DEFLECTION SOLUTIONS OF SPECIAL COUPLED WALL STRUCTURES BY DIFFERENTIAL EQUATIONS

by

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M.Eng. Thesis

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ABSTRACT

An approximate method of analysis is presented for determining the lateral deflections of multi-storey shear wall structures. The method is used to derive differential equations of deflection for wall structures subjected to a general lateral loading. The analysis is based on the continuous medium technique.

The shear wall structures considered consist of the following systems: single coupled-walls; linked walls, that is, linked coupled-walls, series of homogeneous walls linked to single coupled-walls, and series of homogeneous walls linked to linked coupled-walls; and tapered coupled-walls. Deflection formulae for these structures are presented for the conventional loading cases: a concentrated load at the top of the structure, a uniformly distributed load, and a triangularly distributed load. For the tapered coupled-walls, deflection formulae are derived only for the first two loading cases.

The developed formulae can be used for multi-storey structures having symmetrical overall plans, subjected to symmetrical loading. Theoretical predictions of deflection profiles of the linked wall and the tapered coupled-wall structures are compared with results obtained from stiffness matrix computer analyses of these structures for the uniformly distributed loading case.

SOLUTION DES DEPLACEMENTS LATERAUX DE CERTAINES STRUCTURES

A MURS JUMELES PAR EQUATIONS DIFFERENTIELLES

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Mars, 1981

RESUME

Une méthode approximative d'analyse est presentée pour determiner les déplacements latéraux de bâtiments, à murs de cisaillement, à étages multiples. La méthode est employée pour obtenir des équations différentielles pour les déplacements latéraux de bâtiments à murs soumis à des charges latérales quelconques. L'analyse est basée sur la technique de raccordement continu.

Les structures à murs de cisaillement, considérées dans cette étude, comprennent les systémes suivants: murs-jumelés; murs reliés, c'est-à-dire, murs-jumelés reliés, suite de murs pleins reliés à des murs-jumelés, et suite de murs pleins reliés à des murs-jumelés reliés; et des murs-jumelés à variation pyramidale. Des formules de déplacements horizontaux sont présentées pour ces structures soumises aux charges horizontales suivantes: une charge concentrée au sommet de la structure, une charge distribuée uniformément, et une charge distribuée triangulairement. Pour les murs-jumelés à variation d'épaisseur pyramidale, des formules de déplacement sont présentées seulement pour les deux premiéres charges horizontales.

Les formules développées peuvent être employées pour de hautes structures, ayant des plans ainsi que des charges latérales symmétriques. Les prévisions théoriques des déplacements latéraux de structures à murs reliés, et à murs-jumelés à variation pyramidale sont comparées à des résultats obtenus par analyses de structures faites sur ordinateur.

TO MY SISTERS AND PARENTS,

JULIE AND ELIE.

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NOTATION

A_{oj}	Cross sectional area at top of wall j
A_o	$A_{o1} + A_{o2}$
a_j	$\frac{\alpha_j^2 \lambda_j}{1+\lambda_j}$
B_j	Integrating constants
b	Clear span of connecting beams
C_j	Integrating constants
c	Depth of connecting beams
D^n	n^{th} derivative
d_j	Width of wall j
E	Modulus of elasticity
g	Reduction factor for moment of inertia of beam, to include shear effects
H	Height of structure
h	Storey height
I_{oj}	Moment of inertia at top of wall j
I_o	$I_{o1} + I_{o2}$
I_p	Moment of inertia of connecting beams (reduced)
I_{po}	Actual moment of inertia of connecting beams
L_m,Q,R,S,s,CO, SI	Functions necessary for the evaluation of tapered structure
ℓ	Distance between the centroids of the walls, in a coupled wall
M_E	Externally applied moment
M_i	Moment of interaction
N_j	Integrating constants
P_E	Lateral point load at top of structure (external load)
P_i	Lateral point load at top of structure (interaction force)

P	Lateral point load at top of structure
p	Intensity of triangular lateral load
T	Integral shear force
t_{oj}	Thickness at top of wall j
t_{bj}	Thickness at bottom of wall j
v	$1 + \kappa x$
w_E	General distributed loading (external load)
w_i	General distributed loading (interaction force)
w	Intensity of uniformly distributed load
x	Distance along the height from top of structure
y	Lateral deflection of structure
z	x/H
α	Physical parameter of coupled-walls, relating the shear and bending stiffness
$\beta, \gamma, \eta, \mu, \nu, \phi, \psi$	Physical parameters
δ_E	Deformation of cut lamina due to external load
δ_j	Deformation j of cut lamina
θ	Rotation of walls
κ	Measure of linear taper of walls
λ	Physical parameter of coupled-walls, measure of axial flexibility
ζ_j	Dummy variables of integration
Γ	Forcing function
T	Integrating constant

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

1.1 High-Rise Buildings

In modern multi-storey buildings, shear walls are used extensively to resist lateral forces exerted by wind and earthquakes. The term 'shear walls' as used in this work covers both homogeneous walls, and coupled-walls. The main function of the shear walls is to increase the rigidity against lateral loading as well as to resist vertical loading.

In addition to strength and stability requirements the behaviour of a structure under service loading has to be considered when designing a multi-storey building. The most important serviceability criterion for high-rise buildings is the lateral deflection of the structure because, if this is excessive, it affects the integrity of non-structural partitions, cladding and glazing, as well as the comfort of the occupants. Generally, as the height of a building increases, the sway under lateral loading surpasses the strength requirements and becomes the governing factor in the design of a tall structure.

The distinguishing feature of shear walls is that they have much higher moments of inertia than columns, and widths which are comparable to the spans of adjacent beams or slabs. The high in-plane rigidity of shear walls, and economies due to their speed of erection and low reinforcing steel content make then the most feasible and attractive lateral load resisting elements.

Homogeneous shear walls behave as vertical cantilever beams under the action of lateral loading, deflecting predominantly in a bending mode configuration. They may be analysed simply in their elastic range using simple bending theory.

Shear walls are often weakened by vertical bands of openings for doors, windows or corridors, yielding highly redundant structures; such pierced walls or walls with openings are often referred to as coupled shear walls. Coupled shear walls also arise when two coplanar homogeneous shear walls are rigidly connected by coupling beams or floor slabs at each floor level. They can be considered as frames with very high column to beam stiffness ratio, thus deflecting in a shape which is a combination of bending and shear modes.

The structural system of a multi-storey shear wall building generally consists of a number of parallel shear walls, homogeneous and/or coupled, symmetrically arranged in plan and joined by slabs. When the building is subjected to a symmetrical lateral loading, it does not twist, and can be idealized for analysis by an equivalent planar lateral load resisting assembly. If the walls are identical, they will deflect identically and only one wall need be analyzed. Whereas when the walls are nonidentical, the horizontal interaction forces produced by the presence of the slabs must be considered in the analysis. Three types of planar wall assemblies are investigated in this thesis, these are:

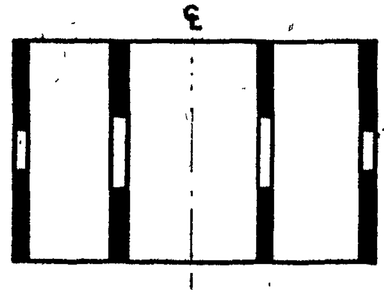
a) two linked coupled-walls

b) a coupled-wall linked to homogeneous walls

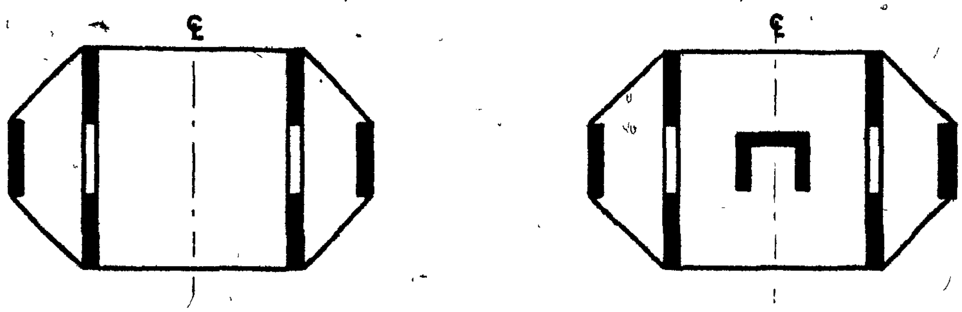
c) two linked coupled-walls linked to homogeneous walls

A great variety of structural systems can be reduced to the above assemblies, a few layouts are illustrated in Fig. 1.

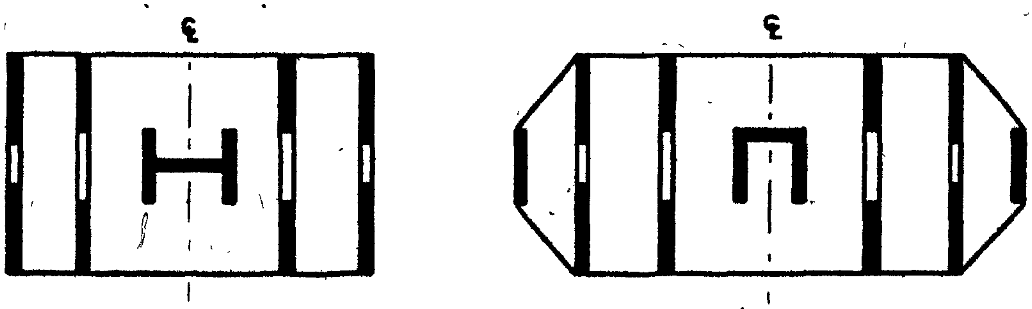
If a building is very tall, it becomes necessary to increase the thickness of the shear walls towards the base. Such an increase in thickness becomes essential in the lower regions of the walls because of the presence



a) Linked Coupled-Walls



b) Coupled Walls Linked to Homogeneous Walls



c) Linked Coupled-Walls Linked to Homogeneous Walls

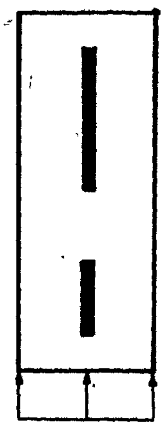
Fig. 1 Structural Systems of Shear Wall Buildings (Plan View) of high gravity forces, and high bending moments; these, result from the accumulation of the vertical loads and the lateral loads respectively.

1.2 Scope of the Thesis

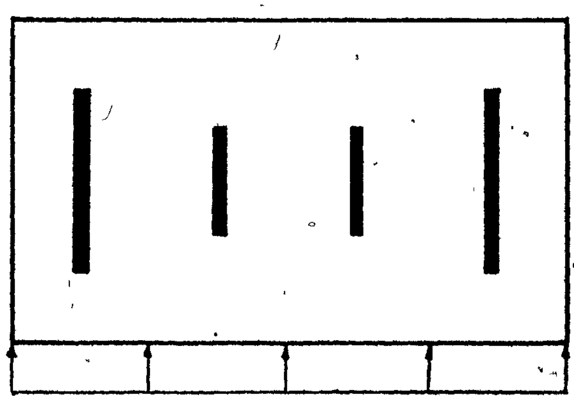
The determination of the lateral drift of a multi-storey building is important and should be undertaken in the early stages of the design to ensure that the deflection limitations are not exceeded. The calculation of the lateral sway of tall buildings can be a long and tedious process because of the high degree of kinematic indeterminancy involved; therefore, simplifying assumptions are often made in the analysis. In the inital stages of design a computer analysis is often not warranted because of the considerable effort and time required in the preparation of the input data and in the relatively high cost of running the complex programs for these multi-storey structures. Approximate deflections and actions obtained by rapid hand methods are usually preferred.

Single coupled shear wall structures have been extensively studied, and deflection formulae along with deflection curves have been obtained by various authors (1,2,3) for the three conventional loading cases. Acceptable deflection formulae for tall single homogeneous shear wall structures may be obtained using ordinary beam theory.

Multi-storey shear wall structures often comprise several coupled and/or homogeneous shear walls connected together in series, Fig. 2a, or in parallel, Fig. 2b, by beams or floor slabs. If, in planar structures consisting of distinct shear walls connected by beams or floor slabs, Fig. 2a, the bending stiffness of the connecting members or their wall connections is low, the connecting members behave effectively as hinged-end links; such structures are often referred to as linked series assemblies. Structures with symmetrical



a) Planar Series Assembly

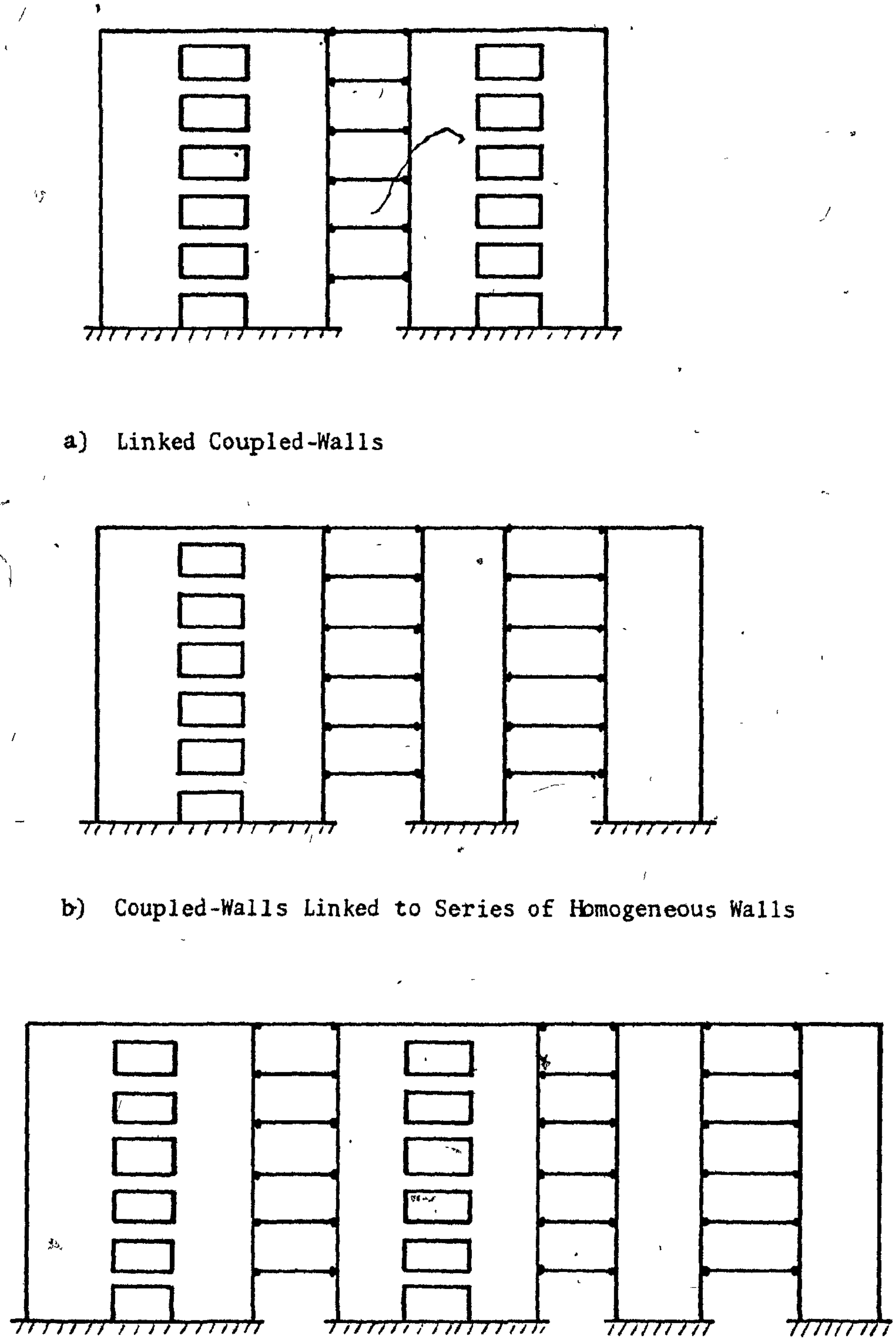


b) Symmetrical Parallel Assembly

Fig. 2 Shear Wall Assemblies (Plan View) overall plans consisting of vertical shear walls in parallel connected by floor slabs and subjected to symmetrical loads such as shown in Fig. 2b, can be idealized by planar linked series assemblies if the floor diaphragms are assumed to be rigid in their own planes and to have negligible rigidity in the normal direction. Therefore both the series assemblies and the symmetrical parallel assemblies can be idealized by linked planar shear wall structures. The pin-ended links simulate the effect of the floor slabs or connecting beams in constraining the assemblies to translate identically.

In this work, a fundamental differential equation of deflection is developed for coupled shear wall structures subjected to a general distributed loading and a top concentrated load. This equation is essential for the study of deflections of linked wall structures. Three types of linked shear wall assemblies are investigated, Fig. 3; one consisting of two coupled-wall assemblies linked together, and two others consisting of either one or two linked coupled-wall assemblies linked to a series of homogeneous walls. These linked structures are analysed for general lateral loading, and deflection formulae are developed for the following lateral loading cases: uniformly distributed loading, triangularly distributed loading, and a concentrated load at the top of the structure. The uniformly distributed and triangularly distributed loadings can be superposed to simulate an equivalent static wind loading, and the triangularly distributed loading with the top concentrated load to simulate an equivalent static earthquake loading. The developed deflection formulae are suitable for design office preliminary calculations.

For the case of coupled shear walls with variable thickness, an approximate method is used to derive deflection formulae for coupled shear



a) Linked Coupled-Walls

b) Coupled-Walls Linked to Series of Homogeneous Walls

c) Linked Coupled-Walls Linked to Series of Homogeneous Walls

Fig. 3 Planar Linked Shear Wall Assemblies (Elevation View) walls with tapered thickness under the actions of uniformly distributed loading, and a concentrated load at the top of the structure.

The method of analysis used in this work is based upon the continuous medium technique which has been applied in several papers, primarily for the analysis of coupled shear wall structures under horizontal loading. This technique is extended to include the shear wall structures investigated in this thesis, namely: uniform coupled-walls subjected to general distributed lateral loading along with top concentrated load, linked walls, and tapered coupled-walls.

1.3 Continuous Medium Technique

The method of analysis, as applied to coupled shear walls assumes, in its most basic form, that the discrete system of connecting beams may be replaced by an equivalent continuous medium. The theory assumes that the sectional properties of the structure remain constant over the height, that the coupling beams have a point of contraflexure at mid-span and that they do not deform axially. Several authors have used this approach to investigate the response of these highly indeterminate coupled-wall structures, but with differing choices of variables; all have yielded essentially the same results.

The method was first used by Chitty (4) in the analysis of a cantilever composed of a number of parallel beams interconnected by cross-beams; later Chitty and Wan (5) applied the technique to the analysis of building frames subjected to wind loading. The technique was applied to the analysis of coupled shear walls by Beck (1), Rosman (6), Coull and Choudhury (1) and a number of other authors of which a selection of papers is listed in the bibliography.

1.4 Organization of the Thesis

The basic governing differential equations for coupled shear wall structures subjected to a general distributed loading and a top concentrated load are developed in Chapter II for structures having walls with linearly varying thicknesses. These equations are also valid for uniform coupled shear walls since they can be considered as tapered coupled shear walls with zero thickness variation.

The governing differential equations are used in Chapter III to derive a fundamental differential equation of deflection for uniform coupled shear walls under a general lateral loading, which is in turn used to obtain a deflection formula for the three conventional lateral loading cases.

The fundamental equation, obtained in Chapter III, is essential for the derivation of the differential equations of deflection as well as the deflection formulae which are developed for linked structures in Chapter IV. The three linked structures considered are: two linked coupled-walls, one coupled-wall linked to a series of homogeneous walls, and two linked coupled-walls linked to a series of homogeneous walls.

In Chapter V, deflection formulae are developed for tapered coupled shear wall structures.

Concluding remarks of the preceding chapters are discussed in the final chapter.

CHAPTER II

DERIVATION OF GOVERNING DIFFERENTIAL EQUATIONS OF COUPLED SHEAR WALL STRUCTURES

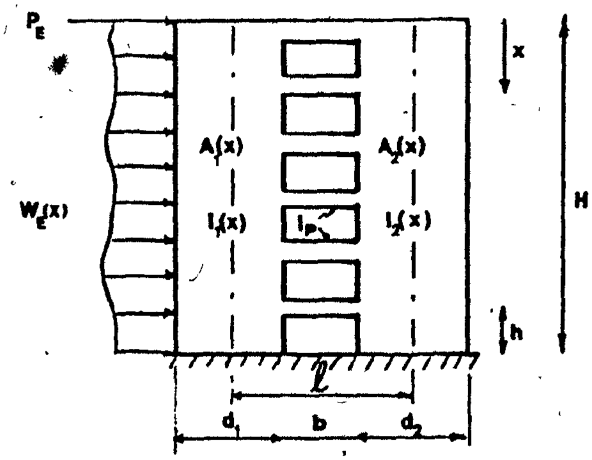
2.1 Introduction

The two basic governing differential equations, necessary for the derivation of deflection formulae of uniform coupled-wall, tapered coupled-wall, and linked wall structures are developed in this chapter. These are the equations of deflection and integral shear force for tapered coupled shear wall structures, subjected to a general horizontal loading. The equations are applied to uniform coupled-wall structures by setting the taper to zero. The continuous medium technique is used for the derivation of the equations.

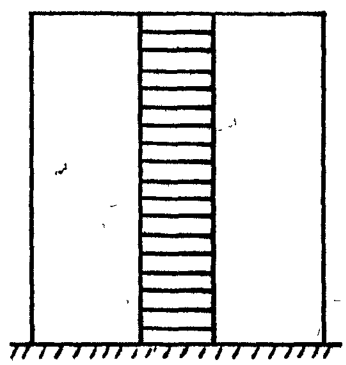
2.2 Formulation of the Problem and Assumptions

Considering Fig. 4a, the coupled-wall structure has a high degree of static indeterminancy. In the analogous structure, Fig. 4b, the discrete connecting beams of flexural stiffness EI_p are replaced by a continuous medium or lamina of flexural stiffness EI_p/h per unit height. This medium has the same storey to storey flexural stiffness as the connecting beams. By cutting the continuous lamina along its midspan and introducing a vertical distributed shear force of intensity T' per unit length acting along the cut section, the coupled structure is reduced to two statically determinate structures. The integral shear force $T = \int_0^x T' \, d\zeta$ becomes the statically redundant function.

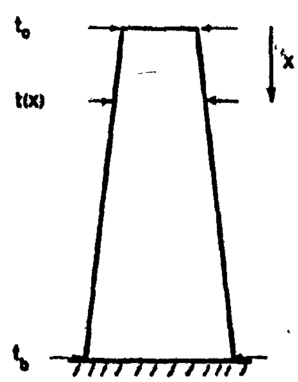
The first of the two basic governing differential equations is obtained for the deflection by considering the moment curvature relationships of the two walls. The second governing differential equation is developed for



a) Actual Structure (Front View)



b) Analogous Structure (Front View)



c) Tapered Thickness (Side View)

Fig. 4 Tapered Coupled Shear Walls

the integral shear force T by considering the compatibility of deformations of the lamina along the cut section.

In the continuous medium analogy, used by the author, the following simplifying assumptions are introduced:

1) All sections, walls and beams, are linearly elastic, and the taper of the vertical shear walls is relatively small, such that simple engineer's bending theory may be applied.

2) The values of:

   a) the storey to storey height h

   b) the clear span b and the cross sectional properties of the connecting beams.

   c) the width of the walls d_1 and d_2

   d) the distance ℓ between the centroid of the cross sections of the walls

   are all kept constant throughout the height H of the structure. The cross sectional properties of the top connecting beam are one half the corresponding values of the lower connecting beams.

3) The points of contraflexure of the connecting beams are located at mid-span. This is reasonable, unless there are large differences in the rigidities of the adjacent walls, since the cross sections of the walls are much greater than the cross sections of the connecting beams.

4) The connecting beams are axially rigid in their longitudinal direction, such that both walls deflect equally.

5) The structures are rigidly fixed to the foundation.

The cross-sectional dimensions of the tapered shear walls over the height of the structure are required prior to the calculation of the deformations of the lamina along the cut section. The thickness of the walls varies linearly with height, Fig. 4c, that is for walls 1 and 2 respectively:

$$t_1(x) = t_{o1}(1 + \kappa x) \qquad 2.1a$$

$$t_2(x) = t_{o2}(1 + \kappa x) \qquad 2.1b$$

where

$$\kappa = \frac{1}{H} \left\{ \frac{t_{b1}}{t_{o1}} - 1 \right\} = \frac{1}{H} \left\{ \frac{t_{b2}}{t_{o2}} - 1 \right\} \qquad 2.2$$

For typical high-rise structures $\kappa H \leq 4.0$, and for uniform walls the value of κ is zero since the top and bottom thicknesses are equal.

The cross-sectional areas and moments of inertias of the walls with tapered thicknesses become:

$$A_1(x) = A_{o1}\ (1 + \kappa x) \qquad 2.3a$$

$$A_2(x) = A_{o2}\ (1 + \kappa x) \qquad 2.3b$$

$$I_1(x) = I_{o1}\ (1 + \kappa x) \qquad 2.3c$$

$$I_2(x) = I_{o2}\ (1 + \kappa x) \qquad 2.3d$$

2.3 Governing Differential Equation of Deflection

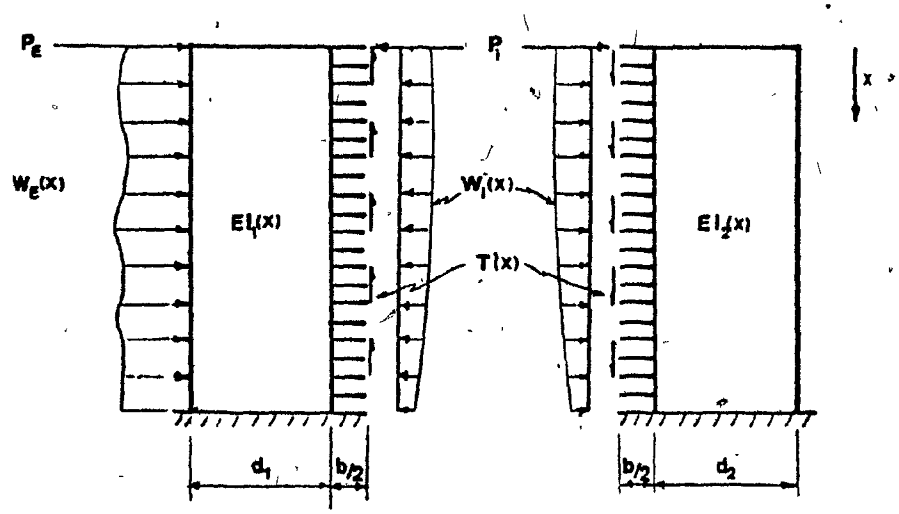


Fig. 5 Analogous Coupled Shear Wall System

The governing differential equation of deflection for tapered coupled shear walls is obtained from the moment-curvature relationship of the two resulting walls in the analogous system, Fig. 5, by virtue of the assumption that both walls deflect equally.

The moment-curvature relationships of wall 1 and wall 2 respectively are:

$$EI_1(x)\,\frac{d^2y}{dx^2} = M_E(x) - \frac{(b+d_1)}{2}\int_o^x T'd\zeta - M_i(x) \qquad 2.4$$

$$EI_2(x)\,\frac{d^2y}{dx^2} = -\frac{(b+d_2)}{2}\int_o^x T'd\zeta + M_i(x) \qquad 2.5$$

where $M_E(x)$ is the external moment resulting from the externally applied lateral loading and is given by:

$$M_E(x) = \int_o^x \int_o^{\zeta_2} W_E(\zeta_1)\, d\zeta_1\, d\zeta_2 + \int_o^x P_E d\zeta \qquad 2.6$$

and where $M_i(x)$ is the bending moment due to the interacting axial forces in the connecting medium:

$$M_i(x) = \int_o^x \int_o^{\zeta_2} W_i(\zeta_1)\, d\zeta_1\, d\zeta_2 + \int_o^x P_i d\zeta \qquad 2.7$$

Adding Eqs. 2.4 and 2.5 and substituting the expressions for $I_1(x)$ and $I_2(x)$ gives the following governing differential equation:

$$EI_o\ (1 + \kappa x)\ \frac{d^2y}{dx^2} = M_E(x) - T\ell \qquad 2.8$$

where

$$I_o = I_{o1} + I_{o2}$$

Tall buildings rigidly fixed to the foundation can be considered as cantilever beams, with zero deflection and rotation at their base. The corresponding boundary conditions are:

$$y(H) = 0 \qquad 2.9a$$

$$\frac{dy}{dx}(H) = 0 \qquad 2.9b$$

2.4 Governing Differential Equation of Integral Shear Force

The governing differential equation of integral shear force T is developed in the subsequent subsections. By considering the various deformations of the cut lamina, due to the external loading and the integral shear force, Fig. 5, and enforcing a compatibility condition, such that no resultant relative deformation of the cut is present, the governing differential equation of integral shear force is then established.

2.4.1 Deformations of the Cut Lamina

The various deformations of the cut lamina, due to the bending moments and the normal forces in the walls and due to the shear forces in the connecting lamina, are derived for an arbitrary location x along the cut in Appendix A. These are:

a) Rotation of walls due to free bending under general external horizontal loading, if no base rotation is present, Fig. 6a.

$$\delta_E(x) = \frac{\ell}{EI_o} \int_x^H \frac{M_E(\zeta)}{(1+\kappa\zeta)} \, d\zeta \qquad 2.10$$

b) Reverse bending deformation in walls due to shear forces in the connecting beams, Fig. 6b.

$$\delta_1(x) = \frac{\ell^2}{EI_o} \int_x^H \frac{T}{(1+\kappa\zeta)} \, d\zeta \qquad 2.11$$

c) Bending and shear deformations of connecting lamina due to the vertical distributed shear force, Fig. 6c.

$$\delta_2(x) = \frac{T' \, h \, b^3}{12EI_p} \qquad 2.12$$

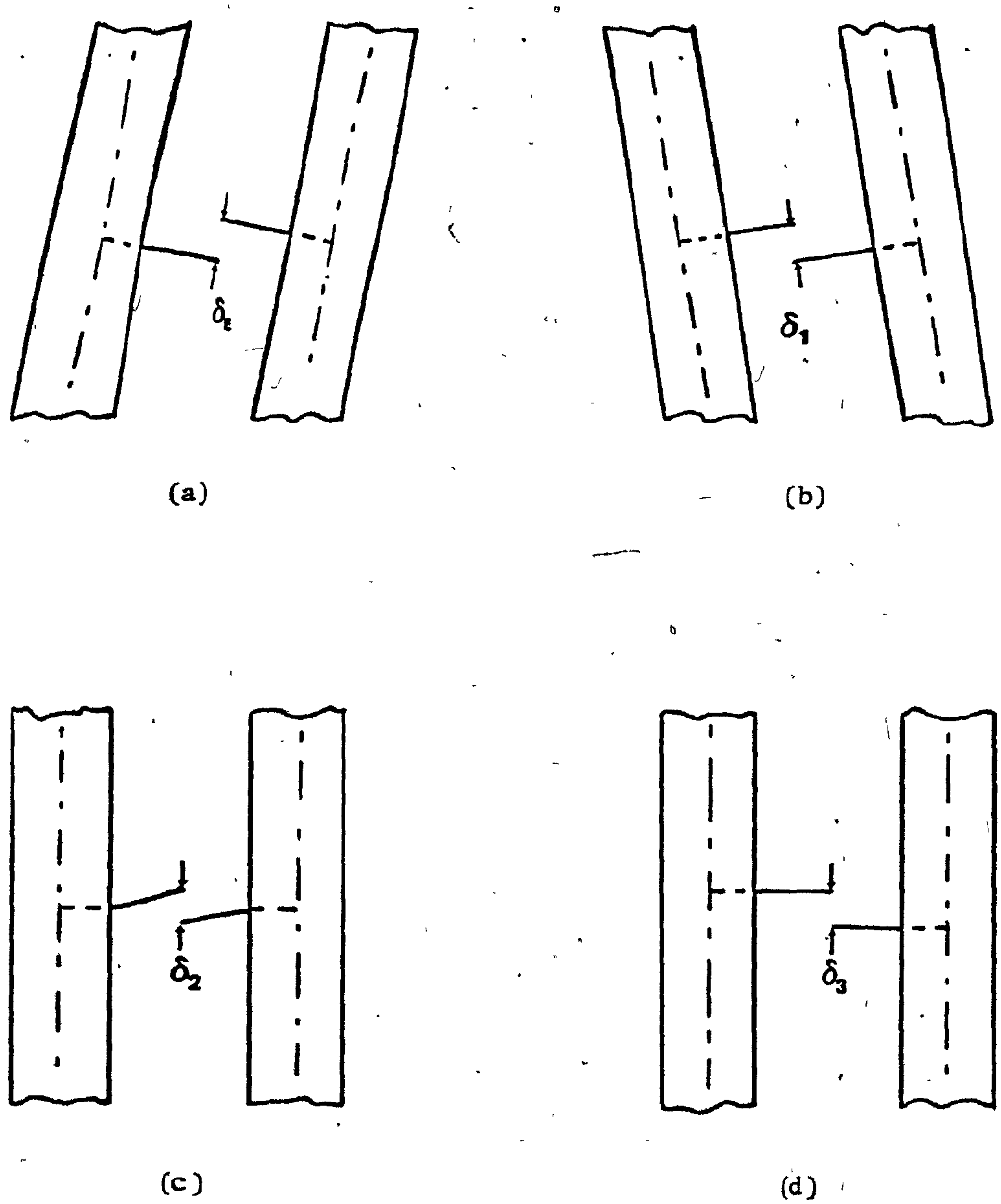


Fig. 6 Deformations of the Cut Lamina

To include the effect of shear in the connecting beams, I_p may be reduced such that

$$I_p = \frac{I_{po}}{(1+g)}$$

where I_{po} is the actual moment of inertia and g is given for rectangular beams in Appendix A. The reduced moment of inertia should be used especially for deep beams.

d) Axial deformation of the walls due to the integral shear force, if no relative displacement of the base is present, Fig. 6d.

$$\delta_3(x) = \frac{A_o}{E(A_{o1}\ A_{o2})} \int_x^H \frac{T}{(1+\kappa\zeta)}\, d\zeta \qquad 2.13$$

2.4.2 Compatibility of Deformations of the Cut Lamina

The compatibility condition requires that no resulting relative deformation be present at the cut section; this leads to the establishment of the following compatibility equation

$$\delta_1(x) + \delta_2(x) + \delta_3(x) = \delta_E(x) \qquad 2.14$$

Substituting the expressions for the deformations of the cut lamina, Eqs. 2.10-2.13, into the above compatibility equation gives

$$\frac{T'hb^3}{12EI_p} + \frac{1}{E}\left(\frac{\ell^2}{I_o} + \frac{A_o}{A_{o1}\ A_{o2}}\right) \int_x^H \frac{T}{(1+\kappa\zeta)}\, d\zeta = \frac{\ell}{EI_o} \int_x^H \frac{M_E(\zeta)}{(1+\kappa\zeta)}\, d\zeta \qquad 2.15$$

where $I_o = I_{o1} + I_{o2}$ and $A_o = A_{o1} + A_{o2}$

dividing through by $hb^3/12EI_p$ and introducing the abbreviations:

$$\alpha^2 = \frac{12 I_p \ell^2}{I_o h b^3} (1 + \lambda) \qquad 2.16a$$

$$\beta = \frac{12 I_p \ell}{I_o h b^3} \qquad 2.16b$$

$$\lambda = \frac{I_o A_o}{\ell^2 (A_{o1} \cdot A_{o2})} \qquad 2.16c$$

Eq. 2.15 can be rewritten

$$T' + \alpha^2 \int_x^H \frac{T}{(1+\kappa\zeta)} d\zeta = \beta \int_x^H \frac{M_E(\zeta)}{(1+\kappa\zeta)} d\zeta$$

Finally differentiating the above equation and using the fact that

$$\frac{d}{dx} \left(\int_x^H f(\zeta) d\zeta \right) = -f(x)$$

the following governing differential equation is obtained

$$\frac{d^2T}{dx^2} - \alpha^2 \frac{T}{(1+\kappa x)} = -\beta \frac{M_E(x)}{(1+\kappa x)}$$

or multiplying through by $(1+\kappa x)$ the above equation becomes

$$(1+\kappa x) \frac{d^2T}{dx^2} - \alpha^2 T = -\beta M_E(x) \qquad 2.17$$

The left-hand side of the above equation relates to the physical parameters of the structure,whereas the right-hand side relates to the external loads applied to the structure.

The boundary conditions for the above equations are:

$$T(0) = 0 \qquad 2.18a$$

$$\frac{dT}{dx}(H) = 0 \qquad 2.18b$$

the first one represents a zero accumulation of T' at the top of the walls, and the second is introduced assuming that the foundation restrains any relative rotation and vertical translation at the base of the walls.

2.5 Discussion

The governing differential equations of deflection and of integral shear force, established in this chapter, Eqs. 2.8 and 2.17, for tapered coupled shear walls subjected to a general lateral loading, are the basic equations necessary for the development of deflection formulae of the shear wall structures considered in the subsequent chapters.

For the case of uniform coupled shear walls ($\kappa=0$) loaded by a uniformly distributed load ($W_E(x) = w$; $P_E = 0$) the governing equations reduce to

$$\frac{d^2T}{dx^2} - \alpha^2 T = -\beta' x^2$$

$$EI_o \frac{d^2y}{dx^2} = \frac{wx^2}{2} - T\ell$$

where

$$\beta' = \frac{w}{2}\beta$$

which are identical to the ones obtained by Rosman (6) and by Coull and Choudhury (1).

2.6 Summary

The governing differential equations of tapered coupled shear walls subjected to a general lateral loading are now established, namely: the differential equation of deflection

$$EI_o(1+\kappa x)\,\frac{d^2y}{dx^2} = M_E(x) - T\ell \qquad 2.8$$

and the differential equation of integral shear force

$$(1+\kappa x)\frac{d^2T}{dx^2} - \alpha^2 T = -\beta\, M_E(x) \qquad 2.17$$

These equations are to be used for the development of the differential equations of deflection, and the deflection formulae of the shear wall assemblies investigated in this thesis.

CHAPTER III

UNIFORM COUPLED SHEAR WALLS

3.1 Introduction

In the previous chapter a differential equation for deflection has been obtained for tapered coupled shear walls, as a function of the external load and of the integral shear force. In this section a fundamental differential equation for deflection is derived for uniform coupled-walls, as a function of only the external load. Such an equation is essential for the development of differential equations of deflection of the linked wall structures investigated in Chapter IV since, in the latter structures, coupled-walls subjected to general horizontal loading are present.

3.2 Derivation of Fundamental Differential Equation of Deflection

For uniform coupled shear walls κ is set to zero in both Eq. 2.8 and Eq. 2.17, which yield

$$\frac{d^2T}{dx^2} - \alpha^2 T = -\beta\, M_E(x) \qquad 3.1$$

$$EI\,\frac{d^2y}{dx^2} = M_E(x) - T\ell \qquad 3.2$$

with boundary conditions given by Eqs. 2.9a, b and Eq. 2.18a,b.

Solving for the integral shear force T in Eq. 3.2

$$T = \frac{1}{\ell}\left\{M_E(x) - EI\,\frac{d^2y}{dx^2}\right\}$$

and, differentiating twice

$$\frac{d^2T}{dx^2} = \frac{1}{\ell} \{\frac{d^2M_E(x)}{dx^2} - EI \frac{d^4y}{dx^4}\}$$

Substituting these expressions into Eq. 3.1 and rearranging and multiplying through by ℓ gives

$$EI \{\frac{d^4y}{dx^4} - \alpha^2 \frac{d^2y}{dx^2}\} = \frac{d^2M_E(x)}{dx^2} - (\alpha^2 - \ell\beta) M_E(x) \qquad 3.3$$

from Eqs. 2.16a and b

$$\ell\beta = \frac{12I_p\ell^2}{Ihb^3} = \frac{\alpha^2}{(1+\lambda)}$$

Substituting the above expression into Eq. 3.3 gives the fundamental equation of deflection for a uniform coupled shear wall under general horizontal loading.

$$EI \{\frac{d^4y}{dx^4} - \alpha^2 \frac{d^2y}{dx^2}\} = \frac{d^2M_E(x)}{dx^2} - \frac{\alpha^2\lambda}{(1+\lambda)} M_E(x)$$

Only two of the boundary conditions are in terms of y; these are as given by Eq. 2.9a and b. The other two, Eqs. 2.18a and b, have to be rewritten in terms of y. By substituting the boundary condition T(0)=0 Eq. 2.18a, in Eq. 3.2 the following boundary condition results

$$EI \frac{d^2y}{dx^2}(0) = M_E(0)$$

For the case of structures loaded by lateral forces $M_E(0) = 0$, consequently the above boundary condition reduces to

$$\frac{d^2y}{dx^2}(0) = 0$$

For the second boundary condition, Eq. 3.2 is differentiated once and evaluated at $x = H$. Substituting $\frac{dT}{dx} = 0$ for $x = H$, Eq. 2.18b, the last boundary condition yields

$$EI \frac{d^3y}{dx^3}(H) = \frac{dM_E(H)}{dx}$$

Introducing for simplicity the differential operator D, where

$$D^n = \frac{d^n}{dx^n}$$

Eq. 3.3 is rewritten

$$EI\, D^2 \{D^2 - \alpha^2\}\, y = \{D^2 - \frac{\alpha^2\lambda}{(1+\lambda)}\}\; M_E(x) \qquad 3.4$$

with boundary conditions

$$y = 0 \qquad \text{at} \quad x = H \qquad 3.5a$$

$$Dy = 0 \qquad x = H \qquad 3.5b$$

$$D^2y = 0 \qquad x = 0 \qquad 3.5c$$

$$EI\, D^3y = D\, M_E \qquad x = H \qquad 3.5d$$

This differential operator is introduced to simplify the derivations of the differential equations for the linked shear wall structures.

3.3 Deflection Formulae

Deflection formulae for uniform coupled shear walls have been derived by various authors for the three common lateral loading cases. Also, curves are available for the rapid evaluation of maximum deflection (1,2,3). These were obtained by solving for the integral shear force T in Eq. 3.1, inserting this result in Eq. 3.2 and integrating twice the latter equation.

For the sake of completeness, deflection formulae are obtained for the three conventional loading cases by solving Eq. 3.4 subjected to the boundary conditions given by Eqs. 3.5.

The most general deflection formula which comprises the three loading cases, viz., concentrated load at top of structure, uniformly distributed load, and triangularly distributed load, is given by:

$$y(z) = T\{B_1\cosh(\alpha H)z + B_2\sinh(\alpha H)z + C_o + C_1 z + N_2 z^2 + N_3 z^3 + N_4 z^4 + N_5 z^5\} \quad 3.6$$

where $z = x/H$

Evaluating the above expression at $z = 0$, the top deflection is obtained

$$y_{TOP} = T\{B_1 + C_o\} \quad 3.6b$$

where

$$B_1 = \frac{-2\ N_2}{(\alpha H)^2}$$

$$B_2 = \frac{N_1}{(\alpha H)\cosh(\alpha H)} - B_1\tanh(\alpha H)$$

$$C_1 = -\{N_1 + 2N_2 + 3N_3 + 4N_4 + 5N_5\}$$

$$C_o = -\{C_1 + N_2 + N_3 + N_4 + N_5 + B_1\cosh(\alpha H) + B_2\sinh(\alpha H)\}$$

The expressions for T and N's are listed in Table I for the three lateral loading cases.

	Concentrated Load at Top	Uniformly Distributed Load	Triangularly Distributed Load
$M_E(x)$	Px*	$\frac{wx^2}{2}$ **	$p\,\frac{x^2}{2}\{1 - \frac{x}{3H}\}$ ***
T	$\frac{PH^3}{EI}$	$\frac{wH^4}{EI}$	$\frac{pH^4}{2EI}$
N_5	0	0	$-\frac{1}{60}\left(\frac{\lambda}{1+\lambda}\right)$
N_4	0	$\frac{1}{24}\left(\frac{\lambda}{1+\lambda}\right)$	$-5N_5$
N_3	$\frac{1}{6}\left(\frac{\lambda}{1+\lambda}\right)$	0	$\frac{(1 - 12\,N_4)}{3(\alpha H)^2}$
N_2	0	$\frac{-(1 - 24\,N_4)}{2(\alpha H)^2}$	$-3N_3$
N_1	$\frac{(1 - 6\,N_3)}{(\alpha H)^2}$	$-2N_2$	$\frac{(1 + 2\,N_2 - 12\,N_4)}{(\alpha H)^2}$

* P is the magnitude of the concentrated load, ** w is the uniform intensity of the load

*** p is the maximum intensity of the load at the top.

TABLE 1 Constants in Eq. 3.6

3.4 Discussion

A fundamental differential equation for the deflection y of uniform coupled shear walls under general horizontal external loading is now available uniquely as a function of the external load.

For the special case of a uniform coupled shear wall structure loaded by a uniformly distributed load w Eq. 3.4 reduces, after rearranging, to

$$\frac{d^4y}{dx^4} - \alpha^2 \frac{d^2y}{dx^2} = \frac{w}{EI} \{1 - \frac{\alpha^2\lambda}{(1+\lambda)} \frac{x^2}{2}\}$$

which is the same differential equation as obtained by Kuster (9) for coupled structures under a uniformly distributed load, by introducing an unknown interaction force.

3.5 Summary

In the foregoing chapter a fundamental differential equation of deflection has been obtained for uniform coupled shear walls under a general external lateral loading. This fundamental equation is

$$EI\, D^2 \{D^2 - \alpha^2\}y = \{D^2 - \frac{\alpha^2\lambda}{(1+\lambda)}\} M_E(x) \qquad 3.4$$

This equation is essential for the development of differential equations of deflection of linked wall structures comprising coupled shear walls.

Also, a general deflection formula is given for uniform coupled shear walls, valid for the three common loading cases.

CHAPTER IV

LINKED SHEAR WALL STRUCTURES

4.1 Introduction

Linked planar shear wall structures, Fig. 3, are idealized structures comprising a number of shear walls, both homogeneous and/or coupled, acting together to resist lateral forces. They arise in the case of planar bents consisting of shear walls connected by beams or floor slabs, when the stiffness of the connecting members or their wall connections is low. They arise also in plan symmetrical sets of parallel shear wall bents connected by floor slabs and subjected to symmetrical lateral loading. In these cases the floor slabs are assumed to be rigid in their own plane and of negligible rigidity transverse to their plane. The pin-ended links in the idealized planar structures simulate the effect of the connecting beams or slabs in constraining the shear walls to deform identically. The horizontal interaction through the links causes redistribution of the external loading amongst the resisting shear walls.

In the previous section a general differential equation for the deflection of a coupled shear wall under a general horizontal loading was obtained. This equation is used to derive deflection formulae for the three loading cases acting upon:

a) linked coupled-walls

b) a single coupled-wall linked to a series of linked homogeneous walls.

c) linked coupled-walls linked to a series of linked homogeneous walls.

4.2 Formulation of the Problem and Assumptions

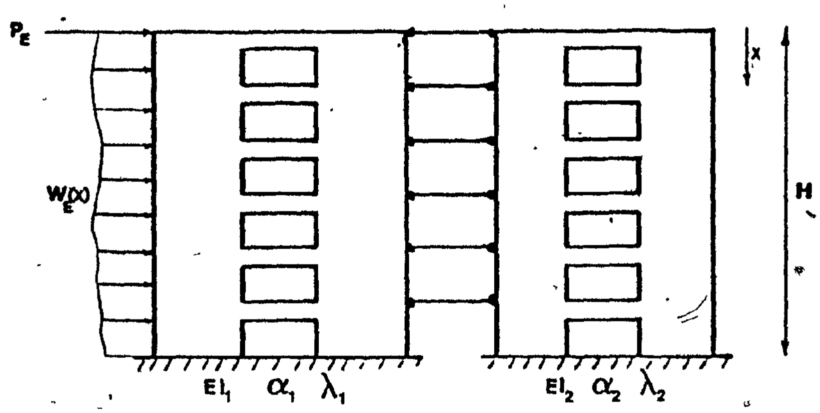


Fig. 7 Linked Coupled Shear Wall System

The planar structure of Fig. 7 has a high degree of static indeterminancy. To solve for the deflection along the height of the assembly the discrete links are replaced by a continuous medium of pin-ended links. This medium constrains the two walls to deflect equally, thus redistributing the external load between the two walls. The medium is cut along its height and equal and opposite distributed horizontal interaction force $w_i(x)$ and concentrated top interaction force P_i are applied to the walls 1 and 2 to maintain compatibility of lateral deflection, Fig. 8. Consequently the linked shear wall system reduces to two individual shear wall systems loaded externally. System 1 is loaded by the external forces minus the interacting forces, and system 2 is loaded by the interacting forces only, with both systems deflecting equally, thus resulting in a system of differential equations.

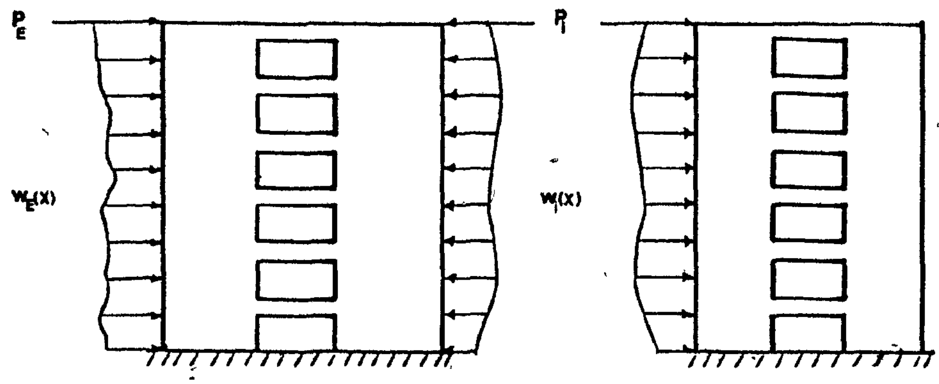


Fig. 8 Interaction Forces in Linked Coupled Shear Wall Systems

In the analysis of linked shear wall structures the following additional assumptions are introduced:

a) the shear wall systems deflect equally

b) the links are pin-ended to the walls

c) the links are axially rigid

d) homogeneous shear walls behave as cantilever beams when loaded laterally in their elastic range - an acceptable assumption for high-rise structures.

4.3 Linked Coupled Shear Walls

Deflection formulae are derived for two coupled shear walls linked together as shown in Fig. 7. This assembly is reduced to two coupled shear walls, loaded as shown in Fig. 8. Each conforms to the differential equation given by Eq. 3.4 with boundary conditions given by Eqs. 3.5, since both behave as coupled shear walls loaded by general lateral loads. They result, therefore, in a system of two differential equations.

4.3.1 Derivation of Governing Differential Equation

The interaction forces P_i and $w_i(x)$ create equal and opposite moment $M_i(x)$, where

$$M_i = M_i(x) = \int_o^x \int_o^{\zeta_2} w_i(\zeta_1)\, d\zeta_1\, d\zeta_2 + \int_o^x P_i d\zeta \qquad 4.1$$

Also

$$M_E = M_E(x) = \int_o^x \int_o^{\zeta_2} w_E(\zeta_1)\, d\zeta_1\, d\zeta_2 + \int_o^x P_E d\zeta \qquad 4.2$$

Coupled shear wall 1 is subject to an external moment equal to $M_E - M_i$, whereas coupled shear wall 2 is subject to M_i.

Now applying the governing differential equation for the deflection of coupled shear walls, as given by Eq. 3.4.

For coupled shear wall 1

$$EI_1 D^2 \{D^2 - \alpha_1^2\}y = \{D^2 - \frac{\alpha_1^2 \lambda_1}{(1+\lambda_1)}\} \quad (M_E - M_i) \qquad 4.3$$

with boundary conditions from Eqs. 3.5

$$y(H) = 0 \qquad 4.4a$$

$$Dy(H) = 0 \qquad 4.4b$$

$$D^2 y(0) = 0 \qquad 4.4c$$

$$EI_1 D^3 y(H) = D(M_E(H) - M_i(H)) \qquad 4.4d$$

For coupled shear wall 2

$$EI_2 D^2 \{D^2 - \alpha_2^2\}y = \{D^2 - \frac{\alpha_2^2 \lambda_2}{(1+\lambda_2)}\} \quad M_i \qquad 4.5$$

with boundary conditions

$$y(H) = 0 \quad 4.6a$$

$$Dy(H) = 0 \quad 4.6b$$

$$D^2y(0) = 0 \quad 4.6c$$

$$EI_2D^3y(H) = D\ M_i(H) \quad 4.6d$$

Adding Eqs. 4.3 and 4.5 and introducing the following abbreviations:

$$a_j = \frac{\alpha_j^2\ \lambda_j}{(1+\lambda_j)} \qquad \text{for } j = 1,2 \quad 4.7a$$

$$EI_T = EI_1 + EI_2 \quad 4.7b$$

$$\phi^2 = \alpha_1^2\ EI_1 + \alpha_2^2\ EI_2 \quad 4.7c$$

gives

$$\{EI_T\ D^4 - \phi^2D^2\}\ y = \{D^2 - a_1\}\ M_E + (a_1 - a_2)\ M_i \quad 4.8$$

When $a_1 - a_2 \neq 0$ or $a_1 \neq a_2$, the interacting moment is obtained and is given by

$$M_i = [\{EI_T\ D^4 - \phi^2D^2\}y - \{D^2 - a_1\}\ M_E]/(a_1-a_2) \quad 4.9$$

The special case when $a_1 = a_2$ will be investigated later in this section.

Substituting back Eq. 4.9 into Eq. 4.8 and rearranging yields

$$D^2\ \{EI_TD^4 - (\phi^2 + a_1EI_2 + a_2EI_1)D^2 + (a_1\alpha_2^2\ EI_2 + a_2\alpha_1^2\ EI_1)\}\ y$$

$$= (D^2 - a_1)(D^2 - a_2)\ M_E$$

Dividing through by EI_T and introducing the following abbreviations

$$\eta^2 = \frac{1}{EI_T} \{(\alpha_2^2 + a_1)EI_2 + (\alpha_1^2 + a_2) EI_1\} \qquad 4.10a$$

$$\gamma^4 = \frac{1}{EI_T} \{a_1\alpha_2^2 EI_2 + a_2\alpha_1^2 EI_1\} \qquad 4.10b$$

yields the governing differential equation of deflection as a function of the external load

$$D^2 \{D^4 - \eta^2 D^2 + \gamma^4\}y = \{D^4 - (a_1+a_2)D^2 + a_1a_2\} \frac{M_E(x)}{EI_T} \qquad 4.11$$

This is a sixth-order differential equation in y, therefore six boundary conditions are required to solve it. Four boundary conditions can be readily obtained by adding the boundary conditions of Eqs. 4.4 and 4.6, yielding

$$y(H) = 0 \qquad 4.12a$$

$$Dy(H) = 0 \qquad 4.12b$$

$$D^2y(0) = 0 \qquad 4.12c$$

$$D^3y(H) = D\, M_E(H)/EI_T \qquad 4.12d$$

Two other boundary conditions are required to get a unique deflected shape y for a given external lateral load.

A fifth boundary condition is obtained by evaluating Eq. 4.8 at x = 0

$$EI_T D^4 y(0) - \phi^2 D^2 y(0) = D^2 M_E(0) - a_1 M_E(0) + (a_1 - a_2) M_i(0) \qquad 4.13$$

but

$D^2y(0) = 0$ from Eq. 4.12c

$M_E(0) = M_i(0) = 0$ from Eqs. 4.1 and 4.2

substituting these in Eqs. 4.13 and dividing through EI_T gives

$$D^4 y(0) = D^2 M_E(0)/EI_T \qquad 4.14$$

To obtain the last boundary condition Eq. 4.8 is differentiated once and evaluated at x = H, which gives

$$EI_T D^5 y(H) - \phi^2 D^3 y(H) = D^3 M_E(H) - a_1 DM_E(H) + (a_1 - a_2)\, DM_i(H) \qquad 4.15$$

but

$$DM_i(H) = EI_2 D^3 y(H) \qquad \text{from Eq. 4.6d}$$

$$D^3 y(H) = DM_E(H)/EI_T \qquad \text{from Eq. 4.12d}$$

Substituting these into Eq. 4.15 and rearranging gives

$$D^5 y(H) = \frac{1}{EI_T} \{D^3 M_E(H) + \psi^2 DM_E(H)\} \qquad 4.16$$

where $$\psi^2 = \frac{1}{EI_T} \left(\frac{\alpha_1^2 EI_1}{1+\lambda_1} + \frac{\alpha_2^2 EI_2}{1+\lambda_2}\right) \qquad 4.17$$

Therefore, for a given lateral loading, for the case $a_1 \neq a_2$, a deflection formula for linked coupled shear walls may be obtained by solving differential Eq. 4.11, subjected to the boundary conditions given by Eqs. 4.12, 4.14, and 4.16.

4.3.1.1 Special Case when $a_1 = a_2$

When $a_1 = a_2$ or $\alpha_1^2\lambda_1/(1+\lambda_1) = \alpha_2^2\lambda_2/(1+\lambda_2)$ Eq. 4.8 reduces to

$$\{EI_T D^4 - \phi^2 D^2\}\, y = \{D^2 - \frac{\alpha_1^2 \lambda_1}{(1+\lambda_1)}\}\, M_E(x) \qquad 4.18$$

By introducing the following abbreviations:

$$\alpha^2 = \phi^2/EI_T \qquad 4.19a$$

$$\frac{\alpha^2}{(1+\lambda)} = \frac{\alpha_1^2 \lambda_1}{(1+\lambda_1)} \qquad 4.19b$$

Eq. 4.18 can be rewritten as

$$EI_T D^2 \{D^2 - \alpha^2\} y = \{D^2 - \frac{\alpha^2\lambda}{(1+\lambda)}\} M_E(x) \qquad 4.20$$

The above equation is identical to Eq. 3.4 and has the same boundary conditions. Therefore, deflection formulae for linked coupled shear walls having $a_1 = a_2$ are equal to the deflection formulae of a single coupled shear wall with the following physical parameters.

$$\alpha^2 = \frac{\alpha_1^2 EI_1 + \alpha_2^2 EI_2}{EI_T} \qquad 4.21a$$

$$\lambda = \lambda_1\lambda_2 \left(\frac{EI_T}{\lambda_1 EI_2 + \lambda_2 EI_1}\right) \qquad 4.21b$$

$$EI = EI_T = EI_1 + EI_2 \qquad 4.21c$$

The above expression for λ is easily obtained by solving for λ from Eqs. 4.19a, and b taking into account the fact that $a_1 = a_2$.

When the linked coupled shear walls are identical, $EI_1 = EI_2$, $\alpha_1^2 = \alpha_2^2$, and $\lambda_1 = \lambda_2$, Eqs. 4.21 reduce to

$$\alpha^2 = \alpha_1^2$$

$$\lambda = \lambda_1$$

$$EI = 2EI_1$$

Substituting these into Eq. 4.20 and dividing through by 2 yields

$$EI_1 D^2 \{D^2 - \alpha_1^2\} . y = \{D^2 - \frac{\alpha_1^2 \lambda_1}{(1+\lambda_1)}\} \; \frac{M_E(x)}{2}$$

Now comparing the above equation to Eq. 3.4, it is noted that the two linked identical coupled shear walls have the same deflection formulae as a single coupled shear wall loaded by one-half of the total external load; this was to be expected for the case of two identical coupled shear walls linked together.

Linked coupled shear walls with $a_1 \doteq a_2$ will not be pursued further in this work since their deflection formulae are equal to the deflection formulae of single coupled shear walls with properties given by Eqs. 4.21. The latter deflection formulae have been obtained for the three common loading cases, and are given in Section 3.3.

4.3.2 Derivation of Deflection Formulae

Eq. 4.11 is rewritten here as:

$$D^2 \{D^4 - \eta^2 D^2 + \gamma^4\} \, y = \Gamma(x) \qquad 4.22$$

where $\Gamma(x)$ is the forcing function equal to

$$\Gamma(x) = \{D^4 - (a_1 + a_2) \, D^2 + a_1 a_2 \} \, \frac{M_E(x)}{EI_T}$$

Solving for the homogeneous solution by setting $\Gamma(x) = 0$ in Eq. 4.22, the indicial roots are

$$0, \; 0 \pm \mu \text{ and } \pm \nu$$

where

$$\mu = \left(\frac{\eta^2 + \sqrt{\eta^4 - 4\gamma^4}}{2}\right)^{1/2} \qquad 4.23a$$

$$\nu = \left(\frac{\eta^2 - \sqrt{\eta^4 - 4\gamma^4}}{2}\right)^{1/2} \qquad 4.23b$$

Therefore, the homogeneous solution for any arbitrary forcing function $\Gamma(x)$ is

$$y_h(x) = T\{B_1\cosh(\mu x)+B_2\sinh(\mu x)+B_3\cosh(\nu x)+B_4\sinh(\nu x)+C_o + C_1'x\} \qquad 4.24$$

thus, for any arbitrary external lateral loading, $M_E(x)$, the homogeneous solution is $y_h(x)$ and the general solution is:

$$y(x) = y_h(x) + y_p(x) \qquad 4.25$$

where $y_p(x)$ is the particular solution to the differential equation with forcing function $\Gamma(x)$.

A complete solution is obtained by solving for the constants in Eq. 4.24 using the appropriate boundary conditions.

The general deflection formula for linked coupled shear walls subjected to the three conventional lateral loading cases used in the static analysis of a tall building is

$$y(z) = T\{B_1\cosh(\mu H)z+B_2\sinh(\mu H)z+B_3\cosh(\nu H)z+B_4\sinh(\nu H)z + C_o + C_1 z$$
$$+N_2z^2 + N_3z^3 + N_4z^4 + N_5z^5\} \qquad 4.26$$

from which the top deflection is given by

$$y_{TOP} = T\{B_1 + B_3 + C_o\} \qquad 4.26b$$

The constants are

$$B_1 = \frac{2}{N_1} \quad \frac{N_6 + (\nu H)^2 N_2}{(\mu H)^2}$$

$$B_3 = \frac{-2}{N_1} \quad \frac{N_6 + (\mu H)^2 N_2}{(\nu H)^2}$$

$$B_2 = \frac{1}{N_1} \quad \frac{N_7 + (\nu H)^2 N_8}{(\mu H)^3 \cosh(\mu H)} \quad - B_1 \tanh(\mu H)$$

$$B_4 = \frac{-1}{N_1} \quad \frac{N_7 + (\mu H)^2 N_8}{(\nu H)^3 \cosh(\nu H)} \quad - B_3 \tanh(\nu H)$$

$$C_1 = - \{2N_2 + 3N_3 + 4N_4 + 5N_5 + \frac{N_7 - (\eta H)^3 N_8}{(\mu H)^2 \ (\nu H)^2}\}$$

$$C_o = - \{C_1 + N_2 + N_3 + N_4 + N_5 + B_1 \cosh(\mu H) + B_2 \sinh(\mu H) + B_3 \cosh(\nu H) + B_4 \sinh(\nu H)\}$$

The constants T, N's are given in Table II for the three loading cases.

	Concentrated Load at Top	Uniformly Distributed Load	Triangularly Distributed Load
T	$\frac{PH^3}{EI_T}$	$\frac{wH^4}{EI_T}$	$\frac{pH^4}{2EI_T}$
N_5	0	0	$-\frac{a_1 a_2}{60\gamma^4}$
N_4	0	$\frac{a_1 a_2}{24\gamma^4}$	$-5N_5$
N_3	$\frac{a_1 a_2}{6\gamma^4}$	0	$\frac{-12(\eta H)^2 N_4 + (a_1+a_2)H^2}{3(\gamma H)^4}$
N_2	0	$\frac{24(\eta H)^2 N_4 - (a_1+a_2)H^2}{(\gamma H)^4}$	$-3N_3$
N_1	$(\mu H)^2 - (\nu H)^2$	$(\mu H)^2 - (\nu H)^2$	$(\mu H)^2 - (\nu H)^2$
N_6	0	$(1 - 24N_4)/2$	$(1 - 12N_4)$
N_7	$(\psi H)^2$	$(\psi H)^2$	$(\psi H)^2 - 2N_6$
N_8	$-(1 - 6N_3)$	$-2N_6$	$-(2N_2 + N_6)$

TABLE II Constants in Eq. 4.26

4.4 Coupled Shear Wall Linked to Homogeneous Shear Walls

The deflection formulae of a coupled shear wall linked to a series of n homogeneous shear walls, Fig. 9, are investigated. By introducing n interacting moments, as given by Eq. 4.1, this assembly can be reduced to a coupled shear wall and n individual homogeneous shear walls, all loaded externally and all constrained to deflect equally. This assembly results in a system of n+1 differential equations, with n+1 unknown functions, the n interacting moments and the deflection y.

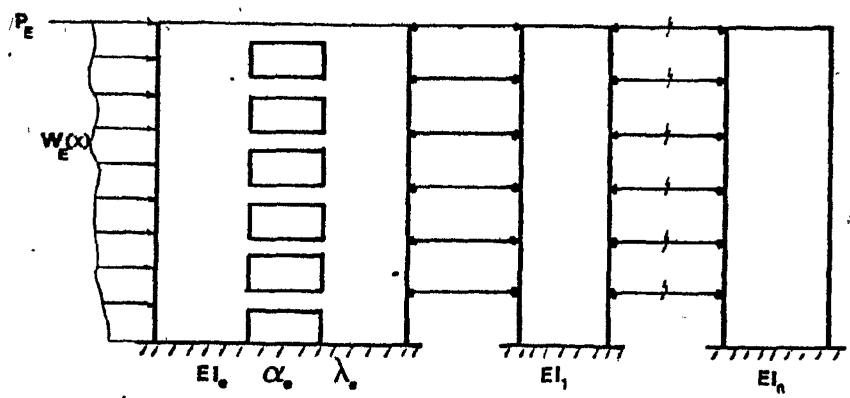


Fig. 9 Coupled Shear Wall Linked to n Homogeneous Shear Walls

4.4.1 Derivation of Governing Differential Equation

The links of the series assembly of Fig. 9 are replaced by a continuum of pin-ended links which are then cut along their height. Now, n interacting moments given by Eq. 4.1 are introduced to constrain the walls to deflect equally, consequently the n+1 differential equations are obtained.

For coupled shear wall 1 from Eq. 3.4

$$EI_e D^2 \{D^2 - \alpha_e^2\} y = \{D^2 - \frac{\alpha_e^2 \lambda_e}{(1+\lambda_e)}\} (M_E - M_1) \qquad 4.27$$

For the n homogeneous shear walls using simple bending theory

$$EI_1 D^2 y = M_1 - M_2$$

$$EI_2 D^2 y = M_2 - M_3$$

$$\vdots \qquad \vdots \qquad \vdots \qquad 4.28$$

$$EI_{n-1} D^2 y = M_{n-1} - M_n$$

$$EI_n D^2 y = M_n$$

where M_1 to M_n are the n interacting moments.

The right hand sides of Eqs. 4.28 form a telescopic series, therefore, by summing Eqs. 4.28 the following simple equation is obtained:

$$EI_s D^2 y = M_1 \qquad 4.29$$

where

$$EI_s = \sum_{j=1}^{n} EI_j \qquad 4.30$$

Which shows that n linked homogeneous shear walls can be idealized for deflection purposes by a single lumped homogeneous shear wall having a stiffness equal to the sum of the stiffnesses of the n individual walls.

The systems of n+1 differential equations has been reduced to a system of two differential equations given by Eqs. 4.27 and 4.29. Substituting the expression for M_1 from Eq. 4.29 into Eq. 4.27 and rearranging, yields

$$\{(EI_e + EI_s)D^2 - \alpha_e^2[EI_e + \frac{EI_s\lambda_e}{(1+\lambda_e)}]\} D^2y = \{D^2 - \frac{\alpha_e^2\lambda_e}{(1+\lambda_e)}\} M_E(x) \qquad 4.31$$

By introducing the following abbreviations

$$EI = EI_e + EI_s \qquad 4.32a$$

$$\lambda = \lambda_e \{1 + \frac{EI_s}{EI_e}\} \qquad 4.32b$$

$$\alpha^2 = \alpha_e^2 (\frac{\lambda_e}{\lambda})(\frac{1+\lambda}{1+\lambda_e}) \qquad 4.32c$$

Eq. 4.31 can be rewritten as

$$EI\, D^2 \{D^2 - \alpha^2\} y = \{D^2 - \frac{\alpha^2\lambda}{(1+\lambda)}\} M_E(x)$$

which is identical to the differential equation for a single coupled shear wall, as given by Eq. 3.4. Consequently, the deflection formulae for a coupled shear wall linked to n homogeneous shear walls are the same as those obtained for a single coupled shear wall with physical parameters as given by Eqs. 4.32. The deflection formulae for a single coupled shear wall, for the three common loading cases used in tall buildings analysis, are given in Section 3.3.

4.5 Linked Coupled Shear Walls Linked to Homogeneous Shear Walls

The deflection formulae of two linked coupled shear walls linked to a series of n homogeneous shear walls and loaded by general lateral forces, Fig. 10, are investigated in this section. By introducing n+1 interaction moments, as was done in Section 4.4, the assembly can be idealized by n+2 discrete structures, n homogeneous shear walls and two coupled shear walls, loaded externally and constrained to deflect equally. This planar assembly results in a system of n+2 differential equations, with n+2 unknown functions, n+1 interacting moments and the deflection y.

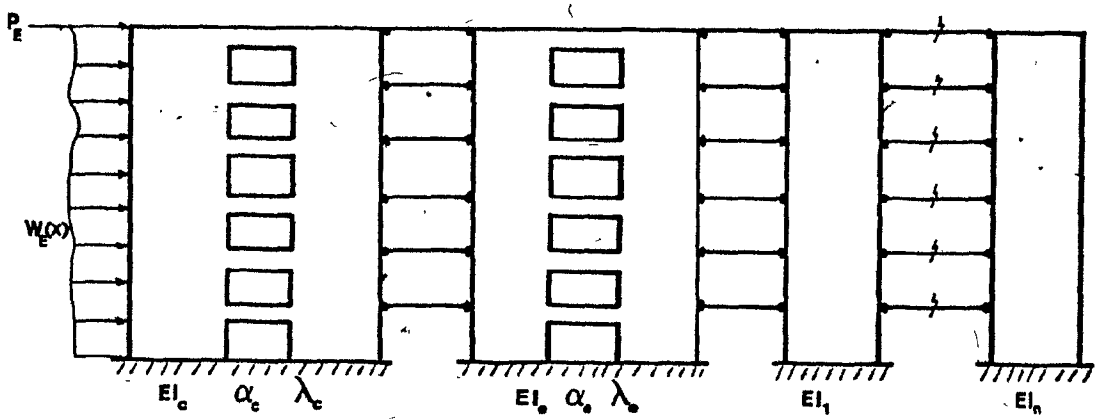


Fig. 10 Linked Coupled Shear Walls Linked to n Homogeneous Shear Walls

4.5.1 Derivation of Governing Differential Equations

Going through the same procedure as followed in Section 4.4.1, that is, replacing the links of the series assembly, Fig. 10, by a continuum of pin-ended links and then cutting them along their height, n+1 interacting moments, each one of the form given by Eq. 4.1, are introduced to constrain the planar assembly to deflect equally. By following this procedure a system of n+2 differential equations is obtained.

For the two coupled shear walls from Eq. 3.4

$$EI_c D^2 \{D^2 - \alpha_c^2\}\, y = \{D^2 - \frac{\alpha_c^2 \lambda_c}{(1+\lambda_c)}\} \quad (M_E - M_i) \qquad 4.33$$

$$EI_e D^2 \{D^2 - \alpha_e^2\} y = \{D^2 - \frac{\alpha_e^2 \lambda_e}{(1+\lambda_e)}\} (M_i - M_1) \qquad 4.34$$

For the n homogeneous shear walls from simple bending theory

$$\begin{aligned} EI_1 D^2 y &= M_1 - M_2 \\ EI_2 D^2 y &= M_2 - M_3 \\ &\vdots \\ EI_{n-1} D^2 y &= M_{n-1} - M_n \\ EI_n D^2 y &= M_n \end{aligned} \qquad 4.35$$

where M_i and M_1 to M_n are the n+1 interaction moments.

As was done in Section 4.4.1, by summing Eqs. 4.35 the following simple equation is obtained.

$$EI_s D^2 y = M_1$$

where

$$EI_s = \sum_{j=1}^{n} EI_j$$

Substituting the above expression for M_1 into Eq. 4.34 and rearranging, reduces the system of n+2 differential equations to a system of two differential equations given by

$$EI_c D^2 \{D^2 - \alpha_c^2\} y = \{D^2 - \frac{\alpha_c^2 \lambda_c}{(1+\lambda_c)}\} (M_E - M_i) \qquad 4.36a$$

$$\{(EI_e + EI_s) D^2 - \alpha_e^2 [EI_e + \frac{EI_s \lambda_e}{(1+\lambda_e)}]\} D^2 y = \{D^2 - \frac{\alpha_e^2 \lambda_e}{(1+\lambda_e)}\} M_i \qquad 4.36b$$

By introducing the following abbreviations

$$EI_1 = EI_c \qquad 4.37a$$

$$EI_2 = EI_e + EI_s \qquad 4.37b$$

$$\lambda_1 = \lambda_c \qquad 4.37c$$

$$\lambda_2 = \lambda_e\{1 + \frac{EI_s}{EI_e}\} \qquad 4.37d$$

$$\alpha_1^2 = \alpha_c^2 \qquad 4.37e$$

$$\alpha_2^2 = \alpha_e^2 \left(\frac{\lambda_e}{\lambda_2}\right)\left(\frac{1+\lambda_2}{1+\lambda_e}\right) \qquad 4.37f$$

Eqs. 4.36 can be rewritten as

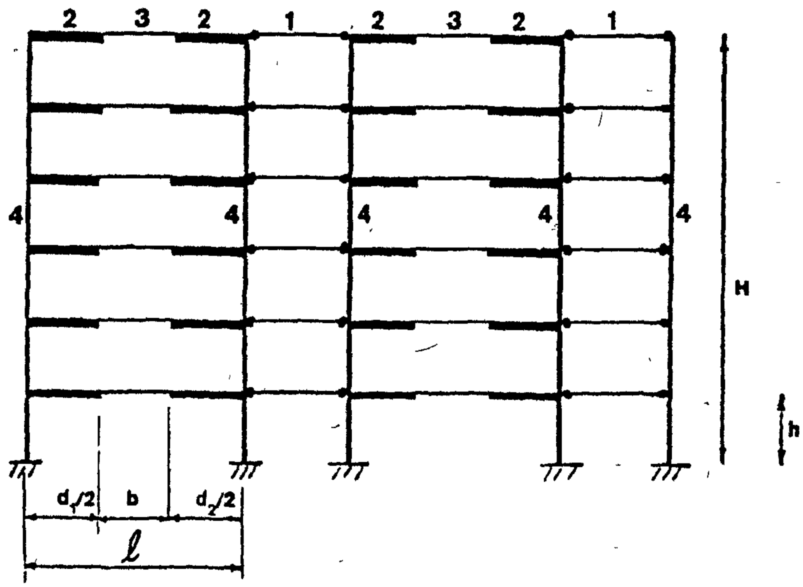
$$EI_1 D^2 \{D^2 - \alpha_1^2\} y = \{D^2 - \frac{\alpha_1^2\lambda_1}{(1+\lambda_1)}\} (M_e - M_i)$$

$$EI_2 D^2 \{D^2 - \alpha_2^2\} y = \{D^2 - \frac{\alpha_2^2\lambda_2}{(1+\lambda_2)}\} M_i$$

The above system of differential equations is identical to the one obtained for two linked coupled shear walls and given by Eqs. 4.3 and 4.5. Therefore, series assemblies consisting of two linked coupled shear walls linked to n homogeneous shear walls can be idealized for deflection purposes by two linked coupled shear walls having physical parameters as given by Eqs. 4.37. Their deflection formulae for the three conventional loading cases are given in Section 4.3.2 for the general case $a_1 \neq a_2$. For the special case when $a_1 = a_2$ the results of Section 4.3.1.1 should be used.

4.6 Examples and Discussion

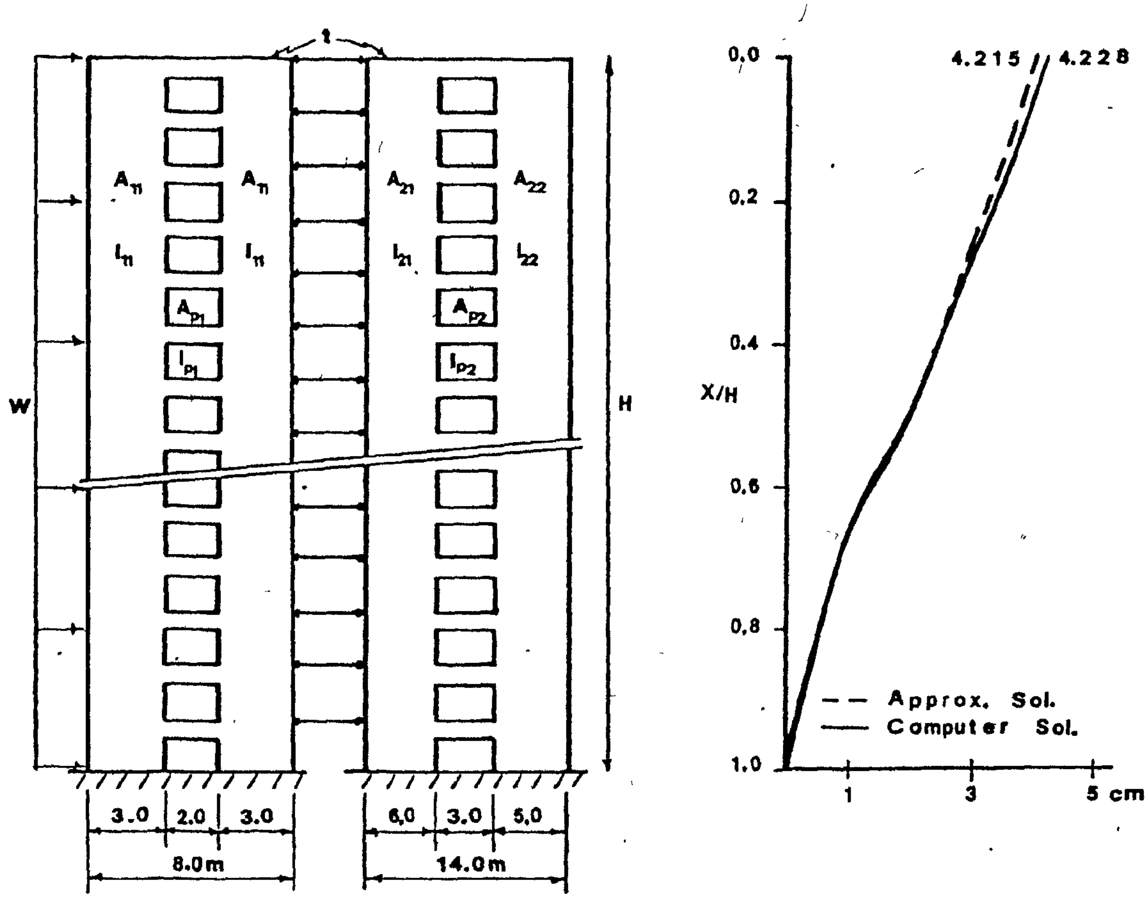
The deflection curves of two assemblies, one consisting of two linked coupled shear walls and the other consisting of a coupled shear wall linked to a homogeneous shear wall, subjected to a uniformly distributed lateral loading are calculated using the deflection equations derived in the foregoing sections. These deflection curves are compared with deflection curves obtained by computer analyses of these assemblies. For the computer analyses, the "SAP IV" (10) structural analysis program has been used. In the computer analysis, coupled shear walls are idealized using the wide-column frame method and links are simulated by pin-ended axially rigid members. The idealized structural elements used in the computer analysis are shown in Fig. 11.



1. Links: pin-ended and axially rigid.
2. Rigid arms: axially rigid and infinite moment of inertia with length equal to one half of width of connected wall.
3. Coupling beams: axially rigid, with actual dimensions and properties.
4. Shear walls: line elements with actual dimensions and properties:

Fig. 11 Idealized Elements Used in Computer Analysis

Example 1, Linked Coupled Shear Walls



(a) Assembly (b) Lateral Deflection

Fig. 12 Deflection Profile of Linked Coupled Shear Walls

Properties:

E = 28,000 MPa

H = 75.0 m; h = 3.75 m; t = 300 mm; w = 15 kN/m

$I_{p1} = 675. \times 10^6\ mm^4$; $A_{p1} = 90. \times 10^3\ mm^2$

$I_{11} = 675. \times 10^9\ mm^4$; $A_{11} = 900. \times 10^6\ mm^2$

$I_{p2} = 5.4 \times 10^9\ mm^4$; $A_{p2} = 180. \times 10^3\ mm^2$

$I_{21} = 5.4 \times 10^{12}\ mm^4$; $A_{21} = 1.8 \times 10^6\ mm^2$

$I_{22} = 3.125 \times 10^{12}\ mm^4$; $A_{22} = 1.5 \times 10^6\ mm^2$

The physical parameters necessary for the evaluation of the deflection formulae are calculated from Eqs. 2.16.

For coupled wall 1

$$EI_1 = 1.35 \times 10^{12}\ E$$

$$\lambda_1 = \frac{1.35 \times 10^{12} \times 1.8 \times 10^6}{5000^2 (900\text{x}10^3 \text{x} 900\text{x}10^3)} = 0.120$$

$$\alpha_1 = \left(\frac{12 \times 675 \times 10^6 \times 5000^2\ (1+0.120)}{1.35 \times 10^{12} \times 3750 \times 2000}\right)^{1/2} = 74.833 \times 10^{-6}\ mm^{-1}$$

$$\alpha_1 H = 5.6125$$

Similarly, for coupled wall 2

$$EI_2 = 8.525 \times 10^{12}\ E$$

$$\lambda_2 = 0.144$$

$$\alpha_2 = 78.780 \times 10^{-6}\ mm^{-1}$$

$$\alpha_2 H = 5.9085$$

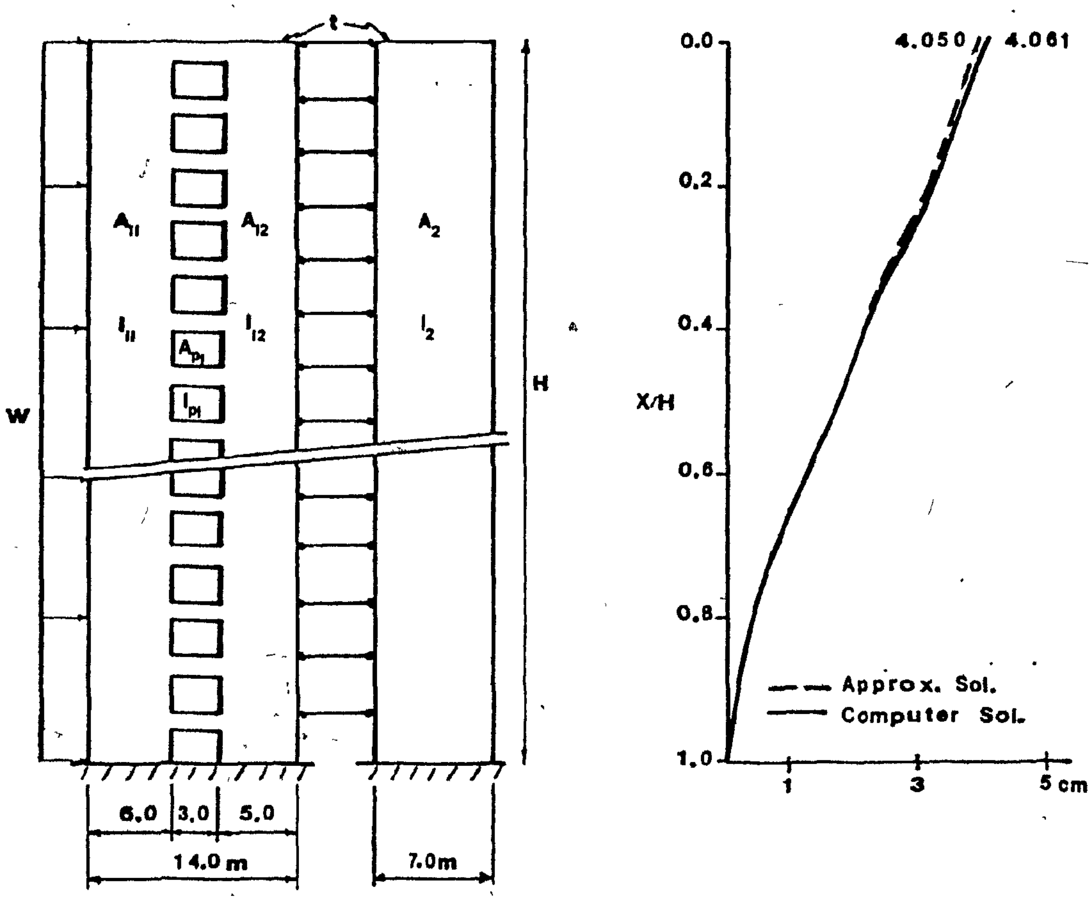
In order to use the appropriate deflection formula for the evaluation of the deflection profile, the condition $a_1 \neq a_2$ must be checked. The formula for linked coupled walls, Eq. 4.26, must be used when $a_1 \neq a_2$, whereas for the special case $a_1 = a_2$ the two coupled-walls reduce to a single coupled-wall with a new set of physical parameters as outlined in Section 4.3.1 and in this case the formula for single coupled-walls, Eq. 3.6, must be used.

$$a_1 = \frac{(74.833 \times 10^{-6})^2 (0.120)}{(1 + 0.120)} = 600.00 \times 10^{-12} \text{ mm}^{-2}$$

$$a_2 = \frac{(78.780 \times 10^{-6})^2 (0.144)}{(1 + 0.144)} = 782.21 \times 10^{-12} \text{ mm}^{-2}$$

Since $a_1 \neq a_2$, the approximate deflection profile of the two linked coupled walls assembly is obtained by using Eq. 4.26 for uniformly distributed loads. The physical parameters are used to evaluate the constants of Eq. 4.26, then this equation is evaluated along the height of the structure, from z = 0 to z = 1, thus obtaining the deflection profile of Fig. 12b.

Example 2 Coupled Shear Wall Linked to Homogeneous Shear Wall



(a) Assembly (b) Lateral Deflection

Fig. 13 Deflection Profile of a Coupled Wall Linked to a Homogeneous Wall

Properties:

$E = 28{,}000$ MPa

$H = 75.0$ m; $h = 3.75$ m; $t = 300$ mm; $w = 15.0$ kN/m

$I_2 = 8.575 \times 10^{12}$ mm^4; $A_2 = 2.1 \times 10^6$ mm^2

The section properties of the coupled wall are the same as those of Example 1.

The physical parameters are

$EI_1 = 8.525 \times 10^{12}$ E

$\lambda_1 = 0.144$

$\alpha_1 = 74.833 \times 10^{-6}$ mm^{-1}

$EI_2 = 8.575 \times 10^{12}$E

This structure is of the type studied in Section 4.4, consequently the results obtained in that section are used to calculate the deflected shape. That is, this linked wall structure can be idealized for deflection purposes by an equivalent coupled shear wall having physical parameters as given by Eq. 4.32. The new physical parameters are

$$EI = 8.525 \times 10^{12}E + 8.575 \times 10^{12}E = 17.10 \times 10^{12}\ E$$

$$\lambda = 0.144\ (1 + \frac{8.575 \times 10^{12}\ E}{8.525 \times 10^{12}\ E}) = 0.289$$

$$\alpha = (74.833 \times 10^{-6})\ (\frac{0.144}{0.289}\ \frac{(1.289)}{(1.144)})^{1/2} = 56.087 \times 10^{-6}\ \text{mm}^{-1}$$

$$\alpha H = 4.207$$

These physical parameters are used to evaluate the constants of Eq. 3.6, which in turn are used to evaluate the equation itself along the height of the structure, from which the deflection curve, Fig. 13b, is obtained.

The results obtained for the deflection of the two sample structures considered, linked coupled shear walls, and a coupled shear wall linked to a homogeneous shear wall, are in close agreement with the results obtained by the more exact stiffness matrix computer analysis. In fact, for the two specific examples the difference between the approximate and the "exact" solution is less than 1.0% throughout the height of the structure. The deflection curves of a number of other structures falling into the categories of linked assemblies considered in this section have also been obtained by the approximate and "exact" methods and these were always in close agreement.

The deflection curves of a greater variety of high-rise shear wall structures may now be evaluated from the results obtained in this section, a few of these were illustrated in Fig. 1.

A practical outcome of this chapter is that, assemblies of single coupled shear walls or linked coupled shear walls which are linked to n linked homogeneous shear walls may be idealized for deflection purposes by single coupled shear walls or linked coupled shear walls, respectively, by changing the physical parameters of the latter two structures.

The evaluation of the deflection formulae obtained in this section may become quite cumbersome, but with the use of a simple computer program of several lines these formulae can be evaluated over the height of a structure by specifying only a few parameters, whereas, if a computer structural analysis were used several hundred input lines would have to be typed. Therefore, an economy of time and effort may be introduced by using the approximate method instead of the "exact" method.

4.7 Summary

The deflection formulae of three types of linked shear wall structures are studied in the foregoing chapter; viz., linked coupled-walls, coupled-walls linked to a series of homogeneous walls, and linked coupled-walls linked to a series of homogeneous walls.

Fundamental differential equations of deflection are developed for these shear wall structures. For the linked coupled-walls, a general deflection formula for the three loading cases is developed. The other two structures, the coupled-wall linked to a series of homogeneous walls, and the linked coupled-walls linked to a series of homogeneous walls, can be idealized for deflection purposes by a coupled-wall, and by linked coupled walls respectively, by introducing new physical parameters. Therefore, the deflection formulae of the former two structures are identical to the ones of the latter equivalent structures.

The results obtained from the approximate deflection formulae developed are in close agreement with the ones obtained by the more exact stiffness matrix computer analysis.

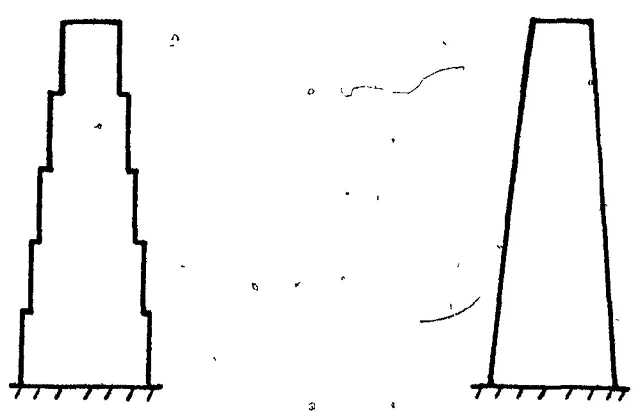
CHAPTER V

TAPERED COUPLED SHEAR WALLS

5.1 Introduction

In high rise shear wall structures, if the building is very tall, it is usual to reduce the walls cross section with height. In practice this reduction is done in steps.

In this section approximate deflection formulae for the particular case of coupled shear walls with variable thickness are obtained. A linear variation of thickness is used to approximate the actual stepped variation of thickness found in practice, Fig. 14. Two lateral load cases are considered: a uniformly distributed loading, and a concentrated load at the top of the structure.



(a) Stepped Variation (b) Linear variation

Fig. 14 Variable Thickness of Walls in Tall Buildings

The deflection formulae being in terms of power series, a variable cross section of the coupling beams would make it impossible to obtain the general terms of these power series, consequently in these approximate solutions of tapered coupled shear walls only the shear walls have varying thicknesses with height, the cross section of the coupling beams is kept constant throughout the height of the structure.

The differential equations obtained in Chapter II for tapered coupled shear walls with constant connecting beams are used in the derivation of the deflection formulae.

5.2 Derivation of Deflection Formulae

The deflection formulae are obtained by first solving for the integral shear force from Eq. 2.17 and, secondly, by substituting this result in Eq. 2.8 and then by integrating this latter equation twice. The two governing differential equations for tapered coupled shear walls as obtained in Chapter II are rewritten here for convenience.

$$(1 + \kappa x) \frac{d^2T}{dx^2} - \alpha^2 T = - \beta M_E(x) \qquad 2.17$$

$$EI\ (1 + \kappa x) \frac{d^2y}{dx^2} = M_E(x) - T\ell \qquad 2.8$$

5.2.1 Integral Shear Force

Eq. 2.17, being a linear differential equation with variable coefficients, can be solved using power series solutions. The solutions will converge at least for:

$$|x| < \frac{1}{\kappa}$$

Since, only the range $0 \le x \le H$ is of interest a converging series solution is obtained for

$$\kappa H < 1$$

or, from Eq. 2.2

$$t_{b1} < 2t_{o1}$$

meaning that for convergence of the series solutions, the thickness at the bottom of the wall should not exceed twice the corresponding thickness at the top of the wall. This is the least radius of convergence.

Solving for the homogeneous part of Eq. 2.17, seeking a solution in the from of a power series.

$$T_h(x) = \sum_{n=0}^{\infty} B_n x^n \qquad 5.1$$

Substituting this series in Eq. 2.17 yields

$$(1 + \kappa x) \sum_{n=2}^{\infty} n(n-1)B_n x^{n-2} - \alpha^2 \sum_{n=0}^{\infty} B_n x^n = 0$$

then rearranging the indices gives

$$(2B_2 - \alpha^2 B_o) + \sum_{n=3}^{\infty} [n(n-1)B_n + \kappa(n-1)(n-2)B_{n-1} - \alpha^2 B_{n-2}]x^{n-2} = 0$$

For this to be true all the coefficients of x^j must vanish, which leads to

$$B_2 = \frac{\alpha^2}{2} B_o$$

and the recurrence formula

$$B_n = -\kappa \frac{(n-2)}{n} B_{n-1} + \frac{\alpha^2}{n(n-1)} B_{n-2} \qquad n=3,4,\ldots\infty$$

Using the above recurrence formula, the general terms in the series can be inferred from the first few terms and then established by mathematical induction. Part of this series solution can be expressed in terms of hyperbolic functions. The homogeneous solution obtained after rearranging is:

$$T_h(x) = B_o\ [\cosh(\alpha x) + S_1(x)] + B_1\ [\sinh\ (\alpha x) + S_2(x)] \qquad 5.2$$

where

$$S_1(x) = \sum_{n,j=1}^{\infty} (-1)^j\ a_{nj}\ (\frac{\kappa}{\alpha})^j\ \frac{(\alpha x)^{2n+j}}{(2n+j)!} \qquad 5.3a$$

$$S_2(x) = \sum_{n,j=1}^{\infty} (-1)^j\ b_{nj}\ (\frac{\kappa}{\alpha})^j\ \frac{(\alpha x)^{2n+j+1}}{(2n+j+1)!} \qquad 5.3b$$

in which the coefficients are given by:

$$a_{nj} = \sum_{j_j=1}^{n} [--[\sum_{j_2=1}^{j_3} (2j_2)[\sum_{j_1=1}^{j_2} (2j_1-1)]]---] \qquad 5.4a$$

$$b_{nj} = \sum_{j_j=1}^{n} [--[\sum_{j_2=1}^{j_3} (2j_2+1)\ [\sum_{j_1=1}^{j_2} (2j_1)]]---] \qquad 5.4b$$

Therefore the general solution for integral shear force of tapered coupled shear walls for the two loading cases under consideration is given by:

$$T(x) = B_o[\cosh\ (\alpha x) + S_1(x)] + B_1[\text{sinch}\ (\alpha x) + S_2(x)] + T_p(x) \qquad 5.5$$

where $T_p(x)$ represents the particular solution of Eq. 2.17. The constants B_o and B_1 are obtained by applying the boundary conditions, Eq. 2.18, to the above equation. The constants B_o and B_1 and the particular solution are listed in Table III for the two loading cases considered.

	Concentrated Load at Top P	Uniformly Distributed Load W
β_e	$P\beta$	$\frac{W}{2}\beta$
T	$\frac{\beta_e}{\alpha^2}$	$\frac{2\beta_e}{\alpha^4}$
$T_p(x)$	Tx	$T(1 + \kappa x + \alpha^2 \frac{x^2}{2})$
B_o	0	$-T$
B_1	$\frac{-T}{\alpha \cdot (\cosh(\alpha H) + S_2'(H))}$	$\frac{T\{\sinh(\alpha H) + S_1'(H) - \alpha H - \kappa/\alpha\}}{\cosh(\ H) + S_2'(H)}$
$S_1'(x) = \sum_{n,j=1}^{\infty} (-1)^j a_{nj} (\frac{\kappa}{\alpha})^j \frac{(\alpha x)^{2n+j-1}}{(2n+j-1)!}$		$S_2'(x) = \sum_{n,j=1}^{\infty} (-1)^j b_{nj} (\frac{\kappa}{\alpha})^j \frac{(\alpha x)^{2n+j}}{(2n+j)!}$

TABLE III Particular Solutions and Constants in Eq. 5.5

For the particular case of uniform walls, $\kappa=0$, and a uniformly distributed load, Eq. 5.5 reduces to

$$T(x) = \frac{2\beta_e}{\alpha^4} \left\{ \left(\frac{\sinh \alpha H - \alpha H}{\cosh \alpha H}\right) \sinh (\alpha x) - \cosh (\alpha x) + 1 + \frac{\alpha^2 x^2}{2} \right\}$$

which is the solution for uniform coupled shear walls as obtained by Rosman (6) and, Coull and Choudhury (1); this serves as a partial check on the developed theory.

5.2.2 Deflection Formulae

The external moment $M_E(x)$ and the integral shear force $T(x)$ are now explicitly available for the two loading cases considered. Therefore, the deflection formulae may now be obtained by integrating twice Eq. 2.8, and by applying the appropriate boundary conditions. Eq. 2.8 is rewritten as

$$\frac{d^2y}{dx^2} = \frac{1}{EI_o} \cdot \frac{1}{(1+\kappa x)} \{M_E(x) - (T_h(x) + T_p(x))\ell\} \qquad 5.6$$

Since $T_p(x)$ is a function of the external moment $M_E(x)$, by introducing the simplifying abbreviations

$$Q''(x) = \frac{M_E(x) - T_p(x)\ell}{(1 + \kappa x)} \qquad 5.7a$$

$$R''(x) = \frac{T_h(x)\ell}{(1+\kappa x)} \qquad 5.7b$$

Eq. 5.6 becomes

$$\frac{d^2y}{dx^2} = \frac{1}{EI_o} \{Q''(x) - R''(x)\}$$

By integrating twice the above equation, the general deflection formula is obtained and is given by

$$y(x) = \frac{1}{EI_o} \{Q(x) - R(x) + C_o + C_1 x\} \qquad 5.8$$

The constants C_o and C_1 are obtained by applying the boundary conditions. The function Q(x) is relatively easy to obtain since it involves two successive integrations of Q''(x) which itself is a rational function. R(x), however, is not so easy to obtain since it involves integrations of hyperbolic functions and power series divided by a linear function, and both result in new power series. The derivation of Eq. 5.8 is shown in Appendix B. The constants of integration, Eq. 5.8, are also evaluated in Appendix B, and are given by

$$C_1 = R'(H) - Q'(H)$$

$$C_o = R(H) - Q(H) - C_1 H$$

The functions Q(x), R(x) and, their first derivatives along with the functions necessary for their evaluation are listed in the subsequent tables for the lateral loadings considered.

	Concentrated Load	Uniformly Distributed Load
$\frac{(1+\lambda)}{\lambda} Q(x)$	$P\ (\frac{1}{\kappa})^3\ \{\frac{v^2}{2} + v - v \log v\}$ *	$\frac{w}{2}\ [(\frac{1}{\kappa})^4\ \{\frac{v^3}{6} - v^2 - v + v \log v\} - \frac{x^2}{\alpha^2\lambda}]$
$R(x)$	$B_o\ell\ \{R_1(x) + R_2(x)\} + B_1\ell\ \{R_3(x) + R_4(x)\}$ **	
$R_1(x)$	$(\frac{1}{\kappa\alpha})\ \{\cosh\ (\frac{\alpha}{\kappa})\ CO\ (\frac{\alpha}{\kappa}\ v) - \sinh\ (\frac{\alpha}{\kappa})\ SI\ (\frac{\alpha}{\kappa}\ v)\}$	
$R_3(x)$	$(\frac{1}{\kappa\alpha})\ \{\cosh\ (\frac{\alpha}{\kappa})\ SI\ (\frac{\alpha}{\kappa}\ v) - \sinh\ (\frac{\alpha}{\kappa})\ CO\ (\frac{\alpha}{\kappa}\ v)\}$	
$R_2(x)$	$(\frac{1}{\kappa})^2 \sum_{j,n=1}^{\infty} (-1)^j\ a_{nj}\ (\frac{\alpha}{\kappa})^{2n}\ \frac{L_{2n+j}(v)}{(2n+j)!}$	
$R_4(x)$	$(\frac{1}{\kappa})^2 \sum_{j,n=1}^{\infty} (-1)^j\ b_{nj}\ (\frac{\alpha}{\kappa})^{2n+1}\ \frac{L_{2n+j+1}(v)}{(2n+j+1)!}$	

* $v = 1+\kappa x$

** B_o and B_1 from Table III

TABLE IV Function Q(x) and R(x)

	Concentrated Load at Top	Uniformly Distributed Load
$\frac{(1+\lambda)}{\lambda} Q'(x)$	$P \, (\frac{1}{\kappa})^2 \{v - \log v\}$	$\frac{w}{2} [(\frac{1}{\kappa})^3 \{ \frac{v^2}{2} - 2v + \log v\} - \frac{2x}{\alpha^2 \lambda}]$
$R'(x)$	$B_o \ell \{R_1'(x) + R_2'(x)\} + B_1 \ell \{R_3'(x) + R_4'(x)\}$	
$R_1'(x)$	$(\frac{1}{\kappa}) \{\cosh (\frac{\alpha}{\kappa}) \, CO' (\frac{\alpha}{\kappa} v) - \sinh (\frac{\alpha}{\kappa}) \, SI' (\frac{\alpha}{\kappa} v)\}$	
$R_3'(x)$	$(\frac{1}{\kappa}) \{\cosh (\frac{\alpha}{\kappa}) \, SI' (\frac{\alpha}{\kappa} v) - \sinh (\frac{\alpha}{\kappa}) \, CO' (\frac{\alpha}{\kappa} v)\}$	
$R_2'(x)$	$(\frac{1}{\kappa}) \sum_{j,n=1}^{\infty} (-1)^j a_{nj} (\frac{\alpha}{\kappa})^{2n} \frac{L'_{2n+j}(v)}{(2n+j)!}$	
$R_4'(x)$	$(\frac{1}{\kappa}) \sum_{j,n=1}^{\infty} (-1)^j b_{nj} (\frac{\alpha}{\kappa})^{2n+1} \frac{L'_{2n+j+1}(v)}{(2n+j+1)!}$	

TABLE V First Derivatives of Q(x) and R(x)

	Function F(v)	First Derivative F'(v)
CO(v)	$v \log v - v + \sum_{j=1}^{\infty} \frac{v^{2j+1}}{(2j)(2j+1)!}$	$\log v + \sum_{j=1}^{\infty} \frac{v^{2j}}{(2j)(2j)!}$
SI(v)	$\sum_{j=1}^{\infty} \frac{v^{2j}}{(2j-1)(2j)!}$	$\sum_{j=1}^{\infty} \frac{v^{2j-1}}{(2j-1)(2j-1)!}$
$L_m(v)$	$(-1)^m v \{\log v - 1\} + \sum_{r=0}^{m-1} (-1)^r \binom{m}{r} \frac{v^{m-r+1}}{(m-r)(m-r+1)}$	$(-1)^m \log v + \sum_{r=0}^{m-1} (-1)^r \binom{m}{r} \frac{v^{m-r}}{(m-r)}$

TABLE VI Functions CO(v), SI(v), $L_m(v)$ and Their First Derivatives

5.3 Example and Discussion

The deflection curve of a tapered coupled shear wall loaded by a uniformly distributed load is calculated using the deflection formulae derived in the preceding section. The deflected shape of the structure is obtained, using a computer program which sums the various power series by "do" loops and evaluates the deflection formula along the height of the structure. This deflection curve is compared with the deflection curve obtained by a stiffness matrix computer analysis of the structure. Here again the "SAP IV" (10) program is used for the computer analysis. In the stiffness analysis the tapered coupled shear wall is idealized using the wide column frame method discussed in Section 4.6. The vertical tapered shear walls are simulated by columns with discrete steps in thickness throughout their height.

Example: Tapered Coupled Shear Wall

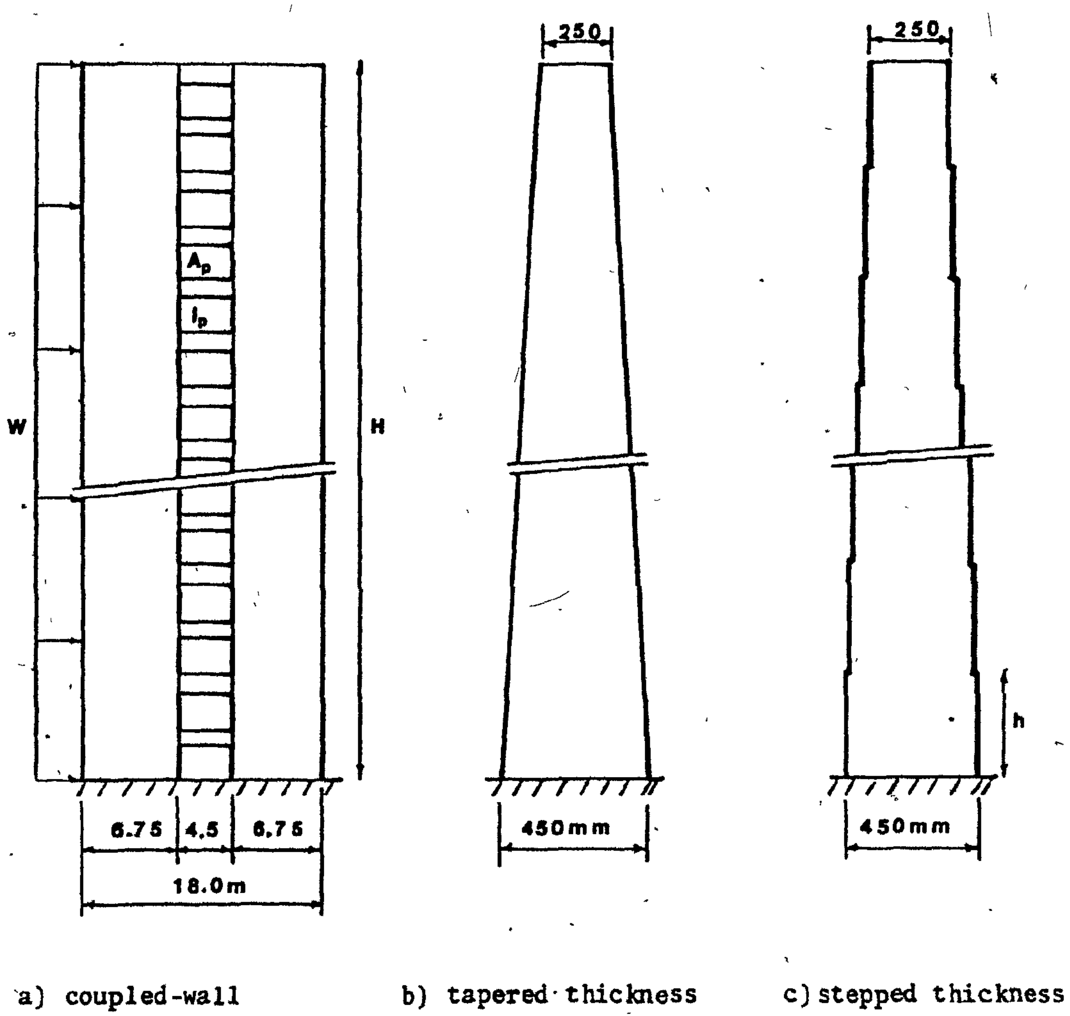


Fig. 15 Sample Tapered Coupled Shear Wall

E = 28,000 MPa

H = 75.0 m; h = 3.75 m ; w = 15.0 kN/m

Coupling beams are of constant size throught the height (175x250x4500 mm) except the top beam which has half the moment of inertia of the lower beams.

$I_p = 111.65 \times 10^6 \text{ mm}^4$; $A_p = 43.75 \times 10^3 \text{ mm}^2$

$I_{o1} = I_{o2} = 6.407 \times 10^{12} \text{ mm}^4$; $A_{o1} = A_{o2} = 1.6875 \times 10^6 \text{ mm}^2$

The approximate deflected curve of the tapered coupled shear wall is obtained from Eq. 5.8 for the uniformly distributed loading.

The physical parameters are from Eq. 2.16 and Eq. 2.2:

$$EI_o = 12.814 \times 10^{12} \text{ mm}^4$$

$$\lambda = \frac{12.814 \times 10^{12} \times 3.375 \times 10^6}{11250^2(1.6875 \times 10^6 \times 1.6875 \times 10^6} = 0.120$$

$$\alpha = \left(\frac{12 \times 111.65 \times 10^6 \times 11250^2 \ (1 + 0.120)}{12.814 \times 10^{12} \times 3750 \times 4500^3}\right)^{1/2} = 6.586 \times 10^{-6} \text{ mm}^{-1}$$

$$\kappa = \frac{1}{75000}\left(\frac{450}{250} - 1\right) = 10.667 \times 10^{-6} \text{ mm}^{-1}$$

or $\alpha H = 0.494$ and $\kappa H = 0.80$

Substituting these into the deflection formula, Eq. 5.8, and using the computer program for the evaluation, the top deflection of the tapered coupled wall, Fig. 15b, is obtained

$$y_{TOP} = 9.712 \text{ cm}$$

When using the computer structural analysis on the stepped model, Fig. 15c, the following top deflection is obtained

$$y_{TOP} = 9.581 \text{ cm}.$$

The deflection profiles from both the approximate deflection formula of the tapered coupled shear wall and the computer structural analysis of the stepped coupled shear wall are shown in Fig. 16.

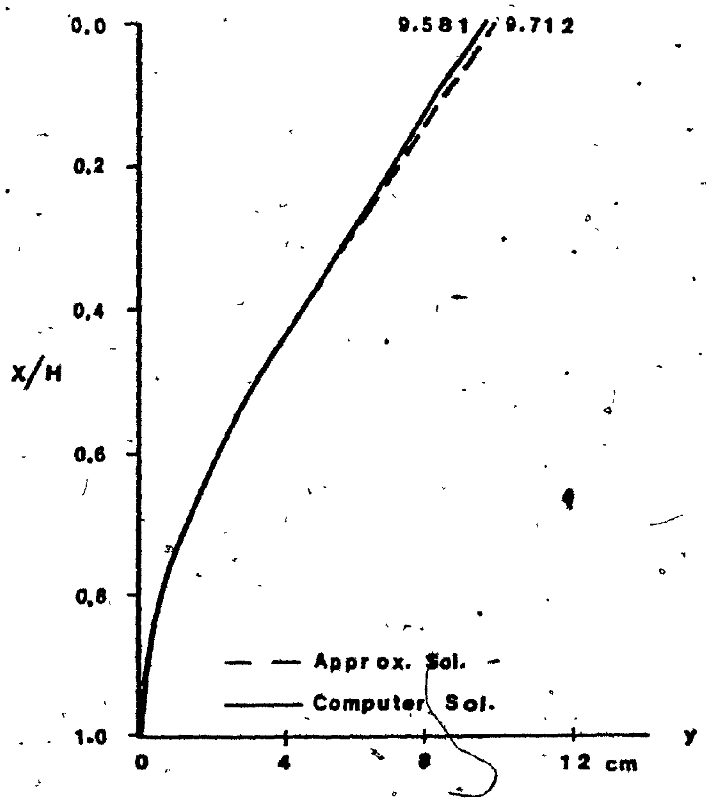


Fig. 16 Lateral Deflection of a Tapered Coupled Shear Wall

Fig. 16 shows close agreement between the approximate and the "exact" deflection profiles, the difference between the two solutions does not exceed 1.5% throughout the height of the structure.

At first glance the evaluation of the deflection formulae of tapered coupled shear walls seems to be almost impossible but, with the use of "do" loops in a computer program, these deflection formulae can be relatively easily evaluated. Once this program is available the deflection profile for each new tapered coupled shear wall is readily obtained by specifying the taper and physical parameters of the top of the structure; whereas, if a stiffness matrix computer analysis were used, this would involve several hundred lines of input for each new tapered shear wall structure. Consequently, for preliminary design purposes, the use of the approximate method becomes more convenient than the "exact" method as the required number of tapered coupled shear walls deflection profiles to be evaluated increases.

Unfortunately a disadvantage of this method is that the approximate solution does not behave well, converging only slowly for $\alpha H \geq 1.0$ and $\kappa H \geq 1.0$, and requiring a great deal of computing time for each evaluation of the deflection formulae. Also, if the computer program is not written with high accuracy in mind it might not even converge. To increase the accuracy of the results of a Fortran Program, which evaluates the deflection formulae of tapered coupled shear walls, a number of steps should be taken; three of these are: to use double precision throughout the calculations; to use logarithmic and gamma functions when summing the power series such that, for example, the Fortran expression for the general term of Eq. 5.3a $(\kappa/\alpha)^j \ (\alpha x)^{2n+j}/(2n+j)$, is

```
DEXP (j* DLOG(κ/α) + (2*n+j)*DLOG (α*x) - DLGAMA (2*n+j));
```

thus preventing the manipulation of exceedingly large numbers; and to properly nest the "do" loops of double summations such that the alternating terms of the double summations, Eq. 5.3, are summed first, so as to avoid the manipulation of large numbers.

5.4 Summary

Deflection formulae for coupled shear wall structures, having walls with tapered thickness, are developed in the chapter for two lateral loading cases: a distributed loading, and a point load at the top of the structure. The formulae are in the form of power series, and converge rapidly for $\alpha H < 1.0$ and $\kappa H < 1.0$. For this range, the results obtained from the approximate formulae are in close agreement with the more exact ones obtained by computer structural analysis.

CHAPTER VI

CONCLUDING REMARKS

A method based on the continuous medium analogy has been presented for the evaluation of the deflection profiles of linked shear wall as well as for tapered coupled shear wall structures. The method of computation presented is an approximation and holds for the elastic range of the structures prior to cracking and inelastic actions.

The worked examples show that the results obtained for the linked shear wall structures are in excellent agreement with the "exact" computer stiffness matrix analysis. The method is suitable for hand calculations as well as for programmable calculators or digital computers. The method becomes more attractive when faced with the alternative of a long and tedious computer structural analysis. The difference in accuracy does not warrant the use of a stiffness matrix analysis over the approximate analysis.

For the case of tapered coupled shear walls, the approximate results also show very close agreement with the "exact" computer analysis. Unfortunately, as discussed in Section 5.3, a disadvantage of the approximate method is that the deflection formulae converge only slowly for $\alpha H \geq 1.0$ and $\kappa H \geq 1.0$ and may require a great deal of computation before convergence is attained. For this reason, it might in such cases become more feasible and accurate to use a computer structural analysis.

The expressions involved in the evaluation of the deflection formulae of tapered coupled shear walls are lengthy and it would become more practical to use a programmable calculator or a small capacity computer to evaluate them. Even if a large capacity computer is available, the evaluation of the deflection profile for the range of αH and κH previously mentioned will be as accurate and less time consuming than by more conventional computer analysis methods.

A practical equation developed in this thesis is the fundamental differential equation of deflection obtained for a coupled shear wall subjected to a general lateral loading. This equation can be used for the analysis of other structural systems comprising one or more coupled shear walls, along with other lateral load resisting assemblies. The procedure to be followed for the analysis of such structural systems would be identical to the one used in Chapter IV, namely by introducing interacting forces.

Alternatively the results obtained in this thesis can be utilized further for solving the deflection profiles of other lateral load resisting assemblies, comprising rigid frames, braced frames, coupled-walls, and homogeneous walls, by reducing the former two structures to their equivalent coupled-walls with parameters αH and λ as developed by Kuster (9).

A valuable result which emanates from the study of linked wall assemblies is that, a structural system comprising m coupled-walls and n homogeneous walls can be idealized for deflection purposes by a new structural system comprising m coupled-walls by changing the physical parameters of one of the coupled-walls.

APPENDICES

APPENDIX A

DEFORMATIONS OF THE CONNECTING LAMINA IN TAPERED COUPLED SHEAR WALLS

Derivation of Deformations of the Cut Lamina

The deformations of the cut lamina of the analogous tapered coupled shear wall system, Fig. 5, are developed here. These are used to derive the governing differential equation of integral shear force in Section 2.4.

a) Rotation of walls due to bending under external loading, and shear forces in the connecting beams.

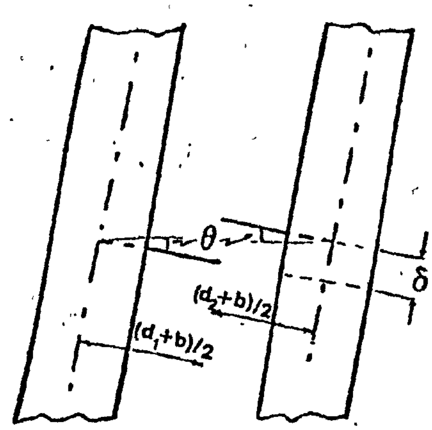


Fig. 17 Bending Deformation of Walls

For small rotations, which is the case here,

$$\delta(x) = \frac{(b+d_1)}{2}\theta(x) + \frac{(b+d_2)}{2}\theta(x) = \ell\theta(x)$$

where the rotation of the walls at level x is

$$\theta(x) = \frac{dy}{dx}$$

The rotation $\theta(x)$ is obtained by dividing Eq. 2.8 by $EI_o(1+\kappa x)$ and integrating from $\zeta = x$ to $\zeta = H$ yielding:

$$\delta(x) = \frac{\ell}{EI_o}\int_x^H \frac{M_E(\zeta)}{(1+\kappa\zeta)}\,d\zeta - \frac{\ell^2}{EI_o}\int_x^H \frac{T}{(1+\kappa\zeta)}\,d\zeta$$

The first term of the above expression is the deformation of the walls due to free bending under external load, while the second is the reverse bending deformation of the walls due to shear in connecting beams.

$$\delta_E(x) = \frac{\ell}{EI_o}\int_x^H \frac{M_E(\zeta)}{(1+\kappa\zeta)}\,d\zeta \qquad 2.10$$

$$\delta_1(x) = \frac{\ell^2}{EI_o}\int_x^H \frac{T}{(1+\kappa\zeta)}\,d\zeta \qquad 2.11$$

b) Bending and shear deformations of connecting beams

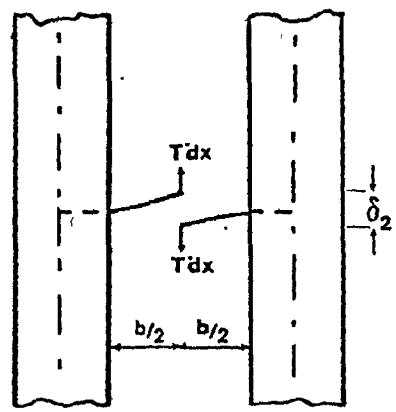


Fig. 18 Deformation of Lamina Due to Bending and Shear in Beams

Assuming the beams to have the same cross section I_p through the height of structure. At level x, for a strip of lamina of height dx, the distributed moment of inertia is given by

$$dI = \frac{I_p dx}{h}$$

The applied force of this strip is

$$dT = \frac{dT}{dx} dx = T'dx$$

For a cantilever beam loaded by a concentrated load at the free end, the deflection at the free end is.

$$\delta = \frac{PL^3}{3EI}$$

Using this expression on the strip of lamina of height dx, the deflection $\delta_2(x)$ is given by

$$\delta_2(x) = \frac{2\,(dT)\,(b/2)^3}{3EdI} = \frac{2(T'dx)\,(b/2)^3}{3E\left(\frac{I_p dx}{h}\right)}$$

from which

$$\delta_2(x) = \frac{T'\ hb^3}{12EI_p} \qquad 2.12$$

To include the effect of shear in the connecting beams, I_p may be reduced such that

$$I_p = I_{po} \quad \frac{1}{(1+g)}$$

where I_{po} is the actual moment of inertia and for rectangular connecting beams

$$g = 2.4\ \left(\frac{c}{b}\right)^2$$

in which c and b are the depth and clear span of the connecting beams. The reduced moment of inertia should be used, especially for deep connecting beams.

c) Axial deformation of walls due to integral shear force.

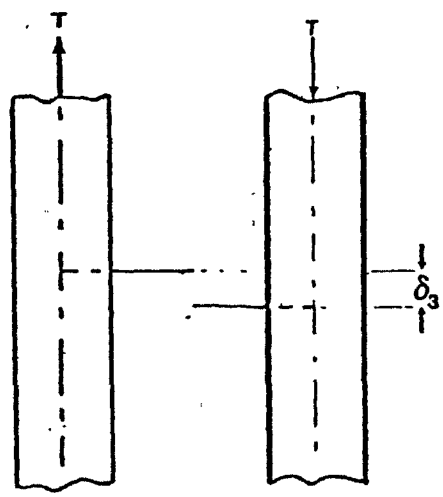


Fig. 19 Axial Deformation of Walls

$$\delta_3(x) = \int_x^H \frac{T}{EA_1(\zeta)} d\zeta - \int_x^H \frac{-T}{EA_2(\zeta)} d\zeta$$

but from Eqs. 2.3 in Section 2.2

$$A_1(x) = A_{o1} \; (1 + \kappa x)$$

$$A_2(x) = A_{o2} \; (1 + \kappa x)$$

Substituting these expressions into the expression for $\delta_3(x)$ and rearranging, the following is obtained

$$\delta_3(x) = \frac{1}{E} \left(\frac{1}{A_{o1}} + \frac{1}{A_{o2}}\right) \int_x^H \frac{T}{(1+\kappa\zeta)}\, d\zeta$$

or

$$\delta_3(x) = \frac{1}{E} \left(\frac{A_o}{A_{o1} \cdot A_{o2}}\right) \int_x^H \frac{T}{(1+\kappa\zeta)}\, d\zeta \qquad 2.13$$

APPENDIX B

DEFLECTION FORMULAE OF TAPERED COUPLED SHEAR WALLS

Derivation of Deflection Formulae

The deflection formulae for two loading cases, a concentrated load at the top of the structure and uniformly distributed loading are derived from the curvature equation in Section 5.1.2.

$$\frac{d^2y}{dx^2} = \frac{1}{EI_o} \{Q''(x) - R''(x)\} \qquad \text{B1}$$

where

$$Q''(x) = \frac{d^2Q}{dx^2} = \frac{M_E(x) - T_p(x)\ell}{(1+\kappa x)} \qquad \text{B2}$$

$$R''(x) = \frac{d^2R}{dx^2} = \frac{T_h(x)\ell}{(1+\kappa x)} \qquad \text{B3}$$

$$T_h(x) = B_o[\cosh(\alpha x) + S_1(x)] + B_1[\sinh(\alpha x) + S_2(x)] \qquad 5.2$$

with boundary conditions:

$$y(H) = 0 \qquad 2.9a$$

$$\frac{dy(H)}{dx} = 0 \qquad 2.9b$$

The constants B_o and B_1 and the function $T_p(x)$ are listed for the two loading cases considered in Table III.

Integrating Eq. B1 gives

$$\frac{dy}{dx} = \frac{1}{EI_o} \{ Q'(x) - R'(x) + C_1 \} \qquad \text{B4}$$

C_1 is evaluated by applying boundary condition, Eq. 2.9a, which yields

$$C_1 = R'(H) - Q'(H)$$

Now integrating Eq. B4 to solve for $y(x)$

$$y(x) = \frac{1}{EI_o} \{Q(x) - R(x) + C_1 x + C_o\} \qquad 5.8$$

By applying boundary condition, Eq. 2.9b, C_o is obtained and is given by

$$C_o = R(H) - Q(H) - C_1 H$$

In order to evaluate the deflection $y(x)$ given by Eq. 5.8 the functions $Q(x)$ and $R(x)$ and their first derivatives have to be solved for the loading cases considered.

1) <u>Q(x) and its first derivative</u>

a) Concentrated load at top of structure

$$M_E(x) = Px$$

from Table III

$$T_p(x) = P \frac{\beta}{\alpha^2} x$$

Substituting these into Eq. B2

$$Q''(x) = P(1 - \frac{\beta\ell}{\alpha^2}) \frac{x}{(1+\kappa x)}$$

Substituting the expression for $\beta\ell$ from Section 3.2

$$Q''(x) = P(\frac{\lambda}{1+\lambda}) \frac{x}{(1+\kappa x)}$$

Introducing the abbreviation

$$v = (1 + \kappa x)$$

and integrating the desired results are obtained and are given by

$$Q'(x) = P(\frac{\lambda}{1+\lambda})(\frac{1}{\kappa})^2 \{v - \log v\}$$

$$Q(x) = P(\frac{\lambda}{1+\lambda})(\frac{1}{\kappa})^3 \{\frac{v^2}{2} + v - v \log v\}$$

b) Uniformly distributed load

$$M_E(x) = w\frac{x^2}{2}$$

from Table III

$$T_p(x) = w\frac{\beta}{\alpha^2}\ \frac{x^2}{2} + w\frac{\beta}{\alpha^4}\{1+\kappa x\}$$

Following the procedure used for the previous loading case

$$Q''(x) = \frac{w}{2}\left(\frac{\lambda}{1+\lambda}\right)\ \frac{x^2}{(1+\kappa x)} - \frac{w}{\alpha^2(1+\lambda)}$$

Now integrating the above expression and using the same abbreviation used in the first loading case.

$$Q'(x) = \frac{w}{2}\left(\frac{\lambda}{1+\lambda}\right)\left(\frac{1}{\kappa}\right)^3\left\{\frac{v^2}{2} - 2v + \log v\right\} - \frac{w}{\alpha^2(1+\lambda)}x$$

$$Q(x) = \frac{w}{2}\left(\frac{\lambda}{1+\lambda}\right)\left(\frac{1}{\kappa}\right)^4\left\{\frac{v^3}{6} - v^2 - v + v\log v\right\} - \frac{w}{\alpha^2(1+\lambda)}\ \frac{x^2}{2}$$

2) R(x) and its first derivative

The function R(x) is the same for all loading cases since it is a function of $T_h(x)$ the solution to the homogeneous part of Eq. 2.17. Eq. 5.2 is identical for all loading cases since it is independent of the external load.

Rewriting the expression for R''(x)

$$R''(x) = B_o\ell\{R_1''(x) + R_2''(x)\} + B_1\ell\{R_3''(x) + R_4''(x)\}$$

where

$$R_1''(x) = \frac{\cosh(\alpha x)}{(1+\kappa x)}$$

$$R''_2(x) = \frac{S_1(x)}{(1+\kappa x)}$$

$$R''_3(x) = \frac{\sinh(\alpha x)}{(1+\kappa x)}$$

$$R''_4(x) = \frac{S_2(x)}{(1+\kappa x)}$$

a) $R_1(x)$ and $R_3(x)$ and their first derivative

$$R'_1(x) = \int \frac{\cosh(\alpha x)}{(1+\kappa x)} dx$$

$$R'_3(x) = \int \frac{\sinh(\alpha x)}{(1+\kappa x)} dx$$

using the hyperbolic identities and v for $(1+\kappa x)$ the following are obtained

$$R'_1(x) = (\tfrac{1}{\kappa}) \{\cosh(\tfrac{\alpha}{\kappa})\, CO'(\tfrac{\alpha}{\kappa} v) - \sinh(\tfrac{\alpha}{\kappa})\, SI'(\tfrac{\alpha}{\kappa} v)\}$$

$$R'_3(x) = (\tfrac{1}{\kappa}) \{\cosh(\tfrac{\alpha}{\kappa})\, SI'(\tfrac{\alpha}{\kappa} v) - \sinh(\tfrac{\alpha}{\kappa})\, CO'(\tfrac{\alpha}{\kappa} v)\}$$

where

$$CO'(\zeta) = \int \frac{\cosh \zeta}{\zeta} d\zeta = \log \zeta + \sum_{j=1}^{\infty} \frac{\zeta^{2j}}{(2j)(2j)!}$$

$$SI'(\zeta) = \int \frac{\sinh \zeta}{\zeta} d\zeta = \sum_{j=1}^{\infty} \frac{\zeta^{2j-1}}{(2j-1)(2j-1)!}$$

Integrating the expressions for $R'_1(x)$ and $R'_2(x)$ the following expressions are obtained

$$R_1(x) = (\tfrac{1}{\kappa\alpha}) \{\cosh(\tfrac{\alpha}{\kappa})\, CO(\tfrac{\alpha}{\kappa} v) - \sinh(\tfrac{\alpha}{\kappa})\, SI(\tfrac{\alpha}{\kappa} v)\}$$

$$R_3(x) = (\tfrac{1}{\kappa\alpha}) \{\cosh(\tfrac{\alpha}{\kappa})\, SI(\tfrac{\alpha}{\kappa} v) - \sinh(\tfrac{\alpha}{\kappa})\, CO(\tfrac{\alpha}{\kappa} v)\}$$

where

$$CO(\zeta) = \int CO'(\zeta)\, d\zeta = \zeta \log \zeta - \zeta + \sum_{j=1}^{\infty} \frac{\zeta^{2j+1}}{(2j)(2j+1)!}$$

$$SI(\zeta) = \int SI'(\zeta)\, d\zeta = \sum_{j=1}^{\infty} \frac{\zeta^{2j}}{(2j-1)(2j)!}$$

b) $R_2(x)$ and $R_4(x)$ and their first derivative

$$R_2'(x) = \int \frac{S_1(x)}{(1+\kappa x)}\, dx$$

$$R_4'(x) = \int \frac{S_2(x)}{(1+\kappa x)}\, dx$$

Since both $S_1(x)$ and $S_2(x)$ are power series in αx both $R_2'(x)$ and $R_4'(x)$ have terms proportional to

$$s_m(x) = \int \frac{(\alpha x)^m}{(1+\kappa x)}\, dx$$

Substituting v for $1+\kappa x$ this reduces to

$$s_m(x) = \left(\frac{1}{\kappa}\right)\left(\frac{\alpha}{\kappa}\right)^m \int \frac{(v-1)^m}{v}\, dv \qquad \text{B5}$$

introducing the abbreviation

$$L_m'(v) = \int \frac{(v-1)^m}{v}\, dv$$

Expanding the term in the bracket and integrating each term, then using the binomial coefficient, gives,

$$L_m'(v) = (-1)^m \log v + \sum_{r=0}^{m-1} (-1)^r \binom{m}{r} \frac{v^{m-r}}{(m-r)} \qquad \text{B6}$$

Substituting this expression into Eq. B5, the following general term is obtained

$$s_m(x) = \left(\frac{1}{\kappa}\right)\left(\frac{\alpha}{\kappa}\right)^m L_m'(v)$$

Using this result for the evaluation of the expressions for $R_2'(x)$ and $R_4'(x)$ yields

$$R_2'(x) = \left(\frac{1}{\kappa}\right) \sum_{j,n=1}^{\infty} (-1)^j a_{nj} \left(\frac{\alpha}{\kappa}\right)^{2n} \frac{L'_{2n+j}(v)}{(2n+j)!}$$

$$R_4'(x) = \left(\frac{1}{\kappa}\right) \sum_{j,n=1}^{\infty} (-1)^j b_{nj} \left(\frac{\alpha}{\kappa}\right)^{2n+1} \frac{L'_{2n+j+1}(v)}{(2n+j+1)!}$$

Integrating the above two expressions gives

$$R_2(x) = \left(\frac{1}{\kappa}\right)^2 \sum_{j,n=1}^{\infty} (-1)^j a_{nj} \left(\frac{\alpha}{\kappa}\right)^{2n} \frac{L_{2n+j}(v)}{(2n+j)!}$$

$$R_4(s) = \left(\frac{1}{\kappa}\right)^2 \sum_{j,n=1}^{\infty} (-1)^j b_{nj} \left(\frac{\alpha}{\kappa}\right)^{2n+1} \frac{L_{2n+j+1}(v)}{(2n+j+1)!}$$

where $L_m(v)$ is obtained by integrating the expression for $L'_m(v)$ given in Eq. B6 with respect to v, this is given by:

$$L_m(v) = (-1)^m v \{\log v - 1\} + \sum_{r=0}^{m-1} (-1)^r \frac{v^{m-r+1}}{(m-r)(m-r+1)}$$

These results are tabulated in Section 5.2.2, Tables IV, V, and VI.

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