax cuts on the share of self-employment. I reduce the tax rate by 10% at a time, from the original 30% to 0. Table 1.4 presents the occupational choice with tax reform. In the second column, you can find the proportion of the self-employed in the economy at a given tax rate indicated in the first column. The third column summarizes how many self-employed people have switched to the wage-employment sector compared to the original 30% tax rate case.

Table 1.4: Corporate tax cut in the benchmark scenario

Tax rate	Prop. of SE	Change in prop. of WE
0.3	38.30%	
0.2	37.70%	+ 0.6%
0.1	36.90%	+ 1.4%
0	36.50%	+ 1.8%

The quantitative exercise shows that corporate income tax cuts have a limited effect on promoting wage employment, consistent with the empirical findings in Pham (2020) but in contrast to the results in Gollin (2006). Figure 1.8 shows that few agents along the indifference curve switch from self-employment (yellow dots) to wage-employment (blue dots), which are highlighted in the green dots.

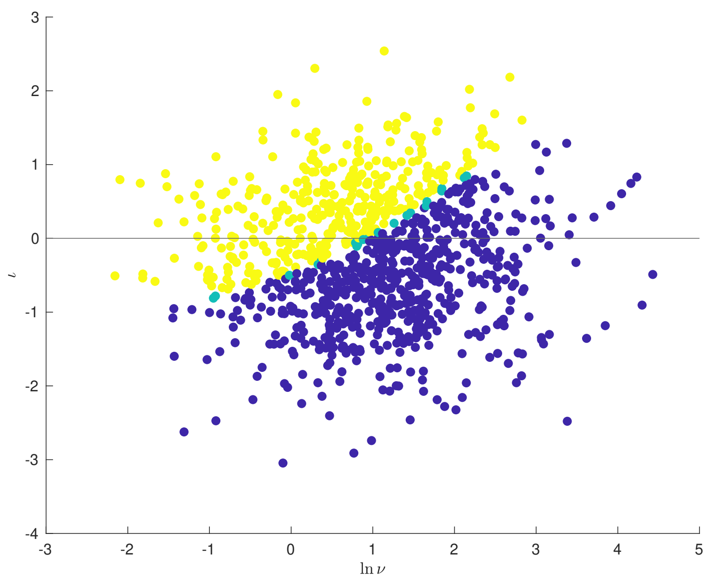


Figure 1.8: Transition of occupational choice with tax reform

Notes. Blue dots represent agents who choose to be wage-employed throughout; yellow dots represent agents who choose to be self-employed throughout; and green dots along the indifference line represent agents who switch from self-employed to wage-employed with tax reforms.

The difference between my results and other quantitative findings in the literature stems from the relative price of home production substitute goods that the self-employed produce. With tax cuts from 30% to 0, the wage rate in the corporate sector increases, as shown on the left axis in Figure 1.9. Higher wage incentivizes the self-employed to switch to the wage-employment sector due to higher income. Now, less self-employed in the economy are producing home production substitute goods. Due to limited supply, the relative price of home-production-substitute goods increases, as the right axis of Figure 1.9 shows. The increase in the price of home production substitute goods makes self-employment more profitable, thus putting a break on the switch of occupations.

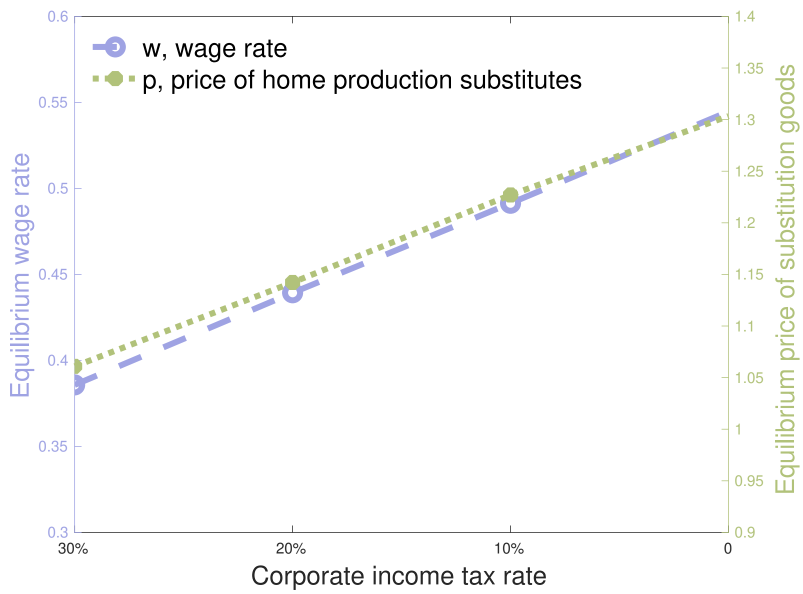


Figure 1.9: Equilibrium prices

Notes. The x-axis represents the corporate tax rate from 30% to 0. The purple dashed line shows the equilibrium wage rate (left y-axis) with the corporate tax cut, and the green dotted line shows the equilibrium price of home production substitutes (right y-axis).

In the quantitative literature that assumes the self-employed and the wage-employed produce homogeneous goods, there is no relative price mechanism. Agents only respond to the wage rate. With higher wage rates in the corporate sector, more self-employed will switch occupations. Thus, the corporate tax cut is more effective in promoting wage-employment in their scenarios.

A higher elasticity of substitution. Now, I increase the elasticity of substitution ϵ to a higher value 10. In the literature, where it assumes homogeneous goods across sectors, it is equivalent to think that the elasticity of substitution between goods produced by the wage-employed and the self-employed is infinity. A higher elasticity of substitution implies that two goods are more similar, thus bringing the analysis closer to the scenario studied in the previous quantitative literature. I recalibrate the model with the new ϵ. See Appendix 1.7.3 for details.

Table 1.5: Corporate tax cut with $\epsilon = 10$

Tax rate	Prop. of SE	Change in prop. of WE
0.3	38.30%	
0.2	36.80%	+ 1.5%
0.1	35.40%	+ 2.9%
0	33.40%	+ 4.9%

When the goods produced by the two sectors are more similar, corporate tax cuts have a more powerful effect on promoting wage employment. Table 1.5 shows that at each level of the tax cut, almost three times as many self-employed would switch to wage-employment compared to the benchmark scenario with a lower elasticity of substitution presented in Table 1.4. As a result, we see more agents switch occupations along the indifference curve, shown in green dots in Figure 1.10.

A less sharp increase in the relative price of home production substitute goods explains why tax cuts are more effective in promoting wage employment when the goods produced in two sectors are more substitutable. Each agent holds an optimal amount of consumption bundle between manufacturing goods and home-production-substitute goods as shown in equation (1.14) for the wage-employed and equation (1.15) for the self-employed. When manufacturing goods and home goods are easier to substitute (ϵ higher) and less self-employed producing home-production-substitute goods c_{ph} due to job switch, agents will replace more home production substitute goods c_{ph} with manufacturing goods c_m. The reduced demand for c_{ph} translates into a slower-growing path of the price of home production substitute goods with tax cuts, as Figure 1.11 shows. Since the home production substitute goods' price increase does not keep up with the wage increase with the tax reform, working in the corporate sector has more comparative advantage for agents near the indifference curve. Therefore, more self-employed will switch

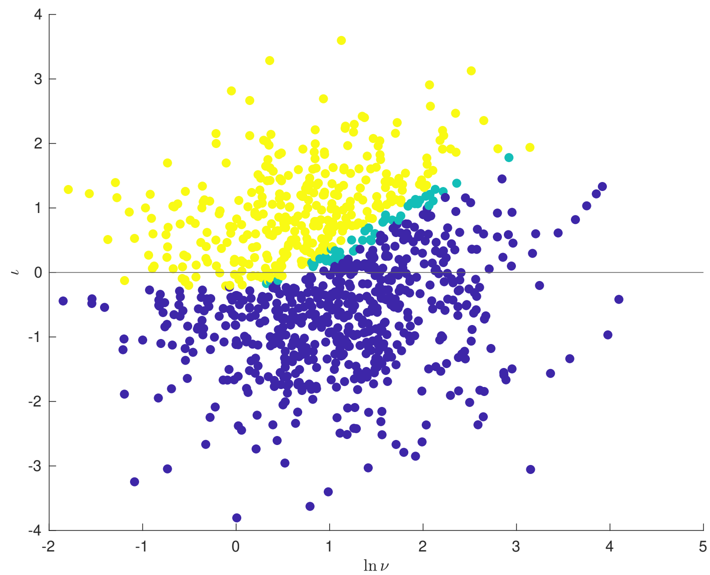


Figure 1.10: Transition of occupational choice with tax reform with $\epsilon = 10$

Notes. Blue dots represent agents who choose to be wage-employed throughout; yellow dots represent agents who choose to be self-employed throughout; and green dots along the indifference line represent agents who switch from self-employed to wage-employed with tax reforms.

to work in the corporate sector. As a result, the corporate tax cuts are more successful in promoting wage employment.

When $\epsilon = 10$, the trajectory of the relative price of home-production-substitute goods, p, is almost flat with tax cuts. Therefore, it closely resembles the one-good scenario commonly studied in the literature, where there doesn't exist a relative price effect.

To summarize, when assessing the impact of corporate tax cuts on promoting wage employment in a model with heterogeneous goods, the effect is minor, in line with some empirical findings. The model suggests that the relative price reacts to the occupation transition and brings unintended consequences. With tax cuts, the wage rate in the corporate sector will increase, attracting the self-employed to switch occupations. In the meantime, due to less supply of home-production-substitute goods as the self-employed left, the relative price of the goods provided by the self-employed will increase, thus attenuating the effect of the tax reform. If the goods produced by the self-employed and the wage-employed are easier to substitute, households demand fewer goods made by

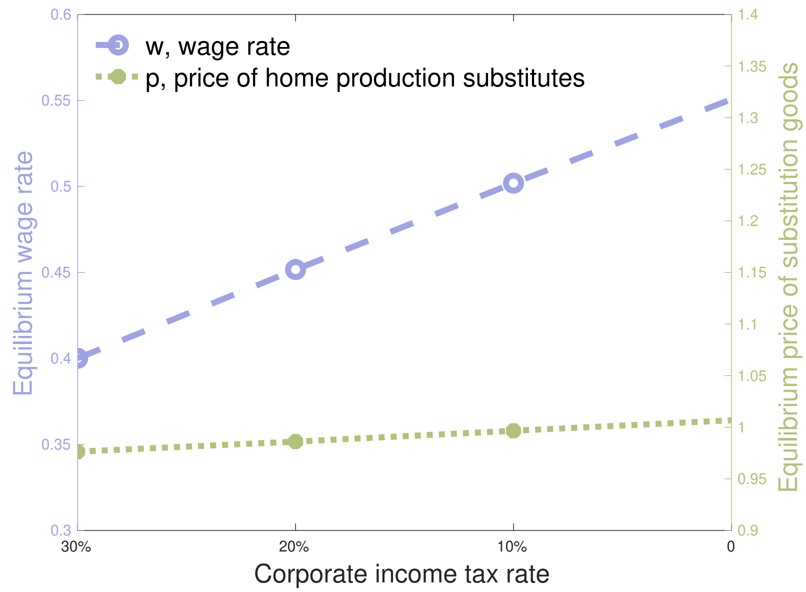


Figure 1.11: Equilibrium prices when $\epsilon = 10$

Notes. The x-axis represents the corporate tax rate from 30% to 0. The purple dashed line shows the equilibrium wage rate (left y-axis) with the corporate tax cut, and the green dotted line shows the equilibrium price of home production substitutes (right y-axis).

the self-employed. As a result, the relative price increase will be slight when the self-employed transit to the corporate sector, and the corporate tax cuts will be more powerful to promote wage employment in this case.

1.5.2 Increased regulation of the informal sector

Self-employment is generally connected with informal activities in low-income countries. Many low-skilled set up small businesses for subsistence and don't have any formal business registration. As a consequence, these informal self-employed escape tax obligations and impair the fiscal capacity of the country. Due to limited government revenues, developing countries often find themselves short of funding to finance projects that promote long-term growth, like infrastructure investments or education.

To eradicate the informal sector and encourage formalization, governments often implement two key measures. The first measure involves increasing surveillance of the informal sector. For example, governments may deploy more enforcement officers to oversee informal activities, with those caught operating without proper registration facing higher fines. Alternatively, they might impose a lump-sum tax on small businesses run by the self-employed. The second measure focuses on reducing the entry costs for the formal sector. This includes policies aimed at simplifying the processes for establishing formal businesses, minimizing bureaucratic hurdles, reimbursing formalization fees, and providing assistance with registration. In a nutshell, these regulations raise the relative costs of remaining in the informal sector, thereby incentivizing formalization.

I model the regulation of the informal sector by adding a lump-sum cost, s, to the operation of self-employment. In this case, the budget constraint of the self-employed becomes:

$$c_m + pc_{ph} = p\xi(\bar{T} - h - l) - s. \tag{1.19}$$

With a higher level of supervision, the informal activities have a chance of being caught and will pay a higher fine; thus s increases. Alternatively, policies that reduce the cost of formalization translate into a higher relative cost for the self-employed who stay in the informal sector ($s \uparrow$). The agents make the occupational choice based on higher utility as before, with a new budget constraint for the self-employed.

Then, I study how increasing the regulation of the informal sector affects the wage employment in the economy. With a higher cost of staying self-employed, people switch to the corporate sector, thus bringing down the equilibrium wage rate. As fewer self-employed produce home-production-substitute goods, the relative price of the goods they make increases. Figure 1.12 shows the transition of equilibrium prices with more strict regulation.

The elasticity of substitution between goods produced in two sectors matters for the effectiveness of regulation on promoting wage employment. When goods are harder to substitute (lower ϵ), the home-production-substitute goods price over wage ratio rises faster with heavier regulation (see panel (a) of Figure 1.13) because consumers have a greater demand for the goods made by the self-employed. Hence, reducing self-employment is less significant, making regulation policies less efficient.

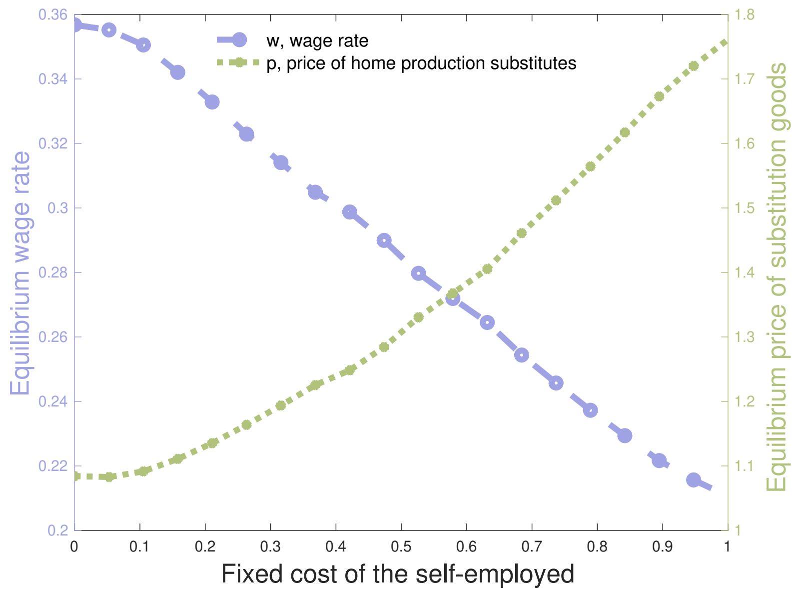
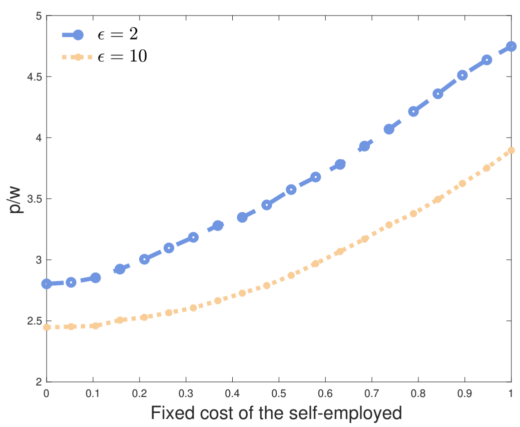
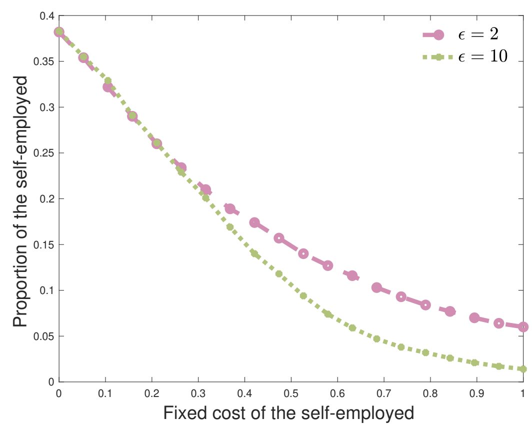


Figure 1.12: Equilibrium prices with regulation of the informal sector

Notes. The x-axis represents the fixed cost of the self-employed. The purple dashed line shows the equilibrium wage rate (left y-axis) with the corporate tax cut, and the green dotted line shows the equilibrium price of home production substitutes (right y-axis).



(a) Price ratio



(b) Prop. of self-employment

Figure 1.13: Regulation of the informal sector

Notes. Panel (a) shows the equilibrium price ratio $\frac{p}{w}$ with different levels of fixed cost of self-employment for two levels of elasticity of substitution. Panel (b) shows the proportion of self-employment with varying levels of fixed cost of self-employment for two levels of elasticity of substitution.

1.6 Summary and Concluding Remarks

This paper explains why policy reforms, like corporate tax cuts, may not successfully boost wage employment in some experiments. Empirically, I find that the self-employed and the wage-employed spread out in different occupations in developing countries, with the self-employed most likely taking jobs that provide home production substitute goods. Given this empirical evidence, I modify an assumption typically used in the quantitative literature that both employment sectors produce the same goods.

In an occupational choice model where the self-employed produce home production substitute goods and the wage-employed produce manufacturing goods, a relative price effect attenuates any policy attempts to increase wage employment. With fewer self-employed in the economy, the reduced supply raises the price of the goods they provide. This unintended consequence weakens the policies that aim to lower the self-employment rate. The result is consistent with empirical literature studying the impact of corporate income tax cuts on wage employment in Vietnam, China, Brazil, etc.

The model also has some limitations. First, I am not considering the job search, which has more frictions in developing countries. Many people flow through self-employment while searching for more formal jobs. Second, I ignore some corporations that provide home production substitutes, like Starbucks and McDonald's, that are popular in Western countries. Therefore, the model is more suitable in a developing country setting, where corporatized home production substitute goods have a small presence. Third, I assume that all self-employed provide home production substitute goods and avoid paying taxes for simplification, which is not valid in reality. In low-income countries, street food vendors may be the most common form of self-employment; there are also self-employed who are not doing home-production-related occupations, follow the tax regulations, and are essential to the functioning of the economy. Future research may address these limitations.

This paper delivers several policy implications. First and foremost, while designing industrial policies that intend to increase employment share in specific sectors, governments should consider the previous sectors where the newly-attracted workers come from, how substitutable the goods/services in different sectors, and how policies affect the prices of goods in various sectors. As the model shows, the relative price change across sectors might dampen the effectiveness of policies promoting employment in specific industries. Second, given that most self-employed in developing countries produce home production substitutes, essential in everyday life and hard to replace, it is advisable to strengthen efforts to formalize these businesses and improve their productivity. Moreover, the emergence of more efficient home appliances, like vacuum robots and washing machines, will lessen household's demand for goods from the self-employed, thus lowering the self-employment rate.

1.7 Appendix

1.7.1 Details on occupations

Table 1.6: Average proportion in each occupation group for low-income countries (%)

	Self-employed	Wage-employed
Group 1: Managers	3.71	4.28
Group 2: Professionals	2.74	22.27
Group 3: Technicians and Associate Professionals	3.29	7.77
Group 4: Clerical Support Workers	0.21	4.82
Group 5: Service and Sales Workers	49.56	17.22
Group 6: Craft and Related Trades Workers	20.06	14.67
Group 7: Plant and Machine Operators, and Assemblers	6.22	9.69
Group 8: Elementary Occupations	14.21	19.27

Table 1.6 presents the occupational distribution by employment status in low-income countries. The 8 occupational groups are classified according to ILO standards, excluding agricultural and armed forces occupations. The proportions within each employment status category sum up to 1.

In developing countries, service-and-sales-related occupations dominate among the self-employed. These jobs include personal service workers, personal care workers, protective service workers, and street and related sales and service workers. The goods and services provided by these occupations—such as street foods, childcare, haircuts, and massages—are closely related to home-production substitutes. Therefore, I aim to consolidate these occupation groups to better understand how the two employment sectors differ in their distribution within home-production-related occupations.

Home-production-related occupations include personal service workers, personal care workers, protective service workers, cleaners and helpers, food preparation assistants, street and related sales and service workers, refuse workers, and other elementary occupations. A challenge in isolating these occupations is that, according to ILO classification, some street food salespersons are categorized under sales occupations, which also include shop salespersons not related to home production. To address this, I use data from Tanzania to estimate the proportion of salespersons involved in street food sales. The Tanzania Integrated Labour Force Survey 2020/2021 indicates that, among self-employed salespersons, 77.80% are stall and market salespersons, while 22.20% are shop salespersons and demonstrators. For wage-employed salespersons, the proportions are 34.35% and 65.55%, respectively. Therefore, I apply these ratios to the ILO sales occupation data to differentiate between stall and market salespersons and other sales roles.

1.7.2 Calibration details

Log hourly wage

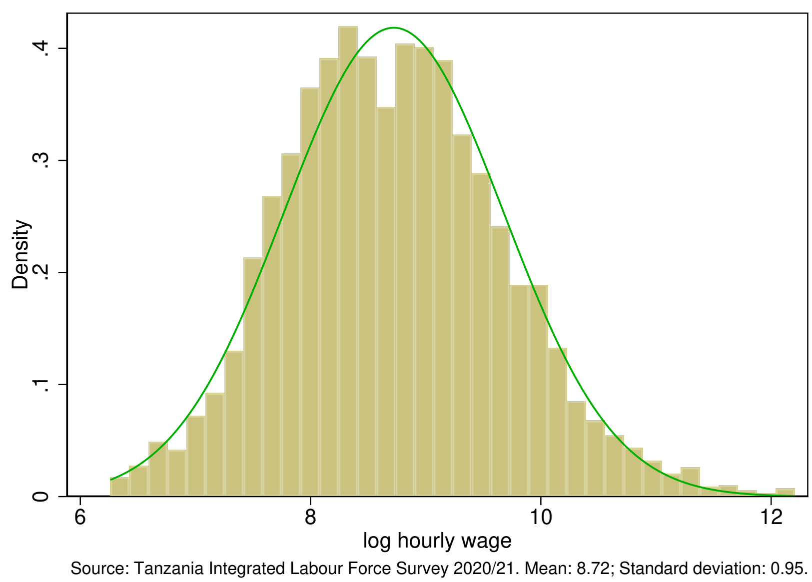


Figure 1.14: Distribution of log hourly wage for the working-age employees

Classification of National Account

For each non-agricultural activity in Tanzania's national account, I categorize it as either home-production-substitute goods or non-home-production-substitute goods to calculate the household's expenditure share in these two categories. The classification of each economic activity follows.

- Non-home-production-substitute goods
  - Mining and quarrying
  - Manufacturing
  - Electricity supply
  - Water supply; sewerage, waste management
  - Construction
  - Information and communication
  - Financial and insurance activities
  - Real estate
  - Professional, scientific and technical activities
  - Administrative and support service activities
  - Public administration and defence
  - Education
  - Arts, entertainment and recreation
- Home-production-substitute goods
  - Wholesale and retail trade; repairs
  - Transport and storage
  - Accommodation and Food Services
  - Human health and social work activities
  - Other service activities
  - Activities of households as employers

1.7.3 Calibration with a higher elasticity of substitution

Predetermined parameter values

Table 1.7: Predetermined parameter value

	Parameter value	Source
α	$\frac{1}{3}$	
ϵ, elasticity of substitution between c_m and c_h	10	
ζ, elasticity of substitution between c_{ph} and c_{sh}	4	Moro et al. (2017)
σ, relative risk aversion	1.4	Fang and Zhu (2017)
z, TFP term	1	normalization
ρ, home production productivity	1	normalization
ξ, NE productivity	1	normalization
$\bar{T}$, total time endowment	1	normalization
μ_ν, mean of $\ln \nu$	1	normalization
τ, corporate income tax rate	0.3	

- Targeted moments

Table 1.8: Targeted moments

	Model	2020 Data
Prop. of self-employment	0.38	0.38
Avg market work time	0.39	0.39
Avg household work time	0.26	0.26
Std log wage	0.95	0.95
Consumption goods expenditure share	0.71	0.71
Average income ratio: worker over NE	2.77	2.77

- Calibrated parameter values

Table 1.9: Calibrated parameter values

	2020
σ_ν, std of $\ln \nu$	0.9328
θ, weight on consumption in the utility function	0.6398
ϕ, weight on c_m in the c_{com} composite	0.3384
ψ, weight on c_{ph} in the c_h composite	0.4618
μ_ι, mean of ι	-0.1032
σ_ι, std of ι	1.1207

1.7.4 Analytical Results

Optimization problem of the wage-employed

$$\max_{h,l,c_m,c_{ph}} u_{we}(c_m, c_h, l) = \theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma} \tag{1.20}$$

$$s.t. \quad c_m + pc_{ph} = w\nu(\bar{T} - h - l) \tag{1.21}$$

where

$$c_{com} = \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{1.22}$$

$$c_h = \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \tag{1.23}$$

$$c_{sh} = \rho h \tag{1.24}$$

The Lagrangian function is:

$$\mathcal{L} = \theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma} + \lambda\left(w\nu(\bar{T} - h - l) - c_m - pc_{ph}\right) \tag{1.25}$$

1. FOC wrt c_m:

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial c_m} = \theta c_{com}^{-\sigma}\frac{\epsilon}{\epsilon-1}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} \phi\frac{\epsilon-1}{\epsilon}c_m^{-\frac{1}{\epsilon}} - \lambda = 0 \\
&\implies \theta\phi c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_m^{-\frac{1}{\epsilon}} = \lambda
\end{aligned} \tag{1.26}$$

2. FOC wrt c_{ph}:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_{ph}} &= \theta c_{com}^{-\sigma}\frac{\epsilon}{\epsilon-1}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} (1-\phi)\frac{\epsilon-1}{\epsilon}c_h^{-\frac{1}{\epsilon}} \\
&\cdot \frac{\zeta}{\zeta-1}\left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} \psi\frac{\zeta-1}{\zeta}c_{ph}^{-\frac{1}{\zeta}} - \lambda p = 0 \\
\implies &\theta(1-\phi)c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_h^{-\frac{1}{\epsilon}} \cdot \psi\left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} c_{ph}^{-\frac{1}{\zeta}} = \lambda p
\end{aligned} \tag{1.27}$$

3. FOC wrt h:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial h} &= \theta c_{com}^{-\sigma} \frac{\epsilon}{\epsilon-1} \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} (1-\phi) \frac{\epsilon-1}{\epsilon} c_h^{-\frac{1}{\epsilon}} \\
&\cdot \frac{\zeta}{\zeta-1} \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} (1-\psi) \frac{\zeta-1}{\zeta} c_{sh}^{-\frac{1}{\zeta}} \rho - \lambda w \nu = 0 \\
\implies \theta(1-\phi) c_{com}^{-\sigma} & \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_h^{-\frac{1}{\epsilon}} \cdot (1-\psi)\rho \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} c_{sh}^{-\frac{1}{\zeta}} = \lambda w \nu
\end{aligned}
\tag{1.28}
$$

4. FOC wrt l:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial l} &= (1-\theta) l^{-\sigma} - \lambda w \nu = 0 \\
\implies (1-\theta) l^{-\sigma} &= \lambda w \nu
\end{aligned}
\tag{1.29}
$$

Combine FOCs:

1. Purchased substitute goods, c_{ph}, v.s. home production goods, c_{sh}

   Combine (1.27) and (1.28):

$$
\frac{\psi}{(1-\psi)\rho} \left(\frac{c_{ph}}{c_{sh}}\right)^{-\frac{1}{\zeta}} = \frac{p}{w\nu} \tag{1.30}
$$

$$
\implies \frac{c_{ph}}{c_{sh}} = \left(\frac{w\nu\psi}{p(1-\psi)\rho}\right)^{\zeta} \tag{1.31}
$$

2. Manufactured goods, c_m, v.s. total home production goods, c_h

Combine (1.26) and (1.27):

$$\frac{(1-\phi)c_h^{-\frac{1}{\epsilon}}\psi c_h^{\frac{1}{\zeta}}c_{ph}^{-\frac{1}{\zeta}}}{\phi c_m^{-\frac{1}{\epsilon}}} = p \tag{1.32}$$

$$\implies \left(\frac{c_h}{c_m}\right)^{-\frac{1}{\epsilon}}\left(\frac{c_h}{c_{ph}}\right)^{\frac{1}{\zeta}} = \frac{p\phi}{\psi(1-\phi)} \tag{1.33}$$

$$\implies \frac{c_h}{c_m} = \left[\frac{p\phi}{\psi(1-\phi)}\left(\frac{c_h}{c_{ph}}\right)^{-\frac{1}{\zeta}}\right]^{-\epsilon} \tag{1.34}$$

$$\implies \frac{c_h}{c_m} = \left[\frac{\psi(1-\phi)}{p\phi}\left(\frac{c_h}{c_{ph}}\right)^{\frac{1}{\zeta}}\right]^{\epsilon} \tag{1.35}$$

3. Manufactured goods, c_m, v.s. self-made home production goods, c_{sh}

   Combine (1.26) and (1.28):

$$\frac{c_m}{c_h} = \left[\frac{w\nu\phi}{\rho(1-\psi)(1-\phi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \tag{1.36}$$

4. Amount of leisure l

   From (1.29):

$$l = \left(\frac{1-\theta}{\lambda w\nu}\right)^{\frac{1}{\sigma}} \tag{1.37}$$

Derive analytical solutions:

1. Since $\frac{c_{ph}}{c_{sh}} = \left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta}$ and $c_{sh} = \rho h$, then we have:

$$\begin{aligned} c_{ph} &= \left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta}\cdot c_{sh} \\ &= \underbrace{\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta}}_{\mathbf{P}}\cdot\rho h \end{aligned} \tag{1.38}$$

2. By definition, $c_h = \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}$, then we have:

$$\begin{aligned} c_h &= \left(\psi(\mathbf{P}c_{sh})^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\psi\mathbf{P}^{\frac{\zeta-1}{\zeta}}c_{sh}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\left(\psi\mathbf{P}^{\frac{\zeta-1}{\zeta}} + (1-\psi)\right)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \\ &= \underbrace{\left(\psi\mathbf{P}^{\frac{\zeta-1}{\zeta}} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}}}_{\mathbf{H}} c_{sh} \end{aligned} \tag{1.39}$$

More specifically, we could write $\mathbf{H}$ as:

$$\begin{aligned} \mathbf{H} &= \left(\psi\mathbf{P}^{\frac{\zeta-1}{\zeta}} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\psi^{\zeta}\left(\frac{w\nu}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}} \end{aligned} \tag{1.40}$$

3. Since $\frac{c_m}{c_h} = \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}$, then

$$\begin{aligned} c_m &= \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} c_h \\ c_m &= \underbrace{\left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}\mathbf{H}}_{\mathbf{M}\equiv\mathbf{XH}} \cdot c_{sh} \end{aligned} \tag{1.41}$$

We could write $\mathbf{X}$ explicitly:

$$\mathbf{X} = \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon}\left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon}{1-\zeta}} \tag{1.42}$$

Thus, $\mathbf{M} = \mathbf{XH}$:

$$\mathbf{M} = \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon} \left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta-\epsilon}{\zeta-1}} \tag{1.43}$$

4. By definition, $c_{com} = \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$, which is also a linear function of c_{sh}.

$$\begin{aligned} c_{com} &= \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\phi\left(\mathbf{M}c_{sh}\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\left(\mathbf{H}c_{sh}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \underbrace{\left(\phi\mathbf{M}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{H}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}}_{\mathbf{G}} c_{sh} \end{aligned} \tag{1.44}$$

To simplify $\mathbf{G}$:

$$\begin{aligned} \mathbf{G} &= \left(\phi\mathbf{M}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{H}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\phi(\mathbf{XH})^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{H}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\phi\mathbf{X}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\right)^{\frac{\epsilon}{\epsilon-1}} \mathbf{H} \end{aligned} \tag{1.45}$$

Since $\mathbf{X} = \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon} \left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon}{1-\zeta}}$ and $\mathbf{H} = \left(\psi^{\zeta}\left(\frac{w\nu}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}}$, then

$$\begin{aligned} \mathbf{G} = &\left(\phi\left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon-1} \left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon-1}{1-\zeta}} + 1-\phi\right)^{\frac{\epsilon}{\epsilon-1}} \\ &\cdot \left(\psi^{\zeta}\left(\frac{w\nu}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}} \end{aligned} \tag{1.46}$$

5. Because $\theta\phi c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_m^{-\frac{1}{\epsilon}} = \lambda$, now we simplify λ.

$$\lambda = \theta\phi\mathbf{G}^{-\sigma}\left(\frac{\mathbf{G}}{\mathbf{M}}\right)^{\frac{1}{\epsilon}} c_{sh}^{-\sigma} \tag{1.47}$$

6. Since $l = \left(\frac{1-\theta}{\lambda w\nu}\right)^{\frac{1}{\sigma}}$, then

$$l = \underbrace{\left(\frac{1-\theta}{w\nu\theta\phi}\right)^{\frac{1}{\sigma}} \mathbf{G}\left(\frac{\mathbf{G}}{\mathbf{M}}\right)^{-\frac{1}{\epsilon\sigma}}}_{\mathbf{L}} c_{sh} \tag{1.48}$$

7. In the budget constraint: $c_m + pc_{ph} = w\nu(\bar{T} - h - l)$, could solve for h, thus everything else.

$$\begin{aligned}\mathbf{M}\rho h + p\mathbf{P}\rho h &= w\nu\bar{T} - w\nu h - w\nu\mathbf{L}\rho h \\ (\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho)\, h &= w\nu\bar{T} \\ h &= \frac{w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho}\end{aligned} \tag{1.49}$$

The other variables could also be solved as:

$$c_{sh} = \rho h = \frac{\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{1.50}$$

$$c_{ph} = \mathbf{P}c_{sh} = \frac{\mathbf{P}\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{1.51}$$

$$c_h = \mathbf{H}c_{sh} = \frac{\mathbf{H}\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{1.52}$$

$$c_m = \mathbf{M}c_{sh} = \frac{\mathbf{M}\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{1.53}$$

$$c_{com} = \mathbf{G}c_{sh} = \frac{\mathbf{G}\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{1.54}$$

$$l = \mathbf{L}c_{sh} = \frac{\mathbf{L}\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{1.55}$$

8. The optimal consumption bundle, the ratio between c_m and c_{ph}:

$$\begin{aligned}\frac{c_m}{c_{ph}} &= \frac{\mathbf{M}}{\mathbf{P}} \\ &= \frac{\left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}\cdot\left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1}+(1-\psi)\right)^{\frac{\zeta}{\zeta-1}}}{\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta}} \\ &= \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}\cdot\left[\psi+(1-\psi)\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{1-\zeta}\right]^{\frac{\zeta}{\zeta-1}}\end{aligned} \tag{1.56}$$

Derive the expressions for time use

1. Simplify $\rho\mathbf{L}$:

   First of all, we know that

$$\begin{aligned}\mathbf{L} &= \left(\frac{1-\theta}{\theta\phi w\nu}\right)^{\frac{1}{\sigma}}\mathbf{G}\left(\frac{\mathbf{G}}{\mathbf{M}}\right)^{-\frac{1}{\epsilon\sigma}} \\ &= \left(\frac{1-\theta}{\theta\phi w\nu}\right)^{\frac{1}{\sigma}}\mathbf{H}\left(\phi\mathbf{X}^{\frac{\epsilon-1}{\epsilon}}+1-\phi\right)^{\frac{\epsilon}{\epsilon-1}}\left(\phi+(1-\phi)\mathbf{X}^{\frac{1-\epsilon}{\epsilon}}\right)^{\frac{1}{\sigma(1-\epsilon)}}\end{aligned} \tag{1.57}$$

   Plug in $\mathbf{X}$, which is

$$\begin{aligned}\mathbf{X} &= \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \\ &= \left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{-\frac{\epsilon}{\zeta}}\end{aligned} \tag{1.58}$$

   Then we have

$$\begin{aligned}\mathbf{L} = \frac{1-\theta}{\theta\phi w\nu}\mathbf{H}&\left(\phi\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon-1}\mathbf{H}^{\frac{1-\epsilon}{\zeta}}+1-\phi\right)^{\frac{\epsilon}{\epsilon-1}}\left(\phi+(1-\phi)\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{H}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1}{1-\epsilon}} \\ &\cdot\underbrace{\left(\frac{1-\theta}{\theta\phi w\nu}\right)^{\frac{1-\sigma}{\sigma}}\left(\phi+(1-\phi)\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{H}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1-\sigma}{\sigma(1-\epsilon)}}}_{\equiv\mathbf{I}}\end{aligned} \tag{1.59}$$

Then,

$$\rho\mathbf{L} = \frac{1-\theta}{\theta\phi w\nu}\mathbf{H}\left(\rho\phi\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon-1}\mathbf{H}^{\frac{1-\epsilon}{\zeta}} + \rho(1-\phi)\right)^{\frac{\epsilon}{\epsilon-1}}$$
$$\cdot\left(\rho\phi + \rho(1-\phi)\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{H}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{I}$$
$$= \frac{1-\theta}{\theta\phi w\nu}\mathbf{H}(1-\phi)^{\frac{\epsilon}{\epsilon-1}}\phi^{\frac{1}{1-\epsilon}}\left(\rho^{2-\epsilon}\underbrace{(w\nu)^{\epsilon-1}\left(\frac{\phi}{1-\phi}\right)^{\epsilon}(1-\psi)^{1-\epsilon}\mathbf{H}^{\frac{1-\epsilon}{\zeta}}}_{\mathcal{R}} + \rho\right)^{\frac{\epsilon}{\epsilon-1}} \tag{1.60}$$
$$\cdot\left(\rho + \rho^{\epsilon}\underbrace{(w\nu)^{1-\epsilon}\left(\frac{\phi}{1-\phi}\right)^{-\epsilon}(1-\psi)^{\epsilon-1}\mathbf{H}^{\frac{\epsilon-1}{\zeta}}}_{\mathcal{R}^{-1}}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{I}$$
$$= \frac{1-\theta}{\theta w\nu}\left(\frac{1-\phi}{\phi}\right)^{\frac{\epsilon}{\epsilon-1}}\mathbf{H}\left(\rho^{2-\epsilon}\mathcal{R}+\rho\right)^{\frac{\epsilon}{\epsilon-1}}\left(\rho+\rho^{\epsilon}\mathcal{R}^{-1}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{I}$$
$$= \frac{1-\theta}{\theta w\nu}\left(\frac{1-\phi}{\phi}\right)^{\frac{\epsilon}{\epsilon-1}}\mathbf{H}(\mathcal{R}\rho^{1-\epsilon}+1)\mathcal{R}^{\frac{1}{\epsilon-1}}\cdot\mathbf{I}$$

Plug in $\mathcal{R}$:

$$\rho\mathbf{L} = \frac{1-\theta}{\theta(1-\psi)}\mathbf{H}^{1-\frac{1}{\zeta}}\left(\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{1-\phi}\right)^{\epsilon}(1-\psi)^{1-\epsilon}\mathbf{H}^{\frac{1-\epsilon}{\zeta}}+1\right)\cdot\mathbf{I}$$
$$= \frac{1-\theta}{\theta}\left[\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}} + \frac{1}{1-\psi}\mathbf{H}^{1-\frac{1}{\zeta}}\right]\cdot\mathbf{I}$$
$$= \frac{1-\theta}{\theta}\left[\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1}+1\right]\cdot\mathbf{I} \tag{1.61}$$

2. Simplify $p\mathbf{P}\rho$:

$$p\mathbf{P}\rho = (w\nu)^{\zeta}\left(\frac{\psi}{1-\psi}\right)^{\zeta}(p\rho)^{1-\zeta} \tag{1.62}$$

3. Simplify $\mathbf{M}\rho$:

$$\begin{aligned}\mathbf{M}\rho &= \mathbf{X}\mathbf{H}\rho \\ &= \left(\frac{w\nu\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{1-\frac{\epsilon}{\zeta}}\rho^{1-\epsilon}\end{aligned} \tag{1.63}$$

4. Derive h:

$$\begin{aligned}h &= \frac{w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \\ &= \frac{\bar{T}}{\frac{\mathbf{M}\rho}{w\nu} + \frac{p\mathbf{P}\rho}{w\nu} + 1 + \mathbf{L}\rho} \\ &= \frac{\bar{T}}{(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{1-\frac{\epsilon}{\zeta}}\rho^{1-\epsilon} + (w\nu)^{\zeta-1}\left(\frac{\psi}{1-\psi}\right)^{\zeta}(p\rho)^{1-\zeta} + 1 + \rho\mathbf{L}} \\ &= \frac{\theta\bar{T}}{\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1} + 1} \cdot \frac{1}{\theta + (1-\theta)\mathbf{I}}\end{aligned} \tag{1.64}$$

5. Derive l:

$$\begin{aligned}l = \rho\mathbf{L}\cdot h &= (1-\theta)\bar{T}\cdot\frac{\mathbf{I}}{\theta + (1-\theta)\mathbf{I}} \\ &= (1-\theta)\bar{T}\cdot\frac{1}{\frac{\theta}{\mathbf{I}} + 1 - \theta}\end{aligned} \tag{1.65}$$

$$\mathbf{I} = \underbrace{\left(\frac{1-\theta}{\theta\phi w\nu}\right)^{\frac{1-\sigma}{\sigma}}}_{(1)}\left(\phi + (1-\phi)\underbrace{\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}}_{(2)}\underbrace{\mathbf{H}^{\frac{\epsilon-1}{\zeta}}}_{(3)}\right)^{\frac{1-\sigma}{\sigma(1-\epsilon)}}$$

6. Derive n:

$$\begin{aligned}n &= \bar{T} - l - h \\ &= \bar{T} - (1-\theta)\bar{T}\frac{\mathbf{I}}{\theta + (1-\theta)\mathbf{I}} - h \\ &= \theta\bar{T}\frac{\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1}}{\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1} + 1} \cdot \frac{1}{\theta + (1-\theta)\mathbf{I}}\end{aligned} \tag{1.66}$$

7. Derive the work-to-home production time ratio, $\mathbf{N}$

$$\mathbf{N} = \frac{n}{h} = \rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1} \tag{1.67}$$

Optimization problem of the self-employed

$$\max_{h,l,c_m,c_{ph}} u_{se}(c_m, c_h, l) = \iota + \theta\frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma} \tag{1.68}$$

$$s.t. \quad c_m + pc_{ph} = p\xi(\bar{T} - h - l) \tag{1.69}$$

where

$$c_{com} = \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{1.70}$$

$$c_h = \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \tag{1.71}$$

$$c_{sh} = \rho h \tag{1.72}$$

The Lagrangian function:

$$\mathcal{L} = \iota + \theta\frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma} + \lambda\left(p\xi(\bar{T} - h - l) - c_m - pc_{ph}\right) \tag{1.73}$$

1. FOC wrt c_m:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_m} &= \theta c_{com}^{-\sigma}\frac{\epsilon}{\epsilon-1}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}}\phi\frac{\epsilon-1}{\epsilon}c_m^{-\frac{1}{\epsilon}} - \lambda = 0 \\ \implies \theta\phi c_{com}^{-\sigma}&\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_m^{-\frac{1}{\epsilon}} = \lambda \end{aligned} \tag{1.74}$$

2. FOC wrt c_{ph}:

$$\frac{\partial \mathcal{L}}{\partial c_{ph}} = \theta c_{com}^{-\sigma} \frac{\epsilon}{\epsilon-1} \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} \frac{\epsilon-1}{\epsilon}(1-\phi) c_h^{-\frac{1}{\epsilon}}$$
$$\cdot \frac{\zeta}{\zeta-1} \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} \psi \frac{\zeta-1}{\zeta} c_{ph}^{-\frac{1}{\zeta}} - \lambda p = 0$$
$$\Longrightarrow \theta(1-\phi) c_{com}^{-\sigma} \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_h^{-\frac{1}{\epsilon}} \cdot \psi \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} c_{ph}^{-\frac{1}{\zeta}} = \lambda p \tag{1.75}$$

3. FOC wrt h:

$$\frac{\partial \mathcal{L}}{\partial h} = \theta c_{com}^{-\sigma} \frac{\epsilon}{\epsilon-1} \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} (1-\phi) \frac{\epsilon-1}{\epsilon} c_h^{-\frac{1}{\epsilon}}$$
$$\cdot \frac{\zeta}{\zeta-1} \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} (1-\psi) \frac{\zeta-1}{\zeta} c_{sh}^{-\frac{1}{\zeta}} \rho - \lambda p \xi = 0$$
$$\Longrightarrow \theta(1-\phi) c_{com}^{-\sigma} \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_h^{-\frac{1}{\epsilon}} \cdot (1-\psi)\rho \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} c_{sh}^{-\frac{1}{\zeta}} = \lambda p \xi \tag{1.76}$$

4. FOC wrt l:

$$\frac{\partial \mathcal{L}}{\partial l} = (1-\theta) l^{-\sigma} - \lambda p \xi = 0$$
$$\Longrightarrow (1-\theta) l^{-\sigma} = \lambda p \xi \tag{1.77}$$

Combine FOCs

1. Purchased substitute goods, c_{ph}, v.s. home production goods, c_{sh}

   Combine (1.75) and (1.76):

$$\frac{\psi}{(1-\psi)\rho} \left(\frac{c_{ph}}{c_{sh}}\right)^{-\frac{1}{\zeta}} = \frac{1}{\xi} \tag{1.78}$$
$$\Longrightarrow \frac{c_{ph}}{c_{sh}} = \left(\frac{\psi \xi}{(1-\psi)\rho}\right)^{\zeta} \tag{1.79}$$

2. Firm-manufactured goods, c_m, v.s. total home production goods, c_h

Combine (1.74) and (1.75), get exactly the same results as in worker's maximization problem.

3. Firm-manufactured goods, c_m, v.s. self-made home production goods, c_{sh}

   Combine (1.74) and (1.76):

$$\frac{c_m}{c_h} = \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)} \left(\frac{c_h}{c_{sh}} \right)^{-\frac{1}{\zeta}} \right]^{\epsilon} \tag{1.80}$$

4. Amount of leisure, l

   From (1.77):

$$l = \left(\frac{1-\theta}{\lambda p \xi} \right)^{\frac{1}{\sigma}} \tag{1.81}$$

Derive analytical solutions

1. Since $\frac{c_{ph}}{c_{sh}} = \left(\frac{\psi\xi}{(1-\psi)\rho} \right)^{\zeta}$ and $c_{sh} = \rho h$, then we have:

$$\begin{aligned} c_{ph} &= \left(\frac{\psi\xi}{(1-\psi)\rho} \right)^{\zeta} \cdot c_{sh} \\ &= \underbrace{\left(\frac{\psi\xi}{(1-\psi)\rho} \right)^{\zeta}}_{\mathbf{Q}} \cdot \rho h \end{aligned} \tag{1.82}$$

2. By definition, $c_h = \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}}$, then we have:

$$\begin{aligned} c_h &= \left(\psi (\mathbf{Q} c_{sh})^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\psi \mathbf{Q}^{\frac{\zeta-1}{\zeta}} c_{sh}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\left(\psi \mathbf{Q}^{\frac{\zeta-1}{\zeta}} + (1-\psi) \right) c_{sh}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \\ &= \underbrace{\left(\psi \mathbf{Q}^{\frac{\zeta-1}{\zeta}} + (1-\psi) \right)^{\frac{\zeta}{\zeta-1}}}_{\mathbf{K}} c_{sh} \end{aligned} \tag{1.83}$$

More specifically, we could write $\mathbf{K}$ as:

$$\begin{aligned}\mathbf{K} &= \left(\psi \mathbf{Q}^{\frac{\zeta-1}{\zeta}} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\psi \left(\frac{\psi\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\psi^{\zeta} \left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}}\end{aligned} \tag{1.84}$$

3. Since $\frac{c_m}{c_h} = \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}$, then

$$\begin{aligned}c_m &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} c_h \\ &= \underbrace{\left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \mathbf{K}}_{\mathbf{N}\equiv\mathbf{Y}\cdot\mathbf{K}} \cdot c_{sh}\end{aligned} \tag{1.85}$$

Simplify $\mathbf{Y}$:

$$\begin{aligned}\mathbf{Y} &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \\ &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon} \mathbf{K}^{-\frac{\epsilon}{\zeta}} \\ &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon} \left(\psi^{\zeta} \left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon}{1-\zeta}}\end{aligned} \tag{1.86}$$

Finally, $\mathbf{N} \equiv \mathbf{Y} \cdot \mathbf{K}$:

$$\begin{aligned}\mathbf{N} &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon} \mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}} \\ &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon} \left(\psi^{\zeta} \left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta-\epsilon}{\zeta-1}}\end{aligned} \tag{1.87}$$

4. By definition, $c_{com} = \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$, which is also a linear function of c_{sh}.

$$\begin{aligned} c_{com} &= \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\phi\left(\mathbf{N}c_{sh}\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\left(\mathbf{K}c_{sh}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \underbrace{\left(\phi\mathbf{N}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{K}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}}_{\mathbf{F}} c_{sh} \end{aligned} \tag{1.88}$$

To simplify $\mathbf{F}$:

$$\begin{aligned} \mathbf{F} &= \left(\phi\mathbf{N}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{K}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\phi(\mathbf{YK})^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{K}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\phi\mathbf{Y}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\right)^{\frac{\epsilon}{\epsilon-1}}\mathbf{K} \end{aligned} \tag{1.89}$$

Since $\mathbf{Y} = \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon}\left(\psi^{\zeta}\left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon}{1-\zeta}}$ and $\mathbf{K} = \left(\psi^{\zeta}\left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}}$, then

$$\begin{aligned} \mathbf{F} = &\left(\phi\left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon-1}\left(\psi^{\zeta}\left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon-1}{1-\zeta}} + (1-\phi)\right)^{\frac{\epsilon}{\epsilon-1}} \\ &\cdot\left(\psi^{\zeta}\left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}} \end{aligned} \tag{1.90}$$

5. Because $\theta\phi c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_m^{-\frac{1}{\epsilon}} = \lambda$, now we simplify λ.

$$\begin{aligned} \lambda &= \theta\phi c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_m^{-\frac{1}{\epsilon}} \\ &= \theta\phi c_{com}^{-\sigma} c_{com}^{\frac{1}{\epsilon}} c_m^{-\frac{1}{\epsilon}} \\ &= \theta\phi\mathbf{F}^{-\sigma}\left(\frac{\mathbf{F}}{\mathbf{N}}\right)^{\frac{1}{\epsilon}} c_{sh}^{-\sigma} \end{aligned} \tag{1.91}$$

6. Since $l = \left(\frac{1-\theta}{\lambda p\xi}\right)^{\frac{1}{\sigma}}$, l can be written as:

$$l = \underbrace{\left(\frac{1-\theta}{p\xi\theta\phi}\right)^{\frac{1}{\sigma}} \mathbf{F} \left(\frac{\mathbf{F}}{\mathbf{N}}\right)^{-\frac{1}{\epsilon\sigma}}}_{\mathbf{L}} c_{sh} \tag{1.92}$$

7. In the budget constraint: $c_m + pc_{ph} = p\xi(\bar{T} - h - l)$, could solve for h, thus everything else.

$$\begin{aligned} \mathbf{N}\rho h + p\mathbf{Q}\rho h &= p\xi\bar{T} - p\xi h - p\xi\mathbf{J}\rho h \\ (\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho)\, h &= p\xi\bar{T} \\ h &= \frac{p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \end{aligned} \tag{1.93}$$

The other variables could also be solved as:

$$c_{sh} = \rho h = \frac{\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{1.94}$$

$$c_{ph} = \mathbf{Q}c_{sh} = \frac{\mathbf{Q}\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{1.95}$$

$$c_h = \mathbf{K}c_{sh} = \frac{\mathbf{K}\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{1.96}$$

$$c_m = \mathbf{N}c_{sh} = \frac{\mathbf{N}\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{1.97}$$

$$c_{com} = \mathbf{F}c_{sh} = \frac{\mathbf{F}\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{1.98}$$

$$l = \mathbf{J}c_{sh} = \frac{\mathbf{J}\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{1.99}$$

8. The optimal consumption bundle, the ratio between c_m and c_{ph}:

$$\begin{aligned}\frac{c_m}{c_{ph}} &= \frac{\mathbf{N}}{\mathbf{Q}}\\ &= \frac{\left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}\cdot\left(\psi\left(\frac{\psi\xi}{\rho(1-\psi)}\right)^{\zeta-1}+(1-\psi)\right)^{\frac{\zeta}{\zeta-1}}}{\left(\frac{\psi\xi}{\rho(1-\psi)}\right)^{\zeta}}\\ &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}\cdot\left[\psi+(1-\psi)\left(\frac{\psi\xi}{\rho(1-\psi)}\right)^{1-\zeta}\right]^{\frac{\zeta}{\zeta-1}}\end{aligned} \tag{1.100}$$

Derive the expressions for time use

1. Simplify $\rho\mathbf{J}$:

   First of all, we know that

$$\begin{aligned}\mathbf{J} &= \left(\frac{1-\theta}{\theta\phi p\xi}\right)^{\frac{1}{\sigma}}\mathbf{F}\left(\frac{\mathbf{F}}{\mathbf{N}}\right)^{-\frac{1}{\epsilon\sigma}}\\ &= \left(\frac{1-\theta}{\theta\phi p\xi}\right)^{\frac{1}{\sigma}}\mathbf{K}\left(\phi\mathbf{Y}^{\frac{\epsilon-1}{\epsilon}}+1-\phi\right)^{\frac{\epsilon}{\epsilon-1}}\left(\phi+(1-\phi)\mathbf{Y}^{\frac{1-\epsilon}{\epsilon}}\right)^{\frac{1}{\sigma(1-\epsilon)}}\end{aligned} \tag{1.101}$$

   Plug in $\mathbf{Y}$, which is

$$\begin{aligned}\mathbf{Y} &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}\\ &= \left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{-\frac{\epsilon}{\zeta}}\end{aligned} \tag{1.102}$$

Then we have

$$
\begin{aligned}
\mathbf{J} = \frac{1-\theta}{\theta\phi p\xi}\mathbf{K}&\left(\phi\left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon-1}\mathbf{K}^{\frac{1-\epsilon}{\zeta}}+1-\phi\right)^{\frac{\epsilon}{\epsilon-1}}\\
&\cdot\left(\phi+(1-\phi)\left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{K}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1}{1-\epsilon}}\\
&\underbrace{\cdot\left(\frac{1-\theta}{\theta\phi p\xi}\right)^{\frac{1-\sigma}{\sigma}}\left(\phi+(1-\phi)\left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{K}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1-\sigma}{\sigma(1-\epsilon)}}}_{\equiv\mathbf{U}}
\end{aligned}
\tag{1.103}
$$

Then,

$$
\begin{aligned}
\rho\mathbf{J} &= \frac{1-\theta}{\theta\phi p\xi}\mathbf{K}\left(\rho\phi\left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon-1}\mathbf{K}^{\frac{1-\epsilon}{\zeta}}+\rho(1-\phi)\right)^{\frac{\epsilon}{\epsilon-1}}\\
&\cdot\left(\rho\phi+\rho(1-\phi)\left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{K}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{U}\\
&= \frac{1-\theta}{\theta\phi p\xi}\mathbf{K}(1-\phi)^{\frac{\epsilon}{\epsilon-1}}\phi^{\frac{1}{1-\epsilon}}\left(\rho^{2-\epsilon}\underbrace{(p\xi)^{\epsilon-1}\left(\frac{\phi}{1-\phi}\right)^{\epsilon}(1-\psi)^{1-\epsilon}\mathbf{K}^{\frac{1-\epsilon}{\zeta}}}_{\mathcal{O}}+\rho\right)^{\frac{\epsilon}{\epsilon-1}}\\
&\cdot\left(\rho+\rho^{\epsilon}\underbrace{(p\xi)^{1-\epsilon}\left(\frac{\phi}{1-\phi}\right)^{-\epsilon}(1-\psi)^{\epsilon-1}\mathbf{K}^{\frac{\epsilon-1}{\zeta}}}_{\mathcal{O}^{-1}}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{U}\\
&= \frac{1-\theta}{\theta p\xi}\left(\frac{1-\phi}{\phi}\right)^{\frac{\epsilon}{\epsilon-1}}\mathbf{K}\left(\rho^{2-\epsilon}\mathcal{O}+\rho\right)^{\frac{\epsilon}{\epsilon-1}}\left(\rho+\rho^{\epsilon}\mathcal{O}^{-1}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{U}\\
&= \frac{1-\theta}{\theta p\xi}\left(\frac{1-\phi}{\phi}\right)^{\frac{\epsilon}{\epsilon-1}}\mathbf{K}(\mathcal{O}\rho^{1-\epsilon}+1)\mathcal{O}^{\frac{1}{\epsilon-1}}\cdot\mathbf{U}
\end{aligned}
\tag{1.104}
$$

Plug in $\mathcal{O}$:

$$\begin{aligned}
\rho\mathbf{J} &= \frac{1-\theta}{\theta(1-\psi)}\mathbf{K}^{1-\frac{1}{\zeta}}\left(\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{1-\phi}\right)^{\epsilon}(1-\psi)^{1-\epsilon}\mathbf{K}^{\frac{1-\epsilon}{\zeta}}+1\right)\cdot\mathbf{U} \\
&= \frac{1-\theta}{\theta}\left[\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}}+\frac{1}{1-\psi}\mathbf{K}^{1-\frac{1}{\zeta}}\right]\cdot\mathbf{U} \\
&= \frac{1-\theta}{\theta}\left[\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}}+\left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{p\xi}{p\rho}\right)^{\zeta-1}+1\right]\cdot\mathbf{U}
\end{aligned} \tag{1.105}$$

2. Simplify $p\mathbf{P}\rho$:

$$p\mathbf{Q}\rho = (p\xi)^{\zeta}\left(\frac{\psi}{1-\psi}\right)^{\zeta}(p\rho)^{1-\zeta} \tag{1.106}$$

3. Simplify $\mathbf{N}\rho$:

$$\begin{aligned}
\mathbf{N}\rho &= \mathbf{Y}\mathbf{K}\rho \\
&= \left(\frac{p\xi\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{1-\frac{\epsilon}{\zeta}}\rho^{1-\epsilon}
\end{aligned} \tag{1.107}$$

4. Derive h:

$$\begin{aligned}
h &= \frac{p\xi\bar{T}}{\mathbf{N}\rho+p\mathbf{Q}\rho+p\xi+p\xi\mathbf{J}\rho} \\
&= \frac{\bar{T}}{\frac{\mathbf{N}\rho}{p\xi}+\frac{p\mathbf{P}\rho}{p\xi}+1+\mathbf{J}\rho} \\
&= \frac{\bar{T}}{(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{1-\frac{\epsilon}{\zeta}}\rho^{1-\epsilon}+(p\xi)^{\zeta-1}\left(\frac{\psi}{1-\psi}\right)^{\zeta}(p\rho)^{1-\zeta}+1+\rho\mathbf{J}} \\
&= \frac{\theta\bar{T}}{\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}}+\left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{p\xi}{p\rho}\right)^{\zeta-1}+1}\cdot\frac{1}{\theta+(1-\theta)\mathbf{U}}
\end{aligned} \tag{1.108}$$

5. Derive l:

$$\begin{aligned}
l &= \rho\mathbf{J}\cdot h = (1-\theta)\bar{T}\cdot\frac{\mathbf{U}}{\theta+(1-\theta)\mathbf{U}} \\
&= (1-\theta)\bar{T}\cdot\frac{1}{\frac{\theta}{\mathbf{U}}+1-\theta}
\end{aligned} \tag{1.109}$$

6. Derive n:

$$
\begin{aligned}
n &= \bar{T} - l - h \\
&= \bar{T} - (1-\theta)\bar{T}\frac{\mathbf{U}}{\theta + (1-\theta)\mathbf{U}} - h \\
&= \theta\bar{T}\frac{\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{p\xi}{p\rho}\right)^{\zeta-1}}{\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{p\xi}{p\rho}\right)^{\zeta-1} + 1} \cdot \frac{1}{\theta + (1-\theta)\mathbf{U}}
\end{aligned}
\tag{1.110}
$$

7. Derive the work-to-home production time ratio

$$
\frac{n}{h} = \rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{p\xi}{p\rho}\right)^{\zeta-1}
\tag{1.111}
$$

2

Quantifying the Impact of Closing the Gender Gap in the Labor Market on Japan's Economy

2.1 Introduction

Debt sustainability is a significant concern for Japan's economy. According to OECD data, Japan's debt-to-GDP ratio reached approximately 260% in 2022, far surpassing the OECD average of 89%. This makes Japan's public debt the most unsustainable in the world. The primary driver of this accumulated debt is the aging population, which necessitates higher government spending on public pensions and healthcare.

Japan has a lower female labor force participation rate compared to other developed countries with similar income levels. Although there has been a noticeable increase in recent years, with the proportion of female employees out of all female population rising from 38% in 1985 to 47% in 2022, it still lags behind the male employment rate. In 2022, female employment was approximately 77% of the male employment rate.

A higher female labor force participation rate is emerging as a key driver of economic growth. Increased female labor participation directly expands production by adding more labor inputs. Additionally, higher female labor income increases the tax base, enabling the government to reduce tax rates while maintaining the previous primary balance level. Lower tax rates reduce distortions and, in turn, benefit economic growth.

More women participating in the labor market also benefits Japan's fiscal policies. With higher tax revenues from labor income and consumption taxes, the government's primary balance will improve. Consequently, the government will have the capacity to reduce tax rates while maintaining the primary balance at its previous level.

In this paper, we extend a neoclassical growth model to include a gender dimension, building on the work by Hansen and İmrohoroğlu (2016). We calibrate this model to Japan's economy and explore the impact of increased female labor force participation on Japan's fiscal policies and economic outcomes. Specifically, we ask:"If the female labor force participation rate were to match that of the male's and the government reduced the consumption tax rate to maintain the primary balance at its previous steady-state level, what would the new consumption tax rate be, and how would other economic variables change in the new steady state?"

Based on our benchmark scenario results, achieving parity in female labor force participation with the male's allows for a 27% reduction in the consumption tax rate, maintaining the primary balance at its previous steady state level. Concurrently, consumption and output expand, while average working hours for both genders decrease. These outcomes persist even when accounting for increased government expenditure aimed at stimulating female labor force participation.

Given the estimated benefits of increased female labor market participation, the government should consider implementing a range of measures to encourage more women to enter the workforce. These measures could include providing greater support for childcare and eldercare, offering more flexible working arrangements, and promoting equal pay and opportunities for career advancement. Additionally, investing in education and training programs specifically targeted at women can help equip them with the skills needed for higher-paying and in-demand jobs. By creating a more inclusive and supportive environment for women in the labor market, the government can not only enhance economic growth and fiscal sustainability but also foster a more equitable and balanced society.

Related Literature. Our paper bridges the literature on fiscal policies and gender gaps in the labor market. It builds on previous works and introduces several contributions:

Firstly, this paper expands on the literature studying Japan's debt sustainability and fiscal policies, including Kitao and Yamada (2021), İmrohoroğlu et al. (2019), Hansen and İmrohoroğlu (2016), İmrohoroğlu et al. (2016). Our contribution lies in exploring how to enhance fiscal capacity in Japan through an innovative approach: increasing female labor force participation.

Secondly, we contribute to the literature on female labor force participation. Notable prior works include Ngai et al. (2024), Dinkelman and Ngai (2021), and Fernández (2013). Our paper examines the fiscal impact of closing gender gaps in the labor market, providing a fresh perspective on the potential economic benefits of gender equality in employment.

Structure. The rest of the paper is organized as follows. Section 2.2 describes the neoclassical growth model with a gender dimension. Section 2.3 summarizes the calibration of the model to the Japanese economy. Section 2.4 outlines the quantitative exercise. Finally, Section 2.5 provides the conclusion.

2.2 A Neoclassical Growth Model with Gender

2.2.1 The environment

This model builds upon the neoclassical growth framework presented in Hansen and İmrohoroğlu (2016) by incorporating a gender dimension. Due to differing social expectations and household responsibilities, men and women often have varying work hours and respond differently to fiscal policies. Consequently, modeling male and female labor decisions separately allows for a more accurate assessment of how government policies influence household labor and consumption choices.

The time period is one year. Upper case variables denote aggregate variables, while lower case variables represent detrended per capita variables. For each aggregate variable M_t, its detrended per capita counterpart are:

$$m_t = \frac{M_t}{A_t N_t}$$

where A_t is the TFP growth factor and N_t is the population at time t.

Demography. The economy is populated by a representative household, with N_t members at time t. The population grows at rate η_t, such that $N_{t+1} = \eta_t N_t$. At each time t, the share of male in the population is $s_{m,t}$, i.e., $N_{m,t} = s_{m,t} N_t$, and the share of female is $s_{f,t}$, i.e., $N_{f,t} = s_{f,t} N_t$, where $s_{m,t} + s_{f,t} = 1$. Among male population, $e_{m,t}$ share is currently employed, so $E_{m,t} = e_{m,t} N_{m,t}$; while among female population, the employment rate is $e_{f,t}$.

2.2.2 The government

There is a government in the economy that collects tax revenues from the household. It has government expenditures and provides transfers to the household. Each period, the government can issue a one-period zero-coupon bond that will be purchased by the household.

Revenue. The government receives the tax revenue from four sources:

(i) consumption tax, at the rate $\tau_{c,t}$, from aggregate consumption C_t;

(ii) labor income tax, at the rate $\tau_{l,t}$, from male earnings $w_{m,t} L_{m,t}$ and female earnings $w_{f,t} L_{f,t}$;

(iii) capital income tax, at the rate $\tau_{k,t}$, from capital income $r_t K_t$;

(iv) debt interest income tax, at the rate $\tau_{d,t}$, from debt interest income $i_t D_t$.

Hence, the total tax revenue is $TAX_t = \tau_{c,t} C_t + \tau_{l,t} \left(w_{m,t} L_{m,t} + w_{f,t} L_{f,t} \right) + \tau_{k,t} r_t K_t + \tau_{d,t} i_t D_t$.

Primary balance and government debt. At each period, the government expenditure is G_t, and the transfer payments to the household is Z_t. Thus, the government primary balance is $B_t = TAX_t - G_t - Z_t$. At time t, the government can issue one-period discount bonds, D_{t+1}, which will be bought by the household.

Budget constraint. At each period, the government pays off its debt and associated interest payment by using its primary balance B_t and issuing new debt, D_{t+1}. The government's budget constraint is $D_{t+1} + B_t = (1 + i_t)D_t$.

2.2.3 The household

Endowments. At each period, each member of the household is endowed with 1 unit of time, that could be spent between market work, $l_{m,t}$ for male, or $l_{f,t}$ for female, and leisure, $1 - l_{m,t}$ for male, or $1 - l_{f,t}$ for female. At time 0, the household is endowed with initial holdings of aggregate capital $K_0 > 0$ and aggregate one-period zero-coupon bond $D_0 > 0$.

Optimization problem. The household values consumption and dislikes working. $\lambda_{m,t}$ and $\lambda_{f,t}$ are the relative disutility of working parameters for male and female, respectively. We put the bonds in the utility as in the previous literature, to account for the fact that even though the government bonds provide a lower return than capital, which was suggested by the data, the household still holds a positive amount of government bonds.

Given a sequence of male wages, female wages, capital rental rates, government bond interest rates $\{w_{m,t}, w_{f,t}, r_t, i_t\}_{t=0}^{\infty}$, tax rates on consumption, labor, capital income, and bond income, and per-capita transfer payments $\{\tau_{c,t}, \tau_{l,t}, \tau_{k,t}, \tau_{d,t}, z_t\}_{t=0}^{\infty}$, the household chooses a sequence of detrended per-member consumption, male working hours, female working hours, capital, and real bond holdings $\{c_t, l_{m,t}, l_{f,t}, k_{t+1}, d_{t+1}\}_{t+0}^{\infty}$ to solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t N_t \left[\ln c_t - e_{m,t} s_{m,t} \lambda_{m,t} \frac{l_{m,t}^{1+1/\psi}}{1+1/\psi} - e_{f,t} s_{f,t} \lambda_{f,t} \frac{l_{f,t}^{1+1/\psi}}{1+1/\psi} + \theta_t \ln d_{t+1} \right]$$

$$s.t. \quad c_t + \eta_t k_{t+1} + \eta_t d_{t+1} + tax_t = w_{m,t} l_{m,t} e_{m,t} s_{m,t} + w_{f,t} l_{f,t} e_{f,t} s_{f,t} + (1 + r_t - \delta_t) k_t + (1 + i_t) d_t + z_t \tag{2.1}$$

where $tax_t = \tau_{c,t}c_t + \tau_{l,t}(w_{m,t}l_{m,t} + w_{f,t}l_{f,t}) + \tau_{k,t}r_tk_t + \tau_{d,t}i_td_t$. The parameter β is the time discount factor, ψ is the Frisch elasticity of labor, θ_t is the relative utility of bonds. η_tk_{t+1} and η_td_{t+1} are next period's per-capita capital and bond holdings, taking into account of the population growth at rate η_t.

2.2.4 The firm

There is a representative firm in the economy that produces the consumption goods using the Cobb-Douglas production function, $Y_t = K_t^{\alpha_t}(A_tL_t)^{1-\alpha_t}$, where L_t is the aggregate of female and male labor, i.e. $L_t = L_{m,t} + \phi_tL_{f,t}$. ϕ_t represents the gender wage gap. The income share of capital is α_t. The labor-augmenting factor productivity A_t grows at the exogenous rate γ_t, i.e. $A_{t+1} = \gamma_tA_t$. The firm maximizes its profit by paying the wage rate and capital interest rate equal to their marginal productivity. Therefore, we have $r_t = \alpha_t\frac{Y_t}{K_t}$, $w_{m,t} = (1-\alpha_t)\frac{Y_t}{L_t}$, and $w_{f,t} = \phi_t(1-\alpha_t)\frac{Y_t}{L_t}$.

Capital depreciates at rate δ_t, and the aggregate gross investment is X_t. From the law of motion of capital, next period's capital is determined by $K_{t+1} = (1-\delta_t)K_t + X_t$.

2.2.5 Equilibrium

Given a sequence of government fiscal policy $\{G_t, Z_t, \tau_{c,t}, \tau_{k,t}, \tau_{l,t}, \tau_{d,t}\}_{t=0}^{\infty}$, the population size, male and female population share, and employment rate $\{N_t, s_{m,t}, s_{f,t}, e_{m,t}, e_{f,t}\}_{t=0}^{\infty}$, and technology $\{A_t\}_{t=0}^{\infty}$, a competitive equilibrium consists of an allocation $\{C_t, l_{m,t}, l_{f,t}, K_{t+1}, D_{t+1}\}_{t=0}^{\infty}$, factor prices $\{w_{m,t}, w_{f,t}, r_t\}$ and the bond interest rate $\{i_t\}_{t=0}^{\infty}$ such that

- the allocation solves the household's problem;
- the allocation solves the firm's profit maximization problem with factor prices given by: $w_{m,t} = (1-\alpha_t)\frac{Y_t}{L_t}$, $w_{f,t} = \phi_t(1-\alpha_t)\frac{Y_t}{L_t}$, and $r_t = \alpha_t\frac{Y_t}{K_t}$;
- the government budget is satisfied;
- the goods market clears: $C_t + [\eta_tK_{t+1} - (1-\delta)K_t] + G_t = Y_t$;
- the bonds market clears.

2.2.6 Detrended Equilibrium Conditions

The detrended equilibrium conditions include:

(i) Euler equation concerning the consumption-capital choice:

$$\frac{(1+\tau_{c,t+1})c_{t+1}}{(1+\tau_{c,t})c_t} = \frac{\beta\left[1+(1-\tau_{k,t+1})r_{t+1}-\delta_{t+1}\right]}{\gamma_t} \tag{2.2}$$

(ii) Euler equation for bonds:

$$\frac{\theta_t}{\eta_t d_{t+1}} + \frac{\beta\left[1+(1-\tau_{d,t+1})i_{t+1}\right]}{(1+\tau_{c,t+1})c_{t+1}} = \frac{\gamma_t}{(1+\tau_{c,t})c_t} \tag{2.3}$$

(iii) first-order condition for male working hours:

$$l_{m,t} = \left[\frac{(1-\tau_{l,t})w_{m,t}}{\lambda_{m,t}(1+\tau_{c,t})c_t}\right]^{\psi} \tag{2.4}$$

(iv) first-order condition for female working hours:

$$l_{f,t} = \left[\frac{(1-\tau_{l,t})w_{f,t}}{\lambda_{f,t}(1+\tau_{c,t})c_t}\right]^{\psi} \tag{2.5}$$

(v) production function:

$$y_t = k_t^{\alpha_t}\left(e_t l_t\right)^{1-\alpha_t} \tag{2.6}$$

$$e_t l_t = e_{m,t}s_{m,t}l_{m,t} + \phi_t e_{f,t}s_{f,t}l_{f,t} \tag{2.7}$$

(vi) law of motion for capital:

$$\eta_t\gamma_t k_{t+1} = (1-\delta_t)k_t + x_t \tag{2.8}$$

(vii) government's budget constraint:

$$tax_t = \tau_{c,t}c_t + \tau_{l,t}(w_{m,t}l_{m,t}e_{m,t}s_{m,t} + w_{f,t}l_{f,t}e_{f,t}s_{f,t}) + \tau_{k,t}r_tk_t + \tau_{d,t}i_td_t \quad (2.9)$$

$$b_t = tax_t - g_z - z_t \quad (2.10)$$

$$\eta_t\gamma_td_{t+1} + b_t = (1 + i_t)d_t \quad (2.11)$$

(viii) market clearing conditions:

$$r_t = \alpha_t\frac{y_t}{k_t} \quad (2.12)$$

$$w_{m,t} = (1 - \alpha_t)\frac{y_t}{e_tl_t} \quad (2.13)$$

$$w_{f,t} = \phi_tw_{m,t} \quad (2.14)$$

$$y_t = c_t + x_t + g_t \quad (2.15)$$

2.3 Calibration

We calibrate the model's steady state to Japan's economy in 2022. We use each variable and parameter's 2022 value as the steady state value. Appendix 2.6.1 shows the details of the steady-state solution.

2.3.1 Data source

Population. The data on total population N_t, population growth rate η_t, male and female population share $s_{m,t}, s_{f,t}$ comes from the World Bank's World Development Indicators (WDI). Our population measurement includes people of all ages.

Gender-specific employment rate. The data on male and female employment rate, $e_{m,t}, e_{f,t}$ comes from the International Labor Organization. The employment rate here indicates the share of the employed out of the total population from all ages.

Working hours. The average male and female working hours data comes from Japan's Statistical Survey Department, Statistics Bureau, Ministry of Internal Affairs and Communications. The original data consists of the average weekly working hours of employed males and females. We normalize the working hours by dividing them by 98, which is the weekly discretionary hours, we assume.

Wage rate. The data on average cash earnings per regular employee by gender comes from Japan's Ministry of Health, Labour, and Welfare. Combining monthly earnings and working hours data, we can calculate the average hourly wage rate by gender. The gender wage gap, ϕ_t, is the average female hourly wage rate ratio to the average male hourly wage rate.

National accounts. The data on output, Y_t, private consumption, C_t, government expenditure, G_t, transfer payments, Z_t, capital stock, K_t, government debt, D_t, government debt interest payment, $i_t D_t$, capital income share, α_t, comes from World Economic Outlook (WEO) July 2023 issue.

We interpolate the debt interest rate i_t using the data on outstanding debt amount and debt interest payments. What's more, the Cobb-Douglas production function permits us to calculate the labor-augmenting productivity factor as $A_t = \frac{1}{L_t}\left(Y_t K_t^{-\alpha_t}\right)^{\frac{1}{1-\alpha_t}}$. Then, we can calculate the productivity growth factor, $\gamma_t = \frac{A_{t+1}}{A_t}$.

Tax rates. We use the average tax rate on consumption $\tau_{c,t}$, labor $\tau_{l,t}$, capital income $\tau_{k,t}$ estimates made by McDaniel (2007), which updated the tax series until August 2023. The tax rate on debt interest income $\tau_{d,t}$ is 20% from Hansen and İmrohoroğlu (2016).

Depreciation rate. The capital depreciation rate, δ_t, is obtained from the average depreciation rate of the capital stock as reported by the Penn World Table (PWT).

2.3.2 Calibration strategy

Four remaining unknown parameters in the utility function are calibrated jointly by matching four moments in the steady state with the data. Male and female average working hours are informative of the parameters on relative disutility of work, λ_m, λ_f. The debt-to-GDP ratio in the economy is instructive of the relative utility of debt θ. And the real debt interest rate helps to pin down the time discount factor β. We calibrate four moments jointly by minimizing the sum of the squared difference between moments in the data and the model counterparts. Using calibrated parameters, the moments in the model match precisely with the data.

As we can see in Table 2.1, on average, Japanese females have a higher disutility of work than male ($\lambda_f > \lambda_m$). The estimation is consistent with the Japanese social norms, which show that women shoulder more household responsibilities. Thus, females are less willing to participate in the labor market and devote fewer hours to working than males. The estimated time discount factor β falls in the reasonable range.

Table 2.1: Calibration of parameters in the utility function and matched moments

Moments	Value	Parameter	Parameter value
l_m, male working hour	0.4163	λ_m, disutility of work (male)	$\lambda_m = 16.0715$
l_f, female working hour	0.3204	λ_f, disutility of work (female)	$\lambda_f = 19.0505$
$\frac{d}{y}$, debt-to-GDP ratio	2.6126	θ, utility of debt	$\theta = 0.2531$
i, debt interest rate	0.0054	β, time discount factor	$\beta = 0.9383$

2.4 Quantitative analysis

2.4.1 Benchmark scenario

To study the impact of closing the gender gap in the labor market on Japan's fiscal policy and economic conditions, we counterfactually equate the female labor force participation rate ($e_{f,t}$) to the male rate ($e_{m,t}$) in the steady state. This hypothetical change could be achieved by promoting gender equality through altered social norms, implementing supportive childcare and eldercare systems to relieve women from household duties, and fostering a flexible working environment. While full equalization may not be immediately realistic, especially in the short term, these results provide an upper bound for potential outcomes.

Admittedly, government expenditures will increase to finance the supportive policies that promote equalization in labor force participation. However, in our benchmark scenario, we assume that government expenditures remain constant. Consequently, the economic results presented represent the most optimal outcomes. In the next subsection, we assume that government expenditure is proportional to the GDP level, thereby capturing the additional costs required to stimulate higher female labor market participation.

With more women working, the government's tax revenue will increase from two sources: labor income tax contributions from women and increased consumption tax due to higher household income and spending. As its fiscal condition improves, the government gains the capacity to reduce tax rates while maintaining its primary balance at the previous steady-state level. In this section, we examine the scenario where the government reduces the consumption tax rate.

Table 2.2: Variable values in the steady state

	Initial SS	New SS	Change in %
Tax rates			
τ_c, consumption tax	0.1333	**0.0965**	-27.60
τ_l, labor income tax	0.2981	0.2981	–
τ_k, capital income tax	0.2782	0.2782	–
τ_d, debt interest income tax	0.2000	0.2000	–
Endogenous variable	SS value	New SS	Change in %
b, primary balance	-0.0060	-0.0060	–
c, consumption	0.2912	0.3155	8.34
y, output	0.4414	0.4699	6.46
d, debt	1.1532	1.2120	5.10
$\frac{d}{y}$, debt-to-GDP ratio	**2.6126**	**2.5792**	-1.28
l_m, male working hours	0.4163	0.4066	-2.33
l_f, female working hours	0.3204	0.3129	-2.34

Table 2.2 summarizes the variable values at the initial and new steady states. Once the female labor force participation rate increases from 47% to 61% (male employment rate in 2022), the government could reduce the consumption tax rate from 13.33% to 9.65% (around 27.6% decline) to keep the primary balance constant in the new steady state. The decline in the consumption tax rate benefits the economy. Private consumption and total output increase in the new steady state, suggesting that the economic activity is

expanding thanks to a higher female labor force participation rate. In the new steady state, the debt-to-GDP ratio decreases slightly (1.28% drop), implying that government debt is becoming more sustainable. At the same time, both male and female working hours decline on the intensive margin. Since the household consumes more and enjoys more leisure, the new steady state is a welfare-improving scenario.

In the Appendix 2.6.2, we consider another policy experiment where we adjust the consumption tax rate to keep the debt-to-GDP ratio, instead of the primary balance, constant in the new steady state once the female labor force participation rate increases. Table 2.4 shows that the magnitude of the decline in consumption tax rate, debt-to-GDP ratio, and the change in other endogenous variables are very similar in this case.

In an ideal scenario, the Japanese economy experiences significant benefits. This is achieved through equal participation of both genders in the workforce and government expenditure stays constant. Therefore, the government can significantly reduce the consumption tax rate, and the public debt is becoming more sustainable. As a result, households enjoy increased consumption and reduced working hours. However, achieving a higher female labor force participation rate requires increased government spending. In the next section, we will assess whether these economic advantages still apply when we relax the assumption that government spending remains constant.

2.4.2 Robustness check

In this section, we examine a scenario where government spending is a constant fraction of the output rather than being fixed as in the benchmark scenario. With a rising female labor participation rate and increasing economic activity, government spending is also growing in proportion. This is a simple way to capture the increase in government spending, such as investments in human capital and the expansion of childcare facilities, to enhance women's participation in the labor force.

With $\frac{g}{y}$ fixed, we re-calibrate four parameters in the utility function such that four model moments match the data. Table 2.5 in the Appendix displays the calibrated parameter values and the corresponding matched moments. These calibrated parameter values resemble those in the benchmark scenario, and females still exhibit a higher relative disutility of work compared to males.

Table 2.3: Quantitative experiment

	Initial SS	New SS	Change (in %)
Tax rates			
τ_c, consumption tax	0.1333	**0.1179**	-11.55
τ_l, labor income tax	0.2981		–
τ_k, capital income tax	0.2782		–
τ_d, debt interest income tax	0.2000		–
Endogenous variable	Initial SS value	New SS	Change (in %)
b, primary balance	-0.0070	-0.0070	–
c, consumption	0.3158	0.3360	6.40
y, output	0.5195	0.5528	6.41
d, debt	1.3574	1.4306	5.39
$\frac{d}{y}$, debt-to-GDP ratio	**2.6126**	**2.5880**	-0.94
l_m, male working hours	0.4163	0.4064	-2.38
l_f, female working hours	0.3204	0.3127	-2.40

Table 2.3 presents the findings when we maintain government expenditure as a constant fraction of the overall output. In our numerical analysis, we raise the female labor force participation rate to match that of males, and the government adjust the consumption tax rate to maintain a steady primary balance in the new equilibrium. The new consumption tax rate required for the new steady state is 11.79%, marking an 11.55% decrease from the previous level. The reduction in the consumption tax rate necessary to maintain the primary balance is less substantial than in the benchmark case, reflecting the higher government expenditure aimed at encouraging more women to enter the labor market.

In Table 2.3, column 3 indicates the variable values in the new steady state compared to the old steady state in column 2, and the percentage change is listed in column 4. The findings align with the benchmark scenario, showing that increased female participation in the labor market leads to higher household consumption and a boost in economic activity. The debt-to-GDP ratio decreases slightly, suggesting the public debt is more sustainable. In terms of time use, both men and women reduce their working hours and have more leisure time.

Table 2.6 in the appendix 2.6.2 displays the outcomes when the government modifies the consumption tax rate to ensure that the debt-to-GDP ratio remains unchanged in the new steady state. The adjustments in the consumption tax rate and other variables are close to those shown in Table 2.3.

In summary, considering government efforts to increase female labor force participation, the economy exhibits similar changes in the new steady state, albeit to a lesser extent.

2.5 Conclusions

In this paper, we extend a neoclassical growth model to estimate the impact of increasing the female labor force participation rate. We calibrate the model to Japan's economy and find that as more women join the labor force, the economy benefits from a lower tax rate, increased consumption and output, a lower debt-to-GDP ratio, and fewer working hours in the new steady state.

For future research, it would be valuable to quantitatively assess the government expenditure required to encourage higher female labor force participation. Additionally, examining the impact of increased female employment on Japan's fertility rate, labor productivity, and overall economic resilience could provide further insights into the multifaceted benefits of promoting gender equality in the labor market.

2.6 Appendix

2.6.1 Steady state solutions

The lower case variables without time subscript are detrended per capita variables in the steady state. This section outlines the steady state solutions of endogenous variables.

1. capital interest rate

$$r = \frac{\gamma/\beta + \delta - 1}{1 - \tau_k} \tag{2.16}$$

2. capital-labor ratio

$$\frac{k}{l} = e\left(\frac{r}{\alpha}\right)^{\frac{1}{\alpha-1}} \tag{2.17}$$

3. investment-labor ratio

$$\frac{x}{l} = (\eta\gamma + \delta - 1)\frac{k}{l} \tag{2.18}$$

4. output-labor ratio

$$\frac{y}{l} = e^{1-\alpha}\left(\frac{k}{l}\right)^{\alpha} \tag{2.19}$$

5. male's wage rate

$$w_m = (1-\alpha)\frac{y}{el} \tag{2.20}$$

6. female's wage rate

$$w_f = \phi w_m \tag{2.21}$$

7. consumption-labor ratio, $\frac{c}{l}$, and labor, l are solved numerically jointly:

$$\frac{c}{l} = \frac{y}{l} - \frac{x}{l} - \frac{g}{l} \tag{2.22}$$

$$l = \left[\frac{(1-\tau_l)w_m}{(1+\tau_c)\frac{c}{l}}\right]^{\frac{\psi}{1+\psi}} \times \left[\left(\frac{e_m s_m}{\lambda_m^{\psi}} + \frac{\phi^{1+\psi} e_f s_f}{\lambda_f^{\psi}}\right)\frac{1}{e}\right]^{\frac{1}{1+\psi}} \tag{2.23}$$

8. output, consumption, investment, and capital

$$y = \frac{y}{l} \times l \tag{2.24}$$

$$c = \frac{c}{l} \times l \tag{2.25}$$

$$x = \frac{x}{l} \times l \tag{2.26}$$

$$k = \frac{k}{l} \times l \tag{2.27}$$

9. hours worked, male

$$l_m = \left[\frac{(1-\tau_l)w_m}{\lambda_m(1+\tau_c)c}\right]^{\psi} \tag{2.28}$$

10. hours worked, female

$$l_f = \left[\frac{(1-\tau_l)w_f}{\lambda_f(1+\tau_c)c}\right]^{\psi} \tag{2.29}$$

11. debt level

$$d = \left[\frac{\beta}{(1-\beta\eta)\gamma}\right] \times \left[\frac{\theta(1+\tau_c)c}{\beta\eta} + \tau_c c + \tau_l w_m el + \tau_k rk - g - z\right] \tag{2.30}$$

12. debt interest rate

$$i = \left[\frac{\tau_c c + \tau_l w_m el + \tau_k rk - g - z}{d} + \eta\gamma - 1\right] \times \frac{1}{1-\tau_d} \tag{2.31}$$

13. primary balance

$$b = (1 + i - \eta\gamma)d \tag{2.32}$$

14. tax revenue

$$tax = \tau_c c + \tau_l(w_m l_m e_m s_m + w_f l_f e_f s_f) + \tau_k rk + \tau_d id \tag{2.33}$$

2.6.2 Quantitative experiments

Keep the debt-to-GDP ratio constant

In this scenario, we are considering a situation where the government lowers the consumption tax rate in order to maintain a constant debt-to-GDP ratio when the female labor force participation rate is increased to be equal to the male participation rate. All other tax rates are kept the same. In Table 2.4, column 2 shows the initial steady state values for the variables, column 3 presents the new steady state values for the variables, and column 4 summarizes the percentage change between the two steady states.

Table 2.4: Variable values at the steady state

	Initial SS	New SS	Change (in %)
Tax rates			
τ_c, consumption tax	0.1333	**0.0998**	-25.13
τ_l, labor income tax	0.2981	0.2981	–
τ_k, capital income tax	0.2782	0.2782	–
τ_d, debt interest income tax	0.2000	0.2000	–
Endogenous variable	Initial SS	New SS	Change (in %)
b, primary balance	-0.0060	-0.0049	
c, consumption	0.2912	0.3152	8.24
y, output	0.4414	0.4695	6.37
d, debt	1.1532	1.2267	6.37
$\frac{d}{y}$, debt-to-GDP ratio	2.6126	2.6126	–
l_m, male working hours	0.4163	0.4062	-2.43
l_f, female working hours	0.3204	0.3126	-2.43

Robustness check: keeping $\frac{g}{y}$ constant

When keeping the $\frac{g}{y}$ constant, we re-calibrate the model. All other parameter values remain the same as outlined in Section 2.3. The remaining four unknown parameters in the utility function are calibrated together by minimizing the sum of the differences between moments in the data and the model. The calibration results are presented in Table 2.5, showing that four moments in the model precisely match with the data.

Table 2.5: Calibration of parameters in the utility function and matched moments

Moments	Value	Parameter	Parameter value
l_m, male working hour	0.4163	λ_m, disutility of work (male)	$\lambda_m = 17.4418$
l_f, female working hour	0.3204	λ_f, disutility of work (female)	$\lambda_f = 20.6749$
$\frac{d}{y}$, debt-to-GDP ratio	2.6126	θ, utility of debt	$\theta = 0.2057$
i, debt interest rate	0.0054	β, time discount factor	$\beta = 0.9565$

Table 2.6 contains the values of variables at the steady state. When the female labor force participation rate increases, the government adjusts the consumption tax rate to maintain the debt-to-GDP ratio at the previous steady-state level. The government expenditure-to-GDP level is also kept constant. Column 2 displays the initial steady state, column 3 shows the new steady state, and column 4 indicates the percentage change.

Table 2.6: Variable values at the steady state

	Initial SS	New SS	Change (in %)
Tax rates			
τ_c, consumption tax	0.1333	**0.1197**	-10.20
τ_l, labor income tax	0.2981		–
τ_k, capital income tax	0.2782		–
τ_d, debt interest income tax	0.2000		–
Endogenous variable	Initial SS	New SS	Change (in %)
b, primary balance	-0.0070	-0.0063	
c, consumption	0.3158	0.3358	6.33
y, output	0.5195	0.5525	6.35
d, debt	1.3574	1.4434	6.34
$\frac{d}{y}$, debt-to-GDP ratio	**2.6126**	**2.6125**	–
l_m, male working hours	0.4163	0.4061	-2.45
l_f, female working hours	0.3204	0.3126	-2.43

3

Housing Investment and Wealth Accumulation

3.1 Introduction

Wealth inequality in the U.S. has increased over the last few decades, with wealth becoming increasingly concentrated in the hands of the wealthiest individuals (Saez and Zucman, 2016). Why do some people accumulate more wealth while others do not? Admittedly, several factors contribute to this disparity. The rich tend to have higher earnings (Kaymak et al., 2022), possess entrepreneurial spirit (Cagetti and De Nardi, 2006), achieve higher returns on their portfolios (Fagereng et al., 2020), and, of course, benefit from substantial inheritances (De Nardi, 2004).

To promote wealth accumulation among low- and middle-income households, many governments encourage homeownership. Various housing policies aim to facilitate homeownership among Americans, generally falling into two categories. The first category involves government assistance in financing home purchases, which includes a variety of loan programs. The second category comprises tax policies favoring homeowners. For example, mortgage interest payments and property tax payments can be deducted from federal income tax, and imputed rent is typically not taxed.

To understand how homeownership affects household wealth, a strand of literature studies its impact on wealth levels. Di et al. (2007) shows that, after controlling for the propensity to save, those who owned homes for longer periods from 1989 to 2001 had higher net wealth in 2001. Similarly, Turner and Luea (2009) supports the view that each additional year of homeownership increases wealth holdings, even after accounting for unobserved heterogeneity. Their study also finds that the increase in wealth associated with each additional year of homeownership is larger for high-income households ($15K) than for low- and moderate-income households ($6K to $10K). Newman and Holupka (2016) highlights that race plays an important role: Black first-time homeowners experience a decrease in net worth, while timing is crucial for White first-time homeowners. Under a difference-in-difference framework, Wainer and Zabel (2020) identifies that timing also matters for low-income households who own homes for the first time. Households experience significant gains in wealth if they purchased homes during periods of relatively stable real house prices (1980s and 1990s), but they gain little if they bought homes during periods of volatile house prices (2000s and early 2010s).

While existing literature focuses on explaining the relationship between homeownership and the *level* of wealth, few has studied the *channel* of wealth accumulation. This paper aims to fill this gap by exploring the differences in wealth accumulation channels between homeowners and renters. We provide a detailed accounting analysis of how homeownership affects wealth, examining the following three wealth accumulation channels.

First, we explore the household's portfolio composition. For homeowners, housing wealth takes up the majority of total wealth. As for other assets' shares of non-housing wealth, homeowners and renters have similar pattern along the wealth distribution.

Second, we investigate the rate of return on wealth. Overall, homeowners achieve higher average returns on non-housing wealth compared to renters. This is because homeowners, on average, invest more heavily in assets that yield higher returns. When examining individual asset returns, renters achieve returns as competitive as those of homeowners. Moreover, renters experience statistically significantly higher returns on business and farm wealth than homeowners.

Third, we study the saving rate. There are two measures of saving rate, gross saving rate and active saving rate, as defined in Dynan et al. (2004). Our results show that homeowners have higher saving rate in terms of both measures, after controlling for demographic differences. Moreover, we identify that mortgage payment serves as a commitment device, which drives up the saving rate for homeowners.

The rest of the paper is organized as follow. Section 3.2 lays out the wealth accumulation accounting framework. Section 3.3 describes the data. Section 3.4 presents the comparison of wealth accumulation channel between homeowners and renters. Finally, Section 3.5 concludes.

3.2 Wealth Accumulation Accounting Framework

For a household i, at time t, its total wealth, $W_{i,t}$, is composed of wealth in different assets, $W^a_{i,t}$. Each asset wealth, $W^a_{i,t}$, takes up $\phi^a_{i,t}$ share of the total wealth:

$$W_{i,t} = \sum_{a=1}^{A} W^a_{i,t} = \sum_{a=1}^{A} \phi^a_{i,t} W_{i,t}. \tag{3.1}$$

And for each asset wealth, $W^a_{i,t}$, it is the difference between the market value of the asset, $V^a_{i,t}$, and the debt outstanding on this asset, $D^a_{i,t}$:

$$W^a_{i,t} = V^a_{i,t} - D^a_{i,t}. \tag{3.2}$$

Household i has labor income, transfer income, and social security income, $Y_{i,t}$, during the period t to $t+1$. Each asset generates capital income at rate $\kappa^a_{i,t}$ out of the asset wealth. Household also enjoys capital gain at rate $\pi^a_{i,t}$ from each asset wealth. Household's consumption, $C_{i,t}$, and saving, $S_{i,t}$, are subject to the budget constraint:

$$\begin{aligned} C_{i,t} + S_{i,t} &= \underbrace{(1-\tau_{i,t})\left(Y_{i,t} + \sum_{a=1}^{A} \kappa_{i,t}^{a} W_{i,t}^{a}\right)}_{I_{i,t}\text{: disposable income}} + \underbrace{(1-\tau_{i,t}) \sum_{a=1}^{A} \pi_{i,t}^{a} W_{i,t}^{a}}_{G_{i,t}\text{: capital gain}}, \\ &= I_{i,t} + G_{i,t}, \end{aligned} \tag{3.3}$$

where $\tau_{i,t}$ is the marginal income tax rate. The after-tax labor income, transfer income, social security income and capital income constitute the household's disposable income, $I_{i,t}$. And the total capital gain, $G_{i,t}$, is the after-tax capital gain from all assets. Among the total income, $I_{i,t} + G_{i,t}$, household chooses to save a fraction, $s_{i,t}^{\text{tot}}$, and consumes the rest part.

At the beginning of time $t+1$, the household's total wealth, $W_{i,t+1}$, would be its initial wealth, $W_{i,t}$, plus its saving, $S_{i,t}$, during the period t to $t+1$. Therefore, the accumulated wealth, $W_{i,t+1} - W_{i,t}$, is how much the household has saved:

$$W_{i,t+1} = W_{i,t} + S_{i,t}, \Longrightarrow \Delta W_{i,t+1} = S_{i,t}. \tag{3.4}$$

Then, household will reallocate its wealth, $W_{i,t+1}$, among different classes of assets. And the wealth accumulation process repeats.

If we take a closer a look at the accumulated wealth:

$$\Delta W_{i,t+1} = s_{i,t}^{\text{tot}} \left[(1-\tau_{i,t}) Y_{i,t} + (1-\tau_{i,t}) \sum_{a=1}^{A} \left(\kappa_{i,t}^{a} + \pi_{i,t}^{a}\right) \phi_{i,t}^{a} W_{i,t} \right], \tag{3.5}$$

it shows that wealth accumulation could be determined by:

(i) saving rate, $s_{i,t}^{\text{tot}}$;

(ii) asset allocation, $\phi_{i,t}^{a}$;

(iii) asset returns, $\kappa_{i,t}^{a} + \pi_{i,t}^{a}$;

Admittedly, labor/transfer/social security income, $Y_{i,t}$, also plays an important role in the wealth accumulation process. However, it is hardly influenced by the house tenure choice. Therefore, for the purpose of our study, we are going to focus on wealth accumulation channels in terms of saving rate, asset allocation, and asset returns, by comparing the differences between homeowners and renters. Next, we will explain how we measure these three wealth accumulation channels.

3.2.1 Measure of Saving Rate

Following Dynan et al. (2004), we define two measures of saving rate.

The first measure of saving rate is the gross saving rate. It is defined as the change in real wealth divided by real disposable income:

$$s_{i,t}^{\text{grs}} = \frac{\Delta W_{i,t+1}}{I_{i,t}}. \tag{3.6}$$

This is a broad measure of saving rate, which includes active savings and passive gains.

The second measure of saving rate is the active saving rate. It is defined as a measure of the "active saving" divided by real disposable income:

$$s_{i,t}^{\text{act}} = \frac{\text{active saving}}{I_{i,t}}. \tag{3.7}$$

Here, active saving is calculated as the change in real wealth, net of capital gains from assets and windfall gains, and adjust for inflation. The measure of active saving closely resembles the traditional way of defining the saving, which is income minus consumption. Appendix 3.6.1 presents how we construct the active saving in details.

3.2.2 Measure of Asset Allocation

We will examine two measures of asset allocation. First, we look at the share of each asset's wealth out of total wealth. Second, we explore the share of each asset's wealth out of non-housing wealth.

3.2.3 Measure of Rate of Return

We measure the return on asset wealth, instead of return on asset value. If there is debt outstanding on the asset, the return will be leveraged. There are also two measures of asset returns. The first measure is the return that includes capital income and capital gain:

$$r_{i,t}^{a,\text{wkg}} = \kappa_{i,t}^{a} + \pi_{i,t}^{a} = \frac{K_{i,t}^{a} + \Pi_{i,t}^{a}}{W_{i,t}^{a}}, \tag{3.8}$$

where $K_{i,t}^{a}$ and $\Pi_{i,t}^{a}$ are the capital income and capital gain from period t to $t+1$, respectively. Capital gain is the change in asset value minus the net investment in the period. By including capital gain, this is a more comprehensive measure of returns.

The second measure is the return rate without capital gain:

$$r_{i,t}^{a,\text{nkg}} = \kappa_{i,t}^{a} = \frac{K_{i,t}^{a}}{W_{i,t}^{a}}, \tag{3.9}$$

which is a measure of returns that are actually realized.

3.3 Data

3.3.1 Data Source

The data comes from the Panel Study of Income Dynamics (PSID), which is the longest running panel household survey in the world and is directed by faculty at the University of Michigan. PSID includes detailed survey information on American household demographics, income, wealth, and other relevant variables. From 1999 onward, the surveys are conducted biennially. In this paper, we use the data from 1999 to 2017.

PSID provides household's pre-tax income information. To estimate the federal income tax, we use NBER TAXSIM32, following Feenberg and Coutts (1993).

In calculating rate of return and saving rate, all wealth variables and income variables are inflation adjusted to the real value in 2019 dollar, using the CPI-U from the Federal Reserve Bank of Minneapolis. Stock variables (i.e., asset price, debt level, wealth level, etc.) are adjusted using the CPI of the interview year, while flow variables (i.e., net investment, inheritance, etc.) are adjusted using the CPI of the year between two interviews.

3.3.2 Measure of Wealth in PSID

In PSID, the total family wealth is the sum of net worth in the following 8 assets: (1) home equity; (2) checks/savings accounts, certificates of deposit, etc; (3) directly hold stocks; (4) annuity/IRA; (5) other real estate; (6) business and farm; (7) vehicles; and (8) other assets, net of debt values. The debt values include credit card debt, student loan debt, medical debt, legal debt, family loan debt, and other debt.

Following the suggestion by Cooper et al. (2019), we also augment wealth in employer-sponsored pension plans. We add pension wealth in both current employer-sponsored plans and leftovers in previous employer-sponsored plans, with missing values imputed if value range brackets are provided. By augmenting PSID measure of wealth with employer-sponsored pension wealth, it will give us a more comprehensive picture of total household wealth in the US.

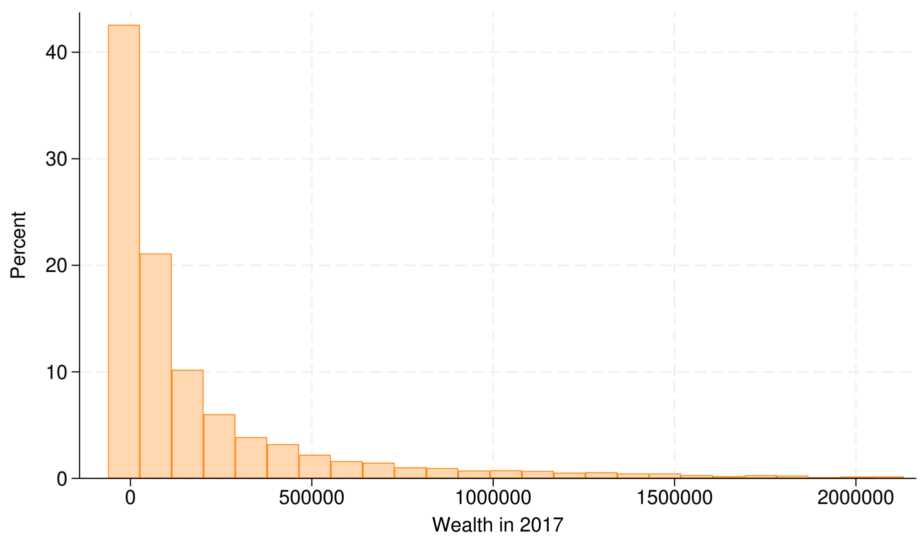


Figure 3.1: Wealth Distribution in 2017

Figure 3.1 illustrates the wealth distribution in 2017, after excluding the top and bottom 2.5% of wealth holders for the purpose of trimming. The distribution is significantly right-skewed, highlighting the considerable wealth inequality present in the U.S.

3.3.3 Sample Selection

We select households where, between two waves of data, the head of the household remains the same and their marital status does not change. This approach helps us avoid complications related to changes in wealth that may arise from variations in family composition. Our sample is limited to households with heads who are under 65 years old. We also exclude households that change their housing tenure between waves or that neither own nor rent their main residence. By doing this, we aim to isolate the impact of homeownership on household wealth accumulation, minimizing the influence of other factors.

3.3.4 Characterization of Homeowner and Renter

Table 3.1: Summary Statistics

	Homeowner	Renter
Age	59.75	46.31
Married?	65%	25%
Family size	2.34	1.95
Education	13.98	13.19
Total family income	$102,527.28	$49,685.39
Total wealth	$708,120.57	$54,599.34

Notes: The table presents weighted average using data in 2017. Income and wealth in 2017 dollars.

Table 3.1 presents the summary statistics that characterize the demographic differences between homeowners and renters. On average, homeowners are older, more likely to be married, have larger family size, and higher years of education. What's more, homeowners have much higher total family income and total wealth.

3.4 Wealth Accumulation Channel

3.4.1 Asset Allocation

Household's assets can be grouped into the following 6 categories: (1) safe financial assets, which includes cash, checking/saving accounts, certificate of deposits, bonds, bills, money market funds, cash value in a life insurance policy, bonds held indirectly in private annuity, IRAs, and employer-sponsored pension plans, etc.; (2) risky financial assets, which includes stocks held directly or indirectly in private annuity, IRAs, and employer-sponsored pension plans; (3) real assets, which includes business and farm; (4) home equity, which is the household's main residence; (5) other real estate, which includes a second home, land, rental real estate, and money owned on a land contract; (6) vehicles.

For the pension wealth composition, in PSID, they have a question asking households how the pension plans are invested. If the answer is mostly (or all) stocks, then we will assign all the pension wealth to risky financial assets; if the answer is mostly (or all) bonds, then we will assign all the pension wealth to safe financial assets; if the answer is some of each, then we will split equally between safe and risky financial wealth. This is following the practice of Cooper et al. (2019).

Portfolio Composition

Figure 3.2 presents the total portfolio composition for all homeowners and renters, using weighted average from 1999 to 2017. As we can see, around 43% of homeowner's wealth is taken up by the home equity. While for renters, the majority (around 47%) of their wealth is made up of vehicles, and safe financial assets also have a significant share (around 32%).

To account for the fact that homeowners are generally wealthier than renters, and wealth plays an important role in household's asset allocation. Figure 3.3 compares the portfolio composition along wealth distribution. The wealth distribution is assigned based on all homeowners and renters in a given wave, therefore, homeowners have comparable wealth with renters within the same wealth percentile. For homeowners in the bottom quintile, their mortgage outstanding are generally greater than the market value

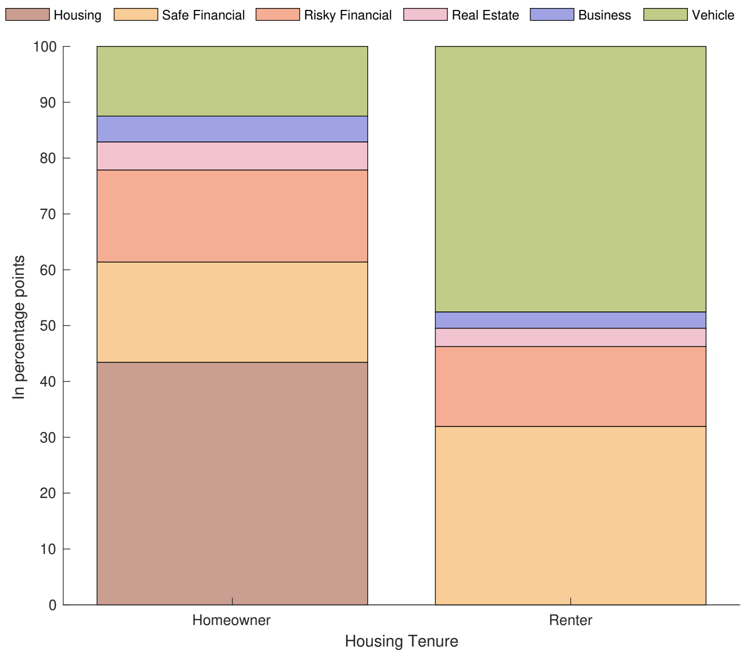


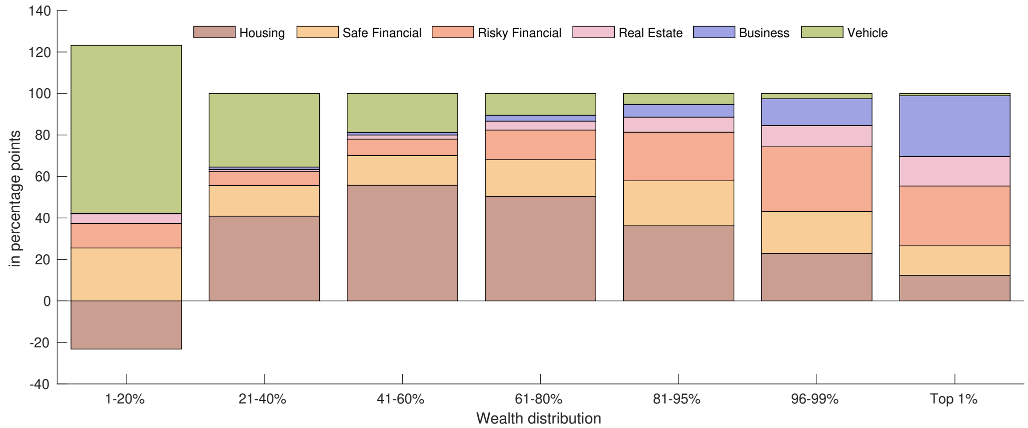
Figure 3.2: Total Portfolio Composition (Weighted Average 1999 − 2017)

of the main home, thus leading to negative housing wealth. Moving up along the wealth distribution, housing's share of total wealth grows larger, reaching the highest at 56% for the middle class households, and then goes down for the wealthiest.

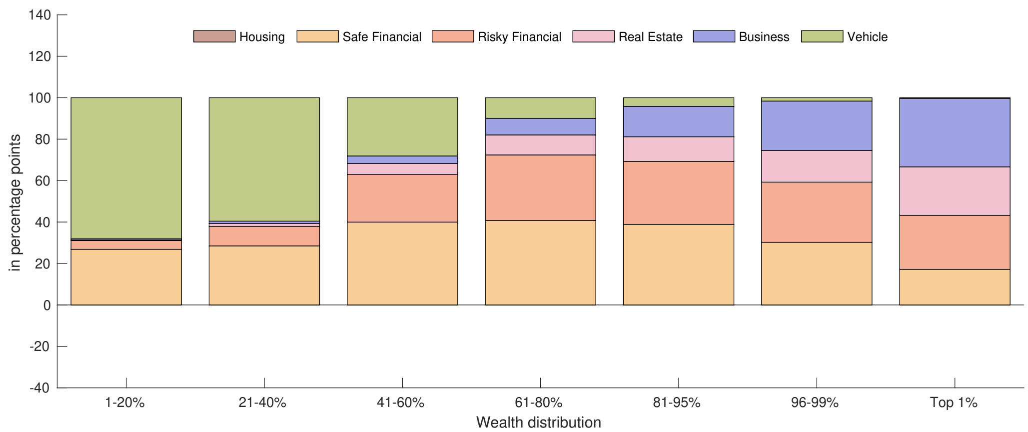
Non-housing Asset Allocation

Figure 3.4 compares homeowner's non-housing asset allocation with the renter's, also along the wealth distribution. Here, we could see some resemblance in the asset allocation between homeowners and renters. For households in the bottom of wealth distribution, non-housing wealth is mainly composed of vehicles and safe financial assets. For wealthier households, non-housing wealth is more dominated by risky financial assets and business, and with a moderate amount of other real estate investment.

Despite the similarity in asset allocation, there are still some discrepancy in non-housing wealth allocation between homeowner and renter. Figure 3.5 plots the difference in share of each asset in homeowner's non-housing portfolio and the renter's. Due to investment in housing, homeowners normally invest less in other financial assets and real assets compared to renters within the same wealth distribution group, with a notable



(a) Homeowner

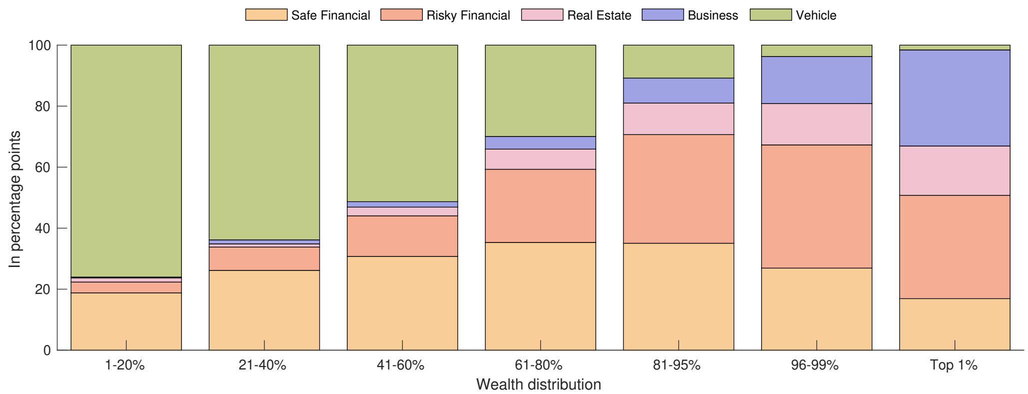


(b) Renter

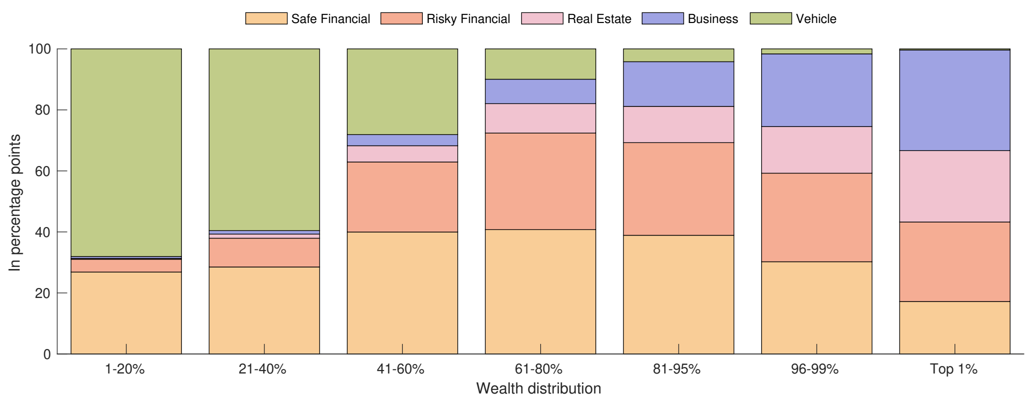
Figure 3.3: Portfolio Composition along Wealth Distribution (Weighted Average 1999 – 2017)

exception of stocks investment in the top quintile.

Asset allocation plays an important role in wealth accumulation since different assets have different return rates. By investing more in assets that have higher returns, households will be able to accumulate more wealth. It remains to study total asset return and individual asset return, which will be the focus of next subsection.



(a) Homeowner



(b) Renter

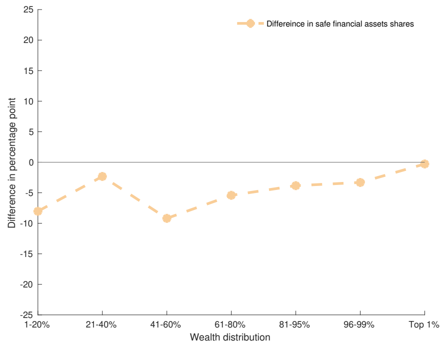
Figure 3.4: Non-housing Asset Allocation (Weighted Average 1999 − 2017)

3.4.2 Rate of Return

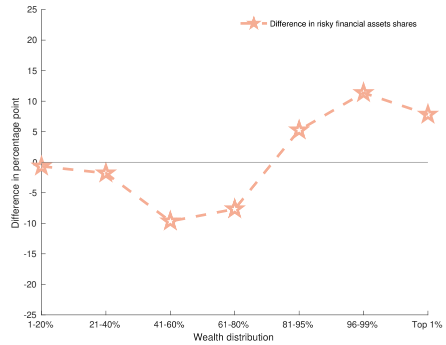
Aggregate Non-housing Return

First, we calculate rate of return on non-housing wealth following Cao and Luo (2017). Non-housing wealth is defined as total wealth excluding home equity and vehicles. The returns are composed of capital income from interests, rent, dividends, trust, loyalties, and asset income from business and farming; and capital gains from stocks, real estate, business, private annuity and IRA.

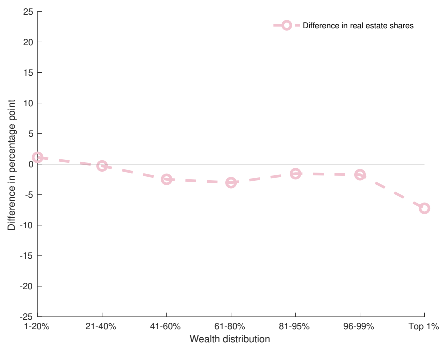
Note that in PSID, they split income from business and farming equally between labor income and asset income, if the household with business income is actively involved in the business and farming. We adjust the asset income share to $\frac{1}{3}$, which is more realistic and is consistent with the finding in Kaymak et al. (2022). Same as in Cao and Luo (2017),



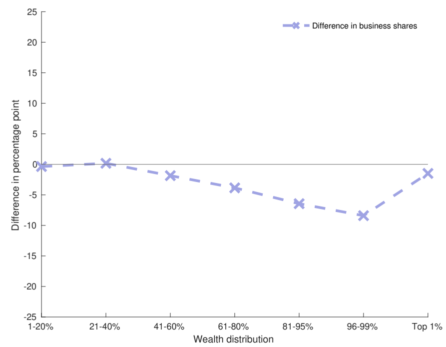
(a) Safe financial assets



(b) Risky financial assets



(c) Real estate



(d) Business

Figure 3.5: Difference in Asset Shares of Non-housing Wealth (owner − renter)

we calculate returns for households with real non-housing wealth greater than $1,000. And we drop observations with annualized real return lower than -100% or greater than 300%.

Table 3.2: Annualized Non-housing Wealth Returns

	Mean	Median	25p	75p	Std	Obs
Panel A: with capital gains						
Homeowner	20.92%	1.28%	0	26.60%	0.56	14,568
Renter	14.11%	0	0	6.09%	0.48	3,694
Total	19.54%	0.65%	0	23.05%	0.54	18,262
Panel B: without capital gains						
Homeowner	6.52%	0.43%	0	2.65%	0.23	15,987
Renter	6.51%	0	0	1.47%	0.25	3,898
Total	6.51%	0.32%	0	2.43%	0.24	19,885

Notes: Returns with capital gains: $t(18260) = -6.752, p = 0.000$, one-tailed. Returns without capital gains: $t(19883) = -0.026, p = 0.489$, one-tailed.

Table 3.2 presents the result. As we can see, the returns on non-housing wealth is highly right-skewed. For the returns with capital gains, homeowners have a statistically significantly higher average return than renters. While for the returns without capital gains, the average returns are not statistically different among two groups.

Table 3.3: Kolmogorov-Smirnov Test on Distribution of Non-housing Wealth Returns

Smaller group	D	P-value
Panel A: with capital gains		
Renter	0.167	0.000
Homeowner	-0.111	0.000
Panel B: without capital gains		
Renter	0.217	0.000
Homeowner	-0.005	0.842

To take a closer look at the whole distribution of non-housing wealth returns, we did a Kolmogorov-Smirnov test (shown in Table 3.3) and plot the empirical CDF (shown in Figure 3.6). For the returns with capital gains, homeowners have more dispersed returns than renters. There are more homeowners cluster at the distribution with large loss and significant gains. For the returns without capital gains, the distribution of homeowner's is to the right of the renter's, and it is statistically significant. Due to the fact that households usually report very little capital income, this measure of return is much smaller and the difference in distribution functions is not very distinguishable from the graph. Probably because of relatively little capital income, the average return is not statistically different (as shown in the t-test in Table 3.2).

The differences in the returns might come from the fact that homeowners and renters have different asset allocations. On average, homeowners invest more in stocks, business, and real estate that are more risky and have large swings in asset price, therefore, the returns with capital gains are more dispersed for homeowners. Also, because homeowners are wealthier and own more assets that bring some capital incomes, the returns in terms of capital income are also (slightly) larger.

Next, we will study whether there is difference in the rate of return in individual asset.

Individual Asset Return

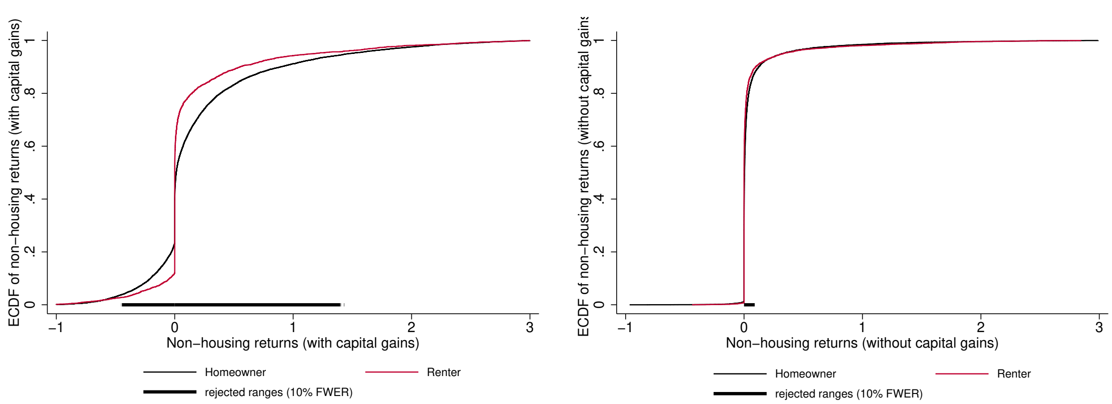
Business and Farm. For households with business and farm returns, the majority (92%) are homeowners. Among these households, 51% of homeowners and 59% of renters have positive capital income from business and farm.

Table 3.4: Annualized Business & Farm Wealth Returns

	Mean	Median	25p	75p	Std	Obs
Panel A: with capital gains						
Homeowner	23.07%	6.38%	−24.35%	51.00%	0.72	1,982
Renter	37.37%	14.93%	−20.88%	77.45%	0.82	162
Total	24.15%	6.87%	−24.35%	53.21%	0.72	2,144
Panel B: without capital gains						
Homeowner	7.66%	0.01%	0	7.73%	0.16	2,174
Renter	13.77%	1.41%	0	18.80%	0.23	185
Total	8.14%	0.04%	0	8.43%	0.17	2,359

Notes: Returns with capital gains: $t(2142) = 2.386, p = 0.009$, one-tailed. Returns without capital gains: $t(2357) = 4.741, p = 0.000$, one-tailed.

Table 3.4 presents the calculation of annualized return on business and farm. Average return on business and farm wealth is quite large, and with large dispersion. An interest-



(a) with capital gains

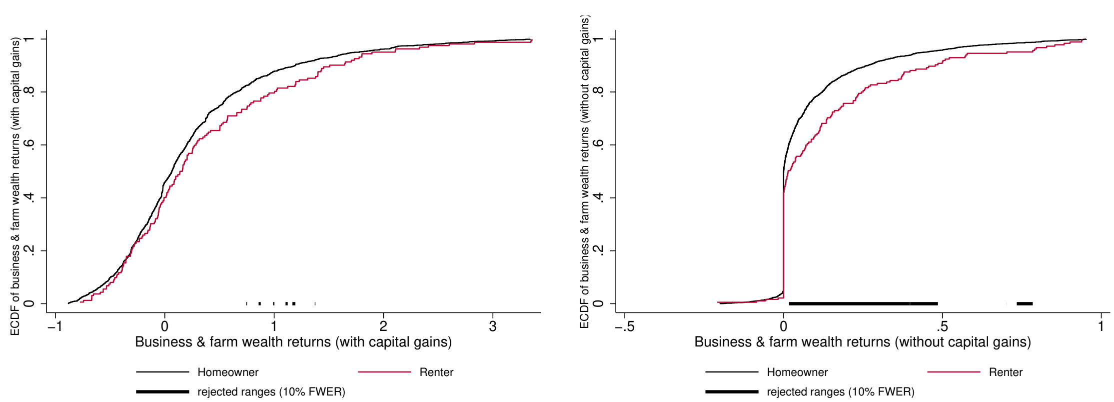
(b) without capital gains

Figure 3.6: Empirical CDF of Annualized Non-housing Returns

ing finding is that on average, renters have significantly higher returns on business and farm, in terms of both measures of returns. The Kolmogorov-Smirnov test (Table 3.5) and the empirical CDF (Figure 3.7) also show that renters enjoy higher returns on business and farm wealth.

Table 3.5: Kolmogorov-Smirnov Test on Distribution of Business & Farm Wealth Returns

Smaller group	D	P-value
Panel A: with capital gains		
Renter	0.009	0.976
Homeowner	−0.093	0.073
Panel B: without capital gains		
Renter	0.005	0.990
Homeowner	−0.153	0.000



(a) with capital gains (b) without capital gains

Figure 3.7: Empirical CDF of Returns on Business & Farm Wealth

There are two possible explanations. First, homeowners are easier to apply for a loan to start a business, because they could use their home equity as a collateral. Since it is harder for renters to get a loan to start a business, they only do it when they are more confident it will be a success. Second, homeowner's mobility is more constrained by their main residence, while renters are easier to relocate. Therefore, renters are more likely to relocate to places where doing business is more profitable. These might help to explain why despite there are fewer renters having business and farm, their returns from business are significantly higher, both on average and in total distribution.

Stocks. Here, we are comparing returns on stocks held directly by households, and not including stocks held indirectly in private annuity, IRAs, or employer-sponsored pension plans. 92.5% of households with stocks returns are homeowners. 77% of homeowners and 71% of renters have positive dividend income.

Table 3.6: Annualized Stocks Returns

	Mean	Median	25p	75p	Std	Obs
Panel A: with capital gains						
Homeowner	13.70%	2.43%	−23.00%	33.75%	0.60	3,725
Renter	15.74%	6.89%	−23.25%	35.32%	0.60	301
Total	13.86%	2.86%	−23.03%	34.03%	0.60	4,026
Panel B: without capital gains						
Homeowner	1.99%	0.76%	0	2.46%	0.03	4,153
Renter	2.44%	0.73%	0	2.97%	0.03	331
Total	2.03%	0.76%	0	2.48%	0.03	4,484

Notes: Returns with capital gains: $t(4024) = 0.562, p = 0.287$, one-tailed. Returns without capital gains: $t(4482) = 2.380, p = 0.009$, one-tailed.

Table 3.6 presents the annualized stocks return. In terms of mean, the returns with capital gains are not statistically different between two groups; and renters have higher returns without capital gains. The Kolmogorov-Smirnov test (Table 3.12) and empirical CDF (Figure 3.11) in Appendix 3.6.2 show that distribution of stocks returns are not statistically different.

Other Real Estate. Among households with real estate returns, 95% are homeowners. For households with real estate returns, 47% homeowners and 37% renters reported positive rental income. In PSID, households report total rental income received in a given year.

Therefore, it is hard to distinguish whether the rental income comes from (part of) main residence or from other real estate for homeowners. We attribute all rental income to other real estate in this case, therefore, homeowner's capital income from other real estate might be exaggerated.

Table 3.7: Annualized Real Estate Returns

	Mean	Median	25p	75p	Std	Obs
Panel A: with capital gains						
Owner	7.86%	1.85%	−16.77%	26.11%	0.42	2,931
Renter	7.11%	1.08%	−21.20%	28.00%	0.44	174
Total	7.82%	1.81%	−16.91%	26.40%	0.42	3,105
Panel B: without capital gains						
Owner	3.19%	0	0	4.17%	0.05	3,207
Renter	2.89%	0	0	2.20%	0.06	183
Total	3.17%	0	0	4.07%	0.05	3,390

Notes: Returns with capital gains: $t(3103) = -0.227, p = 0.410$, one-tailed.
Returns without capital gains: $t(3388) = -0.676, p = 0.250$, one-tailed.

Table 3.7 presents the returns from investing in other real estate. Average returns are not statistically different between homeowners and renters, in both measures. For the tests on the distribution of real estate returns, they are presented in the Appendix 3.6.2. And the distributions of real estate returns are not statistically different.

Home Equity. We follow Flavin and Yamashita (2002) to calculate returns on owner-occupied housing. The returns on housing depends on imputed capital income and capital gains of the house value.

The imputed rental income is set up based on the no-arbitrage condition: the fair price a homeowner would charge if he rented the house to some renters. The imputed rental income is:

$$(r_t + \delta_{i,t})V_{i,t}^h + \text{property tax}, \tag{3.10}$$

where r_t is the real interest rate at time t, $\delta_{i,t}$ is the depreciation rate of housing, $V_{i,t}^h$ is the market value of the house. We assume that landlord would charge renters the opportunity cost of investing in housing and also pass the property tax.

The imputed cost of homeownership includes bearing the depreciation of home equity and paying the property tax:

$$\delta_{i,t} V_{i,t}^h + (1 - \tau_{i,t}) * \text{property tax} \tag{3.11}$$

where $\tau_{i,t}$ is the marginal income tax rate, and the property tax payment could be deducted from the federal income tax.

Thus, the imputed capital income from housing is the imputed rental income minus the imputed cost:

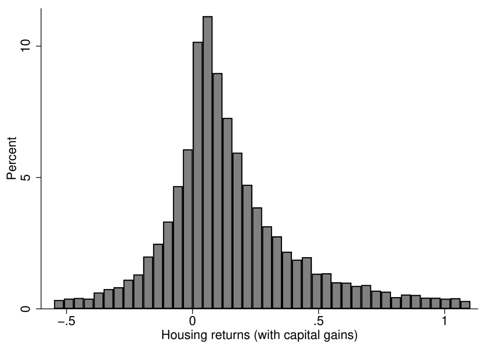
$$r_t V_{i,t}^h + \tau_{i,t} * \text{property tax} \tag{3.12}$$

We follow Flavin and Yamashita (2002) in choosing r_t to be 5%, $\tau_{i,t}$ to be 33%, and the property tax is available in PSID. Note that in Jordà et al. (2019), they used the rent-price approach to calculate the housing return. Their Figure A.27 shows that the rent-price ratio in the US is around 4% to 6% from 2000 to 2015, and it states explicitly that the rent-price ratio for U.S. residential real estate is 4.9% in 2014. Therefore, our choice of 5% for r_t is reasonable.

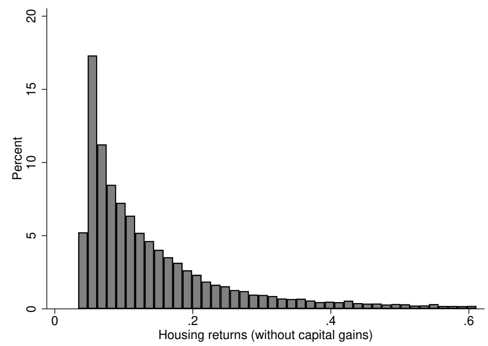
The annualized return on home equity is presented in Figure 3.8 and Table 3.8. Note that we measure returns on housing wealth instead of housing value, when households purchase houses with mortgages (which is usually the case), the returns are leveraged. In general, housing provides considerable returns for homeowners.

Table 3.8: Annualized Returns on Home Equity

	Mean	Median	25p	75p	Obs
With capital gains	14.78%	9.59%	0.27%	25.74%	19,218
Without capital gains	14.28%	10.37%	6.40%	17.85%	19,720



(a) With capital gains



(b) Without capital gains

Figure 3.8: Distribution of Returns on Home Equity

Rate of Return Heterogeneity

Besides heterogeneity in returns within each individual asset, returns are also different across different assets. Table 3.9 compares the average and median returns for different assets using data in PSID from 1999 to 2017. Business and farm have the highest average returns, followed by stocks and then other real estate. Since we imputed capital income for owner-occupied housing, it also has sizable returns.

Table 3.9: Comparison of Individual Asset Returns (1999−2017)

	With capital gains		Without capital gains	
	Mean	Median	Mean	Median
Business & Farm	24.15%	6.87%	8.14%	0.04%
Stocks	13.86%	2.86%	2.03%	0.76%
Real Estate	7.82%	1.81%	3.17%	0
Housing	14.78%	9.59%	14.28%	10.37%

Meanwhile, based on data from the Center for Research in Security Prices (CRSP), the average return on treasury bills, notes, and bonds with different maturities in the same period is shown in Table 3.10. As we can see, safe assets generally carry lower returns compared to risky financial assets and real assets.

Table 3.10: Average Returns on Treasury Bills, Notes, and Bonds

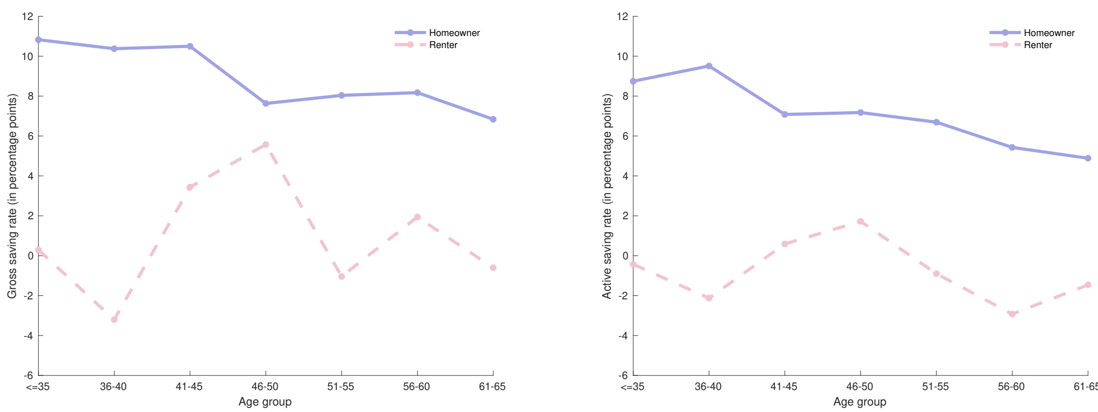
30 Day	90 Day	1 Yr	5 Yr	10 Yr	30 Yr
1.74%	1.94%	2.37%	4.48%	4.92%	7.15%

Returns are heterogeneous among assets, together with different asset allocation, homeowners and renters thus have different returns on their wealth, leading to different wealth accumulation patterns.

3.4.3 Saving Rate

In this subsection, we explore whether there is discrepancy in saving rate between homeowners and renters. We compare two measures of saving rate, gross saving rate and active saving rate, which are defined in Section 3.2.1 following Dynan et al. (2004).

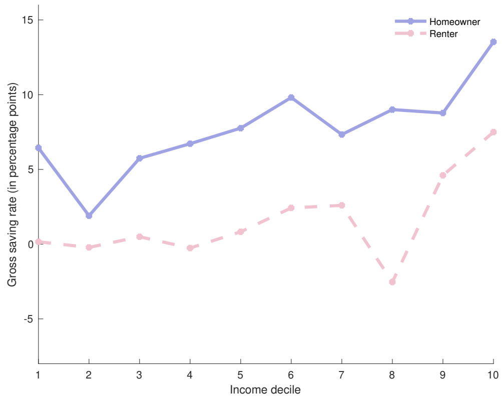
Life-cycle consideration suggests that saving rate might be correlated with age. Figure 3.9 shows that homeowners have higher average saving rate than renters in all age group, in both measures of saving rate.



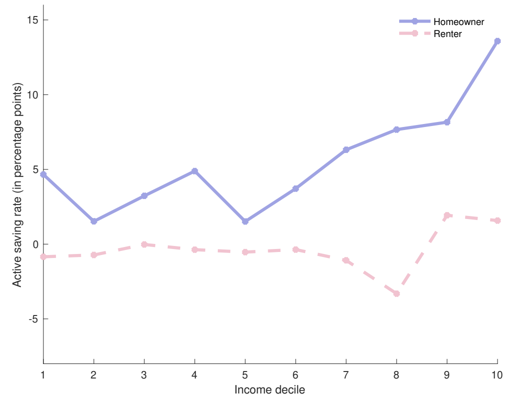
(a) Gross saving rate (b) Active saving rate

Figure 3.9: Average Saving Rate in Different Age Groups

When we compare saving rate after controlling for income, Figure 3.10 shows that homeowners have higher saving rate than renters. The overall pattern is that when households have higher income, the saving rate goes up, which is consistent with the finding in Dynan et al. (2004).



(a) Gross saving rate



(b) Active saving rate

Figure 3.10: Average Saving Rate along Income Decile

To analyze saving rate systematically and control for other variables that might influence the saving rate, we run the following quantile regression:

$$s_{i,t} = \alpha + \beta H_{i,t} + f(x_{i,t}) + \tau_t + \epsilon_{i,t}, \tag{3.13}$$

where $s_{i,t}$ is two measures of saving rate; $H_{i,t}$ is a dummy variable for homeownership; $f(x_{it})$ are control variables, which include age of head, log income, log of lagged wealth, years of education, marital status, family size, number of children; and τ_t is time-fixed effect. The parameter of interest is β, which measures the difference of saving rate between homeowners and renters.

Columns (1) and (4) in Table 3.11 show the results of quantile regression under the specification (3.13). As we can, the coefficients for homeownership are significantly positive. The median homeowner's gross saving rate is 5.89% larger than the median gross saving rate of the renters; and homeowner's median active saving rate is 5.50% larger than that of the renters, after controlling for other demographic and economic differences. The coefficients for other variables also have expected signs.

Mortgage: Commitment Device. One possible explanation for homeowners have higher saving rate is that the majority of homeowners purchases home with mortgages. Therefore, homeowners are obliged to make monthly mortgage payments. Each month, part of the mortgage payments is paid towards interests for outstanding balance, and the other part is applied towards paying down the principal outstanding. The part that pays towards the principal will reduce outstanding debt, increase household wealth, therefore, serves as a commitment device.

To examine whether mortgage serves as a commitment device, we add another variable, which is the mortgage payment towards principal scaled by family income. Columns (2) and (5) in Table 3.11 show that this variable is significantly positive. For households that pay a higher share towards mortgage principal out of their income, their saving rates are significantly higher. Thus, mortgage does serve as a commitment device.

Table 3.11: Quantile Regression for Saving Rate

	Gross saving rate			Active saving rate		
	(1)	(2)	(3)	(4)	(5)	(6)
homeownership	0.0589***	0.0487***		0.0550***	0.0292***	
	(0.007)	(0.005)		(0.005)	(0.003)	
principal ratio		0.4837***	0.4964***		0.7473***	0.7551***
		(0.037)	(0.035)		(0.022)	(0.018)
income	0.0102*	0.0081*	0.0127**	0.0103***	0.0092***	0.0123***
	(0.006)	(0.004)	(0.005)	(0.003)	(0.002)	(0.002)
lagged wealth	0.0009	0.0012	0.0054***	-0.0005	-0.0001	0.0027***
	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
education	0.0042***	0.0049***	0.0055***	0.0020**	0.0021***	0.0022***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
married	0.0140**	0.0203***	0.0309***	0.0117**	0.0167***	0.0184***
	(0.006)	(0.008)	(0.006)	(0.005)	(0.004)	(0.003)
family size	-0.0114***	-0.0117***	-0.0123***	-0.0118***	-0.0104***	-0.0091***
	(0.004)	(0.003)	(0.002)	(0.004)	(0.002)	(0.002)
# child	0.0079*	0.0087***	0.0094***	0.0119***	0.0088***	0.0079***
	(0.004)	(0.003)	(0.003)	(0.004)	(0.003)	(0.002)
age ≤ 35	0.0103*	0.0163***	0.0123***	0.0065*	0.0100***	0.0077
	(0.006)	(0.005)	(0.005)	(0.004)	(0.004)	(0.005)
age $46-55$	0.0012	0.0073	0.0051	-0.0042	-0.0025	-0.0031
	(0.007)	(0.006)	(0.006)	(0.004)	(0.005)	(0.005)
age $56-65$	0.0032	0.0015	0.0059	-0.0028	-0.0056	-0.0060
	(0.006)	(0.009)	(0.012)	(0.006)	(0.006)	(0.005)
Obs	22,208	21,221	21,221	22,222	21,231	21,231
Pseudo R^2	0.0089	0.0214	0.0205	0.0078	0.0585	0.0577

Notes: *** significant at 1% level, ** significant at 5% level, * significant at 10% level. Bootstrapped standard errors in parentheses. All specifications include dummies for the year. Sample uses the saving rate data from 2001 to 2017.

3.4.4 Discussion

In this section, we studied the differences in wealth accumulation channels in terms of asset allocation, rate of return, and saving rate. Homeowners and renters differ in all three channels, leading to different wealth accumulation patterns.

In terms of the overall effect due to differences in wealth accumulation channels, 10-year wealth mobility matrices presented in Appendix 3.6.3 show that homeowners are more likely to move up or stay in the same wealth quintile. However, the relative importance of each channel on contributing wealth accumulation remains unclear right now. And it is also worth exploring whether and how these three channels are jointly related. Moreover, it will be interesting to investigate how would homeownership influence wealth inequality.

3.5 Conclusion

In this paper, we study wealth accumulation by homeowners and renters using PSID data from 1999 to 2017. Our findings reveal notable disparities in the channels through which homeowners and renters amass wealth.

To begin with, there is a discernible distinction in asset allocation between homeowners and renters. Home equity constitutes the predominant share of a homeowner's wealth, with homeowners allocating less to stocks and real assets compared to renters with similar wealth. Nevertheless, despite this discrepancy, homeowners exhibit, on average, greater wealth and a more diversified portfolio.

Moving on to the second point, homeowners enjoy higher average returns on non-housing wealth. In terms of individual asset returns, renters also have competitive returns as homeowners; and renters reap higher on business and farm.

The third aspect highlights the disparity in saving rates, with homeowners exhibiting a higher propensity to save. This discrepancy is attributed to the mortgage serving as a compelled saving mechanism for homeowners.

3.6 Appendix

3.6.1 Active Saving

If a household doesn't move between time a to time b, then the active saving in the period is calculated as:

$$\begin{aligned}
&\text{Active Saving}_{\in(a,b)} \\
&= \text{total wealth at time } b - \text{total wealth at time } a \\
&- (\text{house value at time } b - \text{house value at time } a) \\
&- (\text{real estate value at time } b - \text{real estate value at time } a) \\
&- (\text{business/farm value at time } b - \text{business/farm value at time } a) \\
&- (\text{stocks value at time } b - \text{stocks value at time } a) \\
&+ \text{cost of real estate additions/repairs}_{\in(a,b)} \\
&+ \text{value of real estate purchased}_{\in(a,b)} - \text{value of real estate sold}_{\in(a,b)} \\
&+ \text{value of business/farm invested}_{\in(a,b)} - \text{value of business/farm sold}_{\in(a,b)} \\
&+ \text{value of stocks purchased}_{\in(a,b)} - \text{value of stocks sold}_{\in(a,b)} \\
&+ \text{value of pensions/annuities invested}_{\in(a,b)} - \text{value of pensions/annuities cashed}_{\in(a,b)} \\
&- \text{assets added by movers in}_{\in(a,b)} + \text{debts added by movers in}_{\in(a,b)} \\
&+ \text{assets removed by movers out}_{\in(a,b)} - \text{debts removed by movers out}_{\in(a,b)} \\
&- \text{value of inheritances}_{\in(a,b)},
\end{aligned}$$

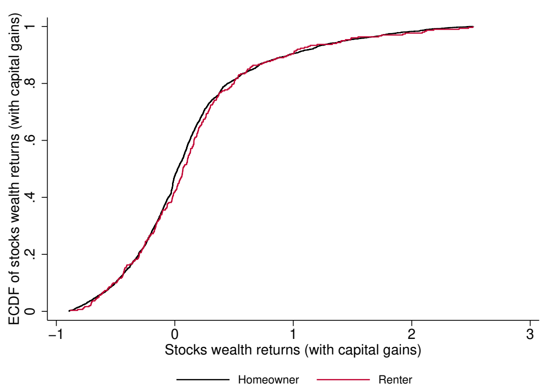
where all the values are inflation-adjusted to the 2019 dollar. If a household moves between time a to time b, then the change in house value is set to 0.

3.6.2 Rate of Return

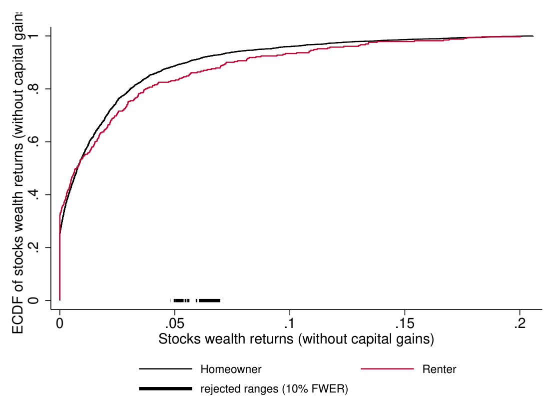
Stocks

Table 3.12: Kolmogorov-Smirnov Test on Distribution of Stocks Returns

Smaller group	D	P-value
Panel A: with capital gains		
Renter	0.017	0.860
Homeowner	−0.063	0.108
Panel B: without capital gains		
Renter	0.071	0.047
Homeowner	−0.059	0.122



(a) with capital gains



(b) without capital gains

Figure 3.11: Empirical CDF of Returns on Stocks

Other Real Estate

Table 3.13: Kolmogorov-Smirnov Test on Distribution of Other Real Estate Returns

Smaller group	D	P-value
Panel A: with capital gains		
Renter	0.059	0.325
Homeowner	−0.043	0.546
Panel B: without capital gains		
Renter	0.124	0.005
Homeowner	−0.026	0.796

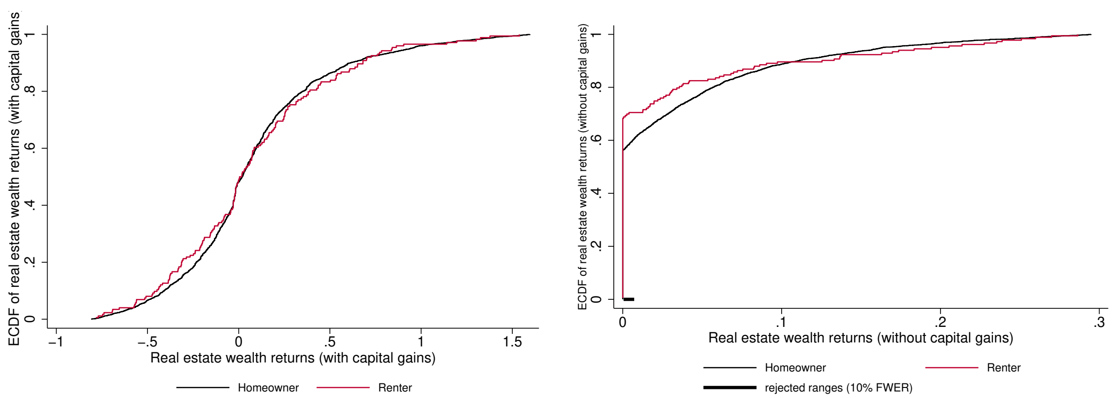


Figure 3.12: Empirical CDF of Returns on Other Real Estate Wealth

3.6.3 Wealth Mobility Matrix

Table 3.14: Wealth Mobility Matrix: Homeowner 1984-1994

		Quintile in 1994				
		1	2	3	4	5
	1	0.0625	0.2500	0.4375	0.1250	0.1250
	2	0.0317	0.2540	0.4286	0.2063	0.0794
Quintile in 1984	3	0.0365	0.0598	0.4319	0.3555	0.1163
	4	0.0120	0.0155	0.1618	0.4578	0.3528
	5	0.0099	0.0011	0.0231	0.1614	0.8046

Table 3.15: Wealth Mobility Matrix: Renter 1984-1994

		Quintile in 1994				
		1	2	3	4	5
	1	0.6231	0.2731	0.0692	0.0192	0.0154
	2	0.3816	0.4058	0.1643	0.0386	0.0097
Quintile in 1984	3	0.1852	0.2716	0.3580	0.1358	0.0494
	4	0.1034	0.3793	0.2414	0.1034	0.1724
	5	0	0.0769	0.0769	0.4615	0.3846

Table 3.16: Wealth Mobility Matrix: Homeowner 1994-2005

		Quintile in 2005				
		1	2	3	4	5
	1	0.0238	0.0952	0.3571	0.2619	0.2619
	2	0.1026	0.1282	0.3590	0.3333	0.0769
Quintile in 1994	3	0.0219	0.0601	0.3607	0.4126	0.1448
	4	0.0054	0.0179	0.1342	0.4275	0.4150
	5	0.0011	0	0.0221	0.1604	0.8164

Table 3.17: Wealth Mobility Matrix: Renter 1994-2005

		Quintile in 2005				
		1	2	3	4	5
	1	0.5478	0.3312	0.0764	0.0446	0
	2	0.3228	0.4567	0.1732	0.0394	0.0079
Quintile in 1994	3	0.3500	0.2667	0.2667	0.0500	0.0667
	4	0.3043	0.2609	0.1739	0.1739	0.0870
	5	0.4000	0	0	0.2000	0.4000

Table 3.18: Wealth Mobility Matrix: Homeowner 2005-2015

		Quintile in 2015				
		1	2	3	4	5
Quintile in 2005	1	0.2727	0.0303	0.3333	0.2879	0.0758
	2	0.1485	0.0792	0.3366	0.3564	0.0792
	3	0.0774	0.0418	0.2531	0.4916	0.1360
	4	0.0301	0.0096	0.0985	0.4432	0.4186
	5	0.0122	0.0010	0.0112	0.0959	0.8796

Table 3.19: Wealth Mobility Matrix: Renter 2005-2015

		Quintile in 2015				
		1	2	3	4	5
Quintile in 2005	1	0.5582	0.2836	0.1194	0.0269	0.0119
	2	0.3147	0.4126	0.2238	0.0315	0.0175
	3	0.2785	0.3418	0.2405	0.1013	0.0380
	4	0.2143	0.2143	0.2143	0.2500	0.1071
	5	0.0714	0	0.2857	0.1429	0.5000

4

Final Conclusion and Summary

In summary, this thesis studies development and inequality at the macroeconomic level.

The first chapter explores the impact of policy reforms aimed at promoting wage employment in low-income countries. The empirical findings reveal that in these economies, wage-employed individuals and self-employed individuals engage in distinct occupations and offer different goods and services in the market. Consequently, policies focused solely on increasing wage rates to stimulate wage employment may not yield the desired outcomes. A critical determinant of the effectiveness of such policy reforms is the elasticity of substitution between the goods produced by the wage employment and self-employment sectors. This factor plays a pivotal role in shaping market dynamics and influencing the consequence of policy interventions. As a policy recommendation based on the empirical findings and the unique characteristics of self-employment in low-income countries, formalizing firms that produce goods competing with those typically produced in the self-employment sector is advised, namely, home production substitute goods. By formalizing these enterprises, policymakers can foster competition that encourages efficiency and productivity growth in the economy.

The second chapter examines the impact of closing the gender gap in the Japanese labor market on fiscal policies and economic outcomes. The quantitative findings indicate that equalizing male and female labor force participation can significantly enhance the government's fiscal capacity, allowing for a substantial reduction in tax rates. This, in turn, would stimulate economic expansion. Consequently, the policy recommendation is to bolster female labor force participation by providing essential support services such as childcare and eldercare. Increasing the female labor force participation rate could serve as a new catalyst for growth and alleviate fiscal pressures.

The third chapter investigates the factors contributing to a higher level of wealth accumulation among homeowners compared to renters in the United States. Empirical evidence suggests that homeownership not only offers a substantial return on housing investment but also acts as a commitment device that encourages higher savings rates. Consequently, the policy recommendation is to promote affordable housing and increase homeownership, particularly for less privileged households, as a means to reduce wealth inequality.

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