COMPUTER GRAPHICS ON JOUKOWSKI AIRFOILS[δ]

by

J.H.T. Wu and P.P. Ostrowski
Department of Mechanical Engineering
McGill University

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ABSTRACT

Steady, incompressible flow about a Joukowski airfoil section is simulated using a IBM 360-75 digital computer and a Calcomp plotter. The effect of various transformation parameters as well as the streamline pattern and pressure distribution about the airfoil is illustrated. The value of the simulation technique as a teaching tool is demonstrated. The method is shown to be particularly useful for flow visualization as all the basic features of the Joukowski transformation and the potential flow streamline patterns are clearly depicted. Since only very unsophisticated computing methods are required, the simulation process provides a flexibility that is conveniently coupled with an obvious time-saving and economic advantage in simulating the process, concept and phenomena of flow about an aifoil together with the variation of parameters in computation for observing responses. For instance, computing results showing the flow about a Joukowski airfoil at an angle of attack ranging from 0 to 16 degrees have sequentially been photographed as a movie.

δ This work was supported by McGill Computing Center and assisted by Mr. K. S. Wong.

INTRODUCTION

The principle of aerodynamic analysis of two-dimensional incompressible fluid flow about an airfoil through conformal transformation from the flow about a circle is well established (Glauert, 1926). Essentially, this method consists of mapping the potential flow about a circular cylinder with circulation in a uniform flow through a kind of mathematical transformation by which the cylinder is transformed into an airfoil shape. The streamlines are also transformed hence the flow pattern about the airfoil is easily obtained for practical applications. By varying the coordinates of the center of the cylinder as well as the parameter of the Joukowski transformation, different airfoils may be obtained.

One of the aims of the conformal transformation is to obtain the velocity and pressure distributions about an airfoil so that the aerodynamic performance of the airfoil can be calculated. The task of performing the calculation manually for just one angle of attack demands about sixteen man-hours (Pope,1951). However, the combination of a computer and a plotter can do the work including streamlines and pressure distributions in a matter of minutes. The time saving and better understanding of this simulation technique are self-evident.

POTENTIAL FLOW ABOUT A CIRCULAR CYLINDER

It is well known that the potential function of a flow about a circle of radius a with a circulation K in a uniform velocity V in a complex coordinates system of $\bar{z}$ as shown in Fig. 1 can be expressed

$$w = V(\bar{z} + a^2/\bar{z}) + (iK/2\pi)\ln(\bar{z}/a) \tag{1}$$

The stream function of Eq. 1 is

$$\psi = V\bar{y}(1 - a^2/(\bar{x}^2 + \bar{y}^2)) + (K/4\pi)\ln((\bar{x}^2 + \bar{y}^2)/a^2) \tag{2}$$

where the circulation K determines the location of two stagnation points. At K = 0, these two points are located on the circle and diametrically opposite each other. As K increases, they move closer. When $K = 4\pi aV$, they coalesced into one. If K is larger than this value, the coalesced stagnation point stays outside the circle.

To illustrate the effect of K on the flow patterns, Figs. 2 and 3 are plotted with K equal to 10 and 18 ft^2/sec respectively. For simplicity, one chooses a = 1 ft and V = 1 ft/sec for Eq. 2. It is noted that $\psi = 0$ gives stagnation streamlines including the circle in Fig. 2 but not in Fig. 3. The critical value of K for this case is 4π. For the present application on the airfoil transformation K is related to the angle of attack for an airfoil. The angle is limited to be less than 16 degreees in order to avoid flow separation and so K is much less than the critical value.

JOUKOWSKI TRANSFORMATION

The simplest transformation which maps the flow about a circle in the z (not $\bar{z}$) plane in Fig. 1 into the flow about an airfoil in the Z plane is the Joukowski transformation

$$Z = z + b^2/z \tag{3}$$

where b is an arbitrary real constant. b is a very important parameter which determines the shape of the airfoil trailing edge. To explain this one has to look into the mapping relationship

$$\frac{dZ}{dz} = 1 - b^2/z^2 \tag{4}$$

Equations 3 and 4 show that z = 0, +b and -b are singular points.

To obtain a sharp trailing edge, one takes S in Fig. 1 as a singular point of $z = -b_o$ (not b) so that the transformation is non-conformal and the trailing edge becomes a cusp. But the velocity at the trailing can be finite if the point S is a stagnation point. To satisfy these two conditions with a given location of x_o and y_o for the center of the circle as in Fig. 1, one obtains

$$b_o = \mathrm{SQRT}(a^2 - y_o^2) - x_o \tag{5}$$

$$K = 4\pi aV \sin(\alpha + \beta) \tag{6}$$

where α = the angle of attack and

$$\beta = \arcsin(\, y_o/a\,) \tag{7}$$

Hence for $b = b_o$, we have a sharp cusped trailing-edged airfoil.

In engineering design of airfoils, the trailing edge is a little rounded. To acheive this, the value of b is chosen a little less than b_o. For the same x_o and y_o, Point S reamins at $z = -b_o$ but the singular point $z = -b$ is within the circle. After transformation, Points S becomes a rounded trailing edge which is conformal with an angle (Glauert 1926) at the trailing edge equal to

$$\theta = \pi(2 - \mathrm{SQRT}(3(b/b_o)^2 + 1) \qquad (8)$$

It can be seen that if $b = b_o$, $\theta = 0$.

To transform the flow about a circle into that about an airfoil, one has to rotate and translate the $\bar{z}$ coordinates system in order to get the z system as shown in Fig. 1 and then transform the z system into the Z system by the Joukowski transformation. The variation of Joukowski airfoil depends on these three parameters: b, x_o and y_o. For instance $x_o = 0$ yields circular arc airfoils and $y_o = 0$ yields symmetrical airfoils. The variation of b will be used as an example as follows:

Given V = 1 ft/sec, a = 1 ft, α = 10 degrees, $x_o = 0.05a$ and $y_o = 0.04a$ (Bairstow, 1946), one obtains K = 2.193 ft^2/sec and $b_o = 0.9588$ from Eqs. 6 and 5 respectively.

Figures 4 and 5 show the streamlines about airfoils with the value of b equal to 0.94868 and 0.91104 and the angle θ (Eq. 8) equal to 2.831° and 13.348° respectively. In these two cases, the trailing edge is observed to be a stagnation point so that the flow proceeds smoothy over the airfoil. The forward stagnation point lies on the underside and near the nose of the airfoil. Leading edge suction effects are evident as the streamline spacing is observed to be smaller.

Figure 6 is a typical chordwise pressure distribution corresponding to an angle of attack of 6 degree for an airfoil as in Fig. 5. It can be seen that the major portion of the lift contribution from the pressure distribution is derived from the suction and compression on the upper surface and underside, respectively, around the leading edge of the airfoil.

Numerical integration of the pressure distribution in the chordwise and normal dirction can lead to the determination of the lift coefficient of the airfoil. This shows that the lift force can exist in a perfect fluid and obeys the Kutta-Joukowski theorem of lift. Further computation can give the moment of the Joukowski airfoil.

CONCLUDING REMARKS

The objective of adequately simulating the performance of Joukowski airfoils with a digital computer and plotter has been partially accomplished. It is evident that the simulation procedure has a high educational value with regard to the visualization of streamlines around circles and airfoils etc. All the essential features of the potential flows are clearly and effectively demonstrated by this technique. The difficult aspects in sigular points as well as the variety of Joukowski airfoils can be easily and aptly illustrated. The computing techniques involved are relatively simple and straightforward for students with an elementary knowledge of comuter programming. Finally, the time saving and self-interest of this simulation technique are evident, and a computer has to go with a plotter for educational stimulation.

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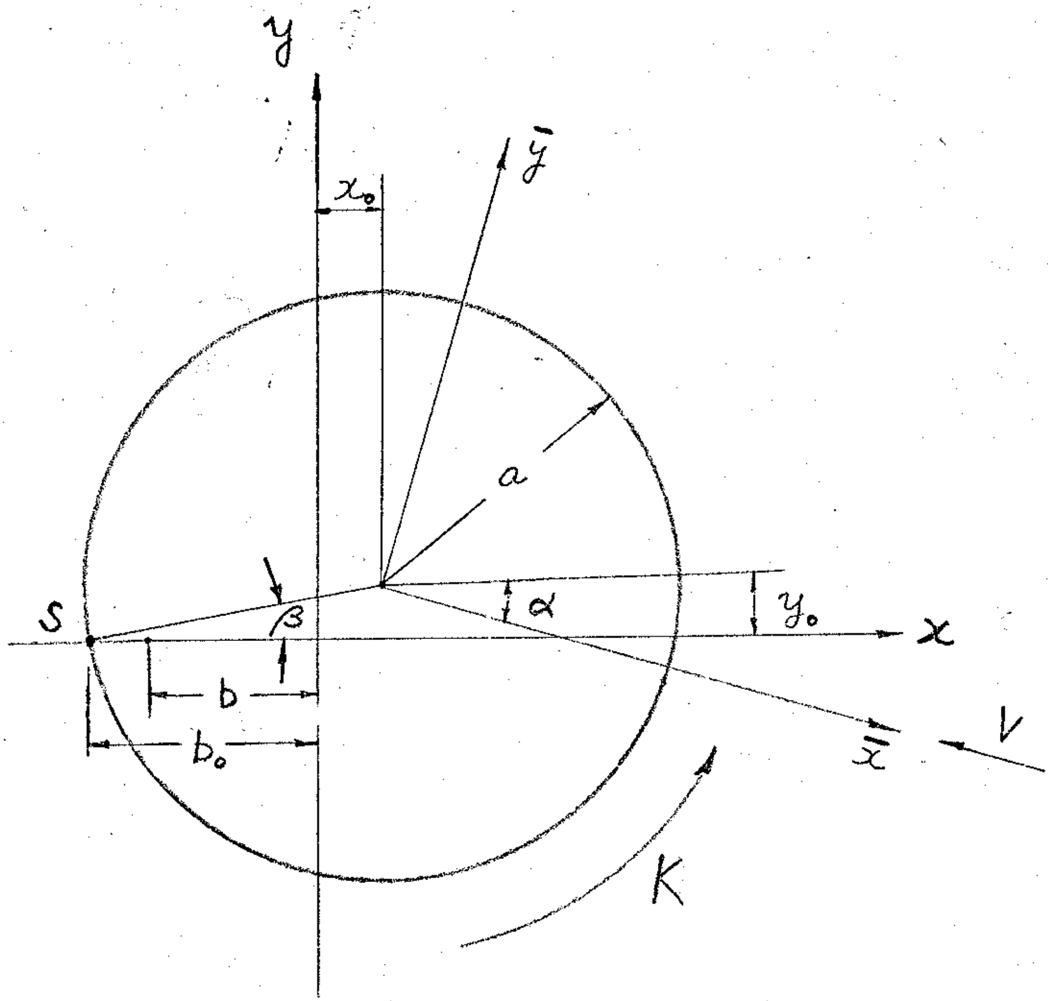

Fig. 1 Geometrical construction

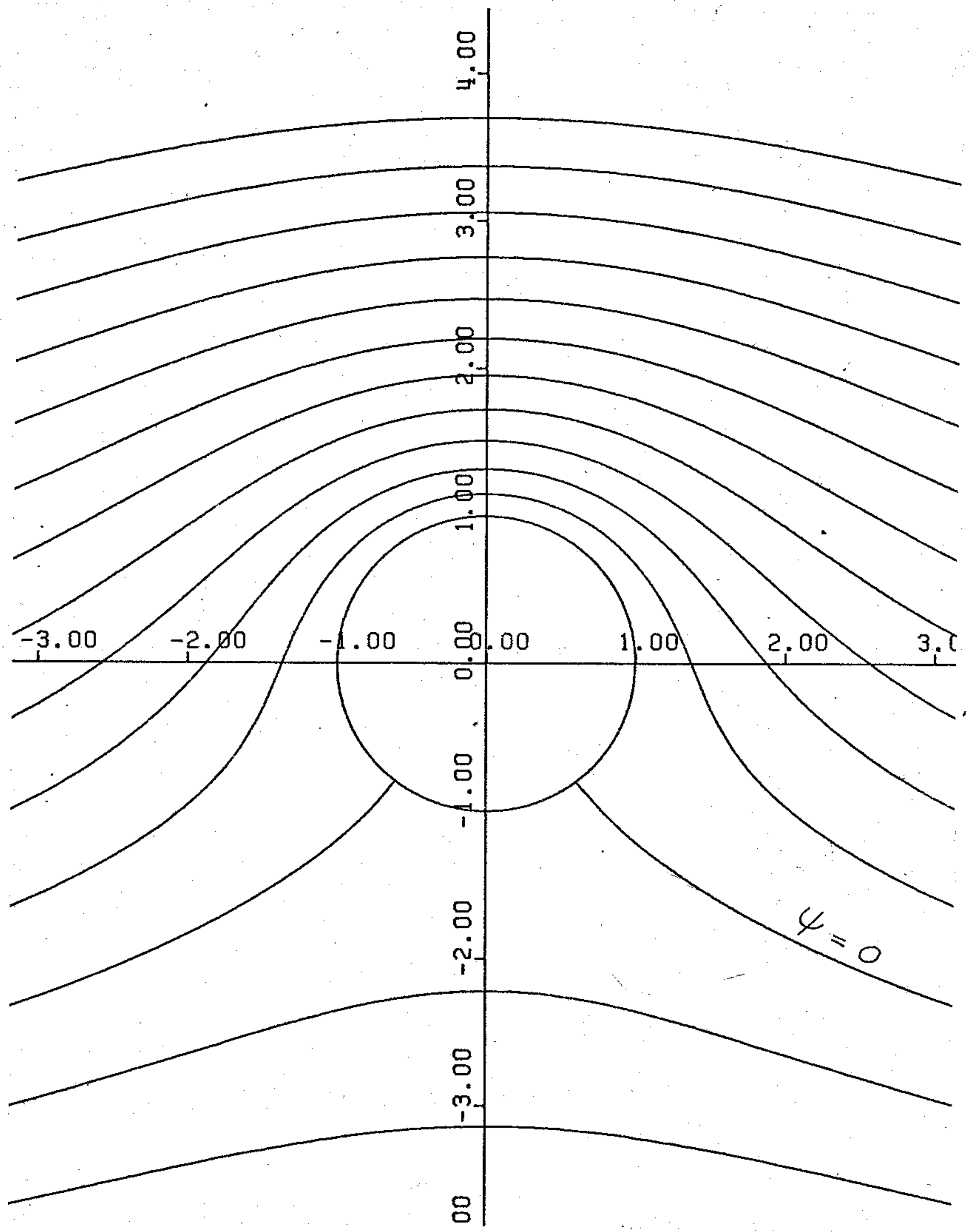

Fig. 2 Potential flow about a circle with $K = 10\ ft^2/sec$, $a = 1$ ft and $V = 1$ ft/sec.

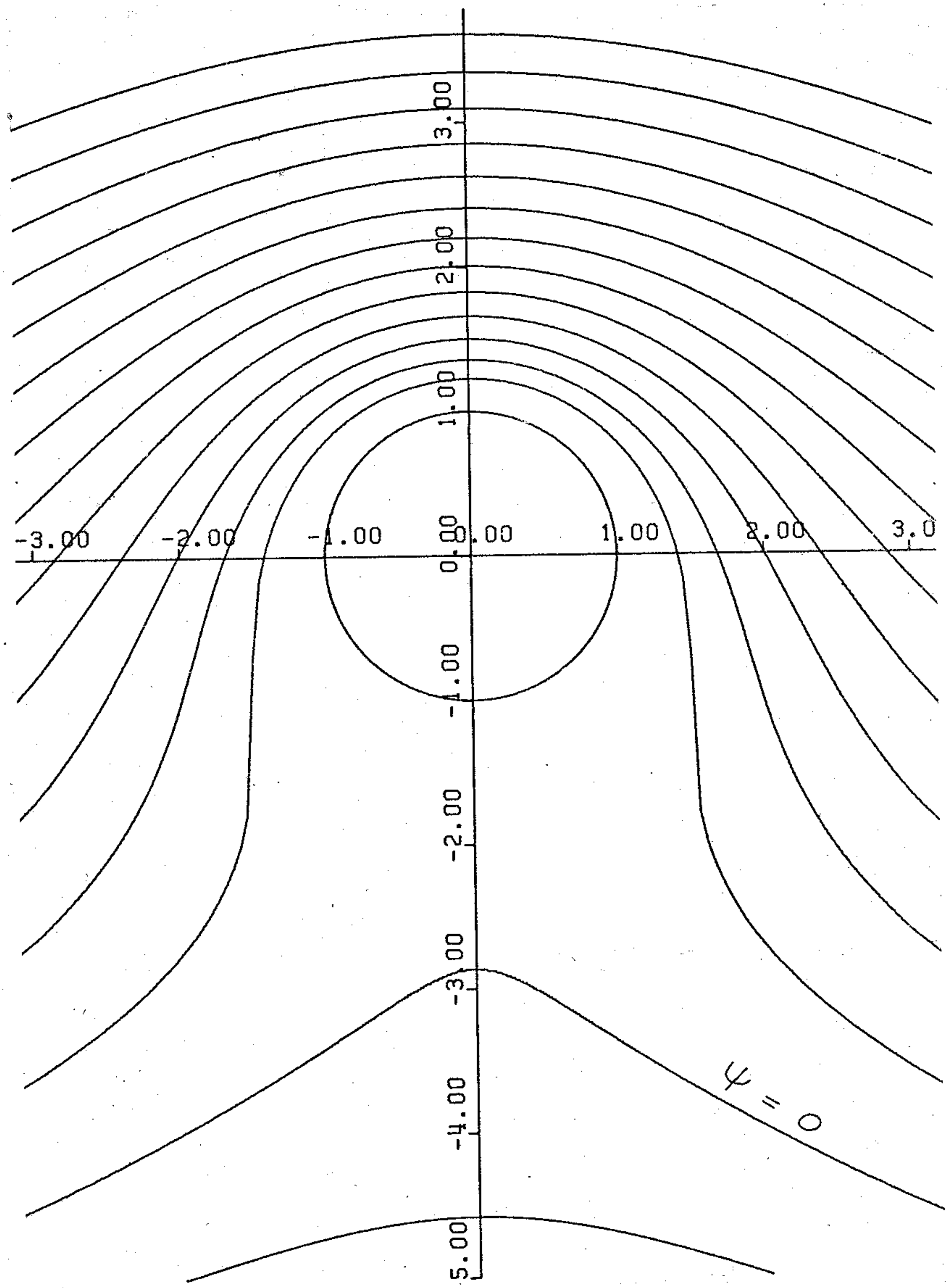

Fig. 3 Potential flow about a circle with $K = 18$ ft^2/sec
$a = 1$ ft and $V = 1$ ft/sec.

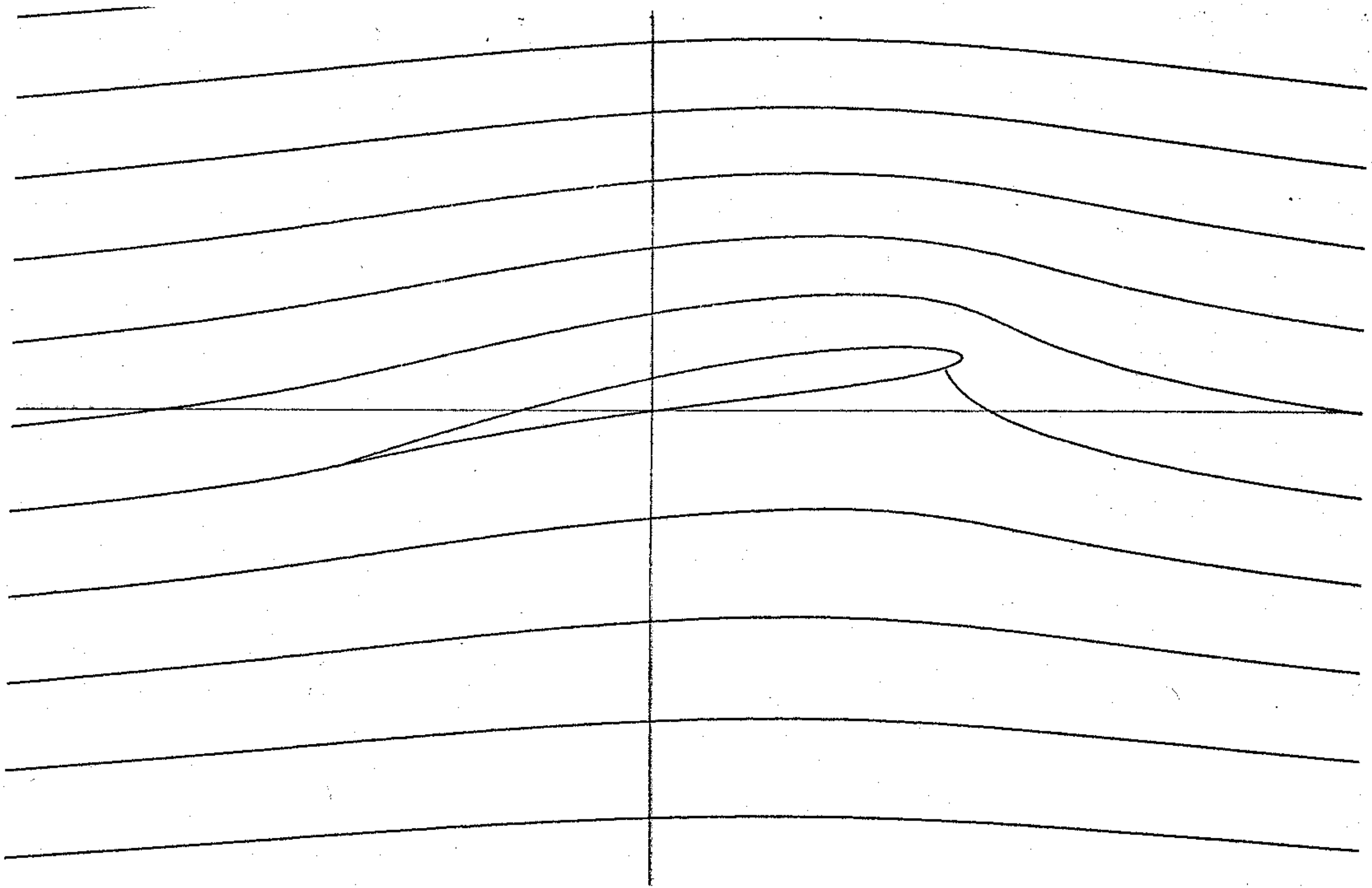

Fig. 4 Potential flow about a Joukowski airfoil with $b_o = 0.9588$, $b = 0.9487$, $\theta = 2.831$ degrees and $\alpha = 10$ degrees.

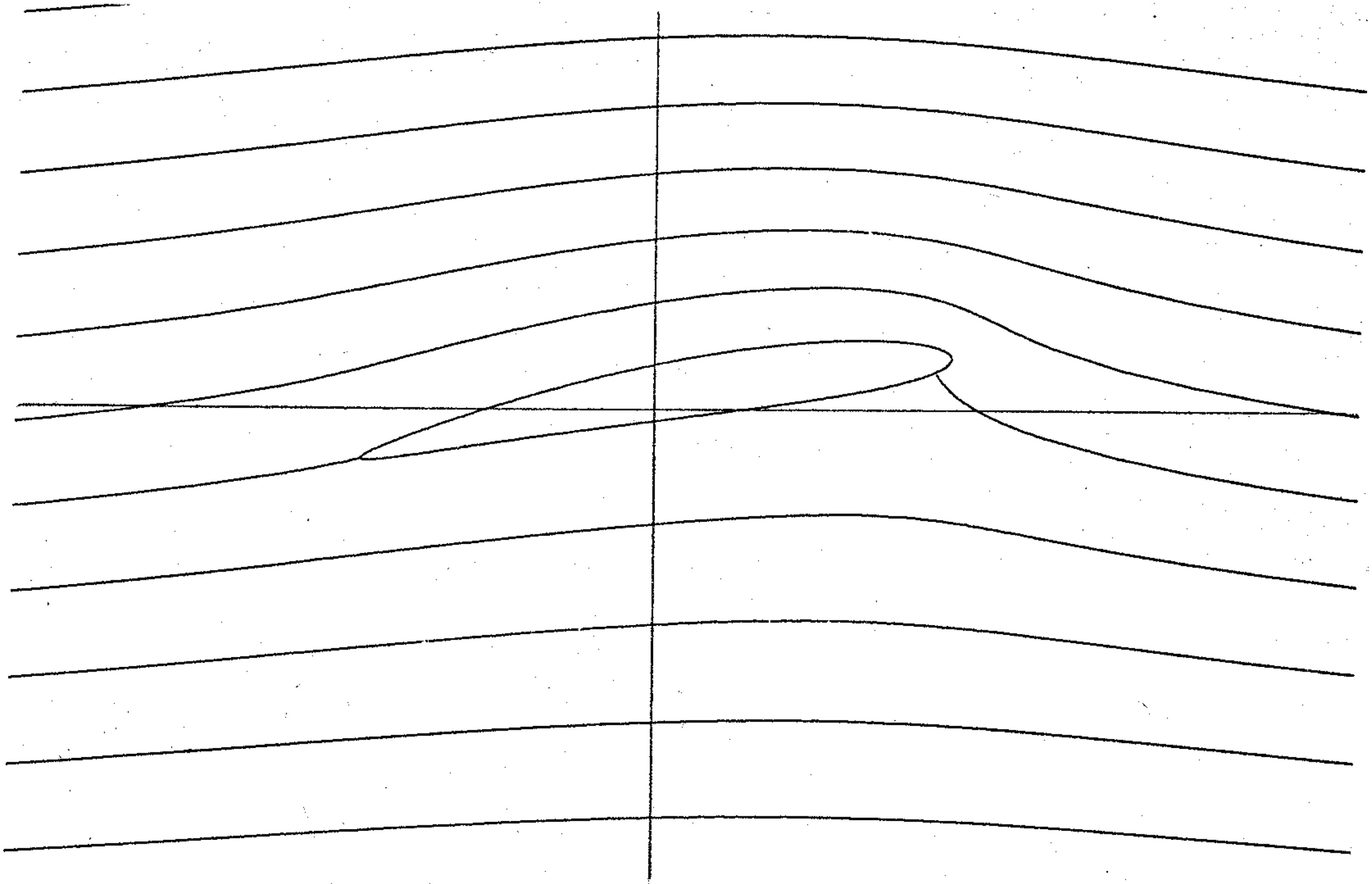

Fig. 5 Potential flow about a Joukowski airfoil with $b_o = 0.9588$, $b = 0.9110$, $\Theta = 13.348$ degrees and $\alpha = 10$ degrees.

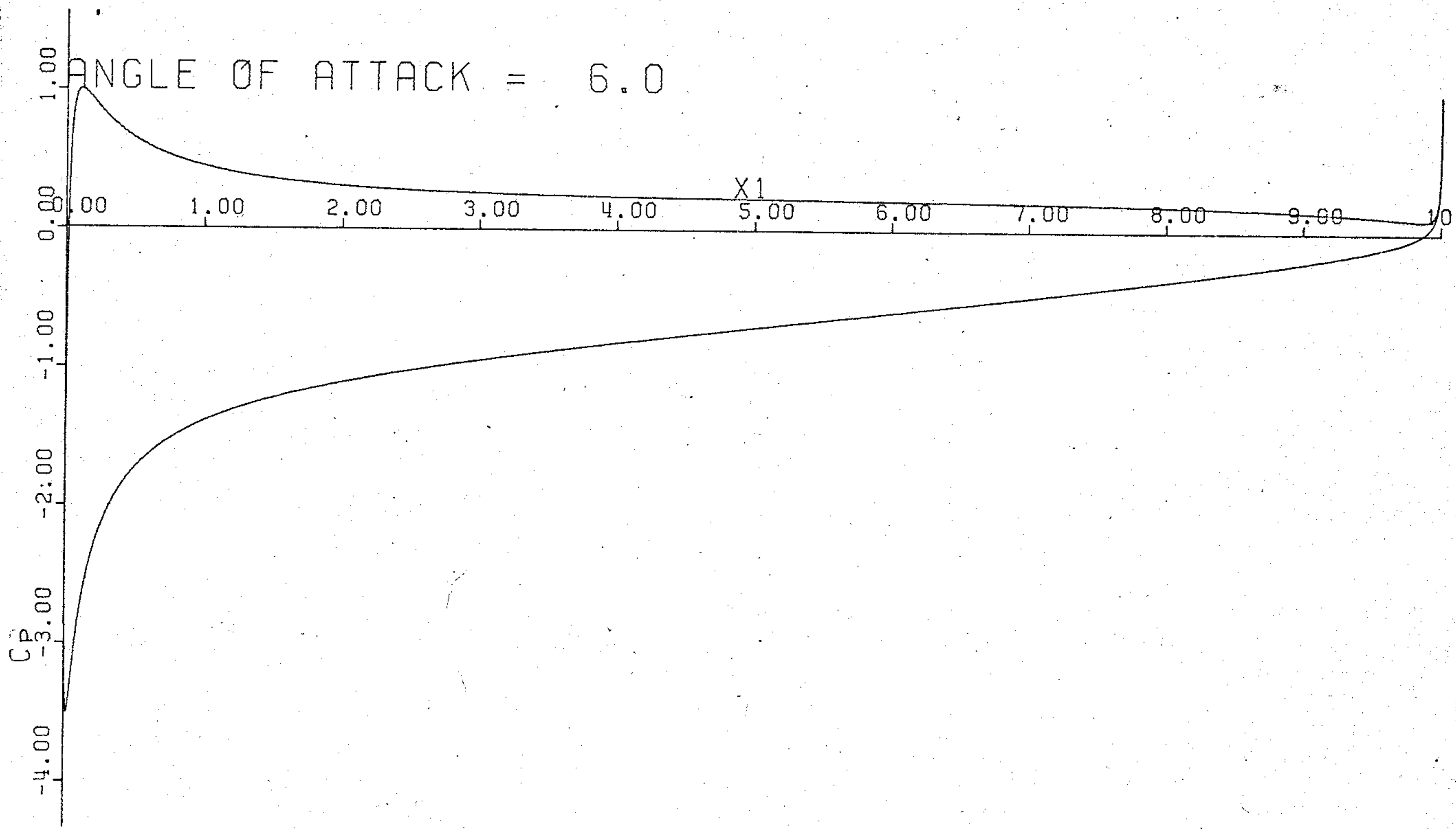

Fig. 6 Chordwise pressure distribution for the Joukowski airfoil as in Fig. 5 except α = 6 degrees.

A COMPUTER-PLOTTER PROGRAM$

ON JOUKOWSKI AIRFOIL

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J.H.T. Wu, K.S. Wong & P.P. Ostrowski

Department of Mechanical Engineering
McGill University

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ABSTRACT

Steady, incompressible flow about a Joukowski airfoil section is simulated at moderate angles of attack using a digital computer and plotter. The effect of various transformation parameters as well as the streamline pattern and pressure distribution about the airfoil is illustrated. Numerical integration of the pressure distribution at 6.50° angle of attack yields a lift coefficient that agrees favorably with known experimental results.

The value of the simulation technique as a teaching tool is demonstrated. The method is shown to be particularly useful for flow visualization as all the basic features of the Joukowski transformation and the potential flow streamline patterns are clearly depicted. Since only relatively unsophisticated computing methods are required, the simulation process provides a flexibility that is conveniently coupled with an obvious time-saving and economic advantage as compared to that done by slide rules or desk calculators.

$ This work was supported by McGill Computing Center.

1. Introduction

The principle of aerodynamic analysis of two-dimensional flow about an airfoil through conformal transformation is well established (Glauert, 1926). Essentially, this method consists of mapping the potential flow about an infinite circular cylinder with circulation into a complex plane in which the cylinder is transformed into an airfoil shape. The streamlines are transformed as well, hence the flow pattern about the airfoil is easily obtained. By varying the origin of coordinates in the complex plane associated with the cylinder and the parameters of the transformation, different airfoil shapes may be obtained.

2. Potential Flow About a Circular Cylinder

In order to successfully apply the technique of conformal transformation, the flow must be steady, two dimensional, incompressible and irrotational. Under these conditions the velocity potential ϕ and the stream function ψ may be defined. Since these functions are harmonic, i.e., everywhere continuous and orthogonal, lines of constant ψ (which represent the streamlines) are also subject to conformal transformation.

The transformation from one coordinate system to another system is accomplished with the aid of the complex potential function ω

$$\omega = f(z) = \phi + i\psi \qquad \text{--------------(1)}$$

which is defined in terms of the complex coordinate

$$z = x + iy = re^{i\theta}$$

For the flow about a circular cylinder with circulation in a uniform stream parallel to the x-axis, the potential function ω is given by

$$\omega = U\left(z + \frac{a^2}{z}\right) + \frac{i\Gamma}{2\pi} \ln \frac{z}{a} \tag{2}$$

where U is the free stream velocity, a is the radius of the cylinder and Γ is the circulation. In the present application, the streamlines are of primary interest and according to (1) the appropriate stream function is (see Figs. 2 and 3)

$$\psi = Uy\left(1 - \frac{a^2}{x^2+y^2}\right) + \frac{\Gamma}{2\pi} \ln\left(\frac{x^2+y^2}{a^2}\right)^{\frac{1}{2}} \tag{3}$$

Thus the line $\psi = 0$ corresponds to the stagnation streamline, part of which is coincident with the boundary of the cylinder $r = a$.

The Joukowski Transformation

The simplest transformation which maps the flow about an infinite circular cylinder in the $z = x + iy$ plane into a flow about an airfoil in the new $\zeta = \xi + i\eta$ plane is the Joukowski transformation

$$\zeta = z + \frac{b^2}{z} \tag{4}$$

where b is an arbitrary, but real, constant. The singular points of the transformation are $z = 0, \pm b$. Therefore the constant b should be chosen so that these points (which lie on the x-axis) fall within or on the circle $r = a$. In the z plane, the point $z = re^{i\theta}$ is transformed into the ζ plane according to the relations

$$\xi = \left(r + \frac{b^2}{r}\right)\cos\theta, \qquad \eta = \left(r - \frac{b^2}{r}\right)\sin\theta \tag{5}$$

Furthermore, a circle of radius a in the z plane corresponds to the equation in the ζ plane

$$\frac{\xi^2}{(a+\frac{b^2}{a})^2} + \frac{\eta^2}{(a-\frac{b^2}{a})^2} = 1$$

hence in the general case the circle is transformed into an ellipse.

The manner in which velocities are transformed can be inferred from equation (1) since the velocity components u and v relative to the x and y axes respectively are given by

$$\frac{dw}{dz} = u - iv$$

The analogous expression in the ζ plane leads to the velocity transformation relation

$$V_\zeta = \frac{V_z}{\left|\frac{d\zeta}{dz}\right|} \qquad (6)$$

where V_ζ and V_z are the velocities at corresponding points in the ζ and z planes respectively. For the present case where the transformation is given by (4) it is evident that

$$\left|\frac{d\zeta}{dz}\right|^2 = \left|1 - \frac{b^2}{r e^{2i\theta}}\right|^2 = \left(1 - \frac{b^2}{r^2}\cos 2\theta\right)^2 + \left(\frac{b^2}{r^2}\sin 2\theta\right)^2 \qquad (7)$$

A more conventional airfoil shape and flow pattern can be obtained by exploiting several features of the transformation. The airfoil will have a finite thickness and camber if the center of the circle $r = a$ is offset by an amount $z_0 = x_0 + i y_0$ from the origin of coordinates (Figure 1). In particular, a circular arc airfoil results (zero thickness) if $x = 0$ and a symmetrical profile (zero camber) is obtained if $y = 0$. The shape of the airfoil section is also dependent upon the choice of the parameter $\frac{b}{a}$, which is more or less arbitrary. Specifically, the angle β controls the camber of the airfoil while the ratio $\frac{b}{a}$ fixes the thickness. Referring to Figure (1) and Equation (5), it can be seen that the point S ($\theta = -\pi$) where the circle cuts the negative x axis will be transformed to the trailing edge of the airfoil. If S is a singularity of the transformation the mapping is no longer conformal at this point, and the airfoil takes on a sharp trailing edge. For this reason b is frequently chosen as the distance b_o in the diagram, and this choice always results in a cusped trailing edge as $d\zeta/dz$ is then zero at S. Although this characteristic is usually avoided in the design of most aircraft wings, this procedure results in an airfoil with a well rounded leading edge, as the other singularities $z = 0, \pm b$ will then lie well inside the circle in the z plane.

Several methods may be employed to eliminate the cusped trailing edge, and these usually involve modifying the transformation so that a finite trailing edge angle is produced with $b < b_o$, i. e., reducing the value of b slightly so that the singularity lies just inside the circle and the trailing edge becomes slightly rounded (Bairstow, 1946).

Angle of Attack and Circulation

The Joukowski transformation does not alter the flow pattern at $z = \pm\infty$, hence if the x, y axes are at some angle α relative to the free stream velocity U, the airfoil will be at an angle of attack α in the ζ plane. However, for arbitrary values of the circulation Γ, the transformation usually leads to the physically inadmissable situation of infinite velocities at the trailing edge of the airfoil. For the case of a cusped airfoil this difficulty is overcome by applying the Kutta condition which states that the circulation should be such that the flow leaves the trailing edge smoothly but with a finite velocity, and is equivalent to demanding that the point S be a stagnation point on the circular cylinder.

This procedure leads to a unique relation between the circulation Γ and the angle of attack α

$$\Gamma = 4\pi a U \sin(\alpha + \beta) \tag{8}$$

where the angle β is defined in Figure 1. Although in this case Equation (6) leads to an indeterminate expression, a simple application of L'Hospitals' rule shows that the velocity of the trailing edge is indeed finite.

When the trailing edge is rounded, some authors remark that the Kutta condition "is lost" (Glauert, 1926; Pope, 1951). Despite this, one can still demand that point S be a stagnation point in the z plane so that Equation (8) remains valid. The only difference is that $d\zeta/dz$ is now finite since b is inside the circle so that the point S (the trailing edge) is also a stagnation point in the ζ plane.

The flow field around the airfoil can then be completely determined. However, the potential and stream functions defined by Equations (1), (2) and (3) correspond to the case where the uniform U stream is parallel to the x-axis. Thus points in the flow field are computed in this coordinate system (Figure 1) and are then rotated through an angle α by the rotational matrix

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \tag{9}$$

and translated by the amount $z_0 = x_0 + i y_0$ into the z plane from which the transformation takes place.

Pressure Distribution and Lift

The ultimate aim of the conformal transformation is to obtain the velocity distribution in the ζ plane so that the aerodynamic performance of the airfoil can be calculated. In particular, the lift is of primary interest, and this is determined by computing the velocity (and hence the pressure) on the surface of the airfoil. This velocity, which is purely tangential to the body surface and corresponds to the $\psi = 0$ streamline, is denoted by the subscript (o) and in the z' plane (relative to the $x' - y'$ axes

$$V_o = -\left.\frac{\partial \psi}{\partial r'}\right|_{r'=a} = 2U \sin\theta' + \frac{\Gamma}{2\pi a}$$

Then according to (6) and (7) the velocity in the ζ plane is

$$V_o = \left(2U\sin\theta' + \frac{\Gamma}{2\pi a}\right) \Big/ \left[\left(1 - \frac{b^2}{r^2}\cos 2\theta\right)^2 + \left(\frac{b^2}{r^2}\sin 2\theta\right)^2\right]^{1/2} \tag{10}$$

where (r , θ) are the coordinates in the $x - y$ system that correspond to the cylinder coordinates (r', θ') in the $x' - y'$ system.

Since the flow is incompressible and irrotational, the pressure distribution is given by Bernoulli's equation

$$p_o + \frac{1}{2}\rho V_o^2 = \text{constant} = p + \frac{1}{2}\rho V^2$$

where p_o is the pressure on the surface of the airfoil while ρ and p are the density and free stream pressure respectively. The pressure coefficient for the airfoil is then given by

$$C_p = \frac{p_o - p}{\frac{1}{2}\rho U^2} = 1 - \left(\frac{V_o}{U}\right)^2 \tag{11}$$

Despite the fact that the lift force in the direction perpendicular to the free stream can be computed directly by the well known Kutta-Joukowski theorem

$$L = \rho U \Gamma$$

The same result can be obtained by integrating the pressure force around the airfoil. In the ξ and η directions (parallel and perpendicular to the airfoil chord) the net forces are (see Fig. 9 for F_ξ)

$$F_\eta = \frac{1}{2}\rho U^2 \int_{\eta_{min}}^{\eta_{max}} (C_{p_+} - C_{p_-})\, d\eta \;, \quad F_\xi = \frac{1}{2}\rho U^2 \int_{\xi_{min}}^{\xi_{max}} (C_{p_+} - C_{p_-})\, d\xi \tag{12}$$

where the subscripts (+) and (-) refer to locations that are positive or negative with respect to the ξ and η axes. However, since the airfoil (i.e., ξ axis) is at an angle of attack α relative to the free stream, the net lift perpendicular to the α direction is

$$L = F_\xi \cos\alpha - F_\eta \sin\alpha$$

and the lift coefficient per unit span is

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 c} = \frac{F_\xi \cos\alpha - F_\eta \sin\alpha}{c} \tag{13}$$

where c is the chord of the airfoil ($c = \xi_{max} - \xi_{min}$). In accordance with the theoretical considerations the lift and drag forces should be equal to $\rho U \Gamma$ and zero respectively.

Computer Simulation

Once the airfoil shape and free stream conditions are decided upon (U , a , α , b , x_0, y_0 are specified) the computational procedure is straightforward. First, the angle β is calculated and the circulation Γ is computed from Equation (8). The streamlines in the z plane are determined by specifiying ψ and varying one of the coordinates in the z plane (say x) and calculating the other (y) from Equation (3) by iteration. These coordinates are then rotated by (9), translated by an amount z_o and finally transformed into the plane according to Equation (5). Velocities are transformed by Equation (10) and the pressure distribution is determined from Equation (11). Finally, the lift force components are calculated by Equation (12) with the aid of a numerical integration scheme and the lift coefficient is computed from Equation (13).

The above procedure is a relatively simple operation on a digital computer and the results can be conveniently displayed by a digital plotter. In this way various parameters associated with the transformation can be varied systematically and visual results obtained rapidly with comparitive ease. Computer simulation of the phenomena therefore has a high instructional value.

A specific airfoil was chosen for the simulation and a description of the physical characteristics as well as the results of wind tunnel tests are presented in the literature, and it is denoted as airfoil " A " (Bairstow, 1946). This airfoil is constructed with the transformation parameters

$$x_o = .04a, \quad y_o = .05a, \quad b = (a^2 - y_o^2)^{\frac{1}{2}} - x_o = .96a, \quad b_o = .91a$$

which yield a slightly rounded trailing edge due to the fact that b is a little less than the value $b_o = 0.96$ required to place the singularity $z = -b$ on the circle $r = a$. The Joukowski transformation associated with these parameters gives an airfoil with a maximum thickness and camber of about 8% and 6% respectively.

For the simulation, the radius of the circle and the freestream velocity were conveniently chosen as

$$a = 1 \text{ ft.}, \qquad U = 1 \text{ ft/sec.}$$

and the angle of attack was varied between 0 and 10 degrees. Following the procedure outlined above, the streamlines in both the z' and ζ planes were constructed and the lift characteristics of the airfoil were determined. The simulation was carried out on an IBM 360 computer with the aid of a CALCOMP digital plotter. A computer-plotter program together with a sampling plot is included in this report.

Results and Discussion

In order to clarify as well as visualize the influence of the parameter b/a on the airfoil shape, streamline patterns were plotted for several values of $\frac{b}{a}$. As noted above, $\frac{b}{a} = 0.96$ will yield a cusped trailing edge while smaller values give a rounded trailing edge. Figures 4 to 8 show this influence rather dramatically for the cases of various $\frac{b}{a}$ at an angle of attack of 10°. Clearly, the airfoil takes on a nearly elliptical shape for relatively small values of $\frac{b}{a}$ and a more conventional shape as approaches the value appropriate to a cusped trailing edge. It is interesting to note that if the critical value $\frac{b}{a} = 0.96$ is exceeded and $b/a = 1$ is used, the airfoil profile becomes that of a flattened figure 8. Figure also illustrate the influence of parameter b/a upon the rounding of the leading edge, thickness and camber. In addition, the validity of the Kutta formula (equation 8) relating circulation and angle of attack is clearly demonstrated. In all cases the trailing edge, although rounded, is observed to be a stagnation point so that the flow proceeds smoothly over the airfoil.

Figure 2 and 3 show the streamline pattern and transformation parameters appropriate to airfoil "A" for the circulation $\Gamma = 5, 10$ ft^2/sec. and these results correspond to the unrealistic values of $\alpha = 20^\circ, 50^\circ$ respectively. They aptly illustrate the effects of circulation on the streamline pattern associated with the potential flow about a circular cylinder. In particular, the downward movement of the stagnation points is clearly and dramatically illustrated by the CALCOMP plotter.

The more interesting situation of the flow pattern in the ζ plane is shown in figure 6 which depicts the streamlines corresponding to airfoil "A" at an angle of attack of 10 degrees. First, it should be noted that the choice of $b/a = .911$ yields an airfoil shape that is quite conventional with only a slightly rounded trailing edge and a well rounded leading edge. As expected, the rear stagnation point occurs at the trailing edge while the forward stagnation point lies on the underside but near the nose of the airfoil. Leading edge suction effects are evident as the streamline spacing is observed to decrease as the angle of attack increases. Here, the usefulness and power of the Joukowski transformation is vividly demonstrated.

The chordwise (ξ direction) pressure distribution corresponding to an angle of attack of 6° is shown in figure (9), and it is evident that the figure is typical of conventional airfoil shapes. Particularly evident is the leading edge suction, although the sudden rise in pressure at the trailing edge (stagnation point) is not realistic owing to the relatively small wake that would be observed at this angle of attack in practice. Clearly demonstrated is the well known fact that most airfoils derive the major portion of their lift from the suction on the upper surface of the airfoil rather than the positive pressure on the underside, as might be supposed by a layman.

Concluding Remarks

The stated objective of adequately simulating the performance of a practical Joukowski airfoil section with a digital computer and plotter has been successfully accomplished for moderate angles of attack. Generally, the theoretical results with regard to the flow pattern and pressure distribution are reasonable, and comparison with the limited experimental results that are available is quite favorable.

It is evident that the simulation procedure has a high educational value, particularly with regard to streamline pattern visualization. All of the essential features of the potential flows involved as well as the lifting characteristics of airfoils are clearly and efficiently demonstrated by this technique. In addition, the finer points as well as the flexibility of the Joukowski transformation can be aptly illustrated. The computing techniques involved are relatively simple and straightforward, and despite the usual debugging problems that arise, the method can be applied by students with an elementary knowledge of computer programming.

Finally, the time saving and economic advantages of this simulation technique are self-evident. Indeed, the task of performing the calculations manually for just one angle of attack demands about sixteen man-hours (Pope, 1951) while the computer does the job in a matter of minutes, and for a fraction of the cost, and provides the streamline patterns as well.

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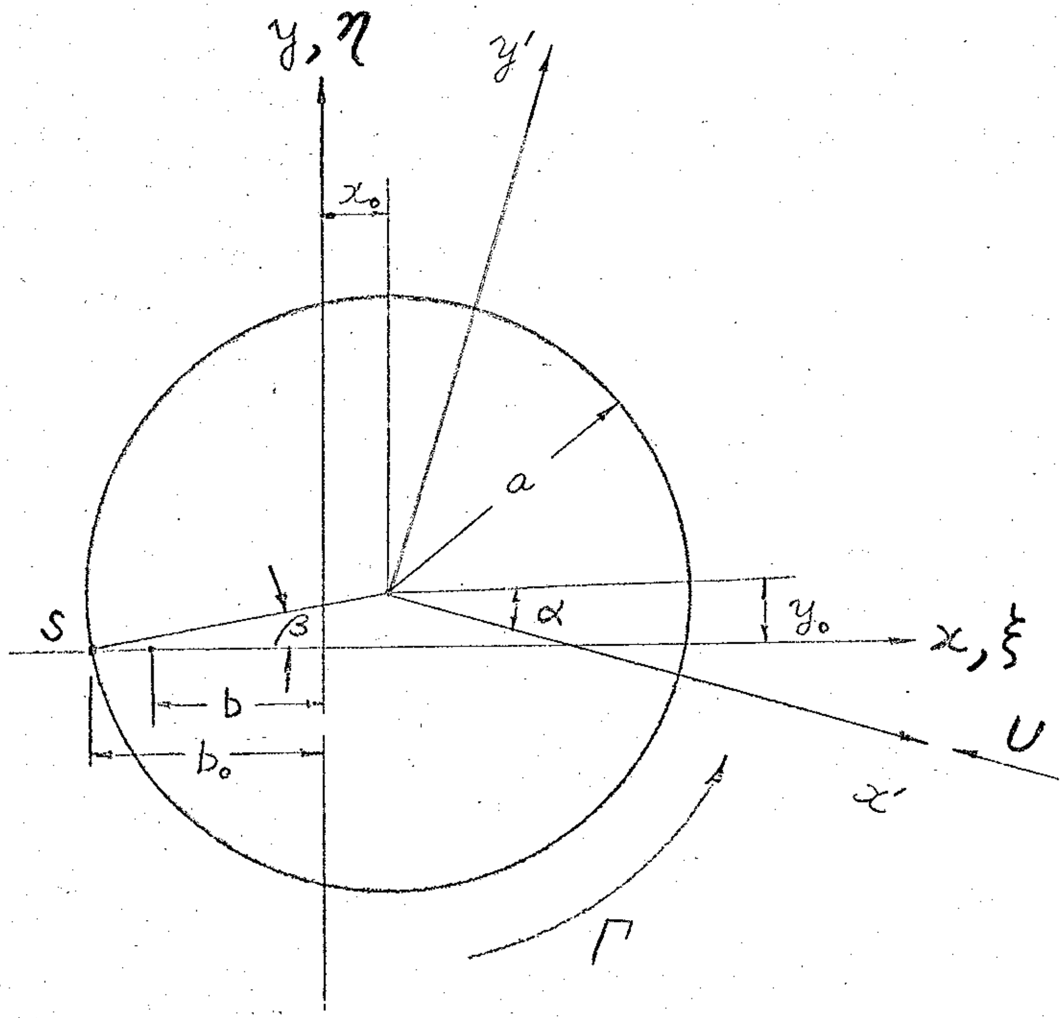


Fig. 1 Geometrical construction

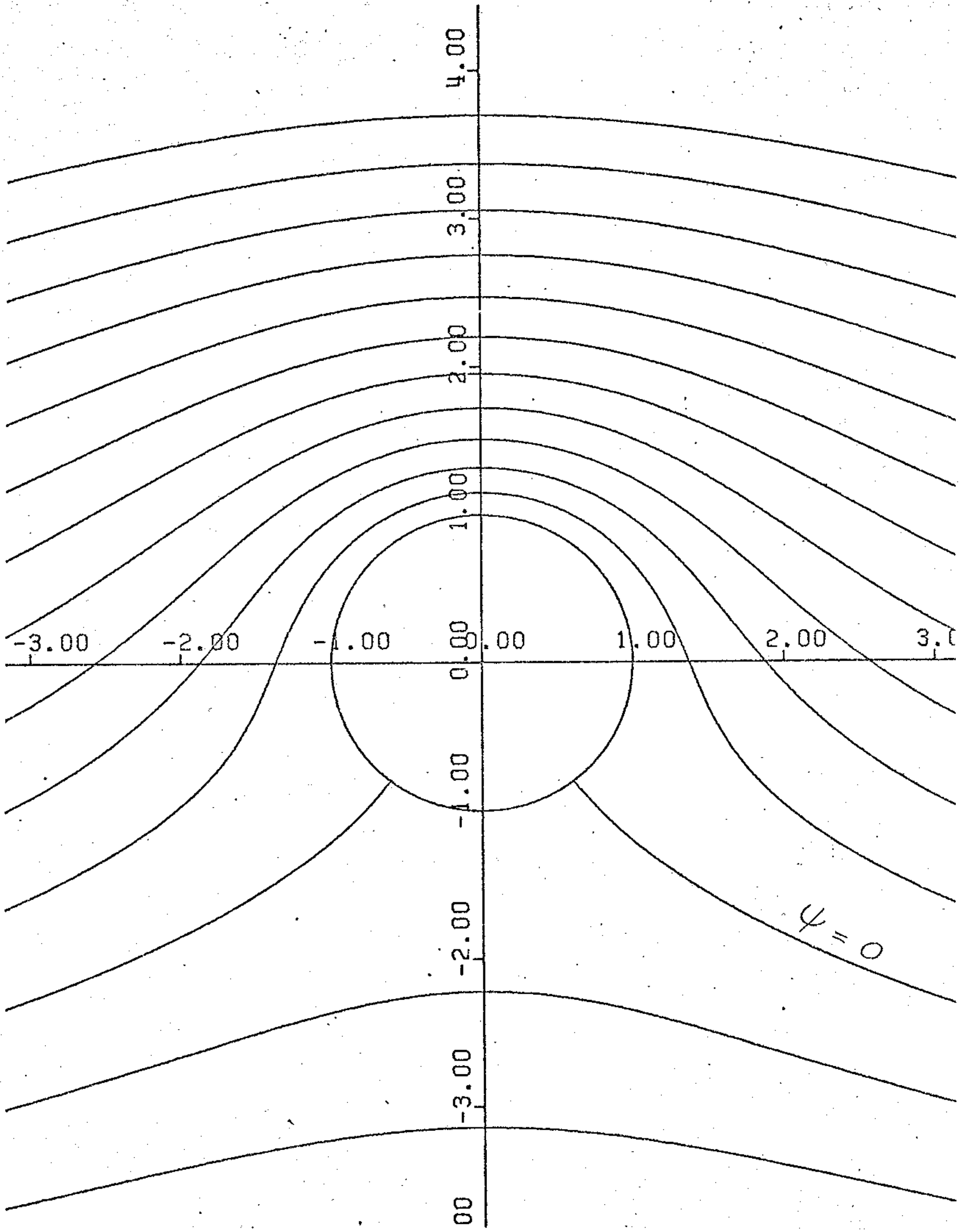


Fig. 2 Potential flow about a circle with $K = 10\ ft^2/sec$, $a = 1$ ft and $V = 1$ ft/sec.

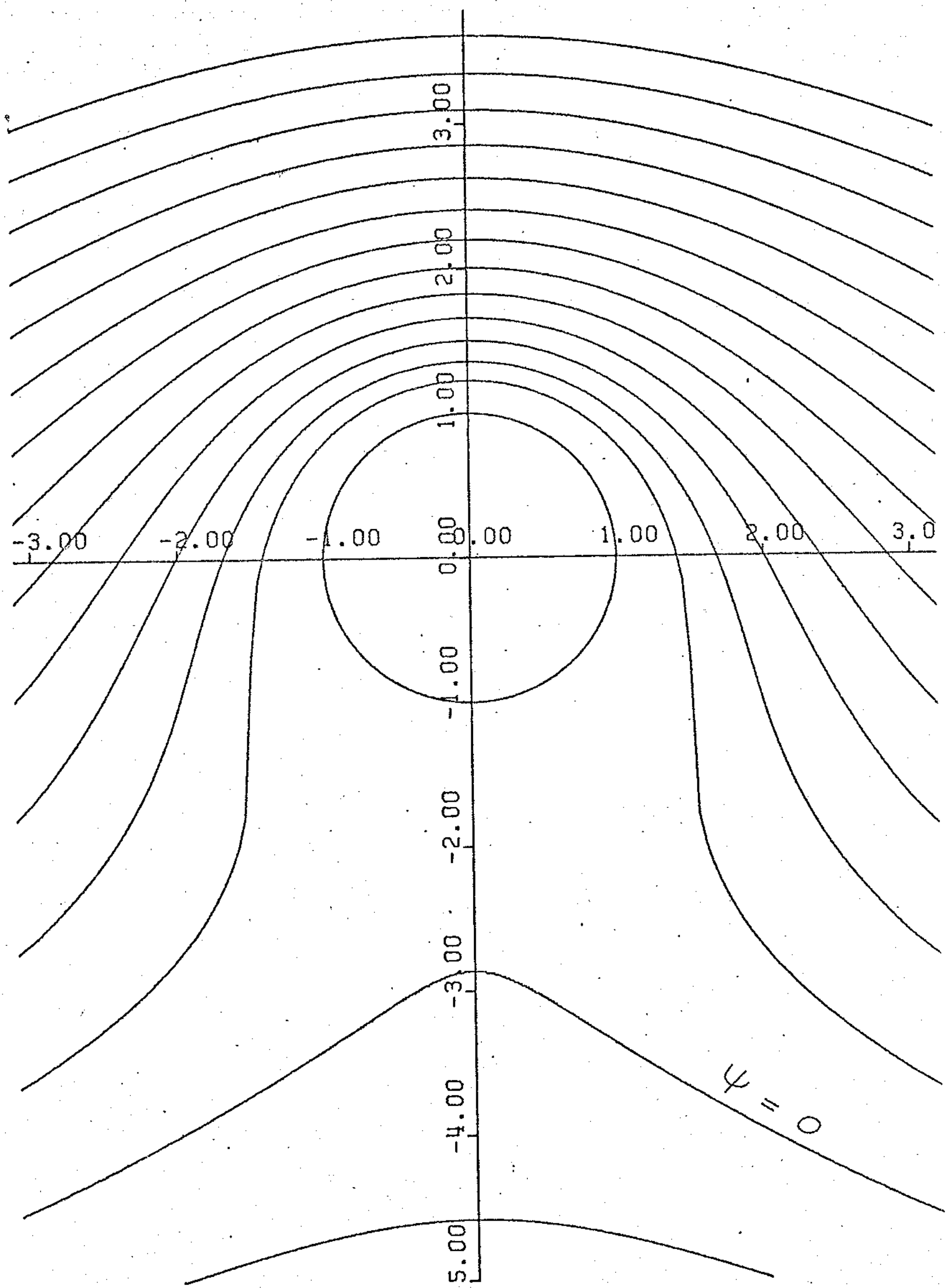


Fig. 3 Potential flow about a circle with $K = 18\ ft^2/sec$ $a = 1$ ft and $V = 1$ ft/sec.

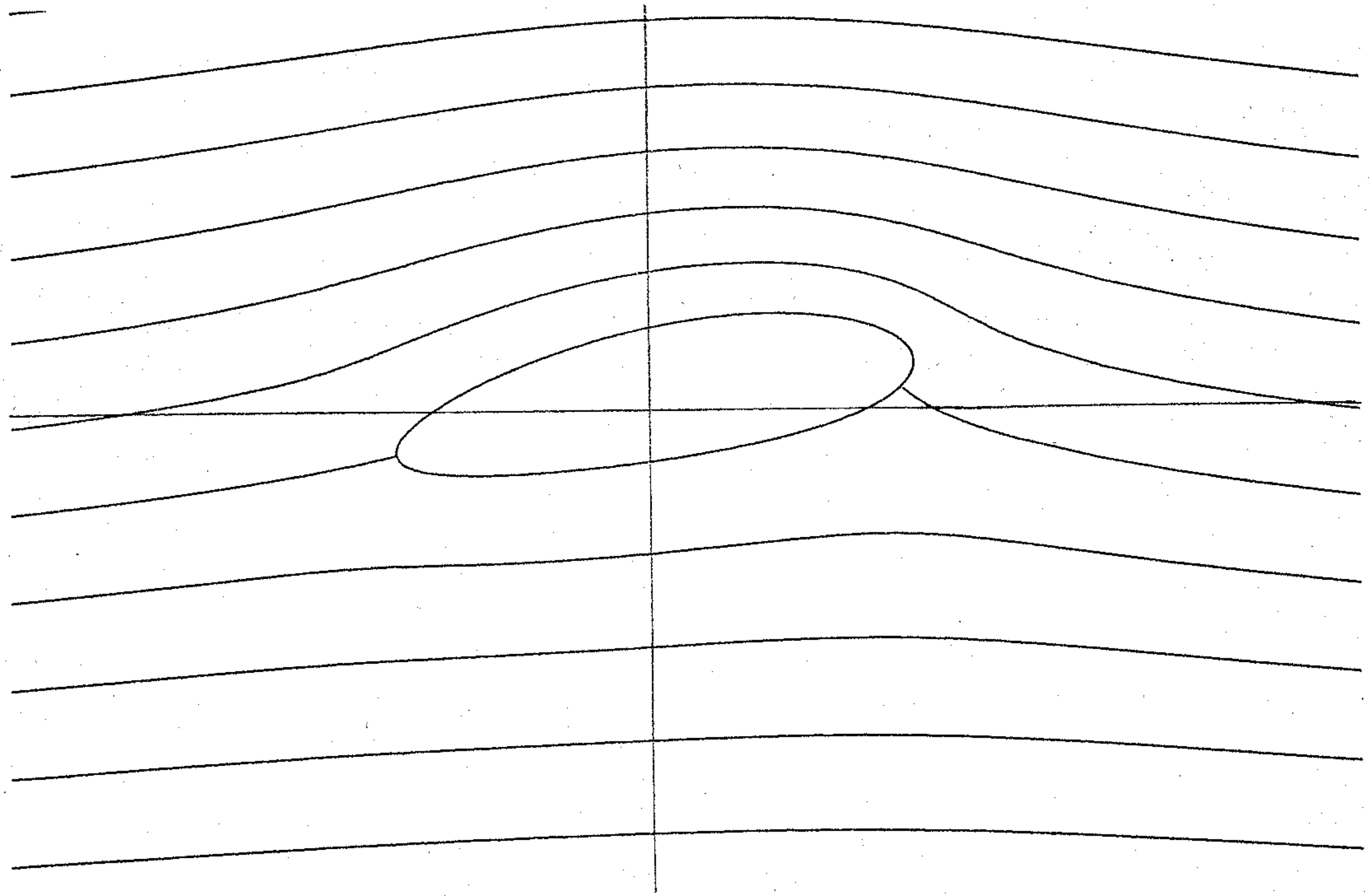


Fig.4 Potential flow about a Joukowski airfoil with $b_o = 0.9588$ and $b = 0.7748$ and $\alpha = 10$ degrees

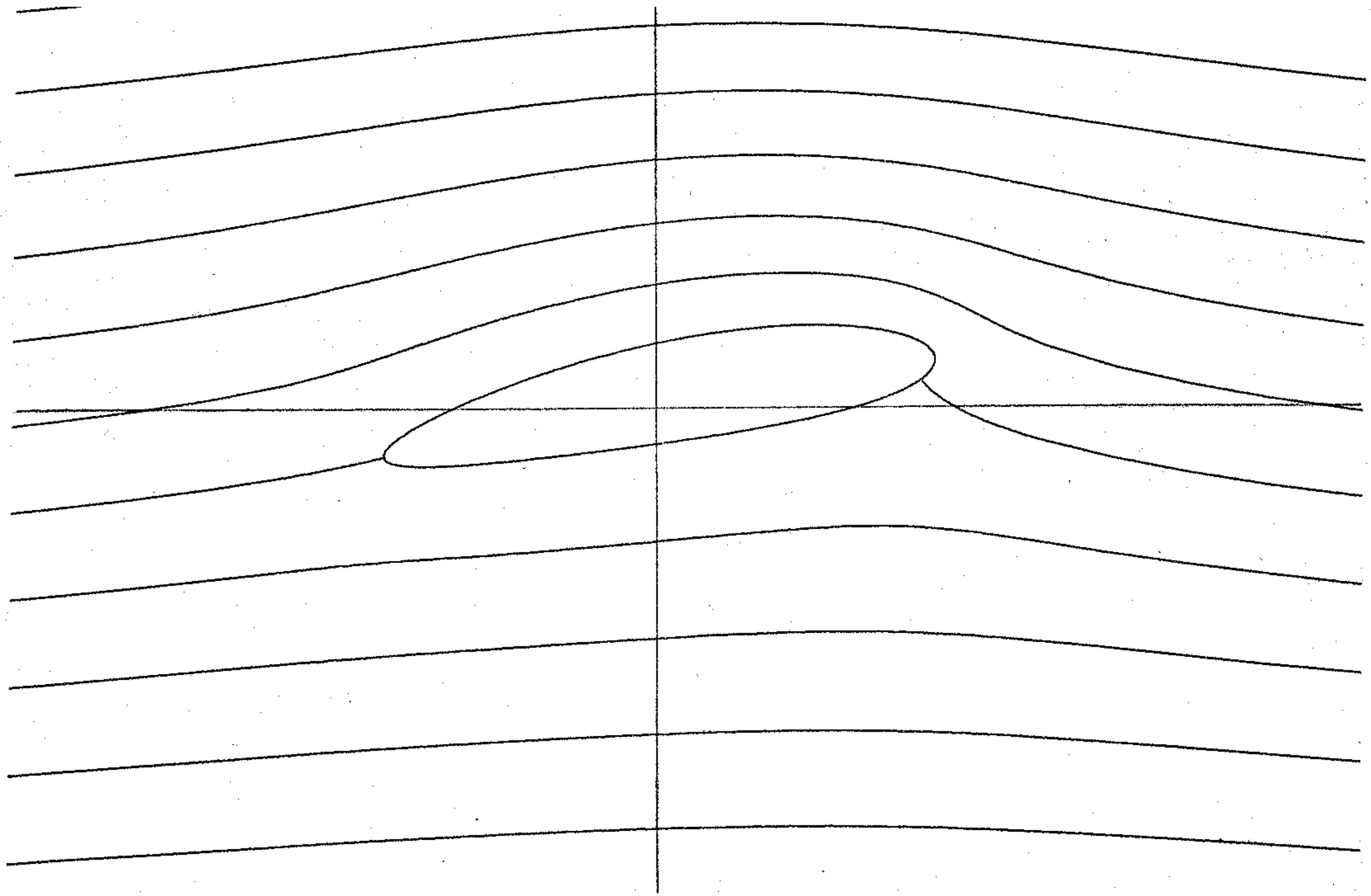


Fig.5 Potential flow about a Joukowski airfoil with $b_o = 0.9588$, $b = 0.8368$ and $\alpha = 10$ degrees

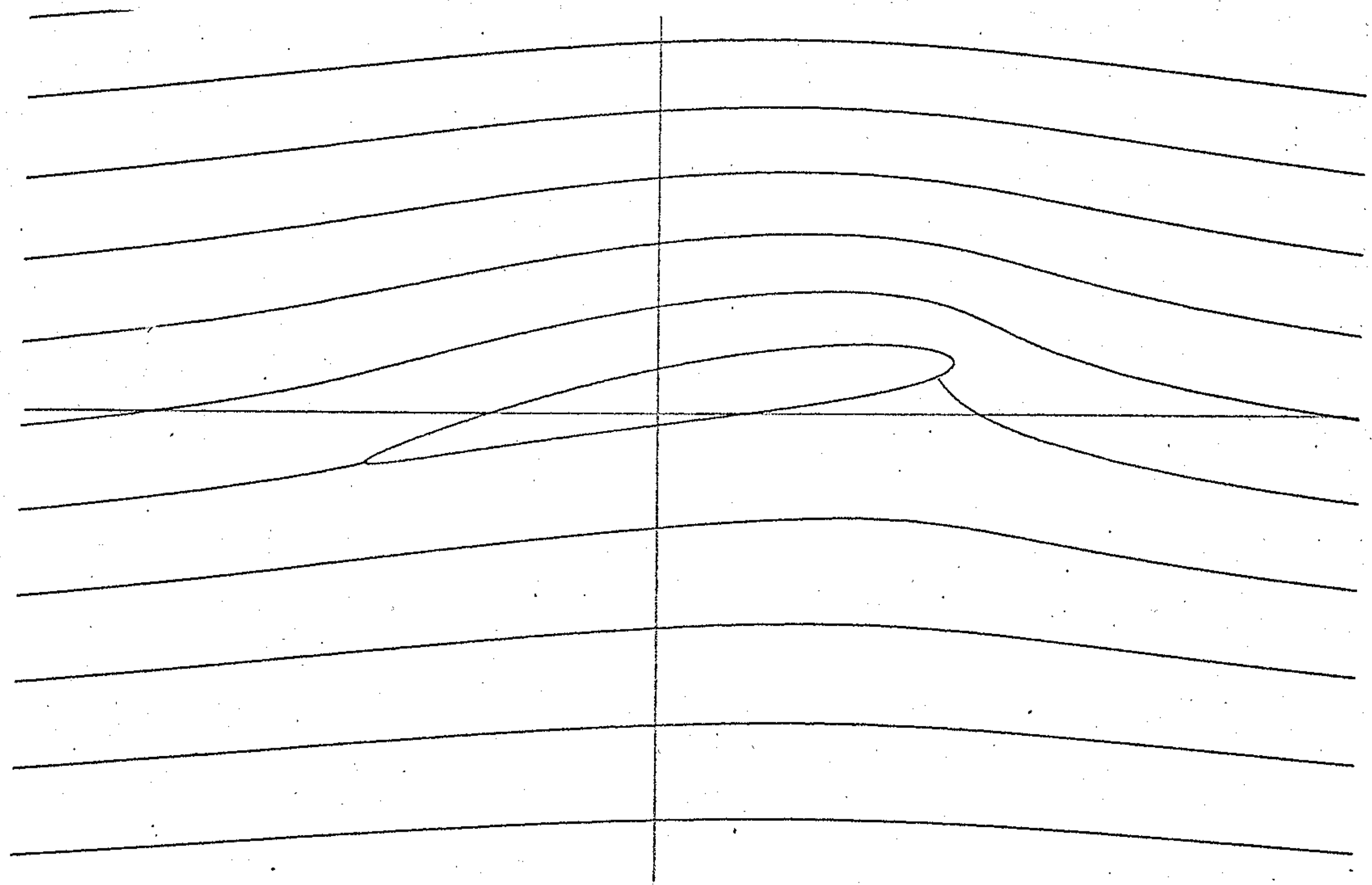


Fig. 6 Potential flow about a Joukowski airfoil with $b_o = 0.9588$, $b = 0.9110$, and $\alpha = 10$ degrees

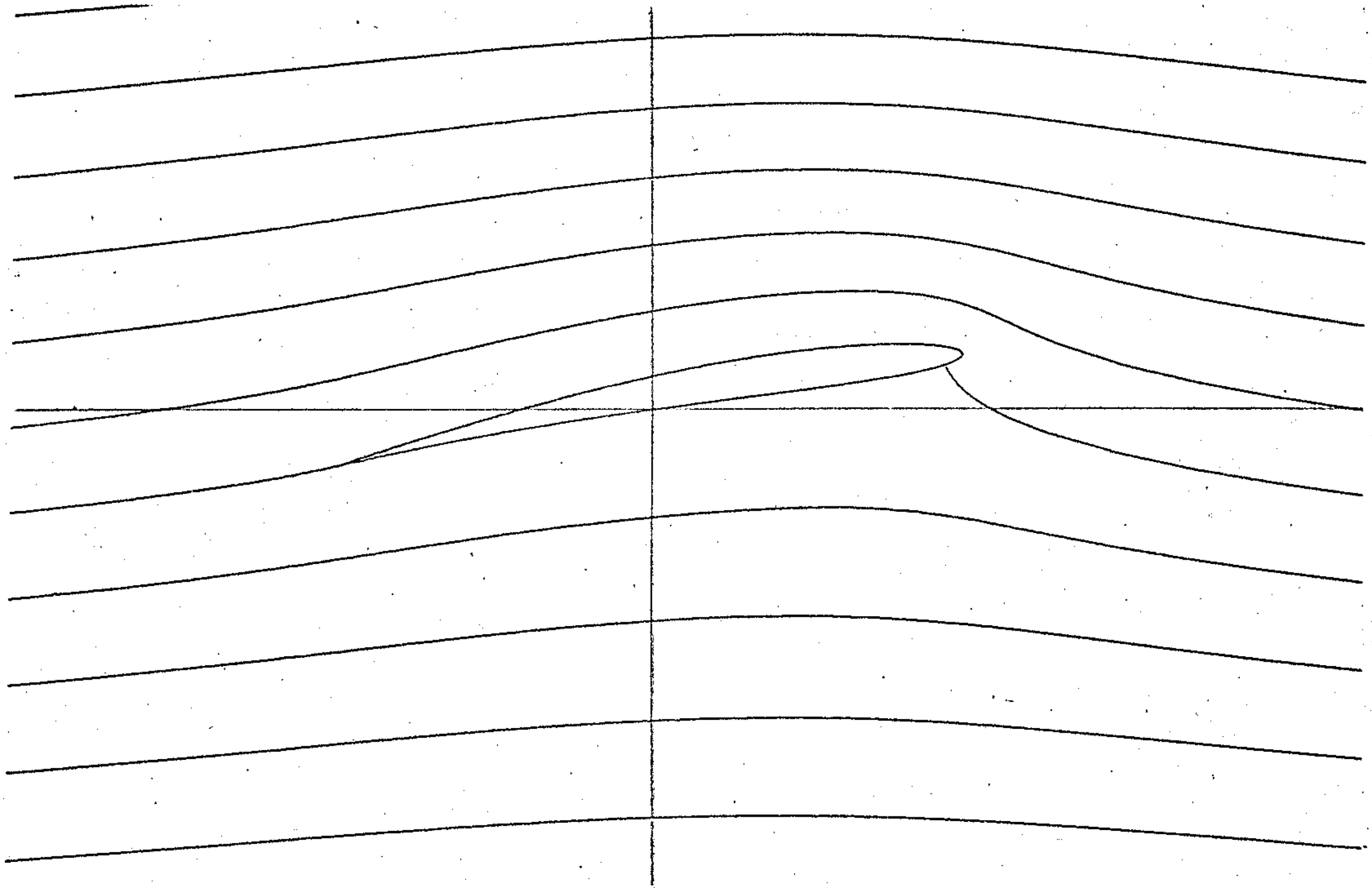


Fig. 7 Potential flow about a Joukowski airfoil with $b_o = 0.9588$, $b = 0.9487$, and $\alpha = 10$ degrees

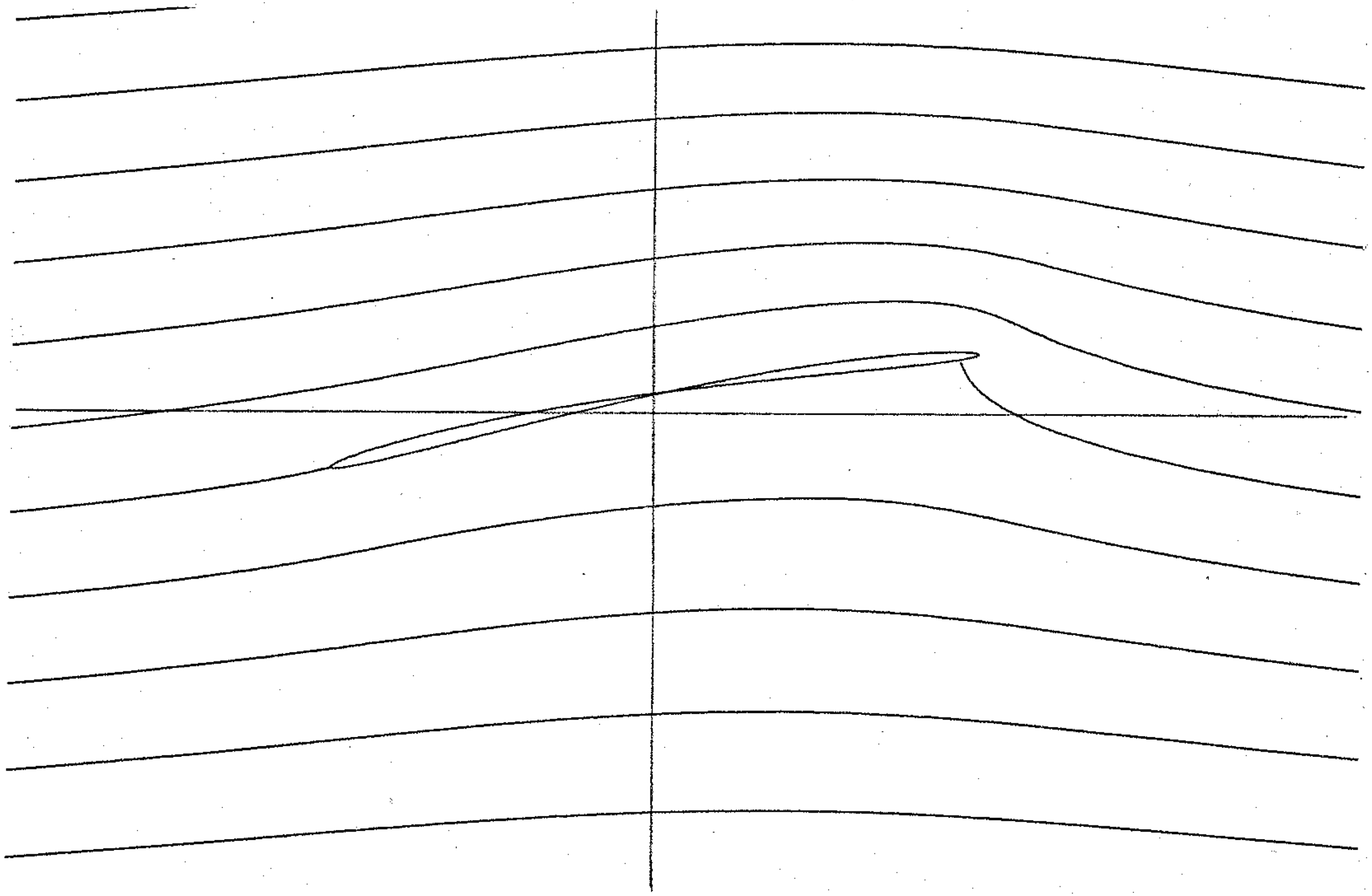


Fig. 8 Potential flow about aJoukowski airfoil with $b_o = 0.9588$, $b = 1.0$ and $\alpha = 10$ degrees

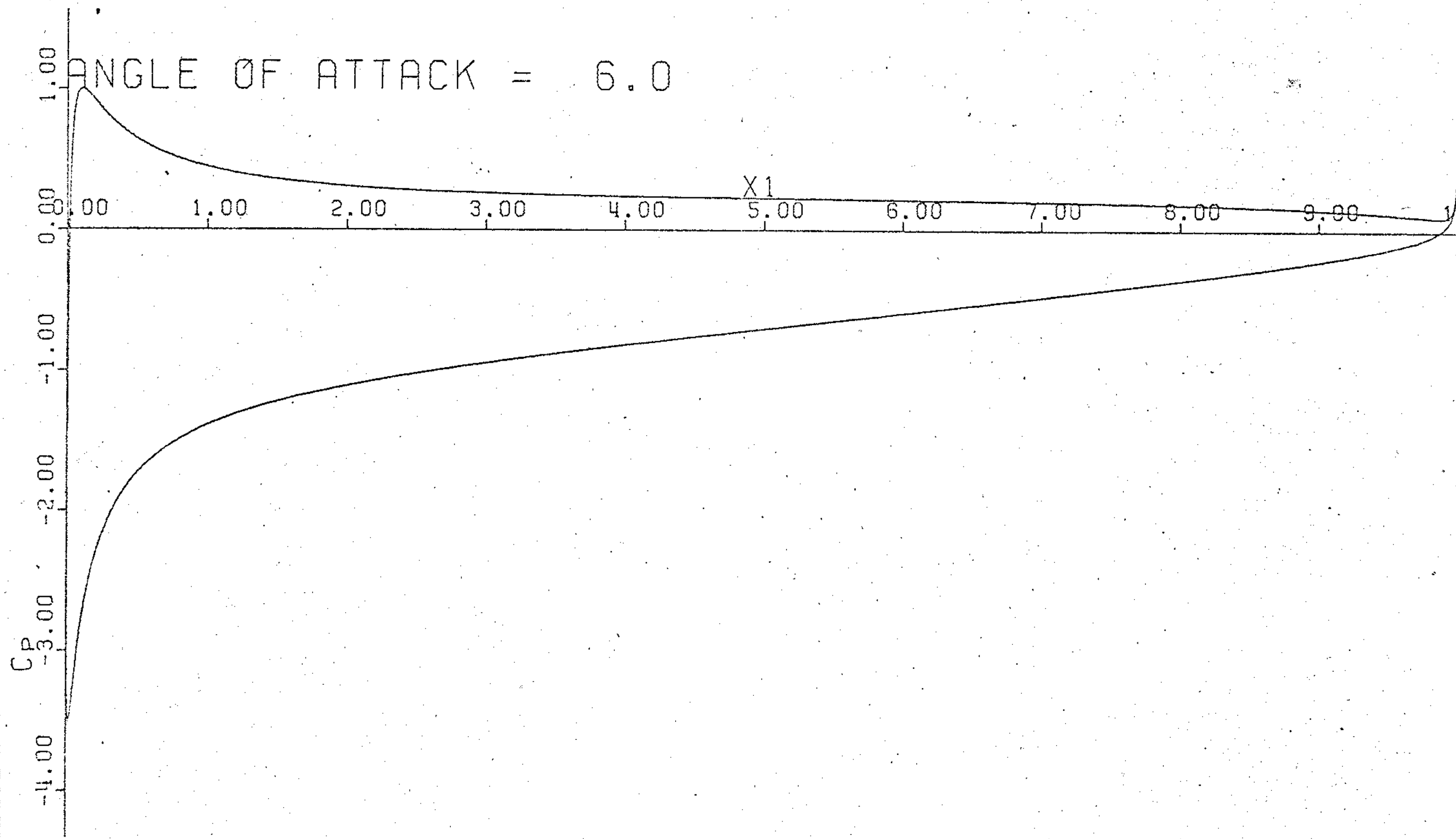


Fig. 9 Chordwise pressure distribution for the Joukowski airfoil as in Fig. 6 except $\alpha = 6$ degrees.

```
      DIMENSICN ITIMES(400),X(400),Y(400),YTRIAL(200),
     1  ZATA(400),ETA(400)
      DIMENSION XP(400),YP(400),XRBODY(400),YRBODY(400)
      CALL PLOTCN
      XLIMIT=8.0
      PSILIM=3.5
      A=1.0
      U=1.0
      CALL SYMBCL(6.0,2.0,0.25,
     1  17HJCUKOWSKI AIRFOIL,00.0,17)
      CALL SYMBCL(7.0,1.5,0.2,
     1  8HANCLE # ,.00,8)
C     ALPHA=ANGLE CF ATTACK(INCIDENCE), GAMMA=CIRCULATION
      XG=0.04*A
      YG=0.05*A
      BATA=ARSIN(YG/A)
C
      CALL REODY(A,XRBCDY,YRBODY,M)
      ALPHAD=10.0
      ABC=.83
      ALPHA=ALPHAD*3.14159/180.0
      GAMMA=4.0*3.14159*U*A*SIN(ALPHA+BATA)
      NSTOP=C
C
      DO 801 IA=1,200
      Y(IA)=C.0
 801  CONTINLE
C
      CALL KCSKI(ABC,XRBODY,YRBCDY,M,A,ALPHA,ZATA,ETA)
      ZATA(M+1)=-8.0
      ETA(M+1)=-5.0
      ZATA(M+2)=1.0
      ETA(M+2)=1.0
      CALL DASHY(8.0,0.0,8.0,10.,4)
      CALL DASHY(-C.2,5.0,16.,5.0,4)
      CALL NLMBER(8.8,1.5,0.2,ALPHAD,0.0,-1)
      CALL LINE(ZATA,ETA,M,1,0,0)
      PSI=0.C
      KX=0
 7C0  I=1
      X(I)=-XLIMIT
      YTRIAL(1)=PSI/U
C
 200  J=0
      IF(I-13) 210,210,220
 210  YL=YTRIAL(I)-(GAMMA+0.4)
      YR=YTRIAL(I)+(GAMMA+0.4)
      GO TO 100
 220  YL=YTRIAL(I)-0.2
      YR=YTRIAL(I)+0.2
 100  YM=YL+(YR-YL)/2.
      FCNY=U*YM-(A**2*YM*U)/(X(I)**2+YM**2)+GAMMA/
     1  (4.0*3.14159)*ALOG(X(I)**2+YM**2)-PSI
      J=J+1
```

```
C
      IF(J-26) 104,104,1000
 1000 WRITE(6,4) I,X(I),YM,FCNY,PSI,GAMMA
 4    FORMAT ('0','CANNOT GET OUT OF LOOP AFTER TRYING 27',
     1  ' TIMES, ***X(', I3, ' )=',F8.3,'    YM=',
     2  F8.3,'  FCNY=',F10.5,'  PSI=',F6.2,
     3  '  GAMMA=',  F6.2)
      WRITE(6,5)
 5    FORMAT('--------------------------------')
      GO TO 20
C
 104  IF(PSI)106,105,106
 105  IF(ABS(FCNY)-1.0E-4) 20,20,10
 106  IF(ABS(FCNY)-1.0E-3) 20,20,10
 10   IF(FCNY) 30,20,40
 30   YL=YM
      GO TO 100
 40   YR=YM
      GO TO 100
C
 20   Y(I)=YM
      IF(Y(1)+5.0) 31,32,32
 31   IF(PSI) 33,33,34
 33   PSI=-PSI
 34   NSTOP=999
      GO TO 49
 32   ITIMES(I)=J
      IF(PSI) 21,201,21
 201  IF(X(I)+(A+0.2)) 21,202,202
 202  RADIUS=SQRT(X(I)**2+Y(I)**2)
      IF(RADIUS-(A+0.01))2000,203,21
 2000 I=I-1
 203  CALL KOSKI(ABC,X,Y,I,A,ALPHA,ZATA,ETA)
      ZATA(I+1)=-8.0
      ETA(I+1)=-5.0
      ZATA(I+2)=1.0
      ETA(I+2)=1.0
      CALL LINE(ZATA,ETA,I,1,0,0)
      WRITE(6,1)
      WRITE(6,3) (X(J),Y(J),ITIMES(J),ZATA(J),ETA(J),J=1,I)
      K=I
      DO 204 L=1,K
      XP(L)=-X(I)
      YP(L)=Y(I)
      ITIMES(L)=0
      I=I-1
 204  CONTINUE
      CALL KOSKI(ABC,XP,YP,K,A,ALPHA,ZATA,ETA)
      CALL LINE(ZATA,ETA,K,1,0,0)
      WRITE(6,1)
      WRITE(6,3) (XP(J),YP(J),ITIMES(J),ZATA(J),ETA(J),J=1,K)
      GO TO 400
 21   I=I+1
      YTRIAL(I)=Y(I-1)
```

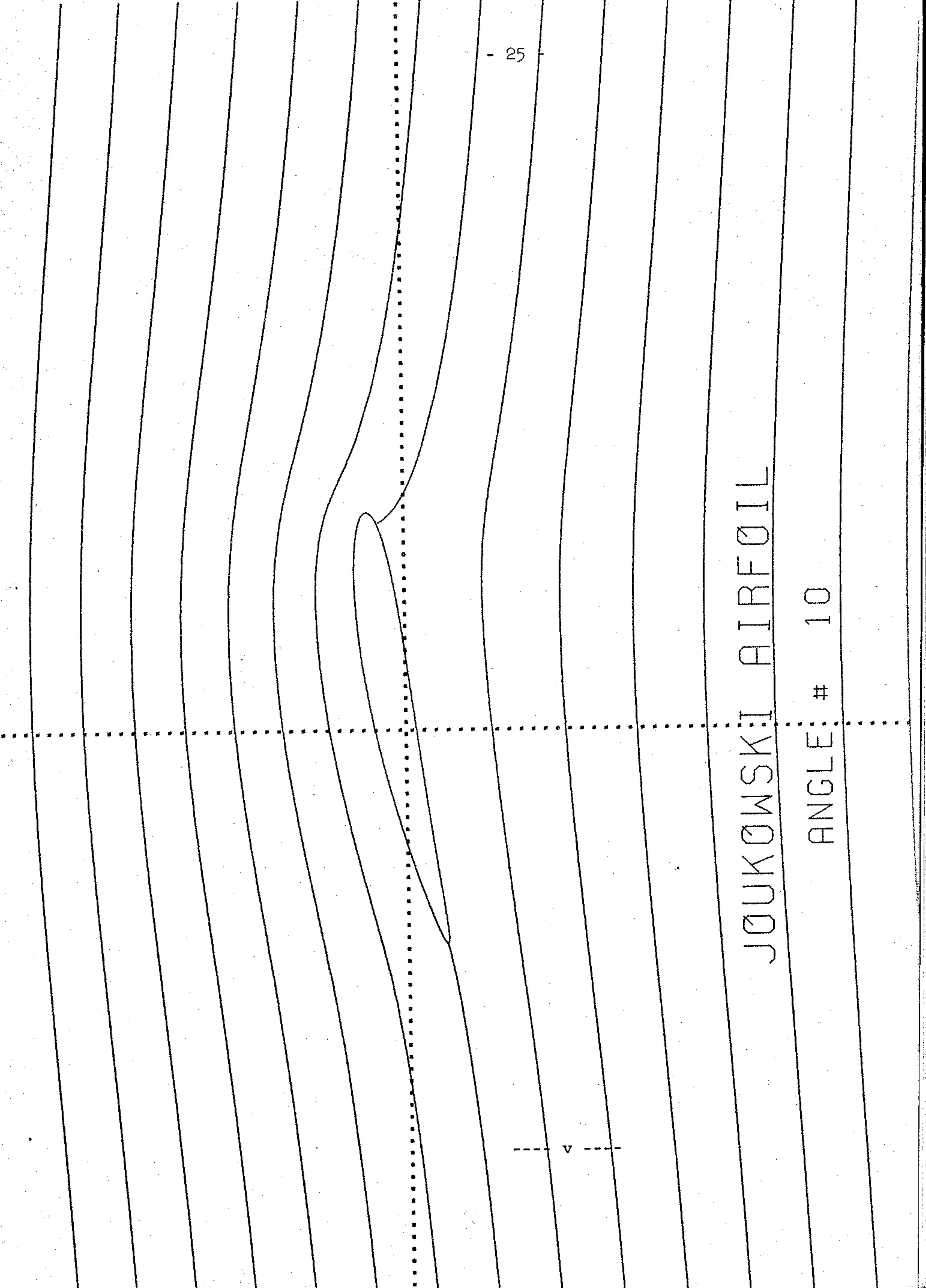
```
      IF(X(I-1)) 22,300,300
   22 IF(ABS(X(I-1))-(A+1.0)) 23,23,24
   23 X(I)=X(I-1)+0.05
      IF(PSI) 200,25,200
   25 IF(ABS(X(I-1))-(A+0.1)) 26,26,200
   26 X(I)=X(I-1)+0.01
      GO TO 200
   24 X(I)=X(I-1)+0.5
      GO TO 200
C
  300 WRITE(6,2) PSI,GAMMA,ALPHAD
    2 FORMAT(//,12X,'(   PSI=',F7.1,10X,'GAMMA=',F7.3,10X,
     1  'ALPHA=',F7.1,'  DEGREES  )')
      N=I-1
      K=2*N
      DO 301 J=I,K
      X(J)=-X(N)
      Y(J)=Y(N)
      ITIMES(J)=0
      N=N-1
  301 CONTINUE
      CALL KCSKI(ABC,X,Y,K,A,ALPHA,ZATA,ETA)
C
      WRITE(6,1)
    1 FORMAT('0',2X,'** X **',22X,'** Y **',21X,'TRIALS',5X,
     1  'ZATA',7X,'ETA')
      WRITE(6,3) (X(I),Y(I),ITIMES(I),ZATA(I),ETA(I),I=1,K)
    3 FORMAT(2F8.3,I4,2F8.3,2X,2F8.3,I4,2F8.3,2X,2F8.3,I4,
     12F8.3)
      ZATA(K+1)=-8.0
      ETA(K+1)=-5.0
      ZATA(K+2)=1.0
      ETA(K+2)=1.0
      CALL LINE(ZATA,ETA,K,1,0,0)
C
      IF(PSI)48,400,48
   48 IF(NSTOP) 49,50,49
   49 PSI=PSI+0.5
      KHALF=K/2
      IF(Y(KHALF)-3.5) 700,700,500
   50 PSI=-PSI
      KX=KX+1
      IF(MOD(KX,2)) 700,400,700
  400 PSI=PSI+0.5
      IF(PSI-PSILIM)700,700,500
  500 CALL PLOT(18.0,0.0,-3)
  900 CALL ENDPLT
      STOP
      END

NS IN EFFECT*  ID,BCD,SOURCE,NOLIST,NODECK,LOAD,NOMAP
NS IN EFFECT*  NAME = MAIN     , LINECNT =        56
STICS*     SOURCE STATEMENTS =        136,PROGRAM SIZE =      19394
STICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE RBODY(A,X,Y,M)
      DIMENSION X(400),Y(400)
      RADIAN=0.0
      I=1
   10 X(I)=A*COS(RADIAN)
      Y(I)=A*SIN(RADIAN)
      RADIAN=RADIAN+3.14159/180.0
      I=I+1
      IF(RADIAN-3.14159) 10,10,20
   20 N=I
      M=2*(I-1)
      L=N-1
      DO 30 I=N,M
      X(I)=X(L)
      Y(I)=-Y(L)
      L=L-1
   30 CONTINUE
      WRITE(6,1)
    1 FORMAT(/,'---------------------------------')
      RETURN
      END
```

```
S IN EFFECT*  ID,BCD,SOURCE,NOLIST,NODECK,LOAD,NOMAP
S IN EFFECT*  NAME = RBODY    , LINECNT =       56
TICS*    SOURCE STATEMENTS =        21,PROGRAM SIZE =        748
TICS*  NO DIAGNOSTICS GENERATED
```



```
      SUBROUTINE KOSKI (ABC,X,Y,N,A,ALPHA,ZATAR,ETAR)
      DIMENSION X(400),Y(400),ZATAR(400),ETAR(400)
C
C     ZATAR=ZATA ROTATED,  --  ETAR=ETA ROTATED
C
      BSQA=ABC*(A**2)
      DO 1 I=1,N
      XROTAY=X(I)*COS(ALPHA)+Y(I)*SIN(ALPHA)
      YROTAY=-X(I)*SIN(ALPHA)+Y(I)*COS(ALPHA)
      XPRIME=XROTAY+0.04*A
      YPRIME=YROTAY+0.05*A
      ROU=SQRT(XPRIME**2+YPRIME**2)
      ZATA=(ROU+BSQA/ROU)*(XPRIME/ROU)
      ETA=(ROU-BSQA/ROU)*(YPRIME/ROU)
      ZATAR(I)=ZATA*COS(-ALPHA)+ETA*SIN(-ALPHA)
      ETAR(I)=-ZATA*SIN(-ALPHA)+ETA*COS(-ALPHA)
    1 CONTINUE
      RETURN
      END
```


JØUKØWSKI AIRFØIL
ANGLE # 10
---- v ----