### Multiuser Detection and Performance Evaluation for Cellular DS-CDMA Networks with Partial Information

Liqing Zhang



Department of Electrical and Computer Engineering McGill University Montreal, Canada

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To my wife Lin and our daughter Bessie, To our parents in both families.

#### Abstract

We consider the design and performance of CDMA-based multiuser detectors with partial information. Specifically, we assume that the multiuser signal presented to the receiver is contaminated by interference from sources whose parameters — power, delay, signature sequence — are unknown. The application we have in mind is to the uplink reception problem in CDMA cellular wireless, it being reasonable in such a setting to suppose that detailed parametric models of out-of-cell interferences may not be available to every base station.

In the first part of the thesis, we propose forms of enhanced multiuser detectors with partial information, whose front end uses the covariance structure of the outof-cell contribution to suppress interference prior to multiuser signal detection. The proposal, amenable to adaptive as well as non-adaptive implementation, features a bank of linear equalizers at the input, and either a maximum-likelihood (ML) sequence detector or a linear detector at the output. Computational complexity, given the size of the detection group, is independent of the total number of active users.

The second part of the thesis evaluates the performance of the proposed receiver structures, both for the AWGN channel and for the multi-path channel with slow Rayleigh fading. The evaluation criteria include mean squared error, effective signalto-noise ratio, probability of bit error, asymptotic efficiency and near-far resistance. The results, functions not only of the channel parameters but of the particular character of the side information assumed available, provide an expanded view of the performance tradeoffs available in the application of multiuser detectors. The proposed receivers are observed to be near-far resistant and superior to enhanced singleuser detectors. The ML-based variants that we studied outperform the linear ones, the performance difference being especially significant when parameter information is incomplete or MAI is severe. Bit error rate decreases with the size of the detection group, and falls off rapidly with the number of resolved multi-path components. Comparison of short and long spreading in our numerical experiments suggest that short spreading is less sensitive to incompletely characterized interference.

#### Sommaire

Nous traitons du design et des performances de détecteurs AMRC multi-utilisateurs à information partielle. Nous supposons plus particulièrement, que le signal multiutilisateurs à l'entrée du récepteur est contaminé par des interférences dont les paramètres de source - puissance, retard, signature - sont inconnus. L'application que nous envisageons est celle liée au problème de la réception des signaux cellulaires sans fil AMRC de la voie montante. Dans ce cas, il est raisonnable de supposer que les modèles paramétriques détaillés de linterférence extra-cellulaire ne sont pas toujours disponibles pour toutes les stations de base.

Dans la première partie de la thèse nous proposons une structure de récepteur dont le pré-traitement exploite la structure de covariance de l'interférence extracellulaire. pour supprimer l'interférence avant le décodage multi-utilisateurs. Cette structure, qui se prête aussi bien à une réalisation adaptative que non-adaptative, comporte une série d'égaliseurs linéaires à l'entrée et d'un détecteur à maximum de vraisemblance (MV) ou d'un détecteur linéaire à la sortie. La complexité du calcul est indépendante du nombre total d'utilisateurs actifs pour une grandeur donnée du groupe de détection.

La seconde partie de la thèse évalue les performances de la structure du récepteur proposé dans le cas du canal BBGA et celui du canal multivoie à évanouissements de Rayleigh. Le critère d'évaluation tient compte de l'erreur quadratique moyenne, du rapport signal-bruit, de la probabilité d'erreur sur les bits, de l'efficacité asymptotique et de l'immunité à l'effet près-loin. Les résultats, qui dépendent non-seulement des paramètres du canal mais également du caractère particulier de l'information latérale que nous supposons disponible, offrent une meilleure vue d'ensemble des compromis disponibles lors de l'emploi de la détection multi-utilisateurs. L'architecture proposée se révèle être résistante à l'effet près-loin et supérieure aux détecteurs complexes à utilisateur unique. La variante MV que nous avons étudiée a surpassé la variante linéaire, l'écart en termes de performances étant particulièrement significatif lorsque l'information disponible sur les paramètres est incomplète ou que l'interférence multi-accès est importante. Le taux d'erreur sur les bits diminue en fonction de la grandeur du groupe de détection et diminue rapidement en fonction du nombre de chemins multivoies isolés. La comparaison entre l'étalement à séquence courte et à séquence longue à l'aide de nos résultats numériques suggère que l'étalement à séquence courte est moins sensible à l'interférence incomplètement caractérisée.

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## List of Symbols

Symbol

### Explanation

$K/K_n/M$	A convention in this thesis to designate both the detection problem and the information structure that characterizes it: $K$ denotes the total number of active users in the system; $K_n \leq K$ denotes the number of users whose parameters are known and used for multiuser detection: $M \leq K$ denotes the size of the <i>target</i> group — the group
	of users selected for multiuser signal estimation and joint detection.
T	Transposition of a matrix or vector.
*	Complex conjugation of a matrix or vector.
*	Convolution operator.
< , >	Inner product operator.
$\mathbf{a}_k$	= $[a_k(0) \ a_k(1) \ \dots \ a_k(N-1)]^T$ , the signature vector of user k.
$lpha_{mk}(t)$	A time-varying fading coefficient of user $k$ .
$\alpha_k(t)$	A time-varying fading coefficient of the $k$ th path component.
$b_k(l)$	Transmitted bit of user $k$ at the $l$ th bit interval.
$\mathbf{b}_{K_n}$	A bit sequence vector of $K_n$ users.
$\mathbf{b}_M$	A bit sequence vector of $M$ users.

$\mathbf{b}_{M_p}$	A bit sequence vector of $M_p$ signal components from $M$ users.
b	$=\mathbf{b}_{M_p}.$
$C_{jk}(i-l)$	$= \int_{-\infty}^{\infty} s_j (t - iT_b - \tau_j) s_k (t - lT_b - \tau_k) dt$ , signature waveform correlation between user j at the <i>i</i> th bit interval and user k at the <i>l</i> th bit interval.
$\mathcal{C}_L$	Symmetric $ML \times ML$ correlation matrix which is a diagonal block matrix with an element matrix of $\mathbf{C}(0)$ .
$d_j(l)$	Multiuser signal aggregate associated with user $j$ at the $l$ th bit interval, which is desirable for multiuser detection and will be estimated by the front-end $j$ th signal equalizer.
$\hat{d}_j(l)$	An estimate of $d_j(l)$ .
â	An output sequence vector consisting of sample vectors from all front- end signal equalizers
$\mathbf{d}_{\mathbf{I}}$	Residual interference vector (from unwanted interfering sources) at the outputs of all front-end signal equalizer.
$e_j(l)$	An estimate error between $d_j(l)$ and the output of equalizer $j$ at the $l$ th time instant.
$F_{j}$	An indecomposable set of error sequences that affect user $j$ .
$\mathbf{g}(t)$	A signal vector formed by time-shifted signature waveforms of resolved signal components from the known users.
$h_j(t)$	An impulse response of the $j$ th front-end equalizer with the linear optimal (MMSE) estimation.
J	The number of resolved multipath components.
K	The number of user population in a CDMA network.
$K_n$	The number of known users in a cell of interest for base-station mul- tiuser detection.

L	A total number of transmitted bits of each user.
M	The number of known users in a joint detection group.
$M_p$	The number of signal components from the $M$ users in the target detection group.
$\mu_j$	The step size of an adaptive algorithm used in the front-end $j$ th equalizer.
Ν	The number of chips per bit interval.
Ν	The filtered noise vector from outputs of all front-end linear equalizers.
$N_0$	Power spectrum density of white noise.
$P_k$	The (large-scaled) received signal power of user $k$ .
$\mathbf{p}_{j}$	An $N \times 1$ cross-correlation vector between the chip-matched sample vector of the received signal and the desired multiuser signal aggregate associated with user $j$ .
$\psi( ext{t})$	A chip waveform.
$\mathbf{Q}(l-i)$	Signal correlation matrix whose entry is $\langle s_k(t-iT_b-\tau_k), h_j(lT_b+T_b-t) \rangle$ , where $s_k(t)$ is the signature waveform of an unknown interfering user k and $h_j(t)$ is an impulse response of a known user j.
$ ho_{jk}$	Signal correlation of signature waveforms between user $j$ and user $k$ .
r(t)	The received signal aggregate at a base station.
$\mathbf{r}_j(l)$	An $N \times 1$ chip-matched filter sample vector of $r(t)$ .
R	Signal correlation matrix whose entries are correlations between sig- nature waveforms and impulse responses of known users.
$\mathbf{R}(i-l)$	Signal correlation matrix whose entries are correlations between sig- nature waveforms at the $l$ th bit interval and impulse responses at the <i>i</i> th interval, both associated with the known users.

$\mathcal{R}_L$	Signal matrix whose element matrices are $\{\mathbf{R}(i-l)\}$ .
$R_I$	Correlation matrix of residual interference vector $\mathbf{d}_{\mathbf{I}}$ .
$r_D(t)$	The signal aggregate from known sources.
$\mathbf{S}_{j}$	An $N \times N$ correlation matrix of the sampled vector of the received signal in the $j$ equalizer.
$\hat{\mathbf{S}}_{j}(X)$	An estimate of $\mathbf{S}_j$ based on a moving (time) average of the received signal over an observation X-bit interval.
$S^D_t(\mathbf{b}_{K_n})$	The signal aggregate from known sources.
$T_{b}$	A bit interval.
$T_c$	A chip interval
$ au_k$	A transmission delay of user $k$ .
$\mathbf{w}_{jo}$	$N\times 1$ optimum coefficient vector of the $j{\rm th}$ equalizer.
$\mathbf{W}(i-l)$	Signal correlation matrix whose entries are correlations between impulse responses at the $l$ th bit interval and impulse responses at the $i$ th interval, both associated with the known users.
$\mathcal{W}_L$	Signal matrix whose element matrices are $\{\mathbf{W}(i-l)\}$ .

### Chapter 1

### Introduction

Mobile phones and wireless data services have greatly influenced our daily lives. Parallel to well-known wireline networks, the widespread deployment of cellular wireless networks and their services has been witnessed almost everywhere in our modern life, which will revolutionize the concept of communication and information processing for business, professional, and private applications. To satisfy the growing demand for global wireless communication services, future mobile systems are required to accommodate a set of the standard service capabilities; for example, the third-generation (3G) wireless systems have been required to satisfy the International Telecommunications (IMT-2000) requirements [1, 2]. Thus, the system design to achieve all these goals in a cellular wireless network is a challenging task. Among the many difficulties imposed by this demand for services are cell-based multiple services for multiple users in a limited radio frequency resource and a time-varying wireless propagation environment; that is, the multiple-access schemes play a crucial role in the system design.

In multiple-access applications, where many users transmit to and from a single point over a limited and shared radio resource, multiple-access techniques are required to manage the sharing of the resource to guarantee the separation of users' signals in simultaneous transmissions. The commonly used, multiple access techniques include time division multiple access (TDMA), frequency division multiple access (FDMA), and code division multiple access (CDMA). CDMA technology has shown more promise to potentially enhance performance than other multiple access technologies; in fact, it has been chosen as the globally adopted standard air interface in 3G wireless communication systems [3, 4]. Our research interest of this thesis focuses on CDMA technology, which will be described in the sequel; information on TDMA and FDMA schemes can be found in many references and textbooks, for example, [5, 6].

CDMA technologies can be broadly divided into two categories: frequency-hoping CDMA (FH-CDMA) and direct-sequence CDMA (DS-CDMA). The FH-CDMA technique can be viewed as a hybrid of TDMA and FDMA. In a FH-CDMA system, the available spectrum (channel) bandwidth is subdivided into a large number of contiguous frequency slots. In any signaling (time) interval, the transmitted signal occupies one or more of the available frequency slots; the selection of the frequency slot(s) is pseudorandom, according to the preassigned unique hoping pattern for each user.

In a DS-CDMA system, all users transmit over the entire available spectrum bandwidth at the same time. The bandwidth of a user's (information) signal is expanded by multiplying it with a spreading waveform, often referred to as **signature** waveform or pseudo-noise (PN) waveform, whose bandwidth is much wider than that of the user's signal. In its simplest, binary form, a signature waveform consists of a chip waveform (usually common to all users) and a PN sequence (unique for each user) whose rate is some large integer multiple of the information bit rate. As a consequence, DS-CDMA is often referred to as a wideband multiple-access technique. The receiver can distinguish a user by its unique PN sequence from the received aggregate signal of multiple users.

In this thesis, we'll address the problems of base-station receiver design in cellular wireless DS-CDMA networks. Because of the sharing of the same radio spectrum, any signal of a user will result in multiple-access interference to other active users (assuming nonorthogonal signature waveforms), and in a fading multipath propagation environment, it will also produce the inter-symbol interference (ISI) to the user itself; if not properly controlled, one user signal can be very strong and mask the weak signals of other users at the receiver. Thus, it is very challenging for the base-station receiver design to take these effects into account in wireless DS-CDMA systems. In a multi-cell setup, the base-station receiver in a cell has the parameter knowledge — timing, signature waveform, and/or signal strength — of the in-cell users. However, the base station usually has no such knowledge of the users who



**Fig. 1.1** Direct-sequence code-division multiple-access (DS-CDMA) communication system (baseband model).

are out of the cell, and the signals of the unknown users are not under control of the in-cell receiver; some out-of-cell mobiles (especially these close to the desired cell edge) can transmit substantially high powers, due to their poor-quality links, thus resulting in the strong out-of-cell interference that can not be ignored for the in-cell signal reception. As a result, it becomes even more challenging for the CDMA receiver design that incorporates all available user information and takes the effects of in-cell and out-of-cell sources into consideration in cellular wireless DS-CDMA networks. This thesis will propose and evaluate forms of enhanced base-station receiver architectures under the partial system parameter information scenarios, which can fully exploit the known parameter knowledge of in-cell users and the available statistical signal characterization of out-of-cell sources for both the multiuser detection of in-cell users and the suppression of unwanted interference in cellular wireless DS-CDMA technique: important benefits, existing impairments, and sensible solutions.

#### 1.1 Direct-Sequence Code-Division Multiple-Access (DS-CDMA)

The block diagram shown in Fig. 1.1 illustrates the basic elements of a typical DS-CDMA baseband communication system for transmitting the binary information sequences of multiple users. At the transmitting end, the information bits of a user, generated at the *bit* rate, are fed into the channel encoder to produce a coded symbol sequence; this sequence (usually performing additional interleaving in many systems of practical interest) is then modulated by the (binary) modulator; and finally, the



Fig. 1.2 Typical cellular multiple-cell systems with a frequency reuse pattern. Cells with same-number designation use the same set of frequencies simultaneously.

modulated signal is multiplied by a preassigned, unique signature waveform of the user. The inverse procedures are made at the receiving end to obtain the estimated information bits of the user. The PN sequence employed in both the transmitter and the receiver for each active user needs to be unique: it furnishes the addressing or identification such that multiple users can be properly separated at the receiver end. Synchronization (or timing) of the signature sequence generated at the receiver with the signature sequence contained in the incoming received signal is required to demodulate the received signal. Moreover, both the timing and the phase recovery are desired at the receiver for coherent reception. The chip rate of PN sequence is usually much higher than information bit rate, and the ratio of the chip rate to the bit rate is often loosely referred to as the processing gain. Note that the processing gain in a coded CDMA system is split into two parts: a coding factor (due to the channel coding) and a spreading factor (due to the signal spreading of the coded symbols); for a CDMA system without channel error coding, the coding factor is considered one, and thus the spreading factor is equal to the processing gain. Further information on the fundamentals of DS-CDMA can be found in [7, 8, 9].

Among many attractive features of DS-CDMA technology are its ideal frequency

reuse and its effective use of silence activity in a bandwidth-on-demand, fair-sharing manner. Frequency reuse is one of the most important cellular parameters in the evaluation of spectrum utilization efficiency and can be described as follows: each cell is assigned a group of distinct radio channels to guarantee the disjoint frequency bands and avoid mutual interference between the neighboring cells. Cellular wireless systems based on TDMA/FDMA technology requires frequency reuse to prevent or reduce co-channel interference [10, 5]. A typical cellular TDMA/FDMA multiplecell network with a frequency reuse pattern is shown in Fig. 1.2. Each hexagon represents a cell and cells marked with the same number designate simultaneous use of the same set of carrier frequencies (channels). Since the number of available frequencies (or channel sets) is 7, the frequency reuse factor is 1/7 [5, 11]. Contrary to TDMA/FDMA, a cellular DS-CDMA network can reuse all seven frequencies (or channel sets); that is, one frequency band can be used by each cell, leading to a frequency reuse factor of 1 and thus, more efficient spectrum use than TDMA and FDMA schemes. Moreover, the ubiquitous frequency usage in all CDMA cellular cells makes frequency assignment and management much easier; for example, when a mobile user migrates from one cell to another, its service can be readily offered by different base stations simultaneously, a technique known as soft hand-off.

Because of the ubiquitous frequency usage in a cellular CDMA system, transmission power becomes a commonly shared resource for all active users and thus, the advantages of the voice activity factor and data burst on-and-off characteristic along with variable bit rates can be effectively (and naturally) used; the effective interference is alleviated at the receiver during the off-periods of other interfering signals, leading to an ultimate system performance improvement. Moreover, since each incoming transmission signal contributes a certain amount of interference, raising the total interference power at both the base station and the mobile users, the effect of increasing the number of active mobile users in a cellular CDMA network is a smooth performance degradation, known as **soft capacity**.

So far, we have described a few key features of cellular DS-CDMA networks. However, there are some impairments embedded in DS-CDMA based systems that must be overcome to achieve full CDMA capacity. The primary drawback is multipleaccess (or co-channel) interference (MAI), due to multiuser communication channels sharing the same frequency bandwidth. Second, the MAI in unbalanced power scenarios results in the well known near-far problem. Third, the received aggregate signal at a base station in a cellular wireless CDMA network involves both the in-cell and the out-of-cell interfering sources whose parameter knowledge is substantially different, and some unlocked out-of-cell interfering sources can be very strong to the in-cell reception. And finally, channels are time varying and characterized by the fading, multi-path effects that are common to all mobile radio propagation environments.

Reasonable techniques employed to overcome these CDMA drawbacks include power control, channel coding, diversity schemes, and multiuser detection, each of which is able to tackle the CDMA problems from a different angle and provide partial solutions. Power control (PC) can effectively manage the shared power resource of the in-cell signals to minimize the transmitted power while maintaining the required quality of service (QoS) for an individual in-cell user, thus keeping the total interference at a minimum level as well as alleviating the near-far problem. For example, to compensate for the deep faded signal in wireless fading multi-path channels, the fast power control strategy has been employed in both forward and reverse links of CDMA2000 systems [3, 4], where the estimated performance metric is compared with the desired target in each PC slot, and the transmitted power is adjusted accordingly (i.e., raising or lowering the power level if the measured metric is below or above the target value). Many power control schemes can be found in [12]-[16]. An effective power control algorithm can reduce the MAI level and alleviate the near-far problem from the in-cell sources; however, it is difficult to resolve the near-far problem resulting from the out-of-cell interfering sources in a multi-cell wireless CDMA network.

Reliable communication hinges on effective ways of adding redundancy to the data to protect it from the random disturbances introduced by the channels. The channel error-correction coding schemes are such an effect way of putting this redundancy in the transmitted bits to correct the received signal errors introduced by transmission medium, and can improve the overall channel bandwidth utilization of random access CDMA channels [17]-[19]. Moreover, the recently-proposed Turbo coding scheme [20, 21], often thought of as a refinement of the concatenated encoding structure and an iterative algorithm for decoding the associated code sequence, can approach performance very close to the Shannon capacity (at the expense of

complexity); specifically, for a bit error probability of  $10^{-5}$  and a Turbo code rate of 1/2, a required  $E_b/N_o$  of 0.7 dB was reported. The powerful error correction capability of channel error control coding has been widely used in practical wireless communication systems. For example, the convolutional code with a code rate of 1/3 and constraint length of 9 was employed in IS-95 digital cellular systems [22, 23]. Convolutional and Turbo coding schemes have both been employed in CDMA2000 cellular systems for fundamental channel and supplemental channel transmissions in radio configurations 3, 4, and above [3]. Since channel error-correction coding requires redundancy, it consumes more spectrum bandwidth in a coded DS-CDMA system, thus reducing the spreading gain for a given frequency bandwidth CDMA system.

Diversity technology is another important performance-enhancement scheme in advanced wireless networks such as the 3G wireless CDMA systems; it is especially applicable when combined with current VLSI technology and well-developed advanced digital signal processing techniques. Diversity techniques can be broadly divided into two categories: non-contrived diversity and contrived diversity schemes. Non-contrived diversity is formed naturally by wireless propagation environments such as fading multi-path channels (e.g., the multi-path diversity in a RAKE receiver [28, 29]) and soft hand-off. Contrived diversity is a controlled (artificially designed) diversity with delicate antenna design and sophisticated signal processing. It includes receiver (spatial) diversity, for example, adaptive antenna array [24]-[27] and beamforming/sectorization [30, 31, 32, 33]); transmitter (spatial) diversity [34], for example, spatial and temporal (ST) coding [35]; and polarization diversity, that is, the polarization division multiple access (PDMA) serving voice and data with different signal polarization.

Multiuser detection (MUD) technology, unlike other schemes, explores the signal infrastructure of multiple access interference (MAI) itself for joint multiple-user signal detection or effective MAI cancellation. MUD can remove the interference limit floor in DS-CDMA systems without occupying more radio spectrum resources, thus increasing spectral efficiency and system capacity; for example, the maximumlikelihood sequence multiuser detector proposed in [36] can approach the single-user performance of an isolated single-user communication channel when the base station has the parameters of all active users and a sufficiently-large computational capability. Consequently, MUD that addresses the CDMA impairments has drawn a great deal of attention in the past decade; such an advanced technology has also been proposed as one of very promising options to enhance performance in 3G and future-generation wireless CDMA systems [37, 1, 2, 38].

In this thesis, our research focuses on the multiuser detection technology in multiple-user, multiple-cell DS-CDMA networks.

### 1.2 Multiuser Detection Issues in Cellular DS-CDMA Networks

We consider the base-station reception for reverse link transmission in a typical cellular DS-CDMA wireless network, where the *partial system parameter information* setting is assumed: the aggregate signal received at any base station consists of components from the in-cell sources, whose parameters — received power, timings and signature sequence — are *known* to the receiver, components from the out-of-cell sources, whose parameters are *unknown*, and background noise. The base-station receiver is interested in the signal detection of the in-cell known users; however, since some undesired signal components from unknown users — for example, the out-of-cell mobiles close to the boundary of the desired cell — can be very strong to the base-station reception, the out-of-cell interfering sources can not be ignored in the signal detection of the in-cell users. As a consequence, the effective base-station multiuser reception in a multi-cell CDMA network becomes the problem of both the joint detection of in-cell users and the suppression of the out-of-cell or unwanted interference.

Now it comes to a question: How have the interference cancellation and multiuser detection problems been treated previously in the literature? A detailed review of CDMA receivers can be found in Section 2.3; here, we briefly examine the general detection strategies. There are two popular methodologies in addressing the problems of multiuser detection and interference suppression. The first methodology assumes the parameter knowledge of all active users at the receiver (i.e., the centralized user information scenario) and involves no signal equalization in the receiver front end. Typically, a bank of conventional matched filters (MFs), one for each active user, can be employed in the receiver front end, and the output sequences of these MF filters can provide a sufficient statistic [36] for the joint detection of the active users; a decision algorithm applied to the statistic to separate users in the second-stage signal processing can be either nonlinear or linear. When the second-stage signal processing employs the maximum likelihood sequence (MLS) decision algorithm, the overall receiver architecture is the well-known optimum multiuser detector proposed by Verdu in [36]; when the second-stage processing employs a linear transformation such as decorrelating or MMSE, the overall architecture becomes a form of sub-optimal, extensively studied linear multiuser detectors. Since the parameter knowledge of the out-of-cell users is usually unknown at the base-station, the multiuser receivers proposed in line with this methodology are appropriate for the base-station reception in a single-cell CDMA system, where the out-of-cell sources are absent. In this thesis, we propose forms of the nonlinear and linear multiuser receivers for the base-station reception in *multi*-cell wireless CDMA settings, where the out-of-cell interference is present and the interference suppression is considered; we relax the the centralized user information requirement, assuming that while the shape of the *chip* waveform for each source is known, signature sequences, received powers, and timings are known for the in-cell sources only at the receiver.

The other methodology involves signal equalization technique in the detector front end for interference cancellation, assuming that certain parameter knowledge of at least one active user is known at the receiver. In such a case, an enhanced frontend filtering in place of the conventional matched filter is employed to suppress the unwanted interference while the signal equalization target is the transmitted bits of an individual user, which is called *single-user*-oriented signal equalization technique in this thesis; the second-stage signal processing simplifies to a binary decision device, leading to another form of extensively studied linear CDMA detectors. Thus, the resulting receiver architecture is most appropriately applied to the case where the parameter knowledge of only one user is known to the receiver. However, the singleuser-oriented signal equalization is not very efficient for the base-station reception in that it can not explicitly use the information on other (in-cell) known users to enhance performance. In fact, directing the equalization toward a single-user signal destroys the well-formed multiuser signal correlation infrastructure of the multiple known users, which otherwise can be used for effective joint detection of the multiple users. The multiuser signal infrastructure of the in-cell sources, or the so-called multiple-

access interference (MAI), is known to the base-station reception, and exploiting it to demodulate the multiuser signals and remove the interference limit floor is a fundamental rationale behind MUD. In this thesis, we'll demonstrate that it is feasible to fully exploit the knowledge of the in-cell users for effective joint detection and for performance enhancement with partial information (e.g., multi-cell) setups at a base station. The crucial strategy to achieve these goals is to direct the front-end signal equalization toward the *multiuser signal aggregate* of the in-cell known users, rather than the *signal component* of each individual known user. Thus, the known signal correlation infrastructure among the in-cell users can be retained, rather than destroyed, during the out-of-cell interference suppression in the front end, and then used for effective MUD in the second stage; the parameter knowledge of in-cell known users can be *explicitly* incorporated into the multiuser detection and the interference suppression to improve performance. Such a signal processing technique is referred to as multiuser-oriented signal equalization throughout this thesis, which distinguishes our proposed multiuser detection scheme from other schemes found elsewhere in the literature.

For the (nonlinear) maximum likelihood sequence multiuser detection, whose implementation complexity increases exponentially with the number of known users in the joint detection group, MUD of all in-cell users may incur an extremely high implementation cost; it may be beyond the affordable computational capacity of a base station when the number of the in-cell users is very large. In such a case, the joint detection for a subset of the in-cell users, able to trade performance for complexity, becomes especially important in a real wireless CDMA system. We also investigate such a detection issue in our research.

In this thesis, we propose and evaluate forms of enhanced multiuser detectors for base-station reception in a cellular DS-CDMA wireless network, assuming partial system parameter information setup. We write K for the total number of in-cell and out-of-cell signals received,  $K_n \leq K$  for the number of in-cell signals whose parameters are known and used for MUD (while the  $K - K_n$  out-of-cell interfering sources are unknown and need to be suppressed), and  $M \leq K_n$  for the size of the *target* group: the group of signals selected for multiuser signal estimation (in the front end) and joint detection (in the second stage). The notation  $K/K_n/M$  designates both the detection problem and the information structure that characterizes it. In addition, J denotes the number of resolved components for each user in fading multipath channels. Detailed system models can be found in Sections 2.1 and 2.2, and the justification of certain important research assumptions is provided in Section 2.6. Part of the research results have been published in [42, 43, 44], and the original thesis contributions are stated in next section.

#### 1.3 Thesis Contributions

The principal, original contributions of this thesis are: Firstly, a linear multiuseroriented signal equalization technique, beating conventional matched filtering and single-user-oriented equalization schemes, is proposed for both interference suppression and multiuser signal estimation in the receiver front end. By directing the target toward the multiuser signal aggregate desirable for joint detection, the proposed equalization technique generalizes the concepts of the single-user-oriented, binary signal equalization schemes; it enables the full exploration of the available knowledge of multiple users to estimate the desired multiuser signals on a symbol-by-symbol basis during the suppression of unwanted interference; it allows for elaborate and effective MUD with partial information to enhance performance. Thus, such a signal equalization scheme distinguishes our proposed multiuser detection scheme from alternatives found in the literature. (Sections 2.5, 3.1, 3.4.1, and 3.4.2 for AWGN channels; Sections 3.5.1 and 3.5.2 for fading multi-path channels).

Secondly, based on the proposed multiuser-oriented signal equalization methodology, multiuser receiver architectures for maximum likelihood sequence (MLS) detection and linear multiuser detection are derived in multi-cell CDMA networks under AWGN and Rayleigh fading multi-path channels. In general, the proposed architectures have two stages. The first stage, forming the front end of the device, amounts to a bank of linear multiuser-oriented signal equalizers on the basis of MMSE criterion, one for each of the target sources for joint detection; its role is to estimate the multiuser signal aggregate by suppressing the unwanted interference. The second stage, acting *jointly* on all outputs from the front-end equalizers, is a maximum-likelihood sequence (MLS) detector or linear multiuser detector; its structure is calculated from the second-order statistics of the equalizer error processes, assumed approximately Gaussian on the basis of a Central-Limit argument. The proposed maximum likelihood sequence (MLS) multiuser detection is a generalization of the MLS multiuser detection [36, 45] to multi-cell, partial information scenarios.

The proposed receivers can be characterized by  $K/K_n/M$  detection that designates the multiuser detection, the interference suppression, and and the information structure that characterizes it. When the number of known-parameter users is set to one, the proposed K/1/1 linear MMSE receiver reduces to a form of enhanced linear single-user detectors [46, 47, 48]. In the particular case of a single-cell system under AWGN channel where out-of-cell interference is absent, the front end reduces to the conventional matched filter bank; the overall architecture for  $K_n/K_n/K_n$  MLS multiuser detection reduces to the one described in [36]; and the architecture for  $K_n/K_n/K_n$  linear detection reduces to the linear MMSE [49] or decorrelating [50] detector.

The multiuser receivers developed in this thesis can be categorized in detail as follows:

- $K/K_n/K_n$  MLS multiuser detector in AWGN CDMA channels, whose computational complexity (in terms of the number of multiplications) is on the order of  $2^{K_n}$  (for synchronous in-cell transmissions), independent of K (Sections 3.1 and 3.2).
- $K/K_n/K_n$  linear decorrelating and MMSE multiuser detectors in AWGN CDMA channels (Section 3.3).
- $K/K_n/M$  ( $M < K_n$ ) MLS detectors for AWGN CDMA channels, whose computational complexity is on the order of  $2^M$  (in the synchronous case) and on the order of  $2^{3M}$  (in the asynchronous case), independent of K and  $K_n$  in both cases. They can provide a configurable trade-off between performance and complexity (Section 3.4).
- K/M/M MLS multiuser detector under multi-path slowly Rayleigh-faded channels, whose computational complexity is in the range of  $O(J2^M)$  (in the synchronous case) and  $O(J2^{3M})$  (in the asynchronous case), independent of the total number of unwanted/unresolved interfering components (Section 3.5).
- K/M/M linear multiuser detection architecture in multi-path slowly Rayleigh-faded channels (Section 3.6).

• A least-mean-squared (LMS) based adaptive implementation of the proposed multiuser detectors (Section 3.8).

And finally, the system performance of the proposed multiuser detectors is analyzed and evaluated with respect to signal-to-interference (plus noise) ratio (SIR), asymptotic efficiency, near-far resistant and bit error rate, and also justified with the benchmark performance of several well-known multiuser receivers, including conventional matched filter, the enhanced single-user based (K/1/1) detector, and the centralized K/K/K multiuser detector. Moreover, the study investigates a variety of issues of practical importance to system design and signal reception, such as the trade-off between performance and implementation cost, and the impacts of user parameter knowledge, detection group size, the number of resolved multi-path components, and so forth.

The analytical and simulation results demonstrate that the proposed front-end linear multiuser-oriented signal equalization scheme is effective; the proposed MLS multiuser detectors are near-far resistant and able to approach the performance bound of an ideal single-user system, as the base station approaches the full parameter knowledge of all active users; the proposed  $K/K_n/K_n$  linear detectors outperform the well-known enhanced single-user K/1/1 detectors; the proposed  $K/K_n/K_n$  MLS detectors outperform the proposed  $K/K_n/K_n$  linear detectors at the price of a higher computational cost.

The performance evaluation details are given below:

- Provided a performance analysis of the proposed multiuser-oriented signal equalization in terms of minimum mean-squared error (MMSE) and effective SNR; quantified the equalization performance by numerical examples (Section 4.2).
- Presented upper and lower bounds to the bit-error probability for the proposed  $K/K_n/M$  MLS detection in synchronous AWGN CDMA channels; provided numerical examples to evaluate the performance of the MLS and linear detectors (Sections 4.3.1 and 4.3.2).
- Provided a bound analysis on bit-error probability for proposed K/M/M MLS and linear multiuser detectors in multi-path slowly Rayleigh-faded CDMA

channels; quantified their performance in multi-cell, partial information setups (Sections 4.3.3 and 4.3.4).

- Analyzed the statistical characteristics of short spread and long spread signals (Section 3.7); quantified their impacts on performance (Sections 4.3.2 and 4.3.4).
- Presented an analysis of the asymptotic efficiency and near-far resistance for the proposed multiuser detectors; quantified the detection performance by numerical examples (Section 4.4); provided an explanation of the saddle-shaped asymptotic efficiency performance in a full-blown MLS multiuser detector (Section 4.4.3).

#### 1.4 Thesis Organization

Chapter 2 provides the fundamentals to this thesis research, including the system model and research assumptions, a thorough review of the literature on CDMA receivers, the problem formulation and the research statements.

In Chapter 3, the multiuser-oriented signal equalization scheme is proposed for the receiver front-end signal processing; forms of enhanced MLS and linear multiuser detectors, applicable to the base-station reception in multi-cell CDMA networks, are derived under Gaussian channels and multi-path Rayleigh fading channels; a form of unifying  $K/K_n/M$  MLS multiuser receiver architecture is developed to provide a trade-off between performance and complexity.

Chapter 4 presents performance analysis and evaluation on the proposed multiuseroriented signal equalizer, linear and nonlinear multiuser detectors developed in Chapter 3. It also quantifies the impacts of a variety of issues of interest, including detection group size, short spreading vs. long spreading, and the number of resolved multi-path components.

Chapter 5 addresses the applications of the proposed multiuser detectors in current and future wireless networks; Chapter 6 summarizes this study and provides future research directions.

Appendices A-F provide necessary definitions, proofs and numerical derivations.

### Chapter 2

### **Fundamentals**

In this chapter, we first present a system model, study assumptions, and background for this thesis research. Then a detailed literature review and a problem analysis of CDMA receivers are addressed to elaborate on what has been done so far in this area. Finally, the multiuser detection rationale in multi-cell CDMA networks and the research statements of this thesis are described.

#### 2.1 System Models for Code Division Multiple-Access

#### 2.1.1 Cellular CDMA network model

We consider the signal reception at a base station (or cell site) in a multi-cell Kuser DS-CDMA wireless network. For the target detection group of M users in a cell with  $K_n$  known users, the issue in question is the  $K/K_n/M$  multiuser signal detection problem in the *partial system parameter* framework, where  $M \leq K_n \leq K$ . The signal presented to the base station includes contributions from the in-cell sources as well as the out-of-cell sources. Assuming that user k  $(1 \leq k \leq K)$  is assigned a signature waveform  $s_k(t)$  of one bit duration  $T_b$ , the signature waveform can be expressed as

$$s_k(t) = \sum_{n=0}^{N-1} a_k(n)\psi(t - nT_c), \quad 0 \le t < T_b, \ k = 1, 2, \dots, K$$
(2.1)

where  $\{a_k(n), n = 0, 1, ..., N - 1\}$  is the kth user's spreading sequence, consisting of N chips that take on the values  $\{\pm 1\}$  and can be written as a vector form  $\mathbf{a}_k =$ 



Fig. 2.1 The multiple-access model of a cellular CDMA network.

 $[a_k(0) \ a_k(1) \ \dots \ a_k(N-1)]^T$ .  $T_c$  is the inverse of chip rate and we have  $T_b = NT_c$ .  $\psi(t)$  is a chip pulse that is a deterministic function defined over  $[0, N_cT_c)$ , where  $N_c$  is an integer that is determined by the system design. The signal waveform  $s_k(t)$  is assumed to be normalized to have *unit* energy, i.e.,  $\int_0^{T_b} s_k^2(t) dt = 1$ .

The transmitted baseband signal of user k can be represented as

$$x_k(t) = \sum_{l=-\infty}^{+\infty} b_k(l) s_k(t - lT_b), \quad 1 \le k \le K$$
(2.2)

where  $b_k(l)$  is the transmitted bit <sup>1</sup> of user k at time  $lT_b$  and takes on a value from  $\{\pm 1\}$ .

After each user's signal passes through its respective propagation channel, the baseband signal received by the base station is the aggregate of K individual transmissions as [51]

$$r(t) = \sum_{l=-\infty}^{+\infty} \sum_{k=1}^{K} \sqrt{P_k} b_k(l) y_k(t - lT_b) + n(t)$$
(2.3)

where  $P_k$  is the received signal power of user k, including the effect of antenna gains, a path loss, and a large-scale loss; n(t) is additive white Gaussian noise (AWGN)

 $<sup>^{1}</sup>$ It may designate either a coded symbol or an uncoded information bit. However, we do not consider the effect of the channel correction coding in this MUD study.
with two-sided power spectral density  $N_0/2$ ;  $y_k(t)$  is the channel output of user k, given by applying  $s_k(t)$  to the channel input as

$$y_k(t) = s_k(t) \star h_k(t) \tag{2.4}$$

in which  $\star$  symbolizes the convolution operator and  $h_k(t)$  is the channel impulse response of user k. The system model of a cellular CDMA network is shown in Fig. 2.1.

From (2.3), the *L*-bit instance of the received signal presented to the receiver can be split into the contributions from in-cell, out-of-cell and background noise, respectively, as

$$r(t) = \sum_{l=1}^{L} \sum_{k=1}^{K} \sqrt{P_k} b_k(l) y_k(t - lT_b) + n(t)$$
  
=  $S_t^D(\mathbf{b}_{K_n}) + S_t^I + n(t)$  (2.5)

where  $S_t^D(\mathbf{b}_{K_n}) = \sum_{l=1}^{L} \sum_{k=1}^{K_n} \sqrt{P_k} b_k(l) y_k(t - lT_b)$  is the received signal aggregate contributed by the  $K_n$  known in-cell users,  $S_t^I = \sum_{l=1}^{L} \sum_{k=K_n+1}^{K} \sqrt{P_k} b_k(l) y_k(t - lT_b)$ is the received signal aggregate contributed by the  $K - K_n$  unknown users, and n(t)is the additive white Gaussian noise.  $\mathbf{b}_{K_n}$  denotes the  $K_n L \times 1$  bit vector of the  $K_n$  known users. We present the system assumptions for (2.5) and the  $K/K_n/M$ detection in next subsection; the multiuser detection and the interference suppression rationale will be described in Section 2.5.

#### 2.1.2 System assumptions

The users in the cellular CDMA network are enumerated so that the indices  $\{1, \ldots, M\}$  identify the target group;  $\{M + 1, \ldots, K_n\}$  identify the other in-cell users whose signal parameters are known;  $\{K_n + 1, \ldots, K\}$  identify the interference whose parameters are unknown.

The transmitted bits of each user take on values from  $\{\pm 1\}$  with equal probability; they are modeled as zero-mean, uncorrelated random variables for the same user and zero-mean independent random variables for different users. The chip waveform  $\psi$ is common to all users and is known to the receiver.

## Channel models:

- For MUD study in AWGN channels, an ideal radio propagation model is employed, whose impulse response h<sub>k</sub>(t) for user k is given by h<sub>k</sub>(t) = δ(t τ<sub>k</sub>), 1 ≤ k ≤ K, in which τ<sub>k</sub> represents the propagation delay of the kth user's signal at the receiver.
- For MUD study in multipath fading channels, the lowpass impulse response of the channel at time instant t to an impulse transmitted by user k at time t - τ is expressed as [6]

$$h_k(\tau; t) = \sum_{m=1}^{J} \alpha_{mk}(t) \delta(\tau - \frac{m}{W} - \tau_k), \quad 1 \le k \le K$$
(2.6)

where  $\{\alpha_{mk}(t)\}\$  are time-varying channel coefficients of user k.  $J = \lfloor T_m W \rfloor + 1$  $(\lfloor x \rfloor$  denotes the smallest integer greater than x, and  $T_m$  is a multipath delay spread, assuming that  $T_m \ll T_b$  is the number of delay taps that can be resolved at the CDMA receiver.  $\tau_k$  denotes the propagation delay of user k.

## Assumptions for $K_n$ in-cell users:

- BPSK modulation and coherent reception.
- The delay, phase, and received amplitude are known a priori.
- In multipath slowly-faded channel, estimations of channel coefficients  $\{\alpha_{mk}(t)\}$  for known users in joint detection group are required.

## Assumptions for $K - K_n$ out-of-cell users:

- The second-order signal statistic of  $S_t^I$  in (2.5) needs to be estimated based on the received aggregate signal or overhead channels, and no other knowledge of out-of-cell users is required.
- Information about the total number of active users, K, is not required for the joint detection schemes; in our numerical examples,  $K K_n$  unknown signal

sources represent the mobiles with relative strong interfering powers (e.g., outof-cell mobiles close to the cell edge of a cell of interest) to the base-station reception that need to be suppressed.

## Additional assumptions:

- Short spreading codes are assumed throughout this thesis for analysis and simulation, unless otherwise stated.
- In theory, long spreading codes can also be employed in MUD, but they will incur a performance loss as compared to the short spreading codes [51, 52]. Since it is a topic of interest, this thesis will present some analyses on the characteristics of long spread CDMA signals (in Section 3.7), and quantify the performance by numerical examples (in Sections 4.3.2 and 4.3.4). Where employed, the long spreading codes will be explicitly stated.

We have assumed that the timings and received amplitudes of (in-cell) known users, as well as the channel parameters (in a slow fading multi-path scenario), are known *a priori* to the receiver. The estimations of these parameters, though not of prime interest in this thesis, can be made available through an overhead channel [53, 54]; for instance, each mobile in 3G systems [3] has provided a pilot channel for reverse-link coherent demodulation and channel estimation. Furthermore, the topic of estimating reception parameters without overhead information has also been extensively studied in the literature, including [55, 56] for timing and carrier phase estimation and [57, 58, 59] for channel estimation.

## 2.2 Cyclostationarity and Channel Equalization

It can be shown that the aggregate CDMA signal at the receiver is a continuous-time cyclostationary process in wireless propagation environments (also, cf. [62]). First, we examine the cyclostationarity of the received signal at a base station under an AWGN channel. In such a channel environment, the received signal r(t) in (2.3) can

be written as

$$r(t) = \sum_{l=-\infty}^{+\infty} \sum_{k=1}^{K} \sqrt{P_k} b_k(l) s_k(t - lT_b - \tau_k) + n(t)$$
(2.7)

where the transmitted symbols  $\{b_k(l)\}$  are modeled as  $E[b_k(l)] = 0$ ,  $E[b_k^*(l)b_{k'}(l)] = \delta_{kk'}$  for different users, and  $E[b_k^*(l)b_k(l')] = \delta_{ll'}$  for different bit intervals of user k,  $\delta_{kj}$  is the Kronecker delta ( $\forall l \neq j$ ,  $\delta_{lj} = 0$  and  $\delta_{ll} = 1$ ). The delays  $\{\tau_k\}$  are modeled as (unknown but) deterministic parameters.  $s_k(t)$  is a spread signal waveform of user k, which is a deterministic function defined over the bit interval  $(0, T_b)$ . It is readily seen that the mean value of r(t) is zero, and the autocorrelation function can be written as

$$\Phi_{rr}(t+\tau,t) = E[r^{*}(t+\tau)r(t)]$$
  
= 
$$\sum_{l=-\infty}^{+\infty}\sum_{k=1}^{K} P_{k}s_{k}^{*}(t+\tau-lT_{b}-\tau_{k})s_{k}(t-lT_{b}-\tau_{k}) + E[n^{*}(t+\tau)n(t)].$$
(2.8)

Since  $s_k(t - lT_b - \tau_k)$  is a deterministic function and n(t) is a Gaussian stationary process, we have

$$\Phi_{rr}(t+\tau+mT_b,t+mT_b) = \Phi_{rr}(t+\tau,t)$$
(2.9)

for  $m = \pm 1, \pm 2, \ldots$  Hence, the autocorrelation function of r(t) is periodic with a period  $T_b$ . Thus, the received CDMA signal r(t) is a cyclostationary process with a period  $T_b$  in an AWGN channel.

Secondly, we consider this issue under a slowly-faded multipath channel described in (2.6); the time-variant tap weights  $\{\alpha_{mk}(t)\}$  are assumed to be constant for a short period, for example, a fast power control group of 1.25 milliseconds [3]. Thus, the channel impulse response can be simplified from (2.6) as

$$h_k(t) = \sum_{m=1}^{J} \alpha_{mk} \delta(t - \frac{m}{W} - \tau_k), \quad 1 \le k \le K.$$
 (2.10)



Fig. 2.2 A linear chip-matched (fractionally-spaced) FIR employed as a single-user-oriented signal equalizer in a cellular CDMA network.

where  $\{\alpha_{mk}\}\$  are assumed to be independent for different users and different multiple path components. By checking the autocorrelation function of r(t) in such a channel condition, it is seen that the received CDMA signal aggregate at the base station is also a cyclostationary process with a period of one bit interval  $T_b$  in a slow-fading multipath channel.

The sampled discrete sequence of the continuous cyclostationary signal may or may not be cyclostationary, depending on the sampling rate's relation to the cyclostationary period. In particular, samples with a sampling rate equal to the inverse of the cyclostationary period are actually a wide-sense stationary sequence and the autocorrelation matrix of the samples is symmetric and *Toeplitz* [63]. On the other hand, samples by oversampling (with respect to the cyclostationary period) are widesense cyclostationary and the autocorrelation matrix of the samples is symmetric, but usually not *Toeplitz*. Such signal cyclostationarity can be used for effective channel identification and signal equalization; in fact, recent research results suggest that exploiting the cyclostationarity characteristic of communication signals can lead to algorithms requiring only second-order statistics for channel amplitude and phase estimation [60, 57, 64, 61], interference cancellation [46, 65, 66] and signal separation [67, 68, 69], which is more attractive than higher order statistic (HOS) techniques.

To take advantage of the cyclostationarity property for effective signal estimation, a so-called linear fractionally-spaced filter is often employed with an observation over the cyclostationary period. A typical chip-matched FIR structure for linear signal equalization is shown in Fig. 2.2, where  $\psi(t)$  is the chip waveform,  $T_b$  and  $T_c$  are the bit interval and the inverse of chip rate, respectively,  $\tau_k$  is transmission delay of user k, the number of tap weight coefficients  $\{w_k(n)\}\$  is equal to the spreading gain N, and  $b_k(l)$  and  $\hat{b}_k(l)$  are the transmitted bit of user k and its estimate at the output of the equalizer at the *l*th bit interval, respectively. By choosing a set of optimal tap weight coefficients  $\{w_k(n)\}\$  on the basis of a certain optimization criterion, the linear signal equalizer in Fig. 2.2 can not only suppress the multiple access interference from other users but also mitigate the effect of inter-symbol interference (ISI) in a dispersive channel. Moreover, the equalization structure can adapt to the slow time variation of wireless propagation channels by periodically updating the tap weight coefficients  $\{w_k(n)\}$ . Note that the linear signal equalization shown in Fig. 2.2 is single-useroriented in the sense that the equalization target is the transmitted bits  $b_k(l)$  of user k only. We'll illustrate in this thesis that by directing the equalization target toward the signal aggregate of multiple users (to be defined in Section 2.5), rather than a single-user signal  $b_k(l)$ , such a linear fractionally-spaced filter architecture can achieve the multiuser-oriented signal equalization in the front end of the proposed multiuser detectors, in which the known parameter knowledge of more than one user can be fully exploited to enhance performance.

## 2.3 DS-CDMA Receivers

In this section, we'll review the literature on DS-CDMA receivers to date. The DS-CDMA receivers can be broadly divided into two categories: single-user receiver and multiuser receiver. The single-user receivers refer to the detectors whose signal detection or estimation is based on the single-user signal of interest. For the interference suppression in most of the detectors in this category, a front-end signal equalization technique is employed to estimate the transmission bits of a single user only on the basis of the second-order statistic of the received signal; thus the multiuser signal infrastructure (desirable for joint multiuser detection) in the received aggregate is not retained after the front-end signal equalization. In contrast, the multiuser receivers refer to the detectors whose signal detection considers the utilization of the well-formed multiuser signal correlation infrastructure. The receivers

in this category usually assume the full parameter information of all active users; that is, the receiver has knowledge of signature sequence, timings, and channel impulse response of each user, as well as the received amplitude for many nonlinear and linear multiuser detectors. The classification of DS-CDMA receivers is presented in Fig. 2.3.

## 2.3.1 Single-User receivers

The single-user CDMA receivers can be further classified into linear and nonlinear receivers on the basis of their detection strategies.

## **Linear Receivers**

## **Conventional MF receiver**

The conventional matched filter (MF) receiver only requires the signature waveform and the timings of a single user of interest for signal detection, and assumes that signals from other users and background noise are a white Gaussian process. Because of its simple implementation, the conventional MF filter is still used in many practical CDMA systems, such as second- and third-generation wireless CDMA systems [3]. However, since it neglects the existence of multiple-access interference (MAI), the receiver performance will incur a severe degradation in a multiuser environment, especially, under *near-far* scenarios [58].

To improve the detection performance, research efforts over the past decades have been focused on CDMA receivers with better performance. There are two directions toward this goal: (1) searching for good PN codes, and (2) applying interference suppression techniques. The research following the first direction is reflected in the early CDMA study in which the orthogonal PN codes or signature sequences with small correlation values are investigated [70, 71]; the CDMA capacity analysis can be found in [72, 73, 74]. In the past decade, extensive research along the second direction has been undertaken, leading to forms of so called enhanced single-user detectors. The CDMA receivers employ a front-end linear single-user based signal equalizer to suppress the interference of unknown or unwanted signal sources and to estimate the transmitted bits of the single user. We focus on the review for such



Fig. 2.3 Classification of DS-CDMA receivers.

enhanced single-user detectors in the sequel.

#### Linear adaptive single-user detectors

The linear adaptive single-user detectors [48, 65, 47] assume that the receiver has knowledge of the signature waveform and timings of a single user of interest for coherent reception, and they also require an initial training sequence (at the start of the transmission) to implement an adaptive detection. These single-user detectors can be characterized as adaptive K/1/1 linear receivers in terms of our definition.

A filter structure, shown in Fig. 2.2, can be employed as the front-end filter signal equalizer in the linear single-user detectors. As a result, the cyclostationary nature of CDMA signals can be explored in the suppression of the MAI from other interfering sources. Moreover, the filter structure can adapt to the time variations of channels by automatically adjusting the coefficients.

The linear adaptive single-user detectors can achieve a significant improvement in MAI and near-far environments as compared to the conventional MF receiver. This type of linear detector can be especially applicable in scenarios where the parameter knowledge of only one user is known; however, the detection is not very effective for a CDMA base-station reception, since these linear detectors do not intend to exploit the parameter knowledge of other known users to improve the detection performance.

## Blind linear single-user detectors

The term *blind* arises from the sense that no training sequence is required in a CDMA receiver. Without a training sequence, a form of K/1/1 non-adaptive linear single-user detector is proposed in [46, 77], where a linear fractionally-spaced FIR architecture was employed in the receiver front end, and the target signal  $b_k(l)$  was considered a random variable to be estimated in the front-end signal equalizer. An MMSE estimation of the single-user signal  $b_k(l)$  leads to optimal equalizer tap coefficients; this can be achieved by solving the Wiener-Hopf filter equation [63] for the coefficient vector (see Appendix D). Thus, the matrix inversion is required in the linear optimal signal equalization and its computational complexity increases linearly with the number of coefficient taps used in the front-end equalizer.

To adaptively implement the detection without the knowledge of a training sequence, the self-recovering single-user detectors are investigated in [78] - [82]. In [78] - [80], a constrained minimum output energy (CMOE) criterion was employed; in [81, 82], the blind adaptive detection was based on the second-order statistics of the received signal. These blind adaptive single-user detectors can also be viewed a estimation-detection architecture [83], in which the front-end signal processing considers the single-user signal as a random variable that needs to be estimated before making the binary detection.

Other schemes used for blind adaptive signal detection include the subspace technique [84, 85, 86]. The signal detection mechanism in [84, 86] maximizes the projection of the received signal into the estimated single-user signal space (or equivalently, minimizes the projection into the noise space) by using the eigenvalue decomposition (ED) or singular value decomposition (SVD) on the estimated correlation matrix of received signal vector; the single-user receiver [85] incorporates both the subspace and space-time diversity techniques into the signal detection and the interference rejection.

Other blind adaptive detectors employ a parallel structure that consists of one conventional MF receiver and one linear adaptive single-user detector; the minimization of the error formed by the difference of the outputs of the two receivers leads to the convergence of tap coefficients in the linear adaptive single-user detector [80].

As a consequence, the training process is not required. However, as expected, the scheme suffers a little performance degradation since the output of the conventional MF receiver is utilized in place of the desired transmitted bits.

To exploit the parameter knowledge of more than one user at the base station to enhance performance, linear detectors [39]-[41] have recently been proposed, which can be described as  $K/K_n/1$  linear detection; their methodology is to explore the available knowledge of known in-cell users to cancel the out-of-cell interfering sources and to estimate the transmitted bits of the user of interest. In [39] and [40], a linear hybrid multiuser detection was proposed based on the subspace tracking technique; its signal equalization was performed by a combination of a decorrelating projection into the known signal space of in-cell known sources and a MMSE suppression of unknown out-of-cell signals. In [41], a linear adaptive detector was proposed on the basis of the signal "orthogonalization", which can be approximately achieved by the inverse of the known signal auto-correlation matrix of the in-cell users and by the stochastic approximation with averaging. Since these linear detectors employ the single-user-oriented signal equalization technique, the well-formed multiuser signal correlation infrastructure of the in-cell sources is not explicitly or fully used for the signal estimation and detection.

## SNR-maximizing single-user detectors

Another criterion to optimize the front-end linear filtering or equalization is to maximize the desired signal-to-interference and noise ratio at the filter output, thus leading to a form of SNR-maximizing single-user detectors [75, 76, 11, 87]. This form of detectors employs a fractionally-spaced linear filter, whose sampling rate is greater than the chip rate, such that the cyclostationarity of the CDMA interfering sources can be fully exploited; these single-user linear detectors are one-shot based and suppress the interference from other users by taking advantage of the coloration of the chip-waveform power spectrum or whitening the multiple-access interference and noise. The spatial-temporal one-shot linear detector based on single-user SNRmaximizing criterion is proposed in [88], where the optimization of the filter response was performed jointly in both spatial and time domains using a receiver antenna array at the base station. Such linear detectors, which can also be characterized as K/1/1 detection, are able to offer good near-far resistance and achieve better performance than the conventional MF filter. They can be considered a good alternative to conventional MF filters since they are applicable to both long-spread CDMA systems and short-spread ones. However, since only the knowledge of one single user is assumed in these receiver design, they are not very efficient for the base-station reception where the receiver has the parameter knowledge of more than one user in terms of user information utilization. The one-shot linear detector [89] exploited the partial information (i.e., chip delay and signal strength) of the known users to enhance performance, which can be considered  $K_n/K_n/1$  linear detection.

## Nonlinear Receivers

## **Optimum detector**

Nonlinear, optimum single-user detector is proposed in [90] to track and detect the signal of the user of interest on the basis of the likelihood-ratio statistic in the presence of unlocked interfering users; its implementation complexity per bit increases exponentially with the total number of active users in the system. However, it is not appropriate for base-station reception in a cellular CDMA network, since the use of available parameter knowledge of in-cell known users for joint detection is not considered in this detection methodology.

## DFE

The decision-feedback equalizer (DFE) consists of two tapped-delay line (TDL) filters [6]: a feed-forward filter and a feedback filter; it is usually employed to mitigate the effects of severe inter-symbol interference to improve performance in the presence of moderate to severe amplitude distortion. The choice of DFE receivers is often motivated by the fact that they are capable of exploiting the cyclostationary nature of the CDMA signal to effectively cancel multiple-access interference [91, 92]. The computational complexity of DFE detectors is a linear function of the number of taps of the feed-forward and feedback filters. As the interference needs to be subtracted from the signal aggregate in the process of receiver equalization and detection, a good estimate of the received signal amplitudes of users of interest is required.

## Neural networks

Another class of nonlinear multiuser detectors is neural networks. Unlike other common interference suppression techniques, neural networks are capable of performing nonlinear filtering [93]. Different schemes with neural networks are proposed for nonlinear multiuser detectors in literature, among which are multi-layer perception (MLP) [94], adaptive detection on the basis of radio basis functions (RBF) [95], and eigenvector neural networks [96]. They can detect signals in non-stationary and non-Gaussian environments, and can achieve near-optimum performance. However, the principal concerns about these detectors are their high implementation costs, propagation errors, and slow convergence rates in their adaptive detection.

## Other types of detectors

Other CDMA detectors include the nonlinear suboptimal single-user maximumlikelihood sequence (MLS) detectors over Rayleigh fading channels presented in [97, 98, 99], and multiuser detectors employing adaptive antenna array and beamforming techniques [24, 30, 35, 33]. In frequency-selective fading channels, the temporal diversity (or path diversity) feature can be utilized by taking advantage of the characteristic of wideband CDMA signals, and thus the well-known RAKE receivers such as [28, 29, 100] can be well applied to CDMA systems. In the scenarios where the multipath components can not be resolved but their inter-delays are known, a form of optimum and suboptimum detectors is proposed in [101, 102], which generalizes the RAKE receivers assuming the path resolvability.

## 2.3.2 Multiuser receivers

In this class of CDMA detectors, the parameter knowledge of multiple users — signature sequence, timings, and received amplitude — is assumed to be available to the receiver, and the well-formed MAI information of the known users is *explicitly* explored for joint multiuser detection. On the basis of their detection algorithm, the multiuser receivers can be further divided into linear and nonlinear receivers. For the linear multiuser detectors, linear transformations are employed to separate the signals of the known users; for the nonlinear sequence detectors, nonlinear multiuser detectors are utilized for joint detection of the known users, both exploiting the parameter knowledge and the signal correlation infrastructure of multiple known

users.

The early research study on multiuser detection started from late 1970s and focused on CDMA-based optimum nonlinear detection, including [103, 104, 105, 36]. It is Verdu's milestone work [36] that has really incited the wide research interest on multiuser detection; his study demonstrates that the CDMA systems are not necessarily interference-limited. Because the maximum-likelihood sequence (MLS) multiuser detection proposed in [36] plays a crucial role in the research and development of multiuser receiver designs, we start our review of this receiver category from it.

#### Nonlinear Receivers

#### **Optimum detector**

We consider the reverse-link receiver design at the base station in a single-cell  $K_n$ -user DS-CDMA system under an AWGN channel, in which the receiver has the parameter knowledge of all  $K_n$  active users, and the problem in such a setup can be characterized as  $K_n/K_n/K_n$  detection. To examine the problem of the optimum multiuser detection in such a centralized information case, we write, from (2.5), the received baseband signal at the base station as

$$r(t) = \sum_{l=1}^{L} \sum_{k=1}^{K_n} \sqrt{P_k} b_k(l) s_k(t - lT_b - \tau_k) + n(t)$$
(2.11)

$$= S_t^D(\mathbf{b}_{K_n}) + n(t), \qquad (2.12)$$

where L symbolizes the total number transmitted bits for each user,  $\tau_k$  is the transmission delay of user k, and the other notations are same as in (2.5).  $S_t^D(\mathbf{b}_{K_n}) = \sum_{l=1}^{L} \sum_{k=1}^{K_n} \sqrt{P_k} b_k(l) s_k(t - lT_b - \tau_k)$  and  $\mathbf{b}_{K_n}$  denotes the  $K_n L \times 1$  bit vector of all active users. The issue of interest is the classical problem of detecting the random signal  $S_t^D(\mathbf{b}_{K_n})$  in the additive white Gaussian noise [106], [107]. By formulation of the likelihood ratio between each of the hypotheses

$$H_{\mathbf{b}}: r(t) = S_t^D(\mathbf{b}_{K_n}) + n(t), \text{ over all bit symbols of the } K_n \text{ users} \qquad (2.13)$$

and a null hypothesis  $H_0: r(t) = n(t)$ , the optimum multiuser detection, in the sense of minimizing the bit error probability, is made in favor of the largest likelihood ratio [106] over all bit symbols, or equivalently, maximizes the likelihood function

$$\ell(\lbrace r(t); \infty < t < \infty \rbrace | \mathbf{b}_{K_n}) = \kappa \exp\{\Omega(\mathbf{b}_{K_n})/2\sigma_N^2\}$$
(2.14)

where  $\kappa$  is a constant,  $\sigma_N^2 = N_0/2$  is the noise variance, and

$$\Omega(\mathbf{b}_{K_n}) = 2 \int_{-\infty}^{\infty} S_t^D(\mathbf{b}_{K_n}) r(t) dt - \int_{-\infty}^{\infty} [S_t^D(\mathbf{b}_{K_n})]^2 dt.$$
(2.15)

The received-signal dependent term on the right-hand side of Equation (2.15) can be further written as

$$\int_{-\infty}^{\infty} S_{t}^{D}(\mathbf{b}_{K_{n}})r(t)dt = \sum_{l=1}^{L} \sum_{k=1}^{K_{n}} \sqrt{P_{k}}b_{k}(l) \int_{-\infty}^{\infty} s_{k}(t-lT_{b}-\tau_{k})r(t)dt$$
$$= \sum_{l=1}^{L} \sum_{k=1}^{K_{n}} \sqrt{P_{k}}b_{k}(l)y_{l,k}, \qquad (2.16)$$

where

$$y_{l,k} \stackrel{\Delta}{=} \int_{-\infty}^{\infty} s_k (t - lT_b - \tau_k) r(t) dt \qquad (2.17)$$

is a sample of the integral at the instant of  $lT_b + \tau_k$ , which is actually the output of a filter matched to the modulation signal at the *l*th bit interval of user *k*. The whole sequence from outputs of a bank of  $K_n$  matched filters,

$$\mathbf{y} = [y_{1,1} \ \dots \ y_{1,K_n} \ y_{2,1} \ \dots \ y_{2,K_n} \ \dots \ y_{L,1} \ \dots \ y_{L,K_n}]^T,$$
(2.18)

is able to provide a *sufficient* statistic for the decision of the information bit vector  $\mathbf{b}_{K_n}$  [51, pp. 155], [108]. Therefore, the optimum  $K_n$ -user multiuser receiver structure consists of a front-end matched filter bank, followed by a maximum likelihood decision algorithm, as shown in Fig. 2.4. To obtain the optimum detection formulation, let  $\mathbf{A}_P$  be the  $K_n L \times K_n L$  diagonal matrix defined as



Fig. 2.4 Optimum  $K_n$ -user multiuser detector.

$$\mathbf{A}_P = \operatorname{diag}\{\sqrt{P_1} \dots \sqrt{P_{K_n}} \dots \sqrt{P_1} \dots \sqrt{P_1} \dots \sqrt{P_{K_n}} \dots \sqrt{P_1} \dots \sqrt{P_{K_n}}\},\$$

and  $\{\mathbf{C}(i-l)\}\$  be the  $K_n \times K_n$  normalized signal correlation matrices whose entries are defined by

$$C_{jk}(i-l) = \int_{-\infty}^{\infty} s_j(t-iT_b-\tau_j)s_k(t-lT_b-\tau_k)dt.$$
 (2.19)

Then the maximization of the likelihood function (2.14) is equivalent to selecting a  $\hat{\mathbf{b}}_{K_n}$  that maximizes the decision metric  $\Omega(\mathbf{b}_{K_n})$  given in (2.15), or the metric in its discrete-time vector form as

$$\Omega(\mathbf{b}_{K_n}) = 2\mathbf{b}_{K_n}^T \mathbf{A}_P \mathbf{y} - \mathbf{b}_{K_n}^T \mathbf{A}_P \mathcal{C} \mathbf{A}_P \mathbf{b}_{K_n}, \qquad (2.20)$$

where

$$\mathcal{C} = \begin{bmatrix}
\mathbf{C}(0) & \mathbf{C}^{T}(1) & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\
\mathbf{C}(1) & \mathbf{C}(0) & \mathbf{C}^{T}(1) & \mathbf{0} & \dots & \mathbf{0} \\
\mathbf{0} & \mathbf{C}(1) & \mathbf{C}(0) & \mathbf{C}^{T}(1) & \dots & \mathbf{0} \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
\mathbf{0} & \dots & \mathbf{0} & \mathbf{C}(1) & \mathbf{C}(0) & \mathbf{C}^{T}(1) \\
\mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{C}(1) & \mathbf{C}(0)
\end{bmatrix}$$
(2.21)

is a symmetric  $K_n L \times K_n L$  matrix. When the Viterbi decision algorithm is em-

ployed, it was shown [51] that the time complexity per bit for the optimal multiuser detection is independent of the total number of transmitted bits for each user, L, and only depends on the number of known users,  $K_n$ , on the order of  $2^{K_n}$ . Notice that the matrix C, whose entry is the signal correlation infrastructure between users j and k described in (2.19), is a key component for the optimum multiuser sequence detection in the decision metric (2.20). An extension of the optimum MLS detection under an AWGN channel to the single-path, frequency-selective Rayleigh-fading channel was proposed in [45], assumed to be a perfect channel estimate; the one-shot maximum likelihood sequence detection with a linear array was proposed in [109] for synchronous DS-CDMA systems.

The optimum maximum likelihood sequence detection has assumed expensive information requirements: the timings, signature waveform, received signal strength of all active users in the system, and a high computational capability at the base station. Such full information requirements are available at the base-station receiver in a single-cell CDMA system; however, they are usually not available at a base-station receiver in a multi-cell CDMA network.

## Interference cancellation (IC) and decision-driven detectors

We'll review several nonlinear multiuser detectors with less complexity than the optimum multiuser detection in the following; more nonlinear multiuser detectors of this type can be found in a few review papers [114, 115, 38].

Nonlinear, estimation and subtractive interference cancellation detectors include the multi-stage nonlinear CDMA detectors and the successive-cancellation detectors. The multi-stage nonlinear CDMA detectors proposed in [111, 112, 113, 116, 117] assume the centralized system parameter setup; that is, the parameter knowledge — signature waveform, timings and signal strength — of all active users is available to the receiver, thus applicable to the base-station reception in a single-cell CDMA network. The group-based detectors [113, 116] intend to reduce the implementation complexity of the optimum  $K_n/K_n/K_n$  detection; joint detection for a subset of  $K_n$  users is proposed with the time complexity per bit on the order of  $2^M/M$ , and  $M < K_n$  symbolizes a group size. Such detectors can be characterized as  $K_n/K_n/M$ nonlinear detectors, which consist of a bank of  $K_n$  MF/decorrelating filters at the front-end signal processing and a nonlinear maximum likelihood sequence decision algorithm. In a cellular CDMA network with full user information scenarios, a decision-based multiuser detector that takes advantage of macro diversity (i.e., soft hand-off) can be found in [118]. Another nonlinear detector, with a reduced and differential parallel interference cancellation technique, has been proposed recently in [119].

The idea behind the successive interference cancellation detectors [104, 110, 114, 120, 121] is to demodulate the active users with respect to their desired signal strengths: the signal of user with the largest SNR is first detected, and then sub-tracted from the received signal aggregate; detection of the user with the second largest SNR follows, and so forth, until the weakest user is detected. Such an idea is capable of achieving the capacity region on the basis of information theory [122]. As long as the bit estimates and signal parameter estimates are reliable, the IC based receivers can deliver near-optimum performance. The complexity per bit of this operation grows linearly with the number of users whose signals of interest are being cancelled.

Other nonlinear multiuser detectors include an adaptive, nonlinear multiuser receiver [123] proposed for the base-station reception in a multi-cell CDMA network. In [123], initial training sequences of the in-cell users are required for the detection, whereas the other parameters of the known users, such as signature waveform and timings, are not explicitly used; since this receiver employs the single-user-oriented equalization, the multiuser signal correlation infrastructure desirable for MUD is not retained in the front-end estimation of individual single-user signal and thus must be regenerated for the nonlinear detection, therefore introducing double estimation errors; the ML decision metric employed in the detector is basically the one described in (2.20) that was developed for the optimal multiuser detection with a full user information setup.

## **Linear Receivers**

Linear multiuser detectors in this category can be characterized as  $K_n/K_n/K_n$  linear detectors. The receiver architecture is same as the one described in Fig. 2.4, except that a linear transformation and a binary decision algorithm replace the nonlinear Viterbi algorithm. Such linear multiuser detectors are suboptimal, but they are still

far superior to conventional MF receivers with respect to their error-probability performance in interference-limited environments; moreover, their computational complexity increases linearly with the number of users. Two important types of the linear detectors are the decorrelator (or decorrelating detector) and the MMSE detector.

## **Decorrelators**

Assuming that the signature waveforms and timings of all active users are available at the receiver, the decorrelators (or decorrelating detectors) choose the linear transformation to have zero output multiple-access interference in the absence of background noise (zero-forcing) [50, 124, 125, 126]. To gain insight into the decorrlating CDMA receivers, we denote the output y from the MF filter bank in (2.18) as

$$\mathbf{y} = \mathcal{C}\mathbf{A}_P\mathbf{b}_{K_n} + \mathbf{N} \tag{2.22}$$

where **N** is the output noise vector of  $K_n$  matched filters that has a zero mean and an autocorrelation matrix  $\frac{N_0}{2}C$ ;  $\mathbf{A}_P$  is the amplitude  $K_n L \times K_n L$  diagonal matrix; Cand other parameters were defined in (2.21). By applying the linear transformation  $C^{-1}$  to both sides of Eq. (2.22), we obtain the estimated bits for individual users as

$$\mathbf{A}_P \hat{\mathbf{b}}_{K_n} = \mathbf{A}_P \mathbf{b}_{K_n} + \mathcal{C}^{-1} \mathbf{N}.$$
(2.23)

It is apparent from (2.23) that such a linear estimation of  $\mathbf{b}_{K_n}$  is unbiased and the linear detectors do not require the estimation of the received signal power for each user. Note that the linear decorrelating detectors are a sensible choice when the received amplitudes are completely unknown, and they can be shown to be near-far resistant [127]. However, these detectors may result in significant noise enhancement and suffer a performance degradation, when the background noise power is not negligible.

A one-shot decorrelating detector is proposed in [128], and other truncated, window-based decorrelators are put forward in [129, 130, 131, 132]. Decorrelating receivers with space-time coding are proposed in [27, 133] for the base-station multiuser signal detection in synchronous DS-CDMA systems. Adaptive versions of decorrelators are investigated in [134, 125, 135, 124]; they are applicable in multipath fading environments.

#### **MMSE** detectors

MMSE detectors are an alternative to the decorrelating receivers when the receiver has additional knowledge of received signal powers of the active users. Such detectors are capable of suppressing both the multiple-access interference and the background noise on the basis of MMSE equalization criterion [49, 80]. Instead of  $C^{-1}$ , the linear transformation

$$\mathcal{K}_{opt} = (\mathcal{C} + \frac{N_0}{2} \mathbf{A}_P^{-2})^{-1}$$
(2.24)

is applied to both sides of Eq. (2.22) such that the mean-squared error (MSE)

$$E[(\mathbf{A}_P\mathbf{b} - \mathcal{K}\mathbf{y})^T(\mathbf{A}_P\mathbf{b} - \mathcal{K}\mathbf{y})]$$

can be minimized. It is noted from (2.24) that such a linear estimation is biased. The linear MMSE detectors are well defined in the sense that the transformation in (2.24) always exists because the matrix therein is the sum of a nonnegative definite matrix C and a diagonal positive definite matrix  $\frac{N_0}{2} \mathbf{A}_P^{-2}$ ; the linear MMSE transformation exists even though the decorrelating solution may not exist. As a consequence, the linear MMSE detectors usually perform better than the linear decorrelators; furthermore, as the level of background noise tends to zero, performance of the linear MMSE detectors converges to that of the linear decorrelating detectors.

A combination of MMSE detector and soft hand-off diversity is proposed in [118] for a multi-cell CDMA network. A linear MMSE detector for demodulating the transmitted bits of all users in a single bit interval, based on a finite length of output (bit-spaced) samples, is proposed in [49], and can be readily extended to an adaptive version due to limited input samples and *one-shot* estimations. In addition, an MMSE adaptive detector that processes the output samples of the decorrelating detector is proposed in [136]; an MMSE detector with maximum-ratio combining (MRC) is proposed for synchronous multi-carrier (MC) CDMA systems in a frequency non-selective Rayleigh fading channel [137].

Both decorrelating and MMSE linear detectors are applicable to base-station reception for a single-cell CDMA system, where the parameter knowledge of all active users is available to the receiver and there is no out-of-cell interference. Notice that the front-end filtering in these linear multiuser detectors extracts the signal correlation infrastructure of the known users, which can be used for "real", joint detection in second stage in that the detection exploits an intact multiuser signal infrastructure imbedded in the original received signal rather than a regenerated one. Therefore, the detection philosophy in these receivers is different from that of the enhanced single-user CDMA receivers, in which the front-end filtering focuses on the single-user-oriented signal estimation and does not retain the well-formed multiuser signal correlation infrastructure (that is actually valuable for MUD).

## 2.4 Performance Criteria

The performance criteria frequently used in the evaluation of CDMA receivers include probability of bit error (or bit error rate), asymptotic efficiency, minimum mean squared error and signal to interference-plus-noise ratio (SIR).

## 2.4.1 Probability of bit error

An accurate performance evaluation for maximum-likelihood sequence multiuser detectors is often with respect to probability of bit error. However, the closed-form expression of the probability of bit error for the detection is usually intractable. Alternatively, the performance appraisal often resorts to the upper and lower bounds of the probability of bit error under specific system settings.

In the performance analysis of MLS multiuser detectors, the indecomposable error vectors and the indecomposable set of error vectors (defined in Appendix C) play an important role since they can more accurately describe the transmitted vectors. For the MLS detection of  $K_n$  users in a CDMA synchronous system, there are a total of  $3^{K_n}$  error sequences; however, the number of the indecomposable error vectors can be very small. For example, we consider a specific system setup, in which all PN code cross-correlations are identical and positive, and the received amplitudes of all users are equal. In such a case, there are no indecomposable error vectors of weight (see Appendix for definition of error vector weight) equal to or greater than

3; specifically, the  $K_n$ -dimensional indecomposable error vectors that affect user 1 can be expressed as [51]

 $[+1, 0, 0, \dots, 0, 0],$  $[+1, -1, 0, \dots, 0, 0],$  $[+1, 0, -1, \dots, 0, 0],$  $\dots,$  $[+1, 0, 0, \dots, -1, 0],$  $[+1, 0, 0, \dots, 0, -1],$ 

plus their antipodal sequences. It is seen that only  $2K_n$  (out of  $3^{K_n}$ ) error sequences are indecomposable in such a special case. This system setting and resulting indecomposable error vectors will be employed for the bound performance evaluation of the proposed multiuser detectors with partial information in Chapter 4.

The performance bounds for the joint detection in a  $K_n$ -user synchronous DS-CDMA system can be found in Appendix C, where the upper bound of the probability of bit error for user k is presented in (C.3) and the lower bound is expressed by (C.6).

## 2.4.2 Asymptotic efficiency

An alternative to the bit error rate (BER) in the evaluation of  $K_n/K_n$  MLS multiuser detection is the asymptotic efficiency, which is defined as [138]

$$\eta_k = \sup\left\{ 0 \le \gamma \le 1 \mid \lim_{\sigma \to \infty} p_k(\sigma) / Q\left(\frac{\sqrt{\gamma E_k}}{\sigma}\right) < +\infty \right\},$$
(2.25)

where  $p_k$  is the BER for user k,  $\sigma$  is a parameter relevant to the signal-to-noise ratio, and  $E_k$  is the actual energy per bit of user k.

## 2.4.3 Near-far resistance

The near-far resistance is the worst-case asymptotic efficiency over the power distributions among all users. The near-far resistance of user k for the  $K_n/K_n$  MLS multiuser detection can be simply written as [51]

$$\bar{\eta}_k = \frac{1}{(\mathcal{C}_1^{-1})_{kk}} \tag{2.26}$$

where  $C_1$  is the correlation matrix of the signature waveforms of  $K_n$  known users.

#### 2.4.4 Minimum mean squared error and SIR

Minimum mean squared error (MMSE) and SIR are other alternatives to evaluate the performance of CDMA receivers, especially the linear detectors in which an interference cancellation scheme is employed. A performance analysis with respect to the minimum mean squared error is presented in Appendix D for enhanced K/1/1linear detectors and their adaptive implementations in a cellular K-user DS-CDMA network. SIR can also be used to evaluate the effectiveness of the interference suppression and the signal estimation for a user of interest. SIR is closely related to MMSE; it can be shown that the minimum mean squared error at the output of linear filtering always leads to the maximum SIR [139, 11].

## 2.5 Problem Formulation and Research Statements

#### 2.5.1 Problem Formulation

We consider the problem of effective base-station receiver design in a multi-cell DS-CDMA network. The received signal components at a base station consist of contributions from in-cell known users, out-of-cell unknown users, and background noise. The received signal can be expressed, from (2.5), as

$$r(t) = S_t^D(\mathbf{b}_{K_n}) + S_t^I + n(t)$$
(2.27)

where  $S_t^D(\mathbf{b}_{K_n})$  is the received signal aggregate of  $K_n$  known in-cell users;  $S_t^I$  is the received signal aggregate of  $K - K_n$  unknown users, which can not be ignored for the signal detection of in-cell users; and n(t) is additive white Gaussian noise. The fundamental problem of the base-station reception is detecting signals of  $K_n$  known users  $(S_t^D(\mathbf{b}_{K_n}))$  in the presence of out-of-cell interfering sources  $(S_t^I)$  and background noise. Notice that  $S_t^I$  is usually a zero mean cyclostationary process with a period of the bit interval for short spread signals (cf. Section 2.2); more precise statement of the signal cyclostationarity is to be addressed in Section 2.6. Our research makes no assumptions about  $S_t^I$  except that its second-order statistic can be estimated at the receiver. Based on this framework whose assumptions are very reasonable in a practical wireless network (e.g., 3G systems), we will demonstrate in this thesis that the effective joint detection of in-cell users at the base station does exist in the multi-cell CDMA network.

In such a partial system parameter setup, the base-station receiver has different knowledge about the in-cell signals and the out-of-cell sources. Signature waveform, timings, and received signal strength of each in-cell user are assumed known or knowable at the receiver, whereas only the second-order statistic of  $S_t^I$  is assumed. As a result, the receiver design for effective base-station reception should treat the two signal aggregates —  $S_t^D(\mathbf{b}_{K_n})$  and  $S_t^I$  — differently such that the known parameter knowledge of the in-cell known users and the statistical knowledge of the out-ofcell unknown users can be fully incorporated into signal detection and interference suppression. In this thesis, we propose a technique to achieve these objectives. The proposed receiver architectures include two processing stages: the front end deals with the suppression of  $S_t^I$ , while retaining the multiuser signal correlation structure of  $S_t^D(\mathbf{b}_{K_n})$ ; the second stage deals with the joint detection of in-cell known users based on the outputs of the front end.

The challenging task in the front-end signal processing is extracting the multiuser signals of the known users in the interference suppression procedure. To see the desired multiuser signal aggregate that is desired for MUD, let us consider a synchronous CDMA system under an AWGN channel for the convenience of problem elaboration. In such a case,  $S_t^D(\mathbf{b}_{K_n})$  in (2.27) can now be written as  $S_t^D(\mathbf{b}_{K_n}) = \sum_{l=1}^L \sum_{k=1}^{K_n} \sqrt{P_k} b_k(l) s_k(t-lT_b)$ . In the absence of  $S_t^I$ , it is known that a bank of front-end  $K_n$  MF can provide a sufficient statistic (cf. (2.18)) for the optimum MUD [36]. The output from the matched filter of user k in the lth bit interval can be expressed as

$$y_{l,k} = \int_{-\infty}^{\infty} s_k(t - lT_b)r(t)dt$$
  
=  $d_k(l) + \int_{-\infty}^{\infty} s_k(t - lT_b)n(t)dt$ ,

where

$$d_{k}(l) = \int_{lT_{b}}^{(l+1)T_{b}} s_{k}(t - lT_{b})S_{t}^{D}(\mathbf{b}_{K_{n}})dt$$
$$= \sum_{j=1}^{K_{n}} \sqrt{P_{j}}b_{j}(l)\rho_{kj}$$
(2.28)

in which  $\rho_{kj}$  is the correlation of signature waveforms between users k and j over a one-bit interval.  $\{d_k(l), k = 1, 2, \ldots, K_n, l = 1, 2, \ldots, L\}$  are sufficient for the optimum multiuser detection when the out-of-cell interference is absent. It is observed from (2.28) that  $d_k(l)$  can be seen as the projection of  $S_t^D(\mathbf{b}_{K_n})$  onto the signature space of user k over one (lth) bit interval; moreover,  $d_k(l)$  has a well-formed infrastructure constructed by signals of the in-cell known users. The set of the quantities  $\{d_k(l)\}$  provides the basis for joint multiuser detection in a CDMA-based system.

In the presence of  $S_t^I$  (i.e., there exist  $K - K_n$  out-of-cell unknown interfering signal sources), we still intend to extract the quantities  $\{d_k(l)\}$  (desirable for the multiuser detection) from a larger received aggregate; each equalization target  $d_k(l)$ is only composed of signals from the in-cell known users. In such a scenario, the estimation of  $d_k(l)$  and the suppression of the out-of-cell interference are required, different from the centralized information (e.g., single-cell) scenario in which a bank of front-end matched filters is sufficient to provide such desirable multiuser signal information. To achieve these goals, we employ a form of linear signal equalizers in place of the matched filters in the front end of our proposed multiuser detection with partial information. Specifically, the front-end signal equalization uses a factionally-spaced chip-matched linear filter architecture and targets the multiuser signal aggregate  $d_k(l)$  (rather than the transmitted bit  $b_k(l)$  of user k) on the basis of MMSE criterion, thus leading to multiuser-oriented signal equalization technique. Notice that the chip-matched filtering can well exploit the CDMA signal cyclostationarity for the linear equalization. Moreover, since the transmitted bits in question are unknown,  $d_k(l)$  is considered a random variable in the front end that will be estimated on a bit-by-bit basis. The optimal linear estimation problem here with respect to MMSE criterion is equivalently to solving the Wiener filter equation [63] to find the optimal tap coefficients for the linear multiuser-oriented signal equalizer. As a consequence,  $\{d_k(l)\}$  can be estimated by a bank of front-end  $K_n$  multiuser-oriented signal equalizers. Moreover, it can be shown (in Section 3.2) that their equalizer outputs can provide a sufficient statistic with minimized residual interference and noise components for joint detection of  $K_n$  in-cell known users in the second stage. Either a linear or nonlinear decision algorithm can be applied for the multiuser detection. Joint detection of  $K_n$  in-cell users in such a scenario can be characterized as  $K/K_n/K_n$  detection. In our study, we also provide a form of  $K/K_n/M$  ( $M \leq K_n$ ) MLS detector that can trade performance for complexity. The proposed linear and nonlinear multiuser receivers and detailed derivations will be presented in Chapter 3.

For the proposed multiuser detectors, we evaluate their performance by a combination of analysis and simulation and in terms of several well-known metrics, such as minimum mean-squared error (MMSE), effective SNR, BER, asymptotic efficiency, and near-far resistance. Moreover, we justify them with a few benchmark receivers. In addition, we also study many performance-impact parameters such as the detection group size, the number of out-of-cell unknown users, the number of resolved multi-paths, the near-far problem, the spreading code types, and the estimation errors. Details of the performance analysis and evaluation will be presented in Chapter 4.

#### 2.5.2 Research Statements

In this thesis, we develop and evaluate forms of enhanced nonlinear and linear multiuser detectors for base-station reception in a multi-cell DS-CDMA wireless network under AWGN and slowly Rayleigh-faded multipath channels. The proposed receiver architectures have two processing stages. The first stage amounts to a bank of linear, multiuser-oriented signal equalizers on the basis of MMSE criterion, one for each of the target sources for joint detection; its role is to estimate the multiuser signal aggregate by suppressing the unwanted interference. The second stage, acting *jointly* on all outputs from the front-end equalizers, is a maximum-likelihood sequence (MLS) or linear multiuser detector; its structure is calculated from the second-order statistics of the equalizer error processes, assumed approximately Gaussian on the basis of a Central-Limit argument (also, cf. [140, 141]).

**Table 2.1** Comparison of relevant multiuser receivers in terms of  $K/K_n/M$  detection structure, where  $K \leq K_n \leq M$ . Knowledge of one user includes signature waveform, timings, and/or received amplitude; for a fading channel, it may also include training or channel estimation.

Multiuser	Front-end	Separation		
detection	linear	rule for	Info	Reference
type	filtering type	known users	Exploited	
(Adaptive)	single-user	binary	one-user	[46] [48]
Linear $K/1/1$	based equal	decision	knowledge	[79]
(Adaptive)	single-user	involved in equal/	$K_n$ -user	
Linear $K/K_n/1$	based equal	binary decision	knowledge	[41] $[39]$
Linear		linear trans/	$K_n$ -user	
$K_n/K_n/K_n$	MF	binary decision	knowledge	[49] [50]
Linear	multiuser	linear trans/	$K_n$ -user	$this \ work$
$K/K_n/K_n$	based equal	binary decision	${ m knowledge}$	
Nonlinear		MLS sequence	$K_n$ -user	
$K_n/K_n/K_n$	MF	decision	knowledge	[36] [45]
Nonlinear	MF/	MLS sequence	$K_n$ -user	
$K_n/K_n/M$	decorrelating	decision	knowledge	[113] [116]
Adaptive MLS	single-user	MLS seq deci/	initial training	
$K/K_n/K_n$	based equal	metric similar to [36]	of $K_n$ users	[123]
Non-linear	$\mathbf{multiuser}$	Derived MLS	$K_n$ -user	this work
$K/K_n/M$	based equal	seq det metric	knowledge	$\checkmark$

The multiuser detection developed in this thesis can fully exploit the known parameter knowledge of the in-cell users and the statistical knowledge of the out-of-cell sources to enhance performance. The front-end multiuser-oriented signal equalization scheme plays a crucial role in the proposed joint detection with partial information; it distinguishes the proposed multiuser detectors from alternatives found in the literature. For linear detectors [46, 48, 79, 41, 39], their front-end equalization target is exclusively the single-user transmitted bit  $b_k(l)$ , and thus the well-formed signal correlation infrastructure (of paramount importance to multiuser detection) of multiple known users can not be retained. For the other types of linear CDMA detectors [49, 50], they assume the full parameter information of all active users, thus applicable to the base-station reception in a single-cell CDMA system (or the cases where the out-of-cell unknown sources are assumed weak or white). The multiuser detection proposed in this thesis is an extension of the multiuser detection [49, 50] in a single-cell setup to a multi-cell CDMA environment.

For nonlinear MLS multiuser detectors [36, 45, 113, 116, 123],  $S_t^I$  is assumed absent or white in [36, 45, 113, 116]. [123] only uses training sequences to combat  $S_t^I$ with the single-user-oriented signal equalization, but it has no intention to explicitly employ the other available parameter knowledge of the in-cell known users to improve performance. The proposed MLS multiuser detectors are a generalization of MLS detection [36, 113, 116] to a multi-cell CDMA network under AWGN channels, and a generalization of [45] to a multi-cell setup under fading multipath channels. A classification of the multiuser detectors closely relevant to our research interest and respective receiver feature description are presented in Table 2.1.

## 2.6 Limitations and Approximations

We make certain assumptions in order to simplify the derivation and performance analysis of our multi-user detector. This section reviews the most important of those assumptions and assesses their impact on the applicability and generality of the results.

The main assumptions are these:

- The modulation format is BPSK.
- The chip waveform is time-limited. In the detailed calculations, it is further assumed that in fact the chip waveform is time-limited to  $T_c$  to the duration of a bit divided by the processing gain.
- The outputs from the front-end equalizers are approximately Gaussian.

The following explains:

• Modulation format: Our model here is consistent with the IS-2000 standard for 3G wireless [3], which calls for BPSK on the reverse link for mobiles with a radio configuration of 3 or greater. IS-2000 does provide for quadrature

modulation and complex spreading on the forward link. The receiver structure we derive here (in Chapter 3) can in fact be amended to accommodate QPSK (or complex PN spreading) with different PN codes for the in-phase and quadrature channels. The signature vector thus becomes complex-valued, as do the tap coefficients that parameterize the equalizers. Matrix transpose in the complex setting becomes Hermitian transpose.

- Chip waveform: Practical channels are frequency-limited, which implies that the chip waveform has infinite time duration. Our assumption in this connection is thus not, strictly speaking, consistent with practice. We claim that the inconsistency does not materially impact our conclusions, mainly because practical chip shapes (the raised-cosine, for example) fall off sufficiently quickly outside of  $[0, T_c)$  that the energy leakage from one chip interval to another can be neglected in the (bit-based) performance calculations. Longer chip shapes can in any case be accommodated both by the receiver architecture (without change either in the front-end equalization or in the multi-user detector) and by the performance analysis (but at the cost of additional computational complexity; see Sections 3.1, 3.7). The particularly simple shape selected for convenience in our derivations is seen elsewhere; see, for example, [36, 46, 39, 41].
- Gaussian approximation: Our assumption here is at least informally suggested by the Central Limit Theorem. The role of the front-end equalizers is to suppress the interference contributed by those sources (which for convenience we describe as "out of cell") whose parameters are unknown at the base station. In the scenario of interest to us, and which we hold to be of practical importance, the out-of-cell interferers are many and weak, except possibly for a small number of strong ones. In such a case, the residuals produced by MMSE equalization are typically roughly equal and small; for example, in a five-user system, where all but one of the sources are known (in-cell), the power in the interference signal following equalization comes in at some four percent of the power in the target signal (see Table 4.4 in Chapter 4). The detailed shape of the distribution of the residual signal is of little import when the associated signal power is so small relative to that of the signals of interest. Reference [140], which considers more carefully the applicability of the Gaussian approximation

in settings such as ours, provides additional confirmation that the assumption is not unreasonable.

There is a final remark that we include here regarding the definition of our model. As spelled out in Sections 2.1.2 and 2.5.1, the unknown interference (the out-of-cell contribution to the received signal) is supposed characterized by its second-order statistics (essentially the covariance function). These are shown to be *periodic*, in the sense of remaining unchanged when the origin of time is shifted by one period. The period in question is  $T_b$ , the duration of a single bit. Such periodicity is described by saying that the unknown interference is (wide-sense) cyclostationary. Cyclostationarity is proved in Section 2.2 subject to three conditions: that all signature sequences are one bit wide (short spreading), that consecutive bits within each source stream are *iid*, and that the user population is plesiochronous — that is, that all bit intervals in all streams have the same length. The point we wish to make here concerns the difference between cyclostationarity and stationarity (both understood as wide-sense, though the basic facts have strict-sense formulations as well). As noted, all transmitted signals are individually cyclostationary. The time offset due to propagation delay from transmitter to receiver does not change that. So individual signals are cyclostationary at the receiver. Because a sum of uncorrelated, cyclostationary processes (all with the same period) is itself cyclostationary, it ensues that the aggregate signal at the receiver is cyclostationary. It can happen, though, that the period of the aggregate is smaller than the common periods of the components. The effect depends on those time offsets due to propagation delay, offsets that in general differ from one component signal to another. If there are M components whose arrival times at the receiver are uniformly spaced across a bit period, then the period of the aggregate is  $T_b/M$ , suggesting that in the limit of large M the aggregate is not merely cyclostationay, but in fact stationary. Similarly, if the time offsets of the M components are drawn randomly, in *iid* fashion from the uniform probability distribution on  $[0, T_b)$ , then in the limit of large M the aggregate is again stationary — both conditionally (with probability one), given the actual values of the time offsets, and unconditionally, where the time offsets are averaged out in the calculation of the covariance. Appendix F supplies the details.

We do not make any specific assumptions about the distribution of time offsets

at the receiver. We do assume that the covariance function for the aggregate "out-ofcell interference, whether cyclostationary or in fact stationary, can be estimated at the receiver. It will be seen (from Section 3.7, Chapter 4 and the comparisons offered there between short and long spreading) that receiver performance can benefit from the cyclostationary signal structure produced by short spreading.

# Chapter 3

# **Receiver Architecture**

In this chapter, we propose and derive forms of nonlinear and linear CDMA multiuser detectors in partial system parameter scenarios under Gaussian channels and multipath slowly Rayleigh-faded channels.

## **3.1** $K/K_n/K_n$ Multiuser Equalization in Gaussian Channels

In this section, we consider the multiuser-oriented signal equalization for base-station  $K/K_n/K_n$  detection in a multi-cell K-user CDMA network under Gaussian channel, in which the receiver knows the signature waveform, timings, and received signal amplitude of  $K_n$  (< K) in-cell active users, but has no parameter knowledge of the  $K - K_n$  out-of-cell interfering sources. Without a loss of generality, we assume that users of interest for joint detection are indexed from 1 to  $K_n$ , and other unknown users from  $K_n + 1$  to K. For convenient analysis of the proposed equalization and detection ideas, we assume that the  $K_n$  users are bit synchronized; thus, we have the propagation delay  $\tau_k = 0, k = 1, 2, \ldots, K_n$ . The signal equalization and detection problems for an asynchronous setup will be addressed in Section 3.5, where the effect of multipath fading is considered.

From (2.5), the received aggregate signal can be written as

$$r(t) = \sum_{l=1}^{L} \sum_{k=1}^{K_n} \sqrt{P_k} b_k(l) s_k(t - lT_b) + \sum_{l=1}^{L} \sum_{k=K_n+1}^{K} \sqrt{P_k} b_k(l) s_k(t - lT_b - \tau_k) + n(t)$$
  
=  $S_t^D(\mathbf{b}_{K_n}) + S_t^I + n(t),$  (3.1)

where l denotes lth bit interval,  $P_k$ ,  $b_k(l)$ , and  $s_k(t)$  are, respectively, the received signal power, transmitted bit, and signature waveform of user k. { $\tau_k$ ,  $k = K_n +$ 1,  $K_n + 2, \ldots, K$ } are the transmission delays of out-of-cell unknown interfering sources, which are assumed to be unknown but deterministic parameters.  $S_t^D(\mathbf{b}_{K_n}) =$  $\sum_{l=1}^{L} \sum_{k=1}^{K_n} \sqrt{P_k} b_k(l) s_k(t - lT_b)$  is the received signal aggregate due to the contribution of  $K_n$  known users of interest;  $S_t^I = \sum_{l=1}^{L} \sum_{k=K_n+1}^{K} \sqrt{P_k} b_k(l) s_k(t - lT_b - \tau_k)$  is the received signal aggregate due to the contribution of  $K - K_n$  unknown users; n(t) is additive white Gaussian noise (AWGN) with two-sided power spectral density  $N_0/2$ .  $\mathbf{b}_{K_n}$  denotes the  $K_n L \times 1$  bit vector of the in-cell users. The role of multiuser equalization is to effectively suppress  $S_t^I$  while retaining the multiuser signal correlation infrastructure of  $S_t^D(\mathbf{b}_{K_n})$  (to be employed for joint MUD in the second stage).

## 3.1.1 Multiuser information for $K_n$ -user joint detection

As discussed in Section 2.5.1, if there were no out-of-cell  $K-K_n$  interfering resources, the signal aggregates of  $K_n$  users —  $\{d_j(l)\}$  described in (2.28) — would provide a sufficient statistic for the in-cell  $K_n$ -user joint detection, which are produced by passing the received signal through a bank of  $K_n$  MF filters. In the presence of the out-of-cell interfering signals, the quantities  $\{d_i(l)\}$  can provide sufficient information of mutual signal infrastructure among the signals of the  $K_n$  in-cell users that is desirable for joint detection of the users. However, a bank of  $K_n$  MF filters can no longer generate these desired quantities; instead, a bank of front-end  $K_n$  signal equalizers is required to suppress the interference from  $K - K_n$  unknown sources and estimate  $\{d_j(l)\}$ . Each of the front-end signal equalizers employs a linear, fractionally-spaced, chipmatched filter in our study such that the cyclostationary characteristics of CDMA signals can be well exploited; each signal equalization targets respective multiuser signal aggregate  $d_j(l)$ , as shown in Fig. 3.1. As a result, each front end forms a linear multiuser-oriented signal equalizer, different from the single-user-oriented signal equalizer proposed in the literature in which the desired multiuser signal aggregate is not retained after the front-end equalization. An estimate of  $\{d_j(l)\}$  by the frontend multiuser signal equalization can provide a decision statistic that enables "real", elaborate multiuser detection in the second stage.

Each desired multiuser signal aggregate  $d_j(l)$  at the *l*th bit interval can be con-

sidered a projection of the received signal aggregate of the  $K_n$  users  $S_t^D(\mathbf{b}_{K_n})$  into the known signal space of user j, which is expressed as

$$d_{j}(l) = \langle S_{t}^{D}(\mathbf{b}_{K_{n}}), s_{j}(t - lT_{b}) \rangle$$
  
= 
$$\sum_{k=1}^{K_{n}} \sqrt{P_{k}} b_{k}(l) \rho_{jk}, \quad j = 1, 2, \dots, K_{n}, \quad (3.2)$$

where  $\langle x(t), y(t) \rangle = \int_{-\infty}^{\infty} x(t)y^*(t)dt$  denotes the inner product of signals x(t) and y(t), and  $\rho_{jk} = \langle s_k(t), s_j(t) \rangle$  is the correlation between signature waveforms of user j and user k at the *l*th bit interval. It is seen that  $d_j(l)$  has a zero mean and a variance

$$E[(d_j(l))^2] = \sum_{k=1}^{K_n} P_k \rho_{jk}^2, \qquad (3.3)$$

where E[\*] denotes an expectation operation over the transmitted bits. Thus, the variance of  $d_j(l)$  is known to the receiver; in fact, the parameter knowledge of the  $K_n$  in-cell users has been explicitly incorporated into this second-order statistic, which can be employed for effective signal equalization and detection (to be described in next subsection and the following section). In our study,  $d_j(l)$  is first treated as a random variable to be estimated by a linear multiuser-oriented signal equalizer, thus leading to a linear optimal front-end filter design; then, an estimate of  $d_j(l)$  is used for second-stage joint MUD. Consequently, the proposed multiuser detection with partial information setups can be considered an estimation-detection problem (also, see [83]).

## 3.1.2 Linear optimal estimation of desired multiuser signals

Our goal is to design a bank of  $K_n$  linear optimal multiuser-oriented signal equalizers, one for each of the known users and using MMSE estimation criterion, such that the mean squared error between the desired multiuser signal  $d_j(l)$  and its estimate  $\hat{d}_j(l)$ at the output of the front-end linear equalizer for user j can be minimized, where  $j = 1, 2, \ldots, K_n$ .



Fig. 3.1 A linear FIR filter for multiuser-user equalization in a decentralized user information scenario to estimate  $d_i(l)$  defined in (3.2).

#### A. Discrete-Time Signal Representation

The *n*th sample in the *l*th bit interval at the output of the chip-matched filter for user j (as shown in Fig. 3.1) can be written as

$$r_j(l,n) = \int_{-\infty}^{\infty} r(t)\psi^*(t - lT_b - nT_c)dt,$$
 (3.4)

where  $0 \leq n \leq N-1$ , N is the processing gain, and l symbolizes the lth transmitted bit of user j. Notice that the chip waveform  $\psi(t)$  in (3.4) can be any deterministic function. For the convinence of symbol notation, we consider a chip waveform that is confined to the interval  $[0, T_c)^1$ ; this particularly simple shape selected for convenient analysis is seen elsewhere, for example, [36, 46, 39, 41]. As a result, we have,  $\int_0^{T_c} \psi^2(t) dt = 1/N$  for the normalized signature waveform  $s_j(t)$ , and the chip-rate sample  $r_j(l, n)$  can be expressed as

$$r_j(l,n) = \sum_{k=1}^{K_n} \frac{\sqrt{P_k}}{N} b_k(l) a_k(n) + \sum_{i=1}^L \sum_{k=K_n+1}^K \sqrt{P_k} b_k(i) u_{jk}^{(n)}(l,i) + n_j(l,n), \quad (3.5)$$

<sup>&</sup>lt;sup>1</sup>Other forms of chip waveforms include a time function defined over a longer duration than  $T_c$  or a bandlimited signal.

where  $a_k(n)$  is the *n*th chip of user k,

$$u_{jk}^{(n)}(l,i) = \sum_{m=0}^{N-1} a_k(m) \int_0^{T_c} \psi(t+lT_b - iT_b + nT_c - mT_c - \tau_k) \psi^*(t) dt,$$

and  $n_j(l,n) = \int_{lT_b+nT_c}^{lT_b+(n+1)T_c} n(t) \ \psi^*(t-lT_b-nT_c) \ dt$ . Let us define an N-dimensional vector at the *l*th bit interval of user *j* as  $\mathbf{r}_j(l) = [r_j(l,0) \ r_j(l,1) \ \dots \ r_j(l,N-1)]^T$ .  $\mathbf{n}_j(l) = [n_j(l,0) \ n_j(l,1) \ \dots \ n_j(l,N-1)]^T$ .  $\mathbf{u}_{jk}(l,i) = [u_{jk}^{(0)}(l,i) \ u_{jk}^{(1)}(l,i) \ \dots \ u_{jk}^{(N-1)}(l,i)]^T$ . From the above definition for  $u_{jk}^{(n)}(l,i)$ ,  $\mathbf{u}_{jk}(l,i)$  can be shown to have the following properties:

Property 1:  $\mathbf{u}_{jk}(l, i)$  depends on the time difference between bit intervals l and i, and the transmission delay  $\tau_k$  of user k relative to user j.

Property 2: Given  $\tau_k > 0$  and l (of user j),  $\mathbf{u}_{jk}(l, i)$  is a non-zero vector only when i = l or i = l - 1.

Using the above properties of  $\mathbf{u}_{jk}(l,i)$ , the sample vector  $\mathbf{r}_j(l)^2$  can be simplified as

$$\mathbf{r}_{j}(l) = \sum_{k=1}^{K_{n}} \frac{\sqrt{P_{k}}}{N} b_{k}(l) \mathbf{a}_{k} + \sum_{k=K_{n}+1}^{K} \sqrt{P_{k}} [b_{k}(l) \mathbf{u}_{jk}(l,l) + b_{k}(l-1) \mathbf{u}_{jk}(l,l-1)] + \mathbf{n}_{j}(l),$$
(3.6)

where  $\mathbf{a}_k = [a_k(0) \ a_k(1) \ \dots \ a_k(N-1)]^T$  and  $P_k$  are, respectively, signature vector and received signal power of user k.  $\mathbf{n}_j(l)$  is an N-dimensional Gaussian vector with zero mean and covariance matrix  $\sigma_N^2 \mathbf{I}_N$  in which  $\mathbf{I}_N$  denotes the  $N \times N$  identity matrix,  $\sigma_N^2 = N_0/2N$ . Notice that the chip-rate sample sequence of user j,  $\{r_j(l,n)\}$ , is cyclostationary with a period of one bit interval  $T_b$ .

## B. Impulse Responses of the Linear Optimum Filters

<sup>&</sup>lt;sup>2</sup>With a chip waveform defined over longer interval than  $T_c$ , it will include more terms characterizing the leakage energy in the non-central chip intervals (usually assuming relative small). However, the analysis is straightforward.

The output of the *j*th equalizer at the *l*th bit interval of user j has the form

$$\hat{d}_j(l) = \mathbf{w}_j^T \mathbf{r}_j(l), \tag{3.7}$$

where  $\mathbf{w}_j = [w_j(0) \ w_j(1) \ \dots \ w_j(N-1)]^T$  is an N-dimensional tap-coefficient vector representing the coefficients in the *j*th equalizer. The linear optimal equalizer is designed such that the mean squared error between  $d_j(l)$  and its estimate  $\hat{d}_j(l)$  can be minimized. Such an MMSE estimation criterion results in the optimum coefficient vector  $\mathbf{w}_{jo}$  in the equalizer, which is the solution to the Wiener-Hopf filter equation [63]

$$\mathbf{S}_{j}\mathbf{w}_{jo} = \mathbf{p}_{j},\tag{3.8}$$

where  $\mathbf{S}_j \stackrel{\Delta}{=} E[\mathbf{r}_j(l)\mathbf{r}_j(l)^T]$  is an  $N \times N$  correlation matrix of the sample vector in the equalizer of user j at any given bit interval, and  $\mathbf{p}_j \stackrel{\Delta}{=} E[\mathbf{r}_j(l)d_j(l)]$  is an  $N \times 1$ cross-correlation vector between the sample vector and the desired multiuser signal aggregate associated with the front-end jth equalizer. It can be shown, from (3.6), that the correlation matrix  $\mathbf{S}_j$  can be expressed as

$$\mathbf{S}_{j} = \frac{1}{N^{2}} \sum_{k=1}^{K_{n}} P_{k} \mathbf{a}_{k} \mathbf{a}_{k}^{T} + \sum_{k=K_{n}+1}^{K} \bar{P}_{k} [\mathbf{u}_{jk}(l,l) \mathbf{u}_{jk}^{T}(l,l) + \mathbf{u}_{jk}(l,l-1) \mathbf{u}_{jk}^{T}(l,l-1)] + \sigma_{N}^{2} \mathbf{I}_{N} = \mathbf{S}_{j}^{K_{n}} + \mathbf{S}_{j}^{U_{n}} + \sigma_{N}^{2} \mathbf{I}_{N},$$
(3.9)

where the transmission delays of unknown sources have been modeled as unknown but deterministic parameters.  $\bar{P}_k$  denotes the average received signal power of an unknown user.  $\mathbf{S}_j^{K_n} = \frac{1}{N^2} \sum_{k=1}^{K_n} P_k \mathbf{a}_k \mathbf{a}_k^T$  is the correlation matrix due to signals from  $K_n$  known users,  $\mathbf{S}_j^{U_n} = \sum_{k=K_n+1}^K \bar{P}_k[\mathbf{u}_{jk}(l,l)\mathbf{u}_{jk}^T(l,l) + \mathbf{u}_{jk}(l,l-1)\mathbf{u}_{jk}^T(l,l-1)]$  is the correlation matrix due to signals from  $K - K_n$  unknown interfering sources, and  $\sigma_N^2 \mathbf{I}_N$ is the correlation matrix of background AWGN noise. Notice that the correlation matrix  $\mathbf{S}_j$  is independent of the bit interval l, since  $\mathbf{u}_{jk}(l,l)$  and  $\mathbf{u}_{jk}(l,l-1)$  in  $\mathbf{S}_j$ only depend on (short) signature sequences and independent of the bit interval l.
The cross-correlation vector can be represented as

$$\mathbf{p}_j = \frac{1}{N} \sum_{k=1}^{K_n} P_k \rho_{jk} \mathbf{a}_k, \qquad (3.10)$$

where  $\rho_{jk}$  is the correlation of signature waveforms between user j and user k. Notice that the cross-correlation vector  $\mathbf{p}_j$  is known to the receiver and includes the multiuser information of  $K_n$  in-cell users; in fact, this second-order statistic has incorporated the known signature waveforms, timings and receive signal amplitudes of the  $K_n$  users in the formulation.

Since the spreading waveforms of different users are independent,  $S_j$  is usually non-singular <sup>3</sup>. From (3.8), the optimum coefficient vector  $\mathbf{w}_{jo}$  can be written by

$$\mathbf{w}_{jo} = \mathbf{S}_j^{-1} \mathbf{p}_j, \tag{3.11}$$

where  $X^{-1}$  denotes the inverse of matrix X. It is important to notice that from (3.10) and (3.11), the parameter knowledge of the  $K_n$  known users has been *explicitly* used in the proposed linear multiuser-oriented signal equalization for both the estimation of the desired multiuser signal aggregate and the suppression of the out-of-cell unknown interfering sources.

Let us define a new quantity, enhanced signature waveform, for user j with respect to the  $N \times 1$  optimum coefficient vector of the jth equalizer,  $\mathbf{w}_{jo} = [w_{jo}(0) \ w_{jo}(1) \ \dots \ w_{jo}(N-1)]^T$ , as

$$w_j(t) = \sum_{n=0}^{N-1} w_{jo}(n)\psi(t - nT_c), \quad j = 1, 2, \dots, K_n, \quad 0 \le t < T_b, \quad (3.12)$$

where  $\psi(t)$  is the chip waveform. The impulse response of the linear optimal equalizer (estimator) for user j can be represented by

$$h_j(t) = w_j(T_b - t), \quad j = 1, 2, \dots, K_n, \quad 0 \le t < T_b.$$
 (3.13)

Thus, a bank of front-end  $K_n$  linear multiuser-oriented signal equalizers with im-

<sup>&</sup>lt;sup>3</sup>If, in some extreme case in which  $S_j$  is singular, the CDMA system is not operating properly and the optimum coefficient vector  $\mathbf{w}_{jo}$  may not exist [81].

pulse responses  $\{h_j(t)\}$ , as defined in (3.13), can replace a bank of  $K_n$  MF filters in a multi-cell CDMA network for the suppression of interference from  $K - K_n$  unknown sources and the estimation of multiuser signal aggregates  $\{d_j(l)\}$  of the known users (desirable for joint MUD in the second-stage) simultaneously in the front-end signal processing. Notice that each of  $K_n$  linear MMSE equalizers also maximizes the ratio of the desired multiuser signal aggregate of  $K_n$  users over the residual interference of  $K - K_n$  unknown sources plus noise at the output of the front-end linear filter [139, 11].

Estimation of  $\mathbf{S}_j$ . In the design of the linear optimal estimators formulated in (3.11), the cross-correlation vector  $\mathbf{p}_j$  is known, but the signal correlation matrix  $\mathbf{S}_j$  defined in (3.9) is usually *unknown* due to the fact that the component matrix  $\mathbf{S}_j^{U_n}$  is unknown. Given that the received powers and relative delays of users are constant over a period of interest for multiuser detection (for example, within a power control group), the correlation matrix  $\mathbf{S}_j$  can be estimated by a moving (time) average based on X recent received signal vectors  $\{\mathbf{r}_j(i)\}$  as [39]

$$\hat{\mathbf{S}}_j(X) = \frac{1}{X} \sum_{i=1}^X \mathbf{r}_j(i) \mathbf{r}_j(i)^T.$$
(3.14)

Eq. (3.14) provides a simple way to estimate the correlation matrix, which is based directly on the input received signal; however, it should be noticed that the information about the known users is ignored in such a case. Alternatively, the correlation matrix  $\mathbf{S}_j$  can be obtained by first measuring the second-order statistic of the unknown interfering sources only or the component matrix  $\mathbf{S}_j^{U_n}$ , through training sequences of the known users that can be found, for example, in the users' traffic frames of 3G wireless UMTS-TDD mode network [4]. The cross-correlation vector described in (3.10) can always fully exploit the parameter knowledge of the known users in the proposed multiuser-oriented equalization.

# 3.2 Maximum Likelihood Sequence (MLS) Multiuser Detection

The output sequences of the bank of  $K_n$  linear optimum filters designed in last section provide a decision statistic for joint detection of the  $K_n$  users. We'll show that such a decision statistic is sufficient. For the *j*th linear equalizer with its impulse response  $h_j(t)$ , the estimate of  $d_j(l)$  at the output of the front end, observed at the time instant  $(l + 1)T_b$ , can be expressed as

$$\hat{d}_{j}(l) = \langle r(t), h_{j}(lT_{b} + T_{b} - t) \rangle$$
  
= 
$$\int_{-\infty}^{\infty} r(t)h_{j}^{*}(lT_{b} + T_{b} - t)dt, \quad j = 1, 2, \dots, K_{n}.$$
 (3.15)

We have assumed that the correlation matrix described in (3.9) is non-singular, and thus the  $K_n$  basis functions  $\{h_j(t)\}$  defined in (3.13) (or equivalently,  $K_n$  orthonormal basis functions derived from  $\{h_i(t)\}\)$  span the signal space of  $K_n$  users whose modulation is BPSK. In the case of the unknown interfering sources and the background noise, the functions  $\{h_i(t)\}$  do not span their signal space. However, since the unknown sources and the noise are independent of the known signal sources, the unknown interference and noise terms that fall outside the known signal space is irrelevant to the detection of the  $K_n$  known users [6, P. 235]. In other words, the joint detection of  $K_n$  users can be based entirely on the equalization output sequences  $\{d_i(l)\}$ . It is important to observe that the  $K_n$  basis functions  $\{h_i(t)\}$ is closely related to the projection energy from the unknown interference and the noise onto the signal space. In our proposed multiuser-oriented equalization, such an unwanted projection energy can be minimized (or the energy of the unknown interference and noise terms that fall outside the desired known signal space can be maximized). Consequently,  $\{\hat{d}_i(l)\}$  can provide a sufficient statistic with minimized residual interference and noise components for detecting the signals of the  $K_n$  known users.

Now define an output sample vector  $\hat{\mathbf{d}} = [\hat{\mathbf{d}}(1)^T \ \hat{\mathbf{d}}(2)^T \ \dots \ \hat{\mathbf{d}}(L)^T]_{K_nL\times 1}^T$  where  $\hat{\mathbf{d}}(l) = [\hat{d}_1(l) \ \hat{d}_2(l) \ \dots \ \hat{d}_{K_n}(l)]_{K_n\times 1}^T$ , and denote the entries of  $K_n \times K_n$  matrices **R** 

and  $\mathbf{Q}$ , and  $K_n \times (K - K_n)$  matrix  $\mathbf{Q}(l - i)$  as

$$\{\mathbf{R}\}_{jk} = \langle s_k(t), h_j(T_b - t) \rangle,$$
  

$$j = 1, 2, \dots, K_n, \quad k = 1, 2, \dots, K_n, \quad (3.16)$$

$$\{\mathbf{W}\}_{jk} = \langle h_k(T_b - t), h_j(T_b - t) \rangle, j = 1, 2, \dots, K_n, \quad k = 1, 2, \dots, K_n,$$
(3.17)

and

$$\{\mathbf{Q}(l-i)\}_{j(k-K_n)} = \langle s_k(t-iT_b-\tau_k), h_j(lT_b+T_b-t) \rangle, j = 1, 2, \dots, K_n, \quad k = K_n + 1, K_n + 2, \dots, K, \quad (3.18)$$

respectively; then, the output sequence vector  $\mathbf{d}$  can be written by

 $\hat{\mathbf{d}} = \mathcal{R}_L A_{K_n} \mathbf{b}_{K_n} + \mathcal{Q}_L A_I \mathbf{b}_I + \mathbf{N}, \qquad (3.19)$ 

where, by definition,

 $\begin{aligned} \mathbf{b}_{K_n} &= [b_1(1) \ b_2(1) \ \dots \ b_{K_n}(1) \ b_1(2) \ b_2(2) \ \dots \ b_{K_n}(2) \ \dots \ b_1(L) \ b_2(L) \ \dots \ b_{K_n}(L)]_{K_nL\times 1}^T. \\ \mathbf{b}_I &= [b_{K_n+1}(1) \ b_{K_n+2}(1) \ \dots \ b_K(1) \ \dots \ b_{K_n+1}(L) \ b_{K_n+2}(L) \ \dots \ b_K(L)]_{(K-K_n)L\times 1}^T. \\ A_{K_n} &= \operatorname{diag}\{A_{K_n}(1), \ A_{K_n}(2), \ \dots, \ A_{K_n}(L)\}_{K_nL\times K_nL}, \\ A_{K_n}(l) &= \operatorname{diag}\{\sqrt{P_1}, \ \sqrt{P_2}, \ \dots, \ \sqrt{P_{K_n}}\}_{K_n\times K_n}. \\ A_I &= \operatorname{diag}\{A_I(1), \ A_I(2), \ \dots, \ A_I(L)\}_{K_nL\times K_nL}, \\ A_I(l) &= \operatorname{diag}\{\sqrt{P_{K_n+1}}, \ \sqrt{P_{K_n+2}}, \ \dots, \ \sqrt{P_K}\}_{(K-K_n)\times (K-K_n)}. \\ \mathbf{N} &= [\mathbf{N}(1)^T \ \mathbf{N}(2)^T \ \dots \ \mathbf{N}(L)^T]_{K_nL\times 1}^T, \ \mathbf{N}(l) &= [z_1(l) \ z_2(l) \ \dots \ z_{K_n}(l)]_{K_n\times 1}^T, \ \text{where} \\ z_j(l) \ \text{in } \mathbf{N}(l) \ \text{is defined as} \ z_j(l) &= < n(t), \ h_j(lT_b + T_b + \tau_j - t) >, \ \ j = 1, 2, \dots, K_n. \end{aligned}$ 

$$\mathcal{R}_{L} = \begin{bmatrix} \mathbf{R} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{R} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}_{K_{n}L \times K_{n}L}, \qquad (3.20)$$

$$Q_{L} = \begin{bmatrix} Q(0) & 0 & 0 & 0 & \dots & 0 \\ Q(1) & Q(0) & 0 & 0 & \dots & 0 \\ 0 & Q(1) & Q(0) & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & Q(1) & Q(0) & 0 \\ 0 & \dots & 0 & 0 & Q(1) & Q(0) \end{bmatrix}_{K_{n}L \times (K-K_{n})L}$$
(3.21)

and

$$\mathcal{W}_{L} = \begin{bmatrix} \mathbf{W} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{W} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{W} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{W} & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{W} \end{bmatrix}_{K_{n}L \times K_{n}L}$$
(3.22)

Notice that  $z_l(l)$  is a Gaussian random variable with zero mean and variance  $\frac{N_0}{2}$ ; N is the output noise vector of the MMSE equalizers, which has zero mean and covariance matrix

$$E[\mathbf{N}\mathbf{N}^T] = \frac{N_0}{2} \mathcal{W}_L; \qquad (3.23)$$

and its vector components  $\{\mathbf{N}(l)\}$  satisfy  $E[\mathbf{N}(i)\mathbf{N}^{T}(l)] = \frac{N_{0}}{2}\mathbf{W}$  for l = i and zero otherwise. By definition (3.18), it can be shown that the  $K_{n} \times (K - K_{n})$  matrix  $\mathbf{Q}(i)$  satisfies  $\mathbf{Q}(i) = \mathbf{0}$ , unless i = 0 and 1.

We rewrite the output sequence vector in (3.19) as

$$\hat{\mathbf{d}} = \mathcal{R}_L A_{K_n} \mathbf{b}_{K_n} + \mathbf{d}_I + \mathbf{N}, \qquad (3.24)$$

where  $\mathbf{d}_I = \mathcal{Q}_L A_I \mathbf{b}_I$  is the residual interference vector from the  $K - K_n$  users. The output residual interference vector  $\mathbf{d}_I$  can be assumed to be approximately Gaussian on the basis of a Central-Limit argument. Such approximate Gaussianity behavior for the residual interference at the outputs of linear MMSE estimators has been well studied under various conditions (see [140, 141]).

Now our problem in (3.24) is reduced to a sequence detection (the sequence in question being  $\mathbf{b}_{K_n}$ ) in coloured Gaussian noise channel. Thus the maximumlikelihood sequence decision algorithm for the  $K_n$  known users selects a  $\mathbf{b}_{K_n o}$  that maximizes the payoff function

$$\exp\left\{-\frac{1}{2}(\hat{\mathbf{d}}-\mathcal{R}_L A_{K_n}\mathbf{b}_{K_n})^T (R_I + \frac{N_0}{2}\mathcal{W}_L)^{-1}(\hat{\mathbf{d}}-\mathcal{R}_L A_{K_n}\mathbf{b}_{K_n})\right\},\qquad(3.25)$$

where  $R_I$  is the correlation matrix of the residual interference of  $K - K_n$  interference defined by  $R_I = E[\mathbf{d}_I \mathbf{d}_I^T]$ . After neglecting the terms that are common to all of the hypotheses in (3.25), we obtain the maximum-likelihood decision metric for multiuser detection in partial information settings that selects a  $\mathbf{b}_{K_n o}$  to maximize the loglikelihood function

$$\Omega(\mathbf{b}_{K_n}) = 2\mathbf{b}_{K_n}^T A_{K_n}^T \bar{\mathbf{d}} - \mathbf{b}_{K_n}^T A_{K_n}^T \mathcal{R}_U A_{K_n} \mathbf{b}_{K_n}, \qquad (3.26)$$

where  $\mathbf{\bar{d}} = \mathcal{R}_L^T (R_I + \frac{N_0}{2} \mathcal{W}_L)^{-1} \mathbf{\hat{d}}$ , and  $\mathcal{R}_U = \mathcal{R}_L^T (R_I + \frac{N_0}{2} \mathcal{W}_L)^{-1} \mathcal{R}_L$ . Notice that the multiuser detection metric described in (3.26) for  $K/K_n/K_n$  MLS detection has been formulated in a way that is similar to (2.20) for  $K_n/K_n/K_n$  MLS multiuser detection. However, the two detection metrics are quite different in that the new formulation considers the suppression of the  $K - K_n$  out-of-cell unknown sources and includes the effect of the residual interference. The architecture for the proposed  $K/K_n/K_n$  multiuser detectors consists of a linear optimal multiuser-oriented signal equalizer bank, followed by maximum-likelihood sequence detection algorithm, as shown in Fig. 3.2. The computational complexity per bit of MLS decision algorithm is  $O(2^{K_n}/K_n)$ , independent of K, in settings where the in-cell sources are synchronized. The proposed  $K/K_n/K_n$  MLS multiuser is a generalization of the MLS multiuser detector [36] to multi-cell wireless networks; in the special case where  $K_n = K$ , the front-end filter bank reduces to a bank of MF filters, and the proposed MLS multiuser detector becomes the well-known architecture for optimum  $K_n/K_n$  MLS multiuser detector becomes the well-known architecture for optimum  $K_n/K_n$  MLS multiuser detector with full parameter information setup [36].



Fig. 3.2 The structure of proposed  $K/K_n/K_n$  MLS multiuser detectors in CDMA AWGN channels.

# **3.3** $K/K_n/K_n$ Linear Multiuser Detection

Recently, linear multiuser detection under additive Gaussian channels has been extensively studied, due to its low-complexity and robust near-far resistance. In the case where full parameter knowledge of all active users is available to the receiver, the  $K_n/K_n/K_n$  linear multiuser detectors [50, 49] have a well-known receiver architecture that consists of a bank of front-end  $K_n$  MF filters, followed by either a zero-forcing or an MMSE linear transformation. In this section, we propose  $K/K_n/K_n$  ( $K_n < K$ ) linear detection with partial information (e.g., multi-cell) setups.

The architecture of the  $K/K_n/K_n$  linear detection can be obtained by replacing the second-stage, nonlinear MLS decision algorithm in Fig. 3.2 with either a zeroforcing or an MMSE decision algorithm, which is shown in Fig. 3.3.

## **3.3.1** $K/K_n/K_n$ decorrelating detectors

We write the output sequence vector in (3.24) as

$$\hat{\mathbf{d}} = \mathcal{R}_L A_{K_n} \mathbf{b}_{K_n} + \mathbf{N}_{int}, \qquad (3.27)$$

where  $\mathbf{N}_{int} = \mathbf{d}_I + \mathbf{N}$ . Notice that  $\mathbf{d}_I$  is the residual interference vector from the  $K - K_n$  unknown users, assumed to be (small-valued) approximately Gaussian with zero means on the basis of a Central-Limit argument;  $\mathbf{N}$  is filtered Gaussian background noise. Using decorrelating (zero-forcing) criterion and neglecting the effect of residual interference and background noise  $\mathbf{N}_{int}$ , the bits of individual users can be estimated by

$$A_{K_n} \dot{\mathbf{b}}_{K_n} = A_{K_n} \mathbf{b}_{K_n} + \mathcal{R}_L^{-1} \mathbf{N}_{int}.$$
(3.28)

The zero-forcing interference cancellation can also be viewed as a maximum-likelihood estimation of the transmitted signal. To examine this, recall the detection problem in (3.26) that the MLS decision algorithm selects a discrete-valued bit vector  $\vec{\mathbf{b}}_{K_no}$  to maximize the metric

$$\Omega(\vec{\mathbf{b}}_{K_n}) = 2\vec{\mathbf{b}}_{K_n}^T A_{K_n}^T \vec{\mathbf{d}} - \vec{\mathbf{b}}_{K_n}^T A_{K_n}^T \mathcal{R}_U A_{K_n} \vec{\mathbf{b}}_{K_n}, \qquad (3.29)$$

where  $\mathbf{\bar{d}} = \mathcal{R}_L^T (R_I + \frac{N_0}{2} \mathcal{W}_L)^{-1} \mathbf{\hat{d}}$ , and  $\mathcal{R}_U = \mathcal{R}_L^T (R_I + \frac{N_0}{2} \mathcal{W}_L)^{-1} \mathcal{R}_L$ . Now consider the scenario where the discrete-valued assumption is dropped for  $\mathbf{\vec{b}}_{K_n}$ , i.e.,  $\mathbf{\vec{b}}_{K_n}$  is thought to be continuous-valued variables. As a consequence, the nature of the problem has changed from a detection to an estimation; the estimator is followed by a simple device that produces the final bit decisions. With this framework in mind, maximization of  $\Omega(\mathbf{b}_{K_n})$  can be accomplished by direct differentiation of (3.29) with respect to  $\mathbf{\vec{b}}_{K_n}$ , which gives rise to a linear transformation with a solution  $\mathbf{\vec{b}}_{K_n}$ 

$$A_{K_n} \vec{\mathbf{b}}_{K_n} = \mathcal{R}_U^{-1} \vec{\mathbf{d}}$$
  
=  $\mathcal{R}_L^{-1} \hat{\mathbf{d}}$   
=  $A_{K_n} \mathbf{b}_{K_n} + \mathcal{R}_L^{-1} \mathbf{N}_{int}.$  (3.30)



Fig. 3.3 The structure of proposed  $K/K_n/K_n$  linear multiuser detectors in CDMA AWGN channels.

Thus, the  $K/K_n/K_n$  linear decorrelating detector consists of a bank of  $K_n$  front-end linear multiuser-oriented signal equalizers, followed by a zero-forcing linear transformation. Such an architecture is an extension of the linear decorrelating detector [50] in a full information (e.g., single-cell) setup to a partial information (e.g., multi-cell) setup.

Notice that the above derivation shows that, in a maximum-likelihood sense, a linear receiver is the best means of estimating continuous-valued bit symbols embedded in additive Gaussian interference plus noise. From the same sense, the zero-forcing detection is suboptimal, since the transmitted bit symbols are actually discretevalued.

## **3.3.2** $K/K_n/K_n$ linear MMSE detectors

The decorrelating detectors generally enhance the residual interference and noise since it neglects them in the linear transformation, and thus suffer from a performance penalty. Alternatively, a linear transformation  $\mathcal{K}$  can be employed in place of  $\mathcal{R}_L^{-1}$  in (3.30) such that the mean squared error  $E[(A_{K_n}\mathbf{b}_{K_n} - \mathcal{K}\hat{\mathbf{d}})^T(A_{K_n}\mathbf{b}_{K_n} - \mathcal{K}\hat{\mathbf{d}})]$  can be minimized, thus suppressing both the interference from the other known users and the residual interference plus background noise in the front end. On the basis of the projection principle [142], minimization of such a mean squared error is equivalent to finding the error variable  $(A_{K_n}\mathbf{b}_{K_n} - \mathcal{K}\hat{\mathbf{d}})$  that is orthogonal to any component of  $\hat{\mathbf{d}}$ ; that is,

$$E[(A_{K_n}\mathbf{b}_{K_n} - \mathcal{K}\hat{\mathbf{d}})\hat{\mathbf{d}}] = 0.$$
(3.31)

Solving the equation (3.31) leads to the optimal choice  $\mathcal{K}$  for  $K/K_n/K_n$  linear MMSE detector, which can be expressed as [51]

$$\mathcal{K}_{opt} = A_{K_n}^2 \mathcal{R}_L^T (\mathcal{R}_L A_{K_n}^2 \mathcal{R}_L^T + R_I + \frac{N_0}{2} \mathcal{W}_L)^{-1}.$$
 (3.32)

Thus the architecture of the  $K/K_n/K_n$  linear MMSE multiuser detector consists of a bank of front-end  $K_n$  linear multiuser-oriented signal equalizers, followed by the linear MMSE transformation  $\mathcal{K}_{opt}$ .

The  $K/K_n/K_n$  linear MMSE detector is well defined in the sense that the transformation in (3.32) always exists because the matrix therein is the sum of a nonnegative definite matrix  $\mathcal{R}_L$  and a diagonal positive definite matrix  $\frac{N_0}{2}\mathcal{W}_L$ ; that is, the linear MMSE transformation exists, even though the decorrelating solution in (3.30) may not exist. Moreover, the  $K/K_n/K_n$  linear MMSE detector performs better than the  $K/K_n/K_n$  linear decorrelators, since it takes the effect of the residual interference plus noise into consideration. It is seen that when particularizing  $K/K_n/K_n$ linear MMSE detection to the case where  $K_n = K$ ,  $R_I = 0$  and  $\mathcal{W}_L = \mathcal{R}_L$ , the front-end linear optimal filters reduce to a bank of conventional matched filters, and the second-stage linear MMSE transformation becomes

$$\mathcal{K}_{opt} = (\mathcal{R}_L + \frac{N_0}{2} A_{K_n}^{-2})^{-1}; \qquad (3.33)$$

thus, the proposed linear MMSE detection architecture reduces to the one proposed in [49]. Moreover, in the case where  $K_n = 1 (< K)$ , the proposed structure reduces to the extensively studied [46], [47], [48] for the linear single-user MMSE detection.

# 3.4 $K/K_n/M$ Equalization and MLS Detection in Gaussian Channels

In Section 3.2, we have proposed the  $K/K_n/K_n$  MLS multiuser detection with implementation complexity increasing exponentially with  $K_n$ . When  $K_n$  is very large, or the receiver is constrained to certain computational capability, the resulting complexity of  $K/K_n/K_n$  MLS multiuser detection can be too high to be affordable for a base-station reception. In this section, we propose a flexible  $K/K_n/M$  ( $M \leq K_n$ ) MLS detection architecture with tradeoffs between performance and complexity. For the  $K/K_n/M$  MLS detection, we assume that the users of interest for joint detection in the target group are indexed first, from 1 to M; the other known users, from M+1to  $K_n$ ; and the unknown interfering sources, from  $K_n+1$  to K. For convenient elaboration of the detection design, we further assume that transmissions of the  $K_n$  in-cell sources are synchronized, whereas signals of  $K - K_n$  out-of-cell unknown sources are asynchronous.

#### 3.4.1 The detection statistic for M users

In the *absence* of multiple access interference (MAI) from K-M sources, the sampled outputs of M MF filters form a *sufficient statistic* for the joint detection of the Musers. Write  $\langle f, g \rangle$  for the usual  $(L^2)$  inner product between functions f and g, and  $\rho_{jk}$  for the correlation  $\langle s_j, s_k \rangle$  between signature waveforms of users j and k on a single bit interval. The multiuser signal aggregate desirable for MUD at the output of the *j*th MF filter (sampled at the *i*th bit interval) can be denoted by

$$d_j(i) = \langle r_M(t), s_j(t - iT_b) \rangle = \sum_{k=1}^M \sqrt{P_k} b_k(i) \rho_{jk}, \qquad (3.34)$$

where

$$r_M(t) = \sum_{l=1}^{L} \sum_{k=1}^{M} \sqrt{P_k} b_k(l) s_k(t - lT_b)$$

represents the received signal from the sources in the target group observed over an L-bit interval. Notice that the multiuser signal defined in (3.34) for joint *M*-user detection is actually a projection of  $r_M(t)$  into the signal space, expanded by the

known signature waveforms of the M users.

In the presence of the K - M sources, the multiuser signal aggregate  $d_j(i)$  of M users in the target group is considered our desired front-end signal equalization target associated with user j. Our approach to the design of a receiver for the  $K/K_n/M$  detection extracts the  $d_j(i)$ , in the form of MMSE estimates  $\hat{d}_j(i)$  (on a bit-by-bit basis), from a larger received aggregate of K sources and background noise; these  $\hat{d}_j(i)$  are then processed by an M-ary multiuser maximum-likelihood sequence detection algorithm.

#### **3.4.2** Linear MMSE estimation of the $d_i(i)$

The estimate of  $d_j(i)$  in the *j*th equalizer,  $\hat{d}_j(i)$ , is obtained by processing the received signal r(t) through a linear equalizer consisting of an *N*-tap fractionally-spaced chipmatched filter (shown in Fig. 3.1), and by directing the linear signal equalization toward the multiuser signal aggregate  $d_j(i)$  on the basis of MMSE criterion. The output of the *j*th equalizer at the *i*th bit interval of user *j* has the form

$$\hat{d}_j(i) = \mathbf{w}_j^T \mathbf{r}_j(i), \tag{3.35}$$

where  $\mathbf{r}_j(i)$  is the chip-rate sample vector of the received signal at the output of chipmatched filter in the *i*th bit interval described in Appendix A.  $\mathbf{w}_j = [w_j(0) \ w_j(1) \ \dots \ w_j(N-1)]^T$  is an N-dimensional coefficient vector of the *j*th equalizer; the optimum coefficient vector  $\mathbf{w}_{jo}$  can be obtained by solving the Wiener-Hopf filter equation

$$\mathbf{S}_j \mathbf{w}_{jo} = \mathbf{p}_j, \tag{3.36}$$

where  $\mathbf{S}_j$  is an  $N \times N$  correlation matrix of a (chip-rate) sample vector over one bit interval in the *j*th equalizer, i.e.,  $\mathbf{S}_j = E[\mathbf{r}_j(i)\mathbf{r}_j(i)^T]$ , and  $\mathbf{p}_j$  is an  $N \times 1$  crosscorrelation vector between the sample vector and the desired multiuser signal aggregate, i.e.,  $\mathbf{p}_j = E[\mathbf{r}_j(i)d_j(i)]$ . The correlation matrix  $\mathbf{S}_j$  can be formulated as described in (3.9). The cross-correlation vector can be represented as

$$\mathbf{p}_j = \frac{1}{N} \sum_{k=1}^M P_k \rho_{jk} \mathbf{a}_k \tag{3.37}$$

where  $P_k$  and  $\mathbf{a}_k$  denote, respectively, the received signal power and the  $N \times 1$  signature vector of known user k, and  $\rho_{jk}$  is the correlation of signature waveforms between user j and user k.

The enhanced signature waveform  $w_j(t)$  and the impulse response of the linear optimum equalizer  $h_j(t)$  can be denoted as given in (3.12) and (3.13), respectively, with respect to the optimum coefficient vector  $\mathbf{w}_{jo}$ . As a result, the estimate of  $d_j(i)$ at the output of the front-end linear filter of user j, observed at the time instant  $(i+1)T_b$ , can be expressed as

$$\hat{d}_{j}(i) = \langle r(t), h_{j}(iT_{b} + T_{b} - t) \rangle = \int_{-\infty}^{\infty} r(t)h_{j}^{*}(iT_{b} + T_{b} - t)dt, \quad j = 1, \dots, M.$$
(3.38)

Define  $\hat{\mathbf{d}}(i) = [\hat{d}_1(i) \ \hat{d}_2(i) \ \dots \ \hat{d}_j(i) \ \dots \ \hat{d}_M(i)]^T$  (*i* being the *bit* index and *j* being the *user* index); all of the output samples from the front-end *M* multiuser-oriented signal equalizers form the  $ML \times 1$  column vector constructed by concatenating the  $\hat{\mathbf{d}}(i)$ 

$$\hat{\mathbf{d}} = [\hat{\mathbf{d}}(1)^T \ \hat{\mathbf{d}}(2)^T \ \dots \ \hat{\mathbf{d}}(L)^T]^T;$$
(3.39)

then with a little bit of work, it can be shown that in fact

$$\hat{\mathbf{d}} = \mathcal{R}_L A_M \mathbf{b}_M + \mathcal{Q}_L A_I \mathbf{b}_I + \mathbf{N}.$$
(3.40)

The symbols in the expression are defined in the Appendix A. Briefly,  $\mathcal{R}_L$  is a matrix of the inner products of signature sequence  $s_k$  (of user k) and impulse response  $h_j$ (of equalizer k), and  $\mathcal{Q}_L$  is a matrix of the inner products of  $s_{k'}$  and  $h_j$ , where j = $1, \ldots, M, k = 1, \ldots, M$ , and  $k' = M+1, \ldots, K$ ;  $A_M, A_I$  are diagonal matrices whose diagonal elements are amplitudes  $\sqrt{P_k}$ , arranged cyclically;  $\mathbf{b}_I$  is the bit sequence contributed by the interfering sources and  $\mathbf{b}_M$ , the heart of the matter, is the bit sequence to be decoded. The symbol  $\mathbf{N}$  (on the far right in the expression for  $\hat{\mathbf{d}}$ ) denotes a zero-mean Gaussian ML-vector with the block-diagonal covariance matrix displayed in the Appendix A.

Let  $R_I$  be the autocorrelation matrix of the residual interference of the K-M

sources, which is defined by  $R_I = E[\mathbf{d}_I \mathbf{d}_I^T] = E[\mathcal{Q}_L A_I A_I^T \mathcal{Q}_L^T]$  where the expectation is performed over the detection-related unknown quantities of  $K - K_n$  signals; then the following result characterizes  $R_I$ .

Proposition 1: The autocorrelation of the residual interference  $R_I$  is an  $ML \times ML$  symmetric matrix that has zero entries outside a band along its diagonal with upper and lower bandwidth 2M - 1.

The proof of Proposition 1 is presented in Appendix E. It is expected that correlations between residual interference components in different bit intervals are relatively small, since the MMSE equalization is made on a symbol-by-symbol basis; that is,  $E[\mathbf{Q}(1)\mathbf{Q}^{T}(0)]$  and  $E[\mathbf{Q}(0)\mathbf{Q}^{T}(1)]$  are matrices with small-valued entries. Moreover, since  $\mathcal{Q}_{L}$  is associated with the signal correlation between the users in the target group and other unwanted sources, Proposition 1 still satisfies in the case where the in-cell sources are asynchronous.

#### 3.4.3 The maximum-likelihood sequence detection

The goal is to recover  $\mathbf{b}_M$  from  $\hat{\mathbf{d}}$ . Rewrite  $\hat{\mathbf{d}}$  in the form

$$\hat{\mathbf{d}} = \mathcal{R}_L A_M \mathbf{b}_M + \mathbf{d}_I + \mathbf{N},\tag{3.41}$$

where  $\mathbf{d}_I = \mathcal{Q}_L A_I \mathbf{b}_I$  represents what remains of the unwanted MAI following the filtering provided by the equalizer front end.  $\mathbf{d}_I$  should be small, to the extent that the filtering is effective. It can be argued ([140, 141]) that  $\mathbf{d}_I$  is approximately Gaussian, in which case, the sum  $\mathbf{d}_I + \mathbf{N}$  (residual MAI + noise) is approximately Gaussian as well. Thus, our problem is reduced to sequence detection (the sequence in question being  $\mathbf{b}_M$ ) in coloured Gaussian noise, for which the optimal estimate  $\hat{\mathbf{b}}_M$  is the vector  $\mathbf{b}_{Mo}$  that maximizes the likelihood function

$$\exp\left\{-\frac{1}{2}(\hat{\mathbf{d}}-\mathcal{R}_L A_M \mathbf{b}_M)^T (R_I + \frac{N_0}{2}\mathcal{W}_L)^{-1} (\hat{\mathbf{d}}-\mathcal{R}_L A_M \mathbf{b}_M)\right\}$$

where  $R_I$  is the correlation matrix of the residual interference of the K-M sources defined by  $R_I = E[\mathbf{d}_I \mathbf{d}_I^T]$ . This simplifies to

$$\hat{\mathbf{b}}_{Mo} = \arg \max_{\mathbf{b}_M} \{ 2\mathbf{b}_M^T A_M^T \bar{\mathbf{d}} - \mathbf{b}_M^T A_M^T \mathcal{R}_U A_M \mathbf{b}_M \},$$
(3.42)

where  $\mathbf{d} = \mathcal{R}_L^T (R_I + \frac{N_0}{2} \mathcal{W}_L)^{-1} \mathbf{\hat{d}}$ , and  $\mathcal{R}_U = \mathcal{R}_L^T (R_I + \frac{N_0}{2} \mathcal{W}_L)^{-1} \mathcal{R}_L$ . The proposed receiver structure can be outlined in Fig. 3.4, and its computational complexity per bit of the *M*-user MLS decision algorithm is  $O(2^M/M)$  in settings where the in-cell sources are synchronized, independent of *K* and  $K_n$ .

#### **Remarks:**

(1) The detector that was just derived has the form described in [36], except that the matched-filter bank there is replaced here by a bank of multiuser-based signal equalizers developed in Sections 3.4.1 and 3.4.2. In the particular case  $K = K_n = M$ , our receiver reduces to the one in [36]; in the case of  $K = K_n$  and  $M < K_n$ , it is an alternative implementation to the subset detection proposed in [113], where an MMSE-based front-end filtering here replaces a decorrelating-based processing there.

(2) It follows from (3.41) that

$$R_I + \frac{N_0}{2} \mathcal{W}_L = \mathbf{E} \left[ \hat{\mathbf{d}} \hat{\mathbf{d}}^{\mathrm{T}} \right] - R_L A_M A_M^{\mathrm{T}} R_L^{\mathrm{T}}, \qquad (3.43)$$

suggesting that the correlation matrix for the variable  $d_I + N$  can be estimated from the outputs of the equalizer front end.

(3) Partitioning strategy: It is found from the numerical results (to be presented in Chapter 4; also, refer to [113]) that users with weak received signals can benefit from joint detection with strong-power users. Thus, one of the recommended partitioning strategies sorts the  $K_n$  known users by their received powers from strongest to weakest; it selects the user with the strongest signal and the user with the weakest signal, the user with the second strongest signal and the user with the second weakest signal, and so forth, putting them in one detection group until the group size is equal to M; it chooses other detection groups similarly until none of the  $K_n$ users remain.



**Fig. 3.4** The structure of proposed  $K/K_n/M$  multiuser MLS detector.

# 3.5 Multiuser Signal Equalization and MLS Detection in Fading Multipath Channels

In this section, we consider the multiuser signal equalization and detection under frequency-selective slowly Rayleigh-faded channels for a cellular CDMA network. We write K for the total number of active users in the network and M (< K) for the number of users of interest to be jointly detected. It is assumed that a J-multipath model is employed in our analysis; all multipath components of the M known can be tracked, estimated and resolved (to be used for joint MUD), whereas multipath signal components from the other users are considered unwanted interfering sources to be suppressed in the front-end filtering. Thus, there are a total of  $K_p = KJ$ signal components in the network, among which  $M_p = MJ$  components are path components of interest to the receiver for multiuser-oriented signal equalization and joint M-user MLS sequence detection.

From (2.3) and (2.6), the aggregate equivalent complex baseband signal at the

## 3.5 Multiuser Signal Equalization and MLS Detection in Fading Multipath Channels

receiver can be represented by

$$\tilde{r}(t) = \sum_{i=-L}^{L} \sum_{k=1}^{K} \sqrt{P_k} b_k(i) \mathbf{f}_k^T(t) \mathbf{s}_k(t - iT_b) + \tilde{n}(t)$$
(3.44)

where  $P_k$ ,  $b_k(i)$  and  $T_b$  are, respectively, the received signal power, the *i*th transmitted bit and the bit interval of user k. Note that the transmitted bit length for each user has been modified to 2L+1 in (3.44) for convenient analysis.  $\mathbf{s}_k(t) = [s_k(t-\tau_{k1}) s_k(t-\tau_{k2}) \dots s_k(t-\tau_{kJ})]^T$  whose element is a normalized signature waveform given by

$$s_k(t) = \sum_{n=0}^{N-1} a_k(n)\psi(t - nT_c), \quad 0 \le t < T_b,$$
(3.45)

assuming that the signature waveforms from different users are independent and the time-shifted signature waveform components of the same user are uncorrelated. J denotes the total number of path components for each user.  $\tau_{kl}$  is the transmission delay of the *l*th path component of user k,  $\{a_k(n), n = 0, 1, \ldots, N-1\}$  is the *k*th user's spreading sequence, consisting of N chips that take on values  $\{\pm 1\}$  and can be written as a vector form as  $\mathbf{a}_k = [a_k(0) \ a_k(1) \ \ldots \ a_k(N-1)]^T$ .  $\psi(t)$  is a chip pulse that is defined over  $[0, T_c)$  (For effect of a chip waveform of a longer duration on chip-rate sample measurements, see comments in Section 3.1.2), and satisfy  $\int_0^{T_c} \psi^2(t) dt = 1/N$ , where  $T_c = T_b/N$  is the inverse of the chip rate. The noise process  $\tilde{n}(t)$  is circularly symmetric, complex-valued, additive, and Gaussian with power spectral density  $N_0$ .  $\mathbf{f}_k(t) = [\alpha_{k1}(t) \ \alpha_{k2}(t) \ \ldots \ \alpha_{kJ}(t)]^T$ ; the fading coefficients  $\{\alpha_{km}(t)\}$  from K - M undesired sources are considered complex-valued, mutually-uncorrelated (over both k and m), zero-mean, stationary Gaussian random processes.

For the total number of signal component of K users,  $K_p = KJ$ , we re-index, for convenience of the analysis, the  $K_p$  component signals as  $1, 2, \ldots, J$  for the multipath components of user 1,  $J + 1, J + 2, \ldots, 2J$  of user 2, and so forth; signal components from M users in the target detection group are first indexed. Thus, the subscript kin  $s_k(t)$  denotes the signature waveform of the kth signal component in the following, where  $k = 1, 2, \ldots, K_p$ , and the same re-indexing notations of k in fading coefficients  $\alpha_k(t)$  and transmission delays  $\tau_k$ . As a result, the received signal can be expressed as

$$\tilde{r}(t) = \sum_{l=-L}^{L} \sum_{k=1}^{K_p} \sqrt{P_k} b_k(l) \alpha_k(t) s_k(t - lT_b - \tau_k) + \tilde{n}(t) = \tilde{r}_D(t) + \tilde{r}_U(t) + \tilde{n}(t)$$
(3.46)

where  $\tilde{r}_D(t) = \sum_{l=-L}^{L} \sum_{k=1}^{M_p} \sqrt{P_k} b_k(l) \alpha_k(t) s_k(t - lT_b - \tau_k)$ , where  $\tau_k$  is a transmission delay of resolved path component k, and  $\tilde{r}_U(t) = \sum_{l=-L}^{L} \sum_{k=M_p+1}^{K_p} \sqrt{P_k} b_k(l) \alpha_k(t) s_k(t - lT_b - \tau_k)$ .

# 3.5.1 Multiuser information for joint detection in fading multipath channels

In a fading multipath channel, each transmitted signal may disperse into many replicas of itself through the propagation environment at the receiver and these signal multipath components from each user can usually be considered to fade independently. The received  $M_p$  aggregate signal components from the M known users,  $\tilde{r}_D(t)$  in (3.46), are of prime interest for joint multiuser detection, whereas signal components of the unwanted K - M users (i.e.,  $\tilde{r}_U(t)$ ) need to be suppressed based on their second-order statistic.

As before, we define the projection of  $\tilde{r}_D(t)$  onto the component signature waveform of the *jth* multipath as the desired multiuser signal aggregate to be estimated in the front-end filtering (and then to be jointly decoded for the M users in the second-stage signal processing). The multiuser signal aggregate  $d_j(l)$  can be written as

$$d_{j}(l) = \langle \tilde{r}_{D}(t), s_{j}(t - lT_{b} - \tau_{j}) \rangle$$
  

$$j = 1, 2, \dots, M_{p}, \qquad (3.47)$$

where  $\langle x(t), y(t) \rangle = \int_{-\infty}^{\infty} x(t)y^*(t)dt$  denotes the inner product. As a result, a bank of  $M_p$  front-end linear equalizers, one for each finger reception, needs to be employed in the presence of unwanted sources  $\tilde{r}_U(t)$  to estimate  $\{d_j(l)\}$ . It is important to

## 3.5 Multiuser Signal Equalization and MLS Detection in Fading Multipath Channels

notice that while  $M_p$  linear filters are employed and thus  $M_p$  estimates of the target quantities  $\{d_j(l)\}$  are produced, the transmitted bits of interest come from only Musers; thus, the second-stage MLS decision algorithm only needs to search in an Mdiementional signal space for joint sequence detection of the M users, leading to an implementation complexity that increases exponentially with M but linearly with J.

#### 3.5.2 Liner optimal multiuser signal estimation

The front-end *j*th linear equalizer (for each finger reception) employs a fractionallyspaced chip-matched filter; an observation window of  $T_b$  is used for the bit-by-bit based signal equalization, thus leading to an *N*-tap filter. For convenient analysis, we further assume that the delay spread  $T_m$  of each known user satisfies  $T_m \ll T_b$ (cf. [116, 143]). It should be noticed that in scenarios where there is a relative large delay spread, the analysis is similar and straightforward. The *n*th sample in the *l*th bit interval at the output of the chip-matched filter in the equalizer for the *j*th component path is given by

$$r_{j}(l,n) = \langle \tilde{r}(t), \psi(t - lT_{b} - nT_{c} - \tau_{j}) \rangle$$
  
= 
$$\int_{-\infty}^{\infty} \tilde{r}(t)\psi^{*}(t - lT_{b} - nT_{c} - \tau_{j})dt$$
  
= 
$$\sum_{i=-L}^{L} \sum_{k=1}^{K_{p}} \sqrt{P_{k}}\alpha_{k}(i)b_{k}(i)u_{jk}^{(n)}(l,i) + n_{j}(l,n), \quad j = 1, 2, ..., M_{p}, (3.48)$$

where

$$u_{jk}^{(n)}(l,i) = \sum_{m=0}^{N-1} a_k(m) \int_0^{T_c} \psi(t + lT_b - iT_b + nT_c - mT_c + \tau_j - \tau_k) \psi^*(t) dt, \quad (3.49)$$

 $n_j(l,n) = \int_{lT_b+nT_c+\tau_j}^{lT_b+(n+1)T_c+\tau_j} \tilde{n}(t) \ \psi^*(t-lT_b-nT_c-\tau_j) \ dt, \ 0 \le n \le N-1, \ i \text{ symbolizes}$ the *i*th transmitted bit of the *j*th component signal, and  $\alpha_k(i) = \alpha_k(iT_b)$ . Let us define an *N*-dimensional vector at the *l*th bit interval of the *j*th component signal as  $\mathbf{r}_j(l) = [r_j(l,0) \ r_j(l,1) \ \dots \ r_j(l,N-1)]^T$ , then the sample vector  $\mathbf{r}_j(l)$  can be represented by

$$\mathbf{r}_{j}(l) = \sum_{i=-L}^{L} \sum_{k=1}^{K_{p}} \sqrt{P_{k}} \alpha_{k}(i) b_{k}(i) \mathbf{u}_{jk}(l,i) + \mathbf{n}_{j}(l), \quad j = 1, 2, \dots, M_{p}, \quad (3.50)$$

where  $\mathbf{u}_{jk}(l,i) = [u_{jk}^{(0)}(l,i) \ u_{jk}^{(1)}(l,i) \ \dots \ u_{jk}^{(N-1)}(l,i)]^T$ .  $\mathbf{n}_j(l) = [n_j(l,0) \ n_j(l,1) \ \dots \ n_j(l,N-1)]^T$  is an N-dimensional Gaussian vector with a zero mean and a covariance matrix  $\sigma^2 \mathbf{I}_N$ , where  $\mathbf{I}_N$  denotes the  $N \times N$  identity matrix and  $\sigma^2 = N_0/N$ . The output of the *j*th equalizer at the *l*th bit interval of the *j*th component signal has the form  $\hat{d}_j(l) = \mathbf{w}_j^T \mathbf{r}_j(l)$ , where  $\mathbf{w}_j = [w_j(0) \ w_j(1) \ \dots \ w_j(N-1)]^T$  is an N-dimensional coefficient vector, representing the coefficients in the *j*th equalizer. Thus, targeting the desired multiuser signal aggregate  $d_j(l)$  defined in (3.47), the linear optimum equalization finds a coefficient vector  $\mathbf{w}_{jo}$  that minimizes the mean squared error

$$\mathbf{w}_{jo} = \arg \min_{\mathbf{w}_j \in \Re^N} E[(\mathbf{w}_j^T \mathbf{r}_j(l) - d_j(l))^2], \qquad (3.51)$$

and the resulting weight coefficient vector is actually the solution to the Wiener-Hopf filter equation given by

$$\mathbf{S}_j \mathbf{w}_{jo} = \mathbf{p}_j, \quad j = 1, 2, \dots, M_p, \tag{3.52}$$

where  $\mathbf{S}_j$  is an  $N \times N$  correlation<sup>4</sup> matrix of the sample vector in the equalizer of user jat any given bit interval, i.e.,  $\mathbf{S}_j = E[\mathbf{r}_j^*(l)\mathbf{r}_j(l)^T]$ , and  $\mathbf{p}_j$  is an  $N \times 1$  cross-correlation vector between the sample vector and the desired multiuser signal aggregate  $d_j(l)$ , i.e.,  $\mathbf{p}_j = E[\mathbf{r}_j(l)d_j^*(l)]$ . The cross-correlation vector of the *j*th equalizer can be written as

$$\mathbf{p}_{j} = \sum_{k=1}^{M_{p}} P_{k} \{ \alpha_{k}^{2}(l-1)\rho_{jk}(l,l-1)\mathbf{u}_{jk}(l,l-1) + \alpha_{k}^{2}(l)\rho_{jk}(l,l)\mathbf{u}_{jk}(l,l) + \alpha_{k}^{2}(l+1)\rho_{jk}(l,l+1)\mathbf{u}_{jk}(l,l+1) \}$$
(3.53)

<sup>&</sup>lt;sup>4</sup>The pseudo-correlation of the complex received signal vector is in fact zero, assuming that the real and image fading parts are zero mean and independent Guassian.

## 3.5 Multiuser Signal Equalization and MLS Detection in Fading Multipath Channels

and the correlation matrix can be written as

$$\mathbf{S}_{j} = \sum_{k=1}^{M_{p}} P_{k} \{ \alpha_{k}^{2}(l-1) \mathbf{u}_{jk}(l,l-1) \mathbf{u}_{jk}^{T}(l,l-1) + \alpha_{k}^{2}(l) \mathbf{u}_{jk}(l,l) \mathbf{u}_{jk}^{T}(l,l) + \alpha_{k}^{2}(l+1) \mathbf{u}_{jk}(l,l+1) \mathbf{u}_{jk}^{T}(l,l+1) \} + \sum_{k=M_{p}+1}^{K_{p}} \bar{P}_{k}[\bar{\alpha}_{k}^{2}(l-1) \mathbf{u}_{jk}(l,l-1) \mathbf{u}_{jk}^{T}(l,l-1) + \bar{\alpha}_{k}^{2}(l) \mathbf{u}_{jk}(l,l) \mathbf{u}_{jk}^{T}(l,l) + \bar{\alpha}_{k}^{2}(l+1) \mathbf{u}_{jk}(l,l+1) \mathbf{u}_{jk}^{T}(l,l+1)] + \sigma^{2} \mathbf{I}_{N}$$

$$= \mathbf{S}_{j}^{D} + \mathbf{S}_{j}^{U} + \sigma^{2} \mathbf{I}_{N},$$
(3.54)

where  $\rho_{jk}(l,i) = \langle s_k(t-iT_b-\tau_j), s_j(t-lT_b-\tau_j) \rangle$  is the partial correlation between signature waveforms of the *j*th and *k*th signal components at their *l*th and *i*th bit intervals, respectively,  $\mathbf{S}_j^D = \sum_{k=1}^{M_p} P_k\{\alpha_k^2(l-1)\mathbf{u}_{jk}(l,l-1)\mathbf{u}_{jk}^T(l,l-1)$  $1) + \alpha_k^2(l)\mathbf{u}_{jk}(l,l)\mathbf{u}_{jk}^T(l,l) + \alpha_k^2(l+1)\mathbf{u}_{jk}(l,l+1)\mathbf{u}_{jk}^T(l,l+1)\}$  is the correlation matrix of the *M* users of interest and the resolved  $M_p$  signal components, and  $\mathbf{S}_j^U =$  $\sum_{k=M_p+1}^{K_p} \bar{P}_k[\bar{\alpha}_k^2(l-1)\mathbf{u}_{jk}(l,l-1)\mathbf{u}_{jk}^T(l,l-1) + \bar{\alpha}_k^2(l)\mathbf{u}_{jk}(l,l)\mathbf{u}_{jk}^T(l,l) + \bar{\alpha}_k(l+1)\mathbf{u}_{jk}(l,l+1)$  $1)\mathbf{u}_{jk}^T(l,l+1)]$  is a correlation matrix of the  $K_p - M_p$  interfering components from the K - M unwanted sources, in which  $\bar{\alpha}_k^2(.)$  is the mean energy of the path *k* at a bit interval. Notice that, since  $\{\mathbf{u}_{jk}(l,l-1)\}, \{\mathbf{u}_{jk}(l,l)\}$ , and  $\{\mathbf{u}_{jk}(l,l+1)\}$  are actually independent of *l* based on *Property 1*,  $\mathbf{S}_j$  and  $\mathbf{p}_j$  are both independent of the bit interval *l* (as long as the channel doesn't vary greatly in the period of interest for joint multiuser detection; for example, a 1.25-millisecond power control slot [3] in a slow fading environment).

Generally,  $\mathbf{p}_j$  is known to the receiver and the correlation matrix  $\mathbf{S}_j$  is not. One way of estimating  $\mathbf{S}_j$  is to use a moving (time) average based on Z recent received signal vectors  $\{\mathbf{r}_j(i)\}$  as

$$\hat{\mathbf{S}}_j(Z) = \frac{1}{Z} \sum_{i=1}^{Z} \mathbf{r}_j(i) \mathbf{r}_j(i)^H.$$
(3.56)

Observe that this estimate is only based on the received signal vectors sampled at different bit interval.

Let us define the enhanced signature waveform in the form for the *j*th  $(1 \le j \le M_p)$  component signal with respect to the  $N \times 1$  optimum coefficient vector as

$$\varpi_j(t) = \sum_{n=0}^{N-1} w_{jo}(n)\psi(t - nT_c), \quad 0 \le t < T_b,$$
(3.57)

the impulse response of the linear equalizer can be denoted by  $h_j(t) = \varpi_j(T_b - t)$ . Thus, the estimate of  $d_j(l)$  after the *j*th linear equalization can be written in terms of the enhanced signature waveform as

$$\hat{d}_j(l) = \langle \tilde{r}(t), \varpi_j(t - lT_b - \tau_j) \rangle.$$
 (3.58)

Now define the data vector for M users with  $M_p = JM$  resolvable component signals as

$$\mathbf{b} = [\mathbf{b}^{T}(-L, M) \ \mathbf{b}^{T}(-L+1, M) \ \dots \ \mathbf{b}^{T}(L-1, M) \ \mathbf{b}^{T}(L, M)]^{T},$$
(3.59)

where

$$\mathbf{b}(l,M) = [\underbrace{b_1(l) \ b_1(l) \ \dots \ b_1(l)}_J \underbrace{b_2(l) \ b_2(l) \ \dots \ b_2(l)}_J \ \dots \underbrace{b_M(l) \ b_M(l) \ \dots \ b_M(l)}_J]^T;$$

the signature waveform vector as

$$\mathbf{s}(t) = [\mathbf{g}^{T}(t + LT_{b}) \ \mathbf{g}^{T}(t - LT_{b} + T_{b}) \ \dots \ \mathbf{g}(t - LT_{b} - T_{b}) \ \mathbf{g}^{T}(t - LT_{b})]^{T}, \quad (3.60)$$

where

$$\mathbf{g}(t) = [s_1(t-\tau_1) \ s_2(t-\tau_2) \ \dots \ s_{M_p}(t-\tau_{M_p})]^T;$$

the enhanced (or effective) signature waveform vector as

$$\mathbf{w}(t) = [\mathbf{h}^{T}(t + LT_{b}) \ \mathbf{h}^{T}(t - LT_{b} + T_{b}) \ \dots \ \mathbf{h}(t - LT_{b} - T_{b}) \ \mathbf{h}^{T}(t - LT_{b})]^{T}, \ (3.61)$$

where

$$\mathbf{h}(t) = [\varpi_1(t-\tau_1) \ \varpi_2(t-\tau_2) \ \dots \ \varpi_{M_p}(t-\tau_{M_p})]^T;$$

and the matrix of fading coefficients as

$$\mathcal{C} = \operatorname{diag}\{\mathbf{C}(-L) \ \mathbf{C}(-L+1) \ \dots \ \mathbf{C}(L-1) \ \mathbf{C}(L)\}, \tag{3.62}$$

where

$$\mathbf{C}(l) = \operatorname{diag}\{\alpha_1(l) \ \alpha_2(l) \ \dots \ \alpha_{M_p}(l)\},\$$

With these symbol definitions, we can readily describe the joint detection of  $M_p$  signal components for M users in the target group, which is given in next subsection.

#### 3.5.3 Maximum likelihood sequence multiuser detection

The output vector at the outputs of the  $M_p$  linear equalizers can be written as

$$\hat{\mathbf{d}} = \mathcal{RC}A_M \mathbf{b} + \mathbf{d}_I + \mathbf{N} \tag{3.63}$$

where  $\mathcal{R}$  is a  $(2L+1)M_p \times (2L+1)M_p$  matrix that is defined as

$$\mathcal{R} = \int_{-\infty}^{\infty} \mathbf{w}^{*}(t) \mathbf{s}^{T}(t) dt$$

$$= \begin{bmatrix} \mathbf{R}(0) & \mathbf{R}(-1) & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{R}(1) & \mathbf{R}(0) & \mathbf{R}(-1) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}(1) & \mathbf{R}(0) & \mathbf{R}(-1) & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{R}(1) & \mathbf{R}(0) & \mathbf{R}(-1) \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{R}(1) & \mathbf{R}(0) \end{bmatrix}$$
(3.64)

with an  $M_p \times M_p$  block element

$$\mathbf{R}(i-l) = \int_{-\infty}^{\infty} \mathbf{h}^*(t-iT_b)\mathbf{g}^T(t-lT_b)dt.$$
(3.65)

 $A_M$  is the  $(2L+1)M_p \times (2L+1)M_p$  amplitude matrix of the received signals from the *M* users of interest and is defined as

$$A_M = \operatorname{diag}\{\underbrace{\mathbf{V} \ \mathbf{V} \dots \mathbf{V}}_{2L+1}\},\tag{3.66}$$

in which

$$\mathbf{V} = \operatorname{diag}\{\underbrace{\sqrt{P_1} \ \sqrt{P_1} \ \dots \ \sqrt{P_1}}_{J} \underbrace{\sqrt{P_2} \ \sqrt{P_2} \ \dots \ \sqrt{P_2}}_{J} \ \dots \underbrace{\sqrt{P_M} \ \sqrt{P_M} \ \dots \ \sqrt{P_M}}_{J}\}.$$
(3.67)

 $\mathbf{d}_I = \int_{-\infty}^{\infty} \tilde{r}_U(t) \mathbf{w}^*(t) dt$  is the residual interference vector of unwanted  $K_p - M_p$  component signals from the K - M users. N is the output noise vector, which has a zero mean and an autocorrelation matrix  $N_0 \mathcal{W}$ , where  $\mathcal{W}$  is given by

$$\mathcal{W} = \begin{bmatrix} \mathbf{W}(0) & \mathbf{W}^{T}(1) & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{W}(1) & \mathbf{W}(0) & \mathbf{W}^{T}(1) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{W}(1) & \mathbf{W}(0) & \mathbf{W}^{T}(1) & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{W}(1) & \mathbf{W}(0) & \mathbf{W}^{T}(1) \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{W}(1) & \mathbf{W}(0) \end{bmatrix}$$
(3.68)

with an  $M_p \times M_p$  the block element

$$\mathbf{W}(i-l) = \int_{-\infty}^{\infty} \mathbf{h}^*(t-iT_b)\mathbf{h}^T(t-lT_b)dt.$$
(3.69)

Notice that after the front-end linear equalization, the output sequence includes the desired multiuser signals of the M users, the residual interference from unwanted sources, and background noise; the first component is what we need for the multiuser detection of the M users, while the latter components  $(\mathbf{d}_I + \mathbf{N})$  can be assumed to be approximately Gaussian on the basis of a Central-Limit argument (also, cf., [140, 141]). With the above analysis in mind, the discrete-time multiuser detection task in (3.63) becomes a sequence detection problem in a coloured Gaussian noise channel. Thus, the maximum-likelihood sequence detector for the M users selects a  $\mathbf{b}_o$  to maximize the likelihood function

$$\mathbf{b}_{o} = \arg \max_{\mathbf{b} \in 2^{(2L+1)M}} \exp\left\{-\frac{1}{2}(\hat{\mathbf{d}} - \mathcal{RC}A_{M}\mathbf{b})^{H}(R_{I} + N_{0}\mathcal{W})^{-1}(\hat{\mathbf{d}} - \mathcal{RC}A_{M}\mathbf{b})\right\} (3.70)$$

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 $R_I$  is the correlation matrix of the residual interference of K-M interference defined by  $R_I = E[\mathbf{d}_I^* \mathbf{d}_I^T]$ . After simple operations and neglecting the terms that are common to all hypotheses in (3.70), we obtain the decision metric that selects a  $\mathbf{b}_o$  to maximize

$$\Omega(\mathbf{b}) = 2\operatorname{Re}\{\mathbf{b}^{T}A_{M}^{T}\mathcal{C}^{H}\mathcal{R}^{T}(R_{I}+N_{0}\mathcal{W})^{-1}\hat{\mathbf{d}}\} - \mathbf{b}^{T}A_{M}^{T}\mathcal{C}^{H}\mathcal{R}^{T}(R_{I}+N_{0}\mathcal{W})^{-1}\mathcal{R}\mathcal{C}A_{M}\mathbf{b}$$

$$= 2\operatorname{Re}\{\mathbf{b}^{T}A_{M}^{T}\mathcal{C}^{H}\bar{\mathbf{d}}\} - \mathbf{b}^{T}A_{M}^{T}\mathcal{R}_{D}A_{M}\mathbf{b}$$

$$= 2\operatorname{Re}\{\mathbf{b}^{T}A_{M}^{T}\mathbf{y}\} - \mathbf{b}^{T}A_{M}^{T}\mathcal{R}_{D}A_{M}\mathbf{b},$$
(3.71)

where  $\mathcal{R}_D = \mathcal{C}^H \mathcal{R}^T (R_I + N_0 \mathcal{W})^{-1} \mathcal{R} \mathcal{C}$ ,

$$\bar{\mathbf{d}} = \mathcal{R}^T (R_I + N_0 \mathcal{W})^{-1} \hat{\mathbf{d}}, \qquad (3.72)$$

and

$$\mathbf{y} = \mathcal{C}^H \mathbf{d}. \tag{3.73}$$

Notice that the expression in (3.73) represents the maximal-ratio combining (MRC). After absorbing the linear optimum filtering and MRC into the decision variable vector  $\mathbf{y}$ , the maximum-likelihood decision function, or detection metric, can be formulated in a way similar to (3.26) for AWGN channel. Thus, the architecture of proposed multiuser MLS detector consists of a bank of  $M_p$  linear optimum filters and MRCs, followed by an maximum-likelihood sequence (MLS) decision algorithm, shown in Fig. 3.5.

When particularizing the proposed multiuser detection to a single-path full information (e.g., single-cell) scenario, in which M = K and thus  $\mathbf{d}_I = \mathbf{0}$ , the front-end linear optimum filtering reduces to a bank of  $M_p$  MF filters; the architecture reduces to one described in [45].

# 3.5.4 Discussions on implementation complexity of the proposed MLS detectors

A. In the absence of  $\tilde{r}_U(t)$  with J=1: We have  $K_p = K$  and  $M_p = M = K$ , and thus  $\mathcal{R}_D$  in (3.71) becomes a correlation matrix between the known signature waveforms in an asynchronous CDMA system, which is a band matrix with upper and lower



Fig. 3.5 Proposed multiuser MLS detector under CDMA fading multipath channels.

bandwidth M - 1. When Viterbi decision algorithm and the recursive state search mechanism are employed, the computational complexity of the multiuser decision algorithm is the well known on the order of  $2^{M}$  [51], since the dimensionality of the state space of the algorithm is  $2^{M-1}$ .

B. In the absence of  $\tilde{r}_U(t)$  with  $J \neq 1$ : We have K = M and  $M_p = K_p = JM$ , meaning that J independent multiuser signal replicas are resolved at the receiver for each user and there are a total of  $M_p$  components; however, since only M signal sources are transmitting, the multiuser detection in such a setup has a computational complexity of  $O(J2^M)$  — increasing linearly with J and exponentially with M.

C. In the presence of  $\tilde{r}_U(t)$  with J=1: We have  $K_p = K$  and  $M_p = M$  (< K). Thus, the computational complexity per bit in the proposed MLS decision algorithm is on the order of  $2^{(2L+1)M}/(2L+1)M$ , in theory. However, it can be substantially reduced because of an excellent characteristic of the correlation matrix  $\mathcal{R}_D$  in (3.71). For the MLS decision algorithm,  $\mathcal{R}_D$  plays a key role in deciding the implementation complexity; while  $\mathcal{R}_D$  is no longer a band matrix with upper and lower bandwidth M-1, it is still a symmetric matrix with its off-diagonal entries becoming smaller as they move away from the diagonal.

To examine it in detail,  $\mathcal{R}_D$  can be expressed as  $\mathcal{R}_D = \mathcal{C}^H \mathcal{R}^T (R_I + N_0 \mathcal{W})^{-1} \mathcal{R} \mathcal{C}$ ,

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where  $\mathcal{C}$  is a diagonal matrix;  $\mathcal{R}$  and  $\mathcal{W}$  are symmetric band matrices with upper and lower bandwidth M-1; and  $R_I$  is a symmetric correlation matrix of residual interference from unknown sources, which can be shown by definition to be a band matrix with upper and lower bandwidth 2M-1. Notice that the off-diagonal entries in these matrices become smaller as they move away from the diagonal, since they are correlations between signals of different users or in different bit intervals. Thus,  $R_I + N_0 \mathcal{W}$  is a symmetric band matrix with upper and lower bandwidth 2M - 1with relative small off-diagonal entries. It can be shown that in such a case, the off-diagonal entries in a band along the diagonal of  $(R_I + N_0 \mathcal{W})^{-1}$  with upper and lower bandwidth 2M - 1 are equivalent to those of  $(R_I + N_0 \mathcal{W})$  [144]. With this provision in mind, the left and right multiplications of  $(R_I + N_0 \mathcal{W})^{-1}$  with band matrices  $\mathcal{RC}$  (i.e.,  $\mathcal{R}_D$ ) result in a matrix whose off-diagonal entries are smaller as they move away from the diagonal; moreover, since the proposed front-end linear equalization is on a bit-by-bit basis, it effectively decreases the overall signal correlation length [145, 146], thus speeding up the decreasing rate (or de-correlation) of these off-diagonal entries as they move, from one bit to another, away from the diagonal. It is seen that the decreasing-valued off-diagonal entries in  $\mathcal{R}_D$  away from the diagonal indicate that the effects of these elements on a (central-positioned) bit detection are becoming negligible. Thus, we apply a cut-off strategy for the proposed MLS multiuser detection such that  $\mathcal{R}_D$  is approximately expressed by a band matrix with upper and lower bandwidth 3M-1; this is equivalent to considering the correlation effects of four adjacent bit intervals, two for each side, on the bit detection, while neglecting effects of the other smaller off-diagonal entries. Such a cut-off correlation length can be justified to have provided enough information for the bit decision as follows: The residual interference correlation matrix  $R_I$  is small-valued to the extent that each front-end linear equalization is effective; moreover,  $\mathcal{R}_D$  approaches a band matrix with upper and lower bandwidth M-1, or correlation matrix in the centralized detection setup [51] as entries of  $R_I$  approaches zeros, meaning that the entries of  $\mathcal{R}_D$  in the bandwidth of M-1 along the diagonal contains the principal correlation information for each bit detection. With the cut-off approximation, the proposed MLS decision algorithm can be shown to have time computational complexity on the order of  $2^{3M}$  [51, pp. 166-173] when Viterbi algorithm is employed. Notice that the computational complexity is related to the cut-off thresholds that can

trade implementation cost for detection performance; for example, we can employ a shorter cut-off length (e.g., 2M - 1) to further reduce the computational complexity of the proposed MLS multiuser detection. Detailed investigations on the reduced complexity and its trade-off issues for MLSD algorithm can be found, for example, in [145, 146]. Other complexity reduction schemes for MLS multiuser detection algorithm can be found in [147, 148] whose computational complexity is polynomial.

D. In the presence of  $\tilde{r}_U(t)$  with  $J \neq 1$ : We have  $M_p = JM$  resolved signal components in the system. However, since each of the M sources has J signal replicas, the dimension of the sequence search state only increases linear with J. As a result, the MLS multiuser detectors in such a case have a computational complexity of  $O(J2^{3M})$ .

In summary, the computational complexity of the proposed maximum likelihood sequence decision algorithm is in the range from  $O(J2^M)$  (synchronous settings or full-blown detectors) to  $O(J2^{3M})$  (asynchronous settings).

#### **Remarks:**

We have presented asymptotic computational requirements above for the secondstage non-linear MLS decision algorithm based on Viterbi decision algorithm and the recursive state search mechanism with an infinite length of the transmitted bits (i.e.,  $L \to \infty$ ). In reality, a finite-*L*-bit frame is chosen for the joint signal detection. Thus, additional computational costs are required for the search of first 3*M* bits of each user in the frame before the recursive algorithm can be applied. Other fixed computational costs include the inverse of the correlation matrix for each frontend equalizer and its (non-adaptive) update, whose complexity is typically on the order of  $N^3$  where *N* is the number of weighting taps (equal to the processing gain in our study) in the equalizer. For an adaptive implementation of the front-end equalization, the computational cost can be reduced since the matrix inverse is not required, and will depend on the adaptive algorithm employed; the complexity of least mean squared (LMS) algorithm we employ in this study is on the order of *N*.

# 3.6 Linear Multiuser Detection in Fading Multipath Channels

In this section, a linear transformation in the second-stage signal processing is applied to the estimates of the desired multiuser aggregates (i.e., the outputs of a bank of front-end  $M_p$  linear MMSE equalizers); it leads to a linear multiuser detector in a partial information (e.g., multi-cell CDMA) setup.

For convenient analysis, we assume that transmissions of all active users are synchronous (an extension to the asynchronous scenarios is straightforward [81]). It is further assumed that the delay spread of multipath signal components from each user is much less than one bit period (cf., [143]). As a result, the linear multiuser joint detection of the M users can be made on a symbol-by-symbol basis, i.e., oneshot sequence demodulation. We particularize the notation (for scenarios where out-of-cell signals are assumed to be asynchronous) described in last section to the synchronous case as follows: Let  $\hat{\mathbf{d}}_{M_p}$  denote the estimate of the  $M_p \times 1$  bit vector  $\mathbf{b}_{M_p}$  (that is,  $\mathbf{b}(i, M)$  given in (3.59));  $\mathbf{R}$ , the  $M_p \times M_p$  matrix  $\mathbf{R}(0)$  in (3.64);  $\mathbf{W}$ , the  $M_p \times M_p$  matrix  $\mathbf{W}(0)$  in (3.68);  $\mathbf{C}$ , the  $M_p \times M_p$  fading matrix  $\mathbf{C}(i)$ ;  $\mathbf{d}_{I_p}$ , the  $M_p \times 1$  residual interference vector (with covariance matrix equal to  $\mathbf{R}_I = E[\mathbf{d}_I^* \mathbf{d}_I^T]$ ); and  $\mathbf{N}_{M_p}$ , the Gaussian background noise vector (with covariance matrix  $\mathbf{W}N_0$ ) at the *i*th bit interval. Thus, the output vector after the  $M_p$  linear MMSE equalizers can be written as

$$\mathbf{d}_{M_p} = \mathbf{RCVb}_{M_p} + \mathbf{d}_{I_p} + \mathbf{N}_{M_p}. \tag{3.74}$$

Note that the first term on the right-hand side of the equation (3.74) is what we need for joint detection of the M users, while  $\mathbf{d}_{I_p} + \mathbf{N}_{M_p}$  can be assumed to be approximately Gaussian on the basis of a Central-Limit argument, with a covariance matrix equal to  $\mathbf{W}_I = \mathbf{R}_I + \mathbf{W}N_0$ . After the front-end equalization, we face the joint detection for the M-user target group. Thus, the decorrelating filter can be employed to eliminate the MAI among the group of desired users. The output of the decorrelating filter is given by

$$\mathbf{z} = \mathbf{R}^{-1} \hat{\mathbf{d}}_{M_p} = \mathbf{C} \mathbf{V} \mathbf{b}_{M_p} + \mathbf{n}_I \tag{3.75}$$

where the residual interference plus noise vector  $\mathbf{n}_I = \mathbf{R}^{-1}\mathbf{d}_{I_p} + \mathbf{R}^{-1}\mathbf{N}_{M_p}$  is Gaussian zero mean with covariance matrix equal to  $\mathbf{W}_v = \mathbf{R}^{-1}\mathbf{W}_I\mathbf{R}^{-H}$ . Without a loss of generality, we consider the component vector of user 1 in (3.75), which is denoted by

$$\mathbf{z}_1 = (\mathbf{CV})_{11}\mathbf{b}_1 + \mathbf{n}_{I1},$$
 (3.76)

where  $(X)_{11}$  denotes the (1, 1) block entry, a square  $J \times J$  sub-matrix, in the matrix X and  $\mathbf{n}_{I1}$  is a Gaussian zero mean J-vector with covariance matrix  $(\mathbf{W}_v)_{11}$  associated with user 1. Since the resolved multipath signal components for each user can be effectively combined to maximize the output SIR, we derive the maximal-ratio combining (MRC) architecture for the linear detection. By Cholesky decomposition  $(\mathbf{W}_v)_{11} = \mathbf{L}\mathbf{L}^H$ , the residual interference plus noise vector can be whitened as

$$\mathbf{z}_{1w} = \mathbf{y}\sqrt{P_1}b_1 + \mathbf{L}^{-1}\mathbf{n}_{I1} \tag{3.77}$$

where  $\mathbf{y} = \mathbf{L}^{-1}\mathbf{c}_1$  and  $\mathbf{c}_1 = \text{diag}\{\alpha_1(i) \ \alpha_2(i) \ \dots \alpha_J(i)\}$ . The output of the coherent MRC for user 1 can be written as

$$d_1 = \mathbf{y}^H \mathbf{y} \sqrt{P_1} b_1 + \mathbf{y}^H \mathbf{L}^{-1} \mathbf{n}_{I1}$$
(3.78)

It is noted that  $\mathbf{y}^H \mathbf{y} = \mathbf{c}_1^H [(\mathbf{W}_v)_{11}]^{-1} \mathbf{c}_1$ . The architecture of proposed linear multiuser detection consists of a linear multiuser equalizer bank, a decorrelator, signal whitening and MRC, which is shown in Fig. 3.6. The proposed linear multiuser detection under multipath fading channel is is an extension of [143] in a full information (e.g., single-cell) setup to a partial information (e.g., multi-cell) setup.

# 3.7 Multiuser Signal Equalization and Detection with Long-Spread Signals

In this section, we study the statistical characteristics of long-spread signals and their resulting interference suppression and joint detection capability. Short spreading codes are often employed for multiuser detection analysis and the receiver design, due to the fact they are not bit-by-bit varying and their correlation characteristics can be well used. In contrast, long spreading sequences vary on a bit-by-bit basis,

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Fig. 3.6 Proposed linear multiuser detection architecture

but they can also be applicable to MUD, in theory. It is expected that multiuser detection in a long-spread system requires a higher computational implementation than a short-spread system, since the former needs to update its signal waveform cross-correlations in every bit interval.

We assume a cellular K-user CDMA network, in which the receiver has parameter knowledge of the  $K_n$  in-cell users and has no parameter knowledge of  $K - K_n$  out-ofcell users. It is further assumed that in-cell signal transmissions are synchronous for convenient analysis. The front end employs a bank of  $K_n$  fractionally-spaced chipmatched filters (one for each user) to estimate the multiuser signal aggregates of  $K_n$  users while suppressing the interference from  $K - K_n$  resources. For the known long spreading codes assigned to in-cell users, the cross-correlation vector can be expressed as (3.10) and is known to the receiver, but the correlation matrix is not known and needs to be estimated. From (3.9), the correlation matrix associated with user j has the form

$$\mathbf{S}_{j} = \mathbf{S}_{j}^{K_{n}} + \mathbf{S}_{j}^{U_{n}} + \sigma_{N}^{2} \mathbf{I}_{N}$$

$$(3.79)$$

where  $\mathbf{S}_{j}^{K_{n}} = \frac{1}{N^{2}} \sum_{k=1}^{K_{n}} P_{k} \mathbf{a}_{k} \mathbf{a}_{k}^{T}$  is the correlation matrix due to signals from  $K_{n}$  known users,  $\mathbf{a}_{j} = [a_{j}(0) \ a_{j}(1) \ \dots \ a_{j}(N-1)]^{T}$  and  $P_{j}$  are, respectively, the signature vector and the received signal power of user j,  $\mathbf{S}_{j}^{U_{n}} = \sum_{k=K_{n}+1}^{K} \bar{P}_{k} E[\mathbf{u}_{jk}(l,l)\mathbf{u}_{jk}^{T}(l,l) + \mathbf{u}_{jk}(l,l-1)\mathbf{u}_{jk}^{T}(l,l-1)]$  is the correlation matrix due to signals from  $K - K_{n}$  unknown interfering sources in which  $\bar{P}_k$  denotes the average received signal power of unknown user k; the expectation  $E[\cdot]$  is over the long spreading sequences that are modeled as random variables taking values equally from  $\{+1, -1\}$ .

Signal detection in a full information (e.g., single-cell) setup: In such a case, there is no interference from out-of-cell sources (e.g.,  $\mathbf{S}_{j}^{U_{n}} = \mathbf{0}$ ) and the front-end linear equalizer bank reduces to a bank of  $K_{n}$  MF filters. Although the parameter knowledge of the  $K_{n}$  in-cell users — including the long spreading codes — is known to the base station, the receiver must update  $\mathbf{S}_{j}^{K_{n}}$  for multiuser detection in every bit interval, because  $\mathbf{S}_{j}^{K_{n}}$  and the cross-correlation vector of user j change on a bit-bybit basis; this leads to a more computational implementation for multiuser detection with long-spread signals than short-spread signals.

Interference suppression and signal estimation in a partial information (e.g., multi-cell) setup: In such a case, both signal suppression of interference from the out-of-cell sources and estimation of the desired multiuser signal aggregate of the  $K_n$  users are required in the front-end linear equalization. Since the matrix component  $\mathbf{S}_j^{K_n}$  is known, the correlation matrix  $\mathbf{S}_j$  is known if the statistical characterization of unknown  $\mathbf{S}_j^{U_n}$  can be estimated (for example, using training sequences or an overhead channel). It can be shown that the matrix  $\mathbf{S}_j^{U_n}$  has a simple, closed-form formulation. To see this, we express  $\mathbf{S}_j^{U_n}$  as  $\mathbf{S}_j^{U_n} = \sum_{k=K_n+1}^K \bar{P}_k C_I^{jk}$ , where  $C_I^{jk}$  represents a component correlation matrix, due to contributions from an unknown user k and a known user j, which can be written by

$$C_I^{jk} = E[\mathbf{u}_{jk}(i,i)\mathbf{u}_{jk}^T(i,i) + \mathbf{u}_{jk}(i,i-1)\mathbf{u}_{jk}^T(i,i-1)].$$
(3.80)

One good characteristic of  $C_I^{jk}$  can be summarized in the following proposition:

Proposition 2: For a given chip waveform of duration  $T_c$  and long spread signals from unknown  $K - K_n$  sources, the correlation matrix  $C_I^{jk}$  is a constant band matrix with upper and lower bandwidth 1, *independent* of the unknown user k  $(k > K_n)$ and the user j  $(j \le K_n)$  of interest. In particular, when a rectangular chip waveform is utilized,  $C_I^{jk}$  can be expressed as

$$C_{I}^{jk} = \frac{1}{N^{2}} \begin{bmatrix} \frac{2}{3} & \frac{1}{6} & 0 & 0 & \dots & 0\\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & 0 & \dots & 0\\ 0 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & \dots & 0\\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots\\ 0 & \dots & 0 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6}\\ 0 & \dots & 0 & 0 & \frac{1}{6} & \frac{2}{3} \end{bmatrix}_{N \times N}$$
(3.81)

The proof of Proposition 2 is presented in Appendix B. Note that the quantity  $C_I^{jk}$  is independent of the unknown interfering sources and the user of interest; moreover,  $C_I^{jk}$  is a band matrix along its diagonal with upper and lower bandwidth 1, demonstrating that the correlation of long spread signals is non-zero only over two adjacent chip intervals. Therefore, the component correlation matrix  $\mathbf{S}_j^{U_n}$  from the contributions of unknown interfering sources with long spreading signals is significantly different from the one with short spreading signals whose  $\mathbf{S}_j^{U_n}$  has non-zero entries. As a result, it can be seen from the above analysis that short-spread signals can provide more accurate statistical information about the unknown interference for signal equalization, leading to a more effective interference suppression capability than long-spread signals. In Chapter 4, we will quantify, by numerical examples, the difference in performance between long-spread signals and short-spread signals when they are respectively employed in CDMA systems for the proposed multiuser detection.

The Proposition 2 can be generalized to the cases where a chip waveform that is time-limited on  $[0, N_cT_c)$  (where  $N_c$  is an integer), which can be described as follows:

Proposition 2': For a given chip waveform defined over  $[0, N_c T_c)$  and long spread signals from unknown  $K - K_n$  sources, the correlation matrix  $C_I^{jk}$  is a constant band matrix with upper and lower bandwidth  $N_c + 1$ , *independent* of the unknown user k $(k > K_n)$  and the user j  $(j \le K_n)$  of interest. This statement can be easily shown by realizing the fact that adjacent chip-rate samples within a period of  $N_c$  chip intervals have a non-zero correlation for such a time-limited chip waveform.

# 3.8 An Adaptive Implementation of Multiuser Equalization and Detection

The front-end multiuser-oriented signal equalization in the proposed multiuser detectors has no requirements for initial training. However, when additional initial training is provided for each known user, the optimal filter coefficients in each front-end linear equalizer can be recursively approached, leading to an adaptive implementation of the MUD and obviating the need to estimate the covariance matrix of the received signal and the matrix inversion. The adaptive implementation can also track channel (slow) time variations, which is important to a practical CDMA system.

Initial training sequences can be used before actual data transmission, and then the decision-directed mode can be employed once an adaptive algorithm has converged. In this section, we apply a simple, but well-known adaptive algorithm least mean squared (LMS) — to each of the front-end  $M_p$  linear MMSE multiuser equalizers for K/M/M detectors developed in Sections 3.5 and 3.6 under fading multipath channels. The LMS algorithm employs unbiased noisy estimates of the gradient vector to adjust the coefficients of an MMSE equalizer, which, in our case, can be written by

$$\mathbf{w}_{j}^{(m+1)} = \mathbf{w}_{j}^{(m)} - \mu_{j} e_{j}(m) \mathbf{r}_{j}(m), \ j = 1, 2, \dots, M_{p},$$
(3.82)

where  $\mathbf{w}_{j}^{(m)}$  is a weight-vector in the *j*th equalizer at the *m*th iteration of coefficient adjustment,  $\mathbf{r}_{j}(m)$  is an input vector in the *m*th time instant,  $\mu_{j}$  is the step size of the algorithm, and  $e_{j}(i) = d_{j}(i) - \mathbf{r}_{j}^{T}(i)\mathbf{w}_{j}^{(i)}$  is an estimate error in the equalizer *j* at the *i*th time instant. Thus, in such an adaptive implementation scenario, the so-called enhanced signature waveform defined for user *j* in (3.12) needs to be based on the steady-state coefficients  $\{\mathbf{w}_{j}^{(s)}\}$ , that is, replacing  $\{\mathbf{w}_{jo}\}$  with  $\{\mathbf{w}_{j}^{(s)}\}$  in (3.12); then the impulse responses of the MMSE filter expressed in (3.13) can be formulated accordingly.

It can be shown that the LMS algorithm converges if and only if the step-size parameter  $\mu_j$  satisfies the condition [63]

$$0 < \mu_j < \frac{2}{tr[\mathbf{S}_j]}, \quad j = 1, 2, \dots, M_p,$$
 (3.83)

where  $tr[\mathbf{S}_j]$  denotes the trace of correlation matrix  $\mathbf{S}_j$ .

## 3.9 Summary

In this chapter, we developed forms of enhanced uplink multiuser detectors with partial information setups that can fully exploit the parameter knowledge of known users and the statistical knowledge of unknown sources to improve performance. The receiver architectures for maximum likelihood sequence (MLS) and linear multiuser detection were derived and formulated in a cellular CDMA network under AWGN and slowly Rayleigh-faded multipath channels. Generally speaking, the proposed architectures have two stages: The first stage, forming the front end of the device, amounts to a bank of linear multiuser-oriented signal equalizers on the basis of MMSE criterion, one for each of the target sources for joint detection; its role is to estimate the multiuser signal aggregate by suppressing the unwanted interference. The second stage, acting *jointly* on all outputs from the first equalization banks, is an MLS or linear multiuser detector; its structure is calculated from the second-order statistics of the equalizer error processes, assumed approximately Gaussian on the basis of a Central-Limit argument. The proposed  $K/K_n/K_n$  MLS multiuser detection for AWGN channel is a generalization of the MLS multiuser detection [36] to partial information, multi-cell CDMA scenarios, whose computational complexity is on the order of  $2^{K_n}$ , independent of K; the proposed K/M/M MLS multiuser detection for Rayleigh-faded multipath channel is a generalization of the MLS detection [45] to partial information, multi-cell CDMA scenarios, whose computational complexity is in the range of  $O(J2^M)$  to  $O(J2^{3M})$ , independent of K and the total number of multipath components JK. The  $K/K_n/K_n$  linear multiuser detection employs a strategy quite different from those proposed in [39]-[41] in that the parameter knowledge of all  $K_n$  known users can be explicitly used in the proposed receivers;

the proposed K/M/M linear detection in fading multipath channel is an extension of the linear detection [143] in full information (e.g., single-cell) settings to partial information (e.g., multi-cell) scenarios.

A form of unifying  $K/K_n/M$  ( $M \leq K_n \leq K$ ) multiuser detection was also proposed under an AWGN channel, providing tradeoffs between performance and complexity with the settable parameter M; the implementation complexity of the multiuser detectors increases exponentially with M — on the order of  $2^M$  in synchronous settings and  $2^{3M}$  in asynchronous settings, but independent of  $K_n$  and K. In the special scenario where the number of known-parameter users is one, the proposed K/1/1 linear MMSE receiver reduces to a form of the enhanced linear single-user detectors [46], [47], [48]. In the particular case of a full information (e.g., single-cell) system setup under AWGN channel, the front end reduces to the conventional matched filter bank; the  $K_n/K_n/M$  MLS multiuser detection is an alternative implementation to [113]; the overall architecture for  $K_n/K_n/K_n$  MLS multiuser detection reduces to the one described in [36]; the architecture for  $K_n/K_n/K_n$  linear detection reduces to the linear MMSE [49] or decorrelating [50] detector.

We have also investigated the second-order statistical characterization of unknown out-of-cell interfering sources (that is associated with the detector's interference suppression capability) and the detection complexity with long-spread signals or short-spread signals. Finally, an adaptive implementation of the proposed multiuser detectors has been addressed, using LMS adaptive signal processing algorithm and assuming that an initial training sequence for each known user is provided. In such a case, the optimal tap coefficients in each front-end multiuser-oriented signal equalizer can be recursively approached and no matrix inversion is required.
# Chapter 4

# **Performance Analysis**

In this chapter, we study, by both analysis and simulation, the performance of the multiuser detectors developed in Chapter 3 for cellular CDMA networks; the evaluation criteria include minimum mean squared error, effective signal-to-noise ratio (SNR), probability bit error, asymptotic efficiency, and near-far resistance. We also justify the proposed multiuser detectors by comparing their performance with that of a few benchmark CDMA receivers, including conventional MF receiver, enhanced single-user-oriented linear detectors, the full-blown multiuser detectors and the single-user reception in an isolated single-user system. Numerical examples are carefully chosen to quantify the performance of the proposed multiuser detectors in different system setups, each trying to address one or two performance aspects.

Section 4.1 introduces the system settings and assumptions used in performance analysis and evaluation. The performance of the proposed linear multiuser-oriented signal equalization is studied in Section 4.2 with respect to minimum mean squared error and SNR; its goal is to examine the effectiveness of the front-end linear multiuseroriented signal equalization. Section 4.3 presents the performance analysis and evaluation on the multiuser detectors in terms of the probability of bit error for both AWGN and dispersive slowly Rayleigh-faded channels. Finally, Section 4.4 addresses the detection performance with respect to asymptotic efficiency and near-far resistance.

# 4.1 Analysis and Simulation Configurations

This section presents assumptions and configurations used for the performance evaluation in this chapter. The notation of the symbols used to describe the proposed multiuser detection is given below: K denotes the total number of active users in a multi-cell CDMA network;  $K_n$  ( $K_n \leq K$ ), the number of (in-cell) users whose parameter information is known at the receiver of a cell of interest and exploited for multiuser detection; M ( $M \leq K_n$ ), the number of (in-cell) users chosen for joint detection (M being a system parameter of the proposed multiuser detection to be studied as well); J, the number of resolved multipath components (another parameter of interest to be studied); N, the number of chips in each bit interval (i.e., spreading factor); and finally,  $\mu$ , a step size of the adaptive algorithm. Thus, the notation  $K/K_n/M$  can designate both the detection problem and the information structure that characterizes it.

# 4.1.1 Spreading sequences

Spreading sequences employed in the numerical examples for performance evaluation include Gold codes of length N = 7 given in Table 4.1 (also used in [113, 114]) and random sequence codes of length N = 31 described in Tables 4.2, 4.6 and 4.7, each with a fixed cross-correlation between paired sequences (The code cross-correlation is 0.2258 in Table 4.6, 0.3548 in Table 4.2, and 0.4839 in Table 4.7). A rectangular chip waveform of duration  $T_c$  is employed.

# 4.1.2 Channel models and system settings

Two types of channel models are used for performance evaluation in this chapter: AWGN channel and 3-path Rayleigh-faded channel. For the multipath fading model, it is assumed that the fading taps from the different paths of each user are i.i.d complex Gaussian with zero mean and roughly the same variance; the fading coefficients of interest can be resolved, tracked, and estimated (for instance, a pilot channel is provided for reverse link coherent demodulation and channel estimation, and one dedicated finger is assigned as a path search engine for Rake reception in 3G systems [3]), where schemes such as [53, 54] can be applied. Moreover, in our simulation study, the quasi-static frequency selective fading setup is assumed; that is, the complex fading coefficient in each path remain constant for a fixed number of symbols (i.e., within the channel coherent period) and is then changed to a new independent value. Our performance analysis is limited to synchronous CDMA settings as [50, 81, 124, 112] for convenient analysis.

# 4.1.3 Other assumptions

In our numerical examples, the  $K - K_n$  out-of-cell unknown sources represent a few interfering sources with significant signal powers, for example, the sources from the out-of-cell mobiles close to the boundary of the desired cell that have strong interference and cannot be ignored for the in-cell signal reception. For benchmark performance, the single-user performance in an isolated single-user system is provided; the performance with the parameter knowledge of all K users, corresponding to the full-blown K/K/K MLS detection, is also evaluated. In the study for near-far scenarios, different received powers or relative received amplitude/power distributions are always with respect to the received amplitude/power of user 1. In an adaptive implementation for the front-end linear multiuser signal equalization, an initial training sequence is assumed for each of the known users, such that the optimal filter impulse response can be recursively approached (thus obviating the need to estimate the covariance matrix of the received signal).

# 4.2 Performance of Linear Multiuser-oriented Signal Equalization

# 4.2.1 Normalized minimum mean square error (NMMSE)

In last chapter, we proposed a multiuser-oriented signal equalization strategy for front-end signal processing to extract desirable multiuser decision statistic and to suppress interference from unwanted sources, based on the linear minimum mean squared error (MMSE) criterion. To evaluate the effectiveness of the front-end equalization, we employ  $K/K_n/M$  ( $K \leq K_n \leq M$ ) detection architecture developed in Section 3.4, and present an analysis on the front-end output mean squared errors in this subsection. For the front-end linear optimal filtering associated with user j, the output MMSE between the desired multiuser signal aggregate and its estimate has the form (cf. Appendix D)

$$J_{min}^{(j)}(e) = \sigma_j^2 - \mathbf{p}_j^T \mathbf{w}_{jo}, \quad j = 1, 2, \dots, M,$$
(4.1)

where  $\mathbf{p}_j$  is the cross-correlation vector between the received signal vector and the target multiuser signal aggregate of the *j*th equalizer, and  $\mathbf{w}_{jo}$  is the optimal coefficients of the equalizer.  $\sigma_j^2$  is the variance of the desired multiuser signal aggregate associated with user *j*, given by

$$\sigma_{j}^{2} = E[d_{j}(i)d_{j}^{*}(i)]$$
  
=  $\sum_{k=1}^{M} P_{k}\rho_{jk}^{2}$   
=  $\mathbf{p}_{j}^{T}\mathbf{a}_{j} \quad j = 1, 2, \dots, M,$  (4.2)

in which \* denotes the complex conjugate,  $d_j(i) = \sum_{k=1}^M \sqrt{P_k} b_k(i) \rho_{jk}$ ,  $P_k$  is the received signal power of user k,  $\mathbf{a}_j$  is the spreading vector of user j, consisting of N spreading chips, and  $\rho_{jk} = \frac{1}{N} \mathbf{a}_k^T \mathbf{a}_j$  represents the correlation between the signature waveforms of users j and k.

From (4.1) and (4.2), we can write  $J_{min}^{(j)}(e)$  as

$$J_{min}^{(j)}(e) = \mathbf{p}_j^T \mathbf{a}_j - \mathbf{p}_j^T \mathbf{w}_{jo}$$
  
=  $\mathbf{p}_j^T \partial_{jo}, \quad j = 1, 2, \dots, M,$  (4.3)

where  $\partial_{jo} = \mathbf{a}_j - \mathbf{w}_{jo}$ . Thus, the optimum coefficient  $\mathbf{w}_{jo}$  can be decomposed into two components: the signature sequence vector  $\mathbf{a}_j$  and its deviation vector  $\partial_{jo}$ , as

$$\mathbf{w}_{jo} = \mathbf{a}_j - \partial_{jo}, \quad j = 1, 2, \dots, M.$$

$$(4.4)$$

The deviation vector  $\partial_{jo}$  is devised to offset the signature vector  $\mathbf{a}_j$  of user j (i.e., the conventional MF filter) in order to effectively estimate the desired multiuser signal aggregate and suppress the unwanted interfering sources. It is seen that the deviation vector becomes a zero vector in the absence of the K - M unwanted users; that is,  $\mathbf{w}_{jo} = \mathbf{a}_j$ , and the front-end linear multiuser signal equalizer reduces to conventional

MF filter. Finally, a normalized minimum mean squared error (NMMSE) is defined (such that the minimum mean squared errors with different detection group sizes M can be fairly compared) as

$$J_{Norm}^{(j)}(e) = 1 - \frac{\mathbf{p}_{j}^{T} \mathbf{w}_{jo}}{\mathbf{p}_{j}^{T} \mathbf{a}_{j}}, \quad j = 1, 2, \dots, M.$$
(4.5)

# 4.2.2 Effective signal to noise ratio

An alternative performance criterion to evaluate the front-end linear multiuser signal equalization is the effective SNR at the output of the *j*th equalizer for  $K/K_n/M$  detection, which can be defined with respect to the above NMMSE as

$$SNR_{e}^{(j)} = \frac{1 - J_{Norm}^{(j)}(e)}{J_{Norm}^{(j)}(e)} = \frac{\mathbf{p}_{j}^{T} \mathbf{w}_{jo}}{\mathbf{p}_{j}^{T} \partial_{jo}}, \quad j = 1, 2, \dots, M,$$
(4.6)

where  $\partial_{jo}$  is a difference vector between the enhanced signature vector and the original signature vector of user j. It is known that minimizing the mean squared error (MSE) also maximizes the desired signal to noise ratio (SNR) [139, 11].

# 4.2.3 Statistical characterization of residual interference

The statistical characterization of the residual interference at the output of each front-end multiuser signal equalizer can offer an important metric for evaluating the effectiveness of the linear equalization. One important characteristic of the autocorrelation matrix  $R_I$  that describes the maximum residual interference energies related to the minimum mean square errors  $\{J_{min}^{(j)}(e), j = 1, 2, ..., M\}$  can be expressed by the following proposition:

Proposition 3: The autocorrelation matrix  $R_I$  of the residual interference, under stationary channel conditions, can be described relative to the minimum mean square errors  $\{J_{min}^{(j)}(e), j = 1, 2, ..., M\}$  from the *M* linear MMSE equalizers as

$$\frac{tr[R_I]}{L} < \sum_{j=1}^M J_{min}^{(j)}(e), \tag{4.7}$$

assuming that the background noise power is nonzero, where  $tr[R_I]$  is the *trace* of  $R_I$  and L is the length of the user's transmission bits.

**Proof**: From definition (3.34), the vector notation of the desired multiuser signal aggregates desirable for *M*-user joint detection can be written as

$$\mathbf{d} = \mathcal{C}_L A_M \mathbf{b}_M,\tag{4.8}$$

where the bit vector  $\mathbf{b}_M = [b_1(1) \ b_2(1) \ \dots \ b_M(1) \ b_1(2) \ b_2(2) \ \dots \ b_M(2) \ \dots \ b_1(L)$  $b_2(L) \ \dots \ b_M(L)]^T$  and the amplitude vector  $A_M = \text{diag}\{A_M(1), \ A_M(2), \ \dots, A_M(L)\}, \ A_M(i) = \text{diag}\{\sqrt{P_1}, \ \sqrt{P_2}, \ \dots, \ \sqrt{P_M}\}.$   $\mathcal{C}_L$  is a symmetric  $ML \times ML$  matrix, defined as

$$C_{L} = \begin{bmatrix} \mathbf{C}(0) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{C}(0) & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}(0) & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{C}(0) & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{C}(0) \end{bmatrix}_{ML \times ML}$$
(4.9)

where  $\{\mathbf{C}(l)\}\$  are  $M \times M$  normalized signal cross-correlation matrices whose entries are given by

$$C_{jk}(l) = \int_{-\infty}^{\infty} s_j(t) s_k(t+lT_b) dt, \qquad j = 1, 2, \dots, M, \ k = 1, 2, \dots, M.$$
(4.10)

By applying the optimum coefficient vectors for each of the front-end linear multiuser equalizers, the output vector from the M linear optimal filters can be obtained and denoted in (3.40), and thus, the auto-correlation matrix of the output error sequence

vector can be written as

$$E[(\mathbf{d} - \hat{\mathbf{d}})(\mathbf{d} - \hat{\mathbf{d}})^T] = (\mathcal{C}_L - \mathcal{R}_L)A_M A_M^T (\mathcal{C}_L - \mathcal{R}_L)^T + R_I + \frac{N_0}{2}\mathcal{W}_L, \qquad (4.11)$$

where we have assumed that the transmitted bits are modeled as random variables that: (1) take on values from  $\{+1, -1\}$  with equal probabilities, (2) are uncorrelated for the same user, (3) are independent for different users, and (4) are independent of background noise. Then we have

$$tr\{E[(\mathbf{d} - \hat{\mathbf{d}})(\mathbf{d} - \hat{\mathbf{d}})^{T}]\} = tr[(\mathcal{C}_{L} - \mathcal{R}_{L})A_{M}A_{M}^{T}(\mathcal{C}_{L} - \mathcal{R}_{L})^{T}] + tr[R_{I}] + \frac{N_{0}}{2}tr[\mathcal{W}_{L}].$$
(4.12)

Under stationary channel conditions, the elements are periodic in M along the diagonal of each of the matrices in (4.12). Since  $tr[(\mathcal{C}_L - \mathcal{R}_L)A_M A_M^T (\mathcal{C}_L - \mathcal{R}_L)^T] \ge 0$ ,  $\frac{N_0}{2}tr[\mathcal{W}_L] > 0$ , and the left-hand side of Eq. (4.12) can be denoted as  $L \sum_{j=1}^M J_{min}^{(j)}(e)$ , we obtain the proposition.

The above proposition demonstrates that the total energy of the residual interference sequences from all M linear MMSE equalizers in each bit interval is upper bounded by the sum of the corresponding M minimum mean square errors. As a result, the MMSE at the output of each multiuser-oriented linear equalizer is a good metric in characterizing the front-end interference suppression capability.

Table 4.1Gold code of length 7.

seq. no.	spreading sequences
1	+1 -1 -1 +1 +1 +1 -1
2	+1 +1 -1 -1 -1 -1 -1 -1
3	-1 +1 -1 +1 +1 +1 -1
4	+1 -1 -1 -1 -1 +1 -1
5	-1 +1 +1 -1 -1 +1 +1

# 4.2.4 Numerical examples

For the proposed  $K/K_n/M$  detection, the front-end signal processing consists of a bank of M front-end linear multiuser-oriented signal equalizers, one for each of the

seq. no.	random spreading sequences
1	-1 -1 1 1 -1 -1 1 1 1 1 -1 -1 1 1 -1 -1
2	1 -1 1 1 1 -1 1 1 -1 -1 -1 -1 1 1 -1 1 1 1 1 1 1 -1 1 1 1 1 -1 1 1 1 1 1 1 1 1 1 1
3	1 -1 1 1 -1 1 1 1 1 1 -1 -1 1 -1 1 1 1
4	1 -1 -1 1 1 -1 1 1 1 1 -1 -1 -1 -1 -1 1 1 1 1 1 -1 1 1 1 -1 -
5	1 1 1 1 -1 -1 1 1 1 -1 -1 -1 -1 -1 -1 1 1 1 -1 1 1 1 -1 1 -1 1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 -1 1 1 -1 1 -1 1 1 -1 -

Table 4.2Random signature sequences of length 31 with a fixed cross-correlation value 0.3548 constructed by the random search.

M users. To evaluate the effectiveness of the linear signal equalization, we examine the minimum mean square error and effective SNR produced at the output of the filter for user j. It is assumed that the parameter knowledge of the  $K - K_n$  unknown sources is not available to the receiver, but their signal second-order statistic can be independently estimated, through pilot channels or training sequences (for example, each traffic frame of known users provides a training sequence in [4]). The procedure for obtaining the numerical results is described as follows:

- 1. Generate information bits of users: The transmitted information bits of active users were independently generated and the bits for the same user took the values of 0 or 1 with equal probability.
- 2. CDMA signal spread: The signals of users were BPSK-modulated  $(0 \rightarrow +1; 1 \rightarrow -1)$  and spread by PN sequences, given in Table 4.1 or Table 4.2, employing rectangular chip waveform.
- 3. Estimate correlation matrix: The  $N \times N$  correlation matrix of the unknown sources,  $\mathbf{S}_{j}^{U_{n}}$ , was estimated based on a moving time average over 200 bit intervals as

$$\hat{\mathbf{S}}_j^{U_n} = \frac{1}{200} \sum_{i=1}^{200} \mathbf{r}_{Un}(i) \mathbf{r}_{Un}(i)^T,$$

where  $\mathbf{r}_{Un}(i)$  is the sample vector of the received  $K - K_n$  unknown sources at the chip-matched filter output of user j at the *i*th bit interval. Since the signal

correlation matrix of  $K_n$  known sources,  $\mathbf{S}_j^{K_n}$ , is available at the receiver, the  $N \times N$  correlation matrix of all received signal aggregates,  $\mathbf{S}_j$ , can be estimated by  $\hat{\mathbf{S}}_j = \mathbf{S}_j^{K_n} + \hat{\mathbf{S}}_j^{U_n}$ .

4. Estimate optimal filters: The optimal coefficient vectors for the front-end linear *M*-user equalization were estimated by

$$\hat{\mathbf{w}}_{jo} = \hat{\mathbf{S}}_j^{-1} \mathbf{p}_j,$$

where  $\mathbf{p}_j$  is a known cross-correlation vector that can be represented as

$$\mathbf{p}_j = rac{1}{N} \sum_{k=1}^M P_k 
ho_{jk} \mathbf{a}_k$$

in which  $P_k$  and  $\mathbf{a}_k$  denote, respectively, the received signal power and the  $N \times 1$  signature vector of known user k, and  $\rho_{jk}$  is the correlation of signature waveforms between user j and user k.

5. **Performance evaluation:** The formulations of MMSE in (4.5) and effective SNR in (4.6) were applied.

**EXAMPLE** 1 — using Gold sequences of N = 7

In this example, we consider a synchronous CDMA system with a total of five active users, whose signals are spread by Gold sequences of N = 7 given in Table 4.1. Among these active users, four are in-cell users whose parameter knowledge is known to the receiver in a cell of interest. The fifth user is out-of-cell user and the base station has no knowledge of its parameters; however, its signal is relatively strong, and it can not be ignored by the base-station receiver for joint detection of the four known users.

We examine the normalized minimum mean square error at the output of the front-end linear optimal multiuser signal equalizer for user 1 as a function of detection group size M, where M is a value from one to four: M = 1 represents user 1 itself only; M = 2 represents the detection group of users  $\{1, 2\}$ ; M = 3 represents the detection group of users  $\{1, 2, 3\}$ ; and M = 4 represents the group of all active known

users  $\{1, 2, 3, 4\}$ . For a given M, the multiuser signal infrastructure of the M users is interested, while the K - M sources are considered unwanted signals that must be suppressed in the front-end signal processing.

**Table 4.3** Normalized minimum mean square errors at the outputs of linear MMSE equalizers for user 1 in a five-user cellular CDMA network: K = 5,  $K_n = 4$  and M = 1 - 4. Case I: Gold sequences of N = 7,  $E_b/N_0 = 14$  dB (user 1), equal received powers:  $P_2 = P_3 = P_4 = P_1$ ; Case II: Gold sequences of N = 7,  $E_b/N_0 = 14$  dB (user 1), non-equal powers:  $P_2 = 10P_1$ ,  $P_3 = P_4 = P_1$ .

	Normalized MMSEs of user 1			
Case	M=1	M=2	M=3	M=4
I	0.0899	0.0814	0.0514	0.0344
II	0.0907	0.0693	0.0448	0.0304

Table 4.3 shows the NMMSE results for two cases: A perfect power control is assumed in Case I; a non-equal power distribution (e.g., due to non-perfect power control) is studied in Case II. In both Case I and Case II, the normalized minimum mean square error of the linear equalizer designed for user 1 decreases as the detection group size M increases; this illustrates that a larger detection group is more favored for interference suppression and signal estimation in the front-end filtering. These observations can be intuitively explained by energy ratio of the desired multiuser signal aggregate over unwanted signal aggregate. In this example, the ratio of the number of desired multiuser signal (power) units to the number of unknown/unwanted signal units is 1:4 for Case I and 1:13 for Case II, when M = 1; such a ratio becomes 4:1 for Case I and 13:1 for Case II, when M = 4. It is important to notice that in the proposed multiuser-oriented signal equalization scheme, multiuser signals from the known users for joint detection are retained rather than suppressed in the front-end signal processing, while only the unknown/unwnated interfering sources are suppressed, thus able to incorporate all available user information into interference suppression and signal estimation (and joint detection as well in the second stage) to improve performance. However, the single-user-based signal equalization, in its original form, doesn't distinguish the unknown sources from the signals of the other known users (both being considered unwanted interference that will be suppressed) in the front end. Thus, the multiuser-oriented signal equalization technique proposed in this study is different from the single-user-based equalization scheme proposed in the literature with respect to processing methodology for interference suppression and signal detection; it opens up our mind for CDMA signal equalization and detection and provides a framework for effective multiuser detection with partial information.

It is also observed from the table that the NMMSE value of user 1 dramatically decreases as M increases from 1 to 2 in Case II, while the value decreases only slightly in Case I. The reason is that user 2 in Case II has a much stronger received signal power than user 1 and other users ( $P_2 = 10P_1$ ), and when detection group size M increases from one to two, the strong signal of user 2 changes from a "hostile" (to user 1), interfering source that must be suppressed to a "friendly", cooperating source that combines the signal of user 1 to suppress the other interfering sources. These phenomena demonstrate that a member with a weak received signal can benefit from members with strong received signals in the same detection group for interference suppression of the unwanted sources (it was also observed in [113]).

# **EXAMPLE** 2 — spread by random sequences of N = 31

In this example, we study the NMMSE of the front-end linear multiuser signal equalization when spreading sequences with a large cross-correlation are employed — that is, there is more MAI between users. The random sequences of N = 31 with an equal cross-correlation value of 0.3548 (given in Table 4.2) are employed in this example, different from the Gold sequences used in Example 1 that have a very small (excellent) cross-correlation. We examine the effects of the proposed linear signal equalization with different group sizes on performance.

We consider a five-user synchronous CDMA system in which four users are in-cell sources whose parameter knowledge is known to the receiver in a cell site of interest, while the fifth user is an out-of-cell unknown source whose signature sequence, timings and received power are not available to the cell site. The detection group size M is from one to four.

First of all, from Table 4.4, a performance similar to Example 1 can be observed: in both equal power and non-equal power cases, the normalized MMSE decreases as the grouping size M increases; the NMMSE in the power-imbalanced Case II

**Table 4.4** Normalized minimum mean square errors at the outputs of the front-end linear multiuser signal equalizers for user 1 in a five-user CDMA system: K = 5,  $K_n = 4$ , and M = 1 - 4. Case I: N = 31 with random spreading and cross-correlation of 0.3548,  $E_b/N_0 = 16$  dB (user 1), equal received powers; Case II: N = 31 with random spreading and cross-correlation of 0.3548,  $E_b/N_0 = 16$  dB (user 1), non-equal powers:  $P_2 = 10P_1$ ,  $P_k = P_1$ , k = 3, 4, 5, 6.

	Normalized MMSEs of user 1			
Case	M=1	M=2	M=3	M=4
I	0.0649	0.0572	0.0508	0.0440
II	0.0652	0.0286	0.0267	0.0241

is smaller than the perfect power control Case I for a given grouping size M > 1, indicating that a user with a weak received signal can benefit from the joint detection group, especially members with strong received powers. Note that our results do not suggest that the power control functionality can be removed from CDMA systems with MUD, since it is required to save the battery life of mobiles and achieve different grades of services (GoS); however, our results do illustrate that the constraining power control requirement in current CDMA systems can be relaxed if the interference suppression and multiuser detection techniques are employed in the receiver.

Secondly, we evaluate the enhanced single-user detection and the MF-filter detection with regard to SNR when M = 1. It can be readily estimated from Table 4.4 that the effective SNR of user 1 is  $SNR_e = 11.58$  dB in Case I, and  $SNR_e = 11.56$ dB in Case II. If the conventional MF receiver is employed for user 1 in such a scenario, it is estimated that the output signal-to-noise ratio is  $SNR_{MF} = 2.98$  dB in Case I, and  $SNR_{MF} = -2.14$  dB in Case II. As a result, the enhanced detection is near-far resistant; moreover, the detection with M = 1 gains more than 8 dB in Case I and more than 13 dB in Case II as compared to the conventional MF receiver in such scenarios.

Finally, compared to the performance bound of an isolated single-user system, i.e.,  $E_b/N_0 = 16$  dB, the enhanced (linear) single-user (M = 1) detection still incurs a performance loss of about 4.5 dB. In this thesis, we'll demonstrate that such a performance gap can be reduced using our proposed  $K/K_n/K_n$  linear and MLS detection schemes by fully exploiting all available knowledge of multiple known users (to be quantified in the following sections).

# 4.2.5 Mean square error in an adaptive implementation

Each front-end linear multiuser signal equalizer can be adaptively implemented when initial training sequences are provided for users in the joint detection group, in which the optimal coefficients of the linear equalizer are recursively approached (thus obviating the inversion of the correlation matrix of the receiver signal that the Wiener solution requires). Toward this goal, an adaptive least mean squared (LMS) algorithm in the family of stochastic gradient algorithms is applied in the following analysis to study the behaviour of adaptive convergence and the mean squared error in stable states.

We consider the K/M/M multiuser signal equalization and detection problem in slowly Rayleigh-faded multipath channel where K - M resources are unwanted signals to be suppressed, and J paths (or fingers) for each of the M known users can be tracked and resolved at the receiver.  $M_p = MJ$  front-end linear multiuser signal equalizers are employed to suppress the signal components from K - M unwanted users and estimate the desired multiuser signal aggregates  $\{d_j(l)\}$  defined in (3.47) (desirable for second-stage joint detection of the M users). Using the LMS algorithm, the optimal coefficients for the *j*th linear equalizer can be recursively approached by

$$\mathbf{w}_{j}^{(l+1)} = \mathbf{w}_{j}^{(l)} - \mu_{j} e_{j}(l) \mathbf{r}_{j}(l), \ j = 1, 2, \dots, M_{p}$$

where  $\mathbf{w}_{j}^{(l)}$  is a weight-vector in the *j*th equalizer at the *l*th bit iteration of coefficient adjustment,  $\mathbf{r}_{j}(l)$  is an  $N \times 1$  vector consisting of N output samples in the *l*th bit interval from the chip-matched filter,  $e_{j}(l) = (d_{j}(l) - \mathbf{w}_{jl}^{T}\mathbf{r}_{j}(l))$  is the estimation error at the *l*th bit interval, and  $\mu_{j}$  is a step size in the algorithm that satisfies the sufficient convergence condition

$$0 < \mu_j < \frac{2}{\sum_{k=1}^N \lambda_k^{(j)}}, \quad j = 1, 2, \dots, M_p, \tag{4.13}$$

where  $\{\lambda_k^{(j)}\}\$  is a set of eigenvalues associated with the correlation matrix  $\mathbf{S}_j = E[\mathbf{r}_j(l)\mathbf{r}_j(l)^T]$ .

The noisy estimates cause random fluctuations in the coefficients around the optimum Wiener solution  $\mathbf{w}_{jo}$  of the *j*th linear equalizer, leading to an increase in the mean square error(MSE) at the output of the equalizer. The MSE associated with the *j*th equalizer will converge to the final MSE  $J_F^{(j)}(e) = J_{min}^{(j)}(e) + J_{ex\infty}^{(j)}(e)$ , in which  $1 \leq j \leq M_p$ , and  $J_{ex\infty}^{(j)}(e)$  is the variance of the measurement noise, or excess mean-squared error. The excess mean-squared error  $J_{ex\infty}^{(j)}(e)$  as [63]

$$J_{ex\infty}^{(j)}(e) = J_{min}^{(j)}(e) \frac{\sum_{k=1}^{N} \mu_j \lambda_k^{(j)}}{2 - \sum_{k=1}^{N} \mu_j \lambda_k^{(j)}}, \ j = 1, 2, \dots, M_p,$$
(4.14)

and the final, stable-state, normalized MSE can be written as

$$J_{min0}^{(j)}(e) = \frac{J_F^{(j)}(e)}{\sigma_j^2}$$
  
=  $(1 - \frac{\mathbf{p}_j^T \mathbf{w}_{jo}}{\mathbf{a}_j^T \mathbf{p}_j})(1 + \frac{\sum_{k=1}^N \mu_j \lambda_k^{(j)}}{2 - \sum_{k=1}^N \mu_j \lambda_k^{(j)}}), \ j = 1, 2, \dots, M_p, \quad (4.15)$ 

where  $\sigma_j^2 = E[d_j(l)d_j^*(l)]$ ,  $\mathbf{a}_j$  is the signature vector of user j, and  $\mathbf{p}_j$  is the cross-correlation vector of the *j*th linear equalizer.

In the following example, we study the equalizer transient behaviour and its (normalized) MSE on an adaptive implementation of the front-end linear signal equalizers.

#### **EXAMPLE** 3 — Equalizer transient behavior and MSE

In this numerical example, the linear equalizer transient behavior and its MSE performance are examined. We consider a five-user CDMA system. Signals of active users are spread by a set of random sequences with a cross-correlation of 0.3548 (given in Table. 4.2); four of the five users are known in-cell synchronous sources; a three-path slow Rayleigh-fading channel model is assumed with the average energies among the three paths: 0.350, 0.335, and 0.315, and the path delays: 0,  $T_c$ ,  $2T_c$  (one chip interval difference). We also assume that all the three path components can be

tracked and resolved (i.e., J = 3).

Figures 4.1 and 4.2 show the transient and convergent behaviour of the three resolved paths in each of the fingers for user 1 on the basis of the LMS adaptive algorithm. A total of 600 bit iterations are employed for each user in the initial training with step size  $\mu = 0.1$ . Figure 4.1 shows single-user-oriented signal equalization whose target is the transmitted bits of user 1. Fig. 4.2 shows multiuser-oriented signal equalization whose target is the multi-component aggregate of all known users; that is,  $d_j(l)$  includes the signal correlation infrastructure of  $M_p$  (= 12) components from M = 4 users. It can be observed from the figures that the convergence rate in Fig. 4.2 is faster than the rate in Fig. 4.1; specifically, the three tracking fingers in Fig. 4.2 almost enter stable states after 300 bit iterations, whereas the three fingers in Fig. 4.1 do not converge to stable states until 400 bit iterations. This observation illustrates that the multiuser-oriented signal equalization performs better than the single-user-based signal equalization with respect to their dynamic transient behaviours for an adaptive implementation; the explanation is that the former can incorporate all known user information including the four training sequences into its signal equalization.

**Table 4.5** Simulated MSEs at the outputs of the proposed linear equalizers for user 1 under a 3-path Rayleigh-fading channel in a five-user CDMA system:  $E_b/N_0 = 20$  dB (user 1),  $P_k = 1, k = 1, 2, 3, ..., 5$ , N = 31, K = 5, M = 4, and step size  $\mu = 0.1$ .

path	M=1	M=4
#1	0.0997	0.0876
#2	0.0994	0.0883
#3	0.1121	0.1047

The MSEs of reception fingers in their stable states as shown in Figures 4.2 and 4.1 are tabulated in Table 4.3. The results demonstrate that the MSE in the stable states for each finger in the case where M = 4 is less than the MSE in the case where M = 1, meaning that the multiuser-oriented signal equalization is more effective in suppression of unwanted interfering sources. Moreover, we can estimate the performance of single-user detection (i.e., M = 1) after the maximal-ratio combining as



Fig. 4.1 Transient and convergent behaviors of the single-user-based signal equalizer of user 1 with a system setup: K = 5, M = 1,  $E_b/N_0 = 20$  dB (user 1), a three-path model, short random spreading of N = 31 and a cross-correlation of 0.3548, and a step size of  $\mu = 0.1$ .

follows: For finger 1 with M = 1, the normalized MMSE (normalized by its average path fading energy 0.350) is 0.0285, and thus  $SNR_{e1} = 3.995$  dB. Notice the fact that the first finger has the best output effective SNR among the three resolved paths of user 1. If the three paths are optimally combined, we have the RAKE output effective SNR < 8.77 dB. Note that for a simple estimation, we have assumed that the other two fingers have the same performance as the first one; thus RAKE output SNR is less than  $SNR_{e1} + 4.77$  (i.e.,  $10*\log 3$ ) dB. Compared to single-user performance limit of  $E_b/N_0 = 20$  dB, this example shows more than an 11-dB gap between the linear single-user-based detection and the performance bound. This leaves a large room (cf. Example 2 under AWGN channel) for possible performance enhancement in the multipath fading channels, motivating us to find other advanced MUD detection schemes in such a partial system parameter information setup; the proposed 5/4/4 linear and nonlinear multiuser detectors can provide one of the solutions achieving this goal.



Fig. 4.2 Transient and convergent behaviors of the multiuser-oriented signal equalizer of user 1 in a system setup: K = 5, M = 4,  $E_b/N_0 = 20$  dB (user 1), a three-path model, short random spreading of N = 31 and a cross-correlation of 0.3548, and a step size of  $\mu = 0.1$ .

# 4.3 Probability of Bit Error

The performance of the proposed maximum-likelihood sequence (MLS) and linear multiuser detectors is analyzed with respect to bit error probability in this section. Numerical examples are also provided to quantify the performance of the proposed multiuser detection under various system setups and channel conditions.

# 4.3.1 $K/K_n/M$ MLS detection performance under AWGN channel

In this subsection, we present an analysis of bit error probability for  $K/K_n/M$  MLS multiuser detection under synchronous CDMA AWGN channels. Since a closed-form expression for the bit error probability of the multiuser MLS detector is usually intractable, we seek upper and lower bounds on the performance measurement for the proposed detection. An upper bound of bit error probability is summarized in the following proposition:

Proposition 4: Given the decision metric described in (3.42), a  $K/K_n/M$  MLS multiuser detector is upper bounded on the probability of bit error of user j (1 <  $j \leq M$ ) by

$$p_j^{ML}(\sigma) \le \sum_{\epsilon \in F_j} 2^{-w(\epsilon)} Q\left(\sqrt{f(\epsilon)}\right),$$
(4.16)

where  $w(\epsilon)$  is the weight of the error vector  $\epsilon$  (see Definition 2 in Appendix C),  $\sigma^2 = \frac{N_0}{2}$ , and Q(\*) is the Q-function, defined as  $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ .  $F_j$  is the set of *indecomposable* error vectors associated with user j (see Definition 3 in Appendix C).  $f(\epsilon)$  can be written as

$$f(\epsilon) = \epsilon^T A_M^T \mathcal{R}_U A_M \epsilon, \qquad (4.17)$$

where  $\mathcal{R}_U$  is the correlation matrix and  $A_M$  is the received amplitude matrix of M known users.

**Proof:** From (3.42), the maximum likelihood function can be expressed as  $\Omega(\mathbf{b}_M) = 2\mathbf{b}_M^T A_M^T \mathbf{d} - \mathbf{b}_M^T A_M^T \mathcal{R}_U A_M \mathbf{b}_M$ , where  $\mathbf{d} = \mathcal{R}_L^T (R_I + \frac{N_0}{2} \mathcal{W}_L)^{-1} \mathbf{d}$ , and  $\mathcal{R}_U = \mathcal{R}_L^T (R_I + \frac{N_0}{2} \mathcal{W}_L)^{-1} \mathcal{R}_L$ . From [51, P.189], we have that the union bound of probability of bit error of user j for the MLS detection can be written by

$$p_j^{ML}(\sigma) \leq \sum_{\epsilon \in V_j} Pr[\epsilon \in \mathbf{A}(\mathbf{b}_M); \Omega(\mathbf{b}_M - 2\epsilon) \geq \Omega(\mathbf{b}_M)],$$

where  $V_j$  is the set of error vectors that affects user j and  $\mathbf{A}(\mathbf{b}_M)$  is the set of error vectors that are admissible with a given transmitted bit sequence  $\mathbf{b}_M$  (see Definition 1 in Appendix C). Since the event  $\{\epsilon \in \mathbf{A}(\mathbf{b}_M)\}$  depends only on the transmitted vector and all transmitted bits are assumed to be equiprobable and equally likely, we have [51, P.189]

$$Pr[\epsilon \in \mathbf{A}(\mathbf{b}_M)] = \prod_k^M Pr[(b_k - \epsilon_k)\epsilon_k = 0] = 2^{-w(\epsilon)}.$$

Moreover, because of symmetry, the event  $\Omega(\mathbf{b}_M - 2\epsilon) \geq \Omega(\mathbf{b}_M)$  is independent of

the transmitted vector  $\mathbf{b}_M$ , due to the fact that

$$\Omega(\mathbf{b}_{M} - 2\epsilon) - \Omega(\mathbf{b}_{M}) = -4\epsilon^{T}A_{M}^{T}\bar{\mathbf{d}} + 2\mathbf{b}_{M}^{T}A_{M}^{T}\mathcal{R}_{U}A_{M}\epsilon + 2\epsilon^{T}A_{M}^{T}\mathcal{R}_{U}A_{M}\mathbf{b}_{M}$$
$$-4\epsilon^{T}A_{M}^{T}\mathcal{R}_{U}A_{M}\epsilon$$
$$= -4\epsilon^{T}A_{M}^{T}\mathcal{R}_{L}^{T}(R_{I} + \frac{N_{0}}{2}\mathcal{W}_{L})^{-1}(\mathbf{d}_{I} + \mathbf{N})$$
$$-4\epsilon^{T}A_{M}^{T}\mathcal{R}_{U}A_{M}\epsilon, \qquad (4.18)$$

where we have used the equation (3.41) in denoting  $\hat{\mathbf{d}}$ . It was shown [140, 141] that the residual interference plus noise (i.e.,  $\mathbf{d}_I + \mathbf{N}$ ) at the output of the front-end linear MMSE equalization is approximately Gaussian; thus, the first term on the right-hand of Eq. (4.18) can be considered a Gaussian variable with zero mean and variance

$$Var(\mathbf{d}_I, \mathbf{N}) = 16\epsilon^T A_M^T \mathcal{R}_U A_M \epsilon, \qquad (4.19)$$

since  $R_I \stackrel{\Delta}{=} E[\mathbf{d}_I \mathbf{d}_I^T]$  (E[\*] being the expectation operator) and  $E[\mathbf{N}\mathbf{N}^T] = \frac{N_0}{2}\mathcal{W}_L$  (also, cf. (3.23)).

As a result, we conclude from (4.18) that

$$Pr[\Omega(\mathbf{b}_M - 2\epsilon) \ge \Omega(\mathbf{b}_M)] = Q\left(\sqrt{f(\epsilon)}\right),$$
(4.20)

where  $f(\epsilon)$  can be written by

$$f(\epsilon) = \epsilon^T A_M^T \mathcal{R}_U A_M \epsilon.$$

Notice that  $f(\epsilon)$  designates an SNR in our case that determines the detection performance. From above analysis, it is seen that the events  $\{\epsilon \in \mathbf{A}(\mathbf{b}_M)\}$  and  $\{\Omega(\mathbf{b}_M - 2\epsilon) \geq \Omega(\mathbf{b}_M)\}$  are independent. Thus, we have

$$p_{j}^{ML}(\sigma) \leq \sum_{\epsilon \in V_{j}} Pr[\epsilon \in \mathbf{A}(\mathbf{b}_{M})] Pr[\Omega(\mathbf{b}_{M} - 2\epsilon) \geq \Omega(\mathbf{b}_{M})]$$
$$= \sum_{\epsilon \in V_{j}} 2^{-w(\epsilon)} Pr[\Omega(\mathbf{b}_{M} - 2\epsilon) \geq \Omega(\mathbf{b}_{M})].$$
(4.21)

Moreover, by removing all the (redundant) decomposable error vectors from  $V_j$ 

[51, P.190] or using the indecomposable set  $F_j$  associated with  $V_j$ , (4.21) is tightly bounded and can be expressed as

$$p_j^{ML}(\sigma) \le \sum_{\epsilon \in F_j} 2^{-w(\epsilon)} Q\left(\sqrt{f(\epsilon)}\right).$$

So, Proposition 4 is derived.

Notice that in our case, the third condition (3) in Definition 3 of Appendix C that is used to identify the decomposable error vectors (such that the indecomposable set of error vectors  $F_j$  can be derived from the set of error vectors  $V_j$ ) needs to be replaced by  $(\epsilon')^T X \epsilon'' \geq 0$ , where  $X = A_M^T \mathcal{R}_U A_M$ . Using the indecomposable set  $F_j$ , the upper bound of probability of bit error (4.16) can provide reasonably accurate worst-case results in AWGN channel if  $f(\epsilon)$  is high [51]. The lower bound of probability of bit error is described by the following proposition.

Proposition 5: Given the decision metric described in (3.42), a  $K/K_n/M$  MLS multiuser detector is lower bounded on the probability of bit error of user j  $(1 < j \le M)$  by

$$p_j^{ML}(\sigma) \ge 2^{1-w_{j,min}} Q\left(\sqrt{f_{j,min}}\right),\tag{4.22}$$

where

$$f_{j,min} = \min_{\epsilon \in F_j} \{ \epsilon^T A_M^T \mathcal{R}_U A_M \epsilon \},\$$

 $w_{j,min}$  is the minimum weight of the error vector(s) in  $F_j$  that achieves  $f_{j,min}$ , and  $\sigma^2 = \frac{N_0}{2}$ .

**Proof**: We derive the lower bound of bit error probability by detecting the multiuser signals with the side information. With the help of a genie, a detection bit error for user j will occur if and only if the transmitted  $\mathbf{b}_M$  and the error sequences affecting user j are such that either of the following events occur [51, P.195] :

$$\{\epsilon \in \mathbf{A}(\mathbf{b}_M)\} \bigcap \{\Omega(\mathbf{b}_M - 2\epsilon) \ge \Omega(\mathbf{b}_M)\};$$

$$\{\epsilon \in \mathbf{A}(-\mathbf{b}_M)\} \bigcap \{\Omega(\mathbf{b}_M + 2\epsilon) \ge \Omega(\mathbf{b}_M)\}$$

The two events are nonoverlapping, since  $\{\mathbf{A}(\mathbf{b}_M)\}\$  and  $\{\mathbf{A}(-\mathbf{b}_M)\}\$  are disjoint; moreover, these events have identical probabilities. It has been demonstrated in the proof of Proposition 4 that the events  $\{\epsilon \in \mathbf{A}(\mathbf{b}_M)\}\$  and  $\{\Omega(\mathbf{b}_M - 2\epsilon) \geq \Omega(\mathbf{b}_M)\}\$  are independent, and so are the events  $\{\epsilon \in \mathbf{A}(-\mathbf{b}_M)\}\$  and  $\{\Omega(\mathbf{b}_M + 2\epsilon) \geq \Omega(\mathbf{b}_M)\}\$ . By choosing error vectors  $\hat{\epsilon}$  and  $-\hat{\epsilon}$  (one for each admissible set) from  $F_j$  that achieve the smallest energy  $f(\hat{\epsilon})$ , denoted by  $f_{j,min}$ , and noting the fact that  $f(\hat{\epsilon}) = f(-\hat{\epsilon})$ , the lower bound of bit error probability can be expressed as

$$p_{j}^{ML}(\sigma) \geq Pr[\hat{\epsilon} \in \mathbf{A}(\mathbf{b}_{M})]Pr[\Omega(\mathbf{b}_{M} - 2\hat{\epsilon}) \geq \Omega(\mathbf{b}_{M})] + Pr[\hat{\epsilon} \in \mathbf{A}(-\mathbf{b}_{M})]Pr[\Omega(\mathbf{b}_{M} + 2\hat{\epsilon}) \geq \Omega(\mathbf{b}_{M})] = 2^{1-w_{j,min}}Q\left(\sqrt{f_{j,min}}\right).$$

Thus, Proposition 5 is derived.

Notice that for the full-blown K/K/K MLS detection where only the additive white Gaussian noise is present, the lower bound (4.22) is tight and close to the upper bound (4.16) in high (enough) SNR region [51]. When  $\sigma \to 0$ , the additive terms in the right-hand side of (4.16) will be dominated by  $Q(\sqrt{f_{j,min}})$  terms, since  $Q(\mathbf{x})$  is a monotonous decrease function in the range  $x \in [0, +\infty]$ . However, the lower bound (4.22) can not always provide tight results for  $K/K_n/M$  MLS multiuser detection with partial information (i.e.,  $K_n < K$ ), since there exist both the additive white Gaussian noise and the residual interference in such detection scenarios. In some cases in which the variance of the residual interference may be relatively large, there could be other error sequences (than  $\hat{\epsilon}$  and  $-\hat{\epsilon}$ ) such that the resulting items  $\{2^{-w(\epsilon)}Q(\sqrt{f(\epsilon)})\}$  can not be neglected as compare with the items  $2^{1-w_{j,min}}Q(\sqrt{f_{j,min}})$ , even if  $\sigma \to 0$ ; in other words, the lower bound (4.22) may provide a very loose performance. Observe that the performance of  $K/K_n/M$  MLS detection will be lower-bounded by the lower-bound performance of the full-blown K/K/K MLS detection.

seq. no.	random spreading sequences
1	-1 1 1 1 -1 1 1 1 -1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 -
2	-1 1 1 -1 1 1 -1 1 -1 1 1 -1 -1 -1 1 1 1 1 1 -1 -
3	-1 -1 1 1 1 -1 -1 1 1 1 1 -1 -1 -1 -1 -1
4	-1 -1 1 1 -1 1 -1 1 -1 -1 -1 1 -1 -1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 -1 1 -1 1 -1
5	-1 1 -1 1 1 1 -1 1 1 -1 -1 -1 -1 -1 1 1 1 1 1 1 1 1 1 1 1 -1 -
6	1 1 -1 1 -1 -1 -1 -1 -1 -1 1 -1 1 -1 1 1 1 1 1 1 -1 1 -1 1 1 1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1
7	-1 1 1 1 -1 -1 1 1 1 1 1 1 1 -1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 1
8	-1 -1 -1 1 1 1 1 1 -1 -1 1 1 -1 1 1 1 1

**Table 4.6**Random signature sequences of length 31 with a fixed cross-<br/>correlation value 0.2258 constructed by random search.

Table 4.7Random signature sequences of length 31 with a fixed cross-correlation value 0.4839 constructed by random search.

seq. no.	random spreading sequences
1	1 -1 -1 -1 -1 1 1 1 -1 1 1 1 1 1 1 1 1
2	1 -1 -1 -1 -1 1 1 1 -1 1 1 -1 1 1 1 1 1
3	1 1 -1 -1 -1 1 1 1 -1 1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 -
4	1 -1 -1 -1 1 1 1 1 -1 1 1 -1 1 1 1 1 -1 -
5	1 -1 1 -1 -1 1 1 1 -1 1 -1 -1 -1 -1 -1 1 1 -1 -
6	1 -1 -1 1 1 1 1 1 1 1 1 1 -1 -1 -1 -1 1 1 1 -1 -

# 4.3.2 Numerical results

For  $K/K_n/M$  detection, the outputs from a bank of M front-end linear multiuseroriented signal equalizers provide a decision statistic that can be used for joint detection of the M users in the second-stage processing; we present several numerical examples in this subsection to quantify the detection performance with regard to bit error probability in AWGN channels. The numerical results are obtained by the following procedure:

1. Generate user information bits: As described in Section 4.2.4.

- CDMA signal spread: The signals of the users are BPSK-modulated (0 → +1; 1 → -1) and spread by PN sequences with rectangular chip waveforms where the random signature sequences have a spreading gain of 31 and a fixed cross-correlation, as given in Table 4.6 or 4.7.
- 3. Estimate correlation matrix: As described in Section 4.2.4.
- 4. Estimate optimal filters: As described in Section 4.2.4.
- 5. **Residual matrix:** The correlation matrix of the residual interference plus noise is estimated at the output of the front-end linear equalizer bank by the time average over a 200-bit interval as

$$R_{I} + \frac{N_{0}}{2} \mathcal{W}_{L} = \frac{1}{200} \sum_{i=1}^{200} \hat{\mathbf{d}}(i) \hat{\mathbf{d}}^{\mathrm{T}}(i) - R_{L} A_{M} A_{M}^{\mathrm{T}} R_{L}^{\mathrm{T}},$$

where  $A_M$  is the amplitude diagonal matrix and  $R_L$  is the signal correlation matrix of the known users.

6. Performance evaluation: For MLS multiuser detection in Examples 4 - 7, the upper bound of bit error probability (4.16) was employed to evaluate the performance of user 1 for the proposed  $K/K_n/M$  (M > 1) joint detection like [116], since it can provide tightly-bounded results; for linear detection, the  $K/K_n/K_n$  linear MMSE and decorrelating detectors developed in Section 3.3 are studied, and the probability of bit error is evaluated by the estimated effective SNR.

**EXAMPLE** 4 —  $K/K_n/K_n$  MLS and linear multiuser detectors

In this example, we evaluate the performance of the proposed  $K/K_n/K_n$  MLS detector and  $K/K_n/K_n$  linear MMSE and decorrelating detectors in terms of the probability of bit error, and compare them with conventional MF filter, the fullblown K/K/K MLS detector and the receiver in an isolated single-user channel. We consider a synchronous cellular CDMA system with six active users, in which the signals of the active users are spread by random spreading sequences with a fixed



Fig. 4.3 Bit error probabilities of user 1 versus  $E_b/N_0$  for conventional MF receiver, 6/6/6 optimum, 6/5/5 MLS multiuser detection, and 6/5/5 linear detectors in a six-user CDMA system with random spreading of fixed cross-correlation value 0.2258, spreading factor N = 31 and perfect power control.

cross-correlation of 0.2258 (given in Table 4.6) and a spreading factor of N = 31. We assume that five out of the six users are in-cell users whose parameter knowledge is known to the receiver at a cell site of interest, and the sixth user is unknown (emulating a scenario when an interfering source with a strong signal is close to the edge of the cell of interest). Equal received powers are also assumed. For the proposed 6/5/5 detection schemes, a bank of five front-end linear multiuser-oriented signal equalizers is used to suppress the interference from the sixth unknown source, while a decision statistic including the multiuser information of the five known users is produced; the second-stage signal processing applies an MLS decision algorithm, a linear MMSE or linear decorrelating transformation to the decision statistic.

Fig. 4.3 shows that the conventional MF receiver experiences a severe performance degradation, and its performance only improves slightly as the SNR increases. However, the performance can be significantly improved by either a 6/5/5 linear MMSE (denoted by the dashed -x- line) or 6/5/5 linear decorrelating (denoted by the dotted -\*- line) detector; moreover, the MMSE linear detection outperforms the



Fig. 4.4 Bit error probabilities of user 1 versus  $E_b/N_0$  for conventional MF receiver, 6/6/6 optimum, 6/5/5 and 6/5/4 MLS multiuser detectors, and 6/5/5 linear MMSE detector in a six-user CDMA system with random spreading of a fixed cross-correlation value 0.4839, spreading factor N = 31 and perfect power control.

decorrelating linear detection, and the performance of the two linear detectors converges as SNR increases. It is shown in the figure that the proposed 6/5/5 linear detectors still incur a 2.3-dB loss at BER of  $10^{-4}$ , as compared with the single-user performance bound (denoted by the solid line). The performance can be further improved when an MLS decision algorithm replaces a linear transformation in the second-stage signal processing; specifically, the proposed 6/5/5 MLS multiuser detector gains additional 1.8 - 2 dB at BER of  $10^{-4}$  over the linear detectors, driving the detection performance very close (i.e., about 0.3-dB) to the full-blown 6/6/6 MLS detection or single-user performance bound.

**EXAMPLE** 5 — Performance/cost trade-off and effect of code cross-correlation

In this example, we examine the performance of the proposed  $K/K_n/M$  MLS detection and the  $K/K_n/K_n$  linear MMSE detection using PN code with a large cross-correlation. As in the last example, we consider a synchronous cellular CDMA

system with six active users, except that the user signals are spread by PN code with a cross-correlation of 0.4839 described in Table 4.7. The large code crosscorrelation means high multiple access interference among the users. In such a setup, we quantify the 6/5/5 MLS detector and 6/5/4 MLS detector to examine a trade-off between performance and implementation complexity. The performance of the proposed linear and MLS multiuser detectors is also compared to that of the conventional MF receiver and the optimum (full-blown) 6/6/6 MLS detector, and single-user bound.

Fig. 4.4 shows that the performance of the MF filter does not improve as  $E_b/N_0$ increases, implying that the interference dominates the performance all the time. The performance can be significantly improved when the 6/5/5 linear MMSE detector is employed instead of MF reception; the proposed 6/5/5 multiuser MLS detector can further gain about 2.1 dB at BER of  $10^{-4}$  over the 6/5/5 linear detection. Moreover, We evaluate the proposed 6/5/4 MLS detection to illustrate a trade-off between performance and cost with respect to the 6/5/5 MLS detection; specifically, the 6/5/4 MLS detection trades a 0.9-dB performance for the implementation complexity of  $O(2^4)$  from  $O(2^5)$ . If a network design can provide the receiver with the parameter knowledge of the out-of-cell source such that the full-blown 6/6/6 MLS detection is performed, an additional 1-dB gain can be obtained over the 6/5/5 MLS detector and single-user bound at BER of  $10^{-4}$ , while the gap decreases as  $E_b/N_0$  increases.

To quantify the effect of PN code with increased cross-correlation on performance, we can compare the relevant results in Fig. 4.4 and in Fig. 4.3; specifically, there is approximately a 1.5-dB performance loss at BER of  $10^{-4}$  for the linear MMSE detection due to an increase in the code cross-correlation from 0.2258 to 0.4839, and a 1.1-dB performance loss for the 6/5/5 MLS detection; this degradation is relatively small, compared to the doubling (i.e., a 3-dB increase) of the code correlation.

**EXAMPLE** 6 — effect of unknown short-spread signals on performance

In this example, we consider an eight-user short-spread synchronous CDMA system in which active users employ random spreading sequences with a fixed crosscorrelation of 0.2258 and a spreading factor of N = 31. We examine the effect of



Fig. 4.5 Bit error probabilities of user 1 versus  $E_b/N_0$  for six-user MLS detection in 8/8/6, 8/7/6, and 8/6/6 detection and information scenarios in an eight-user short spread CDMA system with random spreading of fixed cross-correlation value 0.2258, spreading factor N = 31 and an equal received power is used in the simulation.

unknown interfering sources on performance of the six-user MLS detection as the number of the unknown users varies.

The effect of the number of out-of-cell interfering sources (or the total unknown interfering power level) on the six-user joint detection performance is plotted in Fig. 4.5, where the number of unknown users varies from 0 to 2. The figure illustrates that the performance of the six-user MLS detection degrades very slowly as the number of unknown sources grows from zero to two in a short-spread CDMA system. Specifically, there is a 0.45-dB loss in performance from 8/8/6 MLS detection to 8/7/6 MLS detection at BER of  $10^{-4}$ , and a 0.55-dB performance loss from 8/7/6 MLS detection to 8/6/6 MLS detection. Such an observation demonstrates that the six-user MLS detection has a good capability to cancel the unknown short-spread signals as long as their second-order statistic can be precisely estimated; this can be attributed to the multiuser-oriented signal equalization scheme and the excellent signal characterization of short spread CDMA signals, which is cyclostationary with a period of one bit interval (cf. Section 2.2) and suitable for per-bit based signal



Fig. 4.6 Bit error probabilities of user 1 versus  $E_b/N_0$  for six-user MLS detection in 8/8/6, 8/7/6, and 8/6/6 detection and information scenarios in an eight-user CDMA system with spreading factor of N = 31. User signals are long spread by random sequences with cross-correlation between 0.2258 and 0.097. Equal received power is used in the simulation.

estimation and equalization.

**EXAMPLE** 7 — effect of unknown long-spread signals on performance

In this example, we consider the same setting as the previous example, except that the user signals are long spread by random sequences with cross-correlations between 0.2258 and 0.097. We will examine the effect of long spread CDMA signals and unknown interfering sources on performance for the proposed 8/8/6, 8/7/6, and 8/6/6 MLS detectors.

It is obvious from Fig. 4.6 that the performance quickly degrades as the number of unknown interfering long spread signals increases (as compared to Fig. 4.5); a substantial performance loss was observed from 8/8/6 detection to 8/6/6 detection. Specifically, there was a 1.2-dB loss in performance from 8/8/6 MLS detection to 8/7/6 MLS detection at BER of  $10^{-4}$ , and a 1.8-dB performance loss from 8/7/6 MLS detection to 8/6/6 MLS detection. Such observations imply that the suppression of unknown long spread interfering source(s) in the front-end linear signal equalization is not efficient even if the interference second-order statistic can be precisely measured. The primary reason is that the correlation period for long spread signals is very short — on the order of one chip period; in fact, the correlation matrix of the unknown long spread signals is a band matrix with upper and lower bandwidth of 1 (cf. Section 3.7 for analysis details) because the signature waveform for each user changes on a bit-by-bit basis. As a result, a noise floor is expected to appear for the multiuser detection as more unknown interfering sources with long spread signals are seen at the base-station receiver.



Fig. 4.7 Average bit error probabilities of user 1 versus SNR for conventional MF receiver, three-user MF-based detectors (linear MMSE detector, and MLS detector), single-user-based 4/1/1 linear MMSE detector and three-user equalization-based 4/3/3 detectors (liner MMSE detector and MLS detector) in a four-user synchronous CDMA system with power distribution  $[1,4,4,1]P_1$  over the four active users.

#### **EXAMPLE** 8 — simulated performance for different CDMA receivers

We have evaluated the bounded performance of the proposed multiuser detectors in the previous examples. In this example, we study the performance of several CDMA receivers — by pure simulation; that is, we employ their decision metrics instead of their performance formulation. The simulation results can provide an average detection performance for each of the receivers in given system configurations.

We consider a four-user CDMA system under an AWGN channel with a simulation setup similar to [113]: The first three users are known-parameter users, while the fourth user is an unknown interfering source. Signals of the active users are spread by Gold sequences of N = 7. The received powers are set for a near-far scenario with a power distribution (relevant to user 1) over the four active users as  $[1,4,4,1]P_1$ . To illustrate the effect of unknown interference in such a setting, we simulate the proposed three-user equalization-based MLS and linear MMSE detectors, three-user MF-based MLS and linear MMSE detectors, and two other well-known receivers: the enhanced single-user detectors [46] [48] and the conventional MF receiver. A benchmark performance in an isolated single-user communication channel is also provided.

Fig. 4.7 shows the average bit error probabilities of user 1 versus the SNR for conventional MF receiver, three-user MF-based 4/3/3 detectors (linear MMSE detector and MLS detector), enhanced single-user-based 4/1/1 linear MMSE detector, and the proposed 4/3/3 detectors (linear MMSE detector and MLS detector). The three-user MF-based 4/3/3 detectors, neglecting the fourth interfering source, consist of a bank of three front-end MF filters, and a linear MMSE transformation or MLS decision algorithm in the second stage. The proposed 4/3/3 detectors exploit the full knowledge of users to suppress interference of the fourth known user; they consist of a bank of three multiuser-oriented signal equalizers, followed by either a linear MMSE transformation or an MLS decision algorithm. The single-user-based 4/1/1 linear MMSE detector suppress the interference with the parameter knowledge of only one user, though information about all three users is available at the base-station receiver.

It is evident from Fig. 4.7 that if there is no any interference cancellation technique employed for the detector design, MF-based receivers perform poorly and show interference-limited performance floors; it is interesting to observe that when the MLS multiuser detection (denoted by solid -\*- line) or linear MMSE detection (denoted by dashed -\*- line) is applied to the output sequences from the three MF filters of the known users, their performance is only equivalent to that of the MF receiver (denoted by dashed  $-\Delta$ - line), implying that either the output sequences from MF filters cannot provide a reliable decision statistic for multiuser detection, or relative strong interference from unknown signal source(s) cannot be neglected for effective joint multiuser detection. The enhanced single-user-based 4/1/1 linear MMSE detection (denoted by dashed  $\rightarrow$  line) can greatly improve the system performance in terms of bit error probability, assuming that only parameter knowledge of the desired user is used. Since the receiver has the parameter knowledge of the three known users, it can be fully exploited in the signal equalization and detection to improve performance (one motivation behind this thesis research). The proposed 4/3/3 linear MMSE detector (denoted by dashed -o- line) demonstrated a 0.9dB performance gain at BER of  $10^{-3}$  over the 4/1/1 detector. However, it is shown in Fig 4.7 that the proposed linear MMSE detection still incurs a more than 3-dB loss as compared to the single-user performance bound (denoted by the dashed line) at BER of  $10^{-3}$ . Thus, one question naturally arises: Is there any further performance enhancement (another motivation behind this thesis research)? The answer to this question is yes, namely, the proposed 4/3/3 MLS detection (denoted by solid -oline); specifically, the nonlinear multiuser detection achieves an additional 1.3-dB gain over the 4/3/3 linear MMSE detection at BER of  $10^{-3}$ , and the price of the gain is an increased implementation complexity in the second-stage signal processing — on the order of  $2^3$  in such a setting.

# 4.3.3 Multiuser detection performance under multipath fading channels

In this subsection, we analyze the performance of the proposed MLS and linear multiuser detectors developed in Sections 3.5 and 3.6 under synchronous system settings. It is assumed that there are K active users in the system, and each user has 3 Rayleigh-faded path components whose delay spread is much smaller than one bit interval; there are  $M (\leq K)$  known-parameter users with a total of  $M_p = MJ$  path components that can be resolved for MUD at the base-station receiver. With a synchronous setup,  $\mathbf{R}(1)$  and  $\mathbf{R}(-1)$  in (3.64) and  $\mathbf{W}(1)$  in (3.68) reduce to  $\mathbf{R}(1) = \mathbf{R}(-1) \approx \mathbf{0}$  and  $\mathbf{W}(1) \approx \mathbf{0}$ . The correlation matrix of the residual interference  $R_I$  reduces to a block diagonal matrix. For convenient analysis, we use the same notations as in Section 3.5, simplified here to one-shot *M*-user signal equalization and joint detection.

We derive an upper-bound on the probability of bit error for the proposed K/M/MMLS detection under the dispersive channel conditions. Similar to the proof for Proposition 4 in Section 4.3.1 (also, cf. [45]), it can be shown that the upper bound on the probability of error of user j, conditioned on the fading channel matrix C, is given by

$$p_j(K, M, J \mid \mathcal{C}) \le \sum_{\mathbf{e} \in \mathcal{E}_j} 2^{-w_M(\mathbf{e})} Q(\sqrt{\mathbf{e}^T A_M^T \mathcal{R}_D A_M \mathbf{e}}),$$
(4.23)

where Q(\*) is the Q-function, defined as  $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ .  $\mathcal{E}_j = \{\mathbf{e} \in \{-1, 0, 1\}^{M_p}\}$  (with a dimension of  $M_p = JM$ ) is the indecomposable set of the error vectors resulting from M sources, and each user is associated with J components in  $\mathcal{E}_j$  that take the same value (i.e., the same error pattern). Thus,  $w_M(\mathbf{e})$  now designates the total number of error patterns in the error sequence  $\mathbf{e}$  or the number of users that are affected by the error sequence in such a scenario (also, cf., [51, 45, 149]).  $\mathcal{R}_D$  and  $A_M$  are, respectively, the correlation matrix and received amplitude matrix (c.f., Section 3.5.3 for the definitions). Let us define

$$X = \mathbf{e}^{T} A_{M}^{T} \mathcal{R}_{D} A_{M} \mathbf{e}$$
  
=  $\mathbf{c}^{H} \mathcal{D}^{T} A_{M}^{T} \mathcal{R}^{T} (R_{I} + N_{0} \mathcal{W})^{-1} \mathcal{R} A_{M} \mathcal{D} \mathbf{c}$   
=  $\mathbf{c}^{H} \mathcal{H} \mathbf{c},$  (4.24)

where **c** and  $\mathcal{D}$  are defined as the  $M_p$ -dimensional vector  $\mathbf{c} = [\alpha_1(0) \ \alpha_2(0) \ \dots \ \alpha_{M_p}(0)]^T$ and the diagonal matrix  $\mathcal{D} = \text{diag}\{e_1 \ e_2 \ \dots \ e_{M_p}\}$ , respectively, and  $\mathcal{H} = \mathcal{D}^T A_M^T \mathcal{R}^T (R_I + \mathcal{D}^$   $N_0 \mathcal{W})^{-1} \mathcal{R} A_M \mathcal{D}$ . The characteristic function of X is given as [150]

$$\psi_X(jv) = \prod_{i=1}^{w_M(\mathbf{e})J} \frac{1}{1 - jv\lambda_i}$$
$$= \sum_{i=1}^{w_M(\mathbf{e})J} \frac{\pi_i}{1 - jv\lambda_i}, \qquad (4.25)$$

where  $\{\lambda_i\}$  are the nonzero eigenvalues (assumed to be distinct) of the matrix  $\mathcal{KH}$ , with  $\mathcal{K}$  being the covariance matrix of the complex-valued Gaussian random vector **c**. The coefficients of the partial fraction expansion  $\{\pi_i\}$  can be expressed as  $\pi_i = \prod_{l=1, l \neq i}^{w_M(\mathbf{e})J} \frac{\lambda_i}{\lambda_i - \lambda_l}$ . Thus, the unconditional bit error probability for multiuser MLS detection in such a situation is obtained by averaging over the channel state information as

$$p_{j}(K, M, J) \leq \sum_{\mathbf{e}\in\mathcal{E}_{j}} 2^{-w(\mathbf{e})} \int_{0}^{\infty} Q(\sqrt{x}) f_{X}(x) dx$$
$$= \sum_{\mathbf{e}\in\mathcal{E}_{j}} 2^{-w_{M}(\mathbf{e})} \sum_{i=1}^{w_{M}(\mathbf{e})J} \frac{\pi_{i}}{2} \left[ 1 - \sqrt{\frac{\lambda_{i}}{2 + \lambda_{i}}} \right]$$
(4.26)

where the power density function of the chi-square-distributed random variable X is  $f_X(x) = \sum_{i=1}^{w_M(\mathbf{e})J} \frac{\pi_i}{\lambda_i} e^{-x/\lambda_i}$ . Notice that  $\lambda_i$  can be considered the effective average signal-to-interference (SIR) ratio for the *i*th component. In the particular case when M = K and with a single-path channel model, it coincides with the upper bound proposed in [45].

It is worthwhile to justify our analysis in a particular scenario: an isolated singleuser CDMA system under a *J*-path Rayleigh-faded channel model; the proposed front-end linear equalizer bank reduces to a bank of *J* MF filters. Thus,  $R_I = 0$ , we have a total of two error vectors (i.e.,  $w_M(\mathbf{e}) = 1$  with  $\mathbf{e}$  and  $-\mathbf{e}$ ), and  $\mathcal{H}$  reduces to  $\mathcal{H} = A_M^T \mathcal{R} A_M / N_0$ . It is further assumed that the user's signature waveform is ideal in that its time-shifted versions are orthogonal; then,  $\mathcal{R}$  reduces to a diagonal matrix, and the bound described in (4.26) reduces to the well-known single-user RAKE bound given in [6], denoted by  $\bar{p}_1$  as

$$\bar{p}_1 = \sum_{i=1}^J \frac{\pi_i}{2} \left[ 1 - \sqrt{\frac{\lambda_i^{(0)}}{2 + \lambda_i^{(0)}}} \right], \qquad (4.27)$$

where  $\lambda_i^{(0)} = \frac{P_1}{N_0} E[|\alpha_i(0)|^2]$ . The performance of the proposed K/M/M MLS detection under multipath slowly Rayleigh-faded channels is lower bounded by (4.27).

For the proposed K/M/M linear multiuser detection described in Section 3.6, the second-stage signal processing amounts to separating the signal components of the M known users, and optimally combining the signal replicas of the same user on the basis of outputs from the front-end  $M_p$  linear multiuser-oriented signal equalizers. Thus, an analysis of the detection performance under Rayleigh fading multipath channels is similar to [143]. The unconditional bit error probability upper bound turns out to be

$$p_1(J) = \sum_{i=1}^{J} \frac{\bar{\pi}_i}{2} \left[ 1 - \sqrt{\frac{\lambda_i}{2 + \lambda_i}} \right],$$
 (4.28)

where  $\{\lambda_i\}$  are the nonzero eigenvalues (assumed to be distinct) of the matrix  $\mathcal{KH}$ , with  $\mathcal{K}$  being the covariance matrix of the complex-valued Gaussian random vector  $\mathbf{c}_1$ . The coefficients of the partial fraction expansion  $\{\bar{\pi}_i\}$  can be expressed as  $\bar{\pi}_i = \prod_{l=1, l \neq i}^J \frac{\lambda_i}{\lambda_i - \lambda_l}$ . In the special case where M = K, the performance formulation in (4.28) for the K/M/M linear detection under slow Rayleigh-fading multipath channels is same as [143].

#### 4.3.4 Numerical results

In this subsection, we provide numerical results in the evaluation of the proposed K/M/M detection under three-path slow Rayleigh-fading channels. Different path components for each user experience independent fading, with the average energy distribution among the three paths being 0.350, 0.335, and 0.315, and assume the quasi-static frequency-selective slow-fading setup: the fading coefficients for three paths of a user are generated based on a complex Gaussian distribution, and are not changed for a 200-bit interval once generated. It is equivalent to assuming that

the coefficients are constant in one transmission frame of 20 milliseconds in IS-2000 systems [3]. It is further assumed that in-cell sources are bit synchronous and three path delays are set to  $[0, T_c, 2T_c]$  in the simulation; out-of-cell unknown signals can be asynchronous.

Due to fading and multipath nature, the received amplitudes of path components can be significantly different and thus an indecomposable set of the error sequences is very expensive to calculate; even if an indecomposable error-vector set is found, loosely-bounded performance is possible. Therefore, the detection performance is evaluated using a decision metric instead of bound analysis in fading multipath channel conditions; in fact, such a scheme will provide an average receiver performance that is of more practical interest. The procedure to obtain the numerical results is described as follows:

- 1. Generating the information bits of users: The transmitted information bits of the active users are independently generated and the bits for the same user take a value of 0 or 1 with equal probability.
- 2. Spreading the CDMA signal: The signals of the users are BPSK-modulated  $(0 \rightarrow +1; 1 \rightarrow -1)$  and spread by PN sequences with spreading gain of 31, using rectangular chip waveforms; the spread signals of each user are weighted by the fading coefficients (assuming to be quasi-static) and time shifted by allocated delays.
- 3. Estimating the correlation matrix: The continuous aggregate received signal passes through the chip-matched filter of user j and is sampled at the chip rate to obtain the discrete (sampled) received signal vector,  $\mathbf{r}_j(i)$ , for the user. The  $N \times N$  correlation matrix of the received signal is then estimated based on a moving time average of 100 bit interval (assuming less than the coherence time in a slowly-faded quasi-static channel) by

$$\hat{\mathbf{S}}_j = \frac{1}{100} \sum_{i=1}^{100} \mathbf{r}_j(i) \mathbf{r}_j(i)^T,$$

where  $\mathbf{r}_{j}(i)$  is the received aggregate signal sample vector at the chip-matched filter output of user j at the *i*th bit interval.

4. Estimating the optimal filters: The optimal coefficient vectors for the front-end linear M-user equalization are estimated by

$$\hat{\mathbf{w}}_{jo} = \hat{\mathbf{S}}_j^{-1} \mathbf{p}_j,$$

where  $\mathbf{p}_j$ , the cross-correlation vector, can be represented by

$$\mathbf{p}_{j} = \sum_{k=1}^{M_{p}} P_{k} \{ \alpha_{k}^{2}(l-1)\rho_{jk}(l,l-1)\mathbf{u}_{jk}(l,l-1) + \alpha_{k}^{2}(l)\rho_{jk}(l,l)\mathbf{u}_{jk}(l,l) + \alpha_{k}^{2}(l+1)\rho_{jk}(l,l+1)\mathbf{u}_{jk}(l,l+1) \}$$

in which  $\rho_{jk}(l, i) = \langle s_k(t - iT_b - \tau_j), s_j(t - lT_b - \tau_j) \rangle$  is the correlation between signature waveforms of the *j*th and *k*th signal components at their *l*th and *i*th bit intervals, respectively.  $\alpha_k^2(l)$  is the fading signal component power of path *l*. Elements of  $\mathbf{u}_{jk}(l, l+1)$  were defined in (3.49).

5. Estimating the residual matrix: The correlation matrix of the residual interference plus noise is estimated at the output of the front-end linear equalizer bank by time averaging over 100 bits as

$$R_I + \frac{N_0}{2} \mathcal{W} = \frac{1}{100} \sum_{i=1}^{100} \hat{\mathbf{d}}(i) \hat{\mathbf{d}}^T(i) - \mathcal{R} \mathcal{C} A_M A_M^T \mathcal{C}^T \mathcal{R}^T,$$

where  $\hat{\mathbf{d}}(i)$  is the front-end equalizer output sequence vector,  $A_M$  is the amplitude diagonal matrix of active users and  $R_L$  is the signal correlation matrix.

6. Evaluating the performance: For MLS detection, M-user joint detection is based on the outputs of MJ linear optimal multiuser-oriented signal equalizers by applying the decision metric

$$\Omega(\mathbf{b}) = 2\operatorname{Re}\{\mathbf{b}^T A_M^T \mathbf{y}\} - \mathbf{b}^T A_M^T \mathcal{R}_D A_M \mathbf{b},$$

where  $\mathcal{R}_D = \mathcal{C}^H \mathcal{R}^T (R_I + N_0 \mathcal{W})^{-1} \mathcal{R} \mathcal{C}$ , whose off-diagonal entries beyond a band with upper and lower bandwidth  $3M_p - 1$  are set to zeros in the simulation by


Fig. 4.8 Average bit error probabilities of user 1 versus SNR for multiuser detectors under three-path Rayleigh-fading channels in a six-user CDMA network with short random spreading of N = 31 and equal crosscorrelation of 0.2258, K = 6, M = 3, 5, J = 3 and  $P_k = P_1, k =$ 2,3,4,5,6. The performance is compared with the single-user RAKE bound (J=3).

applying the cut-off strategy discussed in Section 3.5.4, and  $\mathbf{y} = C^H \mathbf{\bar{d}}$ ; other details can be found in (3.71).

For the proposed linear detection, the probability of error is estimated based on the SNR at the output of the RAKE receiver developed in Section 3.6.

Notice that for the given three-path fading model, J denotes the number of resolved paths (RAKE fingers) used for the maximal-ratio combining (MRC); in the case when J = 2, for example, only the two strongest paths from the three signal replica are tracked for the RAKE combining; the third path signal component is considered unknown interfering source that must be suppressed in the proposed multiuser detection (that is, the performance/complexity tradeoff issues and path diversity effect are considered).

**EXAMPLE** 9 — K/M/M detection in dispersive channels

In this example, we quantify the performance of the proposed MLS and linear detection in a six-user (K = 6) CDMA system under a three-path Rayleigh-faded channel. We assumed that five users are parameter-known synchronous users and the three path components for each user can be tracked and resolved (J = 3) at the receiver; the sixth is unknown, out-of-cell asynchronous sources. Signal of all active users are short spread by random sequences with an equal cross-correlation of 0.2258.

4.8 is obtained by simulating the scenario of equal received (large-scale Fig. based) signal powers, and it serves to study the performance of the 6/5/5 linear detection developed in Section 3.6 and the 6/5/5 MLS detection developed in Section 3.5, as well as the 6/3/3 MLS detection, which trades the detection performance for implementation complexity. To benchmark the investigated performance of the multiuser detectors, the single-user RAKE lower bound (cf. (4.26) or [6]) is provided. The results illustrate that the 6/5/5 linear detector (denoted by the dashed -x- line) is efficient since no interference and noise floor appears; the 6/5/5 MLS detector (denoted by the solid -o- line) can achieve an additional gain of about 3.2 dB at BER of  $10^{-4}$  over the linear detector. The 6/3/3 MLS detector (denoted by the dashed -\*- line) simulates a scenario where the base station can not afford a computational complexity of  $3 * 2^{15}$  (i.e., M = 5) but a complexity of  $3 * 2^9$  (i.e., M = 3); in such a case, the nonlinear detector trades a performance loss of about 1.4dB for the reduced implementation complexity (Note that its performance is still better than the linear detection by a 1.8-dB gain at BER of  $10^{-4}$ ). Compared to the single-user RAKE bound (denoted by the dashed - - line), the 6/5/5 MLS detector has a gap of about 1.9 dB, due to the effect of the three unknown interfering fading components from the sixth user.

**EXAMPLE** 10 — effect of the number of resolved paths on performance

In this example, we study the effect of path diversity on performance in a five-user short-spread CDMA system under a three-path slowly Rayleigh-faded channel. The simulation setup is: The in-cell known sources are bit synchronous, while the fifth is unknown, out-of-cell asynchronous sources; we study the 5/4/4 MLS detection, and signals of five users are spread by random sequences with an equal cross-correlation



Fig. 4.9 Average bit error probabilities of user 1 versus SNR for 5/4/4 MLS multiuser detector under a 3-path Rayleigh-fading channel in a 5-user short-spread CDMA system with random spreading with equal code cross-correlation of 0.3548 and N = 31, equal received large-scaled powers, and J = 1 - 3.

of 0.3548 and N = 31; equal received large-scale powers (excluding the small-scale Rayleigh fading) for all users are assumed. Thus, the aggregate received signal at the base station includes a total of 15 signal components, amongst which 12 signal components are from the four known users and the other three are from the unknown (strong) interfering source. For J ( $1 \le J \le 3$ ) resolved multipath components for each known user,  $M_p = 4J$  signal components are of interest for multiuser signal equalization and joint detection; the other 15-4J path components are not resolved and considered unwanted interfering sources to be suppressed in each of  $M_p$  front-end linear multiuser-oriented signal equalizers.

Fig. 4.9 shows the average bit error probability of user 1 for 5/4/4 MLS detection as a function of SNR with a different number of resolved path components (e.g., fingers employed) at the receiver. We observe from Fig. 4.9 that the bit error probability falls off sharply as the number of resolved multipath components J increases from one to three. It can be explained as follows: When J = 1, the number of resolved signal components from the four known users is 4, and the other



Fig. 4.10 Average bit error probabilities of user 1 versus SNR for an adaptive implementation of 5/4/4 MLS detector under a three-path Rayleigh-fading channel in a five-user short spread CDMA system with random spreading with equal cross-correlation of 0.3548 and N = 31. An initial 600-bit training for each user of interest is employed, the step size is set to  $\mu = 0.1$ , and the LMS algorithm is used for recursive estimates of the optimal coefficients.

unresolved 11 path components (plus the three components from the fifth unknown source) need to be suppressed in each front-end linear multiuser signal equalizer; as J increases, more path components are assumed to be resolved at the receiver and can be included in the joint detection of the four users, leading to improved interference suppression capability and increased path diversity gains (since fewer unwanted signal components need to be suppressed in the front-end linear multiuser signal equalizers and more resolved components are combined for joint detection). The best scenario is that all 3 paths (i.e., J = 3) for each of the known users are resolved and used for interference suppression and joint detection; that is, a total of 12 desired signal components are combined to "combat" the 3 interfering signal components from the unknown user and retained in the front-end multiuser signal equalization, which are used for the second-stage nonlinear joint detection.

Fig. 4.10 illustrates an adaptive implementation of the 5/4/4 joint detection

in Fig. 4.9. An initial 600-bit training is employed at the start of transmissions, and the LMS algorithm is used to adaptively search for the optimum coefficients in 4J front-end linear equalizers, thus obviating the estimation of the correlation matrix and avoiding the matrix inversion operation in calculation of the optimum linear coefficients (e.g, Wiener solution). The step size set to  $\mu = 0.1$  (equivalent to 1/(2\*total-signal-power)) in the training process, and the initial equalization coefficients start from zero.

Fig. 4.10 also shows that the probability of bit error of the 5/4/4 MLS detection falls off sharply as the number of resolved paths J increases, which is consistent with Fig. 4.9. Figures 4.10 (i.e., adaptive case) and 4.9 (i.e., non-adaptive case) demonstrate equivalent performance in terms of the probability of error in the 5/4/4 MLS detection. This can be explained as follows: For the non-adaptive implementation, the correlation matrix of the received aggregate signal needs to be estimated by a moving timing average over the observed multiple-bit interval, resulting in a certain estimation error as compared to the true statistical value and a relative higher computational complexity due to the matrix inversion operation. In the adaptive implementation, the optimum tap coefficients in each of the front-end linear equalizers are recursively approached using a gradient of instantaneous error, thus introducing excess mean-squared error in the solutions; however, the scheme has avoided the correlation matrix estimation and inversion, leading to a lower computational complexity.

**EXAMPLE** 11 — effect of errors in parameter estimation on performance

In this example, we examine the performance sensitivity of the proposed 5/3/3 MLS and linear detectors to estimate errors in the received amplitudes and channel fading coefficients required for the joint detection. We consider a five-user CDMA system under a three-path Rayleigh-fading channel, in which the signals of the active users are short spread by random sequences with an equal cross-correlation of 0.2258 and a spreading factor of N = 31. We assume that the receiver has the parameter knowledge of first three users whose transmissions are bit synchronous, but has no knowledge of the other two users whose signal components are asynchronous and must be suppressed in the front-end linear multiuser signal equalization. The estimation of the received signal amplitude (including the fast fading effect) for each



Fig. 4.11 Average bit error probabilities of user 1 versus SNR for multiuser MLS detector under a three-path Rayleigh-fading channel in a five-user short spread CDMA system with a random spreading of N = 31 and an equal cross-correlation of 0.2258. A 5% estimation error of the received amplitude is introduced for each known user to examine its effect on performance.

of the three paths of the known users experiences a 5% error, where the positive or negative error percentage is randomly generated among all multipaths in our simulation.

The simulation results, shown in Fig. 4.11, demonstrate that the proposed 5/3/3 MLS and linear detectors experience a slight performance degradation in such a case. Specifically, the 5/3/3 MLS detection suffers a negligible degradation, and the 5/3/3 linear detection suffers a 0.2-dB loss at BER of  $10^{-4}$ ; this insensitivity to the estimate errors can be probably explained as follows: The joint detection proposed in this thesis is based on the multiuser path-component aggregate of multiple users; an estimate error of one path component may make the signal aggregate value deviate up from its true value, while an estimate error of another path component may cause it to go down. The probability of all the estimate errors changing the the desired signal aggregate value in one direction is very small. As a consequence, the independent estimation errors of all path components involved in the multiuser signal



aggregate tend to cancel each other out.

Fig. 4.12 Average bit error probabilities of user 1 versus SNR for 5/4/4 MLS multiuser detector under Rayleigh-fading 3-path channels in a fiveuser synchronous CDMA systems. The users' signals are long spread by random spreading sequences with the cross-correlations between 0.3548 and 0.097, and with N = 31; the parameter knowledge of the first four users are known and the fifth user is strong, unknown interfering source; both equal power and non-equal (large-scaled) power scenarios are considered for the known users, whereas the average interference (largescaled) power of the fifth, unknown user is fixed relative to user 1 as  $\bar{P}_5 = P_1$ .

**EXAMPLE** 12 — effect of long spread signals on performance

In this example, we study the performance of the 5/4/4 MLS multiuser detector in a long-spread CDMA system under a Rayleigh-faded 3-path channel. There are five synchronous users in a CDMA network, in which first four users are considered known-parameter sources of interest to the base-station receiver; the fifth user is considered an unknown out-of-cell interfering source with a relatively strong transmission power (that cannot be ignored during the joint detection); the user signals are long spread by random spreading sequences with cross-correlations between 0.3548 and 0.097 and with N = 31.

Fig. 4.12 illustrates the average bit error probabilities of user 1 versus SNR for the proposed MLS multiuser detection with different path resolved cases and nearfar scenarios. By comparing Fig. 4.12 to Fig. 4.9, we see a significant performance degradation in this example; in fact, a performance floor forms as the SNR increases, even if there is only one strong (relative to the desired user's signal), unknown signal source; this implies that the residual interference dominates performance as the background noise diminishes. This inefficient interference suppression results from the short-correlation characterization of long-spread CDMA signals (also, see explanations in Example 7). However, the effect of path diversity on performance is evident, shown in Fig. 4.12: The probability of bit error falls off sharply as the number of resolved paths J increases from one to three, and the detection is near-far resistant for any fixed level of unknown out-of-cell interference.

#### 4.4 Asymptotic Efficiency and Near-Far Resistance

Asymptotic efficiency and near-far resistance, alternative criteria over the bit error rate for multiuser detection, characterize the detection performance relevant to single-user bound at high SNR regions.

#### 4.4.1 Asymptotic efficiency

We present an analysis on the asymptotic efficiency of user 1 for  $K/K_n/M$  MLS multiuser detection in an AWGN channel developed in Section 3.4. Let  $F_1$  denote the indecomposable error-vector set defined in an one-symbol interval;  $\mathcal{R}_L$ , a matrix of inner products between signature sequences (of known users in the target detection group) and linear equalizer impulse responses;  $R_I$ , the correlation matrix of the residual interference of unwanted signal sources;  $N_0$ , background noise density; and  $\mathcal{W}_L$ , correlation matrix of front-end linear equalizer impulse responses. Other details can be found in Appendix A. The asymptotic efficiency is a performance relative to the single-user bound at high SNR regions, where in our case, the SNR of user 1 (in a single-user channel) is  $\frac{P_1}{\sigma^2}$  or  $\frac{2P_1}{N_0}$ . We assume that the bounds of (4.22) and (4.16) for effective  $K/K_n/M$  MLS multiuser detection are dominated by a term equal to (modulo a certain factor)  $f_{1,min}$  in these high SNR regions, and then the asymptotic efficiency of user 1 can be expressed as [113, 51]

$$\eta_{1} = \frac{f_{1,min}}{P_{1}/\sigma^{2}}$$
$$= \min_{\boldsymbol{\epsilon}\in F_{1};\epsilon_{1}=1} \frac{1}{P_{1}} \epsilon^{T} A_{M}^{T} \bar{\mathcal{R}}_{U} A_{M} \epsilon, \qquad (4.29)$$

where  $\bar{\mathcal{R}}_U = \mathcal{R}_L^T (2R_I/N_0 + \mathcal{W}_L)^{-1} \mathcal{R}_L$ . For the K/K/K MLS multiuser detection, the asymptotic efficiency (4.29) becomes the optimum asymptotic efficiency described in [51]; for the  $K/K_n/M$  MLS multiuser detection with partial information, the asymptotic efficiency (4.29) is always less than unity since the residual interference is nonzero.

#### 4.4.2 Near-far resistance

The near-far resistance represents the worst-case asymptotic multiuser efficiency in (4.29) over all possible average received powers  $\{P_k\}$  of the signals of the M-1 users (other than user 1 of interest) in the *M*-user detection group. For  $K/K_n/M$  MLS multiuser detection in an AWGN channel, the near-far resistance of user 1 is given by [127]

$$\bar{\eta}_1 = \inf_{\{P_k\} \in E_M} \eta_1, \tag{4.30}$$

where  $\eta_1$  is defined in (4.29).  $E_M$  is the set of all possible received signal powers from the desired users, save user 1; that is,  $E_M = \{P_k; P_k > 0, k = 2, 3, ..., M\}$ .

#### 4.4.3 Numerical results

In this subsection, we evaluate the performance of the proposed multiuser detection in terms of asymptotic efficiency and near-far resistance. We consider a four-user, synchronous CDMA system, the same setting in [113, 125]: The signature sequences of the users are Gold code of length 7; *one-shot* (sequence) multiuser detection is studied. The results were obtained using the procedure (Steps 1 through 6) in Section 4.3.2, except that PN sequences given in Table 4.1 were used, and the performance was evaluated in terms of (4.29) and (4.30).



Fig. 4.13 The asymptotic efficiency of user 1 vs. amplitude ratio for  $K/K_n/M$  in a four-user synchronous CDMA system with full information:  $K_n = K = 4$ , M = 1 - 4, and Gold sequences of N = 7.

#### **EXAMPLE** 13 - 4/4/M multiuser detection

In this example, we examine the performance of the proposed multiuser detection, assuming the parameter knowledge of the four active users is available to the receiver, that is,  $K_n = K = 4$ ; other assumptions include:  $E_b/N_0 = 15$  dB (associated with user 1) and Gold sequences of N = 7. Specifically, we study the asymptotic efficiency and near-far resistance of the 4/4/1 (i.e., 4/4/4) linear MMSE detector, the 4/4/2and 4/4/3 MLS multiuser detectors, and the full-blown 4/4/4 MLS detector; we also examine asymptotic efficiency as a function of received signal amplitude (with regard to user 1), and provide an explanation of the saddle-shaped performance of the nonlinear detection schemes.

Fig. 4.13 demonstrates the asymptotic efficiency of user 1 as a function of received amplitude with different detection group sizes. It is seen that the asymptotic efficiency of the single-user detection (i.e., M = 1) is about 0.47; it increases as M increases, and approaches unity for the full-blown MLS detection (i.e., M = 4). The asymptotic efficiency of user 1 becomes constant when the received amplitude of other users (relative to user 1) is large for each M; this indicates that these multiuser detectors are robust in the face of the power imbalance, and thus, are near-far resistant. These results are consistent with [113], in which a zero-forcing-based linear filtering (an alternative to the MMSE criterion proposed here) was used in the front-end signal equalization, and joint detection with different grouping sizes was investigated. These multiuser detectors show different performance losses in terms of near-far resistance. Specifically, the near-far resistance of user 1 is about 0.47 for the single-user-based (4/4/1) linear MMSE detector, 0.493 for 4/4/2 MLS detector, and 0.57 for 4/4/3 and 4/4/4 detectors.

It is interesting to observe the changes in the asymptotic efficiency over the received power (relative to user 1) distribution for the proposed 4/4/M (M > 1) MLS detectors. The asymptotic efficiency of these multiuser detectors starts to degrade once the relative received amplitude drops below 1.0, and reaches its lowest point at a certain lower level; however, as the relative amplitude continues to decrease (toward zero), the asymptotic efficiency starts to increase (i.e., the detection performance is getting improved). The resulting curve forms a saddle-shape (or "V"-shape) in the relatively low received amplitude region (between 0 and 1). Such a performance phenomenon in MLS multiuser detection is well known in the literature; however, it appears to haven't had a clear explanation. We try to provide an explanation for the saddle-shaped performance, using 4/4/4 MLS detection in this example. For the full-blown nonlinear detection, the front-end linear multiuser-oriented signal equalizers reduce to MF filters, and the second stage uses an MLS decision algorithm. The one-shot output from the front-end MF filter of user j can be written as

$$\hat{d}_j(l) = \sum_{k=1}^4 \sqrt{P_k} b_k(l) \rho_{jk} + e_j, \quad j = 1, 2, 3, 4,$$
 (4.31)

where  $\rho_{jk}$  in (4.31) is the cross-correlation between the signature waveforms of users j and k.  $e_j$  is filtered background noise after the jth front-end filter.  $P_k$  and  $b_k(l)$  are the received signal power and the transmitted bit at the lth bit interval of user k, respectively. It is well known that  $\{\hat{d}_j(l), j = 1, 2, 3, 4\}$  at the lth bit interval can provide a sufficient statistic for the one-shot sequence multiuser detection. This is achieved by searching **all** possible combinations (or vectors) of the four users' transmitted bits,  $(b_1(l), b_2(l), b_3(l), b_4(l))$ , at the lth bit interval, comparing them with

the observation samples  $(\hat{d}_1(l), \hat{d}_2(l), \hat{d}_3(l), \hat{d}_4(l))$ , and finding the "closest" vector in terms of the maximum likelihood. Thus, the resulting performance depends very much on the signal constellations of  $\sum_{k=1}^{4} \sqrt{P_k} b_k(l) \rho_{jk}$  for all (2<sup>4</sup>) different combinations of  $(b_1(l), b_2(l), b_3(l), b_4(l))$ . A combination of the four users' transmitted bits that leads to a unique signal constellation is referred to as an *effective* bit combination in the sequel. The number of total effective bit combinations in  $\sum_{k=1}^{4} \sqrt{P_k} b_k(l) \rho_{jk}$  is bounded by 2<sup>4</sup>, and as a function of the received power distribution among users and the cross-correlation between the signature waveforms of paired users; it is directly related to the nonlinear sequence detection performance. For a tractable analysis, we consider a simple power-distribution and cross-correlations scenario: Assuming that  $P_2 = P_3 = P_4 = xP_1$ , where x is a small-valued signature waveform cross-correlation and equal for any signature pair. As a result, (4.31) can be expressed as

$$\hat{d}_1(l) = \sqrt{P_1}(b_1(l) + x\rho \sum_{k=2}^4 b_k(l)) + e_1,$$
 (4.32)

and

$$\hat{d}_j(l) = \sqrt{P_1}(xb_j(l) + \rho b_1(l) + x\rho \sum_{k=2, k \neq j}^4 b_k(l)) + e_j, \quad j = 2, 3, 4, \quad (4.33)$$

where  $\rho$  (< 1) in (4.32) and (4.33) is cross-correlation between the signature waveforms of any paired users. We discuss how to search through all the effective bit combinations for joint detection of the four users below:

 $x\rho > 1$ : When  $x\rho > 1$ , there are a total of eight effective bit combinations leading to different signal constellations for  $\sqrt{P_1}(b_1(l) + x\rho \sum_{k=2}^4 b_k(l))$  in (4.32); there are a total of 12 effective bit combinations leading to different signal constellations for  $\forall j, j \neq 1$  and  $\sqrt{P_1}(xb_j(l) + \rho b_1(l) + x\rho \sum_{k=2,k\neq j}^4 b_k(l))$  in (4.33). Moreover, the distances between different signal constellations are very large. In such a case, the 4/4/4 MLS detection for user 1 benefits greatly from the multiuser signal samples at the outputs of the other front-end filters (i.e.,  $\hat{d}_2(l) - \hat{d}_4(l)$ ), and can achieve the single-user performance bound.

 $x \sim \rho$ : When x and  $\rho$  take values of same order (both less than one), there are a total

of eight effective bit combinations leading to different signal constellations for  $\sqrt{P_1}(b_1(l) + x\rho \sum_{k=2}^4 b_k(l))$  in (4.32); there are also more chances to form an identical signal constellation in (4.33) with different bit combinations due to the fact that when j = 2, for example, the constellation term can be written as  $\sqrt{P_1}[x(b_2(l) + \rho b_3(l)) + \rho(b_1(l) + xb_4(l))]$ ; mutual cancellation of the bit combinations from four users is then possible, resulting in the number of effective bit combinations for any  $j, j \neq 1$  in (4.33) being (much) smaller than 12. Furthermore, the area of different signal constellations also shrinks significantly. In such a case, the 4/4/4 MLS detection for user 1 probably benefits the least from the multiuser signal samples at the outputs of the other front-end filters, thus resulting in the worst case performance (i.e., near-far resistance).

 $x \to 0$ : When x approaches zero, a total of eight effective bit combinations lead to different signal constellations for  $\sqrt{P_1}(b_1(l) + x\rho \sum_{k=2}^4 b_k(l))$  in (4.32); there are a total of twelve effective bit combinations that result in different signal constellations for  $\forall j, j \neq 1$  and  $\sqrt{P_1}(xb_j(l) + \rho b_1(l) + x\rho \sum_{k=2, k\neq j}^4 b_k(l))$  in (4.33). Although the area of different signal constellations shrinks significantly in such a case, the signal component from user 1 dominates in each of the four observation samples at the front-end equalizer outputs; thus, the detection performance of user 1 can be gradually improved to the single-user bound.

Such an explanation also applies to the asymptotic efficiency in the 4/4/3 and 4/4/2 MLS detection scenarios. However,  $e_j$  in (4.31) should be interpreted as residual interference plus noise, since each front-end filter in these detectors employs the linear multiuser-oriented signal equalization (rather than MF filtering) for interference suppression of the unwanted signal source(s).

### **EXAMPLE** 14 - 4/3/M multiuser detection

In this example, we study the performance of the proposed multiuser detection with partial system parameter information, corresponding to K = 4,  $K_n = 3$  and M = 1 - 3, in a four-user synchronous CDMA system; the same setting as the previous example, except that the fourth user is assumed to be unknown (out-ofcell) interfering source. Signals of users are spread by Gold sequences of N = 7, and the performance of user 1 is evaluated for 4/3/1 linear detector, and the 4/3/2



Fig. 4.14 The asymptotic efficiency of user 1 vs. amplitude ratio for  $K/K_n/M$  detection (with partial information) in a four-user synchronous CDMA system: K = 4,  $K_n = 3$ , M = 1 - 3,  $E_b/N_0 = 15$  dB (user 1), and Gold sequences of N = 7.

and 4/3/3 MLS detectors with  $E_b/N_0 = 15$  dB (in terms of user 1). To obtain the optimum coefficients (or Wiener solution) for each front-end linear equalization, the autocorrelation matrix of the unknown fourth user's signal is estimated (through overhead channels) using a time averaging over 200 received symbols as described in Section 4.2.4, and the cross-correlation vector is calculated using the definition in (3.37) on the basis of the parameters of the known users in target detection group.

It is seen from Fig. 4.14 that the asymptotic efficiency of user 1 is 0.42 for the linear single-user-based detection, 0.51 for the 4/3/2 MLS detection, and 0.75 for the 4/3/3 MLS detection. Since near-far resistance is the worse-case asymptotic efficiency performance over a (relative) power distribution, it is obtained from the figure that the near-far resistance is 0.42, 0.446, and 0.535 for the linear single-user-based detector, the 4/3/2 MLS detector and the 4/3/3 MLS detector, respectively. Compared these results with Example 13, we can find that each corresponding detector for a given M ( $1 \le M \le 3$ ) here incurs some performance loss, which is expected since the receiver assumes no parameter knowledge of the fourth interfering source (i.e., partial information setup).

### 4.5 Summary

In this chapter, we studied the performance of multiuser linear and MLS detectors with partial information, which were proposed for reverse-link base-station reception in cellular CDMA networks under both AWGN and multipath slowly Rayleigh-faded channels; the evaluation criteria included minimum mean squared error, effective signal-to-noise ratio, probability bit error, asymptotic efficiency and near-far resistance. We also justified these detectors by comparing them with several well-known CDMA receivers, including conventional MF receivers, enhanced single-user-oriented linear detectors, the full-blown (centralized) multiuser detectors and the receiver in an isolated single-user channel.

Numerical examples were carefully chosen to quantify the performance of the proposed multiuser detectors. The numerical results illustrated that the front-end linear multiuser-oriented signal equalization proposed in this thesis is effective, beating conventional MF filtering and single-user-based signal equalization. The proposed linear and nonlinear multiuser detectors are efficient and near-far resistant. We also examined performance-impacting parameters such as detection group size, the number of unknown out-of-cell (strong) interfering sources, the number of resolved path components, spreading type and parameter estimation errors. Other important issues we investigated included the trade-off between detection performance and implementation cost, the near-far problem, and performance of an adaptive variants of the proposed multiuser detectors. 

# Chapter 5

# **Application Issues**

The CDMA receivers developed in this thesis can be characterized as a combination of interference cancellation and "real" multiuser detection, and they can provide one feasible way for effective base-station reception in multi-cell CDMA networks, where the receiver has only the parameter knowledge of a subset of user population. Thus, the proposed multiuser detection has potential applications in current and future wireless networks to increase spectrum utilization efficiency and system capacity.

In Section 5.1, we present a brief overview of 3G wireless systems and describe attractive features that are appropriate for use of the multiuser detectors developed in this study. Sections 5.2-5.5 provide a few specific application scenarios using the proposed multiuser detection to improve performance.

### 5.1 3G System Configurations for Advanced Signal Reception

Currently, there are two types of CDMA-based, globally-adopted standards for the third-generation (3G) cellular wireless systems [151]: CDMA2000 [3] and Wideband-CDMA (WCDMA) [4]. CDMA2000 is backward compatible with IS-95A/B, and supports the reuse of existing IS-95A/B service standards; it includes single-carrier direct-spread (DS) systems and multi-carrier (MC) systems. The baseline chip rate for a DS-CDMA system is 1.2288 Mcps (i.e., 1x), and the scalable chip rate and RF bandwidth are Nx1.2288 Mcps and Nx1.25 MHz, respectively, where N =

1,3,6,9,12; a multi-carrier approach can be used to maintain orthogonality (and thus compatibility) with IS-95 carriers. CDMA2000 cellular wireless networks can be configured to satisfy different service requirements. There are six different radio configurations (RCs), each with a different rate (e.g., 9.6 kbps or 14.4 kbps) set, coding and modulation schemes; applications with RC1 and RC2 setups are backward compatible with IS-95A/B mobiles. The Third-Generation Partnership Projects 2 (3GPP2), created at the end of 1998, has been placed in charge of developing the global CDMA2000-based standard. The CDMA2000 Release 0 is basically 1xRTT; Release A is well-known for its 307 kbps support; Release B has light content (mainly the "rescue channel"); the latest release (end of December 2002) is CDMA2000 Release C [3] in which a set of standardized features for 1xEv-DV is provided. The future CDMA release (Release D) will probably include items such as reverse-link enhancement features, cell selection soft hand-off and antenna enhancement technologies [152].

Compared to the IS-95, CDMA2000 systems put a lot of efforts into fast and reliable channel measurements to enhance performance throughout overhead channels, such as reverse-link pilot channels and auxiliary pilot channels; such efforts can guarantee advanced signal estimation and detection to mitigate the effects of fading and interference, and support more voice users and higher data-rate services. New features in CDMA2000 systems include coherent detection (e.g., reverse-link BPSK modulation), fast power control, Turbo coding and spatial diversity. The improved channel estimation and user information in CDMA2000 systems provide an excellent condition for applying the CDMA multiuser receivers developed in this study.

The WCDMA standard, sponsored by Japan's ARIB (Association of Radio Industries and Businesses) and the ETSI (European Telecommunications Standards Institute), is based on wideband Direct-Spread (DS) CDMA technology. WCDMA systems employ an RF bandwidth of 5 MHz with a chip rate of 3.84 Mcps, and can provide high bit-rate services at up to 2 Mbps. WCDMA supports two basic modes of operation: Frequency Division Duplex (FDD) and Time Division Duplex (TDD). FDD mode employs separate RF bandwidths for reverse- and forward-link transmissions, whereas TDD mode uses a single RF frequency band for both reverse- and forward-link transmissions. Equivalent to the 3GPP2 for CDMA2000, the organization of Third-generation Partnership Projects (3GPP) has been created to proceed the detailed standardization work for WCDMA.

One of the most important features of WCDMA systems is their provision for advanced CDMA receiver concepts [37]: its air interface has been crafted so that advanced receiver concepts, such as multiuser detection and antenna technology, can be deployed by the network operator to increase capacity. Toward this goal, pilot symbols and common pilot channels have been provided to enable reliable channel measurements [153, 154] and coherent detection in both reverse- and forwardlink transmissions. As a consequence, using the proposed multiuser detectors in WCDMA-based cellular wireless networks is very attractive, especially in TDD-based systems where short spread CDMA signals are employed and the number of in-cell users is small.

### 5.2 Joint Detection for Multiple SCH Services

In a cellular CDMA network that meets the CDMA2000 standard, high-rate data services are provided by supplement channels (SCHs), and up to two reverse-link SCH (R-SCH) channels for each mobile are supported in radio configuration (RC) 3 and RC4. A high-rate SCH requires a (much) higher transmission power than a low-rate fundamental channel (FCH); the data traffic is bursty and usually lasts a few seconds (e.g., 5 seconds). As a result, signals from one active R-FCH channel of a mobile will usually result in significant interference to the base-station reception for other users as well as the other channels of the mobile itself, leading to severe near-far effects and ISI (in a multipath fading environment). The situation worsens in the case when two R-SCH channels of the mobile are served simultaneously. For instance, if we assume a two-path propagation environment (for one-antenna base-station reception), a mobile with two active SCH channels will have four path components. Even if the mobile time-aligned signal components from the respective SCH channels are assumed to be free of mutual interference (due to their signal orthogonality), any path component (finger) reception of the mobile at the receiver experiences the two time-shifted interfering components from itself: one from the same SCH channel (i.e., ISI) and one from the other SCH channel (i.e., the near-far effect), and other interfering sources (e.g., from other mobile users). If the effects of these interfering sources are not taken into consideration, it will significantly degrade the detection performance of the mobile. In such a case, the signals from the two SCHs of the mobile can be jointly detected to improve performance by using the K/2/2 linear or nonlinear multiuser detectors developed in this research. Since the parameter knowledge of the two SCH channels is known to the base-station receiver, the known signal infrastructure of the resolved components from the mobile can be fully explored for joint multiuser detection; then, the signal components resulting in the ISI and near-far problem are changed from 'hostile", mutual interference to "friendly cooperation" for the base-station reception. Moreover, the joint multiuser detection can also incorporate the full parameter knowledge of the two SCH channels into the suppression of other unwanted interfering sources. In addition, since signals from one mobile experience the same fading pattern, one channel measurement (from the pilot channel) can be used for the joint detection of the two R-SCH channels, improving the hardware utilization. Notice that the multiuser detection of the signal components from multiple mobiles and SCHs is straightforward.

The K/2/2 multiuser detection described for reverse-link applications can be used for forward-link reception at a mobile station in the scenarios where the two forward SCHs (F-SCHs) are simultaneously active (in a forward-link setup with RC3, RC4 or RC5).

### 5.3 Joint Detection for Transmissions in TDD Mode

The WCDMA TDD mode is typically applied to wireless networks with small-cell (i.e., micro-cell) environments because of its discontinuous transmission and timesharing nature. The tight, fast reverse-link power control is hard to perform in a TDD system, since the reverse link is not continuously available. The effect of outof-cell interfering sources becomes evident to any base station because of the small cell sizes. Thus, advanced CDMA receivers such as multiuser detectors are desirable for mitigating the near-far effect and suppressing unknown or unwanted interference. In a TDD system, short spread signals are employed, the number of simultaneously active users is small (i.e.,  $K_n = 1\text{-}16$ ) in each cell and the transmissions of in-cell users are well scheduled. Moreover, the transmission control and channel assignment enable excellent channel estimation for signal reception. Specifically, every transmission frame in TDD mode is divided into 15 time slots, each with 2560 chips or 0.26 microseconds ( $\mu$ s), and one time slot is allocated to either a reverse-link burst or forward-link burst. Each data burst (or time slot) contains two data fields separated by a midamble, followed by a guard period; the midamble in a reverse-link burst is a training sequence of 512 chips that can be used to accurately estimate the reverse-link channel impulse response. Therefore, the proposed  $K/K_n/K_n$  linear or MLS multiuser detectors are applicable to the base-station reception in a CDMA TDD-mode network.

For forward-link transmissions in TDD mode, the enhanced single-user-based K/1/1 multiuser detectors can be applied at each mobile station to suppress the unwanted interfering sources. Moreover, when two or more transmission services are simultaneously enabled in a mobile, their signals can be jointly detected using the proposed K/M/M linear or MLS multiuser detectors to improve performance.

### 5.4 Multiuser Detection for Spatial Diversity and Softer Hand-off

It was shown in Chapter 4 that the bit error rate falls off sharply as the number of resolved multipath components for each in-cell user increases; however, the implementation complexity only increases linearly with the number of resolved paths for a given number of users in the detection group. The results demonstrate that it is worth the cost and effort for CDMA system design to take advantage of path diversity to enhance performance. In CDMA2000 and WCDMA systems, the RAKE receivers that are currently being used can exploit path diversity in wireless fading multipath environments; however, they are usually MF-based, and no countermeasures against interference are considered in each finger reception. One feasible, effective method that is going forward to improve performance at the base station is employing the K/M/M linear detectors proposed in this study for fading multipath channels. Moreover, the base station can offer the required computational capability (the complexity being on the order of  $2^M$ ) and multiuser information.

For the receiver antenna diversity, each antenna is "raking" the desired signal components from a subset of in-cell users at the base station; the signal components of one or more users that are raked from all the base-station antennas can be jointly detected by applying the proposed multiuser detection. Hand-off technology is a sort of antenna diversity that has been employed in current 3G wireless systems to guarantee service continuity and improve performance. Revere-link hand-off schemes fall into two categories: soft and softer hand-offs. The soft hand-off, also called macro path diversity, refers to reverse-link transmissions that have two or more simultaneous radio links between a mobile and different base stations; a base-station controller chooses a good frame among those from the mobile that have been received at the different base stations (i.e., so-called frame selection diversity). In contrast, softer hand-off transmissions have two or more radio links between a mobile and different *sectors* within the same base station; a single RAKE receiver combines all the resolved paths (or fingers) in the softer hand-off links. The resolved path components in the softer hand-off can be fully exploited to enhance performance by applying the proposed multiuser detection.

### 5.5 Multiuser Detection for Hotspot Traffic Relief

In a cellular wireless network, traffic loads in different cells (or base-station sectors) are often not uniformly distributed over the network coverage area; they are also time-varying in each cell. Many cases can be observed in a live cellular network where one or two cells are with heavily loaded traffic, while the surrounding cells have a light traffic load. Overloaded cells, often referred to as hotspots, can result in high blocking rates for new calls and high link dropping rates for active calls; thus, it is difficult to guarantee the grade of service (GoS) of users in these hotspot cells. Relieving hotspot situations and improving their service quality is challenging in dense traffic areas. One way to deal with hotspots is to "wisely" adjust the cell coverage by managing the forward-link channel powers of relevant cells in the network in order to optimize the system spectrum usage and achieve the required GoS. For example, the "cell breathing" algorithm reduces the forward-link coverage of heavilyloaded cells and increases the coverage of lightly-loaded cells such that some traffic (close to the cell edge) is shed to the sounding cells to relieve the overloaded cells. Such a scheme requires excellent coordination among the different base stations and accurate measurements of the traffic loads to trigger power adjustments; this leads to a very high implementation complexity. Alternatively, we can increase the spectrum

utilization efficiency and cell capacity by employing advanced receiver technologies. The multiuser detectors proposed in this thesis, for example, can be applied to relieve the hotspot situations. By incorporating the available parameter knowledge of multiple known users, services and antennas into interference suppression and joint detection, less power will be required for each link to achieve the same GoS and the system (or sector) capacity can be increased, thus resulting in hotspot traffic relief.

# Chapter 6

# **Concluding Remarks**

### 6.1 Research Summary

In this thesis, we proposed and evaluated forms of enhanced multiuser detectors for base-station reception in cellular DS-CDMA wireless networks. The study was based on the partial system parameter configurations; that is, the received aggregate at any base station is composed of: (1) components from the in-cell sources, whose parameters (e.g., received power, timings, signature sequence) are *known* to the receiver; (2) components from the out-of-cell sources, whose parameters are *unknown*; (3) and background noise. In such setups, CDMA receivers that only assume the knowledge of a single user are not efficient, and multiuser detectors that employ a bank of front-end MF filters are not applicable, since the out-of-cell interfering sources can not be ignored. In contrast, the proposed multiuser detectors can fully exploit the parameter knowledge of the in-cell known users and the statistical knowledge of the out-of-cell unknown interfering sources to enhance performance.

As a first step in the receiver design, a multiuser-oriented signal equalization scheme was developed for receiver front-end signal processing to treat in-cell and out-of-cell signal sources differently, since their available knowledge is significantly different. Such a front-end signal equalizer can extract (or estimate) the multiuser signal aggregate of the in-cell known users (desirable for joint multiuser detection) directly from a larger received signal aggregate during the suppression of unwanted interfering sources; it generalizes the concept of the single-user-oriented, binary signal equalization scheme, and allows for "real", elaborate multiuser detection. Thus, the multiuser-oriented signal equalization technique distinguishes our proposed multiuser detectors from alternatives found in the literature.

Based on the front-end multiuser signal equalization philosophy, maximum likelihood sequence (MLS) and linear multiuser detectors have been derived in AWGN channels and slowly, Rayleigh-faded multipath channels. The proposed receiver architectures with partial system information setups have two stages. The first stage, forming the front end of the device, amounts to a bank of linear multiuser-oriented signal equalizers on the basis of MMSE criterion, one for each of the target-group sources for joint detection; its role is to estimate the multiuser signal aggregate by suppressing the unwanted interference. The second stage, acting *jointly* on all outputs from the front-end equalizers, is a maximum-likelihood sequence (MLS) or linear multiuser detector; its structure is calculated from the second-order statistics of the equalizer error processes, assumed to be approximately Gaussian on the basis of a Central-Limit argument. The proposed  $K/K_n/K_n$  maximum likelihood sequence (MLS) multiuser detection for AWGN channels is a generalization of MLS multiuser detection [36] for multi-cell CDMA scenarios, whose computational complexity is on the order of  $2^{K_n}$ , independent of K. The proposed K/M/M MLS multiuser detection for Rayleigh-faded multipath channel is a generalization of MLS detection [45] for CDMA multi-cell, multipath environments, whose computational complexity ranges from  $O(J2^M)$  (synchronous cases) to  $O(J2^{3M})$  (asynchronous cases), independent of K and the total number of multipath components JK. The proposed  $K/K_n/K_n$ linear multiuser detection in AWGN channels is an extension of multiuser detection [49, 50] (in a single-cell full information setup) to multi-cell partial information CDMA setups, and the K/M/M linear detection in multipath Rayleigh-faded channels is an extension of the linear detection [143] to multi-cell partial information CDMA scenarios.

We also proposed a unifying  $K/K_n/M$  MLS multiuser detector architecture that can trade performance for complexity with a configurable parameter M ( $2 \le M \le K_n$ ), where M can be determined by how much computational capability and information the receiver has; the receiver implementation complexity increases exponentially with M — on the order of  $2^M$  in synchronous settings and  $2^{3M}$  in asynchronous settings (for AWGN channels) — but independent of  $K_n$  and K. Moreover, the notation  $K/K_n/M$  designates both the detection problem and the information structure that characterizes it. In the special scenario where the number of known-parameter users is one, the proposed K/1/1 linear MMSE receiver reduces to a form of enhanced linear single-user detectors [46, 47, 48]. In the particular case of centralized information (i.e.,  $K = K_n$ ) setups under an AWGN channel, the front end reduces to the conventional matched-filter bank; the overall architecture of  $K_n/K_n/K_n$  MLS multiuser detection reduces to the one described in [36];  $K_n/K_n/M$  MLS multiuser detection provides an alternative to [113]; the architecture of  $K_n/K_n/K_n$  linear detection reduces to a linear MMSE [49] or decorrelating [50] detector.

In addition, an adaptive implementation of the proposed multiuser detectors was provided, assuming that the initial training sequences for in-cell active users were employed. In such a case, each of the front-end linear multiuser-oriented signal equalizers can recursively approach its optimal filter tap coefficients; using a decisiondirected mode, the adaptive multiuser detectors can also adapt to channel variations in wireless propagation environments.

The performance of the proposed multiuser linear and MLS detectors was evaluated with respect to minimum mean squared error, effective signal-to-noise ratio (SNR), probability bit error, asymptotic efficiency and near-far resistance. Numerical examples were carefully chosen to quantify the performance of the proposed receivers with partial information. We also justified them by comparing the proposed detectors with several benchmark CDMA receivers. Moreover, we studied many performance-impacting parameters that are relevant to the multiuser detection, such as the detection group size, the number of resolved path components, the spreading type and the parameter estimation errors; other important performance issues that were investigated include a performance/complexity trade-off, the nearfar problem and the convergence behaviour of the adaptive multiuser detectors. In addition, the saddle-shaped asymptotic efficiency of MLS multiuser detection as a function of power distributions was intuitively explained in a four-user, full-blown detection scenario.

Numerical results, functions not only of the channel parameters but of the particular character of the side information assumed available, provide an expanded view of the performance tradeoffs available in the application of multiuser detectors. It was shown that the linear and MLS detectors that employ a bank of front-end MF-based filters while neglecting out-of-cell interference incur a severe performance penalty in multi-cell partial information scenarios, demonstrating that the performance enhancement doesn't come by simply increasing the implementation complexity and without giving a proper receiver design. The linear and MLS multiuser detectors developed in this thesis for base-station reception in cellular CDMA networks are near-far resistant; the proposed linear multiuser-oriented signal equalization is effective, beating the conventional MF filtering and single-user-oriented signal equalization techniques. The proposed  $K/K_n/K_n$  linear detectors outperform the enhanced single-user-based (K/1/1) linear detectors, illustrating that exploring the full knowledge of in-cell known users can enhance performance. The proposed  $K/K_n/K_n$  MLS detectors outperform the proposed  $K/K_n/K_n$  linear detectors, demonstrating that the proposed MLS multiuser detection can further enhance system performance over the proposed linear detection at the price of a higher implementation cost; moreover, the performance difference is especially significant when parameter information is incomplete or MAI is severe. The  $K/K_n/M$  (M > 1) MLS detector can offer an improved performance as M increases (i.e., trading performance for complexity), for fixed K and  $K_n$ . The  $K/K_n/K_n$  MLS detector can approach the performance bound of an ideal single-user system as  $K_n$  goes to K; that is, the base station approaches the full knowledge of all K active users. The relevant CDMA receivers investigated in this research can be sorted in terms of performance improvement as the conventional MF (K/1/1) filter, the enhanced single-user K/1/1 detectors, the proposed  $K/K_n/K_n$  linear detectors, the proposed  $K/K_n/M$  MLS detectors and the full-blown K/K/K MLS detector; their computational complexity and information requirements at the receiver increase accordingly.

The results also illustrated that the proposed adaptive detectors perform well in terms of their convergence behaviour and the MMSE at the output of each front-end equalizer; the detectors show a slight performance degradation when a 5% estimation error of the received amplitude is introduced for each of the known users in the detection group. An important observation from the numerical experiments is that short spreading is less sensitive to incompletely characterized interference than long spreading. There is a difference in multiuser detector's ability to suppress interference from out-of-cell unknown sources for short spread and long spread signals; the latter incurs an obvious performance loss as compared to the former. Furthermore, for a fixed detection group size M, the bit error rate falls off sharply as the number

of resolved multipath components for each in-cell user increases; the results suggest that path diversity is an effective way of enhancing performance, and that trading multipath diversity implementation for performance is worthwhile, since the reception cost only increases linearly with the number of resolved multipath components in the multiuser detection.

Application issues of the proposed multiuser detectors in current and future wireless CDMA networks were addressed. Specifically, we provided a few practical application scenarios in CDMA2000 and WCDMA systems, where the implementations of the multiuser detectors developed in this study are feasible and the potential performance enhancements are expected.

### 6.2 Directions for Future Study

Directions for future work based on this thesis research are provided in this section.

#### 6.2.1 Other forms of enhanced nonlinear multiuser detectors

In the proposed multiuser receivers, linear MMSE and decorrelating transformations and MLS decision algorithms have been employed in the second stage for joint detection. In fact, other decision algorithms are also applicable in the second-stage signal processing as long as they can exploit the well-formed multiuser signal correlation infrastructure for MUD. For example, we can use nonlinear decision algorithms such as multi-stage, parallel interference cancellation (PIC) schemes [111], [112], [113], [116], [117] and successive interference cancellation (SIC) schemes [104], [110], [114], [120]. In their original forms, most of these nonlinear detectors assume a centralized information (e.g., single-cell) setup and thus, their front-end filtering is MF-based. For partial information (e.g., multi-cell) configurations, the linear multiuser-oriented signal equalizer developed in this study can replace the MF-based front-end filtering to suppress out-of-cell unknown interfering sources and extract desired multiuser signal aggregate of in-cell sources, leading to other forms of enhanced nonlinear multiuser detectors in cellular CDMA networks. Although the development of a receiver architecture for these nonlinear multiuser detectors is straightforward, detailed performance analysis and evaluation require more research efforts.

#### 6.2.2 A combination of the multiuser detection and channel encoding

Another important issue that needs to be investigated is quantifying the system performance when employing both the multiuser detection and a channel error correction coding, where the common frequency spectrum is shared by the multiuser detection and coding, and the signals of the active users to be jointly detected become coded symbols. Since channel coding schemes are usually employed in practical wireless systems, such an evaluation in a cellular DS-CDMA wireless network would certainly be worthwhile.

# Appendix A

# **Detailed Symbol Notations**

As a front-end optimal filtering for  $K/K_n/M$  multiuser detection, a bank of M linear multiuser-oriented signal equalizers is employed, each of which consists of a chip-matched filter and an N-tap FIR filter (over a bit period).

For the *j*th equalizer, where  $1 \leq j \leq M$ , the *n*th sample in the *i*th bit interval at the output of the chip-matched filter is given as

$$r_{j}(i,n) = \langle r(\cdot), \psi(\cdot - iT_{b} - nT_{c}) \rangle$$
  
=  $\sum_{k=1}^{K_{n}} \sqrt{P_{k}} b_{k}(i) a_{k}(n) / N +$   
 $\sum_{l=1}^{L} \sum_{k=K_{n}+1}^{K} \sqrt{P_{k}} b_{k}(l) u_{jk}^{(n)}(i,l) + n_{j}(i,n),$  (A.1)

where  $u_{jk}^{(n)}(i,l) = \sum_{m=0}^{N-1} a_k(m) \int_0^{T_c} \psi(t+iT_b-lT_b+nT_c-mT_c-\tau_k)\psi^*(t)dt, n_j(i,n) = \int_{iT_b+nT_c}^{iT_b+(n+1)T_c} n(t) \ \psi^*(t-iT_b-nT_c) \ dt, \ 0 \le n \le N-1$ , and *i* symbolizes the *i*th transmitted bit of user *j*. Let us define an *N*-dimensional vector at the *i*th bit interval of user *j* as  $\mathbf{r}_j(i) = [r_j(i,0) \ r_j(i,1) \ \dots \ r_j(i,N-1)]^T$ , then the sample vector  $\mathbf{r}_j(i)$  can be represented by

$$\mathbf{r}_{j}(i) = \sum_{k=1}^{K_{n}} \sqrt{P_{k}} b_{k}(i) \mathbf{a}_{k} / N + \sum_{k=K_{n}+1}^{K} \sqrt{P_{k}} [b_{k}(i) \mathbf{u}_{jk}(i,i) + b_{k}(i-1) \mathbf{u}_{jk}(i,i-1)] + \mathbf{n}_{j}(i), \qquad (A.2)$$

where  $\mathbf{a}_k$  is the signature sequence vector of user k,  $\mathbf{u}_{jk}(i,l) = [u_{jk}^{(0)}(i,l) \ u_{jk}^{(1)}(i,l) \ \dots$ 

 $u_{jk}^{(N-1)}(i,l)]^T$ , and  $\mathbf{n}_j(i) = [n_j(i,0) \ n_j(i,1) \ \dots \ n_j(i,N-1)]^T$  is an N-dimensional Gaussian vector with zero mean and covariance matrix  $\sigma_N^2 \mathbf{I}_N$ , where  $\mathbf{I}_N$  denotes the  $N \times N$  identity matrix and  $\sigma_N^2 = N_0/2N$ .

Let us define entries of two  $M \times M$  matrices, **R** and **Q**, and one  $M \times (K - M)$ matrix  $\mathbf{Q}(i - l)$  as

$$\{\mathbf{R}\}_{jk} = \langle s_k(\cdot), h_j(T_b - \cdot) \rangle,$$
  
$$j = 1, 2, \dots, M, \ k = 1, 2, \dots, M,$$
(A.3)

$$\{\mathbf{W}\}_{jk} = \langle h_k(T_b - \cdot), h_j(T_b - \cdot) \rangle, j = 1, 2, \dots, M, \ k = 1, 2, \dots, M,$$
(A.4)

and

$$\{\mathbf{Q}(i-l)\}_{j(k-M)} = \langle s_k(\cdot - lT_b - \tau_k), h_j(iT_b + T_b - \cdot) \rangle,$$
  

$$j = 1, \dots, M, \ k = M + 1, \dots, K,$$
(A.5)

respectively, where  $\tau_{M+1} = \tau_{M+2} = \ldots = \tau_{K_n} = 0$ . Thus, from (3.37) and (3.38),  $\hat{\mathbf{d}}$  can be formulated as

$$\mathbf{d} = \mathcal{R}_L A_M \mathbf{b}_M + \mathcal{Q}_L A_I \mathbf{b}_I + \mathbf{N}$$

where, by definition,

 $\mathbf{b}_{M} = \begin{bmatrix} b_{1}(1) \ b_{2}(1) \ \dots \ b_{M}(1) \ b_{1}(2) \ b_{2}(2) \ \dots \ b_{M}(2) \ \dots \ b_{1}(L) \ b_{2}(L) \ \dots \ b_{M}(L) \end{bmatrix}^{T}. \\ \mathbf{b}_{I} = \begin{bmatrix} b_{M+1}(1) \ \dots \ b_{K}(1) \ \dots \ b_{M+1}(L) \ \dots \ b_{K}(L) \end{bmatrix}^{T}. \ A_{M} = \operatorname{diag}\{A_{M}(1), \ A_{M}(2), \ \dots, \ A_{M}(L)\}, \ A_{M}(i) = \operatorname{diag}\{\sqrt{P_{1}}, \ \sqrt{P_{2}}, \ \dots, \ \sqrt{P_{M}}\}. \ A_{I} = \operatorname{diag}\{A_{I}(1), \ A_{I}(2), \ \dots, \ A_{I}(L)\}, \ A_{I}(i) = \operatorname{diag}\{\sqrt{P_{M+1}}, \ \sqrt{P_{M+2}}, \ \dots, \ \sqrt{P_{K}}\}. \ \mathbf{N} = [\mathbf{N}(1)^{T} \ \mathbf{N}(2)^{T} \ \dots \ \mathbf{N}(L)^{T}]^{T}, \ \mathbf{N}(i) = [z_{1}(i) \ z_{2}(i) \ \dots \ z_{M}(i)]^{T}, \ \text{where} \ z_{j}(i) \ \text{in} \ \mathbf{N}(i) \ \text{is defined as} \ z_{j}(i) = \mathbf{N}(i) = \mathbf{N}(i)^{T} \ \mathbf{N}(i)^{T} \ \mathbf{N}(i) = \mathbf{N}(i)^{T} \ \mathbf{N}(i$ 

 $\langle n(\cdot), h_j(iT_b + T_b + \tau_j - \cdot) \rangle, \quad j = 1, 2, \dots, M.$ 

$$\mathcal{R}_{L} = \begin{bmatrix} \mathbf{R} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{R} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix},$$
(A.6)

$$Q_{L} = \begin{bmatrix} \mathbf{Q}(0) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{Q}(1) & \mathbf{Q}(0) & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}(1) & \mathbf{Q}(0) & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{Q}(1) & \mathbf{Q}(0) & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{Q}(1) & \mathbf{Q}(0) \end{bmatrix}$$
(A.7)

and

$$\mathcal{W}_{L} = \begin{bmatrix} \mathbf{W} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{W} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{W} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{W} & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{W} \end{bmatrix},$$
(A.8)

where  $\mathcal{R}_L$  and  $\mathcal{W}_L$  are two  $ML \times ML$  matrices and  $\mathcal{Q}_L$  is an  $ML \times (K - M)L$ matrix. From (A.5), it can be shown that the  $M \times (K - M)$  matrix  $\mathbf{Q}(l)$  satisfies  $\mathbf{Q}(l) = \mathbf{0}$  unless l = 0 and 1. Notice that  $z_l(i)$  is a Gaussian variable with zero mean and variance  $\frac{N_0}{2}$ , and **N** is the output noise vector of the MMSE equalizers which has zero mean and covariance matrix

$$E[\mathbf{N}\mathbf{N}^T] = \frac{N_0}{2} \mathcal{W}_L, \qquad (A.9)$$

and for its vector components  $\{\mathbf{N}(i)\}\)$ , we have covariance matrix as  $E[\mathbf{N}(l)\mathbf{N}^{T}(i)] = \frac{N_{0}}{2}\mathbf{W}$  for i = l and zero otherwise.

# Appendix B

# **Proof of Proposition** 2

**Proof**: From (A.1), let's define  $\bar{n}_{jk}$  as  $\bar{n}_{jk} = \lfloor \tau_k/T_c \rfloor$  where  $\lfloor x \rfloor$  denotes the integer part of the real number  $x, \tau_k$  is the transmission delay of out-of-cell user k ( $K_n + 1 \le k \le K$ ), and  $0 \le \bar{n}_{jk} \le N - 1$ , then  $\tau_k = \bar{n}_{jk}T_c + v_{jk}$  where  $v_{jk}$  is a random variable uniformly distributed over  $[0, T_c]$ . Thus,  $u_{jk}^{(n)}(i, l)$  can be written as

$$u_{jk}^{(n)}(i,i-1) = \begin{cases} a_k(N-\bar{n}_{jk}-1+n)\chi_l(\upsilon_{jk}) \\ +a_k(N-\bar{n}_{jk}+n)\chi_r(\upsilon_{jk}) & 0 \le n < \bar{n}_{jk} \\ a_k(N-1)\chi_l(\upsilon_{jk}) & n = \bar{n}_{jk} \end{cases}$$
$$u_{jk}^{(n)}(i,i) = \begin{cases} a_k(0)\chi_r(\upsilon_{jk}) & n = \bar{n}_{jk} \\ a_k(n-\bar{n}_{jk}-1)\chi_l(\upsilon_{jk}) & \\ +a_k(n-\bar{n}_{jk})\chi_r(\upsilon_{jk}) & \bar{n}_{jk} < n \le N-1 \end{cases}$$
(B.1)

where  $\chi_l(v_{jk}) = \int_0^{T_c} \psi(t + T_c - v_{jk})\psi^*(t)dt$  and  $\chi_r(v_{jk}) = \int_0^{T_c} \psi(t - v_{jk})\psi^*(t)dt$ . Now it can be shown that the diagonal entries of the matrix  $E[\mathbf{u}_{jk}(i,i)\mathbf{u}_{jk}^T(i,i) + \mathbf{u}_{jk}(i,i-1)\mathbf{u}_{jk}^T(i,i-1)|\tau_k]$  are  $\chi_l^2(v_{jk}) + \chi_r^2(v_{jk})$ , the sub-diagonal entries are  $\chi_l(v_{jk})\chi_r(v_{jk})$ , and the entries elsewhere are zeros, then  $C_I^{jk}$  can be written by

$$C_{I}^{jk} = \begin{bmatrix} C_{a} & C_{a} & 0 & 0 & \dots & 0 \\ C_{b} & C_{a} & C_{b} & 0 & \dots & 0 \\ 0 & C_{b} & C_{a} & C_{b} & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & C_{b} & C_{a} & C_{b} \\ 0 & \dots & 0 & 0 & C_{b} & C_{a} \end{bmatrix}_{N \times N},$$
(B.2)

where  $C_a = \frac{1}{T_c} \int_0^{T_c} [\chi_l^2(v_{jk}) + \chi_r^2(v_{jk})] dv_{jk}$  and  $C_b = \frac{1}{T_c} \int_0^{T_c} \chi_l(v_{jk}) \chi_r(v_{jk}) dv_{jk}$ . For a rectangular chip waveform, we have  $\psi(t) = \frac{1}{\sqrt{NT_c}}, 0 \leq t \leq T_c$ , thus,  $\chi_l(\tau_{jk}) = \frac{v_{jk}}{NT_c}$  and  $\chi_r(\tau_{jk}) = \frac{T_c - v_{jk}}{NT_c}$ ; Now we can solve explicitly the following two integrals:

$$\frac{1}{T_c} \int_0^{T_c} \left[ \frac{(T_c - v_{jk})^2}{N^2 T_c^2} + \frac{v_{jk}^2}{N^2 T_c^2} \right] dv_{jk} = \frac{2}{3N^2}$$
(B.3)

and

$$\frac{1}{T_c} \int_0^{T_c} \frac{(T_c - \upsilon_{jk})\upsilon_{jk}}{N^2 T_c^2} \, d\upsilon_{jk} = \frac{1}{6N^2}.$$
 (B.4)
# Appendix C

# **BER Bounds for** $K_n/K_n$ **MLS Detection**

The exact analysis for the probability of bit error for MLS detector is usually untractable. However, we can derive the upper and lower bounds of the bit-error-rate (BER) for  $K_n/K_n$  MLS detection under a synchronous CDMA system, which is presented as follows.

Definition 1: error vectors. Let the normalized difference between any pair of distinct transmitted vector be referred to as an *error vector* and denoted as  $\epsilon$ . The set of error vectors that affects the *k*th user is

$$V_k = \{ \epsilon \in \{-1, 0, 1\}^{K_n}, \ \epsilon_k \neq 0 \}$$
(C.1)

and the set of the error vectors for the  $K_n$  users is given by

$$V = \bigcup_{k=1}^{K_n} V_k.$$

Note that for an error vector set  $V_k$  and binary transmitted bits, the total number of the error vectors is  $2*3^{K_n-1}$ . The vectors in  $V_k$  can be divided into an indecomposable subset (see Definition 3) that is important for the performance analysis, and its complement subset.

The set of error vectors that are *compatible* (or *admissible*) with a given trans-

mitted bit sequence  $\mathbf{b}_{K_n} \in \{-1, 1\}^{K_n}$  is defined by

$$\mathbf{A}(\mathbf{b}_{K_n}) = \{ \boldsymbol{\epsilon} \in V, \ \boldsymbol{\epsilon}_k = b_i \text{ or } 0 \}$$
$$= \{ \boldsymbol{\epsilon} \in V, 2\boldsymbol{\epsilon} - \mathbf{b}_{K_n} \in \{-1, 1\}^{K_n} \}.$$

The error vector component in the admissible set is zero if no error occurs at that component or equal to the transmitted vector component otherwise. The compatible (or admissible) error vectors that affect user k can be denoted by

$$\mathbf{A}_k(\mathbf{b}_{K_n}) = \mathbf{A}(\mathbf{b}_{K_n}) \bigcap V_k.$$

Definition 2: weight and energy of an error vector. The number of nonzero components (weight) of an error vector and the energy of a hypothetical multiuser signal modulated by  $\epsilon$  are denoted, respectively, by

$$w(\boldsymbol{\epsilon}) = \sum_{k=1}^{K_n} \mid \epsilon_k \mid$$

and

$$\| S(\boldsymbol{\epsilon}) \|^{2} = \int_{0}^{T_{b}} \left( \sum_{k=1}^{K_{n}} \epsilon_{k} \sqrt{P_{k}} s_{k}(t) \right)^{2} dt$$
$$= \boldsymbol{\epsilon}^{T} A_{P}^{T} \mathcal{C}_{1} A_{P} \boldsymbol{\epsilon}$$
(C.2)

where  $s_k(t)$  and  $\sqrt{P_k}$  are signature waveform and received signal amplitude of user k.  $A_P$  is a  $K_n \times K_n$  amplitude matrix of  $K_n$  users and  $C_1$  is a  $K_n \times K_n$  correlation matrix of signature waveforms of  $K_n$  users (Since the synchronous scenario is considered, one-shot sequence decision is sufficient for the (optimum) MLS multiuser detection).

Definition 3: indecomposable set.  $F_k \subset V_k$  is an indecomposable set for user k, obtained from  $V_k$  in (C.1) by discarding its decomposable error vectors that can be decomposed into non-zero vectors such that (1)  $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}' + \boldsymbol{\epsilon}''$ ; (2) if  $\boldsymbol{\epsilon} = 0$ , then  $\boldsymbol{\epsilon}' = \boldsymbol{\epsilon}'' = 0$ ; (3)  $(\boldsymbol{\epsilon}')^T X \boldsymbol{\epsilon}'' \geq 0$ , where  $X = A_P^T C_1 A_P$ .

With the above definitions, the upper bound of the probability of bit error of the optimum multiuser detector for user k is expressed as [51]

$$p_k(\sigma) \le \sum_{\epsilon \in F_k} 2^{-w(\epsilon)} Q\left(\frac{\parallel S(\epsilon) \parallel}{\sigma}\right)$$
 (C.3)

where Q(\*) is the Q-function, defined as  $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ .

Definition 4: minimum distance, minimum weight. The minimum distance of user k,  $d_{k,min}$ , is defined as

$$d_{k,min} = \min_{\boldsymbol{\epsilon} \in F_k} \parallel S(\boldsymbol{\epsilon}) \parallel \tag{C.4}$$

and the minimum weight of user k,  $w_{k,min}$ , is defined by

$$w_{k,\min} = \min_{\boldsymbol{\epsilon} \in F_k; \|S(\boldsymbol{\epsilon})\| = d_{k,\min}} w(\boldsymbol{\epsilon})$$
(C.5)

The lower bound of the probability of bit error of the optimum multiuser detector for user k is expressed as [51]

$$p_k(\sigma) \ge 2^{1-w_{k,min}} Q\left(\frac{d_{k,min}}{\sigma}\right)$$
 (C.6)

# Appendix D

# Mininum Mean Squared Error for K/1/1 Linear Detection

Consider the single-user based signal equalization and detection of user j, whose architecture is shown in Fig. 2.2, we need to minimize the mean-squared error between the equalization target,  $b_j(i)$ , and equalized output (or a decision statistic),  $\hat{b}_j(i)$ . The output of the equalizer for user j has the form

$$\hat{b}_j(i) = \mathbf{w}_j^T \mathbf{r}_j(i) \tag{D.1}$$

where the N-dimensional coefficient vector  $\mathbf{w}_j = [w_j(0) \ w_j(1) \ \dots \ w_j(N-1)]^T$  is chosen to minimize the mean squared error

$$MSE = E[(b_j(i) - \mathbf{w}_j^T \mathbf{r}_j(i))^2] .$$
 (D.2)

where  $\mathbf{r}_{j}(i)$  is an  $N \times 1$  vector consisting of N output samples in the *i*th bit interval from the chip-matched filter. The minimization of the MSE results in the optimum coefficient vector  $\mathbf{w}_{jo}$ , which is the solution to the Wiener filter equation [63]

$$\mathbf{S}_j \mathbf{w}_{jo} = \mathbf{p}_j \tag{D.3}$$

where  $\mathbf{S}_j$  is an  $N \times N$  correlation matrix of the sample vector in the group-based equalizer of user j at any given bit interval, i.e.,  $\mathbf{S}_j = E[\mathbf{r}_j(i)\mathbf{r}_j(i)^T]$ , and  $\mathbf{p}_j$  is a  $N \times 1$  cross-correlation vector between the received signal sample vector and the desired transmitted bit, i.e.,  $\mathbf{p}_j = E[\mathbf{r}_j(i)b_j(i)] = \frac{1}{N}P_j\mathbf{a}_j$ . As a result, the minimum mean-squared error at the output of the equalizer is

$$J_{min}^{(j)}(e) = 1 - \mathbf{p}_j^T \mathbf{w}_{jo} .$$
 (D.4)

Alternatively, the minimization of the MSE can be recursively achieved using an adaptive algorithm such as LMS or RLS. We employ the LMS algorithm in the following analysis.

The LMS algorithm employs unbiased noisy estimates of the gradient vector to adjust the coefficients of single-user-based signal equalizer. The noise in these estimates causes random fluctuations in the coefficients about their optimal values  $\mathbf{w}_{jo}$ , and thus leads to an increase in the MSE at the output of the single-user-based equalizer. As a result, the MSE for the *j*th single-user-based equalizer converges to the final MSE as  $J_F^{(j)}(e) = J_{min}^{(j)}(e) + J_{ex\infty}^{(j)}(e)$ , where  $1 \leq j \leq M$  and  $J_{ex\infty}^{(j)}(e)$  is the variance of the measurement noise, or *excess mean-squared error*, when the step size parameter  $\mu_j$  is set sufficiently small [63]. The final MSE with the LMS algorithm can be written as

$$J_F^{(j)}(e) = J_{min}^{(j)}(e) + J_{ex\infty}^{(j)}(e)$$
  
=  $(1 - \mathbf{p}_j^T \mathbf{w}_{jo})(1 + \sum_{k=1}^N \frac{\mu_j \lambda_k^{(j)}}{2 - \mu_j \lambda_k^{(j)}})$  (D.5)

where  $\{\lambda_k^{(j)}\}\$  is the set of the eigenvalues of the correlation matrix  $\mathbf{S}_j$ . In practice, a training or known sequence of the desired user can be used to recursively approach the optimum coefficients of the equalizer. After the linear equalizer achieves stable states, its coefficients can be adaptively adjusted using a decision-directed mode [48].

# Appendix E

## **Proof of Proposition** 1

**Proof:** Since  $A_I$  is a diagonal matrix and its entries do not affect our conclusions on checking the zero entries of  $R_I$ , let  $A_I$  be a unit matrix, and thus  $A_I A_I^T = \mathbf{I}$ . Then, from (A.7), we have

$$Q_{L}Q_{L}^{T} = \begin{bmatrix} \mathbf{Q}(0)\mathbf{Q}^{T}(0) & \mathbf{Q}(0)\mathbf{Q}^{T}(1) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{Q}(1)\mathbf{Q}^{T}(0) & \Upsilon & \mathbf{Q}(0)\mathbf{Q}^{T}(1) & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}(1)\mathbf{Q}^{T}(0) & \Upsilon & \dots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{Q}(1)\mathbf{Q}^{T}(0) & \Upsilon & \mathbf{Q}(0)\mathbf{Q}^{T}(1) \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{Q}(1)\mathbf{Q}^{T}(0) & \Upsilon \end{bmatrix}_{ML \times ML}$$
(E.1)

where  $\Upsilon = \mathbf{Q}(0)\mathbf{Q}^{T}(0) + \mathbf{Q}(1)\mathbf{Q}^{T}(1)$ . Since  $\mathbf{Q}(0)\mathbf{Q}^{T}(1)$  and  $\mathbf{Q}(1)\mathbf{Q}^{T}(0)$  are both  $M \times M$  matrices,  $R_{I}(=E[\mathcal{Q}_{L}A_{I}A_{I}^{T}\mathcal{Q}_{L}^{T}])$  is a symmetric matrix that has zero entries outside a band along its diagonal with upper and lower bandwidth 2M - 1.

# Appendix F

# From Cyclostationary to Stationary

This little note describes one way in which a sum of cyclostationary random processes can approach stationary. The notions of cyclostationarity and stationarity are used here in the *wide* (or weak) sense only<sup>1</sup>; prefix "ws" is occasionally inserted as a reminder. All processes are assumed centred, and thus of zero mean.

#### Notation:

• X is a cyclostationary (real) random process with covariance function R and period T; in particular,

$$R(s,t) = \mathbf{E}X(s)X(t), R(s+T,t+T) = R(s,t), \text{ all } s,t$$

- $X_1, X_2, ...,$  are *iid* replicates of X.
- U is a random variable *independent* of X and *uniformly* distributed on [0, T).  $U_1, U_2, \ldots$  are *iid* replicates of U, the U sequence being independent of the X sequence.
- Y is a random process obtained from X by shifting the origin of time by U: Y(t) = X(t + U) (all t).  $Y_i$  is constructed similarly from  $X_i$ ,  $U_i$ :  $Y_i(t) =$

<sup>&</sup>lt;sup>1</sup>This for convenience; there is no doubt a strong-sense version as well of the basic facts

 $X_i(t+U_i).$ 

Fact: Y is (ws)-stationary.

**Proof:** Compute  $\mathbf{E}Y(s)Y(t)$  by conditioning first on U. Recalling that U is uniformly distributed and independent of X, we have

$$\mathbf{E}Y(s)Y(t) = \int_0^T R(s+u,t+u)du = \int_s^{T+s} R(u,u+\Delta)du$$

where  $\Delta \stackrel{\Delta}{=} t - s$ . Define  $f_{\Delta}(u) \stackrel{\Delta}{=} R(u, u + \Delta)$ . The assumed cyclostationarity of X implies that  $f_{\Delta}$  is periodic, with period T. So

$$\mathbf{E}Y(s)Y(t) = \int_{s}^{T+s} f_{\Delta}(u)du = \int_{0}^{T} f_{\Delta}(u)du \stackrel{\Delta}{=} F(\Delta),$$

a function of  $\Delta$  only.

**Remarks:** It follows by the Strong Law of Large Numbers that as  $M \to \infty$ ,

$$\frac{1}{M}\sum_{i=1}^{M}R(s+U_i,t+U_i) \to \mathbf{E}R(s+U,t+U) \stackrel{\Delta}{=} F(t-s).$$
(F.1)

Write  $\mathbf{U}^M$  for the *M*-vector  $(U_1, ..., U_M)$ . Define

$$Z_M(t; \mathbf{U}^M) \stackrel{\Delta}{=} \frac{1}{\sqrt{M}} \sum_{i=1}^M Y_i(t).$$

The cumbersome notation for the argument of  $Z_M$  is meant to serve as a reminder that  $Z_M$  depends on  $\mathbf{U}^M$ . The normalization in the definition of  $Z_M$  is exactly what is needed to support asymptotic *unconditional* Gaussianity in the limit of large M, where "unconditional" means that the U's are averaged out in the computation of the statistics. The *conditional* covariance function for  $Z_M$  is defined by

$$S_M(s,t;u^M) \stackrel{\Delta}{=} \mathbf{E}[Z_M(s;\mathbf{U}^M)Z_M(t;\mathbf{U}^M) \mid \mathbf{U}^M = u^M].$$

For given s, t the prescription defines a random sequence  $S_1(s, t; U^1), S_2(s, t; U^2), \dots$ 

**Proof:** The  $Y_i$  are statistically independent, both with and without the conditioning on the U's. So

$$S_M(s,t;u^M) = \frac{1}{M} \sum_{i=1}^M \mathbf{E}[Y_i(s)Y_i(t) \mid U_i = u_i] = \frac{1}{M} \sum_{i=1}^M R(s+u_i,t+u_i).$$

Combine this with Eq. (F.1) to conclude that

$$S_M(s,t;\mathbf{U}^M) = \frac{1}{M} \sum_{i=1}^M R(s+U_i,t+U_i) \to F(t-s).$$

as claimed.

Summary: Cyclostationary X can be made stationary by including (and averaging out) a random time shift U that is uniformly distributed over one period. X can be made *asymptotically* stationary (F.1) by summing a large number of independent realizations, each time-shifted by a quantity u that is drawn from a distribution uniformly distributed over one period; in that case the averaging ove the time shift is done *spatially* by averaging arithmetically over components, rather than statistically over U.

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