McGill University
Department of Civil Engineering and Applied Mechanics

STATE OF THE ART OF CABLE DYNAMICS IN APPLICATION TO GUYED STRUCTURES

Structural Engineering Series Report

STENG-2013-01

By
Victor Egorov

May 2013
Montréal, Québec

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2.1 Early inextensible cable theory development

Cables are widely used structural elements because of their mechanical efficiency. Their physical and mechanical properties, such as light weight, flexibility, high tension resistance, make them appropriate choice for different engineering applications such as cable-stayed and suspension bridges, rope-ways, modern guyed masts for power transmission lines and towers for the telecommunication industry, offshore and mooring structures (e.g. oil platforms, drilling rigs). Therefore, cable static and dynamic behavior has been an object for detailed and extensive studies for more than a century. Various cable problems have been already looked at since times of Renaissance in Europe and these have become a base for the following development of cable mechanics. Sketches of Leonardo da Vinci [1] from the fifteenth century have described the catenary notion. Stevin [2] in 1586 has performed experiments with loaded strings. Studies accomplished for suspension bridges have stated that a cable hangs in a parabolic arc [2]. In contrast to this view, the Bernoulli brothers (James and John), Leibnitz and Huygens, more or less jointly, have concluded that a hanging cable has a shape of catenary (or chainette) [3]. The first studies of vibrations of a taut string were presented in treatises by Brook Taylor, d'Alembert, Euler, Johan and Daniel Bernoulli during the first half of the eighteenth century. In 1732 D. Bernoulli has performed the analysis of the transverse oscillations of a uniform cable, supported at one end and hanging under gravity. A few decades later, the same problem has been examined by L. Euler. Both Bernoulli and Euler have given the natural frequency solutions studied previously but in the form of infinite series. At that time, considerable work had been focused on the analysis of discrete systems. In 1788 Lagrange et al. [4] have developed solutions for the vibrations of an extensible, massless string, fixed at each end, from which numerous dead weights were hung. Such an arrangement of hung masses represented the cable continuum under uniform gravity loading.

In 1820 Poisson [5] has developed the general partial differential equations of the motion of a cable element under the action of general force system, thus, making a milestone contribution to the theory of cable vibrations. Poisson has used these equations to improve the solutions previously obtained for the vertical cable and the taut string. Therefore, by 1820, correct solutions had been obtained for the linear free vibrations of uniform cables having the limiting static form of a catenary. However, apart from Lagrange's study on the equivalent discrete system, solutions for cables with sag were not known at that time. In 1851 Rohrs [6], in collaboration with Stokes, has given an approximate solution for the symmetric vertical vibrations of a uniform suspended inextensible cable with a small sag-to-span ratio. The solution had been obtained by using a form of Poisson's general equations, correct to first order and, in addition, another equation had been used to describe the continuity of the chain: The chain was assumed to be inextensible, so the continuity equation related only to geometric compatibility. In 1868 Routh [7] has given exact solutions for an inextensible heterogeneous sagging cable, which hung in cycloid. These solutions have described the symmetric vertical in-plane vibrations (and associated longitudinal motion). Like Rohrs before him, Routh assumed the cable to be inextensible and has demonstrated that his results for the cycloidal cable reduced to Rohr's solution for the uniform cable when the ratio of sag-to-span was small. Routh has also obtained an exact solution for the anti-symmetric modes (and associated longitudinal motion) of the cycloidal cable (See Figure 1). The equations of motion proposed by Routh have the general form

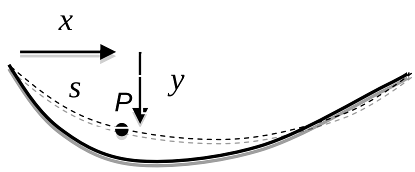

Figure 1. Inextensible chain in motion

$$\frac{\partial^2 \zeta}{\partial t^2} = \frac{1}{m}\frac{d}{ds}\left(T\frac{d\xi}{ds} + U\frac{dx}{ds}\right),$$
$$\frac{\partial^2 \eta}{\partial t^2} = \frac{1}{m}\frac{d}{ds}\left(T\frac{d\eta}{ds} + U\frac{dy}{ds}\right),$$

Where x, y are rectangular coordinates of any point P on a suspension chain when hanging in equilibrium, x being measured horizontally; s is a distance along the arc of the chain to *P* from some convenient origin; m is a mass per unit length of the chain at P. When the chain is executing a small oscillation, *P*, at time t, takes coordinates $x+\xi$ and $y+\eta$ with U being an increase in tension noted by T. If the chain is inextensible, its geometry at any instant must conform to a third equation

$$\frac{dx}{ds}\frac{d\xi}{ds} + \frac{dy}{ds}\frac{d\eta}{ds} = 0.$$

2.2 Modern cable dynamics theory

After Routh's contributions, the subject has not got any further development until 1941 when Rannie and von Karman independently obtained natural frequencies for both the symmetric and anti-symmetric in-plane modes of an inextensible three-span cable [8]. In 1942 Kloppel and Lie [9] for the first time have considered the cable elasticity. In 1945 Vincent [10] has further developed the work by Rannie and von Karman including the effects of cable elasticity on the symmetric modes. However, he had not fully explored the nature of the obtained solutions from an engineering perspective.

In 1950 Bleich et al. [11] have developed a comprehensive study on the theory of vibration in suspension bridges, in which consideration has been given to the effects of cable elasticity. Since the authors of the theory were concerned with long span cables with large ratios of sag/span, the effects of cable elasticity were not appreciable compared to geometric effects. Pugsley's [12] semi-empirical theory for the natural frequencies of the first three in-plane modes was presented in 1949. He has demonstrated the applicability of the results by conducting experiments on cables with deep profiles in which the ratio of sag/span ranged from 1:10 up to about 1:4. In 1953 Saxon and Cahn [13] have presented theoretical solutions for cables with significant sag by assuming again that the cable was inextensible. They have obtained solutions that effectively reduced to the previously known results for inextensible cables of small sag/span and for which asymptotic solutions gave excellent results for large sag/span ratios. Goodey in 1961 in [14] has also investigated deep-sag cables, but by different methods. According to Goodey, the element PP^* undergoes the motion in the principal form with the shape $C = A\sin\omega t$, where A is the arbitrary constant (See Figure 2), ω is the angular frequency. If $x + \xi\sin\omega t$, $y + \eta\sin\omega t$ are the coordinates of P at the time t, s is the arc of chain measured from the chain midpoint and $\psi = \pm\alpha$, the solution for the frequency of the odd modes has the following form

$$\lambda = \frac{n\pi}{p_\alpha}\left\{1 - \frac{(q_\alpha + p_\alpha)p_\alpha}{n^2\pi^2} + O\left(\frac{1}{n^4}\right)\right\},\ n = 1,2,..$$

And for the even modes

$$\lambda = \frac{\left(n + \frac{1}{2}\right)\pi}{p_\alpha}\left\{1 - \frac{(q_\alpha + \sigma_\alpha)p_\alpha}{\left(n + \frac{1}{2}\right)^2 n^2} + O\left(\frac{1}{n^4}\right)\right\},\ n = 1,2,..,$$

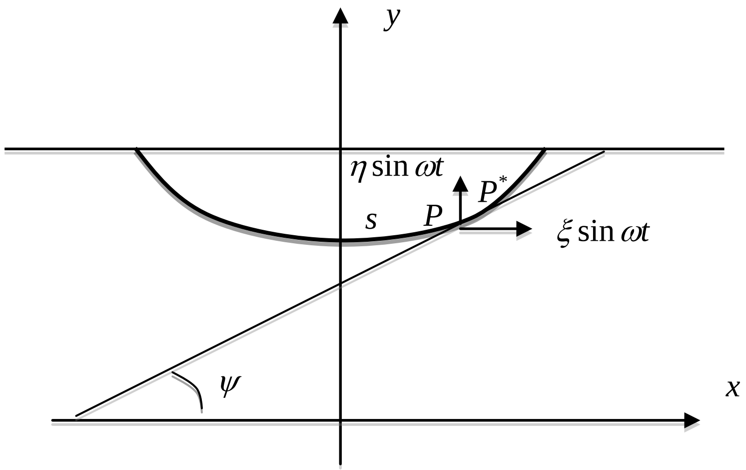

Figure 2. Suspended chain

Where $\lambda^2 = \omega^2 cg$, $c = (1/2) L \cot \alpha$, g is the gravitational force.
p_α , q_α are evaluated as

$$p(\psi) = \int_0^\psi \sec \psi d\psi \ ,$$

$$q(\psi) = \int_0^\psi \frac{1}{8}\left(13 - \frac{3}{4}\tan^2 \psi\right) \cos \psi d\psi \ .$$

σ_α is expressed as

$$\sigma_\alpha = \left(1 - 1/4 \tan^2 \alpha\right) \cos \alpha \cos ec \alpha \ .$$

2.3 Linear Elastic Cable Theory proposed by Irvine and Caughey

However, until the 1970s practically no experimental work had been done to validate the proposed theories that displayed remarkable discrepancies between the natural frequencies of the symmetric in-plane modes of an inextensible sagging cable and those of a taut inextensible string. In this perspective, as shown by Irvine and Caughey in 1974 [15], a proper description of that transition range requires the consistent inclusion of cable elasticity. Their work has revealed an extensive comprehension of the linear theory of free vibrations of a rigidly supported horizontal cable (level span with fixed ends) with a ratio of sag/span from approximately 1:8 to zero (perfectly straight). Their fundamental assumption was that the dynamic cable tension is a function of time alone (i.e. the elastic deformation is assumed to be quasi-static). On that basis, the authors have demonstrated that the dynamic behavior of an elastic cable essentially depends on only one geometric-elastic parameter, now well-known as Irvine's λ^2 elastic cable parameter. The parameter is expressed as

$$\lambda^2 = \left(\frac{mgl}{H}\right)^2 \Big/ \left(HL_e/EA\right),$$

Where mg is the self-weight of the cable per unit length; H is the horizontal component of cable tension; l is the distance between the supports (span length) and L_e is the length of the cable.

Following Figure 3 from [15]

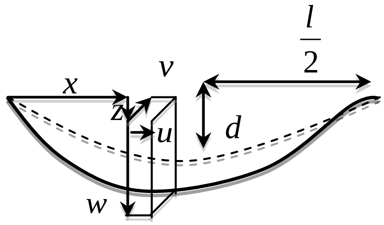

Figure 3. Components of displacement in displaced profile

the equations of motion obtain the form

$$\frac{\partial}{\partial s}\left\{(T+\tau)\left(\frac{dx}{ds}+\frac{du}{ds}\right)\right\} = m\frac{\partial^2 u}{\partial t^2},$$

$$\frac{\partial}{\partial s}\left\{(T+\tau)\left(\frac{dz}{ds}+\frac{\partial w}{\partial s}\right)\right\} = m\frac{\partial^2 w}{\partial t^2} - mg,$$

$$\frac{\partial}{\partial s}\left\{(T+\tau)\right\} = m\frac{\partial^2 v}{\partial t^2},$$

Where u, w are the longitudinal and vertical components of the in-plane motion, respectively, v is the out-of-plane, or swinging component, and τ is the additional tension generated. All are functions of both position and time.

For certain values of this λ^2 parameter, the so-called "crossover" points, the natural frequencies of the symmetric in-plane modes and the respective anti-symmetric in-plane modes coincide. (See Figure 4) This study is considered a milestone in cable vibration theory since it has become the base for an extensive following research. Later on, Irvine has extended his theory to inclined cables [16]. However, in this paper, the weight component parallel to the cable chord has been neglected. Jointly with Griffin [17], Irvine has performed the analysis of cable response to dynamic loading as it occurs in the case of support acceleration due to earthquake.

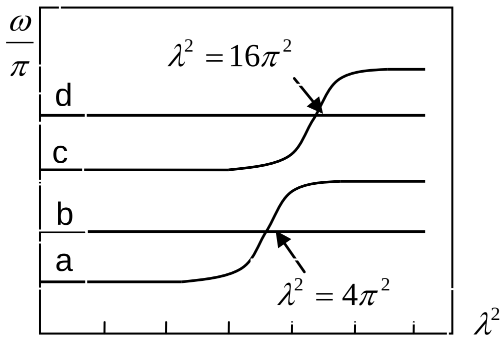

Figure 4. General dimensionless curves for the first four natural frequencies of a flat-sag suspended cable: (a) first symmetric in-plane mode, (b) first antisymmetric in-plane mode, (c) second symmetric in-plane mode, (d) second antisymmetric in-plane mode.

A more precise solution for the free vibrations of an inclined extensible cable has been developed by Triantafyllou in 1984 [18]. For this, the spatial variation of the dynamic cable tension and the weight component parallel to the chord have been duly taken into consideration. As a result, for the frequency curves of inclined cables as a function of λ^2 the "cross-overs" have been replaced by an "avoided crossing" (i.e., nearly a "cross-over", (see Figure 5), while the two modes of nearly coinciding frequencies become hybrid modes, a mixture of symmetric and anti-symmetric shapes, with a significant effect on the dynamic tension. Based on the developed general asymptotic solution, Triantafyllou and Grinfogel in 1986 [19] have proposed simple expressions for natural frequencies and mode shapes for inclined cables neglecting the cable inertia in the longitudinal direction, which is equivalent to a quasi-static stretching assumption originally made by Irvine and Caughey. In 1984 Shih and Tadjbaksh [20] have presented the equations governing small-amplitude free vibrations of extensible elastic cables. The resulting eigenvalue problem has been solved using the Galerkin procedure and the numerical results obtained have agreed with previous results in the inextensible limit.

Yamaguchi and Ito [21], Yamaguchi [22] have also developed the theory of inclined cables, stating that the in-plane natural vibration properties of inclined cables differed from those of horizontal cables.

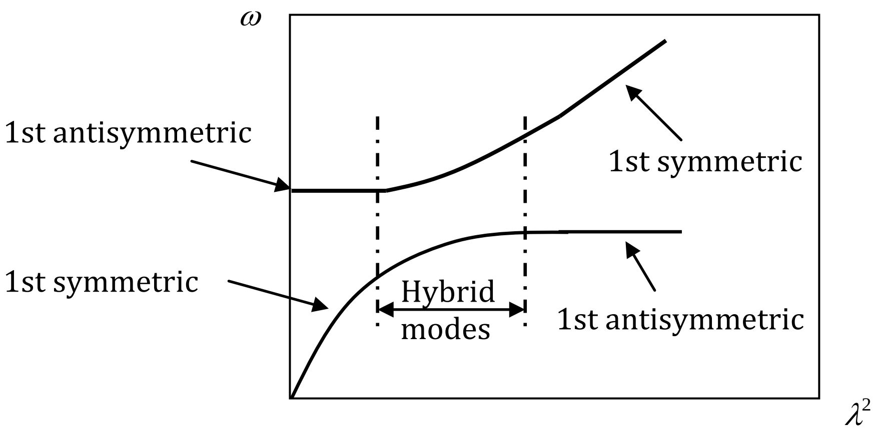

Figure 5. First two natural frequencies as a function of λ^2

Wu et al. [23] have modified the expressions for in-plane natural frequencies of an inclined cable derived by Irvine by considering a geometric parameter, which had been neglected by Irvine. By doing so, additional properties of inclined cables can be captured. The parameter taken into account by Wu et al. is ε^2 ($\varepsilon = 8\beta\cos\theta$), where $\beta = mgL/(8H\sec\theta)$ with mg being a self-weight of the cable, L - the distance between the cable supports, θ - angle between the horizontal line and the line connecting the cable supports. The equations of the cable free motion proposed by the authors take the following form

$$\frac{d}{ds}\left((T+\tau)\left(\frac{dx^*}{ds}+\frac{\partial u^*}{\partial s}\right)\right) = m\frac{\partial^2 u^*}{\partial t^2} - mg\sin\theta\,,$$
$$\frac{d}{ds}\left((T+\tau)\left(\frac{dz^*}{ds}+\frac{\partial w^*}{\partial s}\right)\right) = m\frac{\partial^2 w^*}{\partial t^2} - mg\cos\theta\,,$$

Where T is the initial tension of the cable; τ is the additional tension generated; w^*, u^* are the displacements in the local coordinate system; mg is the self-weight of the cable

However, the bending stiffness of an inclined cable has not been considered in any of the abovementioned studies. Yamaguchi et al. [24] have concluded that the cable bending stiffness will influence its natural frequencies and modal shapes. In order to consider the effect of the bending stiffness, some studies have been focused on the derivation or modification of finite difference solution models [25,26], finite element solution models [27] or simplified solution models such as the lumped mass approach [28]. In 2009 Wu et al. have proposed the theoretical formulae for the in-plane natural vibrations of small-sag inclined elastic cables considering their bending stiffness [29]:

$$\frac{d}{ds}\left(T\,\frac{dx^*}{ds}+EI\,\frac{d^3z^*}{ds^3}\,\frac{dz^*}{ds}\right)=-mg\sin\theta\,,$$
$$\frac{d}{ds}\left(T\,\frac{dz^*}{ds}+EI\,\frac{d^3z^*}{ds^3}\,\frac{dx^*}{ds}\right)=-mg\cos\theta\,,$$

Where x^*, z^* are longitudinal and transversal coordinates, respectively; EI is the bending stiffness. All the other notations are mentioned above.

2.4 Nonlinear Elastic Cable Theory

2.4.1 Small kinematics

The research mentioned so far has been focused on the investigation of small amplitude cable vibrations forming linear elastic cable vibration theory. However, certain resonance phenomena such as cable galloping and large wake-induced vibrations have been observed in different types of cable structures and exposed strongly nonlinear cable behavior. Therefore, it has become essential to study cable vibrations in the framework of nonlinear mechanics. In [30] Irvine has introduced the geometrically nonlinear model of a taut inclined cable, though without solving it. The behavior of a nonlinear model has been also analyzed by Carrier [31,32]. Meirovitch [33] has shown that the undamped nonlinear steady state in a plane response of a taut flat cable is given by the solution of a Duffing equation. Consequently, nonlinear free vibrations of a suspended cable have been studied by a number of researchers: West et al. [34], Hengold and Russel [35], Hagedon and Schafer [36], Luongo et al. [37,38], Rega et al. [39], Benedettini et al. [40]. All of them have investigated simple cable models, with one or two degrees of freedom, developed and utilized to obtain analytical solutions. In the same theoretical research, both single-degree-of-freedom [41] and multiple-degree-of-freedom formulations [42,43,44,45,46] have been studied to explore numerous nonlinear phenomena arising in cable forced vibrations including the effects of nonlinear modal coupling under various external/internal resonance conditions and the possibility of non-periodic responses. Consequently, nonlinear cable dynamics theory has been verified through systematic physical experiments [47,48]. All the theoretical models developed and used for the analysis possessed a certain number of assumptions to simplify the analytical treatment. Among those, the initial static strain has been neglected in order to obtain an inextensible parabolic profile of the cable equilibrium configuration where the sag-to-span ration was of the order 1/8 or less. Besides the initial strain assumption, the dynamic tension has been defined as a function of time only, thus being spatially uniform, which ensued from the inertial force in the longitudinal direction being neglected according to a quasi-static stretching model of the cable in motion. Nevertheless, in 1996 Behbanhi-Nejad and Perkins [49] have demonstrated that the analysis of tension waves propagating freely along the cable length cannot be implemented using those simplifying assumptions. Pakdemirli et al. [50] and Rega et al. [51] have illustrated that the results obtained by analyzing reduced-mode discrete models of cables may be quantitatively erroneous for cables with non-zero sag. Above that, several investigations have demonstrated that the effect of axial strain on the dynamic behavior can be significant depending on elasto-geometric properties. Takahashi and Konishi [52] have examined sagged cables with either horizontal or inclined supports qualitatively, discussing geometrically nonlinear effects, but they have neglected the significance of cable extensibility. Luo and Mote [53] have developed a comprehensive 3-D model governing the steady response of a travelling tension wave in an arbitrarily sagged, elastic cable, and have obtained exact, closed-form solutions for steady motion under various loadings. Nevertheless, a further 3-D nonlinear coupling, as well as the assessment of the variability of dynamic tension during vibration, had yet to be done.

2.4.2 Large kinematics

An extensive research of nonlinear vibrations of inclined cables has been performed by Srinil et al. [54,55,56,57,58]. Initial detailed comprehensive discussion and comparison of large amplitude free vibrations of inclined cables have been presented in [56]. The major findings presented in this paper consisted in describing the extent of two- and three-dimensional couplings, the occurrence of nonlinear dynamic tensions, and the meaningfulness of modal transition phenomena ensuing from the activation of various internal resonance conditions. In [54] Srinil et al. have investigated internally resonant dynamics of the free nonlinear vibrations of horizontal and inclined suspended cables by means of a multi-mode Galerkin-based discretization and second-order multiple scales. The paper has been aimed at the study of planar internal 2:1 resonances. The developed equations of motion have considered the system asymmetry due to the cable inclination and its dynamic extensibility. Closed-form solutions for nonlinear amplitudes, frequencies and dynamic configurations have been obtained with considering higher-order effects due to quadratic nonlinearities.

According to [56] the governing equations of motion in the global coordinate system read:

$$\left(\frac{EA+EA(1+\varepsilon_0)u'}{\sqrt{1+y_0'^2}}-\frac{EA(1+u')}{\sqrt{(1+u')^2+\left(y_0'+v'\right)^2+w'^2}}\right)'=\frac{w_c\sqrt{1+y_0'^2}}{g(1+\varepsilon_0)}\ddot{u},$$

$$\left(\frac{EA+EA(1+\varepsilon_0)v'}{\sqrt{1+y_0^2}}-\frac{EA(y_0+v')}{\sqrt{(1+u')^2+\left(y_0'+v'\right)^2+w'^2}}\right)=\frac{w_c\sqrt{1+y_0^2}}{g}\ddot{v},$$

$$\left(\frac{EA+EA(1+\varepsilon_0)w'}{\sqrt{1+y_0'}}-\frac{EA(w')}{\sqrt{(1+u')^2+\left(y_0^2+v'^2\right)+w'^2}}\right)=\frac{w_c\sqrt{1+y_0^2}}{g}\ddot{w},$$

Where the total strain of the cable centerline at the displaced state read:

$$\varepsilon=\frac{d\bar{s}-ds}{ds}=\frac{(1+\varepsilon_0)}{\sqrt{1+y_0'^2}}\sqrt{(1+u')^2+\left(y_0'+v'\right)^2+w'^2}-1,$$

And where u,v,w are the displacements in the global Cartesian coordinate system; ds is the elemental length of the cable in the unstretched state; y_0 is the coordinate of a cable point in the equilibrium state; E,A are the modulus of elasticity and cross-sectional area of the cable, respectively; g is the gravitational force; w_c is the cable weight per unit of unstreched length. The differentiation of a function by time is indicated by two dots.

The theoretical predictions have been verified by finite difference solutions. In [59] the authors have described the experimental modeling of the linear free and nonlinear forced vibrations of sagged inclined cables followed by experimental testing. The paper has been focused on identifying cable hybrid modes due to system asymmetry, which causes the presence of an avoidance phenomenon in the natural frequency spectrum. Some 3-D nonlinear dynamics under the simultaneous parametric/external excitation due to a harmonically time-varying support movement has been investigated. The effect of non-planar/planar internal resonances producing large –amplitude out-of-plane multi-modal interactions has been experimentally observed and complemented by space-time numerical simulation of the corresponding (geometrically nonlinear) partial-differential equations of parametrically-forced cable motion. The authors have presented the experimental and numerical results that reflect the fundamental linear/nonlinear dynamic characteristics of inclined cables with the presence of the cable asymmetry, its sag and dynamic extensibility. In [57] Srinil et al. have dealt with the analytical investigation of resonant multi-modal dynamics due to 2:1 internal resonances in finite amplitude free vibrations of inclined and horizontal cables. Based on the formulated cable model, approximate partial differential equations of 3-D coupled motion of small sagged cables have been developed. These equations account for both spatio-temporal variation of non-linear dynamic tension and system asymmetry due to inclined sagged configuration. In order to perform the solution of non-planar/planar motion, a multi-dimensional Galerkin's expansion has been performed. A second-order asymptotic analysis under planar 2:1 has been also accomplished by the method of multiple scales. The analysis of approximate closed-form solutions of nonlinear amplitudes, frequencies and dynamic configurations of resonant nonlinear normal modes has presented the cable response on resonant/non-resonant modal contributions. In the following paper [58] Srinil and Rega have investigated the multi-modal dynamics due to planar 2:1 resonances in the nonlinear, finite-amplitude free vibrations of horizontal and inclined cables based on the second-order multiple-scales solution from [54]. In the conclusions of this paper, the dependence of resonant dynamics on coupled vibrations amplitudes, and the significant effects of cable sag, inclination and extensibility on system non-linearity are underlined together with contributions of longitudinal dynamics as it has been done in [54]. Again, the authors have illustrated the spatio-temporal variation of non-linear dynamic configurations together with dynamic tension associated with 2:1 resonant non-linear normal modes. The analytical solutions have been evaluated by finite difference-based numerical investigations of the original partial differential equations of motion as this validation has been applied in [54]. Despite detailed and precise analysis of the nonlinear cable dynamics, Srinil et al. have not reflected, for example, the effect of the supports flexibility, as absolutely rigid fixed supports represent an approximation of reality. Berlioz and Lamarque [60] have also investigated nonlinear vibrations of inclined cables, and have implemented an experimental and theoretical study in order to highlight the nonlinear dynamic behavior of an inclined cable. Their results have presented good agreement between theory and experiment, and several nonlinear motions have been observed and identified under periodic external stresses. Zuo et al. [61] have developed a set of governing equations of nonlinear free transverse in-plane vibration of an inclined cable with small sag. The equations have reflected the effects of the cable's Irvine parameter (λ^2) and vibration modes on its natural frequencies, and internal nonlinear couplings between the cable modes. When single mode vibration occurs, the governing equations are simplified as a classical Duffing equation with a quadratic term. The averaging method has been used to solve approximately the Duffing equation. As an example, dynamic responses of a practical inclined cable have been analyzed. The results have been verified by numerical methods. These, however, do not reflect the out-of-plane behavior. Li et al. [62] have studied the influence of the tension variability on nonlinear natural frequencies of an inclined cable. Considering the bending stiffness of the cable, the authors have formulated a nonlinear vibration equation based on Newton's law neglecting the vibration coupling of in-plane and out-of-plane motion and ignoring the axial vibration along the cable.

3. Dynamics of Cable Structures

3.1 General

As it has been mentioned above, cables are typically used as elements of a more complex structure. Therefore, there is a need for investigating the problem of interaction between cables and other system elements that induce cable vibrations. There is extensive research available on inclined cables considering supports motions, as well as composite analysis of cables and supported structures (guyed towers, cable-stayed bridges). In 1965 boundary induced vibrations have been treated by Davenport and Steels [63], while a more refined theory has been developed by Velestos and Darbre [64].

Starossek has studied the dynamic response of an extensible sagging cable and presented in [65] a dynamic stiffness matrix whose coefficients were functions of the frequency of motion, and that is suitable for dynamic direct stiffness analysis of composite/coupled mechanical systems such as cable-stayed bridges and guyed masts. The study has been restricted to small displacements (linear theory) and considered structural motion within the vertical cable plane only.

In [66] Kim and Chang presented the free vibration analysis and a dynamic matrix derivation for an inclined cable. In the paper the cable was assumed to have an elastic catenary profile, and the chord-wise component of self-weight and viscous damping have been considered. After linearization of the equations of motion, closed-form solutions of free vibrations have been derived. Based on the solution of free vibration of a cable with displaceable boundaries, the dynamic stiffness matrix has been assembled. The dynamic stiffness coefficients and the effects of damping have been investigated. Such a matrix for an inclined cable can be applied to the dynamic analysis of cable-supported structures such as cable-stayed bridges or guyed masts. However, the authors have stated that for tightly stretched inclined cables utilization of more accurate theory was indispensable.

In [67] Li and Wang have dealt with the vibration of a guy that may be excited by small motions of the mast. A theoretical model has been developed for the study of nonlinear vibration of an inclined guy. Numerical analysis has been used to predict the parametric vibration. The authors have shown that reasonably small anchorage motion amplitudes may lead to important guy vibration. It has also been found that excitation amplitude, pretension, tilt angle and internal damping ratio all play very important roles in parametric resonance.

Wei et al. [68] have implemented analysis of the characteristics of the parametric vibrations of the cable in cable-stayed bridge, especially the vibration amplitude and the tension fluctuation by using Finite Elements Method. The nonlinear response of a cable-stay under a synchronous excitation has been studied. It has been observed that the in-plane vibration of the support can induce the out-of-plane symmetric vibration modes leading to coupling vibration. However, the out-of-plane symmetric vibration modes of the cable support can only induce the out-of-plane symmetric modes, and not the in-plane vibration of the cable. The authors have also found that the increase of cable damping dwindles the frequency bandwidth of large-amplitude vibrations, but enlarges the tension fluctuations.

The effects of excitation amplitude, pretension, tilt angle and damping ratio on parametric and external resonances have also been studied by Wu [69]. The study has focused on in-plane nonlinear dynamics of a heavy elastic suspended cable whose lower end is fixed to the ground and the upper end is pin supported and movable horizontally to simulate the bending motion of a guyed stack due to vortex shedding. The parametric cable vibration has been studied extensively as well.

In [70] Cai and Chen have investigated nonlinear dynamics of a heavy elastic suspended cable for application to stack/wire systems. Small oscillations of the cable at the support attached to the stack, resulting from the bending motion of the stack due to vortex shedding, lead to parametric and external excitation. Numerical analysis has been used to predict the parametric and external resonances of the elastic suspended cable, which contained cubic nonlinearities due to cable stretching and quadratic nonlinearities due to equilibrium cable curvature in a tilted configuration. Numerical results agree well with the original stack/wire response, which had been previously by the same authors using an analytical prediction with a finite element code and observation of the stack/wire behavior. Additional parametric analyses have been pursued to distinguish between parametric and external resonances and their couplings. It was found that excitation amplitudes and tilted angles play very important roles in parametric and external resonances.

Sparling and Davenport [71] have studied the nonlinear dynamic behavior of guy cables in turbulent winds in relation to tall guyed telecommunication masts. A numerical study has therefore been undertaken to investigate the dynamic behavior of inclined cables excited by imposed displacements. It has been found that the nonlinear coupling of related harmonic response components has been significant and acted in the plane of the guy. Positive aerodynamic damping has been shown to effectively suppress resonant and nonlinear coupling responses.

Wu [72] has investigated in-plane nonlinear dynamics of a heavy elastic suspended cable having its lower end fixed to the ground and the upper end pin supported. The upper support has been set as movable simulating the motion of the supported structure under vortex shedding. The author has confirmed that the nature of the excitation (amplitude and forcing frequency), pretension, tilt angle and internal damping ratio all play roles in parametric and external resonances. Most of the studies considering cable support movements have been focused on in-plane cable vibrations and none has been found that would consider different base support conditions simulating different soil mechanical properties, if it is a cable anchored to the ground.

3.2 Cable-stayed bridges

Some researchers have focused their investigations on parametric excitations of cables. For example, for cable-stayed bridges, Lilien and Costa [73] have found that very large amplitude vibrations can be induced by low-frequency mechanical tension oscillation of the stay cables in long span bridges. A global/local mode approach, which can analyze the coupled vibration of cables and bridges, has been introduced by Warnitchai et al [74]. Gatulli et al. [75, 76] and Gatulli and Lepidi [77] have developed a cable-stayed beam model and found that vibration energy can transfer from low frequencies to high frequencies through parametric resonance; the numerical results have been confirmed by experimental studies. Gattulli et al. [78] have also studied the coupled stay cable- bridge deck vibration and the influence of strong mode localization on the structural response. Their investigation has shown that a veering phenomenon may occur under particular parameter combinations that enable internal resonance between local and global modes. These conclusions have been confirmed by a refined finite element analysis of the existing Bill Memorial Emerson Bridge, USA.

Georgakis and Taylor [79] have performed finite element analysis of a cable under harmonic excitations and studied the changes of initial cable conditions. A research by Caetano et al. [80] has described several possible mechanisms of cable vibration of the International Guadiana Bridge at the border of Spain and Portugal. On-site measurements and finite element analysis have indicated that internal resonance of the cable-deck system may be a major cause of large-amplitude cable motions.

Fujino et al. [81] have proposed a three-degrees-of-freedom nonlinear model of a cable-stayed-beam and accomplished analytical steady-state solutions of the auto-parametric resonance between the out-of-plane cable vibration and the beam vibration. Using this model, Warnitchai et al. [74] have also examined active tendon control of cable-stayed bridge vibrations. It was found that excitation amplitudes and tilted angles play very important roles on parametric and external resonances. Kang et al. [82] have aimed to investigate the coupled nonlinear vibration response of the cable-deck system by numerical analysis of a proposed three-degree-of-freedom model. The authors have attempted not only to take account of the coupling effect of the in-plane and out-of-plane vibrations of the cable, but also to consider the vertical vibration of the bridge deck as an independent degree of freedom. Implementing analytical and numerical analysis, the authors have made conclusions about several coupling vibration effects of the cable-deck system.

Khadaroui [83] has introduced a one-degree-of-freedom model of an inclined cable subjected to an external excitation of the cable support. This model has been modified and analyzed later by Berlioz and Lamarque [84]. The authors have studied both primary and sub-harmonic resonances by using multiple scales method. Both in-plane and out-of-plane motions have been studied. Despite modifying the original model, Berlioz and Lamarque have neglected longitudinal displacements of the cable. Nevertheless, an experimental validation of the theoretical results has demonstrated qualitative agreement. In conclusion, the authors have suggested studying the effect of external random excitation. An overview of the literature and results on the response of cables under random excitation has been done by Raouf in 2002 [85].

3.3 Guyed telecommunication masts

Many research works have been done to study the static and dynamic behavior of guyed towers. The simplest way to analyze a guyed tower has been to assume the mast of the structure to be a continuous beam on elastic supports with a set of springs to idealize the taut guy-wires attached to the tower mast. Cohen and Perrin [86,87] have made the earliest contributions to the study of guyed masts. Their first paper [86] has investigated wind loading and presented a set of charts that could be used to predict the drag loads produced by wind. The second paper [87] has presented a model that described the dynamic response of a guyed mast. The mast has been treated as a cantilever beam column on elastic supports and the guy cables have been considered to have a parabolic profile. Starossek has also performed similar studies for cables supporting masts or towers [88].

Rowe [89] has investigated the amplification of stresses and displacements in guyed towers when changes in the mast geometry are included. This paper has also developed charts to investigate the structural behavior of a mast based on modeling the guys as bars. Hull [90] has studied the sectional critical moment of inertia corresponding to a critical buckling wind load and conducted a stability analysis of guyed towers in terms of static mast behavior. Goldberg and Myers [91] have investigated the importance of including wind effects on guy cables in the overall tower response. Odley [92] has presented a solution where other effects (ice loads, shear deformations, etc.) have been included in the guyed tower model.

Williamson and Margolin [93] have showed the importance of including mast shear deformations in the analysis of guyed masts.

In [94] Madugula et al. have underlined that due to the flexibility associated with both the slender mast and the guy cables, guyed towers are very sensitive to dynamic excitation from gusty winds, which has been investigated by Sparling et al. [95,96]. In addition, the strong dynamic interaction that exists between the motion of the mast and the relatively massive guys, each of which has frequency dependent stiffness properties, leads to rather complex dynamic behavior. Unlike conventional building structures, guyed towers routinely exhibit 20 or more active vibration modes when excited by turbulent winds. The lowest mode is typically dominated by large amplitude vibrations of one or more guys at the top support level, with little flexural bending in the mast, while the next few modes involve significant vibrations at progressively lower guy support levels in turn. Higher modes, on the other hand, are dominated by flexural vibrations of the mast alone, with the degree of curvature in the mast increasing for progressively higher modes. Intermediate modes feature varying degrees of coupled motion involving both the mast and guys. In [97] a three-dimensional dynamic response of a guyed tower subjected to turbulent buffeting has been investigated by Sparling and Davenport. Nonlinear dynamic response has been determined in the time domain using Newmark's Beta step-by-step integration of the governing equations of motion. A simulated windstorm containing both along-wind and across-wind turbulences has been generated. The authors have focused their analysis mainly on the tower behavior giving some attention to dynamic guy tension.

Peil et al. [98] have also studied the dynamic behavior of guys and guyed masts under wind load. The authors have implemented comparisons of theoretical and experimental results that have shown that a precise prediction of the dynamic response of guyed systems is possible if the dynamic behavior of the guys is taken into account.

A theoretical and experimental research on the dynamic behavior of guyed masts under wind has been accomplished by Ma et al. [99]. Recently Bastos et al. [100] have presented a numerical dynamic analysis of a cable stayed mast under a time domain simulating turbulent wind. Several finite elements models have been studied in order to assess the importance of nonlinear effects associated with the mast slenderness and significant cable sag.

4. Mitigation of cable vibrations

4.1 General

As it has been described above, cable structures are of frequent use in a wide range of practical applications for supplying both support and stability to large structures. Due to their overall slenderness and inherent flexibility, cable structures possess dynamical susceptibility to excitation from surrounding mediums, so, cable vibrations may happen in some circumstances, which eventually may degrade the system performance. For example, large-amplitude amplitude cable vibrations have negative effects on the cables integrity causing the fatigue accumulation and their even possible rupture. Hence, appropriate countermeasures have to be applied to reduce or suppress the large amplitude vibrations. In general, these countermeasures can be classified into three types: aerodynamic, structural and mechanical, and belong to a category of so-called passive techniques of control. Having been used with proven effectiveness in structural performance, these techniques have certain limitations.

Among the first attempts to address vibration control for guyed masts was the paper prepared by Hirsch [101], who summarized some results of full-scale tests involving passive vibration control of a guyed mast vibrations equipped with two tuned-mass dampers on the mast top part (See Figure 6a). The paper has mentioned the optional approach of the guy vibrations mitigation by installing a damper on the cable (see Figure 6b). The observations have been compared with theoretical considerations, and the conclusions have been focused on optimum vibration control. It was derived that natural mode shapes and frequencies could be computed considering linear behavior of the guyed mast, but the results have been judged to be qualitative. In all cases it was considered impossible to ensure the cables to be dynamically stable relating to all possible solutions of vibrations, including galloping and parametric excitation. A full dynamic analysis of a guyed mast exposed to various forms of wind excitations has been considered unfeasible.

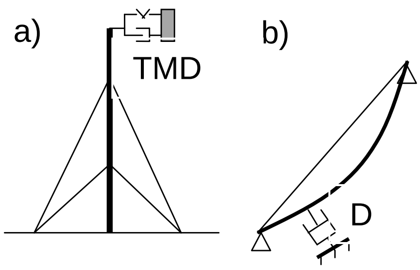

Figure 6. Guyed mast with a tuned-mass damper on top. Cable with a damper.

Several damping devices types installed in practice on guyed masts are described in [94]. However, no detailed investigation on damping devices installed on inclined guy cables have been found in literature.

Recently, several attempts have been made to accomplish active and semi-active control of guy cable vibrations. In particular, cables are essential structural elements in cable-stayed bridges stating their vibration control as a crucial task. Therefore, an extensive research has been done for this critical issue. Aerodynamic control methods have been studied and applied to mitigate large-amplitude vibrations of stay cables in bridges [102], which occur under wind oblique to a cable with or without rainfall. Modifications of circular cylinder surfaces, such as strakes and helical wires, effectively mitigate Karman vortex-induced vibrations normal to wind flow. The authors of the mentioned paper studied a flow around a yawed cylinder with various strake patterns using three-dimensional detached eddy simulation at a specific Reynolds number in order to understand their effectiveness in reducing large-amplitude, low-frequency vibrations of stay cables. The results of the research have demonstrated that a suitable strake pattern causes disturbance of the development of coherent flow structures around an oblique cable. It consequently suppresses or weakens the associated forces at low frequency. However, further investigations on the performance of strakes have been announced necessary in terms of their optimal number, size, and pitch of the windings.

Structural damping for bridge stay cables has been used very frequently as a temporary measure before the installation of damping devices on cables or as a permanent measure. A popular mitigation method consists in installing cross ties connecting several stay cables together. For example, Yamaguchi and Nagahawatta [103] have investigated analytically a cable system comprising two main cables with two cross ties. It has been established that there is more or less damping effect associated with the use of cross ties and this effect can be increased by using more flexible and more dissipative ties compared with the stay cables. The authors have shown that their energy-based method of damping evaluation is very effective and the modal damping of cable systems can be estimated analytically by evaluating the modal strain energy and modal potential energy of the initial cable tension.

4.2 Vibrations of a cable equipped with a viscous damper

The phenomenon of free vibrations of a taut cable with an attached viscous dashpot damper (external mechanical damping) has been studied by a large number of authors. Carne [104] and Kovacs [105] were among the first to determine first-mode damping ratios for damper locations near the cable lower end (See Figure 7). Carne has introduced an approximate analytical solution obtaining a transcendental equation for the complex eigenvalues of the damped cable and an accurate approximation for the first-mode ratio as a function of the viscous damper coefficient and location. Later, Kovacs has developed approximate solutions for the maximum attainable damping ratio and the corresponding optimal damper coefficient.

Several authors have investigated the free-vibration problem using Galerkin's method with the sinusoidal mode shapes of an undamped cable as basic functions (see for example [106]. Pacheco et al. [106] have introduced non-dimensional parameters to develop a "universal estimation curve" (See Figure 8) of normalized modal damping ratio versus normalized damper coefficient, which is applicable in many practical design situations.

Xu et al. [107] have developed an efficient and accurate transfer matrix formulation using complex eigenfunctions in order to estimate nodal damping. In more recent research on cable vibration mitigation several results have been issued by Krenk in 2000 [108] who has proposed an exact analytical solution for the vibrations of a taut cable equipped with a concentrated viscous damper.

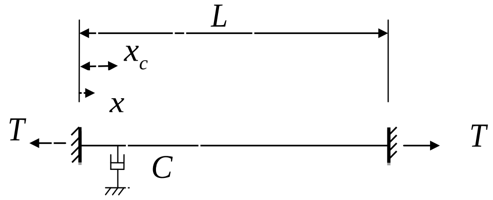

Figure 7. Taut cable with a viscous damper in transverse direction

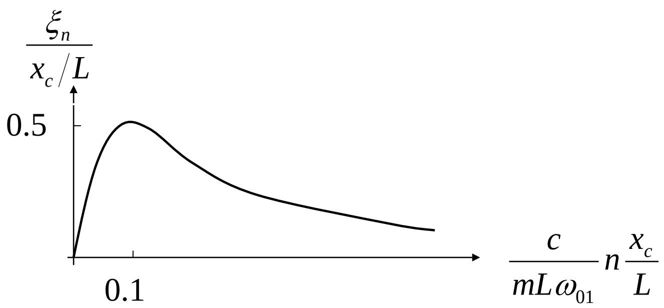

Figure 8. Universal curve relating modal damping ratio ξ_n with damper size c, location of damper x_c, cable parameters m, L and its fundamental frequency ω_{01}.

The solution has been obtained in terms of damped complex-valued modes, leading to a transcendental equation for the complex eigenfrequencies; this formulation has permitted to determine explicitly the optimal location of the damper depending on its damping parameter. In this investigation only a few first modes of vibrations have been studied considering damper locations near the lower cable end, which is a usual practice.

In 2002 Krenk and Nielsen [109] have generalized the previously developed analysis of a taut cable under the effect of a transverse viscous damper installed close to the end of a shallow cable by complex modal superposition analysis. In this continued research it has been found that the effect of the damper on the nearly antisymmetric modes is independent of the sag and axial rigidity. In contrast, the nearly symmetric modes develop regions of reduced motion near the ends, with increasing cable stiffness, which reduces the efficiency of the viscous damper. The resulting modal damping ratio and optimal tuning of the damper have also been presented.

Viscous damper performance in the higher modes has posed a particular interest since full-scale measurements by Main and Jones in 2001 [110] have indicated that vibrations of moderate amplitude can occur over a wide range of cable modes. In 2002 Main and Jones [111] have formulated the free-vibration problem in order to investigate the dynamics of the cable passive linear damper system in higher modes and without restriction on the damper location. In this research, the cable has been modeled as a taut horizontal string with initial sag and bending stiffness being neglected. The authors have developed an analytical formulation of the complex eigenvalue problem in order to derive an equation for the eigenvalues that is independent of the damper coefficient $c/\sqrt{Tm}$, where c is the viscous damper coefficient, T is the tension in the cable, m is the mass per unit length of cable. This equation has the following form

$$\sin(2\pi\varphi l_1/L)\cosh(2\pi\sigma l_2/L)+\sin(2\pi\varphi l_2/L)\cosh(2\pi\sigma l_1/L)=\sin(2\pi\varphi),$$

Where l_1 and l_2 are the lengths of two cable segments; σ and φ are the real and imaginary parts of the non-dimensional eigenvalue, respectively; L is the length of the cable. The solution of the equation has allowed the authors to analyze the behavior of the system damping ratio as a function of the damper location.

In a companion paper [112] the same authors have investigated the dynamic behavior of a taut cable with a passive nonlinear viscous damper attached at an intermediate point using an averaging procedure. An approximate analytical solution for the amplitude-dependent effective damping ratio in each mode has been developed by assuming the same form of solution as for the linear damper. The analysis has revealed potential advantages that may be offered by a nonlinear damper over the traditional linear damper. Hoang and Fujino [113] have studied different types of nonlinear dampers attached to a stay cable (friction damper, bilinear viscous damper, quadratic-linear viscous damper). The equivalent viscous damping over one cycle of cable vibration has been estimated on the basis of the absorbed energy. An equivalent modal damping ratio has been obtained that depends on the damper characteristics and damper motion displacement. Potential design advantages over linear dampers (higher modal damping) have been shown. However, as in all the analytical investigations mentioned above, this study has focused only on horizontal taut string model for the cable.

Koralalage and Cheng [114] have proposed a novel energy-based damping estimation method by introducing the concept of kinetic energy decay ratio. The proposed formulation takes into account the finite flexural rigidity and sag extensibility of the cable. Its application has no restrictions on the damper location. By introducing the kinetic decay ratio as a key index, the relation between the additional damping provided by the external damper and the kinetic energy dissipation rate of the damped cable has been derived by the authors. A numerical example has been presented in the paper, which has provided results favorable with a few earlier studies. However, the case study has been limited to the horizontal cable.

Weber et al. [115] have investigated the optimal tuning of Coulomb friction dampers on cables, where the optimality criterion was maximum additional damping in the first vibration mode. The expression for the optimal friction force level of Coulomb friction followed from the linear viscous damper via harmonic averaging. The authors have concluded that the friction force level had to be adjusted in proportion to cable motion amplitude at the damper location. It has been also stated that the resulting non sinusoidal cable motion clearly violates the assumption of pure harmonic motion and explains why such dampers have to be tuned differently from optimal linear viscous dampers.

Zhou and Sun [116] have studied damping of taut cable by a bilinear viscous damper. Being mode and amplitude dependent the bilinear damper has been approximated by an equivalent linear viscous damper based on equivalent energy dissipation in one cycle of oscillation. The research has shown that the bilinear damper has its advantages over linear damper providing higher damping in higher modes of vibration.

Huang and Jones [117] have recently studied the effects of linear elastic spring supports for an intermediate damper analyzing two types of dampers: linear elastic and friction-threshold. The authors have introduced an effective flexibility coefficient in order to investigate the effect of different values of support stiffness on the effectiveness of the linear viscous damper. The influence of the linear elastic support on a cable-damper system with a friction threshold has also been investigated by using the result of the linear viscous damper and the equivalent energy method. It has been established that, compared to a perfect support, the linear elastic support reduces the effectiveness of the damper.

Recently, Impollonia et al. [118] have presented the results of their investigations on optimal damper design of inclined sagged cables with bending stiffness. The dynamic behavior of a cable has been studied by means of a finite element model considering sag, bending stiffness and inclination of the cable. In particular, the proposed approach has been compared with solutions proposed for the taut horizontal cable with bending stiffness. The results obtained have shown significant differences with respect to horizontal taut strings with bending stiffness. For the case of inclined the phenomenon of the hybrid modes has been noticed. The difference in mode shapes frequency values has been found more evident for higher modes. The proposed refined model would allow a more precise damper design for the case of long stay-cables.

Lan et al. [119] have also recently analyzed the effects of stay cable properties on damper effectiveness in cable-damper system. The researchers have concluded that for a long cable with significant sag, even though damper properties and other parameters are properly selected, the attainable model damping ratio in the first in-plane vibrational mode is much smaller than for tauter cables due to cable frequency avoidance caused by cable sag and inclination (See Section 2.3). The authors have also stated there exists a certain cable length for which the maximum damping will likely be reduced to zero, which means that dampers cannot provide efficient damping.

Hoffman and Distl [120] have discussed the parameters allowing an efficient and economic design of stay cable dampers. Specific details of the stay cable dampers like the stiffness of their support have shown significant influence as demonstrated by a real project and parametric studies. In consequence, it has been concluded that for longer stay cables external dampers on stiff supports provide the highest efficiency. The researchers have outlined that adjustment and optimization of passive dampers to more than one mode still present a challenge. In cases of lack of damping efficiency, adaptive cable dampers with semi-active control could be brought into use.

4.3 Semi-active and active dampers

Several studies have been made in order to analyze the potential for improved damping by semi-active or active dampers, which may have a better performance over traditional passive viscous dampers. For example, in [121] Johnson et al. have investigated damping features of semi-active devices. The equations of motion of a cable equipped with such a device have been derived using the assumed modes approach and a control-oriented model has been developed. The control-oriented model has been shown to be accurate and a comparative study of the cable response between passive, active and semi-active dampers has been implemented. The response with a semi-active damper has been found dramatically reduced compared to the optimal passive linear viscous damper for typical damper configurations, which has demonstrated the potential benefits of using a semi-active damper for absorbing cable vibratory energy.

In another research [122], Hui et al. have investigated the vibration mitigation of a stay cable equipped with a shape memory alloy (SMA) damper, which dissipates energy through its hyperelastic behavior. The solution of the problem has been obtained in the closed form and used to determine the additional equivalent modal damping ratio provided by the SMA damper. The additional damping ratio depended on both the parameters and locations of the SMA damper when the cable vibrated with only a single mode. The responses and the additional equivalent modal damping ratios of the cable with a damper attached at different locations have been studied under harmonic excitation in plane with single-mode only and with coupled multi-mode vibration. The results have demonstrated that the SMA damper can suppress the vibrations significantly and the control effectiveness is influenced by the SMA parameters and locations.

4.4 Stockbridge tuned-mass dampers

4.4.1 Application to overhead line conductors

The task of suppression of cable vibrations exists for overhead transmission line conductors under vortex-induced or Aeolian vibrations. A common mitigation technique to suppress these vibrations is to attach tuned-mass damping devices to conductors (See Figure 9). A very common type of such devices is the Stockbridge damper. (See Figure 10). The performance of this type of dampers has been investigated for decades. In 1969 Claren and Diana [123] have investigated the response of stranded conductors under exciting transverse harmonic forces. As a part of the study, the dynamical behavior of a taut cable with one or more Stockbridge dampers has been investigated.

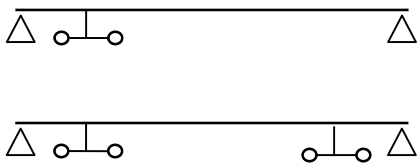

Figure 9. Taut cable with one or two dampers

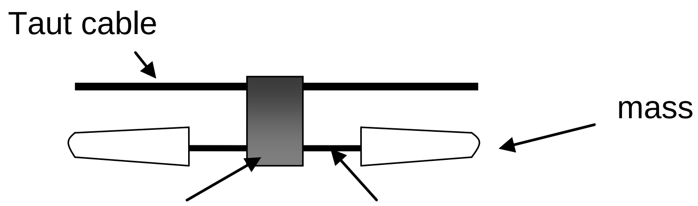

Figure 10. Stockbridge damper

The various damper parameters (damping constant, natural frequency) on the cable response have been analyzed making possible to accurately measure the dissipated energy per cycle. Consequently, the Energy Balance Method has been widely used in order to investigate the aspects of the dynamic behavior of a conductor equipped with this type of damper [124,125].

In [126] the free vibration of a transmission line conductor equipped with a number of Stockbridge dampers has been modeled by a differential equation of motion of a tensioned beam acted on by concentrated frequency dependent forces, and an exact solution of this model is obtained using integral transformation. The authors have obtained the expressions for the mode shapes that could be used to predict accurately the overall strains of lines in operation or under design. The numerical examples presented have shown that the usual sinusoidal mode shapes could be significantly distorted by the dampers to give rise to dangerous strains. A numerical analysis of Aeolian vibrations of a conductor equipped with a Stockbridge damper has been presented by Vecchiarelli et al. [127]. In the study the numerical model has been derived for steady-state mono-frequency vibration of a single overhead conductor. The developed model was capable of accounting for the effects of conductor flexural rigidity, more than one spatial mode of conductor vibration, travelling waves and damper mass. This model was based on empirical data pertaining to wind power input, the conductor self-damping and the energy-dissipation characteristics of the damper.

Recently, Li et al. [128] have applied finite element analysis to investigate vertical, steady-state, mono-frequency aero-vibration of a single transmission conductor with a Stockbridge-type damper attached. Wang et al. [129] have modeled a transmission line conductor equipped with a number of Stockbridge dampers using a differential equation of motion of a tensioned beam acted on by concentrated frequency-dependent forces.

There has been some research performed on optimal design parameters of the Stockbridge dampers. Markiewicz [130] has proposed a method and computational model for the evaluation of the optimum dynamic characteristics of Stockbridge dampers. Richardson [131] has proposed a quantitative prediction of damper requirements in terms of energy or power dissipation as a function of either frequency or wind speed. In [132] Canales et al. have also proposed an optimal design procedure of a Stockbridge damper. Kalombo et al. [133] have mathematically modeled a Stockbridge damper in order to establish an approach to predict the remaining life of the damper studying a taut cable excited by a transverse harmonic force and equipped with a Stockbridge damper. Following [132] the stress on the messenger cable under the dynamic response of the damper is expressed as

$$\sigma = \frac{U_a d}{2I_a}\sqrt{\left(A'_{\max}\right)^2 + \left(B'_{\max}\right)^2} \ ,$$

Where U_a, d, I_a are the amplitude of the damper excitation, diameter and section moment of inertia of a single wire of the messenger cable, respectively. The parameters $A'_{\max}$ and $B'_{\max}$ depend on the damping coefficient for the messenger cable.

An optimization technique has been formed based on a criterion of the mean attenuation or efficiency of the damper. This includes the objective function, which should be maximized and which is expressed as

$$\overline{V}_D = \frac{\sum V_{D_i}}{n},$$

Which is accompanied by inequality constraints related to the mean attenuation of the cable at the no damping side, and the endurance stress due to fatigue of the messenger cable. The first constraint is defined as

$$\overline{V}_U = \frac{\sum V_{U_i}}{n} \geq 0.8.$$

In order to meet the endurance requirement it is necessary to keep the stresses of a wire lower than its endurance. Thus, the second constraint is expressed as

$$\sigma \leq \frac{S_e}{N},$$

Where $\overline{V}_D, \overline{V}_U$ are the mean attenuation or efficiency corresponding to the damping and no damping sides of the cable, respectively; V_{D_i}, V_{U_i} are the attenuation for the ith vibration mode on the damping and no damping sides of the cable; n s the number of modes of vibration to be considered, σ is the stress due to fatigue, S_e is the endurance stress of the messenger cable material; N is the safety factor for fatigue.

The attenuation is defined as function of the amplitude of vibration of the cable Y by the following equations:

$$V_U = 100\left[1 - \frac{Y_{UD}}{Y_{UO}}\right],$$

$$V_D = 100\left[1 - \frac{Y_{DD}}{Y_{DO}}\right],$$

Where, as mention above, the subindex in V and the first subindex in Y are related to the damping side (D) or no damping side (U) of the cable, while the second subindex in Y refers to the presence (D) or absence (O) of the damper.

4.4.2 Application to inclined guy cables

However, despite all the research on conductors equipped with Stockbridge dampers, no studies have been found in literature for Stockbridge dampers performance when they are installed on inclined taut cables such as guy wires. Such an application of this type of dampers is encountered in practice. For example, the largest producer of electricity in Canada, Hydro Quebec, uses Stockbridge dampers on guy wires supporting transmission towers. Several studies on this passive damping solution have been reported in [134,135,136,137] and have focused on the guyed transmission towers used by Hydro Quebec. This technique has been used successfully to suppress guy wire vibrations for wires anchored in rock. However, the dampers themselves have suffered damage and their behavior is not yet fully understood [138,139,140].

The geometry and boundary conditions of inclined cables used for guyed towers differ from those used in cable-stayed bridges and have never been studied as extensively. Guyed transmission towers share similarities with guyed telecommunication towers, especially for shorter telecommunication masts, in terms of overall flexibility and transverse lateral stiffness. A number of studies have been performed on the dynamic response of guyed telecommunication towers. The first detailed finite element modelling study has been realized by Guevara and McClure [141,142] in relation to seismic motion. Subsequently, the study has been continued by Amiri [143], Dietrich [144], Faridafshin [145], and more recently by Ghafari Oskoie [146,147]. However, all these investigations have dealt with the seismic response of the tower to an earthquake and have not examined the effect of cycling loadings in cables leading to fatigue phenomena requiring mitigation solutions.

A recent monograph [148] on communication structures, based on the work of members of Working Group 4 (Masts and Towers) of the International Association for Shells and Spatial Structures, addresses several aspects of guy response to wind effects but does not propose a detailed treatment of guywire fatigue. It essentially recommends relying on regular cable inspections to identify cable damage and subsequent replacement. It is evident that frequent inspection may not guarantee high reliability of the structure. It is possible for a cable to undergo a rupture without exterior signs of damage. Although this approach may be acceptable for telecommunication masts, it is not practical for transmission towers that are far more numerous and typically located in much less accessible areas.

4.5 Marine cable structures

Cables are employed in marine structures and undergo similar dynamic phenomenon as guyed structures or cable-stayed bridges, although in much different frequency ranges.

Among recent studies on mooring cables, the paper by Zhang et al. [149] can be mentioned. The authors have analyzed the dynamic response of mooring systems under heave motion at the end of the cable on the sea bed. The effects of sway and fluid-drag forces have been considered as well. The results have indicated super-harmonic (self-excited through coupling) response of the mooring system, which are caused by the harmonic motion of the cable upper point. The authors have pointed out that this super-harmonic response component has a significant influence of fatigue cumulative damage of the cable and should be considered in fatigue analysis.

Couliard and Langley [150] have reported on the results of an investigation of the statics and dynamics of station-keeping systems for floating production platforms using taut leg configuration and moored in ultra deep waters. The results have shown that lateral motions occur mostly near the top of cables while the lower part is mainly subjected to axial deformations. In [151] Gobat and Grosenbaugh have presented an empirical model for the cable oscillations induced by vertical motions at the top of a catenary mooring. Yu and Tan [152] have proposed an efficient two-dimensional finite element model for a mooring cable and seabed interaction employing a hybrid beam element to simulate the mooring cable.

Modarres-Sadeghi et al. [153] have analyzed vortex-induced vibrations of long distributed cable structures (risers and mooring cables); this is an inherently complicated phenomenon since various combinations of travelling and standing wave patterns develop along the cable length due to the riser multi-mode excitations. The authors have made a comparison in terms of dynamical behavior (travelling waves versus standing waves, amplitudes and frequencies of oscillations) as well as the fatigue life calculations using a Van der Pol wake oscillator model. In the classic Van der Pol model, the dynamics of the rigid cylinder is described using a linear forced oscillator model, in which the external force (external to the rigid cylinder) comes from the wake and the wake itself is described by a forced Van der Pol oscillator equation.

The external force in the Van der Pol oscillator is related to the cylinder oscillation by a coupling term proportional to the cylinder displacement, velocity or acceleration. In the mentioned paper, the Van der Pol model extended to a long flexible structure is used. The research has demonstrated that the theoretical model can predict fatigue damage of the cable fairly well. Although, the study has been dedicated to the phenomenon similar to the one taking place in guyed aerial structures, but in a different frequency band, this approach may be used to investigate the behavior of the guyed masts experiencing Aeolian vibrations.

Vibration suppression is also critical for marine cables because of the adverse effects of fatigue accumulation. Fatigue life estimation techniques have been proposed following cable monitoring. For example, in [154] Makundan at al. have used ambient vibration measurements (typically using strain gages and accelerometers) in order to understand the evolution of the riser VIV (vortex-induced vibrations), with the final aim of estimating the fatigue damage. For this purpose the authors have employed systematic techniques to reconstruct riser VIV response using the data from the available sensors. The reconstructed riser response allows estimation of the dynamic axial stresses due to bending and consequently the estimates of the fatigue damage along the entire riser. The above methods can take into account the fatigue damage arising from complicated riser motions involving the presence of travelling waves even with the use of very few sensors. An alternate approach using a Van der Pol wake oscillator model is also explored to obtain fatigue life estimates caused by riser VIV.

The most popular suppression devices used for marine risers are helical strakes and fairings. [155,156,157]. The length of the surface treatment may vary. At about 40% of the cable length covered with suppression devices, response may reach the model without any suppression technique while 70% or greater of the cable length covered o is effective at suppressing significant response [155]. Although the geometry and excitation frequencies of marine cables differ from the guywires used in transmission towers, the methodology in analyzing the vibration phenomenon and, especially, technique for fatigue accumulation estimation can be very useful if properly adapted to inclined cables used for guyed towers.

5. Conclusions

The above presented literature review has demonstrated that there is no available research on mitigation techniques for vibrations of guyed transmission towers, which would consider different excitation mechanisms, different soil conditions, precise dynamic behavior of inclined cables. Therefore, such a research represents an innovative scientific contribution to the related technical field. As the literature has also presented, the Finite Elements Method is a proven and useful tool for the analysis of complex dynamic problems and this will be used in the mentioned research.

The proposed research will be based on the aspects of the presented literature review such as guyed mast computational modeling under different types of dynamic excitation and mitigation techniques for the guyed structure vibrations and their efficiency.

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