Linear Stability of Plane Couette Flow at Moderate Reynolds Numbers

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Abstract

The linear stability of plane Couette flow is studied by using a symbolic computation package (Mathematica). The power series method is used to obtain the solution of the Orr-Sommerfeld equation. This solution shows that plane Couette flow is unstable.

Résumé

La stabilité linéaire du flux de Couette sur un plan est étudiée par l'intermédiane d'un outil de calcul symbolic (Mathematica). La méthods de série de puissance est utilisée pour obtenir la solution de l'équation de Orr-Sommerfeld. Cette solution montre que le flux de Couette sur un plan est instable.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Hydrodynamic stability

There are two types of fluid motion: laminar motion and turbulent motion. We know that laminar motion occurs only when the Reynolds number is very small. If an infinitesimal disturbance is introduced in the laminar flow and it decays, the flow can maintain its laminar motion. However, if the disturbance grows, the main flow will be disturbed and turbulence may result. In this case we say the laminar motion is unstable. If the disturbance decays we say the flow is stable.

The problem of determining if the laminar motion is stable or unstable with respect to the infinitesimal disturbance is a stability problem. The stability theory that uses the idea of infinitesimal disturbances and does not go beyond the first approximation is called the linear stability theory.

We are, now, investigating in the laminar motion of the simplest form, the plane Couette flow.

Viscosity plays an important role in the flow. Even very small viscosity can make fluid flow unstable. According to Yih [5], Case and Orr have proved that the inviscid plane Couette flow is stable. However, the stability or instability of viscous plane Couette flow has not been firmly established.

Because of the infinite extent, it is impossible to investigate this kind of flow experimentally. However, by neglecting infinite extent, one can do the experiment for the plane Couette flow in the laboratory, as L.S Tuckerman and D. Barkley [16], F. Daviaud *et al* [13], N. Tillmark and P.H. Alfredsson [12], Dauchot and Daviad [14] and S. Bottin *et al* [15]. However, without neglecting infinite extent, we can also still study the stability of the flow from a mathematical point of view.

The analytical solution for the governing equation for the stability problem (The Orr-Sommerfeld equation) for the plan Couette flow is known. The accompanying numerical computation for the solution of the problem is non-trivial. There are a number of numerical computations for this problem, such as G.B. Davis and A.G. Morris [19] and Southwell and Chitty [20]

To study this problem, Deardorff [21] used a coupling finite difference methods with trial and error numerical method to study the stability of the plane Couette flow. In 1973, Davey used a "complete" orthonormalization with a parallel shooting procedure to investigate instability of the flow. Studies up to this date have not shown instability of the plane Couette flow but this has not been discounted.

There is also analytical work for the special cases of instability of plane Couette flow for large values of a non-dimensional parameter in the problem. Hopf [17] used asymptotic methods of approximation to study the problem. His studies did not shown instability of the plane Couette flow. Wasow [18] showed in his work that for a given wave number the plane Couette flow is stable if the product of the wave number and the non-dimensional parameter is sufficiently large.

In numerical works for studying the instability, we have to deal with parameters whose values range over a wide interval, which make the computation exceedingly intensive. Thus, symbolic computations may offer some benefits in solving such problems. With the availability of high performance personal computer in recent years, symbolic computations have facilitated the investigation of other types of flows; for example, Tam [22].

In this thesis, we will show that the plane Couette flow is unstable by using the power series solutions obtained from a symbolic computation.

1.2 The Plane Couette Flow

From Rosenhead [1], the governing equations for incompressible viscous flow, with reference to a general Cartesian coordinate system x, y, z with the velocity field $\bar{v} = (u, v, w)$ where u, v, w are the velocity components in x, y, z direction respectively, are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u \tag{1.1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v \tag{1.2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w. \tag{1.3}$$

These are the Navier-Stokes equation and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1.4}$$

is the Continuity equation.

Here, ρ is the density of the fluid, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity; μ is static viscosity, p is pressure, and $\nabla^2 = \frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 y} + \frac{\partial^2}{\partial^2 z}$. We are interested in the flow called the Couette flow.

Suppose that there are two parallel infinite plates filled in the gap between them with a viscous fluid (Figure 1.1). While we keep the lower plate stationary , we move the upper plate with a constant velocity U in the $x-$direction. The friction between the plate surface and the fluid will induce a fluid flow; and this flow is called the plane Couette flow.

To find the velocity distribution for the laminar plane Couette flow, we suppose the flow is steady (independent of time) and there is no velocity in the y, z direction, i.e. $\bar{v} = (u, v, w) = (u, 0, 0)$. Then equations (1.1),(1.2) and (1.3) and equation (1.4), simplify to

$$u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y} + \frac{\partial^2 u}{\partial^2 z}) \tag{1.5}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \tag{1.6}$$

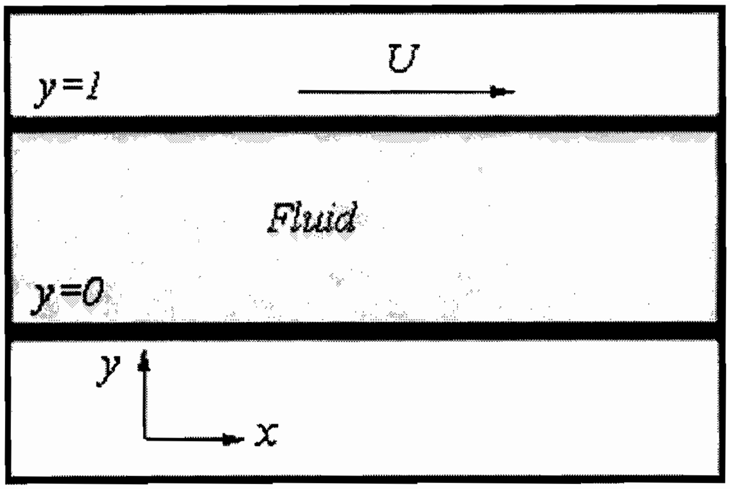

Figure 1.1: The Plane Couette flow.

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = 0 \qquad (1.7)$$

We can conclude from equation (1.6) and (1.7) that $p = p(x)$. Moreover, since there is no pressure gradient in the Couette flow, we have

$$\frac{\partial^2 u}{\partial^2 y} = 0. \qquad (1.8)$$

One can easily solve this ordinary differential equation, equation (1.8), to obtain

$$u(y) = Ay + B$$

where A, B are constant determined by the following conditions: 1. At $y = 0$, $u(y) = 0$ and 2. At $y = 1$, $u(y) = U$. Finally we get the velocity distribution for the plane Couette flow to be

$$u(y) = Uy.$$

1.3 The hydrodynamic stability theory

In this section we provide a brief introduction to the hydrodynamic stability theory in the case of viscous incompressible parallel laminar flow in the absence of external forces.

From Rosenhead [1], we can derive the formulation for the hydrodynamic stability as follows. Let

x denote the distance along the flow,

y denote the distance between the plates and

z denote the distance that is perpendicular to x and y.

And (u, v, w)in direction of x, y, z, respectively.

Introducing a characteristic length, L, and a characteristic velocity, U_0, and defining

$$x = \frac{x}{L}, \qquad y = \frac{y}{L}, \qquad z = \frac{z}{L}$$

$$u = \frac{u}{U_0}, \qquad v = \frac{v}{U_0}, \qquad w = \frac{w}{U_0}$$

$$p = \frac{p}{\rho U_0}, \qquad \tau = \frac{U_0 t}{L}$$

the Navier-Stokes equation and the continuity equation become:

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{R}\nabla^2 u \tag{1.9}$$

$$\frac{\partial v}{\partial \tau} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{R}\nabla^2 v \tag{1.10}$$

$$\frac{\partial w}{\partial \tau} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{R}\nabla^2 w \tag{1.11}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{1.12}$$

Where, we define the Reynolds number, as $R = \frac{U_0 L}{\nu}$.

Suppose that there is a two-dimensional, steady solution of equation (1.9) - (1.12):

$$U = U(y), \qquad V = 0, \qquad W = 0, \qquad p^0 = constant$$

Let an infinitesimal perturbations (u', v', w', p') be introduced. We then have:

$$u = U + u'; v = v'; w = w'; p = p^0 + p'. \tag{1.13}$$

For which we set the direction of the flow to be along the x-axis.

Now, substitute equation (1.13) into equations (1.9) - (1.12) and linearize. We can obtain the linear equations:

$$\frac{\partial u'}{\partial \tau} + U\frac{\partial u'}{\partial x} + v'\frac{\partial U}{\partial y} = -\frac{\partial p'}{\partial x} + \frac{1}{R}\nabla^2 u' \tag{1.14}$$

$$\frac{\partial v'}{\partial \tau} + U\frac{\partial v'}{\partial x} = -\frac{\partial p'}{\partial y} + \frac{1}{R}\nabla^2 v' \tag{1.15}$$

$$\frac{\partial w'}{\partial \tau} + U\frac{\partial w'}{\partial x} = -\frac{\partial p'}{\partial z} + \frac{1}{R}\nabla^2 w' \tag{1.16}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0. \tag{1.17}$$

We let the perturbation functions,u', v', w' and p', take the form $e^{i(\alpha x+\beta z-\alpha c\tau)}$, where,

α is a real wave number along x direction,

β is a real wave number along z direction,

c is a complex number , $c = c_r + ic_i$.

We have

1. If $c_i > 0$, the disturbance grows with time and the flow is unstable (amplified case).

2. If $c_i < 0$, the disturbance decays with time and the flow is stable (damped case).

3. If $c_i = 0$, the disturbance neither grows nor decays (neutral case).

Then we can write :

$$u' = u_1(y)e^{i(\alpha x+\beta z-\alpha c\tau)} \tag{1.18}$$

$$v' = v_1(y)e^{i(\alpha x+\beta z-\alpha c\tau)} \tag{1.19}$$

$$w' = w_1(y)e^{i(\alpha x+\beta z-\alpha c\tau)} \tag{1.20}$$

$$p' = p_1(y)e^{i(\alpha x+\beta z-\alpha c\tau)} \tag{1.21}$$

Now, we substitute equation (1.18) - (1.21) into equation (1.14) - (1.17) and eliminate p_1 and obtain :

$$(U-c)(D^2-\gamma^2)v_1 - v_1 D^2 U = -\frac{i}{\alpha R}(D^2-\gamma^2)^2 v_1 \tag{1.22}$$

$$(U-c)(\beta u_1-\alpha w_1)+\frac{i\beta}{\alpha}v_1 DU=\frac{i}{\alpha R}(D^2-\gamma^2)(\beta u_1-\alpha w_1) \tag{1.23}$$

$$i\alpha u_1+Dv_1+i\beta w_1=0. \tag{1.24}$$

Where $\gamma^2=\alpha^2+\beta^2$, $D=\frac{d}{dy}$ and the boundary conditions for equation (1.22) are:

$$v_1=\frac{dv_1}{dy}=0 \text{ at } y=0 \text{ and } y=1$$

For the two-dimensional theory we have $\beta=0$ and it is similar if we define:

$$u_1=\frac{d\phi}{dy}; \tag{1.25}$$

$$v_1=-i\alpha\phi, \tag{1.26}$$

which provides the stream function:

$$\psi=\phi(y)e^{i\alpha(x-c\tau)}$$

Then equation (1.22) can be written as:

$$\alpha^4\phi-2\alpha^2\phi''+\phi^{iv}=i\alpha R((U-c)(\phi''-\alpha^2\phi)-\phi U'') \tag{1.27}$$

The differential with respect to y is denoted by primes and iv. Equation (1.27) is called the *Orr-Sommerfeld equation* with the boundary conditions:

$$\phi=\phi'=0 \text{ at } y=0 \text{ and } y=1.$$

1.3.1 Hydrodynamic stability for case of the Couette flow

From section 2.1 , the velocity profile for the plane Couette flow is $u(y)=Uy$, it can be seen that the basic flow for the plane Couette flow is $U=y$. Since

$$\frac{d^2U}{dy^2}=0,$$

then we can write the Orr-Summerfeld for a case of the plane Couette flow as

$$(y-c)(\phi''-\alpha^2\phi)=-\frac{i}{\alpha R}(\phi^{iv}-2\alpha^2\phi''+\alpha^4\phi)$$

Now we ask what are the boundary conditions for this equation ? Obviously, they must be homogenous.

Because of the fact that we must have no perturbation velocity at the interface between the plates and a fluid, then at $y = 0$ and $y = 1$, $u^{'}$ and $v^{'}$ have to vanish. From:

$$u_1 = \frac{d\phi}{dy}$$

and

$$v_1 = -i\alpha\phi$$

We can conclude that the boundary conditions for the Orr-Sommerfeld for the case of the plane Couette flow are

$$\phi(0) = 0, \qquad \phi'(0) = 0,$$

$$\phi(1) = 0, \qquad \phi'(1) = 0$$

Then we have to solve the ordinary differential equation

$$(y - c)(\phi'' - \alpha^2\phi) = -\frac{i}{\alpha R}(\phi^{iv} - 2\alpha^2\phi'' + \alpha^4\phi)$$

with boundary conditions:

$$\phi(0) = 0, \qquad \phi'(0) = 0,$$

$$\phi(1) = 0, \qquad \phi'(1) = 0$$

to investigate the hydrodynamic stability of the plane Couette flow.

1.3.2 The analytical solution

In this part we will solve the equation (1.27) analytically with the boundary conditions

$$\phi = \phi^{'} = 0 \text{ at } y = 0 \text{ and } y = 1.$$

First, change the variable y to x and then we write

$$(D^2 - \alpha^2)\phi(x) = \varphi(x)$$

Then from (1.27) and the above equation we can get:

$$(D^2 - \alpha^2)\varphi(x) = i\alpha R((x - c)\varphi(x) \tag{1.28}$$

Then write

$$y = \frac{\alpha^2}{(\alpha R)^{\frac{1}{3}}} + i(\alpha R)^{\frac{1}{3}}(x - c)$$

and then

$$\frac{d}{dx} = i(\alpha R)^{\frac{1}{3}}\frac{d}{dy}, \qquad \frac{d^2}{dx^2} = -(\alpha R)^{\frac{2}{3}}\frac{d^2}{dy^2}.$$

Equation (1.28) becomes:

$$\frac{d^2\varphi}{dy^2} + y\varphi = 0 \tag{1.29}$$

This is the Airy equation which has the solution known as the Airy function $Ai(y)$ and $Bi(y)$.

Then we can get the function ϕ as:

$$\phi(y) = C_1 e^{\frac{i\alpha}{(\alpha R)^{\frac{1}{3}}}y} + C_2 e^{-\frac{i\alpha}{(\alpha R)^{\frac{1}{3}}}y} + \frac{(\alpha R)^{\frac{1}{3}}}{\alpha}\int_0^y sin(\frac{\alpha}{(\alpha R)^{\frac{1}{3}}}(y-t))[K_1 Ai(t) + K_2 Bi(t)]dt. \tag{1.30}$$

This is the analytical solution of the case of the plane Couette flow. In principle, imposition of the boundary conditions on a linear combination of these four solutions will provide the characteristic equation with which to calculate the eigenvalues. However, the difficulty with computations involving integrals of the Airy functions makes this approach prohibitive.

Chapter 2

The Computation

2.1 Formulation and Solution

For the plane Couette flow the basic flow is $U = y$ where $0 < y < 1$, We let the perturbation stream function be

$$\psi(x, y, \tau) = \phi(y)e^{i\alpha(x-c\tau)}$$

From the linear hydrodynamic stability theory (in the previous chapter) we have ϕ governed by the Orr-Sommerfeld equation

$$(y - c)(\phi'' - \alpha^2\phi) = -\frac{i}{\alpha R}(\phi^{iv} - 2\alpha^2\phi'' + \alpha^4\phi) \tag{2.1}$$

Where ,

$c = c_r + ic_i$ is a complex parameter,

α is a real parameter,

and R is the Reynolds number based on the width between the plates and the maximum value of U

For this equation we found that the appropriate boundary conditions at $y = 0$ and $y = 1$ are

$$\phi = \phi' = 0 \tag{2.2}$$

Equation (2.1) has a set of four fundamental solutions satisfying the four initial conditions at $y = 0$:

$$(\phi, \phi', \phi'', \phi''') = (1, 0, 0, 0);$$

$$(\phi, \phi', \phi'', \phi''') = (0, 1, 0, 0);$$

$$(\phi, \phi', \phi'', \phi''') = (0, 0, 1, 0);$$

$$(\phi, \phi', \phi'', \phi''') = (0, 0, 0, 1),$$

respectively.

Now, we construct two linearly independent power series solutions to equation (2.1) in view of the condition (2.2). Let ϕ_1 and ϕ_2 satisfy the condition

$$(\phi, \phi', \phi'', \phi''') = (0, 0, 1, 0) \tag{2.3}$$

and

$$(\phi, \phi', \phi'', \phi''') = (0, 0, 0, 1) \tag{2.4}$$

at $y = 0$ respectively. We set

$$\phi_1(y) = \sum_{k=2}^{M} b_k y^k;$$

$$\phi_2(y) = \sum_{k=3}^{M} d_k y^k,$$

where b_k and d_k depend on the parameter obtained by using the symbolic calculation package. For $\phi_1(y)$ the condition (2.3) gives $b_2 = \frac{1}{2}$ and $b_3 = 0$. Likewise, for $\phi_2(y)$ the condition (2.4) we set $d_3 = \frac{1}{6}$.

We consider first the neutral case where $c_i = 0$

We want to use as many terms as possible because the accuracy of the results increases with increasing number of terms. But we also have to avoid crashing the PC.

First, we calculate the solution ϕ_1, ϕ_2 with as many terms as can be obtained on a personal computer with a Pentium II processor. After that we compute the value at $y = 1$ and plot a graph on $0 < y \leq 1$ in doing this we try computing with a range of Reynolds number, from R=0 to 300 a range of α from 0 to 30 and a range of c_r from 0 to 1. We seek a suitable amount of terms that makes the function converge.

However, we have to impose the boundary at $y = 1$ to obtain the characteristic equation, which is:

$$\begin{vmatrix} \phi_1 & \phi_1' \\ \phi_2 & \phi_2' \end{vmatrix} = 0. \tag{2.5}$$

In this operation, we have to multiply the functions and their first derivatives. Sometimes, the computer crashes if the number of terms used exceeds the computational capacity of the personal computer.

We have found that if we use about 95 terms we can accomplish the convergence of the solutions obtained and at the same time the computational capacity of the personal computer is not exceeded.

It is noted that the convergence of the solution means that the numerical outputs of the solutions converge to a certain value when a specific value of terms has been used. Even if the latter value is exceeded the numerical solution still converge. We do not want to convey the idea that the convergence is proven in the analytical way. For moderate R value (0-300) approximately 95 terms are required on numerical outputs convergence. For R greater than 300 more than 95 terms are needed on convergence of the numerical outputs. Because of this, we can use solutions obtained only for moderate Reynolds numbers. i.e. from 0 to 300.

The following are examples of the value of the functions and their first derivatives at $y = 1$ for $\alpha = 5$, R= 250 and $\alpha = 10$, R= 200, respectively. We note that $\phi_1(1)$, $\phi_1'(1)$, $\phi_2(1)$ and $\phi_2'(1)$ are complex numbers.

Terms	ϕ_1	ϕ_1'
80	139.171 - 950.443 I	25501.1 - 18102.2 I
85	139.184 - 951.777 I	25502.1 - 18209.4 I
90	139.219 - 951.847 I	25505.1 - 18215.4 I
95	139.222 - 951.849 I	25505.4 - 18215.5 I
100	139.222 - 951.849 I	25505.4 - 18215.5 I

Table 2.1: A table of ϕ_1 and its first derivative at y = 1 as α= 5 and R = 250

Terms	ϕ_2	ϕ_2'
80	83.5883 - 109.156 I	4453.89 - 501.896 I
85	83.5316 - 109.073 I	4449.33 - 495.193 I
90	83.5263 - 109.071 I	4448.88 - 494.98 I
95	83.526 - 109.071 I	4448.85 - 494.981 I
100	83.526 - 109.071 I	4448.85 - 494.981 I

Table 2.2: A table of ϕ_2 and its first derivative at y = 1 as α= 5 and R = 250

Terms	ϕ_1	ϕ_1'
80	269990. - 769201. I	$3.02114 * 10^7 - 2.00039 * 10^7$ I
85	260865. - 685983. I	$2.94238 * 10^7 - 1.32526 * 10^7$ I
90	256779. - 688113. I	$2.90769 * 10^7 - 1.34451 * 10^7$ I
95	256906. - 687620. I	$2.9089 * 10^7 - 1.33997 * 10^7$ I
100	256900. - 687621. I	$2.90889 * 10^7 - 1.33998 * 10^7$ I

Table 2.3: A table of the ϕ_1 and its first derivative at y = 1 as α= 10 and R = 200

Terms	ϕ_2	ϕ_2'
80	9645.35 - 64408.8 I	$2.21609 * 10^6 - 1.76299 * 10^6$I
85	10048.5 - 62285.4 I	$2.2436 * 10^6 - 1.58442 * 10^6$I
90	9983.97 - 62927.9 I	$2.23791 * 10^6 - 1.64017 * 10^6$I
95	10061.3 - 62916.3 I	$2.24494 * 10^6 - 1.63906 * 10^6$I
100	10061.0 - 62915.6 I	$2.24482 * 10^6 - 1.6389 * 10^6$I

Table 2.4: A table of the ϕ_2 and its first derivative at y = 1 as α= 10 and R = 200

The functions ϕ_1, ϕ_2 have very very lengthly expressions. The following is ϕ_1, ϕ_2 stated in a Mathematica "Short" form :

$$\phi_1(y) = \frac{y^2}{2} + << 12663 >>,$$

$$\phi_2(y) = \frac{y^3}{6} + << 12287 >> .$$

The number in the parentheses shows the number of terms that follow the first term. These terms are polynomial of α and R.

In the next section we will show how the solution of the characteristic equation can be obtained graphically.

2.2 Graphical solution of the characteristic equation

The characteristic equation is obtained by imposing the condition at $y = 1$ (using equation (2.5)). Because the characteristic equation is in complex form the real part and the imaginary part of the characteristic equation have to be zero. We note that both of these parts of the characteristic equation are made up of many many terms. This can be observed in the "Short" form of Mathematica as following: The first 152 terms of real part of the characteristic equation are:

$$\frac{1}{12} + \frac{\alpha^2}{90} - \frac{67\,R^2\,\alpha^2}{1814400} + \frac{c\,R^2\,\alpha^2}{6720} - \frac{c^2\,R^2\,\alpha^2}{6720} + \frac{\alpha^4}{1260} - \frac{13\,R^2\,\alpha^4}{2395008} + \frac{c\,R^2\,\alpha^4}{45360} - \frac{c^2\,R^2\,\alpha^4}{45360} +$$
$$\frac{29\,R^4\,\alpha^4}{23775897600} - \frac{c\,R^4\,\alpha^4}{99066240} + \frac{c^2\,R^4\,\alpha^4}{32288256} - \frac{c^3\,R^4\,\alpha^4}{23950080} + \frac{c^4\,R^4\,\alpha^4}{47900160} + \frac{\alpha^6}{28350} - \frac{67\,R^2\,\alpha^6}{194594400} +$$
$$\frac{c\,R^2\,\alpha^6}{712800} - \frac{c^2\,R^2\,\alpha^6}{712800} + \frac{4211\,R^4\,\alpha^6}{22865620377600} - \frac{23\,c\,R^4\,\alpha^6}{14944849920} + \frac{71\,c^2\,R^4\,\alpha^6}{14944849920} - \frac{c^3\,R^4\,\alpha^6}{155675520} +$$
$$\frac{c^4\,R^4\,\alpha^6}{311351040} - \frac{17707\,R^6\,\alpha^6}{20071441567457280000} + \frac{487\,c\,R^6\,\alpha^6}{4344467871744000} - \frac{1699\,c^2\,R^6\,\alpha^6}{2896311914496000} +$$
$$\frac{37\,c^3\,R^6\,\alpha^6}{22865620377600} - \frac{227\,c^4\,R^6\,\alpha^6}{91462481510400} + \frac{c^5\,R^6\,\alpha^6}{498161664000} - \frac{c^6\,R^6\,\alpha^6}{1494484992000} + \frac{\alpha^8}{935550} -$$
$$\frac{353\,R^2\,\alpha^8}{27243216000} + \frac{c\,R^2\,\alpha^8}{18918900} - \frac{c^2\,R^2\,\alpha^8}{18918900} + \frac{39649\,R^4\,\alpha^8}{3620389893120000} - \frac{73\,c\,R^4\,\alpha^8}{793945152000} +$$

$$\frac{113\,c^2\,R^4\,\alpha^8}{396972576000} - \frac{c^3\,R^4\,\alpha^8}{2594592000} + \frac{c^4\,R^4\,\alpha^8}{5189184000} - \frac{187043\,R^6\,\alpha^8}{138492946815455232000} +$$

$$\frac{181\,c\,R^6\,\alpha^8}{10453875816384000} - \frac{30509\,c^2\,R^6\,\alpha^8}{334524026124288000} + \frac{61\,c^3\,R^6\,\alpha^8}{241359326208000} -$$

$$\frac{563\,c^4\,R^6\,\alpha^8}{1448155957248000} + \frac{c^5\,R^6\,\alpha^8}{3175780608000} - \frac{c^6\,R^6\,\alpha^8}{9527341824000} +$$

$$\frac{1458473\,R^8\,\alpha^8}{680554340651147010048000000} - \frac{3701\,c\,R^8\,\alpha^8}{1000226838111621120000} +$$

$$\frac{5183\,c^2\,R^8\,\alpha^8}{18754253214592896000000} - \frac{673\,c^3\,R^8\,\alpha^8}{577053945064396800000} + \frac{5617\,c^4\,R^8\,\alpha^8}{1846572624206069760000} -$$

$$\frac{c^5\,R^8\,\alpha^8}{19912144412160000} + \frac{7\,c^6\,R^8\,\alpha^8}{136540418826240000} - \frac{c^7\,R^8\,\alpha^8}{33790305669120000} +$$

$$\frac{c^8\,R^8\,\alpha^8}{135161222676480000} + \frac{\alpha^{10}}{42567525} - \frac{R^2\,\alpha^{10}}{3031426944} + \frac{c\,R^2\,\alpha^{10}}{742996800} - \frac{c^2\,R^2\,\alpha^{10}}{742996800} +$$

$$\frac{19883\,R^4\,\alpha^{10}}{53219731428864000} - \frac{181\,c\,R^4\,\alpha^{10}}{57597111936000} + \frac{11\,c^2\,R^4\,\alpha^{10}}{1129355136000} -$$

$$\frac{c^3\,R^4\,\alpha^{10}}{75785673600} + \frac{c^4\,R^4\,\alpha^{10}}{151571347200} - \frac{2527387\,R^6\,\alpha^{10}}{327346965200166912000000} +$$

$$\frac{27823\,c\,R^6\,\alpha^{10}}{2797837309403136000} - \frac{29411\,c^2\,R^6\,\alpha^{10}}{559567461880627200} + \frac{37\,c^3\,R^6\,\alpha^{10}}{2534272925184000} -$$

$$\frac{c^4\,R^6\,\alpha^{10}}{44460928512000} + \frac{c^5\,R^6\,\alpha^{10}}{54854392320000} - \frac{c^6\,R^6\,\alpha^{10}}{164563176960000} + \frac{161981\,R^8\,\alpha^{10}}{489324195344213139456000000} -$$

$$\frac{35641\,c\,R^8\,\alpha^{10}}{618685764228315463680000} + \frac{3317\,c^2\,R^8\,\alpha^{10}}{7661743210257776640000} - \frac{2003\,c^3\,R^8\,\alpha^{10}}{1091156550667223040000} +$$

$$\frac{20971\,c^4\,R^8\,\alpha^{10}}{4364626202668892160000} - \frac{89\,c^5\,R^8\,\alpha^{10}}{11191349237612544000} + \frac{c^6\,R^8\,\alpha^{10}}{122981859753984000} -$$

$$\frac{c^7\,R^8\,\alpha^{10}}{212878925715456000} + \frac{c^8\,R^8\,\alpha^{10}}{851515702861824000} - \frac{13276927\,R^{10}\,\alpha^{10}}{59909253051867723386286243840000} +$$

$$\frac{23581\,c\,R^{10}\,\alpha^{10}}{4854096017814594343403520000} - \frac{25567\,c^2\,R^{10}\,\alpha^{10}}{539344001979399371489280000} +$$

$$\frac{3649\,c^3\,R^{10}\,\alpha^{10}}{13456415371965861335040000} - \frac{216851\,c^4\,R^{10}\,\alpha^{10}}{21530264595145378136064000000} +$$

$$\frac{349\,c^5\,R^{10}\,\alpha^{10}}{137485725384070103040000} - \frac{1451\,c^6\,R^{10}\,\alpha^{10}}{329965740921768247296000} +$$

$$\frac{79\,c^7\,R^{10}\,\alpha^{10}}{15276191709341122560000} - \frac{c^8\,R^{10}\,\alpha^{10}}{253021808850370560000} + \frac{c^9\,R^{10}\,\alpha^{10}}{564044001575672217600} -$$

$$\frac{c^{10}\,R^{10}\,\alpha^{10}}{2820220007878361088000} + \frac{\alpha^{12}}{2554051500} - \frac{559\,R^2\,\alpha^{12}}{91380033360000} + \frac{c\,R^2\,\alpha^{12}}{40078962000} - \frac{c^2\,R^2\,\alpha^{12}}{40078962000} +$$

$$\frac{283\,R^4\,\alpha^{12}}{33232230484992000} - \frac{c\,R^4\,\alpha^{12}}{13956223276800} + \frac{31\,c^2\,R^4\,\alpha^{12}}{139562232768000} - \frac{c^3\,R^4\,\alpha^{12}}{3322910304000} +$$

$$\frac{c^4 R^4 \alpha^{12}}{6645820608000} - \frac{432961 R^6 \alpha^{12}}{17533604654243758080000} + \frac{23407 c R^6 \alpha^{12}}{734432293718323200000} -$$
$$\frac{41299 c^2 R^6 \alpha^{12}}{244810764572774400000} + \frac{53 c^3 R^6 \alpha^{12}}{112989583648972800} - \frac{43 c^4 R^6 \alpha^{12}}{59468201920512000} +$$
$$\frac{c^5 R^6 \alpha^{12}}{1705760622720000} - \frac{c^6 R^6 \alpha^{12}}{5117281868160000} +$$
$$\frac{3441107 R^8 \alpha^{12}}{1866960006851767055155200000} - \frac{222329 c R^8 \alpha^{12}}{69007258317773647872000000} +$$
$$\frac{6706223 c^2 R^8 \alpha^{12}}{2760290332710945914880000000} - \frac{331 c^3 R^8 \alpha^{12}}{320479546349813760000} +$$
$$\frac{1469 c^4 R^8 \alpha^{12}}{542350001515069440000} - \frac{157 c^5 R^8 \alpha^{12}}{349729663675392000000} + \frac{241 c^6 R^8 \alpha^{12}}{524594495513088000000} -$$
$$\frac{c^7 R^8 \alpha^{12}}{37663194549657600000} + \frac{c^8 R^8 \alpha^{12}}{150652778198630400000} -$$
$$\frac{100031573 R^{10} \alpha^{12}}{29032945709751281333354102784000000} +$$
$$\frac{61338689 c R^{10} \alpha^{12}}{8064707141597578148153917440000000} - \frac{982759 c^2 R^{10} \alpha^{12}}{131668688026082908541288448000000} +$$
$$\frac{176167 c^3 R^{10} \alpha^{12}}{41073120150738875213414400000} - \frac{1313371 c^4 R^{10} \alpha^{12}}{821462403014777504268288000000} +$$
$$\frac{13 c^5 R^{10} \alpha^{12}}{3217121599896207360000000} - \frac{331 c^6 R^{10} \alpha^{12}}{4718445013181104128000000} +$$
$$\frac{367 c^7 R^{10} \alpha^{12}}{44418465124084187136000000} - \frac{1123 c^8 R^{10} \alpha^{12}}{177673860496336748544000000} +$$
$$\frac{c^9 R^{10} \alpha^{12}}{35252750098479513600000} - \frac{c^{10} R^{10} \alpha^{12}}{1762637504923975680000000} +$$
$$\frac{361486819 R^{12} \alpha^{12}}{3183985090097003521266277744115712000000 0} -$$
$$\frac{6175381 c R^{12} \alpha^{12}}{2041016083395515077734793425715200000 0} +$$
$$\frac{49767163 c^2 R^{12} \alpha^{12}}{13606773889303433851565289504768000000} - \frac{769999 c^3 R^{12} \alpha^{12}}{29032945709751281333354102784000000} +$$
$$\frac{14922773 c^4 R^{12} \alpha^{12}}{116131782839005125333416411136000000} - \frac{2356643 c^5 R^{12} \alpha^{12}}{53764714277317187654359449600000000} +$$
$$\frac{995063 c^6 R^{12} \alpha^{12}}{9216808161825803597890191360000000} - \frac{3709 c^7 R^{12} \alpha^{12}}{191674560703448084329267200000000} +$$
$$\frac{337 c^8 R^{12} \alpha^{12}}{13450846365154251531878400000000} - \frac{53 c^9 R^{12} \alpha^{12}}{23186438794771945684992000000} +$$
$$\frac{647 c^{10} R^{12} \alpha^{12}}{46372877589543891369984000000000} - \frac{c^{11} R^{12} \alpha^{12}}{1954412465459704233984000000} +$$
$$\frac{c^{12} R^{12} \alpha^{12}}{117264747927582254039040000000} + << 46005 >> = 0,$$

and the first 117 terms of the imaginary part of the characteristic equation are:

$$\frac{R\,\alpha}{360}-\frac{c\,R\,\alpha}{180}+\frac{R\,\alpha^3}{2520}-\frac{c\,R\,\alpha^3}{1260}-\frac{R^3\,\alpha^3}{3742200}+\frac{7\,c\,R^3\,\alpha^3}{4276800}-\frac{c^2\,R^3\,\alpha^3}{302400}+\frac{c^3\,R^3\,\alpha^3}{453600}+\frac{R\,\alpha^5}{37800}-$$

$$\frac{c\,R\,\alpha^5}{18900}-\frac{29\,R^3\,\alpha^5}{726485760}+\frac{269\,c\,R^3\,\alpha^5}{1089728640}-\frac{c^2\,R^3\,\alpha^5}{1995840}+\frac{c^3\,R^3\,\alpha^5}{2993760}+\frac{307\,R^5\,\alpha^5}{80029671321600}-$$

$$\frac{643\,c\,R^5\,\alpha^5}{16005934264320}+\frac{29\,c^2\,R^5\,\alpha^5}{174356582400}-\frac{89\,c^3\,R^5\,\alpha^5}{261534873600}+\frac{c^4\,R^5\,\alpha^5}{2905943040}-\frac{c^5\,R^5\,\alpha^5}{7264857600}+\frac{R\,\alpha^7}{935550}-$$

$$\frac{c\,R\,\alpha^7}{467775}-\frac{19\,R^3\,\alpha^7}{7783776000}+\frac{59\,c\,R^3\,\alpha^7}{3891888000}-\frac{c^2\,R^3\,\alpha^7}{32432400}+\frac{c^3\,R^3\,\alpha^7}{48648600}+$$

$$\frac{317\,R^5\,\alpha^7}{543058483968000}-\frac{1117\,c\,R^5\,\alpha^7}{181019494656000}+\frac{7\,c^2\,R^5\,\alpha^7}{272209766400}-\frac{151\,c^3\,R^5\,\alpha^7}{2858202547200}+\frac{c^4\,R^5\,\alpha^7}{18681062400}-$$

$$\frac{c^5\,R^5\,\alpha^7}{46702656000}-\frac{10723\,R^7\,\alpha^7}{692464734077276160000}+\frac{13\,c\,R^7\,\alpha^7}{56070018953625600}-\frac{41\,c^2\,R^7\,\alpha^7}{27877002177024000}+$$

$$\frac{857\,c^3\,R^7\,\alpha^7}{167262013062144000}-\frac{23\,c^4\,R^7\,\alpha^7}{2172233935872000}+\frac{47\,c^5\,R^7\,\alpha^7}{3620389893120000}-$$

$$\frac{c^6\,R^7\,\alpha^7}{114328101888000}+\frac{c^7\,R^7\,\alpha^7}{400148356608000}+\frac{R\,\alpha^9}{34054020}-\frac{c\,R\,\alpha^9}{17027010}-\frac{29\,R^3\,\alpha^9}{333456963840}+$$

$$\frac{41\,c\,R^3\,\alpha^9}{75785673600}-\frac{c^2\,R^3\,\alpha^9}{908107200}+\frac{c^3\,R^3\,\alpha^9}{1362160800}+\frac{541\,R^5\,\alpha^9}{15929715529728000}-$$

$$\frac{13\,c\,R^5\,\alpha^9}{36040080384000}+\frac{c^2\,R^5\,\alpha^9}{663075072000}-\frac{281\,c^3\,R^5\,\alpha^9}{90509747328000}+\frac{c^4\,R^5\,\alpha^9}{317578060800}-$$

$$\frac{c^5\,R^5\,\alpha^9}{793945152000}-\frac{4289\,R^7\,\alpha^9}{18004083086009180160000}+\frac{25903\,c\,R^7\,\alpha^9}{7201633234403672064000}-$$

$$\frac{7067\,c^2\,R^7\,\alpha^9}{307762104034344960000}+\frac{7421\,c^3\,R^7\,\alpha^9}{92328631210303488000}-\frac{223\,c^4\,R^7\,\alpha^9}{1338096104497152000}+$$

$$\frac{137\,c^5\,R^7\,\alpha^9}{669048052248576000}-\frac{c^6\,R^7\,\alpha^9}{7240779786240000}+\frac{c^7\,R^7\,\alpha^9}{25342729251840000}+$$

$$\frac{142019\,R^9\,\alpha^9}{592082276366497898741760000000}-\frac{347639\,c\,R^9\,\alpha^9}{74010284545812237342720000000}+$$

$$\frac{2293\,c^2\,R^9\,\alpha^9}{56712861720928917504000000}-\frac{273289\,c^3\,R^9\,\alpha^9}{1361108681302294020096000000}+$$

$$\frac{761\,c^4\,R^9\,\alpha^9}{1200272205733945344000000}-\frac{587\,c^5\,R^9\,\alpha^9}{444545261382942720000000}+\frac{67\,c^6\,R^9\,\alpha^9}{36931452484121395200000}-$$

$$\frac{41\,c^7\,R^9\,\alpha^9}{25852016738884976640000}+\frac{c^8\,R^9\,\alpha^9}{1248889697530675200000}-\frac{c^9\,R^9\,\alpha^9}{5620003638888038400000}+$$

$$\frac{R\,\alpha^{11}}{1702701000}-\frac{c\,R\,\alpha^{11}}{851350500}-\frac{R^3\,\alpha^{11}}{479975932800}+\frac{c\,R^3\,\alpha^{11}}{77138989200}-\frac{c^2\,R^3\,\alpha^{11}}{37892836800}+$$

$$\frac{c^3\,R^3\,\alpha^{11}}{56839255200}+\frac{82139\,R^5\,\alpha^{11}}{734432293718323200000}-\frac{17483\,c\,R^5\,\alpha^{11}}{146886458743664640}+\frac{c^2\,R^5\,\alpha^{11}}{20067772032000}-$$

$$\frac{683\,c^3\,R^5\,\alpha^{11}}{6652466428608000}+\frac{c^4\,R^5\,\alpha^{11}}{9599518656000}-\frac{c^5\,R^5\,\alpha^{11}}{23998796640000}-\frac{69211\,R^7\,\alpha^{11}}{515571470190262886400000}+$$
$$\frac{2099533\,c\,R^7\,\alpha^{11}}{103114294038052577280000}-\frac{7907\,c^2\,R^7\,\alpha^{11}}{6061980837040128000000}+\frac{11351\,c^3\,R^7\,\alpha^{11}}{247990125151641600000}-$$
$$\frac{c^4\,R^7\,\alpha^{11}}{10518185373696000}+\frac{409\,c^5\,R^7\,\alpha^{11}}{3497296636753920000}-\frac{c^6\,R^7\,\alpha^{11}}{12671364625920000}+$$
$$\frac{c^7\,R^7\,\alpha^{11}}{44349776190720000}+\frac{66173\,R^9\,\alpha^{11}}{1779835206532017925914624000 0}-$$
$$\frac{9791387\,c\,R^9\,\alpha^{11}}{13348764048990134444359680000 0}+\frac{16279\,c^2\,R^9\,\alpha^{11}}{2563126737517306920960000 0}-$$
$$\frac{1705601\,c^3\,R^9\,\alpha^{11}}{5382566148786344534016000000}+\frac{109\,c^4\,R^9\,\alpha^{11}}{108541362145318502400000}-$$
$$\frac{43249\,c^5\,R^9\,\alpha^{11}}{2062285880761051545600000}+\frac{c^6\,R^9\,\alpha^{11}}{346398904973721600000}-\frac{193\,c^7\,R^9\,\alpha^{11}}{7638095854670561280000}+$$
$$\frac{c^8\,R^9\,\alpha^{11}}{7833944466328780800 0}-\frac{c^9\,R^9\,\alpha^{11}}{352527500984795136000}-$$
$$\frac{3244009\,R^{11}\,\alpha^{11}}{188714147113383328666801668096000000}+\frac{4144967\,c\,R^{11}\,\alpha^{11}}{99323235322833330877264035840000 00}-$$
$$\frac{15923833\,c^2\,R^{11}\,\alpha^{11}}{349470642802561719753336422400000 0}+\frac{8847907\,c^3\,R^{11}\,\alpha^{11}}{2995462652593386169314312192000 00}-$$
$$\frac{44969\,c^4\,R^{11}\,\alpha^{11}}{35596704130640358518292480000 0}+\frac{66629\,c^5\,R^{11}\,\alpha^{11}}{177983520653201792591462400000}-$$
$$\frac{563\,c^6\,R^{11}\,\alpha^{11}}{717675486504845937868800000}+\frac{29207\,c^7\,R^{11}\,\alpha^{11}}{2511864202766960782540800000 0}-$$
$$\frac{c^8\,R^{11}\,\alpha^{11}}{836869632772600627200000}+\frac{281\,c^9\,R^{11}\,\alpha^{11}}{3464640279678566596608000 00}$$
$$-\frac{c^{10}\,R^{11}\,\alpha^{11}}{305523834186822451200000 0}+\frac{c^{11}\,R^{11}\,\alpha^{11}}{1680381088027523481600000 0}+\frac{R\,\alpha^{13}}{111648537000}-$$
$$\frac{c\,R\,\alpha^{13}}{55824268500}-\frac{73\,R^3\,\alpha^{13}}{2010360733920000}+\frac{227\,c\,R^3\,\alpha^{13}}{1005180366960000}-\frac{c^2\,R^3\,\alpha^{13}}{2175715080000}+$$
$$\frac{c^3\,R^3\,\alpha^{13}}{3263572620000}+\frac{1283\,R^5\,\alpha^{13}}{5245944955130880000 0}-\frac{2927\,c\,R^5\,\alpha^{13}}{112413106181376000000}+\frac{c^2\,R^5\,\alpha^{13}}{917123243904000}-$$
$$\frac{c^3\,R^5\,\alpha^{13}}{445074515424000}+\frac{c^4\,R^5\,\alpha^{13}}{438624160128000}-\frac{c^5\,R^5\,\alpha^{13}}{1096560400320000}-\frac{247777\,R^7\,\alpha^{13}}{5914907855809169817600000 0}+$$
$$\frac{1198307\,c\,R^7\,\alpha^{13}}{18820161359392813056000000}-\frac{1543\,c^2\,R^7\,\alpha^{13}}{377708036769423360000 0}+\frac{8131\,c^3\,R^7\,\alpha^{13}}{566562055154135040000 0}-$$
$$\frac{313\,c^4\,R^7\,\alpha^{13}}{104918899102617600000}+\frac{107\,c^5\,R^7\,\alpha^{13}}{29144138639616000000}-\frac{c^6\,R^7\,\alpha^{13}}{403534227317760000}+$$
$$\frac{c^7\,R^7\,\alpha^{13}}{1412369795612160000}+\frac{78698749\,R^9\,\alpha^{13}}{384033673409408483245424640000000}-$$
$$\frac{44551889\,c\,R^9\,\alpha^{13}}{10972390668840242378440704000000}+\frac{62723\,c^2\,R^9\,\alpha^{13}}{17780571493826352906240000000}+<<46010>>=0.$$

The graphical solution of the characteristic equation is accomplished by using the special package of Mathematica called "ImplicitPlot". This will give a contour plot of the implicit function provided. The solutions are found as follows: Because we know that c_r should lie between zero and one, we fix a value for α, then solve the real and imaginary parts of the characteristic equation to obtain the corresponding values of c_r and R. These are the points where the curves cross. To obtain a better view of the crossing curves, we divide the range of c_r and R into many intervals. To determine the presence of crossing curves, we first plot the curve of the real part of the equation, and then we plot the curve of the imaginary part. Then, we combine them on the same scale, and examine them for the presence or absence of an intersection.

Any crossing curves observed are magnified to obtain a more accurate solution. These processes are shown in figures 2.1- 2.4 (the figures shown are obtained with $\alpha = 16$, the vertical axis is c and the horizontal axis is R).

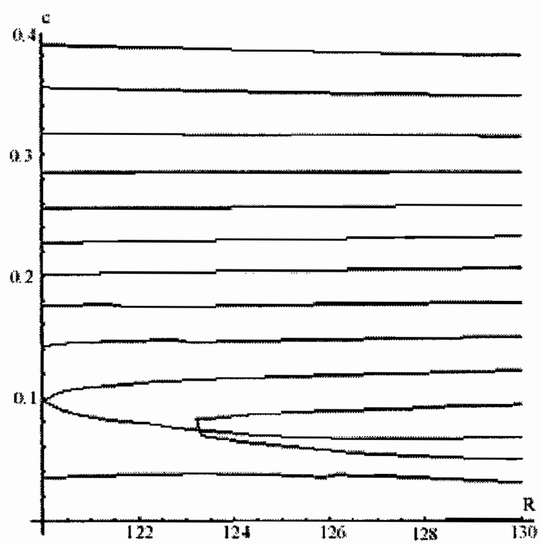

Figure 2.1: Crossing curves.

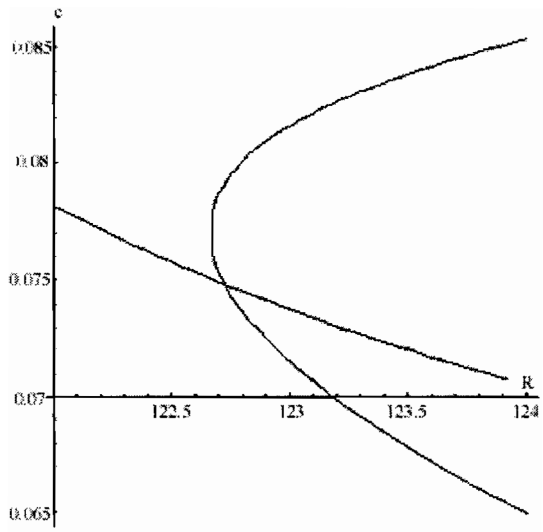

Figure 2.2: The first magnification.

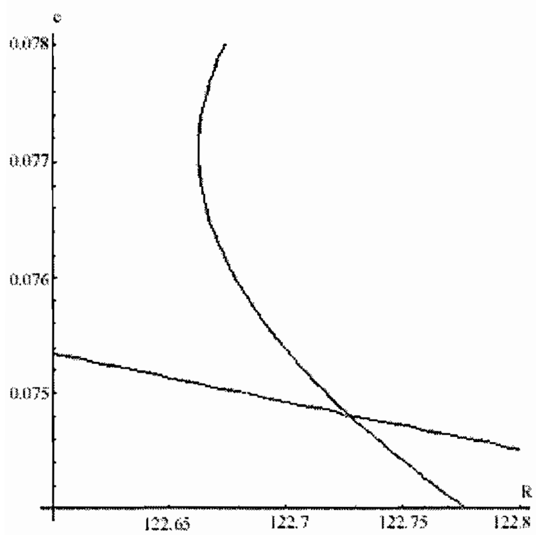

Figure 2.3: The second magnification.

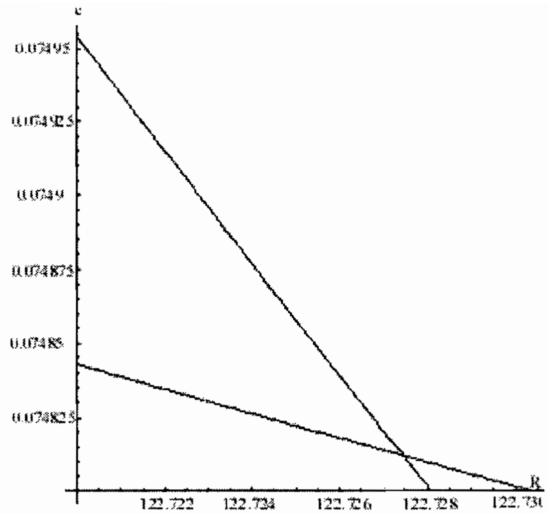

Figure 2.4: A better view of a graphical solution.

We repeat this process using α increasing by 1 from 0 to 30. The result of this calculation is presented in Figure 2.5. We can see the loop structure showing instability of the plane Couette flow.

Now, we consider the case of a damped solution; We compute the program again with $\alpha = 9, 10$ and $c_i = -\frac{1}{1000}$ and solve the eigenvalue problem again to find that there are solutions shown in Figures 2.6 and 2.8.

We also consider a case of amplified solution; we run the program again with $\alpha = 9, 10$ and $c_i = \frac{1}{1000}$. The solutions are presented in Figures 2.7 and 2.9.

The case of amplified, damped and neutral cases are combined when $\alpha = 9, 10$ and are represented in Figure 2.10.

We note that the symbolic computation when $c_i \neq 0$ becomes much more difficult and very time consuming. Thus, we have to set the value of α first before doing the calculations, or the program will terminate before the specific number of terms are obtained. Since our objective is to show that the plane Couette flow is unstable, we have performed computation for a non-zero c_i only for the two values above. If we examine Figure 2.10, in conjuction with Figure 2.5, we see that a disturbance with wave number $\alpha = 10$ admits a damped solution with $c_i = -\frac{1}{1000}$ at $R = 250$; a neutral solution at $R = 252$; and an amplified solution at $R = 253$ with $c_i = \frac{1}{1000}$.

Thus, the disturbance behaves differently at different Reynolds numbers. The same pattern is repeated around $R = 274$.

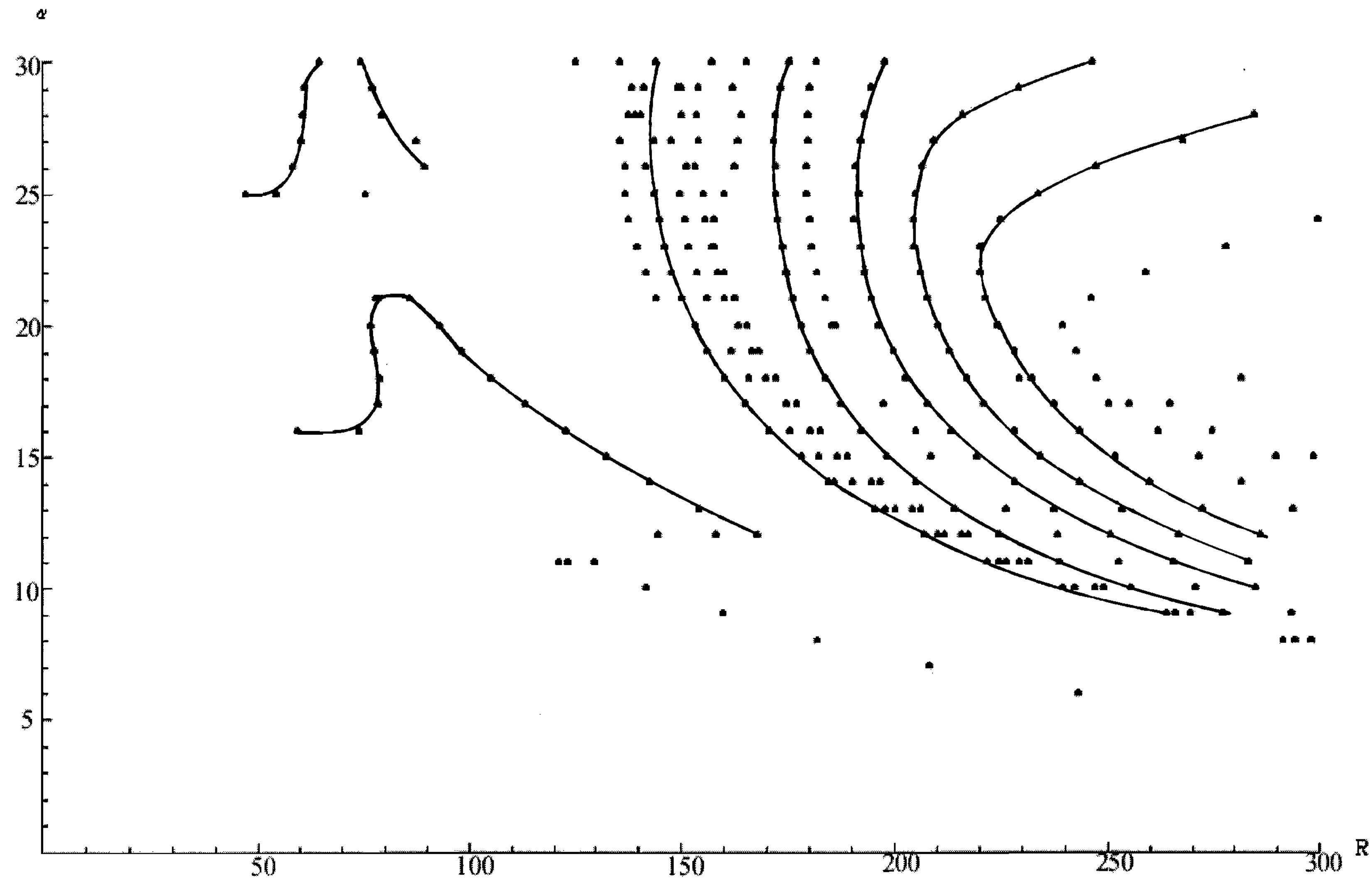

Figure 2.5: The graph for the neutral case (c_i=0).

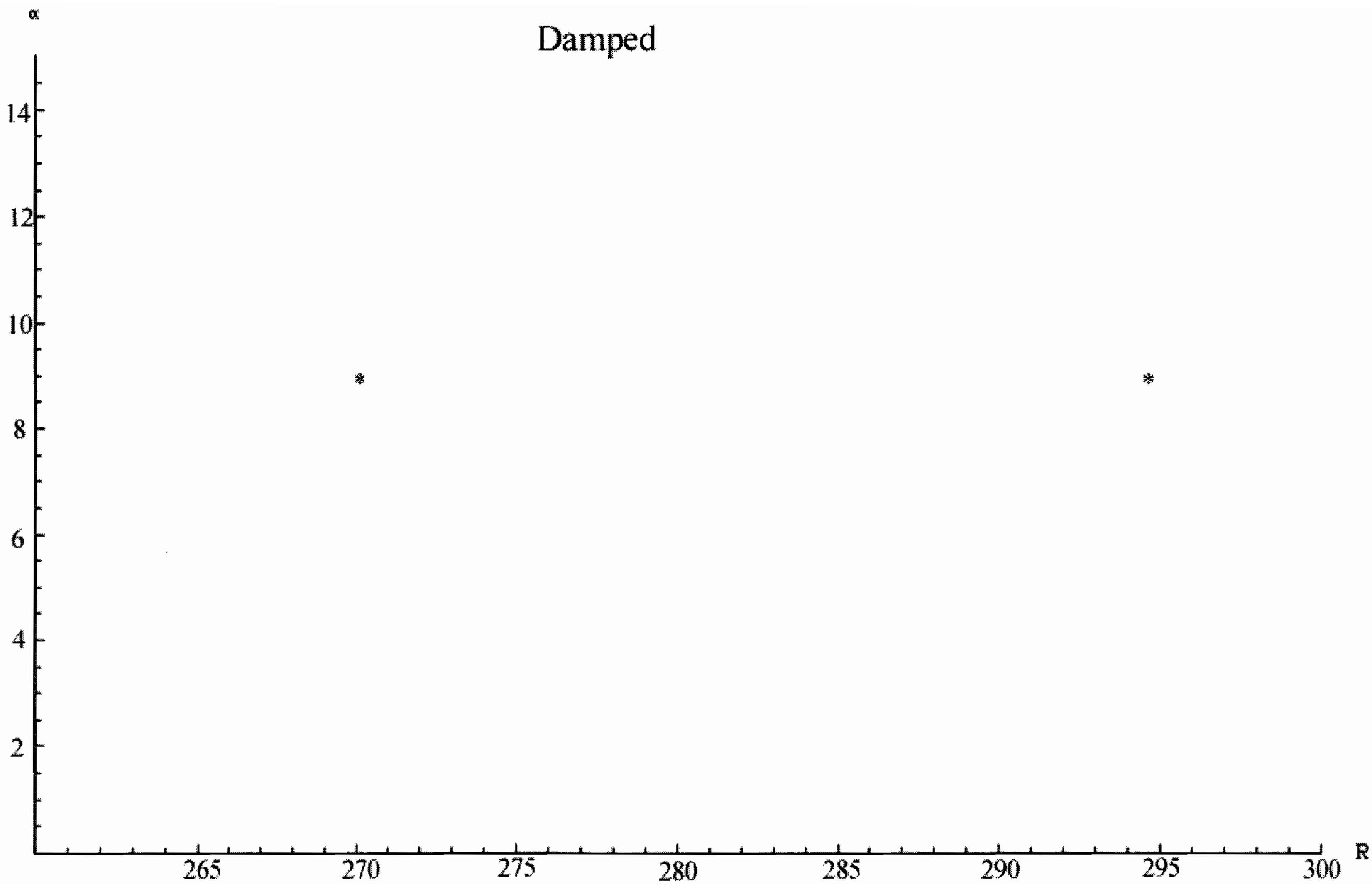

Figure 2.6: The graph for the damped case when $\alpha = 9$.

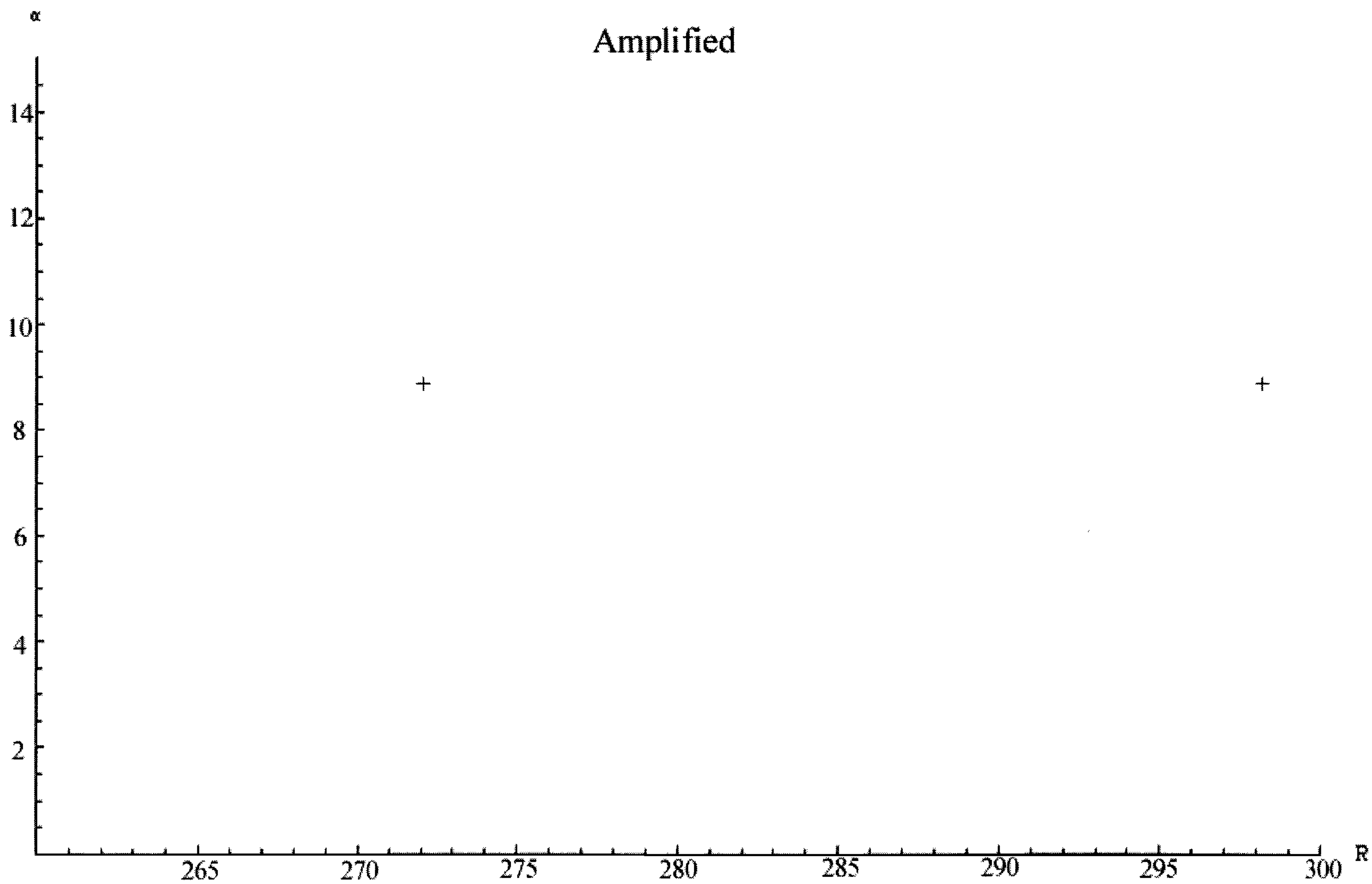

Figure 2.7: The graph for the amplified case when $\alpha = 9$.

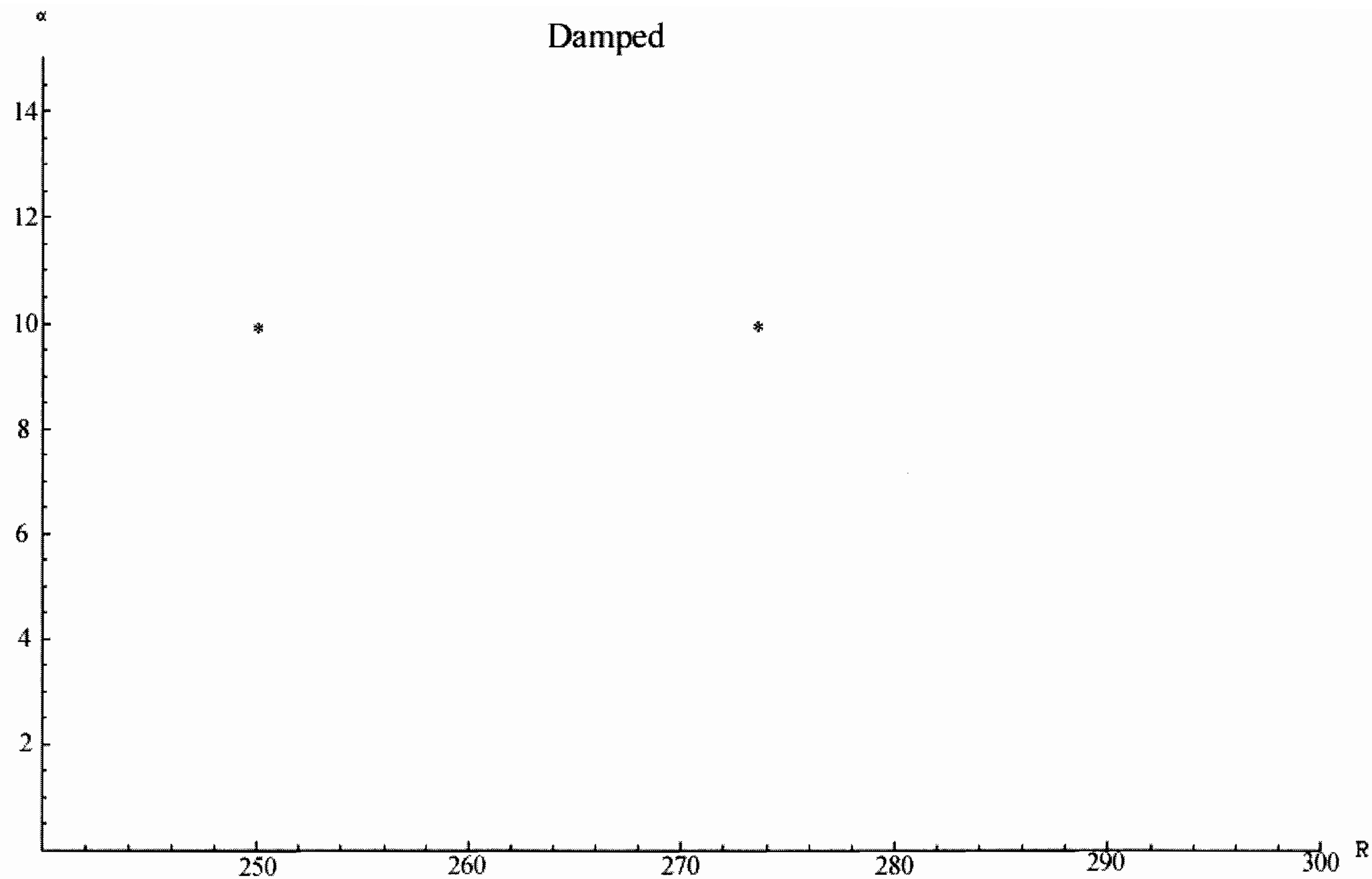

Figure 2.8: The graph for the damped case when $\alpha = 10$.

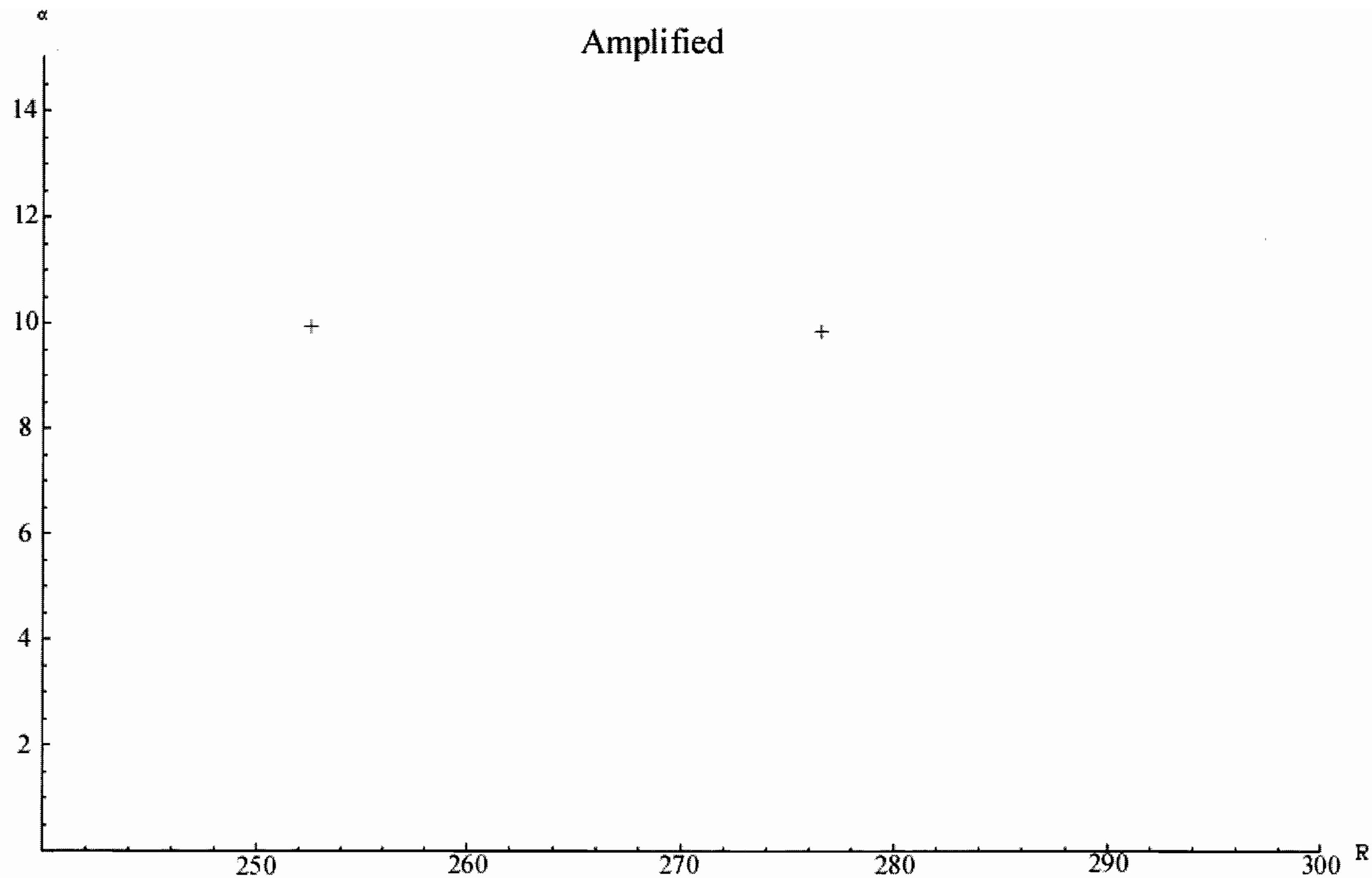

Figure 2.9: The graph for the amplified case when $\alpha = 10$.

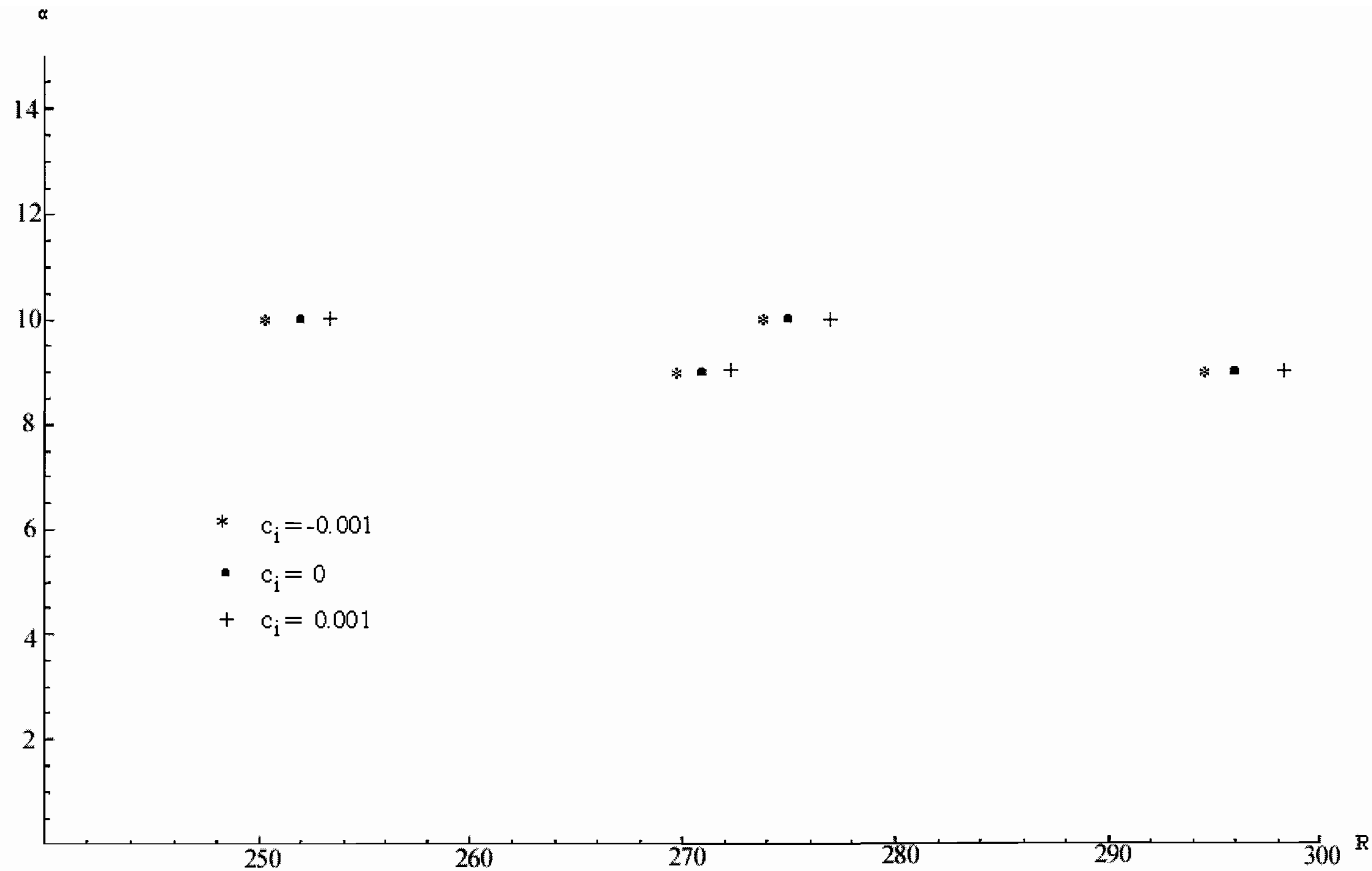

Figure 2.10: The graph for damped, neutral and amplified cases when $\alpha = 9$ and 10

Chapter 3

Conclusion and Summary

3.1 Discussion

We have studied the stability of the plane Couette flow by using the Computational approaches. Having analytic coefficients, the Orr-Sommerfeld equation has the power series solutions that converge to the analytical solutions (See [6]). To obtain the high accuracy power series solution, we have to use as many terms as possible. Moreover, in doing the computation, we do not use floating point operation (i.e. change the rational number to a floating-point number and then do the computation) that means we do not have a round off error. So the solutions obtained have very high accuracy. Using a very high performance computer with very large amount of RAM, more terms can be obtained.

3.2 Conclusion

The calculation has been done on personal computer with Pentium II 350 MHz CPU and 128 Megabytes of RAM. From the results obtained, we can conclude that the Plane Couette flow is unstable (we can see the loop structure in Figure 2.5, in which we have joined points to form a continuous curve. We have only done this for a few curves so as not to make the figure too crowded). More precisely, from Figure 2.10 we can see the region in which the flow is damped, neutral and then amplified. The result obtained is in agreement with experimental work from Tillmark and Alfredson and F. Daviand *et al.* From their experimental results, they have found that there is turbulence in plane Coutte flow when the Reynolds number is about 350 or greater.

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