Six-Dimensional Supergravity Braneworlds and the Cosmological Constant

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Abstract

We review the lore of effective field theories as a background to hierarchy problems in general and the cosmological constant problem in particular. We outline some of the attempted four-dimensional solutions to the cosmological constant problem and conclude that ones based upon the usual assumptions of four-dimensional field theory typically do not work. We argue that one way to relax the assumptions is to seek solutions to the cosmological constant problem which rely on the presence of extra dimensions. We explicitly exhibit that standard compactification techniques fail to solve the cosmological constant problem because they reduce the problem to a four-dimensional one.

We argue that brane-world models may be helpful in solving the cosmological constant problem because standard model loops contribute to the tension and not to the vacuum energy directly, and can fulfill our stated aim of constructing a model which *uses* the extra dimensions to mitigate the cosmological constant problem. We identify necessary (not sufficient) properties a theory must possess to successfully use this observation. These properties are: a scaling symmetry encoded in a dilaton-like scalar, and bulk supersymmetry.

We therefore investigate supersymmetric six-dimensional brane-world models. Our models are imbedded within a 6D supergravity that has many of the features of realistic string models. We explicitly show that the compactification of the 6D theory has many of the same features as string compactifications, including flat four-dimensional space, chiral fermions, moduli, moduli-stabilisation using fluxes, and gluino condensation. We show that by calculating the non-perturbative correction to the superpotential and loop-corrections to the Kähler function that a meta-stable deSitter vacuum can be found. The vacuum energy can be tuned to be $\sim 10^{-6} M_{\text{Planck}}^4$.

We find that *all* solutions of the supergravity equations of motion, under a symmetry *ansatz*, have flat branes. This implies that this property is independent of some of the details of the branes, such as their tensions. The source of the branes' flatness is the required classical scaling symmetry of the action.

We consider whether this class of models may provide a solution to the cosmological constant problem within the large extra dimensions scenario, in which the radius $r \sim 0.1\text{mm}$, and in which the standard-model fields are trapped on a 3-brane. We conclude that it may be possible to produce naturally a cosmological constant that is of order $r^{-4} \sim \left(10^{-3}\text{eV}\right)^4$ due to loops because the supersymmetry-breaking scale in the bulk is $M_{\text{SUSY}} \sim r^{-1}$, although there remains a great deal of work to be done. We comment on recent extensions to cosmological backgrounds.

Further work within these models is outlined, including higher-dimensional models, use of effective field-theory techniques in theories with sharp boundaries, and the treatment of quantum corrections.

Resumé

Nous passons en revue les théories effectives des champs en tant que toile de fond pour les problèmes des hiérarchies en général, et plus particulièrement pour le problème de la constante cosmologique, arrivant à la conclusion que celles qui sont basées sur les suppositions habituelles pour les théories des champs en quatre dimensions ne fonctionnent pas. Nous argumentons qu'une manière de relaxer ces suppositions consiste en la recherche de solutions au problème de la constante cosmologique qui font appel à la présence de dimensions supplémentaires. Nous démontrons de manière explicite que les méthodes standard de compactification ne résolvent pas le problème puisqu'elles ne font que le transcrire en un langage quadri-dimensionnel.

Nous faisons l'argument que les modèles branaires peuvent aider à résoudre le problème de la constante cosmologique puique les boucles provenant du Modèle Standard contribuent à la tension et non pas directement à l'énergie du vide. Ils peuvent donc remplir l'objectif enoncé qui est de construire un modèle *qui fait usage* de dimensions supplémentaires pour atténuer le problème de la constante cosmologique. Nous identifions des propriétés nécessaires (mais non suffisantes) que doit posséder une théorie pour être en mesure de faire usage de cette observation. Ces propriétés sont: une symétrie d'échelle encodée dans un champ scalaire de type dilaton, ainsi que la supersymétrie dans le volume total.

Nous étudions donc des modèles branaires supersymétriques en six dimensions. Nos modèles sont plongés dans une théorie de la supergravité en six dimensions qui possède plusieurs des caractéristiques des modèles réalistes en théorie des cordes. Nous démontrons explicitement que la compactification de la théorie à six dimensions partage plusieurs aspects avec les compactifications en théorie des cordes, notamment un espace quadri-dimensionnel plat, des fermions chiraux, des moduli, la stabilisation des moduli à l'aide de flux, et la condensation des gluinos. Nous montrons par un calcul non-perturbatif de la correction au superpotentiel et des corrections des boucles à la fonction de Kähler qu'un vide deSitter meta-stable peut être realisé. L'énergie du vide peut être ajustée pour donner $\sim 10^{-6} M^4_{\text{Planck}}$.

Nous trouvons que *toutes* les solutions des équations du mouvement de la supergravité, sous un certain ansatz symétrique, ont une géométrie branaire plane. Ceci implique que cette dernière propriété est indépendente des détails de la brane, telles que sa tension. La planarité de la brane provient de la symétrie d'échelle de l'action.

Nous considérons si ce type de modèle peut mener à une solution au problème de la constante cosmologique dans le contexte d'un scénario comportant des grandes dimensions supplémentaires, dont le rayon $r \sim 0.1$mm, et dans lesquels les champs du Modèle Standard sont restreints sur une 3-brane. Nous arrivons à la conclusion qu'il peut être possible de produire de fa con naturelle une constante cosmologique de l'ordre de $r^{-4} \sim \left(10^{-3}\text{eV}\right)^4$ grâce aux boucles puisque l'échelle à laquelle la supersymétrie est brisée dans le volume total est $M_{\text{SUSY}} \sim r^{-1}$, quoiqu'il reste encore bien du travail à accomplir. Nous commentons certaines extensions récentes à un contexte cosmologique.

Nous décrivons des pistes de travail additionnel dans le contexte de ces modèles, incluant des modèles avec un nombre plus élevé de dimensions, l'utilisation des techniques des théories des champs effectives dans des théories possédant des conditions aux frontiéres hautement définies, ainsi que le traitement des corrections quantiques.

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The most important acknowledgement is of my supervisor, Cliff Burgess. He has been a mentor, a teacher and a friend. His depth of knowledge is an inspiration; his excitement towards physics and research are contagious; his kindness and generosity towards his students is unparalleled. It is a debt not easily repaid.

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Without my family I cannot have been in the position to think about doing a PhD. My mother, Judi, taught me to love to learn, bought books even when I didn't want to read, and provided all of the support anyone could want as I started to study. Arshin has been my best friend since he was born, and without him to show me what is really important in life I don't know where I would be. Your strength and your laughter are a constant inspiration to me. Hossein has always been a guide, even from a distance, helping me to achieve.

My friends have helped me keep my sanity: Dave and I discovered Montréal together, to our mutual detriment; Joel has helped me to understand myself better; Neil is always good for a drink, and this, if nothing else, is the mark of a good friend; Pat's conversation is always illuminating, and through counterexample he has helped me realise just how comparitively simple life can be; Amps' facility for engaging conversation is surpassed only by his considerable facility for culinary appreciation: what more could one want in a friend? And Caroline, whose converation makes time fly, is engaging, is challenging, and above all, is so much fun! Here's to *Bourbaki, Newman, Cage: A nightmare for a plate in G#-major.*

The position of honour in these acknowledgements, however, and my very deepest gratitude, is reserved for Nina, my wife, and my love. For always knowing the right thing to say, for always making me laugh, and for being with me, I am forever thankful. Without her I would certainly not have made it through graduate school (mostly) unscathed. Through doubt, disappointment and elation, you have always been there. I can only say, I love you, and thank you!

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As it began, so it ends:

The physicist does not study nature because it is useful,

he studies it because it is beautiful.

If nature were not beautiful, it would not be worth studying,

and if nature were not worth studying,

life would not be worth living.

-Henri Poincaré

Contents

Statement of Original Contributions

The work presented in this Thesis is based on the publications [147, 176, 177], and heretofore unpublished extensions of these papers. All of these papers were collaborative projects involving my supervisor Cliff Burgess, and our collaborators.

Chapters 1–2 are summaries and interpretations of introductory material and the field, along with a few results of my own. There is little original content here; they are naturally strongly influenced by numerous references and by conversations with Cliff Burgess and by his publications.

Chapter 3 contains well-known results in compactification, derived in my own peculiar fashion. More substantively, this chapter contains the details of the compactification performed in [147]. I was involved in all aspects of this work. I helped to establish the four-dimensional supersymmetry transformation rules, and hence the compactification; with guidance from Cliff Burgess, I was responsible for identifying the symmetries responsible for the massless four-dimensional degrees of freedom; I was involved in understanding the issues involved with six-dimensional anomaly-cancellations; and, with guidance from Cliff Burgess, I was soley responsible for performing the analysis of the minimum of the loop- and nonperturbatively-corrected scalar potential.

Chapters 4–6 are based on the work in ref [176] and ref [177]. I was involved in all aspects of this work. I was responsible for finding the generalisation of the sphere-compactification of the Salam-Sezgin supergravity, for determining the supersymmetry of the resulting solution, and for checking the calculations of other members of the collaboration. Susha Parameswaran and I together understood the topological issues involved on manifolds with singularities. Cliff Burgess and Fernando Quevedo were responsible for providing the motivation, interpretation and most of the quantum and string arguments presented, especially as regards the cosmological constant.

In ref. [177] I found and presented the argument tying together the classical scale-invariance of the action and the vanishing of the cosmological constant, which clarified the source of self-tuning in these solutions. I explicitly checked the self-tuning of the solutions and was involved in nearly all of the content of section 4.

The work in the last chapter on higher-order operators and boundary conditions is motivated by work Cliff Burgess and I did together, but is my own. The Appendix "Warped Metrics and Conditions for Flat Metrics" is my own work. It is an extension and generalisation of the conditions found in [177] for brane-flatness in supergravity-like theories.

Chapter 1

The Rise and Fall of the Standard Model

The standard model of particle physics [1] is the most successful and comprehensive model of particle interactions ever constructed, and has been tested to unprecedented accuracy. Our understanding of the standard model underpins our understanding of nearly every aspect of modern physics [2].

In addition to being the means for understanding particle physics, the standard model also underpins our understanding of non-gravitational particle interactions throughout the entire history of the universe after it cooled below about a TeV. The standard model reproduces the correct light-element abundances (nucleosynthesis) from a single input parameter, the baryon density at the big bang; it correctly describes the conversion of these light elements into heavier elements in stars; and it correctly describes the supernova explosions that are responsible for distributing these heavy elements throughout the galaxy, and which are ultimately responsible for providing the correct conditions for life (please see [65] and references therein).

Despite the standard model's phenomenal success, it is known to be incomplete [198]. This is not to say that the standard model is incorrect, in fact it is almost certainly correct at low energies—given the degrees of freedom and their charge assignments, the standard model (without the symmetry-breaking sector) is the unique theory describing the interactions of three families of leptons and three families of quarks interacting through the gauge group $U(1) \times SU(2) \times SU(3)$ spontaneously broken to $U(1) \times SU(3)$ [198].

Furthermore, our knowledge of the standard model's limitations doesn't imply that we know precisely how it will fail, for then we would know a great deal about physics beyond the standard model. What we *do* know is that the standard model is provisional in a particular, very well-defined sense: there is an energy, $M_{\text{newphysics}}$, above which the standard model is no longer an accurate description of physics. The standard model is an *effective* description of physical phenomena at energies far below this scale [198].

What value $M_{\text{newphysics}}$ takes is the subject of a great deal of the research being done at the forefront of modern theoretical physics. Everyone seems to agree that $M_{\text{newphysics}}$ is almost certainly below M_{Planck}, the scale at which the quantum effects of gravity become important [53]. It is believed by the majority of the theoretical physics community that at or below energies $\mathcal{O}(M_{\text{Planck}})$ standard field-theoretic descriptions of physics break down, and a new theory, string theory, provides a valid description, but this scale is so impossibly high that direct measurement at a particle accelerator is beyond the reach of any currently-conceivable technology. Nevertheless, by being the only viable theory of quantum gravity, string theory has forced the physics community to challenge seemingly obvious assumptions, such as the four-dimensionality of space-time [53].

There is hope, however, for finding new physics below the Planck scale; there is compelling evidence that the standard model is replaced or modified at energies as low as a few TeV— within the range of experiments planned to begin at the LHC in a few years. The most obvious and popular contender for a modification of the standard model at this scale is weak-scale supersymmetry [3], but it is now possible to construct string-like models (including the brane-models in the mix) with $M_{\text{string}} \sim \mathcal{O}(\text{TeV})$ [66, 67, 136, 176, 177, 178, 180].

Of course, no argument for a particular extension of the standard model will be perfect, since we currently have very few ways of experimentally differentiating between extensions (an unhappy circumstance that will hopefully change once the LHC begins taking data). The best I hope to do in this introductory chapter and in the next is to explain why we believe so fervently that the method of effective field theories, and in particular the standard model, is almost certainly correct, and how we have come to the conclusion that there is likely new physics on the horizon, with $M_{\text{newphysics}} \sim \text{TeV}$. We will see that the major theoretical motivation for new physics is the presence of hierarchy problems in the standard model, and this will help explain why theoretical physicists have taken the drastic steps of introducing supersymmetry, extra dimensions, and branes. [1]

[1] A brief explanation for the time being: A brane is a $3+1$-dimensional submanifold of a higher-dimensional space (like a piece of paper floating in the middle of a room, except infinite in extent). Branes find their original motivation in string theory but have an independent phenomenological existence in brane-world models. A supergravity is a way of coupling general relativity to a particular kind of particle physics, and supergravities are low-energy approximations to string theories. The 'super' in supergravity comes from the supersymmetry of the theory. Supersymmetry is a symmetry that ties together the bosons and fermions in a theory in a particular way that softens their ultra-violet behavior.

During our discussion of the hierarchy problem we will see that hierarchies have traditionally been solved by postulating (and subsequently verifying the existence of) new symmetries. This will set the stage for the cosmological constant problem, for which no four-dimensional solution is known: no postulated four-dimensional symmetry is known which is both consistent with observations and which successfully explains the small size of the cosmological constant. Following a review of Weinberg [122] I catalogue some of the main contenders for four-dimensional solutions and explain briefly why none of them are adequate. The conclusion to draw from this study may well be that, regardless of how well effective four-dimensional field theories have served us to date, perhaps the ideas embodied therein simply don't apply to the cosmological constant problem.

This thesis takes what may be considered a relatively conservative view: we preserve the tenets of effective field theories and the usual vacuum-selection criteria, but we expand our space of allowed theories to extra-dimensional theories with branes. This is a hypothesis, born of string theory, that has a great deal of currency in the theoretical physics community. The main point of this thesis will be the presentation of a class of models, which, while preliminary, seem to have the relevant features for an extra-dimensional solution to the cosmological constant problem. It must be emphasised: these are preliminary models, and much can yet go wrong with them (as has with every other proposal for a solution to date). There are many usual suspects, which have been the downfall of many solutions to the cosmological constant problem, and a large part of the work involved is in the results of these interrogations. These are presented in Chapter 6, the punchline being that this class of models have passed many nontrivial checks and warrant further study. (But see [205].)

1.1 Effective Field Theories and the Standard Model

The standard model is an effective theory of non-graviational particle physics at energies below a few hundred GeV. It represents the culmination in an extremely successful model of a way of thinking about particle physics, begun by Gell-Mann and Low [18], and developed further by many authors, prominent among them Weinberg and Wilson [13, 14, 197, 16, 17].

Quantum field theories are *the natural way to describe low-energy, quantum-mechanical, Lorentz-invariant particle interactions.*

The reason such broad, general statements can be made is that quantum field theories are based on a set of 'motherhood principles' [197], each of which is extremely well-tested, at least at low energies. *Effective field theories* such as the standard model are quantum field theories which arise from the passage to a low-enegy approximation from an underlying, more fundamental theory. Because they are quantum field theories they rest on the strength of the motherhood principles, which makes them extremely general; because they are effective theories they are simplified descriptions of only the relevant degrees of freedom. The usefulness of effective field theories lies in our ability to perform calculations with controlled theoretical errors, *even when we don't know the underlying theory* from which the effective theory is derived.

The extension of the idea of quantum field theories to the concept of effective field theories explained the prominence of *renormalisable* quantum field theories, and, even more importantly, showed how one could reliably calculate loop-corrections to non-renormalisable theories [12].

Effective field theories are understood to be expansions in powers of energy over some higher mass scale E/M. Once one picks the accuracy with which a given calculation needs to be performed, one may keep the appropriate numbers of operators in order to achieve this accuracy [8], whether or not corrections are made to higher-order operators. In this way sensible, controlled calculations can be made with non-renormalisable theories [197, §12.3].

In the next few sections I will present some of the principles on which are based our belief in quantum field theories in general, and effective field theories in particular. I will work through three examples, each of which illustrates different features of the effective field theory picture. The first example proves in a matter of a few lines that the sky is blue. This example prominently displays the ease with which conclusions may be drawn using effective field theories, and simultaneously shows that the methods may be used in real life situations. The second example illustrates the direct calculation of an effective field theory through the 'integrating out' of heavy degrees of freedom. This process will naturally exhibit the two hierarchy problems in the standard model, the stabilisation of scalar masses and the cosmological constant problem. The last example is an exhibit in the strength of symmetries in effective field theories; it shows how hierarchy problems can be solved through the use of symmetry.

1.2 Quantum Fields

Quantum field theory is a general framework for describing the interactions of quantum particles in situations where particle number is not conserved. The apparatus of quantum field theory is general enough to be applied equally well to thermal systems and to relativistic high-energy particle interactions.

One quick way to get to quantum fields is to use electrodynamics as a jumping-off point. This is a field theory and one may wonder how one should quantise such things. Using this as a template, we guess that a typical quantum field theory contains fields, such as scalars, $\phi(x)$, spinors, $\chi(x)$ and vectors, $V_\mu(x)$, all interacting through a hamiltonian density, *e.g.*,

$$\mathcal{H}_{\rm int}(x) = g\phi\bar{\chi}\chi + V_\mu\bar{\chi}\gamma^\mu\chi + \frac{\lambda^4}{4!}\,\phi^4 + \cdots \tag{1.1}$$

The hamiltonian is constructed as an integral of this density,

$$H = \int d^3\mathbf{x}\,\mathcal{H}(x). \tag{1.2}$$

In order to promote this theory to a quantum field theory we should promote the fields, ϕ, χ and V_μ to operator-valued functions which act on a Fock space. The properties of these fields are most easily derived in the canonical formalism, in which one posits a Lorentz-invariant lagrangian density and adapts the usual Poisson-bracket $\leftrightarrow$ commutator quantisation trick to a field theory: Let $\mathcal{L}(\phi, \dot{\phi})$ be a lagrangian density for a field theory of a collection of fields, $\{\phi\}$. The canonical momentum is given by

$$\Pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \tag{1.3}$$

and we assume the equal-time canonical commutation relations,

$$[\Pi(t, \mathbf{y}), \phi(t, \mathbf{x})] = i\,\delta^3(\mathbf{x} - \mathbf{y}). \tag{1.4}$$

It is often convenient to re-write the fields and the canonical momentum in terms of creation and annihilation operators which are arbitrary (operator-valued) coefficients in the solution to the linearised equations of motions arising from the lagrangian. For a real scalar field, for example, this expansion would take the form

$$\phi(x) = \int \frac{d^3\mathbf{p}}{\sqrt{2p^0(2\pi)^3}} \left[a_{\mathbf{p}} e^{ip\cdot x} + a^*_{\mathbf{p}} e^{-ip\cdot x}\right]. \tag{1.5}$$

The coefficients are chosen to make the Lorentz-transformation properties of a and a^* simple. After applying eq. 1.4, we find that a and a^* satisfy the Heisenberg algebra:

$$[a^*_{\mathbf{p}}, a_{\mathbf{p}'}] = \delta^3(\mathbf{p} - \mathbf{p}') \tag{1.6}$$

This is enough to construct states, to do perturbation theory, and to ask questions about scattering of particles. While this presentation works, it is hardly illuminating.

Below I will present a different point of view, due primarily to Weinberg [197], which clearly expresses the inevitability of the framework of quantum fields for describing physical phenomena. This point of view shows that quantum field theory is the expression of a set of motherhood principles which are so well-tested as to be nearly certainly true, at least at energies that are currently accessible to us.

1.2.1 Motherhood Principles

The apparatus of quantum field theory is based on 'motherhood' principles [197]:

- Quantum Mechanics

- Locality
- Poincaré Invariance
- Cluster Decomposition

Quantum Mechanics

Quantum mechanics sets the stage, in terms of a Hilbert space to be populated with states and acted on by a unitary evolution operator:

$$U = e^{-iHt}. \tag{1.7}$$

Locality

Locality of the interaction implies that particles interact at a single spacetime point. Under this assumption, the hamiltonian must be written as the integral over space of a hamiltonian density, $\mathcal{H}(x)$,

$$H = \int d^3\mathbf{x}\,\mathcal{H}(x). \tag{1.8}$$

Lorentz Invariance

The theory is Lorentz-invariant if the same unitary operators perform Lorentz transformations on both 'in' and 'out' states; this is what we mean by a Lorentz-invariant S-matrix. This condition is guaranteed if (but not only if) $\mathcal{H}$ forms a scalar in the following sense:

$$U(\Lambda, a)\mathcal{H}(x)U^{-1}(\Lambda, a) = \mathcal{H}(\Lambda x + a), \tag{1.9}$$

and if in addition

$$[\mathcal{H}(x), \mathcal{H}(x')] = 0 \qquad \text{for } (x - x')^2 > 0. \tag{1.10}$$

(Although this may look incorrect at first glance it is. Consider for example $\mathcal{H}(x) = \frac{1}{2}(\partial_0\phi)^2 + \frac{1}{2}\partial_i\phi\partial_i\phi$,:

$$U(\Lambda)\mathcal{H}(x)U^{-1}(\Lambda) = \frac{1}{2}(\partial_0'\phi(x'))^2 + \frac{1}{2}\partial_i'\phi(x')\partial_i'\phi(x') \equiv \mathcal{H}(x'), \qquad (1.11)$$

where $x' = \Lambda x$. Similarly, if $\mathcal{H} = i\bar{\psi}(x)\left(\gamma_0\partial_0 + i\gamma_i\partial_i\right)\psi$, and D is the spinor-representation of the Lorentz group, then

$$\begin{aligned} U(\Lambda)\mathcal{H}U^{-1}(\Lambda) &= i\overline{D\psi}(x')\left(\gamma_0\partial_0 + \gamma_i\partial_i\right)D\psi(x') \\ &= i\bar{\psi}(x')\left(\gamma_\mu\Lambda_0^\mu\partial_0 + \gamma_\mu\Lambda_i^\mu\partial_i\right)\psi(x') \\ &= i\bar{\psi}(x')\left(\gamma_0\partial_0' + \gamma_i\partial_i'\right)\psi(x').) \end{aligned} \qquad (1.12)$$

The last requirement above, that the hamiltonian densities commute for space-like separations ensures the Lorentz invariance of the time-ordering operator that arises in the manifestly covariant (modern) perturbative expansion. It can also be shown that these conditions guarantee Lorentz invariance non-perturbatively [197, § 3.5]. (It is usually easier to construct a Lorentz-invariant lagrangian density

$$U(\Lambda, a)\mathcal{L}(x)U{-}1(\Lambda, a) = \mathcal{L}(\Lambda x + a) \qquad (1.13)$$

and to derive the hamiltonian density from it.)

Lorentz invariance (or *co*variance) is also used to define particles. Define a single-particle state as a state of definite momentum which furnishes an irreducible representation of the Lorentz group.

$$U(\Lambda, a)|\mathbf{p}, \rho\rangle = D(\Lambda, a)_\rho{}^{\rho'}|\mathbf{p}, \rho'\rangle \qquad (1.14)$$

where ρ is a composite index enumerating the spin states and where $D(\Lambda, a)$ is a finite-dimensional (and therefore non-unitary) matrix representation of the Lorentz group:

$$D(\Lambda', a')D(\Lambda, a) = D(\Lambda'\Lambda, \Lambda' a + a') \tag{1.15}$$

Since energy and mass are interchangable in a Lorentz-invariant theory, single-particle quantum mechanics is not an adequate description in this setting. We require a Fock space constructed of the tensor product of all n-particle states:

$$\mathfrak{h} = |\mathrm{VAC}\rangle \otimes \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n \otimes \cdots \tag{1.16}$$

where $\mathcal{H}_n$ represents the Hilbert space of n-particle states. Multi-particle (free) states transform as the tensor product of single-particle states under Lorentz transformations.

We may define creation and annihilation operators, which move one between the different Hilbert spaces in the Fock space. An annihilation operator destroys a particle:

$$a_\sigma|\sigma_1 \cdots \sigma_n\rangle \propto \delta_{\sigma,\sigma_1} |\widehat{\sigma}_1, \sigma_2, \cdots \sigma_n\rangle + \cdots + \delta_{\sigma,\sigma_n} |\sigma_1, \sigma_2, \cdots \widehat{\sigma}_n\rangle \tag{1.17}$$

where σ is a composite label enumerating discrete states such as spin and particle type as well as continous labels such as momentum, and $\widehat{\sigma}_i$ means σ_i has been removed from the list.[2] The vacuum is defined as the state for which

$$a_\sigma|\mathrm{VAC}\rangle = 0 \qquad \forall\, \sigma \tag{1.18}$$

Creation operators do the opposite: they add a particle to the list:

$$a^*_\sigma|\sigma_1, \cdots \sigma_n\rangle \propto |\sigma, \sigma_1, \cdots \sigma_n\rangle \tag{1.19}$$

[2]We are assuming for simplicity that all the particles are bosons. Additional minus signs are necessary to correctly treat fermions.

Equivalently, creation and annhilation operators can be defined in terms of the Heisenberg algebra,

$$[a_\sigma^*, a_{\sigma'}] = \delta(\sigma - \sigma') \tag{1.20}$$

where the δ includes discrete as well as continuous labels.

Any hamiltonian density may be constructed from creation and annhiliation operators. This is because an operator is defined by its matrix elements between states and any value can be ascribed to any matrix element by a suitable choice of coefficient in a sum of terms such as

$$a_{\sigma_1}^* \cdots a_{\sigma_n}^* a_{\tau_1} \cdots a_{\tau_m}. \tag{1.21}$$

Cluster Decomposition

Cluster decomposition is the property of well-separated experiments becoming uncorellated (in the absence of EPR-like [71] corellations). In terms of the S-matrix, we require the position-space S-matrix to satisfy

$$S_{(\mathbf{x}_1,\sigma_1)(\mathbf{x}_2,\sigma_2)\cdots(\mathbf{x}_n,\sigma_n)} \longrightarrow S_{(\mathbf{x}_1,\sigma_1)(\mathbf{x}_2,\sigma_2)\cdots(\mathbf{x}_k,\sigma_k)} S_{(\mathbf{x}_{k+1},\sigma_{k+1})\cdots(\mathbf{x}_n,\sigma_n)} \tag{1.22}$$

as the distance between the clusters $\{x_1, x_2, \cdots, x_k\}$ and $\{x_{k+1}, \cdots x_n\}$ becomes infinite.

Requiring the clustering of the S-matrix implies that the coefficient of every term such as eq. 1.21 have one and only one delta function, a delta function that ensures momentum conservation. If $\mathbf{p}_{\sigma_i}$ is the momentum variable associated with the compound index σ_i, then the coefficient G in

$$H = \sum_{\sigma_1,\sigma_2,\cdots,\tau_1,\cdots} G_{\sigma_1\sigma_2\cdots\tau_1\cdots} a_{\sigma_1}^* a_{\sigma_2}^* \cdots a_{\tau_1} \cdots\cdots \tag{1.23}$$

must be of the form

$$G_{\sigma_1\sigma_2\cdots\tau_1\cdots\tau_m} \propto \delta^3(\mathbf{p}_{\sigma_1} + \mathbf{p}_{\sigma_2} + \cdots - \mathbf{p}_{\tau_1} - \cdots - \mathbf{p}_{\tau_m}) \times g_{\sigma_1\cdots\tau_m}, \tag{1.24}$$

with g free of delta functions. This techincal-seeming requirement ensures that the coordinate-space S-matrix factorises as the difference of particle coordinates, x_i, becomes large. If additional delta functions of $\mathbf{p}$'s were present in $g_{\sigma_1\cdots\tau_m}$, then new combinations of momenta would be conserved. Since momentum conservation is equivalent to translational invariance, changing this linear combination of coordinates would therefore not affect the S-matrix. But since there is a new translation invariance in this combination of coordinates, moving this cluster of coordinates away from others will not ensure the clustering of the S-matrix. Therefore only the overall momentum-conserving delta function is allowed.

Already, therefore, we have particles, cluster decomposition and interactions which are developed based on sound principles. This goes a long way toward explaining the inevitability of quantum field theory, except that we still don't have any fields. In order to finish the presentation, we need to know how to guarantee the Lorentz invariance of $\mathcal{H}$.

Fields

To see how to guarantee Lorentz-invariant theories, note that by taking appropriate linear combinations of products of creation and annihilation operators, a Poincaré-invariant density in the sense of eqs. 1.9, 1.10 can be constructed. But how do we construct such a scalar in detail? It is here that fields really come into their own. Fields are a particular linear combination of creation and annihilation operators,

$$\psi(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} \left[u(\sigma) a_\sigma e^{ip\cdot x} + v(\sigma) a_\sigma^* e^{-ip\cdot x} \right] \tag{1.25}$$

which have simple Lorentz transformation properties. The field $\psi(x)$ transforms under a finite-dimensional representation of the Lorentz group (such as the Dirac or vector representations), while a_σ, a_σ^* transform under the unitary, infinite-dimensional representation of the Lorentz group. u and v are Clebsh-Gordon coefficients for accomplishing this translation. For a scalar field, for example,

$$u = v = \frac{1}{\sqrt{2p^0}}, \tag{1.26}$$

which ensures that the measure $d^3\mathbf{p}/\sqrt{2p^0}$ is Lorentz invariant, while for a Dirac fermion,

$$[i\not{p} + m]\, u = [-i\not{p} + m]\, v = 0 \tag{1.27}$$

is the C-G condition. We thus see that the Dirac equation is the condition for projecting the infinite-dimensional unitary spin-1/2 representation onto the finite-dimensional pseudo-unitary spinor representation.

Fields are combinations of creation-annihilation operators that transform simply under Lorentz trasformations and which therefore make simple the construction of invariants (such as lagrangians and hamiltonians) out of irreducible representations of the Lorentz group.

The conclusion of all of this machinery is the same as the usual conclusion in field theory: in order to perform computations of scattering amplitudes or of transition probabilites or anything else, the fastest way is to construct a Lorentz-invariant lagrangian density and to quantise canonically (or to use the path integral). The *reason* the canonical approach is a good description of a vast array of natural phenomena, however, rests on the shoulders of the four principles of field theory: quantum mechanics, locality, cluster decomposition and Lorentz invariance. [3]

1.3 Effective Theories

Effective field theories are theories that are valid only up to some other fundamental mass scale. They are understood to arise from the 'integrating out' (a terminology arising from path integrals) of heavy degrees of freedom. The virtue of an effective field theory is that it focusses attention on the *relevant* degrees of freedom. If one is interested only in low-energy scattering of particles one should not need to worry about the excitation of high-energy particles.

This separation of scales is what makes physics comprehensible. In order to understand the scattering of light from particulate matter (in order to explain why the sky is blue) one does not need to know that the particles are made of leptons and quarks and that the leptons interact through the electroweak interactions, and that the electroweak symmetry is spontaneously broken at $\sim$ 300GeV, *etc. ad nauseam.* In order to explain why the sky is blue one needs only focus on the relevant degrees of freedom: uncharged, heavy particles that can Rayleigh-scatter light [21, 24]

The effective field theory programme is to

[3] One important property of quantum field theories hasn't been demonstrated here. The analyticity properties of the S-matrix are vital to a correct description of particle interactions. It seems that quantum field theory is a general set of rules for easily constructing S-matrices that are Lorentz invariant, are local, that cluster and that have the correct analyticity properties [12].

1. Identify the degrees of freedom,
2. Write every term allowed under the assumed symmetries, and
3. To organise these terms in order of increasing mass-dimension, the higher mass-dimension terms being of lower order in the low-energy expansion.

1.3.1 Brief Prelude on Effective Lagrangians

The modern view of renormalisation allows us to reasonably consider non-renormalisable lagrangians, and to use them in controlled, predictive ways. This is a radical departure from the 'old' view in which only renormalisable lagrangians could be used to make reasonable physical predictions. In this section I briefly outline the modern view and its justification, as it figures prominantly in interpreting quantum corrections to theories containing gravity.

When trying to calculate any quantity in a quantum field theory beyond lowest order, infinities are encountered when integrating over momenta of virtual particles in internal lines of Feynman diagrams. These infinities are dealt with by absorbing them into parameters in the lagrangian. Because only the combination of the original parameter and all of these infinite 'corrections' is ever measured in any experiment this problem can essentially be ignored; one simply measures the parameter in an experiment and assigns this value to the fully 'corrected' one. The *renormalisable* theories are those for which this programme can be carried out without adding new operators and parameters to the original lagrangian. According to the interpretation given in this paragraph there seems no way to reasonably extract information from non-renormalisable lagrangians, since we need to keep adding infinite terms to the lagrangian.

On the other hand it was well-known [23, 197] that non-renormalisable lagrangians could be the consequence of renormalisable ones. In the reference cited above, Euler and Heisenberg found that photon-photon scattering could be described at energies far below the electron mass by the non-renormalisable interactions

$$\mathcal{L}_{\mathrm{EH}} = \frac{1}{m_e^2} \left[c_1 \left[F_{\mu\nu} F^{\mu\nu} \right]^2 + c_2 \left[F^*_{\mu\nu} F^{\mu\nu} \right]^2 \right] \tag{1.28}$$

for some pure numbers c_1 and c_2. These terms arise from integrating out a single electron loop in the box diagram.

The idea of taking such phenomenological lagrangians seriously, and of calculating loop-corrections within them seems to have only taken hold much later [13]. The acceptance of such theories arose partly from attempts to quantise general relativity, which is non-renormalisable [197], partly from the deeper understanding of the content of renormalisation arising from renormalisation-group arguments [12, 14, 16, 17, 18], and partly from experience with phenomenological pion lagrangians [13].

All of this experience showed that indeed non-renormalisable lagrangians can be used to systematically calculate corrections to any order in the loop expansion, as long as one simultaneously performs an expansion in energy over some heavy scale. This reorganisation of perturbation theory allows calculations to be performed systematically to any order in loops and in derivatives, since operator-mixing which requires higher-order operators to be included to 'soak up' the infinities encountered in lower graphs only affects operators higher-order in derivatives.

The philosophical adjustment required to fully appreciate the power of effective lagrangian techniques was not small; it required abandoning the idea of a fundamental quantum field-theoretic description of phenomena. Quantum fields become merely a very convenient way of describing quantum mechanical many-body physics, and the lagrangian formulation serves merely to efficiently encode symmetries and motherhood principles [197] such as quantum mechanics, locality and cluster decomposition. This seems to be enough to encode the analytic structure required in the S-matrix, and seems sufficiently general to allow the expression of phenomena as wide-ranging as superconductivity, QCD and general relativity, to name a few well-tested, disparate applications which can be understood in this way. [4]

If quantum fields only encode symmetries and motherhood principles, then there seems no reason to include only the renormalisable terms in the effective theory. It turns out, in fact, that these are the only operators that are not 'irrelevant' (in a sense made precise by the renormalisation group) at low energies. In this way the justification for considering renormalisable lagrangians arises more naturally as a good approximation, not a new fundamental principle of nature.

The effective lagrangian programme, then, is to decide upon the accuracy to which one wishes to describe phenomena and to include only operators of the required order in the action. These terms all incur additional unknown constants in the theory which must be measured. Once these are measured

[4] There seem to be some theories that do not have lorentz-invariant lagrangian formulations although there is nothing wrong with the degrees of freedom. This seems to be a limitation in the lagrangian formulation of the theory, not of quantum mechanics [43].

an unlimited number of predictions may be made. It is in this sense that non-renormalisable theories are predictive, and thus these are no less a predictive than renormalisable theories.

1.3.2 Mass Dimension

Given a set of degrees of freedom (say the electron and photon), one must decide on the form of $\mathcal{H}$. There will in general be an infinite number of terms satisfying the Lorentz-invariance property eq. 1.9. One may restrict the operators under consideration, however, based on power-counting rules, which state that the most relevant operators at low energies are those with the lowest mass-dimension (in units with $\hbar = c = 1$). The mass-dimension of fields is worked out by requiring that the action be dimensionless (since $\hbar = 1$), and (usually) by requiring a canonical kinetic term. Therefore for scalars we have

$$S = -\frac{1}{2}\int d^N x\,(\partial\phi)^2 + \cdots \tag{1.29}$$

and since $\partial \sim$ (mass),

$$[\phi] = (\text{mass})^{N/2-1} \tag{1.30}$$

while for spinors we have

$$S = -\int d^N x\,\bar{\psi}\not{\partial}\psi + \cdots \tag{1.31}$$

so that

$$[\psi] = (\text{mass})^{(N-1)/2} \tag{1.32}$$

The mass-dimension of an operator is the product of the mass-dimension of all of the fields and derivatives: $(\partial\phi)^2\bar{\psi}\psi$ has mass-dimension 7 in four dimensions. In N dimensions, an operator of mass-dimension D requires a coefficient of mass-dimension $N - D$ to keep the action unitless.

The notion of the mass-dimension of operators is important because it allows controllable approximations in energy to be performed. Suppose that all of the mass scales appearing as coefficients in the interactions in a theory are of the same order, M to some power. In this case, in order to make the dimensions work out in the amplitude, the effects of operators of high mass dimension D (which are the complicated operators with many products, *e.g.* the mass-dimension 7 operator above) are suppressed by powers of $(E/M)^D$, where E is the typical interaction energy of the process. As long as $E/M \ll 1$, we can safely perform a perturbative expansion in high mass-dimension operators. Therefore restricting oneself only to low mass-dimension operators entails a controllable approximation.

1.3.3 Why the Sky is Blue: An Effective Field Theory Analysis

For the scattering of light by nonrelativistic, neutral particles, our degrees of freedom are the photon, A_μ, and a nonrelativistic particle given by a complex wavefunction, χ, describing an uncharged field. The symmetries we require are:

- **Gauge invariance:** This allows terms to be constructed only from $F_{\mu\nu}$. Since χ is not charged, we can't construct terms of the form $(\partial_\mu - qA_\mu)\chi$.

- **Rotational invariance:** Every term in the lagrangian must be a rotational scalar.

- **Parity invariance:** The lagrangian must be invariant under the action:

$$\begin{aligned}\mathbf{x} &\to -\mathbf{x}\\ \mathbf{B} &\to \mathbf{B}\\ \mathbf{E} &\to -\mathbf{E}\end{aligned} \tag{1.33}$$

The mass dimension of the operators are identified from the kinetic terms, which we canonically normalise. We also require unitarity, which implies the reality of the lagrangian, so that each term needs as many χs as χ^*s. We may now write the most general gauge-invariant interaction of light with nonrelativistic, parity-invariant, uncharged matter,

$$\begin{aligned}\mathcal{L} &= -i\chi^*\partial_t\chi - \chi^*\frac{\nabla^2}{2M}\chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\ &- a\,E^2\,\chi^*\chi - b\,B^2\,\chi^*\chi + \cdots ,\end{aligned} \tag{1.34}$$

where the dots indicate operators of higher mass-dimension which are further suppressed by powers of energy.[5] Dimensional analysis shows that the mass dimension of $F = dA$ (and therefore $\mathbf{E}$ and $\mathbf{B}$) is 2, and that of χ is 3/2. This implies that a and b are $(\text{mass})^{-1}$. Since low-energy light ($E \ll$ the excitation energies of the atoms) cannot penetrate the structure of the particulate matter, a and b are on the order of the size of the particles, $\sim a_0$, the Bohr radius.[6] Working out the scattering of light using this interaction

[5]Because this is a nonrelativistic theory there are also relativistic corrections in v/c, in addition to corrections in E/M. I ignore these corrections here, but they can be worked out straightforwardly [24].

[6]a and b receive contributions from all masses in the high-energy description, but these all appear as $1/M$ so the smallest mass dominates. This explains why we don't need to understand quarks to understand the blueness of the sky.

shows that the cross-section for a photon of energy E_γ is

$$\sigma \sim E_\gamma^4 a_0^2, \tag{1.35}$$

which shows that high-energy radiation scatters more frequently than low-energy radiation, and through the usual argument this explains the blueness of the sky. Of course this expression is not valid to arbitrarily high energies. In particular it is not valid at energies near the binding energy of the electrons to the nuclei. A new low-energy theory is valid at these energies, accounting for atomic excitations.[7]

This example vividly displays the strength of the effective field theory technique, albeit in a nonrelativistic setting. The full calculation of the Rayleigh scattering rate beginning with the hydrogenic wavefunction is a long, arduous process, requiring many delicate cancellations [21, 22]. It is only when one passes to the low-energy limit that massive simplifications occur. In the effective field theory method one passes to the low energy limit immediately, bypassing the complicated intermediate steps, all of the effects of which are encoded in the numerical coefficients of a_0 in a and b. One may choose any combination of experiment or calculation one wishes in order to obtain a and b. Once they are in hand any number of calculations can be performed, such as the scattering calculation outlined above, or one can use the above lagrangian to calculate the electric polarisability of the atmosphere: the Maxwell equations from above give (when coupled to a classical source, $\delta\mathcal{L} = -A_\mu J^\mu$),

$$\nabla \cdot [\mathbf{E}(1 + a\chi^*\chi)] = J^0. \tag{1.36}$$

[7]There are also very low-energy rotational resonances that one can worry about. These couple to quadrupole radiation and are further derivative-suppressed, not affecting our conclusion in the visible.

Since $\mathbf{D} = \mathbf{E} + \mathbf{P}$,

$$\mathbf{P} = a\chi^*\chi. \tag{1.37}$$

(This would be one way of measuring a, for example.)

By following the programme of effective field theories: identifying the relevant degrees of freedom, identifying the symmetries, and by using the derivative expansion, we were able to obtain highly nontrivial results with very little work. This is the main practical lesson of effective field theories: life is simpler and the physics more transparent when the correct degrees of freedom are chosen.[8] The standard model is an effective field theory in the same sense. It is the correct description of the relevant low-energy degrees of freedom.

1.4 The Standard Model is a Low-Energy Effective Theory

The standard model is a particular low-energy effective theory. It is the most general renormalisable theory of three families of quarks and leptons invariant under the gauge group $SU(3) \times SU(2) \times U(1)$ (with particular charges)[9] which is spontaneously broken to the subgroup $SU(3) \times U(1)$.

[8] This is an example of Weinberg's second law of theoretical physics: *You can choose any coordinates you like, but if you choose the wrong ones you'll be sorry* [12].

[9] The charges in the standard model can actually be almost entirely fixed (up to an overall scale) by requiring anomaly cancellation. Requiring the cancellation of gravitational anomalies (the trace of the $U(1)$s) as well as the traces of the cubes of the charges (the gauge anomalies) family-by-family constrains the hypercharge assignments (in the absence of any assumption regarding the symmetry-breaking sector) to one of: chiral electromagnetism, in which the left- and right-handed components of the quarks have different charges; a theory in which the right-handed electron is chargeless; and the standard model charge assignments. [19]

By the standard model it is usually meant the renormalisable interactions of the quarks and leptons supplemented by at least one Higgs (a scalar particle which spontaneously breaks the symmetry). The fact that the standard model is an effective theory means that we can reliably calculate the effects of adding higher-order operators which parametrise the effects of higher-energy physics. For example by parametrising beyond-the-standard model effects through dimension-six operators of the form [39]

$$\frac{1}{M^2}\bar{d}u\bar{q}L \qquad \frac{1}{M^2}\bar{q}q\bar{u}l \qquad \cdots \tag{1.38}$$

which violate baryon and lepton number we can put bounds on the high mass scale M by puting bounds on the proton lifetime. (Baryon nonconservation leads to proton decay.)[10] The proton lifetime will be given by

$$\Gamma \sim 8\pi^3 \frac{m_p^5}{M^4} \tag{1.39}$$

Since the proton lifetime is known to be greater than 10^{21} years [72], $M \gtrsim 10^{18}$GeV (under the assumption that the unitless coefficients are $\sim O(1)$).

This is very close to both the Planck scale and to the GUT scale. The Planck scale is the scale at which the quantum effects of gravity become important, while the GUT scale (Grand Unified Theory) is the scale at which the product group $SU(3) \times SU(2) \times U(1)$ is conjectured to unify into a larger group. This conjectured unification is based on the running of the strong, weak and electromagnetic couplings; within a large class of models [70] the couplings all unify at a scale around 10^{16}GeV. This is one example of indirect evidence of physics beyond the standard model. In the next section I will

[10]In eq. 1.38 L and q are the lepton and quark doublets, while u, d and l are, respectively, the up-type, down-type and leptonic $SU(2)$ singlets.

discuss this possibility within the context of observational evidence and of the hierarchy problems.

1.5 Beyond the Standard Model

The evidence pointing to new physics at a scale $M_{\text{newphysics}}$, not-too-far above a few TeV, is based on a mix of theoretical and observational evidence.

1.5.1 Observational Evidence: Precision Cosmology

Besides neutrino masses, all of the direct observational evidence for physics beyond the standard model is a result of precision cosmological tests that have been performed recently; this evidence seems to point unequivically to the existence of cold dark matter and to dark energy, which together form 95% of the mass-energy of the universe. Only 5% is composed of the baryonic matter and leptons which are described by the standard model. This evidence is robust and multi-faceted [190]:

- Measurements of the cosmic microwave background radiation, the light emitted just before the universe became transparent to light, are best fit by a universe with 73% dark energy, 23% dark matter and 4% normal (baryonic) matter [63].
- High-redshift supernovae give a velocity profile of the universe in the past, and suggest that the acceleration of the universe is best fit by a universe which is composed of roughly $\sim$ 75% dark energy, 23% dark matter and 5% baryonic matter[64].

- Dark matter has been known to be needed since the early 70s when Rubin and Freeman performed observations of galactic rotation curves [61]. The objects at the edge of all the galaxies surveyed have a much higher velocity than a simple count of the objects in it would suggest. Thus 'dark' matter is needed to make up the missing mass and allow the objects to have higher orbital velocities.

- Simulation of mass clustering and galaxy formation in the early universe suggest that galaxies would not have formed without dark matter to provide additional gravitational pull in regions of high density, lending further credence to the existence of dark matter.

- Nucleosynthesis correctly predicts the light element abundances. It predicts a baryonic mass density $\sim 5\%$, in agreement with the other methods, and leaves open the composition of the rest of the energy density of the universe.

(For an excellent review with references, please see [191].)

This evidence seems convincingly to require the existence of two additional constituents in the universe beyond what is predicted by the standard model: the universe seems to be *dominated* by dark matter and dark energy, which are only observed through their gravitational effects. Dark matter and dark energy differ in the way their energy densities scale as the universe expands: as the radius of a given region in the universe expands from r to αr the energy density of the dark matter falls as $1/\alpha^3$: it behaves like a fluid of cold massive particles. Dark energy behaves like a cosmological constant: as the universe expands, its energy *density* stays constant.

1.5.2 Theoretical Arguments: Hierarchies and Naturalness

The theoretical evidence for the incompleteness of the standard model is in the form of arguments about naturalness, and it is here that opinions (and estimates for the scale of $M_{\text{newphysics}}$) diverge. A theory, or a parameter in a theory, is unnatural if extreme fine-tuning is required between independent parameters in order to produce a given result. An example of a finely-tuned theory is a condensed-matter system near its critical point; in a theory near its critical point correlations become arbitrarily long (which corresponds to a particle becoming nearly massless). This theory will appear unnatural because the natural scale for such a system is set by its density, ρ, the temperature, T, or the inverse lattice spacing, a^{-1}, while the mass of physical, propagating particles can be several orders of magnitude smaller.

In a classical theory one is able to introduce two widely-separated scales and be done with it; while one may feel uncomfortable in doing so, it is hardly a disaster (buildings and ants are enormously different in scale). This issue of scales becomes a *naturalness problem*, however, when the stability of the hierarchy is threatened by quantum (or, in the critical theory above, thermal) corrections. Thermal and quantum corrections tend to introduce the fundamental scales, corresponding to the lattice spacing and the like, into particle masses through renormalisation. Therefore in order to maintain the hierarchy not only does the large ratio of scales have to be introduced initially, it must be maintained, order-by-order, in perturbation theory.

The fine tunings in the standard model coupled to gravity are very severe. In the Higgs sector retaining a Higgs mass $\sim$ TeV requires fine-tunings of one part in $\sim 10^{15}$ and measurements of the cosmological constant put it at $\sim (10^{-3}\text{eV})^4$, while the natural value for it is $\sim (10^{18}\text{GeV})^4$, a fine-tuning of one part in $\sim 10^{30}$ in energy.

Higgs-Mass Hierarchy

The fine-tuning of the Higgs mass arises because we believe there are physical propagating particles beyond those in the standard model at energies $\gg$ TeV. We believe this is true because the standard model does not unify gravitational interactions, and, while it is possible to sensibly perform calculations of general relativity coupled to the standard model, this theory is not UV complete. We believe there is a sequence of effective field theories starting at or below the Planck scale which reduce at energies $E \ll M_{\text{Planck}}$ to the standard model.

As a toy-model of the Higgs-mass hierarchy problem, consider a scalar field, ϕ, of mass m coupled to another scalar, ψ of mass $M \gg m$:

$$\begin{aligned} \mathcal{L} \;=\; & -\frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \\ & -\frac{1}{2}(\partial\psi)^2 - \frac{M^2}{2}\psi^2 - \frac{g}{4!}\psi^4 \\ & -\frac{\rho}{2}(\psi\,\phi)^2 \end{aligned} \tag{1.40}$$

We are going to integrate out the particle of heavy mass to obtain a low-energy description valid at energies $E \ll M$, and we will see that doing so 'destablises' the mass of the ϕ: under renormalisation $\delta m^2 \sim M^2$, so that in order to retain the hierarchy $m \ll M$ between the masses a fine-tuning of the bare ϕ mass must be performed: $m_0^2 \sim M^2 + m_{\text{phys}}^2$; to ensure that the physical ϕ mass is much smaller that M^2 the bare mass must have a piece much larger than m_{phys}. If, *e.g.*, $M/m \sim 1000$ then m_0 must be tuned to $1/10^{\text{th}}$ of one percent. As stated above, the hierarchy problem is that $M_{\text{Planck}}/m_{\text{Higgs}} \sim 10^{15}$.

In order to see this it is enough to calculate the effective potential for ϕ, whose effective mass is $M^2 + \rho\phi^2$, so that the effective potential after integrating out ψ fluctuations is given by

$$\begin{aligned} \delta V_{\text{eff}}(\phi) &= -\frac{1}{64\pi^2}(M^2+\rho\phi^2)^2 \log\left[(M^2+\rho\phi^2)/\mu\right] \\ &= -\frac{1}{64\pi^2}\left[1+2\log M^2/\mu^2\right] M^2\,\phi^2+\cdots, \end{aligned} \tag{1.41}$$

(we ignore powers of g). This shows that even if the bare masses of ϕ and ψ are arranged so that $m \ll M$, quantum effects will tend to make $m \sim M$. The problem with this is that in order to have physical masses $m \ll M$ we must tune the bare parameters in the lagrangian order-by-order in perturbation theory so that $m_{\text{bare}} \sim M$, but with corrections on the order of m_{phys}/M that survive the cancellation.

As we will describe later in more detail, supersymmetry (see appendix B) can mitigate this problem if is unbroken at energies above a TeV. Supersymmetry imposes equality between masses of particles of opposite statistics. Since opposite-statistic particles enter into the effective potential with opposite signs, a cancellation is possible in the above calculation, 'stabilising' the Higgs mass. Supersymmetry can be broken and still protect the Higgs mass since at very high energies supersymmetry-breaking is irrelevant. Thus, weak-scale supersymmetry (in which supersymmetry is still a good symmetry at energies much larger than the weak scale) is the most popular candidate for a phenomenological theory valid at just-above-standard-model energies, since it allows higher-than-weak-scale energies to cancel in the effective potential, while hiding the superpartners from collider experiments by virtue of their TeV-scale masses.

While it is certainly possible [47] for the standard model to be finely tuned and to remain successful (once the parameter under question, like the Higgs mass, is measured no explanatory power is lost), or for supersymmetry-breaking to be far above the weak scale, there is a wealth of circumstantial evidence that suggests that fine-tunings are the hallmark of new physics—experience has taught that explanations do exist for the appearance of small numbers. The only known ways of taming hierarchies is through the introduction of new symmetries.

1.5.3 The Cosmological Constant Problem

We will have much to say about the cosmological constant problem later in this thesis, but it is appropriate to pause here to describe it briefly in the context of hierarchy problems.

The cosmological constant is a term in the gravitational lagrangian which is not forbidden by symmetry:

$$\mathcal{L}_{\text{grav}} = \sqrt{g}\left(\frac{1}{2}R - \Lambda\right) \tag{1.42}$$

The cosmological constant, Λ, receives contributions from every mass scale in the problem as it is integrated out,

$$\Lambda \sim M^4 \tag{1.43}$$

and since it is measured to be $\sim (10^{-3}\text{eV})$, much smaller than any other mass scale in physics, every naive quantum correction to it makes it unacceptably large.

The quantum cosmological constant problem and the Higgs-mass hierarchy problem are actually (formally) related, both arising from the super-renormalisability of the operators encoding the quantities in question; since both the Higgs mass operator and the vacuum energy are positive mass-dimension operators, renormalisation introduces positive powers of the cutoff in the counterterms associated with them. For a cutoff on the order of any scale above the electroweak symmetry breaking scale (such as the GUT scale $\sim 10^{16}$GeV or the Planck scale $\sim 10^{18}$GeV) a fine-tuning problem develops. This fine-tuning problem is a requirement that the counterterms be 'tuned' to the right value, order-by-order in perturbation theory (to one part in $\sim 10^{82}$ in the worst-case scenario) in order to achieve a physically acceptable cosmological constant. (The cutoff need not have anything to do with the size of expected quantum corrections from higher-energy physics; one should instead speak of integrating out out masses of heavy particles [7, 6]. This use seems to be standard in this context and is harmless here, since the masses of heavy particles really will be on the order of the cutoffs being described, as the cutoffs are physical cutoffs having to do with the introduction of new physical degrees of freedom.)

1.5.4 Symmetry and Fine Tuning

One does not have to look far to see fine-tunings. For example, the masses of the photon and graviton are zero. Their masses are nominally subject to the same difficult-to-tame corrections as the Higgs, and so it is a puzzle why they remain massless. The resolution here is that a symmetry protects the masses; gauge invariance forbids the appearance of mass terms for these particles, ensuring the cancellation of all contributions to their masses.

The pion mass is another example of an apparently finely-tuned dimensionful parameter[11]. The pion mass is $\sim$ 100MeV, while the QCD scale (below which quarks condense into mesons like the pions), is $\sim$ 500GeV. This (relatively mild) fine-tuning of order one part in 10^3 is also explained through symmetry arguments: The pion is a pseudo Goldstone boson of the nearly massless u and d quarks; a risidual approximate chiral symmetry protects the pion mass from aquiring arbitrarily large quantum contributions [198].

In the next section an example is presented in which a hierachy between a massive and massless scalar occurs. As in the case of the pion system, this hierarchy is stable (natural) because of a symmetry. The massless mode is acutally a Goldstone boson of a broken symmetry. The symmetry protects the GB from receiving mass-corrections to all orders in perturbation theory. This exhibits clearly the relationship between symmetries and natural hierarchies in effective field theories.

1.6 The Strength of Symmetries

The only known ways to tame hierarchies with standard four-dimensional effective field theory techniques is through symmetries. In this section we present an example of a symmetry controlling a hierarchy, and briefly present some symmetries that have been used to try to solve hierarchy problems in the standard model.

[11]The pion is a condensate of quark-antiquark pairs. It is the lightest particle in low-energy QCD, and is thus ubiquitous. The explanation for its small mass [2], especially within the framework of field theory [5], remains one of the crowning acheivements of field theory, and represented a significant step towards the modern understanding of effective or phenomenological lagrangians.

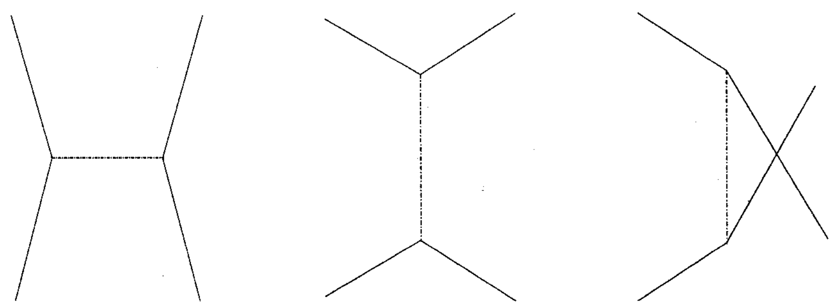

Figure 1.1: Tree-level graphs contributing to σ-σ scattering in the full theory. Solid lines are σs and dashed lines are ρs.

1.6.1 An Example

In this section we present a model in which there is a hierarchy between a massless field and a massive one that is maintained to all orders in perturbation theory because the massless field is a Goldstone boson.

Consider a very simple model [8] with a single complex scalar field, ϕ, of mass m, which enjoys a symmetry under $\phi \to e^{i\theta}\phi$

$$\mathcal{L} = -\left|\partial\phi\right|^2 - \frac{\lambda}{4}\left(v^2 - |\phi|^2\right)^2 \tag{1.44}$$

One may expand this in the two real degrees of freedom about the semiclassical minimum of the potential, $\phi = (v + \rho/\sqrt{2}) + i\sigma/\sqrt{2}$:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(\partial\sigma)^2 \\ &- \frac{\lambda}{4}\left(\sqrt{2}v\rho + \frac{1}{2}(\rho^2 + \sigma^2)\right)^2, \end{aligned} \tag{1.45}$$

which shows that the ρ-mass is λv^2, and that classically the σ is massless.

The calculation of σ-σ scattering at tree-level in this theory requires the computation of the three diagrams in fig. 1.1. After computing the amplitude, and after expanding in powers of momentum over the ρ-mass, $M = \lambda v^2$, we

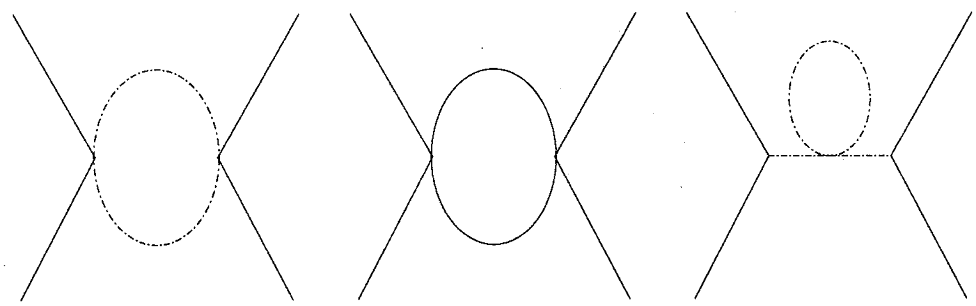

Figure 1.2: Examples of one-loop graphs for σ-σ scattering, including the exchange of σs and ρs in the full theory. In this channel there are a total of 8 graphs. Taking into account crossings gives 24 graphs total.

find that the amplitude goes like $p_{\rm CM}^4$, where $p_{\rm CM}$ is the centre-of-mass energy. In fact if one proceeded to compute loop diagrams such as those in fig. 1.2 one would find the same behavior to all orders in perturbation theory. *At low energies the scattering of σ particles is always suppressed by powers of momentum over M.*

This is quite a general statement, but it is not at all easy to see in the present setup. The reason that this general statement is so hard to see is the presence of the extraneous ρ degrees of freedom. Since we are only asking about processes with σ particles in the 'in' and 'out' states, and since we are only asking questions far below the mass of the ρ, it should be possible to parametrise the effects of ρ particles in an *effective low energy* theory.

One way to proceed is to write the most general lagrangian for σ fields with arbitrary coefficients in front of each of the operators. We are allowed to write terms which are local, which are Lorentz-invariant, and which are polynomial in fields[12]:

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= -\frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 \\ &- \sum_{n\geq 3} M^{4-n} a_n \sigma^n \\ &- \frac{b_1}{M}(\partial\sigma)^2\sigma - \frac{b_2}{M^2}(\partial\sigma)^2\sigma^2 - \cdots \\ &- \frac{c_4}{M^4}(\partial\sigma)^4 - \cdots \end{aligned} \tag{1.46}$$

The powers of M are inserted to ensure that the coefficients a_i, b_i and c_i are pure numbers.

It turns out that our tree-level calculation of the amplitude, above, is enough to simplify the lagrangian, to order p^4/M^2, to only the following terms:

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= -\frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 \\ &- \sum_{n\geq 5} M^{4-n} a_n \sigma^n \\ &- \frac{c_4}{M^4}(\partial\sigma)^4 - \cdots \end{aligned} \tag{1.47}$$

Notice that $a_n = 0$ for $n < 4$. We find this out the hard way by calculating $\sigma-\sigma$ scattering with this lagrangian and matching coefficients to the amplitude calculated with the full lagriangian. If we did not have access to the full lagrangian to begin with, but knew that we were interested in low-energy σ scattering, we would start with 1.46 and perform experiments to obtain the values of the coefficients.

Any calculation of an odd number of σs scattering will find the amplitude to be zero. This is because the high-energy theory enjoys a symmetry under

[12] We are assuming that σ is close to being a free-field, as it was in the original theory, eq. 1.44. For small fluctuations, and away from singularities in field variables, one may always expand in fields about the semi-classical minimum, in which case the quadratic, free-field part remains the most relevant part at low energies.

$\sigma \to -\sigma$, and this symmetry is not broken by integrating out ρ, and so survives to the low-energy theory. Given this additional piece of information at low energies the lagrangian *ansatz* can be simplified:

$$\begin{aligned}
\mathcal{L}_{\text{eff}} &= -\frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 \\
&- \sum_{m\geq 2} M^{4-2m} a_{2m}(\sigma^2)^m \\
&- \frac{b_2}{M^2}(\partial\sigma)^2\sigma^2 - \cdots \\
&- \frac{c_4}{M^4}(\partial\sigma)^4 - \cdots
\end{aligned} \tag{1.48}$$

This can then be further reduced (more straightforwardly) to eq. 1.47 through experiment or comparison with the full calculation.

Now we are able to see the enormous power of effective field theories. By knowing only about a discrete symmetry of σs along with Lorentz-invariance, we were able to restrict the form of the lagrangian to the much simpler eq. 1.48. There are a number of puzzles in the formulation presented above, however.

- Why is the σ massless? Does it remain so even after including loop-corrections due to σs? We saw above that hierarchies between scalar masses are difficult to maintain. Why is the σ mass much smaller than the ρ mass?

- How do we see that all the a_ns are zero? The all-orders result quoted in the previous section suggests that no non-derivative couplings of σ are allowed.

We can exhibit both of these properties in general: *there exists a massless degree of freedom in the original lagrangian 1.44, to all orders. This massless* *degree of freedom couples only through derivative interactions, to all orders.* Both of these results are due to (and are protected from quantum corrections by) a symmetry [5]. We can exhibit them by explicitly representing the symmetries of the original lagrangian in the fields. The original lagrangian contained a $U(1)$ symmetry which was broken by the vacuum (the semi-classical expansion of fields included a constant part, v). We therefore split the field into a degree of freedom which carries the broken $U(1)$ symmetry explicitly, and one which is orthogonal:

$$\phi = r(x)e^{i\theta(x)} \tag{1.49}$$

This choice of field variables is guaranteed to succeed due to the Goldstone theorem [5]. $\theta(x)$ explicitly carries the $U(1)$ symmetry through $\theta \longrightarrow \theta+$const. The original lagrangian in these field variables becomes

$$\mathcal{L} = -\partial_\mu r \partial^\mu r - r^2 \partial_\mu \theta \partial^\mu \theta - \frac{\lambda}{4}\left(v^2 - r^2\right)^2 . \tag{1.50}$$

Even once we expand r about its semiclassical minimum $r = v$, θ only ever appears accompanied by derivatives. Since this implies the symmetry $\theta \longrightarrow \theta + \text{const}$, integrating out the r-field (of mass M) has no effect on this. Therefore the effective theory of θs can only contain terms

$$\mathcal{L}_\theta = -\sum_n M^{4-2n} f_n (\partial\theta)^{2n}. \tag{1.51}$$

Since

$$\theta = \arctan \frac{\sigma}{v+\rho}, \tag{1.52}$$

every instance of σ is accompanied by a derivative. This proves that σ is massless and that it decouples at low energies, its interactions being suppressed by powers of momentum.

As promised, the number of terms that one keeps, in this example, is governed entirely by the accuracy with which one wishes to calculate σ-σ scattering. Assuming $f_n \sim O(1)$, and for energies up to ϵM, one would achieve $O(\epsilon^4)$ accuracy in σ-σ scattering with only the $(\partial\sigma)^4$ term. higher-order scattering can be computed as well; if one wanted to also account for 3-σ scattering one could include the terms $(\partial\sigma)^6, \sigma^2(\partial\sigma)^4, \sigma^4(\partial\sigma)^2$. These terms would allow corrections to σ-σ scattering to be calculated in the form of loops of internal σs, entirely within the low-energy theory. In this way the effects of loops form high-energy phenomena, the ρs, is completely incorporated in the low-energy theory, while the low-energy loops of σs remains to be performed.

* * *

This example shows how powerful the knowledge of symmetries is in restricting the kinds of terms that are allowable in an effective theory. Similarly powerful results can be had for real-world situations, such as electrodynamics and general relativity.

In the modern view of quantum field theories, in which they are expressions of symmetry principles on field content to derive S-matrices with the correct analyticity properties, it is a simple exercise to write down electrodynamics (and general relativity), and to justify their form [15].

Suppose we wish to write a theory for a massless spin-1 particle. It turns out that the tensor representations of the spin-1 fields are the self- and anti-self-dual twoforms:

$$F^{(+)}_{\mu\nu} \qquad \text{and} \qquad F^{(-)}_{\mu\nu} \tag{1.53}$$

Parity invariance requires them to be joined into a single two-form F. If one tries to construct a theory of such a field one finds that the forces exchanged by F are not long-ranged and do not correspond to any known force. There exists a pseudo-representation of the spin-1 field, however, in terms of a vector field, A_μ. Under Lorentz transformations,

$$A_\mu \to \Lambda_\mu{}^\nu A_\nu + \partial_\mu \omega \tag{1.54}$$

for some function ω constructed form the polarisation vectors and creation/annihilation operators making up A. Therefore, under a Lorentz transformation, A undergoes a *gauge transformation.* The only way to salvage Lorentz invariance in this theory is to again construct a two-form, but this time in terms of the underlying A field:

$$F = dA \tag{1.55}$$

and to make the whole theory gauge invariant. The lowest-order gauge-invariant couplings of A are

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A_\mu J^\mu \tag{1.56}$$

where J_μ is required to satisfy

$$\partial_\mu J^\mu = 0 \tag{1.57}$$

in order for the theory to be gauge invariant. Variation of the above lagrangian with respect to A yields the Maxwell equations. A similar (but more involved) analysis for a massless spin-2 particle yields general relativity [15].

1.6.2 Symmetries and the Higgs-Mass Hierarchy Problem

The experience with symmetry's control of hierarchies, as exhibited in the toy-model above, has led to extensions of the standard model through the addition of symmetry, a few of which are listed below. The essence of these extensions are that either the Higgs field is not a fundamental scalar, so that its dynamics are radically different at high energies (thereby protecting the scalar mass by unasking the question), or it is a fundamental scalar (at least at energies a few times a TeV) and there are symmetries amongst other particles that cancel the contributions to the Higgs mass from UV particles (supersymmetry, as mentioned already, does this).

- **Technicolor.** Technicolor theories [49] do not have a Higgs field to break the electroweak symmetry group. These theories depend on an additional confining gauge group, added to the standard model, which dynamically breaks $SU(2) \times U(1) \rightarrow U(1)$. Dynamical symmetry breaking occurs when an asymptotically free theory runs to low energy, the symmetry breaking occuring when the coupling becomes $\sim O(1)$. Since the running of the coupling is logarithmic, exponential hierarchies between the electroweak and GUT (or Planck) scales can easily be accomodated.

- **Weak-scale supersymmetry.** Supersymmetry is a symmetry which relates the bosons and fermions in a theory. As described in appendix B, supersymmetry seems to be the most general notion of symmetry which is consistent with the idea of a scattering matrix and the 'motherhood' principles of quantum mechanics. Supersymmetry can help stabilise the weak scale because the relations it imposes between fermions and bosons ensures that their contributions in loops cancel. Weak-scale supersymmetry [3] enforces relations amongst the masses and couplings of fermions, at scales above the weak scale, ensuring the cancellation of their contributions at high energies to the Higgs mass, thus relating the supersymmetry-breaking and the electroweak symmetry-breaking scales and stabilising the Higgs mass.

- **Little Higgs mechanism.** The little Higgs mechanism [48] uses a large number of gauge groups to mock up a fifth dimension whose 'radion' is the Higgs (the Higgs remains light because it is a pseudo Goldstone boson);

- **Extra-dimensional models.** Extra-dimensional models are special because they do not solve the hierarchy problem through symmetries (at least directly). They solve the hierarchy problem by lowering the fundamental gravitational scale, which is extra-dimensional, to close to the four-dimensional weak scale. The usual four-dimensional Planck scale is reproduced through geometrical effects.

  These models come in two varieties. Warped extra dimensions models [66, 67] solve the hierarchy problem by introducing an exponential metric function, allowing the four-dimensional Planck scale and the weak scale to be exponentially related without the direct addition of artificially large numbers. Large extra dimensions models [136] relate the extra-dimensional Planck scale and the four-dimensional Planck scale through a very large geometrical volume factor arising from the compactification *e.g.* from six dimensions to four.

Each of these proposals for extending the standard model has strengths and weaknesses. It seems that technicolor theories are probably strongly disfavoured because the simplest models have flavour-changing neutral currents, and models which don't are so complicated as to lose much of their initial motivation [198]. Little Higgs theories are an exciting new direction for study and remain viable, testable extensions of the standard model. We will not have any more to say about them, however, and the reader is referred to the extensive literature for further information [48]. One other large body of literature with a great deal of interesting work about which I will have little or nothing to say is the literature associated with Grand Unified Theories (GUTS) [69]. These theories are typically supersymmetric (but see [70]), but are always based on a group like $SO(10)$ or $SU(5)$ which has as a subgroup the standard model gauge group $SU(3) \times SU(2) \times U(1)$. The fundamental gauge group is broken at some higher scale (the GUT scale $\sim 10^{16}$GeV) to $SU(3) \times SU(2) \times U(1)$. These theories are motivated by the convergence of the three running couplings in the supersymmetric standard model at this energy. Please see [69] for modern reviews and references. In the next two subsections I briefly describe weak-scale supersymmetry and brane-world models as solutions to the hierarchy problem.

1.6.3 Weak-Scale Supersymmetry

One of the most actively studied extensions of the standard model is weak-scale supersymmetry, or the supersymmetric standard models. A search for citations of the paper in ref. [42] alone showed over 2600 results on the high energy physics abstract service, SPIRES [13]. This particular extension of the standard model is popular because a relatively simple statement of symmetry has far-reaching and beautiful consequences (see appendix B for a brief review

[13] http://www.slac.stanford.edu/spires/hep

and a presentation of my notation), and because supersymmetry seems to be an integral part of a fundamental theory of gravity (string theory) [53, 54]. For an excellent review with references, please see [3].

The primary reason for the popularity of weak-scale supersymmetry is that it stabilises the Higgs mass. The supersymmetric standard model seems to come with a price, however, since in it one loses the natural explanations for baryon and lepton conservation, and the supersymmetric standard model has problems with flavour-changing neutral currents; in the standard model the suppression of baryon, lepton and FCNCs all arise as consequences of accidental symmetries. This can be fixed by imposing an additional discrete R-parity, which in addition provides a stable dark matter candidate which is lacking in the standard model. There are, however, a litany of other problems with the standard weak-scale supersymmetry picture, including large dimension-five operators of the form $qq\tilde{q}\tilde{l}$, which contribute to proton decay, new flavour violations in the dimension-four gaugino-sfermion interactions, and large superpartner contributions to $(g-2)_\mu$ and $b \to s\gamma$. [47]. Of course, these problems can also be viewed as an opportunity, since experiment has begun to severely constrain the parameter-space of the supersymmetric standard model, and the LHC is likely to verify or exclude it entirely in the next few years.

1.7 Strings, Branes and Extra Dimensions

Partly due to the problems of weak-scale supersymmetry as a method of stabilising the Higgs mass, partly due to the influence of string theory, and partly simply to explore the possible solutions to the problem, extra-dimensional models were proposed and a flurry of activity on them began in 1998-1999. These are the models with which, for the most part, the rest of this thesis is concerned.

Extra dimensions were not new; string theory had been known to require 26 dimensions (the bosonic string [53]) or 10 dimensions (the supersymmetric string [54]) for a number of years. These were usually hidden through the *compactification* of the extra dimensions. The extra dimensions beyond the usual four were made small, of radius $r \sim 1/M_{\text{Planck}}$. Since an energy on the order of M_{Planck} is required to probe this distance, the extra dimensions are effectively hidden from any future experiments.

The bulk of the work on compactification of string models occured in attempting to construct vacua of string theory (solutions to the string equations of motion) containing $\mathcal{N} = 1$ supersymmetry, so that the supersymmetric standard model was reproduced at energies $\ll M_{\text{Planck}}$inheriting the problems of the supersymmetric standard model, the major problem the community seemed to have with this programme was the apparently infinite number of suitable vacua, and that, although one could come very close to finding the standard model particle content in the low-energy models, no concrete solution with $SU(3) \times SU(2) \times U(1)$ spontaneously breaking to $SU(3) \times U(1)$ and with three families of leptons and three families of quarks was found. It seems that the seeming arbitrariness of the solutions found left no hope of finding an explanation for three families and the particular pattern of standard model symmetries in any case.

(It can be proved [56] that the only solutions to the superstring equations of motion which result in flat four-dimensional $\mathcal{N} = 1$ supersymmetric space were solutions with constant dilaton, no flux and a product manifold $M_4 \times CY_3$, where CY_3 is a Calabi-Yau three-fold, a three complex-dimensional manifold of $SU(3)$ holonomy which is Ricci flat. There are an infinite number of Calabi-Yau manifolds, but only a few known metrics. This doesn't hamper string model-building much since the particle content, Yukawa couplings and coupling constants depend only on topological quantities of the CY like the Dirac index and the Betti numbers [11].)

1.7.1 D-branes and Brane-Worlds

String theory, and string model-building experienced a revolution upon the discovery of D-branes [52], within the spectrum of string theory. D-branes are, nominally, surfaces describing the position at which Dirichlet boundary conditions are satisfied by open strings; a D-brane is a place where open strings end. The fundamental insight of ref. [52] (see also [50, 51] for earlier work on extended objects by the same author) was that these objects carry Ramond-Ramond charge, thus explaining the source for these heretofore mysterious gauge fields, and that these objects are dynamical, despite their origins as static boundary conditions.

D-branes carry on their world volume a supersymmetric gauge theory, the Chan-Paton degrees of freedom (gauge fields) at the end of the strings. Thus, from one point of view, particles are trapped on a brane, with only closed-string (gravitational) degrees of freedom able to probe the space between D-branes.

Brane-world models abstract the idea of a D-brane away from the particular string theory context. These surfaces carry tension, thus sourcing the

Einstein equations, and are assumed to trap the standard model particles on them.

D-Branes

A p-brane is a $p+1$-dimensional submanifold with p spatial directions on which open strings can end with Dirichlet boundary conditions. Open strings carry at their ends gauge degrees of freedom which induce a (super) Yang-Mills theory on the world-volume of the brane as the ends move tangentially along the brane. In this way the D-brane 'traps' a Yang-Mills theory on its world-volume.

Because string theory is a dynamical theory of gravity (read geometry) the D-brane does not remain static, but aquires dynamics of its own. Accounting also for the Yang-Mills string-ends, one obtains the Dirac-Born-Infeld action,

$$S = -T \int_{\mathrm{WV}} e^{-\phi} \sqrt{\det\left[g_{MN}\partial_\mu X^M \partial_\nu X^N + \alpha' F_{\mu\nu} + \cdots\right]} \tag{1.58}$$

where WV means the world-volume of the brane, X^M are the coordinates of the brane, F is the field-strength of the Yang-Mills field, ϕ is the dilaton, which controls the string coupling strength, and the dots mean extra terms which we will not worry about here. (For very nice reviews with references the reader is invited to read refs. [55].) Expanding the action using $\log\det = \mathrm{tr}\log$, we get

$$S = -T \int_{\mathrm{WV}} e^{-\phi} \sqrt{\det\left[g_{MN}\partial_\mu X^M \partial_\nu X^N\right]} \left\{1 + \alpha'^2 \frac{1}{4}\,\mathrm{tr}F^2 + \cdots\right\} \tag{1.59}$$

We see that we obtain the bosonic part of a Yang-Mills action. String models can be made which add matter of various sorts [59].

D-brane actions have a second part, their coupling to Ramond-Ramond (RR) charges.[14] RR fields are $p+1$-forms, and couple to branes in the same way the electromagnetism couples to particles—by coupling to the branes' worldvolumes:

$$S = -q \int_{\text{WV}} C_{p+1} \tag{1.60}$$

Indeed the RR charges provided the first clue that D-brane-like objects should exist in string theory; it was a mystery to *what* RR charges coupled for some time [52]. [15] This coupling is analogous to the electromagnetic coupling to a particle,

$$S = -q \int d\tau A_M(X(\tau)) \frac{dX^M}{d\tau} d\tau \tag{1.61}$$

where $X(\tau)$ is the world-line of the charged particle.

D-Branes have been instrumental in understanding the basic structure of string theory. Indeed, the use of the singular (string theory, instead of string theories) in the previous sentence is due to D-branes; before the 'second string revolution' [58] there were five string theories which seemed to have no real connection to each other. D-branes allowed the discovery of dualities between these five theories; the current understanding is that each of these string theories represents a perturbative description around a particular vacuum of

[14]This repetition is not merely a communal stutter in the string community; open strings have two ends, each of which may have either of two conditions imposed on them. These two conditions were discovered by Ramond and by Naveau & Schwarz, so strings can be RR, NS-NS, and chiral strings can be R-NS or NS-R.

[15]For the indicially challenged, the above action is given by

$$S = -\frac{q}{(p+1)!} \int d^{p+1}\tau \sqrt{-g_{ind}}\, C_{M_1 \cdots M_{p+1}} \partial_{\mu_1} X^{M_1} \cdots \partial_{\mu_{p+1}} X^{M_{p+1}} \epsilon^{\mu_1 \cdots \mu_{p+1}},$$

where τ^μ are coordinates on the D-brane, $\partial_\mu \equiv \partial/\partial\tau^\mu$, g_{ind} is the induced metric $g_{MN}\partial_\mu X^M \partial_\nu X^N$, and ϵ is a *tensor.*

a single background-independent theory, which has been given the monicker 'M-theory'.[16]

The construction of true D-brane models within string theory has experienced huge advances; there are intersecting D-brane models that can come very close to producing the standard model degrees of freedom [59]. The huge leap forward for extra-dimensional model building, however, occured with the introduction of brane-world models, in which the idea of a submanifold on which the (non-gravitational) standard-model degrees of freedom could be trapped was abstracted away from the very specific string setting; these objects, termed *branes*, do not necessarily share any of the very special properties of real D-branes, such as partially broken supersymmetry [50, 51, 55], sourcing Ramond-Ramond fields [52], or having a super-Yang-Mills theory [53, 54] on their world volume, they are simply surfaces in spacetime on which particles may be trapped.

1.7.2 Brane Worlds

The discovery of branes in string theory was enormously liberating to phenomenologists and to theorists looking to explain the peculiarities of the standard model. For many years the only really viable extensions of the standard model were GUTs, the MSSM and compactifications of various stripes, which, while very rich and promising subjects, as described above had many problems of their own. The most notable amongst these problems for the present discussion is the problem of *naturalness* or *hierarchies*, about

[16]'M' is a very versatile letter in English: 'Meta', 'Mother', 'Matrix' and many other words have been used when the occasion seems appropriate. The author reserves judgement on the profundity of the choice of 'M' until such profundity can be proved to be language-independent.

which we have already said much. Brane world models' *raison d'etre* is to a large extent to solve the hierarchy problems.

Brane-world models use a geometry consisting of standard model particles trapped on a three-brane (a three-spatial-dimensional submanifold which sweeps out a 3+1-dimensional world volume). The simplest braneworld models will have an action of the form

$$S = \int_{M_{4+n}} \sqrt{-g}\frac{M_{4+n}}{2}R + \text{other bulk fields} + \sum_i T_i \int_{M_4^{(i)}} \sqrt{-g_{\text{ind}}} \qquad (1.62)$$

where gravity and possibly other bulk fields can propagate in the bulk. A specific brane-world model solves the equations of motion arising from this lagrangian and derives the effective four-dimensional description.

There are two quite different flavours of braneworld models: those employing warped internal dimensions (Randall-Sundrum [66, 67] models) and those employing large extra dimensions ('ADD' [136] models), although there exist hybrid models. Both types of model employ the geometry of the 'bulk' (the directions in space perpendicular to the foliations in which the branes are embedded) to help solve the hierarchy problems on the 'branes', where the standard model is trapped.

The simplest warped models are five-dimensional [66, 67]

$$ds^2 = W^2(y)\,\eta_{\mu\nu}dx^\mu\,dx^\nu + dy^2 \qquad (1.63)$$

and exploit the exponential metric factor (the 'warp factor')

$$W \sim \exp\left[-k|y|\right], \qquad (1.64)$$

induced by the Israel junction conditions in the presence of a brane-like singular sources at $y = 0, 1$. Because it is the *induced* metric that sets the scale for masses, the mass scales at the $y = 1$ brane are exponentially suppressed relative to their bare values:

$$m_{\text{phys}} = e^{-k} m_{\text{bare}} \tag{1.65}$$

Therefore the fundamental (extra-dimensional) mass scale can be the Planck mass, 10^{19}GeV, with a 'TeV' brane positioned at $y = 1$. The hierarchy between the Planck mass and the weak scale in this picture is a result of the exponential form of the warp-factor, and no large numbers are required, making the hierarchy 'natural'. This mechanism is exhibited in detail in section 4.2

Large extra dimensions models [136] solve the hierarchy problem by having very large-volume (possibly even flat) internal dimensions. Upon compactification the lower-dimensional Planck scale, M_{Planck}, is related to the fundamental (higher-dimensional) scale of gravity, M_{fund}, through the geometric relation

$$M_{\text{Planck}}^2 = V\, M_{\text{fund}}^{2+n} \tag{1.66}$$

where n is the number of extra dimensions and V is the volume of the extra dimensions. If $VM_{\text{fund}} \gg 1$ then the fundamental scale of gravity can be much lower than the observed (effective) four-dimensional Planck-scale. We will develop these models more fully in section 4.3.

1.8 Roadmap

In the rest of this thesis we develop the point of view that if we retain the principles of effective field theories, and if we account for evidence from string theory that there are extra dimensions, then we are led to construct brane-world models to try to explain the cosmological constant problem.

We will describe some of the failed four-dimensional attempts to solve the cosmological constant problem, and present some posited solutions to the cosmological constant problem which are from without the standard effective field theory paradigm. Restricting ourselves to effective field-theory descriptions forces us to consider extra-dimensional theories. We show that standard compactifications of extra-dimensional theories are not helpful in solving the cosmological constant problem because the compactification scale is too high.

We then describe brane-world models and show that if such a model were to contain a certain set of properties (a dilaton-like scalar coupled conformally), then these solutions can naturally produce flat-space solutions. The rest of the thesis explores models for doing so, and tries to identify the features such a solutions would need to possess.

As we will see in the next chapter, there have been many failed attempts to solve the cosmological constant problem. Some of the failures have been relatively subtle. It is not the purpose of this thesis to provide a final answer to the cosmological constant problem, but to explore one possibility, and to point out that this idea has not failed in at least some of the ways in which it could have. The cosmological constant problem is sufficiently hard that this, in my opinion, is already progress.

Chapter 2

The Cosmological Constant Problems

The effective theory philosophy can also be used to understand gravitation even though it is nonrenormalisable [13, 15, 19, 20, 9]. Because gauge invariance and masslessness are so closely related in quantum field theory, the assumption of a massless spin-two boson implies general covariance of the theory. General covariance in turn can be used to construct the relevant invariants [15], showing that the theory must be constructed from the curvature tensors of a metric function. This guarantees that the usual rules for constructing the Einstein-Hilbert lagrangian and for coupling matter to gravitation by covariantising derivatives is generally valid, and that the usual power-counting arguments allow us to parameterise low-energy gravitational effects [9].

In order to construct the Einstein-Hilbert lagrangian, therefore, we write down all of the invariants that can be constructed from the metric and its derivatives, organising terms by their derivative-suppression:

$$\mathcal{L} = -a_0 M^4 \sqrt{g} + a_2 M^2 R\sqrt{g}$$

$$+a_4^{(1)}\, R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda} + a_4^{(2)}\, R_{\mu\nu} R^{\mu\nu} + a_4^{(3)}\,(R)^2 + \cdots\,, \qquad (2.1)$$

where a_i^j are dimensionless couplings for terms with i derivatives, and M is the relevant mass-scale of gravitation. As is usual, terms with the least derivatives are the most relevant at low energies, so we expect to obtain a good approximation to low-energy gravitational dynamics with the Einstein-Hilbert lagrangian. Performing the matching to the Newtonian case establishes $M^2 \equiv M^2_{\text{Planck}} = 8\pi G_N$, where G_N is the usual gravitational constant, so $M \sim 10^{18}$GeV:

$$\mathcal{L} = \sqrt{g}\left[\frac{M^2}{2} R - a_0\, M^4\right]. \qquad (2.2)$$

All of our experience with effective field theory-techniques suggests that $a_0 \sim \mathcal{O}(1)$. The cosmological constant problem is that $a_0 \sim 10^{-120}$.

As discussed previously, there are two aspects to any fine-tuning problem: the initial imposition of the finely-tuned relationship, and the running of this relationship with energy. The cosmological constant is tuned in both of these senses: semi-classically it must be very finely-tuned to ensure cancellations between different contributions to the vacuum energy (*e.g.* from various particle-physics phase transitons), and quantum mechanically the cosmological constant is a disaster because running to low energies destroys any fine-tuning peformed at high energies. This last observation, that the cosmological constant problem is essentially a failure of dimensional analysis at very low energies—energies even below the electron mass—is what makes the cosmological constant problem so difficult. In the absence of symmetries such as supersymmetry, if there is any interesting physics at all at energies of the order of m, then the cosmological constant receives a contribution of the order of m^4, rendering even the electron a catastrophe.

There have been many attempts at solving the cosmological constant problem, all of which seem essentially to fall into one of five categories, the original four identified by Weinberg [27, 122], plus a few new ideas which involve modifying gravity at extremely low energies.

1. **Self-tuning mechanisms.** Adjustment mechanisms, or 'self-tuning' mechanisms, couple scalar fields to gravity in such a way as to cancel the cosmological constant when the scalars are at the equilibrium value of their potential. Weinberg gave a sort of 'no-go' theorem [122] in which he showed that this mechanism cannot work in four dimensions without fine-tuning of some sort, and in any case is invariably destroyed by any four-dimensional quantum corrections.

2. **'Deep symmetries'**, such as supersymmetry [199], which constrain or control the contributions to the cosmological constant from the usual catastrophic sources [40]. These models seem either to contradict particle-physics experiments because the symmetry is unbroken at the milli-electron-volt scale, or, if the symmetry is broken at a phenomenologically viable energy, do not produce a sufficiently small cosmological constant [122].

3. **Quintessenece.** The quintessence field couples to gravity and matter in such a way as to make the cosmological constant small at late times because it tracks the matter energy-density [118, 119, 120, 121]. Thus quintessence attempts simultaneously to solve the 'old' cosmological constant problem, of why the cosmological constant is so small, and the 'new' cosmological constant problem, of why the vacuum energy den-

sity and the matter energy-density are the same now [27]. Quintessence models tend to suffer the same catastrophic contributions from quantum corrections as a plain vacuum energy density [122].

4. **Anthropic Considerations.** The anthropic principle states that the values of the constants of nature are preconditioned by the critereon that someone exist to measure them. Anthropic models of the cosmological constant focus on various ways of instantiating a probabilistic framework, either through vacuum-to-vacuum transitions through wormholes [31], by using a four-form field to soak up the vacuum energy [35, 34], or through the string 'landscape' [32] (and many more—see the references in [32, 34] for other attempts). In any case, by conditioning the value of the cosmological constant on the observation that galaxies have formed, it has been estimated that the probability for the occurrence of the present value of the cosmological constant to be on the order of $10\% - 15\%$ [29].

5. **Infrared Modifications of Gravity** are generally new ideas, inspired by the likes of holography/black hole entropy [25] and extra dimensions [147]. These proposals hinge on lowering the cutoff for the relevant gravitational physics to $\sim 10^{-3}$eV so that the cosmological constant is naturally of the observed order.

2.1 Statement of the Problem

There are actually *two* 'old' cosmological constant problems, the problem of why the cosmological constant is classically fine-tuned to be small, and the problem of how this fine-tuning is maintained in the presence of quantum corrections. (There is also the *'why now'* problem of why the cosmological constant vacuum energy is just now approximately the same as the matter energy density in the universe. We will briefly mention this problem in relation to our model and quintessence in a later chapter.)

2.1.1 Classical Fine-tuning

The classical cosmological constant problem is a fine-tuning problem, requiring extraordinary cancellations between completely unrelated contributions to the vacuum energy. Consider the contribution of the electroweak symmetry-breaking transition to the vacuum energy. The gravity-Higgs action is given by,

$$S = \int \sqrt{-g}\left[\frac{M_p^2}{2}R - \Lambda_0 - |D\phi|^2 + \mu^2 |\phi|^2 - \frac{\lambda}{2}|\phi|^4\right] \tag{2.3}$$

but the cosmological constant which affects cosmic evolution is not Λ_0, it is the cosmological constant obtained by integrating out all fields with masses greater than the Hubble scale, since this is the relevant scale for cosmology. Semiclassically, the correct low-energy cosmological constant is obtained by evaluating the Higgs-part of the action at its minimum, $|\overline{\phi}|^2 = \mu^2/\lambda$, so that, ignoring fluctuations in ϕ,

$$S = \int \sqrt{-g}\left[\frac{M_p^2}{2}R - \left(\Lambda_0 - \frac{1}{2}\frac{\mu^4}{\lambda}\right)\right]. \tag{2.4}$$

An enormous fine-tuning is required between the bare cosmological constant, Λ_0, and the vacuum energy of the higgs background to produce $\Lambda_0 - \mu^4/2\lambda \approx (10^{-3}\text{eV})^4$. Since the Higgs mass is $\mu^2/\lambda \sim (100\text{GeV})^2$, we require $\Lambda_0 -$

$\frac{\lambda}{2}\left(100\mathrm{GeV}\right)^4 \sim \left(10^{-3}\mathrm{eV}\right)^4$, representing a fine-tuning of order one part in $\lambda^{1/4}10^{14}$ of the required valued of Λ_0.

This problem is exacerbated by the many such phase transitions in the history of the universe, each of which contributes to the bare vacuum energy to produce the low-energy cosmological constant that we observe; there is presumably a contribution from QCD hadronisation, which occurs at $\Lambda_{\mathrm{QCD}} \sim$ 500GeV, from supersymmetry-breaking if such a thing occured, at $M_{\mathrm{SUSY}} \gtrsim$ TeV, from baryogenesis, *etc.* Each of these transitions contributes to the cosmological constant, so that

$$\Lambda \sim \Lambda_0 \pm \Lambda_{\mathrm{QCD}}^4 \pm M_{\mathrm{SUSY}}^4 + \cdots , \tag{2.5}$$

which either requires Λ_0 to be very large in the ultraviolet—and to have precisely the right value to cancel all of these contributions—or requires the individual contributions to (at least partially) cancel with alternating signs.

If only the classical fine-tuning described above was required to explain the smallness of the cosmological constant then perhaps it would not present such a difficult puzzle. The further problem with the cosmological constant is its enormous sensitivity to the details of quantum ultra-violet physics which destabilises any classical fine-tunings one attempts to impose. It turns out that even if all of the phase transitions are somehow cancelled, physics at the electron mass which is well-understood ruins the finely-balanced value of the cosmological constant.

2.1.2 Quantum Corrections

The cosmological constant problem is difficult because it is a *low-energy* problem, and it is a low-energy problem because even if the cosmological constant is made small at some high scale, it does not remain small as we descend to lower scales. For example, let us consider the very well-understood physics of the electron, muon and photon. Even if we manage to explain the smallness of the cosmological constant just below the muon mass, integrating out the electron to one loop leads to a cosmological constant too large by a factor of 10^{36}.

Consider the Wilsonian effective action for a particle-physics model (for simplicity a scalar field of mass $m \ll M_{\text{Planck}}$) coupled to gravity and valid for energies below a cutoff Λ_{UV} with $m \ll \Lambda_{\text{UV}} \ll M_{\text{Planck}}$. Imagine integrating out physics above some scale μ with $m \ll \mu \ll \Lambda_{\text{UV}}$ and fine-tuning parameters to ensure that the mass continues to satisfy $m \ll \Lambda_{\text{UV}}$, and to ensure that the renormalised vacuum energy $\Lambda \ll \Lambda_{\text{UV}}^4$. This renormalisation step cancels all contributions to the cosmological constant from energies between μ and Λ_{UV}.

Now let us pass to very low energies by integrating out the scalar field entirely, to be left with an effective low-energy theory of gravity alone. Because we have renormalised away the contributions to the vacuum energy from energies between μ and Λ_{UV}, only scales between μ and m contribute. But now integrating out particles of spin s_i with $N_d^{(i)}$ degrees of freedom leads to a contribution [40]

$$\delta V = \sum_i N_d^{(i)} \frac{(-)^{2s_i}}{64\pi^2} m_i^4 \log \frac{m_i^2}{\mu^2}, \tag{2.6}$$

where i runs over particles, of spin s_i and mass m_i, and $N_d^{(i)}$ counts the number of real degrees of freedom. Thus, the vacuum energy receives a contribution

$$\delta\Lambda \sim m^4 \tag{2.7}$$

from integrating out particles of mass m. Regardless of what renormalisation and fine-tuning we performed at high scales, low scales contribute to the cosmological constant.

This shows vividly what a problem the calculation of the vacuum energy density is: even the electron—a very low-energy particle by particle-physics standards—gives a contribution to the vacuum energy density that is too large by a factor of $(10^9)^4$. Since we believe we understand *low energy* electrodynamics very well, this is an enormous failure; even if we somehow solve the problem at high energies where we can add new, as-yet unmeasured physics to our heart's content, we cannot materially change electromagnetism in the infrared!

2.2 Deep symmetries

The essence of deep-symmetry-type solutions to the cosmological constant is to impose a relationship between different particles or different sectors in a theory to cancel the quantum contribution to the cosmological constant in 2.6. The best-studied such symmetry is supersymmetry (see appendix B). The important feature of supersymmetry for the cosmological constant problem is the relationship between the couplings and masses of particles of different spins: each bosonic particle has a fermionic partner with the same mass. Since the number of propagating degrees of freedom are given by $N_d^{(i)} = 1(2)$ for real (complex) scalars, $N_d^{(i)} = 2$ for Dirac spin-half particles, photons and gravitons, and $N_d^{(i)} = 3$ for massive vectors and $N_d^{(i)} = 2(4)$ for massless (massive) Rarita-Schwinger (spin-3/2) fields, we see from eq. 2.6 that equal masses between bosons and fermions can cleanly cancel the unwanted contribution to the cosmological constant:

$$\delta\Lambda = (+)N_d\, m^4 \log m^2/\mu^2 + (-)\, N_d\, m^4\, \log m^2/\mu^2 = 0. \tag{2.8}$$

The problem with all such symmetry-arguments is that these symmetries contradict other observations and therefore must be broken (usually quite badly). In the case of supersymmetry, no supersymmetric partners of the same mass as the standard model particles are seen, implying that supersymmetry, if true, is broken at a scale at least on the order of a TeV to ensure that $m_{\text{superpartner}} \approx m_{\text{SM}} + \text{TeV}$ and remain unseen. But this in turn ruins the delicate cancellations in eq. 2.8.

In order for supersymmetry (or indeed any four-dimensional symmetry) to be a solution to the cosmological constant problem it generically must be unbroken until $\sim 10^{-3}$eV. If a symmetry does remain unbroken to this scale, it is difficult to construct models which obey the symmetry but which nonetheless correctly reproduce well-known physics. If, on the other hand, the symmetry is broken at phenomenologically acceptable energies $\gg 10^{-3}$eV in order to recover standard phenomenology, the scale of the cosmological constant gets raised to the symmetry-breaking scale, ruining the solution.

2.3 Anthropic Arguments

The anthropic principle is often stated as 'The constants of nature are measured to be what they are because if they were different we would not be here to measure them.' This is not a completely content-free statement if and only if there is a probabilistic framework in which to phrase the question of the values of the various constants [27].

Before Newton's theory of gravitation the explanation of the earth-sun distance was a question of fundamental significance. Kepler attempted to explain the earth-sun distance using nested Platonic solids [27]. Following these efforts it is not surprising that Newton's theory of gravitation was met with disappointment; it did not predict the earth's orbit uniquely. (If only one solar system is known, it is a miracle that the earth is precisely the right distance away from the sun to allow water to exist in liquid form. Of course we now understand the earth's finely-balance orbit to be an historical accident; there is an ensemble of solar systems from which we may sample in order calculate the number of habitable planets in the milky way ($\sim 4 \times 10^{11}$ [38]).)

The probability of the observation of a particular value of the cosmological constant is preconditioned by the requirement that there exist observers to measure the vacuum energy. The existence of observers is dependent on a sufficient density of baryons condensing into galaxies to form stars and solar systems. Let ρ_V be the the observed vacuum energy and let S be the event that stars form. Let $P(\rho_V)\, d\rho_V$ be the *a priori* probability of the occurance of a vacuum energy between ρ_V and $\rho_V + d\rho_V$, and let $P(S)$ be the probability that sufficient number of baryons condense into galaxies to form stars. Further, let $P(X|Y)$ represent the contingent probability of the occurance of X, given that Y has occured, then

$$P(\rho_V|S)d\rho_V = \frac{P(\rho_V \& S)}{P(S)}\, d\rho_V, \tag{2.9}$$

using Bayes' rule for contingent probabilities. Since $P(\rho_V \& S) = P(S \& \rho_V)$,

$$P(\rho_V \& S) = P(\rho_V)\, P(S|\rho_V) \tag{2.10}$$

so we can re-write 2.9 as

$$P(\rho_V|S)d\rho_V = \frac{P(S|\rho_V)\, P(\rho_V)}{P(S)}\, d\rho_V. \tag{2.11}$$

The probability that the cosmological constant is small given that stars formed is given by $P(S|\rho_V)$, the probability that stars form given the small value of the cosmological constant, multiplied by the *a priori* probability distribution for the cosmological constant, and normalised by $P(S)$, the *a priori* probability for stars to form.

We do not know how to calculate either $P(S)$ or $P(\rho_V)d\rho_V$, but we can calculate $P(S|\rho_V)$ with reasonable astrophysical assumptions [29], and since *some* value of ρ_V is seen $P(S)$ can be subsumed into a normalisation constant for the probability distribution function through

$$\int d\rho_V P(\rho_V|S) = 1. \tag{2.12}$$

Finally, the *a priori* calculation of $P(\rho_V)$ can be circumvented because $P(S|\rho_V)$ is very sharply peaked around very small (by particle-physics standards) values of ρ_V [29], so in this range $P(\rho_V)$ can be taken to be constant. [1] Under these assumptions it can be shown [29] that

$$P(\rho_V \approx (10^{-3}\mathrm{eV})^4|S) \approx 10\% - 15\%. \tag{2.13}$$

More recent contributions to the anthropic proposal involve counting vacua in string compactifications [32]: a typical compactifying manifold has

[1] This is not a trivial assumption, but there exist a large class of models in which it is satisfied [28] (but see Garriga and Vilenkin [37]).

several hundred cycles around which fluxes can be wrapped, leading to estimates for the number of string theory vacua $\sim x^{100}$. These proposals are usually coupled to an idea like eternal inflation that allows the space of these vacua to be populated.

Despite the analogies between the earth-sun distance and the unlikely value of the physical constants of nature, and despite the seeming simplicity of anthropic solutions, they remain controversial for two reasons. The first reason is that it is difficult to do anything but estimate the variety and number of vacua in string theory since a full formulation is still lacking. Another criticism argues that an anthropic explanation is tantamount to a retreat from the standard ontological framework of science, since it rejects a predictive basis for the basic structure of the universe. (See [33] for an historical review of anthropic reasoning with references.)

I will not have anything more to say about the ongoing debate regarding the anthropic principle in this thesis. It seems safe to say that an anthropic vacuum-selection mechanism requires a new, additional structure to be imposed on physical theories that is not well-tested, but that it may offer some insight to the cosmological constant problem. In short, the author remains firmly agnostic on this issue.

2.4 Quintessence

Quintessence models can provide the right equation of state for dark energy,

$$w \equiv p/\rho \sim -1 \tag{2.14}$$

because

$$w = (K - V)/(K + V) \tag{2.15}$$

in these models, where K the kinetic, and V the potential energy. For slowly-varying fields, $K \ll V$, and thus $w \approx -1$.

Quintessence theories naturally have 'tracker' solutions [118] in which the quintessence field's energy tracks the local matter energy-density. As long as this tracking holds, we expect the vacuum energy to be $\sim 10^{-3}$eV, since this is the scale of matter energy-density. The smallness of the matter energy density, in turn, is explained by the age of the universe.

It is difficult to make quintessence models behave appropriately in four dimensions, since the scalar potential suffers from the same instability as the Higgs potential. In essence, these models do not address the cosmological constant problem at all, since quantum corrections tend to lift the quinessence field's mass to be of the same order as all of the other standard model masses. If the quintessence field is very heavy, it cannot hope to track very-low matter energy-densities. Four-dimensional quintessence seems not to help essentially with the fundamental cosmological constant problem [122]. It is worth noting that in higher dimensions, however, viable quintessence models can, in fact be constructed [119, 120, 121].

2.5 Infrared Modifications of Gravity

One class of these models—inspired by the Bekenstein-Hawking bound on entropy—lowers the effective cutoff for the cosmological constant by ensuring that the vacuum energy density of the space within the Hubble horizon, $L \sim 10^{-33}$eV, is less than the Schwarzschild mass for the same length-scale,

$M^2_{\text{Planck}}\, L$ (a proxy for the requirement that the universe not collapse to a black hole) [25]:

$$\mathcal{L}^3 \omega_{\text{UV}}^4 \sim M^2_{\text{Planck}}\, L \tag{2.16}$$

so that the cutoff,

$$\omega_{\text{UV}} \sim \left(M^2_{\text{Planck}}\, L\right)^2 \sim 10^{-3}\text{eV}. \tag{2.17}$$

These models seem to suffer from an incorrect equation of state, however [26] since if the cosmological constant scales as $\Lambda \sim L^{-2}$ then because $L \sim a^{3/2}(t)$, where $a(t)$ is the scale factor,

$$\Lambda \sim a^3(t), \tag{2.18}$$

implying that it has the equation of state of cold dark matter, which is ruled out by WMAP [63].

The proposal for the cosmological constant presented in this thesis hinges on lowering the cutoff relevant for the cosmological constant problem, but does so through a combination of large extra dimensions and supersymmetry.

2.6 Summary

Observations from supernovae and from measurements of the CMB seem unequivocally to require dark energy of some sort, and that the value of the cosmological constant has been approximately 10^{-3}eV since photon-matter decoupling. Within classical general relativity the cosmological constant is simply another parameter, consistent with the assumed coordinate invariance of the theory, that must be included in the low-energy lagrangian; it must simply be measured and then used to make other predictions.

The cosmological constant gets elevated to the level of a *problem*, however, when we couple gravity to theories of particle physics because it is a relevant parameter in the IR. Almost any physics of any sort which occurs at a scale M contributes to the cosmological constant an amount

$$\delta\Lambda \sim M^4 \tag{2.19}$$

This makes even the well-understood physics of electrodynamics problematic for the cosmological constant, because the cosmological constant must be tuned, order by order in perturbation theory to ensure that it remains small despite electron loops, $\sim m_e^4$.

Four-dimensional attempts to solve the cosmological constant are rife with problems of principle and with observational problems. In the rest of this thesis we explore extra-dimensional solutions to the cosmological constant problem. As we will explain in the next chapter, extra dimensions are expected to be a part of a theory of gravity. However, standard compactifications of higher-dimensional gravities do not materially change the analysis presented in this chapter because the compactification scale is so high. This will lead us to consider other higher-dimensional theories which allow a lower compactification scale which is compatible with the scale of the cosmological constant, $\sim 10^{-3}$eV.

Chapter 3

Extra-Dimensional Theories and Compactifcation

We have seen that it is difficult to solve the cosmological constant problem in the standard four-dimensional effective field theory framework. In the first chapter we made the case that the effective field theory programme is based on 'motherhood principles': quantum mechanics, locality and cluster decomposition. Given the difficulty in solving the cosmological constant problem under a given set of assumptions it may be worthwhile to relax some standard assumptions. In this chapter I present motivation for abandoning the assumption that the universe is essentially four-dimensional and will present some of the basic tools used to analyse the four-dimensional consequences of extra-dimensional theories.

As discussed in previous chapters, string theory seems to be the most promising candidate at present for a quantum theory of gravity. Historically, the quest for a consistent quantum theory of gravity has been a story of enlarging the space of possible solutions to the problem. As an example, it was recognised very early that string theory is only consistent in 26 dimensions (bosonic) or 10 or 11 dimensions (supersymmetric), requiring the development of mechanisms to hide these extra dimensions. The methodology for hiding these extra dimensions become a topic of research all its own, acquiring the monicker 'compactification.' Compactification, which we will describe in some detail in the early sections of this chapter, describes making the extra dimensions which are unwanted very small (compact) so as to be unobservable.

As we will see, any string-like theory that compactifies from higher dimensions to four fails to solve the cosmological constant problem, *even if the compactification is to flat four-dimensional space* . We will provide an explicit example in which the compactification is in fact (uniquely) to flat four-dimensional space, but in which the cosmological constant problem nevertheless rears its ugly head.

3.1 String Theory and String-Inspired Models

Depending on her background, the reader may well wonder why it is reasonable to enlarge the search for theories of particle physics to extra-dimensional theories. After all, it is clear the three spatial and one time dimension is all we have ever seen and that these are prefectly adequate for describing all physics ever measured in a laboratory.

As with many things in modern particle theory, this enlargement of the space of possible theories was hard-won, after a perceived failure to find solutions within the standard framework. The loosening of the ties to four-dimensional physics was very heavily dependent on the progress being made in string theory, and indeed nearly all extra-dimensional theories are 'string-inspired', if not true string models. Strings are regarded as being the most viable path towards a quantum theory of gravity, but have the peculiar feature of being consistent only in 26, 10 or 11 dimensions, depending on the particular flavour (not a technical word) of string [53, 54].

The consistency of the superstring in 10 dimensions (or of the bosonic string in 26 dimensions) arises as a conditon for anomaly cancellation. The superstring has a local conformal symmetry whose anomaly is given by the trace of the worldsheet stress tensor and is proportional to the number of sigma-model fields (the number of coordinates of spacetime) minus 10 [56][1]:

$$T^a{}_a \sim (D - 10) \tag{3.1}$$

The eleven-dimensional description of string theory arises from dualities. All of the string theories are believed to be equivalent to each other under various of these dualities, and to be particular vacua of M-theory. One of the low-energy representations of M-theory is the unique 11-dimensional supergravity.

Much of the string-theory model-building literature over the past twenty years has focused on ways of hiding these extra dimensions in ways satisfactory to experimental and phenomenological constraints [4]. The usual method for compactification is to begin with one of the supergravities which describes the low-energy string degrees of freedom, and to seek viable classical background solutions, around which one can quantise. The bosonic sector

[1] This statement is true for the free string in a flat background spacetime. In other, more complicated spacetimes the critical dimension can be different. See for example the linear dilaton spacetimes in ref. [60]. These configurations are unstable to decay to the supersymmetric flat-space configurations.

of the supergravities is typically of the form

$$[-g]^{-\frac{1}{2}}\,\mathcal{L} = \frac{M_{10}^8}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{e^{-2\phi}}{12}G_{MNP}^2 - \frac{e^{-\phi}}{4}F_{MN}^2 + \cdots\,, \tag{3.2}$$

where M_{10} is the ten-dimensional Planck scale, $G = dB + F \wedge A + \omega \wedge R$, and the dots indicate higher-derivative terms required for anomaly cancellation. One seeks an $\mathcal{N} = 1$ supersymmetric four-dimensional vacuum [2] By making the most general metric and field *ansatz* consistent with maximally symmetric four-dimensional space:

$$\begin{aligned} ds^2 &= W^2(y)\, g_{\mu\nu}dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n \\ F &= F_{mn}(y)\, dy^m \wedge dy^n \\ G &= G_{mnp}\, dy^m \wedge dy^n \wedge dy^p \\ \phi(y) &= \phi(y) \end{aligned} \tag{3.3}$$

where x^μ and y^m are the four-dimensional and extra-dimensional coordinates, respectively, and the curvature calculated from $g_{\mu\nu}$ alone, $\widehat{R}_{\mu\nu}(g_{\mu\nu}) = \Lambda_4 g_{\mu\nu}$, the equations of motion become equations for the metric and field-strength functions. The most phenomenologically favourable compactifications are those to flat space $g_{\mu\nu} = \eta_{\mu\nu}$ and which have one supersymmetry.

The conditions which imposes $\mathcal{N} = 1$ 4D supersymmetry impose a number of conditions, which amount to

$$g_{\mu\nu} = \text{AdS or Minkowski}, \tag{3.4}$$

(we impose the further constraint that $g_{\mu\nu} = \eta_{\mu\nu}$ since we definitely don't live in Anti-de Sitter space),

$$W = \text{const} \tag{3.5}$$

[2]More supersymmetry doesn't allow complex representations for the fermions, and it was/is still believed that weak-scale supersymmetry is good phenomenological thing.

(no warping)

$$R_{mn} = 0 \tag{3.6}$$

(Ricci-flatness of the internal manifold),

$$\partial\phi = G = 0, \tag{3.7}$$

and a particular non-zero F. The internal manifolds, g_{mn} have an additional interesting property (if the spin connection is embedded in the gauge connection) that they are manifolds of $SU(3)$ holonomy, which means that spinors parallel-transported around a closed path come back to an $SU(3)$-transformed version of themselves [3] [56, 54].

There are many generalisations of the above methods to more complex spacetimes, but all realistic string models in 4D share backgrounds for which the spacetime is 4D Minkowski times an internal 6D manifold, and they have chiral fermions in 4D with spectrum close to the Standard Model. The simplest models also have important unsolved issues such as supersymmetry breaking and the presence of moduli fields such as the dilaton [73]. Several ideas have been put forward to deal with these issues, including nonperturbative effects, such as gaugino condensation [74], and the introduction of fluxes of antisymmetric tensor fields [75]. Furthermore brane/antibrane systems and intersecting branes have been considered both to obtain realistic models with broken supersymmetry [76] and for the possibility of generating cosmological inflation [77, 78].

[3]This condition arises because the condition for one intact four-dimensional supersymmetry is equivalent to the condition that there be a covariantly constant spinor. Requiring $\mathcal{N} = 1$ 4D supersymmetry requires $R_{mnpq}\Gamma^{pq}\xi = 0$ (Γ are the 10-dimensional Dirac matrices and ξ is the supersymmetry-variation parameter) to leave one component of ξ invariant (so that not all the supersymmetry is broken). The subgroup of $SO(6)$ (the internal space's tangent group) which accomplishes this is SU(3).

However, there is at the moment not a single model that can achieve all the successes simultaneously. For instance, to get inflation it is needed to assume that some of the moduli have been fixed by an unknown mechanism. Fluxes of Ramond-Ramond fields have been used to fix some moduli but not all of them. Supersymmetric models have to face the breaking of supersymmetry, and gaugino condensation and other nonperturbatively generated superpotentials usually lead to runaway potentials [74, 79]. Non-supersymmetric models such as brane/antibrane systems at singularities or intersecting brane models tend to be unstable, with the corresponding scalar potential not under control.

One of the aims of this chapter is the development of a six-dimensional toy-model of string compactifications which can lead to a simpler setting in which to investigate some of these issues. Before doing so, however, compactification is more fully developed in the next few sections.

3.2 A Detailed Example of Compactification

In this section we perform the compactifcation of a very simple model in some detail to exhibit the physical and mathematical characteristics of the procedure. The main points are that by making the internal dimensions small, we decompose one higher-dimensional degree of freedom into an infinite tower in a specific way. The tower of modes have characteristic mass $\sim 1/R$, where R is the typical size of the compactified dimensions. Therefore, at low energies, $E \ll 1/R$, the theory looks entirely four-dimensional. We will see that the four-dimensional theory thus obtained may have additional characteristics beyond those naively expected from the higher-dimensional theory, and we will see that these can nevertheless be understood in terms of additional symmetries of the underlying compactifying manifold.

Consider a complex scalar field in five dimensions,

$$\mathcal{L} = -|\partial_M \phi|^2 - m^2|\phi|^2 - V(\phi, \phi^*) \tag{3.8}$$

where the five dimensions are chosen to be $\mathcal{M}_4 \times S^1$, four-dimensional Minkowski space times a circle of radius R.[4] We wish to demonstrate that for energies small compared to the inverse radius, $E \ll 1/R$, the theory looks like a four-dimensional scalar field theory. For higher energies a tower of higher-mass states will appear, revealing the extra dimension through normal-mode oscillations of the field in the extra dimension. We denote by $M, N, \cdots$ indices in the full five dimensions, by $\mu, \nu, \cdots$ indices in four dimensions and by z and x, respectively, coordinates in all five and in four dimensions. If we refer only to the fifth-dimensional coordinate, we use y.

In the path integral, correlation functions and amplitudes are calculated by summing over all configurations of $\phi(z)$. This can be done in any number of ways, for example by latticising the spactime in which the ϕ's live. Another way to do the path integral is to expand the functions in normal modes of the quadratic operator in the lagrangian, effectively diagonlising the gaussian integration. This latter method provides a means of regularising high-energy contributions in a way specifically tied to the size of the extra dimensions, allowing us to ask questions about the extra dimensions naturally, particularly simplifying the discussion of the low-energy effective theory.

[4]This choice is for now arbitrary and is made for simplicity. In the supergravity theory that is presented below the background metric is a solution to the equations of motion.

Any function of the five-dimensional coordinates, z, can be expanded as

$$\phi(z) = \sum_n \phi_n(x)\chi_n(y) \tag{3.9}$$

where $\chi(y)$ are any set of complete modes in the fifth dimension. If $\chi(y)$ are eigenfunctions of the extra-dimensional quadratic operator,

$$-(\partial_5)^2 + m^2 \tag{3.10}$$

with eigenvalues λ_n, then the lagrangian 3.8 will be given by

$$\mathcal{L} = -\sum_{m,n}\left[\frac{1}{2}\partial_\mu\phi_n^*(x)\partial^\mu\phi_m(x)\chi_n^*(y)\chi_m(y) - \frac{\lambda_n}{2}\phi_m^*(x)\phi_n(x)\chi_m^*(x)\chi_n(x)\right] \tag{3.11}$$

Since the mass operator, eq. 3.10 is self-adjoint,

$$\int dy\, \chi_n^*(y)\chi_m(y) = \delta_{mn}, \tag{3.12}$$

the action is given by

$$S = \sum_n \int d^4x \; -|\partial_\mu\phi_n(x)|^2 - \lambda_n|\phi_n(x)|^2 + \cdots \tag{3.13}$$

where the dots represent terms in V in eq. 3.8 arising from inserting eq. 3.9. We see that the single (free) five-dimensional scalar field has decomposed into a tower of four-dimensional scalar fields, each mode representing a possible normal-mode oscillation of the field in the extra dimension. (This is simply a result of $E = mc^2$, the vibrational energy in the extra dimension being translated into rest-mass energy in four dimensions.)

The dots in 3.13 represent interactions. For example, a five-dimensional interaction term such as

$$\int d^5z\, \frac{g}{24}|\phi(z)|^4 \tag{3.14}$$

becomes

$$\sum \int d^4x \, \frac{\widetilde{g}_{mnpq}}{24} \, \phi_m^* \phi_n^* \phi_p \phi_q(x) \tag{3.15}$$

where

$$\widetilde{g}_{mnpq} = g \int dy \chi_m^* \chi_n^* \chi_p \chi_q(y) \tag{3.16}$$

Let us consider in detail the particular case of a circle of radius R (so that y is periodic with period $2\pi R$), where the eigenfunctions are given by,

$$\chi_n = \frac{e^{in\theta}}{\sqrt{2\pi R}} \tag{3.17}$$

and the corresponding eigenvalues are

$$\lambda_n = \frac{n^2}{R^2} + m^2 \tag{3.18}$$

and which give the mass of the purely four-dimensional mode ϕ_n.

The lowest-energy modes are those with $n = 0$, and it is these modes which appear in the low-energy theory. In order to pass to the low-energy limit we must account for the effects of the massive modes with $n > 0$. We may do so classically by solving the equations of motion for ϕ_n with $n > 0$ in terms of ϕ_0 and inserting the result back into the action. As we will show, in this particular case $\phi_n = 0$ is a minimum of the potential, so that the *truncation* of the theory to the $n = 0$ modes is *consistent*. This is not always the case.[5]

[5]The term *consistent truncation* is used in the supergravity literature to mean any trucation of the lower-dimensional fields such that a solution of the lower-dimensional equations of motion is also a solution of the higher-dimensional equations of motion. Since in supergravity one is usually interested in lifting solutions *exactly* back to the higher-dimensional theory and not necessarily the scattering of particles, one is often more interested in consistent truncations. In low-energy effective descriptions we must integrate out the massive modes to correctly incorporate the effects of heavy modes in the light modes.

Let us see that $\phi_n = 0$ for $n > 0$ is a minimum of the potential arising from $V = \frac{g}{24}\phi^4(z)$. After substituting the expansion for ϕ, eq. 3.9 into V and integrating over y, we get

$$
\begin{aligned}
\mathcal{L}_4 &= -|\partial\phi_0|^2 - \sum_m |\partial\phi_m|^2 - m^2|\phi_0|^2 - \sum_m \lambda_m|\phi_m|^2 \\
&- \frac{g}{24\,R}\left[4\sum_m |\phi_m|^2|\phi_0|^2 + \sum_m \phi_m\phi_{-m}(\phi_0^*)^2 + \sum_m \phi_m^*\phi_{-m}^*(\phi_0)^2\right. \\
&+ 2\sum_{mn} \phi_m\phi_n\phi_{m+n}^*\phi_0^* + 2\sum_{mn}\phi_m^*\phi_n^*\phi_{m+n}\phi_0 \\
&+ \left.\sum_{mnp} \phi_m^*\phi_n^*\phi_p\phi_{m+n-p} + |\phi_0|^4\right] \qquad (3.19)
\end{aligned}
$$

Because of the appearance of $\chi_n \sim e^{iny/R}$ with every occurrence of ϕ_n, a conservation condition has arisen, in which ϕ_n carries n units of an abelian charge. This is directly related to the $U(1)$ isometry of the compactifying manifold, S^1, and is a general feature of compactifications: isometries of the underlying compactified manifold express themselves as symmetries of the compactified theory. When we consider the compactification of gravitational phenomena the global symmetry gets gauged, as we will see in the supergravity example presented later in this chapter. In the next section I will prove both of these assertions.

Clearly $\phi_n = 0$ is an extremum. To see that it is stable (a minimum) we show that the second derivative of the ϕ_m mass matrix is always positive-definite at $\phi_m = 0$. Writing ϕ_m in terms of its real and imaginary parts,

$$\phi_m = \frac{1}{\sqrt{2}}\left(\rho_m + i\sigma_m\right) \qquad (3.20)$$

we find that

$$V_{\rm quad} = m^2|\phi_0|^2 + \frac{1}{2}\sum_m \lambda_m(\rho_m^2 + \sigma_m^2)$$

$$+\sum_m \begin{pmatrix}\rho_m\\ \rho_{-m}\\ \sigma_m\\ \sigma_{-m}\end{pmatrix}^{\mathrm{T}} \widetilde{M} \begin{pmatrix}\rho_m\\ \rho_{-m}\\ \sigma_m\\ \sigma_{-m}\end{pmatrix} \tag{3.21}$$

where

$$\widetilde{M} = \begin{pmatrix} A & B\\ B & D\end{pmatrix} \tag{3.22}$$

and

$$A = \begin{pmatrix} 2|\phi_0|^2 & \mathcal{R}\\ \mathcal{R} & 2|\phi_0|^2\end{pmatrix} \qquad B = \begin{pmatrix} 0 & \rho_0\sigma_0\\ \rho_0\sigma_0 & 0\end{pmatrix} \qquad D = \begin{pmatrix} 2|\phi_0|^2 & -\mathcal{R}\\ -\mathcal{R} & 2|\phi_0|^2\end{pmatrix}, \tag{3.23}$$

where

$$\mathcal{R} = (\rho_0^2 - \sigma_0^2) \tag{3.24}$$

is the real part of ϕ_0^2. Since the term $\sum_m \lambda_m (\rho_m^2 + \sigma_m^2)$ is minimised at $\phi_m = 0$, in order to show that $\phi_m = 0$ is a minimum it is enough to show that it is a minimum of the term containing $\widetilde{M}$. To do this, note that

$$\det\widetilde{M} = \det\begin{pmatrix} A & B\\ C & D\end{pmatrix} = [\det A]\left[\det\left(D - CA^{-1}B\right)\right]. \tag{3.25}$$

Since $\det A = \det D$, and $A^{-1} = D/\det A$, it's easy to show that

$$\det\widetilde{M} = [\det A]\,[\det D]\,\det\left[(1 - \det B/\det A)\right]. \tag{3.26}$$

Now, since

$$\det A = 3\rho_0^4 + 10\rho_0^2\sigma_0^2 + 3\sigma_0^4 \geq 0, \tag{3.27}$$

and since

$$\det B = \rho_0^2\sigma_0^2, \tag{3.28}$$

we have that at

$$\det\widetilde{M} \geq 0, \tag{3.29}$$

with equality *iff* $\phi_0 = 0$. Therefore the low-energy effective action, valid for energies $\ll m + 1/R$ is given by

$$S = \int d^4x \ -\eta^{\mu\nu}\partial_\mu\phi_0^*\partial_\nu\phi_0 - m^2|\phi_0(x)|^2 - \frac{\lambda}{4!}\,|\phi_0(x)|^4, \tag{3.30}$$

where

$$\lambda = \frac{g}{2\pi R} \tag{3.31}$$

is the effective four-dimensional coupling.[6] The $U(1)$ charge that is carried by ϕ_0 is not the $U(1)$ charge arising from the extra dimensions, and which was carried in n units by the n^{th} modes—ϕ_0 carries no units of this charge, it, along with all the higher modes, carries the original $U(1)$ charge conserved by the five-dimensional action, eq. 3.8.

Let us now recap what occured in the previous example, with an eye to generalising what we can:

- Each higher-dimensional mode results in a tower of lower-dimensional modes. If all of these modes are kept, no information is lost.
- We may pass to the low-energy limit by accounting for the effects of the tower of four-dimensional higher-mass states. Classically we may do so by solving the equations of motion for the higher modes in terms of the light modes.
- The lower-dimensional modes are organised by symmetry: Any of the original symmetries of the theory that are unbroken by the background

[6]This action is most easily (and is often) obtained by making the *ad hoc ansatz* $\phi(z) = \widetilde{\phi}(x)$, in which the five-dimensional field is independent of the fifth coordinate. This trick only works for toroidal and other flat geometries in the absence of gauge fields. The framework we present is the general one.

help to organise the lower-dimensional theory. In addition, new symmetries may arise as a result of isometries of the background solution about which we are expanding.

As a result the most efficient way to achieve the compactified low-energy theory is to first identify the symmetries and to count the massless modes that are protected by such symmetries. Examples include any gauge fields whose four-dimensional gauge symmetry is unbroken by the background. Next, one counts the new symmetries arising from the background and organises the states in multiplets of these symmetries. This information is usually enough to severly constrain the theory, reducing the problem of compactification to the calculation of a few parameters.

* * *

It is not always the case that we may set the massive fields to zero. As an example consider the most general two-field four-dimensional scalar potential, given by

$$\begin{aligned} V(\psi,\phi) &= \frac{1}{2}\left[m^2\phi^2 + M^2\psi^2\right] + \frac{1}{2}\left[\mu\phi\psi^2 + \widetilde{\mu}\phi^2\psi\right] \\ &\quad + \frac{\rho}{3!}\,\phi^3 + \frac{\widetilde{\rho}}{3!}\,\psi^3 + \frac{\lambda}{4!}\,\phi^4 + \frac{\widetilde{\lambda}}{4!}\,\psi^4 \end{aligned} \tag{3.32}$$

The stationary point with respect to ψ is given by

$$\frac{\partial V}{\partial \psi} = \psi\left[M^2 + \widetilde{\mu}\phi + \frac{1}{2}\widetilde{\rho}\psi + \frac{\widetilde{\lambda}}{3!}\,\psi^2\right] + \frac{1}{2}\mu\phi^2 = 0, \tag{3.33}$$

whose perturbative solution (the solution that is analytic in all the non-quadratic parameters) is given by

$$\psi = -\frac{\mu}{2}\left(\frac{\phi}{2}\right)\left[1 - \frac{\widetilde{\mu}}{M}\frac{\phi}{M} + O\left(\frac{\phi^2}{M^2}\right)\right] \tag{3.34}$$

Plugging this back into the potential gives

$$V_{\text{eff}} = \frac{m^2}{2}\phi^2 + \frac{\rho}{3!}\phi^3 + \frac{1}{4!}\left(\lambda - 3\,\frac{\mu^2}{M^2}\right)\phi^4 + \frac{1}{8}\,\frac{\mu^2\widetilde{\mu}}{M^4}\,\phi^5 + O(\phi^6). \qquad (3.35)$$

Clearly the effect of integrating out ψ is nontrivial, the cubic term linear in ψ being the culprit.

Decoupling seems to hold in the above potential, as, when $M \to \infty$ all of the effects of the high-mass particle drop out. Decoupling can fail, however, when $\mu = \alpha M$, since then

$$V_{\text{eff}} \to V(\psi = 0, \phi) - \frac{3\,\alpha}{4!}\,\phi^4. \qquad (3.36)$$

This is problematic because the two methods for obtaining the action do not agree in the $M \to \infty$ limit. One must always integrate out as opposed to trucate for valid low-energy descriptions. (This issue can be important, for example, in compactifications of the superstring on Calabi-Yau backgrounds [11].)

3.3 Isometries and Symmetries

Let $\mathcal{L}$ be a lagrangian in $4{+}n$ dimensions depending on fields which we collectively call ϕ, and which may or may not include the metric. We continue to use the notation of the previous section: $M, N, P, \cdots$ refer to indices running over the full $4+n$ dimensions, z referes to coordinates in $4+n$ dimensions, while x^μ and $\mu, \nu, \rho, \cdots$ are coordinates and indices in four dimensions and y^m and $m, n, p, \cdots$ are coordinates and indices in the extra n dimensions. Let

$$< \phi >= \varphi(y) \qquad (3.37)$$

be the background values of the fields, and let them be invariant under an isometry

$$[\mathcal{Z}_i, \varphi(y)] = 0, \tag{3.38}$$

where $\mathcal{Z}_i$ are hermitian differential operators encoding the isometry group of the background:

$$[\mathcal{Z}_i, \mathcal{Z}_j] = iC^k{}_{ij}\mathcal{Z}_k \tag{3.39}$$

(In a gravitational theory the $\mathcal{Z}_i$ would be the Killing vectors and the bracket in eq. 3.38 would refer to the Lie derivative in the Killing direction. For the metric with Killing vectors ξ_i we would have

$$\xi_i^T \partial_M g_{TN} + \xi_i^T \partial_N g_{MT} + \xi_i^T \partial_T g_{MN} = 0, \tag{3.40}$$

so

$$\mathcal{Z}_{i(MN)}{}^{RS} = -i\left[\delta_M{}^T \delta_E{}^R \delta_N{}^S + \delta_N{}^T \delta^S{}_E \delta^R{}_M + \delta_M{}^R \delta_N{}^S \delta_E{}^T\right] \xi_i^E \partial_T \tag{3.41}$$

generates eq. 3.40 through

$$\left[\mathcal{Z}_{i(MN)}{}^{RS}, g_{RS}\right] = 0 \tag{3.42}$$

The same isometry acting on a scalar takes the much simpler form

$$\mathcal{Z}_i = -i\xi_i{}^M \partial_M \tag{3.43}$$

In the example in the previous section, of a five-dimensional field reduced on a circle to M_4, we would have a single generator of isometries,

$$\mathcal{Z} = -i\,\partial_y.) \tag{3.44}$$

The $\mathcal{Z}_i$ naturally organise the four-dimensional fields. Since by assumption the group of isometries is compact there exist a complete set of functions, $\chi_n(y)$, which furnish a representation of the isometry group:

$$[\mathcal{Z}_i, \chi_n] = \mathfrak{Z}_{nm}\chi_m \tag{3.45}$$

where only a finite number of elements of $\mathfrak{Z}$ are non-zero; in other words we may construct finite-dimensional irreducible representations of the group, the representations being labeled by ℓ, say, such that

$$\left[\mathcal{Z}_i, \chi_n^{(\ell)}\right] = \mathfrak{Z}_{nm}^{(\ell)}\chi_m^{(\ell)} \tag{3.46}$$

with $m, n = 1 \cdots N$ for an N-dimensional representation. (The reader should keep in mind the spherical harmonics as a prototypical example, with the $\mathcal{Z}_i$ the three differential angular momentum operators, and $\mathfrak{Z}$ given by the appropriate represenation's matrices.) We may expand any $4{+}n$-dimensional function in terms of the χ:

$$\phi(x, y) = \varphi(y) + \sum \chi(y)\phi(x). \tag{3.47}$$

Let Z_i be the quantum operators that act on the Hilbert space. Then,

$$\begin{aligned} [Z_i, \phi(x, y)] &= [\mathcal{Z}_i, \phi(x, y)] \\ &= [\mathcal{Z}_i, \varphi] + \sum \phi_n(x)\,[\mathcal{Z}_i, \chi_n] \\ &= \sum \phi_n(x)\mathfrak{Z}_{nm}\chi_m \end{aligned} \tag{3.48}$$

but also,

$$[Z_i, \phi(x, y)] = \sum [Z_i, \phi(x)]\,\chi \tag{3.49}$$

Since the $\chi(y)$ are linearly independent, we learn that

$$[Z_i, \phi_m(x)] = \mathfrak{Z}_{mn}\phi_n(x) \tag{3.50}$$

That is, the ϕ_n fill out representations of the isometry group.

If the field being compactified is not a spacetime scalar, we must also account for its *covariance* with respect to coordinate changes. This additional complication is easily dealt with by generalising the expansion 3.47 to include tensorial or spinorial eigenfunctions. For example, a vector $V_M(z)$ would expand as

$$\begin{aligned} V_\mu(x,y) &= \sum V_\mu^{(i)}(x)\chi^{(i)}(y) \\ V_m(x,y) &= \sum V^{(i)}(x)\chi_m^{(i)}(y) \end{aligned} \tag{3.51}$$

where $\chi^{(i)}$ are the usual scalar eigenfunctions and ${\chi^{(i)}}_n(y)$ are spacetime-vector eigenfunctions.[7]

This proves that compactifying on an internal manifold with isometries yields a lower-dimensional theory which represents the isometries as internal symmetries on the fields, since Z_i are coordinate transformations and $\mathcal{L}$, the lagrangian is invariant under Z_i: $[\mathcal{Z}_i, \mathcal{L}] = 0$.

The previous treatment is independent of whether or not we are performing the compactification in a gravitational theory. It is clear, however, that on a non-flat manifold derivatives of fields will not transform simply under the isometries. It turns out that the covariant derivative, which ensures coordinate invariance, also ensures that the charges carried by the Zs, above, remain conserved. The symmetry is lifted from a global symmetry to a local symmetry due to the local coordinate invariance of the theory, with the gauge

[7]The eigenfunctions with more complicated spacetime transformation properties can be constructed straightforwardly by constructing the appropriate differential operators $\mathcal{Z}$ which encode isometries on these objects and by solving the resulting Laplace-like equation that arises from the quadratic Casimir, $\sum_i Z_i^2 = \lambda$. The appropriate $\mathfrak{Z}_i$ are given by the Lie derivative of the objects in the Killing direction, or, equivalently, by requiring the form-invariance of the object under the isometry.

fields arising from the $m\mu$ components of the metric—the metric components with one index in the extra dimensions (an index that is a four-dimensional spacetime scalar but which transforms as a vector under the isometries of the internal manifold) and an index in the four dimensions.

We only show the gauge symmetry encoded in the massless modes, a very small subgroup of the full higher-dimensional diffeomorphism group. As shown for example in [44], there exists an infinite tower of lower-dimensional gauge symmetries which are Higgsed by the compactification procedure.

Gravity and Gauged Isometries

In order to show that the isometries of the internal manifold become encoded as gauge symmetries in the lower-dimensional theory we will look at the transformation properties of the various components of the vielbein, E_M^A (see appendix A). This will lead to an *ansatz* for the vielbein which correctly encodes the gauge symmetry in a four-dimensional vector field $A^{(i)}{}_\mu$, and correctly describes the massless gravitational degrees of freedom in a four-dimensional vierbein, e_μ^α. I adopt the convention for tangent-space indices to match the world index conventions: $A, B, \cdots$ correspond to the full $4+n$-dimensional tangent space, $a, b, \cdots$ correspond only to the internal tangent-space directions, while $\alpha, \beta, \cdots$ correspond to the four-dimensional tangent space.

We assume that

$$\langle E_M^A \rangle = \begin{pmatrix} \langle E_\mu^\alpha \rangle & \langle E_\mu^a \rangle \\ \langle E_m^\alpha \rangle & \langle E_m^a \rangle \end{pmatrix} = \begin{pmatrix} e_\mu^\alpha(x) & 0 \\ 0 & e_m^a(y) \end{pmatrix} \tag{3.52}$$

where $e_\mu^\alpha(x)$ is the vierbein for the observable four dimensions and $e_m^a(y)$ is an invariant vector under the isometries encoded by the Killing vectors $\xi^{(i)m}(y)$:

$$e^a_m(y') = \frac{\partial y^p}{\partial y'^m} e^a_p(y), \tag{3.53}$$

which for infinitessimal transformations, $y' = y + \epsilon^{(i)}\xi^{(i)}$, becomes

$$\xi^{(i)n}\partial_n e^a_m + \partial_m \xi^{(i)n} e^a_n = 0. \tag{3.54}$$

We assume that the Killing vectors form an isometry group with structure functions ${C^k}_{ij}$

$$[{\xi^m}_i \partial_m, {\xi^n}_j \partial_n] = -{C^k}_{ij} {\xi^n}_k \partial_n \tag{3.55}$$

We wish for the four-dimensional gravitational interactions to be encoded in the vielbein, so we expect the graviton to make an appearance in the four-four components of the vielbein

$$E^\alpha_\mu \sim e^\alpha_\mu(x). \tag{3.56}$$

(Usually this is accompanied by a prefactor, $r^s(x)$, the volume modulus to some power. This can be included in the analysis which follows with no changes to the conclusions, but clutters the equations so we leave it out.)

Now let us perform a *local* transformation,

$$y'^m = y^m + \epsilon^{(i)}(x)\xi^{(i)m}(y) \tag{3.57}$$

on E^A_M:

$$\delta E^A_M = \epsilon^{(i)} {\xi^{(i)}}^n \partial_n E^A_M + \delta^m_M \epsilon^{(i)} \partial_m {\xi^{(i)}}^n E^A_n + \delta^\mu_M \partial_\mu \epsilon^{(i)} {\xi^{(i)}}^n E^A_n. \tag{3.58}$$

Examining the components separately, we find that

$$
\begin{aligned}
\delta E^{\alpha}_{\mu} &= \partial_{\mu}\epsilon^{(i)}\xi^{(i)n}E^{\alpha}_{n} && (3.59)\\
\delta E^{a}_{\mu} &= \epsilon^{(i)}\xi^{(i)^n}\partial_n E^{a}_{\mu} + \partial_{\mu}\epsilon^{(i)}\xi^{(i)^n}\,E^{a}_{n} && (3.60)\\
\delta E^{\alpha}_{m} &= \epsilon^{(i)}\left[\xi^{(i)^n}\partial_n E^{\alpha}_{m} + \partial_m\xi^{(i)^n}\,E^{\alpha}_{n}\right] && (3.61)\\
\delta E^{a}_{m} &= \epsilon^{(i)}\left[\xi^{(i)^n}\partial_n E^{a}_{m} + \partial_m\xi^{(i)^n}\,E^{a}_{n}\right] = 0 && (3.62)
\end{aligned}
$$

The E^{α}_{m} may be set to zero self-consitently, since E^{α}_{m} transform amongst themselves, which in turn implies that $\delta E^{\alpha}_{\mu} = 0$. Now inspection of

$$\delta E^{a}_{\mu} = \epsilon^{(i)}\xi^{(i)^n}\partial_n E^{a}_{\mu} + \partial_{\mu}\epsilon^{(i)}\xi^{(i)^n}\,e^{a}_{n}, \tag{3.63}$$

shows that they transform as vector-fields charged under a non-abelian symmetry generated by the Killing vectors $\xi^{(i)}$, and which satisfy eq. 3.55. We make the *ansatz*

$$E^{a}_{\mu} = \xi^{(i)^a}\,A^{(i)}_{\mu}(x), \tag{3.64}$$

which, upon repeated use of eq. 3.55 shows that

$$\xi^{(i)^a}\,\delta A^{(i)}_{\mu}(x) = \xi^{(i)^a}\,\left[\partial_{\mu}\epsilon^{(i)} + C^{i}{}_{jk}\epsilon^{(j)}\,A^{(k)}_{\mu}\right]. \tag{3.65}$$

This implies that A is a four-dimensional gauge field transforming in the adjoint of the group represented by the algebra eq. 3.55. Four-dimensional general covariance and this, new, gauge-invariance together imply that the degrees of freedom expressed in E^{A}_{M} in the *ansatze*

$$
\begin{aligned}
E^{\alpha} &= e^{\alpha}_{\mu}(x)\,dx^{\mu}, && \text{(graviton)}\\
E^{a} &= \xi^{(i)^a}\,A^{(i)}_{\mu}(x)\,dx^{\mu} && \text{(gauge field)} \qquad (3.66)
\end{aligned}
$$

can be expressed more simply and naturally than by the full extra-dimensional Einstein-Hilbert lagrangian,

$$S = \int d^N z \, \frac{1}{2} \, \widehat{R}. \tag{3.67}$$

These symmetries imply that 3.67 can be written as

$$S = \int dx \, V \left[\frac{1}{2} \, R(e) - \frac{1}{4 \, g^2} F_{\mu\nu} F^{\mu\nu} \right] \tag{3.68}$$

where g^2 is the coupling constant for the gauge field and can be calculated as an integral over the extra dimensions, y of the Killing vectors, $R(e)$ is the Ricci scalar constructed only from the four-dimensional vierbein, and $F = dA + A \wedge A$ is the usual non-abelian field-strength for the gauge field A [148].

In the example considered in detail in this section, of the compactification of six-dimensional supergravity to four dimensions, we will see this phenomenon explicitly. The internal manifold in this compactification is a two-sphere which has an $SO(3)$ isometry. This leads to the existence of an effective four-dimensional, confining, non-abelian gauge theory and to interesting consequences.

* * *

3.4 An Example of a String-like Compactification

Our aim is to explore the cosmological constant problem within a string context. In order to fully explore the cosmological constant problem within string theory, a compactification of it to four dimensions must be specified.

Any solution to the string equations of motion which claims to be a solution to the cosmological constant problem must simultaneously solve the problem of producing the correct four-dimensional particle physics, four-dimensional cosmology, and probably also a viable inflationary model.

In this section we show that most of these issues can be investigated in a setting that is much simpler than the explicit string constructions and yet shares the relevant properties of those models. The starting point is minimal 6D gauged supergravity coupled to at least one $U(1)$ vector multiplet. This model was studied in [146], who considered compactification on a two-sphere stabilized by a nonvanishing magnetic flux through the sphere for the $U(1)$ gauge field. This compactification was found to lead to a chiral $N = 1$ supersymmetric model in flat 4D spacetime at scales lower than the compactification scale. We here reconsider this model and study its consequences in more detail. We may see this as a first attempt to extract phenomenological implications to gauged supergravity potentials that are being derived recently in string theory. In this section we concentrate only on general issues concerning the model's low-energy effective action, supersymmetry breaking and moduli stabilization. We investigate how the introduction of branes changes the implications of this 'bulk' physics in subsequent chapters. (We also show in these later chapters in which we consider much more general compactifications that the particular compactification considered here is the unique supersymmetric vacuum that has the topology $M_4 \times M_2$.)

The model has several stringy properties, such as the presence of the standard S and T moduli fields of string compactifications, and we use these to address the issue of lifting the vacuum degeneracy. Furthermore, the low-energy theory has a Fayet-Iliopoulos term which is generated by the Green-Schwarz anomaly-cancelling mechanism. We find this induces a D-term potential in the low-energy theory that can naturally fix the field T, leaving only the S direction flat to all orders in perturbation theory.

The nonabelian $SO(3)$ symmetry associated to the isometries of the two-sphere is asymptotically free and so generates a nonperturbative potential for S through the gaugino condensation mechanism. Together with the tree-level Kähler potential, this leads to a runaway potential for the dilaton field S. This kind of scenario, with fixed T and with runaway S, could provide a natural way for the model to realize inflation, along the lines suggested in ref. [78]. We further argue that if certain perturbative corrections to the Kähler potential arise, then the gaugino-condensation potential need not generate a runaway, and could be used to stabilize S. As is expected on general grounds, this minimum arises at the margins of what can be computed using semiclassical methods.

We will show that the matter content of the resulting theory after compactification is a supersymmetric $SO(3) \times U(1)$ gauge model with two chiral multiplets, S and T. The expectation value of T is fixed by the classical potential, and S describes a flat direction to all orders in perturbation theory. We consider possible perturbative corrections to the Kähler potential in inverse powers of $\mathrm{Re}\, S$ and $\mathrm{Re}\, T$, and find that under certain circumstances, and when taken together with low-energy gaugino condensation, these can lift the degeneracy of the flat direction for $\mathrm{Re}\, S$. The resulting vacuum breaks supersymmetry at moderately low energies in comparison with the compactification scale, with positive cosmological constant. Finally, it is argued that the 6D model might itself be obtained from string compactifications, giving rise to realistic string compactifications on non Ricci flat manifolds. Possible phenomenological and cosmological applications are briefly discussed.

The next section gives a brief review of the relevant features of 6D gauged supergravity, and its supersymmetric compactification to 4D on a sphere. Section (3.4.2) then derives the low-energy 4D supergravity which describes the low-energy limit of this compactification, as well as discussing its likely vacuum. The flat directions of the low-energy theory are the topic of Section (3.4.4), where it is shown that all are lifted. This section shows that some moduli are stabilized at finite values, while others may be stabilized, or run to infinity, depending on the details of the corrections to the model's Kähler function. For a review of superpotential, Kähler functions and for the most general $\mathcal{N} = 1$ 4D supergravity in standard form, see appendix B.

The main results of this section are to show that the six-dimensional supergravity presented is a rich string-theory toy-model, and that, as it shares other features of string compactifications, it also shares the cosmological constant problem of string models. In the next chapter we discuss extensions of the compactifications of this model to ones including branes in order to address the cosmological constant problem within a stringy context.

3.4.1 The 6D Salam-Sezgin Model

We begin by recapping the Salam-Sezgin compactification of the six-dimensional supersymmetric Einstein-Maxwell system [144, 145, 146].

The Model

The field content of the theory consists of a supergravity multiplet – which comprises a metric (g_{MN}), antisymmetric Kalb-Ramond field (B_{MN}), dilaton (ϕ), gravitino (ψ_M) and dilatino (χ) – coupled to a $U(1)$ gauge multiplet – containing a gauge potential (A_M) and gaugino (λ).

The fermions are all complex Weyl spinors – satisfying $\Gamma_7\psi_M = \psi_M$, $\Gamma_7\lambda = \lambda$ and $\Gamma_7\chi = -\chi$ – and they all transform under the $U(1)$ gauge symmetry. For instance, the gravitino covariant derivative is

$$D_M\psi_N = \left(\partial_M - \frac{1}{4}\omega_M{}^{AB}\Gamma_{AB} - igA_M\right)\psi_N, \tag{3.69}$$

where $\omega_M{}^{AB}$ denotes the spin connection. Here g denotes the 6D $U(1)$ gauge coupling, which in fundamental units ($\hbar = c = 1$) has the dimension (mass)$^{-1}$.

The field strength for B_{MN} contains the usual supergravity Chern-Simons contribution

$$G_{MNP} = \partial_M B_{NP} + F_{MN}A_P + \text{(cyclic permutations)}\,, \tag{3.70}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the usual abelian gauge field strength. Notice that the appearance of A_M in this equation implies B_{MN} must also transform under the $U(1)$ gauge transformations, since invariance of G_{MNP} requires

$$\delta A_M = \partial_M\omega, \qquad \delta B_{MN} = -\omega\, F_{MN}\,. \tag{3.71}$$

This transformation allows the gauge anomalies due to the chiral fermions to be cancelled by a Green-Schwarz mechanism, as must happen if this supergravity emerges as a low-energy compactification of string theory (more about this below).

The bosonic part of the classical 6D supergravity action is:[8]

$$e^{-1}\mathcal{L}_B = -\frac{1}{2}R - \frac{1}{2}\partial_M\phi\,\partial^M\phi - \frac{e^{-2\phi}}{12}\,G_{MNP}G^{MNP} - \frac{e^{-\phi}}{4}\,F_{MN}F^{MN} - 2g^2e^{\phi}, \tag{3.72}$$

where we choose units for which the 6D Planck mass is unity: $\kappa_6^2 = 8\pi G_6 = 1$. As usual $e = |\text{det}e_M{}^A| = \sqrt{-\text{det}g_{MN}}$.

The part of the action which is bilinear in the fermions is

$$\begin{aligned} e^{-1}\mathcal{L}_F \;=\;& -\bar{\psi}_M\Gamma^{MNP}D_N\psi_P - \bar{\chi}\Gamma^M D_M\chi - \bar{\lambda}\Gamma^M D_M\lambda \\ & -\frac{1}{2}\partial_M\phi\left(\bar{\chi}\Gamma^N\Gamma^M\psi_N + \bar{\psi}_N\Gamma^M\Gamma^N\chi\right) \\ & +\frac{e^{-\phi}}{12\sqrt{2}}\,G_{MNP}(-\bar{\psi}^R\Gamma_{[R}\Gamma^{MNP}\Gamma_{S]}\psi^S + \bar{\psi}_R\Gamma^{MNP}\Gamma^R\chi \\ & \qquad\qquad - \bar{\chi}\Gamma^R\Gamma^{MNP}\psi_R + \bar{\chi}\Gamma^{MNP}\chi - \bar{\lambda}\Gamma^{MNP}\lambda) \\ & -\frac{e^{-\phi/2}}{4}\,F_{MN}\left(\bar{\psi}_Q\Gamma^{MN}\Gamma^Q\lambda + \bar{\lambda}\Gamma^Q\Gamma^{MN}\psi_Q - \bar{\chi}\Gamma^{MN}\lambda + \bar{\lambda}\Gamma^{MN}\chi\right) \\ & +ige^{\phi/2}\left(\bar{\psi}_M\Gamma^M\lambda + \bar{\lambda}\Gamma^M\psi_M + \bar{\chi}\lambda - \bar{\lambda}\chi\right), \end{aligned} \tag{3.73}$$

where the completely antisymmetric products of Dirac matrices are defined by $\Gamma_{MN} = \frac{1}{2}(\Gamma_M\Gamma_N - \Gamma_N\Gamma_M)$, $\Gamma_{MNP} = \frac{1}{6}(\Gamma_M\Gamma_N\Gamma_P \pm \text{permutations})$ and so on.

The 6D supersymmetry transformations which preserve the form of this

[8]Our metric is 'mostly plus' and like all right-thinking people we follow Weinberg's curvature conventions [80].

action are

$$
\begin{aligned}
\delta e^A_M &= \frac{1}{\sqrt{2}}\left(\bar\epsilon\Gamma^A\psi_M - \bar\psi_M\Gamma^A\epsilon\right)\\
\delta\phi &= -\frac{1}{\sqrt{2}}\left(\bar\epsilon\chi + \bar\chi\epsilon\right)\\
\delta B_{MN} &= \sqrt{2}\,A_{[M}\delta A_{N]} + \frac{e^{\phi}}{2}\left(\bar\epsilon\Gamma_M\psi_N - \bar\psi_N\Gamma_M\epsilon\right.\\
&\qquad\left. -\bar\epsilon\Gamma_N\psi_M + \bar\psi_M\Gamma_N\epsilon - \bar\epsilon\Gamma_{MN}\chi + \bar\chi\Gamma_{MN}\epsilon\right) \qquad (3.74)\\
\delta\chi &= \frac{1}{\sqrt{2}}\,\partial_M\phi\,\Gamma^M\epsilon + \frac{e^{-\phi}}{12}G_{MNP}\,\Gamma^{MNP}\epsilon\\
\delta\psi_M &= \sqrt{2}\,D_M\epsilon + \frac{e^{-\phi}}{24}\,G_{PQR}\,\Gamma^{PQR}\Gamma_M\epsilon\\
\delta A_M &= \frac{1}{\sqrt{2}}\left(\bar\epsilon\Gamma_M\lambda - \bar\lambda\Gamma_M\epsilon\right)e^{\phi/2}\\
\delta\lambda &= \frac{e^{-\phi/2}}{4}\,F_{MN}\,\Gamma^{MN}\epsilon - \frac{i}{\sqrt{2}}\,g\,e^{\phi/2}\,\epsilon\,,
\end{aligned}
$$

where the supersymmetry parameter is complex and Weyl: $\Gamma_7\epsilon = \epsilon$.

Global and Approximate Symmetries

Besides supersymmetry, the $U(1)$ gauge symmetry and the Kalb-Ramond symmetry, $\delta B = d\Lambda$, the model also has a few other symmetries (and approximate symmetries) which are useful to enumerate here for later convenience.

First, if the background spacetime admits an harmonic 2-form, Ω, then because B only enters the action through dB the background has a global symmetry $\delta B = c\,\Omega$, where c is the constant symmetry parameter. Because $\Omega \neq d\Lambda$ for any globally-defined 1-form this symmetry can be regarded as independent of the Kalb-Ramond gauge symmetry.

Second, the field equations obtained from this action have a classical symmetry under the following constant rescaling of the fields:

$$g_{MN} \to \sigma\, g_{MN}, \qquad e^{\phi} \to e^{\phi}/\sigma, \qquad \psi_M \to \sigma^{1/4}\,\psi_M, \qquad \chi \to \chi/\sigma^{1/4}, \qquad \lambda \to \lambda/\sigma^{1/4}, \tag{3.75}$$

with no other fields transforming. This is just a symmetry of the equations of motion, rather than a *bona fide* symmetry because it does not leave the action invariant, but rather rescales it according to $\mathcal{L} \to \sigma^2 \mathcal{L}$. This symmetry reflects the possibility of performing redefinitions to write the Lagrangian density as $\mathcal{L} = e^{-2\phi}\,\mathcal{L}_{\rm inv}$, with $\mathcal{L}_{\rm inv}$ a function only of the invariant 'string-frame' quantities $g^s_{MN} = e^{\phi}\, g_{MN}$, $\psi^s_M = e^{\phi/4}\,\psi_M$, $\chi^s = e^{-\phi/4}\,\chi$, $\lambda^s = e^{-\phi/4}\,\lambda$ and $\partial_M \phi$. In general, since only the dilaton transforms under the scaling symmetry in the string frame, the ℓ-string-loop contribution to the action scales as $\mathcal{L}_\ell \to \sigma^{2-2\ell}\,\mathcal{L}_\ell$.

Anomaly Cancellation

As mentioned above, the fermion content of the 6D model as described so far has anomalies, which must be cancelled if the theory is to make physical sense. In particular, they must cancel if the model is to be considered as the low-energy limit of an underlying consistent theory, such as string theory, at still higher energies. The anomaly cancellation conditions for 6D supergravity coupled to a single tensor- plus n_V vector- and n_H hyper-multiplets are well understood, so we simply summarize here several features which are used below.

These anomaly-cancelling conditions are significant for two separate reasons. First, they provide new nontrivial constraints on the kinds of particles which must appear in the 6D theory. In particular they require the existence of many more 6D matter supermultiplets than are considered above. Second, they show the necessity for specific higher-derivative corrections to the above supergravity lagrangian without reference to any specific, more microscopic, theory like string theory. As we shall see, the neglect of these corrections when using the above 6D supergravity lagrangian ultimately requires for consistency the conditions

$$1/r^2 \ll e^{\phi} \ll 1\,, \tag{3.76}$$

with r defined in terms of the volume of the extra dimensions using the Einstein-frame metric.

The 6D Green-Schwarz Mechanism

Six-dimensional anomalies are described by an 8-form constructed from the gauge and gravitational field strengths [81]. In order for anomalies to be cancelled by a Green-Schwarz type mechanism [82] – involving the shifting of a bosonic field in the theory – this anomaly form must factorize into the wedge product of pairs of lower-dimension forms.[9] In general this imposes a strong set of conditions in six dimensions, some features of which we now summarize [83, 84].

To describe the anomaly cancellation conditions we must first generalize the above field content to potentially include n_V gauge multiplets, as well as n_H matter 'hyper-' multiplets which involve 6D scalars and fermions whose helicity satisfies $\Gamma_7 = -1$. 6D supersymmetry requires the scalars within these hypermultiplets to take values in a quaternionic manifold, and precludes

[9]If several bosonic fields are involved then the anomaly 8-form can be the sum of such products, one for each of the bosonic fields.

them from appearing in the gauge kinetic terms or in the kinetic term for the dilaton field ϕ [86].

A necessary condition for the factorizing of the anomaly 8-form is the vanishing of the coefficient of the $\mathrm{tr}(R^4)$ term. With n_V gauge multiplets and n_H hypermultiplets, this is assured by the condition $n_H = n_V + 244$, which determines the number of hyper-multiplets in terms of the dimension of the 6D gauge group, $n_V = \mathrm{dim}G$ [83]. For simple gauge groups whose quartic casimir invariant is linearly independent of those at lower orders, there is another condition which amounts to requiring the vanishing of the $\mathrm{tr}(F^4)$ term.

If the only gauge group is the $U(1)$ considered above, we have $n_V = 1$ and so $n_H = 245$ hyper-multiplets are required to cancel anomalies. (For our purposes more general gauge groups may also be possible, provided that the compactification solution we introduce in the next section remains a solution to the equations of motion.) In this case anomaly cancellation through a shift in the field B_{MN} requires the anomaly 8-form to be

$$I_8 = k\Big(\mathrm{tr}\, R^2 - vF^2\Big)\,\Big(\mathrm{tr}\, R^2 - \tilde{v}F^2\Big)\,, \tag{3.77}$$

where the trace is over the fundamental representation of $SO(5,1)$ and multiplication represents the wedge product. Here the numbers k, v and $\tilde{v}$ are calculable given the precise fermion content of the 6D theory.

An anomaly of this form may be cancelled by adding the Green-Schwarz term

$$\mathcal{L}_{\mathrm{anom}} = -kvB\Big(\mathrm{tr}\, R^2 - \tilde{v}F^2\Big), \tag{3.78}$$

provided the transformation rule, eq. (3.71), for B_{MN} is modified to $\delta B = -\omega\, F + \alpha_L/v$ where $d\alpha_L = \delta\omega_L$ gives the transformation property of the gravitational Chern-Simons form, ω_L, which is in turn defined by the condition $d\omega_L = \mathrm{tr}\, R^2$. Invariance of the field strength, G_{MNP}, then requires its definition be modified to $G = dB + AF - \omega_L/v$.

The anomaly-cancelling term and the modifications to G are linked by supersymmetry to one another, and to other higher-derivative terms in the 6D action beyond those described above. For instance in the Einstein frame the $U(1)$ gauge kinetic functions get modified to [87, 88]

$$-\frac{1}{4}\left(e^{-\phi} + \frac{\tilde{v}}{v}\, e^{\phi}\right) F_{MN}F^{MN}\,. \tag{3.79}$$

As we see in more detail once the compactification to four dimensions is described below, all of these new terms are suppressed relative to the ones discussed in the previous sections in the sense that they involve higher powers of either $1/(e^{\phi}r^2)$ or e^{ϕ}/r^2 (or both).

The Compactification

The equations of motion for the bosonic fields which follow from the action, eq. (3.72), are:

$$\begin{aligned}
&\Box\, \phi + \frac{1}{6}\, e^{-2\phi}\, G_{MNP}\, G^{MNP} + \frac{1}{4}\, e^{-\phi}\, F_{MN}F^{MN} - 2g^2 e^{\phi} = 0 \\
&D_M\Big(e^{-2\phi}\, G^{MNP}\Big) = 0 \\
&D_M\Big(e^{-\phi}\, F^{MN}\Big) + e^{-2\phi}\, G^{MNP}\, F_{MP} = 0 \\
&R_{MN} + \partial_M\phi\, \partial_N\phi + \frac{1}{2}\, e^{-2\phi}\, G_{MPQ}\, G_N{}^{PQ} + e^{-\phi}\, F_{MP}F_N{}^P + \frac{1}{2}\, (\Box\phi)\, g_{MN} = 0.
\end{aligned} \tag{3.80}$$

The compactification is found by searching for a solution to these equations which distinguishes four of the dimensions – $x^\mu, \mu = 0, 1, 2, 3$ – from the other two – $y^m, m = 4, 5$. The Salam-Sezgin solution is obtained by constructing this solution subject to the symmetry ansatz that the spacetime be separately maximally symmetric in the first four and last two dimensions. This leads to the following Freund-Rubin-type ansatz [89] for the solution: $\phi =$ constant and

$$g_{MN} = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & g_{mn}(y) \end{pmatrix} \qquad \text{and} \qquad F_{MN} = \begin{pmatrix} 0 & 0 \\ 0 & F_{mn}(y) \end{pmatrix}, \tag{3.81}$$

where $g_{\mu\nu}$ is a maximally-symmetric Lorentzian metric (*i.e.* de Sitter, anti-de Sitter or flat space), and g_{mn} is the metric on the two-sphere, S_2. Maximal symmetry implies the gauge field strength is proportional to the sphere's volume form, ϵ_{mn}, and so

$$F_{mn} = f\, \epsilon_{mn}\,, \tag{3.82}$$

where f is a constant. All other fields vanish.

The gauge potential, A_m, which gives rise to this field strength is the potential of a magnetic monopole. As such, it is subject to the condition that the total magnetic flux through the sphere is quantized: $g \int_{S_2} B\, d^2y = 2\pi n$, with $n = 0, \pm 1, ...$ This requires the normalization constant, f, to be:

$$f = \frac{n}{2\, g\, r^2} \tag{3.83}$$

where r is the radius of the sphere.

As is easily verified, the above ansatz solves the field equations provided that the following three conditions are satisfied: $R_{\mu\nu} = 0$, $F_{mn}F^{mn} = 8\, g^2 e^{2\phi}$ and $R_{mn} = -\, e^{-\phi}\, F_{mp}\, F_n{}^p = -f^2 e^{-\phi}\, g_{mn}$.[10] These imply the four dimensional spacetime is flat, the monopole number is $n = \pm 1$ and the sphere's radius is related to ϕ by

$$e^{\phi}\, r^2 = \frac{1}{4g^2}\,. \tag{3.84}$$

[10] In reference [83] the authors construct a similar solution by embedding the monopole in an E_6, also achieving flat four-dimensional space. The solution in [83] breaks supersymmetry.

Useful intuition about this result can be obtained by constructing the scalar potential for r and ϕ which is obtained by substituting our assumed background solution into the classical action. One finds in this way three contributions, coming from the Einstein-Hilbert term, the Maxwell kinetic term for A_m and the explicit dilaton potential. In order to eliminate mixing between these scalars and the fluctuations of the 4D metric, it is necessary to perform a Weyl rescaling to ensure the 4D Einstein-Hilbert action remains r-independent. We take, then:[11]

$$g_{MN} = \begin{pmatrix} r^{-2}\, g_{\mu\nu} & 0 \\ 0 & r^2 g_{mn} \end{pmatrix} \tag{3.85}$$

and find the following potential:

$$V = -\left. \frac{\mathcal{L}_B}{e_4} \right|_{\text{no derivatives}} = \frac{2\, g^2\, e^{\phi}}{r^2} \left(1 - \frac{1}{4g^2\, e^{\phi} r^2} \right)^2 , \tag{3.86}$$

where $e_4 = \sqrt{-\det g_{\mu\nu}}$. From this we see how eq. (3.84) emerges as the minimum of the scalar potential for r and ϕ. Because this potential is minimized at $V = 0$ we also see why the 4D metric must be flat. Finally, we see that the combination e^{ϕ}/r^2 parameterizes a flat direction, since its potential vanishes identically once $e^{\phi} r^2 = 1/4g^2$ has been chosen.

The existence of the flat direction parameterized by r^2/e^{ϕ} may be also inferred from the scaling symmetry of the supergravity equations of motion, eq. (3.75). Since $s := r^2/e^{\phi}$ transforms under this transformation while $t := e^{\phi}\, r^2$ does not, s plays the role of the dilaton for this symmetry. Since this scaling transformation is only a symmetry of the classical equations, and

[11] We implicitly change units when performing this rescaling, switching to the choice $\kappa_4^2 = 8\pi G_4 = 1$, rather than the same condition for the 6D quantity, κ_6^2.

not of the action, the potential for s need not be exactly flat if we go beyond the classical approximation when computing the low-energy theory.

In the present case it happens that the flat direction with vanishing 4D cosmological constant is not lifted order-by-order in perturbation theory, as may be seen because the solution leaves one 4D supersymmetry unbroken. This may be seen by substituting the solution into the right-hand-side of eqs. (3.74) and checking that the result vanishes for a supersymmetry parameter which is independent of the 2D coordinates, y^m. Equivalently, spinors on S_2 which are constants are Killing spinors for this solution. Their existence is a consequence of the choice $n = \pm 1$ for the monopole number, since this ensures the cancellation of the gauge and spin connections in the covariant derivative, $D_\mu \varepsilon$ [146].

Some consistency conditions need be borne in mind if we regard this field configuration as a low-energy solution in string theory. In this case the approximation of weak string coupling requires we take $e^\phi \ll 1$ and the approximation of using a low-energy field theory similarly requires $r \gg 1$. Both of these requirements imply small values for the combination e^ϕ / r^2.

Low-Energy Fluctuations

Fluctuations about this background may be organized into four-dimensional fields according to the usual Kaluza-Klein procedure, with the generic mode having a mass which is at least of order $1/r$. We wish to identify the effective four-dimensional theory which governs the physics below this scale.

Symmetries

As a preliminary to the identification of the light particle content of the 4D theory, we first identify how the background fields transform under the model's symmetries. Given the background fields of present interest — $g_{\mu\nu}, g_{mn}$ and F_{mn} — these are:

- Unbroken 4D Poincaré invariance, as given by the isometries of 4D Minkowski space:

$$\langle \delta g_{\mu\nu} \rangle = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0. \tag{3.87}$$

  These symmetries ensure the masslessness of the 4D graviton.

- Unbroken $SO(3)$ invariance from the isometries of the internal metric,

$$\langle \delta g_{mn} \rangle = \nabla_m Z_n + \nabla_n Z_m = 0 \tag{3.88}$$

  which ensures the masslessness of three 4D spin-one particles.

- Broken local $U(1)$ invariance, broken because of the transformation

$$\langle \delta B \rangle = \omega \langle F \rangle. \tag{3.89}$$

  From this we draw two conclusions. First, the 4D gauge field A_μ is not exactly massless. Second, we see that in the Kaluza-Klein expansion $B_{mn}(x, y) = b\,\epsilon_{mn} + \dots$ (where ϵ_{mn} is the 2D volume form) the field b mixes with the Goldstone Boson for the $U(1)$ gauge symmetry breaking.

- Unbroken 4D Kalb-Ramond symmetry

$$\langle \delta B_{\mu\nu} \rangle = \partial_\mu v_\nu - \partial_\nu v_\mu = 0 \tag{3.90}$$

  for constant v_μ.

- Because the 2-sphere's volume form, ϵ_{mn}, is harmonic, the action has a global symmetry, $\delta B_{mn} = c\,\epsilon_{mn}$, and this is superficially broken by the background, since $\langle \delta B_{mn} \rangle \neq 0$. However because $\langle F_{mn} \rangle = f\,\epsilon_{mn}$, there is a linear combination of this global symmetry and the $U(1)$ gauge symmetry which is unbroken by the background fields:

$$\langle B_{mn} \rangle = c\,\epsilon_{mn} + \omega\,\langle F_{mn} \rangle = 0, \tag{3.91}$$

provided $c = -f\,\omega$. This shows that in the Kaluza-Klein expansion $B_{mn} = b\,\epsilon_{mn}$, the field b becomes massless in the limit when either f or the $U(1)$ gauge coupling vanish.

Particle Content

On symmetry grounds we expect the following bosonic particle content of the effective theory well below the scale $1/r$. (The corresponding fields are also given up to mixing due to the nonzero background flux $\langle F_{mn} \rangle$).

No.	Spin	Field
1	2	$g_{\mu\nu}(x)$
4	1	$A^a_\mu(x)$, 3 combinations of $g_{m\mu}(x,y)$
4	0	$B_{\mu\nu}(x)$, $\phi(x)$, $r(x)$, $B_{mn} = b(x)\,\epsilon_{mn}/e_2$

where we count here real scalar fields.

This counting arises as follows:

- The massless spin-2 particle follows as the gauge particle for the unbroken 4D Lorentz invariance of the background metric.

- The three massless spin-1 particles which arise as combinations of $g_{\mu n}$ are the gauge bosons for the $SO(3)$ group of isometries of the 2-sphere.

- The field $B_{\mu\nu}$ dualizes to a massless scalar, a, according to the definition $\partial_\mu a = e^{-2\phi}\epsilon_{\mu\nu\lambda\rho}G^{\nu\lambda\rho}/e_4$. The symmetry $a \to a$+constant can be broken by anomalies, which can arise after dualization due to the appearance of the Chern Simons terms in the field strength $G_{\mu\nu\lambda}$.

- The fields b and A_μ are *not* massless. The gauge field is not massless because the background field $F_{mn} \neq 0$ breaks the $U(1)$ gauge symmetry, with an expectation value which is of order $f = \pm 1/(2gr^2)$ in size. The covariant derivative for b is $\partial_\mu b + fA_\mu$, indicating that b is the Goldstone boson which is eaten by the gauge boson.[12]

- The combination $t := e^\phi r^2$ is also not massless since it is not a modulus of the background configuration, being fixed by the condition (3.84).[13] We shall see that this scalar's mass is also suppressed by powers of e^ϕ and so can appear in the low-energy theory below $1/r$.

- The orthogonal combination $s := r^2 e^{-\phi}$ is massless in the classical approximation, as we saw from the scalar potential, eq. (3.86).

Light Boson Masses

To see why the masses of the fields A_μ and $t = r^2 e^\phi$ are suppressed by powers of e^ϕ, we must compute their kinetic terms in addition to their mass terms. For instance, for the gauge field, A_μ, the mass term arises from the square of the term $F_{mn}A_\mu$ which appears in the Kalb-Ramond kinetic term. Keeping

[12]The possibility that these fields get a mass appears to have been missed in ref. [146], but was recognised in ref. [83] in a similar context.

[13]The possibility that fluxes could freeze geometric moduli has been noted previously in [115].

in mind the Weyl rescaling of the 4D metric this gives a mass term of order

$$\frac{\mathcal{L}_{\rm mass}}{e_4\, e_2} = -\frac{1}{4} e^{-2\phi}\, G_{mn\mu}\, G^{mn\mu} \sim e^{-2\phi} F_{mn} F^{mn} A_\mu A^\mu \sim \frac{e^{-2\phi}}{g^2 r^2}\, A_\mu A^\mu\,, \tag{3.92}$$

where $e_2 = \sqrt{\det g_{mn}}$. By contrast, the kinetic term is

$$\frac{\mathcal{L}_{\rm kin}}{e_4\, e_2} = -\frac{1}{4}\, e^{-\phi}\, F_{\mu\nu} F^{\mu\nu} \sim r^4 e^{-\phi}\, F_{\mu\nu} F^{\mu\nu}. \tag{3.93}$$

Comparing these gives a gauge boson mass of order:

$$m_A^2 \sim \frac{e^{-\phi}}{g^2 r^6} \sim \frac{1}{g^2}\frac{1}{s\, t^2} \sim \frac{g^2}{s}, \tag{3.94}$$

where we have used the condition $g^2\, r^2\, e^\phi = O(1)$, $t = r^2\, e^\phi$ and $s = r^2/e^\phi$.

The mass for t is found in an identical way. The kinetic term for r arises from substituting the ansatz, eq. (3.85), into the 6D Einstein-Hilbert term. Together with the explicit ϕ kinetic term this leads to the following kinetic terms for t and s:

$$\begin{aligned} \mathcal{L}_{\rm kin} &= -g^{\mu\nu}\left[2\,\frac{\partial_\mu r\, \partial_\nu r}{r^2} + \frac{1}{2}\,\partial_\mu\phi\,\partial_\nu\phi\right] \\ &= -\frac{1}{4}\, g^{\mu\nu}\left[\frac{\partial_\mu s\, \partial_\mu s}{s^2} + \frac{\partial_\mu t\, \partial_\nu t}{t^2}\right]. \end{aligned} \tag{3.95}$$

In terms of s and t the potential, eq. (3.86), becomes

$$V = \frac{2\, g^2}{s}\left(1 - \frac{1}{4 g^2\, t}\right)^2, \tag{3.96}$$

and so $d^2V/dt^2\Big|_{\rm min} = 4\, g^2/(st^2)$. Comparing with the kinetic term gives a mass which is of the same order as was found above for m_A^2:

$$m_t^2 \sim \frac{g^2}{s} \sim \frac{g^2 e^\phi}{r^2}. \tag{3.97}$$

For the purposes of comparison, it is worth also recording here the generic size of Kaluza-Klein masses. For instance, given a massless 6D scalar field, $\Phi(x, y)$, and keeping in mind the metric rescaling, eq. (3.85), we may write

$$g^{MN}\nabla_M\nabla_N\Phi = \left(r^2\, g^{\mu\nu}\nabla_\mu\nabla_\nu + \frac{1}{r^2}\, g^{mn}\nabla_m\nabla_n\right)\Phi\,, \tag{3.98}$$

from which we see $m_{KK} \sim 1/r^2$.

Light Fermions

A similar calculation can be made for the spectrum of light fermions, and leads to the following light fermion spectrum:

No.	Spin	Field
1	3/2	$\psi_\mu(x)$
6	1/2	$\chi(x)$, $\lambda(x)$, 4 combinations of $\psi_m(x, y)$

For our purposes it is fruitful to determine how these fields assemble into multiplets of the unbroken 4D supersymmetry. The identification of these multiplets may be explicitly obtained by using the supersymmetry transformations of eq. (3.74) – such as by following the arguments of ref. [92] – and leads to the following:

- The massless gravitino required by the unbroken supersymmetry is the partner of the graviton.
- Three massless gauginos arise as partners of the $SO(3)$ gauge bosons. These fermions come from the higher-dimensional gravitino due to the simultaneous existence of a Killing spinor and three Killing vectors.
- A massless fermion combines with s and a into a massless chiral multiplet, whose complex scalar part may be written $S = \frac{1}{2}(s + ia)$.

- Two fermions, with masses $m^2 \sim g^2/s$, join t and $A_\mu + \partial_\mu b/f$ to fill out a massive spin-1 multiplet. This massive multiplet can be regarded as the result of a massless spin-1 multiplet 'eating' the chiral multiplet whose complex scalar part is $T = \frac{1}{2}(t + ib)$ *via* the Higgs mechanism.

Once the fermions are chosen to transform in the standard way under $N = 1$ 4D supersymmetry, they do not carry the $U(1)$ gauge charge, even though the 6D fermions did – *c.f.* eq. (3.69). In detail this happens because the 4D supersymmetry eigenstates are related to the 6D fermions by powers of the scalar e^{ib}, which cancel the 6D fermions' transformation properties. Only the gauginos of the low-energy theory transform nontrivially under the $SO(3)$ gauge symmetry.

3.4.2 The 4D Effective Theory

Since the low-energy theory has an unbroken $N = 1$ supersymmetry, it must be possible to write it in the standard $N = 1$ supergravity form. From the previous section we see that the matter superfields in terms of which the action below the compactification scale is expressed are the massless chiral multiplet, S and the three massless gauge multiplets, $A_a, a = 1, 2, 3$.

Since our interest is in exploring the shape of the scalar potential as a function of both r and ϕ, it is useful to extend the effective action to also include the massive chiral field, T, and the massive $U(1)$ gauge multiplet, A, which is related to it by the Higgs mechanism. The effective theory obtained in this way is not a *bona fide* Wilsonian action when evaluated along the flat direction, however. It is not because the mass of these fields are $m_t^2 \sim g^2\, e^{\phi}/r^2$ and $m_A^2 \sim e^{-\phi}/(g^2 r^6)$, which are the same order of magnitude as the generic

Kaluza-Klein mass, $m_{KK}^2 \sim 1/r^4$, when evaluated along the trough of the potential (along which $g^2r^2e^{\phi} \sim O(1)$).[14] We may nevertheless choose to 'integrate in' these modes in the spirits of refs. [100, 99], with the idea that parametrically their masses depend differently on r and e^{ϕ}, and so there can be regions of field space away from the bottom of the potential's trough for which they are systematically light compared to m_{KK}.

In order to completely specify all of the terms of the 4D supergravity action, it suffices to identify the Kähler function, $K(S, S^*, T, T^*, A, A_a)$, the gauge kinetic functions, $H_3(S,T)$ and $H_1(S,T)$ for the $SO(3)$ and $U(1)$ gauge groups, the superpotential, $W(S,T)$, and the Fayet-Iliopoulos term, ξ [93, 94].

The Lowest-Order Action

In this section we determine these functions classically, by comparison with the direct truncation of the 6D action [91, 92], followed by a discussion of the kinds of corrections which may be expected for the result [92, 96].

The Kähler Function

An important constraint on K arises because b is eaten by the $U(1)$ gauge field, A_μ, since this implies its derivatives can only enter $\mathcal{L}$ through the gauge-invariant combination $\partial_\mu b + fA_\mu$. One infers from this that the superfields T and A must enter the Kähler function only through the combination $T + T^* + cA$, for a real constant c to be determined below. Similarly, the shift symmetry $a \to a+$ (constant) implies K can depend on S only through the

[14]We thank G. Gibbons and C. Pope for correcting an error concerning the relative sizes of m_{KK} and m_t, m_A in the original version of this paper.

combination $S + S^*$.

The form for K is most easily read off from the scalar kinetic terms, which in the Einstein frame must take the form $\mathcal{L}_{\rm kin} = -K_{ij^*}\partial_\mu z^i \partial^\mu z^{j*}$ for generic complex scalar fields, z^i. (As usual subscripts here denote derivatives of K with respect to the relevant scalar field, evaluated with all fields except the scalars vanishing.) Comparing this with the direct truncation calculation of the kinetic terms for r and ϕ, eq. (3.95), gives the result

$$K_{tr} = -\log\Big(S + S^*\Big) - \log\Big(T + T^* + cA\Big). \tag{3.99}$$

The Gauge Kinetic Functions

The gauge kinetic functions are more constrained than is the Kähler function since they must depend holomorphically on their arguments. They may be read off from the gauge boson kinetic terms, which must have the general form $\mathcal{L}_{\rm kin} = -\frac{1}{4}\Big[(\mathrm{Re}\ H_1)\, F_{\mu\nu}F^{\mu\nu} + (\mathrm{Re}\ H_3)\, F^a_{\mu\nu}F^{\mu\nu}_a\Big]$. Alternatively, for some purposes they may be more simply obtained from the related terms $\mathcal{L}_\theta = -\frac{1}{4}\Big[(\mathrm{Im}\ H_1)\, F_{\mu\nu}\tilde{F}^{\mu\nu} + (\mathrm{Im}\ H_3)\, F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a\Big]$, since the imaginary parts of S and T appear in more restricted ways in the reduction of the 6D action.

Comparing these with the direct truncation of the 6D action gives the result: $\mathrm{Re} H_1 = e^{-\phi}\, r^2 = s = 2\,\mathrm{Re}\, S$, from which we find $H_1 = 2S$. (That the leading contribution to H_1 must be proportional to S follows from the recognition that H_1 scales like $H_1 \to \sigma^2\, H_1$ under the classical transformation, eq. (3.75), together with the transformations $S \to \sigma^2\, S$ and $T \to T$.) The higher-derivative corrections of eq. (3.79) which follow from anomaly cancellation correct this result to give

$$H_1 = 2\left(S + \frac{\tilde{v}}{v}T\right). \tag{3.100}$$

A similar direct dimensional reduction for the $SO(3)$ gauge fields is more involved, since the massless mode is a linear combination of the fields $A_\mu(x,y)$ and $g_{\mu n}(x,y)$ [95]. (The necessity for mixing between A_μ and $g_{\mu n}$ may be seen by performing a local $SO(3)$ transformation corresponding to the general coordinate transformation $y^m \to \xi^m(x,y) = \omega^a(x)\,K_a^m(y)$, where $K_a^m(y), a = 1,2,3$ are the three Killing vectors which generate the $SO(3)$ isometries of the sphere. Under this transformation the massless 4D gauge potential must transform as $\delta A_\mu^a = \partial_\mu \omega^a + \cdots$.) Consequently, the $SO(3)$ gauge kinetic function acquires contributions from both the 6D Einstein-Hilbert and Maxwell terms of the action.

For our purposes the details of this reduction are not necessary in order to conclude that H_3 is given by an expression very much like eq. (3.100):

$$H_3 = 2(\alpha S + \beta T), \tag{3.101}$$

for constants α and β which are given in terms of the anomaly coefficients k, v and $\tilde{v}$. This conclusion is most easily established by considering $\mathcal{L}_\theta$ and recognizing that for the $SO(3)$ fields these terms are linear in $a = 2\,\mathrm{Im}\,S$ and $b = 2\,\mathrm{Im}\,T$. This is most easily seen from the contribution of the Lorentz Chern-Simons term in $G_{MNP}\,G^{MNP}$ and from the Green-Schwarz anomaly cancelling term, eq. (3.78). Linearity in a is as expected from the scaling property, eq. (3.75), together with the transformation properties of S and T.

The Scalar Potential

The constant c in the Kähler potential, the superpotential, W, and the Fayet Iliopoulos term, ξ, are fixed by considering the scalar potential, eq. (3.96). This must agree with the general supergravity form $V = V_D + V_F$, where

$$\begin{aligned} V_F &= e^K \Big[(K^{-1})^{ij^*} (W_i + K_i W)(W_j + K_j W)^* - 3|W|^2 \Big], \\ V_D &= -\frac{1}{2} \, (\mathrm{Re}\, H_1) \, D^2 - \frac{1}{2} \, (\mathrm{Re}\, H_3) \, D_a D_a \,, \end{aligned} \tag{3.102}$$

where D_a and D are the auxiliary fields for the two factors of the gauge group, which we have not yet integrated out (hence the potential's unusual sign). Here, as usual, $(K^{-1})^{ij*}$ denotes the inverse of the matrix of second derivatives, K_{ij*}.

Given that neither S nor T carry $SO(3)$ gauge quantum numbers, we see that $D_a = 0$ must be used when comparing with the truncated 6D action. Since T does transform under $U(1)$, D can be nonzero and, from the Kähler and gauge kinetic functions found above, the $U(1)$ D terms of the low-energy action arise from the following terms:

$$\begin{aligned} \mathcal{L}_D &= s \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D^2 \right) + D \left(\xi + \frac{\partial K}{\partial A} \bigg|_{A=0} \right) \\ &= s \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D^2 \right) + D \left(\xi - \frac{c}{T + T^*} \right) . \end{aligned} \tag{3.103}$$

Here ξ is the Fayet-Iliopoulos term, which is permitted only for $U(1)$ gauge fields. Notice that consistency requires we use only the lowest-order expression $H_1 = 2S$ when comparing with the action given above.

Integrating out D implies the saddle-point condition

$$D = -\frac{1}{s} \left(\xi - \frac{c}{T + T^*} \right) , \tag{3.104}$$

and so leads to the potential

$$V_D = +\frac{1}{2\, s} \left(\xi - \frac{c}{t} \right)^2 . \tag{3.105}$$

Comparing this with eq. (3.96) we read off:

$$\xi = \pm 2\, g \qquad \text{and} \qquad c = \pm \, \frac{1}{2\, g} \,. \tag{3.106}$$

This appearance of Fayet-Iliopoulos terms when the fermion content has a gauge anomaly is discussed in more general terms in ref. [97].

Since this completely accounts for the scalar potential and supersymmetry is unbroken, we conclude that the superpotential vanishes:

$$W = 0. \tag{3.107}$$

Perturbative Corrections

The above expressions for K, W, H_1, H_3 and ξ are derived by classically truncating 6D supergravity, and so in principle they only apply strictly in the limit that $r \to \infty$ and $e^\phi \to 0$, since it is only in this limit that the corrections to truncation vanish. In this way we see that the truncation results are approximations to the full expressions which work for the region $s, t \to \infty$ of the space of moduli.

For sufficiently large s and t – both of which are large if $1/r^2 \ll e^\phi \ll 1$ – the corrections to the truncation may be computed order-by-order in a low-energy, weak-coupling expansion in powers of $1/s$ and $1/t$. (Some of these corrections have already been computed for the gauge kinetic functions above.) Fortunately, the interplay of 6D and 4D supersymmetry strongly restricts the form which such corrections may take [92, 96]. As usual, these implications are stronger for the holomorphic functions H_1, H_3 and W than they are for K and so we discuss these two cases separately.

Holomorphic Functions

We first discuss the form which perturbative corrections may take for the holomorphic functions of the supergravity action. For the superpotential, W, as has been known for a long time [98], holomorphy completely forbids perturbative corrections from arising within perturbation theory [98], leading to the complete absence of correction to W to all orders in $1/T$ and $1/S$ [92, 96]. This leaves eq. (3.107) as the complete prediction to all orders.

Perturbative corrections to H_1 and H_3 do arise at one loop, and are given by the S-independent terms in eqs. (3.100) and (3.101). No further corrections beyond these are allowed to all orders in perturbation theory, however. This may be seen from the symmetry under shifts in $\mathrm{Im}\, S$, which is broken only by the Chern-Simons terms in the field strength for B_{MN}, since these determine the gauge transformation properties of B_{MN}, eq. (3.71). As we have seen, these Chern-Simons terms are themselves related to the Green-Schwarz action which cancels the gauge anomaly of the 6D fermions, and this connection with the anomaly precludes there being additional terms of this form which are generated beyond one loop. We see that expressions (3.100) and (3.101) are therefore the complete predictions – up to the additions of S- and T-independent constants – for H_1 and H_3 to all orders in $1/S$ and $1/T$.[15]

Perturbative corrections are necessarily concentrated into the Kähler function, and it is to a discussion of these that we now turn. These come in two forms.

[15] As has been noted elsewhere, since this argument relies on holomorphy it strictly applies only to the Wilson action, and not necessarily to the generator of 1PI vertices [99, 100].

Käher Function: Powers of $1/t$

Corrections to the Kähler function can arise, and do so independently as powers of $1/t$ and $1/s$, since these have roots in the more microscopic theory as independent expansions in powers of e^{ϕ} and $1/r$. This may be seen explicitly by considering two types of corrections to the lowest-order action in the 6D theory, as we now do. We start with powers of $1/t$, which play an important role in what follows, and which we now argue correspond to the contributions under dimensional reduction of higher-derivative corrections to the 6D effective theory.

The simplest way to identify corrections to K is to compute the corrections to the Kähler metric by examining the kinetic terms of the scalars r and ϕ in the 4D effective action. Examples of higher-derivative corrections to these kinetic terms are the contributions of higher-curvature terms to the radion kinetic energy. For instance a higher-curvature correction to the Einstein-Hilbert action (in the string frame) in six dimensions

$$\frac{\mathcal{L}_{SF}}{e_6} \sim e^{-2\phi}\Big[R_s + k_n R_s^n\Big] \tag{3.108}$$

becomes, in the Einstein frame

$$\frac{\mathcal{L}_{EF}}{e_6} \sim R + k_n e^{-(n-1)\phi} R^n, \tag{3.109}$$

due to the rescaling $g_{MN} \to e^{\phi} g_{MN}$ which is required to remove ϕ from in front of the Einstein-Hilbert part of the action. On dimensionally reducing we extract one factor of the 4D Ricci tensor, $R_{\mu\nu} \propto \partial_\mu r\, \partial_\nu r/r^2$, from R^n, with the remaining factors being proportional to the two-dimensional curvature:

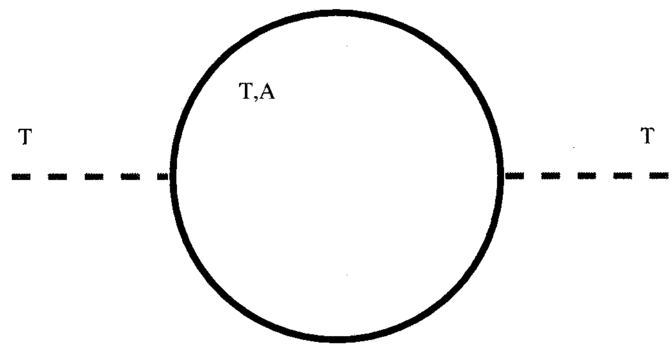

Figure 3.1: One-loop diagram contributing to the kinetic term of the field T. The internal lines are also T and/or the $U(1)$ gauge field.

$R_{(2)}^{n-1} \propto (1/r^2)^{n-1}$, leading to the 4D kinetic term

$$\frac{\mathcal{L}_{\rm kin}}{e_4} \sim \frac{\partial_\mu r \, \partial^\mu r}{r^2} \left[1 + \frac{k_n}{(e^\phi r^2)^{n-1}} \right] \sim \frac{\partial_\mu r \, \partial^\mu r}{r^2} \left[1 + \frac{k_n}{t^{n-1}} \right] . \tag{3.110}$$

For instance, in string theory such corrections could arise from sigma-model corrections at string tree level.

Käher Function: Powers of $1/s$

Powers of $1/s$ can arise due to loops within the 4D theory itself. For instance, consider the one-loop correction to the kinetic term for T which is induced by the graph of Fig. (3.1). Since the 4D loop integrals diverge (quadratically) in the ultraviolet, they are insensitive to the masses of the multiplets in the loop and we can ignore the mixing between the T multiplet and the $U(1)$ gauge multiplet. If we take all internal lines in Fig. (1) to be T fields, then the vertices are of order $\partial^3 K/\partial t^3 \sim 1/t^3$, while each propagator contributes $[K_{TT^*}(p^2 + m_t^2)]^{-1}$ with $K_{TT^*} \sim 1/t^2$. Taking the 4D ultra-violet cutoff to be $m_{KK}^2 \sim 1/r^4 \sim M_p^2/(st)$, we estimate:

$$\delta K_{TT^*}(T \text{ loop}) \sim \frac{m_{KK}^2}{(4\pi)^2 M_p^2} \left(\frac{1}{t^3}\right)^2 \left(t^2\right)^2 \sim \frac{1}{(4\pi)^2\, s\, t^3}. \tag{3.111}$$

Alternatively, if one of the internal lines of Fig. (1) is a $U(1)$ gauge multiplet, then the coupling between V and T, given by the lowest order Kähler function $K = -\log(T + T^* + cV)$, is of order $\partial^3 K/\partial V \partial T \partial T^* \sim c/t^3$, and we see that each vertex of Fig. (1) contributes a factor c/t^3. Taking $s \gg t$ the gauge propagator contributes a factor of $1/s$, so we estimate:

$$\delta K_{TT^*}(T - V \text{ loop}) \sim \frac{m_{KK}^2}{(4\pi)^2 M_p^2} \left(\frac{c}{t^3}\right)^2 \left(\frac{t^2}{s}\right) \sim \frac{1}{(4\pi)^2\, s^2\, t^4}, \tag{3.112}$$

where we use the lowest-order condition $t \sim 1/g^2 \sim c^2$.

Käher Function: Logarithms of $1/s$

Having seen how powers of $1/t$ and $1/s$ control the modifications to K, we next consider the possibility that more subtle types of corrections arise, which depend logarithmically on $1/s$. Indeed, logarithmic dependence on coupling constants is known to arise in 4D physics if the energies of some low-lying states, E_l, are suppressed by powers of coupling constants, g, relative to higher-energy states, E_h: $E_l \propto g^n E_h$. In this case logarithms of couplings can arise as logarithms of energy ratios: $\log(E_h/E_l) \sim n \log(1/g)$. The most well-known example of this type is perhaps the QED prediction for the Lamb shift, which involves a famous factor of $\log(1/\alpha)$ [101]. The potential for these kinds of logarithms exists in the low-energy 4D theory arising from the 6D supergravity compactification considered above because of the existence of hierarchies of mass scales. For instance if string states have masses $M_s \sim M_p$ then these are very different from typical compactification scales, $m_{KK}^2 \sim 1/r^4 \sim M_p^2/(s\,t)$, which are suppressed by powers of the small quantity $1/(s\,t)$. If logarithms of such ratios arise they can give rise to logarithms of s and t, of the form

$$\log\left(\frac{M_p^2}{m_{KK}^2}\right) \sim \log(s\,t), \qquad \log\left(\frac{M_p^2}{m_A^2}\right) \sim \log(g^2 s\,t^2), \qquad \log\left(\frac{M_p^2}{m_t^2}\right) \sim \log(s/g^2)\,, \tag{3.113}$$

all of which are similar in size when evaluated along the bottom of the scalar potential.

Some of these logarithms can arise in the four dimensional theory due to the appearance of large logarithms in the running of the couplings, and if so their appearance can be understood (and often re-summed) using standard renormalization-group arguments [102, 103]. To this end imagine running the 4D effective theory within the 4D theory, where the running of the inverse couplings, $H_1(S,T)$ and $H_3(S,T)$, is given by

$$\begin{aligned} H_1(S,T)|_{\mu} &= H_1(S,T)|_{\mu_0} + b_1 \log\left(\frac{\mu^2}{{\mu_0}^2}\right) \\ H_3(S,T)|_{\mu} &= H_3(S,T)|_{\mu_0} + b_3 \log\left(\frac{\mu^2}{{\mu_0}^2}\right)\,, \end{aligned} \tag{3.114}$$

with b_1 and b_3 are the standard supersymmetric one–loop beta–function coefficients for the $U(1)$ and $SO(3)$ gauge groups, respectively. Using the expressions (3.100) and (3.101), we may solve for the running of s and t

$$s(\mu^2) = s_0 + b_s \log\left(\frac{\mu^2}{\mu_0^2}\right), \qquad \text{and} \qquad t(\mu^2) = t_0 + b_t \log\left(\frac{\mu^2}{\mu_0^2}\right), \tag{3.115}$$

where, as long as the equations are non-singular, b_s and b_t are linear combinations of b_1 and b_3 depending on the coefficients of S and T in H_1 and H_3.

The dependence of large logarithms on masses, m, smaller than m_{KK} may then be traced by running these 4D couplings down to $\mu = m$ from $\mu_0 = m_{KK}$. This gives logarithms of the form $\log(m_{KK}^2/m^2)$, such as

$$\log\left(\frac{m_{KK}^2}{m_A^2}\right) \sim \log\left(g^2 t\right) + \text{constant}\,, \tag{3.116}$$

which are large if $t \gg 1/g^2$ (away from the bottom of the potential).

We see from these considerations that the existence of logarithms of s and t in the corrections to K are not unlikely. Without performing a more sophisticated calculation it is difficult to pin down the precise power of s and/or t which appears inside the logarithm. This is because low-energy logarithms like $\log(m_{KK}^2/m_A^2)$ can in principle combine with other large logarithms which arise purely from the high-energy theory, such as $\log(M_p^2/m_{KK}^2)$ to give new logarithms like $\log(M_p^2/m_A^2)$. We therefore parameterize this possibility by writing the resulting full RG-improved Kähler function as

$$K = -\log\left[s - b_s \log\left(st^a\right) + k_s\right] - \log\left[t - b_t \log\left(st^a\right) + k_t + cA\right], \tag{3.117}$$

where a, k_s and k_t are order-unity constants.

Notice that expanding eq. (3.117) in powers of $1/s$ and $1/t$ gives the corrections to K to leading order in $1/t$ and $1/s$, but to all orders in $(1/s)\log(st^a)$ and/or $(1/t)\log(st^a)$. This observation will become important later when we find minima for the scalar potential.

3.4.3 Nonperturbative Effects in 4D

Given the above semiclassical approximation to the functions K, H, H_{ab} and W, we may use general knowledge of 4D $N = 1$ supersymmetric theories to understand the physics at energies much below the compactification scale.

In particular, our interest is in the existence of any other mass scales at very low energies which might lift the degeneracy of the flat direction described by S.

It is useful to re-instate the Planck mass and to identify the mass scales which arise in the low-energy 4D supergravity. These are:

- The 4D Planck mass: $M_p^2 = 1/\kappa_4^2 = 1/(8\pi G_4)$, as defined by the 4D graviton couplings.
- The 4D cutoff: $m_{KK} \sim 1/r^2 \sim M_p/(st)^{1/2}$, which defines the scale above which the theory is no longer efficiently described by a 4D lagrangian.

To these semiclassical mass scales should be added a new, nonperturbative one: $\Lambda \sim \mu \exp[-(\nu s(\mu) + \lambda\, t(\mu))/3]$, where ν and λ are positive constants which are related to the renormalization-group coefficients for s and t by the condition that Λ be independent of renormalization point μ. Using eqs. (3.115) this implies

$$\nu\, b_s + \lambda\, b_t = \frac{3}{2}\,. \tag{3.118}$$

This new scale arises because the low-energy theory's $SO(3)$ gauge theory is asymptotically free, with Λ defining the confinement scale where its effective coupling becomes strong. At this scale the gauginos of the $SO(3)$ theory condense [100], and because of this condensation (together with the absence of matter fields carrying approximate global chiral symmetries) the $SO(3)$ gauge sector acquires a gap in its spectrum which is of order Λ. The massive energy eigenstates which result are the $SO(3)$-singlet bound states of the gluons and gluinos.

As is well known, this condensation dynamically generates a superpotential in the low-energy theory [74, 100], which is of order Λ^3:

$$W = w_0 \, \exp[-\nu \, S - \lambda \, T], \tag{3.119}$$

for some constant $w_0 \sim \mu^3$.

This superpotential contributes to the scalar potential for s and t by generating a nonzero V_F, which was absent semiclassically. It is this new term which is responsible for the qualitatively new features of the low-energy theory: the lifting of the flat direction for s.

3.4.4 Dynamics of the Flat Directions

We have seen that the strongly-coupled $SO(3)$ gauge couplings dynamically generate a superpotential at low energies, and so the two terms, V_D and V_F conflict in what they would like the fields t and s to do. The semiclassical term, V_D, is minimized when $t \sim 1/g^2$, while the nonperturbative term, V_F, is minimized when $t \longrightarrow \infty$. These cannot be simultaneously minimized and so a compromise must be struck for which at least one of V_F or V_D is nonzero. We find that the vacuum to which this competition between V_F and V_D leads depends in a crucial way on the form of the corrections to K discussed above.

Dilaton Runaway

As a first approximation to the shape of this potential, we consider the superpotential, eq. (3.119), but ignore all corrections to the leading semiclassical Kähler function, eq. (3.99), and gauge kinetic functions. This leads to a scalar potential of the form $V = V_D + V_F$, where V_D may be read off from eq. (3.96), and V_F is given by:

$$V_F(s,t) = \frac{|w_0|^2}{st}\, e^{-\nu s - \lambda t}\, \left[(1+\nu\, s)^2 + (1+\lambda t)^2 - 3\right]. \tag{3.120}$$

If $\Lambda \ll M_p$ we have $w_0 \ll 1$ and the minimum for t is close to the zero of V_D: $1/t = 4g^2 + O(|w_0|^2)$. To linear order in $|w_0|^2$ the potential for s then becomes $V_{\rm eff}(s) \approx V_F(1/t = 4g^2)$, and so

$$V_{\rm eff}\,(s) = \frac{4g^2\,|w_0|^2}{s}\, e^{-\nu\, s - \lambda/(4g^2)} \left[(1+\nu\, s)^2 + \left(1 + \frac{\lambda}{4g^2}\right)^2 - 3\right]. \tag{3.121}$$

We find that the potential for s which is generated in this approximation does not have any minima for positive s besides the runaway solution for which $s \to \infty$. This is the familiar dilaton runaway, with the $SO(3)$ gauge coupling generically driven to zero as s runs off to infinity.

Dilaton Stabilization

The weak part of the previous analysis is the use of the lowest-order Kähler function, eq. (3.99), despite using a nonperturbative expression for the superpotential. We now show that using the renormalization-group-improved expression, eq. (3.117), can generate a potential for s which can have other minima besides the dilaton runaway.

We begin with the Kähler function,

$$\begin{aligned} K(s,t) &= -\log\left[s + \frac{b_s}{2}\log\left(\frac{st^a}{q}\right)\right] - \log\left[t + \frac{b_t}{2}\log\left(\frac{st^a}{q}\right)\right] \\ &\quad + O\left(\frac{\log(st^a/q)}{s^2}, \frac{\log(st^a/q)}{t^2}\right), \\ &\equiv -\log s_0 - \log t_0 + O\left(\frac{\log}{s^2}, \frac{\log}{t^2}\right), \end{aligned} \tag{3.122}$$

where a and q are constants, and where s_0, t_0 are the fields evaluated at the high scale. Notice that changing from s_0, t_0 to s, t in K (and then constructing the scalar potential V) is *not* simply the same as performing a trivial change of variables on the potential V itself. It is not, because this change is not a holomorphic redefinition of S and T.

Under the assumption that $|w_0| \ll 1$ we may compute the effective potential as before, by first minimizing $V_D(s,t)$ to obtain $t = t(s)$ and then examining $V_{\rm eff}(s) \approx V_F[s, t(s)]$. The minimum of V_D occurs when

$$K_T + \epsilon = 0, \tag{3.123}$$

where ϵ is a constant which is of order g^2 and

$$K_T = \frac{\partial K}{\partial T} \approx -\frac{1}{t_0} - \frac{a}{2}\left(\frac{1}{t}\frac{b_t}{t_0} + \frac{1}{s}\frac{b_s}{s_0}\right) \tag{3.124}$$

In the case where $a = 0$ (3.123) can be solved analytically, so that V_D is minimized for $t(s)$ satisfying

$$t_0 \equiv t + \frac{1}{2}\, b_t \log(s/q) \approx \frac{1}{\epsilon}. \tag{3.125}$$

Solving this for $t(s)$ and using the result in $V_F[s, t(s)]$ gives $V_{\rm eff}(s)$. The computation of V_F requires the inverse matrix:

$$\begin{aligned}
(K^{-1})^{SS^*} &= \frac{K_{TT^*}}{||K||}, \\
(K^{-1})^{TT^*} &= \frac{K_{SS^*}}{||K||}, \\
(K^{-1})^{ST^*} &= (K^{-1})^{TS^*} = -\frac{K_{ST^*}}{||K||},
\end{aligned} \tag{3.126}$$

where $||K||$ is the determinant of the matrix K_{ij^*}. To this order in the Kähler function we may ignore the difference between s and s_0 and t and t_0 after taking derivatives, so that

$$K_{TT^*} = \frac{1}{t_0^2}$$

$$
\begin{aligned}
K_{SS^*} &= \frac{1}{s_0^2}\left(1+\beta_t+{\beta_t}^2+3\beta_s+{\beta_s}^2\right)\\
K_{ST^*} &= \frac{\beta_t}{s_0 t_0}\\
||K|| &= \frac{1}{s_0^2 t_0^2}\left(1+\beta_t+3\beta_s+{\beta_s}^2\right) \ ,
\end{aligned}
\tag{3.127}
$$

where $\beta_s = \frac{1}{2}b_s/s_0$, $\beta_t = \frac{1}{2}b_t/t_0 \equiv \frac{1}{2}\epsilon b_t$. We also have the Kähler derivatives

$$
\begin{aligned}
D_S W &= W_S + K_S\, W\\
&= -\left[\nu + \frac{\sigma}{s_0}\right] W\\
D_T W &= W_T + K_T\, W\\
&= -\left(\lambda+\epsilon\right) W,
\end{aligned}
$$

where both results are evaluated at $t = t(s)$ and we have used (3.118), and $\sigma = 2\left(\frac{7}{8}+\beta_t+\beta_s\right)$.

One finds in this way the expression for $V_{\text{eff}}(s) = V_F[s,t(s)]$:

$$
V_{\text{eff}}(s) = \frac{|w_0|^2}{s_0 t_0}\, N(s_0,t_0)\, e^{-2\nu s_0 - 2\lambda t_0}\,, \tag{3.128}
$$

where

$$
N(s_0,t_0) = \frac{\left(\nu s_0+\sigma\right)^2 - 2B\left(\nu s_0+\sigma\right) + C\,t_0^2\left(\lambda+\epsilon\right)^2}{1+\beta_t+3\beta_s+\beta_s^2} - 3 \tag{3.129}
$$

with

$$
B = \beta_t t_0(\lambda+\epsilon) \qquad \text{and} \qquad C = 1+\beta_t+\beta_t^2+3\beta_s+\beta_s^2\,. \tag{3.130}
$$

Notice that this reduces to the previous runaway potential in the limit $b_t \longrightarrow 0, b_s \longrightarrow 0$, as long as we also set $\sigma = 1$ (the conditions (3.118) do not apply in this limit). β_s and σ are both s_0-dependent quantities.

This potential is drawn in Fig. (3.2) for the choice, in (3.101), $\alpha = \beta = 1/800$, and with $\tilde{v}/v = -9/4$, $q = 100$, $b_1 = -1/10$, $b_3 = 6/(4\pi)^2$, $\nu = 0.005$

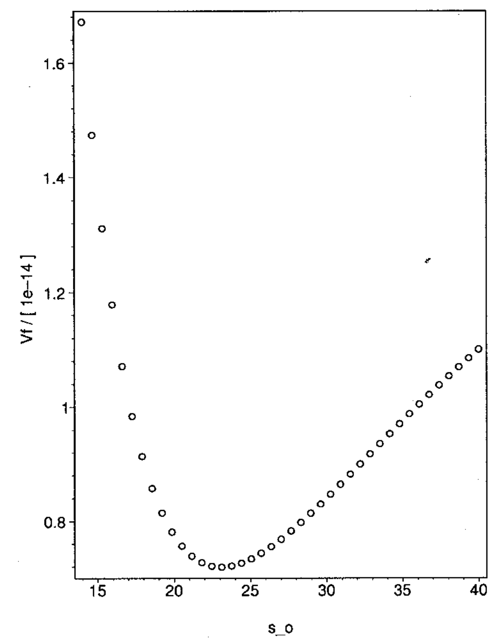

Figure 3.2: The effective potential for s computed using the renormalization-group improved potential. Parameters are chosen as described in the main text.

and $\epsilon = .28$. λ is determined in terms of these by the condition $\nu\, b_s + \lambda\, b_t = 3/2$, while b_t and b_s are determined from b_1 and b_3. For these choices the potential is minimized by the field values $s = 34$ and $t = 8.6$, corresponding to $t_0 = 1/\epsilon \approx 3.6$ and $s_0 = 23$.

It remains to be shown that values this small for α and β can be obtained from realistic string models. The above discussion nonetheless suffices to make our main point that the renormalization-group-improved Kähler function can produce nontrivial minima for the modulus s. We have also found minima having larger values of α and β ($|\alpha|, |\beta| \sim O(0.1)$), albeit for small values of s and t which lie at the limit of what can be understood perturbatively in powers of $1/s$ and $1/t$. The generic existence of such minima can be seen by observing that as long as the potential has a maximum, and increases as one approaches the origin ($s = t = 0$) from the right, then a minimum must exist in between (barring the existence of new singularities in this regime). In the present case of the larger values of α and β, maxima exist for s and t well within the perturbative regime, allowing us to infer the existence of the minima at much smaller field values: $s, t \sim O(0.1)$.

The quantities parameterizing the strength of the supersymmetry breaking are given by the *vev* of the potential, $V_{\text{eff}}(s)|_{\text{min}}$, as well as the expectation values of the auxiliary fields:

$$\begin{aligned} F_i &= e^{K/2} D_i W, \\ M &= e^{K/2} W/3, \end{aligned} \tag{3.131}$$

where i runs over S and T. The mass of S is approximately given by

$$m_s^2 \approx e^{-K} \left. \frac{d^2 V_{\text{eff}}(s)}{ds^2} \right|_{\text{min}}. \tag{3.132}$$

For the minimum described numerically above, we find $\langle V \rangle \approx 7 \times 10^{-15}$, $V'' \approx 9 \times 10^{-17}$, $F_s \approx -9 \times 10^{-8}$, $F_t \approx -2 \times 10^{-6}$, $M \approx 2 \times 10^{-7}$ and $m_s^2 \approx 7 \times 10^{-15}$, all in Planck units. The supersymmetry-breaking scale is therefore seen to be quite low, compared to, say, the compactification scale $m_{KK} \sim M_p/(st)^{1/2} \sim M_p/10$.

Summary

We now summarize our results, and outline some of their potential applications.

We have revisited the Salam-Sezgin compactification of gauged $N = 1$ 6D supergravity, and have computed the $N = 1$ 4D supergravity to which it leads at low energies. The low-energy field content to which we are led is supergravity coupled to a supersymmetric $U(1) \times SO(3)$ gauge theory plus several chiral multiplets which describe the compactification's moduli. The low-energy theory has the following properties:

- The $U(1)$ multiplet 'eats' one of the chiral multiplets *via* the Higgs mechanism at the classical level giving these fields masses which are comparable to the Kaluza-Klein mass scale, $m_{KK} \sim\sim 1/r^2$, when evaluated along the trough at the bottom of the classical scalar potential. This scalar potential arises from the 4D point of view through a Fayet-Iliopoulos term for the $U(1)$ gauge group.
- One combination of scalars parameterizes a flat direction which remains massless to all orders in a semiclassical expansion, and supersymmetry remains unbroken along this flat direction.

- The nonabelian $SO(3)$ gauge multiplet is asymptotically free and once its coupling becomes large its gauginos condense and generate a nonzero superpotential. The scalar potential which results from this superpotential competes with the $U(1)$ D-term potential and lifts the degeneracy of the flat direction.

- The vacuum to which the theory tends depends on the precise form of the perturbative corrections to the Kähler potential. Using the lowest-order result, $K = -\log s + \cdots$, leads to the standard dilaton runaway, in which the massless field parameterizing the flat direction runs off to infinity. In this limit the $SO(3)$ gauge coupling vanishes and supersymmetry remains unbroken.

- If we instead use the renormalization-group-improved version of K, then the runaway can be stabilized for some choices of the parameters. In this context the significance of the present analysis is to identify a type of logarithmic dependence of K on s which would be sufficient to stabilize the runaway. Of course, we do not know yet whether the required parameters can actually arise for low-energy perturbations about a real string vacuum. However, we regard the potential rewards of their discovery to provide sufficient motivation for taking a proper look.

At first sight, this last item appears to run contrary to a standard argument by Dine and Seiberg against the possibility of fixing the dilaton at weak string coupling [105]. This argument essentially states that if the potential is a series in $1/s$, then any minimum – besides the runaway $s \to \infty$ – must balance different terms in this series against one another, and so be incalculable within the context of perturbation theory.

Despite this, we are able to find nontrivial minima in our analysis for two reasons. First, given that t and s are large at the minimum, but $(1/t)\log s$ is $O(1)$, we see that the Dine-Seiberg argument is correct inasmuch as it states that the potential is required to all orders in $(1/t)\log s$ in order to determine its minima. Fortunately, this form is known by virtue of the renormalization-group re-summation.

Second, one may ask how t and s could be large at the minimum in the first place if there are no large parameters in the potential. Although we have not exhaustively searched parameter space for other solutions, it appears that we only obtain large values for s and t when we choose small values for the parameters α and β, and so this may explain the origin of the nontrivial minima within perturbation theory. To the extent that the appearance of these extra parameters which can be tuned to get weak coupling are required, our analysis would be similar to the older racetrack scenarios [106].

In the end, it may be that realistic string models do not provide α and β of the required magnitude. We regard the artifice of exploring the consequences of their being small nonetheless to be of some value, because it allows us to infer some evidence for the existence of minima in the strong-coupled regime even in the cases where α and β are larger. In this case the minima we find would be pushed into the strong-coupling region for which our calculational methods do not directly apply. Nevertheless the existence of these minima still follows from the existence of a maximum for larger values of s and t, together with the general property that the potential is positive and diverging as $s, t \to 0$. Indeed, the existence of the maxima within the perturbative regime can be inferred for a wider range of values for α and β than can the existence of the minima, leaving only the existence of the singularity of the potential at small s, t to be established using more robust arguments.

Supersymmetry Breaking

If the runaway is stabilized the effective 4D model dynamically breaks supersymmetry at a scale which can be naturally very small compared with the compactification scale. If electroweak symmetry breaking occurs at the supersymmetry-breaking scale, and if fundamental scales like M_s and m_{KK} are chosen near the Planck or GUT scales, then this would provide a Kaluza-Klein realization for using dynamical supersymmetry breaking to naturally generate the electroweak gauge hierarchy, along lines initially proposed some time ago [107].

The low-energy implications of such a model may be inferred by regarding the entire theory considered here to be the hidden supersymmetry-breaking sector to which standard-model particles are coupled [108]. As is easily verified, the large hierarchy $s \gg t$ which the stabilization mechanism predicts ensures that the auxiliary field for S is the largest supersymmetry-breaking *v.e.v.*. This makes the phenomenological implications of this kind of supersymmetry breaking the same as for a dilaton-dominated scenario. This has the virtue of being among the most predictive kinds of string-motivated supersymmetry-breaking scenarios, with definite relations predicted for the spectrum of superpartners [108].

3.5 Discussion: The Cosmological Constant

The particular six-dimensional theory we have considered in this chapter has many features in common with realistic string compactifications, such as 4D $\mathcal{N} = 1$ supersymmetry, moduli, flat directions, Fayet-Iliopoulos terms and supersymmetry-breaking through gaugino condensation. It has the advantage, however, of being much simpler than its 10-dimensional cousins, particularly in its compactification to four dimensions; the compactificaton proceeds on an internal two-sphere, as opposed to the Calabi-Yau three folds required in supersymmetric string compactifications.

As this model illustrateed, one of the most difficult tasks of any theory which includes gravity is to produce a small cosmological constant. The particular six-dimensional model that we discussed, in common with string compactifications, compactifies the internal dimensions with flat four-dimensional space, which seems naïvely to solve the cosmological constant problem; also in common with string compactifications, however, the breaking of supersymmetry is a four-dimensional phenomenon, and therefore naturally introduces a hierarchy problem.

Supersymmetry-breaking introduces a hierarchy in all known four-dimensional supersymmetric models. As discussed in previous chapters, the reason supersymmetry is considered an important feature in many models is that if the supersymmetry-breaking scale is $\sim M_{\text{weak}}$ then the Higgs mass is stablised, solving one hierarchy problem. Supersymmetry also has the merit that the cosmological constant is protected from corrections, but this is only valid at scales much larger than the supersymmetry-breaking scale. Since supersymmetry is broken in the real world, the cosmological constant typically becomes on the order of ${M_{\text{SUSY}}}^2 {M_{\text{Planck}}}^2$ or ${M_{\text{SUSY}}}^4$ (it need only go to zero as $M_{\text{SUSY}} \to 0$). Even if supersymmetry was not imposed to solve the Higgs hierarchy problem, since in a supersymmetric context the standard model particles are embedded in supermultiplets, the couplings of superpartners is on the order of standard model self-couplings. The absence of observed superpartners requires $M_{\text{SUSY}} \gtrsim$ TeV [3], which gives an unacceptable cosmological constant in any case . The basic trouble with any such scheme (and all such string-inspired compactifications of which I am aware) is that they effectively reduce the cosmological constant to a four-dimensional problem; the introduction of extra dimensions provides no additional insight or control. Essentially, since the compactification scale in these models is $\sim M_{\text{Planck}}$, and since the cosmological constant problem is a low-energy problem, the four-dimensional cosmological constant problem persists in these models.

In the next chapter we turn to another class of extra-dimensional models, *brane-world models*, which have several features that make them more attractive from the point of view of trying to solve the cosmological constant problem. These models lower the effective higher-dimensional Planck scale by employing one of several techniques that will be outlined in the next chapter, and sequester the standard model from the gravitational sector. We can see immediately the benefit of achieving these two aims: if the standard-model sector is sequestered from the gravitational sector, then standard model loops may not contribute to the macroscopic cosmological constant, disentangling the problems of supersymmetry-breaking, electroweak symmetry-breaking and the QCD scale from questions of cosmology; and second, lowering the natural scale of gravitational physics will, in this case, lower the scale at which a cosmological constant is produced. In the next chapter we will outline the ways in which this sequestering can be accomplished and made physically reasonable, and will give evidence that the gravitational scale may be as low as the weak scale, which may, we will argue, naturally produce a cosmological constant of the right order of magnitude $\sim 10^{-3}$eV.

Chapter 4

Brane-Worlds, the Cosmological Constant, and Supergravity

We have seen that there seems currently to be no viable four-dimensional solution to the cosmological constant problem. We argued that string theory requires extra dimensions, but that compactification does not substantially change the methods of solution available. In this chapter we will present 'brane-world models,' which are alternatives to compactification. We here argue that the combination of a large extra dimensions brane-world scenario and bulk supersymmetry can provide a way to control the cosmological constant. In the next chapter we explicitly construct a class of supersymmetric large extra dimensions models.

Brane-world models are another class of string-inspired models incorporating extra-dimensional gravitational physics to try to understand four-dimensional puzzles. A brane-world model assumes that the standard-model fields are trapped on a four-dimensional surface in a higher-dimensional spacetime. Gravity and other 'bulk' fields propagate in all directions. Brane-world models are inspired by the properties of D-branes, which, in string theory, trap Yang-Mills fields (and can trap charged matter) on their surface and interact with each other *via* gravity.

Brane-world models, as opposed to *bona fide* D-brane models, do not use the specific and special properties of D-branes embedded within a particular string realisation for model-building. There are a number of motivations for attempting to extend the space of theories under consideration at high energies, but without necessarily tying the details to a particular realisation arising from string theory:

- It is believed that each of the string theories is a perturbative description around a different vacuum of UV-complete, fundamental gravitational physics [53, 54]. Any detailed model based on one of the string theories makes an assumption about the vacuum structure of string theory, something about which we are collosally ignorant [32].

- If string theory is correct in any measure there are likely dimensions beyond the usual four. As discussed earlier, there seems no satisfactory way to compactify string theory in the standard way (at least the hope that a unique compactification would be found that was 'the solution' to the string equations of motion seems less and less plausible [32]), so seeking extensions of the methods by which four-dimensional physics is achieved from higher-dimensional physics seems prudent.

- Prior to brane-world models, the only viable solution to the hierarchy problem was some version of a supersymmetric standard model [199]. There is no direct evidence of supersymmetry at present (we all await the LHC eagerly), and alternative solutions should be investigated by all means as a contingency against the non-observation of supersymmetry.

- Even if string theory is wrong, there is no reason to believe that extended objects will play no role in a quantum theory of gravity. Exploring the effects of these objects on low-energy, four-dimensional physics can help to guide the work being done at the more fundamental level, helping to shape the requirements of a theory of gravity.

Braneworld models have also been used to study alternative formulations of inflation [129], cosmology [127, 128], dark matter [130], quintessence [121], and much more. However, the primary goal of the earliest brane-world models was to replace supersymmetry as a solution to the hierarchy problem. (But see [169].)

4.1 Higgs-Mass Hierarchy as a Motivation for Brane-Worlds

Recall that the hierarchy problem exists because both the weak scale $\sim 10^3$GeV and the Planck scale $\sim 10^{18}$GeV are considered to be fundamental scales. The sensitivity of super-renormalisable operators to high-scale physics (if unconstrained by symmetries) then introduces a hierarchy because $M_{\text{Planck}} \gg M_{\text{weak}}$. In this picture the standard model is an effective theory (perhaps one in a long string of such low-energy theories) valid below the Planck scale, and corrections to the standard model can be computed in powers of $E/M_{\text{Planck}} \ll 1$, perhaps providing insight into gravitational phenomena and their interactions with standard model- and standard model-like-fields. This programme is already effective in the calculation of corrections to electroweak processes which can be computed in powers of E/M_{weak}.

There are two broad classes of brane-world models which were constructed to address the hierarchy problem, the Randall-Sundrum (warped) models [66, 67] and the large extra dimensions (LED) models [136]. A broad categorical difference between the two classes of model is their interpretation of the hierarchy problem; the warped models assume that the Planck mass is fundamental and that the explanation lacking is the small size of the weak mass, while the large extra dimensions models take the fundamental scale of gravity to be low $\sim M_{\text{weak}}$, and try to explain the *apparent* anomolously large value of M_{Planck} with a large internal-dimension volume.

4.2 Warped Models

Warped models [66, 67] try to solve the hierarchy problem (specifically posed as the 'exponentially' large discrepency between M_{Planck} and M_{weak}), by using exponential functions in the metric to suppress the apparent masses on the branes, and thus the Higgs mass and the electroweak scale. The metric in models of this sort takes the form

$$ds^2 = e^{-2k|z|} g_{\mu\nu} dx^\mu dx^\nu + dz^2 \tag{4.1}$$

with $z \in [-\bar{z}, \bar{z})$ periodic. The periodicity of z and the discontinuity of the derivative of z at $z = 0, \bar{z}$ ensure the existence of one brane at each of these positions.

The hierarchy is solved in this model because the standard model is located on the brane at $z = \bar{z}$ and the standard model action is constructed using the induced metric $G_{\mu\nu}(z = \bar{z}) = e^{-2k|\bar{z}|} g_{\mu\nu}$. To see why an exponentially-damped metric can solve the hierarchy problem, consider a scalar-field lagrangian with scalar field mass m. When transformed to canonical form, we see that the *physical* mass is exponentially-damped from its bare value, m:

$$
\begin{aligned}
S_{\text{SM}} &= -\frac{1}{2} \int_{M_4} \sqrt{-G} G^{\mu\nu} \left(\partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 \right) && (4.2) \\
&= -\frac{1}{2} \int_{M_4} \sqrt{-g_4} e^{-2k|\bar{z}|} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + e^{-4k|\bar{z}|} m^2 \phi^2 && (4.3) \\
&= -\frac{1}{2} \int \sqrt{-g_4} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + (m e^{-k|\bar{z}|})^2 \Phi^2 && (4.4)
\end{aligned}
$$

where $\phi = e^{k|\bar{z}|}\Phi$. This solves the hierarchy problem because the effective Planck mass is obtained by integrating the bulk Einstein action over the z coordinate:

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} &= \int dz \sqrt{-G_5}\, M_5^3 R^{(5)} && (4.5) \\
&= \int dz \sqrt{-g_4} e^{-2k|z|} M_5 R^{(4)} && (4.6) \\
&= {M_{\text{Planck}}}^2 \sqrt{-g_{\mu\nu}} R^{(4)}, && (4.7)
\end{aligned}
$$

so that

$$
{M_{\text{Planck}}}^2 = \frac{M_5^3}{2k}(1 - e^{-2k|\bar{z}|}). \tag{4.8}
$$

(We have assumed that the non-standard model brane [the Planck brane] is located at $z = 0$.) In detail, by choosing $k\bar{z} \sim 50$ we may introduce a hierarchy of 20 orders of magnitude between the physical higgs mass, $me^{-k|\bar{z}|/2}$ and the the four-dimensional Planck mass, $M_{\text{Planck}} \sim M_5 \sim m$. This solves the hierarchy problem because high-energy contributions are five-dimensional, and so involve integrating out masses on the order of M_5 (the largest scale in this theory). Since the bare mass is already $m \sim M_5$, no fine-tuning is required, and no unnaturally small numbers are introduced.

4.3 Large Extra Dimensions Models

Another extra-dimensional mechanism for solving the hierarchy problem is to construct a brane-world scenario in which the volume of the extra dimensions is very large [136]. We argued in the previous chapter that it was imperitive that the extra dimensions be small so as to ensure an effective four-dimensional description valid at tested energies. Since experiments have been conducted to $\sim$ TeV scales, naïvely it seems impossible to make the extra dimensions larger than this.

In fact, only non-graviational interactions have been tested at energies high enough to rule out deviations from the standard paradigm of four-dimensional general relativity coupled to the standard model. If *only* gravity were to propagate in large extra dimensions, the extra dimensions can be as large as a tenth of a millimeter [131] because short-range graviational phenomena are very-poorly measured.[1] A very-large extra-dimensional model of this sort can be constructed using branes, since this is precisely what branes do: they localise the Yang-Mills degrees of freedom on the brane, making them effectively four-dimensional, while allowing the gravitational degrees of freedom to propagate in all directions.

To see why large extra dimensions can solve the Higgs-mass hierarchy problem, imagine the simple situation of dimensionally reducing a higher-

[1] In fact other fields can propagate in the 'bulk,' it is imperative that 'bulk' fields couple weakly to standard-modelfields.

dimensional theory (with d total dimensions) on torii to four dimensions. The gravitational action (where we use our old notation: z are the coordinates of the full space, x of our usual four dimensions and y are of the internal dimensions and $\widehat{R}$ is the Ricci scalar of the total space and R that of the four dimensions) reduces as

$$\int dz \frac{M_d^{d-2}}{2} \widehat{R} = \int dx \int dy \, \frac{M_d^{d-2}}{2} R = \int dx \frac{V M_d^{d-2}}{2} R \tag{4.9}$$

where $V = \int dy$ is the volume of the extra dimensions. This implies that the effective four-dimensional Planck-scale is given by

$$M_4^2 = \text{Vol} \times M_d^{(d-2)} \sim (\pi r_c)^{(d-4)} \, M_d^{(d-2)}, \tag{4.10}$$

where r_c is the compactification radius. The hierarchy problem is solved if the higher-dimensional Planck mass is TeV, since this would in turn imply that the scale at which the Higgs receives corrections is a TeV, eliminating the fine-tuning required to have a TeV-scale Higgs.

One way to intuitively understand what is happening is to use the extra-dimensional flux law for the potential for a pair of test-masses, μ_1, μ_2 obtained from Gauss' law:

$$V \sim \frac{\mu_1 \, \mu_2}{M_d^{d-2}} \frac{1}{r^{d-3}} \tag{4.11}$$

(because the area of a sphere $\sim r^{d-2}$). As r increases to the size of the extra dimension, r_c, the above law saturates, yielding

$$V_{\text{long-range}} \sim \frac{\mu_1 \, \mu_2}{M_d^{d-2} \, r_c^{d-4}} \frac{1}{r} \sim \frac{\mu_1 \, \mu_2}{M_4^2} \frac{1}{r}, \tag{4.12}$$

recovering the mass formula eq. 4.10.

d	$r(\mathrm{m})$ $t=1$	$r(\mathrm{m})$ $t=10$
5	2×10^{12}	7×10^{11}
6	2×10^{-4}	5×10^{-5}
7	7×10^{-10}	2×10^{-10}
8	2×10^{-12}	3×10^{-13}
9	4×10^{-14}	8×10^{-15}
10	3×10^{-15}	6×10^{-16}
11	6×10^{-16}	1×10^{-16}

Table 4.1: Table listing the radii of large extra dimensions for different total numbers of dimensions and for the two cases in which the fundamental scale of gravitational physics is 1 or 10 TeV.

What size extra dimensions do we need if we required the extra-dimensional Planck-mass to be $t\,\mathrm{TeV}$? If we choose t not-too-different from 1 we have,

$$(\pi r) \sim \frac{t^{-(d-4)/(d-2)}}{2}\,10^{32/(d-4)-19}\,\mathrm{m} \tag{4.13}$$

For $t=1$ and $t=10$ we list the values of the radius for various dimensions in table 4.1

Experimental tests of short-range gravitational effects search for deviations from Newton's Law (or Gauss' Law) (force $\sim 1/r^2$) [131] by fitting extremely fine-tuned measurements of the forces between solid objects placed very closely together to the more general relation

$$F = \frac{m_1\,m_2}{r^2}\left(1+\alpha\,e^{-\lambda r}\right). \tag{4.14}$$

These measurements are notoriously difficult due to Casimir effects, thermal noise and even local inhomogeneities in the Earth's crust [131]. Nevertheless, the best current bounds place

$$\lambda \lesssim 200\,\mu\mathrm{m} \tag{4.15}$$

assuming $\alpha = 1$ (for gravitational-strength tests), thus bounding the size of large extra dimensions of to be less than $O(0.1\text{mm})$. because in these models it is the KK modes of mass $\sim m$ that alter short-distance gravitation—this part of brane-world physics works as described for normal compactifications in the previous chapter. At very short distances we expect to recover the extra-dimensional flux law $\sim 1/r^{n+2}$.)

To summarise, because these models localise all of the very well-measured interactions on a four-dimensional brane, standard model predictions are very easy to retain. Only gravity sees the extra dimensions, and, even though the fundamental scale of gravity and the other forces is the same, gravity is made very weak because it is diluted in a large internal space to which the standard model forces have no access; it is possible to make models with a radius $\sim \mu$m, and which satisfy all current experimental bounds. (There are additional astrophysical bounds from the emission of very light Kaluza-Klein modes, $m \sim 1/r \sim 10^{-3}$eV, in supernovae. This problem can be regarded as a constraint to be imposed on specific models within this framework, however, as opposed to a brush with which to tar the whole paradigm; it is likely much easier to make detailed models satisfying particular supernova bounds than to solve the hierarchy problem [179, 180].)

4.4 The Cosmological Constant and Brane-World Models

Brane-world models as presented above do not provide solutions to the cosmological constant problem, they simply assume the existence of flat 3-branes (and therefore a zero effective four-dimensional cosmological constant). Indeed, as we will see, these extra-dimensional models require fine-tunings in order to ensure the vanishing of the effective four-dimensional cosmological constant; the tensions in the manifold are related to each other through 'consistency conditions' [172]. Because the topology of the underlying manifold is fixed, and because the delta-function singularities induced by branes contribute to the curvature and therefore the Euler number, the tensions of the branes must be related. In the Randall-Sundrum two-brane scenarios, for example, the internal manifold is a circle, so the the sum of the tensions must vanish for the total curvature to vanish. (Additional curvature in the four-dimensional spacetime is required to alleviate the fine-tuning, thus tying the effective cosmological constant to the cancelling of tensions [109].) Similar conditions hold in the ADD scenario.

Nevertheless, as outlined in the next section, brane-world models have one extremely attractive feature from the perspective of the cosmological constant problem, and incorporating this attractive feature into a model which naturally contains flat branes is the purpose of the rest of this thesis. We will find that *static* solutions with flat branes can be made to arise naturally (equivalently, a theory can be constructed which produces naturally flat branes for arbitrary tensions, if the branes in question are pure tension.)

4.5 Tension and Standard Model Vacuum-Energy

The observation on which the rest of the work in this thesis rests is that *it may be easier to solve the cosmological constant problem in brane-world models than in models without branes.* The reason for this is that, since standard model particles are trapped on a brane, purely standard-model loops do not contribute directly to the cosmological constant, but to the tension of the brane on which they reside.

Recall that the cosmological constant problem had two aspects: the problem of making the cosmological constant small at *some* scale, and the problem of retaining the smallness of the cosmological constant as the theory is run down from high to low energies. In particular, all standard model loops, including low-energy loops involving only the electron, destroy any fine-tunings imposed to make the cosmological constant small at energies $\sim$ MeV. We here argue that the running of the cosmological constant as standard-model-scales are intergrated out can be rendered innocuous by brane-world models. We will furthermore argue that supersymmetry can be used to set the cosmological constant to a small value at some high scale, thus naturally allowing a small cosmological constant at all scales.

To see in detail that brane loops do not directly affect the cosmological constant, consider the following simple model, consisting of a brane containing the standard model fields coupled to gravity in the usual way and a bulk action, consisting of gravity and any other fields which propagate in all directions,

$$S = \int_{M_D} d^D x \, \mathcal{L}_{\text{bulk}} + \int_{M_4} d^4 x \, \mathcal{L}_{\text{sm}} \tag{4.16}$$

Now imagine integrating out all standard model fields *exactly* in the path integral. Since the contributions from these fields are localised on the brane the entire result is a potential for the bulk fields which is localised on the brane:

$$S = \int_{M_D} d^D x \, \mathcal{L}_{\text{bulk}} + \int_{M_4} d^4 x \, \left(T_4 \sqrt{-g_4} + \cdots\right) \tag{4.17}$$

where the first term is what *would* have been the troublesome cosmological constant-term, and the extra terms are couplings of bulk fields to the brane. Therefore, *standard-model loops do not directly contribute to the cosmological constant in brane-world models.*

The ways in which this idea fails in actual models is manifold, but the most obvious way is for the brane tension to set the scale for all of the curvatures in the problem, inducing TeV-scale curvatures everywhere, including in the observable four dimensions.

4.6 Fine-Tuning in Randall-Sundrum

To see an example of such a failure, consider again the Randall-Sundrum model discussed above. This model requires a fine-tuning between the brane tensions, the bulk cosmological constant, Λ and the five-dimensional Planck-mass, M in order to ensure flat three-branes on which we may live, and therefore does not provide a satisfactory explanation for the smallness of the effective four-dimensional cosmological constant.

To see this conclusion in detail let us construct the Randall-Sundrum solution. The *ansatz* for the metric is the most general one consistent with maximally symmetric four-dimensional slicings,

$$ds^2 = e^{-2\sigma(z)}\, g_{\mu\nu}(x) dx^\mu dx^\nu + dz^2 \qquad (4.18)$$

with $z \in [-\bar{z}, \bar{z})$ periodic, $g_{\mu\nu}(x)$ is *only* a function of the four-dimensional coordinates x, and $R_{\mu\nu}$, the curvature constructed only from $g_{\mu\nu}(x)$, is given by

$$R_{\mu\nu} = \lambda\, g_{\mu\nu} \qquad (4.19)$$

(that is, we assume $g_{\mu\nu}$ describes deSitter, Minkowski or AdS space). We assume that the action to be minimized is five-dimensional GR coupled to two three-branes

$$S = \int d^5Z \sqrt{-\widehat{g}} \left[\frac{M^3}{2} \widehat{R} + \Lambda \right] - \int d^4x \sum_i \sqrt{-\widehat{g}^{(i)}_{\rm ind}}\, T_i. \tag{4.20}$$

Varying this action and setting $\lambda = 0$ for flat four-dimensional slices yields the equations of motion from the $\mu\nu$ and the 55 components of the Einstein equations, respectively,

$$\begin{aligned} \sigma'' &= \sum \delta(x - x_i) T_i / M^3 && (4.21) \\ 6(\sigma')^2 &= \Lambda / M^3 && (4.22) \end{aligned}$$

(There must be two branes for topological reasons since we take z periodic, as we will see.)

Choose $x_1 = 0, x_2 = \bar{z}$. Integrating the first ($\mu\nu$) Einstein equation over the whole interval implies that

$$T_1 = -T_2, \tag{4.23}$$

and the differential equation itself implies that

$$\sigma = \begin{cases} k_1\, z & \text{for } z \in [0, \bar{z}) \\ k_2\, z & \text{for } z \in [-\bar{z}, 0) \end{cases} \tag{4.24}$$

with

$$k_1 = T_1, \qquad k_2 = -T_1, \tag{4.25}$$

and the 55 equation shows that

$$T_1 = -T_2 = \sqrt{\frac{\Lambda}{6M^3}}. \tag{4.26}$$

The important point from the point of view of the cosmological constant is that the tensions are rigidly related to a parameter in the bulk lagrangian, and that this relation is neither the result of, nor results in, a symmetry: The tensions and the Planck mass need not have anything to do with each other, since the precise values of the tensions are the minima of four-dimensional effective potentials. [2]

This solution is therefore 'fine-tuned,' because in order to have a flat solution we must impose 4.26 and there is no good reason to do so [126]. [3] This fine-tuning makes this setup sensitive to the same issues as four-dimensional theories of particle physics and regular compactifications of higher-dimensional theories:

1. In order to produce a small cosmological constant, we must tune unrelated bare parameters in the lagrangian with no apparent increase in symmetry to protect this tuning.

2. Once we perform quantum corrections the fine-tuning is completely ruined, and must be imposed order-by-order. In other words, none of the fine-tunings are perturbatively stable to quantum corrections—a familiar circumstance from Higgs physics.

This last point arises here because even if we believe bulk physics can be sequestered from the brane cosmological constant problem (bulk physics is clearly disastrous, since it lives at scales $\sim M_{\text{Planck}}$), brane loops are $\sim$ TeV, which induce a TeV-scale cosmological constant.

[2] We can also see the topological explanation, promised above, for two branes: There must be two branes because e^{kz} cannot be made continuously differentiable on a circle.

[3] More generally, it can be shown [128] that assuming an FRW universe with matter and a 'cosmological constant' (tension) on the branes, we must impose eq. 4.26 to replicate the usual matter-dominated expansion of the universe.

However, if we can find a mechanism by which flat branes exist for any choice of brane tensions, the *quantum problem* would seem to be fixed, since we can then claim that four-dimensional geometry is independent of the details of how the electroweak phase trasition occurred. (Without this, the cosmological constant is sensitive to the precise scale at which the EWSB transition occured at the level of 1 part in 10^{12}.)

This is what we wish to explore in the remainder of this thesis: a mechanism that will embed arbitrary-tension branes in naturally-flat slicings of four-dimensional space.

4.7 Self-Tuning and Naturally Flat Branes

In this section we show that flat four-dimensional slices are natural in higher-dimensional dilaton-gravity systems. As long as there is a dilaton in the system—a scalar that has a shift symmetry which encodes a rescaling:

$$\mathcal{L}(\phi - c, e^{\omega\, c} g_{\mu\nu}) = e^{2c}\mathcal{L} \tag{4.27}$$

for a fixed constanta ω, depending on the dimensionality, which we will derive below, *ansatze* of the form

$$ds^2 = W^2(y)\, g_{\mu\nu}(x) dx^\mu dx^\nu + g_{mn}(y) dy^m dy^n \tag{4.28}$$

naturaly yield

$$R_{\mu\nu} = 0, \tag{4.29}$$

where $R_{\mu\nu}$ is computed from $g_{\mu\nu}(x)$ alone. That is, dilaton-gravity systems in higher dimensions naturally yield flat four-dimensional slicings. This wouldn't be interesting except that the relation $(D/2-1)\omega = -2$ precisely reproduces the scaling that naturally occurs in supergravity theories.

This statement requires the following three results:

1. That the *ansatze* 4.28 is the most general possible if we assume maximally-symmetric four-dimensional slices (we do not allow FRW-with-matter cosmological *ansatze*). This theorem is proved, for example in [80];

2. That metrics of the form 4.28 allow the higher-dimensional curvature, $\widehat{R}$, to be expressed simply in terms of the lower-dimensional curvature. In particular, we will show that if we define vielbeins according to

$$E^{\hat{\alpha}}_{\hat{\mu}}(x,y) = W(y)\, e^{\alpha}_{\mu}(x), \qquad E^{\hat{a}}_{\hat{m}}(x,y) = e^{a}_{m}(y) \tag{4.30}$$

such that

$$E_{\hat{\alpha}}{}^{\hat{\mu}} E^{\hat{\mu}}_{\hat{\beta}} = \eta_{\alpha\beta} \qquad e^{\hat{\alpha}}_{\hat{\mu}} e_{\hat{\alpha}\hat{\nu}} = g_{\mu\nu}(x) \qquad e^{a}_{m} e_{an} = g_{mn}(y). \tag{4.31}$$

(α and $\hat{\alpha}$ refer to the same range of indicies but refer to the different tangent-spaces implied by E^{α}_{μ} and $E^{\hat{\alpha}}_{\mu}$), that

$$\begin{aligned} \widehat{R}_{\hat{\alpha}\hat{\beta}} &= \frac{R_{\alpha\beta}}{W^2} - \frac{1}{n\, W^n} \nabla^2 W^n\, \eta_{\alpha\beta} \\ \widehat{R}_{ab} &= R_{ab} - \frac{n}{W} \nabla_a \nabla_b W\, \delta_{mn}, \end{aligned} \tag{4.32}$$

where $\widehat{R}_{\hat{\alpha}\hat{\beta}}$ is the Ricci tensor calculated from the vielbeins E^{A}_{M} and restricted to the index-range $\hat{\alpha}, \hat{\beta}$, and $R_{\alpha\beta}$ is the Ricci tensor calculated purely from the subspace vielbeins e^{α}_{μ} (and similarly for $\widehat{R}_{ab}$ and R_{ab}— we make no distinction between these indices because there is no confusion in this case), and where here $n = \delta^{\alpha}_{\alpha}$ is the number of dimensions in $g_{\mu\nu}(x)$ (taken to be four here, but left general for cosmological applications outlined in the future directions); and

3. Finally, that the equations of motion for systems satisfying eq. 4.27 and under the *ansatz* 4.28 take the form

$$\widehat{R}_{\mu\nu} - \alpha\,\widehat{\Box}\phi\widehat{g}_{\mu\nu} = 0, \tag{4.33}$$

where $\widehat{R}_{\mu\nu}$ is computed from the full metric and $\widehat{\Box}$ is the full d'Alembertian computed from ds^2 in eq. 4.28.

In the previous, D is the total spacetime dimension, $\mu, \nu = 0..n-1$, $m, n, = n...D-1$. We will eventually be interested in $D = 6$, $n = 4$. A hat over a quantity will denote that it is the full D- or six-dimensional quantity.

Let us assume the above and show that $R_{\mu\nu} = 0$. Beginning with eq. 4.33, we expand the D-dimensional $\widehat{\Box}$ as

$$\begin{aligned} \widehat{\Box}\phi &= \frac{1}{\sqrt{\widehat{g}}}\partial_M\left(\sqrt{\widehat{g}}\widehat{g}^{MN}\partial_N\phi\right) \\ &= \frac{1}{W^4}\nabla_m\left(W^4\nabla^m\phi\right)) \end{aligned} \tag{4.34}$$

where ∇_m is the covariant derivative operator constructed from g_{mn}. We can also write the first of eq. 4.32 as

$$\widehat{R}_{\hat{\alpha}\hat{\beta}} = \frac{1}{W^2}R_{\alpha\beta} - \frac{1}{W^n}\nabla_m\left(W^n\nabla^m\log W\right)\eta_{\alpha\beta}, \tag{4.35}$$

so that eq. 4.33 becomes

$$W^{(n-2)}R_{\alpha\beta}(x) - \nabla_m\left[W^n\,\nabla^m\left(\log W - \alpha\,\phi\right)\right]\eta_{\alpha\beta} = 0. \tag{4.36}$$

Integrating over dy shows that

$$R_{\mu\nu}(x) = 0, \tag{4.37}$$

which was to be shown, as long as the internal manifold is compact without boundary.

In appendix C we show that the curvatures can be expressed in the form claimed under the assumptions made. We proceed to prove that scaling in certain scalar-tensor theories of gravity ensures flat branes if we assume maximal symmetry.[4]

Scaling and Gravity-Dilaton Equations of Motion

We now show that under relatively mild assumptions,

$$\widehat{R}_{\mu\nu} = \alpha\,\widehat{\Box}\phi. \tag{4.38}$$

To begin, we assume that the total lagrangian for the system is of the form

$$\mathcal{L} = \sqrt{-\widehat{g}}\,\frac{1}{2}\widehat{R} + \widetilde{\mathcal{L}}(\widehat{g}_{MN}, \phi), \tag{4.39}$$

and that the 'matter' lagrangian, $\widetilde{\mathcal{L}}$ has the scaling property

$$\widetilde{\mathcal{L}}(\widehat{g}_{MN}, \phi) = e^{2c}\,\widetilde{\mathcal{L}}(e^{-\omega c}\,\widehat{g}_{MN}, \phi - c). \tag{4.40}$$

We further assume the *ansatz* of the form 4.28.

The gravitational equations of motion are given by

$$\frac{1}{2}\left[\widehat{R}_{MN} - \frac{1}{2}\widehat{g}_{MN}\widehat{R}\right] + \frac{1}{\sqrt{\widehat{g}}}\frac{\partial\widetilde{\mathcal{L}}}{\partial\widehat{g}^{MN}} = 0. \tag{4.41}$$

Tracing and substituting for the Ricci scalar, $\widehat{R}$ gives

$$\widehat{R}_{MN} + 2\,\tau_{MN} - \frac{1}{\Delta}\,\tau\,\widehat{g}_{MN} \tag{4.42}$$

where $\Delta = D/2 - 1$ and

$$\tau_{MN} = \frac{1}{\sqrt{\widehat{g}}}\frac{\partial\widetilde{\mathcal{L}}}{\partial\widehat{g}^{MN}} \qquad \tau = \widehat{g}^{MN}\tau_{MN}. \tag{4.43}$$

[4] Recently it has been shown that more care must be taken if cosmological FRW-type *ansatze* are allowed. See [205].

By taking the derivative of the scaling relation eq. 4.40 with respect to c and evaluating at $c = 0$ we find that

$$0 = -\omega\,\sqrt{\widehat{g}}\,\tau + 2\widetilde{\mathcal{L}} - \frac{\partial\widetilde{\mathcal{L}}}{\partial\phi} \tag{4.44}$$

Substitution into eq. 4.42 yields

$$\widehat{R}_{MN} + 2\,\tau_{MN} - \frac{1}{\Delta\,\omega}\,\widehat{g}_{MN}\,\frac{1}{\sqrt{\widehat{g}}}\left[2\,\widetilde{\mathcal{L}} - \frac{\partial\widetilde{\mathcal{L}}}{\partial\phi}\right] = 0. \tag{4.45}$$

We now show that when restricted to the four-dimensional sub-manifold, $M = \mu, N = \nu$, eq. 4.45 reduces to eq. 4.38, by arguing that when evaluated on the *ansatz*,

$$\frac{\partial\widetilde{\mathcal{L}}}{\partial\widehat{g}^{\mu\nu}} = -\frac{1}{2}\widehat{g}_{\mu\nu}\widetilde{\mathcal{L}}, \tag{4.46}$$

which will complete the proof of eq. 4.40, since $\omega\Delta = -2$ ensures that

$$\sqrt{\widehat{g}}\,\widehat{R}_{\mu\nu} = \alpha\,\widehat{g}_{\mu\nu}\,\partial_M\frac{\partial\widehat{\mathcal{L}}}{\partial\partial_M\phi}, \tag{4.47}$$

by the ϕ equations of motion,

$$\partial_M\frac{\partial\widehat{\mathcal{L}}}{\partial\partial_M\phi} = \frac{\partial\widehat{\mathcal{L}}}{\partial\phi} \tag{4.48}$$

which was to be shown. As it turns out, $\omega = -\frac{2}{(D/2-1)}$ is precisely the scaling relation in supergravities which allows us to go from the Einstein to the string frame, so there is no dirth of theories which naturally satisfy this constraint.

To see that eq. 4.46 is true we must recall the maximal symmetry with respect to the four-dimensional coordinates. The only possible maximally-symmetric and index-symmetric two-tensor in four dimensions is proportional to the metric tensor, which implies that $(\partial\widehat{\mathcal{L}}/\partial\widehat{g}^{\mu\nu})/\sqrt{\widehat{g}} \propto g_{\mu\nu}$, with a proportionality that is independent of x. To see that the proportionality is constant as claimed we consider that the matter lagrangian is a function of various form-fields. The only maximally-symmetric *ansatze* for form-fields of dimensionality less than four in four dimensions are the constant (for the zero-form) and zero forms (for 2- and 3-forms):

$$\begin{aligned} \phi &= (\mathrm{const}), \\ F_{(2)} = G_{(3)} &= 0 \\ A_{(4)} &= (\mathrm{vol}) \end{aligned} \tag{4.49}$$

Therefore, as long as the lagrangian contains no four-forms (which may be proportional to the volume form in four dimensions), $\partial\widehat{\mathcal{L}}/\partial\widehat{g}^{\mu\nu}$ evaluated on the *ansatz* evaluates to the term which results from the variation of the overall $\sqrt{g}$:

$$\partial\widehat{\mathcal{L}}/\partial\widehat{g}^{\mu\nu} = -\frac{1}{2}\,\widehat{g}_{\mu\nu}\,\widehat{\mathcal{L}}, \tag{4.50}$$

4.8 Bulk Loops and Supersymmetry

We have seen that self-tuning can make brane loops irrelevant for the cosmological constant problem. This is already a significant improvement, if such a model can be constructed. Another crucial ingredient to the success of the self-tuning large-extra-dimensions scenario for solving the cosmological constant problem is that bulk loops not destroy the scaling form of the potential. Supersymmetry, as we have seen, can significanly soften ultraviolet divergences and can ensure non-renormalisation of potentials. In this section we will argue that supersymmetric models may provide a means of controlling bulk loops.

As discussed in previous chapters, loops induce divergent contributions of the form,

$$\begin{aligned} e_6^{-1}\,\delta L_{\rm Bulk} &= (\alpha + \beta\, e^{\phi} + \cdots)M^6 + M^4\left(a_1 + a_2\, e^{\phi} + \cdots\right)\, R \\ &+ M^2\left(b_1 + b_2\, e^{\phi} + \cdots\right) R^2 \\ &+ \log(\mu^2/M^2)\, \left(b_1 + b_2\, e^{\phi} + \cdots\right) R^4 + \cdots \end{aligned} \tag{4.51}$$

with M the relevant UV scale, which we take to be $\sim M_{\rm weak}$ in a LED scenario [180]. (It is e^{ϕ} that appears in the perturbation series here because it appears as the gauge coupling constant $\mathcal{L} \sim e^{-\phi} F_{MN} F^{MN}$.)

Let us deal with each of these terms:

- The terms of order M^6, are clearly a disaster, as they will source all of the Einstein equations and make all curvatures $\sim O(M)$. This is the old cosmological constant problem in a six-dimensional context—this is a renormalisation of the six-dimenisonal cosmological constant.

- The terms of order M^4 are a renormalisation of Newton's constant in six dimensions, and so are not problematic for the usual reasons.

- Terms of order M^2 when evaluated on the equations of motion, where we expect curvatures $\sim 1/r^2 \sim (10^{-3}\mathrm{eV})^2$ yield

$$M^2 R^2 \sim M_{\rm weak}^2 10^{-3}\mathrm{eV} \gg (10^{-3}\mathrm{eV})^4 \tag{4.52}$$

- Terms of order $M^0\, R^4$ provide the correct cosmological constant:

$$R^4 \sim 1/r^4 \sim (10^{-3}\mathrm{eV})^4 \tag{4.53}$$

If we can use supersymmetry to ensure the cancellation of ultraviolet contributions to the operators with positive mass-dimension-coefficients, we can produce a naturally-small cosmological constant.

We expect compactifications with branes to typically break any supersymmetry present in the model, but the supersymmetry-breaking scale in such a compactification would be the compactification scale $M_S \sim M_c \sim 10^{-3}$eV.

- Supersymmetry forbids M^6 terms because bosons and fermions precisely cancel in the calculation of the vacuum energy.
- In calculations performed to date it also seems that the M^4 terms cancel in 6D supergravity [203], although as we have mentioned, these are not dangerous.
- Terms of order M^2 seem not to cancel in the simplest chiral supergravities, but in 6D theories obtained from the compactification of 10D supergravity the field-content is just right to cancel these terms [180, 203]. These terms are analogues of the dangerous $M^2\, M^2_{\rm SUSY}$ terms in four dimensions.
- Terms of order M^0 continue to give the correct cosmological constant when supersymmetry breaks.

Here we see the full power of extra-dimensional covariance being utilized. These same corrections are difficult to organise in four dimensions—indeed a naive four-dimensional analysis would have reached the same (negative) conclusion as reached for 4D field-theoretic solutions to the cosmological constant obtained in chapter 2. Large extra-dimenisonal theories allow us to use extra-dimensional coordinate invariance to simplify and to organise the bulk corrections, while trapping the standard model on a brane and employing self-tuning can nullify the dangerous standard model contributions.

4.9 Supergravity Braneworlds

In this section we make the case that the six-dimensional supergravity theory considered in chapter 3 naturally furnishes the scaling symmetry required to self-tune the four- dimensional effective cosmological constant. Although this model does not contain the multiplet structure to gaurantee control of the M^2 corrections mentioned in the previous section, it provides a simple testing-ground in which to test the self-tuning ideas proposed. This will be the focus of the rest of this thesis. In the next chapter we explicitly exhibit all of the solutions satisfying a warped-product metric *ansatz* with maximally-symmetric four-dimensional slicings, analysed in a previous section. This step is crucial, as we have said, because it is important to exhibit the smoothness of the solutions.

Recall the lagrangian for the six-dimensional Salam-Sezgin supergravity, whose bosonic and fermionic parts are given in eqs. 3.72 and 3.73, which we reproduce here

$$e^{-1}\mathcal{L}_B = -\frac{1}{2}R - \frac{1}{2}\partial_M\phi\,\partial^M\phi - \frac{e^{-2\phi}}{12}\,G_{MNP}G^{MNP} - \frac{e^{-\phi}}{4}\,F_{MN}F^{MN} - 2g^2e^{\phi}, \tag{4.54}$$

and

$$\begin{aligned}
e^{-1}\mathcal{L}_F &= -\bar{\psi}_M\Gamma^{MNP}D_N\psi_P - \bar{\chi}\Gamma^M D_M\chi - \bar{\lambda}\Gamma^M D_M\lambda \\
&\quad -\frac{1}{2}\partial_M\phi\left(\bar{\chi}\Gamma^N\Gamma^M\psi_N + \bar{\psi}_N\Gamma^M\Gamma^N\chi\right) \\
&\quad +\frac{e^{-\phi}}{12\sqrt{2}}\,G_{MNP}(-\bar{\psi}^R\Gamma_{[R}\Gamma^{MNP}\Gamma_{S]}\psi^S + \bar{\psi}_R\Gamma^{MNP}\Gamma^R\chi \\
&\qquad\qquad - \bar{\chi}\Gamma^R\Gamma^{MNP}\psi_R + \bar{\chi}\Gamma^{MNP}\chi - \bar{\lambda}\Gamma^{MNP}\lambda) \\
&\quad -\frac{e^{-\phi/2}}{4}\,F_{MN}\left(\bar{\psi}_Q\Gamma^{MN}\Gamma^Q\lambda + \bar{\lambda}\Gamma^Q\Gamma^{MN}\psi_Q - \bar{\chi}\Gamma^{MN}\lambda + \bar{\lambda}\Gamma^{MN}\chi\right) \\
&\quad +ige^{\phi/2}\left(\bar{\psi}_M\Gamma^M\lambda + \bar{\lambda}\Gamma^M\psi_M + \bar{\chi}\lambda - \bar{\lambda}\chi\right),
\end{aligned} \tag{4.55}$$

This lagrangian *co*-varies under the scaling transformation given in eq. 3.75, which allows the lagrangian to be written in the string frame. Restricting these trasformations to only the bosonic sector reveals precisely the scaling transformation required. It is easy to see that the bosonic part scales as claimed: R and $\partial_M\phi\partial^M\phi$ both scale as $\widehat{g}^{MN}$, and each extra factor of inverse metric in F^2 and G^2 is accompanied by an extra factor of $e^{-\phi}$, while the potential term $g^2\,e^{\phi}$ has an extra e^{ϕ} to compensate for its lack of g^{MN} compared to R and $\partial_M\phi\partial^M\phi$. Accounting also for the $e = \sqrt{\widehat{g}}$, we see that under

$$\phi \;\rightarrow\; \phi - c \tag{4.56}$$

$$g_{MN} \;\rightarrow\; g_{MN}e^{-c}, \tag{4.57}$$

the bosonic action scales as

$$\mathcal{L} \rightarrow e^{-2c}\mathcal{L} \tag{4.58}$$

(recall that we showed that $g_{MN} \rightarrow e^{\omega c}\,g_{MN}$ where

$$\omega = \frac{-2}{\Delta} = -1). \tag{4.59}$$

We now show that *ansatze* of the form asumed in this chapter ensure flat four-dimensional slicings. We will exhibit explicit solutions of the bosonic equations of motion, and will describe how to implement the self-tuning idea. In the final chapter we will discuss the issues of quantum corrections and further work.

Chapter 5

Supergravity Solutions

We have argued that, despite the many four-dimensional attempts to reduce the sensitivity of the cosmological constant to physics at all scales, they all fail in essentially the same way: these attempts require some new low-energy physics which, when directly manifested tames the cosmological constant, but which is required to be hidden in some way by phenomenological considerations of other sorts. Supersymmetry can tame the cosmological constant, but four-dimensional supersymmetry must be broken at $\sim$ TeV because we don't see superpartners; the quintessence field must be very light, essentially recasting the cosmological constant problem as a Higgs-mass-type stability problem; and IR modifications of four-dimensional gravity are difficult to match to existing cosmological knowledge.

We have also argued that standard compactifications do not essentially change the approach to the cosmological constant problem because the compactification happens at a scale high enough to render irrelevant the additional diffeomorphism invariance of the extra dimensions.

Brane world models, however, have a distinct advantage over any of the above methods because they allow extra dimensions as large as 0.1mm. When expressed as an energy, this just happens to be $\sim (10^{-3}\text{eV})^{-1}$— precisely the right scale for the cosmological constant.

We have argued that, as long as flat-brane solutions exist for any choice of brane tensions, brane-world scenarios of this sort can solve the standard-model-part of the quantum cosmological constant problem. Because standard-model-loops contribute only to the brane tension, the details of how the QCD phase transition, or how the EWSB transition occurred are irrelevant for the flatness of our four dimensions.

We argued that the most popular brane-world models did not naturally produce flat branes. We showed that a higher-dimensional self-tuning mechanism furnished by a dilaton-like scalar could provide a mechanism by which to ensure flat branes.

We finally argued that bulk supersymmetry would be important to ensure that the scaling symmetry remained correctly-represented to low energies. Bulk supersymmetry can ensure this because, if the compactification scale $\sim 10^{-3}$eV is also the supersymmetry-breaking scale, corrections to the scalar field potential will be protected above this scale. Below this scale we do not care what happens, because the cosmological constant is $\sim 10^{-3}$eV.

These arguments point directly toward considering warped-product compactifications of 6D supergravity theories. We have analysed one such 6D supergravity in chapter 3, where we constructed its four-dimensional supersymmetric reduction. In this chapter we will present brane-world solutions to this same supergravity. Finding explicit solutions is important because earlier attempts at finding realisations of the extra-dimensional self-tuning idea were unsuccessful because of the singularity-structure of the solutions so realised [141, 142, 143]. In particular, five-dimensional self-tuning solutions exhibited naked singularities which resisted attempts to tame them [142].

The solutions we will present are well-behaved and exhibit the specific form of self-tuning argued for in the previous chapter, namely, that for any given tension, a solution with flat four-dimensional slicings exists, and, among all maximally-symmetric slicings, only flat slicings exist.[1]

Yet another motivation for seeking warped solutions to higher-dimensional supergravities is that, although higher-dimensional models have a long history within supersymmetric theories, there has been less exploration within supergravity theories of the low-energy implications of warped compactifications [169, 170]. This is by contrast with nonsupersymmetric models, described above, for which warped compactifications have been explored in some detail in five spacetime dimensions [66, 67], and in six dimensions [171, 172, 173]. The reason for this difference is partly due to the point of view taken by workers on the 5D models, for whom part of the basic motivation was to provide an approach to the hierarchy problem which is an alternative to supersymmetric models.

In the end Nature may not feel the need to choose to solve the hierarchy problem using only supersymmetry or only warping. Warping may play a role in the hierarchy problem in addition to supersymmetry, rather than in competition with it. In any case, in order to decide whether supersymmetry is a useful for either the cosmological constant or for the hierarchy problem in warped models requires a better theoretical exploration of what is

[1]More general metrics are considered in ref [205].

possible. Certainly if string theory proves to be the correct theory of very-short distances warping is only likely to play a role at low-energies within a supergravity framework.

From the purely model-building point-of-view, six dimensions are also attractive for constructing brane world models with compact internal spaces since the gravitational back-reaction problem for 3-branes (codimension two objects) is soluble in terms of δ-function curvature singularities [174]. Warped examples of this type have been constructed [172, 173], based on the AdS soliton solution [171] to the Einstein equations with negative cosmological constant. Unwarped brane-world solutions have also been constructed, both for nonsupersymmetric [175] and for supersymmetric [176] systems [2].

With these two motivations, then, we forge ahead in this chapter to exhibit the solutions to the Salam-Sezgin model described in chapter 3.

Roadmap

In the following sections we present both unwarped and a warped-compactification solutions of the Salam-Sezgin six-dimensional supergravity. These solutions can be considered to be a proof of the existence of solutions free of excessively-singular points. We discuss possible sources of fine-tuning in these solutions. The warped solutions we present are actually the full set of solutions under a particular symmetry *ansatz* and a restriction to only conical singularities [184, 178].

We construct these solutions by explicitly solving the 6D coupled Einstein-

[2]Codimension two warped solutions of type IIB string theory have also been considered [170], with supersymmetry broken by a global cosmic brane of finite extent.

Maxwell-dilaton equations of the Salam-Sezgin supergravity[144, 145, 146] [3] The non-warped solution provides a very simple example a 'football' [175] or rugby-ball compactification, but permits any tensions whatsoever at the poles, although the two tensions must be equal.

This naturally raises the interesting question of whether the equal-tension constraint may be loosened further by considering warped solutions. In order to investigate this, we proceed to construct warped solutions. We see that in these solutions although there is substantial liberty in choosing the two tensions, there remains a relation between the two tensions. We will comment breifly on even more general solutions found recently which guarantee a flat-brane solution for any two choices of tension, although at the expense of more singular metric and dilaton configurations.

In all of these solutions the warping of the 4D metric goes hand in hand with a nontrivial dilaton configuration, and so these solutions generalize the simpler product-space spherical compactifications of the Salam-Sezgin model [146, 150, 147, 176, 161] presented previously. Unlike the spacetime curvature, the dilaton and electromagnetic fields in our solution *ansatz* are nonsingular at the positions of the 3-branes, and so the solutions can only describe the fields due to 3-branes which do not couple to these fields.

The warped compactifications we present are generalizations of the Randall-Sundrum (RS) warped brane world to codimension two and to a supersymmetric context, which is of interest even divorced from the cosmological constant problem. Warped compactifications have been extensively-studied in five dimensions but not in higher-dimensional spacetimes, so these solutions

[3] These solutions are analytical continuations of the solutions recently found in [183].

can be considered a first effort in seeking more complex brane-world compactifications in codimensions higher than one.

In the warped-compactifiction solutions the dilaton varies over the extra dimensions, and this makes the electroweak hierarchy only power-law sensitive to the proper radius of the extra dimensions (as opposed to being exponentially sensitive as in the RS model). The electroweak hierarchy does not depend exponentially on the size of the internal dimensions in these solutions because the solutions are not asymptotically anti-de Sitter, a property which can be seen to arise from the positivity of the scalar potential. Because the electroweak hierarchy is not exponential we find that some dimensionless combinations of brane tensions and couplings must be chosen to be very large if the hierarchy is to be sufficiently big. Warping changes the phenomenology of these models because the Kaluza-Klein gap can be much larger than the internal space's inverse proper radius. These solutions break all of the supersymmetries of the model.

We organize our presentation as follows. The next section describes the unwarped solution to the Salam-Sezgin supergravity presented in chapter 3. This is requires a simple surgery on the spacetime to place branes at the two poles of the internal two-sphere. It is explicitly shown that the branes are flat and the spacetime so constructed well-behaved. We then proceed to finding a family of warped solutions to the supergravity. These solutions are actually all of the solutions exhibiting conical deficits at the brane positions with maximally-symmetric four-dimensional slicing [184, 178]. We then examine how low-energy features (such as the electroweak hierarchy) depend on the physical properties of the branes involved. The nature of the cosmological-constant self-tuning is then described for both models (see also appendix C). Finally, our conclusions are summarized.

5.1 The Model

Recall that the field content of Salam-Sezgin supergravity consists of a supergravity-tensor multiplet consisting of a metric (g_{MN}), antisymmetric Kalb-Ramond field (B_{MN}, with field strength G_{MNP}), dilaton (ϕ), gravitino (ψ^i_M) and dilatino (χ^i). The fermions are all real Weyl spinors, satisfying $\Gamma_7\psi_M = \psi_M$ and $\Gamma_7\chi = -\chi$ and so the model is anomalous unless it is coupled to an appropriate matter content [81]. The appropriate chiral 6D matter consists of a combination of gauge multiplets, containing gauge potentials (A_M) and gauginos (λ^i), and n_H hyper-multiplets, with scalars Φ^a and fermions $\Psi^{\hat{a}}$. The index $i = 1, 2$ is an $Sp(1)$ index, $\hat{a} = 1, \ldots, 2n_H$ and $a = 1, \ldots, 4n_H$. The gauge multiplets transform in the adjoint representation of a gauge group, G. The $Sp(1)$ symmetry is broken explicitly to a $U(1)$ subgroup, which is gauged.

The matter fermions are also chiral, $\Gamma_7\lambda = \lambda$ and $\Gamma_7\Psi^{\hat{a}} = -\Psi^{\hat{a}}$, but the anomalies can be cancelled *via* the Green-Schwarz mechanism [82], for specific gauge groups and hypermultiplets [150, 84]. An explicit example [150] of an anomaly-free choice is $G = E_6 \times E_7 \times U(1)$, with the hyper-multiplet scalars living on the noncompact quaternionic Kähler manifold $\mathcal{M} = Sp(456, 1)/(Sp(456) \times Sp(1))$.

The bosonic part of the classical 6D supergravity action is:

$$e^{-1}\mathcal{L}_B = -\frac{1}{2}R - \frac{1}{2}\partial_M\phi\,\partial^M\phi - \frac{1}{2}G_{ab}(\Phi)\,D_M\Phi^a\,D^M\Phi^b$$

$$- \frac{1}{12} e^{-2\phi} \, G_{MNP} G^{MNP} - \frac{1}{4} e^{-\phi} \, F^{\alpha}_{MN} F^{MN}_{\alpha} - e^{\phi} \, v(\Phi) \tag{5.1}$$

Here the index $\alpha = 1, \ldots, \dim(G)$ runs over the gauge-group generators, $G_{ab}(\Phi)$ is the metric on $\mathcal{M}$ and D_m are gauge and Kähler covariant derivatives whose details are not important for our purposes. We only require the dependence on ϕ of the scalar potential for $\Phi^a = 0$, which is $V(\phi, \Phi) = 2 \, g_1^2 \, e^{\phi}$. The coupling g_1 denotes the $U(1)$ gauge coupling.

When the hypermultiplets and all but one of the gauge multiplets are set to zero then the supersymmetry transformations reduce to

$$\begin{aligned}
\delta e^A_M &= \frac{1}{\sqrt{2}} \left(\bar{\epsilon} \Gamma^A \psi_M - \bar{\psi}_M \Gamma^A \epsilon \right) \\
\delta \phi &= - \frac{1}{\sqrt{2}} \left(\bar{\epsilon} \chi + \bar{\chi} \epsilon \right) \\
\delta B_{MN} &= \sqrt{2} \, A_{[M} \delta A_{N]} + \frac{e^{\phi}}{2} \Big(\bar{\epsilon} \Gamma_M \psi_N - \bar{\psi}_N \Gamma_M \epsilon \\
&\qquad - \bar{\epsilon} \Gamma_N \psi_M + \bar{\psi}_M \Gamma_N \epsilon - \bar{\epsilon} \Gamma_{MN} \chi + \bar{\chi} \Gamma_{MN} \epsilon \Big) \qquad (5.2) \\
\delta \chi &= \frac{1}{\sqrt{2}} \, \partial_M \phi \, \Gamma^M \epsilon + \frac{e^{-\phi}}{12} G_{MNP} \, \Gamma^{MNP} \epsilon \\
\delta \psi_M &= \sqrt{2} \, D_M \epsilon + \frac{e^{-\phi}}{24} \, G_{PQR} \, \Gamma^{PQR} \Gamma_M \epsilon \\
\delta A_M &= \frac{1}{\sqrt{2}} \left(\bar{\epsilon} \Gamma_M \lambda - \bar{\lambda} \Gamma_M \epsilon \right) e^{\phi/2} \\
\delta \lambda &= \frac{e^{-\phi/2}}{4} \, F_{MN} \, \Gamma^{MN} \epsilon - \frac{i}{\sqrt{2}} \, g_1 \, e^{\phi/2} \, \epsilon \, ,
\end{aligned}$$

where the supersymmetry parameter is complex and Weyl: $\Gamma_7 \epsilon = \epsilon$.

5.2 Unwarped Compactification

The previous sections outline a mechanism which relates a small 4D vacuum energy to brane properties at higher energies $E \sim M_w$, and can explain why this vacuum energy remains small as the modes between M_w and $1/r$ are integrated out. It remains to see if an explicit brane configuration can be constructed which takes advantage of this mechanism to really give such a small cosmological constant.

In this section we take the first steps in this direction, by constructing a simple two-brane configuration within the 2-sphere compactification of the Salam-Sezgin model described earlier, taking into account the back-reaction of the branes. Since our construction also has a constant dilaton field, it furnishes an explicit example of a model for which the classical contributions to ρ_{eff} precisely cancel.

Our attempt is not completely successful in one sense, however, because our construction is built using a non-supersymmetric compactification of 6D supergravity. As such, our general arguments as to the absence of quantum corrections may not apply, perhaps leading to corrections which are larger than $1/r^4$. The model has the great virtue that it is sufficiently simple to explicitly calculate quantum corrections, and so to check the general arguments, and such calculations are now in progress[4].

5.2.1 Branes on the Sphere

The great utility of the spherical compactification of Salam-Sezgin supergravity is the simplicity with which branes can be embedded into it, including their back-reaction onto the bulk gravitational, dilaton and Maxwell fields. Because the solution we find has a constant dilaton, our construction of these brane solutions turns out to closely resemble the analysis of the Maxwell-

[4] D. Hoover and C. P. Burgess, *In progress*

Einstein equations given in ref. [133].

The field equations of 6D supergravity have a remarkably simple solution (when the dilaton does not couple to the branes) for the special case of two branes having equal tension, T, located at opposite poles of the two-sphere. In this case the solution is precisely the same as obtained before in the absence of any branes, but with the two-dimensional curvature now required to include a delta-function singularity at the position of each of the branes. More precisely, the only change implied for the solution by the brane sources comes from the two-dimensional components of the Einstein equation, which now requires that the two-dimensional Ricci scalar can be written $R_2 = R_2^{\rm smth} + R_2^{\rm sing}$, where $R_2^{\rm smth}$ satisfies precisely the same equations as in the absence of any branes, and the singular part is given by

$$R_2^{\rm sing} = -\frac{2T}{e_2} \sum_i \delta^2(y - y_i) \,, \tag{5.3}$$

where as before $e_2 = \sqrt{\det g_{mn}}$.

The resulting solution therefore involves precisely the same field configurations as before: $\varphi =$ (constant), $g_{\mu\nu} = \eta_{\mu\nu}$, $g_{mn}\, dy^m\, dy^n = r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right)$ and $T_\alpha F^\alpha_{mn} = Q\, f\, \epsilon_{mn}$, for a $U(1)$ generator, Q, embedded within the gauge group. As before the parameters of the solution are related by $r^2\, e^\varphi = 1/(4g_1^2)$ and $f = n/(2g_1\, r^2)$ where $n = \pm 1$. The singular curvature is then ensured by simply making the coordinate ϕ periodic with period $2\pi(1-\varepsilon)$ rather than period 2π — thereby introducing a conical singularity at the branes' positions at the north and south poles. The curvature condition, eq. (5.3), is satisfied provided that the deficit ε is related to the brane tension by $\varepsilon = 4\, G_6\, T$.

The 'rugby-ball' geometry[5] so described corresponds to removing from

[5]We use the name rugby-ball to resolve the cultural ambiguity in the shape meant by

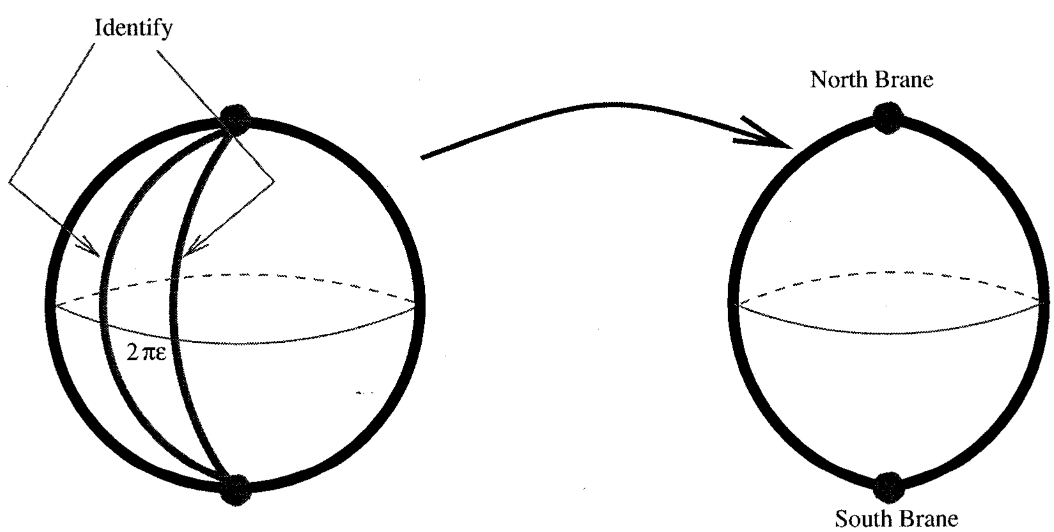

Figure 5.1: The effect of two 3-branes at the antipodal points in a 2-sphere. The wedge of angular width $2\pi\varepsilon$ is removed from the sphere and the two edges are identified giving rise to the rugby-ball-shaped figure. The deficit angle is related to the branes tensions (assumed equal) by $\varepsilon = 4G_6T$.

the 2-sphere a wedge of angular width $2\pi\,\varepsilon$, which is bounded by two lines of longitude running between the branes at the north and south poles, and then identifying the edges on either side of the wedge [154, 153, 125, 133]. The delta-function contributions to R_2 are then just what is required to keep the Euler characteristic unchanged, since

$$\chi = -\frac{1}{2\pi}\int d^2y\,\left(R_2^{\rm smth} + R_2^{\rm sing}\right) = 2\,. \tag{5.4}$$

The singular contribution precisely compensates the reduction in the contribution of the smooth curvature, $R_2^{\rm smth}$, due to the reduced volume of the rugby-ball relative to the sphere.

Finally, the above configuration also satisfies the equations of motion for the branes, which state (for constant φ or vanishing λ) that they move along a geodesic according to

$$\ddot{y}^m + \Gamma^m_{pq}\,\dot{y}^p\,\dot{y}^q = 0\,, \tag{5.5}$$

where Γ^m_{pq} is the Christoffel symbol constructed from the 2D metric, g_{mn}. Consequently branes placed precisely at rest anywhere in the two dimensions will remain there, and this configuration is likely to be marginally stable due to the absence of local gravitational forces in two spatial dimensions.

5.2.2 Topological Constraint

We now show that the above solution is further restricted by a topological argument. This will exclude for instance the possibility of the supersymmetric Salam-Sezgin compactification in which the monopole background is fully

'football', which was used previously in the literature [133]. The name 'periodic lune' has also been used [153].

embedded into the explicit $U(1)$ gauge group factor. But it allows other embeddings, in particular the E_6 embedding of [84] that is non-supersymmetric.

In order to make this argument we write the electromagnetic field strength obtained from the field equations as

$$F = \frac{n}{2\,g_1}\,\sin\theta\;\mathrm{d}\theta \wedge \mathrm{d}\phi\,, \tag{5.6}$$

where $n = \pm 1$. The gauge potential corresponding to this field strength can be chosen in the usual way to be

$$A_\pm = \frac{n}{2\,g_1}\,[\pm 1 - \cos\theta]\;\mathrm{d}\phi\,, \tag{5.7}$$

where the subscript '$\pm$' denotes that the configuration is designed to be nonsingular on a patch which respectively covers the northern or southern hemisphere of the rugby-ball.

Now comes the main point. A_+ and A_- must differ by a gauge transformation on the overlap of the two patches along the equator, and this — with the periodicity condition $\phi \approx \phi + 2\pi\,(1-\varepsilon)$ — implies $A_\pm$ must satisfy $gA_+ - gA_- = N\,\mathrm{d}\phi/(1-\varepsilon)$, where N is any integer and g denotes the gauge coupling constant which is appropriate to the generator Q. In particular $g = g_6$ if Q lies within the E_6 subgroup, as is in ref. [150], or $g = g_1$ if Q corresponds to the explicit $U(1)$ gauge factor, as in ref. [146]. Notice that this is only consistent with eq. (5.7) if g and g_1 are related by

$$\frac{g}{g_1} = \frac{N}{n(1-\varepsilon)}\,. \tag{5.8}$$

In particular, g cannot equal g_1 if $\varepsilon \neq 0$, and so we cannot choose Q to lie in the explicit $U(1)$ gauge factor, as for the supersymmetric Salam-Sezgin compactification.

A deeper understanding of this last condition can be had if the 3-brane action is generalized to include the coupling, of the Maxwell field to the background Maxwell field since in this case the 3-brane acquires a delta-function contribution to the magnetic flux of size $\mathcal{Q} \propto q$. Denoting the flux at the position of each brane by $\mathcal{Q}_\pm$, eq. (5.7) generalizes to

$$A_\pm = \left[\frac{\mathcal{Q}_\pm}{2\pi} + \frac{n}{2g_1}(\pm 1 - \cos\theta) \right] \mathrm{d}\phi \,. \tag{5.9}$$

The same arguments as above then lead to the following generalization of formula (5.8)

$$\frac{\mathcal{Q}_+ - \mathcal{Q}_-}{2\pi} + \frac{n}{g_1} = \frac{N}{g(1-\epsilon)} \,, \tag{5.10}$$

which relates the difference, $\mathcal{Q}_+ - \mathcal{Q}-$, to the integers n and N. This shows that the constraint we are obtaining is best interpreted as a topological condition on the kinds of magnetic fluxes which are topologically allowed in order for a solution to exist (much like the condition that the tensions on each to the two 3-branes must be equal or, in another context, to the Gauss' Law requirement that the net charge must vanish for a system of charges distributed within a compact space). Within this context eq. (5.8) expresses the conditions which are required in order to have a solution with $\mathcal{Q}_+ = \mathcal{Q}_-$. Given its topological (long-distance) character, such a condition is very likely to be preserved under short-distance corrections, and so be stable under renormalization.[6]

Although the choice $\mathcal{Q}_+ = \mathcal{Q}_-$ precludes a solution with $g = g_1$, it does allow solutions where Q lies elsewhere in the full gauge group, such as the E_6 embedding above. This model has the great virtue of simplicity, largely

[6]Note added: This stability is easier to see for the single-brane solutions of ref. [164], where it is very much like the usual quantization of monopole charge.

due to the constancy of both the dilaton and the magnetic flux over the two-sphere. It has the drawback that this simple embedding of the monopole gauge group breaks supersymmetry, and so may allow larger quantum corrections than would be allowed by the general arguments of the previous sections. On the other hand, the choice $g = g_1$ may be possible if $\mathcal{Q}_\pm$ are not equal, and if so would allow a solution with unbroken bulk supersymmetry as in the original Salam-Sezgin model.

It clearly would be of great interest to find an anomaly-free embedding that also preserves some of the supersymmetry, since any such embedding would completely achieve precisely the scenario we are proposing with a naturally small cosmological constant. However, although supersymmetry was required to eliminate the contributions of curvature squared terms, which contribute to $\rho_{\rm eff}$ an amount of order M_w^2/r^2, we see that even without supersymmetry this model achieves a great reduction in the cosmological constant relative to the mass-splittings, M_w, between observable particles and any of their superpartners. A full study of monopole solutions and their quantum fluctuations is presently being investigated.

5.2.3 Conclusions

Even though our scenario has a number of attractive features, it leaves a great many questions unanswered.

First, our attempt to realize the self-tuning in an explicit solution to the 6D equations led to a topological constraint that appears to require a relationship between the brane tension and other (gauge) couplings in the bulk action. It remains to be seen whether this condition is an artifact of the simplicity of our solution (such as being due to our requiring the dilaton and Maxwell fields to be nonsingular at the brane positions) or if it is actually unavoidably required in order to obtain flat 3-branes. In particular, we argue that the topological relation is better interpreted as a constraint on what magnetic fluxes which may be carried by the branes given the topology of the internal space. As such it might be expected to be stable under renormalization, in much the same way as is the condition that the net electric charge vanish for a configuration of charged particles in a compact space.

In the next section we display a more general class of solutions for which the topological constraints are changed, but not removed. For further discussions along this line, and an extensions of this analysis to FRW metrics, see ref. [205].

Note Added: There have been several interesting developments since this paper appeared on the arXiv, which we briefly summarize here.

Ref. [161] provide an interesting analysis of the Salam-Sezgin model without branes, in which they verify the topological condition, eq. (5.8) (as also did ref. [165]), and show that if the Kaluza-Klein scale is of order 10^{-3} eV, then the 4D gauge coupling of the bulk gauge fields must be $g_4 \sim 10^{-31}$ (as opposed to the value of 10^{-15} which is obtained in the absence of a dilaton [136].). Since this follows directly from the large size of the extra dimensions, its explanation rests with whatever physics stabilizes the size of the extra dimensions, and does not represent an additional fine tuning beyond this. The physics of radius stabilization at such a large value remains of course an open question.[7]

Ref. [164] finds the general nonsingular solution to the Salam-Sezgin equations having maximal symmetry in the noncompact 4 dimensions, for arbitrary monopole number. These solutions nicely illustrate many of the arguments made here, since the noncompact 4 dimensions are always flat, as our general self-tuning arguments predict. The 4D curvature which the field equations require for non-constant dilaton is in this case provided by warping in the extra dimensions. Furthermore, the solutions with monopole number greater than 1 provide examples whose topological constraints are very plausibly stable against renormalization inasmuch as they closely resemble the standard monopole quantization condition.

Progress towards embedding our picture into string theory has also been made. Ref. [166] finds a higher-dimensional derivation of a new supergravity which shares the bosonic part of the Salam-Sezgin theory. Ref. [167] obtains exactly the Salam-Sezgin supergravity, by consistently reducing type I/heterotic supergravity on the non-compact hyperboloid $\mathcal{H}^{2,2}$ times S^1.

Ref. [168] provides an explicit recent one-loop string calculation of the vacuum energy within a supersymmetry-breaking framework similar to that considered here. They find a result which is of order $1/r^4$, in agreement with our arguments and with previous calculations [151].

We next discuss warped solutions to the supergravity equations of motion. We find in this case, too, a topological condition relating the tensions of the branes to each other. We then comment on recent developments on the

[7]Notational point: We adopt in this paper a slightly different metric convention than we did in ref. [147] since we here do not work in the 4D Einstein frame. Consequently in this paper KK masses are of order $1/r$ instead of being of order $ge^{\phi/2}/r \sim 1/r^2$, as they are in ref. [147], and as is shown explicitly in ref. [161].

warped solutions.

5.3 Warped Compactification

In this section we present warped compactifications which are solutions of the Salam-Sezgin chiral six-dimensional supergravity-supermatter system.

For our purposes we may set all gauge fields to zero except for a single gauge potential, A, and we also set $\Phi^a = 0$. We derive a warped brane-world solution by continuing a related nontrivial solution for the same system which was found in ref. [183]. The solution in [183] is given by

$$\begin{aligned} ds_6^2 &= -h(\rho)\,\mathrm{d}\tau^2 + \frac{\rho^2}{h(\rho)}\mathrm{d}\rho^2 + \rho^2 \mathrm{d}x_{0,4}^2\,, \\ \phi(\rho) &= -2\,\ln\rho\,, \\ F_{\tau\rho} &= \frac{\hat{\mathcal{A}}}{\rho^5}\,\epsilon_{\tau\rho}\,, \end{aligned} \tag{5.11}$$

where $dx_{0,4}$ denotes a flat 4-dimensional spatial slice, and

$$h(\rho) = -\frac{2\mathcal{M}}{\rho^2} - \frac{g_1^2\,\rho^2}{4} + \frac{\hat{\mathcal{A}}^2}{16\,\rho^6}\,. \tag{5.12}$$

This function has only a single zero for real positive ρ, and $\mathcal{M}$ and $\hat{\mathcal{A}}$ are integration constants which can be positive or negative. This is not a brane-world solution since the point where h vanishes corresponds to a null Cauchy horizon of the geometry.

A warped brane-world solution may be obtained from this one by performing a suitable analytic continuation, in which we first redefine the coordinate

$r = \frac{1}{2}\,\rho^2$ so that the previous solution takes the form

$$
\begin{aligned}
ds_6^2 &= -h(r)\,\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{h(r)} + 2\,r[\mathrm{d}x_1^2 + \mathrm{d}x_{0,3}^2]\,, \\
\phi(r) &= -\ln(2r)\,, \\
F_{\tau r} &= \frac{\hat{\mathcal{A}}}{8\,r^3}\,\epsilon_{\tau r}\,,
\end{aligned}
\tag{5.13}
$$

with

$$
h(r) = \frac{2M}{r} - \frac{g_1^2\,r}{2} + \frac{\hat{\mathcal{A}}^2}{128\,r^3}\,. \tag{5.14}
$$

Here we redefine the integration constant according to $\mathcal{M} = -2M$, in anticipation of our later choice $\mathcal{M} < 0$. The new solution is obtained by performing the analytic continuation

$$
\tau \longrightarrow i\,\theta\,, \qquad x_1 \longrightarrow it \qquad \frac{\hat{\mathcal{A}}}{8} \longrightarrow i\mathcal{A}\,, \tag{5.15}
$$

in which case the it becomes:

$$
\begin{aligned}
ds_6^2 &= 2\,r[-\mathrm{d}t^2 + \mathrm{d}x_3^2] + h(r)\,\mathrm{d}\theta^2 + \frac{\mathrm{d}r^2}{h(r)}\,, \\
\phi(r) &= -\ln(2r)\,, \\
F_{\theta r} &= -\frac{\mathcal{A}}{r^3}\,\epsilon_{\theta r}\,,
\end{aligned}
\tag{5.16}
$$

with

$$
h(r) = \frac{2M}{r} - \frac{g_1^2\,r}{2} - \frac{\mathcal{A}^2}{2\,r^3}\,. \tag{5.17}
$$

This is the desired solution whose properties we now explore.

5.3.1 Singularities and Supersymmetry

Eq. (5.16) describes a Lorentzian-signature solution provided $h(r) > 0$, and so it is useful to enumerate the zeroes of $h(r)$, which occur at

$$
r_\pm^2 = \frac{2\,M}{g_1^2}\left[1 \pm \sqrt{1 - \left(\frac{g_1\,\mathcal{A}}{2M}\right)^2}\,\right]\,. \tag{5.18}
$$

Since $h(r) < 0$ when $r \to \infty$ and $r \to 0$, the regime of interest for a brane-world solution is the interval $r_- < r < r_+$. This interval is not empty provided $M > \frac{1}{2}|g_1 \mathcal{A}| > 0$, a condition which we henceforth assume.

The geometry pinches off at the points $r = r_\pm$, at each of which it generically has conical singularities. We therefore place a 3-brane at each of these points when constructing a brane-world model.

The deficit angles associated with a conical singularity can be obtained either by Taylor expanding the metric in the vicinity of the point and comparing to the canonical form of the metric (rescaling r if necessary):

$$ds_{\text{cone}} = dr^2 + (1-\varepsilon)^2 d\theta^2, \tag{5.19}$$

where $2\pi\varepsilon$ is the conical deficit angle, or by comparing the radius and the circumference of a small circle about that point:

$$(\text{Circumference}) = 2\pi(1-\varepsilon)r. \tag{5.20}$$

We find that the conical defect at $r = r_\pm$ is given by

$$\varepsilon_\pm = 1 - \frac{|h'(r_\pm)|}{2} = 1 - \frac{g_1^2}{2\,r_\pm^2}\left(r_+^2 - r_-^2\right) . \tag{5.21}$$

This last equality is obtained by writing $h(r) = -\frac{1}{2}\,(g_1^2/r^3)(r^2 - r_+^2)(r^2 - r_-^2)$.

These conditions show that the defect angles are completely determined by the two quantities r_-/r_+ and g_1. In particular, one of the conical defects can be smoothed over if r_-/r_+ is chosen appropriately. We find

$$\varepsilon_+ = 0 \quad \Rightarrow \quad \frac{r_-^2}{r_+^2} = 1 - \frac{2}{g_1^2} \qquad \text{and} \qquad \varepsilon_- = 0 \quad \Rightarrow \quad \frac{r_+^2}{r_-^2} = 1 + \frac{2}{g_1^2} . \tag{5.22}$$

Notice that the condition for the removal of the singularity at r_+ requires a large coupling $g_1 > \sqrt{2}$, and so is only of doubtful validity in a perturbative calculation such as ours.

Supersymmetry

This solution generically breaks supersymmetry, as is most easily seen by specializing the χ supersymmetry transformation to it, with the result

$$\delta\chi = \frac{1}{\sqrt{2}}\, \partial_M\phi\, \Gamma^M \epsilon\,. \tag{5.23}$$

This clearly cannot vanish because $\partial_M\phi \neq 0$.

5.3.2 Brane Worlds

In this section we examine the properties of the brane-world scenario constructed from the warped solution given above. In this case the construction requires two 3-branes, respectively located at the conical singularities $r = r_\pm$, allowing us to interpret these singularities as the gravitational back-reaction due to the presence of the branes.

Electroweak Hierarchy

In the present instance the warp factor is $w(r) = 2\,r$ and so the expression for the effective 4D Planck mass becomes

$$M_p^2 = 2\pi \int_{r_-}^{r_+} dr\, w(r) = 2\pi(r_+^2 - r_-^2) = \frac{8\pi\, M}{g_1^2}\sqrt{1-x^2}\,, \tag{5.24}$$

where $x = g_1\mathcal{A}/(2M)$. For comparison, the physical mass of a particle localized on the 3-brane located at $r = r_\pm$ is

$$m_\pm = \mu_\pm \sqrt{w(r_\pm)}\,, \tag{5.25}$$

where the particle action is assumed to be proportional to $g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \mu_\pm^2\chi^2$.

The hierarchy between these scales is therefore

$$\begin{aligned}
\frac{M_p^2}{m_\pm^2} &= \frac{4\pi M}{g_1^2 \mu_\pm^2 r_\pm} \sqrt{1-x^2} = \frac{2\pi \mathcal{A}}{g_1 \mu_\pm^2 r_\pm} \left(\frac{\sqrt{1-x^2}}{x} \right) \\
\frac{m_+^2}{m_-^2} &= \frac{\mu_+^2 \, r_+}{\mu_-^2 \, r_-} = \frac{\mu_+^2}{\mu_-^2} \left(\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \right)^{1/2} ,
\end{aligned} \tag{5.26}$$

so a large hierarchy can be achieved, for example, if all dimensionful quantities are the same order of magnitude, except, say, M, which we take to be much larger. The hierarchy is then controlled by $x \ll 1$, or $g_1 \mathcal{A} \ll 2M$, and in this case the previous formulae for $r_\pm$ reduce to

$$r_+^2 \approx \frac{4M}{g_1^2} \qquad \text{and} \qquad r_-^2 \approx \frac{\mathcal{A}^2}{4M} , \tag{5.27}$$

and so $r_-/r_+ \approx x/2$. Clearly this does not really provide a satisfactory explanation for the electroweak hierarchy, since the desired scales are simply inserted into the higher-dimensional solution.

If the gauge coupling $e^{\phi(r_-)}$ is assumed small, then the solution guarantees the gauge coupling to be even smaller at $r = r_+$ by an amount $e^{\phi(r_+)}/e^{\phi(r_-)} = r_-/r_+$.

5.3.3 Brane Boundary Conditions

To understand what the previous choices for $\mathcal{A}$ and M mean physically it is necessary to connect these integration constants to brane properties.

The counting of boundary conditions proceeds as follows. The smoothness of the dilaton field at the 3-brane positions precludes these branes from directly coupling to the dilaton. Because this is also the choice which preserves the bulk scale invariance, the metric condition at each 3-brane only involves the scale-invariant ratio $\mathcal{A}/M$, implying a topological constraint which relates the two tensions to one another. The Maxwell boundary conditions at each 3-brane then lead to contradictory conditions on the gauge potentials, which imply a final topological restriction, also involving only the ratio $\mathcal{A}/M$.

We are therefore led in this case to three kinds of constraints. The vanishing of the 3-brane/dilaton charge is accomplished by ensuring the smoothness of the dilaton and by setting the coupling of the dilaton to the brane in the Einstein frame to zero. Flux-quantization can be satisfied by adjusting the background gauge coupling, g, in terms of the coupling, g_1, appearing in the scalar potential. The third restriction, relating the 3-brane tensions, arises because of the compactness of the internal two dimensions. (This constraint is the analog of the condition of equal tensions which arises in the unwarped case in a previous section and in reference [176].) In summary, we are led in this model to a picture which is very similar to what was encountered elsewhere for the unwarped solutions to Salam-Sezgin supergravity.

Boundary Conditions and Tensions

In order to relate the tensions to parameters in the solutions, we write dilaton and metric couplings in the brane action as

$$S_3 = -T_3 \int d^4\xi \; e^{\lambda_3 \phi} \sqrt{-\det \gamma_{\mu\nu}}\,, \tag{5.28}$$

Here the induced metric is related to the 6D metric, g_{MN}, and the 3-brane position, $x^M(\xi)$, by $\gamma_{\mu\nu} = g_{MN}\,\partial_\mu x^M \partial_\nu x^N$. For coordinates $\xi^\mu = x^\mu$, this becomes $\gamma_{\mu\nu} = g_{\mu\nu} + g_{mn}\partial_\mu x^m \partial_\nu x^n$, where $\mu, \nu = 0, ..., 3$ and $m, n = 4, 5$. For a brane at rest at $r = r_3$ we also have $x^m = 0$. The quantities T_3 and λ_3 are the physical 3-brane properties which we wish to relate to the bulk geometry.

This action adds source terms to the dilaton and Einstein equations, eqs. (3.80). If the three brane is located at position x_3^m, the source terms are of the form

$$\begin{aligned} \Box\phi + (\cdots) &= \sum_i \lambda_i T_i \frac{e^{\lambda_b \phi}}{e_2} \delta^2(x - x_i) \\ R_{MN} + (\cdots)_{MN} &= T_i \frac{e^{\lambda_i \phi}}{e_2} \left(g_{\mu\nu} \delta^\mu_M \delta^\nu_n - g_{MN} \right) \delta^2(x - x_i) \,, \end{aligned} \tag{5.29}$$

where $e_2 = \sqrt{\det g_{mn}}$, and the sum over i is over the branes and their positions, x_i. These δ-function sources imply nontrivial boundary conditions for the bulk fields at the brane position, as may be determined by integrating the field equations over a small volume of infinitesimal proper radius about the 3-brane position. Assuming the metric, dilaton and Maxwell fields to be continuous at the brane position, we learn how the dilaton derivative and the curvature behave there.

The dilaton derivative at the 3-brane positions, $r_\pm$, becomes:

$$\lambda_\pm T_\pm \, e^{\lambda_\pm \phi} \Big|_{r=r_\pm} = (2r)^2 \, h(r) \, \phi' \Big|_{r=r_\pm} \tag{5.30}$$

which should be read as a condition relating ϕ and ϕ' at the brane positions, given the known couplings $T_\pm$ and $\lambda_\pm$. Since $\phi' = 1/r$ is bounded as $r \to r_\pm$, in the solution of interest, and since $h(r_\pm) = 0$, the rhs vanishes, so

$$\lambda_\pm T_\pm \alpha_\pm^{\lambda_\pm} = 0 \,. \tag{5.31}$$

Since we do not wish to allow either $T_\pm$ or $\alpha_\pm = e^{\phi(r_\pm)}$ to vanish, we take this last condition to require $\lambda_\pm = 0$.

A similar argument applied to the curvature singularity implies the standard relation between the conical defect angle and the 3-brane tension [174], which from eq. 5.21 becomes:

$$
\begin{aligned}
T_{\pm} &= 2\pi\varepsilon_{\pm} \\
&= 2\pi\left[1 - \frac{|h'(r_{\pm})|}{2}\right] \\
&= 2\pi\left[1 - \frac{g_1^2}{2\,r_{\pm}^2}\left(r_+^2 - r_-^2\right)\right] ,
\end{aligned}
\tag{5.32}
$$

from which we see that positive tensions imply that the radii $r_{\pm}$ must satisfy $(r_+/r_-)^2 < 1 + 2/g_1^2$, or in terms of $x = g_1\mathcal{A}/(2M)$: $x^2 > 1 - (g_1^2+1)^{-2}$. Notice that large r_+, $r_+ \gg r_-$, therefore clearly requires $g_1 \ll 1$.

These last two brane boundary conditions determine only one of the two integration constants, M and $\mathcal{A}$, (or equivalently of r_+ and r_-) because they depend on the ratio r_-/r_+, and so can only determine the combination $x = g_1\mathcal{A}/(2M)$. The fact that the two tensions are both determined by the single variable x implies the existence of a constraint relating these tensions. Eliminating r_-/r_+ from eq. (5.32) gives

$$
\frac{T_+ - T_-}{2\pi} - \frac{2}{g_1^2}\left(1 - \frac{T_+}{2\pi}\right)\left(1 - \frac{T_-}{2\pi}\right) = 0 . \tag{5.33}
$$

This is the analogue of the condition that the two 3-brane tensions be equal, which obtains for the unwarped 2-sphere solution [176].

Gauge Fields

A similar condition applies at the position of each brane, which follows from the nature of the brane coupling to the background Maxwell field. The branes considered here carry no flux, and so the flux through a small patch of infinitesimal radius ϵ about each brane position must vanish in the limit $\epsilon \longrightarrow 0$. This condition applied to both branes leads to a topological constraint which the parameters of our solution must satisfy.

To see this, notice that the gauge potential for the magnetic field strength, $F = (\mathcal{A}/r^3)\,\mathrm{d}r \wedge \mathrm{d}\theta$ may be written

$$A = \left(c - \frac{\mathcal{A}}{2r^2}\right) \mathrm{d}\theta\,, \tag{5.34}$$

where c is an integration constant. The condition that F not contain delta-function contributions at $r = r_\pm$ requires A to vanish at these two positions, and this imposes contradictory constraints on c: $c = c_\pm = \mathcal{A}/(2\,r_\pm^2)$. Consequently F can only be nonsingular at *both* $r = r_+$ and $r = r_-$ if eq. (5.34) holds separately for two overlapping patches, $P_\pm$, each of which includes only one of r_+ or r_-.

Although the gauge potential can take different values ($A = A_\pm$ distinguished by constants $c_\pm$) on each of these patches, $A_+ - A_-$ must be a gauge transformation. Periodicity of the coordinate θ on the overlap then requires $c_+ - c_- = n/g$, where g is the gauge coupling appropriate for the background gauge field which has been turned on. Combined with the expressions for $c_\pm$ we find the requirement

$$\frac{\mathcal{A}}{2}\left(\frac{1}{r_-^2} - \frac{1}{r_+^2}\right) = \frac{2M}{\mathcal{A}}\sqrt{1 - \left(\frac{g_1\mathcal{A}}{2M}\right)^2} = \frac{n}{g}\,. \tag{5.35}$$

For the case of large, $r_+ \gg r_-$, this condition simplifies to $2M/\mathcal{A} \approx n/g$, and so $r_-/r_+ \approx g_1\mathcal{A}/(4M) \approx g\,g_1/(2n) \ll 1$. Since the ratio r_-/r_+ is already fixed given T_+ or T_-, we instead read eq. (5.35) as a condition relating g to g_1.

Since all of these conditions only fix the ratio $\mathcal{A}/M$ and none separately determine $\mathcal{A}$ or M, the overall scale of the extra dimensions (say, its volume) remains undetermined. As described in detail in the next section, this is consistent with the scale invariance of the bulk equations which is not broken by the 3-brane. Consequently $\mathcal{A}$ parameterizes a flat direction, for which we expect a classically massless modulus in the low-energy 4D theory. This behavior is in contrast to that of nonsupersymmetric versions of this model, lacking the dilaton, where the volume of the extra dimensions is automatically stabilized in the presence of nonvanishing gauge flux [175].

5.4 Self-Tuning in Six Dimensions

We here present the details of the self-tuning mechanism for the solutions presented. We keep the presentation concise since the arguments are a particular case of the more general arguments for self-tuning presented in sec. 4.7.

For two parallel 3-branes positioned at $y = y_\pm^m$ in the internal dimensions the effective 4D vacuum energy in Salam-Sezgin supergravity is

$$\begin{aligned}\rho_{\text{eff}} &= \sum_{i=\pm} w^2(r_i)\, T_i + \int_M d^2y\; e_2\, w^2 \left[\frac{1}{2} R_6 + \frac{1}{2}(\partial\phi)^2 + \frac{1}{2} G_{ab}(D\Phi^a)(D\Phi^b)\right. \\ &\qquad \left.\left. + \frac{1}{12}\, e^{-2\phi}\, G^2 + \frac{1}{4}\, e^{-\phi}\, F^2 + v(\Phi)\, e^{\phi}\right]_{cl}\right|_{g_{\mu\nu}=\eta_{\mu\nu}} \qquad (5.36)\end{aligned}$$

where $w(r) = 2\,r$ is the warp factor, and M denotes the internal two-dimensional bulk manifold. As before the subscript '*cl*' indicates the evaluation of the result at the solution to the classical equations of motion.

Using the Einstein equation to eliminate the metric gives

$$R_6 = -(\partial\phi)^2 - G_{ab} D\Phi^a D\Phi^b - 3v(\Phi)\, e^{\phi} - \frac{1}{4}\, e^{-\phi}\, F^2 - \frac{2}{e_2} \sum_{i=\pm} T_i\, \delta^2(y - y_i)\,, \qquad (5.37)$$

and using this in ρ_{eff} gives

$$\rho_{\text{eff}} = \int_M d^2y\; e_2\, w^2 \left[\frac{1}{12}\, e^{-2\phi}\, G^2 + \frac{1}{8}\, e^{-\phi}\, F^2 - \frac{1}{2}\, v(\Phi)\, e^{\phi}\right]_{cl}\Bigg|_{cl}\,. \qquad (5.38)$$

The dilaton equation of motion now reads

$$v(\Phi)\, e^{\phi} - \tfrac{1}{4}\, e^{-\phi}\, F^2 - \tfrac{1}{6}\, e^{-2\phi}\, G^2 = \Box\phi - \sum_{i=\pm} \lambda_i T_i e^{\lambda_i \phi}\, \frac{1}{e_2}\, \delta^2(y - y_\pm)\,, \quad (5.39)$$

which gives when inserted into eq. (5.38)

$$\begin{aligned} \rho_{\text{eff}} &= -\frac{1}{2} \int_M d^2y\; e_2\, w^2\, \Box\phi_{cl} + \frac{1}{2} \sum_{i=\pm} \lambda_i T_i\, w^2(r_i) \\ &= \sum_{i=\pm} w^2(r_i) \left[\frac{1}{2}\lambda_i T_i - \pi\, e_2\, n_M\, \partial^M \phi \right]_{r=r_i}, \end{aligned} \quad (5.40)$$

where we evaluate total derivative using the boundary surface ∂M_i, consisting of an infinitesimal region surrounding the 3-brane positions. For the solution considered above this consists of an infinitesimal circle surrounding the brane positions at $r = r_\pm$.

The two contributions to ρ_{eff} therefore vanish when evaluated at the solutions derived in earlier sections. The first term vanishes because we have already seen that the solution described above requires $\lambda_\pm = 0$, and the second likewise vanishes because ϕ' is bounded but $n_M\, \partial^M \phi = \sqrt{g^{rr}}\, \phi' = \sqrt{h}\, \phi'$ vanishes at the brane positions, $r = r_\pm$.

5.5 Discussion

We constructed explicit unwarped and warped, axisymmetric solutions to the dilaton-Einstein-Maxwell field equations arising the Salam-Sezgin supergravity in six dimensions. We identified the circumstances under which they may be interpreted as being generated by simple 3-brane sources, and what geometrical features are required in order for the resulting brane systems to be used as brane-world models having a realistic electroweak hierarchy.

Since all of the solutions have flat 4-dimensional sections regardless of the values of the tensions and couplings on the various branes, we examined in more detail how self-tuning of the 4D cosmological constant arises in these particular solutions. This allows us to identify some of the issues which must be addressed in order to promote these features into a real solution to the cosmological constant problem.

The warped solution to Salam-Sezgin supergravity can have either one or two conical defects, which we interpreted as the position of one or two 3-branes. The number of boundary conditions is larger than the number of integration constants, so the bulk solutions are only produced by the assumed branes if their couplings are adjusted in particular ways.

Nonsingularity of the dilaton requires vanishing dilaton 3-brane couplings: $\lambda_{\pm} = 0$. Furthermore, the 3-brane tensions are subject to a topological condition which generalizes the condition found in the unwarped case (for which the tensions must be equal). Finally, we found a topological condition on the total magnetic flux through the space, whose satisfaction requires the adjustment of one of the couplings, such as the background gauge coupling, g.

Since the required dilaton couplings preserve the classical scale invariance of the bulk theory there is at least one classically flat direction corresponding to the overall volume of the internal dimensions.

Brane-world models based on this solution can have acceptable electroweak hierarchies, but apparently only by inserting the required hierarchies by hand into the 6D theory. Again, this can be chosen to be along the flat direction, pending an understanding of modulus stabilization in this direction.

Unlike the unwarped example, there does not seem to be any compelling numerology which relates the required extra-dimensional sizes to the observed electroweak or cosmological constant hierarchies.

5.6 Open Issues

Our discussion suggests several directions for further exploration. Most notable among these is the solution to the general problem of finding the back reaction of simple 3-brane configurations in six dimensions without the neglect of dilaton or electromagnetic couplings. Given the general configuration it would be possible to identify whether the brane-coupling choices we make play an important role in the low-energy properties and with the self-tuning of the 4D cosmological constant. Some progress toward this has been made recently. In ref [178] it was shown that the warped solutions presented here are in fact identical to all of the solutions catalogued in ref. [184] which have conical singularities at the brane positions.

More generally, in ref [178] it was shown that the other solutions, which have branes with dilaton sources, have non-conical singularities, but also provide a mechanism to explain any pair of brane tensions. That is, for any brane tensions, T_1, T_2, there exists a flat solution with branes of these tensions. Furthermore, as discussed in detail elsewhere, these are the only solutions under the assumption of maximally-symmetric four-dimensional slices.

In ref. [205] more general four-dimensional metrics than maximally-symmetric ones were considered. In order to consistently find solutions, matter on the branes was considered. A different class of solutions which are seemingly disconnected from the flat-brane static solutions presented here were found. In these cosmological solutions self-tuning does not tune away the cosmological constant. The general results we have proved in previous sections do not apply to their result because they employ a different *ansatz* and allow a more general equation of state on the branes.

An equally important issue to be addressed is the extent to which bulk radiative corrections change our results. In particular one would like to address the extent to which supersymmetry helps protect the electroweak hierarchy and 4D cosmological constant, given that these are chosen to be acceptably small at the classical level. We have discussed in the previous chapter that the field content of this supergravity seems not to be sufficiently similar to the 10D supergravity multiplets to ensure cancellation of the dangerous M^2 terms [203]. Construction of solutions within more general supergravities would be interesting.

To this end, it would be useful to know how our solutions may be embedded into a still-higher-dimensional theory like 10D supergravity or string theory. At present this connection can be made more explicit for Romans' supergravity — such as for the explicit lift to ten dimensions described in the appendix (section 6) of ref. [177] — because it is known how to obtain this theory by consistent truncation from higher dimensions. Similar constructions for Salam-Sezgin supergravity are presently being developed, [189], [167].[8]

One of the biggest obstacles to taking this setup seriously as a possible inroad to the cosmological constant problem is Weinberg's old objection [122] to

[8] Ref. [167] obtains an embedding of Salam-Sezgin supergravity by performing a consistent Pauli reduction of 11D/10D supergravity on the non-compact hyperboloid $\mathcal{H}^{2,2}$ times S^1.

self-tuning solutions. In his objection, he noted that self-tuning is equivalent to a dilaton-gravity system in which the dilaton encodes a scaling symmetry, something we have noted. As we will show in the next chapter, he also showed that this does not solve the cosmological constant problem in four dimensions because if the scaling symmetry is broken to allow particle masses, it revives the old cosmological constant problem. In the next chapter we outline Weinberg's argument and show why it can *help* with the cosmological constant problem in a six-dimensional setup.

Chapter 6

Weinberg's Theorem and Self-Tuning

In chapter 1 I outlined the argument that there exists a scale, $M_{\text{newphysics}}$, above which the standard model is no longer a complete description of particle interactions. This argument was based partly on the existence of hierarchies and fine-tunings, and I presented evidence that fine-tunings are typically solved by symmetries. In chapter 2 I outlined the many failed attempts to solve the cosmological constant problem using four-dimensional symmetries. These attempts all ultimately fail because the cosmological constant problem is a low-energy problem; new four-dimensional low-energy symmetries such as supersymmetry and scale-invariance seem to be excluded, unnatural or useless. This led me to extend the assumptions underlying the analysis to include extra-dimensional theories, motivated primarily by string theory.

In chapter 3 I made the case that string compactifications to four dimensions are doomed to fail, precisely because they do not exploit any of the properties of the extra dimensions to solve the cosmological constant problem, and the four-dimensional cosmological constant problem is known to be difficult to solve (chapter 2).

In chapter 5 I made the case that six-dimensional supersymmetric large extra-dimensional theories with branes have an attractive numerology associated with them that makes this a perfect playground for new attempts to solve the cosmological constant problem, and there I presented a class of six-dimensional supergravity models that seem naturally to 'tune' the effective four-dimensional cosmological constant to zero.

In this chapter I try to confront one of the major criticisms of this model. This, first and foremost criticism, is to ask what is happening from a four-dimensional point of view: if the contention is that four-dimensional physics is inadequate for solving the cosmological constant problem, then superficially there appears to be a contradiction: surely effective field theory techniques work here, and we may analyse the purported solution from a four-dimensional point of view. The most pressing such criticism is to deal with Weinberg's theorem, which excludes four-dimensional self-tuning solutions. The resolution to this criticism will be that the six-dimensional models *rely* on Weinberg's theorem to solve the cosmological constant problem, but because of the existence of the very large two-dimensional bulk of size $\mathcal{O}(0.1\text{mm})$, the scales in the six-dimensional model are different, allowing a reasonable solution.

The second and third major criticisms regard explaining the large size of the extra dimensions and controlling the quantum corrections, both of which are key to making this a successful model of the cosmological constant. Although a full treatment of these two issues is still lacking and beyond the scope of this thesis, I present preliminary evidence that both of these issues can be addressed: the size of the extra dimension can be explained naturally by making the six-dimensional radion a quintessence field along the lines of [121], and the dangerous quantum corrections can be controlled by judiciously selecting appropriate (and well-motivated) field content in the bulk. Further important points are cosmological questions regarding over-production of bulk modes during the early universe and imbedding inflation into the model, tasks I leave to future work.

I breifly discuss more general matter content on the branes in the last chapter.

6.1 Weinberg's Theorem

Weinberg has presented a very general argument against self-tuning mechanisms in four dimensions [122], a criticism which any solution to the cosmological constant problem using an 'adjustment mechanism' must address. Weinberg's theorem isolates a classical scale invariance as the underlying mechanism for self-tuning, something we have already identified as important in the solutions to the six-dimensional supergravity equations of motion. The essential criticism in Weinberg's no-go theorem is that scale invariance, if an exact symmetry *does* solve the cosmological constant problem, but at the expense of placing the theory at a scale-invariant point. Scale invariance is clearly badly broken in the real world since particles have masses. In the presence of non-zero masses, quantum corrections ruin classical scale invariance, with

$$T^{\mu}{}_{\mu} \propto M^4 \tag{6.1}$$

where M is a typical particle mass, reviving the old cosmological constant problem.

The key difference between six-dimensional manifestations of self-tuning and others is that in six-dimensional models the scale at which scale-invariance is broken can be very low, $\mathcal{O}(10^{-3}\text{eV})$, something which is phenomenologically impossible in purely four-dimensional models.

Let us proceed with Weinberg's theorem [122]. Consider any four-dimensional gravitational theory coupled to scalar degrees of freedom. We wish to analyse a scenario in which the the excess vacuum energy is 'tuned' away by a scalar degree of freedom. We require that

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}(\phi) \equiv 0, \tag{6.2}$$

when the other fields, collectively denoted by ϕ, are evaluated on solutions to the equations of motion. Consider the case in which there is only one extra degree of freedom, ϕ. [1] For $T_{\mu\nu}$ to vanish naturally on the equations of motion it must be a linear combination of the equations of motion for ϕ,

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = T_{\mu\nu} \equiv (-g)^{-1/2} g_{\mu\nu} f(\phi) \frac{\partial \mathcal{L}}{\partial \phi}. \tag{6.3}$$

Since we seek conditions for zero cosmological constant we can seek space-time-independent solutions to the equations of motion with $\phi = const$ so eq. 6.3 is equivalent to

$$\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} = g_{\mu\nu} f(\phi) \frac{\partial \mathcal{L}}{\partial \phi}. \tag{6.4}$$

Naively it seems that as long as the ϕ equations of motion are satisfied, $R_{\mu\nu} = 0$ is guaranteed. The problem with this argument is that requiring

[1] For an extension to multiple fields see [122]. Aside from a technical requirement that a certain change of coordinates be non-singular, multiple fields do not materially change the argument.

6.3 to be satisfied is equivalent to requiring there to be a scaling symmetry which guarantees that

$$\mathcal{L} = e^{4\phi}\mathcal{L}_0, \tag{6.5}$$

for some $\mathcal{L}_0$ a function of the other fields. This is a problem because 6.5 has no stationary point unless $\mathcal{L}_0$ is tuned to be zero and furthermore, even if $\mathcal{L}_0 = 0$, quantum corrections will break the symmetry and ensure that $\mathcal{L}_0 \sim M^4$ for some too-large mass scale, M. This last problem can be dealt with, too, but at the expense of placing the entire quantum theory at a scale-invariant point, which is phenomenologically unacceptable. For these reasons four-dimensional self-tuning mechanisms are disfavoured.

To see these conclusion, we restate the trace of eq. 6.3 as a symmetry condition $\delta\mathcal{L} = 0$ under

$$\delta g_{\mu\nu} = 2\,\epsilon\, g_{\mu\nu} \qquad \delta\phi = -\epsilon\, f(\phi) \tag{6.6}$$

for constant ϵ. (The trace is the only important part if we only seek maximally-symmetric solutions.) By performing the field-redefinition

$$\Phi = \exp\left[\frac{1}{2}\int^{\phi}\frac{d\phi}{f(\phi)}\right], \tag{6.7}$$

we can re-write the symmetry condition as

$$\delta g_{\mu\nu} = 2\epsilon g_{\mu\nu} \qquad \delta\Phi = -\epsilon. \tag{6.8}$$

This easily exponentiates to imply that

$$\mathcal{L}(g_{\mu\nu}, \Phi) = \mathcal{L}'(e^{2\Phi} g_{\mu\nu}), \tag{6.9}$$

which for constant fields implies that

$$\mathcal{L} \equiv \sqrt{g}\, e^{4\Phi}\, \mathcal{L}_0, \tag{6.10}$$

where $\mathcal{L}_0$ is independent of Φ, as was to be shown. This shows that when the equations of motion are strictly of the form 6.3 either $\mathcal{L}_0$ must be tuned to zero or Φ cannot be stabilised.

In order to provide a viable self-tuning solution to the cosmological constant, we must find a way to make $\mathcal{L}_0 e^{4\Phi} \sim (10^{-3}\text{eV})^4$ in eq. 6.10. This is impossible in four dimensions for the same reasons that the cosmological constant cannot be made that size in four dimensions.

6.2 Weinberg's Theorem in SLED

Supersymmetric large extra dimensions *use* Weinberg's theorem to help solve the cosmological constant problem. We have seen that a scaling symmetry between a dilaton and the graviton is crucial to the self-tuning idea, and that self-tuning is destabilised by quantum corrections. The SLED proposal helps with this problem because *the relevant quantum corrections occur in the bulk, and, there, supersymmetry protects contributions above the supersymmetry-breaking scale.* This means that operators relevant for breaking the scaling symmetry and inducing a cosmological constant are naturally suppressed by the small supersymmetry-breaking scale, 10^{-3}eV. This may allow $\mathcal{L}_0 \sim (10^{-3}\text{eV})^4$.

Naively it seems that brane loops continue to make unacceptable contributions to the vacuum energy, since

$$\delta\Lambda \sim M^4 \tag{6.11}$$

when standard model particles of mass M are integrated out. As we have seen, this is a contribution to the brane tension, however, in the large extra dimensions scenario, and need not induce a concomitantly large cosmological constant. Indeed, any operators originating on the brane, such as standard model particles, contribute only locally and therefore remain brane-based operators [204]. These operators therefore contribute only to the brane tension, which we have argued is irrelevant for the flatness of the four-dimensional slices. The only quantum corrections which can ruin flatness are bulk loops, which need not be small in the SLED scenario.

To correctly gauge the size of the high-energy contributions to bulk loops, and therefore the vacuum energy, we must account for the additional symmetries afforded by the six-dimensional covariance of the theory at scales above a milli-eV. The higher-order bosonic operators that can appear are organised in the derivative expansion, as discussed previously, with the dangerous, divergent contributions given by

$$\delta\mathcal{L}_6\, {e_6}^{-1} \sim a_1\, M^4\, R + a_2\, M^2\, R^2 + a_3\, \log M^2\, R^3 + \cdots . \tag{6.12}$$

The first term is a renormalisation of Newton's constant and is irrelevant for the cosmological constant problem. the other terms, when evaluated on the equations of motion and integrated over the internal dimensions of volume r^2, yield

$$\delta\Lambda_4 \sim \frac{M^2}{r^2} + \frac{\log(Mr)^2}{r^4} + \cdots . \tag{6.13}$$

If we could argue that the first term was zero we would have precisely the correct size for the four-dimensional cosmological constant for $r \sim 0.1\text{mm}$ and $M \sim M_{\text{weak}}$:

$$(0.1\text{mm})^4 \sim (10^{-3}\text{eV})^4. \tag{6.14}$$

Although this is work in progress, it seems that the dangerous term proportional to r^{-2} naturally vanishes in theories with enough supersymmetry: in six-dimensional theories whose field-content is that of a dimensionally-reduced 10-dimensional supergravity the divergent contributions to the six-dimensional term M^2R^2 vanish, ensuring a perturbatively stable small cosmological constant [203]. [2]

6.3 Stability

Finally, we must address the issue of stability. We have argued that the 'cosmological constant' is really a scalar potential of the form

$$\mathcal{L} = m^4\, e^{\phi} \tag{6.15}$$

with $m \sim 10^{-3}$eV, but does this in fact provide a solution to the vacuum energy problem?

As long as the coefficient of the term M^2/r^2 in the four-dimensional potential, described above, is zero, and the scalar potential really is of the form 6.15, this represents a particular model of evolving dark energy and makes a definite prediction for the mass of the relevant scalar mass, $\sim 10^{-3}$eV. Whether such a model can be constructed that passes cosmological tests is an interesting future direction to pursue. See also section 7.2.3.

6.4 Discussion

In conclusion, in this chapter we have presented Weinberg's no-go theorem regarding four-dimensional self-tuning solutions to the cosmological constant

[2]Since the mass-dimension of F, G and $\partial\phi$ are all 3/2 in 6D corrections involving these operators are suppressed at very low energies, and they are zero because of supersymmetry at very high energies.

problem, and showed how it may be possible for extra-dimensional theories to use a self-tuning mechanism to generate a small cosmological constant. We showed that the reason scalar-tensor self-tuning mechanisms do not work in four-dimensions is that they rely on a scaling symmetry which is broken by quantum corrections. Because of the usual particle-physics arguments, this scaling symmetry gets broken in the worst possible way, leading to the usual cosmological constant problem in which the cosmological constant becomes the largest scale in the problem to the fourth power. The difference between large-extra-dimensional and four-dimensional scenarios, however, is that the scale at which the scaling symmetry is broken can be very small in LED scenarios, possibly as low as 10^{-3}eV, implying that the breaking of scale invariance by quantum corrections need not be catastrophic.

Chapter 7

Discussion and Future Directions

In this chapter we summarize and discuss the presentation and explore a few avenues for future work.

7.1 Summary

We presented evidence that the standard model is incomplete. This evidence comes from observations (dark matter, dark energy), and from theoretical evidence (running of the couplings, hierarchies). Focusing on hierarchies, we saw that experience with effective field theories suggests that hierarchies are the hallmarks of new physics.

We focussed, in chapter 2, on the cosmological constant problem, which is a hierarchy problem having to do with the very high scale of gravity relative to the size of the vacuum energy observed. We discussed some of the attempts to solve the cosmological constant problem within the standard four-dimensional effective field theory paradigm, and argued that none are successful. Some solutions outside of this paradigm have had more success, such as an anthropic argument for the small size of the cosmological constant.

We discussed that string theory (the most promising theory for quantum gravity) seems to require extra dimensions beyond the four we see. We suggested that in order to solve the cosmological constant problem, perhaps a conservative view is to retain the principles of effective field theories as they stand, and to enlarge the space of solutions sought to include extra-dimensional solutions.

We showed with an explicit example that standard compactifications, whether supersymmetric or not, cannot help with the cosmological constant problem, because phenomenology requires the compactification scale to be very high. The cosmological constant problem is therefore reduced to a four-dimensional problem, which we know is difficult to solve.

In chapter 4 we described brane-world models which can have a very low compactification scale. These models trap standard model particles on branes, and allow gravity to propagate in the bulk, which severely weakens bounds on the size of extra dimensions. We discussed that in a large extra-dimensional scenario with two extra dimensions, the extra dimensions can be as large as $0.1\mathrm{mm} \sim (10^{-3}\mathrm{eV})^{-1}$.

The novel thing about brane-world models, from the point of view of the cosmological constant problem, is that standard model loops are sequestered on the brane, and so only indirectly affect the bulk geometry. We concluded that if a theory could be constructed which allowed flat four-dimensionsal slicings for arbitrary tensions, this may provide insight into the cosmological constant problem. We showed that a general class of models with scalar fields can naturally have flat branes, under the assumption of maximal symmetry of the four-dimensional slicings.

We also discussed the importance of supersymmetry for controlling the bulk loops. We argued that with sufficient supersymmetry in the bulk, bulk loops would not only preserve the small curvature of the four-dimensional slicings, but because of the numerology of six-dimensional brane-worlds $1/r \sim 10^{-3}\mathrm{eV}$, these loops could naturally produce a cosmological constant with precisely the right size $\sim (10^{-3}\mathrm{eV})^4$.

The two considerations of having a dilaton-like scalar and supersymmetry led us to consider the six-dimensional Salam-Sezgin supergravity. This theory does not have the right field-content to control all of the dangerous bulk corrections (namely the $O(M^2)$ discussed in chapter 6), but provides a great testing-ground for the self-tuning ideas, where we require flat solutions to exist (and only such solutions to exist) for any brane tensions. We constructed unwarped and warped solutions to the equations of motion, and, citing work done in refs. [184, 178], we argued that these were the most general solutions with conical singularities at the brane positions.

We found in each case that there was a topological constraint relating the tensions of the different branes embedded in the spacetime. Further work in this direction, in ref. [178], shows that if we consider branes which couple to the dilaton, we can find a flat-brane solution for any choices of tension. These solutions are more severely singular at the brane positions, the singularities there not being conical.

The most serious criticism of the self-tuning portion of this idea is an old criticism of Weinberg's [122]. He showed that self-tuning solutions do not work in four dimensions. The essence of his argument is that self-tuning requires a scale-invariance, which, if unbroken, yields a zero cosmological constant, but, which when broken by particle masses, reproduces the old cosmological constant problem.

This provides a puzzle for the extra-dimensional self-tuning models, since it seems that an effective field theoretic, four-dimensional description disfavours this mechanism. The way in which brane-world self-tuning models circumvent this criticism is because the scales are different, and because the standard model is trapped on a brane. Standard model particles can have masses without disturbing the scaling of the lagrangian as a whole, because we may integrate out the entire standard model before performing our analysis. Brane loops do not destroy self-tuning (as long as there is enough supersymmetry) because the UV cutoff in the bulk theory is 10^{-3}eV.

7.2 Discussion

The major outstanding questions remaining to be answered in this scenario regard the robustness of the self-tuning idea, and controlling the bulk loops which can ruin the correct size of the cosmological constant. A further pressing problem is to explain the large size of the extra dimension.

7.2.1 Self-Tuning

In deriving our results it was important that we made a symmetry-*ansatz*; this restricted the dilaton from having any space-time dependence except on the internal dimensions, and allowed us to prove that the Ricci tensor contstructed from the four-dimensional metric was zero, assuming only the scaling symmetry of the lagrangian.

The physical question to ask is whether, if one of the tensions changes due to a phase transition, the bulk is able to adjust to maintain flat branes. Because of the result that in the more general solutions of ref. [178], flat-brane solutions exist for any pair of tensions, and these tensions fix the boundary conditions for the dilaton and hence the warp factor, we can imagine a scenario in which there is some intermediate transient behaviour linking the two flat-brane solutions.

In ref. [205] more general, time-dependent, solutions were sought to try to answer this question. It was found that in order to accomodate the extra stress-energy associated with the variation, new sources of matter must be added to the branes. The solutions so found do not seem to exhibit self-tuning. An understanding of this phenomenon in the context of the work presented here is currently under investigation.

7.2.2 Quantum Corrections

As we have already discussed, in order to fully control the bulk loops, we must consider six-dimensional theories with the field-content arising from compactifications of 10D supergravities [203].

7.2.3 Size of Extra Dimensions

The large size of the extra dimensions would seem to imply another hierarchy problem, that between the fundamental scales and the compactification radius. However, six-dimensional quintessence models can be constructed which behave in precisely this way. It is important to note that the cosmological constant produced in these brane-world models is not a constant, but depends on the radion

$$\Lambda \sim 1/r^4 \tag{7.1}$$

This is actually a potential for the radion, the kinetic piece arising from the Einstein equations. Loop corrections to the radion potential produce terms of the form

$$V(r) \sim \frac{M^2}{r^2} + \log\left[M^2 r^2\right]\left(\frac{1}{r^4} + C\right). \tag{7.2}$$

This is the same result as was obtained in six dimensions from considering bulk loop-corrections to the gravitational action. By the same arguments presented there, the dangerous terms are the constant piece, C, and the term proportional to M^2. With sufficient supersymmetry, the M^2 piece can be controlled, and models with a logaritmic potential of the form

$$\log\left[M^2 r^2\right]\left(\frac{1}{r^4} + C\right) \tag{7.3}$$

were discussed first in refs. [119, 120, 121], and can naturally explain the large size of the extra-dimensional radius.

7.3 Future Directions

7.3.1 Neglect of Higher-Order Operators

The existence of singularities in the manifold is problematic from the effective field theory point of view, since it seems to violate the assumptions underlying the derivative expansion. The resolution of this criticism lies in recasting the problem as a boundary-value problem so that we may stay as far away from the singularity as necessary to ensure small enough derivatives (curvatures) and field values.

It is useful to begin with an electromagnetic analogy, and to try to answer the following question: In electrodynamics (an effective field theory of photons and charged particles) what do we mean by the well-known equation

$$\nabla \cdot \mathbf{E} = \frac{q}{4\pi}\,\delta^3(\mathbf{x})? \tag{7.4}$$

After all, the lagrangian for electrodynamics,

$$\mathcal{L} = -\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu + \mathcal{L}_{\text{matter}} \tag{7.5}$$

is valid only in an expansion in fields and derivatives, and is augmented by additional terms at higher orders in this expansion. In four dimensions, for example,

$$\Delta\,\mathcal{L} = -a\,\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - b\,\left({}^*F_{\mu\nu}F^{\mu\nu}\right) \tag{7.6}$$

are the next set of gauge- and lorentz-invariant operators that one may write down. Why is it then that we can make any sensible predictions based on the the delta function approach in 7.4 without including higher-order such as those in 7.6?

An Electromagnetic Example

Expressed more generally, the idea is to try to describe the radiation due to a moving charge as a boundary-value problem, instead of as a sourced-radiation problem. We replace

$$\widehat{S} = \int_M d^N x\; -\frac{1}{4}F_{MN}F^{MN} - A_M(x)J^M(x) \tag{7.7}$$

with an action defined on a new manifold with boundary, which encircles the source. We place a current on the surface that exactly reproduces the solution to the problem with sources away from the boundary.

In other words, we seek an appropriate surface, ∂M, and surface current, j_s, such that the EOMS arising from

$$S = \int_M d^N x \; -\frac{1}{4} F_{MN} F^{MN} - \int_{\partial M} A_M \, \mathrm{j}_s^M \tag{7.8}$$

give the same solution away from the boundary as the solutions to the equations arising from 7.7.

The equations of motion with source $j(x)$ are

$$\partial_M F^{MN} - j^N = 0. \tag{7.9}$$

The equations of motion in the presence of a boundary, with boundary current are given by 7.9 away from the boundary, but integration by parts when performing variations of the fields localised on the boundary yields the following boundary condition

$$n_M F^{MN} + \mathrm{j}_s^N = 0 \tag{7.10}$$

We wish to describe the boundary operators for a surface enclosing a point-charge moving with an arbitrary trajectory $\mathfrak{X}(\tau)$, so we take the action to be

$$\begin{aligned} S &= -\int_M \frac{1}{4} F_{MN} F^{MN} - q \int_M \int d\tau A_M(x) \frac{d\mathfrak{X}^M}{d\tau} \delta^4(\mathfrak{X}(\tau) - x) \\ &= -\int_M \frac{1}{4} F_{MN} F^{MN} - q \int d\tau A_M(\mathfrak{X}) \frac{d\mathfrak{X}^M}{d\tau} \end{aligned} \tag{7.11}$$

Matching

We first perform the matching calculation to get a flavour of how the calculation in gravity should go. We expect to be able to replace 7.11 by

$$S = -\frac{1}{4} \int_M F_{MN} F^{MN} - \int_\Sigma \mathcal{L}_\partial (A, \frac{d\mathfrak{X}}{d\tau}) \tag{7.12}$$

with $\mathcal{L}_\partial$ a series in derivatives and fields on the boundary that will reproduce the solution order-by-order. We fix the boundary action by requiring that for a configuration of sources $J^M(x)$ for which the solution F_{MN} is known, the boundary conditions

$$n_M F^{MN} + \frac{\delta}{\delta A_N}\mathcal{L}_\partial = 0 \tag{7.13}$$

are satisfied.

We choose

$$\mathcal{L}_\partial(A, \frac{d\mathfrak{X}}{d\tau}) = a\, A_M(y)\frac{d\mathfrak{X}^M}{d\tau} + b\, A_M(y)n^M + c\, n_M\,\frac{d\mathfrak{X}^M}{d\tau} + e\,\frac{d\mathfrak{X}^M}{d\tau}n^N F_{MN} + \cdots \tag{7.14}$$

for some $a, b\cdots$ as the most general polynomial function, to first order in A and second-order in derivatives. In the above equation n_M is the normal to the surface $\Sigma \subset M$ which encloses the trajectory, and where each of the $\frac{d\mathfrak{X}}{d\tau}$ is evaluated at some $\tau(y)$ which we must compute. Because we are doing a derivative expansion about the static case we can choose $\tau(y)$ to be the solution to $\min|\mathfrak{X}^M(\tau) - y^M|$ or to $\mathfrak{X}^0(\tau) = y^0$, assuming that the trajectory is timelike. These two perscriptions differ in the coefficients of the higher-derivative terms, but once fixed give the same physical answers. We will choose to fix the time-slicing, $\mathfrak{X}^0(\tau) = y^0$, since it make matching to the static case easy.

Gauge-invariance tells us immediately that $b = 0$, while we can ignore the term involving c, since it is irrelevant for the electromagnetic field; this term corresponds to the interaction of the particle with the boundary. We are therefore left with

$$\mathcal{L}_\partial(A, \frac{d\mathfrak{X}}{d\tau}) = a\, A_M(y)\frac{d\mathfrak{X}^M}{d\tau} + e\,\frac{d\mathfrak{X}^M}{d\tau}n^N F_{MN} \tag{7.15}$$

The lowest-ordest order coefficient, a is most easily found from the static configuration

$$\frac{dX^M(\tau)}{d\tau} = (1, \vec{0}) \tag{7.16}$$

with solution,

$$F^{i0} = q \frac{x^i}{4\pi\, x^3}. \tag{7.17}$$

We choose the surface Σ to be a sphere of radius r enclosing the static charge. The boundary condition reads

$$\frac{q}{4\pi\, r^2} + a\frac{d\mathfrak{X}^0}{d\tau} = 0, \tag{7.18}$$

which in the restframe gives

$$a = \frac{-q}{4\pi} \tag{7.19}$$

so that, to lowest-order, the boundary action is given by

$$\mathcal{L}_\partial = \frac{-q}{4\pi\, r^2} A_M \frac{d\mathfrak{X}^M}{d\tau} \tag{7.20}$$

The next-order term depends on the second derivative of the particle trajectory (the acceleration).

Extension to Gravity

A similar exercise, if carried out in the case of gravity could help to deal with the highly singular solutions with non-conical curvature-singularities. An effective expansion on the boundary would include terms with both the intrinsic and the extrinsic curvature of the boundary:

$$\mathcal{L}_{\partial M} = T + aK + c^\partial R + \cdots \tag{7.21}$$

where K is the extrinsic curvature and ${}^\partial R$ is the intrinsic curvature of the boundary.

7.3.2 Brane Motion

In our models the positions of the branes are fixed, and they are not allowed to move. When considering fully time-dependent solutions, the brane positions should also be allowed to change. Analysing these more general solutions involves including the full DBI-type action arising from broken translational invariance [50, 51] for the branes:

$$\mathcal{L}_{\text{brane}} = \sqrt{Det g_{MN}(X)\partial_\mu X^M \partial_\nu X^N} \tag{7.22}$$

where X are the positions of the branes.

Accounting for brane motions as well as for the other equations of motion means varying the above lagrangian with respect to X, as well as the usual boundary variation of the metric. It would be very interesting to understand the possilbe implications of performing such a calculation in the complicated backgrounds presented in this thesis.

7.3.3 String Theory Derivation?

It is an interesting challenge to derive the 6D theory we started with from string theory. First we note that typically 6D, $N = 2$ supergravity is obtained from K_3 compactifications of type I or heterotic strings. However the supergravity theory obtained in this way is ungauged, whereas ours is gauged supergravity that includes a nontrivial potential $\sim ge^{-\phi}$.

Recently it has been realized that massive 10D and gauged supergravities in lower dimensions can be obtained either from string compactifications on spheres [111] [1] or toroidal and related compactifications, such as K_3, in the

[1] We thank C. Pope for many discussions on these points.

presence of RR or NS-NS fluxes. In the latter case, typically a p-form F_p integrated around non-trivial cycles of the compact space Γ_p can be different from zero, $\int_{\Gamma_p} F_p \neq 0$. The form F_p can be expanded in terms of harmonic forms ω_p^i:

$$F_p = \nu^i \omega_p^i \tag{7.23}$$

where the coefficients ν^i will correspond to the fluxes. In particular these fluxes give rise to potentials precisely of the form we started with, the flux being identified with the gauge coupling constant of the effective gauged supergravity theory. In this way several maximal gauged supergravity theories have been obtained from toroidal compactifications with fluxes [112]. Also compactifications on $K_3 \times T_2$ have been considered with fluxes in both K_3 and T_2. Furthermore, fluxes have also been shown in type IIB and M theory to freeze geometric moduli [115], just as the T field in the present model is frozen.

We have to recall that the general $N = 2$ 6D supergravity has a more complicated spectrum of scalar fields than the one we used, since the gauge group can be much larger than the simplest $U(1)$ that we considered [113]. In that case the potential is quadratic in the hypermultiplet scalars, with overall factors of $e^{-\phi}$, a natural outcome of the fluxes in string theory. It is not completely clear to us how to derive precisely the Salam-Sezgin action from this class of backgrounds yet, although it looks very suggestive.

Furthermore, string compactifications on spheres and related geometries have also been successful in deriving gauged supergravities. In particular the maximal 6D supergravity was derived from an S^4 string compactification and a detailed comparison of the potential was achieved with the $N = 4$ gauged supergravity of Romans [114].[2] We leave as an open question the possible derivation of our $N = 2$ action from this construction as well as any possible relationship with the K_3 backgrounds with fluxes. If such a construction is found then it will be interesting since it will give rise to realistic string compactifications on manifolds which are not Ricci flat (since at least the 2-sphere is not), contrary to standard beliefs.[3]

[2] After completing this work it was pointed out to us that fluxes over $\mathcal{P}^1(\mathcal{C}) = S^2$ were considered in ref. [117] in Heterotic and Type II compactifications.

[3] After finishing this article we became aware of earlier [115] and new [116] work presenting examples of this type. It may be interesting to unravel any connection between these constructions and our work.

Appendix A

Notation and Conventions

Unfortunately there is no universal choice for the various conventional signs that appear in definitions of the metric and curvature, nor for the numerous choices of basis available in defining spinors. I record the conventions used in this thesis in this appendix.

A.1 Metric and Curvature

I use the 'mostly-plus' metric,

$$\eta_{MN} = \mathrm{diag}(-1, 1, \cdots, 1) \tag{A.1}$$

The Christoffel symbols are defined through

$$\nabla_M v_N = \partial_M v_N - \Gamma^S_{MN} v_S \tag{A.2}$$

The curvature corresonds to MTW [62] and Wald [46] conventions:

$$[\nabla_M, \nabla_N] v_P = R_{MNP}{}^Q v_Q \tag{A.3}$$

with

$$R_{MN} = R_{MSN}{}^S \tag{A.4}$$

(Weinberg's [80] conventions are opposite because he chooses to represent covariant derivatives using the semicolon notation; the order of indices is reversed relative to the differential operator notation.)

I also use spin-connections and vielbeins: in n dimensions there always exist n one-forms, $E^A = E^A_M dz^M$, called the vielbein, which square to the metric (E^M_A is the inverse, in the sense given by the equation below, to E^A_M),

$$E^A_M E_{AN} = g_{MN}, \qquad E^M_A E_{BM} = \eta_{AB}. \tag{A.5}$$

The letters $A, B, \cdots$ from the beginning of the alphabet are used to label tangent-space indices, while $M, N, \cdots$ are used to label world indices. The E^A represent a non-coordinate, orthonormal basis which can be used to expand objects on a manifold. In particular, we can expand tensors in 'tangent-space indices':

$$v_A = E^M_A v_M. \tag{A.6}$$

Tangent-space indices are raised and lowered using the flat-space metric, η_{MN}.

The spin connection is defied as

$$dE^A = \Omega^{AB} \wedge E_B \tag{A.7}$$

and enters the covariant derivative as

$$D\psi = d\psi - \Omega^{AB} \mathcal{J}_{AB} \psi, \tag{A.8}$$

where $\mathcal{J}_{AB}$ is a representation of the lorentz group appropriate to the field being differentiated. For a vector we have

$$D_M v_A = \partial_M v_A - \Omega_A{}^B v_B \tag{A.9}$$

while in the conventions given below for spinors,

$$D_M \Psi = \partial_M \Psi + \frac{i}{4}[\Gamma^A, \Gamma^B]\Omega_{ABM}\Psi \tag{A.10}$$

In this language the curvature can be thought of as a two-form field-strength with 'gauge indices' in the tangent space:

$$[D_M, D_N]v^A = R^{AB}{}_{MN}v_B \tag{A.11}$$

A.2 Dirac Algebra and Spinors

The Pauli matrices are given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.12}$$

and we will have occation to use the antisymmetric two-by-two matrix

$$e = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{A.13}$$

and

$$\epsilon = \begin{pmatrix} e & 0 \\ 0 & e \end{pmatrix} \tag{A.14}$$

I use the gamma-matrix conventions of Weinberg, which is chiral basis. A spinor is written as a four-component column vector,

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} \tag{A.15}$$

and its covariant conjugate is given by

$$\bar{u} = iu^\dagger \gamma^0 \equiv u^\dagger \beta \tag{A.16}$$

The Dirac matrices are given by him in his books as

$$\gamma^0 \equiv -i\beta = -i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{A.17}$$

$$\gamma^i = -i \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \tag{A.18}$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.19}$$

The Majorana conditions is given this basis as

$$u^* = -\beta\epsilon\gamma_5 u = \begin{pmatrix} 0 & -e \\ e & 0 \end{pmatrix} u \tag{A.20}$$

So that a Majorana spinor can be written as

$$\psi = \begin{pmatrix} e\chi^* \\ \chi \end{pmatrix} \tag{A.21}$$

Appendix B

Supersymmetry and Supergravity

In this appendix I briefly present an overview of supersymmetry and supergravity as used in this thesis. This is not meant as a pedagogical introduction, for which the reader is referred to the many excellent texts (see eg, [199]) and review articles (see eg, [41]). This appendix is included to fix notation and present some context, but I do outline what I regard to be some of the most compelling *theoretical* evidence (as opposed to phenomenological evidence, which was presented in chapter 1) for supersymmetry. In particular, I depart from a purely superficial description of supersymmetry by presenting a brief introduction to the Coleman-Mandula and the Haag-Lopusanski-Sohnius theorems. I do so because they provides a slightly different motivation for supersymmetry than that quoted several places in the rest of this thesis, the prevaling quoted motivation being to stabilise the electroweak hierarchy or the cosmological constant.

The Coleman-Mandula theorem along with the Haag-Lopuszanski-Sohnius theorem come very close to definitively showing that supersymmetry is the unique extension of the notion of symmetry consistent with the notion of a scattering matrix.

B.1 Overview

Supersymmetric theories are those that are invariant under a $\mathbb{Z}_2$-graded Lie algebra. Graded Lie algebras are exactly like regular Lie algebras except that there are different kinds of commutators for different classes of objects which have different gradings. The grading of a product of two objects is the sum of the gradings (mod 2 in the case of SUSY algebras).

More formally, there are two types of things which are called "bosons" and "fermions" and which are attributed grading 0 and 1 respectivly. The grading of objects A, B is $G(A), G(B)$, and we define a generalised bilinear commutator $\{\cdot, \cdot]$ (no longer strictly antisymmetric), such that

$$\{A, B] = AB - (-)^{G(A)G(B)} BA, \tag{B.1}$$

and

$$G\left(\{A, B]\right) = G(A) + G(B) \bmod 2 \tag{B.2}$$

That is, if either A or B is a boson we recover the usual commutator $\{A, B] = AB - BA$, while if both are fermions we recover the usual anticommutator: $\{A, B] = AB + BA$. We define the hermitian adjoint of operators, A^*, to retain the grading of the original. Operators also satisfy the "super-Jacobi identity":

$$0 = (-)^{G(A)G(C)}\{\{A, B], C] + (-)^{G(B)G(A)}\{\{B, C], A] + (-)^{G(C)G(B)}\{\{C, A], B] \tag{B.3}$$

A supersymmetry *algebra* satisfies commutation relations which look like the ordinary commutation relations of Lie algebras: Let t_a be a set of graded operators. t_a form a super-Lie algebra if

$$\{t_a, t_b] = C^c_{ab} t_c \tag{B.4}$$

with the following properties:

- **Consistency of grading**

$$C^c_{ab} = 0 \tag{B.5}$$

  unless $G(t_c) = G(t_a) + G(t_b)$ mod 2.

- **Consistency of complex-conjugation**

$${C^c_{ab}}^* = -C^c_{ba} \tag{B.6}$$

  (Note the order of indices: $(AB)^* = B^* A^*$.)

Just as Lie algebras correspond locally to Lie groups through the exponential map, super-Lie algebras correspond to super-Lie groups through a generalisation of the exponential map. Separate the t_a into fermionic generators Q_a, of grading 1, and bosonic generators, B_α, of grading 0. Then the super-Lie group elements generatred by the algebra are given by

$$g(\theta, y) = \exp(i\theta^a Q_a + iy^\alpha B_\alpha) \tag{B.7}$$

where the ys are real coordinates on the group manifold and the θs are (anticommuting) coordinates on a Grassman manifold. (For theorems and rigourous definitions, please see ref [194].)

As we shall see, while this seems like one particular generalisaton of the concept of symmetry, it is in fact the only one possible. Under reasonable assumptions two theorems can be proved which together restrict the possible linearly represented symmetries furnished by theories to supersymmetries and the ordinary Lie symmetries. In the next section I will discuss the uniqueness of the supersymmetry algebras, introducing the Coleman-Mandula and Haag-Lopuszanski-Sohnius theorems, which show why supersymmetry is *formally* a natural extension of symmetry. I'll then go on to present my superspace conventions, describe F and D terms, and quote the supersymmetric nonrenormalisation theorem which is used for many arguments throughout this thesis. I will follow this with the lagrangian for the most general $N = 1$ four-dimensional supergravity action, which, if ill-motivated in the context of this appendix alone (it is a very long business deriving it), should at least be comprehensible given the notation set out in the rest of this appendix.

B.2 Uniqueness

The formal significance of this particular extension of the notion of symmetry to *super*symmetry is twofold:

1. Supersymmetry is the most general possible extension of the notion of symmetry compatible with the notion of a scattering operator.

2. Supersymmetry ties together the bosonic and fermionic degrees of freedom in a theory. A supersymmetric variation rotates fermions into bosons and *vice versa*.

There are very few theorems in high energy physics, but the first item above is one of them. It comes in two parts: first, the Coleman-Mandula theorem [195] states that the only Lie algebras (not superalgebras) which leave the S-matrix invariant, and which have the Poincare group as a subalgebra, are direct sums of the Poincaré group with sums of compact semi-simple Lie algebras and $U(1)$s. This means that the only bosonic symmetries of the S-matrix are the ones we already know about; there cannot be new, exotic ones that mix particles of different spins.

The second part of the theorem is the Haag-Lopuszanski-Sohnius theorem [196]. This theorem states that under the same assumptions as the Coleman-Mandula theorem, if we additionally allow *fermionic*, symmetry generators, *i.e.* symmetry-generators of grading 1, the only possible such generators are spin-1/2. It moreover goes on to fix the form of the superalgebra completely. (It should be mentioned that although these two parts together form a very compelling picture of exhaustiveness, loopholes are notoriously difficult to spot in "theorems" in physics. Indeed supersymmetry itself is a loophole to the Coleman-Mandula theorem. This loophole was found by people doing entirely independent research in string theory, and seemingly having no contact with the Coleman-Mandula theorem [199].)

Poor Person's Coleman-Mandula

Here is a version of one part of the Coleman-Mandula argument, due to Weinberg[199], which is presented here in a simplified form. This simple argument will exhibit why simply putting particles of different spin in a multiplet and requesting them to rotate into each other under a semi-simple

Lie group like $SU(N)$ won't work.

Let B_A be bosonic, hermitian generators of a semi-simple compact symmetry of the hamiltonian, so that

$$[B_A, P_\mu] = 0 \tag{B.8}$$

and

$$[B_A, B_B] = i\,C^C{}_{AB} B_C. \tag{B.9}$$

We will assume B_A has a nontrivial transformation under the Lorentz group (allowing it to mix spins of different kinds), and will show that this contradicts the noncompactness of the Lorentz group.

We will need two facts from group theory:

1. Compact semi-simple algebras have real structure constants $C^c{}_{ab}$ from which can be formed a real, positive-definite metric

$$g_{ab} = C^c{}_{da}\, C^d{}_{cb}, \tag{B.10}$$

   and the existence of such a real, positive-definite metric is equivalent to the property of being compact for Lie algebras.

2. The only invariant two-tensor of the Lorentz group (in more than two dimensions) is the metric tensor $\eta_{\alpha\beta}$.

The Bs transform nontrivially under the Lorentz group, so

$$U(\Lambda) B_A U^{-1}(\Lambda) = D^B{}_A(\Lambda) B_B \tag{B.11}$$

where $U(\Lambda)$ is unitary operator of Lorentz transformations associated with the Lorentz matrix Λ and D is the representation of the Lorentz group furnished by the Bs:

$$D(\Lambda_1) D(\Lambda_2) = D(\Lambda_1 \Lambda_2) \tag{B.12}$$

It is easy now show that the metric for the algebra of Bs is an invariant tensor with respect to the Ds. This is because the structure constants are invariant tensors of the transformation and the metric is constructed from the structure constants according to B.10. The structure constants are invariant tensors since the commutation relations of the Bs take the same form both before and after a Lorentz transformation. That this is a problem can be seen immediately, since the only invariant tensors of the Lorentz group are the metric $\eta_{\alpha\beta}$ and the totally antisymmetric tensor $\epsilon_{\alpha\beta\gamma\delta}$. Since η is not positive definite, this contradicts B being a semisimple compact group.[1] [2]

In order to finish the argument we must justify the restriction to compact groups. I will show in the next few paragraphs why *linearly represented* internal symmetries must be compact.

There are various lines of argument against noncompact internal symmetry groups, which make them unfavourable. First, noncompact groups, if represented in a theory, must be global symmetries since the gauge-field hamiltonian is positive-definite *if and only if* the metric of the corresponding lie algebra, constructed from the structure functions, as in eq. B.10, is positive definite (which is equivalent to the compactness of the group) [198]. Second, global symmetries, if non-compact, must be represented nonlinearly. The

[1]Here ref. [199] goes on to explicitly construct a finite-dimensional, unitary representation of the Lorentz group, and then invokes the following fact about groups to prove the theorm: *Noncompact groups such as the Lorentz group have no finite-dimensional unitary representations,* thus achieving the desired contradiction. This construction is unnecessary as we have seen.

[2]The restriction to dimensions large than two is actually necessary, since in $D \leq 2$ the connection between spin and statistics is unclear; in $D \geq 2$ there are only two kinds of statistics possible, fermionic (anticommuting) and bosonic (commuting), having to do with the triviality of all one-cycles in the puctured space. Since this is not true in fewer dimensions, more general statistics ("para-statistics") are possible [197].

proof I present rests on the fact that there exist no finite-dimensional, unitary representations of noncompact groups as linear transformations. Here is the proof. Since the S-matrix must be *invariant* (not *covariant*) with respect to internal symmetry transformations, we must either have an infinite number of particle types or a loss of unitarity. Let T be the transformation of the states Φ_i and Φ_o which commutes with the S-matrix. Then the scattering element is given by

$$
\begin{aligned}
(\Phi_o, S\Phi_i) &= (T\Phi_o, S\,T\Phi_i) \\
&= \left(\Phi_o, T^\dagger S\,T\Phi_i\right) \\
&= \left(\Phi_o, S\,\left(T^\dagger T\right)\,\Phi_i\right)
\end{aligned}
\tag{B.13}
$$

Now, either $T^\dagger T = 1$ and T is infinite-dimensional, or T is finite-dimensional and S is not invariant. Therefore T must generate a compact group of symmetry transformations. [3] [4]

The only examples of which I am aware in which theories succesfully incorporate noncompact internal symmetry groups without violating unitarity

[3]Problems also arise in the effective description of the theory: If low-energy degrees of freedom furnish a linear representation of a noncompact group the hamiltonian does not stay bounded from below. Consider the construction of the free part of the hamiltonian: we assume there exists a non-compact group of transformations $T(\alpha)$, acting linearly on a set of fields, ϕ_i and we wish to find a tensor g_{ij} such that

$$
\mathcal{H}_0 = g_{ij}\Pi^i\Pi^j + g^{ij}\partial_a\phi_i\partial_a\phi_j + \cdots . \tag{B.14}
$$

where $\Pi^i = \partial\mathcal{L}/\partial_0\phi_i$ is the canonical momentum conjugate to ϕ_i, and g^{ij} is the inverse of g_{ij}. It can be shown [198] that for lie groups, the only invariant two-tensor is the metric constructed as in eq. B.10, and that this metric is positive-definite *iff* the group is compact. Since the group is not compact the hamiltonian unbounded from below.

[4]The lorentz transformations do not suffer from this restriction because unitary, infinite-dimensional representations are used for the quantum operators. This does not lead to infinite particle types because the non-compact part generates boosts, and boosted particles are not considered different. (The particle types are generated from the little groups of rotations.)

are theories which represent noncompact groups nonlinearly on their scalar manifold, as supergravities do [200, 201]. The symmetries thus represented within these theories all involve shifts of scalar fields (in an appropriate choice of field coordinates) and have nothing to do with symmetries of one-particle states which are symmetries of the S-matrix[5].

In any case, under the assumption of *linearly represented* symmetries, which are what we are concerned with here, internal symmetries must be compact, and therefore fall under the assumptions of the Coleman-Mandula theorem.

This argument against bosonic symmetries which mix particles of different spins misses the second (and much harder) part of the theorem, that the *only* possible bosonic generators are the Poincaré generators, P_μ, $J_{\mu\nu}$ plus the usual internal symmetry generators, but it shows why simple guesses like trying to put mesons of different spins in a vector furnishing a unitary symmetry is doomed to failure in *relativistic* theories, but not in nonrelativistic ones; the proof by contradiction hinges on the noncompactness of the Lorentz group.

Haag-Lopuszanski-Sohnius

The HLS theorem uses the Coleman-Mandula theorem to prove that fermionic generators of symmetries must be spin-1/2, and must square to give the hamiltonian. According to this theorem, all supersymmetry algebras take the same form, and I will present my notation for the "standard $\mathcal{N} = 1$

[5]These symmetries are now well-understood to arise from symmetries of compactifications from higher-dimensional supergravities, or to encode dualities between different theories[202].

form" of the SUSY algebra. I will again present the flavour of the HLS argument without presenting the details, which can be found in books and reviews on SUSY.

The idea of the HLS theorem is to take super-commutators, $\{\cdot, \cdot]$ of some fermionic generators of symmetries Q_α and analyse them under the action of the Lorentz group. We will use the Coleman-Mandula theorem repeatedly to restrict the right-hand-sides of the commutators, which themselves must be symmetry-generators.

To see how this works, consider the commutator $\{Q_a^*, Q_b]$, where there are unknown Lorentz indices of some sort hanging off the Qs, which belong to the (A, B) representation of the Lorentz group. (Recall that the four-dimensional Lorentz group can be analysed in terms of two sets of independent $SU(2)$ generators

$$\mathbf{A} = \frac{1}{2}(\mathbf{J} + i\mathbf{K}) \qquad \mathbf{B} = \frac{1}{2}(\mathbf{J} - i\mathbf{K}) \tag{B.15}$$

where $J_i = \epsilon_{ijk} J_{jk}, K_i = J_{0i}$ are the Hermitian generators of rotations and boosts, respectively. Any object that transforms under the Lorentz group can be classified by its transformations (and therefore its representation) under the operators A and B.) Since the right-hand-side of the commutator of two Qs is also a symmetry generator and is bosonic, it must belong to the set of generators allowed by the Coleman-Mandula theorem: P_μ, $J_{\mu\nu}$ and T_A. In this notation, P_μ is $(1/2, 1/2)$, since it furnishes a spin-1/2 representation for both of A and B, while $J_{\mu\nu}$ is $(1, 0) \oplus (0, 1)$ and T_A is $(0, 0)$. In this way the problem of finding the transformation properties of the commutator of Qs becomes the problem of the coupling of two independent spins corresponding to $\mathbf{A}$ and $\mathbf{B}$. With a little work we can show that $A + B \leq 1/2$, which proves that the Q is $(0, 1/2)$, while Q^* is $(1/2, 0)$.

Now that we know the Qs are spin-1/2 it is easy to see what their commutator is. $[Q, Q^*]$ is $(1/2, 1/2)$, a vector, and the only one available, by the Coleman-Mandula theorem, is the momentum operator, P_μ. We use the Clebsh-Gordon coefficients for coupling spinors and vectors, the Pauli matrices, to write the commutator of Q with its complex conjugate, Q^* as

$$\{Q_a^{(r)}, {Q^*}_b{}^{(s)}\} = 2\delta^{(r)(s)}\sigma^\mu_{ab}P_\mu \tag{B.16}$$

where δ is the usual Kronecker delta. (We have redefined the Qs through a linear transformation to diagonalise the matrix to the unit matrix on the right-hand side.)

Similar reasoning shows that the Qs commute with P_μ, and restricts the commutator of Q with itself to be a bosonic internal symmetry generator (a 'central charge', since it is the centre of the algebra, commuting with everyone), Z. Once converted from two-component notation using Pauli matrices to four-component notation using Dirac matrices (the translation is in appendix A), we find the four-dimensional suprsymmetry algebra in standard form:

$$\{Q_r, \bar{Q}_s\} = -2\,i\,P_\mu\gamma^\mu + \gamma_L Z^*_{sr} + \gamma_R Z_{rs} \tag{B.17}$$

where $\gamma_L = (1 + \gamma_5)/2, \gamma_R = (1 - \gamma_5)/2$ are the projectors onto left and right helicity. When there is only one supersymmetry generator, we have $\mathcal{N} = 1$ supersymmetry and no Zs:

$$\{Q_r, \bar{Q}_s\} = -2\,i\,P_\mu\gamma^\mu \tag{B.18}$$

Higher supersymmetries, such as $\mathcal{N} = 2, 4$ *etc*will not be used in this thesis except very briefly in our discussion of the breaking of six-dimensional supersymmetry to four dimensions.

B.3 Representation on Fields and Lagrangians

Now that we have the supersymmetry algebra constructing representations of the algebra is easy. We will start with the construction of the simplest (and for our purposes most useful) representation, the *chiral multiplet*, which consists of a single scalar and Weyl fermion, followed by a brief discussion of the *vector multiplet*, which contains vector fields and from which supersymmetric gauge theories are built.

In order to construct the representation of the chiral multiplet it is easiest to work first with the two-component supersymmetry algebra, eq. B.16, then present the fields and algebra in four-component form. We work in $\mathcal{N} = 1$ supersymmetry.

Begin with a complex scalar field, ϕ. We wish the supersymmetry variation of ϕ to give a new field of spin 1/2, ψ, but for ϕ to be the lowest component of the representation. The generator Q is a spin-lowering operator, being $(0, 1/2)$, and its complex-conjugate is a spin-raising operator. Therefore, we require

$$[Q_a, \phi] = 0, [Q^*_a, \phi] = -ie_{ab}\psi_b, \tag{B.19}$$

where the indices are two-component spinor indices. The second commutator must be proportional to antisymmetric two-tensor e_{ab} (see appendix A) because it is $e_{ab}Q^*_b$ which transforms under the $(1/2, 0)$ representation of the

Lorentz group. The super-Jacobi identity gives

$$\{Q_a, \psi_b\} = -i\epsilon_{ac}[\{Q_b, Q_c^*\}, \phi] = 2i(\sigma^\mu e)_{ba}[P_\mu, \phi], \tag{B.20}$$

so that

$$\{Q_a, \psi_b\} = -2\,(\sigma^\mu e)_{ab}\partial_\mu\phi \tag{B.21}$$

Continuing in this manner, and continuing to use the HLS theorem and Lorentz-invariance yields the (off-shell) $\mathcal{N} = 1$ supersymmetry transformations for the chiral multiplet:

$$\begin{aligned} \delta\phi &= \sqrt{2}(\overline{\epsilon_R}\psi_L) \\ \delta\psi_L &= \sqrt{2}\,\partial\!\!\!/\phi\epsilon_R + \sqrt{2}\mathcal{F}\epsilon_L \qquad \text{(B.22)} \\ \delta\mathcal{F} &= \sqrt{2}(\overline{\alpha_L}\,\partial\!\!\!/\psi_R) \qquad \text{(B.23)} \end{aligned}$$

The complex-conjugate of a chiral multiplet is called an anti-chiral multiplet (or right-chiral multiplet). Anti-chiral multiplets satisfy the same variations as those given in B.22, with the interchange $L \leftrightarrow R$.

The direct construction of the particle content of a representation of $\mathcal{N} = 1$ supersymmetry containing a spin-0 particle as its lowest component results in a multiplet with a complex spin-0 particle and a Weyl spin-1/2 particle. The appearance of the additional scalar, $\mathcal{F}$, is a result of the *off-shell* nature of the algebra. $\mathcal{F}$ is an auxiliary field (non-propagating) which is always expressed in terms of other fields in the lagrangian after imposing the equations of motion. The lagrangian so constructed, called *on-shell*, will only be supersymmetric when the fields satisfy their equations of motion. Supermultiplets, whether on-shell or off-shell, always contain the same number of bosonic and fermionic components: off-shell, ψ_L has two complex degrees of freedom, which equals the two complex bosonic degrees of freedom in ϕ and $\mathcal{F}$; on-shell, ψ_L only has one complex degree of freedom (the Dirac equation relates the components of the spinor), which matches the single complex propagating degree of freedom in ϕ.

Two things make $\mathcal{F}$ a very important quantity: if $\mathcal{F}$ gets a *vev* supersymmetry is broken, and the variation of the auxiliary component (the F-term) of a chiral multiplet is always a total derivative (as seen in the last variation in B.22). There are specific multiplication rules for chiral multiplets, with the result that the product of two chiral multiplets (but not a chiral and an anti-chiral multiplet) is again a chiral multipet. A supersymmetric lagrangian can always be constructed by taking the F-term of an arbitrary function of chiral superfields:

$$\mathcal{L}_{\mathrm{F-term}} = [W(\Phi_1, \Phi_2, \cdots)]_{\mathcal{F}} \tag{B.24}$$

A similar statement holds for anti-chiral fields. The function W is conventionally called the superpotential, and gives mass terms and interactions. The scalar potential due to a superpotential is given by

$$\left(\frac{\partial W}{\partial \phi_i}\right)^* \left(\frac{\partial W}{\partial \phi_i}\right) \tag{B.25}$$

(where W is evaluated at $\Phi = \phi$, the scalar-field component of the chiral superfield).

The product of a chiral and anti-chiral field yields a general superfield. General superfields have more complex variations than those given in B.22. The auxiliary field which is the field of highest-weight, and which transforms to a total derivative is conventionally called a D-term. Since the supersymmetric variation of a D-term is a total derivative, supersymmetric actions can also be constructed from them:

$$\mathcal{L}_{\text{D-term}} = [K(\Phi^*, \Phi)]_D \tag{B.26}$$

All F-terms can be written as D-terms, but not all D-terms can be written as F-terms. D-terms tend to give the kinetic parts of actions, and the function K is conventionally called the Kähler function. The simplest Kähler function, $K = \Phi^*\Phi$, gives the free kinetic terms for the chiral multiplet:

$$[\Phi^*\Phi]_D = -\partial\phi\partial\phi^* - \overline{\psi}\partial\!\!\!/\psi + \mathcal{F}^*\mathcal{F} \tag{B.27}$$

F-terms are very special, due to their dependence on either only chrial or only anti-chiral fields. Considering chiral fields as complex coordinates, F-terms are purely holomorphic or purely anti-holomorphic. This restriction provides a very strong constraint on the kinds of corrections that can appear in supersymmetric theories. So strong is this restriction, in fact, that *the superpotential is not renormalised to any order in perturbation theory* as long as supersymmetry remains unbroken. D-terms enjoy no such protection, and are corrected in more-or-less arbitrary ways. A special case in which D-terms are afforded some protection from corrections are those D-terms that can also be written as F-terms. These tend only to receive one-loop corrections.

The non-renormalisation of the superpotential means that the scalar potential B.25 also does not receive any corrections. This is one way to see that the cosmological constant is not renormalised in supersymmetric theories; the effective potential for the fields does not change. (It is also interesting that the scalar potential is non-negative since it is a perfect square.) The stabilisation of the Higgs mass is also clear in this context, since scalar-field masses also arise from the superpotential. All of these results can be seen easily and explicitly by computing the one-loop effective potential for a supermultiplet; the bosonic and fermionic degrees of freedom cancel. The advantage of the present method is that it is true to all orders in perturbation theory (and sometimes even nonperturbatively).

There is one final type of field that is of interest for the supersymmetric arguments made in this thesis, the linear (or vector) superfield. This is the superfield from which Yang-Mills interactions are constructed. The kinetic terms of the Yang-Mills multiplets turn out to be one of the special class of objects that can be written both as the D-term of a general superfield and the F-term of a chiral superfield. It is for this reason that gauge couplings only receive one-loop corrections (if at all) in supersymmetric theories.

The D-term of a vector superfield, if itself gauge-invariant (if and only if the vector carries an abelian charge), can provide a supersymmetric term in the action called a Fayet-Iliopoulos term

$$\mathcal{L}_{\rm FI} = \xi [V]_D \tag{B.28}$$

These terms turn out to also enjoy a non-renormalisaton theorem (the quantum correction to the term is proportional to the trace of all the $U(1)$ generators, which is guaranteed to vanish in the absence of gravitational anomalies). FI terms can also give rise to scalar potentials, as is shown explicitly in chapter 3.

The four-dimensional supergravity multiplet is constructed by using yet another kind of superfield. The very interesting thing about supergravity is that it is the unique extension of global supersymmetry to a gauge symmetry. Because the square of a supersymmetry transformation is a translation, gauging supersymmetry is equivalent to gauging the Poincare group, giving a fully diffeomorphism-invariant theory. The discussion or constructon of $\mathcal{N}=1$ supergravity is lengthy and unilluminating. The terminology set out in this chapter is adequate to understand the most general $\mathcal{N}=1$ supergravity lagrangian, given in [45], and whose bosonic part is reproduced here,

$$\begin{aligned}
\mathcal{L}_B/\sqrt{g} &= \frac{1}{2}R \\
&- \frac{1}{4}\mathrm{Re}\, f_{ab}F^a_{\mu\nu}F^{\mu\nu b} - \frac{1}{4}\mathrm{Im}\, f_{ab}\epsilon^{\mu\nu\sigma\rho}F^a_{\mu\nu}F_{\sigma\rho}{}^b \\
&- K^{ij^*}D_\mu\phi_i D^\mu\phi^*_j \\
&- \frac{1}{2}\mathrm{Re} f^{-1}_{ab}D^a D^{*b} \\
&- e^K\left((W_i+K_iW)(W^*_j+K_{j^*}W^*)K^{ij^*}-3|W|^2\right) \quad \text{(B.29)}
\end{aligned}$$

where D_μ is a covariant derivative, D^a represents the on-shell value of the vector-multiplet D-term, $K_i = \partial K(\phi,\phi^*)/\partial\phi^i$, and $K_{j^*} = \partial K(\phi,\phi^*)/\partial\phi^{i*}$, while K^{ij^*} is the inverse of the matrix

$$K_{ij^*} = \frac{\partial K}{\partial\phi^i\partial\phi^{*j}} \quad \text{(B.30)}$$

f_{ab} are the gauge-kinetic funtions. They are holomorphic functions of the chiral superfields, ϕ_i and give the inverse of the coupling-constant matrix.

The last line of the lagrangian B.29 is called the F-term scalar potential, while the line directly above it is called the D-term scalar potential, due to their origins as the on-shell vevs of the F and D auxiliary fields. The non-renormalisation theorems quoted above for global supersymmetry continue to hold in supergravities: the superpotential is not (perturbatively) renormalised, FI terms are not renormalised in non-anomolous theories, gauge couplings are only corrected to one-loop, but the Kähler function receives general corrections.

Appendix C

Warped Metrics and Conditions for Flat Branes

Consider an extra-dimensional theory of gravity coupled to some bulk and brane matter:

$$S = \int d^D z \, \frac{1}{2} \widehat{R} + \mathcal{L}, \tag{C.1}$$

where we make no assumptions about the structure of the matter lagrangian: it may contain delta-function sources (branes) as well as bulk contributions. We seek solutions with maximally-symmetric four-dimensional slices (which we hope to make flat), fibred over internal dimensions (which will contain branes at various points). The most general possible metric under these assumptions can be written as

$$ds^2 = \widehat{g}_{MN}(z) \, dz^M dz^N = W^2(y) \, g_{\mu\nu}(x) \, dx^\mu dx^\nu + g_{mn}(y) dy^m dy^n, \tag{C.2}$$

where y are the internal coordinates, x are our usual four dimensions, $g_{\mu\nu}$ is one of dS, AdS or Minkowski space (so that the Ricci curvature computed from $g_{\mu\nu}$ is given by $R_{\mu\nu} = \lambda \, g_{\mu\nu}$ with λ positive, negative or zero, respectively). In a following section we show that the Ricci curvatures calculated from this metric *ansatz* are given by

$$\widehat{R}_{\mu\nu}(x,y) = R_{\mu\nu}(x) - \frac{\nabla^2 W^4(y)}{W^2(y)} \qquad \text{(C.3)}$$

$$\widehat{R}_{mn}(x,y) = R_{mn}(y) - 4\,\frac{\nabla_m \nabla_n W}{W}, \qquad \text{(C.4)}$$

where $\widehat{R}$ is the curvature calculated from the full metric $\widehat{g}_{MN}$, $R_{\mu\nu} = \lambda\, g_{\mu\nu}$ and R_{mn} are, respectively the curvatures calculated solely from $g_{\mu\nu}$ and g_{mn}, and ∇_m is the connection calculated solely from the internal metric g_{mn}.

The Einstein equations arising from eq. C.1 are

$$\widehat{R}_{MN} - \frac{1}{2}\widehat{R}\,\widehat{g}_{MN} + 2\,\tau_{MN} = 0. \qquad \text{(C.5)}$$

where

$$\tau_{MN} \equiv \frac{1}{\sqrt{\widehat{g}}}\frac{\partial \mathcal{L}}{\partial \widehat{g}^{MN}}. \qquad \text{(C.6)}$$

Tracing eq. C.5 to replace $\widehat{R}$ gives

$$\widehat{R}_{MN} + 2\left[\tau_{MN} - \frac{1}{2\,\Delta}\,\tau\,\widehat{g}_{MN}\right] = 0. \qquad \text{(C.7)}$$

where $\tau \equiv \tau_M^M$ and

$$\Delta = D/2 - 1. \qquad \text{(C.8)}$$

Substituting eq. C.3 into the above, simplified Einstein equations gives

$$W^2\, R_{\mu\nu} \equiv W^2(y)\,\lambda\, g_{\mu\nu}(x) = -2\,W^2\left[\tau_{\mu\nu} - \tau\,\frac{W^2}{2\,\Delta}\,g_{\mu\nu}\right] + \frac{1}{4}\nabla^2 W^4. \qquad \text{(C.9)}$$

We will say a brane-world scenario gives a *naturally flat* solution if, given the above assumptions and *ansatze*, $\lambda = 0$ is possible for any choice of brane tensions.[1] A necessary and sufficient condition for $\lambda = 0$ is for

$$\int_{M_2} W^2 \left[\tau_{\mu\nu} - \tau \frac{W^2}{2\,\Delta} g_{\mu\nu}\right] + \frac{1}{4}\int_{M_2} \nabla^2 W^4\, g_{\mu\nu} = 0. \tag{C.10}$$

where the integration is performed only over the internal manifold, M_2, given by the metric $g_{mn}(y)$ (not an integration over y with respect to the measure $\sqrt{\hat{g}}$). If, in addition we assume that the internal manifold is compact and complete, so that

$$\int_{M_2} \nabla^2 W = \sum_i \int_{\partial M_2^{(i)}} n \cdot \nabla W = 0, \tag{C.11}$$

where n is the unit normal to the boundary, $\partial M_2^{(i)}$ [2], we may impose the condition on the matter lagrangian that

$$\int_{M_2} W^2 \left[\tau_{\mu\nu} - \tau \frac{W^2}{2\,\Delta} g_{\mu\nu}\right] = 0. \tag{C.12}$$

We now evaluate eq. C.12 in the simplest brane-worlds.

Randall-Sundrum

We have $D = 5$,

$$\mathcal{L} = \sqrt{\hat{g}}\,\Lambda + \sum_i T_i \sqrt{\hat{g}_{\rm ind}}\,\delta(y - y_i), \tag{C.13}$$

with

$$W = e^{-k|z|}, \tag{C.14}$$

[1]Our assumptions expressly exclude FRW-with-matter-type evolutions of the four-dimensional metric, and therfore general matter distributions on the brane with separate pressures and energy densities. For a recent analysis in which this is question is addressed, see [205]. In this paper the authors claim that self-tuning does not work when more general matter than pure tension is placed on the branes. We will discuss this in our concluding chapter.

[2]We include boundary contributions because, as is familiar from electromagnetism, delta functions can be recast as boundary conditions on infinitesimal circles surrounding the singularity—a recasting of the problem that is often convenient.

so that

$$\tau_{MN} = -\frac{1}{2}\,\widehat{g}_{MN}\,\Lambda - \frac{1}{2}\sum_i T_i\delta(y-y_i)\,\delta^\mu_M\,\delta^\nu_N\,\widehat{g}_{\mu\nu}, \tag{C.15}$$

and

$$\tau = -\frac{D\,\Lambda}{2} - 2\sum_i T_i\delta(y-y_i), \tag{C.16}$$

implying that eq. C.10 becomes

$$\int dy\, W^4\left[\Lambda - \frac{1}{2}\sum T_i\delta(y-y_i)\right] \tag{C.17}$$

Similarly analysing the mn components yields the second equation

$$\int dy\, W\left[\Lambda - 2\sum T_i\delta(y-y_i)\right] \tag{C.18}$$

Together, when $W = e^{-k|z|}$ is used, these equations give the relation between tensions and Λ derived in section 4.6.

C.1 Curvatures for Warped-Product Metrics

In this section we will develop a method for deriving the equations of motion in theories of gravity in which the metric takes the 'warped product' form. In order to solve the equations of motion, we reduce D-dimensional derivatives and curvatures to quantities computed only in terms of the metrics and derivatives of the subspaces, without needing to make detailed *ansatz* for the form of the underlying metrics: we wish to calculate the curvature, $\widehat{R}_{MN}$ of the D-dimensional metric

$$ds^2 = \widehat{g}_{MN}dz^M dz^n = W(y)^2 g_{\mu\nu}(x)dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n \tag{C.19}$$

in terms of covariant derivatives and curvatures constructed only from $g_{\mu\nu}$ and g_{mn}, which we denote by ∇_μ and ∇_m, respectively. The covariant derivative constructed from the full metric, $\widehat{g}$ is denoted by $\widehat{\nabla}_M$, the D-dimensional tanget frames are denoted by upper-case letters, E, and the subspace tangent-frames are given by e. (We only use the $\hat{\alpha}$ notation when confusion is likely to arise, or if both tangent spaces are being used in the same formula.)

The spin connection, Ω, is given by

$$dE^\alpha = \Omega^\alpha{}_\beta \wedge E^\beta, \tag{C.20}$$

and

$$[\nabla_\mu, \nabla_\nu]\, v^\alpha = R_{\mu\nu}{}^{\alpha\beta} v_\beta, \tag{C.21}$$

(refer to appendix A for further details). So, in terms of the spin connection (where underlined $\underline{R}$ is Reimann tensor as a two-form), the curvature is,

$$\underline{R}^{\alpha\beta} = -d\Omega^{\alpha\beta} + \Omega^\alpha{}_\gamma \wedge \Omega^{\gamma\beta}, \tag{C.22}$$

Spheres are positively curved.

Define a tangent frame,

$$\begin{aligned} E^a(y) &\equiv e^a(y) = e^a_m dy^m \\ E^{\hat{\alpha}} &= W(y)e^\alpha(x) = W(y)e^\alpha_\mu dx^\mu. \end{aligned} \tag{C.23}$$

where

$$\begin{aligned} E^\alpha_\mu E_{\alpha\nu} &= g_{\mu\nu} \\ e^a_m e_{an} &= g_{mn} \end{aligned} \tag{C.24}$$

Spin Connection

Clearly

$$dE^a = \omega^a{}_b \wedge E^b \tag{C.25}$$

where ω^{ab} is the spin connection for $e^a{}_m dy^m$, so

$$\Omega^{ab} = \omega^{ab}, \tag{C.26}$$

and is not affected by the presence of the rest of the metric.

$$\begin{aligned} dE^{\hat{\alpha}} &= dW \wedge e^\alpha + W de^\alpha \\ &= -\left(e^\alpha \left(\partial_m W\right) E_a^m\right) \wedge E^a + W\omega^{\alpha\beta} \wedge e_\beta \\ &= -\left(e^\alpha \left(\partial_m W\right) E_a^m\right) \wedge E^a + \omega^{\alpha\beta} \wedge E_\beta \\ &= -\left(e^\alpha v_a\right) \wedge E^a + \omega^{\alpha\beta} \wedge E_{\hat{\beta}}. \end{aligned} \tag{C.27}$$

where

$$v_a \equiv E^m{}_a \, \partial_m W. \tag{C.28}$$

We therefore have

$$\begin{aligned} \Omega^{\hat{\alpha}a} &= -e^\alpha v^a \\ \Omega^{\hat{\alpha}\hat{\beta}} &= \omega^{\alpha\beta} \end{aligned} \tag{C.29}$$

where $\omega^{\alpha\beta}$ is the spin connection for e^α.

Reimann Curvatures

$$\begin{aligned} \underline{\widehat{R}^{ab}} &= -d\Omega^{ab} + \Omega^{aD} \wedge \Omega_D{}^b \\ &= -d\omega^{ab} + \omega^{ad} \wedge \omega_d{}^b + 0 \\ &= \underline{R}^{ab} \end{aligned}$$
(C.30)

where $\underline{R}$ is the Reimann curvature calculated from $g_{mn}(x)$.

Similarly,

$$
\begin{aligned}
\underline{\widehat{R}^{\hat{\alpha}\hat{\beta}}} &= -d\Omega^{\hat{\alpha}\hat{\beta}} + \Omega^{\hat{\alpha}D} \wedge \Omega_D{}^{\hat{\beta}} \\
&= -d\omega^{\alpha\beta} + \omega^{\alpha\delta} \wedge \omega_\delta{}^\beta + (-e^\alpha v_d) \wedge (v^d e^\beta) \\
&= \underline{R}^{\alpha\beta} - v^d v_d\, e^\alpha \wedge e^\beta. \qquad \text{(C.31)}
\end{aligned}
$$

The off-diagonal piece is

$$
\begin{aligned}
\underline{\widehat{R}^{\hat{\alpha}b}} &= -d\Omega^{\hat{\alpha}b} + \Omega^{\hat{\alpha}D} \wedge \Omega_D{}^b \\
&= d(e^\alpha v^b) + \Omega^{\hat{\alpha}\hat{\delta}} \wedge \Omega_d{}^b + \Omega^{\hat{\alpha}d} \wedge \Omega_d{}^b \\
&= -e^\alpha \wedge Dv^b. \qquad \text{(C.32)}
\end{aligned}
$$

where

$$
Dv^b = dy^m \left[\partial_m v^b - \omega_m{}^b{}_g v^g\right] \qquad \text{(C.33)}
$$

is the usual tangent-frame covariant derivative for e^a.

Ricci Curvatures

Now we get the Ricci tensors in the tangent frame. We will use the following representation of the Ricci tensor in tangent indices,

$$
\widehat{R}_{AB} = \underline{\widehat{R}}((E_A)^{-1}, (E_C)^{-1})_B{}^C, \qquad \text{(C.34)}
$$

where

$$
(E_A)^{-1} \equiv E_A^M \partial_M \qquad \text{(C.35)}
$$

is the inverse to the tangent one-form:

$$
E_A((E_B)^{-1}) = \eta_{AB} \qquad \text{(C.36)}
$$

So,

$$
\widehat{R}_{\hat{\alpha}\hat{\beta}} = \underline{\widehat{R}}((E_a)^{-1}, (E_C)^{-1})_b{}^C
$$

$$
\begin{aligned}
&= \underline{\widehat{R}}((E_a)^{-1}, E_g{}^{-1})_b{}^g + \underline{\widehat{R}}(E_a{}^{-1}, E_\delta{}^{-1})_b{}^\delta \\
&= R_{ab} - \frac{1}{W}\,\delta^\delta{}_\delta D_a v_b \\
&= R_{ab} - \frac{n}{W}\, D_a D_b W, \qquad \text{(C.37)}
\end{aligned}
$$

where R_{ab} is the Ricci tensor for the spacetime described by $e^a{}_m dy^m$ alone.

$$
\begin{aligned}
\widehat{R}_{\hat\alpha\hat\beta} &= \underline{\widehat{R}}((E_{\hat\alpha})^{-1}, (E_{\hat\delta})^{-1})_{\hat\beta}{}^{\hat\delta} + \underline{\widehat{R}}((E_{\hat\alpha})^{-1}, (E_d)^{-1})_{\hat\beta}{}^d \\
&= \frac{1}{W^2}\left[R_{\alpha\beta} - v^g v_g \left(\delta_{\alpha\beta}\delta_\delta{}^\delta - \delta_\alpha{}^\delta \delta_{\delta\beta}\right)\right] - \frac{1}{W}\,\eta_{\alpha\beta} D_d v^d \\
&= \frac{1}{W^2}\, R_{\alpha\beta} - \eta_{\alpha\beta}\left[\frac{1}{W}\, D_d D^d W + (n-1)\frac{1}{W^2} D_d W D^d W\right] \\
&= \frac{1}{W^2}\, R_{\alpha\beta} - \eta_{\alpha\beta}\frac{1}{n\, W^n}\, D_d D^d W^n \qquad \text{(C.38)}
\end{aligned}
$$

Here,

$$
\begin{aligned}
D_a v_b &= E^m{}_a D_m v_b \\
&= E^m{}_a \left[\partial_m v_b - \omega_{mb}{}^g v_g\right] \qquad \text{(C.39)}
\end{aligned}
$$

is the covariant derivative constructed from $E^a{}_m$ alone. Transforming from the tangent-frame to world-indices proves eq. 4.32.

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