Teaching Intellectually Disabled Students Addition through a Multisensory Approach

Marie Pupo

Department of Educational and Counselling Psychology

McGill University

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Abstract

This study was designed to see if children with intellectual disabilities could be taught to add pairs of single-digit numbers using the Touch Math method. Three intellectually disabled students who could add only by using physical representations of numbers were selected for the study. A multiple-probe design across the 3 students was used to evaluate the effectiveness of the intervention. The intervention consisted of a three-step addition program that was planned to teach students to add by counting the faded touch points of the smaller addend starting from the larger addend. The data show that the 3 children were able to master the program and to retain the Touch Math method from 1 to 5.5 months following completion of the program. Suggestions for future research and for teachers are discussed.

Résumé

Cette étude fut élaborée pour voir si des élèves présentant une déficience intellectuelle pourraient apprendre à additionner des pairs de chiffres allant de 1 à 9 en utilisant la méthode Touch Math. Trois élèves ayant une déficience intellectuelle qui pouvaient additionner seulement en utilisant des représentations concrètes des chiffres furent sélectionnés pour l'étude. L'efficacité de l'intervention fut évaluée en effectuant des sondages selon la méthode du *multiple-probe design across 3 subjects*. Le programme d'intervention était divisé en trois étapes de façon à apprendre aux élèves à additionner des pairs de chiffres en comptant les points effacés (*faded touch points* en anglais) du plus petit chiffre à partir du plus grand chiffre. Les résultats démontrent que les 3 élèves ont été capables de maîtriser le programme et de retenir la méthode de 1 à 5.5 mois après avoir terminé le programme. Des suggestions pour des recherches futures ainsi que pour les enseignants et enseignantes sont discutées.

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Chapter 1

Introduction

Learning basic addition is a crucial skill for anyone wanting to live independently in society. Hanrahan, Rapagna, and Poth (1993) have reported that many intellectually disabled children have difficulty with such basic addition problems as adding single digit numerals. Many of these students cannot add when manipulative materials are unavailable and many of them rely mainly on their 10 fingers to add, a strategy which makes it difficult to add sums beyond 10. Touch Math (Bullock, 1991a) is an approach to addition that is promising in terms of helping intellectually disabled students learn to add numerals with sums beyond 10 without using manipulative materials and without counting on their fingers. Essentially, the Touch Math approach to addition is based on counting concrete objects such as fingers. However, the concrete objects are touch points that are strategically placed onto regular numerals and gradually faded. After the students learn the touch point configurations, they learn to use the count-all strategy using the touch points to add pairs of single digit addenda. Next, they learn to use the count-on strategy, again using touch points. The touch points are gradually faded so that the child learns to transfer the touch point patterns to regular numbers without touch points.

The present study examined the use of the Touch Math approach to teach addition to 3 intellectually disabled students. Data for the study were collected through the use of a Multiple Probe design across subjects as well as through interviews and direct observation procedures.

Definition

Over the past two centuries, there has been considerable change in the treatment of people traditionally categorized as *mentally retarded*. Indeed, one of the most recent changes is to discard the term mental retardation in favour of more positive terms such as children with learning problems, intellectually disabled children, or intellectually handicapped children to name a few. In this thesis, the more positive term *intellectual disability* will be used instead of the term mental retardation. In order to avoid confusion, however, it must be quite clear that the category of children referred to by the term intellectual disability is that which has traditionally been described by the term mental retardation.

According to Kirk, Gallagher, and Anastasiow (1993), the most commonly used definition for children who are mentally retarded is that adopted in 1983 by the American Association on Mental Retardation (AAMR):

> Mental retardation refers to significantly sub-average general intellectual functioning existing concurrently with deficits in adaptive behaviour, and manifested during the developmental period (Grossman, 1983, p.1).

The AAMR has tried to make this definition operational. A *significantly subaverage general intellectual functioning* means that one must have an Intelligence Quotient (IQ) of 70 points or lower on a standardized intelligence test. *Deficits in adaptive behaviour* means that one must also be unable to function normally in the environment i.e. in accordance with standards appropriate for a given chronological age. It is important to note also that the AAMR definition does not require 1) that this be a permanent condition; 2) that the intellectually disabled are a homogeneous group; 3) that the condition have a particular etiology.

IQ has been used to describe different levels of mental retardation (Grossman, 1983). People with Iqs between 50-55 and 70 are categorized as mildly retarded; those with Iqs between 35-40 and 50-55 are classified as moderately retarded while anyone with an IQ below 35 is described as severely or profoundly retarded.

Chapter 2

Review of the Literature

The Intellectually Disabled: A Historical Perspective

Throughout history, intellectually disabled children were, for the most part, dismissed as deviants who were incapable of profiting from educational experiences (Gearheart & Litton, 1979; Haring, 1994). During the 18th and 19th centuries, significant efforts to improve the lot of such individuals came from the work of individual physicians and teachers who suggested that people, marked as intellectually deviant or defective, could be treated. Most prominent among these humanitarians were two European physicians, Phillip Pinel and Jean Marc Gaspard Itard, who in the early 19th century demonstrated that the intellectually disabled should be treated humanely and that they could be partially rehabilitated (Langone, 1986; Repp, 1983). In the mid-19th century, Édouard Séguin, a follower of Pinel and Itard, developed a three-part program for intellectually disabled students, using sensory training techniques. Almost immediately, some American institutions and schools for the intellectually disabled began to offer aspects of Séguin's training program along with custodial care. (Winzer, 1990; Ysseldyke & Algozzine, 1984)

However, in the 1880s, the poor results of sensory training techniques along with the influence of the newly developing theories of Darwin drastically changed society's attitude toward intellectually disabled people. From the two last decades of the 19th century up to the 1930s, the Eugenics Movement, described as Social Darwinism, dominated the social and educational scenes. As a consequence, intellectually disabled people were considered dangerous to society, and were locked up and sterilized for society's protection. (Lilly, 1979; Lusthaus, Gazith, & Lusthaus, 1992)

During the early part of the present century, people began to realize that genetics was more complex than previously suggested by proponents of the Eugenics Movement. Intellectually disabled individuals were seen as less of a threat to society than previously believed and were viewed as having the capacity to adapt to society. At the same time, concerned professionals joined with parents of intellectually disabled children to form pressure groups which demanded that intellectually disabled children receive more humane treatment and be provided with more adequate facilities, instruction and services. As a result of the efforts made by professionals and parents, society's attitudes toward the intellectually disabled gradually improved to the point where institutions housing such children became more comfortable, the treatment practices vastly improved, and provisions were made to de-institutionalize and rehabilitate the intellectually disabled. (Winzer, 1990; Winzer & Grigg, 1992; Ysseldyke & Algozzine, 1984)

During the 1950s and 1960s, most public schools in North America established special education programs or services for the mentally retarded, blind, deaf, emotionally disturbed, and physically handicapped. Teaching methods found to be effective with brain-injured individuals were transformed into special education techniques. Structured approaches to learning such as behaviour modification were also adapted to the special needs of disabled learners (Ysseldyke & Algozzine, 1984).

Special teachers were trained, special programs for the intellectually disabled were established and demonstrated and special classes were rapidly developed to serve the educational needs of disabled learners (Heward & Orlansky, 1988).

In 1978, the U. S. Office of Education passed Public Law 94-142 which guaranteed free public education in the least restrictive environment for all students needing special education due to their impairments. In short, the purpose of PL 94-142 was:

> to assure that all handicapped children have available to them ... a free, appropriate public education which emphasizes special education and related services to meet their unique needs, ... to assist states and localities to provide for the education of all handicapped children, and to assess and assure the effectiveness of efforts to educate handicapped children (Pelosi & Hocutt, 1977, p.3)

In the 1980s, similar policies were implemented in many Canadian Provinces (Andrews & Lupart, 1993; Goguen, 1989). Consequently, considerable attention has been placed on the development of a suitable curriculum for intellectually disabled children who have been mainstreamed into regular classes (Kauffman & Hallahan, 1993; Lewis & Doorlag, 1991; Salend, 1994; Turnbull & Shulz, 1979). In this context, one question that arises concerns the utility of adapting programs used with intellectually normal children to the needs of intellectually disabled students.

Comparing Intellectually Disabled Children with their Normal Counterparts

Hodapp, Burack, and Zigler (1990) report that researchers generally differentiate between two groups of intellectually disabled individuals: familial handicapped, and organically handicapped individuals. Intellectually disabled individuals with no known organic etiology generally have IQs in the mild range of retardation (IQ = 50-70) and have an appearance and socioeconomic background that is generally similar to that of individuals whose IQs fall in the low-normal range (IQ = 80-100). While most intellectually disabled children fall into the familial category, some children who are intellectually disabled are found to have recognizable organic damage (Hutt & Gibby, 1979; Robinson & Robinson, 1976; Schickendanz, Schickendanz, & Forsyth, 1982). For example, epiloia is associated with a dominant gene, phenylketonuria (PKU) is associated with a single recessive gene, Down Syndrome and Fragile X Syndrome are the result of chromosomal abnormalities. Some forms of mental retardation are associated with rubella in the mother, to intrauterine radiation or to lead poisoning; other children who are intellectually disabled have experienced cerebral trauma or have been exposed to other agents that may cause brain damage. Intellectually disabled individuals whose handicap can be associated with some type of recognizable organic damage often appear much different from their non intellectually disabled peers, and have IQs that are below 50.

There is an ongoing debate about whether familial handicapped and organically handicapped individuals follow the same learning patterns or whether they use qualitatively different learning strategies. According to Hodapp, Burack, and Zigler (1990), researchers can be divided into *difference*, *conservative*, and *liberal* camps in relation to this question.

Difference theorists argue that all intellectually disabled individuals, regardless of their etiology, cannot be studied using traditional developmental perspectives because they develop in qualitatively different ways than their intellectually normal peers (Wishart, 1988; Gelman & Cohen, 1988). These researchers suggest that it is useless to look at normal development when trying to determine an adequate intervention program for individuals who are intellectually disabled.

The second group of researchers, conservative developmental psychologists, believe that the familial intellectually disabled follow the same cognitive developmental patterns as do normal individuals except in a slower fashion. These researchers suggest that organically intellectually disabled individuals, such as those suffering from Down Syndrome or those handicapped because of brain damage, may have learning patterns that do not follow normal developmental rules. Consequently, most of their work focuses on similarities between familial intellectually disabled individuals and their intellectually normal peers rather than on organically intellectually disabled individuals.

The third group of researchers, the liberal developmentalists, believe that both familial and organically intellectually disabled individuals follow developmental patterns that are similar to those of their normal peers.

Empirical studies offer some support for the adoption of a conservative developmental model to understand the learning styles of intellectually disabled individuals. Familial intellectually disabled individuals are found to follow the same qualitative developmental stages as do non-handicapped individuals with respect to cognitive and *Piagetian* tasks e.g. seriation, conservation. Organically intellectually disabled individuals are found to follow the same qualitative developmental stages as do normal individuals on some cognitive tasks but not on Piagetian tasks. On Piagetian tasks, organically intellectually disabled individuals are found to function at qualitatively lower levels than their normal peers. (Hodapp, Burack, & Zigler, 1990)

Both the conservative and liberal developmentalists believe that the functioning of familial intellectually disabled individuals fits within the regular developmental formulation. The issues discussed above suggest that developmental knowledge about normal children is useful when trying to design intervention programs with familial intellectually disabled individuals. More specifically, it suggests that these children are likely to acquire learning strategies in the same sequence as do their intellectually normal peers. Thus, when developing interventions for intellectually disabled children whose handicap cannot be associated to some type of recognizable organic damage, it makes sense to take advantage of our knowledge regarding the development of intellectually normal children. However, knowledge concerning the development of intellectually normal children may not be useful when developing intervention programs for intellectually disabled children whose handicap can be associated to some unusual chromosomal patterns like those found in Down Syndrome children.

The Needs of Intellectually Disabled Children for Integration into Society

The successful integration of intellectually disabled individuals into society requires that they be able to care for themselves. This means that they are able to share and look after an apartment, purchase goods, cook, use public transportation facilities, develop intimate relationships, and deal with sexuality. The learning of some of these skills is facilitated by a basic knowledge of arithmetic. As Kirk and Gallagher (1983) point out, an elementary knowledge of the four arithmetic operations is necessary for such basic living skills as making purchases, keeping a budget, and knowing how to save money.

Addition is the simplest operation in arithmetic and serves as the basis for the other operations. A knowledge of addition is useful both within the confines of the classroom and in everyday life. A knowledge of addition contributes significantly to the development of autonomy and independence by school learners. Many of the problems that challenge a learner outside of the classroom may be dealt with independently and effectively if one has a clear understanding of addition. A knowledge of addition should broaden and enhance a learner's problem-solving ability.

Assuming that familial intellectually disabled individuals follow the same developmental stages as their normal counterparts, it may be beneficial to design an intervention program in addition for intellectually disabled children on the basis of our knowledge concerning the normal developmental patterns that children go through when learning addition.

The Strategies that Children Use to Solve Simple Addition Problems

A considerable number of researchers have focused their attention on the processes used by intellectually normal children to solve addition problems (Fuson, 1982; Gelman & Gallistel, 1978; Ginsburg 1986; Hughes, 1986; Kamii & DeClark, 1985; Resnick, 1983; Secada, Fuson, & Hall, 1983). Around 1970, researchers from various fields such as cognitive, language, and perceptual development discovered that memory consisted not only of reproductive processes but also of reconstructive processes (Groen & Parkman, 1972). A reproductive process consists of reproducing a fact by retrieving it from the long term memory store. A reconstructive process consists of reconstructing a fact on the basis of a rule that is retrieved from the long term memory store. At about the same time, researchers such as Suppes and Groen (1967) began to make the same distinctions with respect to the memory processes involved in solving arithmetic problems.

Carpenter and Moser (1984) reviewed and examined studies that investigated the specific strategies children use to solve simple addition problems. They reported that studies made over a period of 50 years consistently revealed that children used 5 strategies to solve simple addition problems. These 5 strategies consist of 3 reconstructive strategies, 1 reproductive strategy, and 1 reproductive/reconstructive strategy. The following is a brief description of the 5 addition strategies.

Reconstructive strategies. The first reconstructive strategy has been described as the *count-all* strategy. Here children usually solve the addition problem by directly modelling the problem with fingers or physical objects. Typically, the children use manipulative objects such as their fingers or blocks to represent each of the addenda in the problem and then they add these numbers by counting all the representative objects or fingers. For example, when asked to add 4+5, they may raise 4 fingers on the left hand and 5 on the right hand and then count all fingers starting with one finger and counting up until they reach the 9th finger.

The second reconstructive strategy is called the *count-on-from-first-addend* strategy. Here the children state the first addend in the problem and then count on from this first addend the number of units represented by the second addend to find the answer. For example, when adding 3+7 the children would say "3," then pause and say "4, 5, 6, 7, 8, 9, 10; the answer is 10". Most children do not require concrete references when using the count-on strategy.

The third reconstructive strategy is called the *count-on-from-larger-addend* strategy. Here children solve the problem much like children who count on from the first addend except that they are more strategic. Instead of counting on from the first addend, they start off by identifying the larger addend in the problem and count on from the larger addend in the problem regardless of its place in the problem. For example, when asked to add 3+7 the children would say "7", pause and then count on saying "8, 9, 10".

<u>The reproductive strategy</u>. This strategy involves *recalling the number facts from long-term memory*. Here children find the sum by recalling previously memorized number facts. This is done with no apparent counting but rather from memory by directly retrieving the stored number fact which has been learned as part of the addition table. For example, when asked to add 3 + 7, children would say "10" without apparently counting on their fingers, counting objects or using counting sequences. It is as though the children have recalled a stored association between the two digits and their sum.

<u>The reproductive/reconstructive strategy</u>. This strategy involves *deriving the*

addition sum by recalling number facts from long-term memory. Here children solve the addition problem by first recalling better-known number facts which have been previously learned and stored in memory as part of the addition table and second, by inferring the addition answer on the basis of these better-known number facts. For example, when asked to add 8 + 9, the child responds "I know that 8 + 8 is 16 and so 8 + 9 must be one more, that is, 17".

Different Approaches to Measuring Addition Strategies

There are several ways in which researchers have studied the strategies that children use to solve addition problems. Groen and Parkman (1972) based their analysis on children's reaction times to addition problems. Riley, Greeno, and Heller (1983) and Briars and Larkin (1984) developed computer models in an attempt to simulate the addition strategies used by children. Carpenter and Moser (1984) relied on interview data to determine the strategies employed by children when learning to add. These approaches are of interest in the present investigation.

Chronometric analysis of mean reaction times. In 1972, Groen and Parkman set out to analyze the long-term memory processes used by children while solving row addition problems of the form $m + n =$ __ with a sum less than or equal to 9. They chose to emphasize reconstructive processes because they found it easier to identify well-defined reconstructive models which predicted systematic differences in reaction time and because their observations of young children suggested that children learn to add by learning to count.

They started off by delineating all the possible counting models. They identified 5 models: counting-all, counting-on from the *leftmost* number, counting-on from the *rightmost* number, counting-on from the smaller number, counting-on from the larger number. Predictions for each counting model were made by considering a counter and two kinds of operations. The hypothesized procedure was as follows: (a) The child sets the counter to an appropriate number, and (b) successively increments the set counter by 1 a suitable number of times.

The following is a description of their 5 counting models.

1. Counting-all: The counter is set to 0 and both numbers are added by increments of one.

2. Counting-on from the leftmost number: the counter is set to the leftmost number and the rightmost number is added by increments of one.

3. Counting-on from the last number: The counter is set to the rightmost number and the leftmost number is added by increments of one.

4. Counting-on from the smallest number: The counter is set to the smaller number and the larger number is added by increments of one.

5. Counting-on from the larger number: The counter is set to the larger number and the smaller number is added by increments of one.

Assuming that these models accurately described the reconstructive processes involved in simple addition, it followed that the mean time required to solve an addition problem was determined by the mean time needed to increase it by one an appropriate number of times (see Figure 2.1).

Figure 2.1. Formula for the mean time required to solve an addition problem.

$t = a + bx$ where

t: mean time to solve a problem

a: mean time to set the counter

b: mean time required to increase the counter by one

x: number of times the counter was increased

A second assumption adopted by Groen and Parkman (1972) was that a and b were constant over problems. In other words, they assumed that "t" varied only as a function of x i.e. $t= f(x)$. By fitting a regression line to the data, they could infer x and so infer which counting model had most probably been used to solve a given addition problem. Subjects in the Groen and Parkman experiment (1972) made their responses by pressing on numbers from 0 to 9 on a response panel. The time required to press a number was also assumed to be constant across problems and across subjects. Correct answers were displayed between problems to encourage the development of efficient counting strategies.

After analyzing the mean reaction time that young children took when solving the problems, Groen and Parkman (1972) concluded that the best explanation for young children's reaction time for solving simple addition problems was a counting-on from larger number strategy. However, their sample of subjects was made up of bright students with an average Binet IQ of 125. As well, these children may have been quite advanced in addition at the time they were studied, since the experiment was conducted at the end of the first grade.

A second chronometric analysis of solution times for simple addition also found that children learn simple addition by first using a counting-on procedure (Svenson, 1975). Svenson basically replicated Groen's study and also found that, for the most part, children were using a count-on strategy. However, as these children were between the ages of 10 and 11, they may also have been quite advanced in addition at the time they were studied.

In 1977, Groen and Resnick repeated Groen and Parkman's 1972 study, but chose a sample of children who had not yet learned addition. From this experiment, they concluded that children started with a counting-all strategy but quickly moved to a counting-on strategy.

Cognitive models using computer programs. Because computer programs can model the addition strategies used by young children when solving simple addition problems, some cognitive psychologists opted to write computer models to enhance their understanding of what children might need to solve simple addition problems. Riley et al. (1983) and Briars and Larkin (1984) designed computer simulations to model different levels of skilled performance in solving word addition problems. Both groups of researchers started by identifying all the possible semantically different types of addition word problems. They identified two types of semantic addition problems: (a) join addition and (b) combine addition problems. Both types of problems can be represented in the form a + b = _ and are presented in Table 2.1.

Table 2.1

Two Types of Semantic Addition Problems.

Join addition problem	You have 4 apples. I give you 6 more apples. How many apples do you have altogether?
Combine addition problem	Charles has 3 peanut butter candy bars. He also has 4 caramel candy bars. How many candy bars does Charles have altogether?

Both models make the following three predictions:

1. There are three levels of skill for solving addition problems. Stage 1 problem solvers rely solely on external representations of the addition problem in order to solve addition problems. Furthermore, they are incapable of considering the part to whole relationship which requires an understanding that the whole is made up of the union of the parts and is greater than the parts. They can perform simple addition only by using a counting-all strategy and by using models such as fingers, blocks, tallies, or pictures to represent and add the addenda in the problem.

Stage 2 problem solvers have a schema that permits them to keep track of the part to whole relationship. In other words, they are able to recognize that an addend is a subset of the sum. According to both groups of researchers, this allows level 2 problem solvers to solve addition problems by counting-on from the first addend, but not by counting-on from the larger addend. The idea is that level 2 problem solvers are incapable of counting-on from the larger addend because they are still incapable of re-representing the problem before attempting to solve it.

Stage 3 problem solvers have a schema that permits them to solve addition problems through a re-representation of the problem. They are capable of constructing the relationships among all the pieces of information in the problem instead of solving the problem by directly representing the action in the problem. This allows level 3 problem solvers to solve addition problems by using a counting-on from the larger addend strategy.

2. Problem solvers will consistently use addition strategies that are available to

them.

3. Among alternative strategies available, they will select the counting strategy that requires the fewest counting steps or that avoids more difficult counting procedures. For instance, a child who knows how to use the count-on and count-all strategies will select a count-on strategy because it involves fewer counting steps.

The data from Carpenter and Moser's (1984) three year longitudinal study provide the most complete empirical test available of these three predictions made about children's performance in simple addition by Riley et al. (1983), and Briars and Larkin (1984). The study by Carpenter and Moser (1984) will be summarized and discussed in terms of the predictions made by these two models.

Direct observation or interviewing procedures. Carpenter and Moser (1984) designed a three-year longitudinal study to determine if their sample of regular classroom children would use the five addition strategies reported in the research on addition over the past 50 years (count-all, count-on-from-larger-addend, count-on-from-first-addend, number facts, derived number facts) to solve simple addition word problems. As well, they were interested in the sequence children would use to acquire the strategies. Interviewers were trained to collect data using coding sheets and clear criteria for each response category. Responses involving modelling with physical objects could usually be classified reliably solely through observation while other responses were classified based on additional probing by the interviewers. With intra-coder and inter-coder reliability coefficients greater than 0.90, they found that children between grade 1 and 3 did use this well-defined set of strategies and acquired these strategies in a specific sequence. Carpenter and Moser (1984) also found that once these strategies were available to children, they used them interchangeably. Children were more likely, however, to use strategies involving counting sequences rather than those involving manipulative materials for large number problems when no physical objects were available.

Overall, they found that for addition problems students learned to use the count-all strategy before the count-on strategy but, after they had developed these two strategies, they used them interchangeably regardless of number size or availability of manipulative objects. All students eventually used the count-on strategy but they did not seem to differentiate between count-on-from-first-addend and count-on-from-larger-addend strategies. The researchers also found that students' ability to use number facts when solving addition problems developed gradually between grades one and three. While only 45% of second graders used number facts with sums less than 10, by the middle of third grade 90% were mastering number facts with sums lower than 10 and 70% were mastering number facts with sums greater than 10. Finally, they also found that 80% of students between grade 1 and 3 used derived facts at least once to solve addition or subtraction word problems.

The Development of Addition Strategies among Intellectually Normal Children

A tentative conclusion that can be made about the developmental strategies used by intellectually normal children is that Carpenter and Moser (1984) have the best description of the stages that children go through when learning simple addition. What is most important in their results is that intellectually normal children appear to use an overt modelling strategy, count-all, at the beginning. They gradually develop more covert strategies such as count-on-from-first-addend and count-on-from-larger-addend. Finally, they use strategies based on recalled number facts.

Not all of the assumptions about children's performance underlying the cognitive models of Briars and Larkin (1984) and Riley et al. (1983) are supported by the empirical data of Carpenter and Moser (1984). It would appear that problem-solvers will consistently use the strategies that are available to them and that they use them interchangeably according to the Carpenter and Moser (1984) data. As predicted by both models, children initially do rely on external representations of the addition problem in order to solve addition problems. At first, they do seem to perform simple addition only by using a count-all strategy based on models such as fingers, blocks and pictures to represent and add the addend in the problem. In a second stage, however, children seem to be ready to learn to perform simple addition by using the count-on-from-first-addend as well as the count-on-from-larger-addend strategies.

The Development of Addition Strategies among Intellectually Disabled Children

There is little research available on how intellectually disabled children learn addition. Baroody (1988) has pointed out that a current belief is that intellectually disabled children learn addition by rote whereas intellectually normal children learn addition by discovering meaningful relationships. However, several recent studies suggest that intellectually disabled children, regardless of etiology, may use the same addition strategies and develop them in the same sequence as do their intellectually normal peers. Irwin (1991) indicated that Down Syndrome children who could add numbers using a count-all strategy were able to learn a count-on strategy. By using direct observation and interviewing procedures, Hanrahan et al. (1993) found that some intellectually disabled children were adding by using a count-all strategy, a count-on strategy and/or by committing number facts to memory. Also, in general, the sequence of acquisition of these strategies appeared to be the same as it is for intellectually normal learners: (a) count-all, (b) count-on, and (c) fact memorization. However, Hanrahan et al. (1993) also observed that many of these children relied on the count-all strategy to add for a long period of time. Similarly, Kirk and Gallagher (1983) concluded that, for intellectually disabled learners, the more advanced addition strategies take much longer to develop and are not as certain.

Problems that Arise for Intellectually Disabled Individuals when Adding

There are at least 4 important problems that arise for those who rely solely on a count-all strategy using physical objects such as fingers, blocks, or tallies to perform simple addition.

1. Students who use this strategy often use their 10 fingers as objects to count and this confines them to adding numbers with sums lower than or equal to 10.

2. Students will often use objects such as blocks, pennies, or buttons to represent each addend in the problem and count the group of representative objects starting from 1. This method is efficient with sums beyond 10 but is impractical since the child has to carry a bag of pennies to every mathematics class.

3. Older children may be too embarrassed to count fingers or objects in front

of others and this may hinder the children from practising addition problems. By the time they reach fourth grade, 70% of their intellectually normal counterparts appear to have memorized addition problems with sums greater than 10. Intellectually disabled children who are still relying on modelling strategies to add will significantly stick out and may have difficulty integrating into the mainstream.

4. Students can use tallies to represent the addenda. Here, children draw tallies on the work sheet to represent each addend and count the representative tallies starting from 1. This method is also efficient for sums greater than 10 but it may take students a considerable amount of time to draw the tallies and count them starting from 1. Using tallies also can take a significant amount of space on the students' work sheets and be cumbersome. Using tallies, students may often need an additional blank sheet of paper to draw the tallies.

To progress in mathematics, these children must be able to overcome these limitations. Intellectually normal children and some intellectually disabled students overcome these problems by developing the more advanced covert addition strategies such as count-on or memorizing the number facts (Hanrahan et al., 1993). It is possible, however, that these strategies involve cognitive abilities that are lacking or take a very long time to develop in most intellectually disabled children. A better solution may be to find an approach to addition which permits these children to add numbers with sums larger than 10 by using an overt count-all strategy inconspicuously.

A Possible Solution to the Addition Problems of Intellectually Disabled Children: The Touch Math Approach

The count-all strategy is well within the cognitive abilities of many intellectually disabled students. Consequently, instead of trying to teach intellectually disabled children the more advanced covert addition strategies such as count-on or recalling the number facts from memory, it may be more appropriate to find an approach to addition which permits these children to add numbers with sums larger than 10 by using a count-all strategy discreetly.

One such approach to addition may be found in the *Touch Math Addition Kit* which was developed by Bullock (1989, 1991a, 1991b, 1991c) and has the trademark of Innovative Learning Concepts, Inc.. It is a *multisensory*, teacher-friendly approach to addition, subtraction, division, and multiplication that aims to address the needs of children in both special education and regular classrooms. The program includes five kits that are made to be taught sequentially, each one providing the teacher and student with a complete math resource and extensive fact mastery activities for a designated math skill. The five kits relate to number concepts, addition, subtraction, sequence counting, multiplication and division.

The *Touch Math Number Concepts Kit* (Bullock, 1991b) is presented before the Touch Math Addition Kit (Bullock, 1991a) as the author points out that for students to develop a strong addition foundation, they must be comfortable with Touch Math numbers and possess strong oral and written counting skills before they begin addition. The Number Concepts Kit provides 175 activity master and teacher aid work sheets that extensively cover (a) counting, (b) number recognition, (c) symbol value, (d) beginning place value, and (e) beginning addition using touch points.

The Touch Math Addition Kit (Bullock, 1991a) is divided into three addition skill levels with respect to single-digit addition. The first skill level of Touch Math addition is called *Beginning Addition* and it involves adding columns of single-digit numbers using a count-all strategy. Students are taught to solve addition problems by touching and counting the touch points that are drawn on the addenda. As discussed above, research on the development of addition strategies suggests that this is the addition strategy that children are able to use in the early stages of development (Briars & Larkin, 1984; Carpenter & Moser, 1984; Groen & Resnick, 1977; Riley et al., 1983). As long as first graders can count touch points in the correct pattern and write double digit numbers, the author estimates that students will master Beginning Addition in about one month (Bullock, 1991c).

The second level skill of Touch Math addition is called *Addition with Continuance Counting* and it covers counting columns of numbers using a count-on procedure. As discussed above, research on the development of addition strategies suggests that this is the second addition strategy that children develop as they progress in addition (Briars & Larkin, 1984; Carpenter & Moser, 1984; Groen & Resnick, 1977; Riley et al., 1983). Students are asked to touch the larger number, say its name, cross it out, and continue counting onto the touch points of the other addend. To make certain that the children memorize these directions, they are taught an addition statement: "I touch the largest number, say its name, and continue counting".

Finally the touch points are faded from all the addenda and the students are taught to count-on from the larger addend without any visible touch points. It is expected that the student will begin to memorize the addition facts after reaching level three. Table 2.2 provides an example of a problem for each of the three simple addition skill levels in Touch Math.

Table 2.2

An Example of a Touch Math Addition Problem at Each Skill Level in Simple Addition.

Beginning addition	Addition with continuance counting	Fact practice
$\begin{array}{r} 8 \\ +5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +5 \\ \hline \end{array}$

The only research on the applicability of the approach to intellectually disabled students has been published by Scott (1993a, 1993b). Working individually with a small number of children with learning and intellectually disabilities, she found that the Touch Math method was suitable for instruction in addition and subtraction. While collecting data for their longitudinal study on the addition strategies used by intellectually disabled children Hanrahan et al. (1993) observed several attempts by special education teachers to apply the Touch Math approach to basic addition. Although these attempts appeared to have limited success and the teachers suggested that progress was slow, one child included in the Hanrahan et al. (1993) sample was making steady progress using the method. Consequently, it was felt that, if the approach were carefully analyzed before individualized instruction began, a teacher might achieve results similar to those reported by Scott (1993a, 1993b).

Research Question

It would seem that the Touch Math approach may be particularly appropriate for children who are intellectually disabled.

First, the Touch Math approach allows them to follow the developmental stages that intellectually normal children go through when learning simple addition. Initially children are taught to count-all, then to count-on-from-larger-addend and gradually to memorize the addition facts.

Second, this approach may be valuable for intellectually disabled children because many of them rely on a count-all strategy using concrete objects such as fingers (Hanrahan et al., 1993). Touch Math provides these students with a way of easily concretizing numbers; it teaches them to treat numbers as though they were marked with touch points.

Third, Touch Math may be useful for these children because the count-all strategy using fingers confines them to adding numbers with sums no greater than 10. Touch Math permits them to add numbers with sums as high as they can count.

Fourth, with Touch Math, the use of the counting-all strategy becomes more discreet. Instead of counting on their fingers the children tap their pencils on the faded touch points on the numerals. This would make their counting behaviour more inconspicuous and facilitate their integration with non-intellectually disabled children.

The approach also seems advantageous as the authors claim that Touch Math reinforces number values, eliminates guessing and reduces arithmetic errors dramatically. Any student who arrives at incorrect sums will either be counting or touching incorrectly. Such problems should be easy to identify and correct (Bullock, 1991c).

Another value of this approach is that it involves the simultaneous use of the visual, kinaesthetic, and auditory channels and consequently increases the chances of learning addition. For instance, when adding, the student is required to look at the dots on the problems, touch the dots on the problems, and after writing the answer, to hear himself repeat the problems and answers orally (Bullock, 1991c).

Finally, this approach is logically developed up to the four arithmetic operations and provides these students with a comprehensive approach to arithmetic.

Given the lack of research on the utility of the Touch Math approach with intellectually disabled children, it was decided to limit the present study to simple addition problems, specifically addition problems involving pairs of addenda with each addend being limited to a single digit value. Children would be taught to add sums up to a maximum value of 18 using the Touch Math approach. This objective, although modest, would provide a useful test of the utility of the Touch Math approach with intellectually disabled children.

Chapter 3

Method

Design

A multiple probe design across subjects was employed to assess the efficacy of the Touch Math approach to teach intellectually disabled students to perform simple addition. The multiple probe design across subjects is a variation of the multiple baseline design across subjects. Both designs are effective in establishing a cause-and-effect relationship between an instructional program and subjects' performance (Horner & Bayer, 1978; Kazdin, 1992; Tawney & Gast, 1984). In both designs, the instructional program is systematically and sequentially applied to one subject at a time. However, the multiple baseline design requires the examiner to collect data on all subjects simultaneously and non-stop throughout the whole study. The multiple probe design does not require the examiner to continuously test subjects who have not yet been introduced to the instructional program. Being tested continuously before instruction may have been boring and frustrating for all but the first subject in the experiment. In order to prevent such negative effects, it was decided to employ the multiple probe rather than the multiple baseline design in the present experiment.

Subjects

Criteria for the selection of subjects. The goal of the present experiment was to use the Touch Math addition approach in order to enhance the addition performance of intellectually disabled students who relied on a count-all strategy using objects to add. The Touch Math addition program has been designed for students who already have developed three specific number skills. In the first place, some of the simple addition problems in the Touch Math Addition Kit have multiple addenda and large sums and so students are expected to be able to count as high as 50 objects (Bullock, 1991a). However, in the present study, students only had to show that they were able to count up to 18 objects since the addition problems always had two addenda (1 to 9) with sums up to 18.

As well, the Touch Math addition program expects students to be able to write the sums for the addition problems. Consequently, before starting the Touch Math addition program students must be able to write numbers 1 to 50 (Bullock, 1991a). However, for the present study sums were no larger than 18. Consequently, students had to demonstrate that they were able to write numbers up to 18.

Finally, the Touch Math method requires students to solve addition problems by counting-on. Hence, it was decided that students had to demonstrate the ability to continue the count from numbers 1 to 9 before entering the study.

In summary, candidates for the present experiment had to demonstrate that they were able to count and write numerals 1 to 18 and to continue the count from numbers up to and including 9. They also had to demonstrate an ability to add with the use of concrete references and an inability to add numbers without the use of concrete references or with the use of touch points.

Screening instruments and procedures for subject selection. Five intellectually disabled students were identified as candidates for the present study. All 5 students attended a special school for children with learning problems located in the greater

Montreal area. The children's counting ability was assessed by requesting that each child count a set of 20 pennies. Their number writing skills were measured by requiring them to write the numbers 1 to 18 on a blank sheet of paper. Each student's ability to continue the count was determined by asking the student to count to the next number for numerals 1 to 9. Finally, addition skills were assessed through the use of a computerized addition test composed of 20 two-addend addition problems with sums no greater than 9. The 20 addition problems are listed in Appendix A. During this addition test, students were given a set of 10 blocks, a pencil and a blank sheet of paper and were instructed to add the numbers that appeared on the computer monitor using any method they wished.

Subjects selected. Four of the five students screened were judged suitable for the study. The fifth candidate indicated that she was already skilled at addition - she could solve the addition problems described in the computerized test without using physical models. The remaining four students were accepted for the study because they possessed the requisite number and addition skills described above and also demonstrated an inability to add without using physical models such as blocks or fingers or by using touch points.

As the multiple probe design requires a sample of 3 subjects, only the 3 students with the most advanced skills as determined by the screening procedures received the intervention program in addition. Each of these students attended a different mathematics class. Thus, the three were unlikely to learn about the Touch Math addition approach from one another prior to their introduction to the intervention program.

Each of the 3 subjects knew how to add using tallies, fingers or blocks but could not add without such external models. They are described as follows:

Subject A was a 12-year-old East-Indian girl who was cooperative and friendly and acted more independently than most of her classmates. She was an attractive child, neatly dressed and popular with her peers. Her most apparent weaknesses were in the areas of speech and in fine motor manipulation skills. Probably as a consequence of her speech impediment, she was very quiet and spoke mcstly with facial expressions and body gestures. On the *Leiter International Performance Scale*, she obtained an IQ score of 57 in 1992. The Leiter scale is a standardized test that was developed for use with children who have motor and physical dysfunctions or who are nonvocal because of a physical handicap. Subject A was sometimes able to solve simple addition problems with sums no greater than 10 by using her fingers to represent each addend and then counting the representative fingers from 1. However, for sums greater than 10 she needed to use objects such as pennies to represent each addend and count the representative objects from 1. Although this method was efficient, it was impractical because she needed to carry a bag of blocks or pennies with her to every mathematics class.

Subject B was a handsome 11-year-old boy. He was usually in good humour and tried hard to succeed in school. When he was upset, he always talked about it with his teachers in a direct and open manner. He tended to get excited and was awkward and clumsy. Subject B obtained a full WISC-R IQ score of 55. Subject B was able to add single-digit numbers by marking tallies on his work sheet to represent each addend and counting the representative tallies from 1. This method was efficient but it would take the student a considerable amount of time to draw and count the tallies. For sums greater than 10 especially, the student needed a significant amount of space on the work sheet to draw the tallies. Overall, the method was cumbersome. An example of the space required by subject B when adding using tallies is presented in Appendix B.

Subject C was a small 12-year-old girl who, while friendly with the examiner, showed signs of severe emotional/behavioral problems and was closely monitored by the school psychologist at all times. Subject C was often taken out of class because she was not adapting to group situations. It was decided that she would need much positive reinforcement to stay interested in the program. Fortunately, most of the time she was very cooperative with the examiner and always seemed happy to see her. Her strength was that she was able to work quickly and neatly when she was in a good emotional state. Subject C obtained a full WISC-R score of 42 in 1993. She was able to solve addition problems as long as the examiner helped her draw the correct amount of tallies to represent each addend. She would proceed to count the representative tallies from 1 on her own. As with subject B, using tallies to represent and count the addenda resulted in correct answers but required a considerable amount of time and space on the work sheets.

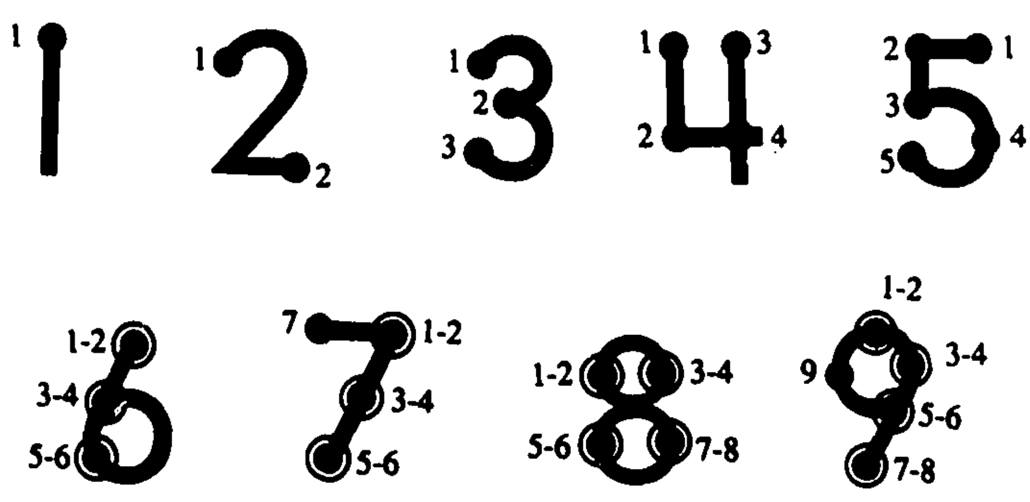
<u>Setting</u>

The 3 subjects who participated in this study all came from the same special school for children with learning problems situated in the greater Montreal area. The study was conducted 4 days a week at the subjects' school in the teachers' lounge during the students mathematics period between 10:40 am and 11:20 am.

Procedures

The instructional program. The Touch Math addition approach (Bullock, 1991a) is based primarily on a system of touch point configurations that are strategically placed on numerals 1 to 9. During the first days, the student needs to learn to touch and count the touch points on numerals 1 to 9 following the correct pattern as indicated in Figure 3.1 below.

Figure 3.1. Touch Math's touching and counting patterns for numbers 1 to 9.



During these days, three paper-and-pencil activities were used at each session to help the student master the touch point configurations: (a) modelling the examiner while she was touching and counting the touch points in the correct patterns for numerals 1 to 9, (b) drawing the touch points on numerals 1 to 9, and (c) writing numerals 1 to 9 with their respective touch points. An illustration of the three activities is presented in Appendix C. Corrective feedback and praise were given during these activities. At the end of each touch point session, the students were presented with a Touch Math number line (Appendix D) and asked to touch and count the touch points following the correct patterns without assistance. Students had to demonstrate 100% mastery of touch points on numerals 1 to 9 on 3 consecutive trials to progress to the addition work sheets.

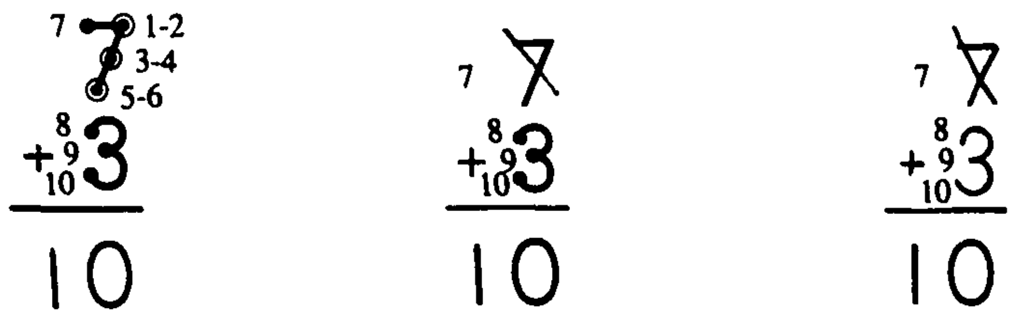
The addition program in the present study was conducted using a set of 14 addition work sheets which are presented in Appendix E. They were designed by using Touch Math Addition Kit work sheets 12A to 38A (Bullock, 1991a). Previous research suggested that students at mild levels of intellectual handicap were able to learn to add using touch points within four to six days (Scott, 1993a). The examiner wanted to give the students the opportunity to go as quickly as Scott's students but also wanted them to complete as many of the Touch Math addition work sheets as possible and in the order prescribed by the Touch Math addition kit. Consequently, Touch Math addition work sheets were presented to the students in the order prescribed by the Touch Math program. However, each work sheet used in this study combined half of two consecutive sheets. Also, the student was presented with one work sheet per session. For instance, for the first session the student was presented with a work sheet made up of the top half of the first Touch Math addition work sheet (12A) and the bottom half of the second Touch Math addition work sheet (13A). If the students obtained the criteria of 85% correct on this combination of problems, during the next session they would be presented with a work sheet made up of the bottom half of work sheet 14A and the top half of work sheet 15A. If the students did not reach criteria on a work sheet, they were presented with the same work sheet during the next instructional session until they reached criteria. The entire program was made up of half of the problems on each of the following three series of work sheets: 12A-19A with touch points on all addenda, 29A-38A with touch points on the smaller addend in each problem, 29A-38A without any touch points on the addenda. The program ended once the student reached criteria on 2 consecutive days with work sheets 29A-38A without touch points on the numerals. Following this procedure, a student did not have to complete the third series of work sheets if she reached criteria before she completed all work sheets. In other words, the program ended when the student reached criteria on any 2 consecutive work sheets in the final series.

The addition program was divided into three sequential steps. During step 1 the subjects were administered work sheets 1 to 4 (half of 12A-19A). Work sheets 1 to 4 each contained 20 addition problems with touch points present on all addenda. The subjects had to learn to add by touching and counting the touch points present on the addenda. An example of a step 1 problem is presented in Table 3.1.

Table 3.1

Touching and Counting Strategy Taught at Each Step in the Addition Program.

Step 1	Step 2	Step 3



During step 2, the subjects were administered work sheets 5 to 9 (half of 29A-38A). Here, touch points were faded from the larger addend in each problem and they had to learn to add by touching and counting the touch points on the smaller addend, starting from the larger addend (see Table 3.1).

Finally, during step 3, the subjects were administered work sheets 10 to 15 (half of 29A-38A with faded touch points). Here, touch points were faded from both the smaller and larger addend in each problem and the students had to learn to add by touching and counting the touch points on the smaller addend, starting from the larger addend. Since the touch points were now faded from both addenda, the students had to learn to touch and count the touch points on the smaller addenda from memory (see Table 3.1).

During steps 1, 2 and 3, the students began each session by practising the touch point configurations on numerals 1 to 9 and the 20 addition problems with the assistance of the examiner. Here, instruction was provided by modelling and providing corrective feedback to the student when necessary. Once practice was over the students were required to solve the same 20 addition problems without assistance. Each instructional session lasted from 20 to 40 minutes until the subjects (1) practised the touch point configurations with the examiner, (2) practised the work sheet with the examiner and (3) completed it afterwards on their own.

Coding during instructional conditions. To reach criteria during each instructional session, the subject had to obtain the correct sum using touch points at least 85% of the time without assistance. While the student was solving a problem, the examiner would identify the processes used to solve a problem and record the information on a specially designed coding sheet (Appendix F). If the student solved a problem by counting-all or counting-on with fingers or touch points, the subject's addition behaviour usually gave a clear indication of the model and strategy used, and consequently no questions were asked, and the student was encouraged to solve the next problem. If the subject solved a problem without using fingers or touch points, and the subject's behaviour gave no clear indication of the strategy and/or model used, the examiner would question the student using Martin and Moser's (1980) general questioning technique (Appendix G). From this information, the examiner was able to compute the percentage of addition problems solved correctly using touch points.

The probe instrument. The objective of the present study was to determine if a sample of 3 intellectually disabled subjects was able to learn how to add numbers 1 to 9 using the Touch Math method. In order to monitor their progress before and after instruction, it was necessary to design probe instruments.

With respect to numbers 1 to 9, there are 81 possible combinations of number pairs with sums no greater than 18. A subset of 21 number pairs was retained for the probe instrument by employing the procedures used by Carpenter and Moser (1984) in their longitudinal study of addition. As such, the following types of number pairs were eliminated: (a) Doubles like 8+8 were eliminated, (b) pairs of consecutive addenda like 6+7 and with sums no greater than 10, (c) number pairs with sums of 10, and (d) addenda of 0 or 1.

The 21 number pairs were arranged on the probe instrument in the following way:

1. The number pairs were ordered randomly.
2. The first 10 number pairs were presented using a horizontal format with 5 of the number pairs having the smallest addend first.
3. The remaining 11 number pairs were presented using a vertical format so that 5 of the number pairs had the smallest addend first.

Both the horizontal and vertical formats were used as addition problems are often presented both ways in mathematics textbooks. A new probe instrument employing the 21 number pairs was designed before each probe session following the procedures described above. An example of a probe instrument that was used in the present study is presented in Appendix H.

The probe conditions. Five probe conditions were conducted to monitor the three students' progress throughout the study.

Probe 1 was conducted at the beginning of the study. The probe test was administered simultaneously to the three subjects to obtain three baseline data series. As soon as the three series of data exhibited stability in level and trend, the first subject was introduced to the intervention program.

Probe 2 was done immediately after subject A mastered the instructional program. The probe test was re-administered simultaneously to the three subjects. When all three data series demonstrated a stable trend and level, the program was applied to subject B.

During probe 3, immediately after subject B mastered the program, the probe test was re-administered simultaneously to the three subjects until a stable trend was established for all three subjects. Then, the intervention was introduced to subject C.

Probe 4 was done after subject C attained criterion-level performance using the Touch Math approach. Again, the probe test was reapplied to each subject simultaneously.

Probe 5 served as a maintenance probe. One month after subject C mastered the program, the probe test was re-administered simultaneously to the three subjects to evaluate the program's long term effect.

The probe sessions. At the beginning of each probe session, students were provided with a pencil or pen, an eraser, and a pencil sharpener and asked to solve the 21 probe problems without any assistance from the examiner.

Coding during probe sessions. While the student was solving the 21 problems without assistance, the examiner recorded the processes used to solve a problem following the same procedures used during the instructional condition. From this information, the examiner was able to compute the percentage of addition problems solved correctly using touch points.

Inter-rater reliability. At least twice for each subject, a second observer independently recorded the strategies used by the subject to solve problems using the coding sheet. From this information, the examiner was able to compute the average inter-observer agreement percentage concerning the percentage of problems solved correctly using touch points. Here the point-by-point method recommended by Tawney and Gast (1984) was used. As such, the formula used was the following:

$$\text{percentage of agreement} = \frac{\text{agreements}}{\text{agreements} + \text{disagreements}} \times 100$$

On average, the inter-rater reliability percentage scores for subjects A, B, and C were 97.5%, 100%, and 100% respectively.

Reinforcements. During each instructional session, students earned a sticker if they practised 100% of the addition problems with the assistance of the examiner.

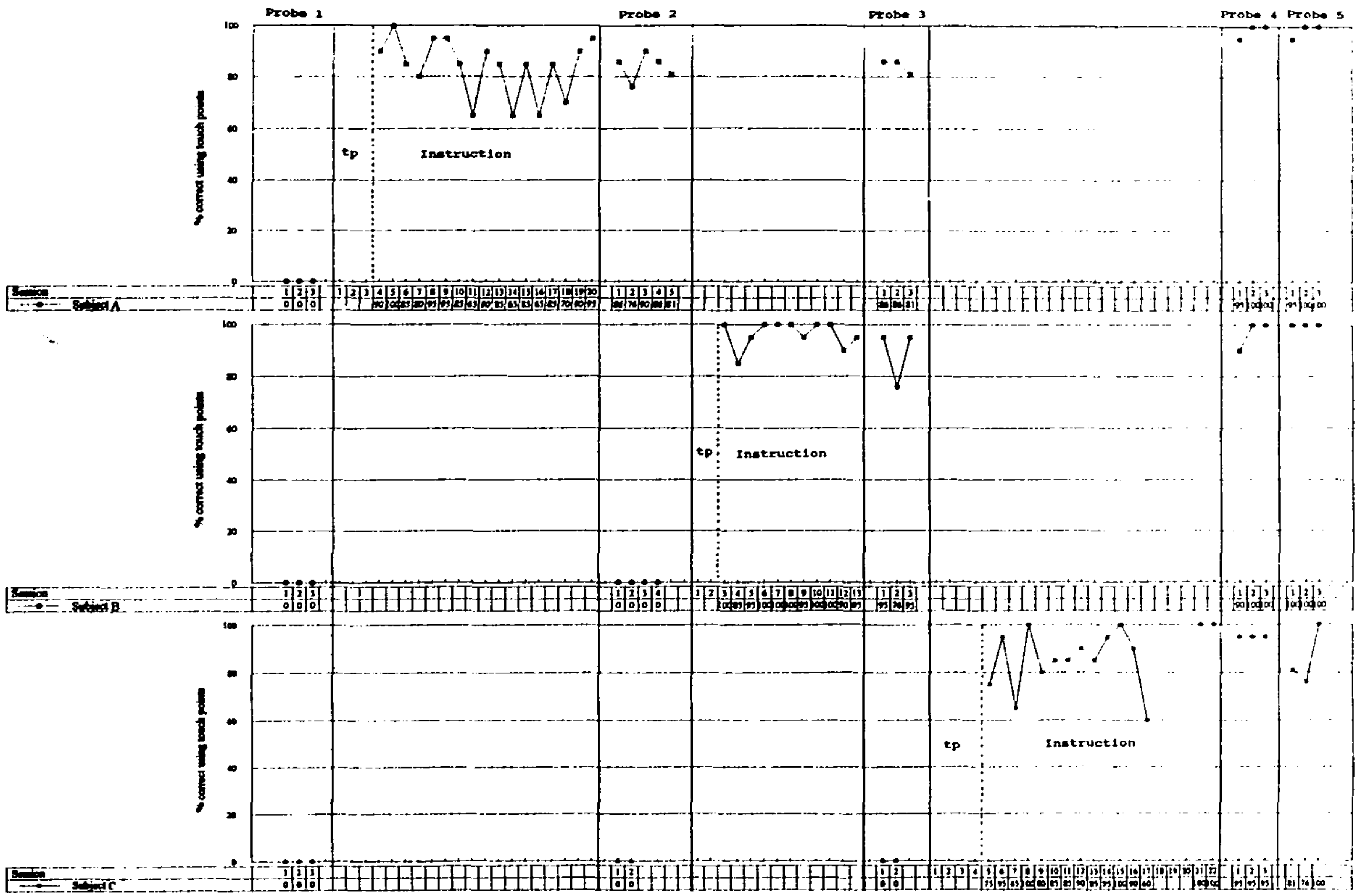
Also, at the end of each instructional and probe session, the students were entitled to a sticker if they attempted to solve 100% of the problems without assistance. Students would usually place their stickers in a sticker book provided to them on the first day by the experimenter. At the end of each instructional and probe session, a prize such as a pencil, a pen, a pin, or a small note pad was given to the students if they obtained 85% or more correct without assistance.

Chapter 4

Results

The dependent measure in the present study consisted of the percentage of addition problems solved correctly using touch points. These data were collected individually across 5 probe conditions for each subject and at the end of each instructional session. During data collection, the examiner offered no assistance to the subjects. The graph (see Figure 4.1) demonstrates the effect of the program on the students' ability to add single-digit numerals using touch points. As can be seen in the graph, the 3 students were able to master the program within 13 to 22 days of one-to-one instruction. Before instruction, the 3 students did not know how to add using touch points and each one of the 3 students solved an average of 0% problems using touch points. After all 3 subjects had been instructed, during the 4th probe, these same students obtained averages of 98.3%, 96.6%, and 95% using touch points to solve the addition problems.

Insert Figure 4.1 about here



tp: touch point training

Figure 4.1. Percentage of addition problems solved correctly using touch points.

During instruction, subject A was able to master the touch point configurations on numerals 1-9 within 3 days and learned to add by counting and touching faded touch points within 17 days. Probes 2, 3 and 4 demonstrated that she was able to remember the method as she obtained average scores of 83.8%, 84.3%, and 98.3% respectively on these probes. On probe 5, administered five and a half months after she completed the program, she was able to solve on average 98.3% problems correctly using touch points.

Subject B was able to master the program in the shortest time period, 13 days. He mastered the touch point configurations on numerals 1 to 9 within 2 days and learned to add by counting and touching faded touch points within 11 days. Probes 3 and 4 demonstrated that he was able to perform up to criteria on the probe instruments. Subject B obtained averages of 88.6% and 96.6% correct responses respectively on probes 3 and 4. Probe 5 indicated that he was able to retain the touch point method up to 3.5 months after completing the program. He solved 100% of the problems correctly during Probe 5 using touch points.

Subject C took 4 days to master the touch point configurations on numerals 1 to 9 and mastered addition by counting and touching faded touch points within 18 days. As demonstrated in Probes 4 and 5 on the graph, she was able to add using touch points on the probe instrument and to retain the method up to 1 month and one week after she completed the program. On Probe 4, on average she solved 95% of the problems correctly using touch points. For Probe 5, she was given the probe test on 3 different days one month after completing the program and obtained an average of 85% using touch points. As recommended by Tawney and Gast (1984) subject C was administered only 2 probe trials in Probes 2 and 3 in order to avoid frustration on her part. As well, during instructional sessions 18, 19 and 20, subject C was required to practice touching and counting the touch points. No addition problems were attempted during these sessions.

Chapter 5

Discussion

Student Success Using the Touch Math Method

The results of the present experiment are encouraging. The study suggests that following less than one month of instruction, moderately intellectually disabled students were able to use the faded touch points to add pairs of single-digit numbers and to retain the method from 1 to 5.5 months following instruction.

The results are also encouraging because, in addition to mastering and retaining the touch point approach to addition, each of the 3 subjects was able to generalize the approach across problem formats. The set of 14 work sheets used for this study always presented the addition problems 9+2, 5+2, 2+6, 5+9, 4+9, 4+2, and 8+6 in a vertical format and the addition problems 3+5, and 2+3 in a horizontal format. Furthermore, the addition problem 8+3 was never present on the instructional work sheets. In other words, the 3 students never had the opportunity to practice the horizontal problems 9+2, 5+2, 2+6, 5+9, 4+9, 4+2, and 8+6, the vertical problems 3+5, and 2+3, or the problem 8+3 with the experimenter. However, subjects were required to solve these problems on the probe instruments. After completing the program, subjects B and C were able to solve 100% of these problems using the Touch Math method correctly. Subject A repeatedly made mistakes solving the horizontal problem 5+9, but this was not because she had difficulty generalizing the method but rather because she tended to confuse the number 9 with the number 10 and because she tended to have difficulty counting the touch points on addend 5.

After completing the program, subjects B and C were often absent from their mathematics classes for personal reasons. Consequently, subject A provided the only true test of generalization to the classroom situation. With the assistance of her teacher, subject A was not only able to master and retain the method but was also able to use the Touch Math addition method to solve all the addition problems in her mathematics textbook. Moreover, because subject A had mastered addition so well, her mathematics teacher decided to use the touch point method to help her master subtraction. Subject A quickly learned to subtract using the method with the assistance of her teacher. Subject A was proud of her newly acquired skills in addition and subtraction. The speedy accomplishments of this teacher with subject A suggest that it may be fruitful for researchers to evaluate the efficiency of the subtraction, division, and multiplication Touch Math programs with larger numbers of intellectually disabled students.

The approach appeared to facilitate the development of addition strategies in the sequence followed by their intellectually normal counterparts : counting-all, counting-on, memory. In fact, subjects A and B started to count-on from the first addend using touch points before they had received formal instruction to do so. Thus, the approach seems to allow students with poor memory skills to develop the addition strategies naturally.

The Touch Math program is also intended to help students acquire addition facts by having them repeat the problems and answers aloud. In this study, subjects would only repeat the problems and sums aloud during correction and with the assistance of the examiner. During instruction, subjects A and B were able to solve doubles such as 5+5 and 6+6 from memory on 2 or 3 occasions. As the examiner spent only 18 days on average teaching addition to each student, it would seem that these students need to spend more time repeating addition problems in order to improve their mastery of addition facts. The Touch Math instructional kit does provide work sheets, strategies, and memory tips to help students practice and memorize their addition facts. These instructional components were not included in this study. Future research is needed to evaluate the efficiency of these components in helping intellectually disabled children memorize the addition table.

The subjects seemed to like the Touch Math approach because it allowed them to see their mistakes. To correct answers, subjects would simply re-count and re-touch the touch points with the assistance of the examiner. Over the course of the program, subject A seemed to become more and more logical when solving problems. On a few occasions, she verified the answers to more difficult problems by comparing them to easier problems. For example, in one instance, she deduced that if 6 + 4 = 10 then 6 + 5 could not be equal to 10. This prompted her to repeat the problem 6 + 5 and obtain the correct answer.

Another reason why students seem to like using the Touch Math approach is that it allows them to count more discreetly than when using fingers, tallies, or blocks. In fact, as they progressed in addition, subjects A and B would try to be more and more inconspicuous while adding. For example, a few months after she had completed the program, as she became very confident using the method, subject A would insist on solving the addition problems without crossing out the larger number. Subject B objected to solving problems by tapping his pencil on the numbers and insisted on counting the touch points quietly in his head. It was as though the two subjects did not want others to know that they were counting. This is consistent with findings by Cobb (1984) who, working with intellectually normal children found many children who were reluctant to admit that they have to count in order to solve a problem. Hanrahan et al. (1993) reported that some intellectually disabled children would rather guess incorrectly than count with blocks or use their fingers.

Student Difficulties Using the Touch Math Method

The touch point configurations are at the basis of the Touch Math method and it is critical that the subject learns the touch point configurations. Each of the three subjects was able to complete touch point training within 2 to 4 daily sessions. Indeed, students who possess the entry skills discussed earlier and who are able to perform one-to-one correspondence should be able to learn to count the touch points. Although it was easy for them to count the touch points, they did have some difficulty in learning and retaining the specific orders in which to count the touch points on each number.

The touch point patterns on numbers 1, 2, and 3 seemed to be the easiest for them to acquire and retain, probably because on these numbers, students are required to count the touch points moving from top to bottom (see Figure 3.1). Most of the touch point configurations on numbers 4 to 9 do not follow a top to bottom order and for this reason seem to be more difficult to remember. During steps 1, 2, and 3 of instruction, subjects tended to forget the order in which they had previously learned to tap and count the touch points and would solve addition problems by using novel counting patterns. See Figure 5.1 for a comparison between a touching and counting pattern used by a student, and the touching and counting pattern suggested by the Touch Math instructional guides. Notice that both solutions lead to the correct sum.

Figure 5.1. An idiosyncratic pattern used to solve an addition problem versus the recommended pattern.

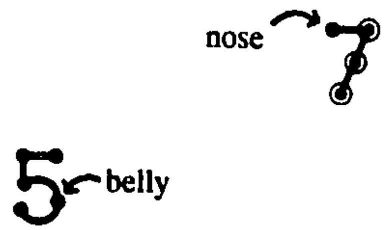
Touching and counting pattern suggested by Touch Math	Touching and counting pattern used by a student

As the use of novel touching and counting patterns did not seem to prevent the students from obtaining the correct answers and because they persisted to touch and count in different orders despite corrective feedback, the examiner decided to allow students to develop their own ways of touching and counting the touch points. Researchers who are interested in further improving the method might find it helpful to conduct an investigation of the touching and counting patterns used by these children spontaneously and to see whether different touch point configurations would be easier for them to retain.

A review of the addition work sheets completed by subjects suggests that they encountered problems with addenda 7, 5, and 6 in that order of difficulty. Subjects seemed to have problems with these addenda because one of their touch points was placed on the middle of each number. It may be that the touch point patterns would be easier for these students to learn if the touch points were always placed at the end points or corner of the numbers. For example, subject C was compelled to count an extra touch point at the bottom of the number 4.

It may also be that figurative descriptions would make it easier for students to retain the touch point patterns. For example, it helped to call the single touch point on addend 7 "the nose of number 7" and the single touch point on the rounded part of addend 5 "the belly of number 5" (see Figure 5.2). Students found these descriptions funny and seemed to find them helpful. These descriptions were used following instructional tips from earlier Touch Math instructional guides (Bullock & Walentas, 1989).

Figure 5.2. Examples of figurative descriptions for a touch point on numbers 5 and 7.



Another observation concerning the touch point configurations is that, although the Touch Math instructional guide recommends that students not draw the touch points on the problems or sums, subjects would sometimes draw the touch points on the addenda when they seemed to be having difficulty remembering the touch points. In fact, subject C drew the touch points on almost all of the problems during the 3 probe sessions following her completion of the instructional program and she obtained 95% on average when doing so (see Appendix I). This seemed to help her when she was still unsure of herself. Nonetheless, drawing touch points on problems did not seem to hinder the progress of subject C since during the following probe condition, she no longer felt the need to draw the touch points on the addenda and obtained an average of 85% using faded touch points.

Ideas for Teachers

Given the speed at which the 3 subjects succeeded when instruction was administered on a one-to-one basis, teachers may want to evaluate the efficiency of this approach when teaching a group of intellectually disabled students rather than providing individualized instruction. The Touch Math instructional guide (Bullock, 1991a) describes dynamic instructional strategies that teachers can use to teach groups of students such as contests, classroom posters, and transparencies. Teachers who intend to use the program with a group of intellectually disabled students should screen students to assure that each student entering the program has strong skills in counting and writing numerals up to 18 since students will not be able to receive as much individualized attention. The Touch Math Number Concepts kit (Bullock, 1991b) provides ample activity work sheets to help students master the counting and writing of numerals.

Educators who are interested in this approach and plan to use the approach with a group of students may be wise to start with the Number Concepts kit and use it with the students well before introducing them to the Addition Kit. Overall, teachers should make certain that their students fit the criteria by which subjects were chosen for the present study.

Additionally, teachers should assure themselves that students are able to identify the larger addend in a problem. During step 2, students are required to identify the larger addend. The addition statement states that the child must "Touch the larger number, say its name and continue counting". In step 2, identifying the larger number in each problem was not difficult because the larger number would always be the one without any touch points on it. In step 3, however, both the larger and smaller addenda no longer have touch points. The Touch Math Addition Kit omits to include this skill in its prerequisites. Subject C could not identify the larger addend in each problem. The examiner had to interrupt the program until she acquired this skill. Once this skill was mastered, she had much more facility completing step 3.

Teachers should be aware that students who are able to pick up the count may still need some assistance picking up the count at the beginning of step 2, when continuance counting is introduced. Here, subjects A and B were provided with a number line (numbers 1-20) so that they could see the number following the larger addend. At the beginning, subject A was able to pick up the count more easily if she started counting from 1.

Despite 100% mastery during touch point training, reviewing the Touch Math configurations on numbers 1 to 9 at the beginning of each instructional session seemed to help students acquire and retain the touch point configurations, especially when they were attempting to solve addition problems in which touch points were faded from all addenda. In fact, subject C resisted practising the touch point patterns at the beginning of many instructional sessions and during step 3, when touch points were totally faded, she demonstrated so much difficulty that it was necessary to interrupt the program so that she could specifically focus on practising touching and counting the touch points. In a classroom, it may be beneficial to post number lines on the classroom walls and on each student's desk. When the touch points were completely faded from the addenda, subjects A and B seemed to practice more autonomously if they had a number line to look at.

Finally, teachers may want to be aware of a variety of other factors that were important to the success and popularity of the program. Tangible reinforcements appeared to be crucial for motivating the students to perform to the level found in this study. Stickers, stars and comments on work sheets, pens, and pencils were their favourite reinforcements. Also, Touch Math colouring activity work sheets seemed to make touch point training considerably more fun for all three subjects. Overall, adding colour to all the work sheets seemed to captivate the students' attention and motivate them to work. Adding colour to the Touch Math numbers and touch points seemed more pleasant to the students than did the regular black and white number line (Appendix J). Teachers might want to consider adding colour to the Touch Math number lines they provide tc students and the ones they choose to post on the classroom walls.

Suggestions for Future Research

Results of the present study suggest that when instruction is administered on a one-to-one basis, the Touch Math method is efficient with intellectually disabled students. However, given that the instruction of these children must often be conducted in groups, it would appear important for researchers to evaluate the effectiveness of the program with a group of intellectually disabled students. The Touch Math Addition instructional guide describes several strategies that may be helpful when teaching groups of intellectually disabled students. Researchers may also want to read carefully the section *Ideas for Teachers* when planning their instructional intervention with a group of intellectually disabled students.

The rapid accomplishments that subject A displayed in subtraction with the assistance of her classroom teacher suggests that researchers may find positive results when evaluating the efficiency of the subtraction, division, and multiplication programs with intellectually disabled students. Researchers may want to investigate the effectiveness of these programs with a small number of intellectually disabled students and to provide instruction on an individual basis.

Since 2 out of the 3 subjects in this study were able to develop, on their own, the same addition strategies that their normal counterparts usually develop on their own, researchers should investigate whether using the program for a longer time period would also help students gradually memorize the addition facts. Students who do not have an intellectual handicap usually have memorized the addition table by the time they reach fourth grade. The Touch Math addition kit has delineated interesting strategies for helping children memorize the addition table. Researchers may want to study the value of these Touch Math instructional strategies in helping intellectually disabled students memorize simple addition facts.

Researchers who are interested in the method might want to investigate ways of further improving the Touch Math method. It may be useful to conduct an investigation of the touching and counting patterns used by these children spontaneously and to see whether different touch point configurations would be easier for them to acquire. A review of the addition work sheets completed by the 3 subjects suggests that the addenda 7, 5, and 6 might be easier for subjects if all of their touch points were always placed on the corners or ends of the line. The examiner also felt that figurative descriptions such as those that were used in earlier versions of Touch Math program (Scott, 1993a) might make it easier for students to retain the touch point patterns.

Finally, it may be worthwhile to try to teach the touch point patterns using a computer assisted approach. *Visage, Inc.* has developed a device called *TouchMate* (1993) that fits under a computer monitor and makes the monitor sensitive to touch. It would be interesting for researchers to try to combine such a computer device with the Touch Math program and to compare the effectiveness of this computerized approach with that found in the present study.

Conclusion

The objectives of the present study were reached. Each of the 3 children participating in the experiment learned how to add simple digit numerals with sums to 18 using the Touch Math method. These results suggest that the Touch Math approach is suitable for teaching addition to intellectually disabled children. Indeed, this multisensory approach to the teaching of arithmetic may represent a uniquely suitable system for teaching the four operations to intellectually disabled students. More research is needed to determine the usefulness of the Touch Math approach with this population.

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Appendices

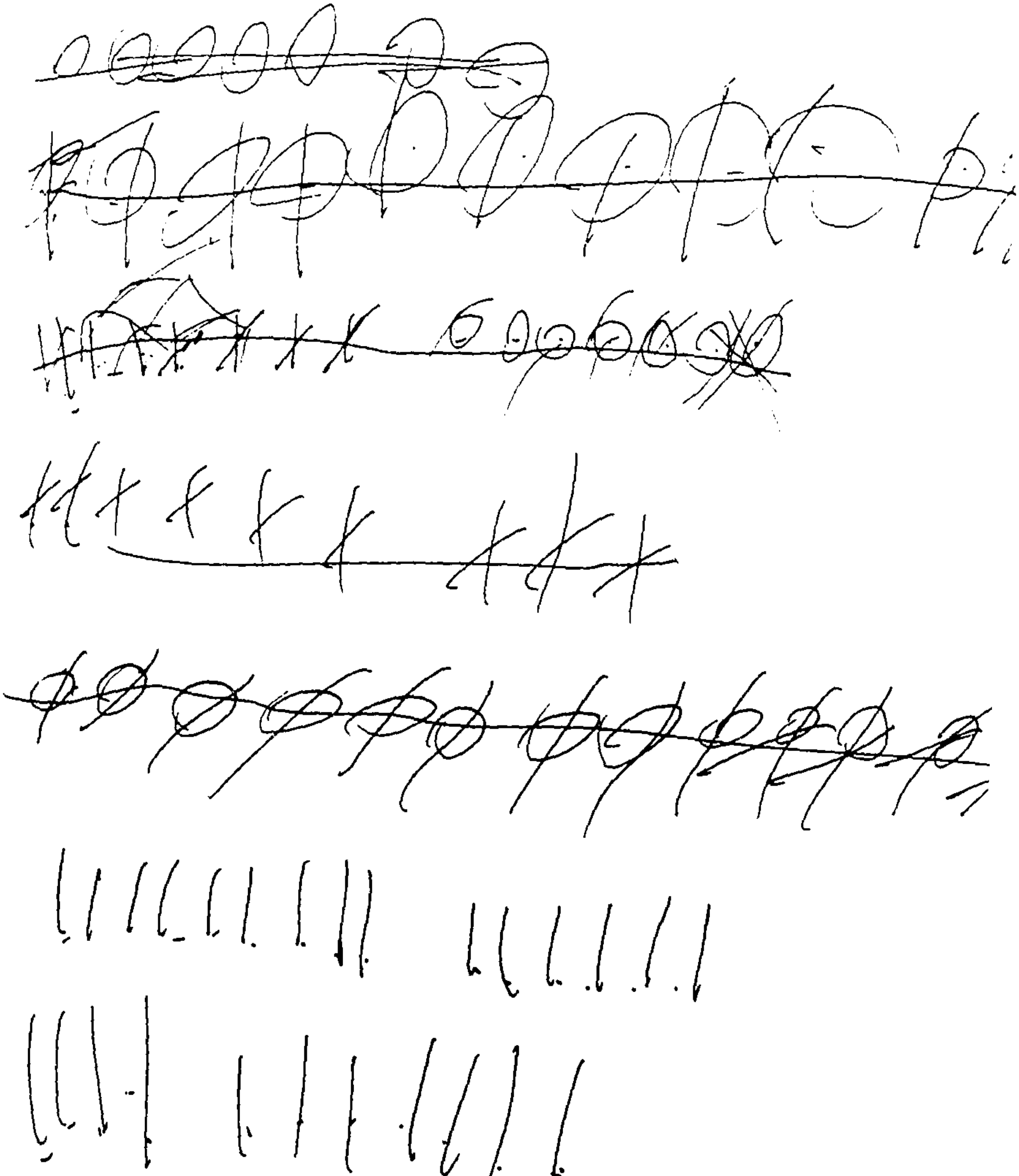
Appendix A

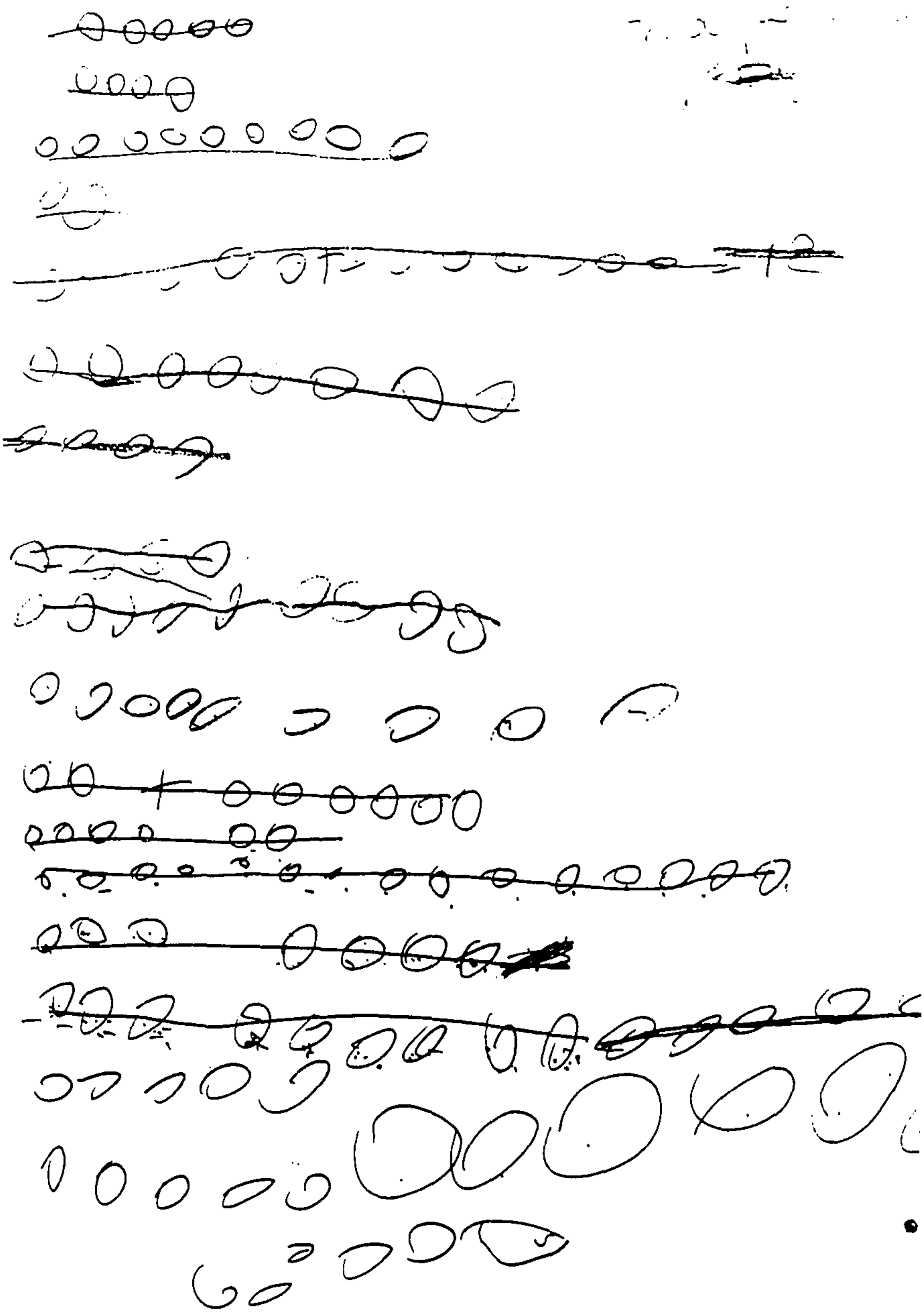
Content: The 20 two-addend addition problems with sums no greater than 9 that were part of the computerized addition test used during screening.

$1 + 8$	$7 + 1$
$2 + 2$	$3 + 1$
$1 + 4$	$3 + 4$
$5 + 3$	$7 + 2$
$1 + 6$	$3 + 6$
$1 + 1$	$6 + 2$
$2 + 3$	$5 + 1$
$5 + 4$	$1 + 2$
$3 + 3$	$4 + 2$
$2 + 5$	$4 + 4$

Appendix B

Content: Illustration of the work sheet space required by subject B when drawing tallies to perform simple addition.





Appendix C

Content: An illustration of the three types of activities used in this experiment to help the subjects learn the touch point configurations on numbers 1-9.

Name: Date: 01-94

Touch and count the touchpoints in the correct pattern

1 2 3 4 5 6 7 8 9

Draw the touchpoints on numbers 1-9

1 2 3 4 5 6 7 8 9

Write numbers 1-9 and draw their touchpoints

1 2 3 4 5 6 7 8 9

Appendix D

Content: The Touch Math number line.

1 2 3 4 5 6 7 8 9

Touch Math® Number Line

Appendix E

Content: The set of 14 addition work sheets used in the addition program of the present experiment. The addition problems were copied from Touch Math Addition Kit work sheets 12A-38A (Bullock, 1991a). The drawings were copied from various Touch Math work sheets (Bullock, 1991a, 1991b).

Name:________________ Date:______________

$4 + 2$	$3 + 1$	$5 + 2$	$1 + 2$	$3 + 4$
$3 + 2$	$2 + 1$	$2 + 2$	$2 + 5$	$4 + 4$
$3 + 4$	$6 + 3$	$7 + 2$	$2 + 6$	$1 + 6$
$3 + 2$	$5 + 4$	$4 + 6$	$6 + 7$	$5 + 5$

Name ________________ Date ________________

7	8	6	9	2
+ 4	+ 2	+ 5	+ 3	+ 9
6	5	9	8	9
+ 4	+ 2	+ 1	+ 6	+ 5
6	7	8	8	4
+ 3	+ 5	+ 4	+ 8	+ 1
8	3	7	6	8
+ 2	+ 8	+ 6	+ 6	+ 7

Name ____________________ Date ________________

3	3	6	3	2
+ 2	+ 4	+ 4	+ 3	+ 5

6	3	9	4	5
+ 6	+ 8	+ 3	+ 2	+ 6

2 + 3 = $\boxed{}$ 6 + 3 = $\boxed{}$

9 + 6 = $\boxed{}$ 9 + 7 = $\boxed{}$

5 + 4 = $\boxed{}$ 2 + 8 = $\boxed{}$

2 + 4 = $\boxed{}$ 4 + 6 = $\boxed{}$

8 + 4 = $\boxed{}$ 6 + 8 = $\boxed{}$

Name: ______________________ Date: ______________________

$4 + 6 = \square$

$7 + 3 = \square$

$8 + 7 = \square$

$7 + 4 = \square$

$8 + 8 = \square$

$6 + 9 = \square$

$7 + 6 = \square$

$8 + 7 = \square$

$2 + 9 = \square$

$5 + 5 = \square$

$5 + 7 = \square$

$6 + 7 = \square$

$6 + 2 = \square$

$7 + 8 = \square$

$3 + 4 = \square$

$7 + 8 = \square$

$9 + 4 = \square$

$7 + 5 = \square$

$4 + 7 = \square$

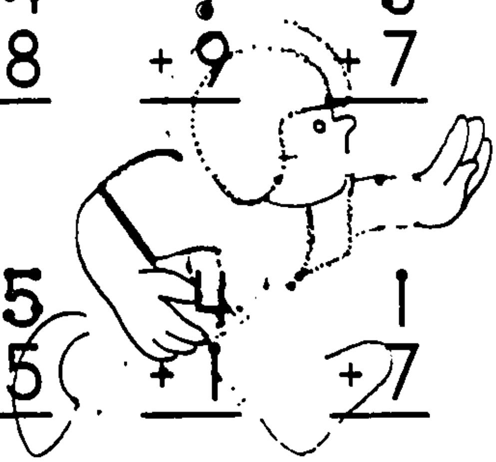
$6 + 6 = \square$

Name ________________ Date ________________

$\begin{array}{r} 8 \\ +\ 7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +\ 3 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ +\ 3 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +\ 6 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +\ 5 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ +\ 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +\ 0 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +\ 8 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +\ 3 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +\ 7 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ +\ 7 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ +\ 8 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ +\ 6 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +\ 8 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +\ 7 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ +\ 8 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ +\ 9 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ +\ 6 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ +\ 7 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ +\ 9 \\ \hline \end{array}$

Name ______________ Date ______________

6 + 2	4 + 9	7 + 8	9 + 1	4 + 6
9 + 2	8 + 4	5 + 8	7 + 7	5 + 6
4 + 8	7 + 9	3 + 7	9 + 4	6 + 7
5 + 5	4 + 1	1 + 7	2 + 6	3 + 8



Name ______________ Date ______________

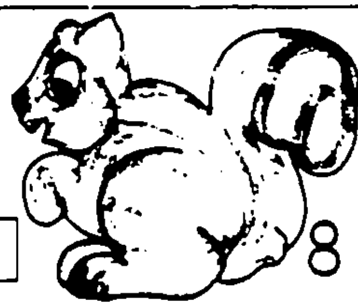
8 + 6	3 + 6	9 + 7	7 + 1	4 + 8
9 + 2	5 + 7	8 + 5	3 + 9	6 + 2
7 + 7	8 + 7	7 + 2	6 + 3	4 + 7
6 + 5	9 + 8	8 + 4	5 + 9	8 + 8

Name ____________________ Date ________________

9 + 3 = ☐	6 + 7 = ☐
7 + 1 = ☐	7 + 9 = ☐
2 + 9 = ☐	2 + 4 = ☐
1 + 5 = ☐	5 + 8 = ☐
7 + 2 = ☐	3 + 4 = ☐

6	6	9	8	6
+ 3	+ 7	+ 3	+ 8	+ 2

8	5	4	7	8
+ 9	+ 4	+ 8	+ 9	+ 4

Name ______________  Date ______________

$4 + 5 = \square$ $8 + 7 = \square$

$6 + 3 = \square$ $6 + 5 = \square$

$4 + 7 = \square$ $9 + 4 = \square$

$8 + 1 = \square$ $5 + 8 = \square$

$9 + 7 = \square$ $7 + 7 = \square$

$6 + 3 = \square$ $5 + 5 = \square$

$5 + 4 = \square$ $7 + 9 = \square$

$6 + 4 = \square$ $9 + 8 = \square$

$7 + 8 = \square$ $3 + 5 = \square$

$6 + 5 = \square$ $9 + 9 = \square$

Name: ______________________ Date: ______________________

8	9	3	9	6
+ 7	+ 3	+ 3	+ 6	+ 5

7	8	9	6	9
+ 5	+ 0	+ 8	+ 3	+ 7

6	1	3	5	5
+ 7	+ 8	+ 6	+ 8	+ 7

7	2	4	2	3
+ 8	9	+ 6	+ 7	+ 9

Name ____________________ Date ____________________

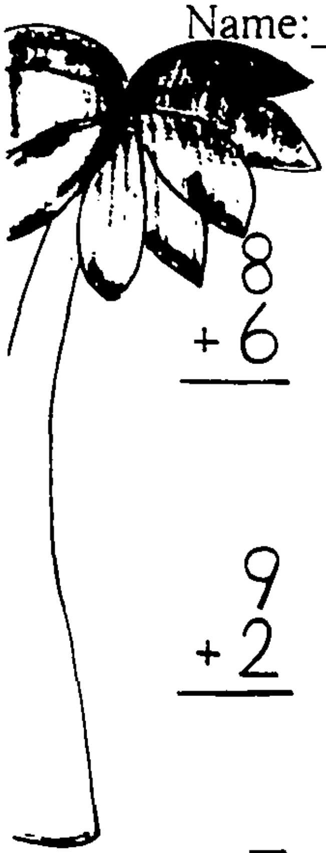
$$\begin{array}{r} 6 \\ +2 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ +9 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ +1 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ +6 \\ \hline \end{array}$$

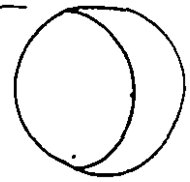
$$\begin{array}{r} 9 \\ +2 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ +4 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ +8 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +7 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ +6 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +8 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +9 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ +7 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ +4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ +7 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ +5 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ +1 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ +7 \\ \hline \end{array} \qquad \begin{array}{r} 2 \\ +6 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ +8 \\ \hline \end{array}$$

Name:__________ Date:__________





$$
\begin{array}{ccccc}
\begin{array}{r} 8 \\ +6 \\ \hline \end{array} &
\begin{array}{r} 3 \\ +6 \\ \hline \end{array} &
\begin{array}{r} 9 \\ +7 \\ \hline \end{array} &
\begin{array}{r} 7 \\ +1 \\ \hline \end{array} &
\begin{array}{r} 4 \\ +8 \\ \hline \end{array}
\end{array}
$$

$$
\begin{array}{ccccc}
\begin{array}{r} 9 \\ +2 \\ \hline \end{array} &
\begin{array}{r} 5 \\ +7 \\ \hline \end{array} &
\begin{array}{r} 8 \\ +5 \\ \hline \end{array} &
\begin{array}{r} 3 \\ +9 \\ \hline \end{array} &
\begin{array}{r} 6 \\ +2 \\ \hline \end{array}
\end{array}
$$

$$
\begin{array}{ccccc}
\begin{array}{r} 7 \\ +7 \\ \hline \end{array} &
\begin{array}{r} 8 \\ +7 \\ \hline \end{array} &
\begin{array}{r} 7 \\ +2 \\ \hline \end{array} &
\begin{array}{r} 6 \\ +3 \\ \hline \end{array} &
\begin{array}{r} 4 \\ +7 \\ \hline \end{array}
\end{array}
$$

$$
\begin{array}{ccccc}
\begin{array}{r} 6 \\ +5 \\ \hline \end{array} &
\begin{array}{r} 9 \\ +8 \\ \hline \end{array} &
\begin{array}{r} 8 \\ +4 \\ \hline \end{array} &
\begin{array}{r} 5 \\ +9 \\ \hline \end{array} &
\begin{array}{r} 8 \\ +8 \\ \hline \end{array}
\end{array}
$$

Name ____________________ Date ____________________

$9 + 3 =$ ___ $6 + 7 =$ ___

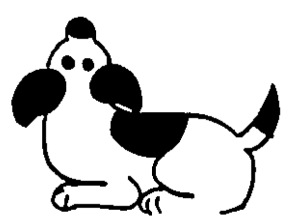
$7 + 1 =$ ___ $7 + 9 =$ ___

$2 + 9 =$ ___ $2 + 4 =$ ___

$1 + 5 =$ ___ $5 + 8 =$ ___

$7 + 2 =$ ___ $3 + 4 =$ ___

$\begin{array}{r} 6 \\ +3 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +2 \\ \hline \end{array}$
$\begin{array}{r} 8 \\ +9 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ +9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +4 \\ \hline \end{array}$



Name: ____________ Date: ____________

4 + 5 = ___

6 + 3 = ___

4 + 7 = ___

8 + 1 = ___

9 + 7 = ___

6 + 3 = ___

5 + 4 = ___

6 + 4 = ___

7 + 8 = ___

6 + 5 = ___

8 + 7 = ___

6 + 5 = ___

9 + 4 = ___

5 + 8 = ___

7 + 7 = ___

5 + 5 = ___

7 + 9 = ___

9 + 8 = ___

3 + 5 = ___

9 + 9 = ___

Appendix F

Content: Coding work sheets on which the experimenter and the second observer recorded the data during instructional and probe conditions.

	Addition problem							
1. MODEL — Problem	1	2	3	4	5	6	7	
1)FINGERS								
2)TALLIES								
3)PICTURES								
4)NUMBERS								
5)OTHER								
6)NOT SEEN								
2. IF INAPROPRIATE STRATEGY								
1)GAVE UP								
2)GUESS								
3)INAPROPRIATE								
3. IF APROPRIATE STRATEGY								
1)COUNT-ALL								
2)SUBITIZING								
3)COUNT-ON FROM FIRST								
4)COUNT-ON FROM LARGER								
5)USE OF NUMBER FACTS								
6)DERIVED FACTS								
3.A. IF ERROR:								
1)MISCOUNT								
2)READING DATA								
3)REPRESENTING								
4)RETRIEVING FACT								

	Addition problem							
1. MODEL Problem	8	9	10	11	12	13	14	
1)FINGERS								
2)TALLIES								
3)PICTURES								
4)NUMBERS								
5)OTHER								
6)NOT SEEN								
2. IF INAPROPRIATE STRATEGY								
1)GAVE UP								
2)GUESS								
3)INAPROPRIATE								
3. IF APROPRIATE STRATEGY								
1)COUNT-ALL								
2)SUBITIZING								
3)COUNT-ON FROM FIRST								
4)COUNT-ON FROM LARGER								
5)USE OF NUMBER FACTS								
6)DERIVED FACTS								
3.A. IF ERROR:								
1)MISCOUNT								
2)READING DATA								
3)REPRESENTING								
4)RETRIEVING FACT								

MODEL	Problem	15	16	17	18	19	20	21	
1)FINGERS									
2)TALLIES									
3)PICTURES									
4)NUMBERS									
5)OTHER									
6)NOT SEEN									
2. IF INAPROPRIATE STRATEGY									
1)GAVE UP									
2)GUESS									
3)INAPROPRIATE									
3. IF APROPRIATE STRATEGY									
1)COUNT-ALL									
2)SUBITIZING									
3)COUNT-ON FROM FIRST									
4)COUNT-ON FROM LARGER									
5)USE OF NUMBER FACTS									
6)DERIVED FACTS									
3.A. IF ERROR:									
1)MISCOUNT									
2)READING DATA									
3)REPRESENTING									
4)RETRIEVING FACT									

Addition problem

Appendix G

Content: Questioning technique used in the present experiment. The technique is based on Martin and Moser's general questioning technique (1980).

First, the examiner would ask the students to explain how they obtained the answer or decided what the answer was. If they said they counted, the examiner would ask them if they counted forward or backward, and with what number they started to count. If the students did not say they counted, the examiner would ask them if they were thinking of any numbers and if so, what numbers and how that helped. Questioning was stopped if the students seemed confused or frustrated.

Appendix H

Content: A probe instrument used in the present study.

NAME: ....

DATE: ....

$2 + 6 = \square$

$5 + 4 = \square$

$5 + 9 = \square$

$9 + 2 = \square$

$4 + 7 = \square$

$8 + 3 = \square$

$5 + 8 = \square$

$5 + 2 = \square$

$5 + 7 = \square$

$9 + 3 = \square$

NAME: ....

DATE: ....

$$\begin{array}{r} 9 \\ +6 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ +5 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ +3 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +3 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ +6 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ +4 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ +9 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +2 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +4 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ +2 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +9 \\ \hline \end{array}$$

Appendix I

Content: Probe test completed by subject C immediately after completing the program. She drew the touch points on almost all of the addition problems and obtained 95%.

NAME: [illegible] DATE: [illegible]

$\frac{20}{21}$

95%

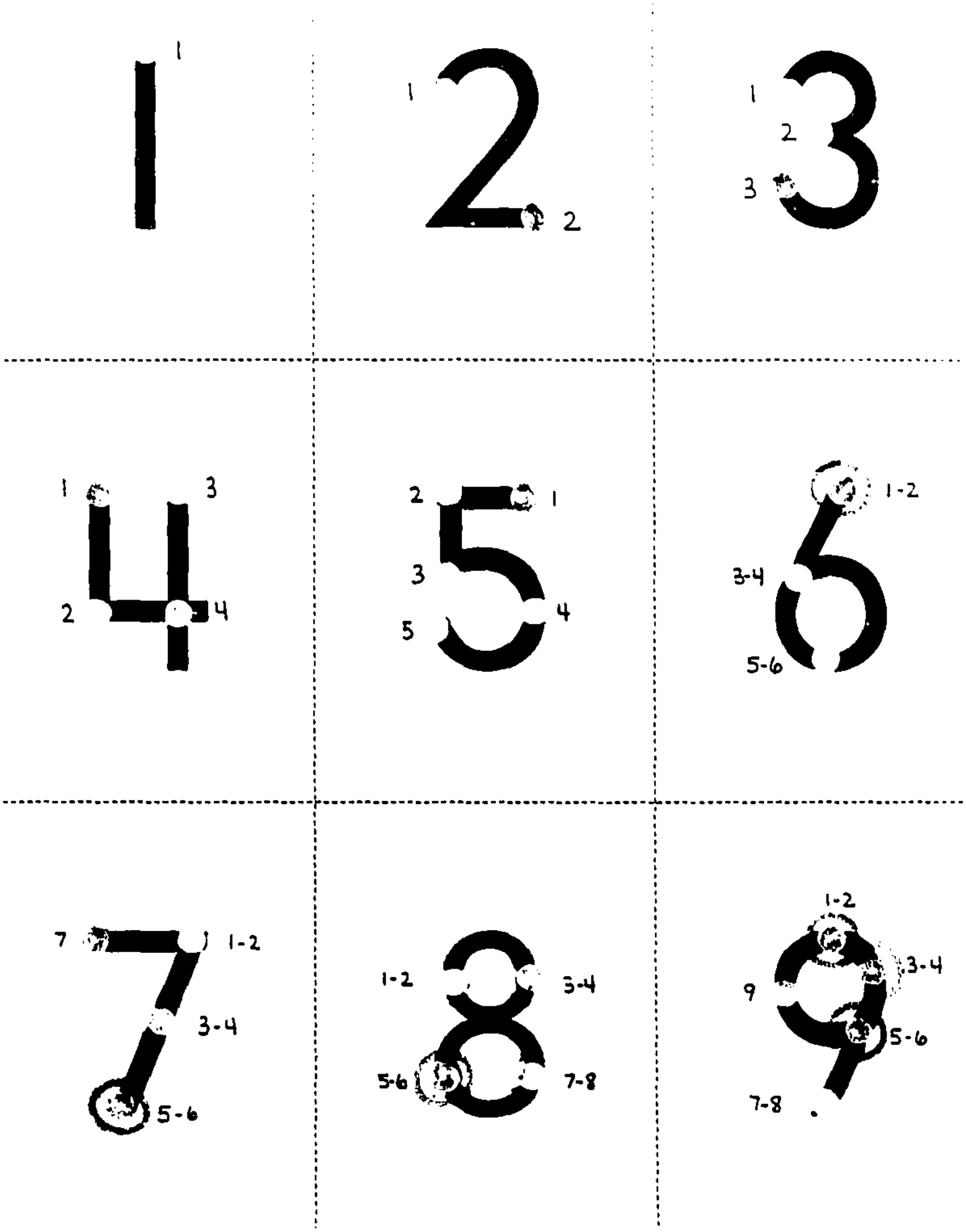
$5 + 4 = \boxed{9}$	$5 + 7 = \boxed{12}$
$9 + 2 = \boxed{11}$	$2 + 6 = \boxed{8}$
$8 + 3 = \boxed{11}$	$5 + 9 = \boxed{14}$
$5 + 2 = \boxed{7}$	$4 + 7 = \boxed{11}$
$9 + 3 = \boxed{12}$	$5 + 8 = \boxed{13}$

NAME: AMG Monday DATE: May 2100

$\cancel{8} + 4 = 8$	$4 + \cancel{9} = 13$	$\cancel{7} + 2 = 9$
$3 + \cancel{4} = 7$	$\cancel{4} + 2 = 6$	$7 + \cancel{9} = 16$
$\cancel{9} + 6 = 15$	$3 + \cancel{5} = 8$	$\cancel{6} + 3 = 9$
$2 + \cancel{3} = 5$	$\cancel{8} + 6 = 14$	

Appendix J

Content: Coloured Touch Math numbers and touch points. Students seemed to prefer these Touch Math numbers to the regular black and white ones.


1
1
2
1
2
3
1
3
2
4
2
1
3
4
5
1-2
3-4
5-6
7
1-2
3-4
5-6
1-2
3-4
5-6
7-8
1-2
3-4
9
5-6
7-8

Appendix K

Content: Permission to use Touch Math materials.