Exploring the Simultaneous Stochastic Optimization of Mining Complexes: Integration of Equipment, Recovery, and Market Uncertainties

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Abstract

A mining complex is an integrated mineral value chain that consists of multiple interconnected components, from the extraction of ore to a set of sellable products. Over the last decade, advancements made in the field of mine planning have led to the development of simultaneous stochastic optimization of mining complexes. The cutting-edge framework of simultaneous stochastic optimization moves away from considering the economic value of blocks to the value of final products, allowing integrated value chain modelling and optimization. This thesis studies the simultaneous stochastic optimization framework through two real-world case studies, applying the methods and assessing the ability, impact, and benefits of incorporating multiple sources of uncertainties.

The first chapter of the thesis presents a literature review on the development of simultaneous stochastic optimization starting with the conventional framework. It details the trail of research of incorporation of multiple components in the optimization process, modelling of different sources of uncertainties, incorporation of uncertainties in mine planning, and finally the establishment of simultaneous stochastic optimization including mathematical modelling of the mining complex with sources of uncertainty, objective formulation, and optimization algorithm.

The second chapter of the thesis presents an application of a stochastic framework that simultaneously optimizes mining, destination and processing decisions for a multi-pit, multi-processor copper mining complex with the incorporation of mining equipment uncertainty in addition to supply uncertainty. The case study assesses the impacts of integrating equipment uncertainty as input that influences all components of the production schedule. It addresses the issue that a conventional schedule is difficult to realize during operation due to the uncertainty of equipment performance. An analysis is provided to demonstrate how the incorporation equipment modifies the production schedule that is capable of producing not only a more realizable schedule but also a schedule with a 2% improvement in terms of economic value from the better capturing of the synergy among components.

The third chapter of the thesis presents an application of the same stochastic framework for a multi-pit, multiprocessor copper mining complex with the incorporation of market uncertainty and recovery uncertainty in addition to supply uncertainty. The study aims to assess the impacts of market and recovery uncertainty that affect the profitability of mining operations. It also details the modelling of different sources of uncertainty at different components of a mineral value chain. The case study provides a comprehensive comparison of the effect of incorporating different combinations of uncertainty on the production schedule. The result shows that the incorporation of additional will improve the project value by around 7% to 12% compared to only including the supply uncertainty. The joint effect of uncertainties as well as the interaction between components within a mineral value chain has to be considered.

Résumé

Un complexe minier est une chaîne de valeur minérale intégrée composée de plusieurs composants interconnectés, depuis l'extraction du minerai jusqu'à un ensemble de produits vendables. Au cours de la dernière décennie, les progrès réalisés dans le domaine de la planification minière ont conduit au développement de l'optimisation stochastique simultanée des complexes miniers. Le cadre de pointe de l'optimisation stochastique simultanée s'éloigne de la considération de la valeur économique des blocs pour se concentrer sur la valeur des produits finaux, permettant ainsi une modélisation et une optimisation intégrées de la chaîne de valeur. Cette thèse étudie le cadre d'optimisation stochastique simultanée à travers deux études de cas réels, appliquant les méthodes et évaluant la capacité, l'impact et les avantages de l'intégration de plusieurs sources d'incertitudes.

Le premier chapitre de la thèse présente une revue de la littérature sur le développement de l'optimisation stochastique simultanée en commençant par le cadre conventionnel. Il détaille le parcours de recherche d'incorporation de multiples composantes dans le processus d'optimisation, la modélisation de différentes sources d'incertitudes, l'incorporation d'incertitudes dans la planification minière, et enfin la mise en place d'une optimisation stochastique simultanée incluant la modélisation mathématique du complexe minier avec des sources d'incertitude, formulation objective et algorithme d'optimisation.

Le deuxième chapitre de la thèse présente une application d'un cadre stochastique qui optimise simultanément les décisions d'extraction, de destination et de traitement pour un complexe minier de cuivre multi-puits et multi-processeurs avec l'incorporation de l'incertitude de l'équipement minier en plus de l'incertitude de l'approvisionnement. L'étude de cas évalue les impacts de l'intégration de l'incertitude de l'équipement comme entrée qui influence tous les composants du calendrier de production. Il aborde le problème selon lequel un calendrier conventionnel est difficile à réaliser pendant l'exploitation en raison de l'incertitude des performances de l'équipement. Une analyse est fournie pour démontrer comment l'équipement d'incorporation modifie le calendrier de production qui est capable de produire non seulement un calendrier plus réalisable mais également un calendrier avec une amélioration de 2 % en termes de valeur économique grâce à une meilleure capture de la synergie entre les composants.

Le troisième chapitre de la thèse présente une application du même cadre stochastique pour un complexe minier de cuivre multi-puits et multiprocesseur avec l'incorporation de l'incertitude du marché et de l'incertitude de la reprise en plus de l'incertitude de l'offre. L'étude vise à évaluer les impacts de l'incertitude du marché et de la reprise qui affectent la rentabilité des opérations minières. Il détaille également la modélisation de différentes sources d'incertitude au niveau de différents composants d'une chaîne de valeur minérale. L'étude de cas fournit une comparaison complète de l'effet de l'incorporation de différentes combinaisons d'incertitudes sur le calendrier de production. Le résultat montre que l'incorporation d'éléments supplémentaires améliorera la valeur du projet d'environ 7 à 12 % par rapport à la seule prise en compte de l'incertitude de l'approvisionnement. L'effet conjoint des incertitudes ainsi que l'interaction entre les composants d'une chaîne de valeur minérale doivent être pris en compte.

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Chapter 1. Introduction and literature review

A mining complex is an integrated mineral value chain that transforms in-situ raw material extracted from mineral deposits into valuable commodities. Components in a mineral value chain can include multiple mineral deposits, stockpiles, processing streams, waste disposal, and transportation to clients or spot markets (Goodfellow and Dimitrakopoulos, 2016, 2017; Montiel and Dimitrakopoulos, 2015, 2017, 2018; Pimentel et al., 2010). The objective of strategic mine planning or life-of-mine planning is to maximize the net present value (NPV) generated by the mining operation by determining various aspects of the production plan such as extraction sequence, processing decisions, transportation, etc., while adhering to operation and environmental constraints specific to each mining complex. The conventional approach for strategic mine planning starts with representing the deposit with a deterministic orebody model. Based on the information provided by the orebody model, different components in a mining complex are optimized separately and sequentially using several optimization algorithms to maximize the economic value (Alford and Whittle, 1986; Dagdelen, 2006; Gershon, 1983; Hustrulid et al., 2013; Lerchs and Grossman, 1965). For example, the mine will be first optimized to produce an extraction sequence, then the processing destination policy will be determined based on the processing destination policy. However, local optimization of individual components, often with misaligned objectives of the final economic outcome of a mining complex leads to suboptimal production plans. Increasing complexity in the mineral value chain and non-linear transfer functions, aggravates such effects. The geological or supply uncertainty of pertinent properties of the mineral deposit, such as attribute grades and material types, is the major contributor to not meeting production targets during mining operations. Baker and Giacomo (2001) reveal that the primary reason for not meeting production targets is the misconception of reserves/resources due to uncertain material supply. 13 out of 48 mining projects in Australia had 20% more than the estimated reserves, and 9 had 20% less estimated reserves. Vallee (2000) also shows that 73% of failed mining projects are attributed to misestimating the ore reserve. Dimitrakopoulos (2011) analyses these studies and highlights that most project failures are due to the inconsideration of supply uncertainty during the conventional strategic mining process.

Over the past decade, significant developments have been achieved in integrating various components within the mineral value chain into a single optimization model while considering uncertainty. This integrated model for mining complexes enables simultaneous optimization of diverse decisions, such as extraction sequence, destination policies, and processing stream utilization, while incorporating uncertainties presented in mining complexes. The primary objective of modelling a mining complex in a single mathematical formulation is to simultaneously determine diverse decisions in a long-term production plan, such as extraction sequence, destination policies, processing stream utilization, waste management, and capital investment decisions. The optimization maximizes the net present value (NPV) of a mining complex, while satisfying operational constraints and managing technical risks, especially those related to supply uncertainty. Simultaneous stochastic optimization leverages the value of final products sold and captures synergies across the value chain, facilitating comprehensive and efficient decision-making under uncertainty for mining operations.

This chapter reviews the technical literature related to strategic mine planning and modelling uncertainty. Section 1.2 covers deterministic approaches to integrate the optimization of mining complexes. Section 1.3 explains the risk associated with supply and market uncertainty, followed by a review of simulation methods for mineral deposits and commodity prices. Section 1.4 reviews the methods for integrating risk into the mine planning process, initial stochastic mine planning formulations and finally the simultaneous optimization of mining complexes. Section 1.5 outlines the objectives and Section 1.6 outlines the remainder of this work.

1.1 Deterministic approach for strategic mine planning

Strategic or long-term mine planning aims to optimize the life-of-mine production decisions of a single mine to maximize the economic outcome (net present value) of assets while respecting economic and operational constraints. Conventional approaches for strategic mine planning start with representing the mineral deposit as a collection of three-dimensional blocks that contain attributes describing the mineral deposit. These attributes may be metal grades, material types, rock properties, tonnage, etc. According to these attributes, blocks are scheduled to be extracted at a certain period (Hustrulid et al., 2013). Conventionally, the block model assumes perfect knowledge of the mineral deposit (deterministic orebody model) as well as fixed mining and processing capacity, recovery, and commodity price (Hustrulid et al., 2013), making the conventional approaches for strategic mine planning deterministic. A deterministic orebody model is a single estimation of the mineral deposit where relevant characteristics of the deposit are considered fixed and certain. Conventional approaches also produce the production schedule of each mine individually, ignoring the interactions between components in a mining complex. Typical steps of conventional approaches in open-pit mine planning generally start with the determination of the optimal cut-off grade (Lane, 1964, 1988), which decides the destination and economic value of a block. Then, the ultimate pit limit is typically determined by the Lerchs-Grossman algorithm (Lerchs and Grossman, 1965) to maximize the cumulative discounted cash flow. Within the ultimate pit limit, pushback designs that account for operational mining widths are determined. The processing stream decision can also be determined based on the material produced by pushback design. A major disadvantage of this approach is the use of deterministic orebody and parameters which can vary significantly in reality, which will be discussed in detail in Section 1.2. Another drawback is the sequential nature of the optimization framework, which will be elaborated in this section. Interrelated components in a mining complex are optimized independently when in reality, their behaviours depend on each other, thus leading to sub-optimal production schedules.

To address the limitation of the sequential approach of conventional mine planning methods, starting in the mid-1990s, researchers started developing optimization frameworks toward joint optimization of multiple interconnected components in mining complexes. Pimentel et al. (2010) provide an important theoretical integrated modelling of a mining complex including mine and milling operations, product blending and railway system, and port operations. Hoerger, et al. (1999a) presented a study that realized the opportunity of including 50 material sources, 8 stockpiles, and 50 destinations in Newmont's Nevada Operations into a single Mixed Integer Programming (MIP) formulation. The goal of the study is to develop an optimizer to capture the synergies between multiple open-pit and underground mines, and processing destinations by simultaneously optimizing material flows. Three continuous variables are used to quantify the material flows: the amount of material from a mine (source) to a processor, the amount of material from a mine to a stockpile, and the amount of material from a stockpile to a processor. This formulation maximizes NPV and satisfies constraints such as the mining rates, processing rates, metallurgical blending limits, cash flow generation rates, and gold production rates. The framework achieves an increased net present value when applied to Newmont's Nevada Operation (Hoerger, et al., 1999b). This linear formulation has several limitations as it only models the material flow between different components. First, it assumes there is a fixed annual production schedule for the mine, which means the extraction sequences need to be predetermined. Also, the blocks are aggregated as pushbacks based on their metallurgical properties, thus losing the selectivity of mining units. The averaging procedure during aggregation might also mislead the material properties and lead to processing and blending issues at processing destinations. This work demonstrates benefits, provides motivations, and shows an example of capturing the value of synergies in a mining complex by simultaneously optimizing multiple components.

Later, BHP developed the Blasor mine planning software (Stone et al., 2018). Blasor generates a production sequence of multiple pits and their ultimate pit limits to maximize the NPV over the life of the operation based on a MIP model. First, it aggregates spatially connected blocks with similar block attributes to reduce the number of integer variables, so that the size of the MIP is reduced and can be solved in a reasonable time. Based on the aggregated orebody model, the optimal extraction sequence of aggregates is generated using the CPLEX MILP engine obeying all mining, slope precedence, processing and market constraints. Then, the optimal aggregate extraction sequence is used to generate pushback designs for each pit. During this step, manual input might be required to assist the algorithm in designing mineable phases. Finally, the optimal panel extraction sequence is generated using the same method and the same mining, processing and marketing constraints to maximize economic value. Blasor was successfully applied to the Yandi Joint Venture, which consists of eleven pits. The optimization result provides long-term feasible extraction sequences that maximize the economic value of the operation. Limitations still exist. For example, the aggregation of blocks misrepresents the selectivity of the mining operation. And the manual adjustment of the pushback designs requires expert knowledge which might produce different results given the same input. Later, BlasorInPitDumping is developed by Zuckerberg et al. (2007) to incorporate in-pit dumping. In-pit dumping is required for some operations that either do not have enough space available outside the pit limits within their permit for dumping or have specific environmental requirements. On top of Blasor, BlasorInPitDumping can model the movement of waste material from the extraction of blocks to in-pit dumping locations based on the haulage network. It also avoids the placement of waste material on top of valuable ore. The model simultaneously optimizes the panel extraction sequence and the placement of waste material in mined-out pit areas while respecting blending and capacity constraints as well as waste repose slope constraints. However, the previously mentioned limitations of Blasor remain in BlasorInPitDumping.

Whittle and Whittle (2007) propose a model that includes more components (multiple mines, stockpile, processing destination) in a mining complex to optimize the extraction sequence of each mine and the processing stream strategies while respecting mining and processing constraints. Although the paper used the term "global asset optimization", it optimizes components sequentially by first determining the extraction sequence and then optimizing the processing and blending strategies for the pre-determined extraction sequence. First, for each mine, nested pit shells are generated from orebody models using a modified version of the Lerchs-Grossman algorithm (Lerchs and Grossman, 1965). Then, nested pits are grouped into pushbacks satisfying the stripping ratio and generating economic value. Within each pushback, mining blocks that share similar grade attributes are aggregated into panels. Blocks belonging to the same panel are also grouped into parcels based on their material types. The sequence of panels and pushback form the extraction sequence of each mine. Aggregations of mining blocks reduce the size of the model allowing the "Prober" to optimize the processing and blending strategies with less computational expense. The "Prober" can solve the optimal processing and blending strategies using a linear solver for a given extraction sequence with a linear formulation. "Prober" repeatedly creates random feasible extraction sequences until convergence. Later, ProberB (Whittle, 2010) is developed to accelerate the optimization process. Starting from randomly generated combinations of feasible extraction depths of each panel of each mine, ProberB optimizes processing and blending decisions. Instead of generating another feasible extraction depth, ProberB modifies the depth based on the optimized result. The modification is repeated until ProberB obtains a local maximum for the randomly generated depths scenario. Then another randomly generated combination of feasible extraction depths can be used to find another local maximum. The process is repeated until the top ten results lie within a small fraction of a percent of the best local maxima. The optimization process can also be conducted with multi-threading to further decrease the computation time. ProberC (Whittle,

2018) further improves the algorithm to comprehend material grade as metal quantities and changes of material by as many procedures modelled as possible, by enhancing the data structure used. It allows increased complexity in modelling material flow for more accurate valuation of different processes. One drawback of the framework is that it employs several optimization procedures throughout the mineral value chain rather than in a single formulation. Each mine is optimized separately first to produce a sequence of pushbacks within its optimized pit limit. Then interactions between components are optimized to improve the outcome of the mining complex. This sequential process reduces the framework's ability to capture synergies between different components and generate higher NPV. Other limitations exist such as the aggregation of blocks into panels, assumptions that parcels are mined with the same proportion, and the use of pre-determined economic value of blocks.

Previously mentioned optimization approaches attempt to optimize many components of the mining complex jointly, however, they all have limitations and are not simultaneous optimization frameworks. First and foremost, they ignore the uncertainty and spatial variability of pertinent properties of the mineral deposit, which is a major source of technical risk in mine planning referred to as supply or geological uncertainty (Dimitrakopoulos et al., 2002; Dowd, 1994, 1997; Ravenscroft, 1992). It misrepresents the actual material along with the impacts on the economic value of the mine production schedule; therefore the integration of supply uncertainty with stochastic simulations into optimization algorithms is required to enhance the robustness of mining complex optimization and its ability to capture synergies. Other limitations result from algorithmic design decisions to speed up the optimization process such as the aggregation of blocks and pre-determination of extraction sequence. There is also a lack consideration of other major decisions for a mining project such as capital decisions and/or operating modes.

1.2 Modelling uncertainties

1.2.1 Modelling uncertainty in mineral deposits

The methods outlined in Section 1.1 provide improvement to the conventional, sequential strategic mine planning approach. A common limitation of the above methods is the neglect of supply uncertainty during the optimization process by using a single estimated orebody model providing a smoothed representation of mineral deposits by averaging the distribution of extreme values and local variability of materials.

Supply or geological uncertainty is the uncertainty and spatial variability of pertinent properties of the mineral deposit, and it is the major contributor to risk in mining operations (Baker and Giacomo, 1998; Vallee, 2000). The primary limitation of estimation techniques and conventional mine planning approaches is their inability to accommodate the inherent spatial variations in deposit grades. Past studies have shown that the long-term mine production schedules produced with an average-type orebody model fail to deliver forecasted production results when supply uncertainty is later applied (Dimitrakopoulos et al., 2002; Dowd, 1994, 1997; Ravenscroft, 1992). This phenomenon arises from what is referred to as the 'smoothing effect' inherent in estimation methods leading to reduced variability in histograms and variograms compared to the actual data distribution (David, 1988; Dimitrakopoulos, 1998; Goovaerts, 1997; Journel and Huijbregts, 1978; Rossi and Deutsch, 2014). These impacts are further accentuated by the nonlinear nature of mining transfer functions (Dimitrakopoulos et al., 2002). This highlights the key takeaway that average-type inputs do not necessarily yield average-type outputs, underscoring the necessity of modelling uncertainty and incorporating it into the strategic mine planning process.

A set of equally probable representations of mineral deposits, referred to as stochastic orebody models/simulations, represent the spatial variability of relevant attributes in a mineral deposit. They are generated by stochastic simulation methods that reproduce the critical statistics, including (i) data value, (ii) histogram, and (iii) spatial correlation of the original data. Stochastic simulations are conditioned on available drill hole data or information obtained from mined-out areas. Among the stochastic simulation methods, a common characteristic is the adaption of the sequential simulation paradigm, which is built upon the underlying theory that a multivariate probability density function (PDF) can be rewritten as the product of univariate distribution functions by recursively applying the definition of conditional probability (Deutsch and Journel, 1997; Journel, 1994; Rossi and Deutsch, 2014). The generic procedure of the sequential simulation follows: (i) define a random path visiting all the nodes, (ii) construct the conditional cumulative distribution function (CCDF) of relevant attributes at each location conditioned on data and previously simulated value, (iii) randomly sample from the CCDF, and (iv) repeat until all nodes have been visited.

When the probability density functions are assumed to be Gaussian, the method is called sequential Gaussian simulation (SGS) (Goovaerts, 1997; Journel, 1994; Journel and Deutsch, 1993; Remy et al., 2009; Rossi and Deutsch, 2014). The Gaussian assumption allows the utilization of the kriging system to estimate the mean and variance for conditional Gaussian distributions. The efficiency of SGS is described by its time complexity, which refers to how the computational time required for conducting the simulation increases with the size or complexity of the geological model or dataset being simulated. SGS has a time complexity of $O(N^4)$ (Luo, 1998). The lower-upper triangular decomposition method (LU) is developed to reduce the computing time to $O(N^3)$ by using vector processing capabilities (Davis, 1987). However, the limitation of high memory requirement is a drawback. Luo (1998) proves that SGS using Screen Effect Approximation (SEA) achieves computational complexity of $O(Nv_{max}^3)$ by limiting the maximum number of conditioning data, v_{max}. Luo (1998) and Dimitrakopoulos and Luo (2004) introduce the Group Sequential Gaussian Simulation (GSGS) that instead of sequential simulation of nodes, is conducted by groups of nodes. GSGS reduces the computing cost to $O\left(\frac{N}{v}(v_{max}^3 + v^3)\right)$, where v is the group size. Furthermore, when $v = 1$, GSGS is equivalent to SGS and when $v = N$, GSGS is equivalent to LU. Dimitrakopoulos and Luo (2004) also explore how to determine the size of the neighbourhood to find a balance between the precision of simulation results and computational efficiency by using screen-effect approximation loss, which is the mean square difference between the simulated values using all conditioning data and local neighborhood data.

LU, SGS, and GSGS generate simulations at the same support as the conditioning data, often require re-blocking to block support. Storing point support results during simulation requires a large amount of memory allocation. To overcome this limitation, Godoy (2003) presents the direct block simulation (DBSIM) which can simulate block models directly at block support. DBSIM utilizes the advantage of GSGS of simultaneously simulating a group of nodes within a block to save computation time; furthermore, DBSIM discards the group of simulated points at each step to output a block directly reducing memory requirements. The consideration of dispersion variance allows the sequential conditional simulation process to use the data points and previously simulated blocks as conditioning data and uses point-block variance and block-block variance even if they are in different support scales. Block sizes as the selective mining unit (SMU) can be used for DBSIM so that re-blocking is not needed, and the result can be directly used as input for stochastic optimizations (Dimitrakopoulos and Ramazan, 2004; Goodfellow and Dimitrakopoulos, 2016, 2017; Leite and Dimitrakopoulos, 2014; Montiel and Dimitrakopoulos, 2015, 2018)

Mineral deposits have multiple correlated attributes of interest for mining operations, including valuable metal content, material types, deleterious elements, etc. Multiple elements can be simulated using previously mentioned SGS methods with the assumption of multi-gaussian distribution and the use of co-kriging (reference). However, computational complexity increases exponentially with the number of attributes to be simulated (Verly, 1992). Methods are developed to reduce the computational requirement and preserve the correlations of the simulated result. These methods first de-correlate the correlated attributes then simulate them separately and then re-correlate them, so the computational complexity will increase linearly, rather than exponentially. The de-correlation of collocated attributes can be carried out by principle component analysis (PCA) (David, 1988). PCA de-correlates correlated variables into independent factors that can be simulated separately without considering correlations of original variables to reduce computing time; however, PCA only guarantees the de-correlation of the covariance matrix at zero lag. Desbarats and Dimitrakopoulos (2000) employ minimum/maximum autocorrelation factors (MAF) (Switzer and Green, 1984) to simulate multi-variate orebodies. MAF applies two PCA decompositions, to de-correlate variables into independent factors at all the lag distances, overcoming the disadvantages of PCA. Furthermore, Boucher and Dimitrakopoulos (2009) develop direct block simulation with Min/Max autocorrelation factors (DBMAFSIM), that combines the benefits of MAF and DBSIM. With a case study on the Yandi iron ore deposit, Boucher and Dimitrakopoulos (2012) demonstrate that the use of DBMAFSIM is able to reproduce both direct- and cross-variogram while demonstrating the benefits of requiring fewer variograms to be calculated and outputting block-support simulation with manageable file sizes.

Previously mentioned Gaussian-based simulation methods are limited by the Gaussian assumption and use of variograms, which are two-point statistical measurements describing the spatial variability of mineral deposits. Although the Gaussian assumption offers conveniences because the distribution of relevant attributes can be described with only mean and variance, natural geological phenomena are known to present complex curvilinear and non-linear spatial patterns (Dimitrakopoulos et al., 2010; Guardiano and Srivastava, 1992). And the maximum entropy property of Gaussian methods results in the 'salt and pepper' effect for extreme values (Journel and Deutsch, 1993). The outcome of strategic mine planning might be impacted by the inability to reproduce these complex patterns and connection of extreme values in mineral deposits, due to spatial connection requirements of extraction sequence and non-linear transfer functions of the mine production schedule.

Multi-point statistic (MPS) methods are one category of simulation methods that address the limitation of two-point statistics and capitalize on the opportunities to infer multiple-point spatial statistics. MPS also follows the sequential simulating paradigm. Different from traditional simulations of a spatial random field, multi-point statistics collect information from training images using data templates, to consider spatial relationships among multiple points simultaneously (Journel, 1989; Remy et al., 2009; Strebelle, 2002; T. Zhang et al., 2006). Training images are a 2D or 3D representation of a geological or spatial phenomenon that serves as a reference for the desired spatial characteristics. They can be obtained from direct observations and measurements from production data, exploration sampling, and mined-out areas. A data template is a search neighbourhood that is defined by a center location and its neighbourhood and is used for pattern matching during the MPS simulation process. It captures multiple data points at the same time. In most MPS simulations, training images are reprocessed and saved as a "pattern database", based on the spatial template. MPS methods search the patterns database for patterns that match the conditioning data. ENESIM (Guardiano and Srivastava, 1992) and SNESIM (Strebelle, 2002; Strebelle and Cavelius, 2014) are MPS that simulate one pixel/node at a time, termed as a pixel-based method. SIMPAT (Arpat and Caers, 2007) and FILTERSIM (Zhang et al., 2006) simulate entire patterns instead of one pixel, termed pattern-based. Compared to ENESIM and SNESIM which only simulate categorical attributes, SIMPAT and FILTERSIM can simulate continuous attributes using different distance measurements for similarity instead of looking for exact match. The storing of a pattern database requires large memory space, and scanning the pattern database is computationally expensive. Direct sampling (Mariethoz et al., 2010) allows sampling from the training image, avoiding storing the data events in a search tree. The process randomly chooses a replicate from the Training Image (TI). Then, a distance function is employed to calculate the degree of similarity between the chosen replicate and the data event. If the replicate is determined to be similar, the simulation node takes on the central value of the said replicate. Otherwise, another replicate is chosen instead. Such random selection of similar data events can accelerate the simulation by running the sampling in parallel. Above mentioned methods need to handle multi-scale spatial characteristics using techniques such as multi-grid to capture spatial phenomena in different scales. WAVESIM (Chatterjee et al., 2016; Gloaguen and Dimitrakopoulos, 2009) utilizes wavelet transform of training image to simulate features in different scales directly, given considerable dependencies between scales as observed from the distribution of the wavelet coefficients. MPS methods demonstrate several limitations. First of all, the use of training images is usually computationally expensive for multivariate models. Also, the use of training images generally requires large amounts of data that are often unavailable. For example, a training image can be a completely mined-out area, which is unavailable at the early stage of a mining operation. Moreover, MPS methods do not always reproduce statistics of exploration data because they are focused on reproducing the pattern in the training image.

Due to the need to reproduce high-order statistics and the limitations of MPS methods, another category of simulation methods has been developed to extend spatial models beyond second-order statistics by accounting for high-order spatial characteristics such as cumulants from conditioning data to simulate complex geological structures and non-linear patterns. Dimitrakopoulos et al. (2010) proposed the high-order stochastic simulation (HOSIM) method that uses spatial cumulants and moments for the description of complex geological structures and non-linear patterns beyond second-order statistics. Cumulants are statistical measures used to describe the properties of a probability distribution (Brillinger and Rosenblatt, 1967; Rosenblatt, 1985), especially for spatial statistics of higher order than mean (first order) and variance (second order). They are mathematical quantities derived from the moments of a random variable and provide useful information about the shape, dispersion, and other characteristics of the distribution. Dimitrakopoulos et al. (2010) demonstrate the ability of high-order cumulants to describe complex geological structures and connectivity and the relations between the higher and the lower-order moments or cumulants. These characteristics show consistency over a series of orders, which cannot be represented or reproduced by multiple-point statistics observed with spatial templates.

However, it also shows that the sequential calculation of cumulants is computationally expensive. Mustapha and Dimitrakopoulos (2010) present a HOSIM method that uses spatial cumulants to build the conditional probability distribution function (CPDF) and high-order Legendre polynomials to approximate said distribution. This framework is capable of reproducing low and high-order spatial statistics and generating more accurate realizations of complex geological, and no prior assumptions about the probability distribution are made. To reduce the high cost of computing high-order spatial cumulant, spatial templates are defined as a priori.

Minniakhmetov and Dimitrakopoulos (2017a) extend the method to jointly simulate spatially correlated variables of deposits. The method first de-correlates the correlated variables into independent factors with the diagonal domination cumulants. Then, factors are independently simulated. The simulated factors are transformed back to generate simulations that reproduce both low and high-order spatial statistics. de Carvalho et al. (2019) further extend the method to directly simulate orebody at block support by estimating the cross-support probability distribution function. The benefits include minimizing the memory requirements and improving the computation time as explained previously. Minniakhmetov and Dimitrakopoulos (2017b) propose a data-driven framework for high-order simulation of correlated variables without using training images. Based on the fact that high-order spatial moments are connected with lower-order moments based on boundary conditions, spatial cumulants can be calculated recursively from hard data using the B-spline approximation of higher-order cumulants under zero-boundary constraints. The method is capable of reproducing with conditioning data and avoiding the necessity of a training image. The option of using a training image is still available in case the conditioning data is sparse. Yao et al. (2018) present a new computational model for high-order stochastic simulation. The method uses spatial Legendre polynomials with location-dependent templates to approximate the CPDF. Doing so avoids the requirement of defining prior templates and reduces the computational time of calculating moments or cumulants by parallelizing the calculation for each replicate. de Carvalho et al. (2019b) compare the effect of using high-order simulations and using two-point SGS simulations in the mine production scheduling optimization. The high-order simulations demonstrate more complex, nonlinear spatial characteristics of the variables and reproduce the connectivity of extreme values. When high-order simulations are used for simultaneous stochastic optimization, the production plan of an open-pit gold mine generates 5% to 16% more NPV when compared to using two-point SGS simulations.

1.2.2 Modelling uncertainty in mining equipment performance

Factors affecting uncertainty in equipment performance include availability, utilization, productivity, breakdown, performance, repair time, cycle time, etc., and these factors vary largely between all the equipment used in a mining operation. Monte Carlo simulations have been used to generate equally probable scenarios that are generated based on historical equipment performance data and can quantify the uncertainty associated with the performance of different equipment in a mining operation. Two major types of Monte Carlo simulation approaches are used to quantify equipment performance uncertainty in the literature (Fioroni et al., 2008; Upadhyay et al., 2015; Upadhyay and Askari-Nasab, 2017). The first type of simulation assumes that equipment performance follows a prior distribution, and then simulations can be done by estimating the parameters of the prior distribution. The second type of simulation follows empirical distribution, which makes no priori assumption, solely based on the historical equipment performance data.

Matamoros and Dimitrakopoulos (2016) simulate shovel and truck availability, by assuming the availability of equipment follows a Gaussian distribution. Quigley and Dimitrakopoulos (2016) simulate cycle time, assuming Gaussian distribution. Both and Dimitrakopoulos (2018) simulate shovel productivity and truck availability assuming a Gaussian distribution. Paduraru and Dimitrakopoulos (2019) simulate the shovel load time, truck cycle time assuming a Gaussian distribution, shovel breakdown time assuming an exponential distribution, and shovel repair time assuming a log-normal distribution. Ozdemir and Kumral (2019) simulate truck fill factor, loading time, dumping time, and truck hauling time by assuming either log-normal, normal, or Weibull distributions. Quigley and Dimitrakopoulos (2020) generate simulations of shovel production, utilization, availability, and truck utilization and availability, assuming multi-Gaussian distribution. However, instead of simulating with a multi-Gaussian distribution, different correlated equipment performance indicators, such as production, utilization, and availability, are first de-correlated into independent factors using Principle Component Analysis (Hotelling, 1933). Then, the PCA factors are simulated separately assuming Gaussian distribution. Finally, the set of independently simulated factors is back-transformed into performance indicators representing the simulated equipment performance. de Carvalho and Dimitrakopoulos (2021; 2023) simulate the shovel loading and truck cycle time assuming normal distribution as well as equipment failure frequencies and repair times assuming Poisson distribution.

1.2.3 Modelling uncertainty in mineral processing recovery

The integration of geo-metallurgical properties, especially recovery rate, into the evaluation and optimization of the mine production plan, is vital for the realization of the economic value of the mining operation (Coward et al., 2009). The uncertainty of processing recovery arises from the interaction between the mineralization (attribute grade, hardness, texture, etc.) and the operation of the mining and processing plant (comminution time, reagent used, etc.). This uncertainty can be modelled by stochastic simulations. Coward et al. (2013), Coward and Dowd (2015), and Jackson et al. (2014) use a set of simulated recovery responses to assess the effect multiple uncertainties have on the economic outcome of mining projects. Coward et al. (2013) propose a method for simulating the recovery curve by bootstrap sampling of the recovery data. The study suggests that these recovery curves can often be reasonably modelled by many models such as the linear, second-order quadratic or logarithm functions, and for each of the input parameters used, it is possible to describe the distribution, based on experiments conducted on the historical production data from the mine. In a case study, the recovery uncertainty is generated by assuming the recovery function of the form $Rec\% = m * \%\ \text{Kimberlite} + \text{c}$. By removing one data point ("bootstrapping") and re-estimating $m\ and\ c$, it was possible to derive a set of $m\ and\ c$ parameters. Repeating the above process, multiple values of $m\ and\ c$ are produced. Then, by computing the mean and variance from the set of m, it was possible to draw random values of m from the normal distributions with the same mean and variance to produce simulated values of m. A similar procedure is followed for simulating the values of c.

Scenario Based Project Evaluation (SBPE), or stochastic risk analysis, is conducted with said recovery uncertainty and geological uncertainty, and the results show significant deviations from production targets due to the joint effect of uncertainties. Jackson et al. (2014) explain the process of SBPE in detail and present a case study incorporating geological, recovery and market uncertainty. Coward and Dowd (2015) present a more comprehensive approach and case study at a copper porphyry mine with flotation processing plants. The study assesses the uncertainty of the net smelter return (NSR) based on the combined effect of geological uncertainty with a set of simulated orebody models, recovery uncertainty by creating multiple simulated grade-recovery curves using the process presented by Coward et al. (2013), and market uncertainty with a set of future metal prices scenarios. The risk analysis reveals that the spread of NPV, the range between P10 and P90, is as large as 70 % of the expected project NPV when the compound uncertainty is considered. These demonstrate the importance of recovery uncertainty by measuring the effect of recovery uncertainty on NPV. Future works are needed to account for the recovery uncertainty in the mine optimization process.

1.2.4 Modelling uncertainty in commodity prices

The commodity price is essential to the economic viability of mining operations as it governs the classification of in-situ resources and profits of sellable products. Also, price uncertainty is generally considered uncontrollable to mining companies over a long-term horizon. Therefore, it is vital to account for the market uncertainty in strategic mine planning (McCarthy and Monkhouse, 2002). Generally, the four main factors governing commodity prices are supply and demand, regulation by cartels or commodity agreements, negotiation between producers and consumers, and fixed prices by a monopoly or oligopoly (Gocht et al., 1988). However, long-term commodity forecasting using an economic model is not common practice for mining companies (Dooley and Lenihan, 2005). Instead of integrating metal price forecasts, the expected commodity price is used for strategic mine planning to produce a long-term plan. This planning process is repeated every year, and the most current fixed price is used. Then, the outcome of the long-term plan is analyzed for its sensitivity with base, upside, and downside case prices as base, best, and worst-case scenarios of the deterministic mine plan. Each commodity is influenced by its characters and markets.

Commodity price forecasting can be formulated as stochastic models like the ones presented by Schwartz (1997), or structural models like the ones presented by Pirrong (2011). Schwartz (1997) presents the two most commonly used models to conduct commodity price simulation, Geometric

Brownian Motion (GBM) and Mean-Reversion (MR). The GBM model incorporates market volatility and long-term interest rates as factors influencing commodity prices. On the other hand, the MR model accounts for a long-term mean price, reverting speed, and volatility as factors. It is commonly accepted that base metal prices are heavily influenced by supply and demand, and are fluctuating around a long-term price, and precious metals tend to be influenced by investment factors such as interest rates (Kernot and West, 1991). Therefore, GBM is commonly used for precious metals price simulation and MR is used for base metal. Many other variants of GBM and MR also exist. For example, Suarez and Fernandez (2009) present an MR model with Poisson jumps (Press, 1967) for modelling sudden and significant changes in price. Del Castillo and Dimitrakopoulos (2014) incorporate copper price simulation into determining ultimate pit limits in addition to the geological uncertainty. The copper prices are simulated using MR with Poisson jumps. Although the integration of market uncertainty has the potential to improve strategic mine plans, the effect of a set stochastic price simulation still requires study.

1.3 Strategic mine planning with uncertainty

1.3.1 Incorporating uncertainty in strategic mine planning

Previous sections describe the conventional method of strategic mine planning, and their need to incorporate multiple sources of uncertainties in the planning process to increase project value while managing risks and exploiting opportunities appropriately. The primary contributor to not meeting project expectations is supply uncertainty. This is due to the fact that optimizing a mine design with deterministic approaches involves complex non-linear transfer functions, and employing an average-type model as input for optimization can yield misleading outcomes. They also demonstrate that including uncertainty can significantly reduce the deviation from the production target while increasing the project value.

One of the first approaches to account for geological uncertainty in the mine production scheduling process is developed by Godoy (2003) and Godoy and Dimitrakopoulos (2004). This stochastic optimization approach manages risks associated with supply uncertainty. First, the stable solution domains for each orebody model are defined based on all feasible stripping ratios, and the stable solution domain is bounded by the best case (pit-shell-by-pit-shell) and the worst case (bench-by-bench). Then the optimum mining rates are determined by a mathematical programming formulation within the overlapping area of the stable solution domain. The optimum mining rate is then used to generate extraction sequences for each one of the orebody simulations. Possible periods for blocks to be extracted are determined for perturbations by considering all extraction sequence generated. For example, if there is a 95% chance a block belongs to the same extraction period, there is only one candidate period for extracting this block, and this block is "freezed" at this period and will not be considered for perturbation. By perturbing the remaining blocks to minimize the deviation from the optimal ore and waste production targets, the optimal production schedule is generated using a simulated annealing (SA) optimization approach (Metropolis et al., 1953). This method generates a single production schedule that aims to minimize deviations from the optimized processing capacity while accounting for supply uncertainty and respecting the production capacities. Certain limitations remain such as the intensive labour required to develop initial schedule for all scenarios. However, it is the first optimization method that produces production schedules accounting for uncertainty. The method is applied to a real-life open-pit gold mine and the production schedule is compared to the base case schedule generated with a single estimated orebody model (Godoy and Dimitrakopoulos, 2004). Compared to the base case schedule, the production schedule generated by the method substantially increases the project NPV by 28% and reduces the risk of deviating from production targets by approximately 9%. Leite and Dimitrakopoulos (2007) apply the method to a low-grade disseminated copper deposit. The result shows an increase of NPV by 26% and a significant reduction in deviations from production targets. Furthermore, Albor and Dimitrakopoulos (2009) study additional aspects of the method. First, it reveals that freezing blocks does not affect the final result other than requiring longer computing time. Also, the method is not sensitive to increasing the number of input simulated orebodies when the number is more than 10. It reveals an important feature that although modelling supply uncertainty requires a number of simulated orebodies to describe the distribution of an individual block, a stable mining production schedule can be obtained with a lower number of orebody simulations. This is caused by the volume-support effect as an annual schedule extracts a large volume of blocks within a single period. As opposed to all previous studies assumed a fixed ultimate pit limit generated by the Lerchs-Grossman algorithm (Lerchs and Grossman, 1965), Albor and Dimitrakopoulos (2009) also improve the method to consider optimal pit limits generated by the stochastic approach. The stochastic approach shows a 10% improvement when letting the optimizer decide the ultimate pit limit due to the input of supply uncertainty. The stochastic optimal pit limit is larger than the one determined by conventional optimization methods due to the optimization of mining rates and using multiple simulated orebody models as input, resulting in a 17% increase in total tonnage and extending the production life by one year.

Stochastic integer programming (SIP) can be used to formulate problems with uncertainty and find a solution that maximizes the desired outcome while satisfying constraints and managing and exploiting associated risk (Birge and Louveaux, 2011). Various technical literature suggests the 2-stage stochastic integer programming model is suitable for the optimization of mining complexes (Dimitrakopoulos and Ramazan, 2008; Ramazan and Dimitrakopoulos, 2005, 2013). The first-stage decisions are taken before observing the uncertainty, and second-stage decisions, also called recourse decisions, are taken after the uncertainty is revealed. SIP formulations provide flexibility to model mining complex optimization. It can maximize the expected NPV of the mining operation, minimize the deviations from production targets including ore tonnage, grade and quality, and create a feasible production schedule at the same time (Albor and Dimitrakopoulos, 2009; Dimitrakopoulos, 2011; Leite and Dimitrakopoulos, 2014). The first stage decisions are the mining and processing decisions describing the production plan of the mining complex. The second stage recourse variables are measurements of deviations from production targets resulting from the first-stage decisions.

Ramazan and Dimitrakopoulos (2005) propose the first application of the two-stage stochastic integer programming (SIP) model. It produces a long-term production schedule using a set of stochastic orebodies as input instead of an average-type model. The objective function aims to maximize the project NPV and minimize deviation from ore, waste, and metal production targets. The first stage extraction decisions are modelled as binary variables. The second stage variables are used to measure deviations from the production target in each scenario. A case study of a hypothetical two-dimensional deposit with a 3-year LOM is conducted with a commercial solver. The use of penalty costs in the objective function generates a balance between the NPV and risk management. The risks are reduced and deferred to later years when more information is available through the use of geological discounting (Dimitrakopoulos and Ramazan, 2004). This overcomes substantial limitations of the maximum-upside and minimum-downside approach. Ramazan and Dimitrakopoulos (2013) extend the framework to incorporate stockpiles to improve the processing strategy and the geological discount rate. The geological risk discount rate enables the optimization to not only reduce the risk but also defer risks into later periods. The optimization model is tested on a gold deposit. The application is computationally expensive. The total number of decision variables exceeds 90,000 with more than 27,000 binary variables and 54,000 constraints. The production schedule is optimized over two time horizons, first years 1-4, with 18,000 binary variables, are optimized, and then years 4-6 with, 9,000 binary variables, are optimized. The combined schedule results in a 10% improvement in NPV and reductions in ore production deviation compared with a conventional schedule. Later, Benndorf and Dimitrakopoulos (2013) proposed a similar formulation to model the blending of material to allow the incorporation of blending constraints for processing destinations requiring their throughput to have certain material properties. The method is applied to the Yandi Central 1 iron ore deposit in Western Australia. Geological uncertainty in iron, silica, alumina, phosphorus, and loss on ignition are modelled through twenty simulated orebodies. Geological risk discounting is used through the blending target deviation penalty, the influence of the material quality on the performance of the production schedule is reflected in the objective function. The case study also explores how to choose appropriate penalty costs for deviating from production targets. $1, $10, and $100 per unit deviation are tested and their impact on schedule dispersion and the achievement of blending targets are demonstrated and compared to determine the most suitable penalty. It finds that medium penalties produced the best results in meeting blending targets while producing a smooth schedule. The result shows that the method can manage the risk of deviating from blending targets while increasing project NPV. However, due to the fact that deviation penalties are modelled in the objective function alongside cash flows, the production target penalty cost and smoothing penalty cost must be calibrated to achieve a satisfactory production schedule.

Multistage stochastic programming mimics real-world scenarios where decision-makers must make choices at different points in time with information already observed in the past and uncertainty that might be observed in the future. Multistage stochastic programming allows decisions to be modelled and optimized in multiple time periods or stages. At each stage, the decision-maker faces a set of decisions and must choose the best course of action based on the information available up to that point. Boland et al. (2008) propose a multistage formulation to allow both mining decisions and processing decisions to adapt to the geological uncertainty observed. It is able to alter mine processing decisions as new information is available. Processing decisions are allowed to change at the same period as uncertainty is revealed, while mining decisions are subjected to a one-year lag to adapt to the uncertainty observed. As opposed to two-stage SIPs that only produce one production schedule, multistage stochastic programming produces a set of production plans corresponding to each one of the input stochastic scenarios. When there are significant differences between the grades of simulated orebodies, the solution is allowed to change the processing and mining decision variables between scenarios. If there are no significant differences, non-anticipativity constraints are enforced to ensure decisions are the same across different scenarios. Such non-anticipativity constraints avoid the set of production schedules overfitting the distribution of grades. The method is applied and compared to a deterministic equivalent base case. The result shows that the proposed approach increases the NPV by 3%. However, there are serval limitations with this method. First, there is no control over deviations because multiple scenarios are considered a set of perfect information; therefore, the set of production schedules will over-fit the corresponding scenario. Consequently, they will perform poorly when tested over a different set of simulations. This suggests that the reality encountered cannot be represented exactly by any of the simulations. Also, the flexibility of the method increases the number of multistage decision variables, which increases computational expense. Lastly, the set of production plans is produced by branching, so there is no unique schedule for implementation.

Approaches based on SIPs provide many benefits for strategic mine planning with uncertainty. A thorough review of the methods, advancement, benefits and drawbacks with examples and summaries of case studies are conducted by Dimitrakopoulos (2011). The stochastic framework addresses the main drawback of estimation methods and conventional planning approaches which is the inability to produce a single optimal schedule to account for the inherent spatial variability of in situ grades. In fact, conventional optimization methods assume perfect knowledge of the orebody in question. Neglecting this fundamental source of risk and uncertainty can result in impractical production projections and suboptimal mine plans. The research presented in this context demonstrates that the stochastic framework adds value to the production plan in the order of 25% (Dimitrakopoulos, 2011). The stochastic framework also produces about 15% more tonnage than deterministic ones. These optimization frameworks also minimize the risk of deviation from production targets with a feasible schedule or deferring them into later periods by the use of geological risk discounting.

1.3.2 Simultaneous stochastic optimization

Simultaneous stochastic optimization incorporates all components of a mining complex in a single mathematical formulation, allowing the modelling of constraints of every component at the same time and any nonlinear transformation so that the solution is able to capitalize the synergies between components (Goodfellow and Dimitrakopoulos, 2016; 2017; Montiel and Dimitrakopoulos, 2015; 2017; 2018). Components in a mining complex can include mineral deposits (open-pit and underground), stockpiles, waste disposal, processing destinations, transportation, products to clients etc. Two-stage SIP formulations with fixed recourse (Birge and Louveaux, 2011) provide a practical formulation that maximizes NPV, while managing technical risks by accounting for supply uncertainty. The simultaneous stochastic optimization framework generates a long-term production schedule that considers the flow of material between the components in the mining complex from the mines to the customers and the spot market. The approach captures the synergies between the different components and determines the optimal production schedule decisions that account for the configuration of a mining complex. The formulation of the mining complex allows the optimization to consider the value of the product sold at the end of the value chain instead of determining the economic value of blocks prior to optimization. This change allows synergies between components to be discovered and capitalized by directly integrating stockpiling, blending, destination policy and capital expenditure decisions into the optimization. The production schedule generated using a simultaneous stochastic optimizer increases the NPV and improves the ability to manage technical risks.

Montiel and Dimitrakopoulos (2013) present an initial approach to model an entire mining complex. The extraction decisions are optimized, followed by the downstream flow of materials. The optimizer modify the extraction sequence first and then assigns a destination to each block based on their material type under different scenarios. The optimizer aims to reduce deviations from production targets and quality requirements using simulated annealing. The method is applied to the Escondida Norte mine in Chile. The case study shows that the method reduces production target deviations from 20% to 5% in the benchmark for the processing and grade targets. Although the NPV is not specifically optimized in this framework, the case study still shows a 4% increase in NPV. Despite only optimizing the extraction sequence, the method integrates important processing decisions into the production schedule by modelling the entire mining complex. However, the misclassification of ore blocks remains an issue because the material type of a block might vary between different scenarios and certain destinations only accept certain types of material. Also, the approach shows better performance when an initial solution is used as input.

Montiel and Dimitrakopoulos (2015) present an optimization model to overcome several limitations mentioned above. The model incorporates multiple mineral deposits, processing steams, and transportation alternatives. The objective function is modelled as a two-stage SIP to explicitly maximize NPV by accounting for the value of the final product sold and operational costs while minimizing deviations from production targets. The block extraction and processing decisions, processing alternatives, and transportation alternatives are optimized simultaneously with supply uncertainty. The simulated annealing solution approach (Metropolis et al., 1953) utilizes three types of perturbations to improve the objective function. The three types of perturbations are used to modify block extraction and processing decisions, processing alternatives, and transportation alternatives respectively (Montiel and Dimitrakopoulos, 2017). First, the block-based perturbations modify the extraction period of a block. Destinations of the blocks are determined by formulating a knapsack problem (Dantzig and Thapa, 2003) based on the overall profitability of the block to a destination across all orebody scenarios. The operating alternative perturbations randomly change the operating modes at processing facilities. The objective value is evaluated for new operating alternatives considering all orebody scenarios. Transportation perturbations randomly modify the proportions of output material from a processing facility at a period to a transportation system. The method is applied to Newmont's Twin Creek operations (Montiel and Dimitrakopoulos, 2018). The simultaneous stochastic optimization increases the NPV by 7% and improves the management of autoclave blending constraints when compared to the deterministic schedule of Twin Creek operations. However, limitations exist for the method. First, destinations are determined by finding the most profitable destination for each block instead of optimizing the destinations of other blocks at the same time. An initial solution is required for reasonable execution time. This would limit the efficient exploration of solution space. The block-based destination decisions are scenario-independent, which reduces the number of decision variable but increase the chance of misclassification of material.

Goodfellow and Dimitrakopoulos (2016) propose a generalized network-based modelling method that allows the inclusion of all components in a mining complex for simultaneous stochastic optimization and considers the value of final products sold at the end of the mining complex instead of block economic values. It is the framework for the research discussed herein. The method aims to simultaneously define the life-of-asset extraction sequences, destination policy, and processing stream decisions while managing the metal production targets. With a two-stage SIP model, the extraction sequence and destination policy are modelled as first-stage decision variables and processing stream decisions are modelled as second-stage decision variables. The scenario-independent extraction decisions determine which period a block is mined. The scenario-independent destination decisions determine the destination of a cluster of blocks after they are mined. This new destination policy avoids misclassification of material by allowing blocks to be assigned to different clusters in different scenarios. The clusters of blocks are decided based similarity of grade attributes using the k-means++ clustering algorithm (Arthur and Vassilvitskii, 2007). Although the destination decisions are scenario-independent, the membership for each block is scenario-dependent due to the supply uncertainty of the simulated metal grade. However, it introduces a user-defined parameter, the number of clusters. A higher number of clusters leads to a higher degree of flexibility for the optimizer and can lead to higher NPV, but it might also create overfitting the distribution of metal grade as the number gets close to the number of blocks and creates difficulty for operation. This multi-variate clustering improves optimization for orebodies with multiple valuable metals as well as better management of deleterious elements. The clustering approach allows the destination policy to be determined jointly with extraction decisions and processing stream decisions during the optimization process. However, the clustering result and the cluster-based destination policy is difficult to visualize or interpret at high dimensions as the boundary becomes complicated. The processing stream decisions determine the proportion of output material from a location to another downstream destination. The scenario-independent decisions are made resilient to the uncertainty observed in the mining complex while scenario-dependent decisions can be made adaptive to the uncertainty. The uncertainty can be characterized through a set of scenarios or a combination of different sets of scenarios, which describes joint uncertainty. The uncertainty and other relevant operation parameters are modelled as attributes such as metal contents, mining cost, recovery, revenue from sales, etc. A multiple neighborhood simulated annealing algorithm is used as the solution approach along with constraint relaxation. This algorithm is an effective strategy that ensures both improving the solution and exploring a wide solution space, making it well-suited for finding global optima instead of getting stuck in local optima (Dimitrakopoulos and Lamghari, 2022). The algorithm will be explained in detail in Section 1.3.3.

A case study was performed on a multi-element nickel laterite mining complex in comparison to the conventional schedule to demonstrate the framework's ability to incorporate supply uncertainty in a mining complex and the benefit of considering the value of the product sold (Goodfellow and Dimitrakopoulos, 2017). The mining complex consists of two orebodies and multiple stockpiles with strict blending requirements. Ignoring supply uncertainty leads to sub-optimal production schedules with destination policy and blending strategies that can be detrimental to the material quality requirements. The simultaneous stochastic optimization must generate an optimal production schedule determining a multi-element destination policy, with clusters on nickel, iron, silica and magnesia grades, and a dry tonnage density factor. The result is compared to a "deterministic-equivalent" design. The "deterministic-equivalent" design is generated by the same formulation but using a single estimated orebody as input. The stochastic design is able to satisfy the key constraints of interest, such as the SiO2:MgO ratio, iron grade and plant feed tonnage, while increasing the NPV by 3%. Another case study of a copper-gold mining complex is also conducted (Goodfellow by Dimitrakopoulos, 2017). This case study incorporates a non-linear grade-recovery relationship for copper and gold grades into the optimization model. In a conventional optimization framework, nonlinear recoveries can only be considered by calculating the economic value of the block assuming each block can be processed separately. This advancement, brought by the consideration of the value of final products sold, enables the calculation of blending of materials at the different processing destination and capture the added value of maximizing recovery through blending. The result of simultaneous stochastic optimization not only better meets the production targets at the sulphide mill and sulphide heap leach while reducing the risk in terms of the quantities sent but also increases the NPV by 22.6%. Goodfellow and Dimitrakopoulos (2015) extend the framework to include capital expenditure (CapEx) options. It allows the modelling of CapEx by modelling concepts such as truck and shovel hours to integrate load, haul fleet purchases etc. It addresses the challenge of determining the optimal timing of large capital investments such as the purchase of shovels and trucks determining the optimal mining rate. The mining rate determines the amount of material and metal to be generated and the mining cost of the mine. Predetermining the mining rate can only obtain a locally optimized production schedule. Hence, the consideration of CapEx decisions is vital to obtain an optimal result. The result of optimization not only shows a 5.7% increase in NPV, when compared to a deterministic design that does not consider risk but also demonstrates the framework's flexibility to model concepts such as truck and shovel hours and haul fleet purchases into the simultaneous optimization.

Farmer (2016) extends the framework introduced by Goodfellow and Dimitrakopoulos (2016) to incorporate processing capacity and mining capacity options by modelling CapEx options. The optimization simultaneously optimizes the sizing of a processing capacity and the mining rate which are two major bottlenecks of mining operations. This provides simultaneous determination of the amount of mining equipment and the size processing facility considering their required capital investment, with the extraction sequence, destination policy, stockpiling, and processing decisions. Additionally, it also incorporates complex revenue calculations such as royalties and metal streaming for copper-gold mining. The calculations of revenues are conducted under price uncertainty, which is modelled by commodity price simulation. However, the price uncertainty is incorporated through a multi-step process. First, the optimization is conducted with only the supply uncertainty incorporated, then the first-stage decisions are frozen and only second-stage decision variables are optimized with both supply and market uncertainty. The resulting production schedule obtains a 12% increase in NPV by producing higher-grade metal during higher-price periods and lower-grade material during lower-price periods.

Kumar and Dimitrakopoulos (2017) incorporate geo-metallurgical uncertainties into simultaneous stochastic optimization in addition to supply uncertainty. The semi-autogenous power index (SPI) and bond work index (BWI) are simulated as input. The non-additive nature of the two attributes makes them difficult to incorporate into the conventional framework. The method classifies the material as soft or hard based on SPI and BWI and then includes geo-metallurgical constraints to limit their ratio at processing mills. Twelve different material types are defined based on the calculation of the hardness of material using geo-metallurgical properties. Operational constraints regarding geo-metallurgical properties are modelled in the processing facility to maximize the utility and recovered metal. A case study is conducted at a copper-gold mining complex. The result indicates a higher chance of meeting production targets at the processing destination especially the constraint regarding the ratio of hard/soft material. In addition to the deduction in deviations from geo-metallurgical targets, the stochastic schedule generates 19.3% higher NPV compared to the conventional schedule, which is used in the current production of the mine.

Zhang and Dimitrakopoulos (2017) present a decomposition method to optimize the upstream mine production schedule (MPS) and downstream material flow plan (MFP) of mining complexes under supply and market uncertainty. Although the optimization of upstream and downstream decisions is separate, the interactions between components are modelled so that the synergy of the two sets of decisions is captured and the combined solution will converge. The optimization process first solves the mine production schedule and the tonnage of each material type is summarized and used as input for optimizing the material flow plan. Then, the result of MPS updates the value of the material and the mine production schedule is re-optimized based on the update value. This process is repeated until a convergence condition is met for the amounts of purchasing and selling of each material type in each period being zero. Using a shadow price for MFP, also shows that ignoring market uncertainty will lead to overestimation of the profitability of the mine plan. Later, Zhang and Dimitrakopoulos (2018) present a two-stage SIP model to integrate forward contracts and non-linear recovery rate into the optimization while considering market and supply uncertainty. An efficient heuristic is used to iteratively optimize the production schedule by gradually moving the lower and upper bound of throughput and head grade until the gap between bounds is small enough for all periods and scenarios. It emphasizes the importance of considering dynamic recovery rates when a forward contract is included as a component of the mining complex. The result of the optimization also provides guidance for accepting a forward contract: (i) the company should reject the contract if the expected NPV and the worst-case NPV would decrease (ii) the company should accept the contract, if the expected NPV and the worst-case NPV would increase, (iii) the company should balance the trade-off of the contract if the expected NPV would decrease and the worst-case NPV would increase.

Saliba and Dimitrakopoulos (2019) present an application of the simultaneous stochastic optimization at a complex gold mining operation in Nevada, restricted by stringent geochemical blending constraints. This study investigates the impact of joint market and supply uncertainties by incorporating commodity price simulations as inputs into the optimization model. This approach allows the simultaneous stochastic optimizer to account for market uncertainty in all three decision variables. Furthermore, the research assesses the efficacy of the simultaneous stochastic optimization framework's cut-off strategies. It does so by considering downstream product values, the intricate non-linear blending of material, and the extraction sequence. This approach replaces the need for a pre-determined cut-off grade optimization used by traditional methods, addressing limitations associated with determining cut-off grades before production scheduling. The proposed approach also reduces the operational complexity, due to the blending requirements in the case study of the mining complex, by reducing the number of material types and stockpiles. The result shows that the joint uncertainty case generates a 3% higher net present value. The reduction of the number of material types and stockpiles to twelve yields similar results when compared to the base case where the number of material types and stockpiles is thirty-eight. This case study demonstrates the flexibility of the framework by integrating commodity price fluctuations and optimizing the mining complex under joint market and supply uncertainty and the ability to reduce the operational complexity in terms of managing the mining complex.

Del Castillo and Dimitrakopoulos (2019) extend the simultaneous stochastic optimization framework developed by Goodfellow and Dimitrakopoulos (2016, 2017) to incorporate CapEx and operating mode decisions with a dynamic optimization approach. The amount and timing of capital investments as well as the operation mode of the processing facility affect the mining and processing capacity of different components in a mining complex. They significantly change the result of the optimal production schedule when they are included. This approach extends the original framework to use a branching mechanism to consider different capital investment decisions under different scenarios. The result comprised a set of alternative investment options with different production schedules. The branching mechanism first performs optimization on every orebody scenario separately by making all three sets of decision variable scenarios dependent. The probability of taking each capital decision in the first year is calculated. When the probability of a capital decision reaches a certain threshold, for example, if 16 out of 20 scenarios will purchase a shovel in the first period, these 16 scenarios will be combined as a branch of the alternative mine plan and the other 4 scenarios will be combined as another branch. Then the first-year extraction decision and destination decisions on each branch are set to be scenario-independent again with two branches separately. Then optimizations are performed on every orebody scenario again with decisions at later periods being all scenario dependent. This dynamic optimization algorithm is repeated until the last period. The result is a set of alternative mine plans that provide a balance between two major benefits. First, a set of alternative mine plans provides flexibility to make capital decisions adapted to the uncertainty observed and a production plan adapted to changing mining and processing capacity. Also, it avoids overfitting the distribution because branching to an alternative mine plan requires a minimum amount of scenarios to choose the same capital investment. A case study is conducted for a copper mining complex considering capital decisions of the purchase of trucks and shovels to increase the mining capacity and a secondary crusher to increase the processing capacity. The solution from the initial framework without the dynamic optimization algorithm is used as a baseline for comparison. The NPV increases by $170 million. Del Castillo (2018) further expands the method to include operating alternatives, such as the processing facility operating mode to increase the recovery at the expense of decreased capacity and the drill-and-blast pattern designs. The result generates 10.5% higher NPV compared to the baseline.

1.3.3 Solving algorithms

The simultaneous stochastic optimization framework mentioned above is computationally expensive, mainly due to the number of integer variables. The number of integer variables can reach the order of billions. Simultaneous stochastic optimization is considered an NP-hard problem (Bienstock and Zuckerberg, 2010; Lamghari et al., 2014), which is challenging for a commercial solver. This challenge is further complicated by the number of scenarios describing uncertainty, which grows exponentially with different sources of uncertainties incorporated. Metaheuristic algorithms are alternative algorithms to provide many benefits for solving simultaneous stochastic optimizations. These benefits include (i) efficient exploration of a large solution space, (ii) handling of complex, non-linear, and multi-modal functions, (iii) flexibility and adaptability to different types of variables, (iv) the ability to hybridize with other optimization techniques, (v) the possibility to parallelize and scale, etc. (Yang, 2010). An effective and efficient metaheuristic should be tailored to the mathematical model at hand to achieve a balance among sufficient optimality, efficient exploration of the solution space, and eventual convergence on the best solution.

As mentioned in the previous section, Godoy and Dimitrakopoulos (2004) propose a simulated annealing algorithm (Metropolis et al., 1953) which performs mine production scheduling problems and is used in many other studies (Goodfellow and Dimitrakopoulos, 2013; Kumral, 2013; Montiel and Dimitrakopoulos, 2013). SA algorithm starts with an initial solution, each perturbation step generates a new solution based on the current solution and its neighborhoods. If the objective value is improved with the new solution, this improving perturbation is accepted. Also, a deteriorating perturbation might also be accepted based on the annealing temperature. A high annealing temperature gives a higher chance of a deteriorating perturbation with large changes being accepted in order to thoroughly explore the solution space. The temperature is gradually reduced until only minor changes are accepted in order to explicitly improve the solution. Cooling schedules and diversifications are used to control the annealing temperature to balance exploration and exploitation. The cooling schedule defines the annealing temperature being reduced after a number of iterations, and the diversification strategy resets the annealing temperature to its initial value, or another defined value. Lamghari and Dimitrakopoulos (2012) propose a diversified Tabu search approach to optimize the extraction sequence to maximize NPV and minimize deviations from production targets. Two variants of the algorithm are tested, long-term memory (LTM) and variable neighbourhood (VN). VN only modify the block extraction period under mining capacity. LTM will try extraction schedules that are infeasible in terms of capacities and then fix them by remembering the search history. Tests of 10 practical case studies are conducted. Both variants of the algorithm generate solutions with less than 5% of the optimality gap and within a fraction of the CPU time needed for commercial optimization software. Both variants of diversified Tabu search perform similarly but the variable neighbourhood is less efficient as the size of applications grows compared to LTM.

Lamghari et al. (2014) propose two variants of variable neighbourhood descent algorithm for a two-stage stochastic formulation to deal with large instances of the mine production schedule with supply uncertainty in a two-stage stochastic integer formulation. The algorithm first generates an initial solution and then uses a decomposition approach to separate the problem into sub-problems, one for each period. The first variant solves the sub-problems exactly using the branch-and-cut algorithm implemented in a commercial solver. The second variant solves the sub-problems approximately with a sequential greedy heuristic. The exact method performs slightly better for solution quality, while the greedy heuristic significantly reduces the solution time.

In the study of simultaneous stochastic optimization of the mining complex presented by Goodfellow and Dimitrakopoulos (2016), the authors utilized multi-neighborhood simulated annealing to solve the problem. The algorithm perturbs for a better solution using three neighborhoods. The first neighborhood changes the mining period of a block, the second neighborhood changes the destination of a cluster of blocks, and the third neighborhood changes the amount of material sent from one destination to another destination. As mentioned previously, simulated annealing has many successes in optimizing extraction sequences; therefore, it is selected as a base algorithm. However, the simultaneous stochastic optimization method by Goodfellow and Dimitrakopoulos (2016) aims to simultaneously define the extraction sequences, destination policy, and processing stream decisions. The ability to optimize both discrete and continuous variables and the ability to escape local optima with three distinct sets of decision variables is essential. To address the concern, the study also compares the multi-neighborhood simulated annealing algorithm with two alternatives, particle swarm optimization (PSO) (Kennedy and Eberhart, 1995) and differential evolution (DE) (Storn and Price, 1997), for their performance and efficiency in optimizing downstream decisions. The study tests the effectiveness of SA when combined with PSO and DE: (i) simulated annealing with downstream decisions optimized by particle swarm optimization (SA-D-PSO) and (ii) simulated annealing with downstream decisions optimized by differential evolution (SA-D-DE). SA-D-PSO and SA-D-DE are tested with a copper-gold mining complex and compared to the SA algorithm. Compared to the base SA, the production schedules obtained by using simulating annealing with PSO and DE increase the NPV by 1.91% and 2.57% but also require 2.4 and 2.9 times longer running time, respectively.

As mentioned previously, metaheuristic algorithms substantially reduce the solution time and improve the solution quality in simultaneous stochastic optimization described in (Goodfellow and Dimitrakopoulos, 2016). However, metaheuristics may involve a relatively large number of parameters and/or algorithm choices, hence it might require fine-tuning for different cases. Lamghari and Dimitrakopoulos (2018) develop a hyper-heuristic approach that provides a learning mechanism that can select or generate heuristics to solve computationally challenging problems. Hyper-heuristics can be self-managed and provide more generality than currently existing methodologies. Hyper-heuristics is an automated process to select low-level heuristics for perturbations. The method uses a combination of reinforcement learning and Tabu search to solve the optimization of a mining complex. A set of 27 low-level heuristics is used to improve the current solution, and three hyper-heuristics, which are score-based online learning mechanisms, are proposed to govern the selection of low-level heuristics. Hyper-heuristics 1 (HH1) selects the non-tabu heuristic having the highest score, which represents the performance of different heuristics. Hyper-heuristics 2 (HH2) can select any heuristic based on a more complicated score representing the performance, pairwise dependencies, and the time elapsed since the heuristic was selected. Hyper-heuristics 3 (HH3) separates heuristics for first-stage decisions and second-stage decisions. For first-stage decisions, HH3 picks a heuristic that is able to improve the solution more efficiently, and for the second stage, the best heuristic is selected with a probability approach. The method is applied to two stochastic mine planning problems. The result demonstrates the methods' abilities to tackle very large instances with multiple destinations including a stockpile and also different classes of problems. Although this method helps choose different heuristics for a specific problem, some hyper-heuristics perform better in one case compared to another. Therefore, it still requires tuning or the ability to switch between hyper-heuristics that best suit the problem at hand. Following the work by Goodfellow and Dimitrakopoulos (2016) and Lamghari and Dimitrakopoulos (2018), Yaakoubi and Dimitrakopoulos (2023) develop the learn-to-perturb (L2P) hyper-heuristic which combines the multi-neighborhood simulated annealing algorithm with reinforcement learning (RL). The goal of the algorithm is to generalize the selection of heuristics across different instances so that no switching between hyper-heuristics is needed. The algorithm starts with an initial solution and then moves to a neighboring solution in search of a better solution. It iterates until no significant improvement can be obtained. In this algorithm, the current solution is modified into a new solution using a heuristic, which is selected from a set of low-level heuristics. The acceptance of the new solution is handled by simulated annealing, and the heuristic selection is based on its past performance and the expected performance predicted by the RL agent. L2P utilizes a score that represents the importance of heuristic h_i and the probability of this heuristic being selected. It consists of two parts. The first part represents the recent performance of different heuristics (Lamghari and Dimitrakopoulos, 2018). A heuristic is given a higher score if it improves solutions more efficiently per unit of execution time and it is given a lower score if it deteriorates the solution and consumes more execution time. The second part is a score provided by a reinforcement learning agent (Yaakoubi and Dimitrakopoulos, 2023). It predicts the performance of future iterations by learning from past performance of different heuristics stored in a replay buffer. The score is able to consider all perturbations and consecutive perturbations. A parameter, λ_{RL}, is used to control the contribution of the reinforcement learning agent toward the final score. With the help of the RL agent, the heuristic selection process is able to learn from past experiences to be self-adaptive and to guide future searches through the solution space more efficiently. The training of the agent is online as to the solution being improved. Three RL algorithms are tested for their ability to predict heuristic performance and improve the solution: advantage actor-critic (A2C), soft actor-critic (SAC), and proximal policy optimization (PPO). Yaakoubi and Dimitrakopoulos (2023) show a comparison between the baseline hyper-heuristic without contribution from the RL agent (L2P-Baseline) and three variants of RL-based hyper-heuristics (L2P-A2C, L2P-SAC, and L2P- PPO). Three stages of numerical testing are conducted in the study. The first stage tests performance, robustness, and generalization capacity by evaluating the number of iterations and execution time required to obtain a solution with an objective function value within 1% of the linear relaxation result computed by CPLEX. The experiment is run multiple times across three different instances ($I_1, I_2,$ and I_3 which are small-medium-size) to ensure generality. All three RL-based hyper-heuristics outperform the baseline hyper-heuristic. L2P-A2C, on average, reduces the number of iterations and execution time by 45%-55%, outperforming the other two variants. However, the performance of L2P-A2C is not as consistent across runs as L2P–PPO which reduces the number of iterations and execution time by 38%-48%. The authors explain the effect as that PPO takes much smaller steps to update its policy, hence presenting a more stable but less adaptive strategy than A2C. The results demonstrate its efficacy across various instances, leading to a notable reduction in the number of iterations by 30–50% and a decrease in computational time by 30–45%. When assessing the performance of variants trained on different instances, typically, the slightly superior variant is the one evaluated on the same instance it was trained on. However, the difference is small enough to indicate that the heuristic selection and performance may exhibit similarity across different instances. It demonstrates the method's ability to generalize. The second stage of testing uses a fourth instance, I_4 which is a real use case, that is significantly larger and more complex than, $I_1, I_2,$ and I_3. The agents are trained and tested on I_4. The L2P–A2C still performs best among the three variants, reducing the number of iterations by 50–80% and the execution time by 40–75%. This indicates the RL agent is able to handle real-life cases with complexity. The third stage of testing uses a fifth instance, I_5, which is a real use case different from I_4 but similar in size. The L2P–A2C agents previously trained on I_4 will be tested on I_5. The L2P–A2C still outperforms the baseline. This indicates the generalizing ability of the RL agent remains effective for real-life size cases. In summary, the L2P hyper-heuristic algorithm is able to not only guide the search for a better solution for simultaneous stochastic optimizations but also generalize previous learning across different problems of different scales.

1.4 Goals and objectives

The goal of the research presented in this thesis is to explore the use of the simultaneous stochastic optimization framework in strategic mine planning of the mining complex by incorporating various sources of uncertainties observed in real-world operations. These different sources of uncertainties may affect the execution and outcome of the production scheduling. The following objectives are set to meet this goal:

- Review the technical literature related to strategic mine planning, conventional approaches for the optimization of mining complexes, and simultaneous stochastic optimization of mining complexes. Review simulation methods for modeling various types of uncertainty related to mining operations.
- Incorporate joint supply and equipment uncertainty directly into the optimization model and analyze their effects on the execution of production schedules and forecasts.
- Incorporate joint supply, recovery, and market uncertainty into the simultaneous stochastic optimization model and analyze their effects on production schedules and forecasts as well as the effect of different combinations of uncertainties
- Summarize the conclusions and contributions of the research presented and provide suggestions for future study.

1.5 Thesis outline

Chapter 1 - Presents the technical literature review on strategic mine planning highlighting the significance of an integrated optimization approach that accounts for the global impact of each component, the resilience and adaptability of production decisions, and the incorporation of uncertainties within the mining complex.

Chapter 2 - Presents an extension of the simultaneous stochastic optimization framework to incorporate equipment uncertainty for creating a realizable mine plan. The case study at a copper mining complex includes two large deposits, multiple crushers, and processing mills. The production schedule generated with the integration of equipment uncertainty is compared to the stochastic schedule generated without equipment uncertainty to highlight the practicality of the production schedule.

Chapter 3 - Presents a major case study an application of a simultaneous stochastic optimization framework, with recovery and market uncertainty in addition to supply uncertainty. The multi-mine, multi-processor copper mining complex is used as a basis for incorporating different sources of uncertainties. Production schedules are generated and the effect of different combinations of uncertainty is demonstrated.

Chapter 4 - Outlines the conclusions obtained from the above contributions, explains the value of the simultaneous stochastic optimization framework, and provides suggestions for future research.

Chapter 2. Simultaneous stochastic optimization of mining complexes with equipment uncertainty: application at an open pit copper mining complex

2.1 Introduction

A mining complex can be considered as an integrated mineral value chain that transforms in-situ raw material extracted from mineral deposits into valuable commodities, with components including multiple mineral deposits, stockpiles, processing streams, waste disposal, and transportation to clients or spot markets (Goodfellow and Dimitrakopoulos, 2016, 2017; Montiel and Dimitrakopoulos, 2015, 2017; Pimentel et al., 2010). Over the past decade, significant developments have been achieved in integrating various components within the mineral value chain into a single optimization model while considering uncertainty. This integrated model for mining complexes enables the simultaneous optimization of diverse decisions, such as the extraction sequence, destination policies, and processing stream utilization, and incorporates uncertainties presented in mining complexes. The simultaneous stochastic optimization leverages the value of the final products sold and captures synergies across the value chain, facilitating comprehensive and efficient decision-making for mining operations.

Conventionally, the components of a mining complex are optimized separately and sequentially (Hustrulid et al., 2013). In the past, several developments have advanced the conventional approach toward joint optimization by incorporating multiple components of a mining complex in the planning process. Hoerger et al. (1999a, 1999b) present a Mixed Integer Programming (MIP) formulation to capture the synergies of jointly optimizing the flow of material tonnage between mines, stockpiles, and processing plants by maximizing the net present value; however, in this case, mine production schedules are a pre-determined input. The Blasor mine planning software of BHP (Stone et al., 2018) simultaneously optimizes the proportion of material extracted from multiple pits for long-term mine planning using a MIP model. However, it aggregates spatially connected blocks to reduce the number of integer variables, such that the size of the MIP is reduced and can be solved in a reasonable time. Whittle and Whittle (2007) propose an approach that includes multiple mines, stockpiles, and processing destinations in a mining complex to capture synergies. However, it employs several optimization procedures throughout the mineral value chain rather than in a single formulation. Each mine is optimized separately first to produce a sequence of pushbacks within its optimized pit limit. Then, interactions between components, modeled by the processing and blending strategies, are optimized to improve the outcome of the mining complex using heuristics approaches (Whittle, 2018). The above-mentioned approaches have limitations, which include aggregate blocks to reduce the computational complexities and the use of estimated (average type) orebody models of the deposit. The latter ignore the uncertainty and spatial variability of pertinent properties of the mineral deposit, which is a major source of technical risk in mine planning, referred to as supply or geological uncertainty (Ravenscroft, 1992; Dowd, 1994, 1997), and affects the grades and material types of the deposit. It misrepresents the impacts on the economic value of the mine production schedule; therefore, the integration of supply uncertainty with stochastic simulations into optimization algorithms is required to enhance the robustness of the mining complex optimization and its ability to capture synergies (Dimitrakopoulos et al., 2002).

In response to the above limitations, the simultaneous stochastic optimization of mining complexes incorporates all of the components of a mining complex into a single mathematical formulation, while accounting for uncertainties (Goodfellow and Dimitrakopoulos, 2016; Montiel and Dimitrakopoulos, 2015). Montiel and Dimitrakopoulos (2015) present a simultaneous stochastic optimization model that can incorporate multiple mineral deposits, multiple processing steams, and multiple transportation alternatives. The block extraction sequence and processing destination, as well as the processing and transportation alternatives, are optimized simultaneously in the presence of supply uncertainty. Montiel and Dimitrakopoulos (2018) introduce an extension that integrates specific operational constraints for creating practical mine production schedules. The study includes a large-scale application and comparisons with conventional methods at the Twin Creeks gold mining complex in Nevada. Montiel and Dimitrakopoulos (2017) introduce a metaheuristic algorithm designed to solve the extensive optimization model of mining complexes. The case study conducted at a copper deposit showed that, compared to a mine plan generated by conventional mine planning software, the simultaneous stochastic optimization model was able to reduce deviations in capacity from 9% to 0.2%, while increasing the expected net present value by

30%. Goodfellow and Dimitrakopoulos (2016) present a generalized network-based modelling method that allows for the inclusion of every component in a mining complex for the simultaneous stochastic optimization and considers the value of the final products sold at the end of the mining complex, instead of block economic values. The method aims to define simultaneously the life-of-asset extraction sequence, destination policy, and processing stream decisions while managing the production targets. The model can be easily adapted for any mining complex. In the given case study, the simultaneous stochastic optimization of mining complexes manages the technical risk associated with geological uncertainty. A case study (Goodfellow and Dimitrakopoulos, 2017) is performed on a copper-gold mining complex, in comparison to the conventional schedule, and resulted in an increase of the NPV by 22.6%, while also more closely meeting production targets and managing the associated geological risk. Due to its generalized and adaptable formulation, subsequent research has tested the framework introduced by Goodfellow and Dimitrakopoulos (2016) through a large variety of applications incorporating various aspects of mining operations. Farmer (2017) expands the generalized model to incorporate capital expenditure (CapEx) and mining capacity decisions, incorporating these aspects into an application with intricate revenue streams, including streaming agreements. Del Castillo and Dimitrakopoulos (2019) expand the mathematical formulation to include a dynamic optimization for strategic planning explicitly including CapEx alternatives and different operating modes. Accounting for the flexibility of an operation, the multi-stage model is able to react and adapt to new information over the life of a mining complex by allowing the solutions to branch. Kumar and Dimitrakopoulos (2019) present an application incorporating geo-metallurgical decisions into the destination policy at a large copper-gold mining complex. Saliba and Dimitrakopoulos (2019) present another application that accounts for both supply uncertainty and market uncertainty through commodity price simulations at a gold mining complex. In summary, simultaneous stochastic optimization offers several advantages, including capturing synergies among different components, managing risk associated with material supply uncertainty, avoiding misrepresentation of mining selectivity by aggregating mine blocks into large volumes, and focusing on the value of the product sold rather than the value of the mining blocks. The documented benefits of simultaneous stochastic optimization include increased metal quantity, higher NPV, reliable production forecasts, integrated waste production and quality management, and the joint integration of supply and demand uncertainty (Dimitrakopoulos and Lamghari, 2022).

The developments discussed above assume constant equipment capacities, such as fixed truck and shovel capacity. However, the impact of uncertain equipment performance on long-term mine production scheduling in simultaneous stochastic optimization remains unexplored. Recent simultaneous stochastic optimization applications that address equipment uncertainty primarily concentrate on the short-term optimization for specific sections of the mine, defined by the long-term plan. The related stochastic optimization approaches for short-term planning and their applications aim, given the long-term plan, to minimize operational costs by optimizing shovel and truck activities, as well as uncertainties related to equipment that are often modelled by making prior assumptions about the distribution models (Both and Dimitrakopoulos, 2018; Matamoros and Dimitrakopoulos, 2016; Paduraru and Dimitrakopoulos, 2019; Quigley and Dimitrakopoulos, 2016). For example, Both and Dimitrakopoulos (2018) use distribution with known means and standard deviations for generating equiprobable stochastic simulations of shovel productivity for modelling equipment uncertainty. The research highlights the benefits of simultaneously optimizing the mining complex with equipment decisions, leading to cost savings from optimized equipment movements and enhanced equipment utilization. However, the short-term production plan is constructed on shorter timescales and designed to meet the targets of the long-term production plan, which does not consider equipment uncertainty.

In the following sections, an extension of the simultaneous stochastic optimization framework proposed by Goodfellow and Dimitrakopoulos (2016) to incorporate uncertain equipment performance in the long-term production planning of mining complexes is presented. Then, the quantification of equipment uncertainty is outlined. Subsequently, a case study of the copper mining complex is presented to incorporate both supply and equipment uncertainty, and to study the potential effect of incorporating equipment uncertainty into the optimization process. Last, conclusions and future work are presented.

2.2 Simultaneous Stochastic Optimization of Mining Complexes

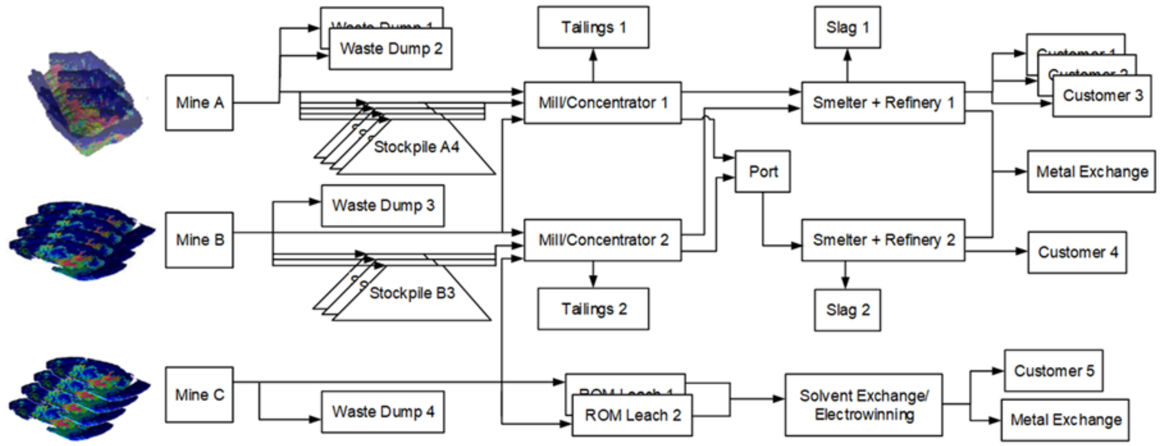

Figure 2.1: An example of a mining complex.

2.2.1 Notation

An example of a mining complex or mineral value chain is shown in Figure 2.1. Sets and indices used for the mathematical formulation of the simultaneous stochastic optimization of mining complexes are presented in Table 2.1. Inputs and parameters are listed in Table 2.2.

Table 2.1: List of sets and indices

Sets/Indices	Description
$i \in \mathrm{N} = \mathbb{M} \cup \mathcal{S} \cup \mathcal{P}$	Location i, within the set of locations N, including mines $\mathbb{M}$, stockpiles $\mathcal{S}$, and processing destinations $\mathcal{P}$
$b \in \mathbb{B}_m$	Block b within the set of blocks $\mathbb{B}_m$ at mine $m \in \mathbb{M}$
$p \in \mathbb{P}$	Primary attributes, sent from one location in the mining complex to another
$h \in \mathbb{H}$	Hereditary attributes, variables of interest at specific locations, not necessarily sent to next locations
$s \in \mathbb{S}$	A set of equally probable scenarios (simulations) describing the geological uncertainty
$s_e \in \mathbb{S}_e$	A set of equally probable scenarios (simulations) describing the equipment uncertainty at location e
$e \in E$	A set of locations that utilize mining equipment

$t \in \mathbb{T}$	Time period within the life-of-mine
$c \in \mathcal{C}$	A set of cluster $\mathcal{C}$ is generated by a pre-processing step using clustering based on multiple elements within each material types

Table 2.2: List of input and parameters

Input/Parameter	Description
$\beta_{p,b,s}$	Simulated block attributes
EC_{e,t,s_e}	Simulated equipment capacity
$\mathbb{O}_b$	Set of overlaying blocks defined by slope angle
$O(i)$	The set of subsequent locations for location i
$O(c)$	The set of available destinations for cluster c
$\theta_{b,c,s}$	Block cluster membership
$p_{h,i,t}$	Unit price or cost of each attribute, discounted based on time periods
$U_{h,i,t}$ and $L_{h,i,t}$	Upper- and lower-bounds for each attribute
$c^{\pm}_{h,i,t}$	Penalty cost for deviations of each attribute

Locations in a mining complex can be described as sources where material is extracted or destinations where the material is sent (Goodfellow and Dimitrakopoulos, 2016). The set of location, $\mathcal{N}$, is comprised of three disjoint subsets: clusters of blocks ($\mathcal{C}$), stockpiles ($\mathcal{S}$), and processing destinations($\mathcal{P}$). $\mathcal{C}$ is a set of clusters of blocks in a mine $m \in \mathbb{M}$ with similar attributes and is served as the sources of material. These clusters of blocks in a mine are generated with the k-means algorithm. $\mathcal{S}$ is a set of stockpiles, which can receive and hold material over time, but do not treat or transform these materials. Stockpiles in a mining complex can be modelled as a source or a destination. $\mathcal{P}$ is a set of processing destinations that treat and process material. Directed arcs, $\mathcal{A}$, in a mining complex define the abilities to send output material from one location $i \in \mathcal{N}$ to a subsequent destination $j \in \mathcal{N}(i)$. Supply uncertainty in a mining complex is described by a set of stochastic orebody scenarios $s \in \mathbb{S} = \{1, \dots, S\}$. The material at each location is characterized by primary attributes and hereditary attributes. Primary attributes ($p \in \mathbb{P}$) are fundamental attributes, such as metal tonnage and total tonnage of materials, which is additive and will flow from location $i \in \mathcal{N}$ to subsequent locations $j \in \mathcal{N}(i)$, if $j \in \mathcal{N}(i)$ exists. Hereditary attributes ($h \in \mathbb{H}$) are the information needed for modelling the mining complex and assist in incorporating non-linearities into the mathematical programming model, such as recovery rates at each processing destination. In a mining complex, different upper and lower production targets and constraints, $U_{h,i,t}$ and $L_{h,i,t}$ can be defined for attribute $h \in \mathbb{H}$ at locations $i \in \mathcal{N}$ in period $t \in \mathbb{T}$. The deviations from a production target associated with property h at location i in period t and scenario s are measured by $d^{\pm}_{h,i,t,s}$, while $c^{\pm}_{h,i,t}$, represent the unit surplus and shortage costs associated with their respective deviations.

The flow of materials in a mining complex are controlled by three sets of decisions: extraction decisions, destination decisions, and processing stream decisions. The extraction decisions $x_{b,t} \in \{0,1\}$ determine whether a mining block $b \in \mathbb{B}_m$ is extracted (1) or not (0) in period $t \in \mathbb{T}$. In mining complexes, materials are supplied by the mines ($m \in \mathbb{M}$), which are described by a group of mining blocks ($b \in \mathbb{B}_m$). Each mining block is described by simulated material types and attributes ($\beta_{p,b,s}, \forall p \in \mathbb{P}, \forall b \in \mathbb{B}_m, s \in \mathbb{S}$). A block may only be extracted if the group of overlaying blocks ($\mathbb{O}_m$) is extracted in the same or earlier period. The group of overlaying blocks is defined by a slope angle required for slope stability. The second set of decisions, destination decisions $z_{c,j,t} \in \{0,1\}$, determines whether a cluster of blocks $c \in \mathcal{C}$ is sent to destination $j \in O(c)$ in period $t \in \mathbb{T}$ after they are mined. A cluster $c \in \mathcal{C}$ is a group of blocks in a mine with similar properties, based on the k-means algorithm. A block in different orebody scenarios $s \in \mathbb{S}$ could belong to different clusters due to different simulated attribute grades and material types. As a result of the k-means algorithm, a set of pre-processed parameters $\theta_{b,c,s}$ will describe whether a block $b \in \mathbb{B}_m$ belongs to cluster $c \in \mathcal{C}$ under orebody scenario $s \in \mathbb{S}$. The available destinations $O(c)$ for cluster c are determined by the material type. The last set is the processing stream decisions $y_{i,j,t,s} \in [0,1]$ which determines the proportion of output material from location $i \in \mathcal{S} \cup \mathcal{P}$ being sent to downstream destination $j \in O(i)$ at period $t \in \mathbb{T}$ under scenario $s \in \mathbb{S}$. The extraction decisions and the destination decision are scenario-independent. This means those decisions have to be made robust to the geological uncertainty in the mining complex. The scenario-dependent processing stream decisions, on the other hand, are made adaptive to the uncertainty. The model assumes that, after the material is sent to an immediate destination from the mine or existing stockpiles, the uncertainties associated with the material are revealed; therefore, the processing stream decisions, which determine downstream operations, can be made and adapted to every scenario.

In a mining complex, for different upper and lower production targets and constraints, $U_{h,i,t}$ and $L_{h,i,t}$ can be defined for attribute $h \in \mathbb{H}$ at locations $i \in \mathcal{N}$ in period $t \in \mathbb{T}$, such as a processing capacity for a flotation mill. The deviations from a production target associated with property h at location i in period t and scenario s are measured by $d^{\pm}_{h,i,t,s}$, while $c^{\pm}_{h,i,t}$, represent the unit surplus and shortage costs associated with their respective deviations. For example, $d^{+}_{h,i,t,s}$ can describe the amount of material the mine feeding to the mill and is above the mill's capacity. With a corresponding penalty cost $c^{+}_{h,i,t}$, the optimization can be guided to control overfeeding the mill.

To extend the general framework defined in Goodfellow and Dimitrakopoulos (2016) and incorporate equipment uncertainties, a set of stochastic equipment capacity simulations $s_e \in \mathbb{S}_e = \{1, \dots, S_e\}$ are included for quantifying the uncertain equipment performance, where $e \in E = \{1, \dots, E\}$ is a set of locations that utilize mining equipment; thus, the capacity of such location mining or processing material is limited by the equipment fleet used. For example, the mining capacity at the mine can be represented as the capacities of truck and shovel, while the crusher capacity at the crushing location affects how much material can be processed after they are mined. EC_{e,t,s_e} represents the capacity of the equipment fleet under scenario s_e at time $t \in \mathbb{T}$. An additional deviation measurement, u_{e,t,s,s_e}, is used to describe the deviation from equipment capacity at location e in period t under equipment scenario s_e when the incoming material is under supply uncertainty s, and the corresponding penalty cost $c^{+}_{e,t}$ will be applied. Equipment capacities are simulated at different locations in a mining complex representing the uncertainties of equipment affecting the mining and processing capability at different locations. The inclusion of equipment capacity penalty and equipment capacity constraints will be detailed in the next section.

2.2.2 Optimization with Equipment Uncertainty

To simultaneously determine the extraction sequence, destination policies, and processing stream decisions for mining complexes under uncertainties, Goodfellow and Dimitrakopoulos (2016) propose a generalized two-stage stochastic optimization model. The objective function is shown in Equation (1) with extensions that incorporate equipment uncertainty.

$$
\begin{aligned}
Max\, & \underbrace{\frac{1}{\|\mathbb{S}\|}\sum_{i\in \mathbb{M}\cup\mathcal{S}\cup\mathcal{P}}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{h\in\mathbb{H}} p_{h,i,t}v_{h,i,t,s}}_{Part\ I} \\
& \underbrace{-\frac{1}{\|\mathbb{S}\|}\sum_{i\in \mathbb{M}\cup\mathcal{S}\cup\mathcal{P}}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{h\in\mathbb{H}}\left(c^{+}_{h,i,t}\, d^{+}_{h,i,t,s} + c^{-}_{h,i,t}\, d^{-}_{h,i,t,s}\right)}_{Part\ II} \\
& \underbrace{-\sum_{t\in\mathbb{T}}\sum_{m\in\mathbb{M}}\sum_{b\in\mathbb{B}_m}\left(c^{smooth}_{m,t}\, d^{smooth}_{b,m,t}\right)}_{Part\ III} \\
& \underbrace{-\sum_{t\in\mathbb{T}}\sum_{m\in\mathbb{M}}\sum_{b\in\mathbb{B}_m}\left(c^{sink}_{m,t}\, d^{sink}_{b,m,t}\right)}_{Part\ IV} \\
& \underbrace{-\frac{1}{\|\mathbb{S}\|}\frac{1}{\|\mathbb{S}_e\|}\sum_{e\in\mathbb{E}}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{s_e\in\mathbb{S}_e}\left(c^{+}_{e,t}\, u_{e,t,s,s_e}\right)}_{Part\ V}
\end{aligned}
\qquad (1)
$$

Part I of the objective function represents the discounted cash flow by accounting for the revenue from metal sales, the mining cost, the processing cost, and other associated costs. Part II represents the cost of deviating from the production target by applying penalties costs, $c^{+}_{h,i,t}$ and $c^{-}_{h,i,t}$, to deviations, $d^{+}_{h,i,t,s}$ and $d^{-}_{h,i,t,s}$, respectively. These penalties are discounted by geological discount rates (Dimitrakopoulos and Ramazan, 2004), $c^{+}_{h,i,t} = \frac{c^{+}_{h,i,1}}{(1+rd)^t}$. With the risk discount rate, the objective function aims to minimize the risk in early periods and defer the risk to later periods when more information is available. Part III and Part IV are smoothing penalties and sink rate penalties to create a minable shape for the schedule. Part III ensures that the extraction sequence is practical in terms of adequate space for equipment access and movement. Extending the formulation of Goodfellow and Dimitrakopoulos (2016), Part V accounts for the additional penalties and related costs specific to equipment capacity, as applied to constraint the optimization to mine and process materials given the simulated equipment capacities.

$$v_{e,t,s} - u_{e,t,s,s_e} \leq EC_{e,t,s_e} \quad \forall\, e \in \mathbb{E}, t \in \mathbb{T}, s \in \mathbb{S}, s_e \in \mathbb{S}_{\mathbb{E}} \tag{2}$$

To enforce the equipment capacity penalty, the additional equipment capacity constraints in Equation (2) are applied in the extend model. For each location e with simulated equipment capacity EC_{e,t,s_e}, a penalty will incur when the production of material under orebody scenario s exceeding each equipment capacity scenario s_e, so the schedule will produce material respecting the equipment capacity.

Mineral complexes have many operational constraints and production targets, such as processing stream capacities and the acceptable ratio of different attributes grade of material in a processor. Equations (3) and (4) calculate the deviations from the upper and lower bounds of each production target at each location under every orebody scenario. By being penalized in the objective, the deviations, $d^{+}_{h,i,t,s}$ and $d^{-}_{h,i,t,s}$, are controlled within upper ($U_{h,i,t}$) and lower ($L_{h,i,t}$) targets respectively.

$$v_{h,i,t,s} - d^{+}_{h,i,t,s} \leq U_{h,i,t} \quad \forall\, h \in \mathbb{H}, i \in \mathcal{C} \cup \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{3}$$

$$v_{h,i,t,s} + d^{-}_{h,i,t,s} \geq L_{h,i,t} \quad \forall\, h \in \mathbb{H}, i \in \mathcal{C} \cup \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{4}$$

To create a minable shape that allows equipment access, the smoothing constraints are included, as shown in Equation (5). $Neigh(b)$ represents the neighbour of blocks surrounding block b within a predefined mining width, as shown in Figure 2.2. The smoothing penalty $d^{smooth}_{b,m,t}$ counts the number of blocks within $Neigh(b)$ that are not mined at the same period as b, then penalizes the objective function. Similarly, the sink rate constraint, Shown in Equation (6), controls how deep the benches can be mined in one period. $\mathscr{b}(b)$ represents the block overlay block b by the height of the predefined sink rate. For example, if the blocks are 10m in height and the sink rate is 30m, then $\mathscr{b}(b)$ is the block 30m above block b, and if they are mined at the same period t, penalty $d^{sink}_{b,m,t}$ will incur.

$$|Neigh(b)| * x_{b,t} - \sum_{n \in Neigh(b)} x_{n,t} - d_{b,m,t}^{smooth} \leq 0 \quad \forall\, b \in \mathbb{B}_{m},, t \in \mathbb{T}, m \in \mathbb{M} \tag{5}$$

$$x_{b,t} + x_{\mathcal{b}(b),t} - d_{b,m,t}^{sink} \leq 1 \quad \forall\, b \in \mathbb{B}_{m},, t \in \mathbb{T}, m \in \mathbb{M} \tag{6}$$

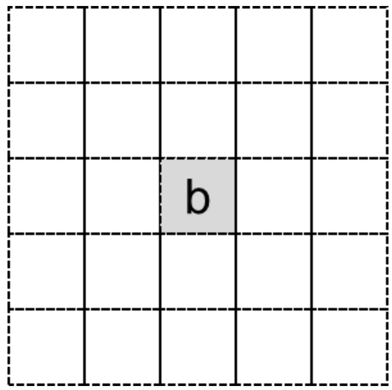

Figure 2.2: Surrounding blocks in smoothing constraints.

Remaining constraints, such as capacity constraints, blending constraints, material type constraints, reserve, slope constraints, and stockpile constraints are detailed in Goodfellow and Dimitrakopoulos (2016).

2.2.3 Solution Approach

The simultaneous stochastic optimization of a mining complex is challenging due to the large number of binary decision variables. Therefore, solution approaches that utilize commercial solvers are often infeasible. Metaheuristics and hyper-heuristics provide solution approaches to efficiently obtain near-optimal solutions for large stochastic optimization models of mining complexes (Goodfellow and Dimitrakopoulos, 2016, 2017; Lamghari and Dimitrakopoulos, 2018). In this work, the solution approach is a combination of multi-neighbourhood simulated annealing with adaptive neighbourhood search, where the selection of heuristic is aided with reinforcement learning (Yaakoubi and Dimitrakopoulos, 2021).

Starting with an initial solution, the simulated annealing algorithm (Kirkpatrick et al., 1983; Metropolis et al., 1953) iteratively explores neighbourhoods (Ropke and Pisinger, 2006; Ribeiro and Laporte, 2012) around the current solution. At each iteration, a new solution ϕ' is generated by modifying either the extraction sequence decisions ($x \epsilon \phi$), destination decisions ($z \epsilon \phi$), or processing stream decisions ($y \epsilon \phi$) of the current solution ϕ using different heuristics. Then, instead of only accepting solutions that improve the objective function value, $g(\phi)$, deteriorating solutions might also be accepted based on a probabilistic acceptance criteria, as shown in Equation (7). This probability is defined by an annealing temperature, $Temp$. Instead of using a fixed temperature, the annealing temperature will gradually cool down to zero based on cooling factors and cooling schedule (Goodfellow and Dimitrakopoulos, 2016). This mechanism balances the exploration, which prioritizes searching through the solution space, and exploitation, focusing on improving the objective function.

$$P(g(\phi), g(\phi'), Temp) = \begin{cases} 1, & if\ g(\phi) \leq g(\phi') \\ \exp\left(\frac{-|g(\phi) - g(\phi')|}{Temp}\right), & otherwise \end{cases} \quad (7)$$

The improvement of the objective function value, $g(\phi)$, depends on the selection of heuristics h_i ($i = 1, \ldots, n,$ where n in the number of low-level heuristics) used to modify the current solution. This selection of heuristics is governed by the learn-to-perturb (L2P) hyper-heuristic, which combines the multi-neighborhood simulated annealing algorithm with reinforcement learning (RL) (Yaakoubi and Dimitrakopoulos, 2021). L2P utilizes a score function, shown in Equation (8), where $S_F(h_i)$ represents the importance of heuristic h_i and the probability of this heuristic being selected (Yaakoubi and Dimitrakopoulos, 2021). The probability of each non-tabu heuristic h_i p_i is computed by normalizing the score function $p_i = \frac{S_F(h_i)}{\sum_{k:H_k} S_F(h_k)}$. This score function is the weighted sum of two scores, $S_1(h_i)$ and $S_2(h_i)$. $S_1(h_i)$ represents the recent performance of different heuristics (Lamghari and Dimitrakopoulos, 2018). A heuristic h_i is given higher $S_1(h_i)$ score if it improves solutions more efficiently per unit of execution time and it is given lower $S_1(h_i)$ score if it deteriorates the solution and consumes more execution time. However, the formulation of $S_1(h_i)$ requires expert knowledge, meaning it might require a different formulation if applied to different instances or during different stage of the optimization process. To generalise the selection of heuristics across different instances, $S_2(h_i)$ is a score provided by a reinforcement learning agent (Yaakoubi and Dimitrakopoulos, 2021). It predicts the performance of future iterations by learning from the past performance of different heuristics stored in a replay buffer.

Compared to $S_1(h_i)$, which only considers the recent performance, $S_2(h_i)$ is able to consider all perturbations and consecutive perturbations. λ_{RL} is used to control the contribution of the reinforcement learning agent toward final score $S_F(h_i)$. When $\lambda_{RL} = 0$, the hyper-heuristic is not aided by the RL agent, referred as the Baseline. The L2P approach refers to the hyper-heuristic aided by the RL agent, when $\lambda_{RL} > 0$. With the help of the RL agent, the heuristic selection process can learn from past experiences to be self-adaptive and can guide future searches through the solution space more efficiently. Figure 2.3 shows a comparison between the baseline hyper-heuristic and L2P approach (with contribution from the RL agent). For the same amount of execution time, the RL approach improves the solution efficiency.

$$S_F(h_i) = (1 - \lambda_{RL})S_1(h_i) + \lambda_{RL}S_2(h_i) \quad (8)$$

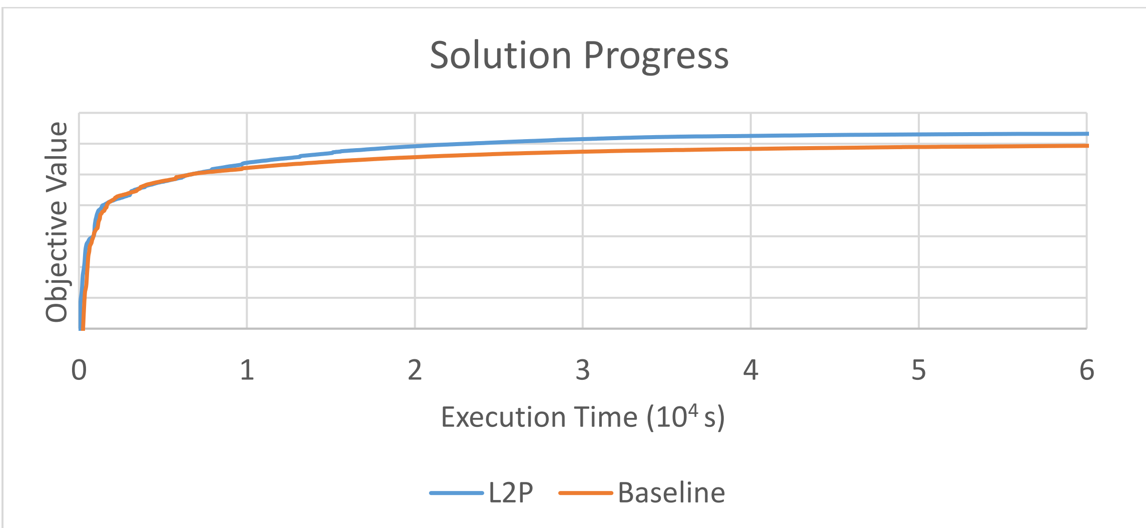

Figure 2.3: Evolution of objective function value, comparison of baseline heuristic selection (red line) and L2P approach (blue line)

2.3 Case Study

2.3.1 Overview of the Mining Complex

The method of simultaneous stochastic optimization described in the previous section is applied to an open-pit copper mining complex. As shown in Figure 2.4, the mining complex consists of two open-pit mines, mine 1 and mine 2, with respectively 414,108 and 157,749 blocks, which are

$25x25x15m^3$ in size. The main attributes of concern are copper total and copper soluble. Each block in the block model belongs to one of the four main material types, sulphide high grade, sulphide low grade, oxide, and waste. Materials produced by the mines can be processed by different processing streams. Two products are produced by the mining complex, copper concentrate, and copper cathodes. As shown in Figure 2.4, Mill 1, Mill 2, and Mill 3 receive high-grade sulphide material from Mine 1 and Mine 2 after materials are crushed by their corresponding crushers and produce copper concentrate as a product. The Oxide Leach pad takes Oxide materials from both mines after the Oxide material is crushed by the Oxide crusher and produces copper cathodes. The Sulphide Leach Pad takes low-grade material from both mines and produces copper cathode as a product. The Sulphide Leach Pad requires the ratio of copper total and copper soluble to be within its operational limit.

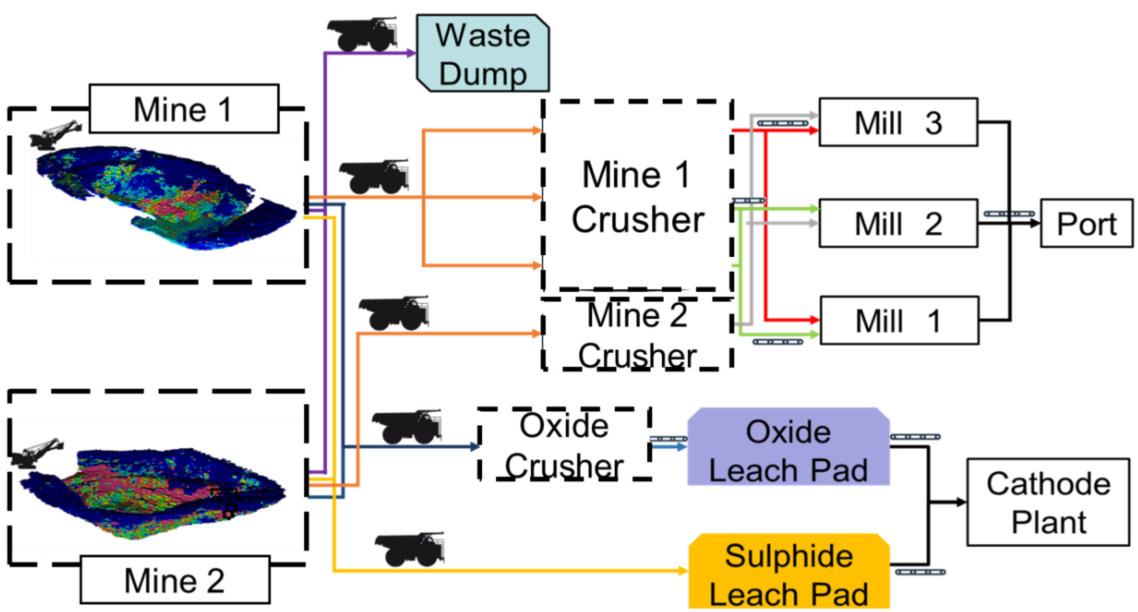

Figure 2.4: Copper mining complex, with 5 locations for modelling equipment uncertainty (highlighted in dash line)

The long-term strategic plan currently used in the mining complex is based on the conventional sequential optimization approach that uses estimated orebodies as input. First, the extraction sequences of each mine are optimized separately based on bench design. Then the destination policy is determined based on the cut-off policy and materials produced by the extraction sequences of both mines. Lastly, another optimization process is used to maximize the utilization of each processing stream. The use of estimated orebodies ignores the geological uncertainties presented in the deposits, and the use of fixed equipment capacity ignores the equipment performance uncertainty.

The simultaneous stochastic optimization framework previously described addresses the disadvantages of the sequential conventional optimization approach. First of all, the stochastic framework considers every component in the mining complex simultaneously to capture the synergies among components. By optimizing the extraction sequence, destination policy, and processing stream decisions of the two mines at the same time, the downstream operation will not be misled by predetermined extraction sequences. Also, the stochastic framework incorporates both supply uncertainty for the two mines and equipment uncertainty for multiple equipment locations. The blending constraint is considered throughout the optimization process.

2.3.2 Modelling Equipment Uncertainty

Equipment capacities can be simulated at different components in a mining complex to represent the uncertainties associated with the different equipment. For modelling realistic equipment uncertainty, Monte Carlo simulation methods are used. Equipment simulation starts with the historical daily production data of each type of equipment. Then, based on the historical production data, the empirical distribution of the equipment performance can be built. Then, by sampling this distribution 365 times and grouping these daily productivities, the annual productivity of a single piece of equipment can be simulated.

Figure 2.5 shows an example of producing multiple simulations for one type of equipment. The blue line represents the empirical distribution constructed from historical production data, and the set of green lines represent multiple sampling results conducted on the empirical distribution. The figure is presented as probability distribution graph where the horizontal axis presents the tonnage per day (tpd) productivity achieved by this type of equipment and the vertical one the frequency of occurrence. Figure 2.5 demonstrates that the equipment simulations are able to reproduce the historical data distribution.

After simulating the annual productivity of each type of equipment, then combined with an equipment plan, which shows how many trucks and shovels of equipment are being used in the mine, the total production capacity of an equipment fleet, such as truck fleet and shovel fleet, can be computed.

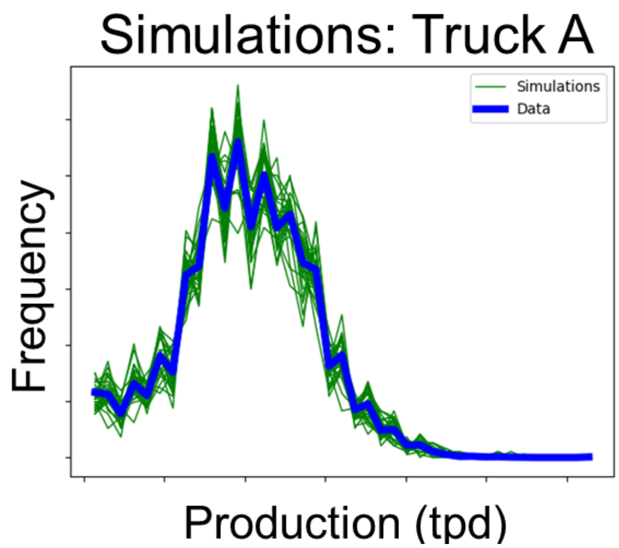

Figure 2.5: Equipment simulation of type A trucks

The production of the mining complex is limited by not only the equipment capacity in the mines for trucks and shovels, but also by the crusher capacity. Therefore, the modelling of equipment uncertainty is conducted at five locations in the mining complex. As shown in Figure 2.4, the uncertainty of the mining capacities of two mines is modelled using truck and shovel uncertainty. Three crushers are used to crush materials from Mine 1 before sending them to the mills, one crusher for crushing Mine 2 materials for the mills, and one crusher for crushing oxide material from both mines for oxide leach.

For the mining capacities of the two mines, equipment simulations for each type of truck and shovel will be produced, and then, based on the equipment plan for each mine, the total simulated mining rate can be computed. Figure 2.6 shows the result of the equipment simulation of six types of trucks. As shown in Figure 2.6, for each type of truck, the equipment simulations (green lines) reproduce the historical data distribution (blue lines). Similarly, Figure 2.7 shows that, for each type of shovel, the equipment simulations (green lines) reproduce the historical data distribution (blue lines).

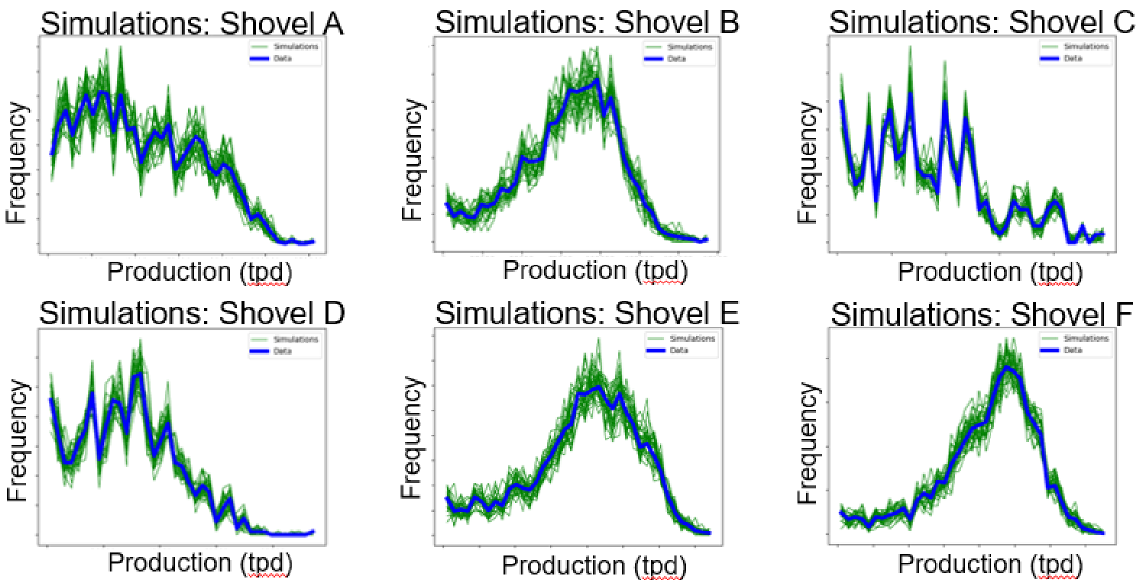

Figure 2.6: Equipment simulations for six types of trucks

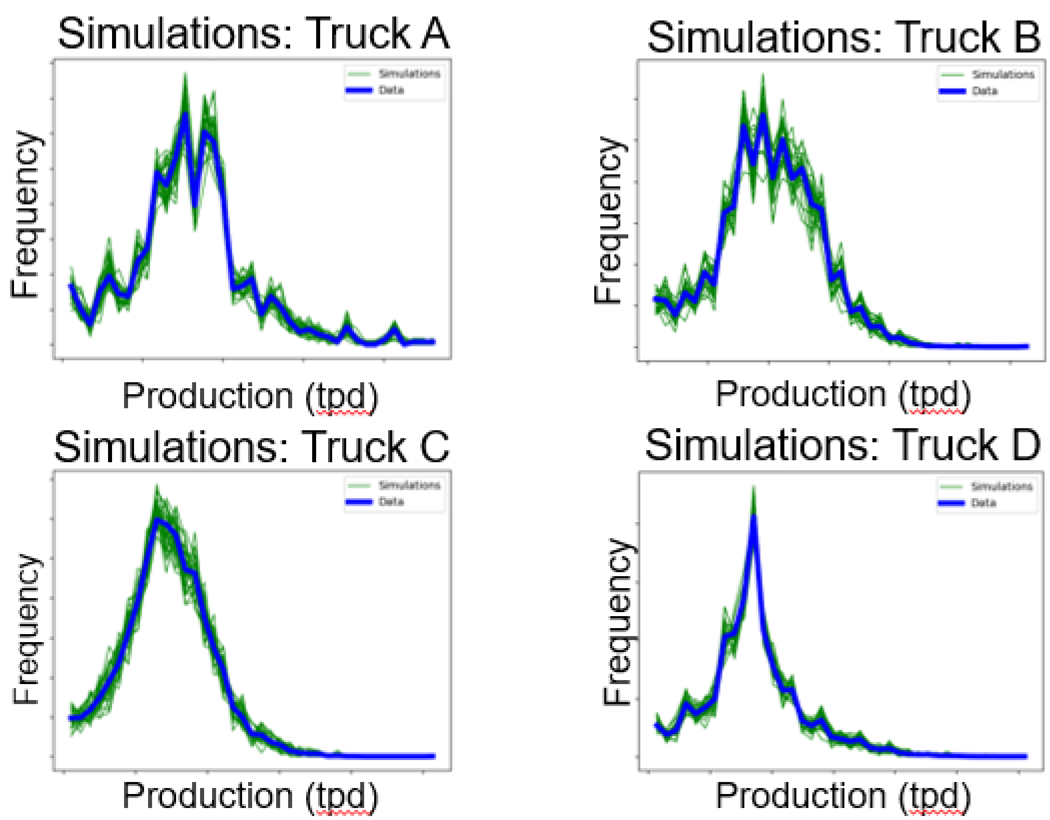

Figure 2.7: Equipment simulation for four types of shovels

For simulating the crusher capacity, Figure 2.8 shows a simulation for five crushers and, similarly to trucks and shovels, simulations reproduce the historical data distribution.

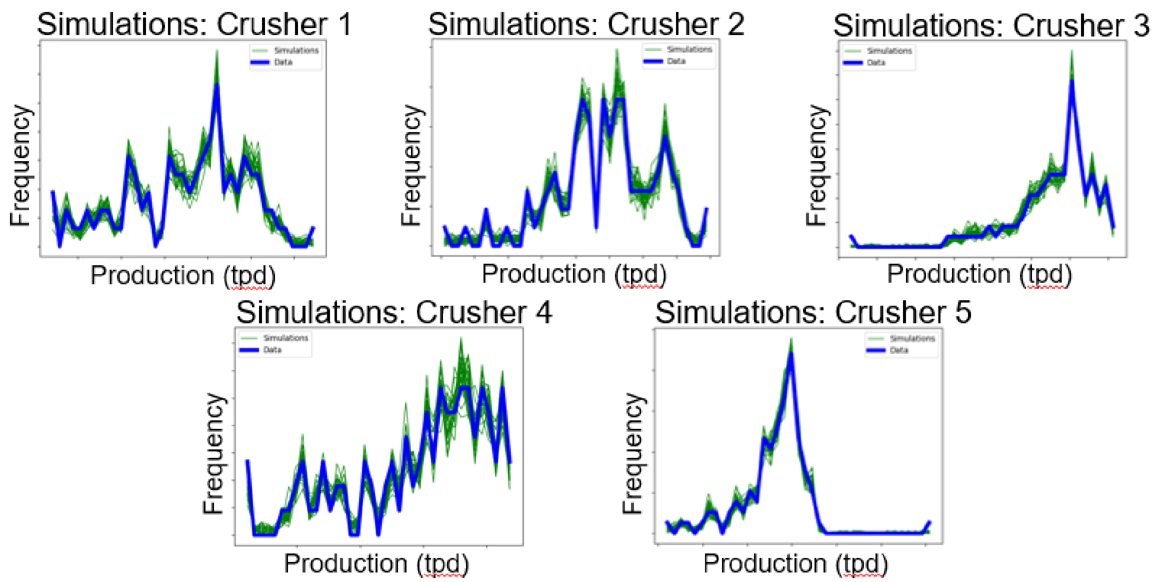

Figure 2.8: Equipment simulation for five crushers

To validate the equipment simulations, statistical comparisons between the simulated and the data productivity are shown in Table 3 and Table 4. Table 3 shows the mean of the historical productivity data of each type of equipment, comparing to the 5 simulations, while Table 4 shows the standard deviation.

Table 2.3: Mean of the equipment productivity (tpd), data and 5 simulations

EquipmentType	Data	Simulation1	Simulation2	Simulation3	Simulation4	Simulation5
Truck A	9086	9214	9106	9031	9020	8897
Truck B	5593	5471	5478	5311	5626	5767
Truck C	7723	7754	7547	7698	7581	7794
Truck D	5325	5428	5170	5415	5469	5354
Shovel A	22021	22441	22496	22665	20970	20288
Shovel B	70593	68817	70416	70140	70871	70453
Shovel C	6169	6083	6225	6197	6566	6147

Shovel D	14430	14315	15004	14479	14473	14587
Shovel E	71428	71151	70817	71913	71934	71063
Shovel F	81331	79259	80052	81181	83323	82218
Crusher1	32818	33284	33554	32874	33229	33336
Crusher2	95717	97071	94630	96218	94994	97362
Crusher3	137753	138329	138495	137130	138934	136767
Crusher4	56625	55805	57662	56221	55801	56426
Crusher5	121269	119753	121523	119281	122430	121362

Table 2.4: Standard deviation of the equipment productivity (tpd), data and 5 simulations

EquipmentType	Data	Simulation1	Simulation2	Simulation3	Simulation4	Simulation5
Truck A	4949	5104	4888	4980	4862	4978
Truck B	4045	4154	4024	4079	4030	3997
Truck C	4435	4410	4409	4527	4275	4360
Truck D	4740	4682	4786	4823	4723	4712
Shovel A	28430	28141	28153	29419	27879	27939
Shovel B	38678	38998	39812	39226	39564	38150
Shovel C	10589	10441	10567	10520	11053	10746
Shovel D	18701	18935	19137	18219	18492	18947
Shovel E	36358	36552	36291	35240	37451	37232
Shovel F	31743	33036	32426	30991	31308	30871
Crusher1	15616	15533	15358	15950	15532	15256
Crusher2	31487	30598	31567	32338	31823	31562
Crusher3	31393	30232	30300	34282	29757	33009
Crusher4	24046	24625	24077	24249	24325	23477
Crusher5	47405	48613	47090	48737	46734	46479

Table 2.5 shows the equipment plan for each mine, the equipment plan shows how many trucks and shovels of each type are being used in the mine. Therefore, using this equipment plan and the

previous simulated productivity of each type of equipment, the total simulated capacity can be computed for the shovel fleet and truck fleet for Mine 1 and Mine 2.

Table 2.5: Number of trucks and shovels being used in two mines

Trucks	Mine 1	Mine 2
A	26	17
B	2	1
C	73	23
D	5	0
Shovels	**Mine 1**	**Mine 2**
A	0	1
B	5	4
C	0	0
D	1	1
E	4	1
F	3	0

2.3.3 Parameters

The case study of simultaneous stochastic optimization for this copper mining complex requires two sets of parameters. The set of parameters includes (i) the economic and (ii) the operational parameters associated with the mining operation. These are summarized in Table 2.6 for the economic parameters and Table 2.7 for the operational parameters. Due to confidentiality reasons, the parameters listed in both tables are scaled.

Table 2.6 summarizes the costs associated with the mining operation and the price of metal produced. Table 2.7 includes the mining width, sink rate, and slope angle limitation, as well as fixed recovery rates, for different processing streams. The penalty costs associated with the objective function are listed in

Table 2.8. They are determined based on a trial-and-error process to achieve an acceptable level of technical risk for different production targets and the economic outcome. The second set is related to the solution algorithm described in Section 3.2.5. The parameters used for the solution approach are summarized in

Table 2.9. The initial temperature specifies the probability of the simulated annealing algorithm accepting deteriorating solutions at the first diversification. The cooling factor, cooling schedule, and the number of perturbations before diversification decide what proportions of the optimization are spent searching in the solution space and improving the solution. The perturbations before diversification and the number of diversifications are specific to the length of optimization.

Table 2.6: Economic parameters

Parameters	Value
Discount rate (NPV)	8%
Geological discount rate	10%
Copper Price ($US)	$US5511
Mining cost (excluding hauling cost, $US)	0.6
Hauling cost (based on location, $US)	0.4 to 1.3
Mill process cost including crushing ($US)	6.4
Oxide leach cost including crushing ($US)	6.6
Sulphide leach cost ($US)	1.1
Stockpile rehandling cost ($US)	0.2
Copper Concentrate Selling Cost ($US)	571
Copper Cathode Selling Cost ($US)	551

Table 2.7: Operational parameters

Parameters	Value
Mining width	200m

Sink rate	100m	
Slope angle	37°	45°
Mill 1 recovery	83%	
Mill 2 recovery	80%	
Mill 3 recovery	82.6%	
Oxide reach recovery	65%	
Sulphide leach recovery	27%	
Number of blocks	414,108	157,749
Number of periods	8	

Table 2.8: Penalty costs

Parameters	Value
Penalty cost – Simulated capacity (Mine 1, Mine 2, Mine 1 Crusher, Mine 2 Crusher, Oxide Crusher)	20, 20, 40, 100, 20 $/ton
Penalty cost – Capacity (Mill 1, Mill 2, Mill 3)	100, 100, 100 $/ton
Penalty Cost – Smoothing Constraint (Mine 1, Mine 2)	50000, 50000 $/ton
Penalty Cost – Sink Rate Constraint (Mine 1, Mine 2)	20000, 20000 $/ton
Penalty Cost – Leach Pad Capacity (Oxide, Sulphide)	20, 20 $/ton
Penalty Cost – CuS/CuT Ratio	4000 $/ton

Table 2.9: Solution approach parameters

Parameters	Value
Initial annealing temperature	0.2
Cooling factor	0.98
Cooling schedule	500
Perturbations before diversification	500,000
Number of diversification	15

2.3.4 Result and Comparisons

The result of the stochastic optimization incorporating both supply and equipment uncertainty will be presented in this section and is referred to as the “joint uncertainty” case. To provide a comparison, the result of only incorporating the supply uncertainty, with constant equipment capacity, will be included and referred to as the "supply uncertainty" case. Therefore, the comparison will demonstrate the effect of including both equipment uncertainty and supply into the optimization framework.

Figure 2.9 shows the risk profile of the amount of material produced by mine 1 and mine 2. Black lines show the simulated equipment capacities, which are the result of Section 2.3.2. As shown in Figure 2.9 (a), the joint stochastic case (blue lines) utilizes more of the simulated capacity of the equipment in mine 1 compared to the supply case (green lines). Mine 2 shows a similar result in Figure 2.9 (b) where the joint stochastic case utilizes more of the capacity of the truck fleet and shovel fleet in the mine.

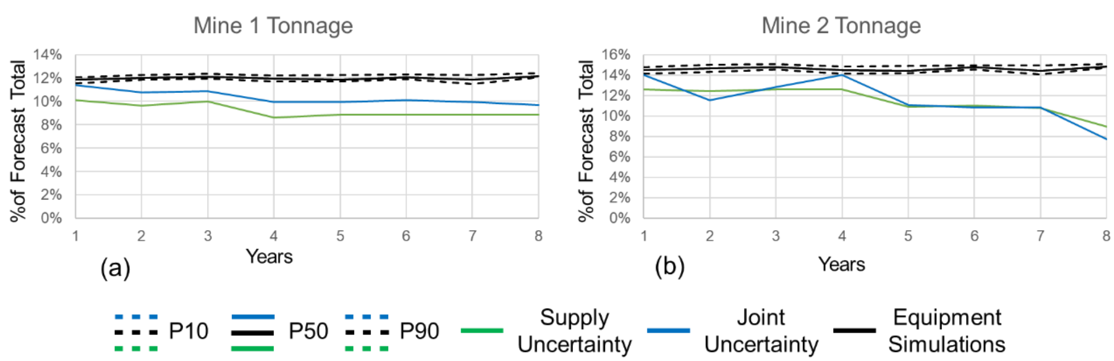

Figure 2.9: Mine 1 tonnages and mine 2 tonnage for supply uncertainty and joint uncertainty case

After the materials are extracted from the mine, they are sent to the crushers for size reduction. The result of crusher production is shown in Figure 2.10. Figure 2.10 (a) shows that the supply uncertainty case is trying to produce slightly more material than what mine 1 crusher can handle. The risk profile for mine 1 shows the crusher production for supply uncertainty would exceed the equipment capacity. In comparison, the joint uncertainty case respects the simulated capacity better than the supply uncertainty case. Although the stochastic optimization tries to push the mine 1 crusher production as much as possible, it still respects the simulated capacity. The supply uncertainty case is likely to send more material to the crusher exceeding the crusher capacity. In another words, the joint case is expected to produce a more realisable schedule as it complies with the simulated production capacity. This can also explain the previous result in which the equipment capacity in the mines was not fully utilized. The mine 1 crusher can only handle a limited amount of high-grade ore from mine 1 based on the simulations. The optimizer is able to identify that the mining complex is restricted by the crusher capacities. For the Mine 2 Crusher shown in Figure 2.10 (b), the production of both supply and joint uncertainty cases respect the simulated capacity. But the joint uncertainty case produces more material compared to the supply case, processing 11% more material. The joint uncertainty case is able to utilize more capacity of the mine 2 crushers. For the oxide crusher shown in Figure 2.10 (c), a similar result is also observed where the production risks exceeding the equipment capacity at years 2, 3, 6, and 7 for the supply uncertainty case. But the joint uncertainty case will be able to respect these limitations.

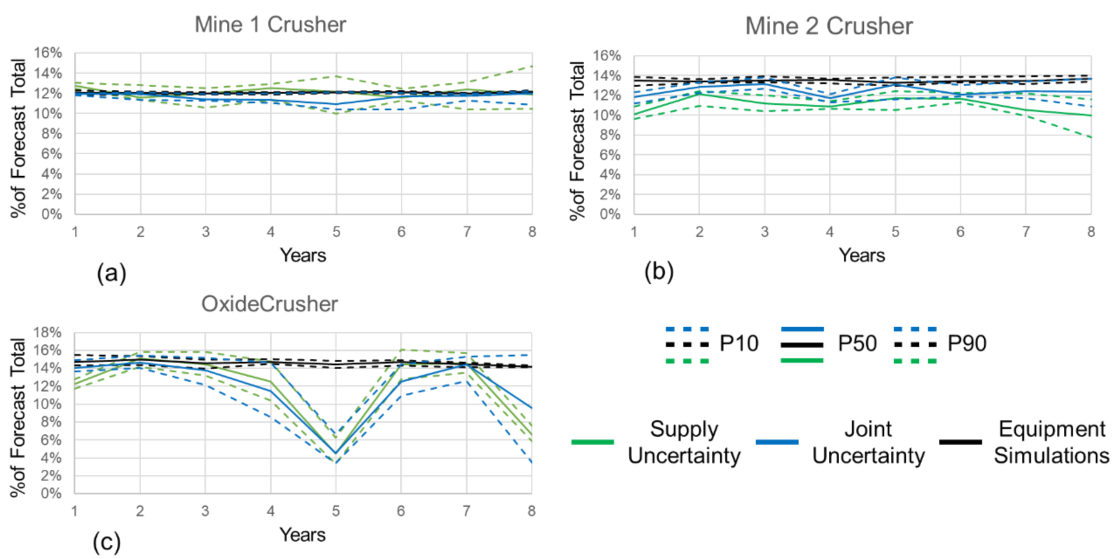

Figure 2.10: Material processed by Mine 1 Crusher (a), Mine 2 Crusher (b) and Oxide Crusher (c)

The result of copper metal recovered from mills 1, 2, and 3 are shown in Figure 2.11. Figure 2.11 (a) and (c) show that, for the joint uncertainty case, Mill 1 and Mill 3 will recover approximately

11% and 16% more more copper in the first two years and a similar amount in later years. Mill 2 will recover 8% less than the supply uncertainty case in the first two year. As for the copper recovered from the leach pad shown in Figure 2.12, the joint uncertainty case gives different results compared to the supply uncertainty schedule. For both Oxide Leach and Sulphide Leach, the joint uncertainty case shows a similar amount of copper is recovered at the earlier periods and slightly more copper at later periods, compared to the supply uncertainty case.

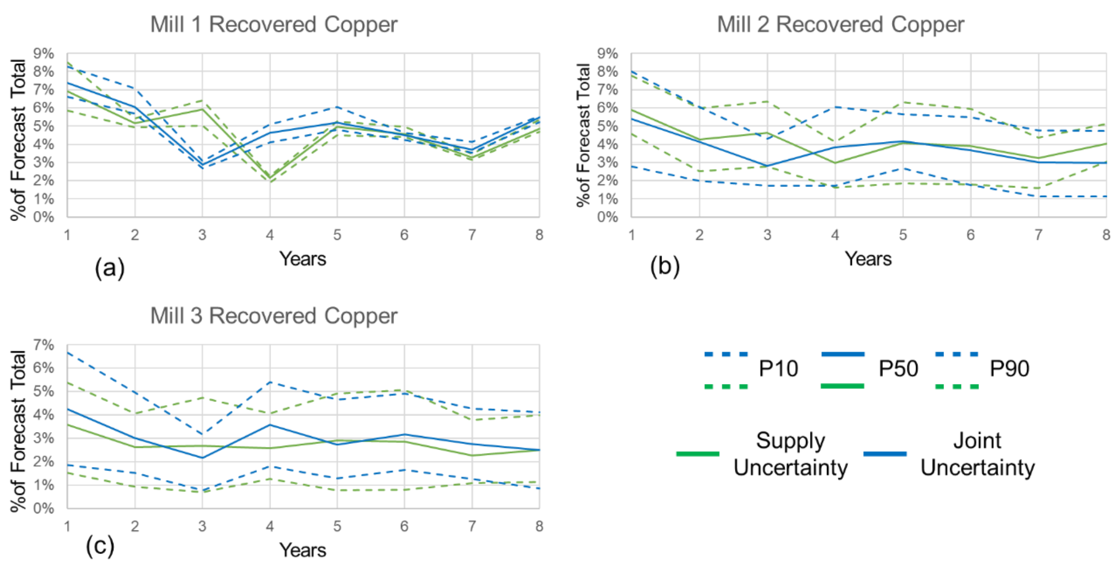

Figure 2.11: Copper recovered by Mill 1 (a) Mill 2 (b) and Mill 3 (c)

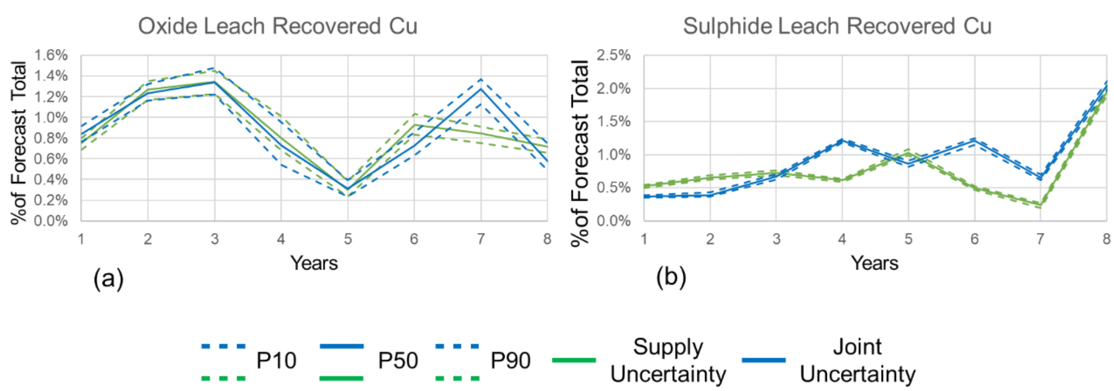

Figure 2.12: Copper recovered by Oxide leach (a) and sulphide leach (b)

For waste management, Figure 2.13 shows the waste tonnages and the copper content in the waste material. The joint uncertainty case shows that it extracts about 13% more waste compared to the supply uncertainty case. However, the copper content in the waste material is less than in the supply case. The joint uncertainty case is capable of improving the waste management in earlier periods.

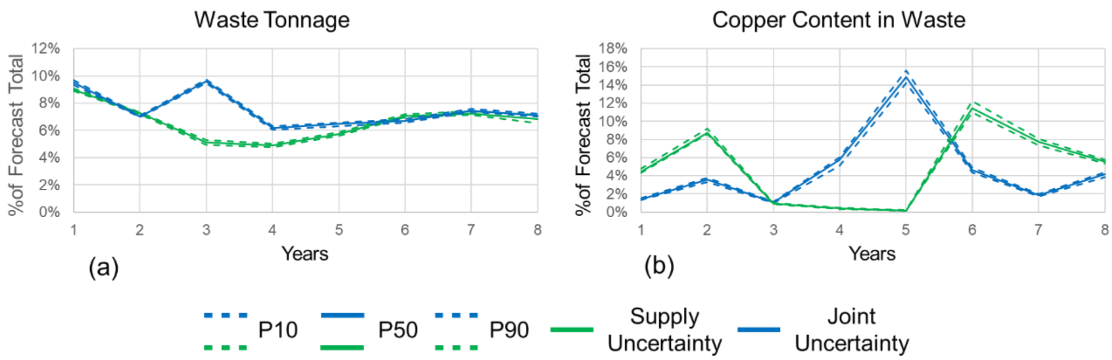

Figure 2.13: Waste management

To demonstrate more clearly the amount of metal recovered, the amount of copper recovered by mills and leach-pads are summed up separately in Figure 2.14. As shown in Figure 16, the mill recovered the majority of the copper metal. About 89% of copper is recovered by the mill, and about 11 % is recovered by the leach pads. The joint uncertainty case recovers about 5% more metal in the first year, and a similar amount of metal in later years, as shown in Figure 2.14 (a). For the Leach Pads, Figure 2.14(b) shows that the stochastic plan recovers a similar amount of copper in earlier years and slightly more copper in later years. As a result, Figure 2.14 (c) shows that the joint uncertainty case can generate a 2% higher NPV compared to what the supply uncertainty case is capable of delivering under equipment uncertainty at the end of eight periods. The production schedules for the two cases are show in Figure 2.15 and Figure 2.16, where each period is represented by a distinct color gradient from blue in period 1 to red in period 8. For Mine 1, the production schedule of the joint uncertainty case extracts slightly more material compared to the supply uncertainty case. This result is consistent with the previous observation: namely, that considering equipment uncertainty improves the use of equipment.

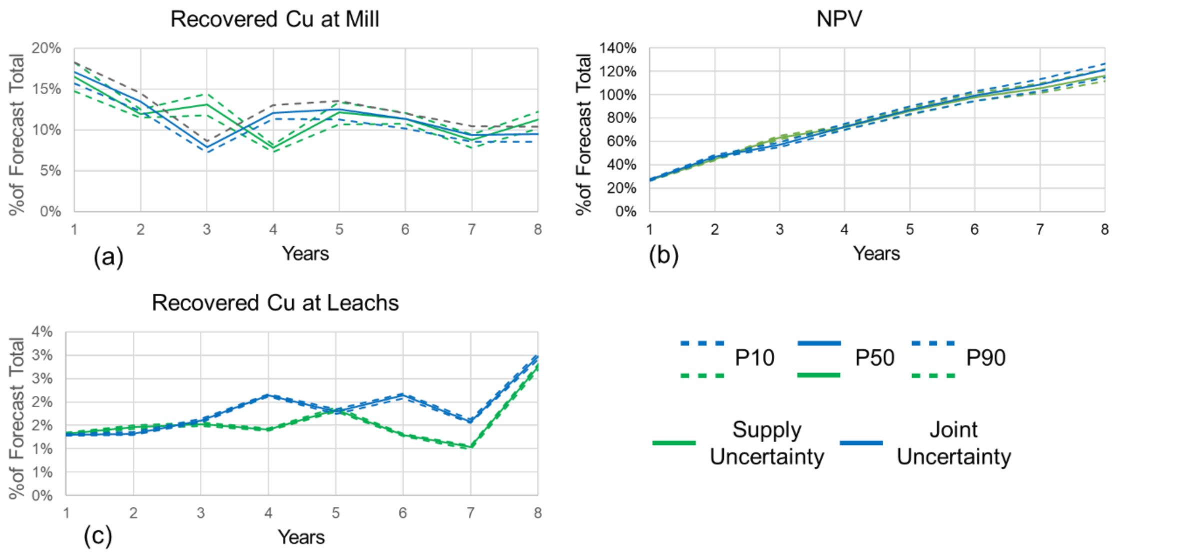

Figure 2.14: Total recovered copper from mills (a) and from leach pads (b), and NPV of the mining complex (c)

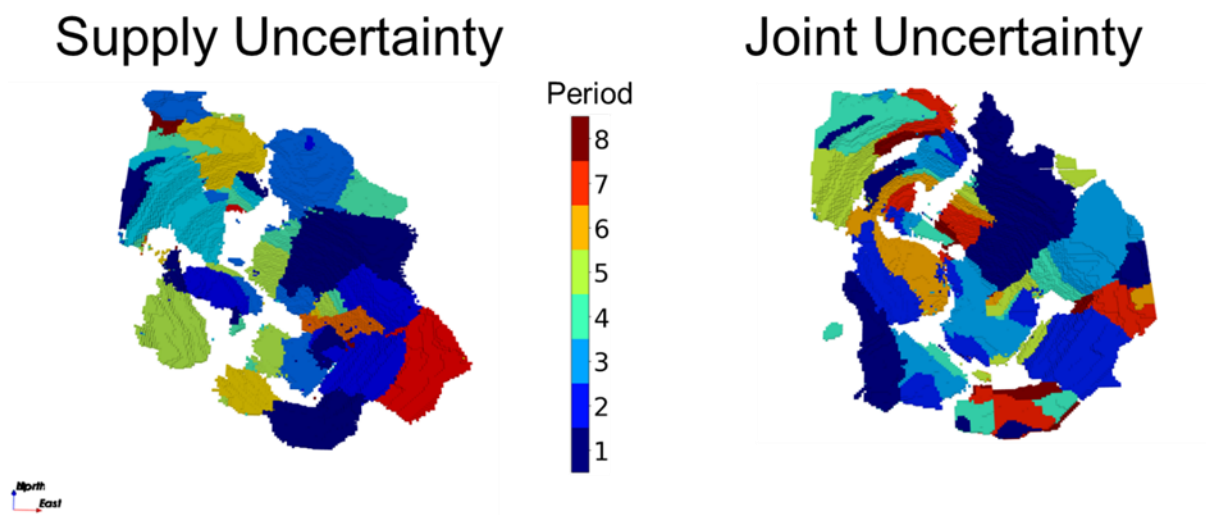

Figure 2.15: Mine 1 production schedule for Supply Uncertainty case and Joint Uncertainty case

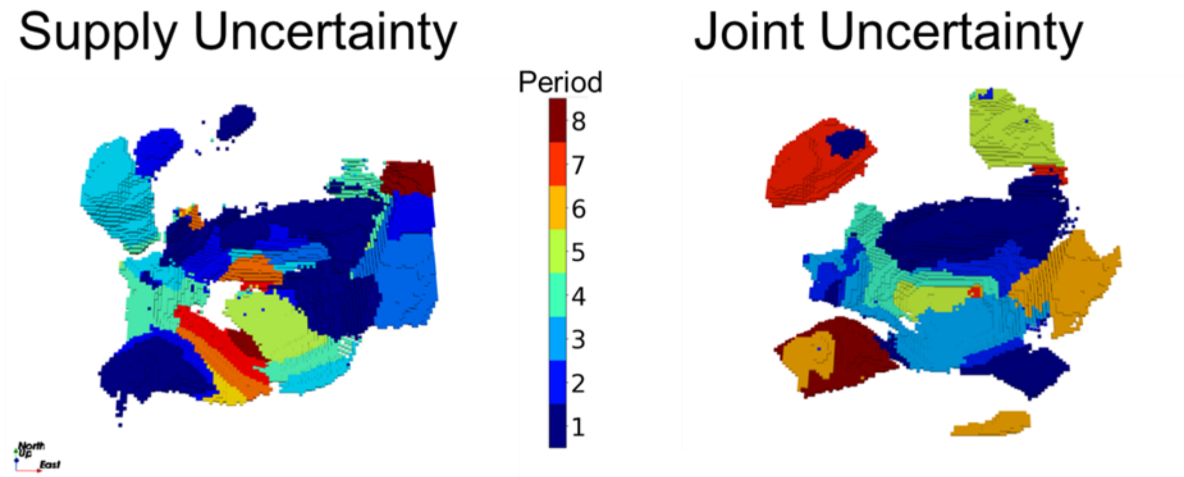

Figure 2.16: Mine 2 production schedule for Supply Uncertainty case and Joint Uncertainty case

In summary, the joint uncertainty case results in a better production schedule with a higher utilization of equipment that better respects the equipment capacity. Also, it can achieve a slightly higher NPV by recovering more copper at earlier periods. This improvement is in addition to the improvement achieved by incorporating supply uncertainty into simultaneous stochastic optimization compared to the conventional method.

2.4 Conclusions

This paper extends the previous simultaneous stochastic optimization framework of mining complexes to include equipment uncertainty and related constraints. The application at a copper mining complex demonstrates the practical aspects of integrating joint supply and equipment uncertainties. The uncertainty of equipment productivity is captured by simulations based on historical production data using Monte Carlo simulations. By integrating equipment uncertainty, in addition to geological (supply) uncertainty, the optimization process respects the simulated equipment capacity, resulting in pragmatic and realistic life-of-asset production schedules. In the case study, the joint uncertainty production schedule produces 5% more copper for the mills in the first year, although the risk profiles show more fluctuations across multiple periods compared to the schedule where only supply uncertainty is considered. The Leach Pads also show higher copper production in later years. Consequently, the proposed extended stochastic schedule now reflects and manages risk regarding equipment productivity, achieves a 2% higher NPV compared to the schedule that considers only supply uncertainty, while it is also capable of improving waste management in the earlier years.

Future research directions could consider haul cycles in the optimization process. Another important avenue could be including factors such as block depth and distance from the mining site, which influence truck productivity and cost structure. Evaluating the feasibility of investing in an additional crusher to alleviate production constraints mentioned previously may significantly impact overall productivity and profitability. In addition, various operating modes can be explored for the mill, such as including options with higher recovery but lower throughput. Additional case studies and comparisons would also be valuable in terms of providing understanding of the effects of different equipment configurations in mining operations.

Chapter 3. Simultaneous stochastic optimization of mining complexes with recovery and market uncertainty: application at an open pit copper mining complex

3.1 Introduction

A mining complex can be seen as an integrated system composed of mines, stockpiles, waste disposal and tailings storage facilities, processing destinations and transportation; all leading to the generation of sellable products delivered to customers and/or the spot market (Dimitrakopoulos and Lamghari, 2022; Goodfellow and Dimitrakopoulos, 2016; Montiel and Dimitrakopoulos, 2015; Pimentel et al., 2010). To deal with this complex system, conventional mine planning approaches optimize each component separately and sequentially (Hustrulid et al., 2013), while ignoring the technical uncertainties related to the corresponding components of the mining complex. Integration of various components within the mineral value chain while considering uncertainty has been a research objective for advanced strategic mine planning over the past decade. A wide range of production decisions, including extraction sequencing, destination policies, and utilization of processing streams can be simultaneously determined by the integrated optimization model for mining complexes while also addressing the uncertainties inherent in mining operations. By employing simultaneous stochastic optimization, the value of the final products sold by the mineral value chain is maximized, synergies between different components are captured, and technical risks are managed.

Over the past decades, developments in optimization models include multiple components of mining complexes advancing the conventional approach toward joint optimization (Hoerger, 1999a, 1999b; Stone et al., 2018; Whittle, 2018; Whittle and Whittle, 2007). However these approaches exhibit several limitations such as the aggregation of blocks to decrease computational complexities in optimization, the requirement of pre-determined mine production schedules as input, the sequential nature of the optimization process, and most significantly the reliance on a single estimated model of the deposits. The use of a single estimated orebody ignores the uncertainties and spatial variability in critical properties of the mineral deposit, which is the primary source of technical risk in mine planning (Dowd, 1994, 1997; Ravenscroft, 1992), referred to herein as supply or geological uncertainty. The geological uncertainty directly impacts the grades and material types within the deposit, leading to a misrepresentation of the actual proportions of material concentration, and their subsequent effects on the outcome of the mine production schedule. To strengthen the robustness of mining complex optimization and enhance its ability to capture synergies, it is necessary to integrate supply uncertainty into the optimization process with stochastic simulations (Dimitrakopoulos et al., 2002).

Simultaneous stochastic optimization of mining complexes overcomes the above-mentioned limitations by considering one single mathematical formulation to simultaneously optimize the components in a mining complex while considering uncertainties (Goodfellow and Dimitrakopoulos, 2016, 2017; Montiel and Dimitrakopoulos, 2015, 2017, 2018). It allows the modelling of constraints of the components at the same time and any nonlinear transformation so that the solution is able to capture the synergies among components. Montiel and Dimitrakopoulos (2015) introduce a model that takes into account multiple mineral deposits, multiple processing steam, and multiple transportation alternatives while considering supply uncertainty. The objective function is modelled as a two-stage Stochastic Integer Programming (SIP) (Birge and Louveaux, 2011) to explicitly maximize the NPV by considering the value of the final product, and operational costs, and minimizing deviations from production targets. The method is applied to Newmont's Twin Creek operations with additional operational constraints to create a practical production schedule (Montiel and Dimitrakopoulos, 2018), and results in a 7% increase in NPV and improves the management of autoclave blending constraints, when compared to the deterministic schedule.

Goodfellow and Dimitrakopoulos (2016) introduced a generalized network-based SIP approach that accommodates all components within a mining complex for simultaneous stochastic optimization. The model aims to simultaneously define the extraction sequence, destination policies, and processing stream decisions over the life-of-mine. The method maximizes the NPV by considering the value of final products at the end of the mining complex, while effectively managing production targets. In a specific case study conducted on a copper-gold mining complex (Goodfellow and Dimitrakopoulos, 2017), simultaneous stochastic optimization was deployed to mitigate the technical risk associated with geological uncertainty. Compared to a conventional schedule, the approach resulted in a 22.6% increase in NPV while effectively managing geological risk. The adaptability of this generalized formulation has led to subsequent research exploring a wide range of applications for mining operations. Farmer (2017) extended the model to incorporate capital expenditure (CapEx) with mining capacity decisions and to incorporate complex revenue calculations such as royalties and metal streaming under price uncertainty. Del Castillo and Dimitrakopoulos (2019) modify the mathematical framework to enable dynamic optimization for strategic planning, explicitly considering CapEx alternatives and various operating modes. This multi-stage model exhibits adaptability over the life of a mining complex by reacting and adapting to new information as needed. Further applications include the work of Kumar and Dimitrakopoulos (2019), who incorporate geo-metallurgical decisions into the destination policy for a copper-gold mining complex. Saliba and Dimitrakopoulos (2019) conduct an application accounting for both supply and market uncertainty through commodity price simulations at a gold mining complex. Levinson and Dimitrakopoulos (2020) applied the method to a gold mining complex to generate a production schedule with active management of the production of acid-generating waste under geological uncertainty. Levinson and Dimitrakopoulos (2023) connect the simultaneous stochastic optimization framework (Goodfellow and Dimitrakopoulos, 2016) for long-term strategic planning and a reinforcement learning approach (Levinson et al., 2023) for short-term production scheduling, to reduce misalignment between schedules of different timescales.

The applications discussed above typically consider constant recovery of metal at the processing destination. However, uncertainty in the metallurgical process is a source of uncertainty in mineral projects in addition to geological and market uncertainty (Coward and Dowd, 2015). Coward et al. (2013), Coward and Dowd (2015), and Jackson et al. (2014) assess the effect of recovery uncertainties on the economic outcome of mining projects. Coward and Dowd (2015) present a comprehensive study evaluating the uncertainty of the net smelter return (NSR) by generating multiple simulated grade-recovery curves (Coward et al., 2009). The risk analysis shows that the range between the 10th and 90th percentile (P10 and P90) of NPV spreads as much as 70% of the expected project NPV when considering compound uncertainty. These findings highlight the significance of recovery uncertainty and its impact on project outcomes. Incorporating recovery uncertainty into the mine optimization process will further enhance the understanding and management of uncertainty in mining operations, and incorporating market uncertainty through the use of commodity price simulations generates a long-term plan that manages and quantifies risk derived from volatile spot markets. Recent simultaneous stochastic optimization applications addressing market uncertainty include the work of Farmer (2016), Zhang and Dimitrakopoulos (2017), and Saliba and Dimitrakopoulos (2019), with the use of commodity price simulation. On the other hand, recovery uncertainty has not been explored for its effect on the simultaneous stochastic optimization of the mining complex.

In the following sections, an application of the simultaneous stochastic optimization method proposed by Goodfellow and Dimitrakopoulos (2016) is presented to incorporate market uncertainty and recovery uncertainty along with geological uncertainty. The process of quantifying market uncertainty through commodity price simulations and recovery uncertainty through recovery curve simulations are outlined. Next, a copper mining complex is employed to conduct case studies exploring the ramifications of different combinations of geological, recovery, and market uncertainties on production schedules. Last, conclusions and possible future work are discussed.

3.2 Simultaneous Stochastic Optimization of Mining Complexes

3.2.1 Definition and notation

Components in a mining complex, $i \in \mathcal{N}$, are classified as three types of locations: clusters of blocks ($\mathcal{C}$), stockpiles ($\mathcal{S}$), and processing destinations($\mathcal{P}$). Clusters of blocks, $\mathcal{C}$, serve as sources of materials or where the materials are extracted from. Clusters of blocks are generated from the block model of mine $m \in \mathbb{M}$ by grouping blocks with similar attributes with the k-means algorithm. Since a block has different properties under a different scenario $s \in \mathbb{S}$, it could belong to a different cluster in a different scenario. $\mathcal{P}$ is a set of processing destinations that receive and transform material. $\mathcal{S}$ is a set of stockpiles which can receive materials from sources or destinations but do not treat or transform those materials. Stockpiles hold material over time for possible future use and can be modelled as a source or a destination. Directed arcs, $\mathcal{A}$, defines the ability to send material from one location to another. Together, locations and directed arcs form a directed graph, $\mathcal{G}(\mathcal{N}, \mathcal{A})$ which mathematically describes a mining complex (Goodfellow and Dimitrakopoulos, 2016).

In a mining complex, materials in a mining complex are first extracted from the mines, and then go through a set of processing destination and stockpile facilities for processing or storage. The materials at the end of the mining complex are the final products sold to customers or spot markets. $v_{i,a,t,s}$ describes the geological and geo-metallurgical attribute a of the material at location i at period t under the scenario s. And, two types of attributes are used, primary attributes ($p \in \mathbb{P}$) and hereditary attributes ($h \in \mathbb{H}$). Primary attributes describe the fundamental properties of materials that can be sent to one location to another, and they are linearly additive. Examples of primary properties include material tonnage and metal content tonnage. Hereditary attributes are calculated from primary attributes or additional operational or financial information necessary for modeling the mining complex. They enable the incorporation of non-linear transformation into the optimization model. Examples include recovery rates and metal prices at each processing destination. A set of stochastic scenarios $s \in \mathbb{S} = \{1, \ldots, S\}$ in employed to comprehensively account for all sources of uncertainty within a mining complex.

3.2.2 Decision variables

Three sets of decision variables control the production schedule of a mining complex and are to be optimized during the simultaneous stochastic optimization. First of all, the extraction decisions $x_{b,t} \in \{0,1\}$ determine whether a mining block $b \in \mathbb{B}_m$ is extracted or not at period $t \in \mathbb{T}$. The destination decisions $z_{c,j,t} \in \{0,1\}$, determine whether a cluster of blocks $c \in \mathcal{C}$ is sent to destination $j \in O(c)$ in period $t \in \mathbb{T}$ after they are mined. The last set of decisions, processing stream decisions $y_{i,j,t,s} \in [0,1]$, determines the proportion of output material from a location $i \in \mathcal{S} \cup \mathcal{P}$ being sent to a downstream destination $j \in O(i)$ at period $t \in \mathbb{T}$ under scenario $s \in \mathbb{S}$. The decision variables can be classified into two groups, scenario-independent or scenario-dependent. The extraction decisions and destination decisions are scenario-independent decisions that have to be made resilient to the geological uncertainty observed in the mining complex. The processing stream decisions are scenario-dependent and adaptive to the uncertainty because the model assumes that uncertainties associated with the material are revealed after the material is sent to its immediate destination. Besides the three sets of decision variables, for the purpose of modeling deviations, $d^{+}_{h,i,t,s}$ is used to represent the surplus exceeding an upper production target, $U_{h,i,t}$, and $d^{-}_{h,i,t,s}$ is used to represent the shortage falling behind a lower production target, $L_{h,i,t}$.

3.2.3 Objective Function

A generalized two-stage stochastic optimization model is proposed by Goodfellow and Dimitrakopoulos (2016, 2017) to maximize the profit of selling products while minimizing the deviation from production targets. It aims to simultaneously determine the extraction sequence, destination policies and processing stream decisions for mining complexes under uncertainties. The objective function is shown in Equation (1).

$$
\begin{aligned}
Max\ & \underbrace{\frac{1}{\|\mathbb{S}\|}\sum_{i\in\mathbb{M}\cup\mathcal{S}\cup\mathcal{P}}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{h\in\mathbb{H}} p_{h,i,t}v_{h,i,t,s}}_{Part\ I} \\
& -\underbrace{\frac{1}{\|\mathbb{S}\|}\sum_{i\in\mathbb{M}\cup\mathcal{S}\cup\mathcal{P}}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}}\sum_{h\in\mathbb{H}}\left(c^{+}_{h,i,t}\,d^{+}_{h,i,t,s}+c^{-}_{h,i,t}\,d^{-}_{h,i,t,s}\right)}_{Part\ II} \\
& -\underbrace{\sum_{t\in\mathbb{T}}\sum_{m\in\mathbb{M}}\sum_{b\in\mathbb{B}_m}\left(c^{smooth}_{b,t}\,d^{smooth}_{b,m,t}\right)}_{Part\ III} \\
& -\underbrace{\sum_{t\in\mathbb{T}}\sum_{m\in\mathbb{M}}\sum_{b\in\mathbb{B}_m}\left(c^{sink}_{b,t}\,d^{sink}_{b,m,t}\right)}_{Part\ IV}
\end{aligned}
\tag{1}
$$

Part I of the objective function accounts for the discounted cash flows from metal sales, the mining cost, the processing cost, and other associated costs. Part II accounts for the cost of deviation from production targets by applying penalties cost, $c^{+}_{h,i,t}$ and $c^{-}_{h,i,t}$, to deviations, $d^{+}_{h,i,t,s}$ and $d^{-}_{h,i,t,s}$, respectively. The geological discount rate is applied to the penalties, $c^{+}_{h,i,t} = \frac{c^{+}_{h,i,1}}{(1+rd)^t}$ (Dimitrakopoulos and Ramazan, 2004; Ramazan and Dimitrakopoulos, 2005, 2013) instead of economic discounting. The risk discount rate minimizes the risk in early periods and defers the risk into later periods when more information is available. Part III and Part IV are smoothing and sink rate penalties. Part III ensures that the extraction sequence creates adequate space for equipment access and movement. Together, the smoothing penalties and sink rate penalties create a practical minable shape for the schedule that allows adequate space for moving equipment and prevents slope instability. For this case study incorporating recovery and market uncertainty in addition to supply uncertainty, please note that Part I accommodates the joint supply, recovery, and market uncertainties. For example, uncertainty recovery rate can be modelled as hereditary attributes at processing mill and market uncertainty can be modelled as hereditary attributes at selling. The objective function will be evaluated across the scenarios describing supply, recovery and market uncertainties.

3.2.4 Constraints

In a mining complex, there are different upper and lower production targets and constraints, denoted as $U_{h,i,t}$ and $L_{h,i,t}$. Examples include mining capacity, processing capacities, and the acceptable ratio of different properties of the material in a mill. They are defined for attribute $h \in \mathbb{H}$ at locations $i \in \mathcal{N}$ in period $t \in \mathbb{T}$, such as the processing capacity for a flotation mill. To quantify deviations from these production targets within each scenario, $d^{\pm}_{h,i,t,s}$ represent the unit deviations from a production target associated with property h at location i in period t and scenario s, while $c^{\pm}_{h,i,t}$, represent the unit surplus and shortage costs associated with these deviations. For instance, $d^{+}_{h,i,t,s}$ can describe the amount of material the mine tries to extract but is above the mine's trucking capacity. With a corresponding penalty cost $c^{+}_{h,i,t}$, optimization can be guided to reduce the amount of material being extracted from the mine. Equations (2) and (3) calculate the deviations from the upper and lower bounds of each production target at each location under every orebody scenario. Then, the deviations are penalized in the objective function.

$$v_{h,i,t,s} - d^{+}_{h,i,t,s} \leq U_{h,i,t} \quad \forall\, h \in \mathbb{H}, i \in \mathcal{C} \cup \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{2}$$

$$v_{h,i,t,s} + d^{-}_{h,i,t,s} \geq L_{h,i,t} \quad \forall\, h \in \mathbb{H}, i \in \mathcal{C} \cup \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \tag{3}$$

$$|Neigh(b)| * x_{b,t} - \sum_{n \in Neigh(b)} x_{n,t} - d_{b,m,t}^{smooth} \leq 0 \quad \forall\, b \in \mathbb{B}_{m'}, t \in \mathbb{T}, m \in \mathbb{M} \tag{4}$$

$$x_{b,t} + x_{\mathscr{b}(b),t} - d_{b,m,t}^{sink} \leq 1 \quad \forall\, b \in \mathbb{B}_{m'}, t \in \mathbb{T}, m \in \mathbb{M} \tag{5}$$

Equation (4) is the smoothing constraints that ensure adequate space for equipment access. Within a predefined mining width, a block b is surrounded by a set of neighboring blocks, denoted as $Neigh(b)$. $d_{b,m,t}^{smooth}$ represents the number of blocks within $Neigh(b)$ that are mined at a different period as b. Then, the objective function is penalized with a smoothing penalty cost $c_{m,t}^{smooth}$. Similarly, the sink rate constraint, Shown in Equation (5), limits the depth a bench can descend within one period. Within a predefined sink rate, $\mathscr{b}(b)$ represents the set of blocks overlaying block b. For example, if blocks are 10m in height and the sink rate is 30m, then $\mathscr{b}(b)$ is the three blocks above block b. A penalty $d_{b,m,t}^{sink}$ will incur if they are mined at the same period t. The comprehensive and detailed explanation of the remaining constraints included in the model, such as capacity constraints, blending constraints, material type constraints, reserve, slope constraints, and stockpiling constraints can be found in (Goodfellow and Dimitrakopoulos, 2016).

3.2.5 Solution approach

The simultaneous stochastic optimization of mining complexes is challenging due to its large number of binary decision variables and the integration of various sources of uncertainty. Utilizing commercial solvers is not feasible for solving the optimization model. Metaheuristics and hyper-heuristics are effective solution methods to obtain near-optimal solutions for large stochastic optimization models of mining complexes (Goodfellow and Dimitrakopoulos, 2016, 2017; Lamghari and Dimitrakopoulos, 2018). The solution approach used in this work is a combination of multi-neighbourhood simulated annealing with adaptive neighbourhood search, where the selection of heuristics is guided by reinforcement learning (Yaakoubi and Dimitrakopoulos, 2023).

3.3 Case Study

3.3.1 Overview of the mining complex

The method of simultaneous stochastic optimization described in the previous section is applied to an open-pit copper mining complex. Shown in Figure 3.1, the mining complex consists of two open-pit mines, Mine 1 and Mine 2, with respectively 414,108 and 157,749 blocks, which are 25x25x15m^3 in size. The main attributes of concern are copper total and copper soluble. Each block in the block model belongs to one of the four main material types, high-grade sulphide, low-grade sulphide, oxide, and waste. Materials produced by the mines can be processed by different process streams. Two products are produced by the mining complex, copper concentrate and copper cathodes. As shown in Figure 3.2, Mill 1, Mill 2, and Mill 3 receive high-grade sulphide material from Mine 1 and Mine 2 after materials are crushed by their corresponding crushers and produce copper concentrate as a product. The recovery uncertainty of Mill 1, Mill 2, and Mill 3 will be modelled as sets of stochastically simulated recovery curves. The recovery uncertainty of Mill 1 and Mill 2 are assumed to be the same and share the same set of recovery scenarios, as given by the mill recovery data from the mining operation. The Oxide Leach Pad takes oxide materials from both mines after the oxide material is crushed by Crusher 4 and produces copper cathodes. The Sulphide Leach Pad takes low-grade material from both mines and produces copper cathode as a product. The Sulphide Leach Pad requires the ratio of copper total and copper soluble to be within its operational limit.

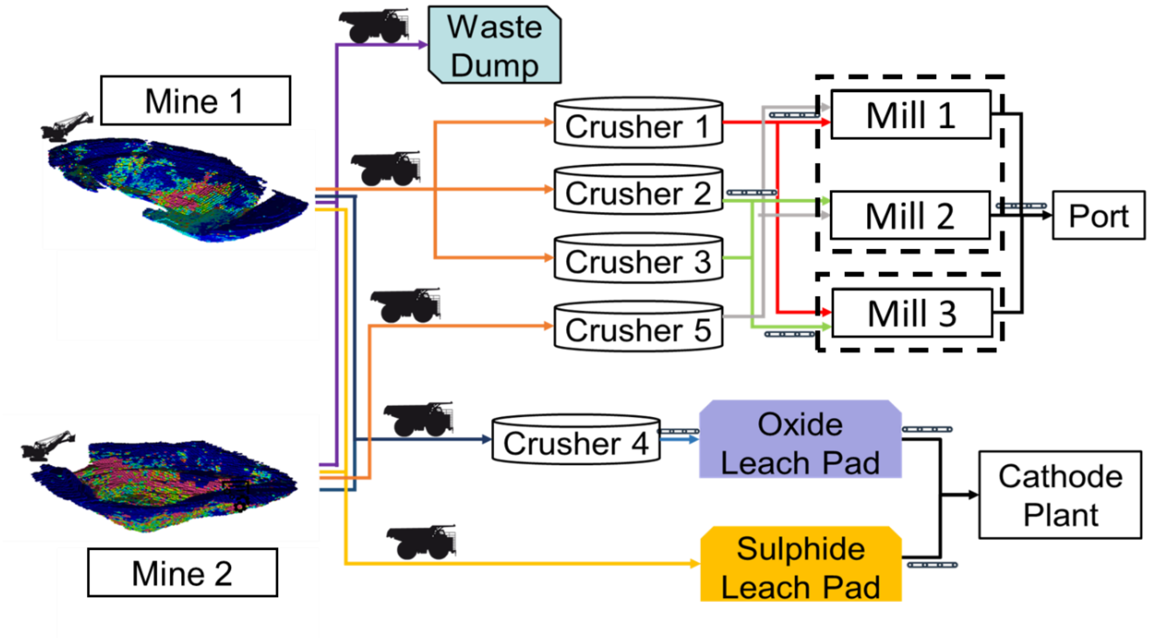

Figure 3.1: Copper mining complex, with 3 mills for modelling recovery uncertainty (highlighted in dash line)

3.3.2 Modelling recovery uncertainty

In the present case study, the group of recovery curves is simulated using the method presented by Coward et al. (2013), with historical recovery data provided by the mining operation. The recovery uncertainty is modelled using the third-order inverse polynomial model of copper grade, $Rec\% = a + \frac{b}{Cu\%} + \frac{c}{(Cu\%)^2}$. By removing one data point ("bootstrapping"), it was possible to fit a regression curve with a set of $a, b, and\ c$ parameters for the model. Repeating the above process by removing different data points, multiple values of $a, b, and\ c$ are produced. Then, by computing the mean and variance from the set of a, it was possible to draw random values of a from the normal distributions with the same mean and variance to produce simulated values of a. A similar procedure is followed for simulating the values of b and c to produce simulated recovery curves. The use of the third-order inverse polynomial model is determined by repeating the simulation process for different models including linear, second-order quadratic, logarithm functions, etc. The third-order inverse polynomial model is chosen because the generated recovery curves best cover the dispersion of the historical data.

Figure 3.2 shows the set simulated recovery curves of Mill 1 and Mill 2in colours which are capable of reproducing the variability of historical recovery data and are comparable to the regression line of all data which is used in the baseline case for comparison purposes. Figure 3.3 shows the set of simulated recovery curves of Mill 3, and similarly, the set of simulated recovery curves reproduces the variability of historical recovery data and is comparable to the regression line of all data.

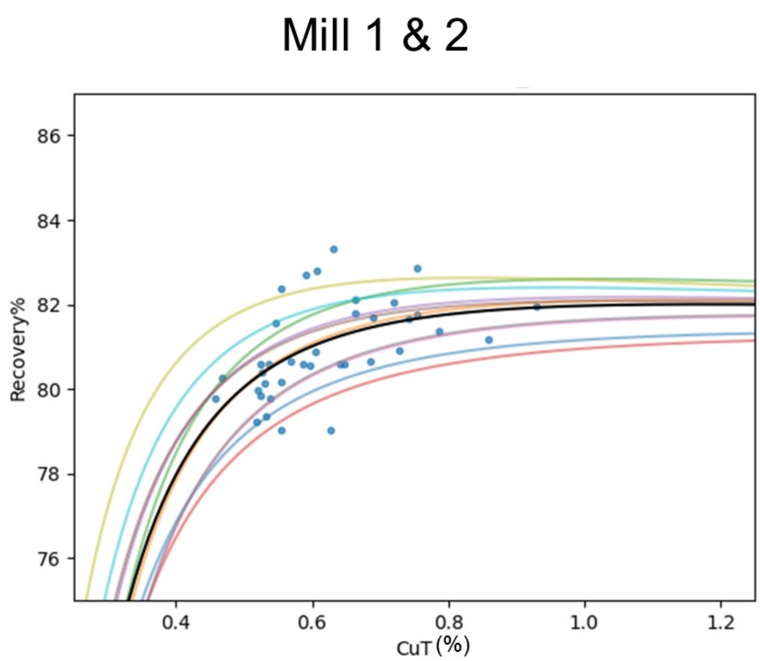

Figure 3.2: Simulated recovery curves (color lines), historical recovery data (blue dots), and the regression line of all data (black line) for Mill 1 and 2

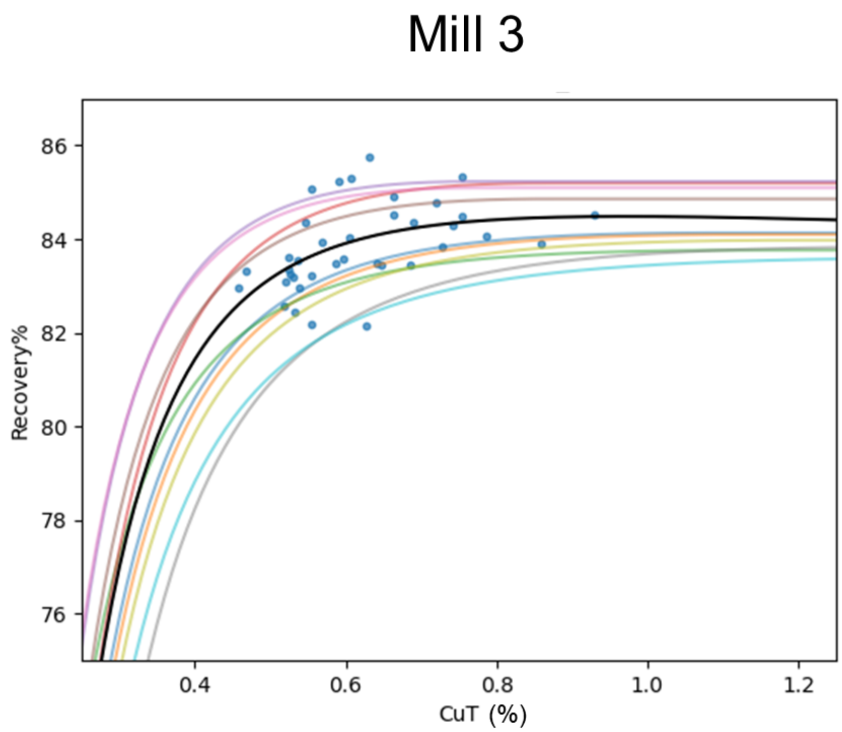

Figure 3.3: Simulated recovery curves (color lines), historical recovery data (blue dots), and the regression line of all data (black line) for Mill 3

3.3.3 Modelling market uncertainty

Copper prices are simulated using an established model for base metals, the Mean Reverting Process (Dixit and Pindyck, 2019; Schwartz, 1997; Suarez and Fernandez, 2009), described by Eq. 8, where x_t is the metal price at time t, dz is the standard normal distribution. Long-term price, $\bar{x}$, is the expected mean price used in price models where the price is reverting around. The speed of reversion is given by parameter η, and σ is the average annual price volatility. The parameters used to model copper price uncertainty in this case study are given in Table 3.1. Figure 3.4 presents the ten simulated copper price scenarios used in this case study. The mean of the simulated prices and the constant price used in the baseline case are comparable. It is typically accepted that the number of simulations necessary to accurately quantify metal price uncertainty can be on the order of hundreds (Briggs et al., 2012). However, it would make the number of joint uncertainty scenarios is in the order of millions for the case study, making the problem intractable. However, the influence of the number of price scenarios on the long-term production scheduling for mineral value chains has not been sufficiently explored. With 10 simulated price scenarios, this case study demonstrates the impact of price uncertainty on the output of simultaneous stochastic optimization of mining complexes.

$$x_t = e^{-\eta \Delta t} * x_t + (1 - e^{-\eta \Delta t}) * \bar{x} + \sigma \sqrt{\frac{(1 - e^{-2\eta \Delta t})}{2\eta}} dz \quad (8)$$

Table 3.1: Parameters for price simulations

Parameters	Value
Initial price, x_0	US$5511/t
Expected mean price, $\bar{x}$	US$5511/t
Reverting speed, η	0.5
Annual price volatility, σ	9%

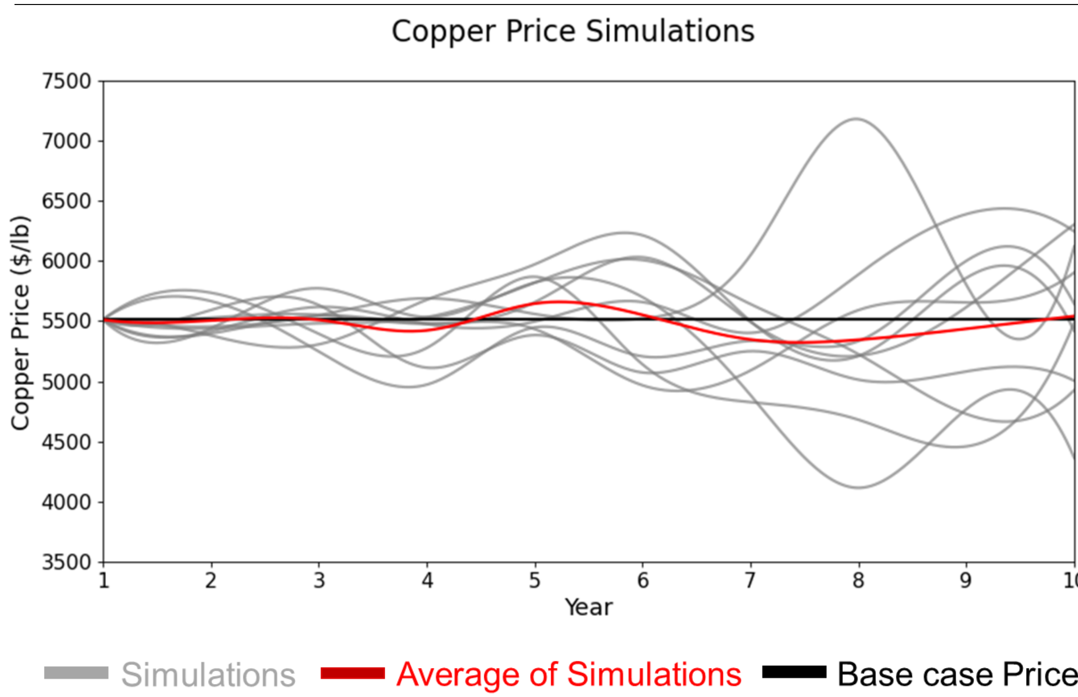

Figure 3.4: Copper price simulations (grey), mean of simulated prices (red), constant copper price (black)

3.3.4 Parameters

The case study of simultaneous stochastic optimization for this copper mining complex requires two sets of parameters. The first set of parameters is the economic and operational parameters associated with the mining operation, which are summarized in Table 3.2 and Table 3.3, respectively. For confidentiality reasons, the parameters listed in both tables are scaled. Table 3.2 summarizes the costs associated with the mining operation and the price of metal produced. Table 3.3 includes the mining width, sink rate, and slope angle limitation as well as fixed recovery rates for different processing streams. The penalty costs associated with the objective function are listed in Table 3.4. They are determined based on a trial and error process to achieve an acceptable level of technical risks for different production targets and the economic outcome (Benndorf and Dimitrakopoulos, 2013).

Table 3.2: Economic parameters

Parameters	Value
Discount rate (NPV)	8%
Geological discount rate	10%
Copper Price ($US)	$US5511
Mining cost (excluding hauling cost, $US)	0.6
Hauling cost (based on location, $US)	0.4 to 1.3
Mill process cost including crushing ($US)	6.4
Oxide leach cost including crushing ($US)	6.6
Sulphide leach cost ($US)	1.1
Stockpile rehandling cost ($US)	0.2
Copper Concentrate Selling Cost ($US)	571
Copper Cathode Selling Cost ($US)	551

Table 3.3: Operational parameters

Parameters	Value	
Mining width	200m	
Sink rate	100m	
Slope angle	37° (Mine 1)	45°(Mine 2)
Oxide reach recovery	65%	
Sulphide leach recovery	27%	
Number of blocks	414,108	
Number of periods	10	

Table 3.4: Penalty costs

Parameters	Value
Penalty cost – Simulated capacity (Mine 1, Mine 2, Mine 1 Crusher, Mine 2 Crusher, Oxide Crusher)	20, 20, 40, 100, 20 $/ton
Penalty cost – Capacity (Mill 1, Mill 2, Mill 3)	100, 100, 100 $/ton
Penalty Cost – Smoothing Constraint (Mine 1, Mine 2)	50000, 50000 $/ton
Penalty Cost – Sink Rate Constraint (Mine 1, Mine 2)	20000, 20000 $/ton
Penalty Cost – Leach Pad Capacity (Oxide, Sulphide)	20, 20 $/ton
Penalty Cost – CuS/CuT Ratio	4000 $/ton

3.3.5 Result and comparisons

Three case studies are conducted to demonstrate the effect of incorporating recovery and market uncertainty into the mining complex. The first case serves as the "baseline," including only supply uncertainty, with two sets of fifteen simulated orebodies for Mine 1 and Mine 2. In the second case, referred to as the "recovery uncertainty" case, it incorporate both supply and recovery uncertainty, characterized by a separate set of ten simulated recovery curves. The last case study, termed the "joint uncertainty" case, includes all three sources of uncertainties: namely supply, recovery, and market uncertainties. The result is presented in two comparisons to highlight the effects of different sources of uncertainties. In the first comparison, the outcomes of the recovery case are compared with those of the baseline case. In the second comparison, the result of joint case are compared to baseline case.

3.3.5.1 Comparison between recovery case and baseline case

Figure 3.5 to Figure 3.8 present the comparison between the baseline case, shown in black lines and the recovery case, shown in orange lines. Figure 3.5 shows the risk profile of recovered copper metal from Mill 1, 2, and 3. Figure 3.5 (a) shows that the recovery uncertainty case extracts more copper in early years and less copper in later years at Mill 1, compared to the baseline. For Mill 2 and Mill 3, shown in Figure 3.5 (b) and Figure 3.5 (c) the recovery uncertainty case extracts more copper in early years and less copper in later years at Mill 1, compared to the baseline. Figure 3.6 shows the throughput grade of the three mills. All three mills behave similarly, with higher-grade material throughput in early years, and lower-grade throughput in later years. The recovery rates achieved by the three mills are shown in Figure 3.7. When recovery uncertainty is incorporated, the risk profile of the recovery rate is wider than the baseline. Incorporating recovery uncertainty in the optimization produces a stable recovery rate compared to the baseline.

Figure 3.8 (a) shows the sum of copper recovered at three mills. It confirms the previous finding that the recovery uncertainty case produces more copper metal in year 2 and a slightly lower amount of copper in some later years when compared to the baseline. Cumulatively, the market case will recover 3.5% more copper and eventually generate 7.8% higher NPV, compared to the baseline case, as shown in Figure 3.8 (b) and Figure 3.8 (c). This result can be visualized when looking at the extraction sequence. Figure 3.9 shows the comparison of the extractions for Mine 1. In the areas indicated by circles, more high-grade materials are extracted for the recovery case in year 2. Similar results are observed when looking at Figure 3.10 showing the extraction sequence of Mine 2, which shows the recovery case extracting more material in year 2.

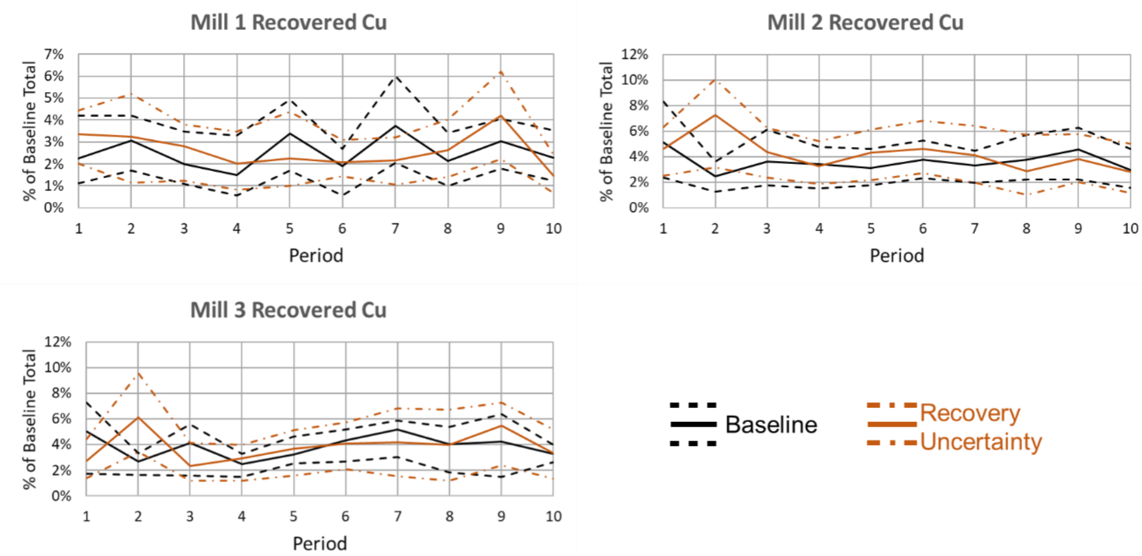

Figure 3.5: Copper metal recovered from Mill 1, 2, and 3 for baseline and recovery uncertainty cases

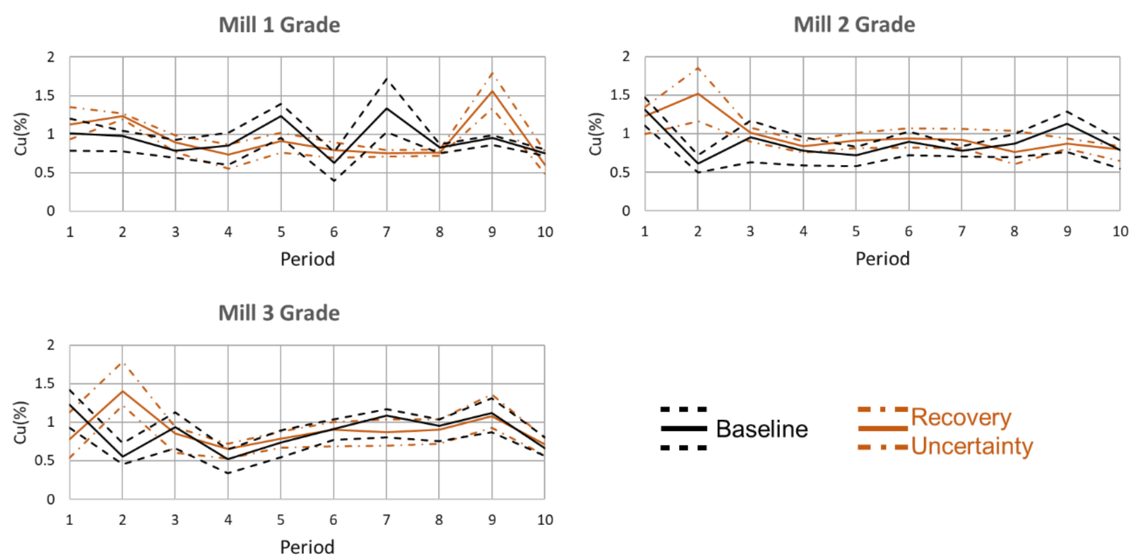

Figure 3.6: Throughput grade of Mill 1 (a), Mill 2 (b) and Mill 3 (c) for baseline and recovery uncertainty case

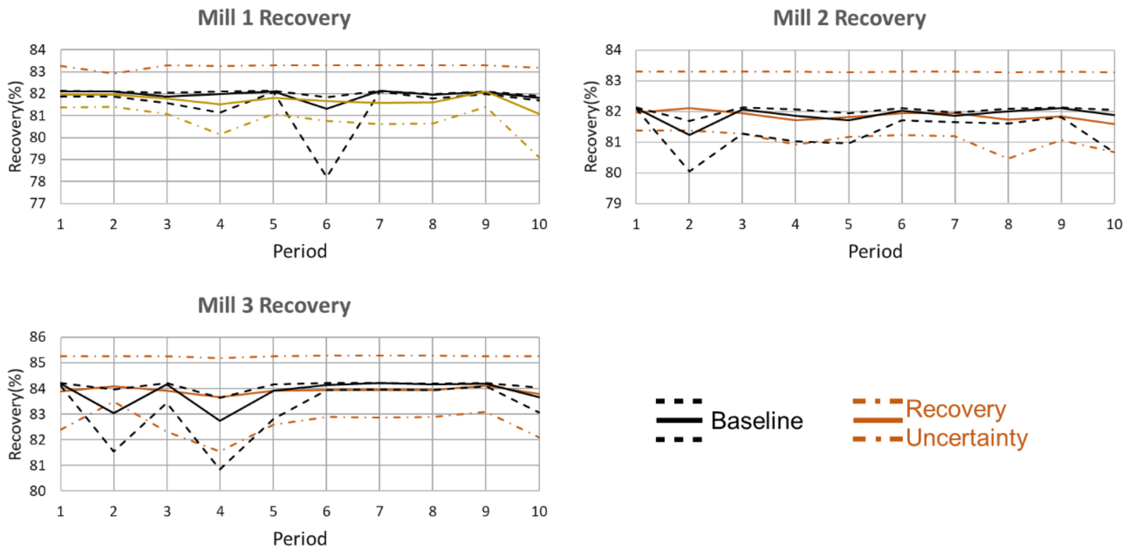

Figure 3.7: Recovery rate of Mill 1 (a), Mill 2 (b) and Mill 3 (c) for baseline and recovery uncertainty case

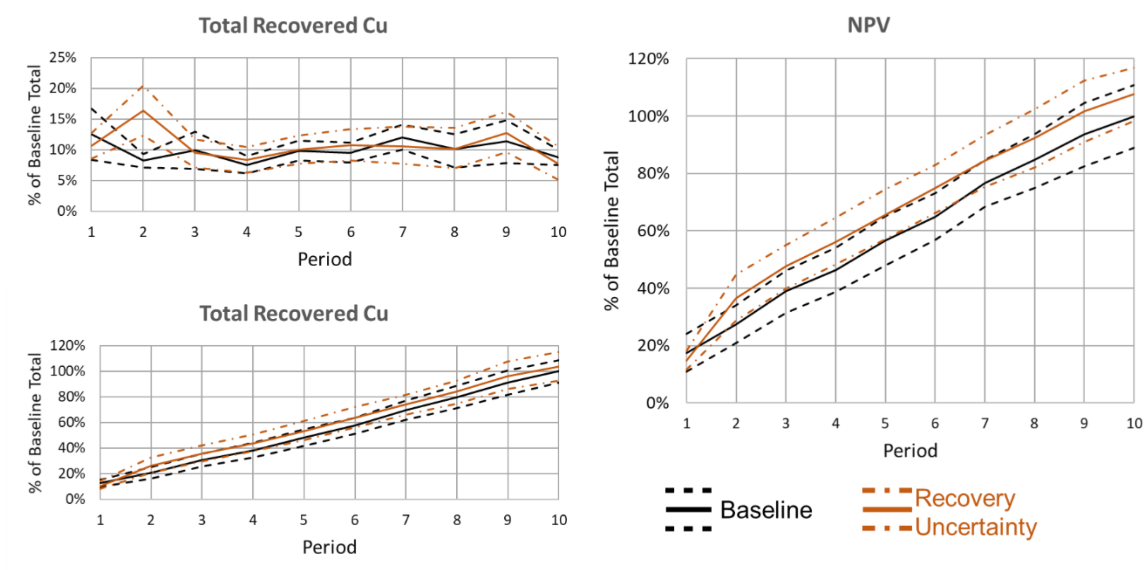

Figure 3.8: Recovered copper from mills (a), cumulative recovered copper from mills (b), and NPV (c) for baseline and recovery uncertainty case

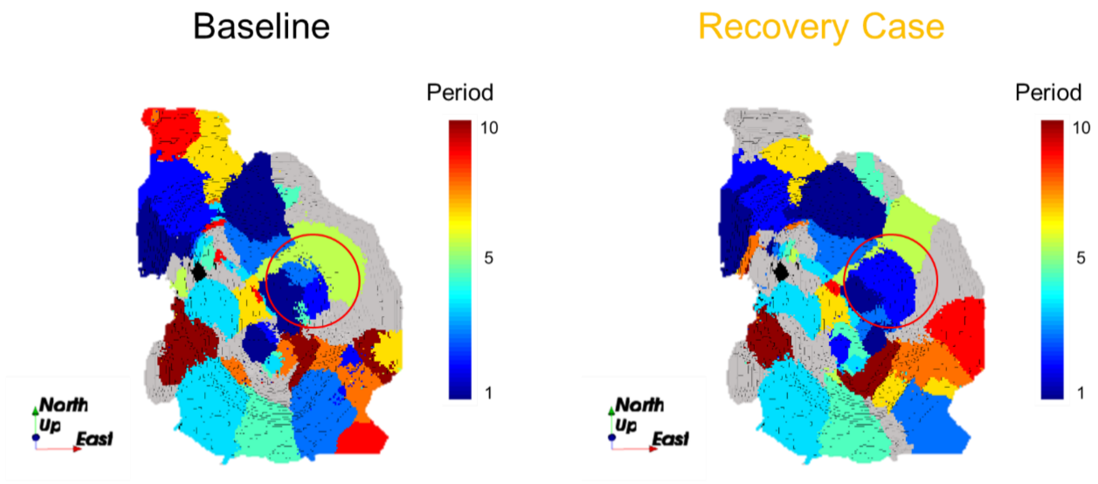

Figure 3.9: Mine 1 production schedule for baseline and recovery uncertainty case

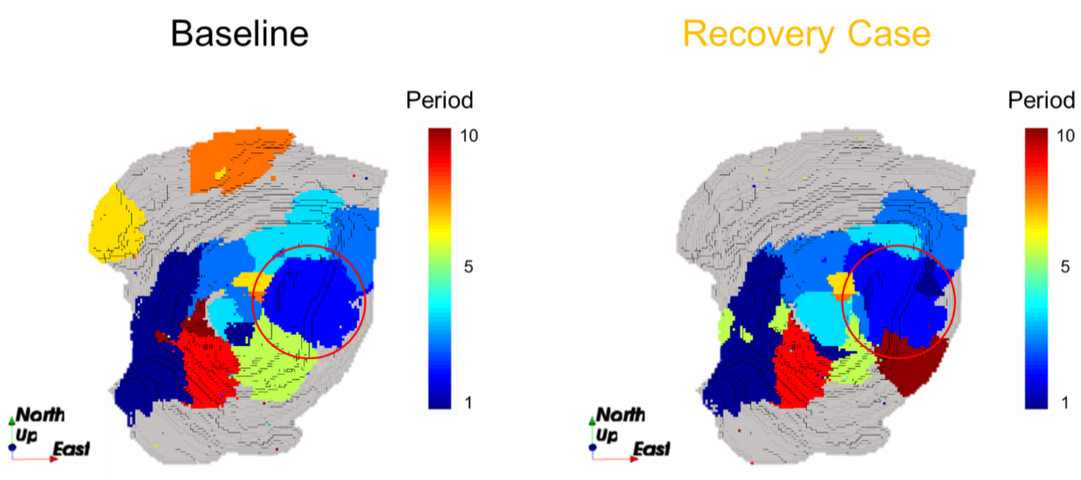

Figure 3.10: Mine 2 production schedule for baseline and recovery uncertainty case

3.3.5.2 Comparison between joint case and baseline case

Figure 3.11 to Figure 3.16 presents the second comparison between the baseline case, shown in black lines and the joint case, shown in blue lines. Figure 3.11 (a) shows that the joint uncertainty case extracts more copper in year 3 and less copper in later years compared to the baseline. For Mill 2 and Mill 3, shown in Figure 3.11 (b) and Figure 3.11 (c), similar results can be observed. Figure 3.12 shows the throughput grade of three mills which behave similarly. The joint case produces higher grade material throughput in year 3 compared to the baseline case, and lower grade input in later years. The recovery rates achieved by the three mills are shown in Figure 3.13. In the joint uncertainty case, the risk profile of the recovery rate is wider than the baseline. This indicates the risk of recovery rate is underestimated in the baseline as recovery uncertainty is not included in the optimization. Also, the 50th percentile of the risk profile for Mill 2 and Mill 3 indicates that the joint case produces a stable recovery rate compared to the baseline.

The difference in performance between Mill 1 and Mill 2 is important for analyzing the production schedules, given Mill 1 and Mill 2 share the same set of recovery uncertainty. This difference in performance arises from the configuration of the mining complex. As shown in Figure 3.2, Mill 1 material is supplied by Crusher 1 and Crusher 5. Crusher 1 receives material from Mine 1 and Crusher 5 receives material from Mine 2. Given Crusher 1 has a capacity half of the capacity of Mill 1, about 50% of the material processed by Mill 1 comes from Mine 2, while Mill 2 and Mill 3 have 15% of the material coming from Mine 2. Therefore, the performance of Mill 1 is dominated by the supply uncertainty and the configuration of the mining complex before being influenced by recovery and market uncertainty. Recovery and market uncertainty exert their effects on top of the underlying supply uncertainty and the synergy encapsulated by the mining complex's configuration as optimized.

Figure 3.14 (a) shows the total copper recovered at three mills. It shows that the mining complex produces more copper metal in year 3 and a slightly lower amount of copper in some later years when recovery and market uncertainty are incorporated. Cumulatively, the joint case will recover 7.0% more copper and eventually generate 12.5% higher NPV, compared to the baseline case, as shown in Figure 3.14 (b) and Figure 3.14 (c). This result can be visualized when looking at the extraction sequence. Figure 3.15 shows the comparison of the extractions for Mine 1. In the areas indicated by circles, more high-grade materials are extracted for the recovery case in year 3. Similar results are observed in Figure 3.16 showing the extraction sequence of Mine 2, which shows the recovery case extracting more material in year 3.

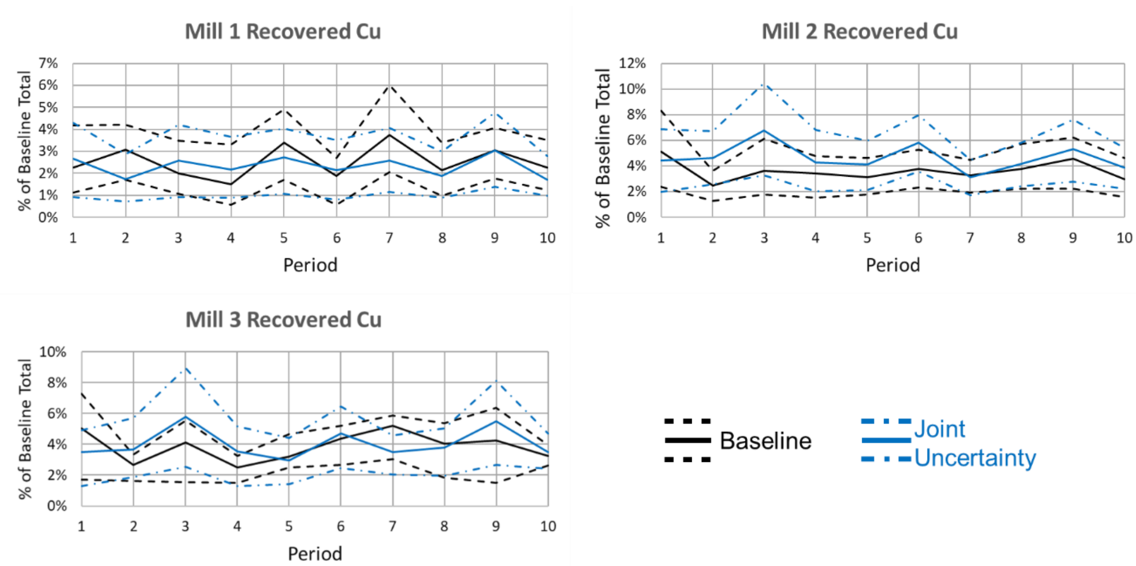

Figure 3.11: Copper metal recovered from Mill 1, 2, and 3 for baseline and joint uncertainty case

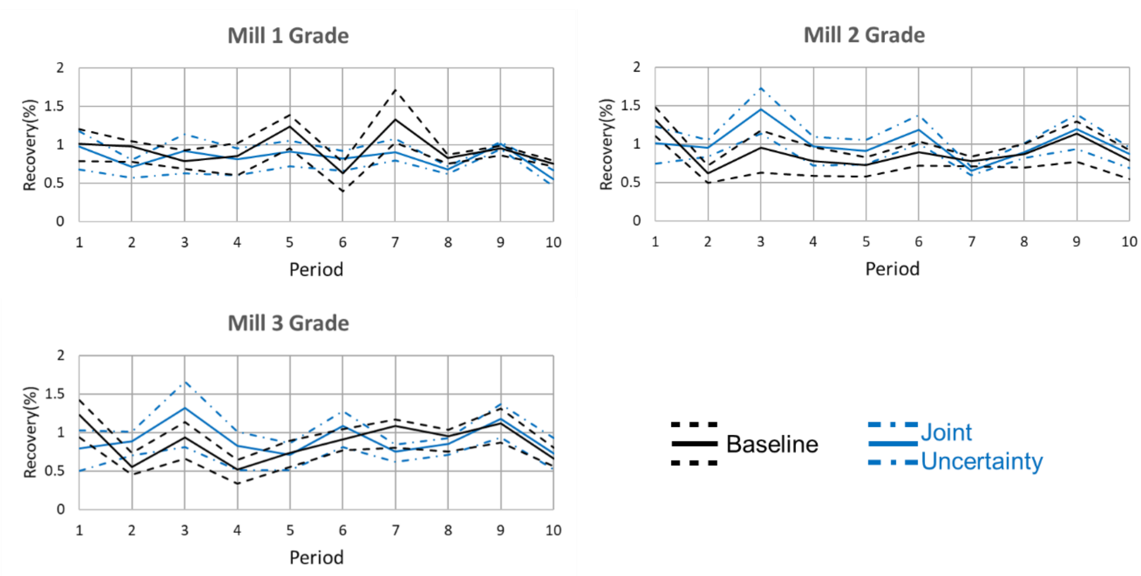

Figure 3.12: Throughput grade of Mill 1 (a), Mill 2 (b) and Mill 3 (c) for baseline and joint uncertainty case

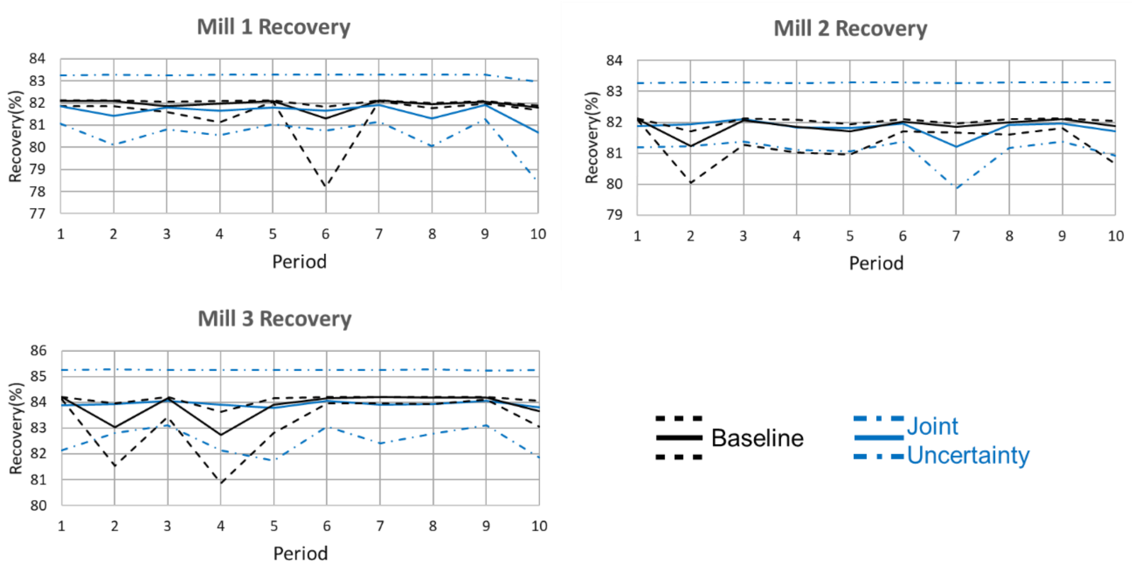

Figure 3.13: Recovery rate of Mill 1 (a), Mill 2 (b) and Mill 3 (c) for baseline and joint uncertainty case

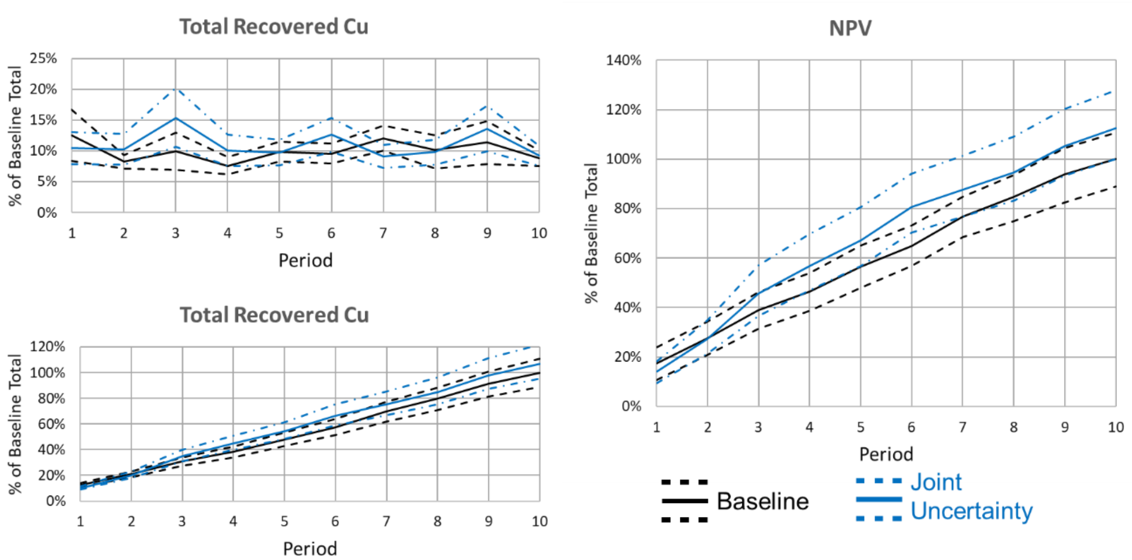

Figure 3.14: Recovered copper from mills (a), cumulative recovered copper from mills (b), and NPV (c) for baseline and joint uncertainty case

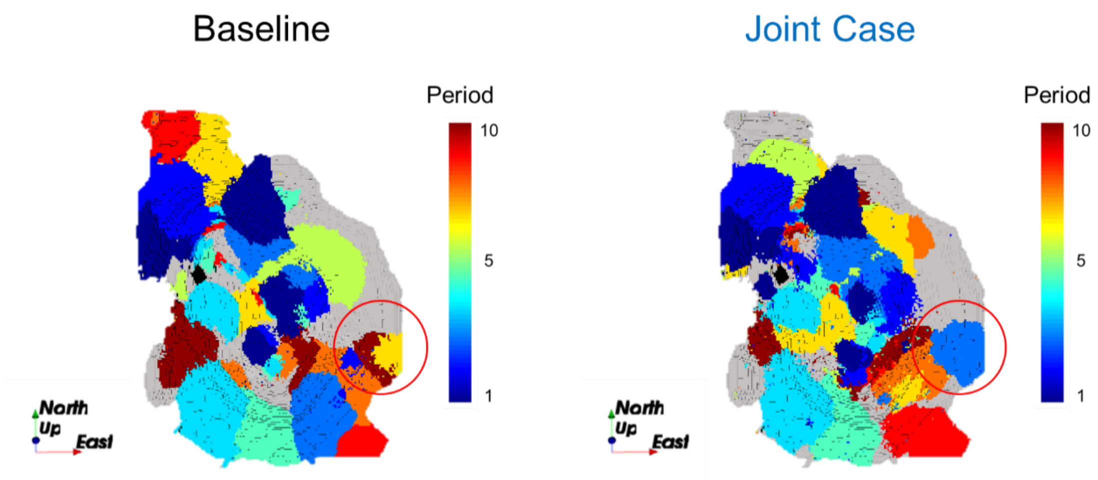

Figure 3.15: Mine 1 production schedule for baseline and joint uncertainty case

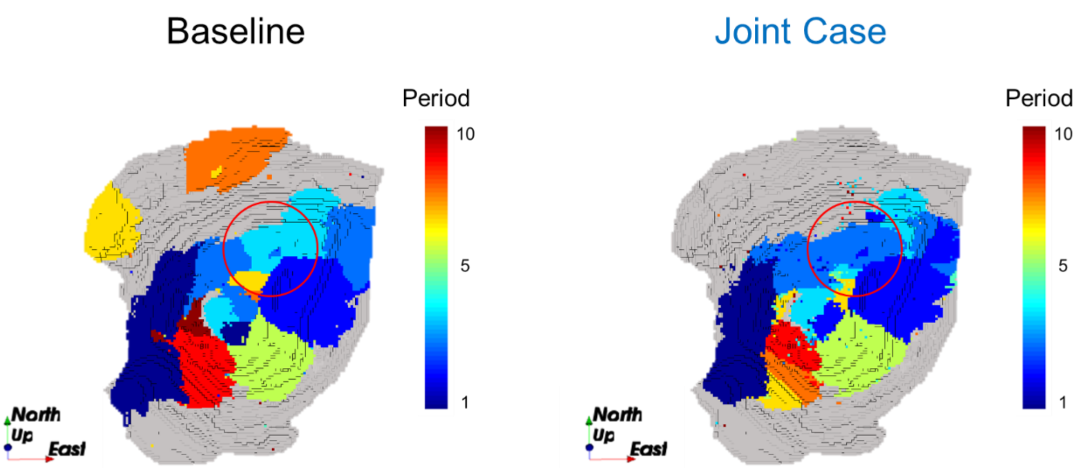

Figure 3.16: Mine 2 production schedule for baseline and joint uncertainty case

3.4 Conclusions

This paper presents an application of the simultaneous stochastic optimization framework of mining complexes that includes recovery and market uncertainty in addition to geological uncertainty. The application is conducted for a copper mining complex, showcasing the framework's ability to supply, recovery, and market uncertainties. The framework searches for a globally optimal solution by optimizing extraction sequences, destination policies, and processing stream decisions simultaneously. The incorporation of recovery uncertainty involves simulations of recovery curves based on historical production data using Monte Carlo simulations of the curve model parameters. Market uncertainty is captured through commodity price simulations based on the mean-reverting process. Through the incorporation of recovery and market uncertainties alongside geological uncertainty, the stochastic optimization process adjusts production schedules, to reflect different combinations of incorporated uncertainties. The recovery case and joint case schedules led to cumulative copper production increases of 3.8% and 7.0%, respectively. In addition, the recovery case and joint case result in higher Net Present Values (NPV) of 7.8% and 12.5% compared to the baseline case. Furthermore, the incorporation of recovery uncertainty leads to production schedules achieving more stable recovery rates, as observed in the recovery case and joint case schedules compared to the baseline. This highlights the impact of the mineral value chain configuration on the production schedule. Importantly, it is shown that the incorporation of multiple sources of uncertainty should not be viewed as a mere sum of uncertainties. Instead, the joint effects and interactions with specific mineral value chains need to be considered for a comprehensive analysis. The presented research provides valuable insights into the effective management of uncertainties and the optimization of production schedules for mining complexes, particularly when dealing with recovery and market uncertainties in addition to supply uncertainty. The framework's versatility and robustness contribute significantly to decision-making processes in the mining operation.

Possible future research may include exploring alternative recovery simulation methods. For instance, considering the correlation between different recovery curve parameters during the simulation to provide more accurate representations of recovery uncertainties. Additionally, conducting case studies and comparisons for mining complexes with varying configurations would be helpful to deepen the understanding of how different mine setups impact production. These research would offer insights into the robustness and adaptability of the optimization framework in different mining scenarios.

Chapter 4. Conclusions and Future Works

4.1 Conclusions

Advancements made in the field of strategic mine planning, stochastic simulations, and operations research during past decades build up to the simultaneous stochastic optimization framework, which is the basis of the thesis (Goodfellow and Dimitrakopoulos, 2016, 2017).

The first application expands upon the existing simultaneous stochastic optimization framework for mining complexes by incorporating both equipment and supply uncertainties. The study's focus is on a copper mining complex, where the framework demonstrates its ability to effectively integrate both equipment and supply uncertainties. To account for equipment productivity uncertainty, Monte Carlo simulations are used based on historical production data, ensuring a more accurate representation. By integrating equipment uncertainty alongside geological uncertainty, the optimizer adheres to the simulated equipment capacity, resulting in a practical schedule. The framework optimizes extraction sequences, destination policies, and processing stream decisions concurrently, enhancing overall efficiency. The stochastic schedule generated by this approach yields a 5% increase in copper production for mills in the first year. However, it is essential to note that risk profiles exhibit greater fluctuations across multiple periods compared to the conventional schedule. Nonetheless, the Leach Pads also display higher copper production in later years. Moreover, the stochastic schedule effectively manages risks associated with equipment productivity, leading to a 2% higher Net Present Value (NPV) compared to the conventional schedule. Furthermore, it showcases improvements in waste management during earlier years. Overall, this chapter demonstrates the value of incorporating equipment and supply uncertainties into the optimization process, providing valuable insights and more robust decision-making for mining complexes.

The second application expands the simultaneous stochastic optimization framework for mining complexes by incorporating recovery and market uncertainties in addition to supply uncertainty. The application is demonstrated in a copper mining complex, showcasing the framework's impressive ability to seamlessly integrate supply, recovery, and market uncertainty. By optimizing extraction sequences, destination policies, and processing stream decisions simultaneously, the framework provides comprehensive solutions. The incorporation of recovery uncertainty involves simulations of recovery curves based on historical production data using Monte Carlo simulations of the curve model parameters. Market uncertainty is captured through commodity price simulations based on the mean-reverting process. By integrating recovery and market uncertainties alongside geological uncertainty, the optimizer generates diverse production schedules, each reflecting different combinations of incorporated uncertainties. The market case, recovery case, and joint case schedules lead to cumulative copper production increases of 2.8%, 3.8%, and 7.0%, respectively. Moreover, they result in significantly higher Net Present Values (NPV) of 8.5%, 7.8%, and 12.5% compared to the baseline case. Furthermore, the incorporation of recovery uncertainty leads to production schedules with more stable recovery rates, as observed in the recovery case and joint case schedules. This highlights the profound impact of different components' configurations within a mineral value chain on the production schedule. Importantly, it is emphasized that the incorporation of multiple sources of uncertainty should not be viewed as a mere sum of uncertainties. Instead, the joint effects and interactions with specific mineral value chains need to be carefully considered for a comprehensive analysis. This research provides valuable insights into effectively managing uncertainties and optimizing production schedules for mining complexes, particularly when dealing with recovery and market uncertainties in addition to supply uncertainty. The framework's versatility and robustness contribute significantly to decision-making processes in the mining industry.

Both of these studies underscore the significance of considering the entire mining complex while effectively managing and quantifying different sources of uncertainty. The simultaneous stochastic optimization framework (Goodfellow and Dimitrakopoulos, 2016, 2017) possesses a highly generalized nature, enabling the modeling of multiple downstream value chain components. Different sources of uncertainty affect different aspect of the mining complex depending on the mining complex at hand. It reveals the importance thoroughly analyzing the operational aspects of a mining complex and incorporating uncertainty modeling tailored to the specific problem at hand.

4.2 Future work

Some possible future work could be first to consider haul cycles. Instead of simulating tonnage productivity and cost structure, blocks that are deeper and further away could consume more truck operating hour and has higher mining cost. Another possible future work is to include the option of capital investment for extra-crusher. As presented in previous sections, the productivity of the mining complex is primarily constrained by the crushers for mine 1. Such options could provide the evaluation of whether or not is worth it to invest extra crusher to release that constraint. Options for operating modes for the mill could also be included. For example, operating modes with higher recovery but lower throughput. These options could provide opportunities to explore different configurations and exploit the synergies amount different components to increase the economic value of the mining complex. Additional case studies and comparisons should also be conducted to understand the effect of different equipment configurations on mining operations. One avenue is to explore alternative recovery simulation methods. For instance, instead of assuming independence, the correlation between different recovery curve parameters could be incorporated during the simulation. This approach may yield more precise and realistic representations of recovery uncertainties. Furthermore, extending the research to encompass additional case studies and comparisons involving mining complexes with diverse configurations would be highly beneficial. By analyzing various mine setups and their influence on production schedules when recovery and market uncertainties are integrated, we can gain deeper insights into the framework's adaptability and effectiveness. These comparative studies would enhance our understanding of how the optimization approach performs under different mining scenarios, contributing to more informed decision-making in the mining industry. Lastly, it is crucial to develop more advanced optimization algorithms capable of efficiently handling the incorporation of additional uncertainties and the increasing complexity of mining complexes

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