A GAME-THEORETIC FRAMEWORK FOR MARKETING DECISION-MAKING USING ECONOMETRIC ANALYSIS

Submitted by: C. Anthony di Benedetto
Department of Management
Mc Gill University, Montreal, Canada

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ABSTRACT

Recent applications of game theory to the oligopoly have characterized the nature of the competition in an industry by examining payoff matrices and the strategies chosen by the players. In this study, a game-theoretic model of an oligopoly is developed, wherein the marketing-mix decisions made by the participating firms are represented as alternate strategic options. Econometric methods are employed to estimate the payoffs in the game matrices. Issues in model operationalization are discussed; then the model is applied to two real situations. In each case, the game matrix derived is used to <u>describe</u> the competitive nature of the industry (by examining the strategic decisions made over time), to <u>evaluate</u> the strategies chosen, given the intentions of the firms, and to <u>recommend</u> desirable strategies for the future.

RÉSUMÉ

La théorie des jeux, appliquée à l'étude des oligopoles, permet de caractériser la nature de la concurrence industrielle grâce à l'examen des sommes à gagner et des stratégies suivies par les joueurs. Cette étude développe un modèle d'oligopole basé sur la théorie des jeux et dans lequel les décisions de marketing prises par les participants sont représentées par des choix stratégiques. Les sommes à gagner sont estimées par des methodes économètriques. Le modèle est operationnel et appliqué à deux situations réelles. Dans chaque cas, on parvient à décrire la nature de la concurrence dans l'industrie; à evaluer les stratégies passées; et à recommander de meilleures stratégies pour l'avenir.

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C. Anthony di Benedetto
Pointe Claire, Canada
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TABLE OF CONTENTS

CHAPTER 1
INTRODUCTION

MAJOR ISSUE

Recent application of game theory to the oligopoly (by Shubik and Levitan 1980, e.g.) have indicated that it is possible to characterize the nature of the competition in an industry by examining the payoff matrices and the strategies chosen by each player. Such work, however, is theoretical in nature, and is not verified empirically. Also, its potential application in a real decision-making situation has not yet been explored.

Translation of this game-theoretical model into a useful decision model posed some interesting problems. In order to determine the behavioural intentions of the players by observing which equilibrium point is reached requires that the modeller know with certainty the appropriate payoffs and decision variables with which to build the game matrix. It also requires that equilibrium be reached, which may indeed not always be so. In addition, all players should have perfect information with regard to the payoffs in the matrix. (These considerations are examined in Chapter 5 of this paper.)

This study takes the approach that such a level of knowledge and information is unreasonable to assume in a typical industry; and thus that it is next to impossible to determine, simply by examining the selected equilibrium points, what the underlying behavioural intentions of the players are, or what in fact motivates the players. The pattern, through time, of the players' strategies (what marketing-mix decisions they made) is all that is known with certainty. However, if one knew both the pattern of strategies with the resulting payoffs, <u>and</u> the players' intentions, a decision-making model could be derived which would select the strategies most conducive to attaining the intended results. In short, where Shubik and Levitan indicate how the intentions of the firms in an industry are revealed through their behaviour, this paper adopts the viewpoint that given the intentions of the players, appropriate strategies may be recommended.

Despite this important philosophical difference, the theoretical model developed in this paper takes, as its base, these recent applications of game theory to economic analysis of competitive behaviour. The logical sequence used in developing the model from simple beginnings into a form which can treat adequately complex carryover and interaction effects is the subject of Chapter 4. This chapter also illustrates how up-to-date methods of econometric analysis can be implemented in estimating the payoffs of the game matrix.

GOALS OF STUDY

The reader will note that a clear distinction is made between the theoretical, mathematical general model in Chapter 4, and issues concerning its operationalization to specific industries, discussed in Chapter 5. This distinction is essential, as the goals of the study (as well as its limitations; see concluding remarks) may be divided this way.

A purely theoretical goal of this paper is the integration of the game-theory model framework (useful in decision-making) with econometric representation of the oligopoly. Such an integration would include adequate treatment of joint marketing-mix decisions (i.e., price and advertising decisions being made simultaneously by each firm); and would provide for carryover effects of such variables if and when appropriate. Ideally, the model should also be easily adaptable to given marketing situations.

Specific operational goals may also be spelled out. The model is visualized as being, basically, a decision-making tool to be used by a firm setting marketing-mix variable levels in an oligopoly. The operational objectives of the study may be easily conceptualized by considering the needs of such a firm. Major questions likely to be asked by the firm include:

1. Is it possible for us to understand better the competitive behaviour of the industry through analysis of previous strategic decisions?

2. How effective have our firm's marketing-mix strategies been in relation to those of our major competitors?

3. Can we make optimal marketing-mix decisions based upon an understanding of the nature of the competition and the payoffs attached to the various strategic combinations?

An ideal theoretical model, once operationalized (i.e., adapted to a specific industry situation), would thus be capable of performing three tasks, corresponding to these three questions:

1. Description of the competitive nature of the industry by examining the strategic marketing decisions taken over time;

2. Evaluation of the strategies chosen, given the intentoins of the firms; and

3. Recommendation for future strategic choice.

The operational model should be able to achieve these objectives. Note that determination of the intentions of the players themselves is not one of the objectives, nor is prediction of strategic choices in the future. The false assumption that econometric analysis of past behaviour will indicate with any certainty what firms will do in the future is not made, nor need it be. All that the model purports to do is to indicate,based on analysis of past behaviour, which strategies appear to be most in line with stated objectives.

THESIS FRAMEWORK

Immediately following this introductory chapter appear two chapters devoted to a review of significant prior research. Two separate literature review chapters are required, as concepts derived from two distinct trains of thought are employed in the development of the theoretical model.

Chapter 2, the literature review on game theory, traces the milestones in the development of this theory from the early work of von Neumann and Morgenstern (1947) to the more complex industry models of Shubik (1959a) and Shubik and Levitan (1980), among others. An illustration of the elementary concepts of game theory is given, as are definitions of the basic terms and simple solution concepts based on the Minimax Principle. Possibilities of cooperation among players in an n-player game ($n \geq 3$) are explored and cooperative solutions are also presented. The issue of rationality in game theory is examined: this leads to a discussion of equilibrium solution concepts for the oligopoly; and finally, to an illustration of how market behaviour could be revealed by examination of selected equilibria.

The literature on econometric modelling is exceedingly large and varied. In Chapter 3, the history of this literature as applied to marketing research is presented, from the classic articles of Vidale and Wolfe (1957) and Nerlove and Arrow (1962) to the empirical models developed by Palda (1964), Weiss (1968), Bass (1969), Beckwith (1972) and others. Among the most current extensions discussed will be those of Jagpal, Sudit and Vinod (1979; 1982), who illustrate the application of some very flexible and adaptable models. Other topics introduced here are: dynamic-adjustment and Koyck-type econometric models, stochastic modelling, pricing considerations, and preliminary game-theory applications.

In Chapter 4, the theoretical model is developed. This chapter may be regarded as the guideline to follow in constructing a game matrix appropriate for a given industry. After a discussion of some preliminary observations, a basic duopoly model based on Shubik and Levitan's game matrices is proposed as a starting point for model development. Subsequent sections of Chapter 4 illustrate how the model would be extended to cover a wider range of decision variables (first, multichotomous, then continuous independent variables); and how carryover effects could be incorporated into the model (alternate approaches are proposed for different circumstances). Finally, extensions for the n-firm oligopoly ($n \geqslant 3$) are developed. Chapter 4 retains a theoretical aspect throughout; problems of data availability and issues of parameter estimation are dealt with in the following chapter. To improve readability, the most complex mathematical manipulations are grouped into an appendix.

Chapter 5 investigates the operationalization of the general model in specific industries. Questions regarding both application and applicability of the theoretical model are raised here. Chapter 5 is divided into three parts. In Part 1, the important distinction between strategies and intentions (alluded to earlier in this introduction) is clarified, and possible drawbacks and problem areas in application are examined. Part 2 assumes that these problem areas have been dispensed with, and shows how the theoretical model would be developed and adapted to a given hypothetical industry. Part 2 culminates in the derivation of a game-matrix representation of the industry. Finally, Part 3 indicates how preferred strategies would be selected and recommended, and how the past behaviour of the firms would be analyzed.

The theoretical model is empirically tested in Chapters 6 and 7. Two industries have been chosen which proved to be relatively easily amenable to the analytical techniques described in

Chapter 4 (problems and considerations in industry selection are discussed in Part 1 of Chapter 5). These chapters also contain the results of the empirical analyses. Complete regression details for the analysis of Industry 1 are given in the appendix to Chapter 6.

Finally, in Chapter 8, the conclusions of the paper are collected and summarized. Difficulties which were encountered in model application are discussed, with implications for applicability in other industries. Theoretical and practical contributions made by the paper are examined. The technical and organizational validity of the model is also examined and practical considerations are discussed. Finally, a number of potentially fruitful avenues for future research are presented.

CHAPTER 2
LITERATURE SURVEY: GAME THEORY

GAME THEORY--INTRODUCTION

Origins and Areas of Development

Most writers on modern game theory trace the "genealogy" of this discipline back to 1944, the year of the appearance of the classic text by von Neumann and Morgenstern, Theory of Games and Economic Behaviour (second edition, 1947). In fact, seminal works by Borel (a series of notes, actually) appeared in a French-language journal in 1927 (English versions available; see Borel 1953a; b; and c). This study spurred further research by von Neumann, who gave his now-famous paper on game theory to the Mathematical Society in Göttingen the following year (von Neumann, 1928).

Nevertheless, the landmark work remains that of von Neumann and Morgenstern. A brief look at the authors' objectives in writing this book is helpful.

> "Our problem is not to determine what ought to happen in pursuance of any set of...*a priori* principles, but to investigate where the equilibrium of forces lies... We think that the procedure of the mathematical theory of games of strategy gains definitely in plausibility by the correspondence which exists between its concepts and those of social organizations." (von Neumann and Morgenstern 1947, pp. 42 - 43)

Thus, as might be expected of a text authored jointly by a mathematician (von Neumann) and an economist (Morgenstern), Theory of Games explored the applicability of a newly-developed branch of mathematics, based on the behaviours and beliefs of rational beings, to a social situation wherein rational individuals strive for the best possible outcomes under given circumstances. As is evidenced by Marshall's notion of utility maximization, and by Pareto's comparative statics and "Pareto-optimality", the concept of optimization of satisfaction given certain constraints has long been an area of investigation by economists (see Hicks 1946 and 1956 for concise histories of economic pursuit).

Economics is not the only discipline wherein the behaviour of rational, payoff-optimizing individuals in studied, however. Soon after the publication of Theory of Games, texts and journal articles appeared applying game theory to political science, warfare, international relations, urban planning and ecology. In addition, a literature on the related topics of gaming and simulation (see below) has emerged (for a pre-1975 literature guide, see Shubik 1975). Indeed, as of the mid-'70's, "literally thousands of articles and books" on game theory and gaming have been published (Shubik 1975, p. 5), including many written from an economic viewpoint. Evidently, clear definitions of all relevant concepts form an essential background to any further discussion. This topic will be returned to later.

A review of the current game-theoretic literature gives some indication of the breadth of application of this topic. Stern (1978) has investigated its applications in simple financial decision processes. Selten and Guth (1982) employ game-theoretical analysis in their business cycle model, developed to evaluate the outcomes of wage bargaining. Smith and Case (1975) model a two-firm sealed-bid auction as a nonconstant-sum game, and determine optimal strategies under conditions of either perfect or imperfect information. Other authors examine a wider range of issues. In Simaan and Cruz (1975), an arms race between two countries is modelled as a game and Nash-equilibrium strategies (see below) are found. Bacharach (1977) models the Battle of the Bismark Sea as a two-person, zero-sum game. This, in fact, is an especially good illustration of an application of game theory and will be returned to later. Bacharach also models management versus union strategies in a wage determination situation using a game in normal form (see below). Batlin and Hinko (1982) use game theory to determine optimal debtor and creditor strategies in a cash-management situation. And Macrae (1982) examines corruption and bribery in underdeveloped countries using a game-theoretic approach.

Game Theory in Economics: a Historical Perspective

Most of the applications of game theory to be examined in this chapter, however, are economic in nature. The history of game theory in economics is interesting in that its popularity among economists has not increased constantly over time. Rather, at least three phases of interest among economists can be delineated:

a) The von-Neumann-Morgenstern theory was immediately applied (with great enthusiasm) to the oligopoly--and in fact development stalled there.

> "As soon as the theory was pigeonholed (as being relevant to the oligopoly)...its popularity waned, for mathematical theorists turned their attention toward the axiomatic analysis of general equilibrium theory." (Schotter and Schwödiauer (S & S) 1980, p. 480)

Luce and Raiffa (1957) present the theoretical developments up to about this point.

b) Two works in the late '50's by Martin Shubik revived interest and investigation. His text Strategy and Market Structure (1959a) was written as an attempt at "a unified approach to the various theories of competition and markets, (where) the main set of techniques employed to achieve this end (were) those of game theory" (1959a, p. xi). A significant improvement of Shubik's models of competition was that they were dynamic in nature. Von Neumann and Morgenstern's theory of games was admittedly "thoroughly static, (although) a dynamic theory would unquestionably (have been) more complete and therefore preferable"(von Neumann and Morgenstern 1947, p. 46). Note the parallel to neo-classical economic theory: during the '30's and '40's, the "comparative statics" of Pareto were being refined into the "comparative dynamics" of Hicks and Keynes (see Hicks 1946).

Shubik's article "Edgeworth Market Games" (1959b) showed the equivalence between the "core" solution concept (discussed below) and the Edgeworth contract curve (see S & S 1980). This discovery sparked some activity in the investigation of the general equilibrium problem, which largely fell from interest by the late '60's.

c) More recently, game theory has been applied to "the design and operation of 'satisfactory' economic and social institutions" and to the "search for voting rules that yield satisfactory results or 'implement' social choice rules": these applications are not in the scope of this paper and the reader is referred to S & S (1980). Additionally, Shubik has expanded and extended his 1959 analysis of the oligopoly situation under linear demand conditions (Shubik and Levitan 1980). Other recent developments are examined later in this paper. It is this latter application of game theory (to the econometric analysis of the oligopoly) which is the starting point of the analysis conducted in this study.

Definitions

As a first step in reducing the huge volume of literature to manageable proportions, formal definitions of game theory and related disciplines are in order.

What is a game? Bacharach (1977) proposes a four-element description of the properties of a game:

"1. A well-defined set of possible courses of action for each of a number of players.

2. Well-defined preferences of each player among possible outcomes of the game and...among probability distributions or mixtures of its outcomes.

3. Relationships whereby the outcome (or at least a probability distribution for it) is determined by the players' choices of courses of action.

4. Knowledge of all of this by all the players.

Elements 1 and 3 are given in the rules of the game and 2 and 4 describe the players." (Bacharach 1977)

One recognizes that these elements (especially 2 and 4) may become restrictive in that they may limit the scope of usefulness of "pure" or von-Neumann-Morgenstern game theory. Knowing what a player prefers (element 2) implies an understanding of what that player considers "rational" behaviour, a major issue of which is returned to later in this chapter, after the basic terms and concepts are discussed.

Game theory is defined by Lucas as "a collection of mathematical models formulated to study decision making in situations involving conflict and cooperation...It is concerned with finding optimal solutions or stable outcomes when various decision makers have conflicting objectives in mind" (1972). Shubik adds that "it provides a formal language for the description of conscious goal-oriented decision-making processes involving one or more than one individual." It is also "a branch of mathematics which can be studied as such with no need to relate it to behavioural problems, to applications, or to actual games" (Shubik 1972b). This observation serves to differentiate game theory from gaming although the two topics are closely interwoven.

Gaming "of necessity employs human beings in some role, actual or simulated" (Shubik 1975). Gaming, then, is more concerned with the actual preparation, implementation and analysis of games for educational, experimental, operations, training, therapy or entertainment purposes (for an excellent discussion of gaming, see Shubik 1972a).

Simulation is frequently confused with gaming in the literature. Shubik makes the distinction that "simulation involves the representation of a system or organization by another system or model which is deemed to have a relevant behavioural similarity to the original system...(hence) all games are simulations. However, (one cannot make) ...the reverse categorization" (Shubik 1972b).

The Lucas definition of game theory, with its emphasis on decision-making, conflict and cooperation, is especially suitable for this study. Using this definition, it is clear that the models constructed in this paper are game-theoretic models, and therefore, according to Shubik (1972b) also simulations of oligopoly situations.

There are a number of game-theory models, distinguished by the number of players, the nature of the competition, the distribution of the players, et cetera. Most such models belong to one of three categories or "formal descriptions". An investigation of these categories is essential to the understanding of the workings and application of game theory.

FORMAL DESCRIPTIONS OF A GAME

Shubik (1964) lists two categories for representing games: the simple, compact normalized form, and the more detailed extensive form. Later writers (Rapoport 1970; Lucas 1972; S & S 1980, e.g.) list a third category named the characteristic function form or coalitional form. All are perfectly acceptable under game theory, and the choice among them depends upon the information requirements of the analyst.

> "When a detailed description of a situation of strategic interdependence is required, we may rely upon...the extensive form of the game...At other times, however, we may...examine only the actions or strategies available to the players and the payoffs associated with such strategies (normalized form) ...At yet other times...we may merely want to know what payoff ...a player or coalition of players can guarantee themselves if they act in concert (characteristic function form)."
> (S & S, 1980)

Rapoport (1970) elucidates this distinction among the categories best. He views the three categories as "levels of abstraction" achieved through "progressive generalization". At the first (extensive) level of abstraction, the "rules of the game" are represented; the extensive form "concentrates on the description of the game's dynamic sequential movement" (S & S, 1980). A game in extensive form is representable by a game tree (see below), indicating all possible outcomes for all possible plays by each player.

The second level of abstraction, the normalized (or <u>normal</u>) form, focuses not on the rules of the game and the game tree itself, but on the strategies available to each player. "The rules are important only to the extent that they determine the structure of the game tree and through it the available strategies and the outcomes associated with the combined strategy choices" (Rapoport 1970). A two-person payoff matrix (see below) is an example of a normal-form representation of a game.

Finally, at the characteristic function level, "the strategies available to the players are also abstracted from. The only givens in the game are now the payoffs which each the several possible coalitions can assure for themselves respectively"(Rapoport 1970). If a three- (or more)- person game is being played, the players can discover the best method of settling the "conflicts of interest" which quite naturally arise, by examining the characteristic functional forms.

> "What then is left? Only the question of how to find the best strategies...To answer this question, one must study the normal form of the game. To describe the strategies in terms of sequential choices conditioned on situations, one must study the extensive form, for...(it displays) the specific decisions which constitute a strategy." (Rapoport 1970)

All analyses in this study are carried out at the second level of abstraction (normal form), since it is at this level that alternate <u>strategies</u> are compared and contrasted by the individual players. However, all three forms are now briefly described.

Games in Extensive Form

The extensive form representation of a game is used when rules and details of play are to be examined. A game tree diagram as is found in Figure 2.1 serves as the best illustration of a game in extensive form. The game tree indicates that players P1 and P2 both must choose one of two strategies. The payoff (gain or loss) obtained by each player is given by the ordered pairs found on the "branches". Reading top to bottom, it can be seen that if P1 chooses Strategy 2 and P2 chooses Strategy 1, P1 will gain 11 units while P2 will lose 9.

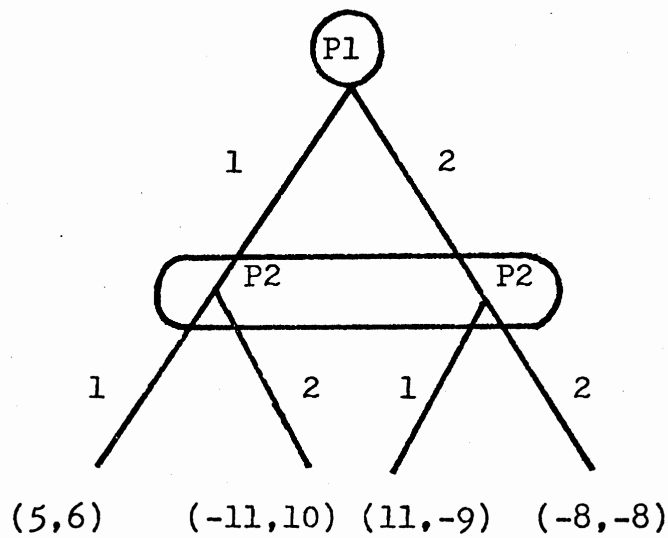


FIGURE 2.1
Source: Shubik (1972b), p. P-41

The game as drawn indicates that the players make their choices simultaneously. To illustrate this effect,

> "...both of the nodes marked P2 (are enclosed) by a curve which portrays an information set. It implies that the second player when called upon to move, cannot distinguish between the two nodes...he does not know what the first player has chosen." (Shubik 1972b)

If P2 were to have information concerning P1's choice before he were asked to move, the game tree would take on a slightly different form (see Figure 2.2). The two P2 circles represent two information sets in this case: i.e., either P2

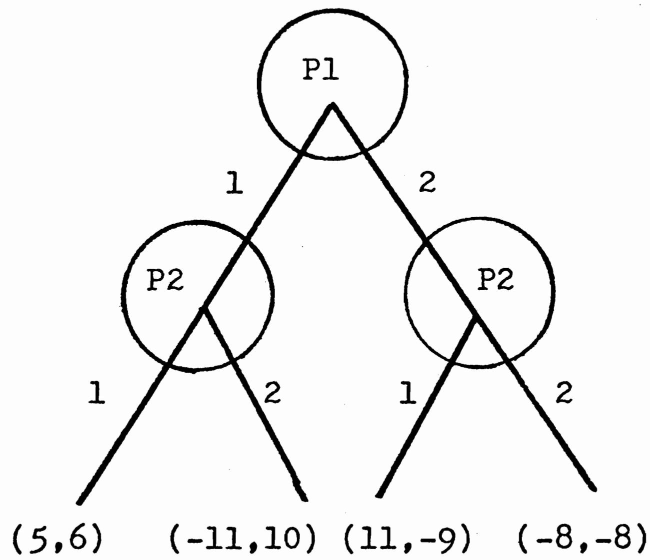


FIGURE 2.2
Source: Adapted from Shubik (1975), p. 14

knows that P1 has chosen Strategy 1, or that P1 has chosen Strategy 2. This game may be said to consist of <u>sequential</u> moves, whereas in that of Figure 2.1, the moves of each player are made <u>simultaneously</u>. Kuhn and Tucker (1950) explain:

> "(One may distinguish) between the occasion of the selection of one among several alternatives, to be made by one of the players or by some chance device, which is called a <u>move</u>, and the actual <u>choice</u> made in a particular play." (Kuhn and Tucker 1950, p. v)

Evidently, fine detail can be worked into a game tree; unfortunately, such trees tend to become large and complex very quickly as the number of strategies and/or players increases (try to imagine a game tree representing every possible strategy in a game of chess, or, for that matter, tic-tac-toe).

Games in Normal Form

Strategies and not rules are represented in the normal form. "The intuitive meaning of a <u>strategy</u> is that of a plan for playing a game" (Owen 1968). A game in normal form is typically represented by a <u>payoff</u> <u>matrix</u>. In a two-person game, the strategies available to each player appear as rows and columns of the matrix, with the outcomes appearing as the corresponding matrix elements. The game tree of Figure 2.1 may thus be rewritten in normal form as in Figure 2.3.

		P2 STRATEGY 1	P2 STRATEGY 2
P1 STRATEGY	1	5, 6	-11,10
	2	11,-9	-8,-8

FIGURE 2.3
Source: Shubik (1972b), p. P-39

Figure 2.3 illustrates a 2 x 2 game, whose properties have been well studied. Of course, 3 x 3 or higher matrices (representing more alternative strategies) and even n x n x n matrices (for more than two players) are also conceivable and follow the same pattern.

The normal form model can also capture the distinction between simultaneous and sequential moves. To convert the game tree of Figure 2.2 to normal form, one must recognize that P1 has two possible strategies, but P2 in fact has four, thanks to his advance knowledge of P1's decision:

1. Choose Strategy 1 regardless of P1's move.
2. Choose Strategy 1 if P1 chooses 1; otherwise choose 2.
3. Choose Strategy 2 if P1 chooses 1; otherwise choose 1.
4. Choose Strategy 2 regardless of P1's move.

The sequential-move game in normal form would appear as in Figure 2.4.

		P2 1	P2 2	P2 3	P2 4
P1	1	5, 6	5, 6	-11,10	-11,10
	2	11,-9	-8,-8	11,-9	-8,-8

FIGURE 2.4
Source: Adapted from Shubik (1975), p. 15

A sequential-move game formulation has been used, for example, to model a duopoly with price leadership (see Basar and Haurie 1982).

A diagrammatical representation of a game in normal form may be used in place of a matrix. The simultaneous-move game of Figure 2.3 may be illustrated by the diagram of Figure 2.5, where the vertices correspond to the cell entries. "Any point in or on the boundary of the area enclosed by the four lines...may represent the average payoff as a result of some extended series of play" (Shubik 1972b, p. P-39).

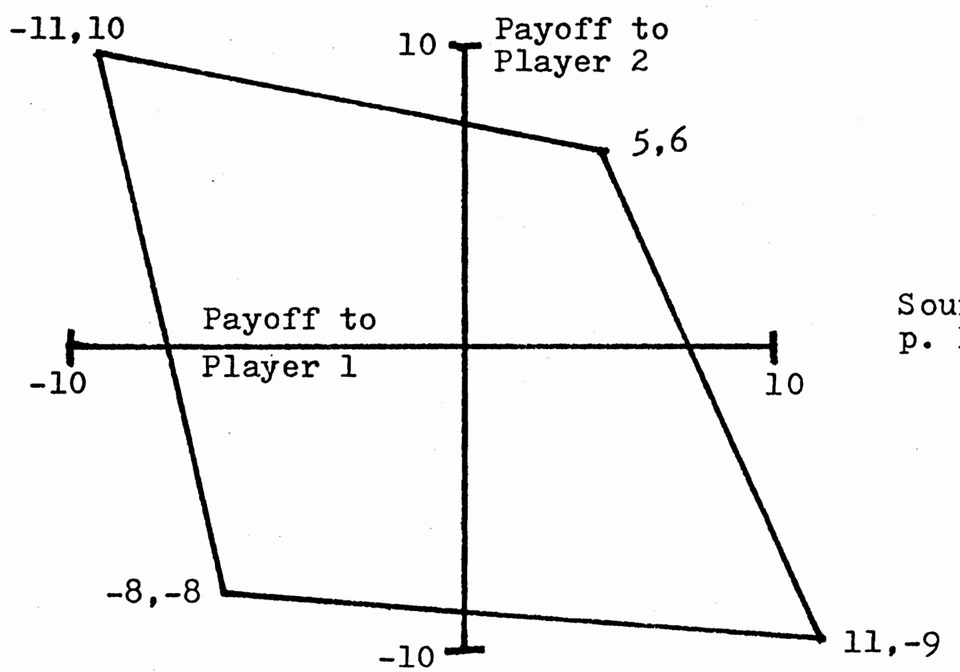


FIGURE 2.5

Source: Shubik (1972b), p. P-40

Games in Characteristic Function Form

The same game may be modelled in such a way as to highlight the coalitional possibilities available to the players. The characteristic function form of a two-person game is somewhat trivial: either the players work together or do not work together. For this simple case, the relevant functions would be as appear in Figure 2.6.

$$v(\ominus) = 0$$

$$v(1) = -8; \quad v(2) = -8$$

$$v(1,2) = 11$$

FIGURE 2.6
Source: Adapted from Shubik (1975), p. 16

Figure 2.6 shows that "a coalition of no one" ($\ominus$) is worth zero. "Player 1 acting by himself can guarantee no more for himself than (-8). Similarly, Player 2 can guarantee no more than (-8). If they act together then can obtain a total of 11." (Shubik 1975, p. 16)

To illustrate a somewhat more complex game: Figure 2.7 contains a 2 x 2 x 2 "matrix" representing the normal form of a three-person game, while in Figure 2.8, the equivalent characteristic function form is presented.

P2

P1	1	2
1	-1,-1,-1	-1,-1,-1
2	-1,-1,-1	5, 5, 5

2

P2

P1	1	2
1	10,10,10	-1,-1,-1
2	-1,-1,-1	-1,-1,-1

1

P3

FIGURE 2.7
Source: Shubik (1975), p. 13

$$v(\emptyset) = 0$$

$$v(1) = v(2) = v(3) = -1$$

$$v(1,2) = v(1,3) = v(2,3) = -2$$

$$v(1,2,3) = 30$$

FIGURE 2.8
Source: Shubik (1975), p. 16

Figure 2.8 clearly shows that cooperation of all three players can result in as much as 30 units of worth to distribute.

Once the game has been modelled, the solution (if one exists) may be found. A large literature has evolved regarding the existence and determination of solutions for the simplest case: the two-person, zero-sum game. By stripping away restrictions (first allowing nonconstant-sum payoffs; then considering more than two participants), succeedingly more complex situations can be visualized, for which additional solution concepts must be developed.

The game matrices developed in later chapters are all in n-person, nonconstant-sum form: therefore, some of the solution methods described in this section (Owen's mathematical method and Vajda's graphical method, e.g.) were not subsequently employed. However, a brief illustration of these and other solution methods serves to introduce many of the essential concepts (domination, equilibrium, stability, optimality) and also to indicate the level of complexity of game analysis to be applied herein.

TWO-PERSON GAMES

Pure Strategies

Consider a two-person zero-sum game; that is, a game wherein one player loses exactly what the other wins, such that the total of the payoffs is zero. The normal form of such a game may be represented as a matrix with only one element in each cell, that being the payoff to Player 1. (It is understood that P2 wins the negative of this amount.) Such a game is given in Figure 2.9.

P1 \ P2			
	5	1	3
	3	2	4
	-3	0	1

FIGURE 2.9
Source: Owen (1968), p.14

P1, the maximizing player, is at cross-purposes with the minimizer P2. Each knows that the payoff depends both on his own and on his opponent's strategic choice. Suppose that P1 and P2 are both rational and somewhat cautious players. Each may be thinking the following:

> "For every choice that I can make, I must fear that my opponent makes that choice which makes my gain... the smallest possible under the circumstances. Hence, if I make that choice which makes this smallest gain as large as possible, then I am as safe as I can ever reasonably expect to be." (Vajda 1956, p. 6)

This amounts to P1 choosing, as his strategy, that row which has the largest minimum value. In Figure 2.9, should P1 choose Row 2, the worst that can happen is that P2 chooses Column 2 and he (P1) gains 2 units. One may say that P1 attempts to maximize his row minima. Similarly, since P2's payoffs are the negatives of the numbers in the matrix, he is trying to minimize his column maxima. In Figure 2.9, P2 would choose Column 2, as the smallest column maximum is 2. This game, then, is easily solved: Element a_{22} (i.e., the second-row, second-column entry) is the largest value in its corresponding column and also the smallest in its corresponding row. It is thereby called a saddle point: the strategy pair chosen is said to be in equilibrium. "A game is in equilibrium if no player has any positive reason for changing his strategy, assuming that none of the other players is going to change strategies" (see Owen 1968, p. 7). The optimal strategies for each player to employ are pure strategies: i.e., each player plays the same strategy and obtains the same payoff no matter how many times the game is played. The game may be said to be stable; and its value, the payoff to P1, is 2.

Mixed Strategies

Consider now the game of Figure 2.10. Here, there is no saddle point, as the largest row minimum (2) is not equal to the smallest column maximum (3). This does not imply, however, that the game is not stable. A kind of stability can still be achieved, but only if combinations of strategies, along with probabilities of selection, are considered. These combinations are known as mixed strategies, or "probability distributions on the set of... pure strategies" (Owen 1968, p. 16). Thus a mixed strategy of (.25, .75) for P1 indicates that, if the game of Figure 2.10

	P2		
P1	4	1	3
	2	3	4

FIGURE 2.10
Source: Vajda (1956), p. 12

were repeated many times, he should choose Row 2 three times as often as Row 1. (See also Vajda 1956, Chapter 1.)

In a mixed-strategy situation, P1 is trying to maximize his "gain-floor" (the weighted average of his expected payoffs against P2's pure strategies), while P2 is minimizing his "loss-ceiling" (vice versa). (See Owen 1968, pp. 16 - 17.) It is easy to show that P1's gain-floor is less than or equal to P2's loss-ceiling (P1 cannot win more than P2 if they are the only two players). What is noteworthy (and in fact was the cornerstone discovery of von Neumann and Morgenstern's <u>Theory of Games</u>) is that these two values are equal. Stated algebraically,

$$\text{Max}_x \text{Min}_y \, \emptyset \, (x,y) = \text{Min}_y \text{Max}_x \, \emptyset \, (x,y),$$

where $\emptyset\ (x,y)$ is the payment of P2 to P1 (see Kuhn and Tucker 1950, p. vi). The original proof of this so-called <u>Minimax Theorem</u> or <u>Principle</u> is given in Owen (1968), with a short alternate proof found in Weyl (1950). This theorem guarantees the existence of an optimum strategy for a two-person game.

In the game of Figure 2.10, the optimum solutions for P1 and P2 are (.25, .75) and (.5, .5, 0) respectively (methods for calculation of these proportions are given later). P2 therefore can flip a coin to determine which of pure strategies 1 or 2 to use at any play of the game. Note that he is advised never to use Strategy 3. This is reasonable, as he can always do better by playing Strategy 2. This illustrates the notion of <u>domination</u>; P2's third strategy is dominated by his second. This kind of game can also have a value; and it will be shown below that the value of the game in Figure 2.10 is 2.5 units (expected payoff to P1 per play in the long run). See also Vajda (1956), p. 10.

Solution Methods

It is desirable to have a method of solution, therefore, for two-person games, which would illustrate both a) the optimum strategies (be they pure or mixed) for each player, and b) the value of the game. A number of solution methods exist, and are listed below (using the framework of Owen 1968).

a) Saddle points: of course, if the game has a saddle point, the corresponding pure strategies are the optimum strategies, and the saddle point's value is the value of the game.

b) Domination as outlined above may be used to simplify a larger matrix. Owen (1968) gives the following example (p. 26).

	P2			
	2	0	1	4
P1	1	2	5	3
	4	1	3	2

Clearly the fourth column is dominated by the second. If it is discarded, one is left with the 3 x 3 matrix

	P2			
	2	0	1	~~4~~
P1	1	2	5	~~3~~
	4	1	3	~~2~~

Note how domination may be used in repetitive fashion. Now Row 1 is dominated by Row 3, leaving behind

$$\begin{array}{c|c|c|c|c|}
 & & P2 & & \\
\hline
 & \cancel{2} & \cancel{0} & \cancel{1} & \cancel{4} \\
\hline
P1 & 1 & 2 & 5 & \cancel{3} \\
\hline
 & 4 & 1 & 3 & \cancel{2} \\
\hline
\end{array}$$

where Column 3 is dominated by Column 2:

$$\begin{array}{c|c|c|c|c|}
 & & P2 & & \\
\hline
 & \cancel{2} & \cancel{0} & \cancel{1} & \cancel{4} \\
\hline
P1 & 1 & 2 & \cancel{5} & \cancel{3} \\
\hline
 & 4 & 1 & \cancel{3} & \cancel{2} \\
\hline
\end{array}$$

The remaining matrix is 2 x 2, which fortunately has a simple algebraic solution.

c) Solution to 2 x 2 games (see Owen 1968, pp. 27 - 29, for the full proof): Consider a 2 x 2 game,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$$

where $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ are the optimum strategies and v is the value of the game. It can be shown that the following theorem holds: "If A does not have a saddle point, its unique optimal strategies and value will be given by

$$X = \frac{JA^*}{JA^*J^t} \; ; \qquad Y = \frac{A^*J^t}{JA^*J^t} \; ; \qquad v = \frac{/A/}{JA^*J^t} \; ;$$

where A* is the adjoint of A, /A/ the determinant of A, and J, the vector (1,1)." (Owen 1968, p. 29)

d) <u>Graphical methods for 2 x n and m x 2 games</u>: Vajda (1956) provides a graphical solution for the matrix of Figure 2.10, which is applicable whenever at least one player has only two pure strategies. Note in Figure 2.11 that three lines are constructed: Q_1R_1, Q_2R_2, and Q_3R_3, each corresponding to one of P2's strategies. P1's mixed strategies are represented by points along the line between I and II. Suppose P1 uses a mixed strategy represented by Point S. "Whatever (P2) does, (P1) will then obtain at least the amount represented by the height of the lowest intersection of the vertical through S with a line corresponding to one of (P2)'s strategies" (Vajda 1956, p. 13). The heavy line therefore represents P1's minimum payoff for any combination of strategies; and P1 maximins this value with mixed strategy M, where the heavy line reaches its maximum. P1's optimum strategy is represented by M, which is at a point three-fourths of the way from I to II; thus, his optimum mix is (.25, .75) as seen before. The value of the game is the length of MV, which is 2.5 in this case.

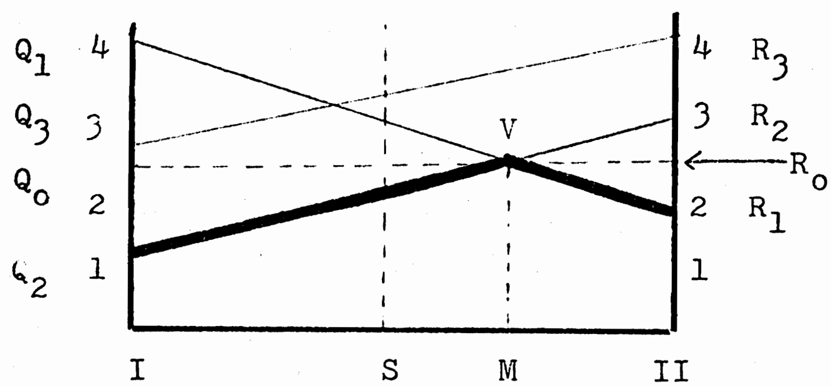


FIGURE 2.11
Source: Vajda (1956), p. 12

P2's optimum mix is represented by the ratio Q_0Q_1/Q_0Q_2 (or, equivalently, R_0R_1/R_0R_2) which equals 0.5 in this case. P2 should therefore use his strategies 1 and 2 in equal proportions. Since strategy 3 is always dominated (i.e., never forms part of the heavy line), P2 will never choose it (as has already been shown). Thus, P2's optimum mix is (.5, .5, 0).

Vajda (1956, pp. 15 - 19) shows another slightly more complex graphical solution for P2's optimum strategy.

e) Linear programming: If no simple method is applicable, the optimum strategies and values of two-person games may be determined using linear programming techniques. Both Vajda (1956) and Owen (1968) demonstrate the use of the simplex method in solving complex games; Owen also describes the simplex algorithm and gives suggested rules for the "pivoting" operation used in the simplex technique. Both authors demonstrate adequately the use of both algebraic and graphical linear-programming techniques. (See Owen 1968, Chapter 3 and Vajda 1956, Chapter 4.)

An Illustration

Bacharach (1977) provides an application of game theory to a military situation: the Battle of the Bismark Sea during the Second World War. The Japanese were to move in a westerly direction from a port in New Britain, sending troops and suppiles in a convoy. They could travel either via a north or a south route. The Americans had to decide whether to reconnoiter along the north or south route. The south route had higher visibility and therefore the Japanese might be subject to an attack of longer duration if spotted immediately. The strategic combinations are modelled as a zero-sum game, where the payoffs listed are the durations of bombing attacks upon the Japanese (in number of days).

		JAPANESE	
		NORTH	SOUTH
AMERICANS	NORTH	2	2
	SOUTH	1	3

If the Americans play N (choose "north"), they receive a payoff of 2 regardless of what the Japanese do. Playing S may result in a gain of 1 or 3. The security level, or assured payoff, of S is 1 in that this is the worst possible outcome for the Americans.

Similar reasoning indicates that the security levels for the Japanese are -2 and -3 for N and S respectively.

The Minimax Principle states that each player maximizes his minimum payoff (i.e., maximizes his security level). By this line of reasoning one would expect both players to play their "north" strategies. This indeed is what happened in the spring of 1942. "In military terms, (the Minimax Principle) focuses on the enemy's capabilities rather than his intentions: The former are known; the latter can at best be guessed. This was the ruling doctrine in American tactics..." (Bacharach 1977) The fact that the "enemy's" intentions are usually unknown in an oligopoly situation also has important implications for the theoretical model to be developed in this paper.

Non-Constant Sum Games

The above discussion of two-person games pertains mainly to the constant-sum situation; that is, when the payoffs of all strategy combinations are equal (in the zero-sum case, of course, the constant is zero). The situation is markedly different when the payoffs are not all equal (as in Figure 2.3). In such a situation, there may be motivation for the players to cooperate or collude (see next section); this may depend upon the rules of the game as well as the players' "personalities". Should they choose not to cooperate, a solution concept such as Nash's Noncooperative Equilibrium Point may be applicable to determine the outcome of the game (Nash 1951; also well-summarized in Shubik 1972b). The main condition of this solution method states that "a point $(\overline{s_1}, \overline{s_2})$ is an equiilibrium point if it satisfies...

$$\text{Max}_{s_1} P_1(s_1,\overline{s_2}) \text{ implies } s_1 = \overline{s_1};$$
$$\text{Max}_{s_2} P_2(\overline{s_1},s_2) \text{ implies } s_2 = \overline{s_2}",$$

where s_i = any strategy of player i; and

$\overline{s_i}$ = a specific strategy of player i. (Shubik 1972b)

Shubik (1962) showed experimentally that if, in addition to this main condition, certain "extra conditions" were also met, the ability of this solution method to predict an equilibrium point was enhanced. These conditions are:

1. The equilibrium strategy for an individual should dominate all other strategies.

2. The equilibrium should be socially rational or Pareto-optimal.

3. The equilibrium should be unique.

4. The equilibrium should not employ mixed strategies. (Shubik 1972b)

The game in Figure 2.3 satisfies all of these conditions except the second, as both players would be better off if each switched strategies (the game in Figure 2.3 is in fact a version of the well-known "Prisoner's Dilemma"). The game in Figure 2.12 clearly satisfies all conditions and is therefore more likely to reach an equilibrium at (5,5).

	P2	
P1	5,5	3,4
	4,3	0,0

FIGURE 2.12
Source: Shubik (1972b), p. P-45.

The Nash noncooperative solution will be revisited later in this chapter and in subsequent chapters.

COOPERATIVE GAMES

In two-person, nonconstant-sum games, and in n-person games ($n \geq 3$), the possibility of cooperation among players must be taken into account; that is, "binding contracts can be made,

...correlated mixed strategies are allowed, and...utility can be transferred from one player to the other (although not always linearly)" (Owen 1968, p. 140).

Whole books have been written on the subject of cooperation among players (see Rapoport 1970, e.g.), as the solution methods are many and varied. For the purposes of this introduction, some of the main concepts are summarized.

Enlarging the Attainable Set

If cooperation among players is illegal or impossible, only a certain number of payoff pairs are attainable. If players play only pure strategies, the attainable set would comprise the payoffs read directly off the game matrix. Intermediate points in the attainable set are added when mixed strategies are also considered. Given a payoff matrix as appears in Figure 2.13, the corresponding attainable set is the shaded region in Figure 2.14.

		PLAYER 2	
		1	2
PLAYER 1	1	2, 1	-1,-1
	2	-1,-1	1, 2

FIGURE 2.13
Source: Bacharach (1977), p. 84

In Figure 2.14, the outcomes of playing pure strategies are shown as points J, K and L. The derivation of the curved line KFGJ is given in Bacharach and is based on consideration of the possible mixed strategies. If each player tosses a fair coin and plays Strategy 1 if it shows heads, the expected utility of each player would be 0.25. Thus, (0.25, 0.25), which is Point F, is on the curve. If each plays Strategy 1 with 0.67 probability, the utilities expected by P1 and P2 are 0.571 and 0.286 respectively: (0.571, 0.286) is the location of Point G.

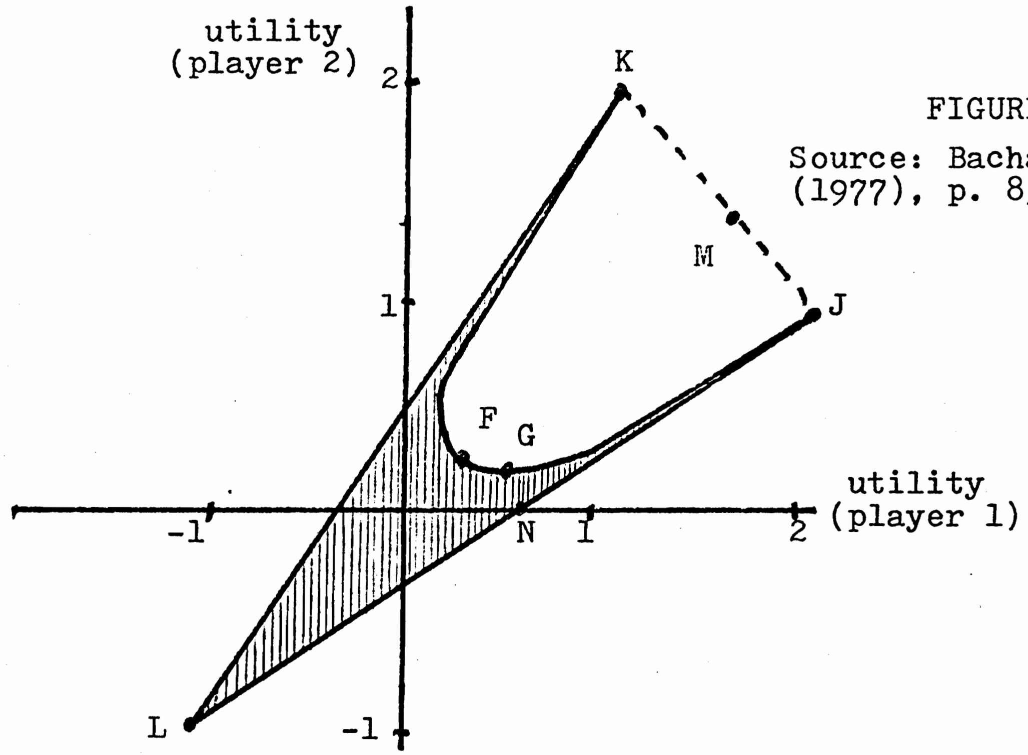


FIGURE 2.14

Source: Bacharach (1977), p. 85

Cooperation between the players increases the size of the attainable set. If the strategies of the two players are perfectly correlated, points on the line JK are attainable; imperfect correlation results in the attainment of points in the intermediate region KFJM. The attainable set under conditions of cooperation between players is thus the <u>convex</u> <u>hull</u> or region enclosed by the points J, K, L: i.e., the points corresponding to pure strategies. One may construct the convex hull by placing pins at each pure-strategy payoff pair and drawing a string around the pins.

> "Equivalently, the convex hull...is the set of all <u>probability</u> <u>mixtures</u> of pure-strategy payoff pairs; or, finally, it is the set of payoff pairs obtained by all probability mixtures of pure-strategy pairs...By cooperation the players can achieve all such points, because they can mix strategy pairs in any way...without cooperation correlated mixtures are ruled out." (Bacharach 1977)

Bargains and the Nash Cooperative Solution

Such a game may be thought of as a simple kind of bargaining game: a game in which the players wish to perform some kind of trade or transaction, and in which the players may choose to make "no trade" or "no transaction". If the players so choose, then their payoffs would equal their security levels; that is, the players "return home" with the same utilities that they had when they started to bargain. Implicit in this line of reasoning is that the players will not consummate a transaction if at least one feels that he would be worse off than had the transaction not taken place. The point in payoff space correspon-ding to the no-trade situation may be called the status quo: the status quo point is of fundamental importance in determining the Nash bargaining solution.

Suppose that R is the cooperatively attainable region and that (u_1, u_2) is a point in R. Nash's arbitration solution to a bargaining game is the point (u_1, u_2) where $(u_1-s_1)(u_2-s_2)$ is maximized, s_1 and s_2 being the status quo levels of players 1 and 2 respectively. The quantities $(u_1 - s_1)$ and $(u_2 - s_2)$ may be called the utility gains. Additionally, the optimum point is subject to the constraints $u_1 \geqslant s_1$; $u_2 \geqslant s_2$.

The Nash arbitration solution is appealing in that it is the only such solution which satisfies all of the four following conditions (adapted from Bacharach 1977):

1. Pareto-optimality (already mentioned);
2. Interpersonal non-comparability: if one player's utility function is rescaled, the solution is not affected;
3. Symmetry: if the game is symmetric, the solution ought to give the players equal payoffs in terms of the sym-metrizing utility indices;
4. Independence of irrelevant alternatives: adding or sub-tracting irrelevant alternatives does not change the solution of the game.

Such an arbitration solution may be applicable in a situation where the players have a clear status quo level equal to their utilities before the bargaining begins. If not, a more general model would have to be constructed. Fortunately, Nash has also provided a general cooperative solution to the two-person game. The two basic ideas behind this solution are as follows:

> "1. All cooperative games are in the final analysis non-cooperative; there is always a latent non-cooperative game behind the cooperative goings-on.
> 2. A bargaining game is a cooperative game in which it is possible to define a determinate rational solution by exploiting the fact that in a bargaining game the strategies of the latent non-cooperative game are singular." (Bacharach 1977)

One attempts therefore to find the latent non-cooperative game, and then to solve it. (Note: much of the following derives very closely from Bacharach 1977, Chapter 5, which gives an excellent description of two-person and n-person cooperative games.)

Interestingly, this non-cooperative game in the general case corresponds to the status quo point in the Nash bargaining case. In this simpler case, "no trade" could be viewed as a <u>threat</u>: in fact, it is the only threat which either player can impose on the other. Thus, the status-quo point is also the unique threat point in this case. In the Nash cooperative solution, this simple threat point is no longer fixed. Each player has a range of threats which he could impose on his opponent. If Players 1 and 2 choose threats t_1 and t_2, and the resulting payoffs would be v_1 and v_2, then (v_1, v_2) may be considered the "status quo" of a <u>pseudo-bargaining game</u> defined by threats t_1 and t_2. This game may be solved as if it were a typical Nash bargaining game, and thereby it has a unique solution. All that remains is to determine what the optimal threats t_1 and t_2 should be. Fortunately, this <u>threat game</u> is non-cooperative--it is indeed the latent non-cooperative game underlying the original cooperative game. Bacharach concludes:

> "Nash shows that the present game--the general multi-threat cooperative game metamorphosed into non-cooperative form--always has as one of its equilibrium pairs a pair $((t_1*,d_1*),(t_2*,d_2*))$ in which the demands d_1*, d_2* constitute the Nash solution of the bargaining game whose status quo is given by t_1*, t_2*! Nash takes this starred equilibrium pair to be his solution of the cooperative game." (Bacharach 1977)

Incidentally, Nash's solution suggests that the threats should never be carried out. Under complete information and assumptions of rational behaviour, players would never carry out the threats because the agreed-upon demands d_1* and d_2* could not by definition make either player any worse off than had the threat been carried out.

Solutions for n Players: the Core Solution

The Nash cooperative solution is applicable in the two-player case. Considering more than two players increases the complexity of the game immensely as players may team up and form factions in many different ways. The core solution method is one of the easiest to apply to an n-person cooperative game and is discussed first.

To understand the "core" solution concept, the term imputation must be introduced. "An imputation is a utility distribution exhausting the worth of (a) coalition and assigning to each player at least the amount that he can guarantee for himself without cooperation" (S & S 1980). Thus, the actual payoff to each player as a result of the coalition will be one of the possible imputations, and will depend on the bargaining power and behaviour of the players involved. Now "the core of a game...is defined as a set of imputations (which)...are not dominated via any coalition" (S & S 1980). Thus, if the core is non-empty, one of its members is likely to be the cooperative solution (i.e., the "split" of payoffs agreed upon by all players in the coalition). The core is best illustrated by numerical example. Figure

2.15 lists a game in characteristic function form. If the players are rational, they will not enter a coalition unless they can assure themselves of at least the payoff each can obtain individually (i.e., the security level); hence, all of the following inequalities must hold:

$$a_1 + a_2 \geqslant 2;$$
$$a_1 + a_3 \geqslant 2;$$
$$a_2 + a_3 \geqslant 2;$$
$$a_1 + a_2 + a_3 = 6.$$

All imputations that can satisfy these inequalities are therefore in the core: it is easily verified that there are many such imputations: (2, 3, 1), for example (see Shubik 1972b).

$$v(1) = v(2) = v(3) = 0;$$
$$v(1,2) = v(1,3) = v(2,3) = 2;$$
$$v(1,2,3) = 6.$$

FIGURE 2.15
Source: Shubik (1972b), p. P-51

Bacharach (1977) summarizes the above considerations into a three-part definition of the core, which he views as a "generalized (von Neumann-Morgenstern) solution set":

"1. (Pareto-optimality) The group must receive at least its security level...

2. Each individual must receive at least his security level...(and)

3. Each coalition must receive at least its security level." (Bacharach 1977)

The similarity to the von Neumann-Morgenstern principle becomes clear: "The principle of rationality...--that a decision-unit should never accept less than its noncooperative security level--is by no means new. All that is new is the application of this idea to decision-units of arbitrary size." (Bacharach 1977) That is the implication, at any rate, of part 3 of the abovementioned definition.

Many games have an empty core. If the second condition of Figure 2.15 were replaced by

$$v(1,2) = v(1,3) = v(2,3) = 5,$$

it can be shown that no division of the six "utils" could satisfy all players at once. In such a situation, other solution concepts must be applied. Among these are: stable sets, the Shapley value, and the bargaining set.

Stable Sets

The stable sets solution, or von Neumann-Morgenstern solution is sometimes applicable and is based on the notions of internal and external stability. "Consider a set of imputations $J = ((x_1, x_2, \ldots, x_n))$ with the following properties:

1. No imputation in J dominates any other imputation in J ('internal stability');

2. If x is an imputation not in J, then there exists at least one imputation in J which dominates x ('external stability'). Such a set of imputations constitutes (a stable set)" (Rapoport 1970). Using an example again to illustrate, consider the game in Figure 2.16.

$$\begin{gathered} v(\emptyset) = 0 \\ v(1) = -2;\ v(2) = -4;\ v(3) = -4 \\ v(2,3) = 2;\ v(1,3) = 4;\ v(1,2) = 4 \\ v(1,2,3) = 0 \end{gathered}$$

FIGURE 2.16

Source: Rapoport (1970), p. 97.

It can be shown that the set of imputations J where $x_3 = -3$ and $x_1 + x_2 = 3$ (x_1 and $x_2 \geqslant 0$) possess internal and external stability and thus constitute a stable set, as does the set J' where $X_3 = -1.5$ and $x_1 + x_2 = -1.5$ (see discussion in Rapoport 1970, pp. 97 - 98). Thus, a game may have more than one stable set.

The Shapley Value

The Shapley value concept, unlike the stable sets method, finds a unique distribution of the payoffs. Again, the starting point of the analysis is the characteristic function form of the game. An axiomatic development is given by Shapley (see Owen 1968, p. 180) which shall not be reproduced here. Rather, it is easier to follow Shubik's simplified explanation:

> "The value is calculated by considering all of the different ways in which a player might enter a coalition. Each player is assigned the increment of wealth that his presence brings to the coalition...All of the increments for each player (are summed)...and (arranged) over all of the coalitions. In other words, the value is a measure of the average incremental worth of each individual." (Shubik 1972b, p. P-51)

Consider the game of Figure 2.17. Also, assume that all orders of forming coalitions are equally probable; i.e., the event "P2 joins P1 first, then P3 joins the coalition" has a probability (1/6), as does each other possible permutation.

$$\begin{gathered} v(\emptyset)=0; \\ v(1)=0;\ v(2)=1;\ v(3)=1; \\ v(2,3)=3;\ v(1,3)=4;\ v(1,2)=5; \\ v(1,2,3)=16 \end{gathered}$$

FIGURE 2.17

Source: Rapoport (1970), p. 106

Now, all the increments (values added) gained through joining coalitions are calculated for each player. In this example, P1 has probability (1/3) of joining an empty coalition (i.e., being first in). The incremental value here is $v(1) - v(\emptyset) = 0$. He has a (1/6) probability of joining P2, a (1/6) probability of joining P3, and a (1/3) probability of joining the coalition P2-P3. The corresponding incremental values are, respectively,

$$\begin{aligned} v(1,2) - v(2) &= 5 - 1 = 4 \\ v(1,3) - v(3) &= 4 - 1 = 3 \\ v(1,2,3) - v(2,3) &= 16 - 3 = 13. \end{aligned}$$

Player 1's Shapley value is therefore

$(1/3)(0) + (1/6)(4) + (1/6)(3) + (1/3)(13) = 5.5.$

The other players' values are calculated similarly as

$v(P2) = 5.5; \qquad v(P3) = 5.$

Note that the sum of the Shapley values equals the value of the "grand coalition" which is 16. (See Rapoport 1970, pp. 106 - 108.)

The Shapley value method thus yields a unique imputation (in this case, (5.5, 5.5, 5)) which gives an indication of the relative strengths of the bargaining positions of the players. This solution concept "has built into it a certain equity principle...(and) might therefore be a strong contender for the status of a 'normative' solution; i.e., one which 'rational players' ought to accept." (Rapoport 1970)

The Bargaining Set

The bargaining set, which is only mentioned in passing here, is "the set of all individually rational payoff configurations in which no player has a justified objection against any other member of the same coalition" (Rapoport 1970, p. 119). Points in the bargaining set possess "a certain form of stability or bargaining stalemate" (Shubik 1972b, p. P-51). Further details are given in Chapter 6 of Rapoport (1970).

This discussion shows that there are a number of ways in which cooperative behaviour may be modelled. In an oligopoly situation, cooperation (as collusion) is illegal. However, collusive solutions are determined for both industries in the empirical-results chapters. This study neither recommends the use of such strategies in real-life situations, nor implies that such behaviour actually exists. Nevertheless, it is instructive to compare the results obtained by the various noncooperative solution concepts to that which could be attained allowing cooperation, to determine how much of an improvement (if any) could be achieved.

RATIONALITY AND THE MINIMAX THEOREM

Rationality Reexamined

Up until now the assumption has been made that all players act "rationally", which is taken to mean that they "maximize the minimum possible gain" or "maximize the security level". The Minimax Principle may be applied extensively to find equilibrium strategies in situations of "rational behaviour". However, as Shubik points out,

> "('Game theory man') has no personality; he really does not learn anything or change his opinion in the course of play. He invariably knows all of the rules of the game; he usually is able to compute and calculate accurately at great speed. He is assumed always to know what he wants and to know what the others want." (Shubik 1972b, p. P-52)

Clearly it is time to return to the unanswered question asked earlier in the chapter: how in fact should rationality be defined? Even neglecting the possibility of cooperation among players, there are a number of conceivable alternative behavioural patterns which may or may not result in the attainment of the so-called minimax equilibrium point. The players may be "sadistically minded" and seek to do as much damage to their opponents as possible (presumably without ruining themselves in doing so). Or the players may play a "cut-throat" game where what matters is not how well they do, but how much better they do than their opponents (this is seen in industry where firms strive to "beat the average" industry profit or market share). Other alternatives are also possible (see Shubik and Levitan 1980, and also the last parts of this chapter).

Evidently potential difficulties may arise from the implicit assumption that the Minimax Principle governs players' actions, or the equivalent assumption that players who do not seek to maximize their security level are acting irrationally.

Bacharach (1977) regards the stringent limitations caused by strict adherence to the Minimax Principle as one of the "so-called failure(s) of game theory".

> "It is possible that the theory has selected the wrong criteria for deciding what is to count as 'rational', and thus as a solution...the reader must judge for himself in the end. What constitutes rational choice is evidently a question a priori; it belongs to philosophy. It certainly cannot be answered by game theory itself, whose results are arrived at by deductive arguments starting from criteria of rationality which have the status of postulates" (Bacharach 1977).

It is necessary, therefore, to take one further step before a useful application of game theory (especially to the oligopoly) can be developed; that is, to acknowledge the existence of alternate "solution concepts" which correspond to the various possible behavioural patterns among players, of which a few have been listed above. Thus, striving to beat-the-average or to do as much damage as possible to the opponent (or even to cooperate with him) are no longer seen as "irrational" behaviour, but entirely plausible rational alternatives under different behavioural assumptions.

Some authors have strived to incorporate further adjustments or improvements to the basic theory. In the case of Kadane and Larkey (1982a; 1982b), the rebuttal they receive from one of the top game theorists is as enlightening (perhaps more so) as their original suggestion. Basing their work on that of Savage (1971), they discuss the merits of implementation of subjective probability in game theory analysis. The subjectivist viewpoint suggests that the "decision-maker has a subjective probability opinion with respect to all of the unknown contingencies affecting his payoffs" (Kadane and Larkey 1982a). According to the subjectivist view, the game theorist employs "rules-of-thumb (in)...forming (his) prior (probabilities) about (his) opponent's likely behaviour in certain simple game situations" (1982a). However, Harsanyi (1982a; 1982b) takes great exception to such a position. He maintains that game theory's strength lies in its use of "normative 'solution concepts' based on suitable rationality postulates and...(assumptions of players' actions) in accordance with the relevant solution concept", and that the use of subjective methods in assigning priors "(amounts) to throwing away essential information; viz., the assumption... that the players will act rationally and will also expect each other to act rationally." (Harsanyi 1982a). He continues:

> "Indeed, their approach would trivialize game theory by depriving it of its most interesting problem, that of how to translate the intuitive assumption of mutually expected rationality into mathematically precise behavioural forms (solution concepts)." (Harsanyi 1982a, p. 121).

The disagreement is, as Harsanyi (1982b) puts it, "about the very foundations of game theory". The point of Kadane and Larkey's paper, that "the empirical data...supports the conclusions that opponents tend to be 'actually or potentially irrational'" is well taken, as is their suggestion that "further psychological research (be made) on actual behaviour of people making decisions in game situations" (1982b). However, normative game theory has been a useful and valuable decision-making model, as Harsanyi points out.

Additional evidence has also been gathered which would cast doubt on the usefulness of the Minimax Principle in finding equilibrium points. Aumann and Maschler (1972) start by stating that "arguments in favour of (the attainment of an equilibrium pair of strategies) are sometimes less than convincing." Taking the simple game shown in Figure 2.18, the equilibrium strategies are easily shown to be mixed: (.75, .25) for P1 and (.5, .5) for P2, with a resulting payoff of (.5, .75). However, to guarantee himself of a payoff of .5, P1 would be advised to play a non-equilibrium mixed strategy, namely (.5, .5); similarly, P2 ought to play (.25, .75). Thus, maximin strategies which are not in equilibrium appear to

	P2	
P1	1,0	0,1
	0,3	1,0

FIGURE 2.18

Source: Aumann and Maschler (1972), p. P-55

be preferred. Aumann and Maschler explain:

> "The reason for the curious phenomenon...is that to achieve equilibrium, each player must play <u>against</u> his opponent rather than <u>for</u> himself...Each player's equilibrium strategy depends chiefly on the magnitude of the entries in the <u>other</u> player's matrix; whereas his maximin strategy depends exclusively on his own matrix" (1972).

They also propose a lengthy example (see original paper) which illustrates that if a time dimension is ignored in constructing the normal form of a game (e.g., if Stackelberg-type price leadership is ignored and players are assumed to make price decisions simultaneously), the minimax solution will not necessarily be the correct one. In their concluding remarks, Aumann and Maschler acknowledge that previous authors have commented on the importance of the time gap, especially in the context of cooperative games. However, this example shows that even in the simplest two-person game with incomplete information, wrong conclusions may be generated if the time gap is not taken into account, or if other behavioural tendencies are ignored.

<u>Other Caveats to Game Theory</u>

Aumann and Maschler have warned about the problems inherent in accepting the behavioural assumptions of the Minimax Principle at face value. Shubik (1975) lists additional precautions which must be taken to ascertain whether a game-theoretic approach is justified at all in a particular modelling situation. He describes, in fact, five difficulties which may arise in the process of modelling strategies and behaviours of players by a game.

1. <u>The definition of rules and problems of wording and coding.</u> Although strategies of play are easy to describe (or pre-

scribe, given certain circumstances), other elements of play such as verbal communication between players may be difficult to code and therefore difficult to model.

2. The definition of rules, the meaning of rationality, and problems of information and data processing ability. It has been shown that "rationality" can have various meanings, depending upon the players and the industry. Furthermore, what is "rational" under incomplete information (or perhaps too much information!) may be quite irrational under other assumptions.

3. The specification of payoffs, goals, and motivation. It is possible that non-numeric goals or payoffs are more important, in some circumstances, than the cell entries in a game matrix. This again depends upon the nature of the players involved, and what motivates them to play the game as they do.

4. The meaning of rationality and the concepts of solution for multiperson games. Individual rationality and collective or "social" rationality are not necessarily consistent. "The different attempts to define a solution to an n-person, non-constant-sum game amount to suggesting different criteria for social rationality if they are offered as normative solutions" (Shubik 1975).

5. The specification of players as individuals or groups. Difficulties may arise when considering the role of the individual players as elements of a bureaucracy. Indeed, the structured firm or group may itself be the "player". Such distinctions are not made explicit in a simple game-theoretic model.

In this study, the definition of what is "rational" given the intentions of the players, and the specification of motivation and payoffs (points 2 and 4) are potential trouble areas. Part 1 of Chapter 5 discusses in detail the importance of careful selection of payoff variables and addresses the issue of strategies versus behavioural intentions.

GAME THEORY AND THE OLIGOPOLY

The first section of this chapter described the scope of game theory and introduced its application in economics. Now that some of the fundamental concepts of game theory have been exposed in detail, it is time to return to the game-theory representation of the oligopoly.

The Oligopoly: Theoretical Background

Neoclassical economic theorists have strived to model the behaviour of each of the elements in the exchange process: the consumer and the firm. The consumer is thought of as attempting to maximize his level of utility derived from the purchase of products, subject to his income constraint. The behaviour of the firm depends upon the competitive nature of the industry it is part of: in this section, the oligopoly (and its special two-firm case, the duopoly) will be examined.

Gould and Ferguson (1980) define the oligopoly as the situation where "more than one seller is in the market but (where) the number is not so large as to render negligible the contributions of each"; also, in oligopoly and duopoly, "each firm must almost surely recognize that its actions affect the rival firm, which will react accordingly"(1980). It is possible to test mathematically for the extent of influence of Firm A's strategies on Firm B's sales by taking partial derivatives. "If the influence of one seller's quantity decision upon the profit of another, $\partial\pi i/\partial\pi j$, is...of a noticeable order of magnitude, (the industry) is duopolistic or oligopolistic...The essential distinguishing feature (of the oligopoly) is the interdependence of the various sellers' actions" (Henderson and Quandt 1980). Econometric techniques may be used to estimate the extent of such influences.

How, though, can the various actions and reactions of the competitors in an oligopoly be modelled, predicted, or explained? Evidently "there is a very large number of possible reaction patterns for duopolistic and oligopolistic markets, and as a result there is a very large number of theories of duopoly and oligopoly." (Henderson and Quandt 1980). Shubik and Levitan (1980) describe four main branches of oligopoly theory: the mathematical approach, the institutional approach (historically the two major branches), a newer, technological-institutional approach (as exemplified by Scherer 1970), and the behavioural approach (some of Shubik's (1959a) work may be classified here). The authors' conclusion indicates the diversity and uncoordinated nature of the study of oligopoly.

> "There is presently no single behavioural theory of the firm...There is not even a single theory of the profit-maximizing firm. In general there is no single theory of oligopoly. There are a host of partially-developed theories based on a mixture of analysis, insight, and to a great extent casual observation" (Shubik and Levitan 1980).

A Game-theoretic Framework: Equilibrium Solution Concepts

As seen before, the classic work Theory of Games by von Neumann and Morgenstern (1947) was an attempt to model mathematically the behaviour of firms within an oligopoly. By now it is evident that the particular mode of behaviour suggested by von Neumann and Morgenstern's Minimax Principle may not always be appropriate. One of the contributions made by Shubik (1959a) in Strategy and Market Structure was the explicit consideration of different solution concepts representing different behavioural patterns; this contribution was returned to and expanded in Shubik and Levitan (1980) and is a focal point of this paper.

First, consider the case of the duopoly where the quantity produced is the only controllable variable. Shubik (1959a) illustrates four possible solution concepts in this situation.

In the Cournot solution, the possibility of collusion (explicit or implicit) is ruled out. Each producer operates believing that the other will not change his current output level. Under this assumption, each firm maximizes profit by adjusting its output quantity accordingly. (For more information, see Henderson and Quandt 1980.) The dynamic (through-time) interpretation of this concept suggests that an equilibrium point may be found by solving the equations

$$\frac{\partial P_i}{\partial q_i} = 0, \qquad i = (1,2), \quad P_i \geq 0$$

simultaneously, where P_i = profit of Firm i, and q_i = output. The nature of the Cournot solution concept is such that readjustment after readjustment occurs until an equilibrium point is reached. A variant of this solution concept is the Stackelberg solution, where one firm is taken as the market leader and the other firm adjusts its production level to the profit-maximizing quantity, given this additional information: this concept will be examined again later.

Another solution concept is called the joint maximal (or Von Neumann-Morgenstern) solution. Here, "the market situation is treated as a two-person, zero-sum, cooperative game. It is assumed that the two firms will cooperate in such a manner as to maximize joint profits" (Shubik 1959a). The two firms then "settle" or divide the total take by using side payments, the amount of which is determined to a great extent by the relative negociating power of the firms involved. Evidently, points which are joint-maximal solutions are also Pareto-optimal, as the players could not improve on their outcome by changing strategies simultaneously.

The Nash cooperative game with side payments solution is similar to the above, except that "the side payments are now determined by evaluating the threats of the duopolists in order to determine a point from which they should agree to work out a 'fair division' of profits" (Shubik 1959a). Here, as before, an absolute joint maximum can be obtained.

Finally, in the Nash cooperative game without side payments, the assumption is no longer made that unrestricted side payments are illegal or acceptable. This does not restrict the firms from reaching the Pareto-optimal surface, but does make attainment of the absolute joint maximum impossible.

Shubik then further distinguishes among these solution concepts by considering the relative threat position of each firm. The outcome of this venture into duopoly theory is that, making different assumptions about the behaviour of the firms, quantities such as production rates, profits, joint profits and final market prices may be calculated (see Shubik 1959a, Chapter 4).

Pricing Strategies for the Duopoly

The Cournot and other solutions described above take quantity produced as the decision variable. Economists such as Bertrand and Edgeworth argued that price variation is a more reasonable strategic variable from an economic point of view. The solution concepts proposed by these (and other) writers for the price-variation and price-quantity-variation situations are also more directly relevant to the marketer making marketing-mix decisions and will be examined below. (A more complete investigation is found in Shubik 1959a.) Furthermore, some of the above solution concepts are capable of being modified into an advertising-expenditure-variation situation. Various solution concepts proposed for this situation are also illustrated in the literature (see, for example, Shubik and Levitan 1980).

The Edgeworth solution is based on the assumptions of "double idiocy" (each player assumes the other will not change his price and adjusts his own accordingly) and "contingent demand", i.e., the leftover demand for the higher-priced product assuming that the firm with the lower-priced product cannot meet all demand. Note that in price variation one must consider the effects of different demand caused by the different price levels: this effect was ignored in the Cournot and other earlier solutions. The Edgeworth solution provides an "Edgeworth range of (price) fluctuation (rather than a single ideal price, which) depends explicitly upon the structure of the contingent demand" (Shubik 1959a). However, Shubik also proves the theorem that "if an Edgeworth duopoly has a pure strategy equilibrium point, then it must be the efficient point", the latter being defined as the "point at which the market demand is just saturated by the quantities offered by the players at that price" (1959a). In other cases, price fluctuations within the given limits will occur.

The Bertrand game or price game is characterized by simultaneous price decisions by each player. It can be shown that a similar theorem as above holds for the Bertrand game as well: under certain circumstances, the efficient point is easily determined as the only equilibrium point. However, in this case, one may also develop an economic interpretation of a mixed-strategy equilibrium. "(Although) the market is unstable in the usual sense of economic theory...if each firm shows that it is willing to vary price in some range, a more general equilibrium may be established." (Shubik 1959a)

Shubik and Levitan (1980) also describe a price-leader solution, where price decisions are not necessarily made simultaneously. Although some of the numerical solutions they derive are admittedly unrealistic, this solution may more adequately describe the price-setting behaviour in many industries than a simultaneous-decision-making solution concept.

This solution has obvious similarities to the Stackelberg solution, wherein each firm examines its own cost functions and determines whether it would prefer to act as price leader or follower.

A disequilibrium occurs when both players would prefer to lead. Basar and Haurie (1982) illustrate how the Stackelberg solution may be represented as a "well-defined" game in extensive form. Given the game of Figure 2.19, with P1 the dominant player, it is easily shown that P1 will choose strategy 2. It is left for P2 to maximize his outcome given that he has this information; thus, he will play Strategy 1 and the circled payoffs will be the values of the game to the respective players. However, this situation can be equally well represented by the matrix (and corresponding game tree) in Figure 2.20, which makes explicit the sequential nature of the price-decision process. There is really nothing new about this representation, as it has been previously seen in Shubik 1975, e.g. What is notable is that, if the game is modelled as shown, with each of the price-follower's strategies made explicit, the Stackelberg price-leadership solution emerges as one of the equilibria (it may not be the only one, as other points may be Pareto-optimal; see discussion in an earlier section).

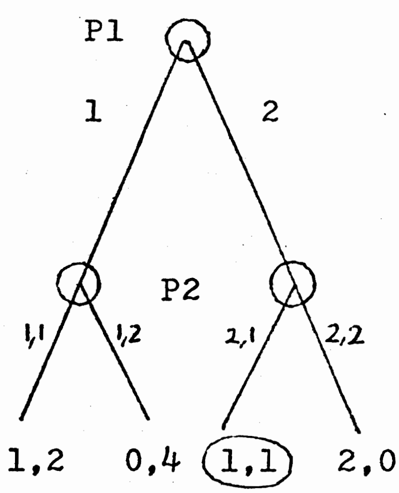
		P2	
		1	2
P1	1	1,2	0,4
	2	1,1	2,0

FIGURE 2.19

Source: Adapted from Basar and Haurie (1982)

FIGURE 2.20

Source: Adapted from Basar and Haurie (1982)



		P2			
		1,1	1,2	2,1	2,2
P1	1	1,2	1,2	0,4	0,4
	2	1,1	2,0	1,1	2,0

Finally, two cooperative price game solutions are also conceivable: mixed-strategy and pure-strategy solutions. They differ from the cooperative quantity-variation games in that contingent demand must be taken into account. In either case, side payments are made to entice the players to cooperate; and the relative threat positions of the players may be a determining factor in deciding the size of the payment.

The Edgeworth game and the price game may both be extended into price-quantity games. Additionally, the Cournot (quantity), Bertrand (price) and price-quantity strategies have all been extended by Shubik (1959a) to the n-player oligopoly, where $n > 2$. The reader is referred to the original work for details.

Mathematical Representations of Solution Concepts

It is useful to categorize the above mentioned solution concepts into cooperative (e.g., Nash-cooperative, joint-maximal or cooperative price games) and noncooperative (Cournot, Edgeworth or Bertrand games) groups. One may also further distinguish two kinds of noncooperative games: those in which each player is intent on maximizing his own payoff, and those in which each player seeks to do the best possible relative to his opponent. This latter distinction is clarified mathematically. In the first instance, the solution conditions may be expressed as

$$\max P_1 (S_1, \overline{S_2}) \text{ implies } S_1 = \overline{S_1}$$
$$\max P_2 (\overline{S_1}, S_2) \text{ implies } S_2 = \overline{S_2}$$

where P_i = payoff to player i;

S_i = possible actions of player i.

"These two conditions call for a type of circular stability. If the first player is aware that the second player is going to select $\overline{S_2}$, the first player will select $S_1 = \overline{S_1}$ when he maximizes his own payoff (and vice versa)." (Shubik and Levitan 1980)

This solution is identical to that described by Shubik (1972b) as the Nash noncooperative solution (see above).

In the second instance, each player essentially wants to maximize the "spread" between his payoff and his opponent's: i.e., to "maximize the difference between the scores". Mathematically, this behaviour is given by

$$\max_{S_1} \min_{S_2} \left[(P_1(S_1,S_2) - P_2(S_1,S_2)) \right]$$

Shubik and Levitan stress that this behaviour is diametrically opposed to joint maximization. Rather than looking after each other's welfare, each player tries to maximize his opponent's "illfare". This may be termed the maxmin or cutthroat solution.

An extension of this solution to the n-player oligopoly ($n > 2$) is the beat-the-average solution. "The idea (here)...is that each firm looks at the rest of the market in aggregate and asks itself: 'Am I doing better than the average?'" (Shubik and Levitan 1980). Evidently this solution is most appropriate when the firms in question are of approximately equal size. This solution has the mathematical expression

$$\max_{S_i} \left[P_i(S_1,S_2, \ldots , S_n) - \frac{1}{n-1} \sum_{j \neq i} P_j(S_1,S_2, \ldots ,S_n) \right]$$

for all i. This expression reduces to the cutthroat solution when $n = 2$. Maximization of profit share (and, with certain caveats, also market-share maximization) may be approximated by beat-the-average behaviour.

Therefore, solutions to non-cooperative games may be mathematically expressed as maximin, beat-the-average, or payoff-maximization solutions. The mathematical representation of a joint-maximal (cooperative) game is

$$\max_{s_1} \quad \max_{s_2} \left[P_1(S_1,S_2) + P_2(S_1,S_2) \right] .$$

These are the major situations to be considered, although Shubik and Levitan illustrate how other (possibly less-likely) behavioural situations may also be mathematically represented. For example, the case where each player is sadistic and intent on doing as much damage as possible may be represented by

$$\min_{s_1} \left[P_2(S_1,S_2) \right] ; \qquad \min_{s_2} \left[P_1(S_1,S_2) \right] .$$

This section (and the preceding one) have focused on price as the decision variable; thus the solutions obtained represent price (or price-quantity) strategies. Shubik and Levitan provide advertising-expenditure strategies as well (1980, pp. 194 - 197). They are similar in concept to the joint-maximization, pure-strategy noncooperative equilibrium, and beat-the-average price solutions and will not be discussed at length here.

Clearly the number of possible game representations is quite large. The above section indicated that the different behavioural possibilities lead to the development of different mathematical representations. Similarly, the behavioural alternatives may in some cases manifest themselves as different matrix solutions. Shubik and Levitan (1980) propose a step-by-step development of the determination of market behaviour through the analysis of the normal-form game matrix. Their investigation merits careful consideration as it serves as the theoretical base for the development of the general analytic model proposed in Chapter 4 (despite the philosophical question raised in Chapter 1).

Determination of Market Behaviour

Examine the payoff matrix of Figure 2.21a. The minimax solution to this game is (6,4). However, what behavioural pattern would have led to this solution? The cooperative equilibrium point is clearly (6,4), as the sum of the payoffs is maximized. However, if each player makes his choice on purely selfish grounds (non-cooperative), each player will always choose his second strategy, leading again to the (6,4) payoff. And if each player plays "cutthroat" (i.e., maximizes the minimum difference in payoffs), again equilibrium is reached at (6,4). "If such a market actually exists, all forms of behaviour previously described are indistinguishable." (Shubik and Levitan 1980).

		P2	
		1	2
P1	1	2,1	2,4
	2	6,1	6,4

(a)

		P2	
		1	2
P1	1	5,5	0,2
	2	2,0	-3,-3

(b)

FIGURE 2.21
Source: Shubik and Levitan (1980), p. 45.

Now consider the game of Figure 2.21b. Here, cooperative and noncooperative (joint-payoff-maximizing) behaviour both lead to payoffs of 5 and 5. However, if both firms play cutthroat, the resulting payoffs would be (-3, -3). If only one player tries to do the other in, one of the other elements of the matrix will give the corresponding payoffs. In this case, one form of behaviour (maxmin or cutthroat) can be distinguished from the others. In a larger-than-two-player oligopoly, the beat-the-average solution would correspond to cutthroat behaviour.

It is also possible to construct a game wherein the cutthroat and noncooperative solutions are identical, while the joint-maximal solution is different; see Figure 2.22a. In this matrix, payoffs may be interpreted as expected returns to each firm. Strategy 2 here may represent a commitment to heavy advertising or product-innovation expenditure. The matrix shows that if only one firm commits itself to advertising or R & D, it gains an advantage over the other; but if both firms commit themselves the payoff to each is slightly lower than had the status quo been maintained (a reasonable result if short-term profit is taken as the returns variable).

> "If both firms are 'peaceful', they can each make a profit of 10, However, there is the possibility of an extra profit, some of which may be obtained by getting a larger market share, for a firm that is willing to lead in sales or innovation. The firm (thereby)...at least...increases its profit expectation." (Shubik and Levitan 1980).

The reader can verify that the joint-maximization solution is (10,10), whereas both noncooperative and cutthroat behaviours lead to (7,7).

		P2: 1	P2: 2
P1	1	10,10	6,12
	2	12,6	7,7

(a)

		P2: 1	P2: 2
P1	1	2,1	3,0
	2	1,2	4,-1

(b)

FIGURE 2.22
Source: Shubik and Levitan (1980), pp. 46 - 47.

In the game of Figure 2.22b, the noncooperative and cutthroat behaviour solutions are also identical: payoff (2,1). Note however in this case that if the players choose to cooperate, none of the four possible combinations is preferred as all yield the same total payoff.

Finally, consider the game of Figure 2.23. Here, the joint-maximal solution is (5,5), the cutthroat solution is (-3,-3), and two noncooperative solutions are found: (-3,-3) and (2,2). "In this market, it is always possible to distinguish between completely cooperative behaviour and the others by observing the outcome. It is sometimes possible to distinguish between noncooperative competi-

		P2		
		1	2	3
P1	1	5,5	1,6	-5,2
	2	6,1	2,2	-4,-2
	3	2,-5	-2,-4	-3,-3

FIGURE 2.23
Source: Shubik and Levitan (1980), p. 47.

tion and the rest depending upon which strategies are employed" (Shubik and Levitan 1980, p. 48).

The matrix representation of a game is richer in information, then, than one may be led to believe in considering simple game theory. The "equilibrium points" discussed above for noncooperative games are just that--they are applicable when noncooperative behaviour patterns are predominant in the industry. Furthermore, the term "noncooperative" has taken on a more focused definition: in this section, its usage has essentially been limited to payoff-maximizing behaviour, clearly distinguishable in definition from cutthroat (maxmin) behaviour. Cooperative solutions had been previously determined in characteristic function form: this section has shown how a normal-form analogue can be applied to find the solution to a game matrix under cooperative behaviour.

Further work by Shubik has also examined the oligopoly from a game-theory point of view. Nti and Shubik (1981) have investigated the issue of entry costs into an oligopoly using a game-theoretic model with price and quantity as the decision variables.

The above discussion, then, summarizes the developments leading up to the market behavioural model of Shubik and Levitan: behavioural solution concepts may be represented mathematically and thereby lead to the selection of certain strategies. It is as this point that development of the theoretical model of this paper begins: Chapter 4 picks up this development. Additionally, Chapter 1 referred to an important philosophical difference between Shubik and Levitan's viewpoint and that of this study: this difference is clarified in Chapter 5. It has been the aim of this last section to demonstrate how Shubik and Levitan's game-theoretical oligopoly model derives directly from the consideration of behavioural alternatives and their mathematical representations.

Before turning to the theoretical model, the other literature branch relevant to this study is examined: namely, econometric modelling of pricing and advertising response and related topics.

CHAPTER 3
LITERATURE SURVEY: ECONOMETRIC MODELLING

MODELLING MARKETING-MIX EFFECTS

The literature on the econometric modelling of sales or market-share response to adjustments in price or advertising expenditure is large and varied. The theoretical and empirical models are too numerous to be all adequately discussed here. Resultingly, this discussion of the historical development of marketing-mix models is restricted to highlighted articles and to models which will be employed in the upcoming section describing the theoretical model of this paper. The chapter is organized as follows: theoretical models, empirical econometric models, stochastic models, and pricing considerations. "Quasi-game-theoretic" approaches to pricing and advertising are also illustrated. A framework similar to that of Sethi (1977) is used to organize and present the relevant theoretical models.

Advertising in Oligopoly Theory

One often sees a tendency in neoclassical microeconomic theory to treat price and quantity offered as the only variables to consider. Complex market models are developed which essentially allow the individual firms to make decisions on only a) whether they choose to enter into, or exit from, the industry, and b) what price they choose to set; in pure competition, of course, even this second decision is never taken by the firm, but prices are instead dictated by industry conditions.

As Shubik and Levitan (1980) rightly point out, there are a number of "weapons in the arsenal" available to firms or producers in the real world: these "weapons" are those strategic elements which forms may take decisions upon, and include "distribution and retailing; legal and institutional factors; production problems;

advertising; public relations;...(and) consumer-information activities" (1980).

Clearly, firms can use advertising (and other elements) as weapons of competition as well as price. In classical oligopoly theory, advertising may be modelled "as a cost and as a method for changing demand" (Shubik 1959a) which evidently is not the whole picture. For one thing, the effects of advertising (as shall be seen in succeeding sections) may be extremely complex and difficult to model or even to understand at times. Secondly, there may be more than one kind of advertising effect as well: Shubik (1959a), using Chamberlin's terms, describes two classes of advertising: manipulative and informative, and argues convincingly that it is very difficult to distinguish between the two.

> "...We have been unable to formulate an operational distinction between the two (classes) because we do not know what we mean by rational or economic action in situations involving incomplete information. What is manipulative and what is informative is open to question." (Shubik 1959a)

Thirdly, the distinction between advertising and other marketing-mix elements (public relations, for example) is not always clear. Quantitative analysis may thus be hindered, as the analyst may have difficulty determining which expenses should be allocated to advertising, and which to other marketing efforts.

Just as there are at least two classes of advertising, there are at least two possible general opinions of advertising effects. Informative advertising may have educational value, thus making people aware of new products. Borrowing from social psychologists, marketers know of the role which advertising plays in the learning process with respect to products or brands. Economists may argue that advertising provides an "external economy" to customers. "After all, information and advice in many areas have a cost attached" (Shubik and Levitan 1980).

Detractors of advertising argue that it may be unnecessary or wasteful, "and not needed at all in a highly educated and 'rational' community. (However, these) opponents of advertising... are referring implicitly to a world of complete information" (Shubik 1959a). Even when situations of incomplete or imperfect information exist, advertising which misleads rather than informs may be used to rectify the information gaps. "The aid to the consumer given by (some advertising) statements is rather dubious, yet it is interesting to note that the value of supplying additional irrelevant, though correct, information is recognized in advertising" (1959a).

This introduction is meant to forewarn the reader to some of the difficulties inherent in the quantitative analysis of advertising effects, as well as to highlight their importance. "Features (such as advertising expenditures) considered merely frictions that do not matter in the long run by many economists have now been regarded as important. Abstract analysis is no substitute for knowledge of the institutional and technological facts of the business being analyzed" (Shubik and Levitan 1980, p. 191). The rest of this chapter investigates the methods applied over the years by various analysts and econometricians in search of industry models which capture some of these "micro-micro"-level advertising and pricing effects.

Note: the special attention paid to price and advertising in this chapter does not imply that other marketing (or even non-marketing) decision variables are irrelevant. However, these are the major marketing decision variables used by the firms under analysis in this study, and a review of the literature on sales response to these particular variables is thereby appropriate.

ADVERTISING CAPITAL MODELS

Empirical analyses such as those of Palda (1964), Tull (1965) and Kuehn, Mc Guire and Weiss (1966) indicate the presence of a carryover advertising effect; that is, as one might expect, the effect of advertising on sales persists through time, lessening as time goes by. It is believed, then, "that advertising expenditures affect the present and future demand for the product and, hence, the present and future net revenue of the firm which advertises" (Sethi 1977). Some analysts have chosen to represent the carryover advertising effect with a capital model, where the advertising capital (or goodwill) is increased "by adding new customers or by altering the tastes and preferences of consumers and thus changing the shape of the demand curve as well as shifting it" (1977). The goodwill can decrease or depreciate as time passes due to brand-switching to competitors' brands, competitive advertising, the entry of new brands to the marketplace, and so on.

A major early advertising-capital model is that of Nerlove and Arrow (1962). They begin with an earlier model, that of Dorfman and Steiner (1954), who maximized net revenue for a firm where: "(a) price and advertising expenditures are the only variables affecting the demand for the product; (b) current advertising expenditures do not affect the future demand for the product; and (c) the decision-maker is a monopolist who can determine both price and advertising expenditures" (Nerlove and Arrow 1962). Their model relaxes condition (b); that is, by specifically incorporating the carryover effect of advertising, they have introduced a dynamic component to the Dorfman-Steiner model.

In their model, "goodwill, a stock related to the flow of current advertising expenditures...depreciates at a constant proportional rate δ, and...the future is discounted at a constant rate of interest, α"(1962). If u is the current advertising expenditure, then the net addition to goodwill due to advertising investment $\dot{A}$ is

$$\dot{A} = u - \partial A;$$

or, "the net investment in goodwill is the difference between gross investment...and depreciation" (Sethi 1977).

Nerlove and Arrow use the calculus of variations to determine optimal prices and advertising (goodwill) levels; the reader is referred to the 1962 article or to Sethi (1977) for numerical details. The major conclusions, however, are as follows: "...changes in α and ∂ affect the optimal goodwill (advertising expenditure) in the same way" (1962); also, "the ratio of goodwill to sales revenue is directly proportional to the goodwill elasticity and inversely proportional to the price elasticity" (Sethi 1977), among other factors. Optimal stationary equilibrium is also calculated for the long run.

The Nerlove-Arrow advertising capital model lends itself to extension and generalization to situations of (a) stochastic fluctuations in goodwill (Tapiero 1975a) and (b) uncertainty (Tapiero 1979). Some of these considerations are revisited below in a section on stochastic models.

The carryover advertising effect on sales is an important consideration in the theoretical model and empirical analyses of this study. Chapter 4 indicates a number of different approaches by which lagged effects may be modelled, and Chapter 5 indicates the scenarios in which each would be preferred. One of these, the goodwill approach, is based on Nerlove and Arrow's advertising capital model.

Whereas Nerlove and Arrow relaxed condition (b) of the Dorfman and Steiner (1954) model, Lambin, Naert and Bultez (1975) relaxed condition (c); that is, they considered the case of the oligopoly. They also examined the distinction between <u>direct</u> and <u>indirect</u> (multiple) <u>reactions</u> ("for example, a competitor may react to a change in price not just by changing his price, but also by changing his advertising and possibly other marketing instruments as well") (Lambin 1976). Their approach involves the construction of a <u>reaction matrix</u> such as that appearing in Figure 3.1.

		2	
		Price	Adv.
1	Price	$\eta_{P_1P_2}$	$\eta_{P_1A_2}$
	Adv.	$\eta_{A_1P_2}$	$\eta_{A_1A_2}$

FIGURE 3.1

Source: Lilien and Kotler (1983), p. 667.

The entries in this figure are elasticities, defined as follows:

$$\eta_{XY} = \text{percentage change in X resulting from a 1\% change in Y.}$$

Thus, $\eta_{P_1P_2}$ indicates how strongly Firm 1's price is affected by Firm 2's price, and so on.

These elasticities may be estimated using equations of the following form:

$$\ln P_1 = \hat{a}_1 + \hat{\eta}_{P_1P_2} \ln P_2 + \hat{\eta}_{P_1A_2} \ln A_2$$

$$\ln A_1 = \hat{a}_2 + \hat{\eta}_{A_1P_2} \ln P_2 + \hat{\eta}_{A_1A_2} \ln A_2$$

where the "hats" indicate estimated parameters (see Lilien and Kotler 1983). The relevance of such a model to the present work is clear in that the reaction matrix may be used to describe the strategic decisions which have been taken through time by the competitors (i.e., a significant $\eta_{P_1P_2}$ indicates that Firm 2's price levels do affect price levels set by Firm 1). The reaction matrix, then, is one way of representing the strategic interplay among firms. The game matrices to be developed herein, on the other hand, use payoffs (rather than elasticities) as the entries; but the intent is still to represent strategic interplay in tabular form.

Lambin et. al. also determine optimal marketing mixes for the participants in an oligopoly, under conditions of both expansible and non-expansible industry demand (see Lambin, Naert and Bultez 1975 and Lambin 1976). Additionally, they show that the Dorfman-Steiner model itself may be viewed as a special case of their model (see discussion in Lilien and Kotler 1983). It should also be noted that in the dynamic extensions of the optimality equations, Lambin (1976) chooses a distributed-lag model based on geometrically-declining weights to represent the carryover effect of advertising, and also proposes the use of the Koyck (1954) transformation. The Koyck carryover approach used in this study (discussed in detail later) is similar to that proposed by Lambin in his study of over 100 European branded goods (1976).

SALES-ADVERTISING RESPONSE MODELS

In this class of models, the carryover effect of advertising is not represented by a fluctuating stock of advertising capital or goodwill. Instead, this "effect is modelled explicitly to obtain a direct relation between sales and advertising in the form of a differential or difference equation" (Sethi 1977).

The operations-research study of Vidale and Wolfe (1957) is one of the important early sales response models. They begin by posing three questions of significance to marketers which may be answered using quantitative means:

"1. How does one evaluate the effectiveness of an advertising campaign?

2. How should the advertising budget be allocated among different products and media?

3. What criteria determine the size of the advertising budget?" (1957)

The answers to these questions (especially the latter two) can only be found if the effectiveness of advertising in improving sales is better understood. With a series of controlled experiments, they determined three parameters which appear to modify the effect of advertising on sales-- the sales decay constant, the saturation level, and the response constant: these parameters make up the basis of their theoretical model. The sales decay constant represents the "decrease (in sales) because of product obsolescence, competing advertising, etc. (in the situation where the firm halts all advertising expenditure). Under relatively constant market conditions, the rate of decrease is, in general, constant: that is, a constant percent of sales is lost each year" (1957). This constant, then, represents the carryover effect of advertising. The saturation level indicates the amount of sales which no amount of advertising could greatly improve upon. The response constant is defined as "the sales generated per advertising dollar when S (the sales level)=0" (1957). It is incorporated to take into account product-to-product differences in sales behaviour. The mathematical model Vidale and Wolfe presented and tested was

$$\frac{dS}{dt} = r\,A(t)\,(M - S)\,/\,M - \lambda S,$$

where S = sales level at time t,

$A(t)$ = advertising expenditure,

r = response constant,

λ = sales decay constant,

M = saturation level.

Note that the concept of a sales saturation level implies diminishing returns to advertising (see Sethi 1977).

Despite the simple nature of their model, Vidale and Wolfe claim that "it has proven useful in the analysis of advertising campaigns" (1957). Further empirical studies have made major refinements to the model such as the introduction of simultaneous-equation techniques and the analysis of elasticities and cross-elasticities: the upcoming section on econometric modelling investigates these improvements. Furthermore, Saseini (1971) and Sethi (1973, 1974) have offered optimal-control extensions of the basic model, and Tapiero (1975b) has presented a stochastic generalization. This last paper will be further discussed later.

Developers of more recent theoretical models have borrowed from the contributions of both early theorists and econometricians. An interesting model of new-product diffusion is proposed by Dodson and Muller (1978). In it, the effects of different information sources (advertising and word-of-mouth) on purchase are modelled, as are repeat-purchase trends. In their model, the market at time t is said to be made up of three types of people: "(those) who are unaware of the existence of the product;...potential customers who are aware of the product but have not yet purchased it; and...current customers who have purchased the product." (Dodson and Muller, 1978) The authors then propose a general model comprising three equations and show how it would be modified in specific cases. What is perhaps most interesting about this model is that Dodson and Muller demonstrate that both the Vidale-Wolfe and the Nerlove-Arrow models are special cases of their model. If trial and repurchase rates are equal, and word-of-mouth effects are zero, then the Dodson-Muller model for repeat sales simplifies to

$$\frac{dS(t)}{dt} = \mu(\gamma N - S(t)) - \phi S(t),$$

where

$S(t) =$ sales at time t,

$\phi =$ the rate of brand switching,

μ and γ = advertising effects, and

N = size of the market.

Note the similarity between this equation and that which was proposed by Vidale and Wolfe (see above).

In the econometric-modelling section below, Palda's (1964) empirical model will be discussed. Dodson and Muller point out that Horsky (1977) had already demonstrated the equivalence between Palda's empirical model and the theoretical model of Vidale and Wolfe; thus, they claim that their model may be considered a generalization of the Palda model as well.

Finally, to obtain the equivalence between the Dodson-Muller and the Nerlove-Arrow models, one must make two further substitutions in the original form of the Dodson-Muller equation. These adjustments result in the revised form

$$\frac{\partial G(t)}{\partial t} = I(t) - \phi G(t)$$

where

$I(t) =$ investment in goodwill and

$G(t) =$ amount in goodwill account.

Note that this expression states the Nerlove-Arrow proposition mathematically: i.e., the overall change in goodwill over time results from the increase due to investment in advertising and the decrease due to loss of sales to other brands.

Subsequent to Vidale and Wolfe's theoretical model, a number of econometric sales- and market-share-response models have been proposed and estimated in the literature. The empirical econometric models literature has a twofold effect on this study. First, the models estimated herein are similar in form to those of Weiss (1968) or Jagpal et. al. (1979) (see below), to name but two. Second, the Koyck and dynamic-adjustment models, derived theoretically below, inspired alternate approaches for capturing advertising carryover effects.

EMPIRICAL ECONOMETRIC MODELS

Classic Models

Important early sales-response models include those of Palda (1964) and Telser (1962); while Banks (1961) provides an early market-share model.

The Palda article was an early attempt to model sales-advertising response using a single-regression-equation model. He used macro sales and advertising data obtained for a proprietary medicine, Lydia Pinkham's Vegetable Compound. The Pinkham data were especially amenable to economic treatment, as the product had virtually no competitors and other marketing-mix elements (e.g., price) were relatively stable through time: many of the extraneous variables which could cause estimation difficulties were thereby irrelevant or had negligible effects.

The Banks study (1961) also used a single-equation model to relate market share of coffee and household cleaners to such decision variables as price, advertising, sales effort, etc. Telser's paper (1962) on cigarette advertising was in the same vein, but with the improvement that advertising elasticities as well as the nature of the returns to advertising (marginal product) were estimated. He accomplished this by proposing four alternative regression forms for the effect of advertising on cigarette sales, "(differing) in their implications regarding the nature of the returns to advertising" (Telser 1962).

Refinements and Improvements

These early efforts encouraged all manner of extensions and improvements, as econometricians and marketers strived to apply better estimation procedures and to formulate more realistic response models. Some of the noteworthy improvements are listed below. (Many of the examples cited are presented in more complete form in Parsons and Schultz 1976.)

Weiss (1968) examined market shares of frequently-purchased consumer products and estimated a multiplicative model, i.e.,

$$\text{Brand Share} = (\text{RP})^{\beta_1} \times (\text{RA})^{\beta_2} \times \ldots$$

where

RP = relative price, or firm's price divided by average industry price;

RA = relative advertising, or firm's advertising expenditure divided by average industry advertising level.

An advantage of a multiplicative model such as that of Weiss is that the parameters β_i may be interpreted as estimates of elasticities, which may then be used in subsequent analyses. This property may be easily demonstrated: the above equation yields a marginal productivity of advertising equal to

$$MP_{adv} = \frac{\partial\ \text{Share}}{\partial\ \text{Adv.}} = (\text{RP})^{\beta_1} \times \beta_2(\text{RA})^{\beta_2 - 1} ;$$

and an average productivity equal to

$$AP_{adv} = \frac{\text{Share}}{\text{Adv.}} = (RP)^{\beta_1} \times (RA)^{\beta_2 - 1} ,$$

so that advertising elasticity is

$$\epsilon_{adv} = \frac{MP_{adv}}{AP_{adv}} = \frac{(RP)^{\beta_1} \times \beta_2 \ (RA)^{\beta_2 - 1}}{(RP)^{\beta_1} \times \ (RA)^{\beta_2 - 1}}$$

$$= \beta_2 .$$

Similarly, the price elasticity may be shown to be β_1. Thus, the obtained coefficients have an economic meaning which is a distinct advantage. Lambin's (1970) market share model including carry-over brand-share effects was also of this form and provided good fit.

Kuehn, Mc Guire and Weiss (1966) made an extension to the simple regression model such that lagged advertising expenditures were incorporated. They defined "advertising shock" as the impact of advertising in a specific period, and postulated that it is a function of last period's advertising shock (carryover effect) and this period's advertising expenditure. Some form of cumulative advertising effect is expected. Advertising may not have immediate effects on purchase behaviour though it may on brand loyalty which may eventually affect purchase behaviour.

A few years later, Montgomery and Silk (1972) estimated the dynamic effect on market share of the three elements of a "communications mix" for a particular product (ethical drugs): journal advertising, direct mail advertising and samples/literature. To obtain the dynamic effect, they utilized a distributed lag model.

This was an improvement on two fronts. Firstly, the effects of each communications variable on market share individually were determined and important differences were found among them. Secondly, the length of the carryover effect of each advertising variable on market share was estimated by determining the number of significant parameters in the model. Short-run, intermediate and long-run elasticities were also calculated for each vehicle.

Bass (1969) was the first to employ simultaneous-equation regression in sales-response estimation. One of the difficulties of earlier studies was their inability to reconcile adequately the "identification problem"; that is, they were inadequate in "identifying the relationship that reflects the influence of sales on advertising, as well as that which reflects advertising's influence on sales" (Bass 1969). Thus, the simultaneous-equation model was designed to deal with the "simultaneous nature of the relationship between sales and advertising...Advertising decision rules, whether rigid or flexible, certainly account for sales. Therefore, single-equation regression models cannot adequately identify advertising-sales and sales-advertising relationships" (Bass 1969).

Bass formed pairs of simultaneous equations made up of one demand function and one advertising-behaviour equation. From these efforts, it was possible to obtain, among other things, sales-advertising cross-elasticities; that is, the effect on sales of one product form of the advertising effort of another. A further benefit of this approach was that optimal advertising expenditures for each product were estimated and in fact were found to be very similar to the actual expenditures. In a followup paper, Bass and Parsons (1969) extended the model such that it was capable of testing aggregate sales and advertising data.

By the time Beckwith (1972) published his article on the multivariate analysis of sales-advertising responses, many authors (Palda 1964 and Telser 1962 among them) had incorporated sales or market-share lag effects into the response function, thereby accounting for repeat-purchase behaviour. Notable about his approach, however, is his use of an iterative estimation procedure (IZEF) and of a two stage estimator (ZEF), both defined earlier by Zellner. He argues that both "are generally more efficient than OLS (ordinary least squares)...the OLS estimators are not usually the most efficient...unless the disturbances...are uncorrelated between brands." (Beckwith 1972). In any case, Beckwith's equation system was "much simpler...to estimate than (that of) Bass's approach" (Clarke 1973). Clarke correctly notes that Bass's equations are underidentified and that the number of brands that could be adequately treated using his method is small.

Telser (1962) had addressed the issue of returns to advertising, and indeed found that "the level of advertising (in the cigarette industry) was high enough to place the companies at the point where there were diminishing returns" (1962). It is also possible that demand may show increasing returns to low levels of advertising, and decreasing returns as advertising expenditure is increased. Jagpal, Sudit and Vinod (1982) cite a study (Krugman 1977) which provides empirical evidence of this effect. Johansson (1973) devised a model which incorporated this modification. He fitted a double-log logistic function to data on hair spray:

$$\ln \left[\frac{\text{Brand share - proportion of repeaters}}{\text{Trial proportion - brand share}} \right] = \ln \beta_0 + \beta_1 \ln (\text{Adv.})...$$

Such a model yields a nonsymmetrical S-shaped curve which would allow for the required shift in returns to advertising.

In his article on cross-elasticities, Clarke (1973) borrows from the previously-cited articles of Bass (1969), Telser (1962) and Beckwith (1972). He uses the independent variable "relative advertising" introduced by Telser (brand's advertising divided by the sum of the brand's competitors' advertising) in setting up his partial-adjustment model, wherein the residuals are correlated. He treats the equation set as a system of seemingly-unrelated regressions (they are all indeed related as market shares should add up to one). He uses an estimation procedure known as SURWADI which is appropriate for seemingly-unrelated equations with autocorrelation. This method is an extension of IZEF used by Beckwith (1972). In a later paper, however, Clarke concedes that "the seemingly-unrelated regression treatment used (here)...had little effect on parameter estimates...A great deal of complexity (was added) to an exploratory demonstration" (Clarke 1978). Nonetheless, a better means of measurement of cross-elasticities (effect of Brand i's advertising on Brand j's sales) was presented.

The Lydia Pinkham data of Palda's (1964) study has been frequently returned to in the literature, due to the desirability of the data: for summaries and model development information, see Weiss, Houston and Windal (1978). Two such studies investigate the serial correlation issue raised by Clarke's 1973 paper (Houston and Weiss 1975; Clarke and Mc Cann 1977). Both papers confirm the importance of the inclusion of autocorrelated errors; however, the presence of autocorrelation does not necessarily mean that the OLS estimates are biased ("although they may be somewhat less efficient than GLS (generalized least-squares) estimation" (Montgomery and Silk 1972)).

The above models have been developed ignoring the possibility of interaction effects: among advertising media, among marketing-mix variables, or through time. They are clearly flexible enough for such modifications to be made: Prasad and Ring (1976) provide an illustration in their analysis of different communications vehicles and price fluctuations.

Not all of these papers are utilized directly in the theoretical development or empirical analysis. At least some relevant ideas are gained from each, however. The model developed for Industry 1 resembles Weiss's (1968) model in that both are multiplicative (although Weiss used shares whereas dollar amounts were employed in this study). Kuehn et.al.'s "advertising shock" (1966) is reminiscent of Nerlove and Arrow's advertising capital model; the method Montgomery and Silk (1972) used to decide upon the correct number to include (determining the number of significant parameters) is borrowed in this paper for the goodwill carryover approach.

Bass's 1969 papers warn about the simultaneous sales-advertising effect. This effect is not directly incorporated into the theoretical model; this omission did not turn out to be serious as empirical fit was still quite acceptable. However, in certain industries where sales levels are known to be a deciding factor in determining future advertising levels, appropriate adjustments might be necessary.

Finally, the issue of diminishing return to advertising (as speculated on by Telser 1962 and examined empirically by Johansson 1973 and Krugman 1977) plays an important role in the discussion of the empirical results of Chapter 6 and has strategic ramifications for one of the firms of the industry.

To conclude this section, note that many concepts which have been examined empirically by the abovementioned authors will be important considerations in the building of the theoretical model of this study, and in its operationalization.

Advanced Multiplicative Models

Jagpal, Sudit and Vinod (1979) developed a sales-response function incorporating advertising interactions through time, by applying a multiplicative nonhomogeneous functional form (MNH) to the Lydia Pinkham data. As is Weiss's multiplicative model, MNH regression coefficients may be interpreted directly as elasticities. But in MNH, interactions between independent variables are modelled explicitly and hypotheses concerning their existence may be tested. A general MNH function may be written (as in Jagpal et. al., 1979):

$$\ln S_t = \gamma + \sum_i \alpha_i \ln A_{t-i} + \sum_{i,j} \beta_{ij} \ln A_{t-i} \ln A_{t-j} + V_t,$$

where S_t = sales in period t,

A_t = advertising expenditure in period t,

$\alpha_i, \beta_i, \gamma$ = parameters,

V_t = disturbance term.

(Note that in the Jagpal formulation, sales and advertising were taken as the relevant variables in the general specification, although other variables, such as market shares or prices, could be substituted or added.)

Three properties of the MNH function are significant:

1. Marginal sales elasticity with respect to advertising may be easily calculated as

$$\epsilon_i = \frac{\partial \ln S_i}{\partial \ln A_{t-j}} = \alpha + \sum_j \beta_{ij} \ln A_{t-j} .$$

2. Marginal sales productivity (i.e., the marginal benefit derived in sales due to advertising) may be determined using

$$MP_i = \epsilon_i \left[\frac{S_i}{A_{t-j}} \right]$$

3. The distributed-lag formulation mentioned earlier may be viewed as a special case of the MNH function: i.e., if all β_{ij}'s are set to zero, the function becomes

$$\ln S_t = \gamma + \sum_i \alpha_i \ln A_{t-i}$$

which is in Cobb-Douglas form.

An extension to the MNH function was applied to the same data in 1982 by Jagpal, Sudit and Vinod. This was the transcendental-logarithmic functional form (translog), which is similar to MNH except that it contains quadratic terms. Thus, as in Johansson's 1973 model, marginal productivities are not held constant. It can be shown that the translog function is simply a more general case of MNH, and in fact collapses to MNH form if all the α_{ii} terms are set to zero. The translog form has the drawback of requiring an unusually large number of parameters to be estimated and therefore will not be returned to in this study.

Price main effects and price-advertising interaction effects could not be studies using the Pinkham data, due to the stable pricing policy used by the manufacturers. Jagpal et. al. admit, though, that "when several marketing-mix variables are included simultaneously in the sales-response function, theory suggests that the policy variables will interact both contemporaneously and over time" (Jagpal, Sudit and Vinod 1982).

The Jagpal et. al. model (1979) illustrates the kind of multiplicative interaction terms which are used in the construction of the theoretical model of Chapter 4.

Although multiplicative models have been used successfully by many researchers, even the more advanced forms are not always appropriate. In an important review article, Little (1979) discusses some of their drawbacks:

> "...zero advertising produces zero sales, and, if lagged advertising terms are included, zero advertising in any lagged period produces zero sales in the current period. The situation is particularly acute for applications with short period lengths (e.g., months or weeks), since zero advertising in such intervals is quite common...S-shaped response (of sales to advertising) is precluded (in the simplest forms; see discussion of translog form). Rise and decay from steady state involve symmetric factors (and the assumption that sales increases due to increasing advertising and sales decreases due to decreasing advertising are symmetrical is not always reasonable)." (Little 1979)

Researchers have sometimes opted for a linear model (Bass and Clarke 1972; Palda 1964). Although linear models avoid the difficulties of modelling the effects of zero advertising, they cause their own set of problems. Most importantly, "linear response is not credible over an indefinite range" (Little 1979). Also, asymmetrical rise and decay times cannot be modelled, so no improvement over the multiplicative form on this issue is gained. Other modelling considerations are given in Little (1979). In this study, some of the model inadequacies indicated by Little concerning multiplicative and linear forms are encountered and dealt with in Chapters 6 and 7.

Koyck and Dynamic-Adjustment Models

The Palda (1964) study, and many other sales-advertising response studies (Bass and Clarke 1972, e.g.) employed geometric distributed lags of the form proposed by Koyck (1954). Also, in many of the aforementioned works (Clarke and Mc Cann 1973; Montgomery and Silk 1972; Houston and Weiss 1975, e.g.), lagged dependent variables appear on the right-hand side of the equation. This section provides a theoretical derivation of two closely-related econometric models which incorporate lagged dependent variables on the right-hand side: the Koyck and dynamic-adjustment models, both of which are developed into approaches for handling advertising carryover. These derivations are due to Johnston (1972).

The Koyck scheme is applicable where lagged independent variables are significant; i.e., where the econometric model is expected to take the form

$$Y_t = \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \ldots + u_t. \tag{3.1}$$

Define a delay operator D such that

$$DX_t = X_{t-1};$$

$$D^2 X_t = X_{t-2};$$

$$D^3 X_t = X_{t-3};$$

etc.

Also define a set of weights w_i which sum to one. Equation 3.1 could thus be rewritten

$$Y_t = \beta(w_0 + w_1 D + w_2 D^2 + \ldots)\, X_t + u_t. \tag{3.2}$$

Now assume that the weights w_i decline geometrically, i.e.,

$$w_i = (1 - \lambda)\lambda^i,$$

$$0 < \lambda < 1.$$

With this assumption,

$$w_0 + w_1 D + w_2 D^2 + \ldots = (1 - \lambda)(1 + \lambda D + \lambda^2 D^2 + \ldots)$$

$$= \frac{1 - \lambda}{1 - \lambda D} \tag{3.3}$$

Substitution of 3.3 into 3.2 gives

$$Y_t = \frac{\beta(1-\lambda)}{1-\lambda D} X_t + u_t \tag{3.4}$$

Rearranging 3.4 gives the general Koyck form,

$$Y_t = \beta(1-\lambda) X_t + \lambda Y_{t-1} + (u_t - \lambda u_{t-1}) \tag{3.5}$$

Using the geometric declining distribution of weights yields the convenient relation for mean lag:

$$\text{Mean lag} = \frac{\lambda}{1-\lambda}\ .$$

In sum, the Koyck scheme for estimating a relationship with lagged independent variables results in a lagged dependent variable on the right-hand side.

Dynamic (or partial) adjustment models are used in econometric analysis in situations where an optimal value for the dependent variable is specified. The optimal value for Y, denoted Y*, given a certain value for an independent variable X, may be expressed by the optimum equation

$$Y_t^* = \phi_o + \phi_1 X_t + u_t \tag{3.6}$$

where u_t is an error term and ϕ_i's are parameters. However, note that:

> "If the income change that has produced X_t has been a large upward (or downward) one, the consumer may not have the requisite knowledge of his utility surface to adjust immediately to the new situation...A reaction or adjustment function (is therefore postulated) which asserts that in the current period he will probably move only part of the way from his starting position (Y_{t-1}) to the optimum position (Y_t^*)." (Johnston 1972)

This adjustment function would take the form

$$Y_t - Y_{t-1} = \gamma(Y_t^* - Y_{t-1}) + e_t \,, \qquad (3.7)$$

where the parameter γ is the coefficient of dynamic adjustment. As Johnston notes, "the closer γ is to unity, the greater is the adjustment made in the current period." (1972) Combining equations 3.6 and 3.7 and performing the required rearrangements yields the operational specification

$$Y_t = \gamma\phi_0 + \gamma\phi_1 X_t + (1 - \gamma)Y_{t-1} + W_t \,, \qquad (3.8)$$

where W_t is the composite error term.

Notable about this specification is that it is quite similar to a Koyck scheme, differing only in the inclusion of a constant term. Koyck models have frequently been used in modelling sales-advertising response functions, owing to their inclusion of a lagged dependent variable on the right-hand side, as has been seen above.

A more complex model of this type, the adaptive expectations model, deals with the situation where optimal Y* may vary over time. These models shall not be considered here (see Johnston 1972).

STOCHASTIC MODELS

The Vidale-Wolfe and Nerlove-Arrow models as well as the Sethi extension mentioned above are deterministic in nature. Tapiero (1975b) makes a strong argument in favour of the application of stochastic techniques to the study of advertising effect on sales. Rather than being able to determine uniquely a sales level "by solution of a difference equation", it is probably more accurate to acknowledge that "the forgetting of past advertising effects and sales response to advertising are in fact probabilistic with parameters that reflect empirical evidence and the time series of sales and advertising effects" (Tapiero 1975b). In this spirit, Tapiero has provided stochastic extensions of many of the important theoretical models.

Although stochastic extensions of the theoretical model of this paper are not within the mathematical scope of this study, they are potentially an interesting avenue for further research: this issue is considered again in Chapter 8. Furthermore, Tapiero's work sometimes involves game-theoretic formulations which merit close inspection here.

Tapiero provides a diffusion approximation to a stochastic, random-walk version of the Vidale-Wolfe model (1975b), and also provides extensions to the Nerlove-Arrow model (1975a; 1978; 1979). In the 1975a paper, he proposes that "additions to goodwill by advertising and the depreciation of it by forgetting are probabilistic effects" (Sethi 1977), and sets up an equation which can be solved as a deterministic optimal control problem. In the 1978 paper, he proposes a stochastic extension of the above model, again carried out by making a diffusion approximation of the relevant equation and by replacing certain probabilities with Taylor expansions (a method also seen in 1975b). He thereby constructs an approximation to the original equation which is in stochastic differential equation form. The mathematical details are not reproduced here; however, some of Tapiero's conclusions are significant:

"1. Advertising is a 'risky investment' in goodwill, substituting current certain expenditures for uncertain profits;

2. Risk-averse firms will advertise less and risk-taking firms more;

3. Large risk-aversion and forgetting rates both lead to the standard competitive result in advertising." (Tapiero, 1978).

Tapiero provides a further generalization of this model, to the case where a number of firms are competing. In this paper (1979), he demonstrates that a Poisson probability distribution may be adequate in representing sales for each of the two firms in a given market. Additionally, if aggregate market sales are Poisson distributed, the market-share advertising model obtained is also Poisson distributed.

A large section of this paper (1979) is devoted to differential games formulations, which is of special relevance here. In fact, a differential-game approach is used in selecting optimal advertising levels under sales-maximization and profit-maximization situations. While the method of differential games may not be appropriate for this paper (more on this topic in Chapter 4), the preliminary observations Tapiero makes in formulating his game models are relevant to this study.

Tapiero highlights some previously-discussed solution concepts not treated in depth by Shubik and Levitan: Nash-equilibrium and Pareto-optimality.

> "...If firms have knowledge of cost structures and reach decisions separately, then depending upon the behaviour and goals of the firms, minimax, Nash, noninferior (Pareto-optimal) or absolutely cooperative strategies may be desirable." (Tapiero 1979)

Of these, Nash strategies are applicable in solving zero-sum games, while the others may be applied to a simultaneous-decision non-zero-sum situation. (The Stackelberg strategy solution would be recommended in the sequential-decision case.)

Unlike Shubik and Levitan, Tapiero proposes strong arguments which would limit the number of behavioural patterns which could actually be manifested in an oligopoly situation.

> "When both firms recognize that there are grounds for cooperation, they may reach "enforced"...agreements... Such solutions are called...Pareto-optimal solutions. When both firms vie for an increase of their market share (when the aggregate market sales remain relatively fixed...), the conflict (or the threat of cutthroat and collusion practices) inherent in the competition for market shares is not likely to lead to these types of solutions." (Tapiero 1979)

Similarly, since firms in a given industry usually have similar knowledge (or ignorance) levels regarding the cost structure, it may be unreasonable to anticipate Stackelberg behaviour, at least in the case of advertising expenditure. Tapiero concludes by considering only minimax and Nash strategies in his paper. Because of the very specialized circumstances which he assumes in his model (only one independent variable, advertising; aggregate sales levels roughly unaffected by advertising; sales or profit-maximizing behaviour), Tapiero's simplifications most likely are valid for his paper, although they may not all be transferable to this investigation. Still, his analysis indicates the importance of considering the game models and behavioural strategies in a realistic context.

In another article, Farley and Tapiero (1981) propose a stochastic model of sales response to different timing patterns of advertising outlay. A treatment similar to that employed before is used to set up and to solve the model. This model is significant in that it borrows elements from both the Vidale-Wolfe and the Nerlove-Arrow models, and also provides a rationalization for the distributed lag empirical models such as those of Palda (1964).

> "These (models) generally involve patterns of market response based on carryover effects and decay of a stock of advertising...Decay effects explicitly due to forgetting ...have been suggested...to account for imperfect measures of buyers and intervening external stimuli that prevent complete extinction in a learning framework" (Farley and Tapiero 1981).

Finally, Tapiero (1983) has also proposed a stochastic diffusion model incorporating both advertising and word-of-mouth effects. He shows that this model is a generalization of the previously-mentioned Dodson-Muller model, itself a generalization of numerous earlier models.

PRICING CONSIDERATIONS

Up until now, the emphasis in this chapter has been mostly on the effects of advertising expenditure on sales. Granted, some of the empirical studies mentioned (Banks 1961; Weiss 1968; Bass 1969 and others) included both price and advertising among the independent variables. However, much of the attention in the literature has been focused on the carryover effects of advertising (Tull 1964; Jagpal, Sudit and Vinod 1979; etc.) or on the differing effects of different media vehicles (Montgomery and Silk 1972).

This is not to say that price effects are of lesser importance or can be ignored. On the contrary, an interesting literature has been built up on the issue of the perceived relationship between price and product quality. An early paper by Leavitt (1954), although flawed by research design, suggested that customers tend to choose a higher-priced brand when the relative qualities of different brands are unknown. A followup study by Tull, Boring and Gonsior (1964) supported this finding. Gabor and Granger (1966) found that many of their "subjects trusted price rather more than the evidence of their senses" in determining product quality. They also suggested the existence of an "acceptable" price range: products priced outside this range are perceived by customers as being either of unacceptable quality or simply too expensive. This notion of an acceptable price range is returned to again later.

In his classic article of pricing psychology, Shapiro (1968) reviews these and other studies, and concludes that there are four reasons why price is frequently taken by customers as a measure of quality.

1. Ease of measurement: price is usually a known, fixed quantity and as such is easily comparable across brands. Also it serves as a proxy quality indicator if the consumer does not have enough expertise to compare brands on "real" quality indices such as durability.

2. Effort and satisfaction: One line of reasoning (see Cardozo 1965, e.g.) suggests that the consumer equates spending more money on a product with expending more "effort". As more satisfaction is derived (according to this theory) through a greater expenditure of effort, the consumer would be most satisfied with his/her purchase if an expensive brand is chosen.

3. Snob appeal: some consumers gain satisfaction by demonstrating that they can afford the most expensive brands.

4. Perceptions of risk: "To reduce the risk of choosing a product of significantly poorer quality, the consumer chooses the higher-priced brand" (Shapiro 1968).

A recent behavioural study of a small-consumer-good product (Mc Gill 1982) indicated that the two highest-priced, highest market-share brands were also perceived by consumers as being the highest in quality. It suggested that perceptions of differences among brands are strongly influenced by marketing-mix variables: the high-price, heavily-advertised brands may be perceived as being of higher quality, whether real quality differences exist or not. By building reputations as "nationally advertised brands", the top brands may effectively maintain high levels of distribution and market share due to risk aversion on the part of some consumers.

Vanhonacker (1983a; 1983b) has recently reanalyzed the nature of price-advertising effects. He notes the existence of two diametrically-opposed schools of thought in the marketing literature on this issue.

> "On the one hand, Kotler (1971) argues that an increase in advertising will have a positive impact on product differentiation which will lead to decreased price sensitivity. On the other hand, Chamberlain (1962) suggests that as advertising expenditures increase, price awareness will increase and ultimately result in higher price sensitivity" (Vanhonacker 1983a).

Interestingly, either viewpoint may be convincingly argued for: the actual effect of advertising on price sensitivity will undoubtedly be product specific. It is therefore likely that it will be difficult to predict, a priori, the direction of price-advertising interaction effects.

Vanhonacker also indicates a potentially more troublesome failing of the pricing literature: its failure to distinguish between changes in price level and minor fluctuations about a (basically stationary) price level. He notes that the abovementioned work of Gabor and Granger (1966), among others, on "acceptable price ranges" examines the effect of setting prices relative to an industry standard. Such works may be characterized as "price perception studies". Attitudinal research into price has focused on the other price issue, namely that of varying price temporarily around a basic level. "By design, experimental studies only capture the interaction with respect to the price level aspect" (Vanhonacker 1983a). Any pricing conclusions drawn from empirical studies should therefore be made with this potentially severe limitation in mind.

The pricing psychology literature can provide insight into some of the results obtained in the empirical analysis of this paper. Some of the results obtained in Chapter 6 concerning price effects, for example, are interpretable using price psychology phenomena, such as that described by Shapiro (1968) and in the Mc Gill study (1982). Having examined some of the contributions of previous writers on price effects will aid, therefore, in explanation and interpretation of price observations made in this study.

GAME-THEORY IMPLICATIONS

Rao and Shakun (1972) develop a "quasi-game-theoretic" pricing model to determine entry price for a new product. The framework is developed using the "acceptable price range" concept of Gabor and Granger and a series of working assumptions. In the two-brand case, they assume the presence of two groups of customers: (a) "quality-conscious" consumers who believe that higher price (within the acceptable range) indicates higher quality, and will therefore purchase the higher-priced brand; abd (b) "price-conscious" consumers who find all products within the acceptable range as of adequate quality, and thus purchase the lower-priced brand. Similar assumptions are made for the three-brand market case. They derive probabilities of purchase for each of the brands as functions of price. They work in a game-theory approach to their model by considering possible behaviour concepts on the part of each brand, and calculating the optimal entry pric e for the new brand under different combinations of behaviour concepts (i.e., maximize payoffs (non-cooperative); maximize joint payoffs; maximize industry sales; minimize opponent's payoffs).

Rao and Shakun emphasize that their approach is only "quasi-game-theoretic" as the extensive forms of the games being played are not developed nor are the specific strategies to be employed by each player. They do show, however, how game-theoretic considerations may be used in modelling the behavious of players in a given market.

Game-theory models had been applied to the issue of advertising expenditure long before Rao and Shakun's pricing model was developed. Montgomery and Urban (1968) trace this application of game theory back to Friedman (1958), who developed five simplified models designed to help answer the questions:

"1. How much of the yearly budget should be allocated to advertising?

2. How should the total advertising budget be allocated by marketing area (if the product or service is distributed over many areas?" (Friedman 1958)

The various models are based on the assumption that the most significant factor in allocating advertising expenditure is the activity of competitors (i.e., how much they spend in relative terms). He demonstrates the potential use of game theory in this regard. In the simplest of his models, for example, he concludes that "the optimal allocation of funds in each area will be proportional to the sales potential in the area" (Friedman 1958). The reader is referred to the original article for details.

An important theoretical extension to the Friedman model was made by Shakun (1965). In this paper, a mathematical game-theoretic approach is taken to develop a model for advertising outlay in "coupled markets" (meaning that "advertising dollars spent in generating sales for one product have an influence on the sales of another product") (Shakun 1965). This model was further extended to take into account dynamic effects (Shakun 1966) and differing organizational structures (Shakun 1968). Although the theoretical models used in these papers are not tested empirically therein, the results are interesting in that the prescribed optimal advertising expenditures (for each player in the industry) resemble those obtained by Friedman (1958). Shakun had used an exponential sales-advertising response function of the type mentioned by Vidale and Wolfe (1957) in his model estimations.

These last three examples have shown very simple applications of game theory to the issues of pricing and advertising, respectively.

One may collect some general observations in summing up this search through the econometric literature. Firstly, the theoretical models of Nerlove and Arrow, and Vidale and Wolfe, provide a starting point for the development of the sales-marketing-mix models used in this study. Secondly, some of the empirical models examined (Jagpal _et_. _al_., for example) illustrate how carryover and interaction effects may be easily accomodated through appropriately-specified functional forms. Thirdly, different ways of accounting for carryover effects have been suggested by Nerlove and Arrow, Koyck, and the dynamic-adjustment formulation: each of these is developed into a carryover approach in Chapter 4. Finally, important psychological considerations concerning the perceptions of price have been discussed by such writers as Shapiro, Gabor and Granger, and Vanhonacker. These considerations will be useful in the analysis of the empirical results of this paper.

CHAPTER 4
THEORETICAL MODEL

INTRODUCTION--CHAPTER OUTLINE

In this chapter, the analytic framework is proposed and the methods used in constructing the theoretical model are discussed. No doubt the reader by this time has become aware of the possible range of economic and marketing territory which could be included in the proposed study. It is not the author's intent to develop and test empirically a model containing a bewildering assortment of marketing, financial and other company-policy variables (both quantitative and qualitative); nor to model all of the "inner workings" of an (almost by definition) highly complex modern industry. On the other hand, the empirical tests to be employed should yield results which have some relevance to marketing decision-making; in other words, the abstraction from reality which is to be considered as "the industry" for empirical testing should contain at least the most significant marketing-mix variables. The problem of careful selection of variables for empirical testing is approached in the next chapter. In any case, in order to walk this middle ground between parsimony and relevance, a stepwise approach is taken which serves to structure this chapter and the next.

In this chapter, a general theoretical model is developed. Herein, the theory considerations of Chapter 2 are expanded into a general game representation where any number of marketing-mix variables may be included, and their effect on any (unspecified) dependent variable (i.e., objective or success measure) may be modelled. Also, using the literature review of Chapter 3 as a guide, general functional forms suitable for calculating the payoffs in the general game representation will be developed. This chapter therefore serves as a guideline for constructing a game matrix appropriate for a given industry.

Chapter 5 opens with a discussion of some of the major problems in modelling which may be encountered. Then, an illustration of how the model would be applied to a hypothetical industry setting is presented, together with a discussion of the relative merits of different proposed methods for treating carryover effects. Finally, the gaming implications of the model are examined. In other words, Chapter 4 shows how the mathematical, theoretical model is developed in general, while Chapter 5 illustrates what can be learned about a specific industry through the application of the game matrix technique.

In Chapters 6 and 7, industries are chosen for empirical analysis, and the general theoretical model of Chapter 4 is adapted and developed into industry-specific "reduced models". Then, the interpretative methods of Chapter 5 are applied to the chosen industries.

This framework allows the author to attain one of the main objectives of the paper: that is, to indicate how the proposed theory may be put into practice in real decision-making situations. The basic model is general enough for application in other industries where data requirements are larger or more marketing-mix (or extraneous) variables need be considered; indeed, even in situations where the buying-and-selling process may be quite different (although extra information would probably be required for the sealed-bid situation; see remarks in concluding chapter). It is the intent of this work to indicate the possible benefits of the game-theoretic approach by keeping the empirical models relatively parsimonious, and also to indicate how the basic model could be extended and thereby perhaps made more applicable.

PRELIMINARY CONSIDERATIONS

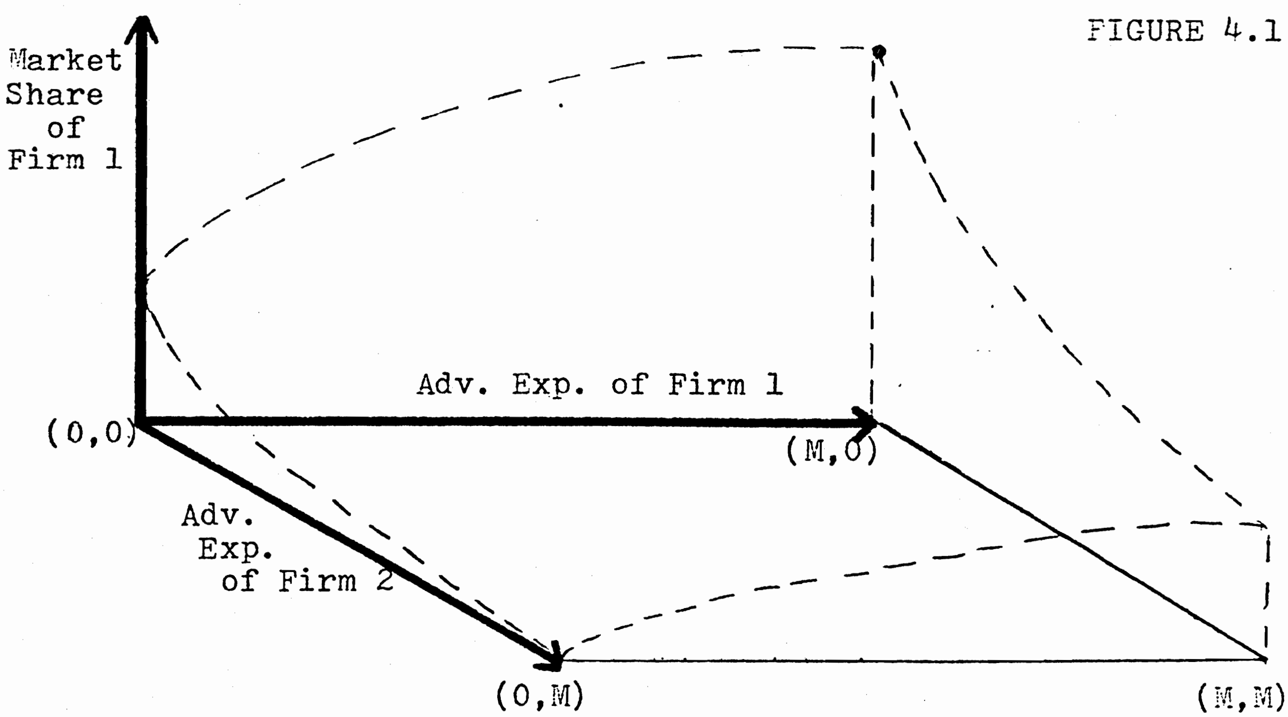
In this section, some of the loose ends of previous chapters are reconciled; specifically, the importance of the research, as outlined in Chapter 1; the determination of market behaviour through matrices by Shubik and Levitan, introduced in Chapter 2; and Tapiero's observations on the sales-advertising relationship in his differential-games generalization of the Nerlove-Arrow model (Chapter 3).

According to Chapter 1, an important objective of this paper is to verify empirically the game-theoretical constructs of writers such as Shubik, Levitan and Tapiero regarding the behaviour of firms in an oligopoly, and to apply the concepts to a real decision-making situation. To accomplish this requires firstly a "translation" of the general strategies in the game matrices of Shubik and Levitan into realistic, quantitative strategies for real marketing-mix decision variables (i.e., pricing levels, advertising expenditures and distribution costs). Secondly, an interpretation of the payoffs associated with each strategic combination is required.

The work of Tapiero (1979) takes steps in this direction: he considers the simplest situation wherein the only strategic variable controlled by the firms is advertising expenditure: furthermore, advertising carryover effect is ignored. The problem is thus reduced to finding the advertising level which optimizes payoff (to be specified later). Through this simplification, Tapiero justifies his use of differential games, which would allow exact calculation of the optimal amount of advertising expenditure (under certain assumptions, evidently, including complete knowledge of the effect of advertising on sales or profits, without uncertainty). That is to say, advertising expenditure is treated as a continuous variable over a given range. Other authors (see Jorgensen 1982a and 1982b, e.g.) have also successfully employed a differential-games approach to advertising and its effect on sales. As for specifying the interpretation of payoffs, Tapiero constructs two models: a zero-sum game of sales optimization; and a non-zero-sum game of profit optimization. (Note that, in assuming constant aggregate sales, maximizing sales becomes identical to maximizing market share.)

The work of Tapiero, although relevant for inclusion here, elicits comments on two fronts. The concept of differential games (as opposed to the discrete strategic options in typical games) is intuitively appealing in that optimal levels can be calculated exactly; however, the model would become extremely cumbersome upon the inclusion of additional decision variables. Figure 4.1 is a conceivable zero-sum market share differential game, where each firm chooses an advertising budget between \$0 and \$M per period. To include even one additional marketing-mix variable (e.g., price level) into the model would add two dimensions, which assuming price and advertising levels to be independent to each other, would have to be mutually orthogonal and orthogonal to the original axes. Thus the market share payoff function would be represented by a hypersurface in five-

FIGURE 4.1



dimensional space. Adding in more decision variables would complicate the situation even more, as would consideration of carry-over advertising effects. Additionally, this representation considers only the two-firm case: a three- or four-firm oligopoly situation would require even more dimensions.

This is not to say that such differential games are theoretically unsolvable or unanalyzable. Rather, the mathematics required would be enormously complex, and thereby not easily applicable in a practical setting. Also, the data requirements needed to construct a differential game with many continuous variables may be restrictive. An approach which uses discrete strategic options is more easily interpretable and allows for independent manipulation of marketing-mix variables with a minimum of complexity. Furthermore, in theory one could approximate a continuous variable's distribution with that of a discrete variable by increasing sufficiently the number of discrete possibilities. Therefore, a typical, normal-game-matrix approach could theoretically be extended to approximate continuous distribution of the decision variables.

The second comment on Tapiero's work is a less severe criticism. Whereas Shubik and Levitan appear to consider profit as the relevant payoff variable, Tapiero constructs a model of profit maximization and also one of sales/market share maximization. The criticism is that these possible objective variables or "success variables" are not the only conceivable ones: sales growth maximization or growth in market share are other possibilities, e.g. In fact, one might expect the relevant objective variable to change over time for a given firm: a new entry into an industry may at first wish to maximize sales growth levels (through heavy investment in advertising and product development) at the expense of short-term profit. As time goes on, sales maximization or profit maximization would take on greater importance (for example, a minimum acceptable target market share may be reached and the firm may concentrate on maximizing profit while maintaining market share). Modelling a "growth" firm into a game matrix having profits as payoffs would yield misleading conclusions (a topic returned to in Chapter 5). Furthermore, there is no guarantee that all firms in an n-firm industry have the same objective. The market leader may very well be striving to maintain its profit levels, while its competitors may be trying to carve out a larger market share. The ideal model, then, would be flexible enough to allow for both changes in objectives through time and divergent objectives across firms. Another corollary of this line of reasoning is that, in setting up the industry-specific game matrices, information concerning the objectives of the various firms should be obtained (for example, through interviews with product managers or industry experts). Resultingly, the theoretical model developed in this chapter leaves the payoff variable (Y) undefined: the issue of choice of dependent variable is returned to in Chapter 5.

Having stated these preliminary objectives, construction of the theoretical model can begin. This step is coupled with the delineation of the proposed econometric analytical techniques.

THE BASIC DUOPOLY MODEL: SIMPLEST FORMS

The simplest version of the general model is based upon Shubik and Levitan's game matrices (despite the important philosophical reservation discussed in Chapter 1 and expanded upon in Chapter 5). Consider the two-firm case, where only one marketing-mix variable is considered, X_1. In this simplest model, carryover effects are negligible, only two values are possible for X_1 (e.g., high/low), and the dependent variable Y is left unspecified (see above). The model would appear as in Figure 4.2, where X_{1j} = level of variable X_1 chosen by player j, which may be either high (denoted by X^H_{1j}) or low (denoted by X^L_{1j}). Also, $Y_j\,(X^{K1}_{11},\ X^{K2}_{12})$ represents the payoff to player j resulting from player 1's choosing level K1 for variable X_1 and player 2's choosing level K2 for variable X_1.

Note that if the dependent variable is market share, it is clear that $Y_1(X^{K1}_{11},\ X^{K2}_{12}) = 1 - Y_2(X^{K1}_{11},\ X^{K2}_{12})$: i.e., the game is zero-sum. The basic model as stated above places no such restriction

PLAYER 2

PLAYER 1		X_{12}^L	X_{12}^H
	X_{11}^L	$Y_1(X_{11}^L, X_{12}^L)$ $Y_2(X_{11}^L, X_{12}^L)$	$Y_1(X_{11}^L, X_{12}^H)$ $Y_2(X_{11}^L, X_{12}^H)$
	X_{11}^H	$Y_1(X_{11}^H, X_{12}^L)$ $Y_2(X_{11}^H, X_{12}^L)$	$Y_1(X_{11}^H, X_{12}^H)$ $Y_2(X_{11}^H, X_{12}^L)$

FIGURE 4.2

on the values of the dependent variables, although it could be adapted to a zero-sum situation.

The model as it stands is nothing more than a generalization of Shubik and Levitan's 2 x 2 game matrices, such as those shown in Figure 2.21a and 2.21b. As such it is hardly an improvement over the differential-games approach of Tapiero and Jorgensen. It is the nature of the extensions which are possible that lends adaptability and applicability to this model.

The first such extension to be made allows for a second

P2

P1		X_{12}^L, X_{22}^L	X_{12}^L, X_{22}^H	X_{12}^H, X_{22}^L	X_{12}^H, X_{22}^H
	X_{11}^L X_{21}^L	Y_1^*, Y_2^*	...	...	...
	X_{11}^L X_{21}^H	$\vdots$	$\ddots$		
	X_{11}^H X_{21}^L	$\vdots$		$\ddots$	
	X_{11}^L X_{21}^H	$\vdots$			$\ddots$

FIGURE 4.3

Note: $Y_1^* = Y_1(X_{11}^L, X_{21}^L, X_{12}^L, X_{22}^L)$

$Y_2^* = Y_2(X_{11}^L, X_{21}^L, X_{12}^L, X_{22}^L)$

decision variable X_2 to be included. Suppose that X_2 can also take on one of two values, high and low. Each player thus has four possible strategic pairings, assuming decisions are made simultaneously. These strategic options may be represented by a game matrix such as in Figure 4.3, where x_{ij} = level of variable i chosen by player j.

As the model undergoes extensions and refinements, it will become handy to think of the strategic pairings as <u>packages</u>. In the above representation, package 1 would be the choice of low values for both X_1 and X_2. Figure 4.3 can thus be rewritten in a much less cumbersome way, as in Figure 4.4. (The notation convention to be used herein is that W_{wj} will represent the choice of package w by player j.) Then, given a set of time-series data, one could calculate average Y_1 and Y_2 values for each strategy combination.

Clearly, more marketing-mix variables (X_3 , X_4 ,...) may also be added to the model. In this paper, a maximum of three marketing-mix variables (corresponding, for example, to pricing, advertising expenditure and distribution costs) will be considered for any industry.

		P2			
		W_{12}	W_{22}	W_{32}	W_{42}
P1	W_{11}	$Y_1(W_{11},W_{12})$ $Y_2(W_{11},W_{12})$	...	...	...
	W_{21}	⋮	⋱		
	W_{31}	⋮		⋱	
	W_{41}	⋮			⋱

FIGURE 4.4

Note: $W_{1j} = X^L_{1j}, X^L_{2j}$; $W_{2j} = X^L_{1j}, X^H_{2j}$; $W_{3j} = X^H_{1j}, X^L_{2j}$; $W_{4j} = X^H_{1j}, X^H_{2j}$

MULTICHOTOMOUS DISCRETE DECISION VARIABLES

As the introduction to the preceding section suggested, there are still a number of quite limiting restrictions in the model. One by one, extensions will herein be developed which make the model more useful.

Firstly, the assumption of discrete, dichotomous decision variables is restrictive and limits applicability in a realistic setting. Reverting back to the two-decision-variable situation (X_1 and X_2), now consider three possible levels for each variable (X_{ij}^H, X_{ij}^M and X_{ij}^L corresponding to high, medium and low). Now nine decision packages are available to each player, as in Figure 4.5.

If an immense data set were available, one could determine the values of the Y_1's and Y_2's by taking averages as before. However, considering that eighty-one Y_1's and eighty-one Y_2's would have to be estimated, one would need a minimum of 81 x 5 = 405 data points to avoid sparse cells (assuming even dispersion and five elements minimum per cell): this requirement could easily be doubled under circumstances of uneven distribution of data points.

This difficulty is overcome by estimating econometrically the effects of variables X_1 and X_2 on Y_1 and Y_2.

P2

P1

	W_{12}	W_{22}	...	W_{92}
W_{11}	Y_1, Y_2	...	...	...
W_{21}	$\vdots$	$\ddots$		
$\vdots$	$\vdots$		$\ddots$	
W_{91}	$\vdots$			$\ddots$

FIGURE 4.5

In this chapter, a multiplicative response model is developed, for the following reasons: (a) the parameters are interpretable as elasticities, as indicated before, so the effects of the different marketing instruments on the dependent variable may be evaluated and compared; (b) by taking logarithms, a multiplicative model can be easily converted to a linear form whose parameters may be estimated using ordinary-least-squares regression or one of its modifications; (c) as it is anticipated that advertising will be one of the major independent variables to consider, it is necessary to construct models which would adequately describe the expected advertising-sales relationship. If, at "reasonable" levels of advertising, sales show decreasing marginal returns to advertising, the model used should be capable of capturing this effect. As is seen in the following discussion, the multiplicative model permits this flexibility.

However, as indicated in Chapter 3, a multiplicative model may not be appropriate. For various reasons (to be considered in Chapter 7), a linear model may be preferred. Linear equations which correspond to the multiplicative-form equations presented in this chapter could easily be derived if necessary. However, for consistency, it is the multiplicative form which is developed in this chapter and the next.

The <u>main</u> effects of X_1 and X_2 on Y_1 and Y_2 may be modelled by a pair of simultaneous equations in Cobb-Douglas form:

$$Y_1 = \alpha_1 X_{11}^{\alpha_{11}} X_{21}^{\alpha_{21}} X_{12}^{\beta_{11}} X_{22}^{\beta_{21}} u_1 \tag{4.1}$$

$$Y_2 = \alpha_2 X_{11}^{\alpha_{12}} X_{21}^{\alpha_{22}} X_{12}^{\beta_{12}} X_{22}^{\beta_{22}} u_2 \tag{4.2}$$

where α_1, α_2 = constants;

$\alpha_{12}, \alpha_{22}, \beta_{11}, \beta_{21}$ = cross main effects of decision variables;

$\alpha_{11}, \alpha_{21}, \beta_{12}, \beta_{22}$ = own main effects of decision variables;

u_1 and u_2 = error terms.

Note that if the β_{ij}'s are less than one, sales will show decreasing marginal returns to decision variables X_{12} and X_{22}.

These equations could be transformed into linear form by taking logarithms:

$$\ln Y_1 = \alpha_1 + \alpha_{11} \ln X_{11} + \alpha_{21} \ln X_{21} + \beta_{11} \ln X_{12} + \beta_{21} \ln X_{22} + u_1 ; \quad (4.3)$$

$$\ln Y_2 = \alpha_2 + \alpha_{12} \ln X_{11} + \alpha_{22} \ln X_{21} + \beta_{12} \ln X_{12} + \beta_{22} \ln X_{22} + u_2 . \quad (4.4)$$

If the <u>interaction</u> effects between decision variables X_1 and X_2 are significant, multiplicative nonhomogeneous extensions of these equations would be appropriate. The general MNH form (given in Chapter 3) would be adapted as follows:

$$\ln Y_1 = \alpha_1 + \alpha_{11} \ln X_{11} + \alpha_{21} \ln X_{21} + \beta_{11} \ln X_{12} + \beta_{21} \ln X_{22} + \xi_{11} \ln X_{11} \ln X_{21} + \xi_{21} \ln X_{12} \ln X_{22} + u_1; \quad (4.5)$$

$$\ln Y_2 = \alpha_2 + \alpha_{12} \ln X_{11} + \alpha_{22} \ln X_{21} + \beta_{12} \ln X_{12} + \beta_{22} \ln X_{22} + \xi_{12} \ln X_{11} \ln X_{21} + \xi_{22} \ln X_{12} \ln X_{22} + u_2; \quad (4.6)$$

where ξ_{11} and ξ_{22} = own interaction effects between decision variables;

ξ_{21} and ξ_{12} = cross interaction effects between decision variables.

Note that "own interaction effects" refers to the effect on Y_1 of the interaction between Firm 1's X_1 level and X_2 level, while "cross interaction effects" refers to the effect of this same interaction on Y_1 (and vice versa). Other interactions could be included (such as the combined effect of Firm 1's price and Firm 2's advertising) but would be more difficult to interpret economically. Therefore, for simplicity's sake these extra interactions are left out in the developments of this chapter.

The appropriateness of interaction terms for a given industry is an application issue and is therefore left to Chapter 5.

A word about the rationale behind the model construction is in order here. Game theory indicates that the decisions taken by a firm are not made in a vacuum. Both own- and competitors'- (cross-) effects should be modelled. The above simultaneous-equations representation indicates only one way in which competitive effects may be worked into a model. An alternative method (which would reduce the number of parameters to estimate in the two-firm case) would be to use shares-of-aggregate expenditure as the independent variables. This procedure has a few drawbacks, however. If X_1 is advertising outlay, then share-of advertising outlay is a meaningful variable: price cannot be treated in this way. Also, if the aggregate outlay per period fluctuates considerably, then a share variable can become difficult to interpret. A firm holding to a stable advertising policy when its competitors' expenditures are fluctuating wildly may mistakenly appear to be deliberately changing its share-of-aggregate expenditure from period to period; or at least it may become difficult to separate conscious strategic changes from effects caused by aggregate fluctuations.

Assuming, then, that the proposed functional forms are appropriate, there remains one step: to fill in the Y_1's and Y_2's in the matrix of Figure 4.5. The high, medium and low levels of variables X_{11}, X_{21}, X_{12} and X_{22} are substituted into the appropriate equation derived through the econometric analysis.

One possible criticism of the model which can be raised at this time is as follows: The game matrix, even for only two firms, two decision variables and three levels of each, is quite large-- a 9 x 9 matrix with 81 possible outcomes. Surely the average decision maker does not follow such a procedure! The answer here is simply that even if the game matrix is large, it is still easily solved for likely equilibrium points using the methods introduced in Chapter 2. Part 3 of Chapter 5 re-examines this issue.

APPROACHING CONTINUITY IN DECISION VARIABLES

The notion that firms have only three possible levels of variables X_1 and X_2 to choose from is still not satisfactory. Most firms presumably have more flexibility in their decision-making than this. Ideally, over a given acceptable range, the distribution of possible levels is continuous.

The difficulties in using differential games in this application have already been discussed. Rather than modelling a continuous distribution for each variable, a discrete-approximation method will be applied in this study. Suppose that the (continuous) distribution of variable X_{11} (between extreme values of 0 and X'_{11}) is to be approximated by three discrete segments, to be called high, medium and low. The distributions of the other variables may be approximated similarly. Exactly the same econometric techniques as outlined above would be employed to estimate the parameters α_1, α_{11}, etc. All that remains is to construct the game matrix as in Figure 4.5.

To accomplish this, the relevant range of variable X_{11} is divided into low, medium and high segments. The midpoint of each segment is taken as the <u>representative value</u> for that segment. In other words, three values for X_{11}, representing the low, medium and high segments, will be used to construct the game matrix: to be consistent with previous notation, these values will be called X^L_{11}, X^M_{11} and X^H_{11} respectively. Given the range of possible values previously noted, it is easy to show that the representative values would be $(X'_{11}/6$, $(X'_{11}/2)$ and $(5X'_{11}/6)$, respectively. Similar treatment is applied to each of the other decision variables. Then, the payoffs are calculated according to the corresponding formulae (Equations 4.3 and 4.4, e.g.). A game matrix identical in form to that of Figure 4.5 would thus be obtained.

The distinction between this procedure and that of the previous section is that, where before X^L_{ij}, X^M_{ij} and X^H_{ij} comprised the entire set of decision possibilities (e.g., only price levels of \$0.80, \$1.00 and \$1.20 are considered), they now represent <u>ranges</u> of decisions (i.e.; below \$0.90; between \$0.90 and \$1.10; and above \$1.10). The reader who feels that such a representation may be too crude to capture the essence of price fluctuation is reminded that the number of discrete segments is not restricted to three. Having more discrete segments would cause the continuous distribution of possible decision-variable levels to be modelled more closely. However, in defense of the simpler model, it may be that a firm setting price levels for a new product might indeed be considering as alternatives (a) the market average; (b) a somewhat higher price (skimming policy); and (c) a somewhat lower price (penetration policy). Having more than three levels in the model might make the decision problem appear more complex than it really is. In any case, the model can be extended to include any number of levels, as the situation warrants.

Up until now, carryover effects of decision variables have been ignored. Three slightly different ways in which the model can incorporate such effects have been considered, each applicable in different situations. Each of these is developed theoretically here; while their relative merits and operationalization considerations are left until Chapter 5.

A GOODWILL MODEL OF CARRYOVER EFFECTS

Return to the two-firm, two-decision-variable, two-level situation (Figure 4.3). Suppose that it is anticipated that variable X_2 will exhibit carryover effects, and that these effects last a maximum of one period's duration. In other words, one need consider only one lagged term, XL_2, which is last period's level of decision variable X_2. The payoffs Y_1 and Y_2 are now a function of six variables:

$$Y_1 = f(X_{11}, X_{21}, XL_{21}, X_{12}, X_{22}, XL_{22})$$

$$Y_2 = f(X_{11}, X_{21}, XL_{21}, X_{12}, X_{22}, XL_{22})$$

The number of possible packages (W's) available to each player is eight. Clearly this number would increase if (a) more than two levels are permissible for either decision variable; (b) longer-lasting carryover effects (i.e., more than one period in duration) are significant; (c) more than one decision variable exhibits carryover effects. A more parsimonious model containing most of the relevant information would be preferred, considering the complexity of the general framework. Two alternate methods are proposed to account for significant carryover effects of one variable: one based on a goodwill-account approach (described in this section); and one employing a Koyck-form econometric approach (described in the next section). Also, a dynamic-adjustment approach is briefly described for the situation where both (or all) decision variables are expected to exhibit carryover effects.

The goodwill-account approach stems from the Nerlove-Arrow advertising capital model (see Chapter 3). The net effect of advertising (or any other decision variable which exhibits carryover effects) on decision variable Y_j is the total of the effects of current- and previous-period levels of advertising, appropriately weighted. The weights could be determined econometrically. For example, the Cobb-Douglas equation pair (Equations 4.1 and 4.2) could be modified to include carryover effects of variable X_2 :

$$\ln Y_{1t} = \alpha_1 + \alpha_{11} \ln X_{11t} + \alpha_{21} \ln X_{21t} + \sum_{\ell=1}^{L} \alpha^*_{\ell 1} \ln X_{21(t-\ell)} + \beta_{11} \ln X_{12t} + \beta_{21} \ln X_{22t} + \sum_{\ell=1}^{L} \beta^*_{\ell 1} \ln X_{22(t-\ell)} + u_{1t} \quad (4.7)$$

$$\ln Y_{2t} = \alpha_2 + \alpha_{12} \ln X_{11t} + \alpha_{22} \ln X_{21t} + \sum_{\ell=1}^{L} \alpha^*_{\ell 2} \ln X_{21(t-\ell)} + \beta_{12} \ln X_{12t} + \beta_{22} \ln X_{22t} + \sum_{\ell=1}^{L} \beta^*_{\ell 2} \ln X_{22(t-\ell)} + u_{2t} \quad (4.8)$$

where $\alpha^*_{\ell j}$ and $\beta^*_{\ell j}$ = lagged effects of variables X_{21} and X_{22} ;

L = the total number of significant lag periods to include;

and subscripts denote time periods. Now, the cumulative effect of variable X_{21} on Y_1 is seen to be

$$\alpha_{21} \ln X_{21t} + \sum_{\ell=1}^{L} \alpha^*_{\ell 1} \ln X_{21(t-\ell)},$$

which upon rearrangement becomes

$$\alpha_{21} \left[\ln X_{21t} + \sum_{\ell=1}^{L} \frac{\alpha^*_{\ell 1}}{\alpha_{21}} \ln X_{21(t-\ell)} \right],$$

or

$$\alpha_{21} \left[\ln X_{21t} + \sum_{\ell=1}^{L} \alpha^{**}_{\ell 1} \ln X_{21(t-\ell)} \right],$$

where $\alpha^{**}_{\ell 1} = \dfrac{\alpha^{*}_{\ell 1}}{\alpha_{21}}$.

One may similarly define:

$$\alpha^{**}_{\ell 2} = \frac{\alpha^{*}_{\ell 2}}{\alpha_{22}} \qquad \beta^{**}_{\ell 1} = \frac{\beta^{*}_{\ell 1}}{\beta_{21}} \qquad \beta^{**}_{\ell 2} = \frac{\beta^{*}_{\ell 2}}{\beta_{22}} \; .$$

Equations 4.7 and 4.8 may thus be rewritten as:

$$\begin{aligned} \ln Y_{1t} = \alpha_1 + \alpha_{11} \ln X_{11t} + \alpha_{21} \left[\ln X_{21t} + \sum_{\ell=1}^{L} \alpha^{**}_{\ell 1} \ln X_{21(t-\ell)} \right] \\ + \beta_{11} \ln X_{12t} + \beta_{21} \left[\ln X_{22t} + \sum_{\ell=1}^{L} \beta^{**}_{\ell 1} \ln X_{22(t-\ell)} \right] + u_{1t} \end{aligned} \tag{4.9}$$

$$\begin{aligned} \ln Y_{2t} = \alpha_2 + \alpha_{12} \ln X_{11t} + \alpha_{22} \left[\ln X_{21t} + \sum_{\ell=1}^{L} \alpha^{**}_{\ell 2} \ln X_{21(t-\ell)} \right] \\ + \beta_{12} \ln X_{12t} + \beta_{22} \left[\ln X_{22t} \sum_{\ell=1}^{L} \beta^{**}_{\ell 2} \ln X_{22(t-\ell)} \right] + u_{2t} \end{aligned} \tag{4.10}$$

Define a new variable Z_{21t}, representing the cumulative effect of variable X_{21} at time t, as follows:

$$Z_{21t} = X_{21t} \prod_{\ell=1}^{L} X_{21(t-1)}{}^{\exp\,(\alpha^{**}_{\ell 1})} \quad ,$$

so that by taking logarithms,

$$\ln Z_{21t} = \ln X_{21t} + \sum_{\ell=1}^{L} \alpha^{**}_{\ell 1} \ln X_{21(t-\ell)} \; .$$

The following variables could be similarly defined:

$$\ln Z'_{22t} = \ln X_{22t} + \sum_{\ell=1}^{L} \alpha^{**}_{\ell 2} \ln X_{22(t-\ell)};$$

$$\ln Z'_{21t} = \ln X_{21t} + \sum_{\ell=1}^{L} \beta^{**}_{\ell 1} \ln X_{21(t-\ell)};$$

$$\ln Z_{22t} = \ln X_{22t} + \sum_{\ell=1}^{L} \beta^{**}_{\ell 2} \ln X_{22(t-\ell)} .$$

Note that Z_{2j} refers to the cumulative effect of firm j's decision variable X_2 on its <u>own</u> dependent variable; while Z'_{2j} represents its cumulative effect on the <u>competitor's</u> dependent variable. Making appropriate substitutions, Equations 4.9 and 4.10 are simplified to:

$$\ln Y_{1t} = \alpha_1 + \alpha_{11} \ln X_{11t} + \alpha_{21} \ln Z_{21t} + \beta_{11} \ln X_{12t} + \beta_{21} \ln Z'_{22t} + u_{1t} \tag{4.11}$$

$$\ln Y_{2t} = \alpha_2 + \alpha_{12} \ln X_{11t} + \alpha_{22} \ln Z'_{21t} + \beta_{12} \ln X_{12t} + \beta_{22} \ln Z_{22t} + u_{2t} \tag{4.12}$$

For game-matrix construction purposes, then, the two decision variables for firm j are taken to be X_{1j} and Z_{2j} , which is a measure of cumulative effect of X_{2j}. The game matrix which would be derived would have the form of Figure 4.6, and the Y_1 and Y_2 values would be estimated using Equations 4.11 and 4.12. (Note: Figure 4.6 assumes that both decision variables are dichotomous; i.e., high/low: extensions for higher levels may easily be made.)

Equations 4.11 and 4.12, and Figure 4.6, are the Cobb-Douglas functional forms with carryover effects and corresponding game matrix (i.e., carryover extensions of Equations 4.3 and 4.4). The situation is only slightly more complex if interaction effects are significant. The Appendix to this chapter shows the derivation of the carryover extensions of the MNH functional forms (Equations 4.5 and 4.6).

P2

P1	X^L_{12}, Z^L_{22}	X^L_{12}, Z^H_{22}	X^H_{12}, Z^L_{22}	X^H_{12}, Z^H_{22}
X^L_{11} Z^L_{21}	Y^*_1 Y^*_2	$\cdots$	$\cdots$	$\cdots$
X^L_{11} Z^H_{21}	$\vdots$	$\ddots$		
X^H_{11} Z^L_{21}	$\vdots$		$\ddots$	
X^H_{11} Z^H_{21}	$\vdots$			$\ddots$

FIGURE 4.6

Note: $Y^*_1 = Y_1(X^L_{11}, Z^L_{21}, X^L_{12}, Z^L_{22})$

$Y^*_2 = Y_2(X^L_{11}, Z^L_{21}, X^L_{12}, Z^L_{22})$

Equations containing X_{1j} and Z_{2j} as the independent variables are thus used to construct the game matrices, in situations where carryover effects are significant. This makes the not unreasonable assumption that, when choosing a level for decision variable X_{2j}, firm j considers not only the instantaneous or short-term effect on the dependent variable, but also the cumulative lag effect of all the previous-period levels. In doing so, the firm chooses to maintain a desired level in the "goodwill account" of variable X_{2j}. The immediate decision to be taken by the firm is how much must be invested in X_{2j} at the moment in order to maintain the desired amount of goodwill.

A KOYCK MODEL OF CARRYOVER EFFECTS

In situations where more than one or two lag periods are significant, the goodwill model of carryover effects may not be appropriate (see section on model application in Chapter 5). The Koyck approach of this study is provided as an alternate method for incorporating lag effects in such situations where only one independent variable exhibits carryover.

The Koyck distributed-lag formulation discussed in Chapter 3 assumes that the weights applied to the lag periods decline geometrically. A similar assumption may be made here as well, in order to simplify estimation of Equations 4.7 and 4.8.

In Equation 4.7, suppose a restriction is placed on the $\alpha^*_{\ell 1}$'s and $\beta^*_{\ell 1}$'s: instead of allowing them to vary independently of each other, these parameters are restricted as follows:

$$\alpha^*_{\ell 1} = \alpha_{21} (1 - \gamma_1)^{\ell} \quad ;$$

$$\beta^*_{\ell 1} = \beta_{21} (1 - \gamma_2)^{\ell} \quad ;$$

where γ_1 and γ_2 vary between zero and one and indicate the significance of the lagged effects. Assuming for simplicity that the lag effects of decision variable X_2 are similar across firms, one may set $\gamma_1 = \gamma_2 = \gamma$. Similar reasoning can be applied to Equation 4.8 as well. Then, Equations 4.7 and 4.8 can be rewritten as

$$\ln Y_{1t} = \alpha_1 + \alpha_{11} \ln X_{11t} + \alpha_{21} \ln X_{21t} + \sum_{\ell=1}^{L} \alpha_{21}(1-\gamma)^{\ell} \ln X_{21(t-\ell)} + \beta_{11} \ln X_{12t} + \beta_{21} \ln X_{22t} + \sum_{\ell=1}^{L} \beta_{21}(1-\gamma)^{\ell} \ln X_{22(t-\ell)} + u_{1t} \qquad (4.13)$$

$$\ln Y_{2t} = \alpha_2 + \alpha_{12} \ln X_{11t} + \alpha_{22} \ln X_{21t} + \sum_{\ell=1}^{L} \alpha_{22}(1-\gamma)^{\ell} \ln X_{21(t-\ell)} + \beta_{12} \ln X_{12t} + \beta_{22} \ln X_{22t} + \sum_{\ell=1}^{L} \beta_{22}(1-\gamma)^{\ell} \ln X_{22(t-\ell)} + u_{2t} \qquad (4.14)$$

which may, of course, be simplified to:

$$\ln Y_{1t} = \alpha_1 + \alpha_{11} \ln X_{11t} + \alpha_{21} \left[\sum_{\ell=0}^{L} (1-\gamma)^{\ell} \ln X_{21(t-\ell)} \right] + \beta_{11} \ln X_{12t} + \beta_{21} \left[\sum_{\ell=0}^{L} (1-\gamma)^{\ell} \ln X_{22(t-\ell)} \right] + u_{1t} \qquad (4.15)$$

$$\ln Y_{2t} = \alpha_2 + \alpha_{12} \ln X_{11t} + \alpha_{22} \left[\sum_{\ell=0}^{L} (1-\gamma)^{\ell} \ln X_{21(t-\ell)} \right] + \beta_{12} \ln X_{12t} + \beta_{22} \left[\sum_{\ell=0}^{L} (1-\gamma)^{\ell} \ln X_{22(t-\ell)} \right] + u_{2t}. \qquad (4.16)$$

Intuitively, this specification makes sense. Since, by definition, $0 < \gamma < 1$, it follows that $(1-\gamma)$ is positive and less than one. Hence, the carryover effect of lagged independent variables becomes smaller and smaller as time goes on. If $\gamma = 0.8$, say, using the third part of Equation 4.15, the net (cumulative) effect of decision variable X_{21} on Firm 1's objective variable Y_1 is

$$\alpha_{21} \left[\sum_{\ell=0}^{L} (1-0.8)^{\ell} \ln X_{21(t-\ell)} \right]$$
$$= \alpha_{21} \left[\ln X_{21t} + (0.2) \ln X_{21(t-1)} + (0.04) \ln X_{21(t-2)} + \ldots + (0.2)^{L} \ln X_{21(t-L)} \right].$$

Now, Z variables representing cumulative effects of decision variables may be defined, as had been done for the goodwill approach:

$$\ln Z_{2jt} = \sum_{\ell=0}^{L} (1-\gamma)^{\ell} \ln X_{2j(t-\ell)}; \quad j = (1,2) \tag{4.17}$$

As before, X_{1j} and Z_{2j} would be taken as the relevant variables for game-matrix construction. The equations used for this purpose would be obtained by making appropriate substitutions in 4.15 and 4.16.

$$\ln Y_{1t} = \alpha_1 + \alpha_{11} \ln X_{11t} + \alpha_{21} \ln Z_{21t} + \beta_{11} \ln X_{12t} + \beta_{21} \ln Z_{22t} + u_{1t}; \tag{4.18}$$

$$\ln Y_{2t} = \alpha_2 + \alpha_{12} \ln X_{11t} + \alpha_{22} \ln Z_{21t} + \beta_{12} \ln X_{12t} + \beta_{22} \ln Z_{22t} + u_{2t}. \tag{4.19}$$

The estimation of the γ's of Equations 4.15 and 4.16 is an operational consideration, and therefore is left for Chapter 5, where all aspects of model estimation and application are dealt with.

A DYNAMIC-ADJUSTMENT CARRYOVER MODEL

Dynamic-adjustment models have been used in dividend policy to model effects of cash flow on size of dividend (Feldstein 1970, 1972; Chateau 1974). The Chateau model yields parameters which are directly interpretable as marginal effects, and also clearly indicates the relative magnitudes of both short-term and long-term effects. Note: This approach is not empirically tested in this study, but is presented as an alternative model applicable where both (or all) decision variables are expected to exhibit carryover.

In a marketing context, effects of decision variables X_1 and X_2 on dependent variable Y may be modelled using equations of dynamic-adjustment form. One must make the assumption that each firm has an optimum Y level (target level) which it wishes to attain. A firm currently carrying 10 to 12% of the market may set a long-term company target, say, of 15% market share: this becomes the Y* of the optimum equation (Equation 3.6). The actual value of Y* does not make any difference in the calculations as the Y* eventually drops out of the model; but to permit application of a dynamic-adjustment model, its existence must be assumed.

Ignoring competitive effects for the moment, a conceivable marketing decision-variable adaptation of the optimum equation for Firm j would be

$$\ln Y^*_{jt} = \phi_{0j} + \phi_{1j} \ln X_{1jt} + \phi_{2j} \ln X_{2jt} + u'_{jt}; \qquad (4.20)$$

while the adjustment equation (Equation 3.7) could be adapted to

$$\ln Y_{jt} - \ln Y_{j(t-1)} = \gamma_j (\ln Y^*_{jt} - \ln Y_{j(t-1)}) + e_{jt}. \qquad (4.21)$$

Combining these equations gives

$$\ln Y_{jt} - \ln Y_{j(t-1)} = \gamma_j(\phi_{oj} + \phi_{1j} \ln X_{1jt} + \phi_{2j} \ln X_{2jt} + u'_{jt} - \ln Y_{j(t-1)}) + e_{jt}$$

which, upon rearrangement, yields the operational equation

$$\ln Y_{jt} = \gamma_j \phi_{0j} + \gamma_j \phi_{1j} \ln X_{1jt} + \gamma_j \phi_{2j} \ln X_{2jt} + (1 - \gamma_j) \ln Y_{j(t-1)} + W_t. \quad (4.22)$$

It remains to incorporate competitive effects into the model. As in the other approaches, firms do not make marketing decisions in a vacuum: Firm 1's payoff depends upon Firm 2's decisions on, say, advertising and price, as well as upon its own. Equation 4.20 must be refined in order to work in the relevant competitive effects, as in Equation 4.23:

$$\ln Y^*_{jt} = \phi_{oj} + \phi_{1j} \ln X_{11t} + \phi_{2j} \ln X_{21t} + \theta_{1j} \ln X_{12t} + \theta_{2j} \ln X_{22t} + u'_{jt} \quad (4.23)$$

Combining Equation 4.23 with Equation 4.21 as before gives

$$\ln Y_{jt} - \ln Y_{j(t-1)} = \gamma_j(\phi_{oj} + \phi_{1j} \ln X_{11t} + \phi_{2j} \ln X_{21t} + \theta_{1j} \ln X_{12t} + \theta_{2j} \ln X_{22t} + u'_{jt} - \ln Y_{j(t-1)}) + e_t ,$$

which yields, upon rearrangement,

$$\ln Y_{jt} = \gamma_j \phi_{oj} + \gamma_j \phi_{1j} \ln X_{11t} + \gamma_j \phi_{2j} \ln X_{21t} + \gamma_j \theta_{1j} \ln X_{12t} + \gamma_j \theta_{2j} \ln X_{22t} + (1 - \gamma_j) \ln Y_{j(t-1)} + W_t . \quad (4.24)$$

Up to here, the carryover effects of the decision variables have not been modelled explicitly; however, an element of carry-over has been introduced in a general sense with the inclusion of the lagged dependent term. One additional step is required in order to model explicitly the carryover effect of both (or all) decision variables: namely, the replacement of the lagged dependent variable $Y_{j(t-1)}$.

In the Appendix to this chapter, Equation 4A.9 is derived which models explicitly the desired carryover effects. It is given below as Equation 4.25: the reader is directed to the Appendix for details of its derivation.

$$\begin{aligned} \ln Y_{jt} = {} & \phi_{0j} + \gamma_j \phi_{1j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{11(t-\ell)} \\ & + \gamma_j \phi_{2j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{21(t-\ell)} \\ & + \gamma_j \theta_{1j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{12(t-\ell)} + \gamma_j \theta_{2j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{22(t-\ell)} \\ & + W^*_{jt}. \end{aligned} \tag{4.25}$$

In the Appendix are also found definitions of the cumulative effects of the decision variables X_{ik} at time t. These are:

$$\ln Z^{(j)}_{ikt} = \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{ik(t-\ell)}; \quad i,k = (1,2).$$

Using these substitutions, the alternate form of Equation 4.25 was derived in the Appendix as Equation 4A.10:

$$\begin{aligned} \ln Y_{jt} = {} & \phi_{0j} + \gamma_j \phi_{1j} \ln Z^{(j)}_{11t} + \gamma_j \phi_{2j} \ln Z^{(j)}_{21t} \\ & + \gamma_j \theta_{1j} \ln Z^{(j)}_{12t} + \gamma_j \theta_{2j} \ln Z^{(j)}_{22t} + W^*_{jt}. \end{aligned} \tag{4.26}$$

Equation 4.26 would be the form subsequently used to estimate the payoffs in the game matrix.

N-FIRM EXTENSIONS FOR THE OLIGOPOLY

The first model presented in this chapter was a basic duopoly model with exactly one discrete, dichotomous marketing-mix variable, X_1. No allowance was made for the possibility of carryover effects; and, given only one decision variable, interaction effects were irrelevant. Succeeding models gradually reduced the many restrictions of this basic model: first, multiple decision variables were permitted; then multichotomous discrete distributions and interaction effects were considered; continuous distributions were approximated; and three alternate methods for representing carryover effects were discussed. All that remains is to allow provision for considering more than two firms at one time.

The basic model for a three-player oligopoly would be a three-dimensional analog of Figure 4.2, as shown in Figure 4.7. This can easily be extended to the case of two independent, multichotomous variables (see Figure 4.8).

The main effects of X_1 and X_2 on Y_j , $j = (1,3)$ may be modelled as before by Cobb-Douglas-form simultaneous equations:

$$\ln Y_1 = \alpha_1 + \alpha_{11} \ln X_{11} + \alpha_{21} \ln X_{21} + \beta_{11} \ln X_{12} + \beta_{21} \ln X_{22} + \gamma_{11} \ln X_{13} + \gamma_{21} \ln X_{23}$$

$$\ln Y_2 = \alpha_2 + \alpha_{12} \ln X_{11} + \alpha_{22} \ln X_{21} + \beta_{12} \ln X_{12} + \beta_{22} \ln X_{22} + \gamma_{12} \ln X_{13} + \gamma_{22} \ln X_{23}$$

$$\ln Y_3 = \alpha_3 + \alpha_{13} \ln X_{11} + \alpha_{23} \ln X_{21} + \beta_{13} \ln X_{12} + \beta_{23} \ln X_{22} + \gamma_{13} \ln X_{13} + \gamma_{23} \ln X_{23} .$$

This can then be generalized to n firms ($n \geqslant 3$) by the set of equations

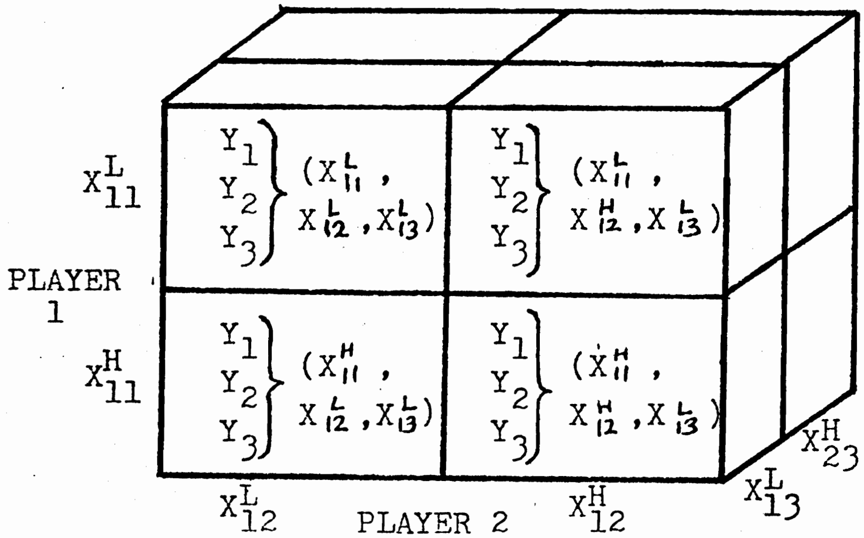


FIGURE 4.7

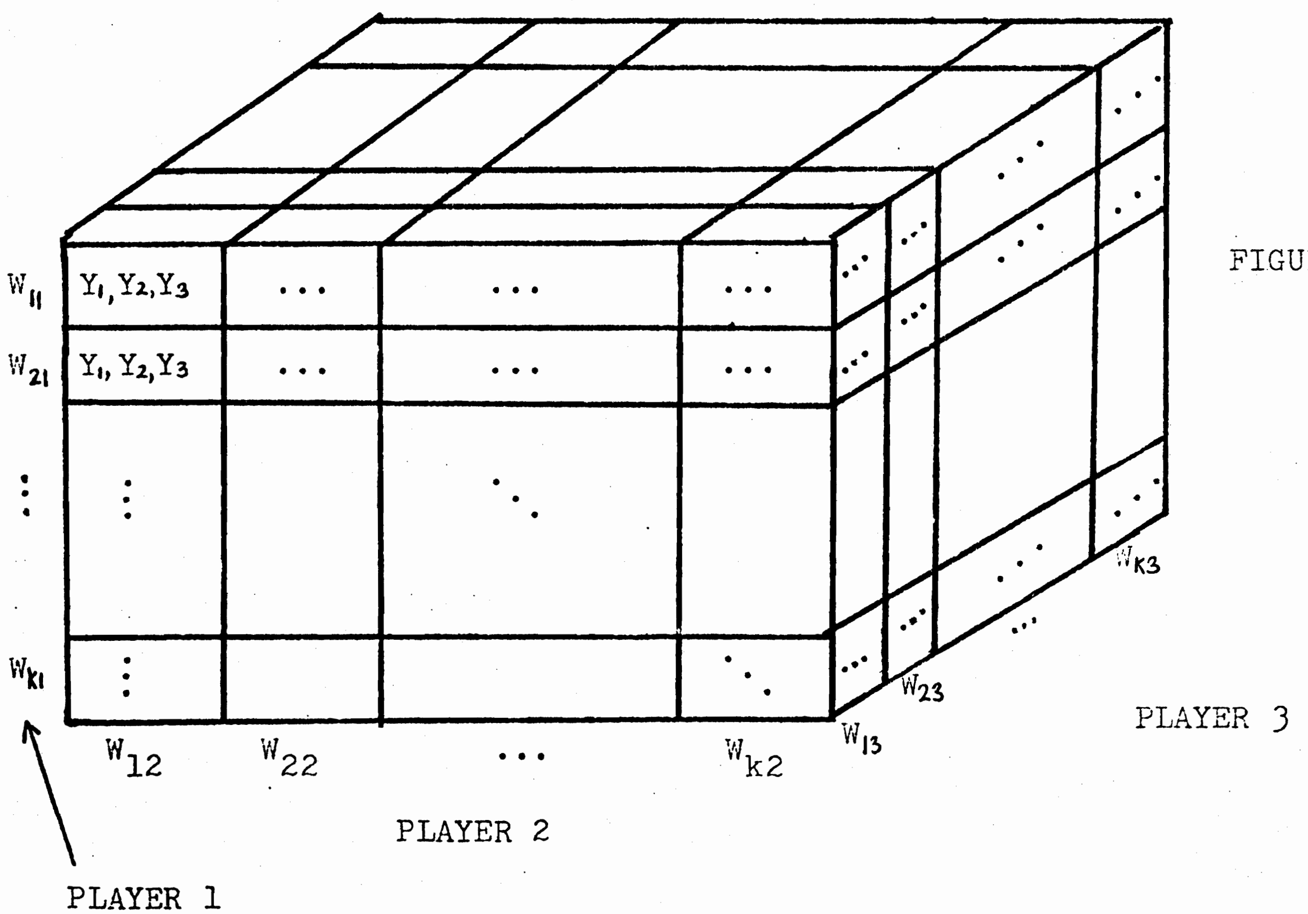


FIGURE 4.8

$$\begin{bmatrix} \ln Y_1 \\ \ln Y_2 \\ \vdots \\ \ln Y_n \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} \alpha_{11}^{(1)} & \alpha_{21}^{(1)} \\ \alpha_{12}^{(1)} & \alpha_{22}^{(1)} \\ \vdots & \vdots \\ \alpha_{1n}^{(1)} & \alpha_{2n}^{(1)} \end{bmatrix} \begin{bmatrix} \ln X_{11} \\ \ln X_{21} \end{bmatrix} + \begin{bmatrix} \alpha_{11}^{(2)} & \alpha_{21}^{(2)} \\ \alpha_{12}^{(2)} & \alpha_{22}^{(2)} \\ \vdots & \vdots \\ \alpha_{1n}^{(2)} & \alpha_{2n}^{(2)} \end{bmatrix} \begin{bmatrix} \ln X_{12} \\ \ln X_{22} \end{bmatrix}$$

$$+ \cdots + \begin{bmatrix} \alpha_{11}^{(n)} & \alpha_{21}^{(n)} \\ \alpha_{21}^{(n)} & \alpha_{22}^{(n)} \\ \vdots & \vdots \\ \alpha_{1n}^{(n)} & \alpha_{2n}^{(n)} \end{bmatrix} \begin{bmatrix} \ln X_{1n} \\ \ln X_{2n} \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix}, \tag{4.27}$$

where α_{ij}'s with superscripts are used instead of β's, γ's, etc. in order to make generalization to the n-firm case easier.

By making the obvious definitions, one could rewrite 4.27 in matrix motation:

$$\ln \mathbf{Y} = \boldsymbol{\alpha} + \boldsymbol{\alpha}^{(1)} \ln \mathbf{X}_1 + \boldsymbol{\alpha}^{(2)} \ln \mathbf{X}_2 + \cdots + \boldsymbol{\alpha}^{(n)} \ln \mathbf{X}_n + \mathbf{W} \tag{4.28}$$

The MNH form, which would be appropriate where interactions are significant, would appear in the n-firm case (ignoring carryover effects for the moment) as the set of equations

$$\begin{bmatrix} \ln Y_1 \\ \ln Y_2 \\ \vdots \\ \ln Y_n \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} \alpha_{11}^{(1)} & \alpha_{21}^{(1)} \\ \alpha_{12}^{(1)} & \alpha_{22}^{(1)} \\ \vdots & \vdots \\ \alpha_{1n}^{(1)} & \alpha_{2n}^{(1)} \end{bmatrix} \begin{bmatrix} \ln X_{11} \\ \ln X_{21} \end{bmatrix} + \cdots + \begin{bmatrix} \alpha_{11}^{(n)} & \alpha_{21}^{(n)} \\ \alpha_{12}^{(n)} & \alpha_{22}^{(n)} \\ \vdots & \vdots \\ \alpha_{1n}^{(n)} & \alpha_{2n}^{(n)} \end{bmatrix} \begin{bmatrix} \ln X_{1n} \\ \ln X_{2n} \end{bmatrix}$$

$$+ \begin{bmatrix} \xi_{11} & \xi_{21} & \cdots & \xi_{n1} \\ \xi_{12} & & & \vdots \\ \vdots & & & \\ \xi_{1n} & \xi_{2n} & \cdots & \xi_{nn} \end{bmatrix} \begin{bmatrix} \ln X_{11} & \ln X_{21} \\ \ln X_{12} & \ln X_{22} \\ \vdots & \\ \ln X_{1n} & \ln X_{2n} \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix} \tag{4.29}$$

which would appear in matrix notation as

$$\ln \mathbf{Y} = \boldsymbol{\alpha} + \boldsymbol{\alpha}^{(1)} \ln \mathbf{X}_1 + \ldots + \boldsymbol{\alpha}^{(n)} \ln \mathbf{X}_n + \boldsymbol{\xi} \ln \mathbf{X}_1 \ln \mathbf{X}_2 + \mathbf{W} \quad (4.30)$$

The goodwill model of carryover effects of variable X_2 may also be extended to the n-firm case. Suppose at first that only one lag period is significant. The appropriate set of equations would be

$$\begin{bmatrix} \ln Y_{1t} \\ \ln Y_{2t} \\ \vdots \\ \ln Y_{nt} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} \alpha_{11}^{(1)} & \alpha_{21}^{(1)} \\ \alpha_{12}^{(1)} & \alpha_{22}^{(1)} \\ \vdots & \vdots \\ \alpha_{1n}^{(1)} & \alpha_{2n}^{(1)} \end{bmatrix} \begin{bmatrix} \ln X_{11} \\ \ln X_{21} \end{bmatrix} + \cdots + \begin{bmatrix} \alpha_{11}^{(n)} & \alpha_{21}^{(n)} \\ \alpha_{12}^{(n)} & \alpha_{22}^{(n)} \\ \vdots & \vdots \\ \alpha_{1n}^{(n)} & \alpha_{2n}^{(n)} \end{bmatrix} \begin{bmatrix} \ln X_{1n} \\ \ln X_{2n} \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha_{11}^{(1)*} & \alpha_{11}^{(2)*} & \cdots & \alpha_{11}^{(n)*} \\ \alpha_{12}^{(1)*} & \alpha_{12}^{(2)*} & \cdots & \alpha_{12}^{(n)*} \\ \vdots & \vdots & & \vdots \\ \alpha_{1n}^{(1)*} & \alpha_{1n}^{(2)*} & \cdots & \alpha_{1n}^{(n)*} \end{bmatrix} \begin{bmatrix} \ln X_{21(t-1)} \\ \ln X_{22(t-1)} \\ \vdots \\ \ln X_{2n(t-1)} \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix}, \quad (4.31)$$

where the $\alpha_{ik}^{(n)}$'s are employed as before, and $\alpha_{\ell j}^{(k)*}$ = parameters corresponding to effect on Y_j of Firm k's level of variable x in period $(t-\ell)$. Since in 4.31 only one lag period is considered, all ℓ's are unity (1) in the second-last matrix.

The corresponding matrix-notation form would be

$$\ln \mathbf{Y}_t = \boldsymbol{\alpha} + \boldsymbol{\alpha}^{(1)} \ln \mathbf{X}_{1t} + \ldots + \boldsymbol{\alpha}^{(n)} \ln \mathbf{X}_{nt} + \boldsymbol{\alpha}_{(1)}^{*} \ln \mathbf{X}_{2(t-1)} + \mathbf{W} \quad (4.32)$$

where $\ln \mathbf{X}_{2(t-1)}$ refers to the matrix comprising each firm's X_2 level last period. Similarly the matrix comprising the X_2 levels ℓ periods ago would be named $\ln \mathbf{X}_{2(t-\ell)}$.

To accomodate more lag periods, additional matrices would be added to 4.32. If the total number of significant lag periods to include is L, this set of equations would be extended as follows:

$$\ln Y_t = \alpha + \alpha^{(1)} \ln X_{1t} + \ldots + \alpha^{(1)} \ln X_{nt} + \alpha_1^* \ln X_{2(t-1)} + \cdots + \alpha_L^* \ln X_{2(t-L)}. \tag{4.33}$$

However, as before, the Koyck model may be more appropriate where more than just a few lag periods are significant. The two-firm model, as described by Equations 4.15 and 4.16, may be adapted to the n-firm situation as follows:

$$\begin{bmatrix} \ln Y_1 \\ \ln Y_2 \\ \vdots \\ \ln Y_n \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} \alpha_{11}^{(1)} & \alpha_{11}^{(2)} \cdots & \alpha_{11}^{(n)} \\ \alpha_{12}^{(1)} & \alpha_{12}^{(2)} \cdots & \alpha_{12}^{(n)} \\ \vdots & \vdots & \vdots \\ \alpha_{1n}^{(1)} & \alpha_{1n}^{(2)} \cdots & \alpha_{1n}^{(n)} \end{bmatrix} \begin{bmatrix} \ln X_{11t} \\ \ln X_{12t} \\ \vdots \\ \ln X_{1nt} \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha_{21}^{(1)} & \alpha_{21}^{(2)} \cdots & \alpha_{21}^{(n)} \\ \alpha_{22}^{(1)} & \alpha_{22}^{(2)} \cdots & \alpha_{22}^{(n)} \\ \vdots & \vdots & \vdots \\ \alpha_{2n}^{(1)} & \alpha_{2n}^{(2)} \cdots & \alpha_{2n}^{(n)} \end{bmatrix} \begin{bmatrix} \sum^{L} (1-\gamma)^{\ell} \ln X_{21(t-\ell)} \\ \sum^{L} (1-\gamma)^{\ell} \ln X_{22(t-\ell)} \\ \vdots \\ \sum^{L} (1-\gamma)^{\ell} \ln X_{2n(t-\ell)} \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix} \tag{4.34}$$

In Equation 4.34, the second last term (of dimension n x 1) may be simplified by rewriting it as the product of two matrices, of dimensions n x L and L x 1 respectively. The resulting explicit form is:

$$\begin{bmatrix} \ln Y_1 \\ \ln Y_2 \\ \vdots \\ \ln Y_n \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} \alpha_{11}^{(1)} & \alpha_{11}^{(2)} & \cdots & \alpha_{11}^{(n)} \\ \alpha_{12}^{(1)} & \alpha_{12}^{(1)} & \cdots & \alpha_{12}^{(n)} \\ \vdots & \vdots & & \vdots \\ \alpha_{1n}^{(1)} & \alpha_{1n}^{(1)} & \cdots & \alpha_{1n}^{(n)} \end{bmatrix} \begin{bmatrix} \ln X_{11t} \\ \ln X_{12t} \\ \vdots \\ \ln X_{1nt} \end{bmatrix} \tag{4.35}$$

$$+ \begin{bmatrix} \alpha_{21}^{(1)} & \alpha_{21}^{(2)} & \cdots & \alpha_{21}^{(n)} \\ \alpha_{22}^{(1)} & \alpha_{22}^{(2)} & \cdots & \alpha_{22}^{(n)} \\ \vdots & \vdots & & \vdots \\ \alpha_{2n}^{(1)} & \alpha_{2n}^{(2)} & \cdots & \alpha_{2n}^{(n)} \end{bmatrix} \begin{bmatrix} \ln X_{21t} & \ln X_{21t-1} & \cdots & \ln X_{21t-L} \\ \ln X_{22t} & \ln X_{22t-1} & \cdots & \ln X_{22t-L} \\ \vdots & \vdots & & \vdots \\ \ln X_{2nt} & \ln X_{2nt-1} & \cdots & \ln X_{2nt-L} \end{bmatrix} \begin{bmatrix} (1-\gamma)^0 \\ (1-\gamma)^1 \\ \vdots \\ (1-\gamma)^L \end{bmatrix}$$

$$+ \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix} .$$

An n-firm analog for the MNH functional form with carryover effects appears in the Appendix to this chapter, as does an n-firm extension to the dynamic-adjustment approach.

CHAPTER 4: APPENDIX
MATHEMATICAL DERIVATIONS

MNH FORM WITH CARRYOVER FOR THE DUOPOLY

The MNH functional forms (Equations 4.5 and 4.6) include interaction effects, and are easily modified to include carryover effects of variable X_2, giving Equation 4A.1.

$$\begin{aligned}\ln Y_{jt} = {} & \alpha_j + \alpha_{1j} \ln X_{11t} + \alpha_{2j} \ln X_{21t} + \sum_{\ell=1}^{L} \alpha^*_{\ell j} \ln X_{21(t-\ell)} \\ & + \beta_{1j} \ln X_{12t} + \beta_{2j} \ln X_{22t} + \sum_{\ell=1}^{L} \beta^*_{\ell j} \ln X_{22(t-\ell)} + \xi_{1j} \ln X_{11t} \ln X_{21t} \\ & + \sum_{\ell=1}^{L} \alpha'^*_{\ell j} \ln X_{11t} \ln X_{21(t-\ell)} + \xi_{2j} \ln X_{12t} \ln X_{22t} \\ & + \sum_{\ell=1}^{L} \beta'^*_{\ell j} \ln X_{12t} \ln X_{22(t-\ell)}\end{aligned} \tag{4A.1}$$

In this equation, $\alpha'^*_{\ell 1}$ and $\beta'^*_{\ell 2}$ =interaction effect between <u>own</u> firm's X_{1t} and $X_{2(t-\ell)}$ levels; $\alpha'^*_{\ell 2}$ and $\beta'^*_{\ell 1}$ = interaction effect between <u>competitor's</u> X_{1t} and $X_{2(t-\ell)}$ levels; and other parameters are defined as before.

Grouping terms as before, the cumulative effect of X_{2j} on Y_j is given by

$$\alpha_{2j} \ln X_{21t} + \sum_{\ell=1}^{L} \alpha^*_{\ell j} \ln X_{21(t-\ell)} + \xi_{1j} \ln X_{11t} \ln X_{21t} + \sum_{\ell=1}^{L} \alpha'^*_{j} \ln X_{11t} \ln X_{21(t-\ell)},$$

or

$$\alpha_{2j} \left[\ln X_{21t} + \sum_{\ell=1}^{L} \frac{\alpha^*_{\ell j}}{\alpha_{2j}} \ln X_{21(t-\ell)} \right] + \xi_{1j} \ln X_{11t} \left[\ln X_{21t} + \sum_{\ell=1}^{L} \frac{\alpha'^*_{j}}{\xi_{1j}} \ln X_{21(t-\ell)} \right],$$

which, upon substituting

$$\alpha_{\ell j}^{**} = \frac{\alpha_{\ell j}^{*}}{\alpha_{2j}} \quad \text{and} \quad \alpha_{\ell j}^{'**} = \frac{\alpha_{\ell j}^{'*}}{\xi_{1j}} ,$$

gives

$$\alpha_{2j}\left[\ln X_{21t} + \sum_{\ell=1}^{L} \alpha_{\ell j}^{**} \ln X_{21(t-\ell)}\right]$$

$$+ \xi_{1j} \ln X_{11t}\left[\ln X_{21t} + \sum_{\ell=1}^{L} \alpha_{\ell j}^{'**} \ln X_{21(t-\ell)}\right] .$$

One may define also

$$\beta_{\ell j}^{**} = \frac{\beta_{\ell j}^{*}}{\beta_{2j}} \quad \text{and} \quad \beta_{\ell j}^{'**} = \frac{\beta_{\ell j}^{'*}}{\xi_{2j}}$$

Equation 4A.1 would thus be written as

$$\begin{aligned} \ln Y_{jt} = \alpha_j &+ \alpha_{1j} \ln X_{11t} + \alpha_{2j}\left[\ln X_{21t} + \sum_{\ell=1}^{L} \alpha_{\ell j}^{**} \ln X_{21(t-\ell)}\right] \\ &+ \xi_{1j} \ln X_{11t}\left[\ln X_{21t} + \sum_{\ell=1}^{L} \alpha_{\ell j}^{'**} \ln X_{21(t-\ell)}\right] + \beta_{1j} \ln X_{12t} \\ &+ \beta_{2j}\left[\ln X_{22t} + \sum_{\ell=1}^{L} \beta_{\ell j}^{**} \ln X_{22(t-\ell)}\right] + \xi_{2j} \ln X_{12t}\left[\ln X_{22t} \right. \\ &\left. + \sum_{\ell=1}^{L} \beta_{\ell j}^{'**} \ln X_{22(t-\ell)}\right] . \end{aligned} \qquad (4A.2)$$

Cumulative-effect variables corresponding to the previously-defined Z's could be introduced as follows:

$$\ln Z_{2jt} = \ln X_{21t} + \sum_{\ell=1}^{L} \alpha^{**}_{j} \ln X_{21(t-\ell)}$$

$$\ln Z'_{2jt} = \ln X_{21t} + \sum_{\ell=1}^{L} \alpha'^{**}_{j} \ln X_{21(t-\ell)}$$

$$\ln Z''_{2jt} = \ln X_{22t} + \sum_{\ell=1}^{L} \beta^{**}_{j} \ln X_{22(t-\ell)}$$

$$\ln Z'''_{2jt} = \ln X_{22t} + \sum_{\ell=1}^{L} \beta'^{**}_{j} \ln X_{22(t-\ell)}$$

Equation 4A.2 could thus be rewritten as

$$\ln Y_{jt} = \alpha_j + \alpha_{1j} \ln X_{11t} + \alpha_{2j} \ln Z_{2jt} + \xi_{1j} \ln Z'_{2jt} + \beta_{1j} \ln X_{12t} + \beta_{2j} \ln Z''_{2jt} + \xi_{2j} \ln Z'''_{2jt} \tag{4A.3}$$

In constructing the corresponding game matrix, X_{11} and Z_{21} will be taken as Firm 1's decision variables, and X_{12} and Z''_{22} as Firm 2's decision variables, as these correspond to the X_{1j} and Z_{2j} of the Cobb-Douglas solution.

DYNAMIC-ADJUSTMENT FORM WITH EXPLICIT CARRYOVER EFFECTS

Equation 4.24 is the operational form of the dynamic-adjustment model previously derived. If this equation is lagged one period, the following is obtained:

$$\ln Y_{j(t-1)} = \gamma_j \phi_{oj} + \gamma_j \phi_{1j} \ln X_{11(t-1)} + \gamma_j \phi_{2j} \ln X_{21(t-1)}$$

$$+ \gamma_j \theta_{1j} \ln X_{12(t-1)} + \gamma_j \theta_{2j} \ln X_{22(t-1)} + (1-\gamma_j) \ln Y_{j(t-2)} + W_{t-1}$$

Substitution into 4.24 yields the equation

$$\ln Y_{jt} = \gamma_j \phi_{oj} + (1-\gamma_j)(\gamma_j \phi_{oj}) + \gamma_j \phi_{1j} \ln X_{11t}$$
$$+ (1-\gamma_j)(\gamma_j \phi_{1j}) \ln X_{11(t-1)} + \gamma_j \phi_{2j} \ln X_{21t}$$
$$+ (1-\gamma_j)(\gamma_j \phi_{2j}) \ln X_{21(t-1)} + \gamma_j \theta_{1j} \ln X_{12t}$$
$$+ (1-\gamma_j)(\gamma_j \theta_{1j}) \ln X_{12(t-1)} + \gamma_j \theta_{2j} \ln X_{22t} + (1-\gamma_j)(\gamma_j \theta_{1j}) \ln X_{22(t-1)}$$
$$+ (1-\gamma_j)(1-\gamma_j) \ln Y_{j(t-2)} + W'_t, \qquad (4A.4)$$

where W'_t is the appropriate error term, i.e.,

$$W'_t = W_t + (1-\gamma_j) W_{t-1}.$$

Lagging 4.24 another period yields

$$\ln Y_{j(t-2)} = \gamma_j \phi_{oj} + \gamma_j \phi_{1j} \ln X_{11(t-2)} + \gamma_j \phi_{2j} \ln X_{21(t-2)}$$
$$+ \gamma_j \theta_{1j} \ln X_{12(t-2)} + \gamma_j \theta_{2j} \ln X_{22(t-2)} + (1-\gamma_j) \ln Y_{j(t-3)} + W_{t-2},$$

and if the substitution for $\ln Y_{j(t-2)}$ is made in 4A.4, one obtains

$$\ln Y_{jt} = \gamma_j \phi_{oj} + (1-\gamma_j)(\gamma_j - \phi_{oj}) + (1-\gamma_j)^2 (\gamma_j \phi_{oj}) + \gamma_j \phi_{1j} \ln X_{11t}$$
$$+ (1-\gamma_j)(\gamma_j \phi_{1j}) \ln X_{11(t-1)} + (1-\gamma_j)^2 (\gamma_j \phi_{1j}) \ln X_{11(t-2)}$$
$$+ \gamma_j \phi_{2j} \ln X_{21t} + (1-\gamma_j)(\gamma_j \phi_{2j}) \ln X_{21(t-1)} + (1-\gamma_j)^2 (\gamma_j \phi_{2j}) \ln X_{21(t-2)}$$
$$+ \gamma_j \theta_{1j} \ln X_{12t} + (1-\gamma_j)(\gamma_j \theta_{1j}) \ln X_{12(t-1)} + (1-\gamma_j)^2 (\gamma_j \theta_{1j}) \ln X_{12(t-2)}$$
$$+ \gamma_j \theta_{2j} \ln X_{22t} + (1-\gamma_j)(\gamma_j \theta_{2j}) \ln X_{22(t-1)} + (1-\gamma_j)^2 (\gamma_j \theta_{2j}) \ln X_{22(t-2)}$$
$$+ (1-\gamma_j)^3 \ln Y_{j(t-3)} + W''_t, \qquad (4A.5)$$

where W_t'' is again appropriately defined as

$$W_t'' = W_t' + (1 - \gamma_j)W_{t-2}.$$

Equation 4A.4 is therefore appropriate where one lag period is significant; and 4A.5 where two lag periods are significant. By extension, one can determine the corresponding expression in the case where L lag periods are significant. This expression is Equation 4A.6.

$$\begin{aligned}\ln Y_{jt} = {} & \sum_{\ell=0}^{L}(1-\gamma_j)^{\ell}\gamma_j\phi_{oj} + \sum_{\ell=0}^{L}(1-\gamma_j)^{\ell}\,\gamma_j\phi_{1j}\ln X_{11(t-\ell)} \\ & + \sum_{\ell=0}^{L}(1-\gamma_j)^{\ell}\,\gamma_j\phi_{2j}\ln X_{21(t-\ell)} + \sum_{\ell=0}^{L}(1-\gamma_j)^{\ell}\,\gamma_j\theta_{1j}\ln X_{12(t-\ell)} \\ & + \sum_{\ell=0}^{L}(1-\gamma_j)^{\ell}\,\gamma_j\theta_{2j}\ln X_{22(t-\ell)} + (1-\gamma_j)^{L+1}\ln Y_{j(t-(L+1))} + W_t^*,\end{aligned} \tag{4A.6}$$

where W_t^* is the corresponding error term. The reader may want to check this equation by substituting $L=1$ and $L=2$, and verifying that 4A.4 and 4A.5 are indeed obtained.

Equation 4A.6 may be suitably rearranged to yield

$$\begin{aligned}\ln Y_{jt} = {} & \gamma_j\phi_{oj}\sum_{\ell=0}^{L}(1-\gamma_j)^{\ell} + \gamma_j\phi_{1j}\sum_{\ell=0}^{L}(1-\gamma_j)^{\ell}\ln X_{11(t-\ell)} \\ & + \gamma_j\phi_{2j}\sum_{\ell=0}^{L}(1-\gamma_j)^{\ell}\ln X_{21(t-\ell)} + \gamma_j\theta_{1j}\sum_{\ell=0}^{L}(1-\gamma_j)^{\ell}\ln X_{12(t-\ell)} \\ & + \gamma_j\theta_{2j}\sum_{\ell=0}^{L}(1-\gamma_j)^{\ell}\ln X_{22(t-\ell)} + (1-\gamma_j)^{L+1}\ln Y_{j(t-(L+1))} + W_t^*.\end{aligned} \tag{4A.7}$$

One may simplify the first term of 4A.7 easily, by using an infinite-series theorem.

It can be shown (see Schwartz 1974, e.g.) that the sum of a geometric series may be expressed as

$$S_L = a + ar + ar^2 + \ldots + ar^L$$

$$= \sum_{\ell=0}^{L} ar^{\ell} = a \sum_{\ell=0}^{L} r^{\ell}$$

$$= a(1+r+r^2+\ldots+r^L)$$

$$= a \left[\frac{1 - r^L}{1 - r}\right]$$

If one takes $r = (1 - \gamma_j)$, this expression becomes

$$a \sum_{\ell=0}^{L} r^{\ell} = a \sum_{\ell=0}^{L} (1 - \gamma_j)^{\ell} = a \left[\frac{1 - (1 - \gamma_j)^L}{1 - (1 - \gamma_j)}\right]$$

and a final simplification in the denominator yields

$$a \sum_{\ell=0}^{L} (1 - \gamma_j)^{\ell} = a \left[\frac{1 - (1 - \gamma_j)^L}{\gamma_j}\right].$$

Finally, substitution into Part 1 of 4A.7 results in the following simplification:

$$\gamma_j \phi_{oj} \sum_{\ell=0}^{L} (1 - \gamma_j)^{\ell} = \gamma_j \phi_{oj} \left[\frac{1 - (1 - \gamma_j)^L}{\gamma_j}\right].$$

With this substitution, 4A.7 becomes:

$$\begin{aligned} \ln Y_{jt} = {} & \gamma_j \phi_{oj} \left[\frac{1 - (1-\gamma_j)^L}{\gamma_j}\right] + \gamma_j \phi_{1j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{11(t-\ell)} \\ & + \gamma_j \phi_{2j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{21(t-\ell)} + \gamma_j \theta_{1j} \sum_{\ell=0}^{L} (1 - \gamma_j)^{\ell} \ln X_{12(t-\ell)} \\ & + \gamma_j \theta_{2j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{22(t-\ell)} + (1-\gamma_j)^{L+1} \ln Y_{j(t-(L+1))} + W^*_t. \end{aligned} \tag{4A.8}$$

Equation 4A.8 is therefore appropriate where the correct number of lag periods to include is L. As L becomes significantly large, the quantity $(1-\gamma_j)^L$ approaches zero, since, by definition of the partial-adjustment variable γ_j, $0 < \gamma_j < 1$. If one were to consider all the lagged effects of the independent variables over a sufficiently long period of time (i.e., as L approached infinity), the first term of 4A.8 would reduce to

$$\gamma_j \phi_{oj} \left[\frac{1-0}{\gamma_j}\right]$$

$$= \phi_{oj}.$$

Furthermore, the last term of 4A.8 (i.e., that one containing the lagged dependent variable $Y_{j(t-(L+1))}$) would vanish, as $(1-\gamma_j)^{L+1}$ would also approach zero. Thus, for a sufficiently large L, 4A.8 becomes

$$\begin{aligned} \ln Y_{jt} = \phi_{oj} &+ \gamma_j \phi_{1j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{11(t-\ell)} \\ &+ \gamma_j \phi_{2j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{21(t-\ell)} + \gamma_j \theta_{1j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{12(t-\ell)} \\ &+ \gamma_j \theta_{2j} \sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{22(t-\ell)} + W_t^*. \end{aligned} \tag{4A.9}$$

The four remaining terms containing summations are not reducible in the same way, but are still easily dealt with. Consider the second term of 4A.9. In expanded form, it could be rewritten

$$\gamma_j \phi_{1j} \left[(1-\gamma_j)^0 \ln X_{11(t-0)} + (1-\gamma_j)^1 \ln X_{11(t-1)} + \cdots + (1-\gamma_j)^L \ln X_{11(t-L)} \right].$$

Define

$$\ln Z_{11t}^{(j)} = \sum_{\ell=0}^{L} (1 - \gamma_j)^{\ell} \ln X_{11(t-\ell)}$$

$$\ln Z_{21t}^{(j)} = \sum_{\ell=0}^{L} (1 - \gamma_j)^{\ell} \ln X_{21(t-\ell)}$$

$$\ln Z_{12t}^{(j)} = \sum_{\ell=0}^{L} (1 - \gamma_j)^{\ell} \ln X_{12(t-\ell)}$$

$$\ln Z_{22t}^{(j)} = \sum_{\ell=0}^{L} (1 - \gamma_j)^{\ell} \ln X_{22(t-\ell)}$$

Substiting these variables into 4A.9 yields the equivalent form

$$\ln Y_{jt} = \phi_{0j} + \gamma_j \phi_{1j} \ln Z_{11t}^{(j)} + \gamma_j \phi_{2j} \ln Z_{21t}^{(j)} + \gamma_j \theta_{1j} \ln Z_{12t}^{(j)} + \gamma_j \theta_{2j} \ln Z_{22t}^{(j)} + W_t^*, \tag{4A.10}$$

which may be used in game-matrix calculation. The γ_j's in 4A.10 ($j = 1,2$) are known to vary between zero and one, by definition. Therefore, an incremental approach (like that used to estimate γ in the Koyck-type model) may be employed here as well to estimate the most appropriate values for γ_1 and γ_2. This incremental approach is discussed in Chapter 5 in the model application section.

N-FIRM MNH FORM WITH CARRYOVER

The n-firm MNH form, ignoring carryover effects, was given previously as Equation 4.29. A set of terms would have to be added to this set of equations, each term representing an interaction between X_{1j} and a lagged X_{2j}. Thus, if L lag periods are significant, there would be L such terms. These terms would take the form

$$\begin{bmatrix} \alpha'^{(1)*}_{\ell 1} & \alpha'^{(2)*}_{\ell 1} & \cdots & \alpha'^{(n)*}_{\ell 1} \\ \vdots & \vdots & & \vdots \\ \alpha'^{(1)*}_{\ell n} & \alpha'^{(2)*}_{\ell n} & \cdots & \alpha'^{(n)*}_{\ell n} \end{bmatrix} \begin{bmatrix} \ln X_{11t} \ln X_{21(t-\ell)} \\ \vdots \\ \ln X_{1nt} \ln X_{2n(t-\ell)} \end{bmatrix}$$

where the more general convention $\alpha'^{(1)*}_{\ell n}$, $\alpha'^{(2)*}_{\ell n}$, $\alpha'^{(3)*}_{\ell n}$ replaces the convention $\alpha'^{*}_{\ell n}$, $\beta'^{*}_{\ell n}$ used previously in Equation 4A.1. If there are L significant lag periods, the MNH carryover form would be as appears in 4A.11.

$$\begin{bmatrix} \ln Y_{1t} \\ \ln Y_{2t} \\ \vdots \\ \ln Y_{nt} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} \alpha^{(1)}_{11} & \alpha^{(1)}_{21} \\ \alpha^{(1)}_{12} & \alpha^{(1)}_{22} \\ \vdots & \vdots \\ \alpha^{(1)}_{1n} & \alpha^{(1)}_{2n} \end{bmatrix} \begin{bmatrix} \ln X_{11t} \\ \ln X_{21t} \end{bmatrix} + \dots + \begin{bmatrix} \alpha^{(n)}_{11} & \alpha^{(n)}_{21} \\ \alpha^{(n)}_{12} & \alpha^{(n)}_{22} \\ \vdots & \vdots \\ \alpha^{(n)}_{1n} & \alpha^{(n)}_{2n} \end{bmatrix} \begin{bmatrix} \ln X_{1nt} \\ \ln X_{2nt} \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha^{(1)*}_{11} & \alpha^{(2)*}_{11} & \cdots & \alpha^{(n)*}_{11} \\ \alpha^{(1)*}_{12} & \alpha^{(2)*}_{12} & \cdots & \alpha^{(n)*}_{12} \\ \vdots & \vdots & & \vdots \\ \alpha^{(1)*}_{1n} & \alpha^{(2)*}_{1n} & \cdots & \alpha^{(n)*}_{1n} \end{bmatrix} \begin{bmatrix} \ln X_{21(t-1)} \\ \ln X_{22(t-1)} \\ \vdots \\ \ln X_{2n(t-1)} \end{bmatrix} + \dots + \begin{bmatrix} \alpha^{(1)*}_{L1} & \alpha^{(2)*}_{L1} & \cdots & \alpha^{(n)*}_{L1} \\ \alpha^{(1)*}_{L2} & \alpha^{(2)*}_{L2} & \cdots & \alpha^{(n)*}_{L2} \\ \vdots & \vdots & & \vdots \\ \alpha^{(1)*}_{Ln} & \alpha^{(2)*}_{Ln} & \cdots & \alpha^{(n)*}_{Ln} \end{bmatrix} \begin{bmatrix} \ln X_{21(t-L)} \\ \ln X_{22(t-L)} \\ \vdots \\ \ln X_{2n(t-L)} \end{bmatrix}$$

$$
+ \begin{bmatrix} \xi_{11} & \xi_{21} & \cdots & \xi_{n1} \\ \xi_{12} & \xi_{22} & \cdots & \xi_{n2} \\ \vdots & \vdots & & \vdots \\ \xi_{1n} & \xi_{2n} & \cdots & \xi_{nn} \end{bmatrix} \begin{bmatrix} \ln X_{11t} \ln X_{21t} \\ \ln X_{12t} \ln X_{22t} \\ \vdots \\ \ln X_{1nt} \ln X_{2nt} \end{bmatrix}
$$

$$
+ \begin{bmatrix} \alpha_{11}^{\prime(1)*} & \cdots & \alpha_{11}^{\prime(n)*} \\ \alpha_{12}^{\prime(1)*} & \cdots & \alpha_{12}^{\prime(n)*} \\ \vdots & & \vdots \\ \alpha_{1n}^{\prime(1)*} & \cdots & \alpha_{1n}^{\prime(n)*} \end{bmatrix} \begin{bmatrix} \ln X_{11t} \ln X_{21(t-1)} \\ \ln X_{12t} \ln X_{22(t-1)} \\ \vdots \\ \ln X_{1nt} \ln X_{2n(t-1)} \end{bmatrix} + \cdots \tag{4A.11}
$$

$$
+ \begin{bmatrix} \alpha_{L1}^{\prime(1)*} & \cdots & \alpha_{L1}^{\prime(n)*} \\ \alpha_{L2}^{\prime(1)*} & \cdots & \alpha_{L2}^{\prime(n)*} \\ \vdots & & \vdots \\ \alpha_{Ln}^{\prime(1)*} & \cdots & \alpha_{Ln}^{\prime(n)*} \end{bmatrix} \begin{bmatrix} \ln X_{11t} \ln X_{21(t-L)} \\ \ln X_{12t} \ln X_{22(t-L)} \\ \vdots \\ \ln X_{1nt} \ln X_{2n(t-L)} \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix}
$$

This may also be written in matrix notation. This form evidently is a generalization of Equation 4.33 which was derived for the Cobb-Douglas functional form:

$$
\ln \mathbf{Y}_t = \alpha_c + \alpha^{(1)} \ln \mathbf{X}_{1t} + \ldots + \alpha^{(n)} \ln \mathbf{X}_{nt} \tag{4A.12}
$$

$$
+ \alpha_1^* \ln \mathbf{X}_{2(t-1)} + \ldots + \alpha_L^* \ln \mathbf{X}_{2(t-L)} + \xi \ln \mathbf{X}_{1t} \ln \mathbf{X}_{2t}
$$

$$
+ \alpha_1^{\prime *} \ln \mathbf{X}_{1t} \ln \mathbf{X}_{2(t-1)} + \ldots + \alpha_L^{\prime *} \ln \mathbf{X}_{1t} \ln \mathbf{X}_{2(t-L)} + \mathbf{W} .
$$

N-FIRM EXTENSION TO THE DYNAMIC-ADJUSTMENT APPROACH

Consider at first the three-firm case. Ignoring the error terms for simplicity, Equation 4.25 would be extended to the following form:

$$\begin{aligned}\ln Y_{jt} = {} & \phi_{oj} + \gamma_j \phi^{(1)}_{1j}\left[\sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{11(t-\ell)}\right] \\ & + \gamma_j \phi^{(1)}_{2j}\left[\sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{21(t-\ell)}\right] + \gamma_j \phi^{(2)}_{1j}\left[\sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{12(t-\ell)}\right] \\ & + \gamma_j \phi^{(2)}_{2j}\left[\sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{22(t-\ell)}\right] + \gamma_j \phi^{(3)}_{1j}\left[\sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{13(t-\ell)}\right] \\ & + \gamma_j \phi^{(3)}_{2j}\left[\sum_{\ell=0}^{L} (1-\gamma_j)^{\ell} \ln X_{23(t-\ell)}\right] \end{aligned} \qquad (4A.13)$$

where j = (1,3) and $\phi^{(1)}_{ij}$, $\phi^{(2)}_{ij}$, $\phi^{(3)}_{ij}$ are used in place of ϕ_{ij} and θ_{ij}, to permit simple extension to n firms ($n > 3$).

Two notational definitions will simplify the following manipulations greatly. Firstly, define

$$1 - \gamma_j = \psi_j \text{ for all } j;$$

also define

$$\gamma_j \phi^{(k)}_{ij} = \theta^{(k)}_{ij} \text{ for all } j \text{ and } k, \text{ and } i = (1.2).$$

Thus, 4A.13 becomes

$$\ln Y_{jt} = \phi_{oj} + \theta^{(1)}_{1j}\left[\sum_{\ell=0}^{L} \psi_j^{\ell} \ln X_{11(t-\ell)}\right] + \theta^{(1)}_{2j}\left[\sum_{\ell=0}^{L} \psi_j^{\ell} \ln X_{21(t-\ell)}\right]$$

$$+\theta_{1j}^{(2)}\left[\sum_{\ell=0}^{L}\psi_j^{\ell}\ \ln X_{12(t-\ell)}\right]+\theta_{2j}^{(2)}\left[\sum_{\ell=0}^{L}\psi_j^{\ell}\ \ln X_{22(t-\ell)}\right]$$

$$+\theta_{1j}^{(3)}\left[\sum_{\ell=0}^{L}\psi_j^{\ell}\ \ln X_{13(t-\ell)}\right]+\theta_{2j}^{(3)}\left[\sum_{\ell=0}^{L}\psi_j^{\ell}\ \ln X_{23(t-\ell)}\right]; \qquad (4A.14)$$

$j = (1,3)$.

To see how this can be converted into matrix notation, consider first the situation where only one lag period is significant. Equation 4A.14 then reduces to

$$\ln Y_{jt} = \phi_{oj} + \theta_{1j}^{(1)}\left[\psi_j^{o}\ln X_{11t} + \psi_j^{1}\ln X_{11(t-1)}\right]$$

$$+\theta_{2j}^{(1)}\left[\psi_j^{o}\ln X_{21t} + \psi_j^{1}\ln X_{21(t-1)}\right] + \theta_{1j}^{(2)}\left[\psi_j^{o}\ln X_{12t} + \psi_j^{1}\ln X_{12(t-1)}\right]$$

$$+\theta_{2j}^{(2)}\left[\psi_j^{o}\ln X_{22t} + \psi_j^{1}\ln X_{22(t-1)}\right] + \theta_{1j}^{(3)}\left[\psi_j^{o}\ln X_{13t} + \psi_j^{1}\ln X_{13(t-1)}\right]$$

$$+\theta_{2j}^{(3)}\left[\psi_j^{o}\ln X_{23t} + \psi_j^{1}\ln X_{23(t-1)}\right]; \quad j = (1,3). \qquad (4A.15)$$

If one were to write each equation explicitly (that is, substituting for all j's), one would obtain the system of equations

$$\ln Y_{1t} = \phi_{o1} + \theta_{11}^{(1)}\left[\psi_1^{o}\ln X_{11t} + \psi_1^{1}\ln X_{11(t-1)}\right] + \cdots$$

$$\ln Y_{2t} = \phi_{o2} + \theta_{12}^{(1)}\left[\psi_2^{o}\ln X_{11t} + \psi_2^{1}\ln X_{11(t-1)}\right] + \cdots \qquad (4A.16)$$

$$\ln Y_{3t} = \phi_{o3} + \theta_{13}^{(1)}\left[\psi_3^{o}\ln X_{11t} + \psi_3^{1}\ln X_{11(t-1)}\right] + \cdots ,$$

where only the first two terms on the right-hand side are shown. To convert the equations into matrix form, notice that the matrix

$$\begin{bmatrix} \theta_{11}^{(1)}\left[\psi_1^0 \ln X_{11t} + \psi_1^1 \ln X_{11(t-1)}\right] \\ \theta_{12}^{(1)}\left[\psi_2^0 \ln X_{11t} + \psi_2^1 \ln X_{11(t-1)}\right] \\ \theta_{13}^{(1)}\left[\psi_3^0 \ln X_{11t} + \psi_3^1 \ln X_{11(t-1)}\right] \end{bmatrix}$$

may be expressed as the product of two simple matrices,

$$\begin{bmatrix} \theta_{11}^{(1)} & 0 & 0 \\ 0 & \theta_{12}^{(1)} & 0 \\ 0 & 0 & \theta_{13}^{(1)} \end{bmatrix} \begin{bmatrix} \psi_1^0 \ln X_{11t} + \psi_1^1 \ln X_{11(t-1)} \\ \psi_2^0 \ln X_{11t} + \psi_2^1 \ln X_{11(t-1)} \\ \psi_3^0 \ln X_{11t} + \psi_3^1 \ln X_{11(t-1)} \end{bmatrix}.$$

Also notice that the right-hand matrix may be further simplified:

$$\begin{bmatrix} \psi_1^0 \ln X_{11t} + \psi_1^1 \ln X_{11(t-1)} \\ \psi_2^0 \ln X_{11t} + \psi_2^1 \ln X_{11(t-1)} \\ \psi_3^0 \ln X_{11t} + \psi_3^1 \ln X_{11(t-1)} \end{bmatrix} = \begin{bmatrix} \psi_1^0 & \psi_1^1 \\ \psi_2^0 & \psi_2^1 \\ \psi_3^0 & \psi_3^1 \end{bmatrix} \begin{bmatrix} \ln X_{11t} \\ \ln X_{11(t-1)} \end{bmatrix}$$

One thereby obtains the desired relation

$$\begin{bmatrix} \theta_{11}^{(1)}\left[\psi_1^0 \ln X_{11t} + \psi_1^1 \ln X_{11(t-1)}\right] \\ \theta_{12}^{(1)}\left[\psi_2^0 \ln X_{11t} + \psi_2^1 \ln X_{11(t-1)}\right] \\ \theta_{13}^{(1)}\left[\psi_3^0 \ln X_{11t} + \psi_3^1 \ln X_{11(t-1)}\right] \end{bmatrix}$$

$$= \begin{bmatrix} \theta_{11}^{(1)} & 0 & 0 \\ 0 & \theta_{12}^{(1)} & 0 \\ 0 & 0 & \theta_{13}^{(1)} \end{bmatrix} \begin{bmatrix} \psi_1^0 & \psi_1^1 \\ \psi_2^0 & \psi_2^1 \\ \psi_3^0 & \psi_3^1 \end{bmatrix} \begin{bmatrix} \ln X_{11t} \\ \ln X_{11(t-1)} \end{bmatrix}$$

This allows the first parts of the equations in 4A.16 to be translated into matrix form as follows:

$$\begin{bmatrix} \ln Y_{1t} \\ \ln Y_{2t} \\ \ln Y_{3t} \end{bmatrix} = \begin{bmatrix} \phi_{01} \\ \phi_{02} \\ \phi_{03} \end{bmatrix} + \begin{bmatrix} \theta_{11}^{(1)} & 0 & 0 \\ 0 & \theta_{12}^{(1)} & 0 \\ 0 & 0 & \theta_{13}^{(1)} \end{bmatrix} \begin{bmatrix} \psi_1^0 & \psi_1^1 \\ \psi_2^0 & \psi_2^1 \\ \psi_3^0 & \psi_3^1 \end{bmatrix} \begin{bmatrix} \ln X_{11t} \\ \ln X_{11(t-1)} \end{bmatrix} + \cdots \tag{4A.17}$$

It is convenient now to convert to matrix notation:

$$\ln \mathbf{Y}_t = \boldsymbol{\Phi}_0 + \boldsymbol{\Theta}_1^{(1)} \boldsymbol{\Psi} \ln \mathbf{X}_{11} + \cdots \tag{4A.18}$$

It is now possible to write 4A.15 completely in matrix notation. Using the conventions set down in 4A.18, one obtains

$$\begin{aligned} \ln \mathbf{Y}_t = \boldsymbol{\Phi}_0 &+ \boldsymbol{\Theta}_1^{(1)} \boldsymbol{\Psi} \ln \mathbf{X}_{11} + \boldsymbol{\Theta}_2^{(1)} \boldsymbol{\Psi} \ln \mathbf{X}_{21} + \boldsymbol{\Theta}_1^{(2)} \boldsymbol{\Psi} \ln \mathbf{X}_{12} \\ &+ \boldsymbol{\Theta}_2^{(2)} \boldsymbol{\Psi} \ln \mathbf{X}_{22} + \boldsymbol{\Theta}_1^{(3)} \boldsymbol{\Psi} \ln \mathbf{X}_{13} + \boldsymbol{\Theta}_2^{(3)} \boldsymbol{\Psi} \ln \mathbf{X}_{23}\,; \end{aligned} \tag{4A.19}$$

where the dimensions of the matrices are as follows:

$\ln \mathbf{Y}_t,\ \boldsymbol{\Phi}_0 \quad = \quad (3,1);$

$\boldsymbol{\Theta}_i^{(j)} \quad = \quad (3,3);$

$\boldsymbol{\Psi} \quad = \quad (3,2);$

$\ln \mathbf{X}_{ij} \quad = \quad (2,1).$

Now, the model can be extended easily to include L significant lag periods. The equation in matrix notation (4A.19) is still valid, but the dimensions of $\boldsymbol{\Psi}$ and $\ln \mathbf{X}_{ij}$ change. These two matrices become

$$\begin{bmatrix} \psi_1^0 & \psi_1^1 & \cdots & \psi_1^L \\ \psi_2^0 & \psi_2^1 & \cdots & \psi_2^L \\ \psi_3^0 & \psi_3^1 & \cdots & \psi_3^L \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \ln X_{ijt} \\ \ln X_{ij(t-1)} \\ \vdots \\ \ln X_{ij(t-L)} \end{bmatrix},$$

respectively. Finally, for the n-firm case with $n \geqslant 3$, 4A.19 must be extended to the following:

$$\ln \mathbf{Y}_t = \mathbf{\Phi}_0 + \mathbf{\Theta}_1^{(1)} \boldsymbol{\psi} \ln \mathbf{X}_{11} + \cdots + \mathbf{\Theta}_1^{(n)} \boldsymbol{\psi} \ln \mathbf{X}_{1n} \qquad (4A.20)$$
$$+ \mathbf{\Theta}_2^{(1)} \boldsymbol{\psi} \ln \mathbf{X}_{21} + \cdots + \mathbf{\Theta}_2^{(n)} \boldsymbol{\psi} \ln \mathbf{X}_{2n} \ ;$$

where the dimensions of the matrices are:

$\ln \mathbf{Y}_t$, $\mathbf{\Phi}_0$ = (n,1);

$\mathbf{\Theta}_i^{(j)}$ - (n,n);

$\boldsymbol{\psi}$ - (n, L+1);

$\ln \mathbf{X}_{ij}$ = (L+1, 1);

and n and L are the number of firms and the number of significant lag periods, respectively.

CHAPTER 5

ISSUES IN MODEL APPLICATION

INTRODUCTION--CHAPTER OUTLINE

Chapter 4 proposed an adaptable, general theoretical model of marketing-mix effects on success variables for firms in a given industry. Despite the level of mathematical sophistication, many questions still remain, however: mostly, these are questions of application. What are the relevant marketing-mix decision variables in the industry? What measures of success are used by the firms in question? Do all firms in the industry have the same objectives? How, indeed, should the industry be defined? And what would be reasonable expectations for application of the model? These are some of the issues which shall be examined in this chapter.

Chapter 5 comprises three parts. In Part 1, various problems which may be encountered in modelling are highlighted. The important distinction between strategies and intentions is made. Awareness of possible drawbacks to the model will aid in the selection of relatively "simple" industries for basic empirical testing. Also, potential problem areas which may be encountered in extensions to more complex situations are discussed.

Part 2 investigates the application of the theoretical model in a specific setting. The steps involved in the building of the industry-specific model are illustrated. This section also considers the appropriateness of including interaction effects, and indicates how one may choose among the different approaches (described in Chapter 4) for dealing with carryover effects. The result of Part 2 is a game matrix representation of a hypothetical industry.

Finally, in Part 3, issues regarding the interpretation of the game matrix are addressed. A game matrix is "solved" for the equilibrium points corresponding to the relevant behavioural patterns; strategies through time by the players are examined; and decision-making implications are discussed. In short, Chapter 5 essentially outlines the development of an industry-specific "theory-in-use" which can be tested empirically.

PART 1--PROBLEMS IN APPLICATION

A number of potential hazards appear when attempting to apply the theoretical model of Chapter 4 to an industry. At the outset of Chapter 5 these pitfalls are introduced, for two reasons: a) to help select industries where the analysis would not be unduly complicated by a myriad of significant independent variables (i.e., "simpler" industries were selected for demonstration purposes); and b) to indicate what difficulties would have to be faced in applying the theoretical model to more complex situations in the future. One may classify the potential hazards into three categories: those concerning choice of dependent variables; those examining the proper specification of decision variables; and those pertaining to the behaviour of the firm.

Dependent Variables

In the discussion of the theoretical model, the payoff variable Y was left undefined. It would require a thorough understanding of the intentions of the firms to know what should be chosen as the objective (i.e., the payoff or dependent variable) for game modelling purposes. The importance of this clear distinction is easily shown.

Suppose that the modeller makes the assumption that both firms in a consumer-goods duopoly are attempting to maximize sales revenue. He also assumes that there is only one relevant decision variable,which is price. (The next section discusses proper selection of decision variables.) Price is taken to be a trichotomous variable: either firm may set a price of $0.80, $1.00 or $1.20 on its product. He decides to find the non-zero-sum game matrix representation of the industry with price as the decision variable, and the sales revenues of Firms 1 and 2 as the payoffs. He may find that the resulting game matrix would appear as in Figure 5.1.

		P2	
P1	0.80	1.00	1.20
0.80	505.6 800	512 940	518.4 1056
1.00	532 808	540 950	548 1068
1.20	518.4 816	528 960	537.6 1080

Note: Upper and lower figures refer to Player 1 and Player 2 respectively; security levels are circled.

FIGURE 5.1
Sales Revenue Maximization

This indeed would be the sales-revenue game matrix obtained if demand for Firm 1's products, expressed as a function of price, were

$$Q_1 = 1000 - 500\,P_1 + 40\,P_2 \tag{5.1}$$

and demand for firm 2's product were

$$Q_2 = 1200 + 50\,P_1 - 300\,P_2 \tag{5.2}$$

Solving for sales revenue using standard microeconomic techniques would yield

$$\begin{aligned} S_1 &= P_1\,Q_1 \\ &= P_1(1000 - 500\,P_1 + 40\,P_2) \\ &= 1000\,P_1 - 500\,P_1^{\,2} + 40\,P_1\,P_2\,, \end{aligned} \tag{5.3}$$

and

$$\begin{aligned} S_2 &= P_2\,Q_2 \\ &= P_2(1200 + 50\,P_1 - 300\,P_2) \\ &= 1200\,P_2 + 50\,P_1\,P_2 - 300\,P_2^{\,2}. \end{aligned} \tag{5.4}$$

Equations 5.3 and 5.4 were used to construct Figure 5.1.

Now assume that both players play minimax. Player 1 maximizes his security level by playing the middle-price strategy, while P2 prefers to set a high price (the preferred strategies are indicated by arrows). Resultingly, P1 would earn a sales revenue of $548; and P2 would receive $1068.

Suppose that the modeller had been wrong in selecting sales revenue as the relevant payoff. The firms strive not to maximize sales revenue, but to maximize profit. The variable and fixed costs incurred by firms 1 and 2 may be as follows:

$VC_1 = \$0.50$ per unit;

$FC_1 = \$200$;

$VC_2 = \$0.45$ per unit;

$FC_2 = \$250$.

If so, total cost expressions for Firms 1 and 2 would be

$$TC_1 = 200 + 0.50\ Q_1$$
$$TC_2 = 250 + 0.45\ Q_2$$

and corresponding profits would be

$$\begin{aligned} \pi_1 &= S_1 - TC_1 \\ &= P_1Q_1 - 200 - 0.50\ Q_1 \qquad (5.5) \\ \pi_2 &= S_2 - TC_2 \\ &= P_2Q_2 - 250 - 0.45\ Q_2 \qquad (5.6) \end{aligned}$$

One may solve for a game matrix with profits as the payoffs, to obtain Figure 5.2.

If the firms were in fact using a minimax strategy, i.e., maximizing their profit security level, both firms would prefer to set high prices. Here is the key issue: if the modeller were

P1 \ P2	0.80	1.00	1.20
0.80	-10.4 100	-8 267	-5.6 410
1.00	66 103.5	70 272.5	74 417.5
1.20	102.4 107	108 278	113.6 425

FIGURE 5.2
Profit Maximization

faced with the game matrix of Figure 5.1 (sales maximization), and noticed that the firms <u>both</u> preferred the high-price strategy, he would have no way of knowing whether a) the firms were using sub-optimal strategies for <u>sales</u> maximization, or b) he had misspecified the model; i.e., chosen the wrong <u>dependent</u> variable. Hence the importance of understanding the intentions of the firms, and thereby of careful selection of the appropriate objective. In order to test the model empirically, the modeller would be wisest to check with industry experts or managers on this issue. (Note: the issue of intentions is taken up again in a general discussion of behavioural issues.)

Chapter 4 indicated the possibility that different firms may have different objectives. This coincides with the view of Porter (1980), who suggests that industries, like products, go through a form of life cycle: he delineates emerging, mature and declining industries. Among the characteristics of emerging industries are uncertainty with regard to appropriate marketing strategies and the tendency of firms to "induce substitution"; i.e., encourage first-time buying. In such an industry, it may be likely that firms are trying to maximize unit sales, in order to gain a share of the market. The game matrix corresponding to the demand equations 5.1 and 5.2 appears in Figure 5.3.

P1 \ P2	0.80	1.00	1.20
0.80	632 1000	640 940	648 880
1.00	532 1010	540 950	548 890
1.20	432 1020	440 960	448 900

FIGURE 5.3
Unit Demand Maximization

In this situation, yet another strategic combination would be preferred: each player would choose to play his lowest price.

Competition in a mature industry is often marked by increased attention to market share and relatively low market growth. Not inconceivably, a well-established firm may be satisfied with its market share and attempt to increase profitability, while one of its smaller competitors (or a new entry in the industry) may place prime importance upon sales growth. Fortunately, such divergent goals may still be represented in the theoretical model. Nowhere is the restriction made that all the payoffs need represent the same success variable.

A further comment on dependent variables concerns short-term versus long-term goals. A long-term objective of profit maximization might involve at first playing a strategy which appears to yield lower short-term profits: for example, a high-advertising strategy where short-term profitability is traded off for investment in advertising which, it is hoped, would lead to even higher sales and profits in the long run. At first this would appear to be a drawback to the model but two observations may be made. The first concerns the issues of strategies versus behavioural intentions. It is not the goal of this research to determine the players' intentions (i.e., what their short-run and long-run objectives are) through examination of the game matrix, but rather, to construct game matrices which will improve a firm's decision-making, given what its intentions are. The discussion of behaviour of the participants takes up this argument again.

The second observation is that the proposed model is dynamic. The brief illustration given above represents a once-only pricing decision. The players make only one decision each; and if they follow minimax behaviour, their one decision is easy to predict. However, having time-series data, the process of repeated decision-making can be observed, and changes in preferred strategies over time may be tracked. Favouring a profit-maximization strategy over time would lend support to the belief that a firm was indeed committed to a long-term profit-maximization objective (even if occasionally the firm would choose another strategy which was potentially more harmful to its competitor(s)).

Independent Variables

Like the success variables, the independent variables are also left unspecified in the theoretical model. The functional forms derived in Chapter 4 contain general independent variables X_1 , X_2 (and possibly X_3). The chapter also illustrates the procedure to follow when one (or both) of the decision variables is expected to exhibit carryover effects, or if interactions prove significant. It had been suggested at various points that X_1 (the non-carryover decision variable) and X_2 (which exhibits significant carryover effects) represent price and advertising levels, respectively. Although this may be a reasonable first assumption in some situations, one must ascertain that these are indeed the appropriate variables to consider for the particular industry under study.

First, consider the relative merits of studying a consumer-goods versus an industrial-goods industry. It may be argued that pricing and advertising strategies are crucial elements of the competition among firms producing consumer goods; in fact, if distribution policies and expenditures are approximately equal (or constant), these two strategic variables will capture much of the interfirm competition. However, the likelihood of extraneous factors complicating the situation is high. Most obviously, the <u>direct</u> effect of either price level or advertising expenditure on sales may not be easy to determine. This may be due to a number of reasons: if "dollars invested" is used to measure the advertising variable, differences in quality of advertising are not modelled explicitly; other promotional efforts (i.e., personal selling and sales promotions) and word-of-mouth communication, which may be just as important in determining brand choice as mass advertising, are ignored; real quality differences across brands are not measured and incorporated into the model; etc. Also, consumers often do not have enough expertise to make the best brand selection (see Shapiro 1968, e.g.) and are liable to make many purchases on impulse. All of these complicating factors make it difficult to derive econometrically the direct effects of price and advertising on sales of a consumer good. Erickson (1981), for one, showed that different estimates for the effect of advertising on sales for Lydia Pinkham's Vegetable Compound (see above) are obtained, depending upon whether the data is reported weekly, monthly or yearly. Given these difficulties, it may be heroic to assume that the direct effects of any decision variables on sales or market share (let alone profit!) are easily estimated.

One might feel somewhat more certain that extraneous factors are less likely to cause difficulty in the case of industrial goods. Industrial buyers are usually a smaller, well-defined group, and are likely to be very well-informed on the relevant characteristics of each firm's products. They would thus have more knowledge upon which to make their choice rationally (i.e., to be less swayed by impulse that the average consumer). However, when studying industrial goods, it is also very likely that marketing-mix variables other than advertising and pricing are far more important in determining brand choice. For instance, product quality or the existence of a well-defined distributor channel may be crucial. Also, manufacturer's advertising to middlemen (i.e., in trade journals) may be more relevant in this context than manufacturer's advertising to final consumers; so some means of distinguishing among advertising targets may be necessary.

Although this paper takes a marketing approach, it should be mentioned here that non-marketing variables may prove to be just as relevant (or even more so) than those variables listed above. The amount of vertical integration, for example, may be an important variable in the industrial-goods market. Turning again to Porter (1980), one sees quite an array of possible strategic variables: these include specialization, brand identification, push-versus-pull, channel selection, product quality, technological leadership, vertical integration, cost position, service, price policy, financial leverage, relationship with parent company, and relationship to home (and host) government (Porter 1980, pp. 127 - 128). One certainly should not be restricted to considering only marketing-mix variables when developing a complete model of an industry.

However, here one must be judicious. Although the strategies represented in the theoretical model of Chapter 4 need not be marketing strategies, the intent of this paper is to develop a <u>marketing-mix</u> decision-making model using the constructs of game theory. Recognizing, then, that many non-marketing strategic variables may be important determinants of a firm's success, one is restricted (for this work, at least) to studying an industry (either consumer-goods or industrial-goods) where the marketing-mix variables are indeed significant. Careful selection of industries is therefore recommended, and interviews with appropriate managers would aid in determining which marketing services are of the greatest importance. This is not to say that non-marketing variables are ignored (see below). Also, this is not a case of "finding data to suit the model"; rather, the industries selected for this paper had the advantage of being relatively easily modelled using a small number of variables, thus facilitating demonstration of the applicability of the theoretical model.

Interviews with management personnel should also help in interpretation of the data. For example, any changes made in the reporting of data (due to accounting changes) should be pointed out; if advertising expenditures include sales promotion efforts or not; etc. Also, there may be "hidden" internal constraints in the setting of decision-variable levels which the modeller must be made aware of: e.g., advertising may be always set at 2% of last period's sales, or the "affordable" method may be used to set the advertising budget (i.e., "How much can we afford to spend this year?").

It may also be that the correct marketing-mix variables are included in the model, but the specification of their effects on the objective variable are wrong. One may assume that price levels have an immediate effect on brand choice; i.e., that <u>current</u> prices are the only relevant price variables to consider. Even if such a model provides good fit to the data, it cannot capture the complete effect on consumer demand as described by Vanhonacker (1983a; 1983b). He argues that there is an immediate effect of price fluctuation about an essentially fixed price level, as well as a long-term effect due to changes in this price level over time. A model which ignores carryover effects of price cannot separate these different pricing effects and essentially can only estimate the effect of the fluctuation effect. It will be noted that the dynamic-adjustment approach proposed in Chapter 4 allows for all the independent variables to exhibit a carryover effect on the dependent variable.

A final point regarding selection of independent variables concerns the masking of a significant underlying variable due to correlation. The modeller may believe, for example, that price and advertising are the relevant decision variables, and may obtain a model with excellent interpretative ability using these variables; but may not have considered that a firm's advertising may be highly correlated to its level of research and development expenditure in this industry. Having collected R & D expenditures at the same time as the price-advertising data would allow the researcher to determine whether R & D was in fact a more significant variable than advertising in influencing sales. Also, sales or profits in the industry may be heavily affected by other, non-marketing variables such as up- and downturns in the economy. One must therefore develop a list of variables (either controllable marketing-mix variables or external variables such as economic indicators) which are potentially significant, <u>before</u> commencing data collection, if for no other reason than to save time in recollection of previously-missed data at a later stage of research.

Behaviour of the Participants

Careful choice of industry and consideration of what strategic variables are likely to be important, in conjunction with in-depth interviews with industry experts or management, would simplify the selection of dependent and independent variables and minimize the likelihood of problems arising from inappropriate variable selection. But even once the model is correctly specified, there still remain problems of interpretation of the results; namely, what the results imply about the behaviour of the participants. It is in this regard that the distinction between this work and that of Shubik and Levitan becomes more pronounced.

In their introduction, Shubik and Levitan (1980) give the following interpretation of their market model (presented above in Chapter 2):

> "It is shown that the specification of the payoffs to each player is tantamount to the specification of the market structure and the goals of the firms. A solution concept may be regarded as the specification of the

> intents of each player. The solution is the outcome resulting from the application of these intents to the market structure." (Shubik and Levitan 1980, p. viii)

Their argument may be interpreted as follows: given a game matrix with accurate, appropriate payoffs, and given perfect information on the part of the players, one could determine the behavioural intentions of the players by examining which equilibrium point is reached.

In this paper, the argument is made that it is difficult, if not impossible, to determine behavioural intentions of individual managers, given only the payoffs of the game matrix. All that is clearly revealed upon examining the game matrix is the pattern, through time, of the players' strategies. Opponent reactions, i.e., analysis of each other's strategies as well as choice of counterstrategies, are potentially partially masked in a numerical game-theory analysis except in very extended gaming. Behavioural factors such as the emotional makeup, intellectual ability and skill of the players are also likely to be masked when considering only the numerical payoffs.

One must therefore be careful in distinguishing between behavioural intentions and strategies. The time-series data is a running record of historical strategies taken by the players: strategies here referring to that which has already been done by the players. Intentions here will be taken to mean what the players had in mind when they chose their strategies, or what they would like to accomplish in the future. Given that the players probably did not have perfect information about the marketplace when playing their (historical) strategies, it is presumptuous to assume that their intentions are clearly revealed by the strategies chosen. Rather, an approach which is opposite to that of Shubik and Levitan is taken. Instead of working backwards to determine behavioural intentions from historical strategies, the intentions will be taken as given through an understanding of the industry and/or interviewing industry managers; thus the appropriateness of the selected strategies can be evaluated in this light.

The danger of taking the approach that intentions are revealed through the players' behaviour is that it leaves unanswered the question of what <u>motivates</u> the players. Players making repeated decisions under changing conditions and without perfect information will not always choose the "optimal" strategies, optimal here referring to that strategic combination which is best suited to attaining the players' objectives or intentions.

To summarize, Shubik and Levitan propose that the <u>intentions</u> of the firms in an industry are revealed through their behaviour; this work takes the approach that <u>given</u> the intentions, strategies which are most conducive to attaining the intended results may be determined.

<u>PART 2--MODEL APPLICATION</u>

Now that Part 1 has indicated some of the potential drawbacks to the theoretical model, and some of the practical considerations which must be made in its implementation, the operational considerations (which were not dealt with in Chapter 4) are examined. A hypothetical scenario is presented for exposition purposes.

In this hypothetical consumer-goods industry there are two firms, A and B. Both firms are well-established and produce many competing small consumer-branded items. One such small consumer product having few substitutes is chosen for analysis. The <u>industry</u> for this product is said to comprise the two firms.

Incidentally, this last statement answers one of the questions posed in the introduction to this chapter: that is, how the industry should be defined. The last paragraph above implies that the industry for a given product is made up of the set of firms which manufacture that product. In the case of some products (small consumer items like peanut butter come to mind), however, the efforts of firms producing close substitutes (like jam) probably cannot be ignored.

Returning to the scenario: Both firms, and the product in question, are in the maturity stages of their respective life cycles. It is decided, through interviews with management personnel, that the relevant marketing decision variables to consider in this industry are pricing and advertising, and that both firms are attempting to maximize sales. Therefore, monthly sales (by firm and aggregate), prices and advertising expenditures are collected for a sufficient number of years. It is hypothesized that advertising may exhibit a carryover effect, and that price-advertising interactions may be significant and should be tested for.

The building of the game matrix involves two steps. First, the functional form (from among those presented in Chapter 4) which models the sales/marketing-mix-response behaviour in the industry most appropriately is determined empirically. Second, the econometric equations are adapted into the forms which will be used to construct the game matrix. These topics are examined here in Part 2.

Once the matrix is constructed, it will be solved for the equilibrium points under the various behavioural assumptions. Then, the relative merits of the strategic choices made by the players over time will be examined, and recommendations on pricing and advertising strategies for the future are made. These issues will be turned to in Part 3 of this chapter.

Determining the Best Functional Form

The first step in data treatment involves checking the distributions of the independent variables for normality. The importance of bivariate normality is explained by Rummel:

> "Although normal univariate distributions are not sufficient for the bivariate distributions to be normal, they increase the likelihood. A bivariate normal distribution has the useful property that the relationship between the two variables is linear;...a sufficient condition for the correlation condition to be a true measure of statistical independence is that the bivariate distribution be normal ...finally, application of tests of significance assume that the distributions of the variables are all normal." (Rummel 1970)

If the data are not univariate normally distributed, it becomes necessary to perform an appropriate transformation. Normalizing the univariate distributions by such means makes bivariate normal distributions more likely; and it is bivariate normality which must not be violated in order for the calculated correlations and coefficients to be meaningful.

One may assume that the histograms of the natural logarithms of the independent variables showed no great deviation from normality. With the assurance that the data are amenable to quantitative analysis, the comparison and selection of models begins. Chapter 4 indicated that the main effects of decision variables may be modelled by Cobb-Douglas-like equations, while an MNH extension would incorporate interaction effects. Using Equations 4.1 through 4.4 as guides, the appropriate functional forms to consider are:

(COBB-DOUGLAS)

$$\ln S_{1t} = \alpha_1 + \alpha_{11} \ln P_{1t} + \alpha_{21} \ln A_{1t} + \beta_{11} \ln P_{2t} + \beta_{21} \ln A_{2t}; \tag{5.7}$$

$$\ln S_{2t} = \alpha_2 + \alpha_{12} \ln P_{1t} + \alpha_{22} \ln A_{1t} + \beta_{12} \ln P_{2t} + \beta_{22} \ln A_{2t}; \tag{5.8}$$

and

(MNH)

$$\begin{aligned} \ln S_{1t} = {} & \alpha_1 + \alpha_{11} \ln P_{1t} + \alpha_{21} \ln A_{1t} + \beta_{11} \ln P_{2t} + \beta_{21} \ln A_{2t} \\ & + \xi_{11} \ln P_{1t} \ln A_{1t} + \xi_{21} \ln P_{2t} \ln A_{2t}; \end{aligned} \tag{5.9}$$

$$\begin{aligned} \ln S_{2t} = {} & \alpha_2 + \alpha_{12} \ln P_{1t} + \alpha_{22} \ln A_{1t} + \beta_{12} \ln P_{2t} + \beta_{22} \ln A_{2t} \\ & + \xi_{12} \ln P_{1t} \ln A_{1t} + \xi_{22} \ln P_{2t} \ln A_{2t}; \end{aligned} \tag{5.10}$$

where S_{jt} = sales in dollars of Firm j in period t;

A_{jt} and P_{jt} = advertising expenditures and retail price level set by Firm j in period t;

and α_i, α_{ij}, β_{ij} defined as previously.

It is hypothesized that MNH will be a more appropriate functional form that Cobb-Douglas, because it incorporates the postulated interaction terms. Thus, the hypothesis to be tested is

H_o: $\xi_{1j} = \xi_{2j} = 0$;

H_a: at least one $\xi_{ij} \neq 0$; $i, j = (1,2)$,

where the ξ_{ij}'s are the interaction coefficients of 5.9 and 5.10.

The addition of interaction terms necessarily improves the fit of any model: more variables always explain more than less variables. Thus, the benefit of increased explanatory ability obtained through the inclusion of interaction terms must be weighed against the corresponding loss of degrees of freedom. Fortunately, standard methods (such as comparing overall-F statistics) are applicable in determining the model with the best explanatory ability.

Suppose the Cobb-Douglas form is chosen (that is, the interaction effects are not found to add significantly to the explanatory ability of the model). As specified in Equations 5.7 and 5.8, carryover advertising effects are still ignored. The next tests to be applied determine which (if any) of the approaches derived in Chapter 4 for the treatment of carryover effects is applicable.

Modelling Advertising Carryover

The goodwill approach, as proposed in Chapter 4, accounts for carryover of advertising by estimating individually each of the weights associated with the lagged variables. Thus, the Cobb-Douglas form is first compared to a similar form containing one-lag-period carryover effects, which is obtained by adapting Equations 4.7 and 4.8:

$$\begin{aligned}\ln S_{jt} = \alpha_j + \alpha_{1j} \ln P_{1t} + \alpha_{2j} \ln A_{1t} + \alpha^*_{1j} \ln A_{1(t-1)} \\ + \beta_{1j} \ln P_{2t} + \beta_{2j} \ln A_{2t} + \beta^*_{1j} \ln A_{2(t-1)} + u_{jt};\end{aligned} \qquad (5.11)$$

where

$j = (1,2)$.

If the advertising effects of period t-1 prove to be significantly different than zero, additional carryover effects may be added in and their significance checked.

Should more than one or two lagged periods be significant, however, difficulties may arise for at least two reasons. First, adding in many lagged effects may introduce substantial levels of multicollinearity which would cause problems in parameter estimation. Second, the number of parameters to estimate escalates rapidly as the number of significant lag periods increases: too many degrees of freedom may be lost with a resulting decrease in the adjusted R-square value and in model usefulness. In these situations, the Koyck approach may be more appropriate.

Chapter 4 derived a pair of general Koyck equations for the two-firm case (Equations 4.18 and 4.19). These could be adapted to the current situation as follows:

$$\ln S_{jt} = \alpha_j + \alpha_{1j} \ln P_{1t} + \alpha_{2j} \ln Z_{1t} + \beta_{1j} \ln P_{2t} + \beta_{2j} \ln Z_{2t} + u_{jt}; \tag{5.12}$$

where

$$j = (1,2);$$

$$\ln Z_{1t} = \sum_{\ell=0}^{L} (1 - \gamma)^{\ell} \ln A_{1(t-\ell)} \; ; \quad \text{and}$$

$$\ln Z_{2t} = \sum_{\ell=0}^{L} (1 - \gamma)^{\ell} \ln A_{2(t-\ell)} \; .$$

One is thereby left with the problem of estimating the size of the γ's. An incremental optimization procedure may be used. The parameter γ is known to be between 0 and 1; therefore, as a first approximation, one-tenth intervals will be estimated ($\gamma = 0.9$, 0.8, 0.7,...). Starting with $\gamma = 0.9$ (recall that <u>high</u> levels of γ indicate that carryover effects are <u>less</u> important), one could substitute for γ in the definitions of $\ln Z_{1t}$ and $\ln Z_{2t}$ to obtain the following:

$$\ln Z_{1t} = \sum_{\ell=0}^{L} (1 - 0.9)^{\ell} \ln A_{1(t-\ell)}$$

$$= (0.1)^0 \ln A_{1t} + (0.1)^1 \ln A_{1(t-1)}$$

$$+ (0.1)^2 \ln A_{1(t-2)} + \ldots + (0.1)^L \ln A_{1(t-L)}$$

and similarly for $\ln Z_{2t}$. Terms would be summed as in the expanded form above, until the point is reached where the inclusion of an additional term causes an insignificant increment to $\ln Z_{1t}$ and $\ln Z_{2t}$.

Both $\ln Z_{1t}$ and $\ln Z_{2t}$ would thus be calculated, then used as independent variables in 5.12, which would then be estimated. This procedure would be repeated for all other levels of γ (0.8, 0.7, etc.) until it is found that lowering γ no longer improves the model. The value of γ which yields the best-fitting econometric equation would be selected. The procedure could also be fine-tuned to obtain even more precise estimation of γ.

The two approaches which have been proposed for the situation where one independent variable exhibits significant carryover effects may thus be compared. The Koyck-type approach allows the modeller to incorporate any number of lagged periods with ease and may therefore be preferable to the goodwill approach, especially where more than one or two lag periods are significant. However, the goodwill approach estimates the carryover effects independently from each other (i.e., geometrically-declining significance of carryover terms is not assumed); thus, if multicollinearity and reduction in degrees of freedom are not serious problems, the goodwill approach is more flexible. The third approach (dynamic-adjustment) would be applicable in this hypothetical situation if both price and advertising exhibited carryover effects (see discussion in Chapter 4).

$$\ln Z_{1t} = \sum_{\ell=0}^{L} (1 - 0.9)^{\ell} \ln A_{1(t-\ell)}$$

$$= (0.1)^{0} \ln A_{1t} + (0.1)^{1} \ln A_{1(t-1)}$$

$$+ (0.1)^{2} \ln A_{1(t-2)} + \ldots + (0.1)^{L} \ln A_{1(t-L)}$$

and similarly for $\ln Z_{2t}$. Terms would be summed as in the expanded form above, until the point is reached where the inclusion of an additional term causes an insignificant increment to $\ln Z_{1t}$ and $\ln Z_{2t}$.

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The so-called Koyck model is not, strictly speaking, in true Koyck form, as the final form of the classic Koyck model has lagged dependent variables on the right-hand side (see Chapter 3). Thus, obstacles normally encountered in using OLS to estimate a Koyck model (inconsistency of the lagged-variable parameter, autocorrelation of error terms) will not necessarily occur (see Taylor and Wilson 1974, Maddala and Vogel 1969, or Clarke 1973 for a discussion of these problems).

Testing for Significance of Effects

Suppose that in the hypothetical industry, only one lagged advertising term was found to be significant, and that the goodwill-form model of Equation 5.11 was selected to represent the industry. Now, hypotheses concerning the magnitude and direction of all effects may be tested individually.

The main effects of own price, current advertising and lagged advertising on A's sales are represented in Equation 5.11 as α_{11}, α_{21} and α_{11}^* respectively. The corresponding effects of the competitor's decision variable levels are β_{11}, β_{21} and β_{11}^*. The significance of each of these effects may be tested for individually by determining whether the values estimated for these parameters are significantly different than zero. The signs of the significant parameters may also be examined to determine if there are any counterintuitive effects, which would decrease the credibility of the selected model. The procedure is then repeated for Firm B. It is conceivable that not all effects are significant for both brands.

Estimation of Payoffs of the Game Matrix

Once these preliminary implications have been drawn, the decision game matrix (to be analyzed in Part 3 of this chapter) may be calculated according to the instructions in the goodwill-model section of Chapter 4. All the terms of Equation 5.11 involving A_1 are grouped, as are all terms involving A_2, giving

$$\begin{aligned} \ln S_{jt} = \alpha_j + \alpha_{1j} \ln P_{1t} + \alpha_{2j} \left[\ln A_{1t} + \frac{\alpha^*_{1j}}{\alpha_{2j}} \ln A_{1(t-1)} \right] \\ + \beta_{1j} \ln P_{2t} + \beta_{2j} \left[\ln A_{2t} + \frac{\beta^*_{1j}}{\beta_{2j}} \ln A_{2(t-1)} \right], \end{aligned} \tag{5.13}$$

where j = (1,2). One may define new Z variables as in Chapter 4:

$$\ln Z_{1t} = \ln A_{1t} + \frac{\alpha^*_{11}}{\alpha_{21}} \ln A_{1(t-1)}$$

$$\ln Z'_{2t} = \ln A_{2t} + \frac{\alpha^*_{12}}{\alpha_{22}} \ln A_{2(t-1)}$$

$$\ln Z'_{1t} = \ln A_{1t} + \frac{\beta^*_{11}}{\beta_{21}} \ln A_{1(t-1)}$$

$$\ln Z_{2t} = \ln A_{2t} + \frac{\beta^*_{12}}{\beta_{22}} \ln A_{2(t-1)} \quad ;$$

Thus, 5.13 may be rewritten

$$\ln S_{1t} = \alpha_1 + \alpha_{11} \ln P_{1t} + \alpha_{21} \ln Z_{1t} + \beta_{11} \ln P_{2t} + \beta_{21} \ln Z'_{2t} \tag{5.14}$$

$$\ln S_{2t} = \alpha_2 + \alpha_{12} \ln P_{1t} + \alpha_{22} \ln Z'_{1t} + \beta_{12} \ln P_{2t} + \beta_{22} \ln Z_{2t} \tag{5.15}$$

Notice that Z_{1t} in the above equations would represent the total cumulative effect of Firm A's advertising upon its <u>own</u> sales, while Z_{2t} would represent the total main effect of Firm B's advertising upon its own sales. Both the Z variables must be calculated for both firms.

To construct the game matrix for the hypothetical firm, the following further assumptions are made:

1. The prices in this industry generally range between P^L and P^H;

2. The advertising expenditures of Firm A normally range between A_1^L and A_1^H per period;

3. The advertising expenditures of Firm B normally range between A_2^L and A_2^H per period;

4. The range of pricing and advertising levels may each be adequately represented by a trichotomous split; e.g., low, medium and high.

Assumption 4 indicates that each firm essentially has nine strategic packages (W's) to choose from. These may be defined according to the diagram in Figure 5.4. With this, the "dummy" game matrix (i.e., still without estimated payoffs) may easily be constructed. It appears in Figure 5.5, where S_A and S_B are the estimated sales levels for Firms A and B respectively under each possible strategic combination. In order to estimate the S_A and S_B values, Equations 5.14 and 5.15 and Assumptions 1 through 3 are employed.

		Firm i's Price		
		LOW	MEDIUM	HIGH
Firm i's Advertising	LOW	W_{1i}	W_{4i}	W_{7i}
	MED.	W_{2i}	W_{5i}	W_{8i}
	HIGH	W_{3i}	W_{6i}	W_{9i}

FIGURE 5.4

		FIRM B				
		W_{12}	W_{22}	W_{32}	...	W_{92}
FIRM A	W_{11}	S_A, S_B	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	W_{21}	$\vdots$	$\ddots$			
	W_{31}	$\vdots$		$\ddots$		
	$\vdots$	$\vdots$			$\ddots$	
	W_{91}	$\vdots$				$\ddots$

FIGURE 5.5

Assumption 1 indicates the relevant price range for which representative low, medium and high prices must be chosen. The selection procedure described in Chapter 4 is employed. In this procedure, the relevant range is divided into three segments of equal size, and the midpoint of each segment is taken to be the representative value. It is easy to derive the following expressions for the representative low, medium and high prices:

$$P^{L*} = P^L + (1/6)(P^H - P^L)$$
$$P^{M*} = P^L + (1/2)(P^H - P^L)$$
$$P^{H*} = P^L + (5/6)(P^H - P^L)$$

Assumptions 2 and 3 indicate the ranges of advertising expenditures of the firms. These variables will not be directly used in the game matrix; rather, the previously-calculated advertising effects (Z_1 and Z_2) are used. The representative values for each of these variables may be selected as above:

$$Z_1^{L*} = Z_1^L + (1/6)(Z_1^H - Z_1^L)$$
$$Z_1^{M*} = Z_1^L + (1/2)(Z_1^H - Z_1^L)$$
$$Z_1^{H*} = Z_1^L + (5/6)(Z_1^H - Z_1^L)$$

and similarly for Z_2.

One further observation on Equations 5.14 and 5.15 need be made. By definition, the variable Z_{1t}' is identical to Z_{1t} with the exception of the multiplier for $\ln A_{1(t-1)}$ (the definitions are given just prior to Equations 5.14 and 5.15). Thus, as a first approximation, it can be assumed that when the cumulative main effect (Z_{1t}) is "low", Z_{1t}' is also "low"; medium and high levels may be regarded similarly. The same statement may be made regarding Z_{2t} and Z_{2t}'. This assumption may be verified by checking the magnitudes of the parameters in the definitional equations.

One is ready now to estimate the payoffs in Figure 5.5 using Equations 5.14 and 5.15. As an illustrative example, suppose that Firm A selects strategy W_{41} (which, according to Figure 5.4, represents medium price and low (cumulative) advertising), and Firm B selects W_{32} (low price, high advertising). The sales levels S_1 and S_2 would be estimated using the equations

$$\ln S_{1t} = \alpha_1 + \alpha_{11} \ln P_{1t}^{M*} + \alpha_{21} \ln Z_{1t}^{L*} + \beta_{11} \ln P_{2t}^{L*} + \beta_{21} \ln Z_{2t}^{\prime H*}$$

$$\ln S_{2t} = \alpha_2 + \alpha_{12} \ln P_{1t}^{M*} + \alpha_{22} \ln Z_{1t}^{\prime L*} + \beta_{12} \ln P_{2t}^{L*} + \beta_{22} \ln Z_{2t}^{H*}$$

In these equations, all parameters and independent variables marked with an asterisk have been estimated. This procedure would be repeated for each cell of the game matrix of Figure 5.5.

In Part 3 of this chapter, a game matrix of simpler form than Figure 5.5 is solved, to keep the explanation as clear as possible. Nevertheless, exactly the same solution procedures may be applied to the game matrix of Figure 5.5, as well as even more complex situations.

PART 3--INTERPRETATION OF THE GAME MATRIX

Part 2 of this chapter culminates in the construction of a game matrix. This part indicates the proposed interpretation method. Firstly, various solution concepts are applied (such as those described in Shubik and Levitan 1980 and in Chapter 2) to determine preferred strategies under different behavioural assumptions; secondly, the strategies actually chosen over time by the players are examined in light of the various solutions; and, thirdly, given the firms' intentions, marketing-mix strategic recommendations are made for the future.

Solving the Game Matrix

To illustrate the application of different solution concepts, a simple example employing one independent variable is borrowed from Bacharach (1977) and adapted to a marketing context. He presents an example of a game matrix applied to a duopoly where each firm adjusts its output level (low, medium or high) to maximize profit. The scenario could be recast such that the players maximize sales revenue through adjustment of price level. The payoffs used by Bacharach are left essentially unchanged; only the definitions of the variables involved have been altered. The resulting game matrix would be as appears in Figure 5.6.

		P2's PRICE	
P1's PRICE	LOW	MEDIUM	HIGH
LOW	3.14 JM 1.06	2.53 1.38	2.16 1.26
MED.	3.16 1.03	2.54 EQ 1.35	2.17 1.23
HIGH	3.21 0.85	2.44 1.01	1.84 MD S 0.78

FIGURE 5.6
Source: Adapted from Bacharach (1977), p.67.

The letters in the upper-right-hand corners of the cells refer to the different possible solutions which are examined below. Two of these (equilibrium-pair and joint-maximum) were obtained by Bacharach; the others were calculated for this study, following the example of Shubik and Levitan (1980). (Note: the mathematical representations of all solution concepts to be considered herein have been presented and discussed in Chapter 2.)

The equilibrium-pair solution (EQ) is obtained if each player plays the strategy which maximizes his minimum possible gain. P1's minimum sales revenues corresponding to low, medium and high price levels are seen to be 2.16, 2.17 and 1.84 respectively; he would thereby choose his medium-price strategy. Similarly, Player 2 would also choose his medium-price strategy.

This is clearly one of the conceivable noncooperative solutions. It is worth noting that this equilibrium pair is not, however, a Nash noncooperative equilibrium point (see Chapter 2) because it is not dominant. Dominance would imply that, no matter what P2 plays, P1 is always best off to choose a medium-price strategy. This is easily refuted: if P2 plays low, P1 is better off to play high. As Bacharach states, having a non-dominant, noncooperative equilibrium point is "weak grounds for judging (the outcome) to be rational, (and) weak grounds too for thinking that it will come about." (1977)

Other noncooperative solutions, of course, are also possible. The cutthroat solution corresponds to the assumption that each player attempts to maximize the minimum distance or "spread" between his payoff and his opponent's; hence the alternate name "maximin-the-difference" (MD). In order to find the MD solution, it is necessary to convert Figure 5.6 into a matrix of differences between payoffs (Figure 5.7). Note that, for the duopoly, this difference matrix essentially represents a zero-sum game.

P1's PRICE \ P2's PRICE	LOW	MED.	HIGH
LOW	2.08	1.15	0.90
MED.	2.13	1.19	0.94
HIGH	2.36	1.43	1.06

FIGURE 5.7

Now, Player 1 maximizes his row minima, and Player 2 minimizes his column maxima. It is seen that a pure-strategy equilibrium point is quickly reached, as the payoff 1.06 is a saddle point; i.e., both players attain their desired objectives by playing their high-price strategies under this behavioural assumption.

If more than two players are involved, the beat-the-average solution (discussed in Chapter 2) would be applied in place of the cutthroat solution. In this case, however, the game would not reduce to an easily-soluble zero-sum game, as it did in the two-player case. In subsequent discussion of this behavioural assumption, the terms "cutthroat", "beat-the-average" and "maximin-the-difference" will be used interchangeably.

If the players were playing sadistically (i.e., choosing strategies which would lead to the worst possible outcome for their opponent), they would each choose their high-price strategies (S). Refer again to Figure 5.6. The worst possible payoff (sales revenue) which could be attained by P2 is 0.78; for this outcome to occur, P1 would have to choose high price. Similarly, P2 must also choose high price.

Thus, three conceivable noncooperative behavioural patterns (EQ, MD and S) have been considered. If the players chose to cooperate (or rather, if they were permitted to cooperate), they would jointly choose the strategies which would result in the highest total sales revenue. In Figure 5.6, this joint-maximal solution (JM) occurs if both players choose low prices. Bacharach picks up the discussion:

> "The duopolists stand to gain from colluding to establish this outcome (JM)...(however, Player 2) only <u>stands</u> to gain from JM--he would actually gain only if there were some kickback forthcoming from (P1)." (Bacharach 1977)

This situation occurs because the "high-high" payoffs (3.14 and 1.01) are not "Pareto-better" than the noncooperative equilibrium payoffs (2.54 and 1.35). If no "kickback" or deal is set up, P2 would still prefer not to cooperate.

<u>Strategies and Recommendations</u>

The above game matrix would have been derived given the information that the two firms have objectives of sales revenue maximization. Now, the pattern of the strategies they have used through time can be evaluated for appropriateness, in light of the solutions obtained above.

Of course, when dealing with firms operating in the real world using perfect information, it is unlikely that an equilibrium point (like EQ above) will be reached and adhered to. As Bacharach (1977) argues,the fact that EQ may be nondominant (as it was in his example), would make the likelihood of its attainment even more remote. What is more likely to be seen is a pattern of moves and countermoves, some of which make the firms better off and some worse off.

Even under circumstances of perfect information, it is unlikely that pure-strategy equilibria will be attained. Bacharach substantiates this observation in his discussion of the repeated playing if the Prisoner's Dilemma game (as in Chapter 2, Figure 2.3) as a <u>supergame</u> (a suggestion originated by Luce and Raiffa 1957).

"Suppose...the prisoner's dilemma is played 100 times in succession...(Define the following supergame strategy:) P1 plays Strategy 2 from (game) t on; up to t, he plays Strategy 1, and as soon as P2 should deviate from Strategy 1, P1 switches to and thereafter sticks to Strategy 2...Notice that though there is a kind of 'temporal collusion' within this 100-long sequence of games, the sequence considered as a single 'supergame' is entirely noncooperative..." (Bacharach 1977) (Note: in the above, the names of the players and strategies have been adjusted to conform to the usage of Chapter 2.)

Bacharach goes on to say that "the plausibility (of such supergame strategies)...casts doubt on the worth of the equilibrium notion for singling out 'solutions' of non-zero-sum games" (1977).

Given, then, that a pattern of moves over time is more likely to occur than a stable equilibrium point, a plan for analysis can be devised. It can be observed how frequently optimal and inferior strategies are played. A firm may have, say, a stated objective of sales revenue maximization, but may consistently choose strategies which are suboptimal for this objective--or which in fact may be better suited to an (implicit) goal of profit maximization (this could be determined by calculating a new game matrix for the industry with profits as the dependent variable). Indeed, if "sales revenue" was the priority variable stated by both firms represented in Figure 5.6, it is not clear whether each firm wishes to maximin its own sales revenue, maximize the minimum spread (cutthroat), or even to do as much harm as possible to the competitor. Different strategies would have resulted in different solution points as indicated in the diagram. Thus the frequencies of selection of the various strategic packages are of interest in analyzing the competition in the industry.

One can combine knowledge about the industry gained extraneously to the results of the game-matrix solution. Suppose it is known that Firm A typically makes marketing decisions which cause the most damage to its competitors (i.e., price undercutting, heavy advertising, etc.) Does Firm A resultingly choose its sadistic solution more often than its equilibrium-pair solution? If the sadistic solution appears to be preferred, it is impossible to tell, using only the game results, whether this preference was intentional (i.e., the firm wanted to play sadistically), or whether ignorance of the market or other factors caused the preference for this solution. However, combined with extraneous information, such an observation would have strong implications for strategic play.

Also of interest is the temporal aspect of play. The responses of Firm A to Firm B's decisions (and vice versa) may be examined for any recurrent pattern, and the relative merits of these response strategies may be judged by noting whether the resulting payoffs to Firm A (or B) are improved.

A trivial example by Bacharach (1977) highlights the temporal aspect of decision-making.

> "Suppose that (in the prisoner's dilemma game of Figure 5.8) the players had, somehow or other, gotten into an (N,N) groove. At the end of Game t, P1 contemplates double-crossing P2 at $t+1$; but he argues that this will induce (P2 to play) C at $t+2$, so he would be forced to play C himself at $t+2$. In one play, he would have more than wiped out his transient gain...so he sticks to N." (Bacharach 1977)

		P2	
		N	C
P1	N	9,9	0,10
	C	10,0	1,1

FIGURE 5.8

Source: Bacharach (1977), p. 61.

Finally, marketing-mix recommendations for the future may be made to both firms, based on preferred strategic combinations derived from the game matrix. In Figure 5.6, the recommendation would be made to either firm to choose (and adhere to) medium price levels in order to maximize future sales. However, if a firm chooses high price, it can increase the spread between it and its opponent (the effect would be lessened if the opponent reacted to this move by raising his own price). Thus, a high-price strategy would be recommended as an alternate choice should the out-distancing of the opponent be a more desirable objective than simple sales maximization. Furthermore, by constructing profit game matrices, strategic combinations which maximize profit levels may also be determined and recommended.

CHAPTER 6

RESULTS AND DISCUSSION: INDUSTRY 1

INTRODUCTION

The theoretical model, proposed in general form in Chapter 4 and explained and further discussed in Chapter 5, is applied to a real-world setting in this chapter. The rationale behind the selection of the econometric model is discussed, as are the preliminary findings of the quantitative analysis; then the game matrix is constructed, solved and interpreted. The discussion follows the pattern of the hypothetical industry analysis of Chapter 5: determining functional forms; discussing significance of effects; construction and interpretation of game matrix.

NATURE OF DATA USED

The manufacturer of one brand of a frequently-purchased small consumer good has provided bimonthly Nielsen sales audit figures (in units and dollars) and advertising expenditures for itself and all competitors over the six-year period from August 1976 to September 1982. Average market prices per bimonthly period were calculated by dividing sales revenue by unit sales for each brand. (Note: the price data actually employed in the analysis were normalized to industry average, in order to adjust for inflation.) Sales in this industry increased slowly for the first 24 bimonths; at this point sales accelerated greatly for each brand, only levelling off near the end of the period under study. To capture the industry sales trends, two additional parameters were added to the estimated models (see ensuing discussion of model-building).

The industry is treated in this study as a triopoly. Two major brands, A and B, account for 12% and 40% market share respectively and are premium priced, compared with the remainder of the brands on the market (manufacturer's and store brands). These remaining brands are combined into a third "brand" labelled "Others" ("O"); and average prices and total advertising expenditures for Others have been estimated.

Previous research by this author has indicated that price and advertising expenditures are the marketing decision variables which have the greatest effect on sales in this industry. Furthermore, it is hypothesized that price carryover effects may be ignored for this industry; i.e., that the theoretical representation discussed in Chapter 4 for the goodwill and Koyck approaches is applicable (one variable may exhibit carryover while the other is assumed to have only current effects on sales).

SELECTION OF ECONOMETRIC MODEL

Preliminary Testing

Chapter 4 dealt with the issue of choice between multiplicative and linear sales-response models. A multiplicative model seemed more reasonable for this industry, as discussed previously, since beyond a "threshold" advertising level, sales would be expected to show decreased marginal returns to increases in advertising. This effect would be represented in a multiplicative model by advertising parameters (exponents) less than one.

As described in Chapter 4, a building-up procedure is used in determining the appropriate functional form for the industry; i.e., a simple model is proposed and successive refinements and extensions to it are attempted.

The basic model tested is a slightly-modified version of Equations 4.3 and 4.4. For <u>each</u> of the three firms (A, B, and Others), the following model depicting sales as a multiplicative function of pricing and (current) advertising levels is taken as a base point.

$$\ln S_{it} = \alpha_o + \alpha_1 Q + \alpha_2 R + \alpha_{11} \ln P_{it} + \alpha_{21} \ln A_{At} + \alpha_{22} \ln A_{Bt} + \alpha_{23} \ln A_{Ot} + W_t; \tag{6.1}$$

where

S_{it} = sales (in hundreds of thousands of dollars) of Firm i at time t;

P_{it} = price of Firm i's brand at time t, normalized to industry average, defined as firm i's price x 100 over industry average;

A_{At}, A_{Bt}, A_{Ot} = advertising expenditure (in thousands of dollars) for Firms A, B and O (Others) at time t;

W_t = error term;

α_i and α_{ij}'s = parameters;

and Q and R are sales adjustment parameters included to isolate the noted industry sales trends, defined as follows:

Q = period number (starting from period 1 and taking on values 1, 2, 3, ... , 37);

R = period number minus 24 (starting from period 25 and taking on values 1, 2, 3, ... , 13).

Note that in this formulation, Firm i's relative price, as well as all firms' current expenditures, are specified as the independent variables, and that carryover advertising (and price!) effects are ignored, as are all other extraneous effects with the exception of the sales trend (represented by Q and R) which is assumed to continue for the short term.

Equation 4.1 was estimated for Firms A, B and O using simple OLS regression. All other regressions reported in the analysis of this industry are OLS as well. To avoid repetition, it is noted here that all regressions were checked for autocorrelation using the Durbin-Watson statistic and were found to be either free of autocorrelation or in the inconclusive range. Residual plots did not indicate any substantial levels of heteroscedasticity. The subject of multicollinearity is taken up in the discussion of the choice between the Koyck and goodwill models). Note: all regressions reported in this study were performed using the MASSAGER '73 package (Statistics Canada, 1973).

This model was then compared to various other models using standard partial F tests (see, e.g., Kleinbaum and Kupper 1978). The significance of the price variable was tested by obtaining ANOVA tables for each brand for both the model of Equation 6.1 and an identical model lacking only the price variable. (Relevant ANOVA tables and partial-F statistics appear in the Appendix to this chapter: see Test 1 of the Appendix for details of this test.) The model containing price was shown to be significantly better than that not containing it for both Brand A and Brand B (F for Brands A and B = 44.27 and 4.40 respectively; both significant at $\alpha = 0.05$ with 1 and 29 degrees of freedom). Additionally, the partial t values for relative price are significant for both Brands A and B and should therefore be left in the model. (Note: t-statistics for the regressions are listed with the corresponding ANOVA tables, where appropriate.)

However, the inclusion of competitors' prices into the model was not justified. The model of Equation 6.1 was next compared to a similar model incorporating the prices of both competitors (e.g., P_{Bt} and P_{0t} in addition to P_{At}) for Brand A only, and the overall model was not significantly improved (partial F = 0.08). This is not altogether surprising, because raw prices were normalized to industry average to account for the price trends in the industry (a raw-materials shortage during 1980 caused prices suddenly to increase, and subsequently to fall dramatically). In other words, the use of normalized, relative prices accounts implicitly for competitive pricing (see Test 2 of Appendix).

This preliminary testing indicated that own advertising, competitive advertising and own adjusted price level appear to be significant factors in determining sales. It still remains to test for the significance of carryover advertising effects; and also for the presence of interaction effects between independent variables.

Carryover Effects: Goodwill Approach

First, the goodwill model (see Chapter 4) was applied. Equation 6.1 was extended such that three new terms, corresponding to one-period lag effects of each firm's advertising expenditure ($A_{A(t-1)}$, $A_{B(t-1)}$, $A_{O(t-1)}$) were introduced; and this new model was estimated and compared to 6.1. It was judged to be an unsatisfactory improvement in explanatory ability due to the inclusion of the lag terms (for brands A, B, and O, the partial F's obtained in Test 3 of the Appendix were 0.932, 0.283 and 0.160 with 3 and 25 degrees of freedom). Secondly, overall F values (indicating overall explanatory ability) as well as adjusted R-square values either decreased substantially or remained unchanged upon introduction of the lag variables. Thirdly, the Durbin-Watson statistics showed that levels of autocorrelation were higher for the lag model. These developments can be explained: the addition of new variables, of course, always increases overall fit (as evidenced by declining SSE values in the ANOVA tables); but at the expense of degrees of freedom and at the risk of introducing multicollinearity into the model. Given that there are only 37 data points to begin with, a model which sacrifices the smallest number of degrees of freedom will, other things being equal, be preferable.

A final indication that the goodwill model is unsatisfactory for this industry: partial t statistical analysis shows none of the lag advertising effects to be strongly significant for any of the three brands (see Appendix). This may be because lagged effects are indeed not significant themselves, or alternatively because high multicollinearity between current and lagged advertising levels makes it impossible to distinguish the effects of each. The goodwill model as described herein cannot distinguish between these two possibilities; therefore, the inability of this model to isolate lagged effects does not necessarily mean that such effects were insignificant; rather, that another means of representing carryover (i.e., the Koyck approach) may be more appropriate.

As a point of interest, another goodwill model, this one containing one-period and two-period lagged effects, was also applied and compared to the basic model. Similar disappointing results were obtained: all partial F values were insignificant (F for brands A, B and O were, respectively, 1.100, 1.096 and 1.110 with 6 and 22 degrees of freedom). Also, many obtained advertising effects were in counterintuitive directions indicating with great likelihood that multicollinearity had become severe (see Test 4 of the Appendix).

This appeared to be a situation as described in Chapter 5: compounding multicollinearity and degrees-of-freedom problems causing difficulties with the goodwill approach. Of the remaining approaches, Koyck is the more applicable, as it can be used where only one variable exhibits carryover effects.

Carryover Effects: Koyck Approach

Central to the application of the Koyck approach is the replacement of the advertising expenditure variable (A_{At}, A_{Bt}, A_{Ot}) in Equation 6.1 with cumulative advertising expenditure variables (to be described ZA_{At}, ZA_{Bt}, ZA_{Ot}), which are obtained by assuming a geometrically-declining advertising effect (see Chapters 4 and 5). The parameter γ indicates the pattern of the geometric sequence. As defined in Chapter 4, high values of γ (i.e., near 1) indicate lower significance of carryover effects; while lowering the value of γ increases the effect of lagged advertising on current sales.

The process is easily visualized by converting Equation 6.1 to its Koyck-form equivalent:

$$\ln S_{it} = \alpha_0 + \alpha_1 Q + \alpha_2 R + \alpha_{11} \ln P_{it} + \alpha_{21} \ln ZA_{At} + \alpha_{22} \ln ZA_{Bt} + \alpha_{23} \ln ZA_{Ot} + w_t; \qquad (6.2)$$

where

$$\ln ZA_{it} = \ln A_{it} + \sum_{\ell=1}^{L} (1 - \gamma)^{\ell} \ln A_{i(t-\ell)}. \qquad (6.3)$$

If γ is set to 1, 6.2 collapses to 6.1 and carryover effects are not introduced into the model.

The value of γ is estimated via an optimization procedure. In this study, the γ in 6.3 was assigned values at decreasing 0.1 intervals from 1.0 (no carryover) to 0.4 (high carryover effects). New independent variables (the ZA_{it}'s) were calculated for each brand according to the patterns as introduced in Chapter 4; i.e., for $\gamma = 0.9$,

$$\ln ZA_{it} = \ln A_{it} + (0.1) \ln A_{i(t-1)} + (0.01) \ln A_{i(t-2)} + \cdots$$

It is seen that this is an infinite series which does not strictly converge (since the values of $\ln A_{i(t-\ell)}$ keep changing). However, the summation was truncated when adding extra terms increased the sum total by less than 0.01.

(Note: in all of the following, the same number of observations and total degrees of freedom were used in order to render the various regression statistics comparable.)

A number of criteria can be used to compare the results of these seven regressions ($\gamma = 1.0, 0.9, \ldots, 0.4$). Most importantly, overall F values may be compared, as they indicate how well the independent variables (considered all at once) explain the dependent variable (sales) (see Kleinbaum and Kupper 1978). Additionally, adjusted R^2 values and error sums of squares (measures of strength of overall relationship) may be compared. Finally, the independent variables determined to have significant effects on sales may be analyzed: there are some a priori predictions as to probable directions of the effects of price and advertising, and, other things being equal, a model which yields interpretable results is preferable.

Figure 6.1 shows the obtained overall F values for all three brands and for all values of γ (including intermediates; see below); while Figure 6.2 indicates the corresponding SSE values. For Brand A, it is clear that, as a first approximation, a γ value of about

FIGURE 6.1
F-values of regressions, γ varying from 1.0 to 0.4, 33 observations

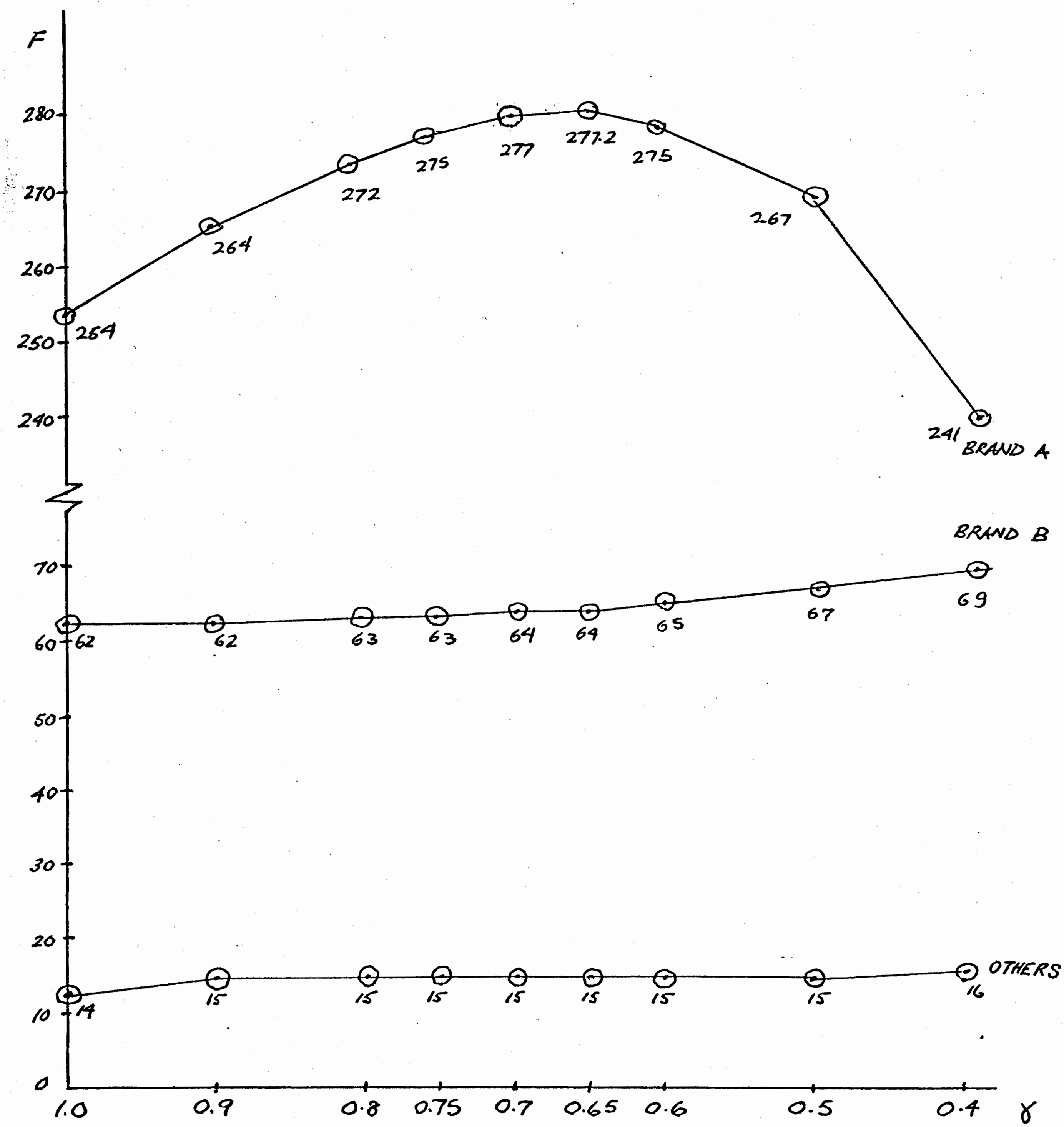


FIGURE 6.2

Error sums of squares, γ varying from 1.0 to 0.4, 33 observations

γ	SSE, BRAND A	SSE, BRAND B	SSE, OTHERS
1.0	0.166	0.350	1.055
0.9	0.160	0.347	1.055
0.8	0.155	0.343	1.052
0.75	0.153	0.341	1.050
0.7	0.153	0.339	1.047
0.65	0.153	0.336	1.041
0.6	0.153	0.333	1.035
0.5	0.158	0.323	1.018
0.4	0.175	0.317	0.995

0.7 seems to be most appropriate. The F statistic reaches its highest point, and the SSE statistic its lowest point, at about $\gamma = 0.7$. (Note: adjusted R-squares are not compared in this example because varying γ caused only insignificant fluctuations to this statistic.) All Durbin-Watson statistics were in the acceptable region.

Having determined that the best combination of explanatory variables appeared at about 0.7, intermediate values of 0.65 and 0.75 were also tried (the results of these regressions also appear in Figures 6.1 and 6.2). The maximum F value for Brand A is obtained at $\gamma = 0.65$; this value is 277.226 which is highly significant with 6 and 26 degrees of freedom. SSE is also at a minimum here for Brand A (SSE = 0.153). Thus, the carryover effect of advertising in this industry apparently is best represented by a Koyck model with geometrically-declining cumulative advertising ZA_{it}, defined as follows:

$$\begin{aligned} \ln ZA_{it} &= \ln A_{it} + \sum_{\ell=1}^{L} (1 - 0.65)^{\ell} \ln A_{i(t-\ell)} \\ &= \ln A_{it} + 0.35 \ln A_{i(t-1)} + 0.123 \ln A_{i(t-2)} \\ &\quad + 0.043 \ln A_{i(t-3)} + 0.015 \ln A_{i(t-4)} \\ &\quad + 0.005 \ln A_{i(t-5)} + \ldots ; \end{aligned} \tag{6.4}$$

i = Brand A, Brand B, Others.

By performing a regression at every γ value, the possibility that the distribution of F's and SSE's are bimodal was investigated and ruled out. There is a clear increase in model fit as γ decreases from 1.0 to 0.65 and a clear decrease afterwards, at least for Brand A. Test 5 of the Appendix compares the major statistics obtained for the different γ values.

A value of approximately 0.65 for γ is reasonable for this industry. Equation 6.4 shows that the effect of lagged advertising expenditures drops off substantially beyond A_{t-3} (the coefficients of $\ln A_{t-3}$ and $\ln A_{t-4}$ are only 0.043 and 0.015, and subsequent effects are even lower). Since the data are bimonthly, three lagged periods corresponds to five or six months. In the model as specified, then, advertising which is "older" than about four months has a weak effect on current sales, and ads "older" than six months have practically no effect at all. It would be difficult to justify a model of this industry with $\gamma = 0.9$ or 1.0 (signifying almost no carryover advertising effects at all); or with $\gamma = 0.4$ or lower (at $\gamma = 0.4$, the coefficient of $\ln A_{t-6}$ is 0.043; effects of year-old advertising would be substantial at this γ level).

The clear maximum in F just described was observed only for Brand A (See Figure 6.1). For neither of the other brands was a clear maximum F value found: in fact, F and SSE statistics remained almost constant, seemingly unaffected by changes in γ. The advertising expenditures of these brands have remained relatively constant, compared to Brand A's: i.e., the variance in their advertising levels through time has been less than that of Brand A. A glance at the original data indicates that Brand B has maintained a relatively high, stable advertising policy, and Others have advertised at consistently low levels, while Brand A has shown great fluctuations in advertising over time. Thus one cannot distinguish among the regressions (for B and Others) because the advertising variables used (the ZA's) were not greatly altered by altering the value of γ. In any case, for Brand B and Others, most of the estimated parameters do not vary much as γ is adjusted, which is, again, as expected.

As a further confirmation of the regression model with $\gamma = 0.65$, the significant independent variables (as determined by partial t values) were also examined. Figure 6.3 contains the major findings. For Brand A, all four decision variables proved strongly significant regardless of the level of γ chosen; furthermore, all effects were in the a priori expected directions: Brand A's sales are positively influenced by its own advertising, and vary inversely with Brand A's price, Brand B's advertising level and Others' advertising level. Only two effects were significant for Brand B at $\gamma = 0.65$ (price and Others advertising) but both were in the expected directions. (It may be

FIGURE 6.3

Parameters with significant effects on sales (with observed direction of effects) as estimated for models with varying γ levels

$\gamma=$	1.0	0.9	0.8	0.75	0.7	0.65	0.6	0.5	0.4
BRAND A	P_A^- A_A^+ A_B^- A_O^-	P_A^- A_A^+ A_B^- A_O^-	P_A^- A_A^+ A_B^- A_O^-	P_A^- A_A^+ A_B^- A_O^-	P_A^- A_A^+ A_B^- A_O^-	P_A^- A_A^+ A_B^- A_O^-	P_A^- A_A^+ A_B^- A_O^-	P_A^- A_A^+ A_B^- A_O^-	P_A^- A_A^+ A_B^- A_O^-
BRAND B	P_B^- A_O^-	P_B^- A_O^-	P_B^- A_O^-	(P_B^-) A_O^-	(P_B^-) A_O^-	(P_B^-) A_O^-	A_O^- $(\underline{A_B^-})$	A_O^- $(\underline{A_B^-})$	(A_O^-) $\underline{A_B^-}$
OTHERS	none (see text)								

Legend: P_i = relative price of Brand i, i = A, B, O.

A_i = cumulative advertising level of Brand i

+, − = direction of effect

Parentheses ( ) = effect only weakly significant ($\alpha = 0.10$); otherwise understood strongly significant ($\alpha = 0.05$).

Underlined effects are in counterintuitive directions.

noted that although overall F was rising slightly as γ was lowered, the results were simultaneously becoming harder to interpret: in Figure 6.3, under $\gamma = 0.4$, note the strongly-significant negative effect on sales of Brand B's own advertising. This is likely caused by excessive statistical weight being put on long-past lagged effects.) (Note: a later section examines in detail the actual values of the estimated parameters.)

Figure 6.3 also indicates another observation made during the data analysis. In none of the models estimated were any individual variables found significant in affecting Others' sales. However, all models were, overall, statistically significant (F values were approximately 14 to 15, while adjusted R^2's ranged at about 0.72), though the fit was admittedly poorer than for either A or B. This is also not unexpected. The "Others" brand is, of course, an amalgam of all smaller brands in this industry, and the independent variables (price and advertising) do not represent strategic choices made by one decision maker. These are, rather, the smaller brands with smaller advertising budgets, and are thereby at a disadvantage in the industry. Their strategic choices may be dictated by either competitive pressures or resource constraints (i.e., cannot advertise as much as desired). Many of these brands are entirely unsupported by advertising, yet still sell (one of the criticisms made by Little (1979) concerning multiplicative models was their inability to explain sales at zero advertising). Nevertheless, the overall model (as judged by overall F) was significant.

To sum up: the Koyck carryover model, with γ set at 0.65, was chosen to represent this data set. It was able to identify, and estimate the size of, a carryover effect which (due to the compounding of multicollinearity and degrees-of-freedom difficulties) the goodwill model was unable to do. Furthermore, no resulting effects were in counterintuitive directions, all overall models were significant with satisfactory adjusted R^2 values, and (in the case of Brand A, which exhibited great variance in advertising expenditure), overall F and SSE statistics were optimized.

Investigating Interaction Effects

Two extensions to the selected model were considered which incorporated interaction effects between price and advertising. In the "all interactions" model, three additional terms were added to Equation 6.2, each representing a price-advertising interaction ($\ln P_{At} \ln A_{At}$; $\ln P_{Bt} \ln A_{Bt}$; $\ln P_{Ot} \ln A_{Ot}$). Note that cross interactions were not considered, in order not to sacrifice too many degrees of freedom. These new regressions (each containing nine variables--the six in 6.2 and the three interaction terms) were run. In each case, overall F was adversely affected, and partial-F tests showed that the model fit was only insignificantly improved by the addition of the interactions (for Brands A, B and O respectively, the partial F's obtained were 0.368, 0.863 and 0.743 with 3 and 23 degrees of freedom). Furthermore, none of the interaction effects was individually significant for any of the brands (no t-values were significant); and the significant main effects of some variables (e.g., Brand A's advertising on its own sales) were no longer isolated. There are just too many insignificant variables hindering accurate interpretation (see Test 6 of the Appendix).

Much the same results were obtained for a second model containing only one set of interactions: those of the firm in question (the "own interactions" model). The addition of only this one interaction term to 6.2 still causes substantial decreases in model appropriateness (as measured by overall F values); furthermore, the partial F values obtained for the addition of the interaction term were of the same insignificant order of magnitude as were those reported above for the "all interactions" case. As a result of these tests, both models incorporating interaction effects were rejected and the model of the form of Equation 6.2 with $\gamma=0.65$ was selected as the most appropriate for this industry. This last test is further discussed in Test 7 of the Appendix to this chapter.

EFFECTS AND THEIR SIGNIFICANCE

The parameters estimated for the model of Equation 6.2, using OLS regression and setting $\gamma = 0.65$, are listed in Figure 6.4. All standard errors are given, and all effects shown to be either strongly ($\alpha = 0.05$) or weakly ($\alpha = 0.10$) significant are marked.

For Brand A the fit is clearly the best. All main effects are strongly significant and in expected directions, and overall F and Durbin-Watson statistics are especially good, as is adjusted R^2.

A section in Chapter 4 discussed the merits of the multiplicative model in a marketing-mix context. These models permit the effect of a decision variable on the criterion variable to be nonlinear. It was previously discussed that, over a normal range, advertising ought to exhibit decreasing marginal returns on sales; and that such an effect would be represented by advertising parameters (exponents) which are less than one. All advertising parameters estimated by the model used here are substantially smaller than one, and in the case of Brand B, two of the three advertising parameters are not significantly different from zero.

It should be remembered that the "advertising" variable discussed here is a measure of cumulative advertising; the strength of carryover effects being determined by the choice of level for parameter γ. In other words, lagged advertising effects are being implicitly included in these models via an advertising stock variable which considers period t's advertising levels affecting sales in period t, period t+1, etc., with diminishing effect as time goes on.

The absolute values of the price parameters are much larger, especially for Brands A and B. This does not necessarily mean, however, that marginal effects of price on sales are higher than proportional (e.g., cutting price in half should cause sales to increase by more

FIGURE 6.4

Estimated coefficients and standard errors for selected model

$$\ln S_{it} = \alpha_0 + \alpha_1 Q + \alpha_2 R + \alpha_{11} \ln P_{it} + \alpha_{21} \ln ZA_{At} + \alpha_{22} \ln ZA_{Bt} + \alpha_{23} \ln ZA_{Ot} + W_t$$

Coeff.	Brand A	Brand B	Others
α_0	23.4358* (3.5447)	12.4012* (6.4100)	-0.1671 (10.9228)
α_1	0.0560* (0.0028)	0.0358* (0.0043)	0.0391* (0.0076)
α_2	0.0230* (0.0067)	0.0219* (0.0097)	-0.0052 (0.0169)
α_{11}	-4.5220* (0.7637)	-1.8330** (1.4004)	0.8681 (2.3463)
α_{21}	0.0192* (0.0088)	0.0136 (0.0133)	0.0193 (0.0237)
α_{22}	-0.1243* (0.0395)	-0.0761 (0.0599)	-0.0529 (0.1048)
α_{23}	-0.0257* (0.0076)	-0.0271* (0.0118)	-0.0013 (0.2100)
F	277.226	64.746	15.204
R^2 (adj.)	0.98	0.92	0.73

Legend: Entries are estimates obtained by OLS regression for each parameter, with standard errors given beneath in parentheses.

* -- significant at 0.05 level.

**-- significant at 0.10 level.

than double). Recall that prices are adjusted to an industry average in this study, in order to allow for erratic price patterns due to industry shortages and inflation. "Prices" essentially refers to "relative prices". The range of relative prices is comparatively low: at no time were any of the brands priced below 90% of the industry average, nor above 110%. Perhaps, then, the larger values determined for price parameters in this study are not only explicable, but indicative that small changes in relative price level do have a noticeable effect on sales, which is as might be expected.

Also, note that for each brand, one (or both) of the time-series parameters is significant. As discussed earlier in this chapter, industry sales exhibited moderate increases through the first four years of observation, then showed large increases thereafter. The results show the significance of these effects. The only time-series parameter which is not significantly different from zero is α_2 for Others, indicating that Brands A and B experienced a larger acceleration in sales in the last two years of the study than did their competitors.

Finally, note again that, except for the continuous sales trend parameter (α_1), none of the effects is significant for Others. This result can be interpreted to mean that sales of these smaller brands are not as greatly influenced by advertising as Brands A and B (which they are not, as many Others brands employ little or no advertising); nor by price levels (all Others brands are uniformly low priced and are seen as "budget" alternatives to the heavily-advertised brands A and B by consumers). Other possible explanations of the lack of significance of individual effects have already been given.

ESTIMATION OF PAYOFFS OF THE GAME MATRIX

The parameters obtained by OLS estimation and presented in Figure 6.4 were then used to construct an estimated game matrix, which indicates what the expected payoffs would be to each player under different combinations of strategic choice.

Chapter 4 described the theoretical rationale behind the use of representative values, and the decision concerning the appropriate number of choice options to employ for each independent variable. In this industry, none of the brands exhibited high variation in (adjusted) price level; thus, a dichotomous split (high/low) was deemed sufficient to represent strategic price decisions. The range of values taken on by the (cumulative) advertising variable was larger for Brand A than for either competitor (see previous section). Thus, high, medium and low representative values were used for A, while advertising strategies employed by B and O were to be represented as "high" or "low".

Next, the ranges of price and cumulative advertising values for each brand were used in order to determine representative values. The procedure described in Chapter 4 (i.e., taking the median value in each representative segment) was employed. Figure 6.5 shows the ranges of each variable (in logarithmic form), as well as the assigned representative values (and the cutoff values, which are not used until a later section). (Note: if any of the observed values had been extreme outliers, they would have been discarded before representative value selection: this situation did not occur with these data.)

The representative values (given as logarithms) were also converted back to the original units using antilogs, for comparison purposes. The range of prices employed by A and B is higher than that of O, and this is reflected in the representative values. The cumulative advertising values selected illustrate an important point: what Brand B considers "low" advertising is in fact higher than what either A or O would consider "high" (this point is crucial to upcoming discussion). The values also clearly show the wider range of Brand A's cumulative advertising levels, and reemphasize the fact that, on the whole, Others advertises comparatively less than either A or B.

FIGURE 6.5

Representative values and cutoffs

Brand, variable	Range	Repr. Values	$e^{(\text{value})}$	Cutoffs	$e^{(\text{cutoff})}$
BRAND A -- ln P	4.607 - 4.700	P^L -- 4.631 P^H -- 4.677	102.6 107.4	← 4.654	105.0
BRAND B -- ln P	4.621 - 4.696	P^L -- 4.640 P^H -- 4.678	103.5 107.6	← 4.659	105.5
OTHERS -- ln P	4.502 - 4.590	P^L -- 4.524 P^H -- 4.568	92.2 96.4	← 4.546	94.3
BRAND A -- ln ZA	1.766 - 8.327	A^L -- 2.860 A^M -- 5.047 A^H -- 7.234	17.5 155.6 1385.8	← 3.954 ← 6.141	52.1 464.5
BRAND B -- ln ZA	6.793 - 8.740	A^L -- 7.279 A^H -- 8.253	1449.5 3839.1	← 7.765	2356.7
OTHERS -- ln ZA	0.000 - 7.021	A^L -- 1.755 A^H -- 5.266	5.8 193.6	← 3.511	33.5

Legend: Representative values and cutoffs are pure numbers since they are defined as logarithms. To convert back to units of pricing and advertising, exponentials were taken. Units of exponential values and cutoffs are as follows: for PRICE: percentage of industry average; for ADVERTISING: in thousands of dollars per period.

These representative values were then substituted into Equation 6.2, using the parameter estimates as listed for each brand in Figure 6.4. Specifically, the equations were solved for the <u>next</u> ensuing period (since values had to be assigned to the trend terms in Equation 6.2). A sample calculation illustrates. To determine Brand A's expected sales in the next period for the situation where <u>all</u> firms employ low price levels and low advertising expenditures, the representative values corresponding to "low" choices are substituted into Equation 6.2 together with the parameters estimated for Brand A, to obtain (subscripts indicate the variable being substituted):

$$\ln S_{At} = 23.4358 + 0.0560\underset{Q}{(38)} + 0.0230\underset{R}{(14)} - 4.5220\underset{\ln P_A}{(4.631)}$$

$$+ 0.0192\underset{\ln A_A}{(2.860)} - 0.1243\underset{\ln A_B}{(7.279)} - 0.0257\underset{\ln A_O}{(1.755)}$$

$$= 4.050.$$

$$e^{\ln S_{At}} = S_{At} = e^{(4.050)} = 57.40.$$

Note that 38 and 14 were assigned to trend parameters Q and R, as these would be the values associated with the next ensuing period. (Note: except for the beat-the-average solution, the actual values assigned to Q and R will make no difference in preferred strategies, as varying Q and R unilaterally amounts to adding a constant to every value in the game matrix. However, in order to convert back from logarithm of sales to sales in $100,000's for purposes of this study, the constants must be added in.)

This procedure was repeated for each combination of representative values and for each brand: resultingly, all the values of the sales-dollars game matrix (which appears in Figure 6.6) were generated: this figure is entirely analogous to the three-dimensional dummy game matrix of Figure 4.8.

FIGURE 6.6

Sales Dollars matrix (entries in hundreds of thousands of dollars per period)

BRAND A ↓ \ BRAND B:	$P^L A^L$	$P^L A^H$	$P^H A^L$	$P^H A^H$
$P^L A^L$	57.40 148.41 126.60	50.86 137.83 120.18	57.40 138.38 126.60	50.86 128.51 120.18
$P^L A^M$	59.86 152.93 132.03	53.04 142.02 125.33	59.86 142.59 132.03	53.04 132.42 125.33
$P^L A^H$	62.43 157.43 137.83	55.31 146.20 130.84	62.43 146.79 137.83	55.31 136.32 130.84
$P^H A^L$	46.62 148.41 126.60	41.31 137.83 120.18	46.62 138.38 126.60	41.31 128.51 120.18
$P^H A^M$	48.62 152.93 132.03	43.08 142.02 125.33	48.62 142.59 132.03	43.08 132.42 125.33
$P^H A^H$	50.70 157.43 137.83	44.93 146.20 130.84	50.70 146.79 137.83	44.93 136.32 130.84

OTHERS: $P^L A^L$

BRAND A ↓ \ BRAND B:	$P^L A^L$	$P^L A^H$	$P^H A^L$	$P^H A^H$
$P^L A^L$	52.46 134.96 125.96	46.48 125.34 119.58	52.46 125.84 125.96	46.48 116.86 119.58
$P^L A^M$	54.71 139.07 131.37	48.47 129.15 124.71	54.71 129.67 131.37	48.47 120.42 124.71
$P^L A^H$	57.05 143.17 137.14	50.55 132.95 130.19	57.05 133.49 137.14	50.55 123.97 130.19
$P^H A^L$	42.61 134.96 125.96	37.75 125.34 119.58	42.61 125.84 125.96	37.75 116.86 119.58
$P^H A^M$	44.43 139.07 131.37	39.37 129.15 124.71	44.43 129.67 131.37	39.37 120.42 124.71
$P^H A^H$	46.34 143.17 137.14	41.06 132.95 130.19	46.34 133.49 137.14	41.06 123.97 130.19

OTHERS: $P^L A^H$

(continued next page)

FIGURE 6.6 (continued)

OTHERS: $P^H A^L$

BRAND A ↓ \ BRAND B:	$P^L A^L$	$P^L A^H$	$P^H A^L$	$P^H A^H$
$P^L A^L$	57.40 148.41 131.50	50.86 137.83 124.84	57.40 138.38 131.50	50.86 128.51 124.84
$P^L A^M$	59.86 152.93 137.14	53.04 142.02 130.19	59.86 142.59 137.14	53.04 132.42 130.19
$P^L A^H$	62.43 157.43 143.17	55.31 146.20 135.91	62.43 146.79 143.17	55.31 136.32 135.91
$P^H A^L$	46.62 148.41 131.50	41.31 137.83 124.84	46.62 138.38 131.50	41.31 128.51 124.84
$P^H A^M$	48.62 152.93 137.14	43.08 142.02 130.19	48.62 142.59 137.14	43.08 132.42 130.19
$P^H A^H$	50.70 157.43 143.17	44.93 146.20 135.91	50.70 146.79 143.17	44.93 136.32 135.91

OTHERS: $P^H A^H$

BRAND A ↓ \ BRAND B:	$P^L A^L$	$P^L A^H$	$P^H A^L$	$P^H A^H$
$P^L A^L$	52.46 134.96 130.84	46.48 125.34 124.21	52.46 125.84 130.84	46.48 116.86 124.21
$P^L A^M$	54.71 139.07 136.46	48.47 129.15 129.54	54.71 129.67 136.46	48.47 120.42 129.54
$P^L A^H$	57.05 143.17 142.45	50.55 132.95 135.23	57.05 133.49 142.45	50.55 123.97 135.23
$P^H A^L$	42.61 134.96 130.84	37.75 125.34 124.21	42.61 125.84 130.84	37.75 116.86 124.21
$P^H A^M$	44.43 139.07 136.46	39.37 129.15 129.54	44.43 129.67 136.46	39.37 120.42 129.54
$P^H A^H$	46.34 143.17 142.45	41.06 132.95 135.23	46.34 133.49 142.45	41.06 123.97 135.23

Brand A's choices are along the six rows;
Brand B's choices are along the four columns;
Others' choices are among the four matrices.

Each cell is to be interpreted as follows:
First entry: Brand A's sales
Second entry: Brand B's sales
Third entry: Others' sales

SOLUTION AND INTERPRETATION OF THE GAME MATRIX

Now that the game matrix has been estimated for this industry, the various "ideal" solutions, each corresponding to different behavioural assumptions, are determined, and their implications for strategic choice are examined.

Equilibrium-Pair Solution

The equilibrium-pair solution concept (EQ) corresponds to a behavioural pattern similar to that assumed by von Neumann and Morgenstern: each player is assumed to maximize his minimum possible payoff. This has been previously described as a risk-averse strategy. Figure 6.7 shows the possible outcomes for each strategic combination and for each player. Minimum payoffs in each case are circled, and the minimax solution is indicated with an arrow.

Brand A shows a clear preference for the low price, high advertising strategic combination (herein abbreviated $P^L A^H$). This is expected, as regression analysis showed that A's sales were strongly influenced by its pricing and advertising levels. Note that the effect of low prices or high advertising on company profits are ignored in this analysis. This is as it should be, under the assumption of sales optimization.

Brand B does not show appreciably improved sales as a result of high advertising: in fact, this brand is apparently slightly better off at low levels of (cumulative) advertising than at high. This observation has strategic implications. First, what had been defined as "low" cumulative advertising for B ($\ln ZA^L_B = 7.279$) would be, for any other brand (A included) considered quite high. Second, the range of variation in B's advertising is comparatively low; thus it is not possible to get a measure of how B's sales would be affected by major cutbacks in advertising using these data. Third, in the

FIGURE 6.7

Equilibrium-pair solution (EQ)

Each player maximizes minimum possible gain under this solution concept. In the chart below, all the outcomes which could be obtained for each possible strategy (depending on the opponents' moves) are listed. The minimum outcome for each strategy is circled, and the maximin solution is indicated with an arrow.

BRAND A:

$P^L A^L$	57.40	50.86	52.46	46.48	
$P^L A^M$	59.86	53.04	54.71	48.47	
$P^L A^H$	62.43	55.31	57.05	50.55	←
$P^H A^L$	46.62	41.31	42.61	37.75	
$P^H A^M$	48.62	43.08	44.43	39.37	
$P^H A^H$	50.70	44.93	46.34	41.06	

BRAND B:

$P^L A^L$	148.41	152.93	157.43	134.96	139.07	143.17	←
$P^L A^H$	137.83	142.02	146.20	125.34	129.15	132.95	
$P^H A^L$	138.38	142.59	146.79	125.84	129.67	133.49	
$P^H A^H$	128.51	132.42	136.32	116.86	120.42	123.97	

OTHERS:

$P^L A^L$	126.60	120.18	132.03	125.33	137.83	130.84	
$P^L A^H$	125.96	119.58	131.37	124.71	137.14	130.19	
$P^H A^L$	131.50	124.84	137.14	130.19	143.17	135.91	←
$P^H A^H$	130.84	124.21	136.46	129.54	142.45	135.23	

Results: A selects $P^L A^H$ and obtains a payoff of 62.43
B selects $P^L A^L$ and obtains a payoff of 157.43
O selects $P^H A^L$ and obtains a payoff of 143.17

econometric testing section, the effect of Brand B's advertising on its own sales was not shown to be significant over this narrow range. Thus, B's sales may decrease substantially if a very low level of advertising is maintained over a long enough period of time; but over the narrow range of advertising expenditure employed by B, only insignificant effects on sales were observed. Brand B is therefore at a point where additional investment in advertising does not increase sales significantly. Hence, B is better off at "lower" levels of advertising (still, recall, comparatively high) than excessively high.

Brand B, like Brand A, in general, makes more sales if prices are lower. For these two brands, "low" and "high" prices may be interpreted as, respectively, approximately 3% above, and approximately 7% above, industry average (see Figure 6.5). High prices are detrimental to sales of either brand, possibly because of the perception that the expensive brands are "pricing themselves out of the market", and the existence of lower-priced alternatives.

Others apparently prefers high prices and lower advertising; however, note that the minimum sales levels attainable for each of the four possible strategic combinations are almost identical (they range from 119.58 to 124.84). None of the four choices is really preferential to Others (which is as it should be, since there is no real "decision maker" nor real "marketing strategy" for Others. However, Others' "decisions" can affect the resulting sales for A and B. For this analysis, assume that the difference among the four outcomes is sufficiently large for Others to prefer $P^H A^L$.

Others does appear, however, to be better off at higher prices than at lower. To understand this, refer back to Figure 6.5. For Others, "low" and "high" prices correspond to approximately 92% and 96%, respectively, of the industry average. If priced too low, Others may be preceived as too cheap or "shoddy goods", with a resultant loss in sales. It may be better for the cheaper brands not to be perceived as being "too" cheap, but to remain close to the industry average.

A final point about the equilibrium-pair solution: the solution point obtained is also a Nash noncooperative equilibrium solution point, because it is dominant; i.e., no matter what the competitors choose, A would always pick low price/high advertising; the same logic holds for B and O. This theoretically would add an extra measure of stability to the solution point: no player would be motivated to change his strategy, even with the knowledge of competitive intentions. As such, the equilibrium solution point is more likely to be reached and adhered to than would be the case without Nash stability (see discussion in Bacharach 1977 and in Chapter 5).

Maximin-the-Difference (Cutthroat) Solution

The previous section assumed risk-averse behaviour on the part of all players. Now, each player tries a different strategy whereby he tries to maximize the difference in sales level between himself and the average of his two competitors. (This is the n-firm analog of the cutthroat solution described and worked out in Chapters 4 and 5.) The formula which expresses this behaviour in mathematical terms was given in Chapter 2 as

$$\max_{s_i} \left[P_i\ (S_1,S_2,\ \ldots\ ,S_n) - \frac{1}{n-1} \sum_{j \neq i} P_j\ (S_1,S_2,\ \ldots\ S_n) \right]$$

For the hypothetical industry of Chapter 5, the payoff matrix was replaced by a matrix giving differences between payoffs (Figure 5.7) in order to find the cutthroat solution. With only two firms in this industry, the matrix of Figure 5.7 was that of a zero-sum game. For the industry studied in this chapter, a payoff-difference matrix was also constructed (see Figure 6.8) using the above equation: since three firms are involved, all three payoff-differences must be shown for each cell (for each strategic combination): note, however, that the values in each cell add to zero, indicating the equivalence between this representation and the zero-sum game derived for the duopoly.

FIGURE 6.8

Payoff-differences matrix for "beat-the-average" solution:
Sales revenue MINUS the AVERAGE of the two competitors' sales figures

BRAND A ↓ \ BRAND B:	$P^L A^L$	$P^L A^H$	$P^H A^L$	$P^H A^H$
$P^L A^L$	-80.11 56.41 23.70	-78.15 52.31 25.84	-75.09 46.38 28.71	-73.49 42.99 30.50
$P^L A^M$	-82.62 56.99 25.63	-80.64 52.84 27.80	-77.45 46.65 30.80	-75.84 43.24 32.60
$P^L A^H$	-85.20 57.30 27.90	-83.21 53.13 30.08	-79.88 46.66 33.22	-78.27 43.25 35.02
$P^H A^L$	-90.89 61.80 29.09	-87.70 57.09 30.61	-85.87 51.77 34.10	-83.04 47.77 35.27
$P^H A^M$	-93.86 62.61 31.25	-90.60 57.82 32.78	-88.69 52.27 36.42	-85.80 48.22 37.58
$P^H A^H$	-96.93 63.17 33.76	-93.59 58.32 35.27	-91.61 52.53 39.08	-88.65 48.44 40.21

OTHERS: $P^L A^L$

BRAND A ↓ \ BRAND B:	$P^L A^L$	$P^L A^H$	$P^H A^L$	$P^H A^H$
$P^L A^L$	-78.00 45.75 32.25	-75.98 42.31 33.67	-73.44 36.63 36.81	-71.74 33.83 37.91
$P^L A^M$	-80.51 46.03 34.48	-78.46 42.56 35.90	-75.81 36.63 39.18	-74.10 33.83 40.27
$P^L A^H$	-83.11 46.08 37.03	-81.02 42.58 38.44	-78.27 36.40 41.87	-76.53 33.60 42.93
$P^H A^L$	-87.85 50.68 37.17	-84.71 46.68 38.03	-83.29 41.56 41.73	-80.47 38.20 42.27
$P^H A^M$	-90.79 51.17 39.62	-87.56 47.11 40.45	-86.09 41.77 44.32	-83.20 38.38 44.82
$P^H A^H$	-93.82 51.43 42.39	-90.51 47.33 43.18	-88.98 41.75 47.23	-86.02 38.35 47.67

OTHERS: $P^L A^H$

FIGURE 6.8 (continued)

BRAND A ↓ \ BRAND B:	P^LA^L	P^LA^H	P^HA^L	P^HA^H
P^LA^L	-82.56 53.96 28.60	-80.48 49.98 30.50	-77.54 43.93 33.61	-75.82 40.66 35.16
P^LA^M	-85.19 54.43 30.76	-83.07 50.41 32.66	-80.01 44.09 35.92	-78.27 40.81 37.46
P^LA^H	-87.87 54.63 33.24	-85.75 50.59 35.16	-82.55 43.99 38.56	-80.81 40.71 40.10
P^HA^L	-93.34 59.35 33.99	-90.03 54.76 35.27	-88.32 49.32 39.00	-85.37 45.44 39.93
P^HA^M	-96.42 60.05 36.37	-93.03 55.39 37.64	-91.25 49.71 41.54	-88.23 45.79 42.44
P^HA^L	-99.60 60.50 39.10	-96.13 55.78 40.35	-94.28 49.86 44.42	-91.19 45.90 45.29

OTHERS: P^HA^L

BRAND A ↓ \ BRAND B:	P^LA^L	P^LA^H	P^HA^L	P^HA^H
P^LA^L	-80.44 43.31 37.13	-78.30 40.00 38.30	-75.88 34.19 41.69	-74.06 31.52 42.54
P^LA^M	-83.06 43.49 39.57	-80.88 40.15 40.73	-78.36 34.09 44.27	-76.51 31.42 45.09
P^LA^H	-85.76 43.42 42.34	-83.54 40.06 43.48	-80.92 33.74 47.18	-79.05 31.08 47.97
P^HA^L	-90.29 48.24 42.05	-87.03 44.36 42.67	-85.73 39.12 46.61	-82.79 35.88 46.91
P^HA^M	-93.34 48.63 44.71	-89.98 44.70 45.28	-88.64 39.23 49.41	-85.61 35.97 49.64
P^HA^H	-96.47 48.78 47.69	-93.03 43.34 49.69	-91.63 39.10 52.53	-88.54 35.83 52.71

OTHERS: P^HA^H

Figure 6.9 gives the minimum payoff differences obtained for each strategic choice, and selects the maximum value for each firm. Firm B would still choose low price, low advertising under this second behavioural pattern. This time, however, both A and O could be persuaded to alter their advertising policies. Brand A's advertising especially appears to have a negligible effect on payoff differences, and in fact, lower advertising is slightly preferable to higher (82.56 below industry average versus 87.87). Brand A's price level appears to have a greater effect on payoff difference than does advertising (for $P^L A^L$, the payoff difference is 82.56 below average; raising price to high level changes payoff difference to 93.34 below average).

Notice also that, by playing cutthroat, Others could be persuaded to advertise more heavily. Its advertising was not shown to have a significant effect on its own sales; but it did affect significantly the sales of both A and B. Therefore, by advertising more, Others could improve its sales position relative to A and B.

Finally, unlike the equilibrium-pair solution, the cutthroat solution is not Nash-noncooperative: Firm B might choose to change its strategy if one or the other opponent would be expected to change. The solution is still Pareto-optimal, though, since no conceivable strategic change would be beneficial to all players involved. If A and O choose to play cutthroat consistently, then B would have no reason to deviate from $P^L A^L$, so the game would still be stable despite its lack of true Nash stability.

As an addendum, it was noted in Chapter 2 that the "beat-the-average" solution concept is best suited to industries where the payoff levels are of approximately the same order of magnitude. The results obtained here may be adversely affected by the discrepancy in sales volume between Brands A and B, although to what extent this discrepancy affects results cannot be determined.

FIGURE 6.9

Maximin-the-difference (cutthroat) solution (MD)

Using the payoff-differences matrix, the following <u>minimum</u> payoffs are obtained.

BRAND A:		obtained if:
$P^L A^L$	-82.56 ←	B plays $P^L A^L$; O plays $P^H A^L$
$P^L A^M$	-85.19	"
$P^L A^H$	-87.87	"
$P^H A^L$	-93.34	"
$P^H A^M$	-96.42	"
$P^H A^H$	-99.60	"

BRAND B:		
$P^L A^L$	43.31 ←	A plays $P^L A^L$; O plays $P^H A^H$
$P^L A^H$	40.00	"
$P^H A^L$	33.74	A plays $P^L A^H$; O plays $P^H A^H$
$P^H A^H$	31.08	"

OTHERS:		
$P^L A^L$	23.70	A plays $P^L A^L$; B plays $P^L A^L$
$P^L A^H$	28.60	"
$P^H A^L$	32.25	"
$P^H A^H$	37.13 ←	"

Results: Not a Nash equilibrium point, since B could be convinced to change his strategy if A or O changed. However, if both A and O play their MD strategies, B would choose $P^L A^L$. Therefore:

A selects $P^L A^L$ and obtains a payoff of 52.46

B selects $P^L A^L$ and obtains a payoff of 134.96

O selects $P^H A^H$ and obtains a payoff of 130.84

Solutions for Sadistic Play

Under this behavioural assumption, each player chooses strategies which lead to the worst possible outcome for his opponents. Figure 6.10 indicates what these worst outcomes are, and what strategic choices would lead to these outcomes.

Due to the symmetry of Figure 6.6, each firm has two strategies which potentially lead to the same "bad" outcome for its opponents. Brand A is indecisive between P^LA^L and P^HA^L, if only sadistic behaviour is taken into account. Similarly, B is indifferent between P^LA^H and P^HA^H, and O is indecisive between P^LA^H and P^HA^H. Under strict sadistic behaviour (i.e., not taking into account what the payoff is to the firm itself, but only considering doing maximum damage to the opponents), one would expect each firm to select each of its two preferred strategies with equal probability. This would yield eight possible combinations, and expected payoffs to each firm would be as calculated and shown in Figure 6.11.

A modified sadistic solution is perhaps more realistic, from a behavioural point of view. In this modification, A recognizes that by playing either P^LA^L or P^HA^L, it can inflict the same amount of damage on B and O. Then, rather than being indifferent between these alternatives, it chooses the one which simultaneously yields the higher level for its own sales. A recognizes that it is always better for its own sales to set prices at a lower level, no matter what the competition does. Thus, P^LA^L would be preferred over P^HA^L. By similar reasoning, B would prefer P^LA^H over P^HA^H, and O would choose P^HA^H over P^LA^H. Resultingly, only one combination (rather than the eight as before) would be selected. The resulting payoffs would be as given in Figure 6.11.

FIGURE 6.10

Sadistic solution (S)

Each player chooses a strategy which would lead to the worst possible outcome for his opponents.

A's worst outcome: 37.75

Reached only if: B plays $P^L A^H$ or $P^H A^H$
O plays $P^L A^H$ or $P^H A^H$

B's worst outcome: 116.86

Reached only if: A plays $P^L A^L$ or $P^H A^L$
O plays $P^L A^H$ or $P^H A^H$

O's worst outcome: 119.58

Reached only if: A plays $P^L A^L$ or $P^H A^L$
B plays $P^L A^H$ or $P^H A^H$

Results: according to <u>strict</u> sadistic behaviour (see text),

A is indecisive between $P^L A^L$ and $P^H A^L$

B is indecisive between $P^L A^H$ and $P^H A^H$

O is indecisive between $P^L A^H$ and $P^H A^H$

Payoffs for strict and modified sadistic behaviour calculated and/or shown in Figure 6.11.

FIGURE 6.11

Expected payoffs under strict and modified sadistic behaviour

Strict sadistic behaviour: 8 possible combinations, each equiprobable:

Brand A plays	Brand B plays	Others plays	Payoff to Brand A	Payoff to Brand B	Payoff to Others
$P^L A^L$	$P^L A^H$	$P^L A^H$	46.48	125.34	119.58
$P^L A^L$	$P^L A^H$	$P^H A^H$	46.48	125.34	124.21
$P^L A^L$	$P^H A^H$	$P^L A^H$	46.48	116.86	119.58
$P^L A^L$	$P^H A^H$	$P^H A^H$	46.48	116.86	124.21
$P^H A^L$	$P^L A^H$	$P^L A^H$	37.75	125.34	119.58
$P^H A^L$	$P^L A^H$	$P^H A^H$	37.75	125.34	124.21
$P^H A^L$	$P^H A^H$	$P^L A^H$	37.75	116.86	119.58
$P^H A^L$	$P^H A^H$	$P^H A^H$	37.75	116.86	124.21

Average (expected) payoffs:

to Brand A: 42.12

to Brand B: 121.10

to Others: 121.90

Modified sadistic behaviour (see explanation in text)

A selects $P^L A^L$ and obtains a payoff of 46.48

B selects $P^L A^H$ and obtains a payoff of 125.34

O selects $P^H A^H$ and obtains a payoff of 124.21

Note that sadistic behaviour (as described in Shubik and Levitan 1980 and in Chapters 2 and 5) would not explicitly lead to this combination being preferred over the other seven, as all are equally "sadistic". But it seems a reasonable expectation that if a firm is faced with two strategic possibilities, each causing identical damage to its opponents, it will choose the one which is also the most beneficial to its own sales.

The expected sales levels for each firm obtained using this modified sadistic solution are higher than had been determined for the strictly sadistic solution, but are still lower than the beat-the-average payoffs. These in turn were lower than the corresponding equilibrium-point payoffs. The players cause themselves the most damage if they choose to play sadistically.

The most noticeable change resulting from the application of the sadistic solution concept is that B can be convinced to raise its (already high, by industry standards) advertising levels, in order to capture as many sales away from its competitors as possible (or to prevent them from improving their relative sales position: this topic is returned to in the last section of this chapter). Unfortunately for B, other firms in the industry have the same idea. This situation is reminiscient of a classic economics scenario: the four gas stations at a busy city intersection engaging in a mutually detrimental price war. Suppose all the players had somehow reached the equilibrium point and were all playing maximin strategies. Now, suppose Firm B decides it wants to play sadistically. Owing to the Nash stability of the EQ solution, the competition would not change their selected strategies (unless, of course, their behaviour patterns also changed). The sadistic strategy of Firm B would indeed cause Brand A's sales to decrease from 62.43 to 50.55, and Others sales to decrease from 143.17 to 124.84; but would hurt itself as well (B's sales would fall from 157.43 to 146.20, assuming low price were maintained).

If Firm B does not want to sacrifice its own sales in order to damage competitive sales levels, it would have no reason to initiate sadistic behaviour in the industry.

Interestingly enough, if B nevertheless played sadistically, A and O are better off not to retaliate as they would push their own sales levels down even further. The effect is very similar to that seen in the price-war situation in classical oligopoly theory: sadistic behaviour hurts everyone in the industry in the long run, and it is understood (even without explicit cooperation) that it is in all participants' best interests to revert back to a more risk-averse strategy (or better still, to leave well enough alone and not to initiate the price war in the first place).

Joint Maximal Solution

So far the "best" noncooperative solution discussed is the equilibrium-pair solution. It results in higher sales levels for each player than any of the others. Could these sales levels be improved upon if the players entered into a situation of explicit collusion or cooperation?

One could visualize two possible scenarios: all three firms could collude to obtain the maximum total industry sales level (three-firm cooperation), or, two firms (A and B, say) could enter into a cooperative agreement which excludes all other brands (two-firm cooperation).

a) Three-firm cooperative solution: Figure 6.12 contains a game matrix which shows the total expected industry sales for each strategic combination. The players, acting jointly, would choose the strategies which lead to the maximum total sales level. This level is 363.03, which occurs if each player plays his equilibrium-pair strategy. In other words, the players would not do better than the equilibrium-pair (noncooperative) solution even with explicit collusion.

FIGURE 6.12

Matrix for joint-maximal solution (three-firm-cooperative)

OTHERS: $P^L A^L$

BRAND A ↓ \ BRAND B:	$P^L A^L$	$P^L A^H$	$P^H A^L$	$P^H A^H$
$P^L A^L$	332.41	308.87	322.38	299.55
$P^L A^M$	344.82	320.39	334.48	310.79
$P^L A^H$	357.69	332.35	347.05	322.47
$P^H A^L$	321.63	299.32	311.60	290.00
$P^H A^M$	333.58	310.43	323.24	300.83
$P^H A^H$	345.96	321.97	335.32	312.09

OTHERS: $P^L A^H$

BRAND A ↓ \ BRAND B:	$P^L A^L$	$P^L A^H$	$P^H A^L$	$P^L A^H$
$P^L A^L$	313.38	291.40	304.26	282.92
$P^L A^M$	325.15	302.33	315.75	293.60
$P^L A^H$	337.36	313.69	327.68	304.71
$P^H A^L$	303.53	282.67	294.41	274.19
$P^H A^M$	314.87	293.23	305.47	284.50
$P^H A^H$	326.65	304.20	316.97	295.22

OTHERS: $P^H A^L$

BRAND A ↓ \ BRAND B:	$P^L A^L$	$P^L A^H$	$P^H A^L$	$P^H A^H$
$P^L A^L$	337.31	313.53	327.28	304.21
$P^L A^M$	349.93	325.25	339.59	315.65
$P^L A^H$	363.03	337.42	352.39	327.54
$P^H A^L$	326.53	303.98	316.50	294.66
$P^H A^M$	338.69	315.29	328.35	305.69
$P^H A^H$	351.30	327.04	340.66	317.16

OTHERS: $P^H A^H$

BRAND A ↓ \ BRAND B:	$P^L A^L$	$P^L A^H$	$P^H A^L$	$P^H A^H$
$P^L A^L$	318.26	296.03	309.14	287.55
$P^L A^M$	330.24	307.16	320.84	298.43
$P^L A^H$	342.67	318.73	332.99	309.75
$P^H A^L$	308.41	287.30	299.29	278.82
$P^H A^M$	319.96	298.06	310.56	289.33
$P^H A^H$	331.96	309.24	322.28	300.26

b) Two-firm cooperative solution: Now consider possible two-firm combinations, e.g., A and B team up and choose the best combination for themselves. Of course, what they actually receive depends on O's choice, so what A and B will try to do is to maximize the worst possible outcome. It can be shown that A and B maximize the minimum joint payoff at their equilibrium-pair strategies; i.e., A would always choose $P^L A^H$; B would always choose $P^L A^L$.

O's choice is uncertain. If, however, O realizes that the other firms are plotting against him, he can at least make the best of a bad situation by playing the strategy which, in combination with the "known" strategies of A and B, yields the better sales payoff to himself: this is $P^L A^H$. Then, the expected payoffs would be identical to the noncooperative and the three-firm cooperative solution: 62.43, 157.43 and 143.17 respectively.

(Note that the above scenario is still somewhat hypothetical, since, as stated before, Others does not actually make price or advertising decisions.)

ANALYSIS AND RECOMMENDATIONS

The ideal solutions and corresponding payoffs which would be obtained under the different behavioural assumptions are summarized in Figure 6.13. As described in Chapter 5 and in Bacharach (1977), of course, it is not expected that one of these ideal solutions will be reached and adhered to in a real-life setting (even though the most preferential combination, the equilibrium pair solution, has the added advantage of Nash stability). This conclusion, indeed, would result from misinterpreting the game matrix. All of the entries therein are <u>estimated</u> payoffs derived from econometric analysis and as such contain a measure of uncertainty. The estimated payoffs are not deterministic and should not be interpreted as such.

FIGURE 6.13

Summary of solution concepts

Solution concept	Strategy chosen by: A	B	Others	Payoff to: A	B	Others
EQUIL. PAIR	$P^L A^H$	$P^L A^L$	$P^H A^L$	62.43	157.43	143.17
MAXIMIN DIFF.	$P^L A^L$	$P^L A^L$	$P^H A^H$	52.46	134.96	130.84
STRICTLY SADISTIC	$\left.\begin{matrix} P^L A^L \\ P^H A^L \end{matrix}\right\}$	$\left.\begin{matrix} P^L A^H \\ P^H A^H \end{matrix}\right\}$	$\left.\begin{matrix} P^L A^H \\ P^H A^H \end{matrix}\right\}$	42.12	121.10	121.90
MODIFIED SADISTIC	$P^L A^L$	$P^L A^H$	$P^H A^H$	46.48	125.34	124.21
3-FIRM COOP.	$P^L A^H$	$P^L A^L$	$P^H A^L$	62.43	157.43	143.17
2-FIRM COOP.	$P^L A^H$	$P^L A^L$	$P^H A^L$*	62.43	157.43	143.17

* -- preferred if Others realizes that A and B are cooperating.

However, it is instructive to analyze the pattern of strategic decisions as made by the players through time. Each firm's price and cumulative advertising levels were classified as low, medium or high, according to the cutoff values listed in Figure 6.5. (These cutoff values are midway between the corresponding representative values.) For example, all periods where Brand A's pricing level fell below 105.0 were considered "low price" periods.

Despite the expected fluctuations, each firm showed a marked preference for the equilibrium-pair option. Brand A, with six strategic options to choose from, picks low-price/high advertising 11 out of 33 times (see Figure 6.14); another 11 observations were low price/medium advertising. (Note: the first four periods were discarded for this analysis, since cumulative advertising levels were only defined for periods 5 through 37.) The $P^L A^L$ combination, solution under both the cut-throat (MD) and sadistic (modified) (S) behavioural patterns, is chosen only five times. Brand A recognizes the effects of comparatively high advertising and low price on its own sales, and appears to make strategic choices with this in mind.

Brand B prefers low prices (still above industry average, though), and the $P^L A^L$ combination (EQ solution) is slightly preferred over the $P^L A^H$ combination (14 times chosen, compared to 12). This latter combination is indeed the modified sadistic solution but choice of this strategic option does not necessarily imply that B is playing sadistically, As discussed before, there is evidence that B is overadvertising (i.e., its "low" advertising level is high enough as it is). The high frequency of selection of $P^L A^H$ is possibly more an indication that B is unconsciously overadvertising than an indication of deliberately sadistic behaviour. This example illustrates one major underlying concept of this study: the selection of a strategic option corresponding

FIGURE 6.14

Frequencies of strategy selection

BRAND A	$P^L A^L$	5 (S) (MD)
	$P^L A^M$	11
	$P^L A^H$	11 (EQ)
	$P^H A^L$	1
	$P^H A^M$	2
	$P^H A^H$	3
BRAND B	$P^L A^L$	14 (EQ) (MD)
	$P^L A^H$	12 (S)
	$P^H A^L$	2
	$P^H A^H$	5
OTHERS	$P^L A^L$	2
	$P^L A^H$	3
	$P^H A^L$	14 (EQ)
	$P^H A^H$	14 (S) (MD)

Legend: (S) -- sadistic solution (modified)
(MD)-- maximin-the-difference (cutthroat) solution
(EQ)-- equilibrium-pair (minimax) solution

to behavioural pattern X does not imply that the firm _intended_ to play according to behavioural pattern X, although the firm is playing _as if_ it intended to follow that behavioural pattern. This conclusion flows from the assertion introduced in Chapter 1 and explained in Chapter 4 and 5, where it was stated that the proposed method of research does not attempt to determine, in retrospect, what the behavioural intentions of the firms were. Instead, the strategic choices made by the firms and the resulting sales levels are used to gain a better understanding of the industry and to make recommendations for the future.

At least one piece of evidence, however, suggests that Brand B had indeed been intentionally playing sadistically. An industry expert had indicated in private communication with the author (Cooper 1984) that B's entire product line is comprised of mature products, usually with the largest market share in their respective product classes. The firm which makes Brand B has not introduced a successful new product into the marketplace in many years. Rather than using a product innovation strategy, B has relied on protecting its big sellers from competitive attack, partially by employing a large advertising budget.

Note that Brand O "selects" its equilibrium-pair strategy 14 times out of 33, as well as its sadistic strategy. The smaller firms appear not to undercut the big brands' prices by too great an extent; but it is difficult to draw any more specific conclusions as there is no unified advertising strategy for the Others brand.

The firms appear to recognize the benefits of staying near the industry average with regard to price. Very high price levels are detrimental to sales for A and B; while very low levels may hurt quality perceptions for the cheaper brands. Brand A realizes that its advertising is effective in increasing sales and thereby usually chooses at least medium advertising levels.

Brand B maintains cumulative advertising at higher levels than any of its competitors do; its "low" advertising level appears to be sufficient to maintain its sales level and should probably be chosen more often. Smaller brands can detract from the sales of A and B through the use of heavier advertising.

Selection of the equilibrium-pair strategies in the future would be highly recommended for Brands A and B. This combination of strategies results in the highest level of sales for each firm: even explicit collusion between some or all firms cannot improve the resulting sales levels. Brand B may require review and adjustment of its advertising policy: "high" advertising expenditure by B does not result in significantly higher sales than "low".

Maintenance of a "high" cumulative advertising expenditure level means: investing enough resources in advertising such that the amount in the "advertising account" (i.e., this period's investment plus the carried-over portions of previous-period investments) is maintained at a minimum level. (Note that this "advertising account" is similar to Nerlove and Arrow's "goodwill account", but since the term "goodwill approach" has already been defined and used herein for another purpose, the tendency to call any cumulative advertising account a "goodwill account" is avoided.)

Smaller brands seem to have a choice. Sales levels of the cheaper brands do not appear to be correlated with the total cumulative advertising levels of these brands. However, if they choose to play sadistically, they can successfully steal sales away from both larger competitors with heavier investment in advertising. It is worth noting that, since the equilibrium-pair solution is also a Nash-noncooperative solution, the preferred choice strategies of A and B do not depend (are not affected) by the strategic choices made by Others: neither would be tempted to change its choice if it had prior knowledge of O's intentions. It is hard to make a case for either the equilibrium point or the sadistic strategic option for Others, but (as has already been seen) this is purely an academic issue. First, there is no real decision-maker for Others; second, the chosen strategies of Others do not affect the preferences of either A or B.

CHAPTER 6: APPENDIX
REGRESSION STATISTICS

FORMAT

Each test is presented according to the following plan:

Reduced Model

A short description of the reduced model is given (e.g., "no price, no lag" indicates which independent variables had been omitted), together with an expression which lists the independent variables included, in the form

$$Y = f(A, B, C, \ldots)$$

The ANOVA tables are then given, as well as the adjusted R-square value and the Durbin-Watson (DW) statistic, for each brand (A, B, Others). Also listed are all independent variables which were found to be significant, together with the corresponding T-value. Significance is indicated as follows:

* -- significant at $\alpha = 0.05$ level.
** -- significant at $\alpha = 0.10$ level.

Also, effects which are in counterintuitive directions are underlined. Note: the sales-trend adjustment factors (Q and R) were found to be strongly significant in all regressions.

Full Model

The description, ANOVA tables, and t-values are given for the full model exactly as for the reduced model.

Partial-F Calculation

Finally, partial-F values are calculated to determine the extent of the improvement on the regression model by the inclusion of the extra variables in the full model. The formula used is as given in Kleinbaum and Kupper (1978).

$$F = \frac{\text{SSE (reduced model)} - \text{SSE (full model)}}{\text{DF (reduced model)} - \text{DF (full model)}} \bigg/ \frac{\text{SSE (full model)}}{\text{DF (full model)}}$$

where SSE = residual sum of squares and DF = degrees of freedom. Partial-F values are marked (*) if significant at $\alpha = 0.05$; otherwise marked (n.s.) for "not significant".

Note: Test 5 is slightly different and is discussed separately.

TEST 1: FOR SIGNIFICANCE OF PRICE EFFECT

Reduced Model: no price, no lag

$$\ln S_{it} = f(K, Q, R, \ln A_{At}, \ln A_{Bt}, \ln A_{Ot})$$

BRAND A	DF	SS	MS	F	
Regression	5	11.542	2.3084	162.221	$R^2_{adj} = 0.958$
Residual	30	0.427	0.0142		DW = 1.757
Total	35	11.969			

Signif. Variables--t-value:

ln(Brand A adv.) 1.56**
ln(Brand B adv.) -2.65*
ln(Others adv.) -2.64*

BRAND B	DF	SS	MS	F	
Regression	5	5.612	1.1225	79.216	$R^2_{adj} = 0.918$
Residual	30	0.425	0.0142		DW = 1.076
Total	35	6.037			

Signif. Variables--t-value:

ln(Brand B adv.) -2.24*
ln(Others adv.) -1.99*

OTHERS	DF	SS	MS	F	
Regression	5	4.414	0.8827	24.673	$R^2_{adj} = 0.772$
Residual	30	1.073	0.0358		DW = 2.005
Total	35	5.487			

Signif. Variables--none.

Full Model: price, no lag

$$\ln S_{it} = f(K, Q, R, \ln P_{it}, \ln A_{At}, \ln A_{Bt}, \ln A_{Ot})$$

BRAND A	DF	SS	MS	F	
Regression	6	11.800	1.9666	336.751	$R^2_{adj} = 0.983$
Residual	29	0.169	0.0058		DW = 2.529
Total	35	11.969			

Signif. Variables--t-value:

ln(Brand A price) -6.64*
ln(Brand A adv.) 2.22*
ln(Brand B adv.) -3.82*
ln(Others adv.) -4.49*

BRAND B	DF	SS	MS	F	
Regression	6	5.667	0.9445	74.139	$R^2_{adj} = 0.926$
Residual	29	0.369	0.0127		DW = 1.251
Total	35	6.036			

Signif. Variables--t-value:

ln(Brand B Price) -2.09*
ln(Brand B Adv.) -2.06*
ln(Others Adv.) -2.75*

OTHERS	DF	SS	MS	F	
Regression	6	4.420	0.7366	20.022	$R^2_{adj} = 0.765$
Residual	29	1.067	0.0368		DW = 2.021
Total	35	5.487			

Signif. Variables--none.

Partial-F Calculation

BRAND A

$$F = \frac{0.427 - 0.169}{1} \Big/ \frac{0.169}{29} = 44.272^*$$

BRAND B

$$F = \frac{0.425 - 0.369}{1} \Big/ \frac{0.369}{29} = 4.401^*$$

OTHERS

$$F = \frac{1.073 - 1.067}{1} \Big/ \frac{1.067}{29} = 0.163 \text{(n.s.)}$$

Conclusion: both advertising and own-price effects must be included in the regression model (see text).

TEST 2: FOR SIGNIFICANCE OF COMPETITORS' PRICES

Reduced Model: price, no lag (as above)

Full Model: all prices, no lag

$$\ln S_{it} = f(K, Q, R, \ln P_{At}, \ln P_{Bt}, \ln P_{Ot}, \ln A_{At}, \ln A_{Bt}, \ln A_{Ot})$$

BRAND A	DF	SS	MS	F	
Regression	8	11.802	1.4752	236.788	$R^2_{adj} = 0.982$
Residual	27	0.168	0.0062		
Total	35	11.970			DW = 2.580

Partial-F Calculation

$$F = \frac{0.169 - 0.168}{2} \Big/ \frac{0.168}{27} = 0.080 \text{ (n.s.)}$$

Conclusion: do not include competitors' prices in model.

TEST 3: GOODWILL APPROACH--LAGGED ADVERTISING EFFECTS (t - 1)

Reduced Model: price, no lag (first 2 observ's deleted)

$$\ln S_{it} = f(K, Q, R, \ln P_{it}, \ln A_{At}, \ln A_{Bt}, \ln A_{Ot})$$

BRAND A	DF	SS	MS	F	
Regression	6	11.023	1.8371	304.660	$R^2_{adj} = 0.982$
Residual	28	0.169	0.0060		
Total	34	11.192			DW = 2.526

Signif. Variables--t-value:

ln(Brand A Price)	-6.53*
ln(Brand A Adv.)	2.18*
ln(Brand B Adv.)	-3.70*
ln(Others Adv.)	-4.23*

BRAND B	DF	SS	MS	F	
Regression	6	5.432	0.9053	69.533	$R^2_{adj} = 0.924$
Residual	28	0.365	0.0130		
Total	34	5.797			DW = 1.242

Signif. Variables--t-value:

ln(Brand B Price)	-2.03*
ln(Brand B Adv.)	-1.80*
ln(Others Adv.)	-2.79*

OTHERS	DF	SS	MS	F	
Regression	6	4.174	0.6957	18.327	$R^2_{adj} = 0.754$
Residual	28	1.063	0.0380		
Total	34	5.237			DW = 2.022

Signif. Variables--none.

<u>Full Model</u>: price, 1 period lag.

$$\ln S_{it} = f(K, Q, R, \ln P_{it}, \ln A_{At}, \ln A_{A(t-1)}, \ln A_{Bt}, \ln A_{B(t-1)}, \ln A_{Ot}, \ln A_{O(t-1)}).$$

BRAND A	DF	SS	MS	F	
Regression	9	11.043	1.2270	202.478	$R^2_{adj} = 0.982$
Residual	25	0.152	0.0061		
Total	34	11.195			DW = 2.589

Signif. Variables--t-value

ln(Brand A Price)	-6.03*
ln(Brand A Adv.)	2.25*
ln(Brand B Adv.)	-2.47*
ln(Others Adv.)	-1.91*
ln(Oth. Adv. t-1)	-1.58**

BRAND B	DF	SS	MS	F	
Regression	9	5.441	0.6046	42.849	$R^2_{adj} = 0.917$
Residual	25	0.353	0.0141		
Total	34	5.794			DW = 1.254

Signif. Variables--none.

OTHERS	DF	SS	MS	F	
Regression	9	4.194	0.4661	11.171	$R^2_{adj} = 0.729$
Residual	25	1.043	0.0417		
Total	34	5.237			DW = 1.924

Signif. Variables--none.

Partial-F Calculation

BRAND A

$$F = \frac{0.169 - 0.152}{3} \Big/ \frac{0.152}{25} = 0.932 \text{(n.s.)}$$

BRAND B

$$F = \frac{0.365 - 0.353}{3} \Big/ \frac{0.353}{25} = 0.283 \text{(n.s.)}$$

OTHERS

$$F = \frac{1.063 - 1.043}{3} \Big/ \frac{1.043}{25} = 0.160 \text{(n.s.)}$$

Conclusion: Goodwill model of carryover effects inappropriate for this industry. Carryover not yet ruled out: try more lag periods.

TEST 4: GOODWILL APPROACH--LAGGED ADVERTISING EFFECTS (t-1 and t-2)

Reduced Model: price, no lag (as above)

Full Model: price, 1 and 2-period lag

$$\ln S_{it} = f(K, Q, R, \ln P_{it}, \ln A_{At}, \ln A_{A(t-1)}, \ln A_{A(t-2)}, \ln A_{Bt}, \ln A_{B(t-1)}, \ln A_{B(t-2)}, \ln A_{Ot}, \ln A_{O(t-1)}, \ln A_{O(t-2)})$$

BRAND A	DF	SS	MS	F	
Regression	12	11.067	0.9222	155.522	$R^2_{adj} = 0.982$
Residual	22	0.130	0.0059		
Total	34	11.197			DW = 2.681

Signif. Variables--t-value:

ln(Brand A Price)	-4.82*
ln(Brand A Adv.)	2.74*
ln(Br. A. Adv. t-2)	1.78*
ln(Brand B Adv.)	-2.16*
ln(Others Adv.)	-1.77*

BRAND B	DF	SS	MS	F	
Regression	12	5.513	0.4594	36.030	$R^2_{adj} = 0.925$
Residual	22	0.281	0.0128		
Total	34	5.794			DW = 1.320

Signif. Variables--t-value:

ln(Brand B Adv.)	-1.59**
ln(Br. B Adv. t-2)	-2.35*

OTHERS	DF	SS	MS	F	
Regression	12	4.421	0.3684	9.932	$R^2_{adj} = 0.759$
Residual	22	0.816	0.0371		DW = 2.108
Total	34	5.237			

Signif. variables--none.

Partial-F Calculation

BRAND A

$$F = \frac{0.169 - 0.130}{6} \bigg/ \frac{0.130}{22} = 1.100(\text{n.s.})$$

BRAND B

$$F = \frac{0.365 - 0.281}{6} \bigg/ \frac{0.281}{22} = 1.096(\text{n.s.})$$

OTHERS

$$F = \frac{1.063 - 0.816}{6} \bigg/ \frac{0.816}{22} = 1.110(\text{n.s.})$$

Conclusion: Goodwill model still not appropriate, degrees-of-freedom loss becoming a problem, yet evidence that lag effects may be significant. Next test investigates appropriateness of the alternative Koyck carryover model.

TEST 5--KOYCK MODEL--OPTIMIZATION OF γ

Note: This test is somewhat different. Nine different models were estimated in order to obtain overall-F, SSE and adjusted-R-square statistics. Partial-F tests are nct relevant, since there are no "full" and "reduced" models (all models have the same number of variables). The only difference among these models is the assigned value cf γ, which is being optimized (see text).

Model Being Estimated: price, Koyck lag

$$\ln S_{it} = f(K, Q, R, \ln P_{it}, \ln ZA_{At}, \ln ZA_{Bt}, \ln ZA_{Ot})$$

where $\ln ZA_{it}$ is defined as in Equation 6.3 in the text.

Rather than giving all nine ANOVA tables for all three firms, the summary statistics are compiled into the following table for comparative purposes.

γ	BRAND A F	SSE	R^2_{adj}	DW	BRAND B F	SSE	R^2_{adj}	DW
1.0	254.1	0.166	0.979	2.376	62.01	0.350	0.920	1.237
0.9	264.3	0.160	0.980	2.436	62.64	0.347	0.920	1.231
0.8	272.6	0.155	0.981	2.490	63.22	0.343	0.921	1.227
0.75	275.4	0.153	0.981	2.515	63.68	0.341	0.922	1.230
0.7	277.1	0.153	0.981	2.533	64.04	0.339	0.922	1.232
0.65	277.2	0.153	0.981	2.543	64.75	0.336	0.923	1.238
0.6	275.8	0.153	0.981	2.547	65.40	0.333	0.924	1.247
0.5	267.4	0.158	0.980	2.521	67.52	0.323	0.926	1.275
0.4	241.3	0.175	0.978	2.381	68.89	0.317	0.927	1.314

OTHERS

γ	F	SSE	R^2_{adj}	DW
1.0	14.95	1.055	0.723	2.033
0.9	15.00	1.055	0.723	2.034
0.8	15.00	1.052	0.724	2.051
0.75	15.05	1.050	0.725	2.055
0.7	15.11	1.047	0.726	2.059
0.65	15.20	1.041	0.727	2.064
0.6	15.32	1.035	0.729	2.071
0.5	15.64	1.018	0.733	2.098
0.4	16.11	0.995	0.739	2.132

TEST 6: FOR INTERACTION EFFECTS (ALL INTERACTIONS)

Reduced model: Koyck, $\gamma = 0.65$

$$\ln S_{it} = f(K, Q, R, \ln P_{it}, \ln ZA_{At}, \ln ZA_{Bt}, \ln ZA_{Ot})$$

where $\ln ZA_{it}$ is defined as in Equation 6.4 in the text.

BRAND A	DF	SS	MS	F	
Regression	6	9.764	1.6273	277.226	$R^2_{adj} = 0.981$
Residual	26	0.153	0.0059		DW = 2.543
Total	32	9.917			

Signif. Variables--t-value:

ln(Brand A Price)	-5.92*
ln(Brand A Adv.)	2.18*
ln(Brand B Adv.)	-3.15*
ln(Others Adv.)	-3.39*

BRAND B	DF	SS	MS	F	
Regression	6	5.019	0.8365	64.746	$R^2_{adj} = 0.923$
Residual	26	0.336	0.0129		DW = 1.238
Total	32	5.355			

Signif. Variables--t-value:

ln(Brand B Price)	-1.30**
ln(Others Adv.)	-2.30*

OTHERS	DF	SS	MS	F	
Regression	6	3.654	0.6089	15.204	$R^2_{adj} = 0.727$
Residual	26	1.041	0.0401		DW = 2.064
Total	32	4.695			

Signif. Variables--none.

Full Model: Koyck, $\gamma = 0.65$, all interactions

$$\ln S_{it} = f(K, Q, R, \ln P_{it}, \ln ZA_{At}, \ln ZA_{Bt}, \ln ZA_{Ot},$$
$$\ln P_{it} \times \ln ZA_{At}, \ln P_{it} \times \ln ZA_{Bt}, \ln P_{it} \times \ln ZA_{Ot})$$

where $\ln ZA_{it}$ is defined as in Equation 6.4 in the text.

BRAND A	DF	SS	MS	F	
Regression	9	9.767	1.0852	171.173	$R^2_{adj} = 0.980$
Residual	23	0.146	0.0063		DW = 2.421
Total	32	9.913			

Signif. Variables--t-value:

ln(Brand A Price) -2.54*

BRAND B	DF	SS	MS	F	
Regression	9	5.052	0.5613	42.751	$R^2_{adj} = 0.922$
Residual	23	0.302	0.0131		DW = 1.395
Total	32	5.354			

Signif. Variables--t-value:

ln(Brand A Adv.) 1.57**
Interaction (ln P x ln A Adv) -1.56**

OTHERS	DF	SS	MS	F	
Regression	9	3.745	0.4161	10.080	$R^2_{adj} = 0.719$
Residual	23	0.949	0.0413		DW = 2.309
Total	32	4.694			

Signif. Variables--none.

Partial-F Calculation

BRAND A

$$F = \frac{0.153 - 0.146}{3} \Big/ \frac{0.146}{23} = 0.368 \text{ (n.s.)}$$

BRAND B

$$F = \frac{0.336 - 0.302}{3} \Big/ \frac{0.302}{23} = 0.863 \text{ (n.s.)}$$

OTHERS

$$F = \frac{1.041 - 0.949}{3} \Big/ \frac{0.949}{23} = 0.743 \text{ (n.s.)}$$

Conclusion: All-interactions model inappropriate. Try own-interactions model.

TEST 7: FOR INTERACTION EFFECTS (OWN INTERACTIONS ONLY)

Reduced Model: Koyck, $\gamma = 0.65$ (see above)

Full Model: Koyck, $\gamma = 0.65$, own interaction

$$\ln S_{it} = f(K,\ Q,\ R,\ \ln P_{it},\ \ln ZA_{At},\ \ln ZA_{Bt},\ \ln ZA_{Ot},$$

$$\ln P_{it} \times \ln ZA_{it}) \quad ,\ i = (\text{Brand A, Brand B, Others}).$$

BRAND A	DF	SS	MS	F	
Regression	7	9.760	1.3943	230.460	$R^2_{adj} = 0.980$
Residual	25	0.151	0.0061		DW = 2.521
Total	32	9.911			

Signif. Variables--t-values:

ln(Brand A Price)	-2.63*
ln(Brand B Adv.)	-3.13*
ln(Others Adv.)	-3.02*

BRAND B	DF	SS	MS	F	
Regression	7	5.021	0.7173	53.412	$R^2_{adj} = 0.920$
Residual	25	0.336	0.0134		DW = 1.241
Total	32	5.357			

Signif. Variables--t-value:

ln(Others Adv.) -2.24*

OTHERS	DF	SS	MS	F	
Regression	7	3.685	0.5264	13.037	$R^2_{adj} = 0.725$
Residual	25	1.010	0.0404		DW = 2.052
Total	32	4.695			

Signif. Variables--none.

Partial-F Calculation

BRAND A

$$F = \frac{0.153 - 0.151}{1} \Big/ \frac{0.151}{25} = 0.331 \text{ (n.s.)}$$

BRAND B

$$F = \frac{0.336 - 0.336}{1} \Big/ \frac{0.336}{25} = 0 \text{ (n.s.)}$$

OTHERS

$$F = \frac{1.041 - 1.010}{1} \Big/ \frac{1.010}{25} = 0.767 \text{ (n.s.)}$$

Conclusion: Interaction term does not significantly improve the model.

CHAPTER 7
RESULTS AND DISCUSSION: INDUSTRY 2

INTRODUCTION

The last chapter presented an in-depth econometric and game-theoretic analysis of a product type: namely, a specific small, branded consumer good. In this chapter, a condensed analysis of a product class--regular cigarettes--is presented, in order to show how the same econometric procedures and game-theory rationale may be applied to a segment of an oligopoly.

THE CIGARETTE INDUSTRY IN CANADA

There are four major Canadian cigarette producers: Imperial, Rothman's, Macdonald and Benson and Hedges. Imperial commands about a 40% share of the market, as it manufactures many of the most popular brands (Du Maurier, Player's, Matinee). The remaining 60% share is approximately equally divided among the other three brands. Furthermore, there is evidence that Imperial is gaining in share at the expense of the other three (see later discussion; more on market share patterns is found in Clifford 1977).

An important aspect of cigarette sales is the predominance of big industrial buyers: department stores (during the period under study, at least; though Eaton's and others currently have halted cigarette sales), supermarkets, pharmacies, etc. However, the example of United Cigar Stores indicates that the industry is also integrated into retailing.

The presence of four major producers is only one aspect of the industry. There are also four major product classes commonly sold in Canada: regular (filter) cigarettes, "light" or "low tar" cigarettes, menthols and plain (unfiltered) cigarettes. All four manufacturers make cigarettes of the first three classes, while Imperial and Macdonald manufacture the two important plain-end cigarette brands. Two product classes (regular and low tar) account for most of the cigarette sales in Canada. As shown in Figure 7.1, the pattern of sales through time for regular cigarettes has been very different from that for low tar brands: the regular brands were all well-established long before 1976, and sales remained relatively constant; while most of the low-tar brands were introduced in the late-'70's and were in the sales-growth stage during the period under study.

The light brands are often advertised heavily upon introduction, with emphasis placed on their lightness (i.e., low tar and nicotine statistics may be emphasized in the ad copy, or a brand name is selected which explicitly describes the brand as being light: for example, Dumont Light or Dumaurier Special Mild). The advertising for such products is informative or persuasive in nature and as such is quite different from the reminder-type ads used for the more familiar regular brands (see, for example, the introductory ads for Vista, Accord or Vantage).

Since the sales patterns and advertising strategies for low tar cigarettes are so different from those for regular brands, it is useful to consider these two product classes as two separate theatres of operation (menthol and plain-ends could also be conceptualized this way). For reasons explained later, the product class of regular cigarettes was chosen for study in this analysis. The point is that it would be incorrect to treat all cigarette sales as homogeneous, since such across-class differences are known to exist.

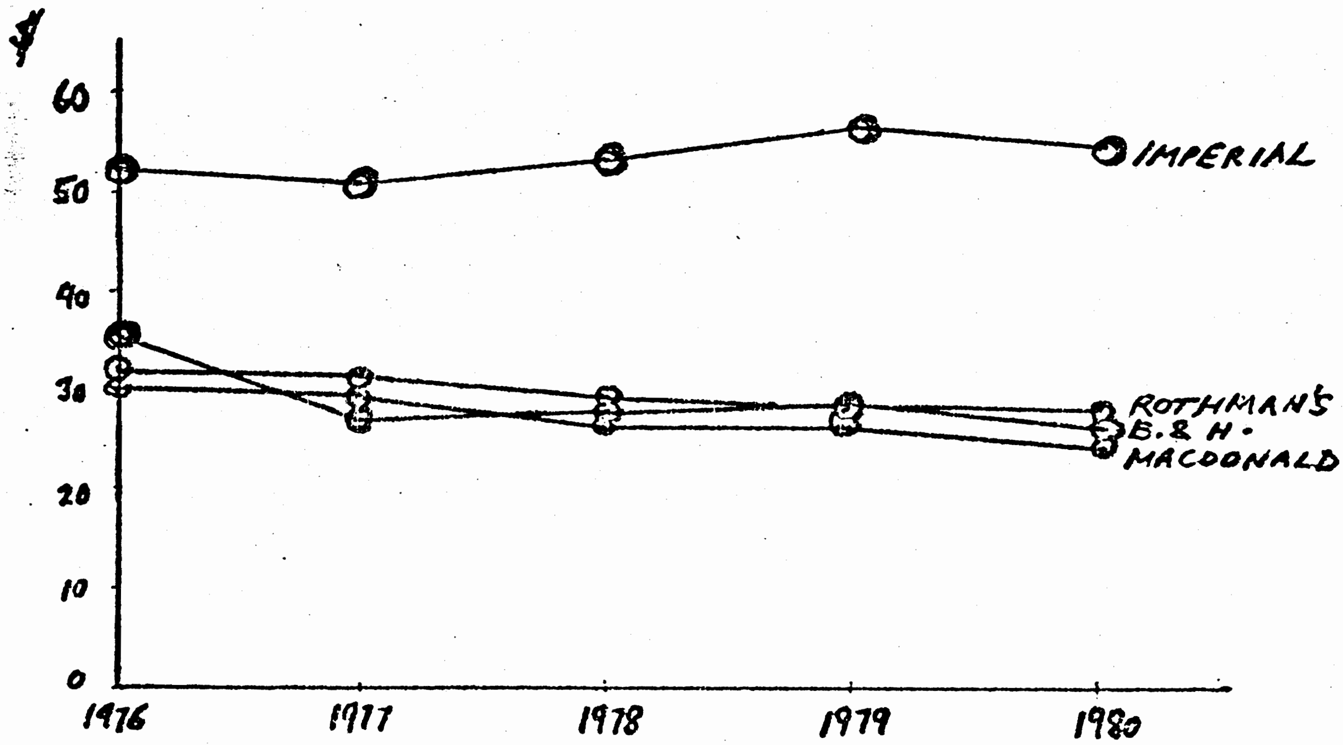
Owing to the nature of the market, there is a particularly strong relation between cigarette sales and advertising. Also, there is much regulation of the sale and advertising of cigarettes in Canada. For the most part, pricing is controlled: most cigarette

FIGURE 7.1

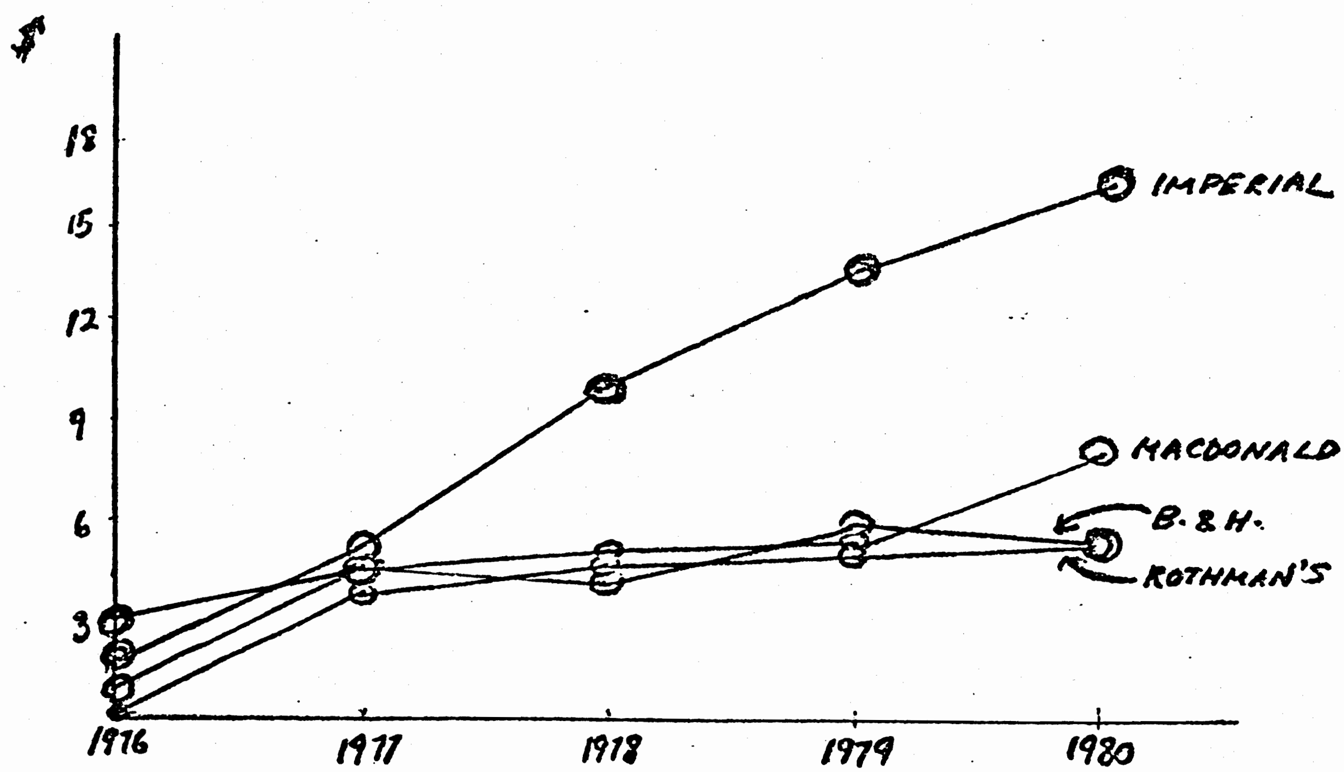
Sales dollars through time--regular and low tar brands

(Note: all reported sales dollars figures have been multiplied by a constant.)

Regular cigarettes



Low-tar brands



packages cost the same for the same number of cigarettes. Advertising is also highly regulated by the government, and the manufacturers have abided by the law (see Clifford 1976). Other advertising restrictions also apply: some media vehicles (e.g., Reader's Digest) prohibit cigarette advertising; and, to avoid further government intervention, cigarette advertising is very much self-regulated by the firms themselves.

NATURE OF DATA USED

Over five years of monthly data on industry sales dollars, print advertising expenditures and outdoor advertising for the Province of Quebec were obtained from the Cigarette Manufacturers' Association and comparison-validated with the largest of the four cigarette manufacturers, for the period between January 1976 and March 1981. In the Province of Quebec, as in the rest of Canada, no television cigarette ads are used, as the manufacturers voluntarily conformed to restrictive legislation in 1971 (see Clifford 1976). All advertising expenditure is therefore invested in either print or outdoor media, or in point-of-purchase displays. Since the analysis was carried out for the Province of Quebec, it should be mentioned that most of the advertising was in French, with the remainder in English. The possibility that French and English submarkets show different sales-response behaviours to advertising was not explored.

During the period under study, many of the currently-popular light brands were introduced. All of the regular brands were well-established long before 1976. For this study, the regular product class was investigated, as there were relatively few complicating factors (i.e., no new product introductions with corresponding ad campaigns, most brands with relatively constant sales levels). A further study on this same industry could investigate the sales of the light brands, especially with regard to the pattern of sales increases from time of introduction; but for the purposes of game-matrix analysis, the more mature, more stable regular product class was more easily modelled.

In-store merchandising (point-of-purchase displays) was not included in the analysis, as the preponderance of advertising expenditure was invested in print and outdoor media. On the subject of advertising in the industry, it should be mentioned that for outdoor and print media (with the possible exception of newspapers), there is generally a very long lead time prior to the appearance of an advertisement; ad campaigns must be planned many months in advance. A firm wishing to react quickly to competitive advertising or economic conditions can only resort to newspaper ads (and point-of-purchase displays).

Price did not appear to be an influential marketing-mix variable in this industry, as the Canadian cigarette smoker has been shown to be extremely price-insensitive (see Clifford 1977).

PRELIMINARY ANALYSIS

A glance at the original data suggested that there may have been a strong seasonal component to the trends of the four manufacturers' regular seasonal sales; however, regression analysis of sales as a function of own- and cross-advertising, month and trend of sales yielded very poor results. It was suspected that there was too much "noise" (random error, extraneous factors such as hoarding) in the monthly data. Therefore the data were compressed into three-month quarters (January to March = Winter; April to June = Spring, etc.) in an attempt to isolate the seasonal effects and to eliminate at least some of the random noise. As the model appropriateness (as measured by adjusted R-square) improved considerably for each brand as a result of this manipulation, the quarterly data were used for the remainder of this analysis.

In one of the first regressions run with the quarterly data, the sales levels for each firm were deseasonalized. Sales were taken to be a linear function of dummy variables representative of three of the four seasons; i.e., the model proposed was

$$S_t = \alpha_0 + \alpha_1 Sp_t + \alpha_2 Su_t + \alpha_3 F_t + w_t ; \qquad (7.1)$$

where S_t = sales dollars (scaled down by a constant factor);

Sp_t, Su_t, F_t = dummy variables set equal to 1 for spring, summer and fall respectively and zero otherwise;

α_i = parameters; and

w_t = error term.

(Note: the rationale of using a linear model here is explained later.)

This regression was revealing on two fronts. First, all seasonal parameters (with one exception: the spring parameter for Macdonald) were highly significant ($\alpha = 0.05$), indicating that decreases in sales across the industry during the post-Christmas winter season cannot be ignored. (It is interesting to speculate as to why January to March sales are lowest. For three of the four brands, the Fall season, October to December, exhibits the highest sales levels, indicating possible pre-Christmas gift buying--leading to less sales to the gift recipients in ensuing months. However, one cannot ignore the possibility that sales decrease during the winter due to the popular New Year's resolution to give up smoking. Presumably many such resolutions do not last into the spring season.)

The second observation from these regressions was derived from the residual analyses. With sales as a function of only seasonal effects, high positive autocorrelation was observed for each of the four brands (Durbin-Watson statistics were, respectively, 0.66, 0.98, 1.63 and 1.09 for Imperial, Rothman's, Macdonald and B & H). Furthermore, the residual plot for Imperial was positively sloped, while it was negatively sloped for the three other firms. These observations suggested that sales trends over time could not be ignored; and that sales appeared to be improving over time for Imperial at the expense of the other firms (the regression ignoring sales trend was, in general, overestimating actual Imperial sales for 1976 to 1978, and underestimating actual sales for 1979 to 1981). Private conversation with an industry expert who has prepared two major reports on the cigarette industry (Gandhi and Zuccaro 1983; Zuccaro 1983) confirms the observation that the industry leader (Imperial) has been increasing its lead over the smaller firms.

The next step in the preliminary analysis was to graph both sales and advertising expenditures for all firms over time. The sales plots revealed that Imperial's sales followed a clear seasonal pattern: almost without exception, winter sales were lowest, followed by a large increase in spring and smaller, additional increases in both summer and fall. The plot is sawtoothed in appearance, with an overall upward trend. Only one of the smaller firms (B & H) exhibited this kind of regular seasonality, except in this case the overall trend was downward. The seasonality was apparent to a lesser extent for Rothman's, which generally followed the pattern but which occasionally had inexplicably high spring sales; while Macdonald's pattern was the most irregular with any one of the four seasons apparently equally likely to be peaks or troughs on the sales plot. This observation of regularity of sales patterns is referred to again later.

The advertising plots indicate that only Imperial appears capable of using advertising expenditure effectively as a means of influencing its own sales. Imperial knows that sales generally peak during fall months: to capitalize on this phenomenon, this firm advertises most heavily during the summer season. Imperial is also the firm with the highest expenditures: in addition to about a 40% market share, it is responsible for about 42% of the total advertising figure in the industry. The effectiveness of its advertising is also suggested by the plots: the only year in which advertising is peculiarly high during winter and spring (1976) is also the only year where spring sales are unusually high (higher in fact than either the following summer or fall seasons). These observations are consistent with Imperial's role as industry leader: it has the resources to increase advertising expenditure when appropriate, with the desired result of improved sales.

The other firms' advertising expenditures appear to be either ineffective or poorly timed. Both Rothman's and B & H advertised heavily in advertising through 1976 and parts of 1977. Rothman's advertising had comparatively little effect on its sales (peaks in sales showed no correspondence to periods of heavy advertising). B & H's sales over the $5\frac{1}{2}$-year period peaked during Fall 1977 which was at the culmination of a two-year period of high advertising expenditure, suggesting that this firm employed a somewhat more successful advertising campaign; however, this strategy appears to have backfired as almost immediately thereafter, advertising budgets were cut drastically (to the point, in fact, where not one cent was spent on advertising in the Province of Quebec in all of 1980). B & H seemingly gambled on a media blitz in the hope of stimulating sales in the long term; then, B & H was unable to stop the sales decline as it could no longer afford to continue heavy advertising.

Macdonald's advertising efforts may be described as "too little, too late". After years of relatively low advertising and falling sales, this firm began heavy advertising in late 1979 and continued through 1980 with high advertising expenditure. These efforts did not, however, stop the decline in sales level.

The reader is invited to verify the preliminary analysis by examining the patterns of sales and advertising through time, which are given in Figures 7.2a and b. The units on the Y-axes have been multiplied by a constant in order to keep the data confidential.

FIGURE 7.2a--Regular cigarette sales through time (figures disguised)

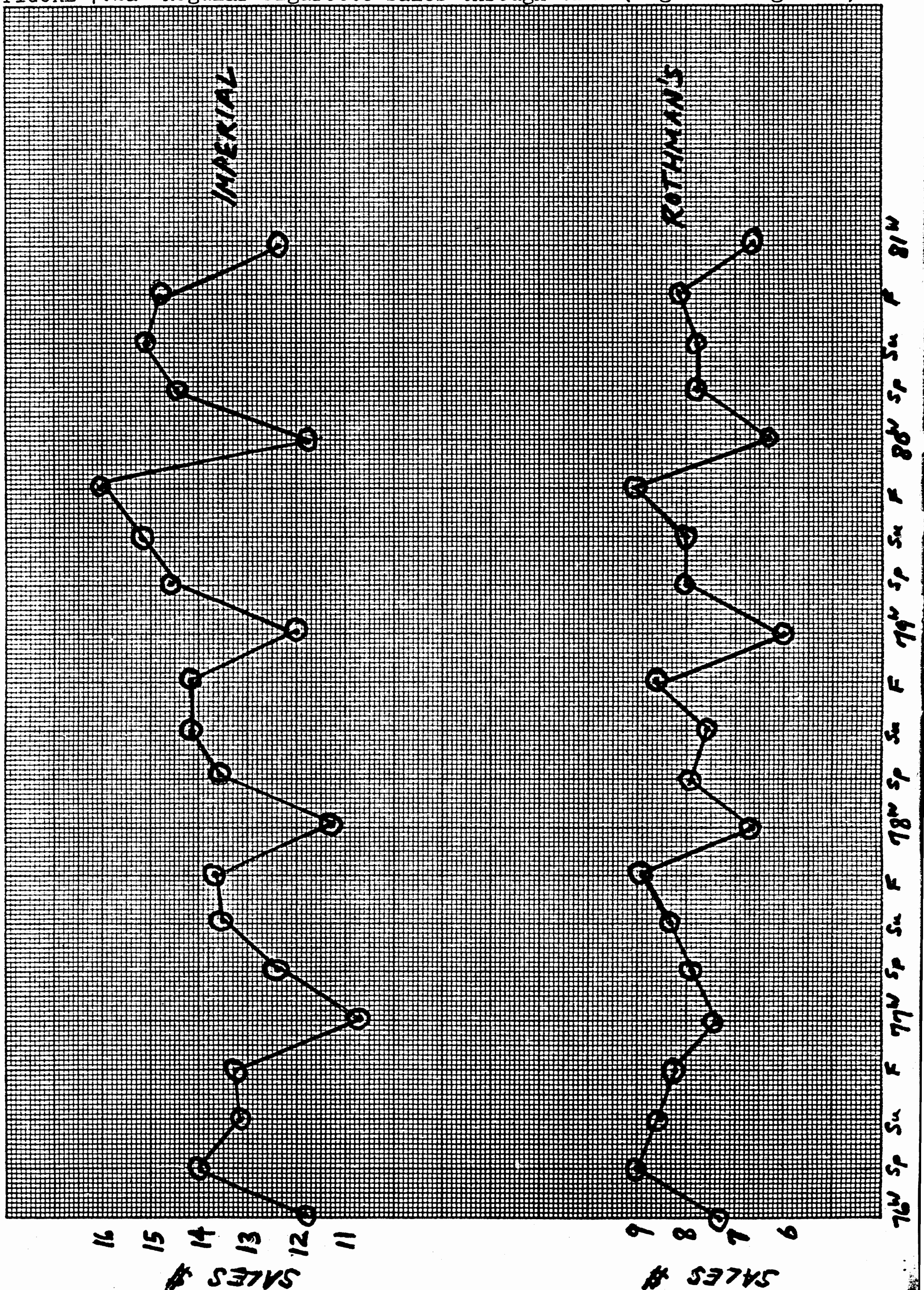


FIGURE 7.2a (continued)

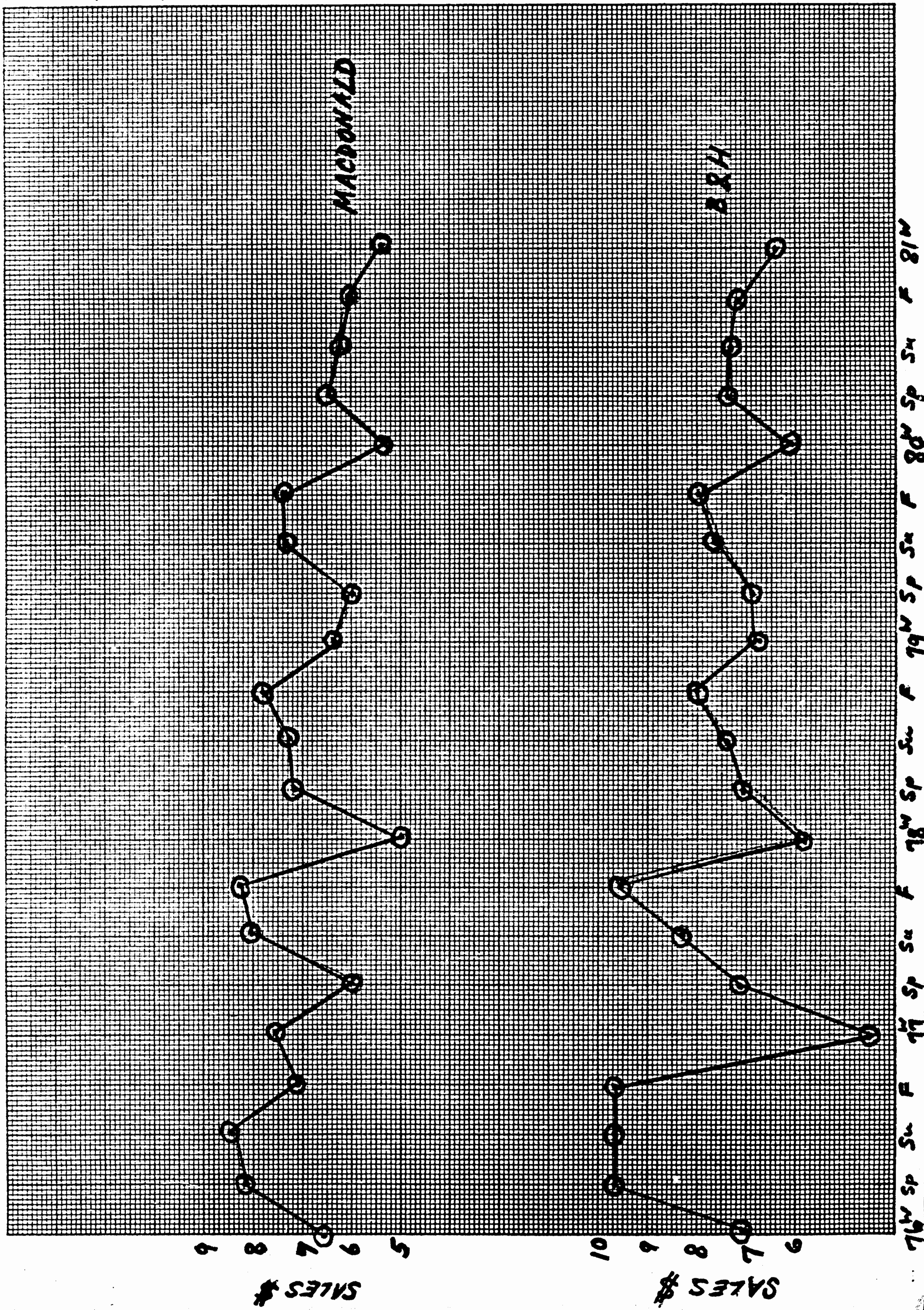
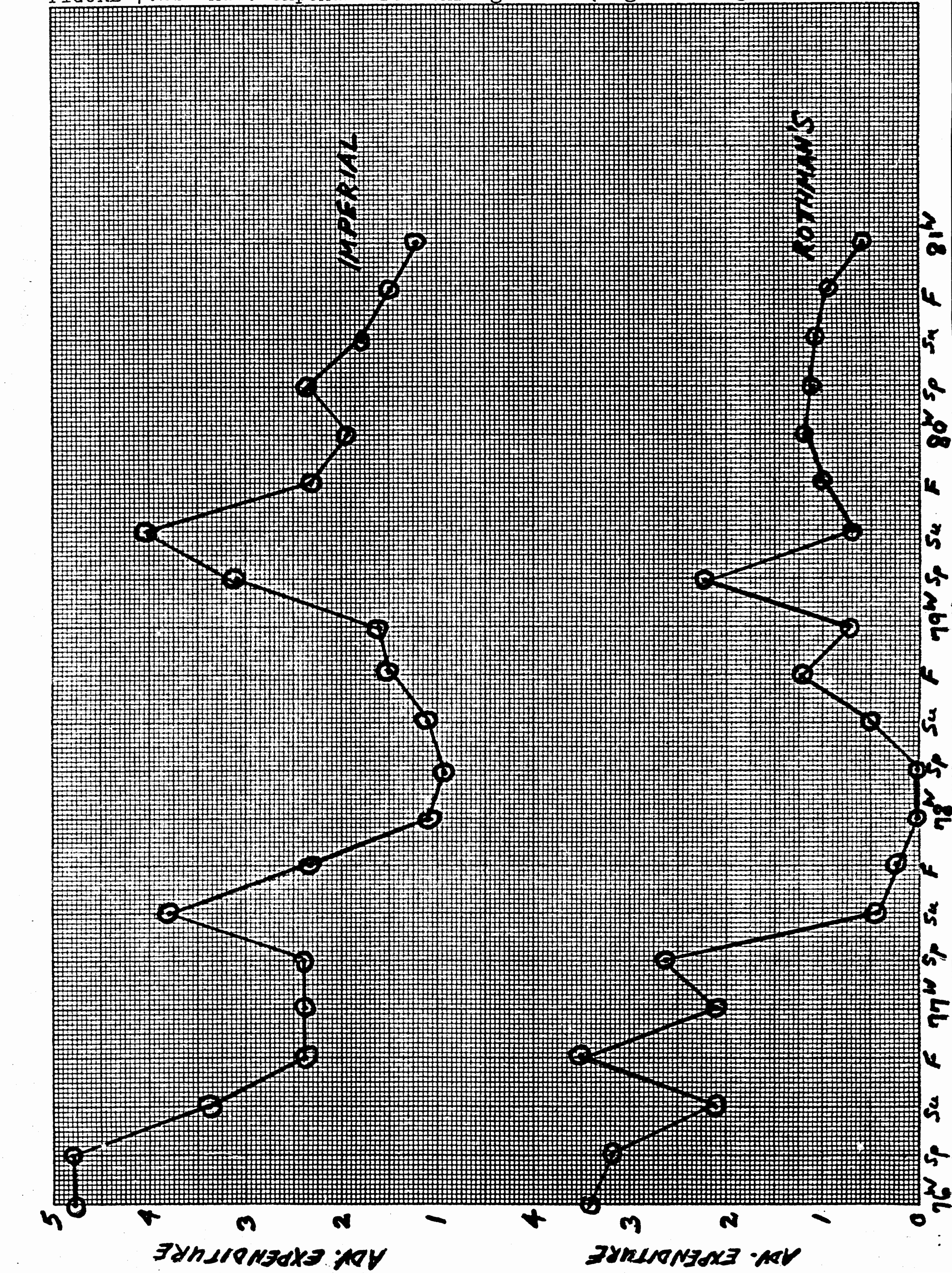


FIGURE 7.2b--Adv. expenditures through time (figures disguised).



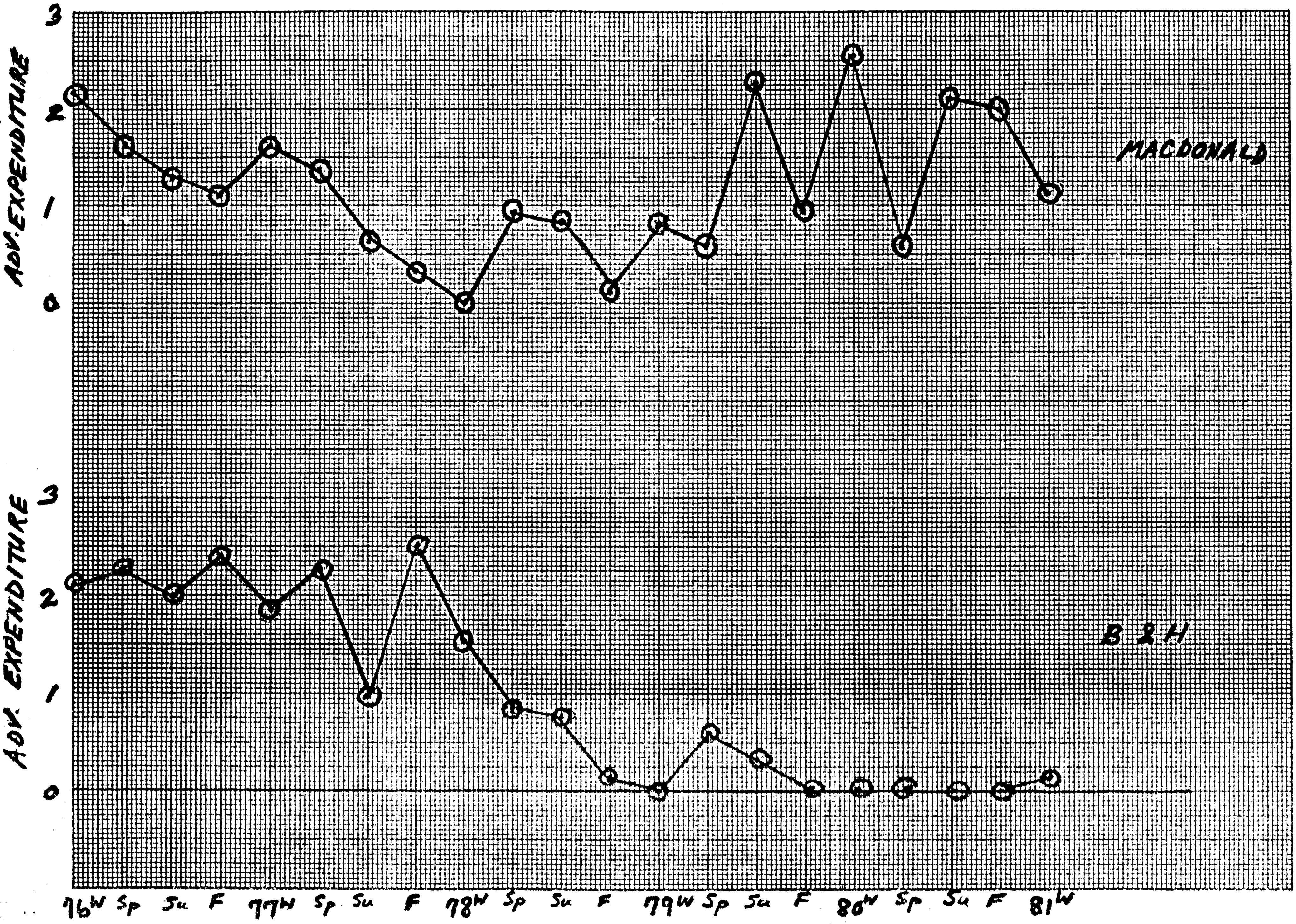


FIGURE 7.2b (continued)

DETERMINING THE ECONOMETRIC MODEL

The previous section indicated that a model of linear form had been used in the deseasonalization of the sales data. All subsequent models used in the analysis of this industry are also linear in form, as opposed to the multiplicative models used in Chapter 6. The use of linear models, as discussed in Chapter 4, has some drawbacks: the parameters cannot be interpreted as elasticities (they are, in fact, not dimensionless as are the multiplicative parameters); and the notion of scale returns is lost (the size of the exponent indicated whether there were decreasing marginal returns to scale over the typical range of independent variables). However, one major fault of the multiplicative model is that nonzero sales at zero advertising cannot be explained; furthermore, in the estimation procedure, logarithms of all variables are taken, and the logarithm of zero advertising is undefined. The advertising variables of Industry 1 were almost all strictly positive. However, as noted earlier in this chapter, zero advertising is often employed by B & H (and occasionally by Rothman's and Macdonald) and therefore a multiplicative model would not be appropriate. In addition, the linear model finally chosen had acceptable explanatory power and thus it was not thought necessary to investigate a more complex functional form to represent the observed sales-advertising relationships in the industry. The subject of adequacy of the model is returned to again later in this section.

It was hypothesized that own- and cross-advertising expenditures would have significant effects on sales. Two basic models were tested, differing only in that Model 2 incorporated a linear sales trend while Model 1 did not.

MODEL 1:

$$S_t = \alpha_0 + \alpha_1 Sp_t + \alpha_2 Su_t + \alpha_3 F_t + \beta_1 A_{It} + \beta_2 A_{Rt} + \beta_3 A_{Mt} + \beta_4 A_{Bt} + w_t; \tag{7.2}$$

MODEL 2:

$$S_t = \alpha_0 + \beta_0 T_t + \alpha_1 Sp_t + \alpha_2 Su_t + \alpha_3 F_t + \beta_1 A_{It} + \beta_2 A_{Rt} + \beta_3 A_{Mt} + \beta_4 A_{Bt} + w_t; \tag{7.3}$$

where α_i, Sp_t, Su_t, F_t, w_t = defined as before;

β_i = parameters;

A_{It}, A_{Rt}, A_{Mt}, A_{Bt} = advertising expenditures (in $100,000) of Imperial, Rothman's, Macdonald and Benson and Hedges in period t, respectively;

T_t = trend constant (set equal to period number).

Each of these models was estimated for all four firms using ordinary least-squares regression.

The overall fit of the models was good (as judged by overall-F and adjusted R-squared values): Model 2 in fact was slightly preferable in this regard. Choosing between the two models, however, necessitated a tradeoff. The correlation matrices of Model 1 showed no evidence of multicollinearity (the highest r value which appeared was 0.61 between Imperial and Rothman's advertising); whereas the trend line proved to be highly negatively correlated with B & H's advertising ($r = -0.89$). This is not surprising, as Figure 7.2 shows that B & H's advertising has been steadily decreasing through time. Now, the trend line had been shown to be significant in at least some of the cases in the preliminary analysis. In Model 1, the variance in sales which is due to underlying trend is erroneously attributed to B & H's advertising. For instance, in the Model 1 estimate for Imperial, sales appear to be positively affected by Imperial's advertising ($t = 2.53$) and negatively by B & H's advertising ($t = -4.04$). When Model 2 is estimated for the same firm, neither the newly-added trend variable nor the B & H advertising variable is highly significant (t's are, respectively, 1.02 and -1.36), which is expected due to the high collinearity between the two (neither adds much explanatory ability to the model given that the other is already included).

Neglecting the trend variable results in overestimating the significance of the B & H advertising variable; introducing it adds a measure of multicollinearity. The choice was made, however, to include the trend variable (i.e., to choose Model 2), as the preliminary analysis indicated that sales trends cannot be ignored.

The parameters estimated for each firm using Model 2 are presented in Figure 7.3, as are two measures of overall fit (overall F and adjusted R-squared) and the Durbin-Watson autocorrelation statistic. Recall that these parameters are not to be interpreted as elasticities; thus, in-depth analysis will not be attempted here. It is notable, however, that Imperial's advertising has a significant beneficial effect on both Rothman's and B & H's sales, as well as on its own. This is possibly due to its market-leader position: its advertising (which at times represents one half of the total advertising expenditure in the industry) may be strong enough to "pull along" the sales of other firms as well as its own. Note that Imperial really does not care if its advertising has this kind of generic or "pulling-along" effect, as long as its sales are increased more than its competitors' (i.e., market share is not lost). All other significant advertising effects are in expected directions.

The F-values for each firm are highly significant ($\alpha = 0.05$); adjusted R-square values range from 0.497 to 0.860. Considering that all marketing variables except advertising, and all extraneous factors such as sudden industry-wide price fluctuations, are ignored, these values are surprisingly high. Note also that in two of the four cases (Imperial and B & H), the Durbin-Watson statistics clearly indicate no autocorrelation (1.95 and 1.90 respectively). A word of explanation is needed for the somewhat high DW statistics for the other two firms (2.75 for both Rothman's and Macdonald). Recall from the earlier discussion that both Imperial and B & H sales plots (see Figure 7.1) showed a regular sawtooth pattern. Linear seasonal dummies were able to explain the regular seasonal variation very well. In the other two cases,

FIGURE 7.3

Estimated coefficients and standard errors for selected model

Coeff.	Imperial	Rothman	Macdonald	B & H
α_0	10.8111* (0.8627)	6.2457* (0.6445)	8.3988* (1.1656)	5.4286* (1.6376)
β_0	0.0499 (0.0491)	-0.0305 (0.0367)	-0.1794* (0.0664)	-0.0072 (0.0933)
α_1	2.0515* (0.3368)	1.1478* (0.2516)	0.7342** (0.4550)	1.0046** (0.6392)
α_2	2.1456* (0.3268)	1.1396* (0.2442)	1.0682* (0.4416)	1.7146* (0.6204)
α_3	2.7061* (0.3343)	1.8286* (0.2497)	1.5232* (0.4516)	2.1714* (0.6345)
β_1	0.3874* (0.1426)	0.2293* (0.1065)	0.2107 (0.1927)	0.3755** (0.2707)
β_2	-0.0782 (0.1714)	-0.0506 (0.1281)	-0.3270** (0.2316)	0.0117 (0.3255)
β_3	-0.0380 (0.2108)	0.0048 (0.1575)	0.0969 (0.2848)	-0.2541 (0.4001)
β_4	-0.3702** (0.2730)	0.0907 (0.2039)	-0.5176** (0.3688)	0.2934 (0.5182)
F	16.346*	12.110*	4.133*	3.471*
$R^2_{adj.}$	0.860	0.816	0.556	0.497
DW	1.95	2.75	2.75	1.90

Legend: Entries are estimates obtained by OLS regression for each parameter, with standard errors given beneath in parentheses.

* -- significant at 0.05 level.
**-- significant at 0.10 level.

however, seasonal variations were much more irregular; thus the linear seasonal dummies were less able to explain these sales fluctuations. Resultingly, there was more error introduced into the model for these two firms, and autocorrelation became more serious. It was decided not to implement more complex estimation procedures (such as GLS) to correct for this autocorrelation, because the OLS-estimated models have acceptable explanatory ability.

It should be noted that the model (which contains no advertising carryover effects) was compared to models with carryover incorporated as before (Koyck approach). With $\gamma = 0.9$ the F-values were almost unaffected (with only a slight improvement for Imperial); but the already-high DW statistics for Rothman's and Macdonald's were adversely affected. Also, the significances of the advertising effects were reduced in almost every case: whereas six advertising effects are starred in Figure 7.3 as being significant, only two proved significant with $\gamma = 0.9$; and these in fact were only marginally significant ($\alpha = 0.10$). Thus, the advertising effects were being confounded using the Koyck approach to model carryover. (Because of the small number of data points, it was decided not to risk too many degrees of freedom by employing the goodwill carryover approach.) Setting $\gamma = 0.8$ caused model significance to decrease even further. Thus, the no-carryover-effects model of Figure 7.3 was retained for further analysis. (Note: this does not imply that there is no carryover advertising effect in this industry. Recall that the periods used are three months long. It is not unreasonable to discover that advertising from four months back does not have a significant effect on today's sales.)

GAME MATRIX CONSTRUCTION AND SOLUTION

Ranges of advertising expenditure for each firm were taken, as had been done for the analysis of Chapter 6; midpoints were found and representative high and low advertising values were estimated. (It was felt that with four firms in the industry, the game matrix would be too unwieldy for presentation purposes if three or more levels were considered; and that the simpler matrix would contain most of the relevant information.) The ranges and representative values are given in Figure 7.4, where it is seen again that Imperial usually advertises to a larger extent than its competitors: Imperial's "low" value is approximately equal to the "high" values of both Macdonald and B & H.

As before, the payoffs of the game matrix were estimated by substituting the representative values into the appropriate regression equations. The difference in this industry is that, since seasonal effects are significant, four different game matrices may be constructed; one for each season. The resulting game matrices are given in Figures 7.5a to d. The winter game matrix (Figure 7.5a) will be solved under the different behavioural assumptions as before; then the other seasons will be discussed.

Equilibrium-Pair Solution (EQ)

It is easily shown, through analysis of Figure 7.5a, that the minimax strategies of the firms are as follows: Imperial, high; Rothman's, low; Macdonald and B & H, high (coded HLHH). If each player plays minimax, the payoffs (estimated sales dollars, multiplied by a constant factor) would be as listed in Figure 7.6.

It can also be shown that all minimax solutions possess Nash stability: no player could be convinced to change strategy regardless of the behaviour of his opponents.

FIGURE 7.4
Advertising ranges and representative values

Imperial

Low: 0.928	Cutoff value: 2.901	Repres. Low: 1.915
High: 4.874		Repres. High: 3.888

Rothman's

Low: 0	Cutoff value: 1.732	Repres. Low: 0.866
High: 3.464		Repres. High: 2.598

Macdonald

Low: 0	Cutoff value: 1.263	Repres. Low: 0.632
High: 2.526		Repres. High: 1.895

B & H

Low: 0	Cutoff value: 1.268	Repres. Low: 0.634
High: 2.536		Repres. High: 1.902

Note: all actual advertising expenditures per quarter have been multiplied by a constant for confidentiality.

FIGURE 7.5
Game matrices

(a): WINTER

Macdonald: L, B & H: L

		Rothman's L	Rothman's H
Imperial	L	12.3244 6.0305 4.3053 6.0248	12.1889 5.9428 3.7390 6.0451
	H	13.0887 6.4829 4.7210 6.7656	12.9532 6.3952 4.5147 6.7859

Macdonald: L, B & H: H

		Rothman's L	Rothman's H
Imperial	L	11.8550 6.1455 3.6490 6.3968	11.7195 6.0578 3.0827 6.4171
	H	12.6193 6.5979 4.0647 7.1376	12.4838 6.5102 3.4984 7.1579

Macdonald: H, B & H: L

		Rothman's L	Rothman's H
Imperial	L	12.2764 6.0366 4.4277 5.7039	12.1409 5.9489 3.8614 5.7242
	H	13.0407 6.4890 4.8434 6.4447	12.9052 6.4013 4.2771 6.4650

Macdonald: H, B & H: H

		Rothman's L	Rothman's H
Imperial	L	11.8070 6.1516 3.7714 6.0759	11.6715 6.0639 3.2051 6.0962
	H	12.5713 6.6040 4.1871 6.8167	12.4358 6.5163 3.6208 6.8370

Legend: Each cell shows expected sales (multiplied by a constant) for each firm as follows:

First entry: Imperial's sales
Second entry: Rothman's sales
Third entry: Macdonald's sales
Fourth entry: B & H's sales

FIGURE 7.5 (continued)

(b): SPRING

		Rothman's L	Rothman's H
Imperial	L	14.3759 7.1783 5.0395 7.0294	14.2404 7.0906 4.4732 7.0497
	H	15.1402 7.6307 5.4552 7.7702	15.0047 7.5430 4.8889 7.7905

Macdonald: L
B & H: L

		Rothman's L	Rothman's H
Imperial	L	13.9065 7.2933 4.3832 7.4014	13.7710 7.2056 3.8169 7.4217
	H	14.6708 7.7457 4.7989 8.1422	14.5353 7.6580 4.2326 8.1625

Macdonald: L
B & H: H

		Rothman's L	Rothman's H
Imperial	L	14.3279 7.1844 5.1619 6.7085	14.1924 7.0967 4.5956 6.7288
	H	15.0922 7.6368 5.5776 7.4493	14.9567 7.5491 5.0113 7.4696

Macdonald: H
B & H: L

		Rothman's L	Rothman's H
Imperial	L	13.8585 7.2994 4.5056 7.0805	13.7230 7.2117 3.9393 7.1008
	H	14.6228 7.7518 4.9213 7.8213	14.4873 7.6641 4.3550 7.8416

Macdonald: H
B & H: H

FIGURE 7.5 (continued)

(c): <u>SUMMER</u>

		Rothman's L	Rothman's H
Imperial	L	14.4700 7.1701 5.3735 7.7394	14.3345 7.0824 4.8072 7.7597
	H	15.2393 7.6225 5.7892 8.4802	15.0988 7.5348 5.2229 8.5005

Macdonald: L
B & H: L

		Rothman's L	Rothman's H
Imperial	L	14.0006 7.2851 4.7172 8.1114	13.8651 7.1974 4.1509 8.1317
	H	14.7649 7.7375 5.1329 8.8522	14.6294 7.6498 4.5666 8.8725

Macdonald: L
B & H: H

		Rothman's L	Rothman's H
Imperial	L	14.4220 7.1762 5.4959 7.4185	14.2865 7.0885 4.9296 7.4388
	H	15.1863 7.6286 5.9116 8.1593	15.0508 7.5409 5.3453 8.1796

Macdonald: H
B & H: L

		Rothman's L	Rothman's H
Imperial	L	13.9526 7.2912 4.8396 7.7905	13.8171 7.2035 4.2733 7.8108
	H	14.7169 7.7436 5.2553 8.5313	14.5814 7.6559 4.6890 8.5516

Macdonald: H
B & H: H

FIGURE 7.5 (continued)

(d) FALL

Imperial		Rothman's L	Rothman's H	Rothman's L	Rothman's H
Imperial	L	15.0305 7.8591 5.8285 8.1962	14.8950 7.7714 5.2622 8.2165	14.5611 7.9741 5.1722 8.5682	14.4256 7.8864 4.6059 8.5885
	H	15.9748 8.3115 6.2442 8.9370	15.6593 8.2238 5.6779 8.9573	15.3254 8.4265 5.5879 9.3090	15.1899 8.3388 5.0216 9.3293
		Macdonald: L B & H: L		Macdonald: L B & H: H	

Imperial		Rothman's L	Rothman's H	Rothman's L	Rothman's H
Imperial	L	14.9825 7.8652 5.9509 7.8753	14.8470 7.7775 5.3846 7.8956	14.5131 7.9802 5.2946 8.2473	14.3776 7.8925 4.7283 8.2676
	H	15.7468 8.3176 6.3666 8.6161	15.6113 8.2299 5.8003 8.6364	15.2774 8.4326 5.7103 8.9881	15.1419 8.3449 5.1440 9.0084
		Macdonald: H B & H: L		Macdonald: H B & H: H	

FIGURE 7.6

Solutions to game matrix with payoffs (winter)

EQUILIBRIUM PAIR (EQ)

Solution: HLHH

Payoffs:

Imperial:	12.5713
Rothman's:	6.6040
Macdonald:	4.1871
B & H:	6.8167

MAXIMIN THE DIFFERENCE (MD)

Solution: HHHH

Payoffs:

Imperial:	12.4358
Rothman's:	6.5163
Macdonald:	3.6208
B & H:	6.8370

SADISTIC (S) *

Solution: HHHH

Payoffs:

Imperial:	12.4358
Rothman's:	6.5163
Macdonald:	3.6208
B & H:	6.8370

JOINT MAXIMAL (JM)

Four-firm cooperative (see text)

Solution: HLLL

Payoffs:

Imperial:	13.0887
Rothman's:	6.4829
Macdonald:	4.7210
B & H:	6.7656

JOINT MAXIMAL (JM)

Three-firm cooperative (see text)

Four possible solutions **

Solution no.:	1	2	3	4
Cooperating firms:	Roth., Macd., B & H	Imp., Macd., B & H	Imp., Roth., B & H	Imp., Roth., Macd.
Solution:	HLLL	HLLL	HLHH	HLHH
Payoffs:				
Imperial:	13.0887	13.0887	12.5713	12.5713
Rothman's:	6.4829	6.4829	6.6040	6.6040
Macdonald:	4.7210	4.7210	4.1871	4.1871
B & H:	6.7656	6.7656	6.8167	6.8167

* Assumption: three smaller firms minimize Imperial's payoff (see text)

** Assumption: non-cooperating firm maximizes payoffs given the strategic choices of the three others.

Beat-the-Average (Cutthroat) Solution (MD)

As before, it was necessary to convert the game matrices to payoff-difference matrices (sales minus the average of all opponents' sales). This procedure was applied to each game matrix, and the winter payoff-differences matrix is given in Figure 7.7.

The minimax solution to this matrix is easily found to be HHHH; that is, it is identical to the EQ solution except that Rothman, under this behavioural assumption, would be convinced to switch to high advertising. As Figure 7.6 shows, all firms do worse under this more risky behavioural pattern, except B & H which shows only a very slight improvement. Like the EQ solution, it can be shown that the MD solution also possesses Nash stability.

Sadistic Solution (S)

Figure 7.8 shows the strategic combinations which result in the worst possible outcomes for each firm. Here, for three firms, no clear sadistic solution emerges which would simultaneously hurt all competitors: this is a different situation than that of Industry 1.

Imperial's advertising has a spillover effect on total industry sales. Cutting advertising hurts its own sales, though, as well as that of all competitors.

Rothman's, Macdonald and B & H do not have unique sadistic strategies. For example: for Rothman's, high advertising hurts Imperial and Macdonald sales, but B & H sales are apparently unaffected by Rothman's advertising. However, if one assumes that the three "followers" would like to "hurt" the industry leader (Imperial), then their strategies are not ambiguous. Each should choose high levels of advertising.

FIGURE 7.7
Payoff-differences matrix (winter)

		Rothman's L	Rothman's H
Imperial	L	6.8709 -1.5210 -3.8213 -1.5286	6.9466 -1.3815 -4.3199 -1.2452
	H	7.0989 -1.7089 -4.0581 -1.3319	7.1746 -1.5694 -4.5567 -1.0485

Macdonald: L
B & H: L

		Rothman's L	Rothman's H
Imperial	L	6.4579 -1.1548 -4.4834 -0.8197	6.5336 -1.0153 -4.9821 -0.5362
	H	6.6859 -1.3426 -4.7202 -0.6231	6.7616 -1.2032 -5.2189 -0.3395

Macdonald: L
B & H: H

		Rothman's L	Rothman's H
Imperial	L	6.8870 -1.4327 -3.5779 -1.8264	6.9627 -1.2933 -4.0766 -1.5928
	H	7.1150 -1.4327 -3.8147 -1.6797	7.1907 -1.4811 -4.3134 -1.3962

Macdonald: H
B & H: L

		Rothman's L	Rothman's H
Imperial	L	6.4740 -1.0665 -4.2401 -1.1674	6.5498 -0.9270 -4.7388 -0.8840
	H	6.4740 -1.2544 -4.4769 -0.9707	6.5498 -1.1149 -4.9756 -0.6873

Macdonald: H
B & H: H

FIGURE 7.8

Lowest possible payoffs (winter)

Firm i	Firm i's lowest payoff	can only be attained if competitors play as follows: (Imp.)	(Roth.)	(Macd.)	(B & H)
Imperial	11.6715	--	H	H	H
Rothman's	5.9428	L	--	L	L
Macdonald	3.0827	L	H	--	H
B & H	5.7039	L	L	H	--

When Imperial realizes that all competitors are playing high, it would make the best of the situation by also playing high (i.e., it would maximize its own sales under the circumstances). The resulting outcome (see Figure 7.6) would be HHHH--identical to that obtained as the MD solution.

Joint-Maximal Solutions (JM)

If all four firms cooperate, they maximize their joint payoffs at HLLL (this can be checked by adding the cell entries in Figure 7.5a).

Additionally, four three-firm cooperative setups are conceivable. If the excluded firm is assumed to maximize its payoff given the strategic combinations of its opponents (who are now acting cooperatively), it is possible to find three-firm cooperative solution points (these are also given in Figure 7.6). Two of these solutions (numbers 1 and 2) are identical to the four-firm cooperative solution (HLLL); the other two are identical to the EQ solution (HLHH). It is also possible to speculate what would happen under two-firm cooperative behaviour: there are six possible pairings. It can be shown, using the same behavioural assumptions as before (non-cooperating firms maximizing payoff given that the others are plotting against them), that in five of the six cases, one of the abovementioned combinations (HLLL or HLHH) would be reached.

As might be expected, the HLLL strategy (where only Imperial advertises heavily) essentially benefits Imperial's sales (Macdonald's sales are also slightly better than EQ with this strategic combination). Indeed, Rothman's and B & H would experience sales declines under this strategic combination; and therefore some kind of equalizing payment or "kickback" would have to be agreed upon among the players to convince these latter firms to join the coalition.

Solutions for Other Seasons

The game matrices of Figure 7.5, corresponding to the four seasons, differ only by constant factors among the payoffs. Thus, although the payoffs are expected to be different from season to season, the selected strategies are not affected. All the solutions obtained for the winter season remain unchanged for the other seasons; and those solutions (EQ and MD) which benefitted from Nash stability in winter are also Nash-stable during the rest of the season.

ACTUAL BEHAVIOUR IN THE INDUSTRY

Under all behavioural assumptions, then, high advertising strategies are predominant. This is perhaps as expected: in Industry 1, there was possibility of tradeoff between price and advertising (e.g., "is it better for our firm to support low price with high advertising or are we better off at high price levels and just sufficient advertising?"). In Industry 2, only advertising is used as a decision variable. Thus, under the reasonable assumption that each of the four firms can afford to hire a good ad agency to prepare its ads, it is expected that "more advertising" should generate more sales than "less".

An exception to this rule, of course, would be the situation where one firm is overadvertising, to the point where additional advertising has a minimal effect (or even a negative effect) on sales. (Recall that "low" and "high" are defined for each firm according to the range of advertising expenditure actually employed by that firm over the period being studied.)

What is observed in the industry, however, does not correspond to what is recommended by game matrix analysis. The plots in Figure 7.2 indicate that, with the exception of a few high-advertising periods in later years by Macdonald, the firms have been steadily <u>reducing</u> their advertising levels over the duration of study. Indeed, if one uses the high/low cutoff values of Figure 7.4 to classify the strategic choices of the four firms through time, it is seen that the firms rarely choose "high" after the end of 1977. Why should the firms choose to cut back in advertising "across the board" on regular cigarettes when advertising is supposedly beneficial to sales (and the choice of high advertising strategies is strongly supported by game-matrix analysis)?

Quite simply, some important external variables have been ignored in this analysis. The seventies were marked by rising opposition to smoking in general and to cigarette advertising. There are at least three factors for this change in public opinion. Firstly, medical associations have for many years warned about the health risks of smoking. Canadian Medical Association representatives made statements to this effect in 1947, as did the British Medical Association in 1956 (see Carroll 1977 and Van Steen 1976 for details). The American Surgeon General's report of 1964 gained more publicity and resulted in warnings placed on cigarette packs. Despite an increased level of health consciousness, however, the health-risk factor might not have been enough to cause the change. Governments tax smokers heavily (smokers in the Province of Quebec, in fact, pay 16 cents a pack, partially to defray cost overruns of the 1976 Olympics).

The third and perhaps most important factor is the social acceptability of cigarette smoking. Van Steen (1976) quotes an American publication (United States Tobacco Journal) which states:

> "If smokers can be made to feel guilty if they do something frowned upon in certain situations, they are less likely to do it...People who smoke and enjoy it are beginning...to enjoy it less and less."

By the mid-'70's, the cigarette manufacturers were admitting they could "do virtually nothing to combat the rising public perception that smoking is anti-social" (Carroll 1977).

This anti-smoking trend has not abated: as recently as 1983 and 1984, the winner of a major Canadian ski competition refused his trophy as a protest against the sponsorship of the sporting event by Macdonald cigarettes; the Canadian Pharmaceutical Association is attempting to halt the sale of cigarettes through pharmacies--including Shoppers Drug Mart (Pharmaprix) owned by Imasco, which controls Imperial Tobacco; and all cigarette manufacturers have been criticized for posting outdoor advertising in close proximity to high schools.

The study duration (1976 to 1981) was, of necessity, a period of great change in the attitudes of the cigarette manufacturers towards the advertising of their products. By the mid-seventies, sales of regular cigarettes were levelling off and the struggle for market shares intensified (see Clifford 1976; Carroll 1977). Additionally, the manufacturers were having difficulty introducing new brands to the marketplace: none of the 34 new (regular) brands introduced between 1970 and 1976 was successful (the industry standard for success being the achievement of a 0.5% market share in one year; further discussion in Cotter 1977 and Smyka 1979). An industry marketing executive suggested in Clifford (1976) that the television advertising moratorium may have lessened the effectiveness of industry advertising for totally new brands. Introducing light versions of already-well-known brands was a less difficult task: of the first 20 light brands introduced since 1976, 13 were successful (Smyka 1979). The observed industrywide decrease in advertising for regular cigarettes is explicable as a logical result of the manufacturers diverting their advertising dollars towards the "growth" product class of light cigarettes.

The cigarette manufacturers may have also felt they were risking even more public outcry by advertising the regular brands heavily in the face of mounting opposition. Rather than fuelling a "bad guy" image, these firms may have adopted a more low-key approach in the advertising of their products. It is a matter of speculation whether the observed advertising cutbacks were made jointly, or if the firms decided independently at about the same time to reduce advertising intensity for regular cigarettes. Certainly one cannot conclude from the data used in this study that any joint decisions had been taken. Additionally, one can visualize falling public opinion to have the opposite effect on advertising: the firms may feel pressure to increase advertising outlay in the face of declining popularity of their products. Evidently the results of this analysis indicate that this was generally not the case in this industry. The failure of the game-theoretic model to anticipate the preference of the cigarette manufacturers for low advertising in later years is due to (a) its inherent assumption that public opinion (like government regulation and economic factors) is assumed to be constant, or to have an insignificant effect on sales, and (b) its neglect of the rise in the importance of the low-tar market, with resulting diversion of advertising funds away from regular brands.

As an aside, note that the use of advertising shares rather than dollars as the independent variables (to account for the industry-wide cutbacks in advertising level) would cause estimation problems. For game-matrix construction purposes, the advertising levels of all four firms must be used as decision variables in the regression model. When shares are used as the advertising variables, they add up, by definition, to 100%--resulting in <u>perfect</u> multicollinearity among the decision variables.

CHAPTER 8
CONCLUSION

A BRIEF REVIEW

Chapter 1 indicated the desirability of developing a general game-theoretic model of industry strategies and payoffs. This model would ideally be capable of: describing the nature of the industry in terms of expected payoffs given past behavioural combinations; evaluating the strategies actually chosen knowing which strategies are preferable, given the intentions and desires of the players involved; and recommending courses of action for the future.

Chapters 2 and 3, in reviewing the game-theory and econometric analysis literature, highlighted important theoretical model-building considerations. Then, Chapter 4 laid the theoretical groundwork necessary for the empirical analyses conducted subsequently. The model developed was quite general in nature and, as such, quite flexible. In theory it would be adapted to much more complex industries than those chosen, although obvious data and estimation difficulties would arise if the modeller attempted to "capture the entire world" within the confines of a n x n game matrix.

After Chapter 5 discussed some of the major issues in model application, the theoretical model was adapted to analysis of two rather different industries. In Industry 1 (Chapter 6), the industry sales patterns appeared to be highly affected by the pricing and advertising decisions taken by the firms, and the results of game analysis showed that (a) "preferred" strategies are already somewhat favoured in the industry (an evaluation of past or historical strategies) and (b) Firm B could benefit by cutting advertising expenditures (a recommendation for future strategic choice). For Industry 2 (Chapter 7), it was shown that high-advertising strategies (generally preferred under the behavioural assumptions) were seldom chosen--but this may have had to do with a neglected third variable, a decline in public opinion which characterized the cigarette industry during the mid- and late-'70's.

APPLICABILITY AND LIMITATIONS OF MODEL

The core of this study is Chapters 4 and 5, wherein the general theoretical model is developed and important issues in operationalizing the model are discussed. A discussion of model relevance and limitations should therefore consider separately the theoretical model of Chapter 4 and its operational equivalents, described in Chapter 5 and implemented in Chapters 6 and 7.

Theoretical Model: Applicability

As developed in Chapter 4, the theoretical model places no limit on the complexity of the industry it can handle. It is purely a mathematical model, designed for the most general case: taken to the extreme, given an infinite supply of data, any number of parameters (the matrix entries in the equations near the end of the chapter) may be estimated.

However, even with such mathematical flexibility, there are still only specific circumstances in which the theoretical model is applicable. Some of these situations have been already discussed. For example, the model has been developed for an n-firm <u>oligopoly</u> (preferably a <u>stable</u> industry; see upcoming discussion). At this point, an operational upper limit on n has not yet been set.

The model also makes the major assumption that the players are acting "rationally". Of course this does not refer to the narrow, maximin definition of rationality employed by von Neumann and Morgenstern: maximin, sadistic, beat-the-average (and even cooperative, where applicable) behaviours are all "rational" in that each is conceivable from the viewpoint of individual players, and each would result in specific courses of action. "Irrational" behaviour would be inexplicable using game-theory analysis.

Two smaller assumptions are also implicit in the theoretical model of Chapter 4. The firms are assumed to be free to set their desired levels for each of the relevant independent variables; and feedback on competitive actions is assumed to be available. The theoretical model is applicable where these assumptions hold: if the firms do not know what their competitors' behaviours have been (even if this knowledge is delayed), or if they are restricted in their setting of, say, advertising expenditures (more on this later), then they cannot respond appropriately to competitive action.

<u>Theoretical Model: Limitations</u>

The first stage of the theoretical model comprises the econometric modelling of the industry. Although econometric limitations are dealt with in the Operational Model section, it should be noted here that the theoretical model assumes that one can develop an adequate model of the industry. This may be difficult where random (i.e., unmodellable) effects are too large.

The theoretical model also assumes away the existence of "hidden internal constraints" (see Chapter 5). If firms do not have free choice on independent variable selection (i.e., if there is a simultaneous sales-advertising effect or if a specific dollar amount is spent on advertising every year regardless of sales), then the game matrix which is developed may have little practical usefulness. A firm may not be permitted to advertise at levels recommended by the analysis. In the case of significant simultaneous effects, the simple, general Cobb-Douglas equations would have to be complemented by additional equations representing advertising behaviour as a function of past sales. These equations would then be solved simultaneously as in Bass (1969).

The model, as developed, would not be directly applicable in oligopolies where competitive actions are not known until it is "too late" (e.g., sealed-bid auctions). In this situation, firms may make educated guesses as to the likely behaviour of their competitors. The Future Research section discusses a possible sealed-bid analysis: however, the theoretical model as developed in Chapter 4 would be insufficient for this application.

Operational Model: Applicability

The stated objectives of the model--description, evaluation and recommendation--are practical in nature. Otherwise put, although a general theoretic model had been constructed, it was still essential to demonstrate its application in real settings. In doing so, some practical issues arise.

The mathematics of the theoretical model had been basically unrestricted. Given a large enough data set, the parameters of even the largest matrices of Chapter 4 could theoretically be estimated. In reality, of course, data is always restricted to some extent and limits on parameter estimation must be reckoned with. A related problem concerns the number of firms in an industry which realistically could be treated using this game-theoretic framework: limits on model complexity need also be considered.

For example, the maximum number of firms which can be handled is probably not much more than about four or five. Although higher-dimensional game matrices could be constructed theoretically, the level of complexity might limit the interpretability of the results. In particular, it may be unreasonable to assume that all firms in a large industry have adequate data-analysis techniques (or even have all the industry data) available. However, as seen in Chapter 6, it is possible to combine the smaller firms into an "Others" brand which the major firms must consider when making their strategic choices.

It is also essential that adequate econometric models of the industry may be constructed ("adequacy" being judged using standard econometric regression statistics). The model is most easily operationalized where a "reasonable" number of independent variables can describe adequately the sales response in the industry. The two industries chosen for study in this paper were, fortuitiously, easily modelled using a minimum of marketing-mix variables (relative price and advertising levels). Some industries may be far more complex and would require many more criterion variables for adequate model fit.

The model is applicable even in situations where non-marketing variables (e.g., uncontrollables like inflation or economic indicators) affect sales, as long as these variables are included in the model. This in fact was seen in Chapter 6, where uncontrollable variables appeared to affect industrywide sales levels and were thus factored out.

Operational Model: Limitations

The above considerations suggest a number of possible operational limitations to the model; some of these are explicitly considered below.

Clearly if data availability is restrictive, degrees-of-freedom problems would preclude estimation of more than a few parameters at a time. Indeed, the Koyck approach to modelling carryover effects is provided as an alternative to the goodwill approach, which would require separate estimation of parameters for each lag period. The degrees-of-freedom considerations with regard to choice between these two models have already been discussed.

The model is probably most suitable for a stable industry (i.e., mature industry with no new competitors or radical innovations). Discussions in Chapter 5 indicated that the theoretical model was flexible enough to allow for both changes in objectives through time and divergent objectives across time. However, it may be difficult to develop useful game matrices for changing objectives through time, due to operational considerations. At what point, for example, does the strategic change take place? (Discussion with company representatives may be able to identify this time with some accuracy.) Furthermore, data requirements would be heavier (should data from the "market-growth" stage, e.g., be combined with that of "maturity", or should two separate matrices be constructed?)

Another limitation of the model involves the implications of strategic recommendations based on historical strategic choices. It was mentioned (in the discussion of Firm B's advertising levels in Industry 1) that one has no way of knowing what would have happened to Firm B's sales had it been constantly advertising at lower levels or had it ceased advertising altogether for any length of time. Under experimental conditions, it may have been possible to test many strategic options and combinations. Using a real-life situations, one is limited by the <u>range</u> of strategic choice employed by the players. The inability of the researcher to find a significant correlation between advertising and sales for Firm B may indicate that the firm is advertising above its threshold limit; but what that limit is, and how sales would be affected below that limit, cannot be determined from the data as given. Nevertheless, the results did yield the useful conclusion that advertising expenditure could be reduced somewhat without seriously harming Firm B's sales level. (Firm B knows, however, that maintaining high advertising expenditure levels protects its product from competitive attack, and thus can justify its advertising strategy.)

Other limitations, usually applicable in econometric analyses, also apply here. One of the most obvious concerns the use of advertising expenditure level (in dollars) as a measure of advertising. In making this simplification, quality and comparative effectiveness of different media or different vehicles are ignored. Additionally, firms may be making minor (or even drastic) changes in their advertising policies, which would not be captured by an expenditure-level variable. For example, when government regulations forced the cigarette advertisers off the television airwaves, the firms undoubtedly invested more in other media, but the overall size of the advertising budgets may not have been affected. If the "replacement" media were less successful in influencing sales than television, a sales decrease would have been observed which could not be accounted for by advertising expenditure level.

Leaving aside these limitations, however, the model was designed to recommend strategic options which would be preferable to the game participants. The deterministic sales payoff estimates used to make these recommendations may contain some margin of error themselves--but it was not intended that these estimates be infallible. Predictions of future sales based on past records might, under some circumstances, be weak--especially given that major new competitors or economic-climate changes may occur. But, other things being equal, small deviations between actual and expected sales would, in most cases, not change the marketing-mix strategies recommended by this analytic technique--and this is an acceptable situation, as recommendation (and not prediction) is the relevant and desired goal.

VALIDITY OF MODEL

A discussion of model validity is called for. The reader is undoubtedly aware of numerous definitions and classifications of validity, so any such discussion must be preceded by careful definition of appropriate terms.

Parsons and Schultz (1976) propose a particularly useful typology of validity pertaining to the building and implementation of empirical decision models. They begin with the concept of a successful model, defined as "one which adequately represents the phenomenon being modelled and is used for the purpose for which it was designed,... (that is,) to make effective decisions" (1976). They suggest that the probability that a model being successful is a function of its technical validity and its organizational validity. It is implied that an empirical model has two aspects of validity, and if the model scores well on both aspects, it has a higher probability of being successful.

Technical validity is the model's "capability of providing some solution, usually an optimal one, to the stated problem". Technical validity may be measured "by the degree to which the model optimizes and the degree to which the model represents the decision situation" (1976). These two concepts pertain to, respectively, the "correctness of (the) representation", and the "closeness with which the model approximates the real market" (1976). According to the industry experts mentioned in Chapters 6 and 7, the model has succeeded in evaluating the market situations and (for Industry 1) in recommending appropriate courses of action. Additionally, "simple" industries (i.e., where a small number of decision variables were capable of explaining a great deal of the sales variance) were chosen, to minimize the occurrence of "spurious correlation between the model's output and the real world"-- a possible consequence of incorrectly specifying a model (1976). This is a specification and measurement problem which was discussed in greater detail in Chapter 5, during the discussion of dependent and decision variable choice.

But a technically-valid model may still not be a successful or useful one. Parsons and Schultz define organizational validity as "(the model's) fit to the organization in terms of structure and behaviour"(1976). Although they intended this term to denote attitudes of employees, flexibility of organization to strategic change, etc., a more narrow aspect of organizational validity will be examined here: that of the intentions of the firms themselves.

Of course the firms are made up of groups of individuals. But even leaving aside the issue of differences of opinion among managers, each firm can still "behave" in different ways, Some of these have been specifically considered herein, as the possible game solutions obtained under different behavioural assumptions. One of the advantages of the model, in fact, is that none of the behavioural assumptions is presumed to be prevalent in the industry; but rather that any one is possible (as are any combinations thereof). The model as presented explicitly considers major noncooperative behaviours which may be likely to occur in real life; and can also speculate as to what would happen under various assumptions of cooperation or collusion (even where such behaviour is unlikely or illegal). So, as far as organizational validity is concerned, the model may be adapted to the structure of, and to the behaviour of the players in, the industry under question.

CONTRIBUTION OF STUDY

A general, flexible theoretical model had been constructed which, when applied to a given industry, would be capable of evaluating past decisions made by the competing firms and of making sound recommendations for the future. Use of a game-theoretic framework allowed the determination of optimum strategies under different behavioural assumptions. The theoretical model (Chapter 4) is developed from basic game-theoretic and econometric techniques. Its flexibility makes it extremely adaptible and therefore useful to marketing managers (given that it is appropriately applied to the industry; see Chapter 5). Its practical usefulness to management is enhanced by its ability to describe the industry in simple terms, to evaluate past strategic choices, and to make recommendations for the future.

The study continues the recent trend towards quantifying the field of marketing. Great headway has been made in the last twenty years by some researchers in the application of econometric techniques to marketing; and over the same span, other authors have concentrated on the game-theoretic modelling of the oligopoly and its ramifications for strategic choice. Both sources of inspiration for this paper (econometric analysis and game theory) may be considered branches of economics. Indeed, the importance of economic considerations within a marketing framework cannot be overemphasized, in this author's opinion: an understanding of the competitive aspects of the industry is essential for proper marketing planning and implementation (e.g., new product development; advertising strategies; allocation of funds across product lines; etc.). Professional marketers can profitably borrow ideas and concepts from both psychology and economics.

The major theoretical contribution of the paper is a synthesis of econometric techniques and the game-matrix framework, culminating in an all-encompassing model theoretically capable of dealing with quite complex industries. On the way to the construction of the theoretical model some additional contributions may also be noted: (a) the estimation of game-matrix payoffs by considering combinations of strategic choices (this concept circumvents the problem of potentially sparse cells which would cause estimation difficulties if payoffs of each cell had to be determined individually); (b) the inclusion of carryover effects of independent variables, as well as current effects, as strategic variables for the game matrix; (c) the consideration of different carryover models including two (Koyck and dynamic-adjustment) which contain an informative parameter (δ), obtained through an incremental optimization procedure.

The practical contributions of the study are threefold and correspond to the desires expressed in Chapter 1: the model, when properly applied, describes the industry in convenient game-matrix form, evaluates different strategic options available to each firm, given its intentions, and makes recommendations as to desirable strategies to follow and likely consequences. The practitioner may prepare a detailed analysis similar to those of Chapters 6 and 7, in developing the marketing strategies for the next year; or may construct only a simple game-matrix representation in order to discover patterns (if any) between his firm's actions, his competitors' actions, and the fortunes of his firm (i.e., what seems to cause sales to rise? When have we appeared to lose market share? Are these questions answerable by investigating strategic combinations?)

The contributions (both theoretic and practical) of this study must be tempered by consideration of its limitations, which have been discussed earlier in this chapter.

AREAS FOR FUTURE RESEARCH

One of the achievements of marketers has been to employ high-level mathematical techniques in their analysis of marketing situations. Aided by computer packages, the marketer employs multiple regression, factor analysis and the like almost routinely. Although out of the scope of this paper, the author can visualize these additional settings for the theoretical model,and improvements in its operationalization:

1. The theoretical model has already been seen to be ill-suited to the sealed-bid situation (see earlier section of this Chapter). However, it would be possible to make adjustments. Based on past performance in bidding situations and perceived risk aversity, competitive behaviour may be approximated. Information on competitive risk-taking would be useful in developing a probability

distribution of possible strategic choices for each opponent: this information could then be used in choosing a strategy which would optimize the expected payoff to the firm.

2. The differential-games approach (used by other writers such as Tapiero 1979) was not employed in this study (see discussion in Chapter 4). Differential games were not ruled out because they were inappropriate: however, the game which would have been constructed would have become too unwieldy and complex for easy analysis. Furthermore, a knowledge of the optimal-control literature would probably be necessary for both the construction and the interpretation of the game matrix. Nevertheless, the payoff function represented as a hypersurface in five-dimensional space is, of course, not impossible in theoretical mathematics. Future research may also be directed at developing differential game-matrix representations of industries, and solving them by optimal-control methods.

3. Finally, stochastic extensions may be developed. The game-matrix entries are, evidently, point estimates, which are probably sufficient for decision-making, comparative purposes. By replacing these point-estimates with confidence intervals, the researcher would be able to determine which inter-cell differences were significant. Casting the whole model into stochastic form is, also, mathematically possible, but not within the scope of this paper.

The (unchanged) theoretical model of Chapter 4 could also be subject to additional tests in its present form. The goodwill model of carryover was tested for Industry 1 and rejected; while the dynamic-adjustment carryover model remained untested in the empirical sections. Future applications to other industries with different decision-making structures could make use of these alternate approaches to determine their usefulness.

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