Control of Coordinated Multiple Robot Manipulators

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Contents

Abstract

This thesis investigates the issues of dynamical modelling, control and load distribution for coordinated multiple robot manipulators An analysis of the load distribution problem for k coordinating robots handling a single payload is carried out and an optimal load sharing algorithm is developed The algorithm calculates the minimum norms of the joint torques and the contact forces The algorithm is based on an optimization scheme which minimizes a quadratic cost function associated with the joint torques and contact force vectors for the coordinating robot arms with the constraint of robot equations for a given trajectory of the payload The developed algorithm is found to be very efficient in terms of computational requirements in comparison with the existing load distribution algorithms Some of the comparative simulation results are provided The developed scheme is very attractive for real time applications

The theory of position control for coordinated multiple manipulator is studied Here, the coordination among k robots is achieved by controlling each of the robot arm in a non-conflicting way as they control the position of the object First, the desired position subspace for the control of the object is defined (the trajectory of the payload) Then a PD control law embedded with the developed optimal load distribution algorithm is designed to assure the stability and trajectory control of the robot arms for a given trajectory of the payload The main objective of this study is to develop a multiple arm load sharing (with minimum norms) position controller The two important aspects of this controller are (i) optimal load sharing, (ii) the application of optimal joint torques and contact forces while maintaining an accurate position control The effectiveness of the developed position control strategy with optimal load distribution for coordinated motion of the robots performing Strawman Tasks (i e a lifting task) is checked by a digital computer simulation It is noted that the controller with the optimal distribution scheme is very effective and the investigated control architecture has a very important and desirable feature from a computational point of view, since is well suited for a distributed computer architecture

Résumé

Cette thèse porte sur les systemes de contrôle et la modèlasation dynamique d'un robot à manipulateurs multiples. Une analyse de la distribution de charge pour k manipulateurs coordonnées est présentee de même qu'un algorithme de distribution de charge optimale. Cet algorithme, basé sur une fonction quadratique des torques aux joints et des vecteurs forces produits par les k manipulateurs sur l'object, dédult les torques minimums et les forces optimales. De plus, l'algorithme proposé a l'avantage d'être plus rapide que le majorite des mèthodes existontes et est donc particulièrement approprié pour les applications en temps reel.

Par la suite, le théorie du contrôle position est appliquée aux k manipulateurs. Ici la coordination du robot est assurée par un contrôle individuel et coherent des manipulateurs. D'abord, la position désirée et les forces à exercer sont obtenues. Par la suite, les forces et les torques à être exercés par chaque manipulateur sont obtenus par optimisation. L'objectif de cette méthode et donc d'obtenir la position et les forces exercées sur l'object et les joints. L'efficacité de cette méthode de contrôle pour un mouvement "petit-à-petit" des manipulateurs pour des taches telles la levée d'objects est obtenue par simulation informatique. La strategie de contrôle proposée s'est avérée très efficace et est particulièrement bien adaptee a une architecture destribuée.

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List of Figures

List of Tables

Chapter 1 Introduction

1.1 Preliminary Remarks

There is growing interest in the development of coordinated multiple manipulator systems Many tasks can be performed with the use of such systems that would be impossible to perform using a single robot manipulator The coordinated manipulator systems promise to bring about significant new developments in the area of assembly tasks in automated manufacturing deep sea explorations and space applications Some other applications may include assembly and transfer of large, heavy or non-rigid loads in the Space Station Program Two or more arms will be necessary to handle complicated and dexterous tasks skillfully for the construction of the Space Station construction However, many technical issues have to be resolved before practical applications can be developed Such issues are divided into three subareas

- Motion planning, obstacle avoidance, and sensing in the coordinated multiple manipulators (CMM),
- Dynamical modeling and control systems in the CMM's
- Software and artificial intelligence in the CMM s

In the following an overview of the 'Dynamical Modelling and Control Systems in the CMM' is provided

1.1.1 Dynamical Modeling and Control Strategies in CMM's

The dynamical modeling and control of CMM systems represent a variety of research problems The current knowledge on these topics is in its infancy, although it is gradually starting to emerge The important research issues on dynamical modelings and control strategies for CMM systems are closely related to the trajectory planning and kinematics of the entire system The following list is in such an order that the subjects considered as high priority research topics are listed first

1.1.1.1 Important Research Issues

(a) Modeling and Simultaneous force/position control of CMM's

When cooperating manipulators grasps a common object, the dynamical model of the system changes from unconstrained to constrained dynamics Indeed, when two or more manipulators hold a common object, a closed chain is formed It imposes nonholonomic and/or holonomic constraints in the system dynamics, which usually results in a loss of degrees of freedom The positions of the grippers of the CMM's are constrained Moreover, the interacting forces (torques) exerted by the cooperating manipulators are related The additional dynamical constraints must be taken into account when control strategies for each individual cooperating manipulator are determined The overall problem needs to be investigated thoroughly

Research topics which are closely related to the aforementioned subjects but are more specific include the systematic determination of the forces and torques acting between the common load and the grippers of the manipulator should be studied These research issues which are limited to the modeling and control of the CMM system need to be studied in great detail Urgent studies on these topics are needed in order to make the CMM system suitable to any applications

(b) Parallel algorithms for control of CMM systems While parallel control algorithms and parallel computational architectures are important research issues for robotics in general, they take on added significance in the CMM context The increase in complexity arising from the use of multiple arms and potential decrease in total number of degrees of freedom will place a major additional strain on conventional computational structures More effort is required on the development of decentralized and parallel architectures which are suitable for this class of problems

(c) Adaptive Control of CMM Systems A particular application determines the autonomy of individual manipulators and the degree of centralization/decentralization of the control in CMM systems The available sensory information needs to be integrated into the control algorithms Research is needed to make control systems utilize the environmental information in an intelligent manner, and thus make the system adaptable and flexible to various situations, for example to those occurring in flexible manufacturing The application of CMM systems to different manufacturing process will be enhanced at the researcher has demonstrated the practicality of adaptive control schemes to multiple manipulator systems Basic studies on the control of redundant CMM systems should be undertaken to shed light on the advantages and disadvantages to this virgin area

1.1.1.2 Other Research Issues

Research issues in dynamics and control which concern not only CMM's but in many respects also single-arm manipulators include

a Planning and reflexes in case of collisions (contact control)

b. Hand Control (fine motion)

c. End-point sensing and control

d Control of flexible arms (flexible links and flexible joints)

e Actuator dynamics and limitations including multiple degrees in spatial actuators

f Specialized architecture of multi-robot control

g Computational speed specifications for direct dynamics, force distribution and overspecified systems

h Control of mobile robots

i Fault tolerances, reliability

1.2 Literature Review

The literature review is carried out based on the nature of the system or of scientific development rather than chronological sequence This approach is believed to be more suitable for describing where the spots of difficulty lie, thus what this thesis must really attack. A survey of the literature shows that, there is a vast body of work published in the area of control of coordinated multiple manipulator It is not possible to discuss every paper in detail However, some of the publications directly related to the subject of thesis primary concern will be discussed to put the present work in perspective

Position control schemes were proposed by Alford [1984] and Fujii [1975]. In this scheme "master slave" coordination strategy is adopted One arm is assigned as the master and follows a preplanned trajectory. The other arm, the slave, follows a trajectory which is derived from the trajectory of the master arm The slave arm is required to satisfy a set of kinematic constraints, which are directed by the grasped object. The master arm is position controlled, the slave is also position servoed to comply with the imposed kinematic control scheme as the forces generated by the deviation of one arm with respect to the other is totally ignored The success of this scheme depends heavily on the precise knowledge of the individual arm kinematics the grasped position of the object and the perfect synchronization of each arm Inherently, assumed in this scheme is that precise trajectory following can be accomplished This is not possible as kinematic and dynamic uncertainties for each manipulator exists In the case of low compliance generated because of the relative error between the two arms this may cause the part to slip in the gripper or even damage to each arm In conclusion, one may say that the master slave is not robust and may even be unreliable in many practical applications

Zheng [1985 a] developed the kinematic constraints between two arms carrying a common load Given the motion of the master arm in terms of position velocity and acceleration, they could determine the motion of the closed chain formed by the two arms and the load Zheng [1985 b] also proposed a dynamic control scheme to solve for the joint torques of the two arms in the context of master slave dual arm coordination

Tarn [1986] developed a robust control scheme based around exact nonlinear output feedback Their simulations show the control scheme is robust to robot arm parameter uncertainties of 20 %

A force control scheme was proposed by Ishida [1977], in this scheme the master arm is position servoed and follows a preplanned trajectory The slave arm is also force servoed to balance and accommodate any interaction force that may arise If high speed operations are desired a nominal trajectory for the slave arm may also be specified

In Hybrid force/position control scheme, forces are controlled in the directions normal to the contact surface, position is controlled in the direction tangential to the contact surfaces (Mason [1981]). When the Hybrid force/position control scheme is applied in the dual arm robot system, a number of problems may arise Errors in programming, modeling or implementation may result in conflict between the force servo goals which could lead to undesirable motions of the object Since interaction forces must also be accommodated even along the directions of motions which are assigned as position controlled directions Mason [1981] suggested that dual arm coordinated compliant motions may be achieved by manipulating each gripper in a manner such that it is compliant in a suitable frame In a non ideal environment such a method may lead to a conflict between each arm As large forces and torques may have to be overcome in the so called position controlled directions

In order to minimize the conflict between individual arms Hayati [1986] suggested the deviation of compliance constraints to suit the multi arm task specification A multi arm dynamic coordination schemed to implement a hybrid force/position control in cartesian coordinates has also been addressed by Hayati

Uchiyama [1987] has shown that Hybrid force/position control schemes can be applied to dual arms and an experiment to support this have been performed

Zapata [1987] addressed the issue of performing an assembly with two arms, he addressed the issues of kinematic exchange of parts and path planning

Ozguner [1987] proposed the application of variable structure system (VSS) to multi arm coordination control In presence of dynamic uncertainities VSS is robust and would be important to deal with any parameter or interaction force variation A decentralized control methodology was also proposed to reduce the complexity of programming a multi arm control task

Nakamura [1987] considered the problem of manipulating an object with multiple fingers, in such a case each finger may apply a force on the object They devised a scheme to minimize the internal forces on the object while still manipulating the part, but they did not consider the dynamics of the individual fingers in the overall control scheme

In the area of software and hardware systems for multi arm control much work is under progress In a paper Zheng [1987] described the software and hardware architecture of their experimental dual robot control system A multiple 68020 microprocessor based multi arm robot control system has been developed by Guptill [1988] An operating environment based around UNIX and C support this multiprocessor based hardware The operating environment allows high level language constructs in terms of transforms and sensory processes to direct the coordination of two or more arms

Raibert [1981] utilized this theory and developed the so called "Hybrid Force Position Control" technique. In this method, force and torque information is combined with position able to achieve the desired position and force in a test related form.

The coordinated motion of two planar robots was studied by Laroussi [1988]. The contact forces were estimated from knowing the trajectory of the load and the input torques. Only linear state feedback was needed for stabilization and control of the system.

On the other hand, Takase [1985] suggested for three manipulators to control two positioned variables of each arm by position control and one positional variable and three orientation by a robot hand, Hanafusa [1977] proposed to grasp an object in such a way that the potential energy stored in the elastic fingers should be minimized.

Salisbury [1982 a],[1982 b] discussed the contact condition between fingers and an object, and suggested that the internal forces should be determined so that they have positive magnitude in the inner normal direction at the contact points on the object.

Hanafusa [1983] and Kobayashi [1985] clarified a kinematical necessary and sufficient condition to manipulate an object arbitrarily by considering the degrees of freedom of fingers and contact points.

Hanafusa [1985] defined the magnitude of grasping forces vectors of three fingers and proposed a performance criterion to determine the optimal internal forces based on the magnitude and the static frictional constraints.

On the other hand, Kerr [1986] approximated the frictional constraints by linear constraints, and proposed to determine the optimal internal forces for the constraints and the joint torque constraints by a linear programming method.

Frederic [1988] discussed an approach to the problem currently under investigation based upon the use of force/torque transducers. In their approach, each arm will contain a separate transducer. Each force/torque unit senses the forces of constraint created by the above enumerated errors, environmental forces created by the object coming into contact with its environment dynamic forces which are a reaction to the acceleration of the arm, and the force of gravity when the robot is earthbound The claimed that by properly modeling the signals available at each sensor, it is possible to distinguish each of the above components and by the utilization of constraint force signals, the magnitude and direction of error is determined and a compensation scheme is synthesized to eliminate them

In the paper by Suh [1988], a new method is developed to coordinate the motion of dual robot arms carrying a solid object, where the first robot grasps one end of the object rigidly an the other end of the object along the object surface while supporting the object It is shown that this flexible grasping is equivalent to the additional one more degree of freedom, giving the second, robot more maneuvering capabilities The load distribution problem for two coordinating industrial robots handling a single object is studied by Zheng [1988] The redundant degrees of freedom are used to optimize certain kind of performance Optimal algorithms are proposed for load distribution with minimum exerted forces on the object

A symmetric non master/slave hybrid position/force coordinated control scheme is presented by Uchiyama [1988] A set of static and kinematic operations for the system is developed and the control scheme is considered as a mutual extension of the hybrid position/force control scheme for a single arm robot A new architecture for compliance control of two cooperating robots is investigated by Kazerooni [1988] using unstructured models for dynamic behaviors of robots In their paper each robot end point follows its position input command vector "closely" when the robots are not in contact with each other When two robots come in contact with each other, one robot controls the position of the contact point, while the other controls the contact force The unified approach of modeling robots is expressed in terms of sensitivity functions

Necsulescu et al [1990-1991] simulated dual arm motion coordination with a string/blanket and a vibrating structure type of payload This study also investigated the aspects of the collision avoidance problem

1.2 Purpose and Scope of the Investigation

From the literature review, it is clear that the control of coordinated multiple manipulators is a complex and challenging problem and has many aspects. The state of the art in the control of multiple interacting manipulators is not very advanced. This thesis mainly deals with the dynamic modeling and control strategies in (simultaneous minimal norm of force-position control) in coordinated multiple manipulators. When cooperating manipulators grasp a common object the dynamical model of the system changes from unconstrained to constrained dynamics. Indeed, when two or more manipulators hold a common object, a closed chain is formed. It imposes nonholonomic and or holonomic constraints in the system dynamics which usually results in a loss of degrees of freedom. The positions of the grippers of the CMM's are constrained. Moreover, the interacting forces (torques) exerted by the cooperating manipulators are related.

This thesis studies aforementioned important research issues in detail. In particular the following carried out. In chapter 2, robot manipulators, kinematics and dynamics are studied and the dynamical model is developed. As a first step the dynamical equations are obtained in one general mating form for all robot architectures and manipulators including the payload object. The force vector exerted by the manipulators on the object is incorporated. Three dimensional rotational and translational dynamics of object is also investigated.

In Chapter 3, optimal(minimum) joint torques and constraint forces are obtained through predetermined tasks of object trajectory. The comparison of these values with the existing literature has been done and the performance of optimization scheme is investigated.

In Chapter 4, the multiple arm load sharing position controller is developed. Two important objectives of this controller are discussed (i) load sharing, (ii) the application of forces and torques to the object while maintaining accurate position control. These objectives are achieved simultaneously by minimizing an interactive force/torque vector. The simulation results are presented, discussed and comparisons with other control strategies are provided. Some closing comments and suggestion for further work are given in Chapter 5.

Chapter 2 Kinematics and Dynamics of Coordinated Multiple Manipulators and Payload.

In this Chapter, the equations governing the general dynamics of coordinated multiple manipulators and payload are derived by Lagrangian formulation. Expressions for kinetic energy T and potential energy V of the whole system are obtained and substituted in Lagrange's equations

Before staring with considerations on kinematics and dynamics of coordinated multiple manipulators, the main parts of robots and manipulator architectures available today should be shortly described in order to study a very generic dynamical model

2.1 Main Parts of An Industrial Robot and Manipulator Architectures

The main parts of an industrial robot are shown in Figure 2.1 in form of a block diagram These parts are the arm, the drives including the gears, the control computer, the gripper including the gripping device, internal sensors or the position measurement system and external sensors

In this thesis, it is assumed that robot manipulators consists of ideal, rigid links which are connected sequentially by joints Each joint allows the connection of two links and the relative motion between the two links occurs only at the joint A joint actuator such as an electric motor coupled directly or indirectly through gears for example, initiates motion by applying a force or torque at the joint In turn the links are accelerated The resulting joint motion is quantified but assigning a joint coordinate at each joint and then measuring the displacement, velocity and higher derivatives Measurement of all of the joint coordinates or translational coordinates of each joint describes the motion of the entire manipulator

There are two types of joints for robot manipulators and they are labeled as either revolute or prismatic In the case of revolute joints, the motion is rotational and the generalized joint coordinate represents angular displacement In this case, the generalized force that is exerted by the actuator at the joint is referred to as the torque For the case of prismatic joints the motion is translational and the generalized joint coordinate represents linear displacement and the associated generalized force is linear force or simple the force

For reaching any point in the working space with a prescribed orientation of the gripper at least six degrees of freedom are necessary Three of them are usually realized by arm These can be either translational (T) or rotational (R)

In figure 2 2 some industrial robot manipulators are illustrated The Unimation PUMA 600 and Cincinnati Milacron have only revolute joints On the other hand, the Stanford manipulator has one prismatic joint Other manipulators exist with various combinations of revolute and prismatic joints called Cartesian, cylindrical, spherical or revolute manipulators

2.2 Coordinated Multiple Robot Manipulator Dynamical Equations

The system under consideration consists of multi-robot arms gripping a single rigid object. A schematic drawing of a multi arm cooperating robot affecting a single object is shown in Figure 2 3. The main assumptions associated with the derivation of the equations of motion are listed below,

(i) the robot arms are rigid

(ii) the contact between each of the grippers and the object is rigid

(iii) the nominal grip points on the object are known

(iv) some information about the object's geometry is available, but its mass distribution is precisely known

(v) the nominal trajectory of all grip points is known, and

(vi) the object may be moving in a gravity field

The free body diagram analysis of the payload (object) shows that each robot arm exert force on the object Thus each robot arm can be studied independently taking into account the constraint force, torque exerted on the object For all robot manipulators, the dynamical equations may be written on a general matrix form, regardless of whether a manipulator has revolute or prismatic joints

Since a robot manipulator consists of a sequence of joints connected to rigid links, each joint-link pair represents one degree of freedom Associated with each joint, there is one generalized coordinate describing the relative motion between the links

Referring to Figure (2 5) the assignment of the ordering of the links begins with the base or inertial link, as the 0^{th} link The corresponding 0^{th} frame of reference is embedded in the 0^{th} link Note that the 0^{th} link is rigidly attached to the "world" and therefore the 0^{th} reference frame is defined to be the inertial frame The ordering of the links and their corresponding reference frames terminates with the n^{th} link or the end effector Therefore, for each link i, there is a corresponding, embedded, link reference frame i Note that each link is invariant (motionless) in its corresponding reference frame

The link reference coordinate frames depicted in Figure (2 5) are described as orthonormal Cartesian coordinate frames represented by the unit vectors $(\tilde{\mathbf{x}}_i, \tilde{\mathbf{y}}_i, \tilde{\mathbf{z}}_i)$ At each joint i the joint coordinate is labeled as either θ_i describing the relative rotational motion or d_i describing the relative linear motion between link i and link $i-1$. Note that the generalized coordinate variable q_i represents either θ_i or d_i. By using one variable q_i to denote motion the notation does not become cumbersome in the dynamical system equations. Since the links are invariant in their corresponding coordinate frames link i and its reference frame move with respect to link i - 1 and coordinate frame i - 1 when joint i causes motion. By convention assume that when joint i causes motion link i moves with respect to link $i-1$.

For most current manipulators there are at least six degrees of freedom with either prismatic or revolute joints. The link reference frames can be determined and established for each link of any manipulator with any number or type of joints. However, to establish a consistent set of reference frames for all of the links in any manipulator as opposed to arbitrary placement of the reference frames, the following three rules must be observed as given in Lee [1982],

1) The z_{i-1} axis lies along the axis of motion of the i^{th} joint

2) The x_{i-1} axis is normal to the z_{i-1} axis and pointing away from it

3) The y_i axis completes the right hand coordinate system

Note that these rules do not restrict the location of the origin of the base (0^{th} frame) as long as it is located along the z_0 axis. The same holds for the n^{th} (end effector) frame.

There are four parameters that are associated with each joint-link pair of a manipulator. These four parameters describe the geometry of joint i link i and its frame of reference with respect to link (i - 1) and its reference frame. Note that the kinematic description of link i is always given with respect to link (i - 1). As it will be shown below these four parameters specify the D-H (Denavit and Hartenberg) transformation matrix completely. The geometric parameters associated with each joint i and link i pair are θ_i, d_i, a_i and α_i. The link parameters a_i and α_i describe the structure of the link and are always constant. The parameters θ_i and d_i describe the motion or the position of the link i relative to link (i - 1) and either θ_i or d_i is constant depending on whether joint i is prismatic or revolute respectively. These parameters are defined in Lee [1982].

With these rules the orthonormal reference coordinate frame are established for each link and the geometric parameters relate the kinematics of adjacent links in a consistent manner. The consistency of such an approach allows for the precise specification of the D-H matrix transform. More importantly, a consistent set of matrix transforms allows the kinematics of all of the links in a robot manipulator to be related to one another by use of linear algebra. A procedure outlining the specification of the assignment of the reference frames and the determination of the four parameters of the D-H transform matrices is given by Lee [1982] in a step-by-step algorithm. This procedure will not be repeated here. The links, the coordinate frame and the geometric parameters are illustrated in Figure (2.5). An example of the application of these rules to a PUMA 600 robot manipulator is shown in Figure (2.6).

Once the coordinate systems and the geometric parameters are established a D-H transformation matrix can be written that relates the i^{th} coordinate frame to the $(i-1)^{th}$ coordinate frame. Any point described by $\tilde{\mathbf{r}}_i$ expressed in the i^{th} coordinate frame can be expressed as a point $\tilde{\mathbf{r}}_{i-1}$ in the $(i-1)^{th}$ coordinate frame. This can be accomplished by performing the following 4 successive operations.

a) Rotate about the z_{i-1} axis by an angle of θ_i to align x_{i-1} axis with the x_i making them parallel.

b) Translate along the z_{i-1} axis a distance of d_i to make the x_{i-1} axis coincident to the x_i axis.

c) Translate along the x_i axis a distance a_i to make the origins coincident.

d) Rotate about the x_i axis an angle of α_i to bring the two coordinate systems into coincidence

These 4 operations are neatly summarized in the D-H transform matrix as

$$A_{i-1}^{i} = \begin{pmatrix} \cos(\theta_i) & -\cos(\alpha_i)\sin(\theta_i) & sin(\alpha_i)\sin(\theta_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\alpha_i)\cos(\theta_i) & -\sin(\alpha_i)\cos(\theta_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (2.1)$$

relating a point in inertial space represented by the vector $\tilde{\mathbf{p}}_{\mathbf{i}}$ in i^{th} coordinate frame and the same point represented by the vector $\tilde{\mathbf{p}}_{\mathbf{i-1}}$ in the $(i-1)^{th}$ coordinate frame

$$\tilde{\mathbf{p}}_{i-1} = \underline{A}_{i-1}^{i}\tilde{\mathbf{p}}_i \qquad (2.2)$$

where

$$\tilde{\mathbf{p}}_i = \begin{pmatrix} \tilde{\mathbf{r}}_i \\ 1 \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \\ z_i \\ 1 \end{pmatrix} \qquad (2.3)$$

Therefore, using the $\underline{A}_{i-1}^{i}$ transform matrix, any point $\tilde{\mathbf{p}}_i$ at rest and expressed in coordinate frame i coordinates in link i can be related to the coordinate frame i - 1 coordinates

Since either θ_i or d_i can be variable representing joint motion for any D-H matrix $\underline{A}_{i-1}^{i}$, the generalized coordinate q_i will be used instead The remaining parameters of $\underline{A}_{i-1}^{i}$ are constant. Therefore, $\underline{A}_{i-1}^{i}$ is a function of q_i and it will be written as

$$\underline{A}_{i-1}^{i} = \underline{A}_{i-1}^{i}(q_i) \tag{2.4}$$

The kinematic transformation for a sequentially linked chain which relates a point $\tilde{\mathbf{p}}_j$ described in coordinate frame j coordinates as a point $\tilde{\mathbf{p}}_i$ in coordinate frame i coordinates is given as

$$\tilde{\mathbf{p}}_i = \underline{T}_i^j \tilde{\mathbf{p}}_j \tag{2.5}$$

where

$$\underline{T}_i^j = \underline{A}_i^{i+1} \underline{A}_{i+1}^{i+2} \underline{A}_{i+2}^{i+3} \cdots \underline{A}_{j-1}^{j} \tag{2.6}$$

For the case of $i = j$, $\underline{A}_i^j$ is simply the 4 by 4 identity matrix, $\underline{1}$. If $i = 0$ and $j = 6$ for a six degree of freedom manipulator, $\underline{T}_0^6$ relates any vector in the end effector frame to the base frame or inertial coordinates. Note that the 3 dimensional position vector $\tilde{\mathbf{r}}_i$ is expressed in homogeneous coordinates $\tilde{\mathbf{p}}_i$.

2.2.1 Differential Kinematic Relationships

The motivation for reviewing the differential kinematics is that the differential forms are used in the Lagrangian form of the dynamical equations of a robot manipulator. The advantage of the Lagrangian form as will be seen is that this form yields a simple matrix closed from description of the manipulator dynamics.

In the systematic D-H notation each D-H transform matrix $\underline{A}^{i}_{i-1}$ describes motion of the (x_i, y_i, z_i) frame with respect to $(x_{i-1}, y_{i-1}, z_{i-1})$ frame The $\underline{A}^{i}_{i-1}$ transformations were set up in a manner such that in the case of a revolute joint the joint variable θ_i corresponds to a rotation about the z_{i-1} axis the z axis of the $(i-1)^{th}$ coordinate frame The same is true in the case of prismatic joints and the prismatic joint variable d_i These facts can be easily exploited in the same systematic manner to develop differential kinematic relationships between the coordinate frames

Consider a given D-H transformation matrix, $\underline{A}^{i}_{i-1}$ from coordinate frame i to coordinate frame (i - 1) and a differential transformation change $\delta\underline{A}^{i}_{i-1}$ The complete transformation with the differential change can be expressed as

$$\underline{A}^{i}_{i-1} + \delta\underline{A}^{i}_{i-1} = Trans(\delta d_i)Rot(\delta\theta_i)\underline{A}^{i}_{i-1} \tag{2 7}$$

where

$Trans(\delta d_i)$ is a transformation representing a differential translation, δd_i along the z_{i-1} axis of the $(i-1)^{th}$ coordinate frame

$Rot(\delta\theta_i)$ is a transformation representing a differential rotation $\delta\theta_i$, about the z_{i-1} axis of the $(i-1)^{th}$ coordinate frame

and the differential transform $\delta\underline{A}^{i}_{i-1}$ is given as.

$$\delta\underline{A}^{i}_{i-1} = (Trans(\delta d_i)Rot(\delta\theta_i) - \underline{1})\underline{A}^{i}_{i-1} \tag{2 8}$$

The differential translation and differential rotation matrices are written as

$$Trans(\delta d_i) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} . Rot(\delta\theta_i) = \begin{pmatrix} 1 & -\delta\theta_i & 0 & 0 \\ \delta\theta_i & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.9}$$

Instead of dealing with two types of variables, d_i and θ_i representing motion, let us revert to using the generalized variable q_i Since each joint allows either translational motion or rotational motion let a general differential matrix be represented as Δ^i_{i-1} and let $\delta \underline{A}^i_{i-1}$ be defined in terms of Δ^i_{i-1} as

$$\delta \underline{A}^i_{i-1} = \Delta^i_{i-1} \underline{A}^i_{i-1} \delta q_i \tag{2.10}$$

Therefore, the partial derivative of the transformation T^j_i with respect to the generalized variable q_k is given as:

$$\frac{\partial T^j_i}{\partial q_k} = A^i_{i+1} \ . \ \Delta^k_{k-1} A^k_{k-1} \cdot \ . \underline{A}^j_{j-1} \tag{2.11}$$

Clearly in the sequence of D-H matrix transformations defining T^j_i, differentiation of the transformation T^j_i with respect to q_k results in a premultiplication of A^k_{k-1} by the matrix Δ^k_{k-1} in that sequence with the proper selection for prismatic or revolute joints.

2.2.2 The Lagrangian Form of the Dynamical Equations

The Lagrangian L is defined as the difference between the kinetic energy T and the potential energy V of a system as;

$$L(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) = T(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) - V(\tilde{\mathbf{q}}) \tag{2.12}$$

In application to robot manipulators the kinetic and potential energy of the system can be expressed in terms of generalized coordinates q_i for $i = 1 \ldots n$ where n is the number of degrees of freedom of the robot manipulator. The dynamical equations in terms of q_i are derived from the Lagrangian as

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \tag{2.13}$$

where the q_i are the generalized coordinates in which the kinetic and potential energies are expressed, the $\dot{q}_i$ are the corresponding velocities and τ_i is the corresponding generalized force including the constraint forces through the object. The force τ_i is a linear force or a torque depending upon whether q_i represents translation or rotation respectively. The right-hand side of equation (2.13) represents the applied force. There is no restriction on the applied forces that excite the system. The left-hand side of equation (2.13) represents all of the intrinsic forces to the robot manipulator system.

2.2.2.1 Kinetic Energy

Given a stationary point position $\tilde{\mathbf{p}}_i$ described in the i^{th} frame its position is described in the base or inertial frame as.

$$\tilde{\mathbf{p}}_0 = \underline{T}_0^i \tilde{\mathbf{p}}_i \tag{2.14a}$$

$$\tilde{\mathbf{p}}_i = \begin{pmatrix} x_i \\ y_i \\ z_i \\ 1 \end{pmatrix} \tag{2.14b}$$

Since $\tilde{\mathbf{p}}_i$ is a stationary point in the i^{th} frame its velocity with respect to the inertial frame is a result of the velocity of the generalized coordinates, q_i. Its velocity, in the inertial frame is given as

$$\frac{d\tilde{\mathbf{p}}_0}{dt} = \{\sum_{j=1}^{i} \frac{\partial T_0^i}{\partial q_j} \dot{q}_j\}\tilde{\mathbf{p}}_i \tag{2.15}$$

The magnitude of the velocity squared is:

$$|\frac{d\tilde{\mathbf{p}}_0}{dt}|^2 = Trace(\frac{d\tilde{\mathbf{p}}_0}{dt} \frac{d\tilde{\mathbf{p}}_0}{dt}^T) \tag{2.16}$$

Substituting equation (2.15) into equation (2.16) yields

$$|\frac{d\tilde{\mathbf{p}}_0}{dt}|^2 = Trace(\sum_{j=1}^{i}\sum_{k=1}^{i} \frac{\partial T_0^i}{\partial q_j}\tilde{\mathbf{p}}_i\tilde{\mathbf{p}}_i^T \frac{\partial (T_0^i)^T}{\partial q_k}\dot{q}_j\dot{q}_k) \tag{2.17}$$

Using equation (2.17) the kinetic energy for link i may be computed. The kinetic energy of a particle of mass dm_i located on link i is:

$$dT_i = \frac{1}{2}Trace(\sum_{j=1}^{i}\sum_{k=1}^{i} \frac{\partial T_0^i}{\partial q_j}\tilde{\mathbf{p}}_i\tilde{\mathbf{p}}_i^T \frac{\partial (T_0^i)^T}{\partial q_k}\dot{q}_j\dot{q}_k)dm_i \tag{2.18}$$

Integrating both sides, the kinetic energy of link i is

$$T_i = \frac{1}{2} Trace\Big(\sum_{j=1}^{i}\sum_{k=1}^{i} \frac{\partial T_0^i}{\partial q_j}\Big[\int_{link\ i} \tilde{\mathbf{p}}_i \tilde{\mathbf{p}}_i^T\, dm\Big]\frac{\partial (T_0^i)^T}{\partial q_k}\dot{q}_j \dot{q}_k\Big) \qquad (2.19)$$

The integral in equation (2.19) is referred to as the pseudo-inertia matrix. Note that the evaluation of the integral is with respect to coordinate frame i established in link i. The pseudo-inertia matrix $\underline{I}_i$, is given as

$$\underline{I}_i = \int_{link\ i} \tilde{\mathbf{p}}_i \tilde{\mathbf{p}}_i^T\, dm \qquad (2.20)$$

$$\underline{I}_i = \begin{pmatrix} \int_{link\ i} x_i^2 dm & \int_{link\ i} y_i x_i dm & \int_{link\ i} x_i z_i dm & \int_{link\ i} x_i dm \\ \int_{link\ i} x_i y_i dm & \int_{link\ i} y_i^2 dm & \int_{link\ i} y_i z_i dm & \int_{link\ i} y_i dm \\ \int_{link\ i} x_i z_i dm & \int_{link\ i} y_i z_i dm & \int_{link\ i} z_i^2 dm & \int_{link\ i} z_i dm \\ \int_{link\ i} x_i dm & \int_{link\ i} y_i dm & \int_{link\ i} z_i dm & \int_{link\ i} dm \end{pmatrix}$$

Finally, the total kinetic energy T of the robot manipulator is given as

$$T = \sum_{i=1}^{n} = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{r=1}^{i} Trace\Big(\frac{\partial T_0^i}{\partial q_j}\underline{I}_i \frac{\partial (T_0^i)^T}{\partial q_r}\dot{q}_j \dot{q}_r\Big) \qquad (2.21)$$

2.2.2.2 Potential Energy

The potential energy of a robot manipulator system is considered only for a gravity field (elasticity of the links are ignored) If the acceleration due to gravity is given as $\tilde{\mathbf{g}}_0$ with respect to the inertial frame then the potential energy of an object of mass m and whose center of mass is given as $\tilde{\mathbf{p}}_{0_{cm}}$ is

$$V = -m\tilde{\mathbf{g}}_0 \cdot \tilde{\mathbf{p}}_{0_{cm}} \tag{2.22a}$$

$$\tilde{\mathbf{g}}_0 = \begin{pmatrix} \tilde{\mathbf{g}}_{0_x} \\ \tilde{\mathbf{g}}_{0_y} \\ \tilde{\mathbf{g}}_{0_z} \\ 0 \end{pmatrix} \tag{2.22b}$$

The total potential energy of the robot manipulator is then

$$V = -\sum_{i=1}^{n} m_i \tilde{\mathbf{g}}_0^T \underline{T}_0^i \tilde{\mathbf{p}}_{i_{cm}} \tag{2.23}$$

where the link mass is m_i and the link center of mass is $\tilde{\mathbf{p}}_{i_{cm}}$, given with respect to the i^{th} link reference frame coordinates

2.2.2.3 The Dynamical Equations

The Lagrangian L, for robot manipulators has the form.

$$L = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{r=1}^{i} Trace\left(\frac{\partial \underline{T}_0^i}{\partial q_j}\underline{L}_i\frac{\partial(\underline{T}_0^i)^T}{\partial q_r}\dot{q}_j\dot{q}_r\right) + \sum_{i=1}^{n} m_i\tilde{\mathbf{g}}_0^T\underline{T}_0^i\tilde{\mathbf{p}}_{i,cm} \tag{2.24}$$

From the Lagrangian, the forces, τ_i at the joints are readily computed using equation (2.13) considering the effect of other manipulators, i.e., constraint forces. Straight forward differentiation of the Lagrangian yields the following result

$$\tau_i = \sum_{j=i}^{n}\sum_{r=1}^{j} Trace\left(\frac{\partial \underline{T}_0^j}{\partial q_r}\underline{L}_j\frac{\partial(\underline{T}_0^j)^T}{\partial q_i}\right)\ddot{q}_r$$
$$+\sum_{j=i}^{n}\sum_{r=1}^{j}\sum_{m=1}^{j} Trace\left(\frac{\partial^2 \underline{T}_0^j}{\partial q_r\partial q_m}\underline{L}_j\frac{\partial(\underline{T}_0^j)^T}{\partial q_i}\right)\dot{q}_r\dot{q}_m$$
$$-\sum_{j=i}^{n} m_j\tilde{\mathbf{g}}_0^T\frac{\partial \underline{T}_0^j}{\partial q_i}\tilde{\mathbf{p}}_{j,cm} \tag{2.25}$$

Equation (2.25) can be rewritten in standard matrix form as

$$\tilde{\tau} = \underline{D}(\tilde{\mathbf{q}})\ddot{\tilde{\mathbf{q}}} + \tilde{\mathbf{H}}(\tilde{\mathbf{q}},\dot{\tilde{\mathbf{q}}}) + \tilde{\mathbf{G}}(\tilde{\mathbf{q}}) \tag{2.26}$$

with

$$\tilde{\mathbf{H}} = \underline{QC}(\tilde{\mathbf{q}})\dot{\tilde{\mathbf{q}}}$$

The elements D_{ij} of $\underline{D}$, the elements, C_{ijr} of C_i, and the elements, G_i of $\tilde{\mathbf{G}}$ are given as

$$D_{ij} = \sum_{p=max(i,j)}^{n} Trace\left(\frac{\partial \underline{T}_0^p}{\partial q_j}\underline{L}_p\frac{\partial(\underline{T}_0^p)^T}{\partial q_i}\right) \tag{2.27}$$

$$C_{ijr} = \sum_{p=max(i,j,r)}^{n} Trace\left(\frac{\partial^2 \underline{T}_0^p}{\partial q_j \partial q_r} \underline{I}_j \frac{\partial (\underline{T}_0^p)^T}{\partial q_i}\right) \tag{2.28}$$

$$G_i = \sum_{p=i}^{n} m_p \tilde{\mathbf{g}}_0^T \frac{\partial \underline{T}_0^p}{\partial q_i} \tilde{\mathbf{p}}_{pcm} \tag{2.29}$$

with

$$\dot{Q} = \begin{pmatrix} \tilde{\mathbf{q}}^T & \tilde{\mathbf{0}}^T & & \tilde{\mathbf{0}}^T \\ \tilde{\mathbf{0}}^T & \tilde{\mathbf{q}}^T & & \tilde{\mathbf{0}}^T \\ & \vdots & & \\ \tilde{\mathbf{0}}^T & \tilde{\mathbf{0}}^T & \cdots & \tilde{\mathbf{q}}^T \end{pmatrix}$$

and

$$C(\tilde{\mathbf{q}}) = \begin{pmatrix} C_1(\tilde{\mathbf{q}}) \\ C_2(\tilde{\mathbf{q}}) \\ \vdots \\ C_k(\tilde{\mathbf{q}}) \end{pmatrix}$$

and the ordering of the elements of $\tilde{\tau}$ and $\tilde{\mathbf{q}}$ are.

$$\tilde{\tau} = \begin{pmatrix} \tau_1 \\ \vdots \\ \tau_n \end{pmatrix} \tag{2.30a}$$

$$\tilde{\mathbf{q}} = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix} \tag{2.30b}$$

$$\tilde{\tau}_i = \tilde{\mathbf{T}}_i + \underline{J}_i^T \tilde{\mathbf{F}}_i \tag{2.31}$$

where $\underline{J}_i$ is $m \times n$ manipulator Jacobian matrix for the i^{th} robot while $\tilde{\mathbf{F}}_i$ is an $m \times 1$ force vector that the i^{th} manipulator exerts on the object.

Standard robotic terminology defines the terms in equation (2 26) as follows

$\underline{D}(\tilde{\mathbf{q}})$ inertia matrix(positive definite symmetric)

$\tilde{\mathbf{H}}$ Coriolis and centripetal force vector

$\tilde{\mathbf{G}}(\tilde{\mathbf{q}})$ force vector due to gravity

τ applied torque and force vector (external and constraint)

$\tilde{\mathbf{q}}$ joint coordinate position

$\tilde{\mathbf{F}}$· force vector exerted on the object (constraint force)

The main feature of matrix $\underline{D}$ is that it is always positive definite symmetric The submatrices of $\underline{C}$, C_i are symmetric Finally, note that the elements of $\underline{D}$ $\underline{C}$ and $\tilde{\mathbf{G}}$ are bounded functions of joint position $\tilde{\mathbf{q}}$ For revolute joints the functions are sums and products of sine and cosines of joint position For prismatic joints the joint variables merely add or subtract linearly Finally, the excursion of prismatic joints is bounded (a physical joint can not extend to infinity)

2.2.2.4 Equations of Motion of the Object

The rotational and translational motion of the object is obtained by Newtonian and Euler Method and the details of this work is not given here for the sake of brevity These equations are given below

The translational equation of motion of the object is

$$\underline{M}\ddot{\mathbf{X}} + m\tilde{\mathbf{G}} = -\sum_{i=1}^{k} \tilde{\mathbf{F}}_i \qquad (2\ 32)$$

and the rotational dynamics of the object is obtained by

$$\underline{I_N}\dot{\omega} + \omega \times \underline{I_N}\omega = \tilde{\mathbf{U}} \tag{2.33}$$

where $\times$ denotes the cross-product, $\underline{M}$ is a 3×3 diagonal matrix whose non-zero elements denote the mass of the object. $\underline{I_N}$ is a 3×3 inertia matrix. $\tilde{\mathbf{U}}$ is a 3×1 constraint moment vector exerted on the object by manipulators.

Now, the equations of motion have been derived, the next step is to study these equations, obtain optimal joint torques and contact forces which will give us the desired trajectory of the object.

Chapter 3 Optimal Load Distribution

The load distribution problem for multi(k) coordinating robots handling a single object is studied in this chapter. When k robots grasp a single object a closed chain mechanism is formed. For a closed chain mechanism, the degrees of freedom are less than the total number of joints. Since for industrial robots in general every joint is installed with an actuator, the number of the actuators will be greater than the degrees of freedom. As a result, there are infinitely many choices of determining the joint torques for a particular load of object.

It follows that constraints need to be introduced to optimize certain kind of performance such that the joint torques can be uniquely determined. The load that an object imposes on the robot end effectors can in general be represented by a three dimensional force and a three dimensional torque. The force and torque which are generated by the k coordinating robots effect the desired motion of the object. The load distribution problem then reduces to the control of the force/torque distribution of the k coordinating robots.

The main objective of this section is to develop a methodology which determines the minimum norm of torques and payload-contact force distribution for each robot once the manipulator dynamics is known(identified/calculated/measured). This methodology includes the constraints associated with the total required torques and the total contact force for the payload.

Consider k coordinating robots holding a single object as shown in Figure 2.3. The robot arms are denoted by subscript $i = 1 \ldots k$. Denote the force the i^{th} arm exerts on the object by an $m \times 1$ vector $\tilde{\mathbf{F}}_i$. Then the following dynamic equation is true for each of the k robots

$$\underline{D}(\tilde{\mathbf{q}})\ddot{\tilde{\mathbf{q}}} + \tilde{\mathbf{H}}(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) + \tilde{\mathbf{G}}(\tilde{\mathbf{q}}) = \tilde{\mathbf{T}}_i + \underline{J}_i^T \tilde{\mathbf{F}}_i \qquad i = 1, \ldots, k \tag{3.1}$$

where $\tilde{\mathbf{q}}_i$ is an $n \times 1$ vector denoting joint positions of the robot. $\tilde{\mathbf{T}}_i$ is an $n \times 1$ vector denoting its joint torques, and $\underline{J}_i$ is its $m \times n$ manipulator Jacobian matrix, $m < n$. Note that the subscript i, refers to the i^{th} robot arm. Here, we assume that k robots have the same degrees of freedom n with $n > m$.

Referring to the equations (2.32) and (2.33) the rotational and translational dynamics of the object can be rewritten as follows

$$\begin{pmatrix} \underline{M} & \underline{0} \\ \underline{0} & \underline{IN} \end{pmatrix} \begin{pmatrix} \ddot{\tilde{\mathbf{X}}} \\ \dot{\tilde{\omega}} \end{pmatrix} + \begin{pmatrix} \tilde{\mathbf{0}} \\ \tilde{\omega} \times \underline{IN}\tilde{\omega} \end{pmatrix} + \begin{pmatrix} m\tilde{\mathbf{G}} \\ \tilde{\mathbf{0}} \end{pmatrix} = -\sum_{i=1}^{k} \tilde{\mathbf{F}}_i \tag{3.2}$$

where $\times$ denotes the cross-product, $\underline{M}$ is a 3×3 diagonal matrix where non zero elements denotes the mass of the object, and $\underline{IN}$ is a 3×3 general motion matrix of the object. $\tilde{\mathbf{X}}$ denotes the position of the object in the inertial frame, $\tilde{\omega}$ is the angular velocities of the object about its three principal axes and $\tilde{\mathbf{G}} = (0 \;\; 0 \;\; g)^T$ with g being the gravitational acceleration.

For a given motion of the object, $\ddot{\tilde{\mathbf{X}}}$, $\tilde{\omega}$, and $\dot{\tilde{\omega}}$ are all known (defined trajectory). Therefore, the left side of equation (3.2) can be calculated by the utilization of inverse kinematics. For the purpose of convenience we denote the left side of equation (3.2) as $\tilde{\mathbf{F}}$, $\tilde{\mathbf{F}} = \sum_{i=1}^{k} \tilde{\mathbf{F}}_i$. Furthermore we assume that there is no relative motion between the object and the end-effectors. Since the motion of the object is predefined, using inverse kinematics, the motions of robot joints are determined. It follows that the left side of equation (3.1) can be calculated and so does the manipulator Jacobian matrices Let the left side of equation (3 1) be denoted as $\tilde{\mathbf{B}}_i$ As a result, one obtains the following equations

$$\underline{J}_i^T \tilde{\mathbf{F}}_i + \tilde{\mathbf{T}}_i = \tilde{\mathbf{B}}_i \qquad i = 1, k \tag{3 3}$$

and

$$\sum_{i=1}^{k} \tilde{\mathbf{F}}_i = \tilde{\mathbf{F}} \qquad i = 1, k \tag{3 4}$$

The above $(k+1)$ vector and matrix equations represent$(kn + m)$ scalar equations, but contain $(kn + km)$ unknowns, $\tilde{\mathbf{T}}_i$ and $\tilde{\mathbf{F}}_i$, $i = 1, k$ Thus equations (3 3) and (3 4) are a set of under-specified equations, which have infinitively many choices of the joint torques and forces

Here, we must decide how much each arm should contribute to the motion of the object This solution can be achieved by positioning the mass and inertia tensor of the object into k parts The redundancy in the force and torque subspace is resolved by minimizing the magnitude of the vector of desired forces and torques subject to equations of motion of k robots and the object as the following,

$$\tau = \frac{1}{2}\tilde{\mathbf{S}}^T \underline{A} \tilde{\mathbf{S}} + \tilde{\lambda}^T \tilde{\mathbf{G}} \tag{3 5}$$

where τ is the functional to be minimized, and

$$\tilde{\mathbf{S}}^T = (\, \tilde{\mathbf{F}}_1^T \quad \tilde{\mathbf{F}}_2^T \quad \ldots \quad \tilde{\mathbf{F}}_k^T \quad \tilde{\mathbf{T}}_1^T \quad \tilde{\mathbf{T}}_2^T \quad \ldots \quad \tilde{\mathbf{T}}_k^T \,)$$

$\tilde{\lambda}^T$ is a $1 \times kn$ vector, known as Lagrangian multiplier, and $\tilde{\mathbf{G}}$ is a $kn \times 1$ constant vector composed of equations of motion of k robots and the object while $\underline{A}$ is a $(kn+km) \times (kn+km)$ weighting matrix which gives us the freedom to select different minimality condition to each force and torque subvectors

$$\tilde{\mathbf{G}} = \begin{pmatrix} \tilde{\mathbf{J}}_i^T \tilde{\mathbf{F}}_i + \tilde{\mathbf{T}}_i - \tilde{\mathbf{B}}_i \\ \sum_{j=1}^{k} F_j - F_i \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{0}} \\ \tilde{\mathbf{0}} \end{pmatrix} \tag{3.6a}$$

$$\underline{A} = \begin{pmatrix} \underline{W}_1 & \underline{0} & \underline{0} & \cdot & \underline{0} & \\ \underline{0} & \underline{W}_2 & \underline{0} & \cdot\cdot & \underline{0} & \\ \underline{0} & & \underline{W}_3 & \underline{0} & & \underline{0} \\ & & \cdot & \cdot & & \\ & & & \cdot & & \underline{W}_k \end{pmatrix} \tag{3.6b}$$

Optimality condition of functional follows that, the partial derivative of τ with respect to each force and torque vector and $\tilde{\lambda}$ vector must be zero An inventory of unknowns and total number of equations show that, totally we have km equations from derivative of τ with respect to force vector and kn equation for derivative of τ with respect to torque vector and another $(kn+m)$ equations by the derivative of τ with respect to $\tilde{\lambda}$, thus $(2kn+(k+1)m)$ equations If we consider number of unknowns, kn unknown torques and km force vectors and $(kn+m)$ unknown Lagrangian multipliers $\tilde{\lambda}$, i e $(2kn+(k+1)m)$. So now, the number of unknowns and the number of equations are equal and form a set of linear algebraic equations one can use very efficient linear system solution to obtain optimal torque and force values without any complexity But here the only problem is the size of matrix, i e. $(2kn+km+m)$ and in the case of coordination of more then 2 robots, the solution of this system, especially in real time causes some problem To this end, we seek some other solution which may reduce the size of system matrix

Referring to equation (3 4) one may solve for $\tilde{\mathbf{F}}_k$ as the following;

$$\tilde{\mathbf{F}}_k = \tilde{\mathbf{F}} - \sum_{i=1}^{k-1} F_i. \tag{3.7}$$

The substitution of this vector into functional τ and reordering it, eliminates $\tilde{\mathbf{F}}_k$ in $\tilde{\mathbf{S}}$ vector, and now the size of the system matrix will be $(kn + km - m)$, (i.e. number of unknowns) This is superior form of optimization to previous approach as far as the number of floating point calculations (computational time) are considered

After some algebraic manipulation, one can obtain

$$\frac{\partial\tau}{\partial\tilde{\mathbf{F}}_i} = \underline{W}_{fi}\tilde{\mathbf{F}}_i + \underline{W}_{fk}[\sum_{j=1}^{k-1}\tilde{\mathbf{F}}_j - \tilde{\mathbf{F}}] + (\underline{0} \quad \underline{0} \quad \underline{J}_i \quad - \underline{J}_k)\,\tilde{\lambda} = \tilde{\mathbf{0}} \tag{3.8}$$

$$\frac{\partial\tau}{\partial\tilde{\mathbf{T}}_i} = \underline{W}_{ti}\tilde{\mathbf{T}}_i + (\underline{0} \quad \underline{0} \quad . \quad \underline{1} \quad .)\,\tilde{\lambda} = \tilde{\mathbf{0}} \tag{3.9}$$

$$\frac{\partial\tau}{\partial\tilde{\lambda}_i} = \underline{G}^* = \tilde{\mathbf{0}} \tag{3.10}$$

where

$$\tilde{\mathbf{G}}^* = \begin{pmatrix} \underline{J}_i^T\tilde{\mathbf{F}}_i + \underline{T}_i - \tilde{\mathbf{B}}_i \\ \underline{J}_k^T[\tilde{\mathbf{F}} - \sum_{j=1}^{k-1} F_j] + \tilde{\mathbf{T}}_i - \tilde{\mathbf{B}}_k \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{0}} \\ \tilde{\mathbf{0}} \end{pmatrix} \qquad i = 1, \ldots, k-1.$$

and from equation (3 9), one can solve for $\tilde{\lambda}$,

$$\tilde{\lambda} = -\begin{pmatrix} \underline{W}_{t1}\tilde{\mathbf{T}}_1 \\ \underline{W}_{t2}\tilde{\mathbf{T}}_2 \\ \\ \underline{W}_{tj}\tilde{\mathbf{T}}_j \\ \\ \underline{W}_{tk}\tilde{\mathbf{T}}_k \end{pmatrix} \tag{3.11}$$

by substitution of $\tilde{\lambda}$ into equations (3.8) and (3 9), one can rewrite these equations as the following,

$$\underline{P}\tilde{\mathbf{Y}} = \tilde{\mathbf{R}} \tag{3.12a}$$

where $\underline{P}$ is

$$\underline{P} = \begin{pmatrix} (\underline{W}_{f1} + \underline{W}_{fk}) & \underline{W}_{fk} & \cdots & \underline{W}_{fk} & -\underline{J}_1\underline{W}_{t1} & \underline{0} & \underline{0} & \underline{J}_k\underline{W}_{tk} \\ \underline{W}_{fk} & (\underline{W}_{f2} + \underline{W}_{fk}) & \cdots & \underline{W}_{fk} & \underline{0} & -\underline{J}_2\underline{W}_{t2} & \underline{0} & \underline{J}_k\underline{W}_{tk} \\ & \cdots & \cdots & \cdots & \cdot & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \underline{J}_1^T & \underline{0} & \underline{0} & \underline{0} & \underline{1} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{J}_2^T & \underline{0} & \underline{0} & \underline{0} & \underline{1} & \underline{0} & \underline{0} \\ & \cdots & \cdots & \cdot & \cdots & & \cdots & \cdots \\ & \cdots & \cdot & \cdots & \cdots & \cdots & \cdots & \cdots \\ -\underline{J}_k^T & -\underline{J}_k^T & -\underline{J}_k^T & -\underline{J}_k^T & \underline{0} & \underline{0} & \underline{0} & \underline{1} \end{pmatrix} \tag{3.12b}$$

$$\tilde{\mathbf{Y}}^T = (\tilde{\mathbf{F}}_1 \quad \tilde{\mathbf{F}}_2 \quad \cdots \quad \tilde{\mathbf{F}}_{k-1} \quad \tilde{\mathbf{T}}_1 \quad \tilde{\mathbf{T}}_2 \quad \cdots \quad \tilde{\mathbf{T}}_k) \tag{3.12c}$$

$$\tilde{\mathbf{R}}^T = (\underline{W}_{fk}\tilde{\mathbf{F}} \quad \underline{W}_{fk}\tilde{\mathbf{F}} \quad \ldots \quad \underline{W}_{fk}\tilde{\mathbf{F}} \quad \tilde{\mathbf{B}}_1 \quad \tilde{\mathbf{B}}_2 \quad \ldots \quad \tilde{\mathbf{B}}_k - \underline{J}_k^T \tilde{\mathbf{F}}) \tag{3.12d}$$

The solution of the system given above will give us optimal (minimum norm of all force and torque vectors) and by changing the weighting matrix one can modify these optimal values(see Figure 3.1).

3.1 Case I: A Simplified Two-Arm Robot

The two robot system with its parameters, shown in Figure (3.2) is made of two identical manipulators. The equations of motion of the robot are described by two similar sets of nonlinear equations. The state of the system consists of angles and angular velocities

$$\tilde{\mathbf{q}} = (\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4)^{\mathbf{T}} \tag{3.13a}$$

and

$$\dot{\tilde{\mathbf{q}}} = (\dot\theta_1 \quad \dot\theta_2 \quad \dot\theta_3 \quad \dot\theta_4)^{\mathbf{T}} \tag{3.13b}$$

The input to the system are four torques acting on the joints

$$\tilde{\mathbf{T}} = (T_1 \quad T_2 \quad T_3 \quad T_4)^{\mathbf{T}} \tag{3.14}$$

let

$$\tilde{\mathbf{F}} = (G_3 \quad F_3 \quad G_4 \quad F_4)^{\mathbf{T}} \tag{3.15}$$

Combining the two sets of equations of motion in matrix form yields to overall dynamic equations of the robot

$$\underline{D}\ddot{\tilde{\mathbf{q}}} + \underline{B}\dot{\tilde{\mathbf{q}}}^2 + \tilde{\mathbf{G}} = \tilde{\mathbf{U}} + \underline{J}\tilde{\mathbf{F}} \tag{3.16}$$

The robot holds a rectangular load with mass M, and width d_1. To simply the analysis somewhat, it is assumed that the load does not rotate around its centre of mass and each robot is in contact with the load at one point

3.1.1 Digital Computer Simulations

The point-to-point motion control is achieved first by defining the desired trajectory of the centre of mass of the load (Figure 3 3) in the x-y coordinate system·

$$x_d = x_f + (x_i - x_f)e^{-50t^3}$$

$$y_d = y_f + (y_i - y_f)e^{-50t^3}$$

$$x_d = -150(x_i - x_f)t^2 e^{-50t^3}$$

$$y_d = -150(y_i - y_f)t^{5} \quad 50t^{3}$$

$$\dot{x}_d = -150(x_i - x_f)(2t \quad 150t^{4}) \quad 50t^{3}$$

$$\dot{y}_d = -150(y_i - y_f)(2t \quad 150t^{4}) \quad 50t^{3} \tag{3.17}$$

where (x_i, y_i) is the initial coordinate of the load and (x_f, y_f) is the final coordinate of the centre of mass of the load. The whole trajectory of the state from its initial to its final value can be computed and is needed as input to the controller.

The trajectory of $x(t)$ and $y_d(t)$ is shown in Figure (3.4). At first using inverse kinematics, all joint variables (including angular positions, velocities and acceleration of each robot) are calculated and plotted in Figures 3.5 -3.7.

The point-to-point motion of the original nonlinear system is simulated, and the optimal desired torques and forces are calculated by the utilization of matrix equation (3.12). Matlab Control Libraries are used to solve this set of algebraic equations and the results are plotted in Figure (3.8) ,(3.9) and (3.10). No actual manipulator dynamics are included - only the desired manipulator dynamics is considered in this section. Section 4.0 stresses the problem of control of actual manipulator dynamics (Fig. 3.1).

The variation of nominal value of optimal torque at joint 1 is plotted with time in Figure (3.8a). A non zero starting value shows the static equilibrium optimal torque value. Similarly in Figure (3.8b, c, d), nominal optimal torque variations are plotted. The magnitudes of joint torque are very impressive. A similar lifting operation was investigated by Laroussi [1988]. The comparison of joint torques for the given same trajectory and robot parameters shows the effectiveness of the developed optimization scheme, (Table 3.1).

The variation of nominal optimal constraint force of robot1 and robot2 in the x and the y direction versus time are plotted in Figures (3 9a-b) and (3 10a-b) The total constraint force vector is equal to inertial force vector of the object due to the acceleration in the x-direction since there is no other force field in this direction On the other hand, along the y-axis of the object, gravity force and inertial force will be effective and constant force will oppose gravity field as well as balance the inertial force due to acceleration in the y-direction of trajectory

3.2 Case II: A Simplified Spatial Two-Arm Robot

Consider two coordinating robot holding a single object in 3-D as shown in Figure (3 11) Let us name one of the two robots as the leader, whose motion is determined in accordance with the required motion of the object, while the other as the follower, whose motion is programmed from the motion trajectory of the leader together with the constraints between the two robots, (Zheng [1988])

For the configuration shown in Figure (3 12), the joint angles are $q_1 = 90$, $q_2 = 330$, $q_3 = 30$ for the leader, $q_4 = 90$, $q_5 = 195$, $q_6 = 207.8$ for the follower, and the payload is 5 kg Utilization of equation (3 12), optimal joint torques and constraint force vectors are obtained

$$\tilde{\mathbf{F}}_1 = (-2.84 \quad -1.83 \quad -23.73)^T \quad N$$

$$\tilde{\mathbf{F}}_1 = (2.84 \quad 1.83 \quad -25.32)^T \quad N$$

and $\tilde{\mathbf{F}} = \tilde{\mathbf{F}}_1 + \tilde{\mathbf{F}}_2 = (0 \quad 0 \quad -49.05)^T$ N, which is equivalent to gravitational force in z-direction The corresponding static joint torque vectors:

$$\tilde{\mathbf{T}}_1 = (\,5.06 \quad -39.12 \quad -8.67\,)^T \quad N-m$$

$$\tilde{\mathbf{T}}_2 = (\,-2.29 \quad -78.01 \quad -10.43\,)^T \quad N-m$$

The magnitude for $\tilde{\mathbf{T}}_1$ and $\tilde{\mathbf{T}}_2$ are calculated as

$$|\tilde{\mathbf{T}}_1| = (\tilde{\mathbf{T}}_1^T.\tilde{\mathbf{T}}_1)^{1/2} = 40.39 \quad N-m$$

and

$$|\tilde{\mathbf{T}}_2| = (\tilde{\mathbf{T}}_2^T.\tilde{\mathbf{T}}_2)^{1/2} = 78.74 \quad N-m$$

and the magnitudes of constraint forces are $|\tilde{\mathbf{F}}_1| = 23.97$ N and $|\tilde{\mathbf{F}}_2| = 25.54$ N

Zheng [1988] has proposed 3 different methods for optimal load distribution for two industrial robots handling a single object, and the same configuration with the equal amount of payload is implemented with these 3 methods. The comparison of results are given in Table (3.1). An analysis of Table (3.2) shows that our optimization technique is superior to these 3 methods (Zheng [1988]) as far as the magnitudes of the force/torque vectors are considered.

Table (3.3) shows constraint force and torque vectors in 3-D and their magnitudes corresponding to several weighting matrices. It may be noted that, one can adjust force/torque magnitudes by the adjustment of these weigthing matrices (positive -definite matrices) according to the objective of the task. Thus, these matrices introduces some new dimensions and degree of freedom to the nature of the optimization problem. The weighting matrices introduce relative importance to some of the elements effective force/torque vectors (consider multiplication of the weighting matrix by the force/torque vector) to be optimized with respect to the others Furthermore, the weighting matrices could also be used as penalty functions to limit the magnitude of certain parameters(i e torque/force vector).

In this chapter, nominal values of optimal joint torques and constraint forces are calculated assuming the object follows the desired trajectory without any error But in practical robotic applications this is not a realistic assumption Thus a control strategy is required in order to maintain the trajectory of the object (desired position) and the constraint force (desired force) Chapter 4 investigates the theory of position control to the case of cooperating multi-arm robots

Chapter 4 Position Control of Coordinated Robot Manipulators with Optimal Load Distribution

The main objective of this section is to develop a multiple arm load sharing (with minimum norms) position controller. Two important aspects of this controller are (i) optimal load sharing (minimum norm of joint torques and payload-contact forces) (ii) the application of optimal forces and torques to the payload while maintaining accurate position control. Here we will assume that, the contact between each of the grippers and the object is rigid. The main difference between this case and that of a single arm is that in the former system additional natural constraints are introduced due to the fact that all the grippers must be connected firmly to a rigid object. Thus, the arms can exert forces or torques on each other without the object containing an external environment. The arms must also move in harmony to induce the desired motion to the object.

The first step is to define a desired trajectory (translational and rotational position, velocity and acceleration) for the object and using inverse kinematics to calculate joint variables, i.e., angular positions, angular velocities and angular accelerations for each joint. These will be the desired joint variables and input to the controller(Figure 4.1). As shown in Figure 4.1, a position controller in joint space is developed with on-line optimization capability which minimizes the magnitude of joint torques and payload contact forces. Λ_v and Λ_p are the coefficient matrices for joint position and velocities to guarantee stability of the motion. D is the inertia matrix while 'H+G' are the non-linear coriolis and gravitational torque vectors. A feed forward scheme for non-linear terms are used to decouple the equations of motions in addition to the position and velocity feedback for control and stability view point Full manipulator and the payload dynamics are investigated The developed optimization scheme is used to distribute the minimum norm of joint torques and contact forces among the manipulators while maintaining an accurate position control of the payload The constraints of the sum of contact forces and joint torques are also satisfied

4.1 Control Dynamics of the Multi-Arm System

In this Section, the equations of motion of multi-arm robotic system will be studied to develop a motion controller The dynamic equation of the i^{th} manipulator in joint space coordinates is given by

$$\underline{D}(\tilde{\mathbf{q}})\ddot{\tilde{\mathbf{q}}} + \tilde{\mathbf{H}}(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) + \tilde{\mathbf{G}}(\tilde{\mathbf{q}}) = \tilde{\mathbf{T}}_i + \underline{J}_i^T \tilde{\mathbf{F}}_i \qquad i = 1, \ldots, k \tag{4.1}$$

$\tilde{\mathbf{T}}_i$ is the joint torque vector and $\underline{J}_i^T \tilde{\mathbf{F}}_i$ is the reactive force from the object

At first, we will obtain the necessary control force/torque equations for the control input Here, $\tilde{\mathbf{T}}_i$ is the equivalent external control torque vector applied to control the multi-arm robotic system while $\tilde{\mathbf{F}}_i$ is the total constraint force which supplies the required force for position control

Our objective is to determine total control input such that, the closed-loop system is stable To start with, equation (4.1) is rewritten in the following form:

$$\underline{D}(\tilde{\mathbf{q}})\ddot{\tilde{\mathbf{q}}} + \tilde{\mathbf{H}}(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) + \tilde{\mathbf{G}}(\tilde{\mathbf{q}}) = \tilde{\tau}_i \qquad i = 1, \ldots, k \tag{4.2}$$

In order to guarantee stability, $\tilde{\tau}_i$ is given as:

$$\tilde{\tau}_t = \underline{D}\ddot{\tilde{\mathbf{q}}}_d + \underline{DK_v}(\dot{\tilde{\mathbf{q}}}_d - \dot{\tilde{\mathbf{q}}}) + \underline{DK_p}(\tilde{\mathbf{q}}_d - \tilde{\mathbf{q}}) + \tilde{\mathbf{H}} + \tilde{\mathbf{G}} \tag{4.3}$$

where $\tilde{\mathbf{H}} + \tilde{\mathbf{G}}$ is the compensation term for Coriolis and gravitational forces Upon substitution for the right side of equation (4 2), one obtains

$$\underline{D}(\tilde{\mathbf{q}})\ddot{\tilde{\mathbf{q}}} + \tilde{\mathbf{H}}(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) + \tilde{\mathbf{G}}(\tilde{\mathbf{q}}) = \underline{D}\ddot{\tilde{\mathbf{q}}}_d + \underline{DK_v}(\dot{\tilde{\mathbf{q}}}_d - \dot{\tilde{\mathbf{q}}}) + \underline{DK_p}(\tilde{\mathbf{q}}_d - \tilde{\mathbf{q}}) + \tilde{\mathbf{H}} + \tilde{\mathbf{G}} \tag{4.4}$$

After some algebraic simplifications, one can rewrite equation (4 4) as follows

$$\underline{D}(\ddot{\tilde{\mathbf{q}}} - \ddot{\tilde{\mathbf{q}}}_d) + \underline{DK_v}(\dot{\tilde{\mathbf{q}}} - \dot{\tilde{\mathbf{q}}}_d) + \underline{DK_p}(\tilde{\mathbf{q}} - \tilde{\mathbf{q}}_d) = \tilde{\mathbf{0}} \tag{4.5}$$

$\underline{D}$ is the inertia matrix, it is a positive definite matrix and if one multiplies by $\underline{D}^{-1}$ the equation (4 5),

$$(\ddot{\tilde{\mathbf{q}}} - \ddot{\tilde{\mathbf{q}}}_d) + \underline{K_v}(\dot{\tilde{\mathbf{q}}} - \dot{\tilde{\mathbf{q}}}_d) + \underline{K_p}(\tilde{\mathbf{q}} - \tilde{\mathbf{q}}_d) = \tilde{\mathbf{0}} \tag{4.6}$$

By defining

$$\tilde{\mathbf{e}} = \tilde{\mathbf{q}} - \tilde{\mathbf{q}}_d \tag{4.7}$$

where $\tilde{\mathbf{e}}$ is the difference between the desired and actual position of the object From the matrix equation (4 6), we have

$$\ddot{\tilde{\mathbf{e}}} + \underline{K}_v \dot{\tilde{\mathbf{e}}} + \underline{K}_p \tilde{\mathbf{e}} = \tilde{\mathbf{0}} \tag{4 8}$$

which indicates that the system can be made stable by selecting $\underline{K}_v$ and $\underline{K}_p$ matrices such that equation (4 8) has poles with negative real parts

4.2 Motion Control

The control block diagram of Figure (4 1) shows the overall system with control The control architecture of Figure (4 1) has a very important and desirable feature From a computational point of view, it is desirable for a distributed computer architecture Thus, each arm can be controlled by a separate microprocessor Using the proposed method, the number of mathematical operations increases only linearly with the number of cooperating arms, therefore computational time does not increase much in the case of very crowded multiple-arm coordination The stability of this system is given by Equation (4 8) that as long as a precise knowledge of the mass property of the arms as well as the object is available The requirement for knowledge of the objects mass property is not that much important if the robot arms are much massive in comparison with the object

The optimization scheme which was developed in chapter 3 is used to distribute torques and contact forces among the robots Therefore, optimal load sharing and the applications of optimal (minimum norm) contact forces and torques while maintaining accurate position control are achieved by the utilization of the proposed control architecture [Figure (4 1)]

4.2.1 Simulation

In this section, a lift operation is simulated from an initial point to a final point with a desired trajectory by two arms (2-DOF). Figure (3 3) illustrates the two arms with the payload The following system parameters are used

$J_1 = J_2 = J_3 = J_4 = 0.005 kg - m^2$(Inertias of links)

$m_1 = m_2 = m_3 = m_4 = 1kg$(masses of links)

$a_1 = a_2 = a_3 = a_4 = 0.25m$(length of links)

$m_g = 0.50kg$(payload)

Two arms, each with 2-DOF, are holding the payload and lifting from the given initial configuration to a final configuration with a desired trajectory. The desired trajectory of the centre of mass of the payload

$$x_d = x_f + (x_i - x_f)e^{-50t^3}$$

$$y_d = y_f + (y_i - y_f)e^{-50t^3}$$

$$\dot{x}_d = -150(x_i - x_f)t^2 e^{-50t^3}$$

$$\dot{y}_d = -150(y_i - y_f)t^2 e^{-50t^3}$$

$$\ddot{x}_d = -150(x_i - x_f)(2t - 150t^4)e^{-50t^3}$$

$$\ddot{y}_d = -150(y_i - y_f)(2t - 150t^4)e^{-50t^3} \qquad (4.9)$$

where (x_i, y_i) is the initial coordinate of the load and (x_f, y_f) is the final coordinate of the centre of mass of the load

The main objective of this simulation is to demonstrate the effectiveness of the position controller (i e error in between actual and desired position) and also to display the magnitude Joint torques and contact forces (minimum possible norm) The control parameters and dynamics of the robots are plotted with time

Figure(4 2a,b) show variations of positions of the payload/and effectors in the X and Y directions Both actual and the desired trajectory are plotted with time As shown in figures, K_v, and K_p matrices are selected to guarantee stability and the actual displacements follow the desired trajectory very closely Figure (4 3) illustrates the variation of velocity (both in the X and Y directions) of the centre of payload and the variation of payload's acceleration with time The actual velocity acceleration profiles are almost identical as the desired velocity and acceleration of the payload both in the X and Y directions

The variations of controlled Joint angles are plotted with time in Figure(4 4) (see the geometry of the robots - Figure 3 3). The variations of the Joint rates (velocity and acceleration) are shown in Figure (4 5)

In Figure (4 6), the variation of total-controlled contact force both in the X and Y direction are plotted with time during the lift operation. As shown in the figure, the total constraint force vector is equal to inertial force vector of the object due to the acceleration in the X direction since there is no other force field in this direction Therefore, it is zero both in $t = 0$ and $t = t_f$ On the other hand, along the Y axis of the object, the gravity force and the inertial force will be effective and constant force will oppose the gravity field as well as the balance the inertial force due to the acceleration in the Y-direction of trajectory Therefore, it is equivalent to the static load $(0.5 \times 9\ 81 \approx 4.9N)$ in the beginning and at the end of the simulations

The distributed contact forces between two robots are plotted with time in Figure(4 7) both in the X and Y directions These contact forces are of minimum norm and obtained by the utilization of on-line optimization scheme (see Figure 4.1).

As shown in the figures, the sum of the individual control forces (both separately, in the X and Y direction) are equal to the total constraint forces The total constraint forces are shared by the two robots in such a way that not only it satisfies the total force constraints equation, but also the dynamic control equations to maintain the desired trajectory

Figure (4 8) shows the variations of (minimum-norm) Joint torques with time These Joint torques both satisfy the dynamic control equations and as well as constraint torque equations

In this chapter, a control architecture for position control of k cooperating arms is developed The redundancy in the force and joint torque subspace are resolved by on-line minimization of a quadratic cost function of the desired constraint force and joint torques The control architecture is well suited for a distributed computer architecture

Chapter 5 General Discussion and Conclusion

5.1 Closing Remarks

Throughout this thesis, the main objective of the investigation has been the study of modeling and simultaneous position control of Coordinated Multiple Manipulators with minimum norms of joint torques and control forces From previous studies, it is noted that there has been very little effort on the distribution of loads to the multiple arms, particularly, the systematic determination of the forces and torques acting between the common load and the grippers of the manipulator The present literature is scarce on these topics This thesis explored a number of fundamental issues dealing with the motion control of a multi-arm robotic system which form a closed kinematic chain Through detailed analysis of a simple multi arm robotic system we developed and implemented a control architecture of position with on-line optimization scheme

5.2 Thesis Summary

During the study of thesis, it is seen that the control of coordinated multiple manipulators is a complex and challenging problem and has many aspects The potential knowledge level in the control of multiple interacting manipulators is not very advanced This thesis mainly dealt with the dynamic modeling and control strategies in coordinated multiple manipulators When cooperating manipulators grasp a common object the dynamical model of the system changes from unconstrained to constrained dynamics Indeed when two or more manipulators hold a common object a closed chain is formed It imposes nonholonomic and/or holonomic constrains in the system dynamics which usually results in a loss of degrees of freedom The positions of the grippers of the CMM's are constrained Moreover, the interacting forces (torques) exerted by the cooperating manipulator are related This thesis studied these afformentioned important research issues

In Chapter 1, a general introduction with a literature survey is given, and the previous work were put in perspective

In Chapter 2, robot manipulators, the kinematics and dynamics of coordinated multiple manipulators were studied The dynamical model including a general shape object was developed As a first step, the dynamical equations were obtained on the general matrix form for all (robot architecture and) manipulators including the payload object

The force vector exerted by the manipulators on the object was incorporated three dimensional rotational and translational dynamics of object was also investigated

In Chapter 3, optimal joint torques and constraint forces were obtained This was accomplished by minimizing a quadratic function of the joint torques and force of both manipulator while maintaining the predefined trajectory The comparison of optimal vector of torques and contact forces obtained in this thesis with existing work were done and the performance of the optimization scheme was investigated and discussed

The multiple arm load sharing position controller was developed in Chapter 4 Two important aspects of this controller were discussed, (i) load sharing, (ii) the application of forces and torques to the object while maintaining accurate position control These objectives were achieved simultaneously by optimization of certain kind of performance, i e , by minimizing an interactive force/torque vector The simulation results were presented, discussed and comparison with other control strategies were done in this chapter

5

5.3 Suggestions for Future Research

The potential applications of coordinated multiple manipulator systems cover a wide range extending from assembly task in automated manufacturing to deep see exploration and service tasks in outer space Two or more arms may be needed to handle complicated and dexterous tasks skillfully However at this time the research issues associated with multiple manipulator systems are not clear Specifically, it is not evident whether the fundamental research questions which must be answered to bring such systems into reality are basically the same as those which arise in single arm manipulators or whether fundamentally new questions arise from the need to coordinate the motion of two or more robot arms

The list of research issues, which can be suggested for future research may be analyzed in three groups (i) Motion Planning Obstacle Avoidance and Sensing in CMMs, (ii) Dynamical Modeling and Control Strategies in CMMs, (iii) Software and Artificial Intelligence in CMM s Although some of the problems related to (ii) (Dynamical Modeling and Control Strategies in CMM's) are tackled in this thesis, there are still very interesting topics left for future work, especially parallel algorithms for control of CMM systems and adaptive control of CMM systems

A particular application determines the autonomy of individual manipulators, and the degree of centralization/centralization of the control in CMM Systems Available sensory information needs to be integrated into the control algorithms Research is needed to make control systems utilize the environmental information in an intelligent manner and thus make the system adaptable and flexible to various situations, for example , to those occurring in flexible manufacturing process of CMM systems to different manufacturing process will be enhanced after the researcher has demonstrated the practicality of adaptive control schemes to mu tiple manipulator systems Basic studies on the control of redundant CMM systems should be undertaken to shed light on the advantages and disadvantages to this virgin area

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Magnitudes Joint Torques (N-m) (Max.)	Different Methods	
	Our Method	Laroussi [1988]
$\tilde{\mathbf{T}}_1$	0 8	3 2
$\tilde{\mathbf{T}}_2$	1 9	1 6
$\tilde{\mathbf{T}}_3$	2 0	4 0
$\tilde{\mathbf{T}}_4$	0 8	1 8

Table 3.1 Comparison of Magnitudes of Maximum Joint Torques

Magnitudes Force/Torque	Different Methods			
	Our Method	Method 1	Method 2	Method 3
$\tilde{F}_1$ (N)	23.97	53 39	0	24 53
$\tilde{F}_1$ (N)	25 54	92.05	50 00	24.53
$\tilde{T}_1$ (N-m)	40.39	24.88	57.81	80.00
$\tilde{T}_2$ (N-m)	78.74	63.63	57.24	53.19

Table 3.2 Comparison of Magnitudes of Constraint Force and Joint Torques

Magnitudes Force/Torque	Different Weighting Values			
	$W_{ii} = 1.0$	$W_{ii} = 2.0$	$W_{ii} = 10.0$	$W_{ii} = 20.0$
$\tilde{\mathbf{F}}_1$ (N)	23.97	23.73	23.70	25.37
$\tilde{\mathbf{F}}_1$ (N)	25.54	26.47	30.32	34.18
$\tilde{\mathbf{T}}_1$ (N-m)	40.39	41.11	43.38	45.47
$\tilde{\mathbf{T}}_2$ (N-m)	78.74	78.48	77.49	76.41

Table 3.3 Effect of Weighting Matrix W

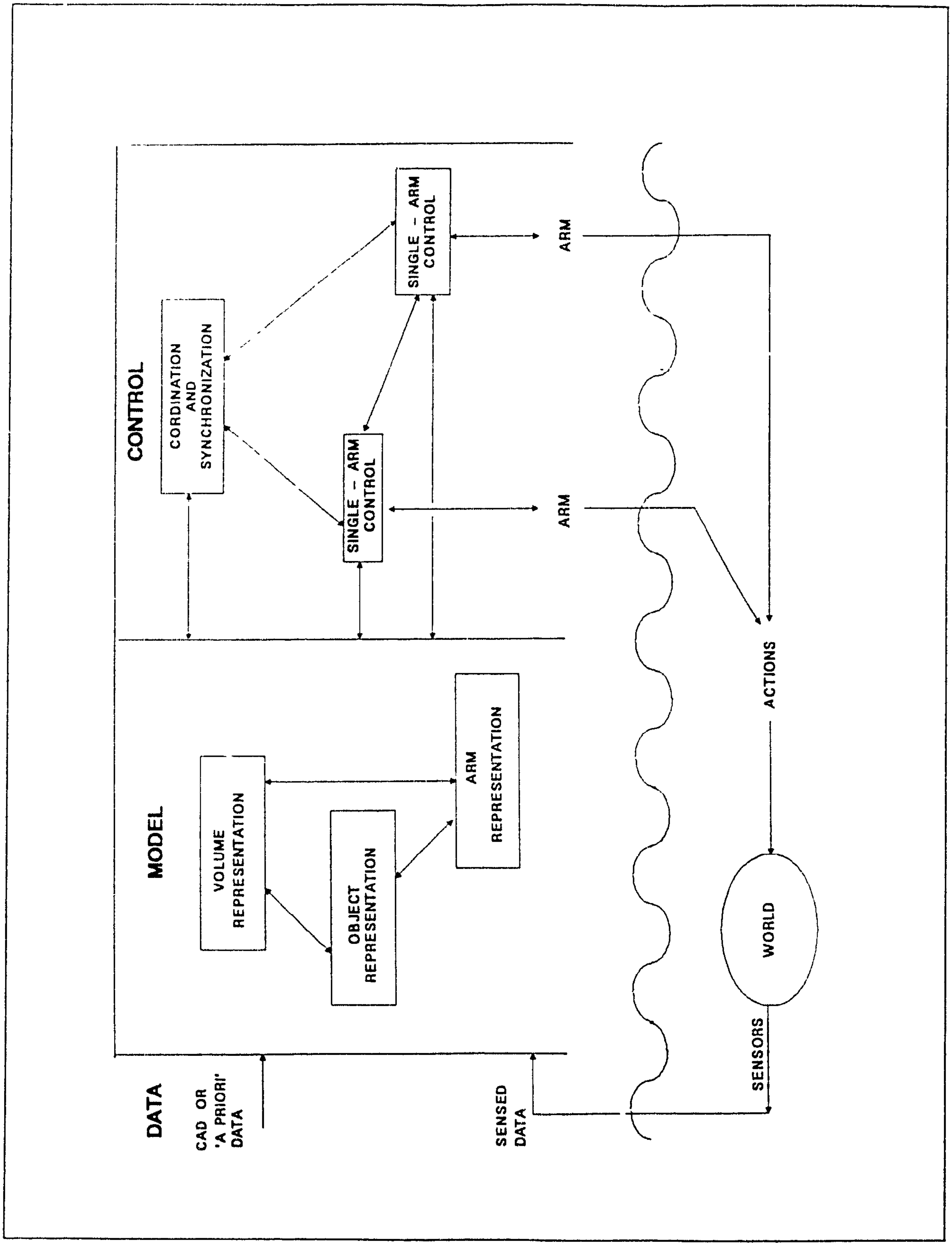


Figure 1.1 Overall System Control Hierarchy of a Dual-Arm Robot

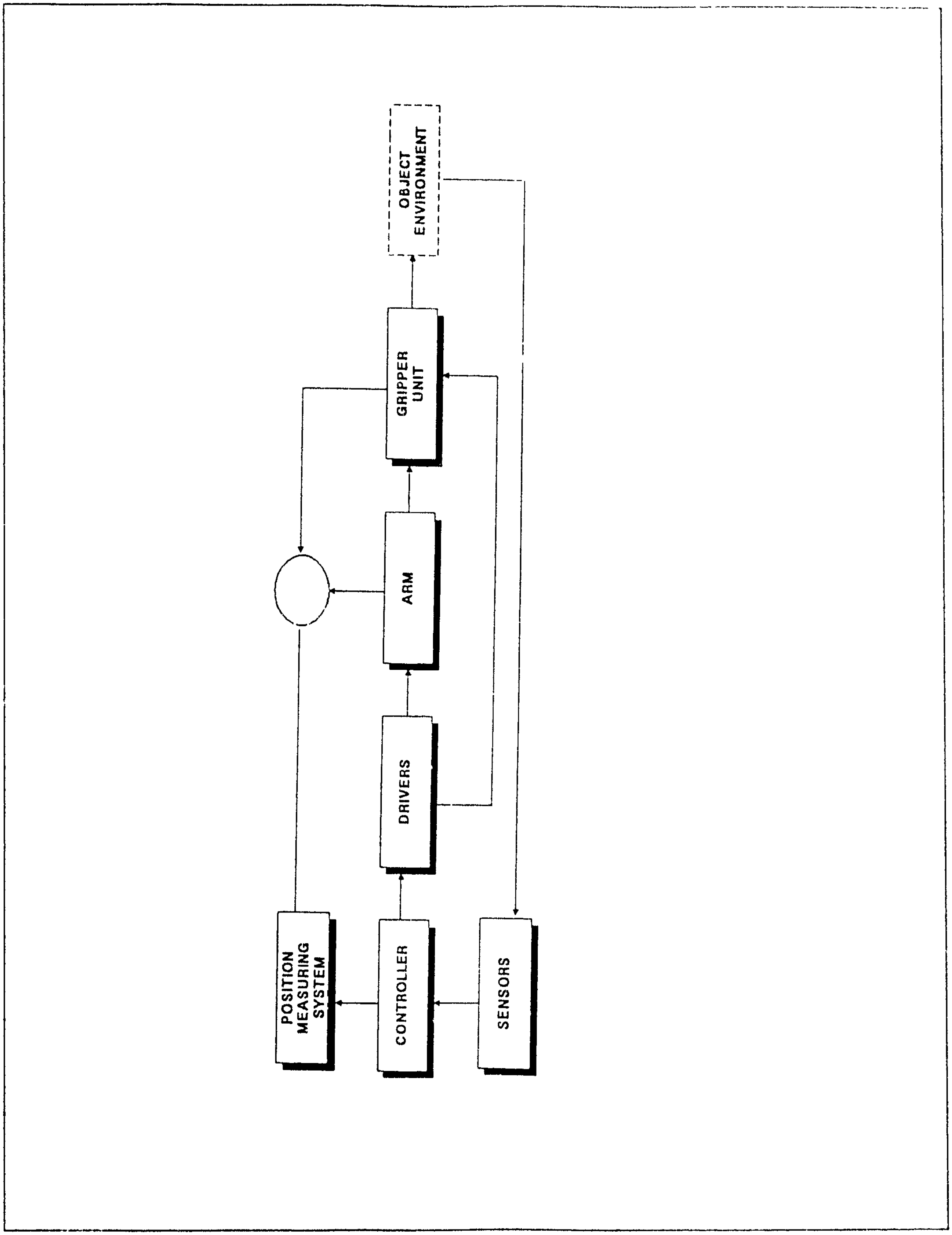
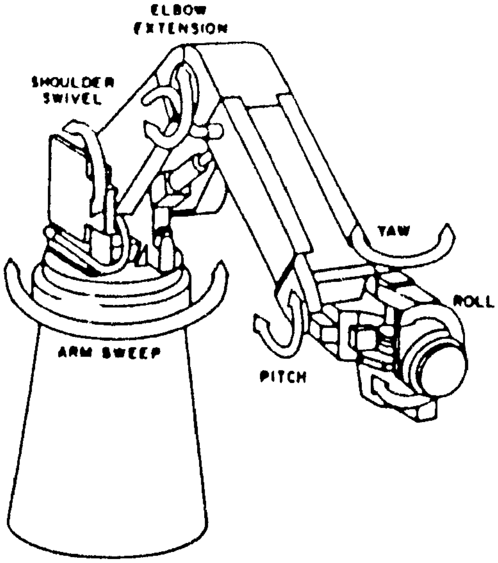
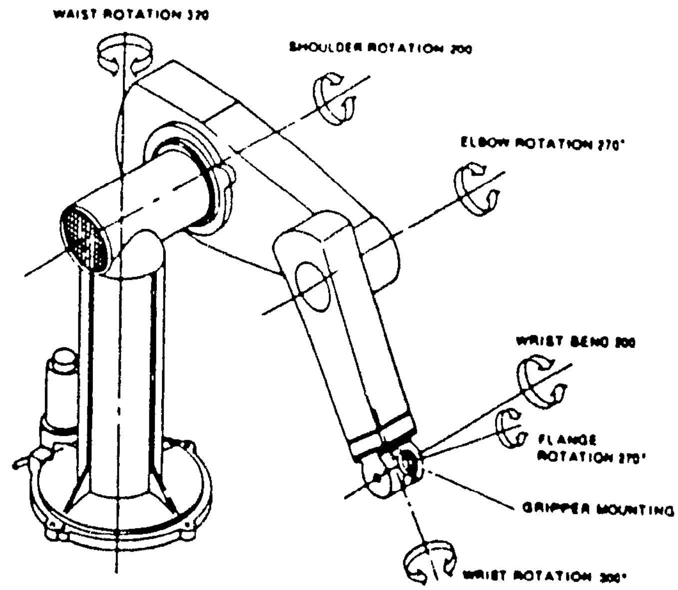


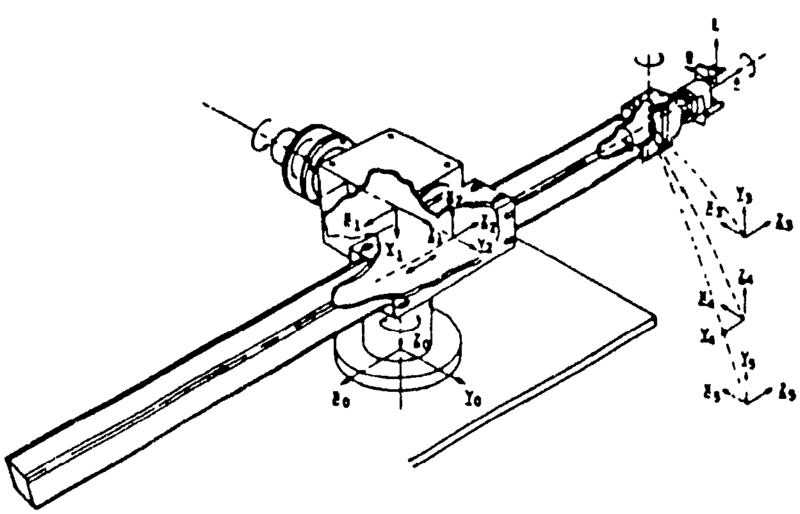
Figure 2.1 Main Parts of an Industrial Robot



Cincinnati Milacron



Unimation Puma 600



Stanford manipulator

Figure 2.2 Typical Industrial Robot Manipulators

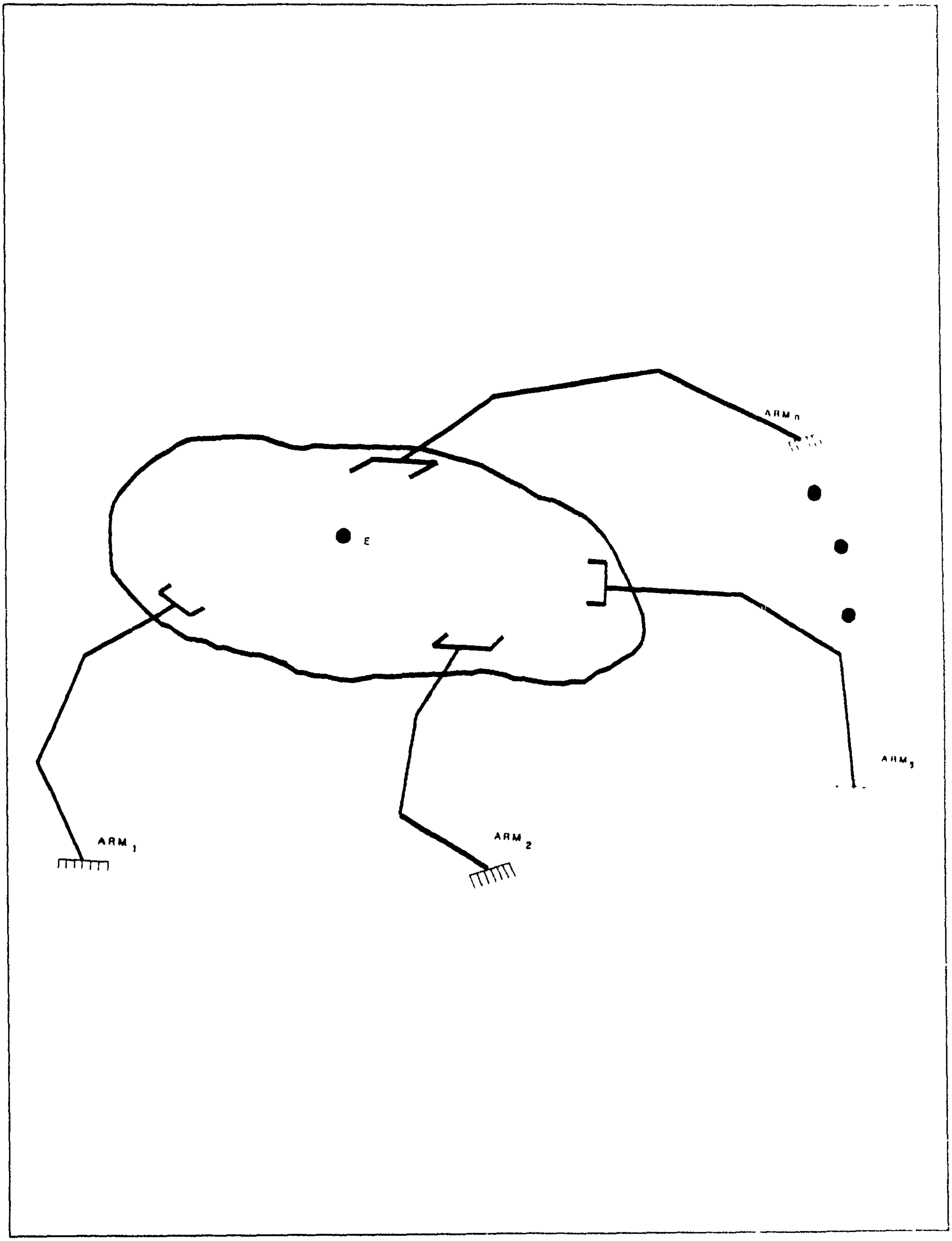


Figure 2.3 Geometry of the System

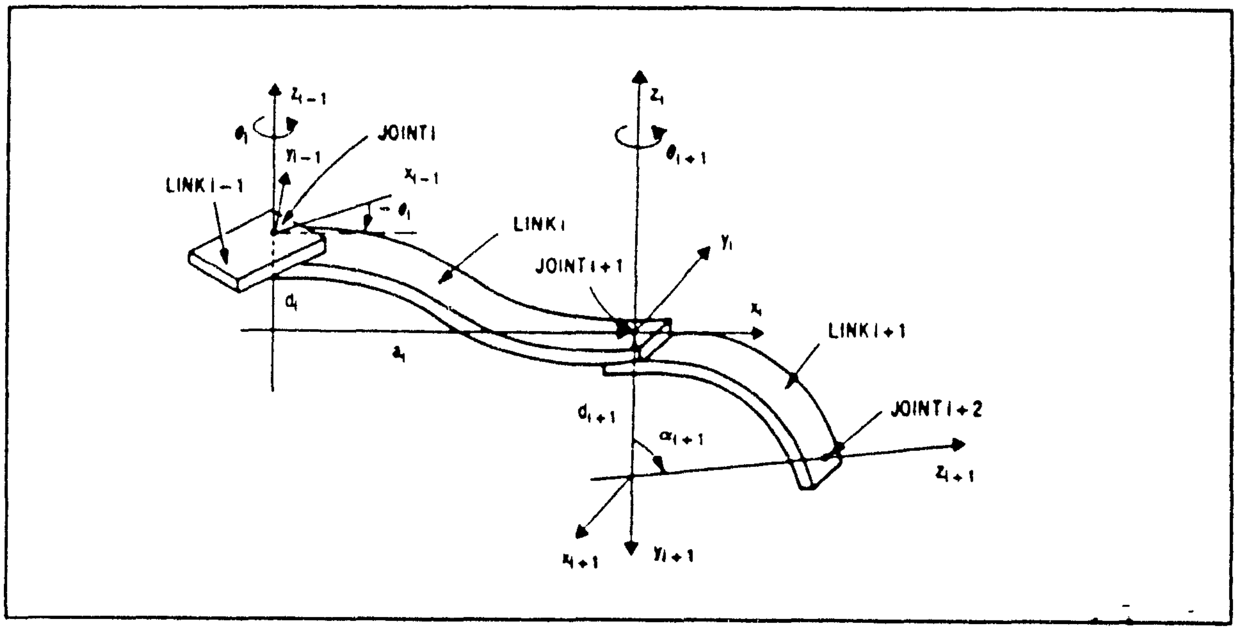
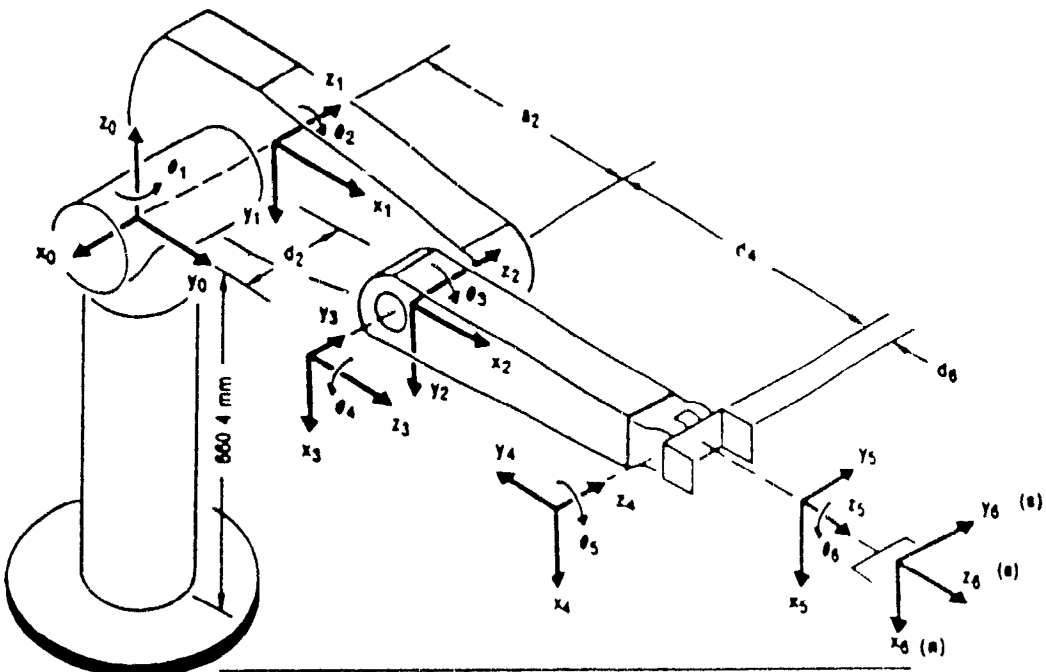


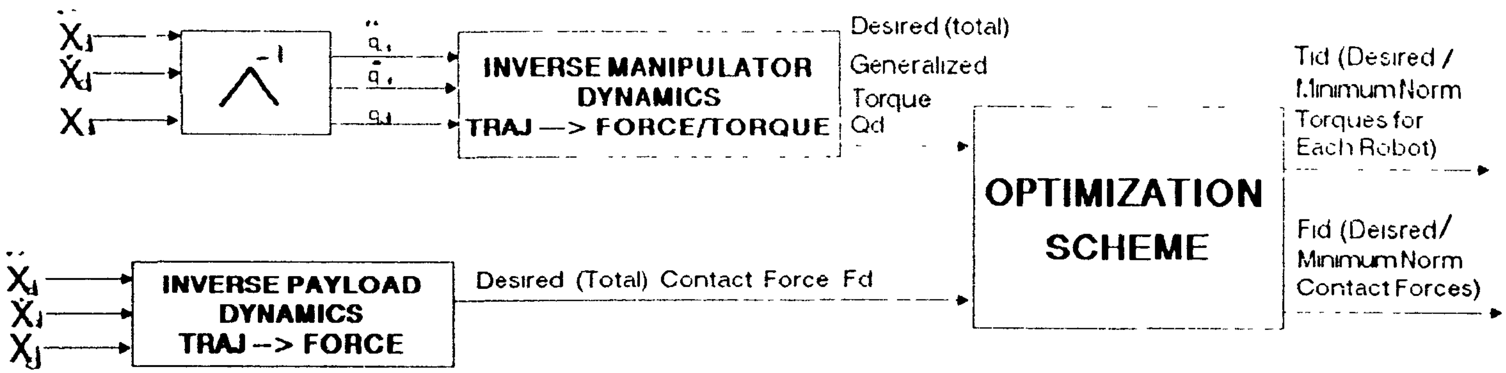
Figure 2.4 D-H Link Coordinate System and Kinematic Description



PUMA ROBOT ARM LINK COORDINATE PARAMETERS

JOINT I	θ_i	α_i	a_i	d_i	RANGE
1	90	-90	0	0	-160 to +160
2	0	0	432 mm	149 5 mm	-225 to +45
3	90	90	0	0	-45 to +225
4	0	-90	0	432 0	-110 to +170
5	0	90	0	0	-100 to +100
6	0	0	0	56 5	-266 to +266

Figure 2.5 D-H Kinematic Description of a PUMA 600



DESIRED TRAJECTORY ----> MINIMUM - NORM JOINT TORQUES (DESIRED) & CONTACT FORCES

Figure 3.1 Optimization Scheme

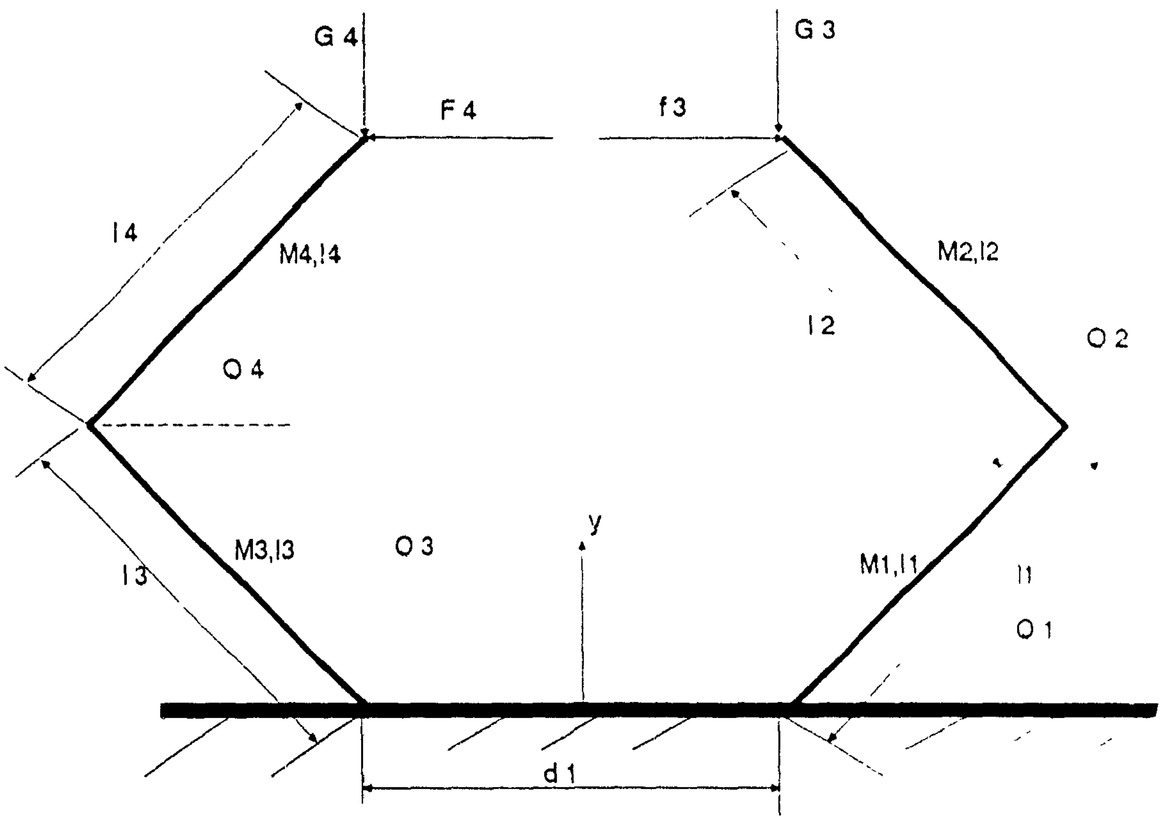


Figure 3.2 Two-Arm Robot and Parameters

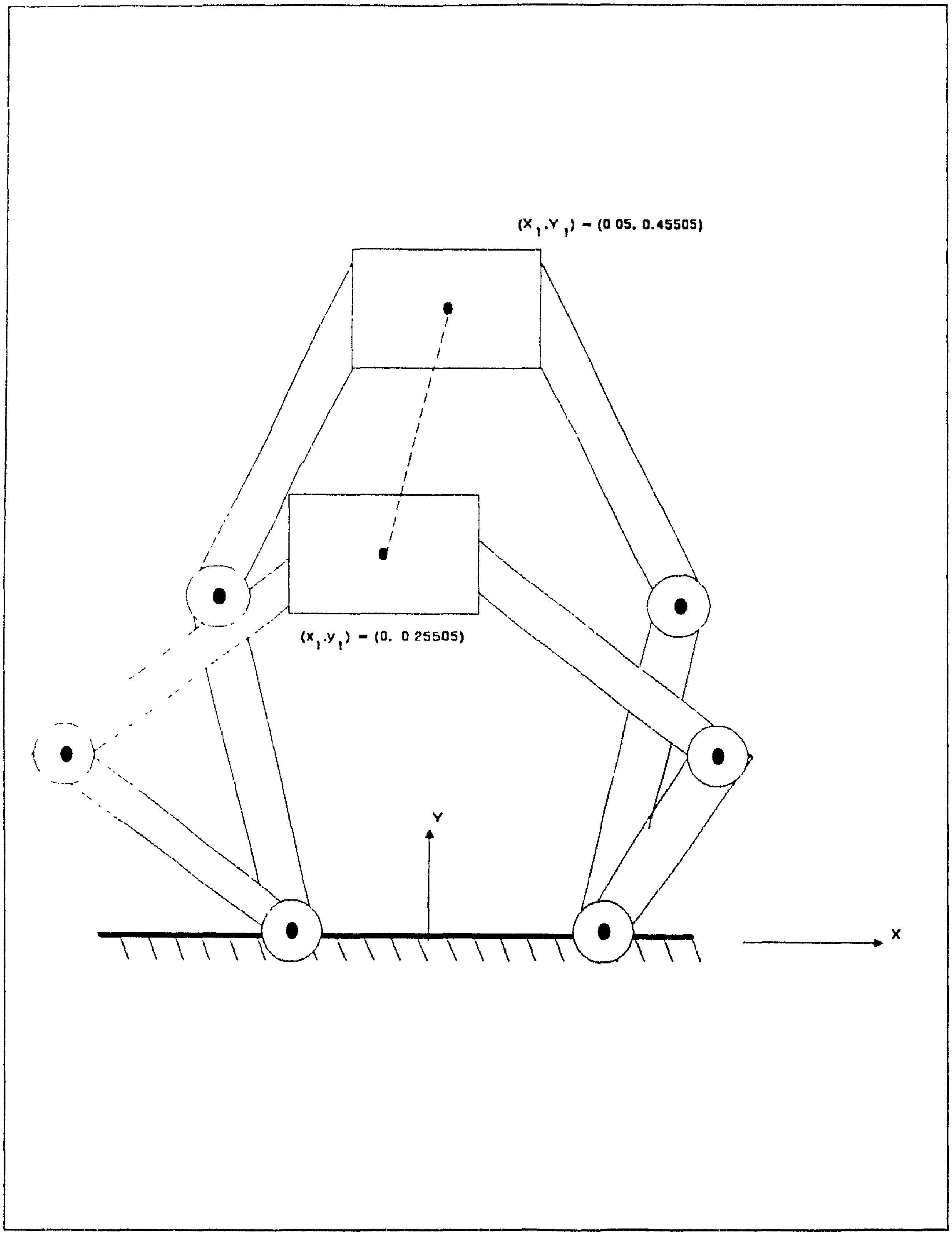


Figure 3.3 Illustration of Point-to-Point Movement

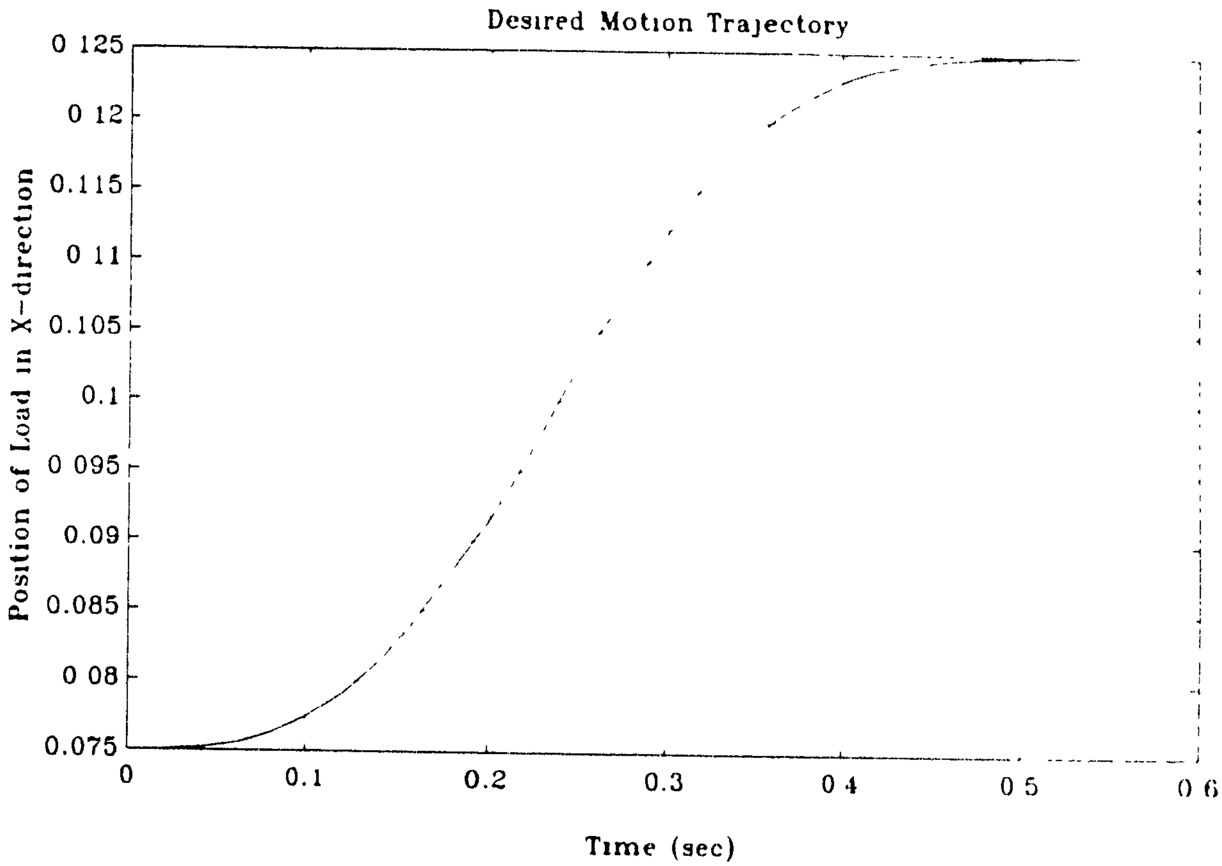


Figure 3.4a The Desired Point-to-Point Motion Trajectory x_d and y_d

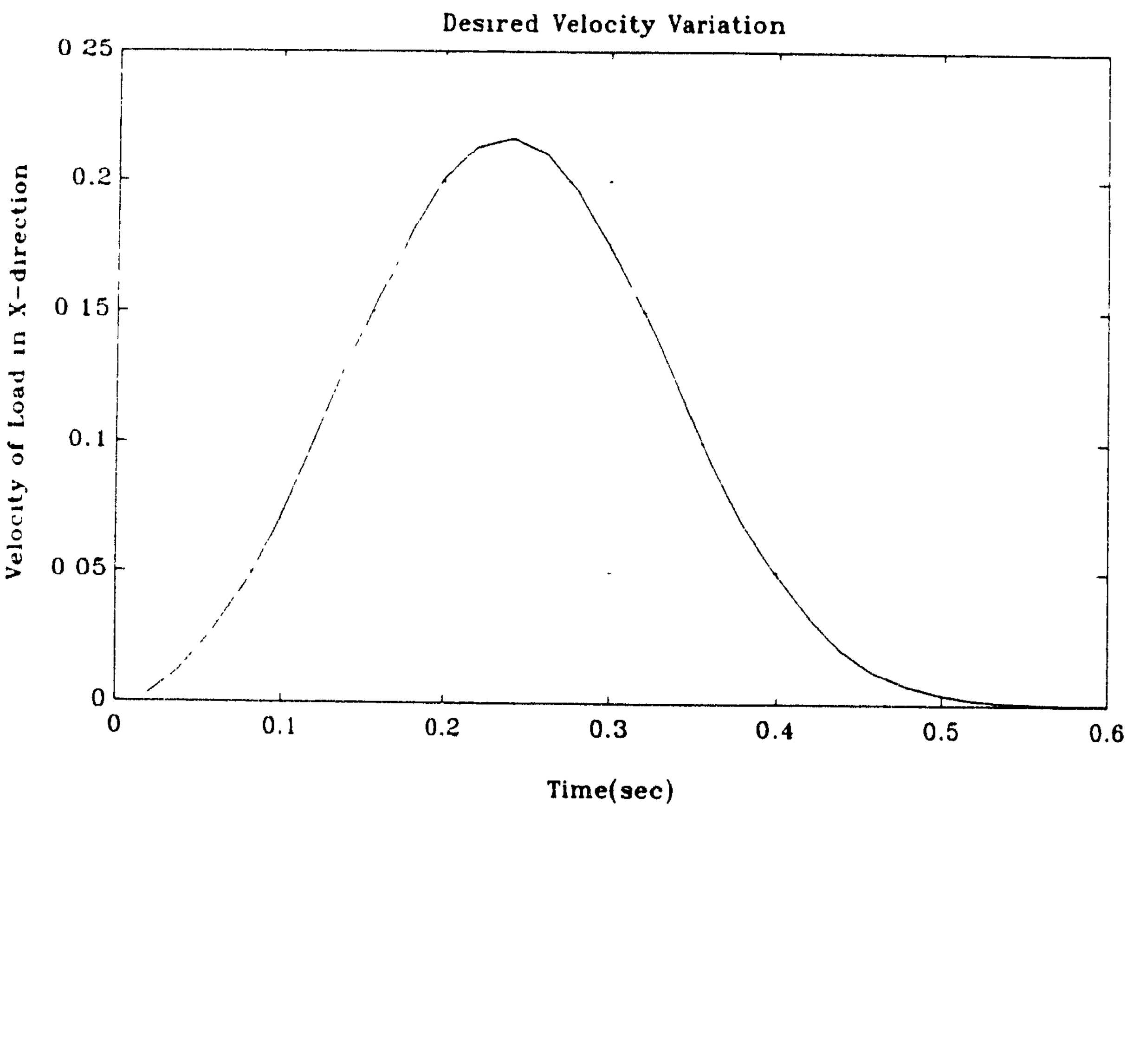


Figure 3.4b Variation of Desired Translational Velocity of the Object with Time

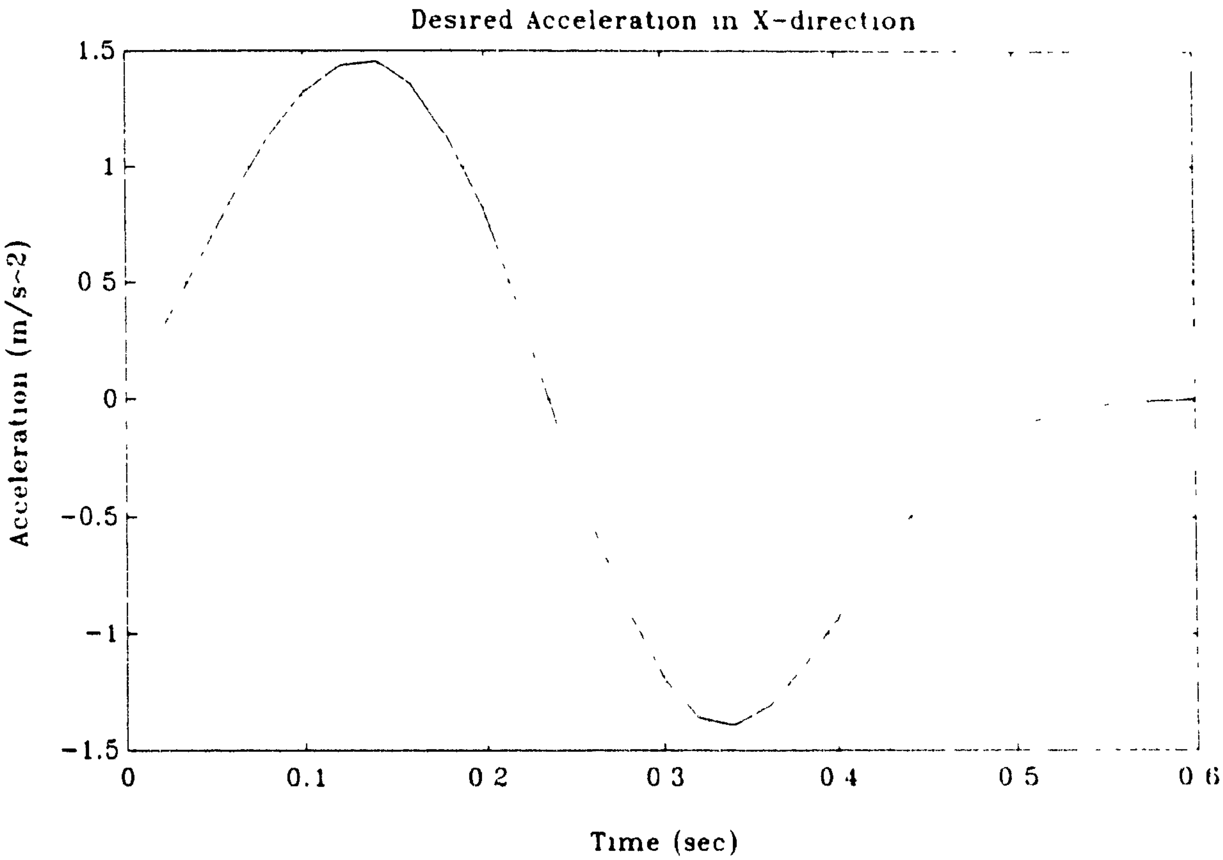


Figure 3.4c Variation of Desired Translational Acceleration of the Object with Time

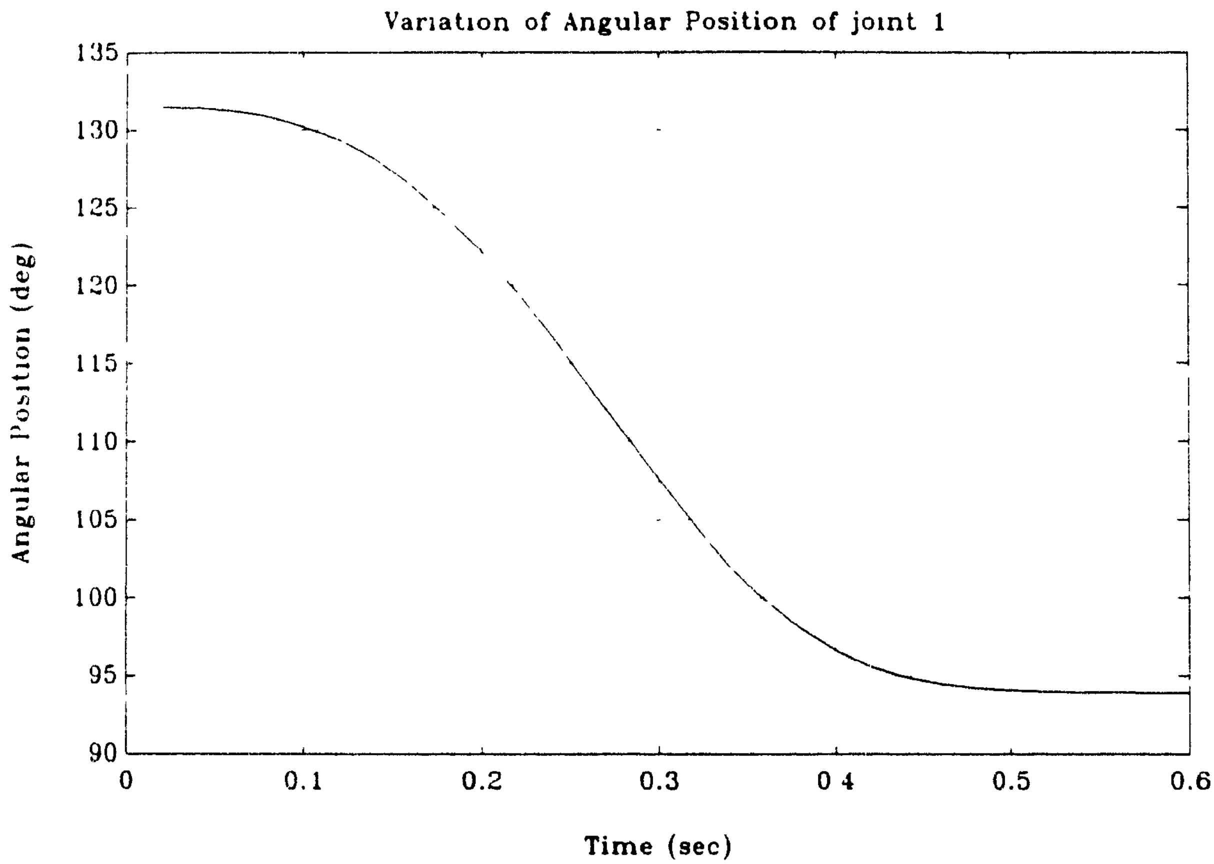


Figure 3.5a Variation of Angular Position of Joint 1 with Time

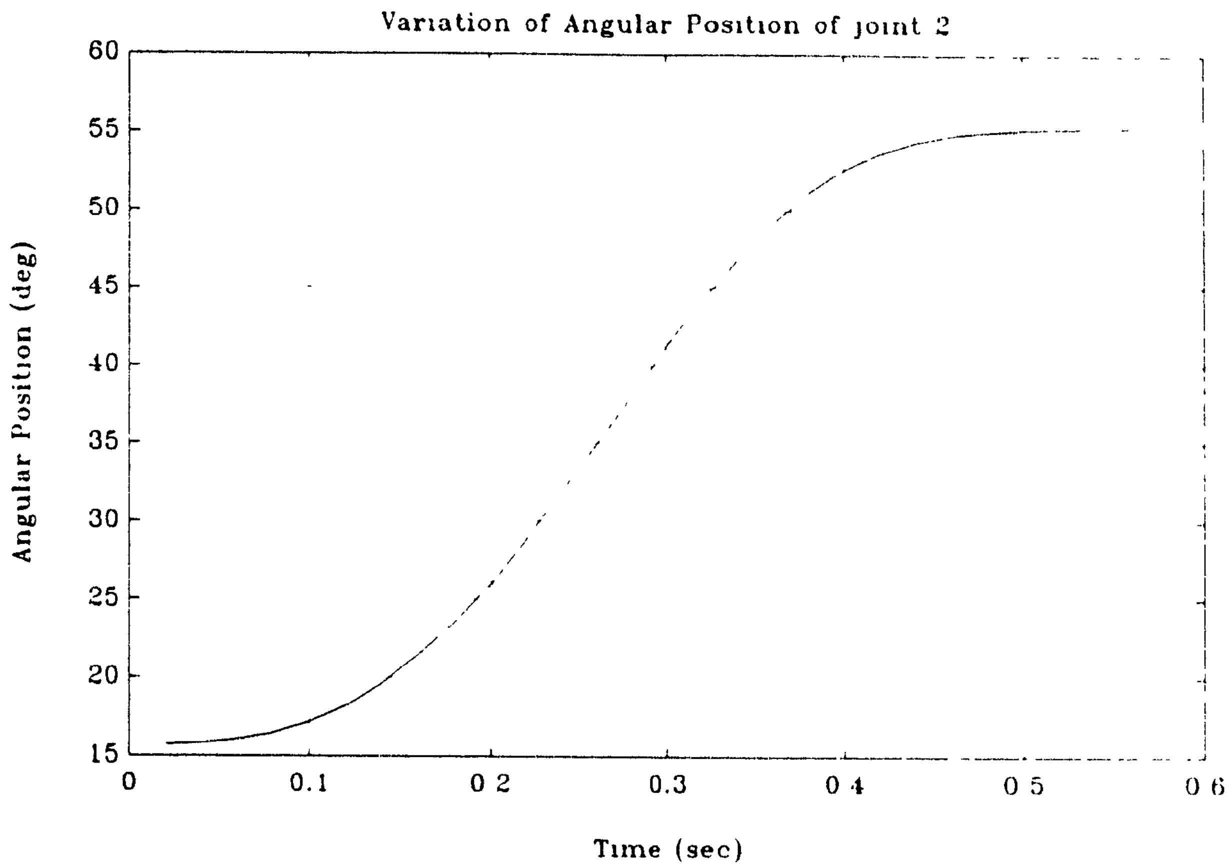


Figure 3.5b Variation of Angular Position of Joint 2 with Time

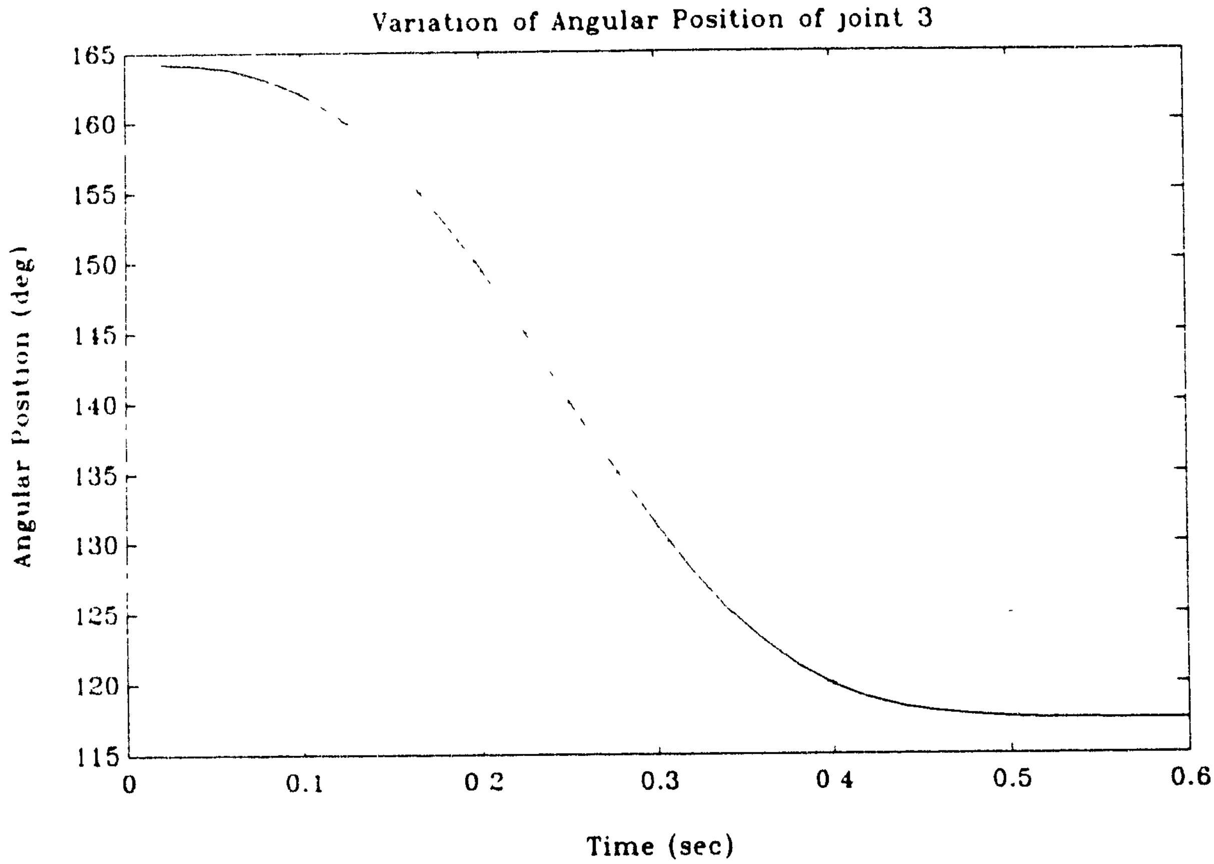


Figure 3.5c Variation of Angular Position of Joint 3 with Time

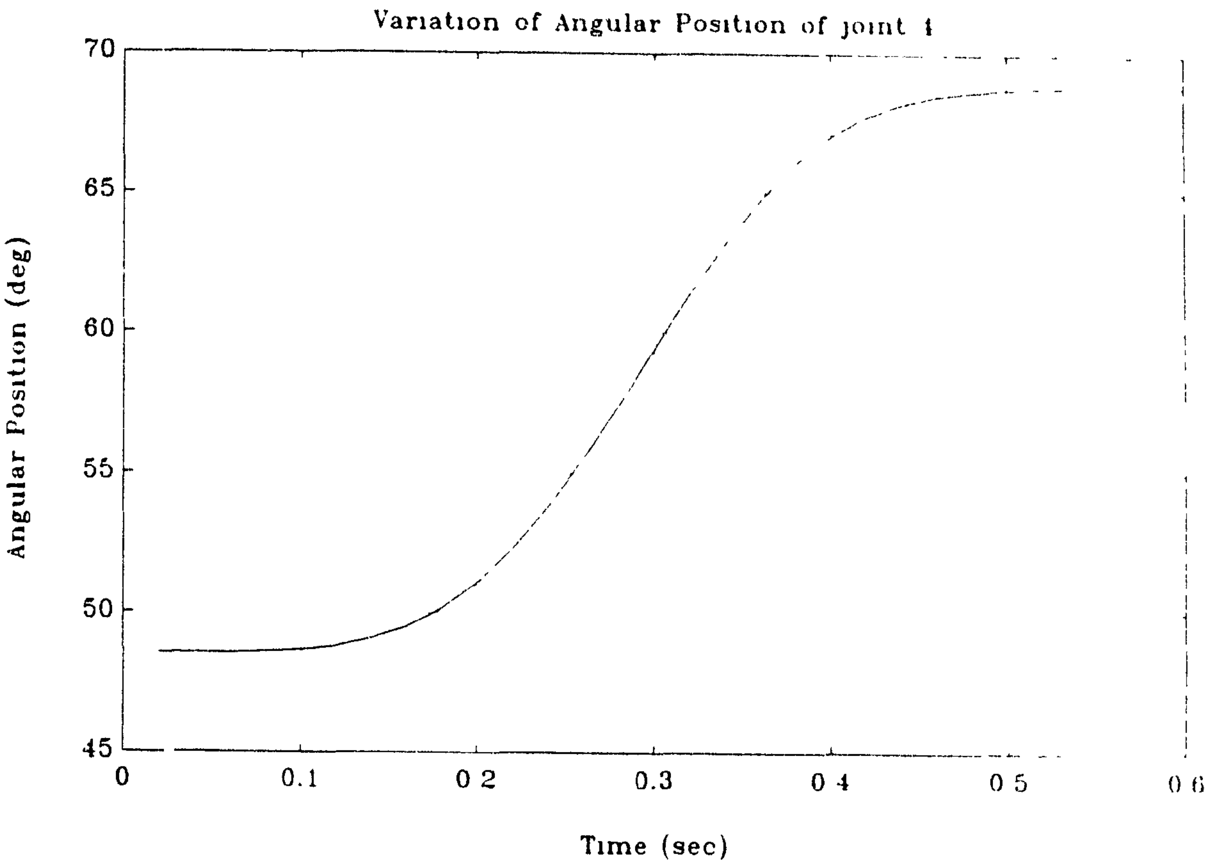


Figure 3.5d Variation of Angular Position of Joint 4 with Time

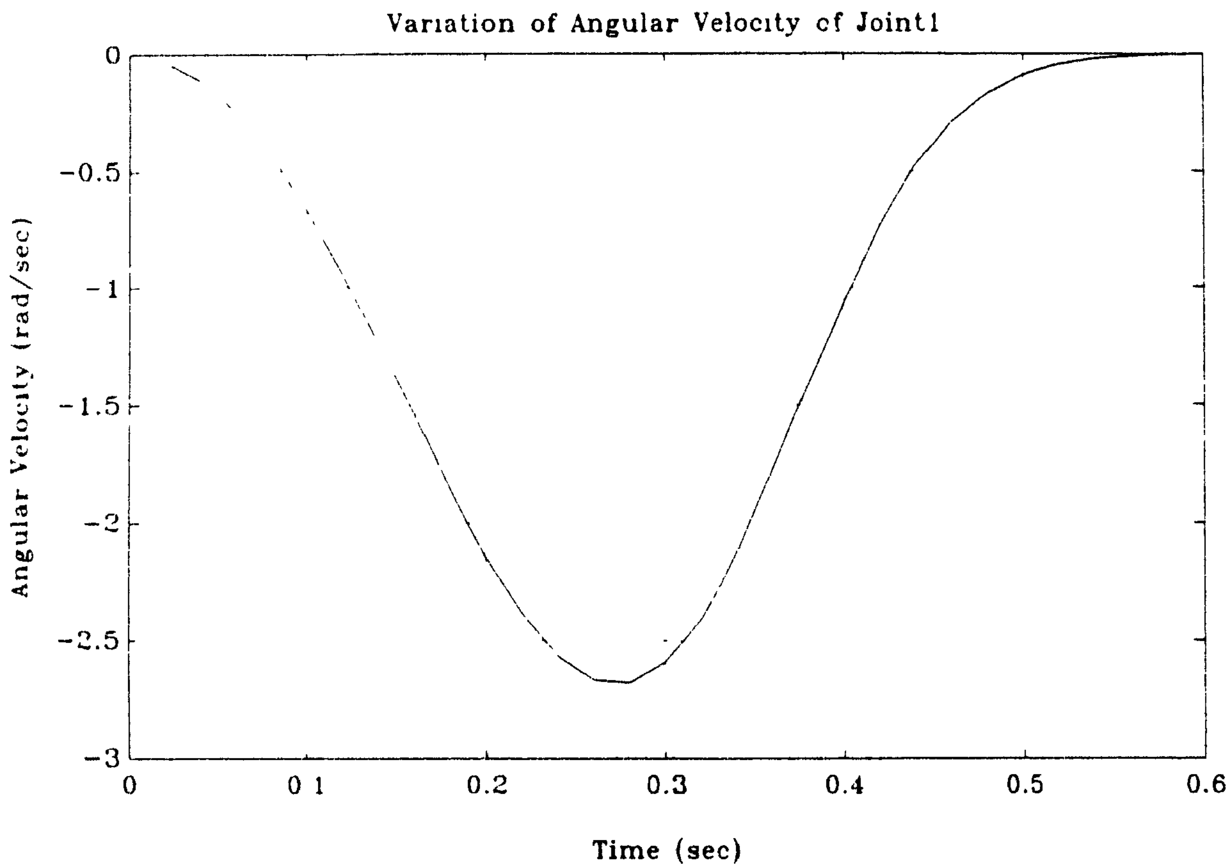


Figure 3.6a Variation of Angular Velocity of Joint 1 with Time

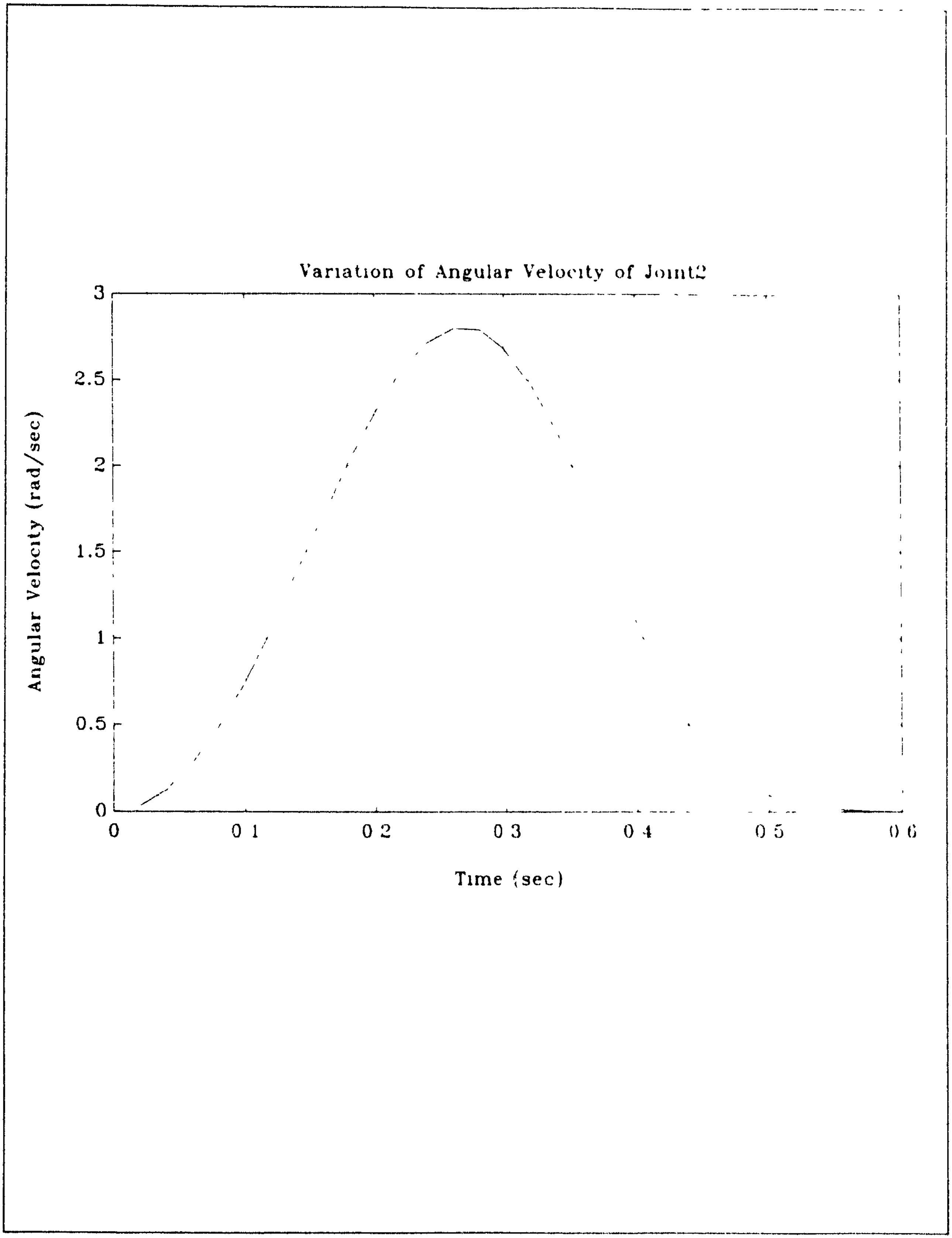


Figure 3.6b Variation of Angular Velocity of Joint 2 with Time

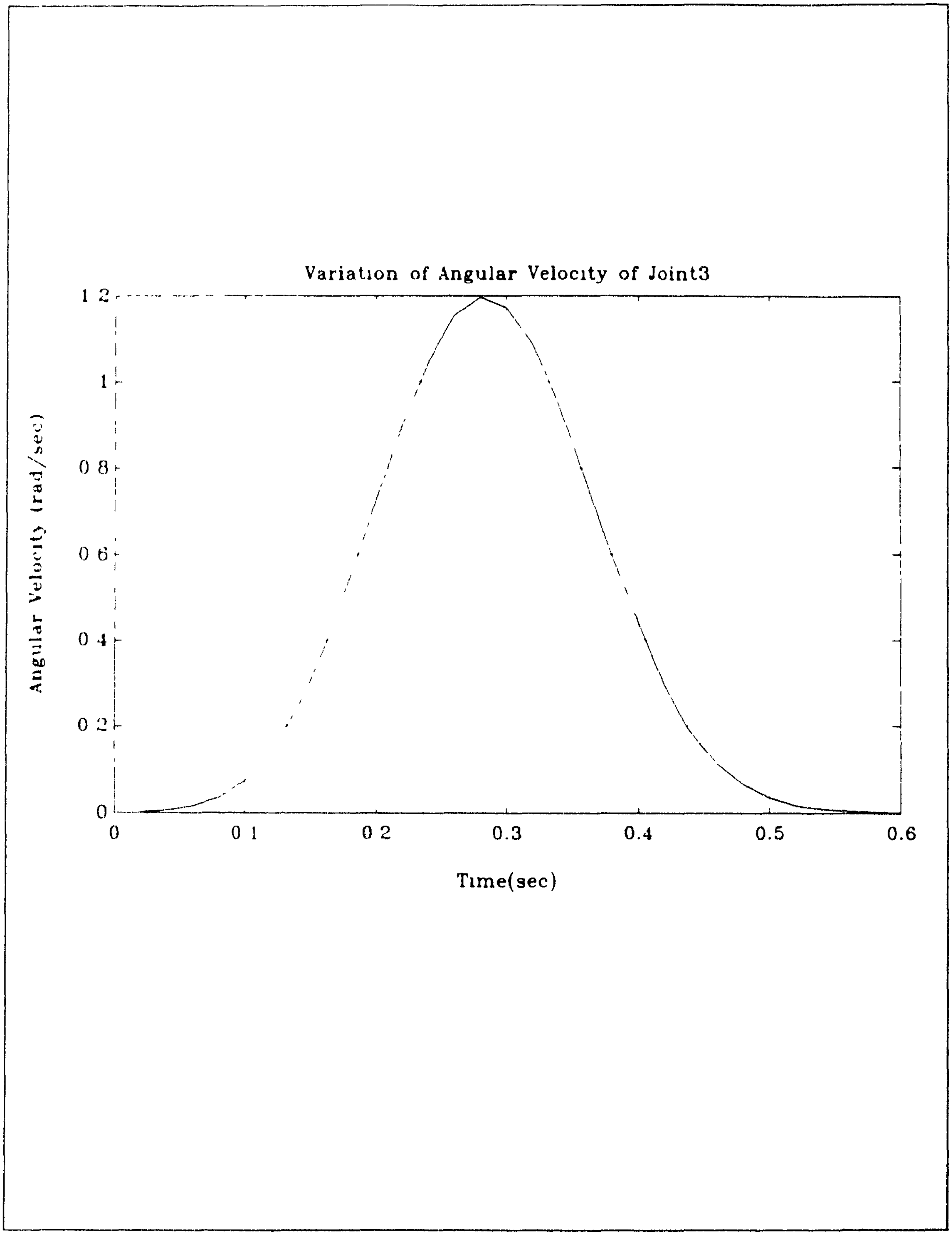


Figure 3.6c Variation of Angular Velocity of Joint 3 with Time

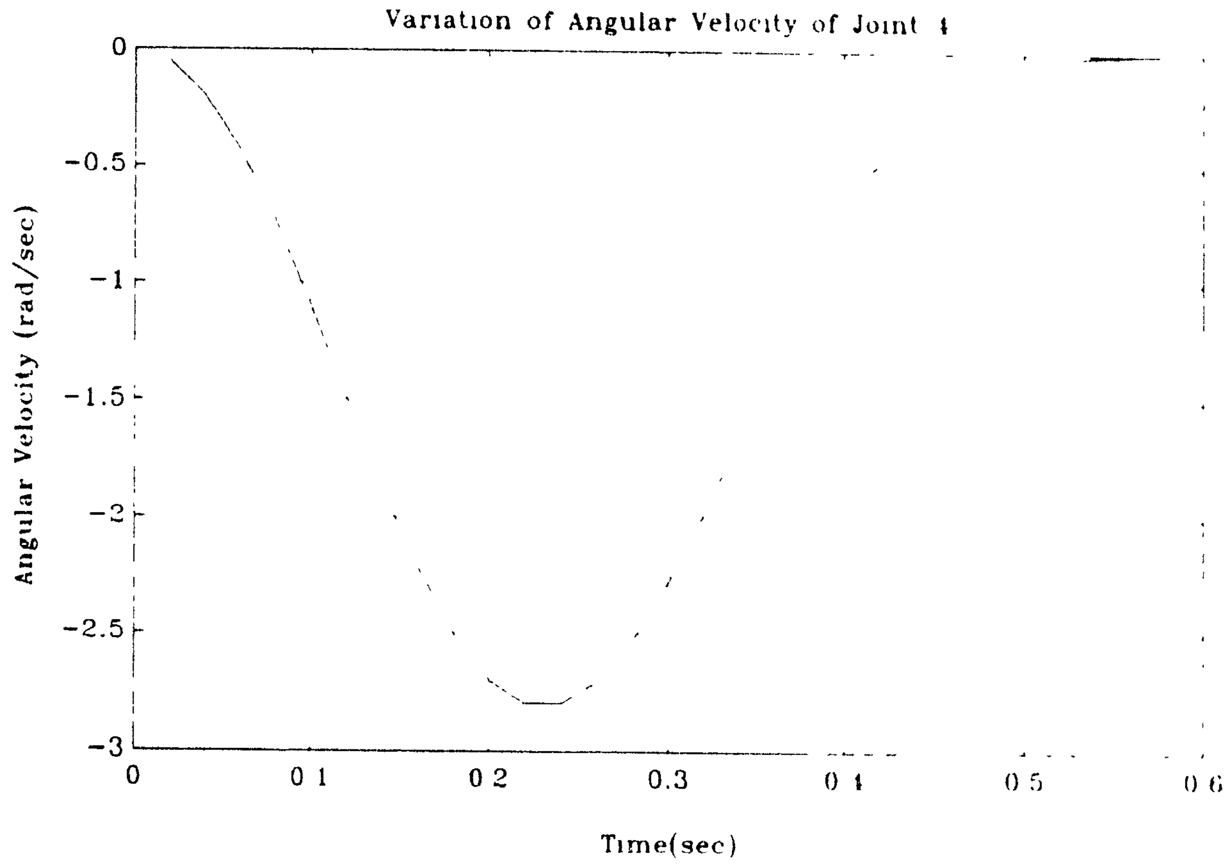


Figure 3.6d Variation of Angular Velocity of Joint 4 with Time

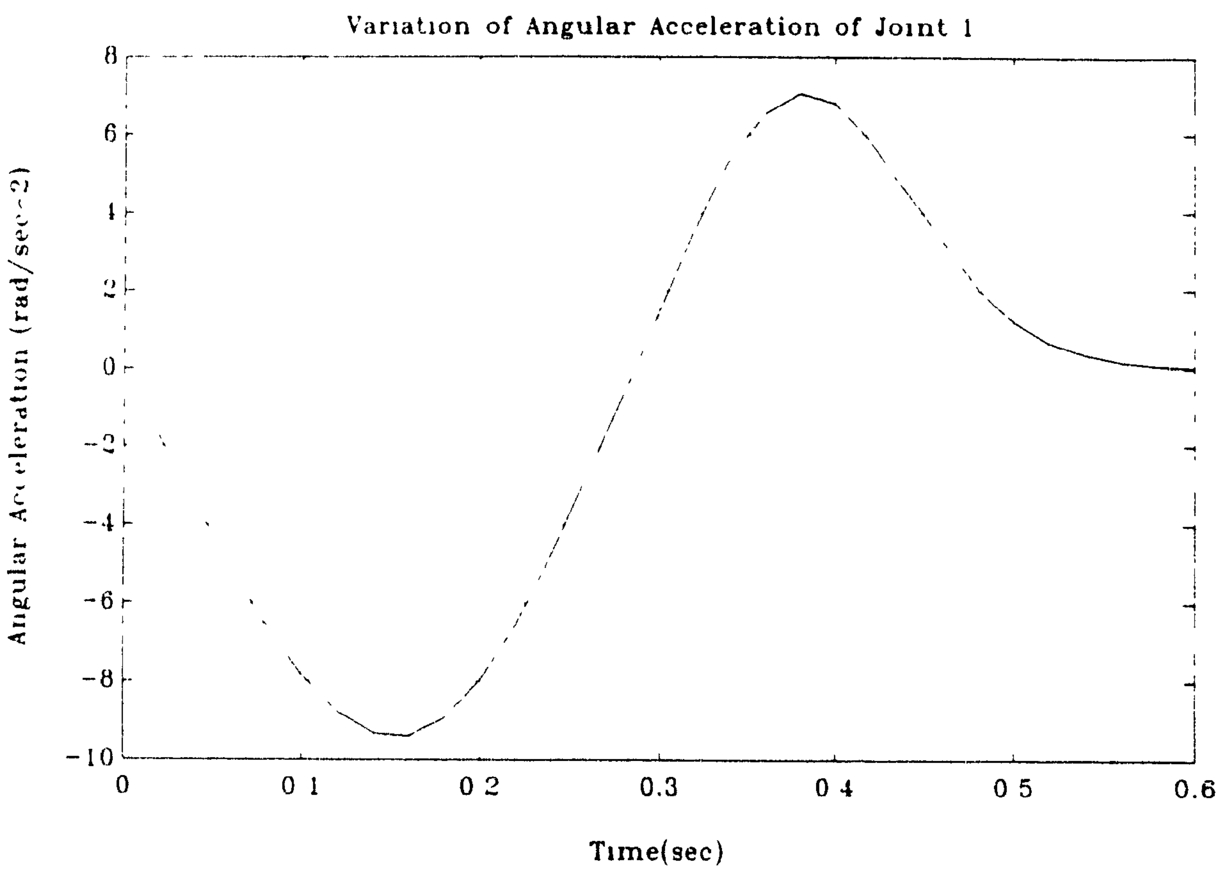


Figure 3.7a Variation of Angular Acceleration of Joint 1 with Time

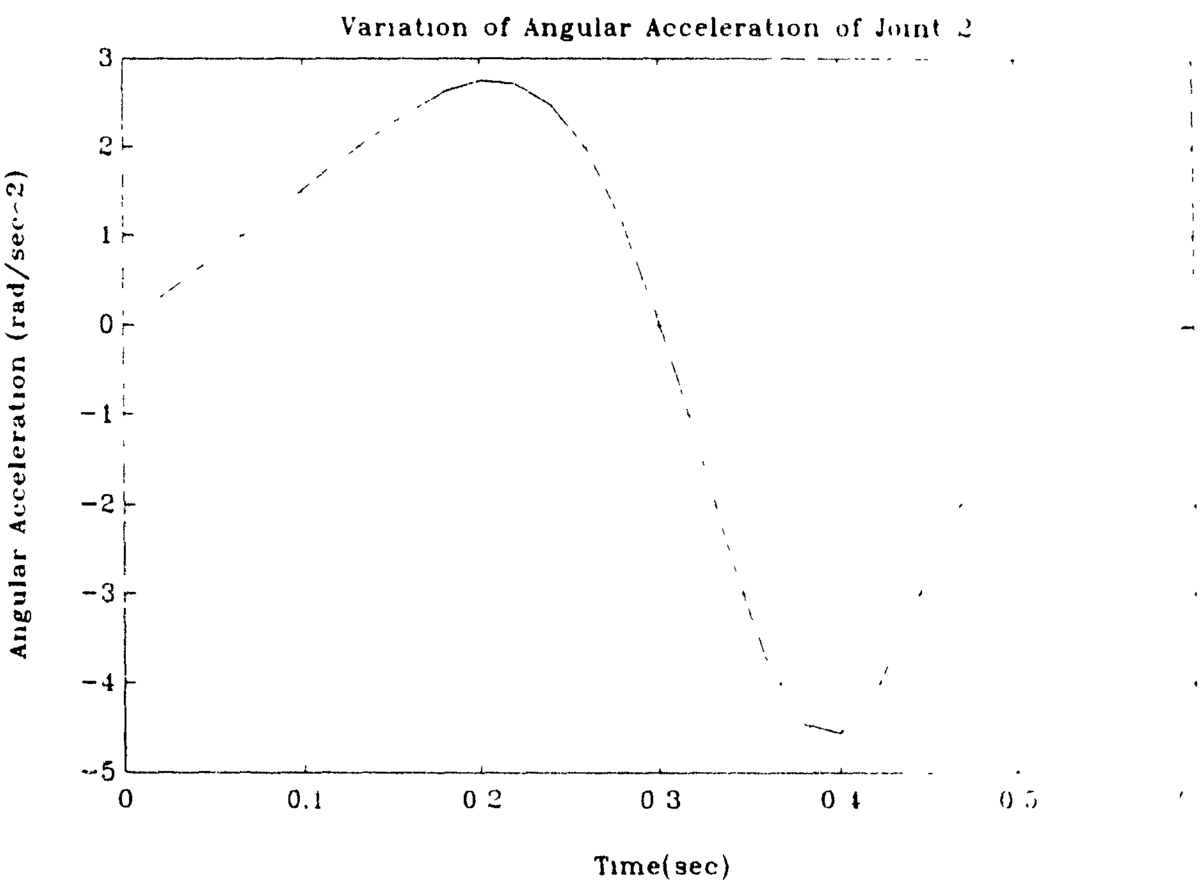


Figure 3.7b Variation of Angular Acceleration of Joint 2 with Time

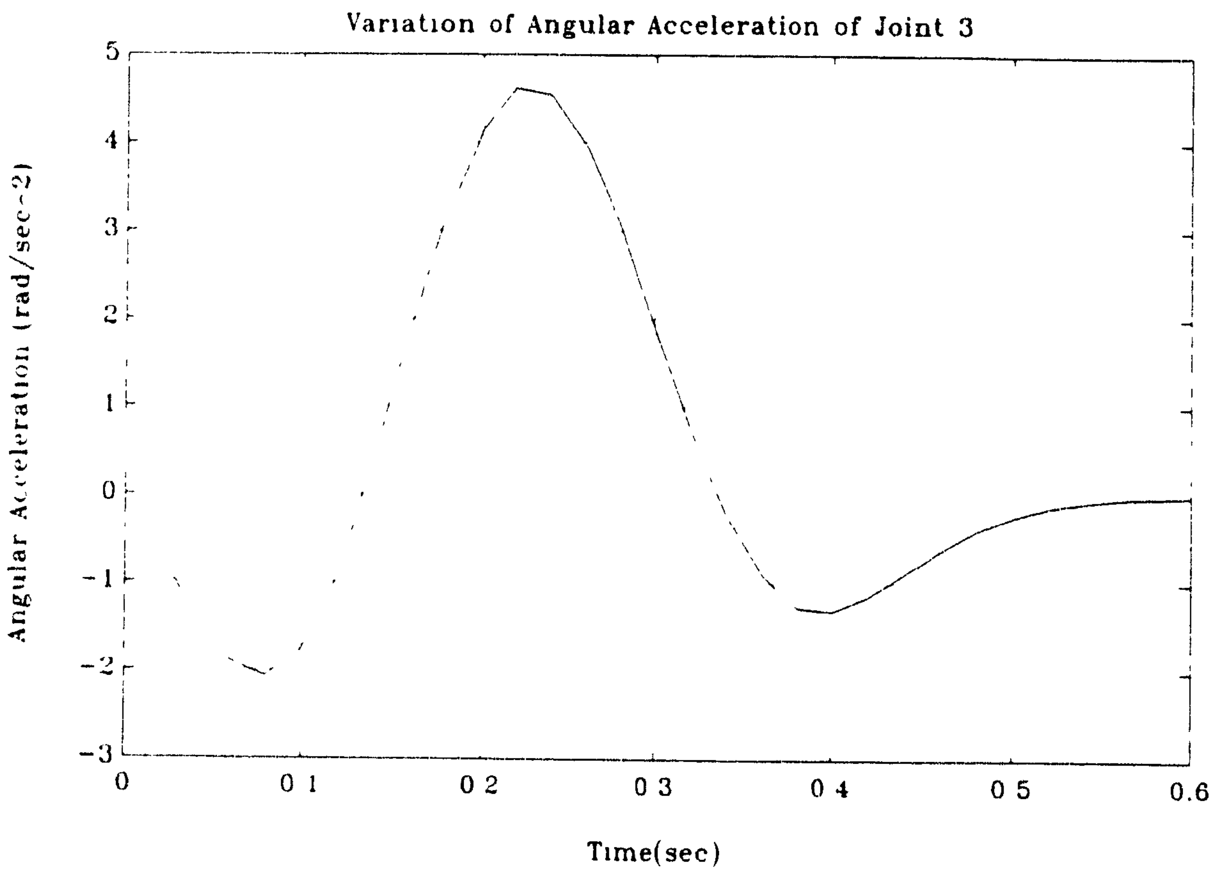


Figure 3.7c Variation of Angular Acceleration of Joint 3 with Time

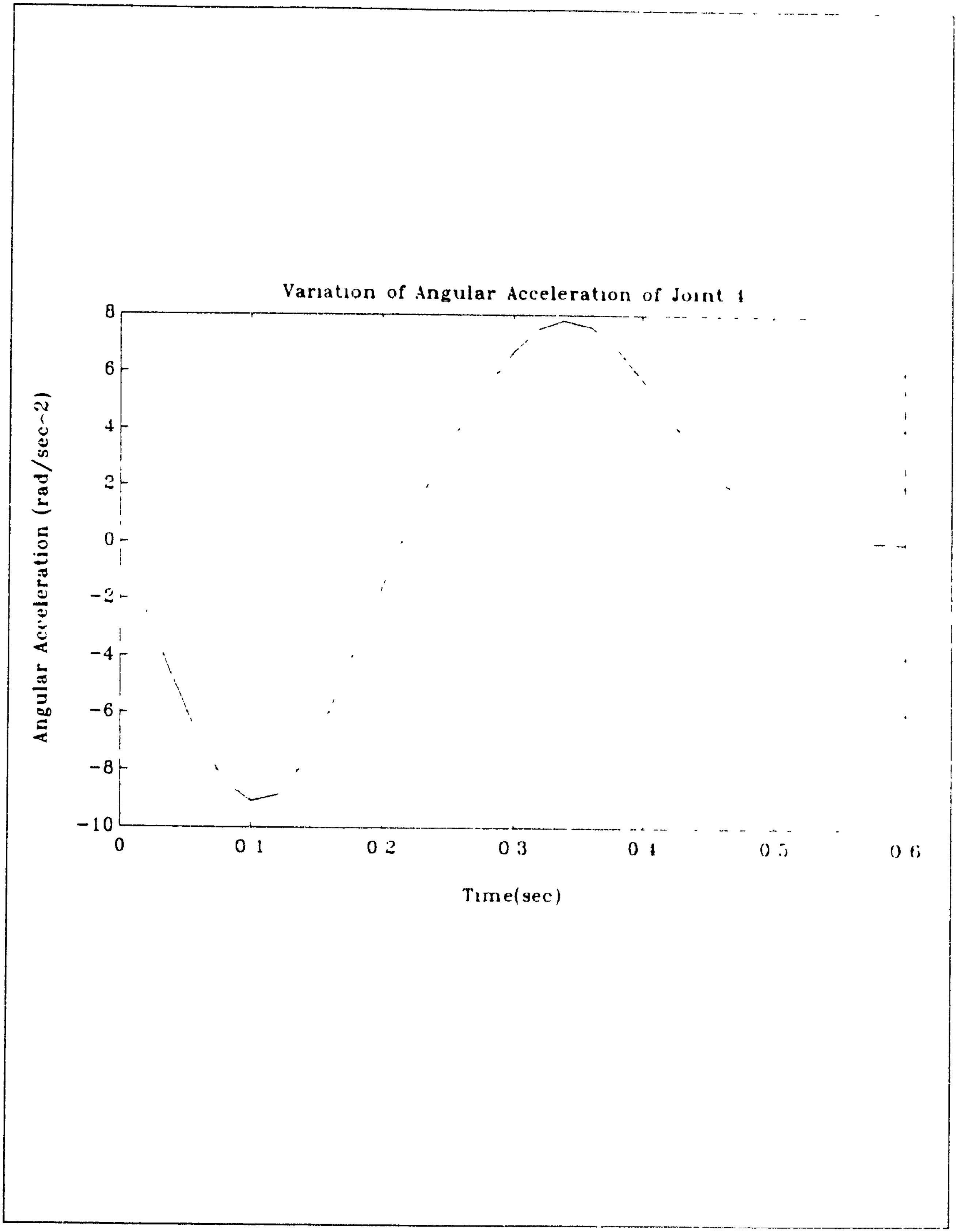


Figure 3.7d Variation of Angular Acceleration of Joint 4 with Time

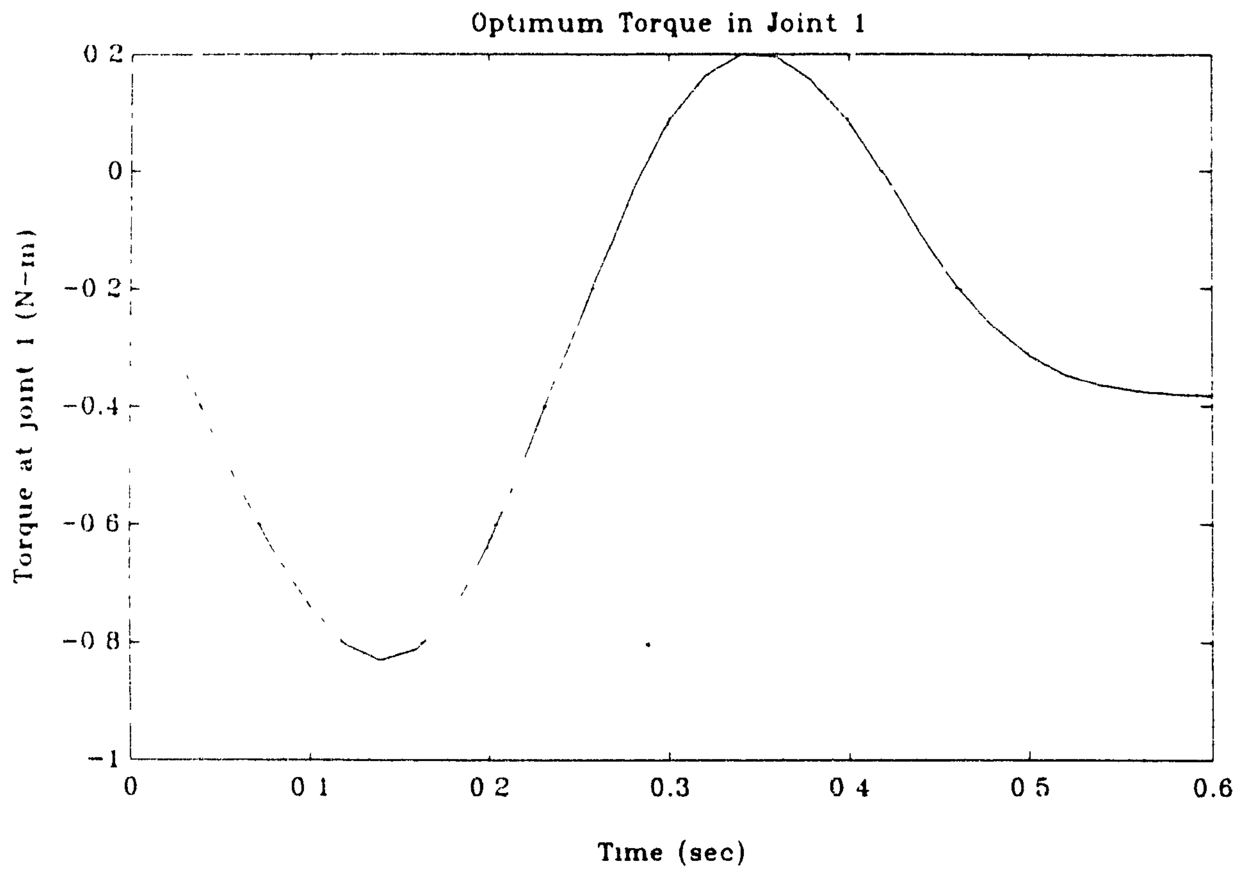


Figure 3.8a Variation of Nominal Optimal Torque at Joint 1 with Time

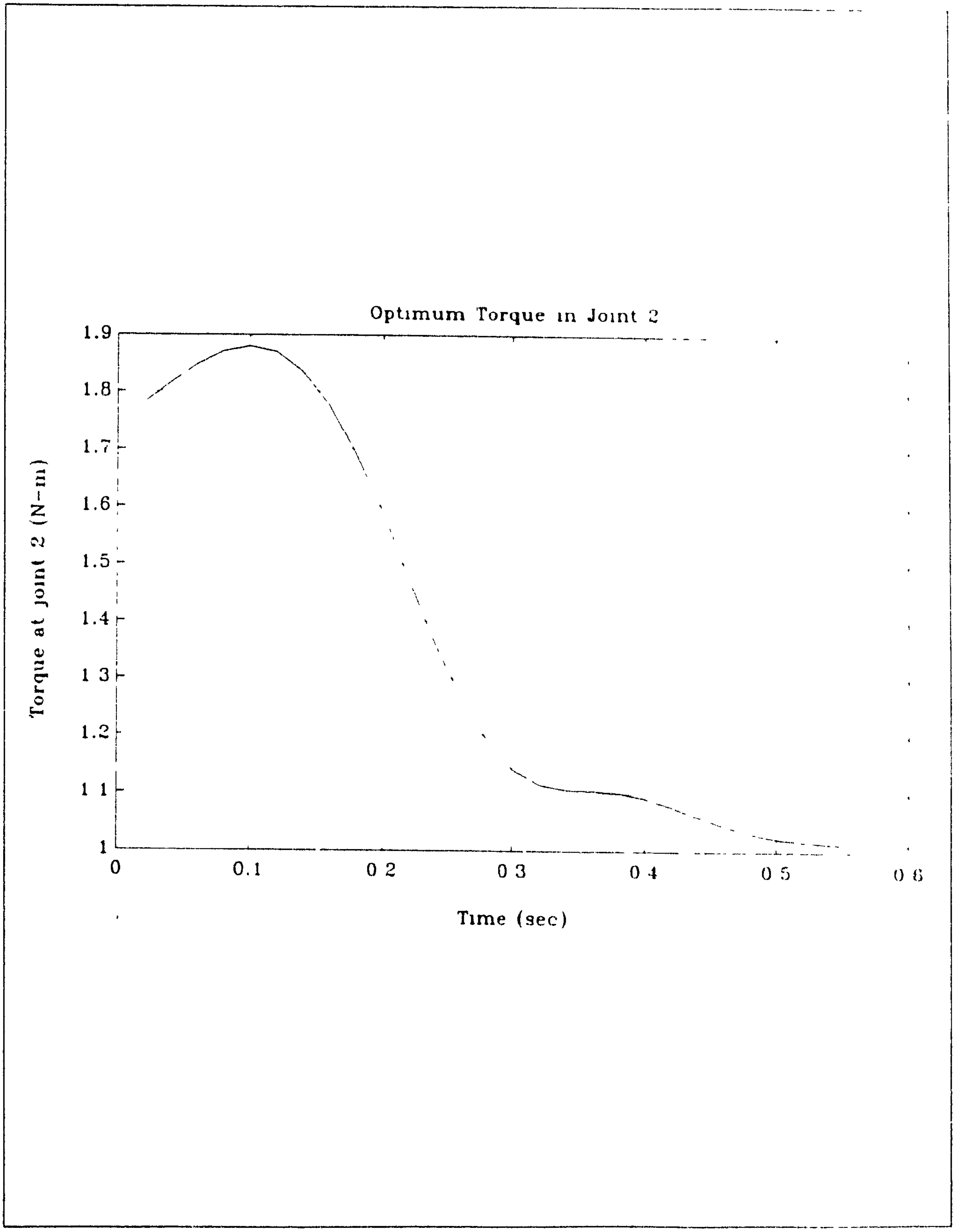


Figure 3.8b Variation of Nominal Optimal Torque at Joint 2 with Time

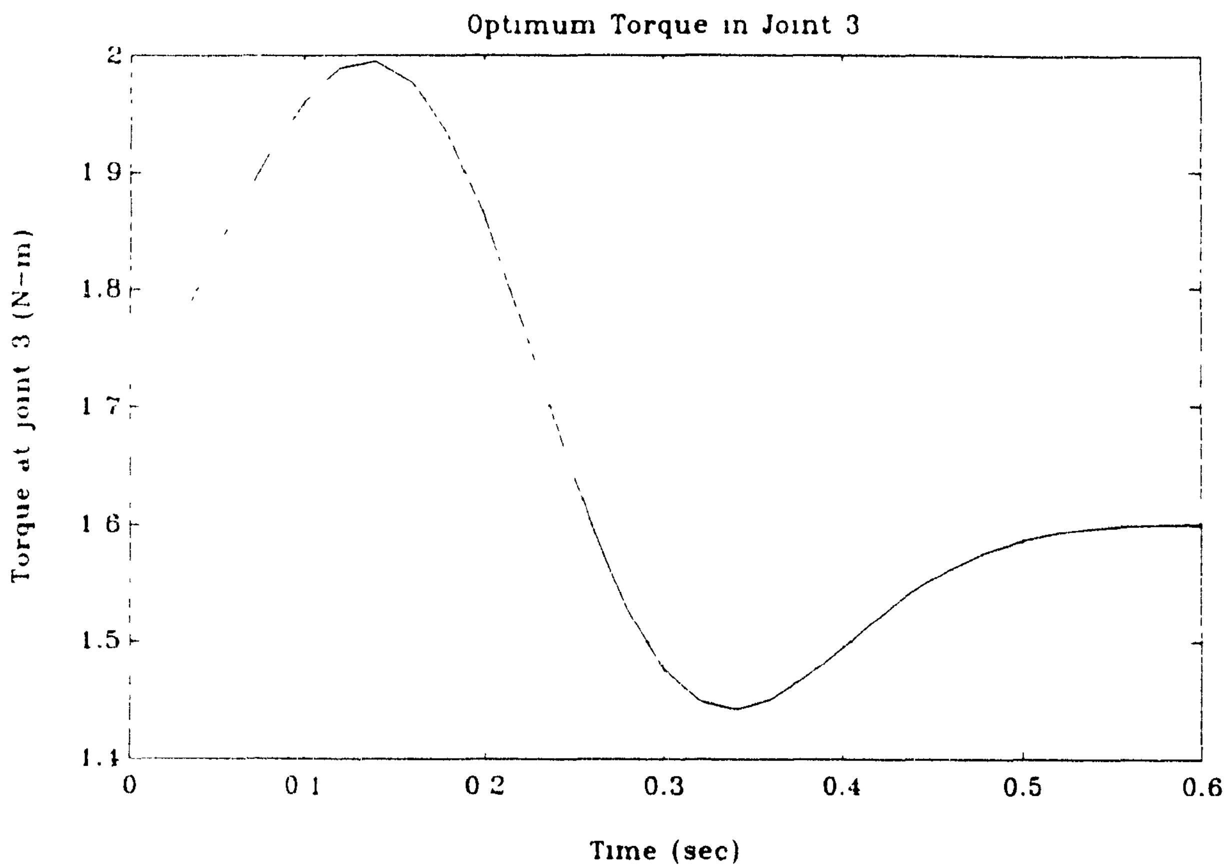


Figure 3.8c Variation of Nominal Optimal Torque at Joint 3 with Time

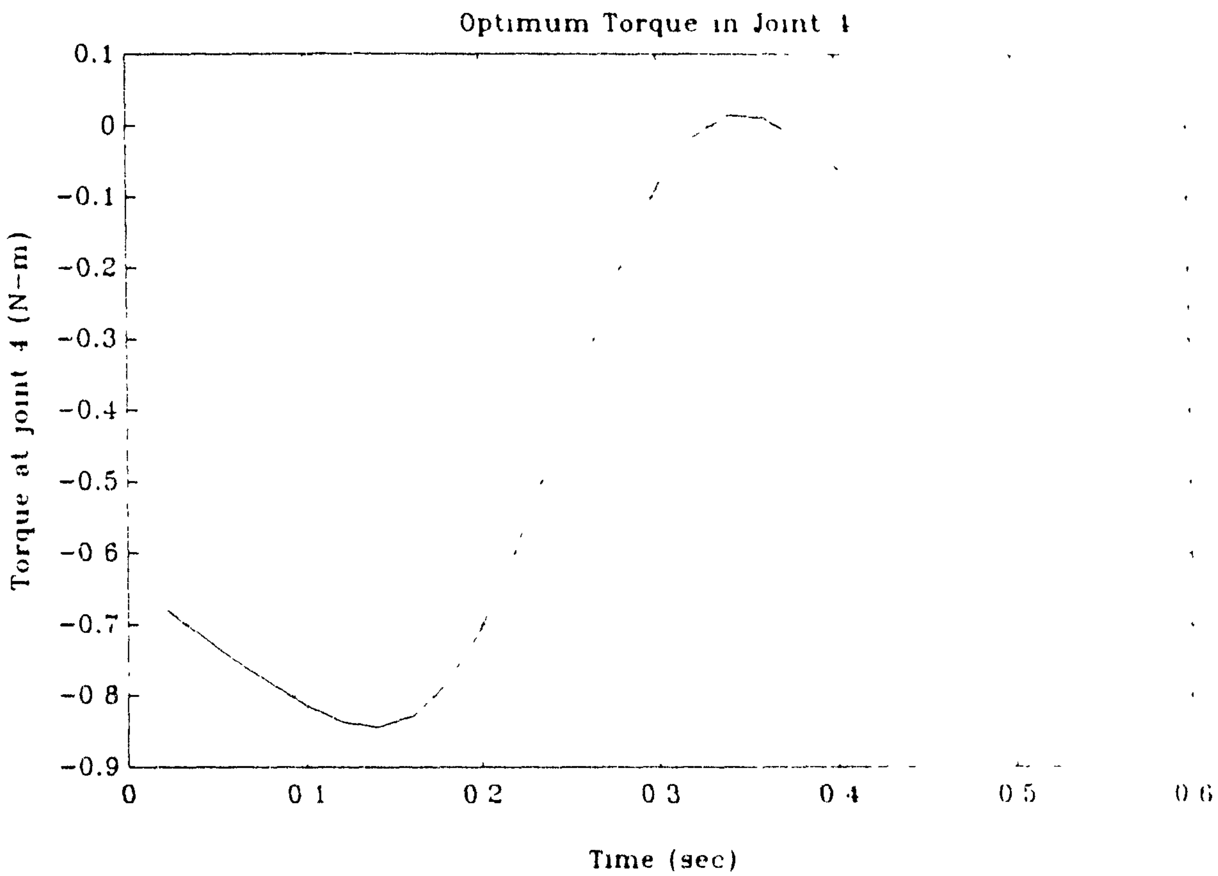


Figure 3.8d Variation of Nominal Optimal Torque at Joint 4 with Time

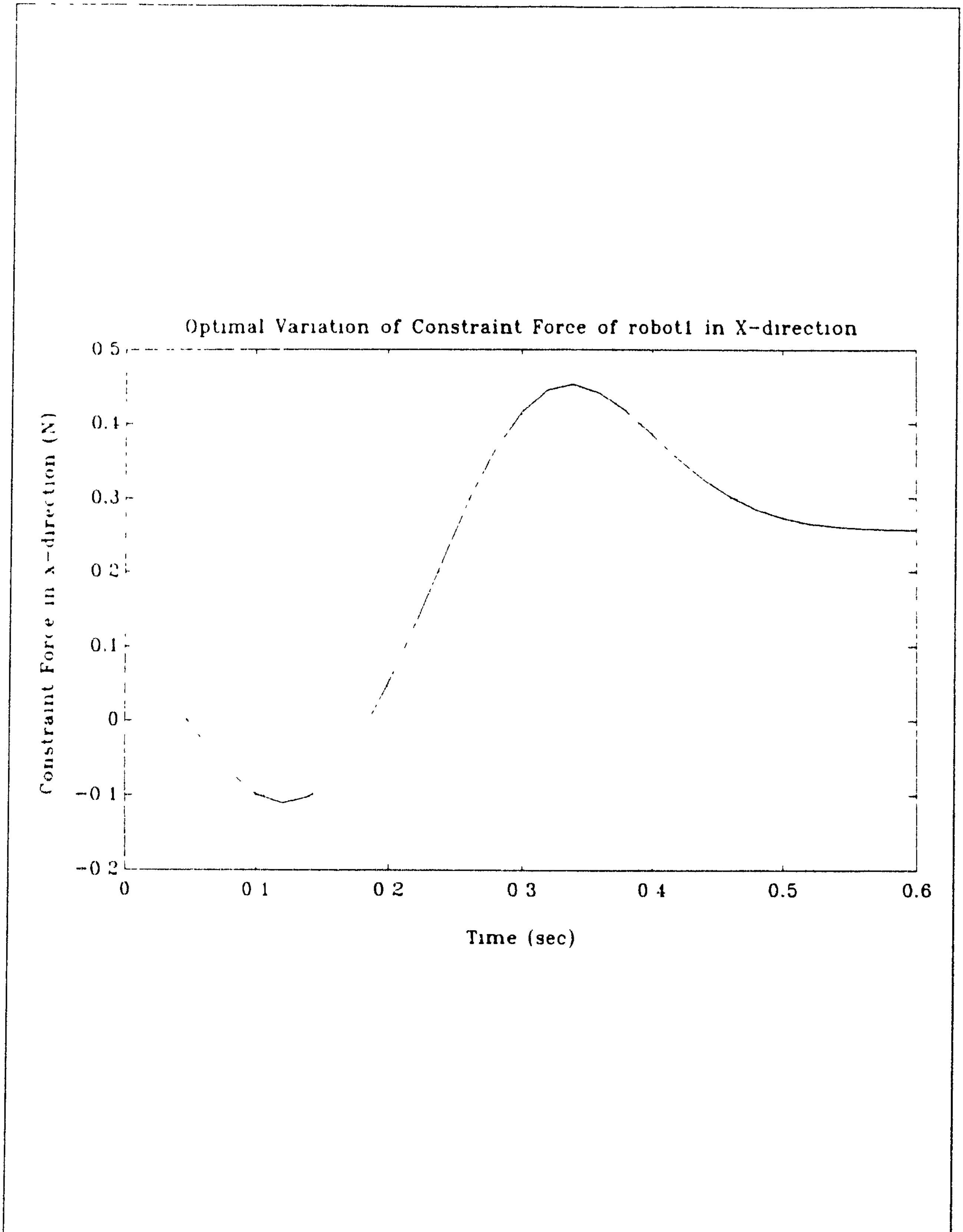


Figure 3.9a Variation of Nominal Optimal Constraint Force of Robot 1 in x-Direction with Time

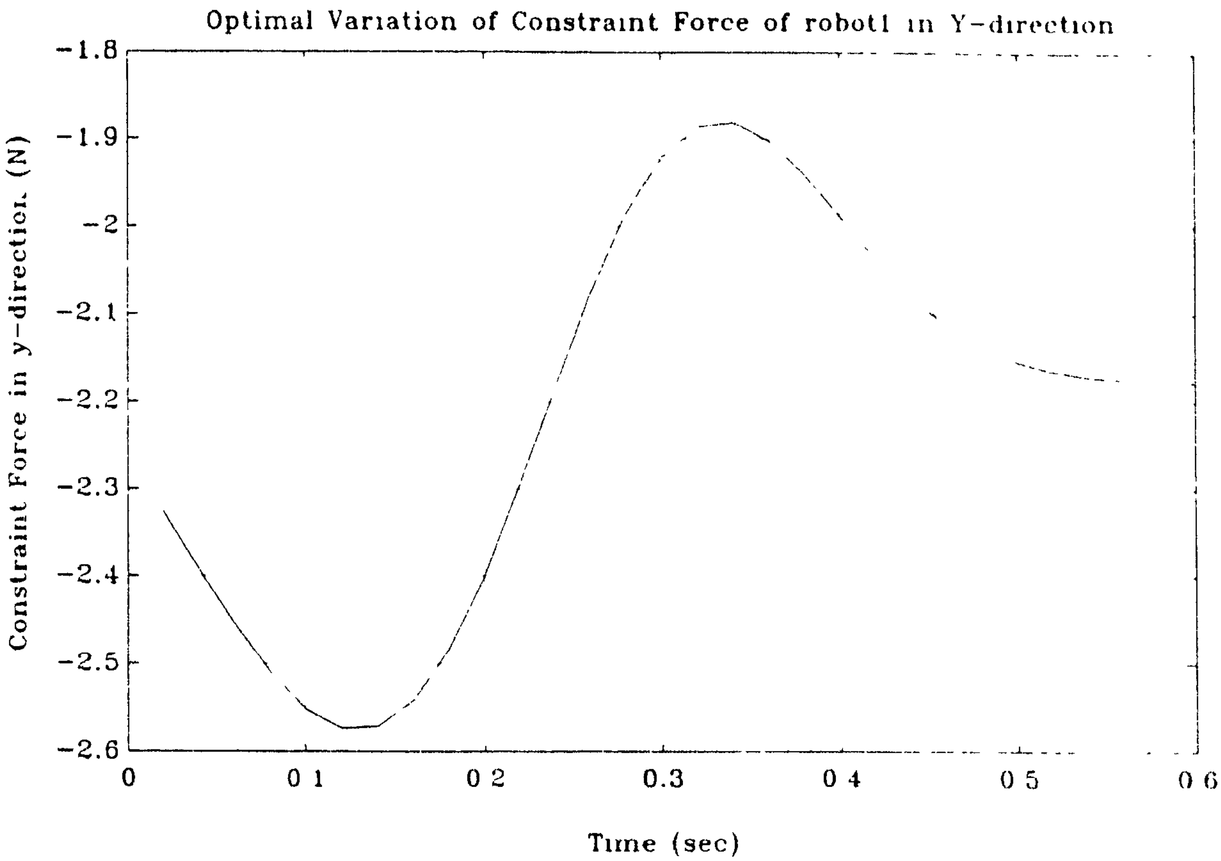


Figure 3.9b Variation of Nominal Optimal Constraint Force of Robot 1 in y Direction with Time

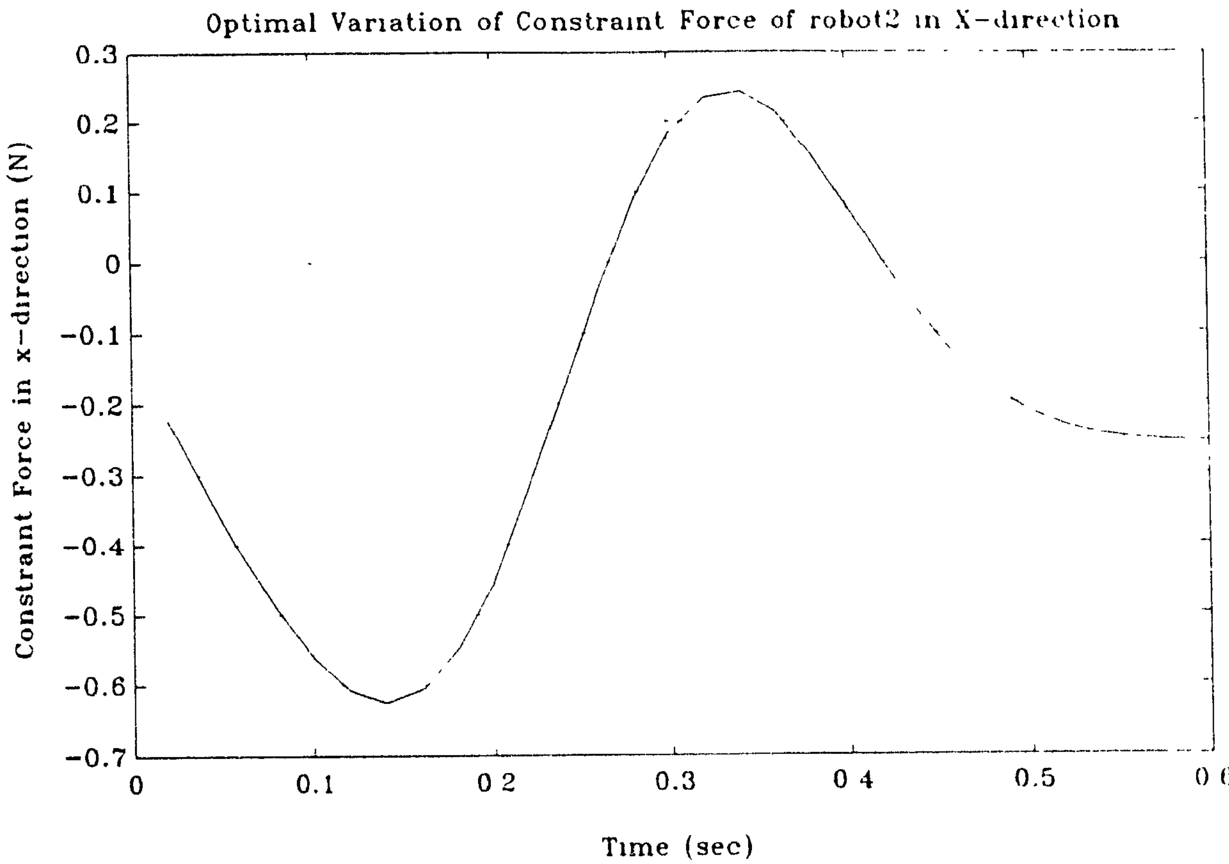


Figure 3.10a Variation of Nominal Optimal Constraint Force of Robot 2 in x-Direction with Time

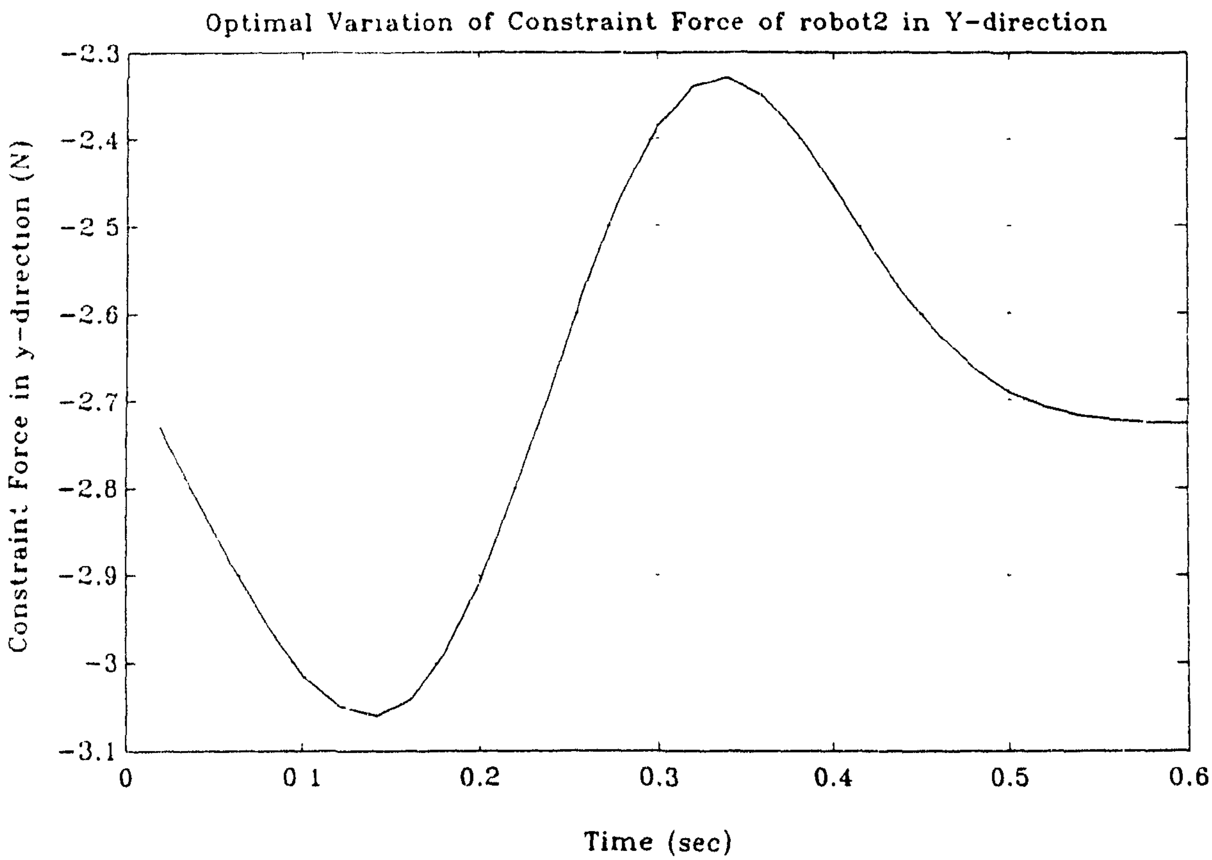


Figure 3.10b Variation of Nominal Optimal Constraint Force of Robot 2 in y-Direction with Time

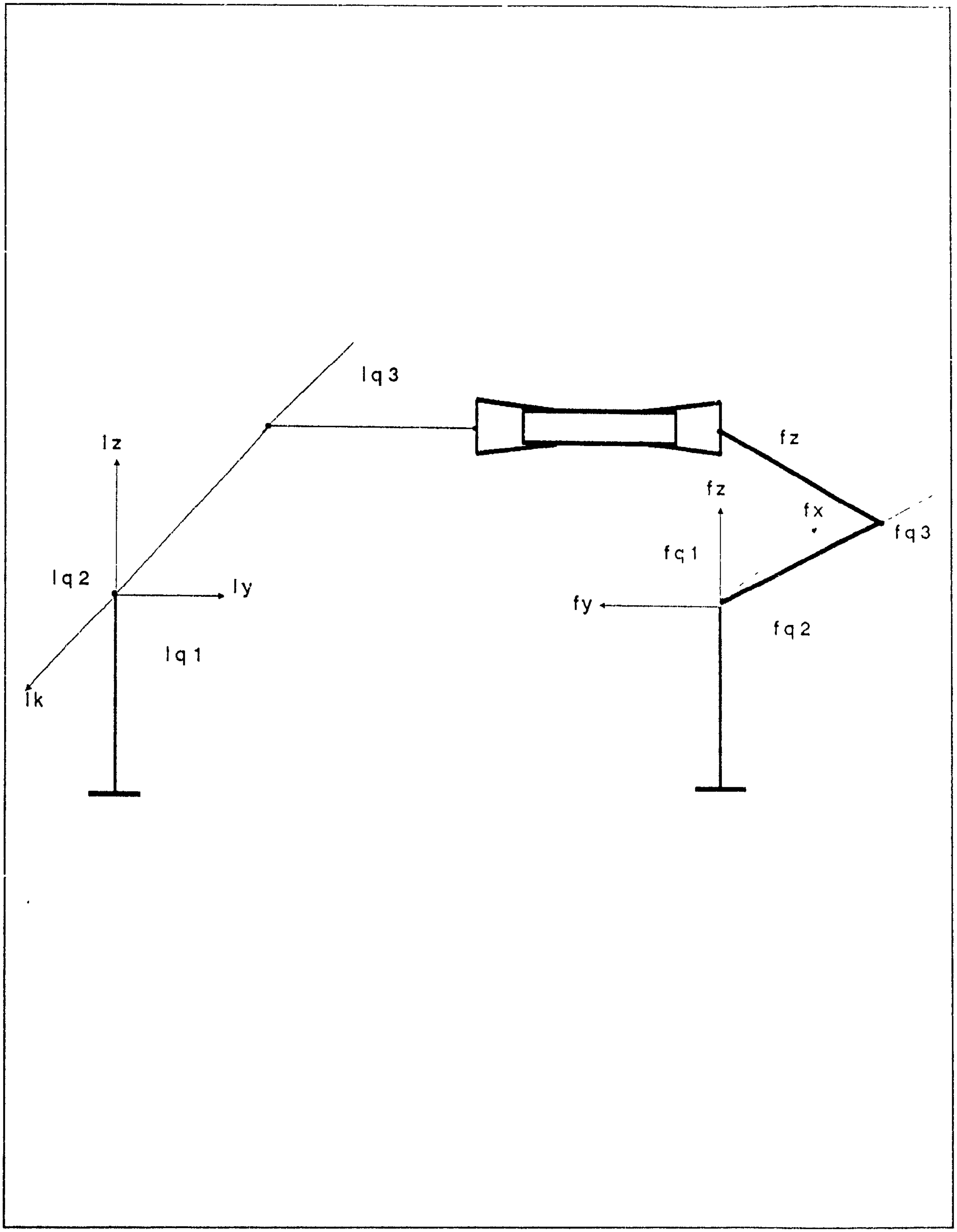


Figure 3.11 Two Industrial Robots Handle One Large Object

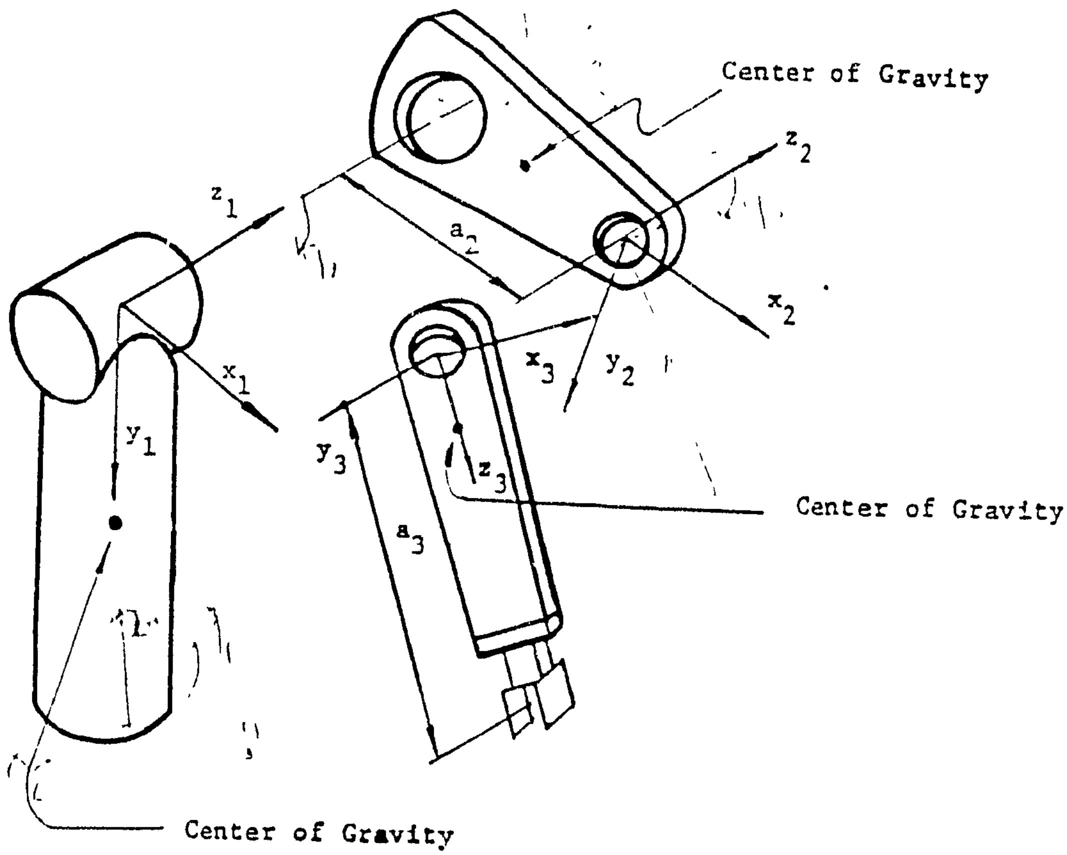


Figure 3.12 The Simplified PUMA 560 Robot and its Parameters

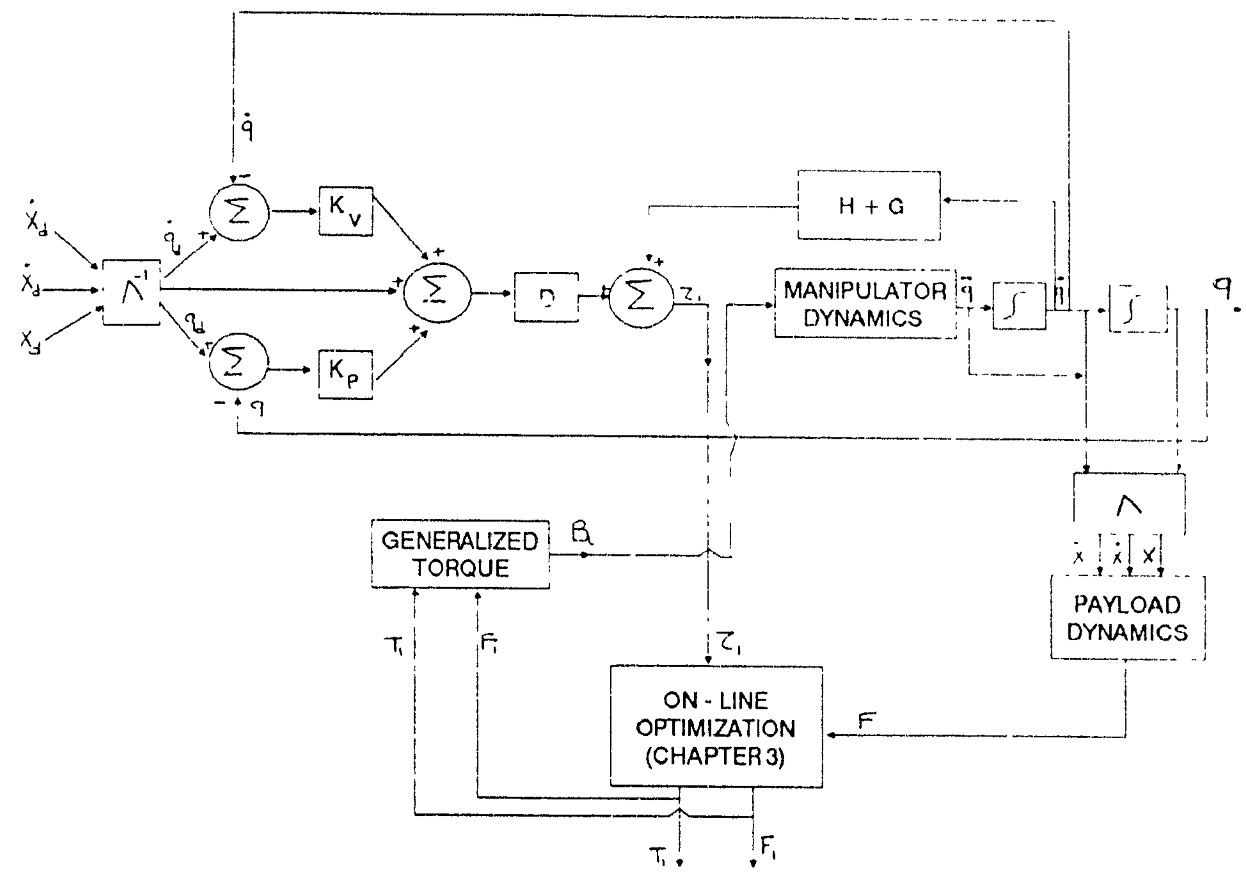


Figure 4.1 Control Block Diagram with Optimization Scheme

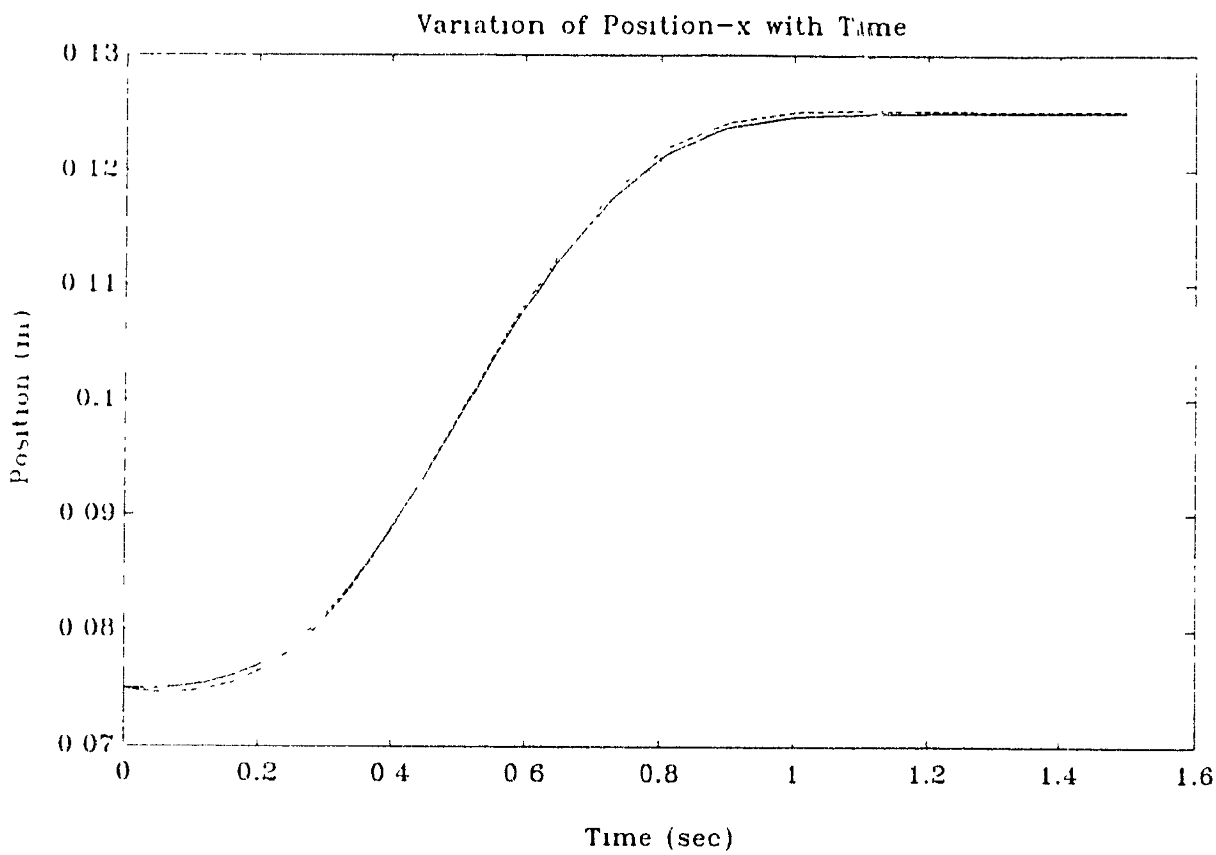


Figure 4.2a Variation of Position of the Payload/End-effector in the X-direction (Desired vs Actual)

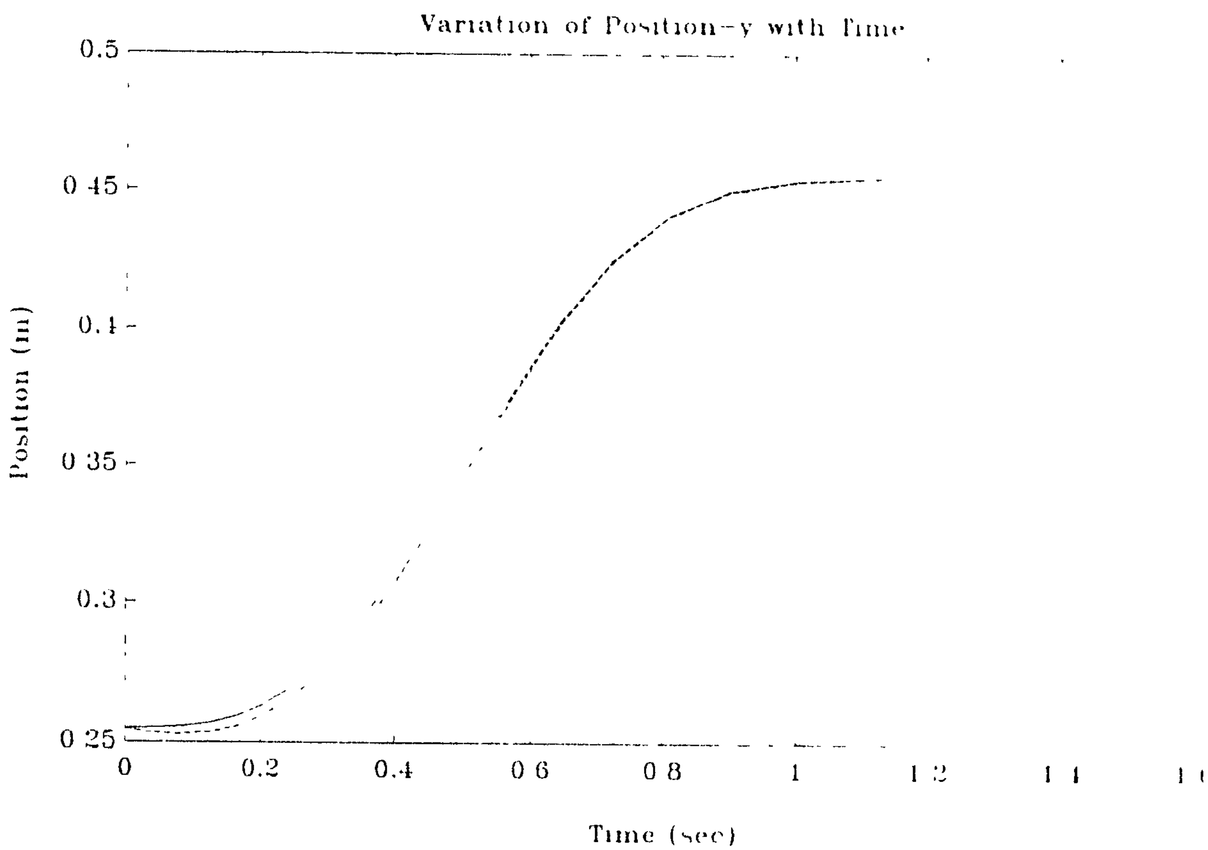


Figure 4.2b Variation of Position of the Payload/End-effector in the Y direction (Desired vs Actual)

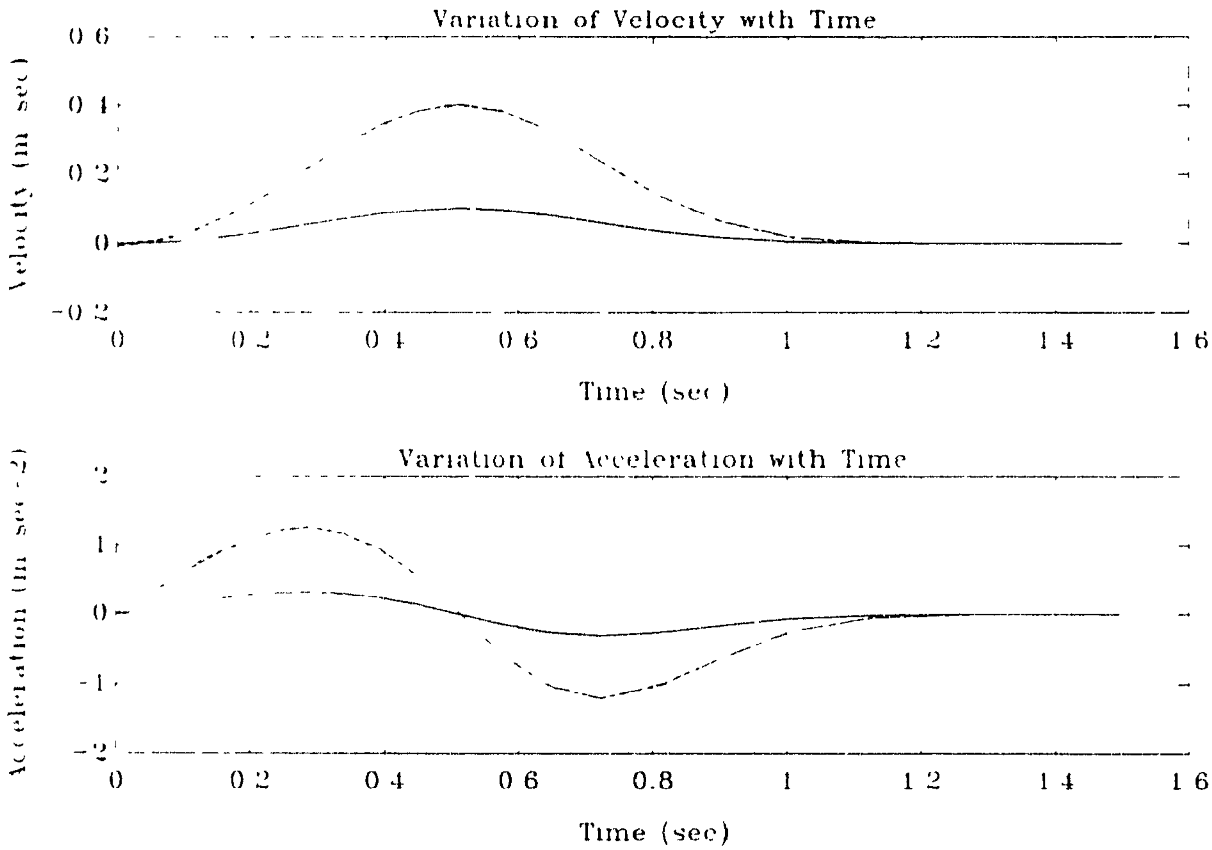


Figure 4.3 Variation of Velocity (Desired vs Actual) & Acceleration (Desired vs Actual) in the X & Y directions

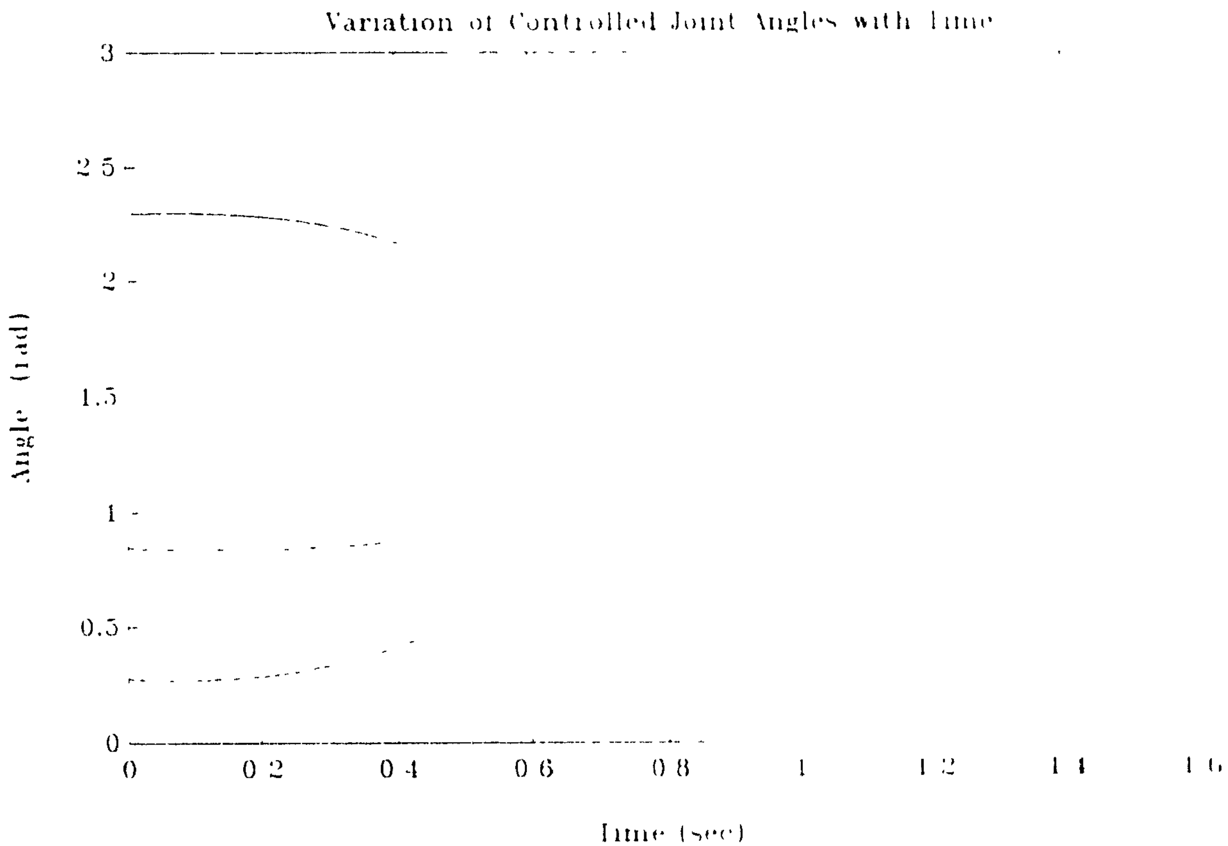


Figure 4.4 variation of Controlled Joint Angles

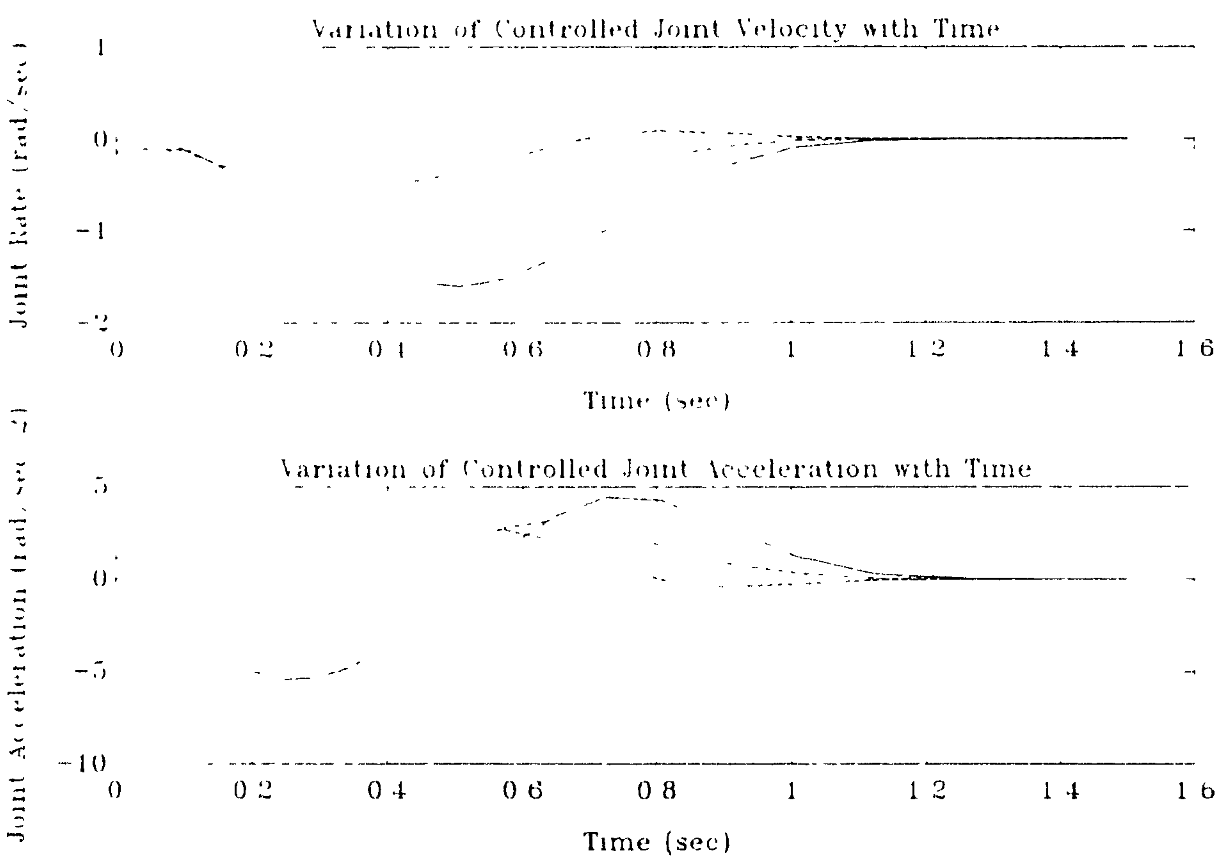


Figure 4.5 Variation of Controlled Joint Rates

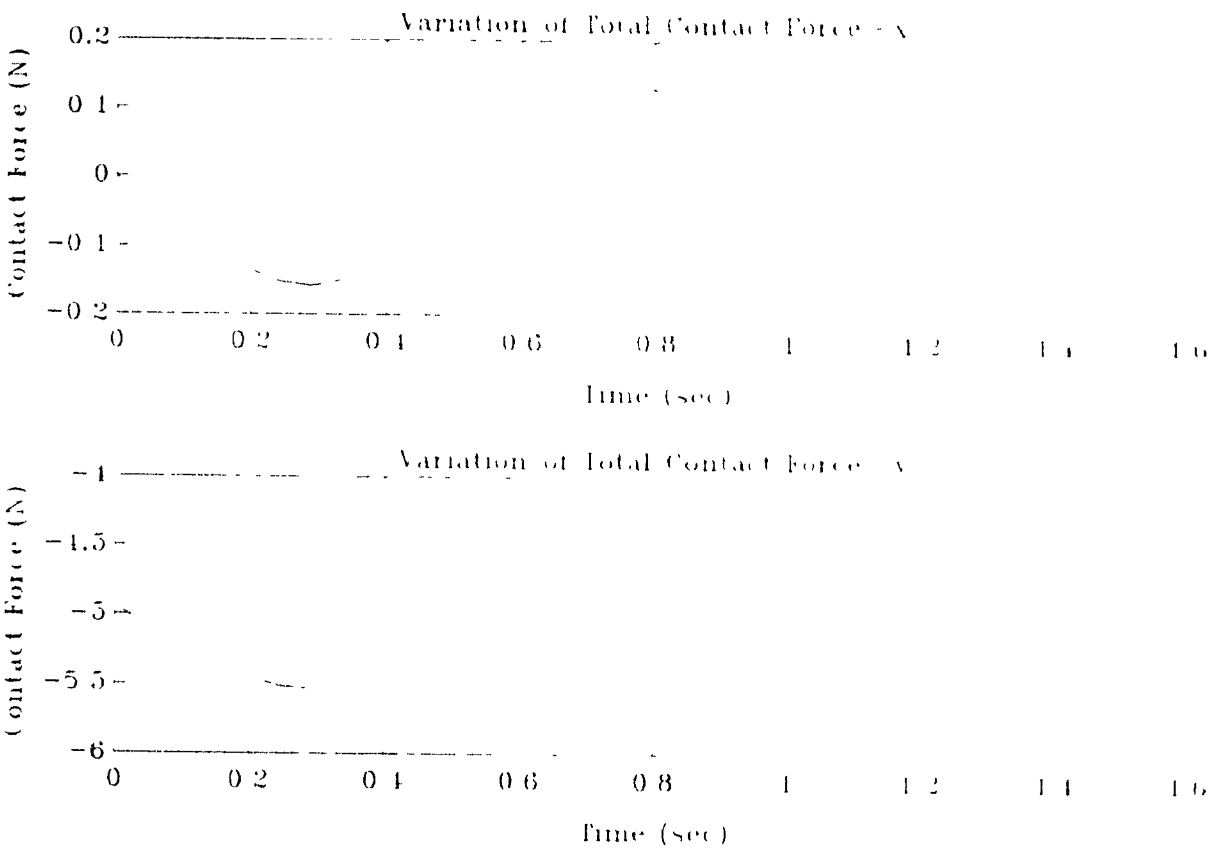


Figure 4.6 Variation of Total-Controlled Contact Force in the X direction & in the Y-direction

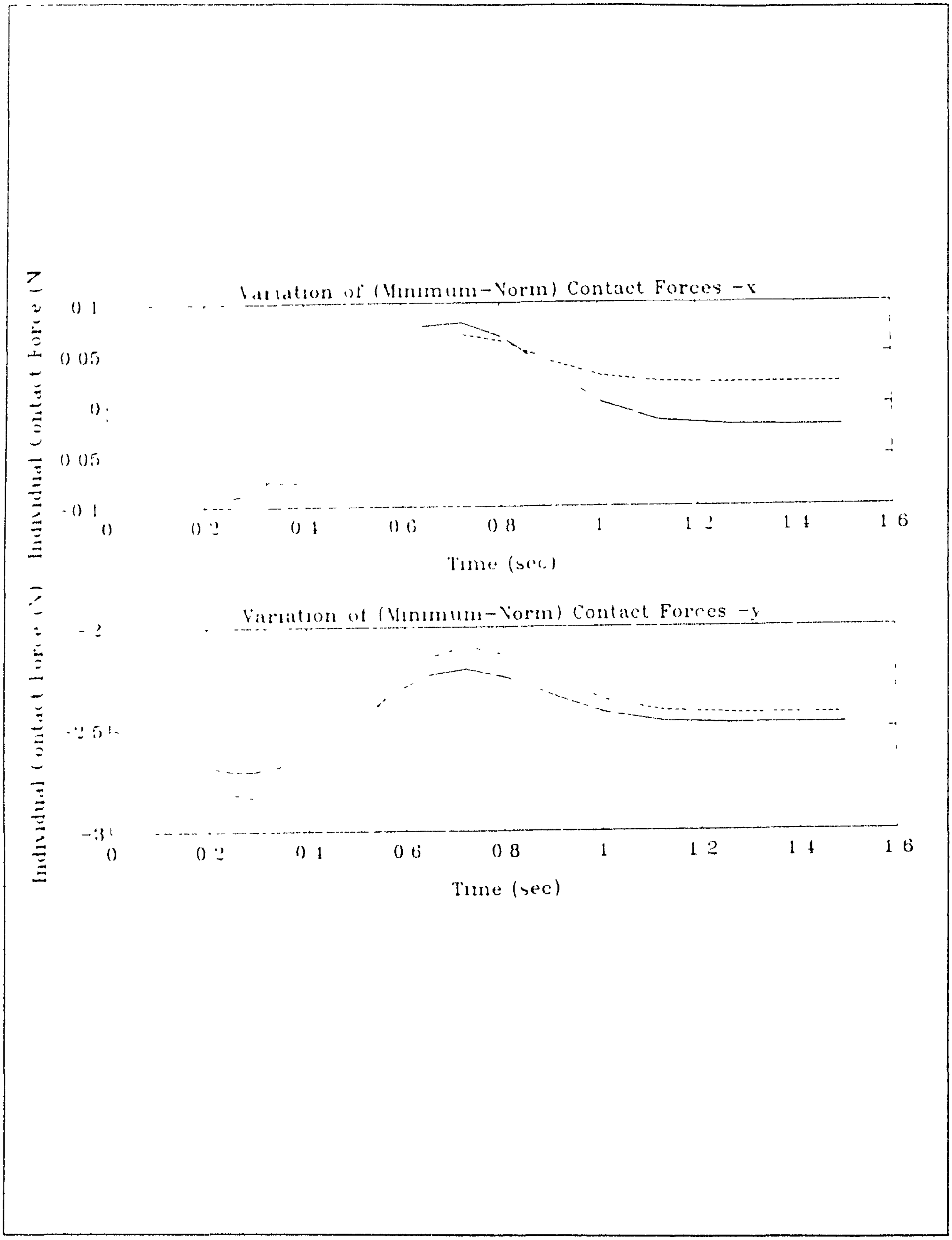


Figure 4.7 Minimum contact Forces (in the X & Y direction)

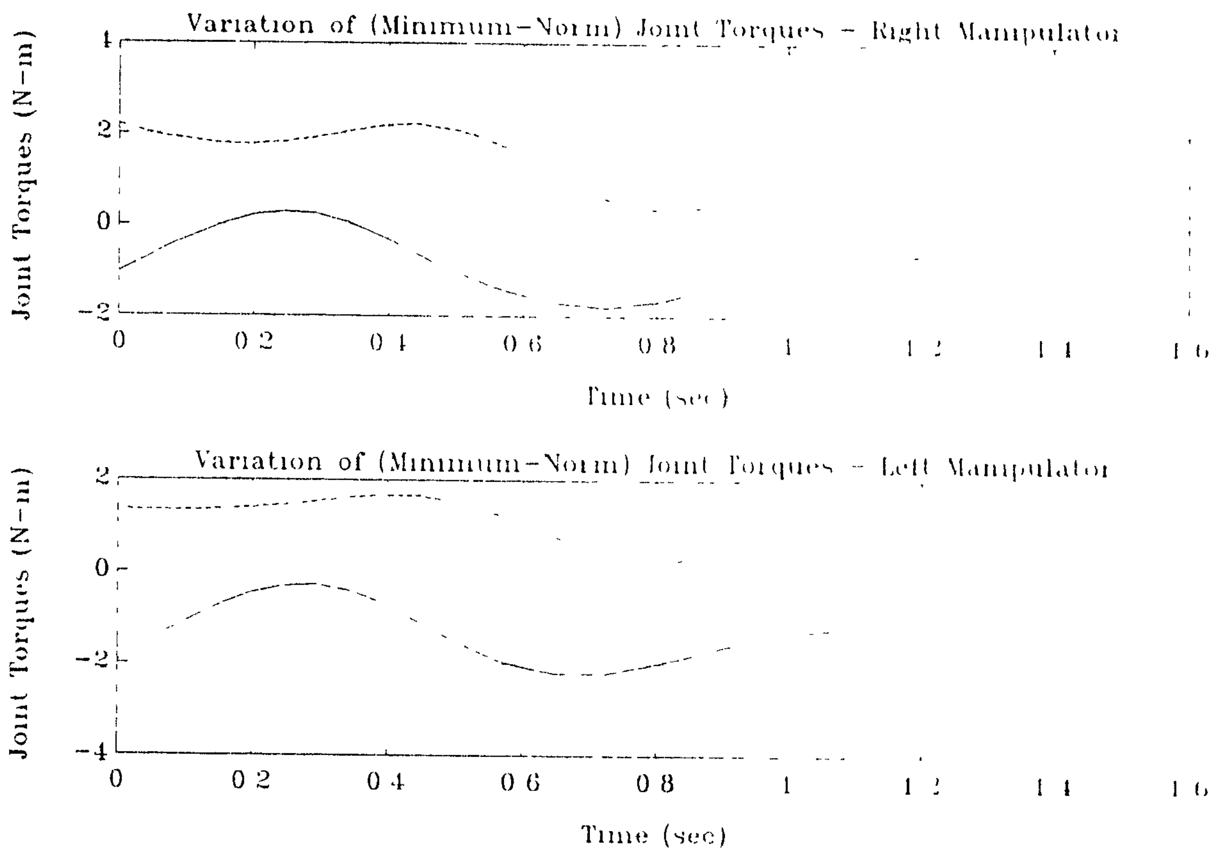


Figure 4.8 Minimum Joint Torques for each Robot