THE ACCELERATION PRINCIPLE

by

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PREFACE

It bears witness at once to my youthful enthusiasm and naivete that this thesis constitutes one section of one chapter of the magnum opus, which I conceived as a freshman, on the cause and cure of depressions. According to all the valedictory clichés, University is supposed to "enrich", "broaden", "introduce fertile fields" and "open up new horizons". Sadly enough, University can also take all the poetry out of the life of the young idealist and leave him at the end of his studentship writing disenchanted prose. Thus, instead of offering the solution to the world's economic ills, my "disenchanted prose" suggests only a few changes -- improvements, I hope -- in the acceleration principle, an analytical tool which may yet prove to be of value in investigating some of our economic problems.

In complying with the regulation that I must tell what original contributions to the subject I have made, I should like to state at the outset that I have attempted throughout to say something new, and have treated summarily those areas of the subject in which I felt that I had nothing to contribute. As a result, this thesis is not a definitive study. The ideas will have to be subjected to the criticism of a wide audience before they can be rejected or incorporated into the accepted doctrine on the acceleration principle. I am swimming against the current of opinion in arguing that the acceleration principle should not be associated with derived demand or "magnification", and that J.M. Clark, not Aftalion, Carver or Bickerdike, gave the first clear statement of the principle. In previous literature, little attempt has been made to relate the acceleration principle to entrepreneurial experience; nor is there to be found an exhaustive treatment of the variables which the

acceleration principle can relate. Although I am not quite alone in arguing that there is a rigorous acceleration principle which is relatively free from the disturbing influence of expectations, I have extended the analysis somewhat and have tried to show that the entrepreneur will be penalized if he permits himself to be influenced by whim rather than by technical necessity. Of the limitations of the acceleration principle which are discussed in Chapter IV, those which apply to the use of the principle in a macro-economic model have received little or no attention in the literature. The critique of Professor Hicks' version of the interaction model is, I believe, important, and my main criticism is original. In the final chapter, I have argued that despite the fact that the acceleration principle is relatively objective, it cannot be used except by those who make it their business to observe the business community at first hand.

I should like to express my appreciation to my tutor, Professor B.S. Keirstead, for his encouragement, his kindly criticism, and the unselfish donation of his time during a period when time, for him, was such a scarce commodity. The thesis owes him a great deal, but I must hasten to add that the inclusion of an idea in the following pages does not necessarily mean that it carries his full approval; I alone must shoulder the responsibility for any heresies. My thanks are also due to the Imperial Oil Company whose generosity has enabled me to devote three reasonably uninterrupted years to the study of the business cycle. Lastly, I should like to record the special debt of gratitude which I owe to one who is at the same time my colleague, my fellow thesis-writer, and my wife. In the matter of thesis preparation, there has been a good deal of reciprocity, but I suspect that the terms of trade have been in my favour.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

The acceleration principle -- a controversial issue

The acceleration principle is a concept which has precipitated a series of controversies about its meaning, its importance, its mathematical form, its applicability, and the variables which it relates, to mention only a few. Like the female principal of the modern novel, the acceleration principle in the modern economic literature enjoys a questionable reputation. Despite the fact that its virtues and vices have been under more or less active consideration for the past thirty years, economists seem no closer to agreement than they were at the beginning on whether the acceleration principle should be granted a place among "respectable" economic theories. Professor A.D. Knox though conceding that "there is an element of truth in the acceleration principle" goes on to add that "it is an element that is so heavily overlaid by other factors that the acceleration principle by itself is inadequate as a theory of investment".[1] For the seminar on current research on business cycles Professor Haberler wrote, "to refute the acceleration principle in its simplest form by means of the econometric apparatus is like constructing an atom bomb for the purpose of killing a mouse, which if not still born has been killed so long ago that its body is by now in an advanced stage of decomposition."[2]

Despite some very severe criticisms of the principle, it has

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[1] Knox, A.D., "The Acceleration Principle and the Theory of Investment: A Survey", Economica, New Series, Vol. XIX, No. 75, August 1952, p. 296.

[2] Haberler, Gottfried, "Seminar: Current Research In Business Cycles", American Economic Review, Papers and Proceedings, Vol. XXXIX, No. 3, May 1949, p. 84.

never been without defenders anxious to protect its good name. R.M. Goodwin warns that to drop the acceleration principle "would surely be mistaken since it is merely the statement of a simple consequence of the one omnipresent, incontestable dynamic fact in economics -- the necessity to have both stocks and flows of goods".[3] R.F. Harrod speaking of the principle states that "its simplicity, ineluctability and independence of all special theories as to the workings of the cyclical process demand for it pride of place".[4]

Since the subject is controversial, the reader would be quite justified in wanting to know at the outset what side the writer is on. I, of course, claim strict neutrality, but recognize that I am probably "neutral" on the side of the acceleration principle, because in the course of this research I have come to believe that despite misadventure in which it was "wrongly used" or in which its underlying principles were "violated", there are still both life and virtue in our principal.

Organization of the thesis

The thesis is divided into seven chapters. We shall concern ourselves in the present chapter with the concept of the acceleration principle. Chapters II and III will carry our study of the principle a little further as we consider the development and origin of the idea. In Chapter IV we shall consider limitations -- conditions under which the principle may not be expected to apply. Chapter V considers the probable operation of the acceleration principle over the cycle, and Chapter VI

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[3] Goodwin, R.M., "The Nonlinear Accelerator and the Persistence of Business Cycles", Econometrica, Vol. 19, No. 1, January 1951, p. 3.

[4] Harrod, R.F., The Trade Cycle, Oxford, Clarendon Press, 1936, pp. 53-54.

is a criticism of the way in which the accelerator has been employed in the literature -- particularly by those writers who (presumably in an attempt to make an honest principle out of it) have married it to the multiplier. As will be apparent in Chapters IV, V, and VI, I do not consider some of the models which have resulted from the union to be either too model or even too legitimate, and hence, in the last chapter, have suggested a method by which the acceleration principle might be used to beget useful and legitimate progeny.

Definition

Before defining the acceleration principle it is traditional to give a demonstration of its operation, and if for this demonstration we choose the shoe industry, we shall be in the very best of tradition. In the time-honoured manner we shall assume that the equipment of our shoe industry consists of one hundred machines each with a fixed output of one hundred shoes a year. The machines may be thought of as the sole producer of the shoes, but if we must worry about cooperating factors, we shall assume them to be in perfectly elastic supply. The life of each machine is ten years, and in order to maintain production at ten thousand shoes per year, ten machines must be replaced annually. Under these assumptions a ten per cent increase in the number of shoes will require a ten per cent increase in the number of shoe machines, and, therefore, in the year in which output is to be increased by ten per cent, twenty new machines must be purchased -- ten for replacement and ten to enable the production of one thousand additional shoes. A ten per cent increase in output of consumption goods has led to a hundred per cent increase in the output of investment goods. If, in the next year, the shoe output is to be increased from eleven thousand to, say, eleven thousand five

hundred, the output of shoe machines in the second year must be ten for replacement (or eleven if one of the ten new machines is to be replaced) and five to permit the output of five hundred additional shoes. The significant thing to note is that although consumption increased in the second year from eleven thousand to eleven thousand five hundred shoes, investment in shoe machines actually decreased from twenty to fifteen (or at most, sixteen). Investment is seen to depend not on the rate of consumption, as is supposed by most theories of derived demand, but on the change of the rate of consumption.

Pausing to do the right thing by the shoe industry has demonstrated that investment is a function of the rate of change of output. In the words of J.M. Clark, "If demand be treated as a rate of speed at which goods are taken off the market... new construction depends upon acceleration".[5] This definition seems quite straight-forward, and it might be supposed that we have succeeded in defining and explaining the concept of the acceleration principle precisely. Unfortunately, we have not. The principle comes in so many editions and versions, all roughly compatible with the above example, that acceleration principle should probably be changed to acceleration principles. First, there are a number of variables which can be employed, and second, there are a number of mathematical forms which can be used to express the acceleration relationship.

The variables of the acceleration principle

The independent variable normally employed in the majority of recent statements of the acceleration principle is output. Output is

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[5] Clark, J.M., "Business Acceleration and the Law of Demand: A Technical Factor in Economic Cycles", The Journal of Political Economy, Vol. XXV, No. 3, March 1917, p. 220.

normally designated by the symbol Y, and may apply to the output of consumption and/or investment goods. Investment, the usual dependent variable, is said to depend on the behaviour of output. In the earlier literature consumption was usually the independent variable. However, in a micro-economic model it should not, strictly speaking, be a question of output or consumption; the accelerator should relate one stage of production to the immediately preceding stage regardless of whether the successive stages involve consumption or investment goods. The production of shoes should be related to the production of shoe manufacturing equipment, in which case we might write that investment is a function of consumption. The production of pig iron should be related to the production of blast furnaces, and in this case, to be consistent, we should probably write that investment is some function of investment! Obviously an increase in output of any good, whether for consumption or investment, will require prior investment, and therefore in a macro-economic model, it is quite in order to use aggregate output as the independent variable. However, it must not be thought that in a macro-economic model it is wrong to relate investment to aggregate consumption; the eventual test of the wisdom of any investment will be provided by the consumption markets, and the logic of a model may require that this be shown by relating investment to consumption.[6]

Our choice of independent variables is not limited to consumption or output; both profits and prices have been suggested either as unique determining factors or at least as co-determinants of

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[6] Or it may be the purpose of the model to explain the greater fluctuation of investment relative to consumption, in which case it is only logical to choose investment and consumption as dependent and independent variables.

investment. On the side of the independent variable, then, we seem to have the choice of using output, consumption, prices or profits -- or any other variable which is likely to apply. On the side of the dependent variable, we may choose investment or capital. This does not, however, exhaust our range of choice, for some of these variables may be expressed first, in money or real terms, second, in terms of actual, anticipated, planned, or desired amounts, and third, in either aggregate or micro-economic terms.

Faced with the choice of working with money or real values, practically all writers on the acceleration principle have chosen to work in real terms.[7] This is not surprising; after all, the acceleration principle purports to express a technical relationship -- the relationship between the quantity of a product and the amount of equipment needed to produce it. If product and/or equipment are expressed in money terms, the relationship is bound to become much more tenuous.

There is less agreement on whether the variables should have actual, anticipated, planned, or desired, values. Any one model may, and likely will, employ more than one kind of value; the independent variable might be assigned anticipated values while the values of the dependent variable could be planned or desired. As far as the dependent variable (investment, say) is concerned, planned values are generally the rule, although the assumption is frequently explicitly or implicitly made that investment plans are always carried out. With regard to the independent variable, the main debate concerns the use of actual or anticipated values.

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[7] The one notable exception is discussed in Chapter VI, p. 116.

As will become apparent, actual or realized values are favoured in the following chapters. The defence of this position will be left until we consider the relationship between the acceleration principle and expectations.[8] For the moment we shall simply note that unless it is otherwise stated, we shall always use realized values of output.

The last choice which we are free to make is between a macro and a micro model i.e. between the use of aggregate and plant values for the variables. The logic of the acceleration principle is most clearly seen when analyzing the investment of a single firm as, for example, the determination of investment in blast furnaces by the output of iron. As we shall see in Chapter IV, there are limitations applicable to the macro accelerator which are not apparent in the analysis of the investment of a firm, but when these limitations are properly taken into account there is certainly nothing illogical about the use of aggregative terms, and the choice between a micro or a macro model will depend upon the purpose of the investigation.

As can be seen, there is a considerable range of choice of variables by which the relationship which we know as the acceleration principle can be expressed. However, in this thesis our independent variable will normally be assumed to have realized values of real output, and our dependent variable will have planned values of real investment.

From the consideration of the variables, we now turn our attention to the various forms which the acceleration principle may take.

The forms of the acceleration principle

1. The mathematical expression of the acceleration principle

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[8] Infra, p.13.

which we shall consider first is a differential equation of the type $dI/dt = f(d^2y/dt^2)$. I is investment, y is output, and t is time. This relationship is sometimes written as $I = f(dy/dt)$, it being understood that I and y are rates. It is on such an expression, presumably, that R.F. Harrod might base his argument that the term "acceleration" is inappropriate. He holds that the rate of flow is dealt with in static theory and, "By analogy, therefore, a steady rate of increase of demand... should be regarded as a velocity. Acceleration would be a rate of change of this."[9] The logic of Mr. Harrod's position depends, of course, on his definition of "static". I should think, however, that pedagogically speaking it would be as well to state time explicitly. Students should be encouraged to think of investment (as well as demand, supply and output) as a flow which must be reported as a velocity of so much per period. With time stated explicitly, it is readily apparent how the acceleration principle received its name, for it is analogous to the acceleration of an object through space. The position of an object will be given by its distance (length) from a reference point. Velocity is distance per unit of time; acceleration will be the second differential of distance with respect to time. Dimensionally, acceleration in space is measured by, say, feet per second per second. Acceleration in economics should be measured dimensionally by dollars or physical units of output per period per period.

The differential equation in this form makes no provision for a lag between output and investment; this means that investment is

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[9] Harrod, R.F., "An Essay in Dynamic Theory", Economic Journal, Vol. 49, March 1939, p. 14.

determined by the present rate of change of output. As it stands such a relationship may be quite unrealistic, but a lag can be introduced by adding another term to the original equation. The acceleration principle is only rarely cast in terms of a differential equation, since most writers seem to prefer to express the relationship as a difference equation, and to this we now turn.

2. The most popular way of expressing the acceleration principle is by means of the difference equation. The acceleration principle as a difference equation could be written $I_t = b(Y_{t-1} - Y_{t-2})$ where b is the acceleration coefficient, and Y_{t-1} and Y_{t-2} refer to outputs of past periods. The expression $(Y_{t-1} - Y_{t-2})$ is sometimes written as ΔY_{t-1} and, using either expression, the equation tells us that investment in period t, I_t, is a (linear) function of a past increment of output, ΔY_{t-1}.

A second variant of this form of the acceleration principle is obtained by multiplying through by b to obtain the equation $I_t = bY_{t-1} - bY_{t-2}$. Since the acceleration coefficient, b, is likely to have a value very close to the capital-to-output ratio,[10] bY_{t-1} and bY_{t-2} may be considered as the amounts of capital existing at times t-1 and t-2. The equation could therefore be written $I_t = K_{t-1} - K_{t-2}$. This formulation of the principle states that businessmen undertake investment in period t which tends to duplicate the realized, and (because there is assumed to be no excess capacity[11]) justified, increase in capital which took place between periods t-2

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[10] Infra, p.45.

[11] Infra, p.59.

and t-1.

When the acceleration principle is cast in this investment-capital form, a desired value for capital is sometimes used. Investment might be said to equal the difference between the desired amount of capital (presumably at the end of period t) and the amount of capital existing at the beginning of the period. This statement makes the acceleration principle applicable to nearly all investment, and while seeming to explain everything, it really explains nothing. This form of the acceleration principle is little more than a way of rephrasing a question. If our problem is to determine the amount of investment in period t, we are no further ahead if we say that I_t will equal the difference between actual and planned capital. Such a statement might serve as a definition of planned investment, I_t, but it is not going to help determine its value.

3. The preceding forms of the acceleration principle have in all cases presented it as a theory of investment. The form which we shall now consider expresses the principle as a theory of capital. This form may be stated $K = f(Y)$ or $K = bY$, where K is capital, Y is output and b is the acceleration coefficient. One normally thinks of output as a result or function of capital, and it may seem surprising that K should be expressed as a function of Y. The acceleration relationship, however, holds that businessmen regulate the amount of capital according to the behaviour of output. K, therefore, is made to depend on Y. This form of the acceleration principle is sometimes cited as the most fundamental -- the form which in some sense represents the technical relationship on which the principle is based. Actually it represents the same idea as is expressed by other forms of the

principle. This can be shown as follows. If $K = b(Y)$ then, with the appropriate assumptions, ΔK will equal $b\Delta Y$. ΔK is of course just another symbol for investment; ΔY can be written as $(Y_{t-1} - Y_{t-2})$ and we are right back to the second form of the acceleration principle. There is in fact no significant difference among any of the mathematical expressions which may be used to state the acceleration principle, and one form may be readily converted into any other.

4. To complete our discussion of its mathematical forms, we might make a case for employing the term acceleration principle to describe any relationship -- economic or otherwise -- which can be expressed in the general form $dy/dt = f(d^2x/dt^2)$ where x and y are any variables, and t is time. Thus when Cassel used marginal analysis to determine the appropriate length of family prayer, he might just as well have used the acceleration principle. The term dy/dt might be considered as the rate of investment in spiritual capital while x might represent sin. The second differential of x with respect to time is the rate of change of sin. Such a relationship is not improbable; after all we quickly become accustomed to our current vices, and any particular rate of sin can be "explained" by such comforting maxims as "none of us is perfect". It is the rate at which new sins are being introduced which causes concern and which clearly calls for increased investment in spiritual capital. Similarly a hypochondriac might equate his investment in doctors' bills not to his rate of suffering or the number of complaints (after all none of us is perfect), but to the rate at which new complaints are cropping up.

Direction of the causal relationship and excess capacity

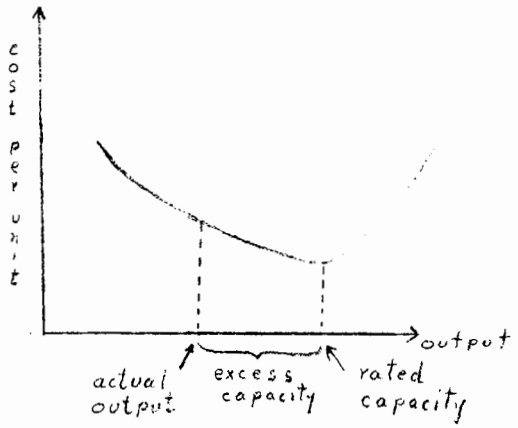
When the third form of the acceleration principle was

considered, we stressed that K depended on Y and not the other way round. This emphasis is important and applies equally to all forms of the principle. The acceleration principle insists on the direction of the causal relationship; Y changes and then K or I changes -- output determines the behaviour of investment.

The direction of the causal relationship suggests that changes of output must precede changes of investment. The problem of how output can increase without a prior increase of capital is solved by (a) postulating a normal reserve capacity but not excess capacity, since the acceleration principle cannot be used to explain investment if there is excess capacity, or by (b) assuming the existence of a stock of finished goods which can satisfy increased demand until the stock of fixed capital can be increased.

Since the operation of the acceleration principle depends on the absence of excess capacity, the principle can be no more exact than the latter concept, so we must pause long enough to make clear what we mean by this term. It is assumed (and this is a reasonably realistic assumption) that every businessman, having considered depreciation and other costs, will have in mind a specific output which he considers to be the "rated capacity" of his plant. If output falls below this mark, the entrepreneur is likely to regard his plant as having "excess capacity" -- the difference between actual output and the output at rated capacity. Although rated capacity is likely to be less than maximum physical capacity, the implication is that to produce beyond this point is in some sense inconvenient, and while output may exceed it for short periods, continued production beyond this level would soon induce additional investment. Rated capacity

may coincide with the lowest point of the average total cost curve, particularly if the curve rises sharply from its lowest point.[12] Excess, rated, and maximum, capacities are defined graphically on the accompanying diagram.[13]



The acceleration principle and expectations

In order to avoid misunderstanding, it is important that the role of expectations in the acceleration principle be made quite explicit. The form of the principle which I would endorse is the "crude form", put forward by Professor B.S. Keirstead, which assumes that "changes in demand cause, post facto, changes in investment".[14] This statement does not entirely exclude expectations as we shall see, but it is in sharp contrast to the form of the principle which recognizes investment as a function of expected increments of output. The latter form is completely general and could apply to virtually all productive (capacity-increasing) investment; even the actions of a "new man" with a new product and a new process are consistent with this statement of the principle.

My contention is that this second interpretation is not useful, except perhaps as a pedagogical device, nor is it what we

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[12] For a similar treatment vide Knox, A.D., op. cit., pp. 277-278.

[13] Rated capacity may be more difficult to identify if the cost curve has a flat "U" shape, although even as a subjective concept it should not prove an insurmountable obstacle to the observers whom we propose in Chapter VII to conduct continuing industry studies.

[14] Keirstead, B.S., An Essay in the Theory of Profits and Income Distribution, Oxford, Basil Blackwell, 1953, p. 68.

commonly understand by the acceleration principle. Few economists would label the investment of the new man as induced, and yet, according to the general statement of the principle, it is just that; the new man invests because he expects output to increase from its initial zero level.

In the crude form of the principle the position of expectations can be expressed as follows: investment is a function of expectations; expectations are a function of past increments of output. Investment still rests on anticipated increments of output, but the expectations which determine the anticipated increments are based in such a simple and direct way on experienced increments that we can skip the expectations stage and relate investment directly to experienced output. This more rigorous form of the principle undoubtedly limits its applicability; but where it is applicable, it may at least serve as a practical and useful tool of analysis.

The acceleration principle is not derived demand or magnification

In order to sharpen our conception of the acceleration principle, it would be useful to touch very briefly on two ideas which are sometimes confused with the acceleration principle, namely the principle of derived demand, and magnification.

Professor Somers has stated: "The acceleration principle is merely a refinement of the principle of derived demand".[15] It is true, of course, that the acceleration principle could be classed as a derived theory of demand, but the reverse is not true; any

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[15] Somers, H.M., Public Finance and National Income, Philadelphia, The Blakiston Co., 1949, p. 72.

observation which merely calls attention to the dependence of investment on consumption does not qualify as a statement of the acceleration principle. There is an extensive literature on derived demand and on underconsumption, which stresses repeatedly the dependence of investment on consumption. Some works hint at an acceleration relationship; some make statements which are consistent with the operation of the acceleration principle; but the conscious distinction between the usual principle of derived demand, which states nothing more than that $dI/dt = f(dC/dt)$, and the acceleration principle, which states that $dI/dt = f(d^2C/dt^2)$, does not seem to have been clearly made until J.M. Clark's 1917 article.[16] Since the derived demand relationship has been frequently suggested as a theory of investment,[17] it would seem to be advisable to maintain a firm distinction between the principle of derived demand and the principle of acceleration.

Another issue which sometimes confuses the concept of the acceleration principle is the so-called magnification which the principle is alleged to involve. The idea that the acceleration principle accounts for the relative and absolute fluctuations in consumption vis-à-vis the corresponding fluctuations in investment plays a large part in the literature of the principle. An acceleration model might be used to demonstrate how fluctuations in investment

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[16] Clark, J.M., op. cit.

[17] For example, Alexander, S.S., "The Accelerator as a Generator of Steady Growth", Quarterly Journal of Economics, Vol. LXIII, No. 2, p. 186. Lerner, Abba P., "A Contribution to the Theory of the Trade Cycle", Econometrica, Vol. 19, No. 4, October 1951, p. 473.

trades could be greater than corresponding fluctuations in consumption trades, but on the other hand, there are other possible explanations of this phenomenon, and there are cases in which the acceleration principle applies which do not involve magnification. We shall consider this matter in greater detail in the following chapter; for the present it should be noted that the acceleration principle involves more than the dependence of investment on consumption (or output), and it involves more than the greater relative fluctuation of investment as opposed to consumption.

The real meaning of the acceleration principle

Let us leave the problem of what the acceleration principle is not, and examine the more positive question of what the principle is in terms of the real world -- in terms of entrepreneurial experience. After all, expressing a principle mathematically may give it precision, but mathematical precision is not sufficient; we must also be sure that we are being precise about the real world. Professor Albert Einstein is reported to have said about his new unified field theory, "The theory is mathematically correct but I have not been able to find out if there are any physical truths in it".[18] In order for our mathematical expressions of the acceleration principle to be convincing, we must be able to find "physical truths" in them; we must be able to show that the equations are meaningful in terms of human behaviour.

The statements of the acceleration principle have indicated that it is a theory of investment, but "theory of investment" is

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[18] The Gazette (Montreal), Wednesday, March 18, 1953.

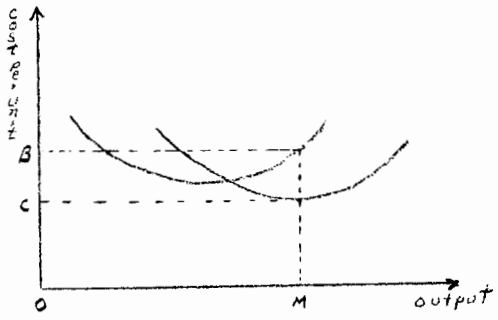
capable of at least two interpretations. First of all, it might be argued that the acceleration principle is descriptive of the process or method by which businessmen arrive at their investment decisions. Entrepreneurs, according to this first interpretation, actually apply some coefficient to a realized or anticipated set of outputs and arrive at the appropriate amount of investment. This interpretation of the acceleration principle could be further subdivided. On the one hand, it might be argued that although the principle describes the process, the actual operation is completely subjective, and, for any of the following reasons, impossible for an outsider to duplicate: (a) it might be that the entrepreneur does not put objective, measurable outputs into the equation -- they may be anticipated, or in some sense ideal, outputs (i.e. the outputs which he thinks could have been sold if certain "unusual" factors had not interfered); (b) the entrepreneur may use actual outputs but employ a subjective acceleration coefficient, which may be a function of his digestion, his desire for prestige in his lodge, and his views on the modern generation; (c) the investment arrived at by the businessman using the process of the acceleration investment equation may not be actual or planned investment but merely the investment "which it would be nice to have". On the other hand, it might be argued that the acceleration principle not only describes the process which the entrepreneur uses but does so in such a way that an outsider might duplicate both the method and the results of entrepreneurial planning. This will mean, by and large, that the variables used in the equation must be objective -- output figures would have to be <u>ex post</u> -- and the coefficient must be based on

technical requirements rather than on entrepreneurial whim. So much for the first interpretation which holds that the acceleration principle describes the investment process. The second interpretation might point out that although the process of investment planning is quite different from that supposed above, or that it is unknowable, nevertheless, entrepreneurial decisions can be predicted by using the acceleration principle.

To maintain that the acceleration principle can describe the investment process is not to suggest that the entrepreneur writes down a difference equation such as $I_t = b(Y_{t-1} - Y_{t-2})$ and then looks about to find values for the unknowns. However, he may use a process which is very nearly the same thing. A few years ago I had occasion to estimate the amount of investment in electrical generating capacity that would be required in a given area for the following five years. The method used to accomplish this task was to plot on a graph the output of the last ten to fifteen years, determine the trend, and extrapolate to find the expected output in five years. Data was collected on the existing capacity, which happened to be quite fully utilized, and by multiplying the real capital-to-output ratio by the expected increase in sales, an estimate was arrived at of the investment that would be required over the next five years. The interesting thing about this procedure (apart from its lack of sophistication) was that both the method followed and the results obtained coincided almost exactly with those of the electrical generating company supplying the area. Moreover the investment plans were actually being put into effect at the rate indicated by this rather crude procedure.

In this case the acceleration coefficient (the capital-to-output ratio) was applied to the increase in output ($Y_{t+5} - Y_t$) where Y_t was the current (trend) value of output and Y_{t+5}, the expected value in five years. This might sound as though we were recanting on the unpleasant things said about using anticipated values for the variables; however, since Y_{t+5} is a simple extrapolative value, $(Y_t - Y_{t-5})$, which is the difference between the realized trend values of output at year t and year t-5, might just as easily have been used. The full investment equation would read $I_t = b(Y_t - Y_{t-5})$ where Y_t and Y_{t-5} are trend values of output as determined graphically.

With the electrical generating company, we found not only that the acceleration principle worked well quantitatively, but also that it was fairly easy to relate the mathematical statement to the real investment-deciding process. In other industries we might find quite a different real phenomenon behind the determination of induced investment. If a company which is without a tradition of growth experiences an increase in output, the entrepreneur will find -- or be told by his engineers -- that production is becoming "uncomfortable"; costs may be up, inventory, down, or the union may be complaining about overtime. The officials in charge of production costs will report, in terms of a cost diagram, that at the existing output, OM, costs are OB, whereas with some investment, costs might be reduced to OC. The entrepreneur will probably find reports on his desk from his plant managers suggesting



additional equipment or building-extensions, and investment will be undertaken without any particular attention being paid to past increments of output.

The two cases may be distinguished by the attitude of the entrepreneur towards growth, which itself will be determined by past events. In an industry with a long history of orderly development, the investment process will probably be rather similar to that of the electric company. If, on the other hand, the entrepreneur considers that he is presiding over a static business in which growth is an exceptional and unpredictable thing, the induced investment is likely to be quite ad hoc. The acceleration principle may still apply as a predictive device, but it will be much more difficult to relate the actual investment process to the mathematical expression, and the mathematical expression operated as a predictive device by an outsider will probably be much less accurate. There are undoubtedly other investment-determining processes which may be covered by one of the mathematical expressions of the acceleration principle, and there are many investment processes to which the principle will not apply at all, for it is not a general theory of investment and it cannot hope to explain either the amount, or the process, of investment in all cases.

Perspective

Perspective is to be the major concern of Chapter VII, but before getting too far along we might discuss briefly the applicability of the acceleration principle. The acceleration principle, as has already been suggested, can be cast in a rigorous, or in a fairly general, form.

In its rigorous form it is of limited applicability; at best it could be expected to apply to only a few firms all of the time, to some firms some of the time and to some firms none of the time. On the other hand the acceleration principle may be expressed in a way which will make it almost universally applicable. We may, for example, say that investment is a function of present and future output whether future output is extrapolative or not. This expression will be generally true of any investment decision to increase capacity, even for a new firm producing a new product. The trouble with this broad expression is that the acceleration principle stops being a predictive device and becomes merely a way of stating a problem.

There is no shortage of general "explanations" of investment. We may be told that investment is a function of the marginal efficiency of capital and the rate of interest, but when we look into the matter more deeply we find that the marginal efficiency of capital depends on technological advance, discovery of new resources, population growth, political situation et cetera. Or we may be told that investment is a function of the natural and market rates of interest; and then we find that the natural rate of interest depends on technological advance, discovery of new resources, population growth, political situation et cetera. Since any theory of investment is left open to criticism, or even ridicule, if any of these factors are ignored, some authors have adopted a generalized form of the relationship and have made either the acceleration coefficient or the expected increase in output a function of everything from technological advance to the weather. Of course total investment is

obviously a function of all these factors, and if we are to crowd all investment decisions under the wings of one theory, we must be sure that no relevant consideration is left out. If we insist on a broad, all-inclusive theory of investment, we might just as well ignore the mathematical expression, say from the very beginning that investment is a function of technological advance et cetera, and leave it at that. If we try to make the acceleration principle a complete theory of investment we must consider all factors and, in effect, make it look very much like every other general theory of investment, and make it just about as useful. However, if there is a component of investment for which the acceleration principle in its more rigorous form works with a reasonable degree of accuracy, it would seem to be more useful to retain the principle in its rigorous form, and apply it to only a limited number of firms. In line with this argument, the acceleration principle discussed in the pages following, unless it is employed in an abstract model, is a <u>partial</u> theory of investment.

This chapter has been concerned with the most fundamental concepts of the acceleration principle -- the definition, the variables, the mathematical forms of the principle and the relation between the mathematical expressions and the "physical truths" in the real world. In the next chapter we shall continue with fundamentals as we consider the origin of the principle.

CHAPTER II

THE ORIGIN OF THE ACCELERATION PRINCIPLE

The names of four men are associated with the origination of the acceleration principle: T.N. Carver, C.F. Bickerdike, A. Aftalion, and J.M. Clark. Our task in this chapter will be to assess the contribution of these men so that we may judge the extent to which they are genuinely originators or merely precursors of the principle.

T.N. Carver

The first statement of the principle in English is attributed to an article written by T.N. Carver in 1903.[1] A careful examination of this ingenious and suggestive article, however, reveals no trace of the acceleration principle. In the best tradition, the article begins with an account of investment in the shoe industry. A plant is assumed to be able to produce one million pairs of shoes at an average cost of $2.00. If the shoes cannot be sold for more than this price, the plant will be worthless; but if they can be sold for $2.25, say, earnings will be $25,000 which, capitalized at five per cent, would impute a value to the plant of $125,000 (sic). If the price of shoes increased to $2.50 (and if the increase was believed to be permanent), the value of the plant would double. "Thus an increase of only one-ninth in the value of the product would double the value of the plant."[2] Generalizing this finding, Professor Carver

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[1] Carver, T.N., "A Suggestion for a Theory of Industrial Depressions", Quarterly Journal of Economics, Vol. XVII, No. 3, May 1903, pp. 497-500.

[2] Ibid., p. 498.

concluded that "a slight fluctuation in the value of the product tends to produce a violent fluctuation in the value of the establishment producing it".[3]

This generalization laid the basis for a theory of the trade cycle. The initial push required to start the process is a slight rise in the value of consumption goods, a rise which automatically increases the value of investment goods and which leads to a higher level of investment. Moreover, the shift of resources from consumption to investment trades tends to increase further the value of consumer goods and therefore increases still more the value of investment goods. There is no check to this process until the new investment begins to flood the market with more articles for consumption. This increased supply will result in a fall in price of consumption goods and, as a direct result, a fall in the price of investment goods. The argument of the article can be summarized in three relationships: investment is a function of the value of investment goods; the value of investment goods is a function of the price of consumption goods; the price of consumption goods is a function of past investment. From the first two relationships it would be possible to derive an "acceleration principle" -- investment, I, could be a function of the rate of change in the price of consumption goods; $I_t = f(P_{t-1} - P_{t-n})$ where P_{t-1} and P_{t-n} are past levels of prices and I_t is the investment undertaken at time t. This relationship would show that investment is a function of the rate of change of price and that there would result a fall of investment whenever the rate of

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[3] Ibid., p. 498.

increase of prices slackened. However, Professor Carver did not make such an assumption; in fact he assumed that the decline in investment would not come until the prices of consumption goods actually declined. In other words $I_t = f(P_t)$ or, more likely, $I_t = f(P_{t-n})$. Investment would continue to increase so long as the prices of consumption goods continued to increase.

One must conclude that although Professor Carver stressed the dependence of investment on consumption, and although he had an explanation of magnification, or the reason why fluctuations in investment should be more violent than fluctuations in consumption, the relationship which he described was not the relationship of the acceleration principle.

C.F. Bickerdike

The next writer who is alleged to be an originator of the acceleration principle is C.F. Bickerdike who published an article in 1914 entitled "A Non-Monetary Cause of Fluctuations in Employment".[4] In this article the author is interested in the violent fluctuations in the investment trades, and asks whether the competitive nature of the market makes any difference to the degree of fluctuation. In the course of showing that competition has the effect of increasing fluctuations in employment, Mr. Bickerdike published a table on the basis of which some might claim that he is a discoveror of the acceleration principle. The table is reproduced below.

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[4] Bickerdike, C.F., "A Non-Monetary Cause of Fluctuations in Employment", Economic Journal, Vol. XXIV, September 1914, pp. 357-370.

Expected, actual demand for total tonnage and yearly investment in tonnage in 000's of tons[5]

	1900	1901	1902	1903	1904	1905
expected demand	20,000	20,500	21,000	21,500	22,000	22,500
actual demand	20,000	20,400	20,600	21,000	22,000	22,500
new and replacement demand		1,400	1,200	1,500 (sic)	2,000	1,500

The first row shows expected demand for capacity on the basis of a two and one-half per cent annual increase; the second row shows actual demand. It is assumed that a ship lasts twenty years so that each year one million tons of new capacity are required for replacement. It can be seen from these figures that when the rate of increase of demand for capacity (shipping services) increases at a decreasing rate (as it does between 1901 and 1902 and again between 1904 and 1905), there will be an absolute fall in the rate of investment required. But Bickerdike does not call attention to these facts and appears to be interested solely in the proposition that minor fluctuations in the demand for the services of durable goods lead to proportionately larger fluctuations in the demand for the durable goods themselves. If Bickerdike noted that an absolute increase in consumption could be accompanied by a decreasing rate of investment, he kept his own counsel on the matter. The idea that investment can decrease while consumption is increasing is such a novel one that it would be remarkable if Bickerdike noted the full acceleration relationship and yet failed to call attention to it. The only thing in the table to which the

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5 Ibid., pp. 359-360.

reader's attention is called is the greater fluctuation in investment as opposed to capacity (which is taken to be identical with the demand for productive services).

In Bickerdike's example, output is so defined (he defines it as a tonnage year) that numerically, but not dimensionally, capital equals output. This relationship could be written $K = bY$ where K is capital, Y is output and b is the acceleration coefficient which, in this case, is equal to one. Alternatively the relationship could be expressed by the form $I_t = b(Y_t - Y_{t-1}) + 1/20 \times 20{,}000$, the first term on the right hand side representing the demand for new investment, and the second, the demand for replacement investment. Investment for the year 1901 is calculated by the equation $I_{1901} = 1(20{,}400 - 20{,}000) + 1{,}000 = 1{,}400$.

While it is possible thus to derive a full acceleration relationship from the table, it is difficult to see how far we should go in crediting the author with the discovery of that principle. To draw an analogy, if an early physicist observed that when two children were balanced on a teeter-totter the process of teetering always sent the lighter child through a greater distance (which, in a sense, is magnification) could we, on this basis, credit the observer with the discovery of the principle of the first-class lever?

A. Aftalion

The next author, A. Aftalion, whose contribution we must assess, poses a similar problem. The works which are alleged to deal with the acceleration principle include a series of articles[6] appearing

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[6] Aftalion, A., "La Réalité des surproductions générales", Revue d'économie politique, 1909, pp. 81-117, 201-229, 241-259.

in the Revue d'Economie Politique in 1909 and his book Les Crises Périodiques De Surproduction.[7] The most relevant passage from one of the early articles is reproduced as an appendix at the end of this chapter. Like Bickerdike and Carver, Aftalion seems to be primarily interested in magnification, and while what he says is quite consistent with an acceleration principle, he certainly gives no clear indication that he noted the full acceleration relationship.

In both the Revue articles and the book, Aftalion stresses at considerable length the fact that the demand for investment goods is a derived demand, and he also shows how the value of consumption goods affects the output and price of investment goods.[8] His argument is very much like that of Carver whose contribution Aftalion recognizes in a footnote. However, Aftalion does go further than Carver and gives a mathematical example which, while difficult to follow, could be construed as being consistent with the acceleration principle. Aftalion assumes that investment output is equal to ten per cent of consumption. If consumption increases by ten per cent (and here there must be some implicit assumptions about the size of the depreciation coefficient and the capital-to-output ratio) Aftalion assumes that a like amount of capital would be required so that a ten per cent increase in consumption would require a hundred per cent increase in the output of capital goods.

As with Bickerdike, the place of Aftalion in the literature in this connection is difficult to assess. I certainly could not agree

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[7] Aftalion, A., Les Crises Périodiques De Surproduction, Paris, 1913.

[8] For example, ibid., p. 357 ff.

with Professor Hansen[9] that the mathematical example to be found in the appendix to this chapter constitutes the first precise statement of the acceleration principle. Aftalion has called attention to the fact that the lighter child travels furthest on the teeter-totter, but does this observation constitute a statement of the principle of leverage? In any event, this much seems clear; without in any way belittling the work and originality of these men, one may say that their contributions do not detract from the originality of J.M. Clark. There is a very big step between the work of Aftalion and Bickerdike on the one hand, and J.M. Clark on the other.

J.M. Clark

The first clear statement of the acceleration principle came with the publication in 1919 of J.M. Clark's famous article "Business Acceleration and the Law of Demand"[10]. In this article it is made explicit that "The demand for maintenance and replacement of existing capital varies with the amount of the demand for finished products, while the demand for new construction or enlargement of stocks depends upon whether or not the sales of the finished product are growing".[11] At this point comes the famous footnote which christened the accelerator. "If demand be treated as a rate of

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9 Hansen, A.H., Business Cycles and National Income, New York, W.W. Norton and Co., 1951, p. 493.

10 Clark, J.M., "Business Acceleration and the Law of Demand: A Technical Factor in Economic Cycles", The Journal of Political Economy, Vol. 25, No. 3, March 1917, pp. 217-235.

11 Ibid., p. 220.

speed at which goods are taken off the market, maintenance varies roughly with the speed, but new construction depends upon the acceleration."[12] Clark also calls attention to the fact that "In order to bring about an absolute shrinkage in the demand for the intermediate product, all that may be needed is that the final demand should slacken its rate of growth."[13] Here, clearly, investment is a function of the rate of change of output, $dI/dt = f\ d^2Y/dt^2$, and the relationship of the acceleration principle has been explicitly recognized.

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[12] Ibid., p. 220n. Italics added.

[13] Ibid., pp. 222-223.

APPENDIX TO CHAPTER II

EXCERPT FROM A. AFTALION

"LA REALITY DES SURPRODUCTIONS GENERALES"[14]

..."C'est du produit final que les capitaux tirent toute leur valeur. Le besoin premier, le besoin direct, est celui des biens de consommation. Le besoin des capitaux est un besoin dérivé, un besoin indirect, soumis aux répercussions du précédent.

Les variations de l'intensité des besoins qui déterminent les phases alternées des cycles périodiques ne peuvent donc être que des variations relatives aux biens de consommation. Mais les mouvements qui ont leur point de depart dans les biens de consommation gagnent aussi les capitaux. C'est même en ce qui concerne les capitaux, en ce qui concerne en particulier le matériel, l'outillage, qu'ils prennent le plus d'intensité et qu'ils sont le plus apparents.

-- Plus d'intensité d'abord. Un excès ou un déficit assez faible d'objets de consommation entraînant des fluctuations modérées de leur valeur amènera un accroissement plus que proportionnel du besoin et de la valeur des instruments de production. Le montant de la fabrication annuelle d'instruments de production, en vue de remplacer le matériel hors d'usage et d'augmenter progressivement l'importance de l'outillage existant, n'égale en effet qu'une portion assez faible du matériel actuellement employé à la production des objets de consommation. Si nous supposons que la proportion est du dixième, et si dans une année donnée la fabrication des objets de consommation devait s'accroître d'un dixième, la production relative

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[14] Loc. cit., pp. 209-210.

au matériel devrait cette année-là doubler: puisqu'à une production normale et annuelle d'un dixième devrait s'ajouter une production supplémentaire égale encore à un dixiéme. Une légère extension des industries de consommation exigera une extension beaucoup plus considérable des industries productrices du matériel.

Un exemple d'ailleurs entièrement hypothétique montrera comment les choses pourront se passer aux époques de prospérité et de dépression. A une industrie d'objets de consommation utilisant 100.000 métiers devrait, je suppose, correspondre une fabrication annuelle moyenne de 10.000 métiers. Mais, comme conséquence d'un déficit de 10 p. 100 des objets de consommation, entraînant le besoin de 10.000 métiers supplémentaires, on constate que, pendant les cinq années de la prosperité, à la production normale par les industries de capitaux de 50.000 métiers, s'ajoutent un contingent de 10.000 métiers rendu nécessaire par le déficit d'objets de consommation, et un second contingent de 10.000 métiers constituant la surcapitalisation de la prospérite. La production est de 70.000 métiers ou de 14.000 annuellement. Dans les cinq années de la depression, par suite d'un excès de 10 p. 100 des objets de consommation, la production, au lieu d'être de 50.000 métiers, se voit diminuée de 10.000 métiers à cause de cet excès d'objets de consommation, impliquant un excès égal de capitaux et de 10.000 autres métiers par suite de la sous-capitalisation de la dépression. La production est de 30.000 métiers ou de 6.000 annuellement. Tandis que la quantité d'objets de consommation produite oscille entre un déficit et un excès de 10 p. 100, entre 90 au début de l'essor et 110 à la crise, le déficit ou l'excès des 10.000 métiers qui en est

la conséquence fait bondir la fabrication des instruments de production de 60 à 140, puis la fait s'effondrer de 140 à 60. Un déficit de 10 p. 100 d'objets de consommation fait plus que doubler, fait croître de plus de 100 pour 100 la production relative au matériel. Un excès de 10 p. 100 d'objets de consommation réduit au chômage plus de la moitié de l'industrie productrice de machines.

CHAPTER III

SOME FUNDAMENTAL PROBLEMS

Three separate problems make up the subject matter of this section. A consideration of these problems -- magnification, the coefficient of acceleration, and the Clark-Frisch controversy on the inevitability of the downturn -- will serve to clarify our concept of the acceleration principle.

Magnification

Magnification has always been closely associated with the acceleration principle. In the previous chapter several references were made to magnification in the works of Carver, Aftalion, and Bickerdike. In 1919 Professor Clark wrote, "it [the acceleration principle] acts as an intensifier of the disturbances it transmits".[1] When H.M. Somers introduced the subject of the accelerator he stated that "in its full glory" it was known as the "principle of acceleration and magnification of derived demand".[2] Professor Boulding also throws the emphasis on the magnification aspect of the principle, for he writes, "The effect of distortions in the age distribution of goods is accentuated by another principle known as the 'acceleration principle'. If one commodity, B, is necessary for the production of another commodity, A, then the fluctuations in the output of A will be reflected by intensified fluctuations in the production of B."[3]

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[1] Clark, J.M., "Business Acceleration and the Law of Demand: A Technical Factor in Economic Cycles", The Journal of Political Economy, Vol. 25, No. 3, March 1917, p. 218.

[2] Somers, H.M., Public Finance and National Income, Philadelphia, The Blakiston Co., 1949, p. 66.

[3] Boulding, K.E., Economic Analysis, New York, Harper & Brothers, 1948, p. 385.

While it might be granted that magnification in some sense is usually involved in cases in which the acceleration principle is applicable, it strikes me that the emphasis which this particular feature of the principle receives is surprising. We may have magnification without the acceleration principle, and we may have the acceleration principle without having magnification. But before showing how this can be, we must pause to consider what magnification is, and what it is that is being magnified.

It is easy to find an explicit definition of magnification in physics, but in economics it seems to be defined only by implication. In physics, magnification means the real or, more generally, the apparent enlargement or exaggeration of an object. The possibility that the phenomenon is apparent or imaginary is usually ruled out in economics by the context. In the case of the acceleration principle the enlargement is quite real and the "object" which is enlarged is normally a change or fluctuation. To complicate the matter still further, these changes may be expressed in absolute or in relative terms, and, since there are a number of variables which might be used to state the acceleration relationship, the changes might apply to a number of different variables.

Let us begin by considering our third form of the linear acceleration relationship, $K = bY$. (K is capital, Y is output and b is the acceleration coefficient.) It is readily apparent that if b is greater than 1 the absolute change in K must be greater than the absolute change in Y. If b is 3, and if Y increases by 100, K must increase by 300. Should b have a value of less than 1, the reverse relationship would hold; the absolute fluctuations in Y would be

greater than the absolute change in K. Hence if b equalled 1/3, and if Y again increased by 100, K would increase by only 33 1/3. If b equals 1, the absolute fluctuation in K and Y are the same. As will be seen later on, we are free to alter the value of b by changing the length of the period, and thus we can always so choose our period that the absolute fluctuations of capital are equal to, less than, or greater than, the fluctuations of output. Obviously, magnification, in the sense of absolute changes, does not play an impressive role as far as the two variables in this particular statement of the acceleration principle are concerned. Nor does magnification loom more important if relative, rather than absolute, fluctuations are considered. In this particular relationship the relative fluctuations in K and Y are always the same regardless of the value of b; a 10 per cent increase in Y will require a 10 per cent increase in the value of K.

One is prompted to ask whether it would have altered our conclusions if, instead of our third form of the acceleration principle which relates output and capital, we had used the second form which is, let us say, $I = b\,\Delta Y$ (I is investment, and ΔY, the increment of output). When it is recognized that I is just ΔK, it is not surprising that our findings with regard to magnification are no different whether we consider $K = bY$ or $\Delta K = b\Delta Y$. Again, values of b can be chosen so that the absolute fluctuation of investment is equal to, greater than, or less than, the absolute fluctuation of ΔY. And again, the relative fluctuations in I and in ΔY (note that we are considering a fluctuation in the increment of output and not in output itself) will be the same regardless of the value of b.

Magnification, in the sense of either an absolute or a relative change, seems to have little to do with the operation of this form of the acceleration principle. And this conclusion is quite general. So long as we consider only the terms which the acceleration principle relates, magnification seems to be completely unimportant.

This being granted, how is it that so much emphasis has been directed towards magnification in articles which supposedly discuss the acceleration principle? One answer, I believe, is to be found in the fact that the changes which have been traditionally considered are fluctuations in variables or terms which do not appear in the acceleration relationship. It will be noted that in our third form of the acceleration principle ($K = bY$) the absolute changes are measured by ΔK and ΔY, while the relative fluctuations are measured by the ratios $\Delta K/K$ and $\Delta Y/Y$. In the second form of the principle ($I = b\Delta Y$) the relevant fluctuations are measured by ΔI and by $\Delta(\Delta Y)$ in absolute terms, and by $\Delta I/I$ and $\Delta(\Delta Y)/\Delta Y$ in relative terms. Writers dealing with the magnification aspects of the acceleration principle, however, usually compare the absolute changes of ΔI and ΔY and the relative fluctuations of $\Delta I/I$ and $\Delta Y/Y$. Yet these terms are not "explained" by the acceleration principle, and while either term may appear in a statement of the principle, there is no single statement of the principle which specifically relates ΔY and ΔI. Lest we be charged with quibbling over the terms that may be ligitimately compared <u>in a discussion of the acceleration principle</u>, let us direct our attention to an even more unusual thing about the traditional treatment of magnification in this context. Most writers when illustrating magnification deal with <u>gross</u> investment. Our

third form of the principle deals with total capital, and our second form (like the first) deals with net investment. To my knowledge no one has ever suggested that the acceleration principle be used to explain gross investment. Even in a model in which all net investment is accounted for by the acceleration principle, we need at least one more theory of investment to account for replacement before we can explain gross investment. Such a theory might be that replacement investment, R, is a linear function of capital; $R = aK$ where a is the rate of depreciation.

Let us now take a numerical example which includes both the acceleration principle and "the depreciation principle" to illustrate the behaviour of the changes of output (Y), capital (K), net investment (I), replacement (R), and gross investment (R + I).

Table 1

0	1	2	3	4	5	6	7	8	9
period	Y	ΔY	K	I	ΔI	R	ΔR	R+I	$\Delta R+\Delta I$
0	0	0	0	0	0	0	0	0	0
1	50	50	100	100	100	10	10	110	110
2	100	50	200	100	0	20	10	120	10
3	100	0	200	0	-100	20	0	20	-100
4	110	10	220	20	20	22	2	42	22
5	120	10	240	20	0	24	2	44	2
6	130	10	260	20	0	26	2	46	2
7	150	20	300	40	20	30	4	70	24
8	180	30	360	60	20	36	6	96	26
9	200	20	400	40	-20	40	4	80	-16
10	210	10	420	20	-20	42	2	62	-18

First we may note that the acceleration equation in this example is $K = 2Y$. The absolute fluctuations in capital (column 4) are twice the absolute fluctuations in output (column 2), and the relative fluctuations in output and capital are the same (e.g. for period four, $\Delta Y/Y = 10/110$ and $\Delta K/K = 20/220$). Since the absolute fluctuations are

a function of the acceleration coefficient, which can be altered by changing the length of the period, we may conclude that as far as output and capital are concerned, magnification does not play an important role.

At this point it might be argued that we are really interested in the absolute and relative changes in output as compared not with capital, but with investment. However, when we compare investment and output, our illustration shows us that there is no necessary pattern of magnification at all. As for the absolute changes, it can be seen that in some periods ΔI is larger than ΔY, and in other periods it is smaller. Looking at the relative changes, one can see that $\Delta I/I$ is sometimes larger, sometimes smaller than the corresponding $\Delta Y/Y$. In period six, the relative change in output is 10/130 -- larger than the corresponding relative change in investment which is 0/20. On the other hand, in period eight, the relative fluctuation in output, 30/180, is only half the corresponding figure for investment, which is 20/60. This lack of pattern is what might be expected; after all the acceleration principle does not explicitly relate ΔY and ΔI (let alone $\Delta Y/Y$ and $\Delta I/I$) and it is not at all surprising that there is no unique relationship between these variables.

The columns to the left of the heavy vertical line in Table 1 contain the values which are derived directly or, in the case of column five, indirectly, from the acceleration relationship. The columns to the right of the heavy line contain those terms which are derived from the "depreciation principle" (columns six and seven) or from a combination of acceleration and depreciation (columns eight and nine). The depreciation principle in this illustration is

represented by the equation R = 0.10 K. The combination of replacement and acceleration investment gives gross investment, and it is this gross investment which is contrasted with output by the majority of writers dealing with the acceleration principle. I do not wish to imply that it is in any way improper to make such a comparison, but I would insist that when we deal with gross investment more is involved than the acceleration principle. Still it is interesting to note that even here there is no unique relationship between the relative magnitudes of ΔY and $\Delta(R+I)$ or of $\Delta Y/Y$ and $\Delta(R+I)/R+I$. In period four both the absolute and relative changes of gross investment exceed the corresponding changes in output (22 as opposed to 10 and 22/42 as against 10/110). In period five, on the other hand, both the absolute and relative fluctuations of gross investment are smaller than the corresponding fluctuations of output (2 as compared with 10 and 2/44 contrasted with 10/120).

Hitherto in our discussion of magnification, we have always picked output as one term of our comparison, either as the term magnified or the term in comparison with which some other variable is magnified. In the role of the "other variable" we have tried capital, net investment, and gross investment, without discovering any "magnification principle" (unless we consider that the capital-to-output relationship (K/Y) involves some magnification when the acceleration coefficient has a value other than unity). However, in a discussion of the acceleration principle, the magnification of at least one other pair of variables has been considered. Professor Knox[4]

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[4] Knox, A.D., "The Acceleration Principle and the Theory of Investment: A Survey", Economica N.S. Vol. XIX, No. 75, August 1952, p. 272.

has observed that if capital is durable and lasts more than one period, the percentage fluctuation in investment must be larger than the percentage fluctuation in capital. In terms of our illustration he appears to argue that $\Delta(R+I)/R+I$ must be larger than $\Delta K/K$. This particular magnification seems to be further removed from the acceleration principle than most, and in any event it does not happen to be true in all cases. Observe that in period six, the relative change in gross investment is 1/23 which is smaller than the corresponding change in capital which, for that period, is 1/13.

Some authors, rather than compare relative and absolute changes in consumption and investment period by period as we have done, have instead considered the behaviour of investment and consumption over a number of periods. We might concede in most cases that such a procedure is favourable to the hypothesis that in an acceleration model the investment industry always fluctuates more than the consumption industry. However the validity of this hypothesis depends on the model and on the range of values chosen for the consumption industry, and a mathematical example can be given which will disprove the generality of even this magnification argument.

Table 2

0	1	2	3	4	5	6	7	8	9
Period	Y	ΔY	K	I	ΔI	R	ΔR	R+I	$\Delta(R+I)$
0	100		100						
1	100	0	100	0		100		100	
2	110	10	110	10	10	100	0	110	10
3	140	30	140	30	20	110	10	140	30
4	150	10	150	10	-20	140	30	150	10
5	100	-50	100	-50	-60	150	10	100	-50

In this table in which $K_t = 1.0\ Y_t$, and $R_t = 1.0\ K_{t-1}$, $\Delta(R-I)$ always

equals ΔY, fluctuations in output are reproduced exactly in the investment industry, and no magnification occurs either period by period or over a number of periods.

If there is no magnification principle associated with the acceleration principle (and if there is such a principle, it is most elusive), what explains its common acceptance in the literature? For one thing, the weight of opinion which links magnification to the acceleration principle is overwhelming. At the beginning of this section we mentioned J.M. Clark, H.M. Somers, K.E. Boulding -- and to this list we could add R.F. Harrod[5], G. Haberler[6], R.A. Lester[7], and a host of others. For another thing, a magnifying process does seem, at first glance, to be very closely associated with the operation of the acceleration principle. Let us consider the typical process which normally leads to the statement or the implication that magnification is an inherent aspect of acceleration. Suppose that a constant rate of output is maintained over a number of periods. During these periods net investment will be zero. Now let us assume that output increases by ten per cent. This increased output will call for net investment the exact amount of which will depend on the capital-to-output ratio. The percentage increase in net investment is infinite, and this does rather look like magnification, for a ten per cent increase in output has led to an infinitely large

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5 Harrod, R.F., The Trade Cycle, Oxford, The Clarendon Press, 1936, p. 56.

6 Haberler, G., Prosperity and Depression, New York, United Nations, 1946, p. 88.

7 Lester, R.A., Economics of Labour, New York, The Macmillan Co., 1941, p. 240.

percentage increase in net investment. If, as is more likely, the increase in net investment is expressed as a percentage of total gross investment the relative increase in investment will be more modest.

By way of summary, there are three important contradictions or qualifications which should be noted about this typical demonstration of magnification. First, the alleged magnification when based on total investment involves more than the acceleration principle. The actual magnification depends on both the acceleration coefficient and the depreciation or replacement coefficient. Second, granting the legitimacy of the comparison of <u>total</u> investment and output, even this magnification is not inevitable. If the replacement coefficient is equal to one (which it may be in the case of stocks or in the case of any perishable asset if a long enough period is taken), the fluctuations in gross investment may mirror, rather than magnify, the sudden increase in output. This point was illustrated in Table 2. Third, the generality of the magnification principle could be easily contradicted by extending the analysis a little further. In terms of Table 3, the "typical demonstration of magnification" considers only period one. In this period absolute and relative changes in investment (31 and 0.76) are in fact larger than the corresponding changes in output (10 and .091). However, in the next period in which output is assumed to increase again by a like amount, the situation is reversed. The absolute and relative changes in investment (2 and 0.048) are smaller than the corresponding changes in output (10 and 0.083).

Table 3

Period	Y	ΔY	$\Delta Y/Y$	K	I	ΔI	R	ΔR	R+I	$\Delta(R+I)$	$\frac{\Delta(R+I)}{R+I}$
	100			300			10				
0	100	0	0	300	0		10	0	10		
1	110	10	.091	330	30	30	11	1	41	31	0.76
2	120	10	.083	360	30	0	12	2	42	2	0.048

A study of magnification may be useful in its own right, and such a study might or might not use a model in which the acceleration principle plays a part, but as far as the acceleration principle itself is concerned, magnification is a red herring. The acceleration principle, contrary to popular belief, does not permit us to state categorically that the output of a good will always fluctuate more or less than capital, net investment or gross investment either in absolute, or in relative, terms.

The coefficient of acceleration

The problems associated with the acceleration coefficient have so far been sidestepped, and they must now be considered. We must deal with the meaning of the term "coefficient of acceleration"; we must consider its dimensions, its relation to the capital-to-output ratio, its value, and the dependence of its value on time and expectations. One of the most important problems, the constancy of the value of the coefficient, has been reserved for separate treatment in Chapter IV.

The acceleration coefficient, like the various coefficients in physics, may be thought of as a factor which somehow relates two variables, or it may be thought of as a kind of multiplier which, with the independent variable, acts as a co-determinant of the dependent

variable. According to the first interpretation, the acceleration coefficient could be viewed as an ex post relationship of investment and output; according to the second, it might be thought of as a multiplier which transforms realized increments of output into investment plans.

It follows from what has been said that the coefficient is not a pure number although we seldom, if ever, see it accompanied by units. If the acceleration principle is written as K dollars of capacity = b times Y dollars of output per period, it is evident that dimensionally b must have the same units as K/Y i.e. dollars of capacity/dollars of output per period. If we may assume that capacity and output can be reduced to a common unit, dollars, the dollars in both numerator and denominator will cancel out leaving only periods. The accelerator coefficient, then, tells us the number of periods that must pass before the value of output can equal the value of capacity; an acceleration coefficient of two tells us that the output of one period is one half the value of capacity and that, therefore, in two periods the value of output will equal the value of capacity. Rather than cancel out the dollars as we have done, it is probably better to retain all units and to think of the acceleration coefficient as representing the amount of capital which is required to produce one dollar's worth of output in one period.

The statement was made in a preceding chapter that the value of the coefficient is likely to be close to the capital-to-output ratio, but no indication was given of why this should be so. The omission must be remedied, and to assist in the task let us consider two cases, the first involving a single increment of output,

and the second, a continuous expansion. Considering the first case, we know that given an increase in output from Y to $Y_{+\Delta}Y$ we shall be able to predict the amount of investment undertaken, provided certain conditions are met. These "certain conditions" include the absence of excess capacity and a knowledge of the capital-to-output ratio. The total net investment which is undertaken if the new higher rate of output is to be maintained (or, more to the point, the investment which will be undertaken if the entrepreneurs <u>believe</u> that the new higher rate will be maintained) is equal to $\Delta Y \times K/Y$. However, although we know the amount of investment, we shall still not be able to asign a value to the acceleration coefficient, because the entrepreneur is still reasonably free to construct his new plant quickly or slowly; in terms of period analysis he is free to spread his investment $\Delta Y \times K/Y$ over many periods or over a fraction of one period. In this case it looks as though the acceleration coefficient need not equal the capital-to-output ratio although the importance of that ratio in determining the size of the accelerator is evident. In the second case, in which the entrepreneur is faced with a continuing expansion, it is another story. Given an increase of ΔY <u>each period</u>, there will be a strong pressure on the businessman to adopt an acceleration coefficient equal to K/Y and a rate of investment of $\Delta Y \times K/Y$ per period. Only this rate of investment would provide for the proper increase of output, ΔY, each period; a lower rate would mean a continuing and increasing pressure on capacity; a higher rate would soon produce excess capacity and discourage the excessive rate of investment. There is a strong tendency, therefore, in a continuing expansion as opposed to a single increment of expansion, for the

acceleration coefficient to equal the capital-to-output ratio.[8]

It might be argued that the relevant capital-to-output ratio is not K/Y but rather $\Delta K/\Delta Y$. This, strictly speaking, is correct, but on the other hand there is some justification for prefering to use K/Y. In the first place, information about K/Y is likely to be more readily available. In the second, K/Y is likely, in the short run at least, to be fairly close to $\Delta K/\Delta Y$ unless the firm is innovating. If it is innovating so much that K/Y differs greatly from $\Delta K/\Delta Y$, the acceleration principle is unlikely to be an important explanation for the investment carried on by that firm. We might indeed conclude that if the acceleration applies at all, the value of the coefficient is likely to be quite close to the value of the capital-to-output ratio.

Because of its close connection to the capital-to-output ratio, the value of the acceleration coefficient must vary with the length of the period chosen. If the technical relationship between capital and output is such that two dollars' worth of capital produces one dollar's worth of output in a period of three months (the value of the coefficient is two), it is apparent that if the period were extended to six months, the same two dollars' worth of capital would be able to produce two dollars' worth of output, and the value of the acceleration coefficient would be reduced from two to one. If we reduced the length of the period to one and one-half months the

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[8] This conclusion is arrived at by R.S. Eckaus, "The Acceleration Principle Reconsidered", The Quarterly Journal of Economics, Vol. LXVII, No. 2, May 1953, p. 214, when he states that the value of the accelerator for the firm "is equal... to the increase in capital required to increase the output by one unit".

coefficient would be increased to four. The value of the acceleration coefficient varies inversely with the length of the period.

In this illustration, despite the fact that the value of the coefficient changed, it is important to note that the technical relationship did not. If confusion arises it is only because we are in the habit of dropping our units in economics, especially when time is concerned. When all units are retained it is apparent that the technical relationship does not change when we say that one unit of output can be produced by two units of capital in three months, by one unit of capital in six months or by four units of capital in one and a half months.

By arguing that the value of the acceleration coefficient is likely to be a function of the technical relationship K/Y, we have taken sides in a debate concerning the effect of expectations on accelerator-induced investment. The minority school of thought on this controversy, which I would support, stresses the technical as opposed to the psychological aspect of the principle. The school of the majority emphasizes the effect of expectations. Of course, a division into schools is quite arbitrary; no one is going to argue that either expectations or technical necessities can be completely ignored; still, a difference in emphasis is quite discernable. For example J.M. Clark wrote, "this circumstance [the acceleration principle] is not psychological, nor does it depend upon the nature of our credit system nor upon the distribution of income, but rather upon the elementary technical necessities of the case."[9] Professor

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[9] Clark, J.M., op. cit., p. 218. Italics added.

Hicks follows this statement very closely when he claims to have explained the cycle in terms of "simple reactions, by entrepreneurs and by customers... based upon the technical necessities of a capital-using economy."[10]

On the other hand, the majority of writers on the acceleration principle seem to give more weight to the role of expectations. Professor Somers has argued that "it is, therefore, the state of expectations rather than purely technical characteristics which determines the size of the Accelerator".[11] D. McC. Wright goes even further and states that it doesn't really matter whether output increases, for the same effect would be achieved if expectations increased.[12]

The truth of the matter probably lies somewhere between the two schools. When dealing with a single increment of output there would seem to be considerable latitude for expectations to make their influence felt. The technical relationship will determine the amount of investment, but the rate of investment can only be fully explained by including the psychological factors. In a continuing expansion, however, expectations are surely much less important, since excesses of pessimism and optimism will be penalized by shortages or by excess capacity. Expectations probably cannot be

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[10] Hicks, J.R., A Contribution to the Theory of the Trade Cycle, Oxford, Clarendon Press, 1950, p. 117. Italics added.

[11] Somers, op. cit., pp. 83-84.

[12] Wright, D. McC., "A Neglected Approach to the Acceleration Principle", Review of Economic Statistics, Vol. XXIII, No.2, may 1941, pp. 100-101.

ruled out of any human decision-making process, but in this case their effect would seem to be at a minimum.

In this connection it is interesting to note the suggestion of R.S. Eckaus[13] that the acceleration principle is improved by the explicit inclusion of expectations. Following his suggestion, we could write the acceleration principle as $I_t = b\eta(Y_{t-1} - Y_{t-2})$ where η is a coefficient of expectations -- an explicit recognition that past increases in output as an entrepreneurial stimulus will not always lead to the same reaction. I should be inclined to argue that given a continuous advance, the technical requirements of the case would make η equal to one. Given a single increment of output, there is no technical value for b; its value will depend on the decision about the rate at which to invest -- a decision which cannot easily be separated from expectations. Mr. Eckaus presents a midway position between the two schools; while suggesting the explicit treatment of expectations, he recognizes that the acceleration principle theory of investment is freer than most from the influence of expectations.

On what factors does the value of the acceleration coefficient depend? It will be influenced, in the first instance, by whether the model is micro-economic or macro-economic. In the case of the firm, as we have seen, the value of the coefficient will be closely associated with the capital-to-output ratio and therefore different for different firms. Furthermore, the concept of capital is quite elastic. We may be interested in only one particular

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[13] Eckaus, op. cit., pp. 222-223.

investment good, or we may be interested in all investment associated with the output of the firm. The value of the cow-to-milk ratio is obviously going to be much smaller than the value of cow+barn+ equipment-to-milk ratio. We should, then, expect a wide variation in the values of micro-acceleration coefficients depending on the nature of the firm, the capital included, and, of course, the length of the period chosen. With a macro-model many additional complications are introduced. If the macro-coefficient is to be an average of individual coefficients, and if aggregate output (Y) is to represent national income, then it is the capital-to-value-added ratio which must be considered for each firm if double counting is to be avoided. In Chapter IV we shall see that there are actually many complications in the use of the macro-acceleration coefficient which will affect its value; for the moment we have probably gone sufficiently far to show that there is no one "logical" or "not improbable" value for the acceleration coefficient.

Despite these problems, writers on the accelerator have sometimes had occasion to place a "reasonable" value on the coefficient. Professor Frisch suggested a value of ten "which means that the total capital stock is ten times as large as the annual production".[14] His estimate, according to an earlier passage, is a numerical value "that may in a rough way express the magnitudes which we would expect to find in actual economic life".[15] In Frisch's article the coefficient

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[14] Frisch, R., "Propagation Problems and Impulse Problems in Dynamic Economics", Economic Essays in Honor of Gustav Cassel, London, Allen and Unwin, 1933, p. 186.

[15] Ibid., p. 185.

relates to a capital-to-consumption, rather than to a capital-to-output, ratio, and using the former ratio naturally gives the coefficient a higher value. But because the relevant period is a year, the coefficient would have a value of twenty for a six-month period -- a very high value even for a capital-to-consumption ratio. Professor Hicks[16] argues that in order to obtain explosive cycles in his elementary case, the value of the coefficient must be greater than $(1+g)^2$ where g is the rate of growth. He accepts Kuznets' estimate which, for a period of six months, assigns to g a value of two. The acceleration coefficient must therefore have a value greater than nine in order for this model to give explosive cycles.

These two estimates seem to me to be very high. There are not many firms with a capital-to-output ratio of nine even for a six-month period. A look through the financial statements of a small sample of manufacturing establishments revealed a capital-to-value-added ratio of between two and three for a twelve-month period, or five for a period lasting only six months.

The Clark-Frisch controversy

One of the innovations introduced by Clark's 1917 article was the statement that a slowing down in the rate of increase of output could lead to an absolute decline in investment. This phenomenon was quickly seized upon as a plausable explanation of the downturn. Although the original statement of the proposition was well hedged by Professor Clark, in an unguarded moment he wrote

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[16] Hicks, op. cit., pp. 93-94.

"the makers of capital equipment are bound... to suffer an absolute decline in the demand for their products... whenever ultimate demand slackens its rate of growth".[17] This statement, which gave rise to the Clark-Frisch controversy[18] on the inevitability of the downturn, was actually inconsistent with Clark's earlier formulation of the principle in which he explicitly stated that demand for capital depended on the amount of demand for consumption goods (depreciation) and on the rate of change of demand.[19] It is only the latter component of investment which must turn down, given a decrease in the rate of increase in output. There is little doubt, however, that Clark thought it more than likely that a slowing down in the rate of increase of output would cause a downturn in total investment.

There are three questions which arise out of the controversy. (1) Was Frisch correct in maintaining that a slowing down in the rate of increase in output is not a sufficient condition for a reduction in investment? (2) Was his qualification, if correct, important? (3) What was the source of the misunderstanding?

With regard to the first question, there cannot be much

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[17] Clark, J.M., Studies in the Economics of Overhead Costs, Chicago, University of Chicago Press, 1923, p. 390.

[18] Frisch, R., "The Inter-relation between Capital Production and Consumer-Taking", Journal of Political Economy, Vol. 39, October 1931, pp. 646-654; also "Capital Production and Consumer-Taking: A Rejoinder", loc. cit., Vol. 40, April 1932, pp. 253-255; and "Capital Production and Consumer-Taking: A Final Word", loc. cit., p. 694. J.M. Clark, "Capital Production and Consumer-Taking: A Reply", loc. cit., Vol. 39, December 1931, pp. 814-816; and "Capital Production and Consumer-Taking: A Further Word", loc. cit., Vol. 40, October 1932, pp. 691-693.

[19] Clark, J.M., "Business Acceleration and the Law of Demand: A Technical Factor in Economic Cycles", loc. cit., p. 220.

doubt about the logical correctness of Frisch's argument if it is applied to gross investment in a micro-economic model. If it is assumed that the demand for capital is a function of the level of output (depreciation or replacement demand) as well as the rate of increase of output, then the decline in demand for new investment caused by a slowing down in the rate of increase of output may be offset by an increase in replacement investment.

We should note exactly what the increase in replacement demand can offset. At best it may mean that employment in the capital goods industry in question will not fall. To suppose, however, that replacement demand can take the place of the demand for net investment in a macro-model is to imagine that the economy is capable of "tailing off" into a stationary state at full employment with no net savings or net investment. Such an occurence is most unlikely. Whereas it is just possible that a particular capital-goods industry could change gradually from a builder of new equipment to a servicer of equipment already built without experiencing a drop in output, such an event is unlikely in an industry or firm, and unthinkable for the whole economy. The Frisch qualification is, after all, of footnote importance, and in a macro-economic model one should be forgiven for failing to mention the qualification at all.

As for the last question, Frisch seemed to feel that the source of the misunderstanding lay in the fact that Professor Clark had forgotten to count and equate equations and variables.[20]

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[20] Frisch, R., "Capital Production and Consumer Taking: A Final Word", loc. cit., p. 694.

The controversy seems to me to stem from the failure (especially on the part of Professor Frisch) to define terms and concepts. It should be clear from Professor Clark's article that the acceleration principle does not explain the behaviour of aggregate investment, and yet this confusion of acceleration and gross investment has plagued the literature for thirty years. (Witness the use of gross investment when demonstrating fluctuation supposedly caused by the acceleration principle.) To return to the statement which started the controversy, Professor Clark could have refuted the criticism not by recanting on the necessity for a decline, but by emphasizing that it is net capital production which must decline.

CHAPTER IV

LIMITATIONS OF THE ACCELERATION PRINCIPLE

Introduction

The relevance of any criticism or qualification of the acceleration principle will depend on how much is being claimed on its behalf. Evidence that the acceleration principle was not used by one firm at one time is relevant as a criticism only if it is claimed that the principle explains all investment for all firms at all times. The general applicability or relevance of a criticism depends not only on the interpretation of the principle, but also on the nature of the criticism itself. Some qualifications are, by their nature, of general applicability, and may be expected to apply to any case in which the acceleration principle is used. Others are more modest, and apply only to particular uses, or misuses, of the principle.

The criticisms which have received most attention in the past are those which apply to the micro-economic case. Writers, when considering the limitations of the acceleration principle, have, for the most part, examined them in the course of relating the accelerator to the firm, but when they have turned to the application of the principle, they have applied it uncritically to the whole economy without always pausing to consider whether there might be further limitations in a macro-economic application which are not present, or obvious, in the case of the single firm.

To compensate in some measure for the amount of attention which has been devoted in the literature to the accelerator as it applies to the firm, we shall devote most of this chapter to examining the difficulties involved in treating the accelerator in a

macro-economic context.

There are many theoretical and practical limitations which must be kept in mind where the acceleration principle is involved. The nine which are considered in this chapter are listed below. (1) The acceleration principle is a crude first approximation. (2) It is of only limited applicability. (3) In the presence of excess capacity the acceleration principle is not applicable. (4) The operation of the principle depends to a certain extent on favourable expectations. (5) The action of the accelerator is modified by a restraint during the downswing. (6) In the absence of excess capacity the accelerator must also operate under a restraint during the upswing. (7) The acceleration coefficient should be applied only to relevant increases in output, and can be applied to changes in total national output only under very unusual assumptions. (8) The accelerator relationship may be upset, and investment decisions adversely affected, by a shortage of cooperating factors. (9) It is probably a good approximation to treat the acceleration coefficient as a constant, but there are several factors which may affect its value.

(1) The acceleration principle -- a first approximation

The acceleration principle has been criticized because it is an inaccurate tool of analysis, and, it must be conceded, there is much truth in this charge. Despite the fact that the principle rests on technical and mechanical relationships it cannot be expected to work with a high degree of precision. The capital-to-output ratio, the basis of the acceleration coefficient, is subject to moderate change especially in the short run: extra shifts can be added; old

machinery can be pressed back into use; replacements can be accelerated or decelerated. All of these factors will permit considerable variation between actual output and the output corresponding to rated capacity. To say, however, that the acceleration principle is inexact, only a tendency, and unreliable under certain circumstances, is not to make a serious reduction in its status as an accepted economic relationship; for these characteristics it shares with most propositions that we have come to call economic laws or economic principles.

(2) Limited applicability

The acceleration principle may be attacked on the ground that it does not apply to all firms. This objection is relevant only if the sweeping assertion has been made that the principle is a general theory of investment. By admitting that the acceleration principle is, at best, only a partial theory of investment, we cut the ground from under this criticism.

However, the admission that the acceleration principle is only a partial theory of investment may not satisfy those critics who carry the argument much further, and claim that the component of investment which the acceleration principle is capable of predicting is such a small fraction of total investment that the principle is completely insignificant. The relative importance of "induced investment" and of the acceleration principle is a question of fact which probably cannot be determined on an a priori basis. The final chapter will deal with this problem more fully and will suggest a method which might enable us to get a better idea of the importance of

the role which the acceleration principle plays in determining total investment.

(3) Excess capacity

It has been generally recognized that the acceleration principle is not applicable to industries in which there is excess capacity. In Chapter I[1] we defined excess capacity in relation to the firm as the difference between rated and actual capacity. This definition will serve for the economy as a whole if we keep in mind the problems of aggregation. For example, we must remember that excess capacity in one industry cannot satisfy additional demand for the products of an unrelated industry. "Capacity" in this context is measured by fixed plant and equipment as opposed to the raw materials and other cooperating factors which we shall consider separately.

This limitation has been cited by L.R. Klein[2] as a reason for rejecting the accelerator as a significant macro-economic relation; for he holds that excess capacity is the rule in our economy and not the exception. Without going so far as to say that excess capacity is the rule, Simon Kuznets[3] has argued that in industries in which demand is variable there will be a chronic tendency to overcapacity. Where the acceleration principle is being incorporated into a model which purports to describe the business cycle, the problem raised by these

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[1] Supra., p.12.

[2] Klein, L.R., "Reply", in Conference on Business Cycles, New York, National Bureau of Economic Research, 1951, pp. 316-317.

[3] Kuznets, Simon, "Relation Between Capital Goods and Finished Products in the Business Cycle", Economic Essays in Honor of Wesley Clair Mitchell, New York, Columbia University Press, 1935, p. 232 ff.

charges of the prevalence of excess capacity may be avoided by assuming, as does Professor Hicks,[4] that the depression is long enough to wear out redundant plant and equipment. Although such an assumption disposes of the problem for the model, it is also in danger of disposing of any real value that the model might have for describing the actual economy. It is doubtful whether the world has ever undergone a depression that was long enough or deep enough to eliminate, or even greatly reduce, excess capacity. Professor A.H. Hansen[5] estimates that the United States emerged from the depression of the thirties with a greater capacity than existed in 1929 despite a net investment which was only nominal.

The wide-spread existence of excess capacity does not, of course, mean that the acceleration principle will be completely inoperative. Even when under-employment of plant and equipment is the general rule, there may be a few industries experiencing an increasing demand which cannot be met comfortably with existing productive capacity and which will, therefore, cause additional investment.

Because the absence of excess capacity is so crucial to the operation of the acceleration principle, there may be some justification for using a form of the principle which takes explicit account of capital. In the case of our electrical generating company[6]

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[4] Hicks, J.R., A Contribution to the Theory of the Cycle, Oxford, Clarendon Press, 1950, p. 105.

[5] Hansen, A.H., Monetary Theory and Fiscal Policy, New York, McGraw-Hill, 1949, p. 111.

[6] Supra., pp. 18.

the equations which were used were $I_t = b(Y_{t+5} - Y_t)$ or $I_t = b(Y_t - Y_{t-5})$. Since these expressions did not include capital, we had to take it into separate account to make sure than the existence of excess capacity would not interfere with our results. We might, however, have used the equation $I_t = K_{t+5} - K_t$, which is obtained by multiplying through by b in the first of the above equations. K_{t+5} is defined as $b(Y_{t+5})$ and is therefore equal to output in period t+5 times the capital-to-output ratio. The product is the equivalent of required capital. K_t, on the other hand, is defined not as bY_t, but as the existing capital; bY_t is only that part of existing capital which is in use. K_t, therefore, includes both used and unused capacity. In this way an allowance for excess capacity has been made an integral part of the statement of the acceleration principle.

(4) Expectations

The operation of the accelerator does not automatically follow past increases in output. Favourable expectations are requisite to its operation to the extent that businessmen will not react immediately to a past increase in output if they do not consider it permanent or indicative of further increases in the future. Where large investments in fixed plant are concerned, the entrepreneurs may be very slow to react. In the case of the United States steel industry which in the post-war boom still had memories of long dismal stretches of redundant capacity, a good deal of business and governmental pressure was required to get more furnaces built despite the fact that the industry was said to be running at over one hundred per cent of rated capacity, and had sufficient orders to keep it fully

occupied for some years to come.

This qualification, however, should not be pushed too far. The steel industry did respond eventually to increased demand; and, as has been pointed out in the preceding chapter, technical necessities will in time correct excesses of optimism and pessimism. This is true even in the case of the single increase in the rate of output -- a case which allows the greatest scope for the free play of expectations. A businessman may at first be very suspicious of a new higher rate of output and may permit his production to exceed what he considers to be his rated capacity, but in time an "uncomfortable" rate of output will convince all but the most lugubrious prognosticator of doom that additional investment is warranted.

(5) The restraint on the downswing

Repeated references have been made in the literature[7] to the proposition that the operation of the accelerator is asymmetrical over the business cycle because of a restraint which operates during the downswing. When output is increasing, it is implied (quite incorrectly -- see the following limitation) that entrepreneurs can invest as rapidly as they choose, but when output is decreasing, it is said (quite correctly) that the entrepreneurs cannot disinvest faster than the rate of depreciation multiplied by the amount of capital in existence. In the legendary shoe industry, a stock of capital of one hundred machines and a depreciation rate of ten per cent would mean that disinvestment could take place at the rate of

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[7] E.g. Tinbergen, J., "Statistical Evidence on the Acceleration Principle", Economica, Vol. V, N.S., May 1938, p. 165.

ten machines per year, and also that in each year output could be cut by ten per cent.[8] A decrease in output of ten per cent or less could therefore be "explained" by the acceleration principle in the sense that the accelerator coefficient (the capital-to-output ratio) multiplied by the decrements of output give the appropriate amount of disinvestment which should be carried out. However, a decrease in output greater than ten per cent could not lead to any more disinvestment, and one would therefore have to assume either that the value of the coefficient is altered or (because there will be excess capacity) that the principle is inapplicable.

A consideration of the restraint operating during the downswing is complicated by the fact that we cannot safely assume that capacity will be reduced by the stock of capital multiplied by the average rate of depreciation. The wearing out of a strategic one per cent of the machinery in some plants might reduce the capacity of the whole plant to zero. On the other hand, and as is more likely, a plant which is adequately maintained may have no reduction in capacity for many years despite a high average rate of depreciation. The failure of capacity to decline will be particularly likely during the early stages of the depression when the plant constructed during the previous boom is still relatively new. Immediately after the downturn, therefore, the restraint on the operation of the acceleration principle is likely to be particularly strong.

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[8] The ten percent in this case must be based on the maximum possible output with the full employment of capacity, which might be larger than the maximum actual output during the boom.

(6) The restraint on the upswing

Despite the restraint which is effective during the downswing, the operation of the acceleration principle is more symmetrical than it is often said to be. Where the acceleration principle can validly be applied, that is, where there is no excess capacity, it must also operate under a restraint during the upswing. If one were to assume (unrealistically) that the upswing followed a depression which had worn out all excess capacity, then the restraint on the upswing would be severe indeed. With no excess capacity available for the production of either consumption or investment goods, prolonged rapid expansion would be impossible. Even if we assumed (more realistically) that at the beginning of the upswing there was excess capacity in the investment goods industries sufficient to provide for a net capital formation of ten per cent of the national income, and if we further assumed a capital-to-output ratio of three-to-one, the restraint operating on the upswing would not permit an increase in the national income of more than three and one-third per cent per year -- still a very severe restraint despite the admission of excess capacity into the argument.

(7) Output relevant for acceleration analysis

In the first chapter of this study, the operation and meaning of the acceleration principle were illustrated by reference to that indispensible tool of economic analysis, the shoe industry. The accelerator applied to the single firm is the concept which was earlier referred to as a micro-economic accelerator. Where its premises hold, it is very easy to conceive of it as providing the

explanation of the investment decisions of the firm. However, the move from a one-firm model to a multi-industry model, and the corresponding move from a micro-economic to a macro-economic conception of the accelerator have opened a trap into which a number of important writers have fallen. To determine the amount of induced investment in the shoe-industry model, the acceleration coefficient was multiplied by the increase in output experienced by the industry. Using macro-models, Professors Hicks[9], Alexander[10], and Schelling[11], to mention only a few, have uncritically applied the accelerator coefficient to increases in aggregate output. The transition from the model of the firm to that of the economy can be made quite safely if the characteristics of the former are preserved in the latter. In the shoe industry there was only one kind of investment -- induced investment; and there was only one reason for undertaking this investment -- past increases in output. For some purposes a "shoe-industry" model of the economy may be quite useful and adequate, and in such cases the application of the acceleration principle will present no additional problems to those already suggested. When, however, it is assumed that there is any kind of productive (i.e. capacity-increasing) investment other than the induced variety, an important modification must be made.

Let us proceed to the modification by way of an example.

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9 Hicks, J.R., op. cit., passim.

10 Alexander, S.S., "The Accelerator as a Generator of Steady Growth", The Quarterly Journal of Economics, Vol. LXIII, No. 2, May 1949, passim.

11 Schelling, T.C., "Capital Growth and Equilibrium", The American Economic Review, Vol. XXXVII, No. 5, December 1947, passim.

In the beginning, following literary, rather than economic, tradition, let us suppose that there is but one industry which is concerned with the production of apples. This industry shall be called "passive" since investment in it will take place only in response to past increases in output. To this peaceful and well-ordered garden economy comes a serpent called "progress" in the person of a New Man intent on establishing a new industry — the manufacture of automobiles. This new industry is "active" since its entrepreneurs do not have, and do not need, the spur of past increases of output, but are actively engaged in creating new investment opportunities. We shall assume that the fruit from the tree of knowledge, i.e. technological advance, is introduced slowly and steadily into the industry. The first automobile is an inefficient and expensive machine and, in consequence, has a very limited market, but by virtue of the steady stream of product-improving and factor-saving innovations, the market is gradually expanded.

Although, when one looks at the output statistics at any point of time, there will be a record of past increases in output in the automobile industry, the acceleration principle cannot explain or predict the investment which is taking place. Expansion, we have assumed, has been brought about by a series of innovations. Each innovation leads the entrepreneur to make a prediction about increased sales and causes him to make a corresponding amount of investment. After the investment has been made and the output expanded (assuming his expectations were correct), it should be apparent to him that the increased output is attributable to a previous innovation. The increased output, by fulfilling the businessman's expectations, may create a

warm glow of satisfaction which will doubtless have a stimulating effect on future investment decisions, but the entrepreneur will not look upon these anticipated increases in output as a reason for further expanding his capacity. If, on the other hand, this same increase in output accrued to a passive industry, it would not constitute the fulfillment of an old expectation, but would be the occasion for the creation of a new expectation which would then lead to induced investment. To imagine that all increases in output lead to induced investment would be to indulge in a sort of double counting as far as the active firms are concerned. The active firm visualizes an increase in output when it innovates; this same increase in output could hardly lead the businessman to repeat his investment. Looked at from a little different point of view, the passive firm, without the extra capacity to enable it to handle the increase in output comfortably, is induced to invest by an increase in sales. The active firm, in effect, creates the excess capacity to begin with, in anticipation of increased output; the anticipated increases in output merely "soak up" this excess capacity.

After this belaboured recital of the reasons why the accelerator does not account for the investment of the active industry, let us look at the passive industry to see what effect the establishment of the new industry has on its investment and output. While investment is being made by the new firms, but before the new products are being offered for sale, there could hardly help but be an increase in output accruing to the passive industries, and, if we assume no excess capacity, this increase will lead to induced investment. What happens to the passive sector after the new products begin to come

onto the market depends on whether the new products are substitutes or "complements" for the old, and on the extent to which they compete indirectly as alternative ways of spending money. It is conceivable that the advance in the boom might be confined almost entirely to the new industries, and it might even happen that the newcomers would expand at the expense of the established industries. Investment in an automobile industry might initially provide the basis for a small boom in the carriage trade, but it would do nothing for the sale of carriages after automobiles began to make their appearance on the road.

From a study of the two-industry model it becomes apparent that after the second industry has been established and has been in operation for some time, aggregate output will have increased. It is equally apparent, however, that not all this increase in output will lead to induced investment; indeed, as we have described the investment-decision process for the industries, only that part of the increase in output which accrues to the passive firms will be relevant to the acceleration process, and only increases which accrue to such firms will lead to induced investment. In terms of our apple-automobile example, the total increase in output may be experienced by both industries, but only that increase in output accruing to apple growers will lead to induced investment.

Now we have reached the place in the argument where a statement can be made of the seventh restriction which should be applied to the operation of the acceleration principle: where there are increases in productive capacity which are caused by other factors in addition to the accelerator, the acceleration coefficient should

be applied only to relevant, and not to aggregate, increases in output. The acceleration principle may be a good explanation of investment by passive firms, but not all firms which experience the increase in output in the economy are passive. The acceleration coefficient, therefore, should be applied only to those changes in output which are actually experienced by passive firms and, of course, by only those passive firms which have no excess capacity.

In order to clarify an important and rather complicated proposition we have engaged in some over-simplification. In the first place, the active sector of the economy has been associated exclusively with innovation. In the second place, it has been stated unequivocally that the acceleration coefficient should not be applied to aggregate output. These statements must now be qualified.

It is probably a good first approximation to say that the active sector of the economy, to which the accelerator <u>does not</u> apply, contains the industries which are innovating, whereas the passive sector, to which the accelerator <u>does</u> apply, contains the industries which are not innovating. Actually it is a little more complicated than this. The essential element which determines whether or not the acceleration principle is applicable is a subtle one, and hinges on the question of which comes first, the increase in capacity or the increase in output. It is customary, though not absolutely necessary, for an innovation to result in an increase in capacity. The introduction of a new firm or a new product suggests an investment which increases capacity, but under certain conditions, an innovation in process might be put into operation without effecting an increase in output potential. It is probably safe to assume that such investment

is not the rule, particularly if it is of any size, since its occurrence would mean that the investing entrepreneur is planning to market a better or cheaper product without preparing to supply a larger market. However, where such investment does occur, it may be treated as unproductive in the very special sense that it is not output increasing. An industry making such an investment should still be considered as a passive industry for our purpose. The unproductive investment indicates that the entrepreneur behaves passively with respect to his output and increases capacity only in response to changes in demand. The increases in output which are experienced by these entrepreneurs will be relevant for determining induced investment; and, it should be pointed out, if all innovations were of this non-productive type, all increases in output would be relevant and the acceleration coefficient could be applied to aggregate changes in output.

While it is true that industries which are innovating may behave passively, it is also true that industries which are not innovating according to the usual meaning of the term may, nevertheless, be active in so far as they build capacity ahead of need and without the stimulus of past increases in output. As an example we might cite the aggressive competition of a chain store which invades a new market area. The building of a new store in an area which is not served by existing stores of that firm must be based on the hope that business can be drawn from competitors, and could not itself be induced by past increases in output. Once the store is constructed, the increase in its output from zero to its rated capacity cannot induce more investment.

Instead of associating active industries with innovation, we would be more accurate if we classed in the active sector of the economy all industries which build rated capacity ahead of need or ahead of that which would be warranted by past increases in output. Nevertheless, it remains a good first approximation to link active industries with innovation, and the approximation can be made more accurate if we add aggressive competition to innovation. In industries which are aggressively competing or are innovating, the acceleration principle is likely to have little relevance, and cannot be relied upon to predict investment decisions accurately. The output experienced by these industries cannot, therefore, be multiplied by a coefficient of acceleration in order to determine the amount of induced investment as is done when the coefficient is multiplied by aggregate increases in output.

The second simplification is that the acceleration coefficient cannot be applied to aggregate changes in output. If the appropriate assumptions are made, this is not necessarily so. Even if the acceleration principle is totally inapplicable to the active sector, the acceleration coefficient might be applied to aggregate changes in output so long as it is assumed that the ratio between output changes in the active and passive sectors remains constant, and, of course, that an adjustment is made in the value of the coefficient. In the real world these are conditions which are unlikely to be realized. At first the boom is likely to be concentrated in the passive industries, and these are likely to have a good deal of excess capacity, but as the new products and the products of the new firms are offered on the market, the expansion in output is likely

to favour the active sector. To apply the acceleration coefficient to aggregate changes in output is to remove the coefficient from the role of a parameter and to treat it as another variable which will have to alter in such a way as to take account of the changing ratio between the relevant and irrelevant increases in output.

(8) Supply of cooperating factors

The acceleration principle has limited applicability in a situation in which the supply of cooperating factors is somewhat less than perfectly elastic. This limitation is of particular relevance when we consider the upper turning point of the business cycle; but before dealing with the turning point we must begin by clarifying a rather fundamental point. Several times throughout this essay it has been stated that the accelerator or the acceleration principle has "explained" or "determined" the amount of induced investment. It would be useful to set forth mathematically and diagramatically the exact meaning of this proposition. The mathematical model which we shall use is given by the difference equation $Y_t = aY_{t-1} + b(Y_{t-1} - Y_{t-2})$ which will be recognized as the familiar interaction between the multiplier and accelerator.[12] Y is the national income at the time period designated by the subscript; a is the propensity to consume, and b is the acceleration coefficient. We shall follow Professor S.S. Alexander[13] in assigning to the

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[12] In introducing the interaction model, we shift from a consideration of investment to a consideration of output. The reason for this change is that cooperating factors are relatively mobile and can more easily be considered as a restraint on total output rather than a restraint on a component of output taken by itself.

[13] Alexander, S.S., op. cit., p. 177.

parameters values which work out rather well; we shall set a at .95, and b at 2.1. Lest we be accused of falling into the same trap in which we thought we recognized several prominent writers, we shall hasten to add that for the moment it is a shoe-industry model of the economy which is being considered. In such a model, induced investment is the only kind of investment, so there is nothing to prevent the application of the accelerator coefficient to changes in aggregate output.

Since the difference equation has two lags, its general solution will have two components and may be written: $Y_t = K_1 m_1^t + K_2 m_2^t$. Professor Alexander has called the m's "growth factors". The K's are constants which are determined only by the values of the variables which start the series. The solution to this particular difference equation is $Y_t = K_1(1.05)^t + K_2(2.00)^t$. Since the difference equation has three variables, two must be specified in order to make it determinate. If $Y_{t-2} = 100$ and $Y_{t-1} = 105$, then Y_t will be "determined" or "explained", and must be 110.25; each succeeding Y will grow at the rate of 5 per cent per period. What was done, in effect, was to choose values of Y so that $K_2 = 0$. The resulting 5 per cent growth, however, is unstable. The slightest displacement would give K_2 a value other than zero, and the larger growth factor would soon dominate the smaller. (The existence of the higher growth factor can readily be seen from the fact that the difference equation is also satisfied by the series 100, 200, 400 etc.; given these values $K_1 = 0$.) If, when $Y_{t-2} = 100$, Y_{t-1} had been given a value of 106 (instead of 105), then Y_t would be 113.30 and the series would grow at first at a rate close to 5 per cent per period; but with succeeding periods, it would

approach the dominant, 100 per cent, rate of growth. Y_{t+1} is less than 123, but Y_{t+8} is about 1200 and Y_{t+9} is over 2200.

As long as we confine ourselves to the realm of mathematics and are not concerned with "physical truths", the mathematical model is sovereign and is perfectly free to come up with any answer it chooses. But when the model is moved conceptually from the discipline of mathematics to the obviously more disciplined discipline of economics, such wanton behaviour cannot be allowed; the model must be disciplined by a restraint.

The problem of a restraint is explicitly dealt with by Professor Hicks in his model of the cycle when he introduces into his discussion the concept of a "ceiling", which represents the inability of the economy to produce an explosive quantity of goods and services. A restraint receives implicit recognition from Professor Alexander when he notes that the explosive "free" path of the difference equation cannot go unchecked for long. Although both authors are aware of a restraint at full employment,they do not acknowledge the operation of any restraint during the upswing. A free upswing might be physically possible if there was excess capacity, but then the acceleration analysis would be inappropriate. Where there is no excess capacity, there will be a limit to the speed with which the upswing can take place.[14] Even though the upswing may be restrained, it remains true that the restraint imposed by full employment will be more serious than the restraint on the upswing; that is, expansion in which there are no unemployed factors to draw on will be more difficult than

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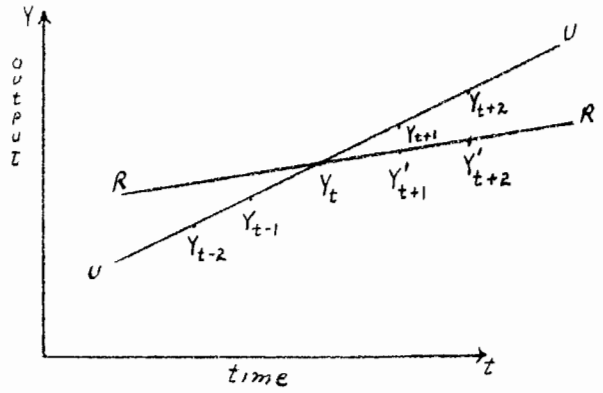
[14] Supra., p.64.

expansion in the presence of unemployed factors.

Let us ignore, for the moment, the restraint on the upswing and assume that at time period t the expansion path of the model encounters the restraint imposed by full employment. Diagramatically, this restraint is represented by the line RR which, we shall say, has a slope of five per cent per period, signifying that at full employment it is not possible for real output to increase faster than five per cent per period. The free path of the unrestrained difference equation is traced by the line UU. Y_{t-2} and Y_{t-1} are initial conditions which must be specified in order to determine the value of Y_t. Once Y_t has been found, Y_{t-1} and Y_t then determine the value of Y_{t+1}. However, in this case, the value of Y_{t+1} "determined" by the difference equation lies in the impossible area above full employment, and the largest value which Y can have at time t+1 is Y'_{t+1}. Once Y'_{t-1} is determined, there are, in effect, two new initial conditions, and the value of Y at time t+2 will be determined by Y_t and Y'_{t+1} and not Y_t and Y_{t+1}. If Y_{t+2} (the value determined by the difference equation) lies above Y'_{t+2} (the maximum value permitted by the restriction), the answer given by the difference equation will be overruled by the answer given by the restraint. If Y_{t+2} lies below Y'_{t-2} the restraint is no longer operative, and the difference equation again determines the value of Y (unless, of course, another restraint becomes effective during the downswing). There will be an <u>effective</u> restraint whenever the accelerator determines a volume of



output greater than the maximum value permitted by the restriction, and so long as there is an effective restraint, the actual level of output will be determined by the restraint (supply difficulties) and not by the free operation of the acceleration principle.

A restriction which overrules the operation of the difference equation will be effective whether it affects capacity or resources. There is an additional condition attached to the capacity restraint however. When this restraint is not effective, the implication is that there must be an excess supply of plant and equipment and therefore that the acceleration principle cannot apply. This is as we would suspect. The accelerator thrives on shortages of plant and equipment, and indeed cannot exist if there is a surplus. This is not true, however, of shortages of cooperating factors. A scarcity of men and material can be an effective restraint on the operation of the accelerator, but an excess supply of these factors will not make the accelerator inapplicable.

(9) Constancy of the coefficient

The final problem arising from a consideration of the acceleration principle is whether the coefficient should be treated as a constant or a variable. The significance of this problem is stressed by A.D. Knox when he states, "The validity of the theory of investment depends upon whether we can really assume the accelerator to be constant."[15] And again, "The crucial problem of the acceleration principle is whether the accelerator is constant."[16] Professor

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[15] Knox, A.D., "The Acceleration Principle, And The Theory of Investment: A Survey", Economica, N.S. Vol. XIX, No. 75, August 1952, p. 272.

[16] Ibid., p. 273.

Kuznets argues[17] that the assumption that the accelerator coefficient is a constant is basic to J.M. Clark's theory and that it is this assumption which makes the acceleration principle important. There is a parallel in monetary theory. The quantity theory of money is said to have fallen into disrepute because doubt was cast on the reliability of the constancy of v. The acceleration principle is in very much the same position. The proposition that induced investment equals the acceleration coefficient, b, times the increment of output has an advantage over the statement that induced investment is a function of innovation, prices, profits, expectations et cetera, only if it can be assumed that b will remain relatively constant. If we must admit that b is a function of innumerable variables, the acceleration principle must go the way of the quantity theory. It is perhaps not too much to say that the constancy of the accelerator is of crucial importance to its usefulness.

The constancy of the acceleration coefficient has been criticized by those who stress the subjectivity of the principle and by those who include in the principle all factors which might be relevant to aggregate investment. H.M. Somers for example has written that "they [technology et cetera] can be considered as factors affecting the magnitude of the Accelerator".[18] On the other hand, the constancy of the acceleration coefficient seems to be supported by those who have stressed the objectivity of the principle.

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[17] Kuznets, Simon, op. cit., p. 213.

[18] Somers, H.M., Public Finance and National Income, The Blakiston Co., Philadelphia, 1949, p. 73.

Elsewhere in this paper[19] we have attempted to show that the value of the coefficient during a continuing expansion on the part of a firm is likely to be very closely tied to the value of the capital-to-output ratio. Furthermore it was suggested that the marginal capital-to-output ratio was likely to be very nearly the same as the average ratio unless the firm was innovating, and in that case the accelerator probably would not apply in the first place. This argument gives some reasonably solid ground for expecting the value of the coefficient to be constant. However some difficulties are introduced when we move from the micro-accelerator which is applicable during an upswing to a macro-accelerator which covers a whole cycle. Let us consider some of the complications introduced by aggregative analysis.

In the first place, in applying the accelerator to the economy as in applying it to the firm, we cannot entirely ignore expectations. In the long run there will be pressure on the businessmen to make their acceleration coefficients, averaged over a number of years, equal to the capital-to-output ratios, but this does not prevent errors of optimism and pessimism from causing entrepreneurs to build too much plant (reflecting a relatively high value of the accelerator coefficient) during the early part of the boom and correspondingly less plant (reflecting a low value of the coefficient) during the later part of the boom. In the second place, the acceleration analysis does not become operative until plants run out of excess capacity, and they are not likely to do this all at once. This factor does not actually affect the value of the coefficient provided that it is applied only to incremental

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[19] Supra., p.

outputs of passive firms without excess capacity, but it does suggest that the increments of output which are relevant to acceleration analysis will grow during the upswing, and, similarly, that the total amount of induced investment will also become more important as the upswing progresses. Third, the fact that the aggregative coefficient is influenced by many individual capital-to-output ratios introduces the possibility that a shift in the relative importance of the industries which comprise the total will affect the value of the aggregate acceleration coefficient. Fourth, A.S. Manne[20] has argued that the Ricardo effect -- the discrimination against factors which have become relatively expensive -- which is so strongly emphasized by Professor Hayek implies that the value of the capital-to-output ratio is likely to drop during the later stages of the cycle. That relative price movements will put pressure on the entrepreneurs to change their production functions may be granted, but the effect that this change will have on the capital-to-output ratio and on the acceleration coefficient is by no means certain. Finally, the operation of the restraints will affect both the aggregate, and the individual firm, acceleration coefficients. This point has been recognized by R.M. Goodwin[21] in the model of the cycle in which he gives the acceleration coefficients two values, one applicable to the middle range of the cyclical swings, and another which applies at the extremes. Professor Somers also makes a distinction between

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[20] Manne, A.S., "Some Notes On the Acceleration Principle", Review of Economic Statistics, Vol. XXVII, No. 2, May 1945, p. 97.

[21] Goodwin, R.M., "The Nonlinear Accelerator and the Persistence of Business Cycles", Econometrica, Vol. 19, No. 1, January 1951, p. 5 ff.

the value of the accelerator at full, and less than full, employment when he writes "$\bar{A}$ is the technical value for A [the acceleration coefficient] appropriate to a level of full employment."[22] It seems fairly clear that the resource restraint must affect the value of the coefficient. Regardless of past increases in output, entrepreneurs will not buy additional equipment if they cannot obtain credit or hire the requisite men and materials.

Of these five factors, the third and fourth are probably not too important, and the second affects the induced, or accelerator, component of investment though not the value of the coefficient itself, if it is used as we suggest. The remaining factors -- the effect of expectations and the effect of full-employment on the value of the accelerator coefficient -- are not as easily dismissed. Where there is a long continuous expansion, one would almost expect that the "technical necessities" (or more specifically, the appearance of excess capacity if he guesses high, and the increasing pressure on existing capacity if he guesses low) would make the entrepreneur trim his expectations to fit the facts. While it might be argued that expectations will not alter the value of the coefficient to any significant degree, the same cannot be said for the resource restraint which appears at full employment. Although it is not inevitable, the resource restraint does suggest a downward revision of the coefficient.

It is my opinion that where the restrictive assumptions of the accelerator are valid, it is a reasonable approximation to treat the coefficient as a constant, at least during the upswing. The

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[22] Somers, H.M., op. cit., p. 82n.

difficulty of acceleration analysis lies not in a shifting value of the acceleration coefficient, but rather in restricting the application of the coefficient to only relevant increments of output.

CHAPTER V

THE ACCELERATOR AND THE CYCLE

Introduction

It is not the purpose of this chapter to present a complete account of the business cycle, for we are interested in only one small aspect of business-cycle analysis -- the operation of the acceleration principle throughout the various phases of the cycle. For the most part the presentation will be verbal, but occasionally it will be convenient to follow the example of Hicks, Samuelson, Alexander, Schelling and others and use difference equations to describe the behaviour of certain variables. Some of the equations, we shall find, are convenient vehicles for illustrating the operation and limitations of the accelerator.

The model

To begin with, let us assume that it is possible to divide industries into two categories, active and passive. The active industries are those to which the acceleration analysis does not apply; conversely, the passive industries include those to which it does apply. In other words, the active industries are those which build capacity which is not directly related to past increases in output. In this category are new firms, old firms making new products, many of the aggressively competitive firms, firms with very long-run projects which are expected to pay for themselves only in the long run, and probably most of the innovating firms. The passive firms are categorized by their passive attitude towards their size of plant. Although such firms may innovate, they do so in such a way as to leave their capacity unchanged.

This classification of industries suggests that output of any period, t, could be broken down into four components: consumption, induced investment, autonomous investment, and unproductive investment. The consumption component, C_t, is not directly relevant to the problem at hand, so we can dismiss it by saying that it might be a function of last period's income, Y_{t-1}; thus $C_t = aY_{t-1}$, where a is the propensity to consume. If there are objections to this consumption function (as undoubtedly there are), any other could be substituted equally well. The induced investment component, I_t, is a function of past increments of output so that $I_t = b(y_{t-1} - y_{t-2})$; b is the acceleration coefficient and $(y_{t-1} - y_{t-2})$ is the sum of all increments of output experienced by passive firms without excess capacity. In this expression investment is lagged by only two income periods, but longer lags could be introduced if desired. If all firms were passive in our sense, y_t would, in the absence of excess capacity, equal total output, Y_t, but usually there are other reasons for productive investment besides past increases in output, and we should normally expect y_t to be only a fraction of Y_t. The third component, which is autonomous investment, A_t, includes all investment which increases rated capacity but which is not induced by past increases in output. The final component, which we have labelled unproductive investment and which we shall designate by the symbol U_t, is unproductive only in the sense that it does not directly increase capacity. It is beyond the scope of this study to explain the behaviour of the components A_t and U_t; all that we need to make explicit about them is that they are not simple functions of past increments of output and that A_t increases capacity while U_t does not. Suffice it to say

that these components result mainly from innovation and aggressive competition and that they will be much influenced by complementarity of capital, strategic innovations, linked advances et cetera. The output components can now be drawn together in the equation

$$Y_t = aY_{t-1} + b(y_{t-1} - y_{t-2}) + A_t + U_t.$$

With the important variables thus specified and defined, let us look at the operation of the accelerator over the cycle.

The Upturn

The question which we would like to answer about the upturn is not so much what causes it, as what part, if any, the acceleration principle plays in bringing it about. On the face of it, one would hardly expect the accelerator to have anything to do with the first increase in output which marks the upturn since according to our definitions induced investment follows, rather than precedes, an increase in output; induced investment is the result and not the cause of increased output. However, the acceleration principle must not be dismissed so rapidly. The acceleration coefficient operates not on the increments of total output, but on the increments of output experienced by passive firms without excess capacity, and it is possible that the latter increments may be positive and that some firms may thus be induced to invest even before aggregate output has increased. Algebraically, we are saying that the expression $(y_{t-1} - y_{t-2})$ may be positive when $(Y_{t-1} - Y_{t-2})$ is zero or even negative. However, although it is theoretically possible that induced investment may have some part to play in the upturn, it is not likely to be an important role because of the existence of wide-

spread excess capacity.

The existence of this excess capacity at the time of the upturn could undoubtedly be verified by looking up the relevant statistics. However, on the theory which is in the best of British traditions that it is easier to think something up than to look it up, we might content ourselves with advancing two theoretical reasons for believing that the upturn must occur in the face of redundant capacity. In the first place, the slump would have to be unrealistically long in order for it to work off all excess stocks of capital. The length of time required is determined by the actual (rather than the bookkeeping) rate of depreciation. It is not likely that the effective average rate of depreciation will be in excess of four per cent per year. Using straight-line depreciation, this would mean an average life of plant and equipment of only twenty-five years, and even with such short-lived equipment, the depression would have to drag on for over eight years with zero gross investment in order to reduce the output potential of the economy by just one-third. Indeed, this eight-year estimate may be too low, for there are two further obstacles to the decline of productive capacity during the early years of the depression. First, even if the capital is subject to an average depreciation charge of four per cent each year, the actual physical depreciation, i.e., the decline in capacity, may be negligible for a number of years. Capacity may be particularly slow in withering away during the early downswing since a good deal of the equipment will be new. Second, we have been assuming straight-line depreciation which is a function of time, but much depreciation will be related more directly to use. If the downswing is rapid during the first few

years, excess capacity will develop, and the effective rate of depreciation on this under-employed capital may be considerably reduced. The depression should be especially good for the longevity of the equipment of the investment-goods industries which, with zero gross investment in the economy, should be completely idle and hence should have a very low rate of physical depreciation.

The second reason for believing that there must be excess capacity at the time of the upturn is that in the real world the upswing takes place faster than the restraint which would be imposed if there were no unused plant would permit. The magnitude of this restraint is found by dividing the capacity of the investment-goods industries in excess of what is required for replacement, by the capital-to-output ratio. If investment-goods industries really had no excess capacity, which with zero gross investment would mean that they had no capacity at all, the numerator (capacity in excess of what is required for replacement) would be negative. Even after entrepreneurs decided to make net investments, output and capacity would continue to decline until this term could be made positive. A numerical example may assist in clarifying the point. Let us assume that the investment-goods industries must have capacity capable of producing $100,000 worth of investment goods each period in order to maintain the economy's capital intact. If, because of a long, severe depression, the capacity output of the investment-goods industries drops to $70,000 per period, total capital will decline at the rate of $30,000 ($100,000 - $70,000) per period, and total output of both investment and consumption goods industries will decline by $30,000 divided by the capital-to-output ratio. If the latter has

a numerical value of, say, 3, total output will decline by $10,000 per period. Even after entrepreneurs decide that they want to increase aggregate output they cannot do so, for output will continue to decline until such time as the capacity of the investment goods industry can be increased to $100,000 per period.

In view of the evidence, I think it must be granted that the upswing is likely to occur in the presence of wide-spread excess capacity, and therefore that induced investment will not be an important factor in bringing it about. Although it cannot be assumed that induced investment is incapable of playing any part whatsoever, we must conclude that the major explanation of the upturn is to be found in the behaviour of autonomous and unproductive investment.

The upswing

Not only is induced investment unlikely to have a large part to play in the upturn because of excess capacity, but it is also unlikely to be much more important during the early part of the upswing, and for the same reason. During the time that autonomous and unproductive investments are being made, but before the active industries begin to offer their products on the market, the passive industries will undoubtedly experience an increase in demand. When the products of the active industries begin to arrive on the market, however, they will affect the demand for the old products directly and indirectly. Directly, the new products may be substitutes for the old products -- nylon may displace cotton; indirectly, they will compete for purchasing power -- television sets may make diamonds harder to sell. If we assume that this direct and indirect substitution is not complete, and that the output of the passive industries

continues to expand, there may come a time when a significant number of the passive industries feel pressed to enlarge their productive capacity, and it is at this time that the induced component of investment becomes important. The exact time that induced investment becomes significant depends primarily on when the passive industries exhaust their capacity, and this, in turn, depends on such things as the productive capacity attained in the previous boom, subsequent gross investment following the downturn, and the rate of depreciation. The manner in which investment in the early part of the boom was divided between the autonomous and unproductive components makes a good deal of difference to the length of time which elapses before the appearance of induced investment. Both components have, dollar for dollar, the same income-generating force, and hence both speed the return of a significant induced-investment component; but by definition the autonomous investment also increases capacity (after an appropriate lag, of course), which works to the detriment of the passive industries and their induced investment. The mere existence of this autonomous and unproductive investment after the upturn will not be a sufficient condition to bring about the removal of excess capacity. If output is to be increased and the acceleration principle is to be brought into play, each period's investment must be larger than that of the preceding period since the multiplier operates not on investment, but on the increment of investment. The "soaking up" of excess capacity will depend not only on the amount of investment but also on its growth.

With zero, but not negative, net investment during the previous depression, there will be no incentive for the passive firm

to undertake induced investment until the "high water mark" of the last boom is reached. If there has been negative net investment, then induced investment will begin somewhat earlier. It is quite possible (especially if population is constant) that the output of the passive industries never exceeds or presses on capacity, and this would mean that the amount by which the output of the present boom exceeds that of the preceding one has all accrued to the industries which comprise the active sector. While this stagnation of the passive sector is possible, and while it is even likely that quite a number of passive industries will not take part in the expansion of the boom, we shall assume that after the upswing has been in progress for some time, excess capacity is exhausted in a number of passive industries, and that induced investment becomes a significant component of total output.

The proposition that induced investment does not become important until the upswing has been in progress for some time has two consequences. In an earlier chapter it was argued that during a long, continuous expansion there would be pressure on the entrepreneur to equate his acceleration coefficient to his capital-to-output ratio; a higher coefficient would mean the appearance, after a lag, of excess capacity, and a lower coefficient would mean a continuing and increasing pressure on productive capacity. In a long-continuing expansion, the difficulty caused by expectations would be at a minimum (remembering that we are speaking only of induced investment by passive industries), and without doing too much violence to reality, we could assume that entrepreneurs are governed directly by the technical necessities which, like a well-

ordered politbureau, would permit very little "expectational deviationism" from the party line laid down by the acceleration principle. However, if the induced component of investment is not going to join the uprising until it is well under way, there is going to be less time to indoctrinate the entrepreneurs and hence more room for optimistic or pessimistic deviationism which will have to be purged by a depression.

The second consequence of the delayed participation of induced investment in the upswing is that there will be less opportunity for maladjustment to develop in the investment goods industries. It has been argued[1] that a rapidly expanding industry will make great demands on the relevant investment-goods industries, and that given time, a freely-operating acceleration principle will cause an over expansion of the investment-goods industries. This over expansion is not something which can develop in a short time, and anything which operates to reduce the length of the relatively free expansion path of the output of passive industries should, one would think, reduce the possibility of a maladjustment caused by the too-rapid expansion of the induced investment component.

In the discussion of the qualifications of the acceleration principle[2] it was maintained that its operation was not free even during the upswing. Without excess capacity, the growth of output would be restrained by the growth in capacity. It remains to be seen where this restraint fits into the present narrative.

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[1] Haberler, G., Prosperity and Depression, New York, United Nations, 1946, pp. 365-366.

[2] Supra., p.64.

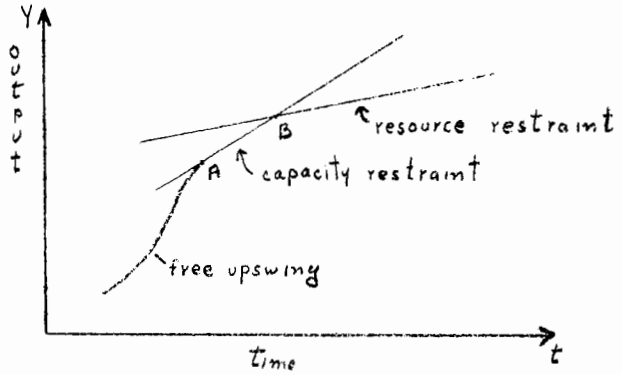
During the early stages of the upswing, businessmen will undertake autonomous and unproductive investment which, through the multiplier, will raise income and output. Because of the unemployed factors and the unused capacity, the increase in output at this stage could take place quite rapidly. The actual rate of increase of output, however, will be controlled by the rate of increase of investment, the size of the multiplier, and the length of the various lags. Although there is nothing to indicate that this initial upswing will take place rapidly, we might still call it "free" since it is operating without a restraint, and regardless of the actual output achieved, the physical plant and cooperating factors are available to enable large increases in output in the comparatively short run. During this free upswing the acceleration principle will have only a minor role to play in the making of investment decisions.

When the upswing has expanded far enough to soak up excess capacity, two new factors are introduced. First of all, the acceleration principle becomes operative on a more significant scale. Second, the growth of output is now subject to a capacity restraint. We shall assume that at this stage there is only a capacity restraint, that is, that there is still under-employment and hence a reasonably elastic supply of cooperating factors. A simplified expression for this capacity restraint would be as follows: the net capital-forming capacity of the investment-goods industries as a percentage of national income, all divided by the capital-to-output ratio. This expression gives the maximum percentage rate of growth of the national income in the absence of excess capacity.

Full employment and the downturn

The occurence of full employment in a strong boom has the effect of introducing an additional restraint into the model of the cycle. The restraint on the upswing was termed a "capacity restraint" since output could not be expanded without limit in the short run because the physical capacity was not available. As new plant was constructed, however, there was no difficulty in putting it into operation because there was assumed to be unemployment in the factor markets, and therefore men and materials could be readily hired to cooperate with the new equipment. If the expansion continued, there would eventually come a time when the cooperating factors would become fully employed. This brings us to the "full employment" restraint.

The expansion path of the economy through the upswing and full employment is illustrated in the accompanying diagram. During the free upswing the accelerator is inoperative. At time A it is assumed that the economy exhausts its excess capacity, and that therefore passive firms will begin to undertake induced investment subject to the capacity restraint. At B the excess supply of cooperating factors is exhausted, and the resource restraint becomes effective. The upper restraint is usually called full employment, implying that it is labour which is in inelastic supply. However, it could just as well be any other factor; it might, for example, be steel; and to be perfectly acurate we should allow for a number of restraints or "ceilings" as they are called by



Professor Hicks. However, labour is in a real sense the ultimate bottleneck, and it would simplify the argument if we assumed that labour was the only cooperating factor which might be in short supply.

It is apparent from the nature of the two restraints that their intersection will almost inevitably form a kink. The slope of the first is restricted not only by capacity but also by shortages of cooperating factors. The kink simply implies that expansion with unemployed factors is likely to be easier than expansion without them. This kink is of the utmost importance to the operation of the acceleration principle, for it indicates a slowing down in the rate of increase of output, and it is commonly held that this is a sufficient reason for a decrease in the amount of investment.

Returning to our legendary shoe industry, we recall that an increase in the sale of shoes from 10,000 to 11,000 a year -- a ten per cent growth -- means an increase in the purchase of shoe machines from ten to twenty per year. If output is then to be raised to 11,500 -- an increase of a little under five per cent -- there will be an absolute reduction in the number of machines ordered in the second year as compared with the first; ten machines will be needed for replacement, and five machines must be purchased in order to produce the extra 500 shoes. A decrease in the rate of increase of output causes an absolute fall in the rate of investment.

Ever since the Clark-Frisch controversy[3] most authors have been careful to qualify the conclusion that a decrease in the rate of increase of output causes an absolute fall in the rate of investment

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[3] Supra., pp. 52-55.

by admitting that the absolute fall in the rate of purchase of investment goods (gross investment) might be prevented by an increase in replacement demand. While the qualification is formally correct and important when thinking of a single industry, it is unlikely that the "Frisch qualification" would prevent a downturn in a model representing the whole economy. As we have argued, the gradual substitution of replacement investment for net investment cannot occur unless there is a convenient increase in the propensity to consume to offset the decreased rate of net investment. With a constant propensity to consume, a reduction in the amount of net investment would, via the multiplier, reduce income, demand and output. Once output began to fall, replacement demand itself would shrink.

We do not have to rely on the Frisch qualification (the increase of replacement demand) to prevent a downturn when the expansion path of the economy encounters the ceiling. It can be shown mathematically that under certain conditions, even when all investment is of the induced variety, a decrease in the rate of increase of output is not a sufficient condition to bring about a downturn. Let us for the moment return to the shoe-industry model of the economy, in which induced investment is the only kind of productive investment. On page *73* such a model was represented mathematically by the equation $Y_t = .95Y_{t-1} + 2.1(Y_{t-1} - Y_{t-2})$. This equation had two growth factors, a minor one of five per cent and a dominant one of one hundred per cent. We noted that the five per cent rate of growth was unstable. If the initial conditions (Y_{t-2} and Y_{t-1}) are given particular values so that $Y_{t-1} = 1.05(Y_{t-2})$, the mathematical expansion will continue indefinitely at the rate of five

per cent per period. The slightest displacement from this path, however, will ultimately cause explosion or break-down, and it might therefore be argued that the five per cent rate of growth is without economic significance. However, the minor growth factor becomes of the utmost importance when we consider the ceiling. If the full-employment restraint will permit a growth of five per cent or more, the expansion path will continue to "creep along the ceiling"; the model will be completely stable in an upward direction and there will be no reason (as far as the mathematical model is concerned) for a downturn. However, if the slope of the full-employment restraint is less than the slope of the minor growth factor, then the model will be unstable downwards, and a downturn will be caused by virtue of the fact that the expansion path will have encountered a severe restraint.

The model is interesting, and the fact that the kink (the slowing down in the rate of increase of output) is not a sufficient cause for a downturn is very suggestive, but we cannot attach much importance to any economic conclusions arrived at by such a model. It would be asking a great deal of a single model to expect it to describe all phases and turning points of the cycle, and although this model might describe the upswing, it will not provide an explanation of the upper turning point since it assumes that at full employment the relationship between output and investment remains the same. At full employment any number of things may happen to upset this relationship; for the moment it is enough to point out that the supply of cooperating factors may become inelastic, and this is sufficient to alter the relationship implied by the acceleration

principle. Although it might be possible to construct a special mathematical model for the upper turning point, the argument would probably be easier to follow in economic, rather than mathematical, terms.

The encounter with a full-employment restraint implies a reduction in the rate of increase in output, and such a reduction, it is claimed (neglecting the Frisch qualification), is a sufficient cause for a downturn in investment. However, there is a major distinction to be drawn between a reduction in the rate of increase of output caused by a slackening of demand, and the same reduction caused by supply difficulties. A demand restraint, with its effect on output and investment through the acceleration principle, is clear and unambiguous; a supply restraint is not. To take a simple case, if the shoe industry wishes to increase its net purchase of machines from ten to twelve, but if, because of supply difficulties, the industry can get delivery of only nine additional machines, the output of shoes will be lower than it would have been if the order for new equipment had been completely filled. But such a reduction in the rate of increase of output would hardly be looked upon by the entrepreneurs as a reason for reducing the amount of investment in subsequent periods. Quite the reverse. The inability of the entrepreneur to have his investment orders filled would probably encourage him to place larger orders in the future.

The actual behaviour of investment plans after the full-employment restraint is reached will depend to a considerable extent on where the supply difficulties first appear. If the shortage first appears in the investment trades themselves, as we assumed when the

order for new machines was cut from twelve to nine, then, of course, firms cannot invest faster than they can get delivery of the plant and equipment that they order. At full employment, however, the capacity restraint is not the only one, and when shortages appear in the supply of cooperating factors, the investment decisions may behave quite differently, and will certainly be influenced by the supply of cooperating factors available. As a limiting case, a perfectly inelastic supply of labour in consumption industries could bring induced investment to an abrupt halt. If a firm is completely passive and has no technological changes of which it can take advantage, and hence can invest only by ordering more of the same kind of equipment, then once an absolute labour shortage appears it will not make further investment regardless of past increases in demand or expectations of future increases. Any additional equipment purchased would have to remain idle solely for want of workers to run it. A labour shortage would thus inevitably lead to a downturn if all investment were in this category, although it would be necessary to postulate in addition that labour was immobile between the consumption and investment trades, since this explanation of the downturn amounts to the paradoxical proposition that unemployment is caused by an excessive demand for labour.

The example chosen, in which there was a perfectly inelastic supply of labour and no innovation, was a limiting case. Induced investment by passive firms is not the only kind of investment; so let us examine in detail what happens to all three investment components at full employment. The induced-investment component will be the hardest hit by supply difficulties. The rate of induced

investment by passive firms will depend on the rate at which the cooperating factor (labour) can be made available to the expanding, passive industries. The supply of labour will depend on the growth of the labour force, the degree to which the unproductive and autonomous investment components are factor releasing, and the rate at which factors are released by declining industries. The active industries will be in much better position to protect themselves against the labour shortage. For the most part, they will be made up of innovating firms, and the innovation may be aimed, in effect, at getting a larger output per worker. If the innovation is very successful, these industries, despite the fact that they are by definition capacity increasing, may also be factor releasing. Furthermore, some of the active industries displaying great sang froid (undeterred by the sight of red in other firms' balance sheets) may be slowly strangling old firms and at the same time picking their pockets of unemployment. Even at full employment the expansion of synthetic textiles may be facilitated by the absorption of employees released from the production of natural fibres.

The unproductive category of investment is least likely to be affected by an encounter with the full employment restraint. Since this type of investment does not aim at increasing capacity, its objective will be either to reduce cost (release factors) or improve the product (which may or may not absorb additional cooperating factors). On balance, one might expect this investment component to be factor releasing, especially at full employment when there will be a strong bias operating in favour of the selection of factor-releasing innovations.

So far we have been inquiring into the behaviour of the investment components at full employment under the tacit assumption that there was no shift in the importance of the various investment components; we have assumed that the active industries remained active and that the passive firms remained passive. However, the restraint, which is so hard on the induced component of investment, might encourage autonomous and unproductive investment, both of which are less vulnerable to the supply difficulties caused by full employment. These supply difficulties, which weaken the accelerator, may cause the passive firms to become at least temporarily active. This is because the shortages may serve as a shock which will encourage innovation. The necessary incentive to change may come from the firm's concern with maintaining its share of the market or with keeping its customers happy. The previous depression may have so conditioned the entrepreneur that the mere thought of turning away business is distasteful. A bottleneck, therefore, may shake up the enterprise, may make the businessman look around for new sources of the scarce factor, and may encourage him to spend time and thought and money on research; it may even make him willing to listen to suggestions from the shop committee on how to overcome bottlenecks! Moreover the price of the bottleneck factor may change and thus alter the relative cost structures. This movement in relative prices may, in turn, make possible, or more attractive, a change in the proportions in which the factors of production are mixed.

To argue that full employment may <u>encourage</u> investment is so at variance with the views of most business-cycle theorists that a short digressive look at the conventional views might be in order.

Three reasons which have been given for believing that the boom discourages investment are: (1) price movements increase risks and make calculation and forecasting more difficult and hazardous; (2) entrepreneurs will expect that the boom will not last for long, and will concentrate on plans to increase output in the very short run in order to cash in on the high prices while they last; (3) costs go up during the boom, and although prices may keep pace, it is expected that investment during the boom will leave the firm saddled with high capital costs which will endanger its competitive position when prices recede.

Granted that all three factors might be important in special circumstances, there is some question about their general validity, and especially about their relevance to a boom in which there is an extensive period of full employment. The generality of the last two points is open to attack since both depend on the expectation of a downturn -- there is a downturn because businessmen expect one. Once the expectation of a decline in costs and prices is replaced by a widely-held expectation of a continuing "creeping inflation", there is no reason for price and cost increases themselves to discourage investment. As far as the first point is concerned, price movements (as opposed to price increases) may increase risks or they may not. Price movements will not increase risks if the entrepreneurs can predict, or think they can predict, the trend of relative prices. The continuation of investment during the boom will require two predictions: a favourable long-run prediction about the price-to-cost ratio i.e. about the long-run profitability of the enterprise, and a prediction about the behaviour of relative costs. For example, if

the businessman is convinced of the long-run profitability of his business and believes that labour costs will increase faster than other costs, he will be encouraged to substitute the services of other factors for those of labour. This will normally mean the investment in labour-saving machinery.

To refer to the very recent situation, which has obviously inspired much of what has gone before, there is evidence that some twenty years of increasing prices have convinced many businessmen that they can count on a gradually increasing price level. This is suggested by the very terms and metaphors in which wages and costs are discussed in the trade journals. The period is referred to as one of "creeping inflation". When cost increases are being reported, they are not "riding up the crest of a wave" but are rather "pushing the economy up to a new higher plateau of costs" thus suggesting the stability of price increases. And not only is it assumed that costs generally are going up, but when consideration is given to the political and economic power of trade union monopoly, it must seem like a fairly safe bet that labour costs will continue to lead the cost increases.

At this point there is a related argument which might be added "in defence of monopoly". Professor Haberler has argued[4] that monopolistic practices encourage depressions because they raise price and reduce output without releasing any compensating expansionary forces. Apart from the question of whether or not increased wages reduce output, there are two things to be said for trade union

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[4] See for example Haberler, G., op. cit., pp. 353, 372-373.

monopoly. In the first place the existence of labour organizations and their position in society mean that there will be sociological and political, as well as economic, considerations assisting the businessman to predict the direction of changes in labour vis-à-vis other costs (if the price of labour were determined by economic considerations alone, prediction of the direction of change of relative costs might be more difficult). In the second place, if labour is the ultimate bottleneck, and if it is the factor whose scarcity is likely to cause the downturn in investment, then it might be a good thing for unions to "shock" entrepreneurs with higher wages before unemployment disappears and labour becomes physically scarce. The wide-spread attempt by businessmen to escape from labour costs at a time of actual or approaching full employment is probably the best guarantee that we can have against a decline in investment. This sort of investment, which may be encouraged by full employment, is not accelerator induced. However, it is important to keep in mind that the inelasticity of supply, which, in itself, will tend to reduce the purely passive output-induced component of investment, will, under certain conditions, tend to increase other types of investment. The encouragement of autonomous and unproductive investment is directly relevant to our problem, because in so far as it results in the release of scarce factors it will increase the amount of induced investment that can be undertaken by the passive firms which, unaided by innovations, have no opportunity to do anything but purchase more of the same kind of equipment.

To recapitulate, if we assume the adequacy of demand, the behaviour of the induced component of investment at full employment,

that is, at the top of a strong boom, will be determined by the difficulties encountered in the supply of capacity and cooperating factors (labour). The capacity restraint will not likely cause an absolute decrease in the amount of investment (the economy is not likely to be able to undertake less investment at full employment), and even if the operation of this restraint does involve a slowing down in the rate of increase of output, it is not likely to have an adverse effect on future investment decisions. The restraint on the supply of cooperating factors will be more harmful to induced investment and may easily cause this component to shrink, regardless of past output or future expectations. Whether the induced investment diminishes or not will depend on (a) the size of this investment component and (b) the rate at which the scarce factor is made available. If the investment component is quite small in relation to the rate at which the scarce factor becomes available, it might be possible that the component would not decline at all at full employment. If the induced component of investment is relatively large the component will behave cyclically, and the encounter with the full employment restraint will produce a downturn of the component. But it is only a downturn of the component that necessarily follows, and the downturn of the component is not sufficient to insure a downturn of aggregate output. In this chapter it has been suggested that under certain conditions, not unlike those prevailing today, the resource restraint, which has the effect of diminishing induced investment, may stimulate other types of investment. These other types of investment not only may compensate for the drop in investment of passive industries but also may increase the supply of the scarce factor available so as to permit the continued growth

of the passive industries.

The downswing

A brief look at the downswing will complete our study of the role of the accelerator during the cycle. Because we have dealt with this phase of the problem before,[5] what we have to say here will be merely by way of review. In considering the limitations, we found that there was a severe restraint on the operation of the accelerator during the downswing. The amount of disinvestment that can be carried out in any period is limited by the rate of depreciation multiplied by the stock of capital; the reduction in output which is possible each period without running into excess capacity is a function of simply the rate of depreciation. If output is reduced each period by less than the rate of depreciation, the acceleration principle will still predict the amount of disinvestment (each period); if, however, the decrease in output is greater than the average rate of depreciation, disinvestment in each period is determined not by the acceleration principle, but rather by the depreciation rate and the stock of capital. This does not mean that in a rapid downswing the acceleration principle is completely inoperative; the principle will still determine the total amount of disinvestment that must take place before excess capacity is removed.

The relevance of the acceleration principle to the downswing is rather similar to its relevance in the case of the single increase of output. In the latter case we found that the principle could predict the amount of investment, but left in doubt the question of

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[5] Supra., p.62.

how the investment would be spread over the succeeding periods. With a rapid downswing the acceleration principle tells us the total amount of disinvestment that is warranted, but cannot itself tell us the amount of disinvestment that will be undertaken each period.

CHAPTER VI

APPLICATIONS OF THE ACCELERATION PRINCIPLE

Introduction

In the literature on the subject, the acceleration principle has been applied in three important ways. It has been used to explain (1) the relative fluctuations in consumption and investment trades, (2) growth, and more particularly, the conditions necessary for steady growth, (3) the business cycle, especially the downturn. The three uses are closely related, but nevertheless each seems to have its own literature. Since we have dealt at some length with the problem of relative fluctuations, we shall confine our attention in this chapter to growth and the cycle.

SECTION I

GROWTH

An extensive literature on the economics of growth has sprung up in the last decade. Our purpose here is not to survey this field, but rather to comment on a few points in the literature which are relevant to a study of the acceleration principle.

Harrod's fundamental equation

In his book Towards a Dynamic Economics Mr. Harrod gives a fundamental equation, $GC = s.$[1] G, which is $\Delta Y/Y$, stands for growth; C (or $I/\Delta Y$) is the symbol for capital; and s (or I/Y) is the fraction of income saved. Although the relationship between the acceleration principle and the fundamental equation is not expressly stated, Mr.

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[1] Harrod, R.F., Towards a Dynamic Economics, London, Macmillan and Co., 1948, p. 77.

Harrod does seem to consider that the equation contains the principle, for he introduces a modification (which we need not consider) "To meet the criticism that this equation gives too much emphasis to the acceleration principle".[2] What, exactly, is the connection between the fundamental equation and the acceleration principle?

It may be recalled that under certain conditions there is justification for substituting the capital-to-output ratio, K/Y or $I/\Delta Y$, for the acceleration coefficient. Thus Mr. Harrod's C has some claim to be considered as the coefficient of acceleration. C_r, which is analagous to C, is defined by Mr. Harrod as "the requirement for new capital divided by the increment of output to sustain which the new capital is required."[3] This is just our old friend the capital-to-output ratio, which is at very least the basis of the coefficient, and which, in a continuing expansion, must equal it. But the acceleration principle is more than a capital-to-output ratio; it is a theory of investment in which the amount of capital outlay is determined by multiplying an increment of output by a coefficient. How closely Mr. Harrod's fundamental equation parallels the acceleration principle can be seen by dividing both sides of the equation by Y to obtain the expression $\Delta Y \times I/\Delta Y = I$. The latter is a recognizable form of the acceleration principle.

Despite the strong family resemblance, it does not appear to be Mr. Harrod's intent to present merely a disguised statement of the acceleration principle. In the acceleration principle the stimulus

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[2] Ibid., p. 79.

[3] Ibid., p. 82.

to entrepreneural action is ΔY, and given any particular increment of output, entrepreneurs then undertake the amount of investment warranted by the increase in output. If the fundamental equation were completely parallel to the acceleration principle, we should expect $\Delta Y/Y$ to be the independent variable or the data given the entrepreneur, and I/Y to be the response. In Mr. Harrod's scheme, however, it is clear that $\Delta Y/Y$ (G or G_w) is not merely data. G_w (which is a special value of G, and which is called the warranted rate of growth) is defined as "that over-all rate of advance which, if executed, will leave entrepreneurs in a state of mind in which they are prepared to carry on a similar advance".[4] This seems to be an inversion of the sequence of events set in motion by the accelerator. We can readily understand entrepreneurs being activated by ΔY, or even $\Delta Y/Y$, but it is difficult to imagine why a particular rate of advance should itself either be "warranted" or be the signal which convinces entrepreneurs that they should carry on as before. Given a particular rate of advance of output, an individual entrepreneur will be forced to adopt a certain rate of investment, and if anything is to merit the term "warranted", it should be the rate of investment. Nor does it seem likely that the entrepreneur would look to $\Delta Y/Y$ to see whether he should carry on at the existing rate of advance.[5] One suspects that the crucial indicator in this regard is C. A high *ex post* capital-to-output ratio means excess capacity or excess stocks, while a low ratio implies shortages and bottlenecks. The comparison of an *ex post*, with a desired, *ex ante*, C would show entrepreneurs

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[4] *Ibid.*, p. 82.

[5] By "rate of advance" Mr. Harrod presumably means rate of investment.

whether past decisions had been overly optimistic or pessimistic. Although the equality of desired, and actual, capital-to-output ratios might leave businessmen in a satisfied state of mind, it is still an open question whether they would therefore be willing to carry on with a similar rate of advance.

The fundamental equation of Mr. Harrod might be caricatured as the acceleration principle standing on its head. This inversion does not follow from the statement of the equation itself, for, as we have seen, the equation can be very easily reduced to a form of the acceleration principle. Nevertheless the direction of the causal relationship seems to be reversed. When developing the fundamental idea of the acceleration principle, we insisted that the sequence of events was important. Once ΔY (or $\Delta Y/Y$) was given, a warranted rate of investment, I, (or, if we prefer, I/Y) necessarily followed. In the fundamental equation, on the other hand, the investment appears to come first; then businessmen examine increases in output (expressed as a fraction of total output) to see not only whether past advances were correct, but also whether similar advances should be carried on in the future.

The fundamental equation is an interesting tool of analysis which deserves more attention, but further attention at this point would lead us away from our main interest. The observations offered here can be summarized as follows. (a) The fundamental equation is a very close relative of the acceleration principle. (b) Unlike the acceleration principle, which takes increments of output as given and then determines a warranted rate of investment, the fundamental equation takes investment (expressed as a fraction of output) as given,

and asks what the warranted increment of output (again expressed as a fraction of output) is. (c) The acceleration principle is a theory of investment which can be related to entrepreneurial experience; it is not easy to relate the fundamental equation to the activities of the firm.

Professor Domar's equation

Unlike Mr. Harrod, Professor E.D. Domar does not suggest that his "fundamental equation" contains any trace of the acceleration principle, but an examination of his very important equation[6] reveals that it, like the fundamental equation of Mr. Harrod, contains a function closely akin to the accelerator. The Domar equation is compounded of two elements: a demand function and a supply function. We may represent the demand function by the equation $\Delta Y_d = \Delta I \times 1/\alpha$; the amount by which this period's income exceeds last period's income, ΔY_d, equals the increase in investment, ΔI, times the multiplier, $1/\alpha$. The supply function, $\Delta Y_s = I\sigma$, tells us that the increase in output, ΔY_s, will be equal to investment, I, multiplied by (ignoring all complications) the output-to-capital ratio, that is, by the inverse of the usual capital-to-output coefficient. If neither excess capacity nor a shortage develops, the increase in output must equal the increase in demand; in other words, $\Delta I \times 1/\alpha$ must equal $I\sigma$.

What is of primary interest for our purpose in Domar's analysis is the supply equation, $\Delta Y = I\sigma$. As the equation stands it does not suggest the acceleration principle, and yet it can be very

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6 Domar, Evsey D., "Capital Expansion, Rate of Growth and Employment", Econometrica, Vol. 14, 1946, pp. 137-147. See also "Expansion and Employment", American Economic Review, Vol. XXXVII, No. 1, March 1947, pp. 34-55; "The Problem of Capital Accumulation", American Economic Review, Vol. XXXVIII, No. 5, December 1948, pp. 777-794.

easily transformed into a recognizable form of it. If we divide both sides by $1/\sigma$, we obtain the expression $I = 1/\sigma \times \Delta Y$. Since σ is the output-to-capital ratio $1/\sigma$ is obviously the capital-to-output ratio, and the whole expression might stand as a statement of the acceleration principle.

This similarity between the supply function and the acceleration principle is very suggestive. When we stop to reflect, it becomes evident that the whole principle of induced investment rests on the question of supply. Businessmen are encouraged to undertake induced investment because increases in output result in a shortage of supply which can be corrected comfortably only by an increase in investment. $1/\sigma \times \Delta Y$ gives the amount of investment which will be required to solve the supply difficulty; $I\sigma$ tells us the amount of supply that will be forthcoming when the investment is undertaken. When we write the acceleration principle $I = b\Delta Y$, the supply condition is implicit. By reversing the equation, as Professor Domar has done, the supply function is made explicit.

It is interesting to find that from the equations of both Harrod and Domar, which are to be found in two important contributions to the literature on growth economics, we can derive the acceleration principle. This suggests that the principle may play a part larger than is generally thought in economic theory.

Professor Alexander's "steady growth"

In an important article written by Professor S.S. Alexander,[7]

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[7] Alexander, S.S., "The Accelerator as a Generator of Steady Growth", The Quarterly Journal of Economics, Vol. LXIII, No. 2, May 1949, pp. 174-197.

a model is introduced which describes the interaction between the multiplier and accelerator, and a question is raised concerning the conditions which must exist before the model can exhibit steady growth. To answer this question Professor Alexander considers different values for the parameters -- the propensity to consume, the acceleration coefficient and the length of the period. Since the difference equation which is used to express the interaction model has both dominant and minor growth factors, and since the minor growth factors are unstable, and tend to become swamped by the larger ones, the problem resolves itself into an investigation of the conditions which are necessary in order to have a dominant growth factor which is low enough to be plausible.

We found that in the particular illustrative example $Y_t = .95Y_{t-1} + 2.1(Y_{t-1} - Y_{t-2})$ [8] which we borrowed from Professor Alexander, the dominant growth factor was a robust one hundred per cent. This, on the face of it, would not appear to be a "reasonable" rate of growth, but the author points out that if income, Y_t, is defined as income in excess of, say, fifty billion dollars, a growth rate of one hundred per cent per year "need not be implausible for a limited number of years".[9] Alexander suggests that the real income sequence in billions of dollars could be 51, 52, 54, 58, 66, 82, 144. However, when we recall that this sequence of incomes must occur in the absence of excess capacity, and that therefore a capacity restraint must operate on the upswing, it becomes apparent that this sequence of

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[8] Ibid., p. 177.

[9] Ibid., p. 178.

incomes is quite improbable after the first two or three years. An increase in output of four billion dollars (from fifty-four to fifty-eight billion), given a capital-to-output ratio of three-to-one, would require an investment of twelve billion dollars; over twenty-two per cent of the fifty-four billion dollar national income would have to be devoted to net capital formation to permit a national income of fifty-eight billion dollars the next year. The sequence of incomes becomes more implausible with each succeeding year. We are led to the conclusion that if the economy is to advance according to the dominant growth factor, the latter should be reasonably small even if output is measured from some arbitrary point.

The quest for reasonable values of the parameters which would permit steady growth (in the absence of price changes) is not successful, and Alexander concludes that "plausible values of accelerator and propensity to consume can indeed permit steady growth, but with constant prices they will lead to so rapid a rate of growth as to suggest that the assumed relationships cannot long persist. Although there are some numerical values of accelerator and propensity that will yield moderate steady growth, those values do not seem consistent with what we know about the facts of our economic system."[10] Although he holds out little hope for steady growth at constant prices, Professor Alexander suggests that "Moderate steady growth of the national income accordingly may be generated by plausible values of accelerator and propensity provided price rises act as stabilizers".[11]

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[10] Ibid., pp. 174-175.

[11] Ibid., p. 175.

There are two comments which I should like to make at this point on Professor Alexander's article. In the first place, the study implicitly assumes that only the dominant rate of growth is of interest in the consideration of the conditions which might lead to steady growth. Actually the minor growth factor is of strategic importance whenever the model is operating under a restraint, and by its very nature the model must always operate under the restriction of a shortage of capacity. The strategic importance of the minor growth factor may be illustrated in the following way. Suppose (as in the example cited) there is a minor growth factor of five per cent and a dominant growth factor of one hundred per cent. And let us say that the effective restraint permits growth at the rate of only six per cent per year. If the initial conditions are such that the rate of growth exceeds five per cent, growth will soon follow the line of restraint and will be kept in check by it. Whereas the rate of growth in an unrestrained model quickly becomes explosive, a restrainted rate of advance never gets the opportunity to run away in an explosive finale, although output may continue to strain at the limitation which holds it in check. If, in our example, the initial conditions in years one and two show a six per cent rate of growth ($Y_{t+2} = 1.06Y_{t+1}$), the free operation of the difference equation results in a value of income in the third year, so that Y_{t+3} is less than $1.069Y_{t+2}$ (instead of $1.06Y_{t+2}$). If $Y_{t+1} = 100$ and $Y_{t+2} = 106$, the restraint will permit an output of 112.36 in year t+3, whereas the free operation of the model would result in an output of 113.30 -- not a large discrepancy. So long as businessmen use realized outputs in their investment equations (rather than the outputs which they might have achieved in

the absence of supply difficulties), the economy will tend to grow along the path traced by the restraint with only a mild upward pressure on the ceiling.

So far we have assumed that the restraint permits a growth greater than the minor growth factor. If the advance permitted by the restraint is less than the minor factor, movement along the restraint will be unstable. If the maximum increase in output is four per cent per year and the minor growth factor is five per cent, the encountering of supply difficulties will automatically cause a downturn. It follows that if the restraint on the upswing is less than the minor growth factor, there would never be an upswing, and the economy would forever bump along the bottom of the depression. Shades of secular stagnation! As soon as the economy showed any signs of expanding it would immediately encounter the restraint which would eliminate the possibility of further expansion -- surely the most stagnant of secular stagnations!

An effective restraint on economic growth raises a number of interesting problems, but its chief importance, so far as we are concerned, lies in the fact that its very existence shifts the emphasis from the dominant to the minor growth factor.

The second comment which might be made on Professor Alexander's article concerns his claim that price increases may act as stabilizers. Price rises, it is suggested, may make plausible a dominant rate of growth which would otherwise be excessive. Increased money income is absorbed partly by an increase in real output and partly by a price increase. In order to demonstrate the stabilizing effect of prices, Professor Alexander appears to abandon the "real"

model with which he has been working and to introduce a model in which consumers and investors react to monetary, rather than to real, phenomena. We are told that "price changes may alter the consumption and investment patterns".[12] But instead of carrying through with this approach, Professor Alexander two paragraphs later announces the assumption "that price changes do not affect either real consumption expenditure or real investment expenditure; and that the latter depend only on the level of real income or on changes in that level".[13] Although this assumption is made "for the moment", there is no indication that it is ever relaxed; and, indeed, it is an assumption that can be easily granted, especially as far as the acceleration principle is concerned. The accelerator, of all things, should be real. If real output is to go up, there must be an increase in real capacity; it is not sufficient that the dollar value of capacity should increase.

Professor Alexander sets up a series of equations in which real income in the t'th period is $\overline{Y}_t$ and the price level is p_t. Money income, Y_t, equals $p_t\overline{Y}_t$. The money income of the t'th period is determined by the real expenditure of the period t-1 valued at the prevailing prices p_{t-1}. These relationships are expressed by the equations:

1 ..... $Y_t = p_t\overline{Y}_t$;

2 ..... $Y_t = p_{t-1}\left[a\overline{Y}_{t-1} - b(\overline{Y}_{t-1} - \overline{Y}_{t-2})\right]$;

3 ..... $\overline{Y}_t(p_t/p_{t-1}) = a\overline{Y}_{t-1} + b(\overline{Y}_{t-1} - \overline{Y}_{t-2})$.

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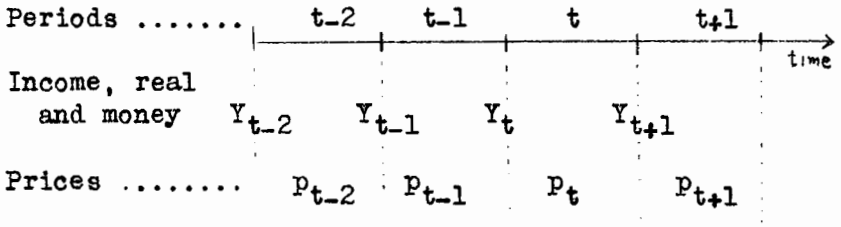
[12] Ibid., p. 188.

[13] Ibid., p. 189.

When these equations are satisfied, it is claimed that the rate of growth will be broken down into a real, and a price, increase. It is my contention that on Alexander's own assumptions these equations are not mutually compatible.

In order to demonstrate this inconsistency, let us define the variables within the framework of three different time charts. The following one is so drawn as to make the second equation true.

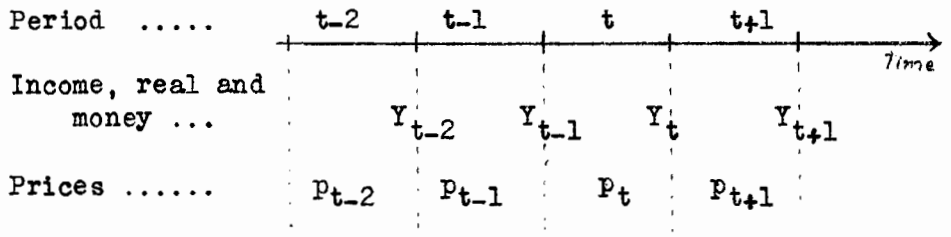


The variables being thus defined by the first time chart, what is the meaning of Y_t? Y_t is the money income which is disposable at the beginning of period t; and Y_t is what it is because people consumed and invested during period t-1 at the prices prevailing in the same period. Disposable income in time t is therefore the result of real income and ruling prices in period t-1. This is precisely what the second equation states. But in what sense is it correct to define $\overline{Y}_t$ as the real income of period t? At the beginning of period t, consumers do not have a stock of goods that can be divided between consumption and investment on the basis of real propensities; there is only a potential flow of goods and, if we like, a stock of money to pay for them. If consumption is a function of real income, what is it that consumers consider to be their real income (as used in the consumption component $a\overline{Y}_t$) at time t? The only real income that has been experienced at this point is the flow of real goods which were

invested and consumed in period t-1. In this sense $\bar{Y}_t$ is real enough, but it is real in period t-1. $\bar{Y}_t$ forms the basis for consumers' (and investors') behaviour in period t, but there is no assurance that $\bar{Y}_t$ will be available in period t. To illustrate this further, one needs only to return to the time chart and imagine that a chain reaction started by a hydrogen bomb snipped off all economic activity at the beginning of period t. $\bar{Y}_t$ would be unaffected, but the real output of the t'th period would be cut to zero.

Although the time chart is consistent with the second equation if we grant this unusual definition of $\bar{Y}_t$, the first equation, $p_t\bar{Y}_t = Y_t$ is clearly wrong. The real output, $\bar{Y}_t$ never has the opportunity to become valued at the prices prevailing during the t'th period because the goods simply don't exist at time t. The money income, Y_t, is equal to the real income $\bar{Y}_t$ valued at prices t-1, not at the prices ruling during period t.

The time chart used above, however, is not the one generally implied by the model. $\bar{Y}_t$ was the real output of period t-1; the subscript t could only be justified on the ground that $\bar{Y}_t$ formed the basis for consumption and investment plans in period t. The following time chart redefines the variables so that $\bar{Y}_t$ is, in fact, the real output of period t.

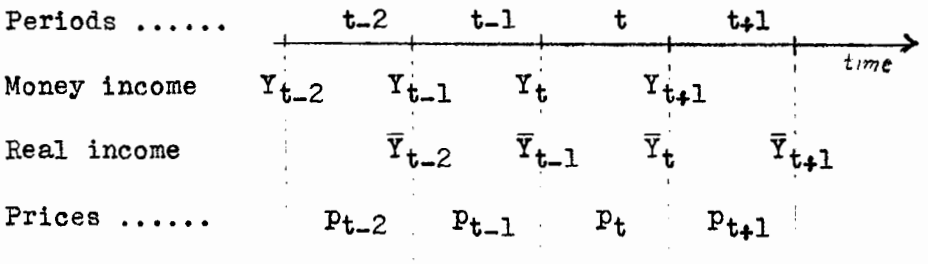


When $\bar{Y}_t$ is defined in the above manner, it becomes apparent that the

real consumption and investment goods which comprise the total real output actually do bear the price tag p_t, and it is quite accurate to state that $Y_t = p_t\overline{Y}_t$. The first equation is consistent with the time chart.

It follows immediately from the above argument, however, that the second equation is now incompatible with the first. $a\overline{Y}_{t-1}$ and $b(\overline{Y}_{t-1} - \overline{Y}_{t-2})$ now refer to consumption and investment in period t but at prices p_t and not p_{t-1} as stated in the second equation.

There is one other possible way of making the equations consistent, and that is to separate the time of reckoning the real, and money, income as follows:



When the variables are defined according to this third time chart, Alexander's equations can be consistent, but they now imply a relationship that he could hardly have intended. $Y_t = p_{t-1}\overline{Y}_{t-1}$ (from equation 2) and $Y_t = p_t\overline{Y}_t$ (equation 1). If these two relationships hold, a rise in real output could be accompanied only by a falling price level. It is, of course, Professor Alexander's intent to have a rising real output accompanied by a rising price level.

This failure to prove that price increases can act as a stabilizer when real propensities are assumed, is not surprising. If consumers and investors are not deceived by the "veil of money", and make their calculations in real terms, or in dollars deflated by a

price index, then price movements should not make any difference. Of course "reality" will be affected by the money phenomenon, but not, I think, on the assumptions made by Professor Alexander.

SECTION 2

BUSINESS CYCLE

Professor Hicks and the Accelerator

From growth we turn to a consideration of the role of the acceleration principle in the literature on the business cycle, and we shall focus our attention on the interaction model of the accelerator and multiplier as used in the recent contribution by J.R. Hicks.[14] Professor Hicks is not the only person, or even the first, to use the interaction model, but for several reasons we seem to be justified in concentrating our attention on his model. In the first place, his discussion of the role of the accelerator has been quite explicit, and this makes the task of evaluation much easier; in the second place, he has probably claimed more for the accelerator than has any other writer on the trade cycle. In his theory of the cycle there can be no mistaking the predominant role enjoyed by the acceleration principle, of which he writes, "It will be my contention, in the following chapters, that the main cause of fluctuation is to be found in the effect of changes in output (or income) on investment... which is ... nothing else but the familiar 'Acceleration Principle'".[15] Finally, despite criticisms which have been raised, the model is still

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[14] Hicks, J.R., A Contribution to the Theory of the Trade Cycle, Oxford, Clarendon Press, 1950.

[15] Ibid., p. 37.

so widely cited that there is some danger that it will become accepted as the General Theory of the business cycle. I say "danger", for I feel that there are criticisms which are so serious as to make the model unacceptable as a theory of the cycle.

The model in outline

Let us begin our critique of the Hicksian model by sketching its central propositions. Professor Hicks attaches a good deal of importance to the proposition, derived from Harrod, that the fluctuations of his model take place around a rising trend. The fluctuations themselves are caused by the interaction of the multiplier and accelerator, and the rising trend is provided by steadily increasing autonomous investment. This model can be represented mathematically by a difference equation similar to the one we have been using $Y_t = aY_{t-1} + b(Y_{t-1} - Y_{t-2})$ with an additional term added to take account of the steadily growing autonomous investment. The values of the parameters of the model -- the acceleration coefficient and the propensity to save -- are so chosen that an unrestrained model will develop explosive cycles. Explosion and breakdown are prevented by two restraints, a "floor" and a "ceiling", between which aggregate output is free to oscillate. The floor is provided by a gradually rising trend of autonomous investment, and the ceiling, by the inelasticity of supply at full employment.

The real national income of the Hicksian model is made up of three components: consumption, induced investment, and autonomous investment. Consumption and induced investment offer no terminological

difficulty. By "induced investment" Hicks means output-induced investment which is brought forth in direct response to past changes in output. His "autonomous investment" consists of "Public investment, investment which occurs in direct response to inventions, and much of the 'long-range' investment... which is only expected to pay for itself over a long period".[16] One difficulty with these definitions of induced and autonomous investment is that although they may be mutually exclusive, they are not necessarily all-inclusive; some kinds of investment will be impossible to classify, according to Professor Hick's definitions, as either autonomous or induced. The investment in a new retail outlet in a new market area is a case in point. Such an investment cannot be stimulated by past increases in output, and yet it could hardly be called autonomous; it is not "long range"; it is not government investment; and it is not innovation in the sense of an applied invention. To be all-inclusive and at the same time mutually exclusive, autonomous investment should include all non-induced investment i.e., all investment which cannot be directly related to past increases in output.

In brief, the Hicksian model may be described as an interaction model imposed on a rising trend of autonomous investment around which there are explosive fluctuations kept in check by a floor of minimum demand and a ceiling of maximum supply.

Recognition of limitations

This model states, or implies, a good deal about the acceleration principle, and our immediate task will be to consider

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[16] Ibid., p. 59.

whether the use of the accelerator has been consistent with the limitations which we discussed in Chapter IV.

(1) The acceleration principle -- a first approximation

A Contribution to the Theory of the Trade Cycle is not, it seems to me, open to the charge that the accelerator is treated as a precise, rather than a crude, tool. The author of Value and Capital would undoubtedly be the first to admit that the acceleration principle is only a rough guide and that induced investment is influenced by many factors. When we say that induced investment depends on past increments of output, we mean, of course, that induced investment depends mainly on past increments of output. However, the admission that the acceleration principle does not determine induced investment precisely does not eliminate its usefulness, and it would be pointless to maintain that we should not use the principle because it is crude. A widespread distaste for crudity would cause serious unemployment among economists and other social scientists.

(2) Limited Applicability

Professor Hicks recognizes that the acceleration principle is of limited applicability in that it does not explain the investment of all firms. "While there can be little doubt that quite a large proportion of the net investment which goes on in normal conditions has been called forth, directly or indirectly, by past changes in the level of output, there is certainly some investment for which this effect is so small as to be insignificant."[17] He goes on to

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[17] Ibid., p. 59.

say that the acceleration principle will not explain those types of investment which he labels as "autonomous".

Because, in the model, he admits that the accelerator does not provide the only explanation of investment, we cannot criticize Professor Hicks on the ground that there are cases in which the acceleration principle does not provide a good explanation of the investment decision. There is the empirical question, however, whether or not Hicks is correct in assuming that the accelerator accounts for "quite a large proportion of the net investment".

(3) Excess capacity

Professor Hicks, of course, is well aware that the acceleration principle will not operate if there is excess capacity[18], and he argues quite explicitly that the downswing must last long enough to wear out any excess plant and equipment.[19] He has not, however, considered how long the depression would have to last in order to accomplish this, or what the effect on the upswing would be if it did.

(4) Expectations

At least two of Hicks' reviewers[20] have criticized him for misplacing, in effect, his copy of Value and Capital and forgetting all about expectations. To me this criticism does not seem to be well taken, at least not in so far as induced investment is

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[18] Ibid., p. 52.

[19] Ibid., p. 105.

[20] Knox, G.A.D., "On a Theory of the Trade Cycle", Economica, N.S. Vol. XVII, no. 67, August 1950, pp. 317-327; Lerner, Abba P., "A Contribution to the Theory of the Trade Cycle", Econometrica, Vol. 19, No. 4, October 1951, pp. 472-474.

concerned, and Professor Hicks does not appear to be really interested in autonomous investment. Expectations are undeniably important to most investment decisions, particularly to the more complicated decisions involving new innovations, but when certain conditions are met, i.e., if we assume a long continuous expansion, such as Pigou's "evenly progressive economy", with no excess capacity, et cetera, we may neglect the effect of expectations on induced investment and relate investment directly to past increases in output. One may take the position that all investment decisions depend on favourable expectations, and in this case "neglecting their effect" may not make much sense. The position taken here is that in the case of the passive firms, which use the acceleration principle, expectations depend in such a simple and direct way on past increments of output that it may be permissible as a first approximation to bypass expectations, as it were, and to relate investment directly to past increases in output.

(5) The restraint on the downswing

It is recognized in Trade Cycle that the accelerator must operate under a restraint during the downswing since there is a limit to the amount of disinvestment in fixed plant which can take place, the amount being determined by the rate of depreciation and the amount of capital.[21]

(6) The restraint on the upswing

So far, there has been substantial agreement with the stand

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[21] Hicks, J.R., op. cit., p. 101.

taken by Professor Hicks on his use of the acceleration principle, but his position with regard to the remaining points is less satisfactory. He is quite explicit, for example, about having a free, unrestrained upswing. He argues that "the expansion in investment, induced by a rise in output, can be as large as it likes, provided that the necessary resources are available".[22] It would seem, however, that the expansion or the rate of expansion -- and it is a rate which is implied in the above passage -- cannot be as large as it likes unless there is excess capacity. Once excess capacity has been absorbed, an additional increase in output of investment or consumption goods might be achieved by extending production into the "inconvenient" range, so to speak, of the cost curve. There is a limit, however, to the increase in production which can be realized in this way. We have tacitly assumed, and I gather that Professor Hicks has done the same, that beyond the output corresponding to rated capacity, more intensive use of plant and equipment is a poor substitute for more investment in fixed plant. Entrepreneurs will be somehow penalized for operating their plants at too high a level,[23] and this is what is implied by the whole concept of rated capacity. The "inconvenience" of operating plant beyond rated capacity means that in the absence of excess capacity, output is subject to a capacity restraint which will be a function of: (a) the ability of the economy to construct additional capacity, and (b) the capital-to-output ratio.

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[22] Ibid., p. 66.

[23] Cf., ibid., pp. 39, 40.

In addition to the capacity restraint there is another output limitation which takes account of shortages of cooperating factors. This restraint, which we have called a resource restraint, will be important during the later stages of an upswing. Both these restraints are likely to be measured in terms of output; thus, building a "ceiling" into a model and defining it in terms of output could presumably be a recognition of a resource, and/or a capacity, restraint. At one place, Professor Hicks shows an awareness of the capacity restraint by stating, "it is the investment ceiling the maximum output of the investment trades which is the impediment to the expansion of output".[24] This statement is the exception; generally Professor Hicks seems to think of the ceiling exclusively in terms of an inelasticity of supply of cooperating factors, and, as a result, the only place where the ceiling enters the argument is at the top of a strong boom. But the capacity restraint should come into the model much earlier, for it is a logically indispensable part of any expansion path which employs the accelerator. If the accelerator is to be used, there must be no excess capacity; if there is no excess capacity, there will be a capacity restraint on output. In other words, if the accelerator is to be used at all, it should (in common with beverages for which tea has a low elasticity of substitution) be used with restraint. Professor Hicks stopped little short of encouraging intemperance when he suggested that the expansion might be as large as it liked!

The existence of the capacity restraint is not, in itself,

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[24] Ibid., p. 129.

sufficient to destroy the usefulness of the model, but it means that the upswing cannot take place faster than is permitted by the restraint, regardless of the values chosen for the parameters. Despite the restraint on the upswing, it will probably remain true that Professor Hicks' full employment ceiling will result in a slowing down in the rate of increase of output. An expansion with unemployed resources but without excess capacity can take place faster than an expansion with no unemployed resources and no excess capacity; expansion which is affected by a capacity restraint alone stands a good chance of being more rapid than expansion checked by both a capacity, and a resource, restraint.

(7) Output relevant for acceleration analysis

The next qualification which deserves special consideration when the acceleration principle is used in a macro-economic model is that the acceleration coefficient should not be applied to increases in output which do not, in fact, lead to induced investment. If, in any model, the possibility is considered that some firms might, because of innovation et cetera, build capacity unrelated to past increases in output, then in such a model, any increase in output which is the product of this capacity must be excluded from the acceleration analysis. In other words, if there is any productive (capacity-increasing) investment other than induced investment, the acceleration coefficient should not be applied to aggregate output. Before undertaking to criticize Professor Hicks on this point, we must first establish whether or not he intended his autonomous investment to be productive in our sense; for if autonomous investment

is productive, the acceleration coefficient should not be applied to aggregate changes in output (as it is in the Hicksian model), but if, on the other hand, it is not productive, the acceleration coefficient can be applied to total output, and some of the logical consistency of the model will be preserved, although it will be a much poorer approximation to reality. It would demand a good deal of the reader to ask him to believe that no autonomous investment increased capacity.

Our task of determining Professor Hicks' position regarding the productiveness or unproductiveness of his autonomous investment is made more difficult because, as Professor Arndt has pointed out,[25] he has not adequately taken into account the dual nature of investment -- its income-generating effects (process effects), on the one hand, and its capacity-generating effects (product effects), on the other. In some sections Professor Hicks seems to hold that all of his autonomous investment is unproductive; at other times he seems to argue just the reverse. In one place he states that he has been reminded by Professor Robertson that "The natural course of events... is that there should be 'some increase in output of final goods, then investment decisions, then the investment process (increase in output of capital goods), and then a large further increase in output of final goods'", and he continues, "It must, I think, be agreed that this increase in capacity is very important; but an increase in output does not follow as a necessary consequence of an increase in capacity. It is necessary that the goods which have now become

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[25] Arndt, H.W., "Mr. Hicks's Trade Cycle Theory", Canadian Journal of Economics and Political Science, Vol. XVII, No. 3, August 1951, p. 402.

capable of being produced without strain, should also be capable of being sold. In this chapter, where we are concentrating on the effects of changes in the demand for output, changes in capacity are not directly relevant. Where they do become relevant is at a later stage of our argument, where we introduce the 'Ceiling'."[26] From this quotation it is apparent that the capacity-increasing effect of investment is recognized; but the subject of this particular passage is induced investment, and it might be argued that the capacity-increasing property is the special attribute of induced investment. At any rate, it is interesting to note that Professor Hicks does not consider that the increase in capacity enters the argument until the ceiling is encountered despite the fact that an increase in capacity without a corresponding increase in demand will mean the emergence of excess capacity and the end of induced investment. Elsewhere, speaking specifically of autonomous investment, the author writes "If the investment is of a productive character, it must clearly have some effect on the ceiling, and this must have some influence on the course of developments, though it cannot (so far as I can see) make any decisive difference to the results obtained".[27] This passage is a recognition of the possibility that autonomous investment is productive, although the importance of the capacity-increasing effect of investment is again discounted.

In his diagram of the business cycle[28] Professor Hicks has

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[26] Hicks, J.R., op. cit., p. 40n.

[27] Ibid., p. 123n.

[28] Ibid., p. 97.

shown the slope of the ceiling to be parallel to the line representing the growth of autonomous investment. This can only suggest that autonomous investment is productive, and that it alone determines the slope of the ceiling. Moreover this diagram indicates that only autonomous investment is productive; for the fluctuations in induced investment appear to have no effect on the ceiling. Actually it is only a capacity restraint which should be directly related to investment; the Hicksian ceiling is a combination of two restraints, and will, therefore, be affected by the size of the investment goods industry and the capital-to-output ratio (capacity restraint), and the availability of cooperating factors (resource restraint). The latter will, in turn, depend on such things as the growth of the labour force, the factor-releasing or factor-absorbing nature of innovations et cetera. The relationship between the rate of growth of autonomous, and induced, investment on the one hand and the slope of the ceiling (the maximum rate of growth of output) on the other is tenuous.

In quite another context, Professor Hicks writes, "the mere property of being closely tied to the movement of current output (which is the distinguishing mark of induced investment) means that induced investment is generally likely to be the more urgent; it is, as a general rule, more necessary for induced investment to be carried through at its own time, if the efficiency of production is to be maintained".[29] If there is autonomous investment, say in "new fangled" automobiles, it may at first increase income and encourage the output

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[29] Ibid., pp. 167-168.

of buggies. In such a case, one wonders why further investment in buggy factories is more important to the efficiency of production than the investment in automobile plants. Only if autonomous investment involved unproductive expenditures such as the erection of pyramids could one agree that in a boom it would be better if such spending was not carried on at the expense of output-induced investment.

These four quotations do not really answer our question about the productivity of Professor Hicks' autonomous investment, but I have the impression that he would probably admit that a large part of his autonomous investment is productive in our sense. The fact that he is not necessarily consistent in his views on the matter, and has not come to our aid with a clear and concise statement on the issue, is probably explained by his avowed belief that the question of capacity is of no great importance to his model except at the top of a strong boom. We have argued that the question of capacity is of great importance to the model over the entire cycle. If autonomous investment is capacity increasing, it will not do to apply a constant acceleration coefficient, as Professor Hicks has done, to aggregate output.

(8) Supply of cooperating factors

Professor Hicks argues that the downturn brings about a transformation of the accelerator, but he does not suggest that the accelerator is altered until the downswing is actually under way. Indeed, he goes even further and argues that the accelerator is not altered at all unless the downswing is rapid. "Suppose then that output has started falling (in absolute magnitude) at such a rate that

it shortly brings the gross investment, which is dependent upon changes in output, to a stop; net induced investment... is therefore a negative quantity, equal to the depreciation on the corresponding part of the capital stock. From this point onwards, so long as the fall in output continues, induced investment ceases to depend on changes in output."[30] It is clear that in describing aggregate output as it "creeps along the ceiling"[31] he has not assumed any decrease in the value of the acceleration coefficient at full employment. If the induced component of investment is reduced by the encounter with the ceiling, it is only because the same acceleration coefficient which was effective during the upswing must now operate on a smaller increment of output. We have argued on the contrary that the acceleration coefficient "at the ceiling" is unlikely to be the same coefficient which operates during the upswing, and, furthermore, that the accelerator is likely to be altered in such a way as to encourage the downturn. However, there will be a number of changes occurring at full employment, and it is likely (though not inevitable) that the induced component of investment will decline; the decline may occur not because of the reduced rate of increase of output, as Professor Hicks argues, but because of the reduced value of the coefficient. Professor Hicks has more to say about the upper turning point, which we shall consider in greater detail a little later. For the moment, suffice it to say that during the upswing the acceleration coefficient (or at least the

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[30] Ibid., p. 102.

[31] Ibid., pp. 98-99.

investment component) will be influenced by the capacity restraint. At the top of the boom, assuming that it is a strong one, the inelasticity of supply of cooperating factors implied by the full-employment ceiling will further alter the relation between past output and future investment. The transformation of the accelerator first occurs during the later stages of the upswing, not during the early stages of a rapid downswing.

(9) Constancy of the coefficient

When considering whether the accelerator coefficient was a constant or a variable, we suggested that it was a reasonable assumption to allow the accelerator to have a constant value provided that it was applied only to the output of passive firms without excess capacity. There is a possibility that the value of the "macro-coefficient" (the acceleration coefficient of a macro-economic model) may fluctuate because of a shift in the relative importance of the output of various industries with different capital-to-output ratios, but with the present degree of approximation this factor is not likely to be too important. Although he does not apply the acceleration coefficient exclusively to relevant increases in output, Hicks treats it as a constant (except for the one change it undergoes during a rapid downswing). He does specify that there should be no excess capacity when the accelerator becomes operative at the first of the upswing, but it is doubtful, on his assumptions, whether there would ever be a depression which would wear out excess capacity. A very long downswing will be required to make a significant reduction in the amount of capacity, even assuming zero gross investment, but in

Professor Hicks' model, gross investment is never zero, and since autonomous investment is maintained throughout the depression there is a very good possibility that there will be more capacity at the end of the Hicksian depression than at the beginning of it.[32] If, at the time of the upturn, excess capacity is admitted into the model, as I think it must be, and if the accelerator coefficient is still applied to aggregate output, then its value will be small at the beginning of the upswing, and will gradually grow as more and more firms exhaust their surplus capacity.

There is another reason why the value of the accelerator should change in the Hicksian model: if autonomous investment, even in part, is capacity-increasing, there will likely be some fraction of the total increase in output which absorbs this capacity, and which does not, therefore, induce further investment. In other words, there are relevant and irrelevant increases in output, and if the acceleration coefficient is to be applied to aggregates, its value must change with the changing proportions of output which <u>will</u> induce investment, and output which will <u>not</u>. So far as I can see, there is not any simple method of determining how this factor is likely to influence the value of a macro-accelerator applied to aggregate output over the cycle. In terms of the active sector (comprising all firms building capacity unrelated to past increases in output) and the passive sector (including firms whose investment

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[32] Cf., A.F. Burns' statement that, "As an empirical matter, we know that the stock of capital in the United States, if not also in other countries, has as a rule continued to grow even in periods of depression". "Hicks and the Real Cycle", <u>The Journal of Political Economy</u>, Vol. LX, No. 1, February 1952, p. 9.

is output-induced), the value will depend in large measure on the gap between aggregate demand and the supply (capacity) of the active industries. There are, however, many complications, not the least of which is the fact that supply is not quite the same thing as capacity.

It was specified earlier that a long and continuing expansion was required in order to give the acceleration coefficient a constant value, and that given such an expansion, there would be pressure on the entrepreneur to equate the accelerator coefficient to the capital-to-output ratio. In a discussion of the expansion path, one might expect Professor Hicks to describe a long, continuing expansion of gradually increasing increments of output, for in an early chapter on the multiplier, he regards as most interesting and most dynamic the case in which investment is steadily changing from period to period.[33] Yet when the accelerator is discussed in the following chapter, we are introduced not to a continuing expansion, but to a single "hump" in output and investment. The possibility, in the following period, of another hump which would prolong the expansion is considered, but we are told that "obviously we cannot count upon this happening".[34] In other words, Hicks considers the accelerator in the "relatively 'static' case of a single change in the rate of investment",[35] but does not extend his treatment to "the cases which, for the theory of the cycle, are much more interesting --

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[33] Hicks, J.R., op. cit., p. 24.

[34] Ibid., p. 43.

[35] Ibid., p. 24.

when investment is continuously expanding, continuously contracting, or changing over from one motion to the other".[36] Actually, when these quotations are read in context, it is evident why the accelerator is considered in a relatively static setting; Professor Hicks is using a "hump" of investment to displace output from its equilibrium path. Still he does not do for the accelerator what he did for the multiplier: he does not examine its behaviour when output is continuously expanding. In a continuing expansion, the businessmen who are slow in building up their capacity will come under increasing pressure and will eventually be compelled to accept the "right"[37] value of the accelerator. However, if the accelerator is considered in the relatively "static" case of a single increase in the rate of output, and if we know that businessmen assume that it is a once-and-for-all change, the accelerator, far from being constant, will be able to range all the way from the capital-to-output ratio (or even higher) down to a very small value, depending on how long the entrepreneurs wish to take to build up their rated capacity.

Summary

Of the nine qualifications which, we argued, must be applied to the use of the acceleration principle, we find that the last four -- which are all of particular interest in the application of the accelerator to a macro-economic model -- have not been adequately recognized by Professor Hicks. He does not take account

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[36] Ibid., p. 24.

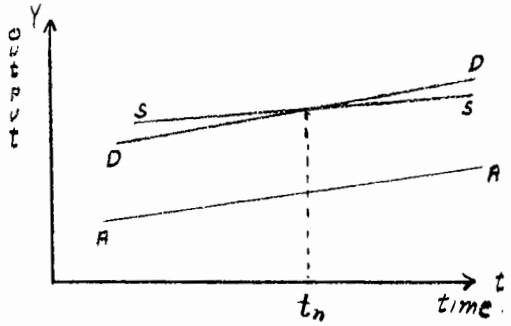
[37] I.e., the value which will eliminate both shortages and excess capacity -- the capital-to-output ratio.

of the restraint on the upswing, nor does he recognize that the acceleration coefficient will be altered when supply becomes inelastic at the full employment ceiling. More important, he applies the coefficient of acceleration to aggregate output, although he does not specify that induced investment is the only capacity-increasing component of investment. Lastly, we might agree with Professor Hicks that the value of the acceleration coefficient is a constant so long as we make the qualification that it be not applied to aggregate changes in output and provided that we are considering a reasonably long expansion, and not a single increment of output.

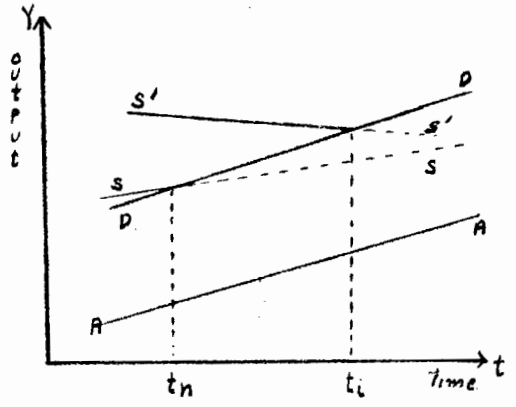
So much for the limitations which Professor Hicks imposes -- or fails to impose -- on the operation of the accelerator. Let us now turn our attention to the application of the acceleration principle to the upturn, the upswing, and the downturn.

The upturn

The unlikelihood that, on Hicks' assumptions, a depression will remove all excess capacity has already been commented on. There is a more fundamental point: even in the absence of excess capacity, a gradually rising trend of autonomous investment will not necessarily cause any induced investment. This can be seen from the accompanying diagram in which time is measured along the horizontal axis, and output (Y), along the vertical axis. AA is the trend of autonomous investment, and DD,



the trend of aggregate demand associated with it. SS represents the supply which can be made available by the firms undertaking the autonomous investment, and can be looked upon as the ceiling for the output of the active firms. It is quite possible that their output may not equal their capacity, and recognition of this possibility can be given by a downward revision of the SS line. In our diagram, demand would not begin to spill over into the passive sector until time t_n, and if SS happened to coincide with DD, or lie somewhat above it, the expansion of autonomous investment would never induce any investment in the passive industries. In such a case the innovating firms are able to satisfy the demand generated by their own investment. The diagram assumes an intersection of SS and DD, which means that the upturn in output of passive firms will begin at time t_n. But the upturn in the output of the passive sector does not mean the beginning of induced investment, for we have not yet made allowance for the excess capacity of the passive firms. This may be done by adding the capacity of these firms to that of the active firms to give an aggregate supply, or capacity, function, S'S', shown in the second diagram. The failure of the passive firms to maintain their plant and equipment during the depression means that S'S' will converge towards SS. When S'S' cuts DD, at time t_i, the acceleration principle comes into play and the upturn in induced investment will begin. Strictly speaking, it would be more accurate to describe t_i as the point at which the operation of the acceleration principle



becomes general. If we follow our own advice and apply the acceleration coefficient to relevant, rather than aggregate, increments of output, it becomes apparent that an accelerator-induced component of investment might exist at all stages of the cycle. Even during the downswing one could undoubtedly find a few passive firms with expanding output.

The upswing

Professor Hicks' application of the accelerator to the upswing raises a number of points of interest. The absence of a restraint on the upswing in his model has already received attention. Then there is the matter of the relative importance of autonomous and induced investment. The device of introducing a steadily growing trend of autonomous investment could be defended if one's avowed purpose was to consider whether or not the accelerator component of investment would itself behave cyclically. But Professor Hicks claims that his model is a good deal more than a device for establishing the cyclical behaviour of induced investment; he argues that it is the leading candidate for "the theory of the cycle"[38] -- a claim which would be justified only if he had demonstrated that induced investment is the sole, or at least the most important, cycle-making factor. At times this seems to be the claim that Professor Hicks is making, although there is little attempt to prove it. He writes, "It will be my contention, in the following chapters, that the main cause of fluctuations is to be found in the effect of changes

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[38] Hicks, J.R., op. cit., p. 2.

in output (or income) on investment",[39] and again, "fluctuations in autonomous investment" may explain "the superficial irregularities"[40] of the cycle. There can be no doubt that Professor Hicks considers the induced component of investment to be the cycle-generating force. However, the overriding importance of the accelerator must be taken as an act of faith, for there is no convincing empirical or theoretical proof in A Contribution to the Theory of the Trade Cycle to show that the fluctuations in autonomous investment are not more important than those in induced investment. Professor Hicks' claims for induced investment are supported only by the argument that his model turns out a "cyclical sequence... remarkably similar to that which is experienced in practice".[41] This parallelism between the model and reality might be coincidence, and in any event, it is open to question whether the model will turn up realistic cyclical sequences after certain necessary changes are made in it, if, for example, one were to take supply into account and apply the accelerator coefficient to relevant, rather than aggregate, increments of output.

Apart from the question of the relative fluctuations of autonomous and induced investment, there is the question of their relative levels. Implicit in Professor Hicks' work is the assumption that the level of autonomous investment is not sufficient to maintain full employment; that is to say, he assumes that technological progress,

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[39] Ibid., p. 37.

[40] Ibid., p. 123.

[41] Ibid., p. 2.

government investment and long-range projects will not maintain prosperity, and he also assumes that the deficiency of investment opportunities cannot be made up by induced investment. The equilibrium rate of growth is therefore assumed to be one which would give chronic under-employment. This "stagnation thesis" may be a very good assumption, but it has not yet achieved the status of an economic law, and to make it the basis of the model is to make the model no more valid than the thesis.

Further modifications need to be made in two of Professor Hicks' analytical tools -- the concept of an equilibrium path, and the super-multiplier. The equilibrium path in his model is derived by applying the super-multiplier to the trend of autonomous investment, AA. In effect, this means that he treats the demand generated by autonomous and induced investment as though it were a simple function of autonomous investment alone. Such a concept as the super-multiplier would be useful only if induced investment bore some simple and direct relation to autonomous investment, but as can be seen, once supply is taken into account, induced investment is <u>not</u> simply and directly related to autonomous investment, but is a function, in terms of our diagram, of the difference between DD and S'S'. Given DD and S'S', there may be some equilibrium rate of growth, but it will not be found by applying a super-multiplier to autonomous investment. We can never be sure that Professor Hicks' equilibrium path, which is found and defined mathematically by the difference equation, will in fact be an equilibrium path, because the model does not keep track of supply except at the ceiling. There is a supply function implicit in the acceleration principle, as we have already

pointed out, and it might be argued that a simple acceleration model does "keep track of supply". However, there is no implicit relation between autonomous investment and supply; therefore the supply arising from productive autonomous investment requires explicit treatment. Ignoring supply might be excusable in some models of the cycle but not in one which depends for its validity on a unique relation between aggregate demand and aggregate supply i.e. on the absence of excess capacity.

To summarize our criticism of Professor Hicks' treatment of the upswing as it relates to the acceleration principle: (1) the upswing instead of being free will take place under a restraint; (2) contrary to Professor Hicks' assumption, the upward slope of autonomous investment is not, itself, a sufficient condition to cause induced investment; (3) since induced investment is not a simple function of autonomous investment, the concept of the super-multiplier is not useful; (4) we cannot be sure that Professor Hicks' equilibrium path is what it professes to be until we are told about the behaviour of capacity; (5) the applicability of the Hicksian model is limited because it is based on an assumption closely akin to a stagnation thesis.

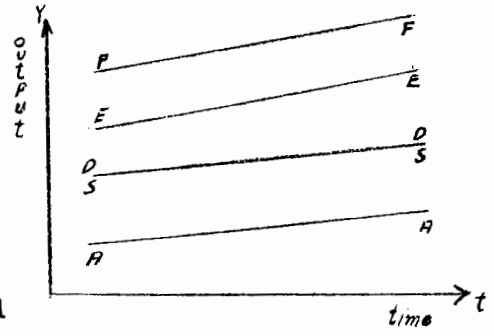
The downturn

Professor Hicks' explanation of the downturn is perhaps the weakest part of his model. There appear to be two main lines of argument. In the first place, he has assumed an equilibrium path of the economy which lies below the full employment ceiling and a ceiling which is, itself, unstable. The author makes explicit the

instability of the ceiling as an expansion path when he writes, "Now the induced investment, which is induced by an increase in output at this rate [permitted by the ceiling], is not sufficient to support a growth of output along the path FF; it is only enough to support an output which expands along the equilibrium path EE. Ouput will therefore rebound from FF back towards EE".[42] As we have seen, Professor Hicks' equilibrium path is not really an equilibrium path unless some additional assumptions are made about the productivity of autonomous investment. The equilibrium path cannot be found by applying a super-multiplier to autonomous investment because there is no unique relationship between the slope of the autonomous investment function and the amount of induced investment. Autonomous investment gives rise to both supply and demand, and, for the sake of simplicity, we might assume that these functions balance, i.e. that the demand created by autonomous investment is satisfied by the supply of the firms undertaking this investment. In terms of our diagram DD = SS. In this case we can conceive of a simple acceleration model of the economy

$[Y_t = aY_{t-1} + b(Y_{t-1} - Y_{t-2})]$

being superimposed on top of the demand and supply generated by autonomous investment. It is interesting to note that if there is one equilibrium rate of growth, EE, which can be drawn above DDSS, there will be an infinite number with the same slope. That is, if the simple acceleration model can grow steadily at five per cent (measured from DDSS), any line with the same slope will represent a possible equilibrium rate of growth. It



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[42] Ibid., p. 99.

will also be stable upwards since the full employment restraint will prevent explosion; it is unlikely that it will be stable downwards. If the slope of the ceiling is greater than the slope of EE, the stability of the full-employment ceiling will be increased because random movements downwards, provided they are not too large, will not cause a downturn. In such a model a downturn will be inevitable only if the slope of the ceiling is less than that of an equilibrium path measured from DDSS. Too much reliance should not be placed on the explanation of the downturn afforded by such a model, because it assumes that the relationships which hold between the variables during the upswing will remain constant when the ceiling is encountered.

The other main line of Professor Hicks' argument concerning the downturn is built on the assumptions of (a) a free upswing and (b) a full-employment restraint and on the proposition that a slowing down in the rate of increase of output will cause a downturn. Despite the fact that we have questioned the free upswing, we might agree that a full-employment restraint could result in a slowing down in the rate of increase of output. This admission, however, does not justify the use of the proposition. The slowing down in the rate of increase of output which leads to a downturn has traditionally been a slowing down in <u>demand</u>. Professor Hicks does not show how a slowing down in the rate of increase of output for reasons of <u>supply</u> causes a downturn. A downturn when the full-employment ceiling is reached will be made more likely, though not inevitable, by the assumption of a decrease in the value of the acceleration coefficient. Such a decrease is a distinct possibility, for it is apparent that a shortage of cooperating factors can cause a decrease in investment;

in fact the decrease in investment may be brought about whether or not there has been a decrease in the rate of increase of output. However, Professor Hicks seems to have rejected the possibility of a reduced accelerator coefficient at the ceiling, for he writes, "It may perhaps be argued that it is the increase in demand, rather than the increase in output, which is ultimately responsible for the stimulation of investment; and therefore that a retardation in the growth of output, solely due to supply difficulties, need have no adverse affect on investment. So far as the industries in which the supply difficulties first appear are concerned, this may be granted. The fact that the effective demand for their products has shot up above the ceiling means that investment in these industries is stimulated to a greater extent than would appear from an examination of their actual output."[43]

Professor Hicks apparently does not rely on a retardation of investment by the bottleneck industries to explain the downturn. Indeed, he seems to argue that the acceleration coefficient continues to grow at the full-employment ceiling, and grows in such a way as to give a larger amount of investment. If investment by the bottleneck industries is maintained, what causes the downturn? His explanation, which directly follows the passage cited above, reads, "But it remains true that the retardation in the growth of output in these industries slows up the demand for the product of other industries; this is the effective way in which the existence of the ceiling imposes a check".[44] It is in this argument that the difficulty

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[43] Ibid., p. 99n. Italics added.

[44] Ibid., p. 99n.

occurs. It seems obvious that a slowing up of the rate of increase in the output of "bottleneck industries" will result in a lower real output than would have been attained if these industries had maintained their pre-ceiling rate of growth. But this slowing up in aggregate output is irrelevant. Makers of other products are not accelerated or decelerated by the behaviour of the output of the "bottleneck industries" or of output generally. When the rate of growth of one item of gross output slows down, it naturally affects aggregate output, but this need have no adverse effect on the demand for the other items which comprise total output. The situation which Professor Hicks describes could even increase the demand for other products, and on two counts. In the first place, the bottleneck industry is stimulated to make even greater investment, and, in the second, in so far as it drains off less purchasing power than it could, other products which compete for purchasing power will benefit. On Professor Hicks' assumptions it would seem that aggregate demand is increased more than aggregate supply, and this is hardly deflationary.

Professor Hicks gives references to Professors Haberler and Harrod to support this "crucial reason for the down-turn".[45] It would seem, however, that he is not justified in claiming the support of either of these authorities. Professor Haberler writes "If at this point [the full-employment ceiling] the level of activity in a number of industries is still dependent on the growth of employment and production in other industries... the volume of output

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[45] Ibid., p. 99n.

will not simply stop expanding and go on at the level which it has reached. It will decline in the capital goods industries, which are geared to the expanding consumers' goods industries".[46] According to Professor Haberler, the investment goods industries are dependent on the rate of growth of output of the consumption goods industries. When the latter reach the full-employment ceiling, the shortage of cooperating factors makes further investment pointless. The bottleneck industries reduce investment. Mr. Harrod also envisages a decrease in investment by firms encountering supply difficulties, for he writes, "The increase of consumption must slow down, once a considerable portion of the unemployment is taken back into work. Consequently a point is bound to come at which the volume of orders for additional capital goods, which it appears profitable to give, is reduced...".[47] Thus both writers state, or imply very strongly, that the bottleneck industries will reduce their level of investment when they hit the ceiling. Professor Hicks argues that the ceiling need have no adverse effect on investment and, indeed, may stimulate it. In other words, he claims that the value of the accelerator coefficient may increase, while Professors Haberler and Harrod argue that it will decrease. So far as I can see, neither of the last two writers has suggested that a slowing down in the rate of output of one commodity would adversely affect the demand for the products of other industries provided that investment in the industry of the first commodity did not slacken.

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[46] Haberler, G., Prosperity and Depression, Geneva, League of Nations, 1941, p. 369. Italics added.

[47] Harrod, R.F., The Trade Cycle, Oxford, Clarendon Press, 1936, p. 165. Italics added.

CHAPTER VII

CONCLUSION

The status of the acceleration principle

Before we attempt to evaluate the present position of the acceleration principle, we must reiterate that we are evaluating the rigorous interpretation of that principle which we introduced in Chapter I. We may recognize, of course, that the principle can be counted as either a general theory of investment or an *ex post* relationship. If the principle is treated as an *ex post* relationship, the coefficient would have whatever value is necessary to equate capital and output. Such a relationship is a truism. It is not a theory of investment. It is neither useful nor interesting. If the principle is interpreted as a general theory of investment, the coefficient must be a function of all factors which are likely in any way to influence investment. I should be in favour of discarding this interpretation of the principle also. We already have general theories of investment which explain everything and nothing and which are in essence simply cataloguing devices for grouping investment-determining factors. The acceleration principle which I intend to evaluate is a rigorous theory which holds that investment is a relatively simple function of past changes in output.

By adopting such a definition we have immediately restricted the applicability of the acceleration principle and must recognize limitations which are indeed severe. We must admit that the operation of the rigorous principle in a firm may be upset by such factors as excess capacity, innovation, shortages of supply (including supply of credit) and aggressive competition. The restricted relevance of the

principle to the firm means that its applicability to the economy as a whole will be especially difficult. However, when all the subtractions from the importance and relevance of the principle have been made in order to take account of the many limitations, there remains a positive and valid residual; there remains a theory of investment which is solidly based on the simple physical truth that more output requires more investment.

Although I believe that the accelerator is a valid theory of investment,[1] I cannot say whether it is important. Validity, which depends on whether or not any firms use (or behave as if they use) the principle, and importance, which depends on how many firms use the principle, can only be determined by analysis of investment decisions, and this brings us to the subject of empirical tests. The reader may very well wonder why more attention has not been paid to the empirical tests which have been made by such writers as Clark,[2] Kuznets,[3] and Tinbergen,[4] and which have played such an important part in the literature of the acceleration principle.

For saying little on the subject of these tests, there are

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[1] Valid in the rather unusual sense that it explains or predicts the investment decisions of some firms.

[2] Clark, J.M., "Business Acceleration and the Law of Demand: A Technical Factor in Economic Cycles", The Journal of Political Economy, Vol. XXV, No. 3, March 1917, pp. 217-235.

[3] Kuznets, Simon, "Relation Between Capital Goods and Finished Products in the Business Cycle", Economic Essays in Honour of Wesley Clair Mitchell, New York, Columbia University Press, 1935, pp. 209-267.

[4] Tinbergen, J., "Statistical Evidence on the Acceleration Principle", Economica, Vol. V, N.S., May 1938, pp. 164-176.

several reasons. First, the very fact that the tests have received so much attention means that they have already been well digested and redigested. Second, the validity of the principle is not in question. I have found that for one company at one time the acceleration principle predicted the results (and described the process) of entrepreneurial planning with a high degree of accuracy. This test, along with some of the favourable tests of others, confirms the validity of the principle that is, it gives an assurance that the principle applies to some firms at some times. Third, a test of the applicability of the principle to a firm, even if it proved unfavourable, would not change my opinion of the principle, because I have not argued in favour of its general validity. An unfavourable test conducted on the investment of a firm would only prove that at that time and for that firm the acceleration principle was not the important explanation of the investment-decision process. A priori reasoning has already indicated that the rigorous acceleration principle is likely to hold only for some firms at some times. The empirical tests on individual firms, whether favourable or unfavourable, are only scraps of evidence as to the importance of the acceleration principle. The reader will not be surprised to find that I am prepared to disregard as any kind of evidence a test which uses aggregates. It would be unusual, in view of the limitations involved in aggregative analysis, if a macro-acceleration coefficient multiplied by total increments of output ever successfully predicted total induced investment.

The status of the interaction model

Before dealing further with the acceleration principle per se, we might pause to assess the status of the interaction model. The Hicksian version of this model is, I believe, untenable on its own assumptions, and even if corrected it would be a, and not the, theory of the cycle. In its present state the model leaves both turning points unexplained; it assumes secular stagnation; and it asserts, without proof, that induced, rather than autonomous, investment is the cycle-generating force.

There are two different ways, which might be suggested, of making the interaction of accelerator and multiplier acceptable as a logically consistent model. First, we might rule out all non-induced productive investment, leaving a simple acceleration, or "shoe industry", model of the economy. If non-induced investment is admitted it must be non-productive in the sense that it does not increase capacity.[5] Such a model could hardly be expected to describe the real world, but it might nevertheless be a useful pedagogical device for studying the behaviour of the acceleration principle. Second, productive autonomous investment might be admitted (and if the model is to represent the real world, this investment must be admitted), and at the same time allowance made for the attendant increase in capacity and output. Probably the easiest and most logical way of avoiding the difficulties which accompany the intro-

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[5] P.A. Samuelson's 1939 statement of the interaction of multiplier and accelerator ("Interactions Between the Multiplier Analysis and the Principle of Acceleration", The Review of Economic Statistics, Vol. XXI, No. 2, May 1939, pp. 75-78) affords an example. The components of this model are consumption, induced investment and government spending.

duction of productive non-induced investment is to apply the acceleration coefficient to relevant $[b(y_{t-1} - y_{t-2})]$, rather than to aggregate $[b(Y_{t-1} - Y_{t-2})]$, increments of output.[6]

The switch from total output to total relevant output for the determination of induced investment underlines the interaction model's major problem -- the importance of induced, as opposed to non-induced, investment. Is induced investment the major cycle-generating force as Hicks and Harrod claim, or does the cycle spring from non-induced investment as Schumpeter maintains? What indirect evidence we have casts doubt on the importance of the interaction model. In Chapter V,[7] two reasons were advanced for believing that at the upturn there must be widespread excess capacity, and from these it follows that the recovery can be explained only by a surge of non-induced investment. However, this evidence does not tell us all we would like to know about the importance of the acceleration principle over the whole cycle. The necessary information can be obtained only by comprehensive and continuing industry studies conducted by a full-time corps of economic observers.

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[6] See page 64ff. The reader may be prompted to ask, could not allowance be made in the coefficient rather than in output? Might we not retain total output as Hicks does, and recognize that the Hicksian b is smaller than our b? The answer is no. First of all we must determine the value of relevant output (y_t) before we can even guess the value of the Hicksian coefficient, and having found what output is relevant we may as well use the information directly. Second, and more important, $(Y_{t-1} - Y_{t-2})$ will sometimes be zero or negative, but we should normally expect $(y_{t-1} - y_{t-2})$ to be positive. No adjustment of the Hicksian b could make up for a zero increment of total output.

[7] Supra, pp. 86-88.

The corps of economic observers

By a "corps of observers" I mean a small group of economists each of whom becomes an expert in one sector of the economy which he keeps constantly under his surveillance. For an observer to qualify as an expert, he should be familiar with the relevant statistics, and should have a rudimentary knowledge of the technical aspects and problems of the industries he is to observe, but first and foremost he would be expected to get to know the key personnel associated with the industries which he studies. The reason for emphasizing the knowledge of personnel will become evident as we procede.

The case for the economic observers

Economists spend much of their time speculating about such matters as businessmen's expectations and entrepreneurial response -- matters which are so completely subjective that we cannot hope to deal with them effectively until we establish machinery for observing businessmen directly. The proper study of economics is man, and a necessary procedure of economics is the observation of the businessman. Despite the fact that the acceleration principle is, by social science standards, a very objective tool of analysis and that therefore it might be expected that by applying it only to "facts and figures" one could use it successfully as a predictive device, I nevertheless maintain that the level and behaviour of aggregate induced investment cannot be even guessed unless we have recourse to a device, such as I propose, for a continuing study of the business community.

There are two alternatives to the observer approach: the

conventional industry study, and the econometric model. The industry study (as opposed to the continuing study), which ends with some sort of report, can do no more than furnish a history of an industry, and though this may be useful, it cannot tell us at any future point of time if the acceleration principle is being used, or if there is excess capacity, or if there has been any change in the value of the current capital-to-output ratio.

The econometric model will not be more successful than the industry study in answering these questions, and even if these questions presented no difficulty, the model would be unlikely to predict the behaviour of induced investment. One example will serve to show why this is so. Let us suppose that the econometrician has solved the problem of distinguishing between output which will, and output which will not, induce investment, and let us agree further that the relationships which obtain during the upswing are known, so that an appropriate model may be devised. Can we rely on such a model to predict a turning point? No. The model will be useful only so long as the underlying relationships remain unchanged but once a disturbance is introduced, we can no longer put much faith in it. Such a disturbance could be the encountering of supply difficulties at full employment. One might imagine that the econometrician could meet this contingency by automatically substituting a "full-employment model". However, there must be at least two such models, for we may recall that two quite different responses to shortages have been suggested in the literature. Full employment might stimulate investment (as Professor Hicks seemed to assume) or it might discourage it (as Professor Haberler and Mr. Harrod assume). Both these

alternative assumptions might be, and probably are, quite correct in different circumstances; the fact that two possible responses exist means that the econometrician must be able not only to build the conflicting full employment models, but also to select the appropriate one. Such a choice could be made only by observers who are in touch with events and who are in a position to judge entrepreneurial response to these events.

The thread of our argument has tangled on several occasions with the intrusion of expectations and other subjective matters, but we have always pleaded that the acceleration principle was relatively free from such awkward considerations. The freedom is only relative, however, and we must not avoid mentioning our bounds just because expectations present a knotty problem; we must now see to the undoing of the expectational tie that binds even the acceleration principle.

First let us review the evidence that supports the position that the accelerator is a "relatively" objective principle. Earlier[8] we attempted to justify the use of actual, rather than anticipated, increments of output as a basis of investment decisions, and we also argued[9] that in a continuing expansion the technical necessities tended to force the acceptance of an objective accelerator coefficient based on the capital-to-output ratio rather than a subjective coefficient based on the state of entrepreneurial digestion. While I believe that these considerations bring an important degree of objectivity to the principle, I would still argue that the remaining

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[8] Supra, p. 13

[9] Supra, pp. 45-46.

subjective elements make it impossible, without having recourse to the proposed observer corps or something very much like it, to use the accelerator as a tool of macro-economic analysis.

There are at least four matters of strategic importance which can be settled only by direct observation of industries and their personnel. First, there is the problem of identifying those industries or firms which do, and those which do not, use the acceleration principle. The initial task of the observer would be to draw up lists which, according to our earlier terminology, could be labelled as "active" or "passive" firms. Since firms may change from one category to the other, these lists would have to be under more or less constant review. Second, excess capacity, which is of crucial importance to the operation of the accelerator, may contain an important subjective element. We have defined excess capacity as the difference between actual output and rated capacity, and while the latter term may be quite objective, it may, if cost curves rise slowly, be little more than an entrepreneurial prejudice which can be brought to light only by someone with a knowledge of the personnel involved. Third, induced investment may be disturbed by shortages of cooperating factors and relative price movements, but the direction and degree of change could be estimated only by an observer. Fourth, the "technical necessity" behind the acceleration principle is the simple fact that increased output encourages investment. However, the investment is encouraged, not forced. Induced investment will flourish only in the climate of confidence in the future. The observers must try to detect whether or not entrepreneurs feel a

"subjective certainty", to use Professor Keirstead's term,[10] about the long-run profitability of business. Given a condition which we might describe as confidence, or optimism, or favourable expectations, I should expect the rigorous acceleration principle to give a good prediction of the investment decisions of passive firms. Confidence is a necessary condition for the operation of the acceleration principle, and confidence can only be detected by persons who make it their business to observe entrepreneurs -- observe their scowls and their tone of voice, measure their blood pressure, and, I am afraid, read their after-dinner speeches.

The case for the use of observers is strong even when we attempt to consider the relatively objective acceleration principle; the case becomes overwhelming if we wish to extend the analysis to less objective matters. The suggestion for a group of observers should receive the support of all economists who stress the importance of autonomous investment or expectations.

The importance of the acceleration principle is in doubt. I suggest that it will remain in doubt until we make provision for the continuing study of the business community. Within the framework of a corps of economic observers the acceleration principle should become a useful tool of aggregative analysis.

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[10] Keirstead, B.S., An Essay in the Theory of Profits and Income Distribution, Oxford, Basil Blackwell, 1953, p.20.

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