Dynamic Simultaneous Optimization of Mineral Value Chains under Resource Uncertainty

M. Fernanda Del Castillo Suarez

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- "Optimal mining rates revisited: Managing mining equipment and geological risk at a given mine setup." Del Castillo, M.F., Godoy, M.C., and Dimitrakopoulos, R. (2015), Journal of Mining Science 51(4), pp. 785-798.

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ABSTRACT

Mining complexes are mineral value chains where extracted material from different mines is transformed into sellable products through a set of processing streams. This value chain is governed by uncertainties at different levels, from the geological attributes of the orebody at the mine(s), to the different operational and processing components that lead the sellable products to the market. Stochastic simultaneous optimization formulations for industrial mining complexes have proven to be effective in generating reliable strategic plans that maximize net present value and, at the same time, manage and reduce risk. However, because of the uncertainties governing a mining complex, particularly the ones related to the geological attributes which define the supply of the system, it has become a priority to integrate flexibility mechanisms that allow a mining project to change and adapt as more information becomes available. Within this adaptability, optimizing the investment timing of high-magnitude capital expenditures throughout the life-of-mine is a priority, due to their high impact on the annual cash-flows and on their effects over the physical mining schedule. Additionally, to improve a mining complex's ability to meet production targets and overall performance, advanced mechanisms should be developed to ensure complex blending constraints are met, managing the geometallurgical variables of the deposit.

This thesis presents a methodology to embed flexibility into mineral value chains, by allowing the strategic mine plan of a mining complex to dynamically consider possible options and alternatives for reacting and adapting to future changes. For this, first, a study on extraction capacity optimization is presented, followed by the development of a mechanism to deal with complex variables of the deposit to meet blending constraints and production

targets. These two components are later integrated into a dynamic optimization model, which optimizes the mining complex's mine plan under geological uncertainty, integrating flexible investment alternatives, as well as operational modes, which allow having a better control over complex, nonlinear geological attributes.

A mixed integer programming formulation to optimize a mining operation's extraction capacity is first developed, which deals with the minimization of risk incurred when optimizing mining production rates, so that production targets are met in the presence of geological uncertainty. The model is developed through the concept of a "stable solution domain", which provides all feasible combinations of ore and waste extraction within the ultimate pit limit of a given deposit, independently of the geological risk. The proposed formulation provides an optimal annual extraction rate, together with the optimal equipment acquisition program (i.e., trucks and shovels). This solution eliminates unnecessary capital expenses and is feasible under all geological scenarios. The developed mathematical programming model is detailed and tested at a gold deposit. The results obtained are used as input to a production schedule design, and compared to the schedule generated using a constant mining rate. The comparison shows that about 40% of equipment acquisition can be delayed for 7 years and mill demand still be met, thus maximizing profit and minimizing risk.

Next, the focus is shifted from a mining operation's extraction capacity definition, to its processing streams, particularly on the decision of where a block is sent once it is extracted, i.e. its destination policy. These decisions are particularly important for complex multi-element mining projects with tight blending and processing constraints. The proposed model is able to simultaneously consider a set of geometallurgical variables that affect the performance of the operation, improving the mineral value chain's ability to meet targets, while maximizing the project's net present value. The proposed destination policy is based on coalition formation clustering, a method developed in game theory, to account for the combined value of groups of mining blocks being processed together (even if these combined values are non-linear), rather than on their individual characteristics. Results of an application at a copper-gold mine with six destinations show significant improvements in meeting processing requirements when compared to a conventional industry approach, reducing, for example, deviations of arsenic concentration from 58% to 7% along the life-of-mine, while increasing the project's net present value by 5.6%.

Both extraction capacity definition and destination policy optimization are integrated next, focusing on the complete mining complex, and expanding the formulation to optimize the annual production schedule, as well as a dynamic equipment investment plan of a mining complex. The dynamic model developed produces a unique initial extraction sequence, while keeping a viable flexible long-term plan for future investment decisions, as may be needed. The flexible long-term plan is obtained through a dynamic optimization which allows making transitioning plans upfront to facilitate change. This method introduces a new adapted multistage stochastic programming model which expands upon the two-stage framework by performing multiple recourse stages that are solved iteratively, allowing parallel designs to be generated in a scenario-tree structure. In this model, dynamic decisions over capital expenditures are made sequentially over time, based on new information that becomes available over production time. The investment decision variables activate costs and effects over the model, letting the optimizer choose the type of investment and timing to be done at the mining and/or processing levels. A case study of a mining complex with two mines is used to test the proposed model, with options to invest in the related truck and shovel fleet, as well as a secondary crusher to potentially increase mining and processing capacities respectively. Results show a substantial probability that the mine design should consider the alternative of investing on the secondary crusher, presenting an increase in expected net present value of over US $170M compared to the two-stage stochastic formulation.

The above model is subsequently extended to include alternatives over operating modes at different levels of the mineral value chain. More specifically, the previously developed dynamic decision-making method is used, and the model is extended to choose optimal operating modes per period, selecting blasting patterns at the mine, and processing relations of throughput and recovery at the plant. This mechanism generates new optimized plans that allow and ease the process of adapting once more information is available. The practical implications of the proposed method are demonstrated through an application over a copper-gold mining complex, where the dynamic model presents a 10.5% increase in net present value compared to a traditional two-stage stochastic formulation.

The dynamic mining complex formulation proposed is able to include flexibility into the optimization of the strategic plan of a mineral value chain. This enables possible developments within the feasible set of alternatives that can be taken, considering the mining complex's configuration, capacities, and constraints. The proposed model is able to generate feasible, operational schedules, while providing a wider view of the mining complex's performance, easing the transition to possible changes due to the periodic unveiling of uncertainty.

RÉSUMÉ

Un complexe minier est une chaîne d'approvisionnement où le minerai est extrait de différentes mines et passe au travers d'un réseau de traitement pour être transformé en un produit commercialisable. Cette chaîne d'approvisionnement est sujette à plusieurs sources d'incertitude à différents niveaux, que ce soit au niveau de la mine et les attributs géologiques du gisement ou au niveau des instances opérationnelles et installations de traitement qui permettent d'amener le produit sur le marché. Les modèles stochastiques d'optimisation simultanée pour les complexes miniers ont démontré leur efficacité à générer des plans stratégiques fiables qui maximisent la valeur présente nette du projet minier tout en contrôlant et réduisant les risques y associés. Cependant, à cause des incertitudes qui gouvernent un complexe minier, en particulier les incertitudes liées aux attributs géologiques qui définissent les ressources du système, il est primordial d'inclure des mécanismes de flexibilité pour permettre au projet minier de s'adapter lorsque plus d'information devient disponible. Dans le cadre de cette adaptabilité, optimiser les décisions d'investissement en capital de grande ampleur est une priorité vu l'impact important de ces décisions sur les liquidités annuelles de la compagnie minière et leur effet sur la planification de l'extraction. De plus, afin de permettre à un complexe minier d'atteindre ses objectifs de production et de performance, des mécanismes avancés devraient être développés pour contrôler les variables géo-métallurgiques du gisement et assurer que les contraintes de mélange complexes soient respectées.

Cette thèse présente une méthodologie dont l'objectif est d'inclure de la flexibilité dans les chaines d'approvisionnement minières en permettant à la planification stratégique de

considérer dynamiquement des options et alternatives réalisables pour réagir et s'adapter aux changements futurs. Pour cela, une étude portant sur l'optimisation de la capacité est tout d'abord présentée, suivie par le développement d'un mécanisme pour appréhender les variables complexes d'un gisement afin de respecter les contraintes de mélanges et atteindre les objectifs de production. Ces deux composantes sont ensuite intégrées dans un modèle d'optimisation dynamique. Ce modèle optimise la planification de l'extraction sous incertitude géologique en intégrant des alternatives d'investissement flexibles, ainsi que des modes opératoires qui permettent un meilleur contrôle des attributs géologiques complexes et non-linéaires.

Une formulation en un programme linéaire mixte en nombres entiers pour optimiser la capacité d'extraction d'une opération minière est d'abord développée. Plus spécifiquement, le modèle développé vise à minimiser les risques associés aux taux de production minière. Il suit le concept du « domaine de solution stable », qui permet d'obtenir toutes les combinaisons réalisables d'extraction de minerai et de rejets miniers au sein de la limite extrême de la fosse, et cela indépendamment du risque géologique. La formulation proposée fournit un taux annuel d'extraction optimal, ainsi qu'un programme d'acquisition d'équipement optimal (i.e., camions, pelles). Cette solution élimine les dépenses en capital superflues et de plus, elle est réalisable sous l'ensemble des scénarios géologiques considérés. Le modèle mathématique développé est détaillé et testé sur un gisement d'or. Les résultats obtenus sont utilisés comme données d'entrée à un design de la planification de la production et ce dernier est comparé à celui obtenu à partir d'un taux d'extraction fixe. La comparaison montre qu'environ 40% de l'acquisition de l'équipement peut être repoussée de 7 ans tout en satisfaisant la demande du moulin, maximisant ainsi les profits et minimisant les risques.

Ensuite, l'étude se focalise sur le réseau de traitement des complexes miniers, en particulier la décision de destination d'un bloc après qu'il ait été extrait, i.e. la politique de destination. Ces décisions sont particulièrement critiques pour un projet minier à multiéléments avec des capacités à faibles marges et des contraintes de mélange. Le modèle proposé permet de considérer simultanément un ensemble de variables géo-métallurgiques pour améliorer la capacité de la chaîne d'approvisionnement à respecter les contraintes opérationnelles, tout en maximisant la valeur présente nette du projet. La politique de destination obtenue est basée sur l'agglomération en formations de coalition (clustering). Cette méthode a été développée en théorie des jeux. Dans le contexte des complexes miniers, elle permet de considérer la valeur combinée de groupes de blocs traités ensemble (même lorsque ces valeurs combinées sont non-linéaires) plutôt que leurs caractéristiques individuelles. Les résultats obtenus avec une application sur une mine de cuivre et or, avec six destinations distinctes, ont montré une amélioration significative quant à l'atteinte des exigences de traitement en comparaison avec une approche industrielle conventionnelle. Par exemple, les déviations en concentration d'arsenic sont passées de 58% à 7% durant la période totale d'opérations, alors que la valeur présente nette du projet a augmenté de 5.6%.

L'optimisation conjointe des capacités d'extraction et de la politique de destinations est par la suite intégrée dans une optimisation globale d'un complexe minier en généralisant un modèle mathématique qui optimise la planification annuelle de l'extraction ainsi qu'une planification dynamique des investissements en équipement. Le modèle dynamique développé produit une séquence d'extraction initiale unique, tout en conservant une planification à long terme flexible et viable pour des décisions d'investissement futures, si elles deviennent nécessaires. La planification à long terme flexible est obtenue à l'aide d'une optimisation dynamique qui permet de planifier les transitions à l'avance afin de faciliter les changements. Cette méthode introduit un nouveau modèle stochastique multi-étapes adapté qui étend l'approche à deux étapes en réalisant plusieurs étapes de recours, optimisées de manière itérative, et qui permet de générer des designs parallèles dans une structure d'arbre de scénarios. Dans ce modèle, les décisions dynamiques sur les dépenses en capital sont faites de manière séquentielle au cours du temps et se basent sur de l'information nouvelle qui devient disponible durant la production. Les variables de décision d'investissement activent des coûts qui agissent sur le modèle, ce qui laisse l'optimiseur choisir le type d'investissement et son moment d'application au niveau de la mine et/ou au niveau du traitement. Une étude de cas portant sur un complexe minier avec deux mines est considérée pour tester le modèle proposé. Les options d'investissement concernent l'achat de camions et de pelles, ainsi qu'un second concasseur pour potentiellement augmenter les capacités d'extraction ainsi que les capacités de traitement. Les résultats montrent une probabilité substantielle que le design de la mine devrait considérer l'alternative d'investir dans un second concasseur puisqu'ils présentent une augmentation de la valeur présente nette de plus de 170M $US, par rapport à un mode stochastique à deux étapes.

Le modèle précédent est finalement étendu pour inclure des alternatives sur les modes opératoires à différents niveaux de la chaîne d'approvisionnement minière. Plus précisément, la méthode de décision dynamique précédemment développée est utilisée. De plus, le modèle permet de choisir des modes opératoires optimaux à chaque période. Ces modes opératoires concernent la sélection du schéma de dynamitage au niveau de la mine et la sélection des modes opératoires aux installations de traitement (compromis entre cadence de production et taux de récupération de minerai). La méthode développée génère des plans optimisés, facilitant le processus d'adaptation lorsque plus d'information devient disponible. Ses principales implications et bénéfices sont présentées dans une application sur un complexe minier de cuivre et d'or, où une augmentation de 10.5% de la valeur présente nette est obtenue comparativement à une méthode basée sur une formulation stochastique traditionnelle à deux étapes.

La formulation dynamique proposée dans cette thèse permet d'inclure de la flexibilité dans l'optimisation de la planification stratégique de la chaîne d'approvisionnement minérale. Cela permet de tenir compte d'éventuelles alternatives qui peuvent être considérées, étant donné les configurations du complexe minier, ses capacités et ses contraintes. Les modèles et les méthodes proposés sont capables de générer une planification réalisable et opérationnelle, tout en procurant une vision plus éclairée de la performance du complexe minier, ce qui facilite la transition à de possibles changements lorsque l'incertitude est révélée.

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CHAPTER 1
General Introduction

1.1 Introduction

A mining complex is a mineral value chain which consists of a set of components, such as mines, stockpiles, waste dumps, and processing plants, that are linked by multiple transportation systems that move material from the supply to the final customers and spot market. Figure 1-1 presents a diagram of the continuous flow of material of a mining complex. The supply of these value chains consists in a set of mines, connected to transitional components involving the stockpiles, waste dumps, and processing streams, such as plants, leach pads or bioleaches, which transform the raw material into sub-products. These sub-products are then transported to their final destination, consisting of ports that deliver the material to customers, or waste dumps and tailings, where material of no value and waste from metallurgical processes is sent.

The goal of optimizing the life-of-mine (LOM) of a mining complex is to maximize value given a set of environmental and operational constraints. Each mine is represented as a set of three-dimensional blocks, which are scheduled to be extracted at a certain period given the mining complex's extraction capacity and operating modes. These blocks are extracted to be processed for profit, and/or to access underlying blocks of the orebody. Processing streams receive this extracted rock, and treat the material selecting an operating mode, usually aiming at maximizing recovery and minimizing processing costs, ensuring that a set of capacity and blending constraints are met. These constraints can become especially complex when dealing with multi-element mines, or in the presence of deleterious elements that need to be controlled. Finally, the produced material is transported, delivered to customers or sold on the open spot market.

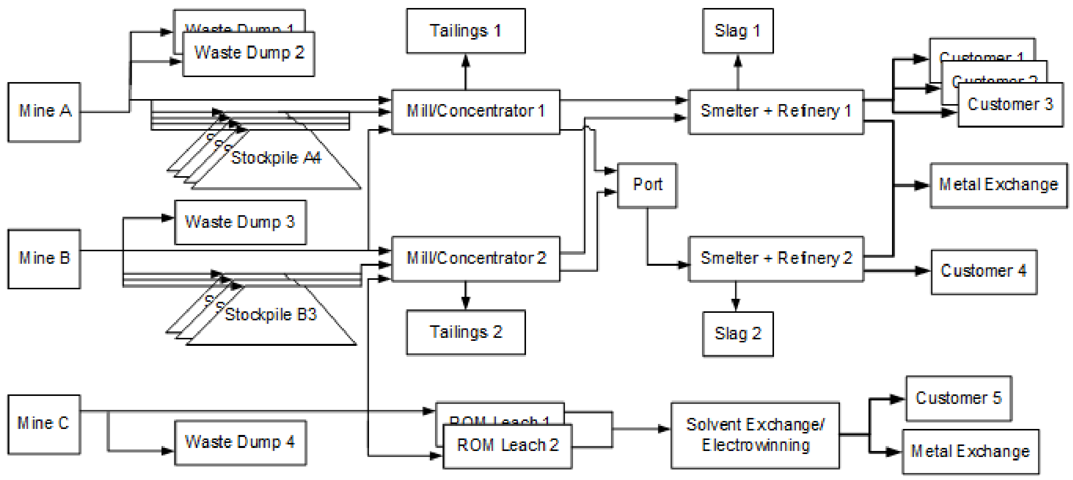


Figure 1-1 Diagram of a mining complex (Goodfellow, 2014)

As Figure 1-1 shows, that a mining complex is a network of interrelated, non-linear components, which strongly depend on each other. As an interconnected value chain, a mining complex should be optimized globally and simultaneously, taking into account the synergies that exist between its components. However, traditional mine planning optimization techniques ignore the interactions between components in a mining complex, optimizing each one independently. New research on global or simultaneous optimization of a mining complex focuses on integrating all its components to generate one global mine plan which accounts for the whole value chain and its interactions. These models aim to coordinate and simultaneously optimize the multi-mine production schedules, destinations policies, as well as the use of the various material handling methods.

Another major limitation of traditional mine planning optimization methods is that they assume that the deposit is known with certainty by using a unique estimated orebody model to represent the characteristics of the material available on the ground. With this, the variability and uncertainty of material concentrations and material types are ignored, and so are the effects they have on the value and operational feasibility of the production schedule.

Multiple optimization models based on stochastic mathematical programming have been developed to include uncertainty. Within the mining industry, stochastic optimization has been present for over a decade, using a set of orebody simulations to take into account the grade variability of the deposit. These methods have proven to produce reliable mine-plans and production schedules that maximize value and manage project's risk, and in recent years, have been extended to consider the whole mining complex. Stochastic simultaneous optimization of a mining complex integrates the effect of geological uncertainty and variability into the global optimization model. By doing so, they produce mine plans and production schedules that can meet blending requirements, manage technical risk, and maximize project value, showing clear benefits both in value and in reliability of meeting production targets when compared to traditional industry practices.

However, so far, the developed models have been limited in dealing with some critical aspects, such as treating complex geometallurgical variables. These variables are crucial in defining the processing performance of a mining complex, and dealing with them becomes especially important in multi-element mines with complex geological structures and processing streams where metallurgical processes demand tighter blending constraints. Another critical aspect of current models is that they produce fixed, static mine plans, which assume that the setting of the mining complex will stay the same, and thus, that the initial solution will be optimal for the life-of-mine (LOM). This assumption is an optimistic simplification. Consequently, conventional mine plans are updated yearly; however, this is a passive solution that can unintentionally inhibit options and alternatives that, though not currently viable, could be profitable in the future. An example of these alternatives is the investment in critical capital expenditures which may affect the value chain's configuration and/or capacity. The conventional way to deal with these alternatives is through sensitivity analyses. However, these analyses are static and do not consider uncertainty, or its effect in the mining complex's context, usually resulting in loss of opportunities and delayed projects, hindering the transition to change, and ultimately resulting in loss of profit. Another type of alternatives is the operating mode alternatives, which exist at different levels of a mining complex, from the mine to the processing streams. Operating mode alternatives can be particularly useful in the optimization of a mining complex when dealing with components of the value chain with specific requirements, designed for certain type of input materials. These requirements are usually ignored by conventional optimizers, and because of the value chain's inherent uncertainties, meeting them can be a hard task. By considering and integrating the different operational alternatives into the optimization, these hard constraints can be better met by tuning the processing streams to the specific characteristics of the material being treated, maximizing the mining complex's performance and its value.

These extensions are crucial, as the performance of these processing streams and of the mining complex as a whole greatly depends on how the different requirements and constraints are met. For example, in a processing plant, blending constraints must be met to maximize metallurgical recovery, which in turn will maximize project value. Maximizing value and meeting these requirements can be aided by considering the mining complex's flexibility alternatives. However, to take full advantage of opportunities, and obtain feasible and reliable mine plans, these alternatives must be considered in the initial evaluation processes, enabling decision-makers to prepare in advance for future changes. To do so, more sophisticated methods must be developed, and flexibilities must be translated into a set of identified, feasible, dynamic alternatives that can be exercised by a particular mining complex, specifically designed upfront to be effectively modelled.

1.2 Literature Review

This section presents a review of the literature pertinent to the topics discussed in this thesis. Section 1.2.1 discusses the strategic optimization of the mine plan, from modelling and including geological uncertainty, to simultaneously optimizing the mining complex under uncertainty, presenting different solving algorithms developed and their extensions, and concluding with limitations to the current state-of-the-art formulations. Section 1.2.2 describes research avenues to include flexibility in a value chain optimization under uncertainty, focusing on real options analysis and stochastic multistage programming formulations.

1.2.1 Strategic or life-of-mine planning

The focus of strategic mine planning is to generate designs and mine production schedules that meet targets and maximize discounted cash-flows over the life of a mining operation. This is done by firstly representing an orebody as a set of three-dimensional blocks, where each block has values for the attributes that define the deposit, such as metal grades, material types, rock properties, or tonnage. According to their characteristics, blocks are scheduled to be extracted at a certain time period (year), to be processed, and/or to access underlying blocks of the orebody (Hustrulid et al., 2013). Traditionally, mining complexes are simplified and optimized in a step-wise fashion, as disconnected components with individual goals (Dagdelen, 2001); for example, the mine may be optimized to minimize cost, while the processing plant may be separately optimized to maximizing recovery (Lerchs and Grossmann, 1965; Johnson, 1968; Kim, 1979; Gershon, 1983; Tolwinski and Underwood, 1996; Hustrulid et al., 2013). This process ignores the synergies that exist between a mining complex's components, producing independent sub-optimal plans and production schedules. At the same time, these conventional plans use estimated orebodies as only input, and ignore the uncertainties that govern the mining complex, such as the deposit's geological attributes of contents, material types, or the market's commodity price.

Because of the high costs associated with exploration, the limited information obtained from exploratory and development drill-holes, and the geological complexity of mineral deposits in general, the characteristics of interest of the orebody considered are highly uncertain at the scale of mining, being one of the main sources of risk in a mining operation (Ravenscroft, 1992; Dowd, 1994, 1997; Vallee, 2000; Godoy and Dimitrakopoulos, 2004). Considering the deposit as known by using a single estimated orebody model to represent the material available in the ground is a major assumption that biases the mine planning process. Traditionally employed geological estimation methods produce a single representation of the deposit, which is smoothed due to the averaging that occurs during the estimation (David, 1977, 1988; Journel and Huijbregts, 1978; Isaaks and Srivastava, 1989; Goovaerts, 1997; Rossi and Deutsch, 2014). These estimated models misrepresent the proportions of material concentrations, and ignore their variability and uncertainty, as well as the effects this has on the value and operational feasibility of the production schedule (Dimitrakopoulos et al., 2002). Godoy (2003) and Godoy and Dimitrakopoulos (2004) state that geological uncertainty is the major contributor to not meeting project expectations. This, because the optimization of a mine design entails non-linear transfer functions, and using a deterministic model to optimize it can provide misleading results. In their study, the authors demonstrate that including this uncertainty significantly reduces the deviation from production targets, and at the same time, it increases the total value of the project.

1.2.1.1 Modelling geological uncertainty

The importance of accounting for geological uncertainty has been documented for decades (Journel, 1974; Dowd, 1976, 1994, 1997; Journel and Huijbregts, 1978; David, 1988; Ravenscroft, 1992; Goovaerts, 1997; Rossi and Deutsch, 2014).

Journel (1974) states that for lack of perfect knowledge of the fluctuations of different characteristics of the extracted material, simulations of the reality are needed to model the deposit's spatial uncertainty of the characteristics of interest, such as the grades or the stripping ratio. Stochastic simulation can be used to generate equally probable representations of the orebody, which respect the spatial correlation and local variability of the deposit, providing a probabilistic assessment of a variable over a group of blocks (Journel and Huijbregts, 1978; Isaaks, 1991; Chiles and Delfiner, 1999; Godoy, 2003; Remy et al., 2009; Rossi and Deutsch, 2014).

Among the different stochastic simulation techniques, an efficient and straightforward method to generate multiple equally-probable representations of a deposit is Direct Block Simulation (DBSim), thoroughly described in Godoy (2003). DBSim combines the upside characteristics of LU decomposition method (Davis, 1987), with the qualities of the well-known sequential Gaussian simulation (SGS) (Goovaerts, 1997), and is a step forward in a more computationally efficient method than the generalized sequential Gaussian simulation (GSGS) described by Luo (1998). LU method (Davis, 1987) is capable of simulating simultaneously and in a fast way a group of nodes; however, it is a computationally expensive method, as the decomposition of the covariance matrix into its lower and upper components (thus the name LU) requires the order of n^3 computations for a matrix of dimensions 'n x n', i.e., with 'n' nodes to simulate. On the other hand, SGS has the upside of being easily implemented but can turn to be very slow as the number of nodes 'n' to simulate increase.

DBSim method starts by dividing the volume to be simulated into groups of nodes, generally accordant with the dimensions of the selective mining unit (SMU) defined by the operation (Godoy, 2003; Dimitrakopoulos and Luo, 2004; Boucher and Dimitrakopoulos, 2009). Subsequently, each of the groups is visited following a random path, sequentially simulating the internal nodes of each group by LU decomposition, which in these conditions is a fast and feasible method given the reduced size of the groups. The main difference between DBSim and GSGS, is that, once the internal nodes of a group are simulated, instead of keeping all the information from each simulated node, as in GSGS, DBSim averages the simulated values of the group, and only stores this resulting value, liberating the memory required to store each individual node. This mechanism aims at saving computational time by accounting in advance for the subsequent re-blocking of the deposit, which is a common first step in the mine planning process, where point-support simulated nodes are averaged into the mine's SMU dimensions. Because of this memory liberation, DBSim becomes computationally inexpensive and simple to implement.

Together with this, DBSim can easily be extended to the simulation of multi-element deposits by using methods such as the minimum/maximum autocorrelation factors (MAF) (Desbarats, 2001; Rondon, 2012). This method de-correlates the variables of interest so that they can be independently simulated (through a method such as DBSim) and then re-joined to generate the multivariate simulations. In this case, however, when using DBSim, a double storage of the data must be done: once the internal nodes of a block are simulated, they must all be back-transformed into their original coordinates, and subsequently averaged and stored outside of the simulation space. On the other hand, the nodes in the simulation space must also be averaged (in their transformed coordinates) to be used to condition the remaining simulation. Clear examples of this process can be found in Benndorf and Dimitrakopoulos (2018) and Boucher and Dimitrakopoulos (2009, 2012).

All previously mentioned methods are Gaussian based, where all conditioning data needs to be transformed into normal-space before being used to simulate the deposit. These methods use variogram models as basis to represent the deposit's statistics, which is a second-order measure of the deposit's spatial continuity. However, second-order statistics are unable to correctly reproduce complex structures of the deposit. Extensive literature can be found on new generations of geological simulation methods. This latest research focuses on exploring the higher-order statistics of the geological data by doing multi-point simulations, which are able to capture complex structures of the deposit, obtaining more information from the conditioning data, and improving the reliability of the simulations generated (Strebelle, 2002; Arpat and Caers, 2007; Remy et al., 2009; Dimitrakopoulos et al., 2010; Mustapha and Dimitrakopoulos, 2010b, a, 2011; Mustapha et al., 2011).

Most high-order methods replace the variogram model by training images, which are a geologic analog of the deposit's mineralized variable (Zhang et al., 2006; Minniakhmetov and Dimitrakopoulos, 2017a). Abolhassani et al. (2017) implements training images in conjunction with machine learning techniques to develop a high-order, nonstationary sequential simulation method, showing promising results. However, generating these training images is an issue in itself (Goodfellow et al., 2012), and thus, new fully data-driven methods are being developed (Minniakhmetov and Dimitrakopoulos, 2017b). All these advancements are done with the goal of producing better quality, and more reliable representations of the deposit to use as input to the mine planning optimization.

1.2.1.2 Strategic mine planning under uncertainty

Conventional mine planning approaches (Hoerger et al., 1999) ignore the non-linear characteristics of the mine design and production scheduling processes, and assume that the optimization of the mine plan using an estimated orebody as input to a deterministic model will provide an average output solution. This is a major assumption due to the non-linear transfer functions that take place during the mine planning optimization (Ravenscroft, 1992; Dimitrakopoulos et al., 2002). Using estimated orebody models as input to the model produces biased mine plans that ignore the data's local variability. Monkhouse and Yeates (2005) note that conventional LOM optimization methods ignore uncertainty and are full of assumptions which will surely be wrong, rendering the supposed optimal solutions suboptimal under the uncertain real world.

A first approach at accounting for geological uncertainty in the mine design and production scheduling process was made by Godoy (2003) and Godoy and Dimitrakopoulos (2004), who use simulated annealing (SA) (Kirkpatrick et al., 1983; Geman and Geman, 1984; Osman and Laporte, 1996). Here, the authors produce a set of schedules, each optimal to one of the geological simulations used, and use them as input to an optimization model which jointly considers them to generate a single schedule that minimizes overall deviations from ore and waste production targets.

A similar analysis is done by Leite and Dimitrakopoulos (Leite and Dimitrakopoulos, 2007) who apply this method to a copper mine obtaining a 15% increase in NPV, even though maximizing project value is not the direct objective of the formulation. As in Godoy (2003) and Godoy and Dimitrakopoulos (2004), the optimization mechanism used consists of three steps. First, the ultimate pit limit and mining rates is optimized. Next, independent schedules are obtained for each simulated orebody, and finally, these independent schedules are combines through an annealing schedule. The final schedule has an increased NPV compared to the traditional schedule, however, because of the assumption of a pre-defined optimal ultimate pit limit in the first step, it has a decreased life of mine compared to the traditional schedule.

Similarly, Albor and Dimitrakopoulos (2009) use SA to optimize an open pit mine production schedule under geological uncertainty, and address the impact of different crucial tuning parameters of the algorithm, such as the effect that the number of initial schedules has on the convergence of the results, and the effect of different initial solutions. An application on a case study shows a 25% increase in NPV when compared to traditional scheduling methods. Just as in Leite and Dimitrakopoulos (Leite and Dimitrakopoulos, 2007), initial results present a shorter life of mine for the stochastic solution, however, all these studies assumed a fixed ultimate pit limit defined based on the Lerchs-Grossman algorithm (Lerchs and Grossmann, 1965) with discounting, for both deterministic and stochastic cases. In their study, the authors prove that by leaving the ultimate pit limit free, the stochastically optimal pit limit is actually larger than the one designed by conventional optimizers (17% larger in the case study presented, in terms of total tonnage), increasing the life of the project in one year when compared to the traditional plan of the case study. The authors show that this effect is consistent throughout multiple case studies.

Another approach is proposed by Meagher et al. (2009) who include price and exchange rate variability as well as geological uncertainty for pushback design formulating the problem as a max flow problem and using a minimum cut algorithm. The authors apply their method to a copper open pit mine, and their results show an increase in project value of 10 to 50% along the life of the mine. Asad and Dimitrakopoulos (2013a) present a parametric maximum flow algorithm to optimize the open pit mine design under uncertain supply (geology) and demand (commodity price). The optimization uses multiple simulations of both uncertainties to generate time-dependent discounted block values, and creates a graph based on this framework. In their paper, the authors focus on the optimization of phase designs and ultimate pit limit, and an application at a case study shows that the stochastic pit limit is 45% larger than the one obtained with conventional methods, presenting both higher NPV and metal production. Asad et al. (2014) extend the work done by Asad and Dimitrakopoulos (2013a) to consider the phase design and ultimate pit limit optimization of a mining complex with multiple ore processing streams under geological uncertainty. Results of a case study show that the stochastic solution produces an almost 9% larger ultimate pit limit, with a 14% higher NPV and a 10% higher copper production when compared to results obtained through current practices. More recently, Chatterjee et al. (2016) introduce an open pit mine design optimization approach under commodity price uncertainty and also focus on the design of production phases and ultimate pit limit definition. The authors present a novel method for commodity price simulations, based on sequential Gaussian simulation with smoothing splines, and present a stochastic formulation which efficiently considers these simulations simultaneously. However, the paper does not consider geological uncertainty or the actual scheduling of the mining operation.

To date, different methods have been developed to allow the integration of geological uncertainty into the design and evaluation of a project, most of them based on stochastic programming. Birge and Louveaux (1997) define stochastic mathematical programming as formulations where parameters of the formulation are assumed to be uncertain (i.e. random fields), with a known probability distribution assigned to them. To include this uncertainty, a set of scenarios is considered simultaneously throughout the optimization process to obtain the production schedule, and not independently as in the previous cases mentioned (Ramazan and Dimitrakopoulos, 2004a; Leite and Dimitrakopoulos, 2007).

Ramazan and Dimitrakopoulos (2004b, a) aim at solving the long-term mine production scheduling optimization under geological uncertainty, and model it as a two-stage stochastic integer program with fixed recourse to minimize deviations from production targets. In their model, the set of orebody simulations is jointly used as input, and decision variables are grouped into extraction and processing decision variables. The first, correspond to 1^{st} stage decision variables taken under uncertainty, and the second, to 2^{nd} stage variables taken as recourse, after some or all of the uncertainty has been uncovered (Birge and Louveaux, 1997), defining where a block is sent after it is extracted. The authors also include extra constraints to generate smooth, operationally feasible schedules for a complex multi-element deposit. Ramazan and Dimitrakopoulos (2013) extend this model and account for leaching, as well as sending and retrieving material from a stockpile. The authors also implement a geological risk discounting rate (GRD) introduced by Dimitrakopoulos and Ramazan (2004), which increases the cost of deviating on the initial years of operation, ensuring a steady feed, and decreases it towards the end of the LOM, when more information is available. To reduce the computational cost of the formulation, the optimization is first done without considering the stochastic constraints related to grade blending, processing, and metal production, and this initial solution is later used as a starting point for the full SIP model. Together with this, to reduce the computational cost even further, waste blocks are considered continuous variables, and only ore blocks are considered integers. However, to ensure slope constraints are respected, a waste block must be completely extracted before its underlying predecessors can be mined.

Lamghari et al. (2013) also formulate the mine extraction scheduling problem of an open pit mine under geological uncertainty as a two-stage stochastic programming model and use a Variable Neighbourhood Descent (VND) metaheuristic algorithm (Mladenović and Hansen, 1997; Hansen and Mladenović, 2001) to solve it. Two different metaheuristic approaches are proposed, differentiated in the way the initial solution is created, and later improved by using VND algorithm. Both methods proposed present very favorable results when compared to the linear relaxation of the problem (solved using CPLEX (2011)), presenting a maximum gap of 2% and 5% respectively. Lamghari et al. (2015) later develop a two-phase hybrid method to optimize the open pit schedule, which uses a series of linear programming models that are sequentially applied to develop an initial solution. This initial solution is later improved by applying a VND heuristic to three neighbourhoods. The proposed formulation is able to solve a large-scale NP-hard problem in a few minutes, presenting less than a 3.2% gap with respect to the linear relaxation of the problem obtained using ILOG CPLEX (2011).

Other applications of stochastic methods in comparison with the deterministic assessment can be found in the literature (Sabour and Poulin, 2006; Musingwini et al., 2007; Dimitrakopoulos and Sabour, 2007; Albor and Dimitrakopoulos, 2010). However, all the mentioned studies aim at optimizing the mining schedule as an independent component of the mining complex and don't account for the whole mineral value chain and the synergies that exist between its different components.

1.2.1.3 Global or simultaneous optimization

During the last few years, mine planning research has evolved to focus on optimizing all stages of a mining complex simultaneously, from the mine to the final costumer, being referred to as global or simultaneous optimization (Hoerger et al., 1999; Whittle, 2007, 2010a, b; Pimentel et al., 2010; Bodon et al., 2011). Pimentel et al. (2010) introduce a mining operation as a supply chain, served by logistic channels, and develop a decision-support system to address a global mining supply chain as an integrated system. In their paper, the authors discuss work done on ultimate pit selection, stochastic optimization, and the importance of blending, among others. In addition, different possible solution approaches to the integrated mining supply chain are discussed, concluding that heuristics would be the best alternative for optimizing the production plan of any real-world mining supply chain, due to its size and complexity. Whittle (2007) introduces the Prober algorithm, a software designed to optimize multiple mines with complex blending constraints and processing destinations. The algorithm works by iteratively creating random feasible solutions, which are locally improved by defining optimal cut-off grades and downstream decisions, such as blending, stockpiling, and general production plan. To reduce the complexity of the model, the algorithm aggregates blocks by material types, which are in turn grouped into panels. In the model, however, it is assumed that blocks in a panel are extracted in the same proportion, indirectly violating slope constraints, as blocks in a lower bench may be partially mined before their overlying blocks are fully extracted. Whittle (2010b) introduces ProberC, an advanced version of the algorithm introduced in Whittle (2007), where more complex costing structures and different configurations of mining operations can be optimized. However, the optimization is still deterministic, using an estimated orebody model which misrepresents the proportions of material grades, and ignores the uncertainty and local variability of the grades and material types of the deposit. Similarly, Whittle (2010a) presents an "enterprise optimization", which consists on ten steps to follow to simultaneously optimize a mining operation using Whittle Software (Whittle, 1999), stressing that decisions made at any point in the mineral value chain can potentially affect the decision for all other points in the chain. However, just as in the previous studies, uncertainty is not considered. Whittle (2014) notes how company policies can often hinder the simultaneous optimization efforts by establishing misleading targets and objectives which are not focused on maximizing value, and highlights the importance of creating integrated teams to optimize a mining complex, consisting of mining engineers, metallurgists, accountants, etc. In his paper the author discusses the difficulties that entail shifting the paradigm of a whole mining company from its traditional optimization methods, but highlights that, based on different case studies, the returns for doing so can be considerable.

Stone et al. (2007) present Blasor, an optimization tool formulated as a mixed integer linear problem, which is able to optimize the extraction sequence of multiple mines, stockpiles and processing streams, and is solved with ILOG CPLEX (2011). The computational cost of the formulation is reduced by aggregating connected blocks by the similarity of their properties, considerably reducing the number of variables in the model. The authors show how their approach improves performance compared to the traditional independent optimization of the value chain's stages. Zuckerberg et al. (2007) present an extension of the Blasor optimizer, BlasorIPD, specifically designed for waste handling through in-pit-dumping. This extended model can flag areas of the pit which have been extracted and can be filled with waste material, ensuring no ore access is lost, and that slope constraints are respected after the waste disposal. However, in addition to the aggregation of blocks, none of these models account for the inherent uncertainties governing the model.

These integrated formulations can simultaneously optimize a mining complex, taking advantage of the synergies that exist between its components. However, due to the size and complexity of the problem, these models have required major simplifications to obtain linear formulations, solvable in a reasonable amount of time. Such simplifications are, for example, avoiding to model stockpiles given their non-linear relations, aggregating blocks, which lead to infeasible schedules and slope violations, or using pre-calculated economic values of blocks. Despite these facts, arguably the strongest assumption of the mentioned simultaneous optimization methods is ignoring the uncertainty related to the geology, which, as mentioned in section 1.2.1.1, is a key parameter of the mining complex.

1.2.1.4 Optimising mining complexes under uncertainty

Stochastic simultaneous optimization of a mining complex integrates the effect of geological uncertainty and variability into the global optimization model, producing mine plans and production schedules that are able to meet complex blending requirements, manage technical risk, and maximize project value, while accounting for all components of the mining complex (Montiel and Dimitrakopoulos, 2015, 2017; Farmer, 2016; Montiel et al., 2016; Goodfellow and Dimitrakopoulos, 2016, 2017).

Montiel and Dimitrakopoulos (2013) present one of the first efforts to model the whole mining complex and optimize the extraction sequence under geological uncertainty, considering both, material type and ore grade uncertainties. The authors consider multiple ore types which, according to their characteristics, can be processed in a set of different processing streams. The proposed method is applied over a world-class copper open-pit mine, reducing production deviations to less than 5%, compared to the original schedule which presented deviations as high as 20%, and, at the same time, increasing the expected net present value by 4%. However, the formulation presents limitations in its destination policy decisions, which are the decision of where a block is sent after it is extracted. This happens when the material type of a block varies between simulations, as some processes accept only certain material types, and misclassification errors must be avoided, making it hard to define a block-based destination decision. Also, no stockpiles or external sources are considered in this mining complex configuration.

To deal with these limitations, Montiel (2014) and Montiel and Dimitrakopoulos (2017) developed a robust destination policy, and optimize the whole mining complex with multiple processing streams under grade and material type uncertainty. In their model, the average profitability of each destination is ranked for each block, and the optimizer defines the final destination of a block as a knapsack problem. This way, the most valuable blocks are sent to their first ranked destination, and as processing capacities are met, remaining blocks are sent to their second ranked destination, and so on, until all extracted blocks have a defined destination. The proposed model is able to develop a mining schedule that defines when each block is mined, and where it is sent.

The previously mentioned stochastic mining complex optimization methods penalize deviations from targets by defining capacity limits as soft constraints. This mechanism minimizes deviations but does not eliminate them. Thus, when executing the mine plan, there may still be cases of lack or excess ore in main processing streams. To deal with this issue, Navarra et al. (2018) present a method to optimize the processing of excess ore, in case processing capacities of a mining complex are exceeded. This mechanism considers adapting the process' cut-off grade to increase ore selectivity in case the ore feed exceeds that given processing plant's capacity.

Menabde et al. (2007) discuss the stochastic Blasor, an extension of the optimization tool presented by Stone et al. (2007) which considers material grade uncertainty through a set of simulated scenarios. The authors also define a robust destination policy based on cut-off grade optimization which accounts for geological uncertainty, and present a MIP formulation where the destination policy is defined to ensure that blocks with similar grades are sent to the same destination on each given period. The destination policy is defined by discretizing the grade distribution of the different orebody simulations into bins and defining a particular bin limit as the optimized cut-off grade for each period, according to the amount of ore and waste in the different scenarios. The proposed method provides a robust overall policy and solves the non-linearity issues of the formulation, and by doing so, it is able to avoid misclassification problems. Here, the authors implement an aggregation mechanism that groups blocks into panels, reducing the number of binary variables in the model considerably. However, this formulation only accounts for a mine with just one element and a single processing stream, defining the cut-off grade that decides if material is ore or waste. Thus, it is limited as it does not allow the classification of multiple attributes or multi-element deposits, where more than one element must be considered simultaneously to meet blending requirements.

One of the main challenges when modelling the stochastic simultaneous optimization of a mining complex, together with the mentioned destination policy problems, is the non-linear transformations that appear when stockpiles and blending constraints are included into the formulation. Goodfellow and Dimitrakopoulos (2016, 2017) propose a general formulation of a mining complex which allows modelling different value chain relations under geological uncertainty, without falling into the simplifications mentioned in the previous section. The authors define primary and hereditary attributes to model the flow of material through the mining complex, where primary attributes correspond to additive characteristics (metal content, tonnages, etc.), whereas hereditary attributes are derived from primary ones (such as grades, which can be calculated as metal tonnage divided by the block's total tonnage, or processing recoveries, economic value, etc.). To deal with the destination policy issues, the authors extend the bin mechanism presented by Menabde et al. (2007), and propose a k-means++ clustering mechanism to pre-process the deposit and classify blocks into different clusters according to their value over multiple variables (such as material type and concentration of different elements). With this clustering mechanism, the destination policy is decided annually for each different cluster, and not for each individual block. The advantage of this method is that it allows accounting for multi-variate relations when defining the destination of a block, which is a necessity when optimizing multi-element mines or complex blending requirements. Also, through clustering, the authors are able to produce a destination policy guide which can provide the optimal destination of a block according to the value of its attributes, by defining its cluster membership.

The destination policy decision can be taken based on multiple aspects, such as defining certain ranges of grades accepted at the different destinations, commonly referred to as cut-off grades (Lane, 1988; Whittle and Wharton, 1995; King, 1999; Rendu, 2014), or based on the general revenues expected from sending a block to each of the possible processing streams. However, this last policy entails a serious oversimplification, which is to assume that a block has a dollar value (Lerchs and Grossmann, 1965; Tolwinski and Underwood, 1996; Ramazan, 2007; Meagher et al., 2009) and not material, which must be blended, treated, processed, refined, etc. in order to actually receive some profit. This last point entails strong biases over the process, as assuming that a block has a pre-defined value disregards, for example, the effect of time value of money. The project's discount rate, not to mention the current market price of the commodity being mined, will ultimately define the value of a block, but only once it has been mined, processed and sold. Together with this, assuming a pre-defined dollar value of a block entails defining when and where this block will be processed, ignoring the effect of non-linear recovery curves and blending requirements at the different processing streams, which depend on the resulting characteristics of the group of blocks being processed together (Ramazan and Dimitrakopoulos, 2004b, a, 2013; Wharton, 2004; Stone et al., 2005, 2007; Whittle, 2010b; Lamghari and Dimitrakopoulos, 2012; Leite and Dimitrakopoulos, 2014).

In models that assume an economic value of the blocks, usually, the optimizer decides where an extracted block should be sent based on its concentration being higher or lower than a cut-off grade of some principal element. In reality, blocks have different attributes and concentration of elements which must be extracted, transported, blended, processed and sold, in order to obtain an actual financial gain from them. This process is also strongly affected by the geological uncertainty present in the deposit, which will ultimately define the performance of processing stream chosen to treat the extracted block. Thus, the actual value of a block depends (i) on the period when it is extracted, (ii) on the quality of the elements contained in it, (iii) on the current price for each of these elements, as well as iv) on the destination where the block is processed, which entails the blending constraints, processing costs, recovery curve of the metallurgical process, etc. Most of which are non-linear aspects which are avoided in most conventional optimizers.

Meagher et al. (2009) work on designing dynamic destination policies, where the destination decisions are updated according to new information that becomes available once a block is extracted. The study considers geological and market uncertainties, as well as the time value of money by calculating the value of a block according to its period of extraction. However, the proposed model only accounts for one mine with one element which can be processed in one processing plant, and the model would grow exponentially if a mining complex with multiple elements, deposits, and processing streams was considered. Together with this, the focus is still placed on assigning an individual dollar value to each block, instead of optimizing the complex as a whole. Meagher et al. (2014) develop a dynamic cut-off grade policy to define block destination, where the optimal cut-off grade is defined on a yearly basis in order to optimize the pushback design and maximize project value. However, as in Meagher et al. (2009), the model only considers one element with one processing facility, and the optimization is done greedily by sequentially maximizing the NPV of each pushback, instead of optimizing the whole deposit simultaneously. Together with this, their method is not extended to the optimization of the mine scheduling problem.

Asad and Dimitrakopoulos (2013b) develop a cut-off grade optimization model for an open pit mining complex with multiple processing streams, under uncertainty in ore supply. The proposed model is defined as an extension of Lane's model (Lane, 1988) to include geological uncertainty, aiming at maximizing NPV while minimizing deviations from production targets. An application at a large copper mine presents increases in NPV of over 13%. However, even though the optimization provides the optimal annual cut-off grade and extraction capacities, it does not provide an actual schedule, and the method is only able to account for one mine with one element.

Montiel and Dimitrakopoulos (2015) propose a global mining complex optimization model under grade and material type uncertainty, with multiple processing and transportation alternatives. The authors test their formulation over a copper mine with two pits, with multiple processing streams and complex blending constraints, and show that their proposed stochastic method not only improves the project's NPV by 5% when compared to the traditional deterministic method (which is shown to strongly violate blending and processing constraints), but it considerably reduces the risk of not meeting blending constraints and processing capacities at the different destinations. The proposed model also includes alternatives over the processing plant's grinding size, as well as two different transportation systems with different assigned costs and capacities, allowing the optimizer to choose the optimal grinding configuration and transportation system. This configuration tuning allows increasing the efficiency of the optimization model, and at the same time, produces more realistic plans which adapt to the characteristics of the material being extracted and treated, tackling some of the most important limitations of previous models.

The different operating modes defined by the authors affect, on the one hand, processing variables, and on the other hand, transportation system alternatives. The first, tackle variables such as metallurgical recoveries, operating costs, blending constraints and throughput of the processing destination, where for example, a processing plant using fine grinding will have higher recovery, but also higher costs and lower throughput than a plant operating at coarse grinding. The second alternatives vary the available capacity and operational costs provided by the different transportation systems. The proposed model aims at maximizing project value and simultaneously minimizes penalties related to deviations from production, transportation, processing and metal targets. Due to the size of the problem, the authors implement a SA algorithm that iteratively perturbs an initial solution at each decision level, until a stopping criterion is met. However, despite the alternatives available, the final solution is still static, and assumes that the setting of the mining complex will stay the same along the LOM.

Considering operational mode alternatives such as the ones mentioned in Montiel and Dimitrakopoulos (2015) allows having a better control over geometallurgical variables that affect the mining complex, such as for example, considering throughput as an active parameter that can be tuned according to the material fed to the plant; or adapting the mines' blasting pattern according to the hardness of the rock being blasted. Dowd et al. (2016), discuss that describing geological, operational and geometallurgical uncertainties and integrating them into the optimization process is one of the main challenges in strategic mine planning nowadays.

It has been seen that, as the complexity of mining projects increase in terms of number of deposits, processing streams and number of elements, traditional optimization methods and destination policies lack in their ability to consider the multidimensional aspect of the mining optimization problem. Recent work on destination policy has extended from cut-off grade optimization to integrate multivariate distributions, which makes them more adept to complex mining projects.

Goodfellow (2014) models the mining complex under geological uncertainty as a two-stage stochastic formulation, and adds the decision variable of investing in capital expenditures (CAPEX). Their work extends from the model proposed by Godoy and Dimitrakopoulos (2004) for production capacity selection and equipment acquisition, and lets the optimizer define the truck and shovel fleet size and purchase plan (i.e., the mine's extraction capacity). In his model, different operational details such as lead times, or the life of the equipment are also included. Farmer (2016) also works on integrating into the model the optimization of mining and processing capacities. Here, the author uses a case study to compare the traditional deterministic plan, with the two-stage stochastic formulation one, showing that the stochastic model with optimized capacities increases the project value by 12% compared to the deterministic solution. Goodfellow (2014) also extends the work proposed by Menabde et al. (2007) on destination policy, and implements a k-means++ clustering mechanism to define the processing destination of blocks considering multiple attributes. K-means clustering is a well-known, fairly robust mechanism to group data which is easy to implement (Arthur and Vassilvitskii, 2007; Gan et al., 2007). This clustering method allows the development of a robust destination policy which accounts for multiple attributes and material types, as well as for geological uncertainty. This way, as in Menabde et al. (2007), blocks with "similar" attributes belong to the same cluster (or bin in Menabde et al. (2007)) and are sent to the same destination. Together with this, the author notes that this clustering mechanism can be implemented for predictive data analysis, to define a trend and classify new data. This predictive mechanism is particularly useful to make decisions over new information obtained during extraction, as each newly extracted block can be represented as a data point according to its characteristics, and plotted on the clustering grid, defining its destination based on its cluster classification. Goodfellow and Dimtrakopoulos (2014) use this mechanism and develop a stochastic optimization model of a mining complex which accounts for geological uncertainty, and considers a multidimensional destination policy.

Though highly interesting, this method does not directly take into consideration the blending constraints that some processing streams entail. For example, if particularly tight metallurgical constraints are required by a process, such as a specific silica-magnesium ratio, if a block has a high magnesium concentration, and another block has a high silica concentration, then it would be preferable to redefine clustering to group these two blocks to be processed together, even if their attributes are not similar.

The goal of creating clusters of blocks is twofold: on the one hand, to reduce the computational cost inherent in mining optimization caused mainly by the very large number of blocks that need to be scheduled. On the other hand, to consider other variables within the processing streams to realistically model the material being processed.

Considering geometallurgical variables in strategic mine planning

A mining project's performance depends on metal production, but also on the management of critical geometallurgical variables. These variables involve any rock property that has a positive or negative effect on the business' ultimate value (Coward et al., 2009; Dunham et al., 2011), such as energy consumptions at the different processes, deleterious elements involved, mineability of the deposit, etc. Dunham et al. (2011) state that geometallurgy is a cross-discipline that combines geology, metallurgy, and mine planning, to design a processing stream fit to the actual characteristics of the resource. Most of these variables are omitted from conventional mine planning methods, not to mention the variability and uncertainty related to them. However, to obtain reliable forecasts of a strategic mine plan, additional details on the rock properties, and specifically, on geometallurgical variables of the extracted material need to be incorporated into the related optimization process. The authors note that, in general, an evaluation of the deposit's geometallurgy must be included in the earliest stages of a mining project, as rock quality and its characteristics affect not only the plant's performance, but also the equipment selection and even the mining method chosen.

Some work has been done in incorporating these variables into the mine design and planning steps. Williams and Richardson (2004) propose a geometallurgical mapping approach which is integrated into the 3D block model of a deposit and can be used to calculate the metallurgical response of certain blocks to forecast their recovery. However, the proposed model does not consider the effects of blending and mixing of material when it is processed. With this 3D model, forecasted project cash flows can be generated by incorporating these recoveries into the mine planning optimization process, but as if each block was being processed by itself. Here, an ore characterization procedure is used as a base for the geometallurgical mapping approach, which corresponds to the quantification of the physical data on orebody samples. However, the authors highlight the importance of correctly defining the sampling and testing methods, and the effect this has on the parameters' range of variability, as well as quality of information.

An alternative approach is proposed by Coward et al. (2009) who classify as "primary" the geometallurgical variables that reflect an intrinsic attribute of the rock, such as mass, grain size, density, etc., and as "response" the ones that reflect the rock's response of an attribute to processes, such as throughput, recovery, grindability, etc. In their model, primary variables are defined to be usually additive or easily managed in a linear fashion; whereas response variables present complex distributions which cannot be easily manipulated, for example, the non-linear quality of metal recovery. The authors state that, because most of the rock properties are non-linear, traditional estimation methods for orebody modelling, such as kriging, are unable to represent them (or characterize their value without serious biases in the results). With this respect, they propose conditional simulation to represent the orebody, which has the advantage of presenting the rocks actual variability and allows the integration of complex non-linear variables into the model. The proposed framework aims at selecting the most important geometallurgical variables to include in the model, given the commodities and deposit at hand, as well as the processing technologies available. However, simulations are done at point support, and the process of upscaling some of these variables into block support entails biases that so far have not been addressed in the literature.

Coward et al. (2013) apply the previous framework to a mining operation, aiming to optimize a mining operation by evaluating geometallurgical recovery factors. Here, the authors note that the main sources of uncertainty must be incorporated into the model and allowed to interact at the correct spatial and temporal scale, to obtain a reliable scenario-based project evaluation. Three sources of uncertainty are specified, and modelling methods are proposed for each (i) spatial uncertainty, meaning pertinent geological attributes of the mine deposit considered, (ii) operational uncertainty, considering the system's flexibilities and process configurations, and (iii) future uncertainty, corresponding to the project's future context, which cannot be predicted with any reliability. In the first case, conditional simulation of the deposit is proposed, to quantify the geological uncertainty by generating multiple representations of pertinent attributes of a deposit. To generate models of operational uncertainty, the authors suggest generating regression analysis mechanisms to simulate multiple recovery curves of a process, based on the available raw data. Finally, future uncertainties are considered by using scenario analysis over multiple forecasts of the different externalities affecting the project, such as prices, costs, taxes, and exchange rates. The authors state that "the importance of the scenarios lies in their use to test the robustness of a strategy, not in their prediction accuracy." However, the proposed model assumes a fixed ultimate pit limit and does not provide any detail on how the production schedule is generated, suggesting that the uncertainty is not incorporated into this crucial part of the optimization process, but rather a robustness study is made over a given mine design.

Macfarlane and Williams (2014) present an optimization model for a copper mine implementing a geometallurgical solution. The case study presented contains uneven presence of cobalt in the deposit, which hinders the blending constraints at the processing destinations. This causes consistent shortfalls on plant feed rates, which also compromise the stability of the mineral value chain in terms of its main performance parameters, such as the throughput rates, acid consumption, and processing recoveries. Analyzing the behavior of these parameters and their effect on the mine production planning shows that mining rate should be increased, and stockpiles should be created to obtain steady processing rates and the required blending targets. It is shown that the costs that must be incurred to increase mining rates and material re-handling are considerably smaller than the increased revenues produced by steady processing rates. However, the mathematical optimization model used is not provided, nor is any detail over the exact changes done over the system; together with this, the focus is placed on obtaining a steady processing rate, and not on directly maximizing cash flows.

With respect to the accurate representation of geometallurgical variables, Van den Boogaart et al. (2014) present a simulation method for representing discrete and continuous geometallurgical parameters. The authors state that "conditional geostatistical simulation of geometallurgical parameters enables the construction of a processing model for computing recovery, equipment usage, processing costs, and other relevant parameters, and thus the monetary value for mining and processing a block with certain parameters." However, they note that traditional geostatistical techniques cannot be directly applied for conditional simulation of some geometallurgical parameters that have non-Euclidean statistical scales, such as grain geometry or mineral composition, which are all non-additive, producing, in some cases, infeasible values in the simulation. In addition, they state that, as the mineral processing is nonlinear, higher order statistics are needed, and not just the mean and variance of these variables. The authors propose a multi-point conditional simulation framework with a training image to jointly simulate dependent variables, ensuring that the simulated points fall within the conditional distributions. They do this through the addition of extra influence functions, to allow including additional predictive variables available at the conditioning locations of the simulating pattern, or at the simulation location itself. In their approach, the authors simulate categorical variables by estimating the conditional probability distribution functions of the training image via multinomial logistic regression. The proposed model allows the use of different scales and data layers (not necessarily categorical) in conditioning locations, through iteratively simulating at different scales, from coarsest to finest. Though a very interesting approach, the proposed model is computationally very expensive to compute, and the authors do not tackle the problems that arise from the change of support of these variables from point to block scales, but propose it as future research.

Geometallurgical variables are crucial in defining the performance of a mining complex. However, both modelling and integrating them into the optimization model are challenging tasks. Some simplifications can be made to obtain block-based values of some geometallurgical variables such as hardness (Coward et al., 2013), and proxy relations can be used to calculate recovery and throughput from these variables (Flores, 2005). With these representations, new methodologies and model extensions must be developed to correctly integrate these variables into the optimization process of a mineral value chain.

1.2.1.5 Solving simultaneous optimization with metaheuristics

With all its extensions, the formulation of the stochastic simultaneous optimization of a mining complex produces models that contain thousands of millions of binary variables, with millions of constraints (Goodfellow, 2014; Lamghari and Dimitrakopoulos, 2015). Furthermore, as more realistic problems are modelled, non-linearities are very hard to avoid without recurring into oversimplifications of the model (Pimentel et al., 2010). Because of this, and due to the complexity of the problem, different metaheuristic methods have been developed to solve the stochastic simultaneous optimization of a mining complex. These algorithmic optimizers produce good quality solutions for non-linear large-scale case studies, in a reasonable amount of time, whereas conventional exact methods are unable to solve them, and they have successfully been used in the past for mine design and production scheduling such as the work described in Lamghari and Dimitrakopoulos (2012, 2016b, a), Lamghari et al. (2013), Goodfellow and Dimitrakopoulos (2017), Montiel and Dimitrakopoulos (2017), amongst others. Even though metaheuristics may not necessarily provide mathematically "optimal" solutions, they are able to solve non-linear formulations, allowing the model to avoid falling into simplification.

Heuristic algorithms are greedy procedures used to solve a mathematical programming model and generate a good solution (Pearl, 1984). The term metaheuristic corresponds to heuristic methods which have incorporated different iterative procedures of search mechanisms to escape from local optima and get closer to the globally optimal solution (Osman and Kelly, 1996; Osman and Laporte, 1996). All these algorithms share a general stage configuration, where the algorithm starts by (i) a global exploration, exploring and (possibly) accepting sub-optimal solutions, and then gradually shift to (ii) a local improvement, where the current solution is improved as much as possible within the current state. This solving mechanism has shown to produce very favorable results when exact methods are unable to provide solutions in a reasonable amount of time.

Simulated annealing (SA) (Kirkpatrick et al., 1983; Geman and Geman, 1984) is a widely used metaheuristic algorithm. Based on the Metropolis algorithm (Metropolis et al., 1953), this method mimics the iterative heat treatment (annealing) of metals performed to increase their ductility. Slow cooling will allow the relocation of particles, increasing the ductility of the element, but taking a long time. On the other hand, fast cooling will cause the material to solidify fast and turn to glass, without time for any structure refinement. From a mathematical programming point of view, SA algorithm starts from an initial solution and moves by searching through its neighbourhood for better solutions. If none are found, inferior solutions can also be accepted with a certain probability (Metropolis et al., 1953), which depends on (i) a dynamic annealing temperature, which is updated by a cooling factor, and (ii) on the size of deterioration in objective function's value. At higher temperatures, more unfavorable solutions are likely to be accepted, allowing the algorithm to diversify the search and escape from local optima. As the search evolves, the temperature is reduced by the predefined cooling factor, tightening the search and restricting the algorithm from choosing unfavorable moves, what forces the solution to converge. Low cooling factors allow for a broader search, usually resulting in better solutions, but taking a long time; on the other hand, applying a high cooling factor will cause the solution to converge fast, but this will likely be a local optimum.

Godoy and Dimitrakopoulos (2004) were the first to use SA for the optimization of the mine production schedule under geological uncertainty. In their paper, the authors use a set of geological simulations to account for the metal grade uncertainty and generate independent optimal schedules for each simulation. Subsequently, a SA mechanism is used to iteratively combine all these schedules into one stochastic solution which minimizes the deviations from ore and waste production targets over all simulations. This method is later implemented and extended by Leite and Dimitrakopoulos (2007), and Albor and Dimitrakopoulos (2009), amongst others. Details on their work can be found in section 1.2.1.2.

Montiel and Dimitrakopoulos (2015, 2017) implement a SA algorithm for the optimization of mining complexes, and propose a solution approach that perturbs the mine plan at different levels of the mining complex under geological uncertainty, in order to generate a stochastic-based production schedule and processing policy. Three different decision variables are implemented in this model, defining (i) the extraction of a block, (ii) the operating mode at the different processing destinations, and (iii) the transportation system implemented. Perturbations of the SA mechanism occur at these three levels, aiming at improving an objective function by modifying the initial solution (which is generated by a traditional mining software). The algorithm works by first performing the scheduling perturbations, which entail favouring valuable blocks to be extracted earlier, and unprofitable ones to be extracted later on the life of the mine (given that all different operational constraints are satisfied); and second, executing operating mode and transportation system perturbations, aiming at minimizing operating and transportation costs and deviations. Perturbations at each different level are done iteratively, fixing solutions at one level to perturb and optimize solutions at the next, until a stopping criterion is met.

Goodfellow (2014) implements a particle swarm (PS) algorithm (Eberhart and Kennedy, 1995) that works in combination with a SA mechanism to simultaneously optimize the components of a mining complex with investment decisions over capital expenditures under geological uncertainty. The author chooses to use PS, a population-based metaheuristic that mimics the behavior of bird flocks or fish schools, because of its properties to optimize both continuous and integer variables. In the proposed formulation, the scheduling variables are considered integers (i.e., mining blocks) from the mine to the first destination (processing plant, stockpile, leach pad, etc.). However, after mined and transported, mining blocks become material, which is a continuous variable for the rest of the destinations down the processing stream. The optimization is divided into two steps that are repeated iteratively until a stopping criterion is met. The initial stage corresponds to scheduling or extraction decisions and destination policy, which are optimized with SA. Later, after a certain number of SA iterations, the solution is frozen, and a PS optimization process decides where the material should be sent next.

Goodfellow (2014) notes that, as in this case not only grade and material type uncertainties are considered, but also investment decisions (which entail large available capacity changes), the value chain's capacity constraints are greatly affected by the investment perturbations. Because of this, perturbations are more likely to converge to local optima, conflicting with the traditional implementation of the SA algorithm. To tackle this problem, the author combines a set of different big and small perturbations along the optimization at different levels of the model, affecting the destination policy, advancing or delaying capital expenditures, and changing the actual schedule. These last perturbations are done at the scheduling level only, through small perturbations at block-support, and also through large perturbations, applying bench-wise modifications, making sure constraints are not violated in the process. This two-stage solution approach (SA with PS) enables detailed modelling of large mining complexes, which include non-linear relations that are typically ignored by conventional optimizers. Experimental results show that the proposed formulation can develop a global optimization of mine production schedule, destination policy and capital expenditure strategy, presenting a risk-based design with an increased NPV compared to the deterministic model, which does not consider risk. However, tuning the different parameters required for both the PS and the SA algorithms is a challenge in itself.

Lamghari et al. (2013) also address the mine production scheduling problem of an open pit mine, and propose two variants to develop an initial solution. This solution is later improved by using a variable neighbourhood search (VNS) algorithm (Mladenović and Hansen, 1997; Hansen and Mladenović, 2001), introducing the concept of stochasticity of the problem into the solution method. Here, the authors formulate the problem as a two-stage stochastic programming model, and propose to solve it with two different metaheuristic approaches, differentiated in the way the initial solution is created. Both cases are based on decomposition, where smaller sub-problems are sequentially solved and later combined to create the initial solution. The first heuristic proposed solves each sub-problem using exact methods (as sub-problems are of manageable size), while the second one uses greedy heuristics. This initial solution is later improved by applying a VND-based procedure. Results show that the first variation presents slightly better results, whereas the second requires considerably less computational time. However, both methods present favorable results when compared to the linear relaxation of the problem (solved using CPLEX (2011)), presenting a maximum gap of barely 2% and 5% respectively. Lamghari et al. (2015) study the same mining scheduling problem, and develop a two-phase hybrid method to solve the problem which uses a series of linear programming models that are sequentially applied to develop an initial solution. This initial solution is later improved by applying a VND heuristic with three neighborhoods. The proposed formulation is able to solve a large-scale NP-hard problem in a few minutes, presenting less than a 3.2% gap with respect to the linear relaxation of the problem obtained using ILOG CPLEX (2011).

Several other metaheuristic methods have been developed over the years. Aiming to increase the quality and efficiency of the searching mechanisms to find good solutions for large, complex mathematical programming models, Ropke and Pisinger (2004) and Pisinger and Ropke (2007) introduce an Adaptive Large Neighbourhood Search (ALNS) algorithm, as an evolution from the traditional Large Neighbourhood Search (LNS) algorithm (Shaw, 1997). In their papers, the authors developed this metaheuristic to solve different variants of the vehicle routing problem, where an initial solution is iteratively fixed and re-optimized, but where several large neighbourhoods compete to be used in an adaptive way to search for the best solution available. The way the variables are fixed and re-optimized and the number of variables chosen to be re-optimized (the neighbourhood) is adapted along the optimization, using several competing heuristic methods during the same search, instead of just one as in the earlier LNS. Ropke and Pisinger (2004) argue that alternating over several heuristics provides a more robust heuristic overall.

Lamghari and Dimitrakopoulos (2015) implement an ALNS algorithm to solve a mathematical programming model of a mining complex's mine plan under geological uncertainty, contemplating almost a thousand million binary variables and millions of constraints. Here, the authors generate an initial feasible solution of the mining complex considering scheduling and destination of blocks under grade and material type uncertainty. ALNS is applied to improve this initial solution by iteratively destroying and repairing it until a stopping criterion is met. In their paper, the authors define fourteen destroying methods and seven repairing methods which alternate as the optimization evolves. These methods are interchanged according to their effectiveness to improve a solution, and their computational requirement, focusing on both intensifying, as well as diversifying the search. In this case, the authors use an acceptance criterion similar to Metropolis' simulated annealing (Metropolis et al., 1953), where a candidate solution that improves the value of the optimization solution is always accepted, whereas a solution that decreases value is accepted with some probability (in order to escape from local maximums).

1.2.1.6 Limitations of current formulations

Many advancements have been made in improving the modelling and solution of a mining complex's mine plan under uncertainty. However, all the aforementioned works have limitations that can be tackled to generate more informed optimization models for strategic mine planning. For example, even though the previously mentioned works include investments and operational alternatives as decision variables in the formulation (Goodfellow, 2014; Montiel, 2014; Farmer, 2016), all current models of stochastic simultaneous optimization of a mining complex assume that the components and setting of the mining complex will stay the same. The optimal solution for a strategic mine plan does not account for changes in the assumed production aspects, and thus, is assumed to be optimal over the LOM.

Assuming the production needs and components of a mining complex will stay the same over the full LOM, and thus, that its corresponding strategic plan will continue being optimal, is a simplification, and one of the main limitations of existing models. Consequently, conventionally, mine plans are re-generated every year with the new information obtained and the updated objectives. This is a "passive" solution that can inhibit options and alternatives that may improve the strategic plan in terms of equipment, infrastructure, locations, etc. (Snowden et al., 2002; Saleh et al., 2009; De Neufville and Scholtes, 2011; Del Castillo and Dimitrakopoulos, 2014). This is most significant for capital expenditures and high impact investments, as they require years of planning, and usually have a deep impact on the mining complex's plan and performance. As a result, substantial improvements and benefits can be provided by quantifying possible options in a strategic LOM plan, such as, for example, investing on processing plant expansions, extra crushers, mining extraction fleet, etc. For example, building a state of the art processing plant may cost over four billion dollars, and may take almost five years to be completed (Mineria Chilena, 2015). Furthermore, considering the significant cost of investments related to the mining industry, optimizing the timing of these options within the strategic plan and evaluating their corresponding probabilities of occurring should be considered in the stochastic simultaneous optimization of a mining complex (including the considerable lead time to get the purchased equipment or build the infrastructure, and its limited life-span).

Traditionally, sensitivity analyses are the main and only studies done to evaluate the viability of these investments (Torries, 1998; Whittle et al., 2007). Monkhouse and Yeates (2005) note that these analyses provide some intuition on the operation's performance through specific changes in some key uncertain parameters. However, they are limited in that these mechanisms do not inherently consider uncertainty, nor do they generate any type of optimized flexible plan to manage it. In addition, and due to it, implementing solely this analysis usually result in loss of opportunities, delayed projects and change-of-plans that impede transitions and result in loss of profit (De Neufville and Scholtes, 2011). Thus, a dynamic probability-based decision tree mechanism can be developed to ensure the value chain is able to plan and actively adapt to feasible, possible changes. As stated by Dowd et al. (2016), one of the main challenges in strategic mine planning in today's world is, amongst others, to develop new ways to include and maximize flexibility in mine design.

1.2.2 Dynamic decision-making in strategic planning

1.2.2.1 Flexibility in value chains under uncertainty

Dixit and Pindyck (1994) state that uncertainty has a decisive effect over a project, and although many times this uncertainty cannot be controlled, it is possible to increase the flexibility of the project in order to be prepared to react timely to it, and overall assess the probability of alternative outcomes. Lavington (1921) was one of the first to relate random changes and uncertainty with the value of flexibility. The term flexibility is a widely used concept; however, as noted by Sethi and Sethi (1990), because of its popularity, over the years, it has been seen to mean different things to different audiences, with over 50 definitions in the manufacturing literature alone. In their research, the authors identify twelve types of flexibility which are grouped into (i) component or basic flexibilities, (ii) system flexibilities, and (iii) aggregate flexibilities. Within the second group, interesting flexibility concepts appear, which could be related to mineral value chains, such as process flexibility, routing flexibility, volume flexibility and expansion flexibility. Mark (2005) defines flexibility as "the ability of a system to maintain the competitive advantage despite environmental change." Hart (1940) notes that "the preservation of flexibility is a fundamental means of meeting future uncertainty," recognizing the value of postponing a decision until more information is available. Kulatilaka and Marks (1988) define production flexibility as the ability to change a process from one operational mode to another, providing the production process with the ability to modify itself in the face of uncertainty. As such, the authors note that flexibility in general only has value in the presence of uncertainty (Kulatilaka, 1988). Moreover, as stated by Merton (1997), there is a clear positive correlation between uncertainty and the value of flexibility, meaning that the higher the uncertainty, the more valuable it will be to have a flexible system. The strategic optimization of a LOM operation is full of uncertainties and assumptions (Monkhouse and Yeates, 2005), showing that including flexibility into a mining operation has the potential of being highly valuable.

Gupta and Rosenhead (1968) state that "the flexibility of a decision must be measured in terms of the number of end states which remain as open options [after a first decision has been made]." Similarly, Mandelbaum and Buzacott (1986) represent flexibility as the relative size of the set of possible decisions to take today, conditioned by the decisions taken on a previous period. Thus, more remaining choices will correspond to higher flexibility. In general, Saleh et al. (2009) note that flexibility can be seen as the potential to change, with the absence of irreversible or rigid commitments. In their paper, the authors identify, amongst others, the flexibility of designs or flexible systems, which go one step further from managerial flexibility, and focus on enabling projects to respond to change with minor time and cost changes. In their definition, a flexible system implies that the system has been designed with some particular characteristics, which may not be necessary or justifiable in the present conditions of the project, but that allows it to adapt if these conditions change, making the design flexible. However, the authors note the lack of literature in this area, specifically in topics such as how to embed flexibility in a system, and on how to evaluate it. Cardin et al. (2013) describe different procedures to generate and include flexibility in engineering systems, compared to traditional benchmarking and sensitivity analyses, stressing the importance of evaluating flexibility options from an early stage of the strategic planning.

In summary, even if a system is entirely flexible and its configuration can be readily adapted, this freedom of action must be controlled and optimized to maximize the project's performance and value. In other words, a flexible system (Saleh et al., 2009) must go hand in hand with an optimization model which is able to integrate these flexibilities in order to provide an optimal plan of action, so that the flexibility doesn't translate into a loss of efficiency due to constant changes, or a mechanism used to constantly put out operational fires (Olsson, 2006).

1.2.2.2 Optimizing with flexibility

The concept of flexibility has not been ignored in the mining industry. In this area, some optimization methods have been developed and adapted to account for both managerial and system flexibilities in a mining project. However, the literature has mostly focused on the evaluation of projects and the value added by flexibility, and not on the optimization of the project's plan itself, or on how to adapt it to facilitate flexibility.

One of the first examples in literature is presented by Singh and Skibniewski (1991), who base on a flexible manufacturing system and present a flexible strip mining operation, aiming at easing the decision making process and its ability to adapt to changes, especially when automation is considered. Recently, Mak and Clarkson (2017) differentiate between adaptability and flexibility, stating that the former must be designed within the system for the uncertainties in the immediate future, whilst the latter focuses mostly on strategy and future developments of the system. Different methods have been developed over the years to identify, include, and evaluate both adaptability and flexibility in value chains under uncertainties. However, to produce feasible strategic plan solutions, a set of complex design constraints must be accounted for, which are imposed over time, and will limit the number of flexibility alternatives available for each specific mining complex. These constraints may consist in space constraints, limited capital, fixed infrastructure, or specific blending constraints, amongst others. Additionally, for a realistic and unbiased valuation of flexibility, the corresponding mine production schedules generated must be clear and operational, which is the main limitation of the work found in the literature so far.

The following sections will review work done on real options analysis, followed by multistage stochastic programming formulations.

Real option valuation

Real option (RO) valuation has shown to provide successful alternative results to account for the value of flexibility, and the effects of uncertainty. This method, developed as an extension of financial options into investment projects, complements the NPV and addresses many of the limitations of traditional discounted cash-flow analysis (Trigeorgis, 1996; Lee and Strang, 2003; Samis et al., 2006, 2011; Savolainen et al., 2017). Just as in most mathematical programs under uncertainty, in a standard RO model, the source of uncertainty is formulated as a stochastic process, enabling the examination of the behavior of the variable, and its effect over a project's performance (Shibata, 2008). With this, the model is capable of quantifying the value of flexibility as a response to the uncertainty (Mun, 2002; Kalligeros, 2006). Dixit and Pindyck (1994) define a RO as the right, but not the obligation, to make an investment in real assets by or at the end of a given period. One of the most interesting concepts that RO points out, is the understanding that flexibility has a value, but also an associated cost, which can be represented as a monetary premium, an opportunity cost, or simply by the time and efforts invested in the preparation and planning required to maintain options available throughout the years of an operation (Amram and Kulatilaka, 1999; De Neufville and Scholtes, 2011). In any case, it highlights the fact that an effort must be made to have access to this flexibility in the future.

This valuation method has been successfully implemented in various industries, with many applications in mining. McCarthy and Monkhouse (2002) state that not considering managerial flexibility in the evaluation process of a mine plan results in underestimations of optimal LOM, which lead to processing plants with extra capacity, higher initial investments, and a loss of capital in general. The authors also clarify that this can be handled by using RO valuation approach. Samis et al. (2006) use RO valuation based on forward contracts for copper to consider the commodity's market variability and obtain better information to select a project to invest in. Sabour and Wood (2009) and Dimitrakopoulos and Sabour (2007) consider commodity price and exchange rate uncertainty and compare the RO valuation method with the traditional static NPV. Results over a case study show over a 15% increase in project value when evaluating through RO. The authors note that this increase in value shows that RO models incorporate the value of accounting for active management, and its ability to react to change based on new information. However, only the option to abandon the project early is considered, and the actual extraction schedule is not optimized under the uncertainty. Cardin et al. (2008) present a model to account for managerial flexibility at the conceptual stage of a mining project, and call it "intelligent management [as a] response to changing operating conditions and market prices." Similarly, Sabour and Dimitrakopoulos (2010) and Del Castillo and Dimitrakopoulos (2014) use RO valuation to incorporate managerial flexibility subject to price variability the first case, and price and geological uncertainty in the second, and calculate a stochastic ultimate pit limit, showing that traditional methods consistently underestimate the size of the ultimate pit. More recently, Haque et al. (2017) study the options of deferring investment, and permanently or temporarily closing an iron ore mine under price and exchange rate uncertainty.

However, none of the previous studies actually optimize the extraction sequence or the mining complex's interactions. In other words, they all account for uncertainty to calculate the project's value distribution, but not to directly optimize the mine plan, as the extraction sequence is assumed fixed. Together with this, as stated by Saleh et al. (2009), traditional RO mostly focus on the valuation of *managerial* flexibility, defined as the ability of management to adjust the course of a project by acting in response to the resolution of uncertainty, but they seldom tackle the problem of how to embed this flexibility in a project or engineering design (Driouchi and Bennett, 2012). That is, they focus mainly on calculating the financial value of flexibility, but not in including flexibility per-se.

An interesting alternative is presented by Kazakidis and Scoble (2003), who present a first approach for a RO model to evaluate a flexible underground mine project under risk of ground related problems, and develop a model to assess and integrate flexibility alternatives to obtain a proactive mine plan. Mayer and Kazakidis (2007) extend on the previous study and show that the higher the volatility of the project, the higher the value of flexibility (Merton, 1997), as the system will be able to quickly adapt to take advantage of big opportunities, but also hedge from big drops. In their paper, the authors evaluate different scheduling options for an underground mine under commodity price and operating cost uncertainty. However, different extraction schedules are compared, but not directly optimized, and no information is provided on the actual mathematical model used.

Tackling the problem of embedding flexibility in a project, Wang and De Neufville (2005) introduce the concept of options "on" and "in" engineering projects. The first being managerial flexibility options taken under uncertainties that are global across the industry (such as exchange rates or commodity price), and the second, options that are inherent to the engineering system and its design (such as changing transportation systems, or investing on a particular expansion). Options "in" projects aim at directly inserting flexibility into the project, tackling the limitation mentioned in Saleh et al. (2009). However, as mentioned by Bowman and Moskowitz (2001), these options require custom tailored optimization models, and thus, a fuller understanding of the system's design, which is probably the reason why studies of these options have been less present in the literature.

Cardin et al. (2007) propose a general screening tool to include flexibility options at the engineering, operational, and management decision levels, especially focusing on the first two, classifying them as options "in" project, which actually modify the system. The authors propose using Design Structure Matrices which represent technical aspects of the engineering system to identify potential sources of flexibility "in" the project. A "Value-at-Risk-Gain" set of curves (Hassan et al., 2005) is used to represent the value of the different options and help discriminate between the most interesting ones. These curves correspond to the representation of the cumulative probability distribution curves of the project's NPVs obtained from Monte Carlo simulations, for the different options of flexibility. Similarly, Lin et al. (2009) develop a screening model to consider flexibility options in capital-intensive systems. In their paper, a case study on an off-shore petroleum project with uncertain reserves is presented, with options on new transportation connections, capacity expansions, and flexibility in operational modes once new information is obtained. The effect of each alternative is also presented with value-at-risk-gain curves.

Groeneveld and Topal (2011) focus on options “in” design (Wang and De Neufville, 2005), and aim to develop a flexible open pit mine design able to adapt under different sources of uncertainty, providing a better risk-return profile. An MIP model is used to determine the optimal mine design under uncertainty, incorporating flexibility into four stages of the system: mine, stockpiles, plant, and capacities. Uncertainties considered are commodity price, capital and operating costs, and plant utilization, which are simulated simultaneously to provide a state-of-the-world used as input to the model. Additionally, cost for executing an option, and for switching configurations are also included, which stabilize the use of alternatives. Groeneveld et al. (2012) extend on the previous study and present a hybrid “operational” method that limits the number of changes happening on the system on the initial periods, as a way of producing a more “operationally feasible” mechanism. However, these studies present several limitations. The case studies consider a deterministic deposit, where blocks are aggregated into parcels to reduce the number of integer variables. These parcels are defined as continuous variables which can be partially mined; however, they must be fully extracted before accessing underlying parcels. This block aggregation and partial extraction of parcels provides untraceable results that may be limiting in the mid to short-term planning. Together with this, technical details provided are limited, particularly in terms of the operation’s production targets, the focus is placed solely on maximizing NPV, and the schedule is assumed fixed. Additionally, by using state-of-the-world scenarios, stochastic parameters are assumed to be known with certainty, what may provide overoptimistic evaluations that translate into higher NPVs.

Ajak and Topal (2015) review RO applications in the mining industry and propose a methodology to assess technical applications of RO in mine design and operational decision making, however, the full mining complex is not considered. The authors present a mine planning RO model with the option to switch extraction between two areas of the mine. The proposed model focuses on a shorter-term optimization, affecting the mining schedule directly, and allowing the mine plan to defer waste and adapt its mining activities depending on commodity price scenarios. Their results show an increase in NPV of around 10% from the traditional base case without flexibility. However, price uncertainty is simplified to a binomial tree, providing no information if the final real price is not one of the forecasted values. Additionally, geological uncertainty is not considered, and the extraction sequence is not exactly optimized, but rather independently fixed within each area, and "paused" by the area-switching option.

More recently, Cardin et al. (2017) propose an approach to assess the value of flexibility and determine the best design of an engineering system, and use a heuristic triggering mechanisms to define when is the best timing to exercise flexibility. In their paper, the authors note that, even though RO analysis has proven to be a useful tool in evaluating flexibility, its practical use has been limited, mostly as it does not provide any straightforward information on the optimal timing of exercising an option, but rather the value of having this option available. To tackle this problem, the authors propose a model based on decision rules and multistage stochastic programming, which aims at providing a guide for dynamic decision making based on the available information as the different uncertainties resolve. Though highly interesting, the study presented is quite general, and the size of the energy system case study used is considerably smaller than the strategic optimization of a mining complex under uncertainty. Together with this, as in most multistage models, final solutions tend to be tailored to the set of scenarios used, providing overoptimistic results, and little information when reality is not exactly represented by the scenarios.

The formulation and optimization of flexibility through stochastic multistage programming is a widely studied mechanism which will be more thoroughly reviewed in the following section.

Multistage stochastic optimization to include flexibility

It has been shown that multiple sources of uncertainty govern a mining complex. In practice, as a production schedule evolves, new information becomes available which affects the decision-making process and promotes active management to react accordingly. However, it is common to see optimization models where the plan designed is tailored exactly for each scenario (Mayer and Kazakidis, 2007; Groeneveld et al., 2012). Assuming that stochastic parameters are known with certainty generates overoptimistic NPVs with potentially unrealistic designs. In reality, there is no upfront information of the future states of the value chain, and multiple scenarios can appear similar at an initial stage and differentiate along the way. This fact is best described mathematically by stochastic multistage programming models, which make use of non-anticipative constraints to ensure that equal scenarios entail equal actions. In other words, these constraints are used to ensure that non-differentiated scenarios (i.e., scenarios that have no apparent difference yet) entail equal decisions. Wang (2005) explains that in reality, the decision maker usually cannot distinguish between any two scenarios passing through the same node and proceeding to different terminal nodes because the state can only be distinguished by information available at that time stage. In his thesis, the author presents examples of multistage formulations on a satellite communication system and on a river basin development under uncertain electricity price. Similarly, Goel and Grossman (2006) note that, in a stochastic system, decisions cannot be based on knowledge that will be revealed in the future, as this is an unrealistic assumption. In their paper, the authors present a standard stochastic program for a linear problem, where, at each time period, the uncertainty is partially resolved based on the path taken. Here, the authors propose a stochastic multistage model to solve a production line problem under endogenous (internal) and exogenous (external) uncertainties, where uncertainty is resolved at each stage depending on a binary decision variable (equal to 1 if uncertainty is resolved and 0 if not), noting that some decision variables are taken at the beginning of each period, before uncertainty is uncovered, and others are implemented at the end of the time period, after the resolution of uncertainty.

Birge and Louveaux (1997) mention a set of elements which justify considering a multistage model, such as the long-term evolution of equipment costs, the long-term evolution of production, the development of new technologies, or the obsolescence of currently available equipment. All these elements are present in mining, where both equipment and mineral processing technologies are constantly being renewed.

However, even though these methods respect the chronological acquisition of new information, they are also limited, mostly because, as opposed to the two-stage stochastic optimization, multistage stochastic optimization models assume that the limited set of stochastic simulations used for the optimization represents all possible scenarios, rather than just a set of possible values obtained from a probability distribution. Thus, because of the way scenarios are used within the optimization, the obtained solution is tailored for this set of scenarios, and if the unveiled uncertainty takes a value that is not accounted for in the set used in the optimization (which is highly likely to happen in reality), then the solution provides no information about what to do. Nevertheless, this modelling mechanism provides interesting characteristics which can be useful to incorporate flexibility alternatives in a value chain, mostly related to the mechanism of gradually obtaining new information through non-anticipative constraints.

Wang and de Neufville (2005) use non-anticipative constraints in their flexible option model to ensure that if two scenarios share the same node in the tree structure used for optimizing, then the decision taken in both cases is the same (i.e., the scenarios are indistinguishable). The multistage programming model proposed by the authors aims at maximizing benefits and minimize costs of different design parameters, subject to technical and economic constraints. The different options available are modelled into these constraints through binary variables, and the average value over all the scenarios is maximized. The paper presents a case study on a river-run hydropower station, a complex system with capacity, environmental, and budget constraints, as well as uncertainty in the price of electricity and seasonal flows (i.e., demand).

Boland et al. (2008) also present a multistage stochastic programming model to solve the open pit mine production scheduling problem under geological uncertainty. The formulation of their problem is an extension of the work done by Goel and Grossman (2006), where they consider two cases, the first, accounting only for processing decisions as scenario dependent (where new information is used to influence processing decision), and the second, considering mining and processing decisions as scenario dependent. Their formulation optimizes the mine production schedule, and aims at maximizing the expected revenue obtained from the metal produced, minus the processing and mining costs. The authors formally define non-anticipativity constraints by introducing a set to identify distinct scenarios, as two scenarios that have differentiated more than a given amount "α", thus, assigning them as “sufficiently different” for the optimizer to branch out and assume two distinct cases. Though an interesting approach, three main limitations can be seen in the authors’ approach, which are strictly related to the core aspects of mine production scheduling. These are (i) the aggregation of blocks to reduce computational cost, which can be partially mined, violating slope constraints, (ii) as mentioned before, the branching mechanism of multistage programming produces solution schedules which are over-fitted to the set of scenarios used, and thus, would have a poor performance when tested over a different set of simulations, and would be irrelevant to reality, as the reality encountered will surely not be represented exactly by any of the simulations. Finally, (iii) multiple possible schedules are provided as an output of the model, which is operationally impractical, as these schedules may be differentiated by a single block. Together with this, despite the aggregation of blocks, the formulation still becomes impracticable for real size operations with millions of blocks, as the multistage formulation is modelled in such a way that, as scenarios differentiate, the design of the system divides into different possible schedules. This occurs successively for each differentiation perceived as the mining evolves, increasing exponentially the size of the problem along the LOM, possibly finishing with as many schedules as number of blocks.

More recently, Apap and Grossmann (2017) present a multistage stochastic program, which simultaneously deals with both endogenous and exogenous uncertainties, and present ways to remove redundant non-anticipativity constraints, significantly reducing the dimensionality of the formulation. The authors apply the model to the capacity expansion of process networks, and to the development of oilfields. Another measure to reduce the size of the problem is presented by Boland et al. (2016), who present a decomposition approach that use scenario grouping to solve large stochastic multistage problems. Here, the related decomposition is performed on clusters of scenarios, and the algorithm searches for feasible, hopefully, good solutions. Zou et al. (2016) also tackle the size and complexity of multistage stochastic integer programs and propose a nested decomposition mechanism. The authors note the difficulty of applying traditional decomposition mechanisms to this type of problems, due to their non-convexity, but prove that when the state variables are binary, their proposed nested decomposition algorithm leads to significant improvement on solving large-scale multistage problems in real-world applications.

In summary, though multiple mechanisms have been developed to account and include flexibility into mining operations, the models developed for both real options and multistage programming have strong limitations. These limitations are related to the specific demands of the stochastic simultaneous optimization of mining complexes, and to the need of producing realistic schedule solutions that can be readily applied in a mining operation. Otherwise, evaluating unrealistic designs will produce biased, misleading NPVs and mine plans with no real use.

1.3 Goal and objectives

The goal of this thesis is to extend the stochastic simultaneous optimization of a mineral value chain onto a dynamic simultaneous optimization model for multi-element mining complexes, capable of reacting to the project's changing environment, while maintaining production targets under material type and element concentration uncertainties. This includes considering active management investment decision making and flexible operational tuning of the components of the mineral value chain. By extending the stochastic two-stage optimization model of a mining complex into an adapted multistage programming model, it is possible to integrate some of the flexibility features of multistage optimization into the strategic optimization plan of a mineral value chain, without falling into the mentioned assumptions or limitations of multistage programming.

To achieve this goal, the following objectives are addressed:

(1) Review past work related to stochastic integer programming, multistage programming and modelling of flexibility options, and outline their limitations when adapting these concepts to optimize the mining complex under uncertainty.

(2) Study the basic mechanisms required to formulate a stochastic integer programming model to optimize the mine production considering capital expenditures, developing ways to optimize the capacities of the value chain.
(3) Develop a destination policy mechanism that integrates complex geometallurgical variables into the optimization model, to assist in accounting for the overall value of the extracted material. This will involve modelling a cooperative game theoretic clustering method initially applied over a simplified model of a mining complex with geological uncertainty.
(4) Develop a flexible model for the global optimization of a mining complex considering the method developed in (2), by extending the mathematical formulations of the stochastic simultaneous optimization of a mining complex into an adapted multistage programming model, able to dynamically consider capital expenditures. Test the developed model on real scale case studies and compare results with conventional industry methods.
(5) Incorporate the flexibility obtained from operational mode alternatives info the dynamic formulation, enabling the optimization model to have a better control over the tuning process at different levels of the mining complex, from mine to port. Subsequently, test the developed methods over real-world, large-scale mining complexes, documenting the results in comparison to traditional optimization methods.
(6) Outline the contributions and limitations of the developed methods, and provide possible future work directions.

1.4 Thesis outline

This thesis is organized into the following chapters:

Chapter 1 presents an introduction to the topics treated in this thesis and includes the literature review in all related subjects considered in this thesis, including traditional and stochastic mine planning optimization frameworks, stochastic simultaneous optimization of mining complexes, operations research techniques to include uncertainty and flexibility, and work related to the dynamic stochastic optimization of a mining complex. Goals and objectives of the work are stated.

Chapter 2 describes a method to optimize extraction capacity and fleet acquisition of a mining operation under geological uncertainty, for a given mining schedule. The proposed model is compared to traditional methods, and its benefits are shown through an application at a gold deposit.

Chapter 3 develops a multi-variate destination policy which implements a coalition formation clustering mechanism to deal with complex geometallurgical variables and blending constraints. The proposed method can account not only for the material's own characteristics but also for the value and metallurgical relation of material treated together. The benefits of this model are demonstrated through an application on a copper-gold mine with multiple destinations.

Chapter 4 introduces the dynamic optimization model for a mining complex under supply uncertainty with capital expenditure alternatives. The dynamic model includes capital expenditures to optimize the mining complex's capacities, and at the same time includes possible high-impact investment alternatives into the optimization, providing full mine plans that can be implemented once more information is known. An application at a multi-mine mining complex demonstrates the benefits of this model.

Chapter 5 expands on the method developed in Chapter 4 and includes operating alternatives to the dynamic model which can act at different levels of a mining complex. A case study over a copper-gold deposit shows the benefits of the proposed extended model, related to keeping alternatives open and allowing for the mining complex to dynamically react to changes. This is done by simultaneously optimizing different operating modes at the extraction and processing levels, as well as optimizing the truck fleet and possible secondary crusher for the mill.

Chapter 6 outlines the contributions and conclusions and recommends future research avenues.

CHAPTER 2
Optimal Mining Rates Revisited: Managing Mining Equipment and Geological Risk at a Given Mine Setup

This chapter studies the optimization of mining extraction capacities through the addition of capital expenditure investments into the mine planning formulation. These capital investments are included considering various realistic parameters, providing an optimized acquisition plan.

2.1 Overview

Production scheduling of open pit mines is a major aspect regarding planning and production streamlining, asset valuation and operations. Production scheduling is a process leading to the determination of a sequence of extraction which involves the removal of at least two types of material: ore and waste. If the production schedule maximizes the project's overall profit, subject to technical, economic and environmental constraints, then it is said to be optimal. Two major technical constraints involved in the determination of such schedule are: (i) the feasible combinations of ore and waste production (stripping ratio), and (ii) the ore extraction rate that meets the mill feed requirements.

Optimization methods have long been used to improve mine design and life-of-mine production schedules (Kim, 1979; Barbaro and Ramani, 1986; Dagdelen and Johnson, 1986; Whittle and Rozman, 1991; Tolwinski, 1998; Whittle, 1999; Godoy, 2003; Stone et al., 2005; Jewbali, 2006; Menabde et al., 2007; Meagher, 2010; Godoy and Dimitrakopoulos, 2011). The common industry practice is to discretize the pit space in a sequence of nested pits (Whittle, 1999), which is accomplished through the repeated use of a parametric ultimate pit algorithm, by successively changing the commodity price. For lower prices, smaller pits are produced (Hustrulid et al., 2013) and will extend toward the area of the highest grade and/or will have a very low stripping ratio. Since early cash flows are subject to less discounting and thus contribute more to the Net Present Value (NPV), it is advantageous to bring income forward and delay expenditure as long as possible.

In dealing with the points rose above on stripping ratios and ore production rates that meet mill feed requirements, the optimization of mine production rates for ore and waste over the life of an open pit mine can only be done within a so-called physically "feasible" domain of solutions. This domain is based on early work (Rzhenevisky, 1968; Tan and Ramani, 1992) revisited by Godoy (2003), and it adopts concepts in the context of open pit scheduling based on nested pits and geological uncertainty. The current mine scheduling framework establishes the feasible domain based on two extreme cases of deferment of waste removal defined by Whittle (1999): the 'worst' and 'best' shown in Figure 2-1.

According to Whittle (1999), the worst case corresponds to mining out each successive bench in a mine before starting the next, without any sequencing optimization. This schedule provides the maximum quantity of waste that can be removed from the pit to recover a certain amount of ore (i.e., the highest stripping ratio). This schedule does not perform well, given that waste is removed from early, and thus discounted little, whereas the income from mining ore at the bottom of the pit is delayed for later periods, and thus heavily discounted. The best case corresponds to the sequential mining of the nested pits, which is, mining each successive bench of the smallest pit possible, followed by the bench of the next pit and so on. This schedule removes the minimum necessary quantity of waste (lowest stripping ratio) that must be removed to provide both the necessary working room and the safety of operations. In economic terms, this schedule then provides the highest NPV. Given the best and worst cases of mining, Figure 2-2(a) shows an example of a feasible solution domain of a gold deposit from Godoy (2003) in the form of a cumulative graph. The solution domain is bounded by the curves of cumulative tonnages of ore and waste of the best and worst mining cases and accounts for all the feasible combinations of stripping ratios for the given orebody being considered and over its life-of-mine. This domain reflects the possible number and spatial arrangement of simultaneous working zones.

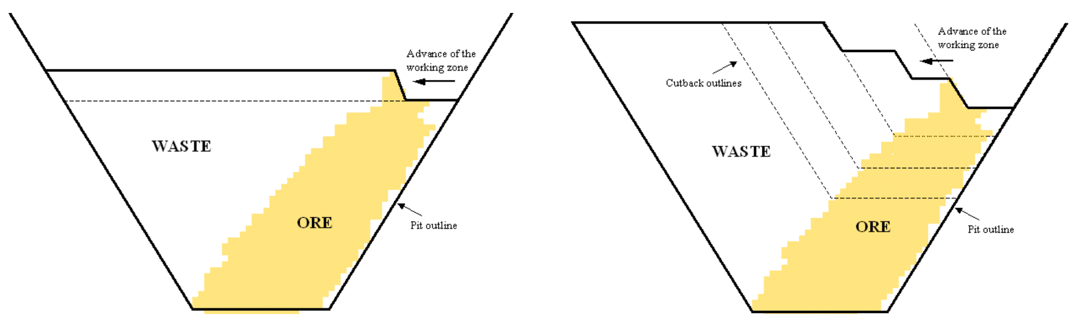


Figure 2-1 Schematic representation from (Whittle, 1999) of the (left) worst and (right) best case mining schedule.

The feasible domain, as presented in Figure 2-2, is a function of two factors: (i) the spatial distribution of ore and waste in the region contained by the ultimate pit limits, and (ii) a specific set of nested pits. The definition of these two factors is subject to a chain of interconnected factors such as geological, economic, technological and environmental. As Dagdelen and Johnson (1986) state, production scheduling can be seen as a prescription of a mine sequence which maximizes cumulative project NPV while satisfying four major constraints: (a) mill feed grade, (b) slope constraints, (c) milling capacity, and (d) mining capacity. The definition of (a) and (b) is all that is required for the derivation of the solution domain in the cumulative graph of ore production and waste removal. However, the specification of (c) and (d) account for the time aspect of the mining sequence formation and can thus further restrict the solution domain. This last aspect can be represented by the cumulative graph of the ore production concerning time, where both extreme mining cases are presented as two separate ore production curves. These curves form the feasible domain of the possible time distribution of the ore production for a given processing capacity.

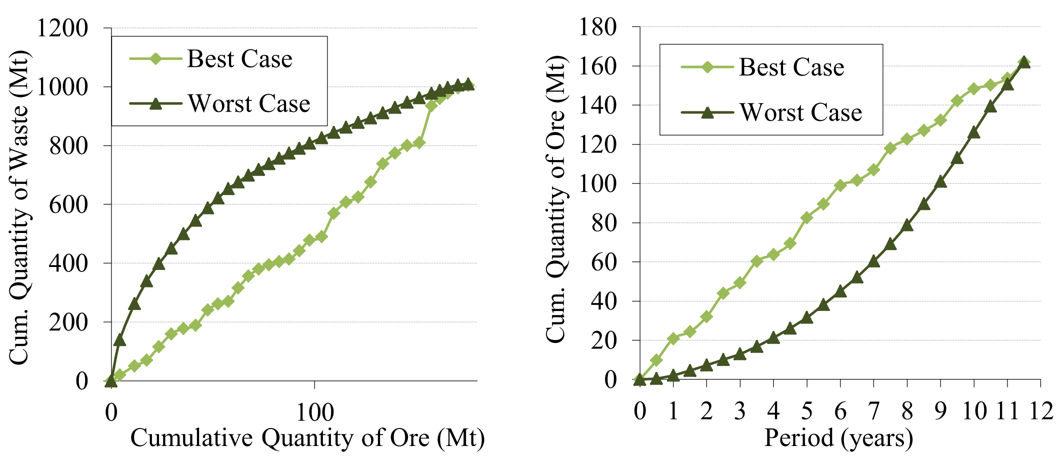


Figure 2-2 From Godoy (2003), the feasible domain of waste removal (left) and ore production (right)

Figure 2-2(b) illustrates this domain, obtained by assuming a constant mining capacity for the extreme cases presented in Figure 2-2(a). It is important to note that the cumulative graphs in Figure 2-2 can account for geological uncertainty (Emery et al., 2014; Boucher et al., 2014) and, in fact, can generate optimal mining rates for a given case which can always be met. Including this uncertainty requires calculating the feasible domain of solution for each stochastically generated scenario of the orebody under consideration (Godoy, 2003). Then, the common intersection of all individual feasible domains provides the 'stable' solution domain (SSD), or domain, where the ore-waste combinations shown, are always available, independent of geological risk. The case study presented in a subsequent section shows this characteristic of the SSD.

The work presented herein aims at optimizing a mine's extraction capacity through the acquisition of mining equipment, constrained by the SSD's feasible ore and waste extraction combinations, in order to maximize NPV and meet production targets. This study builds on the work by Godoy (2003) and Godoy and Dimitrakopoulos (2004), which is limited in that the variables related to the increased and decreased mining capacity are defined as linear. Because of this, the optimization does not produce values of mining capacity that are necessarily multiple of the equipment's total capacity. Thus, the optimal solution may provide a fractional number of equipment, which is ultimately an infeasible solution. While small differences may be accepted, high levels of equipment under-utilization may be practically unviable. Note that, while mining production rates are optimized, a physical mining sequence that meets those rates is not produced, and so, it must be subsequently generated based on them.

The current paper starts by presenting a mixed integer programming (MIP) formulation, which is specific for a mine with one mill, a long-term stockpile, a waste dump, and a mine-owned fleet of mining equipment. Then, an application of the formulation at a gold mine, and comparisons to conventional practices is presented. Discussion and conclusions follow.

2.2 The optimization model

The determination of an optimal combination of ore and waste production consists of selecting a curve, from all the possible curves that fall inside the SSD, which maximizes the corresponding NPV. The optimization model delivers a life-of-mine schedule of ore production and waste removal, as well as a prescription for the formation of mining capacity and the acquisition of equipment, which maximizes total discounted cash flow for a set of economic and technological parameters. This production and acquisition plan is modelled to provide results specifically for a given mine configuration, which includes three material destinations, namely, mill, long-term stockpile (processing low-grade ore after mining stops) and waste. Additionally, it is assumed that the mining fleet is owned by the operation and bought sequentially, equipment is replaced due to its fixed lifespan, while mining rates must remain stabilized for long periods of time (years). The mathematical model includes an objective function and constraints as follow.

2.2.1 Mathematical Formulation

Objective function

The objective function is presented in Eq.(2-1), consisting of 5 parts. The first corresponds to the income obtained from high-grade ore, considering mining, processing, operating, selling and marketing costs. The second section corresponds to the cost of mining low-grade ore. In this case, low-grade ore is destined to a long-term stockpile, which is treated as a temporary waste dump. Thus, there is no income considered from mining it during the evaluated time span, but rather this destination is treated as a buffer, used to possibly extend the life of the mine once ore has been depleted in the pit. The third term corresponds to the cost of mining waste. The fourth term considers the purchase costs, i.e., the cost of adding new equipment of a given type and model in a certain year, to increase the production capacity of the system. Finally, the fifth term corresponds to the ownership costs, such as the cost of unused equipment of a certain type and model, given that the production rate of that year is lower than the maximum available capacity. Eq.(2-1) presents the objective function, where i=1,...,n denotes the time periods to be considered in the production scheduling optimization.

$$\max \quad \sum_{i=1}^{n} d_i \Big[(1-R) \Big[\left(S_i - C_i^{ma}\right)\gamma_i - \left(C_{h,i}^{m} + C_{h,i}^{proc} + C_i^{t}\right)\left(\alpha_{h,i}\right)^{-1} \Big] M_{h,i} -$$

$$-C_{l,i}^{m}(\alpha_{l,i})^{-1} M_{l,i} - C_{w,i} W - \sum_{k=1}^{K}\sum_{v=1}^{V} H_{k,v,i} NE_{k,v,i} - \sum_{k=1}^{K}\sum_{v=1}^{V} U_{k,v,i} DE_{k,v,i} \Big] \qquad (2.1)$$

Equation (2-1) reflects the structure of the NPV of the mining project by discounted cash flow analysis, before taxation and without the treatment of the relevant depreciation and depletion allowances. The depreciation and depletion allowances represent constants in the MIP formulation so as not to affect the optimization of the production schedule. The formulation represents an operating mine where the low-grade ore is stockpiled and not processed. As a result, low-grade ore does not provide revenue, and expression (2-1) takes this into account. Table 2-1 presents the parameters obtained from the SSD. Table 2-2 shows the list of indices and general parameters used in the model. Finally, Table 2-3 presents the variables participating in the objective function and the subsequent constraints.

Table 2-1 Parameters obtained from the SSD

Constant	Definition
PMX_i	maximum cumulative quantity of metal from high-grade ore (tons)
PMN_i	minimum cumulative quantity of metal from high-grade ore (tons)
SMX_i	maximum cumulative quantity of metal from low-grade ore (tons)
SMN_i	minimum cumulative quantity of metal from low-grade ore (tons)
WMX_i	maximum cumulative quantity of waste (tons)
WMN_i	minimum cumulative quantity of waste (tons)
PMB_i	cumulative quantity of metal from high-grade ore in best case (tons)
PMW_i	cumulative quantity of metal from high-grade ore in worst case (tons)
$\Delta SR_{h,i}$	stripping ratio between metal from high-grade ore and waste at year *i*
SMB_i	cumulative quantity of metal from low-grade ore in best case (tons)
SMW_i	cumulative quantity of metal from low-grade ore in worst case (tons)
$\Delta SR_{l,i}$	stripping ratio of metal from low-grade ore at year *i*
$CE_{k,v}$	total capacity of production equipment of k^{th} type, v^{th} model (tons) (k = 1 loading; k = 2 haulage; k = 3 drilling)

Table 2-2 Model's general indices and parameters

Constant	Definition
n	total number of time periods to be considered (i = 1, …, n)
l	subscript to define low-grade ore
h	subscript to define high-grade ore
m	superscript to define mining parameters
ma	superscript to define marketing parameters
$proc$	superscript to define processing parameters
K	number of types of mining equipment (k = 1, …, K)
V	number of models of mining equipment per type k
d_i	discount factor $d_i = 1/(1+r)^i$, where r is the interest rate
S_i	Selling price of metal
C_h^m, C_l^m	unit mining cost of high and low-grade ore respectively
C_w^m	unit mining cost of waste removal
C_h^{proc}	unit processing cost of high-grade ore
C^{ma}	marketing cost per unit of payable metal
R	royalty as % of the net revenue
$\alpha_{h,i}$	basic ore grade
$\alpha_{l,i}$	low-grade ore grade
γ_i	total recovery of the payable metal in year i
C_i^t	time cost due to operating cost of processing support services ($/year)
$C_{k,v}^{max}$	capacity limit in tons of k^{th} type, v^{th} model of production equipment
$H_{k,v,i}$	total purchase cost of k^{th} type, v^{th} model of mine equipment in year i
$U_{k,v,i}$	total ownership cost of k^{th} type, v^{th} model of mine equipment in year i

Table 2-3 Variables

Variable	Definition
$M_{h,i}$	metal from high-grade ore to be removed in year *i* (tons)
$M_{l,i}$	metal from low-grade ore to be removed in year *i* (tons)
W_i	waste quantity to be removed in year *i* (tons)
$NE_{k,v,i}$	new production equipment added of k^{th} type, v^{th} model in year *i* (Integer) (*k* = 1 loading; *k* = 2 haulage; *k* = 3 drilling)
$DE_{k,v,i}$	decreased production equipment of k^{th} type, v^{th} model in year *i* (Integer) (*k* = 1 for loading; *k* = 2 for haulage; *k* = 3 for drilling)
$NC_{k,v,i}$	new capacity added in tons of k^{th} type, v^{th} model of production equipment in year *i* (*k* = 1 loading; *k* = 2 haulage; *k* = 3 drilling)
$DC_{k,v,i}$	capacity decrease in tons of k^{th} type, v^{th} model of production equipment in year *i* (*k* = 1 loading; *k* = 2 haulage; *k* = 3 drilling)

Constraints

The next section presents the constraints for the current formulation.

1. Bounds of metal from high grade ore production:

$$\sum_{j=1}^{i} M_{h,j} \leq PMX_i, \quad \forall i \tag{2.2}$$

$$\sum_{j=1}^{i} M_{h,j} \geq PMN_i, \quad \forall i$$

2. Bounds of metal from low grade ore production:

$$\sum_{j=1}^{i} M_{l,j} \leq SMX_i, \quad \forall i \tag{2.3}$$

$$\sum_{j=1}^{i} M_{l,j} \geq SMN_i, \quad \forall i$$

3. Bounds of waste production:

$$\sum_{j=1}^{i} W_j \le WMX_i, \quad \forall i$$

$$\sum_{j=1}^{i} W_j \ge WMN_i, \quad \forall i \tag{2.4}$$

4. Relationship between waste and metal from high-grade ore production:

$$\text{if } PMB_i \ge PMW_i, \text{ then } \Delta SR_{h,i}\sum_{j=1}^{i} M_{h,j} + \sum_{j=1}^{i} W_j \le \Delta SR_{h,i} PMN_i + WMX_i, \ \forall i$$

$$\text{if } PMB_i < PMW_i, \text{ then } \Delta SR_{h,i}\sum_{j=1}^{i} M_{h,j} - \sum_{j=1}^{i} W_j \le \Delta SR_{h,i} PMN_i - WMX_i, \ \forall i \tag{2.5}$$

where the stripping ratio of high grade ore is: $\Delta SR_{h,i} = \dfrac{WMX_i - WMN_i}{PMX_i - PMN_i}$

5. Relationship between waste and metal from low-grade ore production:

$$\text{if } SMB_i \ge SMW_i, \text{ then } \Delta SR_{l,i}\sum_{j=1}^{i} M_{l,j} + \sum_{j=1}^{i} W_j \le \Delta SR_{l,i} SMN_i + WMX_i, \ \forall i$$

$$\text{if } SMB_i < SMW_i, \text{ then } \Delta SR_{l,i}\sum_{j=1}^{i} M_{l,j} - \sum_{j=1}^{i} W_j \le \Delta SR_{l,i} SMN_i - WMX_i, \ \forall i \tag{2.6}$$

where the stripping ratio of low grade ore is: $\Delta SR_{l,i} = \dfrac{WMX_i - WMN_i}{SMX_i - SMN_i}$

6. Capacity limitation of equipment type $k = 1$:

$$\sum_{i=1}^{n} NC_{kz,i} - \sum_{i=1}^{n} DC_{kz,i} \le C_{kz}^{max}, \quad k = 1; \ \forall z \tag{2.7}$$

7. Distribution of new added capacity among different types of equipment, ensuring that, for example, if hauling capacity is increased, so is the loading:

$$\sum_{j=1}^{i}\sum_{v=1}^{V} NC_{1,v,j} - \sum_{j=1}^{i}\sum_{v=1}^{V} NC_{k,v,j} = 0, \quad k = 2,\ldots,K; \ \forall i \tag{2.8}$$

8. Distribution of capacity decrease among different types of equipment:

$$\sum_{j=1}^{i}\sum_{z=1}^{Z} DC_{1z,j} - \sum_{j=1}^{i}\sum_{z=1}^{Z} DC_{kz,j} = 0, \quad k = 2,\ldots,K; \ \forall i \tag{2.9}$$

9. Capacity disposal given available equipment:

$$\begin{aligned} NC_{kz,i} &\le NE_{kz,i} \cdot CE_{kz}, \quad \forall k; \ \forall z; \ \forall i \\ DC_{kz,i} &\le DE_{kz,i} \cdot CE_{kz}, \quad \forall k; \ \forall z; \ \forall i \end{aligned} \tag{2.10}$$

10. Relationship between added capacity and capacity decrease:

$$\sum_{j=1}^{i} NC_{kz,j} - \sum_{j=1}^{i} DC_{kz,j} \ge 0, \quad \forall k; \ \forall z; \ \forall i \tag{2.11}$$

11. Stable tonnage of material extracted for material type $k = 1$, linked by Eq. (2-8):

$$M_{h,i}(\alpha_{h,i})^{-1} + M_{l,i}(\alpha_{l,i})^{-1} + W_i - \sum_{j=1}^{i}\sum_{z=1}^{Z} NC_{kz,j} + \sum_{j=1}^{i}\sum_{z=1}^{Z} DC_{kz,i} = 0; \quad k = 1; \forall i \tag{2.12}$$

12. Definition of variables

$$\begin{aligned} &M_{h,i} \ge 0, \quad M_{l,i} \ge 0, \quad W_i \ge 0, \quad \forall i \\ &NC_{k,v,i} \ge 0, \quad DC_{k,v,i} \ge 0, \quad NE_{k,v,i} \ge 0, \quad DE_{k,v,i} \ge 0, \quad \forall k, v, i \end{aligned} \tag{2.13}$$

Constraints (2-2) to (2-4) present the bounds on cumulative metal from primary and low-grade ore and waste tonnage, which is limited by the feasible domain defined previously. Constraints (2-5) and (2-6) present the relationship between waste and metal extracted, which considers the different possible geometries of the working zone, dependent on the best and worst cases defined. These cases are the ones that bound the solution domain. Constraint (2-7) ensures that production capacity is no greater than the capacity limit available for loading equipment (type k=1). Constraints (2-8) and (2-9) ensure that the capacity available for one type of equipment is also available for all other equipment types (hauling and drilling), which relation is assumed constant over the whole LOM. Constraint (2-10) links the integer decision variables of new and decreased equipment with their corresponding capacities, which are continuous values. Constraint (2-11) ensures that the cumulative added capacity is higher than the total decreased capacity in each period, what prevents the production rate to get negative values. Finally, constraint (2-12) allows the total extracted rock (primary and low-grade ore, plus waste), to equal the available capacity used, which is provided by the added equipment; and constraint (2-13) identifies the variables as integers or continuous.

2.2.2 Comments

The objective function presented in expression (2-1) shows that the main variables of the model are the time related metal tonnage from high-grade ore, metal from low-grade ore, and waste. While the variable corresponding to the waste quantities allows for the definition of the waste-ore relation over time, the metal variables allow for the optimization of metal quantities. The metal optimization accounts for the ore quality at different parts of the orebody. The remaining variables of the optimization model are the added capacity and capacity decrease of each type and model of mining equipment. The inclusion of these variables deals with the stabilization of the mining rate over time periods as a function of the capacity.

The economic parameters involved in the stabilization of the mining rate are the unit purchase and ownership costs of each type and model of mine equipment. The value of the equipment determines the total purchase cost, and, once bought, its whole capacity can be used as newly added mining capacity of the system. The total ownership cost is the penalty for the capacity decrease that reflects the economic consequences of having the idle equipment. In this context, the stabilization of the mining rate over time periods is determined as a search for the balance between the purchase and ownership costs of the production capacity. This balance represents the direct incorporation of the related capital investments in the production scheduling optimization.

It is important to stress that the definition of proper limit values for the variables related to production capacity is essential to guarantee that the mining rates produced by the optimization formulation are physically mineable. The main reason for that is a possible lack of working space to accommodate a large number of mining-equipment and the corresponding accessibility constraints. If the mining rates remain impractical after tightening the constraints related to maximum allowed capacity, an alternative is to redefine the physical pushbacks. In this case, production periods presenting deviations from the production targets can be flagged in the detailed mining sequence and be investigated further.

2.3 Case Study at a Gold Mine

2.3.1 Generating optimal mining production rates

The present case study aims to demonstrate the technical and practical intricacies of the proposed model in Section 2.2 in a real operation. The case study considers a gold mine with the setup (or mining system) that the previous mathematical model represents, assuming a fixed ultimate pit limit. Thus, high-grade material is processed in a mill, and low-grade material is taken to a long-term stockpile treated as a temporary waste dump, to be maybe processed by the end of the mine's life (thus no profit is obtained from it in the evaluated time span). The deposit consists of an orebody of 170,000 blocks of 15x15x10 meters. The mill cut-off is fixed to 1.2 gr/tonne, which defines the high-grade ore material; low-grade ore corresponds to material with a grade higher than 0.9ppm and lower than 1.2 gr/tonne, and the rest is defined as waste. Accordingly, there is approximately 170Mt of ore (destined to the mill), with an average grade of 2.36 gr/tonne, and 1,000Mt of waste material.

The two types of equipment included in the mathematical formulation correspond to haulage and loaders, which are owned by the mine. The "CAT 793C" model is considered for the former, while the latter case considers two models, the "PC8000" and "FEL 994". Table 2-4 presents the costs and parameters used in this mining operation. Table 2-5 shows the details of equipment's capacity, purchase and ownership costs, amongst others.

The solution obtained from the optimization model is referred to as the "optimal" extraction rate. This solution maximizes the NPV within the SSD, and effectively integrates geological uncertainty into the optimization process by considering the intersection area of the extreme mining cases of 20 geological simulation models of the deposit. Figure 2-3 presents the mill feed demand targets used for the optimization model. The cause of the mill demand's variation between years five and nine is due to the mill being fed by external sources, leaving the presented annual capacity available for the processing of material from the current mine.

Table 2-4 Operation's costs and technical parameters

Parameter	**Value**
Mining Cost (US$/t)	3.00
Processing Cost (US$/t)	8.77
Capital Cost (US$/t)	3.65
Discount Rate (%)	10
Mill Recovery (%)	90
Mill cut-off grade (ppm)	1.2
Breakeven cut-off grade (ppm)	0.9

Table 2-5 Equipment parameters

Type	**Loaders**		**Haulage**
Model	PC8000	FEL 994	CAT 793C
Purchase Cost (MUS$)	4.73	1.92	1.77
Ownership Cost (MUS$/year)	0.68	0.27	0.25
Capacity (Mt/year)	25.0	9.60	3.14
Maximum Availability (units)	5	4	34

Figure 2-4 presents the mining capacity required to meet the presented mill demand in the best and worst mining cases. Only high-grade ore is used to feed the mill to its target, and thus, the worst case needs to remove excessively large amounts of material in the first periods (as mining is done bench-by-bench, to arrive at the in-depth ore). This fact causes a high amount of waste mining in the initial stages of the mine (over 90% of the total material extracted), and mostly pure ore during the last years, which are heavily discounted. On the other hand, in the best case, the ore is made available in the initial periods by mining pit shell by pit shell. During the last years, with more heavily discounted cash flow, the total movement of rock is higher (especially waste, with around 90% of the total extraction in the last period, as the stripping ratio increases for the deepest ore).

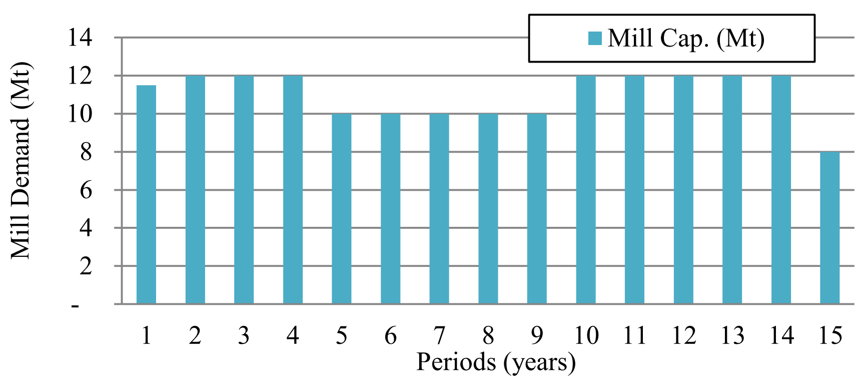


Figure 2-3 Mill's annual available capacity.

Figure 2-5(a) presents the stable solution domain for the pit limit defined for this deposit, created by the intersection of the areas of the cumulative quantities of ore and waste from the "Best Case" and "Worst Case" of 20 orebody simulations, such as the ones presented in Figure 2-4. In this case, the simulations were obtained by direct block simulation (Godoy, 2003). The interior of this domain also presents the solution obtained from the optimization model described in Section 2.2, referred to as "Optimized Case." As mentioned earlier, the optimized mining rate is completely inside of the SSD, what shows that the obtained result is a feasible extraction rate program. Also, the "optimal" mining rate schedule is very close to the "Best Case," particularly before the first 75Mt of extracted ore. The later separation of the optimal case from the best-case limit is likely caused because, as the depth of the deposit increases, the stripping ratio rises, and more waste must be extracted to obtain one ton of ore.

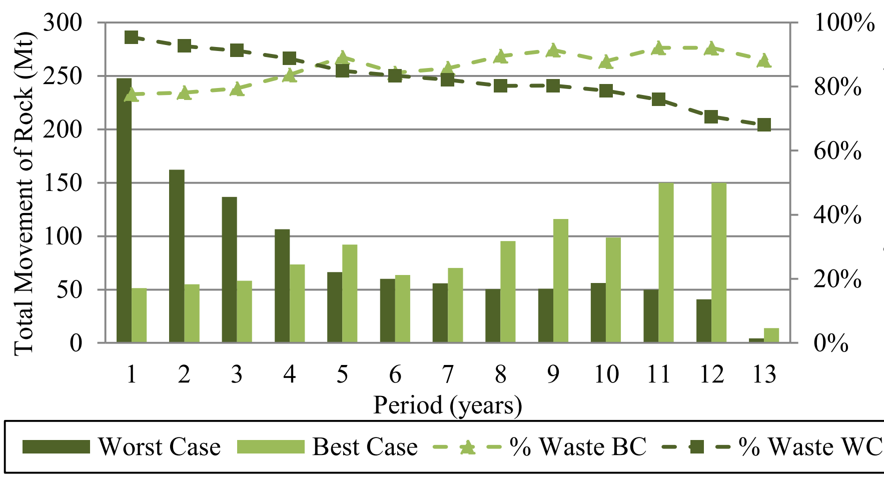


Figure 2-4 Mining capacity required to meet mill demand in best and worst case.

Figure 2-5(b) shows the annual extraction rate defined by the optimizer. Here, the operation starts with a capacity of 63Mt per year, with minor increases in capacity by year 4 and 5, and a major mining rate expansion by year 8, finishing with 100Mt by year 13, when extraction reaches the pit limit. The optimizer aims to maximize profit by three mechanisms. (i) Delaying waste extraction as much as possible while ensuring to maintain a smooth evolution of mining rates. (ii)

Avoiding extreme changes in production rates in consecutive periods, and (iii) ensuring that there is no mining capacity missing or left unused given the existing equipment availability.

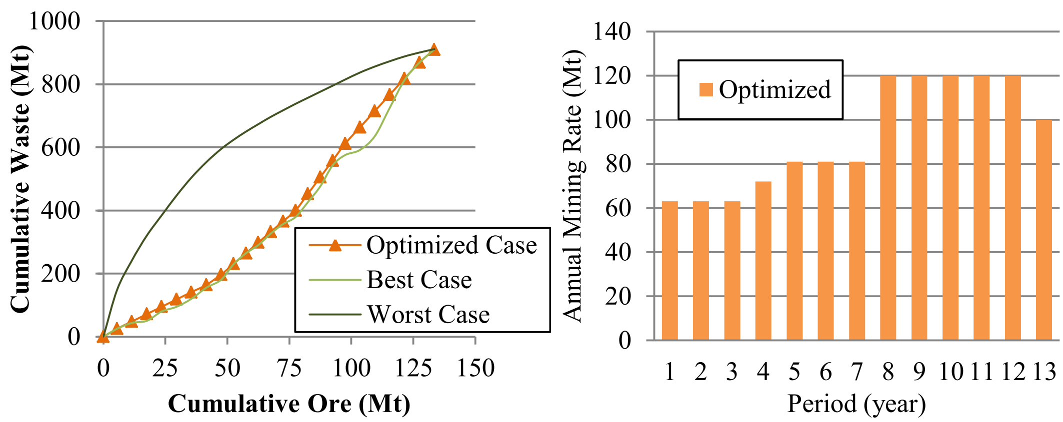


Figure 2-5 (a) Stable solution domain with the proposed model's mining rate solution, and (b) annual production rate plan.

The equipment acquisition program presented in Figure 2-6 confirms this smooth production rate evolution, where for each type of equipment (loaders and haulage), and for each model of a particular type, the figure shows the equipment required per year for the "Optimal" production rate. Here, it shows that in year 4 three haulage trucks are purchased, and six more trucks are added to the fleet in year 8, almost doubling the initial fleet. The PC8000 loader fleet is kept constant along the life of mine and acquiring FEL 994 loaders achieves the capacity increase by more than doubling the initial fleet by year 8, thus delaying capital expenses to maximize the project's NPV.

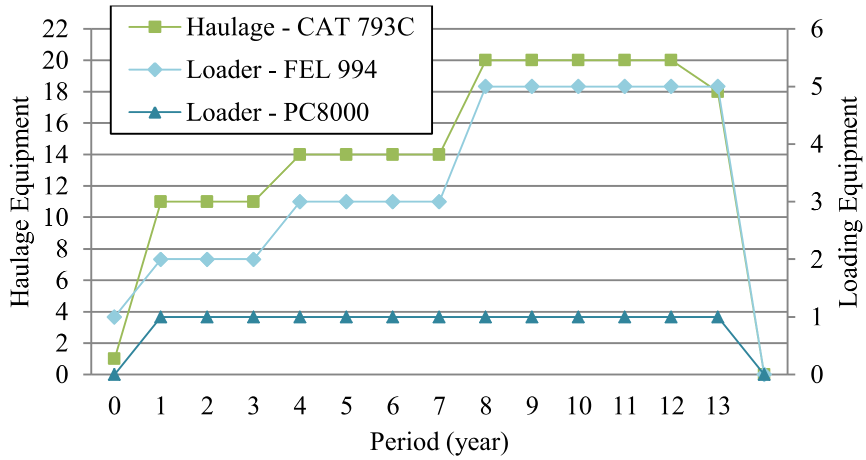


Figure 2-6 Equipment type and model acquisition schedule per period

2.3.2 Using the optimal production rates for scheduling

To further explore the benefits of the proposed optimization model to generate optimal mining rates, the two schedules produced by Milawa Balanced algorithm (Whittle, 1999) available in Whittle Software are compared. One life-of-mine production schedule is based on the mining rates defined by the optimizer presented in Section 2.2, and the other uses a constant mining rate (the traditional approach also practiced at the mine discussed here), equal to the average production of the optimized solution which is 93Mt per year. All the remaining parameters used are identical in both cases, as is the high-grade ore demand destined to the mill. Figure 2-7 presents the annual mining rate for each case.

It is interesting to note the amount of high-grade ore tonnage being extracted in each schedule for the obtained mining rates, as it would be expected that the "Traditional" operation manages to produce more ore due to its higher initial rates. However, the next figures show that this is not the case. Figure 2-8 presents the mill feed demand target in the black dotted line. The actual mill utilization for each of the two generated schedule is presented in this figure by the bars, showing the amount high-grade ore material extracted in each period for the traditional and optimal case. Both obtained schedules manage to meet mill feed demand in every year, suggesting that the traditional schedule has an increased mining rate during the first years only to mine waste, which increases the operation's costs and doesn't generate any profit.

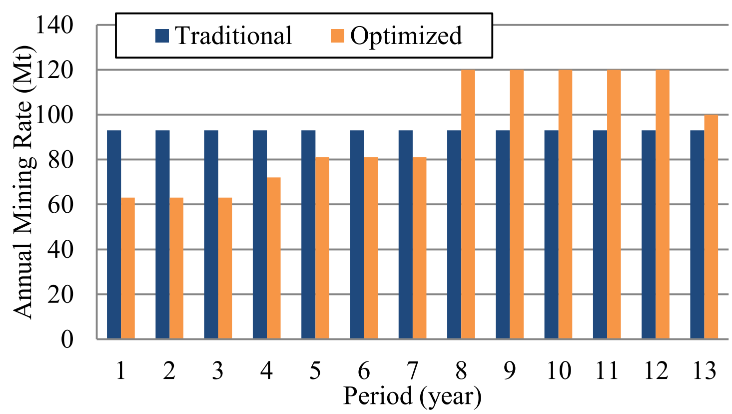


Figure 2-7 Total extraction for schedules based on the traditional and optimal mining rates

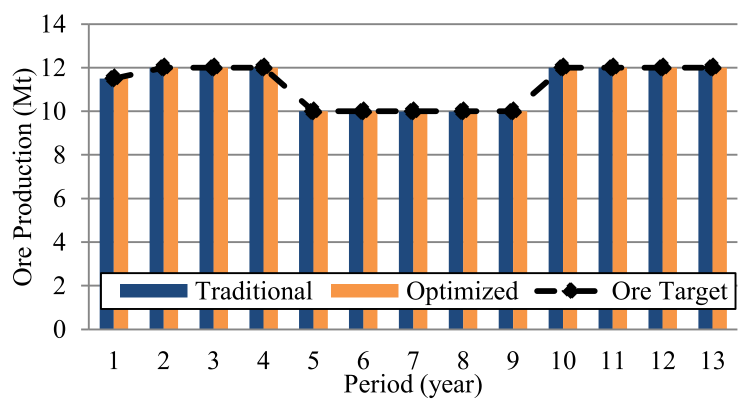


Figure 2-8 Mill's annual available capacity and high-grade ore extracted per period for the traditional and optimized case

The "Traditional" case has a steady extraction rate along the whole project, but from Figure 2-7 and Figure 2-8, it is possible to see that this causes the operation to invest in the unnecessary capital during the initial years (to obtain this steady rate). However, these investments are not necessary to meet mill demand and only results in early waste mining and equipment acquisition. These purchases reduce the profit of the initial years, which are less discounted and thus, have a strong effect on the project's NPV. In comparison, the "Optimal" production rates obtained by the proposed formulation show that during the first half of the mine life there is a lower capacity required, maximizing the profit by meeting mill demand, minimizing waste mining and delaying capital expenses as much as possible.

The previous analysis proves that the optimization model proposed here looks to maximize the NPV of the project by delaying unnecessary expenses and investments and maximizing the metal production. Figure 2-9 shows this value increase, which presents the cumulative discounted cash flow (DCF) for the "Optimized" as well as for the "Traditional" cases, assuming that mill capacity, as well as the mining rates, are perfectly met. By periods 7 and 8, the optimal case incurs on high expenses to increase the equipment fleet and raise the mining rate of the operation. These costs cause a slight decrease in the cumulative DCF, and, as more waste rock is removed at this point, the cumulative cash flow curve flattens in comparison to the traditional case. However, this also allows meeting ore demand, obtaining a 20.7% higher NPV than the stable mining rate case, which decides to extract more waste at the initial years, punishing the cash flow from the beginning of the operation.

The SSD presented in Figure 2-10 illustrates the differences between the traditional and the optimized schedules obtained from Whittle. This figure shows that both extraction sequences are located inside the SSD, proving that they are both feasible mining rates independent of the encountered geology of the deposit. However, the "Traditional" case is consistently further apart from the "Best Case" in comparison with the "Optimized" case, which demonstrates that the traditional mining rates tend to extract higher amounts of waste earlier in the life of mine, only to obtain a fixed, stable mining rate.

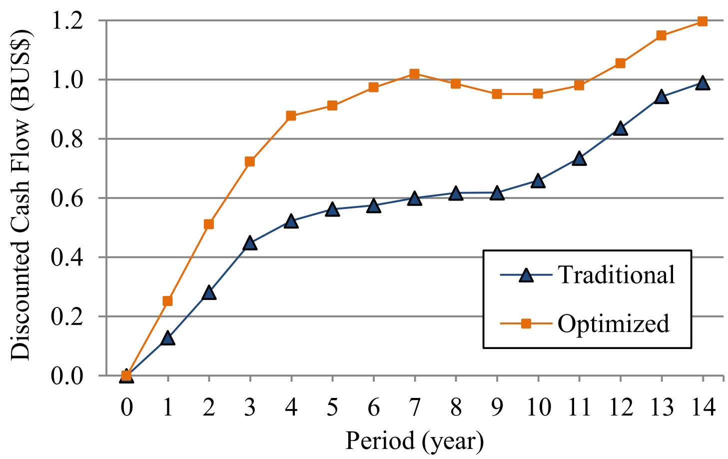


Figure 2-9 Cumulative discounted cash flow for the traditional and optimized case

Even though the estimated model of the deposit was used to obtain the schedule, the mining rates were obtained considering the stable solution domain created from the intersection of the solution domains of 20 different geological simulations of the deposit. This intersection generated the final feasible domain of ore and waste combinations and not the solution domain formed by the estimated orebody model. Figure 2-11 presents the effect of not considering the geological uncertainty to define the stable solution domain and carrying the optimization process over the solution domain defined by the estimated model. In this case, both sequences obtained for the traditional and optimized mining rates (referred to as "Trad – Estimated" and "Opt – Estimated" respectively in Figure 2-10) present infeasible combinations of ore and waste extractions (highlighted in yellow in the graph). These infeasibilities show once again that not considering geological uncertainty in the optimization process results in infeasible mine plans and the impossibility to meet the expected mill demand.

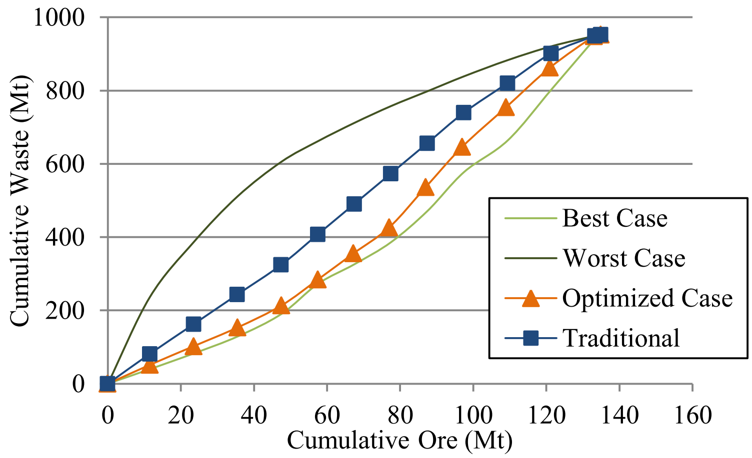


Figure 2-10 Cumulative extraction of ore and waste for the schedules obtained from traditional and optimized mining rates

2.4 Conclusions

In conclusion, the research presented shows that the proposed MIP model not only provides a feasible mining rate which considers equipment acquisition and delaying of capital expenses but also that this mining rate schedule presents clear benefits when used as a starting point for planning a mining schedule. Results were obtained by using the Milawa Balanced algorithm from Whittle Software over a gold mine case study to produce two schedules, with and without optimized mining rates, where the optimal mining rate case presented a 20% increase in NPV. However, the goal of this optimization is to define an optimal mining rate, not an actual mining schedule. The scheduling problem is considered as a separate, complex problem, where the output of the model proposed in this study may be used as an input to the design of an optimal long-term mining schedule. However, this would require assuming a fixed ultimate pit limit *apriori*, which arguably limits the development of an optimal mine production schedule.

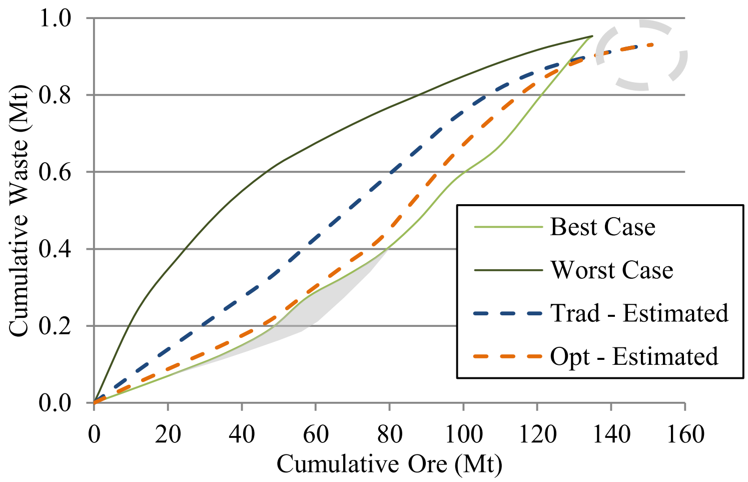


Figure 2-11 Comparison of traditional and optimized schedules obtained over the solution domain (SD), and the feasible stable solution domain (SSD)

Accordingly, an important limitation of the current formulation is that the process is based on the solution domain, which, even though can consider geological uncertainty, it assumes that the ultimate pit limit is defined and fixed. This assumption is not accurate, as after the scheduling is done, and subject to the different uncertainties that govern a mining operation, it is highly possible that the ultimate pit limit will change. Together with this, the different ownership and purchasing costs are assumed constant along the whole life of the equipment, which, once again, is an important simplification in the model.

Further work may focus on extending the proposed formulation to the mining scheduling optimization. Together with this, efforts could be made on increasing the complexity of the mining system considered, i.e., include multiple mines and deposits containing multiple elements, as well as considering the income obtained from the stockpile or other processing streams. Additionally, it would be important to consider different types of uncertainty, including commodity price, costs and equipment variability.

CHAPTER 3
A Multivariate Destination Policy for Geometallurgical Variables in Mineral Value Chains using Coalition-formation Clustering

The previous chapter studied ways to optimize capacities by investing in capital expenditures. Now, the focus is shifted to explore ways to improve processing stream performance, by developing a destination policy that is able to deal with complex geometallurgical variables within the mine planning optimization model.

3.1 Overview

In mining, orebodies define the design and the value of a project, based on the attributes of the rock and the operational characteristics of the project, the processing streams used along the life-of-mine (LOM), and the range of profit produced by the project. The over-simplification that arises in the conventional optimization of mining projects is the assumption that a block of materials (mining block) has an intrinsic dollar value, and that a given cut-off grade will define what is ore (to be processed for profit) or waste. However, there are various other parameters that affect the dollar value of a block. For example, the presence of deleterious elements (such as arsenic), hardness, spatial location (which will define when the material can be extracted), and so on, will all have an effect over block value. Because of this, these pertinent variables should be considered during planning to optimise to which processing stream a mining block will be directed to and with this, to realistically evaluate the project's performance and value.

This issue takes precedence in the increasingly complex deposits being presently developed, where processing plants' and refineries' performance depends greatly on how their different requirements are met (for example, blending constraints must be met in order to maximize metallurgical recovery). These hard constraints force the project to be optimized around them, making it necessary to consider from an early stage not only grade uncertainty, but also all the variability of relevant geometallurgical characteristics (rock hardness, material types, etc.) that affect the configuration of the different processing streams (i.e., energy consumption, metallurgical recovery, etc.). However, because of the high costs associated with exploration, the limited information obtained from sample composites and the inherent flaws in sampling and testing systems, obtaining reliable geometallurgical information is difficult and requires cross-disciplinary efforts (Figueiredo and Piana, 2016). In addition to this, the geology of a deposit (grades, material types, rock properties, other) is highly uncertain, being one of the main sources of technical risk in a mining operation (Godoy and Dimitrakopoulos, 2004). Thus, many efforts have been directed towards developing methods that account for this uncertainty and manage the related rick in the design and evaluation of a project. Two aspects are included in these efforts, stochastic or geostatistical simulation to quantify geological uncertainty and stochastic optimization that uses the quantified uncertainty to manage the related risk while optimizing mine design and planning. Methods developed have been successfully implemented in various mining projects (Goodfellow, 2014; Montiel, 2014).

Geological uncertainty extends to uncertainty in the supply (materials) to various processing streams, giving special importance to the process of re-distributing the extracted rock between the available destinations, so that the different constraints are met. This reordering and delivering process, referred to as *destination policy*, is especially important in poly-metallic mines with multiple processing streams, as there are increasingly complex constraints. Traditional mine planning models define destination policies solely based on the material's concentration with respect to different cut-off grades and treat a block as waste if its (assumed) value is negative. Although it is traditionally used given the methods available to date, this assumption is misleading, as blocks have different attributes and concentrations of elements (other than the grade of the main commodity), which must be extracted, transported, blended, processed, and sold in order to yield a financial gain. All these activities are also strongly affected by the geological uncertainty present in the deposit, which will ultimately define the performance of the mining system. Thus, the actual value of a mining block depends not only on the period when it is extracted, but, in addition, (i) to the quality of the elements and material types contained in it, as well as (ii) on the destination where the block is processed, which entails the blending requirements, processing costs, recovery curve of the metallurgical process, and so on. In other words, the actual "block value" cannot be calculated individually. For example, if the available sulphur content in the processed material is not enough to reach blending constraints in the plant, then lower grade material with higher sulphur content could be sent to the processor from areas of the deposit with higher sulphur content, even if this material is not profitable on its own. Disregarding these non-linear relations would result in failing to meet blending constraints, reducing expected metallurgical recovery, and ultimately decreasing project value.

This paper aims to tackle this problem by developing an optimal destination policy mechanism for polymetallic deposits in order to increase project value and the reliability of project evaluation. This destination policy is based on a multidisciplinary implementation, combining mine planning with coalition formation theory using the "Shapley Value" (which is a line of study of cooperative game theory), and considers within the decision process the deposit's geometallurgical variables, its blending requirements, and the uncertainty related to its geology. These considerations increase project value by improving the performance of the available processes, meeting the project's planned targets, as well as taking maximum advantage of the limited resource.

The next section of this paper reviews the existing literature on mining optimization, focusing on destination policy, as well as on the inclusion of geometallurgical variables. The description of the proposed method follows, with a brief introduction on game theory, and the concepts that will be used. The proposed method is then tested over a real life copper gold deposit with six possible destinations, showing that including complex variables of the processed material in the optimisation not only allows the project to meet blending constrains, but also increases final project value without even changing the extraction schedule. Conclusions and future work follow.

3.2 Literature Review

3.2.1 Mining Optimization and Destination Policy

Thus far, the decision of defining where each block is sent after extraction is based mainly on two aspects: defining certain ranges of grades accepted at the different destinations, commonly referred to as cut-off grades (Lane, 1988; Rendu, 2014), or the general revenues expected from sending a block to each of the possible processing streams. However, these policies are based on a longstanding serious oversimplification in mine planning, which is to assume that a block has an inherent dollar value (Lerchs and Grossmann, 1965; Tolwinski and Underwood, 1996; Ramazan, 2007; Meagher et al., 2009). This results in severe deviations from expected project revenues and performance, as well as clear suboptimal results (Wharton, 2004). By assigning a "dollar value," the formulation assumes a priori when a block will be extracted (i.e., the mining sequence), and what material is ore and what is waste (thus, where it should be sent), before any optimization has been done, bypassing the actual destination policy decision.

Some work has been done in designing dynamic policies, such as in Meagher et al. (2009), where the destination decisions are updated on a yearly basis according to new information that becomes available once a block is extracted. The possibility to re-optimize is considered as valuable flexibility, which is added to the block's value. In this paper, geological uncertainties, market uncertainties, and the time value of money in calculating the value of a block at its period of extraction are accounted for. However, the formulation proposed grows exponentially if multiple elements, deposits, and/or processing streams are considered, and the focus is still placed on assigning an individual dollar value to each block instead of on optimizing the mining complex as a whole. Asad and Dimitrakopoulos (2013b) propose a heuristic approach to select an annual cut-off grade under geological uncertainty, which maximizes the net present value (NPV) of the mining operation and satisfies production constraints. Continuing on this line, Meagher et al. (2014) develop a dynamic cut-off grade policy to define block destination under geological uncertainty. Here, the optimal cut-off grade is defined on a yearly basis in order to optimize the pushback design and maximize project value. However, the model only considers one element with one processing facility and the optimization is done greedily by sequentially maximizing the NPV of each pushback, instead of optimizing the whole deposit simultaneously. Thompson and Barr (2014) generate a dynamic cut-off grade policy under stochastic prices and note the differences between considering uncertainty in the cut-off results when compared to traditional methods. However, the authors still assume an economic value of the block, and do not consider the geological uncertainty of the deposit.

Few methods have been presented in the literature that dynamically account for the destination policy in the optimization process and, at the same time, develop a global mine plan. The multistage stochastic optimization method developed by Boland et al. (2008) presents a destination policy mechanism that optimizes each geological scenario independently once the scenarios have "differentiated enough" along the LOM. Other studies have developed robust destination policies, such as Montiel and Dimitrakopoulos (2015), who proposes a mathematical programming model where destination policies are first stage variables (thus equal over all scenarios). Here, the author considers the optimization of the whole mining complex under geological uncertainty, with multiple material types and processing streams. The method presented is able to develop a mining schedule that defines when each block is mined and where it is sent, avoiding the need for pre-defined cut-off grades, and maximizing project value while meeting production constraints. However, in this case, the destination policy can only be optimized if the material type of a block is the same over all the simulations. In other words, the model might produce misclassification errors (i.e., where oxides are sent to processing streams that only accept sulfides), resulting in infeasible solutions. Menabde et al. (2007) also define a robust destination policy, but it is based on cut-off grade optimization. In their study, the authors account for geological uncertainty and present a MIP formulation where the destination policy is defined by classifying blocks into bins of "similar grades", where each bin is sent to the same destination. By doing so, they are able to avoid misclassification problems, as seen in the previous case. However, their formulation only accounts for a single mine, with one element and a single processing stream, and does not consider the problems that arise with blending requirements that entail more than one element.

As the complexity of mining projects increases (in terms of the number of deposits, processing streams, and elements), traditional destination policies, such as the ones presented in the previous paragraph, lack in their ability to consider the multidimensional aspect of the mining optimization problem. Recent work on destination policy has extended from cut-off grade optimization to integrate multivariate distributions, making them more adept for complex mining projects. Goodfellow and Dimtrakopoulos (2014) developed a stochastic optimization of a mining complex, which accounts for geological uncertainty, and considers a multidimensional destination policy. To do this, the authors implement k-means++ clustering (Lloyd, 1982; Arthur and Vassilvitskii, 2007; Gan et al., 2007), a pre-processing clustering mechanism where the number of groups is defined by the modeller, and all blocks are classified according to their proximity (i.e., similarities in material types, grades, etc. in a Cartesian grid). This way, the destination of a block is defined based on the cluster it belongs to instead of its individual properties for all simulations; this provides a robust destination policy defined under geological uncertainty, and, at the same time, reduces the computational cost of the optimization process.

An example of this clustering process is presented in Figure 3-1, where four clusters of blocks are defined and created by calculating their proximity to each other (considering all scenarios) with regards to their concentration of gold and copper. Thereafter, the cluster's destination decisions are taken accordingly.

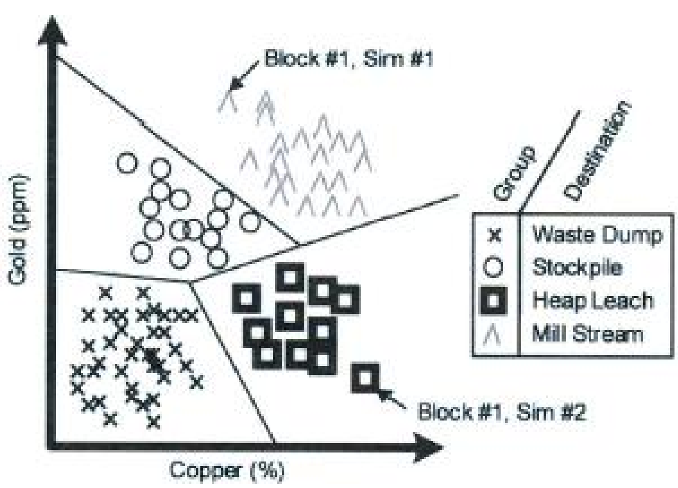


Figure 3-1 Creation of 4 clusters according to gold and copper concentration. From Goodfellow and Dimtrakopoulos (2014)

However, in Goodfellow and Dimtrakopoulos (2014), even though the destination policy is decided over each cluster, the decision of where a given cluster is sent does not directly take into consideration the relation of aggregates of material being processed together, which must altogether meet the complex blending constraints. For example, if particularly tight metallurgical constraints are required by a process, such as a silica-magnesium ratio, if one block has a high magnesium and copper concentration, but is low in silica, and another block has a high silica concentration, but a low copper grade, then it might be preferable to “cluster” and process the two blocks together (even if their attributes are not similar).

Thus far, geometallurcial research has been increasing, with a general agreement from the industry of its importance in a project’s performance. However, most of this information is lost through the planning process, or is not used to its full extent, leaving a gap when it comes to actually using this information in the mine plan to obtain a truly optimized operation that considers the whole flow of material. The method proposed herein aims to integrate this information into the planning process by creating clusters of blocks with two objectives. On one hand, the method aims to reduce the computational cost inherent in mining optimization, which is mainly caused by the large amount of blocks encountered in real-size mining projects; and on the other hand, the method seeks to take into account not only metal grade and recovery, but also the relation of a wider range of geometallurgical variables. The latter variables help to obtain more reliable values of the aggregates of blocks in the different processing streams. This happens because the final recovery and metallurgical performance will depend on the total material being processed together and not on the individual properties of each block. Once material arrives at the plant, there is no longer any perception of a “block”, but rather a blend of extracted material.

3.2.2 Incorporating Geometallurgical Variables

Geometallurgical variables involve any rock property that has a positive or negative effect on the business’ ultimate value (Coward et al., 2009; Dunham et al., 2011). Some of the more critical (and known) properties are recovery, grindability, throughput, power consumption, mineralogy, and content of deleterious materials. As such, geometallurgy is a cross-disciplinary through its combination of geology, metallurgy, and mine planning to design a processing stream that fits the actual characteristics of the resource (Dunham et al., 2011). It is known that a mining project’s overall value and performance depends not only on the ore grade and the plant’s recovery, but also on variables, such as the mineral composition, energy consumption, additives needed, penalties involved, mineability of the deposit, etc. (Van den Boogaart et al., 2011). However, most of these variables are omitted from traditional mine planning methods, not to mention the variability and uncertainty related to them. Given this, in order to obtain a reliable mine plan and a better representation of a project’s value, more detail on the rock properties of the extracted material need to be considered in the optimization.

Some work has been done to incorporate these variables into the mine design and planning steps, such as in Coward et al. (2009), where they are classified as “primary” and “response” geometallurgical variables. Primary variables are defined as additive or easily manipulated to be linear; response variables generally present complex distributions that cannot be easily combined. The authors note that because most of the rock properties are non-linear, traditional estimation methods for orebody modeling (such as kriging), are unable to reproduce them without serious biases in the results. Conversely, conditional simulation methods have the advantage of keeping the variable's spatial variability, avoiding estimation mechanisms. This allows for the integration of complex non-linear variables into the orebody model. Coward et al. (2013) apply this framework to a mining operation and aim to generate a value chain model by evaluating geometallurgical recovery factors.

Van den Boogaart et al. (2011) focus on optimising the mineral processing stage by generating an "adaptive process" using a geomathematical model. This model is developed from conditional expectations and regression models, which adapt the mill's grinding size to the material being processed (if the benefit obtained from doing this exceeds the investment required for having a flexible process). However, the authors note that care must be taken when planning for adaptive processing, as it is a reoccurring mistake to develop models that assume perfect information from limited samples. Here, they define a simplified model to calculate the value generated by different types of ore, depending on their mass, grade, the liberation per grinding size selected, and the corresponding milling energy required. However, the authors neither consider the inherent uncertainty of these variables nor the geometallurgical effect over the whole mining complex, focusing solely on the processing stage, which has been proven to be suboptimal in terms of maximizing project value.

Dunham et al. (2011) comment on the impact that geometallurgy can have over the value and viability of a mining project, transforming the extraction schedule of the deposit compared to traditional evaluations, which do not consider these variables. Accounting for these “non-grade properties” of the rock in the optimization process, together with stockpiling and blending strategies can strongly affect the processing strategies chosen for the operation, and thus the project value. The authors note also that integrating geometallurgy into the mine plan provides a spatial context, which is especially useful to, for example, study the distribution of deleterious elements, which can drastically change the project’s design and improve its performance.

With respect to the realistic representation of geometallurgical variables, Van den Boogaart et al. (2014) present a simulation method for modelling discrete and continuous geometallurgical parameters. The authors state that “conditional geostatistical simulation of geometallurgical parameters enables the construction of a processing model for computing recovery, equipment usage, processing costs and other relevant parameters and thus the monetary value for mining and processing a block with certain parameters.” They also note that traditional geostatistical techniques cannot be directly applied for conditional simulation of geometallurgical parameters for two main reasons. Firstly, many variables have non-Euclidean statistical scales (such as mineralogy), which produce, in some cases, infeasible values in the simulation. Secondly, processing material entails nonlinear transformations of the rock’s properties, and, as such, the conditional distribution of the variables is relevant for the simulation not only their mean and variance, as is done in traditional estimation methods or Gaussian simulations. Due to this, the simulation needs to reproduce the joint conditional distribution of all of the relevant geometallurgical variables being considered. Thus, they propose an adaptation to the traditional conditional simulation procedure by using a joint multipoint conditional simulation framework. In their approach, the authors adapt the single normal equation simulation (SNESIM) proposed by Strebelle (2002) to simulate categorical variables by estimating the conditional probability distribution functions of a training image via multinomial logistic regression (i.e., it is assumed that un-sampled locations follow a multinomial distribution), and extend its application to consider information provided by different scale layers (i.e., other type of variables, not necessarily categorical) in conditioning locations.

Deutsch et al. (2016) adapt different geostatistical and numerical techniques to generate high-resolution simulations of mixed, continuous, and categorical geometallurgical variables, accounting for their non-linearity and for the correlation existing between different jointly- simulated variables. In their study, the authors focus on the grinding index and on the mill's Bond Work index (BWi) to maximize the throughput and metallurgical recovery of an operation. However, the problem with simultaneously simulating geometallurgical variables sampled in different scales (which is often the case with regionalized variables) is yet to be addressed, eliminating the possibility of accounting for the existing correlation between these variables.

It is known that geometallurgical variables directly affect the performance of the downstream processes of a mining complex. Because of this effect, the approach presented herein includes these variables directly into the destination policy mechanism by implementing a multivariate selection method that defines which blocks are processed together at a given place, given their combined multivariable attributes. This is done by implementing coalition formation algorithms (which are an extension of cooperative game theory), which are defined as grouping mechanisms that are able to account for multiple non-linear attributes. This way, the proposed method considers not merely the main elements' grade in a block, but rather a set of properties of the rock, which have an effect on the downstream processes, and groups them according to their processing preferences. This new clustering algorithm will better maximize project value, and will achieve production targets while taking into account the complex blending requirements.

A global overview of game theory and, in particular, the concept of coalition formations is presented next, together with its relation to the destination policy of a mining project.

3.3 Proposed Coalition-based Destination Policy Method

Game theory is the study of strategic interactions between decision makers (Schelling, 1980). Formally, Myerson (1991) defines game theory as the study of mathematical models of conflict and cooperation between rational decision makers, or "players." In order to maximize its utility, a player must make decisions while strategically predicting what the other players will do, as his payoff depends on his own actions, as well as on the other players' actions. This way, game theory provides the techniques for analyzing situations in which two or more agents make decisions that will influence one another's welfare (Aumann, 1976). In particular, cooperative game theory focuses on studying games where players have the opportunity to communicate with each other and form coalitions in order to increase their utility (Osborne and Rubinstein, 1994). This increase of value is obtained given the non-linearity of their utility function. For example, if two agents decide to team up, then their compound value must be higher than or equal to the sum of their individual initial values. The individual contribution of each player to the coalition is usually different and a "solution concept" for a coalitional game is a revenue and/or information sharing mechanism (Brandenburger and Nalebuff, 2002; Von Neumann and Morgenstern, 2007).

3.3.1 Coalition Formation and the Shapley Value

There are multiple ways to divide revenues, but the most recurrent form studied is such that the value allocation is "fair" (Leyton-Brown and Shoham, 2008), making sure that the coalition remains "stable." "Stable" means that none of the players wish to leave the group to form another group as their value is maximized in their current state (Aumann and Dreze, 1974). To solve the problem of "fair value allocation," the most studied revenue sharing mechanism is based on the *Shapley Value* (Shapley, 1953), which is the mathematical evaluation of a player's gain in a game. The Shapley Value is defined by using "characteristic functions", which is the mathematical representation of the value generated by a subset (or coalition) of players in the game (Brandenburger, 2007; Von Neumann and Morgenstern, 2007). By definition, this characteristic function must satisfy three axioms in an N-player game. Given $V(S_i)$, defined as the characteristic function of subset S_i of the N players ($S_i \subseteq N$):

- $V(\varnothing) = 0$
- Given coalitions S_1 and S_2, where $S_1 \subseteq S_2$; $S_1, S_2 \subseteq N$, then $V(S_1) \leq V(S_2)$
- Given disjoint coalitions S_1 and S_2 where $S_1, S_2 \subseteq N$; $V(S_1) + V(S_2) \leq V(S_1 + S_2)$

It can be noted that the third axiom highlights the non-linear nature of the characteristic function (and thus, of the coalition formation process), enabling the representation of complex relations between the players. It can also be noted that, if a parameter in the definition of the characteristic function is negative (i.e. the cost *"c"* of processing higher tonnage will be higher when processing more tonnage, reducing the utility function), then this cost can be normalized and added as a negative (i.e. *"1 – c"*). Translating these three axioms into mathematical form yields the Shapley Value formula, which is a unique payoff allocation that divides the full profit of the grand coalition among the players (Osborne and Rubinstein, 1994) and is calculated in function of their marginal contributions to all possible coalitions (Roth, 1988; Gul, 1989; Brandenburger, 2007; Branzei et al., 2008). This allocation system defines the revenue (or satisfaction degree) obtained by player *i*, $i = 1 \ldots N$ in a game of ***N*** players, for all possible coalitions *C*, where $C \subseteq N$. The mathematical formulation is presented in Eq. (3.1).

$$SH_V(N, i) = \sum_{\forall C} \underbrace{\frac{(|C| - 1)!(n - |C|)!}{n!}}_{\text{Part 1}} \underbrace{\left[V(C) - V(C - \{i\}) \right]}_{\text{Part 2}} \tag{3.1}$$

Equation (3.1) is referred to as "the Shapley Value of player *'i'* "; here, the value is calculated over all possible coalitions ($\forall C$), where $V(C)$ and $V(C - \{i\})$ represent the characteristic function of a coalition with and without player {i}, respectively. This way, "Part 1" corresponds to the summation of all possible permutations of coalitions that can be formed in an *N*-player game, and "Part 2" corresponds to the marginal contribution of player i to each of this coalitions. Also note that $\sum_{\forall i} SH_V(N,i) = V(N)$, i.e., all the value generated in a game is divided among its players.

3.3.2 Defining Priority Groups and Pre-Processing Mechanisms

The main drawback of the Shapley Value is that it cannot be calculated in polynomial time, as the calculation of all permutations of players in a game is computationally very expensive and becomes unmanageable as the number of players increase. Because of this, Liu et al. (2011) present a cooperative game theory approach for multi-objective categorization where the Shappley Value is approximated by "priority groups", which are computed in $O(n^3)$ time, where n corresponds to the number of players in the game. In their paper, the authors present a simple example of three families who must decide where to go on vacation and the value of the different families (players) is represented in a characteristic function. In this case, the function depends on the number of families joining (as economies of scale allow the cost of the trip to be lower for more players), on their holiday destination preference, and on the relative preference of each family to go with each other. Here, the satisfaction of each family is then measured by their Shapley Values and the vacation groups are defined.

The basic idea behind these *priority groups* is that players that present favorable coalitions will still remain together as the coalition gets bigger. This means that the ultimate optimal priority groups can be obtained by recursively combining the latest larger Shapley Values. The authors present the following example: suppose that there are three players (A, B and C), and two available destinations (L and P). All the options are stated and, as a first step, a pairwise combination of the players is done. However, only the combinations that actually create value are kept (as coalitions are formed only if $V(A)+V(B)\leq V(AB)$). These combinations are created recursively, level by level, eliminating unfavorable groups to reduce the computational cost. Algorithm 3.1 presents the pseudo-code to find priority groups presented in Liu et al. (2011).

Algorithm 3.1 Finding priority groups (From Liu et al. (2011))

```
Input:
       N = {P1, ..., Pn}, the set of players
       T = {T1, ..., Tm}, the set of targets
       F, the set of characteristic function
Output: Priority groups
Local variables:
       l, a level number
       gl(TS), gl is a group in l-level and TS the target set that gl belongs to
       Sl, the set of pairwise combinations of members in (l – 1) level
       Cl, the candidate in l-level
       εl, the l-level threshold of priority
       PGi, the set of priority groups
       SHF-t(gl,i, gl-1,k), a Shapley value for the group gl-1,k in target t
       w.r.t. the game (gl,i, F)
Steps:
Step 1. Initialization
       • l ← 1
       • For the l-level, players Pj(j = i, ..., n) form n groups separately.
       g1j(j = 1,...,n): g11 ← P1, g12 ← P2,...,g1n ← Pn
       • Cl ← {g11(T), ..., g1n(T)}
Step 2. Generate candidate set Cl level after level
       Repeat
       (1) l ← l + 1
       (2) Form candidate items (gl-1,j, gl-1,k) from pairwise combinations of the
       members in Cl-1 such that
        SHF-t(gl,i = (gl-1,j, gl-1,k), gl-1,j) > F(gl-1,j),
        SHF-t(gl,i = (gl-1,j, gl-1,k), gl-1,k) > F(gl-1,k), and
        [SHF-t(gl,i = (gl-1,j, gl-1,k), gl-1,j)] * [SHF-t(gl,i = (gl-1,j, gl-1,k), gl-1,k)] > εl
       The set of pairwise combinations is denoted by Se.
       (3) Ce ← Cl – glj(T')
       (4) If there are gli(T), glj(T') in Cl such that T ⊇ T'
       then Cl ← Cl – glj(T')
       Until (l = n or Cl = Cl-1)
Step 3. Generate priority groups
       Select priority groups GPi from Cl such that ∪_{gli ∈ Cl} gli = N
Step 4. Return PGi
```

This procedure is diagramed in Figure 3-2, where two example priority groups are created (PG1 and PG2).

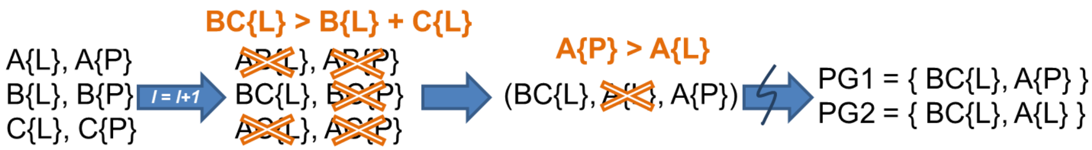


Figure 3-2 Priority group creation process. Adapted from Liu et al. (2011)

In the case of mining, the goal is to define the destinations of blocks being mined on a given year. Therefore, parallel with Liu et al.'s (2011) paper, each family may be represented by each block being extracted on the same period and each holiday destination can be seen as a block's possible processing destination, such as a stockpile, waste dam, mill, leach pad, etc. This leads to a major problem given that a block is considered as a player in the game and mining projects usually entail millions of blocks coming from multiple mines, while the applications presented by Liu et al. (2011) included only up to 60 families (i.e. players). In light of the number of "players" in the mining case, to successfully calculate the Shapley Value of the players in different coalitions, it is crucial to apply a short-cut mechanism (such as the priority groups), that enables a faster and more feasible calculation time. If a mine has one million blocks, which is typical of a medium-sized mine, then calculating all the permutations of thousands of possibilities of coalitions (Eq. 3.1) would make the algorithm impossible to apply in the mining industry.

To reduce the computational cost of the formulation even further, it is proposed to pre-process the deposit by clustering similar blocks together. In this case this is done by using k-means++, where blocks being extracted in the same period which belong to the same initial cluster can be treated as families of blocks and optimized together, as presented in Figure 3-3.

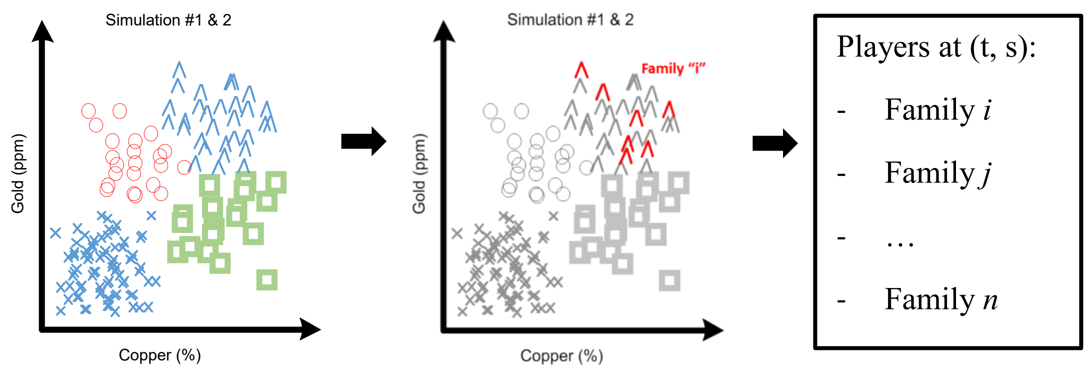


Figure 3-3 Definition of families to pre-process data as input to priority group generation.

Clustering algorithms are usually based on grouping data according to their density and "proximity" in a standardized grid. These algorithms can be classified as centroid-based (such as k-means or affinity propagation clustering), hierarchical (such as spectral clustering), or neighbourhood growers (such as DBSCAN and Agglomerative clustering), where each one has its advantages depending on the type and amount of data being analyzed. K-means++ clustering is implemented here as it allows clustering of qualitative, as well as quantitative, data. Moreover, it is simple to implement considering the large amount of data being represented, and, as shown in Goodfellow and Dimitrakopoulos (2014), it has been successfully applied in the mining optimization process to develop robust destination policies that account for multiple attributes and material types, as well as geological uncertainty. The basic algorithm to perform k-means++ clustering is presented in Algorithm 3.2.

Algorithm 3.2 K-means++ clustering pseudo code

```
Input:
       k, number of clusters,
       D, multi-dimensional data set
Output: set of k clusters
Steps:
Step 1: Initialization
       Arbitrarily choose an object of D as a centroid
       Repeat:
       (1) For each data point x ϵ D, compute d(x) as the distance between x and
           the nearest centroid.
       (2) Using a weighted probability distribution, choose a data point x' as a
           new centroid, where data point x is chosen with probability D(x)^2
       Until k centroids have been chosen
Step 2: Clustering
       Repeat:
       (1) Assign each data point x to the cluster whose centroid's mean has the
           least squared Euclidean distance (i.e. it's nearest mean).
       (2) Calculate the new means of each cluster to be their updated centroids.
       Until there is no change in data assignment;
Return set of data points per cluster
```

Another pre-processing mechanism that could be considered is to remove from the destination policy optimization blocks, which are clearly waste (defined by low concentrations of any or all of the valuable elements encountered), and can be sent directly to the waste dump. However, there are two points that might hinder this removal. Firstly, due to geological uncertainty, a block may appear as waste in some simulations, but not in others, adding ambiguity to the robust definition of a block as "waste". Secondly, even though a block may appear to be waste, as different geometallurgical characteristics are considered, the block can still contain other valuable elements needed for meeting blending constraints.

Together with the clustering mechanism, as a measure to reduce the complexity of the initial formulation, in the following case study the mine production schedule will be assumed fixed. In other words, it will be known which blocks are to be extracted in each period. This way, the focus of the optimization process will be to determine the different coalitions involved on a period, where a coalition represents the blocks that are sent to each destination (as all blocks scheduled to be extracted in one period must be sent somewhere). This will be done over multiple simulations of the orebody model in order to account for the geological uncertainty of the deposit.

3.3.3 Definition of Characteristic Functions in the Mining Context

When applying the coalition formation process to a mining complex, the different possible targets of the cooperative game will correspond to the possible processing destinations of the mining system. In a mining system, the characteristic function of a coalition can be considered as the "willingness" of a given destination to pay for the set of blocks contained in the coalition. This means that each destination may have a different characteristic function that is specified to meet the individual processing constraints, and the players forming the coalition will define the value of the characteristic function V_d, such that $V_d : 2^N \rightarrow \mathbb{R}$, where *N* is the number of players and *d* the destination to which the characteristic function is defined for.

In general, the characteristic function is a linear combination of a set of *j* different parameters (*j=1,…, J* such as preference, cost, targets, etc.), for each available destination $d \in D$ $(p_{j,d})$. A destination depends on each block (b_i)

and/or on the coalition (C) being analyzed, weighted by ($w_j(b_i)$) according to their level of importance in the overall value definition. These parameters (*j=1,…, J*) can be a function of the whole coalition *C* (such as processing costs and recovery), or defined for each specific player b_i's characteristics (such as metal content). As presented in Eq. (3.2).

$$V_{d\in D}(C=b_0,b_1,\ldots,b_k)=\sum_{b_i\in C}\left[\sum_{j\in J}w_j(b_i)\,p_{j,d}(b_i,C)\right],\qquad V_d:2^N\rightarrow\mathbb{R} \tag{3.2}$$

Given this definition, a simple definition of the characteristic function could be to calculate the discounted revenue of the cluster in a given destination as an addition of the parameters that define mining revenue. The following equation presents this formulation:

$$V_{d_t}(C=b_0,\ldots,b_N)=[m(C)\cdot r_d(C)\cdot(price_t-RC(C))-(MC_d(C)+PC(C))] \tag{3.3}$$

Where *m(C)* is the tonnage of cluster *C*, *r(C)* is the recovery, $price_t$ is the commodity price at time *t*, *RC* corresponds to refinery costs, and *MC(C)* and *PC(C)* correspond to the mining and processing costs respectively. However, the transformation of all variables into dollar value through the previous equation causes a loss of information and tractability of the geometallurgical variables that need to be controlled. Given that costs, price, recovery, etc. are not constant, the use of the previous characteristic function should be avoided.

A possible alternative is to generate independent characteristic functions for each set of comparable attributes of a block, obtaining a characteristic function vector for each coalition, referred to as targets (T) in Algorithm 3.1. An example of these independent factors is presented in Eq. (3.4). This independent formulation will improve the tractability of variables of interest, having more flexibility to manage their effect over the coalition formation process. Particularly the relational attributes that are heavily affected by the global material processed together, such as processing costs, blending constraints and recovery. It must be noted, however, that having independent characteristic functions for each destination is equivalent to having one global characteristic function where the weights ($w_j(b_i)$ in Eq. (3.2)) take different values according to different destinations, thus eliminating the effect of some of the parameters.

$$V_d(C) = \begin{bmatrix} \text{T}_1 = \text{processing cost, electricity consumption} \\ \text{T}_2 = \text{recovery, throughput} \\ \text{T}_3 = \text{revenue} \\ \vdots \\ \text{T}_n = \text{distance to meet ore production target, percentage of target met} \end{bmatrix} \quad (3\text{-}4)$$

3.4 Case Study

3.4.1 Overview of the mining complex

The following case study corresponds to a copper-gold deposit, extracted as an open pit with a mining capacity of 25Mtpa and six different processing streams. The deposit, together with gold and copper, contains arsenic and sulphur sulphide concentrations, which must be measured for mill performance. Together with the deleterious elements, the lithology of the deposit presents six different material types, which correspond to high and low grade of oxides, sulphides and transition material with different hardness. These material types affect where a given block can be processed.

Fifteen different geological simulations where generated using direct block simulation (DBSim) (Godoy, 2003) to assess the geological uncertainty of the project. Geological uncertainty is present as different grades, as well as different material types, with variable tonnages per block. Table 3.1 presents the main mining and economical parameters, which are scaled by the mining cost for confidentiality purposes.

Table 3.1 Mining and economic parameters of the copper/gold mine

Processing Costs		**Mining Parameters**	
Sulphide Mill	$11.30 · x	Mining Cost	$1.00 · x
Sulphide Heap Leach	$2.98 · x	Mining Capacity	25 Mt
Sulphide Dump Leach	$1.87 · x	**Economic Parameters**	
Transition Heap Leach	$2.15 · x	Copper Price	$2.9/lb
Oxide Heap Leach	$2.06 · x	Gold Price	$1050/oz
		Discount rate	10%

A diagram of the mining complex is also provided (Figure 3-4), showing the six different destinations available and what they produce (in brackets), as well as the different material types accepted in each case (shown by numbers 1 to 6 in Figure 3-4). The sulphide mill (SM) is the only processing stream that produces both copper and gold, and it has a stockpile available of 1Mt. The sulphide heap leach (SHL) and dump leach (SDL) both produce copper; the transition heap leach (THL) and the oxide heap leach (OHL) produce only gold.

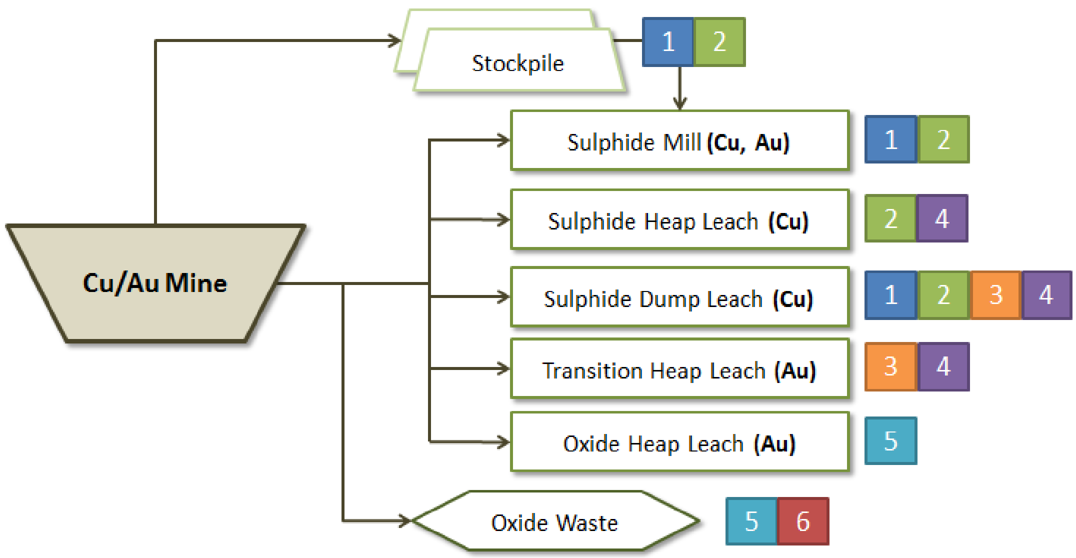


Figure 3-4 Diagram of processing streams available and the material types accepted

Each of these processing streams have a variable recovery curve dependent on the head grade fed to the destination. These curves are presented in Figure 3-5, scaled by the maximum recovery of the SM in this case. Together with this, the two main processing streams (being the SM and the SHL) present a set of processing constraints and blending requirements.

For the Sulphide Mill:

- This destination accepts material types 1 and 2.
- Processing capacity of 3Mtpa, plus a stockpile of 1Mt capacity.
- Sulphur sulphide concentration must be between 6.5 and 8.2%.

- Arsenic content must be below 0.2 to maximize recovery.
- Processing cost of material type 2 is 10% more expensive to process than material type 1 due to rock hardness.

On the other hand, the Sulphide Heap Leach:

- This destination accepts material types 2 and 4.
- There is a processing capacity of 8Mtpa.
- Copper concentration must be over 0.2% at all times.

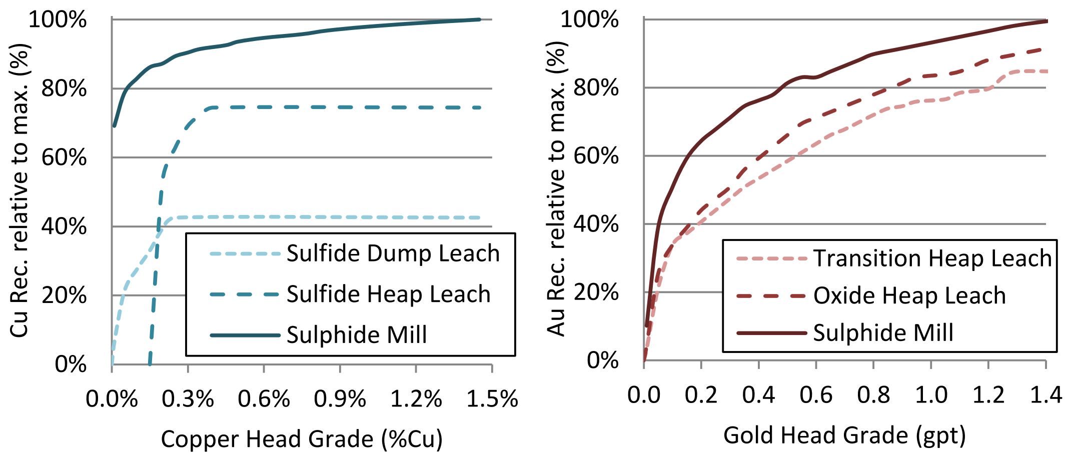


Figure 3-5 Recovery curves of (a) copper and (b) gold per processing destinations

It must be recalled that these requirements must be met simultaneously, where the value generated will proportionally correspond to the quality of material being sent to be processes together in the different destinations. To track the different requirements, a set of characteristic functions will be defined according to the requirements of each main destination, which will help define the most valuable coalitions to process together.

3.4.2 Set of Characteristic Functions

Based on the previous mining complex and the different requirements to maximize processing performance, the following characteristic functions have been defined. In this case, these functions are divided between:

(i) Global functions applied to all destinations, which correspond to:

- Maximize revenue (function of recovery and metal content of material processed together)
- Minimize deviation from production targets

(ii) Sulphide mill functions:

- Minimize deviation from sulphur sulphide concentration limits (6.5%-8.2%)
- Minimize deviations from arsenic maximum concentration (< 0.2%)
- Minimize processing costs (function of material types of rock processed)

(iii) Sulphide heap leach

- Minimize deviations from copper's minimum concentration (> 0.2%)

3.4.3 Pre-Processing Priority Groups and Coalition Generation

Each of these characteristic functions is applied over all the pairwise combinations of players. However, due to the computational intensity of performing all of these combinations, two pre-processing steps are applied over the data to reduce the computing time of the algorithm. Firstly, all scenarios of the whole deposit are clustered together using k-means++ to group blocks into families, making the material type the same for all blocks within a family. In this case, 300 clusters were generated for the 15 simulations of the deposit using Algorithm 3.2, meaning that there are at most 300 players per period, which is less tan the approximately 2000 blocks being extracted per period, but considerably more than the 60 families considered in the formulation presented in Liu et al. (2011). Then, together with k-means++, material-type/destination connections between families and infeasible processing destinations are cut , in order to avoid performing calculations with blocks that are not allowed to be processed in a particular destination due to their material type. This relationship is depicted in Figure 3-6.

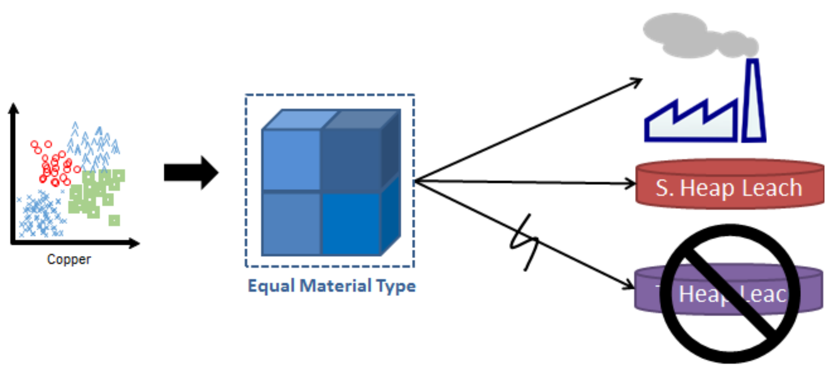


Figure 3-6 Rock/destination linkage pre-processing

In the initial application of the method, the deposit is assumed to have a fixed schedule. Then, the extracted material of each period is optimized into coalitions to be sent to the best available destination given the system constraints and the maximization of revenue.

3.4.4 Numerical Results

To compare the algorithm proposed with current practices, a base case is developed using the traditional method, where blocks are sent to a certain destination given their particular attributes (copper and gold grades in this case).

Figure 3-7 presents (a) the mill tonnage feed per period, and (b) the SHL feed. The orange line shows the expected tonnage feed given the estimated orebody model (base case). The grey lines beneath show the risk analysis of this base case (BC), representing the performance of the proposed schedule and destination policy for the fifteen different geological scenarios.

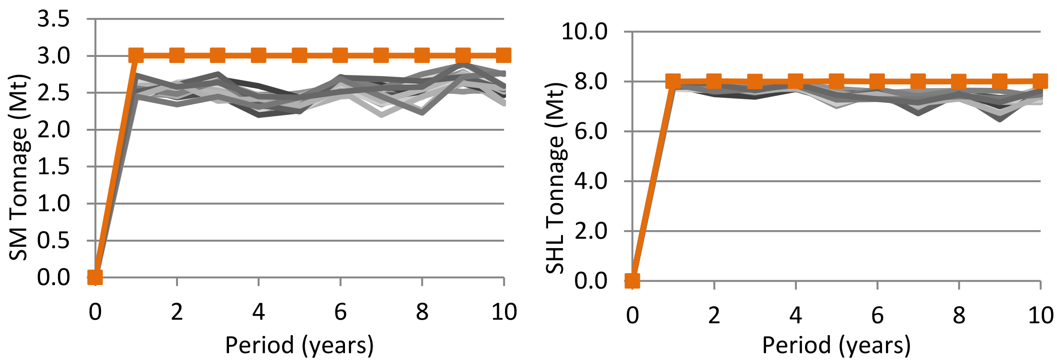


Figure 3-7 Sulphide mill feed (left) and sulphide heap leach (right), for the deterministic case (orange) and the 15 geological scenarios (gray)

It can be seen that, in the case of the mill, there is a 20% shortfall in tonnage along the LOM, showing that the base case is not really realistic when faced with geological uncertainty. In the case of the SHL, the shortfall is less, but there is still difficulty to meet production targets. The other destinations are not presented, as their capacities were unlimited and no geometallurgical constraints where applied over them.

The main constraints considered in this case study were:

i. the sulphur sulphide blending limits in the mill,
ii. arsenic's maximum concentration in the mill,
iii. copper's minimum grade in the SHL.

The comparison between the base case and the proposed method for these three constraints are presented next, where (a) presents the base case's performance (left side figure) and (b) presents the results obtained by optimizing the destination policy with the proposed coalition formation algorithm (right side figure). Figure 3-8(a) presents sulphur sulphide (SS) grade for the deterministic case in orange, where the blending constraints are barely met until Year 8 when the fed SS grade passes the maximum concentration. However, when geological uncertainty is considered (gray lines), the SS exceeds the maximum limit in almost every year. On the other hand, Figure 3-8(b) shows that the priority group coalition method proposed (PG Risk Analysis) manages to considerably reduce SS grade up to the acceptable limits. There are still major deviations in Period 5 and between Periods 7 and 9. However, this is mostly due to the fact that the schedule is assumed fixed for this case study, so the material extracted in that period has a considerably high SS grade. If the algorithm was able to adapt to the schedule, then it would be possible to manage the processed material in order to meet blending constraints by delaying low SS grade material from the initial periods to reduce de feed grade of later periods. This is proposed as future research. .

In the case of arsenic (As), the mill requires a concentration lower than 0.2% in order to maximize metal recovery and obtain optimum processing performance. The base case presented in Figure 3-9(a) shows that the material fed to the mill exceeds the maximum concentration in almost every case, up until Period 7, and again at Period 10. On the other hand, the proposed method improves considerably this processing requirement and is able to keep arsenic concentration below the limit in almost every case, except in Period 3 for some of the geological scenarios.

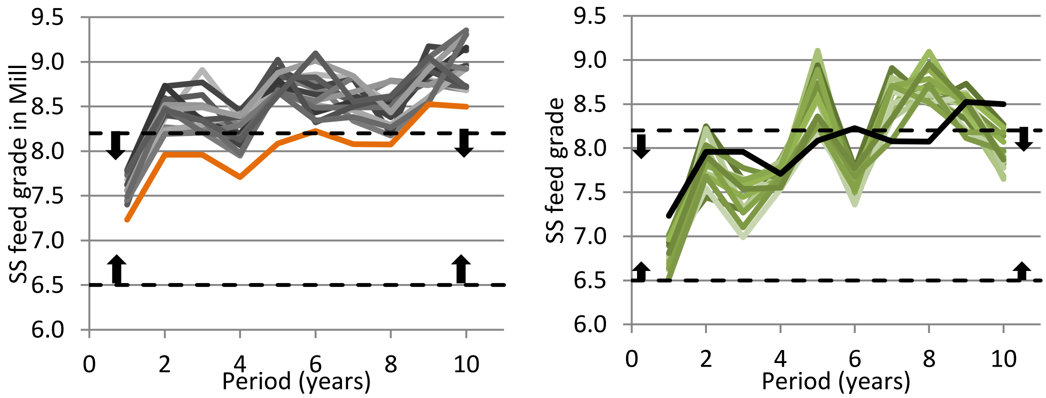


Figure 3-8 Risk profiles for the SS grade fed to the SM in the base case (left), for the deterministic case (orange) and 15 scenarios (gray) and the optimized destination policy (right)

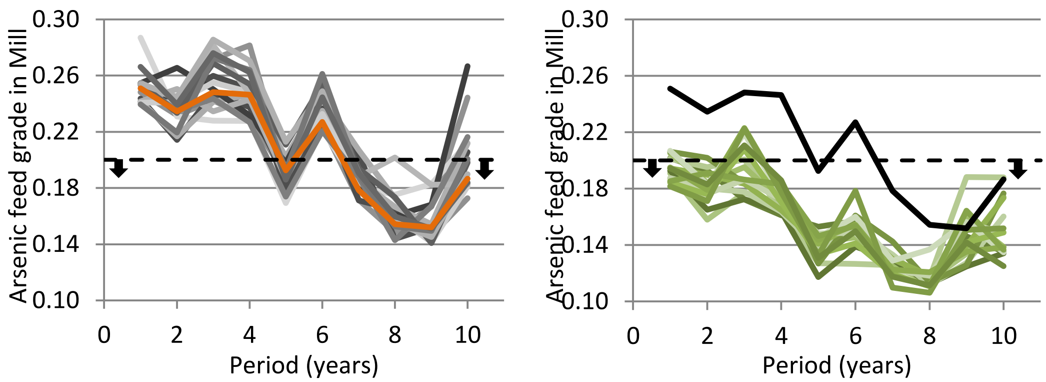


Figure 3-9 Risk profiles for the As grade fed to the SM in the base case (left), for the deterministic case (orange) and 15 scenarios (gray) and the optimized destination policy (right)

Finally, Figure 3-10 shows that the copper (Cu) concentration of the material fed to the SHL is above the required minimum in every scenario for both the base case and the proposed method.

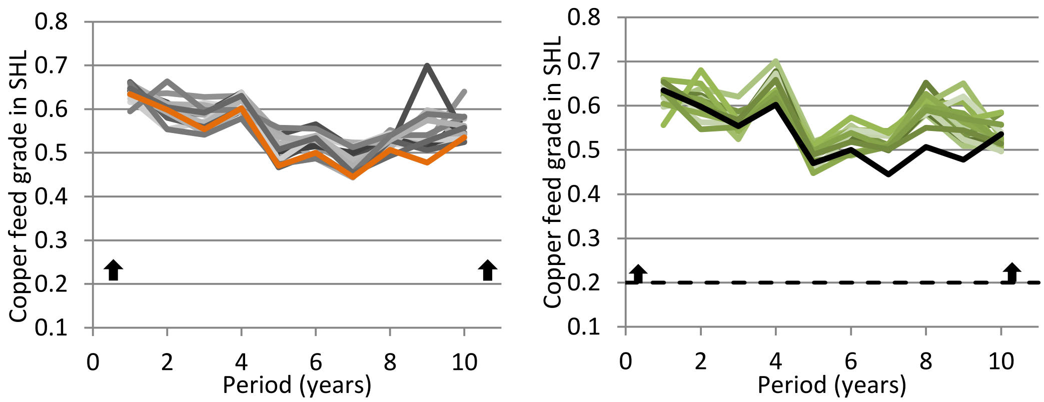


Figure 3-10 Risk profile of Cu grade fed to the SHL in the base case (left), for the deterministic case (orange) and 15 scenarios (gray) and the optimized destination policy (right)

Considering a discount rate of 10% over the 10 year LOM, Figure 3-11 presents the cumulative discounted cash flow for the deterministic base case (in red), which, for confidentiality reasons, is presented as the 100% reference value; for the different geological scenarios over the base case (in gray), which show a 4.8% lower NPV than expected by the deterministic model, and the optimized priority group coalition scenarios (in dotted black), increasing the NPV in an average of 5.6% over the deterministic base case.

3.4.5 Discussion of Results

The case study presented shows a novel contribution to the mining optimization process by developing a formulation that uses cooperative Game Theory techniques to include non-linear relations between the wide ranges of variables that are involved in the optimization of a poly-metallic mining complex.

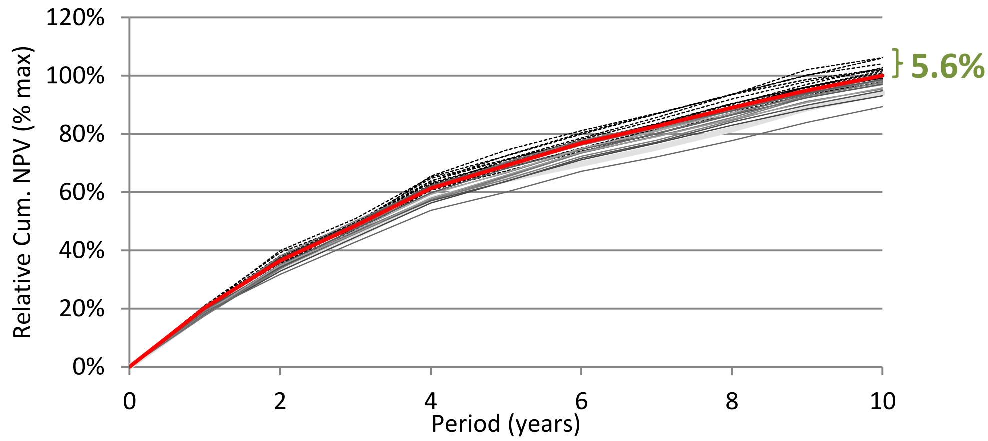


Figure 3-11 Cumulative discounted cash flow (CDF) of the BC (red), and the risk analysis over the BC (gray) and the PG (dotted)

From the previous results, one can see that, by optimizing the distribution of the material being extracted in a multi-variate non-linear manner, it is possible to improve project value and ensure that the processing streams work at their planned targets (i.e. blending constraints, metallurgical recovery, etc.). The distribution mechanism was done by a coalition formation algorithm that considers not only the attributes of each extracted block, but also the characteristics of the cumulative material being processed together (i.e. the interrelation of the blocks' attributes). By doing this, it was shown that all blending constraint requirements are improved without compromising the grade of valuable metals (as shown in Figure 3-8 through Figure 3-11).

In addition, this case study shows that priority grouping can be successfully implemented to reduce the computational cost of calculating the Shapley Value (as in Liu et. al, 2011), making it applicable at the mining scale. It was also shown that coalition formation through the maximization of the Shapley Value is able to generate value by considering multiple categorical and qualitative variables within the grouping process. This finding can be widely applied over different areas, even though computational advances are making it possible to work with increasingly complex models, due to the high amount of data available and being produced, it is crucial to have effective grouping and classifying systems that account for complex non-linear relations of the data at hand.

3.5 Limitations and Future Research

The previous study shows promising results, however, a clear extension of this work would be to consider the scheduling problem within the optimization process. Future research will focus on this by extending the current formulation to consider the selection of material that is extracted in every period and, at the same time, ensuring that blending constraints are met while maximizing project value.

Together with this, the presented destination policy distributes the extracted material as if all of the blocks extracted in a given period where processed together, which is an oversimplification, as a period corresponds to a year and extracted material is treated on the daily. Because of this, future research will seek to apply this method in a shorter-term, making the "cumulative processed material" a more realistic amount.

Another limitation of the presented study is that geological and geometallurgical uncertainties are not directly integrated into the coalition formation, only in the optimization process. To extend this approach, we propose as future research to implement a stochastic Shapley Value (Kargin, 2008) that would represent a set of scenarios. As shown in Table 3.2, the individual contribution of each block in the coalition is calculated over all scenarios (s = 1… S).

This will provide a distribution of the Shapley Value of a block belonging to a coalition C, and the coalition formation process can aim at maximising the *expected Shapley Value* (*E(SH(C,b))*) of a block over the set, and minimize its standard deviation (i.e. the risk of not obtaining that satisfaction level). However, this would greatly increases the complexity and computational cost of the formulation.

Table 3.2 Calculation of the Expected Shapley Value from a set of S scenarios.

$\boldsymbol{C = b_0 \dots b_k}$	***s1***	***s2***	**…**	***sS***
b_0	$V_{d,s1}(b_0)$	$V_{d,s2}(b_0)$		$V_{d,sS}(b_0)$
$b_0 b_i$	$V(b_0 b_i) - V(b_i)$			
…	…			
$b_0 b_1 \dots b_k$	$V(b_0 \dots b_k) - V(b_1 \dots b_k)$			
$E(SH_d(C, b_0))$ ←	$SH_{d,s1}(C, b_0)$	$SH_{d,s2}(C, b_0)$	…	$SH_{d,sS}(C, b_0)$

Applying this stochastic coalition formation method to mining optimization would provide a scenario-independent destination policy, which would facilitate the operational applicability of the method (as geological scenarios do not aim at locally forecasting reality, but rather at representing a set of possible scenarios and particularly, the spatial variability present in the simulated variable). The actual material encountered at the moment of extraction will probably deviate from its simulations, making it necessary to have a fast classification mechanism to define material when it is extracted, and decide its destination based on this classification. A crucial problem with this approach is that must be studied further is the misclassification errors, as the simulated material type of a block can vary from one scenario to another, making a block infeasible to process in a given location in one scenario, but not in others.

3.6 Conclusions

This study presents a multi-objective destination policy for extracted material, which is developed through grouping blocks according to their associative attributes by using a *coalition formation* mechanism. This mechanism presents a novel implementation of Game Theory techniques into the mine planning optimization process to account for the value generated by groups of blocks being processed together and, at the same time, consider complex geometallurgical constraints that are often ignored. The presented procedure develops *characteristic functions* that describe the value of coalitions of blocks being processed together, and defines the optimal destinations by maximizing the Shapley Value (which defines the utility) of each block or cluster of blocks (i.e. the players of this cooperative game).

A case study of a copper-gold deposit with six material types and six possible destinations showed that the proposed PG method is able to account for the value generated from extracted material with multiple categorical and continuous attributes, and optimize its processing destination so that not only all processing and blending constraints are met, but also project value is maximized. This promotes a more realistic representation of the project value. Results from the case study showed that the proposed algorithm was able to reduce As concentrations and improve SS ranges in the mill feed material without reducing Cu grades nor final revenue, as the PG destination policy delivered a project with an NPV 5.6% higher than the base case, which was (developed by traditional methods that violate blending constraints. These results were obtained by redistributing the extracted material, as the schedule was assumed constant for both BC and PG cases.

CHAPTER 4
Dynamically Optimizing the Strategic Plan of a Mining Complex under Supply Uncertainty

The previous chapters study ways to optimize capacities by investing in capital expenditures (Chapter 2), and ways to deal with complex geometallurgical variables (Chapter 3) withing the mine planning optimization model. However, both models are limited, as they assume a fixed schedule. Here, the learnings from the previous chapters are extended into a dynamic model that optimizes the mine production schedule of a mineral value chain, and considers alternatives that allow the model to adapt due to the uncertainty.

4.1 Overview

Mining complexes are mineral value chains consisting of a continuous flow of material with several components: multiple mines, represented by orebody models discretized into mining blocks, mineral processing streams, and transportation systems to deliver products to customers. The performance of each component strongly depends on the other; as a result, these components must be modeled jointly and simultaneously optimized, accounting for profits from the final product(s) sold. The value generated by the synergies that exists between the components of a mining complex, and the need to simultaneously account for them within the strategic plan have been discussed in the technical literature (Hoerger et al., 1999; Whittle, 2007, 2010b; Pimentel et al., 2010; Bodon et al., 2011), being referred to as global or simultaneous optimization. However, these studies are limited, as they require major simplifications, and do not consider the optimization of the mining schedule, nor do they propose a single formulation for the simultaneous mining complex optimization. Additionally, they ignore uncertainty, and provide static plans that are unable to adapt to future information. Within these uncertainties, a major source is the one related to the geological attributes of interest of the mineral deposits, such as grades and material types, which characterize the supply of material from the mines. This uncertainty must be accounted for in the optimization process to generate reliable solutions that manage risk and maximize value (Ravenscroft, 1992; Dowd, 1994, 1997).

Extensions of these optimization models referred to as two-stage stochastic simultaneous optimization of mining complexes are single formulations that use a set of geological scenarios of the mineral deposit mined to quantify the related uncertainties. These models are able to simultaneously generate a life-of-mine (LOM) extraction sequence and destination policy for the mines involved, managing technical risk and maximizing net present value (NPV) (Goodfellow, 2014; Montiel, 2014; Montiel and Dimitrakopoulos, 2015, 2017, Goodfellow and Dimitrakopoulos, 2016, 2017). Montiel et al. (2016) present a heuristic method to solve the LOM production scheduling optimization of a mining complex considering geological uncertainty, including operating alternatives for the processing plants and transportation systems. On the same line of research, Lamghari and Dimitrakopoulos (2016b) propose a network-flow based heuristic algorithm to optimize the mine production schedule under metal uncertainty, also considering the complete mining complex. Other studies have focused on other sources of uncertainty; Kizilkale and Dimitrakopoulos (2014) optimize mining rates under market uncertainty in a mining complex using dynamic programming mechanism, while Zhang and Dimitrakopoulos (2017) account for market uncertainty and develop a decomposition method to optimize both the mining schedule and the downstream material flow plan. Zhang and Dimitrakopoulos (2018) consider both geological and market uncertainty, and propose a model to optimize a mining complex's long-term contract design strategy.

Existing two-stage stochastic optimization models provide a strategic mine plan that allows meaningful assessments and risk quantification, but these plans are static, thus unable to adapt to new information that may be obtained in the future, leading to undervalued strategic plans (Wang, 2005; Eckart et al., 2010). Multistage stochastic programming models (Ahmed et al., 2003; Boland et al., 2008) aim at including operational flexibility into the LOM plan by producing different possible solutions to follow once uncertainty is unveiled. However, these methods generate impractical results, tailored for each scenario, rendering their LOM plans and related financial assessments meaningless. In practice, a unique strategic plan is required to provide a reliable evaluation and facilitate decision making. In this study, a new method is proposed aiming to increase the flexibility of a mining complex by improving the strategic plan's capacity to react and adapt to new information by dynamically deciding on investing in strategic, feasible capital expenditure (CAPEX) options. Flexibility is accounted for in the way of a set of feasible alternatives that can be taken, depending on the configuration and characteristics of the mining complex at hand.

De Neufville et al. (2004) define "flexible designs" as able to pro-actively adapt and reconfigure if needed (De Neufville and Scholtes, 2011). The concept of design flexibility is well known (Siegel et al., 1987), where, by considering a dynamic value chain that accounts for different alternatives, it is possible to assess the overall probability of different outcomes that can lead to higher profits (Dixit and Pindyck, 1994). However, in mining operations, reliable financial assessment can only be obtained if feasible plans are produced, following physical geotechnical restrictions and operational requirements. Developing feasible, operational strategic mine plan is one of the main challenges when considering flexibility in the stochastic optimization of an industrial mining complex, since, as mentioned earlier, a single LOM plan is required to follow and evaluate, such as the one provided by the static two-stage stochastic integer programming (SIP) model. At the same time, accounting for feasible flexibility options can prove to be very beneficial for a strategic plan. It can be assumed that knowing in advance what the possible developments of the mining operation may be allows management to take advantage of opportunities and prepare better for possible changes. Thus, flexible alternatives can be integrated into the existing plan and transitions optimized, maximizing performance without compromising the operational requirements of the strategic plan.

Different efforts at including flexibility in mining operations can be found in the literature, considering uncertainties in commodity price, geology, or operating cost (Singh and Skibniewski, 1991; Kazakidis and Scoble, 2003; Groeneveld and Topal, 2011). Ajak and Topal (2015) focus on flexible decision making at an operational level, considering price uncertainty to decide switching extraction zones. Groeneveld et al. (2012) stress the concept of an "operational schedule", focusing on the development of a strategic plan which can be applied in practice to provide a reliable financial assessment. The authors propose a method which adapts to possible price fluctuations, however, their approach is limited by not considering geological uncertainty, and assuming that all future information is known through the scenarios analyzed. Del Castillo and Dimitrakopoulos (2014) study the effect of commodity price and geological uncertainty in LOM plans, and show the importance of including these uncertainties in strategic decision making. More recently, Montiel and Dimitrakopoulos (2015, 2017) present a stochastic simultaneous optimization of a mining complex model which allows the optimizer to choose between different processing and transportation alternatives, managing throughput versus recovery in the first case, and cost versus capacity in the second. Goodfellow and Dimitrakopoulos (2016) include CAPEX investment decisions to define extraction capacity, letting the optimizer choose when and how many trucks and shovels to purchase. Farmer (2016) also optimizes a mining complex's extraction capacity as part of the optimization process. However, in all these studies, the final solution is still a static design that provides no options, and is fixed for the whole LOM after optimization. Dowd et al. (2016) state that studying ways to integrate flexibility in the design is one of the main challenges in mine planning at present.

In the present study, an operational dynamic simultaneous optimization is developed for the strategic planning of a mining complex under supply uncertainty, where CAPEX alternatives are explicitly included in the formulation. These decisions are taken dynamically along the LOM optimization process, as new information is obtained from the supply's uncertainty, and the strategic mine planning schedule is adapted accordingly. Supply uncertainty is considered by using a set of geological scenarios of the deposit, which represent its spatial variability of grades and material types, and are used as input to the optimization. The proposed method proactively optimizes CAPEX decisions' timing by allowing uncertainty scenarios to differ in these decisions, if doing so adds value to the strategic plan. Thus, if on a given year a representative number of scenarios decide to invest on a CAPEX option, then the optimization will allow the design to "branch" into two parallel feasible plans, divided in annual stages. The model developed herein extends the formulation proposed by Goodfellow and Dimitrakopoulos (2016) to include dynamic decision making, and solved by a metaheuristic method. This mechanism ultimately provides information about the probability of applying different CAPEX alternatives and outlines the optimal period to consider them.

The next section describes the proposed method, mathematical model, and solving procedure implemented. Then, an application to a copper mining complex is presented, and compared to the traditional two-stage stochastic formulation. Conclusions follow.

4.2 Proposed Dynamic Stochastic Optimization of a Mining Complex

4.2.1 Methodology

The proposed method is explained next through the example presented in Figure 4-1. The mine planning schedule is optimized given a set of simulated realizations of pertinent orebody attributes and a unique schedule is produced for years 1 and 2. However, Figure 4-1 shows that in period 3 a major proportion of the simulation scenarios decide to invest on a given CAPEX option. Consequently, the optimization allows the solution process to branch into two feasible designs, one for each subset of orebody simulations, and a unique strategic plan schedule is developed for each of them.

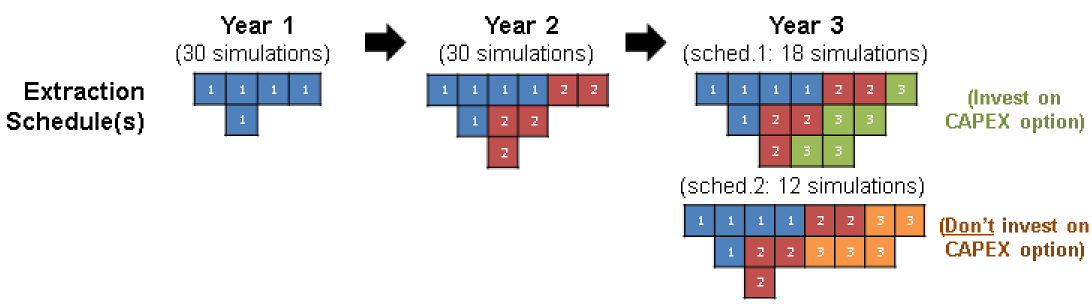


Figure 4-1 Example of including investment alternatives in the optimization process

The previous example shows the importance of clearly defining when to branch during a solution process. This is done by setting a threshold that defines the representative proportion of scenarios which must differ in investment decisions for a solution to be divided into feasible branches, and it is a pre-defined parameter set by the operation's management. This threshold is used to filter irrelevant investment decisions and only consider potentially significant ones, avoiding designing complete mine plans for scenarios that have a relatively low probability of occurring. This parameter is defined in the following relation, where, given a predefined threshold (R), the decision for branching the design is defined as:

$$\begin{cases} do\ not\ invest, & if\ \frac{\sum_{s\in S}\omega_{k*,t,s}}{|S|} < R \\ branch\ solution, & if\ \frac{\sum_{s\in S}\omega_{k*,t,s}}{|S|} \in [R, 1-R] \\ invest, & if\ \frac{\sum_{s\in S}\omega_{k*,t,s}}{|S|} > 1-R \end{cases}, \qquad R \leq 0.5, \forall k^*, \forall t$$

Where $\omega_{k*,t,s}$ represents the binary decision variable that shows if branching investment option k^* is taken at time t, on scenario $s \in S$ or not ($\omega_{k*,t,s} \in \{0,1\}$). From the previous relation, it is clear the importance of defining parameter R, where, for example if R = 50%, then the design would branch only if exactly half of the scenarios choose to invest in a branching alternative during a specific time window. On the other hand, if R = 0%, the design will branch as soon as any scenario decides to invest in a branching decision. In other words, the lower the value of this threshold, the more willing the operation is to consider low probability futures. However, choosing a value that is too low might result in over-fitted plans that perform poorly when tested over new scenarios, falling into the multistage optimization problems mentioned in the previous section. In the previous example presented in Figure 4-1, it is assumed that for the design to branch, R must be ≤ 40% (as 12 out of 30 simulations, i.e. 40% of scenarios branch), so both options (with and without investment) are considered "representative" and the solution process branches (as 60% of scenarios opt on investing, and 40% do not, both ∈

$[R, 1 - R]$). If instead $R > 40\%$, the mine plan would not branch as shown in the figure, and only the design with the investment on CAPEX at year 3 would be kept. It is suggested that a threshold of between 30-35% be used, as these ranges are representative enough to produce robust solutions that are stable under a different set of scenarios, while still detecting possible investment alternatives that might be profitable in the future.

In the proposed approach, it is defined that only a selected sub-set of CAPEX alternatives can cause the mine plan to branch, thus, CAPEX decisions are divided into branching and non-branching alternatives ($K^{<}$ and $K^{=}$ respectively). The first correspond to major investment decisions that are usually taken only once, or once every more than ten years (such as opening a new plant or buying an extra crusher). These options have a significant impact on the mine plan and specifically, over the mining schedule. The decisions of the second group have a relatively reduced impact and/or are multiple small decisions (such as truck purchases). Allowing these minor periodic decisions to branch, the design would make the methodology impractical, as the scenario tree would grow exponentially, increasing the complexity of the model, without important differences between each branch. For example, decisions such as considering buying an extra truck in a fleet of 50 trucks might not be representative or significant enough to generate a separate mine plan. On the other hand, a considerable investment such as an additional crusher would have a high impact on the processing capacity of the mining complex, and thus, on the schedule. However, the final classification of which CAPEX decision falls in each category will depend on the mining complex at hand.

The solving process proposed can be divided into steps that are repeated iteratively. Given that each production year is represented by t, where $t \epsilon \{1,\dots, T\}$, and T is the final production period.

Stage 1: Initialize

Optimize the whole mine plan as a two-stage SIP as in Goodfellow and Dimitrakopoulos (2016), from $t = 1, \dots, T$, where block extraction sequence variables correspond to first stage decisions, unique decisions over all geological scenarios, and the processing stream variables to second stage decisions, scenario-dependent decisions that work as correcting agents of first stage decisions that are made under uncertainty. Set current year $t_i = 1$.

Stage 2: Solve while $t_i < T$, do

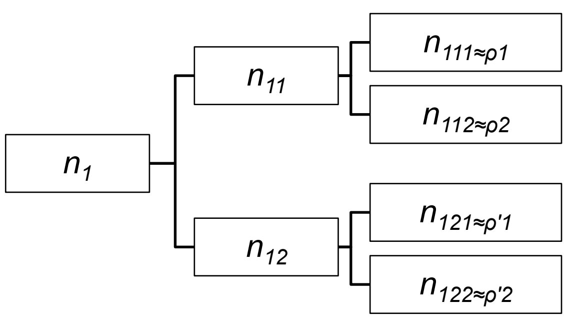
1. Freeze decisions from $t = 1, \dots, t_i$, and consider all future extraction and processing decisions 2nd stage ($t = t_i.+1,\dots, T$). Compare solutions in terms of the investment decisions taken. If a representative number of scenarios (R) perform the same investment in a given year $t^* \in [t_i + 1, \dots, T]$, the optimizer groups those scenarios and branches on that year into two mine plan designs (with and without the investment) with their corresponding subgroup of scenarios. If not, $t^* = 0$.
2. Optimize each active branch from t_i+1 to T as a two-stage SIP (as in Stage 1), fixing the initial year t_i+1 in both, and their corresponding investment decisions.
3. Set t_i equal to the maximum between t^* and t_i +1.

This process is repeated until all periods of the LOM of the mining complex have a unique design assigned to them. The mathematical programming formulation is presented next, and the implementation algorithm is available in Appendix 4.A.

4.2.2 Stochastic Integer Programming Formulation

The proposed model uses the formulation developed in Goodfellow and Dimitrakopoulos (2016), were the authors aim at simultaneously optimizing multi-mine production schedules, destination policies and processing streams under uncertainty, including capital expenditure options. Here, the orebody blocks of all simulations are clustered using k-means++ algorithm, and the destination policy is annually defined per cluster. The main difference in the presented model is that in this case, CAPEX decisions are taken dynamically along the LOM, and the mine plan is adapted accordingly. To do this, the formulation models major strategic investment decisions within the operation as a scenario tree. For the purpose of clarity, this tree is divided by period and events (invest/don't invest), and each node is identified by a *root*, and its event or *leaf* (as presented in Figure 4-2). The root works as an identifier that contains information of the whole decision path of the branch which led to the leaves and is summarized as ρ in Figure 4-2. The leaves correspond to parallel designs with the same "ancestor" or root (for example, in Figure 4-2 a branch corresponds to nodes $\{n_1, n_{12}, n_{121}\}$, where the root of node n_{12} is $\rho=1$, and the root of node n_{121} is $\rho=12$). Note that if two branching alternatives are considered jointly in the optimization, then 4 leaves should be considered per branch (instead of 2), one for each possible investment combinations per year. However, as most of the branching decisions are limited to be taken only "once in the LOM", the tree is simplified considerably.

Three main adaptations are made from the model presented in (Goodfellow and Dimitrakopoulos, 2016). (i) CAPEX decisions have a scenario component (as shown in the following formulation). (ii) As stated in the methodology, after the first period, extraction decisions are also temporarily considered 2nd stage decisions, and (iii) additional dynamic constraints are included in the model (presented next) which control the branching mechanism and act similar to non-anticipative variables (Wang and De Neufville, 2005; Boland et al., 2008). Non-anticipative variables are used to ensure that non-differentiable scenarios entail equal actions (i.e. decisions) over them, and define a tree structure that branches out as scenarios differentiate and, in response, different decisions are taken. The list of sets used is given next, followed by the definition of the decision variables. Finally, the objective function and some main constraints are presented.



- n = node of the tree
- ρ = root "address" of a node
- $\rho 1/\rho 2$ = leaves of a branch

Figure 4-2 Tree structure used to define the branching mechanism of the model presented herein

Table 4.1 Sets and Parameters

P	Primary attributes that are tracked in the supply chain (e.g., metal content, tonnages).
H	Hereditary attributes (derived from primary attributes) that are tracked in the supply chain (e.g., grades, recoveries, value).
T	Time periods in the life of mine, indexed by t = 1…T
S	Set of scenarios, indexed by s = 1, …, S*max*, where $S_{\rho l} \subseteq S$ Sub-set of scenarios in node ρ,l (root ρ, leaf l). Note that $S_1 = S$, $S_{\rho 1} \cap S_{\rho 2} = \emptyset$, and $S_\rho = S_{\rho 1} \cup S_{\rho 2}$
N	Minimum number of scenarios in a branch to allow further branching
K	Set of flexibilities and system options, indexed by k, where $K^< \subset K$ set of options that require branching over the design $K^= \subset K$ set of options that don't require branching
D	Set of locations in the mining complex
$\Theta(j)$	Set of destinations which can receive material from location $j \in D$.
$d^{+/-}_{h,i,t,s} \geq 0$	Deviations from targets on hereditary attribute h, at location i, period t, scenario s
$c^{+/-}_{h,t} \geq 0$	Unitary cost of deviation of attribute h, at period t
$U_{h,i,t}$ / $L_{h,i,t}$	Upper and lower limits for attribute h, at location i, period t
$\kappa_{k,h}$	Unitary extra capacity over attribute h obtained by the purchase of investment k
λ_k	Life of capital option $k \in K$ in years
τ_k	Lead time before an option $k \in K$ is available, in years (since the moment of decision)

Table 4.2 Decision Variables

$x_{b,t,s} \in \{0,1\}$	Defines if block b is extracted at period t, scenario s
$y_{i,j,t,s} \in [0,1]$	Proportion of material sent from i to j in period t, scenario s
$z_{c,j,t,s} \in \{0,1\}$	Defines if cluster c is sent to $j \in \Theta(c)$ in period t, scenario s
$\omega_{k,s,t} \in \{0,1\}$	Defines if investment $k \in K$ is executed in period t, scenario s
$q^{\rho l}_{k^<,t} \in \{0,1\}$	Defines if to branch design over investment option $k^< \in K^<$ in node ρl, period t
$v_{h,i,t,s}$	Value of hereditary attribute h, at location i, period t, scenario s

Mathematical Formulation

Objective function

$$\max \frac{1}{|S|} \sum_{s \in S} \sum_{t \in T} \left(\underbrace{\sum_{h \in H} p_{h,t} \cdot v_{h,t,s}}_{\text{Discounted revenues and costs}} - \underbrace{\sum_{k \in K} p_{k,t} \cdot \omega_{k,s,t}}_{\text{Capital expenditures}} - \underbrace{\sum_{h \in H} c^{+/-}_{h,t} \cdot d^{+/-}_{h,t,s}}_{\text{Risk discounted deviation penalties}} \right) \tag{4.1}$$

Subject to

- Mining complex constraints

$$v_{h,i,t,s} = f_h(p,i,k) \qquad p \in \mathbf{P},\ h \in \mathbf{H},\ i \in D,\ t \in T,\ s \in \mathbf{S},\ k \in K \tag{4.2}$$

- Capacity constraints considering investments

$$\begin{aligned} v_{h,i,t,s} - d^{+}_{h,i,t,s} &\leq U_{h,i,t} + \sum_{t'=t-\lambda_k+\tau_k}^{t} \kappa_{k,h} \cdot \omega_{k,s,t'} \\ v_{h,i,t,s} + d^{-}_{h,i,t,s} &\geq L_{h,i,t} + \sum_{t'=t-\lambda_k+\tau_k}^{t} \kappa_{k,h} \cdot \omega_{k,s,t'} \end{aligned} \tag{4.3}$$

$$h \in \mathbf{H},\ i \in D,\ t \in T,\ s \in \mathbf{S},\ k \in K,\ d^{+/-}_{h,t,s} \geq 0$$

- Dynamic non-anticipativity constraints - Decisions can be different between sets of scenarios (branches), where given $A = \left\lceil \frac{\sum_{k^* \in K^<} q^{\rho}_{k^*,t-1}}{|K^*|} \right\rceil$

Extraction: $x_{b,t,s} - x_{b,t,s'} = M \cdot A$

Cluster destination $z_{c,j,t,s} - z_{c,j,t,s'} = M \cdot A$ (4.4)

Investment $\omega_{k,s,t} - \omega_{k,s',t} = M \cdot A$

$$\forall s \in S_{\rho 1};\ \forall s' \in S_{\rho 2};\ S_{\rho 1} \cap S_{\rho 2} = \varnothing;\ S_{\rho 1} \cup S_{\rho 2} = S_{\rho} \subset S;\ M = \text{big number}$$

Branching in node ρ for t+1 (only if ratio is within the threshold $\in [R_t, 1 - R_t]$)

$$\begin{cases} \frac{1}{|S_{\rho}|} \sum_{s \in S_{\rho}} \omega_{k^*,s,t} \leq (1 - R_t) + 0.5 \cdot (1 - q^{\rho}_{k^*,t}) \\ \frac{1}{|S_{\rho}|} \sum_{s \in S_{\rho}} \omega_{k^*,s,t} \geq R_t \cdot q^{\rho}_{k^*,t} \end{cases} \quad |S_{\rho}| \geq N;\ k^* \in K^<,\ S_{\rho} \subseteq S \qquad (4.5)$$

Set of scenarios per branch

$$S_{\rho 1} = \{\forall s \in S_{\rho} \mid \omega_{k,s,t} = 1,\},\ \text{and} \quad S_{\rho 2} = S_{\rho} \setminus S_{\rho 1} \qquad (4.6)$$

The objective function presented in Eq. (4.2) contains three parts. Each part corresponds to:

1. The first section focuses on maximizing the profits obtained from selling at a discounted price (or cost) of $p_{h,t}$ a quantity $v_{h,t,s}$ of the hereditary attribute *h* in period *t*, scenario *s*. Profit includes material sold from all processing streams.
2. The second term aims at minimizing capital expenditure costs, directly accounting for the cost of flexibility obtained from new investments along the

life of the mine. Here $\omega_{k,s,t}$ is the decision variable that defines if CAPEX alternative k is exercised on period t (within the minimum and maximum purchase limits $L_{k,t}, U_{k,t}$), and $p_{k,t}$ represents the cost of that investment option.

3. The third term presents the penalties for deviating from production target, where $d^{+/-}_{h,t,s} = \sum_{\forall i \in D} d^{+/-}_{h,i,t,s}$, and the costs of deviation $c^{+/-}_{h,t}$ are discounted by a geological risk discounting factor (Ramazan and Dimitrakopoulos, 2013), which aims at deferring risk to later periods.

Constraint (4.2) shows that the hereditary values are a transformation function (which may be linear or non-linear) of primary attributes (such as metal content, and tonnages). Equations (4.3) define the bound constraints for each of the hereditary attributes, setting that the value of the attribute ($v_{h,t,s}$) plus or minus deviations ($d^{+/-}_{h,t,s} \geq 0$) must be within the upper and lower ranges, and if option $k \in K$ is taken (i.e. $\omega_{k,s,t'} = 1$), then this margin is augmented by the amount related to option k ($\kappa_{k,h}$). Thus, for example, if the option of buying a truck is taken, *k* = truck, $\kappa_{k,h}$ = annual capacity of one extra truck, and the limits (*L* and *U*) controlling the extracted tonnage ($v_{h,t,s}$) get augmented by the capacity obtained from that extra truck. Here, also the time required to put the investment in place is considered (lead time τ_k), as well as the life of the investment (λ_k), Thus, to have an increment capacity by period t, an investment decision must have been taken between periods $t - (\lambda_k + \tau_k)$ and $t - \tau_k$. Constraints (4.4) present the dynamic constraints that show that extraction, destination and investment decisions can only be different if the solution has branched (as these constraints disappear if $\sum_{k^* \in K^<} q^{\rho}_{k^*,t-1} = 1$). Constraint (4.5) corresponds to the same branching condition presented in Eq. (4.1), and finally, constraint (4.6) shows how the branching mechanism evolves and scenarios are partitioned accordingly.

4.2.3 Solution Algorithm Process

To develop a solving mechanism, the size of a global mining complex with multiple mines and multiple processing streams under geological uncertainty must be considered, which entails more than a million binary variables, with over a million constraints (Lamghari et al., 2015). Adding investment decisions to the formulation increases the complexity even further, as scenario dependent extraction variables are defined. Because of this, it is infeasible to consider any exact solving mechanism, and instead, a metaheuristic is used to develop a good quality solution in a manageable amount of time. In this case, an adaptive neighborhood search simulated annealing mechanism is implemented (Grogan, 2016), where each decision variable defines a neighborhood, and the solution space is perturbed iteratively, first choosing a neighborhood to perturb, and after, a possible perturbation from that neighborhood (for example adding and/or removing one or multiple trucks if the fleet purchase neighborhood is selected). Here, the probability of selection of a perturbation from its set is constantly being adapted depending on its historical performance in improving the objective function's value.

4.3 Case Study: Cu-Au Mine

4.3.1 Overview

The following case study corresponds to a copper mining complex presented in Figure 4-3, comprised of two mines, two processing destinations (one for sulfides and another for oxides), a waste dump and a sulfide stockpile. As shown in the figure, both mines can feed all destinations. Due to the mine's dimension, it is considered that a maximum of 10 trucks can be operational per period at mine 1, and a maximum of 8 trucks at mine 2. Additionally, each shovel can haul up to 5 trucks, meaning that, for example, if there are six trucks available, but only one shovel, then the actual extraction capacity will correspond to five trucks, as an extra shovel must be purchased to fill that extra truck. The same way, if two shovels are available but only 6 trucks, then the final capacity would also correspond to 6 trucks. It s assumed that this truck/shovel relation remains constant throughout the whole LOM.

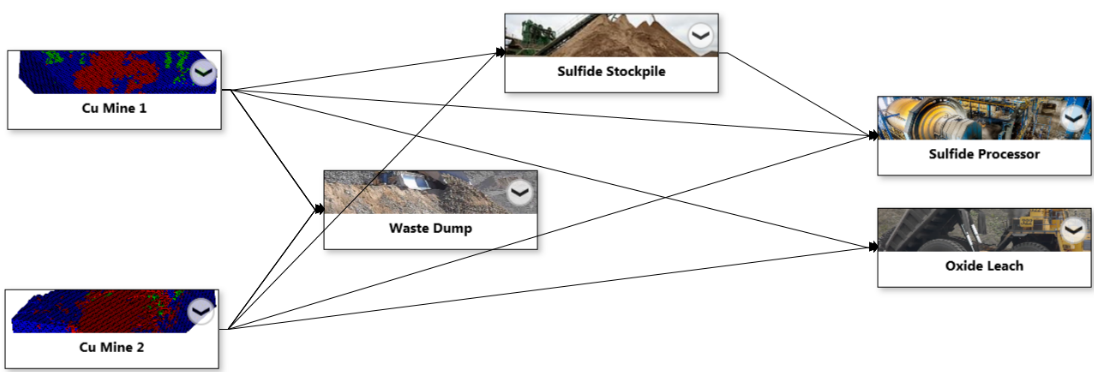


Figure 4-3 Diagram of the mining complex configuration

Mine 1 contains 136,000 blocks, and Mine 2 contains 109,000. The orebody models of both mines are modeled with blocks of 20x20x15m, and have one geotechnical zone, with slope angles of 45° and 40° respectively. Three different material types are considered: waste, oxides and sulfides, and 10 geological simulations of each deposit are used to represent the uncertainty related to copper grade and material type, resulting in 100 scenarios for the mining complex. The main source of profit of this mining complex comes from the Sulfide Processor, which recovers copper, and receives material from both mines and an 8Mt stockpile. This mill has a production capacity of between 30Mt and 32Mt per year, and there is an alternative to invest on a secondary crusher to increase this capacity by 2Mt per year. The operation has an initial truck fleet of 10 trucks assigned to Mine 1, and 8 trucks assigned to Mine 2, with two shovels per mine. Both trucks and shovels are available for the first 2 years of operation, and all further extraction capacity is defined directly by truck and shovel purchase, setting the mining extraction rate. The Oxide Leach is assumed to have unlimited capacity, as does the Waste Dump.

The copper recovery at the Oxide Leach follows a non-linear recovery curve presented in Figure 4-4, and at the Sulfide Processor copper recovery follows a non-linear function defined in Eq. (4.8), both of which depend on the copper head grade of the block being fed. Table 4.3 shows the basic mining and economic parameters of the operation. In this case, a fixed price was assumed for copper, with a discount rate of 10%.

$$\text{Cu Recovery} = \begin{cases} 0.6 \cdot \%Cu & \text{if } \%Cu \leq 0.01 \\ 0.87392 \cdot (\%Cu)^{0.07371} & \text{if } 0.01 < \%Cu \leq 1.45 \\ 0.89 \cdot \%Cu & \text{if } \%Cu > 1.45 \end{cases} \tag{4.8}$$

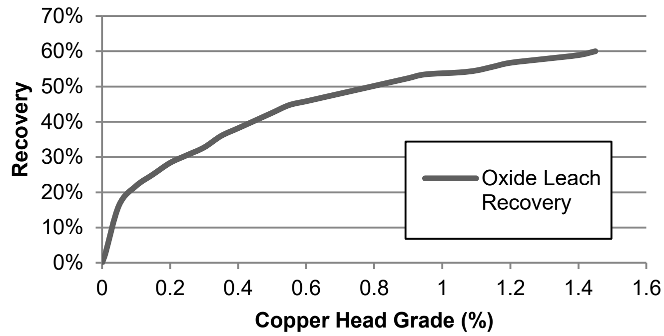


Figure 4-4 Copper recovery curve for the Oxide Leach process

Table 4.3 Mining and economical parameters of the copper/gold mine

Mining Complex Parameters	Mine 1	Mine 2
Base Mining Cost ($/t)	1.52	1.85
Incremental Mining cost ($/bench)	0.025	0.020
Initial Mining Capacity	35 Mt	28 Mt
Sulfide Processor capacity (initial) (Mt)	30 – 32	
Sulfide Stockpile capacity (Mt)	8	
Sulfide Processor cost ($/t)	10.8	
Sulfide Stockpile reclaim cost ($/t)	0.08	
Oxide Leach cost ($/t)	4	
Copper Price	$3800/ton	
Discount rate	10%	

4.3.2 Investment Alternatives Considered

Three CAPEX alternatives (set K in notation) are considered in this case study:

i. **Invest in truck fleet** ($\in K^{=}$, non-branching alternative) (and thus, increasing extraction capacity). Starting from an initial base fleet of 10 and 8 trucks for mine 1 and 2 respectively, available for the first 2 years.

ii. **Invest in shovels** ($\in K^{=}$, non-branching alternative) (and thus, increasing extraction capacity if linked to available trucks). Starting from an initial base fleet of two shovels per mine for the first 2 years.

iii. **Invest on a secondary crusher** ($\in K^{<}$, branching alternative) that allows increasing the processing capacity at the Sulfide Processor.

To test the equipment acquisition mechanism and obtain the optimal extraction capacities at both mines, the initial equipment is available only for the first two periods, enabling the mining operation to extract material during the first periods, but allowing the optimizer to quickly define the optimal capacities for the rest of the LOM. Details of each of these CAPEX alternatives are provided in Table 4.4, including the mining complex's initial configuration capacities, and the changes involved in new investments. For each option, there is a set of operational parameters that must be defined, such as the cost of the option, the periodicity of the decision (that is, how often a decision can be re-taken after an investment is done), the lead time (time between an investment is purchased and it becomes available for the operation), the maximum purchases allowed at once, and the actual increment in capacity constraint obtained per unit if the optimizer decides to invest.

4.3.3 Results

4.3.3.1 Base Case

The base case corresponds to the standard two-stage simultaneous stochastic optimization of the mining complex, were the scenarios are used to define a unique production sequence and destination policy. The truck, shovel, and secondary crusher purchase options are also included in the model, but all as 1st stage non-branching options, assuming that the mines have an initial capacity of ten and eight existing trucks respectively, which are available for the first two years of operation. Here, all scenarios are used to define one global mining and purchase schedule. This way, the solution includes when each block is extracted, where it is sent according to its characteristics (grade, material type, etc.), if a crusher is added, and how many trucks and shovels are purchased per mine and per year (and thus the annual extraction capacity), respecting the parameters mentioned in Table 4.4.

Table 4.4 Information and purchase parameters of each investment option

	Truck ($K^{=}$)	**Shovel ($K^{=}$)**	**2ry Crusher ($K^{<}$)**
Undiscounted cost	US$ 4,800,000	US$ 32,000,000	US$45,000,000
Life of equipment	8 years	10 years	25 years
Periodicity of decision	2 years	3 years	once per LOM
Lead time (years)	1 year	1 years	2 years
Maximum purchase	10 units	2 units	1 unit
Initial Cap. (mine 1&2)	10 & 8 units	2 units per mine	30-32Mt
Tonnage increment	3.5 Mt/unit	Feed for 5 trucks	2.0 Mt/unit

The resulting purchase plan and corresponding total tonnage of extraction capacities is presented in Figure 4-5, where it shows on the left axis the number of trucks and shovels purchased per year for mine 1 (black bars) and mine 2 (grey bars), and on the right axis, the total tonnage available and actual annual extraction for mine 1 (black line), and for mine 2 (grey line), as well as the actual extraction per year in dashed lines. The extraction capacity is consistently respected and at its limit, except at the final years of extraction of mine 2, where extraction decreases. This deviation is acceptable, as it only occurs in the final year of the mine.

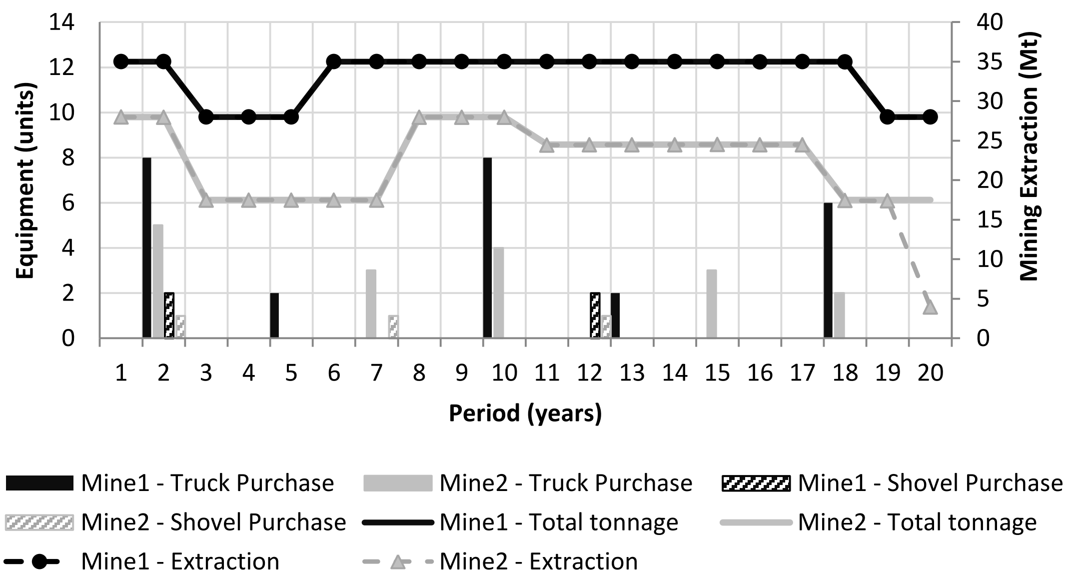


Figure 4-5 Truck purchase plan (left axis) and total mining capacity available (right axis) in the Base Case.

Figure 4-6 shows the risk analysis over the Sulphide Mill feed, where P10 represents the value at which there is a 10% probability of obtaining a value lower than that, P50 represents a 50% probability, and P90 a 90% probability. In this case, the secondary crusher was purchased on year 8, showing an increased processing capacity from year 10 to 20, and a consistent mill feed within the upper and lower bounds, with some deviations on the final 4 periods of the LOM.

The corresponding cumulative discounted cash flow is presented in the left side of Figure 4-7, ranging between US$4.76 and US$5.97 billion, with a P50 value over US$5.41 billion, a P10 of US$5.03 billion, and a P90 of US$5.76 billion. The right side of the figure shows the annual discounted cash flow, which includes all investments presented in Figure 4-5, as well as the crusher in period 8. It can be seen that, despite the large investments incurred, the discounted cash-flow is positive in every period, showing minor differences between the different scenarios.

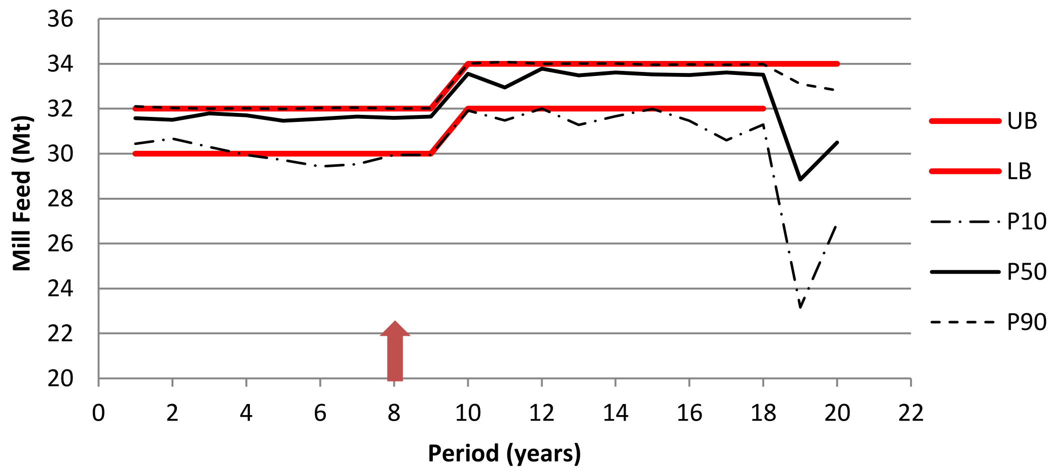


Figure 4-6 Sulphide Mill's lower (LB) and upper (UB) bound capacity and feed per period for P10, P50, and P90 probabilities

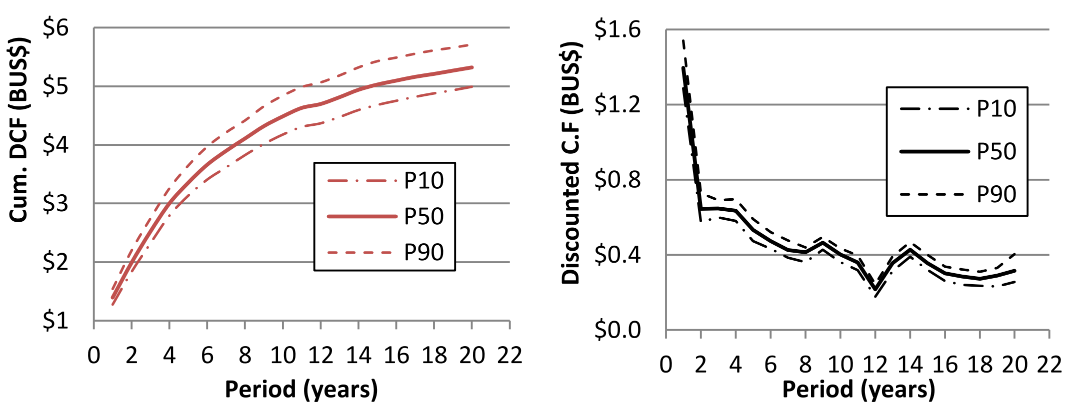


Figure 4-7 Cumulative NPV (left) and Discounted Cash Flow (right) for P10, P50, and P90 probabilities

The previous results are obtained from the schedule presented in Figure 4-8, which shows cross-sections of the schedules obtained for Mine 1 and Mine 2 respectively, where each colour represents the extraction period of each block, from period 1 in dark blue, to period 20 in bright red. From the figure, it might seem that some periods extract considerably more material than others (for example period 10), but this is just due to the cross-section chosen. The purpose of this figure is to show that the schedules produced by the optimizer are smooth and easily adapted to be operational.

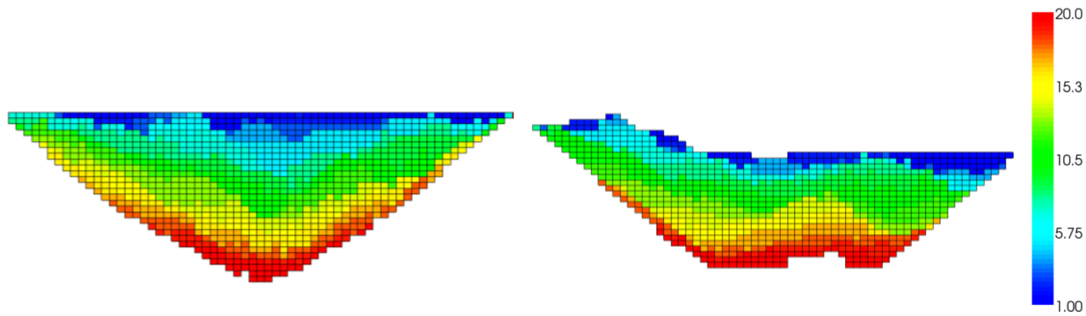


Figure 4-8 Cross-section of the base case schedules for Mine 1 (left) and Mine 2 (right)

4.3.3.2 Proposed Dynamic Case

Following, the dynamic optimization is performed, where, as explained in the method section, first an initial robust optimization is performed using the base case as starting point, and second, an independent 2-stage stochastic optimization of the branches is performed.

Step 1 – Initial 2-stage optimization

In this case, R_t is set to 30% over all periods, meaning that, as explained in Eq. 4.1, if there is between 30% and 70% chance of investing in a secondary crusher, then the solution process branches into two possible mine designs. Additionally, to avoid over-fitted mine plans, the minimum number of scenarios per branch (**N)** is set to 10 scenarios, thus, every branch must have at least 10 scenarios at all periods. With this, the mine plan is optimized as described in Section 4.2.1.

In this case, by re-optimizing the mine plan considering dynamic investments, the optimizer shows a 42% chance to invest in a secondary crusher in year 3. As this 42% is within the representative branching margin (30%-70%), the mine plan is divided at this point, and the two first periods are frozen from the two-stage initial model (i.e. the base case), as presented in Figure 4-9. Here, the two first years' CAPEX purchase plan taken from the two-stage optimization are shown, and the resulting available equipment in each mine.

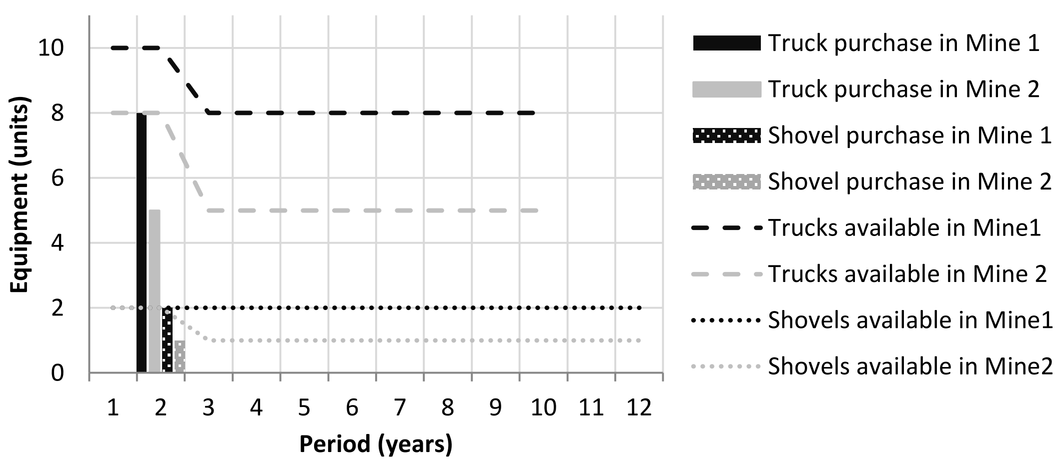


Figure 4-9 Fixed section of purchase plan from initial optimization

As stated in Table 4.4, the trucks and shovels are available one year after they are purchased and are active for 8 and 10 years respectively. Thus, the equipment purchased before the branching period at year 3 are still available for future periods (years 3 through 12 in Figure 4-9), even if the mine plan, as well as the capital investment plan, of these final periods is re-optimized for each branch in the following step.

Step 2 – Branching over the Design

Once the initial stage of the optimization is done, the design options investing and not investing in the secondary crusher are explored. First, the blocks that were already scheduled in Step 1 are removed from the orebody model, and their decisions are fixed (Figure 4-5, for periods 1 to 3). Next, the model is re-solved within the remaining deposit, fixing the corresponding investment decisions for the respective scenarios. Also, note that, as there where truck and shovel purchases during the last period of stage 1 (i.e. period 2), truck purchases are not allowed for the first period of the second stage (i.e. period 3), and shovel purchases are forbidden for the first two periods.

a) <u>Branch 1: No investment in secondary crusher</u>

In this case, the secondary crusher option is removed from the SIP model for period 3, and the model is re-optimized over the final periods of the mine plan (periods 3 to 20). The results obtained with the scenarios that decided not to invest in the secondary crusher in period 3 are shown in the left side of Figure 4-10, where the mill feed is presented, showing that in this case the probability of investing in a secondary crusher is not representative throughout the whole rest of LOM. Thus, Figure 4-10 presents a stable mill feed of around 30-32Mt, with slight deviations occurring in periods 3 and 6; however, there is consistently less than 10% chance of deviating (as the P90 curve is mostly within the upper and lower bounds). Figure 4-11 presents the truck and shovel purchase plan of this branch. Mine 1's purchase plan remains unchanged, but the optimizer decides to reduce the truck fleet of Mine 2 by purchasing one less truck on periods 7, 15 and 18, compared to the base case solution.

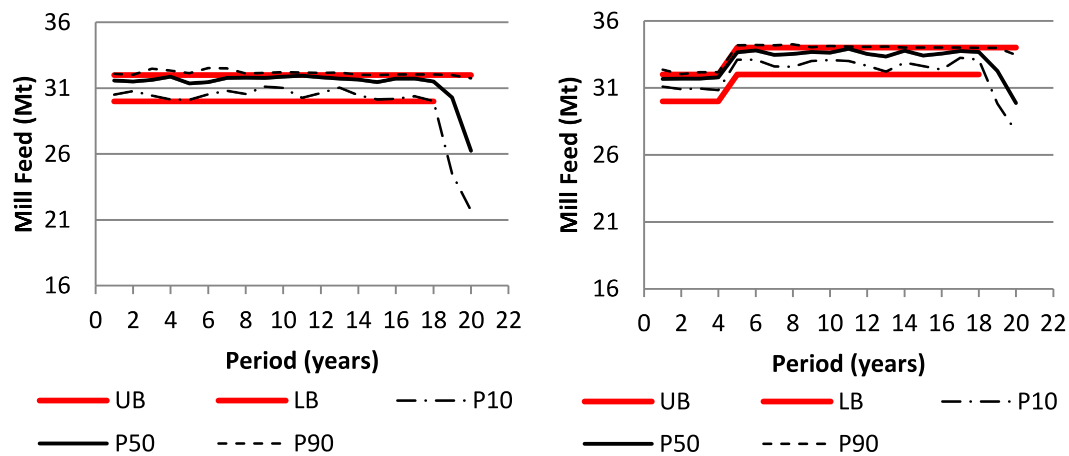


Figure 4-10 Mill feed per period for branch 1(left) and branch 2 (right).

b) Branch 2: Invest in secondary crusher

Next, the option considering the purchase of a secondary crusher is studied. In this case, the cost of the purchase is included in period 3, but the extra capacity is only available at period 5, as the secondary crusher option has a 2-year lead time (Table 4.4). This can be seen on the red lines at the right side of Figure 4-10, which shows the upper and lower bounds of the mill feed target. The equipment purchase plan is presented in Figure 4-12, where, compared to the base case, the optimized decides to get a full fleet of 10 trucks in Mine 1 one period earlier, and keep it stable towards the end of the LOM. The truck fleet in Mine 2 is also increased from period 7 forward, by purchasing a shovel one period earlier, and adding extra trucks in years 6, 14, and 18. As in the first branch, the mill feed in the right side of the figure also presents a tight risk analysis.

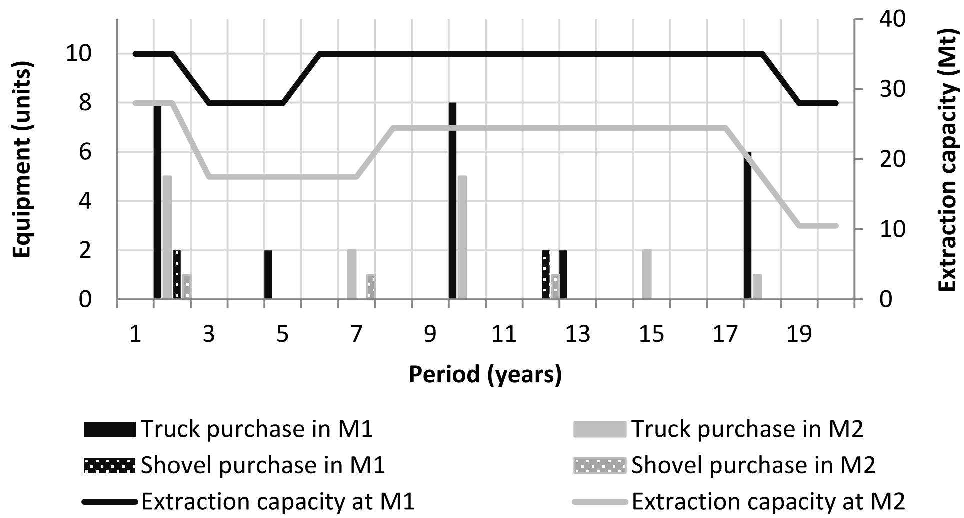


Figure 4-11 Truck and shovel purchase plan and mining capacity for branch 1

The schedules obtained for each branch are presented in Figure 4-13 for branch 1, and Figure 4-14 for branch 2, where it can be seen that the first two periods from the original schedule (Figure 4-8) have been removed, and the remaining blocks have been re-scheduled. The figure show that the generated schedules are physically different, however, they are both smooth and operational.

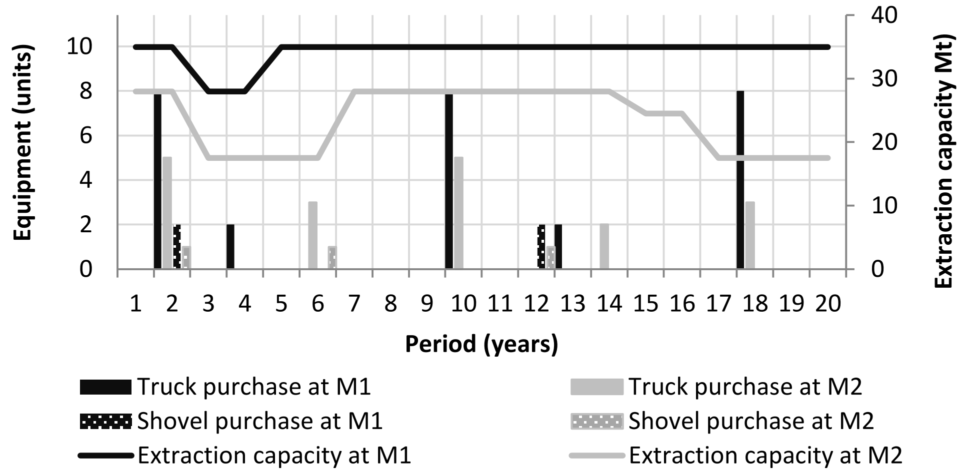


Figure 4-12 Truck and shovel purchase plan and mining capacity for branch 2

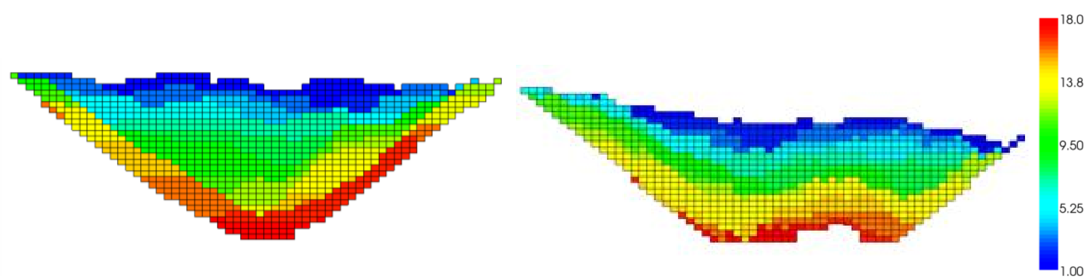


Figure 4-13 Cross-section of the schedules for Mine 1 (left) and Mine 2 (right) for the branch 1 (without the secondary crusher investment)

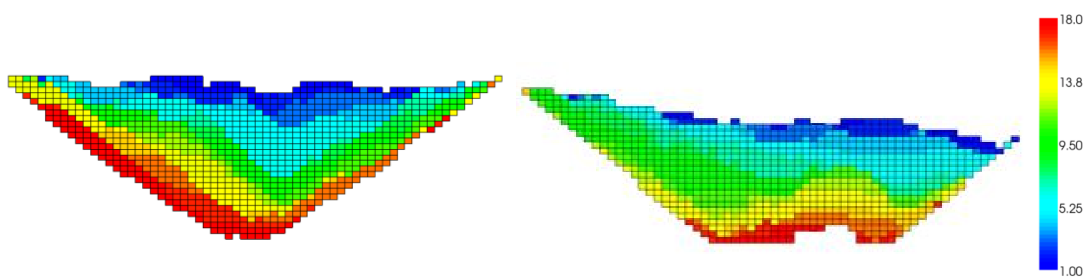


Figure 4-14 Cross-section of the schedules for Mine 1 (left) and Mine 2 (right) for the branch 2 (with the secondary crusher investment)

The biggest scheduling differences can be seen in Mine 1, where the schedule decides to go deeper earlier on the LOM when the secondary crusher is available (left side of Figure 4-14), and leave the stripping of the left wall to the final years of operation. Note that in this case, period 1 is actually period 3 in the original schedule, and period 18 is year 20, the last year of the LOM.

As the scenario partitions of each branch are independent of each other, once the initial periods before the branching are fixed, each branch can be optimized separately, reducing the size of the problem considerably. This way, the final dynamic solution is obtained by joining the results of all branches. This joint model presents a cumulative NPV which ranges between BUS$5.13 and MUS$6.10, as presented in Figure 4-15, with a P50 of BUS$5.58 (3% over the base case NPV), which corresponds to an increase in project value of almost MUS$170 over the initial base case (considering the cost of the investments), and entails a design that provides valuable information by allowing the operation to be prepared to future changes and maximise the project's potential. This is better shown by comparing the P90 values, where the dynamic model presents an 8% higher NPV compared to that of the base case (BUS$5.7 vs. BUS$6.1), meaning that not only on average more value is generated, but the operation is able to considerably increase the value generated by taking advantage of the opportunity to increase production.

This is clearly seen in Figure 4-16, where a Value-at-Risk-Gain graph is presented, showing the cumulative probability distribution of NPVs per simulation for each case studied. For the sake of comparison, this graph also includes the solution for a traditional two-stage optimization without alternatives, where extraction as well as processing capacities are assumed constant (with 10 trucks in

Mine 1 and 8 in Mine 2, and a mill capacity of between 30-32Mt). The corresponding investment costs to obtain these fixed capacities are included in the cash flow of this case.

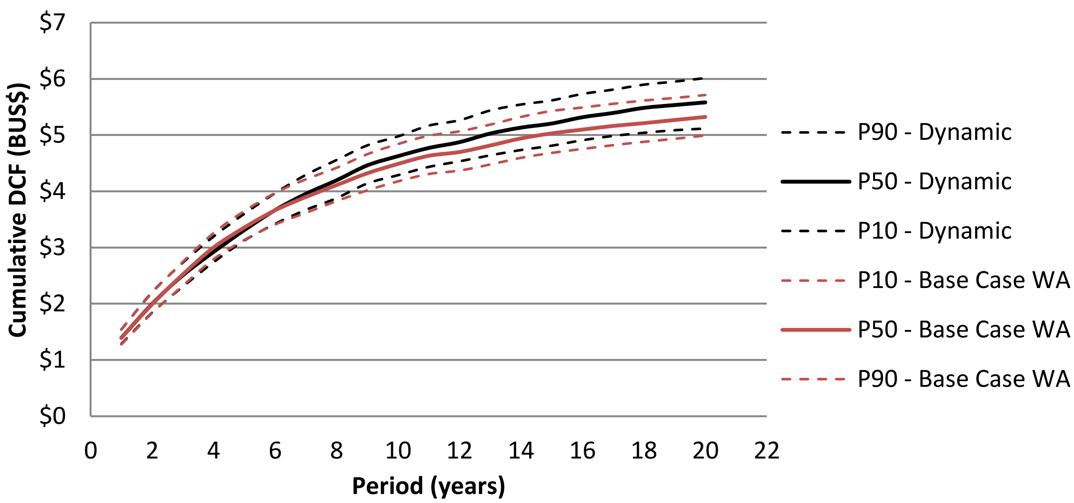


Figure 4-15 Cumulative discounted cash flow for the option of dynamically investing in a secondary crusher (black lines), and for the base case with alternatives (red lines)

Results in Figure 4-16 show that the proposed dynamic case is able to take advantage of favourable scenarios and increase the performance of the project, without any risk of reducing NPV. Additionally, by comparing the obtained results with the ones of the base case without alternatives, it is clear that there is considerable value added by actively including investments within the optimization, defining the optimal capacities at the different levels of the operation while accounting for uncertainty, and for the synergies that exist between the components of the mining complex.

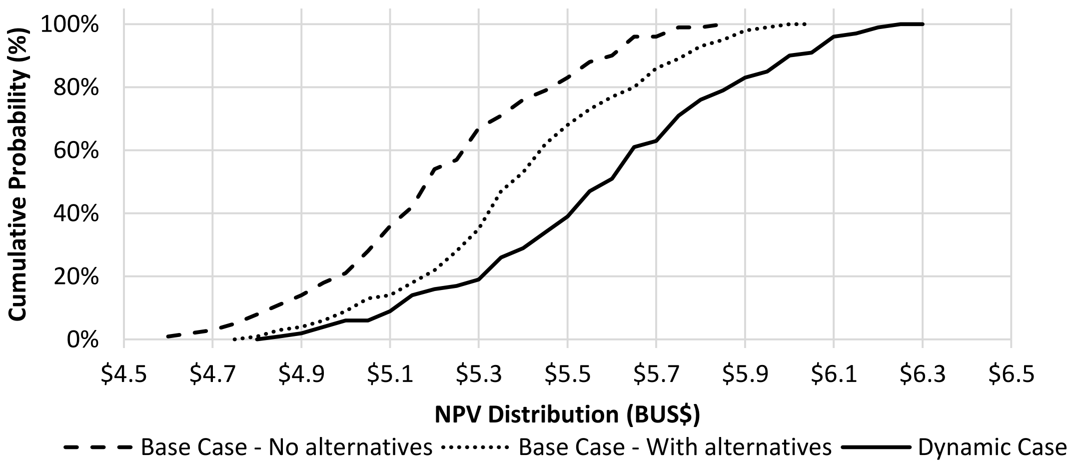


Figure 4-16 Cumulative probability distribution of NPV of (i) 2-stage optimization without alternatives (dashed line), (ii) with alternatives (dotted line), and (iii) proposed dynamic case with alternatives (continuous line).

An interesting fact that can in part explain the overall increase in NPV of the base case with and without alternatives is the equipment acquisition. Figure 4-17 shows the annual extraction in Mine 2 for both cases, compared to the actual capacity available.

The left side shows the case without alternatives, where a fixed fleet of 8 trucks and 2 shovels, or 28Mt capacity is assumed over all periods. However, this capacity is not fully met in most periods after year 5. On the other hand, by including the investment alternatives in the optimization process, the solution produced manages to define the actual capacities needed to maximize value. This can be seen in the right side of Figure 4-17, where the actual extraction in Mine 2 is almost exactly the capacity available, without keeping any equipment unused. Here, the reduced fleet during periods 3 and 7 is caused because the optimizer chooses to delay the purchase of a second shovel to later periods, and thus, only 5 trucks (17.5Mt capacity) are available for one shovel.

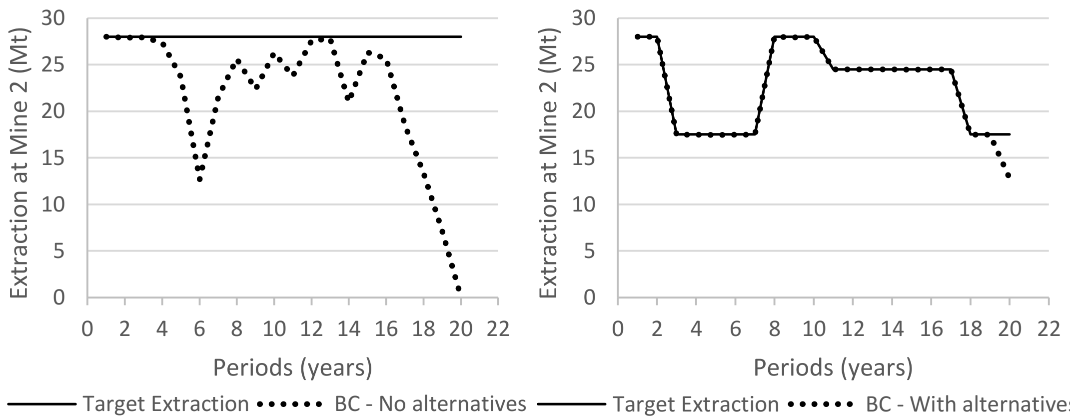


Figure 4-17 Tonnage extraction in Mine 2 relative to the available capacity for the base case without alternatives (left) and the base case with alternatives (right)

4.4 Conclusions

A dynamic SIP model is developed to include flexibility into the strategic mine planning optimization. This is done by a decision-tree structure solving mechanism which allows developing different solution designs, given that a significant percentage of scenarios decide differently over a “branching option”. A case study is presented on a copper mining complex comprised of two pits with two possible processing destinations and one stockpile, which includes two investment options to purchase trucks and shovels for each pit, defining their extraction capacity, and a flexible investment option to add a secondary crusher to the main mill to increase its processing capacity. The first alternatives are considered “non-branching” options, and thus are optimized as 1st stage decisions within a branch, and the second is considered a “branching” option, meaning that scenarios could decide differently over investing in it or not, and thus, the solution design of the mine plan is allowed to branch into two parallel design options. Here, even though scenarios are free to decide differently over branching alternatives if the solution did branch, two-stage optimizations are performed over each branch. This procedure ensures that unique, operational schedules are produced for each of the controlled branches, which can be clearly followed by the operation. Results show that for the case study, there is a 42% chance of investing in the secondary crusher in year 3, representing an overall increase in NPV of over MUS$170 compared to the initial two-stage SIP solution.

In conclusion, by applying this dynamic formulation, it is possible to identify and actively include interesting options that might not be profitable initially but could be valuable in the future. Identifying possibly profitable options on time allows keeping open the flexibility to execute them in the future and, by optimizing their application, the transition to change the mine plan is eased, allowing the project to be better prepared for it. This, to have the flexibility to dynamically alter the initial mine plan as more information is obtained, and implement optimized changes in an efficient manner. The presented mechanism allows the operation to be better prepared for uncertainty, and take full advantage of opportunities while hedging from risk.

Future applications will focus initially on improving the perturbation mechanism used in the adaptive neighbourhood search simulated annealing for investment decisions, as these variables have a major effect on the objective function (for example, the cost of buying a secondary crusher is very large compared to the cost of changing a mining block’s extraction period). This makes the CAPEX neighbourhood highly volatile and thus, requires a considerable amount of corrective perturbations to make them attractive moves. Because of this, the periodicity of choosing this neighbourhood should be adapted to give the optimizer enough time to improve a large change. For this, a mechanism such as a Tabu list can be explored. Also, an in-depth study should be performed over the number of scenarios required to ensure convergence of the solution. Finally, coming work will also concentrate on extending the formulation to include other alternatives within the mining complex, such as operational modes to better control relevant variables affecting the system, particularly geometallurgical attributes.

Appendix 4.A – Algorithm Pseudo Code

```
 = DYMANIC FORMULATION OF THE STRATEGIC OPTIMIZATION OF A MINING COMPLEX =

// Sets and variable definition
BM                     block model of the mine operation
S                      set of simulations of BM {1,...,Smax }
minS                   minimum number of scenarios required to allow branching
T                      set of periods in the LOM {1,...,T_LOM }
K                      set of different investment branching options
R* = [R, 1-R]          defined ratio margin required for branching, R ≤ 0.5
invest(k,t,s)          true if investment k is done on period t, scenario s.
branchPb(k) = {0}      vector of branching periods per investment k = 1..K
branchPeriod(b) = 0    branching period over branch b, initially = 0
InvW(k,t); InvP(k,t)   Investments k done over window t+/-1, and over time t
FinalSol[b][t]         final mine plan solutions per branch b, for period t


// = FUNCTIONS COMPRISED IN THE ALGORITHM =

SLVE_2STG(t_from, T, S*, k, t_inv)
// Solves two-stage SIP over branch b from period t_from to period T, over
scenarios S, with investment k done at t_inv

SOLVE_DYNAMIC (t_from, t_to, S*)
// Defines if and when branching happens, from period t_from to period t_to, over
investment option k, and scenarios S*

SOLVING_PROCESS (S, T)
// Main function, which iteratively solves each of the T periods over the S
simulations and stores the branching


// = ALGORITHM =

SLVE_2STG(t_from, T, S*, k, t_inv)
{

       If t_from > 1,
              For t* from 1 to t_from-1, sol1(t*) ← FinalSol[b][t*]
       Else   sol1    ← empty

       BM'  ← BM without blocks extracted in sol1
       sol2 ← 2-stage SIP opt. from t_from to T, over BM' with set S*, investment
              k on period t_inv, and option to invest in any K

       return sol1 + sol2

}
```

```
// Function SOLVE_DYNAMIC defines if and when branching happens, from period
t_from to period t_to, over investment option k, and scenarios S*

SOLVE_DYNAMIC (t_from, t_to, S*)
{

    //Solve each scenario independently
    For each s_i in S*, do
        sol_s(i) ← SLVE_2ST (t_from, t_to, s_i, NULL, 0)
        For each t from t_from to t_to in sol_s(i)
            If investment k is done in period t, invest(k, t, s_i) ← true

    // Analyse if design should branch over an investment
    For each t from t_from to t_to, do
        For each s_i in S*, and for each k in K; do
            // Store investments done each period
            If invest(k, t', s_i) is true then InvP(k, t') ++;

            // Check investments done during time window t+/-1
            For each t' from t-1 to t+1;
                If invest(k, t', s_i) is true then
                    InvW(k,t) ++; // store investments of window
                    SimSet(k,t) ← s_i // store investing scenarios

    // Choose window with highest probability of investing
    bestT(k) = argmaxt{ InvW(k,t)/|S*| }
    maxR(k) = max{ InvW(k, maxT(k))/|S*| }
    k* = argmin_k( branchPb(k) )

    return { k*, bestT(k*), SimSet(k*) }

}
```

```
//Function SOLVING_PROCESS is the main function, which iteratively solves each of
the T periods over the S simulations and stores the branching

SOLVING_PROCESS (S, T)
{

       b      = 1;           initial branch
       t_frmb = 1;           initial year to start optimization for every branch b
       t_tob  = 2;           year until 2-stage SIP is considered for branch b
       S*(b)  = S;           initial set of geological simulations in branch b
       kb#    = NULL;        investment chosen to perform over branch b
       tb#    = 0;           period chosen to branch at over branch b
       Sb#    = {∅};         set of scenarios that choose to branch over branch b
       activeBranch[b];      defines if branch b is still being optimized (= true)
                             or if it reached the LOM (= false)

       For every active branch b in activeBranch // If activeBranch[b] = true
              While t_frmb < T
                     //Only consider branching if it has min. number of scenarios
                     If |S*(b)|< minS, for every t from t_frmb to T
                            tempS(b) ←SLVE_2ST(b, t_frmb, T, S*(b), NULL, 0)
                            activeBranch[b] ← false;

                     // If there is no branching, keep solving as one design
                     Else if tb# is 0 then
                            tmp(b) ←SLVE_2ST(b, t_frmb, t_tob, S*(b), NULL, 0)

                     // If there is branching, add a new branch to the FinalSol
                     Else
                            FinalSol ← FinalSol[b]  // add branch, cloned from b
                            activeBranch[b+1] ← true // initialize new branch
                            S* ← S*(b+1) = Sb#
                            S*(b) ← S*(b) - Sb#
                            t_tob = t_tob+1 = tb#
                            tmp(b)←SLVE_2ST(b, t_frmb, t_tob, S*(b), NULL, 0)
                            tmp(b+1)←SLVE_2ST(b+1,t_frmb,t_tob,S*(b+1),kb#,t_tob)

                     FinalSol[b] ← tempS(b)

                     t_frmb ← t_tob + 1
                     t_tob  ← t_frmb + 1

                     If t_frmb < T
                            {kb#, tb#, Sb#} ← SOLVE_DYNAMIC(t_frmb, T, S*(b));
                     Else
                            activeBranches[b] ← false;
                            break;

              return FinalSol

}
```

CHAPTER 5
Stochastic Optimization of Mining Complexes Integrating Capital Investments and Operational Alternatives

Chapter 4 introduces a model which integrates dynamic investment alternatives into the stochastic simultaneous optimization of a mineral value chain. This chapter extends the previous formulation to consider operational alternatives that allow optimizing the operational configuration of different components of the mineral value chain.

5.1 Overview

Mining complexes are mineral value chains where material flows from mines to customers through a set of interconnected components. Mines are the source of material, and the components include stockpiles, waste dumps, multiple processing streams (which blend and transform the materials mined into sellable products), and transportation systems, which deliver the products to the market. These interconnected components operate at a cost, with a set of operating mode configuration alternatives, to meet targets and requirements and, thus, they can be optimized simultaneously to capitalize from the synergies that exist between them. The capacity of a mining complex is determined by capital investments, which are known to be irreversible, of high magnitude and with a limited life-span, thus requiring extensive lead times to get the purchased equipment or build required infrastructure. For example, the project to build a new processing plant at the Escondida Mining Complex in Chile had a budget of US$4.3 billion and took over four years to be completed (Mineria Chilena, 2015). Due to their effect and magnitude, investment decisions should be included in the strategic optimization of the mining complex. The focus of strategic mine planning is to generate optimized mine production schedules that meet targets contribute to maximizing the discounted cash-flows. This optimization is based on firstly, representing an orebody by three-dimensional blocks with information on the deposit's pertinent attributes, such as metal grades, material types, geometallurgical variables, and tonnage. Then, these blocks are scheduled to be extracted at a certain time period, to be processed and/or to access underlying blocks within the orebody. Traditionally, mining complexes are simplified, and each component is optimized independently (Hustrulid et al., 2013), with disconnected goals. This procedure ignores the synergies that exist between components, producing independent overall sub-optimal production schedules.

During the last years, mine planning research has evolved from conventional fractured plans to focus on optimizing all components of a mining complex simultaneously, from the mine to the final costumer, being referred to as global or simultaneous optimization (Hoerger et al., 1999; Whittle, 2007, 2010b; Pimentel et al., 2010; Bodon et al., 2011). Pimentel et al. (2010) introduce the concept where a mining operation is approached as a supply chain, served by logistic transportation channels, and develop a decision-support system to address a global mining supply chain as an integrated system. However, no methodology is provided. The paper also discusses different possible (solution) approaches to consider, concluding that heuristics would be the best alternative for optimizing any real-world mining supply chain, due to its complexity. Stone et al. (2007) formulate a model to optimize mines, stockpiles and processing plants, showing improved performance compared to the traditional optimization, but fail to include other elements of the mining complex. Together with this, and to reduce the complexity of this model, blocks are aggregated and grouped into panels. Whittle (2007) introduces the optimizer Prober, which also performs global optimization, but this optimizer is comprised of multiple formulations. Also, as in Stone et al. (2007), blocks are also aggregated into panels.

While the above approaches aim to capitalize from the synergies that exist between the components of a mining complex, due to the size and complexity of the problem, they require major simplifications; for example, avoiding to model stockpiles given their non-linear relations or aggregating mining blocks to larger volumes. A major assumption all previously mentioned studies have is considering the deposit as known and using an estimated orebody model as input to the optimization. With this, they ignore the variability and uncertainty of metal contents, material concentrations, and material types (Goovaerts, 1997), which have been documented in the technical literature to be the main sources of risk affecting mining operations. Not accounting or managing this geological uncertainty has a strong effect over the operational feasibility of the mining schedule, preventing projects from meeting production targets and maximizing their value (Dowd, 1976; Ravenscroft, 1992; Dimitrakopoulos et al., 2002; Ramazan and Dimitrakopoulos, 2013). Dowd et al. (2016), discuss that describing geological, operational and geometallurgical uncertainties and integrating them into the optimization process is one of the main challenges in strategic mine planning, as well as developing new approaches to include and maximize flexibility in mine design. Stochastic optimization has been applied for mine planning obtaining reliable plans and production schedules, showing clear benefits when compared to traditional industry practices (Dimitrakopoulos and Sabour, 2007; Ramazan and Dimitrakopoulos, 2013; Asad et al., 2014; Dowd et al., 2016; Montiel et al., 2016). All these works integrate supply uncertainty into the optimization formulation by using a set of stochastically simulated equally-probable representations of the deposit (Journel and Huijbregts, 1978; Boucher and Dimitrakopoulos, 2009; Remy et al., 2009).

The formulation of the stochastic simultaneous optimization of a mining complex produces models that contain thousands of millions of binary variables, with millions of constraints (Goodfellow, 2014; Lamghari and Dimitrakopoulos, 2015). Because of this, and due to the complexity of the problem, different metaheuristic methods have been developed to solve this problem. Compared to traditional exact methods, which are unable to solve non-linear large-scale case studies, these algorithmic optimizers produce good quality solutions, and they have been successfully used in the past for mine design and production scheduling problems (Lamghari et al., 2015; Lamghari and Dimitrakopoulos, 2016a, 2016b; Montiel and Dimitrakopoulos, 2017; Goodfellow and Dimitrakopoulos, 2017). Thus, they can solve non-linear formulations, allowing the model to avoid falling into simplification.

Stochastic simultaneous optimization of a mining complex integrates the effect of geological uncertainty and variability into the optimization model. By doing so, it produces mine plans and production schedules that are able to meet complex blending requirements, manage technical risk, and maximize project value (Montiel and Dimitrakopoulos, 2015, 2017; Montiel et al., 2016; Goodfellow and Dimitrakopoulos, 2016, 2017). A challenge when modelling the stochastic simultaneous optimization of a mining complex is the non-linear transformations that appear when integrating the different components, such as including blending constraints and stockpiles into the formulation. Goodfellow and Dimitrakopoulos (2016, 2017) can deal with the non-linearities of the model by treating variables as attributes and classifying them as primary or hereditary to model the flow of material through the mining complex. Primary attributes correspond to additive characteristics, such as metal content, and tonnages, whereas hereditary attributes are derived from primary ones, such as recoveries, economic value, among others. The authors also propose a destination policy of the extracted material, which is the decision of where a block is sent after extraction, based on k-means++ clustering mechanism. Blocks are classified into clusters according to their value over multiple variables, such as material type and grade, and the destination policy is decided per cluster, and not for each individual block. This method allows accounting for multi-variate relations when defining the destination of a block, which is essential when optimizing multi-element mines or complex blending constraints. However, the previously mentioned models are limited in the sense that they produce one static model for the whole life of mine. Some extensions to include alternatives to this formulation have been developed. Goodfellow (2014) models the mining complex under geological uncertainty and adds the decision of investing in capital expenditures (CAPEX) to let the optimizer define the fleet size and purchase plan. In the presented model, different operational details such as the life of the equipment are also included, but the plan obtained is fixed. Similarly, Farmer (2016) works on integrating the optimization of mining and processing capacities of the mining complex under geological uncertainty. Montiel and Dimitrakopoulos (2015) include into the optimization operating alternatives for the processing plant and transportation systems. Considering operational mode alternatives also allows having better control over geometallurgical variables that affect the mining complex (Coward et al., 2009, 2013; Boisvert et al., 2013; Sepulveda et al., 2017).

All studies above on stochastic simultaneous optimization of the mining complex optimize the production schedule of a mineral value chain under uncertainty, but they produce one static solution that is assumed optimal for the whole life of mine (LOM). With this, they are limited in accounting that the future is unknown, and strategic plans may change and adapt to new information obtained. Assuming that the initial plan's conditions will stay the same over the full LOM is an over-optimistic simplification. Conventionally, sensitivity analyses are done to evaluate the viability of new investments and processing stream configurations. These are a passive solution that usually results in delayed projects, and loss of opportunities by inhibiting options that, though not currently viable, could be profitable in the future such as locating expensive infrastructure in strategic locations. Also, as several of these high-impact investments require years of planning, a great deal can be gained by preparing in advance for possible changes.

Integrating flexibility into strategic planning has long been a topic of interest in operations research literature (Brennan and Schwartz, 1985; Kazakidis and Scoble, 2003; Deng et al., 2013; Cardin et al., 2015). Multistage stochastic optimization aims at including dynamic decision making into the optimization process, where uncertainty is also represented through a set of scenarios. Multistage optimization uses non-anticipativity constraints to ensure that non-differentiated scenarios entail equal decisions, and thus, the solution is allowed to branch (i.e., divide into parallel possible solutions, as in a scenario tree) if scenarios appear to be sufficiently different. Birge and Louveaux (1997) mention a general set of different aspects which justify considering a multistage model when optimizing a system, such as the long-term evolution of production, the development of new technologies, or the obsolescence of available industrial equipment. All these are present in mining. Boland et al. (2008) present a multistage stochastic optimization model under geological uncertainty for mine production scheduling, where the schedule is branched into parallel solutions as soon as blocks are found to be "differentiable." Though an interesting concept, the formulation becomes impracticable for real size operations with millions of blocks, and the branching mechanism produces schedule solutions which are over-fitted to the set of scenarios used, and thus, would have poor performance when tested over a different set of simulations. Similar efforts have been made to include flexibility in the context of real-options, such as the work of Wang and De Neufville (2005); Lin et al. (2009); Ajak and Topal (2015); Melese et al. (2017), amongst others.

The model proposed in this paper extends the formulation of the stochastic simultaneous optimization of a mining complex, into a dynamic optimization which provides information about the probability of a set of feasible alternatives of being profitable, and thus, that should be considered within the strategic plan. The dynamic model provides an optimized, flexible plan by generating parallel solutions that provide guidance and ease the transition to change the current plan once more information is available. This formulation aims at producing a dynamic evaluation of a set of high-impact CAPEX alternatives, providing a probabilistic analysis of the likelihood of investing in them, as well as the optimized mine production plans to follow in each case. These alternatives are included as a way of increasing a mining complex's flexibility, transforming the strategic plan into a dynamic mechanism that adapts to change.

Three main considerations are included in the formulation: (i) a dynamic investment schedule is developed, which optimizes a set of CAPEX alternatives as a probability-based decision tree. (ii) Operating mode alternatives are included in the mining complex to manage the effect of geometallurgical variables, specifically, rock hardness, throughput, and recovery, at the mine and the processing levels. (iii) Finally, as in previous work, geological uncertainty is considered in the optimization through a set of equally probable simulations of the deposit.

In the following section, the method is outlined, and the mathematical formulation is presented. Next, the formulation is applied over a copper-gold mining complex, and results are compared to the two-stage stochastic method. Finally, conclusions follow.

5.2 Proposed Method

5.2.1 Problem Description

In the proposed mathematical model, decision variables are grouped as (i) extraction, defining when a mining block is extracted; (ii) destination policy, setting where a block is sent once it is extracted; (iii) processing stream decisions, defining what percentage of material passes from one component of the mining complex to the next; (iv) operational mode, describing under which operational mode will the mine or processing stream operate; and finally (v) capital expenditures, defining which investments are acquired at a cost along the life of mine (LOM). The probabilistic analysis will be performed over a subset of these CAPEX decisions, which will be defined as branching decisions. These branching decisions correspond to big irreversible investments that have a decisive effect on the schedule (such as the investment in a new plant).

5.2.1.1 Generating the Probability-Based Decision Tree Solution

The dynamic mine plan produced corresponds to a probability-based decision tree which branches according to investment decisions over high-impact CAPEX alternatives. These branching decisions have two available options, to invest in or not. Thus, the solution is represented as a scenario tree (Safavian and Landgrebe, 1991; Høyland and Wallace, 2001). Traditional decision tree notation is used to keep track of the branching solutions. Here, each node corresponds to the decisions taken during that given period; each possible solution is identified by a branch, and each node can have at most a number of leaves equal to two to-the-power-of branching decision alternatives left to exercise. This shows that branching alternatives increase exponentially with the number of branching decisions. For example, if two investments A and B are available, the partitions would be to invest in A but not in B, in B but not in A, in both, or in none. However, once both A and B are purchased, then there are zero branching alternatives left, and thus, only one possible branch for the rest of the LOM.

This study proposes an adapted multistage formulation to model this problem. Some main differences of the proposed model compared to traditional stochastic multistage formulations (as per in Birge and Louveaux (1997)) correspond to the definition of a "stage," and the reason for branching onto parallel solutions. In conventional stochastic multistage formulations, stages are defined by specific time intervals, and, at each stage, decision variables can differ between scenarios (i.e., branch) if certain differences are encountered within the set of scenarios. This traditional formulation produces a set of parallel solutions over partitions of scenarios (Boland et al., 2008). In the proposed model, parallel solutions, or production plans, are generated depending on the value of a subset of decision variables, and not over differences between the actual individual scenarios. Thus, a stage is defined by the timing of investment decisions, rather than by specific time intervals. Together with this, as mentioned in the previous section, traditional multistage stochastic formulations have some strong limitations. These are mostly related to over-fitting the solutions obtained to the set of stochastic simulations used in the optimization, as the solutions branch exponentially towards later periods (where, by the end of the optimization, there might be as many solutions as scenarios used). This over-fit prevents the solution of being applicable if a different reality is encountered from the ones represented through the set of simulations. The problem occurs because simulations are used as possible realities, and not as a set which, as a whole, represent the probability distribution of the deposit's spatial variability. To overcome these limitations, and to reduce the computational complexity of solving the model, the proposed method uses an iterative mechanism that quantifies the probability of executing these branching decisions, and controls the generation of branching solutions, ensuring that parallel plans are only generated if they have a representative probability of occurring.

This representativity is measured by setting a threshold R, where branching only occurs whenever the probability of investing in a CAPEX alternative during a given time window falls within this threshold. If the probability of investing is lower than the threshold, the solution does not branch, and no investment is made. On the other hand, if the probability is higher than the threshold, there is also no branching, but the full model invests in the CAPEX alternative. The time window is defined here to provide some stabilizing lag within scenarios for the branching investment decision to be taken (i.e., considering all investments during periods $\mathbf{t_\omega} = \{t-\omega,\ldots,t,\ldots,t+\omega\}$ instead of just t). Accordingly, if there is a representative chance of investing on a branching alternative during time window $\mathbf{t_\omega}$, then the final branching period $t^* \in \mathbf{t_\omega}$ is defined as the expected value of the period of investment of the scenarios that invest during that window.

To obtain the probability of investing, which is used to compare against the threshold R defined previously, a look-forward mechanism is used, where a set of sub-problems is iteratively solved. At each iteration, a set of dynamic non-anticipative constraints is used to define the solution's branches, ensuring that 1st stage decisions are equal along all scenarios within a branch. This set of non-anticipative constraints is enforced over an increasing time frame (thus the term dynamic), starting from only being applied during the first period at the first iteration, up to the whole LOM in the last. Thus, when these constraints are not active, decision variables are left free during the last periods of the LOM, allowing the algorithm to quantify the probability of investing in branching alternatives at different periods, and branching if this probability is significant. This mechanism, presented in Section 5.2.2, is able to create mine designs that adapt to possible futures, and through this iterative process, a decision tree is created, where each branch corresponds to a unique production plan, with its corresponding investment schedule, which maximizes project value. Accordingly, the final solution provides a controlled set of possible mine design alternatives that have been probabilistically quantified as being worth considering.

An example of this branching mechanism for one branching decision is presented in Figure 5-1, where, if the branching decision is exercised over a representative number of scenarios, the solution branches, and a unique mine plan is generated for each partition (referred to as branch). Each square (i.e., node) represents the decisions made on a given year (not necessarily at equal time intervals), where the optimizer decides to branch on period t*, generating two parallel future solutions at that period, with and without investment. In turn, by period t**, the top branch decides to do so, producing two parallel designs. Thus, the final solution of the optimization corresponds to three possible production schedules, with their corresponding probabilities of occurring. It must be noted that, even if the scenario tree presents variating time intervals, showing the decisions taken for the branching decisions, the global optimization is still performed at annual time intervals.

5.2.2 Mathematical Formulation

The mathematical formulation presented is based on the two-stage stochastic model for mining complexes proposed by Goodfellow and Dimitrakopoulos (2017), with main adaptations to include operational and investment alternatives, as well as the dynamic branching mechanism described in Section 5.2.1. Similarly to Goodfellow and Dimitrakopoulos (2017), primary and hereditary attributes are used to model the mining complex, where there exists a function that transforms a primary attribute in a given component of the mining complex into a hereditary attribute. For example, the recovery in a plant, a hereditary non-linear attribute of a processing stream, is obtained by the grade of each mining block being fed to it at that period, calculated using metal tonnage and total tonnage, both simulated primary attributes. Also, as in Goodfellow and Dimitrakopoulos (2017), blocks from each orebody are clustered by the similarity of their simulated characteristics using a k-means++ algorithm and the destination policy decisions (defined as $z_{c,j,t,s}$ in Table 5.2) are set annually over each of these clusters, rather than at a block-level.

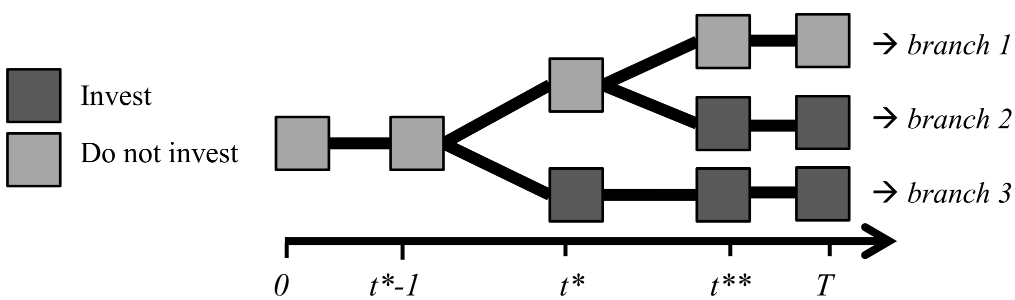


Figure 5-1 Branching mechanism of dynamic stochastic optimization

The different sets used in the mathematical formulation are defined in Table 5.1, followed by the list of decision and state-dependent variables in Table 5.2. Finally, Table 5.3 presents the general parameters, and the parameters used specifically for the flexibility alternatives considered. The full mathematical formulation follows.

Table 5.1 Definition of sets used in the dynamic formulation

Sets and Indices	
P	Primary attributes that are tracked in the supply chain (e.g., metal content, tonnages)
H	Hereditary attributes (derived from primary attributes) that are tracked in the supply chain (e.g., grades, recoveries, economic values)
T	Time periods in the life of mine, indexed by $t = 1 \ldots T$
Ω	Set of scenarios, indexed by $s = 1, \ldots, S$. Where $\Omega_{\rho} \subseteq \Omega$ is the set of scenarios in branch ρ, and $\Omega_{\rho 1}, \Omega_{\rho 2}$ are partitions of Ω_{ρ}, where $\Omega_{\rho 1} \cup \Omega_{\rho 2} = \Omega_{\rho}$, $\Omega_{\rho 1} \cap \Omega_{\rho 2} = \varnothing$
M	Set of mines, indexed by $m \in M$
B_m	Set of blocks in mine $m \in M$, indexed by $b \in B_m$
N(b)	Set of neighbouring blocks of block b located in coordinates i, j, k, where $N(b_{i,j,k}) = \left\{ b'_{i',j',k'} \in B_m : i' \in [i-n, i+n], j' \in [j-n, j+n], k' = k \right\}$
$O(b)$	Set of blocks that overlie block (b)
C	Clusters of blocks with similar attributes, indexed by $c \in C$
Sp	Stockpile destinations that can forward part or all their material to subsequent destinations, indexed by $sp \in Sp$
Pp	Processing stream destinations in the mining complex, indexed by $pp \in Pp$
D	Set of locations in the mining complex: clusters, stockpiles, processing streams $(C \cup Sp \cup Pp)$, where $D_{op} \subseteq D$ is the set of locations containing operational mode alternatives
K	Set of flexibilities and system alternatives, indexed by k. Where, $K^* \subseteq K$ is the set of alternatives that allow branching over the design
Q_j	Set of operational alternatives in location $j \in D_{op}$, indexed by $q \in Q_j$
$\Theta(j)$	Set of locations which can receive material from location $j \in Sp \cup Pp$
$J(j)$	Set of locations which can send material to destination $j \in D$

Table 5.2 Variables used in the model

Decision Variables	
$x_{b,t,s}$	= 1 if block b is extracted at period $t \in \mathbf{T}$, scenario $s \in \Omega$, and 0 otherwise
$z_{c,j,t,s}$	= 1 if cluster c is sent to destination $j \in \Theta(c)$ in period $t \in \mathbf{T}$, scenario $s \in \Omega$, and 0 otherwise
$\mu_{q,j,t,s}$	= 1 if operational mode $q \in Q_j$ is active in location $j \in D$, period $t \in \mathbf{T}$, scenario $s \in \Omega$, and 0 otherwise
$u_{k^*,t}^{\rho l}$	= 1 if design branches over option $k^* \in K^*$ in node ρl, period $t \in \mathbf{T}$, and 0 otherwise
$y_{i,j,t,s}$	$\in [0,1]$. Proportion of material sent from location $i \in Sp \cup Pp$ to $j \in Sp \cup Pp$ in period $t \in \mathbf{T}$, scenario $s \in \Omega$
$w_{k,t,s}$	= 1 if there is a purchase of investment option $k \in K$ executed in period $t \in \mathbf{T}$, scenario $s \in \Omega$, and 0 otherwise
$\sigma_{k,t,s}$	$\in \{0\} \cup \{L_{k,t}, U_{k,t}\}$. Number of investments on option $k \in K$ executed in period $t \in \mathbf{T}$, scenario $s \in \Omega$.
State Variables	
$v_{\varphi,j,t,s} \in \mathbb{R}$	Value of primary attribute $\varphi \in \mathbf{P}$, at location $j \in D \cup M$, period $t \in \mathbf{T}$, scenario $s \in \Omega$. Ex: $\varphi_1 = \psi$ tonnage; then: $v_{\psi,m,t,s}$ tonnage extracted from mine *m,* at $t \in \mathbf{T}$, scenario $s \in \Omega$
$v_{h,j,t,s}^{\mathbf{H}} \in \mathbb{R}$	Value of hereditary attribute $h \in \mathbf{H}$, at location $j \in D \cup M$, period $t \in \mathbf{T}$, scenario $s \in \Omega$. Note that $\mathbf{P} \cap \mathbf{H} = \varnothing$
$v_{h,t,s} \in \mathbb{R}$	Final value of attribute $h \in \mathbf{H}$, at period $t \in \mathbf{T}$, scenario $s \in \Omega$, where $v_{h,t,s} = \sum_{j \in D} v_{h,j,t,s}^{\mathbf{H}}$
$t^* \in \mathbb{Z}^+$	Final branching period within time window (in Table 5.3), dependent on the investment decisions of branching alternatives $w_{k^*,t,s}$
$r_{\varphi,j,t,s} \in [0,1]$	Recovery of attribute $\varphi \in \mathbf{P}$ at location $j \in Pp$, period $t \in \mathbf{T}$, scenario $s \in \Omega$
$d_{h,t,s}^{+},\ d_{h,t,s}^{-} \geq 0$	Surplus or shortage variables (respectively), from deviations over targets of attribute $h \in \mathbf{H}$, period $t \in \mathbf{T}$, scenario $s \in \Omega$

Table 5.3 Set of parameters used in formulation

General Material Flow Parameters	
$\beta_{\varphi,b,s}$	Simulated value of primary attribute $\varphi \in \mathbf{P}$, for block $b \in B_m$ and scenario $s \in \Omega$. Ex.: $\beta_{\chi,b,s}$ = metal content $\chi \in \mathbf{P}$ of block *b*, in scenario *s*
$\theta_{b,c,s} \in \{0,1\}$	Pre-defined cluster classification, = 1 if block $b \in B_m$ belongs to cluster $c \in C$, in scenario $s \in \Omega$, and 0 otherwise
$f_{h,j}(\varphi)$	Function that transforms primary attributes $\varphi \in \mathbf{P}$ into hereditary attribute $h \in \mathbf{H}$ in location $j \in D \cup M$ (defined by the modeler)
$U_{h,i}$, $L_{h,i}$	Basic upper and lower limit of attribute $h \in \mathbf{H}$, in location $i \in M \cup D$. Ex.: $U_{W,i}$ = Upper extraction capacity limit $W \in \mathbf{H}$, at location $i \in M$
$c^{+}_{h,t}$, $c^{-}_{h,t} \geq 0$	Unit cost of positive and negative deviation over targets of attribute $h \in \mathbf{H}$, at period $t \in \mathbf{T}$
$p^{\mathbf{H}}_{h,t}$	Unitary price (or cost) of attribute $h \in \mathbf{H}$, at time $t \in \mathbf{T}$
Option-related parameters	
$\mathbf{N} \in \mathbb{Z}^{+}$	Min number of scenarios in a branch required to allow further branching in $t+1 \in \mathbf{T}$
R	$\in [0, 0.5]$, minimum proportion of scenarios (i.e. threshold) needed to branch design (threshold = [*R, 1 - R*]).
$p^{K}_{k,t}$	Discounted purchase cost of option $k \in K$, period $t \in \mathbf{T}$
λ_k	Life of capital option $k \in K$
τ_k	Lead time before an option $k \in K$ is available (since the moment of decision)
$\kappa_{k,h}$	Per unit increment for constraints that investment option $k \in K$ has on attribute $h \in \mathbf{H}$
ψ_k	Allowed periodicity to decide on option $k \in K$
$\vartheta^{h,j}_{q}$	$\in [-1,1]$. Effect/Adjustment factor over attribute $h \in \mathbf{H}$, at location $j \in D_{op}$ if option $q \in Q$ is taken, where $\vartheta^{h,j}_{k} = 0$ if $j \notin D_{op}$
$\omega \in \mathbb{Z}^{+}$	Time lag to consider branching alternatives at time window $t' \in [t + \omega, t - \omega]$

State variables in Table 5.2 correspond mostly value of the different primary and hereditary attributes along the different components of the mining complex. As mentioned earlier, primary attributes correspond to additive simulated attributes of the rock (i.e., tonnage, metal content, etc.), and hereditary attributes depend on these simulated primary-attribute values, and on the transformation function that defines them (presented in Table 5.3). Together with this, surplus and shortage variables $(d^{+}_{h,t,s},\ d^{-}_{h,t,s})$ are defined to quantify and manage deviations from targets.

With these variables and parameters, the branching threshold described in Section 5.2.1 is defined in relation (5.1), where, for time window $\mathrm{t}^{\omega} = [\mathrm{t}-\omega, \mathrm{t}+\omega]$, the probability of branching must be within threshold $\in [\mathrm{R}, 1-\mathrm{R}]$.

$$\begin{cases} \text{do not invest in } k^* \text{ during } \mathbf{t}^{\omega} & \textbf{if } \textit{probability of investing in } k^* < R \\ \text{branch during } \mathbf{t}^{\omega} & \textbf{if } \textit{probability of investing in } k^* \in [R, 1-R] \\ \text{invest in } k^* \text{ during } \mathbf{t}^{\omega} & \textbf{if } \textit{probability of investing in } k^* > 1-R \end{cases} \tag{5.1}$$

If the probability of branching is within this threshold, then the final branching period $\mathrm{t}^* \in \mathrm{t}^{\omega}$ is defined as the expected value of time of investment occurring within the time window, as defined in Eq. (5.2). Note that t^* is independent of k^*, as, even if there are multiple branching alternatives acting simultaneously, the final solution will branch at most only once per iteration, at time t^* (see Algorithm 5.1 for further details).

$$t^* = E(w_{k^*,t',s} \cdot t'), \qquad t' \in [t-\omega, t+\omega],\ t \in \{\omega+2, \ldots, T\},\ \forall k^* \in K^*,\ s \in \Omega_{\rho} \tag{5.2}$$

5.2.2.1 Dynamic Mining Complex Model

The dynamic mining complex model aims at maximizing the discounted profit obtained from processing the extracted material at the different processing streams and minimizing the cost of different investments overtaken along the life of mine, as well as the deviations from production targets, which act to manage risk and defer it to later periods.

Objective function:

$$\max \frac{1}{S} \sum_{s \in S} \sum_{t \in T} \left\{ \underbrace{\sum_{i \in D \cup M} \sum_{h \in \mathbf{H}} p_{h,t}^{\mathbf{H}} \cdot v_{h,j,t,s}^{\mathbf{H}}}_{\text{net profit from hereditary variables}} - \underbrace{\sum_{k \in K} p_{k,t}^{K} \cdot \sigma_{k,t,s}}_{\text{cost of investments}} - \underbrace{\sum_{i \in D \cup M} \sum_{h \in \mathbf{H}} \left(c_{h,t}^{+} \cdot d_{h,i,t,s}^{+} + c_{h,t}^{-} \cdot d_{h,i,t,s}^{-} \right)}_{\text{penalty for deviations}} \right\} \tag{5.3}$$

Subject to:

a) *Mining Constraints*

Next, mining constraints are presented, which ensure that the extraction is geotechnically and operationally feasible (Eq. (5.4) - (5.7)).

- Slope constraints – ensure that a block b is only extracted once its predecessors O(b) (i.e., its overlying blocks) have been extracted.

$$x_{b,t,s} \leq \sum_{t'=1}^{t} x_{o,t',s} \quad \forall b \in B_m,\ m \in M,\ o \in O(b),\ t \in T,\ s \in \Omega_\rho \tag{5.4}$$

- Mine reserve – a block b can be mined only once in the LOM

$$\sum_{t \in T} x_{b,t,s} \leq 1, \quad \forall b \in B_m, \; m \in M, \; s \in \Omega_\rho \tag{5.5}$$

- All extracted rock must be sent to a single destination – this constraint ensures all clusters C have an assigned destination at each period, which consequently defines the destination of each extracted block of that period. Note that, as the clustering is a pre-processing stage which is independent of the blocks' location, no link is required between the extraction decision variable and the cluster destination, as later the attributes of a block will only be accounted for if this block is extracted (Eq. 5.8).

$$\sum_{j \in \Theta(c)} z_{c,j,t,s} = 1 \;, \; \forall c \in C, \; t \in T, \; s \in \Omega \tag{5.6}$$

- Mineability /Mining width – these constraints ensure that the extraction sequence is smooth and continuous, penalizing the objective function (OF) through state variable $d^-_{h,m,t,s}$ if, for a given block, its neighbouring blocks nb are not extracted at or before that period. Here, *i,j,k* correspond to the coordinates of block b , and n is the number of surrounding blocks that must also be extracted on each direction to ensure mining equipment width requirements.

$$N(b) \cdot x_{b,t,s} \leq \sum_{b' \in nb} \sum_{\tau=1}^{t} x_{b',\tau,s} + d^-_{h,m,t,s} \qquad \forall h \in \mathbf{H}, \; m \in M, \; t \in T, s \in \Omega_\rho \tag{5.7}$$

b) Mining complex constraints

- Stockpile material balance between incoming and outgoing quantities – these constraints ensure that there is a balance between incoming and outgoing material in the stockpiles of the mining complex. This balance is defined by the existing material, plus what is being fed to it, minus what is being taken from that whole amount to forwarding stages of the mining complex.

$$v_{\varphi,i,(t+1),s} = \underbrace{v_{\varphi,i,t,s} \cdot \left(1 - \sum_{j \in \Theta(i)} y_{i,j,t,s}\right)}_{\text{Left-over from previous period}} + \underbrace{\sum_{j \in J(i) \setminus C} y_{j,i,t,s} \cdot v_{\varphi,j,t,s}}_{\text{Incoming form other locations}} +$$

$$+ \underbrace{\sum_{c \in J(i) \cap C} \left(\sum_{m \in M} \sum_{b \in B_m} \beta_{\varphi,b,s} \cdot \theta_{b,c,s} \cdot x_{b,(t+1),s} \right) \cdot z_{c,i,(t+1),s}}_{\text{Coming in from the mines to } i}, \tag{5.8}$$

$$\forall \varphi \in \mathbf{P},\ i \in Sp,\ t \in T,\ s \in \Omega$$

- Processing material balance between incoming and outgoing quantities – these constraints ensure that there is a balance between incoming and outgoing material in the different processing locations of the mining complex. Processors have no left-over material. Thus, current material is defined only by the period's feed from mines and stockpiles.

$$v_{\varphi,j,(t+1),s} =$$

$$\underbrace{\sum_{i \in J(j) \setminus C} r_{\varphi,i,t,s} \cdot v_{\varphi,i,t,s} \cdot y_{i,j,t,s}}_{\text{Coming in from other destinations to j}} + \underbrace{\sum_{c \in J(j) \cap C} \left(\sum_{m \in M} \sum_{b \in B_m} \beta_{\varphi,b,s} \cdot \theta_{b,c,s} \cdot x_{b,(t+1),s} \right) \cdot z_{c,j,(t+1),s}}_{\text{Coming in from the mines to j}} \tag{5.9}$$

$$\forall \varphi \in \mathbf{P},\ j \in Pp,\ t \in T,\ s \in \Omega$$

with

$$\sum_{j \in \Theta(i)} y_{i,j,t,s} \leq 1 \qquad i \in Sp,\ t \in T,\ s \in \Omega$$

$$\sum_{j \in \Theta(i)} y_{i,j,t,s} = 1 \qquad i \in Pp,\ t \in T,\ s \in \Omega$$

- Capacity constraints for Mining/ Equipment – these constraints are global for all components of the mining complex and are affected by both operational and investment alternatives decisions.

$$v_{h,i,t,s} - d^{+}_{h,i,t,s} \leq U_{h,i} \cdot \left(1 + \vartheta^{h,i}_{q} \cdot \mu_{q,i,t,s}\right) + \sum_{k \in K} \sum_{t'=t-\lambda_k-\tau_k}^{t-\tau_k} \kappa_{k,h} \cdot w_{k,t',s}, \tag{5.10}$$

$$v_{h,i,t,s} + d^{-}_{h,i,t,s} \geq L_{h,i} \cdot \left(1 + \vartheta^{h,i}_{q} \cdot \mu_{q,i,t,s}\right) + \sum_{k \in K} \sum_{t'=t-\lambda_k-\tau_k}^{t-\tau_k} \kappa_{k,h} \cdot w_{k,t',s}, \tag{5.11}$$

$$\forall h \in H,\ i \in D \cup M,\ t \in T,\ s \in \Omega_{\rho l} \subseteq \Omega,\ q \in Q_i$$

For example, if the plant changes its operational mode to increase throughput (i.e. $\mu_{q,i,t,s} = 1$), the upper and lower capacity limits will increase by a factor of $\vartheta^{h,i}_{q}$. The same way, if the optimizer decides to invest in an extra crusher at the plant (i.e. $w_{k,t',s} = 1$), then (after the corresponding lead time has passed), the limits also increase by a quantity of $\kappa_{k,h}$. Note that investment alternatives increase the capacity only after the lead time has passed (τ_k), and only for the time defined by the life of the equipment purchased (λ_k).

c) Attribute calculation and state variable definition

The definition of the different primary and hereditary attributes is presented next. These definitions are general to allow the model to adapt to the specific characteristics of different mining complexes, such as the number of elements produced, the set of possible processing streams, geometallurgical variables of interest, to name a phew.

- Value of primary attributes – (defined per scenario) these constraints ensure that the value of any primary attribute is only accounted for if the block is extracted on that period.

$$v_{\varphi,m,t,s} = \sum_{b \in B_m} \beta_{\varphi,b,s} \cdot x_{b,t,s}, \quad \forall m \in M, \ \varphi \in \mathbf{P}, \ t \in T, \ s \in \Omega \tag{5.12}$$

- Final hereditary attribute value – depends on the transformation function $(f_{h,j}(\varphi))$, which is affected by the selection of operational alternative ($\vartheta_q^{h,j}$), activated by the binary decision variable $\mu_{q,j,t,s}$.

$$v_{h,j,t,s} = f_{h,j}(v_{\varphi,j,t,s}) \cdot \left(1 + \vartheta_q^{h,j} \cdot \mu_{q,j,t,s}\right), \quad \varphi \in \mathbf{P}; \ h \in \mathbf{H}; \ j \in D \cup M; \ q \in Q_i; \ \forall t \in T; \ s \in \Omega_\rho \tag{5.13}$$

For example, if operating alternative $q \in Q_j$ affects the recovery of element $\varphi \in \mathbf{P}$ at location $j \in Pp$, then the final recovery ($r_\varphi \in \mathbf{H}$) is defined by $v_{r_\varphi,j,t,s} = f_{h,j}(v_{\varphi,j,t,s}) \cdot \left(1 + \vartheta_q^{r_\varphi,j} \cdot \mu_{q,j,t,s}\right)$.

d) Dynamic / option constraints

These sets of constraints enable the branching mechanism and ensure that equal decisions are taken over all scenarios if no branching has been defined.

- Non-anticipative constraints

These constraints are enforced over a variable time frame T^{α}, which is iteratively augmented (as defined in the algorithm in section 5.2.1). Within this time frame, these constraints are always enforced, except if branching is "activated" (i.e. $u^{\rho}_{k^*,t} = 1$). Here, non-anticipative constraints are defined over extraction (5.14), destination (5.15), investment (5.16), and operational mode (5.17) decisions.

As there can be more than one branching decision in the model, branching can occur over any of them, and thus the value of $u^{\rho}_{k^*,t}$ is considered over all possible branching decisions available (set K*). Note that if all $u^{\rho}_{k^*,t} = 0$, then all decisions must be the same within all scenarios (even branching investment decisions). Which means the solution will remain unique, with or without investment. For ease of notation, $A = \left\lceil \frac{\sum_{k^* \in K^*} u^{\rho}_{k^*,t}}{|K^*|} \right\rceil = \{0,1\}$ in Eq. (5.14) – (5.17).

Given the scenario partition $\Omega_{\rho 1} = \left\{ s \ ; \ w_{k^*,t^*,s} = 1, \forall s \in \Omega_{\rho} \right\}$, $\Omega_{\rho 2} = \Omega_{\rho} \setminus \Omega_{\rho 1}$, the following set of constraints is defined

$$(1 - A)\left(x_{b,(t+1),s} - x_{b,(t+1),s'}\right) = 0, \qquad \forall t \in T^{\alpha};\ b \in M \tag{5.14}$$

$$(1 - A)\left(z_{c,j,(t+1),s} - z_{c,j,(t+1),s'}\right) = 0, \qquad \forall t \in T^{\alpha};\ c \in C;\ j \in D \tag{5.15}$$

$$(1-A)\left(w_{k,(t+1),s}-w_{k,(t+1),s'}\right)=0, \qquad \forall t\in T^{\alpha};\ k\in K \tag{5.16}$$

$$(1-A)\left(\mu_{q,j,(t+1),s}-\mu_{q,j,(t+1),s'}\right)=0, \qquad \forall t\in T^{\alpha};\ q\in Q_j;\ j\in D_{op} \tag{5.17}$$

$$s,s'\in\Omega_{\rho};\ \forall s\in\Omega_{\rho 1};\ \forall s'\in\Omega_{\rho 2}$$

- Branching threshold constraint

These set of constraints define the activation of branching in node ρ, which only occurs if the probability of branching during time window t^{ω} is within the threshold limits$\in[R,1-R]$. The following constraints are used to verify if the branching proportion is within the upper and lower limits of the threshold.

$$\frac{\sum_{t'=t-\omega}^{t+\omega}\sum_{\forall s\in\Omega_{\rho}} w_{k^*,t',s}}{\left|\Omega_{\rho}\right|}\le(1-R)+(1-u^{\rho}_{k^*,t})$$

$$\frac{\sum_{t'=t-\omega}^{t+\omega}\sum_{\forall s\in\Omega_{\rho}} w_{k^*,t',s}}{\left|\Omega_{\rho}\right|}\ge R-(1-u^{\rho}_{k^*,t}) \tag{5.18}$$

$$\forall k^*\in K^*, t\in \mathbf{T}$$

- Stochastic solution stability - there must be enough scenarios in each possible partition (Eq. 5.19).

$$\frac{\left|\Omega_{\rho 1}\right|}{\mathbf{N}}\ge u^{\rho}_{k^*,t^*}, \quad \frac{\left|\Omega_{\rho 2}\right|}{\mathbf{N}}\ge u^{\rho}_{k^*,t^*}, \qquad \forall k^*\in K^*,\ \Omega_{\rho}\subseteq\Omega,\ t\in[T^{\alpha},T] \tag{5.19}$$

Where $\Omega_{\rho 1}=\left\{s\ ;\ w_{k,t,s}=1,\forall s\in\Omega_{\rho}\right\},\ \Omega_{\rho 2}=\Omega_{\rho}\setminus\Omega_{\rho 1}$

- Definition of branching period (t*)

Equations (5.18) and (5.19) define if the system branches during time window t^{ω}. If it does, constraint (5.20) defines the actual branching period t^*, within time window $t^{\omega} = [t-\omega, t+\omega]$, as the nearest integer value of the expected value of period of investment within this time window t^{ω}.

Note that t^* is only activated (i.e. $t^* > 0$), if $u^{\rho}_{k^*,t^*} = 1$.

$$t^* = \sum_{t \in [T^{\alpha}, T]} \left\lfloor \frac{\sum_{t'=t-\omega}^{t+\omega} \sum_{\forall s \in \Omega_{\rho}} t' \cdot w_{k^*,t',s}}{\sum_{t'=t-\omega}^{t+\omega} t'} \cdot u^{\rho}_{k^*,t} + \frac{1}{2} \right\rfloor, \qquad \forall k^* \in K^* \tag{5.20}$$

e) Operational constraints over investment alternatives

These constraints ensure that operational and purchase requirements over the set of investments available are respected.

- Periodicity of investments – decision to invest in CAPEX alternative k is only allowed ψ_k periods after it was previously taken.

$$w_{k,t,s} + \sum_{\tau=t+1}^{t+\psi_k} w_{k,\tau,s} \leq 1, \qquad \forall k \in K \setminus K_{op},\ t \in T,\ s \in \Omega_{\rho} \subseteq \Omega \tag{5.21}$$

- Limits on purchases – These constraints link the activation of the investment decision with the actual number of investments, which must be within the allowed upper and lower limits.

$$\sigma_{k,t,s} \leq U_{k,t} \cdot w_{k,t,s}, \quad \sigma_{k,t,s} \geq L_{k,t} \cdot w_{k,t,s} \qquad \forall k \in K, \forall t \in T, \forall s \in \Omega_{\rho} \tag{5.22}$$

- Limit on branching decisions – as these decisions are defined as high-impact, high-cost decisions, they are allowed "only once in the LOM." This constraint can be replaced by setting the allowed periodicity (ψ_k) in constraint (5.21) big enough to forbid repeating that investment.

$$\sum_{\forall t \in T} w_{k^*,t,s} \leq 1, \ \forall k^* \in K^*, \ \forall t \in T, \ \forall s \in \Omega_\rho \subseteq \Omega \tag{5.23}$$

5.2.2.2 Solution method

The previous problem is iteratively solved as described in Algorithm 5.1. The proposed mechanism is done to obtain the probability value of executing branching decisions, which is found by using a look-forward procedure. The algorithm works by enforcing dynamic non-anticipativity constraints over an iteratively increasing time window $= \{1, \ldots, T^\alpha\}$.

Starting from the first period, non-anticipativity constraints are set to work until an auxiliary time period (T^α), which increases as a moving time window (i.e., equals the first period in the first iteration, and equals the whole LOM in the last). With this, at each iteration, the model is solved, and the partitions are updated.

Algorithm 5.1 Iterative solution mechanism

Initialization

T total the number of periods
Ω total number of simulations
Ω_ρ partition of scenarios in branch ρ, initially equal to Ω
FS final solution containing dynamic production schedule
SP_i, sub-problem defined in Section 5.2.2, solved for the *i*-th iteration
$i = 0$ sub index to define sub-problems
T^α auxiliary time defining final period of enforcement of non-anticipativity constraints
t^* period of branching, defined from Eq. 5.2 and 5.20 (setting $u^\rho_{k^*,t^*} = 1$)

t ← 1, initial period of LOM to optimize
T^α ← $t + 1$

Stage 1

Solve subproblem SP_i over Ω, as described in section 5.2.2 for **T** = [t, T], and $T^\alpha \in \mathrm{T}$.

Stage 2

while $T^\alpha \leq T$ **do**
  t ← T^α
  if $t^* > 0$ **then**
    T^α ← t^*
    Solve sub-problem SP_i for **T** = [t, T], setting $u^\rho_{k^*,t^*} = 1$ with corresponding partitions Ω_ρ calculated from SP_{i-1}
  else do
    T^α ← $T^\alpha + 1$
    Solve sub-problem SP_i for **T** = [t, T]
  end else
  update t^*
  FS ← SP_i
  i++
end while

return FS

5.2.2.3 Solving Mechanism

The solution on the previous model can be represented as solution vector $\mathbf{\Phi} = \left[[x_{b,t,s}],[z_{c,j,t,s}],[\mu_{q,j,t,s}],[u^{\rho l}_{k*,t}],[y_{i,j,t,s}],[w_{k,t,s}],[\sigma_{k,t,s}]\right]$, where most decision variables are binary. Solving a real-size case study would entail almost a billion binary variables (Ex.: a case study with 20 simulations of a mining complex with two mines of 150k blocks each over a 10-year LOM has 20x20x300,000x10 = 1.2B binary variables). However, because of the iterative mechanism described in Algorithm 5.1, this formulation can be decomposed into smaller sub-problems where the extraction sequence variables can be considered scenario independent within each branch, reducing the number of variables to the range of millions to tens of millions. As solving a formulation with millions of binary variables is still a challenge, and due to the non-linearities present in the formulation, a simulated annealing (SA) based metaheuristic algorithm is used to solve it. This algorithm is based on the metaheuristic described in Goodfellow and Dimitrakopoulos (2016), but, instead of integrating two different heuristic mechanisms (simulated annealing and particle swarm), an adaptive multi-neighbourhood simulated annealing is used.

A *neighbourhood* refers to a class of perturbations in the solution vector (i.e. the vector containing decision variables defining the extraction, destination, operating modes, processing stream and investments $[x_{b,t,s}],[z_{c,j,t,s}],[\mu_{q,j,t,s}],[y_{i,j,t,s}],[\sigma_{k,t,s}]$), and *perturbations* correspond to changes in a particular decision variable, such as changing the extraction period of a block, the proportion of material sent from a stockpile to the processing stream, the destination of a cluster on a given period, the operating mode at a plant, or deciding to purchase one extra equipment. To perturb continuous variables, a uniform distribution is plotted along all its possible values, and the cumulative distribution is randomly sampled, defining the new value of the decision variable. Simulated annealing algorithm (Kirkpatrick et al., 1983; Geman and Geman, 1984) works by, starting from an initial solution $\mathbf{\Phi}_0$, perturbing the current, and accepting or rejecting the new solution depending on the annealing probabilities. Adaptive multi-neighbourhood simulated annealing starts from the same basis, but each neighbourhood is selected, first randomly, and next according to an adaptive probability, which is updated according to the performance of that given perturbation in improving the solution.

Perturbations affecting the investment and operating mode decisions correspond to (i) the addition or removal of one or multiple investments at a given year, (ii) the swap of two different investments in two different periods, and (iii) the activation or deactivation of operational modes in different components of the mining complex in a given period. The impact of these perturbations over the objective function value may be drastically different, particularly for the case of branching investment decisions, where a crusher may cost hundreds of millions of dollars and have considerable effect over the schedule and processing capacity of the system. Because of this, to allow the optimizer to stabilize and adapt to these big changes, once a neighbourhood perturbing $[\sigma_{k*,t,s}],\ k* \in K*$ is chosen, this neighbourhood is forbidden to be selected again for a certain number of iterations (2000 iterations in the following case study).

It must be noted that if a perturbation is chosen to modify the current solution, this modification must respect all the constraints of the model, for example, a block cannot be set to be extracted on a period where its predecessors have not been extracted yet. For the case study presented in the next section, the initial solution was obtained by setting all blocks as unmined, and an annealing schedule is set, where the annealing temperature is repeatedly changed after a certain number of iterations, and the solving process stops once a total number of iterations is reached.

5.3 Case Study

The proposed model is applied at an operating mining complex composed of one mine, and six possible processing streams (Figure 5-2). These processing streams are a sulphide mill with a stockpile, three heap leaches for sulphides, oxides, and transition (SHL, OHL, and THL respectively), a sulphide dump leach (SDL) for sulphide low grade and waste, and an oxide dump, for oxide waste. The mining complex produces copper and gold, and the different processing streams have constraints over the type of material received, and the product produced. All this is represented in Figure 5-2 by the squares with numbers beside each destination, which represent the type of material that is allowed in each (defined at the left side of the figure).

The sulphide mill is the only processing stream which produces both gold and copper and is the main source of profit of the mining complex. It has a production capacity of 2.4Mt per year, and the adjoin stockpile can store up to 1Mt. The sulphide heap leach also has a limited capacity of 6Mt, and it is assumed that all other destinations have no capacity restriction. Mining and economic parameters used are presented in Table 5.4. Values have been normalized by the mining cost for confidentiality reasons. In this case, fixed operating costs and commodity prices have been used. Table 5.4 also shows the branching parameters, which define an investment window (ω) of +/- 1 year, and a threshold parameter R_t. Thus, according to the values presented, if between 40% and 60% of scenarios decide to invest within a time window of *[t - 1, t+1]*, then the design branches into parallel solutions.

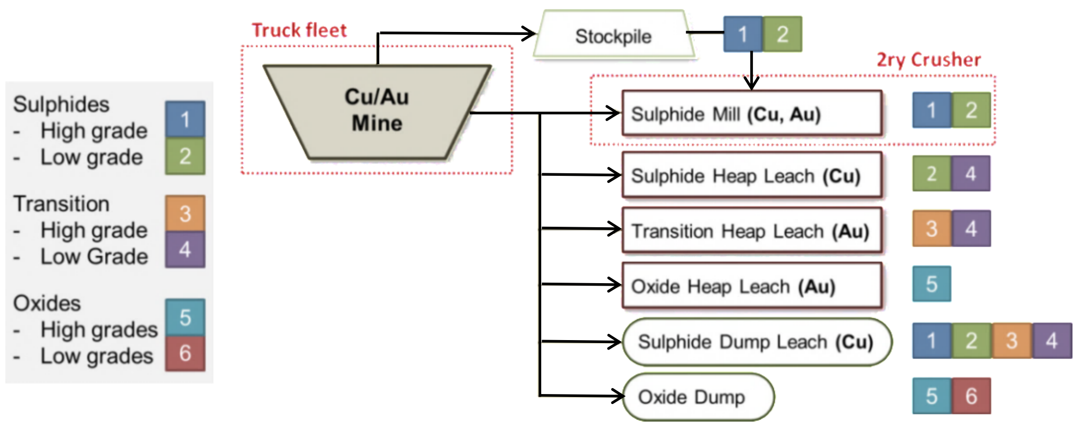


Figure 5-2 Mining complex layout with material allowed and metal produced per destination

Table 5.4 Mining and economic parameters of the copper/gold mine

Mining Complex Param.		**Processing Costs**	
Mining Cost	$1.0 * x	Sulphide mill cost	$11.3*x
Mining Cap.	6 Mt	Sulphide heap leach cost	$3.9*x
SHL Capacity		Sulphide dump leach cost	$1.9*x
SM Capacity	3.4 Mt	Transition heap leach cost	$3.2*x
Mining width	100m	Oxide heap leach cost	$3.1*x
Economic Parameters		**Other Parameters**	
Copper Price	$3.9/lb	ω	1 year
Gold Price	$1450/oz	R_t	35%
Discount rate	10%		

5.3.1 Alternatives Considered

Alternatives added to this case study are divided into “investment” and “operational.” Investment alternatives are included (i) in the sulphide mill, with the possibility of increasing the capacity by adding a secondary crusher to increase the production capacity, and (ii) at the mine, where the optimizer defines the truck fleet, and thus, the annual extraction capacity. Both alternatives are highlighted in Figure 5-2 by dotted lines. Two operational alternatives are included in this case study; (a) one that acts over the mine by adapting the blasting pattern to reduce mining cost, punishing grindability, and (b) an alternative over the sulphide mill’s processing configuration, which increases throughput by reducing the recovery. Operational details on each alternative are presented next.

5.3.1.1 Investment Alternatives

An initial fleet of 2 trucks is available and active for the first six years. This reduced fleet allows testing the optimizer and allowing it to choose the optimal fleet size for later periods, as the optimization process can increase this extraction capacity by purchasing additional trucks. However, it can be assumed that in a real mine this initial fleet may be considerably bigger. Each additional truck has a life of equipment of six years, and once a truck is purchased, it is only available one year later. The decision to purchase a truck can be taken every 2 years, and the maximum purchase quantity is defined to be 5 trucks at a time. These and other operational details are presented in the first column of Table 5.5.

The second CAPEX alternative is the purchase of a secondary crusher at the sulphide mill, which increases the production capacity by 300kt per year. This investment decision is set to be a "branching alternative," which means that the optimizer can branch and develop parallel mine design schedules if a representative number of scenarios differ in this decision variable. These and other operational details are presented in the second column of Table 5.5.

Table 5.5 Purchasing details for the investment alternatives

	Truck (non-branching option)	**Secondary Crusher (branching option)**
Undiscounted cost ($US)	3,800,000	25,000,000
Life of equipment	6 years	25 years
Periodicity of decision	2 years	-
Lead time	1 year	2 years
Maximum purchases	5 units/year	1 unit/year
Tonnage increment per unit	3,500,000 tpa	300,000 tpa
Initial Capacity available	7,000,000 tpa	2,400,000 tpa

5.3.1.2 Operational Alternatives

Mining Mode:

The mining operational mode alternative works by reducing the number of blast-holes in the blasting pattern. By doing so, the overall mining cost is reduced due to less drilling and a reduced amount of explosives but, at the same time, a coarser blasted material is produced, which requires additional work and energy from the system's crusher, reducing the throughput (Figure 5-3). In this case, it is assumed that an 18 blast-hole net is reduced to a 16 blast-hole one (as shown in the left side of Figure 5-3), which reduces the mining costs by 8%, and in turn, reduces the crushing capacity by 3% ($\vartheta_q^{h,j}$ in the formulation in Section 5.2.2). These values were taken from the mine's historical data.

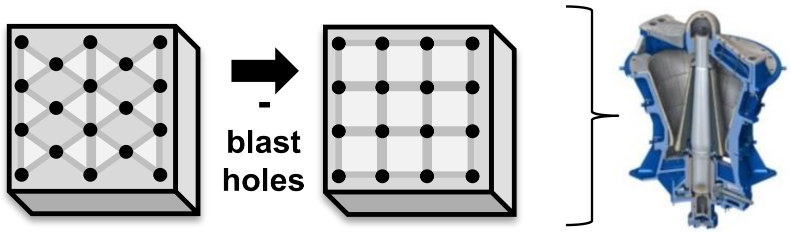


Figure 5-3 Blasting net alternative at the mine level

These operational alternatives allow having better control over the pertinent geometallurgical variables, as, for example, the optimizer may choose to concentrate blasting in areas with harder rock or where the grade is higher, ensuring that that material reaches the processing stream faster.

Processing Mode:

The processing operational alternative at the sulphide mill is defined as the selection between a higher throughput with a lower recovery, or a lower throughput and an increased recovery (Figure 5-4). This metallurgical relation is well known, where the processing time can be reduced by shortening the feed's time at the crusher. Doing so produces a coarser grinding. Processing time can also be reduced by reducing the concentrate's residence time in the different processing stages; however, this will have a negative effect on the metal recovered from processing the material at the sulphide mill.

In this case study, the "activated" operational mode increases the throughput by a 4.4% but reduces the plant's recovery by 0.56% ($\vartheta_q^{h,j}$ in the formulation in

Section 5.2). These values were also taken from historical data of the mine; the plant's recovery curves presented in Figure 5-4 (values are not shown for confidentiality reasons).

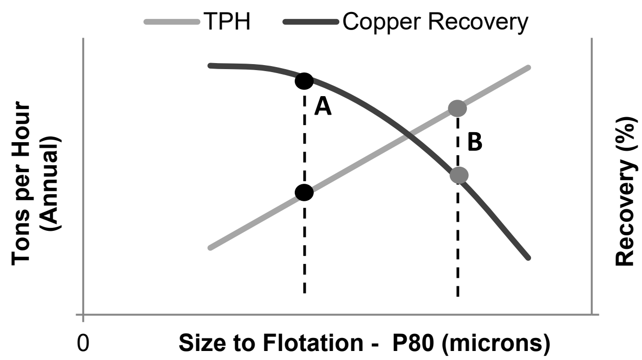


Figure 5-4 Relation of recovery/throughput alternatives (A and B) at the SM

5.3.2 Results

The following section presents the results obtained for the dynamic stochastic optimization method proposed. Subsequently, these results are compared with the traditional two-stage stochastic formulation without alternatives. These results were obtained with a computer with processor Intel® Core™ i7-2600S CPU, with an installed memory of 8.0 GB. Both first and second stages (Algorithm 5.1) were computed with an annealing schedule of 5 million iterations, which took about 40 to 48 hours of solving time in each case. As any metaheuristic algorithm, more iterations represent higher possibilities of an improved solution, but it was found that after 4 to million iterations the solution converges.

5.3.2.1 Proposed dynamic two-stage stochastic formulation with alternatives

The solution of the formulation proposed in Section 5.2.2 shows that there is a 43% probability of investing on a secondary crusher in period 4. As 43% is within the threshold of *R = 40%* defined in Table 5.4 (43% ϵ [40%, 60%]), the design branches at that time. All results obtained from the branch without secondary crusher are presented on the left side of the figures, and the case with secondary crusher is presented on the right side.

The crusher and truck investment plans are presented in Figure 5-5. The right side shows that, when a secondary crusher is purchased, the operation chooses to buy one extra truck by year 5, compared to the branch without secondary crusher. This extra purchase is available in year 6 (also when the crusher is available), which allows balancing the extra mill feed required by the system. Together with this, in both branches trucks are purchased every two years, maintaining an average extraction capacity of 17.5Mt, with a five-year ramp-up. This capacity increases further for the case with secondary crusher, reaching 20-25Mt capacity during years 6 to 13.

Operational alternatives for both branches are presented in Figure 5-6, which shows that when the plan invests in a secondary crusher, the optimizer generally decides against increased throughput (mill mode alternative) and grindability (miner mode alternative). This is clear particularly in the last three periods, where, as there is 300kt extra of processing capacity, the optimizer chooses to maximize recovery and minimize mining costs by keeping both operational alternatives not active.

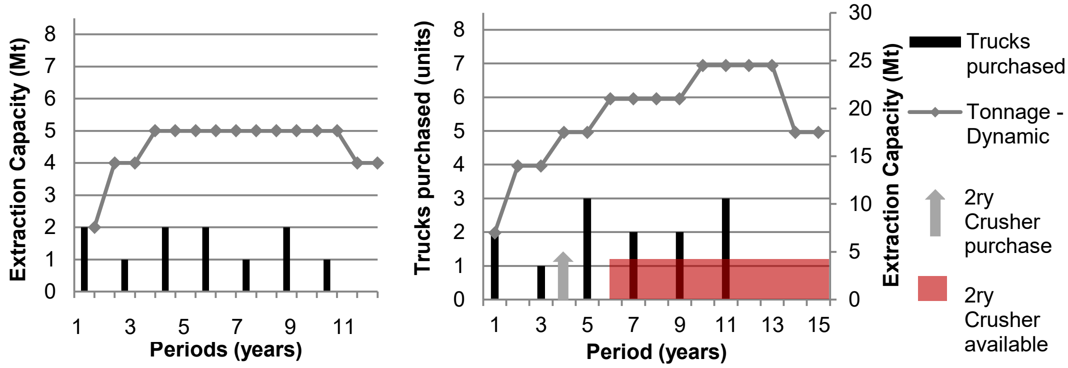


Figure 5-5 Trucks and secondary crusher purchase plan for each branch of the production plan solution, without secondary crusher (left) and with secondary crusher (right)

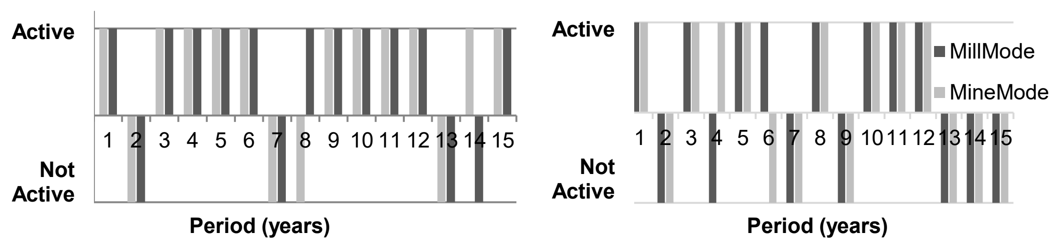


Figure 5-6 Operational mode decisions for the mine (grey) and the mill (black) for each branch of the production plan, without secondary crusher (left) and with secondary crusher (right)

Particularly in the case of operational mode alternatives, the time frame to change them can be considered shorter than a whole year. Because of this, and due to the flexibility of the proposed model, a mid-term analysis is performed by discretizing the first two years in three terms each and defining the corresponding operational mode decision variables (blasting pattern mode and sulphide mill recovery mode) in that schedule's time frame. This analysis provides a more realistic target to guide the short-term plan, considering the actual configuration flexibility that the processing streams have. Figure 5-7 shows the risk analysis over the mid-term feed plan for the sulphide mill, with the initial target in dotted red, and the target adapted by the sulphide mill's operational alternatives in continuous blue. The percentiles 10, 50 and 90 are presented (P10, P50, and P90, respectively), which show there is a 10%, 50% and 90% probability of being under the values presented. It can be seen in the figure that the optimizer decides to apply the mill operational mode to increase the mill's throughput in the last two terms of the first year and on the last term of the second one.

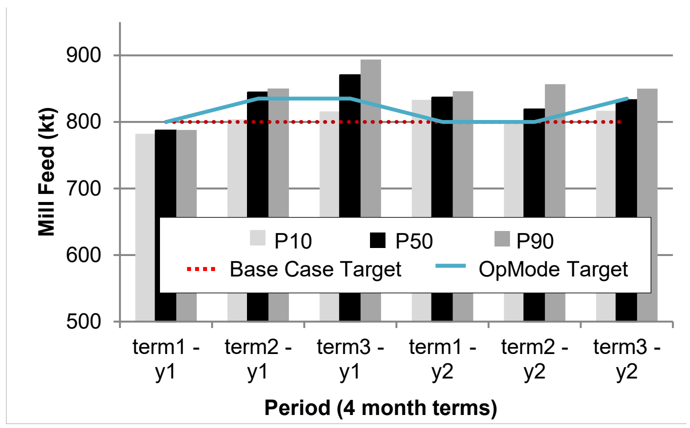


Figure 5-7 Risk analysis over SM feed for the mid-term discretization of the first two years with operational alternatives

This mechanism allows the mining complex to increase the mill feed in those periods and minimize deviations from targets. It must be noted that as the branching occurs on period 4, and the mid-term analysis is done only over the first two periods, this mid-term plan is common for both branches of the production plan (as are all other decisions for the first three years of production).

These operational and investment alternative decisions result in the sulphide mill feed shown in Figure 5-8, where, as in Figure 5-7, the initial target is presented in dotted red line, and the target adapted by the operational mode is presented in continuous blue line. The risk analysis (P10, P50, and P90 values) of the mill feed is also shown, which show that the optimizer does a good job at following the dynamic target in both cases, presenting a tight risk profile with minor deviations mostly on the last five years of the life of mine.

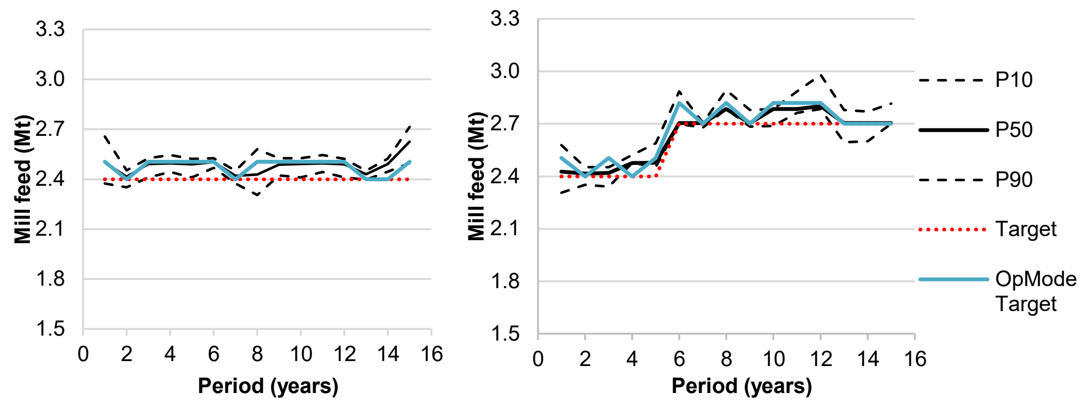


Figure 5-8 Annual SM feed for each branch of the production plan solution (without secondary crusher (left) and with secondary crusher (right))

The left side of Figure 5-8 shows that, even though this branch did not invest in a secondary crusher, the production schedule is using the mill's operational mode flexibility to increase its throughput in most periods, without any cost on investments. On the other hand, the branch with a secondary crusher (right of Figure 5-8) also decides to increase the mill's processing capacity by using the operational modes in some periods, producing relatively tight risk profiles except on the final 3.4 years of production.

The solution obtained from the proposed formulation presents a net present value (NPV) with a P50 of MUS$1460, a P10 of MUS$1320 and a P90 of MUS$1580. The full cumulative discounted cash flow distribution for the dynamic formulation is presented in Figure 5-9, together with the base case (presented next).

5.3.2.2 Comparison to the two-stage stochastic formulation

Results for the two-stage stochastic formulation of the mining complex are presented in this section, where extraction decisions are first-stage decisions, and processing stream decisions are considered second-stage. This case not only ignores the dynamic algorithm presented in the previous section, but also removes all investment and operational mode alternatives from the model; in particular, this corresponds to the second term of the objective function (Eq. 4.2), as well as all investment and operating mode effects and decisions from the set of constraints.

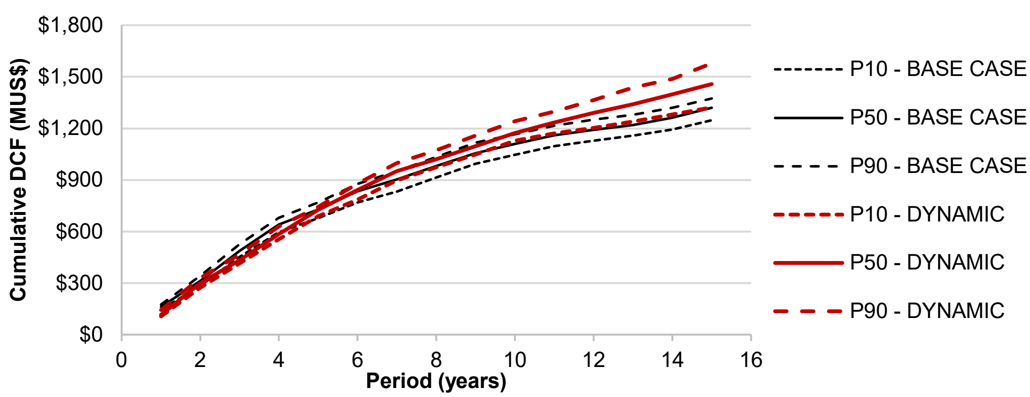


Figure 5-9 Cumulative net present value for the base case (black) and the proposed dynamic formulation (red)

As there are no CAPEX alternatives considered, it is assumed that the mine has a constant extraction capacity of 14Mt per year (i.e., a constant fleet of 4 trucks).

The same way, the mill is assumed to have a constant processing capacity of 2.4Mt per year. Results from this optimization are presented in Figure 5-10, which shows the risk analysis of the annual sulphide mill feed (left), the extracted material (middle), and the cumulative discounted cash flow (right), with P10, P50, and P90 values presented for each case.

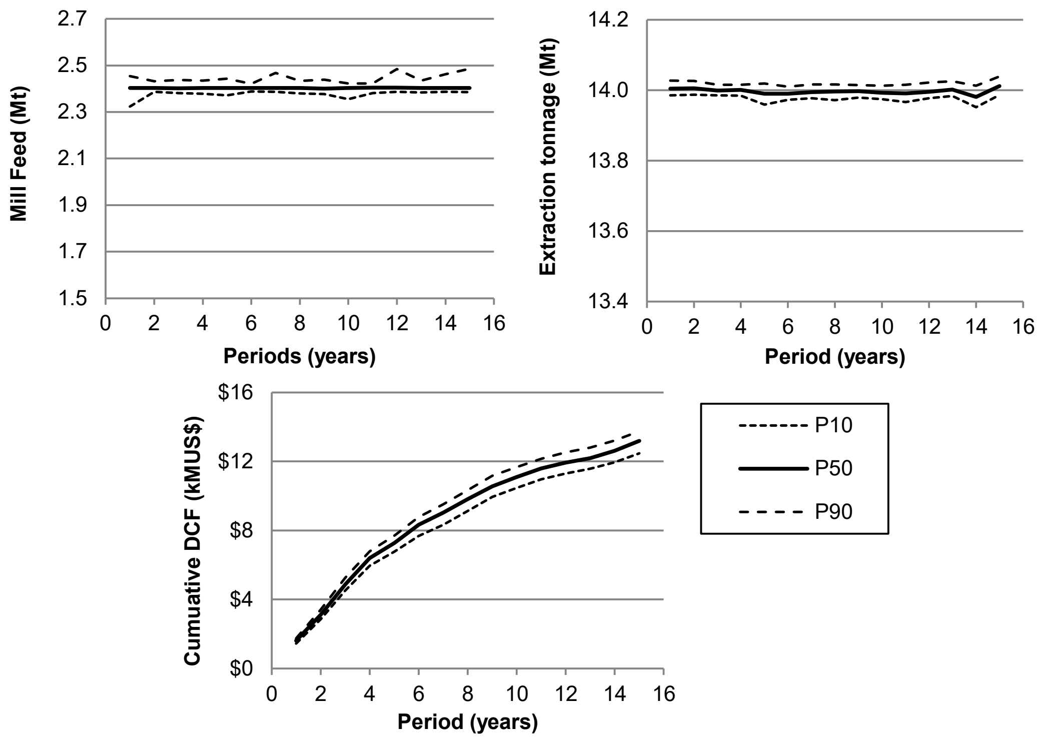


Figure 5-10 (a) Sulphide mill feed (top-left), (b) extraction tonnage (top-right), and (c) NPV (bottom) for the base case two-stage optimization without alternatives

The mine production and extraction plans (top left and right graphs respectively) present very controlled risk, with minimal deviation from production and extraction targets (present mostly at the last three years). The obtained results show that two-stage stochastic optimization can provide production plans that control and manage risk. The cumulative discounted cash flow (bottom graph) presents a NPV distribution with a P50 of MUS$1320, a P10 of MUS$ 1,246, and a P90 of MUS$1,374. Figure 5-10 compares these results, with the ones obtained on the proposed dynamic formulation.

5.3.2.3 Discussion

The two-stage approach presented in the previous section can manage and control risk. However, it assumes that the future is fixed, and does not capitalize regarding value, or takes advantage of changing environments and new information. This value is accounted for in the dynamic formulation by allowing the optimization process to adapt to change by introducing flexibilities in the form of operational modes and dynamic investment alternatives. This can be seen by comparing the different NPV percentiles of both cases in Figure 5-10, where the dynamic model presents a 10.5% higher NPV regarding P50, but almost a 15% higher P90. The obtained results show that the dynamic formulation maximizes value, but, furthermore, provides a production plan that can take advantage of opportunities and capitalizes on the project's possibilities of adapting.

5.4 Conclusions

This paper presents a dynamic two-stage stochastic mixed integer nonlinear programming formulation for modelling and optimizing a mining complex. Mining complexes are value chains where extracted rock from different sources is transformed into sellable products through a set of processing streams. This value chain is governed by uncertainties at different levels, from the geology of the orebody at the mine to the different operational and processing components that lead the sellable products to the market. The presented model aims at considering possible flexibilities in the mine production schedule by including alternatives over capital expenditure investments and operational modes, at different levels of the value chain. More specifically, a dynamic decision-making mechanism is included, where the mine production plan is allowed to branch, and parallel solutions are designed if a representative proportion of geological stochastic simulations agree it is profitable. This model extends from a multistage formulation and prevents the model from producing over-fitted solutions to the set of stochastic simulations used. The proposed method setts a representativity threshold that controls the branching mechanism, and thus, the final solution provides a controlled set of possible mine-plan alternatives that have been probabilistically quantified to be worth considering. This process generates new optimized plans that allow and ease the process of adapting once more information is available.

Due to the size and complexity of the proposed formulation, exact solvers such as CPLEX are unable to provide any solution. Thus, an adaptive multi-neighbourhood simulated annealing metaheuristic is used, which can solve complex, non-linear problems, producing good quality solutions in a relatively short amount of time.

The practical implications of the proposed method are demonstrated through an application over a copper-gold mining complex comprised of one mine and six processing streams. Here, the dynamic model is compared to a traditional two-stage stochastic formulation, presenting a 10.5% increase in net present value in terms of P50, and a 15% higher NPV on a P90 level.

Two main lines of research are proposed as future work; first, in testing new, more sophisticated solving mechanisms such as hyper-heuristics, which can adapt better to solve different mining complex configurations, without the need of setting up the necessary parameters required in the metaheuristic used here. Second, focus on testing the proposed method in a bigger case study with multiple mines, and more than one branching alternative acting simultaneously.

CHAPTER 6

General Conclusions and Future Work

6.1 Conclusions

A mining complex is a mineral value chain formed by interconnected components consisting of a set of mines, stockpiles, waste dumps, processing streams and transportation to final customers. All these components strongly depend on each other and, thus, they must be simultaneously optimized to account for the synergies that exist between them. Simultaneously optimizing the different components of a mining complex under grade and material type uncertainty has been a topic of research of the past years. However, because of these uncertainties, assuming that the setting and production needs of a mining complex will remain unchanged for the life-of-mine, and thus, that the initial optimized solution will remain optimal is a strong limitation of current stochastic simultaneous optimization models. Existing mechanisms to evaluate flexibility have been argued to be useful in their ability to calculate a more realistic range of values of a strategic mining complex plan; however, they fail in focusing on the actual viable feasibility of the designs produced. Thus, if the mine designs and production infrastructure are not feasible, the production forecast and economic valuations will be unreliable.

This thesis presents a methodology to embed flexibility into mineral value chains, by allowing the strategic mine plan of a mining complex to dynamically consider possible options and alternatives for reacting and adapting to future changes. The proposed dynamic optimization mechanism is able to consider a set of feasible flexibility alternatives that will ease the ability of a mining complex to react and adapt to a changing environment, while meeting production targets, and producing production schedules that are feasible at an operational level. This is a challenge for strategic planning, due to the different operational requirements that must be considered to produce feasible mine plans. In addition, these extensions substantially increase the size of the optimization model, requiring the development of more sophisticated metaheuristic mechanisms to obtain good quality solutions in a reasonable amount of time.

Chapter 2 presents a study to actively consider investments and capacity optimization of a mining operation in the stochastic integer programming model, accounting for the different variables that affect equipment acquisition, such as costs, lead time and life of equipment. The method developed is able to optimize the equipment purchase plan under geological uncertainty to minimize costs while meeting production targets. Results from an application at a gold mine presented an increase of 20% in net present value, caused from optimizing ore and waste extraction through equipment acquisition plan across the life of mine, delaying waste extraction towards later periods. This chapter contributes a new mathematical model that realistically includes capital expenditures in the optimization of a strategic mine plan, effectively optimizing the extraction capacity of a mining operation. The actual extraction sequence is not optimized in this case, and the ultimate pit limit is assumed fixed. Together with this, the case study is limited to the basic case of a mining complex with a single mine and one processing plant.

In Chapter 3, the destination policy of a multi-element mining complex is studied, aiming at ensuring that complex blending constraints are met by considering the uncertainty related to the grade and material type of the extracted rock, in addition to other rock properties and processing parameters, such as hardness, throughput, and recovery (also referred to as geometallurgical variables). Here, a fixed schedule is defined, and multi-variate destination policy is generated, which simultaneously accounts for all the variables of interest, and aims at meeting complex blending constraints, while maximizing project value. The method proposed uses coalition formation clustering, a method developed in game theory, to create families of mining blocks which should be processed together due to their interrelated characteristics. Thus, the aim is to meet blending constraints not only by a mining block's own properties, but rather by the joint properties of the group of blocks being processed together, such as their resulting grades of blending elements and sellable metals, simultaneously. At the same time, the coalition clustering is able to deal with both quantitative and qualitative variables, enabling the extension of the method to consider other more complex rock properties. An application at a copper-gold mining complex shows that, for a fixed schedule, deviations from blending constraints and arsenic limits are significantly reduced by up to 30%, while maintaining the level of metal production, and ultimately increasing the net present value by about 6%. The method developed is shown to be highly effective, but for a fixed defined schedule. Reformulation efforts should be placed to expand the model to include the optimization of the full mining complex scheduling problem. These efforts should focus on reducing the computational complexity of the model, as its high computational cost makes it prohibitive to integrate into the simultaneous stochastic optimization of a mining complex.

Chapter 4 extends the model presented in Chapter 2 to optimize the mine plan and extraction schedule of a mining complex, together with the capital expenditure plan, implementing a dynamic mechanism to account for the flexibility of investing on high-impact capital expenditures into the optimization model. The new dynamic formulation proposed is based in multistage stochastic programming, and is solved with an iterative mechanism which resembles the attainment of new information throughout time. This model aims to maximize the net present value of a mining complex and increasing its potential to take advantage of new opportunities, while hedging from risk. Along with the extraction sequence and processing stream, all the components of the mining complex are optimized simultaneously, considering geological uncertainty, as well as a fleet purchase plan, consisting of trucks and shovels that define the operation's extraction capacity, and the option to invest in a secondary crusher to increase processing capacity at the mill. At the same time, the model limits the number of parallel possible schedules developed by ensuring that only investment alternatives that have a representative probability of being profitable are considered, easing the application of the obtained solution plan into a real-life operation. The model is applied at a copper mining complex consisting of two open pits, an oxide leach pad, a sulphide mill with a related stockpile, and a waste dump. Investments over trucks, shovels, and a secondary crusher are included in the optimization model, showing substantial improvements when compared with the standard two-stage stochastic simultaneous optimization of the mining complex, increasing the expected NPV by 4%, corresponding to over MUS$170, with a P90 increase of 11%.

Finally, Chapter 5 extends dynamic formulation presented in Chapter 4 to include operational alternatives, which are introduced to increase the capacity of the optimization to manage the effect of different geometallurgical variables over the mining complex's performance, increasing project value while meeting complex blending and processing constraints, oriented at large-scale deposits. These operational modes are considered at different levels of the mining complex, adapting the blasting pattern at the mine level to increase crushability at higher mining costs, dealing with rock hardness and throughput, as well as at the processing plant level, increasing throughput by punishing metallurgical recovery. These operational alternatives are combined with the investment alternatives in the dynamic model described in Chapter 4, to fine-tune the mining operation for the different possible futures contained in the strategic plan. A solving algorithm is developed, which is iteratively applied, mimicking the sequential acquisition of information obtained with time, as well as the decisions that have already been taken and fixed. The proposed model is applied at a copper-gold mining complex, with alternatives to purchase trucks to define the extraction capacity, as well as a high-impact investment of a secondary crusher to increase the processing capacity at the sulphide mill. Together with this, both operating alternatives mentioned before are included at the mine and plant level. Results show that the option of investing on a secondary crusher has over a 40% probability of being profitable, providing a strategic plan that presented over 10% higher net present value.

6.2 Future Work

There are multiple avenues for future research from the work presented in this thesis. The alternatives considered in the developed model relate mainly to the optimization of capacities of a mining complex, at both the mine and processing level, however, other type of alternatives could be studied, such as investing in integrating different transportation systems within the mining complex, enabling, for example, the connection of all sources of material (mines) to all the different processing streams, in case they are not already in place. In addition, extending the study to decide between competing alternatives which cannot be active simultaneously, such as expansions of different capacity, different transportation systems in a limited space, or investments at different levels of the mining complex under a limited capital context, evaluating the relative impact of each, to choose the optimal one in terms of timing and available capital. The proposed dynamic model can aid the mineral value chain to timely and efficiently shift towards a more efficient operation, evaluating which alternatives would be more profitable and effective to consider, easing the transition and maintaining productivity. This extension is readily obtainable with relatively minor adaptations to the current formulation, but the options considered must be studied within the set of feasible possibilities that can be implemented, according to the configuration, characteristics, and context of the given mining complex.

The solving mechanism employed in this thesis can be further improved. The results obtained are promising, but the adaptive multi-neighbourhood simulated annealing metaheuristic used is time consuming, and the selection and tuning of parameters required by the algorithm is a tedious task based mostly in a trial an error procedure. New hyper-heuristic algorithms have shown to be effective and efficient, especially in solving complex models, such as the global optimization of a mining complex. These algorithms consist of two layers of metaheuristics, where the first is an internal layer, which is iteratively used to select an efficient metaheuristic from the second layer to actually perturb the model; this second layer includes of a set of different simple metaheuristics. This procedure is referred to as a "smart solving mechanism", as the initial layer adapts its probability of choosing a given metaheuristic from the second layer according to its historical performance, and to the type of neighbourhood being perturbed, and because of this adaptability, the parameters required are minimal. A smarter solving mechanism could prove to be highly beneficial in obtaining a faster convergence of the solution, reducing computational times as well as limiting the subjective procedure of parameter selection.

Finally, extending the formulation to optimize and include operational alternatives at a short-term production scheduling could also result in interesting results, affecting the dispatch plans, expanding ore control testing in areas of mixed material, or affecting the blasting pattern on a zone-based definition, and not globally per period, as done in Chapter 5. Especially in the face of new on-line systems receiving real time data, being able to plan for changes is crucial to take full advantage of new information and adapt to it. Machine learning techniques have proven to be highly effective in dealing with real time decision-making; however, a dynamic multistage model could prove beneficial to work as a link between the adaptive short-term and related long-term plan. It must be noted that short and mid-term mine planning entail multiple operational and logistic constraints that are not included in the current strategic long-term model, and thus, an in-depth reformulation of the mathematical model would be needed. Additionally, there are multiple uncertainties governing short-term operation, such as the availability and utilization of the equipment, which should be accounted for. However, these extensions considerably increase the computational cost of the formulation, and thus, more efficient solving mechanisms must be developed.

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