APPLICATION OF DISLOCATION THEORY TO THE YIELD DROP IN MILD STEEL

by

T.R. HSU

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Micromechanics Laboratory

Department of Mechanical Engineering

McGill University

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INTRODUCTION

Many metals, particularly low-carbon steel, have a yield drop from elastic to plastic deformation in the stress-strain curve.

A typical stress-strain curve for mild steel is shown in Figure 1. As shown in the figure, the load increases steadily with elastic strain, drops suddenly, fluctuates about some approximately constant value of load, and then rises with further strain. The load at which the sudden drop occurs is called the upper yield point (point "a"). The constant load is called the lower yield point (point "b"). The elongation which occurs at constant load is called the yield-point elongation (distance "c" in the figure).

The flow curve during the yield-point elongation is irregular because of the appearance of the Lüder bands. Each fluctuation corresponds to the formation of a new Lüder's band. After the Lüder bands have propagated to cover the entire length of the specimen test section, the flow stress increases with strain in the usual manner - signifying the end of yield-point elongation.

The real causes of yield drops in metals have still to be settled. However, the two main theories, proposed by Cottrell and by Hahn, provide the best explanations to date for the above mentioned phenomenon. A recent paper by Bullen, Henderson et. al. shows how both theories are required to explain some of the experimental evidence.

Both Cottrell and Hahn base their theories on the dislocations in the metal playing the main role in the yield drop. It is therefore necessary to mention briefly some of the terminology used in dislocation theory.

<u>Terminology Used in Dislocation Theory</u> (1), (2), (4), (5).

1. Sources of dislocations:

   (a) Heterogeneous Nucleation - dislocations are formed with the help of defects present in the crystal, perhaps impurity particles. One example of this is the Frank-Read source or the multiplication of dislocations.*

Terminology Used in Dislocation Theory (Continued)

(b) Homogeneous Nucleation - formed in perfect lattice by the action of simple stress.

Since Homogeneous Nucleation requires very high stresses, if dislocations are not formed by homogeneous nucleation, then they must be nucleated heterogeneously.

(c) Grown-in Dislocations.

(d) Condensed vacancies.

** ( ) Number in brackets refers to references at the end of the report.

* Reference: Dieter's "Mechanical Metallurgy", 1961, Page 184.

2. Solid Solution:

When homogeneous mixtures of two or more kinds of atoms occur in the solid state, they are known as solid solutions.

Solvent atoms - more abundant atoms in the mixture.

Solute atoms - less abundant atoms in the mixture.

Substitutional solid solution - the solute and solvent atoms are roughly similar and the solute atoms occupy lattice points in the crystal lattice of the solvent atoms.

Interstitial solid solution - the solute atoms are much smaller than the solvent atoms, they occupy the interstitial positions in the solvent lattice.

3. Dislocation Atmosphere.

When a crystal possesses both dislocations and solute atoms, interactions may occur. It is interesting to see that due to this interaction, attractive forces may be produced. A steady-state concentration of solute atoms will build up around the dislocation. This excess of solute atoms is known as the dislocation atmosphere.

The solute concentration can be lowered by increasing the temperature.

The Cottrell Theory of the Sharp Yield Point (3), (10).

As pointed out previously the interaction between solute atoms and dislocations is the cause of "dislocation atmospheres". Cottrell suggested that the atmosphere of solute atoms that collects around dislocations anchors the dislocations. Therefore additional stress is needed to free a dislocation from its atmosphere. This, of course, results in an increase in the stress required to set dislocations in motion, and corresponds to the upper yield-point stress. The lower yield point then represents the stress required to move dislocations that have been freed from their atmospheres.

In mild steel the solute atoms in interstitial solid solution are either carbon or nitrogen.

The yield-drop in mild steel depends strongly on the temperature at yielding. We may visualise that the local concentration of solute atoms near the dislocation, c, is related to the average concentration C_o by the relationship:

$$C = C_o \exp\left(-\frac{U}{KT}\right) \qquad (1)$$

where

U - interaction energy

T - Absolute temperature

K - Boltzmann's Constant

For carbon and nitrogen in steel, U is between 0.5 and 1.0 ev. As the temperature decreases, the solute atmosphere becomes more concentrated and below a critical temperature the atmosphere condenses into a line of solute atoms which is just below the center of a positive edge dislocation and runs parallel to the length of the dislocation. That may explain why the yield point phenomenon is less obvious at high temperatures than it is at lower temperatures.

The amount of shear stress required to break away the dislocation from the atmosphere is a maximum for a small displacement of the dislocation. The movement of the dislocation becomes easier with increasing distance from the atmosphere. Thus as the applied stress is increased past the maximum an avalanche of dislocations is released into the slip plane, and these pile up at the grain boundary. The stress concentration at the tip of the pile up combines with the applied stress in the next grain to unlock the dislocations in that grain, and in this way a Lüder's band propagates over the specimen.

The dislocation movement away from the atmosphere is caused by the applied stress and the effect of thermal fluctuations.

Cottrell and Bilby have also shown that the mean time, tm, before a dislocation is released from its atmosphere is given by:

$$t_m \propto \exp\left[-U\left(\frac{\sigma}{\sigma_o}\right)/KT\right] \qquad (2)$$

where

σ - applied stress

σ_o - yield stress at $0^{o}K$

T - absolute temperature

K - Boltzmann's Constant

The total number of dislocations released at time t can then be expressed as :

$$N_T \propto \int_0^t \exp\left[-U\left(\frac{\sigma}{\sigma_o}\right)/KT\right] dt \qquad (3)$$

where N_T is the total number of dislocations released.

As indicated in Cottrell's theory, the yield-drop will occur when N_T reaches a certain value. Suppose N_T is released in time interval 0 to t_o , then from equation (3) the yielding occurs when

$$\int_0^{t_o} \exp\left[-U\left(\frac{\sigma}{\sigma_o}\right)/KT\right] dt = \text{constant} \qquad (4)$$

Next let us assume that the material is unstressed and has no free dislocations at the initial state of testing. As the test is started and the load is applied to the specimen the stress will be increased while the test is going on. It is therefore reasonable to say that

$$\sigma = f(t) \tag{5}$$

Yokobori investigated the curves of U derived by Cottrell and Bilby and concluded that:

$$U\left(\frac{\sigma}{\sigma_0}\right) = -\frac{1}{n}\ln\left(\frac{\sigma}{\sigma_0}\right) \tag{6}$$

where n - constant.

Substituting equations (5) and (6) into (4) we may obtain the time of yield, t_1 , as given by

$$\int_0^{t_1} [f(t)/\sigma_0]^{\alpha} dt = A \tag{7}$$

where $\alpha = \frac{1}{nkT} \quad (nkT \ll 1)$

A - material constant

Equation (7) enables us to determine the yield stress analytically for any loading condition; it is only necessary to determine the material constant, A.

Campbell (10) applied the above derived theory and compared it with the published experimental results from three different load conditions:

(1) Constant strain - rate test :

$$\sigma = f(t) = E\dot{\epsilon}t \tag{8}$$

where E - modulus of elasticity

$\dot{\epsilon}$ - strain rate.

The yield stress, Y, at constant temperature is then given by:

$$Y = \sigma_0 \left[(\alpha + 1) A \left(\frac{\sigma_0}{E\dot{\epsilon}} \right)^{\alpha} \right]^{\frac{1}{\alpha+1}} \qquad (9)$$

and the time of yielding:

$$t_1 = (\alpha + 1) A \left(\frac{Y}{\sigma_0} \right)^{-\alpha} \qquad (10)$$

(2) Constant Stress Test.

Equation (7) shows that if a constant stress Y is applied at t = 0 the time of yield is:

$$t_1 = c \left(\frac{Y}{\sigma_0} \right)^{-\alpha} \qquad (11)$$

(3) Impact Test.

Assume that the stress-time relation is of the form:

$$\sigma = \sigma_m \sin \omega t \qquad (12)$$

then we find that Y will vary as $\omega^{\frac{1}{\alpha}}$

Campbell concluded that the experimental results showed good agreement with theory for times of loading less than about 1 second. It was indicated that α has to be greater than twenty.

This theory can explain why mild steel may withstand a stress higher than the static yield stress for a short period without yielding.

The Gilman and Johnston Theory of Yield-Drop in Mild Steel (6), (9), (11)

Unlike Cottrell's "unlocking concept" of sharp-yielding for mild steel, Johnston and Gilman found that it is the dynamic resistance to dislocation motion in a crystal that determines the yield stress of LiF (Lithium Fluoride) crystals.

To support their theory, Johnston and Gilman ran several tests on LiF crystals. An etch pit technique was used to measure the dislocation velocity and a magnetic impulser was developed to generate a stress impulse so that the dislocation would move a measurable distance but would not pass out of the specimen.

Johnston's experiment showed the following facts:

A. Dislocation Velocities:

1. The dislocation velocities for screw and edge components are different in the lower ranges of applied stress. The edge components move much faster than the screw components in this particular case, but both velocities end up with the same upper limit (Fig. 2).

2. Experiment showed the motion of dislocation to be non-uniform.

3. The temperature-dislocation velocity relation may be expressed by

$$V_s = f(\sigma) \exp\left(-\frac{U}{KT}\right)$$

where V_s = dislocation velocity.

Hence if the applied stress, σ, is kept constant, the dislocation velocity will increase with temperature. The dependence is very strong.

4. Because of the existence of radiation damage, much higher stress is required to move fresh dislocations into irradiated crystals. In other words, the radiation damage creates a dynamic resistance to dislocation movement.

5. Dislocation velocities are extremely sensitive to applied stress (as shown in equation 13), but stress is very insensitive to dislocation velocities.

6. Macroscopic strain rates can be produced only if there are large numbers of dislocations in motion in a crystal. And also these dislocations must move at high velocities.

B. Dislocation Densities:

1. A large glide band can be formed due to the multiplication of dislocation loops.
   The amount of multiplication of dislocations depends strongly on the applied stress.

2. The relationship between the dislocation density and the applied shear stress is shown in Fig. 3. From this figure one can easily visualize that crystals with low yield stresses (soft crystals) behave differently from crystals with higher yield stresses (hard crystals).

3. The saturated dislocation density in the individual glide bands at the early stages of deformation appears proportional to the initial yield stress.

Johnston's experiments show the following important facts:

1. The dislocation density in an underformed crystal does not determine the initial yield stress.

2. The origin and growth of glide bands have nothing to do with the existing dislocations.

3. The initial yield stress is determined only by the resistance to the motion of the dislocations in deformed crystals. The yield stress is not influenced by the state of pinning, or the geometrical arrangement of the dislocations in as-grown crystals. This is contradictory to Cottrell's yielding theory.

Hahn developed a mathematical model to verify Johnston and Gilman's theory as follows:

Assume $$\epsilon_T = \epsilon_E + \epsilon_P \tag{13}$$

where ϵ_T - total strain in a crystal
ϵ_E - elastic strain
ϵ_P - plastic strain

Differentiating equation (13) with respect to time:

$$\dot{\epsilon}_T = \dot{\epsilon}_E + \dot{\epsilon}_P \tag{14}$$

Also $$\sigma = E \cdot \epsilon_E$$

or $$\dot{\epsilon}_E = \frac{1}{E}\dot{\sigma} \tag{15}$$

Hahn then assigned the plastic strain to be due to the motion of dislocations. From dislocation theory we may say that for a single dislocation of unit length moving through a unit volume a plastic strain of 0.5b can be expected (where b is the Burgers vector). Thus we have

$$\dot{\epsilon}_P = 0.5\, b\, L\, V_s \tag{16}$$

where L - total length of moving dislocations
V_s - average velocity of dislocations

To estimate L, we have to assume the following relation:

$$L = f\rho \tag{17}$$

where f - constant factor
ρ - dislocation density which again can be expressed as

$$\rho = \rho_o\left(1 + \frac{c}{\rho_o}\epsilon_P^{a}\right) \tag{18}$$

where ρ_o - grown-in dislocation density
a, c - parabolic constants (dislocation density in most cases is a parabolic function of plastic strain).

By reviewing Fig. 2, V_s may be approximated by a power function of the applied shear stress:

$$V_s = \left(\frac{\tau}{\tau_o}\right)^n \qquad (19)$$

where τ_o - proportionality constant

n - slope of log-log curve, determined by experiment.

Gilman and Johnston found the relation between the change of flow stress $\Delta\sigma$ and V_s to be:

$$V_s = \left(\frac{\sigma - \Delta\sigma}{2\tau_o}\right)^n \qquad (20)$$

$$\Delta\sigma = q\,\epsilon_p$$

and where q is the macroscopic work-hardening coefficient

σ is the tensile stress corresponding to τ

Combining the derived relations, Hahn obtained a model equation describing the dynamic behaviour of crystalline solids:

$$\dot{\epsilon}_T = \frac{1}{E}\dot{\sigma} + 0.5\,b f \rho_o \left(1 + \frac{c}{\rho_o}\epsilon_p^a\right)(\sigma - \Delta\sigma)^n (2\tau_o)^{-n} \qquad (21)$$

When the deformation proceeds at constant stress, the elastic contribution, $\frac{\dot{\sigma}}{E}$ may be neglected, thus:

$$\dot{\epsilon}_T = 0.5\,b f \rho_o \left(1 + \frac{c}{\rho_o}\epsilon_p^a\right)(\sigma - \Delta\sigma)^n (2\tau_o)^{-n} \qquad (22)$$

or

$$\sigma = \Delta\sigma + 2\tau_o\left[\frac{\dot{\epsilon}_T}{0.5\,b f(\rho_o + c\,\epsilon_p^a)}\right]^{\frac{1}{n}} \qquad (23)$$

By a suitable choice of constants, Hahn obtained solutions of equations (22) and (23). The upper and lower yield points and their dependence on strain rate are shown in Figures 4, 5, 6, 7.

The rate of propogation of a Lüder's band may be found by the following derivations:

Let

$$\mathcal{U} \approx K V_s\,(\sigma = \sigma'_{Ly})$$
$$\approx K\,(2\tau_o)^{-n}(\sigma'_{Ly})^n \qquad (24)$$

where $\mathcal{U}$ - the velocity of the band front

KV_s - the component of V_s normal to the band front.

σ'_{Ly} - lower yield point in non-uniform yielding

Also

$$\frac{d\epsilon_p}{dx} = \frac{d\epsilon_p}{dt}\cdot\frac{dt}{dx}$$

For steady propogation

$$u = \frac{dx}{dt} = \text{constant}$$
$$\frac{d\epsilon_p}{dt} = \dot{\epsilon}_p$$
$$\therefore \quad \frac{d\epsilon_p}{dx} = \frac{\dot{\epsilon}_p}{u} \qquad (25)$$

where x is the distance measured from the band front.

Comparing equations (25) and (22) the stress, σ , at any point along the band is

$$\sigma = \sigma'_{Ly}\,(1 + \epsilon_p) \qquad (26)$$

and

$$\frac{x}{u} = \int \frac{d\epsilon_p}{0.5\,bf(\rho'_o + c\epsilon_p^a)(2\tau_o)^{-n}[\sigma'_{Ly}(1+\epsilon_p) - \Delta\sigma'_{Ly}]^n} \qquad (27)$$

Here ρ'_o is the dislocation density just ahead of the band front. The relationship between ϵ_p and $\mathcal{U}$ is shown in Fig. 8.

Hahn's mathematical model also gives the delay time of yielding at constant stress, i.e.:

$$t = \int \frac{d\epsilon_p}{0.5\,bf(\rho_o + c\epsilon_p^a)(2\tau_o)^{-n}[\sigma - \Delta\sigma]^n} \qquad (28)$$

where ρ_o represents the density of dislocations immediately available to slip.

From the above derived equations and figures, Hahn asserted that the abrupt yield in mild steel is a consequence of the following three factors:

(a) the presence of a small number of mobile dislocations initially.
(b) Rapid dislocation multiplication.
(c) The stress dependence of dislocation velocity.

Hahn further concluded that the properties of mobile b.c.c. metal dislocations (such as their multiplication and velocity characteristics) account for sizeable yield drops without recourse to unlocking.

The Effect of Hydrostatic Pressure on Yielding in Iron

Bullen, Henderson, Hutchison and Wain conducted a very interesting experiment in testing the yield points of coarse-grained Armco iron and both coarse grained and fine-grained high-purity iron wires. Both sets of specimens were subject to some very high hydrostatic pressures (as high as 10^4 atm.).

The results of the tests can be summed up as follows:

1. The higher the hydrostatic pressure applied to the material, the less the sharp-yield phenomenon appears. When the pressure reaches a certain critical point, the sharp-yield point of the material disappears.
2. Plastic deformation of the specimens occurred at stresses much below the lower yield stress under atmospheric conditions. (Bullen showed that the yield stress is reduced by more than half).
3. The yield point disappeared at lower pressures in coarser-grained specimens.
4. The sharp-yield point may disappear at room temperature and remarkably high pressure (say 10^4 atm.), but, by reducing the temperature, the yield point may come back even if the material is still under that pressure.

From the above facts, Bullen drew the following conclusions:

1. Disappearance of the yield point of a material can be caused by free dislocations which are a direct result of pressurization.
2. The gradual disappearance of the yield point with increasing pressure is consistent with the effect of an increasing density of free dislocations.
3. Under the simultaneous multiplication of dislocations and strain-hardening, the stress/strain relationship at yield is a continuous function of time.
4. The temperature dependences of the friction stress and the load drop at yield in pressurized specimens are similar.
5. The effect of pressure on solute atmospheres does not depend markedly on grain diameter.
6. Temperature dependent load drops in pressurized iron could arise from weak interaction between dislocations and precipitated regions.

<u>Conclusions</u>:

After brief studies of Cottrell's and Gilman's theories of sharp yielding in mild steel, together with the help of Bullen's experiment for iron under high pressures, the following conclusions may be drawn:

1. The sharp yield drop in mild steel is the outcome of a dislocation mechanism.
2. Cottrell explained that the sharp yielding of mild steel is due to "unlocking" of dislocations while Gilman and Johnston pointed out that multiplication of dislocations and the stress-dependence of dislocation velocities are responsible for sharp yielding.
3. Hahn developed a mathematical model and verified that both Cottrell's and Gilman's theories are causes of sharp yielding in mild steel. He suggested that a satisfactory description of yielding should involve both the classical unlocking concepts of Cottrell and the dislocation mechanics of Gilman and Johnston.

4. Bullen found through his remarkable experiment that hydrostatic pressure is another important factor which affects the yielding of mild steel.
5. It is the writer's opinion that both Cottrell's and Gilman's theories have their limitations in explaining the sharp yielding of mild steel. Cottrell's "locking of dislocations" may be applied in general, but fails to give a satisfactory explanation under certain conditions. Gilman and Johnston's theory seems to be a solution for these cases, as well as for the case when the specimen is subjected to extremely high hydrostatic pressure. In most cases both theories should be used in explaining the sharp yielding of mild steel.

REFERENCES

1. Read-Hill "Physical Metallurgy Principles".
D. Van Nostrand Company, Inc., 1964.

2. Dieter "Mechanical Metallurgy".
McGraw-Hill Book Co., 1961.

3. Cottrell "Dislocations and Plastic Flow in Crystals"
John Wiley and Sons, Inc. 1962.

4. Cottrell "The Mechanical Properties of Matter"
John Wiley and Sons, Inc. 1963.

5. Weertman's "Elementary Dislocation Theory"
The MacMillan Company, 1964.

6. G.T. Hahn "A Model for Yielding with Special Reference to the Yield-Point Phenomena of Iron and Related B.C.C. Metals", P.727, Vol. 10, Acta Metallurgica, August 1962.

7. F.P. Bullen, F. Henderson, M. Hutchison and H.L. Wain "The Effect of Hydrostatic Pressure on Yielding in Iron" P. 285, Vol. 9, Philosophical Magazine, 1964.

8. W.G. Johnston and J.J. Gilman "Dislocation Velocities, Dislocation Densities, and Plastic Flow in Lithium Fluoride Crystals", Journal of Applied Physics, Vol. 30, No. 2, Feb. 1965, P.129.

9. J.M. Krafft "An Interpretation of Lower Yield Point Plastic Flow in the Dynamic Testing of Mild Steel"

10. J.D. Campbell "Dynamic Yielding of Mild Steel"
Acta Metallurgica, Vol. 1, 1953, P. 706

11. G. Kardos "A Study of Plastic Flow in Steel at High Rates of Strain".
Ph.D. Thesis, Department of Mechanical Engineering, McGill University, September 1965.

12. Notes on Dislocation Theory taken from Dr. J. Jonas' lectures, Spring 1966.

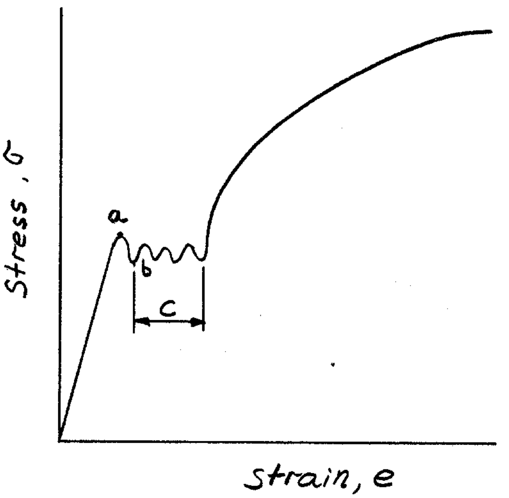

Figure 1

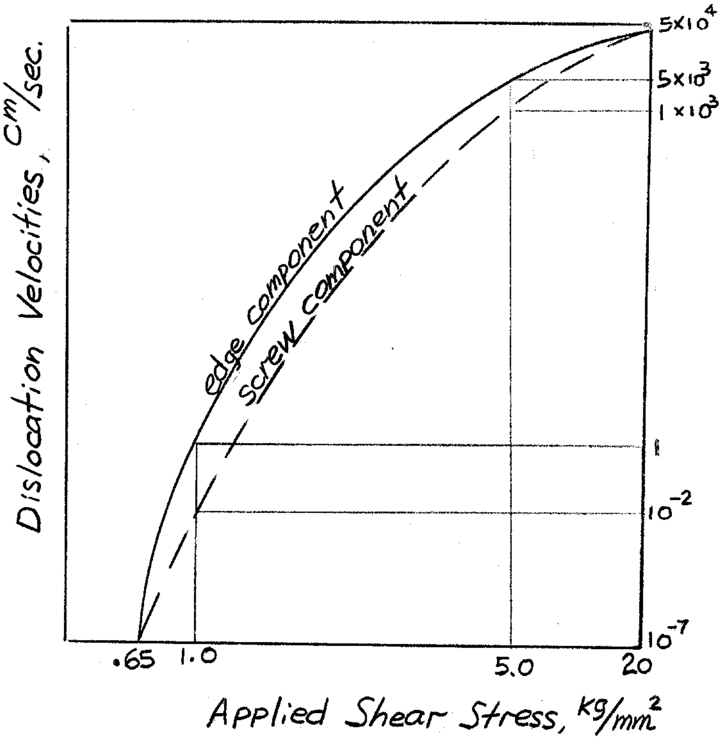

Figure 2

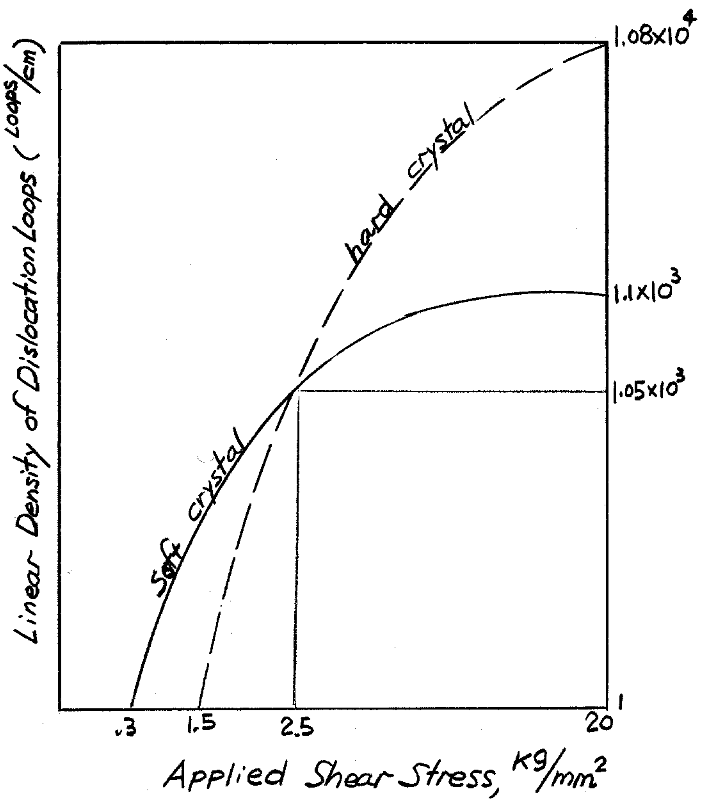

Figure 3

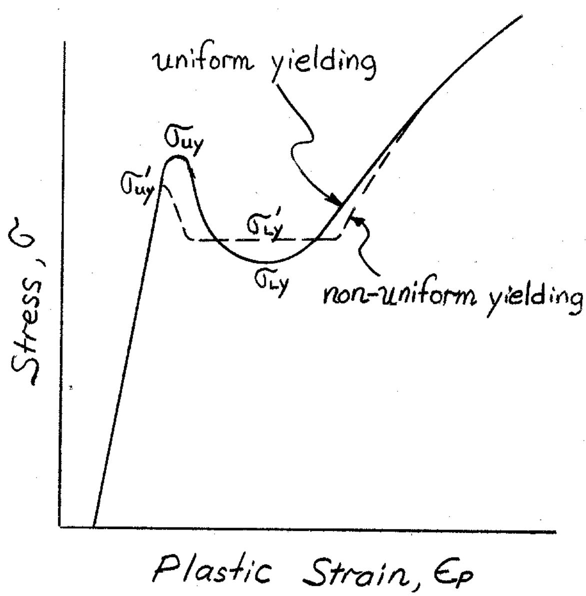

Figure 4

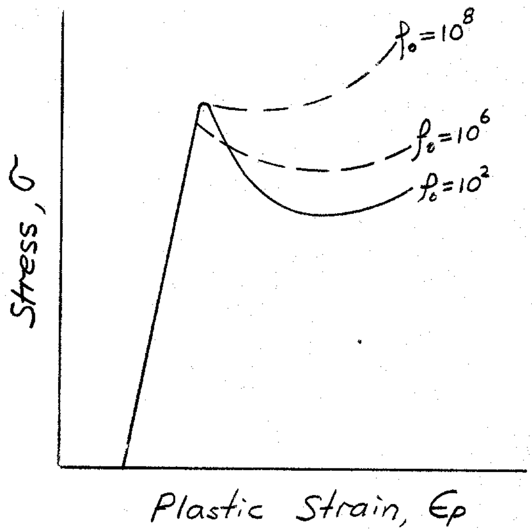

Figure 5. Influence of Initial Dislocation Density.

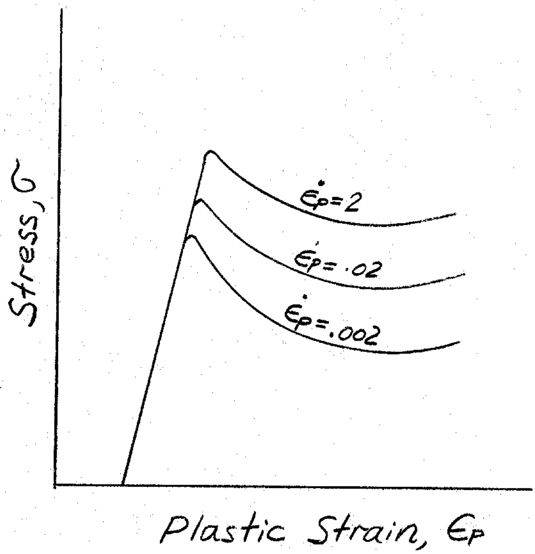

Figure 6. Influence of Strain Rate

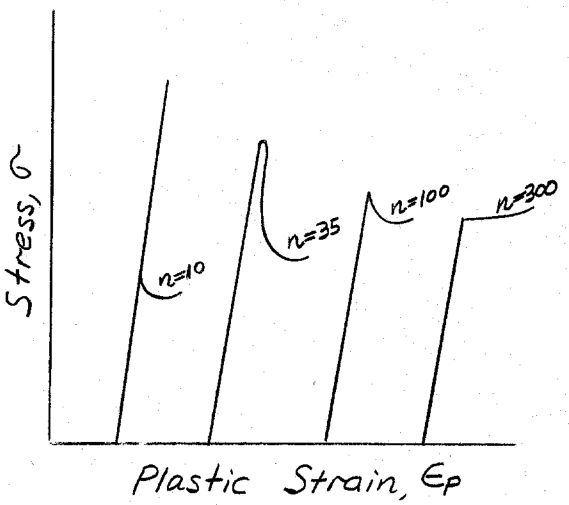

Figure 7.

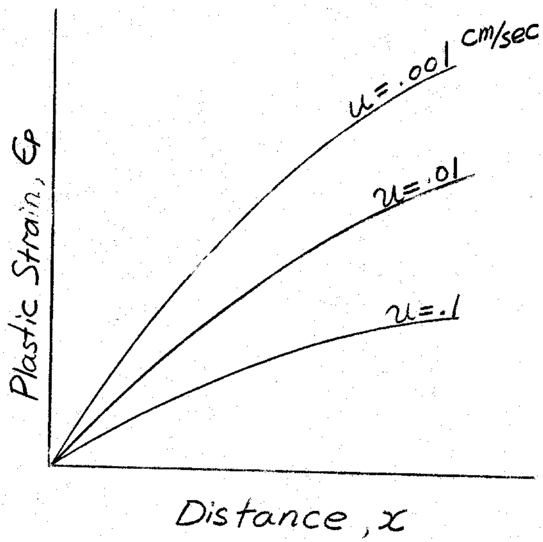

Figure 8.