TORSIONAL RESISTANCE OF A STEEL BEAM HAVING STIFFENERS

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CONTENTS

ABSTRACT

This investigation deals with the behavior of a simply supported 8WF17 beam, fifteen feet long, commonly used as a spandrel when a plate is welded to the bottom flange to carry the eccentric wall.

The test was conducted at three torques equivalent to a two, four, and six inch brick wall ten feet high resting eccentrically on the bottom plate producing bending and large torsional stresses.

Pairs of stiffeners were bolted to the beam at the fifth, quarter, third, and centre points and the effects on the angle of twist, flange, and web stresses studied. The change in position of the neutral plane was also examined.

The observed test data was compared with the theoretical based on the Lyse-Johnston, and Grinter methods of design.

The experiment stopped after permanent yielding occurred in the web and a kink developed in the top flange.

NOTATION

The following symbols were adopted for use in this paper. An attempt was made to conform as close as possible to those commonly found in text books on this subject.

a	torsional bending constant.
A	area.
A_1, B_1, etc.	coefficients of a differential equation.
b	width of flange of beam.
B	torsional flange stress constant of beam.
d, or h	depth of beam.
D	diameter.
e	eccentricity of applied load.
E	Young's modulus of elasticity (29,000,000).
f_t	longitudinal flange stress due to torsion.
f_b	longitudinal flange stress due to vertical bending.
G	modulus of elasticity in shear.
I	moment of inertia of area.
J	polar moment of inertia.
K	torsional constant.
L	length.
M	bending moment.
N	torsional flange shear constant of beam.
r	radius.
S	section modulus.
S_1	torsional shear function.
s_t	web shear stress due to torsion.

s_v	web shear stress due to vertical loads.
s_f	flange shear stress from torsion.
s_q	flange shear stress from lateral bending.
t	thickness.
T	torque or twisting moment.
w	uniform vertical load per foot of beam.
x,y,	Cartesian coordinates.
Z	torsional web shear constant of beam.

Greek Letter Symbols.

μ (mu)	Poisson's ratio, (0.3.).
ψ (psi)	angle of twist.
τ (tau)	shear stress.
σ (sigma)	principal tensile or compressive stress.

CHAPTER I

INTRODUCTION

In the design of a structural steel framework it is considered good practice to avoid subjecting I - beams to the combined effect of bending and torsion, because the torsional resistance of these beams is usually low. It is often easy to arrange a layout in which torsion is either entirely eliminated or greatly reduced. Sometimes however, this is impracticable or impossible, and cases exist where beams carry eccentric loads and consequently are stressed in both bending and torsion.

Many examples of this type of loading can be found in industrial buildings as a result of the complexity of machinery and processes. Since appearance is a secondary feature, no special attention is paid to the fact that these beams may deflect, and rotate slightly more than under pure bending conditions. This however is not the case in building design.

In recent years there has been a marked increase in public demand for a more asthetic appearance in new buildings. Continuous windows have become synonymous with modern architecture and as a result the structural engineer is called upon to design the spandrel beams to support the eccentric wall above the windows which will on the one hand not deflect excessively to destroy the appearance of the structure and possibly crack the masonary, nor on the other hand be overdesigned, thus increasing the cost of the steel work.

It is known that to prevent excessive stresses, and rotation, of the marginal beam loaded eccentrically a member of much larger cross-section is needed than when carrying the same load in pure bending. It is therefore quite logical that a method be found for increasing the torsional rigidity of a beam without increasing the weight appreciably while keeping the fabrication costs as low as possible.

It is the purpose of this experiment to investigate whether or not placing pairs of stiffeners, at the fifth, quarter, third, and centre points of a beam has any beneficial effects in improving the torsional properties. As far as can be learned from published works this has never been attempted before. It is also intended simultaneously to compare the observed results with that predicted on the basis of existing formulae and design procedures commonly in use today.

In the subsequent work use has been made freely of all existing literature on the subject and due acknowledgement is made in the bibliography.

HISTORICAL NOTE

The history of the development of the principles of torsion can be found well documented in any recent literature on the subject, (2, 15),* however to place this work in its proper perspective a brief summary is necessary. Generally the investigations consisted in first determining the value of the torsional constant K,+ by theoretical methods, then by experimental ones and finally after the value of the torsional constant was known experimenters approached the task of calculating the torsional stresses and angle of twist of structural members with particular emphasis on I-beams and channels.

The problem of pure torsion as applied to noncircular sections was first treated correctly by Saint-Venant (13) in 1855, who developed the general solution for the torsional constant applicable to any cross-section. Saint-Venant derived solutions for a number of figures, namely rectangular, triangular, and elliptic. These solutions consisted of complicated mathematical methods often involving infinite series. Many important practical structural sections, such as channels, and I-beams could not even be reduced to mathematical formulae, and it became evident that a new approach was necessary which simplified the original theory.

In 1903, Prandtl (12) showed that if a thin membrane was stretched across a hole having the shape of the cross-section in

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* Numbers in brackets refer to references in the bibliography.

+ J is the polar moment of inertia for a circular cross-section. For noncircular sections, this property is designated as K. K is less than J but has the same units.

question and distorted slightly by pressure on one side, then the differential equation of its surface was identical to the differential equation for a member subjected to torsion. Prandtl further showed that by measuring the volumn and surface properties of the displaced membrane a direct measurement of the torsional rigidity and stress was obtainable, as well as a visual picture of the distribution of stress across the section.

Prandtl's analogy, with a thin soap film as a membrane, was used in many torsion investigations. The outstanding ones were by Taylor and Griffith (5) in England, and in the United States by, Trayer and March (19), and by Lyse and Johnston (9).

Timoshenko (17) shortened the pure torsional theory by slight modifications of Saint-Venant's equations and by mathematical applications of the principals of the membrane analogy. He was also among the first to consider the effect produced by preventing the warping of a cross-section.

A laboratory procedure first used by Young and Huges (20) and later more correctly by Lyse and Johnston (9) to evaluate K, is to subject the member to a shaft torque under uniform torsion conditions. The torque is plotted against the unit angle of twist. The torsional rigidity is obtained from the slope of the straight line portion of the curve.

Lyse and Johnston (9) conducted a series of experiments using the membrane analogy, as well as the actual beams in the laboratory. The tests were on torsionally free, and torsionally fixed ended sections. They derived and checked equations for the evaluation of K for I and wide flange shapes. These have now been generally adopted for use and the value of the torsional constant can be considered as solved. A basic differential equation for the stress in the flanges of a beam subject to torsion was also derived.

Subsequent theoretical work on torsion was done by Sourochnikoff (15) who considered the change in eccentricity of a vertical load on a beam after twisting, and by Goldberg (3) who introduced the angle of warping in an attempt to simplify calculations.

Experimentally Chang and Johnston (2) tested plate girders in torsion by applying a couple at one particular point and studying the effects on rivets, plates etc.

CHAPTER II

THEORY

Limitations of The Simple Torsion Theory.

A member is subjected to torsion if a couple is developed at each cross-section of the member, the plane of the couple being normal to the axis of the member.

The torsional couple is the principal component in a shaft. However, the torsional couple is also an important component in many members of noncircular cross-section.

The well-known torsion formula Eq. (1) for the evaluation of

$$\tau = \frac{Tr}{J} \qquad \text{(1)}$$

the stresses in a shaft having a circular cross-section is based on several assumptions.

1. Statics.

   a. The resultant of the external forces is a couple that lies in a plane perpendicular to the longitudinal axis of the shaft.

   b. The shaft is in equilibrium.

2. Geometry.

   c. The axis of the shaft is straight.

   d. The shaft is circular and free from changes in cross-section.

   e. A plane section normal to the axis of the shaft before twisting remains plane after the shaft is twisted.

   f. Any diameter before twisting remains a straight line after twisting.

3. Properties of the Material.

   g. The material is homogeneous and isotropic.

   h. The stresses do not exceed the proportional limit of the material.

If assumption (a) does not hold, the external forces may be resolved into the components that produce axial loading alone, bending alone, or twisting alone. Each effect may be dealt with separately and the results added. If assumption (b) does not hold, as in a condition of dynamic loading, the stresses may be higher than those given by the torsion formula. If the axes of the shaft is not straight, bending may be introduced even though the loading requirement of (a) is satisfied. Assumption (d) is necessary to eliminate the necessity of considering stress concentration. Assumption (e) is valid for a solid circular shaft or a circular tube of constant diameter and wall thickness but is not valid for other shapes. Assumption (h) places a definite upper limit on the valid range of the formula.

Now consider a plane x,y, of a rectangular cross-section, Fig. 1 subjected to a couple.

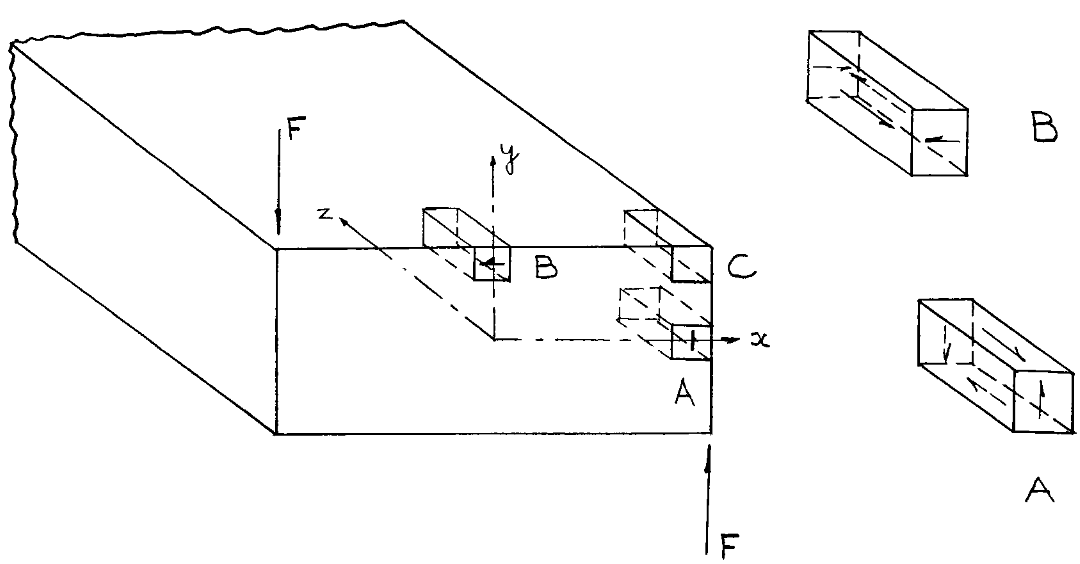

Figure 1

It can be seen that there can be no vertical or horizontal shearing forces on the right hand face of element A. The only shearing forces that can act on block A are a vertical shearing force on the front and back faces, and a longitudinal shearing force on the top and bottom faces. Similarly block B can have no shear on the top face and hence, the shearing force on the front face must be horizontal. Since block C can develope no shearing force on either outside face, it follows that there can be no shearing force on the front face. Or the shearing stress at the corner of the cross-section must be zero, which is not the result that one would obtain from Eq. (1) and consequently it is not valid for any section other than circular.

It is also apparent that in a rectangular shaft the vertical component of the shearing stress near the vertical edges must vary from a maximum at the centre of a side (atA) to zero at the corner (atC), and that the horizontal component near the top must vary from a maximum at the centre (atB) to zero at the corners.

The fundamental reason for the failure of the simple shaft formula to apply to noncircular cross-sections is that they undergo longitudinal warping of the cross-section. That is, the distortion of the cross-section, in such a manner that one corner comes forward along the axis, while the other recedes and plane sections before twisting no longer remain plane after twisting.

The evaluation of the stresses in members of noncircular cross section can be accomplished in several ways, some approximate and some exact.

In the exact method devised by Saint Venant (13), the components of stress are determined so that they satisfy the equations of equilibrium as applied to a isolated block, at any point in the member, and also the stress-strain relationships. However this procedure is limited by the mathematical difficulties involved in solving the differential equation developed.

Another exact method consists of setting up the member in a laboratory and measuring the strains at selected stations corresponding to different conditions of loading. This is physically impossible to do in most cases and we must resort to approximate methods.

In dealing with structural shapes two principal types of section require consideration, the rectangle, and the rectangle modified by sloping sides, as in the flange of an I-beam. In the case of a rectangle an accurate formula was originally derived by Saint Venant (13), determining the torsional constant.

$$K = \frac{1}{3} w^3 t - .21 w^4 \quad - - - - - \quad (2)$$

where w = long side, t = short side, the last term, or end loss effect, is very small and can usually be neglected.

A wide flange beam with parallel sided flanges can be considered as an aggregation of three component rectangles.

Now Eq. (2) can be written

$$T = \frac{G\gamma}{L} \sum \frac{1}{3} w^3 t \quad - - - - \quad (3)$$

in which w is the total width of the rectangle in which the stress is desired and

$$J = \frac{1}{3} \sum w^3 t$$

It was immediately noticed that this equation gave poor results because the value of J was not the polar moment of inertia calculated from the actual areas but less. Furthermore J does not correspond to

$\frac{1}{3}\sum W^3 t$ because of the effect of the fillets.

Lyse and Johnston (9), derived expressions, based on experimental data for a correct value of J. These of course take into account the fillets as well as the effect of tapered flanges.*

In the preceding discussion the evaluation of torsional stresses was based on the assumption that no longitudinal stresses are developed in the torsional member, or that each cross-section is free to warp. However if a cross-section is restrained from warping, the longitudinal stresses developed will add to the longitudinal bending stresses.

As the stresses in a beam are greatly influenced by the connection details we must define very clearly the state of affairs at the ends.

In a "torsionally free" end connection the beam is prevented from rotating throughout its depth i.e. the web remains vertical, but the flanges are pin ended and can develop no moment at their ends. This condition is realized for all practical purposes by the standard framed beam connections and is the case in this experiment. In "torsionally fixed" ends the beam web remains vertical on twisting but in addition, the flanges are fixed, thus developing fixed end moments at their point of support. This condition can only be realized if the end of the beam is welded to a transverse heavy slab and is seldom found in structural work.

A beam may be simply supported with respect to vertical bending, and either free or fixed-ended in torsion; it may be fixed-ended with

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* The value of J (or K for noncircular sections), as well as a,B, C,N, and Z, based on formulae derived by Lyse and Johnston, can be found worked out and tabulated for all beams rolled by the Bethlehem Steel Co. Reference 18.

respect to vertical bending, and either free or fixed-ended in torsion.

A second distinction is to be made between beams subjected to "pure torsion" i.e., loaded by external couples; and beams supporting vertical loads applied otherwise than in the plane of the web.

The resulting torsional stresses are the same for both methods of loading. However in the second case the stress due to pure bending caused by vertical moment and shear must be added to obtain the combined stress.

Another feature of loading the beam eccentrically with vertical load is that the eccentricity increases due to the angle of twist. In this investigation it is assumed that the eccentricity remains unchanged after loading. In fact in our case since the eccentricity is of the order of fourteen, to twenty-three inches the change is extremely small and the effect can be ignored. Calculations have shown that this effect is of the order of two per cent when the eccentricity is two to four inches and getting smaller as e increases.*

Definitions of Torsional Stresses.

The following stresses associated with torsion are clearly defined.

f_t is the longitudinal stress in the flange due to torsion. Across the width of the flange, it is + at one edge, - at the other, varying uniformly in between; but is constant throughout the flange thickness at any given point. To combine the longitudinal stress f_b due to vertical bending, the + and + are added to obtain the

* For a complete rigorous analysis of the change in stress due to a change in eccentricity caused by loading see reference 15.

longitudinal stress on the outer corner of one flange, while - and - are added to obtain the same on the diagonally opposite outer corner of the other flange. The location of the maximum stress f_t along the beam is the same as that of f_b, in the case of a beam free ended in torsion and pin connected, as in our case. However in the case of beams fixed with respect to vertical bending but free-ended in torsion, it is impossible to locate the most highly stressed section by inspection and a trial calculation is necessary. Note that in the case of a shaft with pure torsion between ends, f_t and f_b are both zero.

s_t is the shearing stress in the beam web, due to torsion. It is a maximum at the support where the torque is greatest. The torsional shearing stress is a maximum and + on one surface of the web, a maximum and (-) on the other surface, but does not vary uniformly between. It is slightly greater at mid-depth of the web than elsewhere. The shearing stress from vertical bending s_v is greatest at the neutral axis. In a vertically symmetrical section, the maximum combined web shearing stress will occur at mid-depth, on that surface of the web upon which the shear due to torsion and that due to bending have the same sign.

s_f is the transverse or shearing stress in the flange due to torsion. It is a maximum on the centre line of the top surface of the flange. Along the span it varies with the torque.

s_q is shearing stress due to lateral bending of the flange. This stress varies from zero at the edges to a maximum at the flange centre line, being constant throughout the flange thickness. In the case of rectangular flanges the curve is a parabola hence

$$s_q = 1.5 \frac{V_q}{bt}$$

where V_q = lateral bending shear.

Theories of Design

The two methods used to calculate the stresses in a beam subject to torsion are commonly referred to as the Grinter, and the Lyse-Johnston.

The Grinter Method*

The Grinter (6) method is generally used in design and is considerably simpler than the Lyse-Johnston. It consists essentially of considering the torque per foot, we, as in the case of a continuous eccentric load w Lb/Ft being resisted by the top and bottom flange each having a load of $P = \frac{we}{d}$ Lb./Ft. acting along its length Fig. 2 . The flanges are assumed to be simply supported at the ends and act as

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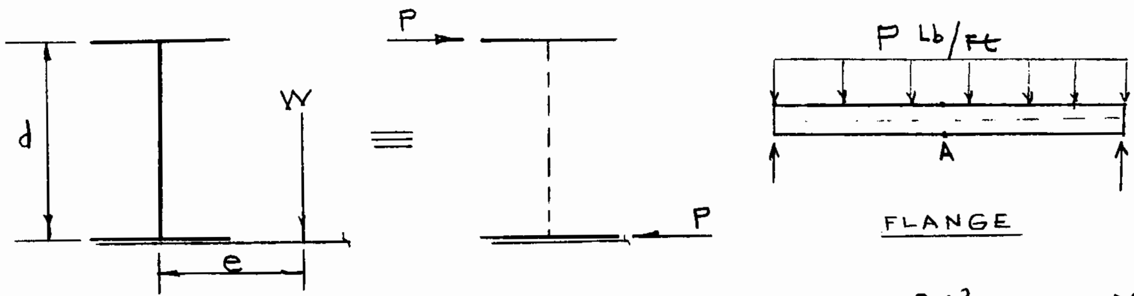
* This method is described in Grinter's book, "Design of Modern Steel Structures" (6). It was not originated by the author nor does he defend its validity. It is referred to as the Grinter method in this thesis for convenience.

FIGURE . 2.



$T/ft = we$ $\qquad$ $P/Ft = \frac{we}{d}$ $\qquad$ $M_{\text{℄}} = \frac{P\ell^2}{8}$ $\qquad$ $f_A = \frac{M_{\text{℄}}\, y}{I}$

simple beams detached from the web. The maximum stress will occur at point A the edge of the flange at the centre line of the beam. The bending stress is added algebraically to this stress, due to the torque, and a beam is chosen so that the final result is under the allowable stress. This method in no way takes into account the web which is of great significance. Also nothing at all is given with respect to the angle of twist of the beam, which in many cases may be even more important than the stresses or the effect of the end connection.

The Lyse Johnston Method.

The Lyse-Johnston (9, 18) is the more rational approach, and as will be seen yields results accordingly.

The basic differential equation for the centre of a beam flange distorted by torsion only, was shown by Lyse-Johnston to be:

$$a^2 \frac{d^3y}{dx^3} - \frac{dy}{dx} = \frac{-2a^2T}{hEI_Y} \qquad \text{WHERE} \qquad a = \frac{h}{2}\sqrt{\frac{EI_Y}{KG}}$$

and $T =$ external torque at point x, y, from the end.

The general solution for any case of loading and end connection is (18).

$$y = \frac{2a^3T}{hEI_Y}\left(A_1 \sinh\frac{x}{a} + B_1 \cosh\frac{x}{a} + C_1 x^2 + D_1 x + E_1\right)$$

denoting by (F) the function

$$\left(A_1 \sinh\frac{x}{a} + B_1 \cosh\frac{x}{a} + C_1 x^2 + D_1 x + E_1\right)$$

It can be shown that:

Deflection. $$y = \frac{2a^3T}{hEI_Y} \cdot (F)$$

Slope. $$\frac{dy}{dx} = \frac{2a^3T}{hEI_Y} \cdot \frac{d(F)}{dx}$$

Flange moment. $$M = \frac{EI_Y}{2} \cdot \frac{d^2y}{dx^2} = \frac{a^3T}{h} \cdot \frac{d^2(F)}{dx^2}$$

Flange stress from torsional bending. $$f_t = M\frac{b}{I_Y} = \frac{a^3T}{h} \cdot \frac{b}{I_Y} \cdot \frac{d^2(F)}{dx^2} \quad \text{---} (4)$$

$$= BTa^2 \frac{d^2(F)}{dx^2} \quad \text{WHERE } B = \frac{ab}{hI_Y}$$

Flange shear from torsional flange bending. $$V_q = \frac{dM}{dx} = \frac{EI_Y}{2} \cdot \frac{d^3y}{dx^3} = \frac{a^3T}{h} \cdot \frac{d^3(F)}{dx^3} \quad \text{--} (5)$$

Or. $$V_q = \frac{T}{h} \cdot \frac{1}{2} \cdot \frac{1}{\left(\frac{L}{2a}\right)} \tanh\frac{L}{2a} \quad \text{AT } x = 0 \quad \text{--} (5a)$$

Flange unit shearing stress. $$S_q = 1.5\,\frac{V_q}{bt} \quad \text{------} (6)$$

Torsional flange shear. $$S_f = S_1 NT, \quad \underset{\text{AT } x=0}{S_1} = \frac{1}{2}\left[1 - \frac{\tanh\frac{L}{2a}}{\frac{L}{2a}}\right] \quad \text{--} (7)$$

Twist angle in radians. $$\psi = \frac{2y}{h} \cdot \frac{4a^3T}{h^2EI_Y}(F) = \frac{aT}{KG}(F) = CT(F) \quad \text{--} (8)$$

Rate of twist in radians per inch. $$\frac{d\psi}{dx} = CT\,\frac{d(F)}{dx}$$

It can be shown that for the solution of the differential equation in our case.

$$A_1 = -\frac{\tanh\frac{L}{2a}}{2.\frac{L}{2a}} \quad ; \quad B_1 = \frac{1}{2.\frac{L}{2a}} \quad ; \quad C_1 = -\frac{1}{2aL}$$

$$D_1 = \frac{1}{2a} \quad ; \quad E_1 = -\frac{1}{2.\frac{L}{2a}}$$

An expression for the torsional web shear suggested by Ref. (18) based on the experimental work of Lyse-Johnston is:

$$s_t = S_1 Z T \qquad - \quad - \quad - \quad - \qquad (9)$$

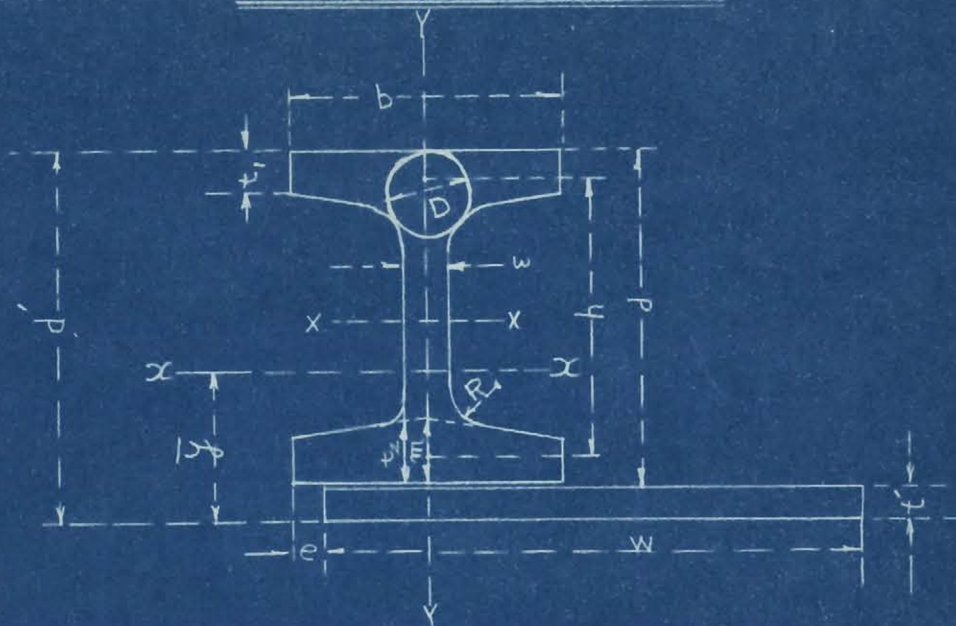
No quantitative theory exists regarding the effect of stiffeners or the bottom plate. In general the plate will increase the moment of inertia and consequently decrease the bending stresses as well as change the centre of gravity of the section. The stress along the bottom of the plate is not uniform since the plate is not welded on symmetrically with the vertical axis of the beam, resulting in slight discrepancies of vertical bending stress from the theoretically calculated. Most designers omit the effect of the bottom plate in their calculations. With respect to the torsional stresses, the Grinter method would not be changed. Since the top flange will govern the choice of beam it is of no concern that the stresses in the bottom flange will drop due to the plate.

In the Lyse-Johnston method it can be seen that for calculating the stresses in the flanges the moment of inertia of the cross-section along the Y axis enters the differential equation as $\frac{I_Y}{2}$ or one flange is only considered with the assumption that the stresses in the other one are similar. All other properties of a particular beam, such as a, C, B, K, Fig. 3 etc., enter the equation for the beam only. Hence to calculate the stresses in the flanges and plate it is necessary first to obtain new values for the above constants as well

FIG. 3

PROPERTIES OF SECTION

8 WF 17 BEAM



FOR ROLLED BEAMS WITH TAPERING FLANGES

$$K^* = \frac{b-w}{6}(t_1+t_2)(t_1^2+t_2^2)+\frac{2}{3}wt_2^3+\frac{1}{3}(d-2t_2)w^3+2\alpha D^4-Et_1^4$$

IN WHICH $\alpha = 0.066 + 0.021\frac{w}{t_2} + 0.072\frac{R}{t_2}$; $E = 0.44104$ FOR 5% FLANGE SLOPE

$$D = \frac{(F+m)^2 + w\left(R+\frac{w}{4}\right)}{F+R+m}$$ IN WHICH

$$F = \frac{R}{20}\left(19.0250 - \frac{w}{2R}\right)$$ FOR 5% FLANGE SLOPE

$$a = \frac{h}{2}\sqrt{\frac{EI_Y}{KG}} = 0.806h\sqrt{\frac{I_Y}{K}}$$

$$B = \frac{ab}{hI_Y} \qquad C = \frac{a}{KG} \qquad \frac{E}{G} = 2.6$$

$$N = \frac{D+t_2}{2K} \qquad Z = \frac{w+0.3R}{K}$$

b	5.250 IN	d	8.00 IN	A	5.00 IN^2
w	0.230 IN	t'	0.375 IN	$\bar{y}$	2.23 IN
t_1	0.25 IN	t_2	0.375 IN	d'	8.375 IN
W	14.00 IN	R	0.30 IN	e	0.25 IN
I_{xx}	56.4 IN^4	I_{YY}	6.72 IN^4	Z	2.00 IN^{-3}
I_{xx}	86.4 IN^4	a	40.11 IN	B	4.081 IN^{-3}
K	0.16 IN^4	C	0(4)22 $(IN\text{-}lb)^{-1}$	N	2.831 IN^{-3}

* SEE REF. 9 AND 18

as I_Y . This would be impossible and inaccurate since the bottom plate is welded to the flange and elastic action is not assured. From practical considerations we must confine our calculations to the stress in the beam only and the added effect of the plate which serves to strengthen the beam can be considered as overdesign material, unless the designer is willing to raise the allowable stress.

From the bending theory $f_b = \frac{My}{I}$ the stiffeners are of no significance as far as the vertical bending stress is concerned. In a plate girder the primary use of intermittent stiffeners is to keep the web from buckling. Bearing stiffeners are used to prevent deformation of the flanges and web at points of very high shear, or where concentrated loads rest. In a beam subjected to torsion caused by an eccentric load the stiffeners have three functions to fulfill. First they hold the flanges together, i.e. they prevent the bottom flange, which holds the load from twisting excessively with respect to the top flange and web. Second they act to prevent warping of the cross-section, and third they prevent the web from distorting.

The first fulfillment is the most important. By holding the flanges together the rigidity of the entire beam is raised with a corresponding drop in the stresses in the flanges at the central part of the beam. Furthermore high vertical components of web stress are prevented. This will be discussed later in the thesis. The second function acts to strengthen the rigidity of the beam but to only a very small extent in our case, since the stiffeners are bolted onto the beam; small longitudinal movements could be expected as the rivets do not fill their holes entirely and warping is not appreciably stopped.

The third consideration becomes more important for thin webs (Ref. 4) where the distortion as in Fig. 4 is pronounced. This is even more

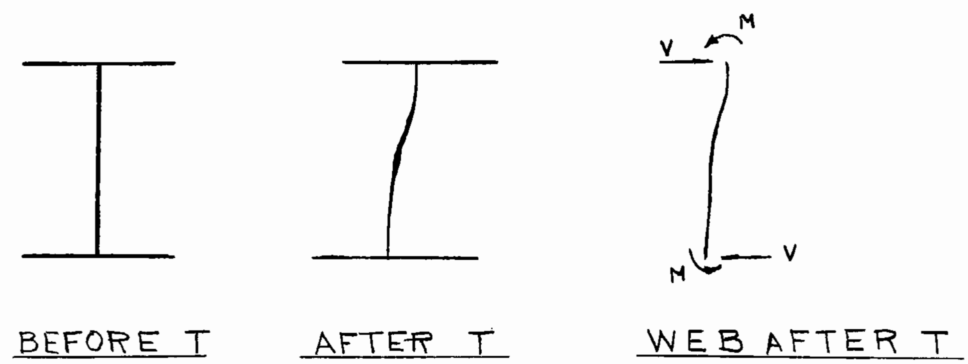


FIGURE 4

prominent when a built in end, or section of symmetry is prevented from warping, causing not only the bending of the flanges in their own planes but also a deformation of the web cross-section. In our beam this phenomena is negligible because the web is not thin i.e. 0.15 inches or less, and the end connection is such as to allow warping. Note that the stress due to this deformation of the web should not be confused with that caused by the twisting of the flanges at different angle due to the applied loading.

The equations duscussed in the theory are based on the assumption that the stresses do not exceed the proportional limit of the material. If the stresses do exceed the proportional limit the maximum stresses are less and the angle of twist greater than the values given by the formulae, and stress may no longer be an adequate criterion of safety.

CHAPTER III

MATERIALS USED IN TEST

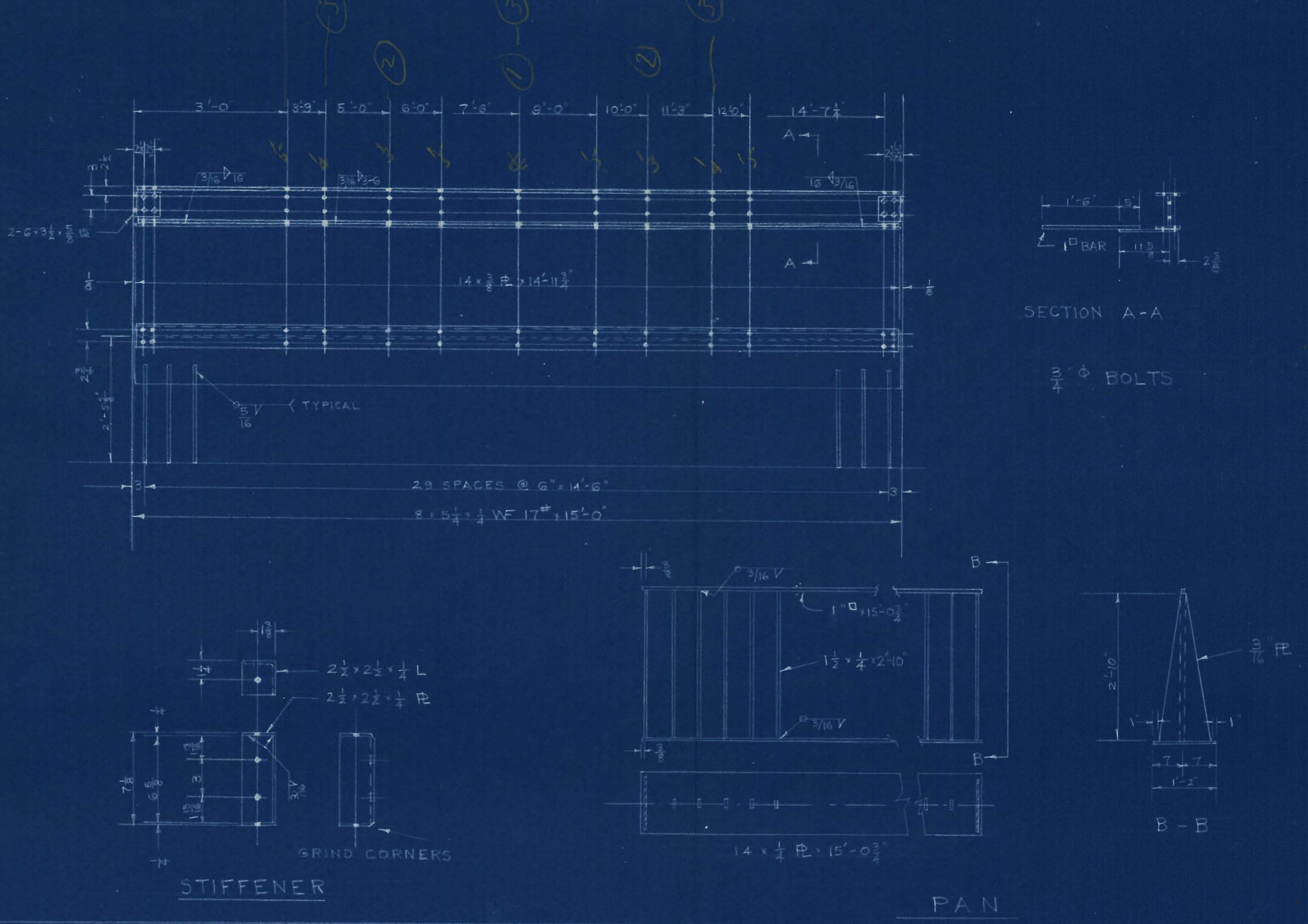
The Beam

When choosing the beam several important factors were to be resolved. On the one hand we wanted a beam of weight and length which is generally used in practice as a spandrel, not designed specifically to resist torsion but used for a particular wall system causing bending in the plane of maximum moment of inertia. On the other hand to achieve the same effects in the laboratory would mean building up a wall section several feet high. The difficulty of doing this was evident. To obtain the proper conditions of loading it became apparent that the available testing machines were not suitable, because we wanted to get a continuous load while the testing machines were only capable of applying one, or two, concentrated loads. Also the beam would have to be very strongly supported at both ends to prevent the large torques from overturning it as well as bearing the vertically applied loads. In addition extensive devices for measuring the angle of twist, and strains, would be fixed on to the beam. Consequently to move all this apparatus safely into the testing machine, and out, would be a very difficult as well as laborious task. As a result it was decided to use concrete blocks as our load. This would have the advantage of assureing a uniform load per foot, and once the beam was in place no other movement was necessary. A beam fifteen feet was chosen as being of average length and practicle for a laboratory experiment.

Consider the range of torques which can act on such a beam in an actual case. It was assumed that a maximum eccentric moment found in practice was an eight inch brick wall ten feet high at a mean eccentricity of about nine inches. This condition could be achieved in the laboratory by placing a load of two hundred pounds per foot at an eccentricity of roughly twenty-eight inches. Now in choosing the beam although the torque per foot applied would produce the same torsional stresses as when the beam was in service, nevertheless the actual load of two hundred pounds plus the weight of the beam, was very small resulting in correspondingly low stresses due to direct bending. If a beam was chosen which would be used in practice on the bases of a ten foot high wall consisting of an eight inch thick eccentric brick exterior facing and six inch of interior block resting directly on the top flange we would probably use an 8WF60, however if this beam was tested in the laboratory with only two hundred pounds per foot of load, although the torque per foot is correct, the fact that the remainder of the load is missing would result in obtaining extremely small deflections, and stresses which would be not only very difficult to measure accurately but highly questionable. It becomes self evident that a beam had to be selected which will also give a reliable set of readings for the range of loadings we were capable of applying experimentally. After careful consideration it was agreed to use an 8WF17, although slightly smaller than would be used for this particular case of loading it was nevertheless a beam used commonly for spandrels supporting walls of fifteen feet length. Figure 5 shows the fabrication details of the beam. Holes were punched in the beam at selected stations for mounting the stiffeners. It was assumed that these holes did not alter the

FIG. 5

FABRICATION DETAILS OF BEAM, STIFFENER, AND PAN.



capacity or characteristics of the beam.

The Bottom Plate

The thickness of the plate was governed by the necessary bending moment with the maximum fibre stress not exceeding 20,000 p.s.i. The width of the plate was such as to hold an eight inch exterior wall on it. Another consideration was that the projecting rods were welded to the top of the first five inches of the plate. The plate was welded to the bottom flange of the beam.

Rods

The rods on which the eccentric load rested consisted of one inch square bars at six inch centres welded on to the top of the plate. There was a small disalignment of the elevation of the rods probably caused by different rates of cooling of the weld material. To assure uniform bearing on all rods minor adjustments were made with shims of thin aluminum foil. This was later shown to be satisfactory and as expected the load applied acted as it was designed to i.e. continuous.

Stiffeners

In practice the stiffeners would naturally be welded on to the beam at the required points. This would have limited the investigation to observations involving only one position of the stiffeners. To avoid this a stiffener was designed which could be removed and affixed to the beam by bolts. It consisted of a $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ angle, with top and bottom plates welded on Figure 5. It was also machined for perfect alignment and shims were forced in to obtain tight fits. The stiffeners were applied in pairs and held by two bolts against the web of the beam, and two at the top, and bottom, flanges. All the bolts used on the beam were high strength type. It was assumed that the bolts acted similar as rivets of the same diameter i.e. 3/4 inch. This assumption is believed to be correct (8). To what extent this system in fact behaved as welded stiffeners is difficult to say. Welded stiffeners would have been much more effective in preventing warping, however the results obtained were consistant and showed relative variations in stress although the magnitude may have been slightly different.

End Connection

At each end a standard connection was used with high strength bolts. Throughout the experiment close inspection indicated that no deformation or slipping took place and the web was held vertically as originally connected.

The Stands

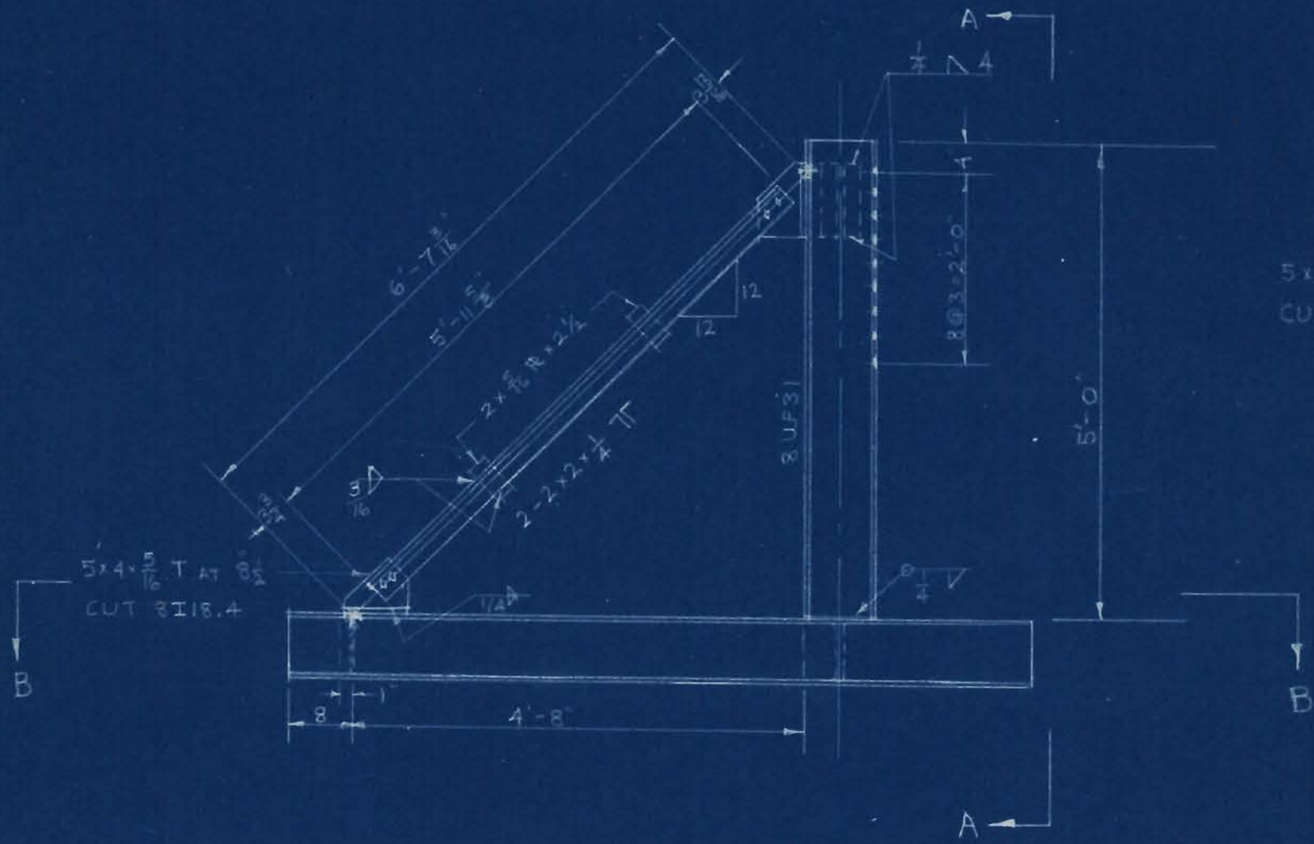
The stands Fig. 6 and Photo 4 were designed and fabricated with the expressed purpose of providing an unyielding support for the beam during this complete test, as well as mounting all types of beams for future investigations.

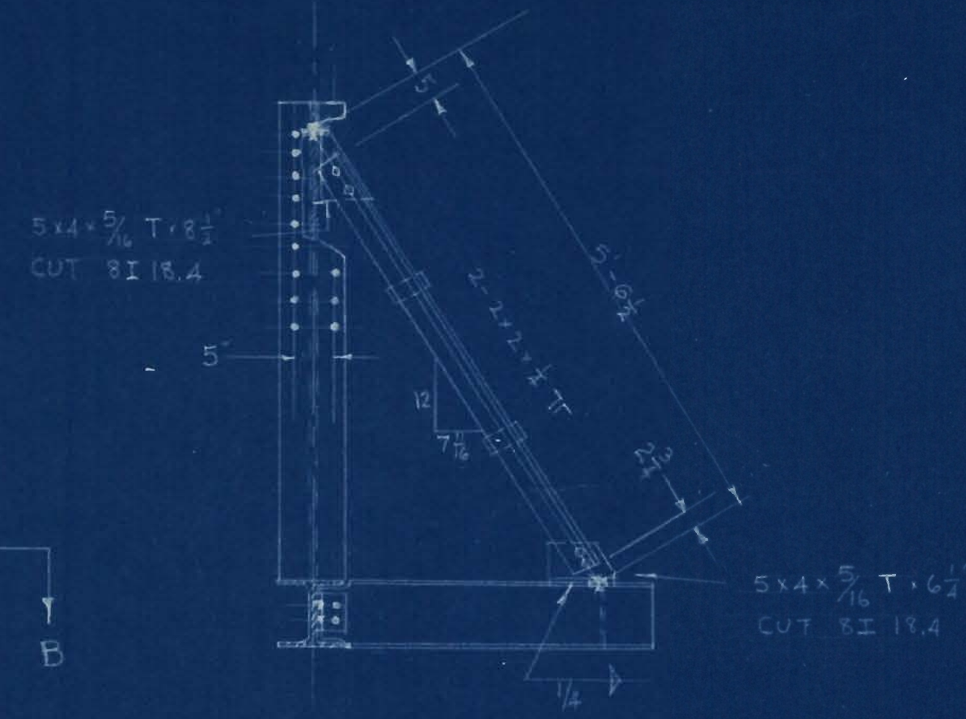
It consisted essentially of a 8WF31 column held vertical by two braces and standing on a stable base. Provisions were made for minute adjustments of the verticallity of the column. Holes were provided all the way up the flange of the column so that the beam could be raised, or lowered, while different clip angles could be used depending on the

FIG. 6

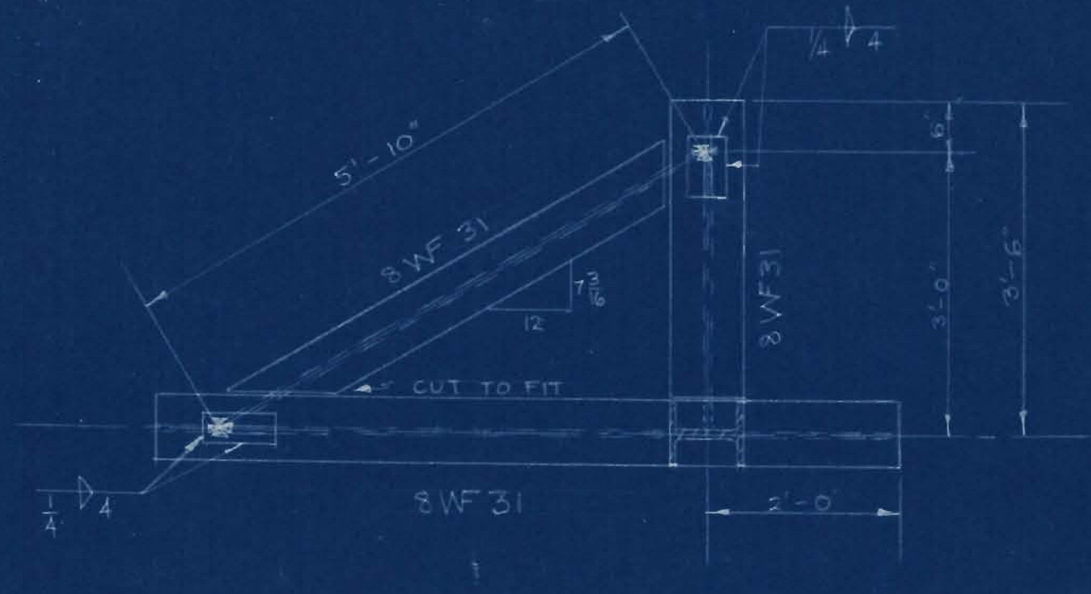
FIG. 6

FABRICATION DETAILS OF BRACKETS





SECTION A-A



SECTION B-B

1 - BRACKET - A2R - RIGHT

1 - BRACKET - A2L - LEFT

depth of beam desired. By moving the stands apart or together any length of beam could be held.

After the beam was connected to the stands a final adjustment was made to get the column exactly vertical with shims and the beam perfectly level. The stand base was then completely filled with grout to assure stability. To prevent overturning four blocks weighing two hundred pounds each were placed on each stand.

Subsequent results proved this arrangement to be entirely satisfactory and periodic inspection showed that the columns remained vertical.

Strain Measurements

Electrical strain gauges were used exclusively to measure the variations in stress along the beam at selected sections, Fig. 7. These consisted of forty-eight linear A-3 gauges, and twenty-four rosettes AR-1 gauges, manufactured by the Baldwin-Lima-Hamilton Corporation.

When using the gauges it is assumed the material is isotropic and homogeneous and strain gradients so small that the strain can be considered as substantially uniform over the area covered by the rosette. The condition of isotropy assumes the modulus of elasticity, and Poisson's ratio to be constant within the elastic limit.

Particular care was taken to place the gauges in the most highly stressed sections of the beam while avoiding areas which will be subjected to local stresses caused by the shifting of the stiffeners. Eight stations were chosen, four on each side and symmetrical at the centre line so that a check on each reading was at all times available

FIG 7

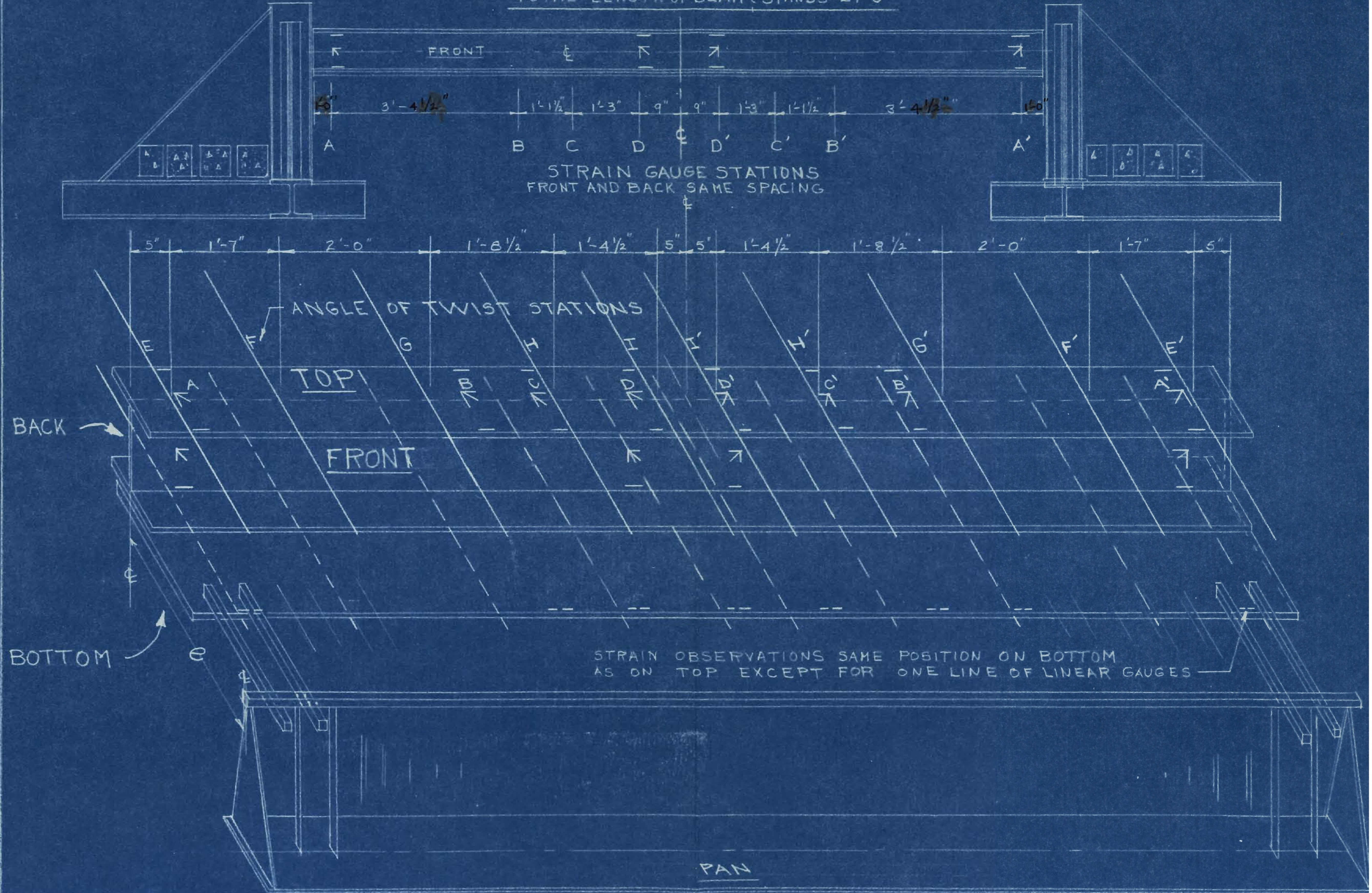
FIG. 7

OBSERVATION STATIONS

AT, A, B, C, D, A', B', C', D' - STRAIN ↖ = ROSETTE, — = LINEAR

E, F, G, H, I, E', F', G', H', I' TOP & BOTTOM - ANGLE OF TWIST

TOTAL LENGTH OF BEAM & STANDS 27'-0"



by comparing it with the exact gauge at the other side of the beam.

On the top flange and bottom plate linear gauges were placed at the edge where the theoretically maximum stress occurs due to the combination of torsional and bending stress. Rosettes were placed at the centre line of the flange where the maximum shear due to torsion occurs.

At mid depth on the front and back of the web rosettes were placed at four stations and linear gauges as close to the flanges as possible. These gauges served not only to give the distribution of stress at that point, but completed the stress picture at the entire cross section making it possible to determine the neutral plane etc.

Angle of Twist

To measure the angle of twist wooden rods one-quarter by one-quarter inches cross-section and forty-seven inches long were placed at ten selected stations along the beam, Fig. 7. At each station a rod was clamped to the top flange, and another to the bottom plate. These rods were not placed directly on the web because the readings would have been erroneous due to web deflection. Furthermore it was felt that a more representative value of twist could be obtained by basing the angular measurements on the movement of the flanges. Pins were pushed into the ends of the rods to provide a fine pointer and to adjust the exact length. Scales were mounted on a light weight steel framework, Photos 1 to 4, at both ends of the rods. The framework was supported directly on the stands and hence completely independent to any movements of the beam. As the beam rotated the projecting rods greatly magnified this rotation and the change of arc recorded as read off the scales. By having scales on both sides a check was obtained on all readings.

Loading

The concrete blocks used as dead weight were poured in the laboratory and each adjusted to weigh fifty pounds, considered to be a weight which could be handled with ease by one man. Minor adjustment of load was done with bricks.

To hold the blocks and set the exact torque, with a constant load, a pan was designed to fit onto the projecting rods at any desired eccentricity. The pan Fig. 5 consisted of a $\frac{1}{4}$ x 14 inch plate, upon which the blocks rested, with hangers every six inches to bring the load up to a one inch square bar resting on the rods from the beam. The pan was prevented from sliding or increasing the eccentricity by clamps.

The pan was made of a large height to allow it to hold up to five hundred pounds per foot. Although the full capacity was not used in this investigation it was designed with an eye to the future. As a result of the high pan, we were obliged to mount the beam three and a half feet above the ground. This arrangement was found very convenient for working.

As the beam twisted more at the centre than at the ends the pan deflected more at the centre but since the pan was very flexable the load continued to act uniformly avoiding concentrations at the ends.

CHAPTER IV

TEST PROCEDURE AND RESULTS.

The apparatus was designed to obtain an almost infinite number of torques simply by changing the load and varying the eccentricity. However to conform as much as possible to practical cases it was decided to test the beam at torques equivalent to a two inch, four inch, six inch, and eight inch brick wall resting eccentrically on the bottom plate. The wall was considered to be ten feet high and weigh eighty-five pounds per cubic foot. These torques amounted to 700, 1600, 2300, and 4200 in.-lbs./ft. i.e. 50 lb./ft. at 14 inches, 100 lbs./ft. at 16 in., and 23 in., and 150 lbs./ft. at 28 in. respectively. The eccentricity being the distance from the centre line of the web, before loading, to the point of application of the load. Further experimentation was stopped after the third torque as the web yielded, and a permanent kink developed in the top flange.

At the start of the experiment only theoretical knowledge was available to guide the procedure. From sample calculations it was assumed that the (-) corner of the top flange at the centre of the beam would be the governing point, with the web stress at the end of the beam being also of immediate but lesser concern. Commencing with the smallest torque the test was carried out with no stiffeners on the beam, then one pair, two pairs etc., were put on. It was observed that the stresses at the critical areas were far below those expected from calculations, and consequently the same order of applying stiffeners i.e. zero, one, two, etc. was used for the 1600 in.-lbs/ft. After analyzing the observations it was noticed that the stresses at the centre line in the top flange fell sharply as more stiffeners were used. With this in mind the procedure for the 2300 in.-lbs./ft. torque was reversed and observations were taken starting with four pairs of stiffeners and decreasing the quantity. This change of order was fortunate because although the previous observations did not indicate that any yielding was imminent the top flange developed a kink when two stiffeners were used, and yielding took place at the web, both at stresses below the expected danger point. The experiment was completed at that torque even after yielding to give a basis for comparison. If the original procedure would have been followed the web would have yielded at the first application of the load making all subsequent results questionable.

A complete cycle of the experiment for one of the two smaller torques consisted of the following steps.

1) All strain gauges and rod deflections were read with no stiffeners on the beam and zero load. This served as the initial zero reading.

2) The loaded pan was adjusted to the required eccentricity and firmly clamped to the rods at several points along the beam to prevent any lateral movement. The pan and weights were then lowered on to the beam.

3) The deflection rods, and then all the strain gauges were read with the beam in this loaded position.

4) The loaded pan was removed from the beam, by jacking up in the case of small loads, and with the aid of the crane in the case of large loads.

5) One stiffener was bolted to the centre and the complete

procedure repeated.

A complete set of observations consisted of following the above steps for zero, one, two, three and four stiffeners.* this cycle required a minimum of five days and at least one zero reading was taken per day as a check. In all trials the load was kept on the beam a maximum of two hours so that any yielding which may be caused by the load hanging a long time was eliminated as much as possible. The load was of course removed over night. The zero readings were checked every day with the previous and a close check was kept to determine if the metal at the gauge station was acting elastically. If differences of zero readings were found to change, a close check was undertaken to observe any evidence of yielding.

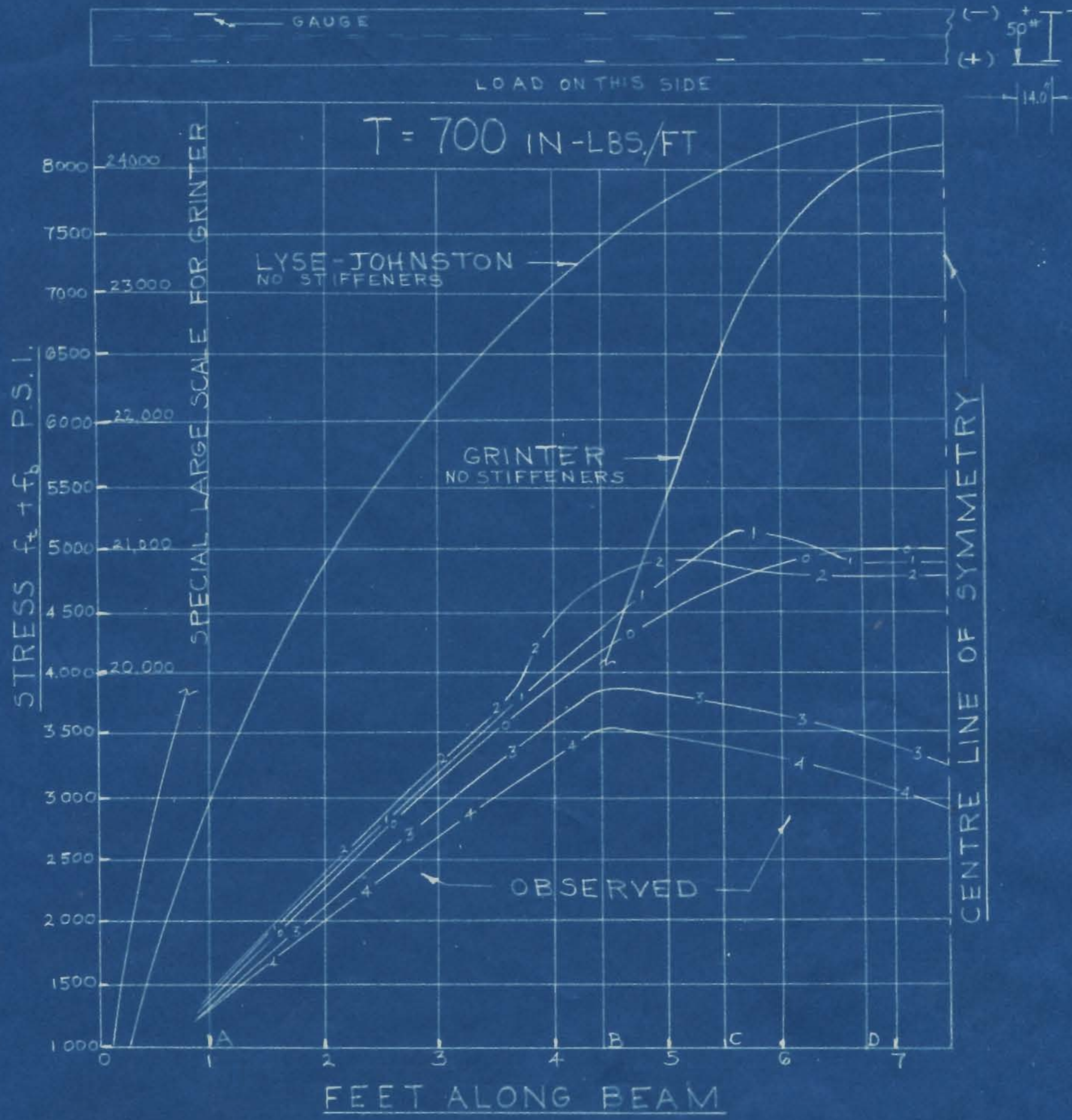
Before the readings were taken for actual use, that is at the initial start of the investigation, numerous trial runs were made at random torques and varying loads to "shake down" the beam and the actual test commenced only after perfect elastic behavior of the beam was guaranteed by numerous readings.

* In the case of the 2300 in.-lbs./ft. torque this procedure was reversed, i.e. four, three, two etc., stiffeners.

FIG. 8

TOP FLANGE

PLOT OF TORSIONAL AND BENDING STRESS, $f_t + f_b$, ALONG LENGTH OF BEAM FOR GRINTER, LYSE-JOHNSTON, AND OBSERVED VALUES USING 0, 1, 2, 3 AND 4 STIFFENERS. OBSERVATIONS TAKEN BY LINEAR STRAIN GAUGES AT STATIONS A, B, C, AND D PLACED ON TOP FLANGE AT (−) EDGE WHERE STRESS IS A MAXIMUM.



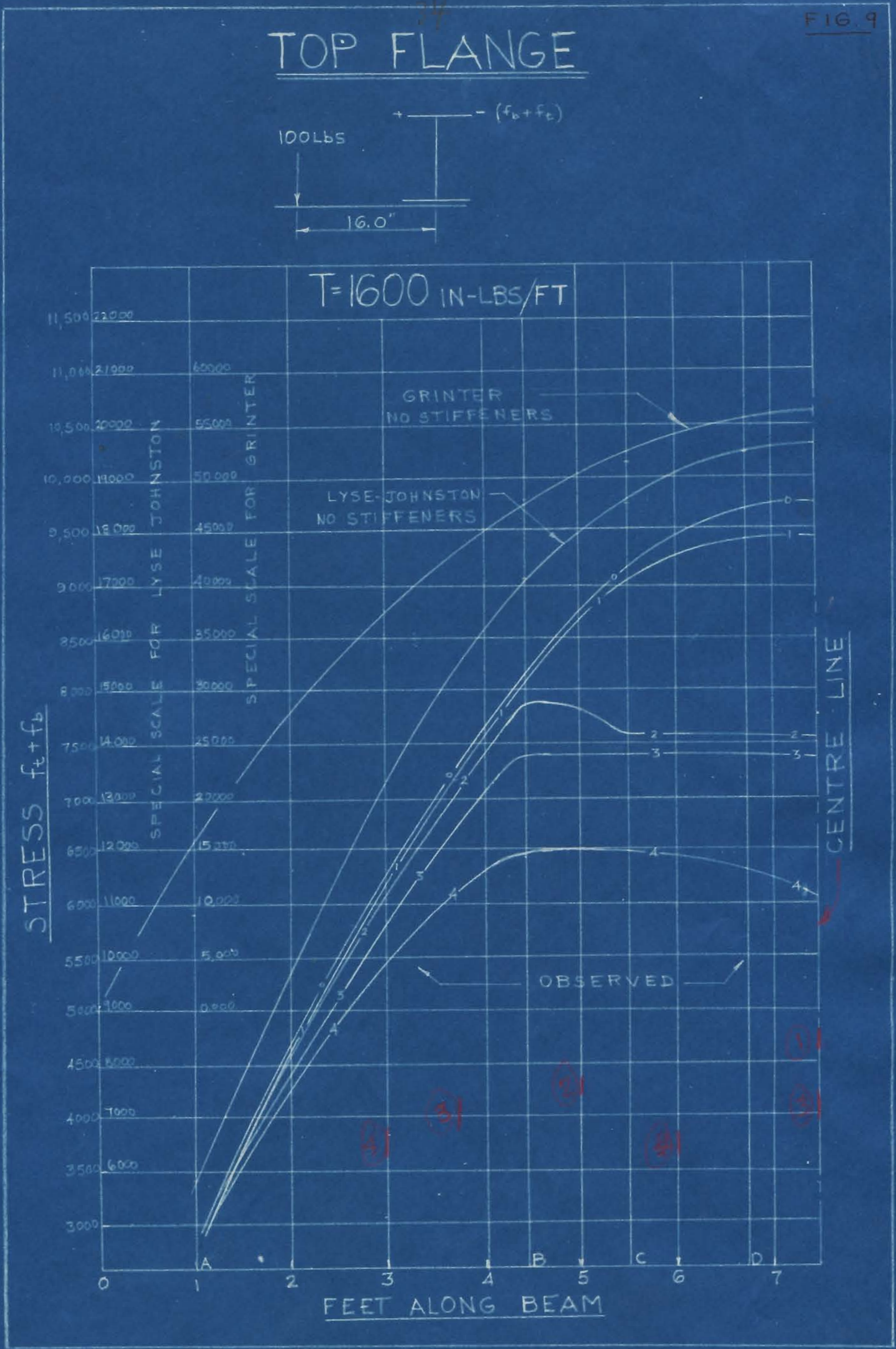


FIG. 9
TOP FLANGE
100LBS
$-(f_b+f_t)$
16.0"
T=1600 IN-LBS/FT
GRINTER NO STIFFENERS
LYSE-JOHNSTON NO STIFFENERS
OBSERVED
SPECIAL SCALE FOR LYSE JOHNSTON
SPECIAL SCALE FOR GRINTER
STRESS f_t+f_b
CENTRE LINE
FEET ALONG BEAM

FIG. 10

TOP FLANGE

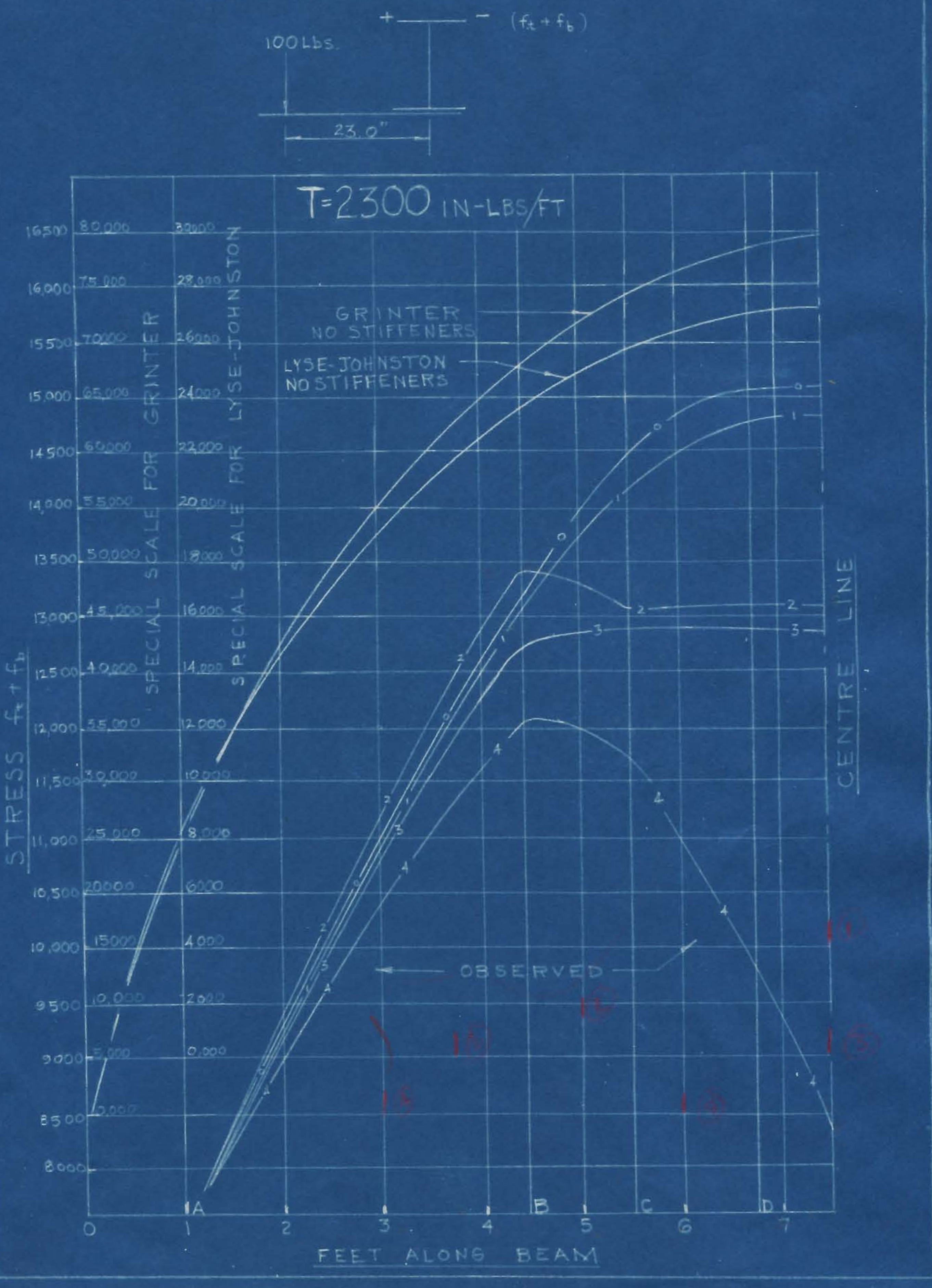


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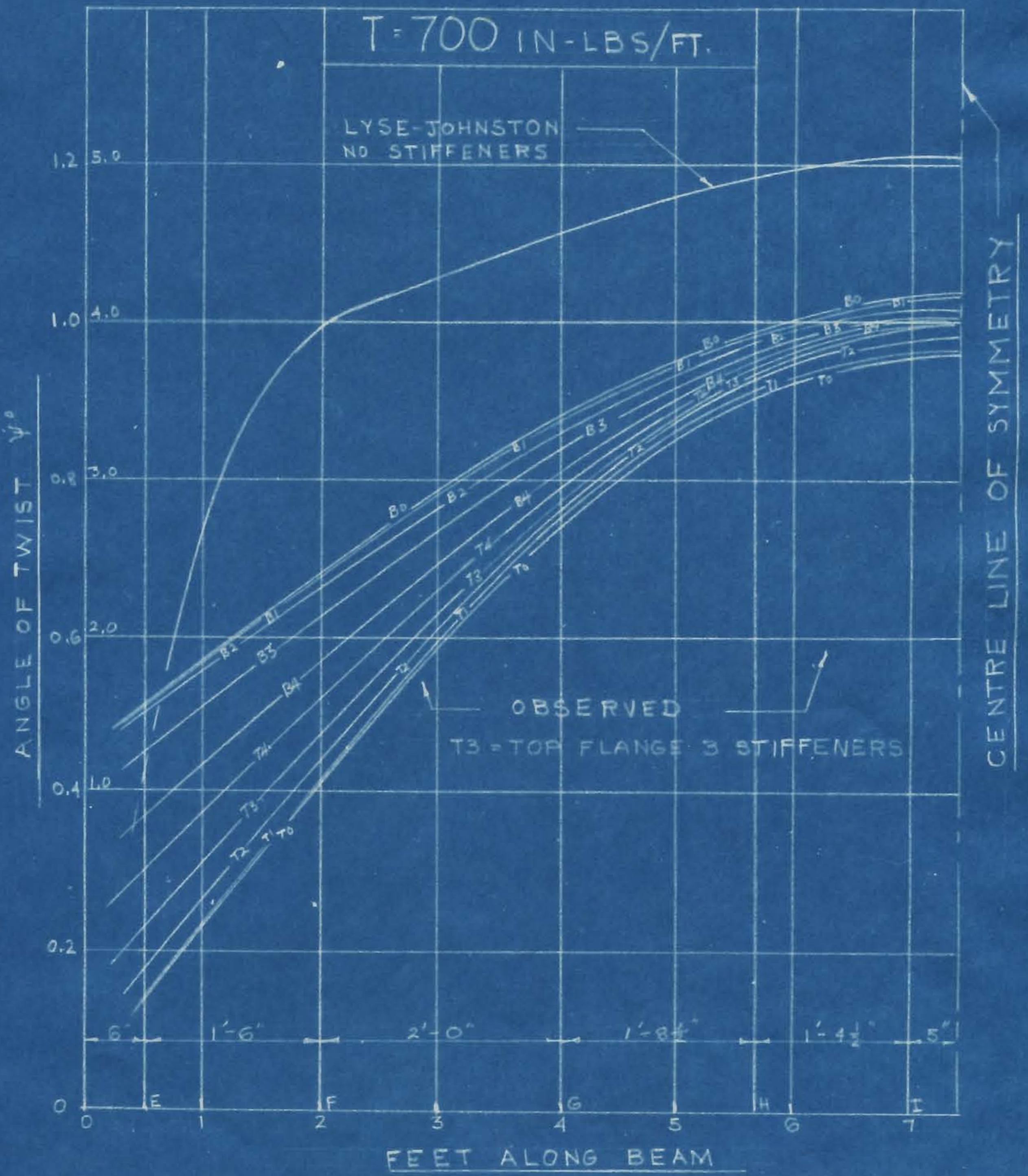
FIG. 11

ANGLE OF TWIST

PLOT OF THE ANGLE OF TWIST, ψ°, ALONG LENGTH OF BEAM FOR LYSE-JOHNSTON AND OBSERVED VALUES. OBSERVATIONS TAKEN AT STATIONS E, F, G, H, AND I, AT TOP FLANGE AND BOTTOM PLATE FOR 0, 1, 2, 3, AND 4 STIFFENERS.



ANGLE OF TWIST

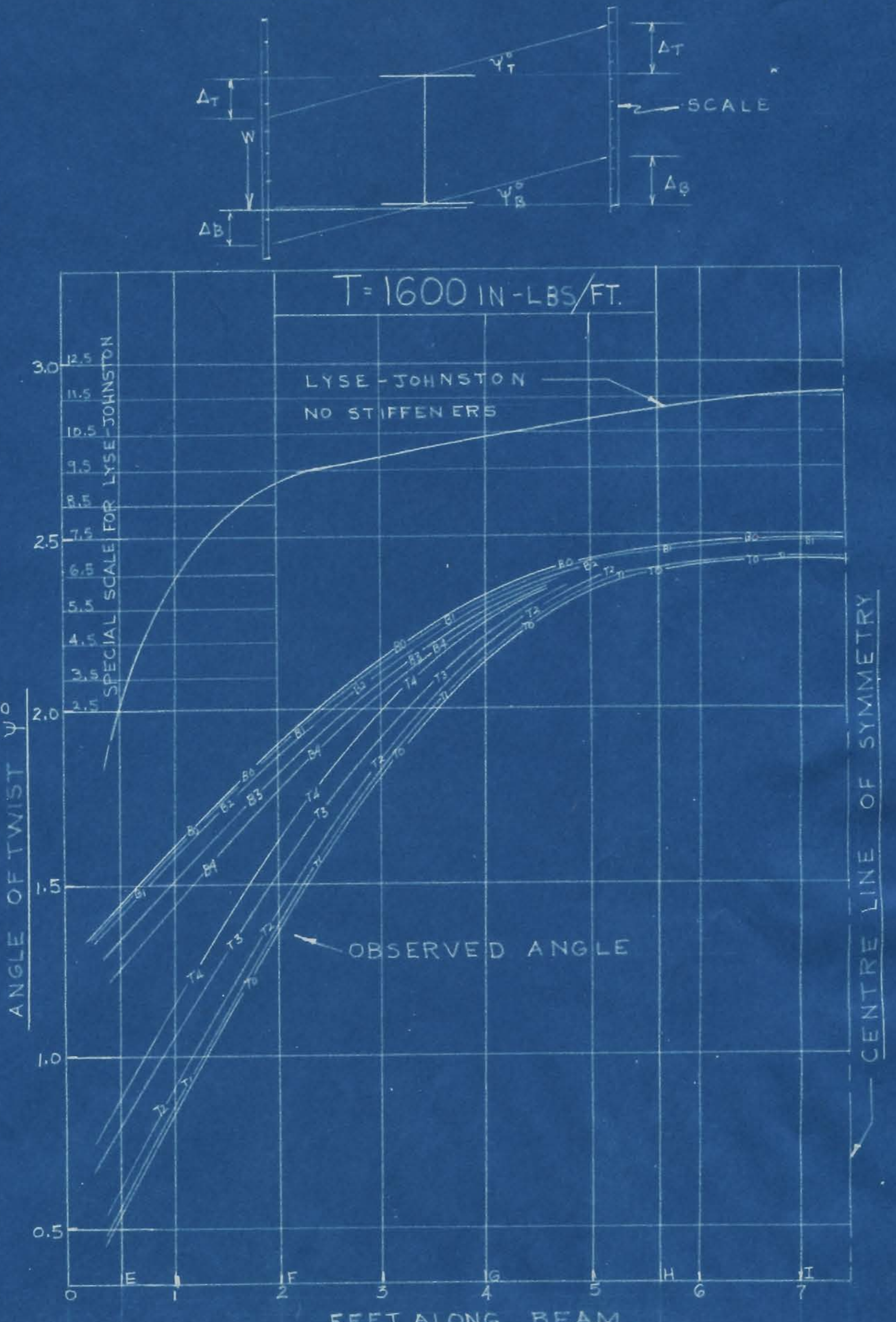


FIG. 13

ANGLE OF TWIST

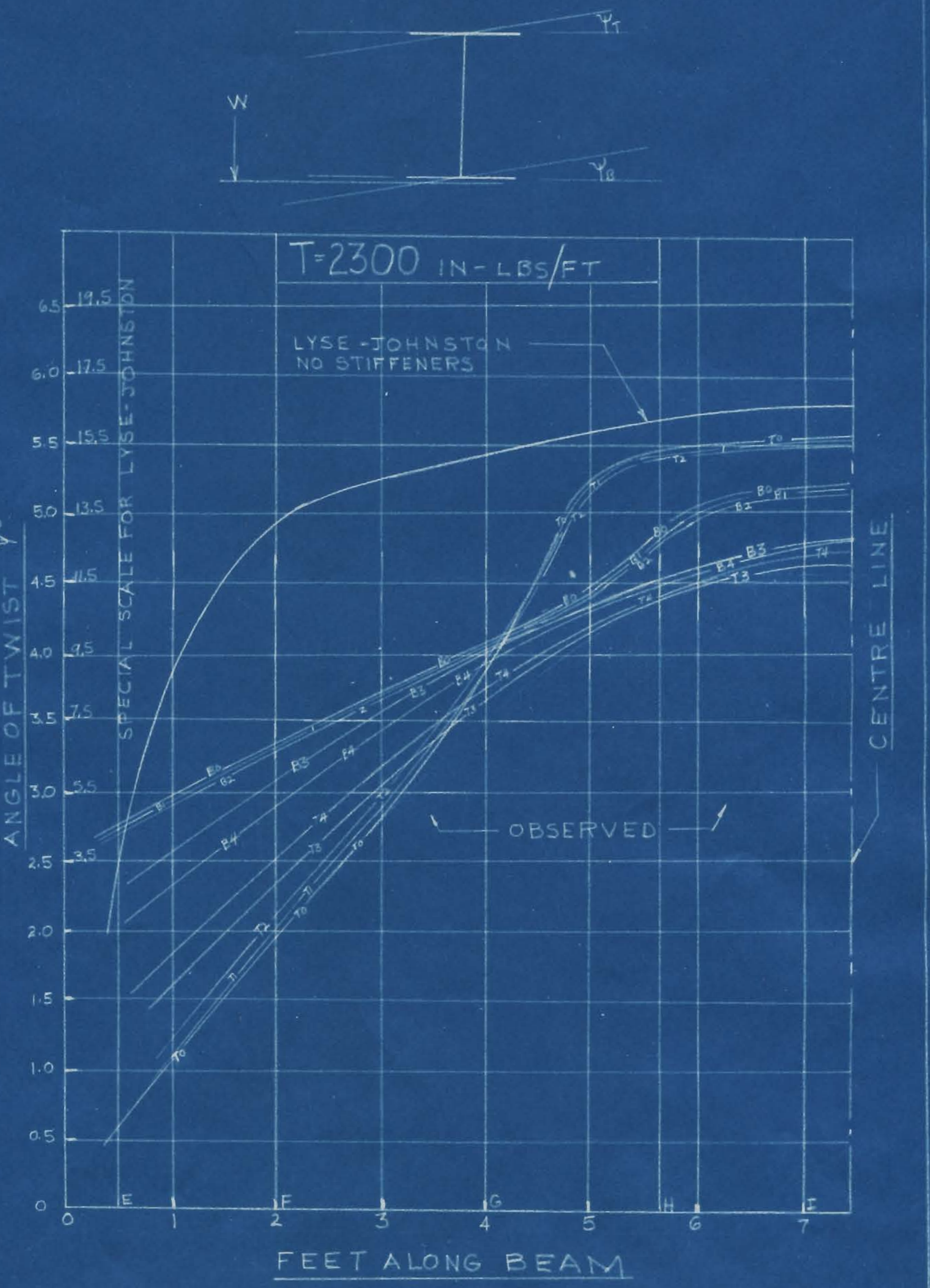


FIG. 14

WEB STRESSES

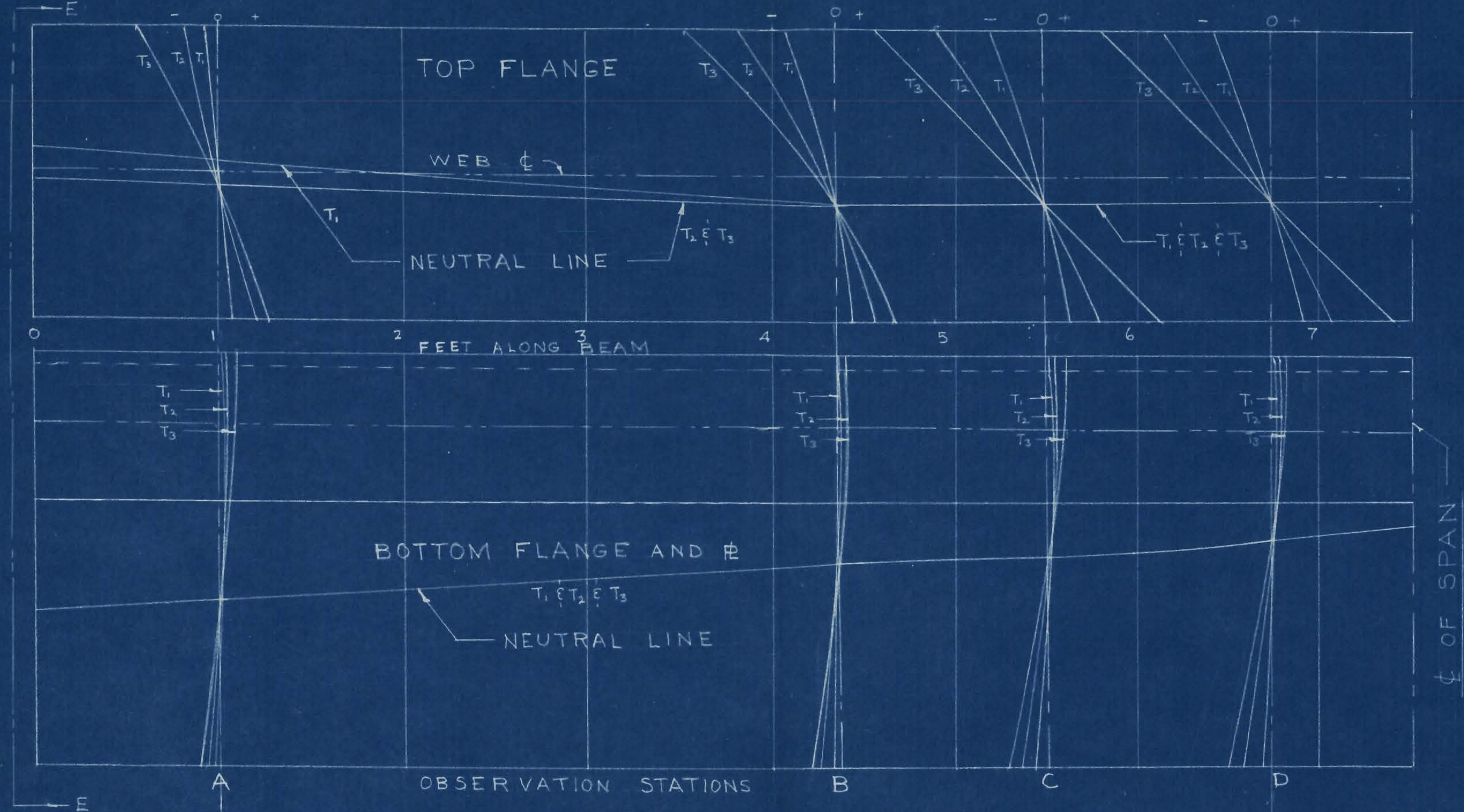
DIAGRAMS SHOW A COMPARISON BETWEEN THE THEORETICAL PRINCIPAL AND MAXIMUM SHEARING STRESSES AND PLANES ALONG WHICH THEY ACT AND THE OBSERVED VALUES USING 0, 1, 2, 3, & 4 STIFFENERS.

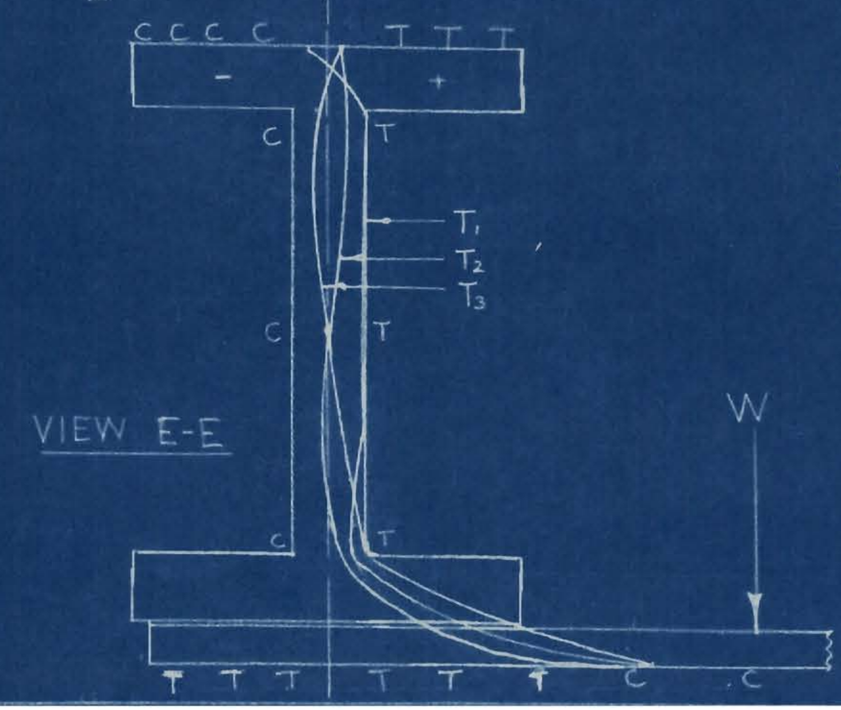
T = 2300 IN-LB/FT.

	STATION A FRONT	STATION A BACK AS SEEN FROM FRONT	STATION D FRONT	STATION D BACK AS SEEN FROM FRONT
THEORETICAL NO STIFFENERS	45°; σ_1 = +17650 psi, σ_2 = -17650, τ = 17650	45°; σ_1 = -16250 psi, σ_2 = +16250, τ = 16250	52°; σ_1 = -2410 psi, σ_2 = +1700, τ = 2080	52°; σ_1 = -2200 psi, σ_2 = +1500, τ = 1900
OBSERVED 0	25°; σ_1 = +28,200, σ_2 = +4600, τ = 11,600	26°; σ_1 = -24,300, σ_2 = -9350, τ = 7500	32½°; σ_1 = +2300, σ_2 = -125, τ = 1200	51°; σ_1 = -1540, σ_2 = +290, τ = 920
1	24°; σ_1 = +27,000, σ_2 = +4160, τ = 11,400	23°; σ_1 = -27,600, σ_2 = -5600, τ = 11,000	21°; σ_1 = +1020, σ_2 = -630, τ = 840	2½°; σ_1 = +1160, σ_2 = +125, τ = 540
2	23½°; σ_1 = +24,200, σ_2 = +4800, τ = 9800	23°; YIELD; σ_1 = -28,400, σ_2 = +6250, τ = 11,500	22½°; σ_1 = +1170, σ_2 = -165, τ = 500	3½°; σ_1 = +1340, σ_2 = -500, τ = 920
3	26°; σ_1 = +20,000, σ_2 = +1240, τ = 9400	24°; σ_1 = -24400, σ_2 = +5000, τ = 14600	36°; σ_1 = +460, σ_2 = -80, τ = 210	
4 STIFFENERS	26°; σ_1 = +19800, σ_2 = +2500, τ = 9300	34°; σ_1 = -25000, σ_2 = +2700, τ = 15,000		OBSERVED STRAIN READINGS TOO SMALL TO BE CONSIDERED ACCURATE.
VECTOR STRESS SCALE	1/10" = 10,000 P.S.I.	1/10" = 10,000 P.S.I.	1/10" = 1000 P.S.I.	1/10" = 1,000 P.S.I.

40

FIG. 15



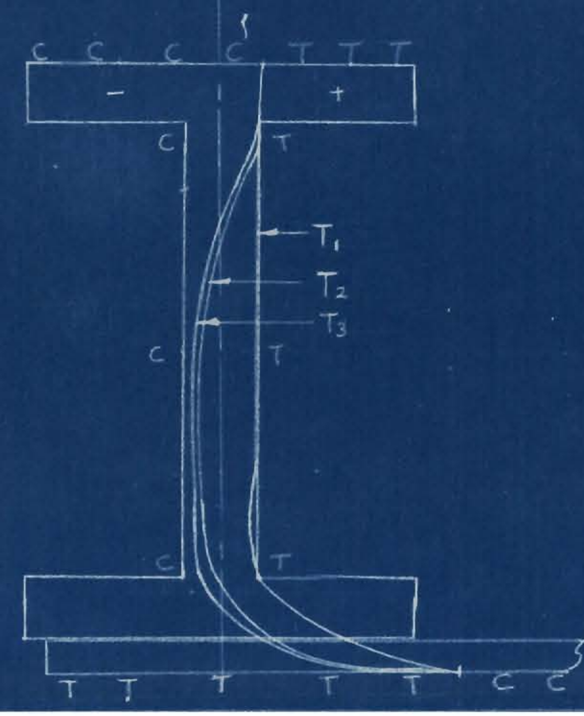


NEUTRAL PLANE

SECTIONS SHOWING THE CHANGE OF POSITION OF THE NEUTRAL PLANE FOR THE THREE TORQUES AND ZERO STIFFENERS ON BEAM.

T_1 = 700 IN-LB./Ft, T_2 = 1600 IN-LB/Ft T_3 = 2300 IN-Lb/Ft

HORIZONTAL STRESS SCALE
1" = 10,000 P.S.I.



CHAPTER V

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

Flange Stresses

In Figs. 8, 9, and 10 comparison is made between the actual observed stresses at the compression edge of the top flange, and the theoretical as calculated by the Grinter, and Lyse-Johnston methods. The (-) top corner was chosen since this is the most highly stressed portion of the top flange, as the longitudinal torsional, and bending stresses, add algebraically to give a maximum here, furthermore in choosing a beam this would be the governing point as far as the top flange is concerned. Only one half of the beam is shown since by symmetry the stresses in the other half are the same. This was found to be true from the results observed where the readings differed a maximum of five to ten micro inches. In order to have a basis of comparison, the theoretical stress due to bending was added to that calculated by the Grinter, and Lyse-Johnston methods, because the observed stresses were a combination of the two. Whether the theoretical bending stress can be added algebraically to the theoretical torsional stress to give a true stress is not certain, however, since the loads are small the bending stress is correspondingly small and even if in error does not appreciably change the values derived from Grinter, and Lyse-Johnston calculations. Also, it must be remembered, that the observed values are for our actual beam with a bottom plate, while the theoretical values are for a beam only.

In general, it is seen that the theoretically calculated stresses average 420% higher than the measured in the case of the Grinter method, and 80% in the Lyse-Johnston.

It is difficult to assign a magnitude to the decrease in stress caused by the presence of the bottom plate, however, the Lyse-Johnston value of 80% is much more reasonable than that of 420% given by the Grinter method.

All the curves have the same general form when no stiffeners are used. Now let us consider the effect of stiffeners. Examination of the graphs for all three torques indicate certain fundamental stress variations taking place as stiffeners are put on and moved about. If one stiffener is placed at the centre line a very small decrease in stress takes place in the vicinity of the centre of the beam. This decrease is negligible, being of the order of 200 psi., however the deformation taking place at that point is of importance. The excess twist of the lower flange and plate as compared to the upper flange acts to pull the front stiffener down and to push the rear one up. This movement will cause the top flange to deform as shown in Fig. 16 . This deformation although not permanent is visible and may give rise to high local stresses. Consider

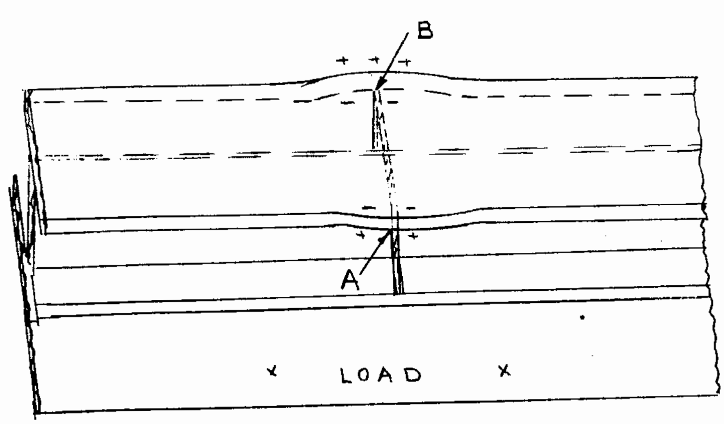


Figure 16.

the region of the stiffener. The front top portion of the flange is pulled down in a concave manner. This will cause compression stress on the top and tension on the bottom. Now, the stress in the top flange due to torsion is uniform across the thickness of the flange and hence at the bottom front corner, point A, the tension stress will add algebraically to give high local stress. Similarly, at the bottom far corner, point B, of the top flange, the compression stress will add to give high local stress.

The same phenomenon will occur at the bottom flange, but since it is reinforced by the plate, the effective stresses will be small and of no concern compared with those of the top flange.

When two stiffeners are affixed, at the third points i.e. five feet from each end, a sudden drop in stress takes place in that portion of the flange between the stiffeners. This is of the order of 15% for the two large torques, and is negligible for the small torque. This effect can be expected and observations show that the stiffeners hold the top and bottom flanges together making them act as one and consequently increasing the torsional rigidity of the beam.

Again there is a tendency for the front part of the flange to be pulled down and the far side to be pushed up. However, this is most severe on the side of the stiffener away from the centre line of the beam, i.e. in the region left of A, and right of B illustrated in Fig. 17.

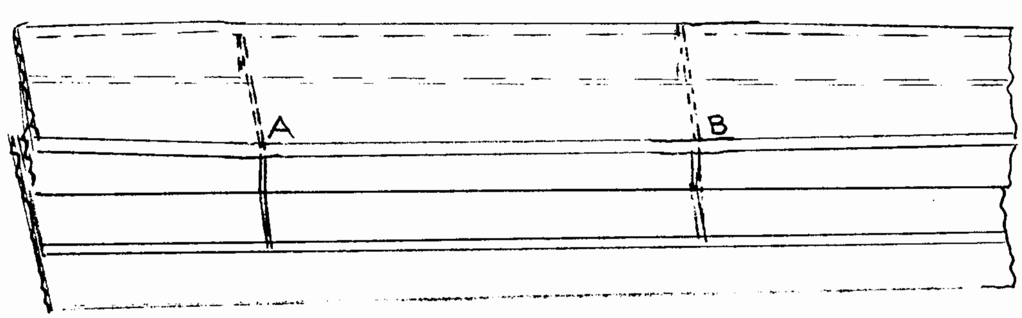


Figure 17.

An increase in local stress of about 400 psi. is recorded in the region of A and B. This stress finally disappears as we go towards the ends. Actually this local stress must be much higher than that recorded since special care was taken when choosing the gauge stations to avoid such regions of high local stress, the strain stations used having been removed from the stiffeners as much as possible. The tendency of the stiffeners to resist the warping effect of the cross-section of the beam may be responsible for some of the increase in stress, but it can be seen from the graph that the increased stress, for the higher torques, is still far below the maximum stress that would occur at the centre line of the beam when no stiffeners are used. Another feature of this is of course the fact that in actual use, a continuous load will give a maximum bending stress at the centre of the beam. Consequently, with the torsional stress reduced in the central portion of the beam, the capacity is actually increased.

In the case of three and four stiffeners, we find that the local stresses between the outside stiffeners continue to drop sharply. The deformation of the top flange is not severe, and the top and bottom flanges act together better than in any of the previous cases. Whether stiffeners would be used to some advantage at a particular job would of course be determined by such considerations as fabrication costs etc.

Angle of Twist

From Figs. 11 ,12, and 13, it is seen that as in all previous cases the striking fact is the large theoretical value obtained from the Lyse-Johnston equation, and the value observed. This difference is 343% in the case of the two smaller torques and 244% in the case of the large torque. The bottom flange twists more than the top, and as the actual load will rest on the bottom plate, it will be the governing factor. The above values indicate that the bottom plate has the effect of reducing the total angle of twist by an average of 310% from the theoretical.

Consider the observed values. The bottom plate and flange acting together have a larger angle of twist than the top flange and this difference is most severe at the ends of the beam where the torque is a maximum and the web is held vertical by the connection. When four stiffeners are on, and the beam is tested, the flanges act together to a greater extent than in any other case. In the central third of the beam, the angle of twist for the top and bottom flange run parallel and differ only by about 0.1 degree which may possibly be due to some slack in the rivets holding the stiffeners, or to slight error in reading the scales.

As the test is run with three, two, one and without stiffeners, the difference of the angle of twist between top and bottom flanges increases especially closer to the ends of the beam. Finally this reaches a maximum of about 0.8 degrees with no stiffeners. It will be noticed, that the maximum angle of twist, occurring at the centre line, is not decreased any worthwhile amount, regardless of the number of stiffeners used. Hence, we may conclude that the only beneficial effect of stiffeners, as far as the angle of twist is concerned, is to make both flanges act together while the maximum actual angle is not changed. Or, in a case in which a beam is chosen on the basis of the angle of twist, it is of no advantage to use stiffeners, and there is no alternative but to choose a larger member.

In the case of the largest torque, i.e. 2300 in.-lbs./ft., it is seen that when two stiffeners were used, a sudden rise in the angle of twist of one degree occurred. This was due to a permanent deformation of the top flange in the vicinity of the central third of the beam. Visual inspection seemed to indicate that this occurred without prior warning and a long thin crank running on each side of the web directly under the flange just below the fillet where the thickness is a minimum, was seen after detailed examination. The experiment was continued and the strain measurements showed that the top and bottom flanges were still acting perfectly elastically while some yielding started to take place on the compression side i.e. back, of the web.

The reason for the permanent kink in the top flange was that there was a very high concentration of downward pull on the front side and upward push on the back side and this force was greater than the top flange could withstand. It is also seen that after the top flange was permanently deformed, the bottom flange angle of twist increased accordingly.

From the above discussion it is advisable that if stiffeners are used at all, they should be used in such quantities as to avoid high concentrations of local stress which may deform the top flange.

Comparison of Grinter, and Lyse-Johnston Methods of Selecting Members From a Series of Beams.

Since the Grinter method is used generally for design, let us further investigate whether the very phenominal stresses obtained in this case is also true for a series of beams.

To resist torsion an ideal section should consist of wide, thick flanges and a depth as small as possible. The C.S.A. specifications for buildings limits the depth of beam to one twenty-fourth the span length. In our case this is eight inches. Now, it may be argued that the Grinter method for design will give more accurate results for beams of larger flanges than the one used in this thesis, i.e. five and a quarter inches wide, since the primary assumption is that the effect of the web is negligible and that all the torque is resisted by the flanges. Let us examine if this arguement is justified. Suppose we choose an arbitrary torque of 1600 in.-lbs./ft. for comparison, and calculate the maximum torsional stresses at the edge of the flange at the centre line of the beam, as shown in table one, as well as the angle of twist from the Lyse-Johnston equations. We are not concerned at this moment with the bottom plate or stiffeners as they have the same effect regardless of method used.

From the theory

$$f_t = BT\, a^2 \frac{d^2(F)}{dx^2} \qquad \text{LET } a^2 \frac{d^2(F)}{dx^2} = Q$$

$$\therefore f_t = QBT \qquad (4)$$

$$\psi = CT\,(F) \qquad (8)$$

$$L = 15.0 \text{ Ft}$$

$$T = 24{,}000 \text{ IN-LBS}$$

Table 1

BEAM	Q	B	f_t L - J p.s.i.	f_t Grinter p.s.i.	% of Grinter over L-J	(F)	C	Ψ degrees
8WF17	.176	4.081	17,300	52,400	206	.384	$.22\times10^{-4}$	11.60
20	.173	3.059	12,650	42,600	237	.415	$.15\times10^{-4}$	8.59
24	.1825	2.082	9,120	23,300	166	.334	$.11\times10^{-4}$	5.06
28	.177	1.542	6,550	20,600	232	.378	$.67\times10^{-5}$	3.50
31	.189	1.443	6,550	14,800	126	.257	$.84\times10^{-5}$	2.98
35	.185	1.125	5,000	12,800	164	.311	$.52\times10^{-5}$	2.23
40	.178	0.874	3,730	11,300	203	.372	$.32\times10^{-5}$	1.64
48	.165	0.594	2,350	9,080	286	.480	$.16\times10^{-5}$	1.06
58	.150	0.417	1,500	7,500	400	.599	$.80\times10^{-6}$	0.66

The table illustrates that regardless of beam weight used, the Grinter method gives stresses of 126% to 400% above the Lyse-Johnston. Hence for the above torque we would be inclined to choose an 8WF28 beam by Grinter or an 8WF17 by Lyse-Johnston i.e. almost half the size. Again, suppose we limit our maximum angle of twist to one and a half degrees. On this basis we would choose an 8WF40, or a larger beam than even by the Grinter method. From the previous discussions it was shown that the angle of twist was theoretically 244% higher than observed. It is unknown to what extent the bottom plate decreased this angle from the calculated but it seems that the theoretical is about 100% too high, or an 8WF31 should be sufficient. It is safe to conclude that the Grinter method is in large error as far as the stresses are concerned, while the angle of twist from the Lyse-Johnston equation is highly questionable. What in fact puts an upper limit on the whole subject is the practical consideration that the torsional stress is usually small contributing only slightly to the combined stresses of bending and torsion. The result being that the overdesign based on stress by the Grinter method is not too serious, and the designs in which the angle of twist is the primary concern is equally small. However, the weaknesses of these methods must be carefully considered when the torsional effects are large, as the amount of overdesign will become exhorbitant.

Web Stresses.

In Fig. 14 a comparison is made between the theoretical, and actual, principal stresses, and the maximum shearing stress as well as the plane along which they act. This comparison is shown for the largest 2300 in.-lbs./ft. torque only because it is of most interest as yielding took place on the back, or compression side. Furthermore the changes in stress of the other two torques was fundamentally the same as this one.

It is recalled that in this cycle the procedure consisted in using four, three, two, one, and zero stiffeners on the beam.

In all existing theory on torsion of beams, channels, angles, etc. where the problem of uniform continuous eccentric load is considered a very important aspect of the problem is usually not mentioned or vaguely dealt with and that is exactly how the load is resting on the beam, or in other words, how does the torque "enter" the beam. In wall systems the method commonly used of supporting the load i.e. bottom or top plate welded to the flange, has an additional effect not considered in any theory. Suppose we isolate an element of surface area at the front rosette at station A Fig. 18a (exactly the same analysis is true for the back side). According to the general theory this element is subject to shears as shown, and a horizontal stress. The shear consists of two quantities added algebraically, namely torsional shear and vertical shear. The horizontal stress is due to bending, tension or compression, depending upon the rosette's position from the neutral axis. In our case this is of the order of -110 p.s.i. and could be neglected.

51

FIG. 18

FIGURE
DISCUSSION OF WEB STRESSES

T = 2300 IN-Lb/Ft.

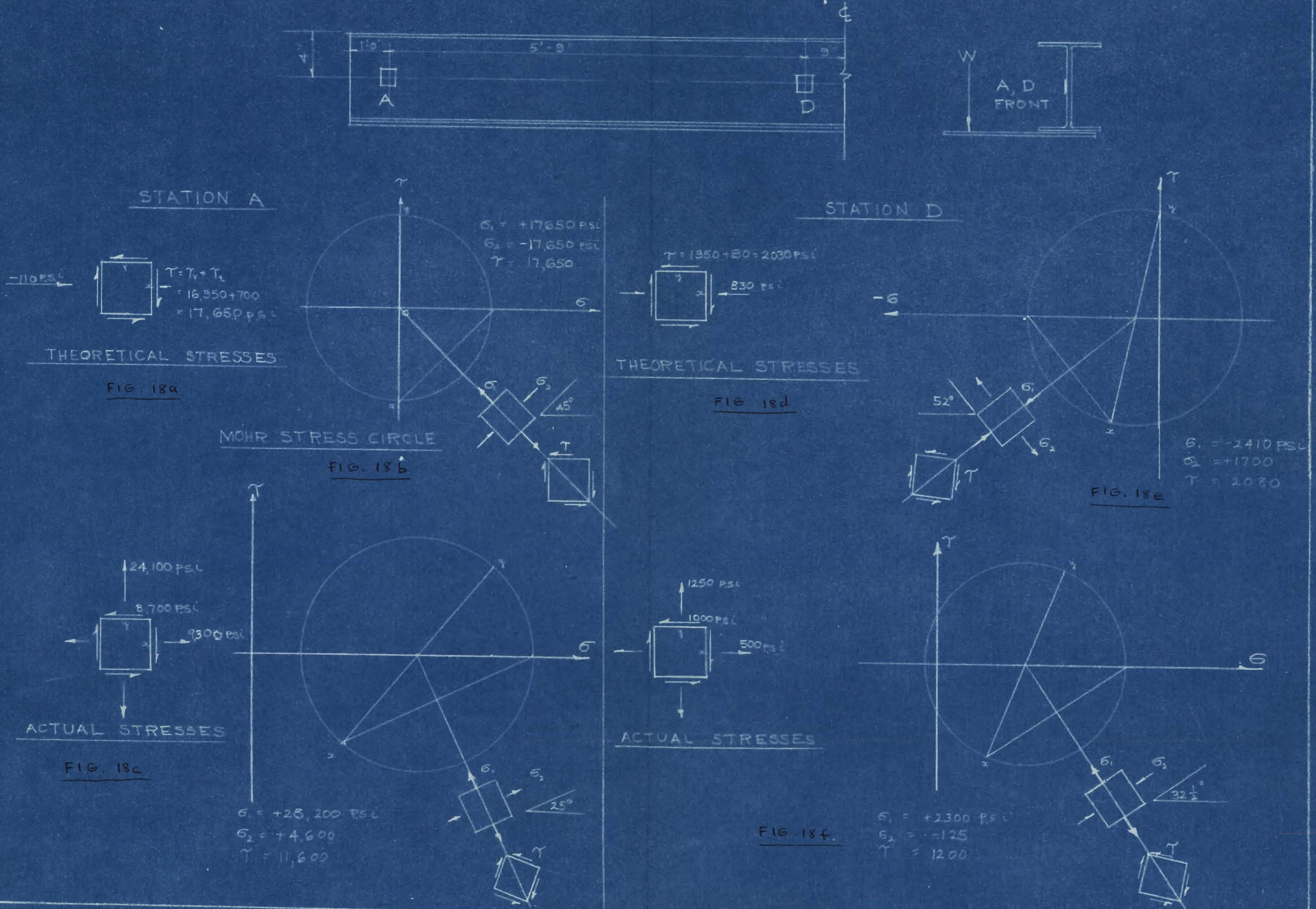


FIG. 18a

FIG. 18b

FIG. 18c

FIG. 18d

FIG. 18e

FIG. 18f

If we draw a Mohr stress circle Fig. 18b we see that the principal, and shear stresses are all of the same magnitude and the direction of their line of action is at 45° to the horizontal. However in the actual case, as a result of applying the load on the bottom plate a large vertical component acts on the element and the true Mohr diagram should be as in Fig. 18c. This added vertical component has the effect of increasing the principal stresses, and the maximum shear. It also causes the angle along which they act to decrease from the horizontal by twenty degrees for the case of no stiffeners on the beam. The maximum positive stress increases by about 10,000 p.s.i. This vertical component is 24,100 p.s.i. for no stiffeners on the beam and decreases sharply as stiffeners are used. Suppose we try to account for this stress by considering one inch length of web as acting as a beam subject to a bending moment OF

$$\frac{2300 \text{ IN-LB/FT}}{12 \text{ IN/FT}} = 192 \text{ IN-LBS/IN. AT THE END OF THE WEB.}$$

$$I = 0.001015 \text{ IN}^4 \qquad f_b = \frac{My}{I} = \frac{192 \times 0.115}{.001015} = 21,800 \text{ P.S.I.}$$

This may be a valid approximation for station A but it fails to answer several important questions. Firstly, is this stress the same all the way up the web, or decreases as the distance from the bottom plate increases. Secondly, is this stress of the same magnitude on both faces of the web, or is it different. If it is the same then the neutral plane should run along the centre line of the web. From Fig. 15 it is seen that for the two large torques this is reasonably true at station A, but is not true for the smallest torque, or for any torques at station D, as can be seen from the observed values.

Consider now the same element of area at station D. The theoretical stresses acting on it are as shown in Fig. 18d . The vertical shear due to torsion is 1950 p.s.i. and that due to vertical loads is very small i.e. 80 p.s.i. However the horizontal stress due to bending is not negligible but is -830 p.s.i. Again the vertical stress on the element is zero and if we draw a Mohr stress circle for this condition Fig. 18e it is seen that the plane of maximum stress is at 52° to the horizontal clockwise. Now in the actual case we have a large vertical component of the order of 1200 p.s.i. acting on the element. The actual observed Mohr circle is as in Fig. 18f . The direction of the action of the maximum stress has now changed to $32\frac{1}{2}^{\circ}$ counter clockwise, and of greater significance is that this maximum stress is not in compression as theoretically shown but in tension. A comparison of the theoretical, and observed values obtained for station A indicate that for the maximum shear stress the theoretical values are greater than observed by 34% and 54% for the front and back respectively while for the maximum principal stresses the difference is 37% and 33% lower.

In previous discussions on the torsional stresses and angle of twist in the flange it was shown that the Lyse-Johnston method gave results of 80% and 244% respectively higher than observed, and basing our predictions of web stress on these results we may conclude that as far as shear is concerned the general trend is continued but is in disagreement with respect to the principal stresses. As pointed out this is caused by the large vertical component on the element of surface discussed previously.

On the basis of the theoretical calculations we are lead to believe that as far as maximum shear is concerned it is a safe criterion for design. Actually this is not quite so. Consider a common case of loading as in Fig. 19a . Theoretically to obtain the

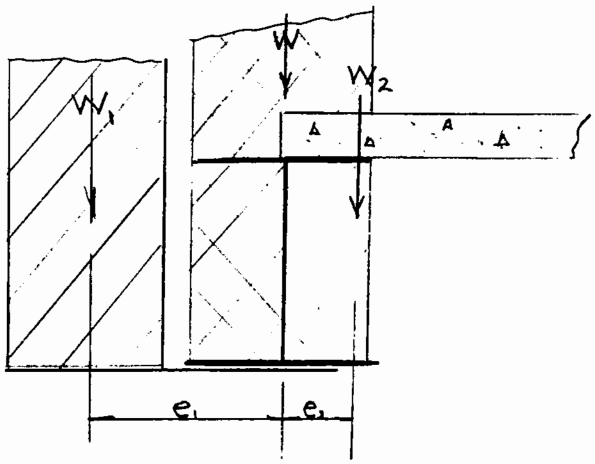


FIG. 19a

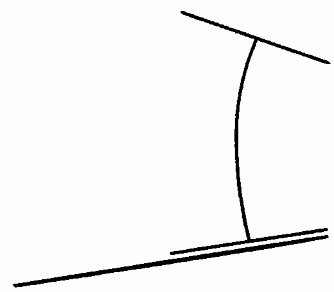


FIG. 19b

resultant torque we would take the conditions so that $(w_1 e_1 - w_2 e_2)$ is a maximum. Now the loading may be of such magnitude that this torque is zero or small and consequently all torsional effects are omitted from the stress calculations. However the web is being deformed as in Fig. 19b and large web stresses are set up, due to the method of loading, which are never taken into account in the calculations. The exact stresses is itself a subject for future investigation.

Observations indicate that the changes of stress in the web are primarily a reflection of the extent to which the stiffeners change this large vertical component, and prevent the web from tending to deform. As this stress is to a large degree dependent upon the angle of twist of the bottom plate relative to the web we would suspect after examining these twist curves that as more stiffeners are used the smaller becomes the vertical component and all the other stresses as well. This is shown to be correct especially at station D where the stresses are much more sensative to the position of the stiffeners than at A. When one stiffener is used at the centre line the maximum principal stress on the front side is reduced by 56%, and 25% on the back side. Two stiffeners on the beam do not change the stress appreciably but a further sharp drop occurs with three stiffeners, because now there are stiffeners on either side of the station and besides holding the flanges together more firmly than with two, they further reduce the stress by bearing against the web and preventing any distortion. With four stiffeners on the observed strain readings were so low, i.e. of the order of five, to twenty micro-inches, that they could hardly be considered accurate.

At the end of the beam or at station A the exact effect of the stiffeners is not too clear. The proportion of decrease in stress due to yielding, and that due to the stiffeners is not known. On the front side they show a gradual decline as more stiffeners are on the beam, however no trend is seen to exist on the back or yielding side. In fact the stresses with four stiffeners are slightly greater than with three. This may indicate that some yielding started immediately on application of the high load.

In originally planning the entire experiment it was decided to confine the observations to loads below the yield point of the material, or within the elastic limit. As a result of this decision the exact values of Poisson's ratio, Young's modulus of elasticity, and the yield point of the material were not found by independent tests in the laboratory. As the web yielded below expected theoretical values, and as part of the experiment was completed after yielding had occurred it is considered important to examine the observed stresses in the light of existing failure theories, however to do this we must assume values for μ, E, and σ elastic limit, as 0.3, 29,000,000, and 33,000 p.s.i. respectively. These values are generally used for structural steel but they cannot be considered as exact as small impurities etc. in the metal may change these slightly. Furthermore in our case yielding occurred on the compression side of the web where the yield point may be different than the assumed 33,000 p.s.i.

On the basis of the maximum principal stress theory, and the maximum shearing stress theory, no yielding should have occurred, because the highest principal stress recorded at station A was -28,400 p.s.i. (when using two stiffeners) and 15,000 p.s.i. (four stiffeners) as maximum shearing stress. Both values below predicted by these theories i.e., 33,000 p.s.i. and 16,500 p.s.i. respectively. Note that the theoretical maximum shearing stress of 16,250 p.s.i. comes close to the yield point and would suggest that the maximum shearing stress theory is fairly accurate for states of stress in which relatively large shearing stresses are developed, however whether this theory is valid for this problem must be confirmed by future tests.

The most accurate of the energy theories of failure or the energy of distortion theory usually referred to as the Huber, von Mises, and Hencky theory states that inelastic action at any point in a body begins only when the strain energy of distortion per unit volume absorbed at the point is equal to the strain energy of distortion absorbed per unit volume at any point in a bar stressed to the elastic limit under a state of uniaxial stress as occurs in a simple tension (or compression) test.

The value of the maximum strain energy of distortion, or energy absorbed in changing shape, as determined from the tension test is

$$\mu w_{de} = (1+\mu)\frac{\sigma_e^2}{3E} \quad - - - - - \quad (10)$$

where σ_e is the yield stress.

The energy of distortion, or the elastic strain energy absorbed by the unit volume as a result of its change in shape is

$$\mu w_d = \frac{1+\mu}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \quad - - \quad (11)$$

where σ_1, σ_2, and σ_3 are the principal stresses in the x, y, and z, directions. Note that in our case σ_3 is zero.

An examination of the observed and theoretically calculated stresses, Fig. 14 indicate that a maximum value for Eq. (11) will occur when the readings on the compression side at station A using two stiffeners are used.

HENCE $\sigma_1 = -28{,}400$ p.s.i. $\quad \sigma_e = 33{,}000$ p.s.i.

$\sigma_2 = +6250$ p.s.i. $\quad E = 29{,}000{,}000$

$\mu = 0.30$

FROM Eq. (10) $\quad W_{de} = (1+\mu)\dfrac{\sigma_e^2}{3E}$

$$W_{de} = \frac{1+0.3}{3 \times 29 \times 10^6}(33,000)^2 = 16.25$$

FROM Eq. (11) $\quad W_d = \dfrac{1+\mu}{6E}\left[(\sigma_1 - \sigma_2)^2 + \sigma_2^2 + (-\sigma_1)^2\right]$

$$W_d = \frac{1+0.3}{6 \times 29 \times 10^6}\left[(-28,400 - 6250)^2 + (6250)^2 + (+28,400)^2\right]$$

$$W_d = 15.30$$

For yielding to occur W_d must be greater than W_{de}, which is not so from the above. However it is safe to say that yielding did occur for this case of loading with two stiffeners on the beam, because the assumed values for the μ, E, and σ_e may not be exact. For example if σ_e is slightly smaller, say 32,000 p.s.i., W_{de} becomes 15.2 indicating yielding. Again on reexamining the observed readings it may be seen that for any set of values, W_d will be much smaller than that for the case chosen and consequently yielding could have occurred only at this point in the experiment. Furthermore it is known definitely that a kink occurred in the top flange when two stiffeners were used. This had the effect of releasing much of the load held by the top flange and shifting it back to the bottom plate as in the case of no stiffeners. This resulted in the vertical component of stress rising sharply to the point where its added influence raised the stresses up to the yield point.

It can also be seen from the observed readings that after yielding had occurred with two stiffeners on the beam a readjustment of stress took place in such a manner as to reduce the stress at the strain rosette. This is shown by the readings for one stiffener, and no stiffeners on the beam where the principal stresses and maximum shearing stress decrease.

Actually the stresses in the web where the major yielding took place must have been much higher than recorded. The observed Luders lines were much more numerous in that section of the web just above the lower fillet, while the measuring strain rosette was at mid depth. This agrees well with what would be visibly seen from the membrane analogy, because as the fillet is approached the slope of the soap film increases sharply indicating torsional shearing stress concentrations due to the sharp curvature of the fillet. In general these high stresses are mostly of a local nature and do not greatly influence the yielding of the beam as a whole. Actually it is unknown to what extent the yielding in the web effected the angle of twist, flange stresses and the yielding of the beam at other places. Whether or not this yielding should govern the design of a beam should be further investigated using many more strain gauges over the web to measure exactly the highest stress concentrations etc.

Further evidence of yielding was seen from the fact that the strain gauges changed their original zero position by about fifty micro-inches.

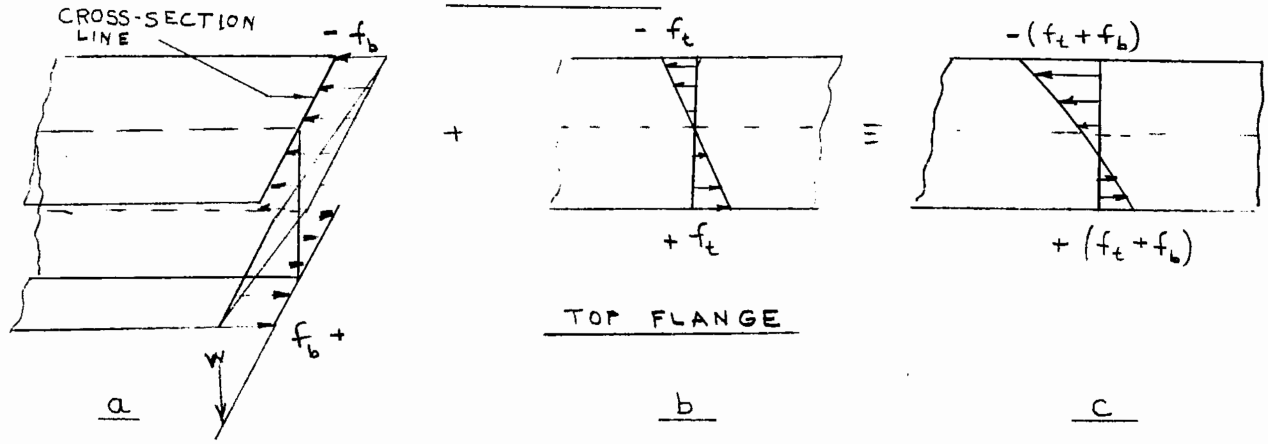
Further discussions of web stresses is limited by the fact that no test data is available as only one rosette was placed on each side of the web. Actually future investigators should also consider the stresses in the light of the theory of beams subjected to unsymmetrical bending, (10) i.e. bending in which the plane of the moment is not perpendicular to a principal axis. This may give a more complete explanation for the high vertical component of stress. Furthermore in the simple bending theory it is assumed that the resultant of the external forces is a couple that lies in, or is perpendicular to, a plane of symmetry of the cross-section, which is not valid in our case. This would also help to explain the location and direction of the neutral plane.

The Neutral Plane.

In a member of constant cross-section subject to pure bending, the neutral plane, or surface, implies a fibre in that cross-section of zero stress, while the stress of any other fibre varies as its distance from the neutral surface, being of opposite sign depending on whether the fibre is above or below. It can also be shown that the neutral plane for pure bending is located at the centre of gravity of area of the section.

In the case of bending, combined with torsion, we can no longer think of the neutral plane as in the old sense because the torsional stresses no longer vary as the distance from the centroid, or if they do vary as the distance from some other plane then this new plane is not necessarily the same as for bending. Hence some new concept of the neutral plane is necessary. In this thesis it is found convenient to define this new plane as the locus of a point which moves along the cross-section such that there is tension stress on one side, compression on the other, and zero at the point itself. As can be seen from Fig. 15 this line traced out is not straight but curves its way through the web. Its exact location at any cross-section depends on the torque and bending stresses at that point. Since these change independently the neutral line changes its location at every section along the beam. In Fig. 15 the location of the neutral line is traced out for the top flange, and bottom plate. Let us examine the influencing factors for this neutral line. At section A, if only pure bending existed the theoretical distribution of stress would be as shown in Fig. 20a i.e. constant

FIGURE 20



along the top surface. If only pure torsion existed the distribution of stress would be as in Fig. 20 b i.e. varying from + at one edge to − at the other. Adding the two gives a theoretical distribution as in Fig. 20 c. Examination of this line shows that it is not necessarily a straight line because it is the combination of a constant stress with a variable stress. However it is almost a straight line as seen from Fig. 15 since the torsional stress is high compared to the bending. Another feature of this line is that it crosses the cross-section line not on the centre line of the web but slightly removed so as to place more area in compression than tension. This agrees with the observed results plotted in Fig. 15 . The neutral line on the top flange is located about one-half inch from the centre line of the web for all three torques and runs almost parallel to it in the central portion of the beam. The smaller torque separates from the other two at the end third of the beam. The exact position of the neutral line depends on the extent to which the components of stress are caused by torsion, and bending. At station D the compression stress is larger than the tension in the upper half of the section and hence more area is in compression. This is not so at station A and the neutral line runs about midway along the web. Note that for the smallest torque the bending stress was half of that for the other two, as the load was only fifty pounds per foot. This may account for the separation of the neutral line from the other torques.

For the bottom plate the theory is the same as before except perhaps it is doubtful whether the bending stress varies uniformly across the entire bottom surface or diminishes towards the edge of the plate. Also the torsional stress would probably not vary across the bottom as in the case of the top flange but would vary as in Fig. 21a , i.e. point C′ would be located near the centre of area of

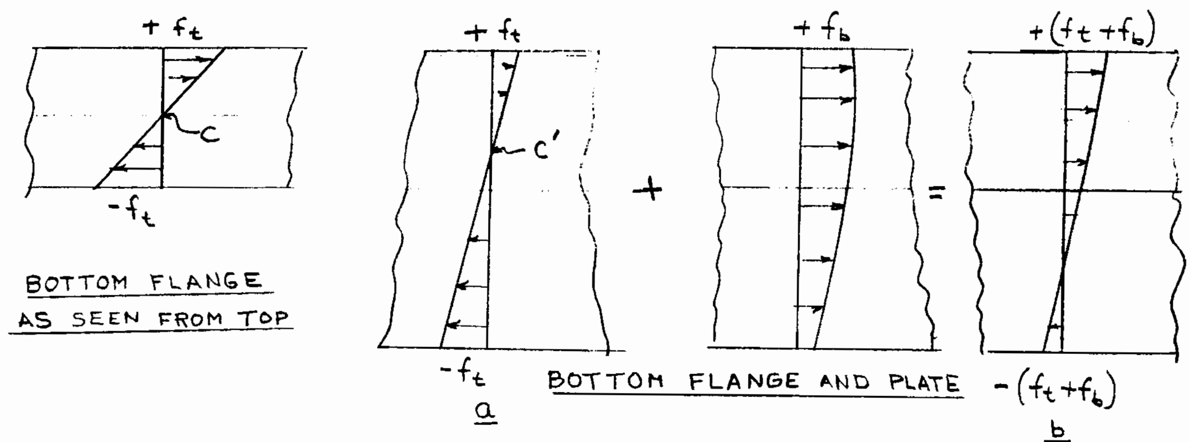


FIGURE 21

the bottom flange and plate. The combination would give a curve as in Fig. 21b. This is shown to be true by the plotted results and furthermore these results indicate that the bottom plate very strongly influences the distribution of torsional stress since the neutral point lies in the plate itself and the surface under the bottom flange of the beam is in tension throughout. It was shown that the actual flange stresses were 80% lower than those calculated by the Lyse-Johnston method with no bottom plate. The above discussion tends to indicate that the bottom plate is responsible for most of this reduction and the accuracy of the Lyse-Johnston approach is strengthened. It is also seen that the neutral line at the bottom surface for all three torques is the same, and this line approaches the centre line of the beam as it runs towards the middle. The fact that the position of the neutral line at the top and bottom surface are identical especially for the larger torques tends to suggest that its position may be also a property of the cross-section independent of the magnitude of the torque. This seems reasonable but must be confirmed by future tests.

In an actual case in practice the torsional stresses will probably be small compared to the bending stresses and the neutral line will move outwards to the edge or even leave the plate entirely i.e. giving only tension along the bottom.

The significance of the discussion on the neutral plane is to illustrate that in a case of loading in which the torsional stresses are high relative to those caused by bending the beam no longer can be thought of as behaving in the old pure bending manner, and the distribution of stress must be thought of in this new light.

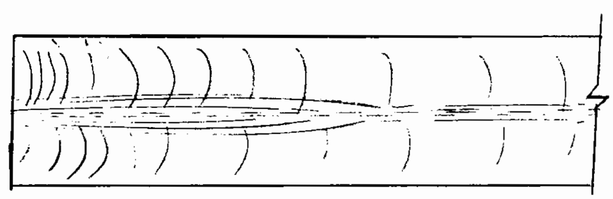
Visual Observations.

When brought from the shop the surface of the beam was covered with a thin grayish coat of shop scale and some rust. At the selected stations this scale was removed with a polisher and the strain gauges glued to the cleaned areas, the remainder of the beam was untouched throughout the entire experiment. As the test progressed cracks developed in this inelastic scale which traced out the stress pattern over the entire beam. All observations were of necessity only visual and no attempt was made to measure or study quantitatively these lines.

When loading commenced the first strain readings indicated that the bottom plate was not acting perfectly elastically with the beam. This was evident from the fact that the measurements on one side did not correspond exactly with those symmetrically opposite. This was due, to a large extent, to the inelasticity of the weld material, the inexact spacing of the welds, and slight variations in the length of welds. On the surface of the bottom plate adjacent to the welds a series of lines was seen to radiate outwards in all directions. These lines were also noticeable on the bottom plate at the points of highest stress i.e. near the ends of the beam directly under the web. At higher torques the lines became more numerous and pronounced.

On the top flange lines developed which ran outwards from the top web centre line and parallel to each other Fig. 22 . These were denser at the ends and diminishing towards the central portion

FIGURE 22



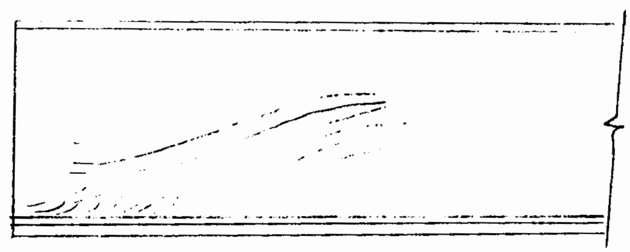
TOP FLANGE

of the beam. Longitudinal lines were also present running over the web top. These were caused by the high torsional shear and their direction, longitudinally corresponded with the directions of the maximum strain as calculated from the observed readings.

After the kink occurred in the top flange a long crack ran under the top flange along both sides of the thinnest section of the web. There was no way of knowing how deep this crack was but the strain readings of the gauges on the top flange showed elastic behavior still in progress and consequently it was assumed that the crack was not a serious warning of impending failure but rather due to the kink and only surface deep.

After yielding occurred at the ends of the beam on the compression side of the web, Luders lines formed which traced out a pattern as in Fig. 23 . These consisted of long sharp lines on the finely polished

FIGURE 23



surfaces. On the untouched areas the mill scale flaked off and fresh scratches appeared on the yielding surface. These layers of slip, or Luders lines showed regular orientation with respect to the directions of the principal stresses.

Suggested Future Experiments.

Since this was the first investigation of the torsional properties of a beam and the influence of stiffeners on the stresses and angle of twist it was impossible to cover more than a few highlights of the problem.

Considerable time and effort was devoted to solving many practical problems which will not hinder future investigations.

Some of these were the design and fabrication of the beam, stands, stiffeners, and pan, as well as pouring concrete blocks for load, and making equipment to measure the angle of twist. Again in placing the strain gauges we had only theoretical considerations to work by. Needless to say errors were made involving the loss of much time and effort, however a firm foundation for further studies was laid.

As was noticed the practical considerations demanded a width and thickness of bottom plate which will remain constant regardless of the size of beam. Hence if a larger beam is used, say an 8WF31 and the size of plate remains the same the influence of the plate on the system will tend to decrease and a correct appraisal of the effects of the plate will only be possible after several more tests using larger beams.

In future tests more consideration should be given to the stresses and deformation of the web since contrary to expectance yielding took place, also it is advisable to pre-determine the exact values of Poisson's ratio, Young's modulus of elasticity, the yield point, and the torsional constant K.

Future investigations should consider the design of a stiffener which will prevent warping of the cross-section as well as the theoretical considerations involved.

The arching effect of walls suggests an experiment in which the load will vary from zero at the ends to a maximum at the centre. This perhaps is really a closer approximation to the actual problem than the use of continuous load.

Conclusions

In this paper the general problem of a beam subjected to torsion caused by a continuous eccentric load was investigated. The effect of stiffeners on the torsional properties was also examined. An effort was made to discuss thoroughly those topics of particular interest to design engineers, i.e. flange stresses, web stresses, and angle of twist. The position of the neutral plane was also dealt with to illustrate that when the torsional stresses are very high in a member it can no longer be thought of as behaving in the old pure bending manner. The observed results were compared with those predicted on the basis of the Grinter, and Lyse-Johnston methods of design.

In all cases it was found that the Grinter method gave extremely high values of stress leading to the choice of uneconomical sections. In general it should not be used unless the proportion of torsional stress is small relative to bending. The Lyse-Johnston approach is by far more accurate and should be used for all torsion problems, although with respect to the angle of twist it was found to be too far on the safe side.

Careful analysis showed that the primary use of stiffeners is to hold the flanges together, reducing the web and flange stresses. Stiffeners should be used where there is a tendency for the flanges to twist excessively due to the nature of the applied loading. It is still impossible to give an exact rule for the quantity or spacing of those stiffeners but the author feels that four pairs evenly spaced should be a minimum. Stiffeners do not change the angle of twist and hence it is no advantage to use them for this purpose.

In the opinion of the author torsional stresses should not be considered as secondary and designers are not justified in raising the allowable 20,000 p.s.i. as set down in the C.S.A. specifications.

The author has noticed in some texts formulae for the rapid solution of the torsional stresses in a beam. Most of these equations do not apply for the conditions of eccentric loading and if used may lead to large errors.

It should be pointed out that in this thesis only the idealized solution was considered. The effects of the continuity and rigidity of connections and the restraints imposed by slabs, walls, and fire proofing are left to the judgement of the engineer.

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PHOTOGRAPHS

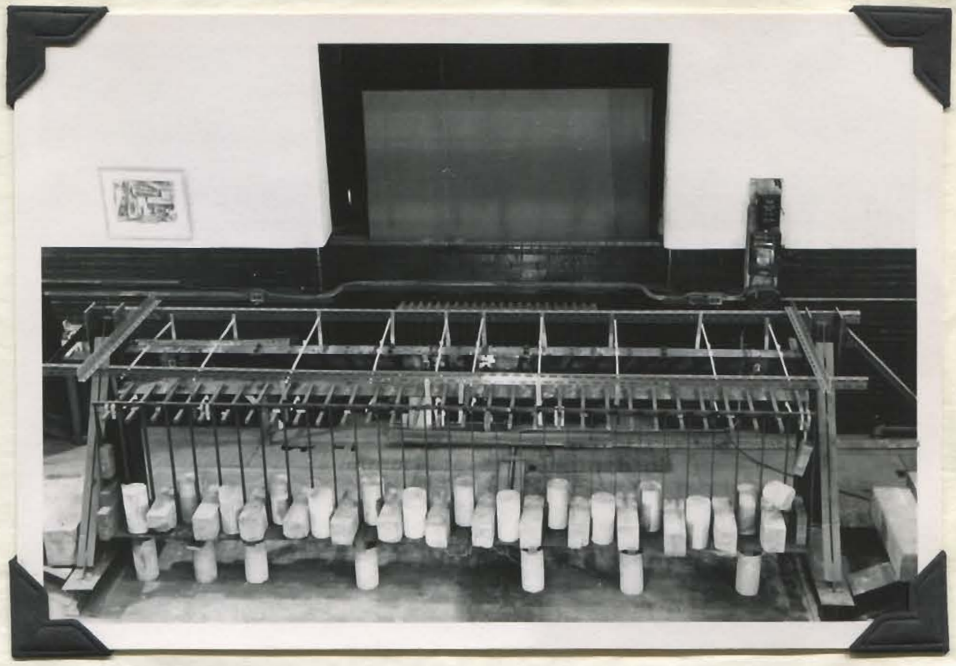


PHOTO I. Overall front view showing load jacked up and resting on concrete cylinders.

PHOTO II. Overall front view showing four stiffeners on beam, and 100 pounds per foot load on pan at an eccentricity of 23 inches.

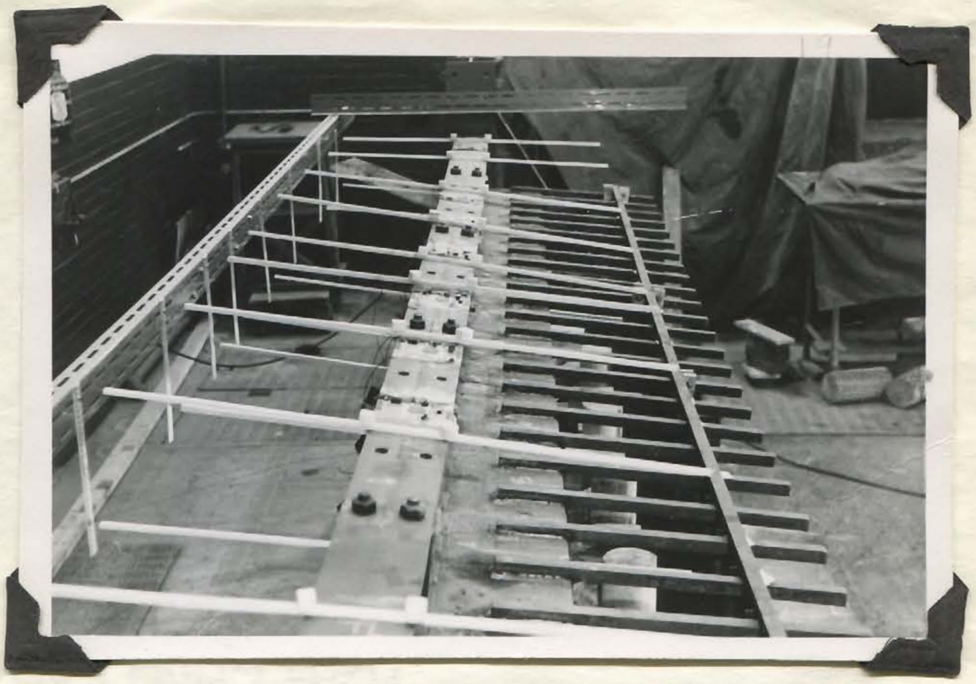
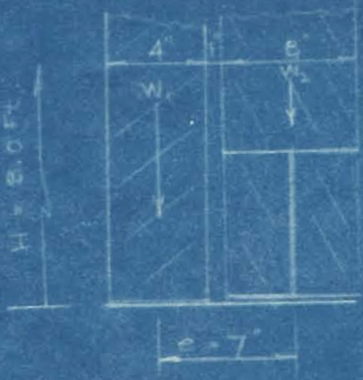


PHOTO III. Longitudinal top view illustrating the angle of twist when beam is loaded.

PHOTO IV. Left side view showing stand base and angle of twist when beam is loaded.

SAMPLE CALCULATION

GIVEN A WALL SYSTEM AS SHOWN. SIMPLE BEAM. TORSIONALLY FREE ENDS. SELECT A SPANDREL.



LENGTH = 16.0 Ft

W_1 = 4" BRICK AT 120 Lb-CU.Ft = 320 Lb/Ft
W_2 = 8" BLOCK AT 80 " " = 430
WT OF BEAM @ Ft = 80
ω = 830 Lb/Ft

$T = 7 \times 320 \times 16.0 = 35{,}800$ IN-Lbs
$M = \frac{\omega L^2}{8} = \frac{830 \times 16^2}{8} = 26.4$ Ft-K

$\frac{L}{2a} = \frac{16 \times 12}{2 \times 45.46} = 2.12$

TRY 8 WF 35		
a*	= 45.46	IN^{-1}
B	= 1.125	IN^{-3}
C	= .(5)52	$(IN\text{-}Lb)^{-1}$
N	= .788	IN^{-3}
Z	= .559	IN^{-2}

FORMULAE

$f_t = BTa^2\frac{d^2(F)}{dx^2}$ LET $a^2\frac{d^2(F)}{dx^2} = Q$ $\therefore f_t = QBT$, $S_f = S_1 NT$,

$S_t = S_1 ZT$, $\psi = CT(F)$, $V_\psi = \frac{T \times 1}{h\ 2} \cdot \frac{1}{\left(\frac{L}{2a}\right)} \tanh\frac{L}{2a}$

$S_\psi = \frac{1.5 V_\psi}{bt}$, $\underset{AT\ x = 0}{S_1} = \frac{1}{2}\left[1 - \frac{\tanh\frac{L}{2a}}{L/2a}\right]$ $S_1 = \frac{1}{2}\left[1 - \frac{\tanh 2.12}{2.12}\right] = .272$

$Q = 0.181$ $(F) = 0.344$

TORSIONAL BENDING $f_t = 0.181 \times 1.125 \times 35{,}800$ = 7300 PSI AT CENTRE
VERTICAL BENDING $f_b = \frac{26{,}400 \times 12}{31.1}$ = 10,200
COMBINED FLANGE STRESS (ON OUTER CORNER) = 17,500 PSI AT CENTRE

TORSIONAL WEB SHEAR = $S_t = 0.272 \times .559 \times 35{,}800$ = 5450 PSI AT END
VERTICAL WEB SHEAR = $\frac{830 \times 16}{2 \times 8.12 \times .315}$ = 2600
COMBINED WEB SHEAR (ON WEB SURFACE, MID-DEPTH) = 8050 PSI AT END

TORSIONAL FLANGE SHEAR = $S_f = .272 \times .788 \times 35{,}800$ = 7650 PSI AT END
$V_\psi = \frac{35{,}800}{8.12 \times 2} \cdot \frac{1}{2.12} \times .97 = 1010$ LbS.
LATERAL BENDING FLANGE SHEAR = $S_\psi = \frac{1.5 \times 1010}{8.027 \times .493}$ = 380
TOTAL FLANGE SHEAR = 8030 PSI AT END

TWIST ANGLE ψ = .(5)52 × 35,800 × 0.344 = 0.064 RADIANS
= 0.064 × 57.3 = 3.68°

NOTE. A SMALLER SECTION CAN BE USED BUT ψ MAY BECOME TOO LARGE.

USE FOUR PAIRS OF STIFFENERS EVENLY SPACED

* REF. 18