REAL TIME STEADY STATE SECURITY ASSESSMENT

IN ELECTRIC POWER SYSTEMS

by

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ABSTRACT

The present thesis tackles the problem of on-line steady state security assessment in electric power transmission networks. The contingencies examined include generation shift as well as line (transformer) outages.

The methodology developed is Pattern Recognition-motivated although not entirely within the frame of traditional statistical Pattern Recognition.

Due to the fact that training samples are rather expensive to obtain in electric power engineering, our first concern was to develop and implement algorithms carrying out the task of intelligently acquiring training points. It is found that these algorithms, permit to substantially reduce the amount of off-line computational effort while, at the same time, the coherency and impartiality of the information contained in the training sets is enhanced.

A new scheme for security assessment (equally applicable for real time security screening) was developed based on the concept of the hyperellipsoids of confidence. It is shown that by proper utilization of the hyperellipsoids of confidence, uncertainty in real time decision making (directly related to the misclassification error) is circumvented. The results of the new methodology were verified by full scale ac simulations.

Finally, the usefulness and potential applicability of the

new scheme is demonstrated for EHV equivalents. Its merits are simplicity and reliability in real time environment.

RESUME

La présente thèse s' attaque au problème de l' évaluation des contingences en temps réel dans les réseaux de transmission d' électricité. Les contingences examinées incluent les variation des génération, ainsi que les pannes de lignes et des transformateurs.

La méthodologie développée est basée sur la méthode de la reconnaissance des formes, bien qu' elle ne se situe pas entièrement dans le cadre traditionel de cette discipline.

Comme les échantillons d' apprentissage sont dispendieux à obtênir dans le domaine de l' ingénierie des systèmes de puissance, notre premier souci a été de développer et implanter des algorithmes permettant l' acquisition intelligente de tels échantillons. Ces algorithmes entrainent une réduction substantielle des calculs en temps différé, tout en améliorant la coherence et l' impartialité de l' information contenue dans l' échantillonage.

Une nouvelle méthode d' évaluation de la sécurité (applicable également a l' évaluation des contingences en temps réel) a été developpée d' après le concept des hyperellipsoides de confiance. Il est démontré que l' utilisation adequate de ces hyperellipsoides permet de contourner le problème de l' incertitude associé à la prise des décision en temps réel: cette incertidute est directement reliée a l' erreur de classification. Les

résultats de cette nouvelle méthodologie ont été verifiés a l' aide de simulations de reseaux.

Nous avons finalement démontré l' utilité et l' applicabilité de la nouvelle methode dans le cas des équivalents-reseaux a très haute tension. Ses principaux avantages sont la simplicité et la fiabilité dans un environnement de calculs en temps réel.

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My dear parents, may this research be dedicated to both of you.

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CLAIM OF ORIGINALITY

To the best of the author's knowledge, the following contributions presented in this thesis are original.

1. Development and implementation of algorithms for training point selection based on the grounds of both selectivity and off-line computational effort economization.

2. Indirect identification of the steady state stability domain of electric power systems utilizing the newly developed algorithms.

3. Development of techniques for constructing hyperellipsoid(s) of confidence.

4. Development of techniques for reliable, real-time, steady state stability and load variability assessment in electric power systems, based on the concept of the hyperellipsoid(s) of confidence.

5. Development of techniques for reliable, real-time, steady state security assessment (for both generation shift and line outages) based on the concept of the hyperellipsoid(s) of confidence.

6. Development of a technique for steady state security screening purposes with direct extensions to contingency ranking, directly applicable for real-time operation.

7. Development of techniques and methodologies to:
   - Free electric power engineers and control strategies, from the limitations of the concept of the misclassification error.
   - Acquire and utilize training sets valid for decision making under variable power system topology.

CHAPTER ONE

INTRODUCTION

1.1. Background

By the term "electric power system" we mean "a network of one or more electrical generating units, loads and/or transmission lines, including the associated equipment electrically or mechanically connected to the network"(1).

Power systems left their regional character long ago and became larger and larger with numerous interconnections. Since then, power system engineers familiarized themselves with the unavoidable fact that the disturbance free mode of operation for a large electric power system is the most rarely encountered one.

However, this fact is of no importance to the customers whose main concern is uninterrupted supply of electrical energy, characterized by high quality voltage and frequency margins. The engineering challenge then, is to meet those stringent demands for service reliability at a cost justifying the result.

The various disturbances a power system can experience during operation, range from simple load demand variations (smooth or abrupt), to the very severe cases of repeated equipment outages (creating "islands" in the power system topological structure). The various events that, eventually, have to be analyzed are called "contingencies".

The ultimate result of any contingency analysis is to determine whether, or not, the system can withstand the impact of the contingency and maintain service continuity. If this is the case, we say that the system is "secure" against the given contingency.

Depending however on the severity of the incoming contingency, the system will invariably experience "transients" until it reaches (if it does so) a new operating point. When the system is able to withstand such transients, we say that it is "transiently secure with respect to the given contingency". An assessment of the post transient state of the system is the object of the "steady state security analysis".

One very important class of system contingencies that attracted much attention over the years, is the one which contains "equipment outages", i.e. loss of generating units, transmission lines or transformers. More specifically, much effort has been devoted to predicting the state of the system (post transient) following a contingency of this nature. The research that follows pertains to steady state security evaluation.

However, a single contingency is rarely, if ever, to be taken into account. The fact that power systems are large, and have been built gradually for many decades, accounts for large amounts of diversity in both the existing equipment and its reliability. Maintenance is mandatory on an everyday basis and this is the source of the so called "scheduled outages". Unexpected failures, of course, do occur and "forced outages" also have to be considered. It should be mentioned at this point that, associated with any potential forced outage, is a measure of the likelehood of its occurrence.

As a consequence, at any operating instance, a set of contingencies is put forward, against which system robustness is to be tested. They can either be considered one at a time (single contingency security analysis) or, if of interest, more than one concurrently (multiple contingency analysis). It is the opinion of the researcher that the proposed methodology for security analysis is best suited for single contingency analysis under the contemplated assumptions.

1.2. Contingency selection.

How those contingency lists are prepared is a very important problem by itself. Scheduled outages are the first entries, but when it comes to forced outages many factors have to be considered.

Climatological conditions, for instance, existing in the various areas of the system can be of great help.

Unsuccesful reclosing of faulted lines or severe permanent damages to the towers (55,56) are, almost invariably, the results of severe storms .

Load demand variations, occuring on a local basis during peak hours, will give a good picture of the power flow paths and help spot possible overloads. The lines most likely to be tripped are also included in the contingency list. Of course the ever present "system troublespots" cannot be omitted either.

It is obvious that, in preparing a list of contingencies, a good amount of off-line work is necessary. We have, furthermore, tacitly assumed the availability of both reliable and quickly implementable weather forecast and short term load forecast schemes (the later also assumes sharp state estimation). A good and experienced operator, familiar with the system, is also indispensable.

It is obvious, from the above short discussion, that contingency lists are liable to changes and hierarchical updating at the beginning of every monitoring interval. Research towards automating contingency selection is in progress and ideas have been reported (2,3,69,70,71,72, 73,74,75,76,77,78,79).

This research assumes that a contingency list is available and, furthermore, it is not concerned at all with real time automatic contingency selection.

1.3. Approaches to steady state security assessment. Statement of the problem.

The steady state security assessment problem is considered as a difficult one in the discipline of power system operation. The main difficulty lies in the fact that electric power systems are higly nonlinear.

Under given loading conditions and system topology, the solution of a nonlinear system of equations (named the load flow equations) is necessary in order to determine the power flow pattern and the voltage profile of the system. The problem is further complicated by the fact that reactive power limits, interchange capability limitations for the tie lines, and off-nominal transformer taps ,have to be included in order to come up with a realistic model for the system.

Simulating a contingency, simply means considering a different system configuration which will reflect the effect of the simulated outage. Due to the nonlinearity of the response of the system to such changes, (assuming that the load demand remains unaltered) a new full scale simulation seems necessary if we are to reveal possible violations of the operating standards.

Given the monitoring interval, to exhaust a given list of contingencies and, therefore, claim that the chosen operating point is "secure", can be a formidable task, if the above obvious approach is adopted.

The unfeasibility of such an approach is demonstrated by the fact that, neither the enormous computational power needed is available, nor will any operator ever be able to implement and screen the results of the voluminous printouts of full scale simulations.

During the early sixties, only marginal computational power was available, despite the pressing needs for security assessment. However, the popular, and quite successful for many systems, method of precalculated "distribution factors" was presented (4). The method uses superposition of power flows on basic normal power flow studies by means of distribution factors. Two types of such "distribution factors" (one type for generation shift and one for circuit outages) are precalculated and then used to predict the post contingency power flow profile. The obvious shortcoming of this method is, and has long ago been recognized as such, the rigid linearity assumptions that it imposes on the network.

1.4. Approaches to steady state security assessment. Linear non iterative methods.

During the early sixties, the recognition of the fact that the digital computer would be of paramount importance in the future, gave birth to a great deal of research efforts to modernize the electric power systems engineers arsenal with good software packages. The modeling had to be suitable for digital simulations and emphasis was given in employing linear algebra, diakoptics and network matrices oriented techniques.

As a byproduct of the extensive research concerning the solution of the load flow equations, the d-c load flow idea emerged (5) for approximate load flow studies. Considerable simplification of the load flow equations can be achieved from the fact that the reactive component of the series impedance of the transmission lines is dominant over the resistive one, and that power lines normally operate under rather low angle difference (due to stability considerations). This modified form of the load flow equations was used for security calculations, its main merit being this very simplicity. Still used by many utilities, under various philisophies, this method permits the detection of possible post contingency real power line overloadings.

The usefulness of the impedance matrix (Z matrix) in computing "changes of state" as applied to short circuit analysis was demonstrated. The idea of utilizing the Z matrix for security analysis was conceived (6), and all methods proposed so far, based on this concept, require the availability of a well solved load flow, called the "base case", as a starting point. The post contingency system profile is predicted by superimposing the incremental changes (in both line flows and bus voltages) to the precontingency "base case" one.

Several variations of the Z matrix methods are available today (7,8,9). Techniques for increasing their efficiency for on-line computations, both in speed and storage requirements, were developed (7,8,9). These improvements work on the assumption that the so called "system troublespots"(lines, transformers, and generally pieces of equipment that are more likely to be overloaded as a result of a contingency) are known to the operator. Those methods present an attractive and high speed alternative.

The backbone of those methods is the utilization of the entries of the Z matrix to predict changes in the network, an idea emanating from Thevenin theorem in linear network theory. The model then is still linear. However, in spite of the network linearity assumption, the results are more reliable than the ones given by either the "distribution factor" method or the "d-c load flow".

The techniques described so far have a common qualitative characteristic from a methodology point of view. They are all non iterative. This inherent feature makes them fast, easily implementable and suitable for on line applications. The fact that their results often show considerable discrepancy when compared with the results of elaborate ac analysis is offset in practice by their simplicity.

1.5. Approaches to steady state security assessment. Linear iterative methods.

Another category of methods which also gained widespread acceptance are the so called linear iterative methods. They emerged from the high degree of sophistication ac load flow solution techniques had reached during the early seventies. In solving the load flow problem, the so called "elimination methods", based on the Newton-Raphson formulation of the nonlinear load flow equations (linearization of the equations around the initial guess of the desired solution), replaced the previously used Gauss (11) oriented techniques (based on iterating on bus voltages until voltage mismatches meet prescribed tolerances). The reason for this is that elimination methods were found to be more reliable from a convergence point of view and, most important of all, "system proof". After computational difficulties (need for inverting the Jacobian at every iteration, and storage problems arising when working with the full inverse of the sparse Jacobian) were bypassed (12) via triangular decompositions and optimal bus reordering, the N-R schemes were fully endorsed.

A concept that has been put forward from the early sixties (13) is the "decoupled" load flow. The main idea is to facilitate computations of the N-R schemes (making the Jacobian block-diagonal) by assuming independence between the real and reactive power flow channels. This kind of modeling is based on the fact that the transmission of real power is higly sensitive to voltage angle variations, and that reactive power flow is mainly dictated by the bus voltage magnitudes. Relative initial inaccuracies were bypassed (14,15) and faster versions became available (16).

In security analysis, the idea of decoupling the real and reactive power models was applied to give a fast iterative method for outage simulation. Efficiency was achieved by simulating the outage via the matrix inversion lemma and by avoiding retriangularization of the characteristic matrices of the two power models (17).

The most attractive feature of the method is its higher accuracy over the Z-matrix methods. However, it is considerably slower for real time computations and requires much more coding.

Another approach to the security problem, via Newton-Raphson, is to utilize the inverse of the Jacobian and look at the system from a sensitivity point of view (18). The contingency is simulated by varying the injections at the buses where the outaged element is connected, in such a way that the post contingency voltages are generated with the precontingency system topology. Adopting this approach, voltages of the system have to be iterated upon, because the entries of the Jacobian are to be computed at the post contingency operating point.

Apart from its accuracy, iterative ac contingency analysis has the major advantage that a complete post contingency profile of the system is available, while with the Z-matrix methods (the higly efficient versions) only the selected "troublespots" are checked. As a result, they can be a great help when the analyst has lost its "feel" for the system or is still "debugging" it in the planning stage.

The immediate adverse result of equipment outage is usually the overloading of a certain portion (or portions) of the remaining transmission network, depending on the severity of the outage (if the precontingency load is to be met). Many of the methods existing for assessing the post contingency system profile stress this aspect of the problem.(6,7,8)

However, there are cases where the system is not so robust when it comes to retaining its precontingency voltage profile. This is a situation encountered in many systems still under expansion (mainly in under-developed countries) and assessing the post contingency voltage profile of the system may be imperative (9).

1.6. Approaches to steady state security assessment. Recent trends. Pattern recognition-oriented methods.

A good security monitoring scheme must be able to assess both overloading and voltage changes: That goal gives another extra credit to linear iterative methods against the Z-matrix oriented ones.

As a rule in today's power system operation practice, the longer the monitoring interval, the better the contingency analysis. This is due to the fact that, on the one hand, more contingencies can be examined (exhausting, if possible, the contingency list) and, on the other hand, (if not many contingencies are being put forward) there is time for utilization of methods of higher accuracy.

The trend today is to operate with shorter and shorter monitoring intervals. As a consequence, time unavailability is the crucial extra operating constraint the operator has to cope with.

It is to be mentioned here that not only security considerations are to be taken into account when choosing the operating point at the beginning of every monitoring interval. Normally, more than one operating point will have to be considered, and selecting the appropriate one will be a compromise between economics and future system robustness. The choise of the operating point is, quite often, dictated by guesswork and policy rather than by optimization of operating indices, because a countless number of factors has to be considered.

Quite frequently, choosing a specific operating point can be a very poor decision if the short term load forecast is found to be inaccurate or, if the miscalculation of the probability of occurrence of a given contingency leads the operator to decide to "take chances".

A shift in the operating point requires preventive control to be put into effect, and a quick answer to the question of security must be provided by the security monitoring scheme, if we are to endorse the new operating point (most likely suggested by economics).

From the above discussion it can be seen that there is a pressing need for both fast and at the same time reliable security monitoring schemes. What one class of methods seems to offer in accuracy lacks in speed and ease of implementation. Reasearch has been directed towards bridging those conflicting but desirable features. Considerable help emerged from the availabilty of high quality hardware which increased considerably the much needed on-line computational power.

At the same time though, the cost of off-line computations has been reduced and this fact gave engineers the idea to look at methods relying on a fair amount of off-line computations. The idea of applying "Pattern Recognition" for power system security analysis was born (19,20,21,22).

The main reason for pursuing development of "Pattern Recognition", and generally simulation oriented techniques, is the fact that information gathered via off-line computations can be used, very quickly, for on line decisions. This way, the real time unavailability constraint is circumvented.

As the term "Pattern recognition" suggests, those methods consist in discriminating a secure from an insecure operating point by simply taking a glance at the operating state of the system. There are "system configurations under specific load" or "system loads under given configurations" that will constitute safe or unsafe operating points.

The operator then understands the word "pattern" as: "a set of morphological and parametric characteristics of the power system, which contains the necessary information to predict a certain outcome."

This dissertation proposes a security monitoring scheme which is "Pattern Recognition" motivated.

The methodology and the philosophy of utilization of Pattern Recognition, as well as how it can be applied to electric power system security evaluation are treated in detail in chapter II.

CHAPTER TWO

METHODOLOGY AND OBJECTIVES

2.1. On the engineering methodology

Generally speaking, when a specific problem is to be tackled, in any engineering field, the mathematics necessary for facing it will always be a direct consequence of both the physical picture and of the extent of modeling required to meet certain operational or design standards.

However, the methodology used is not only a function of how refined the solution will be. This is certainly the objective, but, very often, we are faced with problems that are hard both to formulate and solve.

Either the complexity of the problem is such that modeling is formidable (if not impossible in realistic terms), or the mathematical models that have been suggested are such that they either distort or obscure the problem in such a way as to render any potential solution invalid.

Before the explosion in computer capability, during the seventies and eighties, the trend in devising engineering mathematical models was to get a clear physical understanding of the problem and then to construct a good analytical model describing the phenomenon in question. The basic motivation was, and still is, to obtain quick, exact, elegant and easy- to-use results.

The computer, over the years, familiarized engineers with numerical analysis, converging iterative techniques, and with the availability of considerable, and still growing, computational power.

Under the circumstances, simulation methods, which up to now have been put aside due to the lack of both hardware and software availability, become more and more popular.

Their validity both in the exploratory stage of many hard problems, and in design has been repeatedly proven.

However, analytical methods are always more preferable. As a rule, when one resorts to simulation methods, he simply confesses his inability to face the problem analytically.

A drawback of the simulation methods is that they can be lengthy, and require a lot of coding and computational power. Their greater advantage lies in the fact that a reasonably good physical understanding of the problem can lead to very useful results.

Analog simulation served electric power engineers for decades for power flow studies, economic dispatch, transient and steady state security assessment.

From the early sixties, digital simulation received a lot of attention and today power engineers are equipped with a good variety of reliable software to accomodate their needs for planning and off-line studies.

Problems encountered in power system operation are best suited for simulation-based techniques because, on the one hand, they are very hard and complicated to model extensively, and, on the other hand, accumulated experience over the years is available.

Pattern Recognition methods are classified as simulation oriented methods because, firstly, they require considerable computational effort and, secondly, they base their conclusion on "raw" data processing.

2.2. Context of Pattern Recognition.

Attempting to define the term "Pattern recognition" in a generally accepted way is not easy at all. Many authors refrain from giving formal definitions and rather resort to examples suggesting the type of problems that are usually tackled by Pattern Recognition. Others doubt that a definition even exists, because they support the opinion that Pattern Recognition is not a field of discipline but is evolving into a multidiscipline, due to the diversity of its applications and the variety of developed methods by many scientists working on specific problems in virtually every domain.

In (23) Watanabe suggests that: " Pattern-Recognition is a vast and explicit endeavour at mechanization of the most fundamental human function of perception and concept formation.".

Probably, the difficulty resides in the fact that neither the term "pattern" nor the term "recognition" are formally defined and generally understood. Sayre(23) stresses the fact that: "We simply do not understand what recognition is. And if we do not understand the behaviour we are trying to simulate, we cannot reasonably hold high hopes of being successful in our attempts to simulate it." And he adds "to recognize an object is to perceive it, or have perceived it, and in addition to be able to identify it".

Meisel(23) defines a "pattern" as follows: "In their widest sense patterns are the means by which we interpet the world."

Attempting to define the term "pattern" in a rigid axiomatic way often leads to obscurity and one feels not comfortable (if not a mathematician) to pursue further mathematical modeling. On the other hand, verbal definitions trying to achieve generality, even skilfully, often tend towards vagueness.

The context of Pattern Recognition is so vast that, as Nagy points out (24) it is relatively easy for the experienced pattern recognizer to describe almost any field of scientific and humanistic activity in terms of Pattern Recognition".

Nowadays we are not surprised to assess the fact that, Pattern Recognition motivated techniques found applications in countless fields, ranging from simple differential diagnosis in medicine to most sophisticated target recognition systems for modern missile guidance technology.

Working with Pattern-Recognition provides a certain framework for both terminology and methodology, and researchers in various fields face essentially the same problems, but almost always greatly intensified or simplified by the peculiarities of the situation.

This is the seat of another difficulty. One method that seems to be working very well for a specific application can be very misleading for another, in spite of the fact that both problems can be perfectly formulated as problems solvable by Pattern Recognition.

A direct conclusion of the above fact, for anyone who tries to approach a specific problem using P.R, is that first of all he should be very reluctant in transposing techniques from other fields that gave spectacular results. The philosophy of the utilization of the method, the basic motivations behind its dynamics and mathematical arsenal, should always be weighted against the particular situation the analyst has in mind. If this is not done the best he can hope for is meaningless results.

2.2.1. General Methodology of Pattern Recognition.

The ultimate objective of any P.R. technique is classification. By classification we mean the assignment of specific samples to categories possessing distinct qualitative attributes. In the terminology of P.R those categories are called "classes".

The way to achieve mechanization in classifying, is to accumulate enough "experience", and finally use it to make decisions concerning class membership. The amount of data that play the role of accumulated experience are called "training sets".

The criterion by which classification is achieved is called the "decision rule" or the "classifier". The process of developing the "decision rule", taking into account the "training sets", is called "classifier design" and is accomplished during the "training stage" of any P.R. process. In other words, "classifier design", usually, consists in developing a method for condensing the intelligence scattered in the "training sets" and presenting it in a form convenient for future reference.

From the above short discussion, it is obvious that, since the classifier is designed with direct reference to the "training sets", the degree of relevance of the information contained in the training set will be the crucial factor governing future classifier performance. To achieve data relevance in the training set, means must be found for extracting the most prominent information out of the mass of collected measurements or observations. This task by itself is the objective of the so called "feature extraction" stage of any Pattern Recognition problem.

2.2.2. Feature Extraction.

It has already been pointed out that the stage of "feature extraction" is a very important one. Usually, "raw" data are collected and the "feature extraction" process task is to "filter out" the useful information, discarding the rest. The reason for doing this is to avoid handling redundant data bases that render the process cumbersome and uneconomical. In many cases, redundant information can mask the importance of sensitive factors and lead to erroneous results. The "feature extraction" problem is sometimes a formidable task for the pattern recognizer and many ways of tackling it have been proposed.

When we have two or more classes, feature selection becomes the choosing of those features which are most effective for showing class separability. The strategy for succeeding in this activity lies in mathematically expressing indices of class separability, and implementing them via optimization methods (25). Statistical analysis, based on the correlation of features, can also provide insight concerning the possible redundancy of the entries of the originally selected pattern vector.

If terminology from state space system analysis is used, feature extraction can consist in lowering the dimensionality of the original pattern vector, by retaining the most effective variables. It can also be viewed as a mapping procedure to a lower dimensional space. This new space of reduced dimensionality is called "feature space".

In applying Pattern Recognition to engineering, however, familiarity of the analyst with the system, the existence of good mathematical models, or simply intuition can be of great assistance.

2.2.3. Classifier, classifier design and performance.

The classifiers can either take the form of direct criteria for deducing the class membership of a candidate pattern from simple metric considerations (nearest neighbor rules 26,27,28) or can consist in arriving at expression describing analytically the border lines between classes in the feature space (25).

One should distinguish between probabilistic and deterministic decision making. If the probabilty densities of the populations constituting the classes are known, the problem can be formulated in terms of statistical decision making (29). In case the densities are not known, we can either resort to techniques of density estimation (and reduce the problem to statistical hypothesis again), or apply the so-called non parametric decision making, where distribution-free classification criteria have to be invoked (30,31,32,33,34).

In deterministic decision making, no probability densities govern the class populations (there is no overlapping between them) and we enjoy complete class separability. However, the separatrix (or separatrices), i.e the curve (or surface in higher dimensions) expressing analytically the class borders, can be quite complex and have a higly irregular shape. Fitting schemes, based on either least squares, or penalty factors aspects, have been utilized in applications of this sort but the main difficulty resides in the fact that an "a priori" assumption has to be made for the mathematical expression of the separatrix. As a consequence, any accuracy claimed in training the classifier will simply refer to its parameters while the mathematical expression by itself still remains questionable.

Once a classifier is designed, the main concern of both the designer and, especially, of any potential user, is its degree of correctness (35,36,37). Criteria have to be selected that characterize impartially the performance of the classifier. Obviously, if something is to be avoided as much as possible, it is wrong decision making caused by misclassification.

A "test set" is used to assess classifier performance. The analyst provides the classifier with a set of patterns (normally not used during training) and lets the classifier decide their class membership. The true class membership of the elements of the "test set" has already been determined by simulation or other means. The number of improperly assigned elements of the "test set" when compared to its cardinal number, gives a measure of the "misclassification error" which is to be expected in the application field.

The "misclassification error" is a widely recognized performance index for classifier performance. Research indicates that both the impartiality of "test set" selection and its cardinal number are of significant importance.

The above mentioned "Monte Carlo" motivated way is still the widely applied simulation alternative to the formidable problem of determining analytically the misclassification error. So far, analytical expressions giving the misclassification error exist but, unfortunately, refer to a very small numbers of cases related to probabilistic decision making (25,29).

Misclassification will always occur, and the goal is its minimization rather than its elimination. Good classifiers result in small misclassification errors, but optimal ones give the minimum possible.

Simplicity in classifier design is a very desirable property and, very often, analysts are more than willing to trade ultimate accuracy in exchange with inexpensive, simple and fast decision rules.

2.3. Pattern Recognition and Power Systems. Methodologies and modeling.

Assume that there exist in the state space of an electric power system one or several surfaces having the property of separating the states of the system possessing a certain attribute (being steady state stable, transiently stable, etc) from the states that do not. This surface can be called a "separatrix" because it separates those two classes of system states.

Assume, further, that an analytical expression for the separatrix is available (the variables being the coordinates of the chosen space). If such an analytical expression is available, any state can be checked for its location with respect to the separatrix by simple substitution in the given surface equation. Negative, positive or zero values will imply that the given state is located in the interior, exterior or on the separatrix.

The enormous potential applications of the availability of such separation surfaces is obvious. Decision making is done with the negligible cost of simple function evaluation and, most important of all, extremely quickly.

In transient stability studies, for instance, it is well known that assessing the stability of a multimachine system with respect to a specific disturbance, frequently requires the numerical integration of a system of differential equations for a period of 2-3 minutes. Real time numerical integration is, of course, out of the question and Pattern Recognition is an attractive alternative.

The relevance of the methodology has been pointed out (38,39,40) and results from the industry (41) indicated the feasibility of the approach. Direct extensions of those methods to the problem of steady state security evaluation became available concurrently (39,40).

It is to be mentioned here that the separation surface aspect was also used for identifying, via simulation methods, steady state and transient stability domains and for control purposes (42,43,44,45,46).

However, tackling operation and control problems in power systems via P-R is not as straightforward as it may seem from the above discussion.

Being forced to work and make decisions in a state space, questions concerning the dimensionality of such a space are the first to arise. Any power system has a large number of variables the exact knowledge of which is vital (bus voltage magnitudes, angles, active loads of the buses, reactive loads, reactive source capability for compensation, active source power limits, etc.) in deciding whether or not a given load demand pattern is feasible. If transients are analyzed, other variables such as, generator reactances, parameters representing voltage regulators etc, have also to be considered.

Generally, the original pattern vector can turn out to have numerous entries. If all of the variables initially considered are kept, the dimensionality of the feature space becomes very large and, at the same time, the number of training points required for decent classifier design becomes prohibitively large. Implementing a good feature extraction scheme (38,39,40) is not always a cure to this problem because power systems are large and a good number of variables is needed for proper representation. As a result, the variables selected are often application oriented and engineering judgement as well as knowledge of systems peculiarities are of great assistance. In our case, for a given system configuration, the load demand will be the determining factor for the voltage profile. As a consequence, the bus voltages are not included in the feature vector (being considered dependent variables). Another reason for not considering the precontingency bus voltages as features is the fact that, in today's systems, transmission bus voltage magnitudes are controlled (by either static or rotating devices) and do not experience appreciable variations. In a further effort to reduce the number of features the buses were assumed to have constant load power factors. This means that the pattern of variation of the reactive component of the load, for a specific bus, follows the one of the active component by a proportionality constant. This assumption has been successfuly utilized in the past by the author and others (47,48,49) under similar modeling circumstances.

2.3.1. Pattern Recognition in power systems. Classifier considerations.

The training points are obtained from actual simulations (load flows) and labeled accordingly. If the load flow converges the operational point under consideration is considered to be steady state stable, if not it is considered to be steady state unstable and located in the exterior of the sought separatrix.

The classifier itself is a very interesting aspect of the problem of Pattern Recognition application in power systems. Our problem, being a two class problem, consists in discriminating between, say, steady state stable and steady state unstable states in the feature space. Membership in either of those two classes is not determined by probabilistic arguments, but by actual operating constraints uniquely determining the state under consideration as being stable or not. What the analyst is forced to do, at first, is to guess the analytical form the separatrix could take and then, secondly, to train it. Predicting the analytical expression of the separatrix in a high dimensional space is a very daring undertaking since often the very existence of such a separatrix will not have been proved.

Linear classifiers were at first examined (38,39,40) but at the same time the higher accuracy of second order classifiers was pointed out. Linear classifiers have also been used for identification and control purposes but, again, their performance was found to be inferior to the one higher order functions exhibited (44,45).

However, increasing the order of the classifier does not necessarily mean better accuracy (i.e further reduction of the misclassification error). Problems caused by overfitting may render the classifier useless and unreliable for points located near the surface it analytically represents. In spite of the initial appeal of polynomial discriminant functions (51), experience shows that they can be quite misleading. A wide variety of fitting schemes was also examined (least squares, generalized least squares, etc) as well as potential function methods (50). The latter approach is based on the idea that the separatrix can be expressed as the equilibrium border between stable and unstable points, each exerting an influence (in the gravitational sense) around its vicinity. Although promising, this method relies heavily on the so called "smoothing parameters" of the potential function, parameters which are determined by experience and are very much application oriented.

From the above discussion, it is obvious that classification will be poor in the vicinity of the separatrix although good results are to be expected (irrespective of classifier choice) for points located in nonsensitive areas of the state space. As already pointed out (52), it pays to know exactly the sets used for testing purposes when high classifier accuracy is claimed (one may pick up a testing set in which the secure and insecure points, say, are so far apart that artificial discrimination is easy).

The quest for reasonably good classifiers did not omit nearest neighbor rules either. The experience of the author is in accordance with the results obtained by other researchers (52). Nearest neighbor rules performed spectacularly well (in assessing the steady state stability region) for a 3-bus system, but very poorly in higher dimensions in spite of the fact that generous quantities of training points were provided.

2.3.2. Pattern Recognition in power systems. Tentative application for security screening.

Early recognition of the above capital difficulties led the industry to be rather pessimistic when evaluating the performance of such methodologies (52).

It is of interest to mention that efforts in employing Pattern Recognition for security analysis were concentrated in determining directly, with the aid of the separatrix, whether a given system state is secure or insecure. To the author's knowledge, no breakthrough is forthcoming in circumventing the inherent difficulties associated with this line of approach and, accordingly, our expectations from Pattern-Recognition were far too great.

It is the objective of this dissertation to propose another use of Pattern Recognition when tackling the problem of steady state security of power systems. This dissertation is not concerned with any sort of security function evaluation but it, rather, introduces a Pattern-Recognition motivated security screening technique.

It is generally recognized that comprehensive security analysis should include equipment overload computations as well as the effects of contingencies on bus voltages. Those tasks can be accomplished concurrently on the condition that ac power flow analysis be used. Those methods, however, are still too time consuming for real time computations in spite of the degree of sophistication they have reached due to the development of the decoupled load flow technique (17).

The dilemma can be solved in several ways. One very popular approach among utilities is to limit the number of cases put forward for investigation by the contingency list. This is called security screening. The cases which, as shown by preliminary calculations, require special attention are referred to more elaborate ac analysis, while the ones whose effect on the system is obvious enjoy no special treatment (53). This screening procedure is initiated at the beginning of every monitoring interval.

The proposed screening method classifies a given state in one of the following classes.

- Definitely secure
- Definitely insecure
- Ambiguous

Cases classified as ambiguous are referred to more elaborate ac analysis. The advantages of such a scheme are obvious:

- The operator does not have to be concerned with any sort of misclassification error, a quantity on the one hand practically impossible to compute accurately and on the other hand of virtually no use whatsoever, when assessing the correctness of a decision.

- The scheme is faster than today's screening methods because it is P-R based.

- Decisions as to whether the state under consideration is secure or not are based on data obtained by real, accurate system simulation studies. In todays schemes, however, in order to achieve speed we have to rely on linear non iterative methods whose accuracy has long ago been recognized to be questionable (see chapter I).

2.4. The proposed security screening scheme.

The system is considered in its precontingency topology. The state space of the system is defined as the space of real power injections of the buses experiencing significant load variations. The steady state stability region (load flow equations feasibility region) is identified indirectly via two sets of points, one containing steady state stable training points and the other containing steady state unstable training points. This is accomplished via special selection algorithms (see chapter III).

- 1. A hyperellipsoid is constructed containing as many steady state stable training points as possible but no unstable ones. This set will henceforth be called "confidence hyperellipsoid of the stable states".

- 2. A second hyperellipsoid is constructed enclosing all steady state stable points but as few unstable training points as possible. This set will henceforth be called "confidence hyperellipsoid of the unstable states". The motivation behind such definitions is the following: A state falling in the interior of the "confidence hyperellipsoid of the stable states" is definitely stable, while a state located in the exterior of the "confidence hyperellipsoid of the unstable states" is unstable. Any state found in the zone located between the two hyperellipsoids, is characterized as ambiguous.

-3. A contingency (line or transformer outage) is simulated as a change in the injections of the power system without altering its precontingency topology (18,54). The location of the new state, as defined by the new injections to reflect the outage, is simply checked with respect to the two above mentioned hyperellipsoids, already determined for the system at precontingency topology. Thus, with this technique, the load flow feasibility region of the base case system is used to evaluate the effects of the outage.

CHAPTER THREE

ALGORITHMIC PROCEDURES FOR TRAINING POINT SELECTION.

3.1. Introduction. Objectives of the selection scheme.

As pointed out in the previous chapter, the implementation of the proposed security screening scheme relies heavily on the knowledge of the feasibility region of the precontingency transmission system. For reasons explained earlier (chapter 2), the effort, when identifying the load flow feasibility region, has not, in this study, been directed towards deriving an analytical expression for the separatrix in the space of injections. Rather, two sets of points have been selected to characterize indirectly the sought feasibility region. One set contains steady state stable training points (points that represent feasible injections for the power system). These points are, obviously, located in the interior of the feasibility region. The other set of points contains steady state unstable training points (points that represent injections which either cause the load flow not to converge, or impose violations of the operating constraints). These points are located outside the feasibility region.

When such an approach is to be adopted the following questions are in order:

- How are those points to be selected in order for them to provide a fair outlook of the region under investigation ?
- How many of those points are needed for proper region identification ?

The second of the above questions is not only a very hard one to answer but, also, one that a power system analyst regards with special interest. This is because training samples are expensive to obtain. A training point, in our case, is the result of a full scale ac load flow analysis, which requires a substantial amount of effort. There are cases where the problem of costly training sets can become much more acute. For instance, training points needed for transient stability region identification, in the case of a multimachine system, would be far more costly because numerical integration of the equations describing the rotor oscillations requires far more computational effort, than solving a set of nonlinear load flow equations.

The fact that training samples are costly to obtain is one of the peculiarities of P-R when applied to power systems. It becomes obvious that, since power engineers do not enjoy the privilege of training point availability (taken for granted in virtually every other domain of scientific research), training point economization is imperative. It is very interesting to mention that, this fact has not attracted the attention it deserves, and, to the author's knowledge, very little has been done to come up with cost-sensitive algorithms for training sample selection.

Redundancy in training point availability would render any region identification scheme uneconomical and cumbersome. On the other hand, if fewer training points than needed are available, reduced accuracy renders the scheme useless. Proper engineering, consists in finding the best compromise between accuracy and cost, a question very difficult to tackle especially when the number of dimensions is high.

The scheme proposed herein for training point selection provides a performance index of this kind. As explained, the number of training points needed (the minimum required) is reached when the orientation of the confidence hyperellipsoids is not likely to change in any appreciable manner when additional training points are included in the training sets (see Appendix II).

3.2. Algorithm design guidelines.

When selecting points for region identification it pays to bear in mind that:

- The points should be as close as possible to the separatrix. As a matter of implementation, the distance (consider any metric) between a stable and its corresponding unstable point (the algorithms provide the training points by pairs, i.e one stable one unstable) should be kept within specified limits (thus minimizing the zone of uncertainty for the location of the separatrix).

- The fewer unexplored areas the surface possesses the more complete our knowledge about it will be. As a consequence, it is imperative that the selection algorithm, supplies the training sets with elements representing every area of the separatrix. Overidentification of a region will provide, perhaps, better results locally but, if the operating point is shifted towards least identified areas, the results will be very misleading.

Obviously, the same considerations apply equally well in the case where an analytical expression for the separatrix is sought. The basic motivation behind this line of research is to design classifiers with a certain touch of optimality. By optimality we do not mean the minimization of any statistically motivated performance index of any kind. What the researcher is actually seeking, in the paragraphs to follow, is training sets that make the most out of the least possible number of training samples, thus minimizing the design cost of any potential classifier, with the least compromise concerning its accuracy.

The above brief assessment constitutes the design guidelines for the selection algorithms derived and implemented by the author. Accordingly we will be looking at:

1) Developing algorithms for the selection of training samples.

2) Testing those algorithms: assessment of steady state

stability will be attempted for simple power transmission systems using the training sets resulting from the selection algorithms.

Two algorithms for training sample selection have been devised and implemented. They are examined in detail in the paragraphs to follow.

3.3. Algorithm 1. Description.

This algorithm is a Monte Carlo based "random walk" scheme. It consists in tracking the separation surface and collecting stable and unstable training points in the most interesting part of the state space, i.e in the vicinity of the separation surface. The algorithm reads as follows:

1. Start with a stable (s) and an unstable (u) point.

2. Generate randomly a "knee" point (a) having coordinates such that:
   The ith coordinate of the point (a) lies in the interval spanned by the ith coordinates of (s) and (u).

3. Generate a second "knee" point (b) as above. The "knee" points (a) and (b) define two (2) trajectories.

4. For the first trajectory:
   Examine the "knee" point (a) as to whether is stable or not.
   -If stable exit with a pair of training points one stable (the knee point) and one unstable (u). Define a'=a.
   -If unstable bisect the segment sa and define the new point a'.
   Exit with a pair of training points, one stable (s) and one unstable (the "knee" point).
   -Store the training pair in data sets.

5. Define point b' for the second trajectory exactly as a'.

6. Points a' and b' are stable, by construction, and uniquely, although randomly, defined. Conduct a search first along direction a'b' and then along b'a'. Let the step for the search be mod(a'b').
   Two pairs of points result from the searches:
   - s',u' (s' stable, u' unstable) from the search along a'b'.
   - s", u"(s" stable, u" unstable) from the search along b'a'.

7. Select the pair which is the more remote from, say, a'. Assume that pair is (s',u').

8. Replace s and u with s' and u' respectively. Return to step 2.

It is apparent that this algorithm provides training points by pairs (2 pairs per iteration, one pair per trajectory). Each pair contains one stable and one unstable point. Fig.3.1 illustrates the algorithm in two dimensions.

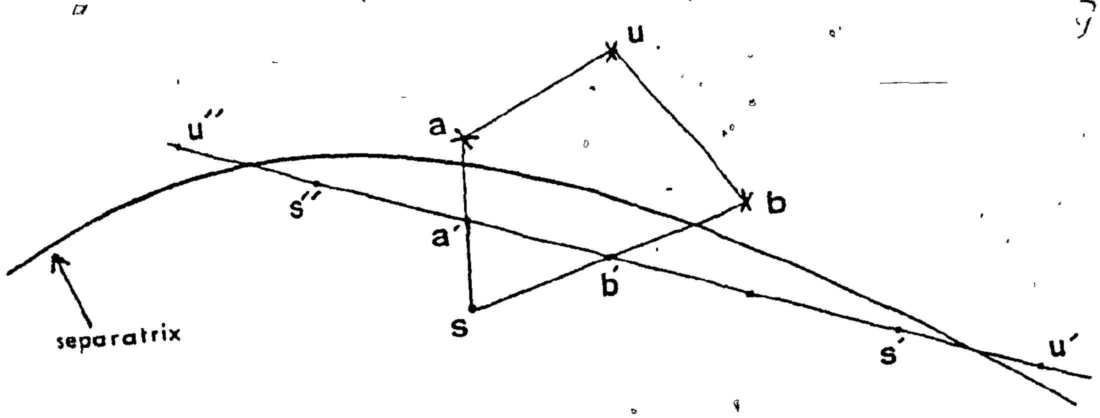


Fig.3.1: Random walk "Monte Carlo" scheme illustrated in two dimensions.

3.4. Testing algorithm 1. Results.

The algorithm has, at first, been tested in two and, later, in higher dimensions. In two dimensions two testing procedures were used. At first, the tracking capability of the algorithm has been tested on given geometrical figures (known separation surfaces).

A circle of a certain radius, and an ellipse were used. Fig.3.2, shows the circle which was to be tracked. The "dots" represent the "steady state stable" training points (in this particular case they are simply points located in the interior of the geometrical figure) and the "crosses" represent the "steady state unstable" training points, as produced by the two dimensional version of the proposed algorithm. The plots were performed in the ZETA plotter at McGill (IBM AMHDAL). Fig.3.3 illustrates the ellipse which was to be tracked. The "dots" and "crosses" bear the previous interpretation.

The second testing procedure in two dimensions was to generate meaningful test sets (the testing points had to be located close to the training points) and assess the relevance of the selected training samples. The test points are also indicated in fig.3.2 and fig.3.3 (shaded triangles). The bounding squares indicate the area inside which random generation of the test points was restricted.

The classifier used for the prelabeled test points was the nearest neighbor rule. Many reasons contributed towards that selection. At first, since testing procedures were under implementation, simplicity was a very desirable factor. However, reliability was also attained at the same time, because the separatrices under consideration complied with the conditions for admissibility of the decision rule. To be more specific, the surfaces (curves in our case) under identification were smooth and the two classes enjoyed complete separability (26).

The misclassification error for the generated test sets is also indicated in figs.3.2 and 3.3 and, as seen, is quite low.

As a final test for the two dimensional case, the actual feasibility region of a 3-bus system was identified. The data for the system are given in Appendix I. The two axes representing the two variables of the state space refer to the real power injections at buses 2 and 3. Bus 1 is considered to be the slack bus. Buses 2 and 3 were assumed to be voltage controlled buses (unlimited capability of reactive compensation)

Fig.3.4 illustrates the training points that resulted after applying algorithm 1 (the "dots" represent feasible injections and the "crosses" correspond to non-feasible injections) as well as the test points generated for misclassification error assessment. The decision rule used was again the single nearest neighbor rule. The misclassification error was, again, found to be quite acceptable in spite of the fact that, the region under identification somewhat violates the admissibility requirements of the chosen decision rule (there is no guarantee whatsoever that the feasibility region will be smoothly shaped, especially when the system consists of transmission lines with rather diverse electrical parameters and lengths).

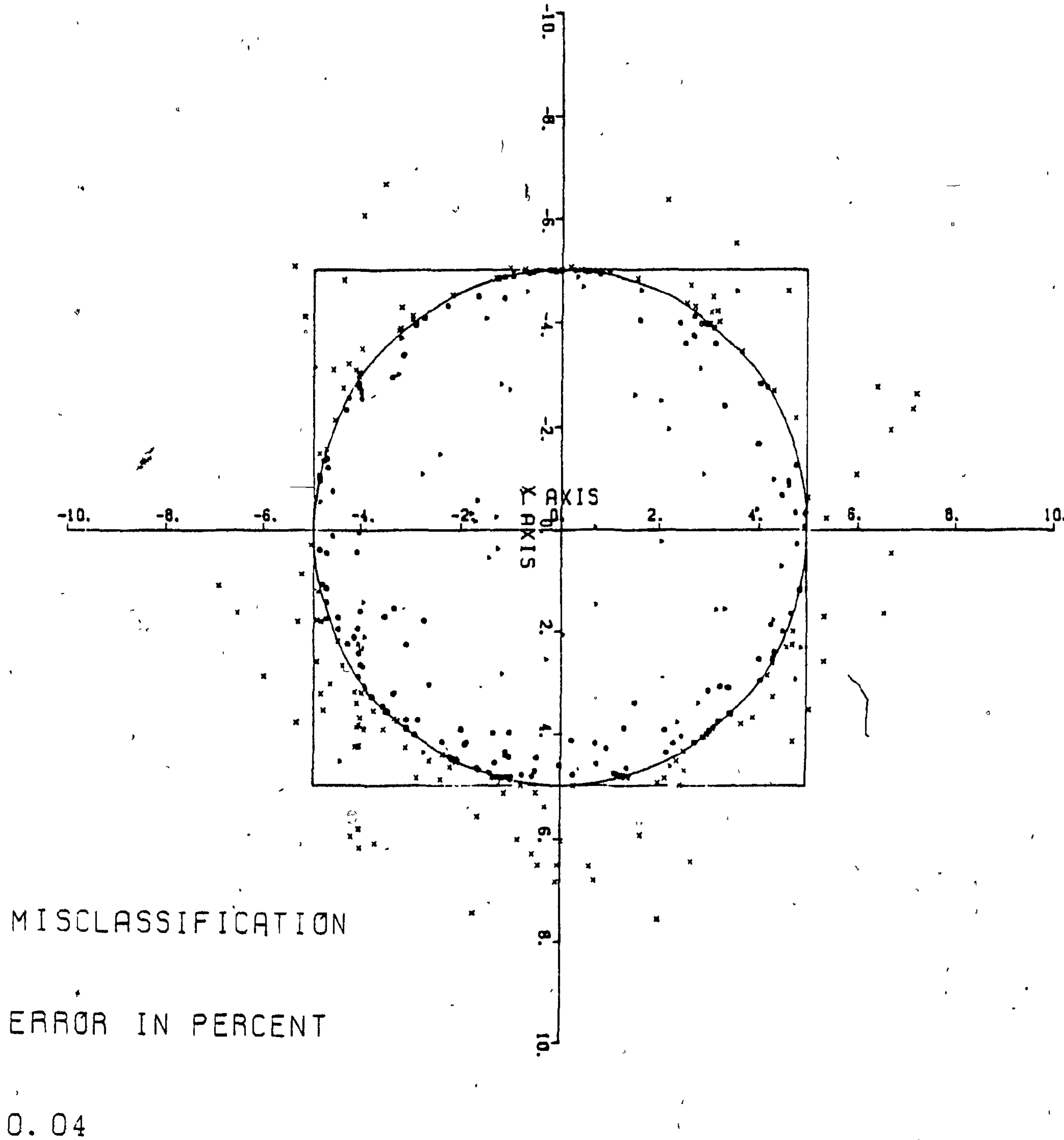


Fig.3.2 : Algorithm 1 applied to circle identification.

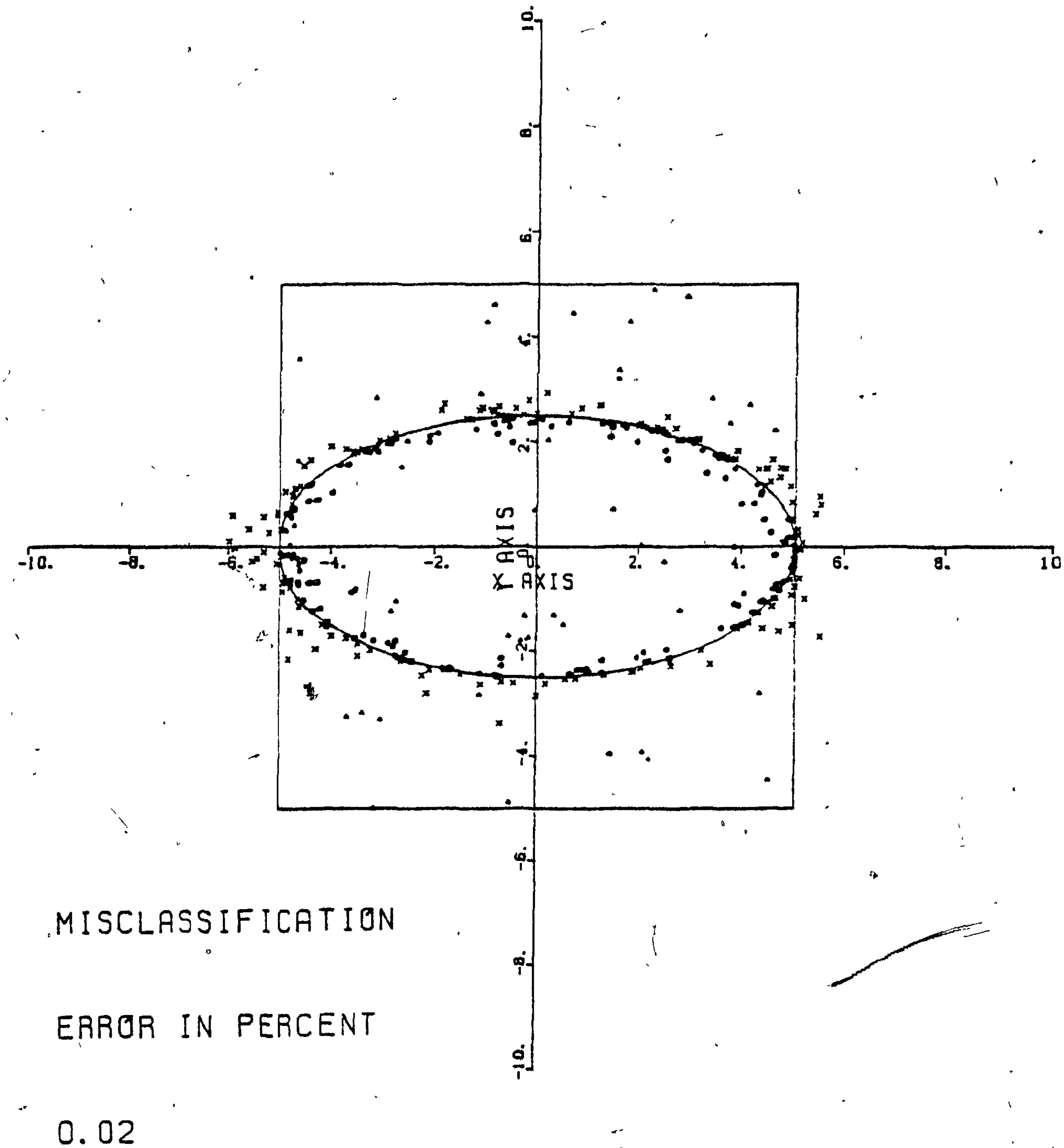


Fig.3.3 : Algorithm 1 applied to ellipse identification.

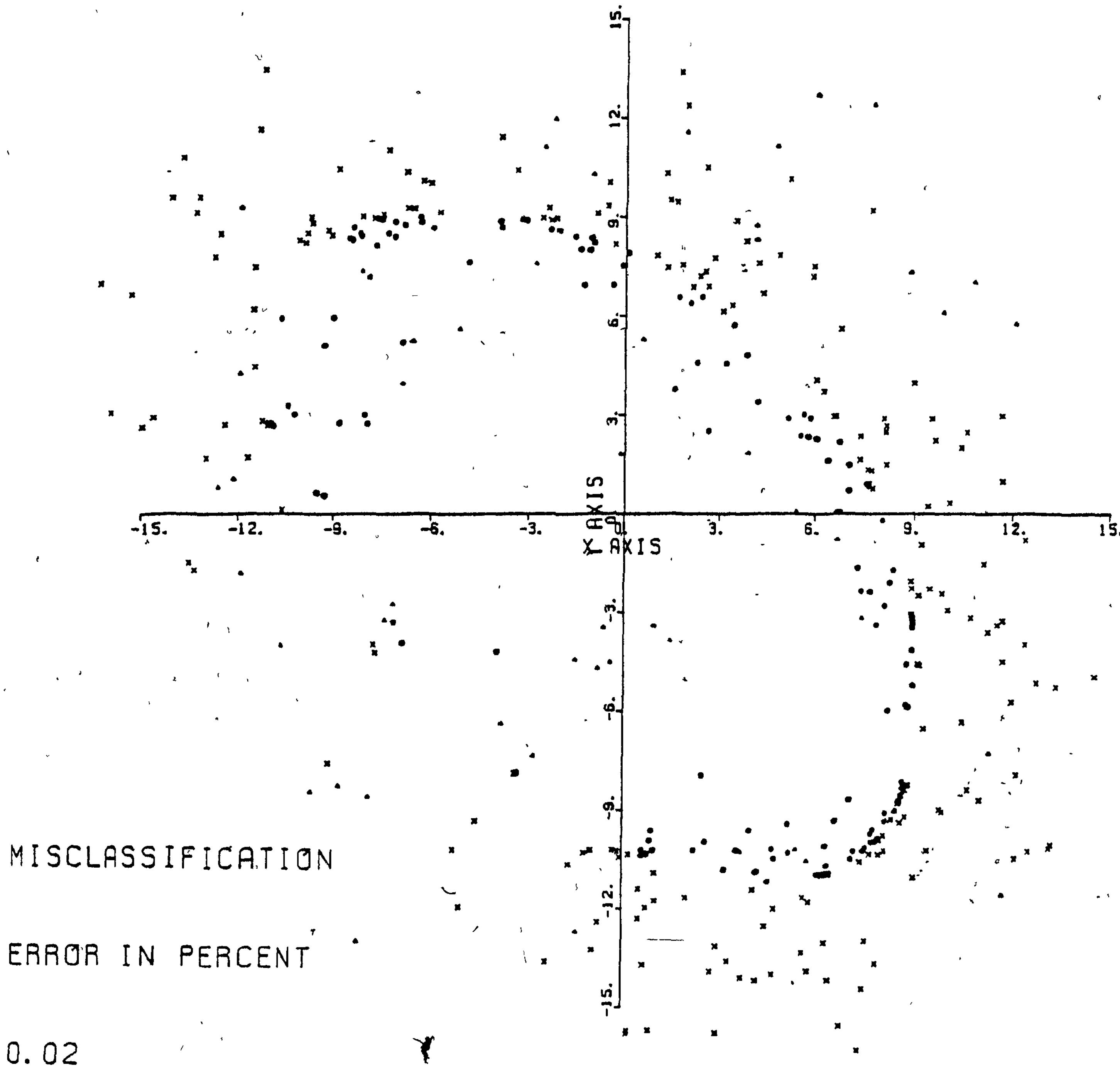


Fig.3.4: Algorithm 1 applied in the case of a 3-bus system.

3.5. Improving the misclassification error. Condensation.

When nearest neighbor decision rules are employed metric considerations are ultimately used for decision making concerning class membership. In our case, for instance, the usual Euclidian metric was used.

By construction, as can be seen from figs.3.2, 3.3 and 3.4, there is a zone of uncertainty between the set of stable and unstable training points. Any test point located in this zone of uncertainty runs a higher risk of being misclassified because, unfortunately, the previously described algorithm cannot tell which point from a specific pair (the stable or the unstable) is closer to the separation surface.

Any remedial measure to minimize this zone of uncertainty will eventually reduce the misclassification error. As a matter of implementation, "condensation" of the training sets was introduced before computing the misclassification rate.

The condensation procedure consists in reducing the distance (below a prespecified threshold) between the stable and the unstable point of every pair. The condensation procedure, as implemented here, reads as follows:

1. Consider the first stable (s) as well as the first unstable (u) training point from the already available uncondensed training data sets.
2. Compute the Euclidian distance between them.

3. If the computed distance is less than the prespecified threshold go to 5. Otherwise continue to 4.

4. Bisect the line defined by the points of the pair. Let the new point resulting from the bisection be the point (n).
   - If point (n) is found stable replace point (s) with point (n). Return to step 2.
   - If point (n) is found unstable replace point (u) with point (n). Return to step 2.

5. Store the new stable and unstable training points in new data sets.

6. Move to the next pair of points in the data sets.

The condensation procedure has been implemented and applied in all the above mentioned test cases. Fig.3.5 refers to circle identification after the data sets pictured in fig.3.2 have been condensed. Fig.3.6 refers to ellipse identification with the data sets pictured in fig.3.3 condensed. Finally, fig.3.7 illustrates the condensation procedure in the case of the actual 3-bus system (compare with fig.3.4). Computation of the misclassification error (with the same test sets) using as training sets the condensed data sets, indicates an improvement in the case of the circle. In fact for this particular case the misclassification error has been halved. However, in the remaining cases the condensation procedure had no immediate effect, in the sense that the misclassification error remained unaltered. In spite of the fact, though, that the condensation procedure has not affected the misclassification error, simple inspection indicates that the condensed training sets are superior candidates for any potentially applicable fitting scheme.

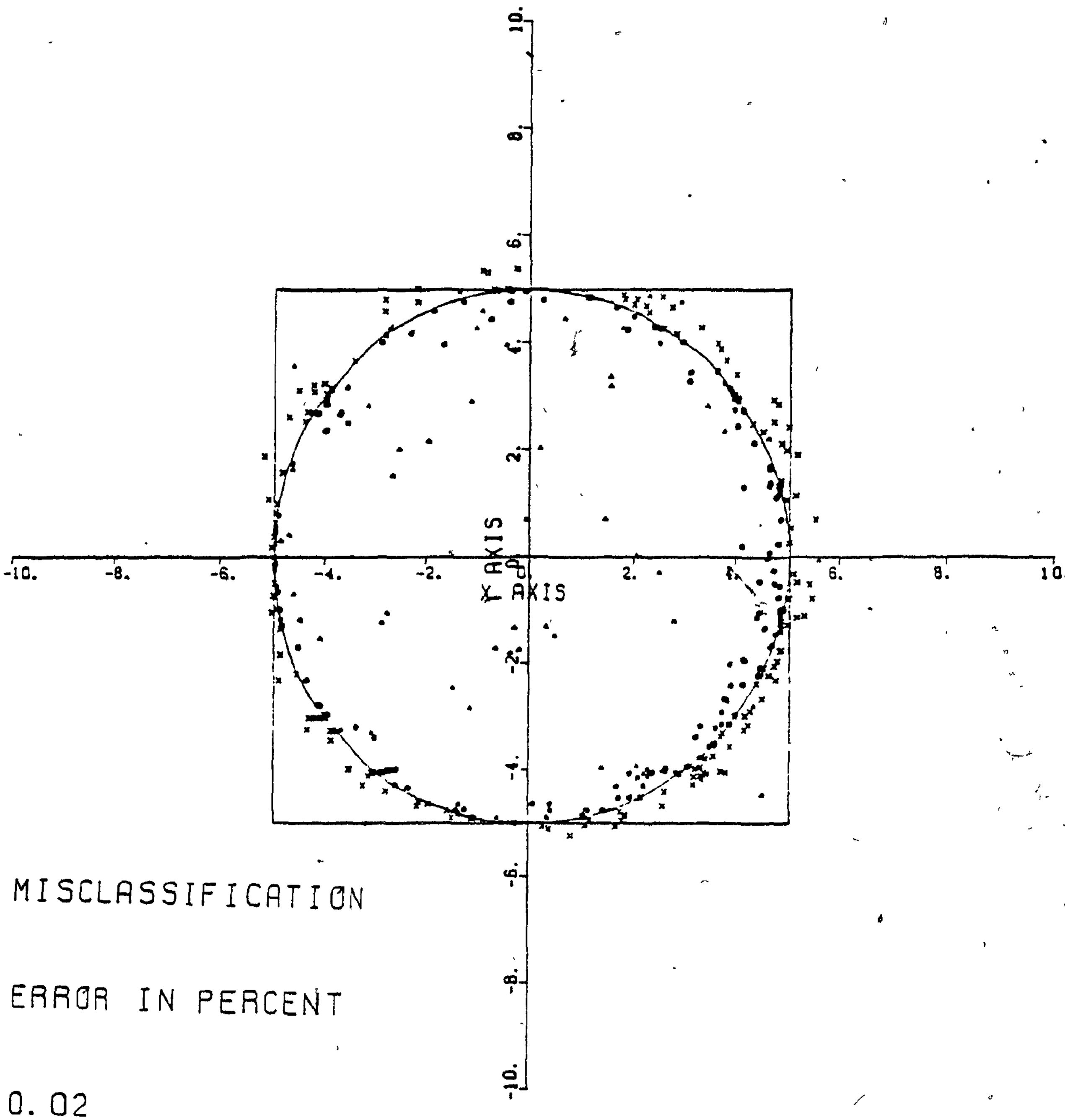


Fig.3.5: Circle identification. Condensed training sets.

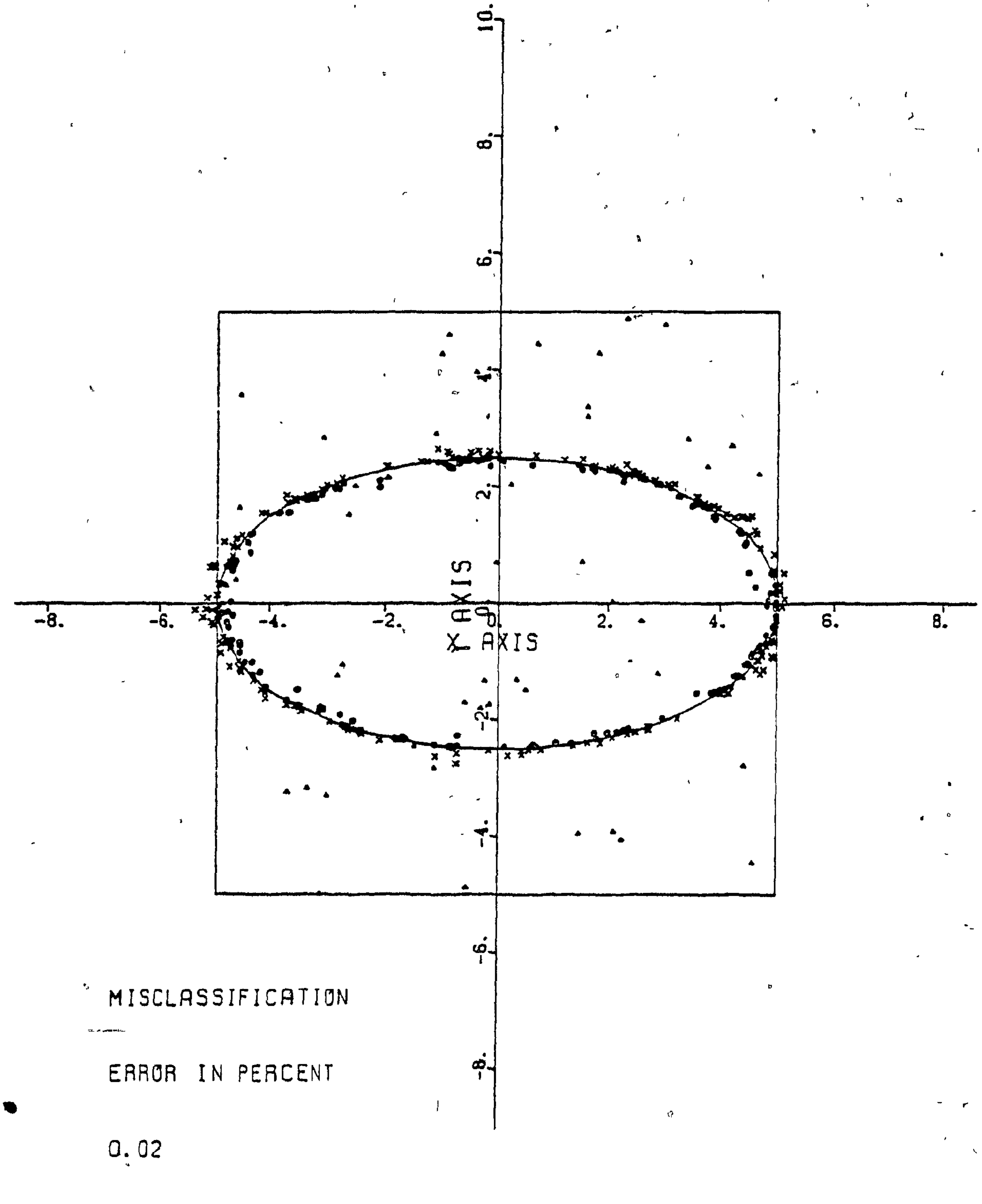


Fig.3.6: Ellipse identification. Condensed training sets.

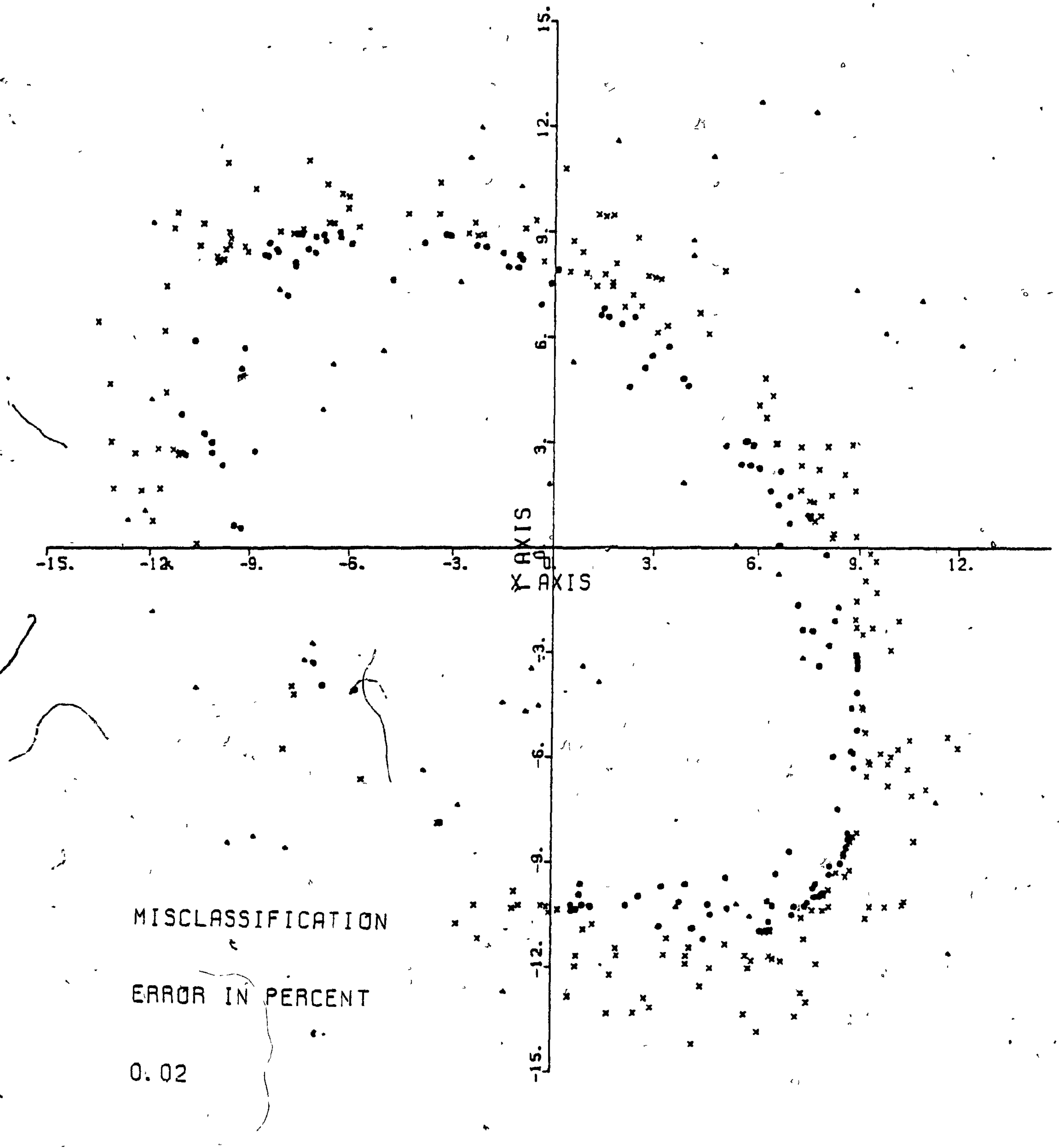


Fig.3.7: 3-bus system feasibility region identification.
Condensation of the training sets.

3.6. Algorithm 1. Numerical experience. Shortcomings and limitations. Search direction triviality.

It is apparent from the description of the algorithm that the "knee" points are randomly generated. After identifying a' and b' the searches proceed with a steplength of mod(a'b'). Numerical experience in two and higher dimensions demonstrates that, frequently enough, the points a' and b' end up being located very close to each other. This causes difficulties during the searches because the search direction itself is ill-defined.

In the event of such a situation, one is forced to generate new "knee" points that will, in turn, produce points a' and b' positioned in a well-conditioned manner. It has been observed that this phenomenon is encountered when the distance between the initially chosen points s and u is not substantial.

If one wants to avoid excessive condensation, it pays to choose the initial points (s) and (u) close to each other. Such a choice, however, increases the risk of encountering "search direction triviality". On the other hand, trying to avoid the above mentioned numerical degeneracy, excessive condensation may be imperative increasing the cost of the selection scheme .

As a matter of implementation, when coding algorithm 1, special sentinels (mod(a'b')< threshold) were used to detect "search direction triviality".

3.7. Algorithm 1. Numerical Experience. Shortcomings and limitations. Subspace restriction.

Assume that points a' and b' share one common coordinate. In two dimensions, this simply means that the two points are located on a line parallel to either one of the coordinate axes. (Sharing one common coordinate in higher dimensions means that the two points lie on a hyperplane of similar attributes).

If points a' and b' have one common coordinate, points s' and u' will share the very same coordinate. Entering the next iteration, the newly generated "knee" points will, by construction, share the same coordinate as well.

Thus, the algorithm will keep providing training points located in a subset of the space of interest (a straight line in two dimensions, a plane in three, a hyperplane in higher than three). Fig.3.8 illustrates the "subspace restriction" degeneracy as portrayed in the two dimensional case.

If points a' and b' share one common coordinate, we are forced to generate new "knee" points defining new trajectories until the resulting points a' and b' enjoy complete coordinate independence. Numerical experience shows that "subspace restriction" occurs, frequently enough, in higher than two dimensions.

3.8. Algorithm 1. Numerical Experience. Shortcomings and limitations. Proximity effect.

It is very probable, due to the nature of randomly conducted searches, that point s' is extremely close to the separation surface. Assume that this is the case. Then, points a' and b' will be found to be located even closer.

A simple glance at the algorithm reveals that, training points will continue to be provided but, they will be collected from a very restricted region of the separation surface, i.e the region in the neighborhood of s'. The algorithm will not be able to transfer the search to other areas of the state space and the result will be overidentification of the specific part. After exhausting the predetermined number of iterations, the training sets will contain redundant information, to say the least. The seriousness of the "proximity effect is further enhanced by the fact that is very hard to detect. One effective and easy to implement method to deal with the problem was found to be "bactracking". If "proximity effect" is detected point s' is replaced by point s". Point s" is located on the line defined by s' and u' in the direction of the stable training samples and mod(s's")=mod(s'u'). Fig.3.9 illustrates the effectiveness of this "backtracking" manoeuvre in two dimensions.

Early detection of the "proximity effect" is imperative because, this degeneracy, literally neutralizes a considerable number of iterations, rendering the procedure both inefficient and incomplete. Detection rests on the fact that an immediate consequence of the "proximity effect" is that points a' and b' will be located extremely close to point s'. This can be revealed by direct comparison of both mod(s'a') and mod(s'b') with prespecified thresholds.

Increasing the threshold values, the scheme will simply become more conservative with more "backtrackings" than needed. But this was found to have no implications whatsoever.

As a final comment on the "proximity effect", let us mention that, it appears with various degrees of severity, and it was found to be almost unavoidable in higher than two dimensions. Severe versions of it can render the algorithm virtually useless.

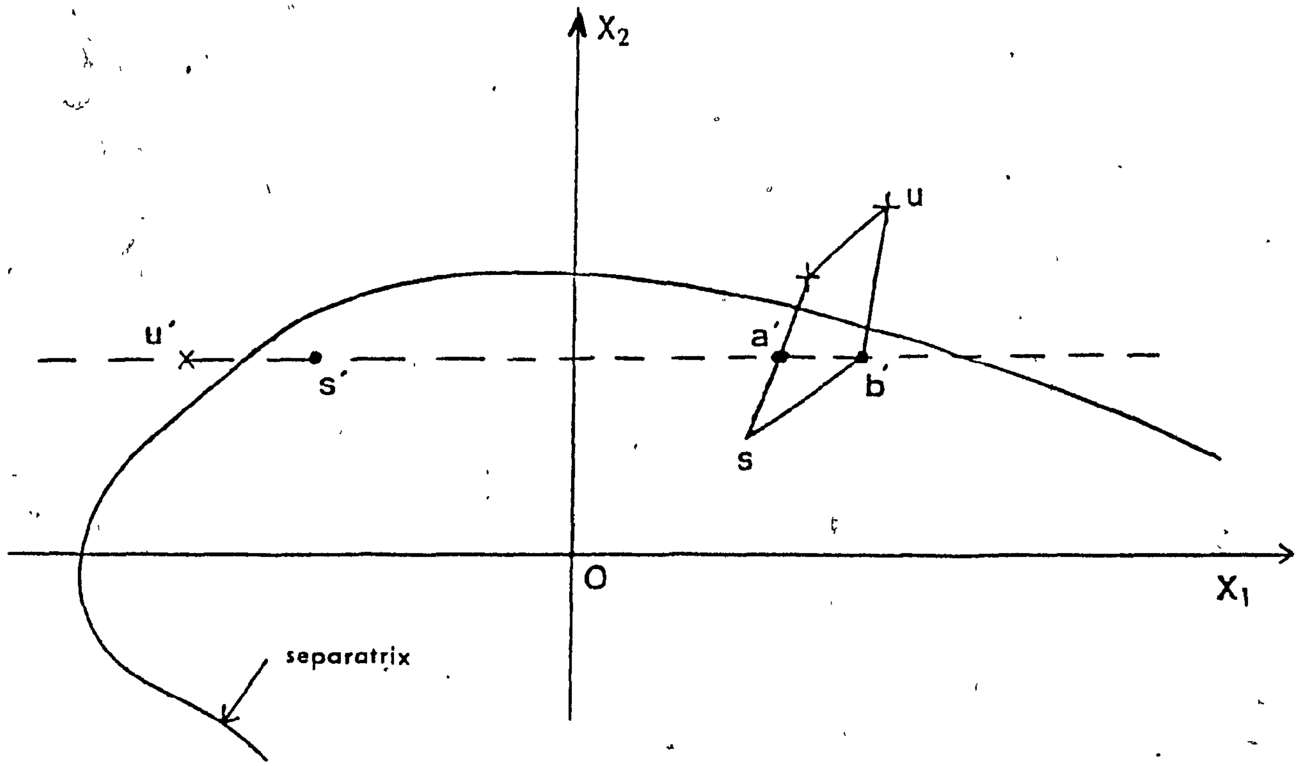


Fig.3.8: "Subspace Restriction" in two dimensions.

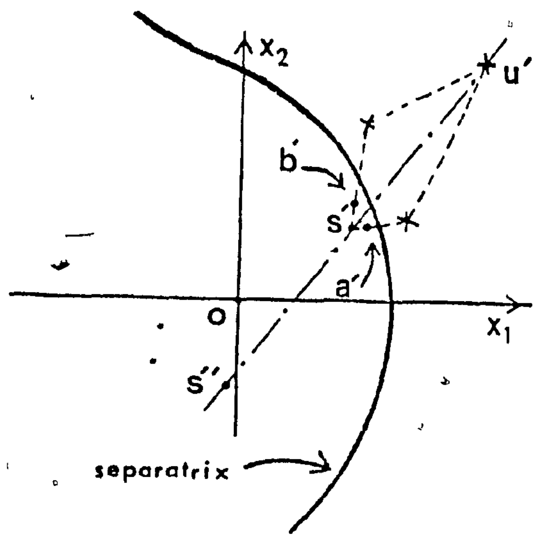


Fig.3.9: "Backtracking" and "proximity effect" as appear in a two dimensional space.

3.9. Algorithm 1. Overall Assessment.

When conceived, this algorithm seemed simple and easily implementable. That was the reason the author pursued its performance to a considerable detail. Unfortunately, the complications associated with the algorithm make the author reluctant to apply it to real life power system applications.

It can be seen from the description of the algorithm that the essential step is the transfer of searches to less known parts of the separation surface (step 6). The main motivation behind search selectivity was to achieve impartiality in training point selection. It has been observed, however, that such impartiality is not always achieved, especially in the case where the region under identification portrays ill-conditioned characteristics.

To be more specific, if one omits the rather obvious complications arising from local separation surface irregularity, the algorithm has a tendency to overidentify certain regions.

One extra reason for unattractiveness, when evaluating algorithm 1, is the fact that a considerable number of simulations could be lost if any of the above mentioned degeneracies are repeatedly encountered during the course of training point selection.

Under the circumstances, the author was more than motivated to explore other algorithms for training point selection.

Algorithm 1 was simply a reference pole for further research and as such should be viewed hereinafter.

3.10. Algorithm 2. Potential functions.

The concept of potential function in the context of Pattern Recognition engineering has assumed different meanings over the past two decades. The idea can perhaps be best traced to Parzen (59) and his attempts to construct probability density functions from the so-called "kernel" functions. Later, the term was used in conjuction with "stochastic approximation" techniques for regression functions (55,56,57,58). It is interesting to mention that, in all the above works, there seems to be no direct relevance of the term "potential function" with its intuitive meaning.

In Pattern Recognition engineering, when designing discriminant functions, one popular approach consists in emphasizing the differences between classes, and thus obtain characteristic discriminant functions. Such methods are, often, called "error correction" methods. Another proposed methodology to approach the discriminant function identification problem consists in utilizing only samples from one class and, then, yield discriminant functions for that class utilizing only "interclass" information. The later approach gave another dimension to the applicability of "potential functions" in Pattern Recognition. Generally, the techniques emerged from this research orientation can be viewed as aspects of the more general problem of reconstructing a probability density from samples of the process, or, of constructing a characteristic function from a fuzzy set.

In the present dissertation, however, "potential functions" will be used to mathematically express the decreasing relevance of a sample point $\underline{Y}$ upon point $\underline{X}$ as the distance $d(\underline{Y},\underline{X})$ increases.

Certain properties are desirable for the "potential function" if it is to be used in the above mentioned sense. These properties can be mathematically phrased as follows:

- $f(\underline{X},\underline{Y})$ should be maximum when $\underline{X}=\underline{Y}$.
- $f(\underline{X},\underline{Y})$ should be approaching zero for $\underline{X}$ distant from $\underline{Y}$ and in the region of interest.
- $f(\underline{X},\underline{Y})$ should be a smooth (continuous) function and tend to decrease in a monotonous fashion with the distance between $\underline{X}$ and $\underline{Y}$.
- if $f(\underline{X},\underline{Y})=f(\underline{Z},\underline{Y})$ where $\underline{Y}$ is a sample point, then the patterns $\underline{X}$ and $\underline{Z}$ should have approximately the same "degree of similarity" to $\underline{Y}$.

The function selected here is $f(\underline{X},\underline{Y})= e^{-(\underline{X}-\underline{Y})^T(\underline{X}-\underline{Y})}$, where T stands for "transpose".

3.11. Algorithm 2. Principle.

The potential at point $\underline{M}$ due to the presence of point $\underline{X}i$ is, as defined:

$$P_i(\underline{M}) = e^{-(\underline{M}-\underline{X}i)^T [C](\underline{M}-\underline{X}i)} \tag{3.1}$$

where [C] is the identity matrix of order n (no smoothing parameter considered), and $\underline{M}$ and $\underline{X}i$ are viewed as n-dimensional vectors (n=dimension of the state space).

With r points in the vicinity of $\underline{M}$, the potential evaluated at $\underline{M}$ becomes:

$$P(\underline{M}) = \sum_{i=1}^{r} P_i(\underline{M}) = \sum_{i=1}^{r} e^{-(\underline{M}-\underline{X}i)^T (\underline{M}-\underline{X}i)} \tag{3.2}$$

where Σ denotes summation over the total number of points $\underline{X}i$, i=1,2...r. Expanding eq.(3.2) gives:

$$P(\underline{M}) = \sum_{i=1}^{r} P_i(\underline{M}) = \sum_{i=1}^{r} e^{Q} \quad , \; Q=-\sum_{j=1}^{n}(M_j-X_{ij})^2 \tag{3.3}$$

where the outer sum is defined over the total number of points r and, the inner sum is defined over the dimensionality of the state space. M_j represents the jth component of vector $\underline{M}$, and X_{ij} the jth component of vector $\underline{X}i$.

Evaluating the gradient at $\underline{M}$ gives:

$$\nabla P(\underline{M}) = \left[\frac{\partial P}{\partial M_1} \;\ldots\ldots\; \frac{\partial P}{\partial M_n} \right]^T$$

where the gradient is an nX1 column vector with

$$\frac{\partial P}{\partial M_i} = 2 \sum_{\kappa=1}^{r} e^{L_\kappa} (X_{\kappa i}-M_i) \quad , \; i=1,2,\ldots,n$$

where:
- the sum Σ is defined over the total number of points.

- $L_K = -\sum_{j=1}^{n} (X_{Kj}-M_j)^2$, n= number of dimensions.

Note that L_K can also be viewed as: $L_K = -d^2(\underline{X_K},\underline{M})$

The orientation of the gradient vector indicates the steepest ascent direction for the scalar potential function at a given point. Recalling the meaning behind the mathematical definition of the "potential function" as used here, it is concluded that the gradient vector points towards areas of the state space which contain training points. Accordingly, the negative gradient direction will be directed towards areas which contain very few training points, or none at all. Those areas of the state space are the least explored and, consequently, of high interest. Any other search direction, but the gradient, is suboptimal .

3.12. Algorithm 2. Description.

The initial form of the algorithm implemented by the author reads as follows:

1. Find a stable (s) and an unstable (u) point.
2. Randomly generate n directions.(n=dimensionality of the state space).
3. Conduct searches along the n-randomly generated directions with point (s) as a starting point. Obtain 1 stable and 1 unstable training point for every search.

4. Find the gradient of the potential function generated by the training points at (s).

5. Generate n random directions in the state space.

6. Select, among the n randomly generated directions, the one with the largest inner product with the gradient vector.

7. Conduct a search along this direction with (s) as a starting point.

8. Return to step 4.

Steps 1,2 and 3 are considered to be the initialization steps of the procedure. At starting, when no training points are available at all, any search direction is acceptable and no special selectivity criteria have to be invoked.

In step 6 the selection of a random vector close to the negative gradient rather than the gradient itself was found to prevent an otherwise oscillatory behaviour of the algorithm.

Step 4 deals with the evaluation of the negative gradient vector at (s). Due to the nature of the "potential function" used, it is obvious that, points distant from (s) will have a negligible contribution to the evaluated potential. As a consequence, points in more remote areas of the state space will be of minor importance.

At first, the use of "weighting factors" was considered to increase the relative influence of distant points. This approach however was found to have two major shortcomings:

- The choice of the "weighting factors" had to be more or less empirical and, rather, application oriented.

- The scheme became computationally less attractive.

Instead, evaluating the potential at the mean value of the available stable training points, was found to be a simple and complication-free solution to the previously mentioned problem. What follows, is the final version of algorithm 2.

1. Find a stable (s) and an unstable (u) training point.
2. Randomly generate n directions in the state space of the system (n= dimensionality of the state space).
3. Conduct n searches along the randomly generated directions with (s) as a starting point, until n unstable points are found.
4. Find the mean value of the available stable training points and evaluate the negative gradient vector at the mean.
5. Generate n directions, randomly, in the state space.
6. Select as search direction the one, that gives the the largest internal product with the direction of the negative gradient.
7. Search along the selected direction with (m) as a starting point, until an unstable point is found.
8. Return to step 4.

3.13. Algorithm 2. Test Results.

The test procedure and data sets adopted for algorithm 2 are identical to the ones used for algorithm 1. The motivation behind such an approach is obvious.

At first, the algorithm has been tested on simple two dimensional geometric figures with well determined boundaries, in order to assess the impartiality of the training point selection scheme. Fig.3.10 demonstrates the results in the case of a circle of radius of 5 units. Dots represent points located in the interior of the circle and crosses represent points located out of the circle. Shaded triangles represent the test points used. It is seen that the impartiality of the training point selection is undisputable (no particular area near the separation surface is overidentified) while, at the same time, a fairly low misclassification error has been achieved. Fig.3.11 pictures the same data sets after the condensation procedure has been applied. Observe that the misclassification error has been halved.

Fig.3.12 represents the results for the case of an ellipse. "Dots", "crosses" and "shaded triangles" bear the same interpretation as before. Note, that in fig.3.12 condensation has already taken place. It is seen that, again, the misclassification rate is low. If instead of adopting the final version of algorithm 2, the initially proposed one is applied, the results are as in fig.3.13. It is seen that, in spite of the condensation effort, the misclassification error is doubled and, furthermore, local overidentification is more pronounced.

Finally, the ~~experimental~~ 3-bus system was used for comparison purposes again. Fig.3.14 indicates the training sets as resulting from the application of algorithm 2. Condensation has also been carried out. "Dots" represent steady state stable injections, "crosses" represent unfeasible injections and "shaded triangles" represent the test set. It is seen that, at first, very low misclassification error has been achieved (only one out of 50 test points used was misclassified) and, at second, impartiality of the selected training samples is attained. Compare Fig.3.14 with Fig.3.7 (identification of the feasibility region via the initially considered "random walk" scheme).

Although the misclassification error is not altered (for the given test set) it is seen that the training samples obtained via the "potential function" approach (Fig.3.14) are far more preferable than the ones portrayed in Fig.3.7.

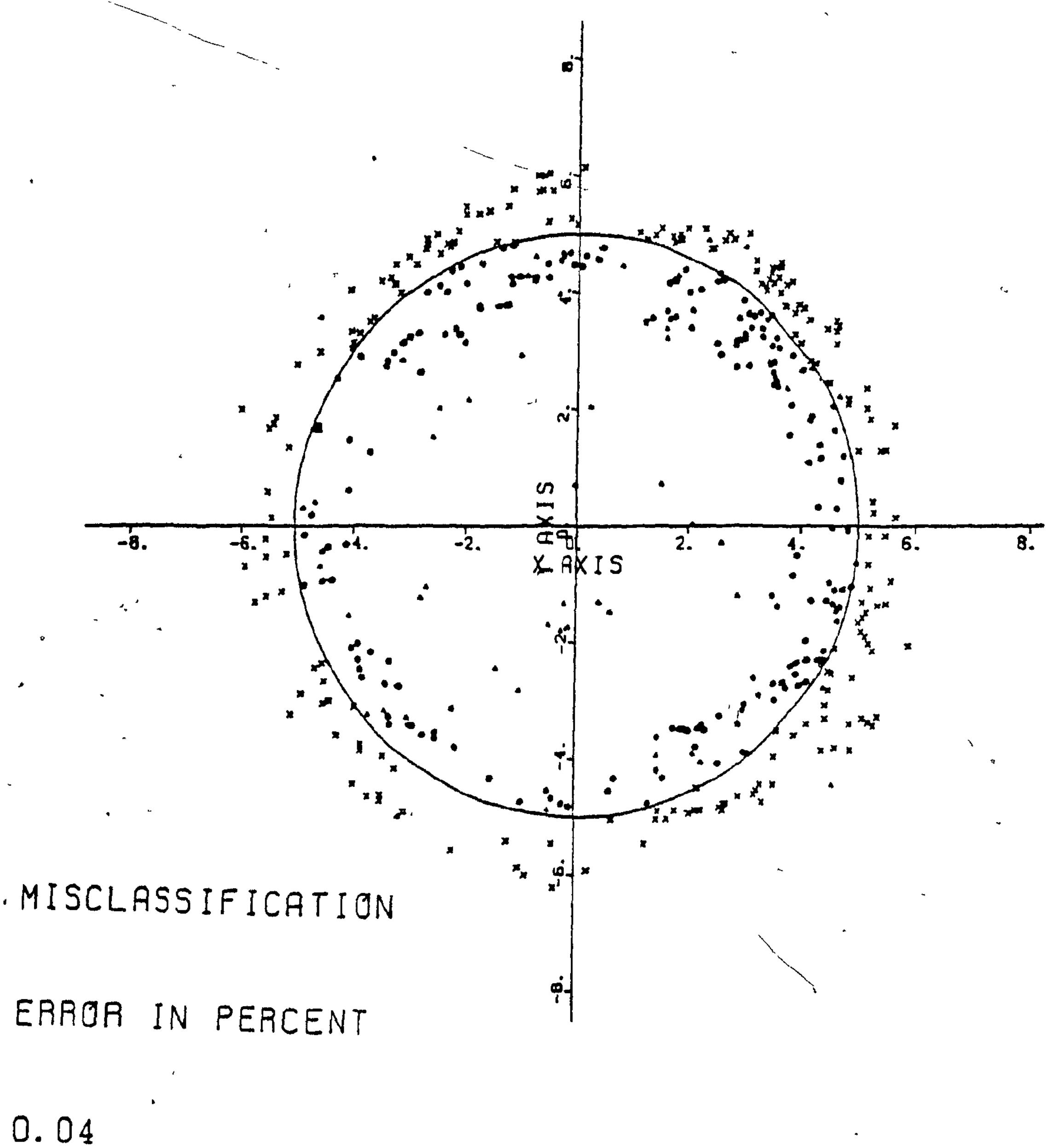


Fig.3.10: Algorithm 2. Circle of radius 5. Training sets.

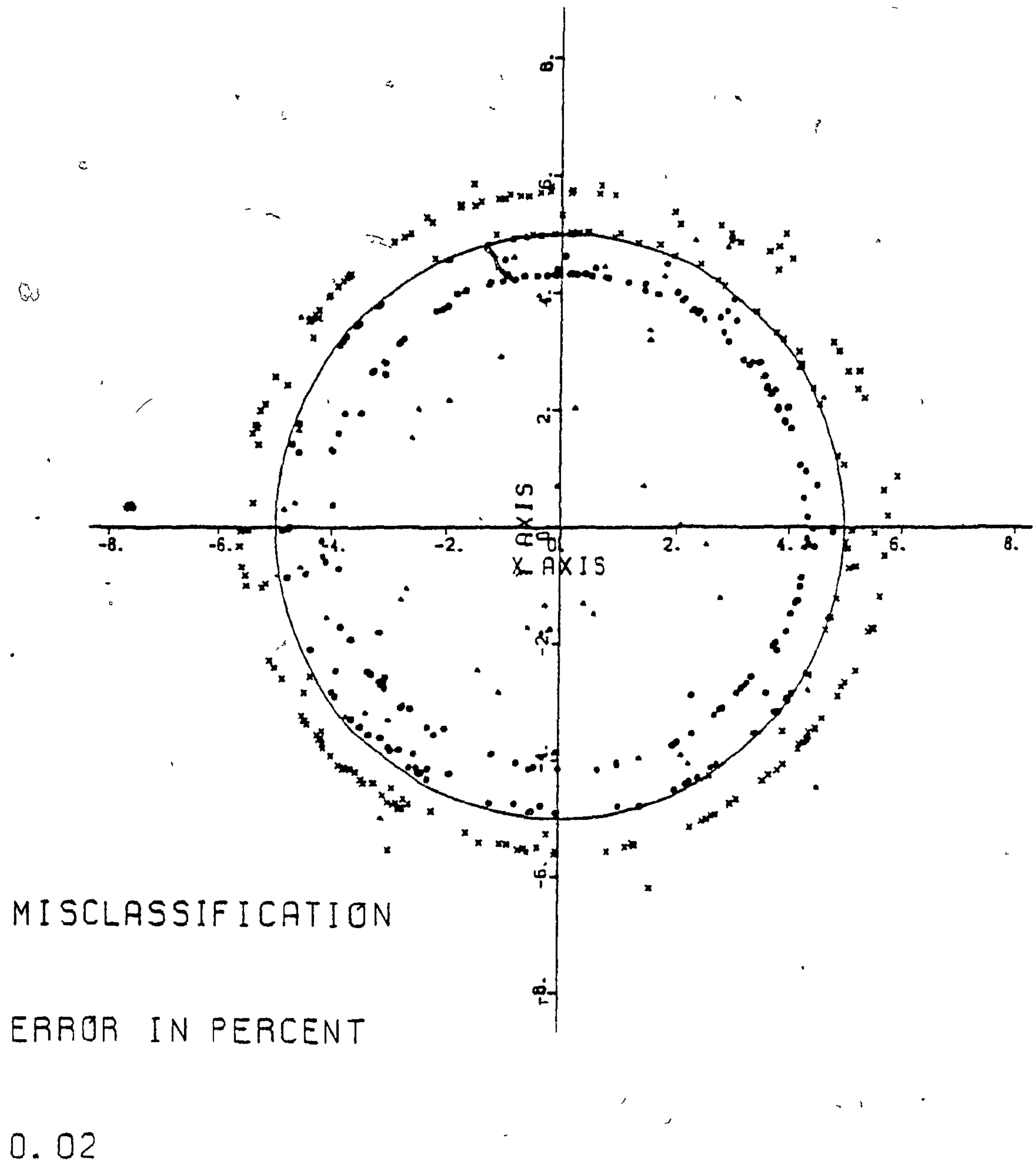
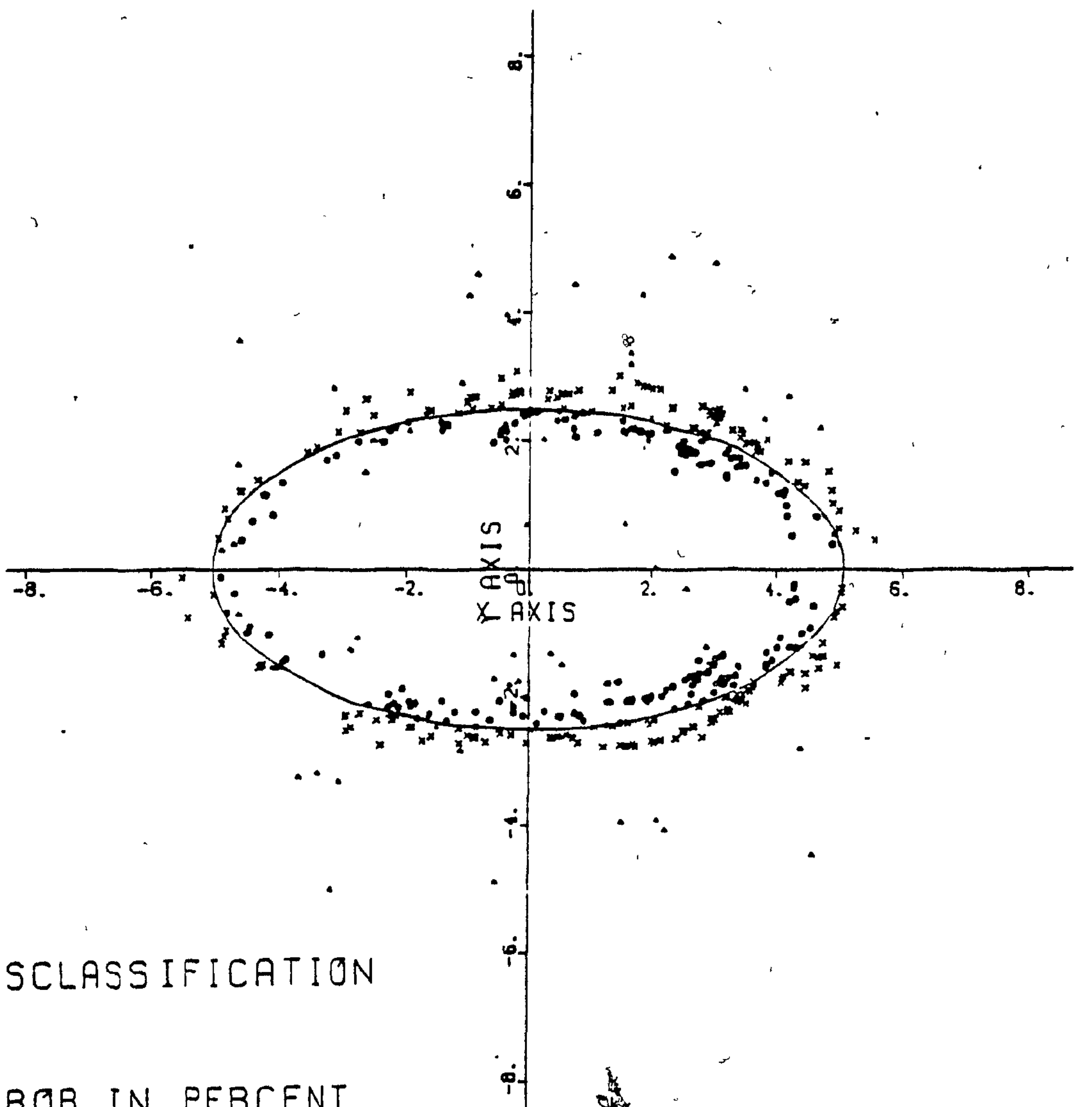


Fig.3.11: Algorithm 2 . Circle of radius 5. Condensed sets.



MISCLASSIFICATION

ERROR IN PERCENT

0.02

Fig.3.12: Algorithm 2. Ellipse in two dimensions.
Condensation of the training sets applied.

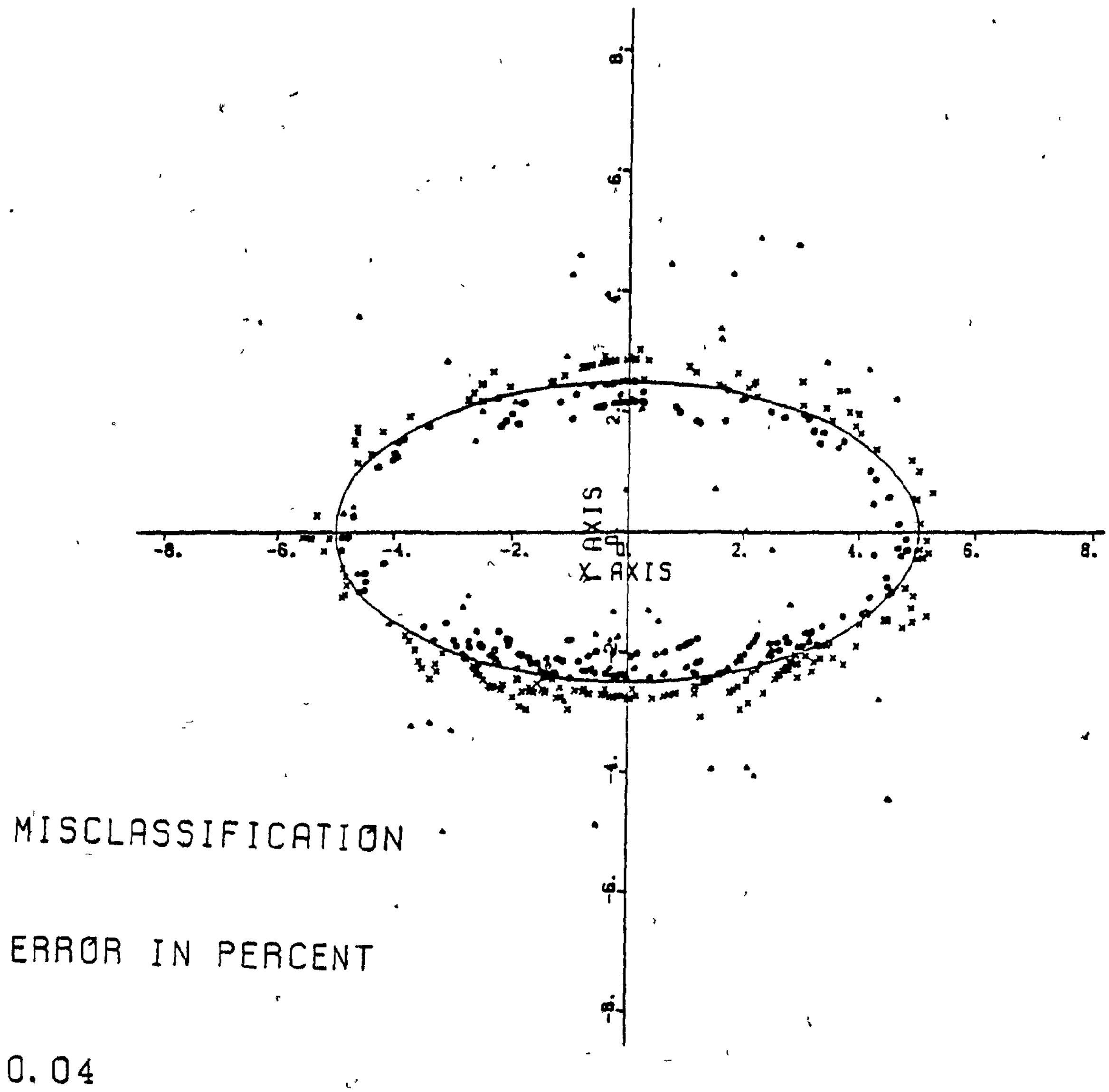


Fig.3.13: Algorithm 2 initial version. Ellipse in two dimensions. Training set condensation applied.

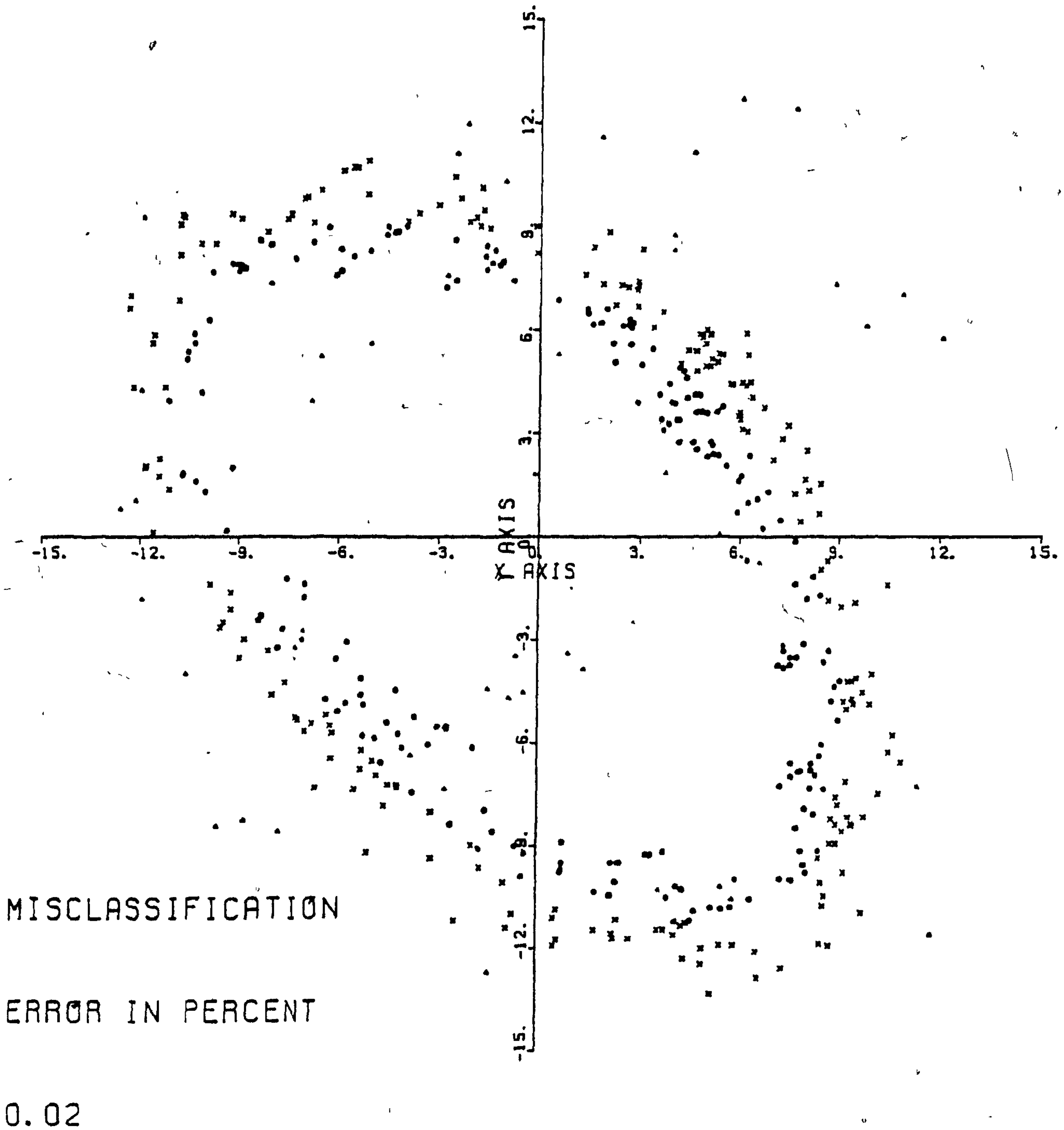


Fig.3.14: Algorithm 2. Final version. 3-bus system.
Condensation applied to the training sets.

CHAPTER FOUR

CONSTRUCTION OF THE HYPERELLIPSOIDS OF CONFIDENCE.

4.1. Introduction. Statement of the problem.

In the previous chapter, algorithms for training point selection have been presented and examined under the criterion that they satisfy two major requirements:

- Training point economization.
- Training point impartiality for global indirect region identification.

Algorithm 2, employing "potential functions", was found to be the best compromise among the selection schemes the author has been experimenting with. Applying algorithm 2, two sets of training points resulted. One set containing as elements points located in the interior and, another set containing as elements points located in the exterior of the region of interest. It is, again, emphasized that it is not the intention of the author to seek analytical expression for the separatrix but, rather, to establish regions of error free decision making, while at the same time minimizing regions of ambiguity.

Consider a hyperellipsoid (ellipsoid in three and ellipse in two dimensions) having the property that every point located in the interior of the hyperellipsoid is also a point located in the interior of the region under identification. Obviously, such a hyperellipsoid has to be inscribed in the region of interest. The motivation behind choosing a hyperellipsoid is rather apparent. A hyperellipsoid has a very simple analytical expression and yet retains a flavour of generality. It enables one to approximate a general region without resorting either to oversimplification (by assuming that the region can be approximated by a hypersphere) or to unnecessary and arbitrary complicating assumptions by assuming a form of higher degree.

However, if such a hyperellipsoid is to be contemplated, one needs to know its center, the orientations of its axes in the state space as well as their lengths. Such an information will, obviously, be obtained from the available training samples. The method of principal component analysis was utilized to infer the necessary information, as explained in the paragraphs to follow.

4.2. Hyperellipsoid Identification.

The method that follows is closely related to a very popular technique used in feature extraction when samples from one probability distribution are available (60,61,62). In this case, the problem of feature extraction is reduced to the one of efficiently representing the already available samples, in a space of lower dimensionality. The main attractiveness of the methodology lies in the fact that the effective features representing the data can be obtained by a linear transformation of the original variables. The technique is commonly known as the "discrete Karhunen-Loeve expansion" (25), and is the basis for numerous variations of feature extraction schemes.

Assume that a given random vector $\underline{X}$ is represented with respect to a certain basis in the state space. Assume that the matrix [L] has as columns the vectors chosen as basis vectors ($\underline{L}_1,\ldots,\underline{L}_n$) and, furthermore, that the basis vectors form an orthonormal set of vectors. Let $Y=(Y_1,\ldots.,Y_n)$ be the coordinate (row) vector of the random vector $\underline{X}$ with respect to the above chosen basis.

Consider that from the sum $\underline{X} = \Sigma\ (Yi.\underline{L}i)$, $i=1,2,\ldots,n$ a certain number of terms, say m, are omitted. In this case, an error is introduced in properly representing the random vector X with reference to the chosen basis. The problem is to choose the orthonormal set of vectors in such a way that, omitting a certain number of terms, minimum mean square error is achieved. It is proven (25) that, the set of orthonormal vectors required is the one containing the eigenvectors of the covariance matrix [S] of $\underline{X}$. The terms that are omitted from the summation are the terms pertaining to the eigenvectors corresponding to eigenvalues of [S] with the smaller magnitude. If no eigenvalue is omitted, then the corresponding eigenvectors will give a complete and accurate representation of the random vector $\underline{X}$ in a new coordinate system. However, the eigenvectors of the covariance matrix [S] bear a particular geometrical interpretation. They denote the direction of data point concentration in the state space. In other words, the eigenvectors of [S] indicate the directions along which the bulk of the available training points are to be found. Thus, detecting the modes of the previously non-structured data can be done by eigenanalysis of their covariance matrix. This reasoning is valid in any number of dimensions.

Among the directions indicated by the eigenvectors of [S] one may distinguish among the ones indicating major data concentration modes and the ones indicating rather minor and unimportant ones. The eigenvectors associated with major modes are the ones corresponding to large eigenvalues. Similarly, minor modes are indicated by eigenvectors corresponding to small in magnitude eigenvalues. In fact, it is proven (25) that the larger the eigenvalue the more important is the mode associated with it.

In our case, no mapping of data to a lower dimensionality space, with prescribed mean square error tolerance, is attempted. However, the principle behind the methodology of the discrete Karhunen-Loeve expansion has been fruitfully used to determine the orientation of the axes of the hyperellipsoid we, ultimately, seek to construct.

4.3. Hyperellipsoid of confidence of the feasible states.

This hyperellipsoid is required to contain as many feasible (in our case steady state stable) points as possible in the state of injections while, at the same time, containing no non-feasible points at all. At first (bearing in mind the reasoning outlined in the previous paragraph) an eigenanalysis of the covariance matrix of the feasible states is carried out. The eigenvectors denote the data modes and will be used as the directions of the axes of the hyperellipsoid. Furthermore, we shall consider the square roots of the eigenvalues, as lengths of the semiaxes of a hyperellipsoid. The equation of such an ellipsoid reads:

$$\frac{[X_1]^2}{\lambda_1} + \ldots\ldots + \frac{[X_n]^2}{\lambda_n} = 1$$

or in matrix form

$$X^T [L] X = H \qquad (4.1)$$

where [L] is the diagonal matrix:

$$[L] = \begin{bmatrix} \prod_{\substack{j=1 \\ j\neq 1}}^{n} \lambda j & & \\ & \ddots & \\ & & \prod_{\substack{j=1 \\ j\neq n}}^{n} \lambda j \end{bmatrix}$$

Eq.(4.1) describes a hyperellipsoid of a given size which is centered at the origin and whose principal axes are aligned with the axes of the coordinate system with respect to which the training samples are represented.

Eigenanalysis of the covariance matrix, however, provided the eigenvectors which, in turn,suggest the orientation of the axes of the ellipsoid. As a consequence, a rotation is in order to align the axes of the originally centered ellipsoid with the directions suggested by the set of the found eigenvectors. At this stage note that during the eigenanalysis of the covariance matrix, it is physically sound to expect distinct (real valued) eigenvalues equal in number to the dimension of the state space. On the other hand, the fact that eigenvectors corresponding to distinct eigenvalues are orthogonal to each other (for the case of a symmetric matrix) reduces the problem to that of aligning two orthogonal systems of coordinates via a rotation matrix.

Let $(\underline{X}_1,..,\underline{X}_n)$ be the old coordinate system, $(\underline{X}_1',..,\underline{X}_n')$ the new system of coordinates as the eigenvectors of the covariance matrix suggest. If [P] is the sought rotation matrix then:

$$(\underline{X}_1,....,\underline{X}_n) = [P]\ (\underline{X}_1',....,\underline{X}_n')$$

Assume, further, that both systems have been normalized. Then,

$(\underline{X}_1,...,\underline{X}_n) = [I]$, $[I]$ being the n-order unit matrix

and

$(\underline{X}_1',...,\underline{X}_n') = [Z]$

where: [Z] is the matrix whose columns are the normalized eigenvectors of the covariance matrix of the data.

We immediately conclude that: $[P] = [Z]^T$

Equation (4.1) then, becomes:

$$[Z^T \underline{X}']^T [L] [Z^T \underline{X}'] = H$$

Let: $$[Z]^T [L] [Z] = [B]$$

and eq.(4.1) becomes:

$$\underline{X}'^T [B] \underline{X}' = H \qquad (4.2)$$

This equation, is the equation of an ellipsoid whose size is identical to the one represented by equation (4.1) but of different orientation. Note, however, that the new ellipsoid is still centered at the origin of the original system of coordinates.

As a final step the center of the sought ellipsoid is to be found. The mean value of the data used to construct the sample covariance matrix is considered to be the center of the sought ellipsoid. In general, the sample mean of the data will not coincide with the origin of the original system of coordinates. Accordingly, a translation is to be carried out, in order to achieve the proper positioning.

Denote by $\underline{X}''$ the coordinates of a point after the translation has been carried out, and by $\underline{X}'$ its initial coordinates. Then:

$\underline{X}' = \underline{X}'' - \underline{T}$, $\underline{T}$ vector of the sample mean coordinates.

Accordingly, eq.(4.2) becomes:

$$(\underline{X}''-\underline{T})^T [B] (\underline{X}''-\underline{T}) = H$$

or

$$\underline{X}''^{T} [B] \underline{X}'' - 2 \underline{T}^{T} [B] \underline{X}'' = H - \underline{T}^{T} [B] \underline{T}$$

Let: $\underline{a}^{T} = -2 \underline{T}^{T} [B]$ and $R = H - \underline{T}^{T} [B] \underline{T}$

The final equation then, for the hyperellipsoid reads:

$$\underline{X}''^{T} [B] \underline{X}'' + \underline{a}^{T} \underline{X}'' = R \qquad (4.3)$$

There is no guarantee, however, that the hyperellipsoid constructed using the eigenvalues of the covariance matrix of the data will satisfy the joint requirements of not containing unstable training states and containing as many feasible states as possible. It has been the experience of the author that, while the first requirement is always met, the second falls short from being fulfilled. This is due, in our opinion, to the fact the data set used to construct the covariance matrix is "hollow" (Recall that the algorithms providing the training points provide points located in the vicinity of the separation surface). Fig 4.1 illustrates such a hyperellipsoid for the case of the data pertaining to the 3-bus experimental system.

At this point we conclude that, although both the center and the orientation of the seeked ellipsoid are reasonably well determined its volume still remains an open question. An algorithmic procedure dealing with the expansion of the available ellipsoid is needed. The author has tried several computerized expansion schemes. The finally adopted one is illustrated in the flow chart of fig. 4.2.

The flowchart pertains to the iterative procedure for one eigenvalue. The procedure illustrated in the flowchart of fig.4.2 is repeated for all the eigenvalues. The already determined eigenvalues are utilized in the iterative procedure for the computation of the subsequent eigenvalues.

The scheme is based on the well recognized fact that the volume of the ellipsoid given by equation (4.3) is critically dependent on the numerical values of the eigenvalues utilized in equation (4.1). In fact, any increase in the magnitude of an eigenvalue will cause the resulting ellipsoid to expand in the direction of the corresponding eigenvector.

The presented flow chart shows that the mechanism of expansion is based on the fact that one should assess one principal direction at a time until a barrier of unstable states is found towards that direction. This mechanism allows more flexibility in manipulating the volume of the sought ellipsoid, and results indicate that the ellipsoids found are superior candidates when compared to others found from schemes adopting concurrent expansion of more than one principal direction. The reason is that the axes of the sought ellipsoid will not necessarily be comparable in magnitude. Such an assumption is obviously arbitrary and cannot be tolerated. Any computerized scheme based on concurrent expansion will eventually stop short towards one principal direction (where further expansion could be possible) simply because a barrier of unstable states was found towards a rather minor mode. The undesirable consequence is that the resulting ellipsoid will turn out to be rather undersized. In chapter V the consequences of operating with such a result are fully explained. Fig.4.3 pictures the ellipsoid of confidence of the stable states (ellipsoid of type 1) as deduced applying the algorithmic procedure illustrated in the flow chart of Fig.4.2. "Dots" represent feasible states and "crosses" represent unstable states. The coordinate axes represent the real power injections for buses 2 and 3 of the 3 bus experimental system utilized in chapter III.

The equation of the hyperellipsoid (ellipse in this particular case) is of the form:

$$\underline{X}^T [A]\, \underline{X} + \underline{b}^T \underline{X} + C = 0$$

where: $\underline{X}$ is the two-dimensional vector, having as entries the real power injections at buses 2 and 3.

[A] is the matrix for the quadratic term. In this particular case the matrix (2 by 2) reads:

$$\begin{bmatrix} 83.87392 & 35.85168 \\ 35.85168 & 69.83078 \end{bmatrix}$$

$\underline{b}^T$ is the coefficient for the linear term. In this particular case (1 by 2) it reads:

[-89.24890 33.75565]

C a scalar. In this particular situation it reads:

-4524.18359

4.4. Hyperellipsoid of confidence of the non feasible states.

The necessary requirements this ellipsoid has to meet are, as stated in the beginning of this chapter, the following:

- The ellipsoid should contain all the stable training points in the space of injections.
- The ellipsoid should contain as few unstable training points as possible in the space of injections.

The techniques and the ideas used to construct the ellipsoid are, essentially, based on the already contemplated arguments used for the construction of the ellipsoid of type I. More specifically, principal component analysis (via eigenanalysis of the covariance matrix of the data) is used here as well to determine the orientation of the axes of the ellipsoid and, finally, the sample mean value of the data is, again, assumed to be the center of the sought ellipsoid.

One should recall at this point that, for reasons already stated, the ellipsoid resulting from eigenanalysis of the covariance matrix of the data is rather undersized and, as a rule, needs to be expanded. Accordingly, an algorithmic procedure to determine the size of the ellipsoid has to be implemented as previously.

Nevertheless, the strategy adopted in the case of ellipsoid of type I cannot be used here because the requirements the ellipsoid has to comply with are rather different.

As far as the first requirement is concerned, any ellipsoid containing all the stable training points is a potential candidate for the one we seek. However, the size of the ellipsoid which is to be finally adopted is of great importance (see chapter V). To state the problem differently, one looks for the ellipsoid of the minimum size among the ones containing the stable training points. It is in order to note at this point however that both the directions of the axes as well as the center of the ellipsoid are predetermined.

The first question that arises is whether an expansion scheme based on the idea of concurrent eigenvalue increase (see paragraph 4.2) is to be adopted, or whether one should resort once more to expansion schemes relying on the concept of modifying only one eigenvalue at a time (i.e, exerting influence on the size of the ellipsoid towards one axis at a time).

The alternative of concurrent expansion (expanding the ellipsoid towards more than one eigendirection at the time by adjusting the size of the corresponding eigenvalues) will invariably lead to suboptimal solutions. This was verified by numerical experience, and is due to the fact that while one may still have stable training points along one of the eigendirections (fact implying that further expansion is possible towards that direction), the stable training points towards one (or more) of the remaining eigendirections may have been exhausted. Consequently, any further expansion along the dominant eigendirection (during the course of the attempt to include all the stable training points as initially requested) will cause unnecessary expansion along the remaining ones. Thus, the suboptimality of the resulting solution.

It is apparent, therefore, that one should rather restrict himself in exploring the remaining alternative, i.e, to devise an expansion scheme that is based on the strategy of adjusting the magnitude of only one eigenvalue at a time.

The next question one is faced with, is to determine which axis, should be adjusted at every iteration. The criterion for selecting the appropriate axis must be such that, on the one hand, the resulting hyperellipsoid possesses the required properties and, on the other hand, for the sake of efficiency, the smallest possible number of iterations be required.

The criterion applied for axis selection reads as follows:

- Expand along the axis whose length modification, by a specified increment, will cause the resulting ellipsoid to contain the minimum number of unstable training points, among the number of unstable training points.

The above criterion is to be applied during every iteration of the procedure. The next iteration will be performed on the condition that not all of the stable training points have been included in the ellipsoid resulting from the current iteration.

Nevertheless, the criterion listed above will, almost invariably, cause the algorithm not to converge in a finite number of steps, in spite of the fact that it is an excellent guideline to follow in search for ellipsoid size optimality.

Failure to converge occurs because the criterion does not require that the number of included stable training points be definitely increased from iteration to iteration. Such a provision would guarantee convergence in a finite number of steps. If no preventive action is taken and the algorithm is designed with only the above listed criterion in mind, the resulting ellipsoid (if any) will be of such size as to have no practical importance whatsoever. Numerical experience showed that in many cases, no convergence at all was attained while in the cases a result was obtained, the resulted ellipsoid was unnaturally distorted along the more dominant eigendirection.

In an attempt to avoid such continuous ellipsoid expansion along algorithm-preferred directions, the construction of a "sentinel" ellipsoid was found to be a very satisfactory, and simple, solution. Before entering the procedure for ellipsoid expansion, a "sentinel" ellipsoid is constructed which will act as a safeguard against the previously mentioned ill-situation. The "sentinel" ellipsoid has as center the center used for the sought ellipsoid and its axes will have the same orientation as the ones of the ellipsoid which ultimately is to be constructed (eigenvectors of the covariance matrix of the data). It is constructed using the simple to implement concurrent axis expansion philosophy (all eigenvalues are increased at every iteration by a specific increment till all the stable training points are contained in it). Obviously, no ellipsoid with an axis larger than the corresponding axis of the "sentinel" ellipsoid is acceptable.

Fig.4.5 pictures the ellipsoid of type II in the case of the 3-bus system after the algorithmic procedure illustrated in fig.4.4 was applied. Note that in this particular case, the covariance matrix of the unstable training points was utilized.

The parameters for the equation of the hyperellipsoid (ellipse here) read as follows:

[A] the matrix for the quadratic term (2 by 2 here)

$$\begin{bmatrix} 173.646 & 17.565 \\ 17.565 & 165.018 \end{bmatrix}$$

$\underline{b}^T$ the coefficient vector for the linear term. (1 by 2 in this case)

[-276.986 196.483]

C a scalar, the constant term of the equation.

-28158.4844

Fig.4.6 pictures the ellipsoid of type II that would result (for the data sets pertaining again to the 3-bus system) if the covariance matrix of the stable training points were used instead. As expected, no substantial difference was found (due to the fact that the data sets have been condensed). Note though that, the ellipsoid pictured in fig.4.7 is slightly reduced in size if compared with the one of fig.4.6.

The parameters for the ellipsoid in question read as follows:

[A] the matrix for the quadratic term (2 by 2 here).

$$\begin{bmatrix} 162.236 & 31.116 \\ 31.116 & 150.048 \end{bmatrix}$$

$\underline{b}^T$ the vector coefficient for the linear term. (1 by 2 in this case)

$$[-223.071 \qquad 147.291]$$

C a scalar, the constant term of the equation.

$$23235.7539$$

It is our opinion that the stricter the condensation, the more resemblence will be achieved in the two ellipsoids. However, the covariance matrix of the stable training points is recommended for use.

An equally appealling criterion for ellipsoid expansion is the following:

- Expand along the axis whose length modification, by a specific increment, will cause the resulting ellipsoid to contain the maximum number of stable training points, when compared with the number of stable training points other ellipsoids would contain had the expansion taken place along any other eigendirection.

The ellipsoid found by adopting the above given criterion is pictured in fig.4.7. Note that no substantial difference is found if this ellipsoid is compared with the one shown in fig.4.6, apart from the fact that it is slightly increased in size. Again the covariance matrix of the stable training points was utilized.

The parameters of the equation of the ellipsoid in this case read:

[A], the matrix for the quadratic term (2 by 2 here).

$$\begin{bmatrix} 176.320 & 14.007 \\ 14.007 & 170.833 \end{bmatrix}$$

$\underline{b}^T$ the vector coefficient for the linear term. (1 by 2 in this particular situation)

$$[-268.569 \quad 202.564]$$

C a scalar, the constant term of the equation.

29748.9766

In all the above considered algorithmic variations the eigenvalue increment was taken to be equal to one half of the square root of the smallest in magnitude eigenvalue of the covariance matrix of the data. If smaller increments for the eigenvalues are used in the procedures for ellipsoid expansion, more refined solutions will be obtained. Naturally, more iterations are needed to reach the solution. For large data sets this may require excessive computational effort.

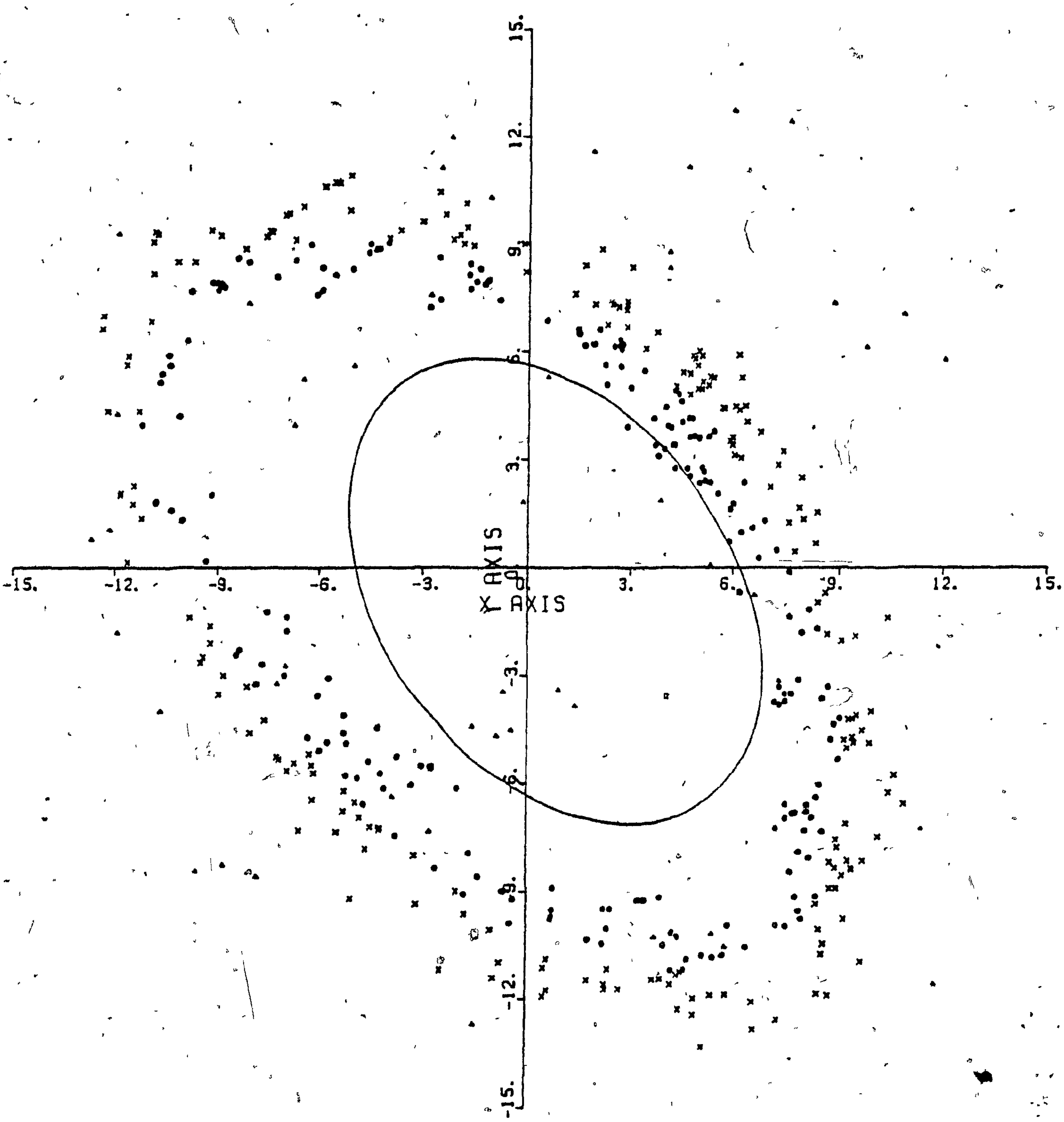


Fig.4.1: Covariance matrix hyperellipsoid in 2 dimensions.

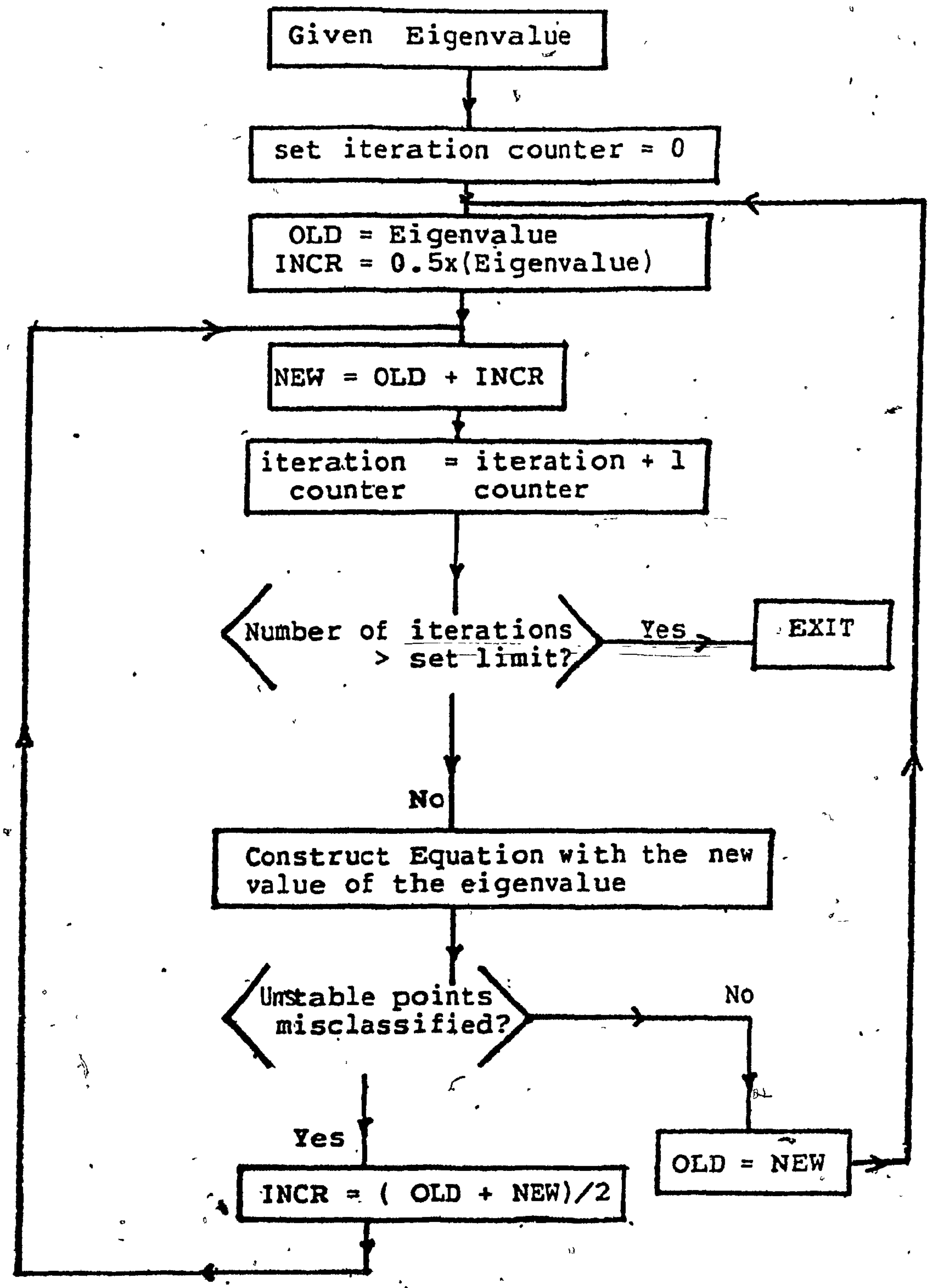


Fig.4.2: Flowchart for hyperellipsoid of type I.

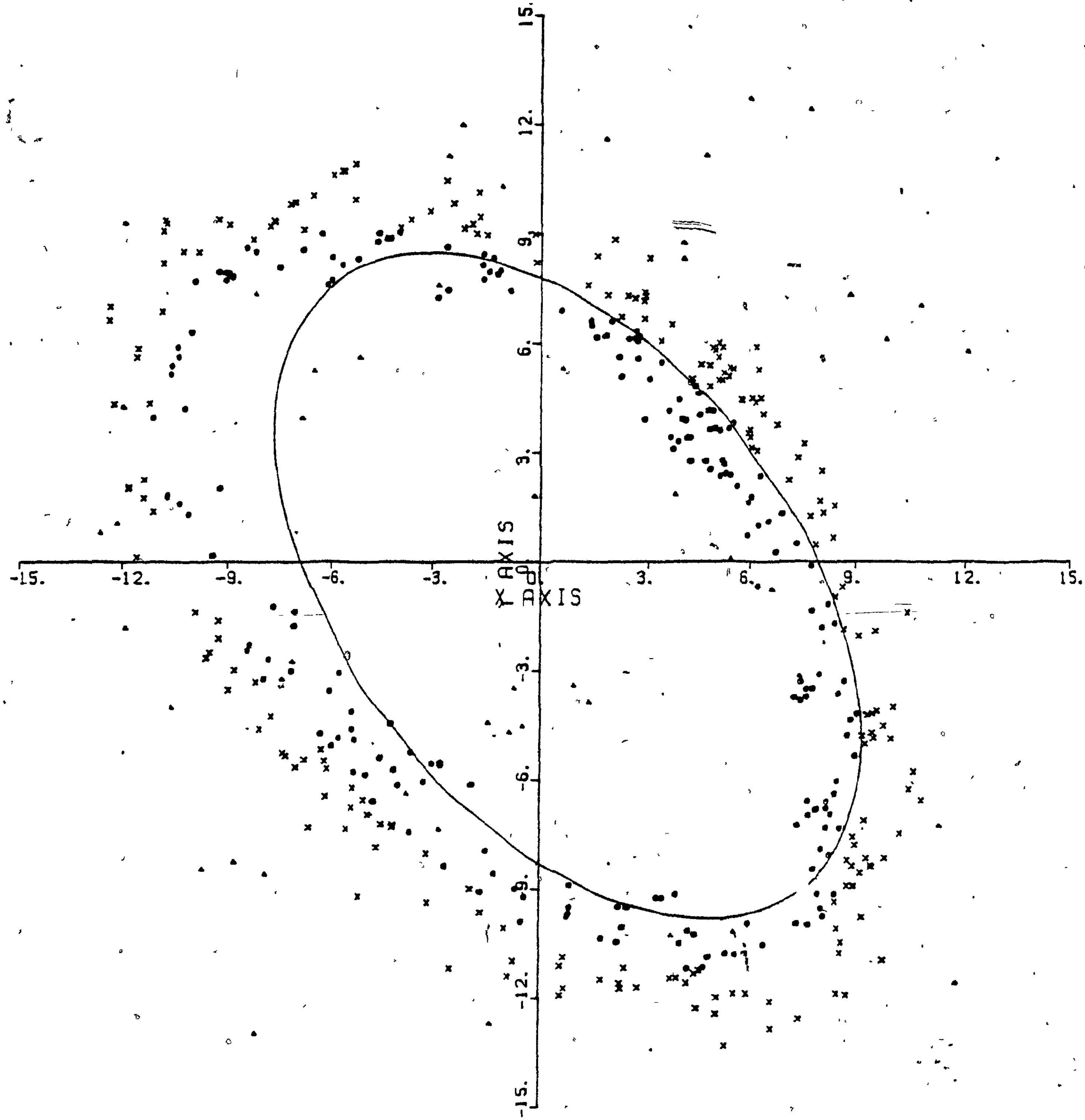


Fig.4.3: 3-bus system. Hyperellipsoid of type I.

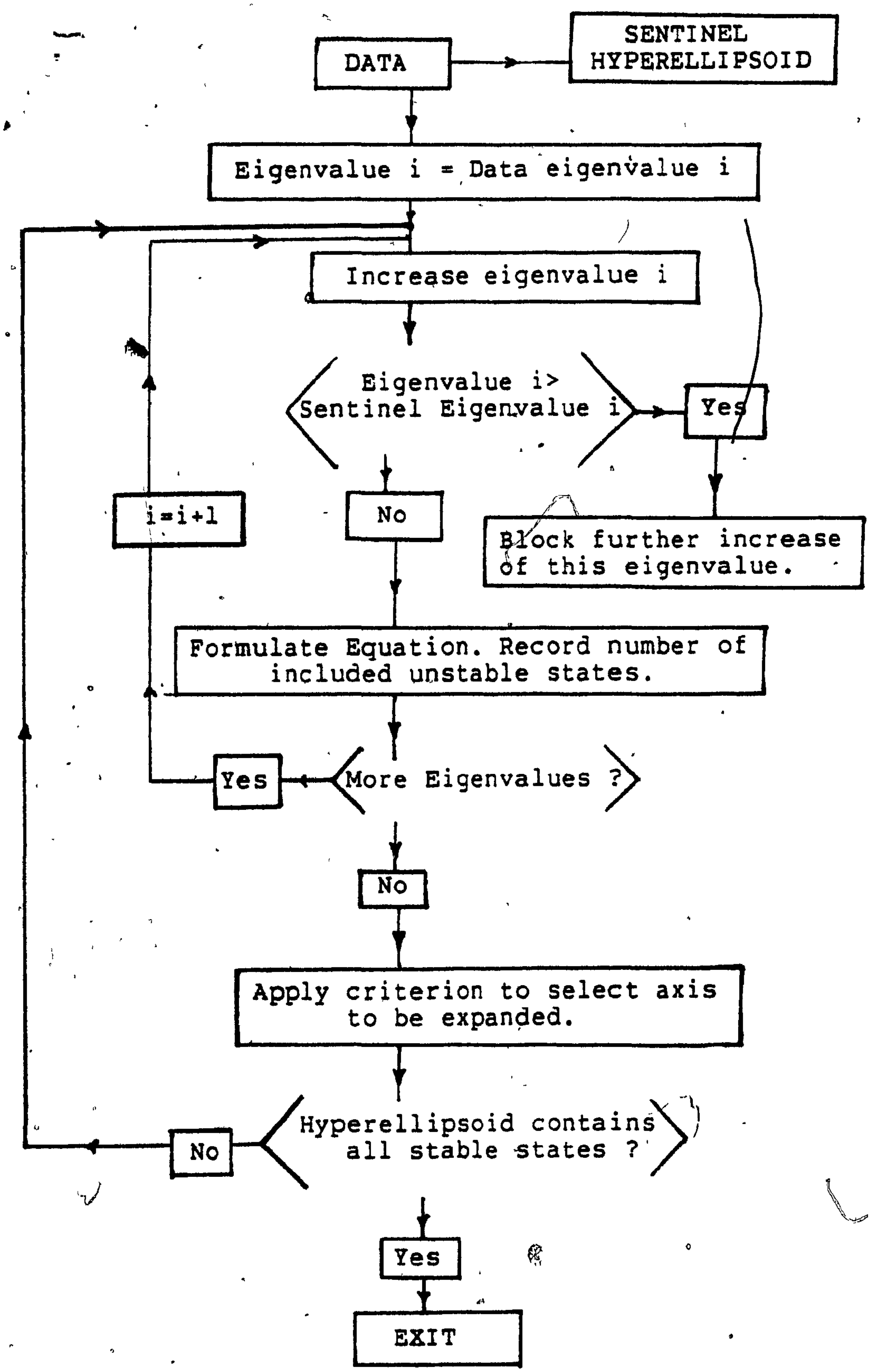


Fig.4.4: Flowchart for hyperellipsoid of type II.

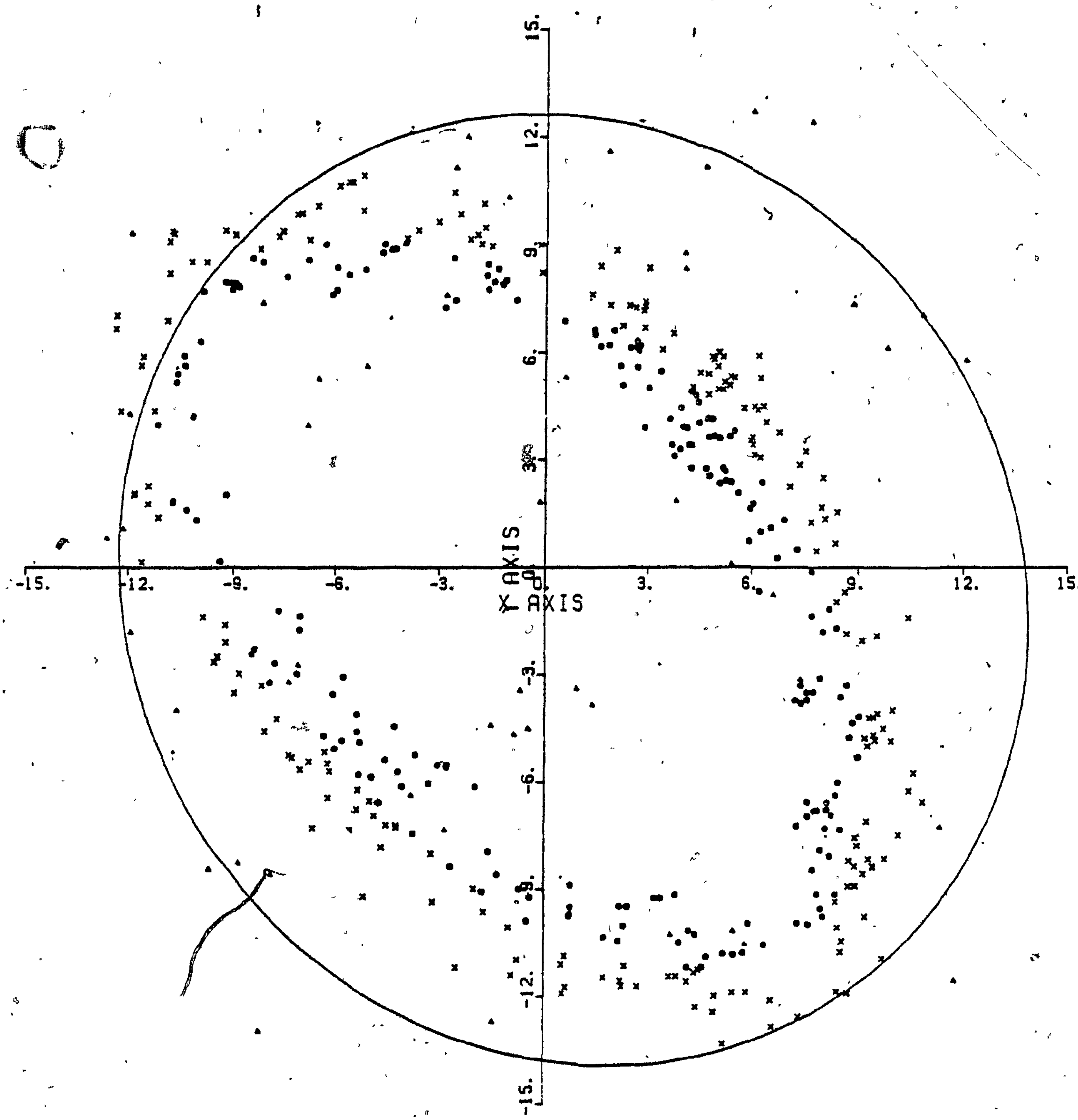


Fig.4.5: 3-bus system. Hyperellipsoid of type II.
Unstable training states utilized.

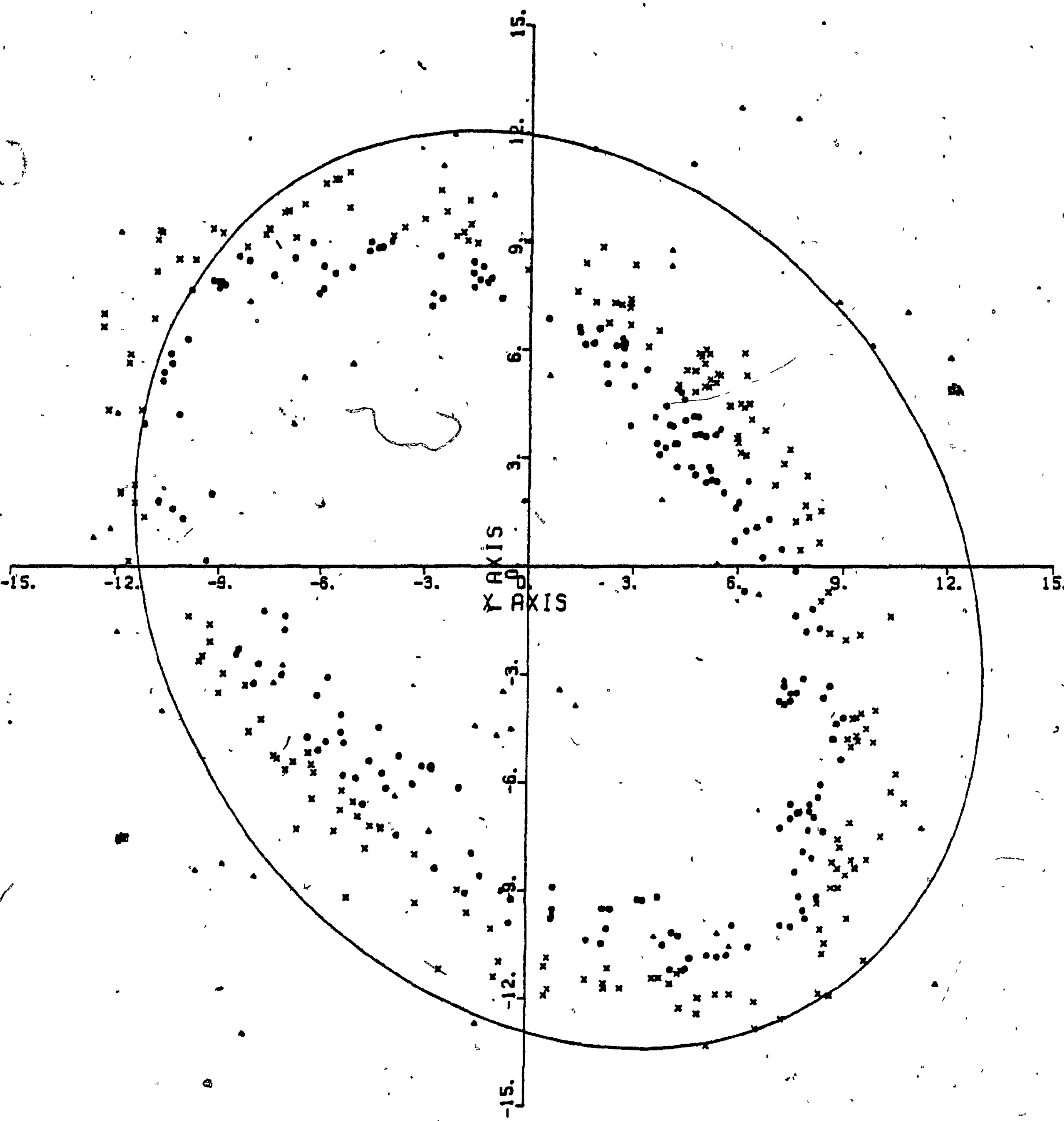


Fig.4.6: 3-bus system. Hyperellipsoid of type II.

Stable training states utilized.

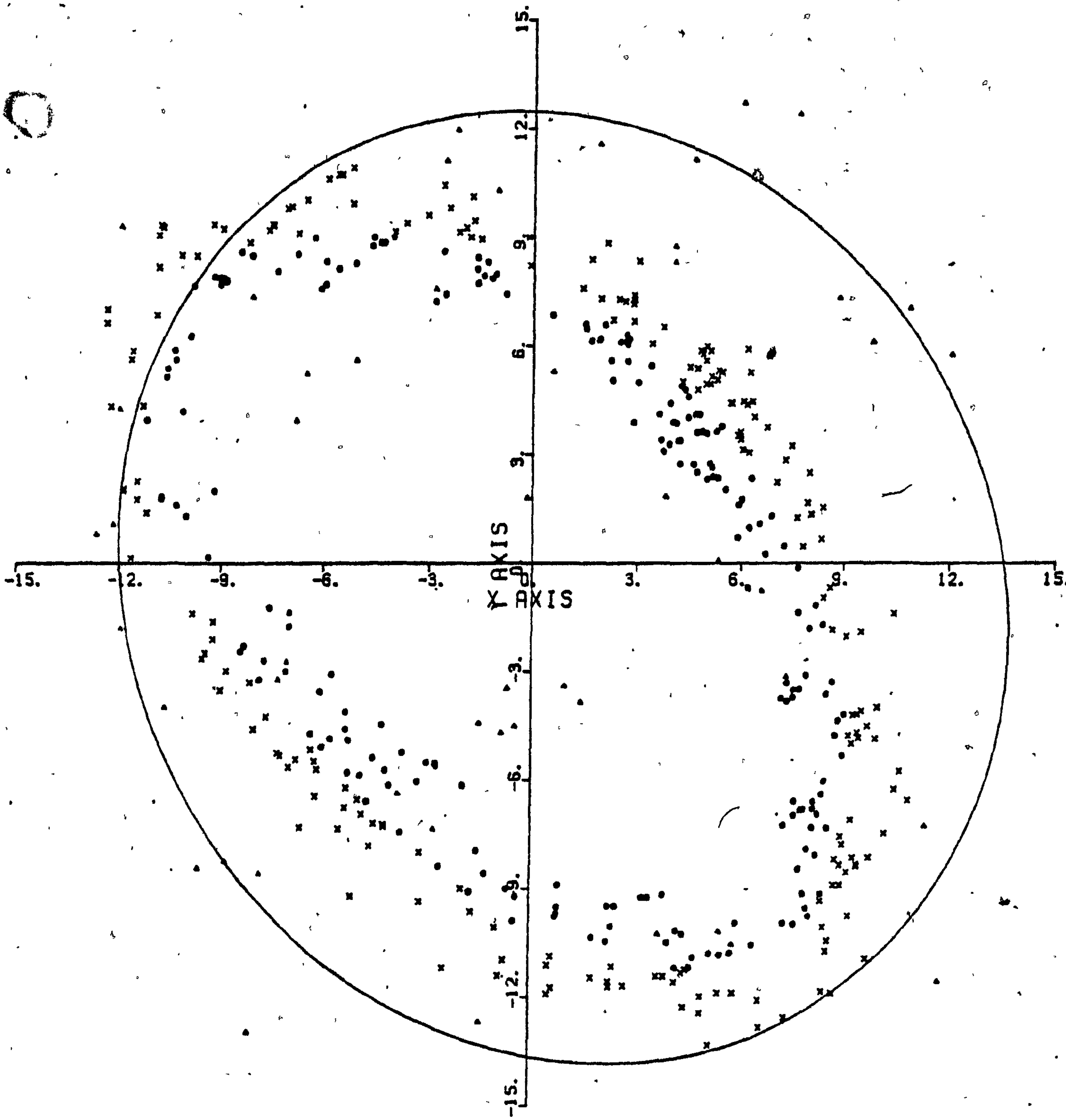


Fig.4.7: 3-bus system. Hyperellipsoid of type II. Stable training states used. Alternate criterion.

CHAPTER FIVE

APLLICATIONS TO POWER SYSTEM OPERATION.

5.1. Introduction.

The main objective of a real time power system control system is to assure that, at any instant, the system operates in a secure and economical manner. Load demand not only is to be accommodated but the cost of power generation has to be minimized as well (96).

The first problem therefore one has to cope with is "load variability". There is no guarantee whatsoever that the load demand pattern will remain unchanged throughout the day and, in fact, it does not. It is a well known fact that the load demand pattern can sharply change from one instant to the next, especially during the so called "peak hours" of the daily load demand curve. As a consequence, the first question to be answered is whether or not the system will be able to satisfy the load demand while retaining certain preset operating standards. Should the system be found capable of supplying the load, optimization considerations enter the scene such as minimization of transmission losses and generating costs, considerations which will to a great degree determine the operating point for the system during the next time period of the monitoring interval.

Taking into account, on the one hand, that there is a trend towards shorter monitoring intervals and, on the other hand, that ambiguity in system capability can lead to incorrect decision making and possible violation of the operating standards, it becomes rather apparent that a fast and accurate method for assessing load flow feasibility is needed. In fact the faster the better because more time is allowed for equally vital tasks such as contingency analysis and economic dispatch.

One other problem for the operator is that he has to account for "system variability". The topology of the system varies due to both scheduled and forced equipment outages. When it comes to predicting load variations one may come up with a load forecast, schemes that utilize feasible statistical modeling and turn out to be satisfactorily accurate. Unfortunately the same cannot be said for the case of contingency analysis because a large number of factors have to be considered if one is to identify and list them all to begin with. Furthermore, as explained in greater detail in chapter I, methods for contingency analysis are rather time consuming if accuracy is to be retained. Despite the inherent difficulties associated with contingency analysis at all levels (for every monitoring interval the problem repeats itself to the full extent) time limitation is also a very severe constraint to the quality of decision making needed for proper real time control. One therefore has to be both fast and accurate on all fronts at all times. The problem is further enhanced in cases where sequential decision making is needed. Experience shows that being "fast" and "accurate" are rather conflicting goals. One therefore settles for efficient "screening". The fewer cases are referred to more time consuming methods for resolution, the better.

The previous chapter of this dissertation was devoted to develop the concept of the hyperellipsoids of confidence in the state space of injections of the power system. Methods for constructing these hyperellipsoids were devised with emphasis on "training point economization", an aspect not meticulously treated in the past.

This chapter indicates how the hyperellipsoids of confidence can be used for both load variability as well as steady state contingency analysis. It is demonstrated that they can chiefly be used for screening purposes, in the quest for speed and accuracy in power system operation.

It is of importance to emphasize that the ideas presented in the paragraphs to follow have been centered, from a methodological point of view, on the recognition of the fact that the concept of misclassification error introduces an uncertainty in real time power system operation. As a consequence it has been the intention of the author to develop methodologies freed from this concept.

5.2. Load Variability Assessment.

The fastest, yet most approximate, method available so far for load flow studies is the D.C. load flow. The simplified mathematical model and the computational ease associated with it made it very attractive. Nevertheless, its accuracy limitations have long ago been recognized.

Assume that in the space of injections of a given power system, the two hyperellipsoids of confidence (of type I and type II) are available. By construction, the states located in the interior of the hyperellipsoid of type I represent steady state stable states, i.e states which constitute a feasible loading condition for the system in question. Similarly, any state not located in the interior of the Hyperellipsoid of type II represents, by construction again, an unfeasible loading for the system. Any point of the state space located in the exterior of the hyperellipsoid of type I while at the same time is found to be located in the interior of the hyperellipsoid of type II is said to represent an "ambiguous" loading state for the system. Such a point, is said to be located in the "uncertainty region" if one is to utilize decision making terminology.

Accordingly, in order to assess the feasibility of a specific load pattern, one need not necessarily resort to full scale simulation if the two hyperellipsoids of confidence are available.

For the various loading conditions the system may encounter only three mutually exclusive cases are possible.

Case 1. The state in question is found to be in the interior of the hyperellipsoid of type I. This is mathematically manifested as:

$$\underline{X}^T [A] \underline{X} + \underline{b}^T \underline{X} + C < 0 \qquad (5.1)$$

where: $\underline{X}$ represents the load pattern of the system (Every entry pertains to a bus loading)

$[A]$, $\underline{b}$, C are the parameters of the equation of the hyperellipsoid of type I..

Case 2. The state in question is found to be located in the exterior of the hyperellipsoid of type II. The situation is mathematically manifested as :

$$\underline{X}^T [A_1] \underline{X} + \underline{b}_1^T \underline{X} + C_1 > 0 \qquad (5.2)$$

where: $\underline{X}$ bears the same interpretation as before.

$[A_1]$, $\underline{b}_1$, C_1 represent the parameters of the hyperellipsoid of confidence type II.

Case 3. The state in question is found to be located in the "uncertainty region". This is mathematically manifested as:

$$\underline{X}^T [A] \underline{X} + \underline{b}^T \underline{X} + C \geq 0 \qquad (5.3a)$$

and

$$\underline{X}^T [A_1] \underline{X} + \underline{b}_1^T \underline{X} + C_1 \leq 0 \qquad (5.3b)$$

For the first two cases decision making is carried out without any difficulty. In the first case, for instance, the operator knows that the given state is permissible for operation. Similarly, in the second case the operator is definitely to avoid such an operational point. In the third case however, no immediate decision making is possible with absolute certainty. The state in question may represent a feasible loading condition for the system or may not. In this case one has to resort to a more detailed analysis if the decision making (concerning load flow feasibility) is to be error free. Viewed from this perspective, the method of the hyperellipsoids presents interesting potential capabilities for "screening" purposes.

Out of a very large number of probable system loadings (provided by short term load forecast) actual simulations are needed only for the states that find themselves located in the "uncertainty region". The behaviour of any other loading state is clear and, most of all, accurately predictable in real time.

5.3. Examples of Load Variability Assessment.

At first the rather simple 3-bus system utilized in previous chapters (III and IV) is used to illustrate the above presented ideas. The data as well as the topology of the system are given in Appendix I. Bus 1 is considered to be the slack bus, and the only source of real power in the system. Note that no generators were assumed to be connected to buses 2 and 3. This would simply increase the loading capability of each bus (inflating the hyperellipsoid of type I). The argument, however, can still be lucidly presented without this extra complication. Buses 2 and 3 are considered to be both type 2 buses (voltage magnitude restrained) with infinite reactive compensating capability (recall that the technique is intended for networks of primary transmission where voltage profile control is very robust). As a consequence, the real power injections at the buses represent the variables of interest and, in fact, constitute the state variables. The results are presented in table 5.1. Ten (10) representative loading conditions (positive load means power flowing out of the system) are examined. The loading conditions of table 5.1. scan the loadability range of the system from concurrent light load to concurrent heavy loading of both buses. The second column of the table indicates the result of an actual load flow study for the load under consideration (the Gauss-Seidel iterative procedure was used). The third column of the table indicates the result one will arrive at if the screening technique of the hyperellipsoids is used.

The proposed methodology for steady state stability assessment can be directly utilized for generation shift assessment, on the condition that the bus which experiences the change in generation is included in the state space for the grid. For the state spaces discussed in this study, only real generation shift can be accommodated with the proposed technique. Generator outages (loss of tie lines transmitting power into the grid) can be modeled as subsequent load increase for the bus in question. Similarly, generation surplus can be modeled as a subsequent load reduction experienced by the bus in question.

LOADING CONDITION FOR THE SYSTEM		LOAD FLOW	SCREENING TECHNIQUE
+1.0	+1.0	Feasible	Feasible
+2.0	+3.0	Feasible	Feasible
+3.0	+3.0	Feasible	Feasible
+3.5	+3.5	Feasible	Feasible
+4.0	+3.0	Feasible	Feasible
+4.0	+4.0	Feasible	Feasible
+3.0	+5.0	Feasible	Feasible
+1.5	+6.5	Feasible	Feasible
+5.5	+3.0	Feasible	Feasible
+6.0	+6.0	Unfeasible	Ambiguous

Table 5.1: 3-bus system. Load Variabilty Assessment.

As can be seen in table 5.1, the only ambiguous case detected by the screening technique is for the heaviest loading. If the suggested simulation is carried out, the result will be that the state under consideration represents an unfeasible load pattern for the system. It becomes apparent that if heavier loads are considered the method of the hyperellipsoids will still demand full scale simulation for their resolution. This is due to the size of the hyperellipsoid of type II which, in turn, is dependent upon the criteria adopted for its construction. Since the construction criteria were intentionally conservative (the goal of the construction being error free decision making) the hyperellipsoid of type II will, as a rule, be oversized. If one is to encounter loading patterns which will be located outside of the hyperellipsoid of type II, the system has to work with its lines loaded either very close or well above their steady state stability limits. The later practice is, of course, a technical impossibility the former is a practice never followed. In either case the conclusion is that practical load patterns will invariably find themselves located either within the boundary of the hyperellipsoid of type I or in the zone of uncertainty. The end result is that the term "ambiguous" in Table 5.1 can just as well be replaced by the term "unfeasible", for first hand decision making. The reasoning presented above suggests that since only one hyperellipsoid is really needed the second hyperellipsoid need not be computed at all. Nevertheless, the lighter computational burden associated with such an approach seems, at first, not to provide the analyst with a tool for discriminating which loading patterns located outside of the boundary of the hyperellipsoid of type I need further investigation and which ones do not.

It is seen from table 5.1 that no case classified as steady state feasible by the method of the hyperellipsoids is found to be non feasible by the corresponding a.c simulation analysis. Nevertheless, cases classified by the screening technique as unfeasible can in reality be feasible. In this particular case no such state was found, among the ones examined. This is due to the fact that the hyperellipsoid constructed turns out to be surprisingly close to the actual separation surface (see fig.4.16) for first quadrant states (note that all injections considered were assumed to be first quadrant injections, i.e real loads which are supposed to be supplied by the system via real power flowing out of the grid).

This is a very desirable coincidence because accurate decision making (for load variability assessment purposes) can be extended to rather heavy loadings. Nevertheless, in practice, heavy loadings which are considered to be mathematically acceptable (the load flow converges) do not conform with everyday operational practice. Lines are normally loaded well below their steady state stability limit, their loading being dictated by transient stability considerations more than anything else. For more complex systems, the region encompassed by the hyperellipsoid of confidence of the type I, does not bear the ambition to approximate the separation surface itself at all. Instead, it rather represents a "core of confidence" of the feasibility region, close to the real life operating conditions (see comment above).

To further illustrate this, a more complex power system was considered next. The system was taken to be a 5-bus system. The network data as well as its topology are given in Appendix I. One slack bus was again considered. This bus, in this case as well, is supposed to be the only source of real power in the system. All four remaining buses are taken to be voltage controlled buses; their reactive compensating capability is considered infinite. Table 5.2 contains the results of the Load Variability assessment technique utilizing the method of the hyperellipsoid of type I (see comments in previous section on the philosophy of utilizing only one hyperellipsoid).

LOADING OF THE SYSTEM				LOAD FLOW	SCREENING METHOD
+0.2	+0.2	+0.2	+0.2	Feasible	Feasible
+0.2	+0.3	+0.35	+0.4	Feasible	Feasible
+0.4	+0.4	+0.4	+0.5	Feasible	Feasible
+0.6	+0.6	+0.6	+0.6	Feasible	Feasible
+1.0	+1.0	+1.0	+1.0	Feasible	Feasible
+1.0	+2.0	+2.0	+2.0	Feasible	Feasible
+1.0	+3.0	+2.0	+2.0	Feasible	Feasible
+1.0	+4.0	+3.0	+3.0	Feasible	Unfeasible
+1.0	+4.5	+3.5	+3.5	Feasible	Unfeasible
+1.0	+4.0	+4.0	+4.0	Feasible	Unfeasible
+1.0	+5.0	+4.0	+5.0	Unfeasible	Unfeasible
+1.0	+6.0	+5.0	+4.5	Unfeasible	Unfeasible
+1.5	+6.5	+5.5	+4.5	Unfeasible	Unfeasible
+2.0	+7.0	+6.0	+4.5	Unfeasible	Unfeasible
+2.5	+6.5	+5.5	+5.5	Unfeasible	Unfeasible

Table 5.2: 5 bus system. Load variability assessment.

As seen in the results displayed in table 5.2, cases classified as nonfeasible by the method of the hyperellipsoid, are found to be feasible by the corresponding ac simulation analysis. Those cases pertain, as expected, to medium loading. Heavy loading as well as light loading are characterized by consistent decision making.

The philosophy of utilization of the method should be clear by—now. By not attempting to produce an analytical expression for the separation surface, we have bypassed (at a cost) the questions related to the misclassification error. Apart from the fact that a reasonable estimation of such a quantity is hard to obtain computationally (and a real challenge to model mathematically), the misclassification error is of no practical significance to real time decision making.

The potential of the method presented is crucially dependent on the volume of the hyperellipsoid utilized. For rather irregularly shaped feasibility regions, it may be of a size not lending itself to applications (accommodating only very light loading for the system). In that case, (apart from the fact that such a system will have acute operating problems no matter what technique is used) the analyst may have to resort to techniques different from the ones presented here.

5.4. Steady state Contingency Analysis. Line outages.

It is well known that associated with any equipment outage, there is a transient which has to be investigated when overall system performance is assessed. Nevertheless, even before the transient and dynamic responses of the system are investigated, it is advantageous to know whether or not the system will be stable from steady state considerations alone. This kind of analysis is known as steady state contingency analysis.

Generally speaking, changes in the system can be of various kinds. One may, for instance, consider generation shifts (changes in generating capability at various buses or at interconnecting nodes) or actual line (transformer) outages. These outages, alter the topology of the system, and this alteration is the seat of the difficulty for fast post contingency power flow assessment.

The question which is frequently of prime concern for power system operators is whether or not there exists such a post contingency power flow in the first place. Presenting it in greater detail is a question which is resolved by applying any of the available load flow methods (so long as the feasibility of the undertaking is assured).

The methods available so far answer those questions concurrrently. In fact, they conjecture the feasibility of the post contingency case after having obtained the detailed power flow pattern indirectly. They can therefore be considered as indirect methods. What this dissertation proposes is a direct method based on the concept of the hyperellipsoids of confidence. In the examples to follow line and transformer outages have been treated.

The approach is based on the recognition of the fact that steady state contingency analysis is essentially a load flow analysis applied to a different system. But load flow cases can be accommodated, as shown previously, via the method of hyperellipsoids on the condition the hyperellipsoids utilized pertain to the current system topology.

An obvious way of implementing such an approach would be the following:

1) For every line outage (single or multiple) the hyperellipsoids are constructed and their equations are stored in a manner suitable for fast on line retrieval.

2) For any contingency examined from the contingency list of the current monitoring interval a simple function evaluation answers the question of the load flow feasibility for the post contingency system status.

In the case security screening is desired one knows immediately (for all contingencies) which cases are to be referred for resolution to more elaborate ac analysis schemes. Furthermore, the level of accuracy for security screening is brought up to the level of load variability assessment.

Thus, it is seen that a feasible approach to the steady state contingency analysis (security screening) problem can be implemented using the hyperellipsoid of confidence pertinent to various post contingency topological structures. Naturally, one needs to construct as many hyperellipsoids, as there are contingencies to be examined. This seems rather cumbersome and discouraging from a computational point of view. Indeed, for many systems (even at the level of reduced transmission equivalents, equivalents this study assumes for the techniques presented hereinafter) the amount of off-line work needed to obtain the hyperellipsoids pertaining to the contingencies put forward may be prohibitevely large.

A tempting question is whether or not precontingency hyperellipsoids can be used for post contingency load flow feasibility, thus circumventing the need to obtain new ones. We emphasize, again, that a given hyperellipsoid pertains to a given topological structure. Accordingly, if one is to utilize the precontingency hyperellipsoids, precontingency topology has also to be retained. Exploring this possibility, we are forced to conclude that the only remaining option is to simulate the line outage, while retaining precontingency topology.

5.5. Outage simulation Retaining Precontingency topology.

It is proposed that the outage of a line be simulated by changing the power injected into the system at the buses connected by this line. The concept has been put forward in the past by several authors (18,54,89,90,91,92,93). One of its latest manifestations has been the efficient simulation of line and transformer outages when assessing steady state security via the sensitivity approach (18,54,91,96). The argument is that the post contingency voltages will not change if precontingency topology is retained and power equal to the one flowing into either end of the line (as the post contingency voltage profile dictates) is injected at the end buses. The situation is pictured in figs. 5.1, 5.2 and 5.3. Fig.5.1 depicts the system at its precontingency state, Fig:5.2 depicts the system at its post contingency state and, finally, Fig.5.3 illustrates the simulation procedure (post contingency voltage profile under precontingency topology is achieved by appropriately varying the power injections at the line ends).

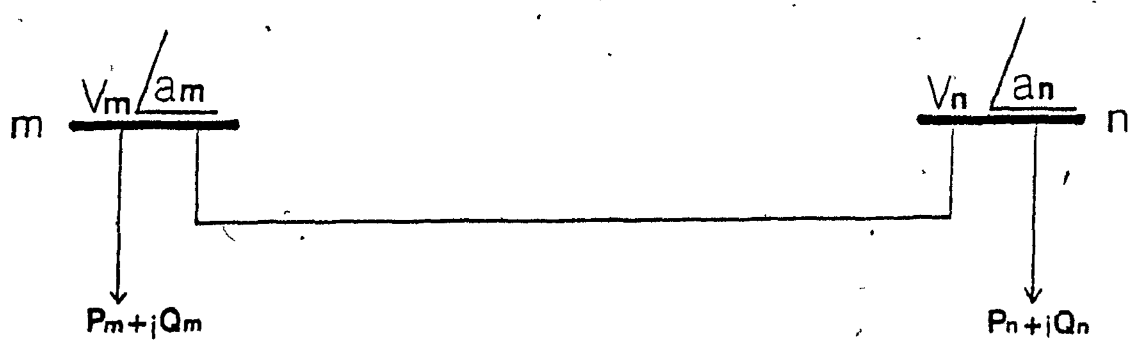


Fig. 5.1: Line mn in precontingency state.

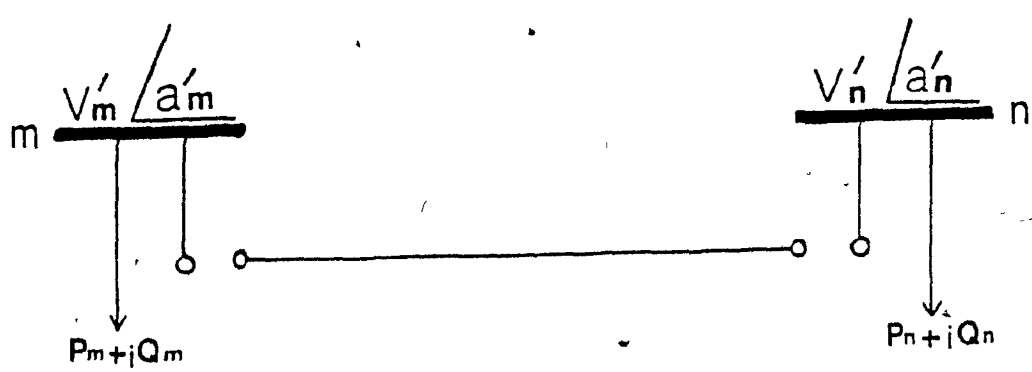


Fig. 5.2: Line mn in post contingency state.

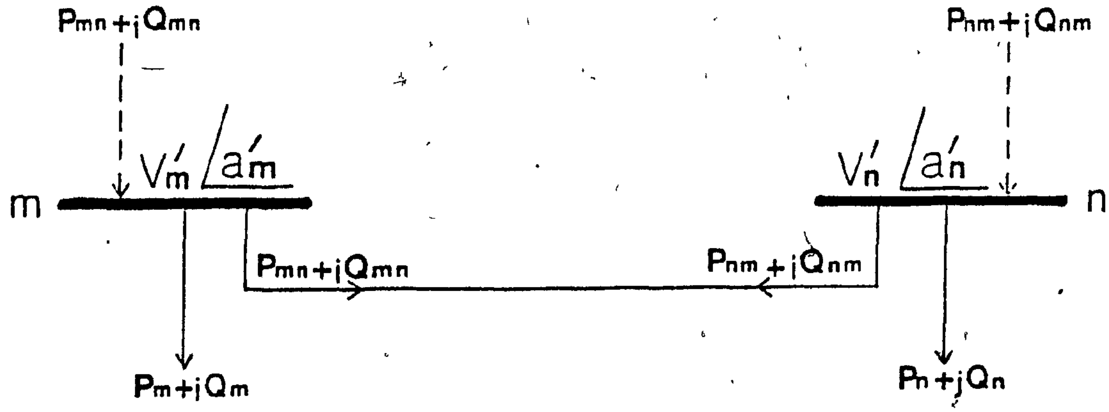


Fig.5.3: Simulation of the post contingency case.

Assuming that the line mn is to be outaged, one has for the needed power injections at the end buses (see Fig:5.3):

$$\Delta Pm+j\Delta Qm = Vm'.Iinm \quad \text{and} \quad \Delta Pn+j\Delta Qn = Vn'.Iinn \qquad (5.4)$$

Similarly for the power flowing in the line mn

$$\Delta Pm+j\Delta Qm = Vm'.Im \quad \text{and} \quad \Delta Pn+j\Delta Qn = Vn'.In \qquad (5.5)$$

where: ΔPm, ΔPn and ΔQm, ΔQn are the real and reactive components of the power injections needed (and by constraint the real and reactive components of the power at the line ends).

V'm, V'n vector quantities denoting bus voltages

Iinm, Iinn vectors for the current injections at the buses m and n respectively.

Im, In vectors denoting the currents at the line ends.

It follows that:

$$Iinm = Im \quad \text{and} \quad Iinn = In \tag{5.6}$$

Fig.5.4 illustrates the above expression. Approximating:

$$In = - Im = I \tag{5.7}$$

the situation becomes the one depicted in Fig.5.5.

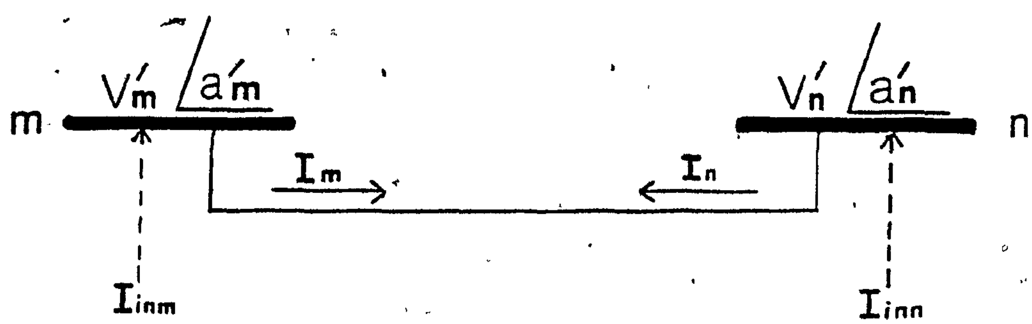


Fig.5.4: Current injections needed for the simulation.

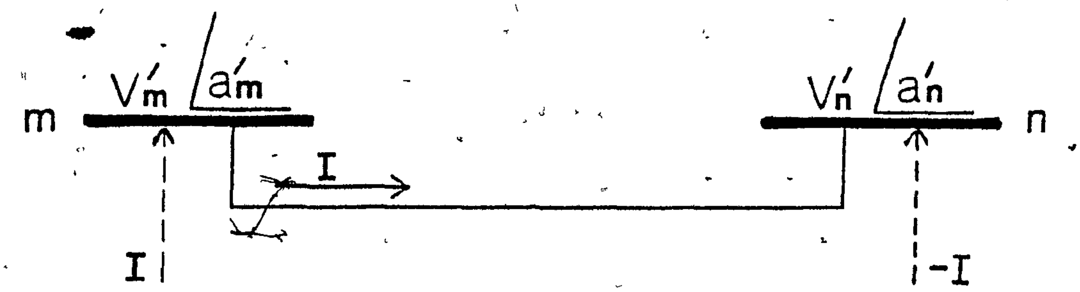


Fig.5.5: Final form of the needed current injections.

Considering the precontingency as "base case" the change in the injections of the buses m and n respectively (due to the outage simulation) will have an effect on the precontingency voltage profile which can be found with the aid of the base case (precontingency) Z matrix of the network. The changes on the voltages are:

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots\cdots\cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots\cdots\cdots & Z_{2n} \\ \cdot & & \cdots\cdots\cdots & \\ \cdots & \cdots & \cdots\cdots\cdots & Zmn \\ \cdot & & \cdots\cdots\cdots & \\ Z_{n1} & Z_{n2} & \cdots\cdots\cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \cdot \\ I \\ \cdot \\ -I \end{bmatrix} = \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \cdot \\ \Delta Vm \\ \cdot \\ \Delta V_n \end{bmatrix} \qquad (5.8)$$

It is seen from (5.8) that:

$$\Delta Vm = (Zmm-Zmn).I \quad \text{and} \quad \Delta Vn = (Zmn-Znn).I \qquad (5.9)$$

But $Vm' = Vm + \Delta Vm$ and $Vn' = Vn + \Delta Vn$ (5.10)

Since: $Vm'-Vn' = Zline\text{-}mn\,.I$, we have that:

$$Vm'-Vn' = Zline\text{-}mn.I = (Vm-Vn)+(\Delta Vm-\Delta Vn) \qquad (5.11)$$

But $$Vm-Vn = Zline\text{-}mn.Imn \qquad (5.12)$$

where: Imn is the precontingency line current.

and $$\Delta Vm-\Delta Vn = (Zmm+Znn-2Zmn).I \qquad (5.13)$$

Solving (5.11) with respect to I we obtain:

$$I = \frac{Imn.Zline\text{-}mn}{Zline\text{-}mn + 2Zmn - Zmm - Znn} \qquad (5.14)$$

Equation (5/14) is rather important because it expresses the current injection needed to simulate the outage using "base case" network quantities.

If one wants to find the power injection needed in order for the outage to be simulated:

$$Sinm = V'm.I \quad \text{and} \quad Sinn = V'n.(-I) \qquad (5.15)$$

where: Sinm and Sinn are the complex power injections needed at buses m and n respectively.

V'm and V'n are the post contingency voltages at buses m and n respectively.

It is desired however that the system retains its precontingency voltages in magnitude. This means that if the post contingency case is to be feasible the voltages at the buses will differ only in angle and not in magnitude. Accordingly, one can approximate Sinm and Sinn in (5.15) as follows:

$$\text{Sinm} = \text{Vm}.(\text{I}) \quad \text{and} \quad \text{Sinn} = \text{Vn}.(-\text{I}) \qquad (5.16)$$

where: Vm and Vn are the precontingency voltages at buses m and n respectively (vectors).

Let $Z = \text{Zline-mn} + 2\text{Zmn} - \text{Zmm} - \text{Znn}.$

If eq(5.16) is decomposed into real and reactive components one has:

$$\text{Pinm} = \frac{\text{Pmn Zline-mn}}{Z}, \quad \text{Pinn} = \frac{\text{Pnm Zline-mn}}{Z} \qquad (5.17)$$

$$\text{Qinm} = \frac{\text{Qmn Zline-mn}}{Z}, \quad \text{Qinn} = \frac{\text{Qnm Zline-mn}}{Z} \qquad (5.18)$$

where: Pmn and Pnm precontingency power flows in line mn from end m to end n, and vice versa, respectively.

It can be seen that any discrepancy between the quantities computed via (5.15) and the ones computed via (5.16) is due to the difference in the angles of the voltages. Nevertheless, the great advantage of (5.16) is that it gives a rather fair indication (under the assumptions it has been derived under) of the injections needed utilizing known "base case" quantities available from any "base case" load flow.

Those injections superimposed to the bus loads reduce the problem to the problem of load variability assessment since the topology of the system is retained.

5.6. Steady State Contingency Analysis. Examples.

At first the small experimental 3-bus system will be used to demonstrate the feasibility of the approach. The line outage to be considered is line 2-3. A couple of case studies will be treated with relative detail at first, in order to illustrate the proposed method.

Case study 1: Heavy Precontingency loading.

Assume the system at precontingency topology with loads (real components) at buses 2 and 3, of 1.5 and 6.5 p.u.. Since those demands represent actual powers flowing out of the grid they are represented as negative injections in the load flow equations. Therefore from a load flow point of view the load pattern is -1.5 and -6.5 for buses 2 and 3 respectively. This is considered the base case and a load flow analysis is carried out to determine both the voltage profile as well as the line flows in the system. It is found that power is transferred from bus 2 to bus 3. At the sending end 2 the power sent to end 3 (via the link 2-3) is 2.0573 p.u and the similar quantity at end 3 has a value of -1.9674 p.u.

The [Z] matrix of the system is constructed (80,81,82,83,84,85,86) and it is found to be:

$$[Z] = \begin{bmatrix} j0.1333 & j0.0666 \\ j0.0666 & j0.1333 \end{bmatrix}$$

(for the sake of simplicity the resistive components of the transmission line impedances were omitted)

Applying directly (5.17) and (5.18) one has:

Pin2 = 2.0573x3.058104 = 6.2912522 p.u (real power)

Pin3 = -1.9674x3.058104 = -6.0163367 p.u (real power)

where:

3.058104 = j0.2/j(0.2+2x0.0666-0.1333-0.1333)

j0.2 = reactance of line 2-3

Superimposing the injections needed to simulate the outage to the bus loads we obtain:

P (at bus 2) = 6.2912522-1.5 = 4.7912522 (generation)

P (at bus 3) = -6.5-6.0163367=-12.516337 (load)

The values just found for the compound powers are substituted into the equation of the hyperellipsoid of type 1 for the 3-bus system. One has to note that in constructing the equation of the hyperellipsoids, actual system loads (being modeled as negative injections for load flow purposes) were considered to be positive. Accordingly the state $\underline{X}$ reads:

$$\underline{X}^T = [\ -4.7912522 \quad 12.516332]$$

Substituting in (5.1) we obtain $f(\underline{X}) = 4890.9688$ ($f(\underline{X})>0.$) This denotes that the state is located outside of the hyperellipsoid and therefore it is considered to be unfeasible. But, unfeasibility for the compound injections means that the system will not survive the outage. A load flow study with post contingency network configuration and precontingency bus loads led us to the same conclusion.

Case 2. System Heavily Loaded

Consider this time the precontingency real loads to be -5.5 and -3.0 p.u for buses 2 and 3 respectively (in network convention negative loads at the buses denote power flowing out of the grid). The base case load flow gives that the power flow in line 2-3 is:

$$P2\text{-}3 = -1.0239 \text{ p.u} \qquad \text{and} \qquad P3\text{-}2 = 1.0469 \text{ p.u}$$

Utilizing (5.16) again we obtain as before:

$$Pin2 = -1.0239\text{x}3.058104 = 3.1311927 \text{ p.u}$$

and

$$Pin3 = 1.0467 x 3.058104 = 3.201529 \text{ p.u}$$

Superimposing the required injections found to the precontingency loads we obtain as compound injections:

$$P2 = -3.1311927 + (-5.5) = -8.6312 \quad \text{p.u}$$

and

$$P3 = 3.201529 + (-3.0) = 0.201529 \text{ p.u}$$

Note that the signs pertain to network conventions. The opposite signs will be used for the state space representation and the state becomes:

$$\underline{X}^T = [\ 8.6312 \quad -0.201529\ ]$$

Substituting in (5.1) we obtain $f(\underline{X}) = 1088.24871$ ($f(\underline{X})>0.$) The method of the hyperellipsoids thus suggests that the system will not be in a position to withstand the outage of line 2-3 under the prescribed loading pattern. A detailed load flow analysis with post contingency data gave the same result.

<u>Case 3:</u> Light Loading for the system.

Let us suppose that the load pattern for this case is -2.0 p.u and -3.0 p.u at buses 2 and 3 respectively. The precontingency base case gives that P2-3=0.3508 p.u and P3-2=-0.3482 p.u Application of (5.16) gives for this particular case:

$$Pin2 = 0.3508 x 3.058104 = 1.0727829 \text{ p.u}$$

and

$$Pin3 = -0.3482 x 3.058104 = -1.0648318 \text{ p.u}$$

Superimposing we have:

$$P2 = -2.0 + 1.0727829 = 0.9272171 \text{ p.u}$$

and

$$P3 = -3.0 - 1.0648318 = -4.0648318 \text{ p.u}$$

For $\underline{X}^T = [\ -0.9272171 \quad +4.0648318\ ]$, equation (5.1) gives $f(\underline{X}) = -4265.3667$ ($f(\underline{X})<0.$). This suggests that according to the methods of the hyperellipsoids the system will withstand the outage. This indeed has been confirmed, again, with a full scale ac analysis.

In the context of security assessment via the method of the hyperellipsoids only the hyperellipsoid of confidence of type I can be used with good enough results (see section 5.3.)

In other words one function evaluation (eq.5.1) will suffice. If the state of the compound injections is found to be in the exterior of the hyperellipsoid of type I, it is considered to represent a fatal contingency. This consideration stems from the fact that due to the rather conservative criteria established for the construction of the hyperellipsoid of type II one rarely finds a state outside its boundary. This means that the vast majority of the cases will be classified as "ambiguous", in reality being unfeasible. On an automated scheme this can lead to redundant ac simulation cases. On the other hand, if only the hyperellipsoid of type I is used for decision making there may be cases where the operating point is considered nonsecure, while being in fact secure. Table 5.3 lists (in its first two columns) the results of full scale ac simulations with precontingency topology. The outage to be investigated is the outage of line 2-3. The third column of the same table provides the results of a Gauss-Seidel load flow algorithm with post contingency network topology. The fourth column gives the results one should arrive at, if the method of the hyperellipsoid(s) is used.

SYSTEM LOADS BUS 2	BUS 3	AC LOAD FLOW SIMULATION	METHOD OF THE HYPERELLIPSOID
+1.0	+1.0	Feasible	Feasible
+2.0	+3.0	Feasible	Feasible
+3.0	+3.0	Feasible	Feasible
+4.0	+4.0	Feasible	Feasible
+4.0	+3.0	Feasible	Feasible
+3.5	+3.5	Feasible	Feasible
+3.0	+5.0	Feasible	Unfeasible
+1.5	+6.5	Unfeasible	Unfeasible
+6.0	+6.0	Unfeasible	Unfeasible
+5.5	+3.0	Unfeasible	Unfeasible

Table 5.3: 3-bus system. Line 2-3 outaged.

It is seen from the results displayed in table 5.3 that no case classified as feasible by the screening technique is classified as unfeasible by the corresponding ac simulation. Nevertheless, some remarks are in order. This specific example, illustrates one limitation of the proposed methodology. The injections needed at the end buses to simulate the outage are given by eqs(5.16). These equations are approximate because they utilize the precontingency bus angles (recall that the system was assumed to be voltage robust) and not the post contingency ones.

Accordingly, the error introduced in estimating the needed injections is transferred directly to the compound injections, which are estimated by direct superposition of the needed injections with the precontingency bus loads.

Suppose that the point suggested by the computed compound injections happens to be located near the separation surface of the feasibility region. Assume, furthermore, that the surface of the hyperellipsoid, in the neighborhood of the compound state, happens to be virtually identical with the separation surface itself. In the case of the load variability assessment this coincidence was highly beneficial but, for security assessment it may very well lead to wrong decision making. In other words, the total absence of the uncertainty zone may cause the state (which otherwise woud be classified as unfeasible) to be classified, wrongly, as feasible.

It is exactly this very existence of the uncertainty zone that, in our opinion, more than compensates for the error introduced by using the eqs 5.16. In this specific example, as illustrated in fig.4.16, the zone of uncertainty is virtually non existent for first quadrant states. As a consequence, erratic decision making is probable for certain load patterns under the concurrent influence of the conditions stated.

As a further illustration of the technique the more complex 5-bus system will be used for the cases to follow. The procedure is the same as the one used for the 3-bus system. Only single outages are examined. Table 5.4. pertains to the case where line 2-3 is considered to be out of commission. Assuming the same precontingency loading as in table 5.3 the system's response (steady state) is assessed with both full scale ac simulations (column 2) as well as with the method of the hyperellipsoid (column 3)

LOADING OF THE SYSTEM				OUTAGED LINE : 2-3	
				LOAD FLOW	SCREENING METHOD
+0.2	+0.2	+0.2	+0.2	Feasible	Feasible
+0.2	+0.3	+0.35	+0.4	Feasible	Feasible
+0.4	+0.4	+0.4	+0.5	Feasible	Feasible
+0.6	+0.6	+0.6	+0.6	Feasible	Feasible
+1.0	+1.0	+1.0	+1.0	Feasible	Feasible
+1.0	+2.0	+2.0	+2.0	Feasible	Feasible
+1.0	+3.0	+2.0	+2.0	Feasible	Feasible
+1.0	+4.0	+3.0	+3.0	Feasible	Unfeasible
+1.0	+4.5	+3.5	+3.5	Feasible	Unfeasible
+1.0	+4.0	+4.0	+4.0	Feasible	Unfeasible
+1.0	+5.0	+4.0	+5.0	Unfeasible	Unfeasible
+1.0	+6.0	+5.0	+4.5	Unfeasible	Unfeasible
+1.5	+6.5	+5.5	+4.5	Unfeasible	Unfeasible
+2.0	+7.0	+6.0	+4.5	Unfeasible	Unfeasible
+2.5	+6.5	+5.5	+5.5	Unfeasible	Unfeasible

Table 5.4: 5-bus system. Line 2-3 outaged.

LOADING OF THE SYSTEM				OUTAGED LINE : 3-4	
				LOAD FLOW	SCREENING METHOD
+0.2	+0.2	+0.2	+0.2	Feasible	Feasible
+0.2	+0.3	+0.35	+0.4	Feasible	Feasible
+0.4	+0.4	+0.4	+0.5	Feasible	Feasible
+0.6	+0.6	+0.6	+0.6	Feasible	Feasible
+1.0	+1.0	+1.0	+1.0	Feasible	Feasible
+1.0	+2.0	+2.0	+2.0	Feasible	Feasible
+1.0	+3.0	+2.0	+2.0	Feasible	Feasible
+1.0	+4.0	+3.0	+3.0	Feasible	Unfeasible
+1.0	+4.5	+3.5	+3.5	Feasible	Unfeasible
+1.0	+4.0	+4.0	+4.0	Feasible	Unfeasible
+1.0	+5.0	+4.0	+5.0	Unfeasible	Unfeasible
+1.0	+6.0	+5.0	+4.5	Unfeasible	Unfeasible
+1.5	+6.5	+5.5	+4.5	Unfeasible	Unfeasible
+2.0	+7.0	+6.0	+4.5	Unfeasible	Unfeasible
+2.5	+6.5	+5.5	+5.5	Unfeasible	Unfeasible

Table 5.5: 5-bus system. Line 3-4 outaged.

LOADING OF THE SYSTEM				OUTAGED LINE : 4-5	
				LOAD FLOW	SCREENING METHOD
+0.2	+0.2	+0.2	+0.2	Feasible	Feasible
+0.2	+0.3	+0.35	+0.4	Feasible	Feasible
+0.4	+0.4	+0.4	+0.5	Feasible	Feasible
+0.6	+0.6	+0.6	+0.6	Feasible	Feasible
+1.0	+1.0	+1.0	+1.0	Feasible	Feasible
+1.0	+2.0	+2.0	+2.0	Feasible	Feasible
+1.0	+3.0	+2.0	+2.0	Feasible	Feasible
+1.0	+4.0	+3.0	+3.0	Feasible	Unfeasible
+1.0	+4.5	+3.5	+3.5	Feasible	Unfeasible
+1.0	+4.0	+4.0	+4.0	Feasible	Unfeasible
+1.0	+5.0	+4.0	+5.0	Unfeasible	Unfeasible
+1.0	+6.0	+5.0	+4.5	Unfeasible	Unfeasible
+1.5	+6.5	+5.5	+4.5	Unfeasible	Unfeasible
+2.0	+7.0	+6.0	+4.5	Unfeasible	Unfeasible
+2.5	+6.5	+5.5	+5.5	Unfeasible	Unfeasible

Table 5.6: 5-bus system. Line 4-5 outaged.

LOADING OF THE SYSTEM				OUTAGED LINE : 2-5	
				LOAD FLOW	SCREENING METHOD
+0.2	+0.2	+0.2	+0.2	Feasible	Feasible
+0.2	+0.3	+0.35	+0.4	Feasible	Feasible
+0.4	+0.4	+0.4	+0.5	Feasible	Feasible
+0.6	+0.6	+0.6	+0.6	Feasible	Feasible
+1.0	+1.0	+1.0	+1.0	Feasible	Feasible
+1.0	+2.0	+2.0	+2.0	Feasible	Feasible
+1.0	+3.0	+2.0	+2.0	Feasible	Feasible
+1.0	+4.0	+3.0	+3.0	Feasible	Unfeasible
+1.0	+4.5	+3.5	+3.5	Feasible	Unfeasible
+1.0	+4.0	+4.0	+4.0	Feasible	Unfeasible
+1.0	+5.0	+4.0	+5.0	Unfeasible	Unfeasible
+1.0	+6.0	+5.0	+4.5	Unfeasible	Unfeasible
+1.5	+6.5	+5.5	+4.5	Unfeasible	Unfeasible
+2.0	+7.0	+6.0	+4.5	Unfeasible	Unfeasible
+2.5	+6.5	+5.5	+5.5	Unfeasible	Unfeasible

Table 5.7: 5-bus system. Line 2-5 outaged.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS.

6.1. CONCLUSIONS.

The problem of the determination of operating regions has been of considerable interest during the past years. The usefulness and potential of the exact knowledge of such operating regions for real time operation and control has long ago been recognized.

Nevertheless, the difficulties associated with analytical approaches are formidable, due to the fact that the equations governing the performance of power systems are nonlinear. Trading computational power for analytical elegance seems, for the moment, to be the only attainable engineering means, for implementing practical control and operating strategies in the real time power system environment.

This dissertation approached the problems of steady state feasibility region identification and steady state contingency analysis from a simulation-based, Pattern Recognition-motivated, point of view.

The following concluding remarks apply for chapters II - V.

i) Direct transposition of Statistical Pattern Recognition methodology to Power Systems engineering is not recommended. Methodologies, for both formulating and solving power system problems, are to be developed on an independent basis. Concepts applicable and extensively used in a statistical environment (such as the concept of the misclassification error) should be addressed from an entirely new point of view in the strive for reliability in real time operational practice.

ii) Training point selectivity as well as training point economization are imperative attributes of the training sets used in power system region identification. Random selection of training samples is to be avoided and the fact that training points are very costly to power system engineers has to be properly accounted for.

With the algorithms presented for training point selection, the desired attributes of selectivity and economization have concurrently been achieved, thus substantially reducing the amount of "off-line" computations needed. The superiority of those algorithms over random selection schemes was demonstrated.

iii) Training points should be located in the immediate vicinity of the separation surface. As a consequence, data set condensation is rather imperative if realistic hyperellipsoids are to be obtained. It was also shown that even for classification purposes (in the traditional Pattern Recognition context) the misclassification error is, as a rule, reduced whenever condensed data sets are used. Algorithms for data set condensation were presented and their validity tested.

iv) An error free method has been developed for state identification based on the concept of the hyperellipsoid of confidence. Methods for constructing the hyperellipsoids needed were also developed, and criteria for optimizing their size were given. The method, makes no use of the concept of the misclassification error, concept which is rather hard to cope with if real-time, error- free decision making is desired.

v) Direct applications of the method of the hyperellipsoid in the real time power system environment were suggested. Load flow feasibility can be assessed on line. By utilizing the hyperellipsoids for screening purposes, the amount of ac simulations needed for error free decision making is greatly reduced. The applicability of the approach was demonstrated for actual power grids.

vi) A method for steady state contingency analysis was developed based on the concept of the hyperellipsoids of confidence of the precontingency feasibility region of the power grid. The method relies on the Z matrix concept and poses the following advantages, as demonstrated by the cases examined.

- Being Pattern Recognition motivated it retains the advantage shared by those methods, i.e, it is very fast and particularly suitable for on line applications.

- The method is very reliable because it relies on the hyperellipsoid of confidence idea. The uncertainty, associated with the misclassification error, which is the main weakness of any potentially applicable Pattern Recognition based operating strategy, has been circumvented. Reliability in real time power systems operation and control is, now, within reach in the context of the proposed methodology.

- The method utilizes the precontingency training data sets to conjecture post contingency system behaviour. As a consequence, once the training sets for the feasibility region of the precontingency power grid are available, one may examine a good variety of potential outages without having to resort to different training sets in order to construct the much needed hyperellipsoids of confidence.

vii) Stability and security indices bearing a definite quantitative and physically explainable meaning were suggested. Quantitative indicators of the degree of uncertainty for the operating environment can be directly contemplated.

viii) The method developed for contingency analysis circumvents one of the major (perhaps the most severe) drawbacks of Pattern Recognition methods as applied so far in Power system engineering: the fact that the slightest modification in network topology usually renders the acquired data sets completely useless. This is no longer a limitation with the suggested methodology.

6.2. RECOMMENDATIONS FOR FURTHER RESEARCH.

It is the opinion of the author that Pattern Recognition based methodologies can be fruitfully applied in power system engineering, especially in real time environment.

Nevertheless, a necessary presupposition is that methods to be used are freed from concepts and methodologies which may suit very well others but present serious limitations in Power System engineering.

Training Point Selectivity and Economization.

Training points in Power system engineering are very costly to obtain and selectivity as well as their economization must be the guidelines on which procedures for their acquisition should be based. Accordingly, research is needed to provide answers, of ever increased sophistication, to the question: " Where should one seek the next best training sample?".

Increasing the volume of error free decision making region.

For many practical power systems, the hyperellipsoid of confidence of type I may represent but a small subset of the actual feasibility region. It would be very useful if techniques for utilizing the unused parts of the feasibility region were developed. New hyperellipsoids of confidence (especially useful for load variability assessment) would emerge (they can be very well viewed as satellite hyperellipsoids) which would complement our global knowledge for the feasibility region. One promising methodological possibility for such an undertaking would be to apply "clustering techniques" to the states not enclosed by the already existing hyperellipsoids.

Improving the accuracy of the method for outage simulation.

The need for more accurate determination of the required injections at the line ends for proper simulation of the considered outage has already been pointed out. Any effort towards increasing the volume of the regions of error free decision making, leads to an uncertainty region characterized by an ever decreasing volume. Accordingly, it can no longer compensate for computational discrepancies when estimating the needed injections.

Multiple line outages.

Only single contingencies have been examined in this dissertation. It is the opinion of the author that the methods presented here are best suited for single outage analysis. Nevertheless, it would be beneficial if an assessment of the limitations of the method were carried out for multiple contingencies.

Power system planning.

Our numerical experience suggests that the more interconnecting links a power grid possesses the more unrestricted the power flow becomes. This indicates to a certain extend, that the buses of such a grid can withstand a maximum loading of the same order of magnitude. This attribute, incidentally, is a very desirable property of EHV backbone networks (the equivalents of which this dissertation mainly addresses). If uniform maximum loading can be accommodated, the hyperellipsoid of type I will be a fair approximation of the feasibility region itself. If the relative volumes of the hyperellipsoid of confidence, on the one hand, and the feasibility region itself are compared, their ratio (in conjuction with the grid bus loading limitations for the currently considered topology) could lead to specific recommendations for branch addition or deletion.

APPENDIX I

LINE AND BUS DATA FOR THE UTILIZED POWER SYSTEMS.

Table I.1 presents the line data of the 3-bus system utilized, while fig.I.1 depicts its topology. It is assumed that the slack bus is bus 1. Buses 2 and 3 are both considered to be voltage controlled nodes (type 2) with infinite reactive compensating capability. Their nominal voltage are assumed to be 1.00 p.u. The voltage at the slack bus was takend to be 1.05 p.u. Bus 1 has been assumed to be the only source of real power in the system.

BUS	BUS	SERIES IMPEDANCE	SHUNT ADMITTANCE
1	2	0.0210+j0.200	0.000+j0.000
1	3	0.0210+j0.200	0.000+j0.000
2	3	0.0210+j0.210	0.000+j0.000

Table I.1: Line data for the 3-bus system.

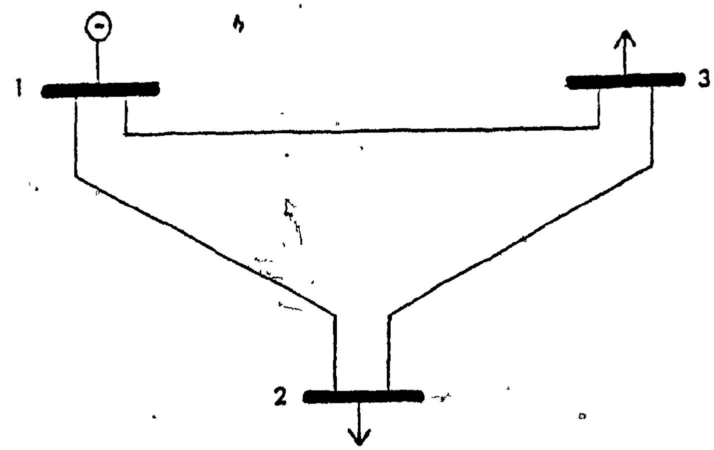


Fig I.1: 3-bus system.

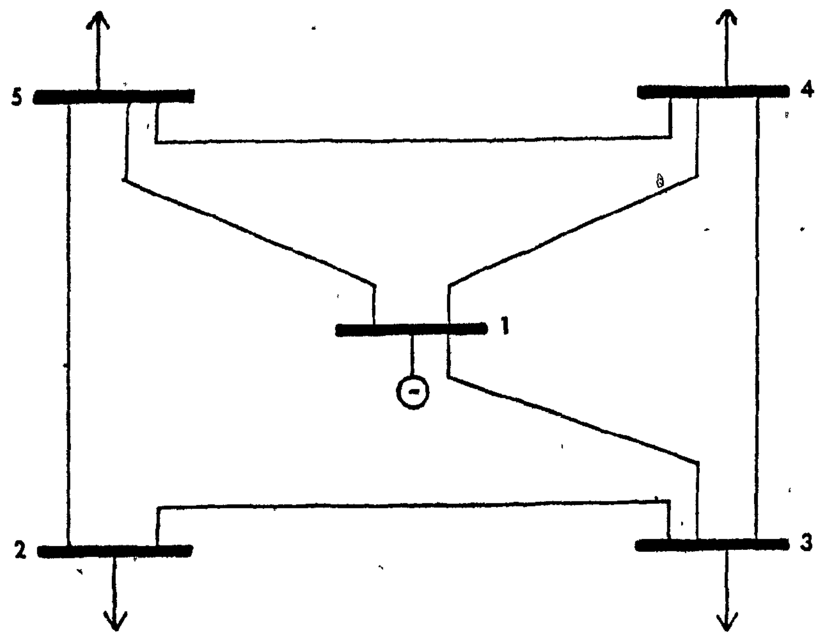


Fig.I.2: 5-bus test system.

Table I.2 presents the line data for the 5-bus test system utilized in the last chapter of this dissertation. Bus 1 is considered to be slack bus. Buses 2, 3, 4, 5 are considered to be type 2 (voltage controlled) buses with infinite reactive compensating capability. Their voltage magnitudes are held at the level of 1.00 p.u. The voltage of the slack bus was assumed to be 1.05 p.u.

BUS	BUS	SERIES IMPEDANCE	SHUNT ADMITTANCE
1	3	0.0208+j0.1367	0.000+j0.000
1	4	0.0237+j0.1823	0.000+j0.000
1	5	0.0309+j0.2372	0.000+j0.000
2	3	0.0574+j0.4397	0.000+j0.000
2	5	0.0419+j0.3212	0.000+j0.000
3	4	0.0366+j0.2806	0.000+j0.000
4	5	0.0410+j0.3149	0.000+j0.000

Table I.2: Line data of the 5-bus test system.

The impedance matrix for the 5-bus test system with the above presented data is found to be as follows (the resistive component of the impedance of the transmission lines has not been taken into account):

$$\begin{array}{cc} & \begin{array}{cccc} \text{Bus3} & \text{Bus4} & \text{Bus5} & \text{Bus2} \end{array} \\ \begin{array}{c} \text{Bus3} \\ \text{Bus4} \\ \text{Bus5} \\ \text{Bus2} \end{array} & \begin{bmatrix} j0.0950750 & j0.0347386 & j0.0270268 & j0.0557513 \\ j0.0347386 & j0.1030543 & j0.0428355 & j0.0394168 \\ j0.0270068 & j0.0428355 & j0.1345723 & j0.0891729 \\ j0.0557513 & j0.0394168 & j0.0891729 & j0.2606760 \end{bmatrix} \end{array}$$

APPENDIX II.

ON ALGORITHMIC PERFORMANCE INDICES.

In chapter III, "cost concious" algorithms were presented for training point selection, the objective being indirect region identification. The question one has to face, is, how many points are actually needed for reasonably accurate indirect region identification. The next immediate question would be how the dimensionality of the state space affects the training point requirements. Generally, if structure of any sort is not assumed (for the surface to be identified) questions of this sort are very difficult to tackle. Any assumption concerning structure (especially in higher than three dimensions) in order for the mathematical modeling to become simpler have to be well founded from a physical point of view if simplification of the problem is not to lead to its distortion..

In this dissertation our interest has been focused on the steady state feasibility region of power systems.

It is in order to mention here that, in spite of the fact that, eventually, the effort of acquiring the training points will be crowned with hyperellipsoid construction, the problem is not reduced to hyperellipsoid identification. To our knowledge, there is no evidence whatsoever for the feasibility region having any affinity for special structure. Any "a priori" assumption, therefore, on the matter will rather be misleading than helpful. Furthermore, it is a well known fact that the number of the buses (directly related to the dimensionality of the state space) in a power system is only one of the complicating factors. The steady state feasibility region is known, from experience, to be far more sensitive to other factors such as, system topology and system parameters. Recognizing the fact that various systems with the same number of nodes, can have radically different feasibility regions (due to their different topological structure) we conclude that, attempting to establish a performance index based solely on the number of the buses (dimensionality) is not really meaningful.

In practice, however, the problem is put more mildly. The power system is given and the dimensionality of the state space known. The analyst, therefore, faces the problem of the "a priori" determination of the number of iterations needed for its computer runs.

To be sure, after a suitable (but still undetermined) number of iterations no more training points will be needed. At that stage the hyperellipsoid resulting from the covariance matrix of the data will have reached its "final form". In other words, there will be a point beyond which acquisition of more training points will have no appreciable effect on the resulting hyperellipsoid(s).

Minor variations in size are to be expected for the hyperellipsoid(s) but its orientation (and its location) in space should be fairly well known. An eigenanalysis of the covariance matrix of the available, to that point, data can immediately reveal such variations. If changes in the directions of the eigenvectors are observed, when more training points enter the data sets, then, being still in the formative stage, more training points are needed. If, on the contrary, a pattern of convergence is observed one feels confident enough to stop.

A direct consequence of such a reasoning is that, being not able to specify "a priori" the number of iterations needed, does not necessarily mean that no stopping criteria are available.

What we, accordingly, propose is a sentinel procedure which if put in effect periodically (every prespecified number of iterations) can prevent redundancy in the data set contents. The scheme relies on the fact that, the eigenquantities of the covariance matrix of the data (utilized for hyperellipsoid construction), will not experience substantial variation if redundant information is added to the training sets.

For the 3-bus system table II.1 shows the results of such a procedure. The covariance matrix utilized for the eigencomputations pertains to the one obtained by utilizing the stable training points. Column 1 of the table depicts the training points collected. Column 2 depicts the eigenvectors of the covariance matrix of the data (in column form) and, finally, column 3 the corresponding eigenvalues.

NUMBER OF TRAINING POINTS UTILIZED	EIGENVECTORS 1	EIGENVECTORS 2	EIGENVALUES 1	EIGENVALUES 2
30	-0.856 -0.517	-0.517 0.256	19.78	55.33
60	-0.856 -0.516	-0.517 0.856	23.98	49.61
90	-0.845 -0.535	-0.535 0.845	25.30	44.66
120	-0.838 -0.546	-0.546 0.838	25.97	47.56
150	-0.833 -0.553	-0.553 0.833	23.80	50.82
180	-0.791 -0.623	-0.623 0.791	24.76	52.40
200	-0.772 -0.636	-0.636 0.772	24.68	53.10

Table II.1: 3-bus system. Variation of eigenquantities.

It is seen that, very rapidly, not appreciable changes are introduced in the eigendirections. Physically, this means that the algorithm utilized to acquire the training points (potential function concept) achieves the desired training point economization. Furthermore, the same facts lead us to the conclusion that the quality of the training points collected at the early stages is rather high, and therefore the algorithm is characterized by high selectivity. In this specific example, 200 training points were, in total, obtained. Nevertheless, one could as well collect 100 only and still feel confident enough.

The column 3 of table II.1. displays similar tendencies for the eigenvalues. One, however, should bear in mind that while eigenvectors are very indicative for the orientation of the hyperellipsoid(s) in space, the eigenvalues account for the actual lengths of the semiaxes. Taking into account the fact that, the square root of the eigenvalue (and not the eigenvalue itself) is a measure of of the length of the semiaxes, one sees that convergence is attained here as well.

Table II.2 is similar in significance to table II.1 but it refers to the 5-bus test system utilized in chapter 5. Column 1 indicates the number of available training points and column 2 shows the eigenvectors of the covariance matrix of the data (stable training points). The eigenvectors (normalized) are sorted in such a way as to correspond one to one with the eigenvalues of the covariance matrix sorted in ascending order. Similarly, table II.3 shows the variability of the eigenvalues with respect to the number of available training points.

NUMBER OF TRAINING POINTS UTILIZED	EIGENVECTORS 1	2	3	4
200	-0.945	0.321	-0.051	0.026
	0.202	0.613	-0.260	-0.718
	-0.074	-0.038	0.919	-0.386
	-0.245	-0.721	-0.293	-0.579
400	0.970	-0.210	-0.040	-0.113
	-0.043	-0.627	0.454	0.638
	-0.024	-0.184	-0.876	0.445
	0.236	-0.685	0.774	0.625
600	-0.959	-0.247	-0.034	0.136
	0.096	-0.742	0.291	-0.596
	0.003	-0.081	-0.932	-0.353
	-0.267	0.618	0.214	0.708
700	-0.963	0.201	0.080	0.162
	0.058	0.775	0.029	-0.629
	-0.054	-0.319	0.876	-0.358
	-0.258	-0.508	-0.475	-0.671
800	-0.956	0.224	-0.006	0.189
	0.051	0.760	0.113	-0.638
	-0.044	-0.153	0.987	-0.011
	-0.285	-0.591	-0.113	-0.746
900	-0.947	0.256	-0.020	0.194
	0.051	0.717	0.012	-0.695
	-0.062	-0.091	0.991	-0.082
	-0.312	-0.642	-0.135	-0.687
1000	-0.948	-0.256	-0.044	-0.184
	0.028	-0.660	0.121	0.741
	-0.045	0.068	0.992	-0.100
	-0.314	0.703	0.002	0.638

Table II.2: 5-bus test system. Variation of eigenvectors.

NUMBER OF TRAINING POINTS UTILIZED	EIGENVALUES 1	2	3	4
200	14.73	17.14	19.45	27.07
400	15.15	19.95	22.48	29.24
600	15.19	22.05	24.16	28.20
700	14.86	22.67	26.42	29.13
800	15.04	22.90	26.49	28.53
900	15.15	22.73	26.10	28,70
1000	15.33	22.75	26.26	28.23

Table II.3: 5-bus system. Variability of the eigenvalues.

In the tables presented above the fact that rather fast convergence is attained for both eigenquantities, in both cases, suggests that indeed high selectivity is achieved when collecting training points. As a further substantiation of this claim the 3-bus experimental system was chosen to illustrate the effectiveness of the adopted algorithmic procedure. Fig II.1, Fig II.2, Fig.II.3, Fig.II.4, depict the two training sets for the system but for various numbers of training points. Dots represent stable states crosses represent unfeasible states and shaded tringles randomly generated test states.

It is seen that (Fig.II.1) for very few training points the misclassification error is very low and that selectivity is achieved. For ever increasing numbers of wanted training points, (fig.II.2, Fig.II.3, Fig.II.4) the misclassification error is, again, kept at very low levels. Nevertheless, the feasibility region is more and more identifiable.

The actual number of training points one should obtain is, naturally, strictly dependent on the degree of accuracy desired. Every system is to be treated separately since it presents an entirely new case. The criterion presented here, is rather heuristic and definetely not objective. The stricter the criteria for convergence the larger the required effort will be.

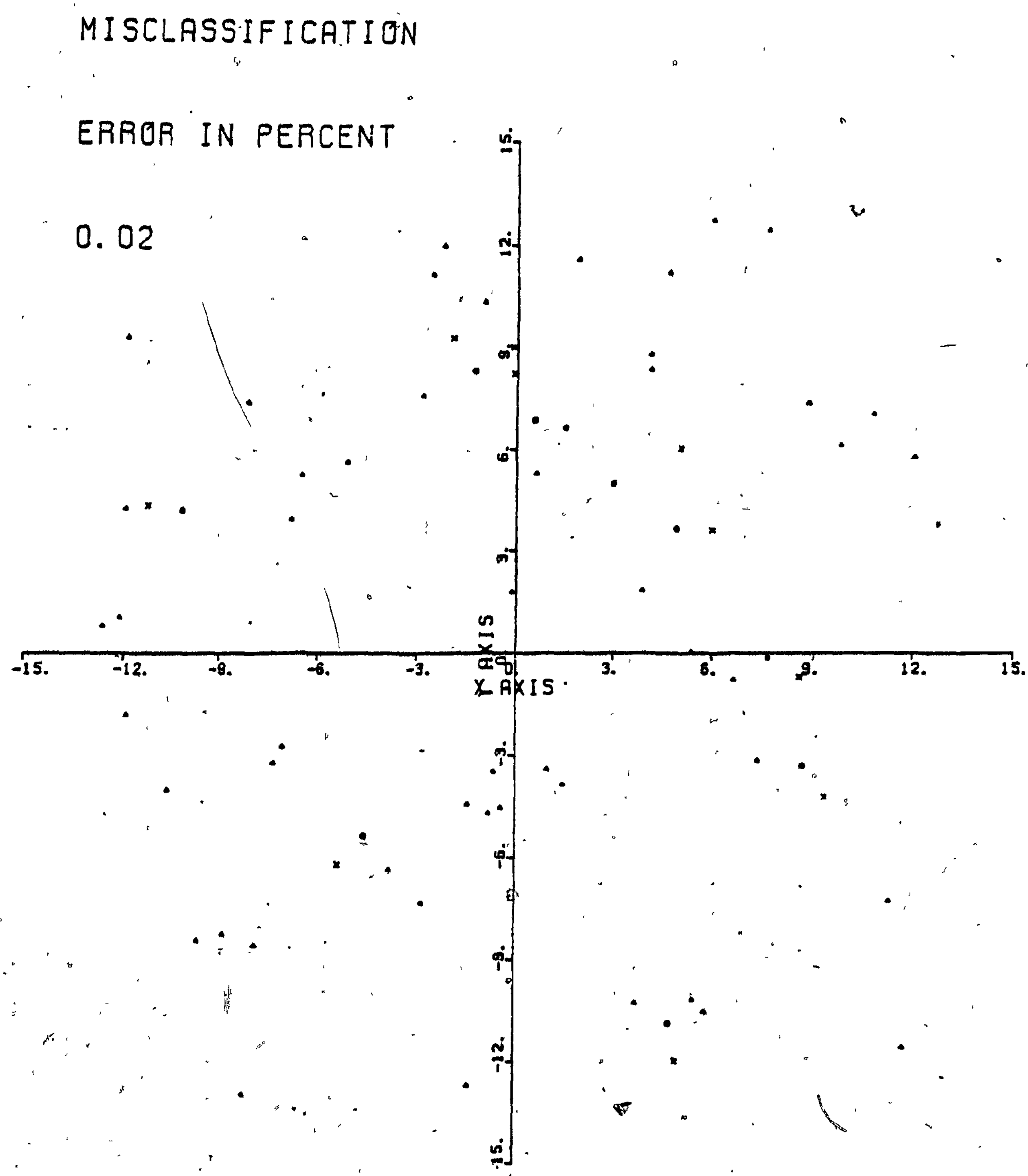


Fig.II.1: 3-bus system. 10 training points collected.

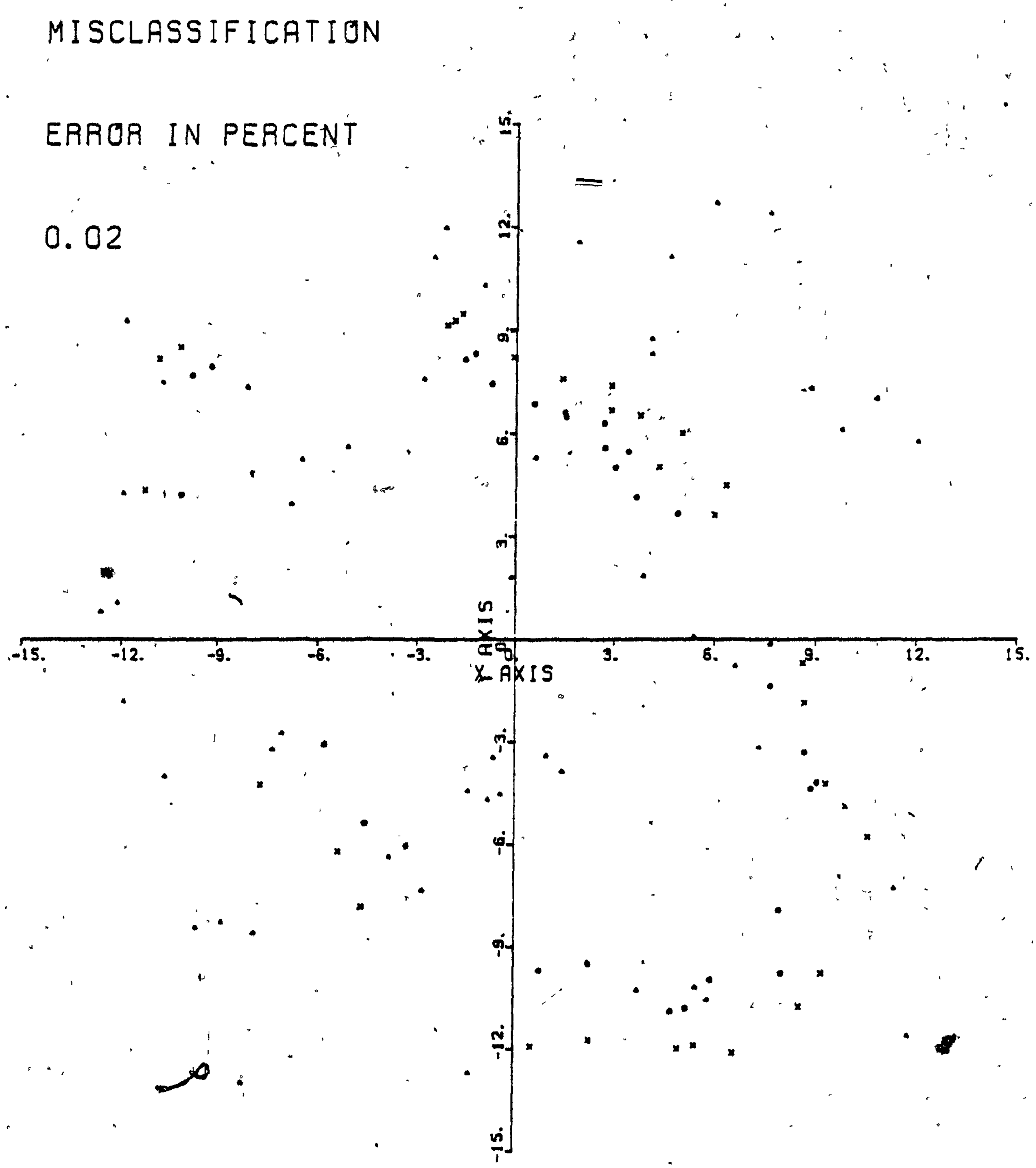


Fig.II.2: 3-bus system. 30 training points collected.

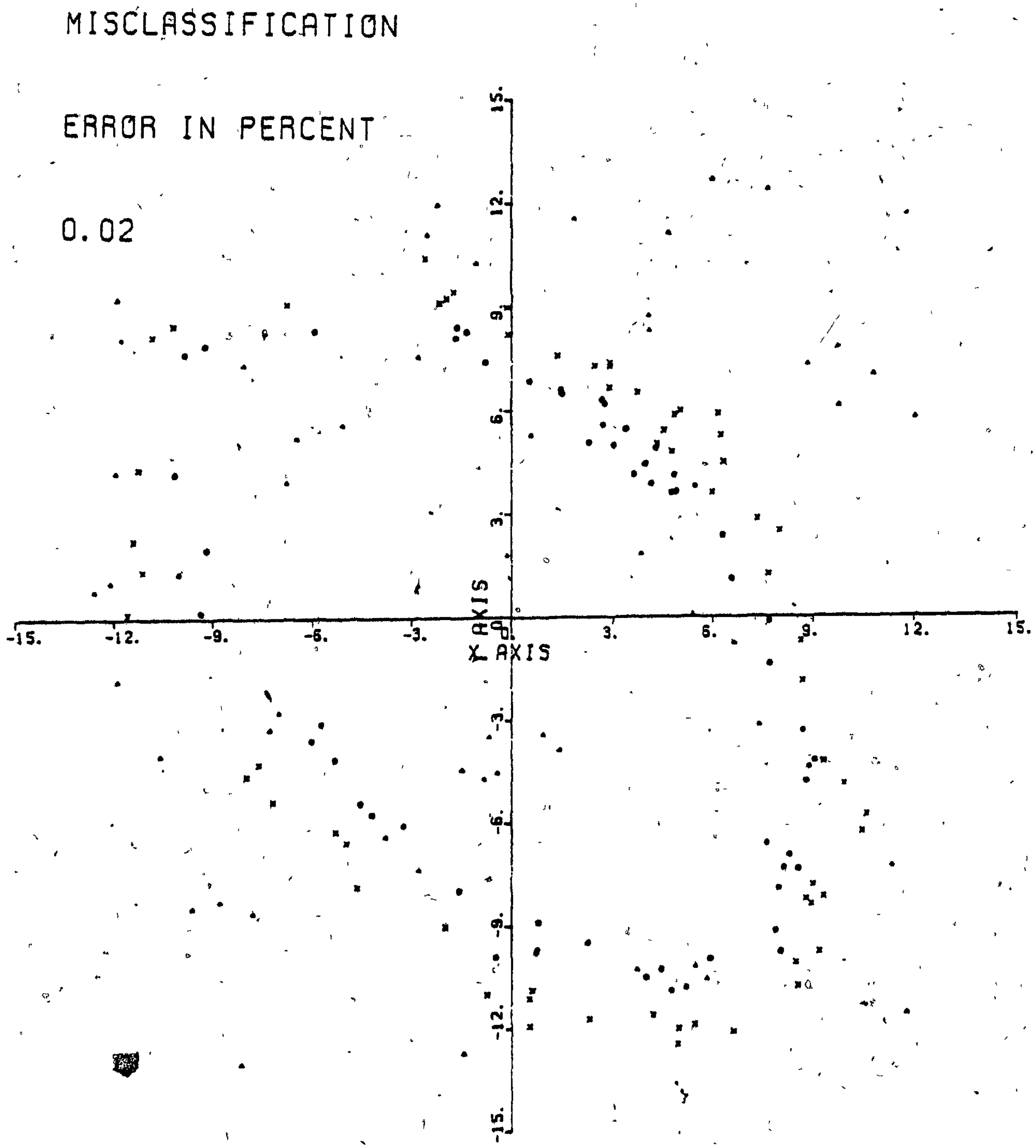


Fig.II.3: 3-bus system. 60 training points collected.

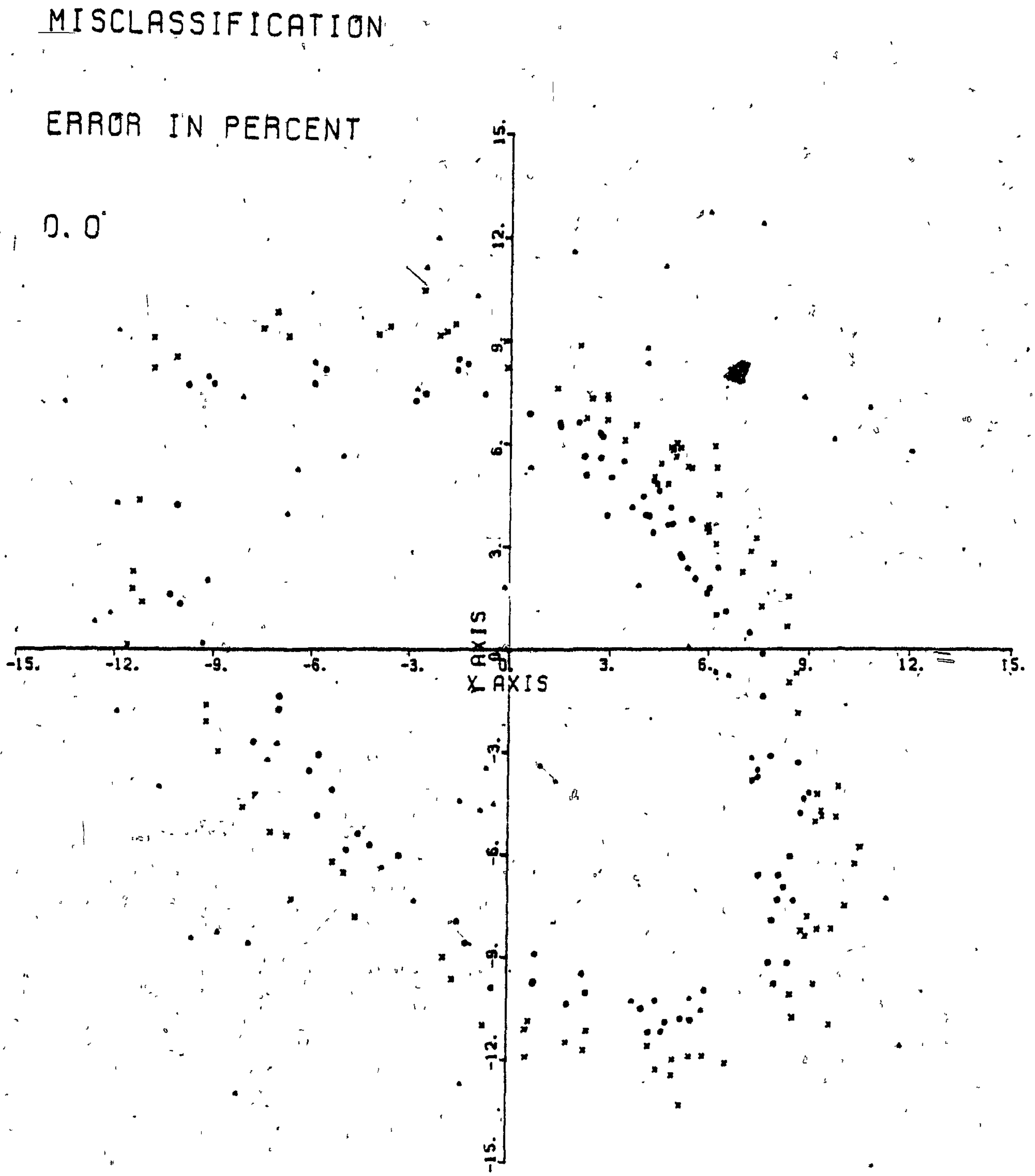


Fig.II.4: 3-bus system. 100 training points collected.

APPENDIX III

STABILITY AND SECURITY MARGINS.

A by product of the concept of the hyperellipsoids of confidence, as utilized here for both steady state stability assessment as well as for steady state contingency analysis, are quantitative indices concerning the degree of stability and security for the system under the considered operating conditions.

Recall that in the detailed analysis of the three sample case studies in art.5.6, the value of eq.(5.1) (for the state vector of the compound injections) has been found to be monotonously dependent on the degree of severity of the precontingency load.

On the other hand, the value eq.(5.1) assumes for a given state, is a direct measure of the location of the state with respect to the surface, eq.(5.1) analytically represents.

For a given system is a matter of scaling, to assess any proximity of a given state and the surface of the hyperellipsoid. This fact in conjuction with the fact that the hyperellipsoid represents but a subset of the actual feasibility region, may lead to various operating strategies.

Assume, for instance, that assessing the effect of an outage (or generation shift) via the presented techniques, one finds the state of the system to be located to the exterior of the region of the hyperellipsoid. Assume, furthermore, that numerical acquaintance with the equation of the hyperellipsoid (for the given grid), suggests that the given state falls just short from being located in the interior of the region.

In this case the operator has every right to decide to endorse the analyzed contingency as a nonfatal one. It should, nevertheless, be mentioned that decision making of this kind is purely subjective and, generally, not justifiable. In this dissertation, no "confidence intervals" were suggested for the grids analyzed. For actual power grids, however, one could assess such margins on an empirical basis, if operating in a slightly uncertain environment is acceptable.

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