AN ANALYTIC APPROACH TO

UNIT COMMITMENT

BY

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ABSTRACT

In the present study a new analytic approach, based on the generalized lagrange multipliers method for the solution of the static unit commitment of thermal units has been investigated. The conditions for optimum unit commitment in this approach are reduced to a set of analytic conditions defining curves, termed switching curves which govern the switching of the system units, and characterize a relatively small number of combinations of constant unit commitment. The optimum schedules can be analytically studied in terms of the load, the reserve margins and the associated lagrange multipliers corresponding to the system incremental cost and the reserve incremental cost respectively. This analytic approach, termed the switching curve concept provides a new and unique insight into the unit switching mechanism not available from purely numerical techniques as well as gaining a physical interpretation for optimum unit commitment in terms of the system incremental costs.

A relatively fast algorithm based on simple numerical techniques and the branch-and-bound method has been implemented using the above concept and encouraging results were obtained. Even though, the static unit commitment does not consider many practical constraints, its solution could be useful as a lower bound or as a basic engine for the general dynamic case.

RESUME

Cette etude presente une approche analytique pour resoudre le problem de repartition a court terme d'un ensemble de moyens de production thermique. La version statique du probleme a ete consideree et les principaux resultats ont ete obtenus a partir de la methode generalisee des multiplicateurs de Lagrange .Les conditions d'optimalite obtenues sont reduites a un ensemble de conditions analytiques definissant des courbes appelees courbes de commutation qui regissent le mechanisme de la mise en marche et d'arret des groupes et basee sur celles-ci Une interpretation physique est deduite en fonction des couts marginaux donnant des resultats satisfaisant economiquement.

L'etude des variations de la gestion optimale peut-etre aussi etudiee en fonction des parametres du system telles que la charge, la reserve tournante et les couts marginaux. Cette methode analytique appelee la methode de la loi des courbes de commutation nous donne un nouveau et unique apercu sur le mechanisme de mise en marche et d'arret des groupes. Un algorithm rapide utilisant des techniques numeriques simples et la methode du type "branch-and-bound" a ete implementee pour la resolution du cas statique et des resultats encourageant ont ete reportes. Cette solution du problem statique peut-etre utile comme limite par defaut ou comme une base pour resoudre le cas dynamique.

ACKNOWLEDGMENTS

I would like to take this opportunity to express my gratitude to my director F.D. GALIANA for his continual guidance, unceasing encouragement and help he provided throughout the course of this research.

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Finally, my sincere thanks to my government who provided me the necessary financial support.

To my departed parents.

TABLE OF CONTENTS

LIST OF SYMBOLS

a_i , b_i , c_{0i}	coefficients in the total generation cost
$C_i(P_i)$	convex quadratic cost function of plant i
P_d	hourly load demand or the power produced
R	actual reserve
R^{Min}	specified reserve margin
λ	system incremental cost
α	Lagrange multiplier corresponding to the reserve constraint
μ, γ	Lagrange multipliers
S_i (.)	switching curve for unit i
$\underline{U}$	vector of switching variables
u_i	switching variable of unit i
k	time variable
P_i	generation of unit i
P_i^{Min}, P_i^{Max}	minimum and maximum limits on generation P_i
. $(\underline{x})$	any scalar function of the vector $\underline{x}$
$\mathcal{L}$ (.)	the Lagrangian function

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTODUCTION

1.1 Background

The large daily variations of the electric energy demand at the different nodes of a power system and the necessity to satisfy at each instant this demand with maximum security and at minimum cost requires the determination of an optimum planning of the different available plants in a predetermined time horizon which ranges usually from one day to one week.

The problem is therefore to determine which units should be kept on-line and which should not, in order to achieve maximum economy.

This is known as a unit scheduling problem, which involves two separate, but mutually connected problems. The first, usually called the unit commitment or pre-dispatch, consists of the selection of units to be placed in operation and of the determination of the instants of their start-up and shut-down in a given period of time, usually ranging from 24 to 168 hours. The second, called the economic dispatch problem, deals with the allocation of the load among generating units which are already running. The general objective is to achieve minimum operating costs, subject to demand, spinning reserve, downtime, up-time and other operationnal constraints. While the problem of economic dispatch has been studied extensively for some time [1,2], the unit commitment problem, which normally has to be solved first, has received less attention, probably because it is a less straightforward problem than the economic dispatch.

The scheduling of generating units is a mathematical programming problem which involves a large number of both continuous (generations) and discrete (unit commitment) variables, and generally it can be stated as a mixed-integer nonlinear programming problem, which is one of the most difficult problems in the area of mathematical programming.

Depending on the cost function and on the number and kind of constraints imposed on the unit commitment, various types of problem formulations are possible. Generally it is useful to divide these into two categories, static and dynamic, characterized as follows :

Static unit commitment problem characteristics:

-- Running costs

-- Generation limits

-- Area reserve margins

-- Variable fuel types

-- Variable operating strategies

Dynamic unit commitment problem characteristics:

-- Start-up and shut-down costs

-- Minimum up and down times

-- Energy and ramp constraints

Different approaches have been developed and applied for both problems. In this thesis the principal topic will be the static thermal unit commitment problem, and how its solution leads to new insight and new approaches to the dynamic problem.

1.2 Review of solution techniques:

In recent years, there has been an increase both in magnitude and complexity of power systems. The variation between the peak and off-peak power demands has become more important and the increase in the costs of some fuel types has been substantial in the past decade. Hence, a systematic approach is needed for the determination of the generating units to be committed to service. It has been shown that the solution of the unit commitment problem results in substantial savings [3,4].

Past approaches to the problem of unit commitment can be grouped into four types of methods as follows

- Priority-list and heuristic methods;
- Integer programming and branch-and-bound;
- Dynamic programming;
- Lagrangian relaxation;

The priority list method [3,5,6] is the most popular because of its simplicity. In this method the order in which units were brought up or shut-down is strictly specified according to their efficiency, for example, and heuristics were used to determine if it was worthwile to bring a unit up or down at a given time. A number of refinements have been added to this original priority list method, including energy interchange modeling, different start-up and shut-down orderings, unit response rates, minimum-up and down-time among many others [7,8]. Even though the method gives a feasible solution which may be far from the optimum, it has remained one of the most used methods. Its approximations reduce the dimensionality and complexity seen in the most sophisticated scheduling mechanisms. Some integer programming and branch-and-bound [9,10,11, 12,13,15] methods were developed to solve the unit commitment. The difficulty here is basically one of dimensionnality. When the system consists of more than some tens of generating units, the above methods become unmanageable and far beyond the capabilities of present computational procedures.

Dynamic programming when applied in a straightforward manner [16] is only applicable for a very limited number of generating units, because the computing time and memory requirements increase factorially with additional units. However, several approaches were developed that included approximations, relaxations and iterative methods, in order to make this procedure tractable, thereby reducing the huge number of combinations to search [17,18,19,20,21,22].

The primary advantages of dynamic programming is in the ease of handling complex coupled constraints, and its ability to model delays, time varying parameters, probabilistic variables, and nonlinear cost curves.

The lagrangian relaxation method [23,24,25,26,27] seems to be the most appropriate for solving the unit commitment problem. As developed recently [23,24,25,26] it has been reported to give accurate results in a reasonable amount of time, with many practical constraints taken into account. The method appears to be very flexible and could be applied to solve very large scale systems (over 200 units).

In chapter 2, after the presentation of the mathematical formulation of the general unit scheduling problem, it will be shown how the above methods are applied to solve this problem.

1.3 The present thesis:

All the methods outlined above solve the unit commitment problem in an algorithmic approach, that is, their output consists of a single numerical solution to the given problem. Although these methods are necessary for solving the general unit commitment problem, they do not easily provide insight of an analytic nature about the mechanism governing the switching of units.

The present thesis will therefore investigate an analytic approach for the study and solution of the static unit commitment problem. This methodology will complement the purely numerical approaches mentioned above. Even if the static unit commitment problem does not consider various practical constraints, the analytic nature and the simplicity of the method add some unique insight basic for the study of the more general dynamic unit commitment formulation. From Everett's theorem [30] and the Kuhn-Tucker optimality conditions [5,36], the following results were derived :

(1) A set of analytic conditions giving the optimum unit commitment for the static case.

(2) The mechanism of how generating units are brought up or down is intrepreted physically in terms of the average unit cost and the system incremental cost giving new insight into the switching mechanism.

(3) This method defines a relatively small number of regions of constant unit commitment compared to the number of possible combinations.

(4) The effect of system load, and reserve margin on the solutions of optimal schedules, can be analytically studied for specific or for general cases.

(5) Some relatively fast algorithms based on simple numerical techniques and the branch-and-bound method were developed to solve the static unit commitment, and were tested on large-sized power systems.

(6) An attempt to solve the dynamic case from the results of the static unit commitment consisting of adding the start-up costs to the fixed costs of the cycling units has been shown to be unstable and therefore giving unsatisfactory results. The second heuristic method used to solve approximately the dynamic unit commitment consisted of perfoming an economic dispatch to generate feasible solutions so that the minimum up-time and down-time constraints are satisfied. Some results were obtained and discussed.

In chapter 2, all the important aspects of the unit scheduling problem for thermal units will be presented. The statement and mathematical formulation of the problem for the dynamic case will be discussed with the various constraints implied by the requirements of the system operation.

In chapter 3 the static unit commitment, as proposed, will be formulated and studied from an analytical point of view and all the main results will be stressed through simple examples.

In chapter 4 an algorithm based on simple numerical techniques for the solution of the static unit commitment problem is proposed. Then, this procedure is applied for practical power systems and the results of the simulations carried out are presented and discussed.

CHAPTER 2

THE UNIT COMMITMENT PROBLEM AND SOLUTION TECHNIQUES

2.1 Introduction:

As was mentioned in the preceding chapter, the scheduling of thermal units consists basically of solving two mutually connected problems. The solution of the first, which is called the unit commitment problem, gives the best combination of the available units; and the solution of the second, called the economic dispatch, consists of allocating the load optimally among units of this best combination, so that the total system cost is minimized.

Almost all known methods applied to solve the unit scheduling problem are a decomposition of the problem into the above two problems and many authors refer to it as a unit commitment/economic dispatch problem (UC/ED).

To have an elementary idea of the scheduling problem [5], suppose that we have N available units, that any one unit can supply the load, and that any combination of these units can also supply the load in a period of time which is usually taken as one hour. The total number of combinations is then 2^N-1, and by performing an economic dispatch for each of these combinations we evaluate the corresponding cost. The least costly combination will thus be our optimal solution. Now, as an example, if the number of units N=10 and the number of time periods is 24 (one day), the total number of combinations of units will be $(2^{10}-1)^{24}$ which is a huge number. In practical power systems, the number of regulating units could be over 100, and the time horizon ranges from 1 to 10 days, so that the total number of combinations reaches astronomical proportions. In practice, many of these combinations will be discarded by the constraints imposed on the sytem, and even by the ordering of the units. Approximations and heuristics are used in almost all known methods, in order to avoid all the alternative combinations.

The main factors influencing the scheduling of thermal units are the daily shape of the consumer demand and the start-up and shut-down costs of each unit. A typical continuous time load demand curve is shown in fig. 2.1. In this chapter a mathematical model of the scheduling of thermal units for the general dynamic case will be presented. The economic load dispatch based on the equal incremental cost criterion will be outlined and then the basics of the existing techniques for the solution of the unit scheduling problem will be reviewed.

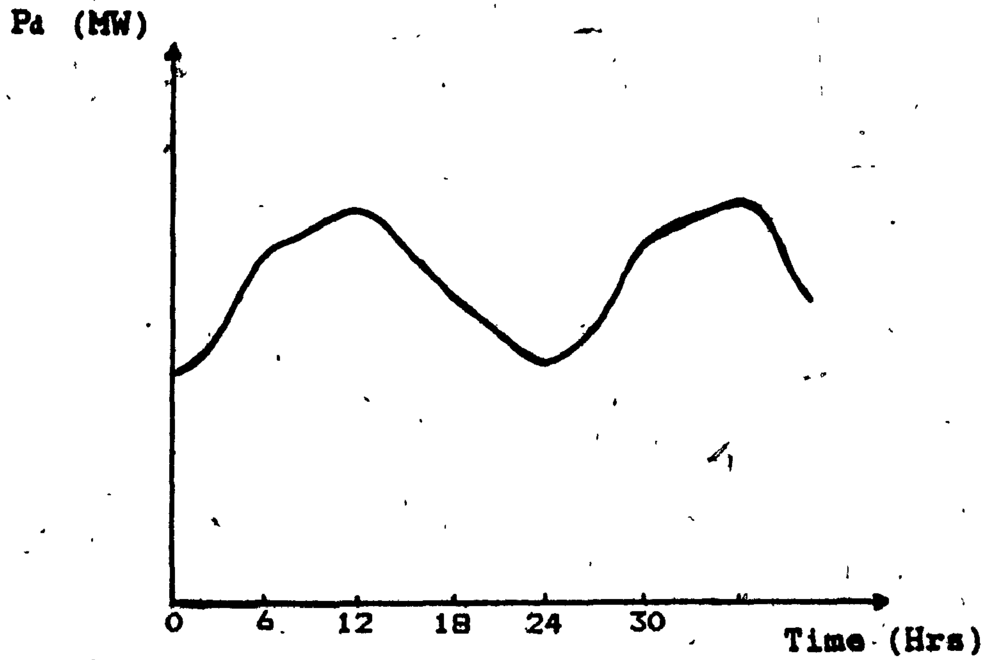

Fig. 2.1 Typical load demand curve

2.2 Start-up costs:

The operation of a thermal unit, consisting of a boiler, turbine and generator will involve a start-up cost. This cost is not due to the megawatt generation from the unit, but to the expenses of bringing the unit on-line, i.e to its operating temperature and pressure. This cost depends on the condition of the boiler after the unit was shut-down, i.e, whether the boiler was allowed to cool, or whether it was banked, i.e, the boiler pressure and temperature was maintained while shut-down. The latter alternative will only be economic if the unit is required for service again after a short time. If a boiler is allowed to cool, its temperature can be approximated by an exponential drop with time. The following expression is commonly used to represent the start-up cost of a unit :

$$C_{SU} = C_C(1-e^{-t/\tau}) + C_{OSU} \qquad (2.1)$$

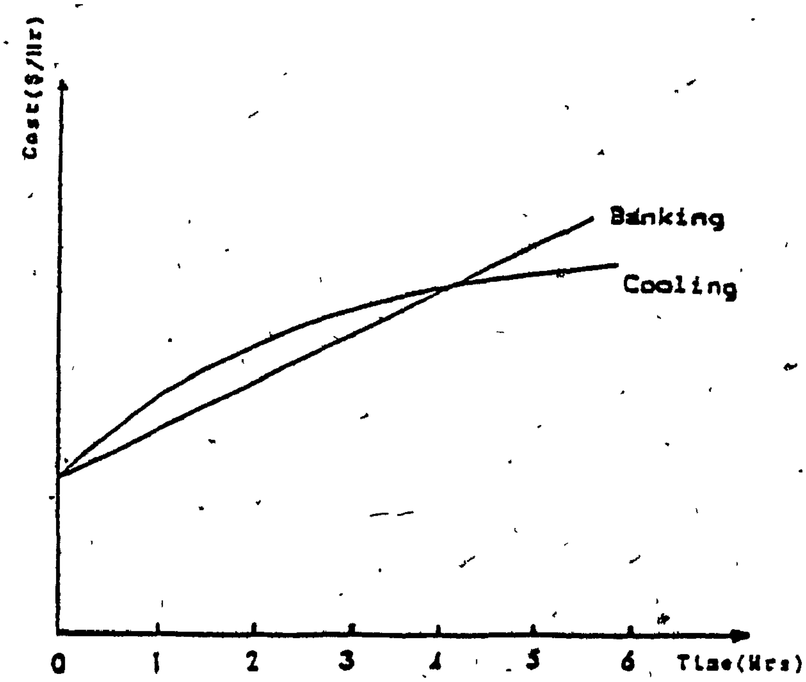

Fig. 2.2 Start-up costs

where C_C is the cold start-up cost, C_{OSU} the cost of starting the turbine alone including the maintenance and operation cost due to start-up, τ the cooling time constant of the boiler in hours and t is the cooling time after the unit was shut down, in hours. When a boiler is banked, the fuel cost per hour of banking is constant, and consequently the cost attributable to the next start-up is given by :

$$C_{SD} = (C_B)t + C_{OSU} \qquad (2.2)$$

The decision whether to shut-down or bank a boiler is determined by the length of the shut-down period. Figure 2.2 shows typical curves of the start-up costs.

2.3 Running costs:

The running costs of a thermal plant are usually represented in terms of the fuel input F needed to produce a certain power output P. Thus, they are represented by a nonlinear curve $C = C(P)$, where the cost C is expressed in terms of fuel cost consumption per hour (dollars/h), and the output P in Megawatts. A common expression used to represent the running cost is :

$$C(P) = c_0 + aP + 0.5bP^2 \qquad (2.3)$$

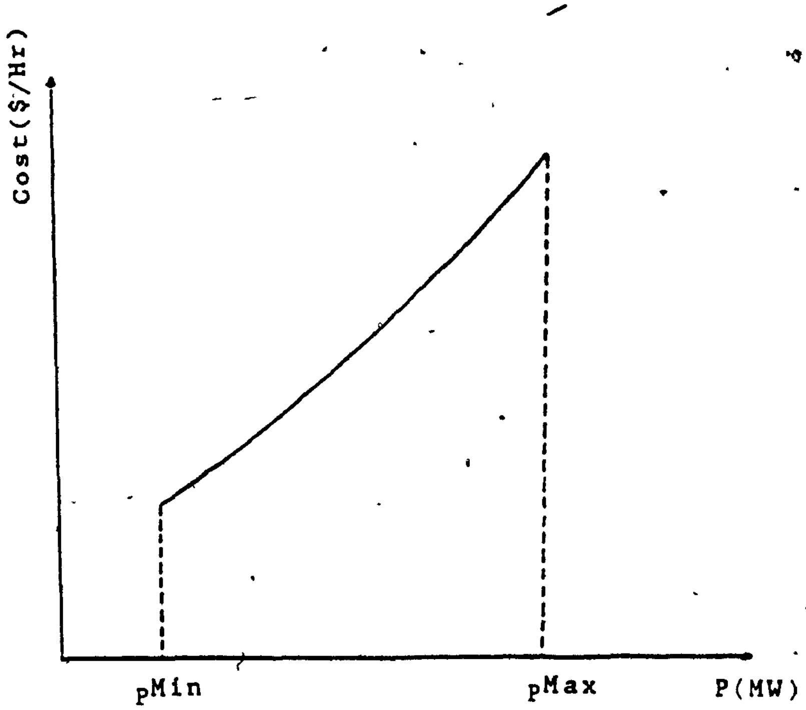

Fig 2.3 Typical cost curve

where the coefficients a, b and c_0 are determined in the procedure of curve fitting through discrete points obtained by field tests. Figure 2.3 shows a typical cost curve versus Megawatts ouptput. The fuel costs C_T for a unit over a commitment period T can therefore expressed as :

$$C_T = \int_0^T C(P(t))\, dt \qquad (2.4)$$

Where P(t) specifies unit output as a function of time. Expressions (2.1) and (2.3) show that the production cost of a unit is a nonlinear function of the output when the unit is running, and a nonlinear function of the cooling time when the unit is to be put into operation It is also discontinuous because it contains fixed terms representing start-up and minimum costs of the unit.

2.4 Total generation costs :

The function of the system operating costs consists of the three different costs which follow :

(i) Start-up and banking costs :

$$C_{SU} = \sum_{k=1}^{K} \underline{C}_S^t \, \underline{V}_k = \sum_{i=1}^{N} \sum_{k=1}^{K} C^i_S \; V^i_k \qquad (2.5)$$

Where

$\underline{C}_S = (C^1_S, C^2_S, \ldots\ldots, C^N_S)^t$ is the vector of the start-up costs. Its elements are given by equation (2.1) or (2 2) and N is the number of units in the system.

$\underline{V}_k = (V^1_k, V^2_k, \ldots\ldots, V^N_k)^t$ is the vector of the unit start-up status, such that : $V^i_k \in (0,1)$. $V^i_k = 1$ corresponds to the start-up of the unit i in the interval k ; $V^i_k = 0$ means there is no start-up of the unit in the interval k, ranging from 1 to K where K is the number of time intervals in the considered period T, such that $T = K\Delta T$ and usually Δt is taken as 1 hour.

(ii) The running costs :

The following expression will represent the running costs of the units :

$$C_R = \sum_{k=1}^{K} (\underline{C}_k(\underline{P}_k))^t \, \underline{U}_k = \sum_{i=1}^{N} \sum_{k=1}^{K} C^i_k(P^i_k) \, U^i_k \qquad (2.6)$$

where

$(\underline{C}_k(\underline{P}_k)) = (C^1_k(P^1_k),\ C^2_k(P^2_k), \ldots C^N_k(P^N_K))^t$ is the vector of running costs in the interval k and its elements are given by (2.3).

$\underline{U}_k = (U^1_k,\ U^2_k, \ldots U^N_k)^t$ is the switching state vector of sytem units in interval k $U^i_k \in (0,1)$. If the unit in interval k is in operation then $U^i_k = 1$, otherwise $U^i_k = 0$.

$\underline{P}_k = (P^1_k,\ P^2_k, \ldots P^N_K)^t$ is the vector of unit outputs in the interval k.

(iii) The shut-down costs:

They could be written similarly as follows

$$C_{DW} = \sum_{k=1}^{K} \underline{C}_D^t \, \underline{W}_k = \sum_{i=1}^{N} \sum_{k=1}^{K} C^i_D \, W^i_k \qquad (2.7)$$

where

$\underline{C}_D = (C^1_D, C^2_D, \ldots\ldots\ldots C^N_D)^t$ is the vector of shut-down costs. The elements C^i_D characterize the shut-down costs for each particular unit.

$\underline{W}_k = (W^1_k, W^2_k, \ldots\ldots\ldots W^N_k)^t$ is the vector of the unit shut down such that $W^i_k \in (0,1)$; $W^i_k = 1$ corresponds to the shut-down of unit i in interval k, and $W^i_k = 0$ otherwise. The vectors U_k, V_k and W_k are mutually connected by the relation :

$$U^i_k - U^i_{k-1} = V^i_k - W^i_k \qquad \begin{matrix} i = 1,2,\ldots\ldots,N \\ k = 1,2,\ldots\ldots,K \end{matrix} \qquad (2.8)$$

The total system cost is obtained by summing up the above defined terms :

$$C_T = C_{SU}+C_R+C_{DW} = \sum_{k=1}^{K} (\underline{C}_S{}^t\ \underline{V}_k + \underline{C}_k(\underline{P}_k)^t\ \underline{U}_k + \underline{C}_D{}^t\ \underline{W}_k)$$

$$= \sum_{i=1}^{N} \sum_{k=1}^{K} (\ C^i_S\ V^i_k + C^i_k\ V^i_k + C^i_D\ W^i_k\ ) \qquad (2.9)$$

2.5 Operating constraints:

In the following sections the various constraints required by the units and the system operation will be defined. The unit constraints are connected to technical limitations of the generating units, and the system constraints on the other hand, are the constraints imposed on the whole system, in order to satisfy the proper operation of the system. The unit constraints will be discussed first and then will follow the system constraints.

2.5-1 Minimum and maximum generating limits:

Each unit must be loaded between two specified limits, which may be varying with time. The constraint could be written as:

$$U_i^k \; P_i^{Min} \leq U_i^k \; P^i_k \leq U_i^k \; P_i^{Max} \qquad (2.10)$$

2.5-2 Minimum up-time and minimum down-time :

Once the unit is running, it should not be turned off before a specified time known as the minimum up-time. Similarly, the minimum down-time, is defined as the time required before a unit can be turned on, once it is decommitted. These defined times are necessary in order to provide time for temperature equalisation within the turbine unit so as to maintain stresses due to temperature differentials within safe limits. The minimum down-time could vary from 3 to 8 hours. These constraints can be formulated as inequalities or handled explicitly

2.5-3 Crew constraints:

If a plant consists of two or more units ,they cannot always be turned on or at the same time. The following constraints limit the number of units that can be simultaneously in the start-up stage to 2.

$$\sum_{i \in P_j} u_i^k (1 - u_i^{k-1}) \tag{2.11}$$

$$j = 1,2.........n$$
$$k = 1,2,........K$$

Where P_j is the set of units of plant j, and n the number of units.

2.5-4 Fuel constraints:

These constraints are imposed on a system in which some units have limited fuel, or have constraints that require them to burn a specified amount of fuel in a given time [27].

2.5-5 Load constraints:

For almost all power systems the daily load curve has the shape shown in figure (2.1). It differs, of course, in magnitude from a small power system to a large one. This load curve is predicted in advance and could be assumed as probabilistic or deterministic for a given problem .Usually for the unit scheduling problem, the load pattern is assumed deterministic and discretized over the 24 hours in intervals of 1 hour.

The load balance for each area is then given for each interval as :

$$\sum_{i=1}^{N} P_i^k = P_{Dk} + P_{Lk} \qquad k = 1,2,\ldots\ldots K$$

where P_{Dk} is the net load after transfers have been allowed for, and P_{Lk} is the area transmission loss at time k. The load balance for the reactive power could also have been taken into account, but usually this is considered as another subproblem. If we use P_{D0k} to represent the load forecast with the losses included in period k we could write the preceding equation as :

$$\sum_{i=1}^{N} P_i^k = P_{D0k} \qquad k = 1,2\ldots\ldots\ldots K \qquad (2.12)$$

2.5-6 Spinning reserve:

Spinning reserve is defined to be the extra generation available on demand from the generators connected to the power system. The function of the spinning reserve is three-fold [32] :

(i) To provide regulating spinning capacity that will cope with errors in load prediction either due to errors inherent in the method or arising from a variation of the load pattern. In the event of frequency deviations, the spinning reserve is taken up by increasing the outputs of the generators,

(ii) To provide step spinning capacity in the event of the loss of a generator.

(iii) Loading spinning capacity is required to allow the reallocation of generator outputs following a generator loss such that the frequency may be restored to its normal value.

Some utilities maintain a spinning reserve capacity of 15 to 25% of the expected load, while others prefer to maintain just enough to cover the loss of the largest unit in operation. The reserve constraint can be formulated as follows:

$$\sum_{i=1}^{N} P_i^{Max} u_i^k \geq P_{D0k} + R_{Mink} \quad k = 1, 2, \ldots K \quad (2.13)$$

where R_{Mink} designates the prescribed spinning reserve in interval k.

2.5-7 Ramp constraints:

Another constraint implied by system load demand is the requirement relative to the rate of generation change, expressed as follows:

$$\left(\frac{dP_i}{dt}\right)_{Min} \leq \frac{dP_i}{dt} \leq \left(\frac{dP_i}{dt}\right)_{Max} \qquad i = 1, 2, \ldots\ldots N \qquad (2.14)$$

and

$$\sum_{i=1}^{N} \left(\frac{dP_i}{dt}\right)_{Max} (u_i{}^k) \geq \frac{d}{dt}(P_{D0k}) \qquad k = 1, 2, \ldots\ldots K \qquad (2.15)$$

where subscripts Min and Max represent minimum and maximum values of the indicated variables respectively. P_{D0k} is the load forecast defined in (2.13).

2.5-8 Network security constraints:

If initially a system is operating satisfactorily and there is an outage, maybe scheduled or forced, some of the constraints of the system may be violated. The complexity of these constraints is increased when a large system is under study. In this case a study is to be made with the outage of one branch or generator at a time and then more than one branch or generator at a time.

Generally these constraints can be expressed as:

$$S_k(\underline{X}_k) \leq 0 \qquad (2.16)$$

$$k = 1, 2, \ldots\ldots K$$

where

$\underline{X}_k$ is the state variables vector of the network in period k.

Additionnal constraints such as the allowable number of start-ups and shut-downs of units, scheduled way of operation of units, etc, can also be formulated and taken into account in the unit scheduling problem.

The unit scheduling or commitment problem, therefore will be formulated as follows:

Minimize C_T

Subject to: i) constraints (2.10)-(2.16)

ii) the vectors $\underline{U}$, $\underline{V}$ and $\underline{W}$ must be integer

iii) and the minimum-up and down-time constraints

where C_T is given by (2.9). If we introduce the new vectors $[X_k]$ and [CC] such that:

$$\underline{X}_k = \begin{bmatrix} \underline{V}_k \\ \underline{W}_k \end{bmatrix} \qquad k = 1,2,\ldots\ldots\ldots\ldots K$$

$$\underline{CC} = \begin{bmatrix} \underline{C}_S \\ \underline{C}_D \end{bmatrix}$$

The objective function C_T could be written as:

$$C_T = \sum_{k=1}^{K} (\underline{C}_k(\underline{P}_k)^t\ \underline{U}_k + \underline{CC}^t\ \underline{X}_k) \qquad (2.17)$$

2.6 Economic load dispatch:

In this section, the classical economic dispatch based on the equal incremental cost criterion [1,2,5] will be briefly discussed.

Given a system with N available generating units the basic economic dispatch problem consists of solving the following mathematical program:

$$\text{Min } C_R = \sum_{i=1}^{N} C_i(P_i) \qquad (2.18)$$

$$\text{S.t} \qquad P_D = \sum_{i=1}^{N} P_i \qquad (2.19)$$

$$\text{and} \qquad P_i^{Max} \le P_i \le P_i^{Min} \qquad (2.20)$$

where C_R is the total fuel cost input to the system, C_i the fuel cost input to the i^{th} unit given by (2.3), P_D the total demand and P_i the generation of the i^{th} unit. By multiplying equation (2.20) by the lagrange multiplier λ and adding it to the cost function (2.19) we form the auxiliairy function known as the lagrangian:

$$\mathcal{L}(P_i,\lambda) = C_R + \lambda(P_D - \sum_{i=1}^{N} P_i) \qquad (2.21)$$

Differentiating $\mathcal{L}(P_i,\lambda)$ with respect to the generation P_i and equating to zero gives the condition for optimal operation of the system if all P_i are within limits.

$$\frac{\partial C_R}{\partial P_i} = \frac{\partial C_R}{\partial P_i} + \lambda(0 - 1) = 0$$

$$= \frac{\partial C_R}{\partial P_i} - \lambda = 0$$

Since $C_R = C_1 + C_2 + \ldots\ldots\ldots + C_N$

$$\frac{\partial C_R}{\partial P_i} = \frac{dC_R}{dP_i} = \lambda$$

therefore the condition for optimum operation is:

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \ldots\ldots\ldots\ldots = \frac{dC_N}{dP_N} = \lambda \qquad (2.22)$$

Here $\frac{dC_i}{dP_i} = \lambda$ is the incremental cost production of unit i in dollars per megawatthours. If as before we use a quadratic eqation to represent the cost, then :

$$\frac{dC_i}{dP_i} = a_i + b_i P_i \qquad (2.23)$$

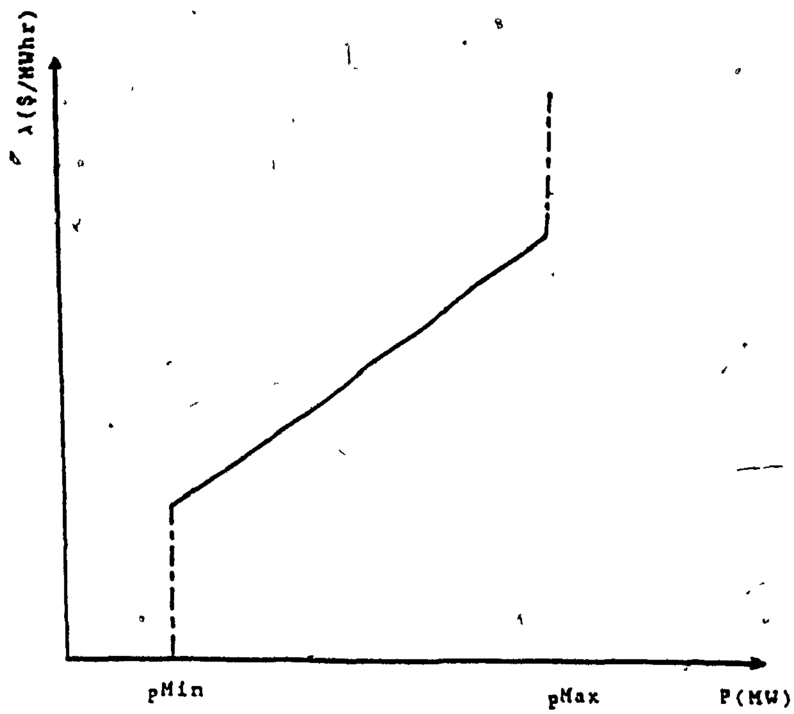

Fig 2.4 Incremental cost curve

The equations (2.22) mean that the machines have to be loaded so that their incremental costs of production are the same. The active power generation contraints (2.20) are taken into account while solving the equations which were derived above. If a generator violates a constraint, in trying to meet the system incremental cost, it is set to the violated limit and the rest of the load is distributed to the remaining generating units according to the equal incremental cost criterion (2.22). Therefore the solution of the simultaneous equations (2.23) will give the economic operating schedule and any good technique for solving a set of linear equations can be used taking into account the inequality constraints (2.20). The analytic expressions for the units output will be derived latter in section 3.4 of the next chapter. Figure 2.4 shows a typical linear incremental cost versus megawatts output of a unit having a quadratic cost.

2.7 solution techniques for the unit scheduling:

Many utilities today use enhanced priority-list methods; this technique was outlined before and will not be discussed here again. In the following sections the techniques based on optimization methods will be outlined.

2.7-1 Integer programming and branch-and-bound :

The unit commitment can be formulated as an integer programming problem. In this case it is assumed that each generating unit has a constant incremental cost and, if the actual cost is nonlinear, a piece-wise representation will be used to give constant incremental costs for the different operating levels. If there are M such levels then the cost of running the single unit is represented by the sum of the individual linearized components [32].The optimization problem is therefore to determine the binary values of V, U, W and the quantized generation levels such that the total operating cost is minimized subject to the necessary constraints. Much of the work in this area was done by Garver [9] who solved this problem using the all-integer method of Gomory for linear integer programs.

Further research has lead to the formulation of the unit commitment as a mixed-integer programming problem [10,13]. However the techniques were not widely used to solve practical problems and no computationnal experiments were reported.

The branch-and-bound method [28,29] is a general approach to the solution of constrained optimization problems; it consists of searching in an intelligent, strucured way the space of all feasible solutions. This space is repeatedly partitioned into smaller and smaller subsets, and a lower bound is calculated for the cost of the solutions within each subset. After each partitioning, those subsets with a bound greater than the cost of a known feasible solution are excluded from all further partioning (fathomed). The branching is continued until a feasible solution is found such that its cost is no greater than the bound for any subset. This basic idea of branching and bounding was applied for solving linear integer, mixed-integer programs and nonlinear programs among many other applications [28].

When applying branch-and-bound to the unit commitment [11,12,15], a relaxed problem, simpler than the given one, is formulated; its solution will be a lower bound (L_{B0}) because its space of feasible solutions is included in the space of the feasible solutions of the original problem. Now, any feasible solution of the original problem is an upper bound (U_{B0}) and the optimal solution must therefore lie between these two bounds. The procedure of branching and bounding can therefore be done as follows:

Initially if V_0 represents the initial set where all the binary variables are not fixed ,the two bounds (L_{B0}) and (U_{B0}) are found as indicated above. A binary variable is chosen for branching and the set V_0 will be separated into two subsets V_1 and V_2 such that $V_1 \cup V_2 = V_0$; V_1 is obtained by setting the binary variable to zero and V_2 is otained by setting the same variable to one. To the subset V_1 we compute (L_{B1}) and (U_{B1}), and to the subset V_2 (L_{B2}) and (U_{B2}) the lower and the upper bounds respectively; We have necessarily (L_{B1}) $\geq$ (L_{B0}) and (L_{B2}) $\geq$ (L_{B0}). If we choose one subset and repeat the same process we will obtain a tree where all the hanging nodes are to be partitioned. If (U^*_{Br}) represent the smallest feasible solution already found and if for a hanging node V_q ,we obtain a lower bound (L_{Bq}) which is larger than(U^*_{Br}),the node is fathomed. If for the same node V_q, (L_{Bq}) = (U_{Bq}) the node is terminal, the optimum for this branch which must lie between these two values is necessarily equal to one of them, and if V_q is empty the node is also terminal. Thus at any step, the tree contains a subset of nodes which are condidates for branching and others which are fathomed. Therefore we see that for large scale problems the method becomes unmanageable unless the branches are fathomed very quickly, which requires the computation of closer lower and upper bounds initially.

2.7-2 Dynamic programming approach.

Like the branch-and-bound method, dynamic programming [33,17,18,19,20,21,22] is an approach which seeks all feasible solutions in a structured way The principal feature of a dynamic programming approach is that the problem of determining the optimum generation of available units for a given load is replaced by an optimization of the outputs of the units for all loads between the minimum and maximum capacity of the units Thus, if the optimum commitment for i units is known, then the optimum for i+1 can be easily computed. The solution of the minimization problem of the cost function (2.17) for the whole optimization period K can be replaced by the minimization of the system cost for each interval k = 1,2,.....K Then a recursive formula of the following type can be obtained:

$$C_{Tk}(U_k) = \underset{\substack{X_k \\ P_k}}{Min}(C_{Tk-1}(U_{k-1}) + [CC_k(U_k, X_k]^t[X_k] + [C_k(P_k)]^t[U_k]) \quad (2.25)$$

where relation (2.8) between U_k, V_k and W_k holds. Assuming the problem of economic dispatch for each interval $k = 1,2,\ldots\ldots K$, is solved (U_k, P_k known) the recurssive expression (2.25) is replaced by:

$$C_{Tk}(U_k) = \underset{X_k}{Min}(C_{Tk-1}(U_{k-1}) + [CC_k(U_k, X_k)]_t[X_k] + f_k(P_k) \quad (2.26)$$

$$k=1,2,\ldots\ldots\ldots K$$

Then the cost (2.26) is function of X_k and U_{k-1} only, and for each combination of generation sets, satisfying the constraints, X_k (i.e V_k and W_k) and P_k can be found inside the whole interval time period K. It is necessary to start with k=1 and for $C_{T0}(U_0)=0$ expression (2.26) becomes:

$$C_{T1}(U_1) = [CC1(U_1, X_1)]^t[X_1] + f_1(P_1)$$

Then successively for $k=2,3\ldots\ldots K$, the minimum operating cost of the set of units $C_{Tk}(U_k)$ are found, using the recursive formula (2.26). The generation $P^i{}_k$ should be quantized into say 1 MW steps, inside each interval. Then, for each load level P_{Dk}, the function $f_k(P_k)$ is calculated together with the combination of n units giving rise to minimum running costs $f_k(P_k)$. The shortcoming of the method is the large memory requirements in case of large systems, which has been termed the "curse of dimensionality". In the unit commitment problem, orderings of the system units and some heuristics have been always used in order to make the problem tractable.

2.7-3 The Lagrangian relaxation technique:

In this method, instead of solving the given problem, another problem, the dual, is formed by incorporating some selected constraints into the objective function via Lagrange multipliers [23,24,25,26,27,34,36]. In general given the following optimization problem called the primal:

$$\begin{aligned} & \text{Min } F(\underline{x}) \\ \text{s.t} \quad & \underline{h}(\underline{x}) = \underline{b} \\ & \underline{g}(\underline{x}) \geq \underline{c} \\ \text{and} \quad & \underline{x} \in X \end{aligned} \qquad (2.27)$$

The dual problem is given as follows:

$$\begin{aligned} & \underset{\underline{\lambda},\underline{\mu}}{\text{Max}} \left\{ \text{Min}(F(\underline{x})+\underline{\lambda}^t(\underline{b}-\underline{h}(\underline{x}))+\underline{\mu}^t(\underline{c}-\underline{g}(\underline{x})) \right\} \\ \text{s.t} \quad & \underline{\mu} \geq \underline{0} \\ & \underline{x} \in X \end{aligned} \qquad (2.28)$$

where

$\underline{x}$ is an n vector, $\underline{h}(\underline{x})$ and $\underline{g}(\underline{x})$ are m and l vectors respectively such that m≤n and l≤n; and $\underline{\lambda}$ and $\underline{\mu}$ are vectors of lagrange multipliers associated with the above constraints and having the same dimensions respectively. Remark that the λ_i are not restricted in sign. If the primal problem (2.27) is convex then the solution of the dual prolem (2.28) will yield the optimal solution of the primal, otherwise it will only be a lower bound. Unfortunately the cost function (2.17) in the unit commitment problem is a non-differentiable and also a non-convex function due to the presence of the binary valued valued variables $U^i{}_k$ and $X^i{}_k$, and thererefore the solution of the corresponding dual problem will only lead to a lower bound which is useful to the branch-and-bound method. The constraints incorporated into the cost function in this case are the load balance (2.13) and the reserve margin (2.16) requirements. While the minimization problem is an easy problem because of its decomposable structure, it is not the same for the dual which requires non-differentiable optimization techniques. In the known methods using this technique to solve the unit commitment, either the dual is approximated by convex functions and solved using differentiable optimization [24,25] or it is solved directly by the subgradient algorithm [23,26,27]. The feasible solutions are generated using heuristics. It has been reported [24,25,26] that the lower bound and the feasible solution generated by these methods for large scale systems are always within 0.5%, and that the branch-and-bound procedure is not required if we are satisfied with this very good near optimal solution.

2.8 Summary:

In this chapter the principal features of the unit scheduling problem of thermal units were presented, and the most talked about methods for its solution were outlined. In chapter 3, a new analytic method will be presented, which differs in nature from all the above presented methods. The principal theoretical results obtained for the static case will be discussed.

CHAPTER 3

ANALYTIC PROPERTIES OF THE STATIC UNIT COMMITMENT

3.1 Introduction

In this chapter the basic static unit commitment problem will be formulated and studied by the method of generalized lagrange multipliers [30]. This analytic approach [31] differs from the ones presented in the preceding chapter in that the mechanism governing the switching of units is explained by a set of analytic conditions, and the nature of the optimum schedules in terms of the system load and reserve margins can be analytically studied. In the following sections, the model for the static unit commitment, deduced from the general dynamic case by neglecting all time coupling terms and constraints, will be formulated. Everett's theorem and the Kuhn-Tucker conditions for optimality will be presented next and it will be shown how they were applied to solve our problem. All the principal results derived from the proposed approach will be applied to an illustrative example consisting of 5 units to highlight the approach. In the next chapter the numerical implementation of this method will be presented with simulation test results of larger systems.

3.2 The static unit commitment:

If we neglect the start-up and shut-down costs in the expression of the total system cost (2.18) we obtain the following expression :

$$C_T = \sum_{i=1}^{N} \sum_{k=1}^{K} U_{ik}\, C_i(P_{ik})$$

Now, since time-coupling constraints are neglected, we can minimize C_T by minimizing for each time interval. This hourly cost is,

$$C = \sum_{i=1}^{N} U_i\, C_i(P_i)$$

The only constraints considered in this basic problem are the load balance equation, the reserve margin requirement and the generation limits for each unit. Therefore our problem will be limited to the following mathematical problem, P1.

Problem P1:

$$\underset{\underline{U},\underline{P}}{\text{Minimize}} \quad C = \sum_{i=1}^{N} U_i \, C_i(P_i) \tag{3.1}$$

$$\text{s.t} \qquad \sum_{i=1}^{N} U_i \, P_i = P_d \tag{3.2}$$

$$\sum_{i=1}^{N} U_i \, P_i^{Max} = P_d + R \geq P_d + R^{Min} \tag{3.3}$$

$$U_i = 0 \text{ or } 1 \tag{3.4}$$

$$P_i^{Min} \leq P_i \leq P_i^{Max} \tag{3.5}$$

To this set of constraints, some others cited in the definition of the static unit commitment problem in chapter 1 could be added, however, as stated above, the mathematical program which is a mixed integer nonlinear problem is not easy to solve and, therefore, we will consider only the above problem.

3.3 General solution:

The above problem can be solved by the general approach consisting of its decomposition into the economic dispatch and the unit commitment problems. The economic dispatch as we know consists of minimizing the cost with respect to $\underline{P}$ by fixing the vector of the unit status, $\underline{U}$, constant. The optimum obtained for the generations $\underline{P}$ will be a function of

$\underline{U}$ and those expressions are used in the unit commitment problem to get the optimum vector $\underline{U}^*$ and consequently the generations $\underline{P}^*$. The economic dispatch problem is relatively easy to solve and analytic expressions for the generations as explained in chapter 2 could be obtained for a given cost function. It is the unit commitment problem, consisting of finding the binary values for the U_i's, which the most difficult problem to solve.

In the following sections the economic dispatch problem outlined in the preceding chapter is presented in more detail and the analytic approach for solving the unit commitment problem is discussed.

3.4 The classical economic dispatch:

Assuming that the unit operating costs are convex and quadratic, given by:

$$C_i(P_i) = C_{i0} + a_i P_i + b_i P_i^2$$

for $$P_i^{Min} \leq P_i \leq P_i^{Max}$$

If we define the minimum and maximum unit incremental costs by:

$$\lambda_i^{Min} = a_i + b_i P_i^{Min}$$
$$\lambda_i^{Max} = a_i + b_i P_i^{Max}$$

Following the reasoning given in chapter 2, we obtain the analytic expressions for the generations P_i's in terms of the system incremental cost λ :

$$P_i = \begin{cases} P_i^{Min} & \text{for} \quad \lambda \le \lambda_i^{Min} \\ \dfrac{\lambda - a_i}{b_i} & \text{for} \quad \lambda_i^{Min} \le \lambda \le \lambda_i^{Max} \\ P_i^{Max} & \text{for} \quad \lambda \ge \lambda_i^{Max} \end{cases} \qquad (3.6)$$

Now that all the P_i's are expressed in terms of λ we could get from equation (3.2) an analytic expression for λ in terms of the U_i's, however for systems consisting of more than a few units a very cumbersome expressions would be obtained. Therefore, the problem of unit commitment instead of being a minimization problem only over the vector $\underline{U}$, will consist, instead, of minimizing the cost over $\underline{U}$ and λ with the power balance equation (3.2) taken explicitly into account. Therefore the problem will be reduced to finding the binary values of $\underline{U}$ and the system incremental cost λ instead of the vectors $\underline{U}$ and $\underline{P}$. If we call this problem P_2 we could formulate it as follows:

<u>Problem P_2</u>:

$$\text{Minimize} \sum_{i=1}^{N} U_i \, C_i(P_i(\lambda)) \qquad (3.7)$$

$$\text{s.t} \quad \sum_{i=1}^{N} U_i \, P_i(\lambda) = P_d \qquad (3.8)$$

and constraints (3.3) and (3.4)

The generation limits, constraints (3.5) are satisfied when $P_i = P_i(\lambda)$ and λ is such that :

$$\lambda^{Min} = \min_i \lambda_i^{Min} \le \lambda \le \max_i \lambda_i^{Max} = \lambda^{Max} \qquad (3.9)$$

Some of the methods outlined in chapter 2 such as the lagrangian relaxation technique or the branch-and-bound method could be applied to solve this problem, however in this work an analytic approach to the unit commitment will be investigated by using the generalized lagrange multipliers method and the Kuhn-Tucker optimality conditions.

3.5 Generalized Lagrange multipliers and the Kuhn-Tucker conditions:

The generalized lagrange multiplier method [30] is a general approach for solving optimization problems in the presence of constraints. Everett's theorem [30] gives the sufficient conditions for optimality, making his results a powerful tool for solving such problems. Actually, either using Everett's theorem or the Kuhn-Tucker conditions for solving our problem of unit commitment the result will be the same. Whenever we talk about sufficient conditions we will refer to Everett's theorem otherwise the necessary conditions provided by the Kuhn-Tucker theorem will be used. The Kuhn-Tucker theorem [34,35,36] provides a set of necessary conditions for optimality for the inequality constrained problems. We summarize them here since they will be used later to solve our problem.

Given the following optimization problem :

$$\text{Minimize} \quad f(\underline{X})$$

$$\text{s.t} \quad h_i(\underline{X}) = 0 \qquad i = 1,2,\ldots .m$$

$$g_i(\underline{X}) \le 0 \qquad i = 1,2,\ldots ..r$$

With $\underline{X}$ being an n dimensional vector.

The lagrangian for this problem is given by :

$$\mathcal{L}(\underline{X},\underline{\lambda},\underline{\alpha}) = f(\underline{X}) + \sum_{i=1}^{m} \lambda i \; h_i(\underline{X}) + \sum_{i=1}^{r} \alpha_i \; g_i(\underline{X})$$

Then for an optimum $(\underline{X}^*,\underline{\lambda}^*,\underline{\alpha}^*)$ the following conditions must hold:

a) $\dfrac{\partial L}{\partial x_i}(\underline{X}^*,\underline{\lambda}^*,\underline{\alpha}^*) = 0 \qquad i = 1,2.........N \quad (3.10)$

b) $\dfrac{\partial L}{\partial \lambda_i} = h_i(\underline{X}^*) = 0 \qquad i = 1,2..........m \quad (3.11)$

c) $g_i(\underline{X}^*) \le 0 \qquad i = 1,2.........r \quad (3.12)$

$$\text{d)}\quad \begin{aligned} &\mu_i^* \, g_i(\underline{X}^*) = 0 \\ &\mu_i^* \geq 0 \end{aligned} \qquad i = 1,2.........r \quad (3.13)$$

Note that these conditions are necessary, that is at an optimum point, they must hold, but the converse may not be generally true. The first two conditions are the well known lagrange equations used in the economic dispatch problem, the third is only the restatement of the inequality constraints, while the fourth, called the complimentary slackness condition, provides a means for handling the binding and nonbinding constraints. The product $\mu_i^* g_i(\underline{X}^*)$ being zero, μ_i^* or $g_i(\underline{X}^*)$ must be zero or both, which means that if $\mu_i^* = 0$ then $g_i(\underline{X}^*)$ is free to be nonbinding, but if μ_{i*} is positive, the constraint $g_i(\underline{X}^*)$ must be zero. The above results will used in the following sections to derive conditions for the optimum static unit commitment.

3.6 Analytic solution for the static UC:

To apply the Kuhn-Tucker conditions to our problem P_2, we consider the relaxed version where the u_i's are assumed continuous, i.e may take values between 0 and 1 . However values of u_i different from 0 or 1 will be infeasible for problem P_2 and thus one must find solutions which are feasible. Neverthless the optimum obtained by the application of the Kuhn-Tucker conditions will yield a lower bound in the case where the u_i's are not at their limits. This lower bound will be used in the branch-and-bound technique to get better feasible solutions Thus, at the optimum, we could have three differents possibilities which could be identified by the following index sets:

U = {i / u_i = 1} i.e the set of the units ON.

L = {i / u_i = 0} i.e the set of the units OFF.

I = {i / 0 < u_i < 1} i.e the set of infeasible unit states.

If we adjoin the load balance equation, the reserve margin and the active u_i inequalities to the system cost function via lagrange multipliers, we obtain the following augmented lagrangian :

$$\mathcal{L} = \sum_{i=1}^{N} u_i C_i(P_i(\lambda)) - \gamma\{\sum_{i=1}^{N} u_i P_i(\lambda) - P_d\} - \alpha\{\sum_{i=1}^{N} u_i P_i^{Max} - P_d - R\}$$

$$- \sum_{i \in U} S_i(1-u_i) + \sum_{i \in L} S_i\, u_i \qquad (3.14)$$

Here, we suppose that constraints (3.9) are inactive, implying that there is not the possibility that all the available units could operate at their minimum or at their maximum, this is usually true due to the presence of base units and the reserve margin.

Using the Kuhn-Tucker conditions we derive the following:

1) $$\frac{\partial L}{\partial u_k} = C_k(P_k(\lambda)) - \gamma P_k(\lambda) - \alpha P_k^{Max} + S_k = 0 \quad (3.14)$$

with
$$S_k \geq 0 \quad \text{if} \quad u_k = 1 \quad (3.15)$$
$$S_k = 0 \quad \text{if} \quad 0 \leq u_k \leq 1 \quad (3.16)$$
$$S_k \leq 0 \quad \text{if} \quad u_k = 0 \quad (3.17)$$

2) $$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{N} u_i \frac{dC_i}{dP_i}\frac{dP_i}{d\lambda} - \gamma \sum_{i=1}^{N} u_i \frac{dP_i}{d\lambda}$$
$$= \sum_{i=1}^{N} u_i \lambda \frac{dP_i}{d\lambda} - \gamma \sum_{i=1}^{N} u_i \frac{dP_i}{d\lambda} = \sum_{i=1}^{N} u_i \frac{dP_i}{d\lambda}(\lambda - \gamma) \quad (3.18)$$

The analytic expressions for P_i are given by equations (3.6) and therefore we get:

$$\frac{dP_i(\lambda)}{d\lambda} = \begin{cases} 0 & \text{for} \quad \lambda \leq \lambda_i^{Min} \\ \frac{1}{b_i} \geq 0 & \text{for} \quad \lambda_i^{Min} \leq \lambda \leq \lambda_i^{Max} \\ 0 & \text{for} \quad \lambda \geq \lambda_i^{Max} \end{cases}$$

Thus the above equation (3.18) shows that :

$$\lambda = \gamma \quad (3.19)$$

Combining equations (3.15) through (3.19), the following set of analytic conditions, termed the switching curve law are obtained:

$$\alpha\ P_k^{Max} \geq C_k[P_k(\lambda)] - \lambda\ P_k(\lambda) \quad \text{if} \quad u_k = 1 \qquad (3.20)$$

$$\leq C_k[P_k(\lambda)] - \lambda\ P_k(\lambda) \quad \text{if} \quad u_k = 0 \qquad (3.21)$$

$$= C_k[P_k(\lambda)] - \lambda\ P_k(\lambda) \quad \text{if} \quad 0 \leq u_k \leq 1 \qquad (3.22)$$

If we denote the switching curves by $S_k(\alpha,\lambda) = 0.$, their expressions are given by:

$$S_k(\alpha,\lambda) = C_k[P_k(\lambda)] - \lambda\ P_k(\lambda) - \alpha\ P_k^{Max} = 0. \qquad (3.23)$$

A typical switching curve resulting from a quadratic cost is shown in Fig (3.1). The space above the switching curve $S_k = 0.$, represents the condition $u_k = 1$, while, $u_k = 0$ occurs in the region below it. Anywhere on the curve u_k can take any value between 0 and 1 including the limits. Therefore for a given pair (λ,α) of the lagrange multipliers, the switching curve concept will provide us the state of each available unit for optimum unit commitment. The lagrange multiplier λ represents the increased cost in \$/MWHr needed to supply the next MW of load and α could be interpreted as the increased cost needed to supply the next MW of minimum reserve margin. Once the pair (λ,α) of Lagrange mutipliers is known the coresponding load P_d, and reserve margin R are easily found. However the most common specified quantities for system operators are the load and reserve margin. Based on this above concept an efficient and relatively simple algorithm could be implemented; this will be the topic of the next chapter.

3.7 Analytic properties of the switching curves:

When the units cost data is given ,the analytic expressions for the switching curves is obtained by using equations (3.20) through (3.22). If in our case we consider quadratic costs of the form :

$$C_i(P_i) = C_{0i} + a_i P_i + .5 \, b_i P_i^2$$

We obtain easily the following expressions for the switching curves as follows :

$$\alpha P_k^{Max} = C_k(P_k^{Min}) - \lambda P_k^{Min} \quad \text{for} \quad \lambda \le \lambda_k^{Min} \qquad (3.24)$$

$$= C_{k0} - \frac{a_k - 2 \lambda a_k + \lambda^2}{2 b_k} \quad \text{for} \quad \lambda_k^{Min} \le \lambda \le \lambda_k^{Max} \qquad (3.25)$$

$$= C_k(P_k^{Max}) - \lambda P_k^{Max} \quad \text{for} \quad \lambda \ge \lambda_k^{Max} \qquad (3.26)$$

We note that the switching curves are composed of three segments, two of them are linear and one is quadratic in

λ. The intersection of the switching curves with the α-axis is given by the non-negative value :

$$\alpha_k = \frac{C_k(P_k^{Min})}{P_k^{Min}} \tag{3.27}$$

And the intersection with the λ-axis is given by the following expressions :

$$\lambda_k = \frac{C_k(P_k^{Min})}{P_k^{Min}} \qquad \text{if } \lambda_k \leq \lambda_k^{Min} \tag{3.28}$$

$$= a_k + (2c_{0k}\, b_k)^{1/2} \qquad \text{if } \lambda_k^{Min} \leq \lambda_k \leq \lambda_k^{Max} \tag{3.29}$$

$$= \frac{C_k(P_k^{Max})}{P_k^{Max}} \qquad \text{if } \lambda_k \geq \lambda_k^{Max} \tag{3.30}$$

We note that if $C_k(P_k^{Min}) = 0.$, both intersects α_k and λ_k will be zero, and in this case the switching curve will be completely below the first quadrant. Such a unit having the above characteristic will always be on, producing zero power with zero fixed running cost until the system incremental cost λ increases to a higher value, where this unit would produce power economically. For units with non-zero fixed cost the state of each of them will be given according to the switching curve concept, i.e for a given operating point on the (λ,α) plane all the units having their switching curves below this point will be on and those having them above will be off. We see from fig 3.1 that α decreases monotonically with increasing λ, and that the fixed operating cost of a unit, c_{i0} shifts the switching curve upward. Therefore units with large fixed costs are brought on only when λ is large (large load P_d), or when α is large (large reserve margin).

3.8 Regions of constant unit commitment:

If we have all the switching curves of the available units plotted in the α vs λ plane, a finite number of regions are defined where the optimum unit combination ,$\underline{U}$, is constant and integer valued (fig 3.2). We note that for different values of α, with increasing λ the order in which the units are brought up, will be different. For example from fig 3.2 we see that for $\alpha = 0$ the units are brought up in the sequence (2,3,1,4,5), while for $\alpha = 1.6$ the sequence is (2,1,4,3,5). The number of regions of constant unit commitment will depend on the unit operating costs data. If the given data is such that the coresponding switching curves do not intersect, then the number of regions of constant unit commitment is N. If on the other hand ,each curve intersects every other curve once then the number of such regions will be N^2, where N represents the

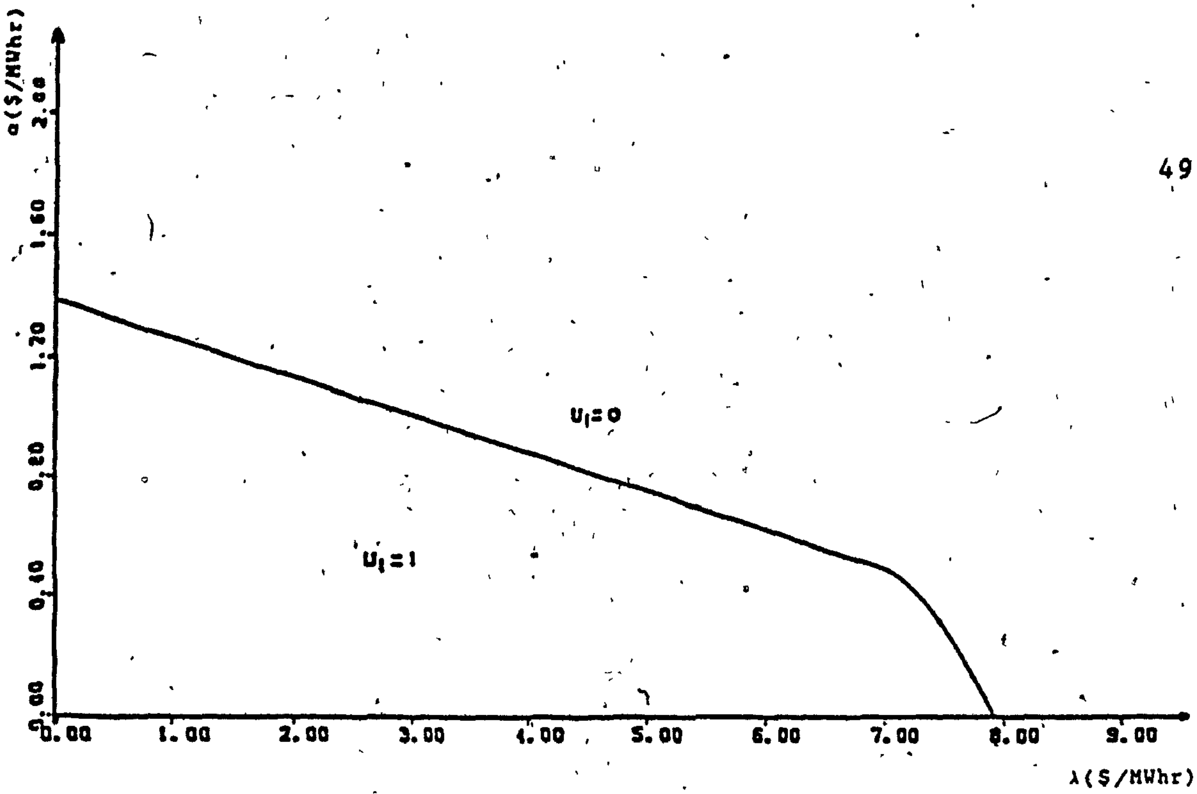

Fig. 3.1 Typical Switching Curve

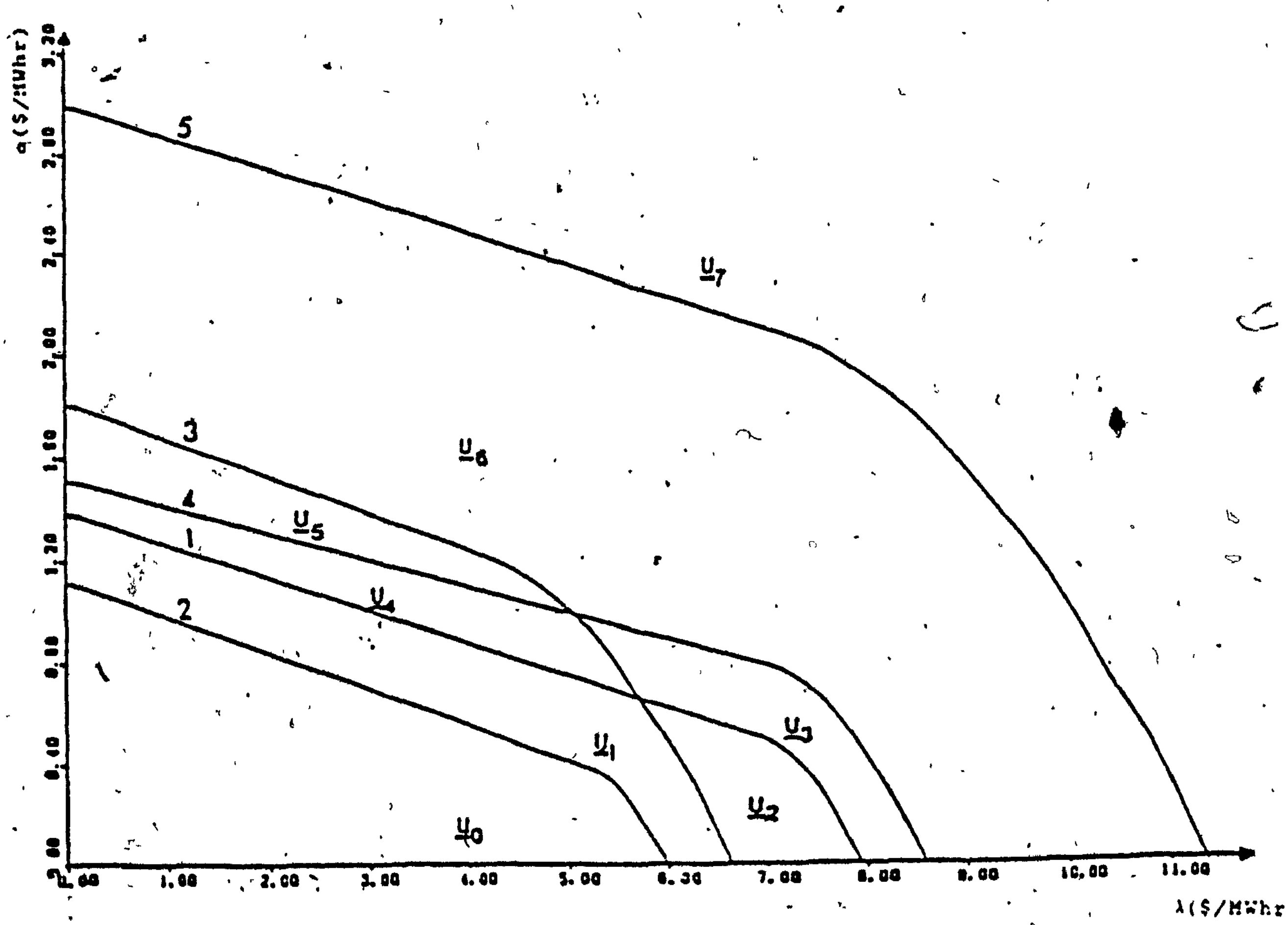

Fig 3.2 Switching curves for the 5 units example

number of units. In fig 3.2 the five switching curves corresponding to the data given in table 3.1 define eight regions of constant unit commitment represented by the following combinations :

$$
\begin{aligned}
\underline{U}_0 &= (0,0,0,0,0)\\
\underline{U}_1 &= (0,1,0,0,0)\\
\underline{U}_2 &= (0,1,1,0,0)\\
\underline{U}_3 &= (1,1,1,0,0)\\
\underline{U}_4 &= (1,1,0,0,0)\\
\underline{U}_5 &= (1,1,0,1,0)\\
\underline{U}_6 &= (1,1,1,1,0)\\
\underline{U}_7 &= (1,1,1,1,1)
\end{aligned}
\qquad (3.30)
$$

Note that these combinations are only the ones wich satisfy the sufficient conditions given by the switching curve law. Remark that the enumeration of all such combinations will be impractical for solving the problem of unit commitment; however the identification of the subset of these combinations which satisfy the load and reserve margin, will be helpful for generating feasible solutions.

Unit	P_i^{Min}	P_i^{Max}	c_{0i}	a_i	b_i
1	45	350	180	6.72	0.0040
2	50	350	135	5.08	0.0030
3	50	350	437	3.72	0.0097
4	47	450	360	6.75	0.0045
5	45	350	743	6.42	0.0165

Table 3.1 Characteristics of the 5 units system

3.9 Physical interpretation of the switching mechanism:

The switching curve characteristics given by equations (3.20) and (3.21) could be rewritten as follows :

$$\text{If } u_k = 1 \quad \text{then} \quad \frac{C_k[P_k(\lambda)]}{P_k(\lambda)} < \lambda + \alpha \frac{P_k^{Max}}{P_k(\lambda)} \tag{3.31}$$

$$\text{if } u_k = 0 \quad \text{then} \quad \frac{C_k[P_k(\lambda)]}{P_k(\lambda)} > \lambda + \alpha \frac{P_k^{Max}}{P_k(\lambda)} \tag{3.32}$$

If we consider the case where the reserve constraints are inactive i.e $\alpha = 0.$, then inequalities (3.31) and (3.32) give the conditions for a unit to be on or off. In this case a unit should be on if its average operating cost is less than the system incremental cost λ ,and should be off if it is greater. Thus a unit having large fixed cost will have large average cost for low loads and will not be turned on unless the load is sufficiently large. If α is greater than zero the right-hand side of the inequalities (3.31) and (3.32) will be larger implying that a unit will be turned on at a higher average cost. In this case the unit is required not only to satisfy the given load but also the higher reserve as well. These physical interpretation of the conditions for optimum unit commitment give satisfactory results from an economic point of view.

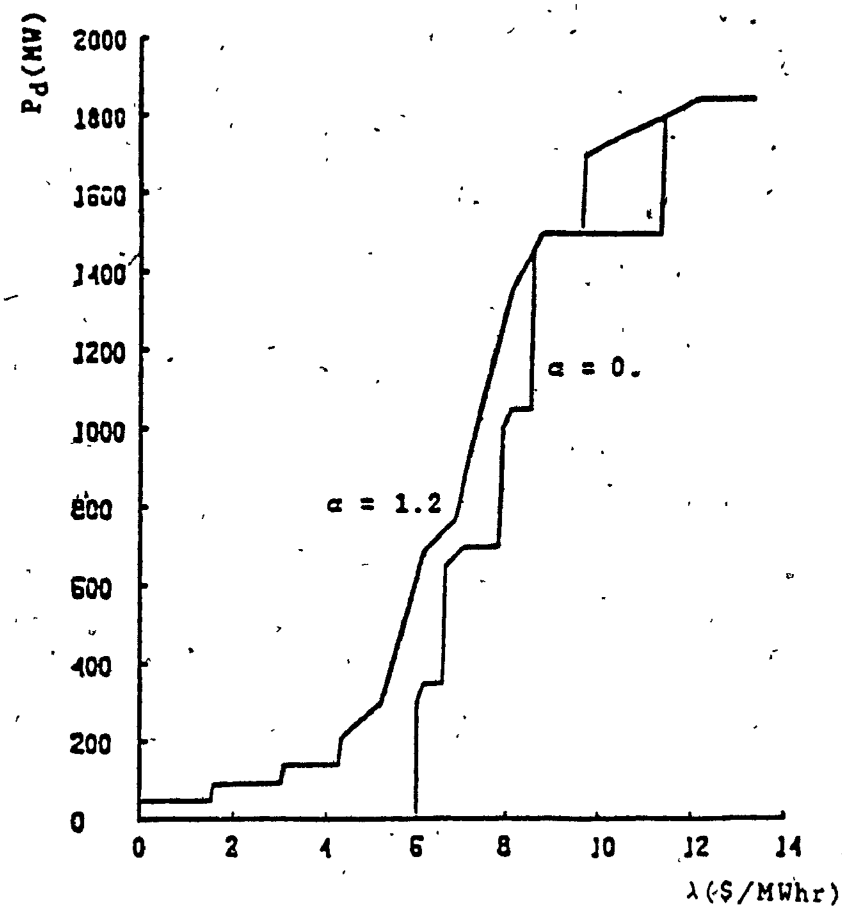

Fig. 3.3 Load Vs λ at constant α for the 5 Units

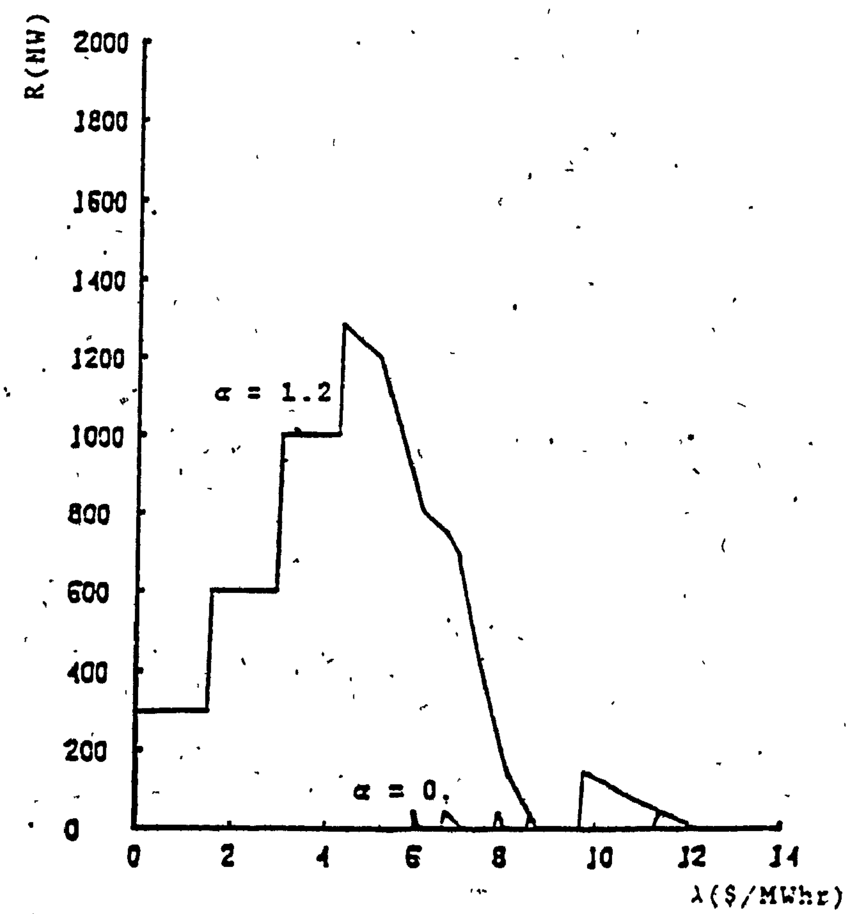

Fig. 3.4 Reserve Vs λ at constant α for the 5 Units

3.10 Load and reserve margin characteristics:

In the above analysis, the mechanism governing the switching of the units has been interpreted in terms of the lagrange multipliers λ and α ; in the following, the behaviour of the load P_d and the reserve R, which are the common quantities specified, will be analysed as functions of λ and α. Specifically, the trajectories of P_d and R versus λ at constant α, and versus α at constant λ will be analysed for our preceding example consisting of 5 units. This will help to understand, how to associate for a given pair (λ,α) of the lagrange multipliers, the corresponding pair (P_d,R) of the load and reserve margin.

a- Trajectories of (P_d,R) Vs λ at constant α :

For any given α, the switching sequence of the available units could be found by inspection, therefore using equations (3.8) and (3.3) giving the load P_d and the reserve R, respectively, one can obtain the characteristics shown in fig 3.3 and fig 3.4 for λ varying from 0 to λ^{Max} and α = 0. The load P_d is a non-decreasing monotonic function of λ, and presents discontinuities whenever λ reaches a new switching curve curve, where a new unit gets turned on. The reserve R, on the contrary, decreases monotonically with increasing λ, until a new unit switches on, where it jumps to a higher value. The trajectories shown in fig 3.3 and fig 3.4 correspond to $\alpha = 0.0$ and $\alpha = 1.2$. Similar characteristics could be obtained for any value of α. They will have the same shape, but they will be shifted depending on the switching sequence of the units.

b- Trajectories of P_d and R Vs α at constant λ:

For λ constant the outputs of each unit $P_i(\lambda)$ will be consequently constant, and therefore from equations (3.8) and (3.3) we see that the variations of P_d and R will be governed only by the switching variables u_i. In the (λ,α) plane, this will correspond to bringing a new unit on, i.e reaching a switching curve for which the corresponding u_i takes the value 1. In this case both P_d and R will increase by the corresponding values $P_i(\lambda)$ and P_i^{Max} respectively. It has to be noted that as long as we remain in a region of constant unit commitment, corresponding to a certain range of α, both P_d and R will remain constant. Depending on the given value of λ, the switching sequence will be completely determined by varying α from zero to α^{Max}.

3.11 Optimum unit commitment for a given load:

The trajectory of α Vs λ for a constant load P_d ,will give a range of (λ,α) operating points satisfying the given load. Some portions of this trajectory will lie on the switching curves corresponding to non-integer solutions, and only the other portions lying inside the regions of constant unit commitment yield integer valued solutions. Fig 3.5 represents such a trajectory for our example, the straight vertical segments of the trajectory are those which correspond to the integer solutions, while the other portions lying on the switching curves are non-integer

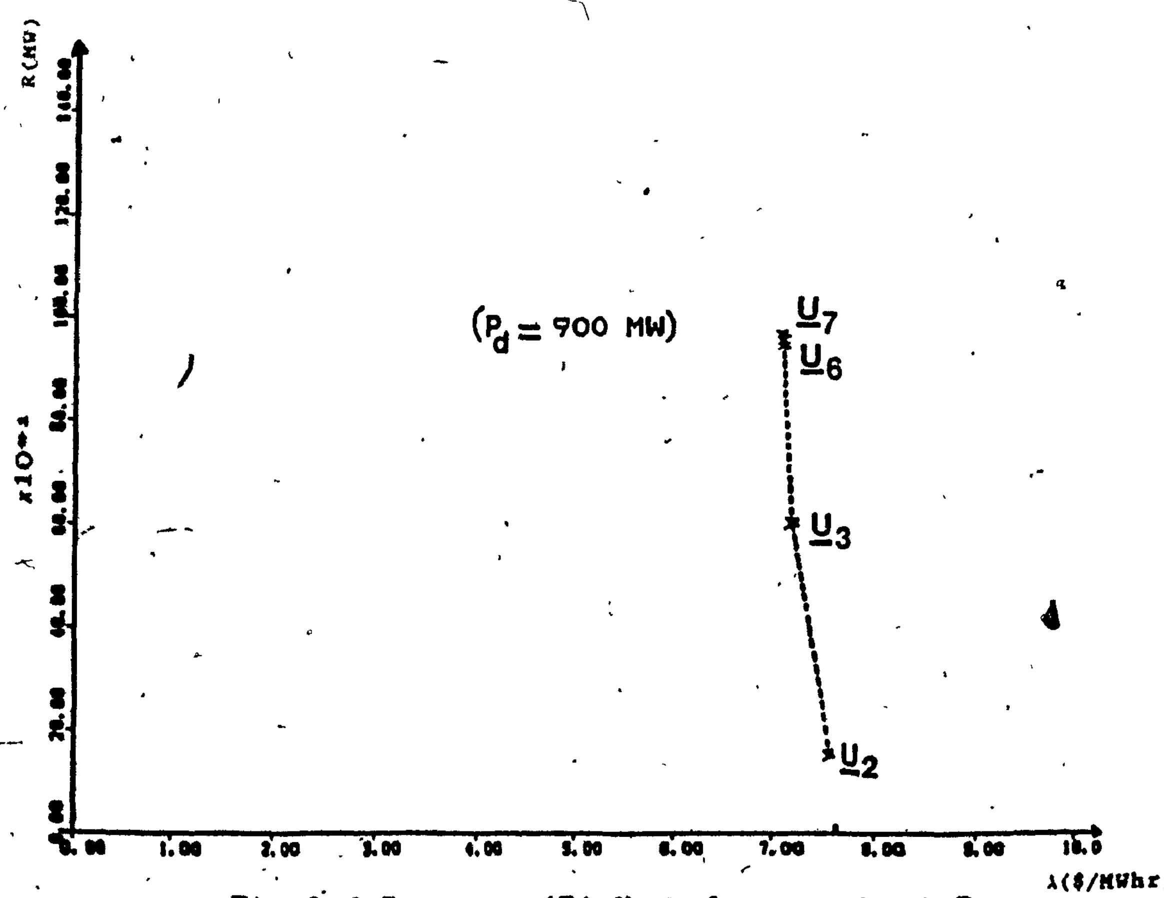

Fig 3.6 Reserve (R) Vs λ for constant P_d

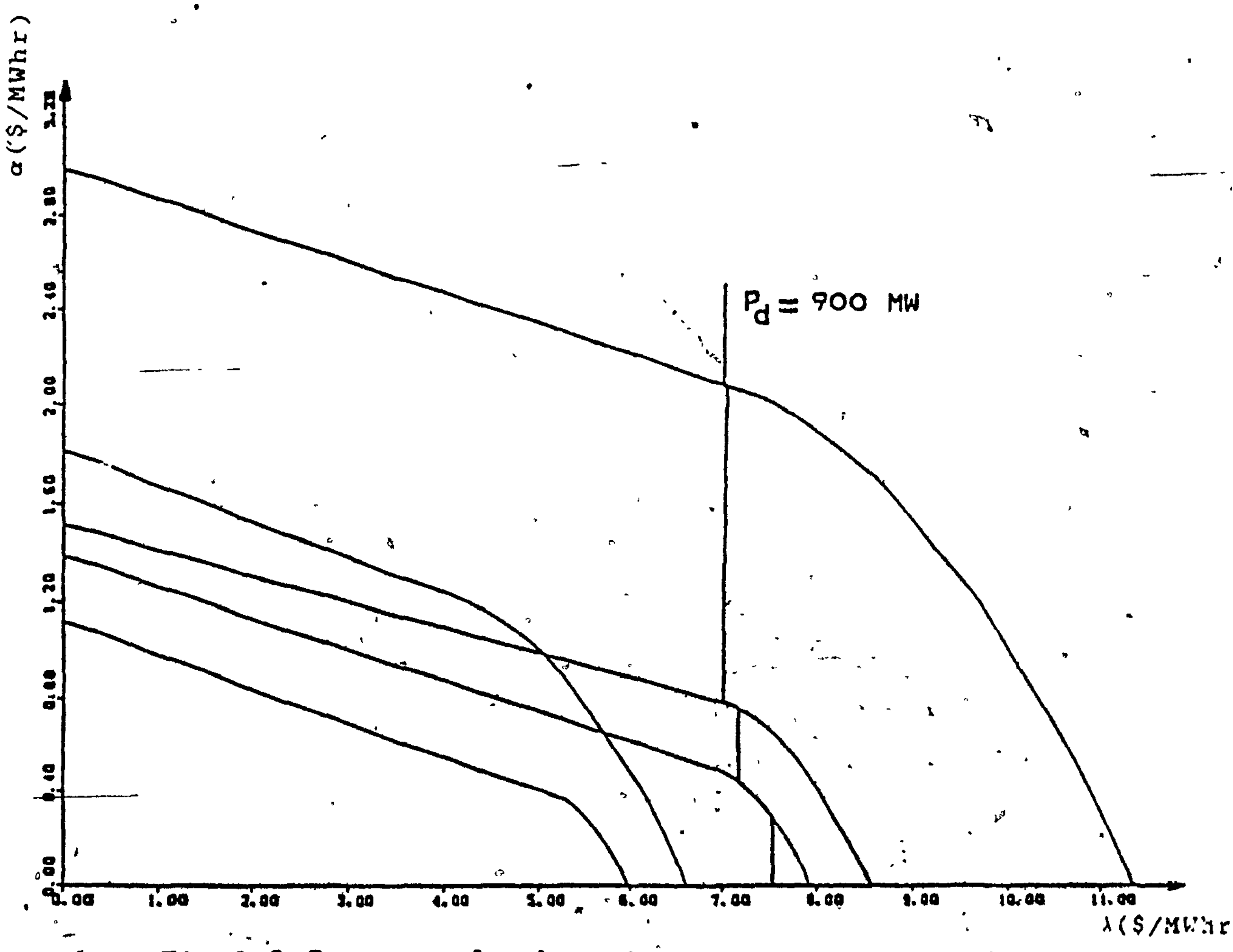

Fig.3.5 Constant load trajectory on the λ α plane

solutions. We remark from the figure that in order to maintain P_d at its specified value, the system incremental cost λ should decrease as α increases, bringing more units on-line. If we enumerate the corresponding integer solutions ($\underline{U}_2$, $\underline{U}_3$, $\underline{U}_6$, $\underline{U}_7$), the values computed for the reserve margin R at these solutions are increasing as λ decreases; the behaviour of R Vs λ for constant P_d is shown in fig 3.6. In this example the lowest computed feasible reserve margin corresponds to the combination $\underline{U}_2$ and has the value R = 148 MW, and if the specified reserve margin is not equal to one of those found above, it will be infeasible for our given problem. Because the conditions derived for optimality are sufficient (Everett's theorem), it is only when the specified reserve margin is equal to one of the feasible reserves found above that we can claim that our solution is optimal. If the sufficient conditions are not satisfied we have to search for better solutions using the branch-and-bound method. However when the computed reserve R is greater than the minimum reserve margin R^{Min} for $\alpha = 0.0$ the solution obtained is a feasible upper bound. If one of the u_i's happens to be non-integer, the infeasible optimum obtained gives a lower bound.

Summary:

In this chapter, the underlying theory of the analytic approach to the static unit commitment has been explained. The optimum unit commitment is reduced to a set of analytic conditions defining the switching curve concept from which a physical intrepretation of the switching mechanism was carried out. The correspondence between the (λ,α) plane of the lagrange multipliers and the load and reserve margin, (P_d,R) was explained enabling us to carry the analysis in either plane. A simple instructive example consisting of five units was studied to derive the principal results obtainable from the switching curve concept.

In the next chapter, an algorithm based on the results of this chapter will be presented and applied to solve practical power systems consisting of 10 and 100 units.

CHAPTER 4

ALGORITHMS FOR THE STATIC CASE AND TEST RESULTS

4.1 Introduction:

In this chapter, an algorithm based on the switching curve concept will be discussed, and the results of the unit commitment of practical power systems consisting of 10 and 100 units will be presented. The algorithm basically seeks the optimum unit commitment by localizing the solution point on the (λ,α) plane of the lagrange multpliers, which satisfies the given load P_d and the reserve constraint. If this point happens to be feasible with the vector $\underline{U}$ of the switching variables having its elements set to their limits, the solution is optimal and we are done, otherwise a feasible integer solution has to be found which is not guaranteed to be optimal. This solution will be used as an upper bound in the branch-and-bound technique, where we try to get better feasible integer solutions. Note that the computed reserve R is not necessarily equal to the specified reserve R^{Min}, but is always the smallest which could be found by the proposed method. The technique used to perform the economic dispatch, i.e to satisfy the load, and to compute the required reserve is based on two binary searches along the λ and α axes which will be discussed in the following sections. A binary search is a general technique among many others for finding zeros for monotonicalyl increasing functions which is satisfied in our problem. It relatively simple to implement, fast and reliable.

4.2 Satisfying the load P_d on the (λ,α) plane :

In this case it is required to find the value of the system incremental cost λ ,such that the load balance equation (3.8) is satisfied. Therefore by fixing α to a constant value say zero, a binary search on the λ-axis could be performed between the minimum and maximum incremental costs of the system to get the desired value of λ. For each value of λ the outputs of the units on are computed using equations (3.6) derived from the equal incremental cost criterion. However it could be seen from Fig 4.1 where a typical P_d versus λ is shown ,that for a given value λ it corresponds a whole range of values of P_d ;these jumps are present whenever new units are brought on; and the corresponding values of λ are given by the intersections of the switching curves with the λ-axis for $\alpha = 0$. Hence whenever a given load P_d lies on the vertical segments, at least one of the switching variables u_i is non-integer and should be computed. This value of u_i corresponds to the switching curve which has the particular value of λ as a root. Thus in each step of the binary search or after it has converged to some $\lambda = \lambda^0$ we check if a non-integer value of u_i could be found such that the given load is satisfied; such a u_i is found by rewritting equation (3.8) as follows:

$$u_k P_k(\lambda^0) + \sum_{i \in Non} u_i P_i(\lambda^0) = P_d \qquad (4.1)$$

From which :

$$u_k = \frac{P_d - \sum_{i \in Non} u_i P_i(\lambda^0)}{P_k(\lambda^0)} \qquad (4.2)$$

Where Non represents the set of the units "on".

Hence, for any given α, the system incremental cost λ can be found by performing a binary search between the values λ^{Min} and λ^{Max} such that the load balance equation (3.8) or (4.1) is satisfied. If λ^0 is found to be different from the intersections of the switching curves with the λ-axis for the given α, then the corresponding switching vector $\underline{U}$ is integer valued and the solution is optimal if the reserve constraint is satisfied. If on the contrary λ^0 is found to be equal to one of these intersections, equation

(4.2) should be used in order to satisfy the load constraint, and in this case even if the reserve constraint is satisfied a feasible solution which is not guaranteed to be optimal has to be found.

Another scheme which could be used to satisty the load balance equation is to identify all the intersects λ_k of the switching curves with the λ-axis or to any parallel line given by α = constant and to check if one of these intersects satisfy equation (4.1) .If yes we got a lower bound as a solution and a feasible solution has to be found, otherwise a reduced binary search could performed between the two bounds found above to get the correct value of λ. The value of α which will satististy the reserve constraint can be computed by using another binary search scheme on α between 0 and the maximum value of α as discussed in the following.

4.3 Satisfying the reserve constraint:

Generally, the value of α which is considered first is zero and if for this value the computed reserve R is greater than the specified reserve R^{Min}, no binary search is required on the α-axis (since R increases with α for a given P_d). If R = R^{Min} the solution obtained is integer feasible then it is also optimal, otherwise if R = R^{Min} and $\underline{U}$ is not integer valued, a lower bound only is obtained. If on the

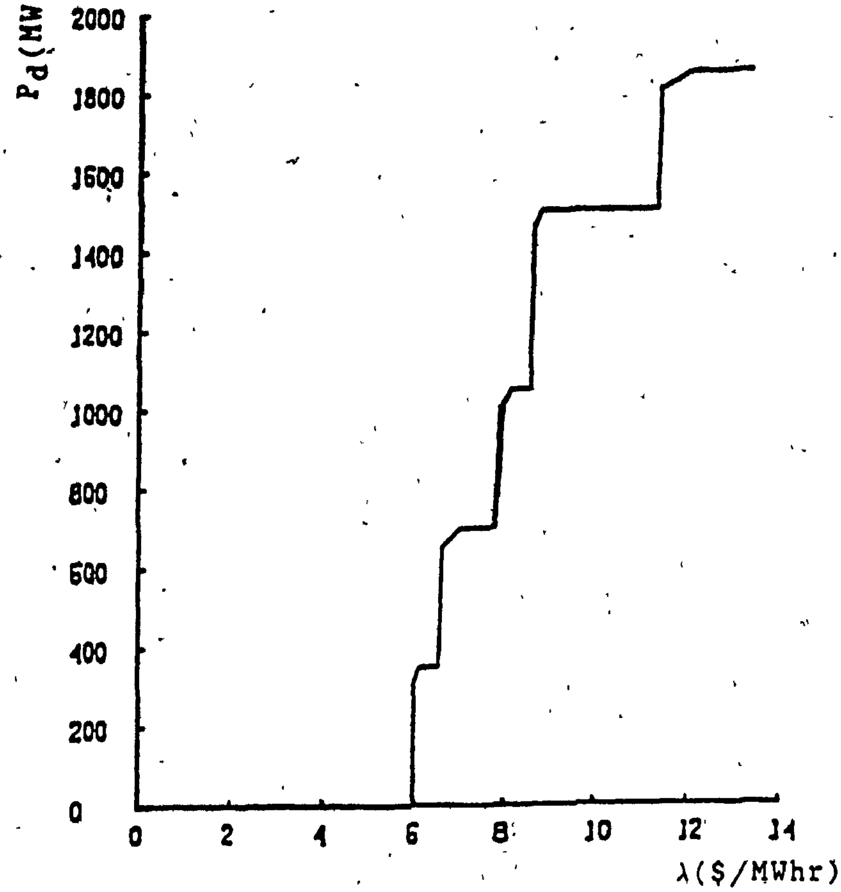

Fig 4.1 Typical load Vs λ at constant α

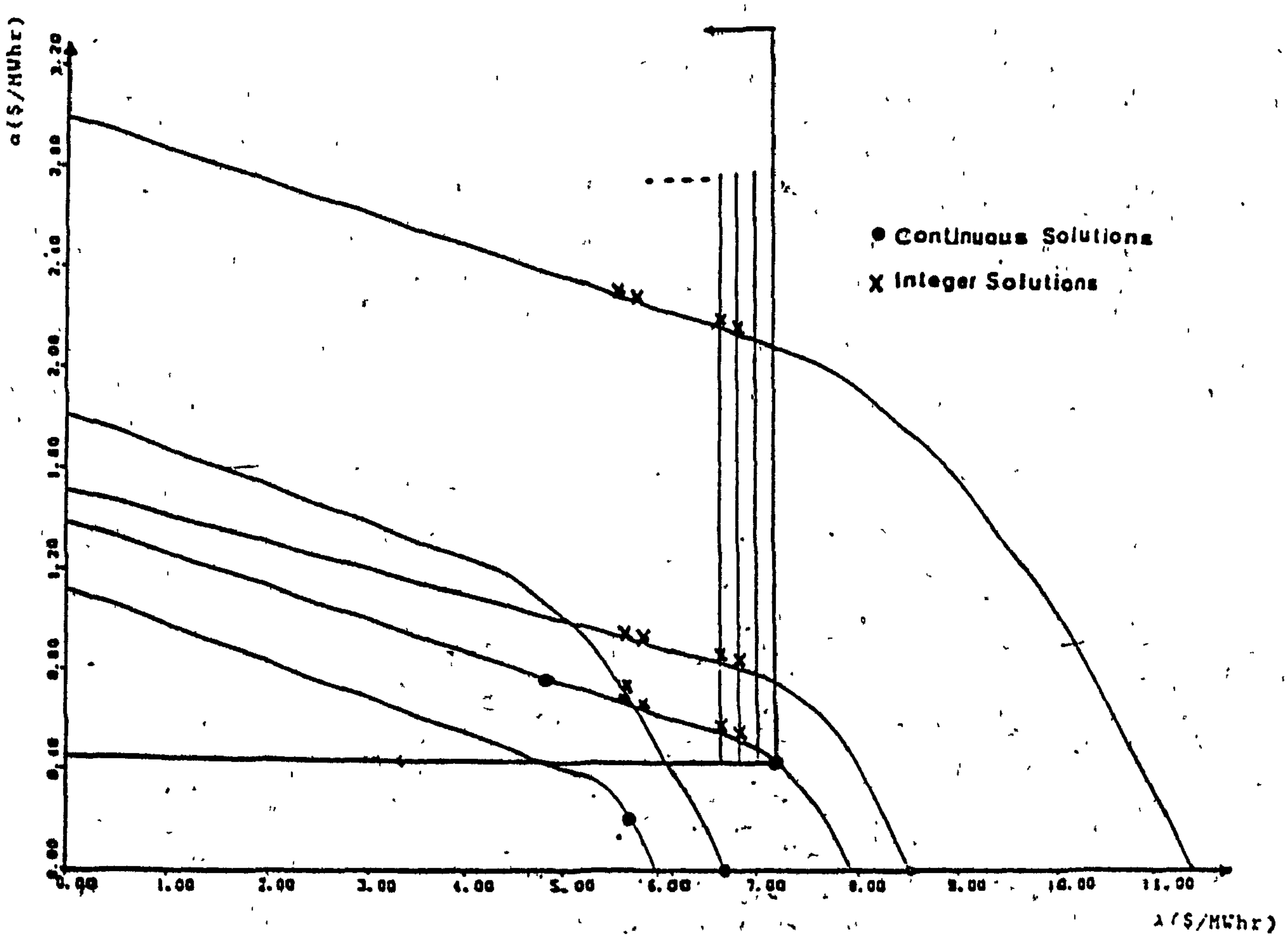

Fig. 4.2 Generating feasible solutions

other hand the computed reserve R is less than the specified reserve R^{Min} for $\alpha = 0.$, a binary search on the α-axis is performed to satisfy the reserve constraint; and as in the first case either of the cited situations could happen.

However from results of the simulations, the case where a solution lies on the intersection of two switching curves could also arise, to get the lower bound in this case the constraints (3.8) and (3.3) on the load and reserve margin respectively must be used as follows to compute the required two switching non-integer variables u_1 and u_2 :

$$u_1 P_1(\lambda^0) + u_2 P_2(\lambda^0) + \sum_{i \in Non} u_i P(\lambda^0) = P_d \tag{4.4}$$

$$u_1 P_1^{Max} + u_2 P_2^{Max} + \sum_{i \in Non} u_i P_i^{Max} = P_d + R^{Min} \tag{4.5}$$

Solving the above linear system we get u_1 and u_2 such that the load balance equation is satisfied and the computed reserve R is equal to R^{Min}. Such a solution is found whenever the binary search on the α-axis converges in α, but not in R within a specified number of iterations. If we are not interested in computing the true lower bound, as above, we could use the solution having the nearest reserve R to R^{Min} ($R < R^{Min}$) as the lower bound in the branch-and-bound technique. This will reduce slightly the computing time of the algorithm, because in this case to compute the lower bound only one non-integer u_i need to be found. The case where the binary search converges to the intersection of more than two switching curves is very unlikely and was not observed.

4.4 Generating feasible solutions:

In the algorithm developed, the method used to generate feasible solutions consists of evaluating the integer solutions starting from the continuous solution point. We know that in order to increase the reserve R, we should decrease the system incremental cost, however to satisfy the constant load requirement we must must bring on new units to compensate this decrease in λ, which could be achieved by increasing α. Therefore once a continuous solution is found, to generate a feasible solution with a greater reserve, we move to the left of the continuous solution by small increments $\Delta\lambda$ and evaluate all the integer solutions above the line α = constant coresponding to the continuous solution. See fig 4.2. A feasible solution found as above with the least reserve R will be a candidate for the optimum; specifically, it will be used as an upper bound in the branch-and-bound technique. A great improvement would be achieved if an analytic expression could be obtained to generate the trajectory of constant load P_d with increasing reserve R. The feasible solutions in this case would be readily available, hence suppressing the need of an iterative scheme such as the one used in this algorithm which takes an important amount of computer time.

In summary, whenever a feasible solution is found with a computed reserve R different from the specified reserve R^{Min}, the solution is not guaranted to be optimal and branch-and-bound techniques should be used to find better solutions if these exist.

4.5 The branch-and-bound method:

Because the conditions for optimum unit commitment derived earlier are only sufficient (Everett's theorem), a feasible solution generated by the above method is optimal only if its corresponding reserve R is equal to the specified reserve R^{Min}, otherwise it is possible that a better solution exists which could be found using a branch-and-bound technique.

In chapter 2 the principles of this technique have been explained and here we shall apply them to solve our specific problem. Generally, the specified reserve margin, R^{Min}, is found to be such that :

$$R_i < R^{Min} < R_{i+1} \qquad (4.6)$$

Where R_i and R_{i+1} correspond to two successive feasible reserve margins found by the proposed method. Instead of using the upper bound R_{i+1} as the approximate solution to our problem a better solution could be found such that :

$$R_i < R^{Min} \le R^s < R_{i+1} . \qquad (4.7)$$

Where R^s represents the computed reserve corresponding to the feasible solution found by using the branch-and-bound algorithm. The cost associated with such a solution if it exits will satisfy:

$$C(R_i) < C(R^{Min}) \le C(R^s) < C(R_{i+1}) \qquad (4.8)$$

Figure 4.3 illustrates the above relation in the cost Vs reserve plane. Note that the solid curve corresponds to the minimum cost with continuous switching variables u_i. The problem is then: Given R_i, R_{i+1}, R^{Min} and their associated costs, we will seek a solution satisfying relations (4.7) and (4.8) . We know that for a given set A of available units the solution found to the continuous problem is always a lower bound to the integer feasible problem. If we define any subset A' of A, the solution to the same continuous problem with A' as the set of available units will have a cost such that :

$$C(A) \le C(A') \qquad (4.9)$$

Where C(A) and C(A') represent the costs associated with the optimal solutions to the continuous problem with A and A' as the set of available units respectively. Thus each time a new subset is formed by removing units from the initial set a better lower bound is found and the branch-and-bound algorithm could perform as follows:

1) set the upper bound C_{high} = large number and the lower bound C_{low} = 0 ; with A (the initial set of units, or a subset of it) as the set of available units check if :

$$\sum_{i \in A} P_i^{Max} \geq P_d + R^{Min}$$

If yes, go to 2, otherwise the branch is infeasible and go to 4.

2) Compute C(A). if C(A) < C_{high} go to 4 otherwise find the corresponding closest integer feasible solution with the cost $C_{int}(A)$ (see section 4.4) and check if:

$$C_{int}(A) < C_{high}$$

If yes, set C_{high} = $C_{int}(A)$ and go to 3, else, the branch is stopped and go to 4.

3) Check if

$$C_{high} - C(A) \leq \epsilon$$

If yes, the branch is stopped and C_{high} will be a candidate for the optimum, else go to 4.

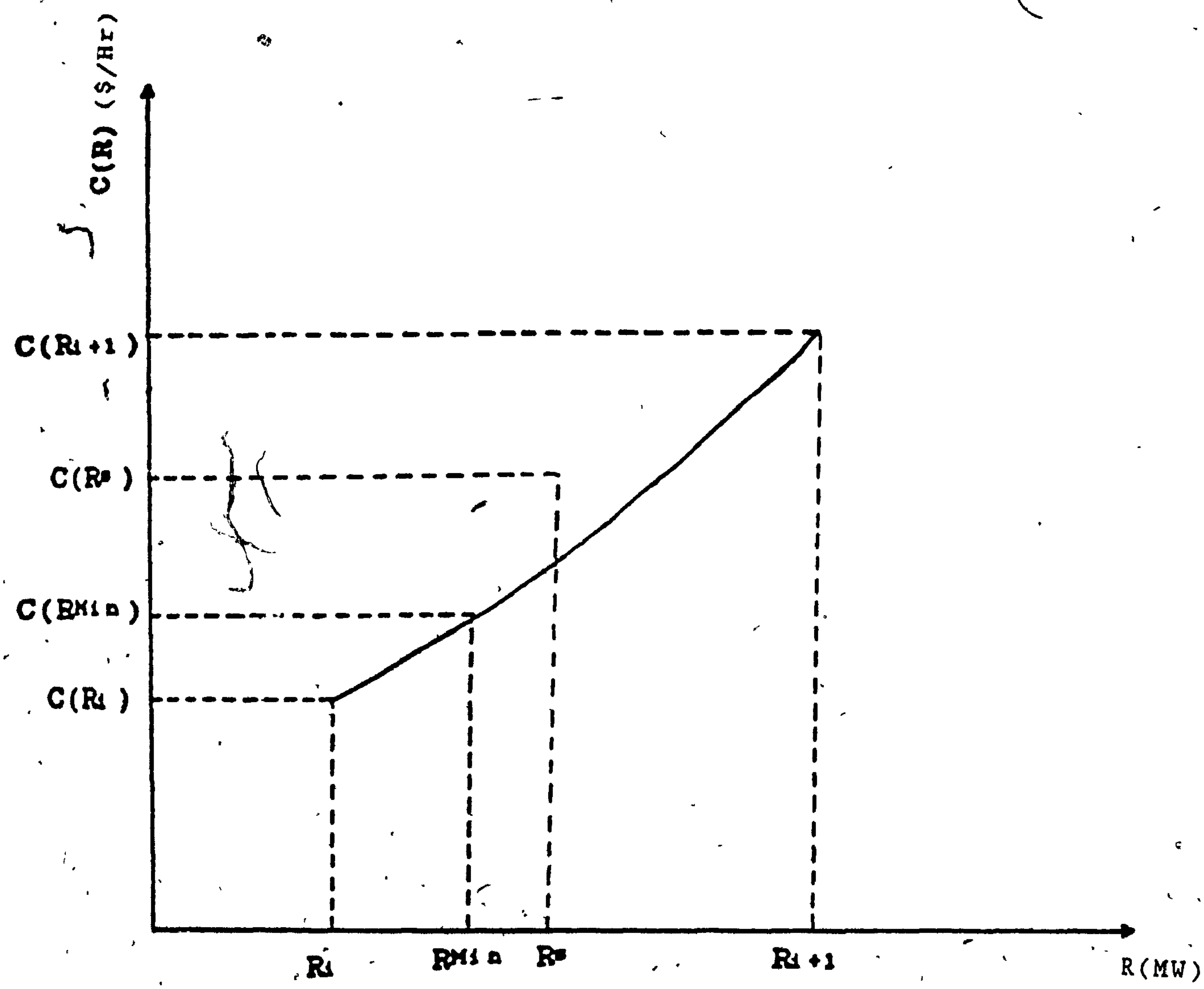

Fig. 4.3 Cost vs. reserve margin for P_d = constant

4) Start a new branch or sub-branch by forming a new subset of A and go to 1.

The branching is continued until a feasible solution with a cost no greater than the bound for any subset is found.

An exhaustive tree for a system of 3 units is shown in fig 4.4 ,and the flowchart of the branch-and-bound algorithm is illustrated in figure 4.5.

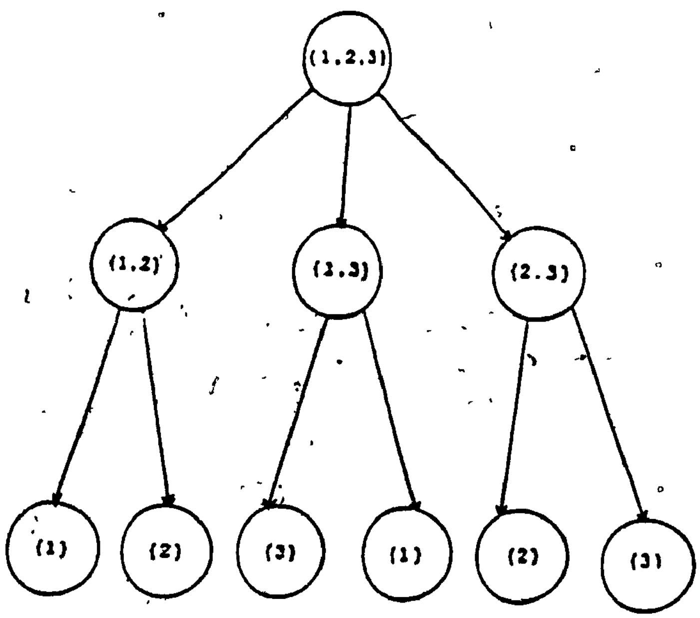

Fig 4.4 A branch-and-bound tree example

4.6 The general algorithm:

All the different steps of the algorithm discussed above will be summarized in the following and will be referred to as the general algorithm.

Set 1) $\alpha_1 = 0.$, $\alpha_2 = \alpha^{Max}$, $\alpha_3 = 0.$

2) perform a binary search on the λ-axis for $\alpha_3 = 0$.

If $abs(\lambda_{i+1} - \lambda_i) < eps1)$ go to 3.

Else continue.

3) Check if abs(P_dGiven - P_dCalculed) < eps2)

If yes go to 5. Else identify and compute a non-integer value of u_i given by equation 4.2 and go to 5.

4) If for $\alpha_3 = 0$. R calculated > R^{Min} go to 10.

Else go to 5.

5) Perform a binary search on the α-axis repeating steps 2) and 3) to satistisfy the load balance.

Chech if $(abs(\alpha_{i+1} - \alpha_i) < eps3)$

If yes go to 6. Else continue.

6) Check if abs(R calculated - R^{Min}) < eps4

If yes go to 10. Else go to 7.

7) Check if 1 or 2 u_i's are not set to their limits and compute them in order to satistisfy both the load and the reserve costraints.

8) We have a continuous solution. Compute its cost

(lower bound). Generate a feasible solution (section 4.4) and compute its cost (upper bound).

9) Call the branch-and-bound routine to find better feasible solutions if any. Go to 11.

10) We have an optimal feasible solution. Calculate its cost and go to 11.

11) Print necessary results and stop.

4.7 Test results and discussions :

A system consisting of 10 units from reference [14] table 4.1 (page 86), is simulated using the proposed algorithm and the results obtained are shown in tables 4.2 and 4.3. These results are obtained by using the general algorithm ,i.e with the branch-and-bound technique, whereas table 4.4 shows the results of the simulation of the same system without using the branch-and-bound algorithm. We could see from these tables that only an improvement of about 0.085% in the total cost is achieved, which suggest that the use of the branch-and-bound technique is probably not necessary and could be dropped from the general algorithm. Remark that the branch-and-bound algorithm is performed only when the initial solution found is not feasible or when the bounds are not within a specified tolerance, which in almost all hours is not the case. Even when the branch-and-bound algorithm is used the branches are

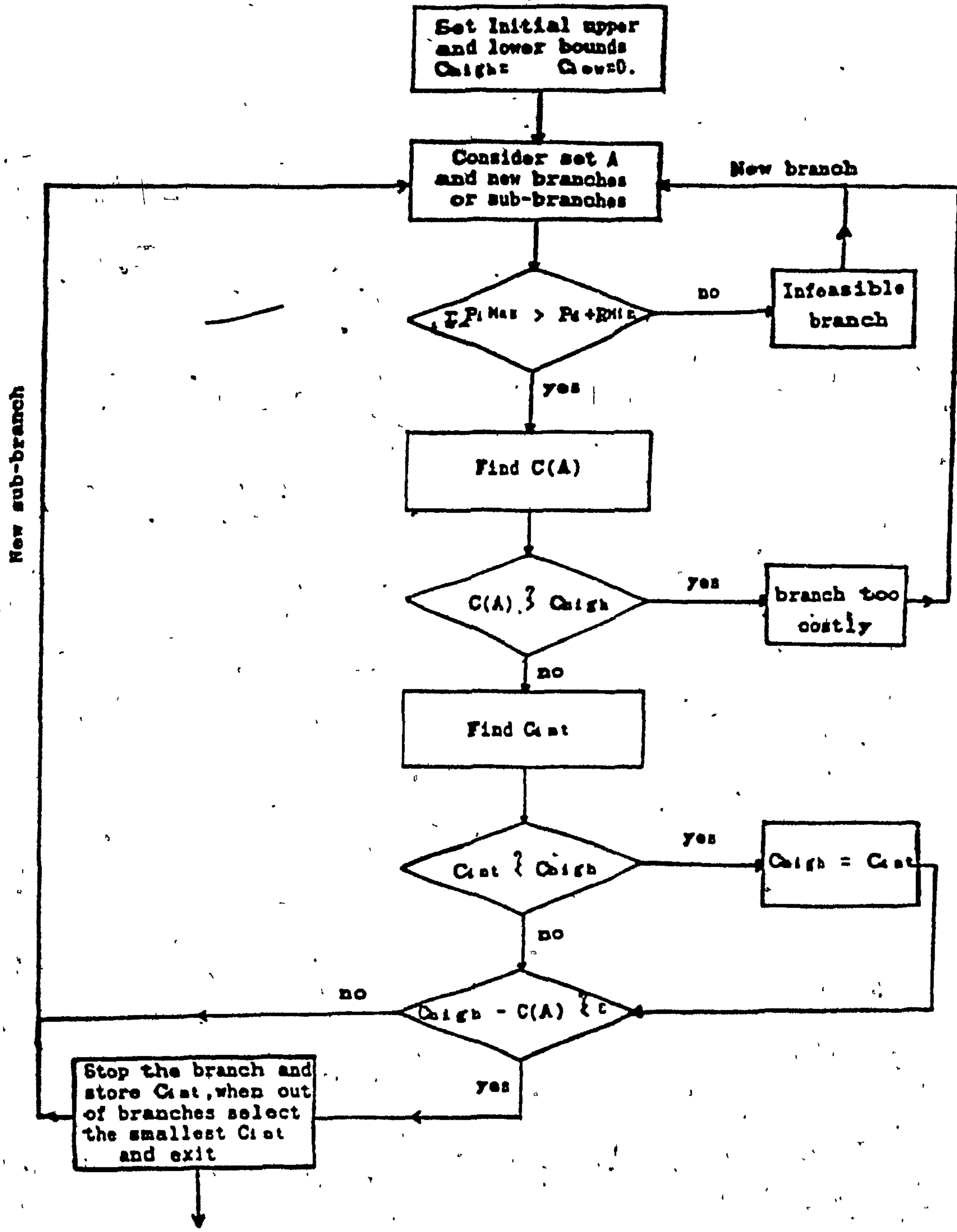

Fig 4.5 Flowchart of the Branch-and-Bound Algorithm

fathomed very quickly, either because the costs are going higher or the available reserve is too small; a typical tree for one hour is shown in fig 4.7 (page 86). The incremental costs of the units are shown in figure 4.6 (page 75), whereas figure 4.8 (page 82) shows the discrete load demand curve for the given data. Figure 4.9 (page 76) represents the switching curves of the units and figures 4.10 and 4.11 (page 77) give the trajectories of the load and the reserve Vs λ at constant α respectively for $\alpha = 0.$ and $\alpha = 0.4$. The results of table 4.2 (page 79) giving the status of the 10 units for the whole time interval of 24 hours, is schematically represented in figure 4.12 (page 78) where it could be seen that the peaking units are only units 2,3,6 and 8, while the remaining are base load units. It could be seen from figure 4.12 that all these units are cycling very quickly. Practically this is not always possible due to the minimum down-time and up-time time constraints which are not taken into account in this basic problem.

For this small system, it took approximately 38s of computer time on the Micro Vax II to solve the problem when the complete algorithm is used and only about 16s when the branch-and-bound algorithm is dropped. It could be seen from table 4.3 (page 80) that the load balance equation is satisfied within less than 0.3% but the computed reserve is always very different from the specified reserve margin taken here as 25% of the load. It was also observed from the

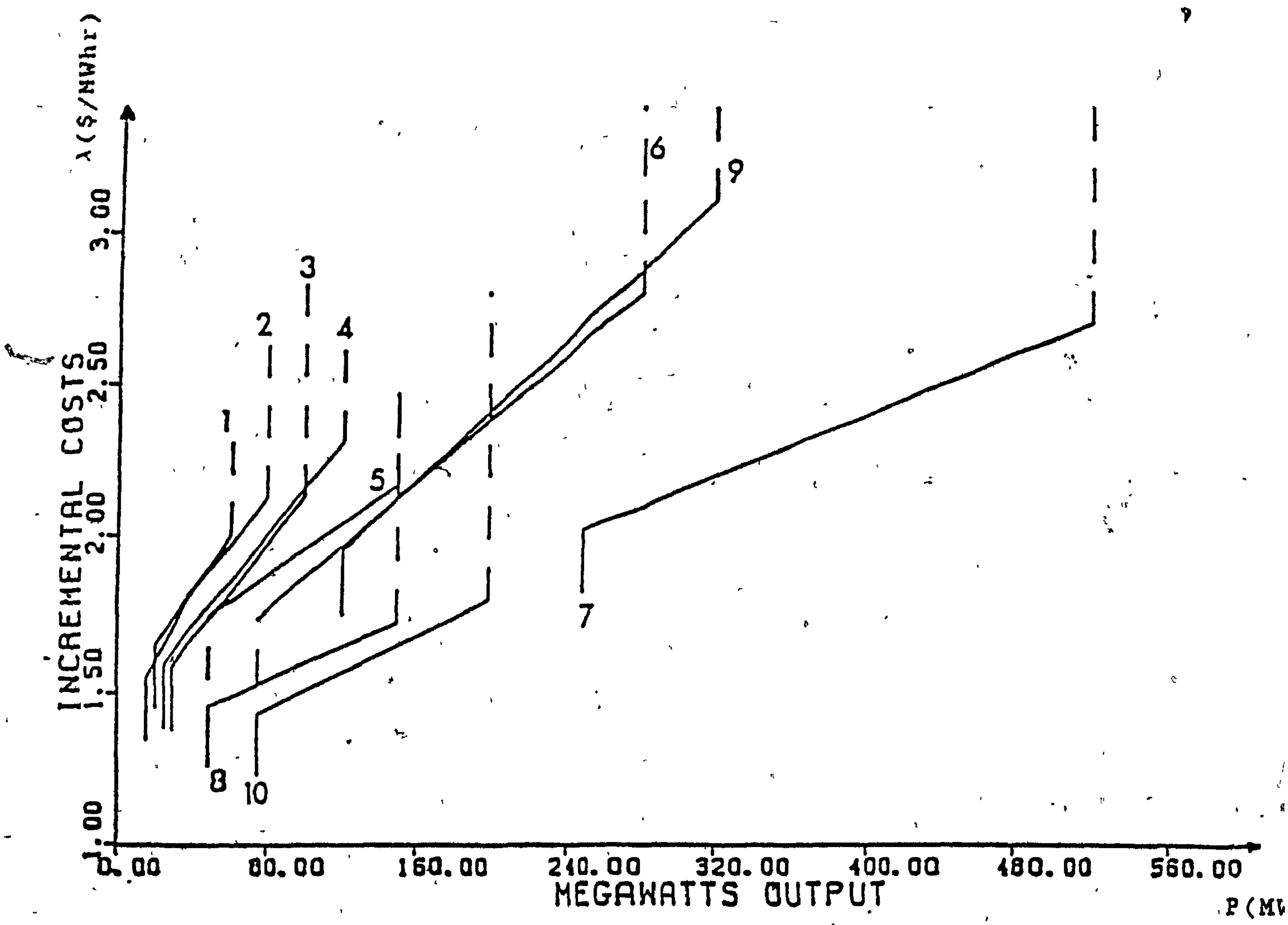

Fig. 4.6 Incremental costs of the 10 units example

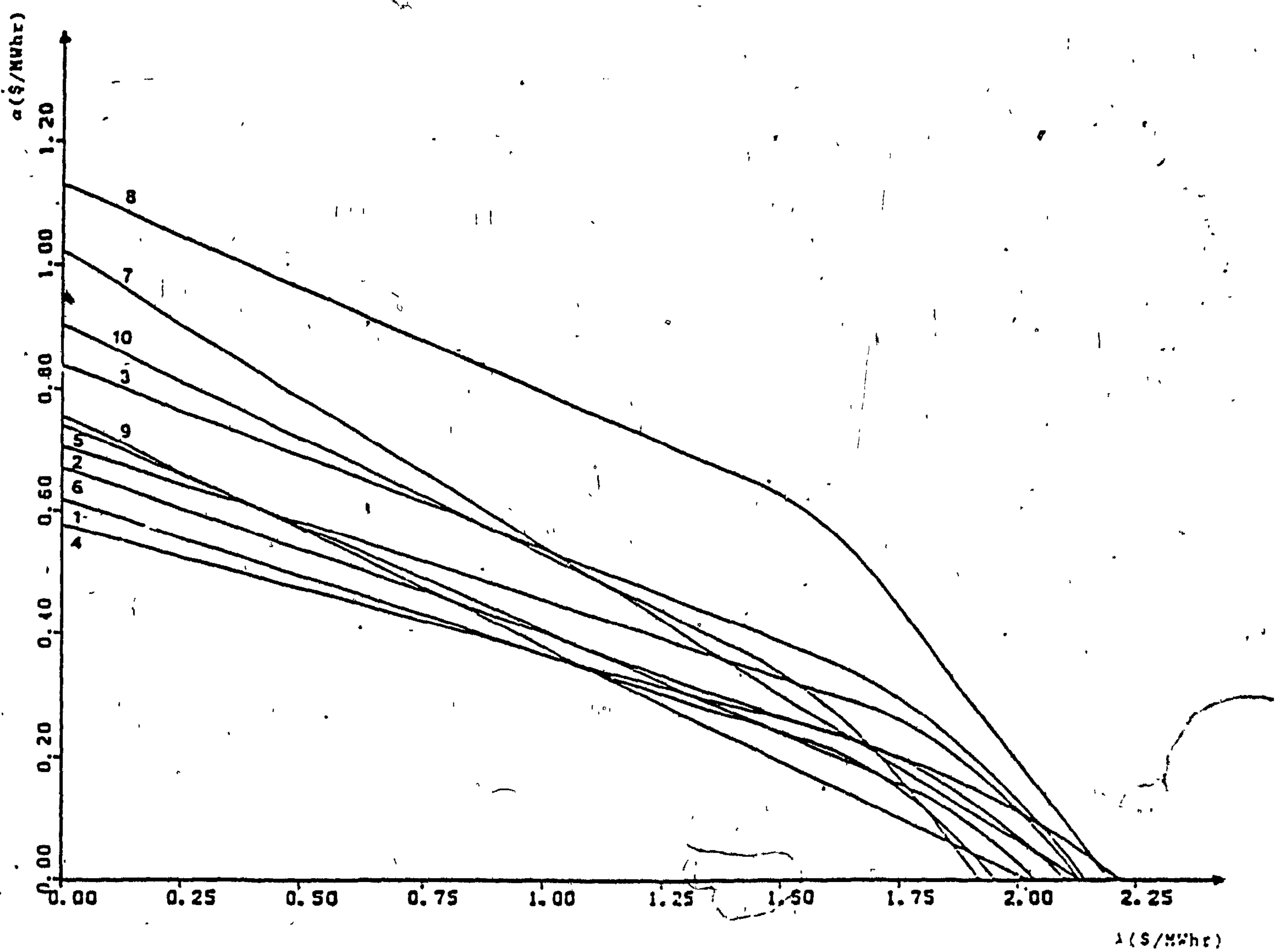

Fig 4.9 Switching Curves of the 10 Units Example

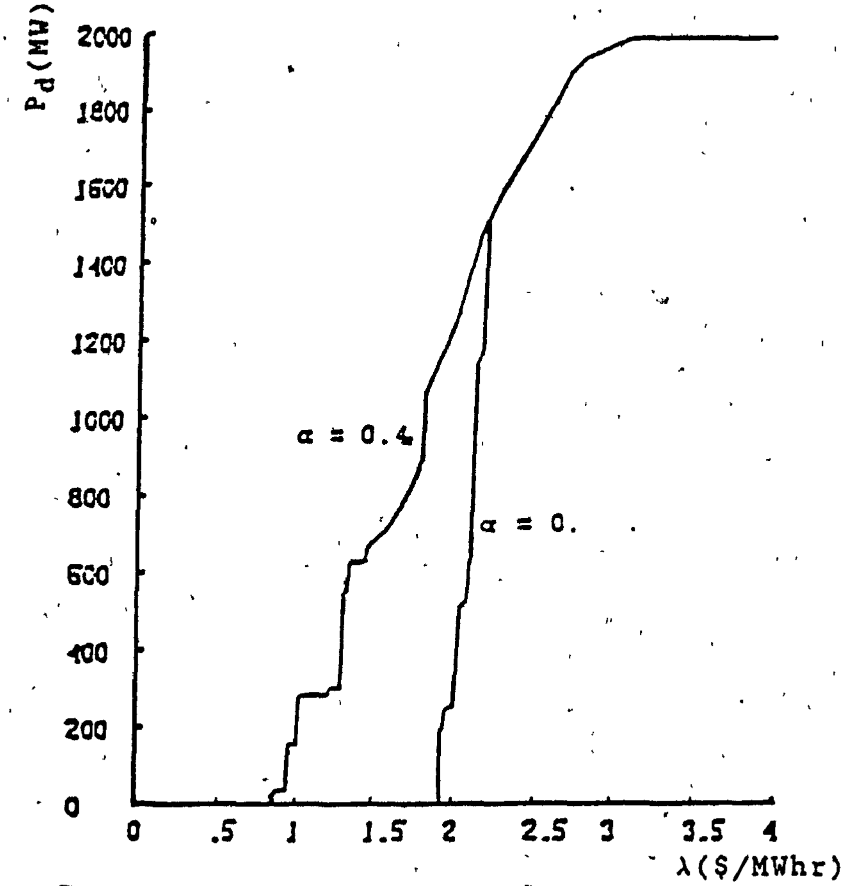

Fig 4.10 Load Vs λ at constant α for the 10 Units

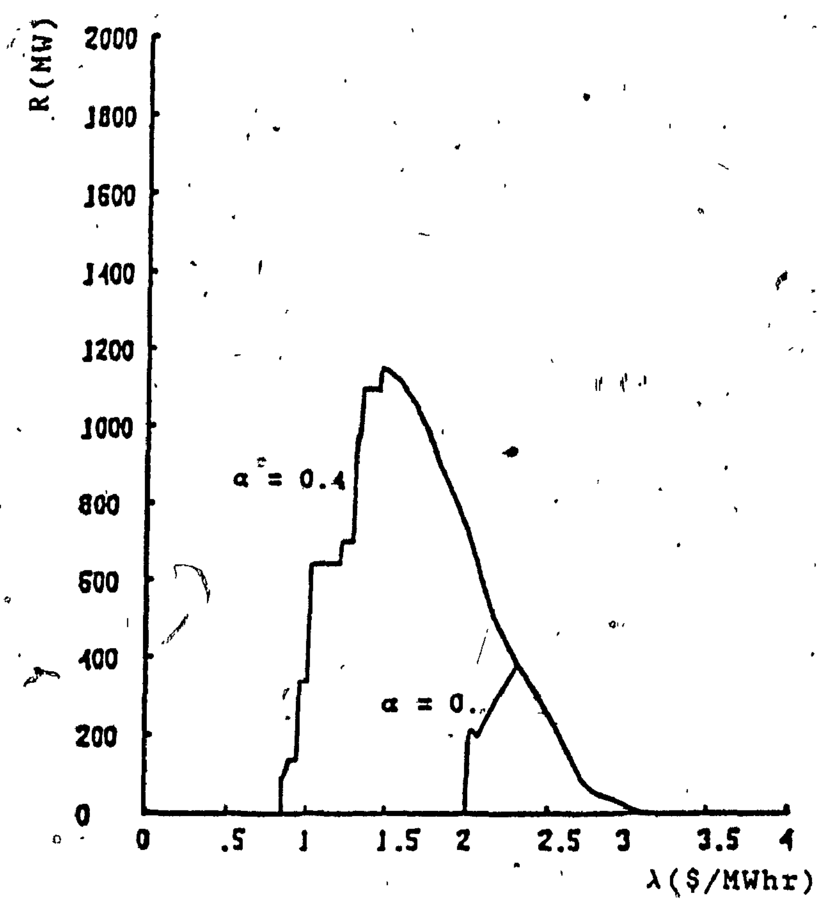

Fig 4.11 Reserve Vs λ at constant α for the 10 Units

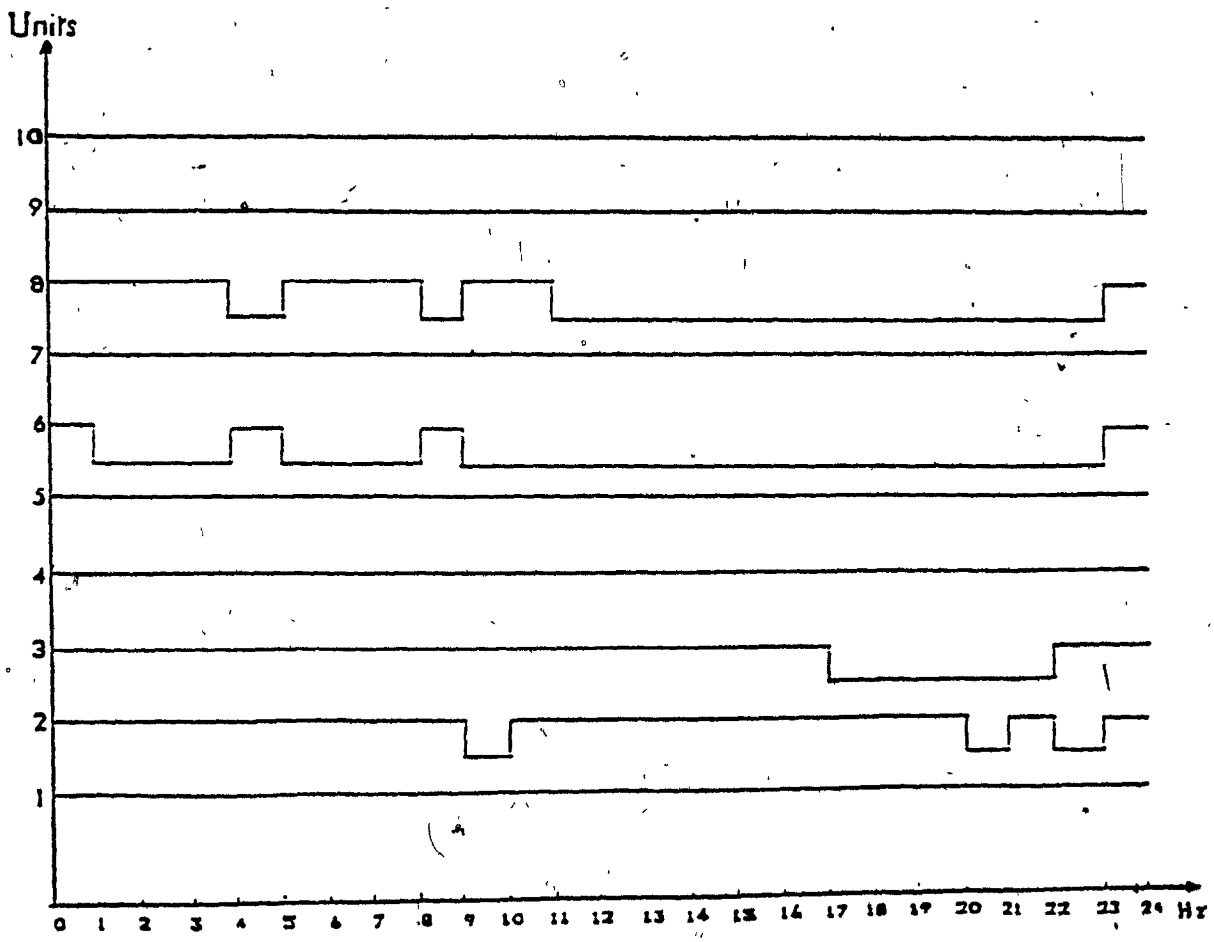

Fig 4.12 Status of the 10 Units for the 24 Hours Period

HOUR	1	2	3	4	5	6	7	8	9	10	INC. COST	COST	CUMULATIVE COST
1	1	1	1	1	1	1	1	1	1	1	2.170976	0.30496834e+04	0.3049683360e+04
2	1	1	1	1	1	0	1	1	1	1	2.269679	0.28562587e+04	0.5905942090e+04
3	1	1	1	1	1	0	1	1	1	1	2.169616	0.26980465e+04	0.8603988601e+04
4	1	1	1	1	1	0	1	1	1	1	2.153616	0.26657622e+04	0.1126975093e+05
5	1	1	1	1	1	1	1	0	1	1	2.136976	0.26376422e+04	0.1390739314e+05
6	1	1	1	1	1	0	1	1	1	1	2.187616	0.27286744e+04	0.1663606755e+05
7	1	1	1	1	1	0	1	1	1	1	2.269679	0.28562537e+04	0.1949232628e+05
8	1	1	1	1	1	0	1	1	1	1	2.187616	0.27286744e+04	0.2222100069e+05
9	1	1	1	1	1	1	1	0	1	1	2.136976	0.26376422e+04	0.2485864290e+05
10	1	0	1	1	1	0	1	1	1	1	2.198026	0.25742543e+04	0.2743289724e+05
11	1	1	1	1	1	0	1	1	1	1	2.075616	0.24802131e+04	0.2991311030e+05
12	1	1	1	1	1	0	1	0	1	1	2.212367	0.24370097e+04	0.3235012000e+05
13	1	1	1	1	1	0	1	0	1	1	2.173185	0.23756309e+04	0.3472575086e+05
14	1	1	1	1	1	0	1	0	1	1	2.156027	0.23409624e+04	0.3706671326e+05
15	1	1	1	1	1	0	1	0	1	1	2.140967	0.23107583e+04	0.3937747159e+05
16	1	1	1	1	1	0	1	0	1	1	2.114967	0.22484278e+04	0.4162589939e+05
17	1	1	1	1	1	0	1	0	1	1	2.090967	0.21885215e+04	0.4381442092e+05
18	1	1	0	1	1	0	1	0	1	1	2.154952	0.21244983e+04	0.4593891922e+05
19	1	1	0	1	1	0	1	0	1	1	2.111241	0.20314712e+04	0.4797039041e+05
20	1	1	0	1	1	0	1	0	1	1	2.097241	0.20002527e+04	0.4997064312e+05
21	1	0	0	1	1	0	1	0	1	1	2.161354	0.19670517e+04	0.5193769485e+05
22	1	1	0	1	1	0	1	0	1	1	2.138870	0.20922687e+04	0.5402996359e+05
23	1	0	1	1	1	0	1	0	1	1	2.182251	0.22205026e+04	0.5625046615e+05
24	1	1	1	1	1	1	1	1	1	1	2.170976	0.30496834e+04	0.5930014951e+05

Table 4.2 Unit commitment of the 10 units with the branch-and-bound routine

INC. COST = System incremental cost ($/MWhr)
COST = Hourly cost ($)

HOUR	1	2	3	4	5	6	7	8	9	10	LOAD	RSOL	R MIN
1	60.0	80.0	100.0	100.9	148.8	157.3	305.3	150.0	156.9	200.0	1459	520.8	291.8
2	60.0	80.0	100.0	113.8	150.0	0.	344.2	150.0	173.9	200.0	1372	328.0	274.4
3	60.0	80.0	100.0	100.7	148.5	0.	304.8	150.0	156.6	200.0	1299	399.3	259.8
4	60.0	80.0	100.0	98.6	144.7	0.	298.5	150.0	153.9	200.0	1285	414.3	257.0
5	60.0	80.0	100.0	96.5	140.8	150.8	292.0	0.	151.0	200.0	1271	559.0	254.2
6	60.0	80.0	100.0	103.1	150.0	0.	311.9	150.0	159.7	200.0	1314	385.3	262.8
7	60.0	80.0	100.0	113.8	150.0	0.	344.2	150.0	173.9	200.0	1372	328.0	274.4
8	60.0	80.0	100.0	103.1	150.0	0.	311.9	150.0	159.7	200.0	1314	385.3	262.8
9	60.0	80.0	100.0	96.5	140.8	150.8	292.0	0.	151.0	200.0	1271	559.0	254.2
10	60.0	0.	100.0	104.5	150.0	0.	316.0	150.0	161.5	200.0	1242	378.0	248.4
11	60.0	72.7	92.3	88.4	126.3	0.	267.8	150.0	140.4	200.0	1197	502.1	239.4
12	60.0	80.0	100.0	105.3	150.0	0.	321.6	0.	164.0	200.0	1182	368.0	236.4
13	60.0	80.0	100.0	101.2	149.3	0.	306.2	0.	157.2	200.0	1154	396.0	230.8
14	60.0	80.0	100.0	99.0	145.3	0.	299.5	0.	154.3	200.0	1138	412.0	227.6
15	60.0	80.0	100.0	97.0	141.7	0.	293.5	0.	151.7	200.0	1124	426.1	224.8
16	60.0	77.6	97.3	93.6	135.6	0.	283.3	0.	147.2	200.0	1095	455.4	219.0
17	60.0	74.6	94.3	90.4	129.9	0.	273.8	0.	143.0	200.0	1066	483.9	213.2
18	60.0	80.0	0.	98.8	145.0	0.	299.0	0.	154.1	200.0	1037	413.0	207.4
19	60.0	77.2	0.	93.1	134.7	0.	281.8	0.	146.5	200.0	993	456.6	198.6
20	60.0	75.4	0.	91.3	131.4	0.	276.3	0.	144.1	200.0	978	471.5	195.6
21	60.0	0.	0.	99.7	146.5	0.	301.6	0.	155.2	200.0	963	407.0	192.6
22	60.0	80.0	0.	96.7	141.2	0.	292.7	0.	151.3	200.0	1022	428.0	204.4
23	60.0	0.	100.0	102.4	150.0	0.	309.8	0.	158.8	200.0	1081	389.0	216.2
24	60.0	80.0	100.0	100.9	148.8	157.3	305.3	150.0	156.9	200.0	1459	520.8	291.8

Table 4.3 Outputs of the units (MW)
RSOL = computed reserve (MW)
R^{MIN} = specified reserve margin (MW)

HOUR	1	2	3	4	5	6	7	8	9	10	INC. COST	COST	CUMULATIVE COST
1	1	1	1	1	1	1	1	1	1	1	2.170976	0.30496834e+04	0.3049683360e+04
2	1	1	1	1	1	1	1	1	1	1	2.100976	0.28633109e+04	0.5912994213e+04
3	1	1	1	1	1	1	1	0	1	1	2.163616	0.27020871e+04	0.8615081257e+04
4	1	1	1	1	1	1	1	0	1	1	2.149616	0.26681198e+04	0.1128320115e+05
5	1	1	1	1	1	1	1	0	1	1	2.137616	0.26391804e+04	0.1392238159e+05
6	1	1	1	1	1	0	1	1	1	1	2.187616	0.27286744e+04	0.1665105600e+05
7	1	1	1	1	1	1	1	1	1	1	2.100976	0.28633109e+04	0.1951436685e+05
8	1	1	1	1	1	0	1	1	1	1	2.187616	0.27286744e+04	0.2224304126e+05
9	1	1	1	1	1	1	1	0	1	1	2.137616	0.26391804e+04	0.2488222170e+05
10	1	1	1	1	1	1	1	0	1	1	2.115616	0.25762084e+04	0.2745843011e+05
11	1	1	1	1	1	1	1	0	1	1	2.083616	0.24835395e+04	0.2994201957e+05
12	1	1	1	1	1	1	1	0	1	1	2.071616	0.24492212e+04	0.3239124084e+05
13	1	1	1	1	1	0	1	0	1	1	2.173185	0.23756309e+04	0.3476687170e+05
14	1	1	1	1	1	0	1	0	1	1	2.156027	0.23409624e+04	0.3710783410e+05
15	1	1	1	1	1	0	1	0	1	1	2.140967	0.23107583e+04	0.3941859243e+05
16	1	1	1	1	1	0	1	0	1	1	2.114967	0.22484278e+04	0.4166702022e+05
17	1	1	1	1	1	0	1	0	1	1	2.090967	0.21885215e+04	0.4385554176e+05
18	1	1	1	1	1	0	1	0	1	1	2.066967	0.21292989e+04	0.4598484070e+05
19	1	1	0	1	1	0	1	0	1	1	2.111241	0.20314712e+04	0.4801631189e+05
20	1	1	0	1	1	0	1	0	1	1	2.097241	0.20002527e+04	0.5001656460e+05
21	1	1	0	1	1	0	1	0	1	1	2.082241	0.19692419e+04	0.5198580654e+05
22	1	1	0	1	1	0	1	0	1	1	2.138870	0.20922687e+04	0.5407807528e+05
23	1	1	1	1	1	0	1	0	1	1	2.104967	0.22233838e+04	0.5630145904e+05
24	1	1	1	1	1	1	1	1	1	1	2.170976	0.30496834e+04	0.5935114240e+05

Table 4.4 Unit commitment of the 10 units system without the branch-and-bound routine

results of the simulations that the branch-and-bound algorithm does not improve substantially the initial costs and could be practically suppressed, in which case the computing time will be greatly reduced as it was pointed above. The unit characteristics of the second simulated system consisting of 100 units are given in table 4.5 (page 90,91). In this case the simulation is carried out without using the branch-and-bound algorithm, and tables 4.6 (page 83) and 4.7 (page 84) show the results obtained. For the one day schedule the bounds obtained are within 0.56% and the computer time was only about 4.15 mn on the Micro Vax II.

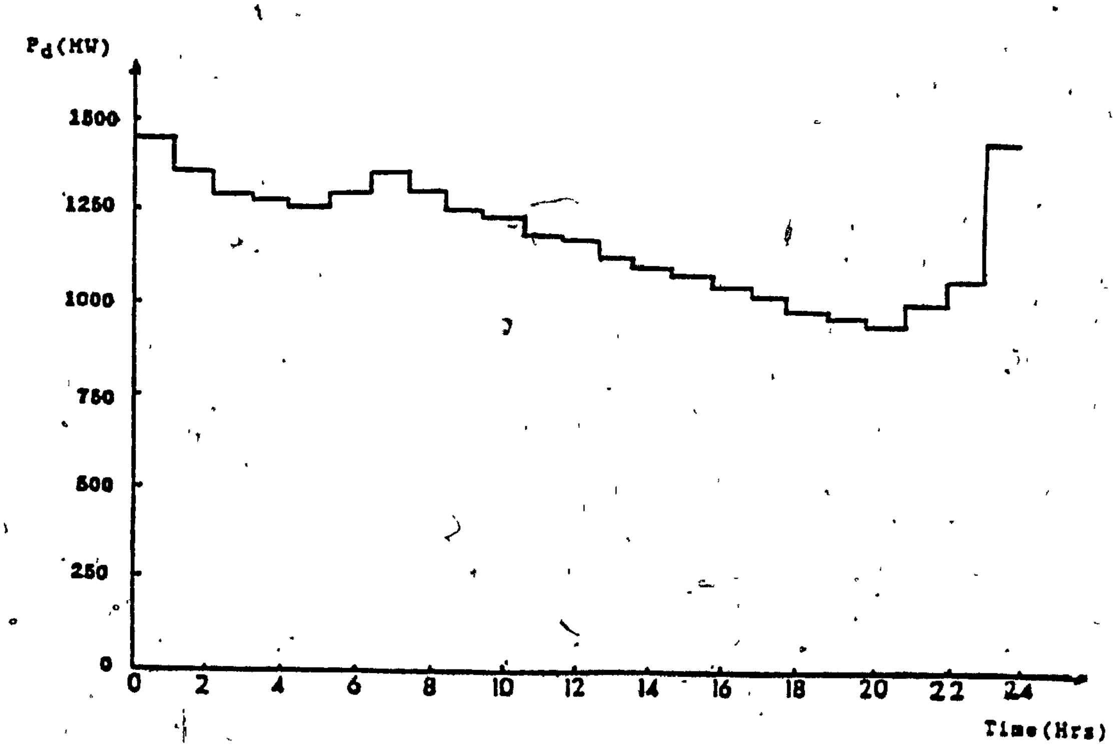

Fig. 4.8 discretized load demand curve

HOUR	UNIT STATUS
1	1111111100111111111111001000000000000000000000000011000000011111111111100000011111111111100001111111111111111
2	1111111100111111111111100000000000000000000000000000000001111111111000000011111111111000011111111111111111111
3	1111111100111111111111001000000000000000000000000000000001111111110000000111111111110000101111111111111111
4	1111111100111111111111001000000000000000000000000000000001111111110000000111111111110000001111111111111111
5	111111110011111111111100000000000000000000000000000000000111111110000000111111111110000101111111111111111
6	1111111100111111111111001000000000000000000000110000000001111111110000000111111111110000111111111111111111
7	1111111100111111111111111000010000000000000011100000001111111111100001111111111111110000111111111111111111
8	1111111111111111111111111111010110000001110111111000111111111111000011111111111111100111111111111111111
9	111111111111111111111111111111110001011111111111111111111111111111001111111111111111011111111111111111
10	11111111111111111111111111111111101101111111111111111111111111111100111111111111111111111111111111111
11	1111111111111111111111111111111110111111111111111111111111111111100111111111111111111111111111111111
12	11111111111111111111111111111111001101111111111111111111111111111100111111111111111111111111111111111
13	11111111111111111111111111111111000101111111111111111111111111111001111111111111111011111111111111111
14	11111111111111111111111111111111100110111111111111111111111111111001111111111111111111111111111111111
15	11111111111111111111111111111111100110111111111111111111111111111001111111111111111111111111111111111
16	11111111111111111111111111111111100010111111111111101111111111111001111111111111111011111111111111111
17	11111111111111111111111111111111100110111111111111111111111111111001111111111111111011111111111111111
18	111111111111111111111111111111111000001111111111111011111111111111001111111111111100111111111111111111
19	1111111111111111111111111111111011100000111111111111001111111111111001111111111111100111111111111111111
20	11111111111111111111111111111111100010111111111111101111111111111001111111111111111011111111111111111
21	11111111111111111111111111111111100010111111111111101111111111111001111111111111111011111111111111111
22	111111111111111111111111111111111000001111111111111001111111111111001111111111111100111111111111111111
23	111111111011111111111111111111100010000000101001111100011111111111110000111111111111110011111111111111111
24	1111111100111111111111111111110001000000000000011100000001111111111100001111111111111100001111111111111111111

Table 4.6 Unit commitment of the 100 units

HOUR	GIVEN LOAD	COMPUTED LOAD	RESERVE MARGIN	COMPUTED RESERVE
1	17118	17116.3	2567.70	2683.69
2	16235	16232.5	2435.25	2567.50
3	15664	15662.5	2349.60	2937.52
4	15497	15497.5	2324.55	2702.48
5	15711	15714.3	2356.65	2685.69
6	16549	16548.1	2482.35	2851.93
7	18200	18203.2	2730.00	3196.79
8	21051	21049.3	3157.65	3250.66
9	23209	23206.4	3481.35	3493.61
10	23793	23793.5	3568.95	3706.53
11	24021	24022.0	3603.15	3677.97
12	23624	23626.3	3543.60	3673.73
13	23214	23214.0	3482.10	3486.04
14	23665	23668.2	3549.75	3631.76
15	23484	23486.8	3522.60	3813.24
16	22893	22893.7	3433.95	3606.30
17	22601	22599.4	3390.15	4300.61
18	22324	22324.0	3348.60	3576.04
19	22027	22026.4	3304.05	3473.59
20	22545	22548.1	3381.75	3951.87
21	23023	23021.3	3453.45	3478.68
22	22091	22093.4	3313.65	3606.62
23	20335	20333.7	3050.25	3116.27
24	18330	18328.6	2749.50	3271.39

Integer cost = 0.17469101e+07
Continuous cost = 0.17370114e+07
Relative error (%) = 0.566645

Table 4.7 Computed load, reserve and costs for the 100 unit sytem over 24 hours

I	HOUR	1	2	3	4	5	6	7	8	9	10	INC. COST	COST	CUMULATIVE COST	I
I	1	1	1	1	1	1	0	1	1	1	1	2.412200	0.30594e-04	0.30594e+04	I
I	2	1	1	1	1	1	0	1	1	1	1	2.269616	0.28562e-04	0.59156e+04	I
I	3	1	1	1	1	1	0	1	1	1	1	2.169616	0.26980e-04	0.86136e+04	I
I	4	1	1	1	1	1	0	1	1	1	1	2.153616	0.26655e-04	0.112791e-05	I
I	5	1	1	1	1	1	0	1	1	1	1	2.137800	0.26340e+04	0.139131e-05	I
I	6	1	1	1	1	1	0	1	1	1	1	2.137616	0.27286e+04	0.664170e+05	I
I	7	1	1	1	1	1	0	1	1	1	1	2.229679	0.28562e-04	0.194979e-05	I
I	8	1	1	1	1	1	0	1	1	1	1	2.187615	0.27286e+04	0.222265e-05	I
I	9	1	1	1	1	1	0	1	1	1	1	2.137800	0.26340e+04	0.248605e-05	I
I	10	1	1	1	1	1	0	1	1	1	1	2.112799	0.25725e+04	0.274331e-05	I
I	11	1	1	1	1	1	0	1	1	1	1	2.075616	0.24802e-04	0.299133e-05	I
I	12	1	1	1	1	1	0	1	0	1	1	2.212367	0.24370e-04	0.323503e-05	I
I	13	1	1	1	1	1	0	1	0	1	1	2.173185	0.23756e-04	0.347259e-05	I
I	14	1	1	1	1	1	0	1	0	1	1	2.156027	0.23409e-04	0.370668e-05	I
I	15	1	1	1	1	1	0	1	0	1	1	2.140967	0.23107e-04	0.393775e-05	I
I	16	1	1	1	1	1	0	1	0	1	1	2.114967	0.22484e-04	0.416259e-05	I
I	17	1	1	1	1	1	0	1	0	1	1	2.090967	0.21885e+04	0.438144e-05	I
I	18	1	1	0	1	1	0	1	0	1	1	2.154952	0.21244e+04	0.459388e-05	I
I	19	1	1	0	1	1	0	1	0	1	1	2.111241	0.20314e+04	0.479702e-05	I
I	20	1	1	0	1	1	0	1	0	1	1	2.097241	0.20002e-04	0.499704e-05	I
I	21	1	1	0	1	1	0	1	0	1	1	2.082600	0.19679e+04	0.519382e-05	I
I	22	1	1	0	1	1	0	1	0	1	1	2.138870	0.20922e+04	0.540304e-05	I
I	23	1	1	1	1	1	0	1	0	1	1	2.103499	0.22197e+04	0.562501e-05	I
I	24	1	1	1	1	1	0	1	1	1	1	2.412200	0.30594e+04	0.593095e-05	I

Table 4.9 Dynamic unit commitment of the 10 units system

Unit	P_i^{Min}	P_i^{Max}	c_{0i}	a_i	b_i
1	15	60	15	1 4	0 0102
2	20	80	25	1 5	0 00792
3	30	100	40	1 35	0 00786
4	25	120	32	1.4	0 00764
5	50	150	29	1 54	0 00424
6	75	280	72	1 35	0 00522
7	250	520	105	1 3954	0 00254
8	50	150	100	1 3285	0 00270
9	120	320	49	1 2643	0 00578
10	75	200	82	1 2136	0 00296

Table 4 1 Characteristics of the 10 units system

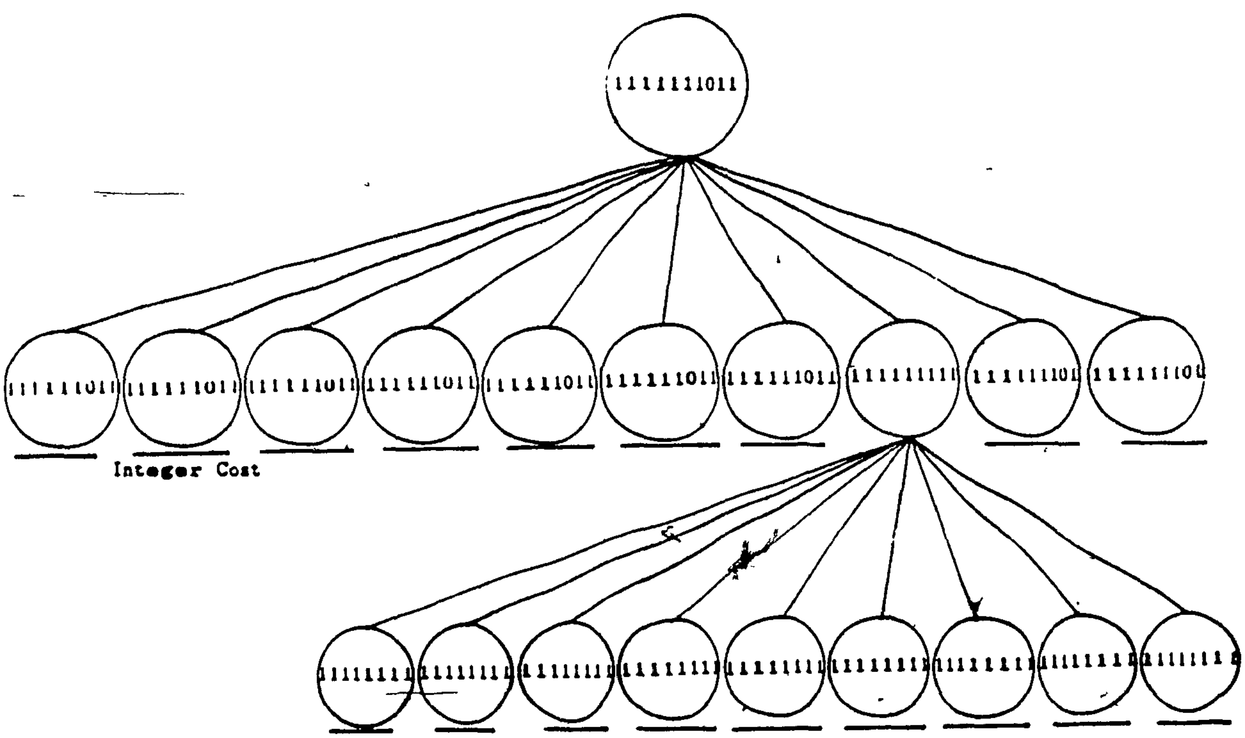

Fig. 4.7 Typical branch-and bound tree

A first approach which has been tried to solve the unit commitment problem by taking into account the minimum up and down times, consisted of incorporating the start-up costs into the fixed costs of the units which were found to cycle very quickly However this method was shown to be unstable and consequently does not give satisfactory results regarding the cycling of the units

The second approach which gave satisfactory results, consisted of solving an economic dispatch problem in each hour where the cycling units do not respect the minimum up and down times constraints The units are committed or decommitted depending on their previous states, so that feasible solutions are found and a physical cycling is obtained Table 4.8 shows the results obtained for the 10 unit system, when this second approach is used It could be seen that the total computed cost for the dynamic solution is not very different (<0 1%) from the one obtained by the general algorithm in the static case The minimum up and down times are chosen for all units to be greater or equal to 5 hours

<u>Summary</u>

In this chapter the application of the switching curve concept to solve practical power systems was exposed The implemented algorithm using simple numerical techniques and the branch-and-bound method is relatively simple and fast and could handle any number of units It was applied here to solve two different power systems and encouraging results were reported

Unit	P_i^{Min}	P_i^{Max}	c_{0i}	a_i	b_i
1	12	50	19.35	3.567	0.004380
2	12	50	19.888	3.454	0.004778
3	12	50	19.924	2.898	0.005132
4	12	50	20.530	2.845	0.005544
5	12	50	20.901	2.585	0.004890
6	12	50	21.539	2.560	0.005264
7	12	50	21.357	2.988	0.005682
8	12	50	22.365	3.290	0.005404
9	12	50	22.280	3.803	0.005510
10	12	50	22.041	4.179	0.004838
11	50	200	67.436	3.069	0.001044
12	50	200	69.310	2.972	0.001140
13	50	200	69.434	2.493	0.001222
14	50	200	71.546	2.448	0.001320
15	50	200	72.841	2.224	0.001164
16	50	200	75.841	2.203	0.001254
17	50	200	74.431	2.571	0.001354
18	50	200	77.943	2.831	0.001288
19	50	200	77.647	3.272	0.001312
20	50	200	76.812	3.596	0.001152
21	50	200	71.724	4.004	0.001120
22	50	200	73.809	3.953	0.001176
23	50	200	66.485	3.874	0.001332
24	50	200	64.984	4.052	0.001228
25	50	200	71.465	3.948	0.001280
26	50	200	65.814	4.223	0.001298
27	50	200	74.936	4.724	0.001256
28	50	200	71.062	4.520	0.001394
29	50	200	67.894	4.615	0.001460
30	50	200	66.086	4.115	0.001482
31	50	200	68.320	4.496	0.001598
32	50	200	71.188	4.937	0.001716
33	50	200	70.934	5.182	0.001770
34	50	200	64.908	5.051	0.001718
35	50	200	66.054	4.898	0.001744
36	50	200	58.265	5.239	0.001784
37	50	200	57.563	4.829	0.001838
38	50	200	57.640	4.450	0.001836
39	50	200	63.499	4.596	0.001882
40	50	200	64.772	4.352	0.001988
41	50	200	60.082	4.757	0.002096
42	50	200	58.202	4.778	0.002146
43	50	200	56.013	4.293	0.002046
44	50	200	48.949	4.235	0.001918
45	50	200	55.351	4.143	0.001850
46	50	200	61.964	4.454	0.001834
47	50	200	57.517	4.681	0.001780
48	50	200	61.836	4.825	0.001644
49	50	200	57.291	5.201	0.001534
50	50	200	64.255	4.932	0.001534

51	50	200	63.372	4.432	0.001438
52	50	200	61.673	4.037	0.001396
53	50	200	61.260	4.074	0.001416
54	50	200	60.467	3.619	0.001396
55	50	200	68.936	3.232	0.001536
56	50	200	62.380	2.817	0.001568
57	50	200	62.620	2.974	0.001612
58	50	200	64.509	2.982	0.001736
59	50	200	69.571	2.786	0.001778
60	50	200	77.877	3.127	0.001748
61	50	200	75.289	3.103	0.001746
62	50	200	71.741	3.464	0.001734
63	50	200	105.429	4.797	0.001636
64	50	200	108.359	4.645	0.001734
65	50	200	108.552	3.896	0.001918
66	50	200	111.854	3.825	0.002072
67	50	200	113.879	3.476	0.001828
68	50	200	117.354	3.443	0.001968
69	100	400	115.534	2.891	0.000740
70	100	400	118.744	2.799	0.000796
71	100	400	118.956	2.348	0.000854
72	100	400	122.574	2.305	0.000922
73	100	400	124.793	2.095	0.000814
74	100	400	128.602	2.075	0.000876
75	100	400	127.517	2.421	0.000946
76	100	400	133.534	2.666	0.000900
77	100	400	133.023	3.387	0.000916
78	100	400	131.596	3.771	0.000806
79	100	400	122.881	2.891	0.000784
80	100	400	180.936	2.799	0.001140
81	100	400	185.964	4.386	0.001246
82	100	400	186.296	3.679	0.001336
83	100	400	191.962	3.612	0.001442
84	100	400	195.437	3.282	0.001274
85	100	400	201.401	3.251	0.001370
86	150	600	179.993	2.923	0.000206
87	150	600	184.994	2.831	0.000226
88	150	600	185.325	2.375	0.000242
89	150	600	190.962	2.331	0.000262
90	150	600	194.418	2.118	0.000232
91	150	600	200.352	2.098	0.000248
92	150	600	198.662	2.449	0 000268
93	150	600	208.036	2.696	0 000256
94	150	600	207.248	3.117	0.000260
95	150	600	205.017	3.425	0.000228
96	150	600	191.439	3.814	0.000222
97	150	600	197.003	3.765	0.000234
98	150	600	177.456	3.690	0.000264
99	320	800	309.123	2.590	0.000722
100	320	800	317.713	2.508	0.000788

Table 4.5 Unit characteristics of the 100 units system(cont'd)

Chapter 5

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH:

5.1 Conclusions:

The objective of this work has been to investigate an analytic approach to the solution of the static unit commitment problem, Analytic solutions are not available from the existing approaches to this problem. The optimum unit commitment in this approach is provided by a simple set of analytic conditions called the switching curve law. Relatively simple and fast numerical implementations could be devised using the switching curve concept for large systems. The advantage of this method is that the effect of the system parameters ,such as the load and reserve margin, on the optimum schedules can be studied for a given problem. An analytic study of a small system has been carried out. From the (λ, α) plane of the lagrange multipliers, a relatively small number of unit combinations defined as regions of constant unit commitment are characterized ,hence reducing the high dimensionality of the problem. Note that λ corresponds to the system load incremental cost and α to the incremental cost of minimum reserve margin. A physical interpretation for optimum unit selection was derived from the switching curve concept giving new insight into the switching mechanism in terms the unit average cost and the system incremental cost.

The algorithm developed based on the switching curve concept is relatively simple and fast, and could handle any number of units. Tests for the simulation of sytems consisting of 10 and 100 units were carried out and encouraging results were obtained.

In conclusion, the static unit commitment problem has been studied from a different approach and some unique and useful results were derived which were not available from the previous approaches to this problem. Even though the static unit commitment does not take into account many of practical constraints which depend on time, its solution can be useful as a lower bound for the general dynamic case to use in the branch-and-bound techique, or as a quick way to obtain a sub-optimal feasible unit commitment to the dynamic case.

5.2 Suggestions for further research:

- To solve the general dynamic case using the switching curve concept would have been a much more difficult task but the inclusion of the start-up and shut-down costs considered as constants could be incorporated in the algorithm to get a better lower bound and would lead to more realistic results concerning the cycling of the regulated units

- The network losses, which were neglected in this simulation could be incorporated in the economic dispatch problem or estimated and represented by the load demand curve.

- The derivation of an analytic expression for the trajectory of the constant load with increasing reserve, i.e the identification of the feasible solutions would be a great achievement, which would improve greatly the efficiency and time of the developed algorithm.

- The algorithm developed in this work is one among many others wich could have been devised using the switching curve concept, the development of an algorithm relying completely on an analytic approach would be faster and desirable, thus suppressing the multiple iterative processes used in this work.

- The branch-and-bound approach itself is very time and memory consuming, and its suppresion would relieve greatly any kind of algorithms which could generate very near feasible optimal solutions

- Heuristics schemes for jumping from the static to the dynamic solution.

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