FLIEGNER NUMBERS AND THEIR APPLICATIONS

by

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SUMMARY

A brief note describes the use of the Fliegner numbers (functions of Mach number and ratio of specific heats) for one-dimensional compressible fluid flow. It is shown that the introduction of the Fliegner numbers simplifies theoretical derivations for (1) isentropic one-dimensional flow, (2) normal shock waves, (3) adiabatic one-dimensional flow in a constant area duct with wall friction and (4) frictionless one-dimensional flow in a constant-area duct with heat transfer.

CONTENTS

NOTATION

A - Flow area (sq. ft.)

C_p - Specific heat at constant pressure (6012 ft.-lb./slug-$^{\circ}$R for air)

$F(M)$ - Fliegner (total) number (Eq. 1)

$f(M)$ - Fliegner (static) number (Eq. 2)

$I(M)$ - Fliegner (impulse) number (Eq. 3)

M - Mach number

m - Mass flow (slug/sec.)

P - Total pressure (lbs./sq.ft.)

p - Static pressure (lbs./sq.ft.)

R - Gas constant (1716 ft.-lb/slug-$^{\circ}$R for air)

S - Entropy (ft.-lb./slug-$^{\circ}$R)

T - Total temperature ($^{\circ}$R)

t - Static temperature ($^{\circ}$R)

v - Velocity (ft./sec.)

ρ - Density (slug/cu.ft.)

γ - Ratio of specific heats (1.4 for air)

Subscripts

o - inlet condition

t - throat condition

x - condition upstream of a normal shock

y - condition downstream of a normal shock

$*$ - choking condition

I. INTRODUCTION

Compressible flow derivations and calculations are in general tedious, but can be much simplified by using non-dimensional flow parameters of the main functional relationships. The use of a mass flow parameter (ref. 1) was shown to expedite calculations, and the present note illustrates the extension to some problems. The Fliegner number (Ref. 2) represents the mass flow parameter in honour of the experimentor who first realized the choked nozzle flow phenomena (Ref. 3)

II. FLIEGNER NUMBERS

In steady, one-dimensional compressible flow of a perfect gas, the Fliegner number based on the total pressure is defined as (Ref. 2),

$$\text{Fliegner (total) No.} \equiv F(M) \equiv \frac{m\sqrt{C_p T}}{AP} = \frac{\gamma M}{\sqrt{\gamma-1}}\left[1 + \frac{\gamma-1}{2} M^2\right]^{-\frac{\gamma+1}{2(\gamma-1)}} \quad ..(1)$$

where m is the mass flow, C_p the specific heat at constant pressure, T the total temperature, A the flow area, P the total pressure, γ the ratio of specific heats and M the Mach number. Analogously, the Fliegner number based on the static pressure and that based on the impulse function per unit area can be obtained (Ref. 4).

$$\text{Fliegner (static) No.} \equiv f(M) \equiv \frac{m\sqrt{C_p T}}{A\,p} = \frac{\gamma M}{\sqrt{\gamma-1}}\left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{1}{2}} \quad ..(2)$$

$$\text{Fliegner (impulse) No.} \equiv I(M) \equiv \frac{m\sqrt{C_p T}}{A(p+\rho V^2)} = \frac{\gamma}{\sqrt{\gamma-1}}\,\frac{M}{1+\gamma M^2}\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{1}{2}} \quad ..(3)$$

where ρ is the density and V the flow velocity. Figure 1 is a plot of Fliegner numbers versus Mach number at $\gamma = 1.4$ for air. The total and impulse Fliegner numbers reach their maxima at $M = 1$,

$$\left[F(M)\right]_{max} = F(1) = 1.281 \qquad \ldots(4)$$

$$\left[I(M)\right]_{max} = I(1) = 1.010 \qquad \ldots(5)$$

When the Mach number approaches infinity, the limit of the impulse Fliegner number is:

$$\lim_{M \to \infty} I(M) = 0.707 \qquad \ldots(6)$$

The static Fliegner number increases monotonically without limit as the Mach number increases.

III. ISENTROPIC FLOW

For isentropic one-dimensional flow, in which m, C_p, T, P and γ are constant, the pressure ratio and area ratio can be written by applying Eqs. 1 and 2:

$$\frac{p}{P} = \frac{F(M)}{f(M)} \qquad \ldots(7)$$

$$\frac{A}{A_t} = \frac{F(1)}{F(M)} \qquad \ldots(8)$$

where A_t is the throat area at sonic conditions. In a de Laval nozzle, Eq. 7 and Fig. 1 show that for a given area ratio, F(M) corresponds to two Mach numbers; one is subsonic for the convergent portion and the other is supersonic for the divergent portion of the nozzle. Knowing C_p, T, P and A_t, the mass flow is determined from Eqs. 1 and 4:

$$m = \frac{1.281\ A_t\ P}{\sqrt{C_p\ T}} \qquad \ldots(9)$$

IV. NORMAL SHOCK WAVES

Across a normal shock, for which m, C_p, T, A, $p + \rho V^2$, and γ are constant, the following equation can be formed by applying Eq. 3.

$$I(M_x) = I(M_y) \qquad \ldots(10)$$

where the subscripts x and y denote upstream and downstream conditions respectively. Equation 9 and Fig. 1 show that the impulse Fliegner number, lying between 0.707 (Eq.6) and 1.010 (Eq. 5) corresponds to two Mach numbers; one supersonic and the othersubsonic, upstream and downstream of the shock respectively. Equation 7 can be alegebraically simplified to

$$M_y^2 = \frac{(\gamma-1)\ M_x^2 + 2}{2\gamma\ M_x^2 - (\gamma-1)} \qquad \ldots(11)$$

The static and total pressure ratios across the shock are (Eqs. 1 and 2.)

$$\frac{p_y}{p_x} = \frac{f(M_x)}{f(M_y)} \qquad \ldots(12)$$

$$\frac{P_y}{P_x} = \frac{F(M_x)}{F(M_y)} \qquad \ldots(13)$$

The Rayleigh supersonic pitot-tube formula becomes:

$$\frac{P_y}{p_x} = \frac{f(M_x)}{F(M_y)} \qquad \ldots(14)$$

The entropy increment due to the shock is

$$\frac{S_y - S_x}{R} = \ln \left(\frac{P_x}{P_y}\right)\left(\frac{T_y}{T_x}\right)^{\frac{\gamma}{\gamma-1}} = \ln \frac{F(M_y)}{F(M_x)} \qquad \ldots(15)$$

where R is the gas constant, and $T_x = T_y$. Hence, knowing M_x, Fig. 1 can be used to determine M_y, pressure ratios and entropy increment for the normal shock.

V. FANNO FLOW

For the case of adiabatic one-dimensional flow in a constant-area duct with wall friction (Fanno flow at constant m, T, A, C_p and γ), if the entrance conditions (subscript o) are known, and if the static pressure is obtained at a section, the Mach number and the total pressure at that section can be determined. Eqs. 1 and 2 give:

$$f(M) = \frac{F(M_o)}{\left(\frac{p}{p_o}\right)} \qquad \ldots(16)$$

Entering Fig. 1 with this value of $f(M)$ uniquely determines M and $F(M)$. From this, the total pressure ratio is

$$\frac{P}{P_o} = \frac{F(M_o)}{F(M)} \qquad \ldots(17)$$

And the entropy increment along the duct is

$$\frac{S - S_o}{R} = \ln\left(\frac{P_o}{P}\right) = \ln\frac{F(M)}{F(M_o)} \qquad \ldots(18)$$

Equation 18 gives a family of Fanno lines which may be plotted on an entropy temperature diagram.

When the Fanno flow is choked ($M = 1$) at exit, the exit conditions (subscript $*$) can be written as:

$$\frac{p_o}{p_*} = \frac{f(1)}{f(M_o)} \qquad \ldots(19)$$

$$\frac{P_o}{P_*} = \frac{F(1)}{F(M_o)} \qquad \ldots(20)$$

$$\frac{S_* - S_o}{R} = \ln\left(\frac{P_o}{P_*}\right) = \ln\frac{F(1)}{F(M_o)} \qquad \ldots(21)$$

Equation 21 for adiabatic flow shows that for a given M_o (either subsonic or supersonic), the entropy increment along the duct reaches its maximum at $M = 1$ since $F(1)$ is a maximum value (Eq. 4).

VI. RAYLEIGH FLOW

For the case of frictionless one-dimensional flow in a constant-area duct with heat transfer (Rayleigh flow at constant m, A, $p + \rho V^2$, c_p and γ), the entrance conditions can be used to determine the exit conditions at $M = 1$ (Eqs. 1, 2 and 3).

$$\frac{T_*}{T_o} = \left[\frac{I(1)}{I(M_o)}\right]^2 \qquad \ldots(22)$$

$$\frac{p_o}{p_*} = \frac{f(1)I(M_o)}{f(M_o)I(1)} \qquad \ldots(23)$$

$$\frac{P_o}{P_*} = \frac{F(1)I(M_o)}{F(M_o)I(1)} \qquad \ldots(24)$$

The entropy increment becomes

$$\frac{S_* - S_o}{R} = \ln\left(\frac{P_o}{P_*}\right)\left(\frac{T_*}{T_o}\right)^{\frac{\gamma}{\gamma-1}} = \ln\frac{F(1)}{F(M_o)}\left[\frac{I(1)}{I(M_o)}\right]^{\frac{\gamma+1}{\gamma-1}} \qquad \ldots(25)$$

Since F(1) and I(1) are at their maxima (Fig. 1), Eq. 25 gives the maximum entropy increment for a given M_o (subsonic or supersonic). In general the entropy change along the duct is:

$$\frac{S - S_o}{R} = \ln\frac{F(M)}{F(M_o)}\left[\frac{I(M)}{I(M_o)}\right]^{\frac{\gamma+1}{\gamma-1}} \qquad \ldots(26)$$

Equation 26 gives a family of Rayleigh lines which may be plotted on an entropy temperature diagram.

VII. CONCLUSION

In conclusion, Fliegner numbers, as introduced, do not add anything new to the theory of one-dimensional compressible fluid flow, but the application of Fliegner numbers simplifies the methematical derivations and calculations for ordinary flow (Ref. 5). In addition, as long as the flow is one-dimensional at constant specific heats, Fliegner numbers can be applied to flow processes with injection or extraction of gases (Ref. 2)

VIII. REFERENCES

1. Jamison, R.R. Mordell, D.L. — The Compressible Flow of Fluids in Ducts. ARC Rm 2031, 1945.

2. Wu, J.H.T. — On a Two-Dimensional Perforated Intake Diffuser. Aerospace Engineering, July 1962

3. Shapiro, A.H. — The Dynamics and Thermodynamics of Compressible Fluid Flow. Ronald Press, New York, 1953.

4. Dailey, C.L. Wood, F.C. — Computation Curves for Compressible Fluid Problems. John Wiley, New York, 1949.

5. Wu, J.H.T. Patel, R.P. — A Simple Experiment for a de Laval Nozzle and a Fanno Tube. TN No. 62-4, Mechanical Engineering Research Laboratories, McGill University, July 1962.

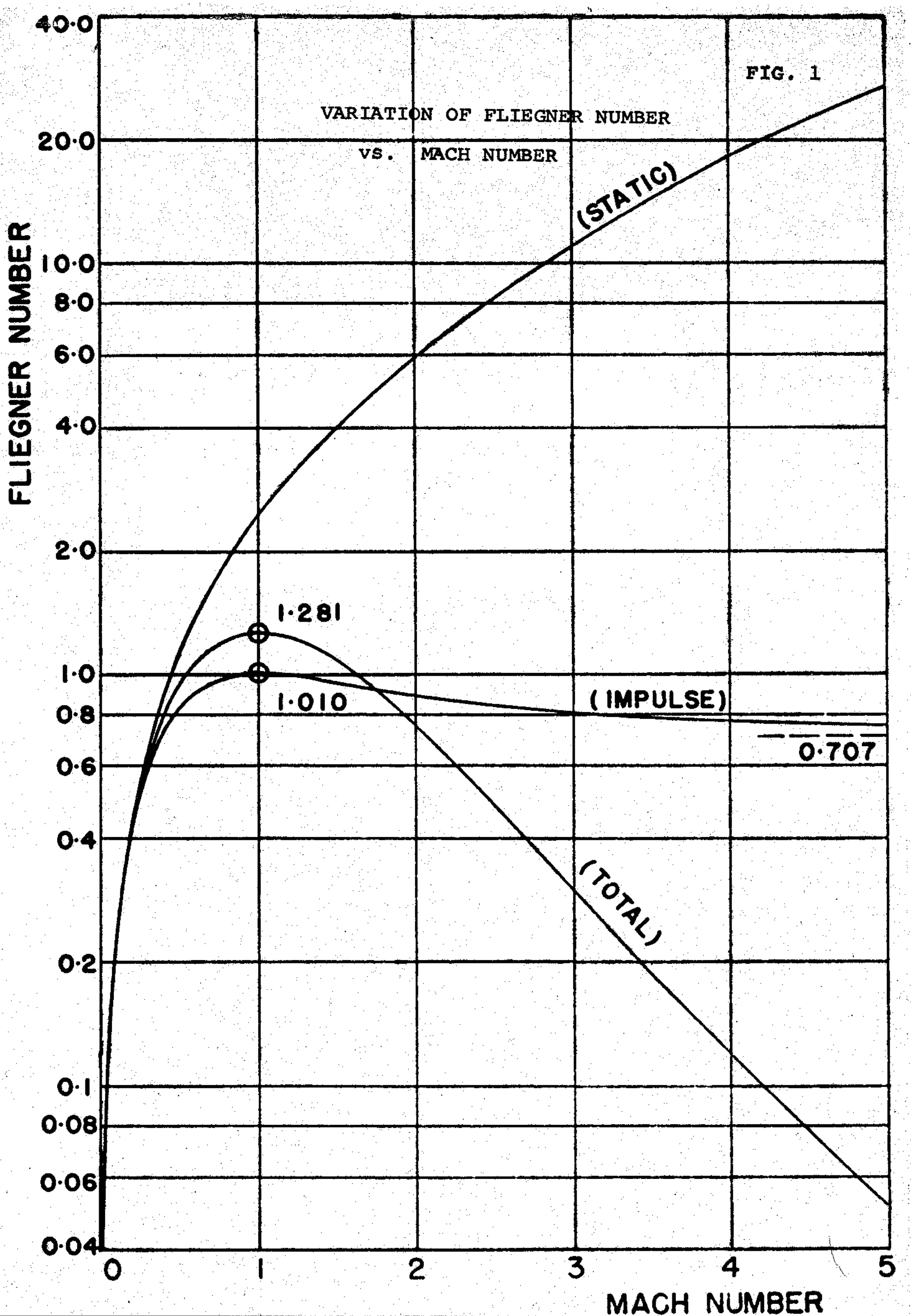
FIG. 1
VARIATION OF FLIEGNER NUMBER
vs. MACH NUMBER
FLIEGNER NUMBER
MACH NUMBER
40·0
20·0
10·0
8·0
6·0
4·0
2·0
1·0
0·8
0·6
0·4
0·2
0·1
0·08
0·06
0·04
0
1
2
3
4
5
(STATIC)
(IMPULSE)
(TOTAL)
1·281
1·010
0·707