Concerto for Piano and Orchestra

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Volume 1: Text

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Abstract

This thesis contains two volumes. The first is a written text that describes my compositional techniques in the context of an analysis of my *Concerto for Piano and Orchestra*. The second volume is the score of this work.

Volume one is divided into two parts. Part I describes my compositional techniques and the original contributions of the thesis. These include the incorporation of psychoacoustic models of hearing into the creative process, as implemented by a computer program written by the author. I give detailed descriptions of models for dissonance and pitch commonality, and discuss my use of contour theory. Part II of the first volume illustrates these techniques through an analysis of the *Concerto for Piano and Orchestra*. The main topics of this analysis are the creation of background harmonic regions based on high pitch commonality to a referential sonority, and the integration of a basic shape or contour into all parameters and structural levels of the music.

Volume two is the full score of the *Concerto for Piano and Orchestra*. This work has a duration of 23 minutes. It is scored for solo piano and a small orchestra consisting of flute (doubling on piccolo), oboe, B-flat clarinet, bassoon, horn in F, trumpet in C, trombone, two percussion and strings (44332). The Concerto is divided into four movements, played without pause, and two brief opening and closing sections.

Cette thèse contient deux volumes. Le premier est un texte qui décrit mes techniques compositionnelles dans le contexte d'une analyse de mon *Concerto pour piano et orchestre*. Le deuxième volume est la partition de cette oeuvre.

Le premier volume se divise en deux parties. La partie I décrit mes techniques compositionnelles et les contributions originales de la thèse. Au nombre de celles-ci, on compte l'incorporation des modèles psychoacoustiques de l'ouïe dans le processus créateur, tel que mis en application par un logiciel écrit par l'auteur. Je décris en détail un modèle de dissonance, un autre pour la persistance de hauteurs, ainsi qu'une théorie des contours. La partie II du premier volume illustre ces techniques par une analyse du *Concerto pour piano et orchestre*. Celle-ci discute principalement la création de régions harmoniques basées sur la communauté de hauteur par rapport à une sonorité de référence, et l'intégration d'un contour de base dans tous les paramètres et niveaux structuraux de la musique.

Le volume deux est la partition complète du *Concerto pour piano et orchestre*. Cette oeuvre de 23 minutes est écrite pour piano solo et petit orchestre, composé de flûte (jouant aussi le piccolo), hautbois, clarinette en Si♭, basson, cor en Fa, trompette en Do, trombone, deux percussions et cordes (44332). Le Concerto se divise en quatre mouvements, joués sans pause, et deux brèves sections d'introduction et de conclusion.

Acknowledgements

I wish to express my gratitude to the many people who have helped me during the preparation of this thesis. My advisor, Dr. John Rea, made invaluable comments, and his piece *Treppenmusik* was one of the original sources of inspiration for my research. Dr. Richard Parncutt answered many questions and provided the code for C programs that implement the models of dissonance and pitch commonality. Dr. Heinrich Taube made numerous changes to his Enved program based on my requests; I also use the MIDI functions of his Common Music software package in my own programming. Dr. Mikael Laurson and Dr. Marcus Castrèn of the Sibelius Academy (Helsinki) kindly gave me permission to incorporate portions of their Macset program into my software. Finally, Dr. François de Medicis assisted me with the translation of the abstract.

The *Concerto for Piano and Orchestra* was commissioned by Radio-Canada for the *Société de musique contemporaine du Québec*. I would like to thank Laurent Major, producer at Radio-Canada, and Walter Boudreau, artistic director of the SMCQ, for this opportunity. I would especially like to thank piano soloist Marc Couroux for his wonderful performance at the premiere of the work on December 9, 1999, in Montreal.

On a personal note, I am extremely grateful to Dale and Kimm Stammen for their "random act of kindness." And above all, I wish to thank my family - Sherry, Aidan, and Brynne - for putting up with everything. This Concerto is dedicated to them.

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Volume 2: Score

Chapter 1: Introduction

1.1 Description

The *Concerto for Piano and Orchestra* has a duration of 23 minutes. It is scored for solo piano and a small orchestra consisting of flute (doubling on piccolo), oboe, B-flat clarinet, bassoon, horn in F, trumpet in C, trombone, two percussion and strings (44332). The piece contains four movements, played without pause, as well as brief opening and closing sections. The first and last movements feature the piano and orchestra, while the second movement is for orchestra alone and the third movement is for solo piano with very little orchestral participation.

1.2 Original Contribution

The original contribution of this thesis is the incorporation of psychoacoustic models of hearing into the compositional process, as implemented by a computer program written by the author.

1.2.1 Objectives

In recent years, one of my principal goals has been to increase the expressive potential of my music by strengthening the connection between my creative intentions, my compositional techniques, and the listener's reactions. I want to be more confident that the basic musical responses I wish to elicit will take place. I believe that improving this connection will help my compositional ideas be more clearly and effectively communicated. In order to realize this objective, one must first attempt to understand the fundamental ways in which people perceive their auditory environments, and then find ways to integrate this understanding into the compositional process.

1.2.2 Psychoacoustic models

The first step in this process is quite challenging. It is clear that many of our instinctive ideas about the way we perceive the world do not correspond to reality. This is why we can be tricked by, say, auditory illusions; we predict that what we hear reflects the actual state of the world of sound, when the truth may be quite different. I have therefore based my compositional practice on my research into the domain of psycho-acoustics, which studies the relationship between the objective qualities of sound in the environment, the physiology of the auditory system, and the subjective perception of sound by the listener.[1] My investigations have revealed a number of exciting develop-

[1] Richard Parncutt, *Harmony: A Psychoacoustical Approach* (Berlin: Springer-Verlag, 1989), 16.

ments in this area that seem highly relevant to musical composition. In particular, I incorporate the following psychoacoustic models into my compositional technique.

Contour Theory

Contours determine many aspects of the piece. This is clearest in motivic techniques, where the use of contours helps ensure recognizable connections between related melodic ideas. Contours are not restricted to horizontal pitch, however, but also affect rhythm, form, and harmonic parameters.

Sensory Dissonance

The psychoacoustic model of dissonance measures the amount of rapid beating or "roughness" that is perceived in a sonority. This is done in two steps. First, I emulate the physical spectrum of the sonority, and then I determine the degree to which the spectrum has components within a critical band of each other.

Pitch Commonality

Pitch commonality is an estimation of the extent to which two sonorities are heard to have spectral components in common. In general, sonorities with high degrees of pitch commonality will be heard as being more closely related than sonorities with low pitch commonality. The model uses virtual pitch theory and the saliences of pure and complex tone sensations to generate the perceptual spectrum of a sonority. In the Concerto this model is used to determine the relationship between successive chords, and between foreground chords and a background referential sonority.

Other Areas

Other aspects of psychoacoustics also play a less fundamental role in the Concerto. These include, for example, theories of auditory streaming from the field of auditory scene analysis.

1.2.3 *Apprentice*

The second step is also challenging for it tries to incorporate these scientific findings into the creative process. Due to the complex mathematical nature of some of these models, I must rely on calculations done by machine rather than time-consuming reckonings by hand. During the development of my compositional technique in the *Concerto for Piano and Orchestra*, therefore, I have written a number of functions in the Lisp programming language that implement these models. As these compositional tools grew more numerous, they were grouped into a larger software package named *Apprentice*. This title indicates that the software is not a self-contained compositional environment that makes independent or autonomous choices, but is merely an aid in the realization of my own ideas. The focus of this thesis is the composition of the Concerto, not the details of the software design. I make reference to *Apprentice* only where it clarifies the process of composition.

1.3 Organization

This thesis is in two volumes. Volume 1 is a written text describing the *Concerto for Piano and Orchestra*, and Volume 2 is the score of this work. Since many of the techniques I use are employed for the first time in this piece, I cannot assume familiarity with the concepts and terminology that underlie them. Volume 1 is therefore divided into two parts. Part I describes in general terms the new compositional techniques I have developed and the psychoacoustic models on which they are based. Chapter 2, *Horizontal Aspects of Pitch,* discusses the combinatorial theory of contour I use in the Concerto, and my use of break-point functions and positional contours to generate foreground and background aspects of horizontal pitch. In chapter 3, *Vertical Aspects of Pitch,* I present in detail the psychoacoustic models of dissonance and pitch commonality that I use. I also describe my approach to the creation of outer voices for harmonic progressions, an algorithm I have created for evaluating the adjacent interval variance of chords, the constitution of the harmonic vocabulary of the Concerto, and the manner in which these aspects interact to govern the harmonic stability of harmonic progressions. Chapter 4, *Duration,* describes my techniques for creating and manipulating foreground and background durations using rhythmic break-point functions and rhythmic profiles. I finish the chapter by discussing contributions of rhythm to overall stability.

Once the reader is acquainted with the terms and ideas presented in Part I, Part II analyzes the *Concerto for Piano and Orchestra* in the light of these new compositional techniques. Chapter 5, *Overview,* describes the overall form of the Concerto and presents two fundamental elements of unity: the referential sonority of the work and the basic shape or contour that determines almost all aspects of the music. I also show the overall background harmonic progression that underlies the foreground harmony throughout the piece. The following chapters discuss the parts of the Concerto in the order in which they occur. In each chapter I describe the form of the section being discussed, and I highlight applications of the basic shape in both foreground and background aspects of horizontal pitch, vertical pitch and duration. In Chapter 10, *Conclusion,* I summarize the original contributions of the thesis and discuss possible future directions for musical applications of psychoacoustics. The score of the *Concerto for Piano and Orchestra* then follows as Volume 2.

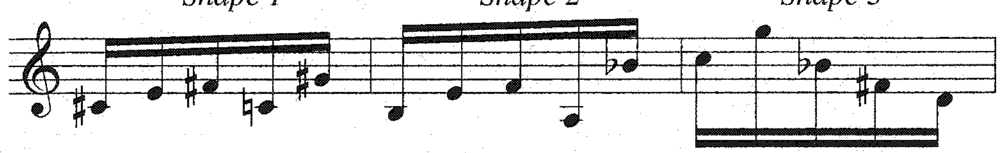
Part I: Compositional Technique

Chapter 2: Horizontal Aspects of Pitch

2.1 Contour Theory

In the *Concerto for Piano and Orchestra*, contours generate both surface melodic material and large-scale registral movement. My goal is to create recognizable similarities between related linear pitch structures. Contour theory is concerned with the relative positions of notes in a pattern - above, below, or the same - and not the actual musical intervals between them. There are two principal ways to keep track of these positions: by noting the relationships between adjacent notes in a pitch pattern only, or between each note and all of the other notes in the pattern. I use the latter approach, called the combinatorial model of contour.[2] There is a large body of experimental evidence to suggest that contour information is an important factor in the long and short term memory of melodies.[3] While contour and musical interval are both important for the retention and recognition of tonal melodies, contour becomes significantly more important in non-tonal contexts, regardless of the listener's musical training.[4] In other words, intervallic changes to an atonal melodic idea which retain the contour of the original are heard as relatively closely related, while changes which do not retain the contour are heard as relatively less closely related.

The use of contours to create melodic similarity differs from the use of pitch-class set relationships. According to Marvin and Laprade, "listeners are generally able to perceive equivalence or similarity among musical contours more easily than among pitch-class sets in melodic settings."[5] The following three melodic shapes illustrate the difference between these two approaches:



Example 2-1: Identical Contours Compared With Identical Prime Forms

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[2] L. Polansky & R. S. Bassein, "Possible and Impossible Melodies: Some Formal Aspects of Contour," *Journal of Music Theory* 36 (1992): 259-284.

[3] For an overview of the current state of research in contour theory, see Mark A. Schmuckler, "Testing Models of Melodic Contour Similarity," *Music Perception* Vol. 16 No. 3 (Spring 1999): 295-326.

[4] Eric Freedman, "The Role of Diatonicism in the Abstraction and Representation of Contour and Interval Information," *Music Perception* Vol. 16 No. 3 (Spring 1999): 365-387.

[5] Elizabeth West Marvin & Paul A. Laprade, "Relating Musical Contours: Extensions of a Theory for Contour," *Journal of Music Theory* 31 (1987): 226.

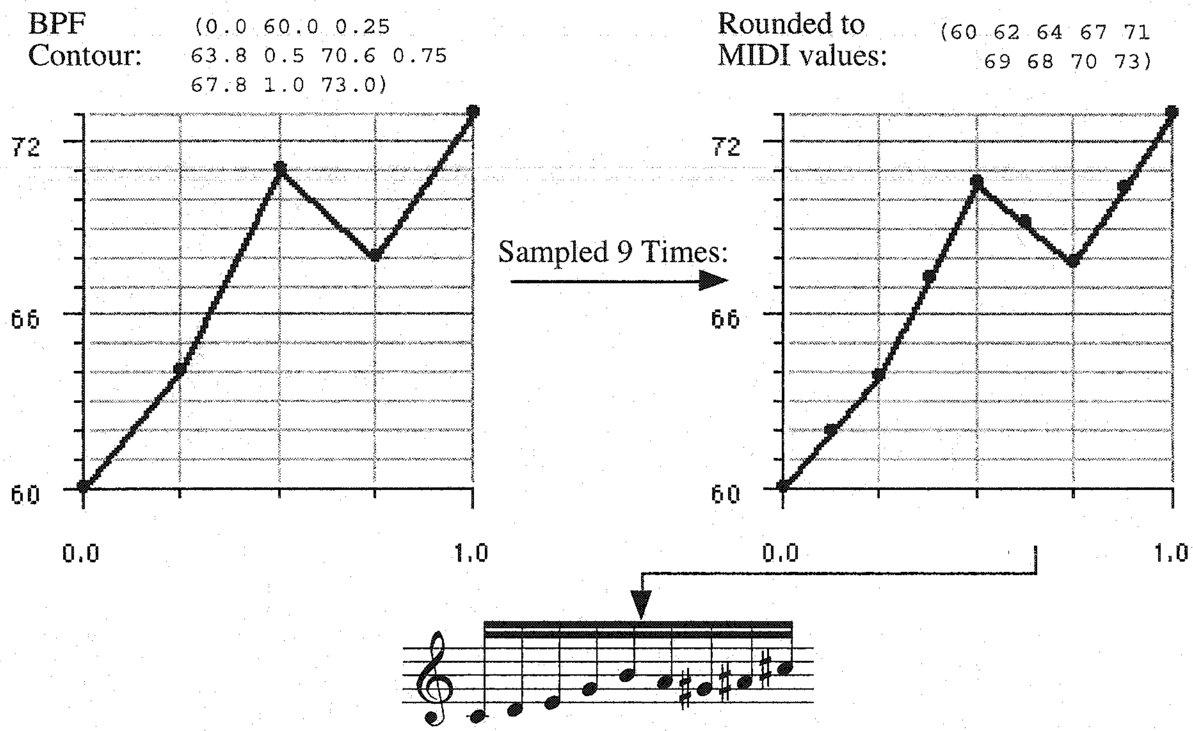
The first shape is the reference. The second shape has the same relationships between the registral positions of its notes as the first, and therefore has an identical combinatorial pitch contour. In both cases, for example, the second note is above the first note, below the third, above the fourth, and below the fifth. The first two shapes do not, however, have the same musical intervals between their notes or reduce to the same prime form. The third shape shares the same prime form as the first, but has a dissimilar contour. According to contour theory, the second motive should be heard as being more clearly derived from the first than is the third. In non-tonal music, therefore, contour resemblance between melodic ideas appears to be more important for the perception of similarity than is membership in a common set class.

In the *Concerto for Piano and Orchestra*, I implement contours in two ways. The first is a relatively straightforward mapping of points on a graphic curve onto two-dimensional pitch space, while in the second I apply a combinatorial pattern of contour relationships to different pitch collections.

2.2 Break-Point Function Contours

In the first technique, a contour is constructed as a break-point function (BPF), consisting of pairs of *x* and *y* values in the form *(x1 y1 x2 y2 x3 y3 . . . xn yn)* where the *x* axis represents the time dimension and the *y* axis represents the pitch dimension. [6] In *Apprentice,* break-point functions may be entered by hand or may be created and viewed graphically using the Lisp program "Enved".[7] A BPF pitch contour is sampled at equal intervals along the *x* axis, and the values are rounded off to the nearest whole number. These values are interpreted as pitches using MIDI key numbers (middle C = 60). The following example shows a BPF contour that is sampled 9 times, with the resulting values transcribed into musical notation. Since BPF contours are mapped onto the total chromatic, it is not possible to predict the set-class identity of pitch patterns generated in this way.

[6] The use of break-point functions in the concerto is not restricted to the pitch domain, but is also applied to other parameters, such as duration, dissonance and pitch commonality.
[7] Enved is part of the Common Music program, which is public domain software written by Heinrich Taube. Apart from this program, all functions related to contours in *Apprentice* were written by the author.



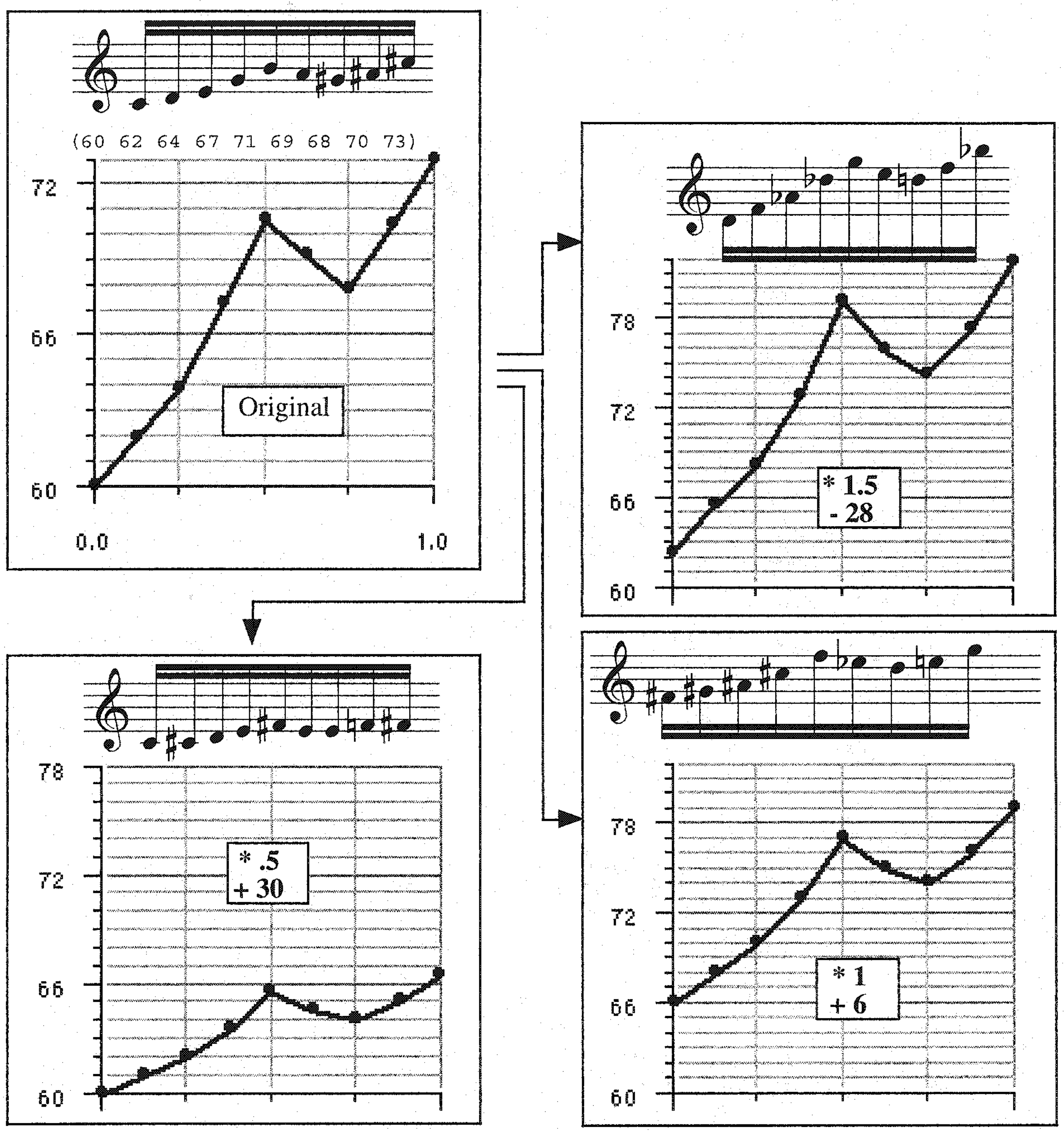
Example 2-2: Sampling a BPF Contour.

2.3 Variation of Break-Point Function Contours

A compelling feature of BPF contours is the large number of related melodic shapes that can be created through the application of a limited number of transformational techniques. Since such a collection of shapes is based on the same contour, its members should all be perceived as having a noticeable "family resemblance." Transformations of BPF contours may be applied to either the *x* or *y* axis, and may be fixed or may vary over time.

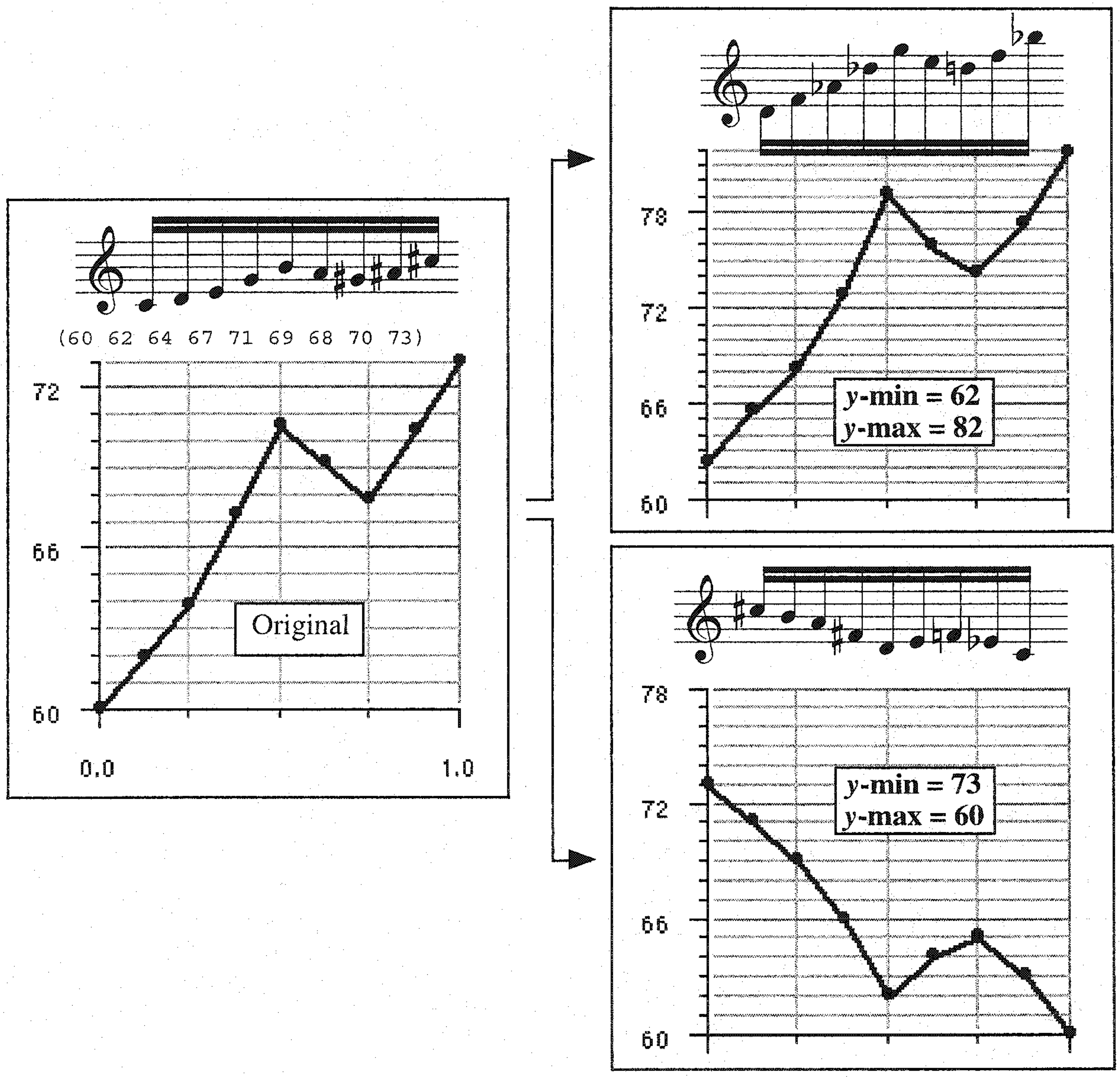
2.3.1 Transformations of the *Y* Axis

In the *Concerto for Piano and Orchestra*, the variation of points on the *y* axis (pitch) involves a combination of multiplication and transposition. Multiplication by a fixed value greater than 1 results in a uniform expansion of the contour, while values less than 1 result in a compressed contour. Multiplicative transformation is always combined with transposition. The addition of a fixed positive value results in an upwards transposition of the expanded or contracted contour, while addition of a negative value results in a downwards transposition. A combination of multiplication by 1 and addition of 0 returns the original pitch pattern unchanged. Example 2-3 gives several examples of the transformation of a BPF contour by multiplication and addition of fixed values. Note that this technique maintains the approximate relative size of musical intervals but not the actual intervals themselves, and that the relative combinatorial positions of the notes of the variants are maintained to a high degree.



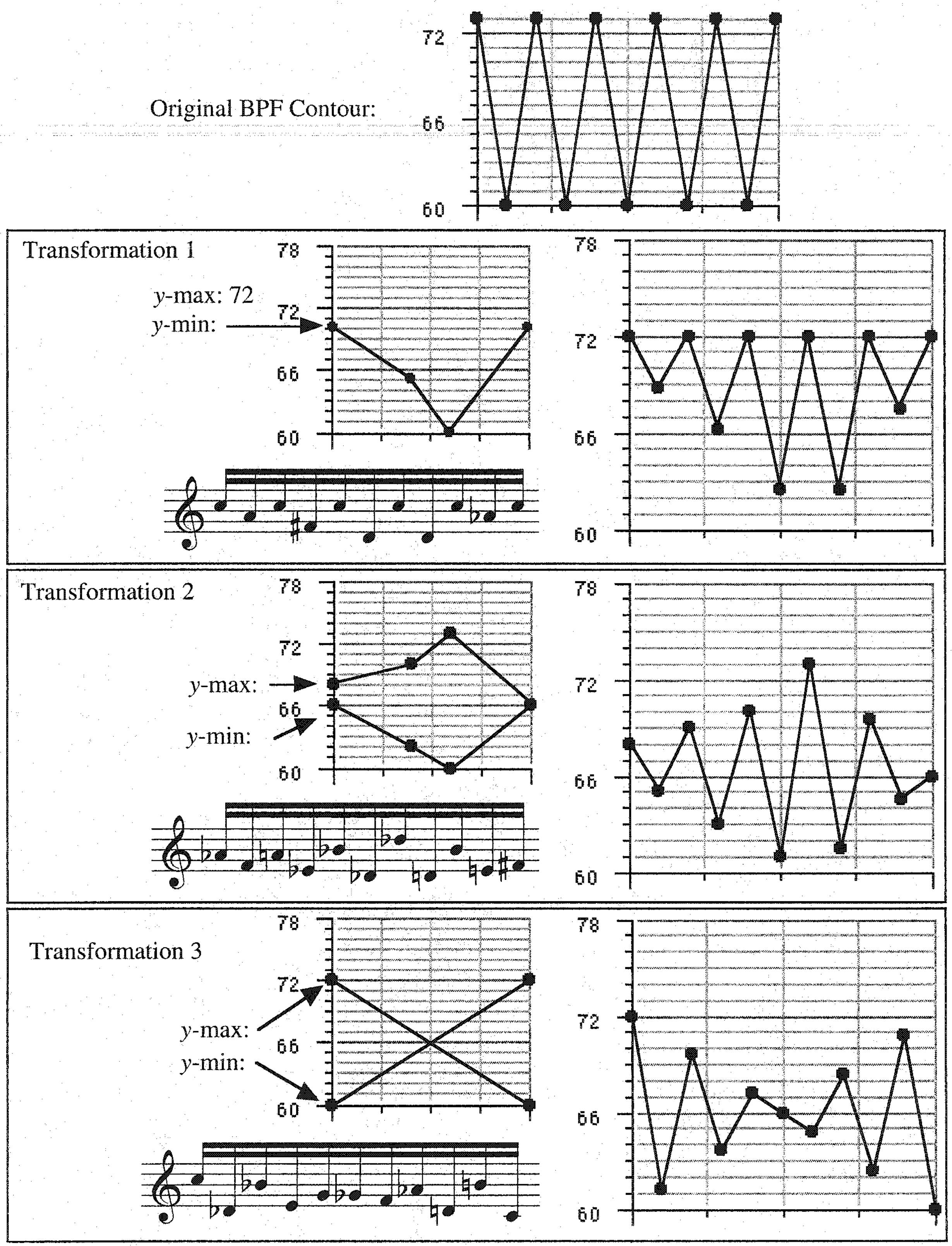
Example 2-3: Transforming *Y* Values by Fixed Amounts

With *Apprentice* it is also possible to vary a BPF contour by specifying new maximum and minimum *y* values. The program then determines the levels of multiplication and addition required to generate the new version. Specification of a maximum *y* value that is less than the minimum *y* value results in inversion of the contour.



Example 2-4: Transforming *Y* Values by Specifying New Maximums and Minimums.

The values used to transform the *y* axis of a BPF contour may also be defined dynamically, as time-varying functions. In this way, a contour may be gradually



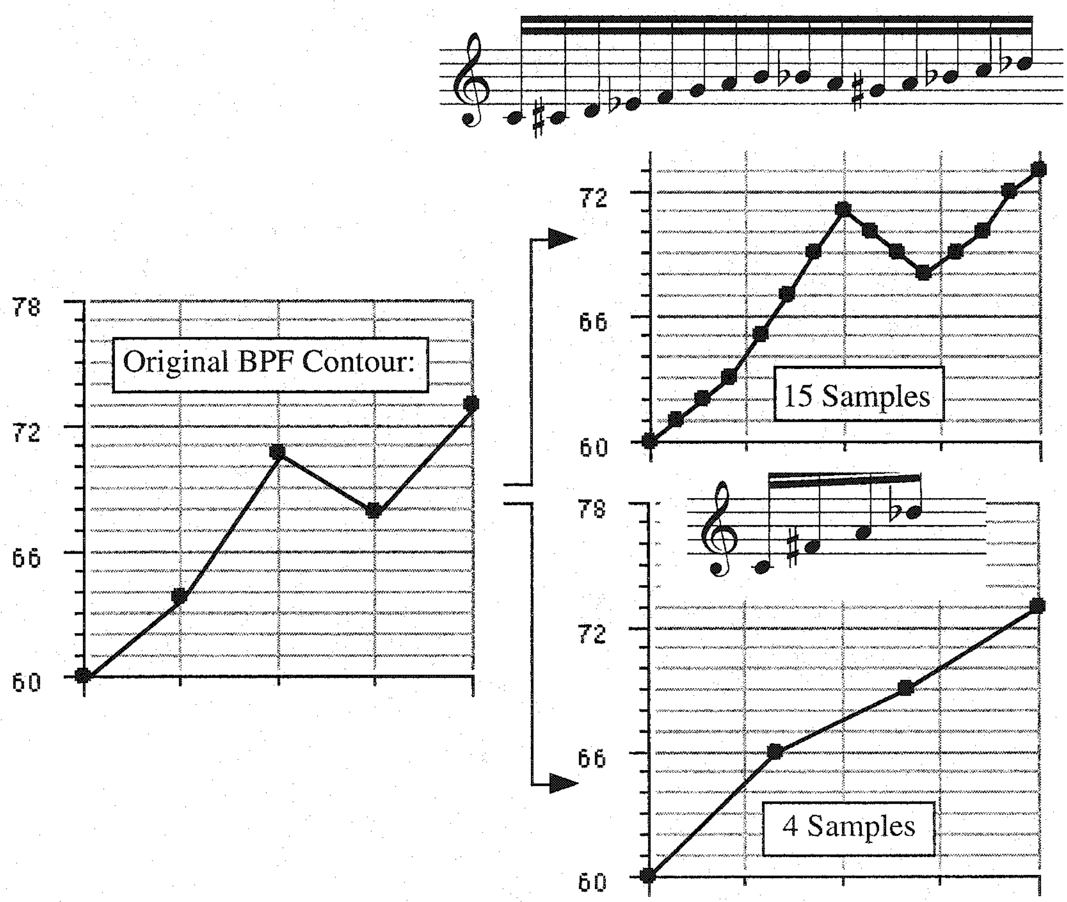
Example 2-5: Dynamic Scaling of *Y* Values

compressed or expanded over time. In *Apprentice*, one varies y values dynamically by using separate functions for maximum and minimum y values. These functions may be arbitrarily complex and operate independently of each other. Example 2-5 gives several samples of this type of transformation.

2.3.2 Transformations of the *X* Axis

Sampling Interval

Transformations of the x axis of a BPF contour affect the sampling interval. The simplest type of transformation uniformly increases or decreases the interval in order to change the number of samples. The result is a pitch pattern with a different number of notes than the original, but with a similar overall contour. The following example gives two versions of the same contour with different sampling intervals:

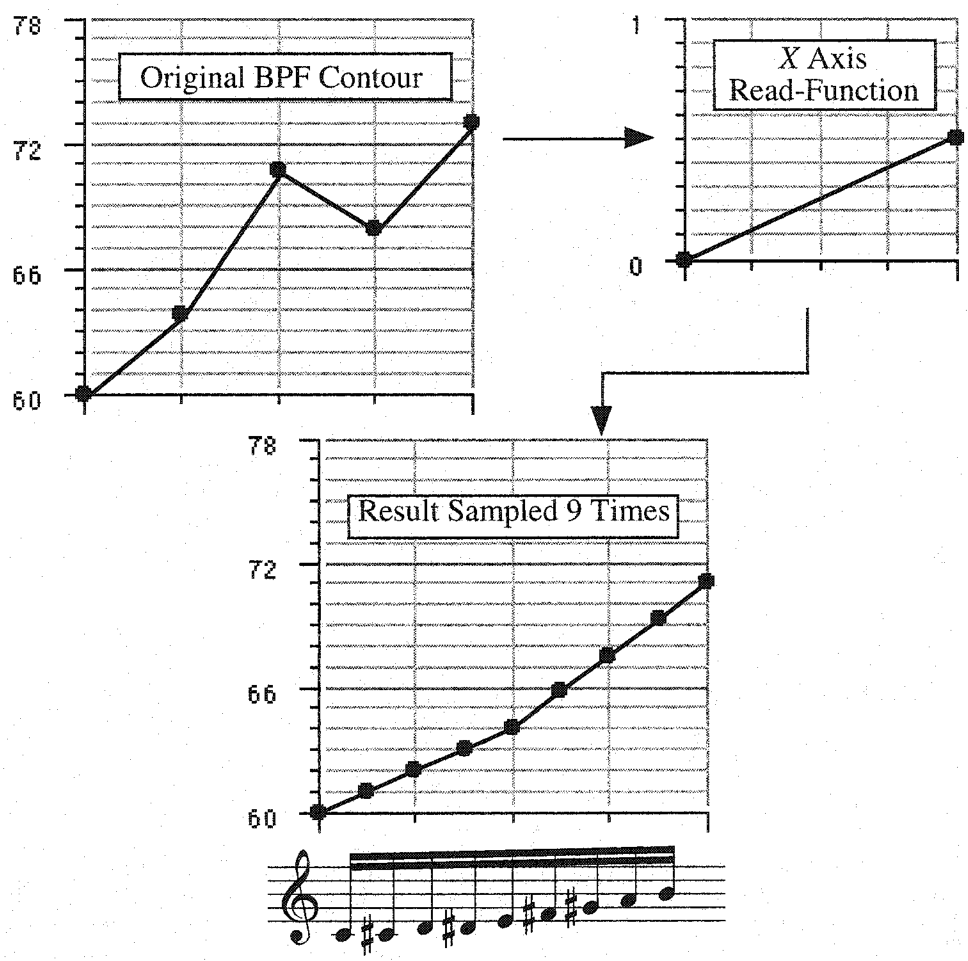


Example 2-6: Varying the Sample Interval

Note that a large reduction in the number of samples tends to distort a contour through omission of identifying features, as in the second transformation of this figure. The BPF contour is the same as in example 2-2, where it was sampled 9 times.

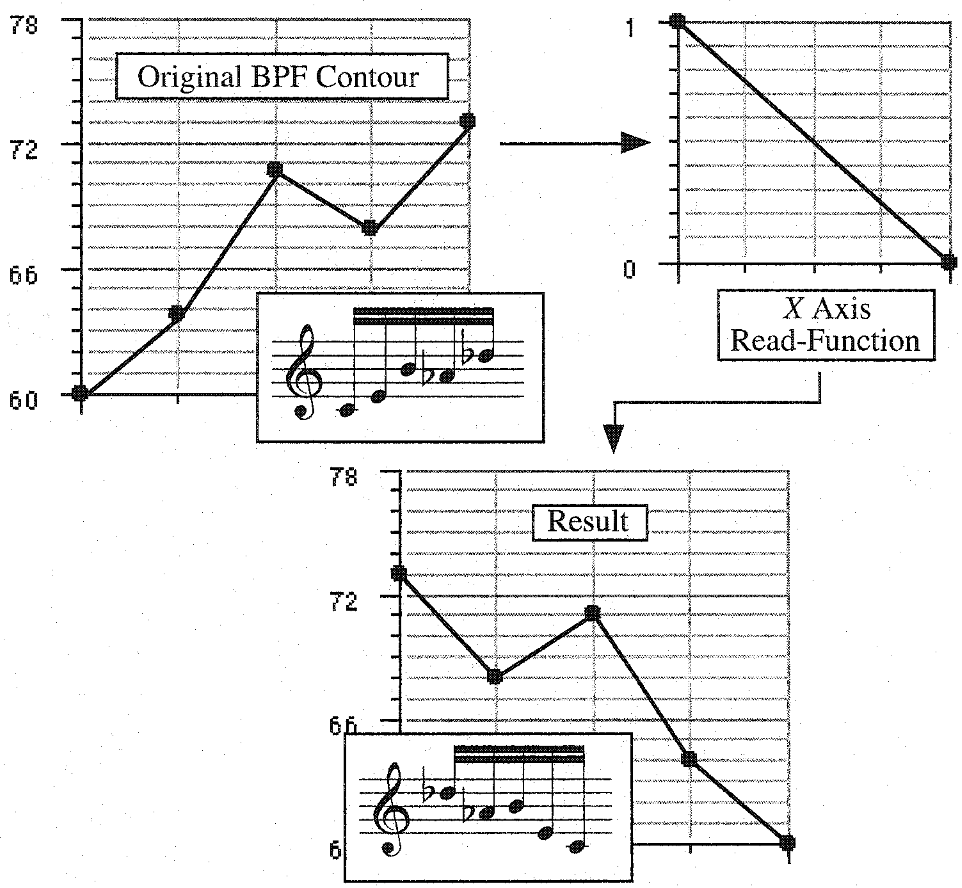
Read-Functions

A more complex type of transformation uses a second function to determine the movement of the sampling point along the *x* axis. Points on the *y* axis of this "read-function" determine how far into the BPF contour to sample. Values range between 0, which indicates the beginning of the BPF contour, and 1, which indicates the end of the contour. A read-value of .33 returns the y value located one third of the way through the original BPF contour, for instance, while a read-function with a straight slope from 0 to .5 indicates that the BPF contour should be read linearly from the start to its half-way point:



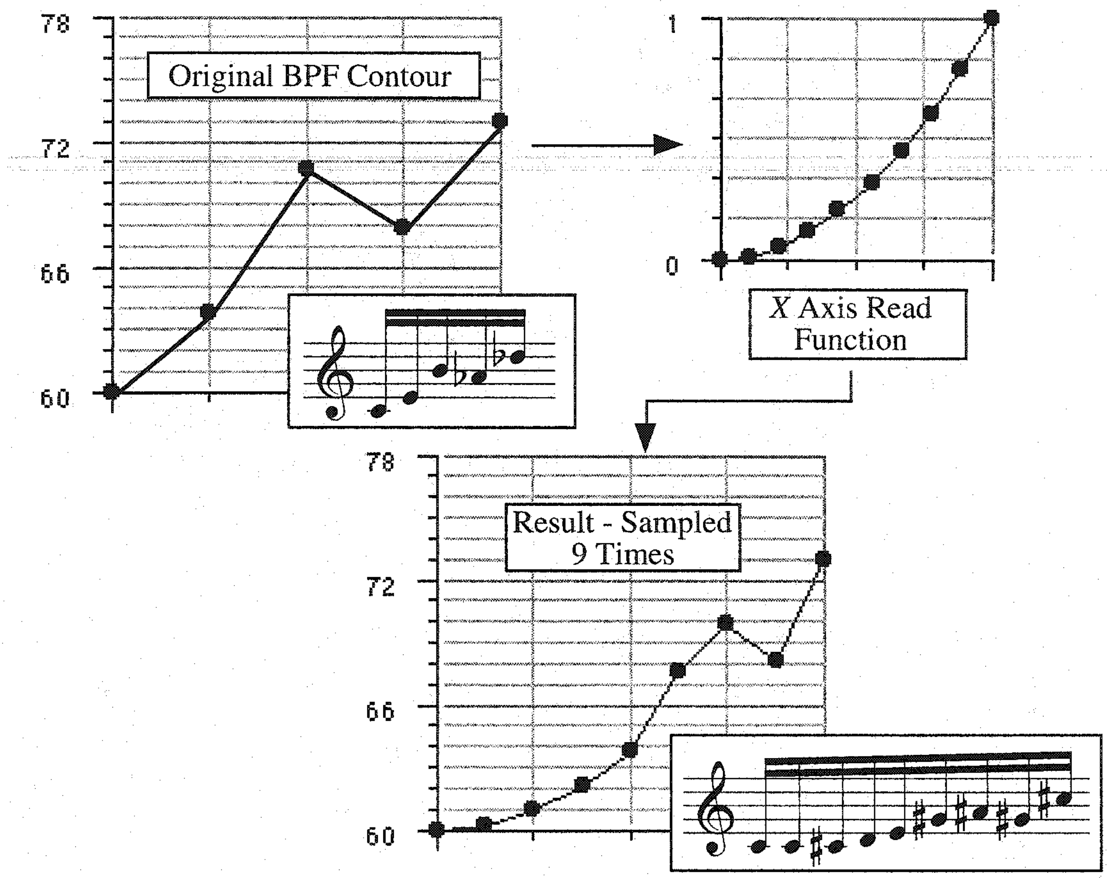
Example 2-7: Varying the Read-Function

To reverse the original contour, I use a read-function with a downward slope:



Example 2-8: Downward Sloping Read-Function

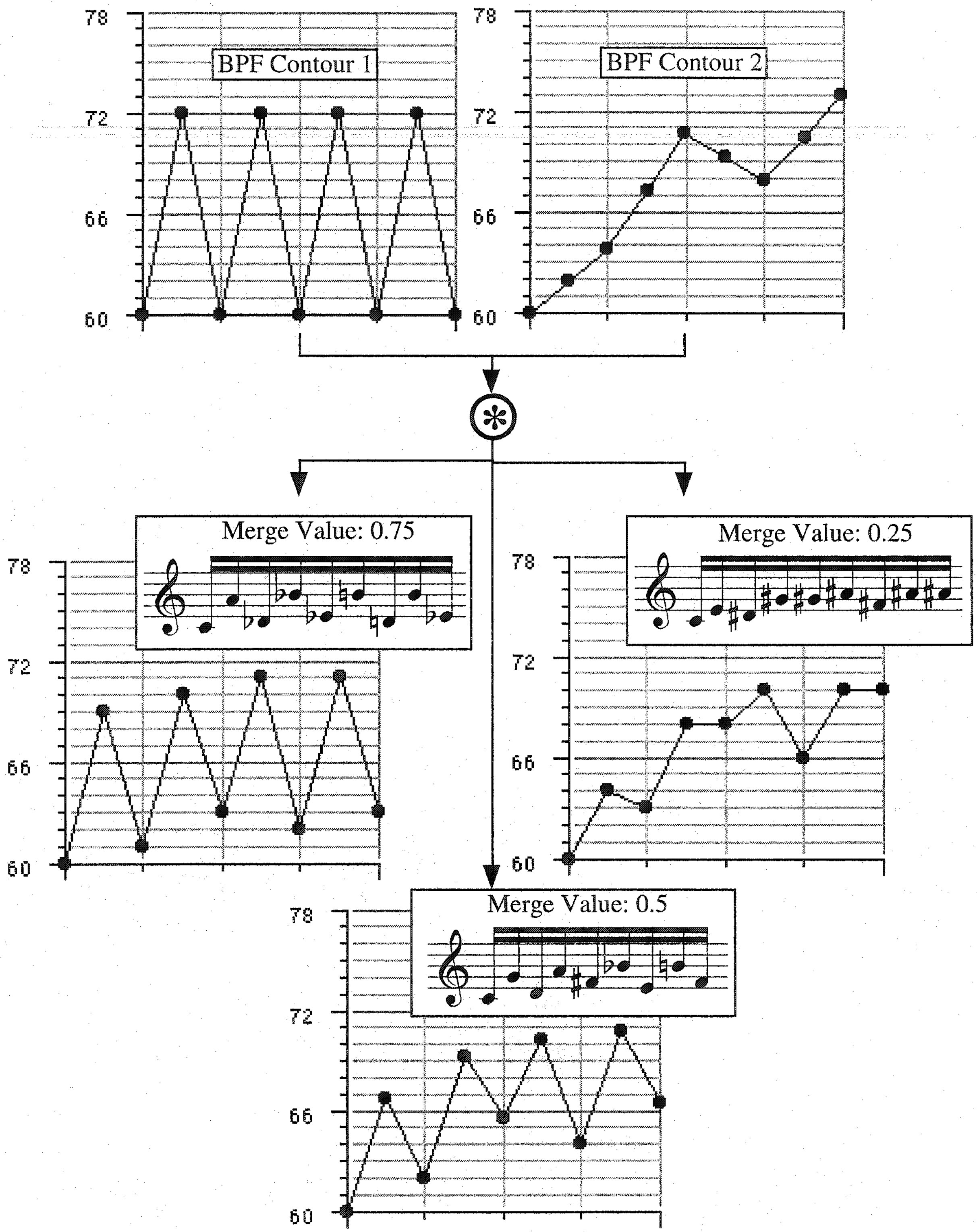
Read-functions may be arbitrarily complex. The following example uses an exponential curve; more samples are taken at the beginning of the BPF contour and fewer are taken at the end. This has the musical effect of changing the perceived speed with which we proceed through the original contour.



Example 2-9: Exponential Read-Function

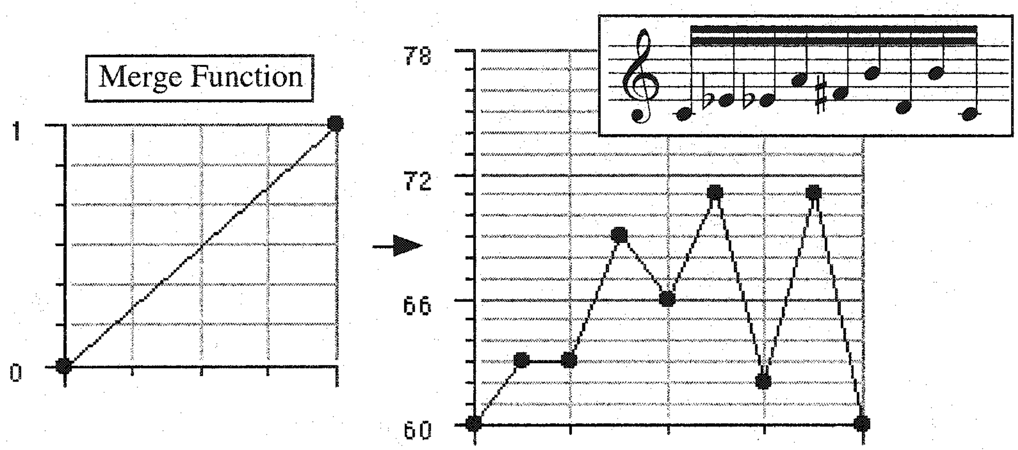
2.3.3 Merging BPFs

The above techniques apply to single BPF contours. Another type of transformation merges two different contours into a single one. In this technique, a weighting value specifies the relative emphasis of one contour against the other. A value of 1 passes points from the first contour unchanged, a value of 0 passes points from the second, and values between 0 and 1 interpolate between the two as a weighted average. For example, given a *y* value from the first contour of 48, a *y* value from the second of 62, and a weighting factor of .75, the result of the merge would be (48 * .75) + (62 * .25) = 51.5, which is rounded off to 52. The next example clarifies this process. Here, the first BPF contour is simply an alternating octave between C_4 (middle C) and C_5, while the second is the same as in example 2-2 above, sampled 9 times.



Example 2-10: Merging Two Contours

The weighting value may also be defined as a break-point function, resulting in the dynamic blending of the two contours. The following example merges the contours from the previous example, this time beginning with contour 2 and gradually transforming the result into contour 1.

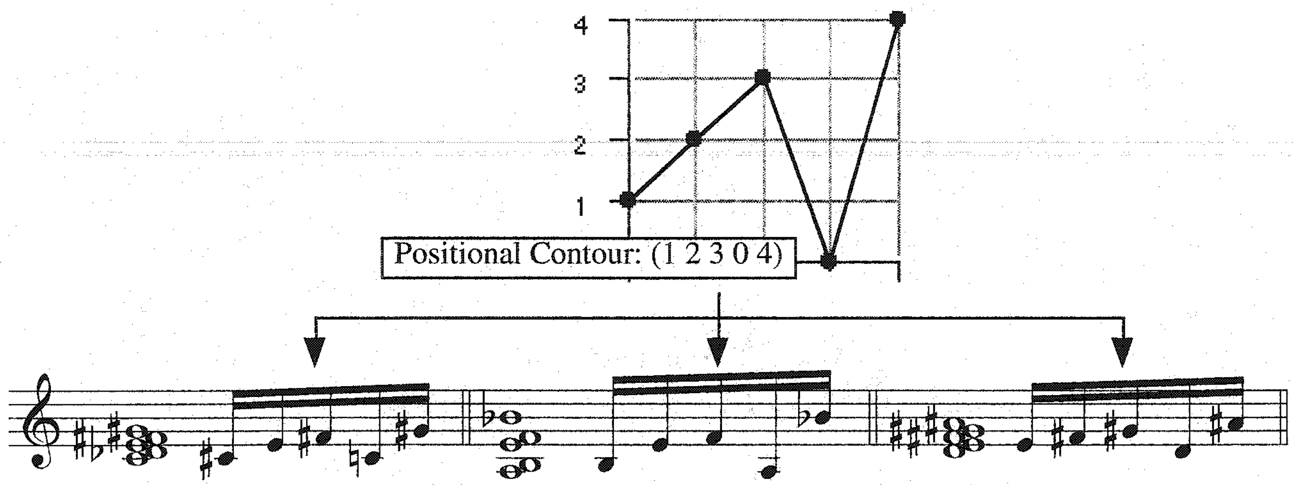


Example 2-11: Dynamic Merging of Two Contours

In the third section of the Concerto, between measures 209 and 273, I use this technique extensively. This will be discussed in more detail in Part II of this thesis.

2.4 Positional Contours

The second approach to contours in the *Concerto for Piano and Orchestra* is the application of a single list of order positions to different chords, resulting in linear pitch patterns with identical combinatorial contours. In other words, the position of a given note relative to all the other notes in each of the resulting patterns will always be the same. The lowest note of a chord is indicated by the order position 0, the next lowest by 1, and so on up to n - 1, where n is the number of notes, or cardinality, of the chord. The values of the contour determine the sequence of chord tones that make up the pattern. The positional contour [0 2 1 3], for example, indicates that the notes of a four-note chord are heard in the following order: first, third, second, and fourth. When one applies this contour to different harmonies, the resulting pitch patterns should be perceived as similar, since they have the same combinatorial contour. In the following example I apply the positional contour [1 2 3 0 4] to three different chords, resulting in three different yet perceptually similar melodic shapes.



Example 2-12: Applying a Positional Contour to Different Pitch Collections

This technique ensures that the relative position of notes in different instances of a contour will always be the same but, unlike BPF contours, the relative size of the musical intervals between notes may vary greatly. In the Concerto, contours of this type are varied primarily by being applied to different collections of pitches. Shorter contours are also linked together to create larger patterns which are applied to chords with greater numbers of notes. This technique is discussed in detail in the second part of this thesis.

Chapter 3: Vertical Aspects of Pitch

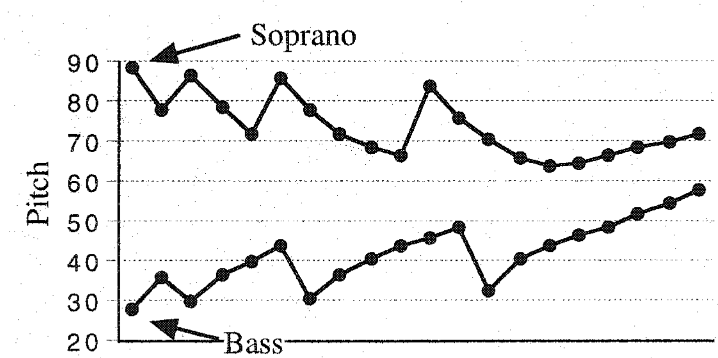
3.1 Approach to Harmony

In the *Concerto for Piano and Orchestra*, I compose harmonic progressions by adding inner voices to a framework of outer voices. I control five parameters: (1) cardinality; (2) sensory dissonance; (3) referential pitch commonality; (4) pitch commonality between successive chords; and (5) adjacent interval variance. The interaction of these properties influences the listener's perception of overall harmonic stability in a progression. In this chapter I discuss these harmonic parameters and, in the case of sensory dissonance and pitch commonality, the psychoacoustic models on which they are based. As well, I describe the exact constitution of the harmonic vocabulary of the Concerto. This vocabulary ensures a coherent, unified harmonic surface. First, however, I discuss my techniques for composing outer voices.

3.2 Outer Voices

3.2.1 Combining Melodic Lines

The simplest way to create outer voices for a harmonic progression is to combine two independently conceived melodic lines. These are created using the techniques discussed in the previous chapter. The outer voices of the piano in the first main section of the Concerto, beginning in measure 20, were made in this way. Here two similar inversionally-related contours were created separately, then transposed and combined to create the outer voice framework:

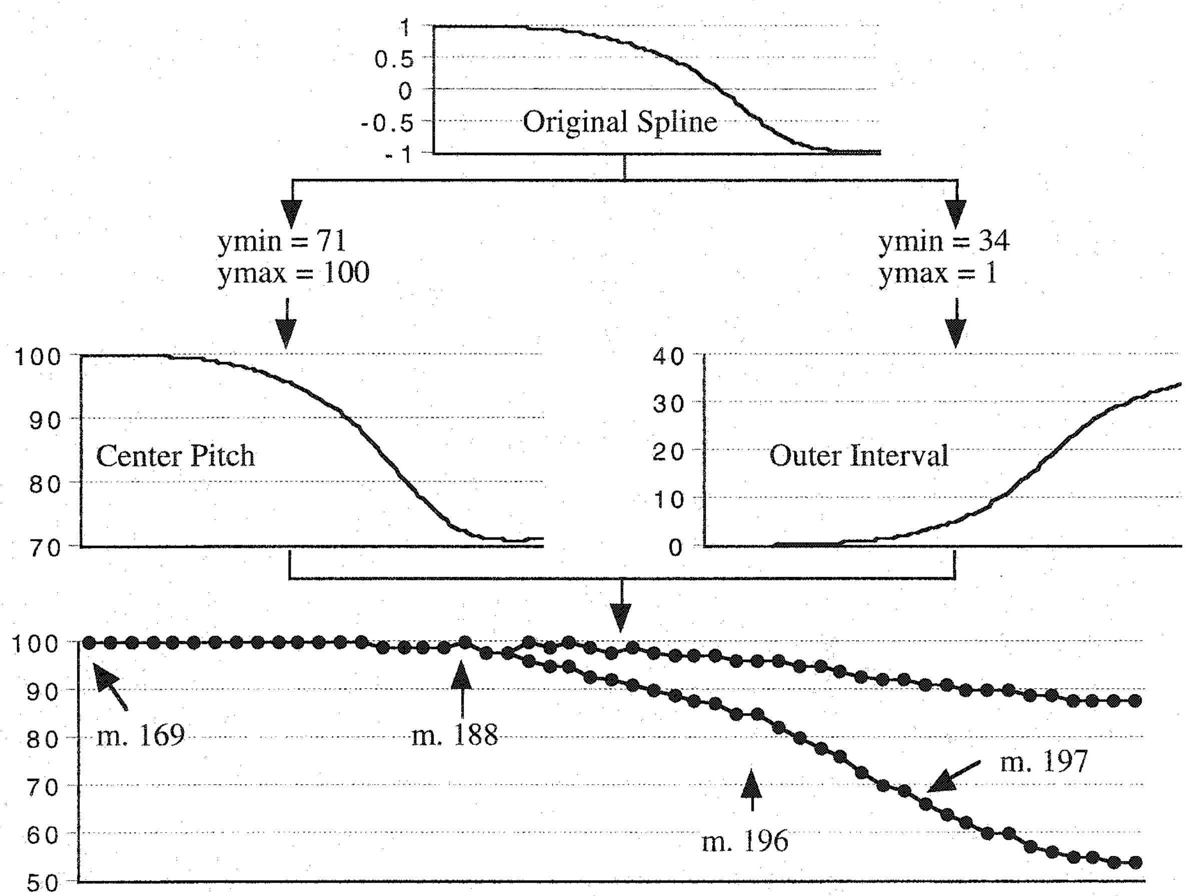


Example 3-1: Combining Melodic Lines to Create Outer Voices

When two lines are combined in this way, one or the other may be altered in order to avoid the interval of an octave, which is not part of the harmonic vocabulary of the Concerto. For example, the interval may be altered to a major seventh or minor ninth, depending on the musical context, by moving one of the voices a semitone higher or lower.

3.2.2 Using Control Functions

A less direct approach uses two control functions - one for center, soprano, or bass pitch contour and the other for outer interval - to create a series of outer voices. The following example from letter H (measure 196) of the Concerto illustrates this process:



Example 3-2: Generating Outer Voices With Control Functions

At this point in the piece there are two musical elements in the orchestra: (1) rapid figurations confined within a narrow pitch range; and (2) a slow descending progression of sustained chords beginning with a unison E7 and ending with an arrival on the chord in measure 199. The outer voices of this progression were created by using a cubic spline to determine the contour of the central pitch of the outer voices. A cubic spline is a mathematical function that traces a smoothly curving exponential path between discrete control points.[8] A second curve determines the outer interval for each successive pair of voices. In this example the pitch curve is for center pitches; outer intervals are calculated so that they are evenly distributed on either side of the center pitch, half above and half below. For example, if the center pitch is middle C (MIDI note 60[9]), and the outer interval value is 6, the resulting outer voices would be A below middle C (60 - 6/2, or 57) and E-flat above middle C (60 + 6/2, or 63).[10] The outer interval curve in this case is derived from the same cubic spline as the center pitch curve, with the new minimum y value set higher than the new maximum y value, resulting in inversion. I then sampled the two curves and used the resulting values to generate the outer voices.

3.3 Sensory Dissonance

3.3.1 Definition

In music, the term "dissonance" has two principal meanings. The first is "stylistic instability, tension, 'disagreement.'"[11] In the context of this definition, consonance refers to elements that are "stylistically 'stable,' 'reposed,' or 'in agreement'."[12] The second meaning, which may be termed "sensory" dissonance, refers to the sensation of rapid beating, or roughness, within a sonority. Consonance is the absence of such beating. These two definitions do not necessarily agree. The interval of a perfect 4th, for example, is considered to be dissonant in common-practice tonal syntax, since it is unstable when it occurs in a two-voice texture or above the bass in chords with three or more notes. In terms of sensory dissonance, however, it is consonant since it is has a very low degree of perceived roughness. The minor second, on the other hand, is dissonant according to both definitions. In the *Concerto for Piano and Orchestra*, the control of sensory dissonance is an important aspect of my compositional technique. The following section describes the psychoacoustic model of sensory dissonance used in the Concerto, and its implementation in *Apprentice*. Harmonic stability will be discussed later in section 3.6, which deals with the interaction of musical parameters.

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[8]For a formal definition, see Robert Sedgewick, *Algorithms* (Reading: Addison-Wesley, 1988). In the Concerto, cubic splines were created using the freeware Macintosh application "Spliner," by Jeff Bellsey.

[9] Indications of pitch in this document use the following conventions. References to pitch involving letter names use the notation of the American Standards Association, where notes are named by their usual musical letter names with a subscript indicating their octave. Thus middle C is notated as C_4, the C an octave higher is C_5, and the note a semitone below middle C is B_3. References to pitch written as numerical values use MIDI key numbers. In this case, middle C is 60, C_5 is 72, and B_3 is 59.

[10] If the outer interval is odd, the outer voices are rounded off to the nearest value. In the case of a soprano or bass curve, the outer interval is simply either subtracted or added to the pitch value, respectively.

[11] W. Hutchinson & L. Knopoff, "The Significance of the Acoustic Component of Consonance in Western Triads," *Journal of Musicological Research* Vol. 3 (1979): 21.

[12] Ibid., 20.

3.3.2 The Psychoacoustic Model

The model of sensory dissonance used in the *Concerto for Piano and Orchestra* originates with Helmholtz,[13] who was the first to experimentally confirm the relationship between beating and dissonance. It was further developed by Plomp and Levelt[14], and was formalized in the research of Hutchinson and Knopoff[15] . The implementation used in the Concerto is based on Hutchinson and Knopoff, with modifications by Richard Parncutt.

In this model, beating is considered to be the source of dissonance. The roughness caused by rapid beating is difficult to describe in words, but is easily recognized when heard. If one plays the interval of a perfect fifth above middle C on the piano, for example, very little roughness is perceived. The interval of a minor second above the same note, on the other hand, contains a great deal of roughness, or sensory dissonance. The degree of roughness of an interval or chord depends on the extent to which it has spectral components within the same critical band. The critical band is the smallest frequency difference that will allow two pure tones to be perceptually identified as two autonomous tones rather than as one single buzzing unit. To illustrate, if two sine waves are generated at the same pitch, only one tone is heard. As the frequency of one of the waves is gradually raised or lowered in a continuous glissando, a periodic variation in the amplitude of the perceived tone is heard. As the frequency distance between the two pure tones gradually becomes greater, the rate and prominence of the beating in the single perceived tone increases until it takes on a buzzing character. The sensation of beating then gradually diminishes until finally two distinct pure tones are heard. This point - roughly the interval of a minor third in the middle register - marks the limit of the critical band. The greatest sensation of roughness occurs when the frequencies of the two pure tones are approximately one quarter of a critical band apart.

In simplest terms, the calculation of sensory dissonance involves the modeling of the combined spectrum of an interval or chord - including the pitch and amplitude of each spectral component - and the determination of the extent to which each pair of pure-tone spectral components is less than a critical bandwidth apart. The degree of dissonance depends on the "closeness and strength of adjacent pure tones in the overtone structure of complex sounds."[16] The following discussion describes the details of this procedure.

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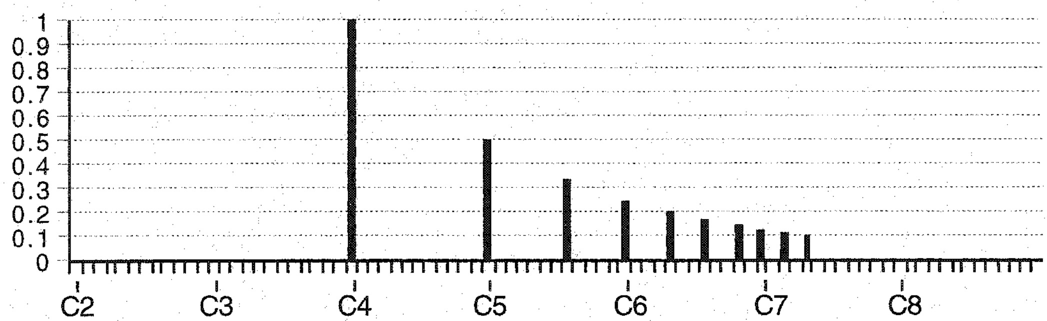
[13] H. Helmholtz, *On the Sensations of Tone* (New York: Dover, English Ed. 1954, Orig. 1877).
[14] R. Plomp & W. J. M. Levelt, "Tonal Consonance and Critical Bandwidth," *Journal of the Acoustical Society of America* 36 (1965): 1526-33.
[15] W. Hutchinson & L. Knopoff, "The Acoustic Component of Western Consonance," *Interface* Vol. 7 (1978): 1 - 29.
[16] Ibid., 3.

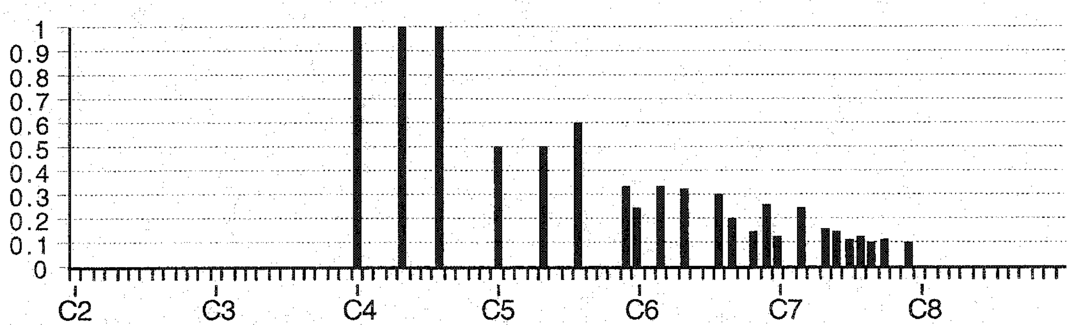
3.3.3 Merged Spectrum

To determine the merged spectrum of an interval or chord, one first creates the spectrum of each individual complex tone. The model uses only the first 10 harmonics, and the amplitude of each is set to 1/N, where N is the position within the harmonic series. The first harmonic (the fundamental) is thus assigned an amplitude of 1/1, the second an amplitude of 1/2, the third an amplitude of 1/3, and so on up to 1/10. Pitches are rounded off to the frequency of the nearest note in equal temperament. This means that the 7th harmonic above a fundamental of C, for instance, which should be noticeably flat, is raised to B-flat. The harmonic spectrum of C_4 would be:



Example 3-3: Spectrum of a Single Complex Pitch

The spectra of each of the complex tones making up the interval or chord are then combined, and the amplitudes of elements at the same pitch are combined by finding the square root of the sum of the squares of each amplitude. The merged spectrum of a C major triad built on middle C combines the above spectrum with those of E_4 and G_4 with the following result:



Example 3-4: Merged Spectrum of a Triad

In this case, as an example, the third harmonic of C_4 coincides with the second harmonic of G_4 at the pitch G_5; the amplitude of the spectral component at this frequency in the merged spectrum has been adjusted accordingly. Note that this model assumes that each note of the chord has the same intensity and harmonic spectrum.

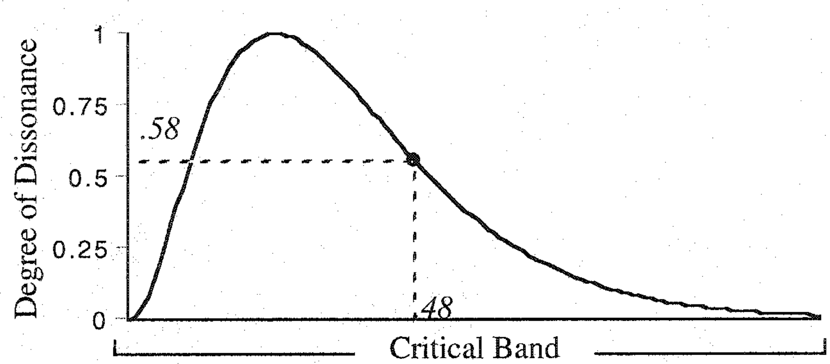
3.3.4 Calculating Roughness

Once the merged spectrum is complete, each pair of pure-tone components is examined in turn to see whether its members are within a critical bandwidth of each other. If they are not, they do not contribute to the overall roughness of the sonority. If they are, the ratio of their frequency difference to the critical bandwidth is determined by dividing the interval in Hz between the two by the critical bandwidth of their center frequency. The center frequency is one half of the sum of the two frequencies. As Plomp and Levelt discovered, the critical bandwidth is not a constant musical interval, but is larger in lower registers than in higher. To derive a critical bandwidth that varies with register, Hutchinson and Knopoff use the formula:

$$\mathrm{CBW}(\bar{f}) = 1.72(\bar{f})^{0.65}$$

where $\bar{f}$ is the center frequency. Given two pure tones at C_4 and D_4, for example, the center frequency, or $\bar{f}$, is $\frac{1}{2}(293.66 + 261.62) = 277.62\ \mathrm{Hz}$, and the critical bandwidth is 66.65 Hz. The frequency difference between the two tones is 32.03 Hz, which is .48 of the the critical band.

As mentioned previously, the amount of perceived roughness varies throughout the critical band. In order to determine the degree of dissonance, the interval of the critical band is plotted against the following curve, derived from Hutchinson and Knopoff:



Example 3-5: Determining the Degree of Dissonance

Here we see that maximum sensory dissonance occurs at roughly .25 of the critical band, and that at .48 of the critical band, the amount of sensory dissonance is .58 of the total possible. This value is scaled by multiplying it by the product of the amplitudes of each tone. So if the amplitude of C_4 in the previous example is 1/3 and the amplitude of D_4 is 1/5, the scaled dissonance of the two pure tones will be $.48 * \left(\frac{1}{3} * \frac{1}{5}\right) = .032$. To determine the overall dissonance of the entire merged spectrum, all of the scaled dissonances between each pair of spectral components are summed, and the result is divided by the total amplitude of the sonority. The greater the value, the greater the estimated sensory dissonance.

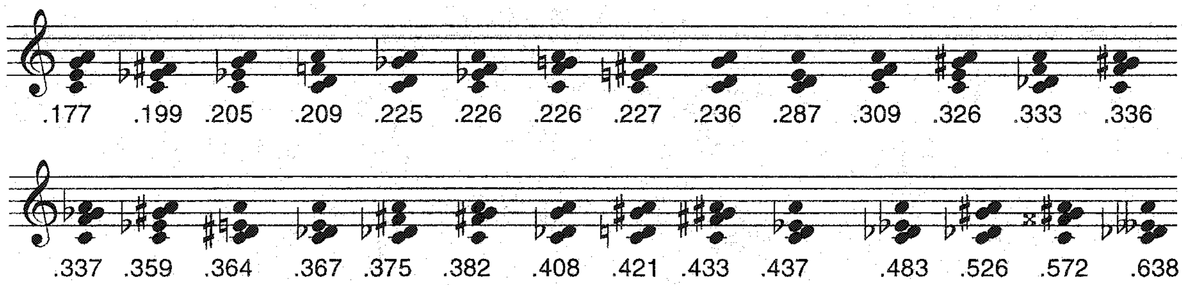
3.3.5 Discussion

Hutchinson and Knopoff use the algorithm on which this model is based to find dissonance factors for all intervals within two octaves[17] and for the triads of Western tonal music[18]. Gregory Danner uses the same algorithm to characterize the dissonance levels of pitch-class sets.[19] Richard Parncutt has written a program in the C language based on a modified version of the algorithm. As mentioned above, in *Apprentice* I incorporate Parncutt's changes. These relate to the curve of perceived roughness across the critical band. Hutchinson and Knopoff find values on this curve by consulting a look-up table, while Parncutt describes it functionally. This change is necessary because Hutchinson and Knopoff do not provide the actual values used in their table. Disagreement between the results obtained by the two models is small, however. In any case, in *Apprentice* the precise value returned by the model is less important than its ability to distinguish relative differences in dissonance between sonorities. The reason for this will be discussed later in the section on combining parameters in harmonic progressions.

This model is of course only useful in so far as it reflects the perception of a typical listener. Hutchinson and Knopoff have found that the results of the model are "in remarkable agreement"[20] with the experimental results of Malmberg's 1918 investigation of the perceived dissonance of musical intervals.[21] In the context of composition using non-tonal harmonic materials, the model must be verified by comparing its results with one's own perception. The following example lists all possible 4-note chords with an outer interval of a major 6th above middle C, and their sensory dissonance. The chords

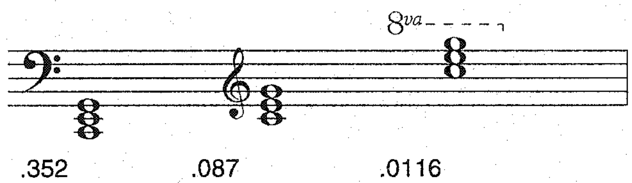
[17] W. Hutchinson & L. Knopoff, "The Acoustic Component of Western Consonance," *Interface* Vol. 7 (1978): 1 - 29.
[18] W. Hutchinson & L. Knopoff "The Significance of the Acoustic Component of Consonance in Western Triads, *Journal of Musicological Research* Vol. 3 (1979): 5-22.
[19] Gregory Danner, "The Use of Acoustic Measures of Dissonance to Characterize Pitch-Class Sets," *Music Perception* Vol. 3, No. 1 (Fall 1985): 103-122.
[20] W. Hutchinson & L. Knopoff, "The Acoustic Component of Western Consonance," *Interface* Vol. 7 (1978): 13.
[21] C. F. Malmberg, "The Perception of Consonance and Dissonance," *Psychological Monographs* 25 (1918): 93-133.

are arranged by increasing dissonance. The effectiveness of the model may be judged by playing the progression on the piano and determining whether one agrees with its ranking:



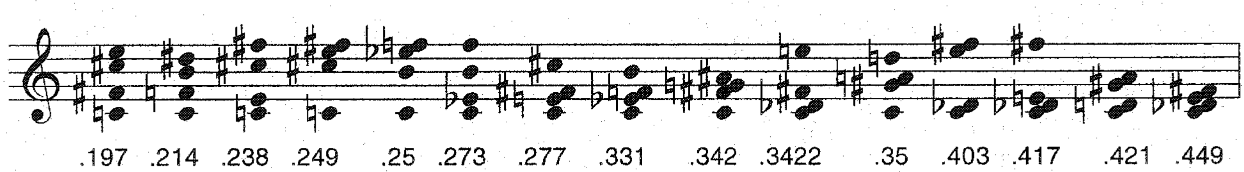
Example 3-6: Tetrachords Sorted by Increasing Sensory Dissonance

The transposition level of chords influences sensory dissonance, since the size of the critical band varies with register. The following three transpositions of a C major triad differ greatly in terms of sensory dissonance. This reflects the experience of many pianists, who often judge the placement of close-position triads in the lower register to be an error on the part of the composer.



Example 3-7: Dissonance Varying with Register

The dissonance of chords is not only affected by transposition level, but also by the specific arrangement of their notes, or their voicing. The following example contains all possible voicings - within 18 semitones starting on middle C - of chords that reduce to the normal form [0 1 4 6]. These are arranged by increasing sensory dissonance. From this example, we see that although chords may share "the same harmony," they are not necessarily equivalent in terms of sensory dissonance.



Example 3-8: Dissonance Varying with Chord Voicing

Since the register and voicing of chords affects its output, this model of sensory dissonance must be applied to chords as they actually appear in the music, not to abstract representations such as prime form.

3.4 Pitch Commonality

3.4.1 The Psychoacoustic Model

In common-practice tonal syntax, the number of common notes between successive chords is an important factor in determining the strength of a progression. In general, a greater number of common tones results in a "weak" or smooth progression, while a lesser number results in a "strong" or uneven progression.[22] In musical set theory, a similar emphasis is placed on the number of invariant pitch-classes between transpositions of a set or its inversion.[23] Both of these approaches to chord succession are restricted to the actual notated pitches of the chords. The model of pitch commonality used in the Concerto differs in that, like the model of sensory dissonance, it utilizes the spectra of chords. Qualitatively, pitch commonality is the degree to which sonorities are heard to share pure and complex tones. Pairs of chords with high pitch commonality are perceived to be more closely related than are chords with low pitch commonality.

The model of pitch commonality used in the Concerto was originally developed by Terhardt, and has been extended and modified by Richard Parncutt. Each author uses slightly different vocabulary. Terhardt calls the pure-tone components of a spectrum "spectral pitches," while Parncutt refers to "pure tone sensations." Similarly, complex tones that are perceived as single entities, but are made up of several pure-tone components, are called "virtual pitches" by Terhardt and "complex-tone sensations" by Parncutt. Since the two sets of terms are more or less equivalent,[24] in this document I use both interchangeably.

3.4.2 Merged Spectrum

To calculate the degree of pitch commonality between two sonorities, one must first generate a merged spectrum for each. These are "perceptual spectra,"[25] as opposed to those used to determine sensory dissonance, which are physical spectra. The two most significant differences between the two types are the utilization of the perceptual salience (the probability of being noticed) of spectral components, as opposed to amplitude, and the incorporation of virtual pitch theory. As a result of these differences, determining pitch commonality is a considerably more complex process than is sensory dissonance. To keep the following discussion manageable, certain details of the model have been

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[22] E. Aldwell & C. Schachter, *Harmony and Voice Leading, Second Edition* (San Diego: Harcourt Brace Jovanovich, 1989), 136.

[23] A. Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973), 29.

[24] R. Parncutt & H. Strasburger, "Applying Psychoacoustics in Composition: 'Harmonic' Progressions of 'Nonharmonic' Sonorities," *Perspectives of New Music* 32 (1994): 90.

[25] Ibid., 109.

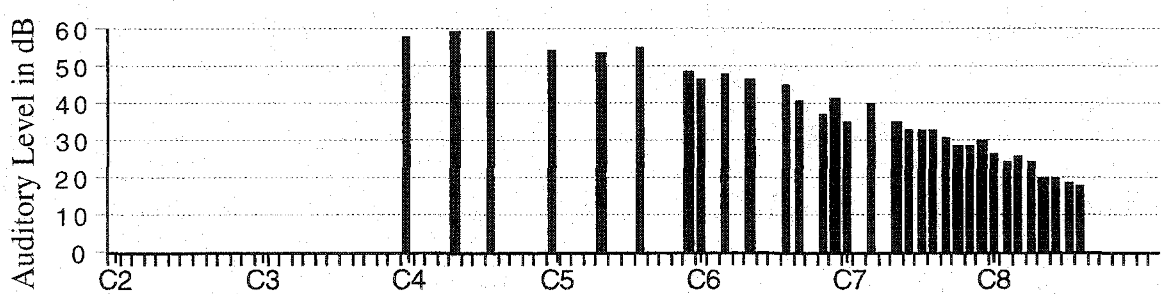
omitted. For an exhaustive description, see chapter 4 of Parncutt's *Harmony, A Psycho-acoustical Approach*,[26] and the Lisp code in Appendix B of this thesis.

3.4.3 Auditory Levels

The spectrum of a single tone used in this model contains 16 harmonics. Rather than amplitude, each spectral pitch is assigned an auditory level, which is the level of the tone in dB relative to the threshold of hearing in quiet. This threshold is not constant, but is generally elevated in the higher and lower registers compared to the middle register. Assuming equal amplitudes, pure tones in the middle register will tend to have higher auditory levels than those in extreme registers. The auditory level of each harmonic is calculated according to the equation:

$$\frac{x*(120-x)}{60}\left(1-\frac{\mathrm{I}(x)}{120}\right)$$

where x is the MIDI pitch of the harmonic minus 12, and $\mathrm{I}(x)$ is the interval in semitones between the harmonic and its fundamental. Once the pitch and auditory levels of the spectral pitches of each complex tone in the chord have been determined, the spectrums are combined and the auditory levels of spectral components occurring at the same pitch are summed. The merged spectrum of a C major triad beginning on middle C (C_4, E_4, and G_4) using auditory levels would be:



Example 3-9: Merged Spectrum with Auditory Levels

This spectrum may be compared with that of the same chord as calculated in the sensory dissonance model (example 3-4).

3.4.4 Pure Tone Audibility

Once the merged spectrum has been generated, the audibility or perceptual prominence of each pure-tone component is estimated. Pure tone audibility depends on

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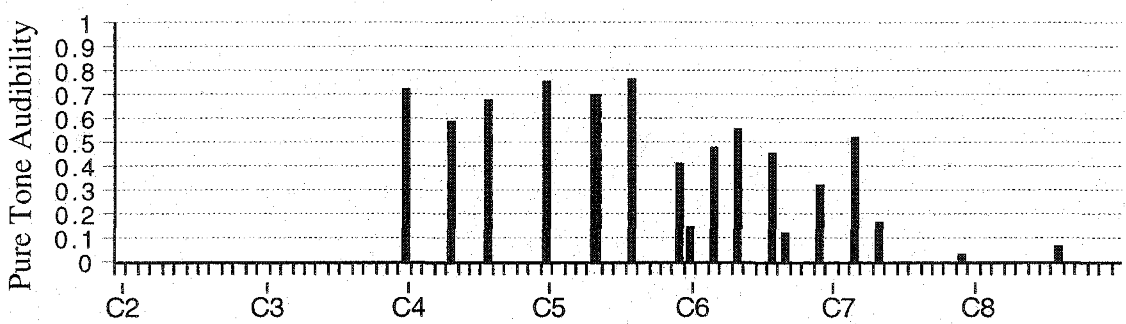
[26] R. Parncutt, *Harmony: A Psychoacoustical Approach* (Berlin: Springer-Verlag, 1989): 77-97.

the audible level (AL) of the spectral pitches, which is the level of each component above the masked threshold. Masking is "the mutual drowning out (or inhibition...) of one sound by another."[27] Pure-tone components in the merged spectrum will partially mask each other if their frequencies are within a distance of approximately 3 critical bands.[28] The closer two pure tones are within this distance, the more they will mask each other. In simplest terms, the masking level of each spectral pitch is found by determining the amount in dB to which its auditory level is reduced due to masking by all the other components of the spectrum. The audible level is determined by subtracting the masking level from the auditory level. If this value is negative, the audible level is set to zero. The audibility of a spectral pitch varies according to its audible level. At low audible levels, audibility is approximately 1/20 of AL, while at higher levels it approaches 1. We now have a list of all the pure-tone components of the spectrum and their audibilities:



Example 3-10: Pure Tone Audibilities

A comparison of this spectrum with the previous reveals that many spectral components have been eliminated because their masking levels were greater than their auditory levels.

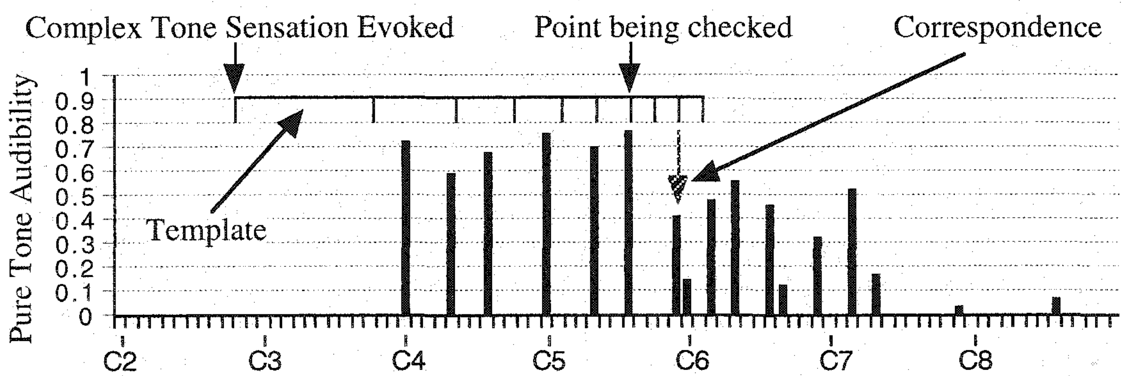
3.4.5 Complex Tone Audibility

Up to this point, we have been dealing only with pure tone sensations, or spectral pitches. The next step is to calculate complex tone audibilities. This is the degree to which complex tone sensations, or virtual pitches, are perceived. A complex tone is made up of a varying number of pure tones, but is heard as a single sound. Examples are notes played by pitched musical instruments and the vowels of human speech. The perceived pitch of a complex tone corresponds to the fundamental frequency of its harmonic spectrum, even if this frequency is not actually present. According to Terhardt,[29] complex-tone sensations are a consequence of conditioning resulting from familiarity

[27] Ibid., 87.
[28] Ibid.
[29] E. Terhardt, "Pitch, Consonance and Harmony," *Journal of the Acoustical Society of America* 55 (1974): 1061-1069.

with the harmonic spectrum of human speech since infancy. Terhardt's model of virtual pitch is thus "based on *familiarity* with the pitch pattern produced by ordinary complex tones."[30] Collections of simultaneous pure tones whose frequencies correspond to this pattern are spontaneously grouped into a single complex tone sensation. Even if some members of this pattern are missing we still hear a single sound whose pitch is that of the fundamental frequency. In a square wave, for instance, every second harmonic is missing, yet the sound is perceived as a single complex tone. In music played from very small speakers, such as portable radios, we hear bass notes even though they lay outside the frequency range of the speaker. This is because the presence of the upper harmonics of these notes is enough to trigger a complex tone sensation at the fundamental frequency.

To determine the extent to which the pure tones of a merged spectrum evoke complex tone sensations, or virtual pitches, a harmonic template is used. This template consists of 10 elements separated by the intervals of the spectrum of a normal pitched sound. Each element of this template in turn is aligned to each pure tone in the spectrum. Any pure tones above a spectral pitch that coincide with the template will generate a complex tone sensation whose pitch corresponds to the position of the lowest element of the template. This procedure is repeated in its entirety for each spectral pitch. Multiple complex tone sensations at the same pitch strengthen the perception of a virtual pitch at that location. The following example clarifies this process:



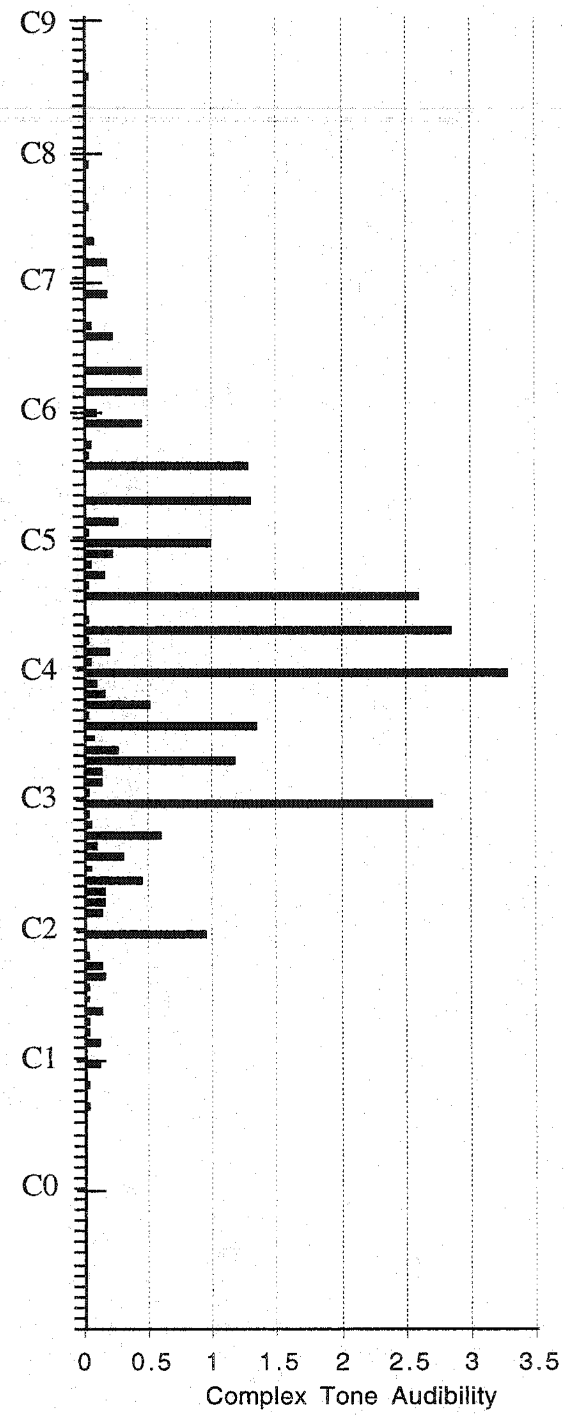
Example 3-11: Determining Complex Tone Sensations

Here the previous merged spectrum is being examined for virtual pitches. At this stage of the procedure, the pure tone at G_5 is being checked against the 7th element of the template. We see that there is a correspondence between the pure tone at B_5 and the 9th element of the template. As a result, a complex tone sensation is evoked at the pitch of the lowest element of the template, which is A_2. Note that this complex tone is not

[30] R. Parncutt, *Harmony: A Psychoacoustical Approach* (Berlin: Springer-Verlag, 1989): 36.

present in the original chord, and that it occurs below the bottom note of the chord.

A large number of complex tone sensations with varying degrees of audibility are normally evoked by this process, with multiple sensations occurring both at the fundamental frequencies of the notes that make up the chord and at other related pitches. In example 3-12, the complex tone audibilities that result from template checking are given for the same C major triad we have been using. Note the large number of sensations evoked, and that the maximum audibility of complex-tone sensations tends to be approximately 3 times that of pure tone sensations. At this stage, we now have two sets of data: (1) a listing of all the pure tone sensations in the spectrum with their audibilities; and (2) all the complex tone sensations with their audibilities. The final step before calculating pitch commonality is to determine the salience, or the probability of being noticed, of each of the spectral and virtual pitches.



Example 3-12: Complex Tone Audibilities

3.4.6 Salience

The salience of pitch sensations depends on the multiplicity of a chord. Multiplicity is defined as "the number of tones consciously perceived (or noticed)"[31] in a sonority. This value changes depending on the way one listens. In relaxed, holistic listening, one might only be aware of the root of a

[31] R. Parncutt & H. Strasburger, "Applying Psychoacoustics in Composition: 'Harmonic' Progressions of 'Nonharmonic' Sonorities," *Perspectives of New Music* 32 (1994): 105.

chord. In closer, more analytical listening, one may be able to discern each of the chord tones, and with effort even spectral pitches. This range of listening styles may be seen as a continuum between 0 and 1, with 1 being the most analytical. In Parncutt's model, this "simultaneity perception parameter" is set independently by the user. For the Concerto, it is fixed at .5, which is considered typical.[32]

Unscaled Multiplicity

There are two measures of multiplicity: unscaled and scaled. As implemented in *Apprentice*, unscaled multiplicity is determined in the following way. First we find the maximum tone audibility that appears in the two sets of data . Then, for each chromatic pitch category, we divide the greater of the tone audibilities that occur in the list of spectral and virtual pitches by the maximum tone audibility. We then sum the results of this division for each pitch category to find the unscaled multiplicity of the chord.

Scaled Multiplicity

The scaled multiplicity is found by raising this value to the power of the simultaneity perception parameter. For the current chord, the unscaled multiplicity is 8.16, and the scaled multiplicity is 2.86.

The salience of each spectral element can now be found. We first divide the audibility of every component in each list by the maximum tone audibility. This value is then multiplied by the unscaled multiplicity and the result is divided by the scaled multiplicity: the result is the salience. We compare the saliences of each pitch category in both lists, and take the larger value for the final answer. The result is the perceptual spectrum of the chord, containing all tone sensations along with their saliences. Example 3-13 shows the pitch saliences for the C major triad we have been examining, as well as for two other non-tonal harmonies.

3.4.7 Calculating Pitch Commonality

To find the degree of pitch-commonality between two sonorities, a correlation coefficient is calculated between them.[33] A coefficient of 1 indicates exact correspondence (ie. the same chord), while -1 indicates complete complementarity. The pitch commonalities between the chords in example 3-13 are: (1) first and second: 0.39; (2) second and third: 0.62; and (3) first and third: 0.73. Higher values indicate greater pitch commonality.

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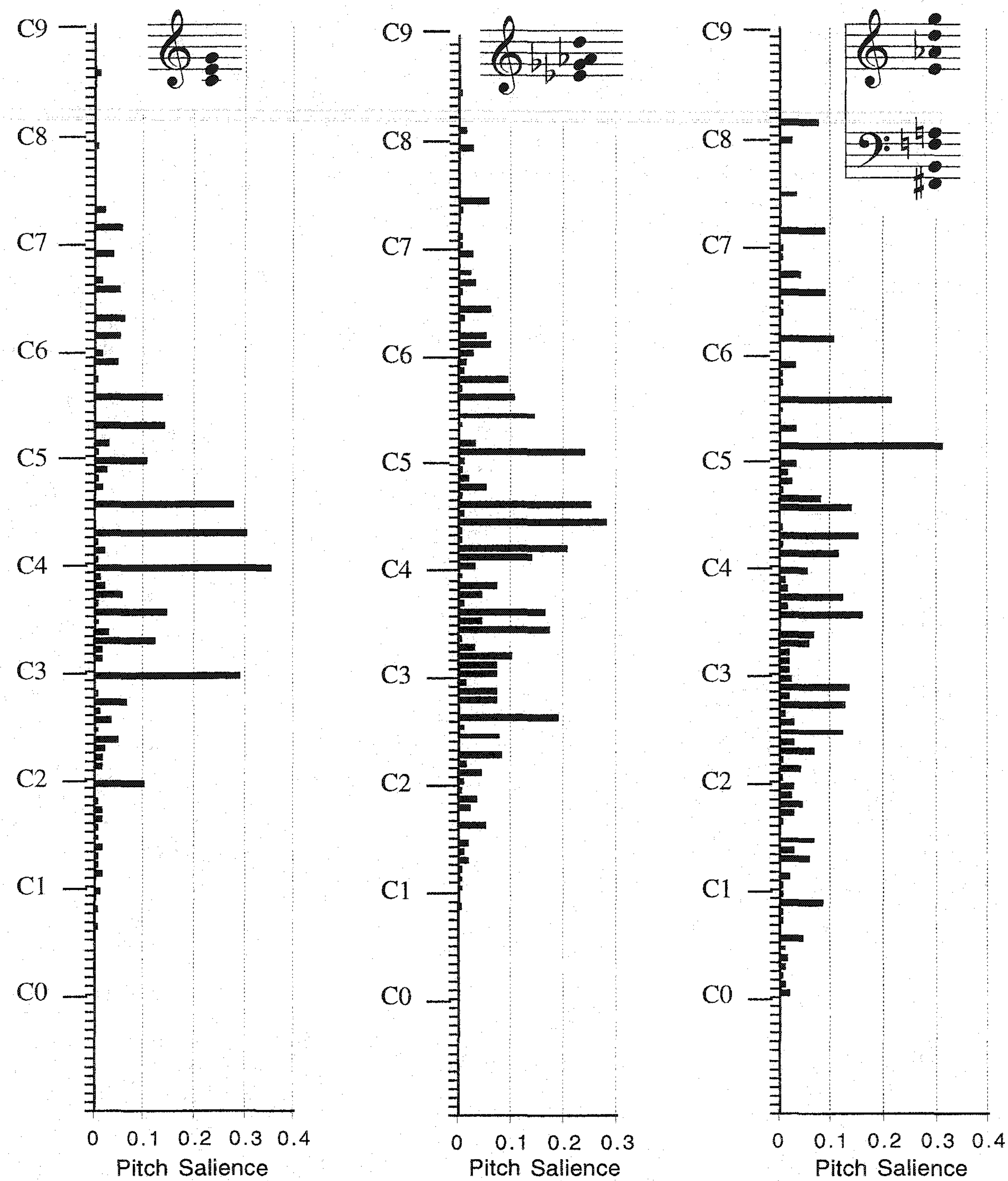
[32] Ibid., 106.

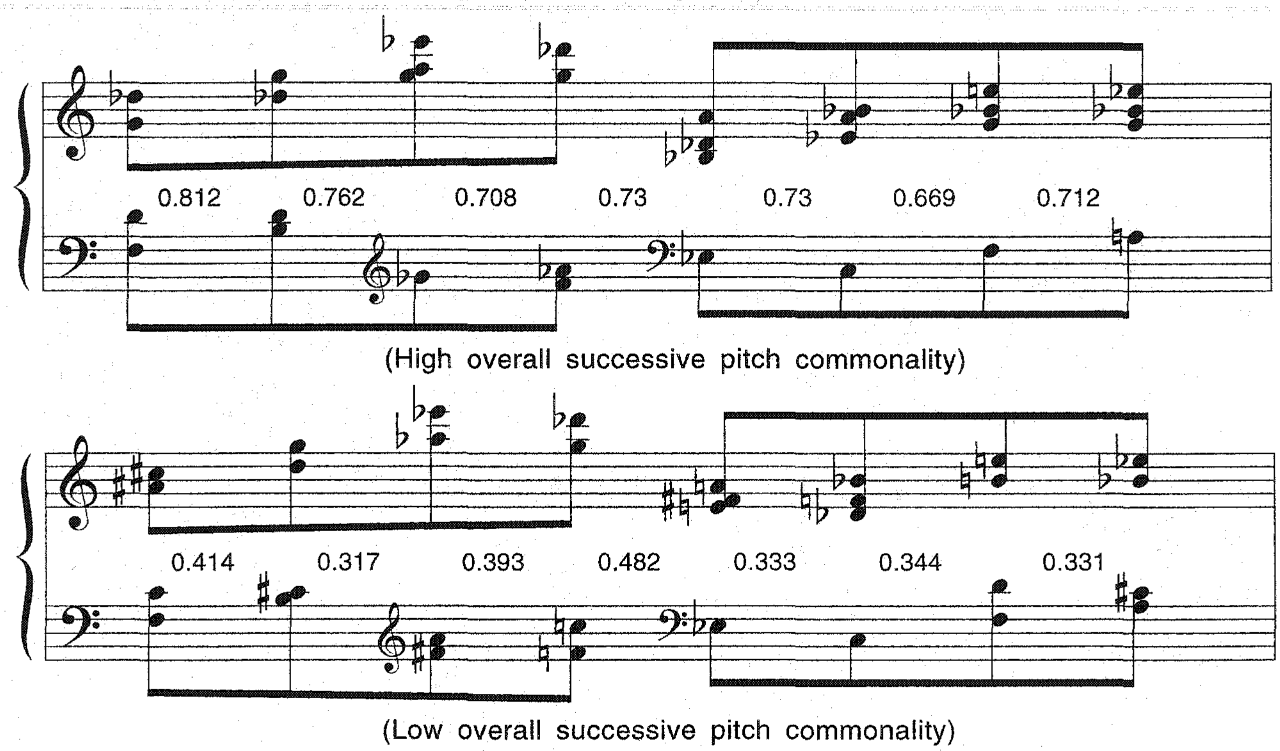
[33] For a description of correlation coefficients, see R. Plutchik, *Foundations of Experimental Research* (New York, Harper and Row, 1968), 135-7.



Example 3-13: Pitch Saliences for Three Different Chords

3.4.8 Successive Pitch Commonality

In the *Concerto for Piano and Orchestra*, pitch commonality is applied in two ways: successively and referentially. Successive pitch commonality is calculated between consecutive chords in a harmonic progression. In example 3-14, each series of chords has the same outer voices. In the first harmonization, consecutive chords have high pitch commonality, while the second version has low successive pitch commonality. The pitch commonality between each pair of chords is given between the staves.



Example 3-14: Pitch Commonality Between Successive Chords

Since it has higher pitch commonality, the connection between chords in the first progression should sound generally smoother than in the second. As we will see, this is an important factor in determining the overall harmonic stability of a passage.

3.4.9 Referential Pitch Commonality

Referential pitch commonality is calculated between the chords of a progression and a middleground referential sonority. The referential sonority may be present in the music, as a sustained chord for example, or may be only suggested. A progression of chords all with a high degree of pitch commonality to the same chord should imply this referential sonority, even if it is not actually sounding. This effect will be stronger if the sonority has been previously emphasized in the piece. In example 3-15 the eight-note chord is the reference and the following four-note chords are sorted by decreasing pitch commonality with this sonority. These are examples of chords as they might appear in a piece of music, not of pitch-class sets.The maximum and minimum pitch commonality values are given below the staff. Chords at the beginning of the list should sound related

Example 3-15: Referential Pitch Commonality

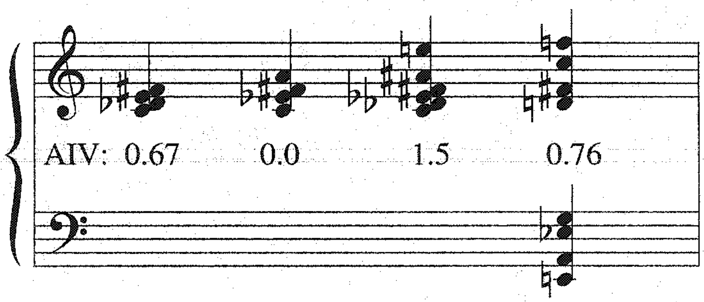
to the referential sonority since they share many components, while chords at the end of the progression should sound less related.

Simultaneously-sounding chords with high pitch commonality will result in relatively little dissonance, since they have many components at the same pitch level. Chords with low pitch commonality normally result in greater dissonance, since their components do not coincide. If the referential sonority is sustained, therefore, the above progression - which has high pitch commonality at the beginning and gradually less as it continues - should give the impression of increasing harmonic tension, since the chords sound gradually less related to the referential sonority and there is increasing dissonance.

3.5 Adjacent Interval Variance

In the Concerto, the voicing of a chord is expressed as the degree to which its notes are evenly distributed across its range. The following model determines the amount of variance in the size of the intervals between adjacent notes in the chord. First, the interval between adjacent notes is calculated as a fraction of the outer interval of the chord. Thus the chord C_4-$C\#_4$-E_4-$F\#_4$ has adjacent intervals of 1/6, 1/2 and 1/3 of a total of six semitones. Then the absolute difference between each pair of intervals is found. In this chord there are three pairs of intervals. The differences are $|(1/6 - 1/2)| = 1/3$; $|(1/6 - 1/3)| = 1/6$; and $|(1/2 - 1/3)| = 1/6$. The adjacent interval variance of the chord is the sum of these differences, in this case $(1/3 + 1/6 + 1/6) = 2/3$ or .67. Example 3-16 gives the adjacent interval variance for a number of different chords. Note that a chord whose notes are all the same distance apart, such as the diminished seventh chord in this example, will have an adjacent interval variance of zero.

This measure of voicing is useful in several ways. It is a good rough predictor of sensory dissonance - low adjacent interval variance often indicates low dissonance - but it is much more efficient to compute. When a subset of chords with a certain degree of dissonance is being extracted from a very large superset, one can make an initial selection using adjacent interval variance, and then examine the resulting subset more closely for



Example 3-16: Adjacent Interval Variance

dissonance. This process results in a substantial reduction in computation time, especially when dealing with thousands of chords. As well, since chords with low adjacent interval variance have their notes more or less evenly spaced between their outer voices, a progression of such chords can ensure that each voice stays within a desired range when orchestrating the progression.

3.6 Interaction of Parameters: Harmonic Stability

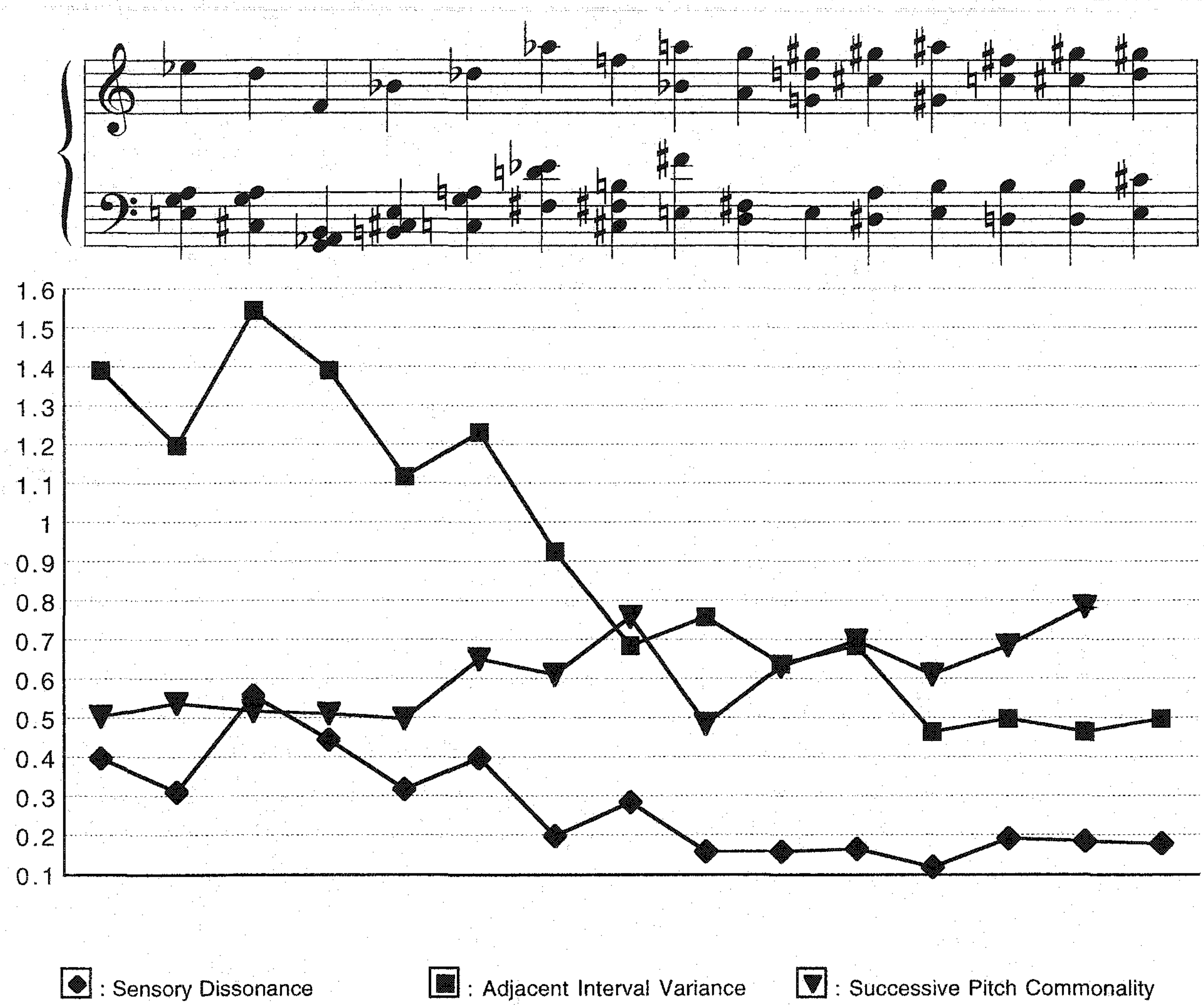
It would be possible to base a composition on only one of the parameters discussed above. One could, for instance, write a harmonic progression in which the amount of dissonance is strictly controlled in absolute terms. In the Concerto, however, I apply these parameters simultaneously. As a consequence, it is difficult to ensure that the precise desired degree of successive and referential pitch commonality, dissonance and adjacent interval variance will be found. Even so, the interaction of these properties has a powerful effect on the overall harmonic stability of a passage. As the following example indicates, stability varies through the combined influence of the four harmonic parameters discussed in this chapter. These parameters are applied in relative, rather than absolute terms. In other words, if the dissonance of the chords of a progression is relatively higher at the end than it is at the beginning, this contributes to a progressively lower relative harmonic stability. In general, stability decreases as dissonance and adjacent interval variance increase, and as successive and referential pitch commonality decrease.

High Harmonic Stability ◄——► Low Harmonic Stability

High Harmonic Stability	Low Harmonic Stability
•Low Adjacent Interval Variance •Low Sensory Dissonance •High Linear Pitch Commonality •High Referential Pitch Commonality	•High Adjacent Interval Variance •High Sensory Dissonance •Low Linear Pitch Commonality •Low Referential Pitch Commonality

Example 3-17: Interaction of Harmonic Parameters

The following example gives an analysis of the amount of adjacent interval variance, dissonance, and successive pitch commonality for a progression of 15 chords. Since no referential sonority has been established, referential pitch commonality is not included.



Example 3-18: Interaction of Parameters in a Harmonic Progression

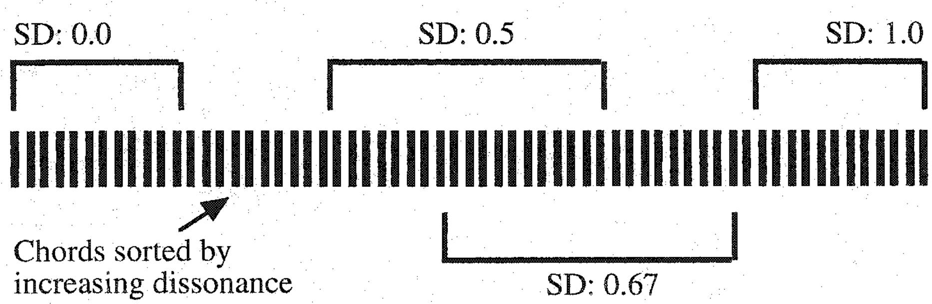
As the chart shows, the absolute value of these properties does not increase or decrease linearly. In general, however, adjacent interval variance and dissonance become gradually lower while successive pitch commonality rises. As a result, the overall harmonic stability of this chord sequence increases.

Other musical features can also influence these parameters. At a high dynamic level, for instance, the upper overtones of most musical sounds become more prominent. This tends to increase the degree of dissonance, since pure tone components within the critical bandwidth of each other will have greater amplitude and will therefore contribute a greater degree of roughness to the sonority. If the above progression were to begin at a dynamic level of *fortissimo* and gradually diminish to *piano*, the dissonance would be emphasized at the start and de-emphasized at the end. Another influencing factor is cardinality. In general, for any two chords with the same outer voices, the chord with the greater number of notes will tend to have a higher degree of dissonance. As I discuss later, the rhythm of a passage can also have an impact on its overall stability.

3.7 Composing Progressions in *Apprentice*

As previously mentioned, in the Concerto I compose chord progressions by harmonizing an outer-voice framework. I add inner voices to each pair of outer voices according to specific criteria. These include cardinality (number of notes), adjacent interval variance, dissonance, successive pitch commonality, and referential pitch commonality. The first step in this process is to find all the chords with the desired cardinality, outer interval, and transposition level. These are not selected from all possible chords, but from the specific harmonic vocabulary of the Concerto. I discuss the precise constitution of this vocabulary in the next section. The list of chords is then analyzed for the previously-mentioned parameters and chords with the desired degree of each are returned. I will now describe the process for making this selection.

I use values between 0 and 1 to express the desired degree of each parameter, with 0 being the least possible and 1 the greatest. These values indicate the relative amount of each parameter, not the absolute value. For instance, a value of 0 for dissonance is equivalent to the following expression: "Of the chords with the desired number of notes, outer interval and transposition level, which ones are the least dissonant?" A value of .5, on the other hand, is equivalent to the expression: "Of these chords, which ones have a moderate amount of dissonance?" To answer this question, we sort the list by increasing dissonance and retain only that segment of the list that corresponds to the desired amount. The approximate portions of the list that would be returned for several different values of sensory dissonance are given in the following example.



Example 3-19: Selecting Regions from a List of Chords
According to Different Degrees of Sensory Dissonance (SD)

Note that very high and low values return slightly fewer chords than intermediate values. In these cases the composer is indicating a strong preference: "extremely dissonant" rather than "somewhat dissonant." This procedure is repeated in turn for each parameter. The result of each successive "slice" is analyzed and sorted, and the corresponding portion is used for the next parameter. In the Concerto the order of analysis is: (1) adjacent interval variance, (2) dissonance, (3) referential pitch commonality, and (4) successive pitch commonality. This process returns a varying number of chords, depending on the length of the original list. For instance, in the harmonic vocabulary of the Concerto there are fewer possible trichords with an outer interval of a major 9th than there are hexachords with the same outer interval. As a result, fewer trichords would be returned at the end of the process than hexachords. In no case, however, does *Apprentice* return less than three chords.

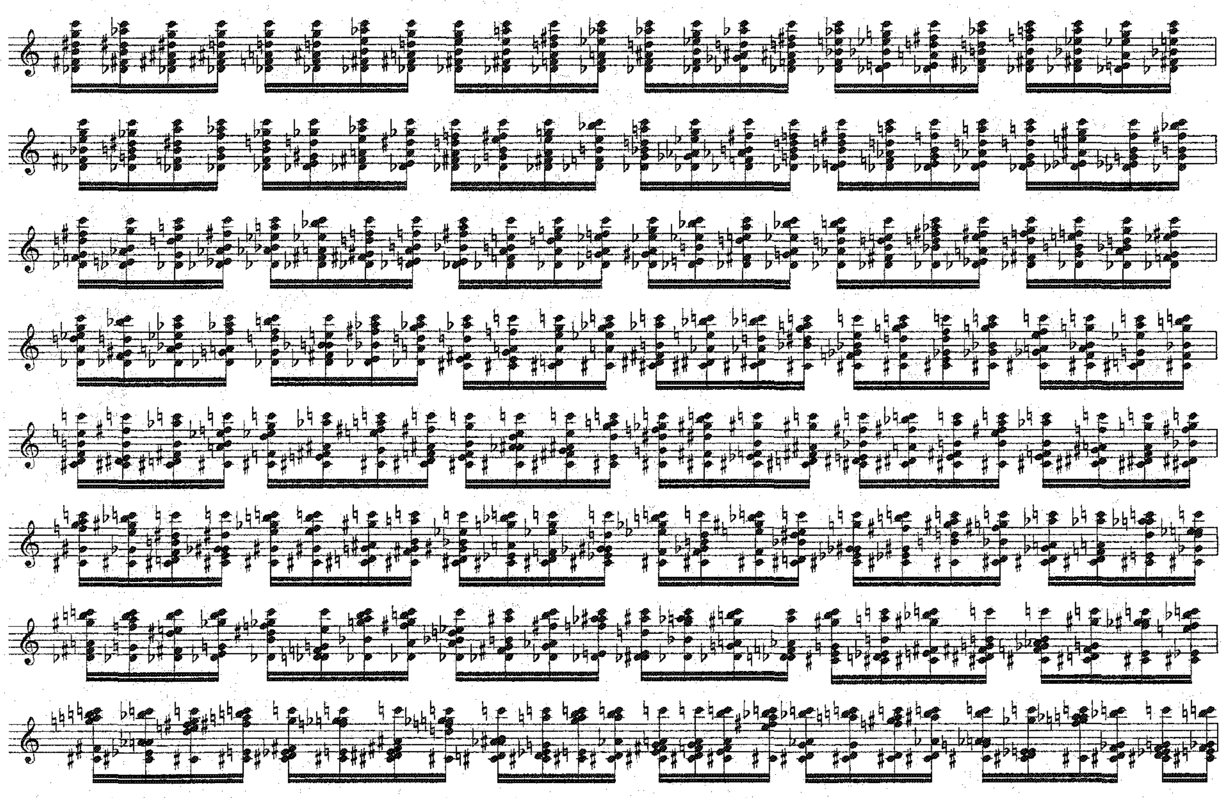
The following example clarifies the harmonization process. Imagine that we are harmonizing the fourth chord out of a progression of eighth chords, and that at this point the current values of the harmonic parameters are:

- referential sonority: (60 61 64 66) or C_4, $C\#_4$, E_4, $F\#_4$
- cardinality: 6
- transposition level: 61 ($C\#_4$)
- outer interval: 23 semitones
- adjacent interval variance: 0.5
- dissonance: 0.33
- referential pitch commonality: 0.67
- successive pitch commonality: 0.75

The program returns five progressively shorter lists of chords, notated as MIDI key numbers (middle C = 60). The first list contains all the chords in the harmonic vocabulary of the Concerto with the desired cardinality and outer interval, transposed to begin on MIDI note 61. The second list includes those chords whose relative adjacent interval variance is moderate. The third list contains those chords from the second list whose relative dissonance corresponds to the indicated amount of 0.33. The fourth list is the result of choosing chords from the previous list that have the desired degree of referential pitch commonality, and the fifth list is the result of analyzing these chords for successive pitch commonality. The composer makes a final choice from this list. Depending on the musical goals of the passage, one could choose the chord which is least dissonant, or more closely related to the referential sonority. Or, since the chords of the list are essentially equivalent in terms of the specified harmonic parameters, one can simply make a random choice.

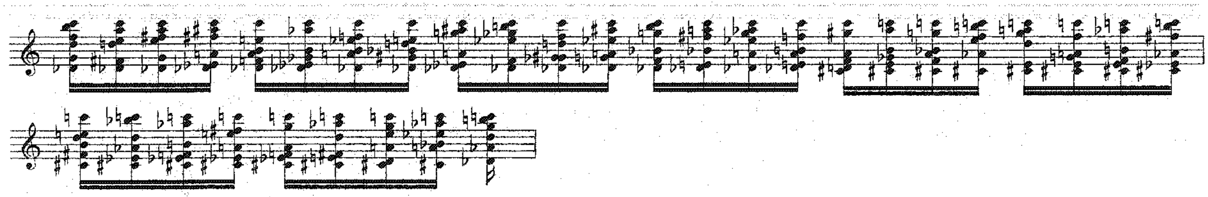
For the above progression, the five lists (in musical notation) would be as follows:

List 1: All chords



List 2: Chords selected by adjacent interval variance

List 3: Chords selected by sensory dissonance



List 4: Chords selected by referential pitch commonality

List 5: Chords selected by successive pitch commonality

3.8 Harmonic Vocabulary

One of the particularly compelling features of the above technique is that the models of dissonance and pitch commonality may be effectively applied to any collection of chords.[34] In the Concerto, however, I do not use all possible chords. Instead, I restrict

[34] In the literature, these models have been applied to both tonal and non-tonal harmonic materials. For an example of the former see E. Bigand, R. Parncutt, & F. Lerdahl, "Perception of Musical Tension in Short Chord Sequences: The Influence of Harmonic Function, Sensory Dissonance, Horizontal Motion, and Musical Training, *Perception & Psychophysic* Vol. 58, No. 1 (1996): 125-141. For an application to non-tonal harmonies, see N. Dibben, "The Perception of Structural Stability in Atonal Music: The Influence of Salience, Stability, Horizontal Motion, Pitch Commonality, and Dissonance," *Music Perception* Vol. 16, No. 3 (1999): 265-294.

my choices to a bank of chords that serves as the harmonic vocabulary of the work. This is necessary because as the number of notes and the size of the outer interval increases, the number of possible chords quickly becomes extremely large. For example, the number of all possible tetrachords with an outer interval of 11 semitones is 45, while the number of hexachords with an outer interval of 23 semitones is 7,315. Although the code that implements the psychoacoustic models in Apprentice has been highly optimized, the time required to compute pitch commonality between tens of thousands of chords, for instance, is not conducive to the creative process. I have therefore created a harmonic lexicon containing a selection of chords with cardinalities from 3 to 10 notes, and outer intervals up to 4 octaves.

3.8.1 Criteria

Unfortunately, to my knowledge there are currently no psychoacoustic models for assessing perceived harmonic similarity.[35] Furthermore, the set-theoretical approaches to similarity that do exist, such as Robert Morris' "similarity index"[36] or Marcus Castrén's RECREL,[37] do not necessarily correspond to the listener's experience.[38] I have therefore based my harmonic vocabulary on the hypothesis that two chords may sound related if they share one or more of the following relationships:

- They have the same normal form, or are transpositionally equivalent. (This is the relationship musicians use when they say that two different collections of pitches are major triads, for example.)
- They have the same prime form, or are equivalent under some combination of inversion and transposition.
- Both are subsets of the same larger chord. In this case they should sound related both to this superset and to each other.
- Both contain the same chord as a subset.
- They have the same adjacent interval content. For instance, the chords C_4-D_4-F_4 and E_4-G_4-A_4 both contain the intervals of a major second and a minor third between adjacent notes, in different order.

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[35] For a discussion of the current state of research in similarity see E. J. Isaacson, "Issues in the Study of Similarity in Atonal Music," *Music Theory Online* 2.7 (1996): http://smt.ucsb.edu/mto/issues/mto.96.2.7/mto.96.2.7.isaacson.html

[36] R. Morris, "A Similarity Index for Pitch Class Sets," *Perspectives of New Music* 18 (1979-80):445-460.

[37] M. Castrén, "Recrel: A Similarity Measure for Set-Classes" (Ph.D. diss., Sibelius Academy, 1994).

[38] C. L. Bruner, "The Perception of Contemporary Pitch Structures," *Music Perception* Vol. 2, No. 1 (Fall 1984): 25-39.

- They have the same total interval class content, or the same interval vector.[39] An interval vector is a measure of all the interval classes that are present in a particular set.

3.8.2 Creating the Harmonic Lexicon

The harmony of the Concerto is based on the all-interval tetrachord whose prime form is [0 1 4 6]. All of the chords used in the piece can be related in one or more of the above ways to this fundamental set. I use this tetrachord for two main reasons. First, it has been an important element of my harmonic language for approximately 10 years. I am attracted to its sound and am familiar with its implications. Second, the fact that its interval vector contains 1 occurrence of each interval class means that for intervals of a major 6th or greater, any pair of outer voices may be harmonized using a version of this set. This does not include chords whose outer interval is a multiple of an octave, which are not used in the Concerto. The following discussion describes how I use these criteria to create the lexicon of chords that serves as the harmonic vocabulary of my Concerto.

Tetrachords

The four-note chords used in the Concerto are based on the set class [0 1 4 6] and the only other tetrachordal set class with the same interval vector, [0 1 3 7]. I generate additional 4-note chords by finding all permutations of the adjacent intervals of each of these sets. For the set [0 1 4 6], this results in the following chords: [0 1 4 6] [0 3 5 6] [0 2 3 6] [0 2 5 6] [0 3 4 6] and [0 1 3 6]. The adjacent intervals of each of these chords, from lowest interval to highest, are: 1-3-2, 3-2-1, 2-1-3, 2-3-1, 3-1-2, and 1-2-3. This list includes both the original chord and its inversion, as well as all other chords with different arrangements of the same adjacent intervals. Permutation of the adjacent intervals of [0 1 3 7] results in another related group of chords: [0 1 3 7] [0 2 6 7] [0 4 5 7] [0 4 6 7] [0 2 3 7] and [0 1 5 7]. For each of these 12 source chords, I find all possible voicings up to four octaves. To illustrate this last step, here are all possible voicings within two octaves of [0 1 4 6], transposed to middle C:

[39] An interval class is denoted by the smaller of an interval and its inversion, reduced to within one octave. The interval class is expressed in semitones. Thus the intervals of a major 10th and a minor 6th are both members of interval class 4. The interval vector is a summary of all the interval classes in a given set, expressed in vector form. The values in the vector refer respectively to the number of occurrences of interval classes 1, 2, 3, 4, 5, and 6. For example, the interval vector of the chord C4-F4-G4-D5 is <0 2 1 0 3 0>, indicating 2 occurrences of interval class 2, 1 occurrence of interval class 3 and 3 occurrences of interval class 5. For more information see A. Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973), 14.

Example 3-20: All Voicings of [0 1 4 6] Within Two Octaves

Note that each of these chords is equivalent under some level of transposition to [0 1 4 6]. Furthermore, this list is exhaustive: there are no other possible arrangements of four notes within two octaves that are transpositionally equivalent to this chord. This procedure is repeated (up to four octaves) for each of the 12 source tetrachords. The result is a large collection of four-note chords with outer intervals up to 48 semitones.

Trichords

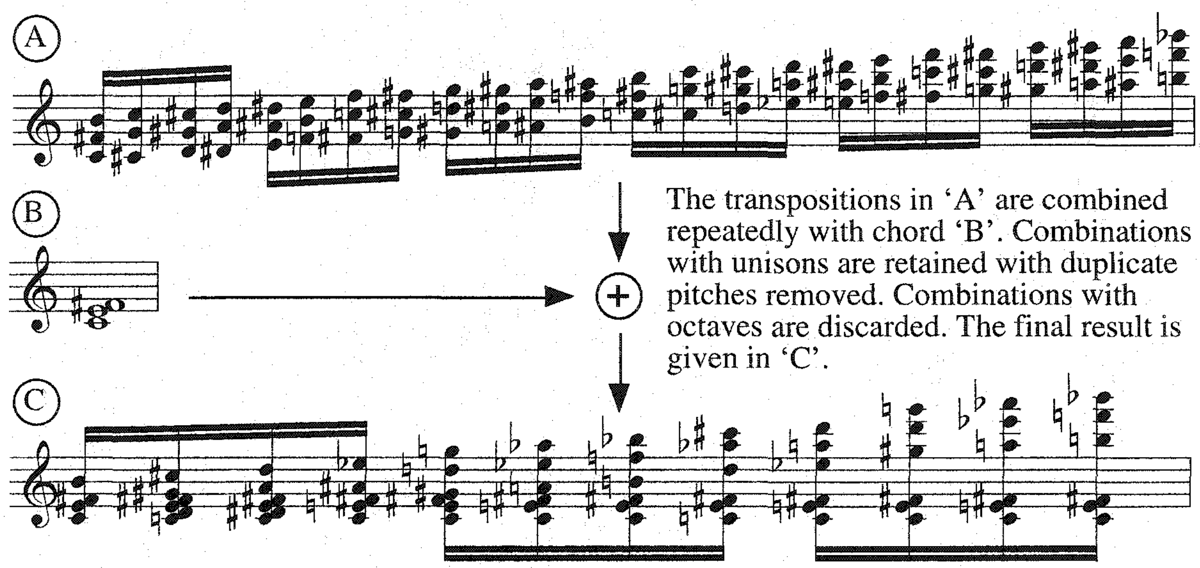
To generate three-note chords, I first find all trichordal subsets of the 12 source tetrachords. For example, the subsets of [0 1 4 6], transposed to begin on pitch class 0, are [0 1 4] [0 1 6] [0 4 6] and [0 3 5]. Duplicates are removed, and I generate all possible voicings of these three-note chords up to 4 octaves. After I once more remove any duplicates, the result is the final list of trichords.

Pentachords

The five-note chords used in the Concerto all contain either [0 1 4 6] or its inversion [0 2 5 6] as a subset. The normal forms of all the possible pentachords that contain [0 1 4 6] as a subset are [0 1 2 5 7] [0 2 3 6 8] [0 1 4 6 9] [0 1 4 6 8] [0 1 4 6 7] [0 1 4 5 6] [0 1 3 4 6] and [0 1 2 4 6]. Pentachords containing [0 2 5 6] are [0 1 3 6 7] [0 2 4 7 8] [0 2 5 6 9] [0 2 5 6 8] [0 2 5 6 7] [0 2 4 5 6] [0 2 3 5 6] and [0 1 2 5 6]. I use all possible voicings of these pentachords within four octaves.

Hexachords

The technique for deriving six-note chords is somewhat circuitous. The first step is to find all the trichordal subsets of [0 1 4 6] and [0 2 5 6]. Following this, I generate all voicings of these trichords within 3 octaves and remove any duplicates. The chords in this list are then combined with each other in the following way. Each chord in turn - transposed to pitch-class 0 - serves as a fixed lower subset. All the members of the list, including the current fixed chord, are combined with this chord by transposing each progressively a semitone at a time, starting on pitch-class 0 and ending when the top note reaches pitch 48. The notes of each transposition are combined with the notes of the unvarying chord. Combinations that contain unisons are retained with the duplicate pitch removed, while those that contain octaves are discarded. Each chord is combined in this way with all the other chords of the trichord list, including itself. The following example demonstrates this procedure using [0 4 6] as the fixed chord ('B') and [0 6 11] as the transposed chord ('A'), which is transposed 24 times. To simplify the notation, the original chords are transposed to middle C.



Example 3-21: Combining Chords

The result of this process is an extremely large list of chords with cardinalities between 3 and 6. The final step is to remove chords whose cardinality is not equal to 6 and that do not contain [0 1 4 6] or its inversion [0 2 5 6] as a subset.

Septachords and octachords

Chords with cardinalities of 7 and 8 are both produced at the same time. First, I generate all possible voicings within 3 octaves of [0 1 4 6] and its inversion [0 2 5 6]. The members of this list are then combined with each other using the above technique. Any combination whose cardinality is greater than 4 will automatically contain either two versions of [0 1 4 6], two versions of [0 2 5 6], or one version of each. For cardinalities less than 8, one or more notes of these subsets will overlap. I incorporate members of this list that have 7 or 8 notes into the harmonic vocabulary of the Concerto.

Nonachords and decachords

The previously-created pentachords with outer intervals less than two octaves are combined with each other using the same technique as for 6, 7, and 8-note chords. Since all pentachords include at least one version of [0 1 4 6] or [0 2 5 6] as a subset, combinations with 9 or 10 notes will contain at least two versions of one or the other of these chords, or one of each. The nine- and ten-note chords used in the Concerto are derived from this list of combinations.

3.8.3 Contents of the Harmonic Lexicon

The following table is the output of an *Apprentice* function that analyzes the harmonic vocabulary of the Concerto as it stands after the above process.

```
Analyzing Harmonic Vocabulary...

Checking for chords of cardinality 3...
  3-note chords in octave 1:
    0 chords with outer interval 0
    0 chords with outer interval 1
    0 chords with outer interval 2
    2 chords with outer interval 3
    2 chords with outer interval 4
    4 chords with outer interval 5
    5 chords with outer interval 6
    6 chords with outer interval 7
    6 chords with outer interval 8
    8 chords with outer interval 9
    7 chords with outer interval 10
    8 chords with outer interval 11
  3-note chords in octave 2:
    0 chords with outer interval 12
    8 chords with outer interval 13
    7 chords with outer interval 14
    12 chords with outer interval 15
    10 chords with outer interval 16
    14 chords with outer interval 17
    15 chords with outer interval 18
    16 chords with outer interval 19
    14 chords with outer interval 20
    18 chords with outer interval 21
    14 chords with outer interval 22
    16 chords with outer interval 23
  3-note chords in octave 3:
    0 chords with outer interval 24
    16 chords with outer interval 25
    14 chords with outer interval 26
    22 chords with outer interval 27
    18 chords with outer interval 28
    24 chords with outer interval 29
    25 chords with outer interval 30
    26 chords with outer interval 31
    22 chords with outer interval 32
    28 chords with outer interval 33
    21 chords with outer interval 34
    24 chords with outer interval 35
  3-note chords in octave 4:
    0 chords with outer interval 36
    24 chords with outer interval 37
    21 chords with outer interval 38
    32 chords with outer interval 39
    26 chords with outer interval 40
    34 chords with outer interval 41
    35 chords with outer interval 42
    36 chords with outer interval 43
    30 chords with outer interval 44
    38 chords with outer interval 45
    28 chords with outer interval 46
    32 chords with outer interval 47
Checking for chords of cardinality 4...
  4-note chords in octave 1:
    0 chords with outer interval 0
    0 chords with outer interval 1
    0 chords with outer interval 2
    0 chords with outer interval 3
    0 chords with outer interval 4
    0 chords with outer interval 5
    6 chords with outer interval 6
    6 chords with outer interval 7
    6 chords with outer interval 8
    6 chords with outer interval 9
    12 chords with outer interval 10
    12 chords with outer interval 11
  4-note chords in octave 2:
    0 chords with outer interval 12
    12 chords with outer interval 13
    12 chords with outer interval 14
    22 chords with outer interval 15
    14 chords with outer interval 16
    22 chords with outer interval 17
    46 chords with outer interval 18
    40 chords with outer interval 19
    32 chords with outer interval 20
    40 chords with outer interval 21
    48 chords with outer interval 22
    48 chords with outer interval 23
  4-note chords in octave 3:
    0 chords with outer interval 24
    48 chords with outer interval 25
    48 chords with outer interval 26
    72 chords with outer interval 27
    48 chords with outer interval 28
    72 chords with outer interval 29
    126 chords with outer interval 30
    102 chords with outer interval 31
    78 chords with outer interval 32
    102 chords with outer interval 33
    108 chords with outer interval 34
    108 chords with outer interval 35
  4-note chords in octave 4:
    0 chords with outer interval 36
    108 chords with outer interval 37
    108 chords with outer interval 38
    150 chords with outer interval 39
    102 chords with outer interval 40
    150 chords with outer interval 41
    246 chords with outer interval 42
    192 chords with outer interval 43
    144 chords with outer interval 44
    192 chords with outer interval 45
    192 chords with outer interval 46
    192 chords with outer interval 47
Checking for chords of cardinality 5...
  5-note chords in octave 1:
    0 chords with outer interval 0
    0 chords with outer interval 1
    0 chords with outer interval 2
    0 chords with outer interval 3
    0 chords with outer interval 4
    0 chords with outer interval 5
    6 chords with outer interval 6
    4 chords with outer interval 7
    4 chords with outer interval 8
    16 chords with outer interval 9
    22 chords with outer interval 10
    28 chords with outer interval 11
  5-note chords in octave 2:
    0 chords with outer interval 12
    28 chords with outer interval 13
    34 chords with outer interval 14
    40 chords with outer interval 15
    60 chords with outer interval 16
    72 chords with outer interval 17
    138 chords with outer interval 18
    108 chords with outer interval 19
    120 chords with outer interval 20
    176 chords with outer interval 21
    200 chords with outer interval 22
    224 chords with outer interval 23
  5-note chords in octave 3:
    0 chords with outer interval 24
    224 chords with outer interval 25
```

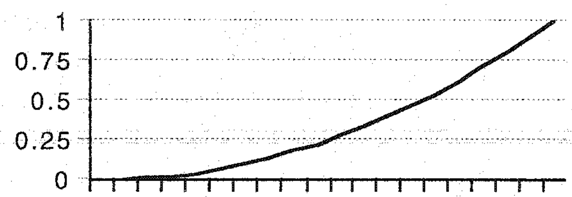
248 chords with outer interval 26
272 chords with outer interval 27
344 chords with outer interval 28
380 chords with outer interval 29
630 chords with outer interval 30
480 chords with outer interval 31
516 chords with outer interval 32
648 chords with outer interval 33
702 chords with outer interval 34
756 chords with outer interval 35
5-note chords in octave 4:
0 chords with outer interval 36
756 chords with outer interval 37
810 chords with outer interval 38
864 chords with outer interval 39
1020 chords with outer interval 40
1092 chords with outer interval 41
1722 chords with outer interval 42
1288 chords with outer interval 43
1360 chords with outer interval 44
1600 chords with outer interval 45
1696 chords with outer interval 46
1792 chords with outer interval 47
Checking for chords of cardinality 6...
6-note chords in octave 1:
0 chords with outer interval 0
0 chords with outer interval 1
0 chords with outer interval 2
0 chords with outer interval 3
0 chords with outer interval 4
0 chords with outer interval 5
5 chords with outer interval 6
10 chords with outer interval 7
17 chords with outer interval 8
45 chords with outer interval 9
82 chords with outer interval 10
129 chords with outer interval 11
6-note chords in octave 2:
0 chords with outer interval 12
129 chords with outer interval 13
180 chords with outer interval 14
229 chords with outer interval 15
335 chords with outer interval 16
442 chords with outer interval 17
721 chords with outer interval 18
796 chords with outer interval 19
1016 chords with outer interval 20
1312 chords with outer interval 21
1704 chords with outer interval 22
2064 chords with outer interval 23
6-note chords in octave 3:
0 chords with outer interval 24
1712 chords with outer interval 25
1864 chords with outer interval 26
1744 chords with outer interval 27
2004 chords with outer interval 28
2064 chords with outer interval 29
1879 chords with outer interval 30
2268 chords with outer interval 31
2356 chords with outer interval 32
2263 chords with outer interval 33
2433 chords with outer interval 34
2479 chords with outer interval 35
6-note chords in octave 4:
0 chords with outer interval 36
2479 chords with outer interval 37
2545 chords with outer interval 38
2443 chords with outer interval 39
2600 chords with outer interval 40
2652 chords with outer interval 41
2159 chords with outer interval 42
2724 chords with outer interval 43
2744 chords with outer interval 44
2592 chords with outer interval 45
2752 chords with outer interval 46
2752 chords with outer interval 47
Checking for chords of cardinality 7...
7-note chords in octave 1:
0 chords with outer interval 0
0 chords with outer interval 1
0 chords with outer interval 2
0 chords with outer interval 3
0 chords with outer interval 4
0 chords with outer interval 5
0 chords with outer interval 6
2 chords with outer interval 7
3 chords with outer interval 8
8 chords with outer interval 9
27 chords with outer interval 10
44 chords with outer interval 11
7-note chords in octave 2:
0 chords with outer interval 12
44 chords with outer interval 13
69 chords with outer interval 14
84 chords with outer interval 15
145 chords with outer interval 16
186 chords with outer interval 17
372 chords with outer interval 18
432 chords with outer interval 19
632 chords with outer interval 20
720 chords with outer interval 21
1200 chords with outer interval 22
1408 chords with outer interval 23
7-note chords in octave 3:
0 chords with outer interval 24
1408 chords with outer interval 25
1872 chords with outer interval 26
1936 chords with outer interval 27
2856 chords with outer interval 28
3184 chords with outer interval 29
5220 chords with outer interval 30
5166 chords with outer interval 31
6759 chords with outer interval 32
6804 chords with outer interval 33
9963 chords with outer interval 34
10692 chords with outer interval 35
7-note chords in octave 4:
0 chords with outer interval 36
4860 chords with outer interval 37
5589 chords with outer interval 38
3726 chords with outer interval 39
5427 chords with outer interval 40
4374 chords with outer interval 41
1620 chords with outer interval 42
3726 chords with outer interval 43
4293 chords with outer interval 44
2754 chords with outer interval 45
4131 chords with outer interval 46
3240 chords with outer interval 47
Checking for chords of cardinality 8...
8-note chords in octave 1:
0 chords with outer interval 0
0 chords with outer interval 1
0 chords with outer interval 2
0 chords with outer interval 3
0 chords with outer interval 4
0 chords with outer interval 5
0 chords with outer interval 6
0 chords with outer interval 7
0 chords with outer interval 8
3 chords with outer interval 9
4 chords with outer interval 10
13 chords with outer interval 11
8-note chords in octave 2:
0 chords with outer interval 12
13 chords with outer interval 13
20 chords with outer interval 14
35 chords with outer interval 15
48 chords with outer interval 16
84 chords with outer interval 17
112 chords with outer interval 18
180 chords with outer interval 19
240 chords with outer interval 20
464 chords with outer interval 21
512 chords with outer interval 22
832 chords with outer interval 23
8-note chords in octave 3:
0 chords with outer interval 24
832 chords with outer interval 25
1024 chords with outer interval 26
1488 chords with outer interval 27
1680 chords with outer interval 28
2388 chords with outer interval 29
2736 chords with outer interval 30
3708 chords with outer interval 31
4320 chords with outer interval 32
6723 chords with outer interval 33
6804 chords with outer interval 34
9477 chords with outer interval 35
8-note chords in octave 4:
0 chords with outer interval 36
6561 chords with outer interval 37
5832 chords with outer interval 38
6966 chords with outer interval 39
5832 chords with outer interval 40
6966 chords with outer interval 41
2268 chords with outer interval 42
6966 chords with outer interval 43
5832 chords with outer interval 44
6966 chords with outer interval 45
5832 chords with outer interval 46
7146 chords with outer interval 47
Checking for chords of cardinality 9...
9-note chords in octave 1:
0 chords with outer interval 0
0 chords with outer interval 1

0 chords with outer interval 2
0 chords with outer interval 3
0 chords with outer interval 4
0 chords with outer interval 5
0 chords with outer interval 6
0 chords with outer interval 7
1 chords with outer interval 8
5 chords with outer interval 9
28 chords with outer interval 10
90 chords with outer interval 11
9-note chords in octave 2:
0 chords with outer interval 12
100 chords with outer interval 13
169 chords with outer interval 14
238 chords with outer interval 15
321 chords with outer interval 16
510 chords with outer interval 17
761 chords with outer interval 18
1210 chords with outer interval 19
1796 chords with outer interval 20
2594 chords with outer interval 21
3870 chords with outer interval 22
5260 chords with outer interval 23
9-note chords in octave 3:
0 chords with outer interval 24
4266 chords with outer interval 25
3902 chords with outer interval 26
2976 chords with outer interval 27
3487 chords with outer interval 28
3585 chords with outer interval 29
2101 chords with outer interval 30
3250 chords with outer interval 31
3122 chords with outer interval 32
2599 chords with outer interval 33
2744 chords with outer interval 34
1936 chords with outer interval 35
9-note chords in octave 4:
0 chords with outer interval 36
2446 chords with outer interval 37
2009 chords with outer interval 38
1313 chords with outer interval 39
1408 chords with outer interval 40
1213 chords with outer interval 41
575 chords with outer interval 42
712 chords with outer interval 43
611 chords with outer interval 44
448 chords with outer interval 45
314 chords with outer interval 46
0 chords with outer interval 47
Checking for chords of cardinality 10...
10-note chords in octave 1:
0 chords with outer interval 0
0 chords with outer interval 1
0 chords with outer interval 2
0 chords with outer interval 3
0 chords with outer interval 4
0 chords with outer interval 5
0 chords with outer interval 6
0 chords with outer interval 7
0 chords with outer interval 8
1 chords with outer interval 9
3 chords with outer interval 10
18 chords with outer interval 11
10-note chords in octave 2:
0 chords with outer interval 12
22 chords with outer interval 13
43 chords with outer interval 14
76 chords with outer interval 15
85 chords with outer interval 16
170 chords with outer interval 17
266 chords with outer interval 18
479 chords with outer interval 19
715 chords with outer interval 20
1183 chords with outer interval 21
1760 chords with outer interval 22
2433 chords with outer interval 23
10-note chords in octave 3:
0 chords with outer interval 24
2242 chords with outer interval 25
1937 chords with outer interval 26
1608 chords with outer interval 27
1720 chords with outer interval 28
2100 chords with outer interval 29
1280 chords with outer interval 30
2197 chords with outer interval 31
1993 chords with outer interval 32
2117 chords with outer interval 33
1945 chords with outer interval 34
2069 chords with outer interval 35
10-note chords in octave 4:
0 chords with outer interval 36
2795 chords with outer interval 37
2375 chords with outer interval 38
2029 chords with outer interval 39
1934 chords with outer interval 40
2404 chords with outer interval 41
1425 chords with outer interval 42
2483 chords with outer interval 43
2196 chords with outer interval 44
2306 chords with outer interval 45
2071 chords with outer interval 46
2157 chords with outer interval 47

Total number of chords is 415534.

3.8.4 Reducing the Harmonic Lexicon

As the cardinality and the outer interval increase, some harmonic "slots" contain a very large number of chords. For example, there are over 10,000 seven-note chords with an outer interval of 35 semitones. In early trials, it became clear that such large numbers were both musically unnecessary and computationally inefficient. An acceptable balance between harmonic variety and performance occurs when the maximum number of possibilities for each outer interval is roughly 200. I reduce the above harmonic vocabulary in the following way. First, I sort the chords in each slot by increasing adjacent interval variance. Slots with a large number of chords are sampled exponentially 200 times, retaining more chords from the beginning of the sorted list (ie. with less adjacent interval variance). To do this, I sample the exponential break-point function given below 200 times, with the maximum *y* value set to the number of chords in the current slot. The resulting values, rounded off to the nearest integer, indicate the position of chords to be retained.



Example 3-22: Exponential Curve Used to Reduce Harmonic Lexicon

Sampling this curve with the maximum *y* value set to 300 gives the following list:

```
(1 1 1 1 1 1 1 2 2 2 2 2 2 2 3 3 3 3 4 4 4 4 5 5 6 6 6 7 7 7 8
8 9 9 10 10 11 12 12 13 13 14 14 15 16 16 17 18 19 19 20 21 21
22 23 24 25 26 27 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41
42 44 45 46 47 48 50 51 52 53 54 56 57 58 60 61 62 64 65 66 68
69 71 72 74 75 77 78 80 81 83 84 86 88 89 91 93 94 96 98 99 101
103 104 106 108 110 112 114 115 117 119 121 123 125 127 129 131
133 135 137 139 141 143 145 147 149 151 153 156 158 160 162 164
166 169 171 173 176 178 180 183 185 187 190 192 194 197 199 202
204 207 209 212 214 217 219 222 225 227 230 232 235 238 240 243
246 249 251 254 257 260 262 265 268 271 274 277 280 282 285 288
291 294 297 300)
```

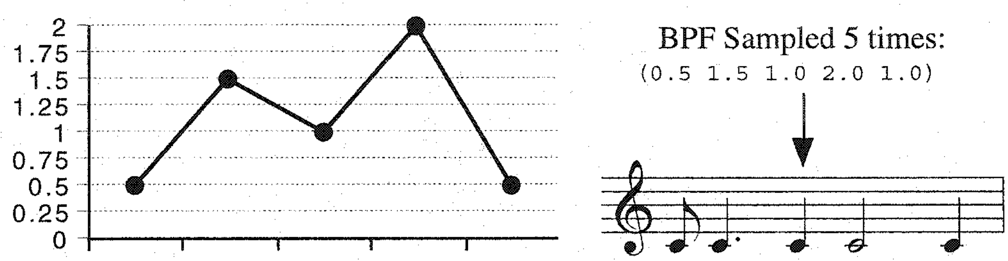
Due to rounding, some values at the beginning of this list are repeated. These are removed, and the number of chords that are extracted from the slot is reduced accordingly. This is acceptable since the maximum value of 200 is only intended to be an approximate guideline. The more chords in a slot, the greater the number of chords in the reduced slot (up to 200). A larger number of chords from the beginning of the sorted list - where voicing is more balanced - are selected than from the end. The total number of chords in the original vocabulary is 415,534. After applying the above technique to all harmonic slots, this number diminishes to 47,603.

Chapter 4: Duration

In the *Concerto for Piano and Orchestra*, I use the same procedures to create and manipulate both foreground rhythms and background formal durations. In this chapter, I use the term rhythm to refer to both of these aspects of musical time. I compose rhythms using two related tools: rhythmic contours and rhythmic profiles.

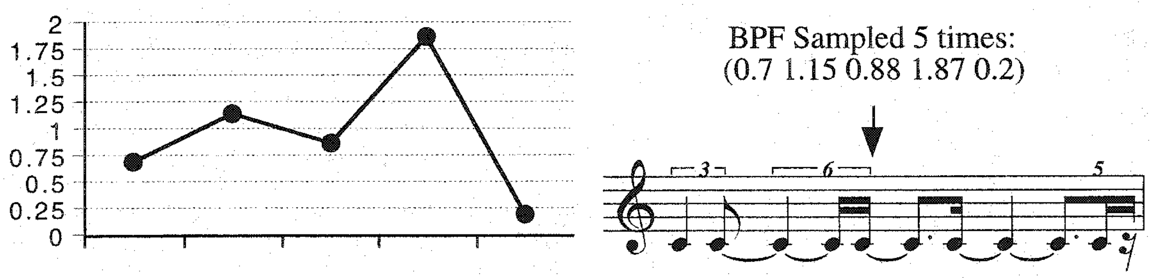
4.1 Rhythmic Contours

Values on the *y* axis of a break-point function may be used as durations in a fairly straightforward way. Sampling the break-point function (0.0 0.5 0.25 1.5 0.5 1.0 0.75 2.0 1.0 0.5) five times results in the list (0.5 1.5 1.0 2.0 0.5). These values are interpreted as multiples of a quarter note. The following example demonstrates this relationship:



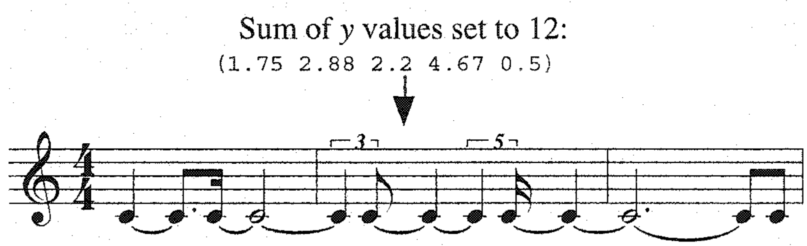
Example 4-1: Sampling a Rhythmic Contour

In the case of a more complex result, values are quantized, or rounded off to the nearest reasonable rhythmic subdivision:



Example 4-2: Rhythmic Quantization

A feature of break-point functions in *Apprentice* not previously mentioned is the ability to specify the sum of the y values that are returned. Although this is not particularly appropriate for melodic contours, it is very useful for duration, where this sum corresponds to the total time span of the resulting rhythm. The user specifies the desired sum in quarter notes, and the program automatically scales the y values. For example, the break-point function used in example 4-2 could be sampled so that the resulting rhythm has a total duration of twelve quarters:

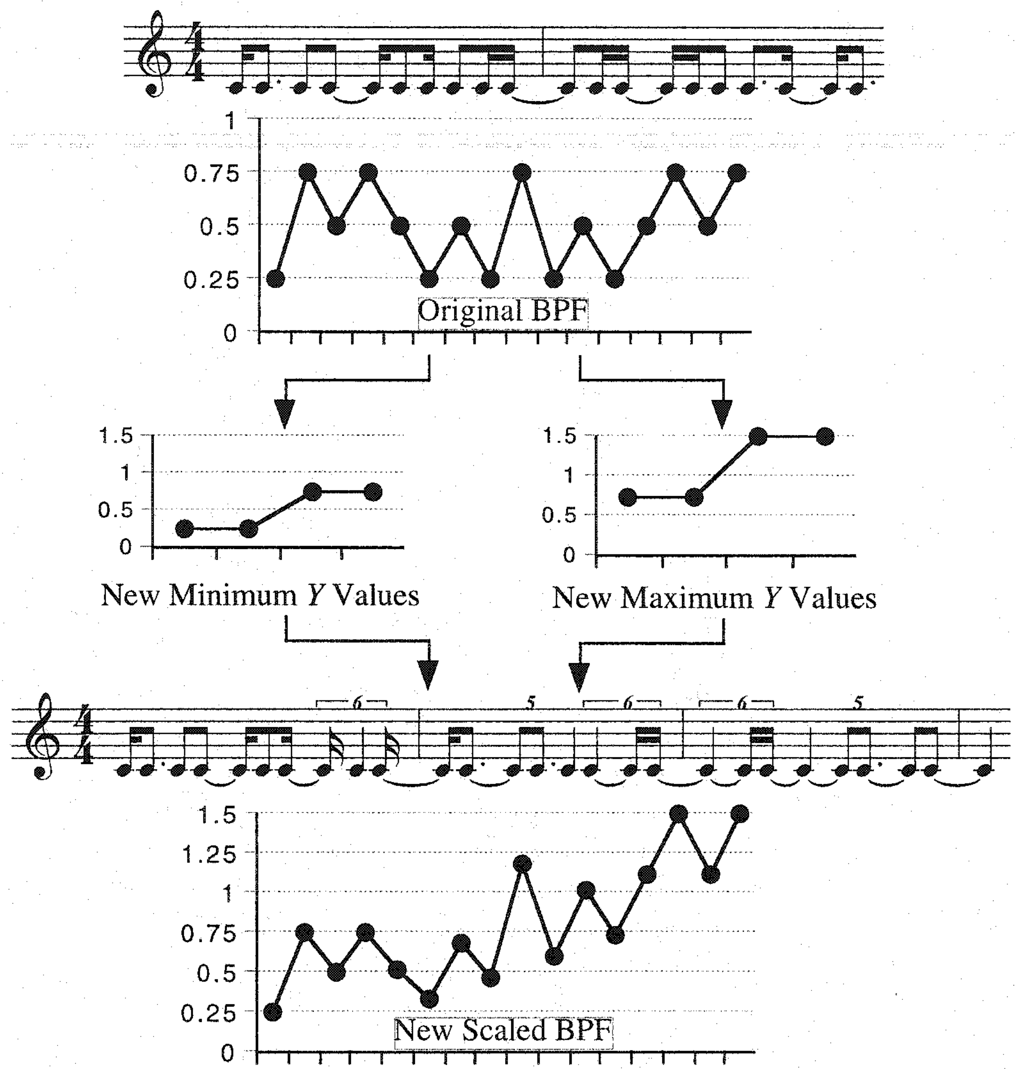


Example 4-3: Setting the Total Duration of a Rhythm

The rhythmic proportions used on the surface of the music are thus easily mapped onto a much larger time scale. This technique is useful for determining formal durations, as in the third movement of the Concerto, for example. This passage is discussed in more detail in Part II of this document.

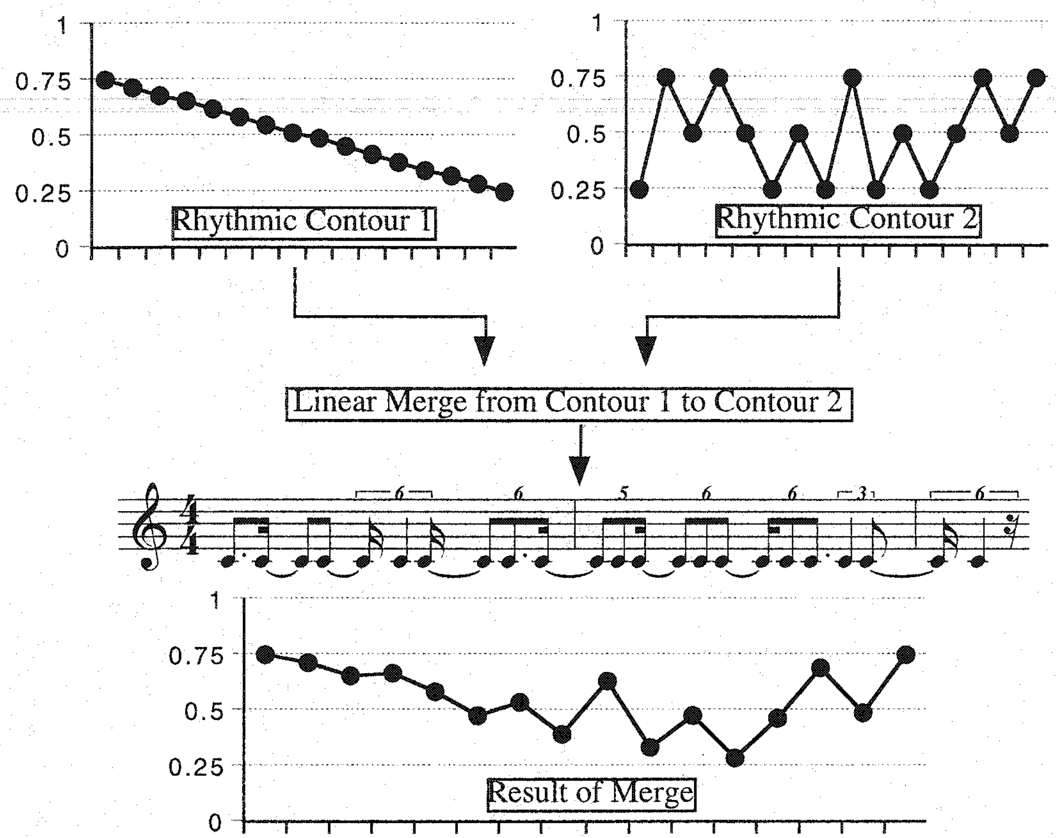
4.2 Transformations of Rhythmic Break-Point Functions

All of the techniques for varying melodic contours in Chapter 2 may be applied to rhythmic contours. Two in particular are frequently utilized in the Concerto: dynamic scaling of y values and merging contours. The first technique is useful for creating notated accelerandos and ritardandos. This is illustrated in example 4-4. Here I have sampled the original rhythmic contour using dynamic scaling of the minimum and maximum y values. The result in this case is a progressive slowing down of the original rhythm.



Example 4-4: Dynamic Scaling of Rhythmic Contour

The second technique permits a gradual metamorphosis between two rhythms. In example 4-5, for instance, the original rhythm from example 4-4 is blended with a contour that decreases linearly from 0.75 to 0.25. The merge begins with 100% of contour 1 and ends with 100% of contour 2. The musical result is the gradual transformation of a fairly steady rhythm to one with greater variance.



Example 4-5: Dynamic Merging Between Rhythmic Contours

4.3 Rhythmic Profiles

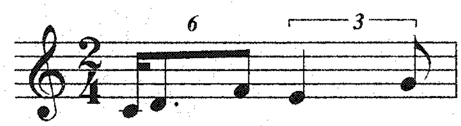
A rhythmic profile is a type of contour that is a more abstract representation of a rhythmic structure. The difference between a profile and a standard rhythmic contour is subtle but significant. The *y* axis of a normal rhythmic contour indicates the precise duration of a rhythm in quarter notes. Values on the *y* axis of a rhythmic profile, on the other hand, are multipliers of an unspecified rhythmic subdivision. The profile may thus be thought of as a class or template of a rhythm, as opposed to a contour, which translates directly into musical notation. This difference allows profiles to be manipulated in powerful ways that would be more difficult using regular contours.

4.3.1 Creating Rhythmic Profiles

A profile is defined using values that determine the number of multiples of a rhythmic subdivision in a given note. This definition has two components. The first is the pattern of multiples, with a minimum value of zero. The second is the number of subdivisions to be added to the members of first list. For example, the rhythm used in example 4-1 could be expressed as a profile in this way:

(1) pattern = (0 2 1 3 1)
(2) amount to add = 1

In example 4-1 the basic subdivision is an eighth note. Each successive rhythmic value is found by multiplying this basic subdivision by each value in the pattern and then adding the appropriate number of subdivisions as given in the profile definition. In the following example I create an instance of the same profile using a basic subdivision of a sextuplet sixteenth note:



Example 4-6: Rhythmic Profile

At first glance, the utility of this way of defining a profile is not obvious, but it will become clearer during the following discussion.

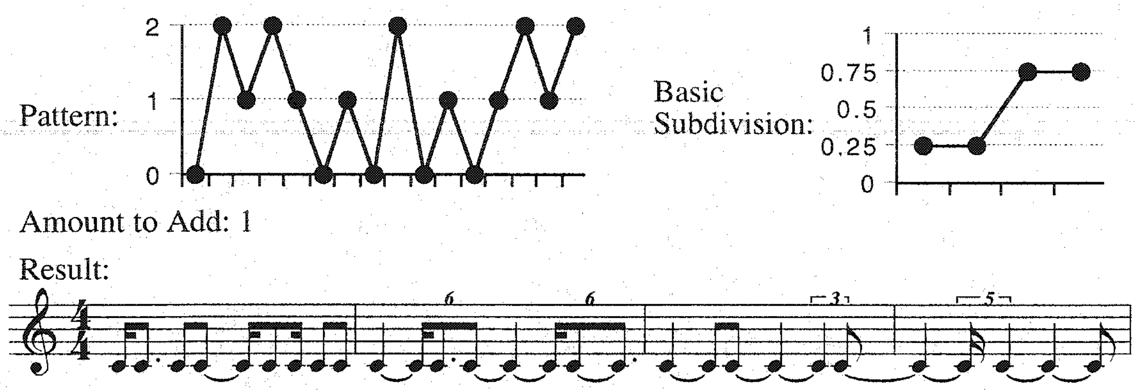
4.3.2 Transformations of Rhythmic Profiles

Varying the Basic Subdivision

The basic subdivision of a profile may be set to different single values, or may be varied dynamically. In example 4-1, for instance, the basic subdivision is an eighth note, while in example 4-6 I apply a different basic subdivision to the same profile. In example 4-7 below, I define the basic subdivision as a break-point function. I apply this function to the following profile:

(1) pattern = `(0.0 2.0 1.0 2.0 1.0 0.0 1.0 0.0 2.0 0.0 1.0 0.0 1.0 2.0 1.0 2.0)`
(2) amount to add = 1

This profile is equivalent to the rhythmic contour marked "original bpf" in example 4-4. The result of the scaling is given below.



Example 4-7: Dynamic Scaling of Basic Subdivision

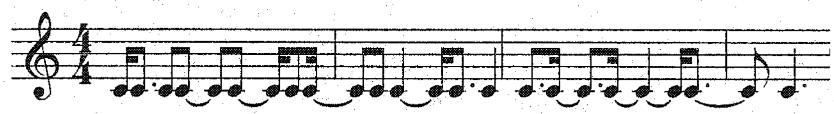
Varying the Amount to Add Independently of the Pattern

Much of the usefulness of rhythmic profiles lies in the ability to define the subdivision used for the pattern separately from the subdivision for the amount to add. In the following example the subdivision for the pattern is a triplet eighth note, while the value to add is a normal eighth note. The profile is (0 2 1 3 1), which is the same as we saw in example 4-6 :

Example 4-8: Separate Values for Pattern and Amount to Add

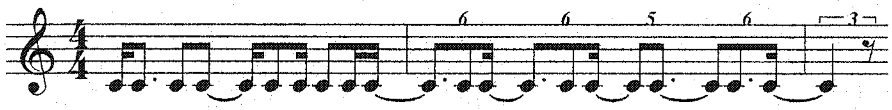
The rhythmic values of example 4-8 are: (1/2 7/6 5/6 3/2 5/6) or (0.5 1.167 0.833 1.5 0.833). The first value is equal to a triplet eighth note multiplied by 0 plus one eighth note, while the second is equal to a triplet eighth note multiplied by 2 (.667) plus one eighth note (.5).

One may also define the amount to add and the subdivisions of the pattern dynamically as break-point functions. In the following examples the pattern is (0 2 1 2 1 0 1 0 2 0 1 0 1 2 1 2), which is the same as example 4-7. The amount to add may be varied in two ways. First, the amount itself may be defined as a break-point function. In example 4-9 the basic subdivision is a sixteenth note, but the amount to add increases linearly from 1 to 4. This has the effect of gradually slowing and "flattening" the rhythm, since the ratio difference between successive notes decreases.



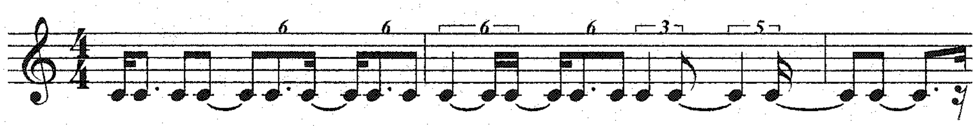
Example 4-9: Increasing the Amount to Add

Example 4-10 shows the second way to vary the amount to add. The number of values added is always 1, but the type of value increases from a sixteenth note to a triplet eighth:



Example 4-10: Scaling the Value to Add

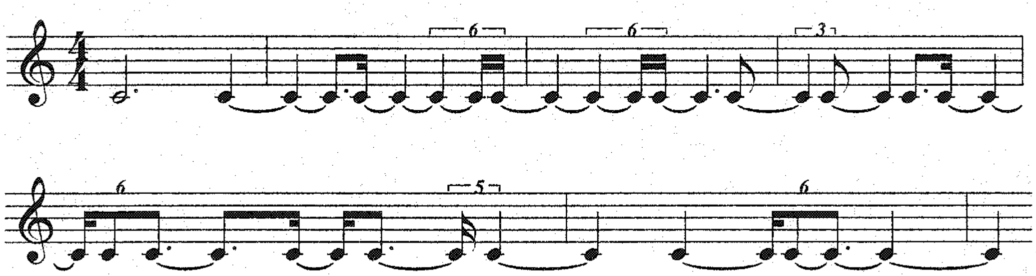
It is also possible to dynamically vary the subdivision of the pattern while adding a constant value. Here the pattern's subdivision increases from a sixteenth note to an eighth note, while the amount being added remains a sixteenth:



Example 4-11: Scaling the Pattern Subdivision

Finally, any combination of the above techniques is possible. In example 4-12 the same rhythmic profile as above has the following time-varying parameters:

- Pattern: `(0 2 1 2 1 0 1 0 2 0 1 0 1 2 1 2)`
- Pattern subdivision: sextuplet sixteenth -> quarter tied to triplet eighth.
- Amount to add: 3 -> 1
- Value to add: quarter note -> sixteenth



Example 4-12: Scaling All Aspects of Rhythmic Profile

The first note has a duration equal to a sextuplet sixteenth multiplied by 0, plus a quarter note multiplied by 3. The last note, on the other hand, is equal to a quarter tied to a triplet eighth note multiplied by 2, plus a sixteenth note multiplied by 1. This last value is 2.667 + .25 = 2.917. It has been quantized to the nearest reasonable rhythmic subdivision: 2.833 or 2 quarter notes plus 5 sextuplet sixteenth notes.

4.4 Rhythmic Contributions to Overall Stability

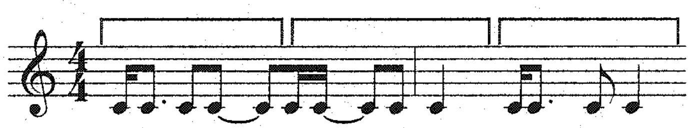
As mentioned in chapter 3, in the Concerto I use rhythm to affect the perceived stability of a harmonic progression. The two main aspects of rhythm that influence stability are predictability and rate.

4.4.1 Rhythmic Predictability

I consider that rhythms which are more predictable will contribute to the stability of a progression, while rhythms that are less predictable will detract from stability. I control three factors that contribute to predictability: periodicity, length, and variety.

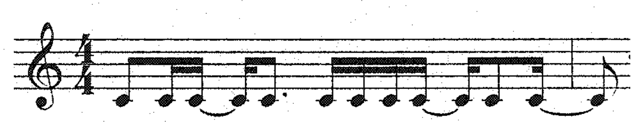
Periodicity

A rhythm with high periodicity contains repeating patterns. The following rhythm, for instance, contains a repeating pattern of one, three, two and four sixteenths:



Example 4-13: High Rhythmic Periodicity

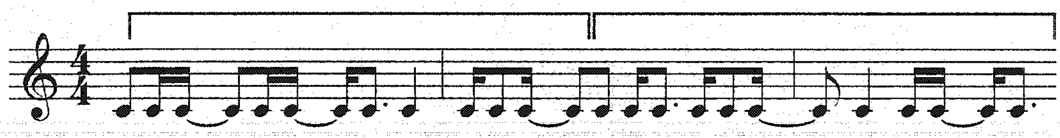
The next example contains no repeating patterns. As a result it is less predictable and should detract from the overall sense of stability of a progression.



Example 4-14: Low Rhythmic Periodicity

Length

The length of a repeating pattern affects predictability. The longer the pattern, the more difficult it is to remember and therefore to predict. Example 4-13 above is a repeating pattern of 4 notes. The following example contains a repeating pattern of 10 notes and is therefore less predictable:

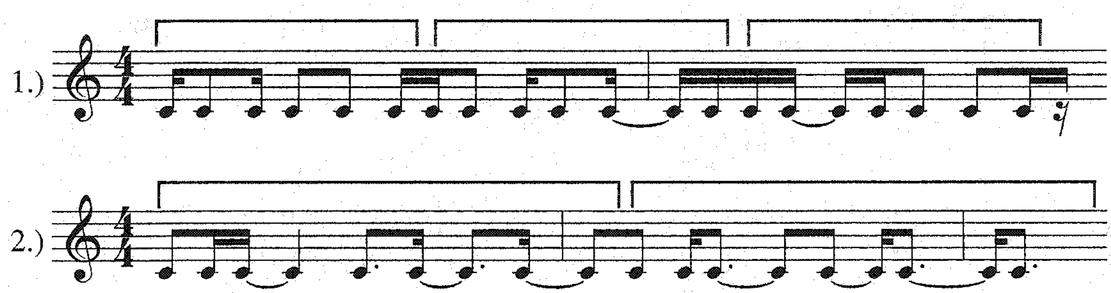


Example 4-15: Long Repeating Rhythmic Pattern

This rhythm is still more predictable than one with no periodic pattern at all.

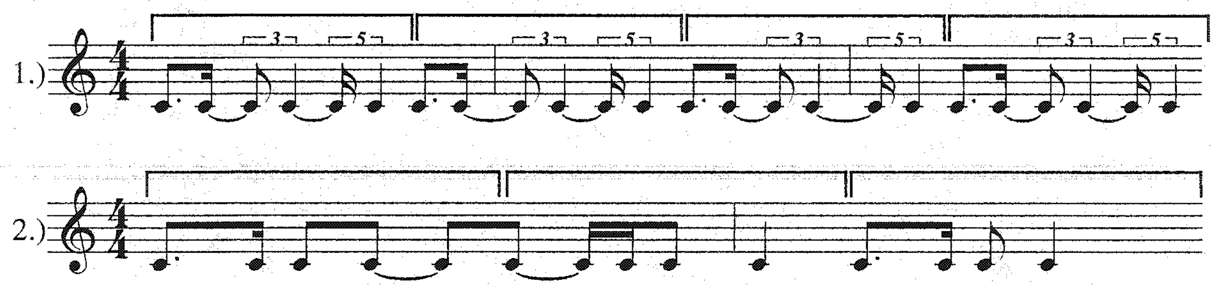
Variety

Two types of variety impact rhythmic predictability: the variety of rhythms used and the variety of subdivisions. In general, a smaller degree of variety will increase stability while a greater degree of variety will decrease stability. The following example has two rhythms, each containing patterns of 6 notes. The first uses only two different rhythmic values: a sixteenth and an eighth. The second uses five values: a sixteenth, an eighth, a dotted eighth, a quarter and a quarter tied to a sixteenth. Since it has greater variety, this rhythm should be less predictable than the first.



Example 4-16: Variety of Rhythmic Values

The next example also has two rhythms. Both contain patterns of 4 notes and have 4 different rhythmic values. However, all the durations of the second rhythm are multiples of the same subdivision while the first rhythm subdivides the beat into sixteenths, triplet eighths and quintuplet sixteenths. The greater variety of subdivisions in the first rhythm should cause it to be less predictable.



Example 4-17: Variety of Rhythmic Subdivisions

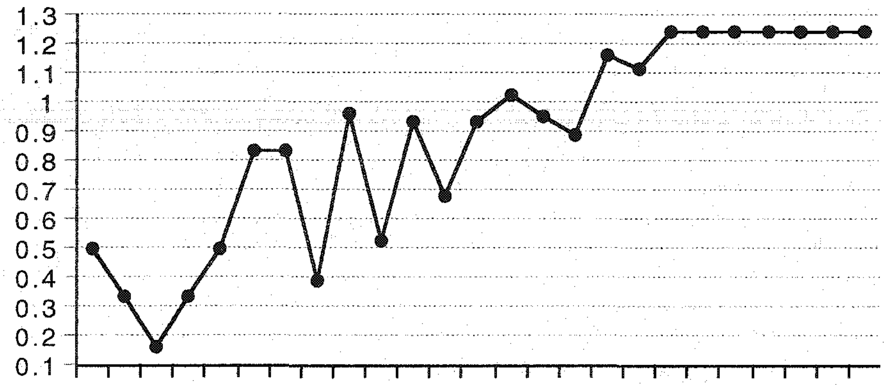
It is clear from the above discussion that the type of rhythm that is most predictable will have the highest possible periodicity, the shortest pattern length and the least variety. The rhythm that corresponds to all these criteria is a constant repeating value or pulsation.

4.4.2 Rate

The slower the rhythm of a harmonic progression, the longer the ear has to absorb each chord and the more comprehensible and stable the progression should sound. Faster rhythms, on the other hand, make it more difficult to clearly hear the harmonic progression. The harmony is therefore less comprehensible and as a result less stable. This is especially true on a first hearing. On subsequent listenings the effect is reduced as one becomes more familiar with the piece. In the Concerto I use slower rhythms to increase stability, and faster rhythms to reduce stability. As well, accelerandos or gradual reductions in the durations of a rhythm tend to decrease stability and increase tension, while ritardandos or gradual increases in duration tend to increase stability and decrease tension.

4.4.3 Combining Rhythmic and Harmonic Stability

In section 3.6 of chapter 3, I discuss the control of harmonic stability through the interaction of harmonic parameters. The progression in example 3-18 has gradually increasing overall stability. It is possible to reinforce this perception rhythmically. According to the criteria described above, the following rhythmic contour has low rhythmic stability at its beginning, and becomes gradually more stable towards its end:



Example 4-18: Increasingly Stable Rhythmic Contour

If we combine this rhythm with a chord progression that has decreasing dissonance, decreasing adjacent interval variance, and increasing successive pitch commonality, the sense of increasing harmonic stability is reinforced:

Example 4-19: Combining Harmonic and Rhythmic Stability

Part II: Analysis of *Concerto for Piano and Orchestra*

Chapter 5: Overview

5.1 Overall Form

The *Concerto for Piano and Orchestra* has six formal divisions as indicated below:

Opening Section:	mm. 1-19
Movement 1:	mm. 20-168
Movement 2:	mm. 169-208
Movement 3:	mm. 209-295
Movement 4:	mm. 295-415
Closing Section:	mm. 416-426

Apart from a short pause between movements 2 and 3, there are no breaks between the movements. The second movement is for orchestra alone and the third movement emphasizes the solo piano. The other parts of the work feature both the piano and orchestra. The Concerto is unified by a characteristic referential sonority, a coherent background harmonic progression and a fundamental basic shape or contour that affects all aspects of the piece.

5.2 Referential Sonority

In the Concerto, I define harmonic regions by consistently choosing chords that have high pitch commonality with a referential sonority. This sonority may be actually present in the music or may be only suggested. If the sonority is not sounding, it will normally have been previously emphasized. The following chord, whose prime form is [0 2 3 4 5 6 7 9], is the primary referential sonority of the Concerto:

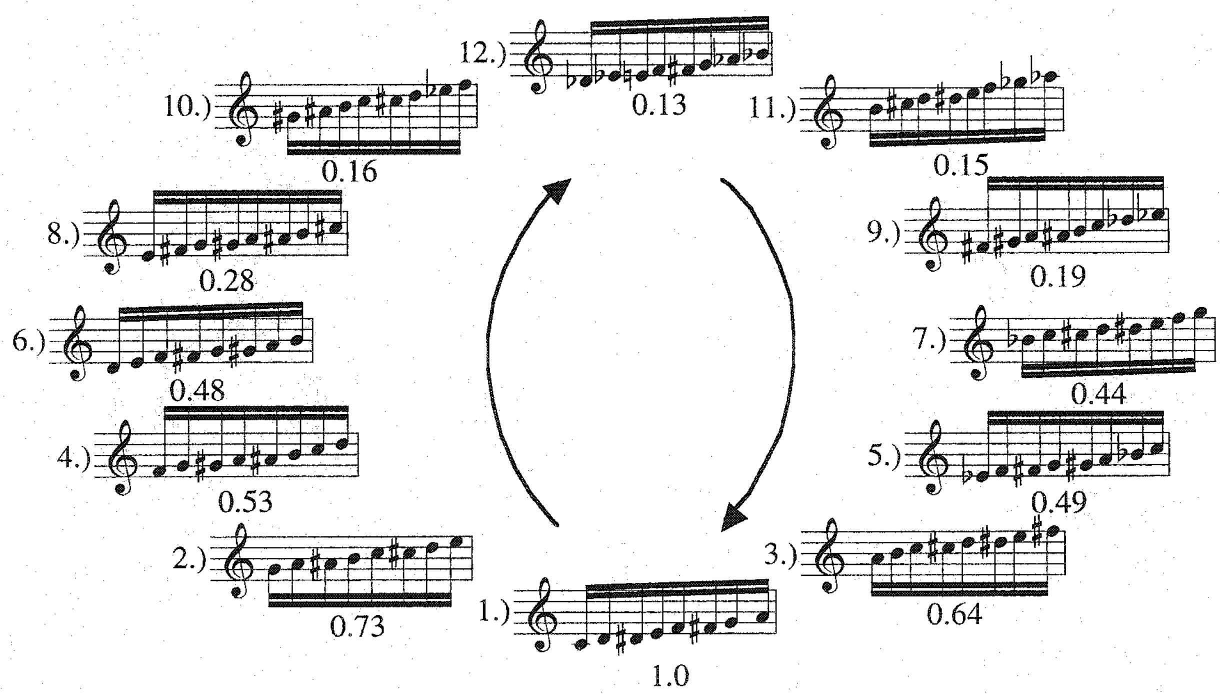
Example 5-1: The Referential Sonority of the Concerto

This chord is inversionally symmetrical: it maps onto itself under inversion and transposition. It is not transpositionally symmetrical, however, since there exist 12 unique transpositions of the chord.

This referential sonority is almost always voiced as above. The outer voices of the chord are in registers that allow them to be comfortably doubled at the octave. In the strings, for instance, the double basses are able to play the low E an octave lower (sounding) and the violins are capable of doubling the F an octave higher. As well, the low E in the basses is an open string, which allows for a resonant bass in loud tutti statements of the chord, as at measure 20. I selected this voicing from among all possible chords with the same outer voices and prime form of [0 2 3 4 5 6 7 9]. A number of criteria contributed to this choice: (1) this voicing has mirror symmetry: the adjacent intervals are the same ascending as descending; (2) the lower and upper tetrachords are the all-interval tetrachord [0 1 4 6] and its inversion [0 2 5 6], which form the basis of my harmonic vocabulary as discussed in chapter 3; (3) it has a low dissonance value of 0.56; and (4) it has a comparatively low degree of adjacent interval variance of 0.76. The last criterion indicates that the notes of the chord are relatively evenly spaced. Given the wide range of the outer intervals, this allows the chord to be scored for the entire orchestra.

5.3 Background Harmonic Progression

Other transpositions of [0 2 3 4 5 6 7 9] act as secondary referential sonorities in the Concerto. In the following example, the prime form of the referential sonority is transposed to every chromatic note in the octave between C_4 and B_4:

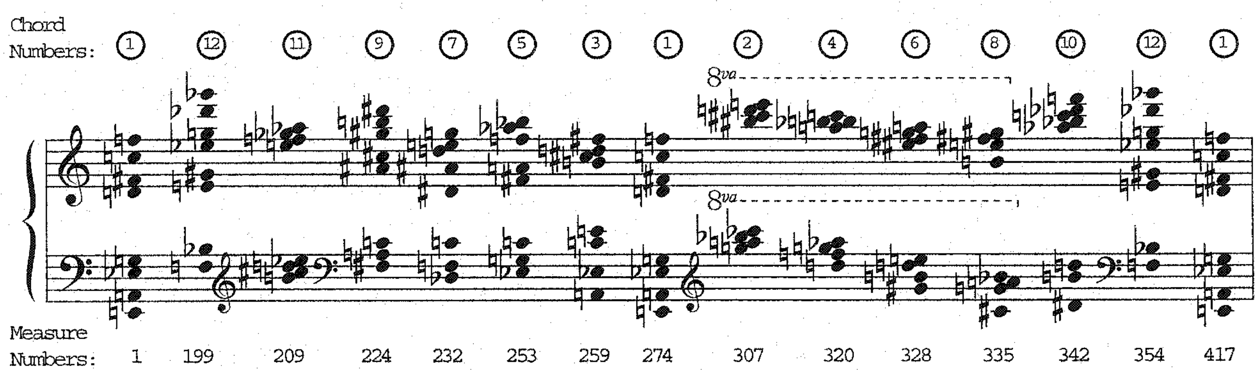


Example 5-2: Relative Pitch Commonality to the Referential Sonority

These chords are arranged by their degree of pitch commonality with the transposition that contains the pitch classes of the referential sonority as it appears in example 5-1 above, [60 62 63 64 65 66 67 69]. The degree of pitch commonality is given below the staves. The numbering in this example indicates the position of each chord in the sorted list. Chord 1 corresponds to the referential sonority, while chord 12 is the transposition most distant from the referential sonority in terms of pitch commonality. In the Concerto these two chords serve as the primary harmonic areas. Since they act in a sense as opposite poles, I refer to chord 1 as the referential sonority and to chord 12 as the antipodal sonority.

The other chords in example 5-2 are used mainly to move between these two harmonic poles. This occurs twice in the Concerto. From measure 199 to measure 274 the background harmonic progression moves from the antipodal sonority to the referential sonority through chords 11, 9, 7, 5, and 3. Each of these chords has a gradually increasing degree of pitch commonality with the referential sonority that arrives in measure 274. In the orchestra from measure 307 to measure 354 the progression is from the referential sonority to the antipodal sonority using chords 2, 4, 6, 8, and 10. In this case the chords have gradually less pitch commonality with the referential sonority. The arrows in example 5-2 indicate these movements.

The details of this background harmonic progression and the actual voicings of the secondary referential sonorities as they appear in the piece are given in example 5-3.

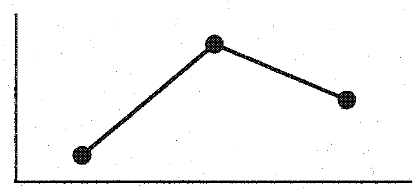


Example 5-3: Background Progression

Note that the third chord in this example, which serves as a secondary referential sonority beginning in measure 209, is not stated on the surface of the music, but is implied by high pitch commonality with the piano chords of the section. As mentioned above, these chords all reduce to the same prime form as the referential sonority, and were chosen to create a smooth progression of pitch commonality between the referential sonority and the antipodal sonority. I discuss the other criteria underlying my choice of the voicings of these sonorities in the following chapters.

5.4 Basic Shape

The following abstract contour generates almost all of the musical material of the Concerto:



Example 5-4: Basic Shape of the Concerto

This contour has just three points and is characterized by a an initial ascent followed by a change of direction to a point midway between the first two. The apparent simplicity of the contour is belied by the degree to which it permeates all aspects of the work. In the pitch domain it functions as a kernel from which I derive the melodic material of the Concerto. In this sense it is related to traditional techniques of motivic development. It differs from these techniques in that it also appears at all structural levels of the work, from the foreground to the background, and in all parameters, including horizontal pitch, vertical pitch, and duration. For instance, in harmonic progressions I use this contour to control aspects such as outer interval, cardinality, dissonance, adjacent interval variance, and pitch commonality. I apply all of the variation techniques discussed in Part I of this thesis to the contour. I also combine multiple versions of this shape to create more complex compound contours. The impact of this basic shape on virtually all facets of the *Concerto for Piano and Orchestra* is one of the principal topics of the following analysis.

Chapter 6: Opening Section (MM. 1-19)

6.1 Description

The role of this brief opening section is expository: it introduces the principal compositional ideas that I develop throughout the rest of the Concerto. For example, the material of the first four measures appears in the third movement of the Concerto in a considerably expanded form, while the piano gesture in measures 5-10 foreshadows the compositional procedures of the fourth movement. Passages similar to the harmonic progression in measures 11 to 19 also occur frequently in the the orchestra during the first and second movements. As we will see below, this section also introduces the referential sonority and the basic shape of the Concerto. Formally, the section begins with solo piano and concludes with the orchestra alone. The transition between the two takes place almost imperceptibly in measure 11, where high string harmonics take over the notes of the chord that is arpeggiated in the piano. This chord masks the attack of the strings, which sounds like a prolongation of the resonance of the piano.

6.2 Background Harmonic Progression

One of the most important functions of this section is to introduce the referential sonority. Although it does not appear in its characteristic voicing until the beginning of the first movement, this chord is nevertheless constantly present. The harmonies of the first three phrases in the piano have high pitch commonality with the referential sonority, and the rising gesture in the piano beginning in measure 5 consists entirely of the notes of this chord. The harmonies underlying this gesture are given below:

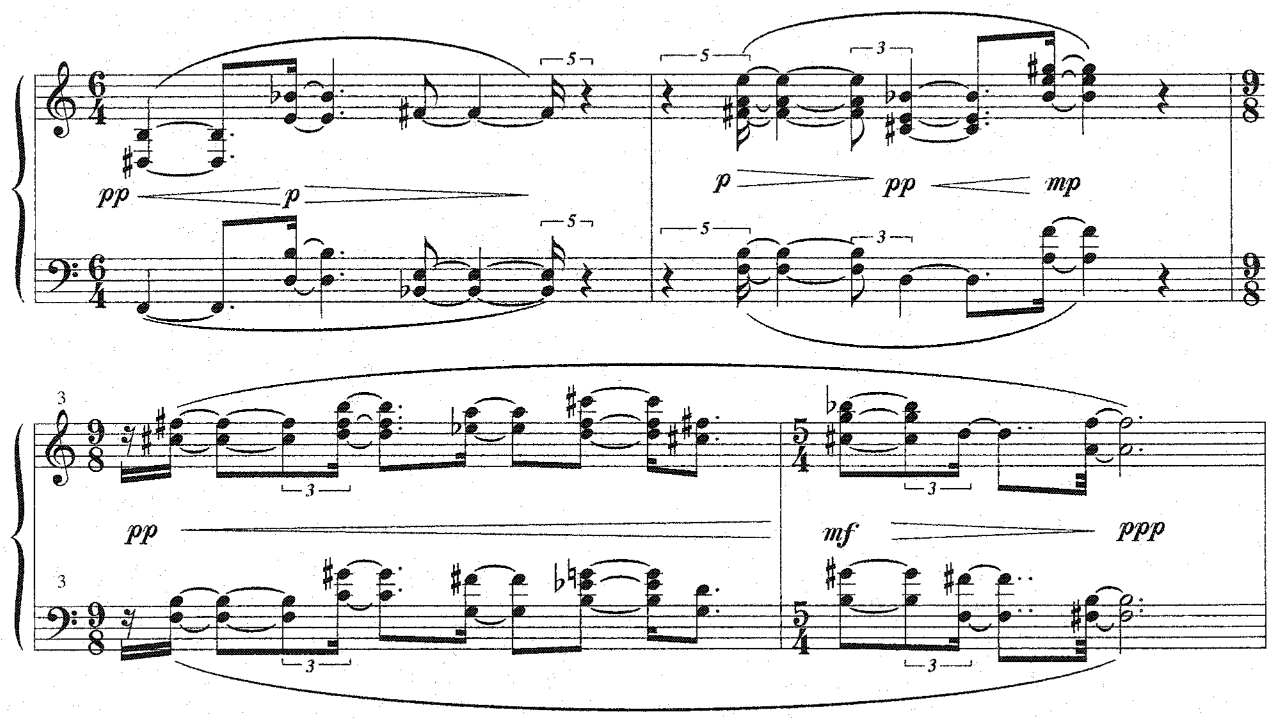
Example 6-1: Harmonic Progression of Measures 5-11

The first chord is the referential sonority transposed an octave lower. Each of the other chords contains all eight notes of the referential sonority in a different voicing. The register of the chords gradually rises while the outer interval gradually decreases up to the final chord in measure 11. This chord has the same pitches in its outer voices as the first - 'E' and 'F' - creating a large-scale transfer of register. A similar registral shift over an even larger time scale occurs in the first movement. The process by which I derive the piano figuration from these harmonies is discussed below. The chords of the orchestral passage from measures 11 to 19 also have high pitch commonality with the referential sonority, which arrives in its typical voicing at the beginning of the first movement in measure 20.

6.3 Applications of the Basic Shape in the Opening Section

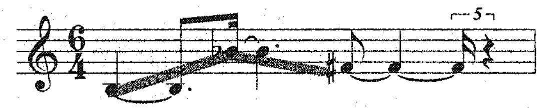
6.3.1 Horizontal Pitch

In the pitch domain, the basic shape may be thought of as an abstract melodic contour (ie. one that does not specify exact pitches) that generates motivic material. I illustrate this approach in the following detailed analysis of the first three phrases of the piano:



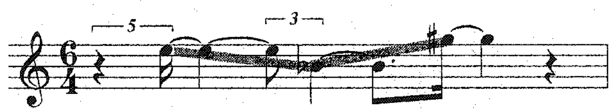
Example 6-2: Opening Piano Phrases

The outer voices of the first phrase both have melodic outlines based on the basic shape. Example 6-3 shows this for the soprano line:



Example 6-3: Basic Shape in Phrase 1

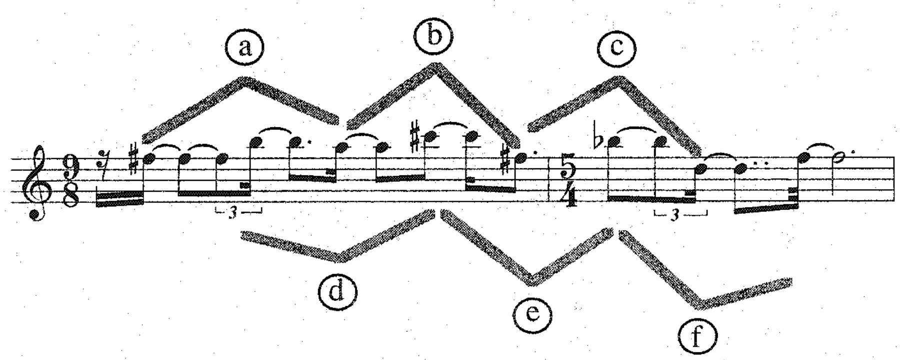
Here the basic shape has been sampled three times as a break-point function with a minimum *y* value of 59 and a maximum *y* value of 70; the resulting values (59 70 66) are interpreted as MIDI note numbers (middle C = 60). The contour of the second phrase is the retrograde inversion of the basic shape. In terms of the techniques for transforming break-point functions discussed in part I of this thesis, this is the result of setting the new minimum *y* value to 80, which is greater than the new maximum *y* value of 70, and using a read-function of (0 1 1 0):



Example 6-4: Basic Shape in Phrase 2

Since the outer voices always move in parallel directions throughout these phrases, the same comments apply to the bottom notes of these progressions.

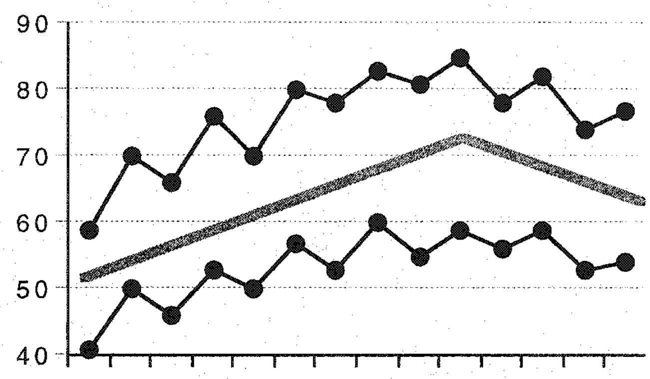
The third phrase of the piano, in measures 3 and 4, demonstrates another way of developing the basic shape. In this phrase, several different versions of the shape are combined to create a compound melodic contour:



Example 6-5: Compound Melodic Shape

Shape 'a' maintains the contour of the original shape, while 'b' and 'c' are retrogrades. The shapes shown below the staff are all inverted; version 'd' is a retrograde inversion.

All of the previously mentioned contours occur at the foreground level, between chords. If we examine the overall shape of these three phrases, however, we see that they also constitute a middleground statement of the basic shape. Example 6-6 shows the outer voices of the 14 chords from the first four bars of the Concerto. The overall contour of the voices is indicated by the line between them:



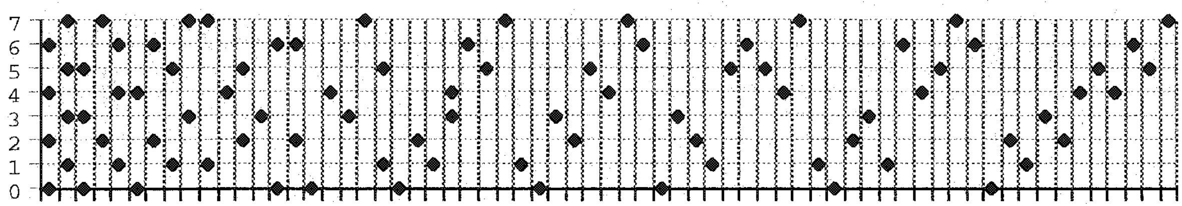
Example 6-6: Background Basic Shape

A similar background contour emerges if we combine the rising piano gesture in measures 3 and 4 with the descending orchestral gesture that follows. In this case the basic shape generates the center pitch of the progression.

The previous examples treat the basic shape as a break-point function. Measures 5 to 19 illustrate the use of the basic shape as a positional contour. This passage was composed in two steps. The first was to create the chord progression previously seen in example 6-1. This progression consists of eight-note chords containing only the pitches of the referential sonority. The second step is to apply the following list of ten positional contours to these chords:

```
((0 2 4 6) (1 3 5 7))
((0 3 5) (2 7) (1 4 6))
((0 4) (2 6) (1 5) (3 7))
((1 7) 4 (2 5) 3 (0 6))
((2 6) 0 4 3 7 (1 5))
(0 2 1 (3 4) 6 5 7)
(1 0 3 2 5 4 7 6)
(0 3 2 1 5 6 5 4 7)
(1 0 2 3 1 6 4 5 7 6)
(0 2 1 3 2 4 3 5 4 6 5 7)
```

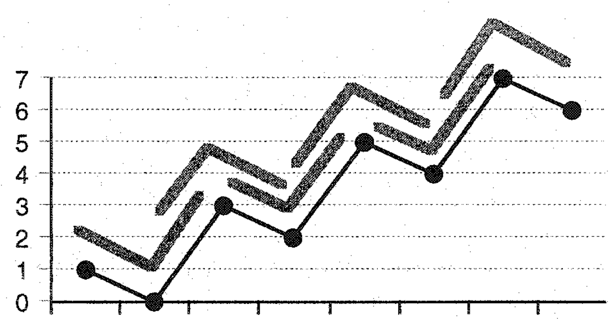
I apply each line in this list to each chord of the progression in turn, except the last chord, which is simply arpeggiated at measure 11. To illustrate, the fourth line of this list indicates that the notes of the fourth chord of the progression appear in the following order: second and eighth notes together, fifth note, third and sixth notes together, fourth note, and first and seventh note (remember that order positions begin at zero). I show this list graphically in the chart below:



Example 6-7: Graphical Representation of List of Positional Contours

The *y* axis of this chart indicates order positions, and the *x* axis represents successive attacks.

If we examine the pattern that is applied to chord 7, we see that it contains multiple versions of the basic shape, combined to create a compound contour:



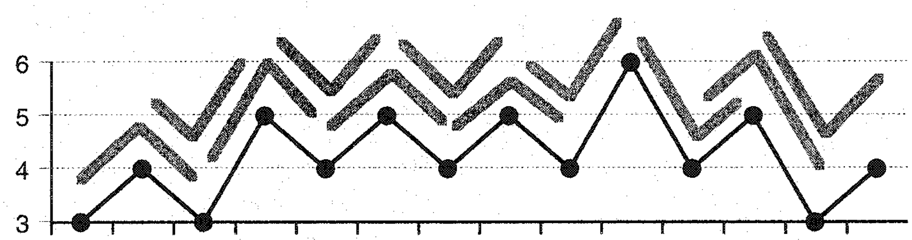
Example 6-8: Positional Contour of Seventh Chord

This is true of the entire list of positional contours. To create contours with simultaneously-sounding notes, I group portions of a contour into sublists whose members will all be played at the same time. For instance, the original positional contour for the first chord was `(0 4 2 6 1 5 3 7)`, which I altered by grouping the first and last four values into single attacks. The final list of positional contours contains a progressive increase in the number of attacks for each chord, from two for chord 1 to eleven for chord 10, and a concurrent decrease in the number of notes in each attack, from four to one.

In the preceding discussion we have seen the basic shape applied to horizontal aspects of pitch at all structural levels, from the surface of the music to the background. We will now examine the application of this shape to other parameters.

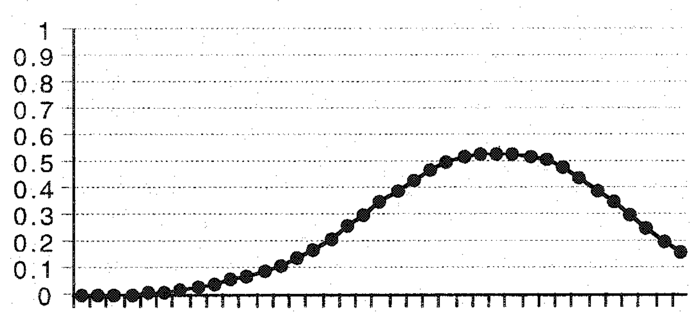
6.3.2 Vertical Pitch

The basic shape is applied to many aspects of vertical pitch in this section. These include, for example, cardinality, dissonance, and outer interval. Let us turn our attention again to the first four measures of the piece. Example 6-9 is a graph of the cardinality of each chord in these measures. The *y* axis of example 6-9 indicates the number of notes in each chord and the grey lines show the segments of the contour that correspond to the basic shape:



Example 6-9: Cardinality Contour

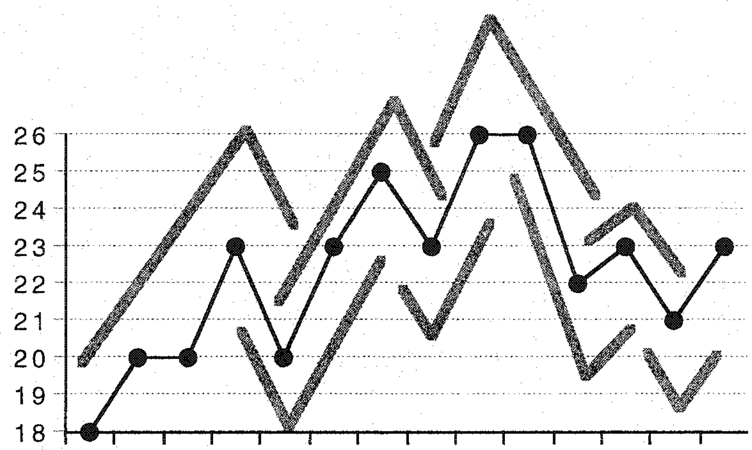
The curve that controls the degree of dissonance for the chords of this progression also derives from the basic shape. In this case the three points of the shape are joined using a cubic spline, resulting in an exponential, rather than linear, curve.



Example 6-10: Dissonance Contour

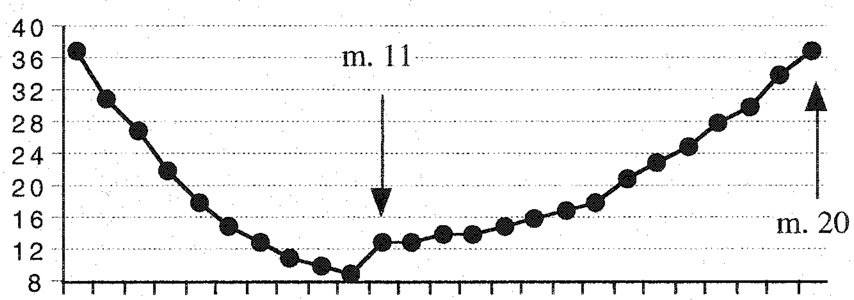
This curve describes the overall relative dissonance of the chords in the first four measures. It was sampled 14 times and the resulting values were used to determine the degree of dissonance for each chord during harmonization as described in section 3.7 of chapter 3.

A similar process determines the outer interval of each of these chords. The *y* axis of example 6-11 indicates the interval in semitones between the outer voices of each chord. Segments of this contour that correspond to the basic shape are indicated in grey. Furthermore, the overall curve of the contour also suggests the basic shape: a gradual increase followed by a decrease to a value somewhere between the initial value and the greatest value.



Example 6-11: Outer Interval Contour

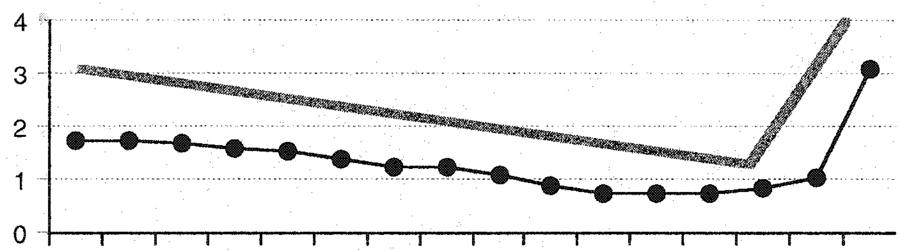
The outer intervals of the piano gesture in measures 5 to 11 and of the orchestral gesture that follows in measure 11 to 19 together form a contour that derives from the inversion of the basic shape:



Example 6-12: Outer Intervals of Measures 5-20

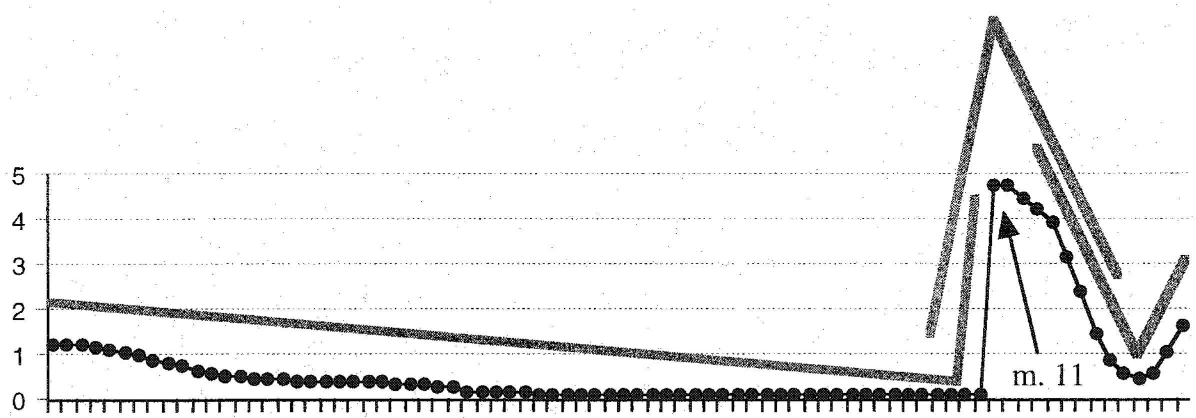
6.3.3 Duration

The durations of this section are generated by the basic shape. The rhythms of measures 0 to 4, including rests, are expressed as multiples of a quarter note in the following list: (1.75 1.75 1.7 1.6 1.53 1.42 1.25 1.25 1.08 0.92 0.75 0.75 0.75 0.83 1.04 3.12). These values are indicated on the *y* axis of example 6-13. The resulting contour is the retrograde inversion of the basic shape, using a read-function that takes more samples from the first half of the shape than from the second:



Example 6-13: Rhythmic Contour of Measures 0-4

The durations of measures 5 to 19 are shown in example 6-14. The rhythms of the orchestra starting in measure 11 are an inversion of the cubic spline version of the basic shape from example 6-10. These rhythms combine with the rhythms of the piano in the preceding passage to create other variations of the basic shape as indicated below:



Example 6-14: Rhythmic Contour of Measures 5-19

Chapter 7: Movement 1 (MM. 20-168)

7.1 Description

The first movement is the most substantial of the *Concerto for Piano and Orchestra*. Its duration is approximately 12 minutes, making up over half of the total duration of the Concerto. The principal compositional idea of the movement is the gradual transference of a background harmonic progression and rhythm to the foreground. At the beginning of the movement these background aspects form the basis of the formal subdivisions, while at the end they constitute the surface of the music. My goal is to turn the music "inside-out" by bringing structural elements that are normally perceived subconsciously to the forefront of the listener's attention. This process is accompanied by the compression and eventual dissipation of the original foreground material.

7.2 Musical Material

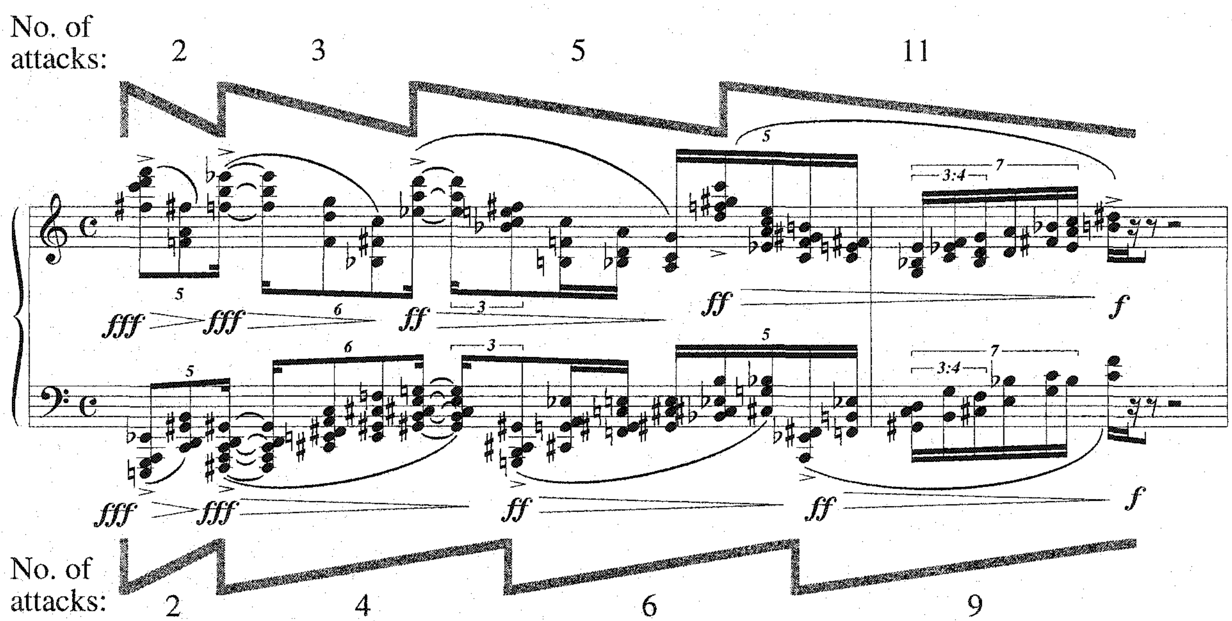
There are four main types of musical material in this movement: (1) prolonged chords in the orchestra; (2) homophonic progressions in the piano; (3) lyrical melodic material in the woodwinds; and (4) gradually accelerating harmonic progressions in the strings. Each of these supports the overall idea of the movement by reinforcing both the background harmonic progression and large-scale rhythmic pulse. I describe the characteristics of these different types in the following sections.

7.2.1 Prolonged Chords

The beginning of each formal section in this movement is marked by a strongly attacked chord that is sustained by the orchestra. These chords are important because they articulate the arrival of each new formal unit and because they introduce the referential sonorities of each section. As we will see, these chords make up the background harmonic progression of the movement. They may be repeated several times during a section. An example is the chord that arrives in measure 20, which recurs at measures 23, 26, and 30. The dynamic level of these chords gradually diminishes throughout their duration. The orchestration reinforces this diminuendo. We see this, for instance, with the tutti chord in measure 30, from which instruments are gradually removed beginning in measure 31. The last instrument to drop out is the double bass in measure 39.

7.2.2 Piano

The material of the piano consists entirely of solid chords with a flexible accelerating and decelerating rhythm. Although the texture of the piano is homophonic, the outer voices of these chords suggest a two-part counterpoint. The soprano contains a series of descending gestures, while the bass contains ascending gestures. These are reinforced by the dynamics: the beginning of each is accented and is followed by a diminuendo. The number of attacks in each gesture increases at different rates for the two voices. In example 7-1, taken from measure 20, the gestures in the right hand of the piano contain 2, 3, 5, and 11 attacks, while those of the left hand contain 2, 4, 6, and 9 attacks. The result is a series of figures that gradually move out of phase with each other, suggesting a note-to-note canon in inversion where the time interval between the voices gradually increases.

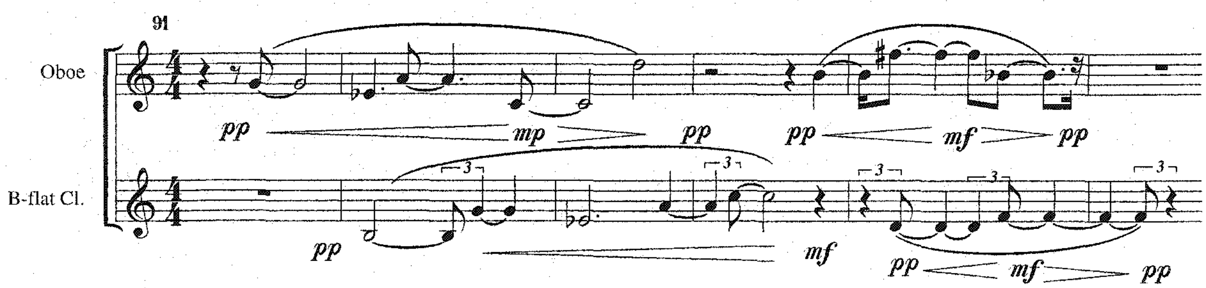


Example 7-1: Initial Piano Gesture of Movement 1

The overall character of the piano material is of strong attacks followed by a dispersal of dynamic energy. We see this both in the descending and ascending gestures, and in the general decrease in dynamic level of the entire phrase shown in example 7-1. The harmonies support this character as well. The above phrase, for instance, has lower successive pitch commonality and greater dissonance and cardinality at its beginning than at its end. The harmonic stability thus gradually decreases, although this is somewhat offset by the accelerating rhythm. Passages of this type are linked to each other by phrases that feature increasing energy. For example, the strong attack in measure 20 connects with that in measure 23 through the bridge in measures 21 and 22. The wave-like ebb and flow of this alternation of increasing and decreasing energy mirrors the general development of the formal subdivisions. I discuss this relationship in more detail in section 7.4 below.

7.2.3 Woodwinds

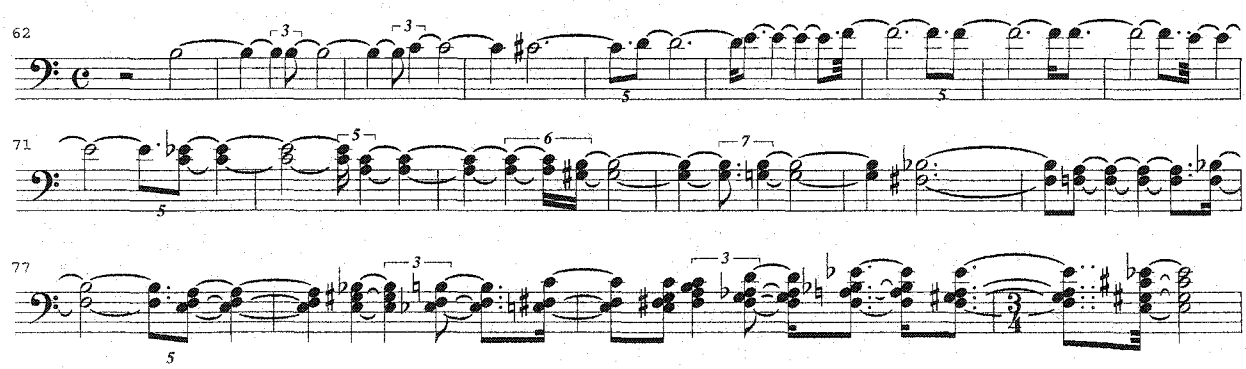
In the first movement, the woodwind material consists of lyrical melodic lines. These develop into a two-part counterpoint during the course of the movement. A characteristic example from measure 91 is given in example 7-2. Note that each part is distinguished by its rhythmic subdivision: sixteenths in the oboe and triplet eighths in the clarinet. Within each formal section this material begins quietly and slowly and gradually increases in intensity and speed leading into the arrival of the next section. I discuss the derivation of this material below in the section on applications of the basic shape.



Example 7-2: Woodwind Material from Movement 1

7.2.4 Strings

Besides taking part in the tutti statements of the prolonged chords at the beginning of each formal section, the strings also play accelerating harmonic progressions that lead into the beginning of the next section. These progressions begin quietly with narrow outer intervals and low cardinality and gradually become louder with wider outer intervals and greater cardinality. A characteristic example from measures 62 to 80 is given below:



Example 7-3: String Material from Movement 1

The rhythm of these passages derives from a rhythmic break-point function. The details of the rhythmic and melodic contours of this material are discussed in section 7.4.

7.2.5 Other Material

In the first movement, the horn and timpani have material that does not derive from the four types described above. Phrases in both instruments feature strong attacks that always coincide with the beginnings of phrases in the piano, followed by decelerating rhythms and decrescendos. The role of this music is secondary: it serves only to support the piano. Examples of this material and its correlation with the piano can be found throughout the movement, as at measures 106, 108, and 112.

7.3 Form

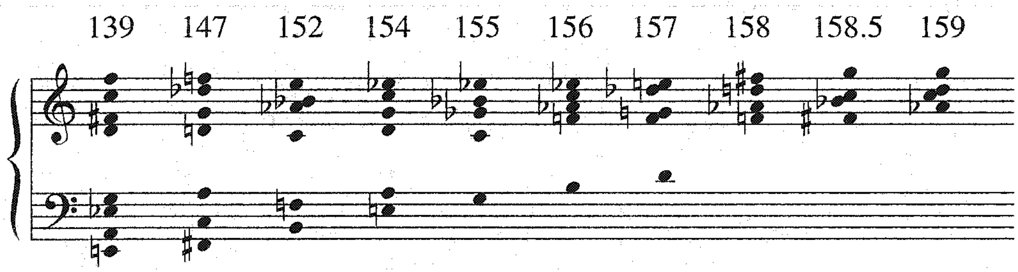
7.3.1 Background Harmonic Progression

The background harmonic progression of the first movement has two parts. The first is given in example 7-4. It begins with a statement of the referential sonority of the Concerto at the start of the movement in measure 20. The next five background chords comprise a symmetrically expanding progression whose goal is a return to the original referential sonority in measure 139:

Example 7-4: Background Harmonic Progression, Measures 20-139

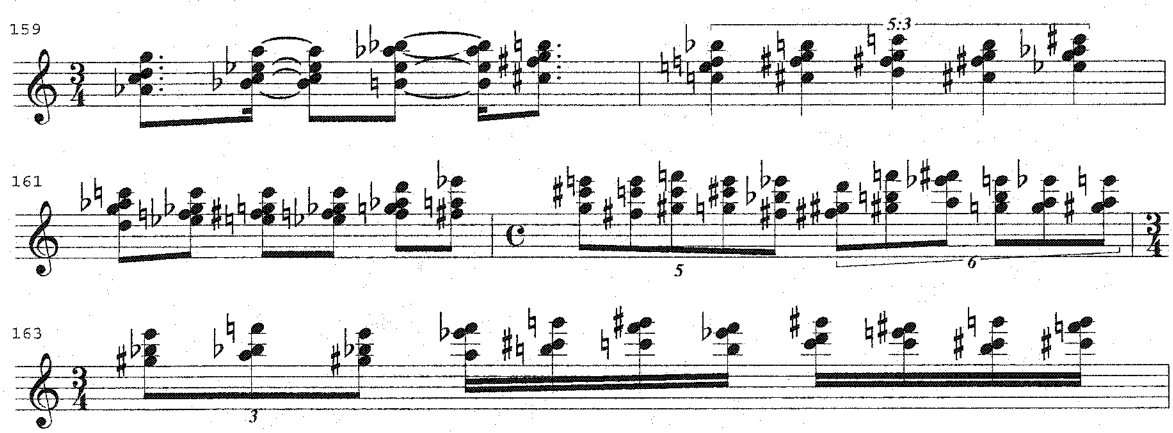
Each chord of this progression is the basis of a formal section of the movement: all the chords of the section have relatively high pitch commonality with this reference. Since the chords of the background progression itself all have high pitch commonality with the referential sonority of the Concerto, the entire movement is a prolongation of this sonority. Although the chords of example 7-4 all have high pitch commonality with the same chord, they have relatively low successive pitch commonality. This allows the formal sections to be clearly articulated, since the harmonic connections between them are unsmooth.

Once the referential sonority returns in measure 139, the second part of the background progression gradually rises in register while the outer interval and cardinality of the chords decreases:



Example 7-5: Background Harmonic Progression, Measures 139-159

Each of these chords also acts as the basis of a formal section, although the duration of these sections becomes gradually much shorter. By measure 159 the duration of each chord becomes so short that the progression moves from the background to the foreground:

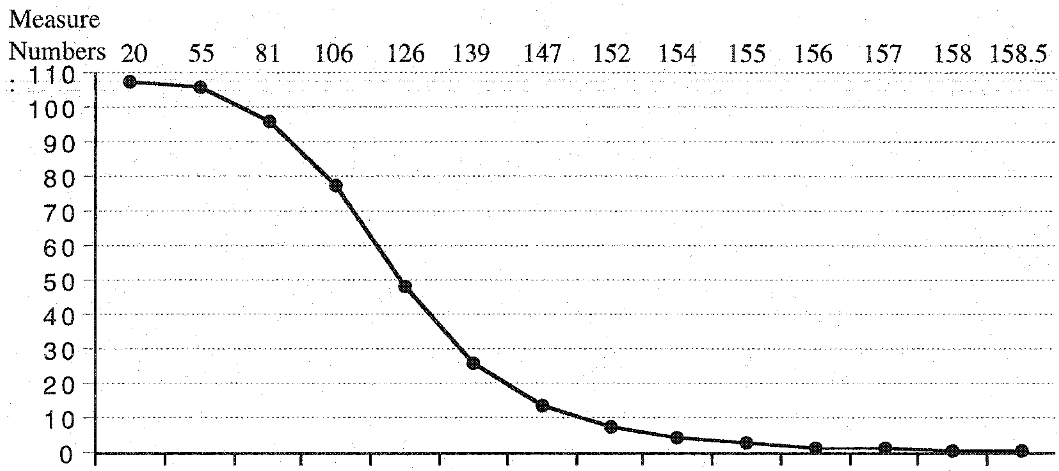


Example 7-6: Continuation of Harmonic Progression in the Foreground

This surface progression continues to rise and become more narrow until it arrives on a single repeated E_7 in measure 167. This pitch is the final arrival point of the entire harmonic structure of the movement, which ends in measure 169.

7.3.2 Formal Durations

The referential sonorities of each formal section were given in examples 7-4 and 7-5 above. The durations in quarter notes of these sections are: 108, 106, 96.5, 77.5, 48.5, 26.5, 14, 8, 4.5, 2.75, 1.75, 1.25, 1, and 1. These durations were obtained by sampling the following cubic spline:



Example 7-7: Background Formal Durations of Movement 1

Cubic splines of this type are effective for generating natural-sounding accelerating rhythms. Although the process of acceleration is quite gradual, by measure 159 the rhythm has moved definitively from the background to the foreground as shown previously in example 7-6. The acceleration continues until half way through measure 164, from which point it remains at the same speed until the end of the movement.

7.3.3 Formal Development of Material

One of the roles of the musical material of this movement is to emphasize the arrival of the sustained chords that mark the beginning of each new formal section. Between the poles represented by these chords, the music is either in the process of receding from or preparing for these formal dividing points. For example, in the section that begins in measure 55, measures 55 to 64 gradually diminish in intensity, while measures 65 to 80 contain a gradual build up to the arrival of the next section in measure 81. The diminuendo occurs in the sustained chord and the piano, while the crescendo occurs in the woodwinds and the strings. My goal is to create a noticeable background pulsation, despite the relatively large durations of the early formal sections. This is a large-scale manifestation of the wave pattern described in connection with the piano material in section 7.2.2 above.

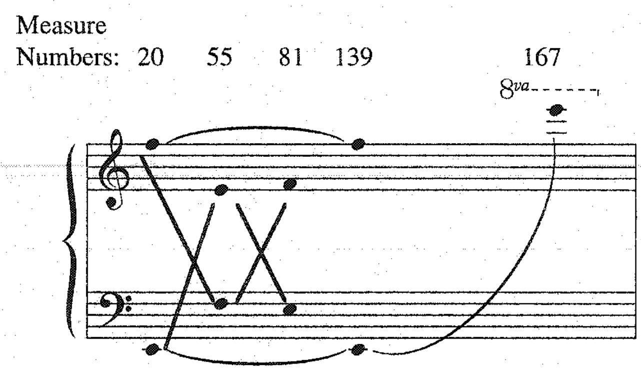
In the passage described above, the point of lowest energy occurs in measure 65. The middle C played by the first cello in this measure belongs to the string material, which began in measure 62. The entrance of this string material overlaps briefly with the final diminuendo of the sustained chord. There is no overlap between the end of the piano material in measure 62 and the beginning of the woodwind material in measure 65. As the duration of the formal sections becomes shorter during the course of the movement, the "fade-out" and "build-up" begin to overlap more and more. In the section beginning in measure 81, the piano and woodwind materials do not overlap either. But in the next section, beginning in measure 106, the clarinet enters at the end of measure 115 while the piano does not finish until the first beat of measure 116. This overlap of two beats becomes two measures in the next section: the clarinet and bassoon enter at the beginning of measure 134 and the piano finishes at the end of measure 135. At this point it is only 3 more measures until the next section begins at measure 139.

This new section is significant because it marks the return of the referential sonority of the Concerto in the background harmonic progression. It is also significant because it marks the end of the woodwind material. This is the first foreground element to disappear as the formal durations are compressed and the background begins to impose itself little by little. As well, it is the first section in which the piano has no pause between the end of one section and the beginning of the next. These recurring periods of rest in the piano part are another aspect of the foreground structural level that disappears with the compression of the formal time scale. The piano material itself is increasingly truncated until it becomes an element of the background progression in measure 159. It then detaches itself from the continuing accelerando and begins to slow down, coming to an end in measure 163. The string material finishes in measure 158. By measure 164, all of the foreground material has dissipated under the pressure of the formal contraction and only the surface manifestation of the former background progression remains. The string harmonics that begin in this measure are elements of the next movement that enter imperceptibly.

7.3.4 Large-Scale Transfer of Register

If we examine the outer voices of the background progression given in example 7-4, we see that not only are the notes of the referential sonority prolonged, but so are the outer voices. In the middle of this prolongation, there is a shift an octave higher in the bass and an octave lower in the soprano. The change of octave is decorated by a middleground incomplete neighbor figure, which would be called a voice exchange in tonal theory.[40] In other words, the F in the soprano passes to the bass while the E in the bass passes to the soprano. After the outer voices return to their original register in measure 139, the remainder of the structural harmonic progression contains a transfer of the bass note from E_2 up 5 octaves to the E_7 that arrives in measure 167 and is prolonged into the next movement. A reduction of this process is given in example 7-8.

[40] E. Aldwell & C. Schachter, *Harmony and Voice Leading, Second Edition* (San Diego: Harcourt Brace Jovanovich, 1989), 97.



Example 7-8: Large-Scale Transfer of Register

The emphasis on E and F in this example is significant since E is the bass note of the referential sonority and F is the bass note of the antipodal sonority, as shown in the first two chords of example 5-3 in chapter 5.

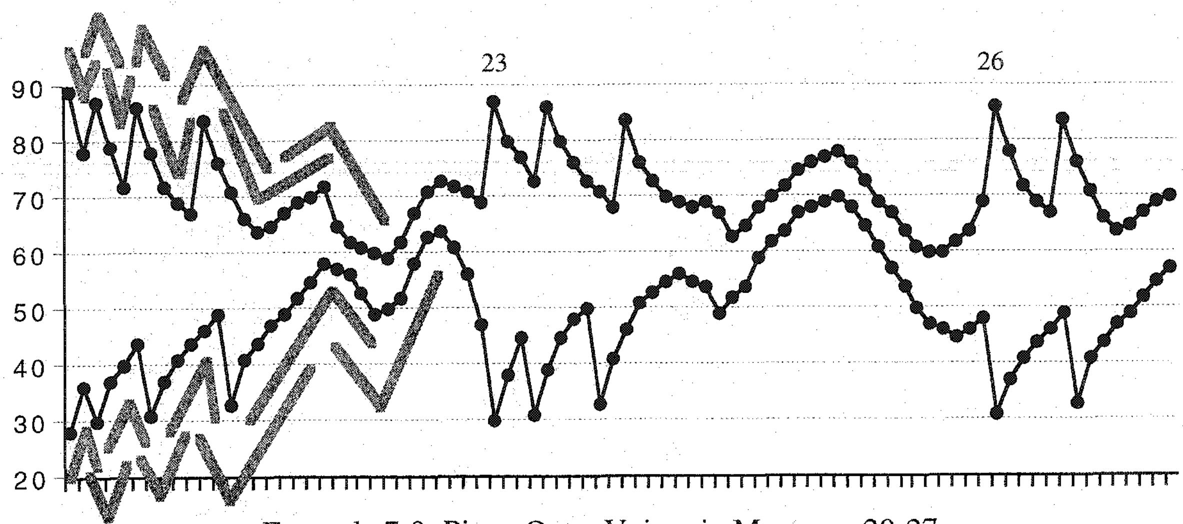
7.4 Applications of the Basic Shape in Movement 1

In this movement, as in the entire Concerto, the basic shape occurs in all parameters and structural levels of the music. The following section discusses the use of the basic shape in the construction of the musical material of the movement.

7.4.1 Horizontal Pitch

Piano

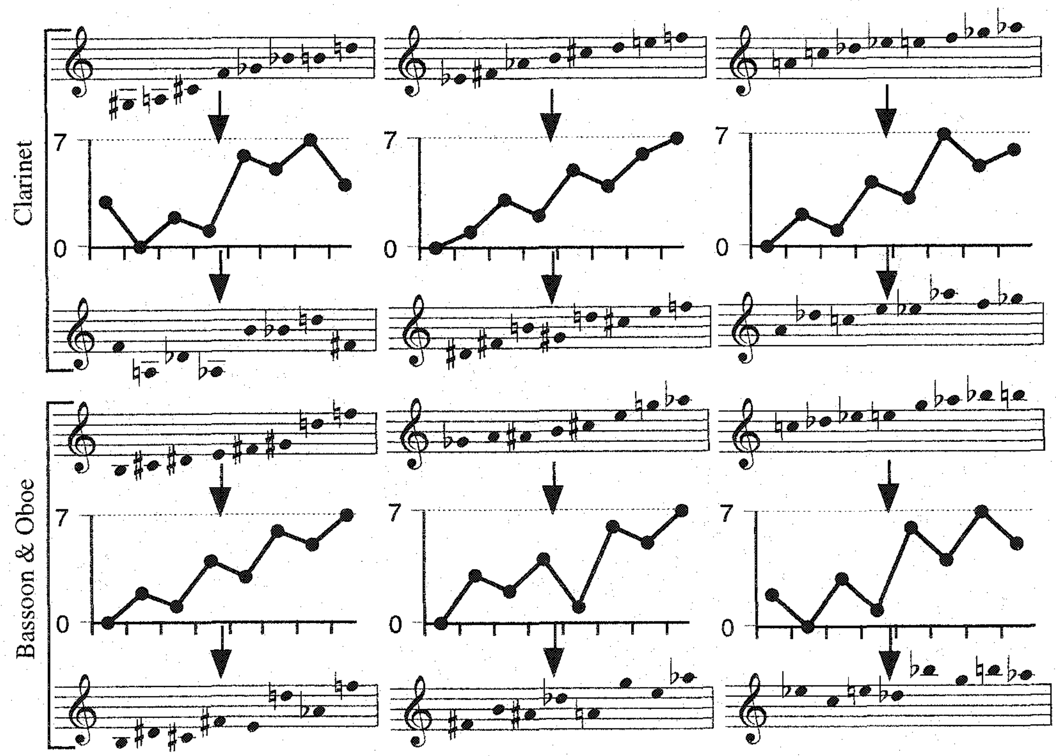
Since each of the outer voices in the homophonic texture of the piano functions as a single voice in a two-part contrapuntal framework, I discuss them in this section rather than in section 7.4.2 on vertical pitch. The following graph of the outer voices of the piano from measure 20 to the end of the second beat of measure 27 clearly shows the large number of occurrences of the basic shape. The first 20 of these outer voices were previously discussed in section 3.2.1 of chapter 3. Measure numbers are indicated above the graph in example 7-9, and instances of the basic shape are shown for the first part of the example.



Example 7-9: Piano Outer Voices in Measures 20-27

Woodwinds

I generate the woodwind melodic figures in this movement by applying positional contours to chords with high referential pitch commonality. The positional contours contain multiple versions of the basic shape that are linked together to create compound contours. In the example below, I illustrate the process of composing the woodwind material from measures 65 to 80.



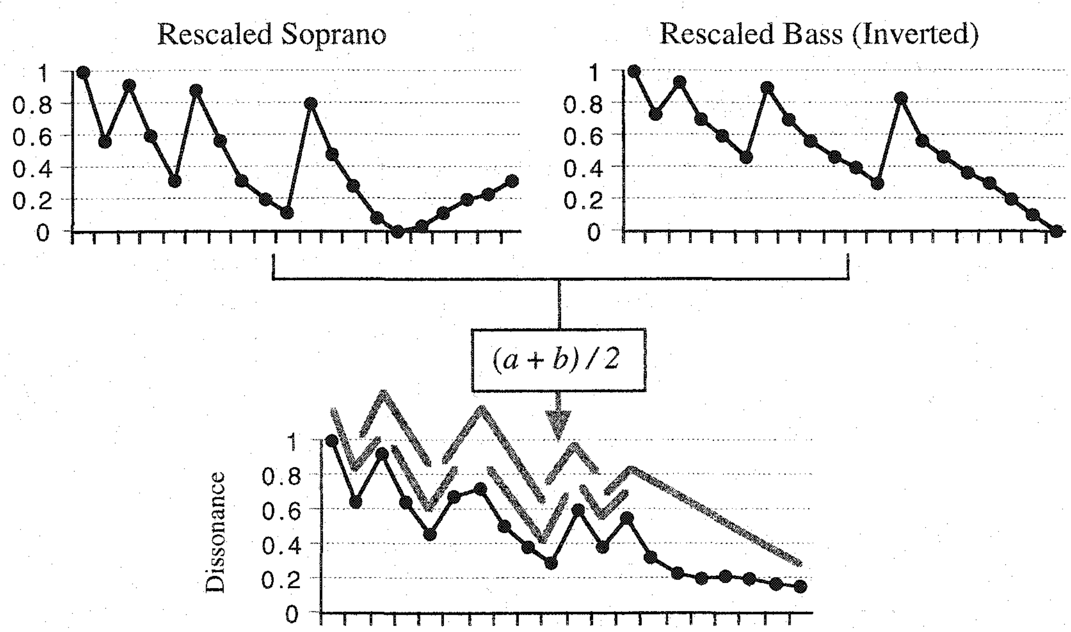
Example 7-10: Woodwind Positional Contours, MM. 65-80

This passage contains two contrapuntal lines, one in the clarinet and the other beginning in the bassoon and ending in the oboe. The example shows the eight-note chords that serve as the harmonic basis of the melodies, the positional contours I apply to these chords, and the resulting melodic lines. These eight-note cells follow each other successively in the music, although they are occasionally separated by rests. The rhythms of this passage are discussed in section 7.4.3 below.

7.4.2 Vertical Pitch

Piano

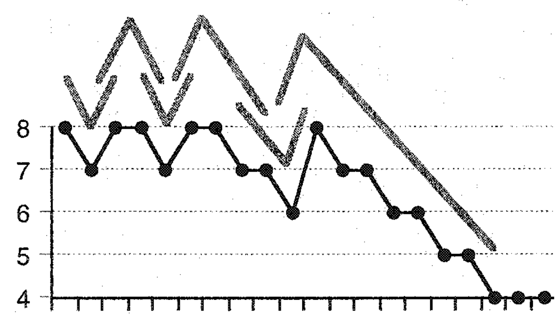
The harmonic parameters of the piano in this movement reinforce the dynamic contour of the gestures in the outer voices. This is particularly true for sensory dissonance. In order to maintain a relationship between the gestures in the outer voices and the dissonance of the harmonization, I derive the dissonance curves from the outer voice contours. The first phrases of the piano in this movement illustrate this process (see example 7-1). In this passage, I first scale the soprano as a break-point function with a minimum y value of 0 and a maximum of 1. I also scale the bass voice with a minimum y value of 1 and a maximum of 0, resulting in an inversion. Then I find the average of the corresponding y values in each of these break-point functions and use the result to determine the dissonance of the harmonized progression:



Example 7-11: Deriving Dissonance from Outer Voices of Piano, MM. 20-21

The dissonance curve follows the contours of both voices, ensuring that the beginning of a gesture in either will be marked by a relative increase in dissonance. One result of this relationship with the outer voices is that the dissonance curve also contains segments derived from the basic shape, as indicated in the above example.

Other harmonic parameters in this passage also relate to the basic shape. For instance, the following example gives the cardinality control function from the same passage:

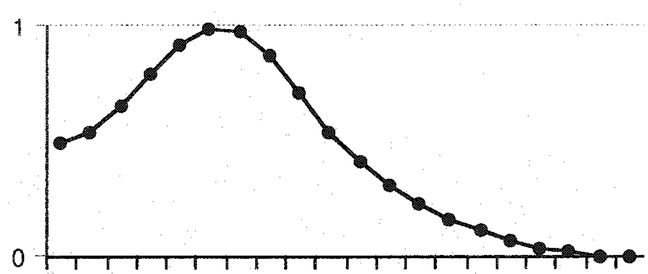


Example 7-12: Piano Cardinality, MM. 20-21

As indicated above the graph, this contour contains subsegments that derive from the basic shape.

Strings

The center pitch curve of the harmonic progressions in the strings in this movement are based on a version of the basic shape whose points are joined by a cubic spline, rather than straight lines:



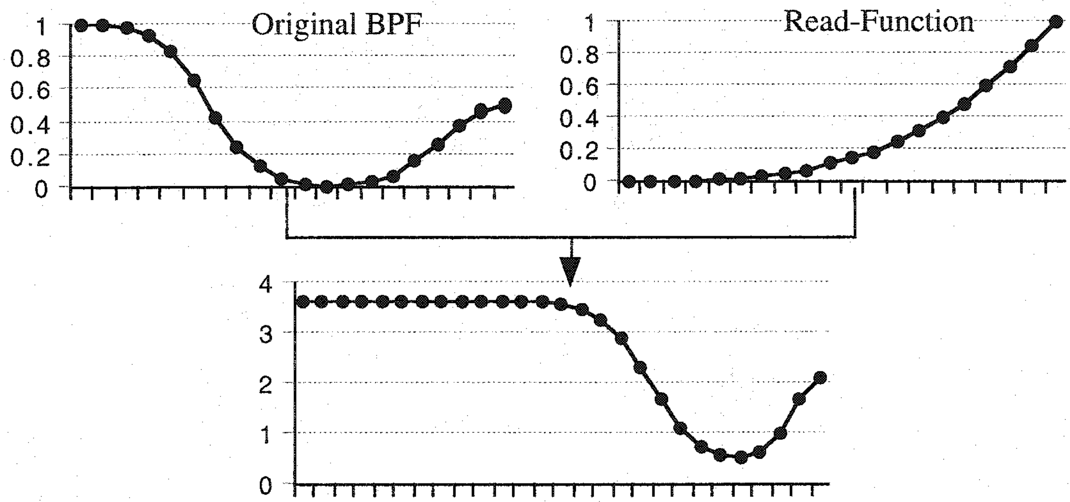
Example 7-13: Center Pitch Curve for String Progressions

This curve varies from one progression to another throughout the movement by having different maximum and minimum *y* values defined. In the string material of measures 34 to 54, for example, the *y* minimum is 55 and the *y* maximum is 65.

7.4.3 Duration

Strings

The rhythms of the string material in this movement derive from another cubic spline. I vary this curve in a number of ways. In the string passage from measures 62 to 80 (as shown in example 7-3), for instance, I alter the original rhythmic break-point function by using an exponential read-function:

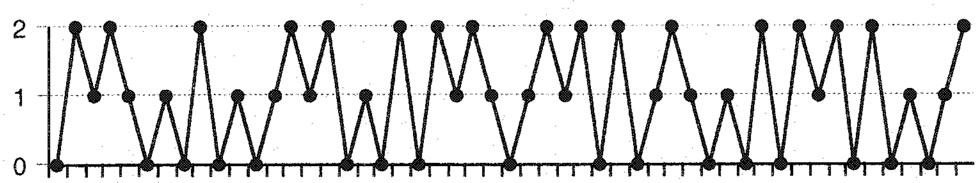


Example 7-14: String Durations, MM. 62-80

The resulting rhythmic contour progresses slowly through the beginning of the original break-point function and more quickly through the end. To determine the actual rhythmic values of the passage, I set new minimum and maximum *y* values and then sample the new rhythmic contour.

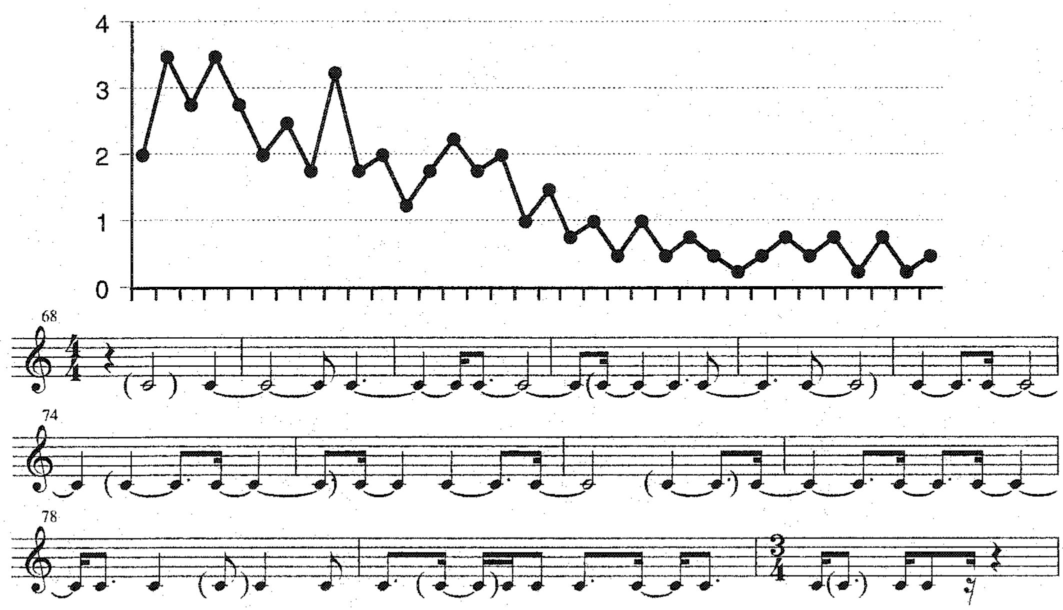
Woodwinds

The rhythms of the woodwind melodic figures are based on the following rhythmic profile:



Example 7-15: Woodwind Rhythmic Profile

This profile is transformed by varying both the pattern subdivision and the amount to add.[41] In the woodwind passage beginning in measure 68 (see example 7-10), the pattern's basic rhythmic subdivision gradually decreases from three sixteenth notes to 1 and the amount added decreases from 6 sixteenth notes to 1. The resulting rhythmic contour is given below. Not all values from the profile are used in this particular passage: its end is truncated. Also, some of these rhythmic values become rests in the music. I indicate these with parentheses in example 7-16. The result is the rhythm of the bassoon and oboe material in measures 68 to 80. The rhythm of the clarinet was composed in a similar manner, but the basic rhythmic value is a triplet eighth note. The use of different rhythmic subdivisions for the two contrapuntal parts increases the independence of the parts.



Example 7-16: Transformation of Woodwind Rhythmic Profile: MM. 68-80

[41] See section 4.3.2 of chapter 4 for a discussion of transformation techniques for rhythmic profiles.

Chapter 8: Movements 2 & 3 (MM. 169-295)

8.1 Description

The second movement (mm. 169-208) is the shortest of the Concerto. It is scored for the orchestra alone and its main function is to introduce the antipodal sonority. This occurs in measure 199, with a triple *forte* attack in the entire orchestra. The arrival of the antipodal sonority is the main formal division of the movement and is the culmination of an extended harmonic progression - mainly in the strings - that begins with the E_7 in measure 169.

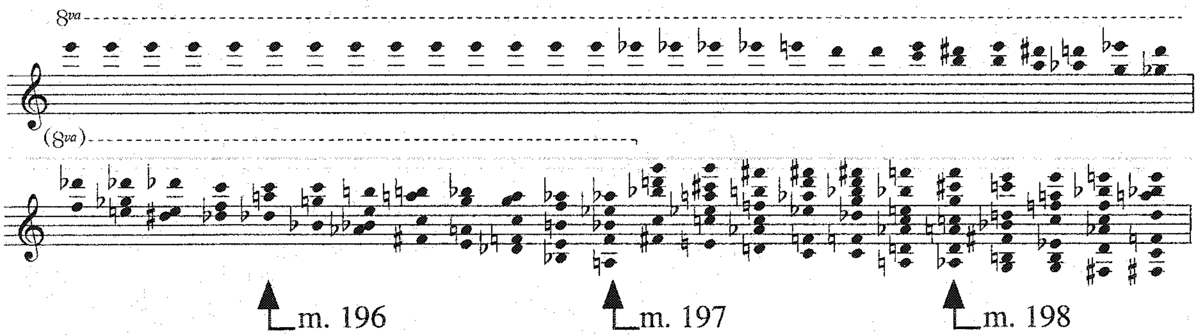
In contrast with the second movement, the third (mm. 209-295) focuses on the piano soloist. Its musical material recalls the piano solo that opens the Concerto. The third movement has six main sections, each of which emphasizes a different referential sonority. The background harmonic progression consists of a gradual return to the referential sonority of the Concerto in measure 274. This point also marks the return of the full orchestra, which performs the material of the piano.

8.2 Musical Material of Movement 2

The second movement contains two principal types of material: a long harmonic progression in the strings and an intricate triple canon. Both of these gradually increase in intensity until the arrival of the antipodal sonority in measure 199. This harmony is prolonged with a gradually decelerating rhythm until the end of the movement in measure 208. I discuss this rhythm below in the section dealing with applications of the basic shape.

8.2.1 Harmonic Progression

The harmonic progression takes place predominantly in the strings from the beginning of the movement. It overlaps with the end of the previous movement, but the string harmonics that enter with the E_7 in measure 163 are so soft that they are almost completely masked by the stronger repeated notes. They are thus not heard until the second movement proper begins in measure 169. The progression leads from this single note to the tutti in measure 199 through a series of chords with gradually increasing cardinality and outer interval:



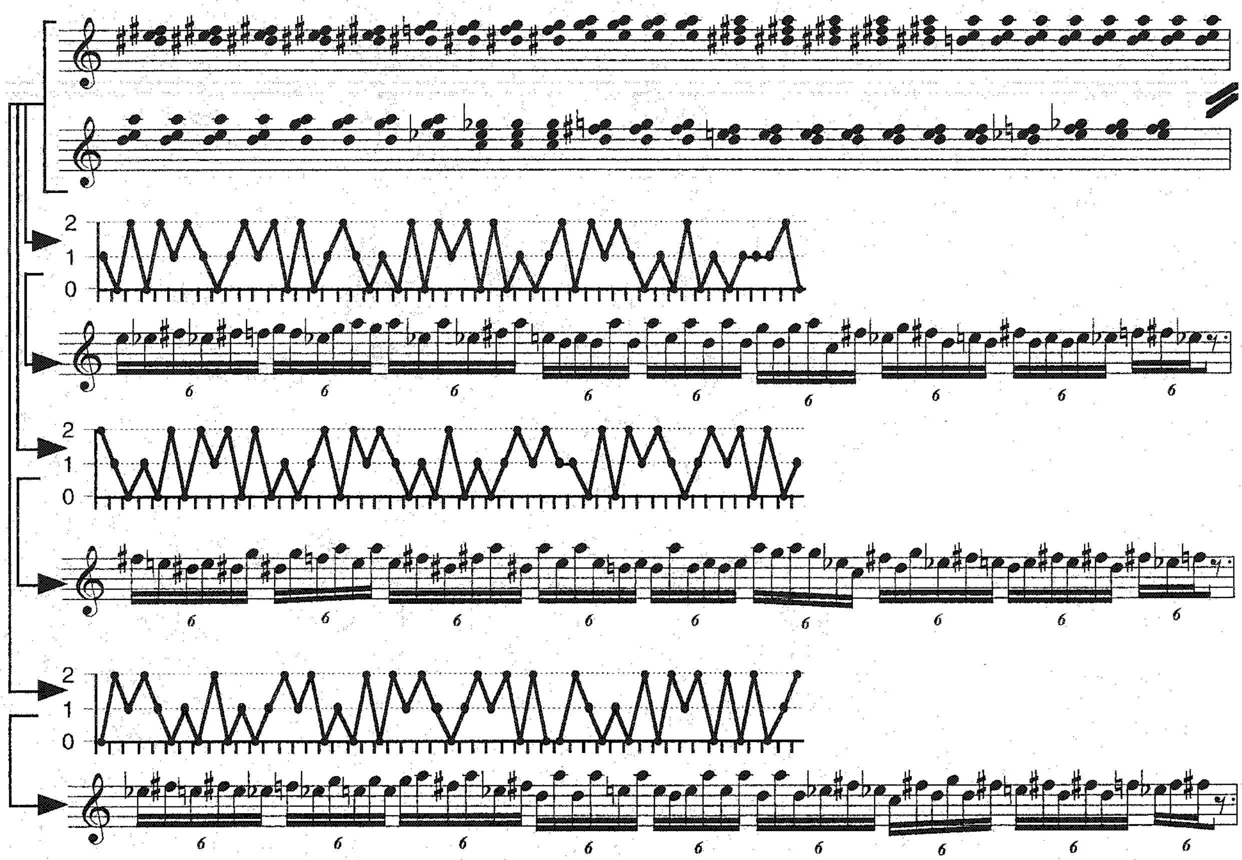
Example 8-1: Harmonic Progression MM. 169-198

These chords have high pitch commonality with the referential sonority of the Concerto, high successive pitch commonality, and relatively low dissonance. I discuss the rhythm of this progression later.

8.2.2 Melodic Canons

In measure 176, the piccolo introduces a melodic idea that spawns three three-voice canons in the woodwinds, brass, and first chairs of the violin I, violin II, viola, and cello sections. These canons are differentiated by their register, speed, and instrumentation. The high canon contains sextuplet sixteenth notes and is played by the flute, first violin and second violin. The middle canon is in sixteenths and is played by the oboe, trumpet and viola. Finally, the low canon has a rhythm of triplet eighths and takes place in the bassoon, horn and cello. The entrance of the leading voice of each canon occurs in measures 181, 187, and 194 respectively. The rhythmic interval of imitation is 51 attacks in the high canon, 34 in the middle, and 17 in the low.

Progressions of trichords form the basis of all of these canons. Three different paths are traced through the progressions such that the combination of all three reconstructs the notes of the original progression. In example 8-2, I demonstrate this process using the high canon. The progression appears in the top two staves. To avoid ledger lines, in this example all pitches have been written an octave lower than they sound. The trajectories taken by each of the voices are given as positional contours with three possible values: 0, meaning the bottom note of a chord; 1, meaning the middle note; and 2, meaning the top note. Rather than being applied to a single chord, however, the values in these contours apply to each successive chord in the progression. Underneath each contour, I give the melodic line that derives from its application to the progression. In the first melodic line, for instance, the first value of the contour is 1, which indicates that the middle note of the chord will be chosen. The top note of this chord is the first note of the second melodic line and the bottom note is the first of the third line. In the actual music, some notes are replaced by rests. This is a practical consideration to allow breaths to be taken in the winds and brass.



Example 8-2: High Canon (Sounds 8ve higher)

In the high canon, the leading voice enters in the piccolo with the first of these pitch collections in measure 181. It continues without pause with the second pitch pattern beginning in the second half of the third beat of measure 183. At this point the first following voice enters with the canon subject in the violin I. The derivation of the melodic lines guarantees that these two voices will have a clear harmonic character and will not contain unisons. The leading voice continues with the third pitch pattern from the example on the fourth beat of measure 185. Here the first following voice commences the second pattern while the second following voice enters with the canon subject in the violin II. We now hear all the notes of the harmonic progression, but the actual melodic figures leap constantly between the soprano, alto, and bass voices of the chords.

The other canons in this section use the same technique as above with different underlying harmonic progressions and different rhythmic intervals of imitation. The middle canon, which first enters in measure 187, is based on a progression of 34 chords:

Example 8-3: Middle Canon Harmonic Progression

This example is notated at sounding pitch. Although the harmonies are different from those of the high canon, the positional contours are the same. Only the first 34 values of each contour are used. The entrances of the canon occur at the following places: in the oboe in the second half of the fourth beat of measure 187; in the trumpet at the beginning of measure 190; and in the first chair viola in the second half of the first beat of measure 192. The canon subject moves in steady sixteenth notes, and the rhythmic interval of imitation is 34 sixteenths, or 8 and a half quarter notes.

The low canon uses the following harmonic progression and the first 17 values of the positional contours:

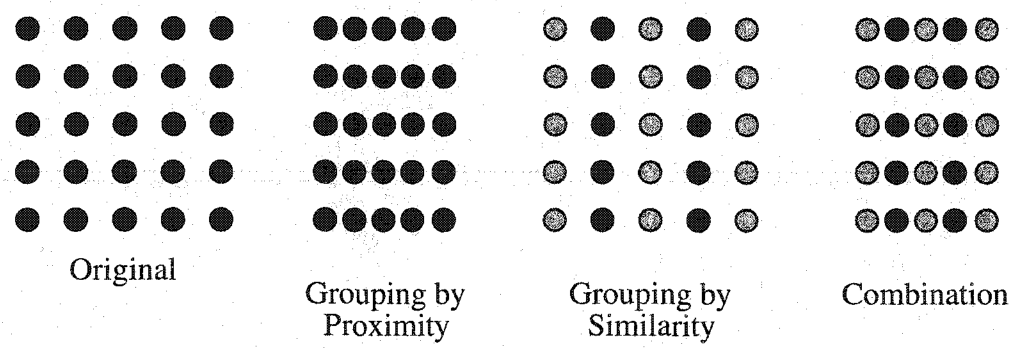
Example 8-4: Low Canon Harmonic Progression

The entrances of the canonic voices occur in the bassoon in measure 194, the horn at the end of the second beat of measure 195, and the first chair of the cello section in the last beat of measure 196. This canon moves at the speed of a triplet eighth note.

Auditory Streams

Part of the interest of this material lies in the tension that exists between two possible ways in which it may be perceived. Once all three canonic voices have entered, we are unsure whether we should follow the voices of the harmony - soprano, alto, and bass - or the instrumental lines. In other words, there is a rivalry between two conflicting streams. An auditory stream is a perceptual mechanism that "serves the purpose of clustering related qualities."[42] The two ways of interpreting this passage relate to two principles of visual perception discovered by the Gestalt psychologists: grouping by proximity and by similarity.[43] These are illustrated in the following example:

[42] A. S. Bregman, *Auditory Scene Analysis: The Perceptual Organization of Sound* (Cambridge: MIT Press, 1990), 10.
[43] Ibid., 12.



Example 8-5: Visual Grouping Strategies

In the first grid, we can see the square as consisting of either rows or columns of dots. In the second grid, however, we tend to see 5 rows of dots. This is because we normally group together visual elements that are close to each other. In music, this is similar to assigning notes that are close in pitch to a single stream. For example, the soprano voice of a progression is identified as a stream partly because its notes all occupy the same register. In the third grid, we tend to see columns because we group together visual elements that are similar. This is analogous to distinguishing auditory streams by their timbre. For instance, in a two-part texture we would normally hear a flute part as belonging to a different stream than a violin part, since its notes all have a similar tone colour. In the canons of this section, however, the smooth voice leading of the underlying harmonic progression competes with the overlapping instrumental timbres for the listener's attention. This is similar to the situation that exists in the fourth grid of example 8-5. Here our attention is drawn simultaneously to the rows (proximity) and the columns (similarity).

8.3 Form of Movement 2

As mentioned in 8.1 above, this short movement divides into two main sections. The first, containing the descending harmonic progression and the three canons, lasts from measure 169 to measure 198. The second section prolongs a single chord from measures 199 to the end of the movement in measure 208.

8.3.1 Background Harmonic Progression

The harmonic progression that underlies this movement has only two elements, the referential sonority and the antipodal sonority: The referential sonority is emphasized in the first part of the movement in two ways. First, the chords of the harmonic progression in the strings have high pitch commonality to it. Second, the notes of the three canons use the pitch-classes of the referential sonority exclusively, effectively saturating the harmony with this chord. The particularly strong presence of the

Measure
Numbers: 169 199

Example 8-6: Referential Sonorities of Movement 2

referential sonority helps to emphasize the background change of harmony in measure 199. This is the first point in the piece that the referential sonority or a chord with high referential pitch commonality to this sonority does not serve as the basis of the harmony. The movement concludes with a sustained prolongation of the antipodal sonority. The derivation of this chord was discussed in section 5.3 of chapter 5.

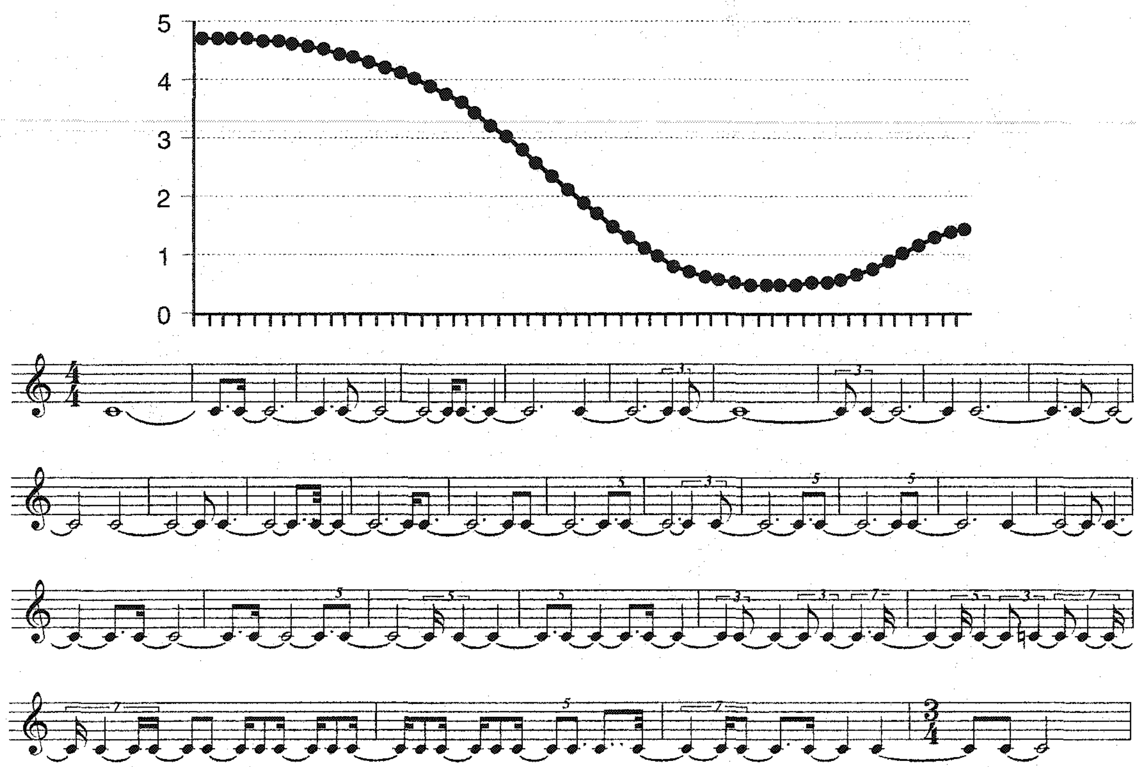
8.4 Applications of the Basic Shape in Movement 2

8.4.1 Horizontal Pitch

The most obvious melodic application of the basic shape occurs in the positional contours that generate the three canons of this movement. These contours are shown in example 8-2 above. They all contain a large number of subsegments that derive from the basic shape. Since the same contour is used for all three canons - even though it is applied to different harmonic progressions - there is a high degree of melodic similarity among the 9 parts of the complex contrapuntal texture.

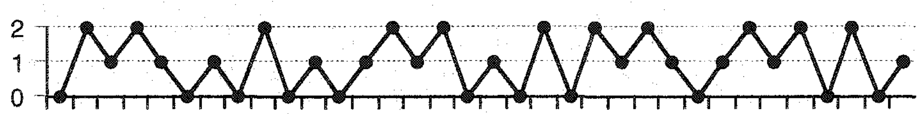
8.4.2 Duration

The basic shape occurs in the durations of this movement in two places: the large-scale rhythmic contour of the expanding harmonic progression in the first part of the movement, and the rhythmic profile of the tutti chord in the second part of the movement. The former is a cubic spline version of the basic shape, as shown in example 8-7. Here the rhythmic break-point function is shown above the rhythm that results from sampling this function. This rhythm begins with the string harmonics of measure 169.



Example 8-7: Rhythm of Harmonic Progression in measures 169-198

The decelerating rhythmic motto that starts with the arrival of the antipodal sonority in measure 199 derives from the following rhythmic profile:



Example 8-8: Rhythmic Profile MM. 199-208

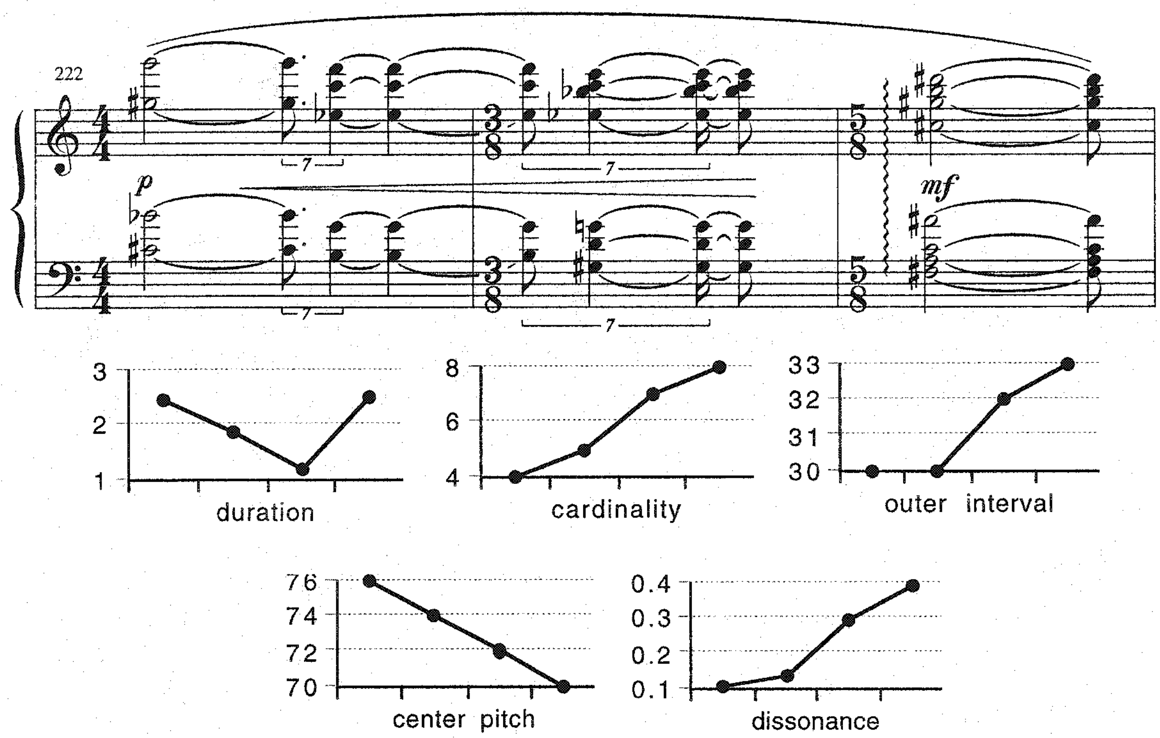
This profile contains multiple versions of the basic shape that join to create a complex compound contour. The deceleration that occurs in this passage is a result of gradually increasing the basic subdivision from one sextuplet sixteenth to three, and the amount to add from one sextuplet sixteenth to eight. The resulting rhythm occurs mainly in the woodwinds, brass and percussion while the strings sustain the antipodal sonority.

8.5 Musical Material of Movement 3

The music of the third movement consists essentially of the homophonic material that appears first in the piano. At measure 174 this material is also played by the orchestra as a counterpoint to the piano. The hands of the piano also divide into two separate homophonic lines at this point.

8.5.1 Piano

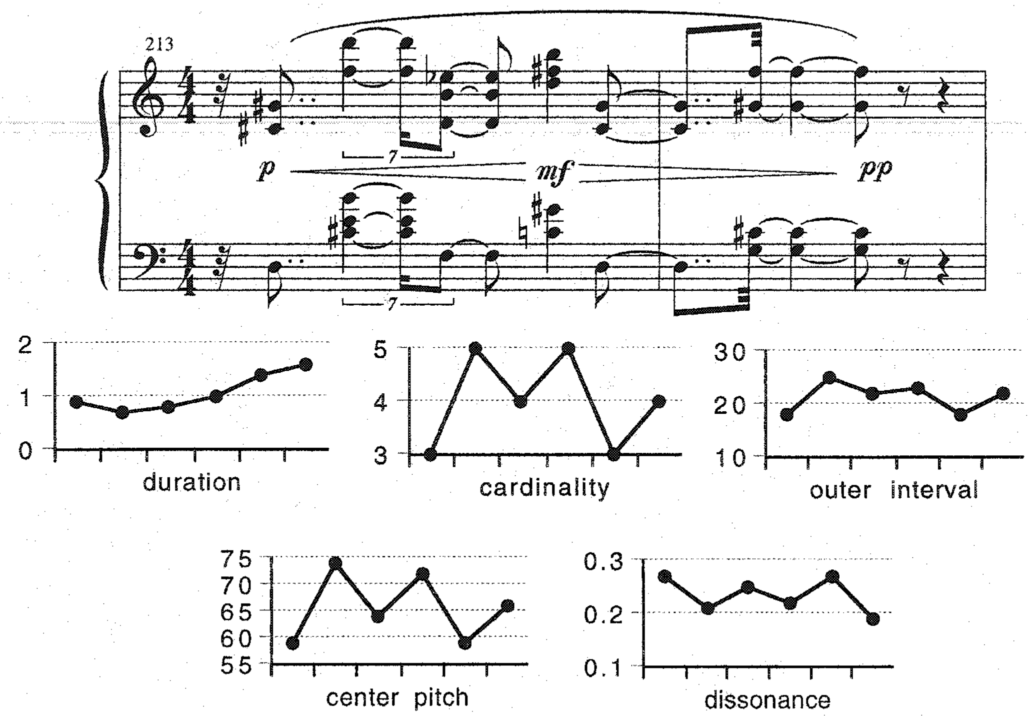
The melodic, harmonic and rhythmic parameters of the piano's harmonic progressions contrast smooth and disjunct contours. These two types are constantly transformed and merged throughout the movement. The following piano phrase from measures 222-224 is an example of the use of smooth contours:



Example 8-9: Smooth Piano Progression

The contours of a number of parameters are given below the staff. Since they contain no changes of direction, I consider them to be smooth. The final value of the rhythmic contour is misleading, since the final chord of the phrase is simply being sustained along with the orchestral chord that enters at the same point. All harmonies throughout this movement have high referential pitch commonality, and the chords of this particular progression are chosen to have high successive pitch commonality.

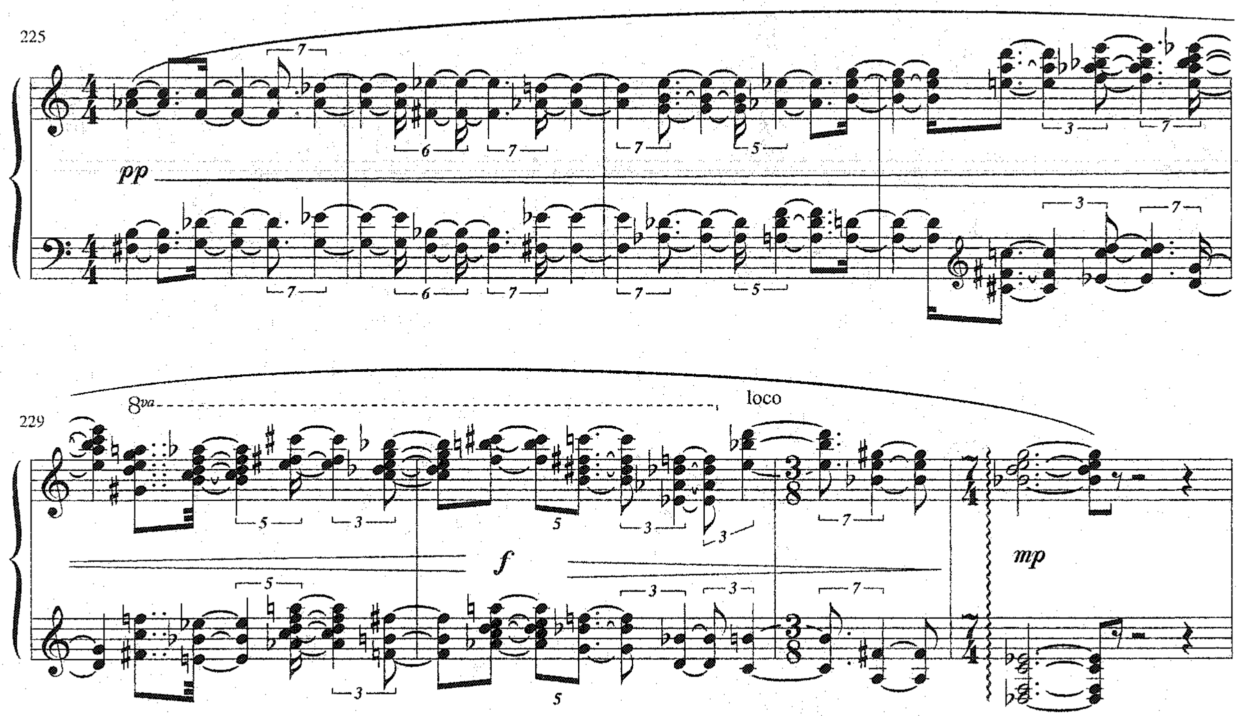
Example 8-10 is taken from measures 213-214. In contrast to the previous example, the contours of this passage contain many changes of direction. As a result, the progression is more disjunct than the previous example.



Example 8-10: Disjunct Piano Progression

Although the chords of this progression have high referential pitch commonality, they have low successive pitch commonality, which contributes to the "jaggedness" of the passage.

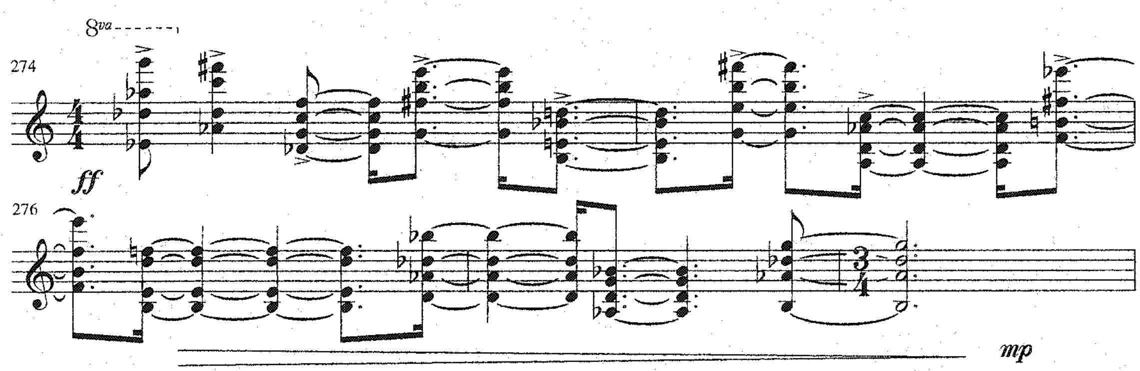
The other phrases throughout this movement take place along a continuum between these two extremes. To accomplish this, I use the technique of merging break-point functions, as discussed in section 2.3.3 of chapter 2. In this movement, merging is done using either a fixed value that determines the degree of smoothness or disjunction, or a break-point function that creates a time-varying merge. In the piano phrase given in example 8-11, which begins in measure 225, the contours that generate the center pitch and outer interval are initially weighted towards smooth contours. During the course of the progression, however, they merge gradually with more disjunct contours, and then return to smooth contours. The cardinality and dissonance increase and decrease as well, while successive pitch commonality begins high, becomes gradually lower and then higher again at the end of the phrase. The interaction of the parameters and their evolution throughout the progression make this passage a good example for the control of harmonic stability. The beginning of the phrase has relatively high harmonic stability. The stability of the progression then decreases up to the second beat of measure 230, and increases again to the end of the progression.



Example 8-11: Merging Smooth and Disjunct Contours

8.5.2 Orchestra

Between the beginning of the movement and measure 274, the orchestra is restricted to sustaining chords that double those arpeggiated by the piano. This happens in measures 224, 232, 253, and 259. At measure 274, however, the orchestra enters with a harmonic progression based on the piano's material, using disjunct contours:



Example 8-12: Orchestra Material, MM. 274-278

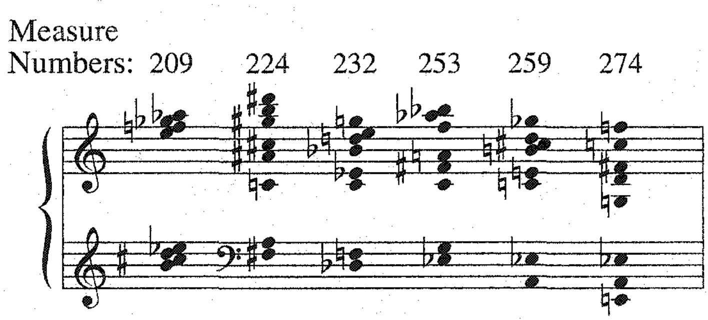
There are three successive phrases of this material that coincide with the phrases of the piano at measures 274, 279, and 284. This material will be discussed in more detail in the section on applications of the basic shape in this movement.

8.6 Form of Movement 3

The third movement has two main formal parts. The first, from measures 209 to 273, features the solo piano with only a few sustained chords in the orchestra. These chords help to articulate the five subsections of this part of the movement. The second part of the movement, starting in measure 274, is distinguished from the first in a number of ways. It marks a change from a homophonic to a polyphonic texture, the orchestra enters, and the goal of the background harmony is reached with a clear statement of the referential sonority of the Concerto.

8.6.1 Background Harmonic Progression

The background harmonic progression moves gradually from the antipodal sonority that arrives at the end of the second movement to the referential sonority in measure 274. This move takes place using five secondary referential sonorities that have gradually increasing pitch commonality to the referential sonority. The entire progression is shown in the following example. The creation of these background chords was discussed in section 5.3 of chapter 5.



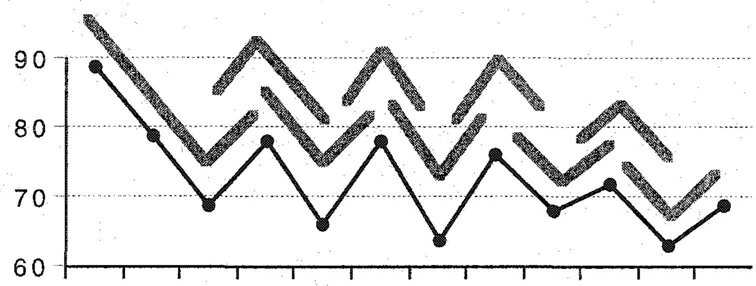
Example 8-13: Background Harmonic Progression of Movement 3

The first of these chords does not appear in the music, but is implied by the high pitch commonality of the piano harmonies. The voicing of this chord as given in the example is the same as that of example 5-2 in chapter 5. It serves as the basis of the harmony beginning in measure 209. The other chords are arpeggiated by the piano at the indicated measures and are also sustained by the orchestra. The harmony of the piano part always has high pitch commonality with the sonority that underlies each subsection.

8.7 Applications of the Basic Shape in Movement 3

8.7.1 Vertical Pitch

Since all the material of this movement is entirely homophonic, applications of the basic shape in the pitch domain control vertical aspects. As example 8-10 above shows, the disjunct contours of the piano part all derive from a combination of multiple versions of the basic shape. This applies both to the piano and the orchestra. For example, the center pitch curve of the string progression previously shown in example 8-12 contains numerous instances of the basic shape, as shown below in example 8-14:

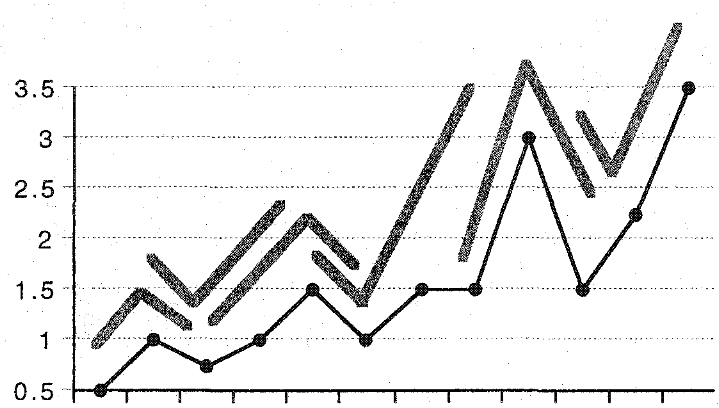


Example 8-14: Orchestral Center Pitch, MM. 274-278

The piano progression that we examined previously in example 8-10 also illustrates the presence of the basic shape in its contours for dissonance, outer interval and cardinality.

8.7.2 Duration

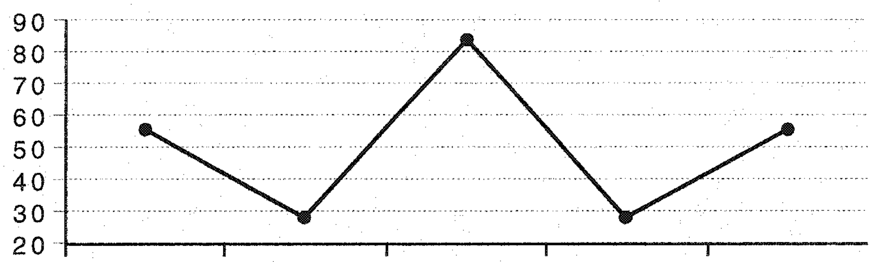
In the following graph of the rhythmic contour of the orchestra passage in measures 274 to 278, there also exist many versions of the basic shape:



Example 8-15: Orchestral Rhythmic Contour, MM. 274-278

The piano rhythms in this same passage are also derived from the basic shape, but are significantly more complex. In the orchestra the rhythmic contour is defined in terms of multiples of a single subdivision: a sixteenth note. In the piano, however, the contours are sampled as break-point functions and the values are quantized (rounded off to the nearest rhythmic subdivision). This difference reinforces the separation of the contrapuntal lines.

The basic shape also occurs in the background formal durations of the first part of the movement. The durations of the five subsections in beats are: 56, 28, 84, 28, and 56. These values are represented graphically in the following large-scale rhythmic contour:



Example 8-16: Durations of Formal Subsections, MM. 209-273

The durations of the subsections contain two linked versions of the inverted basic shape. The first of these is in retrograde, resulting in a symmetrical overall design.

Chapter 9: Movement 4 & Closing Section (295-426)

9.1 Description

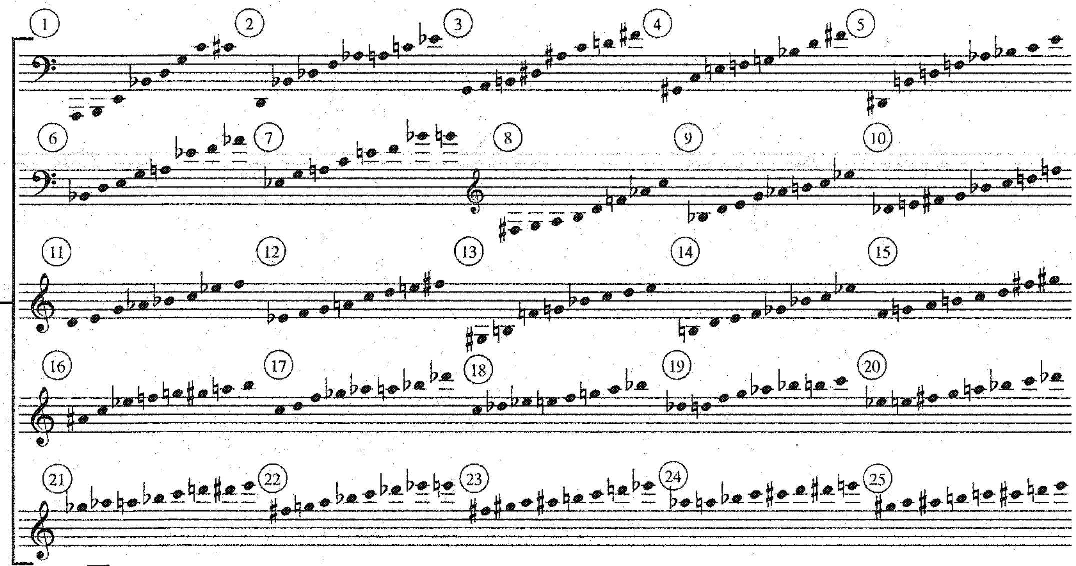
The fourth movement (mm. 295-416) begins almost imperceptibly with the subtle change from two parts to one in the piano on the third beat of measure 295 and the staggered entrances of the high sustained chord in the second violins. The fundamental formal process of the movement is the gradual divergence of the referential sonorities of piano and the orchestra. From the beginning of the movement until measure 374 the orchestra becomes gradually louder and more dense until it almost totally masks the piano. Starting at measure 374, the orchestra performs a gradual diminuendo in a decelerating rhythm until it dies out completely in measure 396. The piano finishes the movement alone. The material of the piano develops the music from measures 5 to 10 of the opening section.

Measures 416 to the end of the piece constitute a brief closing section. The most important aspect of this section is the return of the referential sonority of the Concerto in the orchestra. Almost all of the section is taken up by a prolongation of this chord. The piano figure in measure 416 recalls the material from measure 10 of the opening section.

9.2 Musical Material of Movement 4

9.2.1 Piano

All of the piano material in this movement derives from measures 5 to 10 of the beginning of the Concerto. I use the same technique of applying positional contours to chord progressions. Example 9-1 shows the derivation of the piano part in measures 320 to 327. The top part of this example is the harmonic progression on which the passage is based, with the notes of the chords arranged in ascending order. The progression contains eight-note chords with rising center pitches and decreasing outer intervals. All the chords have high pitch commonality to the referential sonority of the Concerto. Since the notes of the chords are not attacked simultaneously, dissonance and successive pitch commonality play less of a role. The middle part of the example is a list of the positional contours that I apply to these chords, and the bottom part is the result as it is found in the piece. I use this technique throughout the fourth movement.



1) (4 (6 3) 1 (2 5) (0 7))
2) (2 4 5 3 1 6 (0 7))
3) (1 (0 3) 2 5 (4 7) 6)
4) (0 (1 3) 2 5 (4 6) 7)
5) (4 3 (1 6) 5 2 7 0)
6) (2 1 3 (0 6) 5 7 4)
7) (6 (1 4) 3 5 (2 7) 0)
8) (0 (2 1) 4 3 (6 5) 7)
9) (0 (1 3) 2 5 (4 6) 7)
10) ((6 5) 4 7 (1 2) 0 3)
11) ((0 2) (1 3) (7 5) (6 4))
12) (6 (7 4) 5 (2 3) (0 1))
13) (7 5 (6 3) 4 1 2 0)
14) (7 6 (4 5) 2 3 1 0)
15) (0 1 (3 2) 5 4 (6 7))
16) (2 1 (0 3) (4 6) 7 5)
17) ((7 5) 6 4 3 1 (2 0))
18) (2 (1 3) 0 6 (5 7) 4)
19) ((6 1) (3 4) 5 (2 7) 0)
20) (6 7 4 5 (2 3) 0 1)
21) (5 7 (4 6) (1 3) 0 2)
22) (5 (4 6) 7 1 2 0 3)
23) (0 (2 1) 4 3 6 5 7)
24) (5 3 2 4 6 1 0 7)
25) (7 6 4 5 2 3 1 0)

Example 9-1: Application of Positional Contours, MM. 320-327

9.2.2 Orchestra

The orchestra has two types of material in the fourth movement: sustained chords and harmonic progressions. The sustained chords prolong the referential sonorities of the background harmonic progression. These chords are not simply held: each note has a different rhythm that derives from the same basic pattern. I use the rhythmic profile from the woodwind melodic material of the first movement (see example 7-15). The pattern is distributed to the eight notes of the chord in the following way (notes are numbered from 1 for the top to 8 for the bottom):

-Note 1: Original pattern. Basic subdivision: 5 sixteenths
-Note 2: Retrograde of pattern. Basic subdivision: 6 sixteenths
-Note 3: Original pattern. Basic subdivision: 7 sixteenths
-Note 4: Retrograde of pattern. Basic subdivision: 8 sixteenths
-Note 5: Original pattern. Basic subdivision: 9 sixteenths
-Note 6: Retrograde of pattern. Basic subdivision: 10 sixteenths
-Note 7: Original pattern. Basic subdivision: 11 sixteenths
-Note 8: Retrograde of pattern. Basic subdivision: 12 sixteenths

The amount added is always equal to one of the basic subdivision. An example of this material occurs on the fourth beat of measure 307, played by the first violins, violas 1a and 2, and cellos 1a and 2. Despite the simple techniques that generate this material, the result is surprisingly complex.

The second type of material played by the orchestra in this movement is a four-part homophonic progression that contains large leaps. The following example is played by the second violins in measure 329:

Example 9-2: Orchestra Material, MM. 329-334

These progressions relate to the piano solo of the third movement, especially those piano phrases that use disjunct contours, as in example 8-10 of chapter 8. The same techniques generate the two types of material, but the rhythm contours of the orchestral material in this movement are defined as profiles whose basic values are always multiples of a sixteenth note. I do this partly to facilitate the performance of the unison rhythms by more than one player. This profile is the same as that used in the other orchestral material discussed above and in the woodwinds in the first movement. In order to increase the rhythmic variety of the progressions, however, I do not always start progressions at the beginning of the profile as I do in the sustained chords.

9.3 Form of Movement 4

The fourth movement divides into two large sections. The first, from measure 295 to 373 features the interaction of the three types of material discussed above. It divides into six smaller subsections, each of which emphasizes a different referential sonority in the orchestra. The second large section begins in measure 374. From this point, the piano has five phrases of increasing length while the orchestra sustains a single chord with a gradually decelerating rhythm and a large-scale diminuendo to nothing in measure 396. The piano phrases begin in measures 374, 376, 379, 385, and 396. The last phrase is performed solo, with no orchestral accompaniment.

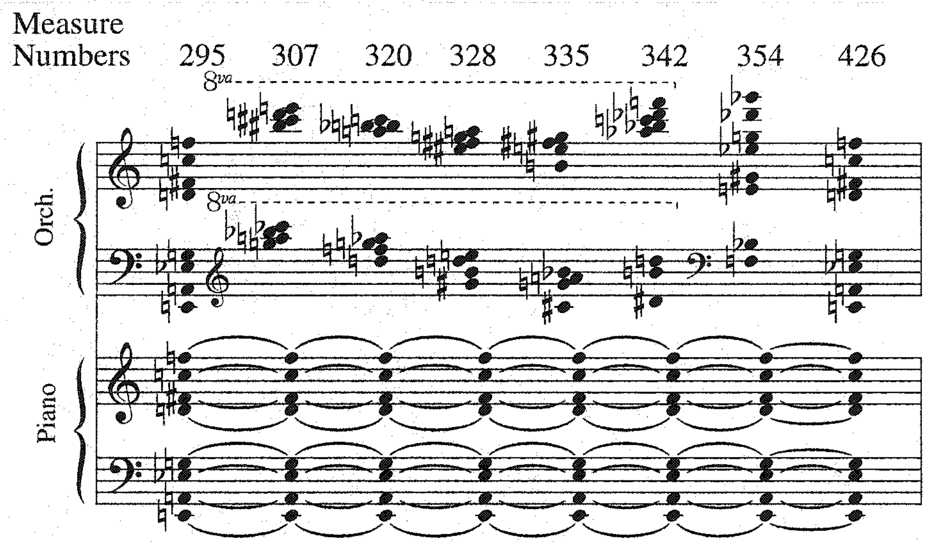
9.3.1 Background Harmonic Progression

The most important aspect of this movement is the harmonic distance that gradually develops between the orchestra and the piano. Throughout the movement the harmony of the piano has high pitch commonality with the main referential sonority of the Concerto. At the beginning of the movement, the orchestra also has high pitch commonality to this chord. During the rest of the movement, however, the background harmonies of the orchestra become gradually less and less related to the referential sonority, leading up to the emphatic arrival of the antipodal sonority in measure 354.[44] This process might be described metaphorically as the piano remaining obstinately in the same place while the world of the orchestra changes around it.

At measure 354 the piano and orchestra are as far apart in terms of pitch commonality as is possible in the harmonic system shown in example 5-2 of chapter 5. As I mentioned in section 3.4.9 of chapter 3, the combination of harmonies that have low pitch commonality increases harmonic tension because the chords imply different referential sonorities and because they create relatively high dissonance. Measure 354 is therefore the point of greatest harmonic tension of the Concerto and represents the formal

[44] For the details of this progression, see section 5.3 of chapter 5.

climax of the work. The resolution of this tension occurs with the return to the referential sonority of the Concerto in the closing section, beginning in measure 426. The entire background progression of the fourth movement and closing section is given below:

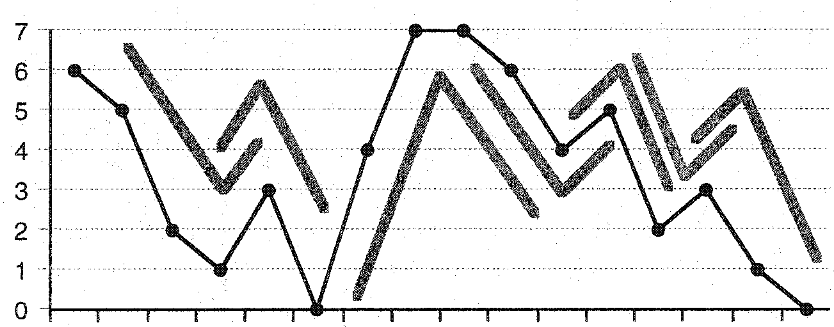


Example 9-3: Background Harmonic Progression
Movement 4 and Closing Section, MM. 295-426

9.4 Applications of the Basic Shape in Movement 4

9.4.1 Horizontal Pitch

As in measures 5 to 10 of the opening section, the positional contours of the piano material in this movement contain multiple versions of the basic shape. The last two positional contours that I previously illustrated in example 9-1 are shown graphically in example 9-4. I indicate instances of the basic shape in grey.

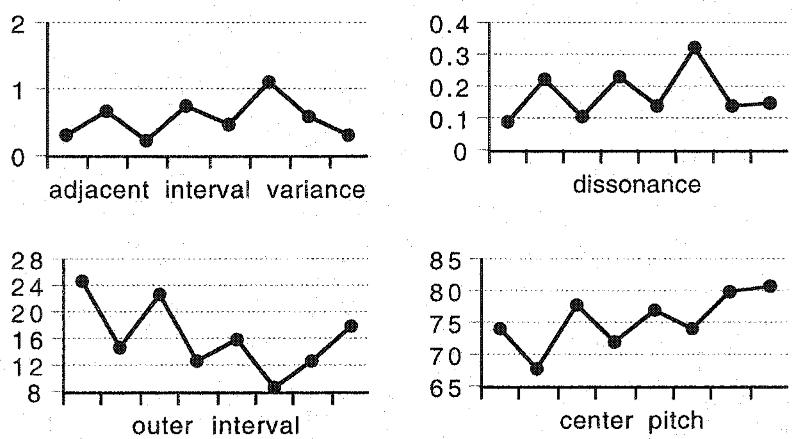


Example 9-4: Basic Shape in Piano Positional Contours

Simultaneous notes in the fourth movement result from grouping elements of a contour together using the same technique as measures 5-10. For instance, in the first positional contour of my previous example of the application of positional contours in the piano (example 9-1), the contour (7 3 5 1 0 4 2 6) becomes (7 (3 5) 1 (0 4) (2 6). Notes that correspond to the positions in brackets are played at the same time.

9.4.2 Vertical Pitch

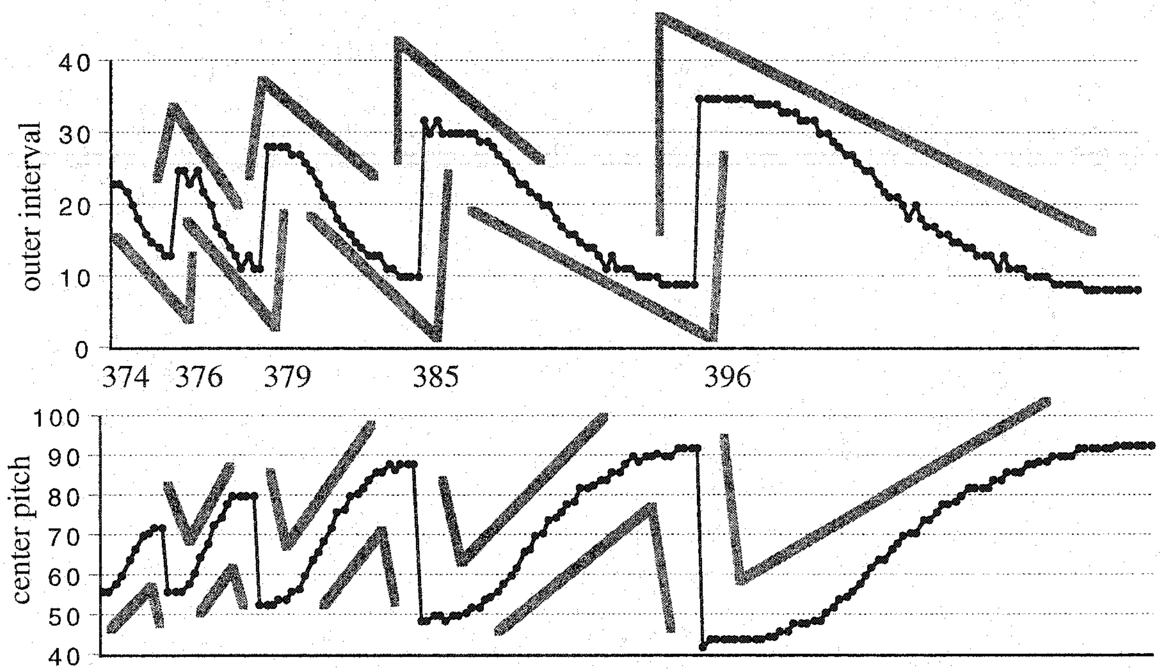
The contours that control the harmonic parameters of the music of this movement contain numerous versions of the basic shape. On the surface of the music, this occurs in the orchestral harmonic progressions for instance. The passage that I previously illustrated in example 9-2 uses the following contours for its harmonization:



Example 9-5: Harmonization Contours, MM. 329-334

All of these contours contain multiple versions of the basic shape, linked to create a compound contour.

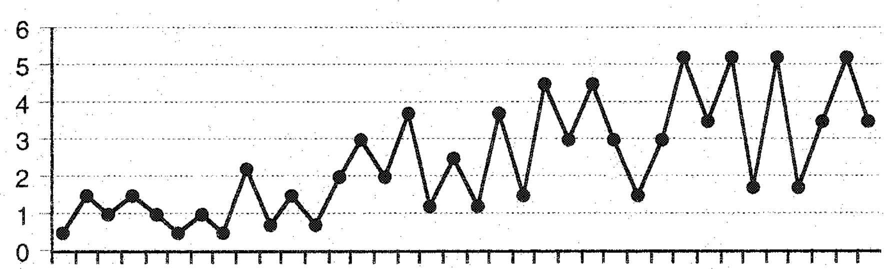
The basic shape also appears in the middle ground during the five piano phrases that end the movement. The total register covered by each of these phrases increases gradually, as does the difference between the outer interval at the start and end of each phrase. The center pitches and outer intervals of these chords are given in the following example. I indicate occurrences of the basic shape in grey and the measure numbers appear between the two graphs.



Example 9-6: Middleground Piano Contours, MM. 374-415

9.4.3 Duration

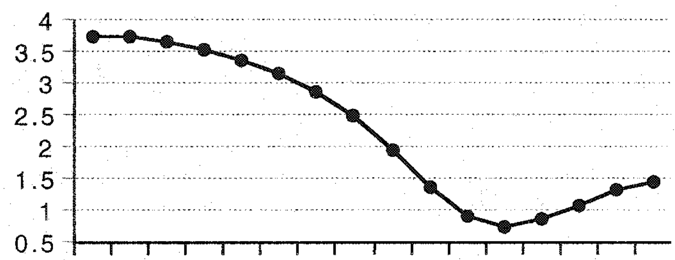
Both the prolonged chords and the harmonic progressions in the orchestra during the fourth movement are based on the same rhythmic profile that was used for the melodic woodwind material in the first material. This profile to the basic shape of the Concerto was discussed in section 7.4.3 of chapter 7. The large-scale decelerating rhythm in the orchestra starting in measure 374 also uses this profile. It begins with a basic subdivision of 2 sixteenths and gradually increases to 7 sixteenths:



Example 9-7: Orchestral Rhythmic Contour, MM. 374-396

9.5 Closing Section

The closing section resolves the harmonic tension that results from the contrast between the antipodal sonority in the orchestra and the referential sonority in the piano. After a brief flourish in the piano that recalls the material of measure 10, the orchestra takes up the referential sonority in the same voicing as its initial entrance in measure 11. Rather than being followed by a harmonic progression, as in the opening section, this chord is simply prolonged to the end of the piece. The notes of the high voicing are gradually transferred down to lower octaves until the characteristic voicing of the referential sonority appears in measure 425. The aggregate rhythm of these attacks derives from the same cubic spline as the rhythm of measures 11-19, sampled 16 times:



Example 9-8: Rhythm of Closing Section, MM. 417-425

The last chord of the orchestra is held with a large crescendo that cuts off suddenly. The piano arpeggiates the referential sonority quietly just before the end of the orchestra's chord, so that we hear the resonance but not the attack. This is the first time in the Concerto that the piano plays the referential sonority in its characteristic voicing. Since the orchestra enters for the first time in the resonance of the piano, we could consider that, in a metaphorical sense, the entire Concerto takes place "inside" the sustain of this chord.

Chapter 10: Conclusion

10.1 Summary

The written part of this thesis describes my compositional techniques from two points of view. Part I of the written text discusses the theoretical background of these techniques and Part II exemplifies them through an analysis of my *Concerto for Piano and Orchestra*. I summarize the main topics of these two parts below.

10.1.1 Part I

The original contribution of this thesis is the integration of psychoacoustic models of hearing into the creative process. These include models for assessing the dissonance of chords and for determining the pitch commonality between two chords. I also incorporate techniques for creating and transforming contours into my treatment of duration and pitch. A software program that I have written, called *Apprentice*, helps me to harmonize outer voices by selecting chords according to several criteria: cardinality, adjacent interval variance, dissonance, successive pitch commonality, and referential pitch commonality. These choices are made from a large bank of related chords that forms the harmonic vocabulary of the Concerto.

10.1.2 Part II

In my analysis of the Concerto, I focus on two main features of the piece. The first is the use of pitch commonality to create harmonic regions. With this technique I am able to imply a referential sonority by choosing harmonies with high pitch commonality to it. I also create background harmonic progressions that move between the main referential sonority of the piece and the antipodal sonority, which is the transposition of the referential sonority that has the least degree of pitch commonality to the original. The second feature that I emphasize is the utilization of a simple contour as the basic generative shape for many elements of the piece. This contour appears in all musical parameters (duration, horizontal and vertical pitch) and at all structural levels (foreground, middleground, background). It is the principal unifying factor of the Concerto.

10.2 Future Directions

Since the *Concerto for Piano and Orchestra* is the first piece that I or anyone else has composed using the psychoacoustic models described above, it is natural for there to be room for further refinements to the compositional techniques I have developed. These improvements result from my experience in composing this work and from the rapid developments currently taking place in the field of psychoacoustics. In the rest of this chapter I discuss possible future directions for the implementation of these models and for their use in composition and other areas.

10.2.1 Contour Theory

In the Concerto I use the combinatorial theory of contour to create variations of the basic shape. According to this theory two pitch structures have the same contour if all of the elements of each have the same relative positions to all their other elements.[45] In my implementation of contours in this piece, however, I have no way of determining the degree of similarity of different contours. For example, I am not able to sort a series of contours by their relative similarity to a basic shape. I am aware of two particularly compelling approaches to this problem. The first, developed by Ian Quinn, involves the application of fuzzy set theory to contour theory.[46] A fuzzy set is "a class with a continuum of grades of membership,"[47] as opposed to sets to which elements may only either belong or not belong. Quinn uses this concept to modify contour theory to allow for the evaluation of partial similarity between contours. The second approach treats a contour as if it were a wave form and reduces it to its constituent harmonics using Fourier analysis.[48] The resulting pitches and amplitudes are then compared to those of other contours to determine the degree of similarity. Future versions of *Apprentice* will likely use one or the other of these approaches, although my work in this area is still ongoing.

10.2.2 Pitch Commonality

The model of pitch commonality used in the Concerto uses a correlation coefficient to compare the perceptual spectra of chords. A correlation coefficient considers that the mutual absence of an element contributes to the similarity of two collections of data. It is not clear that we hear in this way, however. A more appropriate model might be one that considers only the mutual presence of elements. A method developed by Parncutt[49] fulfills this requirement. It calculates the number of pitches a pair of sounds have in common and their salience using the following formulation:

$$C = \frac{\sum_P \sqrt{S_1(P)S_2(P)}}{\sqrt{\sum_P S_1(P) \sum_P S_2(P)}}$$

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[45] See section 2.1 of chapter 2 for a more detailed explanation.
[46] I. Quinn, "Fuzzy Extensions to the Theory of Contour," *Music Theory Spectrum* 29 (1997): 232-263.
[47] L. A. Zadeh, "Fuzzy Sets," *Information and Control* 8 (1965):338-89
[48] M. A. Schmuckler, "Testing Models of Melodic Contour Similarity," *Music Perception* Vol. 16, No. 3 (Spring 1999): 295-326
[49] Richard Parncutt, *Harmony: A Psychoacoustical Approach* (Berlin: Springer-Verlag, 1989), 94.

In this equation *S1* is a list of the saliences of the first sound, *S2* is the saliences of the second, and *P* stands for all pitches within the range of hearing. In more recent compositions, I use this method of determining pitch commonality rather than a correlation coefficient.

A related concern is that in this model only the degree of commonality between the pitches of two chords is considered. It does not take into account the differences between pitches. Parncutt proposes another model that calculates the total intervallic distances between all pitches. This model essentially finds the distance in semitones between every pair of pitches and scales these values by the saliences of the pairs.[50] Using this model, identical sounds have a pitch distance of 0 and the pitch distance between non-identical sounds will always be greater than 0. A useful expansion of the current method of evaluating successive pitch relations in *Apprentice* would be to incorporate both pitch distance and pitch commonality. One possibility is to calculate both values and use the average of the two to evaluate successive pitch relations.

The model of successive pitch commonality used in the Concerto applies to pairs of chords heard in isolation, not to harmonic progressions in actual music. As a result, the model does not take into account the effects of auditory streaming. The soprano voice of a progression, for example, is normally perceived as a stream. Melodic leaps in this voice tend to have more impact than those between the pure-tone components of sonorities. A model of horizontal motion between harmonic voices has been recently proposed.[51] One possible extension to the evaluation of connections between successive chords might be to combine pitch commonality, pitch distance, and horizontal motion into a single value.

10.2.3 *Apprentice*

Apprentice is currently a program with a user base of one: the programmer. As a result, there is almost no distinction between the acts of programming and of using the program. In order to perform certain tasks, it is sometimes necessary to make major changes to the software itself. The analytical and compositional techniques that are facilitated by this software are quite general and are not necessarily linked to my own compositional style. It thus seems likely that they may be of interest to others. One of my major goals in developing this software is therefore to improve the user interface and to organize related activities into more logical modules so that the program becomes easier to use both for myself and others.

[50] Ibid., 95

[51] Bigand, Emmanuel, Richard Parncutt, and Fred Lerdahl. "Perception of Musical Tension in Short Chord Sequences: The Influence of Harmonic Function, Sensory Dissonance, Horizontal Motion, and Musical Training," *Perception & Psychophysics*, Vol. 58, No. 1 (1996): 125-141.

Other possible changes to *Apprentice* relate to the way in which parameters interact during harmonization. Each value is currently computed separately, with no consideration of other parameters. Selecting chords by dissonance, for example, does not take successive pitch commonality into account. And once a selection of chords with the desired amount of dissonance has been made from a larger group, the dissonance values are discarded. The impact of these two parameters may be linked, however, since it is possible that very smooth voice leading between chords may decrease the amount of perceived dissonance due to streaming. In other words, "horizontal motion affects the consonance of chords."[52] In the future, there may be a way to take this interaction into account. As well, it might be possible to designate a single value to describe the harmonic stability of a chord succession that combines the values for dissonance, adjacent interval variance, successive pitch relations, and referential pitch commonality.

10.2.4 Musical Analysis

The process of composing using these psychoacoustic models involves the analysis of thousands of harmonic candidates to find those with the desired degree of parameters including dissonance and pitch commonality. Since these models may be applied to any collection of pitches, there is no reason why they could not be used to analyze the music of composers other than myself. My intuition is that an analysis of the psychoacoustic qualities of chords would be particularly appropriate for the early free atonal works of the composers of the Second Viennese School. *Apprentice* already contains functions to analyze and display the amounts of each parameter graphically. All that is necessary is to convert the pitches of a piece to MIDI key numbers.

10.2.5 Psychoacoustic Research

The models used in this piece are representative of the current state of research in the domain of psychoacoustics. Since *Apprentice* contains the means to enter the notes of a sonority, to analyze the psychoacoustic qualities of the sonority, to play back the sonority to subjects in a study and to record their responses, it could prove to be a useful tool for psychoacoustic research.

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[52] Ibid., 131.

Appendix A: Sensory Dissonance

This appendix contains the Lisp code that implements the model of sensory dissonance used in the Concerto. I am providing this information to allow others to experiment with the psychoacoustic model and to clarify the algorithm. The actual code as it is occurs in Apprentice is somewhat optimized, but is less clear. My goal is to provide a functional, portable implementation in ANSI Common Lisp. No other code from *Apprentice* is required.

```
(defun midi->pitcat (midi-pitch)
  "Converts MIDI pitch numbers (middle c = 60) to Parncutt's pitch
categories (middle c = 48)"
  (declare (type fixnum midi-pitch))
  (- midi-pitch 12))

(defun pitch->Hz (midi-pitch)
  "Translates a MIDI pitch number (middle c = 60) into a frequency
given in Hz. So (pitch->Hz 69) returns 440 Hz."
  (declare (type fixnum midi-pitch))
  (float (* 440 (expt 2 (/ (- (midi->pitcat midi-pitch) 57) 12)))))

(defun mean-freq (f1 f2)
  "Returns the mean frequency of two pure tones in Hz.  Equals 1/2(f1 + f2)."
  (float (/ (+ f1 f2) 2)))

(defun critical-bandwidth (f)
  "Returns the critical bandwidth in Hz for the given frequency,
also in Hz (a4 = 69 = 440 Hz)."
  (declare (type float f))
  (* 1.72 (expt f .65)))

(defun cbw-interval (f1 f2)
  "Gives the interval between two partials in units
of the critical bandwidth. Frequencies should be given
in Hz."
  (declare (type float f1 f2))
  (float (/ (abs (- f2 f1))
     (critical-bandwidth (the float (mean-freq f1 f2))))))

(defun standard-curve (cbw-int)
  "Parncutt's function for g(y) of H&K (p. 4). Gives the dissonance
weighting factor of the frequency difference of two pure tones in
units of the critical bandwidth."
  (declare (type float cbw-int))
  (if (> cbw-int 1.2) ;if critical bandwidth interval > 1.2, *no* roughness
    0
    (let ((ratio (float (/ (the float cbw-int) .25)))) ;.25 is interval for max roughness
      (expt (* (* 2.7182818 (the float ratio)) (exp (* -1 (the float ratio)))) 2))))

(defun pure-tone-dissonance (f1 f2)
  "Gives the pure tone dissonance of two partials given in Hz without
considering amplitude. That is, amp for both f1 and f2 = 1."
```

```
  (declare (type float f1 f2))
  (float (standard-curve (the float (cbw-interval f1 f2)))))

(defun harmonic-interval (harmonic-number)
  "Gives the interval in semitones between fundamental and harmonic."
  (declare (type fixnum harmonic-number))
  (floor (+ (/ (* 12 (log harmonic-number)) (log 2)) .5)))

(defun harmonic-series-pitch (fundamental-pitch &optional (no-of-harms 10))
  "Give harmonic series rounded off to chromatic pitches."
  (declare (optimize (speed 3) (safety 0)) (type fixnum fundamental-pitch no-of-harms))
  (loop for x from 1 to no-of-harms
        collect (+ (harmonic-interval x) fundamental-pitch)))

(defun harmonic-series-frequency (fundamental-pitch &optional (no-of-harms 10))
  "Gives the harmonic series in frequency rounded off to chromatic pitches.
Fundamental pitch is given as MIDI note number (middle c = 60)."
  (mapcar #'pitch->Hz (harmonic-series-pitch fundamental-pitch no-of-harms)))

(defun sum-amplitudes (amp1 amp2)
  "Returns the summed amplitudes of arguments.  Amplitudes are
given as fractions of harmonic numbers (1/n). For example,
the amplitude of harmonic 3 is 1/3."
  (float (sqrt (+ (* amp1 amp1) (* amp2 amp2)))))

(defun get-harm-amps (harm-series)
  (loop for harm in harm-series
        for x = 1 then (+ x 1)
        collect (cons harm (/ 1 x))))


(defun get-amps-of-overlaps (overlaps spect1+amps spect2+amps)
  "To be called by merge-spectrums only."
  (let ((result nil))
    (dolist (x overlaps result)
      (push (cons x (the float (sum-amplitudes
                      (rest (assoc x spect1+amps))
                      (rest (assoc x spect2+amps)))))
            result))))


(defun remove-overlaps (orig final pool)
  (cond ((null orig) final)
        ((member (caar orig) pool)
         (remove-overlaps (rest orig) final pool))
        (t (remove-overlaps (rest orig)
                            (push (first orig) final)
                            pool))))

(defun merge-spectrums (fund-pitch1 &optional (pitches nil))
  "Returns a list of dotted pairs with car = pitch of spectral
component (in MIDI numbers) and cdr = relative amplitude.
Components which occur more than once have their amplitudes
scaled appropriately."
  (let ((result (get-harm-amps (harmonic-series-pitch fund-pitch1))))
    (if (null pitches) result
```

```
        (labels ((rec1 (final current pchs)
                   (let* ((spect+amps (get-harm-amps (harmonic-series-pitch current)))
                          (overlaps (mapcar #' car
                                            (intersection final spect+amps :key #'car)))
                          (overlap+amps (get-amps-of-overlaps overlaps final spect+amps))
                          (merged-spects+amps (nconc final spect+amps))
                          (gapped-spects+amps (remove-overlaps merged-spects+amps
                                                               nil
                                                               overlaps))
                          (re-merged (sort (nconc overlap+amps
                                                  gapped-spects+amps)
                                           #'< :key #'car)))
                     (cond ((null pchs) re-merged)
                           (t (rec1 re-merged
                                    (first pchs)
                                    (rest pchs)))))))
          (rec1 result (first pitches) (rest pitches)))))

(defun diss-numerator (pch-amp1 pch-amp2)
  "Pitches must be given as MIDI notes."
  (declare (type fixnum pch1 pch2))
  (let ((pch1 (first pch-amp1))
        (pch2 (first pch-amp2))
        (amp1 (rest pch-amp1))
        (amp2 (rest pch-amp2)))
      (* (* amp1 amp2) ;Numerator
         (the float (pure-tone-dissonance (pitch->Hz pch1)
                                          (pitch->Hz pch2)))))) ;Numerator

(defun acoustic-dissonance (pitches)
  "Given a list of one or more midi pitches, returns the
acoustic-dissonance, or 'roughness.' For example:
? (acoustic-dissonance '(60 61 62 73))
0.7287
? (acoustic-dissonance '(60))
0.0012
? (acoustic-dissonance '(47 49 53))
0.3846
? (acoustic-dissonance '(62 67 72 77 82))
0.1592"
  (let* ((combined-spectrum (merge-spectrums (first pitches) (rest pitches)))
         (denominator (float          ;denominator is total amplitude of spectrum
                        (reduce #'+
                                (mapcar #'(lambda (x) (* x x))
                                        (mapcar #'cdr combined-spectrum))))))
    (do* ((comb-spect combined-spectrum (rest comb-spect))
          (lowest (first combined-spectrum) (first comb-spect))
          (higher (rest combined-spectrum) (rest comb-spect))
          (interim-list (mapcar #'(lambda (x) (diss-numerator lowest x))
                                higher)
                        (mapcar #'(lambda (x) (diss-numerator lowest x))
                                higher))
          (result interim-list (append interim-list result)))
         ((null (rest comb-spect))
          (round-off 4 (/ (reduce #'+ result)    ;add up all numerators and
                          denominator))))))      ;then divide
```

Appendix B: Pitch Commonality

This appendix contains the Lisp code that implements the model of pitch commonality used in the Concerto. I am providing this information to allow others to experiment with the psychoacoustic model and to clarify the algorithm. The actual code as it is occurs in Apprentice is highly optimized (it runs approximately 50 times faster), but is much less clear. My goal is to provide a functional, portable implementation in ANSI Common Lisp. No other code from *Apprentice* is required. Optimization is left to the reader as an exercise.

```
(defvar *kM* 18
  "Masking gradient.  Typical value 18 dB/critical bandwidth")

(defvar *kT* 3
  "Typical relative importance of pure tone sensations.")

(defvar *kS* 0.5
  "Tendency to hear simultaneous tones.")

(defun pitch->kHz (midi-pitch)
  "Translates a MIDI pitch number (middle c = 60) into a frequency
given in kHz. So (pitch->Hz 69) returns .44 Hz."
  (declare (type fixnum midi-pitch)
           (optimize (speed 3) (safety 0)))
  (* .44 (expt 2 (/ (- (midi->pitcat midi-pitch) 57) 12))))

(defun pure-tone-height (MIDI-note)
  "Gives the pure tone hight (or ERB-rate, Moore and Glasberg, 1983)
in critical bands.  Argument is a MIDI pitch (middle c = 60)."
  (let ((freq (pitch->kHz MIDI-note)))
    (+ 43
       (* 11.17 (log (/ (+ freq .312)
                        (+ freq 14.675)))))))

(defun partial-auditory-level (MIDI-note harmonic-number)
  (declare (type fixnum MIDI-note harmonic-number) (optimize (speed 3) (safety 0)))
  (let ((x (midi->pitcat MIDI-note)))
    (if (< MIDI-note 121)
      (float (* (/ (* x (- 120 x))
                   60)
                (- 1 (/ (harmonic-interval harmonic-number) 120))))
      0)))

(defun partial-auditory-levels (MIDI-note)
    "Returns a list of dotted pairs of pitches of pure-tone
components of a complex pitch with fundamental frequency
at the given MIDI pitch (middle c = 60) and auditory levels
given in dB.  Argument is fundamental pitch."
    (let ((cnt 0)
          (result nil))
      (dolist (x (harmonic-series-pitch MIDI-note 16) (nreverse result))
        (incf cnt)
```

```
        (push (cons x (partial-auditory-level x cnt))
              result))))

(defun add-auditory-levels (PYL-1 PYL-2)
  "Adds two auditory levels.  Equivalent to Parncutt's deriving
YL from PYL, if already a level at a given pitch."
  (* 10 (log (+ (expt 10 (/ PYL-1 10))
                (expt 10 (/ PYL-2 10)))
             10)))

(defun merge-two-spectrums-db (spect1 spect2)
  "Merges two lists of dotted pairs of harmonic components
and their auditory levels into one, adding the auditory
levels of any elements which appear twice."
  (declare (optimize (speed 3) (safety 0)))
  (let ((list1 spect1)
        (list2 spect2))
    (dolist (pair list1 (sort
                          (remove-duplicates (nconc list2 list1) :key #'car)
                          #'< :key #'car))
      (let ((test-case (assoc (first pair) list2)))
        (if test-case
          (setf (rest pair)
                (add-auditory-levels (rest test-case)
                                     (rest pair)))
          (cons test-case list1))))))

(defun merge-spectrums-db (list-of-MIDI-pitches)
  "Given a list of lists containing dotted pairs of MIDI pitches
and the auditory levels (YL) in dB of all harmonics, merge them
together into a single harmonic spectrum, adding the levels
of elements occuring in more than one spectrum."
  (cond ((atom list-of-MIDI-pitches) (partial-auditory-levels list-of-MIDI-pitches))
        ((= (length list-of-MIDI-pitches) 1)
         (partial-auditory-levels (car list-of-MIDI-pitches)))
        (t (labels ((rec (lst result current)
                      (cond ((null (rest lst))
                             (merge-two-spectrums-db
                              (partial-auditory-levels (first lst)) result))
                            (t (setf current
                                     (partial-auditory-levels (first lst)))
                               (rec (rest lst)
                                    (merge-two-spectrums-db current result) nil)))))
             (rec list-of-MIDI-pitches nil nil)))))

(defun pure-tone-height-difference (pitch1 pitch2)
  "Finds pure tone height difference between two pitches.  Used
in finding the partial masking level between the two. Arguments
are given as MIDI pitches (middle c = 60)."
  (abs (- (pure-tone-height pitch1) (pure-tone-height pitch2))))

(defun partial-masking-level (pitch1 pitch2 YL2)
  "Gives the partial masking level of two pure tone components
in a complex tone.  Pitches are given as MIDI notes (middle c =
60).  First argument is first pitch, second argument is second
pitch and third is auditory level in dB of second pitch."
```

```lisp
  (declare (type fixnum pitch1 pitch2) (optimize (speed 3) (safety 0)))
    (let ((pth-dif (pure-tone-height-difference pitch1 pitch2)))
      (if (< pth-dif 3)
        (- YL2
          (* *KM*
             (pure-tone-height-difference pitch1 pitch2)))))) ;*KM* is masking gradient.

(defun masking-level (pitch spectrum &optional (result 0.00000000001))
  "Pitch is a MIDI note.  Spectrum is a list of
dotted pairs of pitch and auditory levels in dB."
  (dolist (x spectrum result)
    (unless (= (first x) pitch)
      (let ((n (partial-masking-level pitch (first x) (rest x))))
        (if n (setf result (+ (expt 10 (/ n 20)) result))))))
  (max (* 20 (log result 10)) 0))

(defun audible-level (pitch-YL spectrum)
  "First argument is a dotted pair of MIDI pitch and auditory
level in dB taken from the second argument, which is a
list of dotted pairs of pitch and YL resulting from merging
spectrums using merge-spectrums-db.  Result is the level
above the masked threshold of the pitch of the first argument."
  (max (- (rest pitch-YL) (masking-level (first pitch-YL) spectrum)) 0))

(defun pure-tone-audibility (pitch-YL spectrum)
  (- 1 (exp (/ (audible-level pitch-YL spectrum) -15))))

(defun pure-tone-audibilities (list-of-pitches &optional (result nil))
  "Argument is a list of MIDI pitches or a 'chord.' Result
is a list of dotted pairs of pure tone components and
their audibilities after considering masking among
all the pure tone components."
  (let ((spectrum (merge-spectrums-db list-of-pitches)))
    (dolist (x spectrum (nreverse result))
      (push (cons (first x)
                  (pure-tone-audibility x spectrum))
            result))))

(defun maximum-pure-tone-audibility (list-of-ptas)
  (let ((list (coerce list-of-ptas 'array)))
    (reduce #'max list :key 'cdr)))

(defun complex-tone-audibilities (list-of-ptas)
  (let* ((result nil)
         (ptas (coerce list-of-ptas 'array))
         (len (length ptas)))
    (dotimes (i 119 result)
      (let ((sum 0)
            (chrom-pc (+ i 13)))
        (dotimes (j 10 sum)
          (let* ((har-num (+ j 1))
                 (har-int (harmonic-interval har-num))
                 (harm (+ chrom-pc har-int)))
            (dotimes (k len)
              (let ((pair (aref ptas k)))
                (if (= harm (first pair))
```

```lisp
                (setf sum (+ sum (sqrt (/ (rest pair) har-num))))))))
        (when (> sum 0) (push (cons chrom-pc (/ (expt sum 2) *kt*)) result))))))

(defun maximum-complex-tone-audibility (implied-fundamentals)
  "Receives output from function 'complex-tone-audibilities'."
  (let ((list (coerce implied-fundamentals 'array)))
    (reduce #'max list :key #'cdr)))

(defun maximum-tone-audibility (ptas ctas)
  "Arguments are complex- and pure-tone audibilities for
the same chord or merged spectrum."
  (max (maximum-complex-tone-audibility ctas)
       (maximum-pure-tone-audibility ptas)))

(defun unscaled-multiplicity (ptas ctas max-ta)
  (let* ((result nil)
         (list1 (coerce ptas 'array))
         (list2 (coerce ctas 'array))
         (len (length list2)))
    (dotimes (i len (reduce #'+ result))
      (let* ((pair (aref list2 i))
             (x (first pair))
             (y (find x (the simple-vector list1) :key #'car)))
        (if y
          (push (/ (max (rest pair) (rest y)) max-ta) result)
          (push (/ (rest pair) max-ta) result))))))

(defun multiplicity (mulX)
  "Receives arguments from functions 'pure-tone-audibilities'
and 'complex-tone-audibilities.' Returns the multiplicity,
or the number of simultaneously noticed tones or pitches,
for the chord resulting in the merged spectrum (the pure
tone audibilities)."
  (expt mulX *kS*)) ;*kS* is a global variable: tendency to hear simultaneous tones.

(defun pure-tone-salience (pitch-pta max-ta mul mulX)
  "Pta is a dotted pair of a pure-tone component as MIDI-pitch
and its audibility, from function 'pure-tone-audibilities.'
Max-ta is from 'maximum-tone-audibility.' Mul is multiplicity.
MulX is unscaled-multiplicity."
  (/ (* (/ (rest pitch-pta) max-ta) mul) mulX))

(defun pure-tone-saliences (ptas max-ta mul mulx &optional (result nil))
  "Ptas is a list of dotted pairs of pure-tone components as MIDI
pitches and their audibilities, from function 'pure-tone-audibilities.'
Max-ta is from 'maximum-tone-audibility.' Mul is multiplicity.
MulX is unscaled-multiplicity."
  (dolist (pair ptas result)
    (let ((x (first pair)))
      (push
       (cons x (pure-tone-salience pair max-ta mul mulX))
       result))))

(defun complex-tone-salience (pitch-cta max-ta mul mulX)
  (/ (* (/ (rest pitch-cta) max-ta) mul) mulX))
```

```
(defun complex-tone-saliences (ctas max-ta mul mulX &optional (result nil))
  (dolist (pair ctas result)
    (let ((x (first pair)))
      (push
       (cons x (complex-tone-salience pair max-ta mul mulX))
       result))))

(defun overall-tone-salience (pitch-pts pitch-cts)
  "Arguments are two dotted pairs of pitch and pure tone
salience and the same pitch and complex tone salience."
  (max (rest pitch-pts) (rest pitch-cts)))

(defun make-ots (list-p-cts list-p-pts)
  (let* ((result nil)
         (list1 (coerce list-p-cts 'array))
         (list2 (coerce list-p-pts 'array))
         (len (length list1)))
    (dotimes (i len result)
      (let* ((pair (aref list1 i))
             (x (first pair))
             (y (find x list2 :key #'car)))
        (if y
          (push (cons x (overall-tone-salience pair y)) result)
          (push pair result))))))

(defun overall-tone-saliences (chord &key (verbose nil))
  "Given a list of MIDI pitches, computes the overall
pitch saliences for entire pitch spectrum."
  (declare (optimize (speed 3) (safety 0)))
  (let* ((ptas (pure-tone-audibilities chord))
         (ctas (complex-tone-audibilities ptas))
         (max-ta (maximum-tone-audibility ptas ctas))
         (mulX (unscaled-multiplicity ptas ctas max-ta))
         (mul (multiplicity mulX))
         (pts (pure-tone-saliences ptas max-ta mul mulX))
         (cts (complex-tone-saliences ctas max-ta mul mulX)))
    (when verbose
      (progn (print `(max-ta is ,max-ta))
             (print `(mulx is ,mulX))
             (print `(mul is ,mul))))
    (make-ots cts pts)))

(defun most-salient-pitch (chord)
  "Given a list of MIDI pitches, identifies which pitch
is the most 'noticeable', or salient."
(first
 (first
  (sort (overall-tone-saliences chord) #'> :key 'rest))))

(defun p->pc (MIDI-note)
  (mod MIDI-note 12))

(defun chromatic-pitch-salience (ots &optional (result nil))
  "Argument is a list of dotted pairs of midi pitch and
overall tone salience.  Ots for each chromatic pitch
class is returned."
```

```
  (let ((n (make-array 12 :initial-element 0)))
    (dolist (pair ots)
      (let* ((x (p->pc (first pair)))
             (y (rest pair))
             (z (aref n x)))
        (setf (aref n x) (+ y z))))
    (dotimes (i 12) (push (cons i (aref n i)) result)))
  (reverse result))

(defun sum-cdrs (list)
  "Sums all the cdrs of a list of dotted pairs."
  (reduce #'+ list :key #'cdr))

(defun sum-of-squares-cdrs (list)
  "Sums the squares of the cdrs of a list of dotted pairs."
  (reduce #'+ (mapcar #'(lambda (x) (expt (rest x) 2)) list)))

(defun multiply-common-elements (list1 list2 &optional (result nil))
  "Multiplies together cdrs of elements in two dotted lists
which have the same cars."
  (dolist (x list1 result)
    (let ((y (assoc (first x) list2)))
      (when y
        (push (* (rest x) (rest y)) result)))))

(defun sum-xy (list1 list2)
  (reduce #'+ (multiply-common-elements list1 list2)))

(defun sum-root-xy (list1 list2)
  (reduce #'+ (mapcar #'(lambda (x) (expt x .5))
                      (multiply-common-elements list1 list2))))

(defun correlation-coefficient (list1 list2)
  "Correlation coefficient of two dotted lists of MIDI pitches
and their overall tone saliences, or ots. Describes the degree
to which the two spectrums are perceived to have common pitches."
  (let* ((size 120) ;That is, there are 120 possible pitches...
         (sumx (sum-cdrs list1))
         (sumy (sum-cdrs list2))
         (sumxx (sum-of-squares-cdrs list1))
         (sumyy (sum-of-squares-cdrs list2))
         (sumxy (sum-xy list1 list2))
         (sx (sqrt (/ (- sumxx (/ (* sumx sumx) size)) (- size 1)))) ;is precedence correct?
         (sy (sqrt (/ (- sumyy (/ (* sumy sumy) size)) (- size 1))))
         (sxy (/ (- sumxy (/ (* sumx sumy) size)) (- size 1))))
    (/ sxy (* sx sy))))

(defun pitch-commonality (chord1 chord2)
  "Calculates the pitch commonality between two lists of
MIDI pitches according to Parncutt (1989)."
  (let ((list1 (overall-tone-saliences chord1))
        (list2 (overall-tone-saliences chord2)))
    (let ((sumx (sum-cdrs list1))
          (sumy (sum-cdrs list2))
          (sumrootxy (sum-root-xy list1 list2)))
      (/ sumrootxy (expt (* sumx sumy) 0.5)))))
```

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Concerto for Piano and Orchestra
("Inside Passage")

Sean Ferguson
Faculty of Music
McGill University, Montreal
December, 2000

Volume 2: Score

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements of the degree of Doctor of Music

Concerto for Piano and Orchestra
("Inside Passage")

Sean Ferguson

Instrumentation:

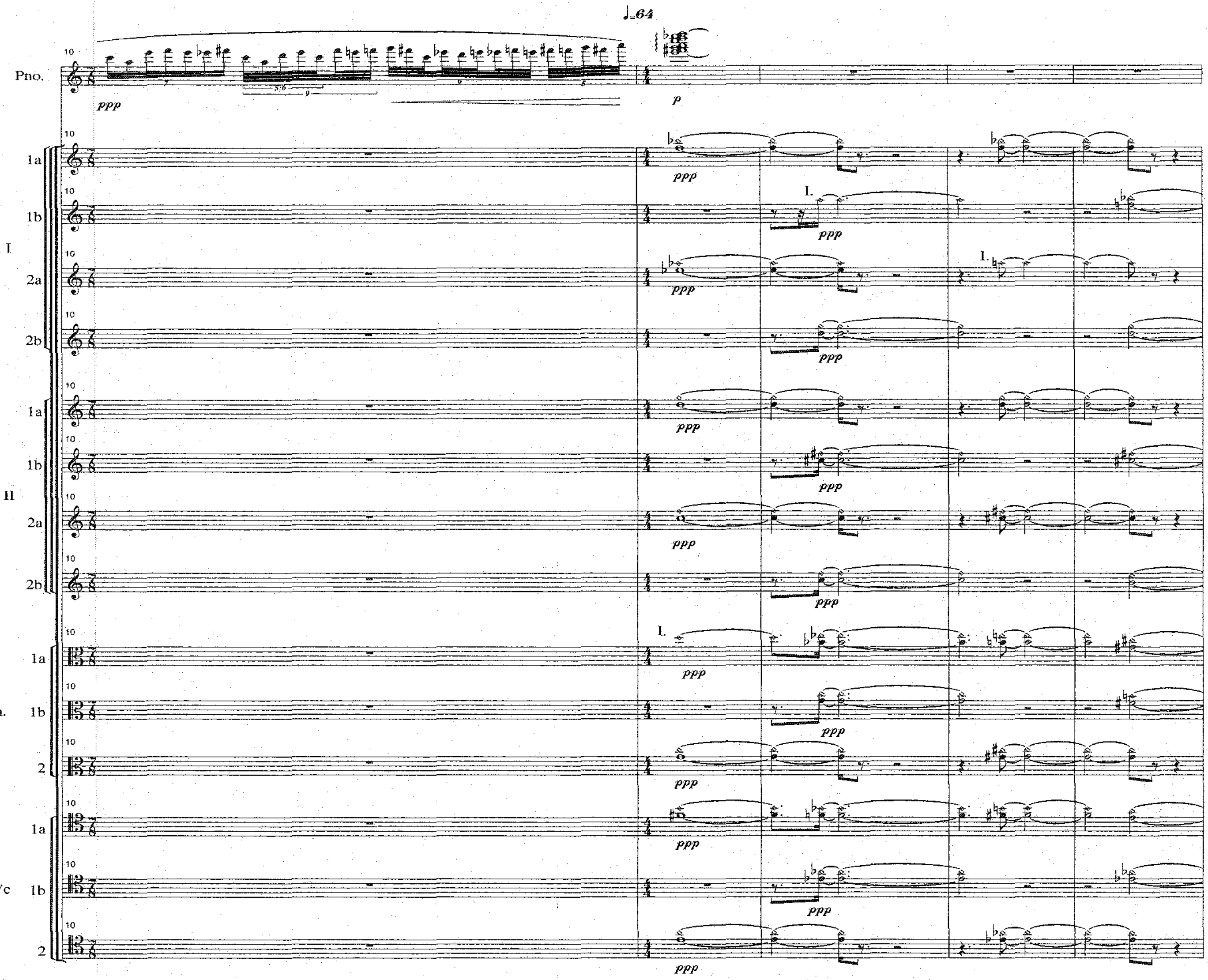
Flute, doubling on Piccolo
B-flat Clarinet
Oboe
Bassoon
Horn
Trumpet in C
Trombone
Percussion I
  -6 Tom-Toms, Marimba
Percussion II
  -Timpani, Marimba
Solo Piano
Strings (44332)

* This work was commissioned by Radio-Canada (Laurent Major, Producer) for the Société de musique contemporaine du Québec, Walter Boudreau, Artistic Director, and Marc Couroux, pianist.

Concerto for Piano and Orchestra
("Inside Passage")

Sean Ferguson (1999)

NB: Score notated in "C." All notes sound as written except for Bass, which sounds an octave lower.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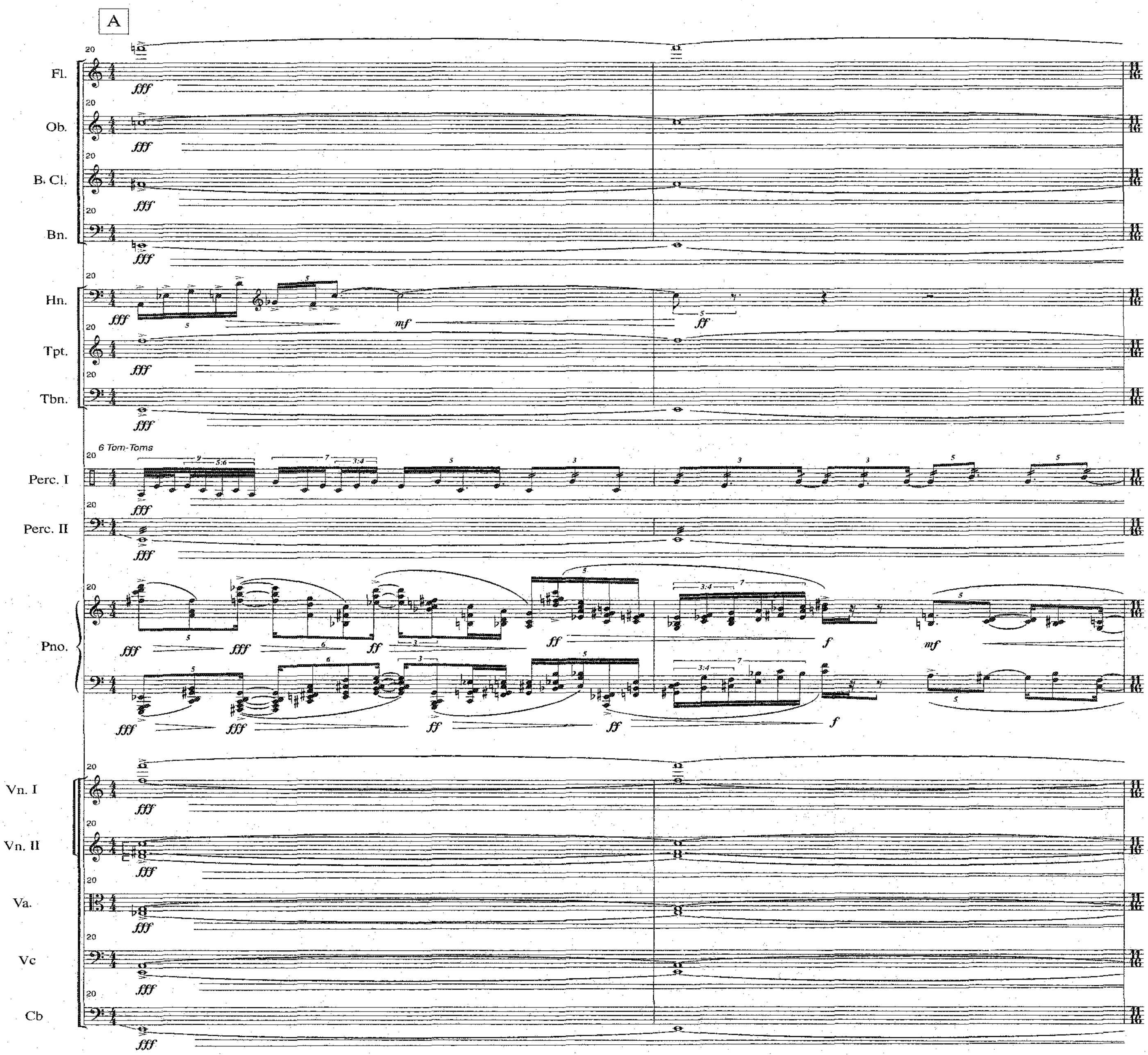
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
I.
ppp


Fl.
Ob.
B. Cl.
Bn.
Hn.
Tbn.
Perc. II
Vn. I
Vn. II
Va.
Vc
Cb
ppp
pp
p
mp
mf
Timpani
gliss.
I.
II.
III.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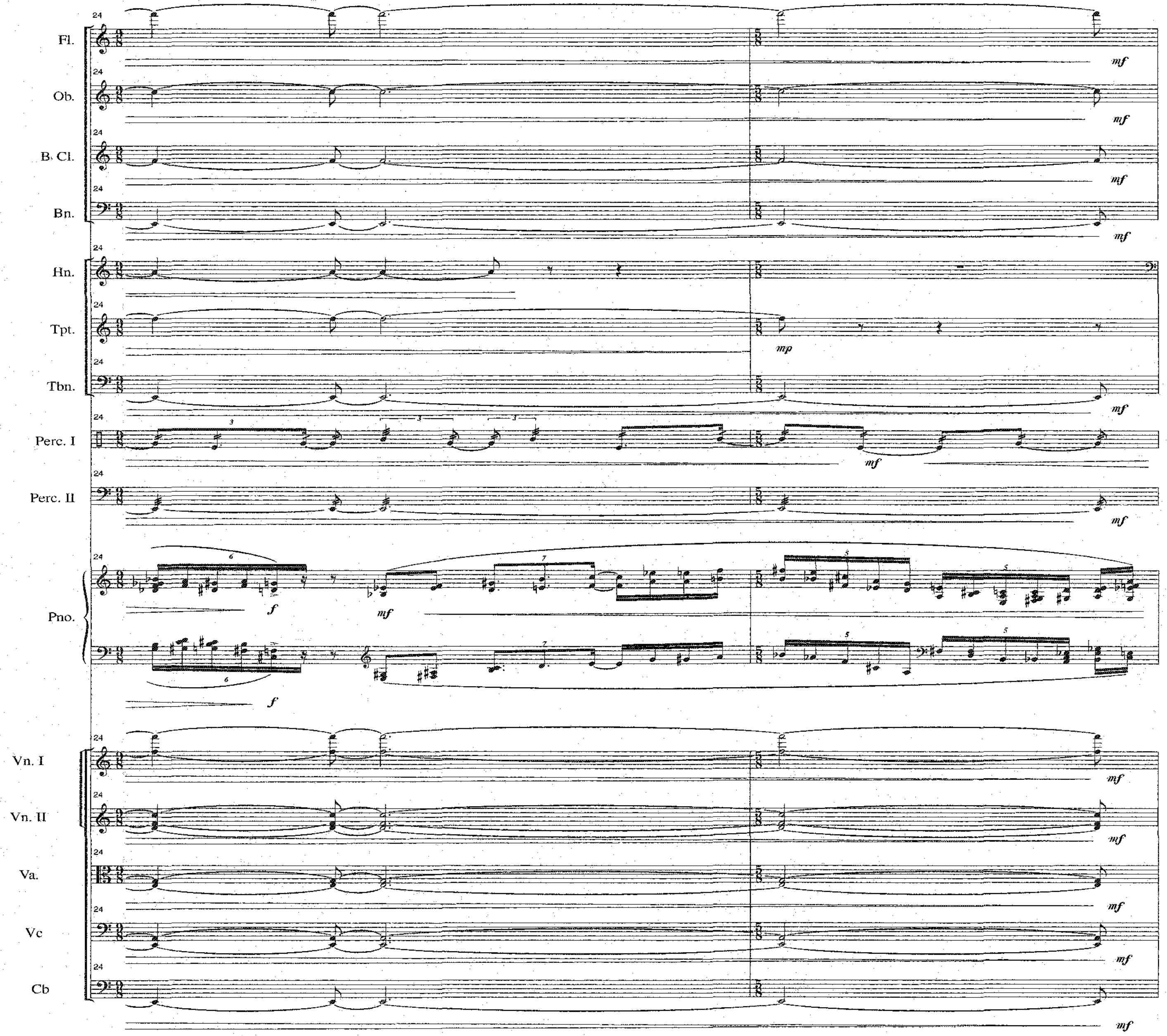
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
6 Tom-Toms
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb


22
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb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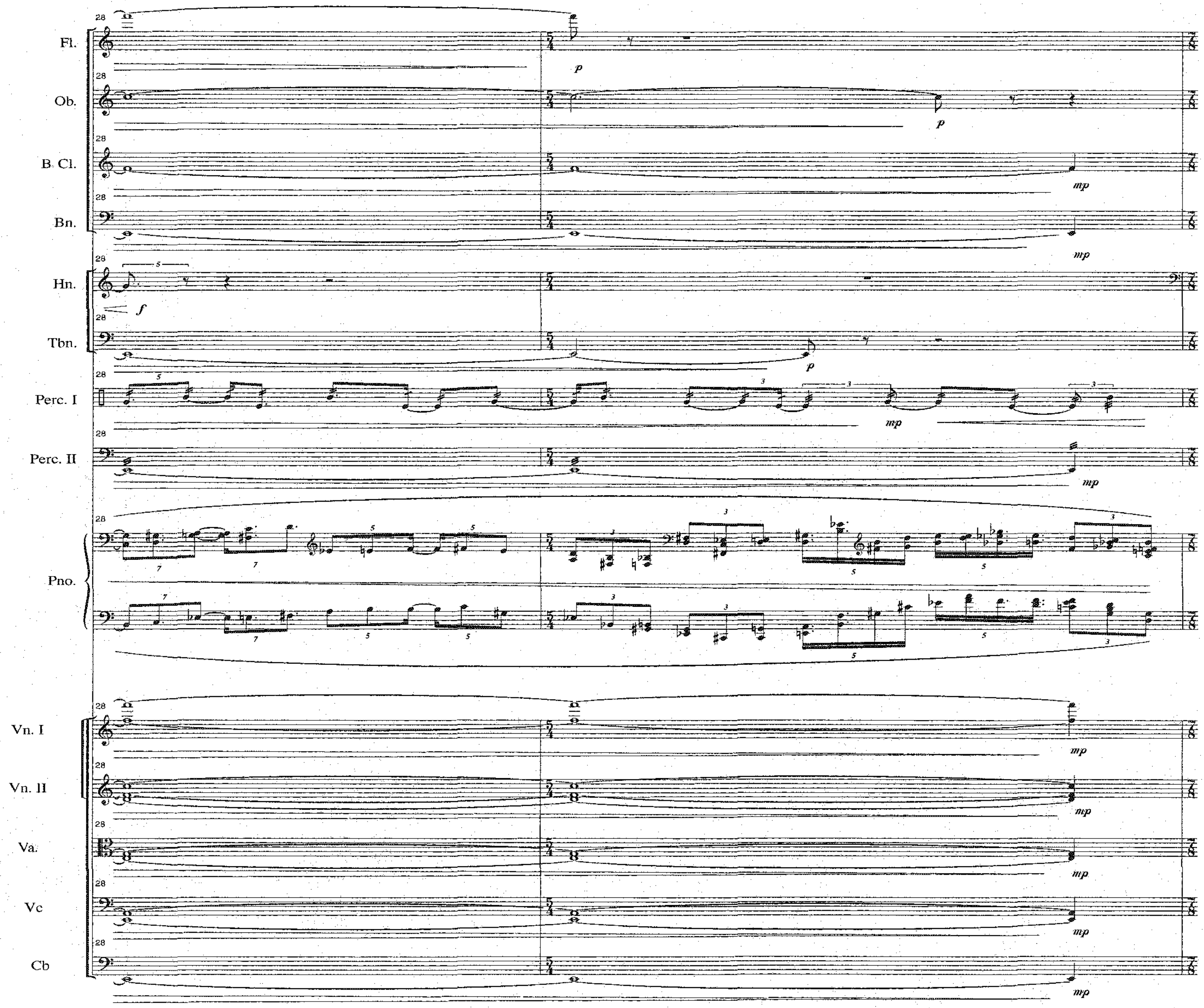
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
mf
mp
f


26
Fl.
ff
Ob.
ff
B. Cl.
ff
Bn.
ff
Hn.
ff
mp
Tpt.
ff
Tbn.
ff
Perc. I
ff
Perc. II
ff
Pno.
ff
mp
ff
ff
Vn. I
ff
Vn. II
ff
Va.
ff
Vc
ff
Cb
ff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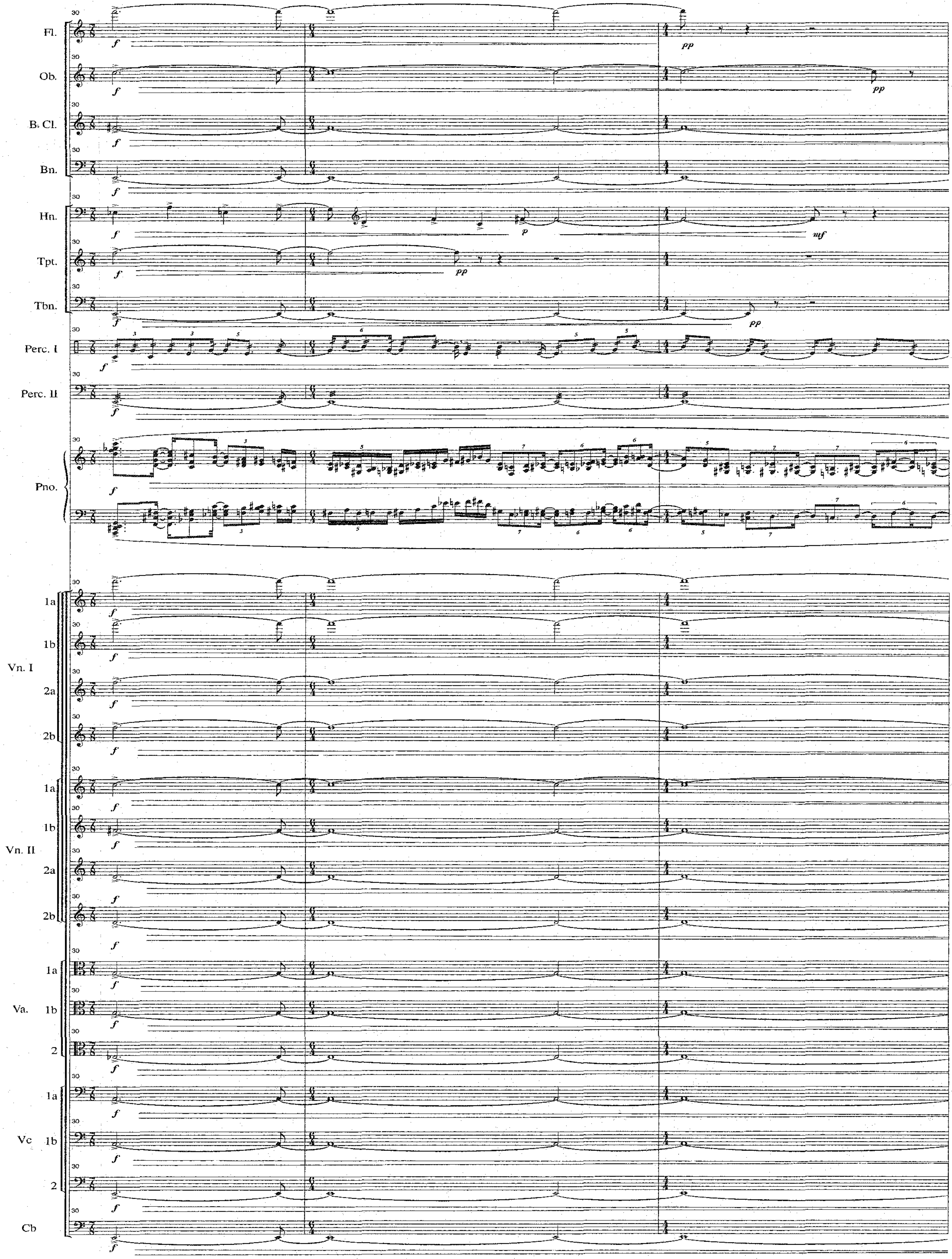
Ob.
B. Cl.
Bn.
Hn.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb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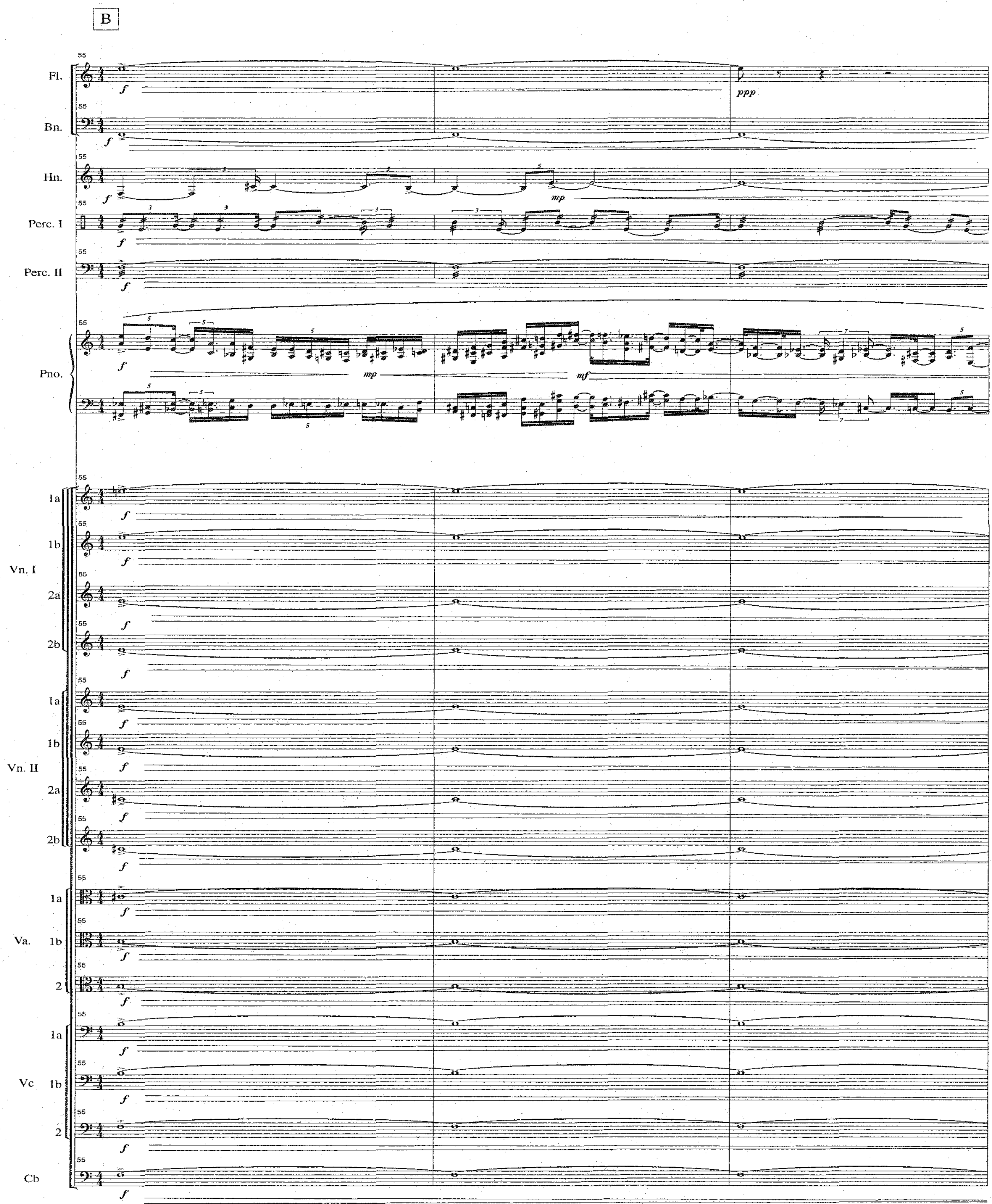
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
30

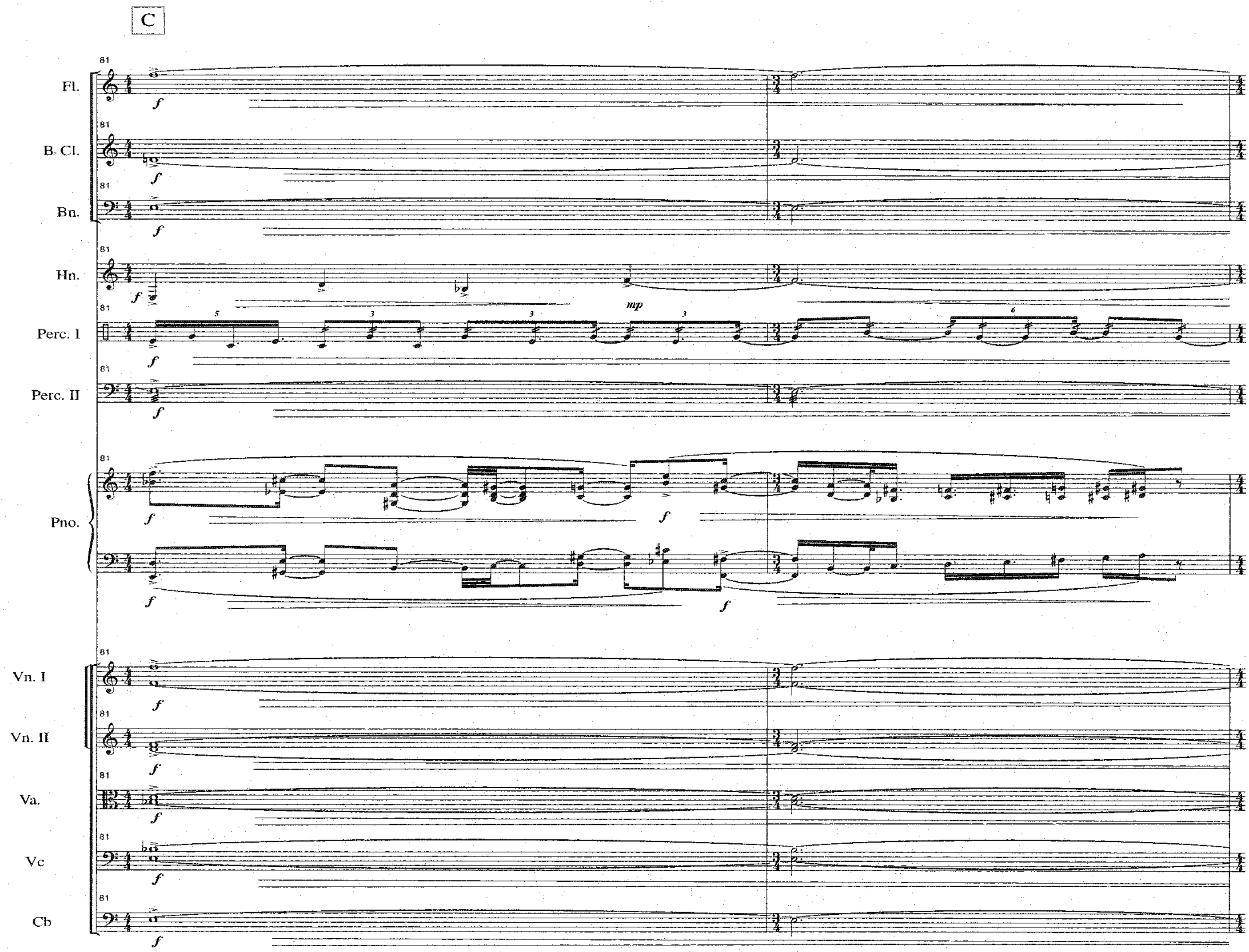

33
B♭ Cl.
Bn.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
Va.
Vc
Cb
espr.

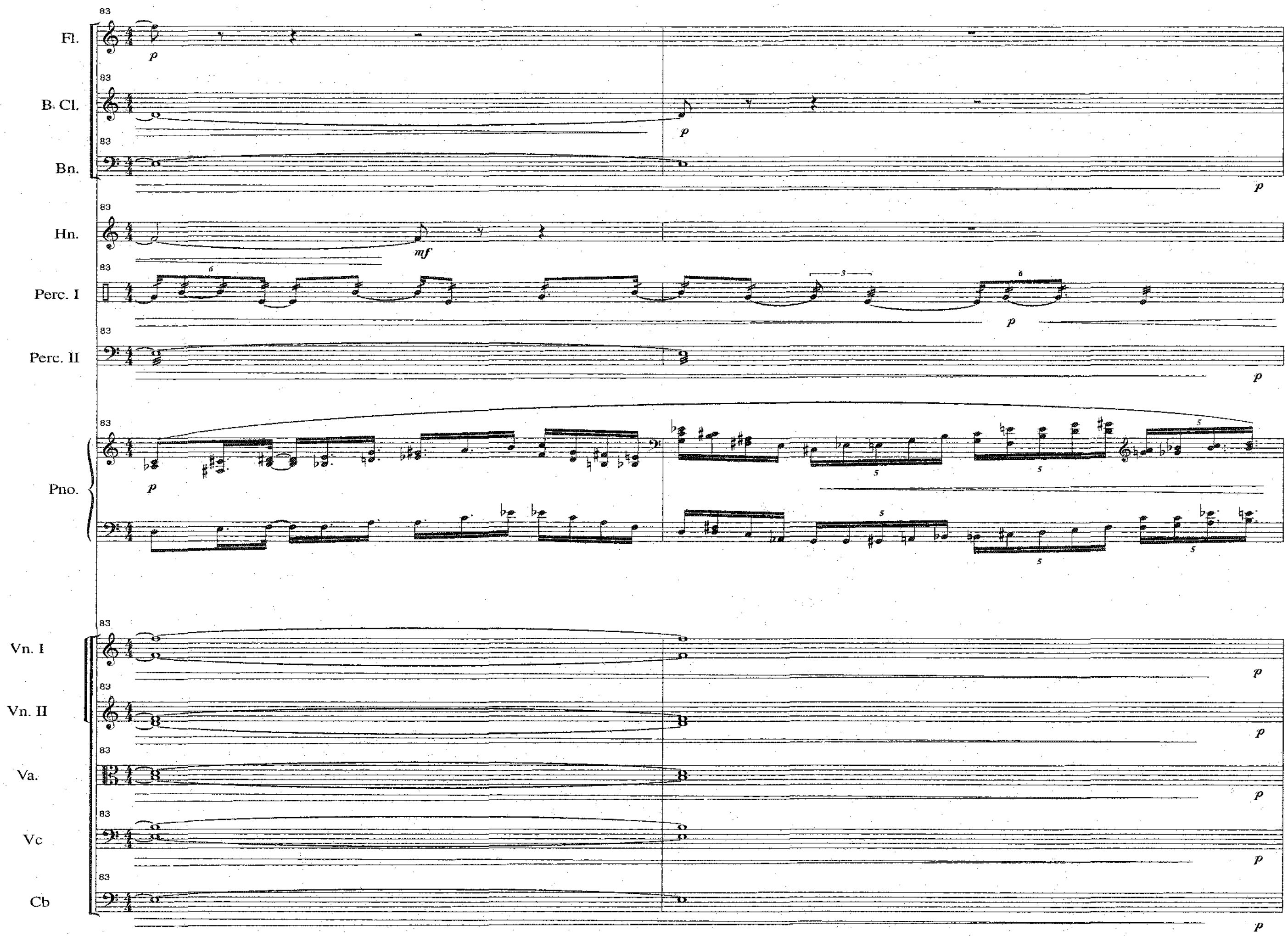

B. Cl.
Va. 1a
Vc 1a
Cb
espr.
ppp
mp
pp
Perc. II
gliss.
Vn. I 1a
2a
Vn. II 1a
Va. 1a
1b
2
Vc 1a
1b
2
p
f

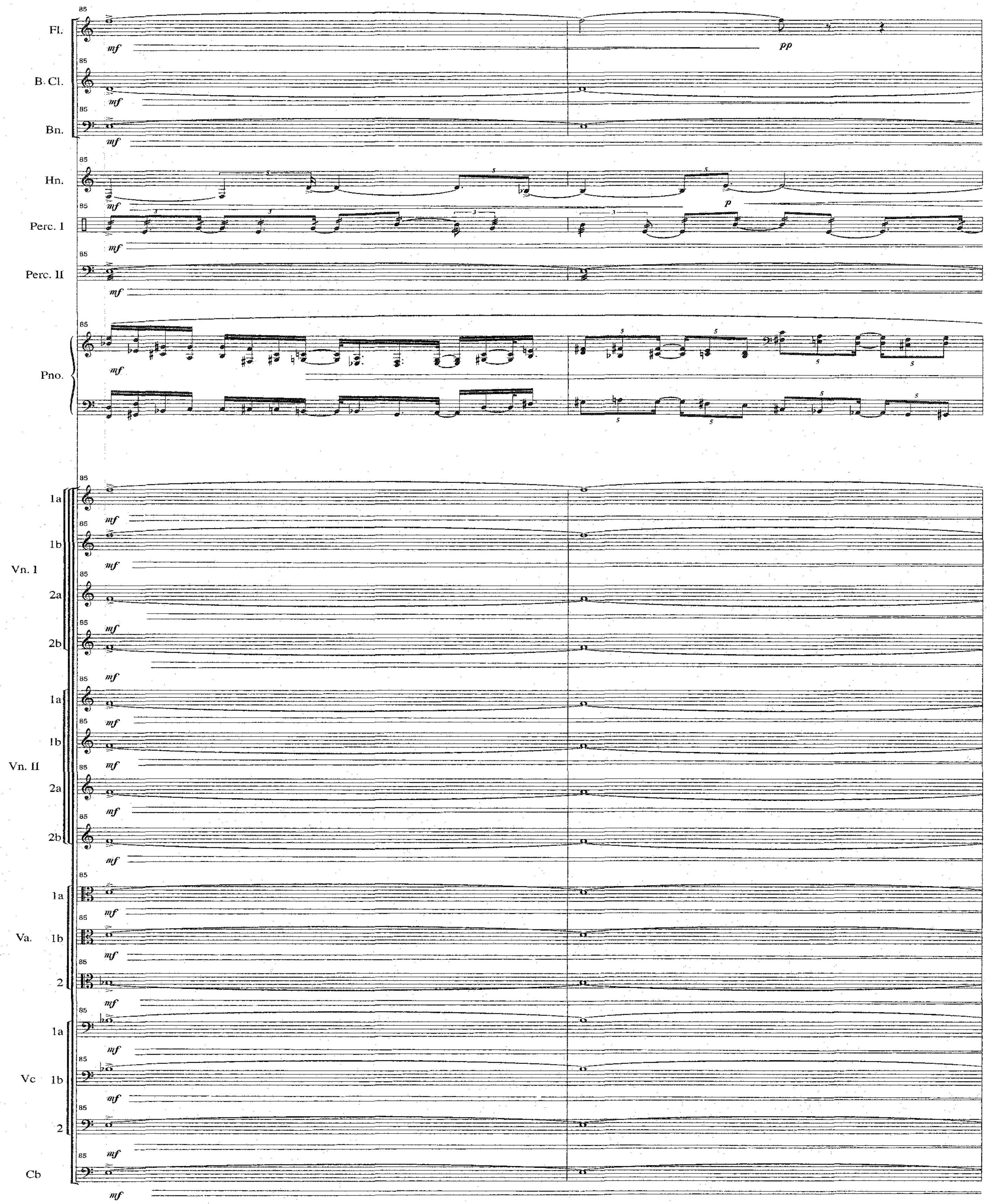

B
55
Fl.
Bn.
Hn.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
Cb
f
ppp
mp
mf


Bn.
Hn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
espr.


63
B♭ Cl.
Bn.
Perc. II
Vc 1a
Vc 2
Cb
espr.
ppp
mp
pp
70
Ob.
B♭ Cl.
Bn.
Va. 1a
Vc 1a
77
Ob.
B♭ Cl.
Perc. II
Vn. I 1a
Vn. I 1b
Va. 1a
Va. 1b
Va. 2
Vc 1a
Vc 1b
gliss.
p
mf

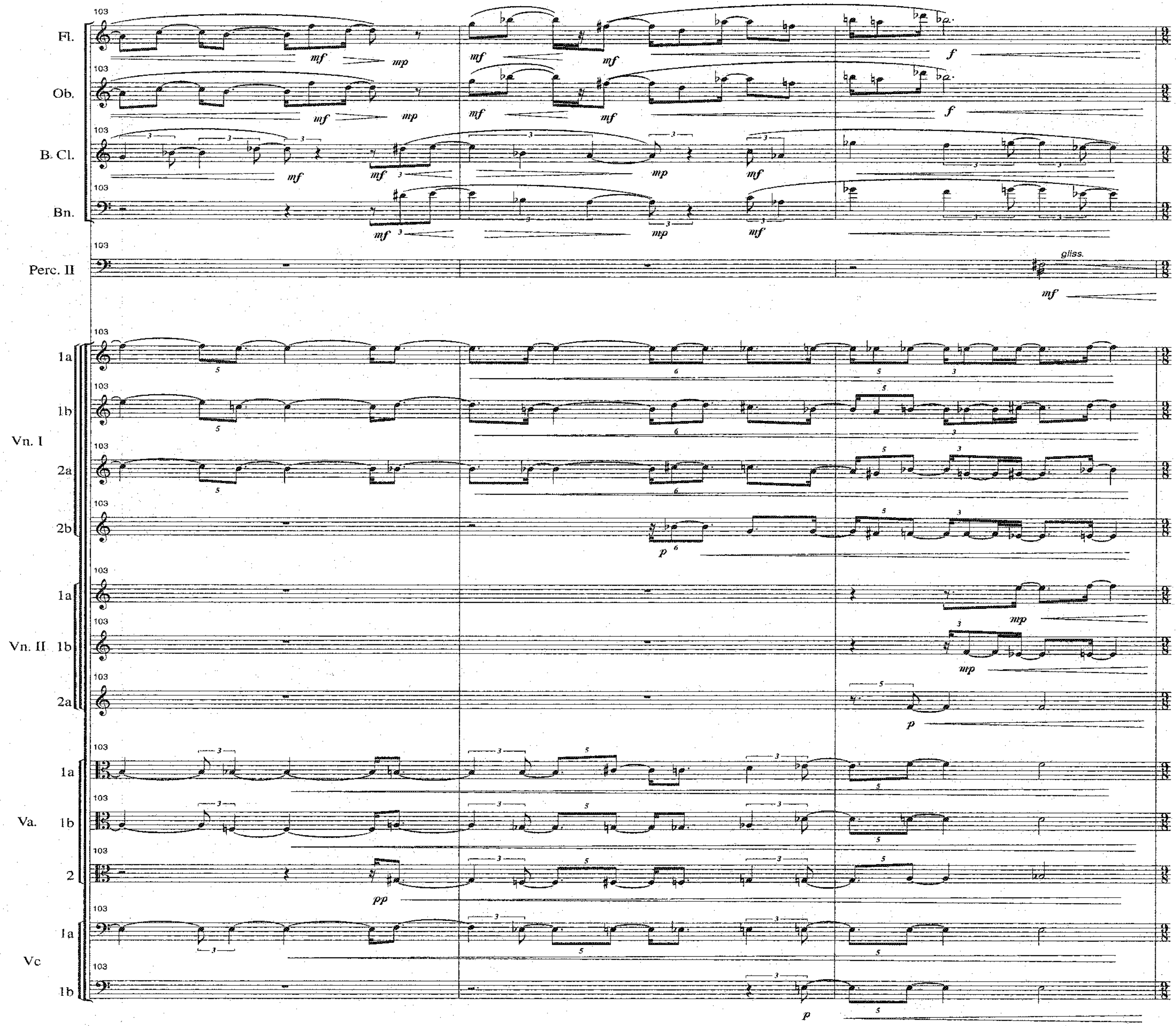

C
81
Fl.
B. Cl.
Bn.
Hn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
f
mp

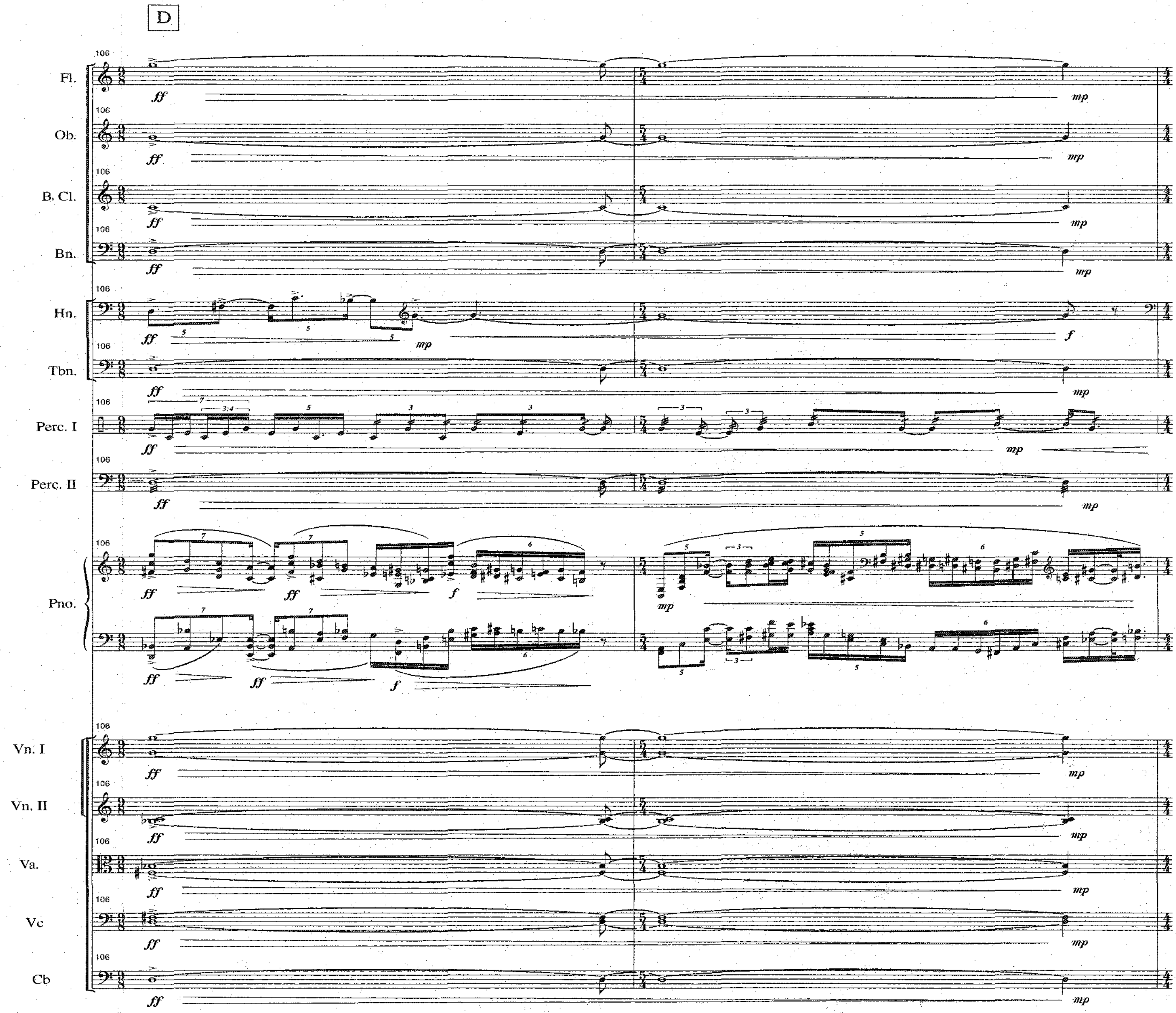

83
Fl.
B♭ Cl.
Bn.
Hn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
p
mf

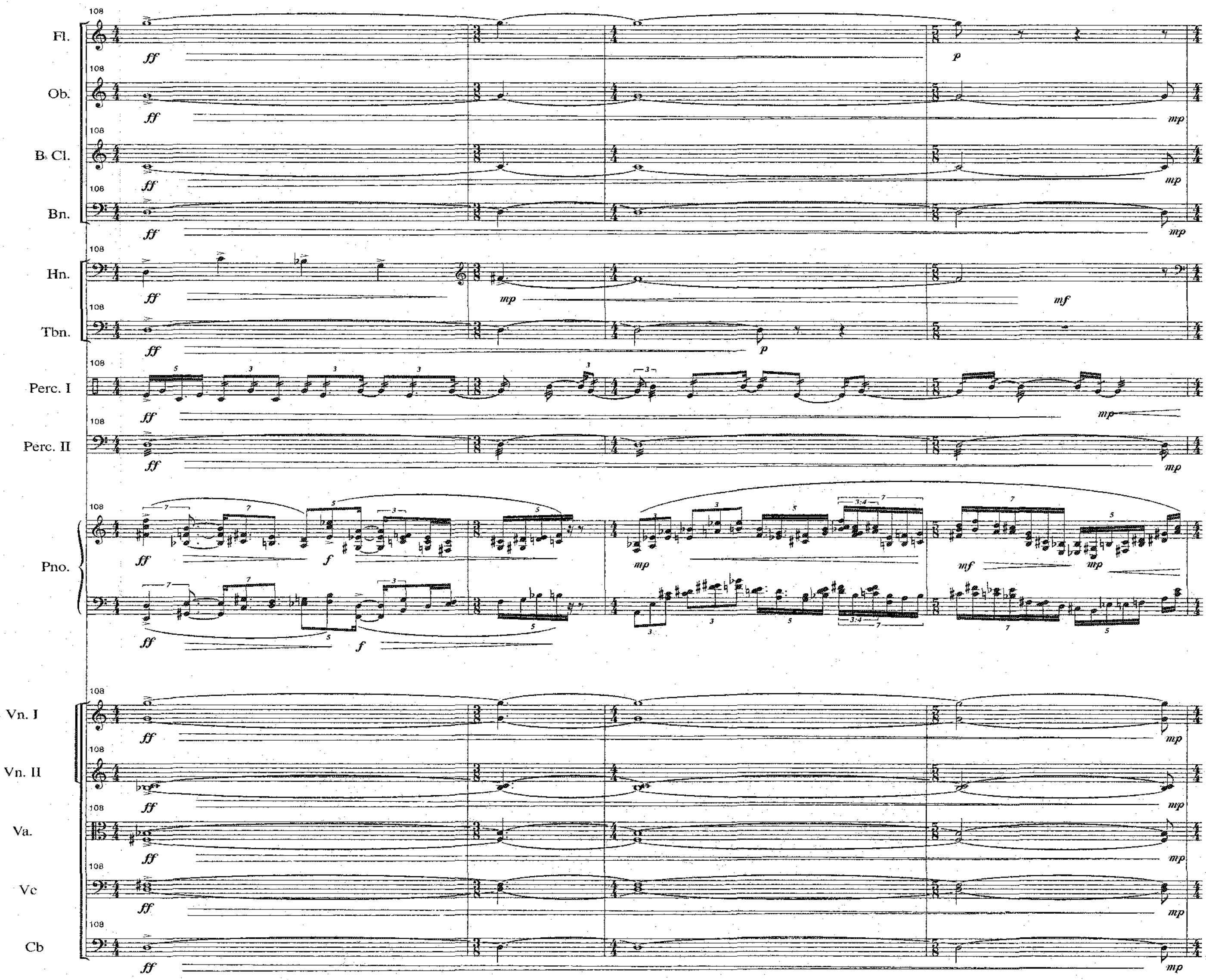

85
Fl.
mf
pp
B. Cl.
Bn.
Hn.
p
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
Va.
2
Vc
Cb

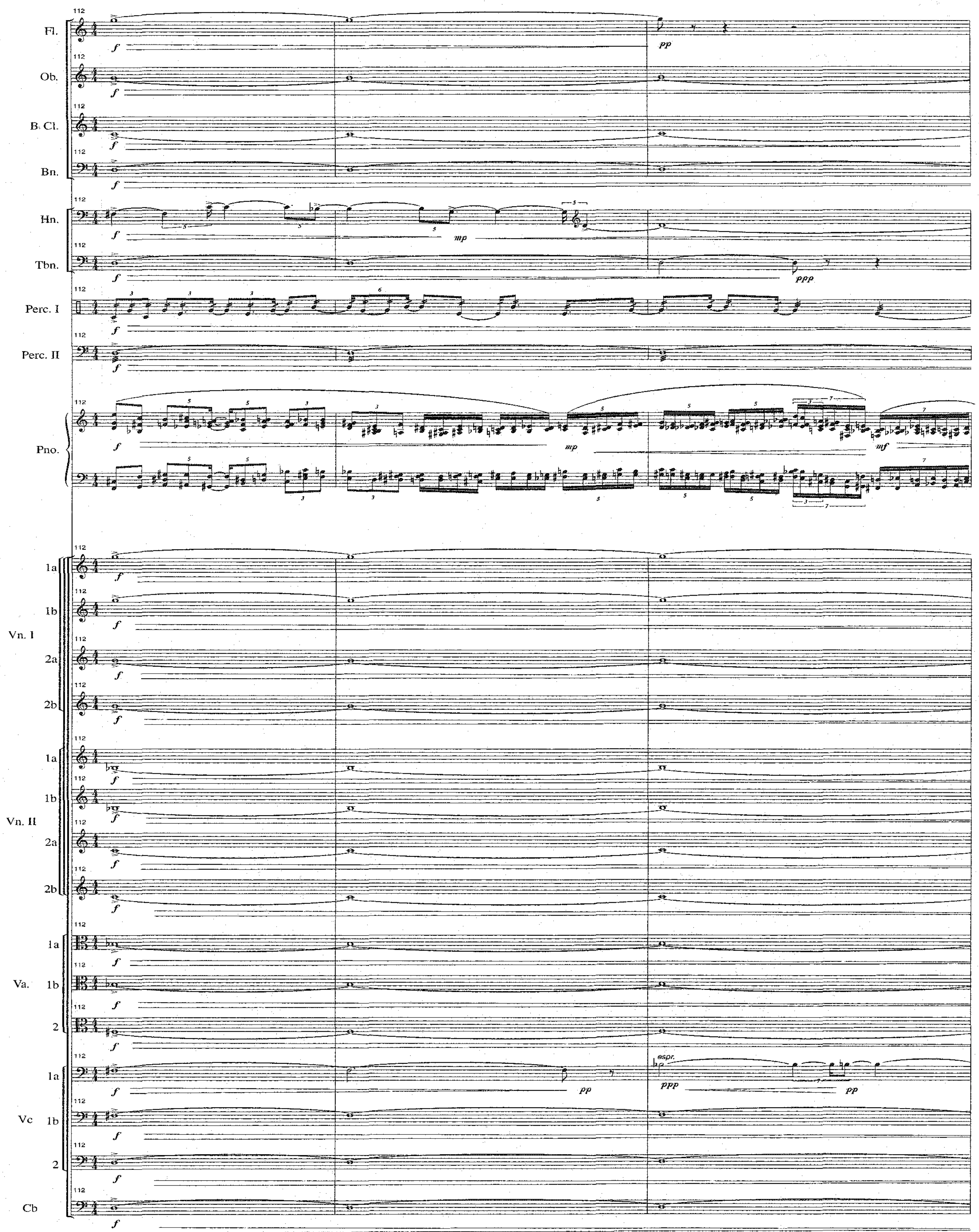

87
B♭ Cl.
pp
Bn.
pp
Hn.
mf
Perc. I
Perc. II
Pno.
p
mf
pp
Vn. I
1a
ppp
1b
ppp
2a
ppp
2b
ppp
Vn. II
1a
ppp
1b
ppp
2a
2b
Va.
1a
1b
2
Vc
1a
ppp
1b
2
Cb


B. Cl.
Perc. I
Perc. II
Vn. I 1a
Vn. II 2a
2b
Va. 1a
1b
2
Vc 1a
1b
2
Cb
espr.
ppp
mp
pp
Fl.
Ob.
B. Cl.
Vn. I 1a
1b
2a
Va. 1a
1b
Vc 1a
mf


103
Fl.
Ob.
B. Cl.
Bn.
Perc. II
gliss.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
Va.
1a
1b
2
Vc
1a
1b
mf
mp
f
p
pp


D
106
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
ff
mp
f

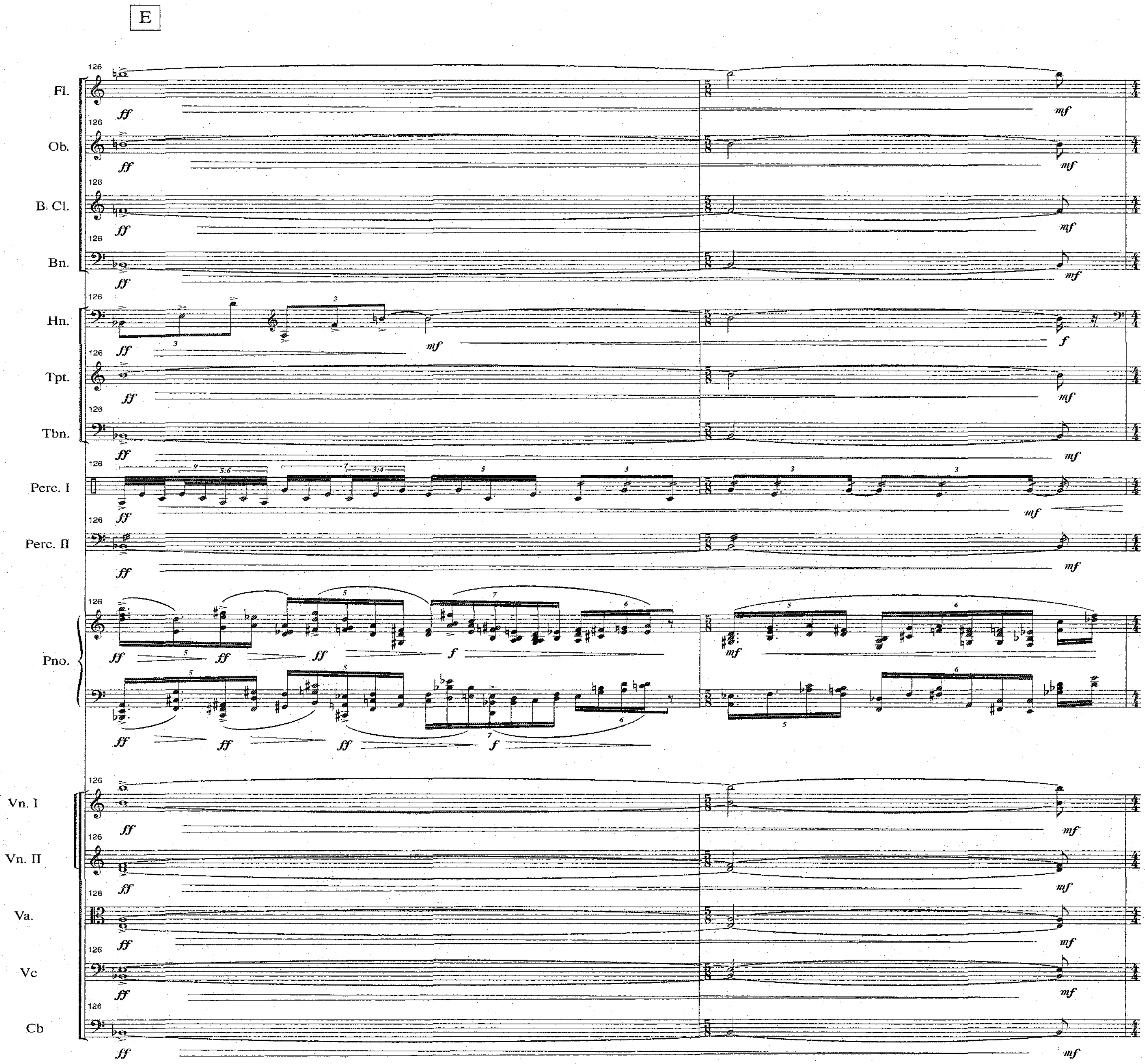

108
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
ff
f
mp
mf
p


Fl.
Ob.
B. Cl.
Bn.
Hn.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
espr.

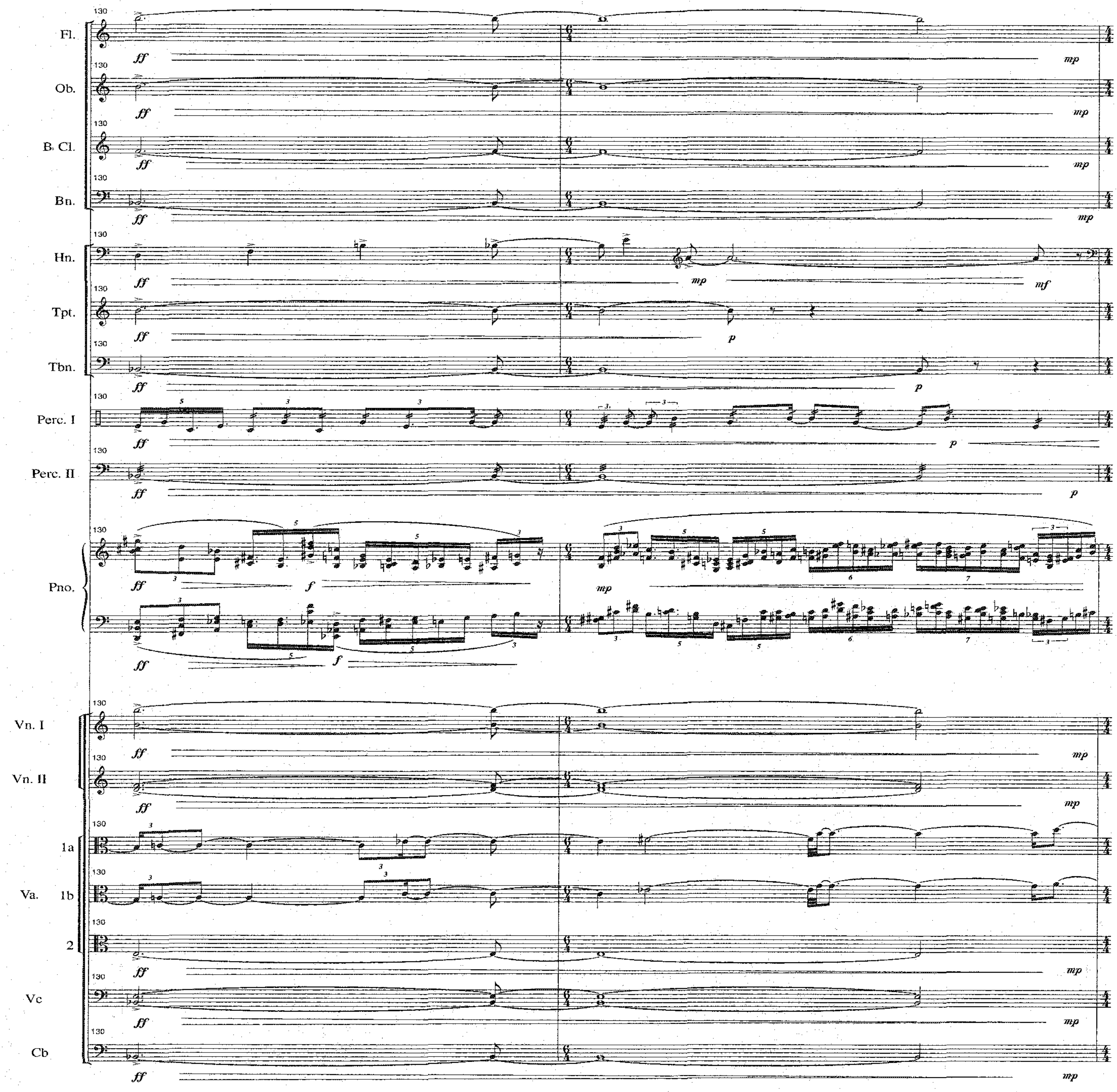

115
Ob.
B. Cl.
Bn.
Hn.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
Cb
espr.
ppp
pp
mf
p

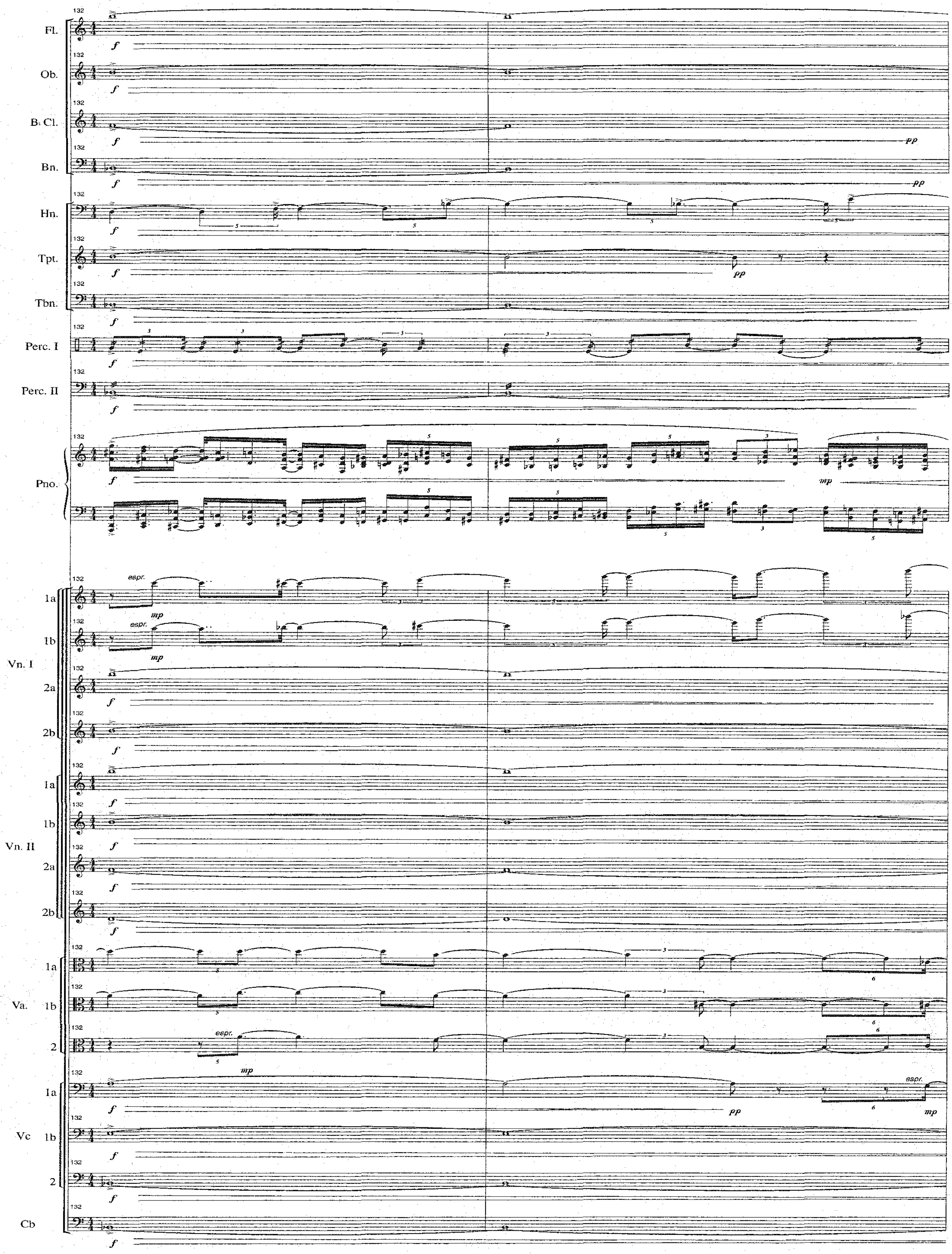

Fl.
Ob.
B♭ Cl.
Bn.
Vn. I
1a
1b
Vn. II
2b
Va.
1a
1b
Vc
1a
1b
2
Cb
espr.


122
Fl.
Ob.
B♭ Cl.
Bn.
Perc. II
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
gliss.
mf
f
pp
p
mp


E
126
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
ff
mf
f

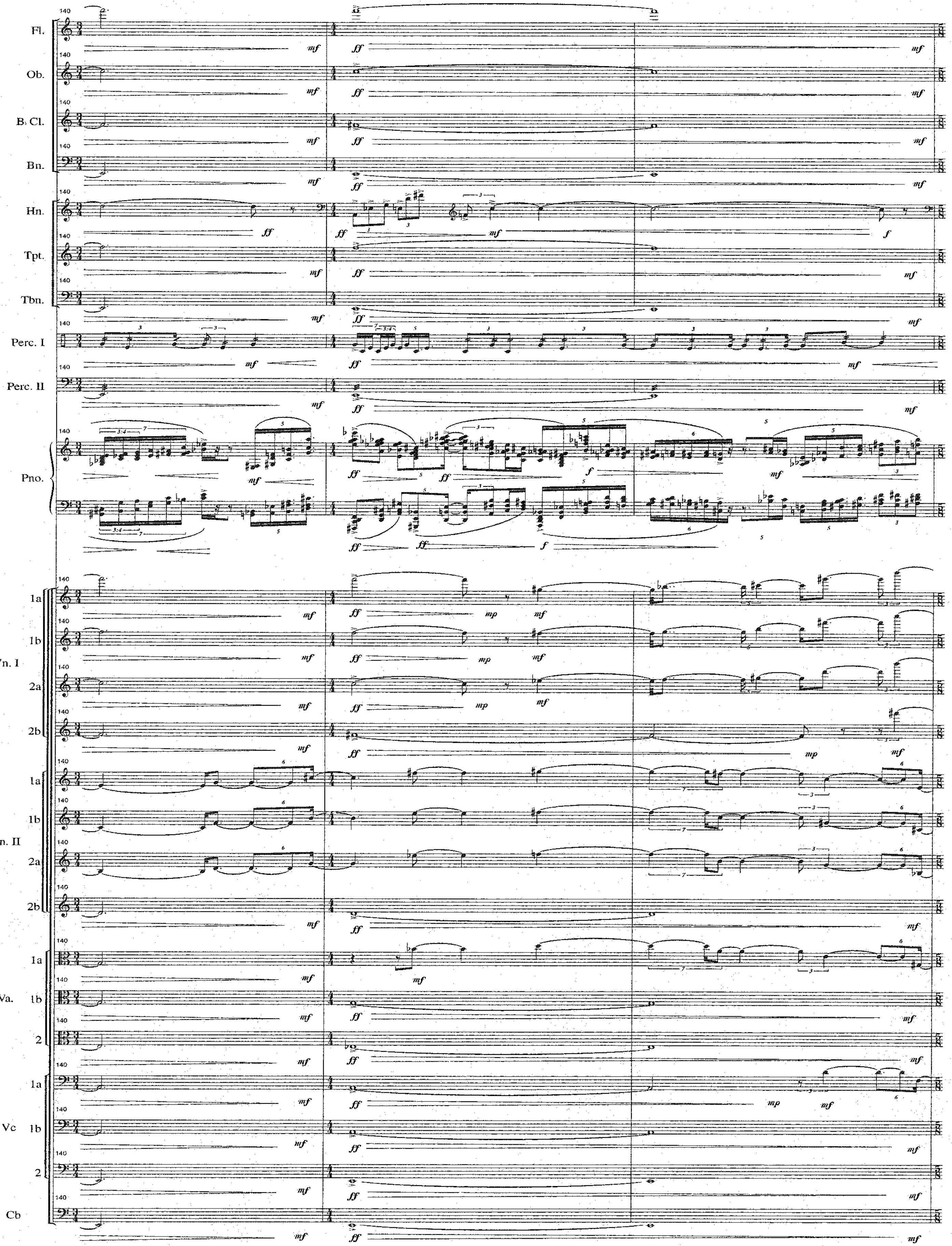

128
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
1a
Va. 1b
2
Vc
Cb
ff
mp
f
mf
pp
espr.

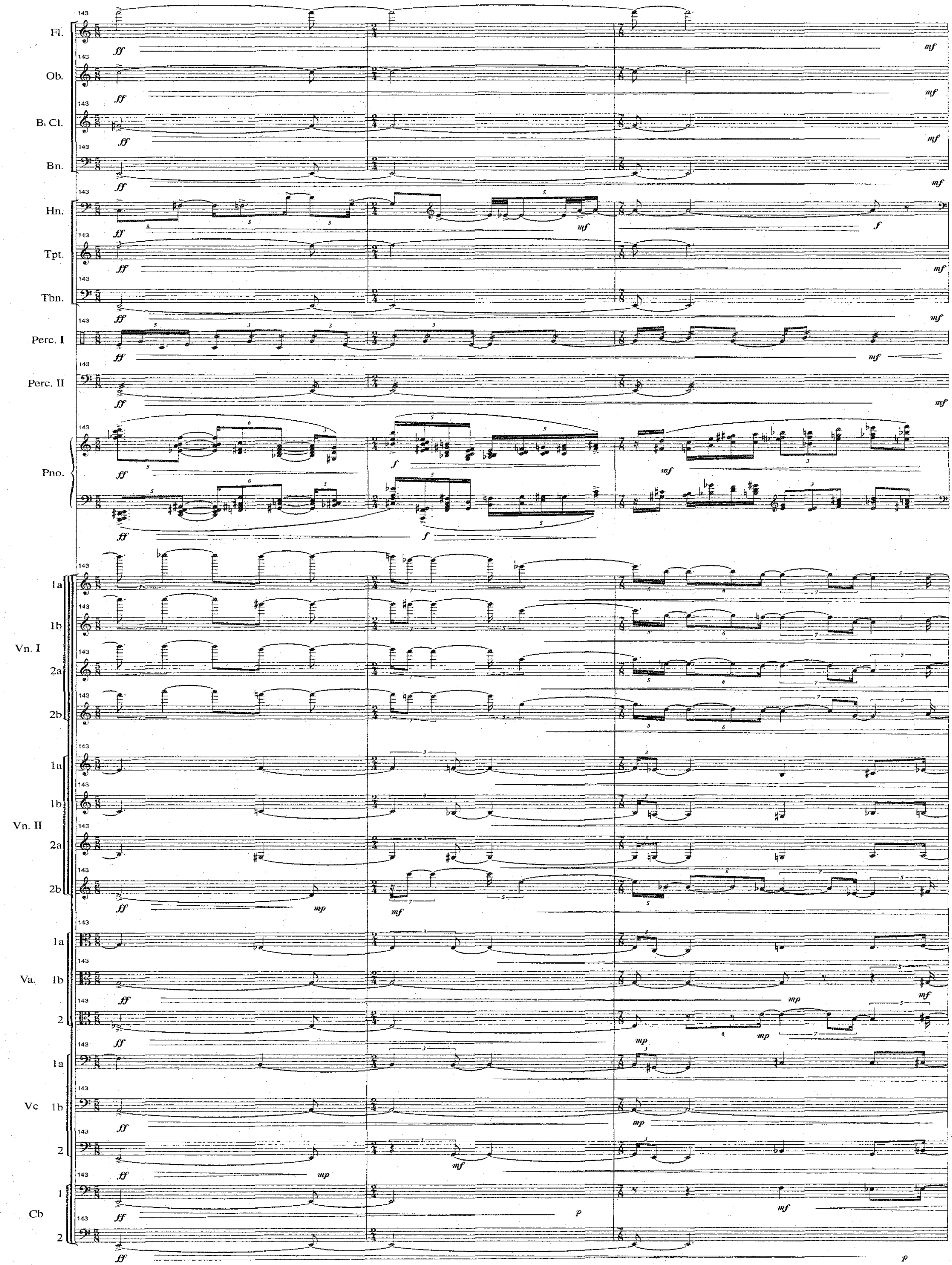

130
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
1a
Va.
1b
2
Vc
Cb
ff
mp
mf
p
f


132
Fl.
Ob.
B♭ Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
Cb
espr.
f
mp
pp


134
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
espr.
Vn. II
Va.
Vc
Cb

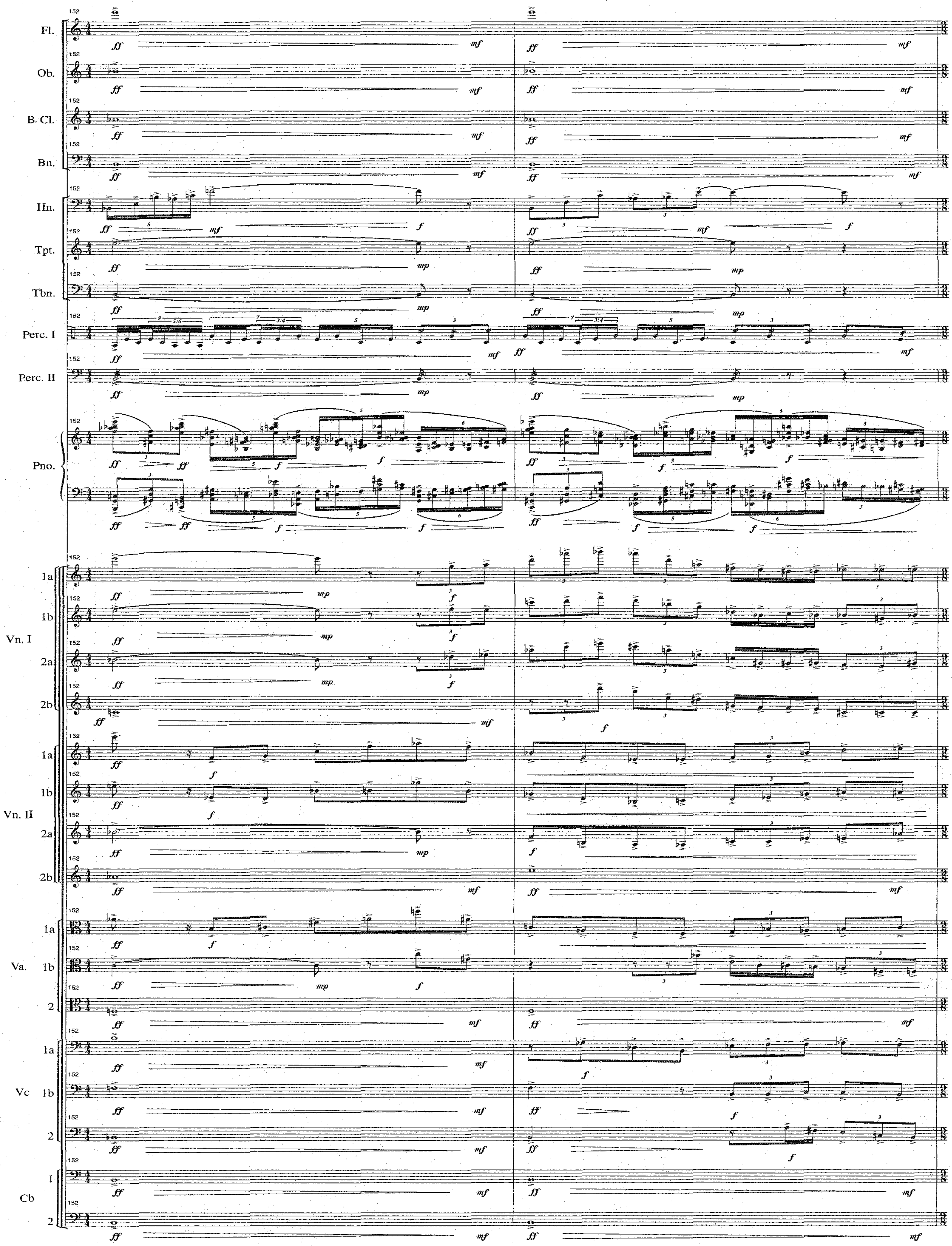

F
137
Fl.
Ob.
B♭ Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
Cb


Fl.
Ob.
B♭ Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb


143
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
Cb
1
2
ff
mf
f
mp
p


Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb

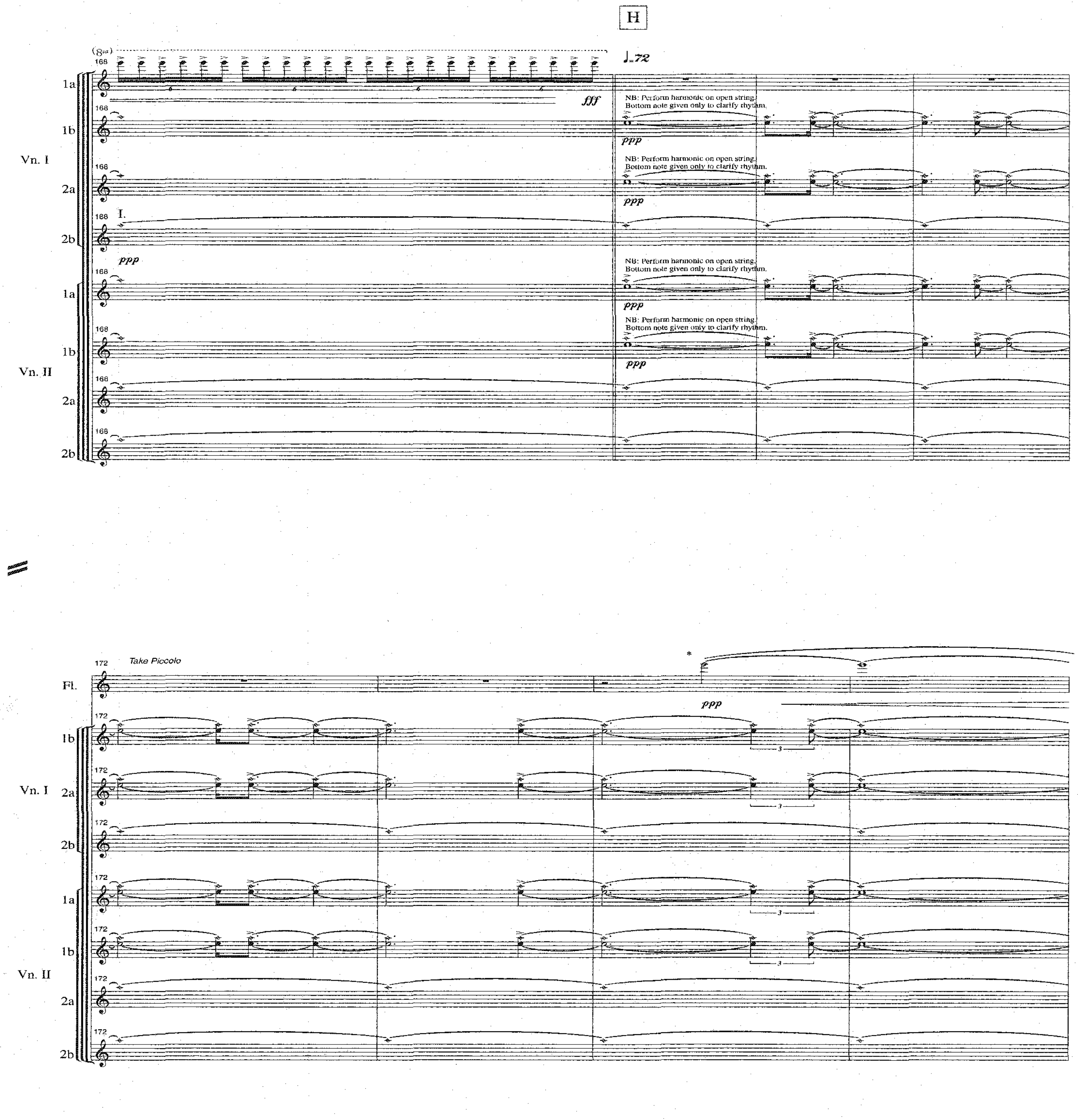

149
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
Cb
1
2
ff
mf
f


Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb

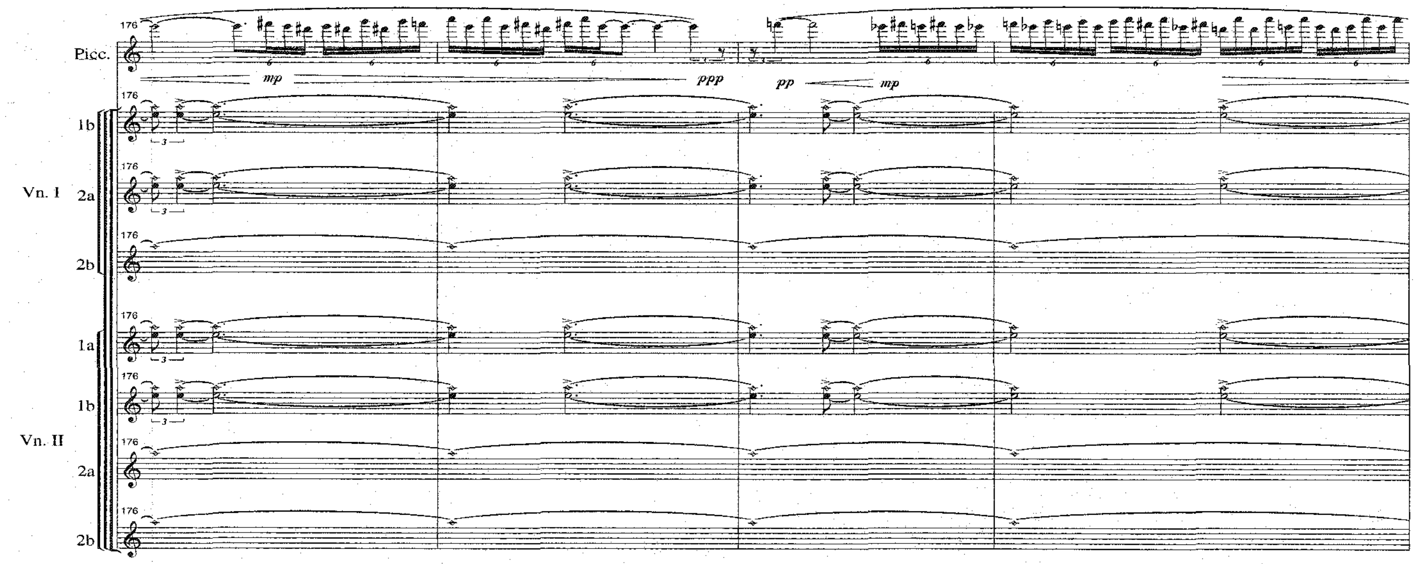

G
Fl.
Ob.
B♭ Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
pizz.

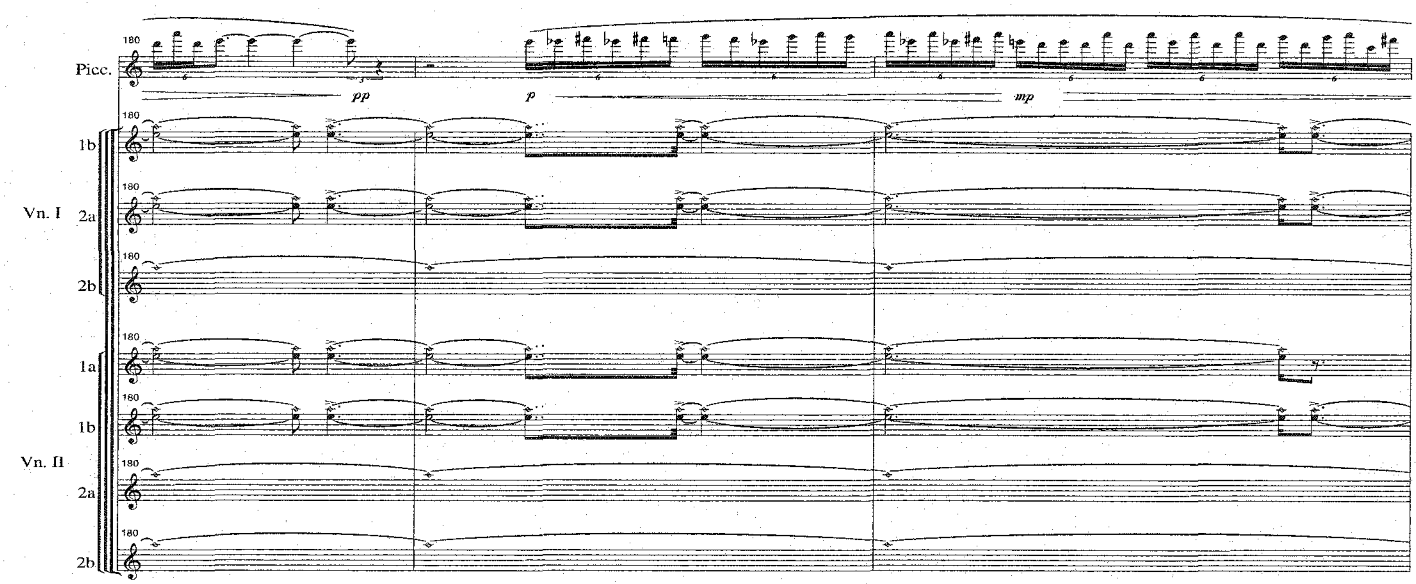

Fl.
Ob.
B. Cl.
Bn.
Hn.
Perc. I
Perc. II
Pno.
diminuendo
Vn. I
Vn. II
Va.
Vc

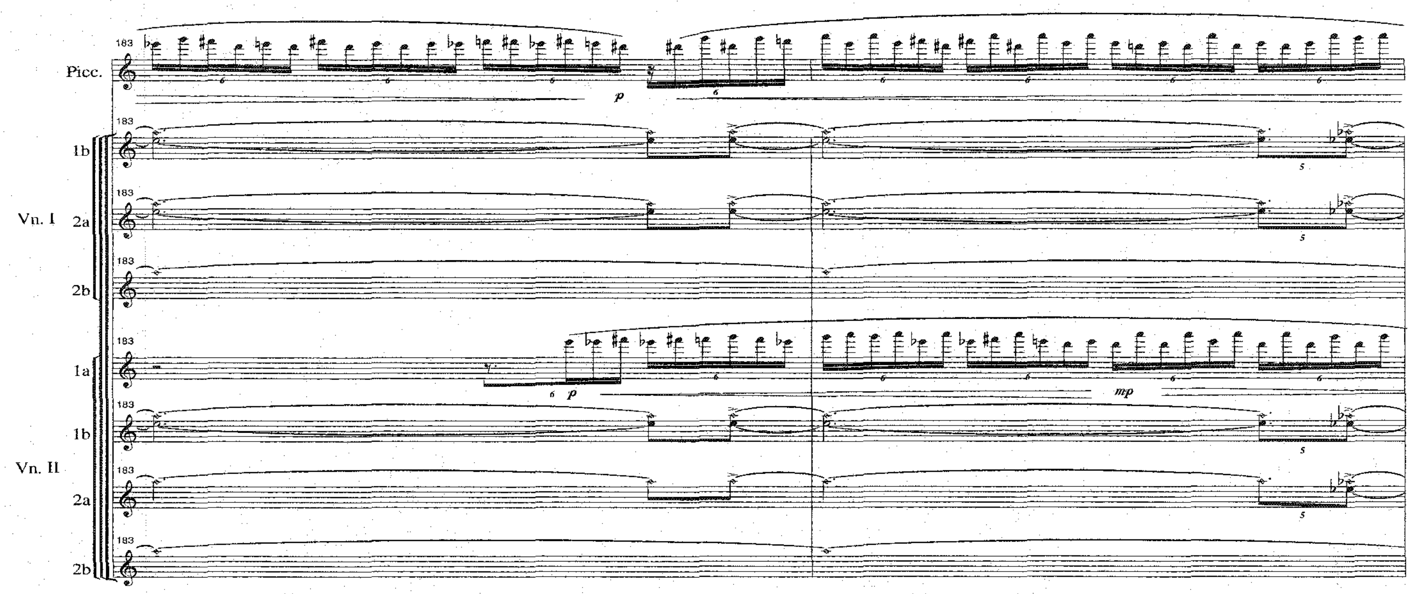
Fl.
Ob.
B. Cl.
Hn.
Perc. I
Pno.
Vn. I
Vn. II
1a
1b
2a
2b
ppp
pp
loco
I.

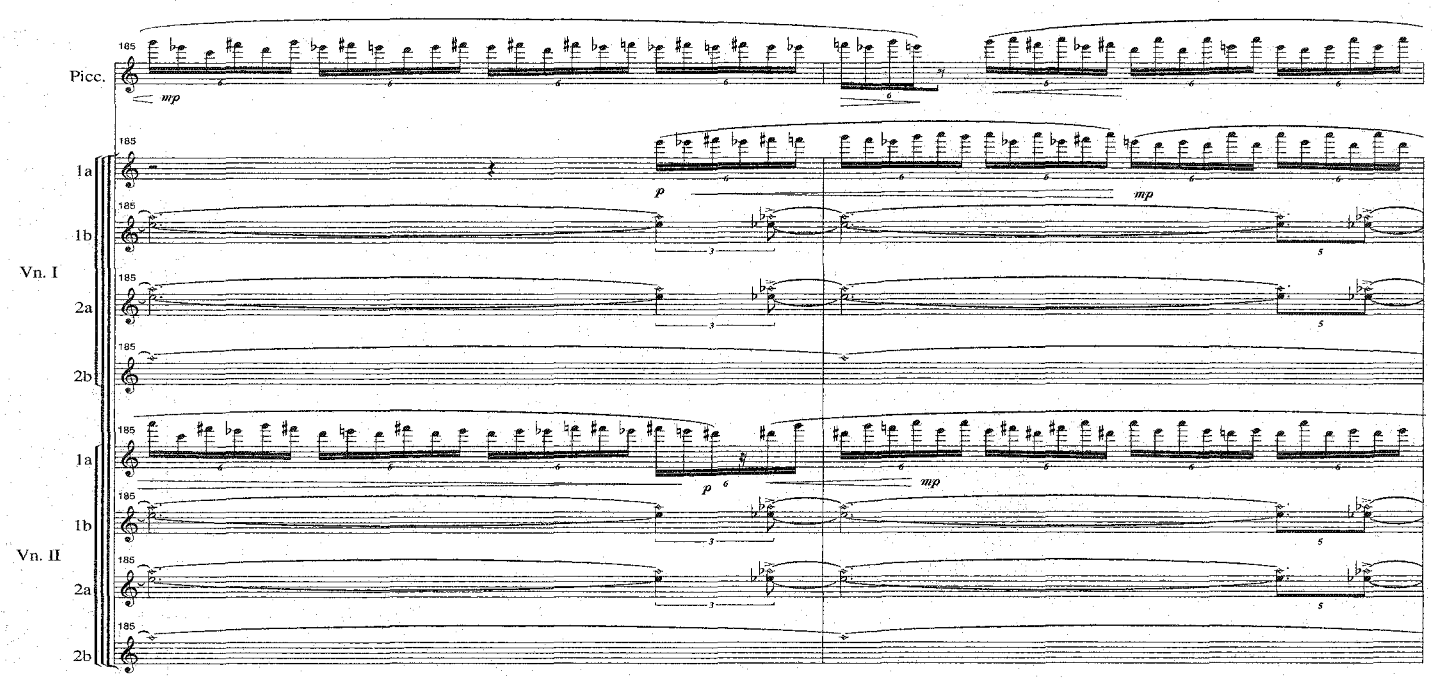


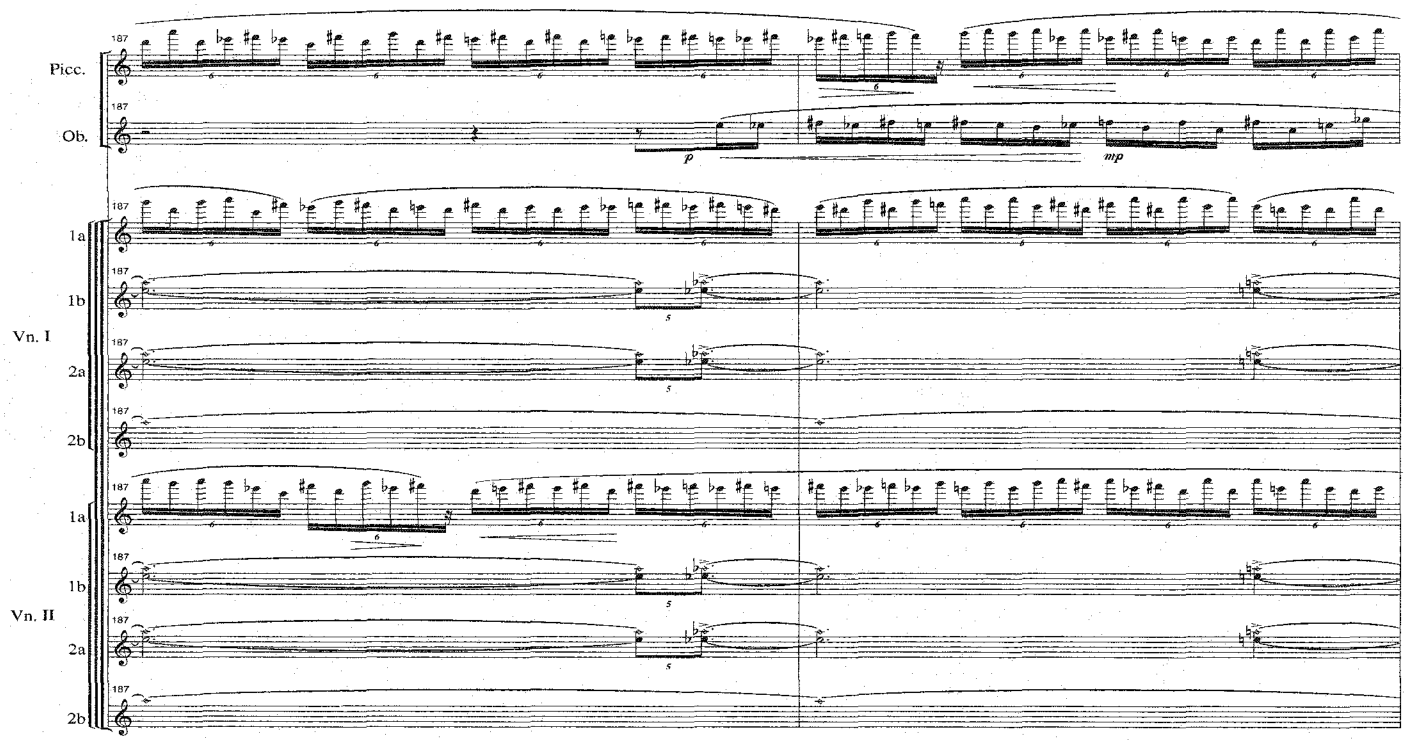
* NB: Piccolo sounds as written

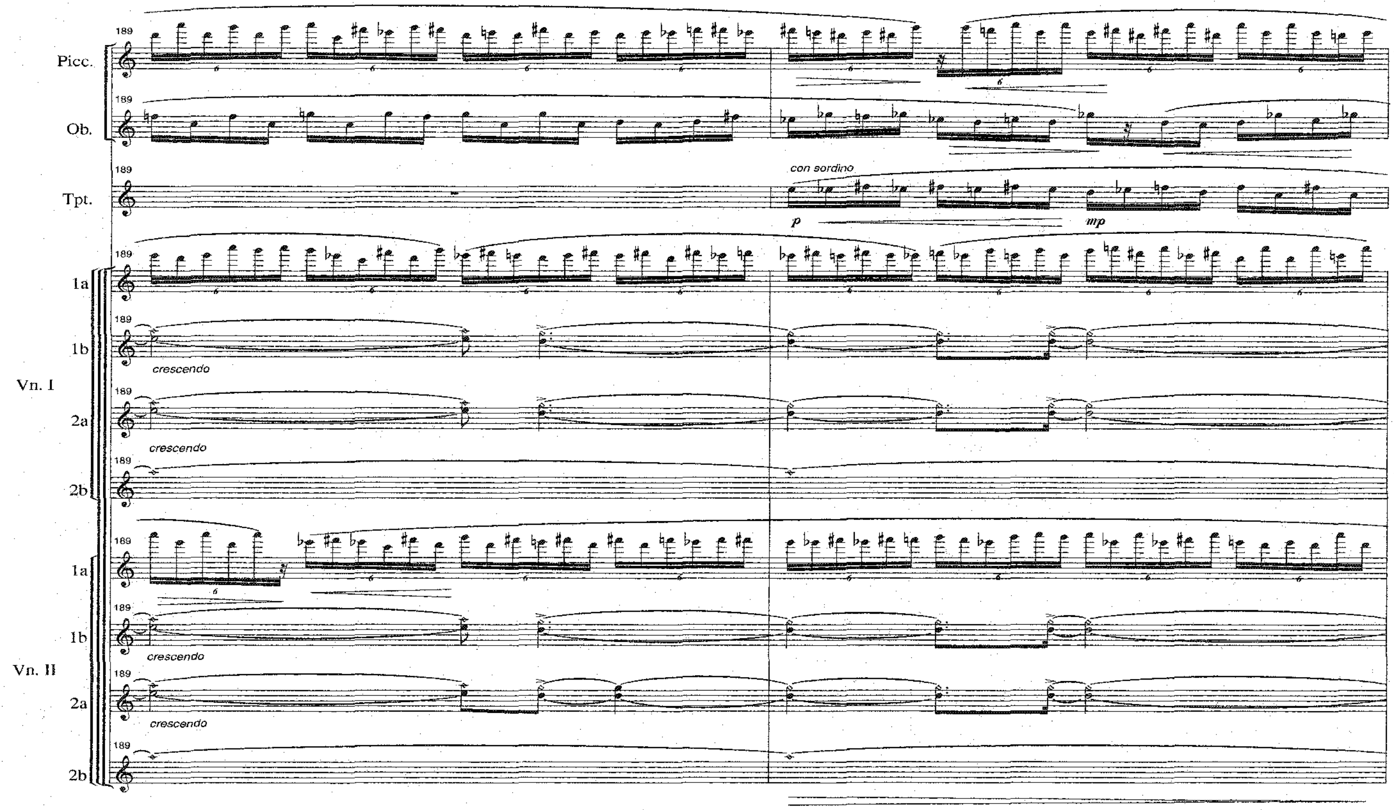

176
Picc.
mp
ppp
pp
mp
Vn. I
1b
2a
2b
Vn. II
1a
1b
2a
2b

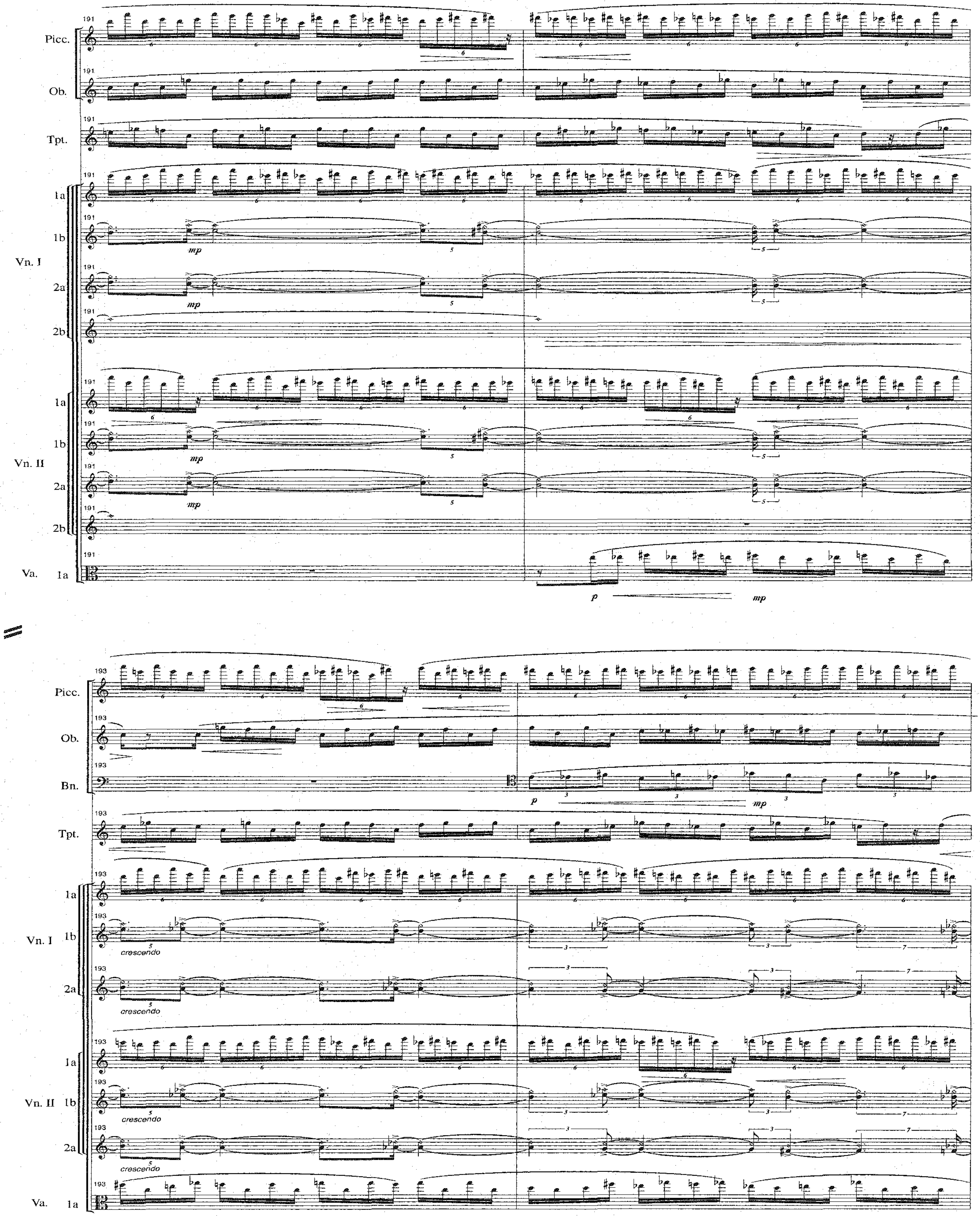

180
Picc.
pp
p
mp
Vn. I
1b
2a
2b
Vn. II
1a
1b
2a
2b

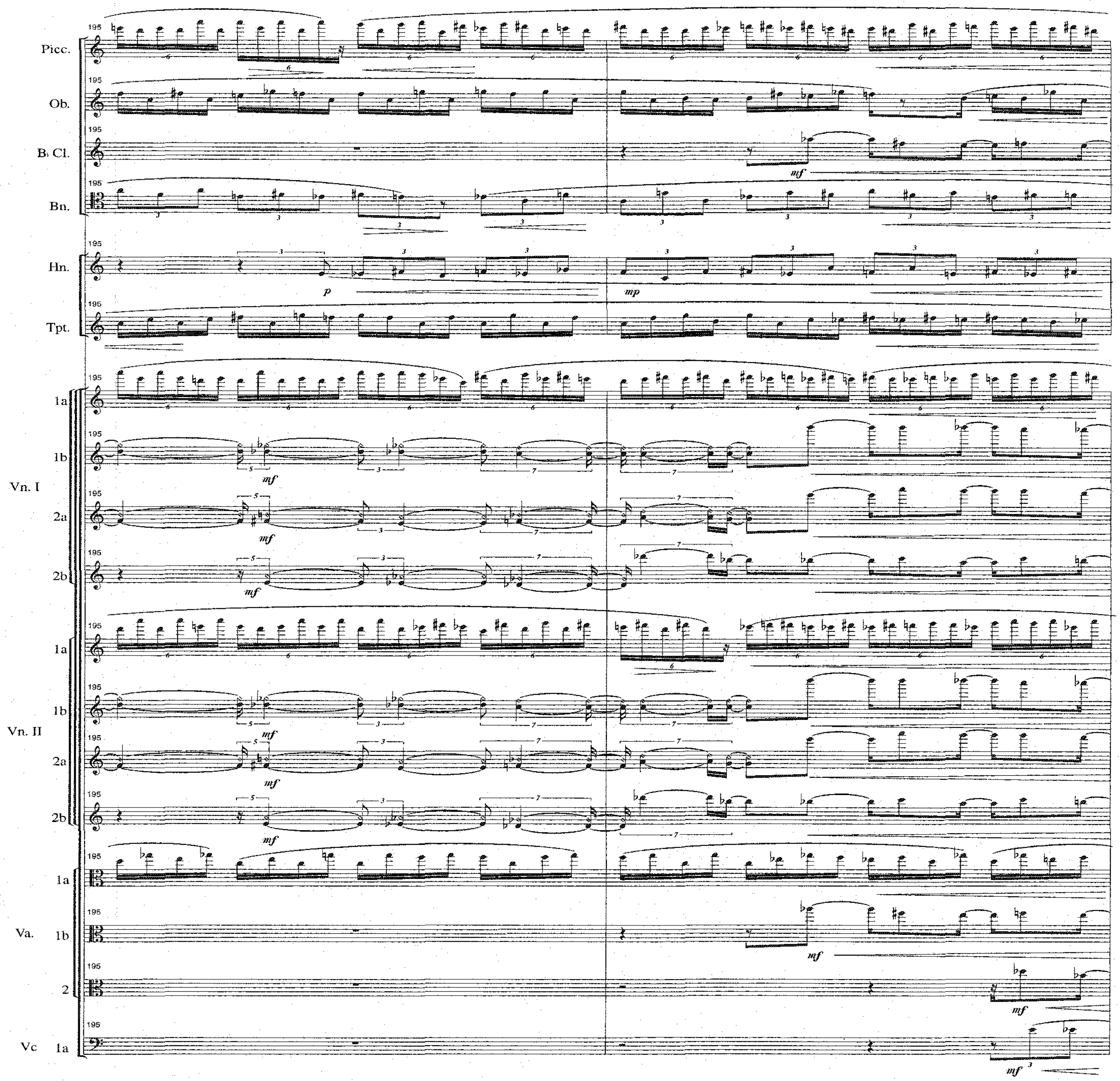

183
Picc.
Vn. I
1b
2a
2b
Vn. II
1a
1b
2a
2b
p
mp


185
Picc.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
mp
p
mp


187
Picc.
Ob.
p
mp
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b

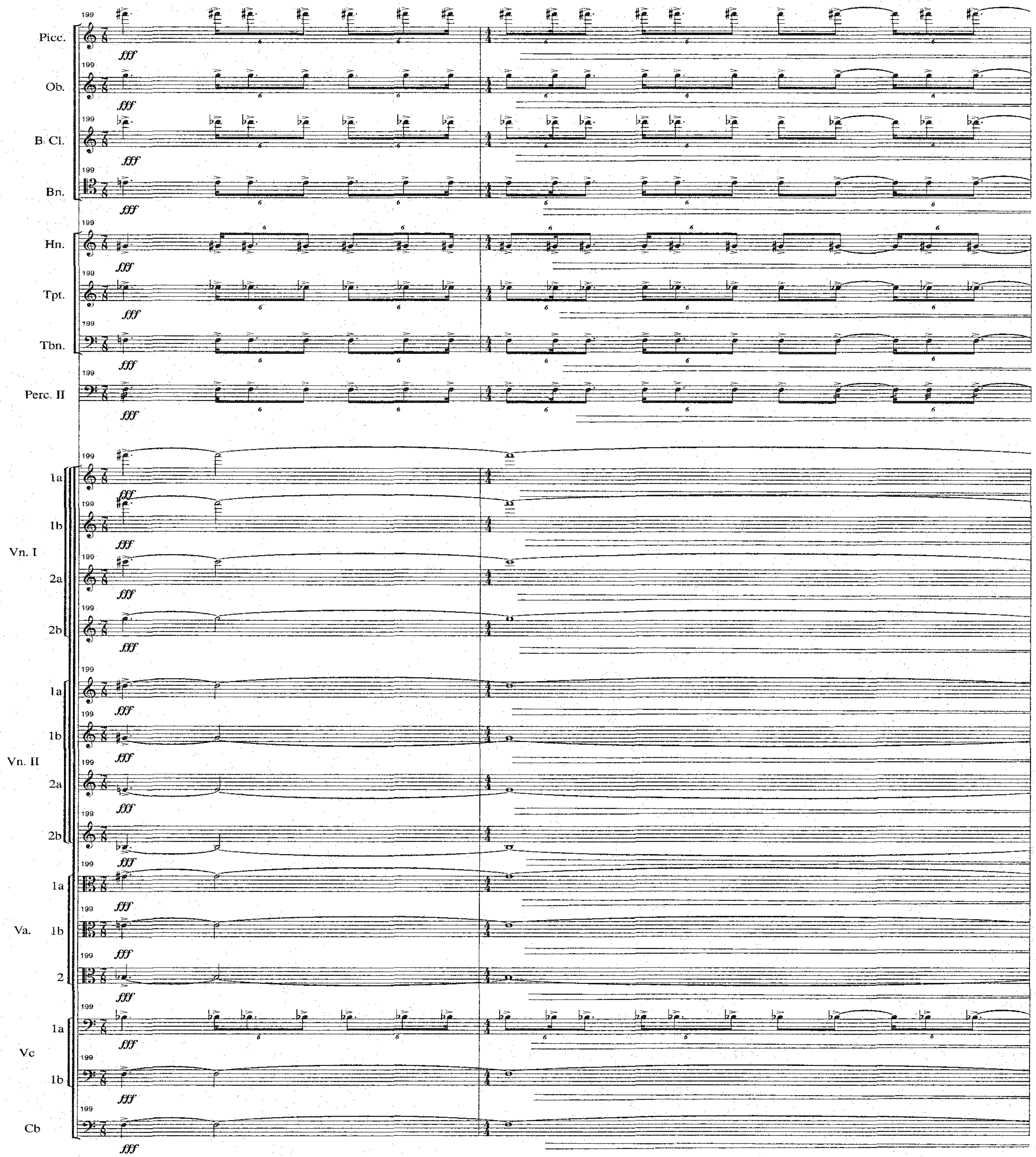

189
Picc.
Ob.
Tpt.
con sordino
p
mp
Vn. I
1a
1b
2a
2b
crescendo
Vn. II
1a
1b
2a
2b

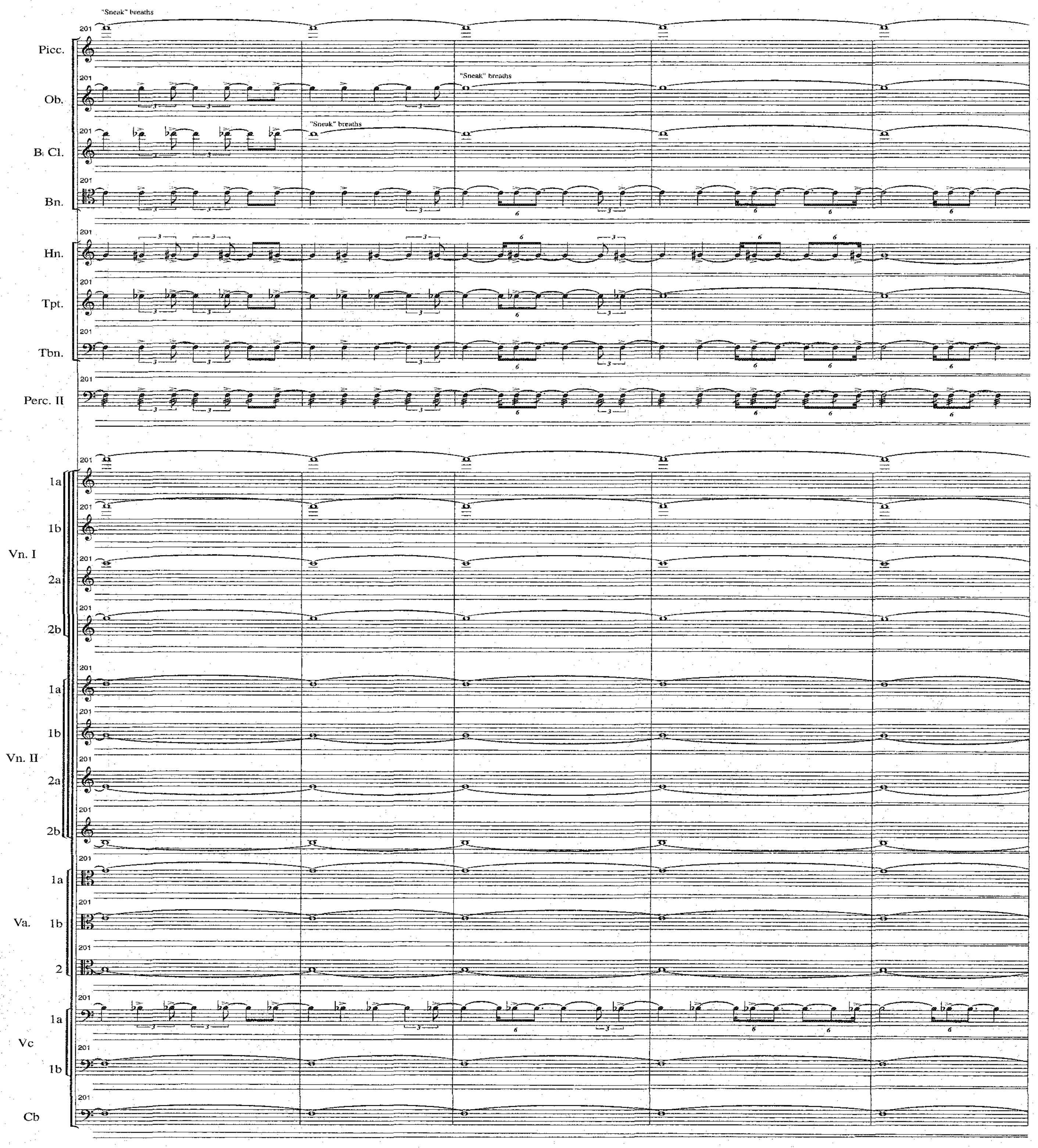

Picc.
Ob.
Tpt.
Vn. I
1a
1b
2a
2b
Vn. II
Va.
mp
p
Bn.
crescendo



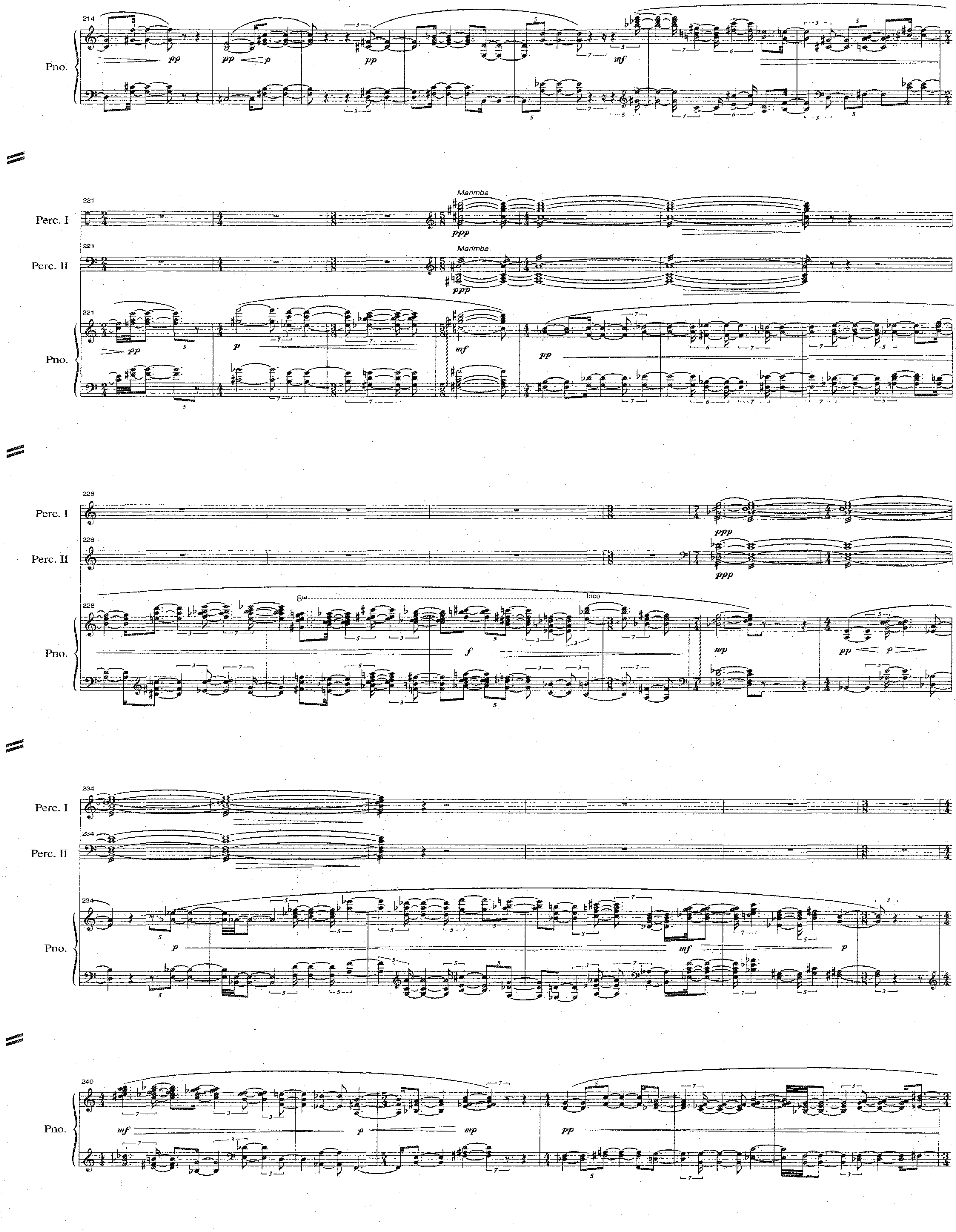
195
Picc.
Ob.
B♭ Cl.
Bn.
Hn.
Tpt.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
p
mp
mf


197
Picc.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. II
gliss.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
Cb
1
2

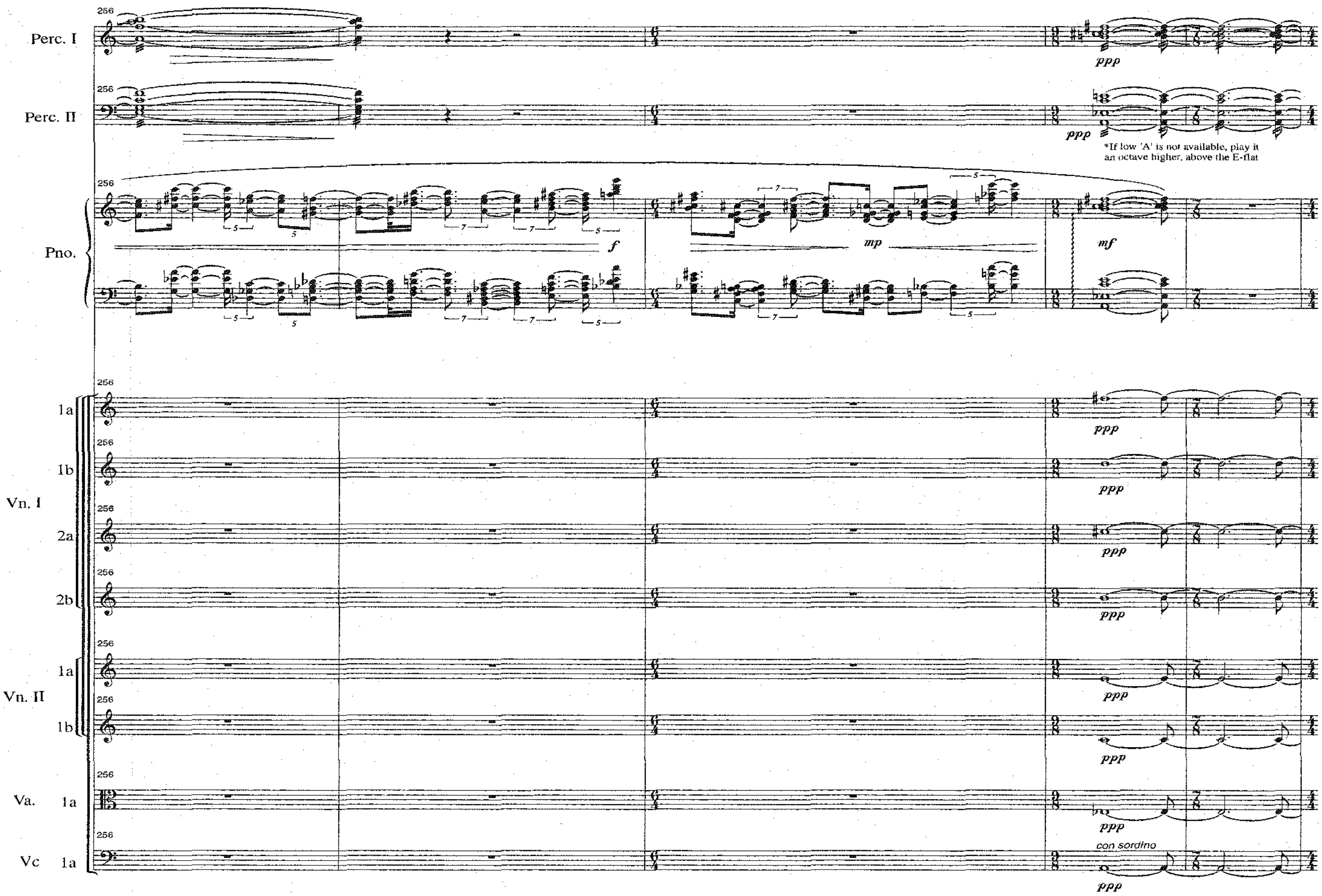

Picc.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. II
Vn. I
Vn. II
Va.
Vc
Cb
fff


"Sneak" breaths
"Sneak" breaths
"Sneak" breaths
201
Picc.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. II
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
Cb


I
Take Flute
remove mute
Picc.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb

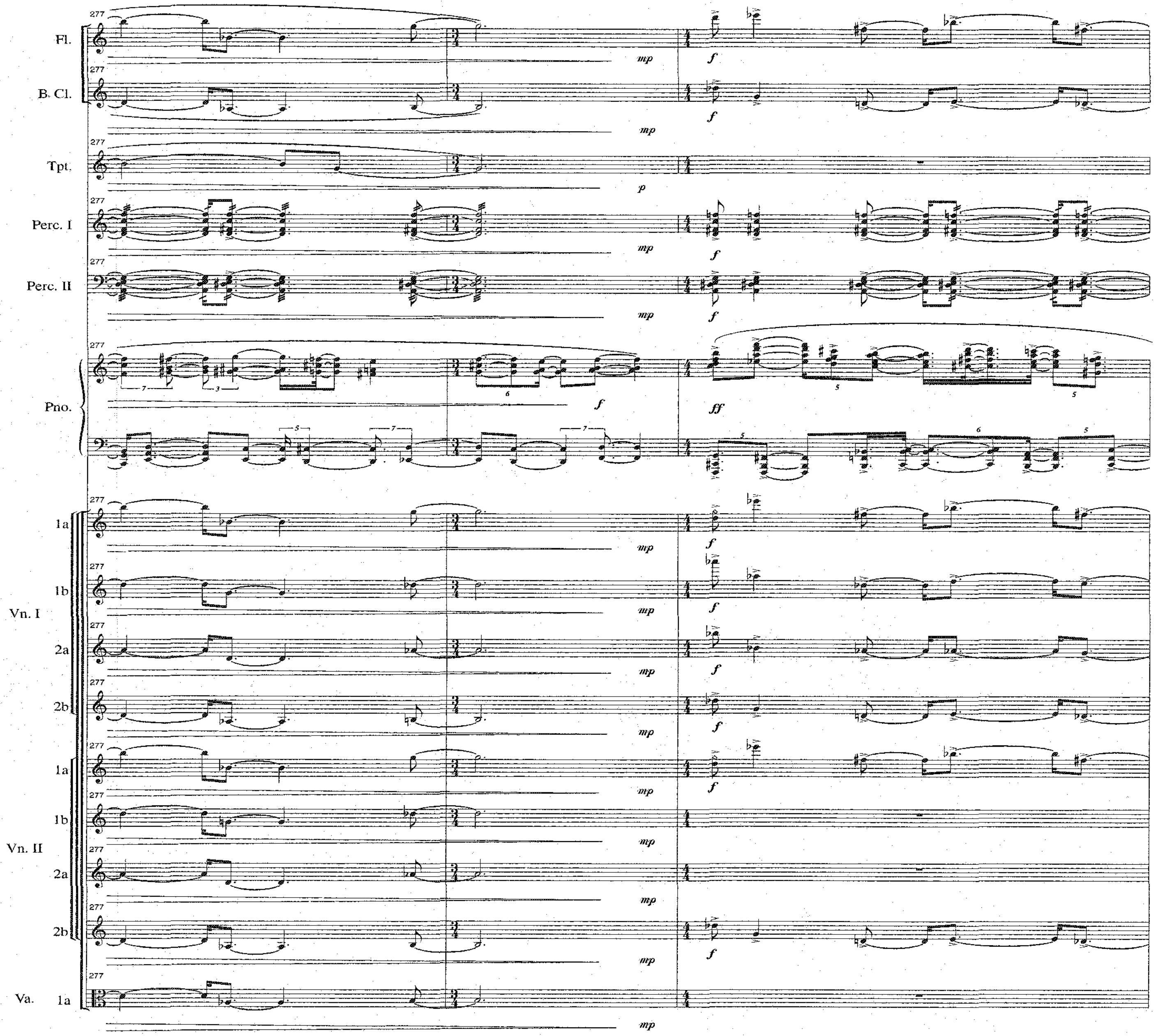

Pno.
Perc. I
Perc. II
Marimba
Marimba
Pno.
Perc. I
Perc. II
8va
loco
Pno.
Perc. I
Perc. II
Pno.
Pno.


246
Pno.
mp
p
mf
f
251
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
Va.
con sordino
ppp


Perc. I
Perc. II
*If low 'A' is not available, play it an octave higher, above the E-flat
Pno.
Vn. I
Vn. II
Va.
Vc
con sordino

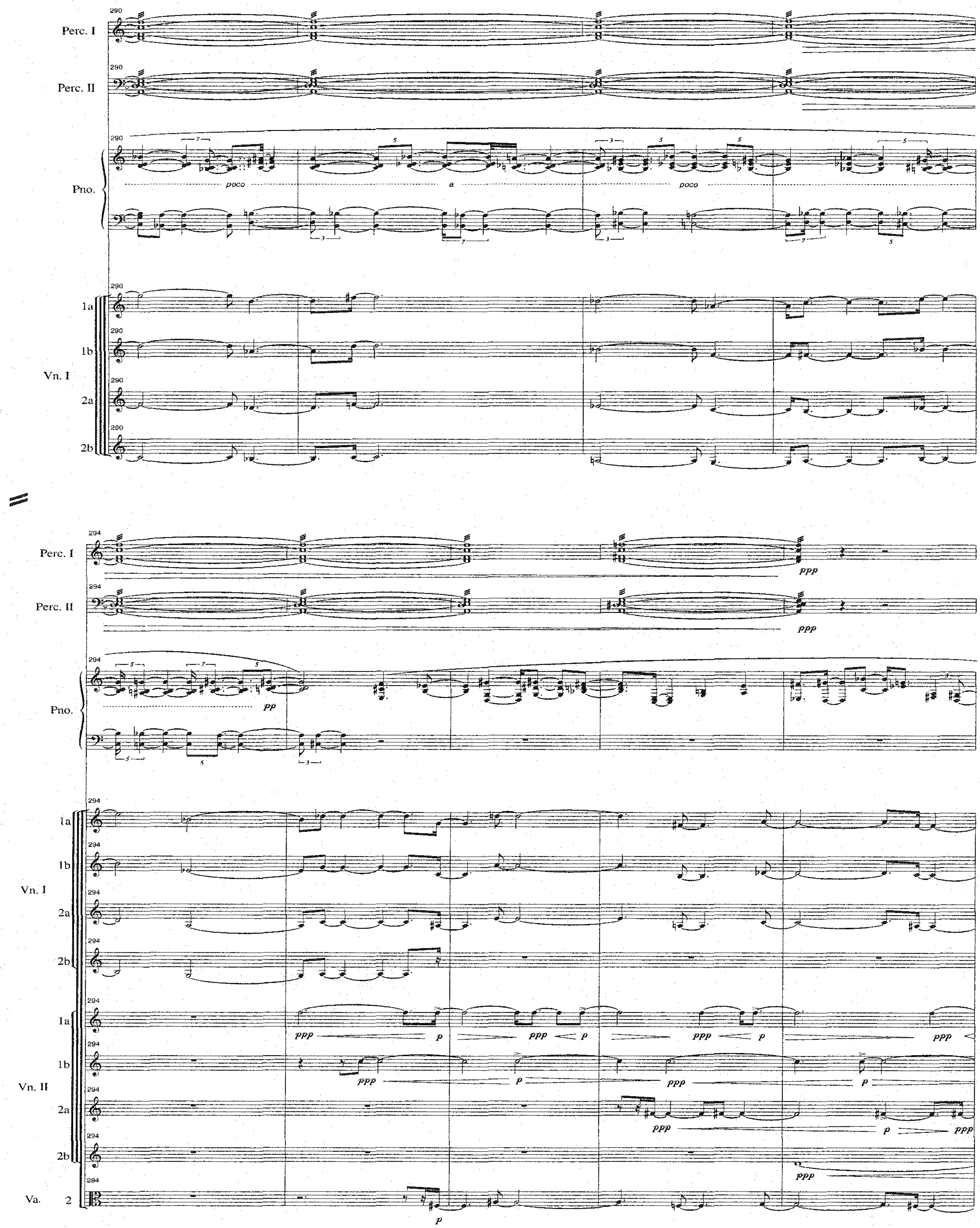

261
Perc. I
Perc. II
Pno.
pp
crescendo
mf
ff
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
Va. 1a
Vc 1a
senza sordino
senza sordino
265
Perc. I
Perc. II
Pno.
p
pp
crescendo
f
270
Pno.
ff

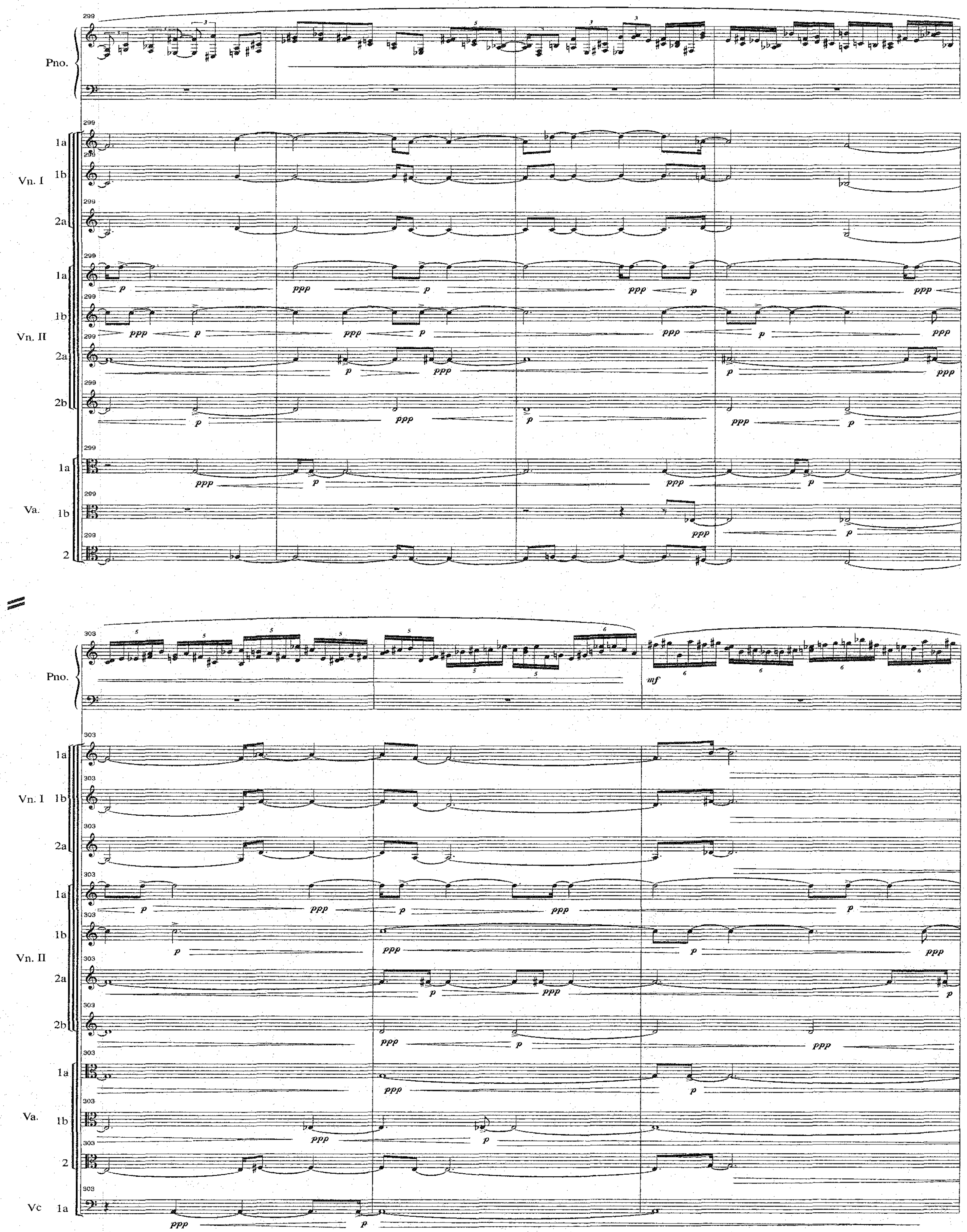

♩=96
274
Fl.
B. Cl.
Tpt.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
ff
mf
fff
senza sordino
8va


277
Fl.
B. Cl.
Tpt.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
mp
p
f
ff


Fl.
B. Cl.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
2b
280


283
Fl.
B♭ Cl.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
p
mp
mf
f
286
pp
diminuendo


Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
Va. 2
290
294
poco
a
poco
pp
ppp
p

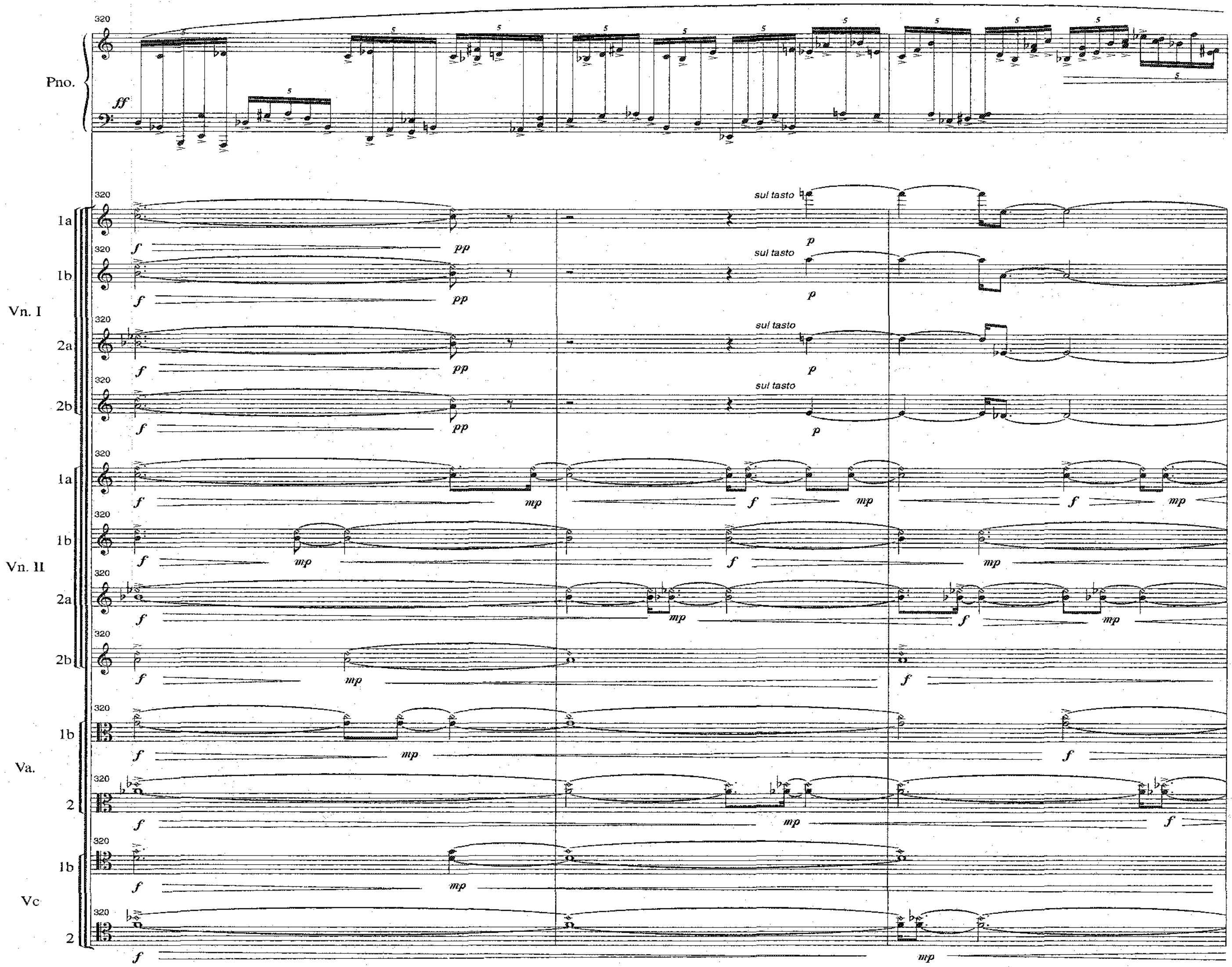


299
Pno.
1a
1b
Vn. I
2a
1a
1b
Vn. II
2a
2b
1a
Va.
1b
2
ppp
p
303
mf
Vc

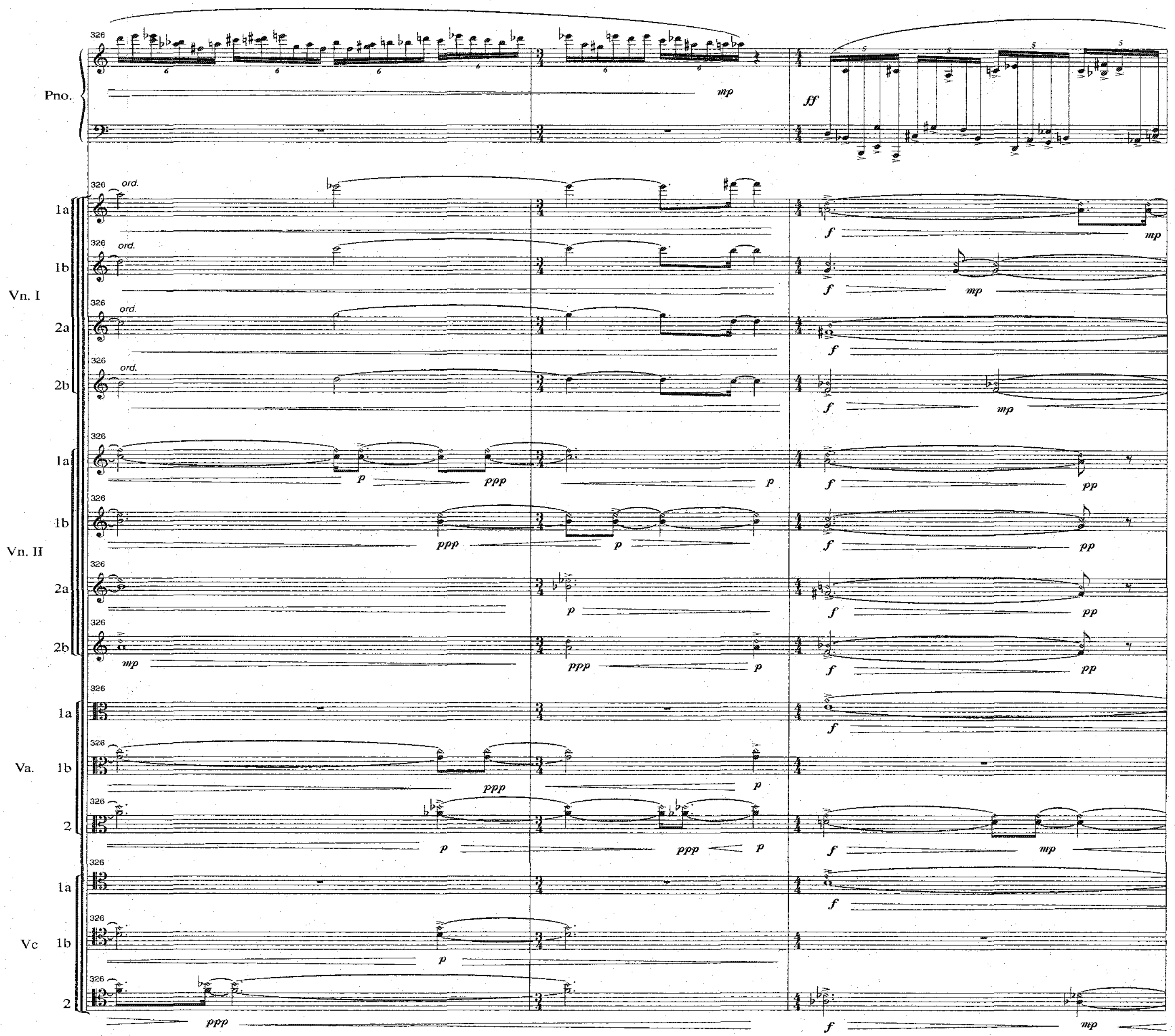

306
Pno.
Vn. I
Vn. II
Va.
Vc

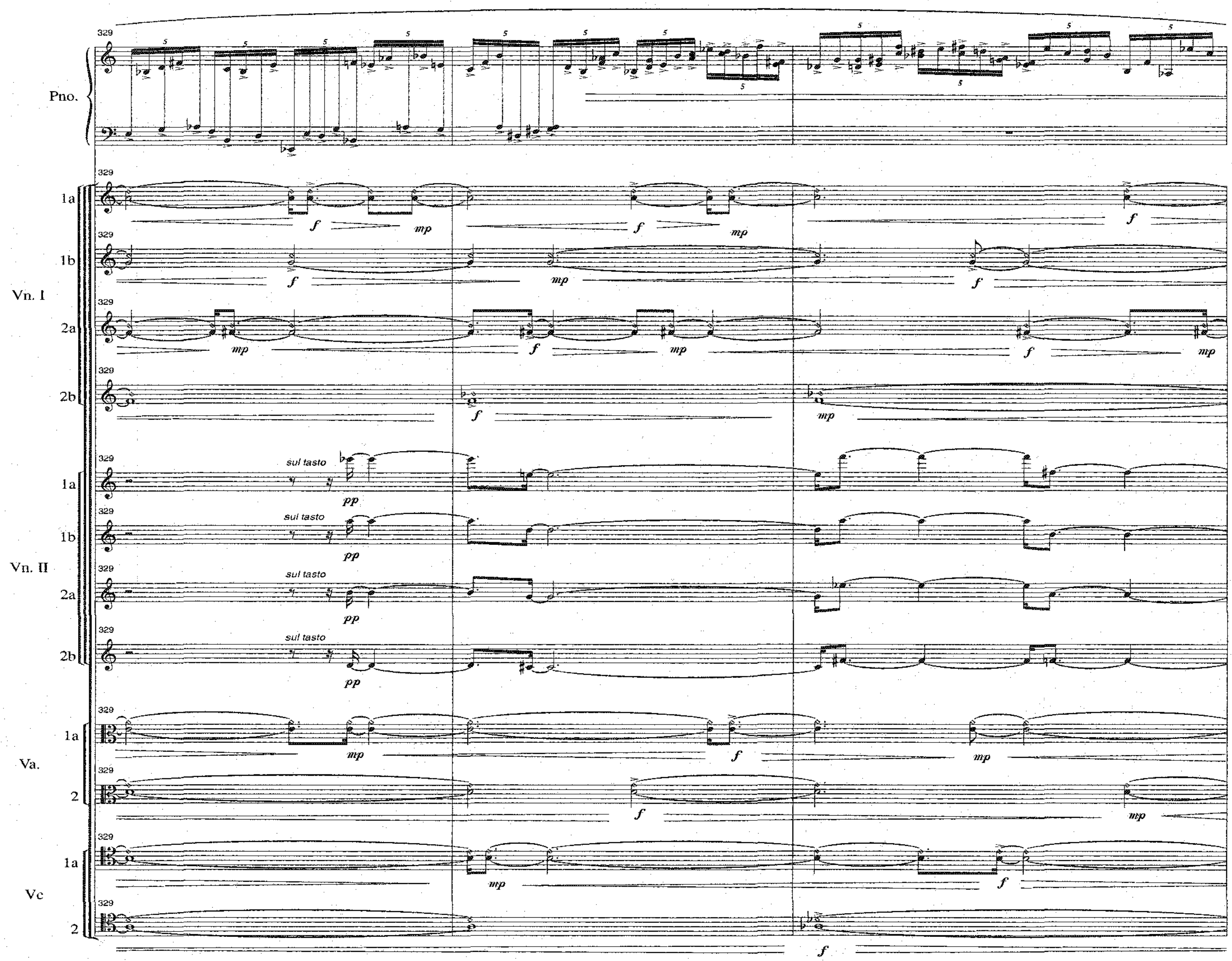

Pno.
1a
1b
Vn. I
2a
2b
1a
Va.
2
1a
Vc
2
crescendo poco a poco

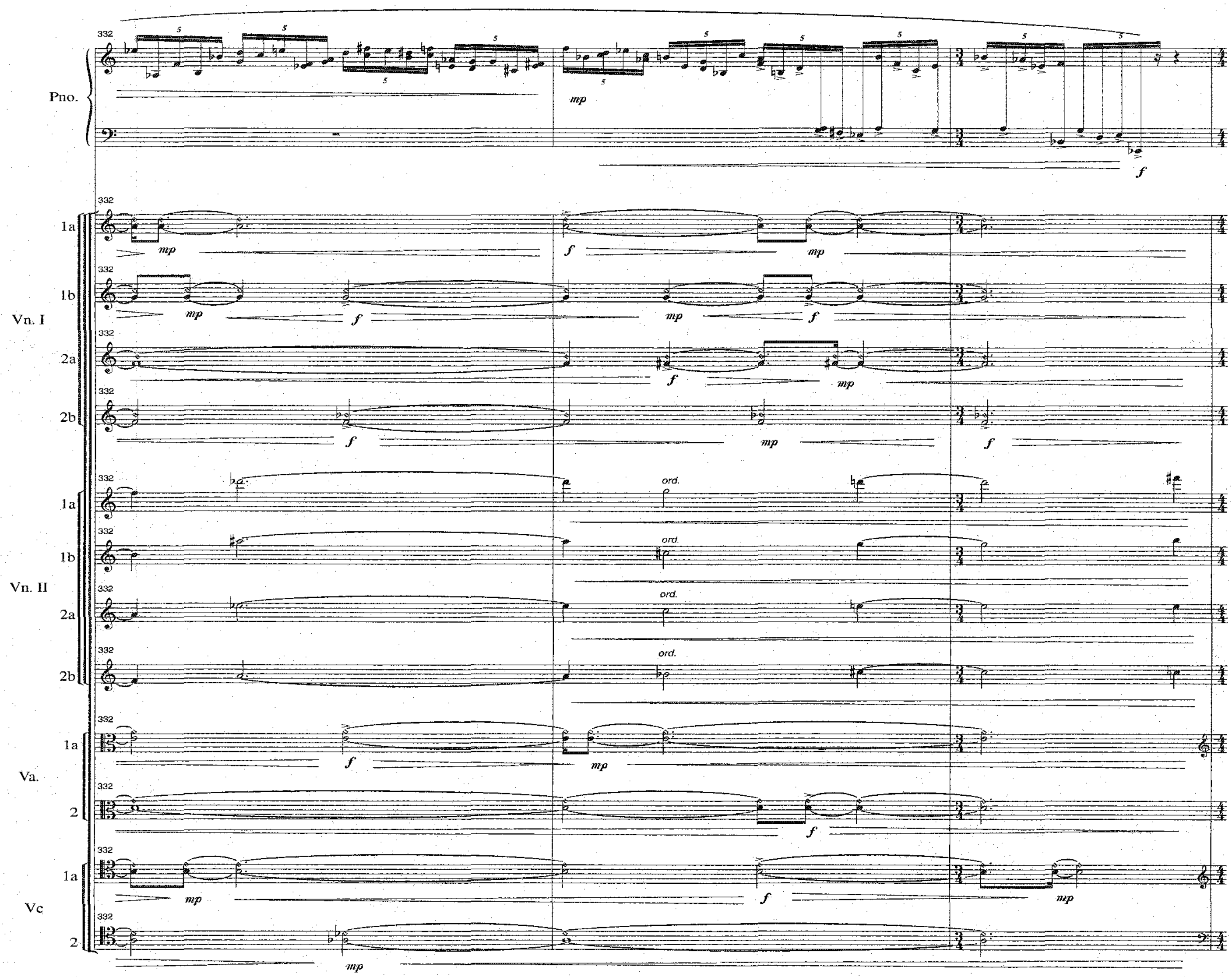

Pno.
Vn. I
1a
1b
2a
2b
Va.
Vc
(cresc)


320
Pno.
ff
Vn. I
1a
1b
2a
2b
Vn. II
Va.
1b
2
Vc
sul tasto
f
pp
mp
p

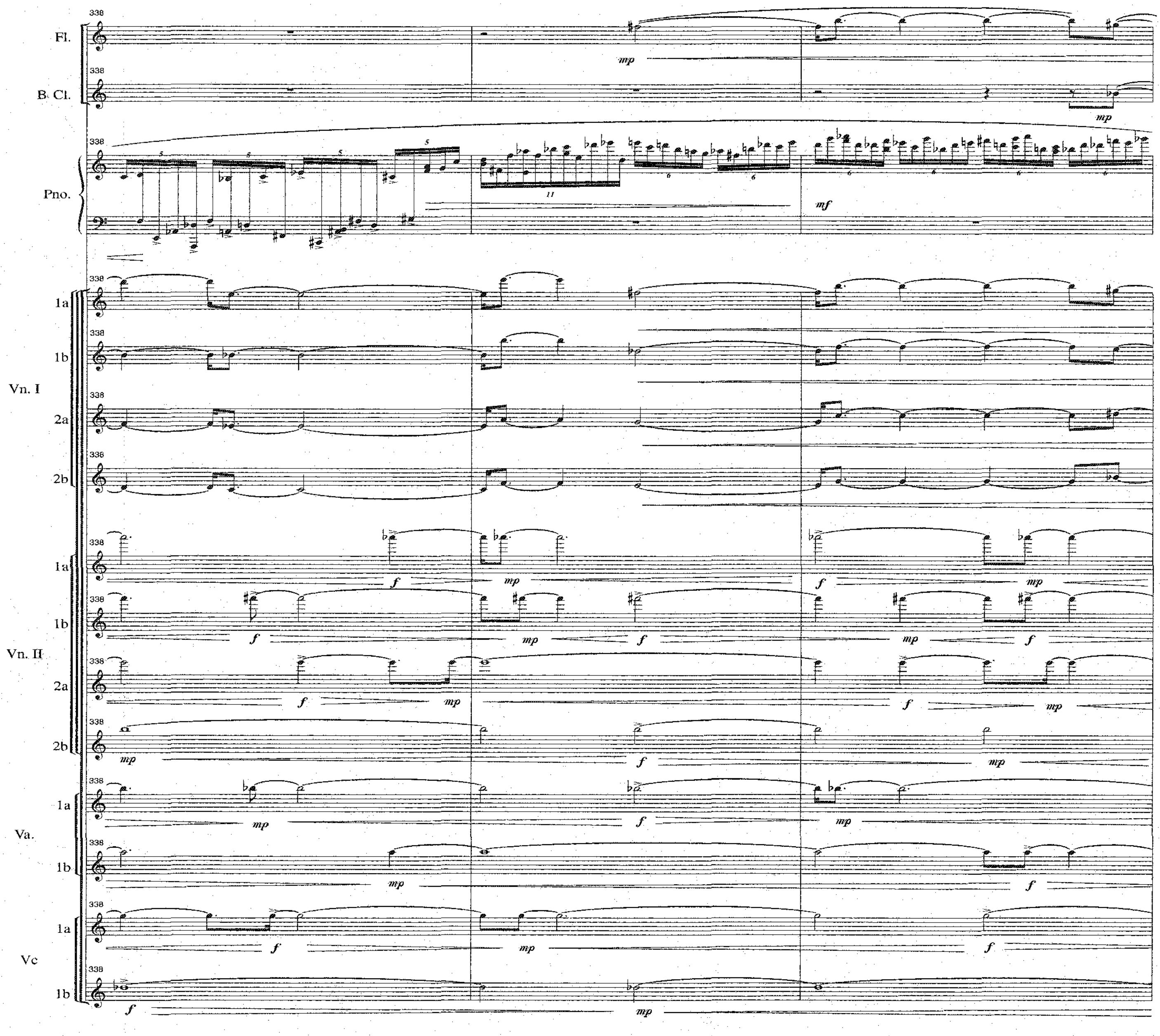

323
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
Va.
Vc
mp
mf
p
pp
f

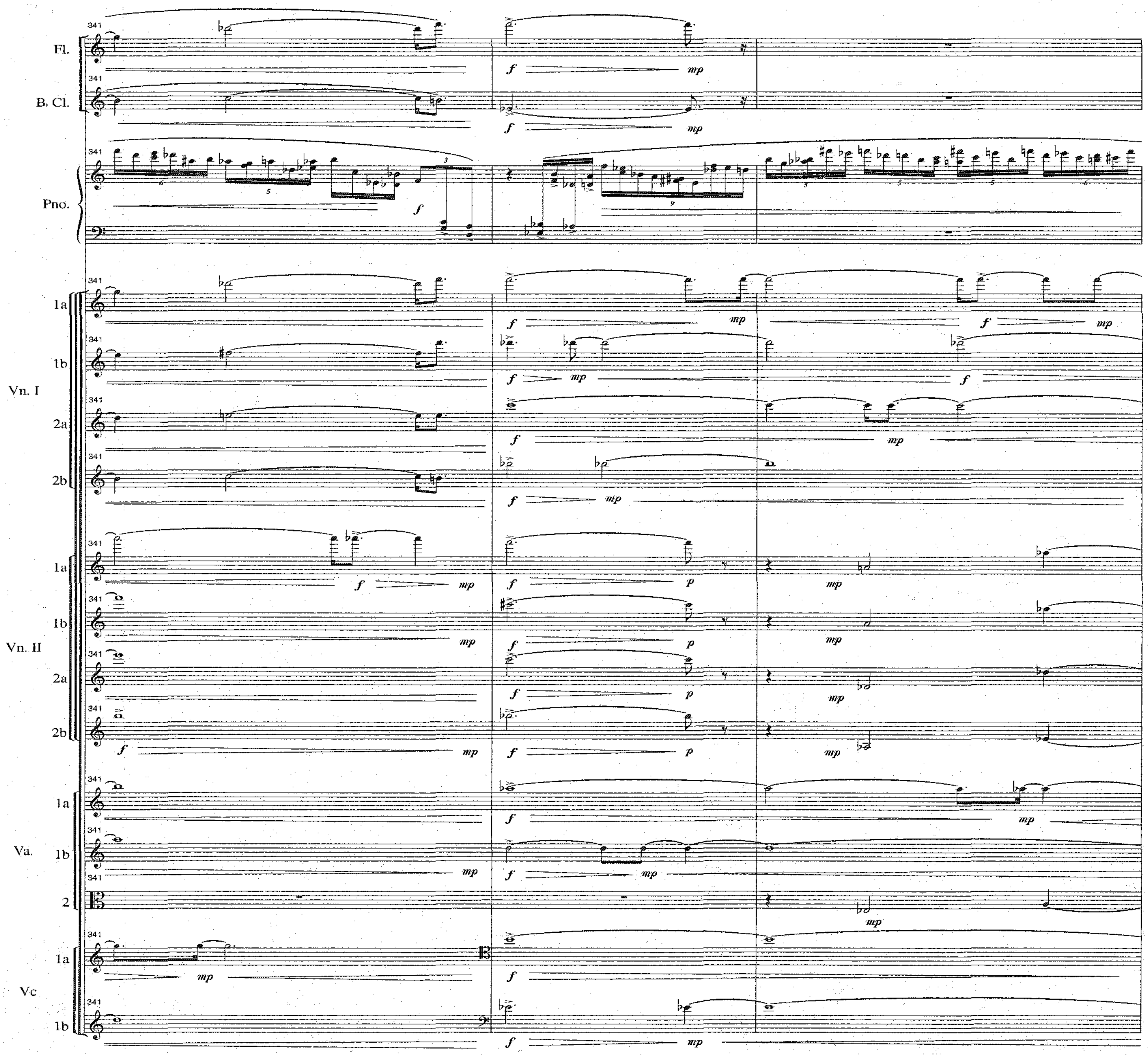

326
Pno.
mp
ff
Vn. I
1a
1b
2a
2b
ord.
Vn. II
Va.
Vc
f
mp
p
ppp
pp

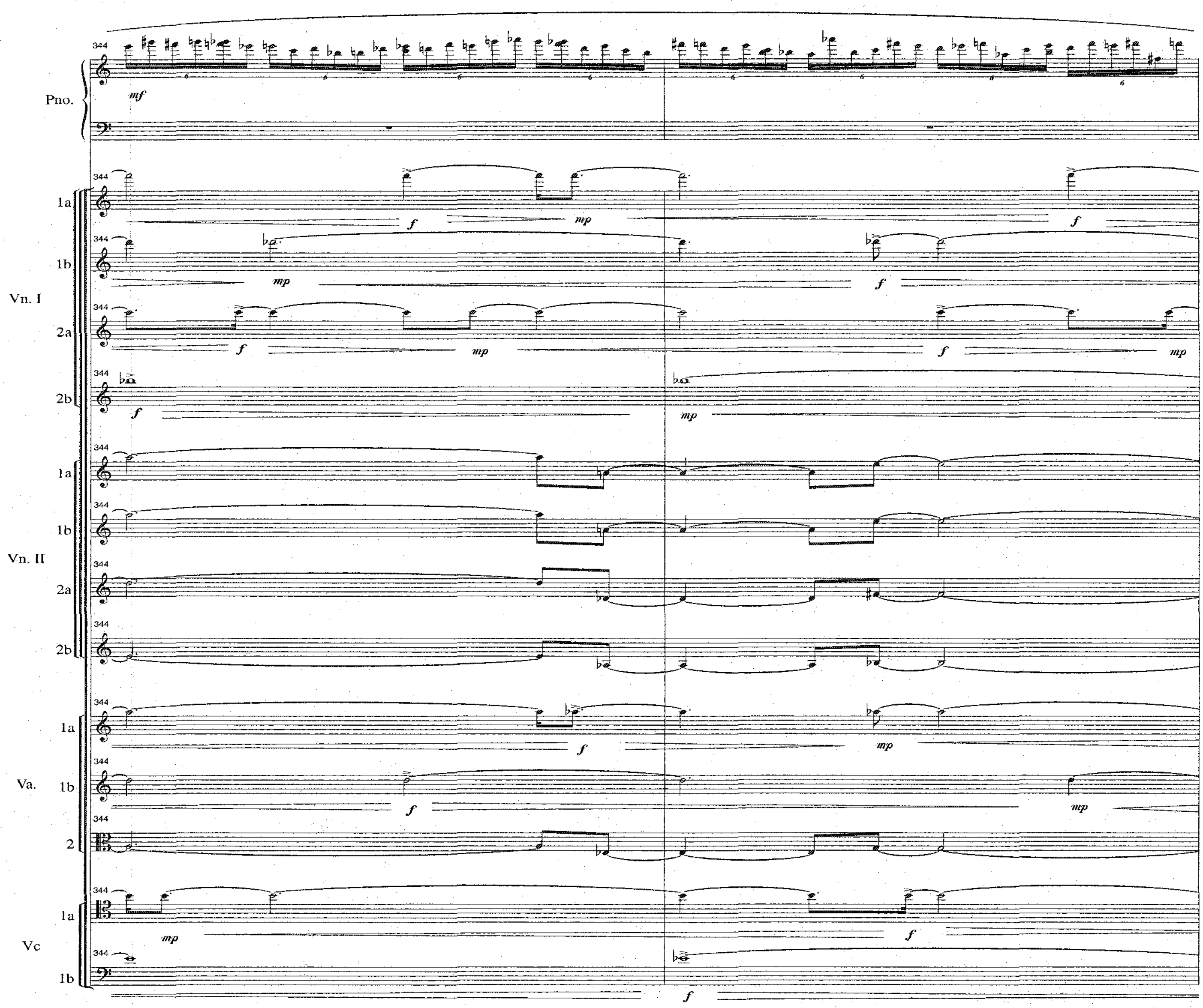

329
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
sul tasto
Va.
Vc
f
mp
pp


332
Pno.
mp
f
Vn. I
1a
1b
2a
2b
Vn. II
ord.
Va.
2
Vc


335
Pno.
Vn. I
Vn. II
Va.
Vc

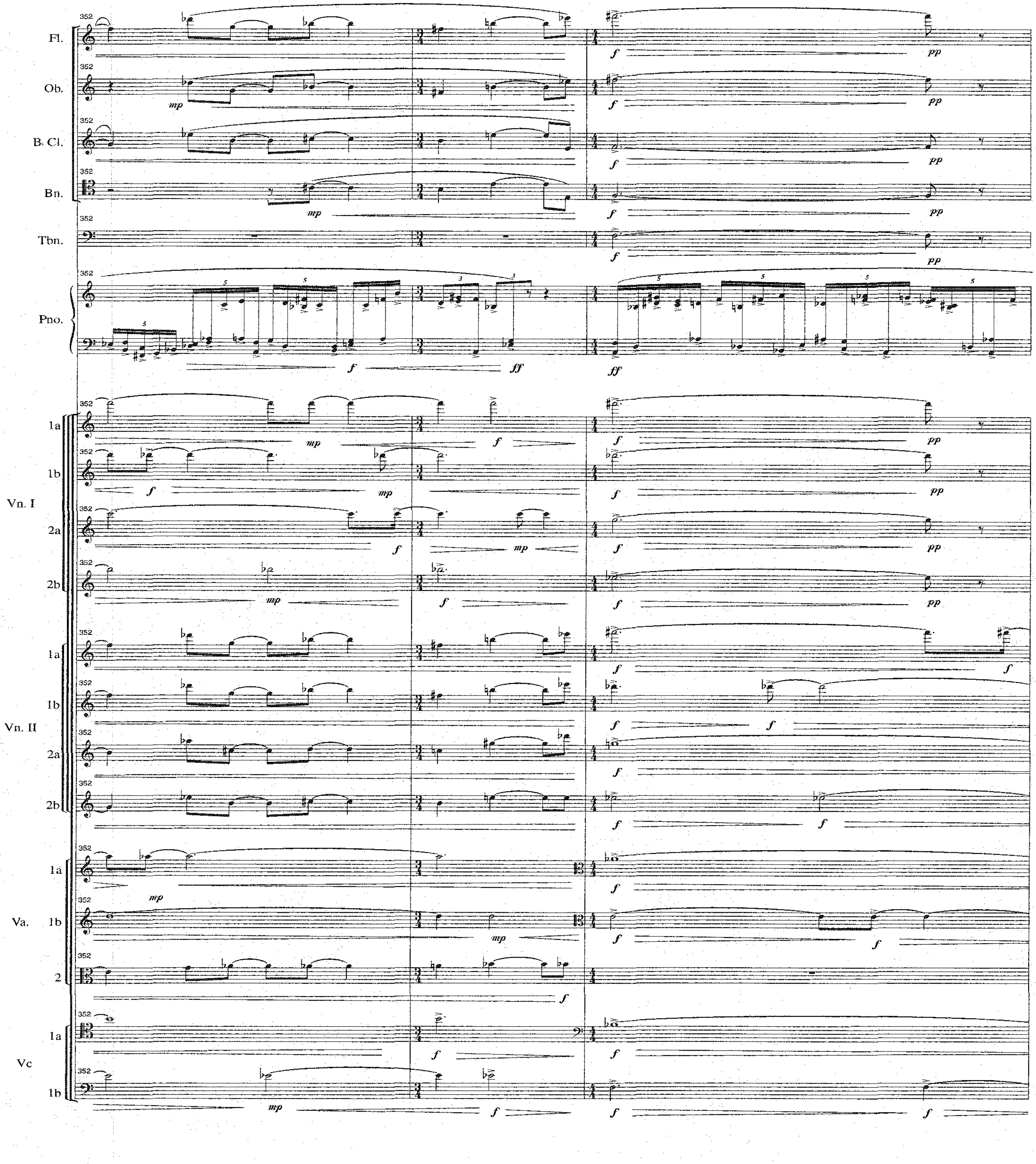

338
Fl.
B. Cl.
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
Vc
1a
1b
mp
mf
f

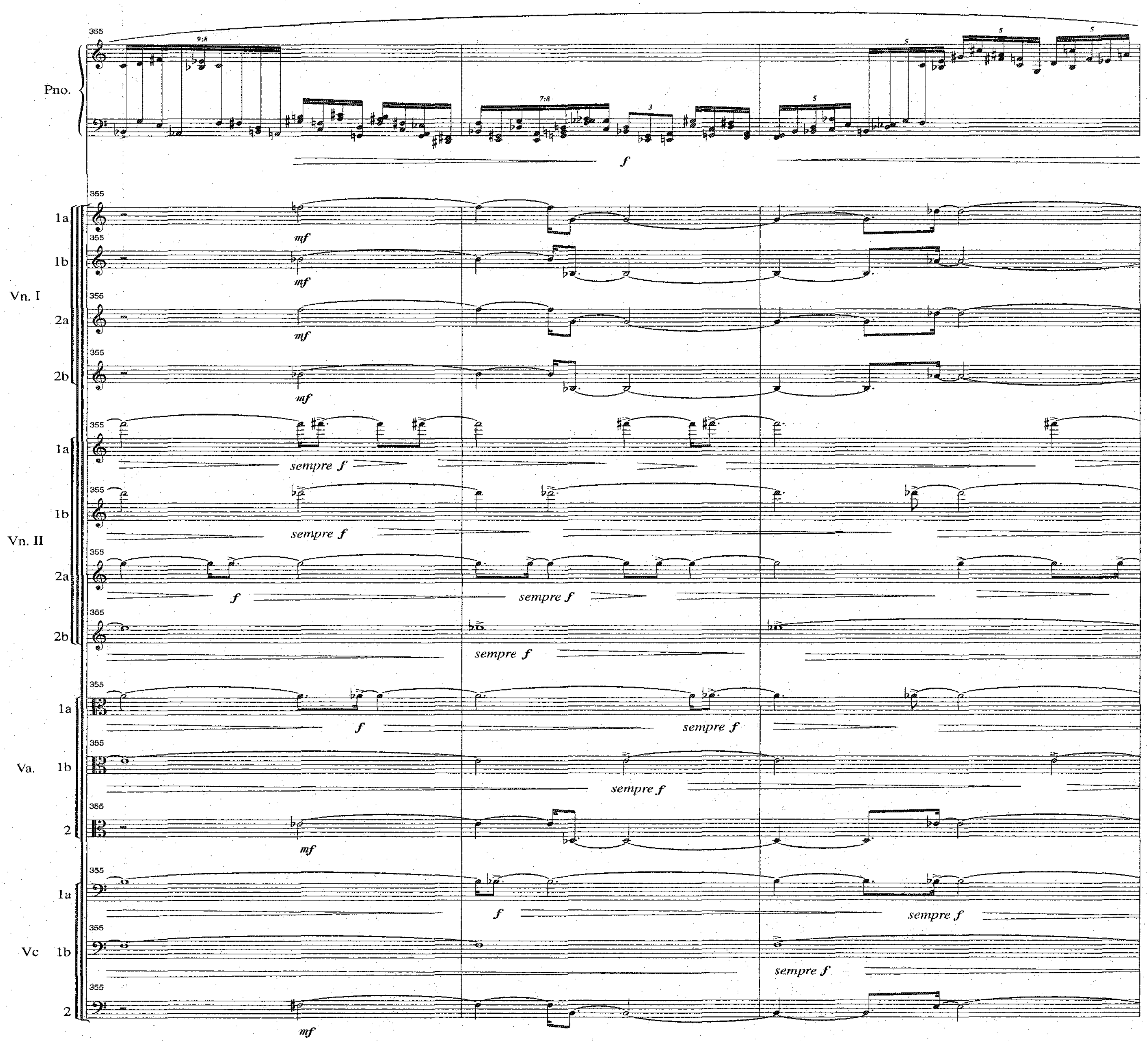

341
Fl.
B. Cl.
Pno.
Vn. I
Vn. II
Va.
Vc
1a
1b
2a
2b
2
f
mp
p

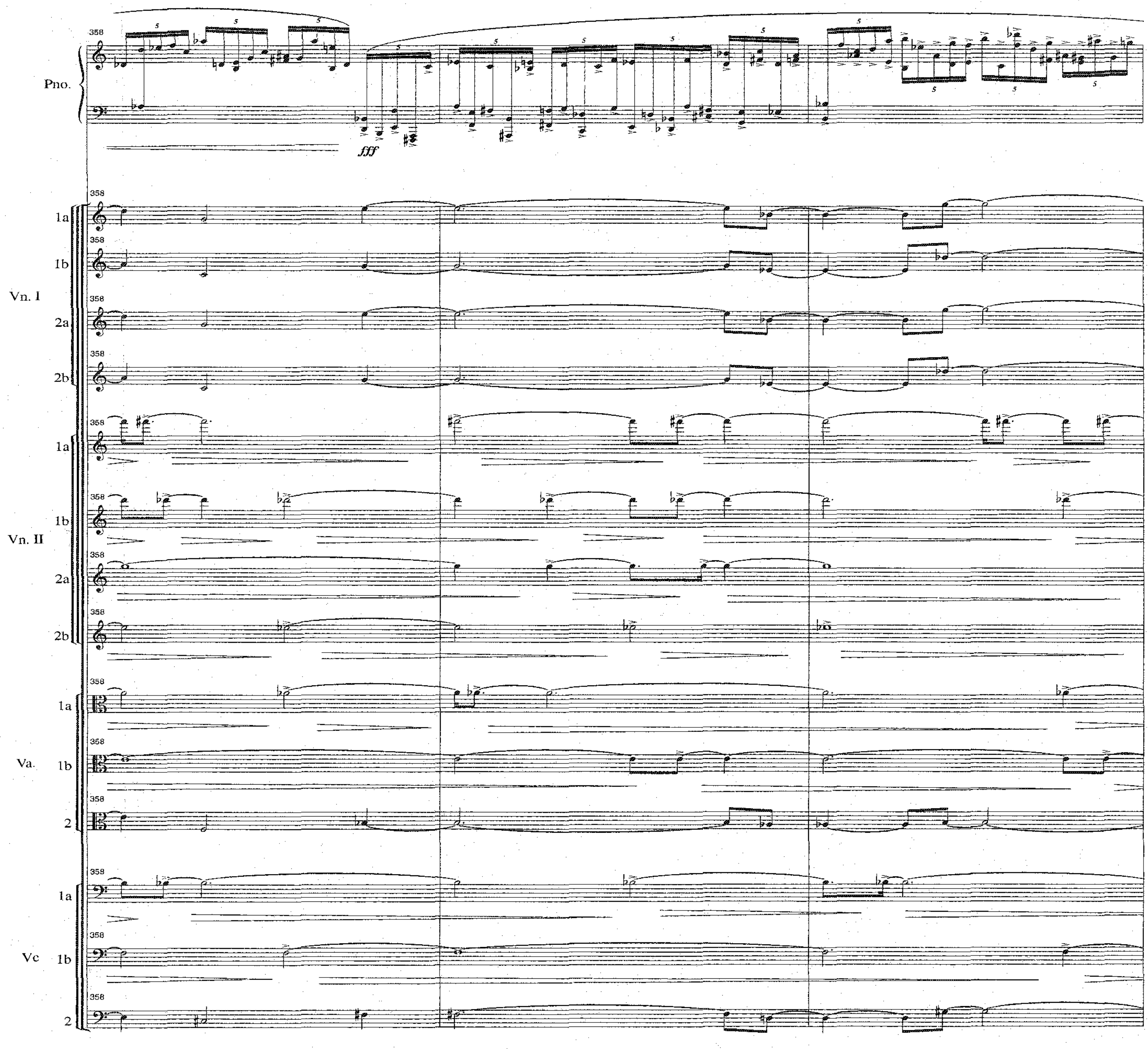

344
Pno.
mf
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
f
mp


346
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
Va.
Vc


349
Fl.
B. Cl.
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
mp
f
ff


352
Fl.
Ob.
B♭ Cl.
Bn.
Tbn.
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b


355
Pno.
9:8
7:8
3
5
f
Vn. I
1a
1b
2a
2b
mf
Vn. II
sempre f
Va.
2
Vc


358
Pno.
fff
Vn. I
1a
1b
2a
2b
Vn. II
Va.
2
Vc

361
Pno.
mf
fff
Vn. I
1a
1b
2a
2b
Vn. II
Va.
2
Vc


364
Pno.
1a
1b
Vn. I
2a
2b
1a
1b
Vn. II
2a
2b
1a
Va. 1b
2
1a
Vc 1b
2


367
Fl.
mf
Pno.
8va
11:8
ff
6
Vn. I
1a
1b
2a
2b
Vn. II
Va.
2
Vc


370
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Pno.
Vn. I
Vn. II
Va.
Vc
1a
1b
2a
2b
2
mf
(8va)
loco
6
11:8


373
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
Cb
mf
ff
fff
f
9:8
11:8


376
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. II
Pno.
9:8
5
11:8
6
f
fff
Vn. I
1a
1b
2a
2b
Vn. II
Va.
2
Vc
Cb

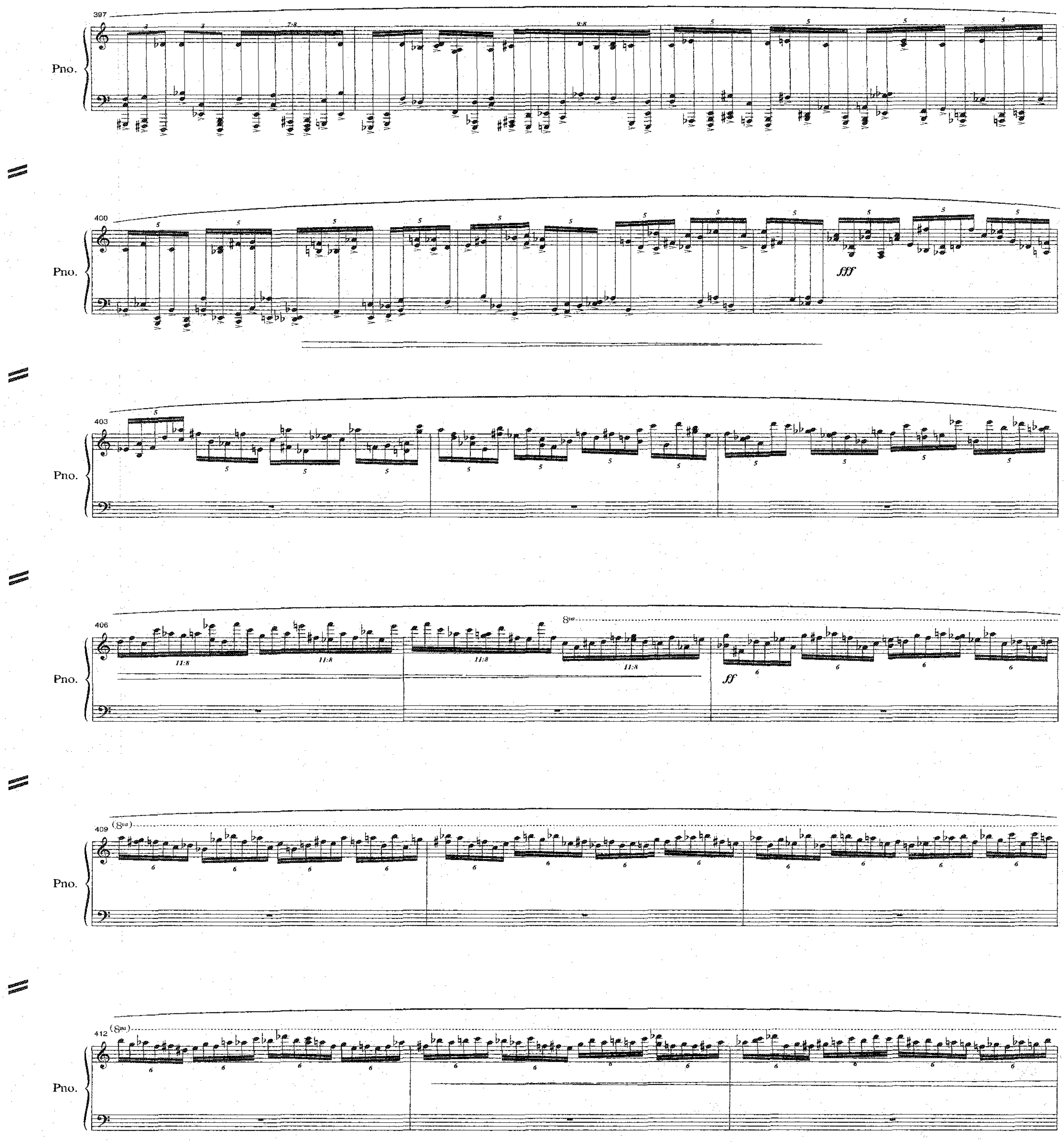

379
Fl.
Ob.
B. Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
Va.
Vc
Cb
f
p
fff
3
7:8
9:8
5


Fl.
Ob.
B♭ Cl.
Bn.
Hn.
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
382

385
Fl.
Ob.
B♭ Cl.
Bn.
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
Cb
1a
1b
2a
2b
2
mf
p
pp
ffff
5:4
7:8
9:8

Fl.
B. Cl.
Bn.
Perc. II
Pno.
Vn. I
Vn. II
Va.
Vc
pp


391
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
Va.
Vc
11:8
fff
pp
394
loco
ff
ffff
ppp


Pno.
fff
ff
8va
11:8


♩.64
Pno.
Vn. I
Vn. II
Perc. I
6 Tom-Toms
Va.


425
Fl.
Ob.
B♭ Cl.
Bn.
Hn.
Tpt.
Tbn.
Perc. I
Perc. II
Pno.
Vn. I
1a
1b
2a
2b
Vn. II
1a
1b
2a
2b
Va.
1a
1b
2
Vc
1a
1b
2
Cb
mf
ff
mp
p