Tonality and Transposition in the Seventeenth-Century Trio Sonata

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Abstract

This dissertation compares snapshots of seventeenth-century harmonic practice in 142 instrumental works by three northern Italian composers: Girolamo Frescobaldi (1583–1643), Giovanni Legrenzi (1626–1690), and Arcangelo Corelli (1653–1713). All three composers wrote large amounts of music in trio-sonata texture, i.e., two melody instruments plus *basso continuo*. I explore how tonality is expressed in this iconic genre and how that expression changes over the course of the seventeenth century. I group each composer's works by tonality (according to final and signature) and use custom-built music analysis software to observe the distribution and syntactic behaviour of chords in each tonality. I compare statistical representations of chord distributions (histograms) and chord progressions (n-grams) to determine how closely tonalities resemble one another both within and between the oeuvres of the three composers, working under the assumption that some tonalities are transpositions of others.

This project engages with existing research on seventeenth-century conceptions of mode and key (Allsop 1999; Barnett 1998, 2002, 2008; Bonta 1984; Dodds 1999; Lester 1977, 1989; Long 2020; Pedneault-Deslauriers 2017; Powers 1998; Stein 2002) and provides new empirical input by looking at how tonalities can be differentiated using their chord content. By examining chord progressions, I am responding to Lester, who calls for a "study of seventeenth and early eighteenth-century theories of harmonic progression" (Lester 1989). My work also engages with data-driven studies of tonality (Albrecht & Huron 2012, 2014; Quinn 2010; Quinn & White 2013; Tompkins 2017; White 2015), applying tested methodologies to new repertoire.

Throughout the project, I wrestle with the notion of the incomplete key signature, which has been problematised by several theorists including Allsop (1999) and Barnett (2002, 2008). My findings corroborate theories of key in French and English music put forward by Pedneault-Deslauriers (2017) and Long (2020), respectively. I show that while signatures that we might call incomplete often do appear in bona fide major or minor tonalities (e.g., G major with an empty signature), there are still measurable differences between some incomplete keys and their "complete" forms (e.g., between G minor with one signature flat and G minor with two signature flats).

Ultimately, this dissertation shows that while the number of available finals increases over the course of the seventeenth century, and the number of distinct tonalities (defined by chord content) decreases, there is still not a universal major or minor mode by the time of Corelli, whose tonalities are not perfect transpositions of one another. We observe that all three composers have different conceptions of tonality and that each of them seeks to differentiate the tonalities they use in a unique way.

Résumé

Cette thèse compare des instantanés de la pratique harmonique du XVIIe siècle dans 142 œuvres instrumentales de trois compositeurs du nord de l'Italie : Girolamo Frescobaldi (1583–1643), Giovanni Legrenzi (1626–1690) et Arcangelo Corelli (1653–1713). Les trois compositeurs ont écrit de grandes quantités de musique dans une texture sonate-trio : deux instruments mélodiques plus la basse continue. J'explore comment la tonalité s'exprime dans ce genre caractéristique et comment cette expression change au cours du XVIIe siècle. Je regroupe les œuvres de chaque compositeur par tonalité (selon la finale et l'armure) et j'utilise un logiciel d'analyse musicale créé sur mesure pour observer la distribution et le comportement syntaxique des accords dans chaque tonalité. Je compare des représentations statistiques de distributions d'accords (histogrammes) et de progressions d'accords (n-grams) pour déterminer à quel point les tonalités se ressemblent à la fois dans et entre les œuvres des trois compositeurs, en partant du principe que certaines tonalités sont des transpositions d'autres.

Ce projet s'inscrit dans le cadre de recherches existantes sur les conceptions du mode et de la clé au XVIIe siècle (Allsop 1999; Barnett 1998, 2002, 2008; Bonta 1984; Dodds 1999; Lester 1977, 1989; Long 2020; Pedneault-Deslauriers 2017; Powers 1998; Stein 2002) et fournit de nouvelles données empiriques en examinant comment les tonalités peuvent être différenciées à l'aide de leur contenu harmonique. En examinant les progressions d'accords, je réponds à Lester, qui appelle à une « étude des théories de la progression harmonique du XVIIe et du début du XVIIIe siècle » (Lester 1989). Mon travail implique également des études de la tonalité guidées par les données (Albrecht & Huron 2012, 2014; Quinn 2010; Quinn & White 2013; Tompkins 2017; White 2015), en appliquant des méthodologies testées à un nouveau répertoire.

Tout au long du projet, je suis aux prises avec la notion de l'armure incomplète, qui a été problématisée par plusieurs théoriciens dont Allsop (1999) et Barnett (2002, 2008). Mes découvertes corroborent les théories de la tonalité dans la musique française et anglaise proposées par Pedneault-Deslauriers (2017) et Long (2020), respectivement. Je montre que si les armures que nous pourrions qualifier d'incomplètes apparaissent souvent dans des tonalités majeures ou mineures véritables (par exemple, sol majeur avec une armure vide), il existe encore des différences mesurables entre certaines tonalités incomplètes et leurs formes « complètes » (par exemple, entre sol mineur avec une armure d'un seul bémol et sol mineur avec une armure de deux bémols).

Enfin, cette thèse montre que alors que le nombre de finales disponibles augmente au cours du XVIIe siècle et que le nombre de tonalités distinctes (définies par le contenu harmonique) diminue, il n'existe toujours pas de mode majeur ou mineur universel à l'époque de Corelli à qui les tonalités ne sont pas des transpositions parfaites les unes des autres. On observe que les trois compositeurs ont des conceptions différentes de la tonalité et que chacun cherche à différencier les tonalités qu'ils utilisent de manière unique.

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1. Introduction

During my second year as a master's student at Western University, I took a course in Schenkerian analysis. For our final project we were allowed to analyse a tonal piece of music of our choice, as long as it was written by a composer between Corelli and Brahms. I chose to analyse a piece by Haydn, but as a violinist having played lots of music by Corelli, one question stayed with me: why Corelli? What makes him the first "tonal" composer?

Arcangelo Corelli (1653–1713) was famously cited as the first tonal composer by Manfred Bukofzer in 1947 (Bukofzer 1947). Bukofzer writes that "Corelli… was the first to put the tonal formulas to systematic use" (Bukofzer 1947, 408) and that "Arcangelo Corelli (1653–1713) can take the credit for the full realization of tonality in the field of instrumental music." (Bukofzer 1947, 410). In his 1999 biography of Corelli, Peter Allsop writes that

> it is highly improbable that Corelli would have regarded himself as the inaugurator of the 'tonal' system in Bukofzer's terms. As Noske remarks, 'Could it be that Bukofzer, despite his careful wording, allowed himself to apply an anachronistic concept of tonality to Corelli's music? Whatever the reader's verdict, it is surely right to challenge, if only to qualify, the received opinion that Corelli established "full" tonality and functional harmony.' (Allsop 1999)

My dissertation aims to qualify exactly this—what Frits Noske refers to as "the received opinion that Corelli established 'full' tonality and functional harmony." (Noske 1982).

To determine whether or not Corelli was really the "first" composer to write fully tonal music, I will compare him with two earlier composers: Girolamo

Frescobaldi (1583–1643) and Giovanni Legrenzi (1626–1690). To make comparisons as fair as possible, I have chosen composers from the same geographical location (northern Italy) who all wrote large amounts of music in trio-sonata texture, i.e., two melody instruments plus *basso continuo*. My corpus comprises 142 instrumental works.

1.1. Tonal Practice

In order to determine whether or not Corelli's music is really "tonal," we need to know what qualifies music as tonal. According to Bukofzer,

> Tonality established a gradated system of chordal relations between a tonal center (the tonic triad in major or minor) and the other triads (or seventh chords) of the diatonic scale. None of these chords was in itself new, but they now served a new function, namely that of circumscribing the key. (Bukofzer 1947, 406)

And according to Carl Dahlhaus, tonal harmony is characterised by (1) the use of the chord as the primary, indivisible musical unit; (2) a well defined syntactical relationship between chords, relative to a single tonal centre; and (3) the treatment of intervallic dissonances according to their membership in different chords (Dahlhaus 2001).

Both of these definitions focus on the same musical object: the chord. Because chords are integral to our conception of tonal harmony, I take the chord as the primary unit of musical information in my analyses. To allow for comparison on a very large scale, my analyses are carried out with the help of custom-built software that is designed to detect chords in seventeenth-century trio sonatas and canzonas.

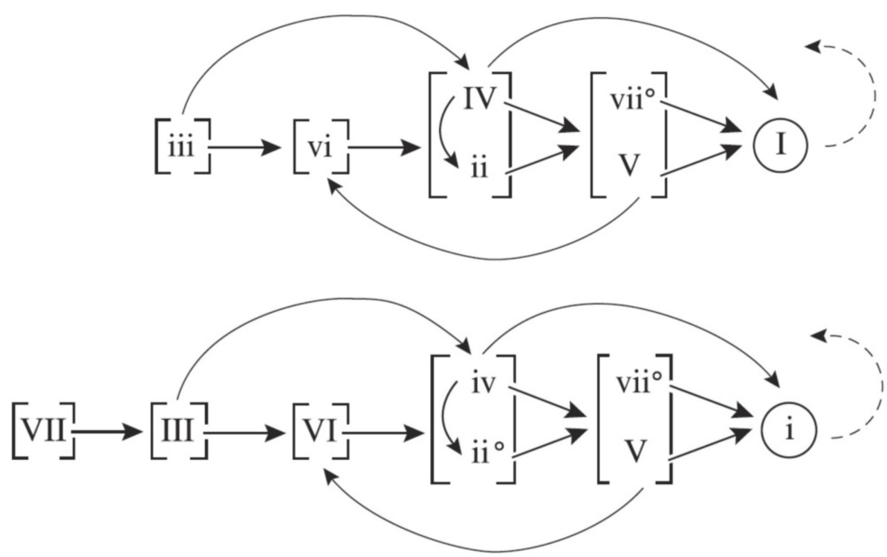
One may wonder about the analytical validity of chords in music as early as Frescobaldi's, which is not generally accepted as tonal. Although chords are a prerequisite for tonal harmony, they are not exclusive to tonal music; scholarly work by Thérèse de Goede-Klinkhamer shows that the notion that a bass line (figured or not) automatically implies chordal harmony was not foreign to Frescobaldi and his contemporaries (de Goede-Klinkhamer 1997). De Goede-Klinkhamer tells us that accompaniment treatises by Agazzari (1607), Banchieri (1607), and Bianciardi (1611) all begin with the same general rule: play consonances on every bass note (de Goede-Klinkhamer 1997, 85). She also asserts that, because of their knowledge of counterpoint, good organists (like Frescobaldi) would have been able to accompany motets and psalm settings just by looking at an unfigured bass line (de Goede-Klinkhamer 1997, 83).

Accepting chords as the "primary, indivisible musical unit," in our chosen repertoire, we now ask: what is it that chords are supposed to *do* in tonal music? According to Bukofzer and Dahlhaus, chords are supposed to relate to "a single tonal centre," or "circumscribe the key." From this, we understand that chords have a *standard* relationship to their tonal centre, i.e., no matter what the tonal center is, "the other triads of the diatonic scale" should have some *fixed* relationship to it.

To quantify the relationship between the triads of a diatonic scale and the tonal centre of that scale, and to determine whether or not Corelli differs from his predecessors in this regard, I look at each tonal centre used by each composer to see if the triads around it relate to it in a predictable way. I define the "diatonic triads" of a piece as the triads that occur using only the natural notes plus any alterations indicated in the signature of the piece. I define the "tonal centre" of a piece as the diatonic triad that has the piece's final as its root. I operationalize the "relationship between diatonic triads and tonal center" as statistical distributions of both individual chords and pair-wise chord progressions represented using Roman numerals (i.e., distances above the final).

Some readers may wonder about the significance of chord progressions that are only two chords long, believing that longer spans of music are necessary in order to determine whether or not harmony is tonal. Recently, Megan Long has shown how tonality is dependent on expectations created by harmonic and metrical regularity in small forms around the turn of the seventeenth century. Long asserts that "tonal expectation is a feature of large-scale harmonic frameworks rather than surface-level chord syntax" (Long 2020, 18). While Long argues against the importance of surface-level chord syntax, she also acknowledges that it is important to "narrow our focus to consider single panels in the tapestry of tonality and tug on the particular threads that combine to form one compelling figure" (Long 2020, 3). The threads that I am tugging on in this project are surface-level chord syntax and total chord content. I do not argue that pair-wise chord progressions create tonal expectation in the music in my corpus, nor that we are able to hear or expect harmony in relation to the total chord content of a musical work, but I do believe that both of these are reliable markers of tonality.

I am not the first to treat pair-wise chord progressions as markers of tonality: in their well-known textbook *Tonal Harmony*, Stefan Kostka, Dorothy Payne, and Byron Almén attest to the importance of pair-wise chord progressions in their Roman-numeral flowcharts, one for major keys and one for minor keys, each of which comprises roughly ten different pair-wise chord progressions (Kostka, Payne, and Almén 2018, 105). Their flowcharts are shown in Example 1.1 (below) and are discussed in greater depth in Sections 2.9.2 and 3.3.



Ex. 1.1. The Kostka-Payne-Almén flowcharts showing standard chord progressions in major keys (above) and minor keys (below)

An important distinction between my work and the theories presented in *Tonal Harmony* is that I do not treat certain progressions as acceptable and others as abnormal: whereas the flowcharts shown below are prescriptive, the flowcharts that I produce in Sections 3.3.3, 4.3.4, and 5.3.3 are completely empirical.

Instead of a key or a mode (both of which have theoretical implications that I wish to avoid), I say that each piece in my corpus is in a "tonality," which is defined only by its final, its signature, and the quality (major or minor) of the diatonic triad having the piece's final as its root. For example, d/♮ signifies a tonality with a final of D, an empty signature, and a minor triad (indicated by the lower-case letter) built on the final, while F/♭ signifies a tonality with a final of F, a single flat in the signature (i.e., B♭) and a major triad built on the final.

If chords in tonal music have a *standard* relationship to their tonal center, and if Corelli's music is the most tonal of the three composers, then the chords in Corelli's tonalities should relate to their finals in a more predictable way than the chords in Legrenzi's and Frescobaldi's tonalities do. This implies that Corelli's tonalities should be more similar to *one another* than the tonalities of his predecessors are to one another.

What the results of my study show is that a higher degree of internal similarity *does* appear among Corelli's tonalities. While distributions of chords and chord progressions follow as many as five or six different paradigms in the music of the two composers before Corelli, the number of different paradigms in Corelli is essentially reduced to two: one for tonalities having a major triad built on their final and one for tonalities having a minor triad built on their final. The different paradigms that appear among the tonalities of the two earlier composers correlate with the different species of octave that appear above the finals of those tonalities, as defined by the signatures of those tonalities. In Corelli, we find that the paradigm that a tonality follows does not correlate very closely with its signature-defined octave species, but instead correlates with the quality of the triad built on the tonality's final (major or minor).

The other substantial change that we observe within the corpus is that as we approach Corelli and tonalities become more internally similar, the number of different finals also increases: Frescobaldi uses only six different finals (F, C, G, D, A, and E), while Legrenzi uses seven (C, G, D, A, E, B, and B♭) and Corelli uses ten (E♭, B♭, F, C, G, D, A, E, B, and F♯). What this implies is that some of Corelli's tonalities must be transpositions of others. Example 1.2 (below) tabulates all of the tonalities appearing in my corpus, organized according to composer and signature-defined octave species. A comprehensive list of all of the works that comprise my corpus is given in Appendix 1.

Composer (finals, tonalities)	**Octave Species**					
	lydian	major	mixolydian	dorian	minor	phrygian
Frescobaldi (6, 10)	F/♮	C/♮, F/♭	G/♮, C/♭	d/♮, g/♭	a/♮, d/♭	e/♮
Legrenzi (7, 15)	B♭/♭	C/♮	G/♮, D/♯, A/♯♯, c/♭	d/♮, g/♭, c/♭♭, e/♯♯	a/♮, e/♯, b/♯♯, g/♭♭	e/♮
Corelli (10, 18)	B♭/♭, E♭/♭♭	C/♮, F/♭, D/♯♯	G/♮, A/♯♯, E/♯♯♯, f/♭♭	d/♮, g/♭, c/♭♭	d/♭, a/♮, e/♯, b/♯♯, f♯/♯♯♯	e/♮

Ex. 1.2. Table of tonalities (final/signature) appearing in my corpus

Following the practice of Beverly Stein (Stein 2002), I keep the Greek names for the octave species in lower case so as not to imply the authentic modes of the same names in either the octenary or dodecachordal system of church modes. I also use "major" and "minor" instead of "ionian" and "aeolian," respectively. Stein's work on tonality in the first half of the seventeenth century is discussed in Section 3.1.

Another useful concept in categorizing seventeenth-century tonalities is the distinction we can make between *Ut*, *Re*, and *Mi* tonalities, as explained by Cristle Collins Judd (Collins Judd 1992). This distinction is essentially the same as the one that we make between tonalities whose finals support major triads (Ut tonalities) and tonalities whose finals support minor triads (Re tonalities), with the addition of the Mi category, for tonalities whose finals support minor triads and in which the first interval above the final is a half step. Collins Judd shows that this relatively simple three-part categorization goes back at least as far as Heinrich Glarean, who says that "The same men teach in this way concerning modes: Every song ends either on *re* or on *mi* or on *ut*" (Glarean 1574, 70). My results show that the tonalities used in seventeenth-century Italy can be understood to fall into these same three categories.

I have mentioned modes and keys, which are often juxtaposed as the two ends of a continuum that spans the seventeenth century. This is a misleading assumption that I hope to eschew by using the more general term "tonality." To fully understand the nature of tonality in the seventeenth century, however, we must also familiarize ourselves with a third system of tonal categorization that was used: what Joel Lester refers to as the "church keys" (Lester 1989) and Harold Powers calls "psalm tone tonalities" (Powers 1998). Of these, Gregory Barnett writes,

> The theoretical and practical significance of this third, psalmodically based system perhaps outweighs any other conception of *tuono* during the seventeenth century. The eight tonalities that arose as accompaniments or substitutes for the eight psalm tones… not only form the basis for the early eighteenth-century notion of keys, but also shape tonal practices particular to the seventeenth century far more substantially than did the modes. (Barnett 2002, 419–20)

The eight church keys, which were first laid out by Adriano Banchieri (Banchieri 1614, 71), are shown in Example 1.3 (below). The reader will notice that some octave species are better represented than others, namely the dorian, major, and minor, while the lydian octave species does not appear at all.

Church Key	Final/Signature	Octave Species
1	d/♮	dorian
2	g/♭	dorian
3	a/♮	minor
4	e/♮	phrygian
5	C/♮	major
6	F/♭	major
7	d/♭	minor
8	G/♮	mixolydian

Ex. 1.3. The eight church keys according to Banchieri

Barnett is a strong proponent of the church keys and their explanatory power for the tonalities that appear in collections of published music by Italian composers of the seventeenth century. He even goes as far as to say that "the so-called incomplete key signatures that proliferated during the seventeenth and early eighteenth centuries… may be properly understood as church keys or their transpositions rather than as incomplete signatures of as yet unrecognized major or minor keys" (Barnett 2002, 428–29). With the methodology I use in this project, Barnett's claim becomes testable: we can determine the extent to which tonalities with "incomplete key signatures," such as c/♭♭, behave like the tonalities of which they are purportedly transpositions, in this case, d/♮ and/or g/♭. What we find is that, while some tonalities really do behave like transpositions of others, many tonalities with "incomplete signatures" behave as if their signatures really are incomplete, i.e., their most likely transpositional relatives are tonalities that have different signature-defined octave species, or "complete" signatures.

Some of the tonalities in my corpus are difficult to compare with any of Banchieri's church keys, e.g., Frescobaldi's F/♮ tonality and the B♭/♭ tonalities that appear in Legrenzi and Corelli. Barnett gives historical rationale for these tonalities, citing Bononcini, and explains that they have "irregular key signature[s] for the transposition[s] indicated" (Barnett 2008, 267). Here I argue that Barnett contradicts himself somewhat, by arguing that tonalities with incomplete key signatures are transpositions of the church keys (and not of major or minor keys), but also that some tonalities use "irregular" key signatures compared to the tonalities of which they are transpositions.

I do not discount the importance of the church keys, much less of transposition, as forces that shaped tonal practice in the seventeenth century, but one of my aims in this project is to show that key signatures do, at some point, begin to look incomplete when we consider tonalities from an empirical perspective. By the time we reach Corelli, many of the tonalities with lydian and mixolydian signature-defined octave species behave more or less as if their octave species were major. There is always a greater distinction to be made between dorian and minor tonalities, however, even in Corelli. The notions of incomplete signatures and irregular transpositions are further discussed in Section 5.1 as they relate to the tonalities used by Corelli.

On the topic of incomplete key signatures, Julie Pednault-Deslauriers has contributed a detailed summary from the perspective of seventeenth-century French music theory (Pedneault-Deslauriers 2017). Her careful discussion of the different factors that motivated composers and theorists to use complete signatures (or not) helps us understand the implications of some of the stranger signatures in Legrenzi and Corelli. Pedneault-Deslauriers shows that French theorists around the turn of the eighteenth century understood the Re tonalities as a single category, but that the signatures for Re tonalities were only just beginning to be standardized—it was not until around 1730, for example, that the signatures of flat-side minor keys consistently included the flat inflecting the sixth degree of the scale, e.g., E♭ in g/♭♭ or A♭ in c/♭♭♭ (Pedneault-Deslauriers 2017, 5.5). Pedneault-Deslauriers's work helps us understand that signature completeness, especially with regard to the sixth scale degree in Re tonalities, was not a priority, even as late as the first two decades of the eighteenth century. My results corroborate this notion, showing that signature-defined octave species is no longer a good predictor of either the total chord content or the pair-wise chord progressions that appear in Re tonalities around the turn of the eighteenth century. Pedneault-Deslauriers's work is discussed again in Section 4.1 as it relates to the tonalities used by Legrenzi.

In an article titled "What do Signatures Signify?", Megan Long discusses the tendency for signatures that we would call "incomplete" to acquire their "missing" signature flats or sharps through a process called "signature creep" in seventeenth-century English music. In this process, a pre-existing Re tonality such as g/♭ "gains an additional flat that reflects the consistent lowering of scale 6 in practice" and a new final/signature combination is produced: g/♭♭. This new signature then makes other tonalities possible: c/♭♭ and B♭/♭♭ (Long 2020b). While B♭/♭♭ does not appear in my corpus (Legrenzi and Corelli both use B♭/♭), a third option for a new final in the two-flats signature appears in Corelli: f/♭♭. Long's work helps us understand that Corelli's signatures are still very much in the process of "creeping."

Another scholar to whom I am responding in this project is Joel Lester, who calls for a survey of "seventeenth- and early eighteenth-century theories of harmonic progression" (Lester 1978). Lester's work focuses on major and minor tonality in German music from the seventeenth and early eighteenth centuries and

I see my research as a first step towards understanding harmonic progressions in Italian music from the same time period. Lester discusses seventeenth-century theoretical conceptions of tonal organization, focusing on the various groupings, orderings, and transpositions of the 12 church modes that eventually led German theorists to propose the system of 24 major and minor keys. My work differs from Lester's in that I observe the beginnings of major-minor tonality empirically.

1.2. Music Information Retrieval

This project provides a new perspective on seventeenth-century tonal practice by analysing a large corpus of instrumental music using custom-built music analysis software. With this approach, I am able to provide empirical evidence in support of existing scholarship on seventeenth-century tonal practice for the first time.

McGill University has a strong community of faculty and graduate students working in the field of music information retrieval (MIR). This community fostered my interest in automated harmonic analysis, ultimately leading to my decision to include it in my dissertation. In a previously published study of the French motet repertoire ca. 1300–1350 (Desmond et al. 2020), my co-authors and I devised a way of classifying sonorities algorithmically using vertical intervals. The algorithm explained in Chapter 2 is based on that work. My work also builds on the ideas and methods of other scholars working in the field of MIR.

In an article written in 2014, Joshua Albrecht and David Huron trace changes in the major and minor modes (what I call the Ut and Re categories) by looking at the distributions of individual scale degrees in music from 1400 to 1750. Using agglomerative hierarchical clustering (a method I explain and employ in Section 3.2.1), they cluster together the works that have similar distributions of scale degrees and find that there is one relatively stable distribution of scale degrees for major-mode works from about 1450 to 1750 (only between 1400 and 1450 do they find a second major-mode distribution), while for minor mode works they find that there are two stable distributions between 1400 and 1600 and that these two distributions collapse into a single stable distribution that persists from 1600 to 1750.

The results presented by Albrecht & Huron make sense, broadly speaking, but at first glance they do not appear to be compatible with my own results. I find that differences between the two kinds of Re tonalities (which I call dorian and minor) are observable in Frescobaldi (ca. 1620), and that these do not completely disappear by the time we reach Corelli (ca. 1690). The differences between their findings and mine likely have to do with differences in methodology: while Albrecht & Huron use pitch classes as their base unit of musical information, I use chords. This suggests that harmonic changes (i.e., changes pertaining to chord vocabulary and syntax) to the Re category of tonalities happened later compared to the changes that occurred to the distributions of individual pitch classes.

A 2018 article by Christopher Wm. White and Ian Quinn discusses chord function in three different corpora of tonal music, one of which is the Bach chorales (White and Quinn 2018). White and Quinn use hidden Markov models (HMMs) to determine how many harmonic functions there are in tonal music, challenging the traditional three-function model. Although I do not discuss harmonic function, this work was influential for me because White and Quinn found that allowing "dissonances and non-triadic structures" into their models (in addition to chords) actually produced tighter clusters. A good example is what they call the "1, 2, 4, 5" sonority, a dominant seventh chord in second inversion with scale degree 1 substituting for scale degree 7 in an inner voice. White and Quinn found that this sonority accounts for more than half of all the sonorities that fill in the progression $\mathrm{I}\rightarrow ?\rightarrow \mathrm{I}^6$ in the Bach chorales.

My harmonic analysis algorithm was designed with non-triadic sonorities in mind. I make an effort to identify dissonant sonorities that are common in seventeenth-century trio sonatas: 5/4, 7/4, 9/4, 9/7, 9/3, and 5/2 sonorities number among the sonorities that I consider. After running my analyses, I find that 5/4 sonorities are actually the fourth most common sonority used by all three composers in my corpus (after major triads, minor triads, and sonorities comprising open fifths and/or octaves).

Another scholar who uses HMMs to determine the number of harmonic functions in different corpora is Daniel Tompkins (Tompkins 2017). In his dissertation, Tompkins examines harmonic practice in seventeenth-century guitar music written in *alfabeto* tablature. His methodology overlaps substantially with mine (as I discuss in Section 2.9.2): his data comprises distributions of individual chords (represented as Roman numerals) as well as pair-wise chord progressions, which he represents using flowcharts (Tompkins 2017, 98), but his premise is fundamentally different from mine. Tompkins does not distinguish between different octave species when subdividing his corpus according to mode: instead, he treats all pieces as either major or minor. While it is true that the Ut and Re categories are relevant throughout the seventeenth century, there are changes happening within these categories that I am able to pin down. I provide a more granular analysis, showing that the Ut tonalities are more closely related to one another than the Re tonalities.

1.3. Summary of Findings

There are two main findings that come out of my work, both of which suggest that Corelli may not be the best candidate for "first tonal composer." The first of these findings is that there is still not a standard version of the minor mode by the time of Corelli. While tonalities in the Ut category are more uniform (i.e., more like transpositions of one another), the Re tonalities still fall into two sub-categories: dorian and minor.

The second of these findings is that Corelli does not make a substantial break from his more immediate predecessor, Legrenzi. The differences between

Frescobaldi and Legrenzi, both in terms of chord distributions and pair-wise chord progressions, are significantly greater than the differences between Legrenzi and Corelli. This does not weaken Corelli's claim as "first tonal composer" per se, but it does reorient us substantially. We now know that the change that occurred over the course of the seventeenth century was not linear: more change happened in the first half of the century than in the second half.

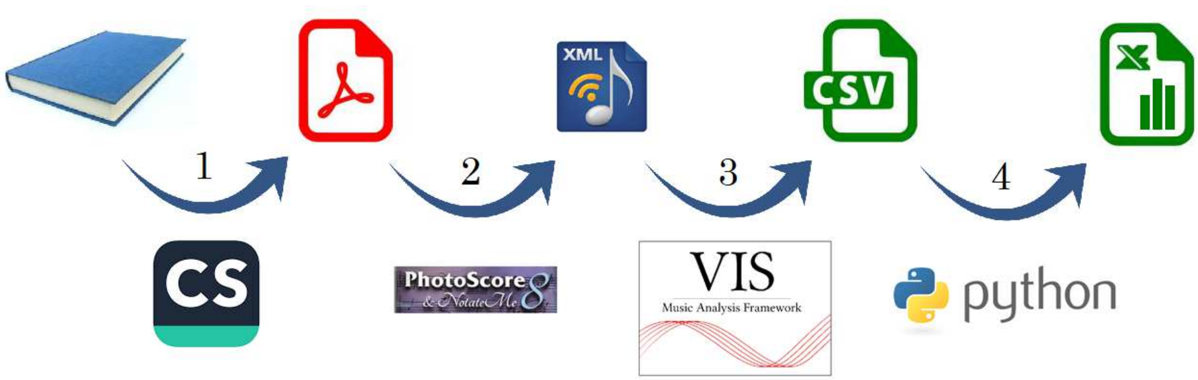
We also learn that certain types of tonalities, according to signature-defined octave species, undergo different amounts of change. The dorian tonalities change more over the course of the century than the minor tonalities do. What this means is that the dorian tonalities gradually become more like minor tonalities, while the minor tonalities remain relatively stable. The same can be said for the mixolydian and major tonalities, but the difference in the amount of change is smaller when we compare these two octave species.

While many of my results suggest clear trends (not always linear) over the course of the century, some of my results point to idiosyncrasies of individual composers or tonalities within the corpus. For example, certain tonalities with the same octave species in Legrenzi can sometimes have significant differences in their distributions of individual chords. Similarly, certain chords and chord progressions in Frescobaldi remain common regardless of the tonality. These results are discussed in Chapters 3, 4, and 5, as well as in the Conclusions section.

2. Methodology

2.1. Workflow Overview

The analytical work for this project was carried out in four steps, which are illustrated in Example 2.1 (below). All four steps were carried out for one composer before proceeding to the next composer.



Ex. 2.1. Analysis proceeded in four steps: (1) scanning printed music using CamScanner to obtain PDF files; (2) OMRing PDF files using PhotoScore to obtain symbolic music files (XML); (3) analysing XML files using web-based music-analysis software; (4) analysing the results of the previous step using Python scripts to obtain data in Excel spreadsheets

2.2. Scanning and OMR

Pictures of scores (mostly collected works) were taken using CamScanner (Step 1 in Example 2.1), an image-to-PDF smartphone app with colour correction. PDFs were then OMRed using PhotoScore, the music-scanning software included with Sibelius (Step 2 in Example 2.1). For PhotoScore, electronically generated PDFs (e.g., canzonas from Frescobaldi's *Primo libro delle canzoni* available on IMSLP) were much easier to OMR than PDFs scanned using CamScanner (e.g., all of the Legrenzi sonatas).

Within PhotoScore, considerable editing (20–30 minutes per work) was required in order for the OMRed music to be exported in XML format. Once XML files were exported from PhotoScore, further corrections (5–10 minutes per work) were made using MuseScore, an open-source music-notation program. For all sonatas by Corelli, symbolic files (MIDI) were already available in online repositories (KernScores[1],CCARH musedata[2]), so the scanning and OMR steps could be skipped and MIDI files converted directly to XML using MuseScore, with minimal corrections. A few works by Frescobaldi were transcribed manually in MuseScore because high-quality images could not be obtained, making OMR impossible.

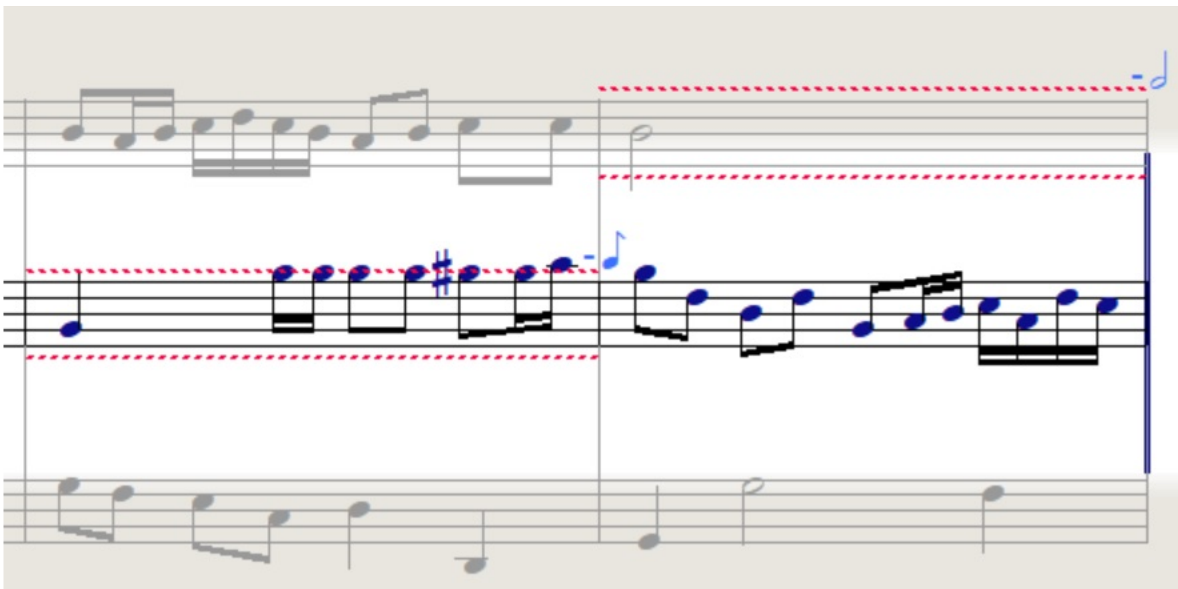
2.3. Types of Corrections

In the process of generating XML files using PhotoScore, a range of corrections was necessary. Works for which high-quality PDFs were available (e.g., files generated by notation software and uploaded to IMSLP) took much less time to correct at the PhotoScore stage. These included four canzonas from Frescobaldi's *Primo libro delle canzoni* (1628). Works for which only printed music was available took a lot of time to correct using PhotoScore before XML files could be exported. These included most of the works by Frescobaldi and all of the works by Legrenzi.

Working in PhotoScore, the main concern was rhythm. In order to export an XML file that can be interpreted by other software, every measure of the music (excluding pickups) must be "full" according to its time signature. Helpfully,

[1]http://kern.humdrum.org/search?s=t&keyword=Corelli
[2]https://github.com/musedata/corelli

PhotoScore will highlight (like a spell-checker) any sections of the music where the note values do not add up to satisfy this requirement. Measures with rhythm errors often contained other errors as well (e.g., pitch), so these were corrected simultaneously (see Example 2.2 below). Measures that were rhythmically correct, even if they contained other errors, were ignored during the PhotoScore step and corrected later in MuseScore.



Ex. 2.2. *Two incorrect measures in a Legrenzi sonata (Op. 2, no. 6, mm. 3–4)—the first quarter note in the first highlighted measure (middle staff) should be dotted and the second highlighted measure (top staff) is missing a half-note rest—the G-sharp in the first highlighted measure is also wrong: it should be an F-sharp.*

Working in MuseScore, corrections were made using a combination of visual and aural editing. Editorial dynamics, articulations, and performance instructions were removed all at once (using the "select all similar elements" command), but tempo indications were preserved (e.g., *Adagio*). Staves that provided editorial realization of the continuo were removed, e.g., those in Albert Seay's editions of

Legrenzi's Opus 4 and Opus 8 (Seay 1968; 1979), as were the staves for melodic bass instruments such as violone or bassoon. This decision was made in order to privilege the continuo part as the fundamental, undecorated version of the bass line in each composition. Accidentals and other pitch-related errors accounted for the bulk of the remaining corrections, especially when ficta notated above the staves (instead of beside the notes) were not detected by PhotoScore during OMR, as was the case for most of the music by Frescobaldi and about half of the music by Legrenzi.

Polished XML files were the main goal of these corrections, but a second file type was also used to encode the finished corpus. MSCZ files (the default file type for MuseScore) preserve more formatting information than XML files, making them easier to read (like a digital version of a printed book, where a fixed page layout helps with memory). Both types of files will eventually be made publicly available.

2.4. Analysis Software

From the finished XML files, five different features (discussed in Section 2.5.1) were extracted: offset, beat strength, bass note, vertical intervals (with quality), and horizontal intervals (without quality). These were obtained using the VIS-Framework (Antila and Cumming 2014), a suite of music-analysis software developed at McGill University under the leadership of Prof. Julie Cumming (Step 3 in Example 2.1). The VIS-Framework came out of the ELVIS project,[3] a larger digital humanities initiative for which Prof. Cumming was the Principal Investigator. The version of the VIS-Framework that was used (which we called

[3]https://diggingintodata.org/awards/2011/project/electronic-locator-vertical-interval-successions-elvis-first-large-data-driven

"The Rodan Client") was developed in Prof. Ichiro Fujinaga's research lab, DDMAL (Distributed Digital Music Archives & Libraries Lab; https://ddmal.music.mcgill.ca/) by Ryan Bannon, and included a GUI (Graphical User Interface) that made the analysis tasks much easier to carry out in batches. Data for the five extracted features were saved in CSV files, which were then analysed using my own Python scripts `tblmkr.py` and `continuo.py` (Step 4 in Example 2.1).

2.5. Features of the Data

Example 2.3 (below) shows the first 26 rows of a table generated by `tblmkr.py` for the final movement of Legrenzi's Op. 8 no. 6, *La Bevilacqua*. Each row of the table is a "slice" of the music, that is, a snapshot of all the voices sounding at a given point in time. Whenever there is an attack in at least one voice, the vertical sonority at that moment is registered as one slice, no matter how many voices are sounding or resting. Each column of the table is a "feature," either extracted by the VIS-Framework (offset, beat strength, bass note, vertical intervals with quality, and horizontal intervals without quality) or generated by `tblmkr.py`. The three features generated by `tblmkr.py` (measure number, dissonant-ness, figured-bass class) will be discussed in section 2.5.2.

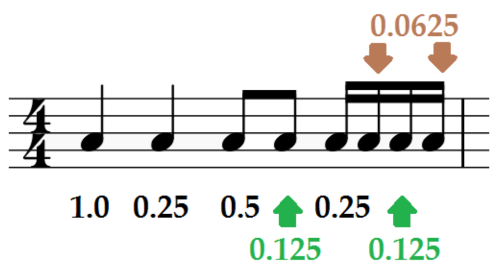
2.5.1. Features Extracted by the VIS-Framework

Offset (Column A) is a number that refers to a specific time point in the music, measured in quarter notes. The first slice occurs at offset 0, the slice two quarter notes after that occurs at offset 2, the slice one eighth note after that occurs at offset 2.5, and so forth. If there is a pick-up note, that note occurs at offset 0 and subsequent slices are measured from the pick-up, not the first downbeat. Offset values are used to calculate the lengths of slices and eventually of chords.

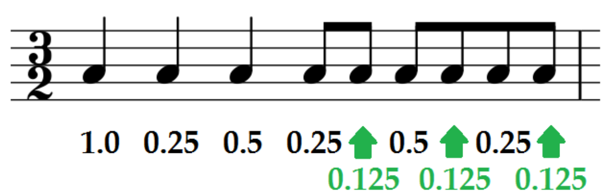
	A	B	C	D	E	F	G	H	I
1	Offset	Measure	BeatStr	BassNote	VertIvls	FBC	Diss	HorizAp	HorizLv
2	320	73	0.5	Rest	['Rest', 'Rest']	__	B	[3, 'Rest', 'Rest']	[-5, 'Rest', 'Rest']
3	321	74	1	E-3	['M3', 'Rest']	3	C	[-5, 'Rest', 'Rest']	[2, 0, 0]
4	322.5	74	0.25	E-3h	['A4', 'Rest']	__	D	[2, 0, 0]	[2, 0, -4]
5	323	74	0.5	B-2	['P8', 'Rest']	8	C	[2, 0, -4]	[-2, 'Rest', 2]
6	324	75	1	C3	['M6', 'Rest']	63	C	[-2, 'Rest', 2]	[-2, 'Rest', 2]
7	325	75	0.5	D3	['P4', 'Rest']	__	D	[-2, 'Rest', 2]	[-2, 'Rest', 0]
8	326	75	0.5	D3h	['M3', 'P8']	83	C	[-2, 'Rest', 0]	[2, -5, 2]
9	327	76	1	E-3	['M3', 'M3']	3	C	[2, -5, 2]	[-4, 0, -4]
10	328	76	0.5	B-2	['M3', 'M6']	63	C	[-4, 0, -4]	[0, 2, 0]
11	328.5	76	0.25	B-2h	['M3', 'M7']	73	D	[0, 2, 0]	[8, 2, 0]
12	329	76	0.5	B-2hh	['M3', 'P8']	83	C	[8, 2, 0]	[-2, -2, 2]
13	330	77	1	C3	['P8', 'M6']	63	C	[-2, -2, 2]	[-2, -2, 2]
14	331	77	0.5	D3	['m6', 'P4']	64	D	[-2, -2, 2]	[-2, -2, 0]
15	332	77	0.5	D3h	['P5', 'M3']	53	C	[-2, -2, 0]	[2, 2, 4]
16	333	78	1	G3	['m3', 'P8']	83	C	[2, 2, 4]	[0, -4, 0]
17	334	78	0.5	G3h	['m3', 'P5']	53	C	[0, -4, 0]	[6, 8, 0]
18	335	78	0.5	G3hh	['P8', 'P5']	53	C	[6, 8, 0]	[-3, -2, -5]
19	336	79	1	C3	['m3', 'P8']	83	C	[-3, -2, -5]	[1, -2, 0]
20	337	79	0.5	C3h	['m3', 'm7']	73	D	[1, -2, 0]	[-2, -2, 2]
21	338	79	0.5	D3	['P8', 'P5']	53	C	[-2, -2, 2]	[1, 2, -5]
22	339	80	1	G2	['P5', 'm3']	53	C	[1, 2, -5]	[0, -3, 0]
23	340	80	0.5	G2h	['P5', 'P8']	53	C	[0, -3, 0]	[0, 2, 5]
24	341	80	0.5	D3	['P8', 'P5']	53	C	[0, 2, 5]	['Rest', 2, 4]
25	342	81	1	G3	['Rest', 'm3']	3	C	['Rest', 2, 4]	['Rest', -3, 0]
26	343	81	0.5	G3h	['Rest', 'P8']	8	C	['Rest', -3, 0]	['Rest', 0, -3]
27	344	81	0.5	E-3	['M6', 'M3']	63	C	['Rest', 0, -3]	[2, 'Rest', -2]

Ex. 2.3. Table generated by `tblmkr.py`*—the first 26 "slices" shown in this table account for the nine measures of music shown below the table*

Beat strength (Column C) is a value between 0 and 1 (inclusive) that is assigned to each slice according to its metrical weight in the measure, where 1 is the strongest part of a measure (downbeat) and weaker beats decrease by a factor of 2 (0.5, 0.25, 0.125, etc.). Examples 2.4 and 2.5 (below) show beat strength values for duple and triple metres. Note that in triple metres, beats two and three are weighted equally: both have a beat strengths of 0.5. Beat strength values are used to determine which slices should be combined to obtain chords.



Ex. 2.4. Beat strength values in duple metre



Ex. 2.5. Beat strength values in triple metre

Bass note (Column D) is the pitch class and octave number of the note sounding in the continuo part for the given slice. In this column, `tblmkr.py` will also append the character "h" to indicate that a bass note is "held" for more than one slice (as in the third slice in Example 2.3).

Column E is a list of **vertical intervals** measured above the bass note, reported with quality (M = major, m = minor, A = augmented, d = diminished) and with all compound intervals reduced to simple (10 becomes 3, 9 becomes 2, but 8 and 1 remain separate interval classes). For every voice sounding above the continuo in a given slice, there is one vertical interval in the list in Column E. In a work with two upper voices, the first vertical interval in the list is always the vertical interval sounding between the topmost voice (e.g., Violin I) and the continuo, even if the upper voices are crossed. If an upper voice is resting, the word “Rest” will appear in the list. If two upper voices are sounding the same pitch class, duplicate intervals will appear in the list. The list of vertical intervals, along with the bass note in column D, is used to determine the roots and qualities of chords.

Horizontal intervals are reported twice for each slice: first, the intervals of approach for each active voice (from highest to lowest) in the current slice (Column H) and second, the intervals by which those voices leave the current slice (Column I). In these two columns, 0 means no melodic motion, 1 means a step (whole step or half step), 2 means a skip of a third (any quality), and so forth. Descending melodic intervals have negative values and ascending melodic intervals have positive values. These values are used to determine how dissonances behave, e.g., whether or not sevenths resolve downwards by step.

2.5.2. Features Generated by `tblmkr.py`

In addition to tabulating the data extracted by the VIS Framework (see Section 2.5.1), `tblmkr.py` adds three more features to the table, which are obtained algorithmically: measure number, figured-bass class, and dissonant-ness.

Measure number (Column B) is calculated on the basis of beat-strength (Column C). Measure numbers begin at 1 and whenever a “1” appears in the beat-strength column (i.e., whenever a new downbeat occurs), the measure number is increased by 1. This feature was added to make the tables produced by `tblmkr.py` more readable for humans.

Figured-bass class (Column F) is a feature that is distilled from the list of vertical intervals in Column E. Just as a pitch class is a general category for many specific pitches, a figured-bass class is a general category for many specific combinations of vertical intervals measured above the bass. The combinations [P5, M3], [P5, m3], [d5, m3], [P5, P5], and [P5, Rest], for example, all belong to the figured-bass class "53." The music-theoretical logic behind the algorithm that generates this feature is explained in Section 2.7. Figured-bass class is important because subsequent processing with `continuo.py` relies heavily on this feature.

The **Diss** feature (Column G) is a label given to each slice depending on its vertical interval content (Column E). Consonant slices are labelled "C," dissonant slices are labelled "D," and slices without any vertical-interval content (i.e., only one voice sounding) are labelled "B." For this feature, any number of seconds, fourths, or sevenths (of any quality) appearing in the list of vertical intervals qualifies the slice as "D." The diminished fifth, whose quality is dissonant but whose step size (5) is understood to be consonant from a contrapuntal perspective, is not considered dissonant in this feature.

2.6. From Tables to Chord Labels

The finished tables (CSV files) are passed to the final step of the analysis workflow: the Python script `continuo.py`. Passing different subsets of the corpus to `continuo.py` is what allows us to compare those subsets. If we want to know how many G minor triads there are in all of Legrenzi's g/♭ sonatas, for example, we would pass all of four of Legrenzi's g/♭ sonatas to `continuo.py`, which counts the triads and returns aggregated results for that batch of four sonatas. If we wanted more granular results, e.g., the number of G minor triads in a single sonata, we would have to pass that sonata alone to `continuo.py`. If we wanted to know how many G minor triads there were in all of Legrenzi's 47 sonatas, we would have to pass all 47 sonatas to `continuo.py` at once.

The custom script `continuo.py` was designed with seventeenth-century instrumental music in mind. It uses information about vertical and horizontal intervals, beat strength, and the presence of dissonance to assign a chord label (root and quality) to each slice of the music. In most cases, this is as simple as combining information about the bass note, vertical intervals, and figured-bass class already tabulated for each slice, for example, "G3" in the bass with vertical intervals [M3, M6] and a figured-bass class of "63" gives an E minor triad. If a triad is complete, i.e., if it comprises three different pitch classes, we do not indicate its inversion. If, however, a triad comprises only two pitch classes, as would be the case for a bass note of "G3," vertical intervals of [Rest, M6], and a figured-bass class of "63," we indicate that this sonority is an "empty sixth" which could be either an E minor triad or an E diminished triad. For cases like these, we append a "6" to the chord label—the "6," therefore, does not denote a first inversion triad so much as it denotes an inverted triad of ambiguous quality.

In some situations, not all of this information is available within a slice of music and `continuo.py` must dig deeper to produce a satisfactory analysis. These "extra-work" situations can be separated into two cases, discussed below.

2.6.1. Harmonically Insufficient Slices

In the first case, the figured-bass class for the given slice is left blank by the algorithm in `tblmkr.py`, as in the first, third, and sixth slices in Example 2.3 (above). This happens when the combination of vertical intervals is not sufficient to produce any of the figured-bass classes that we define in Section 2.7. These slices, termed "harmonically insufficient," can sometimes be grouped together to form longer spans of music (especially when they contain only consonant intervals) and their vertical-interval contents can be added together in order to obtain less ambiguous harmonies.

2.6.2. Dissonant Slices

In the second case, the given slice has been marked "D" for dissonant (i.e., it contains some number of seconds, fourths, or sevenths of any quality) and the figured-bass class may or may not be blank, as in slices 3, 6, 10, 13, and 19 in Example 2.3 (above). In this case, `continuo.py` must check the horizontal motion of the voices (Columns H and I) to see how the dissonances behave.

For a dissonant interval like a second, we first check the horizontal interval by which the second is left. If the second is left by any interval other than a descending step, the slice in question keeps its blank figured-bass class and no chord label will be generated. If the second is left by a descending step, we then check the horizontal interval of approach. If the second is approached by a unison (i.e., if the second is "prepared"), the slice is given a figured-bass class of "9X," denoting a 9-8 suspension). Some dissonant slices will receive a figured-bass class based on the horizontal intervals by which they are approached and left, but many will remain blank.

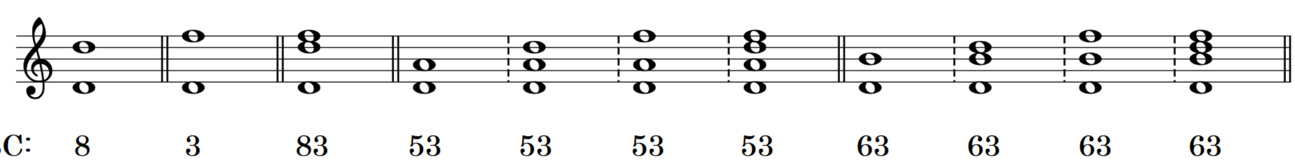
2.7. Python Script **`tblmkr.py`** and the Sixteen Figured-Bass Classes

The Python script `tblmkr.py` creates a tabular representation of a piece of music, which it outputs as a CSV file. As explained in Section 2.5, most of the features in this table are extracted by the VIS-Framework and simply collated into the table, but some are actually generated by `tblmkr.py`, most importantly, the figured-bass classes (FBCs). Example 2.6a tabulates all of conditions for the sixteen figured-bass classes that we have chosen to identify in this repertoire. Example 2.6b shows the five consonant figured-bass classes notated on a staff.

The algorithm that generates figured-bass classes is modeled on a similar algorithm developed by the author for a co-authored study of the French motet repertoire ca. 1300–1350 (Desmond et al. 2020). The algorithm assigns a figured-bass class to each slice based on the vertical intervals above the bass. If the bass crosses with one or both of the upper voices (this happens about one time in every twenty pieces), the figured-bass class will be left blank for that slice.

Explanation of sonority	Position of chord implied	FBC	Necessary intervals	Intervals that must not appear
Consonant, open octave	Root pos.	8	P8	7 / 6 / 5 / 4 / 3 / 2
Consonant, third only	Ambiguous	3	3	8 / 7 / 6 / 5 / 4 / 2
Consonant, octave with third	Root pos.	83	P8 + 3	7 / 6 / 4 / 2
Consonant, fifth, open or not	Root pos.	53	P5 / d5	7 / 6 / 4 / 2
Dissonance of a 4-3 suspension	Root pos.	54	(P5 + P4) / (P8 + P4)	7 / 6 / 3 / 2
Dissonance of a 9-8 suspension	Root pos.	9X	2 + 3	7 / 6 / 4
Dissonance of a double suspension	Root pos.	94	P5 + P4 + M2	7 / 6 / 3
Consonant, sixth, open or not	First inv.	63	6	7 / 5 / 4 / 2
Suspension in bass, option 1	First inv.	52	P5 + M2	7 / 6 / 4 / 3
Dissonant, six-four	Second inv.	64	6 + 4	7 / 5 / 3 / 2
Dissonant, seventh	Root pos.	73	7	6 / 4 / 2
Dissonant, seventh with fourth	Root pos.	74	7 + P4	6 / 3 / 2
Dissonant, seventh with ninth	Root pos.	97	7 + 3 + M2	6 / 4
Dissonant, six-five	Second inv.	65	6 + 5	7 / 4 / 2
Dissonant, four-three	Second inv.	43	6 + 4 + 3	7 / 5 / 2
Suspension in bass, option 2	Third inv.	42	(6 + 2) / (4 + 2)	7 / 5 / 3
All other sonorities	N/A	__		

Ex. 2.6a. *The sixteen figured bass classes—Boolean operators "and" and "or" are represented using plus signs and slashes, respectively—vertical intervals where quality matters are listed in the format P8, M3, etc., while those where quality does not matter are listed in the format 3, 6, etc.—shaded rows denote consonant FBCs*



Ex. 2.6b. *The five consonant figured-bass classes—double bar lines separate different figured-bass classes—dotted bar lines separate different possible versions of the same figured-bass class—FBCs 53 and 63 can contain octaves and/or thirds, or not—FBCs 8, 3, and 83 cannot contain any other vertical intervals—octaves must be perfect, fifths must be perfect or diminished, thirds and sixths can have any quality*

The logic behind the requirements for the three consonant figured-bass classes is founded on continuo practice. Any time a fifth appears, we assume that a third can be sounded with it. This is because as a seventeenth-century keyboard player accompanying a singer or violinist from an unfigured bass part, you would add a third above the bass in your accompaniment if you saw that the melody was sounding a fifth above the bass. In fact, you would almost always play diatonic thirds above the bass, unless you had a reason not to (e.g., a "4" was marked in the figures). For this reason, any perfect or diminished fifth, without a sixth or a dissonant interval sounding simultaneously, is understood to mean 53. A fifth still implies a third if a seventh is also present, but this is a separate figured-bass class because it contains a dissonance.

An octave above the bass does not have the same implications as a fifth, since it can appear in both root-position and first-inversion triads (i.e., an octave by itself could just as easily imply 85 or 83 as it does 86.) Octaves, however, appear often enough in seventeenth-century music that to disregard them completely would mean losing a significant amount of information, so it was decided that, in case this sonority plays an important syntactical role, the figured-bass class "8" would be assigned to all slices that comprise only a perfect octave (and/or its compounds) above the bass.

Like the octave, the third is not sufficient to imply a figured-bass class. Two notes a third apart are usually, but not necessarily heard as the root and third of the active harmony. Unless an adjacent slice can disambiguate the third, which is often the case, the final figured-bass class for this slice will be "__" (blank).

One might wonder: why not consider the figures provided by composers? These are here for good reason: to disambiguate ambiguous harmony. There are three reasons we do not consider figured bass included in the continuo parts: (1) reading (via OMR) and digitizing (as XML) Arabic numerals in printed music is beyond the current capabilities of our software, although the technology does exist; (2) some editions contain only editorial figured bass, some contain only figured bass written by the composer, and some contain a mix of the two, which would complicate the transcription and editing process substantially; and (3) certain composers write more figures than others, which would skew the amount of data we obtain for one composer (Corelli) over the others.

2.8. Python Script `continuo.py`

The second Python script iterates through a table (generated by `tblmkr.py`) one slice at time, but often considers pairs of slices when making analytical decisions. The music-theoretical and logical merits of this method will be discussed at length in the next subsection, after we introduce one more analytical parameter.

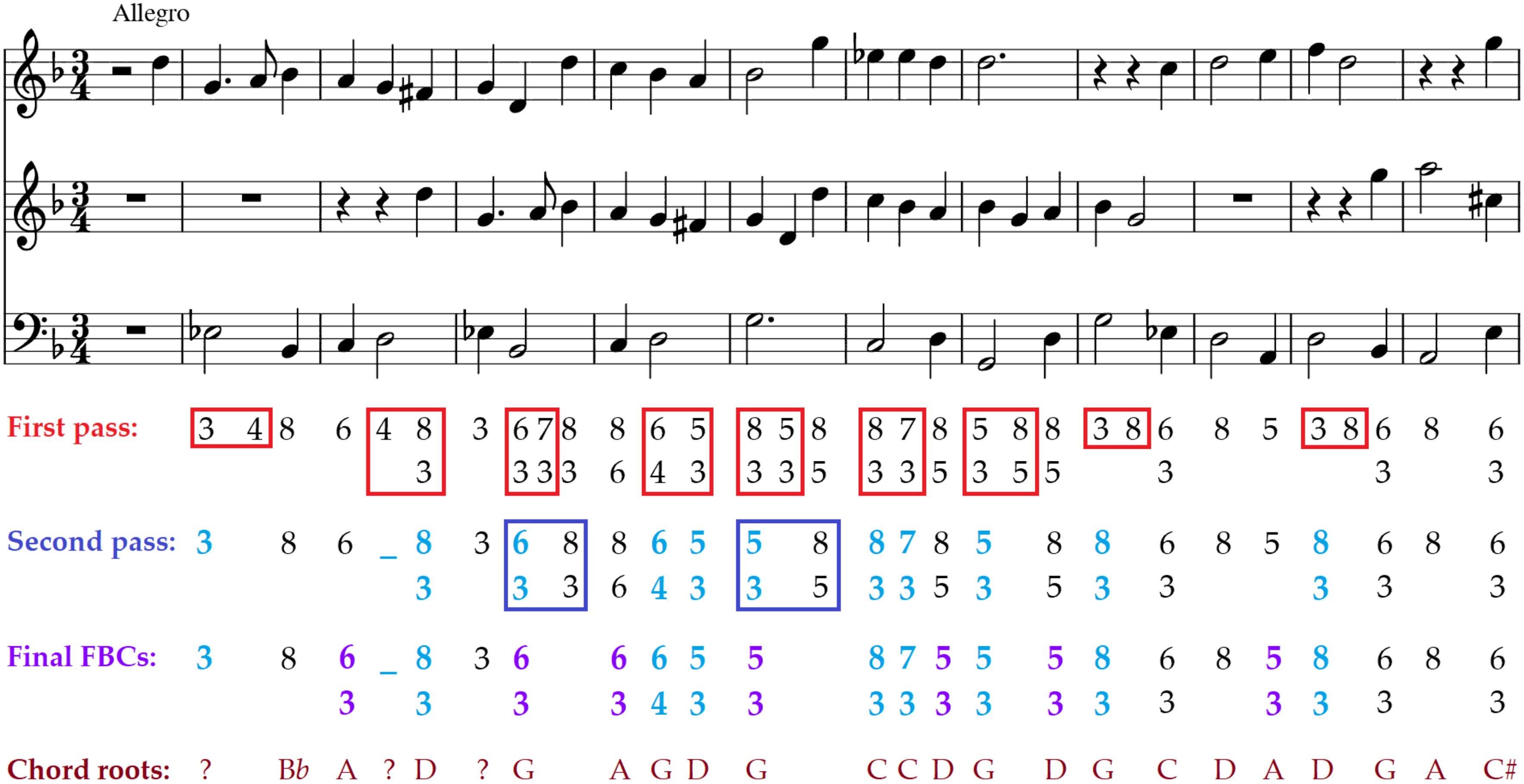
The user-defined "smallest slice" parameter refers to the smallest note value (typically a sixteenth note, but often longer in Frescobaldi) that can be considered a unique harmony, by which we mean any harmonic event that would warrant a new figure in the continuo, e.g., the ♯6 in a 5–♯6 motion. This parameter is set by the analyst, who must look at the music and decide which note value is best. If too large a note value is chosen, information could be lost (depending on the metre), but if too small a note value is chosen, the consequences are less severe. For this reason, it is recommended that a very small note value be chosen unless the user can be sure that a larger note value is appropriate.

The smallest slice parameter is included in the `continuo.py` script for the convenience of the analyst who might want to systematically ignore chord changes that happen below a certain note-value threshold (e.g., when analysing a Bach chorale, deciding to include chord changes on eighth notes vs. quarter notes might produce a vastly different analysis). If, for whatever reason, the analyst does not wish to specify the smallest slice for every work in a corpus (e.g., due to time constraints), the script will simply assign the smallest note value that appears in the work to the smallest slice parameter.

2.8.1. Analysing Pairs of Slices

While many of the individual slices in a seventeenth-century trio sonata will imply chords on their own, some will be harmonically insufficient, with only one or two different pitch classes. Others will be harmonically sufficient, but will also be able to combine with adjacent slices to give more specificity. After much trial and error using methods that sample the music at one or more fixed time intervals, we discovered that analysing pairs of slices was the best way to capture all of the necessary harmonic information with minimal computation.

Example 2.7 (below) shows the first twelve measures of the final movement of the same Legrenzi sonata in Example 2.3, *La Bevilacqua*, with pairs of slices shown



Ex. 2.7. The first twelve measures of the final movement of Legrenzi's sonata La Bevilacqua *(Op. 8, no.6)—pairs of slices are shown in coloured boxes—this section of music would require two "passes" to analyse all of the pairs because some pairs are nested within others—the final figured-bass classes are converted to chord roots and qualities are determined later*

in coloured boxes. In order for two slices to be considered a "pair," they must be: (1) adjacent, (2) in the same measure, and (3) they must occur over the same pitch class in the bass. The two slices of a pair do not need to have any specific relation to one another in terms of length (e.g., they do not need to be the same length) and they do not need to have the same voices sounding. A slice that occurs in a measure all by itself (e.g., the last whole note of a section in 4/4 time or a single pick-up note of any length) will never be considered a member of a pair, so its chord label (if applicable) must be determined only from the information provided by `tblmkr.py`, which is almost always possible.

In many cases, a single bass note will last for more than two slices. So as not to omit any of these slices, `continuo.py` analyses the music in multiple "passes." The excerpt in Example 2.7 (above) requires two passes of analysis in order to catch every pair. In m. 6 of this excerpt, for example, there is a dotted half note in the bass with three quarter-note-length slices above. In the first pass, the first of the two quarter-note slices in m. 6 is treated as a pair. In the next pass, the result of that first pair becomes a single half-note-length slice, which pairs with the remaining quarter-note slice. The algorithm loops over the music as many times as it needs to, which is usually about three times.

2.8.2. Beat Strength and Dissonant-ness

For any pair of slices, there are three possible relationships between the beat-strength values of the two slices: (1) the first slice is stronger (has a larger beat-strength value) than the second slice, as is the case for the first pair in Example 2.7 (with vertical intervals of 3 and 4); (2) the first slice is weaker than the second slice; or (3) the two slices have the same beat-strength value, as is the case for the second pair in Example 2.7 (with vertical intervals of 4 and 8/3). Any pair that satisfies the second condition (first slice is weaker) is discarded. Pairs that satisfy conditions 1 (first slice is stronger) or 3 (same strength) are then analysed to determine whether or not their slices should be combined.

In order to make this decision, we look at pairs of slices according to their dissonant-ness values in (Column G). There are four possibilities: (1) both slices are consonant (CC), as is the case for the last pair in Example 2.7 (with vertical intervals of 3 and 8); (2) the first slice is consonant and the second slice is dissonant (CD), as is the case for the first pair in Example 2.7; (3) the first slice is dissonant and the second slice is consonant (DC), as is the case for the second pair in Example 2.7; and (4) both slices are dissonant (DD), the least common case. Any pair containing a slice with a dissonant-ness label of "B" is discarded. Regardless of their dissonant-ness, all pairs in which both slices have identical vertical-interval content are automatically combined.

For all "CC" pairs (e.g., the last pair in Example 2.7) the intervals of the second slice (here, a single 8) are added to the intervals of the first slice (here, a single 3) to see if the requirements can be met for one of the consonant figured-bass classes (i.e., 83, 53, or 63). For the last pair in Example 2.7, the slices can be combined to produce a figured-bass class of 83: the slice on the downbeat (the 3) is not sufficient to imply a harmony on its own, but the following weak slice (the 8) provides enough information to disambiguate the harmony. Most CC pairs will easily add together to produce a consonant harmony, but some CC pairs will receive two different chord labels (e.g., when the first slice contains a 5 and the second slice contains a 6, or when the two slices contain different qualities of thirds above the bass).

When the slices of a pair are combined, their note values are also combined. This is important because when we look for chord progressions later on, we want to know which chords our new chord is adjacent to. When we combine the last pair in Example 2.7 to produce an 83 sonority, we want to know that this sonority is adjacent to both the open fifth that precedes it and the 63 sonority that follows it. The note value of our 83 sonority, then, must be equal to one half note.

Pairs that are not CC contain one or more dissonant intervals. The dissonant slices of CD pairs are often non-chord tones (e.g., passing, neighbour, échappée, anticipation, cambiata, the first half of a double neighbour, or the preparation of a fake suspension), but the specific type of non-chord tone does not have any effect on the harmony, so no effort is made to identify it, unless it is a seventh above the bass.

Because the introduction of a seventh can result in a new chord label (e.g., a D major chord becomes a D dominant-seventh chord), the horizontal intervals by which a seventh is approached and left are carefully observed. In order for a seventh to produce a new chord, it must be left in one of two ways: (1) proceed down by step over a stationary or moving bass or (2) be held and become another dissonance while the bass moves.

The third pair, in Example 2.7 contains a seventh that is left by ascending step, so this slice does not become its own harmony and the pair takes on the figured-bass class of the first slice (63). The sixth pair in Example 2.7 also contains a seventh, but this seventh is approached by unison (i.e., prepared) and left by descending step, so these slices are not combined: the second slice gets a figured-bass class of 73 and the first slice gets a figured-bass class of 83. Whenever it is decided that the slices in a pair should not combine, both slices are marked "DNC" (do not combine) so that the algorithm does not waste time checking them again on subsequent passes. Apart from the cases where the dissonant slice contains a seventh, CD pairs are always labelled according to the first (i.e., consonant) slice, regardless of the treatment of the dissonant slice.

DC pairs require careful consideration: `continuo.py` checks the horizontal motion (in both directions) of the voices that create dissonant intervals with the bass. Dissonances that resolve on the very next slice (e.g., accented passing tones, accented neighbour tones, appoggiaturas, and some suspensions) can be analysed easily. Pairs that satisfy this requirement are generally labelled like CD pairs: according to the consonant slice. When the first slice (the dissonant slice) contains a seventh, however, different situations can arise. When the seventh in a 73 sonority moves down to the fifth in a 53 sonority, we combine the slices and keep the dissonant label (73). When the seventh in a 73 sonority moves up to the octave in an 83 sonority (a consonant échappée before the seventh resolves), we combine the slices and keep the dissonant label (73). When the seventh in a 73 sonority moves down to the sixth in a 63 sonority, we do not combine the slices: each slice is labelled as a different harmony.

There is one DC pair in Example 2.7 (the fourth pair), which comprises a 64 sonority and a 53 sonority. The horizontal intervals on either side of the dissonant interval (the fourth) are checked to make sure the dissonance is properly treated. As long as the fourth in a 64 sonority has a step (in any direction) on both sides, we treat it as a separate harmony, so for this pair, each slice keeps its original label.

Finally, DD pairs, which are rare, are almost always discarded, except for a handful of built-in paradigms that are most likely to occur in strong-weak pairs: long chains of seventh chords (73 to 73), 4-3 suspensions that resolve to seventh chords (54 to 73), cadential six-four chords that resolve to seventh chords (64 to 73), and other combinations of these three sonorities that appear in succession during a *cadenza doppia* (most commonly 73, 64, and 54, followed by 53).

2.8.3. Unpaired Slices

Many slices in the music will not belong to a pair, especially when the bass is relatively active, which makes held bass notes less common. Consonant unpaired slices keep the figured-bass classes assigned to them by `tblmkr.py` and slices with blank figured-bass classes remain blank. Dissonant unpaired slices are checked in the same way as the dissonant slices in DC pairs, with the added complication that the bass note may change as the dissonance resolves.

Once all slices have been analysed, a list of final figured-bass classes is generated. From these, chord roots are extrapolated and all that remains to be determined are the qualities of the chords.

2.8.4. Determining chord qualities

The quality of each chord is determined by looking at the list of vertical intervals (Column E) for the slice or combination of slices that produced the final figured-bass class for that chord. For the very last sonority in Example 2.7, our bass note is E and our vertical intervals are M6 and m3. This means our chord root is C♯ and our quality is "dim" for diminished. This process is uncomplicated as long as there are at least three different pitch classes in the given sonority, but many sonorities in seventeenth-century trio sonatas do not have three different pitch classes. In the third-to-last measure of Example 2.7, for instance, there is an A in the bass with only an E above it. Because it contains a perfect fifth, the figured-bass class for this sonority is 53. It therefore implies a triad, but because we cannot be certain of the quality of that triad we assign it a quality of "no3."

A similar situation arises for sonorities that comprise only a sixth, like the one on the downbeat of the third measure of Example 2.7. Here, the sixth is major, and the third, which will be provided by the continuo player, will be either major (if the continuo player chooses to play E) or minor (if the continuo player chooses to play E♭). The quality of the resulting triad will be either minor or diminished, so we assign it a quality of "m/d" (minor or diminished). If the quality of the sixth were minor, we could safely assume that the third added by the continuo player would also be minor, resulting in a major triad.

Quality	Meaning	Commonly results from FBCs	Shorthand
maj	major triad	53, 83, 63 with or without 3	E
min	minor triad	53, 83, 63	e
dim	diminished triad	63, 53 with or without 3	eo
aug	augmented triad	63	Ex
sus	dissonant sonority in a 4-3 suspension	54	
no3	triad with no third	8, 53 without 3	
m/d	triad that is either minor or diminished	63 without 3	e6
dom	dominant seventh chord	73, 74, with or without 5	E7
mj7	major seventh chord	73	
mn7	minor seventh chord	73	
hdm	half-diminished seventh chord	73 with or without 3	
m/h	minor seventh, minor third, and no fifth	73 without 5	e73
D/m	minor seventh, perfect fifth, and no third	73 without 3	e75
Dhm	only a minor seventh above the bass	73 without 5 or 3	
dm7	diminished seventh chord	73 with or without 5 and 3	
52	suspension in the bass	52	
???	quality unknown	3 or blank	

Ex. 2.8. List of all qualities that `continuo.py` *can identify—qualities that require at least three different pitch classes are shaded grey*

Example 2.8 (above) tabulates the 16 qualities assigned by `continuo.py`. Many of these qualities never appear in the data presented in Chapters 3, 4, and 5 because we only look at the 10–15 most common chords or chord progressions used by each composer. Most of the chords we will see in subsequent chapters are major or minor triads. After these, the next most common qualities are "no3" and "sus," followed by "dim," "dom," "m/d," "m/h," "D/m," and "aug."

We choose to exclude "no3" sonorities from our discussion simply because they are so common. Every scale degree in a tonality supports either a major, minor, or diminished triad (sometimes more than one of these), which we represent using Roman-numerals. If we add another quality (no3), we increase the number of different "versions" of each Roman numeral to the point where it becomes difficult to compare them all. If we take the ten commonest chords in Frescobaldi's C/♮ tonality, for example, we get:

Cmaj, Gmaj, Amin, Dmin, Fmaj, Emin, <u>Cno3</u>, <u>Gno3</u>, Gdom, and <u>Dno3</u>

This leaves us with only six of seven Roman-numerals to compare, as well as three different versions of the "dominant" harmony in this tonality. If we take the ten commonest chords excluding "no3" and "sus" chords, we get:

Cmaj, Gmaj, Amin, Dmin, Fmaj, Emin, Gdom, Amaj, Bdim, and Dmaj

By excluding "no3" and "sus" chords from our observations, we obtain a greater variety of Roman numerals (vii^{o} is now included), as well as two more major triads (Amaj and Dmaj), which are likely "applied dominants" of other triads. The decision to exclude certain qualities from our discussion was also made with our chosen modes of data representation in mind. These are discussed in the final section of this chapter.

2.9. Representing Data

With an average of about 410 chord labels per piece, or 19,680 chord labels per composer, the question of how to represent data in a way that is both accessible to readers and robust enough to make statistically valid comparisons is an important one. After receiving feedback at conferences and lab meetings, we settled on three main representations for our data: histograms, tornado charts, and flowcharts. To represent a tonality in one of these formats, we take a sample of the 10–15 commonest chords or chord progressions in that tonality and arrange them on the graph in a visually intelligible way. Explanations of these representations are given below, along with examples of how they are used in music-theoretical contexts.

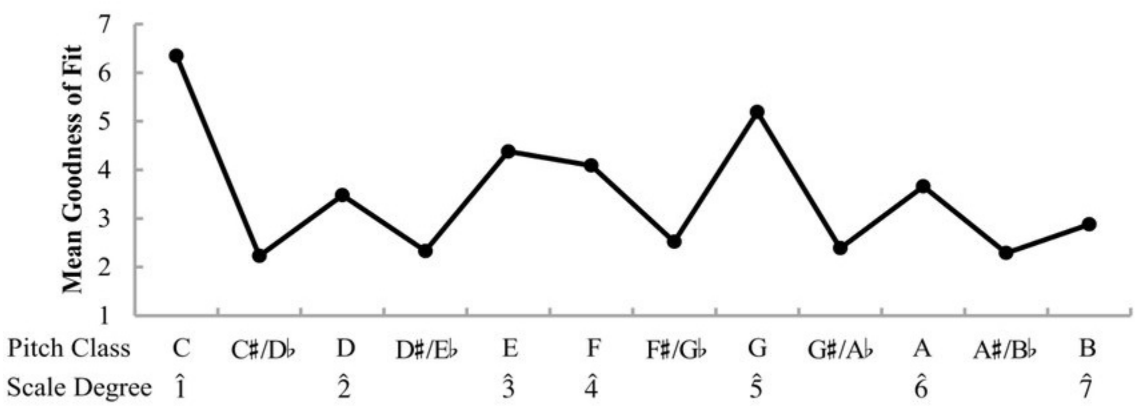
An important distinction that we make when representing our chords is that between Roman-numeral and root/quality representations. A Roman-numeral representation assigns a Roman numeral to every chord according to its distance above the final of the tonality in which it occurs, e.g., I is the triad that has the final of a tonality as its root. Roman-numeral representations are useful for comparing tonalities that have different finals. In contrast, a root/quality representation is preferred when we want to compare chords to other chords. In this representation, each chord is identified using its root and quality, e.g., Amaj.

2.9.1. Histograms, Heat Maps, and Dendrograms

Tallies of individual chords are easily represented using histograms. A histogram is an ordered list of numbers, where each number in the list is a number of times something happens. The most common kind of ordered list in the chapters that follow is a list of Roman numerals (e.g., I, ii, iii, IV, V, vi, vii^{o}). In a Roman-numeral histogram, we are counting how many times each Roman numeral happens within a tonality (e.g., 10, 1, 1, 3, 5, 4, 1). Because we have different amounts of data (i.e., different numbers of pieces) for different tonalities, we represent the numbers in histograms as percentages of the total number of observations made (e.g., 40%, 4%, 4%, 12%, 20%, 16%, 4%). We also use root/quality histograms, in which the order of the list is a ranking. The first item in the list is the most commonly occurring item, the second item the second most commonly occurring, etc. (e.g., Cmaj: 9.08%, Fmaj: 7.78%, Dmin: 6.95%, Gmaj: 5.55%, Amin: 4.93%, Gmin: 4.59%, Bbmaj: 4.16%). The root/quality histogram is useful when we want to compare tonalities that do not contain the same Roman numerals (e.g., major tonalities with minor tonalities).

When we compare tonalities based on their chord content, we actually calculate the similarity between the histograms that represent those two tonalities. Because histograms contain only numbers, we can use them in mathematical and statistical formulas, such as the formula for Euclidean Distance (explained in Section 3.2.1). Histograms can be compared in this way as long as they contain the same number of items. For example, a histogram that tallies the occurrences of twelve different Roman numerals cannot be compared with a histogram that tallies the occurrences of thirteen different Roman numerals.

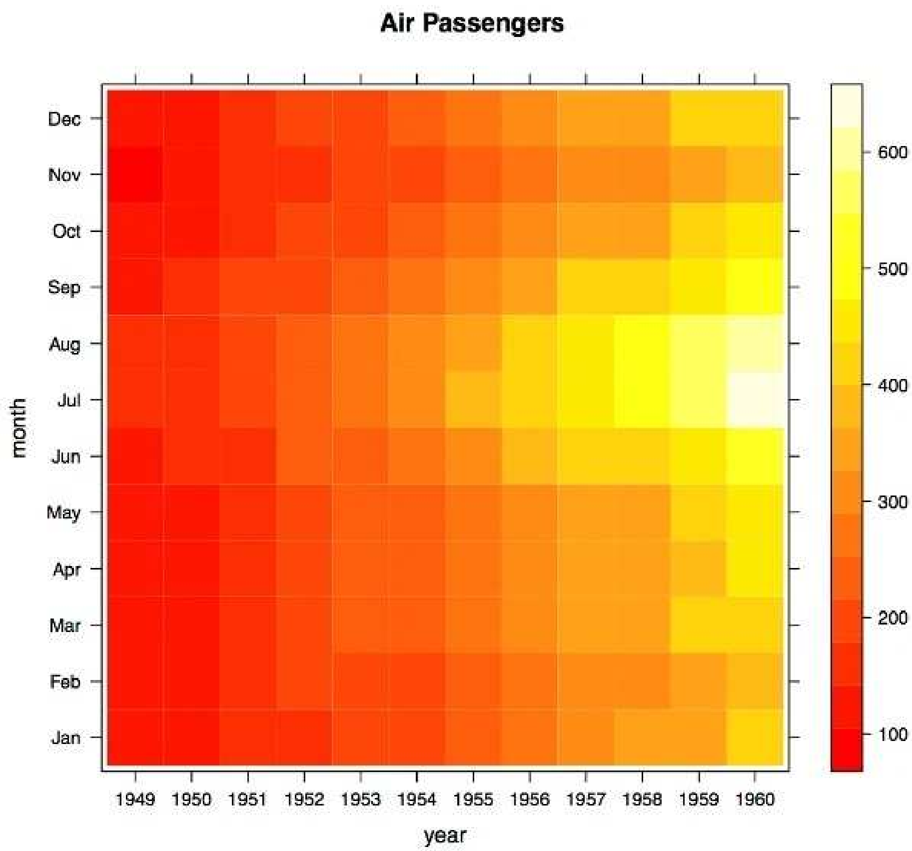
Perhaps the most famous example of a histogram in a music-theoretical context is the "key profile," created by Carol Krumhansl and Edward Kessler (Krumhansl and Kessler 1982). The key profile is an ideal distribution of pitch classes within a key, against which an observed distribution of pitch classes can be measured in order to determine the key of the music from which the observed distribution was obtained. The Krumhansl-Kessler key profile was constructed by having human participants rate the "goodness of fit" of different pitch classes after having heard a IV-V-I chord progression. Example 2.9 (below) shows the result.



Ex. 2.9. Krumhansl & Kessler's original "key profile" for C major (1982)

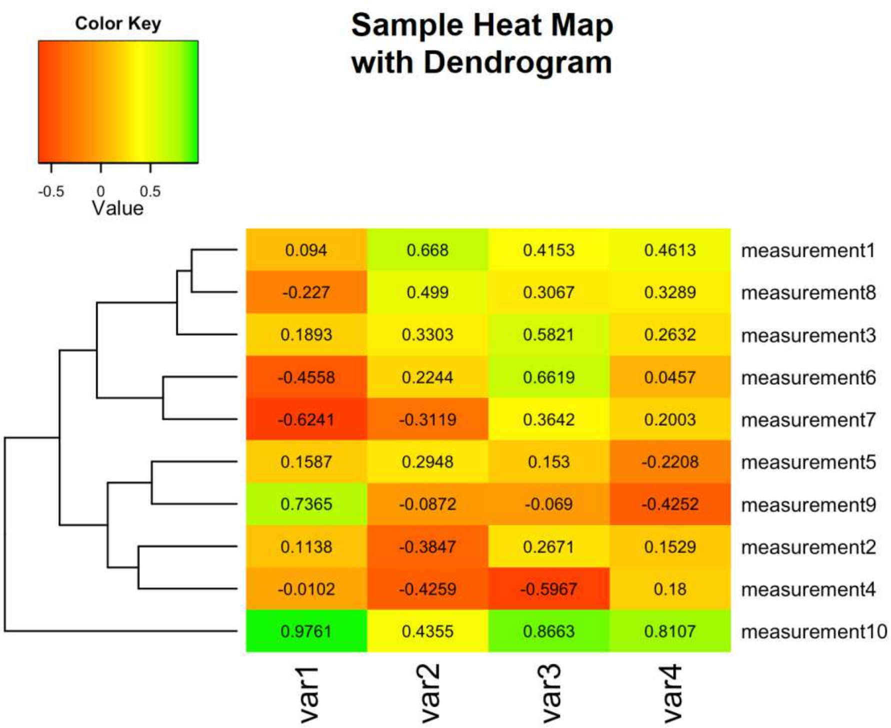
This idea was later taken up by Temperley (Temperley 1999) and Quinn (Quinn 2010), both of whom make adjustments to the original distributions proposed by Krumhansl & Kessler in order to optimize them for the purpose of key finding. Our work differs from these scholars in that we do not compare our observed histograms to an "ideal" distribution representing a major or minor tonality. Instead, we only compare histograms to *one another* in order to determine how similar they are and how this amount of similarity changes over the course of the seventeenth century.

The similarities and differences between a collection of histograms are often represented using heat maps and dendrograms (also known as tree diagrams). A heat map is a table that shows the prevalence of one or more events using a colour scale.



Ex. 2.10. Heat map representing number of air passengers in two dimensions of time: years (x-axis) and months (y-axis)

Example 2.10 (above) shows a simple heat map that plots numbers of air passengers over time. Here, the colour scale helps us quickly ascertain two things: that air travel becomes more common from one year to the next (moving to the right along the x-axis) and that people are more likely to fly in June, July, and August, compared with other months (comparing rows along the y-axis). In our heat maps, each item on the x-axis (i.e., each column of the table) is a different tonality and each item on the y-axis (i.e., each row of the table) is a different chord (see Sections 3.2.2, 4.2.1, and 5.2.1 for examples).

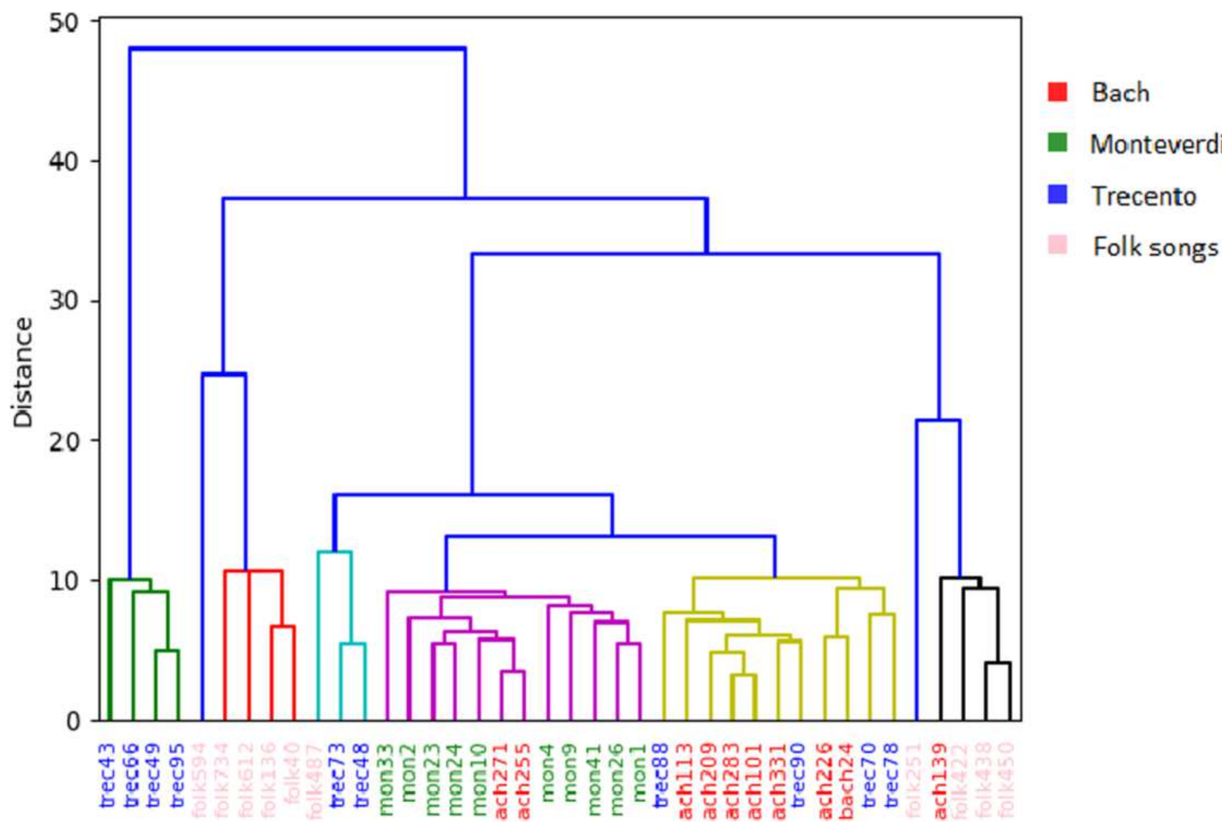


Ex. 2.11. Heat map plus dendrogram—the dendrogram to the left of the heat map shows correlations between rows of the heat map—the most similar rows are the top two rows—the row that is least similar to any other row or group of rows is the bottom row

Heat maps are often paired with tree diagrams, or "dendrograms," another way of visually representing the similarity of items in a group. In a dendrogram, items that are more similar appear closer together on the tree. In our case, the items are tonalities represented as chord histograms. Example 2.11 (above) shows how heat maps and dendrograms typically interact: the dendrogram to the left of the heat map shows correlations between the rows of the heat map. In our case, each row of the heat map is a different tonality, each column is a different chord (for this reason, we need more than four columns), and each coloured rectangle displays the number of times we count a certain chord in a certain tonality. For examples of dendrograms resulting from heat maps, see Sections 3.2.2, 4.2.1, and 5.2.1.

In music-theoretical contexts, dendrograms are often used to represent correlations between different composers or genres of music. This kind of representation has become especially useful in the development of music-recommendation algorithms. One such algorithm, developed by Laskowska and Kamola (Laskowska and Kamola 2020), groups individual musical works based on their motivic similarity. The dendrogram in Example 2.12 (below) represents the correlations between individual pieces from four of their corpora and shows that pieces from the same corpus are often (but not always) grouped together.

In her PhD dissertation, Cecilia Taher used dendrograms to represent correlations between individual pieces written by one composer (Pierre Boulez) in terms of their motivic content (Taher 2016). Our work differs from these examples in that we do not compare data for individual pieces (except for the pieces in Frescobaldi's G/♮ tonality; see Section 3.2.3). Instead, we group pieces together according to their tonalities, i.e., d/♮ pieces with d/♮ pieces, e/♮ pieces with e/♮ pieces, etc., and compare these groups to one another.



Ex. 2.12. Dendrogram from Laskowska & Kamola showing correlations between individual musical works—the group coloured black on the far right comprises folk songs and one work by Bach—the larger group coloured yellow comprises works by Bach and by trecento composers—the purple group comprises works by Bach and Monteverdi

2.9.2. N-grams and Tornado Charts

When comparing tonalities on the basis of individual chords, we make a list of chords (e.g., I, ii, iii, IV, V, vi, vii°) that we will tally in every tonality and thus obtain histograms that always tally the same seven chords. When comparing tonalities on the basis of chord progressions, however, it does not make sense to use a predetermined list of chord progressions because we cannot be sure that every progression on our list will occur in every tonality that we wish to be able to compare. Instead, we observe the 10–15 most common chord progressions in each tonality, which may or may not overlap from one tonality to the next.

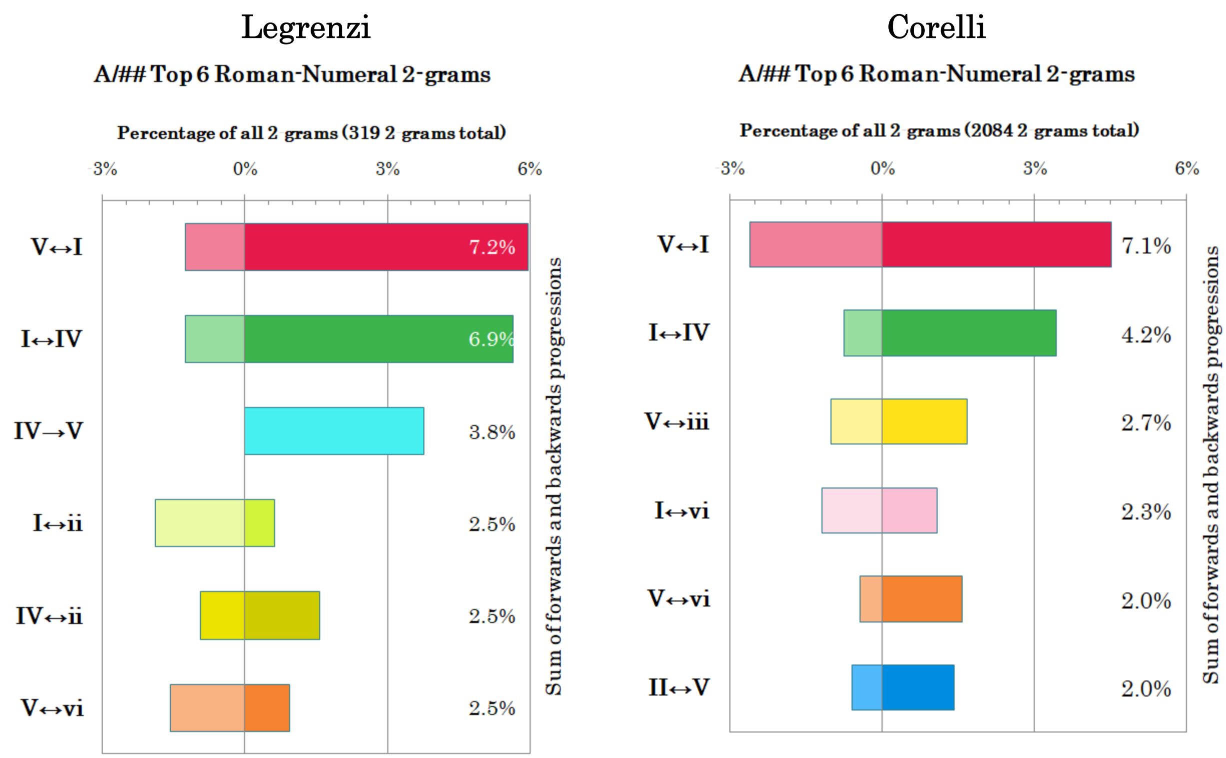
Because all of our chord progressions are only two chords long, we often refer to them as "2-grams," which are a type of n-gram. An n-gram is an ordered sequence of items that is *n* items long. The sequence of chords "Gmaj, Cmaj" is a 2-gram and, by the same token, a "sequence" of chords that is only one chord long, e.g., "Gmaj," is a 1-gram. N-grams are used in a variety of computational applications, including search engines (Broder et al. 1997), author profiling (Keselj et al. 2003), and genome sequencing (Tomovic, et al. 2006).

In music theory n-grams are used in much the same way as they are in the field of computational linguistics: as a way of representing the style of a composer (or author), a time period, or an entire genre of music (or literature). Most of these applications appear in the field of music information retrieval (MIR).

In an article from 2008, Wolkowicz et al. describe a method of profiling composers according to their melodic n-gram vocabulary. (Wolkowicz et al. 2008). Their corpus consisted of works for piano by five composers: Bach, Mozart, Beethoven, Schubert, and Chopin. Different sizes of n-grams were tested and the best results were obtained for an *n* value of 6. When their algorithm is given a new piece, it counts all occurrences of each 6-gram and creates a profile of that piece. It then compares that profile to the 6-gram profile of each composer to see which is the closest match. Their method was able to arrive at the correct answer about 84% of the time. The main differences between their project and ours are that we look at chord n-grams, not melodic n-grams, and all of our conclusions regarding similarity are made by humans, not the algorithm. Our Python script `continuo.py` is capable of finding chord n-grams with *n* values from 1 to 6, but for reasons of scope we only discuss n-grams with *n* values of 1 (individual chords) and 2 (chord progressions that are two chords long).

Because we believe that the order of chords in a chord progression is important (e.g., in tonal harmony, the progression from I to ii is much less striking than the progression from ii to I), we represent all of our chord progressions in two directions, i.e., we count both "forwards" and "backwards" versions of every progression. To represent these data, we use tornado charts. Example 2.13 (below) shows two tornado charts, one for Legrenzi and one for Corelli, plotting the six commonest chord progressions in each composer's A/♯♯ tonality.

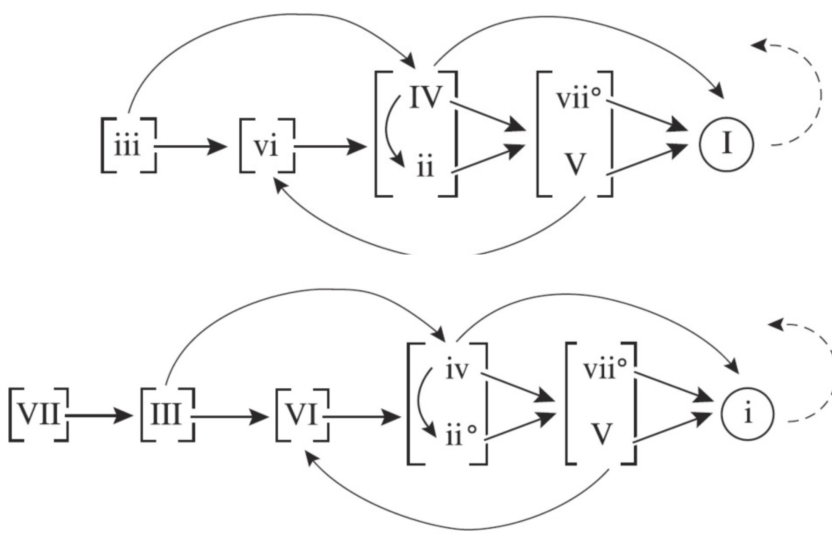
These charts allow both analyst and reader to quickly see which progressions are most common (progressions are ordered from most to least common with the commonest progression at the top), which progressions are more likely to go forwards than backwards and by how much (rightwards-extending bars show the number of "forwards" progressions and leftwards-extending bars show the number of "backwards" progressions), and which progressions are used by more than one composers (colour coding helps us see that Legrenzi and Corelli both use V↔I, I↔IV, and V↔vi progressions in their A/## works).



Ex. 2.13. Tornado charts plotting the six commonest progressions in Legrenzi's and Corelli's A/## tonalities

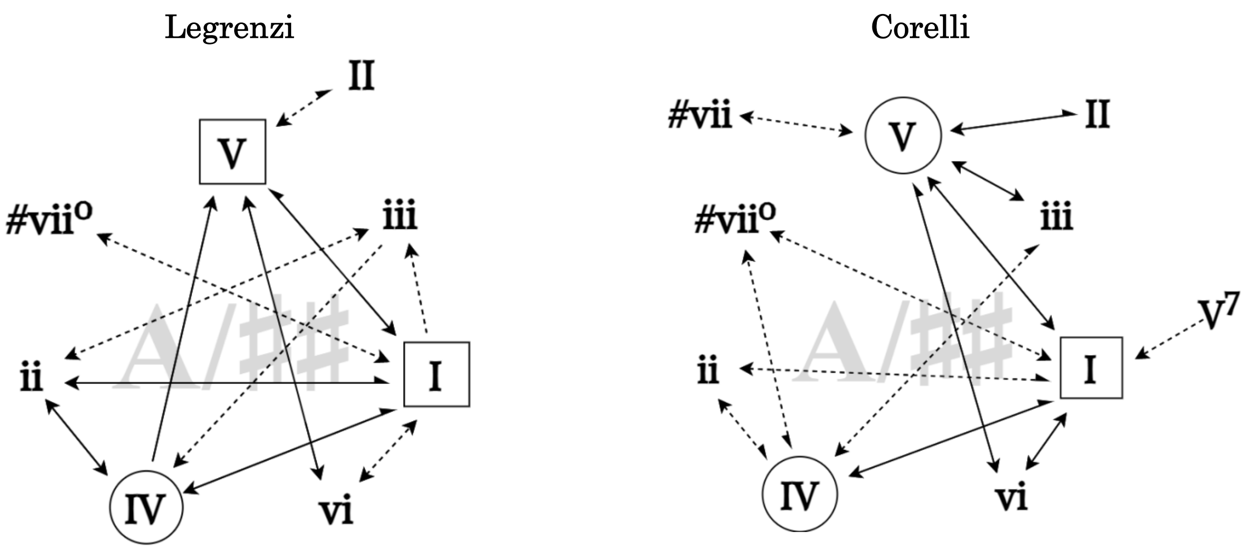
Once we have enough data for individual chords (1-grams) and chord progressions (2-grams), we can combine these data to produce flowcharts that simultaneously represent the prominence of each chord in a given tonality and the likelihood with which it will interact with (i.e., appear directly before or directly after) all other chords in that tonality. This type of representation brings to mind the Roman-numeral flowcharts from *Tonal Harmony* (Kostka, Payne, and Almén

2018) that we saw in Example 1.1. These flowcharts are reproduced in Example 2.14 (below) and again in Example 3.12.



Ex. 2.14. The Kostka-Payne-Almén flowcharts showing standard chord progressions (2-grams) in major keys (above) and minor keys (below)

Our flowcharts differ from the two in Example 2.14 (above) in four ways: (1) whereas the flowcharts above group Roman numerals according to function (IV with ii and vii° with V), ours do not; (2) the flowcharts above show only unidirectional progressions (when in fact, it is also likely that many of these progressions also go backwards) while ours show bidirectional progressions; (3) the flowcharts above do not indicate the likelihood of either individual chords (i.e., which chords are most common) or chord progressions (i.e., whether some progressions are more common than others); and, most importantly (4) the flowcharts above do not represent actual tonalities, but rather ideal tonalities (major and minor) that conform to a set of norms established by the authors of *Tonal Harmony*. Our flowcharts, in contrast, are the products of the data that we have gathered: empirical snapshots of chord syntax for individual tonalities in the works of individual composers. Example 2.15 (below) shows Roman-numeral flowcharts for Legrenzi's and Corelli's A/♯♯ tonalities.



Ex. 2.15. Roman-numeral flowcharts for Legrenzi's and Corelli's A/♯♯ tonalities

In our flowcharts, solid arrows indicate progressions that are the most common (i.e., the forwards and backwards versions of these progressions account for at least 2% of all the progressions in the given tonality) and dashed arrows indicate less common progressions (i.e., progressions below the 2% threshold). Arrow heads indicate the "directionality" of a progression: unidirectional arrows (→) indicate progressions that never go backwards (e.g., the I→iii progression in Legrenzi's A/♯♯ tonality); bidirectional arrows with one half head and one whole head indicate progressions that are at least twice as likely to proceed in one direction over the other (e.g., the V↔I progression in Legrenzi's A/♯♯ tonality); and bidirectional arrows (↔) indicate progressions that are less than twice as likely to proceed in one direction over the other (e.g., the ii↔iii progression in Legrenzi's A/♯♯ tonality).

The boxes and circles in our flowcharts show the prevalence of individual chords: boxed Roman numerals are extremely common, circled Roman numerals are common, and Roman numerals without a box or a circle are less common. Like all of our flowcharts, the two flowcharts in Example 2.15 show only the twelve commonest bidirectional progressions in the given tonality, where the commonness of a progression is obtained by taking the sum of the numbers of forwards and backwards versions of that progression (as percentages of the total number of progressions in that tonality).

In the fifth chapter of his PhD dissertation, Daniel Tompkins uses several of the same methods that we have just explained: dendrograms, chord 2-grams, and flowcharts (Tompkins 2017). Tompkins counts chord 2-grams in a corpus of seventeenth-century guitar music in *alfabeto* notation and his goal is fundamentally different from ours: to determine the extent to which the harmonic progressions in this repertoire are "functional." Tompkins concludes that

> alfabeto symbols were not carelessly sprinkled onto publications for the sake of popularity but progressed in such a way that had clear harmonic function similar to later codified theories. On a local level, there are certainly harmonic movements that may seem jarring, but the corpus overall is built upon a system of regular harmonic function (Tompkins 2018, 118)

Tompkins's methodology is robust, but it is difficult to engage with his work on a more granular level because he assumes from the outset that there are only two modes in his *alfabeto* corpus: major and minor (which are analogous to the larger categories of tonalities that we call Ut and Re, respectively). Were we more interested in harmonic functions, or less interested in the harmonic implications of different modal octaves, Tompkins's work would be more relevant to our own endeavour.

With our computational and analytical methodology clearly laid out, we can now apply these methods to the three different composers in our corpus. Chapters 3, 4, and 5 discuss tonalities, chords, and chord progressions in the music of Frescobaldi, Legrenzi, and Corelli, respectively, recapitulating and expanding on the explanations in this chapter as needed.

3. Frescobaldi

In this chapter, I will apply to Frescobaldi the methods outlined in Chapter 2. The analyses proceed in three phases. First, I compare the distributions of 1-grams (i.e., individual chords) in different tonalities using histograms (i.e., bar graphs), Euclidean distance, and agglomerative hierarchical clustering. Next, I compare the 2-grams (i.e., chord progressions that are two chords long) in each tonality using tornado charts and flowcharts. This chapter explains each phase in detail for the 47 canzonas by Frescobaldi. The next two chapters (Chapters 4 and 5) follow the same three analytical steps, but for Legrenzi and Corelli, respectively.

3.1. Frescobaldi's Tonalities

In the 47 Frescobaldi canzonas that appear in my corpus, ten different tonalities are used. These are tabulated in Example 3.1 below. I have arranged these tonalities according to three theoretical concepts. The first is system; all of Frescobaldi's canzonas use either *cantus durus* (works with an empty signature) or *cantus mollis* (works with a single flat in the signature). Frescobaldi never uses a signature with more than one flat, and never puts sharps in the signature. Arranged in this way, we see that he uses ten out of fourteen possible combinations of final and signature in the *durus-mollis* universe. The missing combinations are B♭/♭, a/♭, b/♮, and e/♭.

The second organizing concept is octave species; each of the four tonalities in *cantus mollis* has the same octave species above the final as the *cantus durus* tonality directly above it. In this chapter and the ones that follow, I refer to these octave species using the Greek names, "dorian," "phrygian," etc., but I use "major" and "minor" for the octaves from C to C and A to A, respectively. The Greek names are kept in lowercase so as not to imply the authentic modes of the same names in either the octenary or dodecachordal system. In her article "Carissimi's Tonal System and the Function of Transposition in the Expansion of Tonality," Beverly Stein follows this same convention (Stein 2002). In Section 3.2, I compare the four *durus-mollis* pairs of tonalities to see whether and how they are similar.

	Ut			Re		Mi	Total
Cantus durus	F/♮(2)	C/♮(5)	G/♮(9)	d/♮(4)	a/♮(3)	e/♮(1)	24
Cantus mollis		F/♭(7)	C/♭(1)	g/♭(13)	d/♭(2)		23
Total	24			22		1	47

Ex. 3.1. Tonalities in 47 canzonas by Frescobaldi

The third concept, demarcating the larger columns of the table, is the distinction between so-called Ut, Re, and Mi tonalities. This distinction is discussed comprehensively by Cristle Collins Judd (Collins Judd 1992) and to a lesser extent by Beverly Stein (Stein 2002), who apply it to the works of Josquin des Prez and Giacomo Carissimi (a contemporary of Frescobaldi's in Rome), respectively. Stein explains that this concept "goes back at least as far as Glarean, who writes that some people teach that 'every song ends on either re or on mi or on ut' and elsewhere that 'even now we have commonly only three modes in frequent use'" (Glarean 1574). When comparing tonalities on the basis of chord content, the Ut, Re, and Mi categories are useful because the tonalities within each category have many triads in common. For example, all four of the Re tonalities (d/♮, g/♭, a/♮, and d/♭) have diatonic i, III, v, and VII triads, and a non-diatonic but commonly occurring V triad.

While the Ut-Re-Mi grouping is both historically sensitive and methodologically expedient, it might elide some important distinctions between individual tonalities. As we will see in Sections 4.2, 4.3, and 5.2, there are some tonalities that appear to straddle the line between Ut and Re, namely Legrenzi's c/♭ tonality and, to a lesser extent, Corelli's f/♭♭ tonality, both of which behave like dorian tonalities even thought their signatures look mixolydian. Section 3.2.2 further discusses the empirical validity of the Ut-Re-Mi grouping in the context of Frescobaldi.

The numbers in the bottom row and rightmost column of Example 3.1 are tallies of canzonas written in each tonal category and system, respectively. Unlike Legrenzi and Corelli, Frescobaldi does not order or organize the works in his published collections of instrumental music according to tonality. While tonalities seem to appear at random within each of the four Frescobaldi collections I chose to digitize, there is a nearly even division between *cantus durus* and *cantus mollis*, as well as between Ut-tonality and Re-tonality canzonas in the aggregate of all four collections. This suggests that, even though order is not important, Frescobaldi (or perhaps his publishers) struck a balance, consciously or not, between the *durus* and *mollis* systems and between the Ut and Re categories.

The Mi-tonality category is the only one of the three that is not well represented, lagging behind Ut and Re by a large margin, with only a single canzona written in e/♮. Stein mentions that "[b]y Carissimi's time the phrygian mode had largely been subsumed under the aeolian" (Stein 2002, 291). In his dissertation, Liam Hynes-Tawa argues that the peculiar relationship between phrygian and aeolian modes results from the failure of the phrygian final (E) to obtain the status of "tonic" during the sixteenth and seventeenth centuries (Hynes-Tawa 2020). I investigate Stein's and Hynes-Tawa's claims in Section 3.2.2 and again in Section 6.2.

Within the Ut and Re categories, certain tonalities are much more popular than others; in 1635, we see that the minor tonalities a/♮ and d/♭ are not as common as the dorian tonalities d/♮ and g/♭. In the Ut category, the lydian tonality F/♮ is not strongly represented. This tonality, along with e/♮ is treated with caution in the analyses that follow and different groupings of the Ut-tonality and *cantus-durus* works are analyzed with and without the lydian and phrygian canzonas.

While it is possible to imagine that the relative popularity of different tonalities can tell us something about tonal practice at any given timepoint in this study, I do not draw any conclusions from the distributions of tonalities within the corpora I have assembled. Instead, I focus on comparing tonalities to one another, both within the oeuvre of a single composer and across the works of all three composers.

3.2. Comparing Chord Histograms

Individual chords were counted and tallied for all canzonas in each of Frescobaldi's ten tonalities. In order to determine which tonalities are most similar both within and between the Ut, Re, and Mi categories, different pairs of tonalities are compared in terms of chord distribution, that is, which chords "make up" these tonalities by percentage. The number of different chords that make up any one of Frescobaldi's tonalities is anywhere between 80 and 180. So, for the sake of simplicity, only a subset of these chords is considered.

When considering which chords make up a tonality, there are two ways of identifying the chords themselves. One is as scale steps (usually diatonic) or "functions" within the tonality, which we represent using Roman numerals. The other way of representing chords is by root and quality. Both representations are used in the analyses that follow and different observations can be made using different representations. For each representation, a different set of histograms was generated from the data. In the Roman-numeral histograms, chords are ordered from "tonic" up to "leading-tone," while the root/quality histograms have chords ordered from most to least common. Although each of the two kinds of histogram tallies a different subset of chords, the logic for selecting these subsets is the same:

among the *most commonly occurring chords* in all tonalities, find a subset of chords that is *shared* by the largest number of tonalities. This allows us to compare "like with like" for a maximum of data points. For the sake of simplicity, both the Roman-numeral and root/quality histograms tally only sonorities that represent complete triads; seventh chords, "sus" chords, and triads of ambiguous quality are not counted.

lydian	major	mixolydian
I	I	I
ii	ii	ii
II		
iii	iii	iii
	iii°	iii°
♭IV	IV	IV
iv°		
	v	v
V	V	V
vi	vi	vi
vii		
	♭VII	VII
vii°	vii°	♯vii°

dorian	minor	phrygian
i	i	i
I	I	I
ii	ii	
		II
ii°	ii°	♯ii°
III	III	III
iv	iv	iv
IV	IV	
v	v	v
V	V	V
		v°
♭VI	VI	VI
vi°	♯vi°	♯vi°
		vii
VII	VII	VII
♯vii°	♯vii°	

Ex. 3.2. *Triads to be tallied in Roman-numeral histograms—non-diatonic triads are shaded*

Example 3.2 shows which chords I have chosen to count in each type of tonality, organized by octave species, for the Roman-numeral histograms. The seven most commonly occurring chords in any given tonality are almost always the diatonic triads (see Exs. 3.4 and 3.5 for exceptions). These seven triads, plus a few other triads that result from the addition of common accidentals (especially the raised leading tone) account for, on average, about two thirds of all the chords observed in each tonality. In Example 3.2, shared triads appear on the same row, diatonic triads are un-shaded, non-diatonic triads requiring no more than one accidental are shaded light grey, and triads requiring two or more accidentals are shaded dark grey.

For the Ut tonalities (left side) the number of tallied chords is ten, so the histograms for these tonalities will have ten bars each. For the Re and Mi tonalities (right side), that number is thirteen because these tonalities have a greater variety of triads. In addition to the most common triads, less common triads were chosen as well so that the mixolydian-major and dorian-minor pairs of tonalities could be compared at as many data points as possible. For example, the non-diatonic iii^{o} triad in C/♮ (Edim) is included so that it can be compared with Bdim, which is diatonic in G/♮. Likewise, Bmin is counted in G/♮ so that it can be compared with Emin in C/♮. Several non-diatonic triads are counted in the lydian and phrygian tonalities as well, although these two tonalities, by virtue of their octave species, are more difficult to compare with the other tonalities in their tables from a Roman-numeral perspective. Still, F/♮ and e/♮ share seven out of ten and ten of thirteen triads with the other tonalities in their tables, respectively.

Example 3.3 shows which chords I have chosen to count in the Ut (left side) and Re-plus-Mi (middle) categories for the root/quality histograms. These chords are the top thirteen most common chords in each category, but "top" is defined in a special way. Working with the Re-plus-Mi category as an example, if we simply take the most commonly occurring chords in the aggregate of all 23 canzonas in that category, our numbers will be skewed by the large proportion of g/♭ canzonas (13 out of 23 pieces). So, instead, we calculate the occurrence of each chord as an average of the occurrences of that chord over each of the five tonalities in either category. These averages are listed the right-hand column of each table.

Chords in Ut	Average Occurrence
Cmaj	15.02%
Fmaj	12.25%
Gmaj	9.27%
Dmin	8.17%
Amin	7.11%
Bbmaj	5.04%
Gmin	3.42%
Emin	2.98%
Dmaj	2.12%
Bdim	1.55%
Amaj	1.50%
Cmin	1.08%
F#dim	0.89%

Chords in Re and Mi	Average Occurrence
Cmaj	10.76%
Dmin	10.31%
Fmaj	10.09%
Amin	10.00%
Gmin	4.90%
Bbmaj	4.40%
Emin	3.82%
Gmaj	3.40%
Emaj	3.38%
Amaj	3.02%
Dmaj	2.43%
Cmin	1.40%
Bdim	1.07%

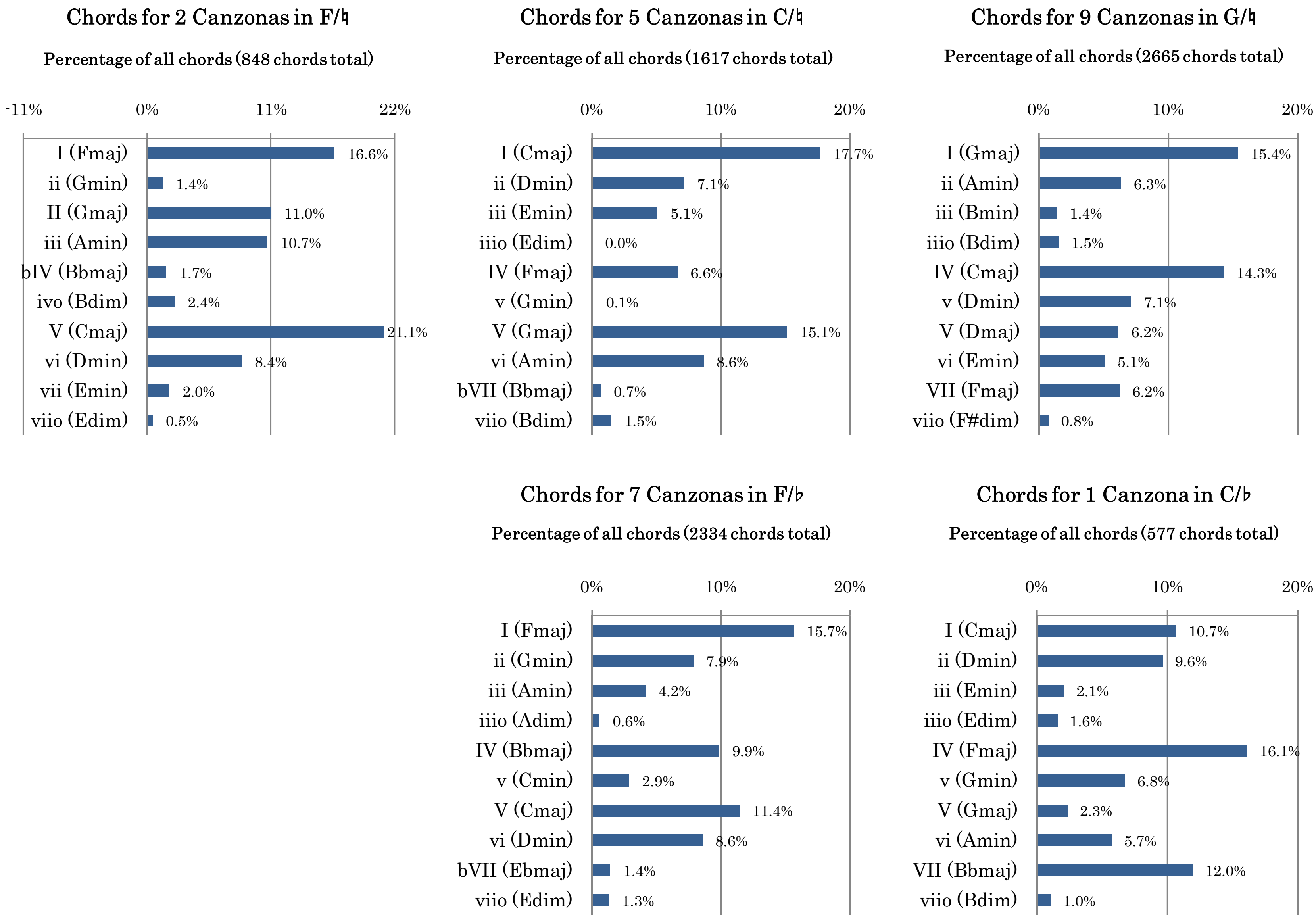
Combined Order	Average Occurrence
Cmaj	12.93%
Fmaj	11.19%
Dmin	9.22%
Amin	8.52%
Gmaj	6.40%
Bbmaj	4.73%
Gmin	4.14%
Emin	3.39%
Dmaj	2.27%
Amaj	2.25%
Bdim	1.32%
Cmin	1.23%

Ex. 3.3. Triads to be tallied in root/quality histograms—shaded chords are not shared

When making comparisons, there are three different characteristics that can be shared by any two tonalities: (1) the octave species, e.g., C/♮and F/♭; (2) the system, e.g., F/♭ and C/♭; or (3) the final, e.g., C/♮and C/♭. In the analyses that follow, I compare each possible pair using the Euclidean distance (see formula and examples in the next section between Roman-numeral and root/quality histograms, and then cluster the tonalities based on the resulting distances.

3.2.1. Comparing Roman-Numeral Histograms

I begin by comparing tonalities on the basis of Roman-numeral content. This representation of chords within a scale (as opposed to root and quality) can tell us which scale steps or "functions" make up a given tonality. We are interested to know whether or not tonalities with the same octave species are made up of the same functional triads (e.g., I, IV, and V) in the same proportions. The pairs of tonalities that share the same octave species are: C/♮ with F/♭ and G/♮ with C/♭ in the Ut category and d/♮ with g/♭ and a/♮ with d/♭ in the Re category. These four pairs are arranged vertically in Exs. 3.4 and 3.5, with each *cantus-durus* tonality directly above its *cantus-mollis* partner. At first glance, these pairs appear to have much in common, with many "peaks" and "valleys" in the same places in their bar graphs. This relationship is most striking between the two major tonalities, C/♮ and F/♭, both of which have peaks on the I and V triads. Both of these tonalities also have very few non-diatonic triads compared to the other Ut tonalities: 7.7% of the tallied triads in F/♭ and only 1.3% of the tallied triads in C/♮ are not diatonic.



Ex. 3.4. Roman-numeral histograms for Frescobaldi's lydian, major, and mixolydian tonalities

The mixolydian tonalities G/♮ and C/♭ have more non-diatonic triads than the major tonalities, as they require accidentals in the formation of the iii and, more importantly, V triads; 13.1% of the tallied triads in G/♮ and 8.0% of the tallied triads in C/♭ are not diatonic. The peaks of G/♮ and C/♭ appear on the I and IV triads, and to a lesser extent, the VII triad. These peaks are of different relative sizes: G/♮ has more I chords than C/♭, which, surprisingly, favours both IV and VII over I. In the analyses that follow, we will see that C/♭ is not alone in having more "non-tonic" triads than "tonic" triads. In fact, four other tonalities (F/♮, a/♮, d/♭, and e/♮), for a total of five tonalities out of ten, behave similarly. In the two chapters that follow, we will see how Roman-numeral distribution changes over time and try to pinpoint when (and in which tonalities) the "tonic" decisively overtakes the other triads.

An important thing to remember is that the C/♭ tonality is represented by only a single canzona in this corpus, so although the percentages between G/♮ and C/♭ are comparable, the actual numbers of observed chords are not. For this reason, the mixolydian octave species, and in fact, all octave species are represented using the weighted average of their component tonalities: the nine G/♮canzonas are added together with the single C/♭canzona and the result is divided by ten. The "average" mixolydian tonality for Frescobaldi, then, is nine parts G/♮ and one part C/♭. This idea is important when combining tonalities into "clusters."Statistically, we need a good way to represent a cluster of tonalities as if it were a single tonality, so that it can be compared with individual tonalities, as well as other clusters; the weighted average, or "centroid" of a cluster is the standard statistical representation.[1]

We begin by measuring the "distance" between each pair of histograms. The simplest way to quantify the distance between two histograms is by taking the Euclidean distance between them.[2] This measurement has already been used in music theory for modeling chord progressions (Paiement, Eck, and Bengio 2005), as well as for content-based music similarity search (Li and Ogihara 2004) and algorithmic recognition and correction of rhythms and rhythmical evenness (Toussaint 2003). The Euclidean distance between two objects p and q is expressed as:

$$d(p,q) = \sqrt{\frac{(p_1-q_1)^2+(p_2-q_2)^2+\cdots+(p_n-q_n)^2}{n}}$$

where each expression in round brackets is the difference between the percentages for a given Roman numeral in a pair of histograms p and q, and n is the number of different Roman numerals we are including in the calculation. As we saw in Example 3.2, different pairs of histograms can have different numbers of Roman numerals in common (7, 10, or 13), so we divide by n before taking the square root to correct for disproportionate numbers of Roman numerals, which would otherwise inflate the Euclidean distance. A sample calculation is shown for the Euclidean distance between C/♮ and F/♭, with percentage values taken from the vertical pair of

[1]StatQuest: Hierarchical Clustering https://www.youtube.com/watch?v=7xHsRkOdVwo&t=4s
[2]StatQuest: Hierarchical Clustering https://www.youtube.com/watch?v=7xHsRkOdVwo&t=4s

bar graphs in the middle of Example 3.4 and an *n* value of ten, since all ten of the triads in this pair of histograms can be compared:

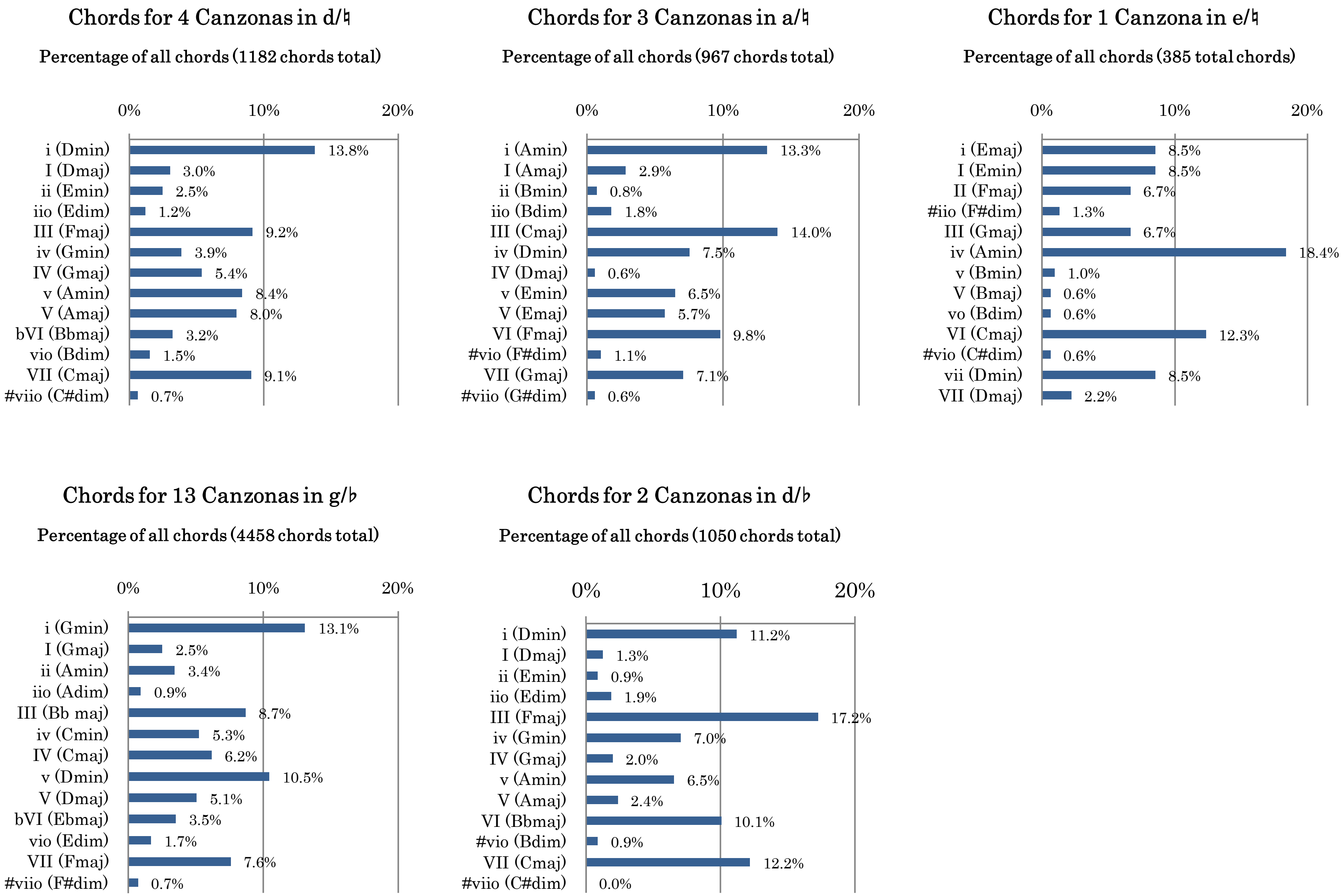
$$d(\mathrm{C}/\natural, \mathrm{F}/\flat) = \sqrt{\frac{(17.7-15.7)^2+(7.1-7.9)^2+\cdots+(1.5-1.3)^2}{10}}$$

$$= \sqrt{\frac{38.76}{10}}$$

$$= 1.97$$

On its own, this number (1.97) has no meaning, but when compared to the distances between other *durus-mollis* pairs, it tells us how much *more* or *less* similar the pairs are. This measurement serves as the basic means of comparing different tonalities in the works of the same composer, as well as similar tonalities in works by different composers (see Chapter 7). The Euclidean distance between the pair of mixolydian tonalities is 2.91, which is greater than the distance between the pair of major tonalities, so the mixolydian pair can be said to be further apart, or *less similar*, than the major pair. What follows then, is that C/♮ and F/♭ can be understood as *transpositions* of one another with greater certainty than C/♭ can be understood as a transposition of G/♮ or vice versa.

One final point about the Euclidean distance: when measuring the distance between a pair of histograms, the Euclidean distance gives *equal weight* to each Roman numeral. This means that differences between rarely occurring triads like vii° are just as weighty as differences between commonly occurring triads like I.



Ex. 3.5. Roman-numeral histograms for Frescobaldi's dorian, minor, and phrygian tonalities

In the Re category (Example 3.5), the dorian tonalities d/♮ and g/♭ have peaks on the i, III, v, and VII triads, and these are of different relative sizes: d/♮ has more III and VII chords than g/♭, while g/♭uses more v chords than d/♮. The Euclidean distance for the dorian pair is 1.22. The minor tonalities, a/♮ and d/♭, also have peaks on the i, III, and VII triads, but instead of v, the next most common triad is VI. As was the case for the C/♭ tonality, we see that the most common triad in both of the minor tonalities is a "non-tonic" triad: III. The biggest difference within this pair is in the amount of VII triads, of which d/♭ has many more than a/♮. The only other *durus-mollis* pair where we saw such a large difference (more than 5% for any pair of corresponding bars) was the mixolydian pair, for which there was a large disparity in the numbers of canzonas representing each tonality. The minor tonalities, in contrast, are represented by roughly equal, if small numbers of canzonas: three works in a/♮ and two in d/♭. The Euclidean distance for the minor pair is 2.10.

Example 3.6 tabulates the Euclidean distances for all pairs of tonalities in the Ut and Re-plus-Mi categories. Distances between *durus-mollis* pairs with the same octave species are shaded light grey and these four distances are the smallest in both tables. Distances between pairs of tonalities that share the same final are shaded dark grey and these distances are not particularly small, especially those between the two F-final tonalities and the two C-final tonalities. The reason they are highlighted here is because, as we progress through the rest of the corpus (and the rest of the seventeenth century), we want to observe these pairs of same-final tonalities carefully to see if they grow closer together from a Roman-numeral perspective. The two Re tonalities with D finals, d/♮ and d/♭, are of special interest since they both appear in Corelli, wherein the former is often understood to be an instance of D minor with an "incomplete key signature."

Euclidean Distances Between Frescobaldi's Ut Tonalities

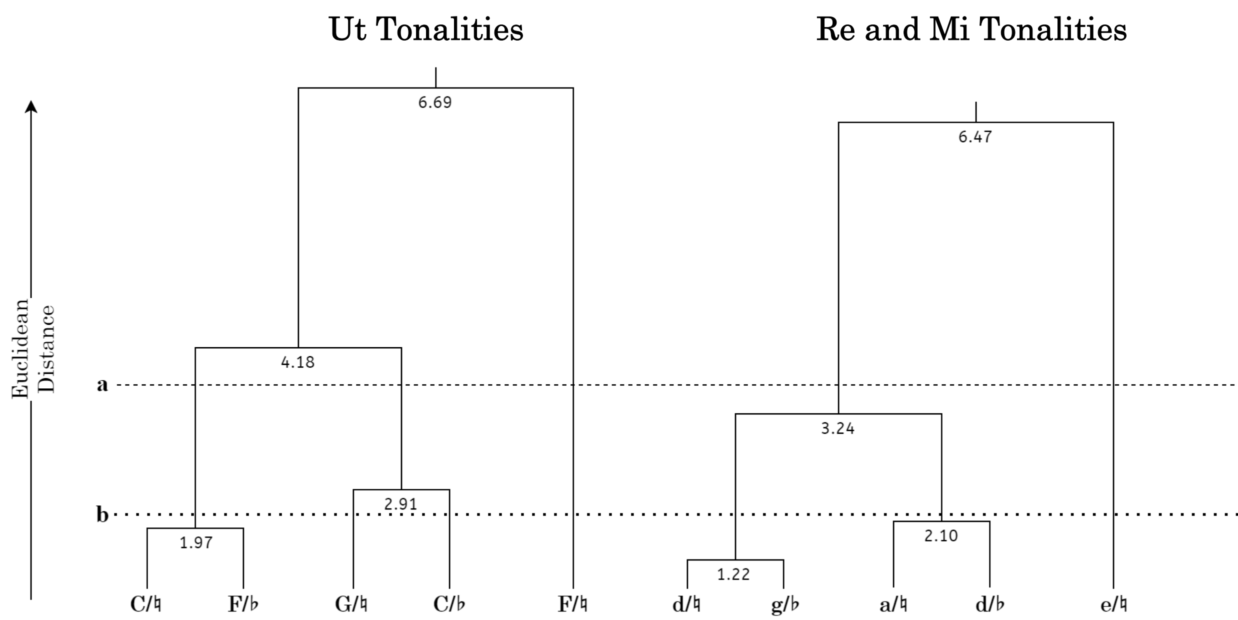
	F/♮	C/♮	F/♭	G/♮	C/♭
F/♮	0	4.26	5.93	8.49	10.31
C/♮		0	1.97	5.04	7.08
F/♭			0	3.35	5.38
G/♮				0	2.91
C/♭					0

Euclidean Distances Between Frescobaldi's Re and Mi Tonalities

	d/♮	g/♭	a/♮	d/♭	e/♮
d/♮	0	1.22	3.03	3.84	7.17
g/♭		0	3.15	3.86	6.71
a/♮			0	2.10	5.59
d/♭				0	6.63
e/♮					0

Ex. 3.6. *Roman-numeral Euclidean distances between all pairs of Ut tonalities (left) and all pairs of Re and Mi tonalities (right)—lightly shaded cells mark durus-mollis pairs with the same octave species, darkly shaded cells mark pairs with same the final*

The final step in understanding our Roman-numeral histograms and the distances between them is a process called "clustering." Based on the distance values in Example 3.6, we can cluster together the tonalities that are most similar using the statistical technique Agglomerative Hierarchical Clustering (AHC). Agglomerative (as opposed to divisive) hierarchical clustering works from the bottom up, combining objects (in this case, tonalities) pair-wise based on some measure of similarity (in this case, Euclidean distance). Once a pair is combined, it is treated as a single object and then compared against the remaining objects until everything has been paired.[3] When considering Roman numerals, this clustering procedure is done on a per-category basis: one analysis for Ut tonalities, one for Re and Mi tonalities. This separation exists for the same reason that it does in Example 3.6: we cannot compare between two larger categories (e.g., Ut with Re) because there are not enough Roman numerals in common.



*Ex. 3.7. Dendrograms based on Roman-numeral distances within the Ut and Re-plus-Mi categories—decimal numbers are Euclidean distances between linked tonalities (from Ex. 3.6) or clusters—cutting the trees at different lines (line **a** or line **b**) produces different numbers of clusters (5 clusters or 7 clusters, respectively)*

The results of these analyses are represented in Example 3.7 (above) as tree diagrams or "dendrograms" which plot all of the distances relative to one another along a single vertical axis. Because our clustering is agglomerative (i.e., bottom-up), the dendrograms are shown with the "leaves" at the bottom and the "roots" at

[3]StatQuest: Hierarchical Clustering https://www.youtube.com/watch?v=7xHsRkOdVwo&t=4s

the top. If we "cut" the trees horizontally at different heights (line a or line b), different relationships (discussed below) become visible. The heights of the solid, horizontal connecting lines in these diagrams are additive. The connecting line labeled 4.18 (just above line a on the left of Example 3.7), for example, is at a height (measured up from the bottom of the dendrogram) of 7.09, which is the sum of 4.18 and the largest number below it, 2.91. Likewise, the connecting line labeled 3.24 (on the right of Example 3.7) is at a height of 3.24 + 2.10, or 5.34. These sums are not displayed as numbers anywhere in the dendrogram because they are already shown visually: the height of each connecting line represents the sum of the numbers below it. The numbers that do appear below each connecting line are the distances between the tonalities and/or clusters that are being connected.

The distances at the first four linkages in Example 3.7 (between d/♮ and g/♭, C/♮ and F/♭, a/♮ and d/♭, and G/♮ and C/♭) are taken directly from Example 3.6, but the distances at linkages above those must be generated in the clustering analysis. The numbers 3.24, 4.18, 6.47, and 6.69 are distances between the "centroids" (i.e., the weighted averages) of the clusters below them. Just as we represented the mixolydian cluster as nine parts G/♮ and one part C/♭, we represent every other same-octave-species pair as the weighted average of its component tonalities.

From the dendrograms, it is clear that the closest pair of tonalities, from a Roman-numeral perspective, is the dorian pair (Euclidean distance = 1.22). The next closest pairs are the major (Euclidean distance = 1.97) and minor (Euclidean distance = 2.10) pairs. If we "cut" the dendrogram by drawing a horizontal line across all the branches at any given point, the number of vertical lines that the horizontal line crosses is the number of clusters at that distance. At line b, we have seven clusters: major, G-mixolydian, C-mixolydian, lydian, dorian, minor, and phrygian. From this, we see that the mixolydian pair is the last pair to form; of all the pairs that share an octave species, it is the most distant. Cutting instead at line a, we now have five clusters: major, mixolydian, lydian, *dorian-minor*, and phrygian. From this, we see that the dorian-minor cluster is the first "pair of pairs" to form; from a Roman-numeral perspective, the dorian and minor pairs are closer together than the major pair is to the mixolydian pair.

This similarity between dorian and minor tonalities is likely due, at least in part, to the fact that the sixth scale degree is often lowered in dorian tonalities. Looking back at Example 3.5, we see that in both d/♮ and g/♭canzonas, Frescobaldi prefers to build a major triad on the lowered sixth scale degree (♭VI) than to use the diatonic vi^{o} triad or the minor vi triad (not plotted), which requires that the third scale degree be raised by a half step and is even less common than the diminished triad. Frescobaldi also opts, about half of the time, for a minor iv triad in both dorian tonalities, even though major IV is natural to the scale (we will see what syntactical roles the IV and iv triads play in the n-gram analyses in Section 3.3). The routine deployment of these two altered triads (♭VI and iv) bring the dorian tonalities very close to the minor from a Roman-numeral perspective.

The lydian and phrygian tonalities, F/♮ and e/♮, are the most distant tonalities in their respective categories; the pairs of pairs (major-mixolydian and minor-dorian) are closer together than the lydian and phrygian tonalities are to any other octave-species pair within their category, so F/♮ and e/♮ remain “clusters” of their own. Suffice to say, these two tonalities are difficult to compare with anything else from a Roman-numeral perspective. From a root/quality perspective, however, we are able to make different observations, as we will see below.

It is important to keep in mind that, like most of the findings in this chapter, the dendrograms serve not as an end result, but as the first “time point” in a series of observations we will make throughout the seventeenth century. The comparison of Frescobaldi’s Roman-numeral dendrograms with those of Legrenzi and Corelli is the ultimate goal.

3.2.2. Comparing Root/Quality Histograms

Roman numerals are a powerful conceptual device for comparing tonalities, but they are also restrictive because no two octave species have the same set of seven diatonic triads (at most, any two octave species share only four diatonic triads). Root/quality histograms provide a different way of comparing tonalities and order chords based on prevalence instead of function. In root/quality histograms, chords are labeled with a root (i.e., letter name) and quality. Because all of the chords in the histograms for Frescobaldi are triads, there are only three possible qualities: major (maj), minor (min), and diminished (dim). For a complete list of qualities used in this project, refer to Example 2.8. The same twelve triads are tallied for all ten tonalities, allowing us to compare tonalities in different categories (e.g., major with minor). A thirteenth triad (based on prevalence) is tallied for each of the Ut and Re-plus-Mi categories, but in each category, that triad is different: F#dim is included for the Ut category, while for the Re-plus-Mi category, Emaj is included.

	F/♮ (2)	C/♮ (5)	F/♭ (7)	G/♮ (9)	C/♭ (1)	AVG
Cmaj	21.07%	17.69%	11.43%	14.25%	10.65%	15.02%
Fmaj	16.64%	6.64%	15.65%	6.24%	16.10%	12.25%
Gmaj	10.99%	15.12%	2.52%	15.37%	2.34%	9.27%
Dmin	8.40%	7.14%	8.56%	7.13%	9.61%	8.17%
Amin	10.69%	8.64%	4.16%	6.33%	5.71%	7.11%
Bbmaj	1.68%	0.66%	9.85%	1.07%	11.95%	5.04%
Gmin	1.37%	0.08%	7.85%	1.07%	6.75%	3.42%
Emin	1.98%	5.07%	0.70%	5.08%	2.08%	2.98%
Dmaj	0.76%	1.41%	0.70%	6.15%	1.56%	2.12%
Bdim	2.44%	1.50%	1.23%	1.54%	1.04%	1.55%
Amaj	0.61%	1.58%	1.06%	3.49%	0.78%	1.50%
Cmin	0.00%	0.17%	2.87%	0.00%	2.34%	1.08%
F#dim	1.68%	1.08%	0.94%	0.75%	0.00%	0.89%

Ex. 3.8a. Top 13 commonest triads in the Ut tonalities—tonalities are listed across the top row of the table and triads are listed on the left in descending order with the commonest triad (Cmaj) at the top—all numbers are percentages of the total number of chords in the given tonality—larger numbers are shaded darker

Separate tallies for each category are displayed as heat maps in Examples 3.8a (Ut tonalities) and 3.8b (Re and Mi tonalities). The un-coloured averages in the rightmost column are the same averages listed in Example 3.3 that were used to determine which chords would be tallied.

	d/♮ (4)	g/♭ (13)	a/♮ (3)	d/♭ (2)	e/♮ (1)	AVG
Cmaj	9.06%	6.19%	14.01%	12.20%	12.34%	10.76%
Dmin	13.81%	10.46%	7.53%	11.19%	8.54%	10.31%
Fmaj	9.17%	7.59%	9.79%	17.23%	6.65%	10.09%
Amin	8.41%	3.43%	13.25%	6.54%	18.35%	10.00%
Gmin	3.88%	13.11%	0.45%	7.04%	0.00%	4.90%
Bbmaj	3.24%	8.69%	0.00%	10.06%	0.00%	4.40%
Emin	2.48%	0.73%	6.48%	0.88%	8.54%	3.82%
Gmaj	5.39%	2.53%	7.08%	2.01%	6.65%	3.40%
Emaj	2.59%	0.03%	5.72%	0.00%	8.54%	3.38%
Amaj	7.98%	1.57%	2.86%	2.39%	0.32%	3.02%
Dmaj	3.02%	5.06%	0.60%	1.26%	2.22%	2.43%
Cmin	0.00%	5.26%	0.00%	1.76%	0.00%	1.40%
Bdim	1.51%	0.52%	1.81%	0.88%	0.63%	1.07%

Ex. 3.8b. *Top 13 commonest triads in the Re and Mi tonalities— tonalities are listed across the top row of the table and triads are listed on the left in descending order with the commonest triad (Cmaj) at the top—all numbers are percentages of the total number of chords in the given tonality—larger numbers are shaded darker*

From a root/quality perspective, the Ut tonalities that appear most similar are those that use the same system (*cantus durus* or *cantus mollis*). For the Re and Mi tonalities (below), the same trend is not as apparent; the only two tonalities that are visually similar in this heat map are a/♮ and e/♮.

Now, the same analytical steps explained in Section 3.2.1 are carried out for the root/quality histograms. Euclidean distances are calculated for all 45 possible pairs of tonalities and tabulated in Example 3.9 (below).

Distances between same-system pairs are shaded light grey and distances between "relative" pairs are shaded dark grey. Looking at Example 3.9, we notice that the distances between same-system pairs (shaded light grey) are, on average, smaller than the distances between different-system pairs (un-shaded). Looking back at the heat maps, we also see that the presence or absence of a signature flat has a large effect on the presence or absence of certain triads like Bbmaj, Gmin, and Cmin, which occur infrequently or not at all in the *cantus-durus* tonalities.

Root/Quality Euclidean Distances Between Frescobaldi's Tonalities

	F/♮	C/♮	F/♭	G/♮	C/♭	d/♮	g/♭	a/♮	d/♭	e/♮
F/♮	0	3.39	5.03	4.33	5.23	5.28	7.48	3.52	4.98	5.06
C/♮		0	6.03	1.86	6.23	4.93	7.21	3.09	6.41	4.20
F/♭			0	5.91	1.03	4.34	3.55	5.34	1.29	6.66
G/♮				0	6.08	4.41	6.50	3.70	6.34	4.72
C/♭					0	4.37	3.80	5.38	1.12	6.51
d/♮						0	4.41	3.62	3.92	4.83
g/♭							0	6.47	3.98	7.34
a/♮								0	5.23	2.20
d/♭									0	6.63
e/♮										0

Ex. 3.9. Root/quality Euclidean distances between all pairs of tonalities—lightly shaded cells mark same-system pairs, darkly shaded cells mark "relative" pairs

The three smallest distances in the table are between F/♭ and C/♭ (1.03), C/♭ and d/♭ (1.12), and d/♭ and F/♭ (1.29), which suggests that these three *cantus-mollis* tonalities together form a sort of "complex" of tonalities, in which the same triads appear in the same relative quantities, but the finals are in three different places. This more than anything we have seen so far in Frescobaldi suggests that the tonalities this composer uses are not related to one another in any way that resembles what we would associate with the common practice. Looking back at the bar graphs in Examples 3.4 and 3.5, we see that C/♭ and d/♭ are two of the five tonalities that have "non-tonic" triads as their commonest triads, and that Fmaj is the commonest triad in both. In all three of these tonalities, the four commonest chords are Fmaj, Cmaj, Bbmaj, and Dmin. The only other *cantus-mollis* tonality that Frescobaldi writes, g/♭, escapes this complex by using far fewer Cmaj and Fmaj triads and introducing Dmaj and Cmin—the latter via the lowered sixth scale degree discussed at the end of Section 3.2.1—in large amounts.

The F/♭-d/♭ pair (Euclidean distance = 1.29) is also what I am calling a "relative" pair, as we do for certain pairs of modern major and minor keys. Today, we explain relative keys in much the same way as I have explained the F/♭-C/♭-d/♭complex: two keys that share the same seven diatonic triads but whose tonics are different. Frescobaldi's C/♭ tonality, then, is also a kind of "relative" to F/♭ and d/♭: the "relative mixolydian." Another feature that we ascribe to relative pairs nowadays is that it is easy to move between them, whether by tonicizing or modulating to the relative major from the minor, or vice versa. This idea will come back into focus when we discuss chord progressions in Section 3.3. The other "relative" pair is the C/♮-a/♮ pair (Euclidean distance = 3.09), which is not as close as the F/♭-d/♭ pair, but still among the six closest pairs in Example 3.9 and is the third closest pair of *cantus-durus* tonalities.

The most famous pair of related (but not necessarily "relative") tonalities in the literature is the a/♮-e/♮ pair (Euclidean distance = 2.20). Referring to the harmonic consequences of this relationship, Gregory Barnett writes that "several sonata composers of the late seventeenth century produced… concluding passages that modern tonal theory would classify as A-minor compositions that end on their dominants", giving examples by Legrenzi and Corelli (Barnett 2008, 245). A similar observation was made in 1657 by Christoph Bernhard, who wrote that the sixth mode (i.e., E plagal) is "very closely connected with the twelfth mode [i.e., A plagal], just as its authentic counterpart [i.e., E authentic] is associated with the eleventh [i.e., A authentic], so that these can hardly be distinguished from one another save at the end" (Hilse 1973, 129). For Bernhard, this affinity is "due to the fact that the close on b♮ is not fully acceptable, or else demands substantial alterations in the semitones" (Hilse 1973, 129). From this, it would appear that Bernhard expects a cadence to the minor dominant, which almost never happens in the e/♮ tonalities we will look at.

This same relationship was discussed about 100 years before Bernhard by both Glarean and Zarlino. For Glarean, it has to do with the complementary ranges of adjacent voices in a polyphonic composition: "whenever the tenor is Phrygian, the bass and cantus often fall into the Aeolian" (Glarean 1574, 250). For Zarlino, modes on E and A—specifically the third (E authentic) and tenth (A plagal)—are closely related because they are "contained within the same diapason, E to e… in one mode this diapason is mediated harmonically and maintains the form of the third mode, whereas in the other it is divided arithmetically and maintains the form of the tenth mode" (Zarlino 1983, 89). Zarlino finishes his explanation by making the same observation we see in Bernhard and Barnett after him: "[T]he composition that we judge to be in the third mode no longer has anything by which we can judge it to be in this mode, except for the final, for the composition ends on the note E" (Zarlino 1983, 89).

For Carissimi, a composer close to Frescobaldi both temporally and geographically, e/♮ is no longer a stand-alone tonality, at least in vocal compositions; it does not appear at all in the corpus of cantatas discussed by Stein (2002). Stein's assertion that "[b]y Carissimi's time the phrygian mode had largely been subsumed under the aeolian" again alludes to the same relationship observed by Barnett, Bernhard, and the sixteenth century authors: that two tonalities can comprise the same musical materials (in this case, the triads Amin, Cmaj, Dmin, and Emin),but still have different finals (A and E). Among all of Frescobaldi's tonalities, a/♮ is the closest to e/♮ from a root/quality perspective. The next closest tonality to e/♮ is C/♮ (Euclidean distance = 4.20).

The F/♮ tonality is closest to C/♮ from a root/quality perspective (Euclidean distance = 3.39). F/♮ is another tonality whose "tonic" is less common than another one of its diatonic triads—in this case, V (Cmaj)[4]—and whose relationship to its closest root/quality partner (the C/♮ tonality) mirrors the relationship between a/♮and e/♮: the less common tonality can be said to be "subsumed" by the more common. These relationships will be examined from another angle in Section 3.3

[4] The F/♮ tonality has so many Cmaj chords that the horizontal axis of its bar graph in Example 3.4 had to be adjusted to accommodate the V bar.

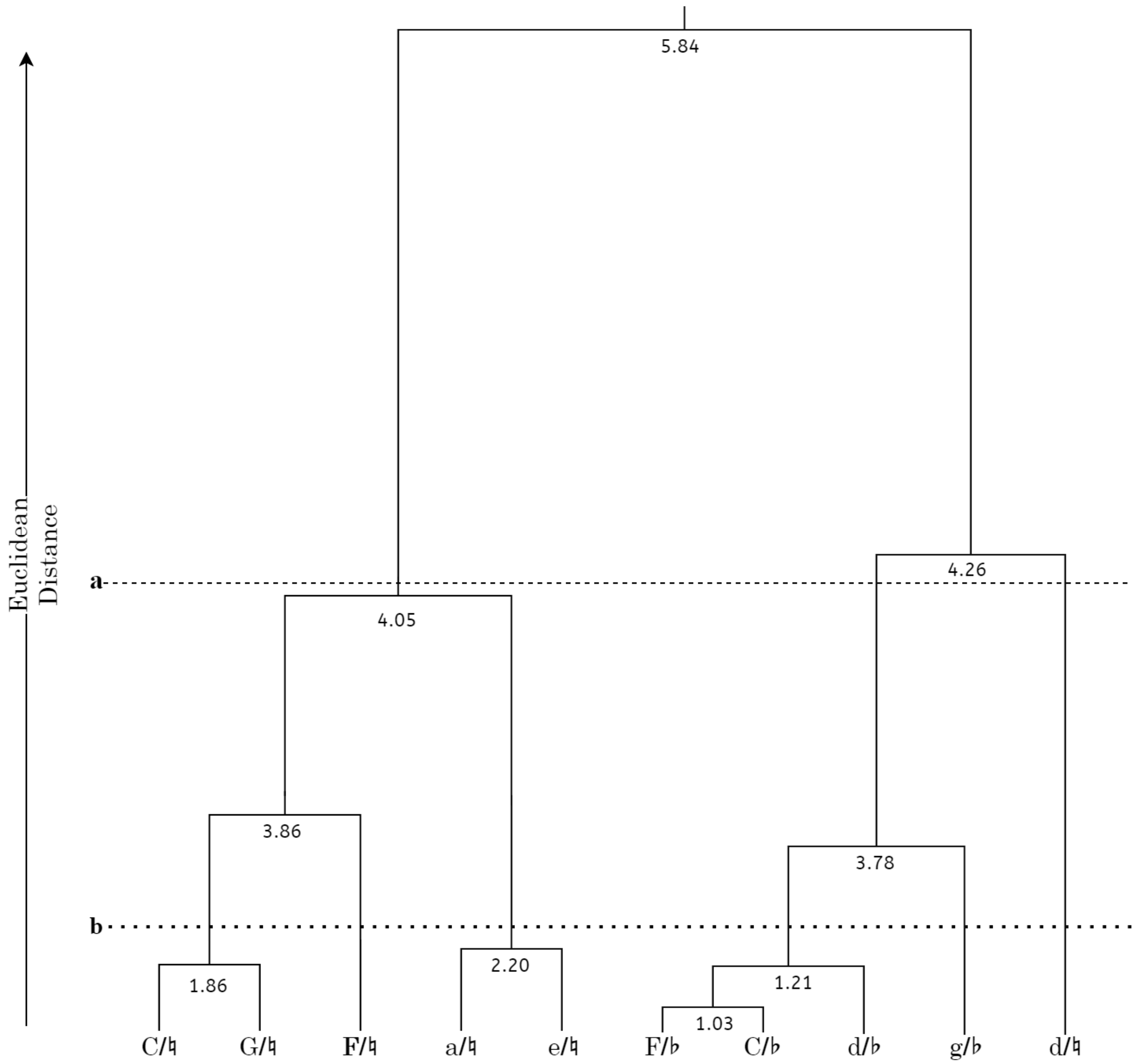
where we show how chord progressions help differentiate lydian from major and phrygian from minor.

One final pair of tonalities is of interest before we discuss the clustering results. The d/♮-d/♭ pair (Euclidean distance = 3.92), which serves as a sort of "barometer" in this project for measuring the similarity between minor and dorian tonalities more generally, is the *durus-mollis* pair (un-shaded cells of Example 3.9) with the smallest distance from a root/quality perspective. Again, the widespread use of accidentals lowering the sixth scale degree in the dorian tonality is the most likely explanation for this similarity. Because the d/♮ tonality uses B-flats more freely than any other *cantus-durus* tonality, it actually sits between *durus* and *mollis* from a root/quality perspective. Surprisingly, the three tonalities farthest from d/♮ are F/♮ (Euclidean distance = 5.28), C/♮ (Euclidean distance = 4.93), and e/♮ (Euclidean distance = 4.83), all of which are "same-system" pairs with d/♮. The closest tonality to d/♮ is a/♮ (Euclidean distance = 3.62), but on average d/♮ is closer to the *cantus-mollis* tonalities than it is to the *cantus-durus* tonalities. This shows that the use of accidental B-flats to lower the sixth scale degree in d/♮ is enough to make this tonality sound more like a *cantus-mollis* tonality from a root/quality perspective.

The dendrogram in Example 3.10 illustrates the root/quality distances between all ten of Frescobaldi's tonalities. Like the dendrograms in Example 3.7, Euclidean distances are plotted along a vertical axis, with distances increasing as we move upwards. Unlike the dendrograms in Example 3.7, there is now a single tree for all ten tonalities. This is because we are able to compare and obtain a distance value between any two tonalities (or clusters of tonalities) from a root/quality perspective by tallying the same twelve triads for all ten tonalities.

In Example 3.10, the *cantus-mollis* tonalities form a clear group; F/♭ and C/♭ are the first two tonalities to be joined in a cluster (Euclidean distance = 1.03) and d/♭ is close behind (i.e., above) them (Euclidean distance = 1.21). The g/♭ tonality eventually joins the *cantus-mollis* cluster, but not until much further up (Euclidean distance = 3.78). The last tonality to join the *mollis* cluster is actually a *durus* tonality: d/♮. The d/♮ tonality, as suggested earlier, has slightly more in common with the *mollis* tonalities (Euclidean distance = 4.26) than it does with the *durus* tonalities (Euclidean distance = 4.61, calculated for comparison but not shown in Example 3.10) because of the use of accidental B-flats.

On the *cantus-durus* side, C/♮ and G/♮ are the first to join (Euclidean distance = 1.86) and next are a/♮ and e/♮ (Euclidean distance = 2.20), but the a/♮-e/♮ cluster does not join with the C/♮-G/♮ cluster until after the latter has been joined by F/♮. This hierarchy stands in contrast to what we saw in the Roman-numeral dendrograms in Example 3.7: the phrygian and lydian tonalities, which were the most distant from the other tonalities in their respective categories from a Roman-numeral perspective, are now grouping together with other tonalities at Euclidean distances of only 2.20 (a/♮ with e/♮) and 3.86 (F/♮ with C/♮ and G/♮) from a root/quality perspective. Also worth noting is the fact that the first three pairs to form (F/♭-C/♭, C/♮-G/♮, and a/♮-e/♮) are all fifth-related pairs.



*Ex. 3.10. Dendrogram based on the root/quality distances between all tonalities (from Ex. 3.9)—decimal numbers are Euclidean distances between linked tonalities or clusters—cutting the trees at different lines (line **a** or line **b**) produces different numbers of clusters (**4** clusters or **6** clusters, respectively)*

Cutting the tree at different horizontal lines, we create different "thresholds" of similarity that allow us to compare clusters. Line b crosses the dendrogram at six points, so cutting the tree here creates six clusters, half of which are individual tonalities: the d/♮, g/♭, and F/♮ tonalities remain single, while C/♮-G/♮, a/♮-e/♮, and F/♭-C/♭-d/♭ are already clusters. Using this threshold, we can compare the two rarest octave species: phrygian and lydian. The joint between a/♮ and e/♮ is below the threshold, while the joint between F/♮ and the C/♮-G/♮ cluster is not. This comparison provides some empirical basis for Stein's claim that by about 1640 "the phrygian mode had largely been subsumed under the aeolian" (Stein 2002, 291) and may offer a hint as to why the e/♮ tonality appears in the works of Legrenzi and Corelli,[5] while the F/♮ tonality does not. At this point, we could say that F/♮, although closer to other tonalities from a root/quality perspective than from a Roman-numeral perspective, was still not similar enough to some more common tonality to have been "subsumed under" or assimilated into it.

Cutting instead at line a gives us four clusters, all of different sizes: F/♭-C/♭-d/♭-g/♭, C/♮-G/♮-F/♮, a/♮-e/♮, and the singleton d/♮. This comparison highlights, once again, the special place of d/♮ as a tonality that is neither very close to nor far from any other group of tonalities from a root/quality perspective. Although its signature is empty, it has enough Edim, Gmin, and Bbmaj triads to put it closer to the *mollis* tonalities than to the *durus*.

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[5] In Corelli's trio sonatas, a single e/♮ sonata falls into Barnett's category of "A-minor compositions that end on their dominants" (Barnett 2008, 245).

3.2.3. A Closer Look at the G/♮ Tonality

In his 1998 article "From Psalmody to Tonality," Harold Powers writes that a "contrast between two different G tonalities both with the major third—two kinds of "G major"—is found in a number of sets of compositions throughout the seventeenth century" (Powers 1998, 286). The contrast Powers is referring to is one that he finds in Pontio's *Ragionamento di musica* of 1588. Pontio explains that the two G modes (octenary modes 7 and 8) have the same principal and final cadences, but that mode 8 should use "the degree C *sol-fa-ut* as principal cadence, so that this mode may be the better recognized and distinct from mode 7, in that it [i.e., C] is the degree that makes the reciting tone of the psalm tone for the mode in question" (Powers 1998, 285). Powers then shows Pontio's example duos for modes 7 and 8 and contrasts the importance of C in the mode 8 duo with the importance of D in the mode 7 duo. Because I do not look at actual cadences, the analyses in this section make the assumption that we can measure the "importance" of a scale degree by observing the prevalence of triads that have that scale degree as their root.

To see whether or not Frescobaldi makes the same distinction as Pontio, I counted the eight commonest triads in all nine of the G/♮ canzonas. These data are visualized as a heat map in Example 3.11 (below) with the largest values coloured green and the smallest values coloured yellow. All numbers in coloured cells are percentages of the total number of chords in one canzona. The first thing that stands out is the prominence of Gmaj and Cmaj in every one of the nine works; in all but one of these canzonas (F8.36a), Gmaj and Cmaj are the two commonest triads (not necessarily in that order). Comparing the prevalence of D with that of C, we see that neither Dmaj nor Dmin ever outstrip Cmaj, however if we combine the percentages for Dmaj and Dmin, the four canzonas on the left (F8.34c, F8.10, F10.11, and F8.36a) actually do use more D triads than C triads.

	F8.34c	F8.10	F10.11	F8.44	F8.36a	F8.37a	F8.35	F8.30c	F8.48	MEAN	VAR
Gmaj	9.75%	9.61%	15.89%	19.39%	6.69%	10.80%	9.17%	16.00%	15.77%	12.56%	18.19
Amaj	4.72%	5.34%	0.85%	3.57%	3.04%	2.82%	0.92%	3.20%	2.51%	3.00%	2.25
Amin	5.03%	5.69%	6.14%	5.61%	4.26%	3.76%	3.06%	4.00%	7.89%	5.05%	2.17
Cmaj	7.55%	11.03%	10.59%	15.31%	10.33%	12.68%	9.79%	14.00%	15.41%	11.85%	7.16
Dmaj	6.29%	4.98%	4.03%	12.24%	3.65%	2.82%	2.75%	5.60%	5.02%	5.26%	8.30
Dmin	6.29%	8.19%	7.20%	3.57%	6.38%	7.04%	4.59%	3.20%	3.58%	5.56%	3.42
Emin	4.09%	5.34%	3.18%	9.18%	3.04%	2.82%	1.83%	4.00%	5.73%	4.36%	4.77
Fmaj	3.46%	5.34%	4.45%	3.06%	7.90%	6.10%	3.36%	3.60%	7.89%	5.02%	3.65

Ex. 3.11. Heat map showing differences in the prevalence of the eight commonest triads in Frescobaldi's G/♮ canzonas—each column of the table is a canzona, each row is a triad—the leftmost canzona has the greatest preference for Dmaj and Dmin over Cmaj, the rightmost canzona has the greatest preference for Cmaj over Dmaj and Dmin, canzonas in the middle use equal amounts of D and C triads

Should we treat these four canzonas as a separate group then? I believe we should not divide the G/♮ canzonas into two different groups because it is impossible to find a grouping (four plus five, three plus six, two plus seven, etc.) where both groups are more internally similar than the group of nine canzonas that we started with. I determined this using a statistical measurement called variance.

The variances for each of the eight commonest triads in Frescobaldi's G/♮ canzonas are shown in the uncoloured column (labeled VAR) on the far right of Example 3.11; these values are calculated based on the mean values in the column just to the left (labeled MEAN). Once we have obtained variance values for each triad, we can use the sum of these values to describe the variance of the group as a whole.[6] For all nine canzonas, the total variance is 49.91.

Comparing the individual variance values in the VAR column, we see that the triad that varies most in terms of prominence is, surprisingly, Gmaj. The next two most variable triads are, as Pontio might have predicted, Cmaj and Dmaj. If we separate the nine canzonas according to their preference for C vs. D, i.e., four canzonas where D is more prominent than C plus five canzonas where C is more prominent than D, we might expect the total variances for both groups to decrease, but this is not the case. When we remove the canzonas that prefer D, the C-preference group does become less variable, with a total variance of 38.60. The D-preference group, however, becomes more variable (i.e., less internally consistent) than the original G/♮ group; the total variance of the D-preference group increases to 63.09. This four-plus-five grouping makes the most sense according to the contrast described by Powers and Pontio, but even if we try different groupings of the G/♮ canzonas, there is never a case where both groups have lower total variances than the group of nine. So, while I recognize that certain G/♮ canzonas use more C triads than D triads, and vice versa, and that this difference may be due to the different modal origins of these canzonas, I choose not to make any distinction between them for the reasons described above.

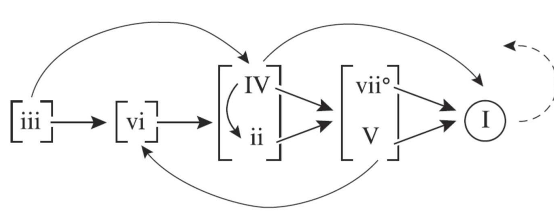
[6] http://www.stat.yale.edu/Courses/1997-98/101/rvmnvar.htm

We have seen how certain groups of Frescobaldi's tonalities are related on the basis of chord content and how different conceptions of chords (Roman numerals within a scale or root/quality labels) reveal different relationships between the tonalities that contain them. The next step is to determine which chords are likely to precede and follow one another. To do this, we observe chord progressions that are two chords long.

3.3. Comparing Chord Progressions

From the data we have gathered for individual chords (1-grams), we can generate 2-grams by combining adjacent, non-identical chords. I consider chords to be non-identical if they have different roots or qualities (or both). Like 1-grams, 2-grams can be labeled using either Roman numerals (e.g., I↔IV) or root and quality (e.g., Cmaj↔Fmaj). For a complete list of the chord qualities used in this project, refer to Example 2.8.

As explained in Section 2.9.2, the data for 2-grams are presented in two ways: first, as histograms that rank progressions by popularity and second, as chord-syntax flowcharts inspired by the diagrams from *Tonal Harmony* (Kostka, Payne, and Almén 2018, 105).



Ex. 3.12. Flowchart for normative harmonic functions in major keys from Kostka, Payne, and Almén

3.3.1. 2-Grams as Roman-Numeral Tornado Charts

We begin with a discussion of the Roman-numeral tornado charts. Examples 3.13 and 3.14 (below) rank the most common progressions in each tonality in the Ut and Re-plus-Mi categories, respectively. Like the histograms in Examples 3.4 and 3.5, each tornado chart represents one tonality. Each tonality has a different ranking of 2-grams, so unlike the data for 1-grams, there is no standard order from graph to graph. Ranked 2-grams are represented with Roman numerals (and later, root and quality) on the left side of each graph and each pair of Roman numerals is connected with an arrow. For most 2-grams, this arrow is bidirectional, signifying the bidirectionality of the progression. A few progressions are unidirectional (i.e., they never go backwards) and these are represented using unidirectional arrows.

The unidirectionality or bidirectionality of a progression, as well as the degrees of bidirectionality of similar progressions, are of interest to us because chord progressions in tonal music are often represented as unidirectional, unless they involve the tonic triad. With this, we have an objective measurement with which we can qualify some of the historical expectations surrounding Corelli. If Corelli was really the first "tonal" composer—and if we accept a model of tonal harmony like the one in Example 3.12—then we expect to find that Corelli's commonest 2-grams are less bidirectional than those of either Frescobaldi or Legrenzi.

The x-axis of each tornado chart goes from negative to positive with its zero value near the middle. "Positive" bars (those that extend to the right of zero) represent the forwards version of a given progression, while "negative" bars (those extending to the left of zero) represent the backwards version. In reality, none of the numbers are negative; both bars represent positive numbers of counted progressions, which are reported as percentages of the total number of progressions in a given tonality. The "tornado chart" style, with forwards and backwards bars radiating from a central axis helps us quickly see the directionality of a progression. For example, the tenth and eleventh (bottom two) progressions in each tonality in Example 3.13 are, on average, more unidirectional than bidirectional. The sums of the forwards and backwards progressions for each 2-gram are indicated on the far right of each graph. These sums are the numbers that produce the ranking order for each tonality.

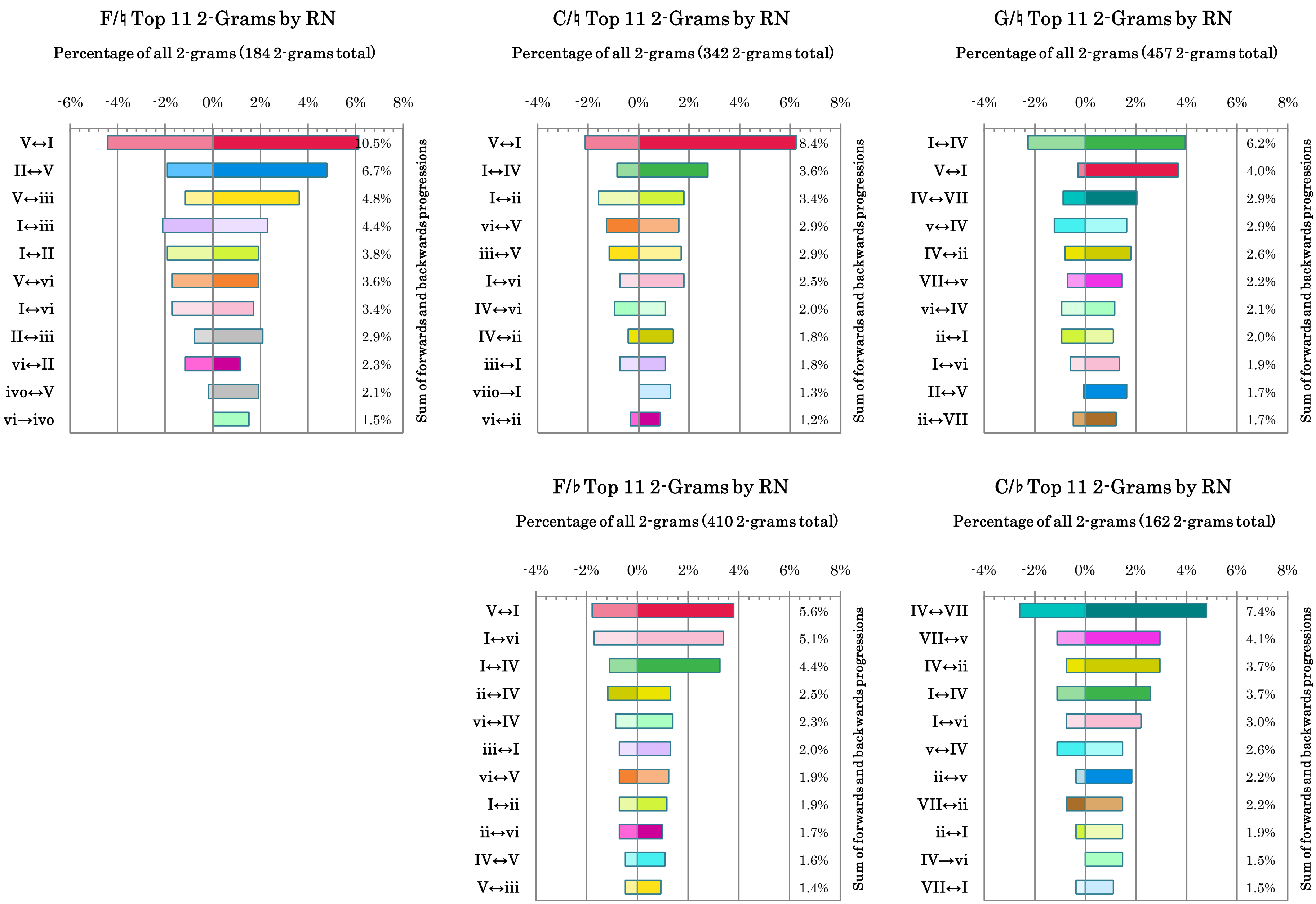
Because each tonality has a different ranking of 2-grams, colour coding is used to help common progression stand out. In Examples 3.13 and 3.14, Roman-numeral 2-grams that appear in at least two different tonalities receive a colour (e.g., the progression V↔I is always red), while 2-grams that appear in only one tonality are grey. For the Roman-numeral 2-grams (Examples 3.13 and 3.14), colour coding ignores the qualities of non-tonic triads, for example, ii↔V and II↔V are both coloured blue. This is not the case for the root-quality 2-grams shown in Examples 3.16 and 3.17.

The forwards and backwards versions of a progression are also coloured in a systematic way: the darker of the two bars indicates a root progression that moves by ascending fourth, ascending second, or descending third, while the corresponding lighter bar indicates a descending fourth, descending second, or ascending third. Since the larger of the two bars is always extending to the right (i.e., "positively"), this difference between dark and light helps us notice when progressions are "flipped" from one tonality to another. For example, in both the F/♭ and F/♮ tonalities, the progression involving V and iii (coloured yellow) is more likely to descend from V to iii (darker yellow is on the right), but in the C/♮ tonality, this progression is most likely to ascend from iii to V (lighter yellow is on the right).

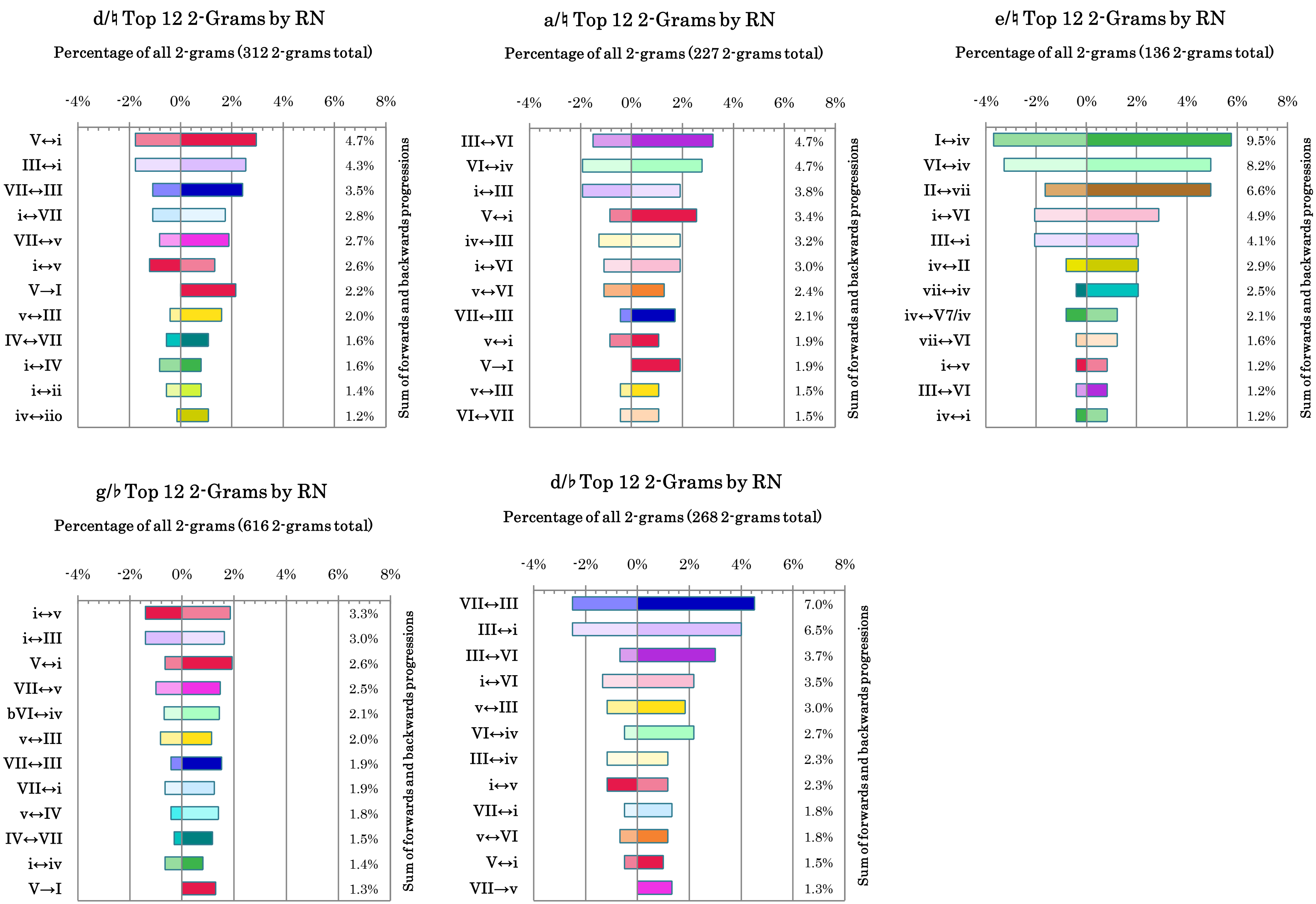
It must be mentioned that, for the sake of simplicity, I do not consider chordal inversions in any of my analyses. Although data for inversions were obtained, it was ultimately determined that the number of different possibilities introduced by inversions would dilute the results to the point where they were no longer useful for making comparisons. If we consider that each triad has three possible bass notes (these appear in different relative amounts depending on the triad), then there are nine different versions of each 2-gram (5/3↔5/3, 6/3↔5/3, 6/4↔5/3, 5/3↔6/3, etc.), about five of which are likely to occur for any given 2-gram in Examples 3.13 and 3.14 (but these are not always the same five versions for every 2-gram). While there are certainly patterns to be found among these permutations (e.g., descending-fifth progressions are likely to include more 6/3↔5/3 variants than other types of progressions), a proper investigation of them is beyond the scope of this project.

In each of the Ut tonalities (Example 3.13), there are 11 different 2-grams. The forwards and backwards versions of these 11 progressions account for 34% of all the progressions in a given tonality, averaged over the five tonalities in the Ut category. In the Re-plus-Mi category (Example 3.14), a twelfth 2-gram is added to keep the percentage of counted progressions the same (ca. 34%). Nearly all of the triads in these 2-grams require no accidentals, the only exceptions being: V, which appears in G/♮ (but not C/♭) and in all of the Re tonalities; II, which only appears in G/♮; I, which appears in all of the Re and Mi tonalities; and the three triads created when the sixth scale degree of a dorian tonality is flattened (ii°, which appears in d/♮, ♭VI, which appears in g/♭, and iv, which appears in both d/♮ and g/♭).

In general, progressions involving chromatically altered triads are more unidirectional. The progressions II↔V and V↔I in G/♮, for example, almost never go backwards. This creates a large contrast between G/♮ and the other Ut tonalities whose V↔I progressions are relatively bidirectional. In the Re tonalities, there are three kinds of V↔I progressions: v↔i, V↔i, and V↔I. These three progressions are hierarchically related in terms of directionality, with v↔i being the most bidirectional, V↔I being the most unidirectional, and V↔i somewhere in between. Although all three progressions do not appear in all four of the Re tonalities, the ones that do appear are always hierarchized in this way. Although its Roman



Ex. 3.13. Commonest progressions by Roman numeral in Frescobaldi's Ut tonalities



Ex. 3.14. Commonest progressions by Roman numeral in Frescobaldi's Re and Mi tonalities

numerals are different, the I↔iv progression in e/♮ has the same qualities and root distance as V↔i in the Re tonalities. Accordingly, its dirctionality is also comparable with that of V↔i (slightly forwards-leaning). This suggests that I↔iv in e/♮ could fulfull a similar role to the V↔i progression in Re tonalities.

The chromatically altered triads discussed so far require sharps; in Examples 3.13 and 3.14, triads with sharpened thirds are only involved in ascending fourth progressions, the majority of which are likely cadential. Chromatically altered triads that require flats behave differently. The i↔iv and ♭VI↔iv progressions, both of which appear in g/♭, are relatively bidirectional. Only the iv↔ii° progression (Gmin↔Edim), which appears in d/♮, is almost completely unidirectional. This is probably because the B♭-E tritone in the ii° triad desires a resolution to an F-A sixth, which could be harmonized with Fmaj (III) or Dmin (i), but not Gmin (iv).

I have shown how V triads interact with I triads, but are V triads approached by other triads besides I? The bidirectionality of V↔I progressions in the Ut tonalities and of V↔i progressions in the Re tonalities indicates that V is most often approached not from IV, ii, or II, but instead from I or i. Pedagogical abstractions of common-practice harmony, such as the one from Kostka, Payne, and Almén shown in Example 3.12 (above), might lead us to believe that this bidirectionality precludes tonal harmony in Frescobaldi, but previous empirical work on chord 2-grams by Christopher White and Ian Quinn (White & Quinn 2018) and David Temperley (Temperley 2009) shows that I→V is generally a more common progression than predominant→V. These observations made by White, Quinn, and Temperley are corroborated by my own findings in this and subsequent chapters.

While a handful of "predominant" triads do interact with V triads in the Ut tonalities (IV↔V in F/♭, II↔V in G/♮, and vi↔V in C/♮), none of the commonest twelve 2-grams in the Re and Mi tonalities are predominant↔dominant progressions. This is something that we expect to change over time, if the tonic→predominant→dominant model of harmony accurately describes the music of Corelli.

Certain "predominant" chords do in fact interact with V in the Re modes, but only rarely. In the a/♮ tonality, for example, Emaj and Esus chords are approached from Bm/h (the sonority B-D-A) and Ddom twice each in three canzonas as well as Bdim and Dmin once each in three canzonas. The only other chord that interacts consistently with the V triad is the iii triad in the Ut tonalities (the progressions coloured yellow in Example 3.13) and this happens far more often than a model like the one in Example 3.12 would predict.

As we did for the 1-gram histograms, we can compare the 2-gram tornado charts to see how many and which progressions are shared among different tonalities. Example 3.15 (below) tabulates the most widely shared 2-grams, with 2-grams that are more common in the Ut tonalities towards the top and 2-grams that are more common in the Re and Mi tonalities towards the bottom of the table.

2-Gram	Ut tonalities where it occurs	Re and Mi tonalities where it occurs	Total number of tonalities where it occurs
I↔II	5	1	6
I↔VI	5	3	8
IV↔II	4	2	6
I↔IV	4	3	7
V↔VI	3	2	5
V↔I	4	4	8
VI↔IV	4	4	8
VII↔V	2	3	5
IV↔VII	2	3	5
V↔III	3	4	7
VII↔III	0	4	4
III↔I	3	5	8

Ex. 3.15. 2-grams shared by different tonalties—lightly shaded rows show the most widely shared progressions and darkly shaded rows show progressions that appear very often in one category but not the other

The leftmost column of this table uses the same colour coding as Examples 3.13 and 3.14 and shows 2-grams without specifying quality (e.g., both the V↔iii progression in the Ut tonalities and the v↔III progression in the Re and Mi tonalities are represented as V↔III), a convention that I adopt in the discussion that follows. The four lightly shaded rows highlight the most widely shared 2-grams (those that appear in eight out of ten tonalities): I↔VI, which is slightly more common in the Ut tonalities; III↔I, which is slightly more common in the Re and Mi tonalities; and V↔I and VI↔IV, both of which appear in just as many Ut tonalities as Re and Mi tonalities. Three out of these four are progressions with root distances of a third and three out of four involve the "tonic" triad. The two darkly shaded rows highlight the 2-grams that are the most specific to their respective tonal categories:

the I↔II progression occurs in all five of the Ut tonalites and only one of the Re tonalities (d/♮), while the VII↔III progression occurs in all four of the Re tonalities and none of the Ut or Mi tonalities.

Compared to the model put forward by Kostka, Payne, and Almén (Example 3.12), these progressions paint a very different picture of the interactions between the diatonic triads in both "major" and "minor" tonalities. The three main differences are: (1) progressions involving the unaltered "tonic" triad in any tonality are bidirectional, in other words, the "tonic" triad can be approached from anywhere, not just from V and VII triads (the most striking example of this is the bidirectional I↔II progression, which is ubiquitous in the Ut tonalities); (2) any two triads that are a third apart and that exist within the same system (*durus* or *mollis*) interact with each other bidirectionally (with the exception of the vii° triad in F/♮, C/♮, and F/♭); and (3) these progressions with root distances of a third—compared with the "fifth progressions" and "step progressions" that predominate in Example 3.12—account for a substantial part of Frescobaldi's 2-gram vocabulary (about one third of it, on average). Interestingly, the ii→vii° progression is one of the only two third progressions "allowed" by the Kostka-Payne-Almén model; the other is IV→ii, which Frescobaldi uses often. As we move towards the end of the seventeenth century, assuming that tonal practice will conform increasingly to the model prescribed by Kostka, Payne, and Almén, we expect there to be some decline in the proportion of third progressions, and that the proportion of fifth progressions will increase.

As in the analyses of 1-grams, the representation of triads using Roman numerals makes tonalities with the same octave species look similar. Each same-octave-species pair of tonalities shares a minimum of nine Roman-numeral 2-grams: in the Ut category, the major tonalities share ten and the mixolydian tonalities nine out of eleven of their commonest 2-grams, while in the Re category, the dorian tonalities share nine and the minor tonalities ten out of twelve of their commonest 2-grams. Each of these pairs has a few 2-grams that set it apart from the other octave-species pairs: the major tonalities are characterised by V↔vi, iii↔I, and V↔iii progressions, while the mixolydian tonalities are characterised by IV↔VII, VII↔v, and v↔IV progressions.

In the Re category, the minor tonalities are characterised by the III↔VI, III↔iv, and i↔VI progressions, while the dorian tonalities are less reliant on progressions that are unique to that octave species; of the nine progressions shared by the two dorian tonalities, only the IV↔VII progression does not appear in either of the minor tonalities. These observations echo the conclusions we drew from Example 3.7: from a Roman-numeral perspective, the dorian tonalities overlap with the minor tonalities more than the major tonalities overlap with the mixolydian.

From a Roman-numeral perspective, we have seen which progressions best characterise which tonalities, how chromatic alterations affect the directionalities of similar progressions, and that many of the most common progressions in Frescobaldi's music defy the norms of common-practice harmony. Next we turn to a root/quality representation of the same data in order to better compare pairs of tonalities like a/♮-e/♮ and F/♭-d/♭ that do not share the same octave species.

3.3.2. 2-Grams as Root/Quality Tornado Charts

Examples 3.16 and 3.17 present the same data as Examples 3.13 and 3.14, in the same tornado-chart format, but now chords are labelled using root and quality. As a result, the colour coding of the 2-grams has changed, for example, the Cmaj↔Fmaj 2-gram is now always coloured red, regardless of the "functional" roles of those two triads within each tonality.[7] To make the charts more legible, chord qualities are represented in the following abbreviated way: major triads receive an upper-case letter (e.g., F), minor triads receive a lower-case letter (e.g., a), diminished triads receive a lower-case letter and an 'o' (e.g., bo), and all other chords receive a three-character quality label as outlined in Example 2.8. The only chord in these charts that is not a simple triad is the Edom7 chord in the e/♮ tonality.

Whereas the Roman-numeral colour coding revealed close relationships between tonalities with the same octave species, the root/quality colour coding reveals similarities between tonalities that use the same system. For example, all four of the *cantus-mollis* tonalities (F/♭, C/♭, g/♭, and d/♭) have substantial amounts

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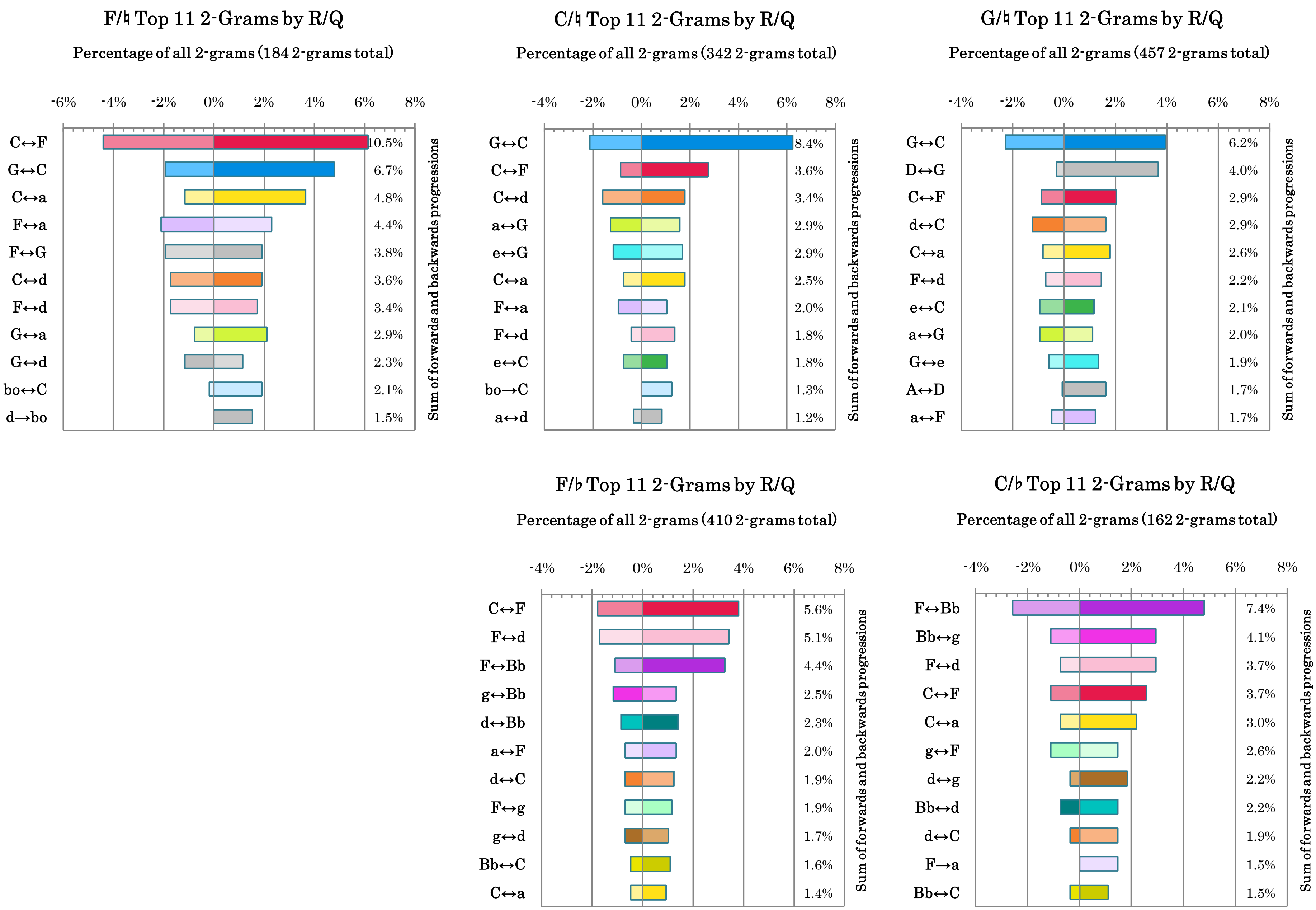
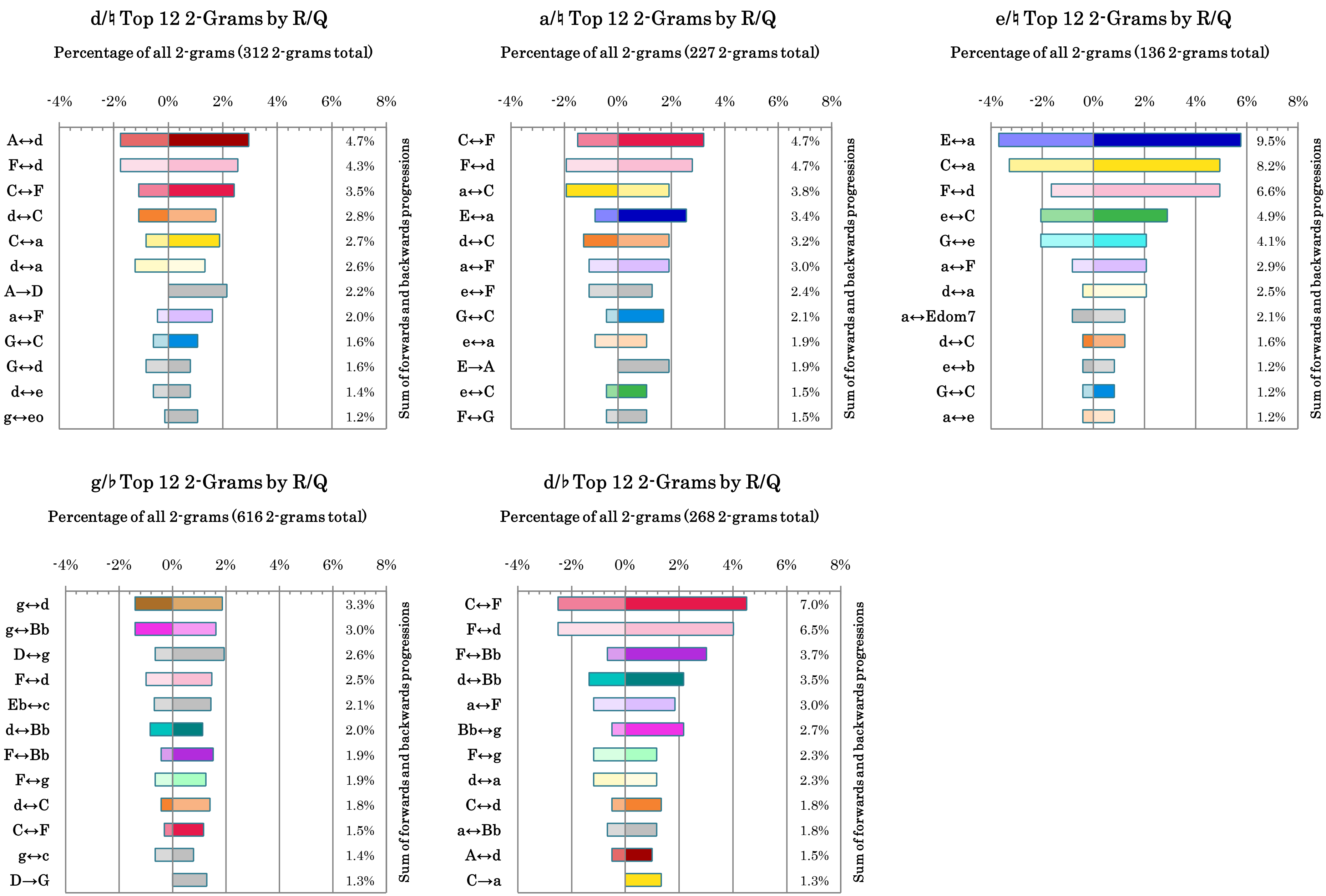
[7]The colour coding of the root/quality charts and the colour coding of the Roman-numeral charts have nothing to do with each other and should not be compared.



Ex. 3.16. Commonest progressions by root and quality in Frescobaldi's Ut tonalities



Ex. 3.17. Commonest progressions by root and quality in Frescobaldi's Re and Mi tonalities

of F↔d (pink), C↔F (red), F↔B♭ (purple), g↔B♭ (magenta), d↔B♭ (teal) progressions. Looking again at the 1-gram heat maps in Examples 3.8a and 3.8b, we see that the five different triads involved in these progressions (B♭, F, C, g, and d) are the five commonest triads in each of the four *cantus-mollis* tonalities.

The *cantus-durus* tonalities also use large numbers of C↔F progressions (except for the e/♮ tonality, in which C↔F does not appear), making this progression the most common in the Frescobaldi corpus. The other progressions that characterize the *cantus-durus* tonalities are: C↔d (orange), C↔a (yellow), G↔C (blue), and F↔d (pink), which was also among the commonest progressions in the *cantus-mollis* tonalities and is the second most common progression in the Frescobaldi corpus.

The comparisons in which root/quality representations are most revealing are those between tonalities for which we have already theorized some other kind of close relationship: the a/♮-e/♮ pair and the F/♭-d/♭-C/♭ group. Comparing a/♮ with e/♮, we see that these two tonalities share eight of their twelve commonest progressions and that the three commonest progressions in the e/♮ tonality (E↔a, C↔a, and F↔d), which account for nearly one quarter of all progressions in that tonality, are the fourth, third, and second commonest progressions in the a/♮ tonality. This overlap is one clear embodiment of the affinity between the a/♮ and e/♮ tonalities described by both early and modern theorists.

The F/♭-d/♭ “relative” pair is another pair of tonalities that overlap substantially from a root/quality perspective. The three commonest progressions in both of these tonalities are the same and appear in the same ranking (C↔F, F↔d, and F↔B♭), even though only one of them (F↔d) involves the “tonic” triad of d/♭; the next three commonest progressions are also the same in both tonalities (d↔B♭, g↔B♭, and a↔F), but their rankings are juggled. Of the remaining progressions in both charts, three more are shared (F↔g, C↔d, and C↔a), for a total of nine shared progressions between F/♭ and d/♭.

Like the a/♮-e/♮ pair, but to an even greater extent because the order of common progressions is better preserved, the F/♭-d/♭ pair represents a single, well-defined vocabulary of chord progressions in which multiple tonalities exist. As it did when comparing root/quality 1-grams, this affinity brings to mind the modern concept of relative major and minor keys. What we see in Frescobaldi, however, is that more than two tonalities can be “relatives” of one another: the C/♭ tonality also uses the same 2-gram vocabulary as F/♭ and d/♭, having a slightly stronger affinity with F/♭ than with d/♭. With F/♭, C/♭ shares all eleven of its commonest 2-grams, but in different ranking order. This highlights a subtle, but important difference: from a 1-grams perspective, F/♭ and C/♭ were the closest within the F/♭-d/♭-C/♭ group, but from a 2-grams perspective (where the order of triads matters), we can say that F/♭ and d/♭ are the closer pair because their shared progressions have a more similar ranking. This shows us that 1-grams and 2-grams do not always tell the same story and it is important to consider both perspectives.

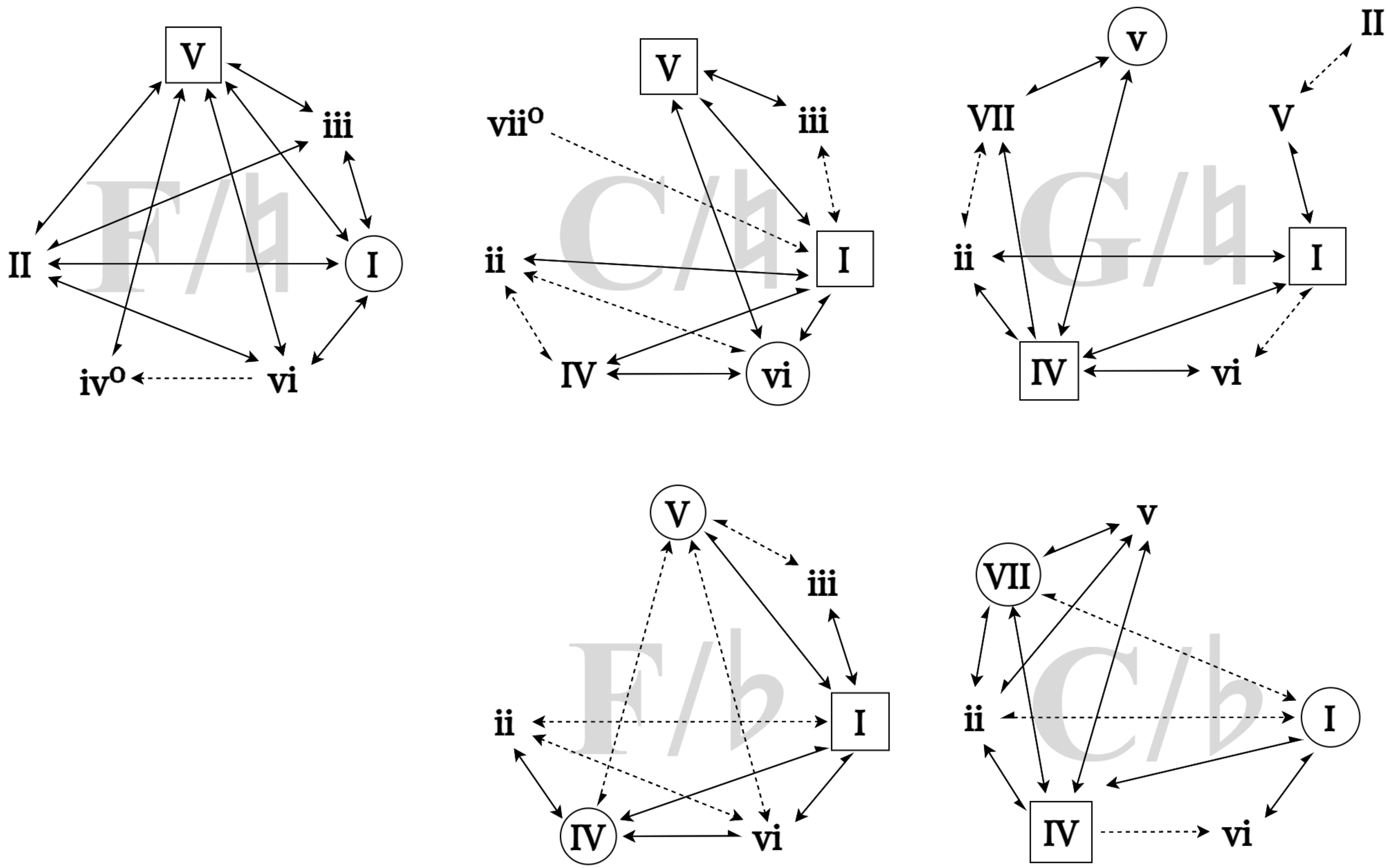
While certain pairs or groups of tonalities have many progressions in common, the tonalities that share the fewest progressions with other tonalities are the dorian tonalities, d/♮ and g/♭. Both of these tonalities have the largest numbers of uncoloured progressions (i.e., progressions that are not shared with at least one other tonality): four out of twelve of their commonest progressions are unique. For the g/♭ tonality, two of these are among the five commonest progressions, while for the d/♮ tonality, they are closer to the bottom of the chart. Furthermore, the commonest progression in the d/♮ tonality (A↔d) is shared with only one other tonality (d/♭). Uniquely occurring progressions like E♭↔c and g↔c in g/♭ and g↔e^{o} in d/♮ are another result of the chromatically flexible sixth scale degree in dorian tonalities. Other tonalities, which use the seven diatonic notes of one system plus a chromatic "leading tone" if required, are confined to a smaller vocabulary of seven or eight different triads. The dorian tonalities make good use of their slightly expanded gamut (seven diatonic notes plus the raised seventh and lowered sixth scale degrees) and regularly use nine (as in g/♭) or ten (as in d/♮) different triads, a feature that can be said to characterize these tonalities.

3.3.3. 2-Grams as Flowcharts

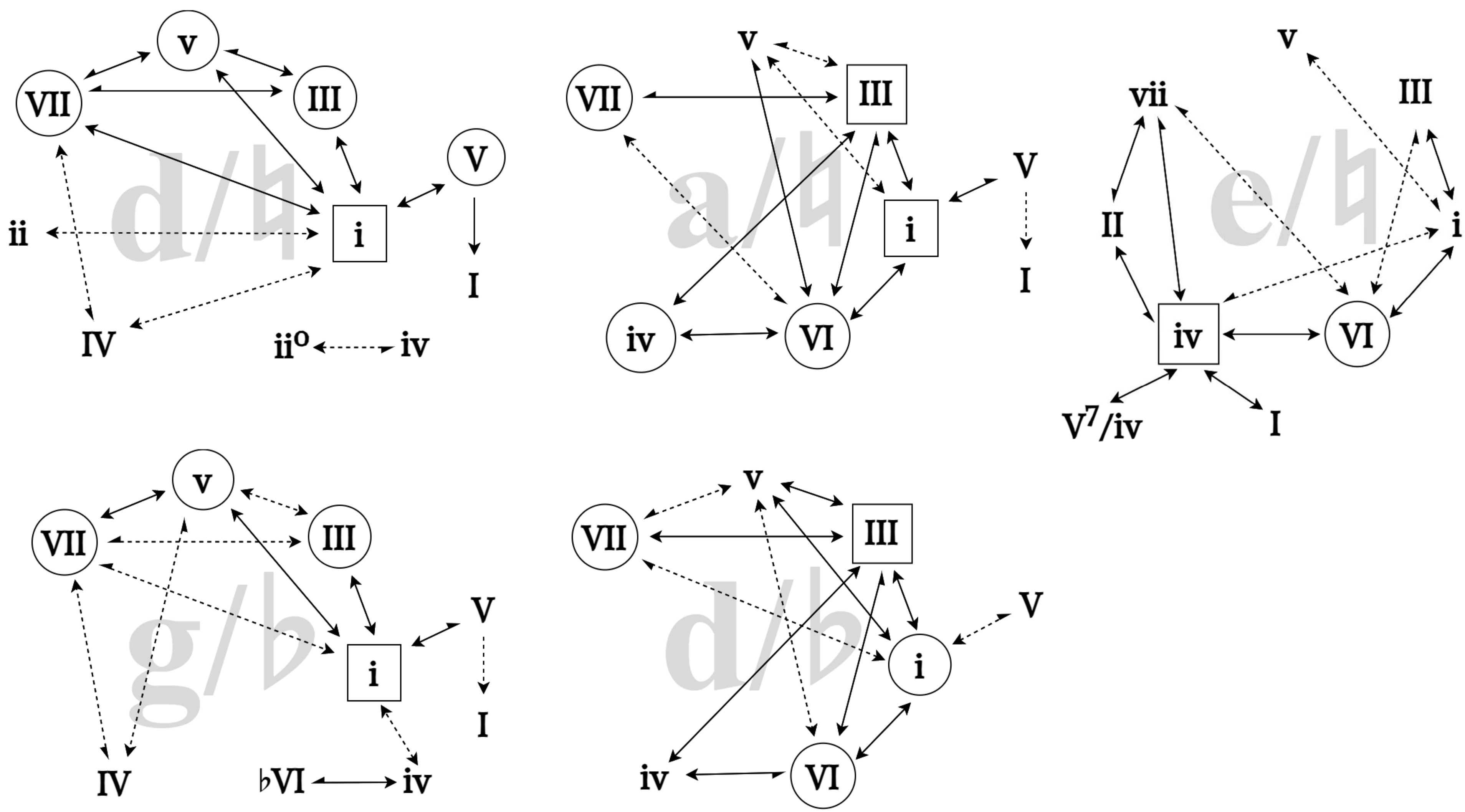
While histograms and tornado charts provide a detailed view, they can be difficult to read and compare. To simplify the cross-composer comparisons made in Chapter

7, Examples 3.18 and 3.19 condense all of the data in the 1-gram and Roman-numeral 2-gram graphs (Examples 3.4, 3.5, 3.13, and 3.14) into flowcharts (inspired by the one from Kostka, Payne, and Almén in Example 3.12). In these diagrams, frequencies of individual chords (1-gram data) are represented using boxes and circles: boxed Roman numerals are extremely common, circled Roman numerals are common, and un-circled Roman numerals are less common. Frequencies of chord progressions (2-gram data) are represented using arrows: solid arrows indicate common progressions (i.e., progressions whose forwards and backwards versions together account for 2% or more of all the progressions in that tonality) and dashed arrows indicate less common progressions (i.e., progressions whose forwards and backwards versions together account for less than 2% of all the progressions in that tonality).

Arrows in these diagrams also have different kinds of heads: unidirectional arrows (→) indicate progressions that never go backwards (e.g., the vii°→I progression in C/♮); bidirectional arrows with one half head and one whole head indicate progressions that are at least twice as likely to proceed in one direction over the other, with the whole head pointing in the direction of the more likely progression (e.g., the V↔I progression in C/♮); and bidirectional arrows (↔) indicate progressions that are less than twice as likely to proceed in one direction over the other (e.g., the ii↔I progression in C/♮).



Ex. 3.18. Roman-numeral flowcharts for Frescobaldi's Ut tonalities—boxed Roman numerals are extremely common, circled Roman numerals are common, and un-circled Roman numerals are less common—solid lines represent the commonest progressions (those that account for at least 2% of all observed 2-grams) and dotted lines represent less common progressions (those that fall below the 2% threshold)



Ex. 3.19. Roman-numeral flowcharts for Frescobaldi's Re and Mi tonalities—boxed Roman numerals are extremely common, circled Roman numerals are common, and un-circled Roman numerals are less common—solid lines represent the commonest progressions (those that account for at least 2% of all observed 2-grams) and dotted lines represent less common progressions (those that fall below the 2% threshold)

Because no new information is presented in these diagrams, we will not devote a great deal of space to their discussion. The only point left to emphasize is that all of these flowcharts, even those representing pairs of tonalities with the same octave species, look quite different from one another. What we expect to see as we approach the end of the seventeenth century is that chords and chord progressions that occur within a comparable number of different tonalities can be modeled using only a few flowcharts—perhaps only two different flowcharts by the time of Corelli, one major and one minor. The two chapters that follow present and analyse all of the same kinds of data (1-grams and 2-grams using Roman-numeral and root/quality representations) gathered from works by Legrenzi and Corelli.

4. Legrenzi

As we did for Frescobaldi, we will explore Legrenzi's tonal practice using three tools: (1) histograms of individual chords (1-grams), between which we measure Euclidean distances to find the most transpositionally similar tonalities; (2) tornado charts showing the relative commonness and directional tendencies of two-chord progressions (2-grams); and (3) flow charts that combine the information from (1) and (2) to give a general overview of the interactions between chords, represented as Roman numerals, within each tonality.

4.1. Legrenzi's Tonalities

As we move from the Frescobaldi corpus (1628–45) to the Legrenzi corpus (1655–73), one thing we notice right away is that signatures now contain not only flats, but also sharps. The D/♯ tonality, for example, appears at least once in each of Legrenzi's four collections of instrumental music (Op. 2, Op. 4, Op. 8, and Op. 10). Another thing we notice is that four sonatas in Op. 10 use signatures with two sharps or flats, instead of one.

In Frescobaldi, the clearest transpositional relationships were found between *mollis-durus* pairs of tonalities, those that used different systems but shared the same octave species. Now with tonalities like g/♭♭ and A/♯♯ in use, we ask: must any tonality with a multi-flat or multi-sharp signature be a transposition of some other tonality? Which transpositional relationships are most common? Which of

Legrenzi's transpositions best preserve the empirically salient characteristics (e.g., the ranking of Roman-numeral 1-grams) of the parent tonality? In what ways are Legrenzi's transpositions different from Frescobaldi's? To answer these questions, I begin by dividing Legrenzi's thirteen tonalities into two groups: "natural" tonalities and "transposed" tonalities, terms used by several seventeenth-century theorists including Nivers (Nivers 1665), Penna (Penna 1679), Rousseau (Pedneault-Deslauriers 2017), and Degli Antonii (Degli Antonii 1687).

Legrenzi orders the sonatas in his first book of instrumental music (Op. 2, 1655) according to the eight church keys (Bonta 1984, xiv). In keeping with this widespread practice (Barnett 2008, 264), I order the tonalities that appear in my Legrenzi corpus in the same way in Example 4.1 (below).

Church Key	Natural Tonality	Transposed Tonalities: Step Down	Step Up	By Fifth
1	d/♮(10)	c/♭♭(1)[1]	e/♯♯(1)[2]	
2	g/♭(4)			
3	a/♮(5)	g/♭♭(1)[2]	b/♯♯ (1)	e/♯(1)[3]
4	e/♮(4)			
5	C/♮(7)	B♭/♭(1)	D/♯ (4)	
7	d/♭(0)	c/♭(3)		
8	G/♮(5)		A/♯♯ (1)	

Ex. 4.1. Tonalities in 47 sonatas by Legrenzi—shaded cells indicate transpositions where signature-defined octave species is not preserved

[1] This sonata, Op. 10, no. 18, appears in two tonalities: c/♭♭ and e/♯♯, both of which we should consider as transpositions of the same parent tonality.
[2] This sonata, Op. 10, no. 17, appears in two tonalities: g/♭♭ and e/♯, both of which we should consider as transpositions of the same parent tonality.

The seven tonalities in the first column on the left are the untransposed church keys,[3] with the sixth church key (F/♭) omitted. For whatever reason, the F/♭ tonality appears in none of the 47 sonatas I chose to analyse, nor in any of Legrenzi's remaining instrumental music published in the four collections from which I assembled my corpus. This omission, likely intentional on the part of the composer, remains a topic for further musicological inquiry. Moving towards the right, the next two columns show what I am calling stepwise transpositions of the original church keys. Some church keys (e.g., a/♮) have both an upwards and a downwards transposition, while others (e.g., G/♮) have a transposition in only one direction, and others (e.g., e/♮) are not transposed at all.

The organization of Example 4.1 also reveals the main hypothesis of this chapter: that step-related (not fourth-related) tonalities are the most transpositionally similar, and that, in several cases, the octave species indicated by the signature does not need to be preserved when a tonality is transposed. It can be argued, for example, that the D/♯ tonality is a transposition of G/♮—both tonalities appear to have a mixolydian octave species—but I have placed D/♯ in the same row as C/♮ because I believe that stepwise transpositions will be more exact than other transpositions (e.g., those by fourths), even when the transposed tonality is "missing" one flat or sharp from its signature.

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[3] The eight church keys are actually the result of a set of transpositions of the eight psalm tones originally put forward by Adriano Banchieri in his *Cartella musicale* of 1614 in order to reduce the vocal range of their reciting tones. See Harold Powers, "From Psalmody to Tonality," in *Tonal Structures in Early Music*, ed. Cristle Collins Judd (New York: Garland, 1998), 294–95.

My belief is founded on one seventeenth-century source in particular: Giovanni Battista Degli Antonii's *Versetti* (Opus 2) of 1687, a collection of 20 organ pieces, twelve of which use tonalities that are transpositions (*una voce piu alta* or *piu bassa*) of the eight church keys[4] (Degli Antonii 1687). Degli Antonii's practice of transposing by step appears to align with Banchieri's: when he first lays out the church keys in his *Cartella musicale* (1614), Banchieri offers alternate versions of church keys 3 (a/♮) and 8 (G/♮) both down a step, i.e., transposed to g/♭ and F/♭, respectively (Banchieri 1614, 71). Barnett discusses Degli Antonii's collection as an example of church key transposition and remarks that it

> provides a rationale for the so-called incomplete key signatures that proliferated during the seventeenth and early-eighteenth centuries. For example, c/two flats [and] A/two sharps… may be properly understood as church keys or their transpositions rather than as incomplete signatures of as yet unrecognized major and minor keys (Barnett 2002, 428–29).

Here, Barnett rightly cautions us against conflating seventeenth-century tonalities like c/♭♭ and A/♯♯ with our modern C minor and A major, respectively, but in doing so, he also glosses over what I believe to be an essential point: half of Degli Antonii's twelve transposed tonalities use signatures that do not preserve the octave species of their "natural" tonality. Three of these six "irregular" transpositions, as Barnett calls them, replace a dorian octave species with a minor one, or vice versa; two more replace major with mixolydian, or vice versa; and one replaces major with lydian.[5]

[4] The composer does not transpose the fourth and seventh church keys (e/♮ and d/♭).
[5] Barnett also discusses transposed church keys and the Degli Antonii versets in *Bolognese Instrumental Music*, 264–70, but again avoids the topic of transpositions that do not preserve the octave species, apart from labeling them "irregular."

Are all of the signatures for these six transposed tonalities "incomplete" then? Do "missing" or "extra" signature flats or sharps actually have an effect on the chords and chord progressions of the transposed tonalities? I will answer these questions for similar transpositions in Legrenzi and determine the extent to which his signatures can be called "incomplete" or not.[6]

In an article in *Music Theory Online*, Julie Pedneault-Deslauriers explains how French music theorists at the end of the seventeenth century went to great lengths to standardize key signatures (Pedneault-Deslauriers 2017). According to the early eighteenth-century theorist François Campion, "all those Italians do not agree on how to notate signatures. Some write more or fewer sharps and flats than the others… I do not approve of that." (Pedneault-Deslauriers 2017) The same Campion, however, would later write that certain accidentals, like the one inflecting the sixth scale degree of a minor-third tonality, "must not be written near the clef", acknowledging that this, the only demarcation between the dorian and aeolian octave species, should remain flexible (Pedneault-Deslauriers 2017). What Campion's instructions mean then, is that sharp-side dorian tonalities, like a/♯ and e/♯♯ should be written with one less sharp, i.e., a/♮ and e/♯, making them look minor, and flat-side minor tonalities like g/♭♭ and c/♭♭♭ should be written with one less flat, i.e., g/♭ and c/♭♭, making them look dorian.

[6]One other place where stepwise transposition appears to be important is in Johann Mattheson's presentation of his 24 major and minor keys in *Das neu-eröffnete Orchestre* (1713, 60–64). As both Powers and Barnett remark, each of the keys in Mattheson's second group of eight is a stepwise transposition of a different key in the first group of eight, whose finals (but not octave species) are derived from the church keys.

Campion's rule also explains the inconsistencies we noticed in Degli Antonii's transpositions. Taking the d/♮ tonality as an example: when transposing up (adding sharps), Degli Antonii uses one less sharp (e/♯) than an exact transposition (e/♯♯) would require; and when transposing down (adding flats) he does not need to "omit" a flat from the signature because the last flat in c/♭♭ does not fall on the sixth scale degree (A). If he were transposing the seventh church key (d/♭), however, Degli Antonii would likely have omitted the third signature flat (A♭) from c/♭♭, which would have made the transposition exact. In other words, Degli Antonii's transpositions of minor tonalities are more likely to be exact when they are step-up transpositions (adding sharps) and likewise, his transpositions of dorian tonalities are more likely to be exact when they are step-down transpositions (adding flats).

Campion's view does not explain the "loose" transpositions we observe from major to mixolydian or major to lydian octave species, but it does provide some clarity on the topic of "incomplete" key signatures and shows us how octave species as taxonomical categories, at least for some musicians, begin to blur together at the end of the seventeenth century.

With a better understanding of Degli Antonii's and Campion's "incomplete" signatures, there is still one tonality of Legrenzi's that stands out as unusual: the c/♭ tonality, which appears in Opus 2 (1655), Opus 4 (1656), and Opus 10 (1673). In Frescobaldi, the C/♭ tonality was a major-third tonality, belonging squarely in the Ut category. In Legrenzi, however, it behaves like a Re tonality, with accidental E-flats written in everywhere and for this reason, I have chosen to include it among the Re tonalities. The missing E♭ from this tonality's signature is surely an example of what Campion meant when he said "Some [Italians] write more or fewer sharps and flats than the others." In fact, Legrenzi even uses c/♭ and c/♭♭ in the same collection of sonatas (Opus 10), albeit in different contexts, which will be discussed later on.

In his edition of Legrenzi's Opus 2, Stephen Bonta explains c/♭ as a transposition of the seventh church key, d/♭, writing that "[t]he seventh mode was identified with both c [minor] and D [major] at the time" (Bonta 1984, xiv). Lorenzo Penna also writes that the seventh church key can be either minor (d/♭) or major (D/♯), but lists cadences only for the minor version and its most common transposition, e/♯ (Penna 1679, 121–22). Following Bonta, I have listed c/♭ as a transposition of the seventh church key (d/♭) in Example 4.1, but unfortunately, we cannot know the extent to which c/♭ and d/♭ are transpositionally related because the latter does not appear in my corpus.[7] We will still compare c/♭with the other Re tonalities and see that it is actually quite similar to d/♮.

Legrenzi uses the c/♭ tonality three times in 47 sonatas, and the c/♭♭ tonality only once. The c/♭♭ tonality appears in one of two versions of the final sonata (no. 18) of his Opus 10 (1673). This sonata, as well as the one just before it (no. 17) is

[7]Bonta never uses the word "transposition" to describe the c/♭ tonality, and it may be the case that Legrenzi also saw c/♭ as more of a "natural" tonality than a "transposed" one. Since there is no d/♭ tonality in his published instrumental works, perhaps the composer's insistence on a signature of only one flat was meant to emphasize the "primary" status of c/♭ as the "untransposed" seventh church key.

presented in an unusual way. Both sonatas "*a quattro, viole da gamba o come piace*" are notated using two different sets of clefs and signatures (Bonta 1984, xv). That the viol players have the option of playing in c/♭♭ or e/♯♯, for example, tells us something about how the composer himself conceived of transposition.

Legrenzi's notated transpositions at the end of his Opus 10 are what we today would call exact, that is, both pairs of finals and signatures imply the same octave species: the dorian sonata (no. 18) can be played in c/♭♭ or e/♯♯, while the minor sonata (no. 17) can be played in g/♭♭ or e/♯. Because both versions of either sonata must sound exactly the same, Legrenzi was obligated to use unambiguous signatures, rather than follow the convention outlined by Campion, in which the sixth scale degree of a Re tonality is left uninflected (the C-sharp in e/♯♯ and the E-flat in g/♭♭ are the places where Campion would expect there to be ambiguity). In the transpositional relationships that we will uncover in Legrenzi's other sonatas, however, this kind of ambiguity is common. We will see that in several cases, tonalities that appear to have different octave species actually behave as transpositions of one another: Legrenzi uses lydian signatures for sonatas that behave as if they were major, mixolydian signatures for sonatas that behave as if they were dorian, and dorian signatures for sonatas that behave as if they were minor, and vice versa.

This practice of transposing by means of "incomplete" signatures might have something to do with the psalmodic practice on which the church keys are based.

Adriano Banchieri, who in his *Cartella musicale* of 1614 supplied the original transpositions of the eight psalm tones that led to the eight church keys known to Legrenzi and other seventeenth-century composers (Barnett 2002, 422), indicates that the third psalm tone, which has its final on A, can be transposed down to G if we change the system from *cantus durus* to *cantus mollis*. In this case, the addition of only a single flat to the signature effects an exact transposition of the psalm tone melody because the third psalm tone does not contain the sixth degree above the final. The same can be said of psalm tones 2, 4, 6, and 8: all of them lack the sixth degree of the final. Banchieri gives a similar example for the eighth psalm tone, showing that it can be transposed "exactly" from G/♮ down to F/♭.

Whatever the rationale for "incomplete" key signatures, their apparently widespread and non-standard usage would eventually become a concern for late seventeenth-century theorists and composers, especially French theorists and composers (Pedneault-Deslauriers 2017). As we will see, Legrenzi's music represents a cross-section of the seventeenth century instrumental repertoire in which harmonic practice is now relatively standard—reminding us in many ways of the common practice—but the notation of key signatures is not.

4.2. Comparing Chord Histograms

Following the same rationale outlined in Section 3.2, tallies of the most common chords were made for each of Legrenzi's thirteen tonalities and Euclidean distances calculated between the resulting histograms. Tonalities are compared using both Roman-numeral and root/quality representations of their commonest chords. In the Re-plus-Mi category, thirteen chords were tallied, while in the Ut category, twelve chords were tallied. These include the seven diatonic triads for as many different octave species as there are in each category, plus the most common chromatically altered chords (e.g., I in the Re tonalities and II in the Ut tonalities).

4.2.1. Comparing Roman-Numeral Histograms

Roman-numeral histograms are presented in Examples 4.1 and 4.2 as heat maps, with larger numbers shaded darker. All numbers are percentages of the total number of observed chords in each tonality, except for the AVG column, which simply averages all of the columns to the left of it.

	g/♭ (4)	g/♭♭ (1)	a/♮ (5)	b/♯♯ (1)	c/♭ (3)	c/♭♭ (1)	d/♮ (10)	AVG		e/♮ (4)
i	7.96%	9.47%	10.51%	7.83%	12.20%	13.83%	10.10%	10.27%	i	4.10%
I	2.81%	2.56%	2.64%	1.79%	1.91%	2.96%	1.91%	2.37%	I	7.87%
ii	0.78%	1.38%	1.08%	2.01%	1.91%	1.98%	1.57%	1.53%	II	6.21%
ii°	2.03%	2.76%	1.86%	1.79%	2.51%	2.96%	1.91%	2.26%	♯ii°	0.53%
III	7.90%	5.72%	7.38%	7.38%	5.26%	6.52%	6.12%	6.61%	III	6.01%
iv	6.40%	6.11%	5.67%	5.59%	5.38%	6.92%	4.04%	5.73%	iv	9.98%
IV	2.15%	2.76%	2.25%	1.79%	2.87%	1.19%	2.56%	2.22%	IV	1.72%
v	3.77%	4.73%	4.06%	2.24%	6.70%	4.35%	4.87%	4.39%	v°	2.05%
V	6.22%	3.55%	7.48%	4.70%	6.70%	6.13%	6.03%	5.83%	v	0.93%
(♭)VI	5.27%	6.51%	4.74%	8.50%	4.31%	4.35%	3.24%	5.27%	V	0.73%
(♯)vi°	1.20%	0.99%	1.61%	0.67%	1.32%	2.57%	1.59%	1.42%	VI	8.72%
VII	4.01%	5.33%	6.21%	3.80%	3.95%	5.14%	4.30%	4.68%	vii	6.61%
♯vii°	1.08%	0.20%	1.27%	1.79%	1.91%	1.78%	1.59%	1.37%	VII	1.98%

Ex. 4.2. Thirteen commonest triads (by Roman numeral) in Legrenzi's Re and Mi tonalities—largest number in each column is shaded darkest—numbers in parentheses show how many sonatas per tonality

All seven Re tonalities have large amounts of i triads, but the next most common triad is not consistent: in the g/♭ and c/♭♭ tonalities, III and iv are very common, while for the c/♭ tonality, both v and V are more common than either III or iv. The a/♮ and d/♮ tonalities have nearly equal amounts of V and III, while the g/♭♭ and b/♯♯ tonalities favour VI over the other diatonic triads. In the case of b/♯♯, the VI triad (G major) is even more common than the i triad. The rankings of the various diatonic triads are but one of many dimensions in which tonalities can be compared. These rankings are summarized, along with many other findings, in the final examples of this chapter, Examples 4.14.a, 4.14.b, and 4.15.

Legrenzi's single Mi tonality, e/♮, which is on the far right of Example 4.2, will not be compared with the Re tonalities from a Roman-numeral perspective because it shares only four diatonic triads with them: i, III, iv, and VI. Instead, it will be compared from a root/quality perspective in Section 4.2.2 below.

	G/♮ (5)	A/♯♯ (1)	B♭/♭ (1)	C/♮ (7)	D/♯ (4)	AVG
I	16.32%	12.11%	13.70%	13.67%	13.70%	13.90%
ii	3.75%	5.08%	3.94%	3.37%	2.94%	3.82%
II	1.85%	3.71%	3.56%	3.87%	2.64%	3.13%
iii	2.23%	2.15%	6.94%	4.08%	3.00%	3.68%
iii°	1.09%	0.00%	0.00%	0.13%	0.30%	0.30%
IV	8.65%	8.79%	5.44%	5.38%	6.67%	6.99%
v	1.96%	1.37%	0.19%	0.04%	0.84%	0.88%
V	9.47%	10.94%	11.07%	13.00%	12.32%	11.36%
vi	4.84%	3.71%	6.19%	6.23%	4.39%	5.07%
vii	0.00%	0.00%	2.44%	1.35%	0.00%	0.76%
(♭)VII	1.85%	0.59%	0.19%	0.13%	1.02%	0.76%
(♯)vii°	2.61%	1.56%	2.06%	2.61%	1.98%	2.16%

Ex. 4.3. Twelve commonest triads (by Roman numeral) in Legrenzi's Ut tonalities—largest number in each column is shaded darkest—numbers in parentheses show how many sonatas per tonality

Compared to the Re tonalities, the Ut tonalities appear to have slightly more in common with one another from a Roman-numeral perspective. All five tonalities have I and V as their first and second most common triads and all contain comparable amounts of the non-diatonic triads ♯vii° and II (the dominant of V), which help us hear I and V as the primary tonal areas of these tonalities. If there is a division in this category, it is between the mixolydian and non-mixolydian tonalities. In the G/♮, D/♯, and A/♯♯ tonalities, which require accidentals in order to raise their leading tones, we see more VII, v, and iii° triads (especially in G/♮), as well as a strong preference for IV over vi. The C/♮ and B♭/♭ tonalities, conversely, prefer vi over IV. This shows that Legrenzi's mixolydian tonalities still resemble those of Frescobaldi in some ways.

Roman-Numeral Euclidean Distances Between Legrenzi's Re Tonalities

	g/♭	g/♭♭	a/♮	b/♯♯	c/♭	c/♭♭	d/♮
g/♭	0	1.26	1.05	1.23	1.74	1.85	1.26
g /♭♭		0	1.43	1.39	1.58	1.75	1.42
a/♮			0	1.78	1.33	1.27	1.04
b/♯♯				0	2.27	2.33	1.91
c/♭					0	1.23	0.99
c/♭♭						0	1.51
d/♮							0

Ex. 4.4. Roman-numeral Euclidean distances between all pairs of Re tonalities—lightly shaded cells mark lowest numbers (smallest distances), darkly shaded cells mark largest distances

Euclidean distances between all pairs of tonalities are shown in Examples 4.4 (above) and 4.5 (below). Lightly shaded cells mark pairs of tonalities whose Roman-numeral distributions are very similar. Darkly shaded cells mark pairs of tonalities whose Roman-numeral distributions are very different. Distances are calculated only between pairs of histograms in the same category (i.e., Re to Re or Ut to Ut) so that histograms always have the same number of dimensions. For an explanation of the mathematics involved in these calculations, refer to Section 3.2.1.

The smallest Euclidean distance in the Re category is between d/♮ and c/♭ (Euclidean distance = 0.99), and very small distances are also found between a/♮ and d/♮ (Euclidean distance = 1.04) and between a/♮ and g/♭ (Euclidean distance = 1.05). Of these three pairs, not one comprises two tonalities that have the same signature-defined octave species, but two of the three pairs (d/♮ with c/♭ and a/♮ with g/♭) have finals that are a whole step apart. We also notice that the tonalities with more flats or sharps in their signatures are farther away from other tonalities on average. This suggests that transposition is perhaps less exact the farther away we get from an empty signature.

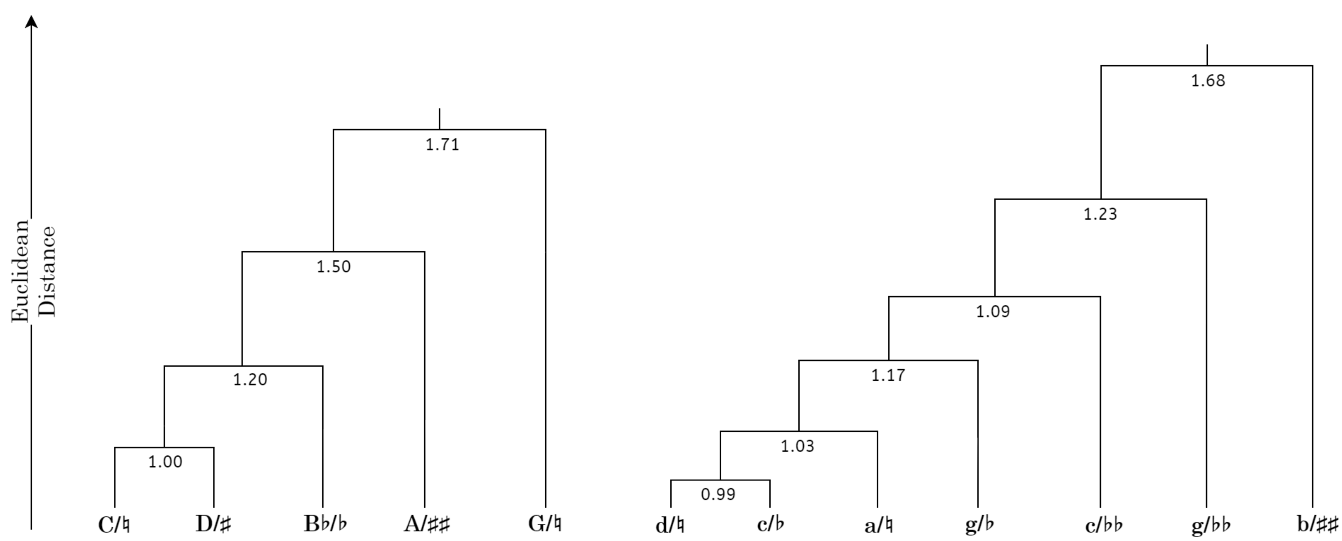
Roman-Numeral Euclidean Distances
Between Legrenzi's Ut Tonalities

	G/♮	A/♯♯	B♭/♭	C/♮	D/♯
G/♮	0	1.60	2.24	2.02	1.41
A/♯♯		0	2.08	1.74	1.18
B♭/♭			0	1.07	1.60
C/♮				0	1.00
D/♯					0

Ex. 4.5. *Roman-numeral Euclidean distances between all pairs of Ut tonalities—lightly shaded cells mark lowest numbers (smallest distances), darkly shaded cells mark largest distances*

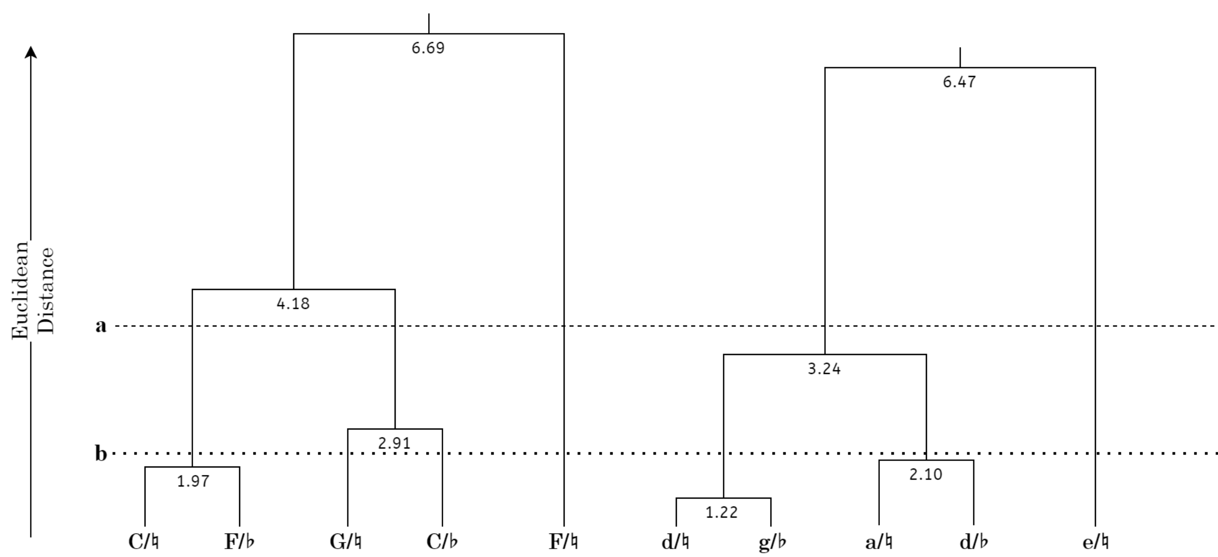
In the Ut category, the two closest pairs are C/♮ and D/♯ (Euclidean distance = 1.00) and B♭/♭ and C/♮ (Euclidean distance = 1.07). Again, neither of these pairs comprises two tonalities with the same signature-defined octave species, but both pairs have finals that are a whole step apart. On average, the tonality that appears to be the farthest away from the other four is the G/♮ tonality, and this is reflected in the Roman-numeral dendrograms below.

Clustering the tonalities based on the Euclidean distance values in Examples 4.4 and 4.5 using hierarchical agglomerative clustering (HAC) produces the two dendrograms (one for each category) shown in Example 4.6. For an explanation of this clustering algorithm, refer to Section 3.2.1.



Ex. 4.6. Dendrograms based on Roman-numeral distances within the Ut (Ex. 4.5) and Re (Ex. 4.4) categories—decimal numbers are Euclidean distances between linked tonalities or clusters

What these dendrograms illustrate is that Legrenzi's tonalities are more like transpositions of one another than Frescobaldi's. In the Roman-numeral dendrograms from Example 3.7 (reproduced below), we saw that Frescobaldi's tonalities group together according to octave species: the major, mixolydian, dorian, and minor pairs all form before any other inter-species clusters form.



Ex. 3.7. Reproduced form Chapter 3—Roman-numeral dendrogram for Frescobaldi's tonalities

In Legrenzi, not only are the first pairs that form inter-species pairs (C/♮ and D/♯ in the Ut category and d/♮ and c/♭ in the Re category), but after these initial pairings there are no other pairs. This means that no two Ut tonalities or Re tonalities, from a Roman-numeral perspective, are more similar to one another than is each individual tonality to the first pair in its category. In the Re category, for example, the first pair to form is the d/♮-c/♭ pair[8] (Euclidean distance = 0.99) and

[8] In Frescobaldi, the d/♮ tonality was also involved in the first pair to form, which was d/♮-g/♭.

based on the distance values in Example 4.4, we might expect that somewhere else in the tree there would be a g/♭-a/♮ pair (Euclidean distance = 1.05). Instead, though, we see that the a/♮ tonality clusters with the d/♮-c/♭ pair (Euclidean distance = 1.03) rather than pairing with the g/♭ tonality, to which it is just slightly less similar. This process then repeats for every other tonality in the Re category: each tonality joins the cluster, rather than first pairing with another tonality with which it is more similar from a Roman-numeral perspective.

If historical expectations for Corelli hold true, then we would expect Corelli's Roman-numeral dendrograms to have shapes similar to those of Legrenzi (more like the "staircases" in Example 4.6 than the "trees" in Example 4.7), but with smaller distances between adjacent tonalities (or smaller "steps").

4.2.2. Comparing Root/Quality Histograms

Next, we compare tonalities on the basis of chord content with chords labeled according to root and quality. This representation allows us to compare tonalities from different categories (e.g., Ut with Re) because we no longer need to consider the functions of the chords we choose to tally. Example 4.7 presents the resulting histograms as a heat map, with larger values shaded darker and tonic triads shown in boxes. Following the same rationale set out in Section 3.2.2, chords in the leftmost column are ordered from most to least common according to the averages of their occurrences in all tonalities, which are listed in the rightmost column.

	c/♭ (3)	c/♭♭ (1)	g/♭ (4)	g/♭♭ (1)	B♭/♭ (1)	d/♮ (10)	C/♮ (7)	a/♮ (5)	e/♮ (4)	G/♮ (5)	b/## (1)	D/# (4)	A/## (1)	AVG
Gmaj	6.70%	6.13%	2.81%	2.56%	1.50%	2.56%	13.00%	6.21%	6.01%	16.32%	8.50%	6.67%	0.59%	6.12%
Dmaj	3.11%	0.00%	6.22%	3.55%	1.50%	1.91%	3.87%	2.25%	1.98%	9.47%	7.38%	13.70%	8.79%	4.90%
Cmaj	1.91%	2.96%	2.15%	2.76%	3.56%	4.30%	13.67%	7.38%	8.72%	8.65%	2.46%	1.02%	0.39%	4.61%
Fmaj	2.87%	1.19%	4.01%	5.33%	11.07%	6.12%	5.38%	4.74%	6.21%	1.85%	0.00%	0.12%	0.20%	3.78%
Dmin	1.91%	1.98%	3.77%	4.73%	6.94%	10.10%	3.37%	5.67%	6.61%	1.96%	0.00%	0.42%	0.00%	3.65%
Amaj	0.48%	0.00%	1.32%	1.18%	1.69%	6.03%	0.80%	2.64%	1.72%	1.85%	3.80%	12.32%	12.11%	3.53%
Cmin	12.20%	13.83%	6.40%	6.11%	3.94%	0.00%	0.00%	0.10%	0.00%	0.00%	0.00%	0.00%	0.00%	3.28%
Amin	0.00%	0.00%	0.78%	1.38%	2.44%	4.87%	6.23%	10.51%	9.98%	3.75%	1.34%	0.84%	0.20%	3.26%
Bbmaj	3.95%	5.14%	7.90%	5.72%	13.70%	3.24%	0.13%	0.59%	0.07%	0.16%	0.00%	0.00%	0.00%	3.12%
Gmin	6.70%	4.35%	7.96%	9.47%	6.19%	4.04%	0.04%	0.44%	0.13%	0.16%	0.00%	0.12%	0.00%	3.05%
Emaj	0.00%	0.00%	0.00%	0.20%	0.00%	2.56%	1.72%	7.48%	7.87%	0.33%	1.79%	2.64%	10.94%	2.73%
Ebmaj	5.26%	6.52%	5.27%	6.51%	5.44%	0.09%	0.00%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	2.24%
Emin	0.00%	0.00%	0.30%	0.00%	0.19%	1.57%	4.08%	4.06%	4.10%	4.84%	5.59%	2.94%	1.37%	2.23%
Bmin	0.00%	0.00%	0.06%	0.00%	0.00%	0.26%	1.35%	1.08%	0.93%	2.23%	7.83%	4.39%	5.08%	1.79%

Ex. 4.7. Fourteen commonest triads (by root and quality) in Legrenzi's thirteen tonalities—largest number in each column is shaded darkest and tonic triads are boxed—bracketed numbers show how many sonatas per tonality

Unsurprisingly, triads requiring flats (Cmin, B♭maj, Gmin, and E♭maj) are more common in tonalities with signature flats and triads requiring sharps (Dmaj, Amaj, Emaj, and Bmin) are more common in tonalities with signature sharps. The E♭maj and Bmin triads are essentially mutually exclusive. This division, however, is not perfectly balanced: some of the sharp-requiring triads, like Dmaj and Amaj, appear relatively often in flat-side tonalities, whereas flat-requiring triads are extremely rare in both sharp-side and natural tonalities. This imbalance suggests that Legrenzi did not use accidental sharps and flats for the same reasons. While accidental sharps could be used to form dominant-function triads (secondary or not) in flat-side, sharp-side, and natural tonalities, accidental flats were perhaps more structural, functioning as the "missing" flats in tonalities with "incomplete" signatures.

Euclidean distances can now be calculated between all pairs of tonalities (for thirteen tonalities, there are 78 possible pairs). These distances are tabulated in Example 4.8 (below).

Root/Quality Euclidean Distances Between Legrenzi's Tonalities

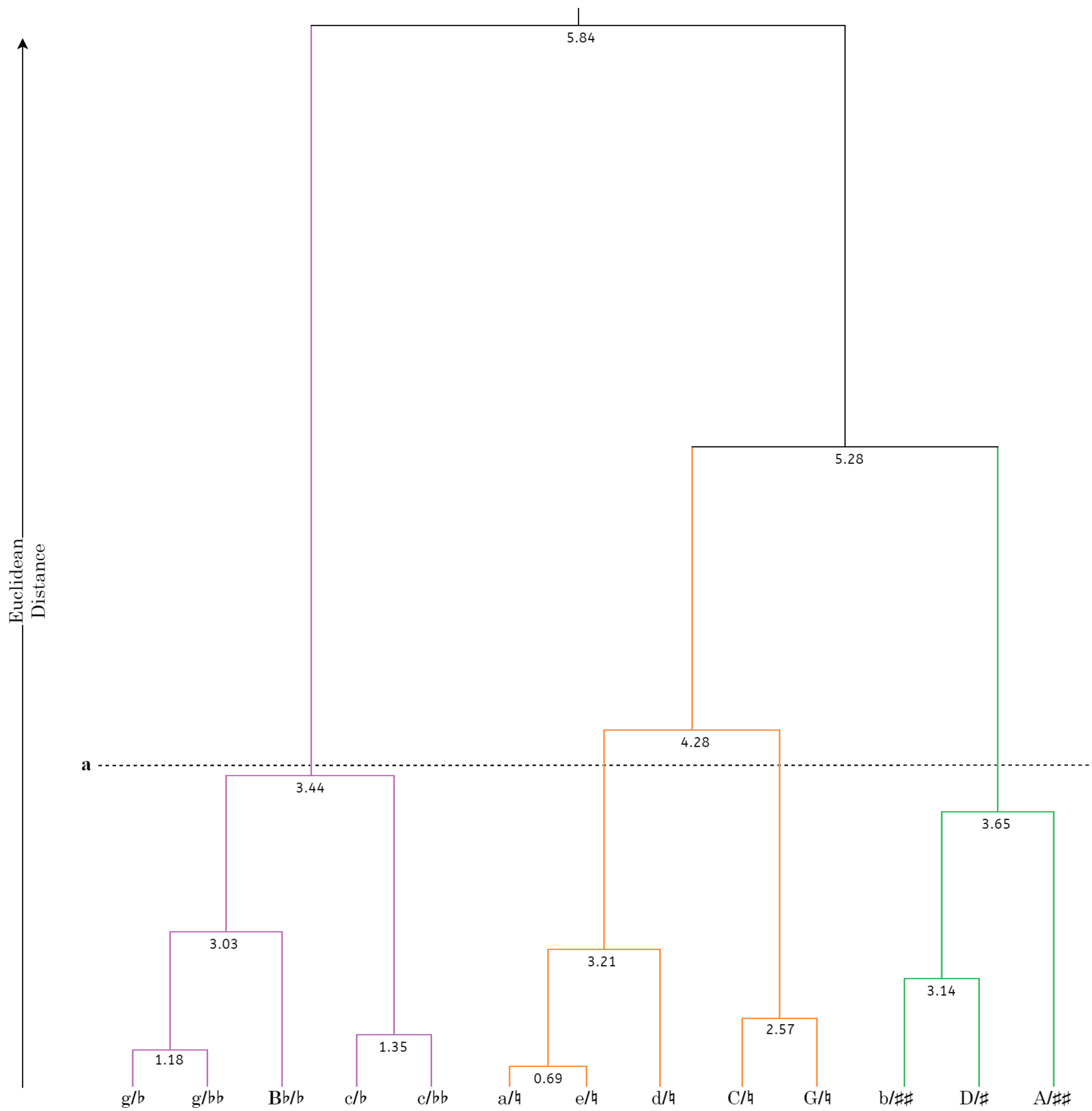
	c/♭	c/♭♭	g/♭	g /♭♭	B♭/♭	d/♮	C/♮	a/♮	e/♮	G/♮	b/♯♯	D/♯	A/♯♯
c/♭	0	1.35	2.42	2.45	4.60	4.99	5.87	5.74	5.96	5.72	5.20	6.18	6.56
c/♭♭		0	3.12	3.16	4.86	5.44	6.18	6.03	6.23	6.26	5.71	6.87	7.03
g/♭			0	1.18	3.07	3.96	5.88	5.37	5.59	5.78	5.05	5.64	5.98
g /♭♭				0	2.99	3.71	5.82	5.21	5.37	6.04	5.38	6.13	6.23
B♭/♭					0	4.03	6.48	5.77	5.82	7.13	6.69	7.38	7.38
d/♮						0	4.70	3.15	3.26	5.43	4.83	5.32	5.23
C/♮							0	3.27	3.17	2.57	4.44	6.01	6.88
a/♮								0	0.69	4.45	4.45	5.63	5.40
e/♮									0	4.64	4.80	6.03	5.75
G/♮										0	3.33	4.68	6.40
b/♯♯											0	3.14	4.21
D/♯												0	3.09
A/♯♯													0

Ex. 4.8. Root/quality Euclidean distance between all of Legrenzi's tonalities—darkly shaded cells mark largest distances (> 6.50), cells shaded red contain distances between relative major/minor pairs, cells shaded blue contain distances between fifth-related tonalities belonging to the same category (Ut, Re, or Mi), and cells shaded orange contain other small distances (< 3.50)—the yellow cell shows the distance between the a/♮-e/♮ pair

Darkly shaded cells mark pairs of tonalities whose root/quality distributions are very different (Euclidean distance > 6.50). Cells shaded blue mark pairs of tonalities whose finals are a perfect fifth apart, while cells shaded red mark Ut-Re pairs of tonalities with the same signatures, i.e., "relative" tonalities. Cells shaded orange mark other pairs of tonalities whose root/quality distributions are very similar (Euclidean distance < 3.50). The a/♮-e/♮ pair, which is the closest pair of tonalities from a root/quality perspective, is shaded yellow.

From the shading, we see that the tonalities B♭/♭ and A/♯♯ have at least four other tonalities from which they are very distant from a root/quality perspective. We also notice that the d/♮ tonality is neither very far from nor very close to any other tonality, except for the a/♮and e/♮tonalities (Euclidean distances 3.15 and 3.26, respectively).In contrast to Frescobaldi's d/♮ tonality, which was relatively ambivalent, but slightly more on the flat side of the root/quality spectrum, Legrenzi's sits closer to the other natural tonalities a/♮ and e/♮.

From these data, we construct the dendrogram in Example 4.9. Like Frescobaldi's tonalities, Legrenzi's tonalities, when analysed from a root/quality perspective, cluster according to signature. Colours are used to highlight the three distinct clusters of the dendrogram: purple for flat-side tonalities, orange for natural, and green for sharp-side tonalities. Of the three large clusters, the natural and sharp-side clusters are the first to join (Euclidean distance = 5.28) and this is most likely because all eight of these tonalities lack flat-requiring triads like Cmin, Bbmaj, Gmin, and Ebmaj.



*Ex. 4.9. Dendrogram based on root/quality distances between all tonalities—decimal numbers are Euclidean distances between linked tonalities or clusters—cutting the tree at line **a** produces four different clusters*

If we "cut" the root/quality dendrogram at the dotted line **a**, we end up with four clusters: the flat-side cluster, the a/♮-e/♮-d/♮ cluster, the C/♮-G/♮ cluster, and the sharp-side cluster. What this reveals is that the natural cluster (in orange) is actually the most diverse: it is composed of two sub-clusters, a/♮-e/♮-d/♮ and C/♮-G/♮, both of which are about as internally similar as the flat-side and sharp-side clusters are on their own.

The five pairs in this dendrogram that are the first to form (g/♭-g/♭♭, c/♭-c/♭♭, a/♮-e/♮, C/♮-G/♮, and b/♯♯-D/♯) can be grouped into three types: same-final pairs, fifth-related pairs, and relative pairs. The same-final pairs g/♭-g/♭♭ and c/♭-c/♭♭are quite close (Euclidean distances of 1.18 and 1.35, respectively) and this suggests that in minor-third tonalities, at least from a root/quality perspective, the difference between one signature flat and two signature flats is not substantial. The fifth-related pair a/♮-e/♮ is the closest pair among all of Legrenzi's tonalities (Euclidean distance = 0.69), which reminds us again of the symbiotic relationship between these two tonalities: the e/♮ tonality uses all of the same chords in the same proportions as the a/♮ tonality (and as we will see later, even the chord progressions are nearly identical from a root/quality perspective). The other fifth-related pair is the C/♮-G/♮ pair, which can be understood—like the a/♮-e/♮ pair, but to a lesser extent—as an authentic-plagal pair because of the shared root/quality vocabulary between C/♮ and G/♮. These two major-third tonalities have much in common from a root/quality perspective because the C/♮ tonality emphasizes G major (with applied

V and ♯vii° chords) and the G/♮ tonality emphasizes C major (with the same two applied chords, neither of which requires an accidental in the G/♮ tonality).

The third type of pair is the relative pair, which comprises one minor-third and one major-third tonality that share the same signature. In Frescobaldi, we saw close relative relationships between the d/♭ and F/♭ tonalities and between the a/♮ and C/♮ tonalities (refer to Example 4.9). In Legrenzi's expanded tonal sphere, we now see relative relationships between b/♯♯ and D/♯ (Euclidean distance = 3.14) and between the G-minor tonalities and B♭/♭ (Euclidean distance = 3.03), which is not a pair, but a group of three. The relative relationship, like the authentic-plagal relationship, ensures a high degree of root/quality similarity because it involves two tonalities that have the same signature or similar signatures and whose seven diatonic triads are rotations of one another.[9] If Legrenzi had chosen to write sonatas with finals on F♯ or E♭, or had not written any sonatas in e/♮, it would be interesting to see how many other relative pairs would form early on in the root/quality dendrogram.

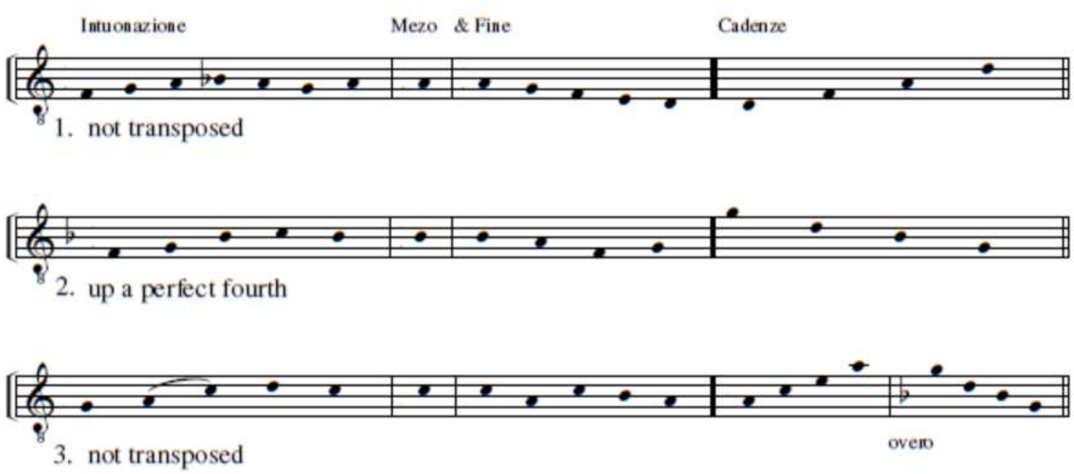
4.3. Comparing Chord Progressions

The next step in understanding Legrenzi's tonal practice is an analysis of chord progressions, or 2-grams. As we did for Frescobaldi, we will use 2-grams to

[9] In our modern relative-major-minor relationship, we say that a minor mode "moves easily to" its relative major because it already contains all of the same diatonic triads as its relative major (including the dominant seventh of the latter): III becomes I, ii° becomes vii°, etc. The same is true of a major-mixolydian pair like C/♮-G/♮, where the G/♮ tonality moves easily to C/♮ because it already contains all of the same diatonic triads as C/♮: IV becomes I, iii° becomes vii°, etc. The same can be said again for the minor-phrygian pair, a/♮-e/♮.

model the syntactical norms of each tonality and determine the extent to which chord progressions, like distributions of individual chords, are transposable. In Legrenzi, we ask: which chord progressions push Re tonalities with different octave species closer together? Which progression push Ut tonalities with different octave species closer together? Which progressions retain the sound or character of a non-major or non-minor tonality?

Because Legrenzi ordered his sonatas according to the eight church keys, and because he worked as an organist in Bergamo for some time (Bonta 2001), we are also interested in how the psalm tones (on which the church keys are based) might have affected Legrenzi's choice of chord progressions. In this chapter, we understand the d/♮ and g/♭ tonalities as church keys derived from the first and second psalm tones, shown in Example 4.10 (below).



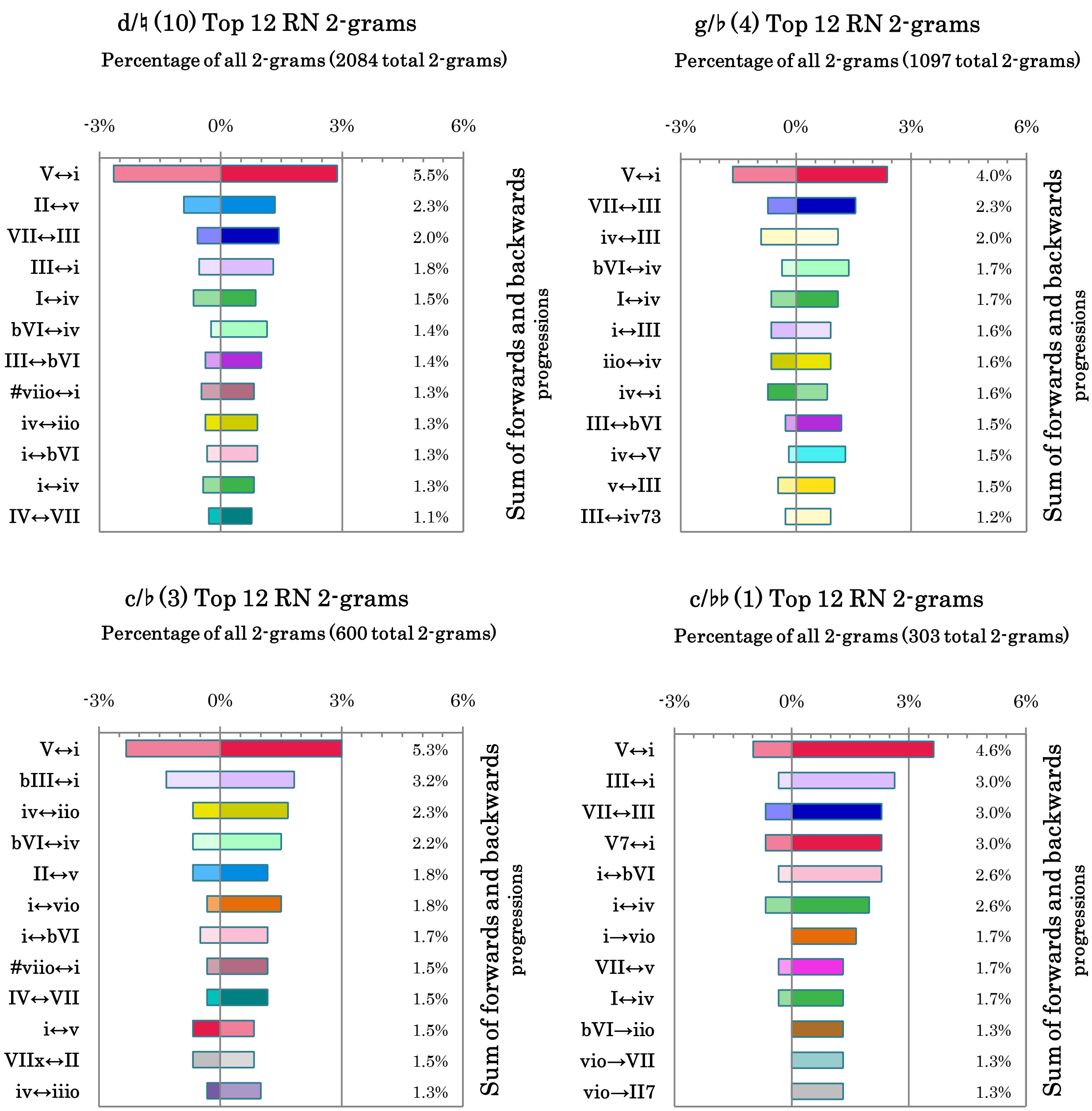
Ex. 4.10. Banchieri's first three psalm-tone transpositions (only the second psalm tone is transposed)—figure taken from Barnett's "Tonal Organization in Seventeenth-Century Music Theory" in Cambridge History of Western Music Theory*—for more information on Banchieri's psalm-tone transpositions, and a list of all eight psalm tones, see Powers (1998).*

While both the d/♮ and g/♭ tonalities have a dorian octave species, the psalm tones from which they are derived (tones 1 and 2) have their reciting tones in different places: the first psalm tone recites on the A a perfect fifth above its final, while the second recites on the B♭ a minor third above its final. The a/♮ tonality, in comparison, has a minor octave species, but the psalm tone from which it is derived (tone 3) bears a very close resemblance to the second psalm tone, also reciting a minor third above its final. Banchieri perhaps acknowledges this similarity when he lists g/♭ as a standard alternate transposition of the third psalm tone. So, for these tonalities we might ask: do chord progressions correlate with octave species? With reciting-tone position? Or do all tonalities within one category (Ut or Re) appear to use similar progressions, regardless of octave species and reciting-tone position?

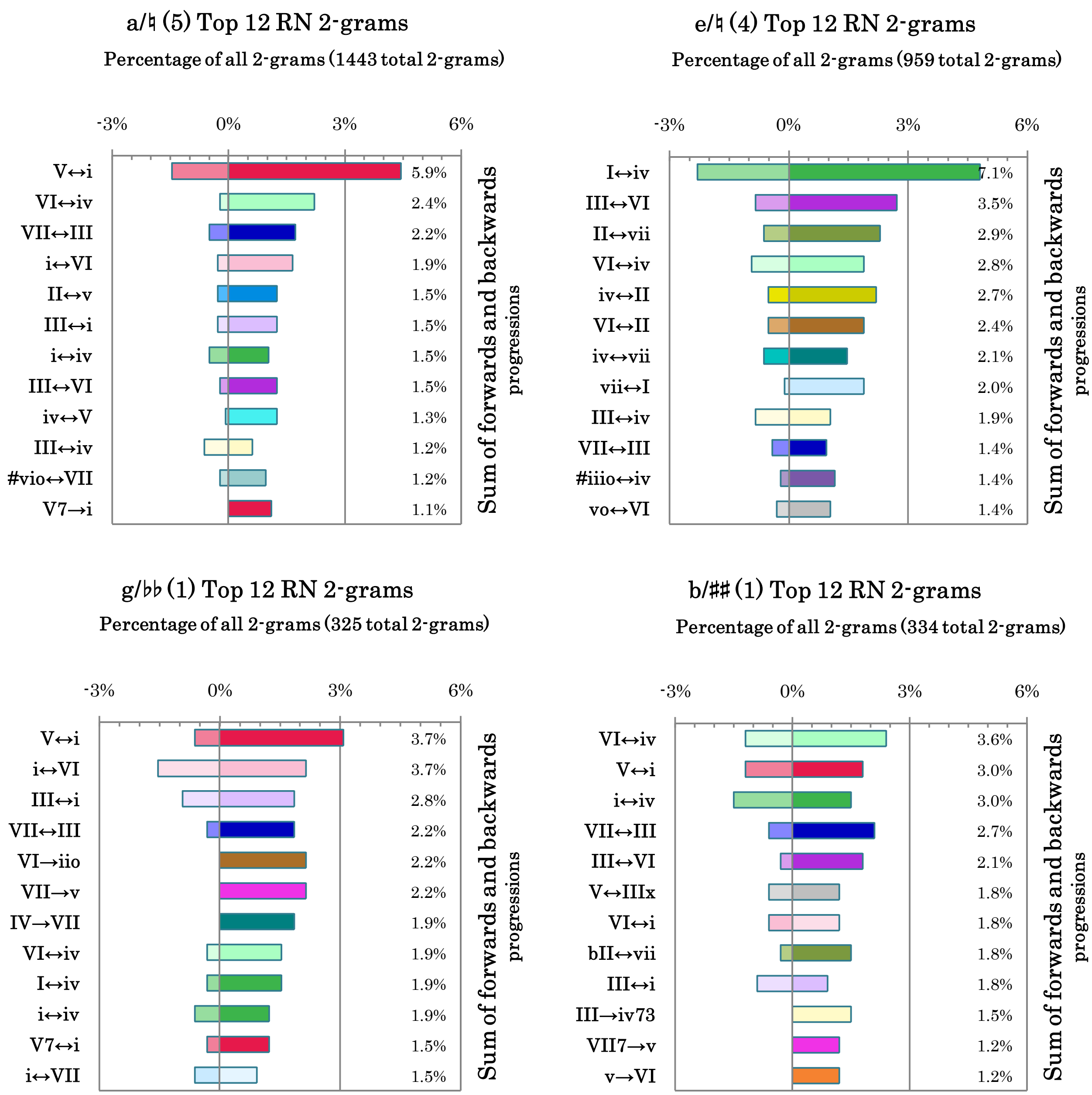
4.3.1. 2-Grams as Roman-Numeral Tornado Charts

Examples 4.11.a, 4.11.b, and 4.12 (below) present, in the same fashion as Section 3.3.1, colour-coded Roman-numeral tornado charts for the twelve commonest chord progressions in each of Legrenzi's thirteen tonalities. These charts illustrate forwards and backwards versions of the same progression using rightwards and leftwards-extending bars (respectively), with the totals of all forwards and backwards progressions listed on the far right. All numbers are percentages of the total number of progressions in one tonality.

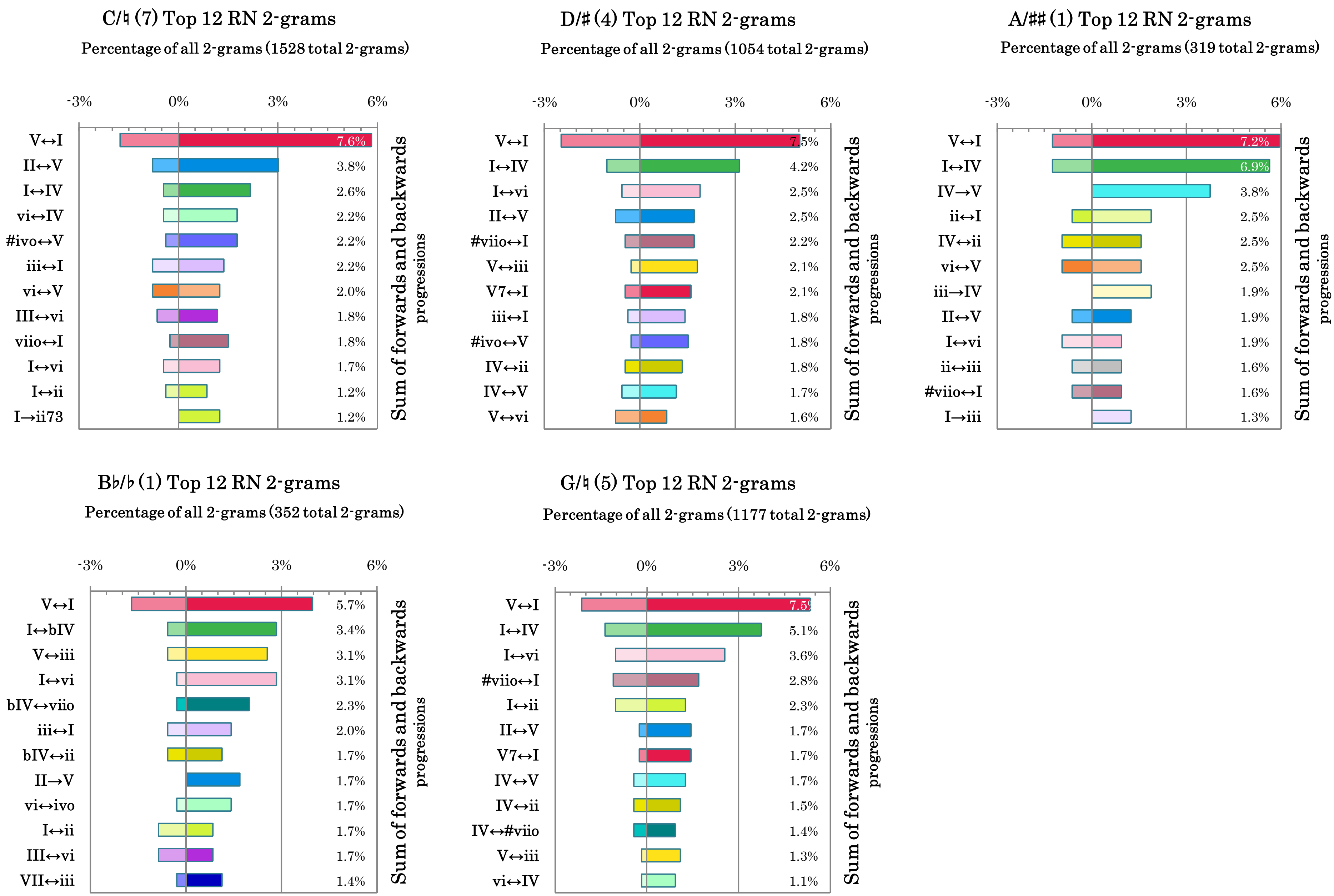
The colour-coding system assigns a colour to any progression that appears in at least two different tonalities, while unshared progressions remain grey. In



Ex. 4.11a. Commonest progressions by Roman numeral in Legrenzi's dorian tonalities—Arabic numerals in brackets show numbers of sonatas per tonality



Ex. 4.11b. Commonest progressions by Roman numeral in Legrenzi's minorand phrygian tonalities—Arabic numerals in brackets show numbers of sonatas per tonality



Ex. 4.12. Commonest progressions by Roman numeral in Legrenzi's Ut tonalities—Arabic numerals in brackets show numbers of sonatas per tonality

Frescobaldi, we needed twenty different colours (plus grey) to colour all of the Roman-numeral progressions. For Legrenzi, we need to add four more colours, for a total of 24 (plus grey) to colour all of the Roman-numeral progressions. This suggests that Legrenzi is reusing (i.e., transposing) more progressions than Frescobaldi. Surprisingly, the Legrenzi tonalities that we might have expected to inflate the number of shared progressions—namely, the two g and two c tonalities in the Re category—do not have very many progressions in common: the two g tonalities share only five of their twelve commonest progressions and none of these five are unique to the g tonalities; the c tonalities share only four of their twelve commonest progressions and only one of these four is unique to the c tonalities (the i↔vi^{o} progression, coloured dark orange).

Comparing first the seven Re tonalities (all of the graphs in Examples 4.11a and 4.11b, except for the one labeled e/♮), we notice that certain progressions are ubiquitous, while others are rare. The V↔i progression occurs in every Re tonality and is always the commonest progression, except in the b/♯♯ tonality, where it is the second most common (after the VI↔iv progression).

The VII↔III progression appears in all of the Re tonalities except for the c/♭ tonality (the only Re tonality in which the III chord is not a diatonic triad) and the VI↔iv progression appears in all of the Re tonalities except for the c/♭♭ tonality, in which neither VI nor iv are diatonic triads. The i↔iv progression appears in all of the Re tonalities except for the c/♭ tonality, in which the iv triad requires and accidental A♭ and the i↔VI progression occurs in all of the Re tonalities except for the g/♭ tonality, in which the VI triad is not diatonic. These four progressions (VII↔III, VI↔iv, i↔iv, and i↔VI) can be said to characterise Legrenzi's Re tonalities as a category and the latter three have an important feature in common: they all require a lowered sixth scale degree. We might expect these latter three progressions to appear more often in minor tonalities (where they do not require accidentals) than in dorian tonalities, but in fact, all of them are evenly distributed between the two octave species (i.e., each progression can be found in three minor and three dorian tonalities). Accidentals are also used in the formation of the ii° triad in dorian tonalities: the d/♮, g/♭, and c/♭ tonalities all use the iv↔ii° progression and in the c/♭ tonality, this is the third most common progression.[10] This shows that accidental flats are being used consistently in dorian tonalities to lower the sixth scale degree.

In the cases we have seen so far, the sixth scale degree of a dorian tonality is flattened to create a ♭VI, iv, or ii° triad, but in one case, we actually find the opposite alteration. The IV↔VII progression, a hallmark of the dorian octave species, which occurred in both of Frescobaldi's dorian tonalities (d/♮ and g/♭) and two of Legrenzi's (d/♮ and c/♭), makes a conspicuous appearance in Legrenzi's g/♭♭ tonality requiring an accidental E♮ to cancel the signature flat.

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[10] The c/♭ tonality uses an enormous number of accidentals: only three of its twelve commonest progressions do not require one or more accidentals on the third, fourth, sixth, and/or seventh scale degrees. The three diatonic progressions are i↔vi°, IV↔VII, and i↔v.

By keeping the sixth scale degree flexible (as Campion advises), Legrenzi has thoroughly blurred the line between the two Re-category octave species, i.e., dorian and minor. By looking at the twelve commonest 2-grams in any given Re tonality, we cannot reliably guess its signature-defined octave species. This conclusion corroborates the one we drew in Section 4.2.1, which was that signature-defined octave species is not a good predictor of the chord content of a tonality (and, by extension, the transpositional similarity between tonalities), and that within one category (Ut or Re) no two tonalities stand out as very similar. If we had to choose the most similar Re tonalities on the basis of 2-grams, we might choose d/♮ and a/♮ (who share nine out of twelve of their commonest progressions) or perhaps c/♭♭ and g/♭♭ (who share the same number, but are each represented by only a single sonata). Both of these pairs, like the pairs we discussed in Example 4.6, are different-octave-species pairs and this again suggests that there are no clear octave-species-defined subcategories within the Re category.

Looking now at the Ut tonalities, we find that many progressions are ubiquitous: the V↔I, I↔IV, II↔V, and I↔vi progressions appear in all five of Legrenzi's Ut tonalities and the iii↔I, (♯)vii°↔I, I↔ii, and IV↔ii progressions appear in four out of five of the Ut tonalities. This means that each of the five Ut tonalities has at least seven progressions in common with every other tonality in its category. As we saw in the Re category, but to an even greater extent, octave-species-specific progressions are almost non-existent. The mixolydian-specific progressions IV↔VII and VII↔v are absent from all mixolydian tonalities, which now use V and ♯vii° in place of v and VII. The lydian-specific progression vi↔iv° is the only species-specific progression that remains intact among the twelve commonest progressions in the B♭/♭ tonality and although it can be argued that the II↔V progression has its origins in the Lydian mode, this progression is not specific to the lydian octave species in Legrenzi's tonal world: it appears in all five of his Ut tonalities.

After considering the relationship between octave species and Roman-numeral progressions, it would appear that Legrenzi's tonalities are of only three types: Ut, Re, and Mi.[11] Let us now consider the relationship between the same Roman-numeral progressions and the features of the psalm tones on which the church keys are based in order to determine what other differences, if any, might separate the tonalities within the Ut and Re categories.

4.3.2. Roman-Numeral 2-Grams in Relation to Psalm Tones

For our purposes, the primary criterion for differentiating between the psalm tones is the distance between final and reciting tone. When a psalm is intoned using one of the eight psalm tones, most of the psalm is sung not on the final, but on a single note, which we call the reciting tone, some interval above the final. In Legrenzi's time, it was common for an organist to accompany the psalms when they were sung (Dodds 2012) and these accompaniments represented tonalities of their

[11] The single Mi tonality, e/♮, is vastly different from any other tonality from a Roman-numeral perspective and remains its own category in Legrenzi, on par with Ut and Re.

own, which we call the church keys. Because most of a psalm-tone melody consists of a single repeated note, we can expect a harmonization of that melody to include many chords and chord progressions that contain that repeated note. For Legrenzi's tonalities, we want to know: do chords and chord progressions that can be used to harmonize a psalm tone's reciting tone appear more frequently in the church keys (and their transpositions) that are based on that psalm tone?

For the second church key (g/♭), for example, the corresponding psalm tone's reciting tone is a minor third above the final, so we are curious to know if there are more triads containing the third scale degree (B♭), namely III, i, and (♭)VI, and whether or not these triads are more likely to appear in close succession, as potential harmonizations of a repeating B♭.

Tonality (Church Key)	Reciting Tone	Triads that Could Harmonize Reciting Tone	Commonest Triads from Ex. 4.2
d/♮ (1)	scale degree 5	i, I, III, ♯iii°, v, V	i, III, V, v
g/♭ (2)	scale degree 3	i, III, (♭)VI, (♯)vi°	i, III, iv, V
a/♮ (3)	scale degree 3	i, III, (♭)VI, (♯)vi°	i, V, III, VII
c/♭ (7)	scale degree 4	ii, ii°, IV, iv, VII, ♯vii°	i, V, v, iv

Ex. 4.13. Triads that can be used to harmonize reciting tones in Legrenzi's "natural" Re tonalities, i.e., those that may represent untransposed church keys

The church keys that we understand as Legrenzi's Re tonalities are the first (d/♮), second (g/♭), third (a/♮), and seventh (c/♭) church keys, and among these there are three different positions for the reciting tone: a fifth above the final (as for d/♮), a minor third above the final (as for g/♭ and a/♮), and a perfect fourth above the final (as for c/♭). Example 4.13 (above) tabulates the triads that could be used to harmonize each of these different scale degrees in the natural tonalities, as well as the four commonest triads in each tonality (taken from Example 4.2).

Of the four Re tonalities, the d/♮ tonality fits this model best: its four commonest triads (i, III, V, and v) can all be used to harmonize the reciting tone of the psalm tone on which it is based, and nine of its twelve commonest progressions include one of these four triads. The g/♭ and a/♮ tonalities fit the model relatively well, having two triads (i and III) among their commonest four that can be used to harmonize the reciting tones of their respective psalm tones. Of these two, the g/♭ tonality fits the model slightly better because it uses an unusually large number of III triads—there are almost as many III triads in g/♭ as there are i triads, and six of the twelve commonest chord progressions in g/♭ involve the III triad, including i↔III and III↔♭VI, which can be used to harmonize a held B♭. The c/♭ tonality stands out as the least likely tonality to harmonize its psalm tone's reciting tone, having only one triad (iv) among its four commonest triads that can harmonize the right scale degree. In terms of chord progressions, three out of the twelve commonest progressions in c/♭involve the iv triad, only one of which (iv↔ii°) can be used to harmonize a held F.

For the remaining Re tonalities, we ask: which scale degrees are they most likely to harmonize, and from this perspective, which untransposed church keys are they most similar to? The commonest triads of the three remaining Re tonalities (c/♭♭, g/♭♭, and b/♯♯) are tabulated in Example 4.14 (below). Based on these data and the data in Examples 4.11a and 4.11b, we might understand these tonalities as transpositions of either the first, second, or third church keys.

The c/♭♭ tonality (which is paired with e/♯♯ as an optional transposition of Legrenzi's Op. 10 no.18) looks most similar to the d/♮ tonality, with scale degree 5 as its most harmonizable scale degree and three of its four commonest triads that can harmonize it. The g/♭♭ and b/♯♯ tonalities have the same four commonest triads, but in different order. The most harmonizable non-tonic scale degree in both of these tonalities is scale degree 3, placing them closest to the second and third church keys.

Tonality	Commonest Triads from Ex. 4.2	Most Harmonizable Scale Degrees	Most Likely Church Key
c/♭♭ or e/♯♯	i, iv, III, V	5, 3, 1	d/♮ (1) transposed
g/♭♭ or e/♯	i, VI, iv, III	1, 3, 5, 6	g/♭ (2) or a/♮ (3) transposed
b/♯♯	VI, i, III, iv	3, 1, 5, 6	a/♮ (3) transposed

Ex. 4.14. Legrenzi's "transposed" Re tonalities and the church keys of which they are most likely transpositions, based on the reciting-tone harmonization model

From this, it follows that we can understand each of these three tonalities as an exact, stepwise transposition of some untransposed Re tonality: c/♭♭ or e/♯♯ as a transposition of d/♮ and b/♯♯ and g/♭♭ as two different transpositions of a/♮. Looking back at Example 4.11b, we see that the b/♯♯ tonality shares eight of its twelve commonest Roman-numeral progressions with the a/♮ tonality (a likely match), while the g/♭♭ tonality shares only half of its twelve commonest Roman-numeral progressions with the a/♮ tonality. For this reason, we might instead understand g/♭♭ as another version of the second church key (g/♭)—with which it also shares half of its twelve commonest Roman-numeral progressions—but with a second signature flat. The g/♭♭ tonality is paired with e/♯ as an optional transposition of Legrenzi's Op. 10, no. 17, which means that if g/♭♭ is a stepwise transposition of a/♮, then e/♯ is best explained as a transposition of a/♮ by fifth; if g/♭♭ is a version of g/♭, then e/♯ is a transposition of the latter by minor third.

If Legrenzi's c/♭ tonality really does represent the seventh church key, as Bonta says, it does not appear to undergo transposition in the same way as the first, second, and third church keys. What is perhaps more likely is that c/♭ is itself a stepwise transposition of d/♮, with which it shares three of its four commonest triads (i, V, and v) and eight of its twelve commonest Roman-numeral progressions.

Tonality (Church Key)	Reciting Tone	Triads that Could Harmonize Reciting Tone	Commonest Triads from Ex. 4.2
C/♮ (5)	scale degree 5	I, iii, V	I, V, vi, IV
G/♮ (8)	scale degree 4	ii, IV, ♯vii°	I, V, IV, vi

Ex. 4.15. Triads that can be used to harmonize reciting tones in Legrenzi's "natural" Ut tonalities, i.e., those that may represent untransposed church keys

The church keys that we understand as Legrenzi's Ut tonalities are the fifth (C/♮), sixth (F/♭), and eighth (G/♮). Although there is no untransposed example of the sixth church key (F/♭) in my Legrenzi corpus, we will still analyse the Ut tonalities to determine which tonalities, if any, might behave like transpositions of the missing F/♭ tonality. In the Ut tonalities, there are also three possible reciting-tone positions: scale degree 5 (as for C/♮), scale degree 4 (as for G/♮), and scale degree 3 (as for F/♭). Example 4.15 (above) tabulates the triads that could be used to harmonize each of these different scale degrees in the natural tonalities, as well as the four commonest triads in each tonality (taken from Example 4.2).

While Legrenzi's Ut tonalities are, from a Roman-numeral perspective, more homogenous than his Re tonalities, there is still a small difference between the two natural tonalities C/♮ and G/♮, which we might attribute to their difference in reciting tone position. The G/♮ tonality uses more IV triads than the C/♮ tonality, while the C/♮ tonality uses more V triads than the G/♮ tonality. Five of the twelve commonest progressions in G/♮ involve the IV triad, while only four involve the V triad. In the C/♮ tonality, there are twice as many progressions involving the V triad (4) as there are progressions involving the IV triad (2) and the V triad in C/♮ interacts with two different applied dominants (♯iv^{o} and II) whereas the V triad in G/♮ only interacts with one (II). These features lend the G/♮ tonality a plagal quality and support the idea that reciting-tone position has some effect on chords and chord progressions in Legrenzi's untransposed church keys.

Tonality	Commonest Triads from Ex. 4.2	Most Harmonizable Scale Degrees	Most Likely Church Key
B♭/♭	I, V, iii, vi	5, 3, 1, 7	C/♮ (5) transposed
D/♯	I, V, IV, vi	1, 5, 3, 6	C/♮ (5) transposed or G/♮ (8) transposed
A/♯♯	I, V, IV, ii	5, 1, 2, 4, 6	G/♮ (8) transposed

Ex. 4.16. Legrenzi's "transposed" Re tonalities and the church keys of which they are most likely transpositions, based on the reciting-tone harmonization model

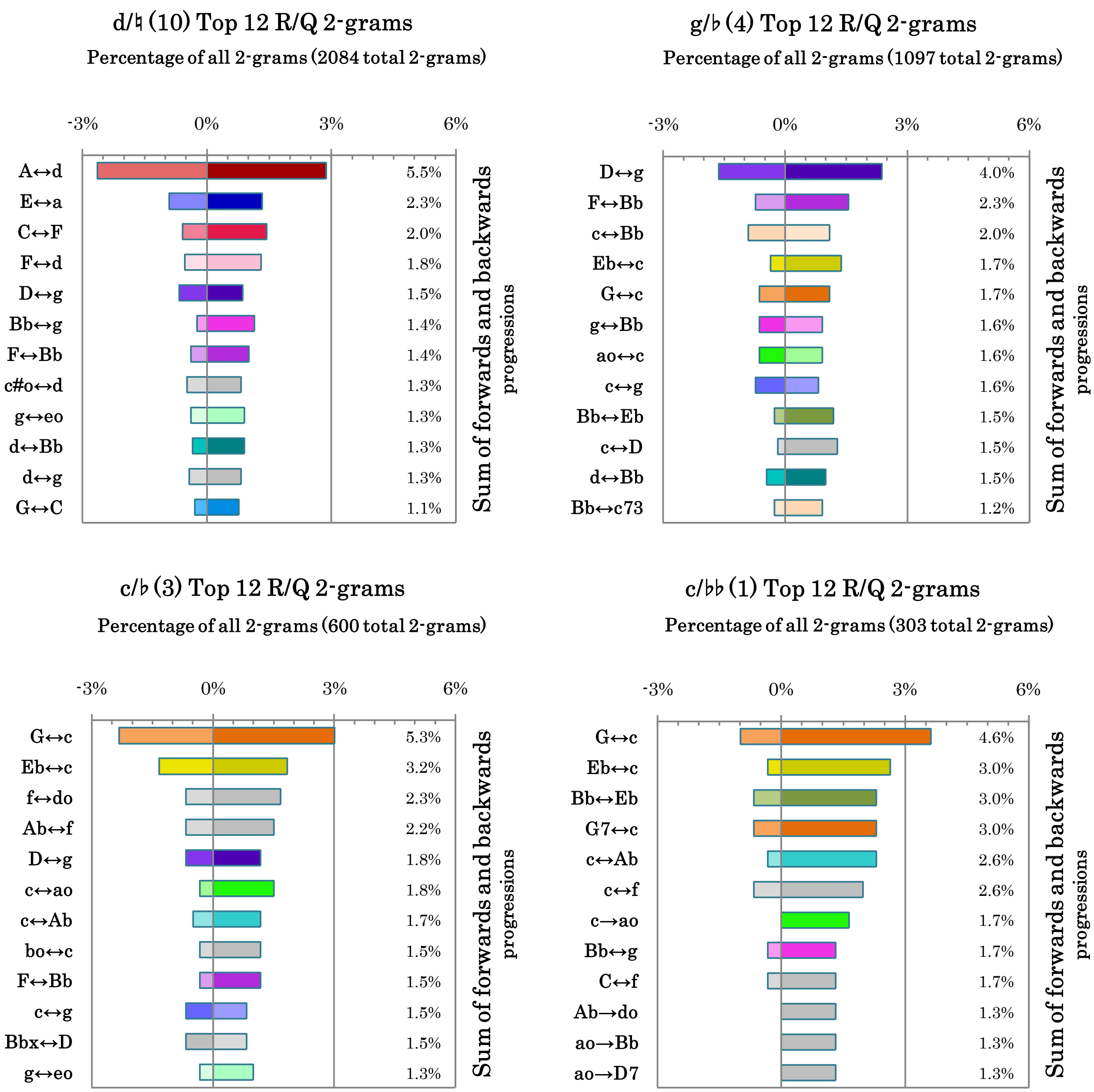
Legrenzi's transposed Ut tonalities are listed in Example 4.16 (above) along with their commonest triads and most harmonizable scale degrees. While all three of these tonalities have scale degree 5 as their most harmonizable non-tonic scale degree, which suggests that all three can be understood as transpositions of the C/♮ tonality, certain small differences suggest that the plagal G/♮ tonality is represented in this group as well.

The B♭/♭ tonality, despite its lydian signature, best represents the C/♮ tonality: both have the same four commonest triads, three of which can be used to harmonize scale degree 5 (the reciting tone of the fifth psalm tone), and they share eight of their twelve commonest Roman-numeral progressions. The D/♯ tonality, with mixolydian signature, appears at first to fall into the C/♮ category as well, sharing all four of its commonest triads and eight of its twelve commonest Roman-numeral progressions with C/♮, including two different progressions that emphasize the V triad using applied dominants (♯iv^{o} and II). But the D/♯ tonality also shares its four commonest triads with the G/♮ tonality, which, like D/♯, ranks IV above vi (whereas the C/♮ tonality ranks vi above IV) and from a 2-grams perspective, the D/♯ and G/♮ tonalities share nine of their twelve commonest Roman-numeral progressions. On these bases, an argument could be made in favour of classifying Legrenzi's D/♯ tonality as a transposition (by fifth) of his G/♮ tonality, rather than a stepwise transposition of his C/♮ tonality (which does not preserve octave species), as we hypothesized in Example 4.1.[12] This would make D/♯ one of only two tonalities in Legrenzi that we might understand as an exact transposition by fifth of some other tonality.[13] The A/♯♯ tonality is much less ambiguous than the D/♯ tonality, emphasizing scale degree 4 with many more IV and ii triads than the other Ut tonalities and sharing eight of its twelve commonest Roman-numeral progressions with the G/♮ tonality.

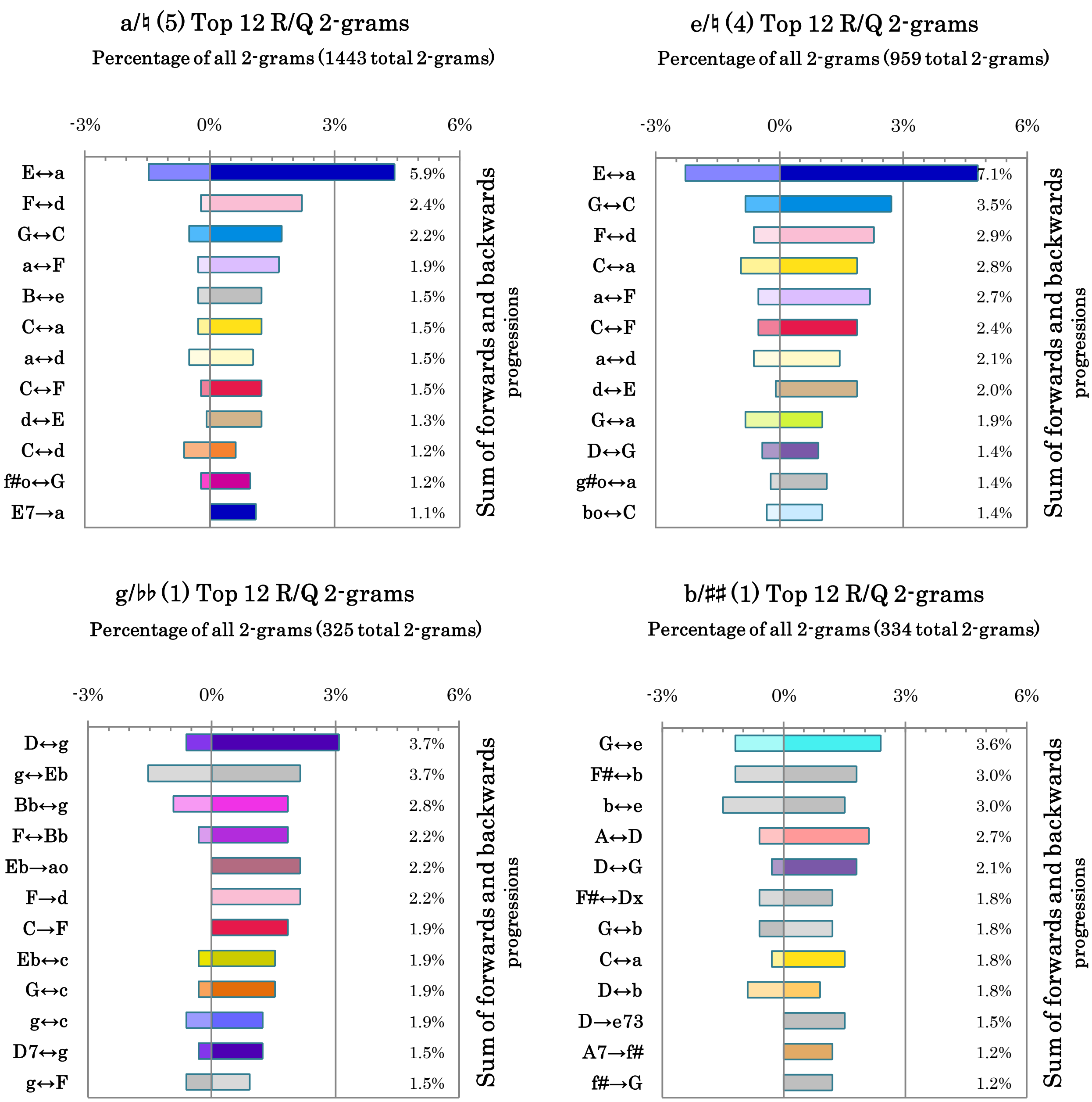
For the most part, these findings corroborate the transpositions laid out in Example 4.1. In some cases, however, we see that transpositions by fifth are more easily explained than transpositions by step, at least from the perspective of Roman-numeral progressions. Specifically, the D/♯ tonality is more likely to be a transposition by fifth of the G/♮ tonality, rather than a stepwise transposition of the C/♮ tonality. In Section 4.2, we saw that the closest transpositional relationships on the basis of chord content (1-grams) were never between tonalities with the same signature-defined octave species. On the bases of chord progressions (2-grams), however, the closest transpositional relationships appear between tonalities with the same signature-defined octave species, apart from the C/♮→B♭/♭ transposition, which does not preserve octave species.

[12] Or, as Bonta mentions, Legrenzi may have conceived of the D/♯ tonality as one of two (likely unrelated) versions of the seventh church key, rather than a transposition of either the fifth or the eighth church key.

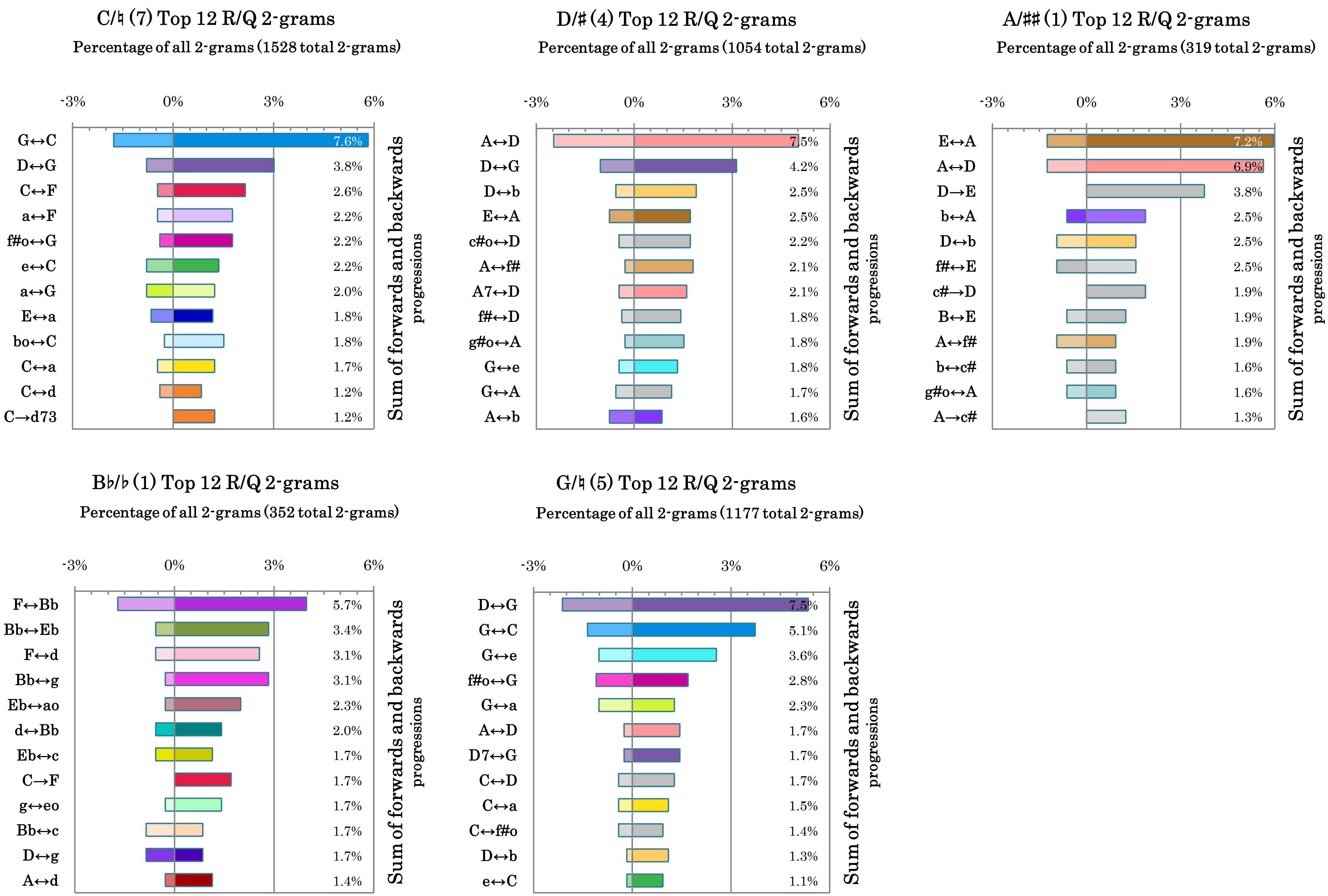
[13] The other tonality that might be understood as an exact transposition by fifth is the e/♯ tonality (paired with g/♭♭), whose parent tonality is either a/♮ (implying transposition by fifth) or g/♭ (implying transposition by minor third).



Ex. 4.17a. Commonest progressions by root and quality in Legrenzi's dorian tonalities—Arabic numerals in brackets show numbers of sonatas per tonality



Ex. 4.17b. Commonest progressions by root and quality in Legrenzi's minorand phrygian tonalities—Arabic numerals in brackets show numbers of sonatas per tonality



Ex. 4.18. Commonest progressions by root and quality in Legrenzi's Ut tonalities—Arabic numerals in brackets show numbers of sonatas per tonality

4.3.3. 2-Grams as Root/Quality Tornado Charts

We turn now to another representation of the same data presented in Examples 4.11a, 4.11b, and 4.12, which will help us understand how Legrenzi's tonalities group together from a root/quality perspective. For our purposes, root/quality similarity and Roman-numeral similarity are opposite and mutually exclusive. If two tonalities have many Roman-numeral progressions in common, it means they are likely transpositions of one another. But if two tonalities have many root/quality progressions in common, it means they are not transpositions of one another. Instead, they form a pair or group of tonalities that we call "relatives" (like the F/♭-d/♭ pair in Frescobaldi), in which the root/quality progressions remain constant while the final changes (typically, finals differ by a minor third). Pairs of tonalities that we call "authentic-plagal" (like the a/♮-e/♮ pair) are another example of the same-progressions-different-finals phenomenon, but here the finals differ by a perfect fourth. In this section, we will see that none of Legrenzi's thirteen tonalities are very similar from a root/quality perspective, except for the a/♮-e/♮ pair.

The colour-coding scheme in Examples 4.17a, 4.17b, and 4.18 (above) is internally consistent, but colours in these three examples should not be compared with those in examples 4.11a, 4.11b, and 4.12. The interested reader can also compare the colours in Examples 4.17a, 4.17b, and 4.18 with those in Examples 3.16 and 3.17 (root/quality tornado charts for Frescobaldi's ten tonalities). In this scheme, a colour is assigned to any root/quality progression that appears in at least two different tonalities, while unshared progressions remain grey. The progression F↔d(F major to D minor), for example, is always coloured pink. In Frescobaldi, we needed 20 different colours (plus grey) to colour all of the shared root/quality progressions. For Legrenzi, we need to add 15 more colours, for a total of 35 (plus grey) in order to colour all of the shared root/quality progressions. This suggests that Legrenzi is using a wider variety of progressions than Frescobaldi.

What we notice first is that no two tonalities with different finals have the same top progression, except for the a/♮ and e/♮ tonalities, in which E↔a is the commonest progression. While in Frescobaldi we saw that certain progressions like C↔F and G↔C were ubiquitous, appearing as the commonest progressions in six out of ten tonalities (half of whose finals were neither C nor F), in Legrenzi we see that the commonest progression in any tonality is always V↔I in that tonality[14] and that the root/quality version of that progression does not appear first in any other tonality unless it has the same final as another tonality (as is the case for Legrenzi's two c and two g tonalities in the Re category).

The C↔F progression, which was the most common in Frescobaldi (appearing among the twelve commonest progressions in nine out of ten tonalities), is also the most common progression in Legrenzi, appearing among the twelve commonest progressions in six out of thirteen of his tonalities. All other root/quality

[14] The V↔i progression in b/♯♯ is actually the second most common, after the G↔e progression (VI↔iv).

progressions in Legrenzi appear among the twelve commonest progressions in five tonalities or fewer.

Example 4.19 lists the thirteen root/quality progressions that appear in at least four different tonalities. Unsurprisingly, progressions requiring at least one accidental or signature flat (the first five progressions in Example 4.19) appear most often in flat-side tonalities. The next two progressions (C↔F and F↔d) are evenly spread between flat-side and natural tonalities, while the E↔a and G↔C progressions best characterise the five natural tonalities. The D↔G and C↔a progressions appear in four out of five natural tonalities plus the b/♯♯ tonality (which is unusual because both triads in the C↔a progression require that the signature C♯ in b/♯♯ be cancelled). The last two progressions in Example 4.19 (A↔D and D↔b), each of which require one or more sharps, appear only in the sharp-side tonalities plus the G/♮ tonality.

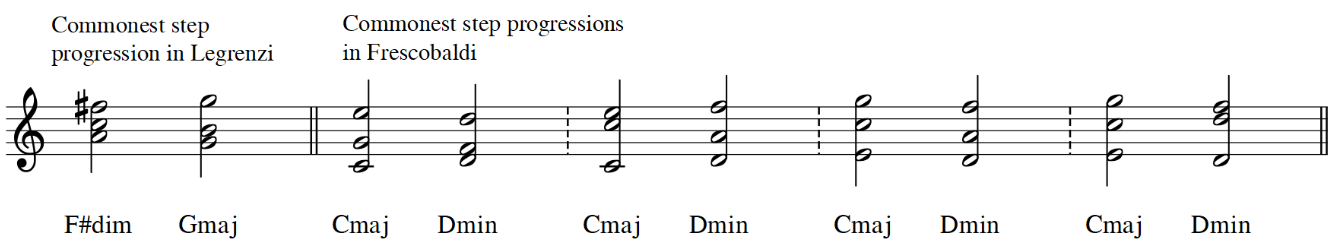
Looking at the rightmost column of Example 4.19, we see that Legrenzi's shared root/quality progressions are evenly spread. There is no single progression that appears in all or most tonalities, as there was in Frescobaldi. That no single root/quality progression appears in more than six out of thirteen tonalities, and that most appear in only five or four out of thirteen tonalities, indicates that Legrenzi is transposing progressions from one tonality (or group of tonalities) to the next, rather than leaving certain "favourite" progressions (like Frescobaldi's C↔F progression) untransposed.

	c/♭(3)	c/♭♭(1)	g/♭(4)	g/♭♭(1)	B♭/♭(1)	d/♮(10)	C/♮(7)	a/♮(5)	e/♮(4)	G/♮(5)	b/♯♯(1)	D/♯(4)	A/♯♯(1)	Total
G↔c	✔	✔	✔	✔										4
E♭↔c	✔	✔	✔	✔	✔									5
D↔g	✔		✔	✔	✔	✔								5
F↔B♭	✔		✔	✔	✔	✔								5
B♭↔g		✔	✔	✔	✔	✔								5
C↔F				✔	✔	✔	✔	✔	✔					6
F↔d				✔	✔	✔		✔	✔					5
E↔a						✔	✔	✔	✔					4
G↔C						✔	✔	✔	✔	✔				5
D↔G						✔	✔		✔	✔	✔			5
C↔a							✔	✔	✔	✔	✔			5
A↔D										✔	✔	✔	✔	4
D↔b										✔	✔	✔	✔	4
Total	4	3	5	7	6	8	5	5	6	5	4	2	2	

Ex. 4.19. Root/quality progressions that appear in at least four different tonalities—rightmost column tallies total tonalities in which each progression appears—bottom row tallies total number of shared progressions per tonality—bracketed numbers show how many sonatas per tonality

Legrenzi's transpositions become more apparent when we consider the types of progressions that are recurring. We can understand the G↔c, D↔g, and E↔a progressions as transpositions of one another, and the same can be said of the F↔B♭, C↔F, G↔C, D↔G, and A↔D progressions. The third progressions, likewise, can be understood as transpositions of a single 2-gram: E♭↔c, B♭↔g, F↔d, C↔a, and D↔b. In this way, every progression in Example 4.19 is one of only three types of progression: Maj↔min by descending fifth, Maj↔Maj by descending fifth, or Maj↔min by descending third.

In Frescobaldi, we saw that step progressions also characterised certain groups of tonalities (e.g., the C↔d progression in the *cantus durus* tonalities). Legrenzi uses far fewer step progressions and those that he does use are of a different kind than Frescobaldi's. Among the twelve commonest root/quality progressions in Legrenzi's tonalities, the most common type of step progression is a diminished triad resolving up by step to a major or minor triad, whereas Frescobaldi's step progressions (as well as his fifth and third progressions) comprised major and minor triads almost exclusively. The most common diminished triad in Legrenzi has F♯ as its root and the f♯°↔G progression occurs in three different tonalities: G/♮, C/♮, and a/♮. In six out of thirteen tonalities we find either a vii°↔I or vii°↔i progression and, if we still accept Corelli as an exemplar of tonal harmony, then we expect the proportion of vii°↔I progressions to increase again in Corelli's sonatas. Example 4.20 (below) shows the commonest step progressions used by Legrenzi and Frescobaldi.



Ex. 4.20. Commonest step progressions used by Legrenzi and Frescobaldi—Legrenzi's f♯°↔G progression appears almost exclusively in the voicing shown—Frescobaldi's commonest step progression, C↔d appears in several different voicings, four of the most common of which are shown

Looking at the bottom row of Example 4.19 (above), we see that the d/♮ tonality has the most shared root/quality progressions of any tonality (eight in total). This suggests that while Legrenzi's tonal world has certainly expanded sharpwards (via transposition) compared to that of Frescobaldi, it still centres on the natural and flat tonalities. Although it is a natural tonality according to its signature, the d/♮ tonality sits half way between the flat and natural groups, sharing just as many progressions with flat-side tonalities (D↔g, F↔B♭ B♭↔g, C↔F, and F↔d) as with natural tonalities (C↔F, F↔d, E↔a, G↔C, and D↔G). In a similar way, the G/♮ tonality groups with both the sharp-side and natural tonalities, using progressions like A↔D and D↔b, despite its natural signature.

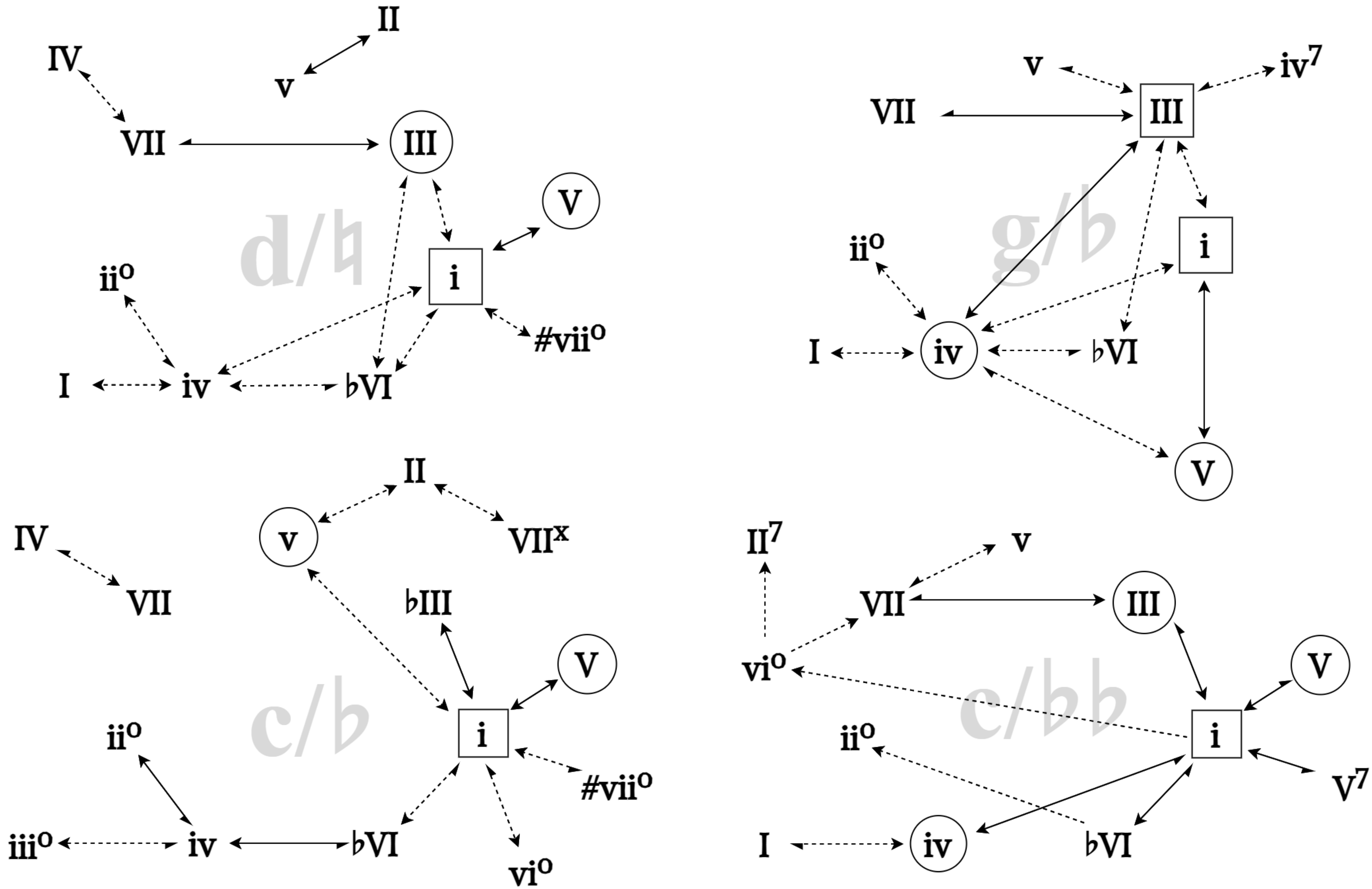
When we compare the top two tornado charts in Example 4.17b, we see how the a/♮ and e/♮ tonalities have grown even closer from a root/quality perspective since Frescobaldi's time. In these two tonalities, the three commonest root/quality progressions (E↔a, F↔d, and G↔C) are the same, but the order of the second and third progressions is switched. The next two root/quality progressions in both tonalities, excluding the B↔e progression in a/♮, are a↔F and C↔a, but again in different order. The next three root/quality progressions in both tonalities (a↔d, C↔F, and d↔E) are also the same, but the order of the a↔d and C↔F progressions is switched. Together, these eight root/quality progressions underpin the authentic-plagal relationship between a/♮ and e/♮, two tonalities with different finals but the same vocabulary of root/quality progressions. This makes it difficult to hear Emin (or Emaj) as the "tonic" triad of the e/♮ tonality. Instead, we are likely to hear Emaj as the "dominant" triad of a tonality whose "tonic" is actually a perfect fourth above the final.

Hearing an Emin triad as the tonic in Legrenzi's e/♮ tonality makes even less sense when we consider that none of the twelve commonest root/quality progressions in that tonality include an Emin triad (although Emaj triads are used in two of the twelve commonest root/quality progressions). According to Example 4.7, only 4.1% of the triads in the e/♮ tonality are Emin triads, making it the seventh commonest triad in that tonality. In Corelli, we expect the root/quality similarity between the a/♮ and e/♮ tonalities to increase again, perhaps to the point where they are indistinguishable when comparing their twelve commonest root/quality progressions.

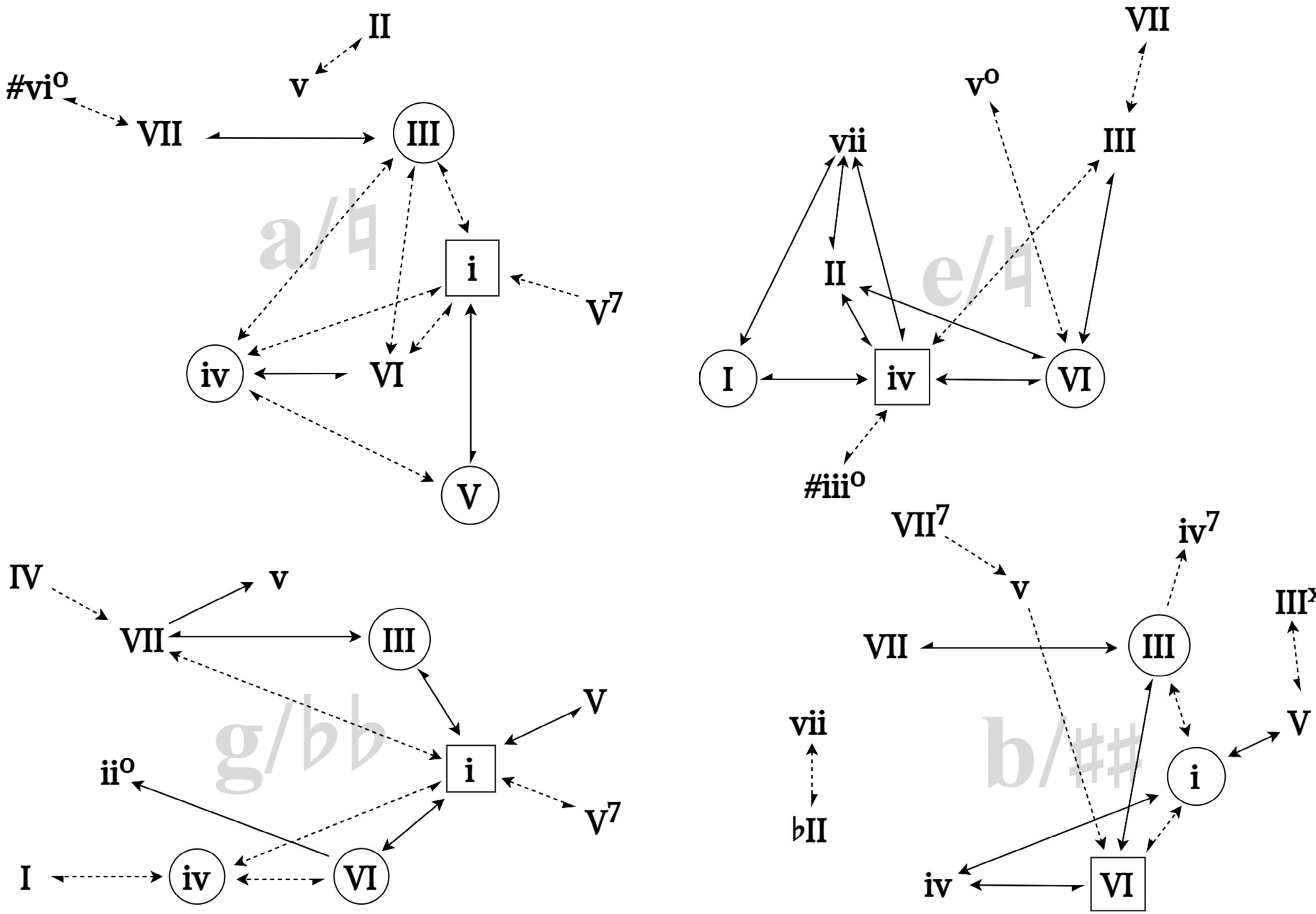
4.3.4. 2-Grams as Flowcharts

The flowcharts in Examples 4.21a, 4.21b, and 4.22 summarize the 1-gram and 2-gram Roman-numeral data for all thirteen of Legrenzi's tonalities. While it is convenient to have these two types of data synthesized in a single format, it is important to remember that Roman-numeral 1-grams and Roman-numeral 2-grams tell different stories on their own. Roman-numeral 1-grams produced the dendrograms in Example 4.6, in which we saw that no two tonalities in Legrenzi are more similar to one another than they are to their category (Re or Ut) as a whole. Roman-numeral 2-grams, on the other hand, revealed that transpositional relationships do still exist between tonalities with the same octave-species, especially between the G/♮, D/♯, and A/♯♯, tonalities.

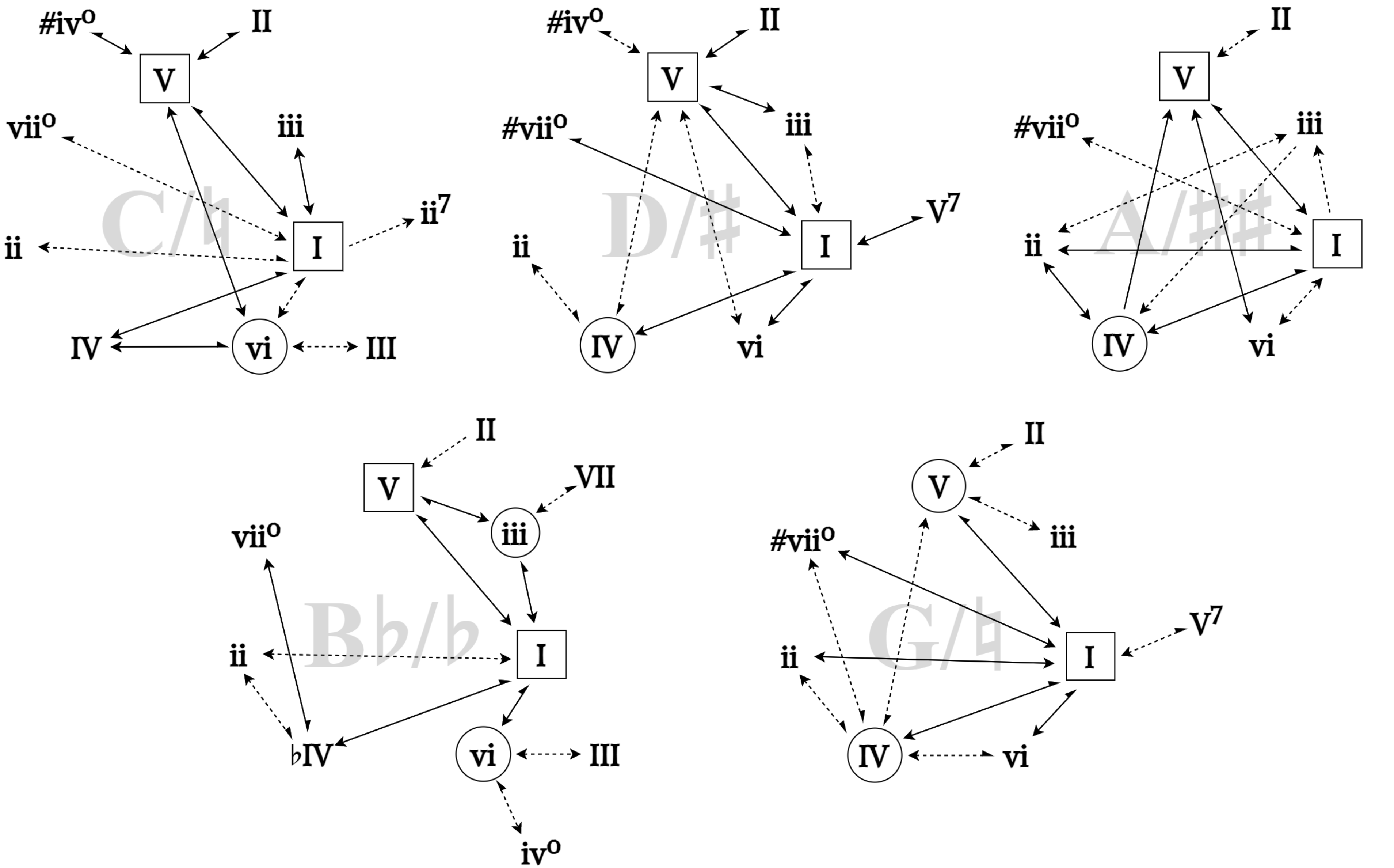
In the same fashion as Section 4.3.3, the flowchart representations below allow for tonalities to be compared in several dimensions at once. The a/♮ and e/♮ tonalities, for example, are revealed to be rotations of the same basic flowchart: if



Ex. 4.21a. *Roman-numeral flowcharts for Legrenzi's dorian tonalities—boxed Roman numerals are extremely common, circled Roman numerals are common, and un-circled Roman numerals are less common—solid lines represent the commonest progressions (those that account for at least 2% of all observed 2-grams) and dotted lines represent less common progressions (those that fall below the 2% threshold)*



Ex. 4.21b. *Roman-numeral flowcharts for Legrenzi's minor and phrygian tonalities—boxed Roman numerals are extremely common, circled Roman numerals are common, and un-circled Roman numerals are less common—solid lines represent the commonest progressions (those that account for at least 2% of all observed 2-grams) and dotted lines represent less common progressions (those that fall below the 2% threshold)*



Ex. 4.22. Roman-numeral flowcharts for Legrenzi's Ut tonalities—boxed Roman numerals are extremely common, circled Roman numerals are common, and un-circled Roman numerals are less common—solid lines represent the commonest progressions (those that account for at least 2% of all observed 2-grams) and dotted lines represent less common progressions (those that fall below the 2% threshold)

we rotate the a/♮ flowchart 90 degrees clockwise and superimpose it on the e/♮ flowchart, we see that most of the arrows, boxes, and circles fall in the same places, while the Roman-numerals change (e.g., Vin a/♮ becomes I in e/♮, i in a/♮ becomes iv in e/♮, etc.). Another tonality that resembles a/♮ in flowchart form is g/♭. Both tonalities have i, III, iv, and V as their commonest triads, and both stand out as Re tonalities with “predominant” iv triads. None of Legrenzi’s other Re tonalities have predominant↔dominant progressions among their twelve commonest progressions. This similarity between a/♮ and g/♭ reminds us of Banchieri’s original psalm-tone transpositions, where he specifies that g/♭ can easily substitute for a/♮ as the third tone.

The major/mixolydian division within the Ut category is apparent in Example 4.22. Two of the five Ut tonalities, C/♮ and B♭/♭, use I, V, and vi as their commonest triads, with vi or II acting as “predominants.” Both tonalities include tonicizations of the vi triad (III↔vi) among their twelve commonest progressions. While it is most similar to C/♮, the B♭/♭ tonality still retains traces of the lydian tonality we saw in Frescobaldi, specifically the vi↔iv° progression. The other three Ut tonalities (G/♮, D/♯, and A/♯♯) use I, V, and IV as their commonest triads, with IV acting as a “predominant” triad. Unlike C/♮ and B♭/♭, none of these three tonalities tonicize the vi triad.

Although the D/♯ tonality resembles the other mixolydian tonalities most closely, it also shares some features with the C/♮ tonality, corroborating the stepwise transposition theory outlined in Section 4.1, but not outweighing the similarity between octave-species pairs evidenced by the Roman-numeral 2-grams (Section 4.3.1).D/♯ is similar to C/♮ in that it uses the same two "secondary dominants" as the latter (II and ♯iv°) and it lacks progressions involving the IV triad to the same extent as the C/♮ and B♭/♭ tonalities.

We might also speculate about the role of the B♭/♭ tonality as a substitute for the missing sixth church key (F/♭). A polyphonic setting of the sixth psalm tone, whose reciting tone lies a major third above its final, would require large amounts of I, iii, and vi triads, all of which are found in the B♭/♭ tonality. This theory offers an explanation for the large percentage of iii triads that is peculiar to the B♭/♭ tonality, although it is impossible to verify without an untransposed F/♭ tonality for comparison.

The observations we have made so far place Legrenzi's tonal style in stark contrast to that of Frescobaldi. Apart from a handful of vestigial features, Legrenzi's tonalities look and sound like major and minor keys, even when their key signatures indicate octave species other than major or minor. As we proceed to analyse Corelli's trio sonatas, we expect to uncover a tonal practice in which the Ut and Re categories become completely homogenous, leaving only two real modes, and a harmonic vocabulary that is fully transposable from one key to another.

5. Corelli

Using the same methods outlined in Chapter 3, (chord 1-grams, chord 2-grams, and Roman-numeral flow charts), we now examine the 48 trio sonatas by Corelli.

5.1. Corelli's Tonalities

Of the three composers in this study, Corelli uses the largest number of different tonalities in his instrumental works. Example 5.1 tabulates the eighteen tonalities (each represented by final and signature) found in the Corelli corpus: eight in the Ut category, nine in the Re category, and one in the Mi category. I have chosen to organize Corelli's tonalities according to the Ut, Re, and Mi categories outlined in Section 3.1 (Judd, 1992), with untransposed or "natural" tonalities in the top row and transposed tonalities (those requiring at least one signature flat or sharp) in the rows below.

Category	Ut		Re		Mi
Natural Tonalities	C/♮(4)	G/♮(4)	d/♮(3)	a/♮(2)	e/♮(1)
Transposed Tonalities	D/♯♯(4)	A/♯♯(3)	f/♭♭(1)	b/♯♯(4)	
	E♭/♭♭ (1)	E/♯♯♯(2)	g/♭(4)	d/♭(1)	
	F/♭(4)		c/♭♭(2)	e/♯(3)	
	B♭/♭ (4)			f♯/♯♯♯(1)	

Ex. 5.1. Corelli's eighteen tonalities organized according to category (Ut, Re, Mi) and octave species (major, mixolydian, dorian, minor, phrygian)—three tonalities appear to be missing one signature flat when compared with their natural tonalities (E♭/♭♭,B♭/♭, and f/♭♭)

Each column contains tonalities with the same signature-defined octave species, with three exceptions: the E♭/♭♭ and B♭/♭ tonalities appear in the C/♮ column and the f/♭♭ tonality appears in the d/♮ column. According to the scheme in Example 5.1, these three tonalities would appear to be missing one signature flat when compared with their natural tonalities. Historical rationale for "missing" flats and other kinds of "incomplete" signatures is given in the discussion that follows.

Sonata	Tonality	Category
1	F/♭	Ut
2	e/♯	Re
3	A/♯♯	Ut
4	e/♮	Mi[1]
5	B♭/♭	Ut
6	b/♯♯	Re
7	C/♮	Ut
8	c/♭♭	Re
9	G/♮	Ut
10	g/♭	Re
11	d/♮	Re
12	D/♯♯	Ut

Ex. 5.2. Tonalities of the twelve sonatas in Corelli's Op. 1—sonatas alternate between major-third and minor-third tonalities (except for last two) and finals proceed in ascending order (except for sonatas 1, 2, and 4)

In the previous chapter, the ordering of sonatas in Legrenzi's Op. 2 (1655) gave us some idea as to how the composer conceived of the tonalities he used. While Legrenzi likely had an excellent understanding of modal theory, he chose to order the pieces in his first book of instrumental music according to the church keys. For

[1] This fourth sonata behaves as if it were in a/♮, but its final is E, so I analyse it here as an e/♮ piece in keeping with my final/signature convention from previous chapters.

this reason, we analysed Legrenzi's music specifically through the lens of psalmody (see Section 4.3.1).

I believe that the twelve tonalities in this collection (tabulated in Example 5.2 above) tell us three things: (1) that Corelli wanted to use all of the natural notes (plus B♭) as finals[2]; (2) that he wanted to have a major-third and a minor-third tonality for each final (leaving out the very distant E-major and f-minor); and (3) that he wanted to keep signature flats or sharps to a minimum (no more than two sharps or flats). In this way, Corelli's Op. 1 is actually a sort of "ordered collection," but rather than following the order of some pre-existing theoretical construct (e.g., modes or church keys), Corelli puts his sonatas in an order that highlights how each different final has its major-third and its minor-third tonality.[3]

From this, one might be inclined to interpret Corelli's tonal world as a simple dichotomy between Ut and Re tonalities, but according to some other Corelli scholars, this is an oversimplification. Corelli's foremost biographer, Peter Allsop, believes that "Corelli himself… in all probability observed the twelve-mode system" (Allsop 1999, 102). Allsop cites a passage from one of Corelli's pupils, Francesco Gasparini, as evidence: "I leave out consideration of the third and fourth modes," writes Gasparini in his *L'Armonico pratico al cimbalo* (1722) "which must be read *Mi, fa, sol* since this is not applied rigorously by present-day composers in its

[2] Corelli uses as many different finals in this one book of sonatas as we saw in all 47 of the Legrenzi sonatas.

[3] The note B is only ever the final of a minor-third tonality in Corelli, while the note B♭ is only ever the final of a major-third tonality.

original form, but with transpositions that would make explanations necessary" (Allsop 1999, 102). Allsop believes that "[t]his can only refer to the two E Tones, each with an initial semitone in their scale structure (*mi, fa*) as expounded by Bononcini…" (Allsop 1999, 102).

Allsop draws on Bononcini often in his discussion of Corelli's tonal practice. Allsop believes that Corelli's tonal world comprises seven modes, a subset of Glarean's twelve, derived by Bononcini in his *Il musico prattico* (1673). These are listed, along with cadences proper to each mode, in Example 5.3 (below), which is adapted from Allsop's Table 6.6 (Allsop 1999, 103). Bononcini eliminates modes III through VII to leave only the modes on D, A, C, and one on G (Allsop 1999, 101). Barnett makes reference to this same set of seven modes, saying: "Bononcini taught that certain modes had fallen out of use in late-*Seicento* practice. Furthermore, the surviving seven modes sufficed because some of these could be used by means of transposition to replace those no longer in use" (Barnett 2008, 256).

Tone	Principio	Mezzo	Finale
I	A	F	d
II	A	F	d
VIII	D	C	G
IX	e	C	a
X	e	C	a
XI	G	a	C
XII	G	e	C

Ex. 5.3. *Bononcini's seven modes "ordinarily used by composers"—according to Allsop, these are the seven modes used by Corelli—the cadences proper to each mode are taken from Bononcini, but also appear in Penna (Allsop 1999, 103)*

Allsop continues to give evidence for the modal bases of Corelli's music in his discussion of "key-relationships" in Corelli. He remarks that Corelli uses the "proper" cadences for each mode, and that these create relationships that we would expect from a tonal standpoint: i, V, and III or i, v, and III in the Re tonalities and I, V, and vi in the Ut tonalities. In two of Bononcini's modes (VIII and XII), however, fewer "tonal" key-relationships are created if the proper cadences are used: a Mode-VIII sonata should prefer IV over vi and sometimes even v over V, while a Mode-XII sonata should prefer iii over vi. When he compares Corelli with some of his contemporaries (Cazzati, Bononcini, and Stradella), who all use either a I-V-IV or a I-IV-v scheme in their Mode-VIII works, Allsop asserts that "it must be regarded with utmost significance that… Corelli's G compositions unequivocally oppose tonic with major dominant as its *cadenza principale* and relative minor as the *cadenza di mezzo*" (Allsop 1999, 104). Allsop goes on to say that while Corelli's G compositions eschew the I-V-IV paradigm in favour of a more tonal I-V-vi paradigm, his C compositions do not: "Corelli shows a particular liking for the even less 'tonal' cadence structures of Tone XII, which characteristically contrasts the tonic of C major with E minor as its *cadenza di mezzo*" (Allsop 1999, 104).

Allsop's theories about Mode-VIII and Mode-XII works in Corelli will serve as foils for my own investigations of Corelli's tonal practice. Using 1-gram and 2-gram data, I look at how closely Corelli's G/♮ sonatas follow the expected I-V-vi paradigm and whether or not a preference for subdominant and/or minor dominant triads still sets this tonality apart from the others in the Ut category. Likewise, I examine Corelli's C/♮ sonatas for any special prevalence of iii triads.

Allsop is also interested in how Corelli's use of signature flats and sharps seems to adhere to a set of transposition rules laid out by Bononcini. Example 5.4 (below), adapted from Allsop's Table 6.4 (Allsop 1999, 101), lists Bononcini's rules for transposing the modes.

Signature	Transposition	Implications
♭	fourth above or fifth below	d/♮ becomes g/♭ a/♮ becomes d/♭ C/♮ becomes F/♭
♭♭	one tone below	d/♮ becomes c/♭♭
♭♭♭	minor third above	d/♮ becomes f/♭♭[4]
♯	fourth below or fifth above	a/♮ becomes e/♯
♯♯	one tone above	G/♮ becomes A/♯♯ a/♮ becomes b/♯♯ C/♮ becomes D/♯♯
♯♯♯	minor third below	a/♮ becomes f/♯♯♯ C/♮ becomes E/♯♯♯
"sometimes the rules are varied"		
♯	for Tone I a tone above	d/♮ becomes e/♯
♯♯	for Tone VIII a third below	G/♮ becomes E/♯♯[5]
♭	for Tone XI a tone below	C/♮ becomes B♭/♭
♭♭	for Tone XII a minor third above	C/♮ becomes E♭/♭♭

Ex. 5.4. *Bononcini's rules for transposing the modes—for Allsop, these rules explain all of Corelli's signatures, including the seemingly incomplete signatures of the B♭/♭ and E♭/♭♭ tonalities—when Bononcini says "sometimes the rules are varied" (Allsop 1999, 101) he means that sometimes the signature-defined octave species is not maintained*

[4] I believe that Corelli's f/♭♭ tonality is, like Legrenzi's c/♭ tonality, a dorian tonality that is missing one signature flat, but Allsop does not comment on this.

[5] The E/♯♯ tonality does not appear in Corelli; only E/♯♯♯ is used.

The rightmost column is my addition, showing how these transpositions manifest in Corelli. Some of these relationships are not exclusive, e.g., the e/♯ tonality would appear to be a transposition of both a/♮ and d/♮. The statistical analyses carried out in the following section will help us refute one or the other of these relationships. For Allsop, these rules are the key to Corelli's signatures, especially the seemingly incomplete signatures of the B♭/♭ and E♭/♭♭ tonalities.

5.2 Comparing Chord Histograms

Using the same methodology as we did for Frescobaldi and Legrenzi, tallies of the commonest chords are made for all of Corelli's tonalities (excluding e/♮). Euclidean distances are then calculated between the resulting histograms. Tonalities are compared using both Roman-numeral and root/quality representations of their commonest chords. In the Re category, fourteen chords are tallied, while in the Ut category, thirteen chords are tallied. These numbers are different so that we obtain roughly the same percentage of the total number of progressions (~53%) in Ut and Re tonalities. These include the seven diatonic triads for each of the different octave species in each category (minor and dorian in the Re category; major, lydian, and mixolydian in the Ut category), plus the most common chromatically altered chords (e.g., I in the Re tonalities and II in the Ut tonalities). Corelli's Mi tonality (e/♮) is compared with other tonalities from a root/quality perspective, but not from a Roman-numeral perspective.

5.2.1 Comparing Roman-Numeral Histograms

Roman-numeral histograms are presented in Examples 5.5 and 5.6 (below) as heat maps, with larger numbers shaded darker. All numbers are percentages of the total number of observed chords in each tonality, except for the **AVG** column, which averages all of the columns to the left of it.

	c/♭♭ (2)	d/♮ (3)	f/♭♭ (1)	g/♭ (4)	a/♮ (2)	b/♯♯ (4)	d/♭ (1)	e/♯ (3)	f♯/♯♯♯ (1)	AVG
i	10.15%	9.75%	9.23%	10.43%	10.66%	10.35%	11.14%	10.05%	12.32%	10.45%
I	1.49%	1.45%	1.58%	1.73%	1.51%	1.37%	1.14%	1.61%	1.18%	1.45%
ii	0.87%	1.12%	0.23%	0.81%	0.43%	0.91%	1.36%	0.80%	0.24%	0.75%
II	2.45%	1.32%	1.13%	1.15%	1.83%	1.42%	2.95%	1.74%	0.24%	1.58%
ii°	0.79%	2.70%	1.58%	1.21%	1.18%	2.49%	1.82%	1.34%	3.32%	1.83%
III	7.87%	9.55%	4.95%	6.22%	7.10%	8.73%	5.91%	5.43%	7.58%	7.04%
iv	5.25%	5.20%	6.08%	6.51%	8.18%	5.99%	3.86%	5.09%	6.64%	5.87%
IV	0.96%	0.99%	1.13%	1.15%	1.40%	0.86%	2.05%	1.00%	0.47%	1.11%
v	4.46%	3.62%	4.50%	4.09%	4.95%	5.78%	5.91%	4.29%	1.66%	4.36%
V	7.17%	6.85%	10.36%	7.60%	6.46%	5.63%	7.27%	7.03%	9.00%	7.49%
(♭)VI	4.37%	4.61%	4.73%	5.88%	5.06%	6.39%	5.00%	4.09%	5.92%	5.12%
(♯)vi°	0.17%	0.26%	0.00%	0.40%	0.00%	0.20%	0.45%	0.47%	0.00%	0.22%
VII	3.85%	5.14%	2.93%	4.55%	3.77%	4.92%	2.95%	3.01%	4.50%	3.96%
♯vii°	1.49%	2.31%	2.48%	2.07%	0.75%	1.78%	2.73%	2.41%	1.66%	1.96%

Ex. 5.5. Fourteen commonest triads (by Roman numeral) in Corelli's Re tonalities—largest number in each column is shaded darkest—bracketed numbers show how many sonatas per tonality—Roman numerals shaded green are diatonic in dorian

Example 5.5 (above) shows a heat map for Roman numerals in Corelli's Re tonalities. The Roman numerals shaded green are for triads that are diatonic within the dorian octave species: the ii, IV, and vi° triads. Of the fourteen tallied chords in Corelli's Re tonalities, these three have the lowest averages and no single tonality includes large numbers of all three triads. The d/♭ tonality, whose octave species is *not* dorian, uses the largest numbers of ii and IV triads (1.36% and 2.05%, respectively), while the e/♯ tonality, also not dorian, uses the largest number of vi° triads (0.47%), only slightly more than the d/♭ tonality (0.45%). What these numbers suggest is that signature-defined octave species is no longer a reliable predictor of Roman-numeral content in this repertoire. Tonalities with dorian signatures like d/♮ and g/♭ do not necessarily use more “dorian-sounding” triads than other Re-category tonalities like d/♭ and e/♯.

In all of Corelli’s Re tonalities, the commonest triad is the i triad. On average, the next most common triads are V and III, but this is not the case for every tonality in the Re category: in the f/♭♭, g/♭, and a/♮ tonalities, iv is more common than III, while in the b/♯♯ tonality, the III, VI, iv, and v triads (in that order) are more common than the V triad. This suggests that there is still some division to be made among Corelli’s Re tonalities from a 1-grams perspective. The particularly low percentage of V triads (i.e., F♯-major triads) in the b/♯♯ tonality is likely due, at least in part, to a tuning system that prioritized triads with fewer accidentals. Although they are beyond the scope of this project, the tuning systems that were in use at the end of the seventeenth century are likely to have had a large effect on the relative amounts of different triads in keys with more than one signature flat or sharp.

Example 5.6 (below) shows a heat map for the Roman numerals in Corelli’s Ut tonalities. In six of the eight Ut tonalities, the I and V triads are the first and second most commonly occurring (only in E♭/♭♭ and A/♯♯ does the V occur more often than I). In six of the eight tonalities, the IV triad is next most common, but in the

E♭/♭♭ tonality II is slightly more common than IV, while in the E/♯♯♯ tonality vi is more common than IV. These data situate E♭/♭♭ as an outlier among the Ut tonalities.

	B♭/♭ (4)	C/♮ (4)	D/♯♯ (4)	E♭/♭♭ (1)	F/♭ (4)	G/♮ (4)	A/♯♯ (3)	E/♯♯♯ (2)	AVG
I	14.65%	14.06%	13.96%	18.77%	16.99%	12.71%	15.85%	16.67%	15.46%
ii	5.14%	4.64%	4.07%	5.75%	4.22%	3.37%	3.96%	3.47%	4.33%
II	2.80%	1.49%	2.11%	6.51%	2.34%	1.89%	2.38%	3.07%	2.82%
iii	5.24%	3.97%	3.11%	4.60%	3.81%	2.81%	4.01%	2.13%	3.71%
III	1.93%	2.30%	2.26%	1.92%	1.97%	1.99%	0.85%	1.87%	1.89%
iii°	0.00%	0.05%	0.15%	0.00%	0.09%	0.10%	0.05%	0.67%	0.14%
IV	7.43%	7.98%	6.58%	6.13%	9.23%	6.43%	8.08%	5.73%	7.20%
v	0.25%	0.05%	0.05%	0.00%	0.23%	0.05%	0.16%	0.53%	0.17%
V	12.46%	8.47%	11.55%	20.31%	11.71%	10.82%	16.11%	14.00%	13.18%
vi	4.93%	5.41%	5.37%	6.13%	5.65%	5.72%	5.55%	6.27%	5.63%
vii	0.61%	0.27%	0.90%	1.53%	0.60%	0.71%	1.16%	0.27%	0.76%
(♭)VII	0.05%	0.41%	0.30%	0.00%	0.28%	0.31%	0.11%	0.13%	0.20%
(♯)vii°	2.09%	2.70%	2.41%	0.38%	2.25%	1.28%	2.06%	2.13%	1.91%

Ex. 5.6. Thirteen commonest triads (by Roman numeral) in Corelli's Ut tonalities—largest number in each column is shaded darkest—bracketed numbers show how many sonatas per tonality—Roman numerals shaded red are diatonic in lydian, those shaded blue are diatonic in mixolydian

The Roman numerals shaded red in Example 5.6 are those that are diatonic within the lydian octave species: the II and vii triads. The E♭/♭♭ tonality, whose signature-defined octave species is lydian, uses the largest numbers of II and vii triads of any Ut tonality, as we expect it to. Corelli's other lydian tonality, B♭/♭, uses a relatively large number of II triads (2.80%), but not very many vii triads (0.61%) compared with other Ut tonalities.

The Roman numerals shaded blue in Example 5.6 are those that are diatonic within the mixolydian octave species: the iii°, v, and (♭)VII triads. Both the iii° and v triads appear most often in the mixolydian tonality E/♯♯♯ (0.67% and 0.53%, respectively), while the (♭)VII triad appears most often in the C/♮ (0.41%), G/♮ (0.31%), and D/♯♯ (0.30%) tonalities, only one of which (G/♮) is mixolydian according to its signature. While we observed little or no correlation between signature-defined octave species and the prevalence of dorian-specific triads in the Re category, certain lydian and mixolydian tonalities in the Ut category appear to use more triads that are specific to their octave species, especially the E♭/♭♭ and E/♯♯♯ tonalities. I believe that, more than an actual division within the Ut category (i.e., that lydian and mixolydian are still relatively easy to distinguish from major on the basis of 1-grams), this suggests that the transpositional relationship between Corelli's Ut tonalities breaks down at the extremes of the transpositional spectrum (i.e., two flats and three sharps).

With all of the multidimensional data in Examples 5.5 and 5.6 tabulated, we now use the Euclidean distance to measure the similarity between tonalities. While it can be productive to compare visual representation of tonalities on the basis of two or three different Roman numerals at a time, the Euclidean distance allows us to compare Re-category tonalities in fourteen dimensions (thirteen for Ut-category tonalities) at once. Euclidean distances between all pairs of tonalities are shown in Examples 5.7 and 5.8 below.

Lightly shaded cells mark pairs of tonalities whose Roman-numeral distributions are very similar (Euclidean distance < 1.00). Darkly shaded cells mark pairs of tonalities whose Roman-numeral distributions are very different (Euclidean distance > 1.70). Distances are calculated only between pairs of tonalities in the same category (i.e., Re to Re or Ut to Ut) so that histograms always have the same number of dimensions. For an explanation of the mathematics involved in these calculations, refer to Section 3.2.1.

Roman-Numeral Euclidean Distances Between Corelli's Re Tonalities

	c/♭♭	d/♮	f/♭♭	g/♭	a/♮	b/♯♯	d/♭	e/♯	f♯/♯♯♯
c/♭♭	0	0.89	1.34	0.84	0.93	1.03	1.02	0.78	1.53
d/♮		0	1.74	1.15	1.35	0.90	1.52	1.32	1.37
f/♭♭			0	1.06	1.49	1.86	1.39	1.02	1.60
g/♭				0	0.83	1.06	1.19	0.81	1.20
a/♮					0	1.05	1.46	1.14	1.53
b/♯♯						0	1.39	1.39	1.63
d/♭							0	0.84	1.91
e/♯								0	1.61
f♯/♯♯♯									0

Ex. 5.7. Roman-numeral Euclidean distances between all pairs of Re tonalities—lightly shaded cells mark lowest numbers (smallest distances), darkly shaded cells mark largest distances

The smallest Euclidean distance between any two of Corelli's Re tonalities is 0.78, between c/♭♭ and e/♯. Interestingly, these two tonalities are comparably close to many other tonalities in the Re category: c/♭♭ is very close to g/♭ and d/♮ (both Euclidean distances < 0.90) and also quite close to a/♮ (Euclidean distance = 0.93), while e/♯ is very close to g/♭ and d/♭ (both Euclidean distances < 0.90). Small distances also appear between g/♭ and a/♮ (Euclidean distance = 0.83) and between d/♮ and b/♯♯ (Euclidean distance = 0.90). This suggests that many different pairs of tonalities in Corelli's Re category are near-transpositions of one another. As was the case in Legrenzi, the only Re tonalities of Corelli's that are not particularly close to any others are those that have the maximum number of signature flats or sharps: f♯/♯♯♯, which has three and f/♭♭, which has only two, but uses accidental A-flats throughout. Again, this shows how the transpositional relationship between breaks down at the extremes of the transpositional spectrum (i.e., three sharps and two flats).

Also reminiscent of Legrenzi is the fact that many of the closest pairs of tonalities in Example 5.7 have different signature-defined octave species: c/♭♭ and e/♯ (Euclidean distance = 0.78), g/♭ and e/♯ (Euclidean distance = 0.81), and g/♭ and a/♮ (Euclidean distance = 0.83). It is not until the fourth and fifth closest pairs that we see tonalities with the same signature-defined octave species: c/♭♭ and g/♭ (Euclidean distance = 0.84) and d/♭and e/♯ (Euclidean distance = 0.84). These data corroborate the earlier claim that signature-defined octave species is no longer a reliable predictor of Roman-numeral content in this repertoire.

As in the Re category, the two closest pairs in the Ut category are different-species pairs: D/♯♯ and G/♮ (Euclidean distance = 0.57) and B♭/♭ and D/♯♯ (Euclidean distance = 0.82). Corelli's D/♯♯ tonality is also quite close to his C/♮ tonality (Euclidean distance = 1.02) and this is the only same-species pair where the Euclidean distance is less than 1.10. Other pairs with Euclidean distance < 1.10

include B♭/♭ and F/♭ (Euclidean distance = 1.00) and C/♮ and G/♮ (Euclidean distance = 1.08), both of which are different-species pairs. This shows that Ut tonalities in Corelli do not need to have the same signature-defined octave species in order to be likely transpositions of one another.

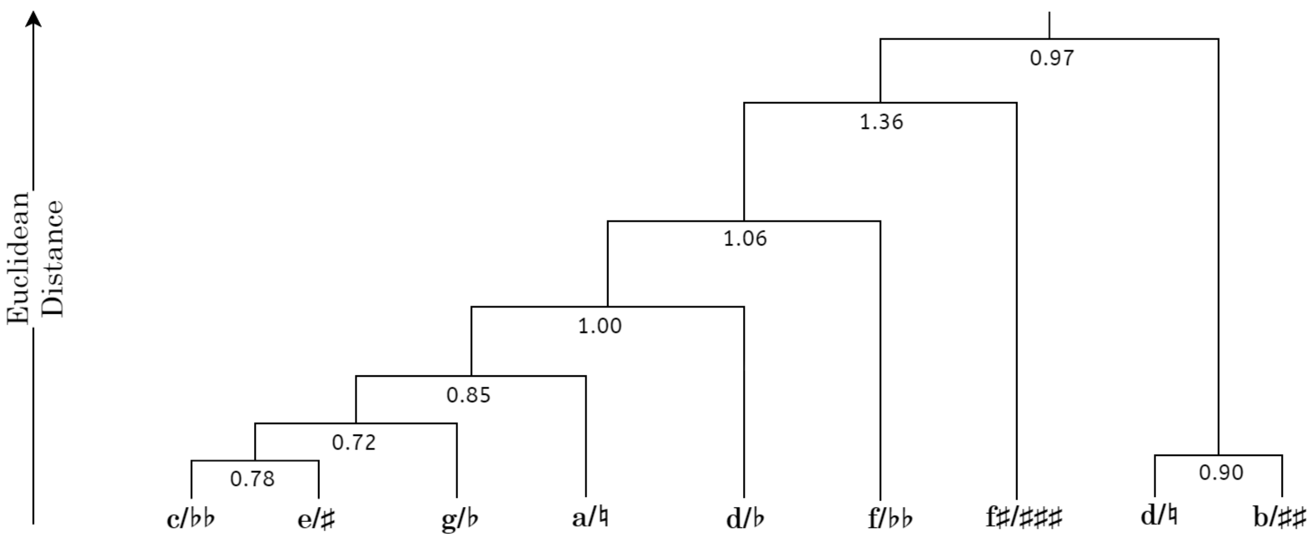
The tonality that stands out as very distant from all the others is the E♭/♭♭ tonality, which is never closer than a Euclidean distance of 2.06 (with A/♯♯) to any other Ut tonality. As we observed in the heat maps (Examples 5.5 and 5.6), Corelli's Ut tonalities appear to be less internally consistent than do his Re tonalities. The Euclidean distances in Example 5.8 are, on average, larger than those in Example 5.7, which suggests that the Re category is more homogenous than the Ut category.

Roman-Numeral Euclidean Distances Between Corelli's Ut Tonalities

	B♭/♭	C/♮	D/♯♯	E♭/♭♭	F/♮	G/♮	A/♯♯	E/♯♯♯
B♭/♭	0	1.28	0.82	2.78	1.00	1.20	1.24	1.37
C/♮		0	1.02	3.93	1.30	1.08	2.26	1.99
D/♯♯			0	3.16	1.15	0.57	1.51	1.19
E♭/♭♭				0	2.93	3.50	2.06	2.37
F/♮					0	1.52	1.35	1.32
G/♮						0	1.86	1.52
A/♯♯							0	1.17
E/♯♯♯								0

Ex. 5.8. Roman-numeral Euclidean distances between all pairs of Ut tonalities—lightly shaded cells mark lowest numbers (Euclidean distances < 1.10), darkly shaded cells mark largest distances (Euclidean distance > 2.00)

An agglomerative hierarchical clustering (AHC) analysis of the Euclidean distances in Examples 5.7 and 5.8 produces the two dendrograms (one for each category) shown in Examples 5.9 and 5.11 below (for an explanation of this clustering algorithm, refer to Section 3.2.1).

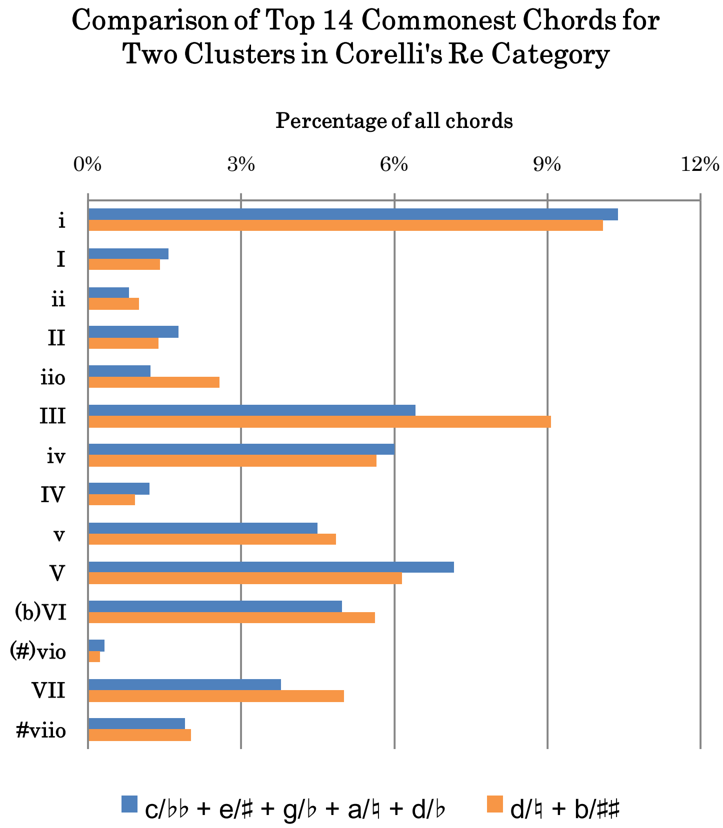


Ex. 5.9. Dendrogram based on Roman-numeral distances within the Re category (Ex. 5.7)—decimal numbers are Euclidean distances between linked tonalities or clusters

This dendrogram contains a surprising result: unlike Legrenzi's Re-category Roman-numeral dendrogram, Corelli's has more than one cluster. Before the d/♭ tonality joins the c/♭♭-e/♯-g/♭-a/♮ cluster (Euclidean distance = 1.00), the two tonalities on the far right of the dendrogram (d/♮ and b/♯♯) join together (Euclidean distance = 0.90). What sets these two tonalities apart? What makes them more similar to one another than to the growing cluster on the left of the dendrogram? Example 5.10 (below) compares the c/♭♭-e/♯-g/♭-a/♮-d/♭ cluster with the d/♮-b/♯♯ cluster in terms of Roman numeral content. We leave f/♭♭ and f♯/♯♯♯ out of this comparison because we want to compare groups of tonalities that are more internally consistent. Each of the c/♭♭, e/♯, g/♭, a/♮, d/♭, d/♮, and b/♯♯ tonalities is a Euclidean distance of 1.00 or less from the tonality or cluster to which it is joined, whereas f/♭♭ and f♯/♯♯♯ exceed this threshold.

What Example 5.10 illustrates is that the tonalities belonging to the larger cluster (c/♭♭-e/♯-g/♭-a/♮-d/♭) use more i, I, II, iv, IV, and V chords, while the tonalities belonging to the smaller cluster (d/♮-b/♯♯) use more ii^{o}, III, VI, and VII chords. From a Roman-numeral 1-grams perspective, these differences suggest that some of Corelli's Re tonalities (d/♮ and b/♯♯) are much more likely to tonicize the relative major than others.

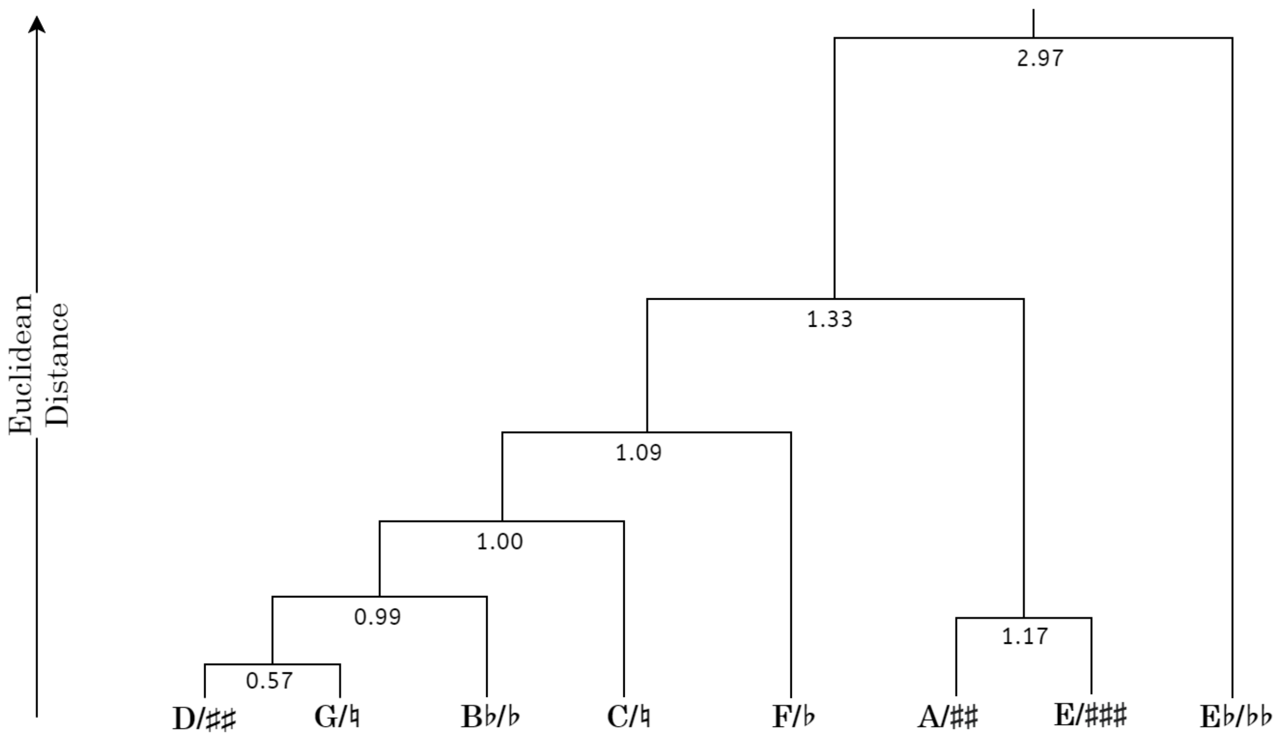
This result is surprising because we initially assumed that Corelli's music, being the most tonal, would use only two modes: one major and one minor. If all of the tonalities in Corelli's Re category really were transpositions, or even near transpositions of one another, the dendrogram in Example 5.9 would be one single cluster: a series of branches making a uniform "staircase" that ascends from left to right, where each new tonality adds on to the right side of the growing cluster. This was what we observed in Legrenzi's Re tonality, where no two tonalities (after the initial pair) were more similar to one another than they were to the cluster. Corelli's



Ex. 5.10. Analysis of Roman numeral content of two clusters of tonalities in Corelli's Re category—the c/♭♭, e/♯, g/♭, a/♮, and d/♭tonalities cluster together because they use more iv, IV, II, and V chords—the d/♮ and b/♯♯tonalities cluster together because they use more ii°, III, VI, and VII chords

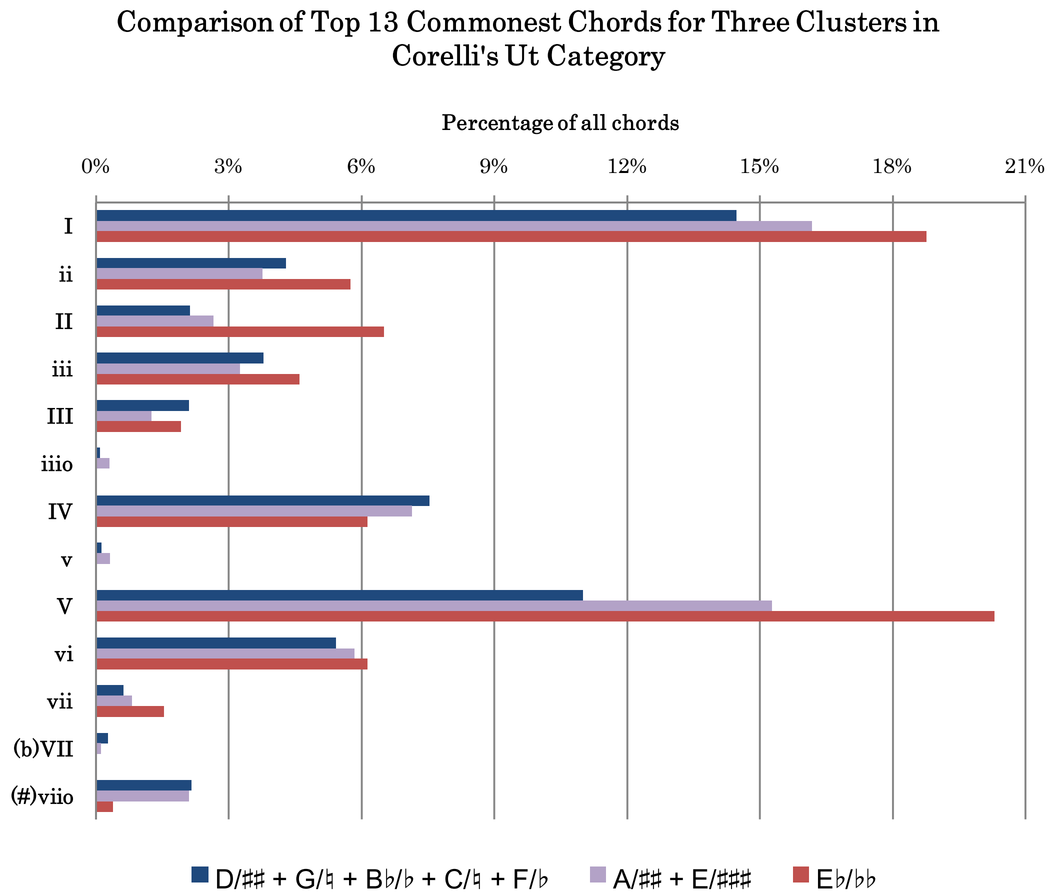
d/♮ and b/♯♯ tonalities, however, are more similar to one another than they are to the c/♭♭-e/♯-g/♭-a/♮-d/♭ cluster, which results in two unequal clusters instead of one. This means that, from a Roman-numeral 1-grams perspective, Corelli's Re category actually comprises two different versions of the minor mode: one that appears to be more likely to tonicize III (the d/♮-b/♯♯ cluster), and another that is perhaps more likely to tonicize the major dominant (the c/♭♭-e/♯-g/♭-a/♮-d/♭ cluster). Importantly, these two versions are not necessarily exclusive to different signature-defined octave species: both clusters contain both dorian and minor tonalities.

Corelli's Ut category appears to follow a similar logic. Example 5.11 (below) illustrates the clustering result for the eight Ut tonalities. Here we see that the A/♯♯ and E/♯♯♯ tonalities, like d/♮ and b/♯♯in the Re category, are more similar to one another than they are to the larger cluster on the left (D/♯♯-G/♮-B♭/♭-C/♮-F/♭). In addition to these two unequal clusters, there is also the singleton E♭/♭♭ tonality, which is the most unique of any tonality in the Ut category.



Ex. 5.11. Dendrogram based on Roman-numeral distances within the Ut category (Ex. 5.8)—decimal numbers are Euclidean distances between linked tonalities or clusters

To understand what sets these clusters apart, we compare their Roman numeral content in Example 5.12 (below). The tonalities belonging to the larger cluster (D/♯♯-G/♮-B♭/♭-C/♮-F/♭) use more ii, iii, III, and IV chords, while the tonalities belonging to the smaller cluster (A/♯♯-E/♯♯♯) use more I, II, and V chords. The E♭/♭♭ tonality amplifies these differences, using even more I, II, and V chords than the A/♯♯-E/♯♯♯ cluster.



Ex. 5.12. Analysis of Roman numeral content of three clusters in Corelli's Ut category—the D/♯♯, G/♮, B♭/♭, C/♮, and F/♭ tonalities cluster together because they use more ii, iii, III, and IV chords—the A/♯♯ and E/♯♯♯tonalities cluster together because they use more I, II, V, and vi chords—the E♭/♭♭ tonality uses many more II and V chords than the other Ut tonalities

While it is not surprising to see "peaks" on the II and V triads in E♭/♭♭, a tonality with a lydian signature (Frescobaldi's lydian tonalities emphasized these same two triads), it is strange that the A/♯♯ and E/♯♯♯ tonalities, both mixolydian according to their signature, emphasize II and V as well. Rather than having different options for "secondary keys," like III vs. V in the Re category, the clustering in the Ut category depends primarily on how much time is being spent in the dominant.

If all of Corelli's tonalities were transpositions of one another, we would have expected their Roman numeral distributions to be nearly identical within a single category (Ut or Re), which was not the case. We move now to a root/quality representation of chords so that we can compare tonalities from different categories, including the Mi category.

5.2.2. Comparing Root/Quality Histograms

With a total of eighteen different tonalities appearing in the Corelli corpus, a large sample of root/quality data was needed to make good comparisons between tonalities with signatures ranging from two flats to three sharps. Corelli's twenty commonest triads are tallied in descending order from most to least common in Example 5.13 (below). When comparing a larger number of different tonalities (eighteen in Corelli versus thirteen in Legrenzi and ten in Frescobaldi), we use more data twenty commonest triads in Corelli versus fourteen in Legrenzi and twelve in Frescobaldi) in order to avoid "false positives," i.e., very small Euclidean distances between tonalities that are very different in terms of their root/quality content.

In the f/♭♭ and E/♯♯♯ tonalities, for example, the nine triads Fmaj, Dmaj, Gmaj, B♭maj, Dmin, Amin, Emin, Gmin, and Bmin are all relatively uncommon. If

	Flat-Side Tonalities							Natural Tonalities					Sharp-Side Tonalities						
	f/♭♭ (1)	E♭/♭♭ (1)	c/♭♭ (2)	B♭/♭ (4)	g/♭ (4)	F/♭ (4)	d/♭ (1)	d/♮ (3)	C/♮ (4)	a/♮ (2)	e/♮ (1)	G/♮ (4)	e/♯ (3)	D/♯♯ (4)	b/♯♯ (4)	A/♯♯ (3)	f♯/♯♯♯ (1)	E/♯♯♯ (2)	AVG
Fmaj	1.58%	6.51%	0.96%	12.46%	4.55%	16.99%	5.91%	9.55%	7.98%	5.06%	7.24%	0.31%	0.27%	0.00%	0.00%	0.00%	0.00%	0.00%	4.41%
Cmaj	10.36%	0.38%	1.49%	2.80%	1.15%	11.71%	2.95%	5.14%	14.06%	7.10%	5.23%	6.43%	4.09%	0.30%	0.51%	0.00%	0.00%	0.00%	4.09%
Dmaj	0.00%	0.00%	2.45%	1.93%	7.60%	0.05%	1.14%	1.45%	1.49%	1.40%	0.80%	10.82%	3.01%	13.96%	8.73%	8.08%	5.92%	0.13%	3.83%
Amaj	0.00%	0.00%	0.00%	0.00%	1.15%	1.97%	7.27%	6.85%	0.09%	1.51%	0.60%	1.89%	1.00%	11.55%	4.92%	15.85%	7.58%	5.73%	3.78%
Gmaj	1.13%	1.92%	7.17%	1.12%	1.73%	2.34%	2.05%	0.99%	8.47%	3.77%	4.43%	12.71%	5.43%	6.58%	6.39%	0.11%	0.47%	0.27%	3.73%
B♭maj	1.13%	20.31%	3.85%	14.65%	6.22%	9.23%	5.00%	4.61%	0.41%	0.54%	0.20%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.68%
Emaj	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.95%	1.32%	2.30%	6.46%	7.85%	0.31%	1.61%	2.11%	0.86%	16.11%	4.50%	16.67%	3.50%
Dmin	0.45%	1.53%	0.87%	5.24%	4.09%	5.65%	11.14%	9.75%	4.64%	8.18%	4.43%	0.05%	0.27%	0.00%	0.00%	0.00%	0.00%	0.00%	3.13%
Amin	0.00%	0.00%	0.17%	0.61%	0.81%	3.81%	5.91%	3.62%	5.41%	10.66%	8.85%	3.37%	5.09%	0.05%	0.20%	0.00%	0.00%	0.40%	2.72%
E♭maj	2.93%	18.77%	7.87%	7.43%	5.88%	0.28%	1.36%	0.26%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.49%
Emin	0.00%	0.00%	0.00%	0.00%	0.00%	0.60%	1.36%	1.12%	3.97%	4.95%	3.62%	5.72%	10.05%	4.07%	5.99%	0.16%	0.00%	0.00%	2.31%
Gmin	0.23%	4.60%	4.46%	4.93%	10.43%	4.22%	3.86%	5.20%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.11%
Bmin	0.00%	0.00%	0.00%	0.00%	0.00%	0.05%	0.23%	0.00%	0.27%	0.43%	0.60%	2.81%	4.29%	5.37%	10.35%	3.96%	6.64%	0.53%	1.97%
Cmin	4.50%	6.13%	10.15%	5.14%	6.51%	0.23%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.81%
F♯min	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.71%	0.80%	3.11%	5.78%	5.55%	12.32%	3.47%	1.78%
Bmaj	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.01%	1.99%	7.03%	0.70%	1.37%	2.38%	0.47%	14.00%	1.77%
Fmin	9.23%	5.75%	5.25%	0.25%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.14%
Abmaj	4.95%	6.13%	4.37%	0.00%	0.35%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.88%
F♯maj	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.74%	2.26%	5.63%	0.42%	1.18%	3.07%	0.79%
C♯min	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.20%	0.00%	0.00%	0.00%	0.00%	0.00%	0.90%	0.91%	4.01%	1.66%	6.27%	0.78%

Ex. 5.13. Twenty commonest triads (by root and quality) in Corelli's eighteen tonalities—non-zero values are shaded orange with the largest number in each column shaded darkest—values of 0.00 are shaded blue and tonic triads are boxed—bracketed numbers in top row show how many sonatas per tonality

we measure the Euclidean distance between f/♭♭ and E/♯♯♯ using only the top fourteen commonest triads, it will appear that these two tonalities are very closely related because they are both lacking in nine out of those fourteen triads.

To remedy this, six more triads are added below: F♯min, Bmaj, Fmin, A♭maj, F♯maj, and C♯min. These six triads are very common in either the flat-side or the sharp-side tonalities, but never both, which helps differentiate very flat tonalities from very sharp ones. This addition also means that every tonality in Example 5.13 now has a maximum of ten and a minimum of seven triads with values of 0.00% (shaded light blue).

Looking at the order of triads in Example 5.13 (commonest at the top), we see that, on average, major triads are more common than minor triads in Corelli. This is because major triads play an important role in this repertoire: they are used as dominants to make cadences. Every tonality has an equal number of major and minor triads (three of each) that we call diatonic. The most common non-diatonic triads in Corelli are dominant harmonies like V in the Re tonalities, III (V/vi) in the Ut tonalities, and II (V/V) in both the Ut and Re categories, all of which are major triads. The commonest minor triads, Dmin and Amin, do not appear until the eighth and ninth rows of Example 5.13, respectively. Comparing these triads with Frescobaldi, where Dmin and Amin were the third and fourth commonest triads, and Legrenzi, where they were the fifth and eighth commonest, we can trace a gradual decline in the relative popularity of these two minor triads—and yet they are still the most common of all the minor triads in Corelli.

The seven major triads that now outrank them have roots on each of the seven letter names (with B♭ instead of B). Three of these use only “natural” notes (Fmaj, Cmaj, and Gmaj) and the other four require one sharp or flat (Dmaj, Amaj, B♭maj, and Emaj). These are the only seven major triads that can be formed using one or zero sharps or flats. Interestingly, the three natural-note major triads are not the three commonest ones: Dmaj and Amaj are both more common than Gmaj, which was not the case in either Frescobaldi or Legrenzi. This is most likely due to the fact that Amaj is the tonic of one tonality and the dominant of three tonalities (D/♯♯, d/♮, and d/♭), while Gmaj is the tonic of one tonality and the dominant of only two tonalities (C/♮ and c/♭♭). If we collapse the d/♮ and d/♭ tonalities together (as we will see, these two tonalities turn out to be the most similar from a root/quality perspective), the Gmaj triad would surpass the Amaj triad in the ranking, the latter being only 0.05% higher than the former in the current ranking. Although it is still less common than Dmaj, Gmaj is the only triad that has a non-zero percentage in every one of Corelli’s tonalities. This positions Gmaj as a “safe” triad to use in any tonality, perhaps because of tuning.

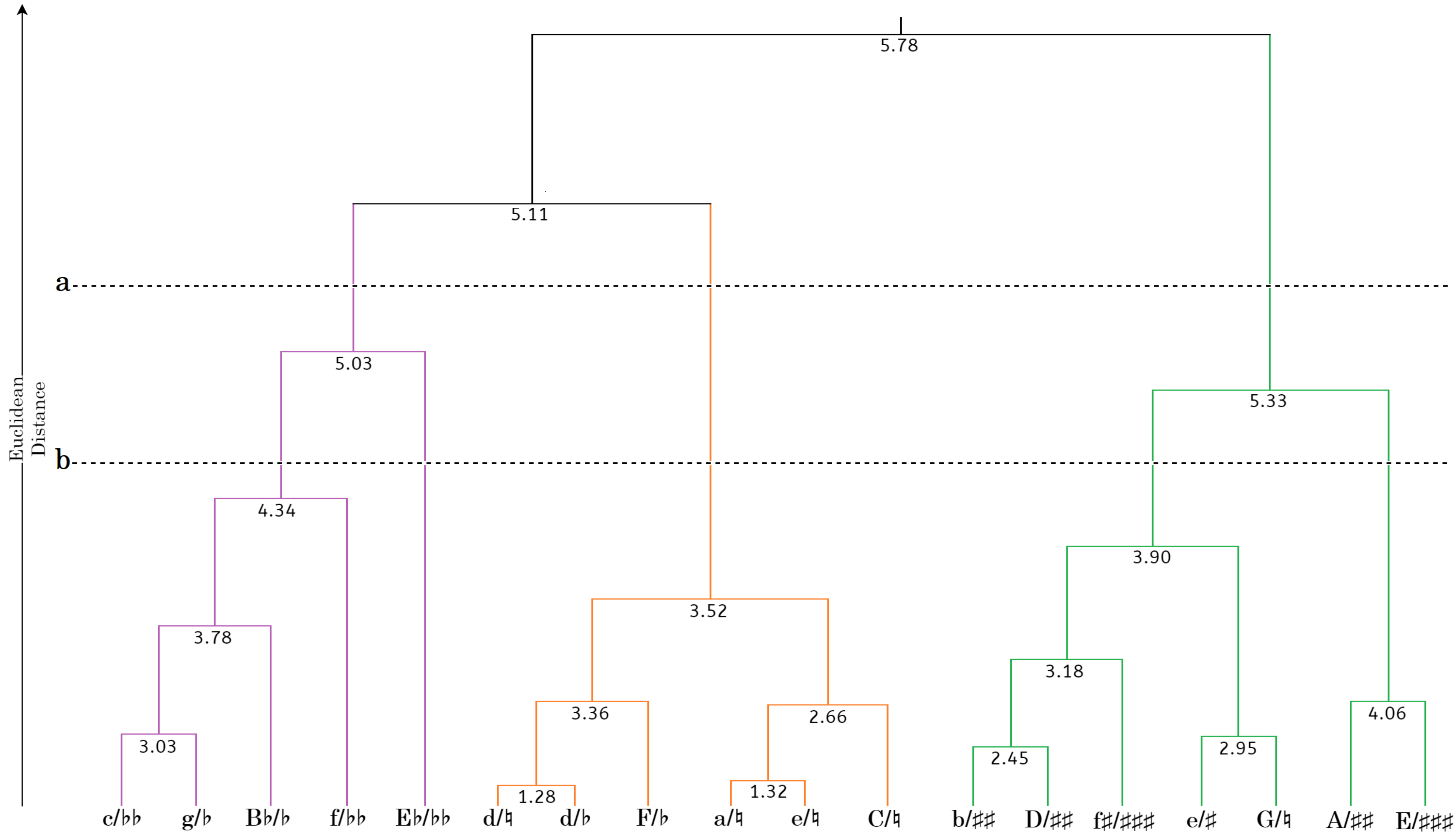
This brings us to another interesting trend in the root/quality data for Corelli’s eighteen tonalities: that some triads are relatively evenly distributed among all tonalities, while for other triads it’s “feast or famine.” Compare the evenly spread Gmaj with the triad ranked just below it: Bbmaj. While Gmaj can be found at least once in every tonality, Bbmaj does not appear at all in seven tonalities (G/♮, e/♯, D/♯♯, b/♯♯, A/♯♯, f♯/♯♯♯ and E/♯♯♯), and only rarely in three more (C/♮, a/♮, and e/♮). As we continue down the ranked list of triads, we see that the "hot areas" (shaded dark orange) move further and further away from the middle (i.e., natural) section of the table and that zero values (shaded light blue) form a mountain in the middle of the table. This trend brings to mind the idea that, in certain tuning systems (e.g., quarter-comma meantone), certain triads (e.g., Gmaj, Dmaj, Cmaj, Dmin) sound good no matter which tonality you are in, while others (e.g., Ebmaj, Gmin, Bmin, Cmin) do not. A study of the instruments and tuning systems used by Corelli and his contemporaries would certainly shed light on the root/quality data presented in Example 5.13, but for reasons of space, such a study is beyond the scope of this project.

The values in Example 5.13 are used to obtain the matrix of Euclidean distances in Example 5.14 (below). These distances range from 1.28 (between d/♭ and d/♮) to 8.91 (between A/♯♯ and E♭/♭♭). As we did for Legrenzi, we used different colours to highlight pairs of tonalities having certain kinds of relationships: dark grey for pairs that are very distant (Euclidean distance > 6.50), blue for fifth-related pairs, red for "relative" pairs, orange for other pairs that are very similar (Euclidean distance < 3.50) and yellow for the a/♮-e/♮ pair.

Root/Quality Euclidean Distances Between Corelli's Tonalities

	f/♭♭	E♭/♭♭	c/♭♭	B♭/♭	g/♭	F/♭	d/♭	d/♮	C/♮	a/♮	e/♮	G/♮	e/♯	D/♯♯	b/♯♯	A/♯♯	f♯/♯♯♯	E/♯♯♯
f/♭♭	0	6.24	3.31	5.16	4.56	5.02	4.74	4.56	3.99	4.54	4.38	4.84	4.53	5.78	5.38	6.69	5.31	6.40
E♭/♭♭		0	4.88	3.80	5.18	6.50	6.50	6.57	7.65	7.59	7.36	7.95	7.65	8.23	7.97	8.91	7.92	8.69
c/♭♭			0	4.34	3.03	5.76	4.74	4.88	5.08	5.17	4.88	4.64	4.78	5.39	5.12	6.75	5.41	6.60
B♭/♭				0	3.21	3.35	4.05	3.69	5.26	5.31	5.04	6.24	5.96	6.65	6.40	7.52	6.35	7.36
g/♭					0	4.77	3.73	3.60	5.12	4.88	4.71	4.85	4.97	5.01	5.06	6.37	5.15	6.62
F/♭						0	3.82	2.89	3.50	4.33	4.16	5.93	5.72	6.83	6.58	7.63	6.58	7.50
d/♭							0	1.28	3.92	2.71	3.03	4.94	4.44	4.98	5.09	5.53	4.85	5.89
d/♮								0	3.65	3.14	3.26	5.07	4.67	5.17	5.25	5.86	5.03	6.19
C/♮									0	2.64	2.67	3.67	3.86	5.64	5.25	6.88	5.79	6.66
a/♮										0	1.32	4.24	3.53	5.38	5.08	5.92	5.23	5.77
e/♮											0	4.10	3.24	5.22	4.85	5.57	4.94	5.05
G/♮												0	2.95	3.28	3.28	6.06	4.82	6.48
e/♯													0	4.22	3.26	5.86	4.54	5.17
D/♯♯														0	2.45	4.06	3.38	6.02
b/♯♯															0	5.02	2.98	5.89
A/♯♯																0	3.68	4.06
f♯/♯♯♯																	0	5.04
E/♯♯♯																		0

Ex. 5.14. Root/quality Euclidean distance between all of Corelli's tonalities—darkly shaded cells mark largest distances (>6.50), cells shaded red contain distances between relative major/minor pairs, cells shaded blue contain distances between fifth-related tonalities belonging to the same category (Ut, Re, or Mi), and cells shaded orange contain other small distances (<3.50)—the yellow cell shows the distance between the a/♮-e/♮ pair



Ex. 5.15. *Dendrogram based on root/quality distances between all tonalities—decimal numbers are Euclidean distances between linked tonalities or clusters—cutting the tree at line **a** produces three different clusters—cutting at line **b** produces five clusters*

One question we can answer with these data is "are relative pairs the most similar from a root/quality perspective?" In Frescobaldi, this was not the case: pairs of fifth-related tonalities with the same signatures, which we called authentic-plagal pairs (i.e., C/♮ and G/♮, a/♮ and e/♮, and F/♭ and C/♭), were more similar from a root/quality perspective than were the relative pairs C/♮ and a/♮ or F/♭ and d/♭. In Legrenzi, we saw a mix of authentic-plagal and relative pairings: C/♮ was closer to G/♮ than it was to a/♮ and a/♮ was closer to e/♮ than it was to C/♮, but D/♯♯ was closer to b/♯♯ than it was to A/♯♯ and the G-minor cluster was closer to B♭/♭ than it was to the C-minor cluster. If this trend continues, we would expect to see fewer or no authentic-plagal pairs in Corelli, and mostly relative pairs instead.

What Example 5.14 reveals is that relative pairs in Corelli (shaded red) are indeed the closest, on average, but not all relative pairs are more similar than competing fifth-related pairs. For example, Corelli's D/♯♯ tonality is closer to b/♯♯ (its relative minor) than it is to either of its fifth-related tonalities (G/♮ or A/♯♯); the same is true from the perspective of b/♯♯, which is closer to D/♯♯ (its relative major) than it is to either e/♯ or f♯/♯♯♯. This pattern holds for the G/♮-e/♯ pair, and the F/♭-d/♮ pair,[6] but the other three potential relative pairings (C/♮-a/♮, B/♭-g/♭, and E♭/♭♭-c/♭♭) are all displaced by fifth-related tonalities: a/♮ is closer to e/♮[7] than it is to C/♮, g/♭ is closer to c/♭♭ than it is to B♭/♭, and E♭/♭♭ is closer to B♭/♭ than it is to c/♭♭. So,

[6] The fact that neither of these pairs is consistent in terms of signature supports the idea that some of Corelli's key signatures really can be thought of as "incomplete."
[7] The a/♮-e/♮ pair is the only authentic-plagal pair left in Corelli.

while Corelli's relative tonalities are more closely related than are those of his predecessors, this relationship is still not fully transposable: it breaks down on the flat-side with tonalities like E♭/♭♭.

The dendrogram in Example 5.15 (above), which is derived from the Euclidean distance values in Example 5.14, shows that Corelli's tonalities, much like Legrenzi's, cluster into three large groups when using an agglomerative hierarchical clustering (AHC) method. If we cut the tree at line ***a***, we get a flat cluster (purple), a natural cluster (orange), and a sharp cluster (green). Some flat-side tonalities (d/♭ and F/♭) cluster with the natural tonalities, and the G/♮ tonality clusters with the sharp-side tonalities, which suggests that this tonality (at least) could be understood to have an "incomplete" signature. As mentioned above, Allsop claims that Corelli's G/♮ tonality is more modern than that of his contemporaries (Allsop 1999, 104) in that it completely eschews the minor dominant and the subdominant as cadence points, so he would likely agree with the argument that Corelli's G/♮ tonality is not unlike a G/♯ tonality from a root/quality perspective.

If we cut the tree instead at line ***b***, we see that certain clusters are much more internally similar than others. The natural cluster (orange) is the most internally similar, remaining undivided, while both the flat cluster (purple) and the sharp cluster (green) become divided: the flat cluster loses the E♭/♭♭ tonality, while the sharp cluster loses the A/♯♯-E/♯♯♯ pair. This tells us that the flat and sharp clusters are less internally similar than the natural cluster and that the E♭/♭♭

tonality in particular is the least similar to any of Corelli's seventeen other tonalities from a root/quality perspective. The A/♯♯-E/♯♯♯ pair is also quite dissimilar from the rest of the sharp cluster, but the A/♯♯ and E/♯♯♯ tonalities are at least somewhat similar to one another.

Here it bears mentioning that the notion of exact transposition between keys is itself a chimera. As shown by Ian Quinn and Christopher White (Quinn & White 2017), even keys in music that we consider to be thoroughly tonal (e.g., keyboard works by Haydn, Mozart, and Beethoven) are not transpositionally equivalent when examined empirically. Like the Roman-numeral flowcharts proposed by Kostka, Payne, and Almén, transpositional equivalence turns out to be a pedagogically expedient oversimplification of a much messier truth. Quinn and White found that keys with more signature sharps or flats tended to have more chromaticism than keys with fewer signature sharps or flats. Our results in this and previous chapters corroborate Quinn and White's findings in that tonalities with more sharps or flats in their signatures (e.g., Corelli's E/♯♯♯ and E♭/♭♭) are generally the farthest outliers in terms of transpositional similarity.

In Corelli's dendrogram (Example 5.15 above), the first pairs to form are the d/♮-d/♭ pair and the a/♮-e/♮ pair. As we saw in Legrenzi, when there are two tonalities in the same category (Ut or Re) with the same final but different signatures, they are very close together from a root/quality perspective. In fact, we might interpret Corelli's two D-minor tonalities as just slightly different versions of the same tonality. Although it is impossible to know for sure, Corelli may have conceived of both of these tonalities as “D-Re” or ‘D-minor-third” rather than D-dorian and D-minor.

In his first book of sonatas (1681), Corelli uses d/♭, but every subsequent appearance of the D-minor tonality lacks a signature flat: d/♮ appears in Corelli’s Op. 2 (1685), Op. 3 (1689), and Op. 4 (1694)—as well as in the Op. 5 violin sonatas (1700), which are not discussed in this project—while d/♭ does not. Whether or not Corelli’s decision to avoid d/♭ in favour of d/♮ after 1681 is meaningful, the numbers show that these two tonalities are the closest of any of his eighteen tonalities from a root/quality perspective (Euclidean distance = 1.28) and that they are on par with Legrenzi’s same-final pairs: g/♭-g/♭♭ (Euclidean distance = 1.18) and c/♭-c/♭♭ (Euclidean distance = 1.35).

The practice of these two later composers stands in contrast to that of Frescobaldi, whose d/♮ tonality is very different from his d/♭ tonality, so much so that d/♮ and d/♭ do not form a pair in the root/quality dendrogram (Example 3.10). Frescobaldi’s d/♭ tonality clusters instead with his other *cantus-mollis* tonalities, F/♭ and C/♭, before being joined by g/♭ and then finally d/♮. This shows that signature plays a much larger role in determining root/quality similarity for Frescobaldi than it does for Legrenzi or Corelli. In Corelli, only two of the six pairs that form first in Example 5.15 are same-signature pairs (a/♮-e/♮ and b/♯♯-D/♯♯).

The a/♮-e/♮ pair remains very similar from a root/quality perspective. Corelli's a/♮-e/♮ pair (Euclidean distance = 1.32) sits in between Frescobaldi's (Euclidean distance = 2.20) and Legrenzi's (Euclidean distance = 0.69). While Legrenzi's a/♮-e/♮ pair turns out to be more similar than Corelli's from a root/quality perspective, Corelli's is still the second most similar pair in all of his eighteen tonalities. The changing relationship between the a/♮ and e/♮ tonalities will be discussed further in the Conclusions section of this project.

5.3. Comparing Chord Progressions

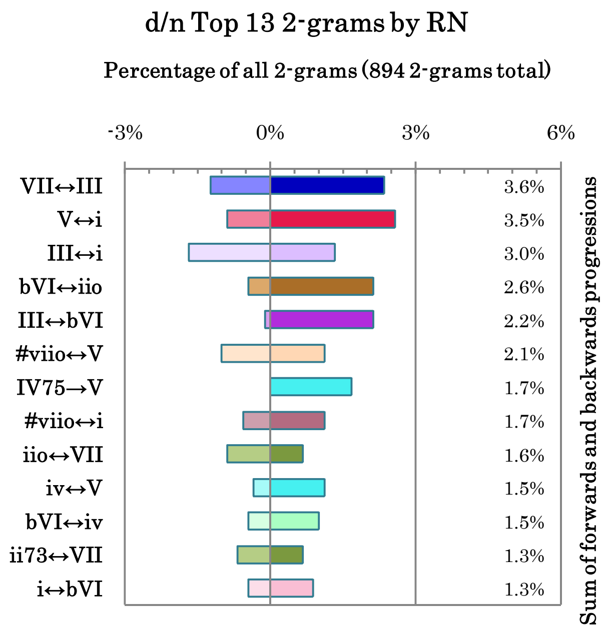
Contrary to our expectations, the 1-gram data presented above did not suggest that there is a universal Re or universal Ut tonality of which all other tonalities can be understood to be transpositions. Instead, we saw that both Re and Ut tonalities clustered into at least two different groups from a Roman-numeral perspective (d/♮ and b/##formed a separate group from the other Re tonalities, while A/## and E/### formed a separate group in the Ut category). We now examine 2-grams from a Roman-numeral perspective to determine whether or not these same groups have any chord progressions in common.

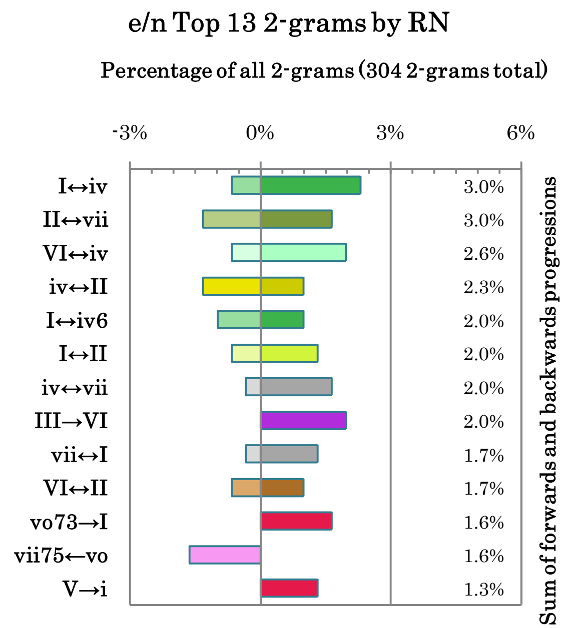
5.3.1. 2-Grams as Roman-Numeral Tornado Charts

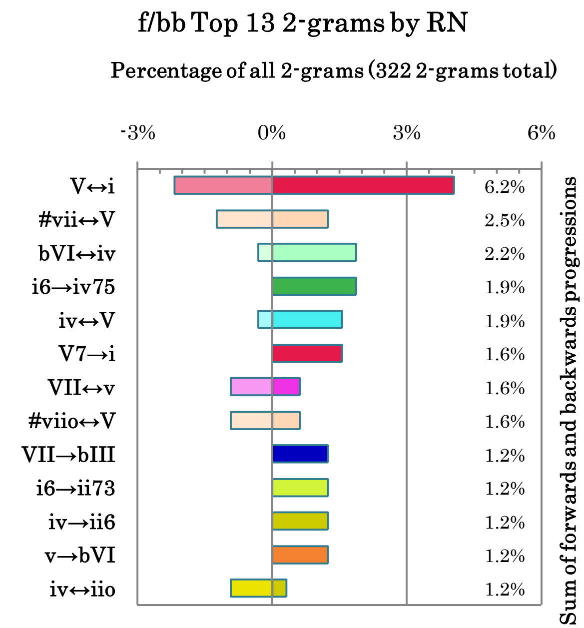
Examples 5.16 and 5.17 (below) display the thirteen commonest 2-grams in Corelli's Re tonalities.These tornado charts illustrate forwards and backwards versions of the same progression using rightwards and leftwards-extending bars (respectively), with the totals of all forwards and backwards progressions listed on the far right of each chart. All numbers are percentages of the total number of progressions in one tonality. Progressions are colour coded according to the Roman numerals they comprise, with qualities and sevenths ignored (e.g., iv↔V, IV↔V, and iv↔V^7 are all coloured cyan).

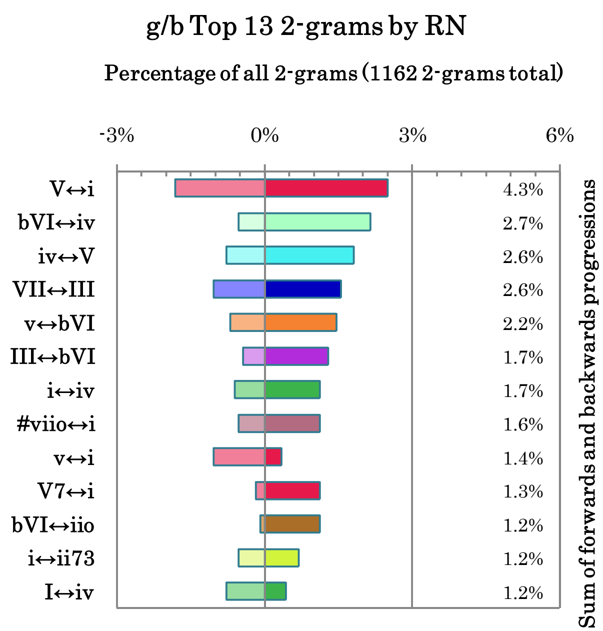
The colour-coding system assigns a colour to any progression that appears in at least two different tonalities, while unshared progressions are left grey. In Legrenzi, we needed a total of 24 different colours (plus grey) to colour all of the Roman-numeral progressions. For Corelli, we need only twenty different colours (plus grey) to colour all of the progressions. This suggests that, compared to Legrenzi, Corelli is reusing a smaller vocabulary of 2-grams, especially in his Ut tonalities, where every progression appears in at least two tonalities. For Frescobaldi, we also needed twenty different colours (plus grey) to colour all of the Roman-numeral progressions, which suggests that Corelli's and Frescobaldi's 2-gram vocabularies are roughly the same size, whereas Legrenzi's is slightly larger.

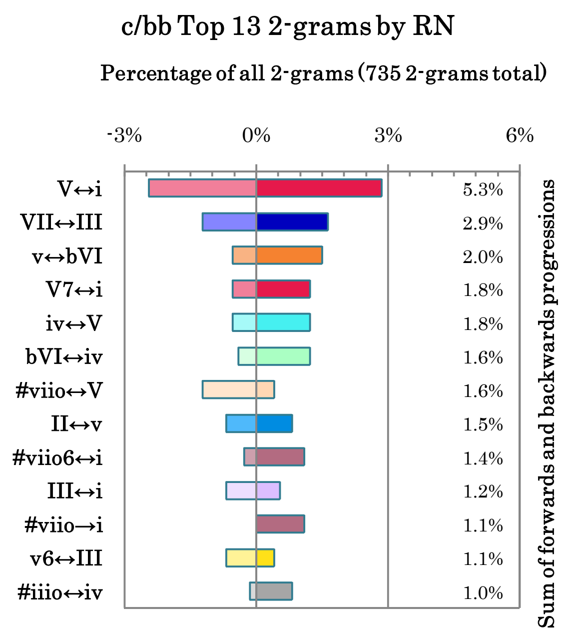
Unlike the tornado charts in Chapter 4, the charts in Example 5.16 and onwards now reflect the same three orderings for all progressions: fifth progressions are always down a fifth, third progressions are always down a third, and step progressions are always up a step. This means that if a fifth progression is actually more likely to go "backwards," i.e., up a fifth, then its leftward-extending bar will be much longer than its rightward-extending bar. This helps "backwards" progressions



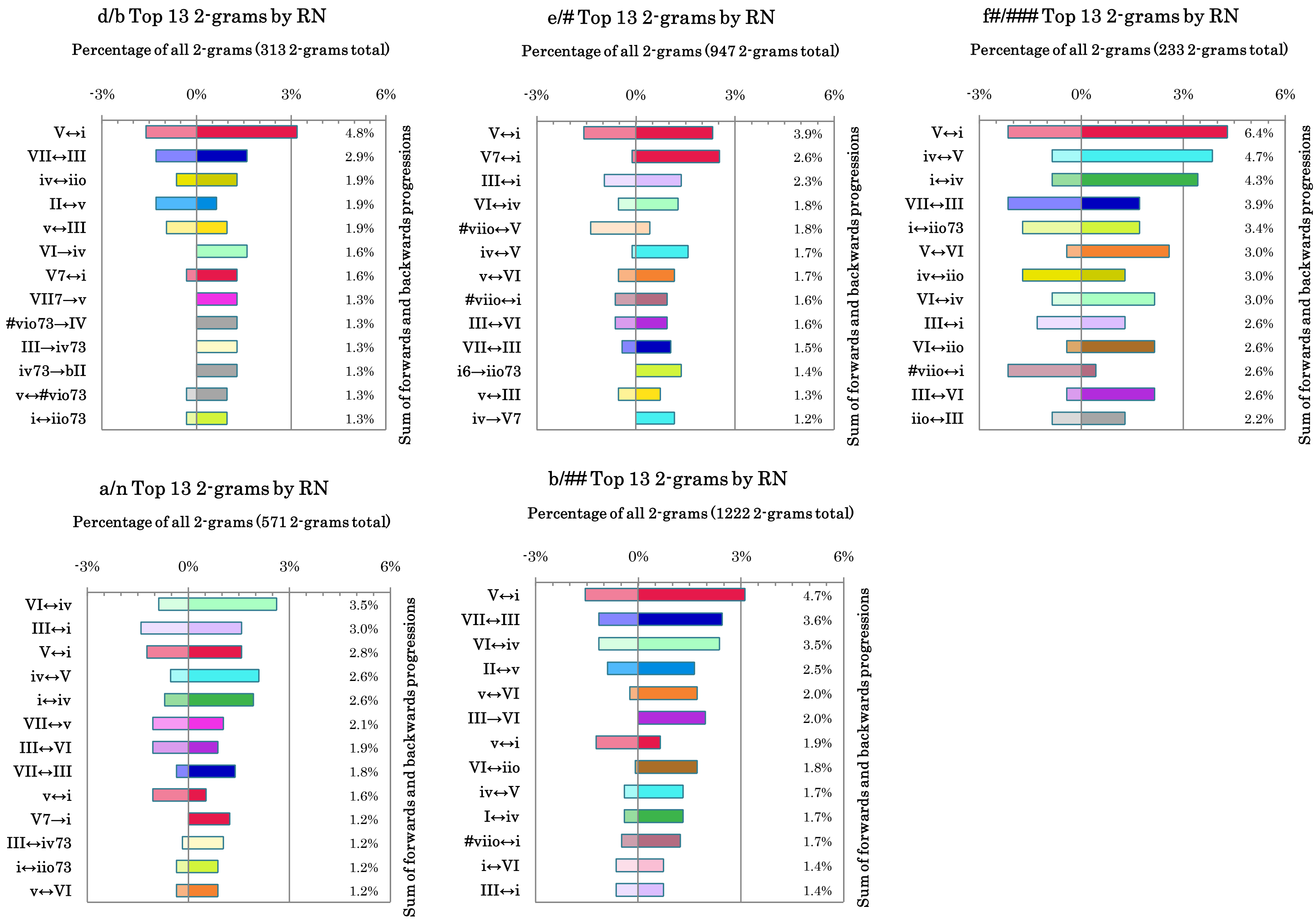








Ex. 5.16. TopRoman-numeral progressions in Corelli's dorian and phrygian tonalities



Ex. 5.17. Top Roman-numeral progressions in Corelli's minor tonalities

stand out in a repertoire where we expect the directedness of chord progressions (especially fifth progressions) to be standard.

For Corelli, we also count thirteen progressions per tonality instead of twelve as we did for Legrenzi, so that the observed percentage of the total number of progressions will be comparable between composers. In Legrenzi, we counted the twelve commonest progressions in each tonality and this meant that we observed an average of 27.96% of all the progressions in any given tonality. In Corelli, we increase the number of counted progressions to thirteen in order to obtain a similar average for the percentage of observed progressions: 28.27%. In Frescobaldi, we counted eleven or twelve progressions per tonality and observed an average of 34.55% of all progressions. What these numbers mean is that Corelli and Legrenzi use comparable numbers of different progressions and that Frescobaldi uses fewer different progressions than either of the two later composer, in other words, a larger percentage of the total number of progressions in Frescobaldi can be accounted for by a smaller number of observed progressions.

In the Re category, some progressions are ubiquitous, but perhaps fewer than we might have expected. The V↔i, VII↔(♭)III, and (♭)VI↔iv progressions are the only three progressions that appear among the thirteen commonest 2-grams in all nine of Corelli's Re tonalities; iv↔V appears in eight out of the nine (in every tonality except for d/♭). Other common progressions include v↔(♭)VI, which appears in seven out of nine Re tonalities, and III↔i, III↔(♭)VI, ♯vii°↔i, and i↔ii°, all of which appear in six out of nine Re tonalities. These results are tabulated in Example 5.18.

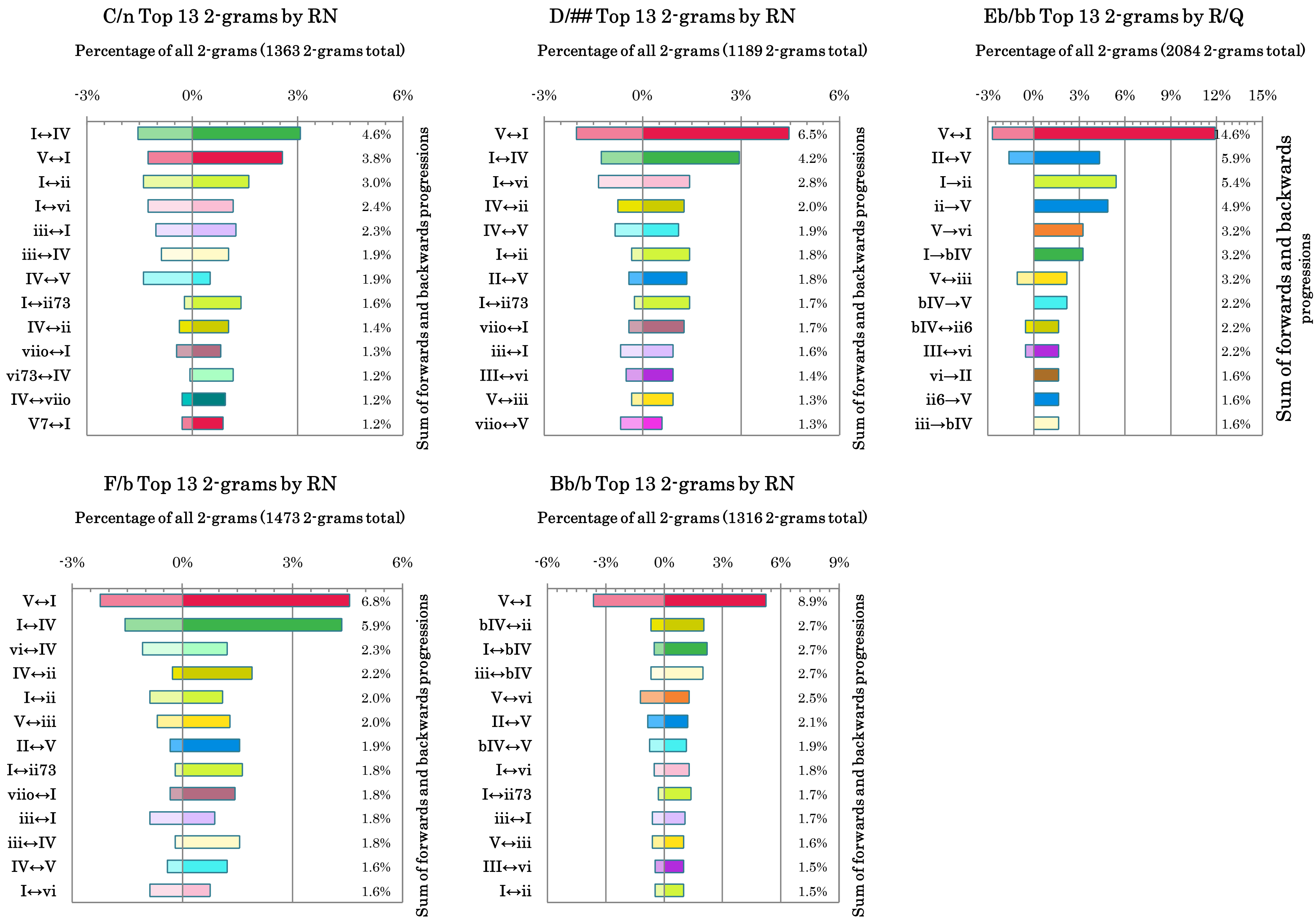
Progression	Number of Re tonalities in which it appears	Re tonalities in which it does not appear
V↔i	9 / 9	
VII↔(♭)III	9 / 9	
(♭)VI↔iv	9 / 9	
iv↔V	8 / 9	d/♭
v↔(♭)VI	7 / 9	d/♭, d/♮
(♭)III↔i	6 / 9	d/♭, f/♭♭, g/♭
♯viiº↔i	6 / 9	d/♭, f/♭♭, a/♮
(♭)III↔(♭)VI	6 / 9	d/♭, f/♭♭, c/♭♭
i↔iiº	6 / 9	b/♯♯, d/♮, c/♭♭

Ex. 5.18. Most ubiquitous Roman-numeral progressions in Corelli's Re tonalities

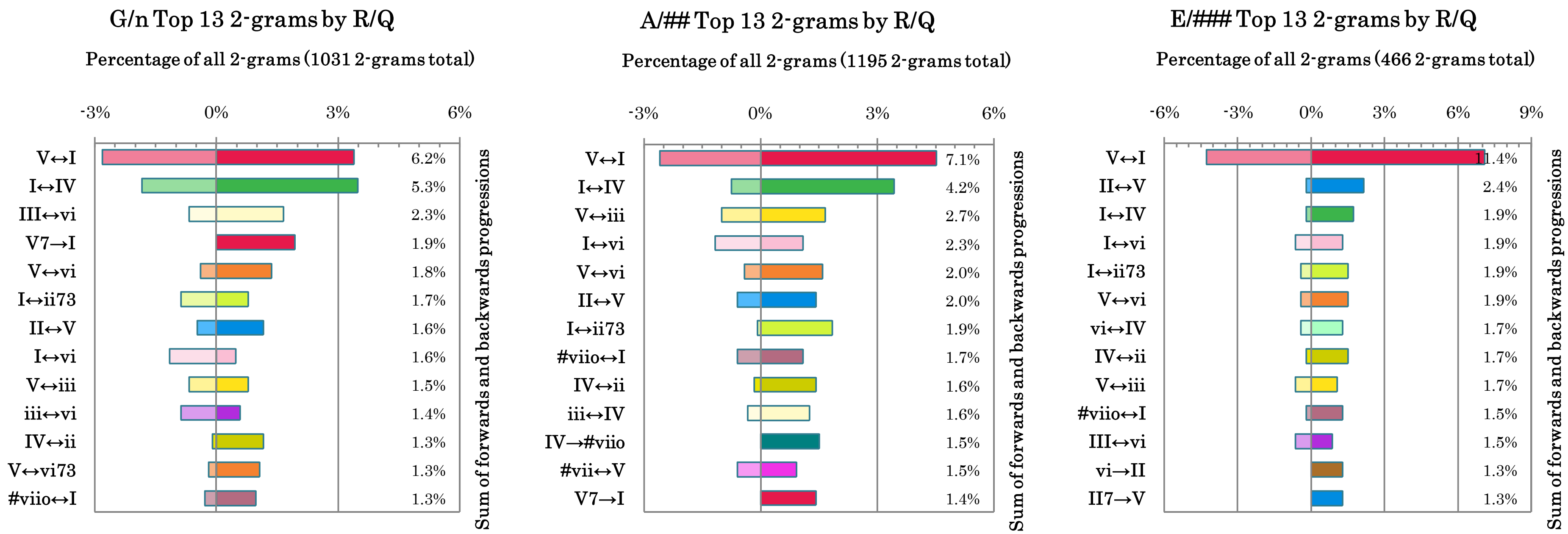
In the rightmost column of Example 5.18, we see that the tonalities that most frequently lack the most ubiquitous progressions are d/♭ and f/♭♭. The d/♭ tonality also stands out from the other Re tonalities for having the largest number of unique (grey) progressions, two of which involve its ♯viº (Bdim) triad. The large number of progressions involving Bdim triads in d/♭ helps explain the extremely high root/quality similarity we observed between d/♭ and d/♮ in Section 5.2.2.

In section 4.3.1, we saw that four progressions characterised Legrenzi's Re tonalities: VII↔III, VI↔iv, i↔iv, and i↔VI. In Corelli, there are two main differences. The first is that Corelli's Re tonalities do not rely very heavily on the i↔iv, and i↔VI progressions: i↔iv appears among the thirteen commonest progressions in only four of Corelli's nine Re tonalities (f/♭♭, g/♭, f♯/♯♯♯, and a/♮) and i↔VI appears in only two (d/♮ and b/♯♯). The second difference is subtle but important: in Legrenzi, the tonalities that lacked these four ubiquitous progressions were ones in which one or both of the triads comprising the 2-gram did not occur naturally (e.g., i↔iv did not appear in c/♭, which does not have a naturally occurring iv triad). Now, in Corelli, accidentals are added much more freely to allow certain progressions to take place in tonalities in which they are not, strictly speaking, diatonic. Tonalities like f/♭♭ and c/♭♭ have 2-grams containing iv and/or ♭VI triads among their most common progressions. That Corelli adds accidental flats to allow these progressions to take place is a sign that this composer's harmonic practice is moving towards incorporating a more fully transposable minor mode.

In the Ut category, Roman-numeral 2-grams are less varied. Examples 5.19 and 5.20 (below) display the thirteen commonest 2-grams in Corelli's Ut tonalities. In the Ut category, there are no unshared (grey) progressions: every Roman-numeral progression that appears among the thirteen commonest progressions in any of the eight Ut tonalities appears in at least one other Ut tonality. Whereas nineteen different colours (plus grey) were required to colour all of the Roman-numeral progressions in Corelli's Re tonalities, only seventeen colours (without grey) are required to colour all of the Roman-numeral progressions in Corelli's Ut tonalities. The most ubiquitous of these progressions are tabulated in Example 5.21 (below).



Ex. 5.19. Top Roman-numeral progressions in Corelli's major and lydian tonalities—the lydian tonalities B♭/♭ and E♭/♭♭ have x-axes with different scales



Ex. 5.20. Top Roman-numeral progressions in Corelli's mixolydiantonalities—the E/### tonality has a x-axis with a different scale

Progression	Number of Ut tonalities in which it appears	Ut tonalities in which it does not appear
V↔I	8 / 8	
I↔(♭)IV	8 / 8	
I↔ii	8 / 8	
(♭)IV↔ii	8 / 8	
II↔V	7 / 8	C/♮
V↔iii	7 / 8	C/♮
I↔vi	7 / 8	E♭/♭♭
(♯)viiº↔I	6 / 8	E♭/♭♭,B♭/♭
iii↔(♭)IV	6 / 8	D/♯♯, E/♯♯♯
(♭)IV↔V	5 / 8	G/♮, A/♯♯, E/♯♯♯
V↔vi	5 / 8	C/♮, F/♭, D/♯♯
III↔vi	5 / 8	C/♮, F/♭, A/♯♯

Ex. 5.21. Most ubiquitous Roman-numeral progressions in Corelli's Ut tonalities

Example 5.21 shows that Corelli's Ut tonalities are more homogeneous than his Re tonalities (there are more progressions shared by all Ut tonalities than there are progressions shared by all Re tonalities) and that there are no glaring outliers from a Roman-numeral 2-grams perspective within the Ut category. In the Re category, d/♭ stood out as the tonality in which five of the nine most ubiquitous progressions did not appear. In the Ut category, the only tonality that appears more than twice in the rightmost column of Example 5.21 is C/♮, which appears a total of four times.

Two of the progressions that are missing from C/♮ are progressions that land on vi: V↔vi and III↔vi. The fact that C/♮ appears to avoid landings on vi (at least among its thirteen commonest 2-grams) corroborates an idea of Peter Allsop's that we discussed earlier. Allsop claims that Corelli's C/♮ tonality is special because it is based on mode XII, which uses iii as its *cadenza di mezzo* instead of vi, while vi is the *cadenza di mezzo* in the other major-third modes that Corelli uses (i.e., modes VII and IX). Corelli's C/♮ tonality uses two bidirectional progressions that involve its iii triad, iii↔I and iii↔IV, but it lacks the V↔iii progression that appears in all of the other Ut tonalities. Corelli's C/♮ tonality also stands out as the only one of his Ut tonalities in which V↔I is not the commonest 2-gram. Instead, I↔IV is the most common.

There is also more of a pattern among the signature-defined octave species of the tonalities appearing in the rightmost column of Example 5.21. Whereas tonalities in the rightmost column of Example 5.18 (Re category) were always a mixture of minor and dorian tonalities, we now see that certain groups of tonalities with the same signature-defined octave species collectively exclude certain progressions. The (♯)vii°↔i progression does not appear in either of the lydian tonalities (E♭/♭♭ and B♭/♭), the IV↔V progression does not appear in any of the three mixolydian tonalities (G/♮, A/♯♯, and E/♯♯♯), and the V↔vi progression is absent from only major tonalities (C/♮, F/♭, and D/♯♯). This is not because there are fewer different octave species in the Ut category—there are just as many as in the Re category, so this suggests that signature-defined octave species is perhaps slightly more useful in differentiating between Corelli's Ut tonalities, and not as important in differentiating between his Re tonalities.

This pattern also strengthens our earlier claim that common progressions are no longer restricted from tonalities in which they are not diatonic, because accidentals are added more freely. The (♯)vii°↔i progression, for example, is not diatonic in mixolydian tonalities, but it appears in all three of them. Likewise, the IV↔V progression is not diatonic in lydian tonalities, but it appears in both of them.

Of the progressions in Example 5.21, we notice that some are less bidirectional than others. Progressions like V↔iii and I↔vi are among the most bidirectional: they are often just as likely to go backwards as they are to go forwards. Fifth progressions like I↔IV and V↔I are slightly less bidirectional, usually going forwards more often than backwards. Other progressions are more unidirectional: IV↔ii is not likely to go backwards (which reminds us of the unidirectional arrow in the flowchart from *Tonal Harmony*), whereas I↔ii, which we might expect to be more unidirectional than IV↔ii, is actually less unidirectional, especially in the F/♭ and G/♮ tonalities. Progressions like V^7↔I, when they do appear, are usually completely unidirectional, which means that dominant seventh chords in Corelli's Ut tonalities are almost always approached by non-tonic harmonies.

Thinking back to the 1-gram results in Section 5.2.1, we want to know whether the same similarities that appeared between tonalities from a 1-gram perspective also appear from a 2-gram perspective. In the Re category, the two tonalities that stood out were d/♮ and b/♯♯. These tonalities clustered together based on their Roman numeral content because they both contain more ii°, III, VI, and VII triads than Corelli's other Re tonalities. Looking at the tornado charts for these tonalities, we see that both use relatively large numbers of progressions that include the four triads in question: VII↔III, which is so common in d/♮ that it supersedes V↔i as the most common progression in that tonality; III↔(♭)VI, which appears among the top six commonest progressions in both d/♮ and b/♯♯; (♭)VI↔ii°, which is a relatively uncommon progression that appears in only two other tonalities besides d/♮ and b/♯♯ (i.e., g/♭ and f♯/♯♯♯); and III↔i, which is unusually common in d/♮, but not as common in b/♯♯.

Of these two tonalities, the one that looks more peculiar from a 2-gram perspective is d/♮, in which the prevalence of ii°, III, VI, and VII is immediately apparent when we look at the five commonest progressions in that tonality (i.e., VII↔III, V↔i, III↔i, ♭VI↔ii°, and III↔♭VI). In addition to these five progressions, the d/♮ tonality uses one other progression that makes use of VII and ii°: the ii°↔VII progression, which likely serves the same purpose as the #vii°↔V progression, but in the "relative" F major.

In our discussion of Example 5.10, we hypothesized that the difference between the d/♮-b/♯♯ cluster and the other Re tonalities was likely the result of a greater emphasis on III as a tonal area in the d/♮ and b/♯♯ tonalities. Our 2-gram results now corroborate this theory, especially in the case of the d/♮ tonality, which appears to have several pairs of transpositionally related progressions that exemplify this "relative" relationship: ii°↔VII and #vii°↔V; VII↔III and V↔i; III↔i and i↔♭VI; and i↔♭VI and ♭VI↔iv.

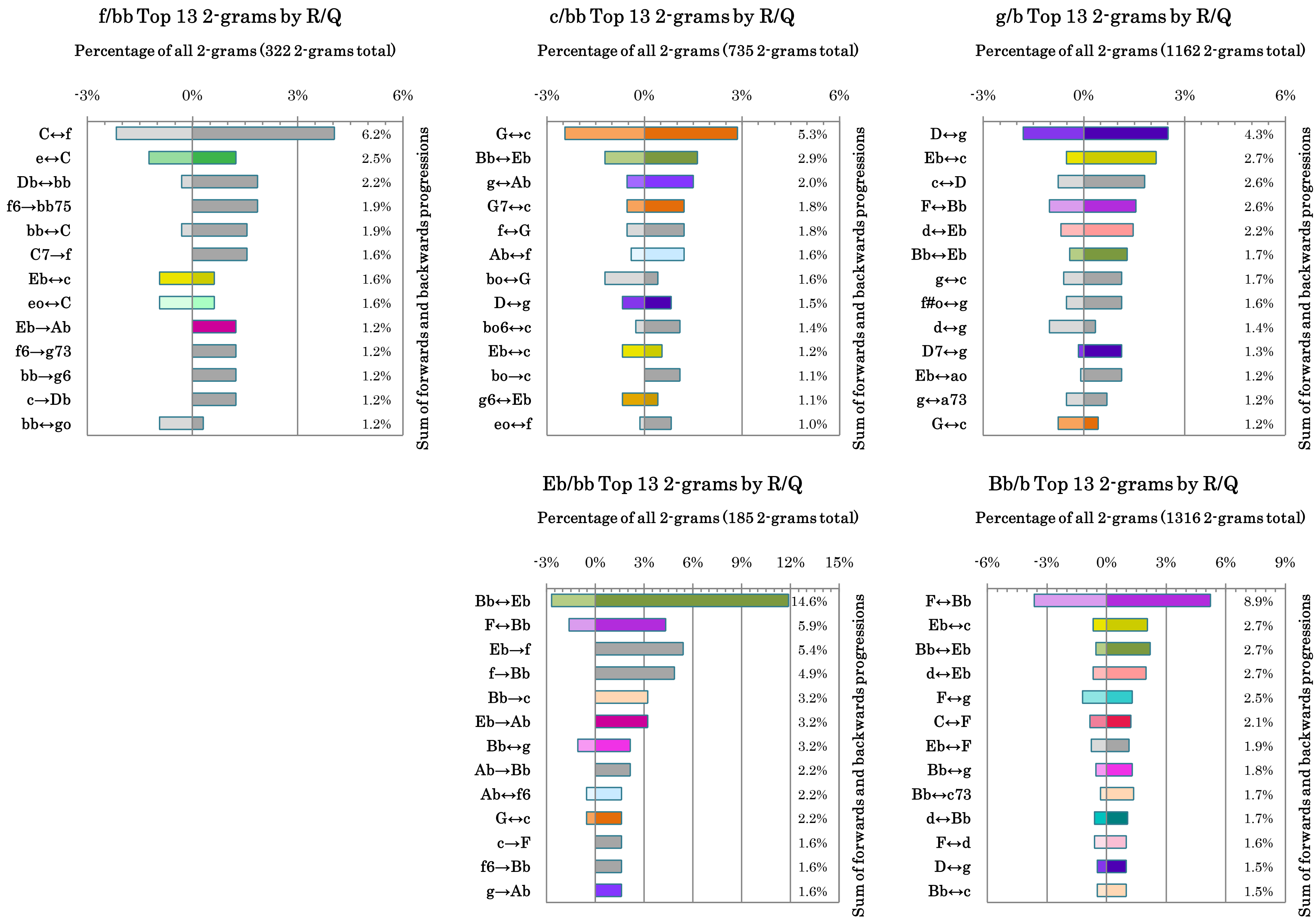
In the Ut category, the tonalities that stood out were A/♯♯ and E/♯♯♯, which clustered together from a Roman-numeral perspective, and E♭/♭♭, which was the last tonality to join the cluster of Ut tonalities (i.e., the least similar to any other Ut tonality). All three of these tonalities use more I, II, V, and vi triads than the other Ut tonalities (refer back to Example 5.12) and we can see that these four triads are relatively common in the 2-gram results as well. The II↔V and V↔vi progressions occur in all three of the A/♯♯, E/♯♯♯, and E♭/♭♭ tonalities, with the II↔V progression occupying second place in the tornado charts for both E/♯♯♯ and E♭/♭♭. These are the only two tonalities where II↔V appears as the second most common progression and also the only two tonalities that have more than one version of II↔V among their thirteen commonest 2-grams (E/♯♯♯ uses II↔V and II^7↔V, while E♭/♭♭ uses II↔V, ii↔V, and ii^6↔V). The prevalence of II↔V in E♭/♭♭ can be attributed to the lydian signature of that tonality, but a rationale for the commonness of this progression in E/♯♯♯ is not immediately apparent.

Another thing we notice about E/♯♯♯ and E♭/♭♭ is that their x-axes use a different scale than the other Ut tonalities (B♭/♭'s also uses a different scale). This is because the V↔I progression in these tonalities is so common that the x-axis must be expanded to show the entire bar. For whatever reason, Corelli uses many more V↔I progressions in tonalities that use more flats or sharps (both in the signature and as accidentals). There are also more V↔I progressions in flat-side Ut tonalities (B♭/♭ and E♭/♭♭) than in sharp-side Ut tonalities (D/♯♯, A/♯♯, and E/♯♯♯), which reminds us of the large number of V↔I progressions in Frescobaldi's F/♮ tonality, the only other lydian tonality we have looked at.

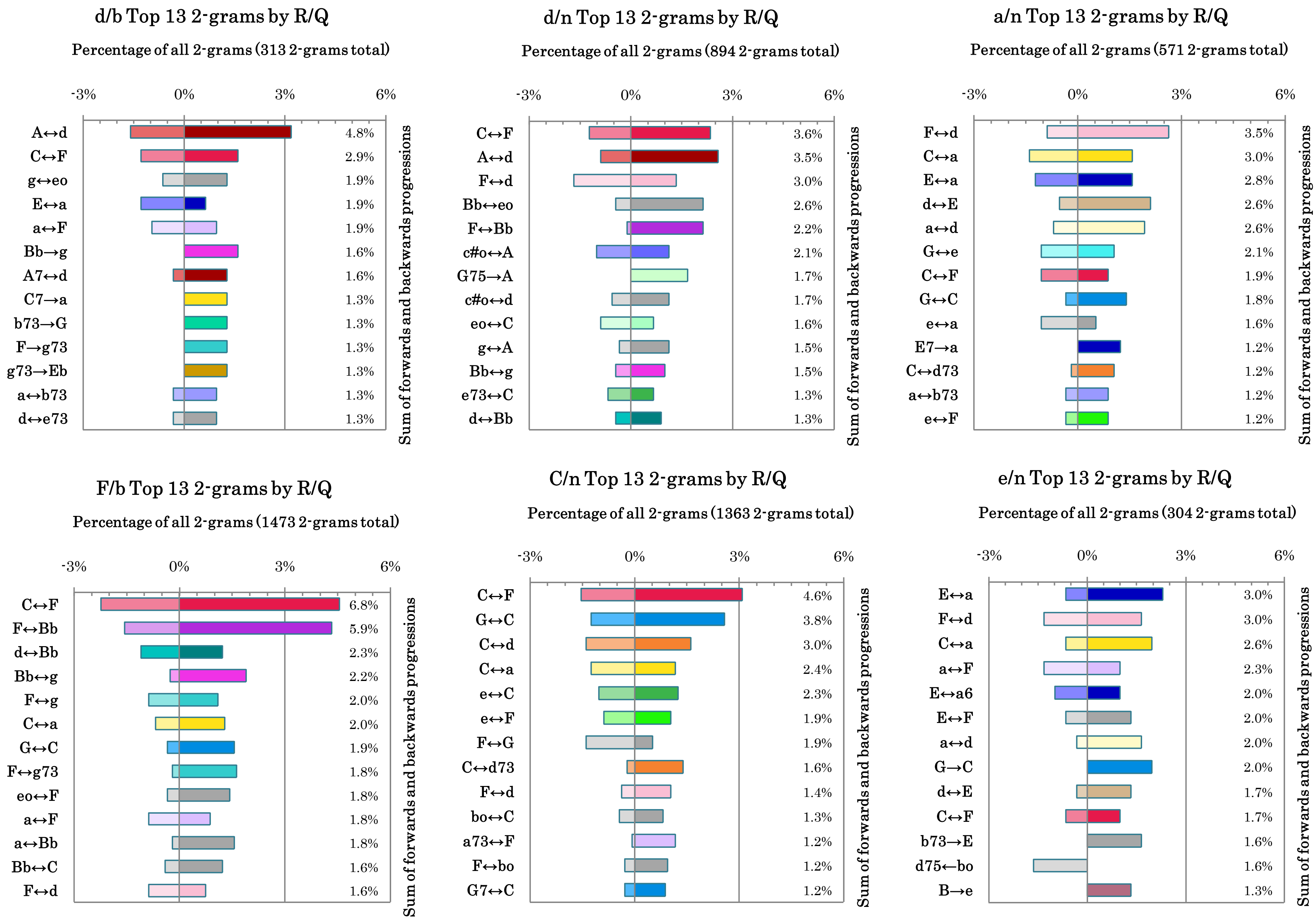
5.3.2. 2-Grams as Root/Quality Tornado Charts

While the Roman-numeral 2-gram results suggest the beginnings of standardized chord progressions in Corelli's tonalities—fewer colours were required to colour all shared progressions compared to Legrenzi and no unique (grey) progressions appeared among Corelli's Ut tonalities—the root/quality 2-grams show something different: that Corelli uses more unique root/quality progressions than either of his predecessors.

Examples 5.22, 5.23,5.25a, and 5.25b(below) contain the same tornado charts found in Examples 5.16, 5.17, 5.19, and 5.20 (above), but in different order and with progressions now labeled and coloured according to the roots and qualities of the chords they comprise (e.g., C↔F is always red, G↔C is always blue, G↔c is always orange, etc.). To colour all of the shared root/quality progressions in Corelli (a progression is shared if it appears in at least two different tonalities), we need 49 different colours. In Legrenzi, we needed 35 colours, while in Frescobaldi, we needed only 20 different colours. This shows that Corelli is transposing his most common progressions further than either of his predecessors. For example, in Legrenzi, B↔E was an unshared progression, appearing only in the A/♯♯ tonality (II↔V in that tonality), but now in Corelli, B↔E appears in both A/♯♯and E/♯♯♯, so it



Ex. 5.22. Commonest root/quality progressions in tonalities in the flat cluster



Ex. 5.23. Commonest root/quality progressions in tonalities in the natural cluster

receives a colour: in Example 5.25b (below), the B↔E progression is coloured "charcoal."

In the root/quality 2-gram examples, tonalities are arranged according to how they clustered in the root/quality 1-gram dendrogram (Example 5.15): tonalities in the flat cluster (which was coloured purple in Example 5.15) are shown together in Example 5.22 (above), tonalities comprising the natural cluster (which was coloured orange in Example 5.15) are shown together in Example 5.23 (above), and tonalities comprising the sharp cluster (which was coloured green in Example 5.15) are shown together in Examples 5.25a and 5.25b (below). This allows us to make visual comparisons between tonalities with similar root/quality content more easily, especially between "relative" pairs of Ut and Re tonalities.

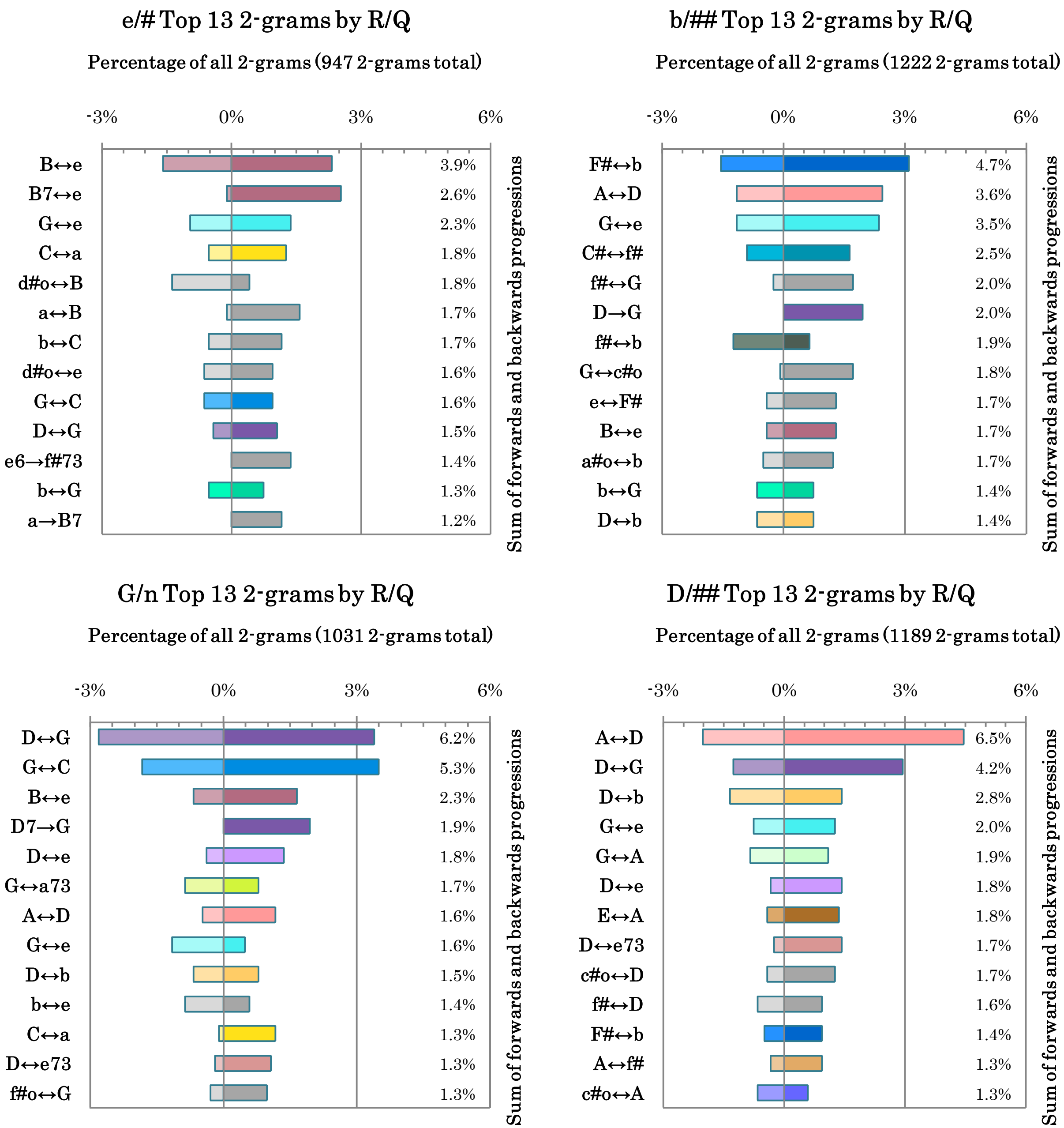
Re Tonality	Relative Ut tonality	Shared root/quality progressions	More similar to
c/♭♭	E♭/♭♭	3/13	g/♭ (4/13)
g/♭	B♭/♭	5/13	none
d/♭	F/♭	5/13	none
d/♮	F/♭	5/13	none
a/♮	C/♮	6/13	e/♮ (7/13)
e/♯	G/♮	5/13	none
e/♮	G/♮	3/13	a/♮ (7/13)
b/♯♯	D/♯♯	5/13	none
f/♯♯♯	A/♯♯	6/13	none

Ex. 5.24. Pairs of relative tonalities in Corelli—if a Re tonality is more similar to another tonality than it is to its relative Ut tonality, that other tonality is displayed in the rightmost column

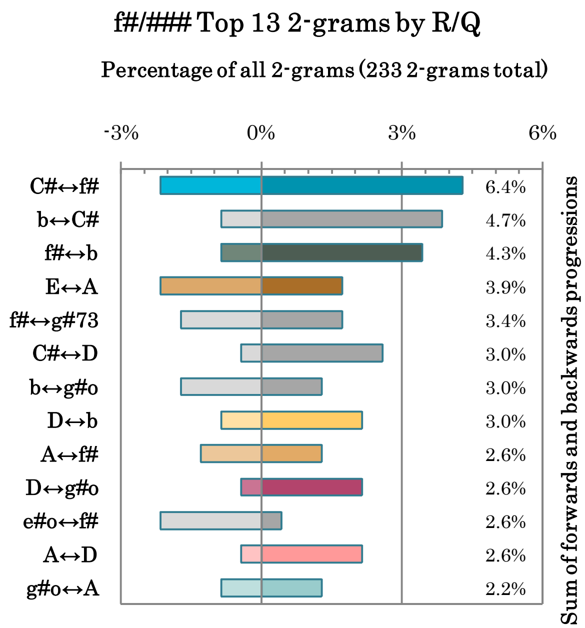
Example 5.24 (above) tabulates the numbers of root/quality progressions shared by the tonalities in each relative pair. Relative pairs comprise one Re tonality and one Ut tonality whose finals are a minor third apart. We also check the Mi tonality (e/♮) against G/♮, whose finals are also a minor third apart.

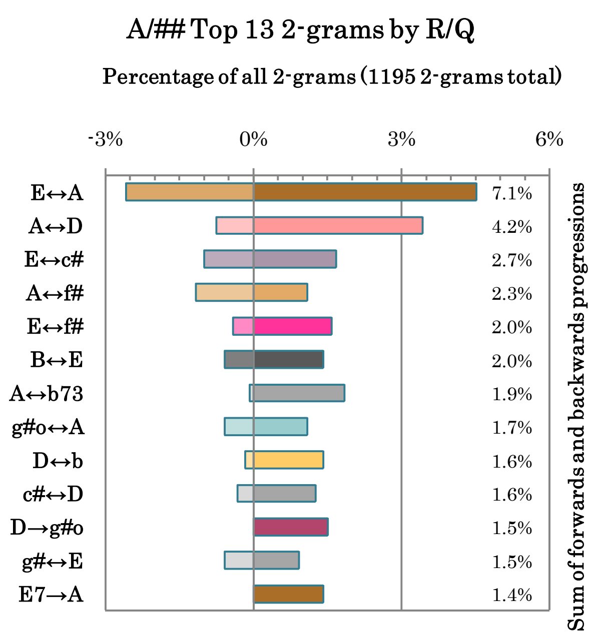
What example 5.24 helps us see is that relative relationships in Corelli are among the closest relationships from a root/quality perspective. This was suggested by the root/quality dendrogram in Example 5.15, where many (but not all) of the first pairs to form were relative pairs. Now, from a 2-gram perspective, relative pairs look even more similar: each Re tonality in the leftmost column of Example 5.24 is more similar to its relative tonality from a root/quality perspective than it is to any other tonality. This is true for every relative pair except for the c/♭♭-E♭/♭♭ pair, an exception which was forecasted by the 1-gram results. The c/♭♭ tonality actually shares more progressions with g/♭ than it does with E♭/♭♭.

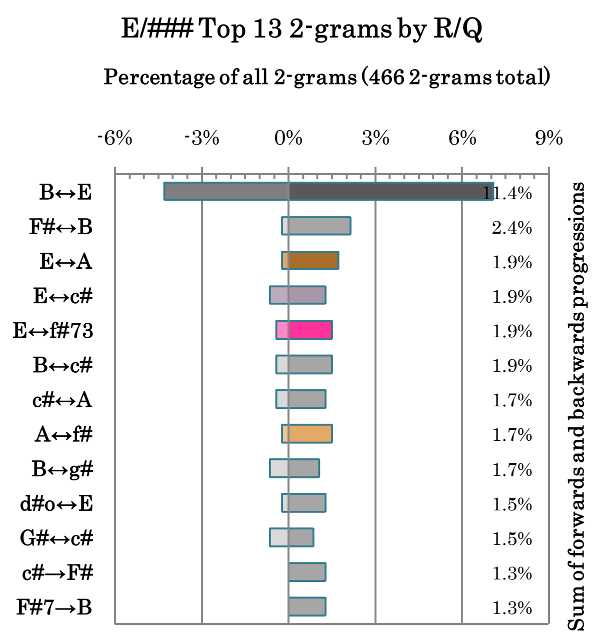
Although the a/♮ and e/♮ tonalities, which are not relatives, are the closest pair of tonalities in Example 5.24 (they share seven of their thirteen commonest progressions), the a/♮ tonality still has a close relationship with its relative, C/♮, with which it shares six of its thirteen commonest progressions. All of the other Re tonalities have six (shaded dark grey in Example 5.24) or five (shaded light grey) out of thirteen progressions in common with their relative Ut tonalities, except for the c/♭♭ tonality, which shares only three of its thirteen commonest progressions with E♭/♭♭. According to this metric, the c/♭♭ tonality is actually more similar to the g/♭ tonality, with which it shares four of its thirteen commonest progressions. This shows that the relative relationship among Corelli's tonalities is quite stable



Ex. 5.25a. Commonest root/quality progressions in tonalities in the sharp cluster







Ex. 5.25b. Commonest root/quality progressions in tonalities in the sharp cluster

(relatives almost always share either five or six of their thirteen commonest progressions) but still not fully transposable: tonalities at one extreme of the transpositional spectrum are not as likely to exhibit the same relationships as those at the other extreme, or those in the middle.

Another tonality having a weak relationship to its relative Ut tonality is e/♮, which shares only three of its thirteen commonest progressions with G/♮. Of course, we know that the stronger relative relationship is between G/♮ and e/♯, not G/♮ and e/♮, but we compare G/♮ with e/♮ to highlight, once again, the fact that signature has less to do with root/quality similarity in Corelli than it does in either of his predecessors, especially Frescobaldi, for whom signature similarity was an excellent predictor of root/quality similarity.

Comparing the signatures of the tonalities that make up the eight relative pairs in Example 5.24, we see that five of Corelli's relative pairs are same-signature pairs (i.e., c/♭♭-E♭/♭♭, g/♭-B/♭, d/♭-F/♭, a/♮-C/♮, and b/♯♯-D/♯♯), while the remaining three are different-signature pairs (i.e., d/♮-F/♭, e/♯-G/♮, and f♯/♯♯♯-A/♯♯). If we average the number of progressions shared by same-signature pairs vs. different-signature pairs, we see that different-signature pairs actually share more progressions, on average, than do same-signature pairs. This comparison emphasizes the unimportance of signature flats and sharps in Corelli's tonal practice and supports the idea that some of his signatures can actually be understood as "incomplete."

The d/♭-F/♭ and d/♮-F/♭ pairs make for another interesting comparison. F/♭ shares five of its commonest progressions with both d/♭ and d/♮, which we might have expected based on the results discussed in Section 5.2.2, since d/♭ and d/♮ were found to be the most similar tonalities (even more similar than a/♮ and e/♮) from a 1-grams perspective. From a 2-grams perspective, however, d/♭ and d/♮ are not very similar at all: they share only three of their thirteen commonest progressions and each one shares a *different* set of five progressions with F/♭. This shows that different representations of a tonality (commonest chord 1-grams vs. commonest chord 2-grams) can produce different, occasionally conflicting results, and that it is often best to consider two or more different representations in order to obtain a fuller picture of any given tonality.

One other trend that we notice in the root/quality tornado charts is that the IV↔V progression (which is iv↔V in Re tonalities), if it appears, is always grey, i.e., never shared. Fifth progressions like A↔D are often shared because most tonalities have II↔V, V↔I, and I↔IV among their thirteen commonest progressions. The A↔D progression, for example, is coloured pink and appears in Corelli's G/♮, D/♯♯, and A/♯♯ tonalities (in which it is the II↔V, V↔I, and I↔IV progression, respectively) and also in b/♯♯ (where it is VII↔III) and f♯/♯♯♯ (where it is III↔VI). Stepwise progressions, however, are shared less often because there are only two places in each Ut tonality where two major triads can be found a whole step apart (I↔II and IV↔V) and two places in each Re tonality where a minor triad might appear a whole step below a major triad (ii↔III and iv↔V). The progression I↔II does not appear among the thirteen commonest progressions in any of Corelli's major tonalities, so the IV↔V progression, if it appears, is always unique from a root/quality perspective. Similarly, the progression ii↔III does not appear in any of Corelli's Re tonalities, so the iv↔V progression, when it appears, is always unshared from a root/quality perspective.

In this way, the progression from "subdominant" to "dominant" turns out to be the single best tonality-defining progression in Corelli's vocabulary, e.g., if you hear the progression a↔B, you can be fairly certain that you are in a Re tonality whose final is E; if you hear the progression A↔B, you can be fairly certain that you are in an Ut tonality whose final is E. This tonality-defining progression appears among the thirteen commonest progressions in two thirds of Corelli's eighteen tonalities: eight out of nine Re tonalities and four out of eight Ut tonalities. The progression d↔E, which appears in a/♮, is technically not tonality-defining because it also appears in the Mi tonality, e/♮.

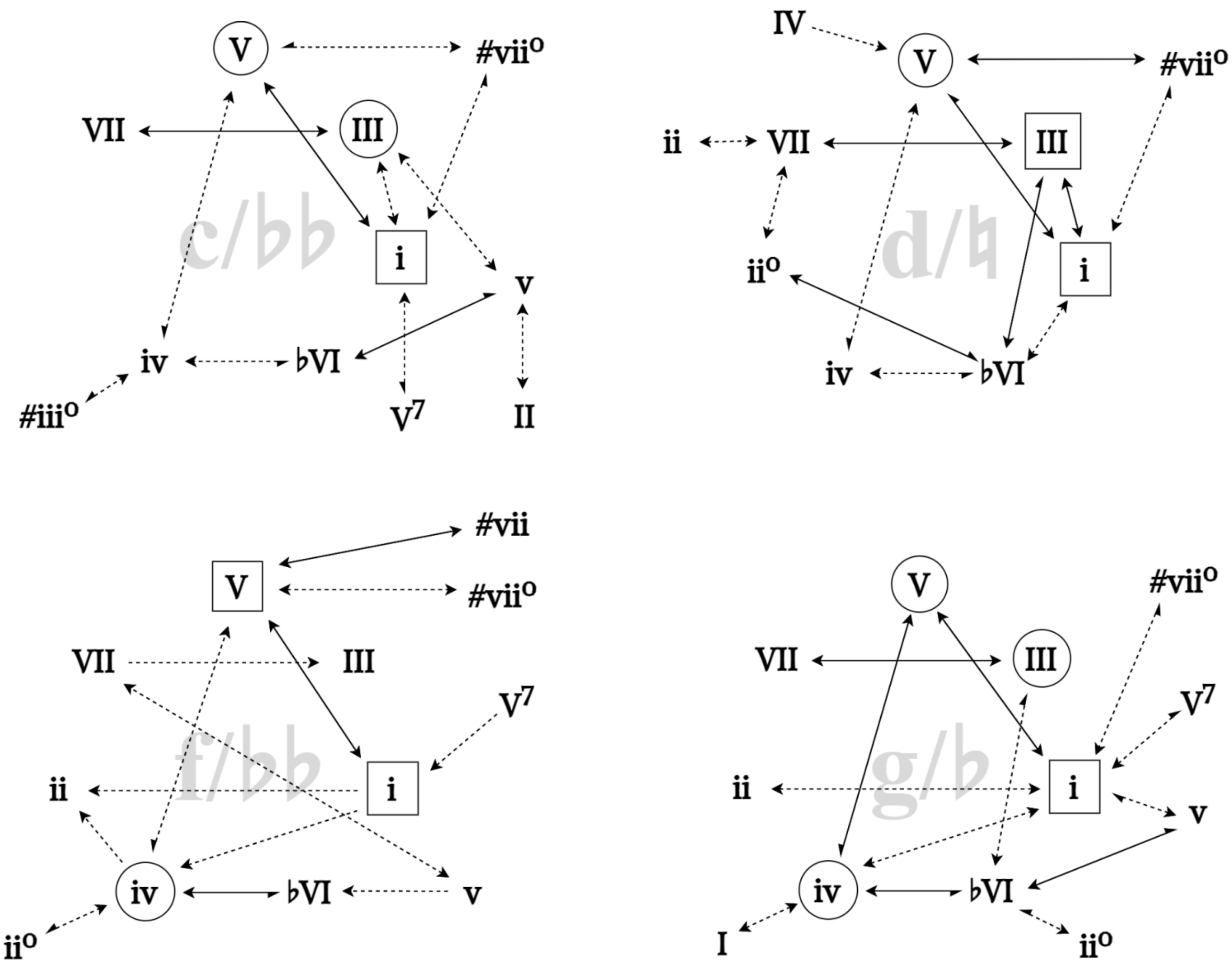
5.3.3. 2-Grams as Flowcharts

We now combine 1-gram and 2-gram data to generate flowcharts in the style of Kostka, Payne, and Almén (Kostka, Payne, and Almén 2018). If similar progressions appear in the Roman-numeral tornado chart for a given tonality (e.g., i↔iv, i↔iv^6, i↔iv^7), they are represented in the flowchart for that tonality using a single bidirectional arrow that takes all three progressions into account. The only progression that is never combined with similar progressions in this way is $V^7 \leftrightarrow i$ ($V^7 \leftrightarrow I$ in Ut tonalities) because of how common it is on its own. Other less common versions of the $V \leftrightarrow i$ progression (e.g., $V^6 \leftrightarrow i$, $V \leftrightarrow i^6$) are combined represented using a single bidirectional arrow.

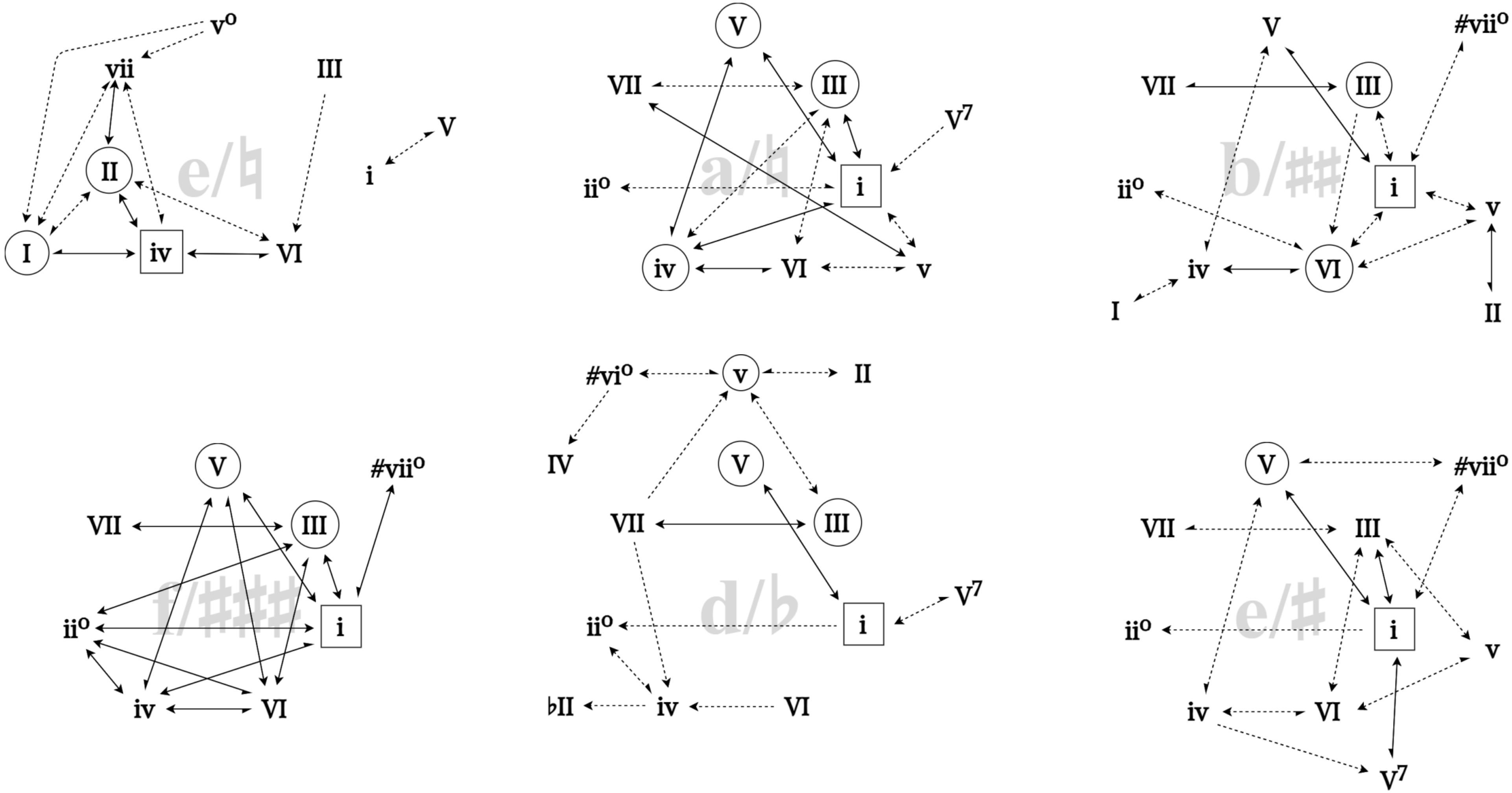
In Examples 5.26 and 5.27 (below) tonalities are grouped according to signature-defined octave species: dorian tonalities are visualized in Example 5.26 and minor tonalities (plus the e/♮ tonality) are visualized in Example 5.27. Example 5.28 (below) shows all of Corelli's Ut tonalities. The first and most important takeaway from these visualizations is that Corelli's Roman-numeral 2-grams are not standard from one tonality to the next, much less standard within a single category (Ut or Re). This suggests that Corelli's tonalities are not perfect transpositions of one another from a 2-grams perspective.

The next most important takeaway is that the flowcharts representing Corelli's tonalities are actually quite similar to those representing Legrenzi's tonalities. Upon observing large differences between Frescobaldi and Legrenzi, we expected to observe comparable differences between Legrenzi and Corelli, but instead, we observe only small differences between the two later composers.

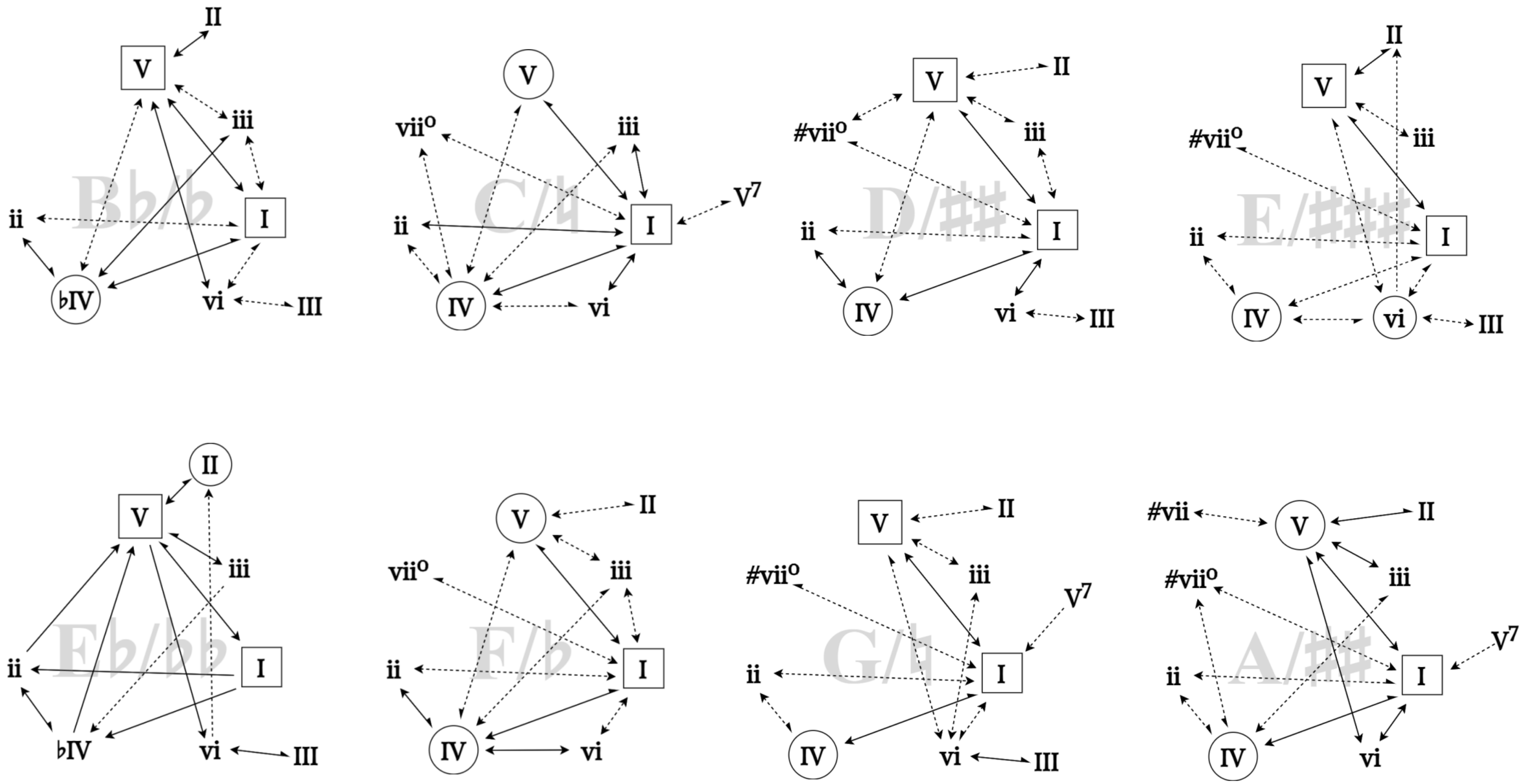
Many of the ways in which Corelli's Ut tonalities appear to be standardized are ways in which Legrenzi's Ut tonalities were also standardized: all of Corelli's Ut tonalities use the applied dominant II except for C/♮; roughly half of Corelli's Ut tonalities use the applied dominant III (V of vi); the (♯)vii$^{o} \leftrightarrow$I progression appears in



Ex. 5.26. Roman-numeral flowcharts for Corelli's dorian tonalities—boxed Roman numerals are extremely common, circled Roman numerals are common, un-circled Roman numerals are less common—solid lines represent the commonest progressions (those that account for at least 2% of all observed 2-grams) and dotted lines represent less common progressions (those that fall below the 2% threshold)



Ex. 5.27. Roman-numeral flowcharts for Corelli's minor tonalities and Mi tonality—boxed Roman numerals are extremely common, circled Roman numerals are common, un-circled Roman numerals are less common—solid lines represent the commonest progressions (those that account for at least 2% of all observed 2-grams) and dotted lines represent less common progressions (those that fall below the 2% threshold)



Ex. 5.28. Roman-numeral flowcharts for Corelli's Ut tonalities—boxed Roman numerals are extremely common, circled Roman numerals are common, un-circled Roman numerals are less common—solid lines represent the commonest progressions (those that account for at least 2% of all observed 2-grams) and dotted lines represent less common progressions (those that fall below the 2% threshold)

all of Corelli's non-lydian Ut tonalities; and the I triad interacts with at least five other chords in each of Corelli's Ut tonalities except for E♭/♭♭. Compared to Legrenzi, Corelli does not make substantial changes to Roman-numeral 2-grams in the Ut tonalities. What Corelli does do is transpose those 2-grams to produce three more Ut tonalities than Legrenzi: E♭/♭♭, E/♯♯♯, and F/♭.

One way in which Corelli's Ut tonalities are actually much more standard than Legrenzi's is in terms of 1-grams. The ranking of individual chords (not chord progressions), as indicated by the boxes (for extremely common chords) and circles (for intermediately common chords) around certain Roman numerals, is completely uniform for six of Corelli's eight Ut tonalities. The three commonest triads in every Ut tonality are I, V, and IV, and always in that order. The only tonalities that do not conform are E/♯♯♯, in which vi is more common than IV and E♭/♭♭, in which V is more common than I and II is slightly more common than IV. Like the standard relationship we observed between relative tonalities (see Section 5.2.2), this pattern breaks down at the extremes of the transpositional range: Corelli's farthest sharp-side and farthest flat-side tonalities do not conform to the same I-V-IV hierarchy as do his other Ut tonalities.

Corelli's Re tonalities are more variable than his Ut tonalities, but compared to Legrenzi's, Corelli's Re tonalities have become more standard. The main difference between Corelli and Legrenzi in the Re category is that Legrenzi's V chords are not likely to interact with other chords besides i, whereas Corelli's V chords always interact with iv (except in d/♭). Corelli also uses ♯vii° much more regularly than Legrenzi. We find ♯vii° in seven of Corelli's ten Re tonalities and only two of Legrenzi's seven Re tonalities. Finally, Corelli eschews certain dorian-specific progressions that were present in Legrenzi. The progressions IV↔VII and i↔vi° were shared (appeared in at least two different tonalities) in Legrenzi, but are completely absent from Corelli's flowcharts. Corelli still uses one dorian-ism in the d/♮, f/♭♭, and g/♭ tonalities, however: the ii chord appears instead of (or in addition to) the ii° chord. This shows that there is still at least one measurable difference between dorian and minor tonalities (according to signature) in Corelli.

Many more comparisons remain to be made between Corelli and his predecessors. In the next and final section of this project (Conclusions), I will discuss trends and developments that span the entire seventeenth century.

6. Conclusions

Now that we have discussed each of the three composers individually, we can compare them, hoping to identify trends and developments. Our conclusions address three things in particular: (1) general trends in the popularity and distribution of chords and chord progressions in each composer's output as a whole; (2) developments in the six tonalities that are used by all three composers (d/♮, e/♮, G/♮, g/♭, a/♮, and C/♮); and (3) future directions and potential research questions that have emerged from this work.

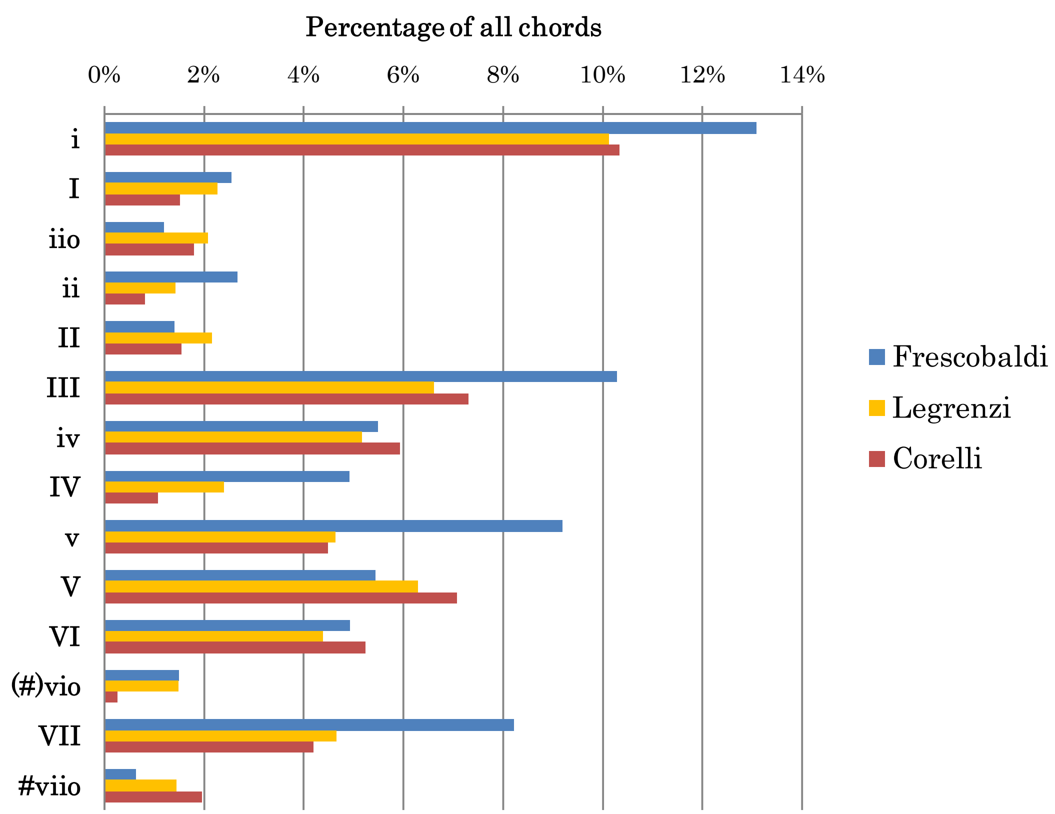
6.1. General Trends

To begin, we aggregate all of the 1-gram data in the following ways: (1) Roman-numeral 1-grams are aggregated for each category, i.e., we obtain an aggregated Roman-numeral distribution for the Ut, Re, and Mi categories for each of the three composers (3 categories × 3 composers = 9 aggregates); and (2) root/quality 1-grams are aggregated for the total output of each composer (3 aggregates).

6.1.1. Aggregated Roman-Numeral 1-Grams

Example 6.1 (below) displays Roman-numeral histograms for each composer's "average" Re tonality. Frescobaldi's average Re tonality is shown in blue, Legrenzi's in orange, and Corelli's in red. Each histogram is a weighted average of all of a composer's tonalities in the Re category: Frescobaldi's average Re tonality is four parts d/♮, three parts a/♮, thirteen parts g/♭, and two parts d/♭, Legrenzi's is ten parts d/♮, five parts a/♮, four parts g/♭, one part g/♭♭, three parts c/♭, one part c/♭♭, and one part b/♯♯, and Corelli's is three parts d/♮, two parts a/♮, four parts g/♭, one part d/♭, two parts c/♭♭, four parts b/♯♯, three parts e/♯, one part f♯/♯♯♯, and one part f/♭♭.

Comparsion of Average Re Tonalities



Ex. 6.1. Comparison of Roman-numeral 1-grams in the "average" Re tonalities of all three composers—some accidentals are in parentheses because only one type of tonality (i.e., minor) require an accidental sharp in the formation of the vi° triad

The reader will notice that Frescobaldi appears to use many more i, III, IV, v, and VII chords than either Legrenzi of Corelli. These large differences are likely caused by two things: (1) changes that happen in the Re category as a whole (these are discussed below) and (2) Frescobaldi's preference for more clearly defined harmony. What we must remember when interpreting 1-gram results in this section is that our analysis software carefully differentiates between chords that are fully expressed (i.e., at least three different pitch classes are present) and chords that are not fully expressed. The sonority comprising only the third (in the bass) and root (in one or more upper voices) of a chord, labeled "m/d" by our software (minor or diminished), becomes increasingly common in Legrenzi and Corelli. For this reason, fewer sonorities are counted as triads in Legrenzi and Corelli when compared to Frescobaldi. For a complete list of ambiguous-quality sonorities, see Example 2.8.

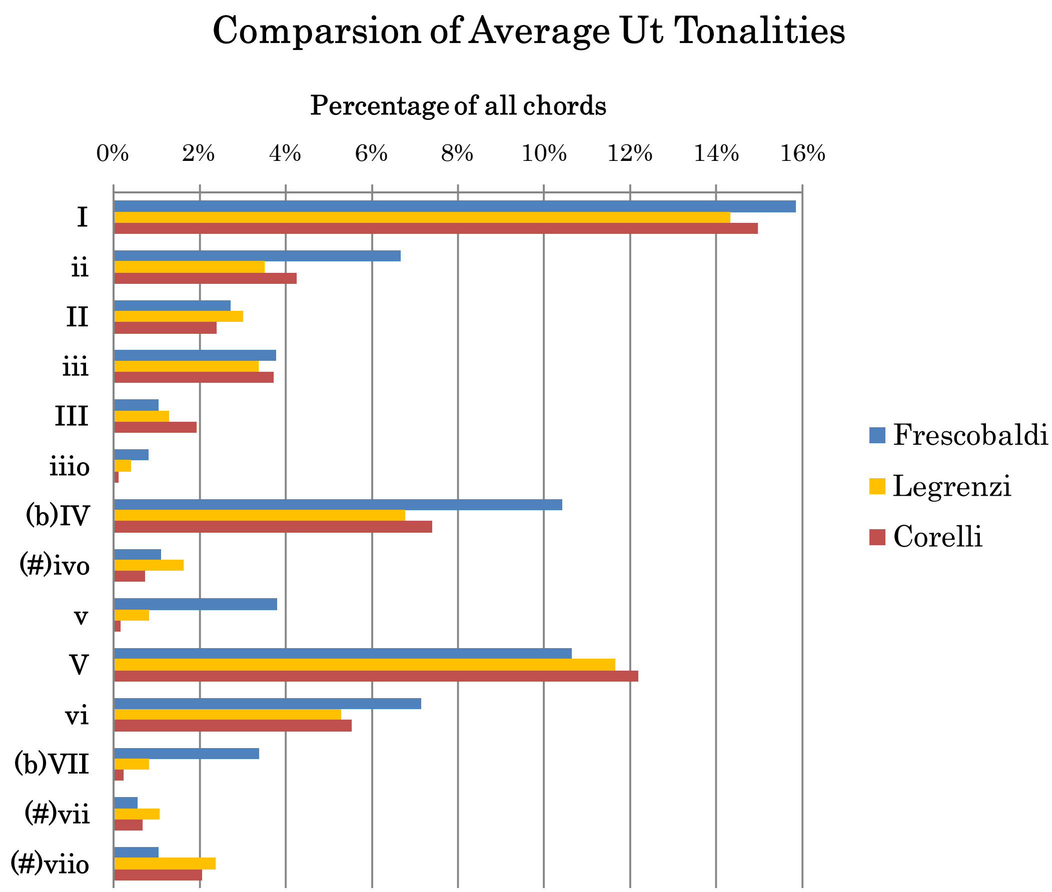
Another factor that likely contributes to greater harmonic ambiguity in Legrenzi and Corelli is the growing importance of continuo in ensemble playing. Because Legrenzi and Corelli both write many more figures in their bass parts than Frescobaldi (Corelli writes the most by far), they are able to write melodic lines that are less harmonically explicit than Frescobaldi's, and instead rely on the continuo to "fill in" whatever is missing.

The changes that happen in the Re category as a whole are marked by the increase or decrease in popularity of certain triads over time. In this Re-category comparison, three of the triads that decline over time are those that are naturally occurring within the dorian octave species: ii, IV, and vi^{o}. This is most likely caused by a move away from the dorian octave species and towards the minor octave species. The VII triad also declines over time, which could mean that Corelli's Re tonalities are spending less time in their relative major tonalities than Legrenzi's or Frescobaldi's. Certainly, the large amount of III and VII chords in Frescobaldi's Re tonalities suggest that the relative major is more common therein. Another triad that declines over time is the I triad. We know from Frescobaldi's flowcharts (Example 3.19) that the majority of these I triads follow V triads, so their decline suggests that the later composers have increasingly less preference for "tierce de Picardie" endings.

The only other triad that declines over time is the v triad, which is very common in Frescobaldi's Re tonalities. I believe this decline is linked to the move away from the dorian octave species because v is more difficult to tonicize in a minor tonality than it is in a dorian tonality. The v triad is diatonic in both minor and dorian tonalities, but the II triad, which acts as an applied dominant of v, is easier to obtain in a dorian tonality, where the supertonic triad is minor, not diminished. In Legrenzi's b/♯♯ tonality, for example, we would need two accidentals to create the II triad, C♯maj. This C♯maj triad never actually appears in Legrenzi's b/♯♯ tonality, and this tonality also has the smallest number of v triads of all of Legrenzi's Re tonalities.

The triads that increase steadily over time are the two "dominant" triads: V and ♯vii°. From this, we might conclude that the "dominant" as a harmonic function becomes more important as we move towards the end of the seventeenth century.

Example 6.2 (below) displays Roman-numeral histograms for each composer's average Ut tonality. Like the averages in Example 6.1, these averages are weighted by numbers of pieces: Frescobaldi's average Ut tonality is two parts F/♮, five parts C/♮, seven parts F/♭, nine parts G/♮, and one part C/♭; Legrenzi's is five parts G/♮, one part A/♯♯, one part B♭/♭, seven parts C/♮, and four parts D/♯; and Corelli's is four parts B♭/♭, four parts C/♮, four parts D/♯♯, two parts E/♯♯♯, one part E♭/♭♭, four parts F/♭, four parts G/♮, and three parts A/♯♯.



Ex. 6.2. Comparison of Roman-numeral 1-grams in the "average" Ut tonalities of all three composers—accidentals are in parentheses because not all types of Ut tonalities (i.e., lydian, major, and mixolydian) require the accidentals in the formation of certain triads

Here again, Frescobaldi uses more of certain triads compared to the later composers, likely for the same two reasons discussed above. The iii°, v, and (♭)VII triads, which occur naturally within the mixolydian octave species, decline to the point where they are essentially non-existent in Corelli. Other large differences between Frescobaldi and the later composers (i.e., ii, (♭)IV, and vi) are likely caused by Frescobaldi's preference for more clearly defined harmony and less of a reliance on the continuo as harmonic support.

The other non-major octave species in the Ut category is the lydian. The triads associated with this octave species, II, (♯)iv°, and (♯)vii, do not decline as sharply as the ones associated with the mixolydian octave species. This is likely because they can be used in the tonicization of the dominant. The II and (♯)iv° triads can act as applied dominants to V and the (♯)vii triad can act as iii of V.

The only triads that increase in popularity over time are III, V, and (♯)vii°. The increase in (♯)vii° suggests a standardization of the seventh scale degree in the Ut category. The increase in V suggests that this triad might benefit from the opposite of the process that affects triads like ii, (♭)IV and vi: instead of becoming less harmonically well defined (e.g., root and third only), it becomes more harmonically well defined in Legrenzi and Corelli. The III triad acts as the dominant of vi in the Ut tonalities and the gradual increase in the popularity of this triad suggests that later composers tonicize the relative minor more often.

6.1.2. Aggregated Root/Quality 1-Grams

We now aggregate the 1-gram data for each composer from a root/quality perspective. Examples 6.3a (below) tabulates the fourteen commonest major and minor triads used by all three composers. Each of the four sections of this table includes the same fourteen triads, but in different order. The leftmost section (labeled "ALL") is simply the sum of the other three sections, not the average. Triads are colour coded (major triads are dark colours and minor triads are light colours) for easy comparison.

The numeric columns display four kinds of data: (1) "Count," the actual number of triads counted (e.g., there are 1232 Fmaj triads in 48 sonatas by Corelli); "CL%," the percentage of all chord labels accounted for by a given triad (e.g., 4.96% of all of the chords in Corelli are Fmaj triads); "Dur%," the percentage of the total duration of the composer's oeuvre accounted for by a given triad (e.g., if Corelli's 48 sonatas were one continuous work, 5.06% of the duration of that work would be Fmaj triads); and "AvgDur," the average duration of a given triad, where a quarter note is equal to 1.0 (e.g., the average length of an Fmaj triad in Corelli is 0.94 quarter notes). The maximum and minimum values in the AvgDur column are shaded grey (dark for maximum, light for minimum).

The first row of data in all four sections of Example 6.3a is not a triad, but a class of sonorities that we have labeled "????." These sonorities, which we will call "unknown," are more common than any single major or minor triad for all three

ALL

Chord	Count	CL%	Dur%	AvgDur
????	11536	19.11%	17.23%	0.9
Cmaj	3782	6.26%	7.32%	1.17
Fmaj	3339	5.53%	5.88%	1.06
Gmaj	3337	5.53%	6.72%	1.22
Dmin	2795	4.63%	5.12%	1.11
Dmaj	2635	4.36%	5.27%	1.21
Amin	2303	3.81%	4.15%	1.09
Amaj	2075	3.44%	4.02%	1.17
Bbmaj	1982	3.28%	3.57%	1.09
Gmin	1787	2.96%	3.58%	1.21
Emin	1525	2.53%	2.46%	0.97
Emaj	1496	2.48%	3.00%	1.21
Cmin	1015	1.68%	1.89%	1.13
Bmin	889	1.47%	1.50%	1.02
Ebmaj	862	1.43%	1.46%	1.03

Frescobaldi

Chord	Count	CL%	Dur%	AvgDur
????	3639	21.64%	18.36%	0.94
Cmaj	1527	9.08%	11.04%	1.35
Fmaj	1309	7.78%	8.25%	1.18
Dmin	1168	6.95%	7.87%	1.26
Gmaj	934	5.55%	7.59%	1.52
Amin	829	4.93%	4.89%	1.1
Gmin	771	4.59%	5.48%	1.33
Bbmaj	699	4.16%	4.59%	1.23
Dmaj	461	2.74%	4.01%	1.62
Emin	359	2.13%	1.84%	0.96
Amaj	305	1.81%	2.59%	1.59
Cmin	269	1.60%	1.79%	1.24
Ebmaj	171	1.02%	1.09%	1.2
Emaj	154	0.92%	0.96%	1.17
Bmin	58	0.34%	0.29%	0.93

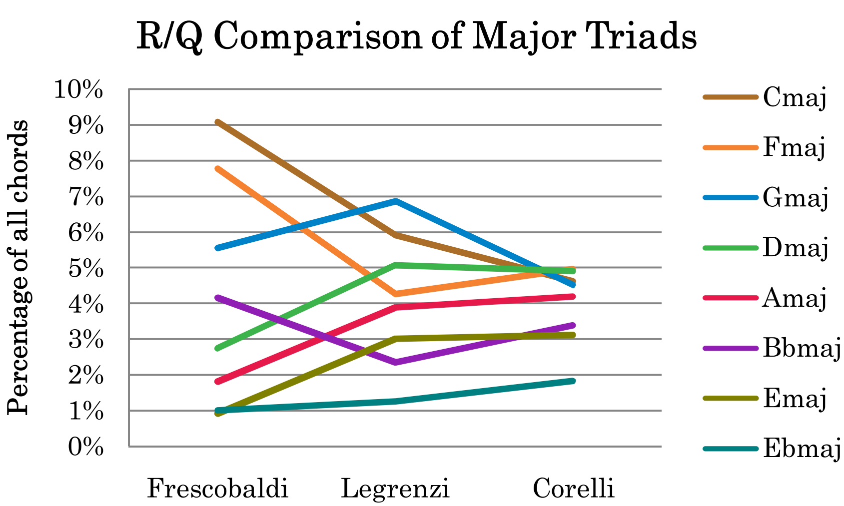
Legrenzi

Chord	Count	CL%	Dur%	AvgDur
????	3394	18.15%	16.13%	0.9
Gmaj	1282	6.86%	8.42%	1.24
Cmaj	1105	5.91%	6.65%	1.13
Dmaj	950	5.08%	6.03%	1.2
Dmin	874	4.67%	4.89%	1.05
Amin	824	4.41%	5.30%	1.21
Fmaj	798	4.27%	4.53%	1.07
Amaj	728	3.89%	4.33%	1.12
Emaj	564	3.02%	3.86%	1.29
Emin	485	2.59%	2.62%	1.02
Gmin	469	2.51%	2.79%	1.12
Bbmaj	440	2.35%	2.56%	1.1
Cmin	342	1.83%	2.05%	1.13
Bmin	258	1.38%	1.42%	1.04
Ebmaj	235	1.26%	1.28%	1.03

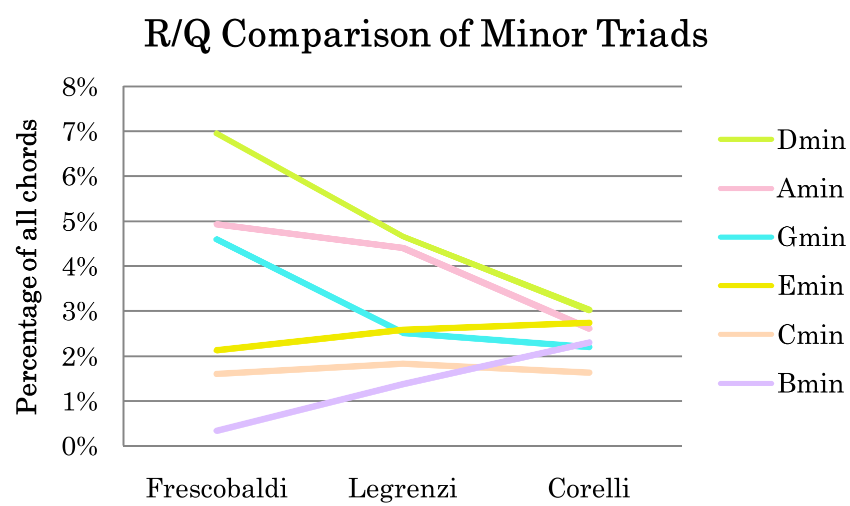
Corelli

Chord	Count	CL%	Dur%	AvgDur
????	4503	18.11%	17.20%	0.88
Fmaj	1232	4.96%	5.06%	0.94
Dmaj	1224	4.92%	5.68%	1.06
Cmaj	1150	4.63%	4.83%	0.96
Gmaj	1121	4.51%	4.60%	0.94
Amaj	1042	4.19%	4.92%	1.08
Bbmaj	843	3.39%	3.55%	0.97
Emaj	778	3.13%	3.94%	1.16
Dmin	753	3.03%	3.07%	0.94
Emin	681	2.74%	2.83%	0.95
Amin	650	2.61%	2.59%	0.91
Bmin	573	2.30%	2.56%	1.03
Gmin	547	2.20%	2.69%	1.13
Ebmaj	456	1.83%	1.91%	0.96
Cmin	404	1.63%	1.85%	1.05

Ex. 6.3a. Tallies of the fourteen commonest triads in all three composers



Ex. 6.3b. Major triads compared across three composers



Ex. 6.3c. Minor triads compared across three composers

composers. In Chapters 3, 4, and 5, we did not discuss unknown sonorities because it was not productive to compare them with other (known) harmonies within the oeuvre of a single composer. Now that we wish to compare composers, it is appropriate to compare percentages of unknown sonorities along with known sonorities (i.e., major and minor triads). We define an unknown sonority as any vertical slice of the music that does not comprise a set of vertical intervals above the bass that matches one of the sixteen figured-bass classes in Example 2.8. The number of different kinds of unknown sonorities is enormous and was not something we wished to quantify in this project, but perhaps the most common causes of unknown sonorities in this repertoire occur when passing tones and neighbour tones appear in the bass.

As a single type of sonority, unknown sonorities account for just under 20% of all vertical sonorities in the corpus (slightly more than 20% in Frescobaldi), making them roughly three times as common as the commonest triad. Each composer's unknown sonority has an average duration that is shorter than any major or minor triad used by that composer (except for Frescobaldi's Bmin triad), which is likely because many unknown sonorities are only one eighth note or one sixteenth note long. Of the three composers, Frescobaldi uses the most unknown sonorities, a finding that seems to be at odds with our earlier observation that Frescobaldi prefers to write harmonies that are more clearly defined. What this tells us is that *when* Frescobaldi writes a tertian sonority, it is more likely to have three different pitch classes, but he actually writes *fewer* tertian sonorities overall (by a margin of about 3.5%) compared to the later composers. One possible reason for this is that Frescobaldi is more likely to put contrapuntal motives in the bass, creating more non-chord tones in that voice (see Example 6.4 below), which create unknown sonorities.

Ex. 6.4. *Frescobaldi's* Canzon prima *(F.9-11), in which the opening soggetto appears in the bass a total of four times in eight measures*

What stood out in Chapter 5 was that major triads had become extremely common in Corelli's vocabulary, with seven different major triads (Fmaj, Dmaj, Cmaj, Gmaj, Amaj, B♭maj, and Emaj) outranking the commonest minor triad (Dmin). Example 6.3a reproduces the same ranking we saw in Chapter 5, but ranking is only half of the story.

While it is true that major triads are more common than minor triads in Corelli, we now also observe that some major triads have actually become less prevalent than they were in Frescobaldi and Legrenzi. Example 6.3b plots the eight major triads appearing in Example 6.3a (Fmaj, Dmaj, Cmaj, Gmaj, Amaj, B♭maj, Emaj, and E♭maj) with quantity (as a percentage of the total number of chords) on the y-axis and time on the x-axis. What we see in this graph is that the major triads that were ranked highest in Frescobaldi (Cmaj, Fmaj, and Gmaj) actually *decline* by the time we reach Corelli. Conversely, the major triads that were least common in Frescobaldi (Emaj, E♭maj, Amaj, and Dmaj) have all *increased* in popularity by the time we reach Corelli.

More generally, we observe that the percentage values for all major triads plotted in Example 6.3b grow closer together as we approach Corelli. Looking back at Frescobaldi's root/quality heat maps (Examples 3.7 and 3.8), we recall that some triads (i.e., Cmaj and Fmaj) were so common that they outranked the "tonic" triads in three of Frescobaldi's ten tonalities (a/♮, d/♭, and C/♭). As more new tonalities appear in Legrenzi and Corelli, the root/quality hierarchy that was apparent in Frescobaldi is gradually dismantled: a larger number of different finals means that triads must be more evenly distributed.This change is even more pronounced for the minor triads plotted in Example 6.3c.

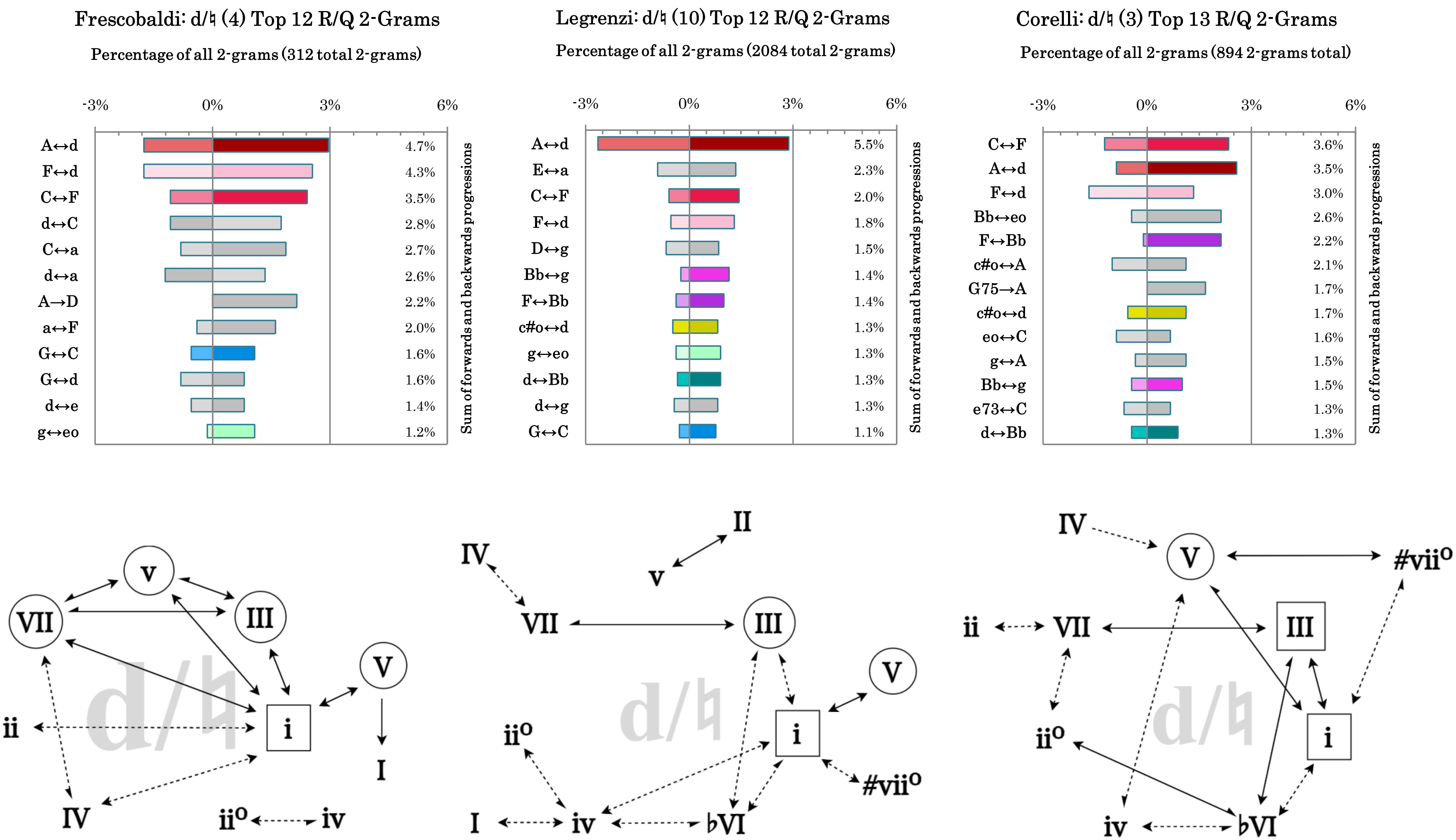
We also observe that triads requiring at least one sharp in their formation (Dmaj, Amaj, Emaj, and Bmin) appear to make the largest gains over the course of the seventeenth century, while triads that require no accidentals (Cmaj, Fmaj, Gmaj, Dmin, and Amin) appear to suffer the largest losses in terms of popularity. The growth of sharp-requiring triads is likely caused by the introduction of signature sharps as we move from Frescobaldi to Legrenzi, while the decline in white-note triads is likely caused by the expansion of the number of different tonalities to include tonalities with one or more signature flats or sharps.

The triads that have the shortest average durations are always minor (Bmin in Frescobaldi, Emin in Legrenzi, and Amin in Corelli) and the triads with the longest average durations are always major (Dmaj in Frescobaldi, and Emaj in both Legrenzi and Corelli). I believe this is because triads having a "dominant" function are longer, on average, than non-dominant triads. Minor triads can never have a dominant function, so they are shorter on average. Dmaj and Emaj stand out as triads that are likely to act as dominants in their respective tonal worlds. In Frescobaldi, Dmaj is the dominant in both g/♭ and G/♮ (Frescobaldi's commonest Re and Ut tonalities, respectively), and only sometimes the tonic in d/♮; in Legrenzi, Emaj is the dominant in both a/♮ and A/♯♯, and its function in e/♮ is more dominant that tonic (the commonest 2-gram in Legrenzi's e/♮ tonality is E↔a); in Corelli, Emaj is the dominant in both a/♮ and A/♯♯, but it is also the tonic in E/♯♯♯. Overall, this suggests that dominant harmonies are likely to last longer than tonic harmonies, on average.

6.2. Six Tonalities in Common

We now turn to the comparison of specific tonalities that appear in the works of all three composers: d/♮, e/♮, g/♭, G/♮, a/♮, and C/♮. Together, these six tonalities are a representative sample of the corpus as a whole, comprising five of the six octave species (all but lydian) that appear in the corpus. We reproduce the root/quality 2-grams and Roman-numeral flowcharts from each of the three analysis chapters and compare them side-by-side to determine how tonalities changed and which tonalities changed the most over the course of the seventeenth century. The bars in the tornado charts representing root/quality 2-grams have been recoloured to show only progressions that are shared between at least two composers. Progressions that are unique to one composer remain grey.

The first tonality to examine is d/♮. Comapring the three tornado charts, the reader will notice that only the A↔d, C↔F, and F↔d progressions are shared by all three composers. Legrenzi's tornado chart has the most coloured bars because he shares two more progressions with Frescobaldi (G↔C and g↔e°) and four more progressions with Corelli (B♭↔g, F↔B♭, c♯°↔d, and d↔B♭). In this way, Legrenzi serves as a sort of "bridge" between the other two composers.



Ex. 6.5. Root/quality 2-grams and Roman-numeral flowcharts for the d/♮ tonality

One triad that tells an interesting story is Cmaj. In Frescobaldi's d/♮ tonality, Cmaj (VII) is one of the most common triads, interacting with Fmaj (III), Dmin (i), Amin (v), and Gmaj (IV). In Legrenzi, the d↔C and C↔a progressions have completely disappeared and the G↔C progression is less common than it was in Frescobaldi: Cmaj remains a part of Legrenzi's d/♮ tonlaity, but it does not interact as freely with the other diatonic triads. In Corelli too, Cmaj remains a part of the d/♮ tonality, appearing in the commonest 2-gram, C↔F. The commonness of the C↔F progression in Corelli helps us understand the transformation that Cmaj has undergone. In Frescobaldi, it was the default triad constructed on the seventh scale degree that could both precede and follow Dmin, a good option for harmonizing scale degrees 2 and 4. By the time of Legrenzi, Cmaj becomes perhaps a less common choice compared to C♯dim, which can be used to harmonize the same two scale degrees. In Corelli, Cmaj has essentially become an applied dominant, still technically diatonic, but only ever interacting with Fmaj and other triads that have a dominant function relative to Fmaj (Edim and Emin73).

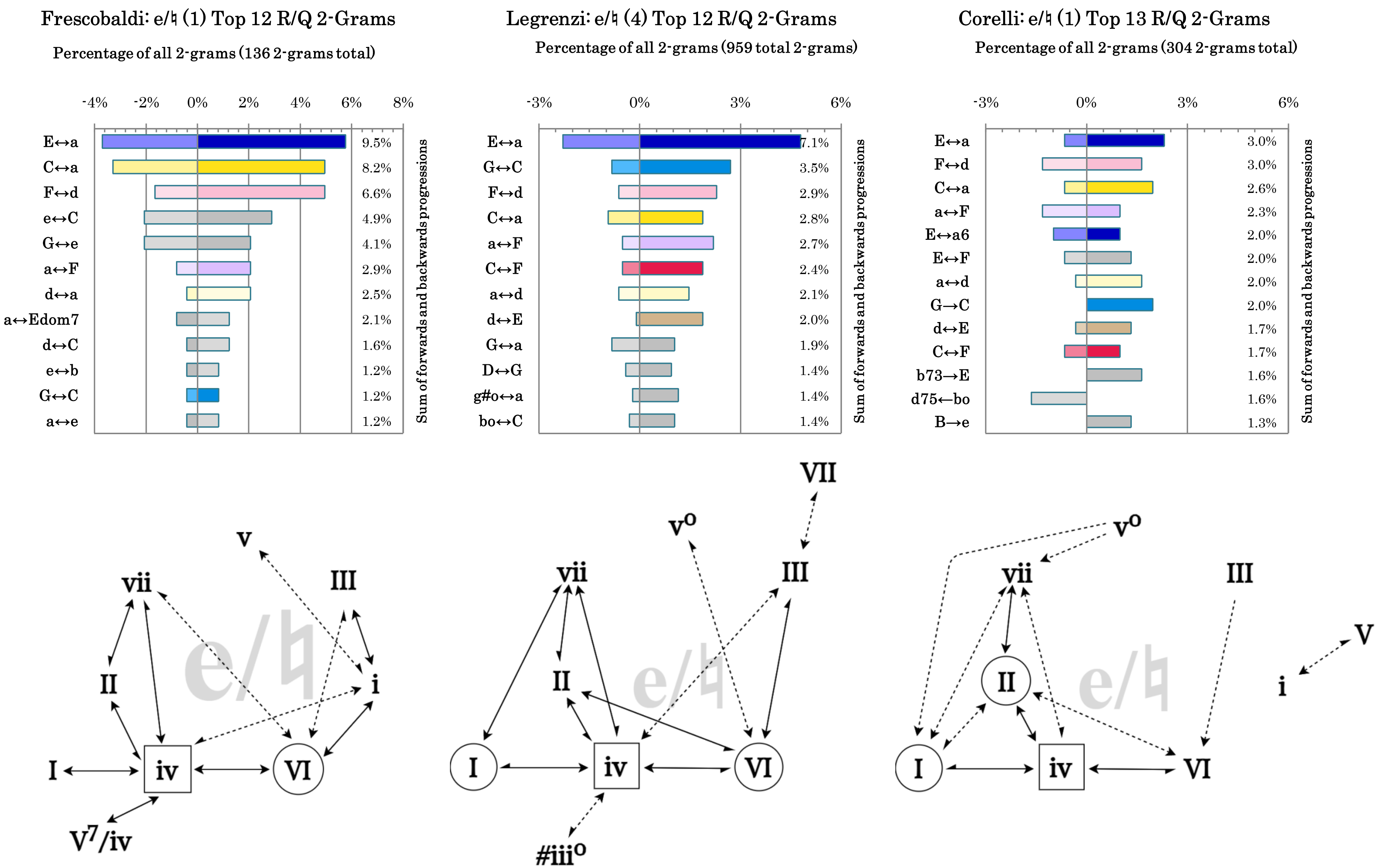
The similarities between Legrenzi and Corelli have mostly to do with the B♭maj triad, which does not appear at all in Frescobaldi's twelve commonest 2-grams. B♭maj features prominantly in the d/♮ tonalities of both Legrenzi and Corelli, suggesting that this triad, like C♯dim, has been accepted as a good alternative to the white-note triad (Bdim or Cmaj) built on the same scale degree. The pitch class B♭ is not totally absent from Frescobaldi's d/♮ tonality, however, and can be seen in the Gmin triad, which interacts only with Edim. Both Gmin and Gmaj continue to appear in the d/♮ tonalities of the later two composers, with Gmin becoming more common after Frescobaldi, suggesting that the d/♮ tonality leans more towards the minor octave species over the course of the century.

Two other triads that disappear by the time we reach Corelli are the Dmaj triad, a favourite of Frescobaldi's when following an Amaj triad, and the Amin triad. Like Cmaj, Amin was one of the commonest triads in Frescobaldi's d/♮ tonality, at liberty to interact with several other diatonic triads (Dmin, Cmaj, and Fmaj). In Legrenzi, it appears to have become an important cadence point and was approached very often from Emaj (a practice that we argued in Section 4.3.1 was psalmodically based). In Corelli, Amin (v) does not appear in any of the thirteen commonest 2-grams, which sets Corelli's d/♮ tonality apart from his other Re tonalities, namely g/♭ and a/♮, that still use both v and V. The lack of Amin in Corelli's d/♮ tonality corroborates Allsop's assertion that Corelli's tonalities are modally based: according to Penna, Berardi, and Bononcini, d/♮ should cadence on III and V, but never v, whereas a/♮ should cadence on III and v, not V (Allsop 1999, 103). According to our findings, Corelli appears to follow these "rules" exactly.

Although much changes in the d/♮ tonality over the course of the seventeenth century, one thing that stays more or less constant is the prevalence of the i, III, and V triads. In the flowcharts, the commonest triads are boxed and other common triads are circled. So while Frescobaldi's VII and v triads decline significantly, the i, III, and V triads remain the pillars of the d/♮ tonality.

The next tonality to compare is e/♮, which is not very well represented in either Frescobaldi or Corelli, both of whom write only one piece in e/♮ in this corpus. Example 6.6 (below) shows that compared to the d/♮ tonality, the e/♮ tonality has more progressions that are shared by all three composers: in the d/♮ tonality, only three 2-grams were shared by all three composers, while inthe e/♮ tonality there are six 2-grams that are shared by all three composers. This suggests that the e/♮ tonality changes less over the course of the seventeenth century than does the d/♮ tonality. Another fact that supports this claim is that there are fewer progressions shared by only two composers: no progression shared by Frescobaldi and Legrenzi is not also present in Corelli and only two progression (C↔F and d↔E) are shared by Legrenzi and Corelli but not Frescobaldi.

What does change as we approach the end of the seventeenth century is the role of the Emin triad, which we label i in the e/♮ tonality. In Frescobaldi, Emin is relatively common and interacts with four other triads: Gmaj, Amin, Bmin, and Cmaj. In Legrenzi's e/♮ tonality, the Emin triad does not appear among the twelve commonest 2-grams, but Emaj is now the third commonest triad (after Amin and Cmaj), interacting with both Amin and Dmin. Together, the disappearance of Emin and the arrival of Emaj suggest that Legrenzi's e/♮ tonality is beginning to treat



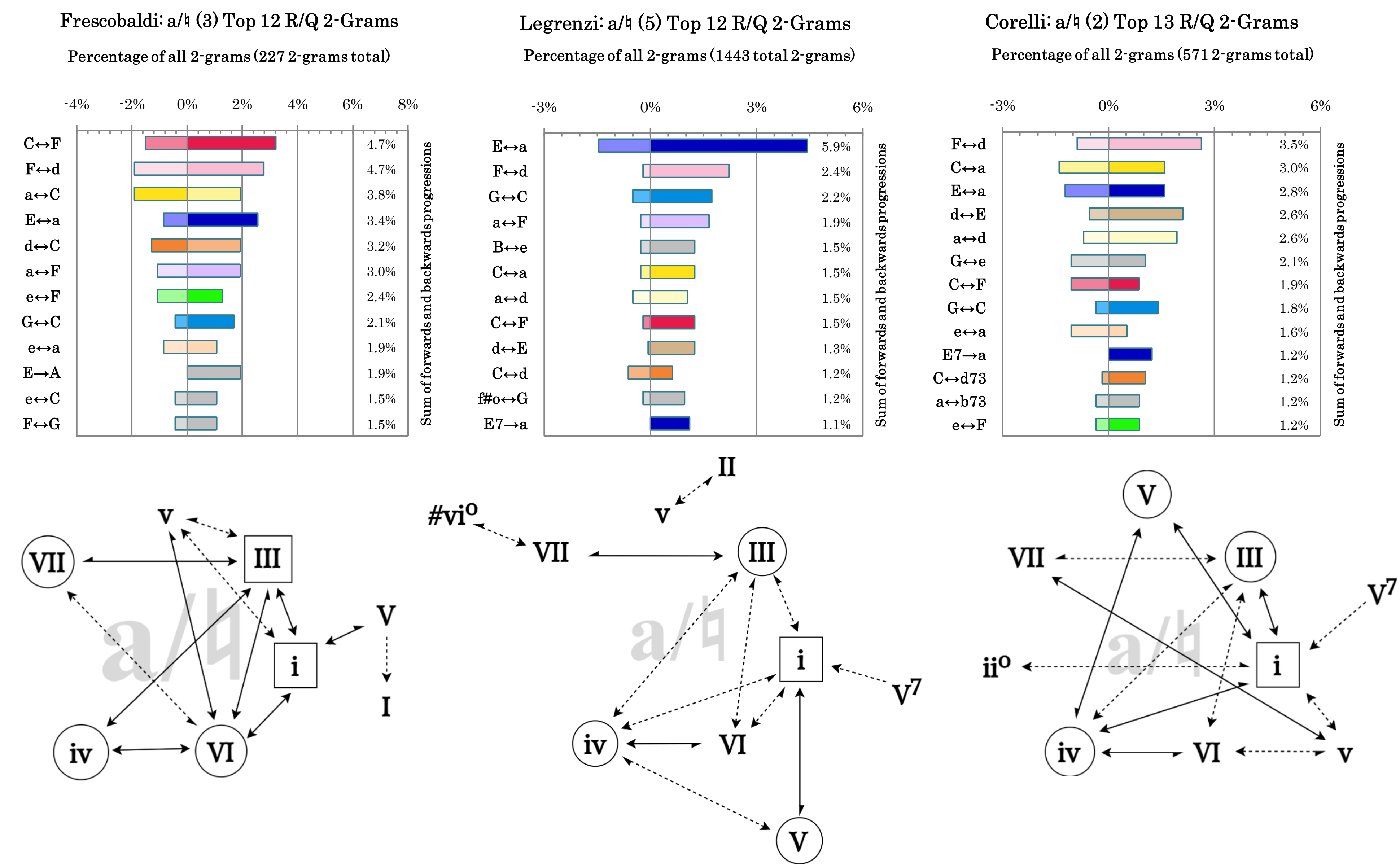
Ex. 6.6. Root/quality 2-grams and Roman-numeral flowcharts for the e/♮ tonality

Amin as a sort of "tonic" harmony. Legrenzi also introduces Bdim and G♯dim triads, which make Amin sound even more like the tonic of a Re tonality.

Corelli continues to use the Bdim triad as "supertonic," connecting it to both Dmin ("iv") and Emaj ("V"). Corelli also reintroduces Emin, but his Emin is different from Frescobaldi's. With Amin now acting as tonic, it is likely that Emin, which is only ever approached from Bmaj, functions as a secondary tonal area: the goal of a modulation. For Frescobaldi, Amin is an important triad, but it has not completely replaced the diatonic triad built on the final, Emin, as the tonal centre of the e/♮ tonality. For Legrenzi and Corelli, however, Amin has become the "tonic" of the e/♮ tonality, even though E remains the final.

Many of the "constellations" that appear in Legrenzi's flowchart reappear in Corelli's, including the vii↔I↔iv triangle (i.e., iv↔V↔i with Amin as tonic) and the VI↔iv, iv↔II, and II↔vii progressions (which also appear in Frescobaldi). A total of eight progressions are shared by Legrenzi and Corelli, while Frescobaldi and Legrenzi share only six. This shows that of the three composers, Legrenzi and Corelli are the most similar in how they use the e/♮ tonality.

Like the e/♮ tonality, the a/♮ tonality appears to undergo relatively little change. Example 6.7 (below) shows that six progressions are shared by all three composers (C↔F, F↔d, C↔a, E↔a, C↔d, and G↔C), three more progressions are shared by Legrenzi and Corelli (a↔d, d↔E, and E^7↔a), and one more progression is shared by Frescobaldi and Legrenzi (a↔F). Interestingly, there are also two more



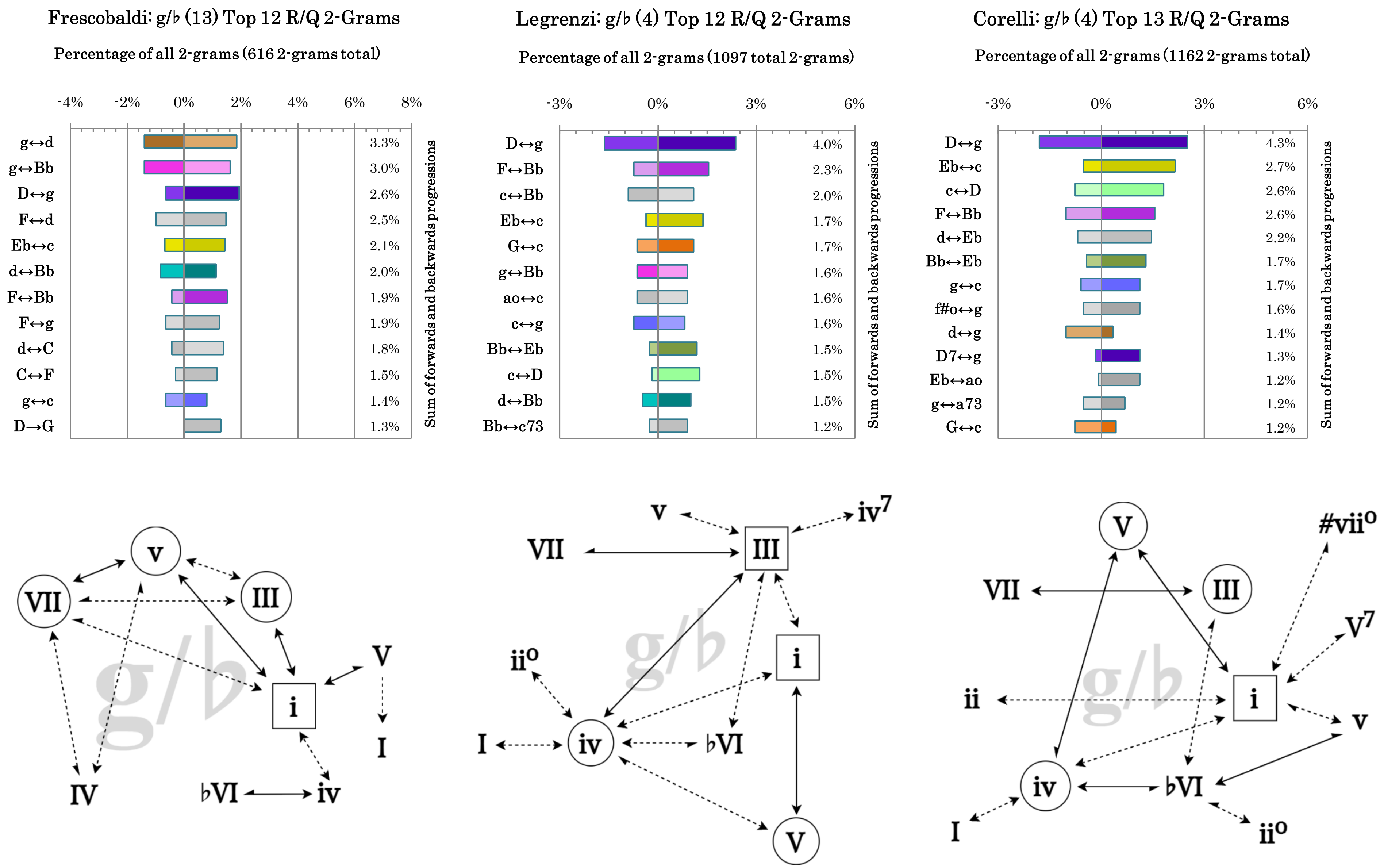
Ex. 6.7. Root/quality 2-grams and Roman-numeral flowcharts for the a/♮ tonality

progressions shared by Frescobaldi and Corelli, but not Legrenzi (e↔F and e↔a). This suggests that the a/♮ tonality is perhaps the most stable of the three tonalities we have discussed so far. From the flowcharts, we see that all three composers make use of both V and v. The i↔iv↔V triangle is visible in Legrenzi and Corelli, but not in Frescobaldi, who does not use "predominant↔dominant" progressions in his Re tonalities (as we discussed in Section 3.3.1).

The III triad (Cmaj) remains prevalent over the course of the seventeenth century, interacting consistently with VII (Gmaj), VI (Fmaj), iv (Dmin), and i (Amin), while the ii° triad (Bdim) remains rare. Frescobaldi and Legrenzi do not include any kind of supertonic (ii° or ii) in this Re tonality and Corelli's only supertonic harmony is b73, a sonority comprising only B, D, and A, avoiding the dininished fifth between B and F. Another triad that remains prevalent in the a/♮ tonality is iv (Dmin). The prevalence of iv is one thing that sets a/♮ as a minor tonality apart from d/♮, in which the "subdominant" triad is sometimes major and sometimes minor.

The only triad that appears in Frescobaldi but not in Legrenzi or Corelli is Amaj (I). Frescobaldi's I triads always follow V triads, just as they do in his d/♮ and g/♭ tonalities. Legrenzi and Corelli prefer to keep the triad built on the final minor, avoiding "tierce de Picardie" endings, as the did in the d/♮ tonality.

Having found substantial differences between the d/♮ tonalities of all three composers, we are struck by how little change there appears to have been in the a/♮



Ex. 6.8. Root/quality 2-grams and Roman-numeral flowcharts for the g/♭ tonality

tonality. This discrepancy between d/♮ and a/♮ supports the theory that dorian tonalities (like d/♮) changed over the course of the seventeenth century to become more like minor tonalities, while minor tonalities (like a/♮) did not undergo substantial changes as their octave species became the prefered octave species for all Re tonalities.

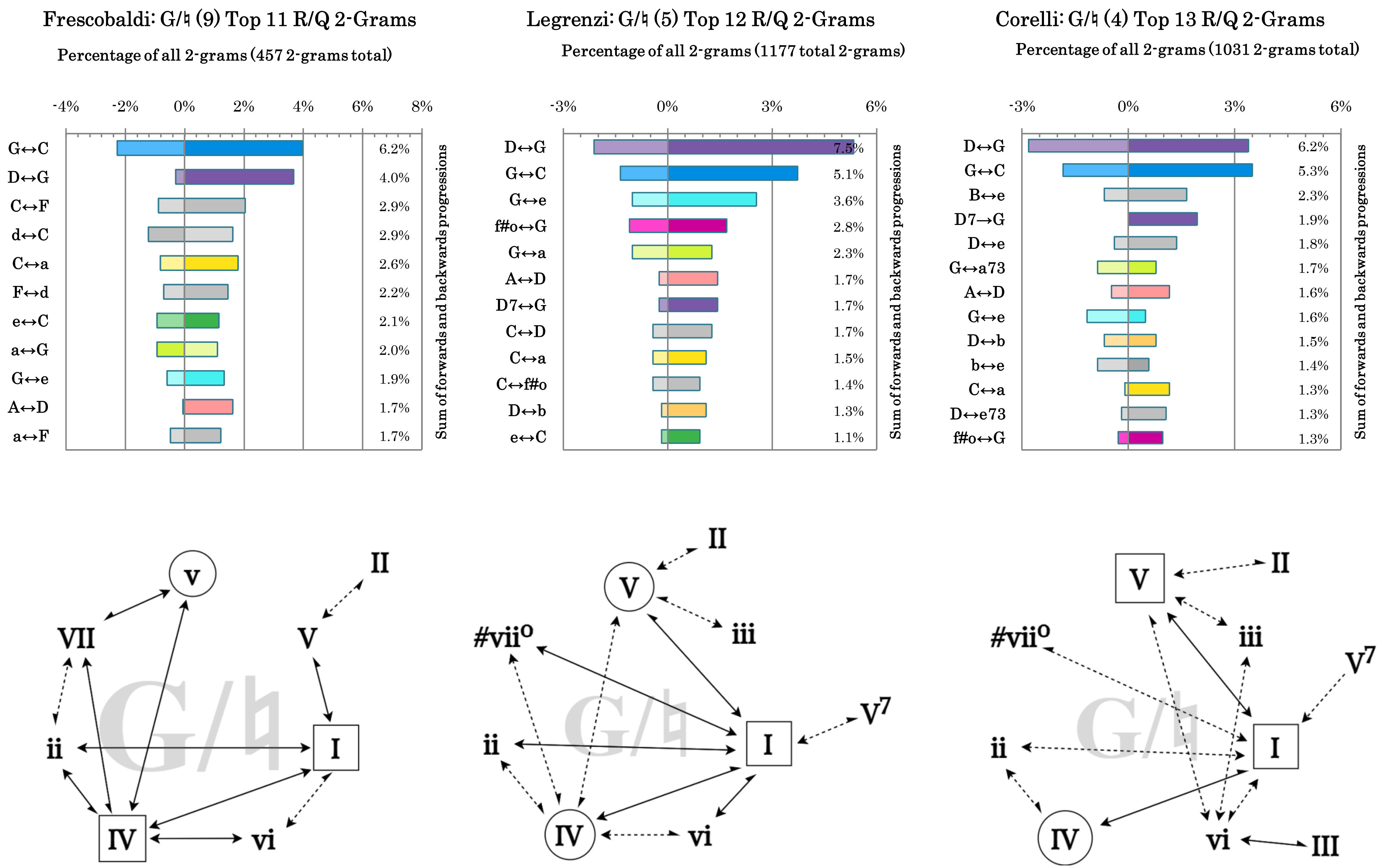
The g/♭ tonality, another dorian tonality, changes more than the a/♮ tonality but less than the d/♮ tonality. Example 6.8 (below) shows that only four 2-grams are shared by all three composers (D↔g, F↔B♭, E♭↔c, and g↔c), while two more are shared by Frescobaldi and Legrenzi (g↔B♭ and d↔B♭), and three more are shared by Legrenzi and Corelli (B♭↔E♭, G↔c, and c↔D). Comparing g/♭ to d/♮, the most striking difference is that all three composers appear to embrace scale degree ♭6 in g/♭ more than they did in d/♮. Frescobaldi, who did not use a ♭VI triad in d/♮, now shares the ♭VI↔iv progression with Legrenzi and Corelli in g/♭. Legrenzi and Corelli both used a major and a minor version of the "subdominant" triad in their d/♮ tonalities, but in the g/♭ tonality, they both use iv and not IV. This suggests that the g/♭ tonality begins to sound minor faster than the d/♮ tonality, which holds onto its dorian sound for longer.

Looking at the commonest triads in each tonality, g/♭ appears to be more similar to a/♮ than d/♮ is to a/♮. For all three composers, the commonest triads in the a/♮ tonality are i, III, V (v for Frescobaldi), and iv. The larger number of iv triads in Legrenzi's and Corelli's g/♭ tonalities puts them closer to a/♮ from a 1-grams ranking perspective. Both Legrenzi's and Corelli's g/♭ tonalities fit the i-III-V-iv paradigm, whereas their d/♮ tonalities never quite acquire a stable iv triad.

Another trend that emerges in the g/♭ and d/♮ tonalities is that Corelli, although he appears to fully embrace the iv and ♭VI triads, is still ambivalent about the supertonic triad. Whereas Legrenzi uses only ii° in both g/♭ and d/♮, Corelli still includes ii, along with ii° in these two tonalities. In this way, Corelli appears to reintroduce some of the ambiguity that characterizes the dorian octave species, which does not substantiate his standing as "first tonal composer."

Having discussed the Re and Mi tonalities, we turn now to the first of two Ut tonalities that remain in use throughout the seventeenth century: G/♮. In terms of how much it changes, G/♮ is similar to e/♮, changing more than the a/♮ tonality, but less than the dorian tonalities, d/♮ and g/♭. Example 6.9 (below) shows that there are six progressions that are shared by all three composers (D↔G, G↔C, G↔a, C↔a, G↔e, and A↔D), one more progression shared by Frescobaldi and Legrenzi (e↔C), and two more progressions shared by Legrenzi and Corelli (f♯°↔G and D↔b).

As it does in the other tonalities we have looked at, the largest change happens between Frescobaldi and Legrenzi. In his G/♮ tonality, Frescobaldi uses VII and v triads (along with V), and completely avoids the diminished triad on B, just as he does in his a/♮ tonality. By the time of Legrenzi, the VII triad appears to have been replaced by ♯vii°, the v triad no longer appears, and the accidental F♯ also serves to create a minor triad on the third scale degree (Bmin instead of Bdim).



Ex. 6.9. Root/quality 2-grams and Roman-numeral flowcharts for the G/♮ tonality

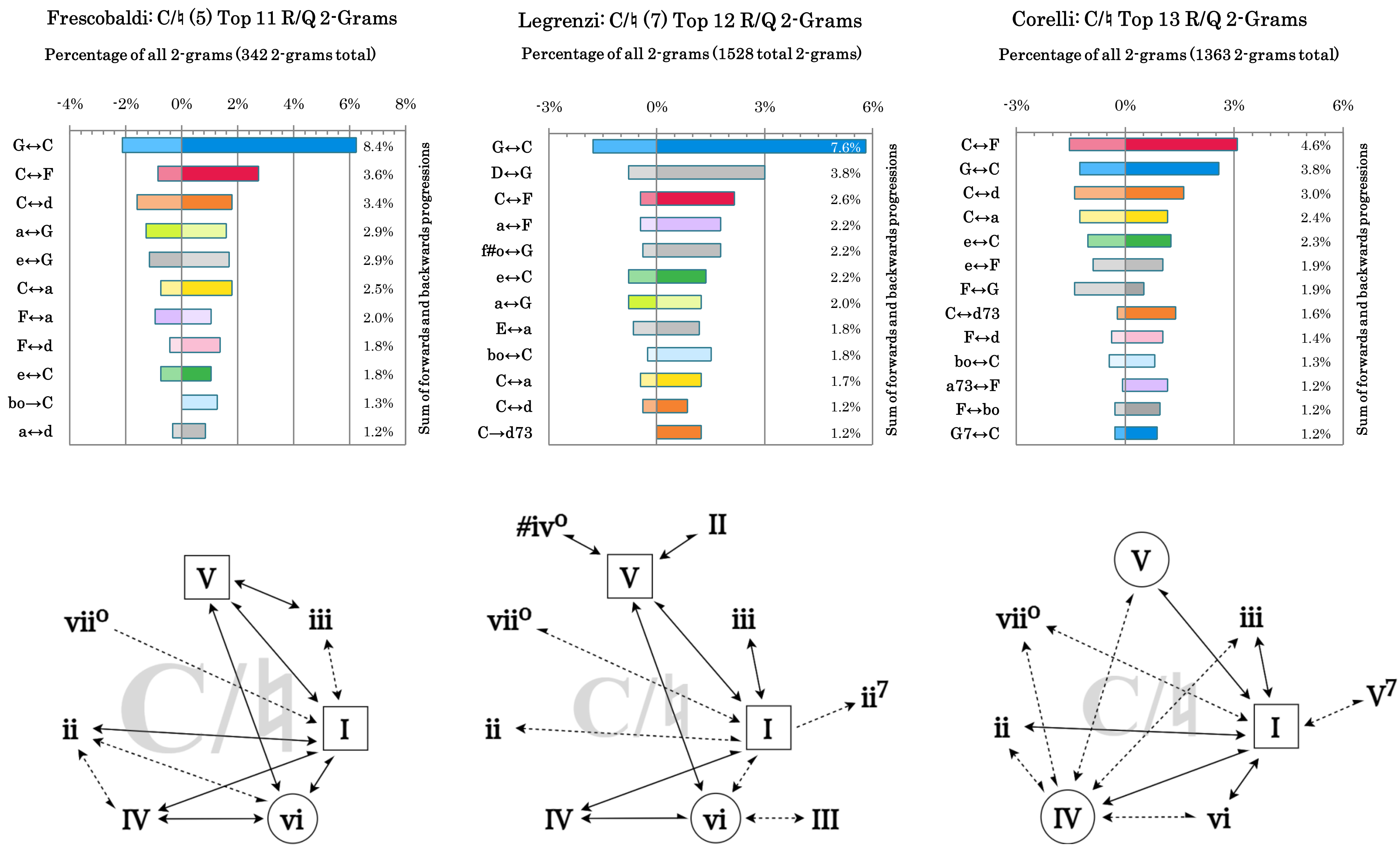
Looking more closely at the Bmin triad in Legrenzi and Corelli, we see that there is a small but significant change from one composer to the next. In Legrenzi, Bmin (iii) interacts only with Dmaj (V), while in Corelli, iii also interacts bidirectionally with vi. This is significant because the original purpose of an accidental F♯ in the mixolydian mode is to make a cadence to G. When Legrenzi writes iii, he only uses it next to a V triad (where we expect there to be an F♯), which is then likely to proceed to I. In this way, the F♯ in the iii triad is a sort of "preview" of the F♯ in the V triad. In comparison, Corelli uses iii next to V, but also next to vi. If a iii triad in Corelli proceeds to vi, which then proceeds to i, or perhaps back to iii, I believe the accidental F♯ acquires a slightly different status: because it is not adjacent to the cadential F♯ of a V triad, it becomes a "free-standing" F♯, a seventh scale degree that sounds diatonic, even though it is an accidental.

From a 1-grams perspective, the G/♮tonality undergoes minimal change. The I and IV triads remain among the commonest triads (i.e., triads that are boxed and circled in the flowcharts) for all three composers. To these, Frescobaldi adds v, while Legrenzi and Corelli use V. By the time of Corelli, V also outranks IV as the second commonest triad. In his discussion of Corelli's G/♮ tonality, Allsop calims that "Corelli's G compositions unequivocally oppose tonic with major dominant as its *cadenza principale* and relative minor as the *cadenza di mezzo*", and that Corelli makes a significant departure from his contemporaries by avoiding cadences on Cmaj and Dmin in the G/♮ tonality (Allsop 1999, 104).

Here I argue that Allsop is wrong on two counts: first, Corelli's avoidance of Dmin is not a departure because Legrenzi also avoids Dmin in his G/♮ tonality. Secondly, while it may be true that Corelli avoids *cadences* to Cmaj, he certainly does not avoid Cmaj as a non-cadential harmony. This raises the difficult question of whether or not a single cadence in the middle of a work can "define" that work's tonality with greater accuracy than a histogram of the most common triads (or chord progressions) in the work as a whole. In any case, the 1-gram data for Corelli's G/♮ tonality suggests that IV remains an important triad therein.

Something else that stands out in the G/♮ tonality is that all three composers use Amaj as a secondary dominant to Dmaj. This is the only one of Frescobaldi's tonalities (Ut or Re) where a secondary dominant appears among the twelve commonest chord progressions (secondary dominants become more common in Legrenzi and Corelli). Although Frescobaldi uses more v than V in his G/♮ tonality, his use of Amaj (V/V) suggests that V is already behaving as if it were a diatonic triad in that tonality.

Finally, comparing the C/♮ tonalities of the three composers, we find that this is the tonality that changes the least from Frescobaldi to Corelli. In Example 6.10 (below), we see that seven progressions are shared by all three composers (G↔C, C↔F, C↔d, C↔a, a↔F, e↔C, and b°↔C), one more progression is shared by Frescobaldi and Legrenzi (a↔G), and one more is shared by Legrenzi and Corelli (C↔d73). Here again, Frescobaldi and Corelli share one progression that does not



Ex. 6.10. Root/quality 2-grams and Roman-numeral flowcharts for the C/♮ tonality—note that some of Corelli's G↔C progression has become G⁷↔C and if we combine these two bars (3.8% + 1.2%), we get a dominant↔tonic progression that is still the most common 2-gram in the C/♮ tonality

appear in Legrenzi (F↔d), suggesting that this tonality does not change very much over the course of the seventeenth century.

What stands out in the flowcharts (above) is that Legrenzi appears to be more similar to Frescobaldi than he is to Corelli in terms of 1-grams. The three commonest triads in both Frescobaldi and Legrenzi are I, V, and vi, whereas Corelli uses I, V, and IV most often. Not only does vi become less common in Corelli, but it also interacts with only two other triads (I and IV). In the two earlier composers, vi interacts with four other triads, including V. What this means is that if Frescobaldi and Legrenzi are using "predominant" harmonies, they are likely using vi triads, whereas Corelli is likely using IV as a predominant.

The commonness of I, V, and IV in Corelli does not come as a surprise, as these are the three commonest triads in six of Corelli's eight Ut tonalities (only E/♯♯♯ and E♭/♭♭ break the pattern). What does come as a surprise is that this change—from I-V-vi to I-V-IV—occurs between Legrenzi and Corelli. This is the only one of the six tonalities we have discussed in this section where Legrenzi and Corelli are not in agreement from a 1-grams perspective (except for e/♮, where no two composers are in agreement). What this tells us is that while the most noticeable changes in the C/♮ tonality took place between Frescobaldi and Legrenzi, there are still changes happening between Legrenzi and Corelli as well.

To summarize our findings in this section, we present the numbers of 2-grams shared between composers as a chart in Example 6.11 (below). The tonalities with the largest numbers of shared 2-grams are C/♮ and a/♮, the major and minor tonalities. That these two tonalities change the least, and that the other tonalities change to become more like them suggests that the major and minor octave species are in the process of becoming the standard octave species for Ut and Re tonalities, respectively.

Tonality	**Number of 2-Grams Shared by…**			
	Frescobaldi & Legrenzi	Legrenzi & Corelli	Frescobaldi & Corelli	All Three Composers
d/♮	5	7	3	3
e/♮	6	8	6	6
a/♮	7	9	8	6
g/♭	6	7	5	4
G/♮	7	8	6	6
C/♮	8	8	8	7

Ex. 6.11. Shared progressions in each tonality—a larger number of shared progressions suggests that a tonality changes less over time—larger numbers are shaded darker

The two composers that consistently share the largest number of 2-grams are Legrenzi and Corelli, which tells us that harmonic syntax, at least from a 2-grams perspective, changes less between ca. 1664 and 1694 than it does between 1628 and ca. 1664. This difference is consistent across all tonalities, i.e., Legrenzi always shares one or two more 2-grams with Corelli than he does with Frescobaldi (except in the C/♮ tonality, where each pair of composers shares the same number of 2-grams). What this tells us is that, regardless of how each composer conceived of his own tonal practice (Ut vs. Re tonalities, modes, church keys) and the extent to which he sought to differentiate between his own tonalities, the relationship between the three composers is more or less constant across all tonalities: Legrenzi is always slightly closer to Corelli than he is to Frescobaldi.

6.3. Major Findings

As indicated at the end of the Introduction section, there are two main findings that come out of this project. The first is that, while tonalities in the Ut category are relatively uniform in terms of their 1-gram and 2-gram content (i.e., more like transpositions of one another), there is still not a standard version of the minor mode by the time of Corelli. Corelli's Re tonalities, like those of his predecessors, still fall into two clear sub-categories: the dorian and the minor.

The second main finding is that, according to our measurements, the largest difference between any two consecutive composers is found between Frescobaldi and Legrenzi, not between Legrenzi and Corelli: the differences between Frescobaldi and Legrenzi, both in terms of chord distributions and pair-wise chord progressions, are significantly greater than the differences between Legrenzi and Corelli. If we understand Corelli's music as a good example of tonal harmony, then our results encourage us to consider Legrenzi's music as (mostly) tonal as well. From this, we understand that the change that occurred over the course of the seventeenth century was not linear: more change happened in the first half of the century than in the second half.

Both of these findings suggest that Corelli may not be the best candidate for "first tonal composer" as we (and other scholars) had initially assumed. This opens

Corelli up for further investigations, both theoretical and musicological, and the question of "what makes Corelli so special" remains to be answered.

We also learned that certain types of tonalities, according to signature-defined octave species, undergo different amounts of change. For example, the dorian tonalities undergo more change over the course of the century than do the minor tonalities. From this, we gather that the dorian tonalities gradually become more like minor tonalities, while the minor tonalities remain relatively stable. The same trend can be observed in the mixolydian and major tonalities, but the difference in the amount of change is smaller when comparing these two octave species.

The differences in the amounts of change we see for different octave species paint a relatively clear picture of the development of major-minor tonality over the course of the seventeenth century, but some of the results in previous chapters highlight idiosyncrasies of individual composers and tonalities that go against the notion of a "clear progression" from one end of the seventeenth century to the other. In Legrenzi, we saw that two tonalities with the same signature-defined octave species, d/♮ and g/♭, had significant differences in their distributions of individual chords (see Example 4.13). We concluded that this was likely a result of Legrenzi's adherence to the system of eight church keys, in which the d/♮ and g/♭ tonalities represent two different church keys, each based on a different psalm tone (psalm tones 1 and 2, respectively).

In Frescobaldi, we saw that certain chords and chord progressions remained common, regardless of the tonality in which they appeared. For example, the three progressions F↔C, F↔d, and F↔B♭ were extremely common in each of the F/♭, C/♭, and d/♭ tonalities. Together, these three progressions accounted for at least 15% of all the 2-grams we observed in each of those three tonalities (see Examples 3.16 and 3.17). This is an example of what sets Frescobaldi apart from the later composers most clearly: the possibility that all of the tonalities in a given system—in this case, the *molis* system—can share the same 2-gram vocabulary even though their finals and octave species are all different.

6.4. Future Directions

In taking a first step towards understanding tonalities and their transpositions in seventeenth-century Italy, this project asks just as many questions as it answers. Some of these would make good follow-up projects to my dissertation, while others are dissertation-length projects in and of themselves.

The fact that Legrenzi was found to be more similar to Corelli than to Frescobaldi helps us identify the period between the two earlier composers (ca. 1630–1655) as a promising focal point for a follow-up project. One northern Italian composer who fits perfectly between Frescobaldi and Legrenzi is Tarquinio Merula (1595–1665), who was active in Cremona. Merula wrote in the same genre as Legrenzi and his output of canzonas and sonatas is substantial: his Op. 9 *Il secondo libro delle canzoni da suonare* (ca. 1631), Op. 12 *Canzoni overo sonate concertate* *per chiesa e camera, libro terzo* (1637), and Op. 17 *Il quarto libro delle canzoni da suonare a 2 e 3* (1651) would make for good comparisons with Frescobaldi and Legrenzi. Merula's Op. 17 (1651) is an ordered collection, like Legrenzi's Op. 2 (1655) and would give us a clear picture of the variety of tonalities in use just before Legrenzi.

One might wonder when (if ever) tonalities finally became perfect transpositions of one another. To answer this question, we might look at later Italian composers like Antonio Vivaldi (1678–1741), Pietro Locatelli (1695–1764), or Baldassare Galuppi (1706–1785), or at composers who were known to have published collections of pieces in every key, such as J. S. Bach's Well-Tempered Clavier (vol. 1 published 1722, vol. 2 ca. 1740) or J. C. F. Fischer's *Ariadne Musica* (1702).

In Section 4.3.2, we argued that different tonalities can be understood as different kinds of transpositions: some tonalities seem to be stepwise transpositions (e.g., Legrenzi's c/♭ and d/♮ tonalities), while others seem to be transpositions by fifth (e.g., Legrenzi's D/♯ and G/♮ tonalities). An investigation of the factors that contribute to these different derivations for different kinds of tonalities would be a worthwhile next step. Such an investigation would likely consider the instruments and performance practice associated with the music in question.

When comparing music written for different instruments, or combinations of instruments, we might also ask whether the tonalities in music written for keyboard instruments (e.g., Bach's preludes and fugues) are more exact transpositions of one another than the tonalities in music written for strings (e.g., Vivaldi concertos), where flat-side tonalities are more difficult to play. More generally, we might ask: what effect does the physicality of an instrument have on the 1-grams and 2-gramsthat we observe? Is music written by violinist-composers (Corelli, Vivaldi) more violinistic compared to music written by organists (Frescobaldi, Legrenzi)? Are there more harmonies that use open strings? Are there fewer flat keys? To what extent do sharp keys and flat keys exhibit different "asymmetrical" behaviours?

The issue of the capabilities of different instruments also overlaps with the major issue of tuning in the seventeenth century. While it was methodologically expedient to treat all transpositions as equal in this project, this was almost certainly not the reality with which seventeenth-century composers were faced, even after Corelli. A close look at the different tuning systems that were in use, which keys or chords they prioritized, and how these align (or not) with the data gathered from automatic harmonic analysis would be invaluable to our understanding of seventeenth-century tonal practice. Two theses on seventeenth-century tuning and temperament by Eric Jarlin Wang (Wang 2011) and Adam Wead (Wead 2014) would serve as promising starting points.

This concludes my study of tonal practice and transpositional relationships in the instrumental music of Frescobaldi, Legrenzi, and Corelli. I would like to invite critiques and corrections to my methodology and to the ways in which I have interpreted my results. It is my hope that the information presented in this dissertation can serve as a reference or a jumping-off point for future studies of seventeenth-century tonal practice.

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Appendix 1

Frescobaldi (47)	Original Publication	Modern Edition	For	F Number	Final/Sig.	Notes
F 8 (26)	*Il primo libro delle canzoni* (Rome: Giovanni Battista Robletti, 1628).	Etienne Darbellay, ed., *Monumenti Musicali Italiani Girolamo Frescobaldi Opere Complete* (Milano: Edizioni Suvini Zerboni 1975).	C/C/Bc (5)	F 8.10	G/n	*Gualterina*
				F 8.11	C/n	*Henricuccia*
				F 8.12	A/n	*Plettenberger*
				F 8.13	G/b	*Todeschina*
				F 8.14	G/b	*Bianchina*
			C/C/B/Bc (5)	F 8.31	D/n	*Lanberta*
				F 8.32	F/b	*Boccellina*
	Canzoni da sonare (Venice: Allessandro Vincenti, 1634).			F 8.30c	G/n	
				F 8.33c	D/n	
				F 8.34c	G/n	
	Il primo libro delle canzoni (Rome: Giovanni Battista Robletti, 1628).		C/C/B/B/Bc (7)	F 8.35	G/n	*Cittadellia*
				F 8.36	G/n	*Arnolfinia*
				F 8.37	G/n	*Altogradina*
				F 8.38b	C/n	*Rovellina*
	Canzoni da sonare (Venice: Allessandro Vincenti, 1634).			F 8.38c	C/n	
				F 8.39c	F/b	
				F 8.40c	F/b	
			C/A/T/B/Bc (9)	F 8.42c	F/b	
				F 8.45c	G/b	*sopra Romanesca*
	Il primo libro delle canzoni (Rome: Giovanni Battista Robletti, 1628).			F 8.41	G/b	*Alessandrina*
				F 8.43	F/b	*Sardina*
				F 8.44	G/n	*sopra Ruggiero*
				F 8.46	D/n	
				F 8.47	D/b	
				F 8.48	G/n	
				F 8.52	G/b	
F 9 (5)	*Il primo libro di capricci, canzon francese, e recercari* (Venice: Alessandro Vincenti, 1642).	Etienne Darbellay, ed., *Opere Complete* (Milano: Edizioni Suvini Zerboni 1975).	C/A/T/B	F 9.11	G/b	
				F 9.12	G/b	
				F 9.13	G/b	
				F 9.14	F/n	
				F 9.15	A/n	
F 10 (11)	*Canzoni alla francese in partitura, Libro 4* (Venice: Alessandro Vincenti, 1645).	Etienne Darbellay, ed., *Monumenti Musicali Italiani Girolamo Frescobaldi Opere Complete* (Milano: Edizioni Suvini Zerboni 1975).	C/A/T/B	F 10.1	G/b	*Rovetta*
				F 10.2	D/b	*Sabbatina*
				F 10.3	G/b	*Crivelli*
				F 10.4	C/b	*Scacchi*
				F 10.5	F/n	*Bellerofonte*
				F 10.6	G/b	*Pesenti*
				F 10.7	G/b	*Tarditi*
				F 10.8	F/b	*Vincenti*
				F 10.9	F/b	*Querina*
				F 10.10	A/n	*Paulini*
				F 10.11	G/n	*Gardana*

F 12 (5)	*Fiori musicali* (Venice: Alessandro Vincenti, 1635).	Alexandre Guilmant, ed., *Frescobaldi Fiori Musicali* (Paris: Maurice Senart, 1922).	C/A/T/B	F 12.14	D/n	*dopo l'Epistola*
				F 12.17	C/n	*post il Comune*
				F 12.27	C/n	*dopo l'Epistola*
				F 12.33	E/n	*dopo il post Comune*
				F 12.41	G/b	*dopo l'Epistola*

Legrenzi (47)	Original Publication	Modern Edition	For	Op-No	Final/Sig.	Notes
Op. 2 (14)	*Sonata a due, e tre. Op. 2* (Venice: Francesco Magni, 1655).	Stephen Bonta, ed., *The Instrumental Music of Giovanni Legrenzi: Sonate a due, e tre. Op. 2. 1655* (Cambridge: Harvard University Press, 1984).	Vln/Vln/Bc (6)	2-1	D/n	*La Cornara*
				2-2	G/b	*La Spilimberga*
				2-3	A/n	*La Frangipana*
				2-4	C/n	*La Strasolda*
				2-5	D/#	*La Col'Alta*
				2-6	G/n	*La Raspona*
			Vln/Vln/Vc/Bc (8)	2-10	D/n	*La Zabarella*
				2-11	G/b	*La Mont'Albana*
				2-12	A/n	*La Porcia*
				2-13	E/n	*La Valvasona*
				2-14	C/n	*La Querini*
				2-15	C/b	*La Torriana*
				2-16	G/n	*La Justiniana*
				2-17	G/n	*La Manina*
				2-18	D/n	*La Savorgnana*
Op. 4 (12)	*Sonate dà Chiesa, e dà Camera, Correnti, Balletti, Alemane, Sarabande a tre, doi violini, e violone. Libro Secondo. Op. 4* (Venice: Francesco Magni, 1656).	Albert Seay, ed., *Giovanni Legrenzi: Sonate Da Camera Opus 4* (London: Oxford University Press, 1979).	Vln/Vln/Bc (6)	4-1	D/n	*La Brembata*
				4-2	G/b	*La Secca Soarda*
				4-3	D/#	*La Benaglia*
				4-4	A/n	*La Tassa*
				4-5	C/n	*La Fini*
				4-6	G/n	*La Pezzoli*
			Vln/Vln/Bc (6)	4-7	D/n	*La Calcagni*
				4-8	D/n	*La Bassi*
				4-9	C/n	*La Terzi*
				4-10	C/n	*La Strozzi*
				4-11	C/b	*La Forni*
				4-12	G/n	*La Tognini*
Op. 8 (6)	*Sonate a due, tre, cinque, a sei stromenti. Libro 3. Op. 8* (Venice: Francesco Magni, 1663).	Albert Seay, ed., *Giovanni Legrenzi: Sonate Da Chiesa Op.4–Op. 8* (Paris: Heugel & Cie,1968).	Vln/Vln/Vc/Bc (4)	8-5	D/n	*La Rosetta*
				8-6	G/b	*La Bevilacqua*
				8-7	G/n	*La Bonacossa*
				8-8	D/#	*La Boiarda*
			Vln/Vln/A/T/Bc (2)	8-11	D/n	*La Fugazza*
				8-12	E/n	*La Marinona*

Op. 10 (15)	*La Cetra. Libro Quarto di Sonate a due tre e quattro stromenti. Op. 10* (Venice: Francesco Magni Gardano, 1673).	Stephen Bonta, ed., *The Instrumental Music of Giovanni Legrenzi: La Cetra. Sonate a due, tre e quattro stromenti. Libro quattro, Op. 10. 1673* (Cambridge: Harvard University Press, 1992).	Vln/Vln/Bc (3)	10-1	D/n	
				10-2	A/n	
				10-3	B/##	
			Vln/Vln/Vla/Bc (6)	10-7	C/n	
				10-8	Bb/b	
				10-9	A/##	
				10-10	C/b	
				10-11	D/#	
				10-12	E/n	
			Vln/Vln/Vln/Vln/Bc	10-13	A/n	
			Vln/Vln/Vla/Vc/Bc (3)	10-14	E/n	
				10-15	D/n	
				10-16	C/n	
			a 4 (2)	10-17	G/bb	Also appears in E/#
				10-18	C/bb	Also appears in E/##

Corelli (48)	Original Publication	Modern Edition	For	Op-No	Final/Sig	Notes
Op. 1 (12)	*Sonate a tre, doi Violini, e Violone, o Arcileuto, col Basso per l'Organo* (Rome: Giovanni Angelo Mutij, 1681).	"Opp. 1–4 PDF Scores" (Center for Computer Assisted Research in the Humanities: 2010). https://wiki.ccarh.org/wiki/MuseData:_Arcangelo_Corelli#Opp._1-4_PDF_scores	a 3 "da chiesa"	1-1	F/b	
				1-2	E/#	
				1-3	A/##	
				1-4	E/n	
				1-5	Bb/b	
				1-6	B/##	
				1-7	C/n	
				1-8	C/bb	
				1-9	G/n	
				1-10	G/b	
				1-11	D/b	
				1-12	D/##	
Op. 2 (12)	*Sonate da camera a tre, doi Violini, e Violone, o Cimbalo* (Rome: Giovanni Angelo Mutij, 1685).		a 3 "da camera"	2-1	D/##	
				2-2	D/n	
				2-3	C/n	
				2-4	E/#	
				2-5	Bb/b	
				2-6	G/b	
				2-7	F/b	
				2-8	B/##	
				2-9	F#/###	
				2-10	E/###	
				2-11	Eb/bb	
				2-12	G/n	*Ciaccona*

Op. 3 (12)	*Sonate a tre, doi Violini, e Violone, o Arcileuto, col Basso per l'Organo* (Rome: Giovanni Giacomo Komarek, 1689).	"Opp. 1–4 PDF Scores" (Center for Computer Assisted Research in the Humanities: 2010). https://wiki.ccarh.org/wiki/MuseData:_Arcangelo_Corelli#Opp._1-4_PDF_scores	a 3 "da chiesa"	3-1	F/b	
				3-2	D/##	
				3-3	Bb/b	
				3-4	B/##	
				3-5	D/n	
				3-6	G/n	
				3-7	E/#	
				3-8	C/n	
				3-9	F/bb	
				3-10	A/n	
				3-11	G/b	
				3-12	A/##	
Op. 4 (12)	*Sonate a tre, Opera Quarta* (Rome: Giovanni Giacomo Komarek, 1694).		a 3 "da camera"	4-1	C/n	
				4-2	G/b	
				4-3	A/##	
				4-4	D/##	
				4-5	A/n	
				4-6	E/###	
				4-7	F/b	
				4-8	D/n	
				4-9	Bb/b	
				4-10	G/n	
				4-11	C/bb	
				4-12	B/##	