An Application of Simultaneous Stochastic Optimization in Mining Complexes and Integrating Mine-to-Port Transportation

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Contribution of Authors

The author of this thesis is the primary author for both manuscripts contained herein. Professor Roussos Dimitrakopoulos is the supervisor of the author's M.Eng., and is a co-author for both manuscripts. Professor Jacques Ferland, a collaborator of the COSMO Laboratory is listed as co-author for one of the manuscripts.

Chapter 2 – LaRoche-Boisvert, M. and Dimitrakopoulos, R. An Application of Simultaneous Stochastic Optimisation of a Large Open Pit Gold Mining Complex under Supply Uncertainty (to be submitted)

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Abstract

A mineral value chain or mining complex is an integrated system representing all components of a mining operation for the extraction, transportation and transformation of material, from sources (open pit and underground mines) to customers or the spot market. Simultaneous stochastic optimization aims to optimize all components of a mineral value chain, including extraction schedules for the mines, stockpile management, processing and transportation scheduling, jointly to capitalize on the synergies that exist within the system. Additionally, the simultaneous stochastic optimization approach incorporates material supply or geological uncertainty using equally probable geostatistical (stochastic) simulations of the attributes of interest of the deposits. The incorporation of material supply uncertainty allows the approach to manage the related major technical risks. In this thesis, a study of simultaneous stochastic optimization is completed on a 3-mine open pit gold mining complex focusing on material hardness management and its effects on the processing facilities. A mathematical formulation jointly optimizing extraction and mine-to-port transportation is also presented. Mine-to-port transportation is an important aspect of certain mining complexes, such as iron ore complexes, that has not been included in previous simultaneous stochastic optimization formulations.

The first contribution of this thesis is the application of simultaneous stochastic optimization at a three-mine open pit gold mining complex, incorporating material supply uncertainty using stochastic simulations of the gold grades of each deposit. The case study maximizes the net present value of the operation by generating life-of-mine schedules for each deposit considered and stockpile management plans, which maximize gold production and minimize the associated costs. The study also assesses the impacts of material hardness on the processing facilities, notably the SAG mill, and the recovered gold. This assessment indicates that the SAG mill is the bottleneck of the operation; due to the lack of availability of soft material in the considered deposits, the throughput of material at the SAG mill is significantly lowered.

The second contribution of this thesis is a new stochastic mathematical programming formulation jointly optimizing long-term extraction scheduling and mine-to-port transportation scheduling for mining complexes under supply uncertainty. Mine-to-port transportation systems represent an important component of certain mining complexes, such as iron ore mining complexes, ensuring that extracted products reach their intended clients. This component of the mineral value chain has not been included in previous simultaneous stochastic optimization formulations, ignoring the interactions between the transportation system and the other components of the mining complex. The proposed model simultaneously optimizes extraction scheduling, stockpile management, mine-to-port transportation scheduling and blending under material supply uncertainty. It aims to minimize the costs associated with meeting quantity and quality demand for the products at the port, managing the risks associated with the material supply uncertainty using stochastic simulations of grades. The model is applied to an iron ore mining complex consisting of two open pit mines, each with a waste dump, a stockpile and a loading area, connected to a single port by a railway system. Material is transported by two trains. At the port, demand for two products are considered, each with quality constraints relating to five elements. Stochastic simulations of the five elements considered are used to represent the material supply uncertainty. By optimizing the extraction and the mine-to-port transportation jointly, the case study is able to determine that only the first train is necessary to transport material to meet demand at the port for the first three years of mine life; for the remainder, the second train is also needed. As such, the second train could be allocated to another operation for better use during the first three years of operation or its purchase could be delayed. The model provides decision makers with a realistic use of the mine-to-port transportation system.

Résumé

Une chaine de valeur minière ou un complexe minier est un système intégré représentant toutes les composantes d'une opération minière pour l'extraction, le transport et la transformation de matériel, des sources (mines à ciel-ouvert et souterraines) aux clients et marché au comptant. L'optimisation simultanée stochastique vise à optimiser conjointement toutes les composantes d'une chaine de valeur minière, telles que la séquence d'extraction des mines, le stockage et traitement de minerai et le transport des matériaux, afin de prendre avantage des synergies qui existent au sein du système. De plus, l'optimisation simultanée stochastique incorpore l'incertitude des matériaux (incertitude géologique) en utilisant des simulations geostatistiques (stochastiques) équiprobables des attributs d'intérêt des gisements. L'incorporation de l'incertitude des matériaux permet à l'approche de gérer les risques majeurs qui y sont liés. Dans cette thèse, une étude de cas est complétée, appliquant l'optimisation simultanée stochastique à un complexe minier d'or à trois mines. Cette étude se concentre sur la gestion de la dureté des matériaux et son effet sur le fonctionnement des installations de traitement de minerai. Une nouvelle formulation mathématique optimisant conjointement les séquences d'extraction des mines et le transport des mines au port est aussi présentée. Le transport des mines au port est un aspect important de certains complexes miniers, tels que les complexes de fer, qui n'est pas inclut dans les précédentes approches d'optimisation simultanée stochastique.

La première contribution de cette thèse est l'application de l'optimisation simultanée stochastique à un large complexe minier d'or, incorporant l'incertitude des matériaux grâce aux simulations stochastiques des teneurs d'or des gisements. L'étude de cas maximise la valeur actuelle nette de l'opération en produisant les séquences d'extraction de chaque mine considérée ainsi qu'un plan de gestion du stockage de minerai; ceux-ci maximisent la production d'or et minimisent les coûts associés. L'étude évalue aussi l'effet de la dureté des matériaux au sein des installations de traitement de minerai, notamment le broyeur SAG, et l'or récupéré. Cette évaluation indique que le broyeur SAG est le bouchon de l'opération; grâce au manque de disponibilité de matériel souple, le débit de matériel au broyeur est réduit.

La deuxième contribution de cette thèse est une nouvelle formulation mathématique optimisant les séquences d'extraction à long-terme et le transport des mines au port des complexes miniers conjointement, sous incertitude géologique. Les systèmes de transport des mines au port représentent une composante importante de certains complexes miniers, tels que les complexes de fer, assurant que les produits extraient se rendent à leurs destinations. Cette composante n'est pas incluse dans les approches d'optimisation simultanée stochastique précédentes, ignorant les interactions entre cette celle-ci et les autres composantes du système. Le modèle proposé optimise les séquences d'extraction, la gestion du stockage de minerai, le transport des mines au port et le mélange de minerai sous incertitude géologique. Il vise à minimiser les coûts encourus afin de satisfaire la demande (quantité et qualité) des produits au port, gérant les risques associés à l'incertitude géologique en utilisant des simulations stochastiques des teneurs de minéraux. Le modèle est appliqué à un complexe minier de fer composé de deux mines, qui ont chacune une halde à stériles, une aire de stockage et une zone de chargement, connecté au port via un système de rails. Le matériel extrait est transporté par deux trains. Au port, la demande pour deux produits est considérée, avec des contraintes de qualité pour chacun des cinq minéraux considérés. Des simulations stochastiques des cinq minéraux sont utilisées afin de représenter l'incertitude géologique. En optimisant l'extraction et le transport des mines au port ensemble, l'étude de cas révèle qu'un seul des deux trains est nécessaire pour transporter le matériel requis pour satisfaire la demande au port pendant les premières trois années d'opérations; pour le reste, le deuxième train est nécessaire. Donc, le deuxième train pourrait être alloué à une autre opération pendant ces trois premières années, ou son achat pourrait être retardé. Le modèle fournit aux décideurs l'utilisation réelle du système de transport des mines au port.

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1. Introduction and Literature Review

1.1. Overview

A mineral value chain, or mining complex, describes a mining operation, from the extraction of raw materials to the sale of final products (Pimentel et al, 2010; Goodfellow and Dimitrakopoulos, 2016). Figure 1 presents a hypothetical depiction of a mineral value chain including multiple mines, stockpiles, processing streams, waste dumps, tailings facilities and sales options. Traditional approaches to life-of-mine planning (or strategic mine planning) consider each component of a mining complex individually with the aim of maximizing the operation's net present value (NPV). Early linear and mixed integer programming approaches to life-of-mine planning required a separate optimization of the components of a mineral value chain due to the lack of computing power available to solve such large problems (Johnson, 1968; Kim, 1968, 1978; Barbaro and Ramani, 1986; Tan and Ramani, 1992; Kim and Zhao, 1994). This optimization by silos fails to capitalize on the synergies that exist within the mining complex, preventing it from obtaining truly optimal results. In addition, the components of a mineral value chain represented in this optimization approach are often simplified to maintain tractability, further misrepresenting results. For example, non-linear components such as stockpiles are often represented by simplified linear approximations or are omitted altogether. As a mineral value chain increases in size and complexity, the loss of value due to the combination of stepwise optimization and simplifying components also increases (Montiel and Dimitrakopoulos, 2015, 2017, 2018; Goodfellow and Dimitrakopoulos, 2016, 2017). As such, a global or simultaneous optimization method which optimizes all components of a mining complex jointly and includes non-linear elements is required.

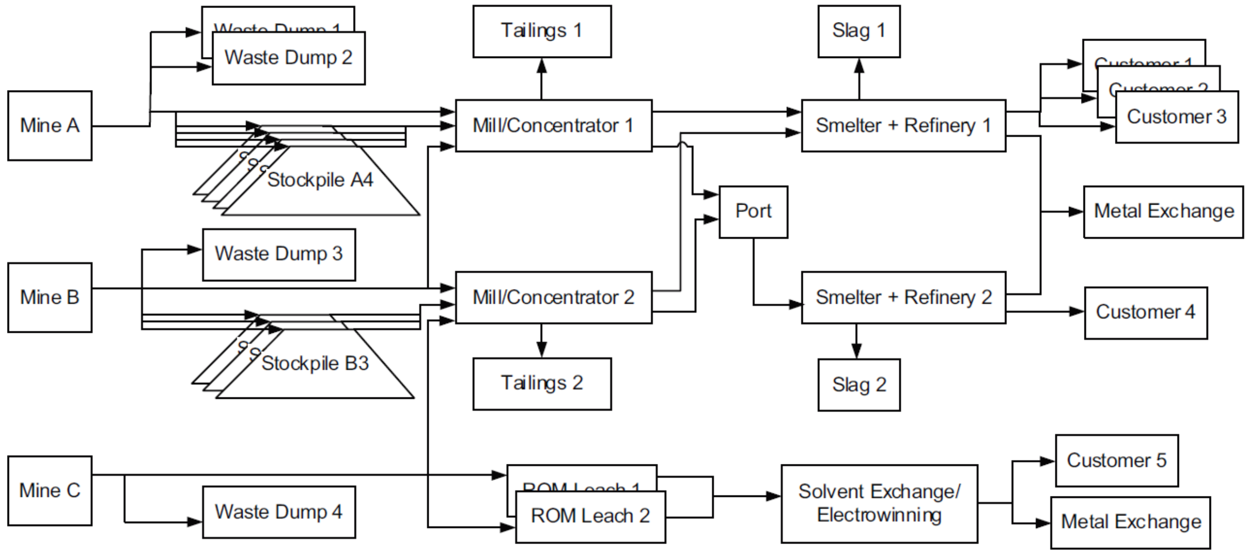


Figure 1 An example of a mining complex with multiple mines, stockpiles, processing streams, and sales options (Goodfellow, 2014)

Traditional and global optimization methods are also deterministic, ignoring various sources of uncertainty present within a mineral value chain. Most notably, geological (material supply) uncertainty, which has been shown to be the leading cause of mining project failure (Baker and Giacomo, 1998; Vallée, 2000), is not considered. Material supply uncertainty refers to a deposit's grade and material type uncertainty. Baker and Giacomo (1998) analyze 48 Australasian mining projects and show that 27% underestimated their reserves by 20% and 19% overestimated their reserves by 20%. Vallée (2000) shows that over 70% of studied mining projects were closed prematurely due to erroneous mineral reserves estimates. Indeed, major differences were noted between the published ore reserve estimates and those realized in the first years of production.

Deterministic optimization methods use an estimated orebody model, generated using geostatistical methods such as kriging (David, 1977; Journel and Huijbregts, 1978; Goovaerts, 1997), to describe a mineral deposit. However, estimated orebody models are a smoothed representation of the deposit. By assigning a value to a point based on the average of the surrounding data, estimation methods cannot reproduce the geostatistical characteristics, such as the local variability, of the data on which they are based (Rossi and Deutsch, 2014). The risks associated with the use of estimated orebody models for mine optimization have been well studied (Ravenscroft, 1992; Dowd, 1994, 1997). To address these risks, orebody (or geological) geostatistical simulations have been developed to be used in lieu of estimated models. These simulations reproduce the statistical characteristics of the data, including spatial patterns and local variability, and are used as a group to quantify the material's uncertainty (Goovaerts, 1997). Geological simulations have been used to assess the risks associated with traditionally developed production schedules and have been used as a direct input into stochastic optimization methods, which actively manage the risk associated with an uncertain material supply (Dimitrakopoulos et al, 2002; Godoy, 2003; Dimitrakopoulos, 2011; Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016). Stochastic optimization methods, and therefore the inclusion of uncertainty in the mine optimization process, have been shown to increase the value of a mining project and reduce its associated risks (Montiel and Dimitrakopoulos, 2015, 2016; Goodfellow and Dimitrakopoulos, 2016, 2017).

1.2. Deterministic Approaches to Mine Optimization

As noted in the previous section, deterministic long-term optimization methods use a single, estimated representation of each uncertain component of a mineral value chain; these representations are unlikely to accurately characterize the true, unknown reality of the components they are representing. This section presents an overview of deterministic optimization methods for long-term mine planning, with a focus on opportunities for improvement. First, stepwise optimization approaches are presented, followed by global optimization approaches.

1.2.1. Stepwise Optimization

Stepwise optimization approaches to long-term production scheduling separate the problem into smaller partitions which are solved sequentially to ensure that each partition can be solved to optimality within a reasonable amount of time. A common approach to stepwise optimization begins by defining an optimal cut-off grade policy (Lane, 1964, 1988; Whittle 1988, 1999; Rendu, 2014), which determines each mining block's destination (e.g. processing facilities, waste dumps or stockpiles) once extracted, given its estimated grade. A block's economic value is defined as the value of its material, based on the estimated grade, at its assigned destination. These economic values are then used to determine the deposit's ultimate pit limit (UPL), often using the Lerchs-Grossman algorithm (Lerchs and Grossman, 1965; Whittle, 1988, 1999). Then, pushbacks are delineated within the UPL, grouping blocks that can be mined in a single continuous operation to obtain a practical and logical mining sequence (Hustrulid et al, 2013). Finally, an extraction sequence is determined for the blocks within the pushbacks (Dagdelen, 2001). Because the cut-off grade policy and production schedules are dependent on one another, a recursive approach to the previously described steps is often necessary to obtain a good final solution, as shown in Figure 2.

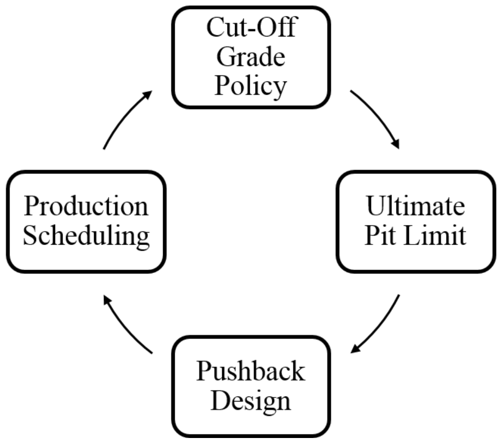


Figure 2 Traditional production scheduling procedure

Stepwise approaches to production scheduling aim to maximize the value of the operation (i.e. metal produced) while respecting resource and technical constraints. However, omitting the interactions between the different components leads to suboptimal results. For example, this approach does not account for the blending of material from different sources (i.e. mines) at the processing facilities since each source is optimized on its own. Methods which optimize all components of a mineral value chain and capitalize on the synergies that exist are therefore required in order to redeem the loss of value resulting from stepwise optimization. Such methods are presented in the following section.

1.2.2. Global Optimization

Efforts to optimize multiple components of a mining complex simultaneously began in the 1990s when Newmont commissioned Urbaez and Dagdelen (1999) to develop a mixed-integer programming (MIP) model for the long-term scheduling of a multi-mine, multi-processing stream operation. The method was successful for small-scale applications; however, changes were required for realistically sized problems. Hoerger et al. (1999a, 1999b) expanded on this work, developing an MIP that schedules pushbacks and models the flow of 50 materials types through a complex mineral value chain including 8 stockpiles and 60 processing streams. This method improved the project's value compared to the traditional stepwise approach, however there are significant drawbacks: it relies on predetermined extraction sequences to describe precedence relationships and it schedules aggregates of blocks (pushbacks) rather than the blocks directly.

BHP Billiton developed Blasor, a strategic mine optimization tool using an MIP formulation to determine ultimate pit limits and extraction sequences for multiple pits (Stone et al, 2007). To reduce the number of integer variables required for this optimization, Blasor aggregates spatially connected blocks with similar material attributes. Rather than generating an extraction sequence from pushbacks and a defined ultimate pit limit, Blasor uses the aggregates to generate the optimal ultimate pits and extraction sequences before generating pushback designs. The pushbacks are then separated into panels, defined as the intersection between a pushback and a bench, which are sequenced in the same manner as the aggregates. Blasor optimizes destination policy decisions simultaneously with the extraction sequence decisions, eliminating the need for an a priori classification of ore and waste blocks and allowing the optimizer to consider the blend of material sent to different processing streams. Stone et al. (2007) applied Blasor to BHP's Yandi iron ore operation, jointly optimizing 11 pits to meet the blending requirements at the processing streams and maximize the NPV. Blasor was later expanded by Zuckerberg et al. (2007) to include waste handling constraints. This expanded model, BlasorIPD, determines which mined-out areas of the pit should be re-filled with waste material by tracking which areas have been depleted. Li et al. (2016) also presents a model for the optimization of the waste material dumping; however, it requires a fixed extraction schedule. Fu et al. (2019) present a mixed-integer programming model to optimize the extraction and waste-dump schedules for a mine. However, the model allows the fractional extraction of blocks, leading to potential slope constraint violations.

Whittle (2007) discusses the need for the joint optimization of mineral value chain components and notes that including multiple components significantly increases the difficulty of solving the problem. Whittle (2010) presents a global optimization framework, Prober C, which aims to solve the previously mentioned joint long-term optimization problem. Though it considers a mining complex as a whole, it optimizes portions of the mineral value chain sequentially. Blocks are aggregated a priori into panels based on geological attributes to reduce the size of the optimization formulation. Pushbacks and ultimate pit limits are also pre-determined in order to reduce the size of the problem. The panels are scheduled jointly with underground sequences to ensure blending and processing constraints are respected. However, the method allows panels to be partially mined, which reduces the difficulty of the problem but can also lead to infeasible results. Mining blocks, or selective mining units, are the smallest unit which can be selectively mined. Therefore, the partial mining of aggregates can lead to the partial mining of blocks, which is not operationally feasible. Additionally, Prober only allows a single panel to be mined at a time in order to ensure that slope constraints are respected, which is an overly conservative approach. Finally, Prober does not guarantee an optimal solution; it generates a set of randomly feasible extraction sequences and optimizes the downstream components of the operation such as transportation, processing and blending (Whittle, 2010, 2014). Though it has many limitations, Prober represents a commercial software that considers different components of a mining complex to a greater extent than its competitors.

The long-term global optimization methods presented in this section improve upon stepwise optimization, however they nonetheless present significant limitations. First, certain components of a mineral value chain, such as transportation systems, are not integrated into the optimization process, and non-linear components such as stockpiles require a linear simplification. Second, the methods use block aggregates for scheduling to reduce the size of the formulation rather than scheduling blocks directly. Finally, sources of uncertainty, and specifically material supply uncertainty, are not incorporated; therefore, the solutions do not account for alternative scenarios. The exclusion or misrepresentation of material supply uncertainty, or uncertainty in a deposit's pertinent attributes such as grades, material types and so on, is the leading cause of mining project failure (Vallée, 2000). Therefore, material supply uncertainty must be included in long-term mine optimization. Methods incorporating uncertainty are presented in the following section.

1.3. Simultaneous Stochastic Optimization of Mining Complexes

This section highlights the improvements in incorporating sources of uncertainty, notably material supply uncertainty, into the strategic mine optimization process using equally probably simulations of the deposit attributes of interest. Methods used to generate stochastic orebody simulations representing material supply uncertainty are presented in Section 1.5. Additionally, this section highlights improvements in the representation of non-linear mineral value chain components such as stockpiles and their integration into the optimization process.

1.3.1. Integration of Uncertainty

Early attempts to incorporate uncertainty into long-term mine optimization focused on adapting existing technologies and methods. Dimitrakopoulos et al. (2007) consider a maximum upside/minimum downside approach to mine planning under uncertainty where an extraction schedule is developed for each orebody simulation in a set using Whittle 4X software. Each schedule is then subject to material supply uncertainty by testing it against the remaining simulations in the set. Key performance indicators (KPI) are used to evaluate the performance of the schedules when subjected to material supply uncertainty; the schedule which maximizes the upside potential for the KPIs is chosen for the operation. This method evaluates the risk associated with a schedule; however, it does not actively manage it nor does it generate an optimal life-of-mine plan. Instead it only determines which of the produced schedules performs best under uncertainty.

Godoy and Dimitrakopoulos (2004) propose using traditional optimization methods to determine a stable solution domain (SSD) for each geological scenario and determine their best- and worst-case ore and waste mining rates; once graphed, the SSD is defined as the area common to all scenarios. A mathematical programming formulation is then used to determine the optimal extraction rates within the SSD, and a production schedule is generated for each scenario according to these targets. The schedules are then combined and modified using simulated annealing (SA) in order to minimize the deviations from the targeted extraction rates as well as from the targeted processing capacity. One case study application of the method resulted in a 28% improvement in NPV and a 9% reduction in deviations from targets when compared to the traditional schedule, and another case study yielded similar results (Leite and Dimitrakopoulos, 2007), validating the approach's efficacy. The method was then subject to a sensitivity analysis in which the initial solution, the number of mining sequences used, and the extension of the ultimate pit limits are evaluated (Albor and Dimitrakopoulos, 2009). It was determined that the sequence which exhibits the least amount of risk and highest NPV when tested with other geological simulations should be used as the starting sequence for the SA to obtain the best results. As for the number of sequences required to obtain a stable solution during the SA process, it was determined that 10 sequences are sufficient. Finally, the ultimate pit limits generated by traditional schedulers were determined to be significantly smaller than those produced when the SA algorithm is free to continue mining until cashflows are no longer positive. Del Castillo et al. (2015) then built on the previous formulation by proposing a mechanism which optimizes the mining rates by directly optimizing the mining fleet, producing an equipment production schedule. The equipment's purchase and underutilization costs are incorporated into the mining rate optimization process. A case study application resulted in a 40% reduction in equipment purchases.

Dimitrakopoulos and Ramazan (2004) move away from the modified use of traditional optimizers and develop a probabilistic approach to the long-term mine optimization problem. Every block is assigned a probability of having relevant attributes above the cut-off grade or within a specific range, according to a set of orebody simulations. The approach then aims to maximize the probability of meeting production and quality targets and introduces smoothing constraints to ensure the resulting schedule is operationally feasible. In order to maximize the probability of meeting targets, the authors apply penalty costs to deviations from said targets. These costs are subject to a geological discount rate (GDR), which reduces the magnitude of the penalty costs over time, ensuring that the targets are well met in earlier periods and deferring the risks of missing targets to later periods. To produce a mineable schedule, the authors ensure that blocks are mined, as much as possible, at the same time as their neighbouring blocks. This allows the schedule to delineate larger areas to be mined in the same period. Though it does not directly account for material supply uncertainty, a case study application of the approach led to 6% reduction in risks associated with meeting production and quality targets while maintaining a mineable schedule.

Groeneveld and Topal (2011) then develop a discrete event simulation approach to optimize capital expenditure decisions at mining complexes under financial and equipment uncertainty. Each scenario is optimized individually, and the results are analyzed to determine which capital expenditure options were chosen in which periods by the most scenarios. The most popular options are then combined into an "optimal" solution; however, this solution fails to consider the impact the production schedule will have on these decisions and vice versa. Groeneveld et al. (2012) then improve the model by fixing the earlier periods to ensure the operational feasibility of the solution while allowing later periods to adapt to uncertainty. Jélvez et al. (2019) present a methodology to incorporate uncertainty in the determination of ultimate pit limits, pushback selection and production scheduling. It is shown that incorporating uncertainty at these different steps reduces the risks of not meeting production targets and that 56% of the value added comes from the direct incorporation of uncertainty in the production scheduling. Ajak et al. (2018) present a predictive data mining algorithm to create real options at the mine operations level and manage clay uncertainty. The algorithm was applied at a case study to determine the probability of encountering problematic ore blocks (those with clay content) during extraction and generate real options to manage them. The algorithm predicted the occurrence of problematic ore blocks with 78% precision.

Goodfellow and Dimitrakopoulos (2013) present two methods to modify, using simulated annealing, an existing pushback design to account for grade and material type uncertainty. The first formulation aims to generate pushbacks such that the absolute deviations from target tonnages at multiple destinations are minimized in all uncertainty scenarios. The second formulation aims to minimize the square of the deviations from the tonnage targets in all scenarios. When applied to a case study, both methods were shown to improve upon the initial pushback design, reducing the risks of not meeting tonnage targets at the different destinations. However, the second formulation was shown to outperform the first. Farmer and Dimitrakopoulos (2018) propose a method for schedule-based pushback design. The approach designs pushbacks from a block-based schedule generated using stochastic integer programming, ensuring mineable shapes.

In addition to the integration of uncertainty in the long-term optimization of open pit mines, advancements have been made to integrate uncertainty when optimizing underground and transitioning mines. Opoku and Musingwini (2013) present a stochastic approach to analyze the transition from open pit to underground under geological uncertainty. Uncertain transition indicators, such as the stripping ratio and NPV, are used to determine whether a mine should be mined entirely open pit, begin as an open pit before transitioning to underground, or be mined entirely underground. The method is applied to four mines, determining whether they should transition from open pit to underground, and under what conditions. MacNeil and Dimitrakopoulos (2018) present a stochastic integer program, a method which will be discussed in further detail in the next section, to determine the depth at which the open pit to underground transition should be made while considering material supply uncertainty. The method considers a set of candidate depths based on the geotechnical constraints of the crown pillar. For each candidate depth, the open pit and underground portions of the mine are optimized. The value of each portion is then combined to determine the value of the mine with the proposed transition depth; the transition depth with the highest value is then chosen. Villalba Matamoros and Kumral (2019) present a stope layout optimization method under material supply uncertainty, aiming to maximize profits and minimize internal dilution. For each orebody simulation, the method aggregates blocks into stopes, penalizing the inclusion of waste blocks. The stope layouts are combined to create an average stope layout design. This average layout is then used as the initial solution for a genetic algorithm to determine the near-optimal stope layout. Bouffard and Boggis (2019) develop the Simplified Linear Integrated Capacity Estimate (SLICE) model, a discrete event simulation model, to estimate the production capacity of an underground mining operation under different uncertainty conditions. The method identified the bottleneck of the operation in each event. Benndorf (2020) suggests a closed-loop approach to mine planning, updating geostatistical simulations using real-time data, revealing the uncertainty of the deposit, throughout the life of the operation. These updated simulations are then used as an input in the mine planning process, allowing decisions to be adjusted according to new information and ensuring that production targets are met.

Though the approaches presented in this section incorporate sources of uncertainty in the mine optimization process, they present several limitations, such as the individual optimization of certain components of a mining complex. The following sections present methods which attempt to overcome these limitations.

1.3.2. Stochastic Integer Programming

Stochastic integer programming (SIP) methods incorporate uncertainty in the long-term optimization of mines through the direct use of simulated scenarios, maximizing the value of the mine and explicitly managing risks (Birge and Louveaux, 2011). Two-stage SIPs consist of first-stage scenario-independent decisions and second-stage scenario-dependent decisions; in the mining context, first-stage decisions often include block extraction decisions while second-stage decisions represent those decisions taken to manage the risks associated with meeting production and quality targets (Ramazan and Dimitrakopoulos, 2007, 2013; Dimitrakopoulos and Ramazan, 2008; Dimitrakopoulos, 2011; Benndorf and Dimitrakopoulos, 2013).

Ramazan and Dimitrakopoulos (2007) present a direct-block scheduling SIP aiming to maximize the NPV of a mine as well as the quality of delivered material while minimizing deviations from ore tonnage, grade and metal quantity and quality targets. The model is applied to a two-dimensional deposit, allowing the problem to be solved with a CPLEX, a commercial software, and a sensitivity analysis is conducted to determine appropriate penalty costs associated with deviations from targets over time. Dimitrakopoulos and Ramazan (2008) then apply the model to two case studies. The cases are solved using two time horizons to maintain a reasonable solution time. The method highlights the value of the stochastic solution: the first case exhibits a 10% increase in NPV while the second case exhibits a 25% increase in NPV when compared to the base cases, generated using an estimated orebody model. Leite and Dimitrakopoulos (2014) then combine the model with a probability cut-off to differentiate ore and waste material. This classification is used strategically to reduce the number of integer variables: ore block extraction decisions are binary while waste block extraction decisions are relaxed. Though this reduction in integer variables reduces the problem's difficulty, allowing the partial mining of blocks is infeasible and the pre-classification of material leads to sub-optimal results. The model is applied to a copper deposit and resulted in a significant increase in NPV and reduction in the risks associated with meeting targets when compared to a traditional schedule. Ramazan and Dimitrakopoulos (2013) further extend the model to include a linear representation of stockpiles. It is applied to the same deposit as in Dimitrakopoulos and Ramazan (2008) and resulted in a 10% increase in NPV. Benndorf and Dimitrakopoulos (2013) also extend the model to allow for multivariate deposits and include smoothing constraints, as introduced in Dimitrakopoulos and Ramazan (2004). The extended model is applied to a multivariate iron ore deposit, managing the blending of five elements: iron, silica, alumina, phosphorus and loss on ignition. A sensitivity analysis is conducted to determine the effect of deviation penalty costs on the quality of the resulting schedule and blending. It was found that an increase in penalty cost decreases the magnitude of deviations from blending constraints; however, there is a penalty cost at which point the reduction in deviations for the blending constraints plateaus while having an adverse effect on the smoothness of the resulting extraction schedule. This plateau is more prominent for the silica than the other elements considered and indicates that the deposit cannot meet the targets and blending with material from other sources is required. Rimelé et al. (2018) present a similar model, incorporating in-pit dumping of waste materials. The model is applied to a multivariate iron deposit and is solved in two time horizons. The resulting schedule reduces the operation's environmental impact while meeting its production and quality targets.

Menabde et al. (2007) present a mixed integer program for long-term mine scheduling and cut-off grade optimization under an uncertain material supply. The model does not directly manage uncertainty using of recourse variables; indeed, it uses the average value of the simulations to ensure production targets are respected. The cut-off grades are optimized for each production period for the single element and destination considered, effectively distinguishing ore and waste material. Asad and Dimitrakopoulos (2013) extend Lane's method for cut-off grade optimization to include multiple processing facilities and material supply uncertainty. The method generates a unique cut-off grade policy while maximizing NPV. Khan and Asad (2019) present a two-stage SIP formulation for the joint optimization of extraction scheduling and cut-off grades under material supply uncertainty. The model maximizes NPV while minimizing deviations from production targets and generates an optimal cut-off grade policy. When applied to a case study, the method generated relatively higher cut-off grades than those generated using Lane's approach. In addition, the method capitalized on the uncertainty, resulting in a higher NPV. Githiria and Musingwini (2019) extend Lane's method for cut-off grade optimization to include both material supply and economic uncertainty; this cut-off grade policy is then used to guide extraction schedule optimization for a mineral deposit. A case study application of the method yielded an increase in NPV when compared to other cut-off grade optimization models.

Kumral (2011) presents an SIP aiming to minimize the extraction costs and maximize the material's expected value while incorporating geological uncertainty as well as uncertain recovery rates and mining and processing costs. The block extraction decisions are scenario-independent; however, the block destination decisions are based on scenario-dependent ore/waste classifications, assuming all uncertainty is revealed once the block is extracted. The model does not allow the solution to deviate from production and quality targets; therefore, the model may fail to yield a feasible solution if the deposit's material does not have the desired characteristics. The model is applied to a case study, resulting in a reduction in NPV when compared to the expected model. However, only four geological simulations are used to quantify the material's uncertainty, which has been shown to be insufficient to produce a stable solution (Albor and Dimitrakopoulos, 2009). Kumral (2013) then proposes a SIP which aims to maximize the value of the operation while minimizing deviations from targets, improving upon the previous model. The model includes a constraint which mandates that the NPV be greater than a pre-determined minimum expected NPV, which can lead the optimization process to sacrifice the meeting of targets to increase the NPV instead of finding the true optimal solution.

Moreno et al. (2017a) propose a scalable stochastic model under geological uncertainty. However, the model allows fractional extraction decisions, which is infeasible in the case where blocks are scheduled. Navarra et al. (2018a) present a SIP relating the mineralogical features of extracted material to plant performance, incorporating geometallurgical models into long-term mine planning. The model aims to maximize the NPV of the mine but does not explicitly minimize risks. Navarra et al. (2018b) present a model to generate a mining schedule which will strictly respect processing constraints while considering material supply uncertainty. The model penalizes excess ore sent to the processing facility in order to improve ore selectivity. Mai et al. (2019) present a SIP method to optimize extraction scheduling at mines under geological uncertainty. The method uses the TopCone algorithm to aggregate blocks (Mai et al, 2018), reducing the size of the problem, and aims to maximize the expected economic value of the project by using the expected grades of the blocks. The method is applied to a case study at a multivariate iron ore deposit, increasing the value of the operation when compared to a traditional scheduler. Morales et al. (2019) present a direct block scheduling approach for mine and mill optimization incorporating grade, recovery and mill throughput uncertainty. The method is applied to a copper deposit, increasing the operation's value by 9.4% compared to traditional scheduling; however, manual smoothing of the schedule is required in order to ensure mineability.

Sepúlveda et al. (2020) present a model for the life-of-mine planning of polymetallic deposits under geological and economic uncertainty. The method adds a "pre-classification" stage after the material is extracted in order to determine whether it should be sent to a processor, stockpile or waste dump, according to a dynamic cut-off grade policy. It also considers potential changes in mining, processing, or refining capacities throughout the life of mine. A genetic algorithm is used to solve the model, aiming to maximize NPV and minimize the risks of not meeting production targets. The model is applied to a case study at a multivariate deposit; however, blocks are aggregated and separated into phases before the optimization process begins, leading to suboptimal results. Chatterjee and Dimitrakopoulos (2020) propose a method to schedule a mine under geological uncertainty by solving sequential sub-problems using the rolling time horizon approach. A combination of the minimum cut algorithm, Lagrangian relaxation and branch-and-cut algorithm is used to solve the sub-problems. When applied to a case study, the method yielded a 26% increase in NPV and 12% increase in metal production when compared to the schedule generated using a conventional method.

Zhang and Dimitrakopoulos (2017) present a "dynamic-material-value-based decomposition method" which iteratively optimizes a mineral value chain under market uncertainty, ensuring that the operation's profitability is not overestimated. The optimization problem is separated into two sections: mine production scheduling in which the extraction schedule is determined, and material flow planning in which the downstream (processing) decisions are determined. These are solved iteratively until convergence, allowing the model to consider the interactions between the components of the mining complex. Zhang and Dimitrakopoulos (2018) then propose a nonlinear SIP incorporating dynamic recovery rates and forward contracts. The approach focuses on optimizing the downstream decisions of a mining complex, using a fixed mine production schedule as an input.

Boland et al. (2008) propose a multistage SIP model which allows the schedule to adapt to uncertainty as it is revealed rather than make fixed decisions as is the case with the previously discussed approaches. Using a set of geological simulations as an input, decisions such as block extraction can differ between scenarios if the scenarios exhibit significant differences; otherwise, non-anticipativity constraints ensure decisions are the same for similar scenarios. Allowing scenario-dependent extraction decisions leads to the creation of one overfitted schedule for each scenario. However, strategic mine planning does not have the flexibility to change schedules as uncertainty reveals itself and therefore requires a single course of action to be determined in advance. Additionally, the approach requires block aggregation techniques to reduce the size of the problem, leading to the partial mining of blocks and slope constraint violations.

1.3.3. Metaheuristic Solution Approaches

Mine optimization practices have been improved significantly with the advent of SIPs, maximizing an operation's NPV and minimizing the risks associated with meeting production and quality targets (Dimitrakopoulos, 2011). However, SIPs are solved using CPLEX (Menabde et al, 2007;

Ramazan and Dimitrakopoulos, 2007; Dimitrakopoulos and Ramazan, 2008; Kumral, 2011), a commercially available software, and are limited in terms of the size of the problem that can be solved within a reasonable amount of time. Metaheuristic solution methods have been developed and applied to mine optimization problems in order to find good solutions for large SIPs. Metropolis et al. (1953) developed the simulated annealing (SA) algorithm based on the metallurgical cooling process of the same name. The algorithm perturbs a solution until a stopping criterion is met and a perturbed solution improving the objective function value is always accepted. Perturbed solutions which worsen the objective function are accepted based on an evaluation of the magnitude of the difference between the perturbed and unperturbed objective function values and the annealing temperature (Kirkpatrick et al, 1983). Non-improving perturbations are accepted in order to escape local optima and attempt to find the global optimum. The SA algorithm was first applied to mine optimization problems by Godoy and Dimitrakopoulos (2004) and has been used in multiple subsequent studies (Godoy, 2003; Leite and Dimitrakopoulos, 2007; Albor and Dimitrakopoulos, 2009; Kumral, 2013; Montiel and Dimitrakopoulos, 2013; Goodfellow and Dimitrakopoulos, 2015).

Lamghari and Dimitrakopoulos (2012) employ the Tabu search (TS) algorithm to the mine optimization problem, applying it to a two-stage mine optimization SIP under geological uncertainty. It was shown to produce solutions with values within 4% of that generated by CPLEX 12.2, a commercial solver, within a significantly shorter time. Senécal and Dimitrakopoulos (2020) parallelize the approach, reducing the solution time proportionally to the number of threads used. Brika (2019) uses the TS algorithm in conjunction with the Bienstock-Zuckerberg algorithm (Bienstock and Zuckerberg, 2009) and a rounding heuristic, yielding solutions within 3.5% of the upper bound limit. Additional metaheuristic methods such as the variable neighbourhood descent (Lamghari and Dimitrakopoulos, 2014; Lamghari et al, 2014, 2015), progressive hedging (Lamghari and Dimitrakopoulos, 2016b), network-flow algorithms (Lamghari and Dimitrakopoulos, 2016a), ant-colony colony optimization (Gilani and Sattarvand, 2016), genetic algorithms (Paithankar and Chatterjee, 2019) etc. have been applied to the mine optimization problem under uncertainty. Additionally, hyper-heuristic methods have been developed for mine optimization, choosing or generating the heuristics used to solve large-size formulations (Lamghari and Dimitrakopoulos, 2018).

1.3.4. Simultaneous Stochastic Optimization

Simultaneous stochastic optimization (SSO) frameworks extend the SIP approaches and combines them with metaheuristic solution methods, allowing for the joint the long-term optimization of various components of a mineral value chain rather than a single mine (Montiel and Dimitrakopoulos, 2015, 2017; Goodfellow and Dimitrakopoulos, 2016, 2017). This holistic approach to mine optimization moves away from using pre-determined economic values of blocks and cut-off grades to focus on maximizing the value of the final products delivered to clients and markets. This shift in focus allows the frameworks to more accurately represent and optimize the mineral value chain, whose processes have elements through which the value of individual blocks is lost.

Montiel and Dimitrakopoulos (2013) present a method which optimizes the long-term extraction schedules at the mines while modelling the flow of material through different processing streams under grade and material type uncertainty. The method aims to minimize deviations from production and quality targets, using simulated annealing to make changes to block extraction periods and destinations. A block's destination can only be changed if its material type does not vary between simulations; otherwise, its destination must correspond to that indicated in the initial schedule. A case study shows that this method successfully reduces deviations from production and quality constraints and increases the value of the operation. The method was then expanded to include the optimization of downstream processes, transportation and operating modes (Montiel and Dimitrakopoulos, 2015, 2017, 2018) and additional sources of material (Montiel et al, 2016). The model explicitly maximizes the operation's NPV while minimizing deviations from production, processing and quality targets. The SA algorithm is used to solve applications, with three perturbation neighbourhoods. The first neighbourhood considers block-based perturbations, modifying the extraction period and destination of blocks. The second neighbourhood considers perturbations to operating modes (e.g. grind size at a processing facility); they are randomly selected and analyzed to determine whether the change will improve the objective function. The third and final neighbourhood considers perturbations to transportation modes, which determine the proportion of material to be transported to its next destination by a certain transportation system. These perturbations are also randomly selected and analyzed. The method, applied to a complex mining operation, is shown to increase the operation's NPV as well as improve its ability to meet its targets (Montiel and Dimitrakopoulos, 2018). In addition, the case study manages blending constraints at the autoclave, utilizing the throughput while ensuring the acid consumption constraints are respected.

Goodfellow and Dimitrakopoulos (2016, 2017) propose a model for the long-term simultaneous optimization of mining complexes, a formulation that acts as the basis for the case study present in Chapter 2. The approach uses a combination of metaheuristics to solve large, complex instances in a reasonable amount of time. The model defines three main variables: scenario-independent binary block extraction variables, scenario-independent binary destination policy variables, and scenario-dependent continuous processing stream variables. The block extraction variables determine a block's extraction period whereas the processing stream variables define the proportion of material sent from one value chain destination to another. The k-means++ algorithm (Arthur and Vassilvitskii, 2007) is used to group blocks into clusters according to their attributes. Cluster membership is scenario-dependent; however, each cluster's destination is governed by the destination policy variables. The use of the clusters reduces the size of the optimization problem while allowing the process to manage secondary and deleterious elements, avoiding the need for pre-determined cut-off grades. Goodfellow and Dimitrakopoulos (2016) explores three different metaheuristic solution approaches for the framework, determining that differential evolution (DE) and particle swarm optimisation (PSO) work well when applied only to the downstream decisions. These methods are then applied in conjunction with SA and compared; the combinations of SA with the other approaches did not yield significant improvements compared to SA on its own while significantly increasing the computation time.

The Goodfellow and Dimitrakopoulos (2016) framework has been applied to a number of case studies, showcasing its ability to incorporate a variety of mineral value chain components, increase an operation's value and manage technical risks. Kumar and Dimitrakopoulos (2019) apply the framework to a case study which incorporates non-additive attributes: the semi-autogenous power index and the bond work index. These attributes represent the material's hardness, which is managed through a hard-soft ratio at the processing facilities. The case study increases the operation's value when compared to the conventional mine plan. Saliba and Dimitrakopoulos (2019a) apply the framework to a case study incorporating market uncertainty in addition to material supply uncertainty. Incorporating market uncertainty allowed the resulting schedule to capitalize on periods of high market prices to produce higher grade ore while managing technical risks. Levinson and Dimitrakopoulos (2019) apply the framework to a case study incorporating waste management constraints. Simulations of the material's acid-generating potential are used to ensure the material is disposed of appropriately. The case study resulted in a schedule that mines acid-generating material strategically, reducing its environmental impact while increase the NPV.

Goodfellow and Dimitrakopoulos (2015) expand the framework to include capital investment decisions using scenario-independent capital expenditure (CAPEX) decision variables. The variables represent the purchase of mining equipment, increasing the operation's mining capacity. Farmer (2016) extends the framework to include decisions to optimize the size of the processing facility as well as market uncertainty. Due to the size of the problem, the optimization was done in two stages: the first optimized the production schedule under a fixed selling price while the second fixed the extraction schedule and optimized the downstream decisions under market uncertainty. The resulting schedule capitalized on the high price periods, increasing the operation's NPV. Saliba and Dimitrakopoulos (2019b) also extend the framework, incorporating tailings management and dam expansion decisions. In a case study, it was shown that the tailings dam capacity, limited by environmental regulations relating to the storage of acid-generating waste material, was the operation's bottleneck. Therefore, the inclusion of the tailings dam expansion option allowed the life-of-mine to increase by 25% and increase the gold production and NPV of the operation. Del Castillo and Dimitrakopoulos (2019) further expand the framework to allow a dynamic optimization of CAPEX alternatives through the integration of a branching mechanism that allows it to explore different CAPEX options in parallel. The branching mechanism allows the schedule to adapt to the uncertainty; branching occurs when the probability of investing in a CAPEX option meets certain criteria. However, the schedule before branching is identical in both branches ensuring that there is a clear schedule to be followed. Levinson and Dimitrakopoulos (2020) applies this extended framework to a case study analyzing investment options in mining equipment, processing facility upgrades and tailings dam expansions; the latter two options have the potential to branch. The case study resulted in significant increases in NPV compared to the base case schedules.

Paithankar et al. (2020) present an SSO framework which considers the long-term extraction sequence at the mines, stockpiling, and cut-off grade policies under geological uncertainty. The model is solved using the maximum flow algorithm for the extraction sequencing optimization and a combination of genetic algorithms for the cut-off grade policy optimization. These solutions methods allow the model to maintain the non-linear characteristics of the mineral value chain, such as the stockpiles. The model is applied to three cases: the first without a stockpile, the second including a stockpile and utilizing the genetic algorithm for the cut-off grade optimization, and the third including a stockpile and utilizing Lane's theory for the cut-off grade optimization. The results indicate that the inclusion of a stockpile increases flexibility and value for an operation, and that the inclusion of material supply uncertainty increases the project's value significantly when compared to a deterministic approach.

SSO frameworks represent the state-of-the art in terms of mine optimization, significantly improving upon previous methods. However, they still present some limitations. For example, stockpiles are modelled using the perfect blending representation (Goodfellow and Dimitrakopoulos, 2016; Montiel et al, 2016; Paithankar et al, 2020). This approach improves upon linear stockpiles representations; however, it misrepresents the variability and uncertainty of the material in the stockpiles (Dirkx and Dimitrakopoulos, 2018). The perfect blending stockpile model averages the characteristics of the material, assigning these averages to represent the entire stockpile. Linear stockpile models are commonly used in mine optimization since they allow the optimization process to include stockpiles while ensuring the problem remains tractable for commercial solvers. Moreno et al. (2017b) present an overview of linear methods used to represent stockpiles in mine optimization formulations. Caccetta and Hill (2003) propose a model including stockpiles; however, the mathematical formulation is not presented. Asad (2005) proposes a model optimizing cut-off grades and stockpiles throughout the life of mine, but the method does not generate a production schedule simultaneously.

Sarker and Gunn (1997) and Koushavand et al. (2014) present deterministic models in which material with grades within pre-defined ranges can be stockpiled. This stockpiled material is assumed to be homogenous, with a pre-defined fixed average grade. This assumption encourages the optimizer to stockpile material with grades below the pre-defined average as it will be "upgraded" to the assigned grade, creating artificial value. However, in order to reduce the amount of “upgrading”, the stockpile’s assigned grade is tuned recursively to minimize the differences between the actual and assigned values. This method has a significant limitation: it assumes the stockpile material is homogenous. Dirkx and Dimitrakopoulos (2018) show that stockpile material is in fact highly variable, therefore the homogeneity assumption will mislead the optimization results. Smith and Wicks (2014) present a model which assumes that material can be reclaimed from a stockpile with perfect selection, overcoming the homogeneity assumption. The method allows the stockpile material to retain its properties and variability; however, it fails to consider the physical barriers associated with stockpile reclamation. Tabesh et al. (2015) present a model that separates the stockpile into bins, each with its own range of acceptable grades and assigned reclamation grade. This allows the model to overcome the homogeneity assumption; however, it does not accurately represent reality since mining operations will not have the large number of stockpiles required for the method and its use of assigned reclamation grades will lead to material “upgrades”. Finally, Brika (2019) proposes a method creating separate stockpiles according to the periods in which the material is stockpiled and reclaimed and its intended destination, allowing the material to retain its properties.

Another limitation of the previous discussed long-term simultaneous stochastic optimization methods is the exclusion of transportation scheduling, specifically for mine-to-port transportation systems. These systems represent an important component of certain mining operations, such as iron ore mining complexes. Therefore, their exclusion from the optimization process fails to capitalize on synergies and leads to a loss of value (Pimentel et al, 2010; Leite et al, 2019). Methods to integrate mine-to-port transportation systems when optimizing mining complexes are presented in the following section.

1.4. Transportation in Mining Complexes

Mine-to-port transportation systems represent an integral part of some mining operations, ensuring that products reach clients and markets. However, these transportation systems are not included in the production scheduling optimization and are instead optimized individually. Typically, long-term SSO frameworks represent mine-to-port transportation systems with a single fixed capacity over the period considered. This failure to capitalize on the synergies that exist between transportation systems and other mining complex components leads to a loss in value for the operation (Pimentel et al, 2010; Leite et al, 2019). Everett (2001) discusses the importance of optimizing all aspects of a mine-to-port transportation system at both the short- and long-term planning horizons to ensure product uniformity.

Research in the integration of mine-to-port transportation optimization into production scheduling has been focused on the short-term planning horizon. Abdekhodaee et al. (2004) present a mixed-integer programming (MIP) model which schedules rail and stockyard activities for a coal mining operation. The model is developed specifically for a multi-mine, single port operation connected by a tree-structure railway network. It considers periods of one week, in which the demand for material from particular mines must be met while ensuring the stockyard can manage the material between the time it arrives and the time it is loaded onto a ship. The model considers the interactions between the railway system and the port; however, it assumes a fixed extraction schedule at the mines. Liu and Kozan (2011) present a model scheduling a single-track railway system connecting two mines to a port over the course of one week. The authors model the problem as a job-shop problem, where a train travelling from one location to another is considered a job. A job is composed of tasks, i.e. travelling on a specific railway segment, which are completed by machines, i.e. railway segments. A machine can only complete a single task at a time, therefore only one train can travel on a railway segment at once. The model optimizes the mine-to-port transportation system; however, it assumes a fixed extraction schedule at the mines, thereby ignoring the interdependencies of these two components. Similarly, Thomas et al. (2013) also model the short-term mine-to-port transportation problem using job-shop scheduling but the authors include extraction scheduling at the mines.

Kameshwaran et al. (2013) propose a multi-stage, integrated operations mine-to-ship scheduler for short-term optimization. The authors acknowledge the strong interdependencies of the different components of a value chain and the importance of optimizing them simultaneously. Additionally, a multi-stage framework is used to allow the scheduler to adapt to equipment underperformance or breakdowns. In terms of railway optimization, the authors consider a macro view of the problem, for example using bounds on the number of trains permitted on a rail segment at any one time. Though the approach models the railway system and port operations, it requires a fixed extraction schedule at the mines as an input. Blom et al. (2014) present a non-linear model which optimizes a multi-mine, multi-port operation over the course of 13 weeks. The problem is split into two subsections, notably the mine portion and the port portion, which are solved iteratively. The railway transportation system linking the sub-problems is not directly incorporated into the optimization process. Singh et al. (2014) suggest a model which optimizes the railway transportation system connecting a complex multi-mine, multi-port iron ore mining operation over a medium-term planning window. Though the model assumes a fixed extraction schedule at the mines, its implementation successfully increased the tonnage delivered to the ports by up to 1 million tonnes per year. This highlights the importance of including the optimization of the transportation systems into the overall optimization of a mining operation. Bodon et al. (2011) present a model which incorporates uncertainty into the short-term mine-to-port optimization problem. Discrete event simulation is used to optimize the extraction scheduling at the mines, the mine-to-port transportation system (an overland conveyor) and the port operations simultaneously. The model was applied to a case study at a coal mine highlighted the method's benefits, such as examining the trade-offs between various capital expenditure options and assessing alternative operational practices.

There have been limited attempts to incorporate mine-to-port transportation scheduling to the long-term mine optimization framework. Montiel and Dimitrakopoulos (2015) incorporate transportation alternatives for the material output from the processing facilities, allowing the optimizer to determine the proportion of material to be transported by different means. However, the possible proportions are pre-determined, and the model does not directly optimize the transportation systems themselves. Belov et al. (2020) present a model which schedules activities on a short-term planning horizon in order to guide long-term scheduling and capacity planning. The method uses a rolling time horizon approach to produce daily train and vessel schedules as well as manage port stockpiles over the course of one year. This approach allows the method to maintain the level of detail of short-term scheduling over a long-term horizon. However, the approach does not include extraction scheduling at the mines, nor does it directly include sources of uncertainty. Chapter 3 presents a new long-term simultaneous stochastic optimization framework incorporating the optimization of a mine-to-port transportation system under supply uncertainty.

1.5. Modelling Geological Uncertainty

Material supply or geological, that is to say grade and material type, uncertainty and variability has been shown to be the largest source of risk in a mining operation (Vallée, 2000). This uncertainty impacts the metal produced by a mining operation, subsequently affecting its cashflows and profitability. Traditional methods for mine optimization do not account for this source of uncertainty, instead relying on a single, estimated representation of the orebody. The use of a single representation of the orebody implies that the deposit is perfectly known; however, orebody models are based on sparse data (i.e. drillholes) and therefore cannot be perfectly known until extraction (Goovaerts, 1997). In addition, estimated orebody models, often produced using methods such as kriging (David, 1988; Journel and Huijbregts, 1978; Goovaerts, 1997), assign a grade value to a point based on an average of the surrounding data, resulting in a smoothed representation of the deposit. This "smoothing effect" prevents estimation methods from reproducing the geostatistical characteristics of the data on which they are based, such as the grade histograms and variograms, misrepresenting the variability of the orebody and misinforming optimization processes which rely on them (David, 1977, 1988; Journel and Huijbregts, 1978; Goovaerts, 1997; Dimitrakopoulos, 1998; Rossi and Deutsch, 2014). Stochastic simulation methods have been developed to overcome the shortcomings of estimation methods. These simulations reproduce the geostatistical properties of the data, providing a better representation of the orebody. They have been used to assess the risks associated with life-of-mine plans and schedules produced using estimated orebody models and deterministic methods (Ravenscroft, 1992; Dowd, 1994, 1997; Dimitrakopoulos, 1998; Dimitrakopoulos et al, 2002) as well as an input for stochastic optimization methods, which directly manage these risks (Dimitrakopoulos and Ramazan, 2004; Godoy and Dimitrakopoulos, 2004; Ramazan and Dimitrakopoulos, 2007; Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016).

Sequential simulations methods are commonly used in geostatistical applications for both continuous (e.g. grade distributions) and categorical (e.g. material type boundaries) variables (Journel and Alabert, 1989; Goovaerts, 1997; Rossi and Deutsch, 2014). Generally, these methods divide the volume to be simulated into a grid of nodes and define a random path visiting each node. At each node on the path, a value is sampled from a conditional cumulative distribution function (ccdf), which is built using the data and the previously simulated nodes. A neighbourhood is often defined in order limit the number of points used to create the ccdf; points located nearer to the current node will have more influence on its value than those at a larger distance, a phenomenon known as the screen effect (Isaaks and Srivastava, 1989; Goovaerts, 1997).

Sequential Gaussian simulation (SGS) has been widely used to simulate continuous variables, such as metal grades (Goovaerts, 1997). The method requires the data to be transformed to a normal distribution, allowing it to use Kriging to determine the estimated mean and variance and build the ccdf (Rossi and Deutsch, 2014). The method has a relatively simple implementation; however, it is computationally expensive when deposits require large number of nodes to be simulated. Dimitrakopoulos and Luo (2004) present the generalized sequential Gaussian simulation (GSGS) method, which overcomes some limitations of SGS by simulating groups of nodes simultaneously. Nodes with similar neighbourhoods of conditioning data are grouped and simulated together using the Lower-Upper (LU) decomposition method (Davis, 1987). The optimal grouping size is determined by minimizing the screen-effect approximation loss; when groups consist of a single node the method is equivalent to SGS, while when a single group encompasses all nodes the method is equivalent to LU. Both SGS and GSGS require the simulation results to be converted from the point-support scale to the block-support scale for mine optimization purposes. Godoy (2003) proposes the direct block simulation (DBSIM) method, which combines the simulation of point-support values and the conversion to the block-support scale. The method simulates the nodes within a mining block using GSGS, then determines and stores the block-support value for the group of nodes before discarding the individual nodes. This block-support value is then added to the set of conditioning data for the next block to be simulated. The method is used to generate the geological simulations used in the case study in Chapter 2. This method reduces the memory requirements when compared to SGS and GSGS and eliminates the need to transform the simulations from the point-support scale to the block-support scale afterwards. Boucher and Dimitrakopoulos (2009) extended the DBSIM method for multivariate simulations by combining it with the minimum/maximum autocorrelation factors (MAF) method (Desbarats and Dimitrakopoulos, 2000), creating the DBMAFSIM simulation method. The spatially correlated variables are decorrelated using the MAF method and for every block in the random path, each variable is simulated according to the DBSIM approach. This approach therefore maintains the computational benefits of the DBSIM approach while ensuring that the geostatistical properties of multivariate deposits are respected.

The previously discussed simulation methods can only reproduce second-order statistics and rely on Gaussian assumptions. However, many geological attributes exhibit complex spatial patterns that cannot be reproduced using only two-point statistics (Guardiano and Srivastava, 1993; Strebelle, 2002; Journel, 2003, 2005). Multi-point simulation (MPS) frameworks, attempt to overcome the limitations of the Gaussian methods by using information which is not present in the conditioning data by using training images (Journel, 2005; Zhang et al, 2006; Remy et al, 2009; Mariethoz and Renard, 2010; Chatterjee and Dimitrakopoulos, 2012; Strebelle and Cavelius, 2014; Mariethoz and Caers, 2015). Training images (TI) describe the spatial data distribution of the geological attribute of interest and can be used for both continuous and categorial variables (Guardiano and Srivastava, 1993; Strebelle, 2002). However, TIs are difficult to produce as they often requiring unavailable data and can rely on geologists' interpretation of the patterns. As such, when conditioning data is sparse, MPS simulations tend to reproduce the TI's properties instead of the data's (Goodfellow et al, 2012a). To overcome the challenges of using TIs, Mustapha and Dimitrakopoulos (2011) present a high-order simulation (HOSIM) method. The method uses TIs in conjunction with high-order statistics such as cumulants, limiting the impact of the TI on the reproduction of the data's properties. Goodfellow et al. (2012b) propose a method to approximate high-order spatial statistics by decomposing them into a set of weighted sums. This allows the method to use only the available data, avoiding the use of TIs. Experimental results indicate the proposed method generates better spatial moment maps than the maps created using the actual sparse spatial moments. Minniakhmetov and Dimitrakopoulos (2017b) extend the HOSIM framework for multivariate deposits using a novel method to decorrelate the variables using both the covariance matrix and high-order statistics. Minniakhmetov and Dimitrakopoulos (2017a) then extend the HOSIM framework for use in deposits with multiple categorical variables using a recursive B-spline approximation algorithm to approximate the high order cumulants. De Carvalho et al. (2019) extend HOSIM, allowing it to simulate at the block-support scale directly, reducing memory requirements. Finally, de Carvalho and Dimitrakopoulos (2019) use the previously described simulation method to generate the orebody simulations used as an input for the simultaneous stochastic optimization of a mining complex. The results are compared to the simultaneous stochastic optimization of the same mining complex using simulations generated with SGS. The case study demonstrates that the HOSIM extension presents a better continuity of high grades and the life-of-mine plan generated favours the extraction of these zones. This leads to a 7% increase in recovered gold and a 5% to 16% increase in NPV when compared to the optimization using SGS simulations.

1.6. Goal and Objectives

The production of this thesis is to further simultaneous stochastic optimization for the long-term planning optimization of mining complexes through the major application at a gold mining complex and the extension of the method integrating mine-to-port transportation. To achieve this goal, the following objectives are set:

1. Review strategic mine planning optimization methods, approaches to integrate mine-to-port transportation scheduling in the long-term production planning optimization of mining complexes as well as an overview of the methods generating conditional orebody simulations used to quantify material supply uncertainty.
2. Apply simultaneous stochastic optimization to life-of-mine planning at a three-mine gold mining complex and quantify the risk of not meeting production forecasts as a result of material supply uncertainty.
3. Develop a new method which incorporates mine-to-port transportation into the long-term simultaneous stochastic optimization of mining complexes.
4. Summarize the main contributions and conclusions of the research conducted and present suggestions for future work.

1.7. Thesis Outline

The thesis is organized as follows:

1. Chapter 1 presents a literature review of optimization methods used in the long-term planning of mining complexes, the integration of mine-to-port transportation into the long-term optimization of mining complexes and the use of simulations methods to quantify and manage material supply (geological) uncertainty.
2. Chapter 2 presents a case study application of a simultaneous stochastic optimization framework at a large gold mining complex. The case study highlights the method's ability to manage production risks at various points throughout the value chain.

3. Chapter 3 proposes a new model which integrates mine-to-port transportation scheduling into the long-term optimization of ore mining complexes under geological uncertainty. A case study at an iron ore mining complex presents the model's benefits.
4. Chapter 4 presents a summary of the contributions of the thesis and recommendations on future work.

2. An Application of Simultaneous Stochastic Optimization at a Large Open Pit Gold Mining Complex under Supply Uncertainty

2.1. Introduction

Mining complexes or mineral value chains are systems composed of various components, including mines, stockpiles, processors, waste dumps, tailings facilities, transportation and so on (Montiel and Dimitrakopoulos, 2015, 2017, 2018; Goodfellow and Dimitrakopoulos, 2016, 2017). The simultaneous stochastic optimization (SSO) approach integrates all components of a mineral value chain into a single mathematical formulation to maximize the production and net present value (NPV) over the life of the related mining complex. By considering the interactions between the components of a mineral value chain, the SSO approach defines the extraction sequences for the mines considered, cut-off grades, stockpile management and blending at the processing facilities to maximize NPV over the life of the assets involved. The approach also manages material supply uncertainty and the related risk based on geostatistical simulations of the relevant properties of the mineral deposits involved, which reproduce the local variability and uncertainty of the available material (Goovaerts, 1997).

Traditional mine planning and optimization methods treat the major components of the related mineral value chain separately in a sequential fashion and are deterministic, ignoring the interdependencies of the components and failing to manage the technical risks arising from an uncertain material supply. Global optimizers have been developed to jointly optimize different components of a mining complex (Hoerger et al, 1999b; Stone et al, 2007; Whittle, 2007, 2010, 2014; Pimentel et al, 2010), however they are deterministic and make certain approximations. Stochastic optimization frameworks have been developed to incorporate and manage the uncertainty in the optimization of a mineral value chain. These approaches use stochastic simulations of mineral deposits to quantify the material supply uncertainty and manage the associated risks (Ramazan and Dimitrakopoulos, 2007, 2013; Paithankar and Chatterjee, 2019) as well as determine optimal cut-off grade policies (Menabde et al, 2007; Githiria and Musingwini, 2019; Khan and Asad, 2019). Ramazan and Dimitrakopoulos (2007, 2013) present a stochastic integer program to optimize a mineral deposit's extraction schedule, maximizing NPV and minimizing deviations from ore tonnage, grade and metal quantity and quality targets. Mai et al. (2019) propose a stochastic integer programming model to maximize the NPV of a mineral deposit and minimize the risks of not meeting production targets under geological uncertainty. However, the approach aggregates blocks to reduce the computational requirements of the optimization problem. Morales et al. (2019) present a method to optimize mine and mill operations under grade, recovery and mill throughput uncertainty. Menabde et al. (2007) extend Blasor, the optimization tool presented by Stone et al. (2007), to include geological uncertainty and cut-off grade optimization based on grade bins. In addition, the optimization tool sequences multiple pits, as opposed to a single deposit as in the previously discussed methods, to ensure product and quality requirements are respected while maximizing NPV.

The above previously discussed stochastic optimization methods aim to maximize the value of the mine by maximizing the economic value of the blocks extracted under an uncertain material supply. These economic values are determined prior to the optimization process under the assumption that each block will be processed individually, ignoring transformations caused by the blending and non-linear interactions of material. The simultaneous stochastic optimization (SSO) approach shifts focus from the maximization of the economic values of blocks to the maximization of the value of the final product generated by the mining complex. This allows the approach to account for the effects of blending and non-linear interactions of material on the products generated. Montiel and Dimitrakopoulos (2015, 2017) present a model for the simultaneous stochastic optimization of mining complexes, including production scheduling, destination policy, processing, operating modes at the processing facilities and transportation alternatives under material supply uncertainty. In addition, the approach considers material supply from underground mines (Montiel et al, 2016). Goodfellow and Dimitrakopoulos (2016, 2017) develop a generalized simultaneous stochastic optimization approach to jointly optimize the different components of a mineral value chain under geological uncertainty. The method easily accommodates different types of mining complexes of varying sizes and their relevant components, such open pit mines, stockpiles, processing facilities, waste dumps, tailings dams and so on. However, it does not directly incorporate operating modes, transportation alternatives or material supply from sources other than open pit mines. This approach has been applied to different case studies incorporating market supply uncertainty (Saliba and Dimitrakopoulos, 2019a), waste management (Levinson and Dimitrakopoulos, 2019), tailings management (Saliba and Dimitrakopoulos, 2019b) and non-additive attributes such as hardness (Kumar and Dimitrakopoulos, 2019). In addition, the approach has been extended into a dynamic simultaneous stochastic optimizer to include capital investments (Del Castillo and Dimitrakopoulos, 2019; Levinson and Dimitrakopoulos, 2020). Finally, Paithankar et al. (2020) propose a model for the simultaneous stochastic optimization of extraction schedules and cut-off grades considering grade uncertainty and stockpiling.

The work herein presents a case study of SSO (Goodfellow and Dimitrakopoulos, 2016, 2017) at the Rosebel Gold Mines (RGM) mining complex in Suriname, owned by the IAMGOLD Corporation. The case study considers three RGM deposits for a total of 1.07 million blocks. The case study includes three stockpiles, a waste dump and a processing facility. Each mine considers four material types, which are treated differently throughout the mineral value chain. The direct block simulation (DBSIM) method (Godoy, 2003; Boucher and Dimitrakopoulos, 2009) is used to generate geostatistical simulations of gold grades within each deposit considered, representing the material supply uncertainty of the mining complex. The case study considers an elaborate haulage cost scheme to accurately represent the costs related to transporting the materials from sources to destinations. Finally, the case study integrates material hardness management at the semi-autogenous grinder (SAG) mill. In the subsequent sections, an overview of the SSO method is presented. Then, the case study at the above-mentioned gold mining complex demonstrates the practical aspects of the SSO method. Conclusions follow.

2.2. Method

The stochastic mathematical programming model for the simultaneous stochastic optimization (Goodfellow and Dimitrakopoulos, 2016) applied in the present study is outlined in this section.

2.2.1. Definitions and Notation

Throughout the mineral value chain, the materials are defined as the products extracted from mines and/or that are the results of blending, separation or processing activities. These materials are described by attributes which represent their properties, such as mass or metal quantity. These attributes can be separated into two categories. Primary attributes ($\mathscr{p} \in \mathcal{P}$) are additive and can be passed on from one location to another; their value is denoted as $v_{\mathscr{p}its}$. Hereditary attributes ($h \in H$) are of interest at a specific location; their value is denoted as $v_{hits} = f_{hi}\left(v_{\mathscr{p}its}\right)$. Hereditary attributes facilitate the inclusion of non-linear transformation functions within the SSO framework.

Material is obtained from the mines, $\mathscr{m} \in \mathbb{M}$, by extracting a set of blocks, or selective mining units (SMU), $b \in \mathbb{B}_{\mathscr{m}}$. Each block is assigned a bin or cluster, $c \in \mathcal{C}$, according to geological attributes such as grade and material type. Cluster membership is scenario dependent, $\mathscr{s} \in \mathcal{S}$, and defines the destinations, $i \in \mathcal{O}_{(c)}$, to which extracted blocks can be sent; the destination policy is scenario independent. The blocks are mined at a cost of $mc_{bt} = \frac{mc_{bt_0}}{(1+d)^t}$. The set $\mathbb{T}$ represents the number of scheduling periods or years, $t \in \mathbb{T}$. A block b is eligible to be extracted if its set of predecessors according to the related slope constraints, $\mathbb{O}_b$, is fully extracted. Mineability constraints ensure the schedules produced by the optimization process are feasible. As such, blocks within block b's smoothing window, $\mathbb{W}_b$, are subject to a penalty cost, $c_{\mathscr{m}t}^{smooth} = \frac{c_{\mathscr{m}t_0}^{smooth}}{(1+r)^t}$, applied to the number of blocks within the window mined in a different period than that of b. Additionally, a block $\bar{b} \in \mathbb{Q}_b$ lying at a certain vertical distance (i.e. the sink rate distance) above another block b is subject to a penalty cost, $c_{\mathscr{m}t}^{sink} = \frac{c_{\mathscr{m}t_0}^{sink}}{(1+r)^t}$, applied when the blocks b and $\bar{b}$ are mined in the same period (further discussion of smoothing and sink rates can be found in Section 2.2.4). The total amount of material mined cannot exceed the mining capacity; any excess will incur a penalty cost, $MC_{ht} = \frac{MC_{ht_0}}{(1+r)^t}$. The extracted material is hauled its destination $i \in \mathcal{O}_{(\mathscr{m})}$ at a cost of $hc_{\mathscr{m}it} = \frac{hc_{\mathscr{m}it_0}}{(1+d)^t}$. Material sent to a stockpile $i \in \mathbb{S}$ can be reclaimed at a cost of $rc_{hit} = \frac{rc_{hit_0}}{(1+d)^t}$ and sent to an eligible destination, $j \in \mathcal{O}_{(i)}$. The material sent to a processor $i \in \mathbb{P}$ is processed at a cost of $pc_{hit} = \frac{pc_{hit_0}}{(1+d)^t}$. Deviations from quantity and quality constraints at the processor are penalised by $c_{hit}^{+} = \frac{c_{hit_0}^{+}}{(1+r)^t}$ and $c_{hit}^{-} = \frac{c_{hit_0}^{-}}{(1+r)^t}$, according to whether the deviations exceed the bound or are in deficit, respectively. The revenue generated by the final products delivered by the mining complex is represented by $p_{hit} = \frac{p_{hit_0}}{(1+d)^t}$.

2.2.2. Decision Variables

The formulation proposed by Goodfellow and Dimitrakopoulos (2016) defines four critical decisions variables. First, the scenario-independent binary block extraction decision variable, x_{bt}, holds a value of one if block $b \in \mathbb{B}_m$ is extracted in period $t \in \mathbb{T}$, and holds a value of zero otherwise. Second, the processing stream decision variable, y_{ijts}, is a real number between zero and 1 indicating the proportion of material being sent from location $i \in \mathbb{S} \cup \mathbb{P}$ to location $j \in \mathcal{O}_{(i)}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathcal{S}$. This decision variable is scenario-dependent since the model assumes that once the material is extracted and sent to a destination, its uncertainty is revealed. This assumption allows the processing stream decision variables to adapt to each uncertainty scenario. Third, the scenario-dependent binary cluster membership variable, θ_{bcs}, holds a value of one if block $b \in \mathbb{B}_m$ belongs to cluster $c \in \mathcal{C}$ in scenario $s \in \mathcal{S}$, and holds a value of zero otherwise. Finally, the scenario-independent binary destination policy variable, z_{cjt}, holds a value of one if the cluster $c \in \mathcal{C}$ is sent to destination $j \in \mathcal{O}_{(c)}$, and holds a value of zero otherwise. It is important to note that a cluster can be assigned to a single destination, however multiple clusters can be assigned to the same destination.

Additional decision variables include the scenario-dependent surplus, d^+_{hits}, and deficiency, d^-_{hits}, variables. These represent the quantity exceeding an upper-bound target (U_{hit}) or the shortage from a lower-bound target (L_{hit}), respectively, for attribute $h \in H$ at destination $i \in \mathbb{S} \cup \mathbb{P}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathcal{S}$.

2.2.3. Objective Function

The objective function (1) of the Goodfellow and Dimitrakopoulos (2016) two-stage stochastic integer programming model maximizes the expected profits of the products generated by the mineral value chain while minimizing the risks of failing to meet capacity, blending and mineability requirements.

$$\max \frac{1}{|\mathcal{S}|}\sum_{s\in\mathcal{S}}\sum_{t\in\mathbb{T}}\Bigg(\underbrace{\sum_{i\in\mathbb{P}}\sum_{h\in H} p_{hit}v_{hits}}_{Part\ I} - \underbrace{\sum_{i\in\mathbb{P}}\sum_{h\in H} pc_{hit}v_{hits}}_{Part\ II}$$
$$- \underbrace{\sum_{m\in\mathbb{M}}\sum_{b\in\mathbb{B}_m}\sum_{i\in\mathcal{O}_{(m)}}\sum_{c\in\mathcal{C}} hc_{mit}x_{bt}\theta_{bcs}z_{cit}}_{Part\ III} - \underbrace{\sum_{i\in\mathbb{S}}\sum_{h\in H} rc_{hit}v_{hits}}_{Part\ IV} \tag{1}$$
$$- \underbrace{MC_t d_{ts}^{mine}}_{Part\ V} - \underbrace{\sum_{i\in\mathbb{P}}\sum_{h\in H} PC_{hit}^{+} d_{hits}^{process}}_{Part\ VI} - \underbrace{\sum_{i\in\mathbb{P}}\sum_{h\in H} SAG_{hit}^{+} d_{hits}^{SAG}}_{Part\ VII}\Bigg)$$
$$- \sum_{t\in\mathbb{T}}\sum_{m\in\mathbb{M}}\sum_{b\in\mathbb{B}_m}\Bigg(\underbrace{mc_{bt}x_{bt}}_{Part\ VIII} + \underbrace{c_{mt}^{smooth} d_{bt}^{smooth}}_{Part\ IX} + \underbrace{\sum_{\bar{b}\in\mathbb{Q}_b} c_{mt}^{sink} d_{b\bar{b}t}^{sink}}_{Part\ X}\Bigg)$$

Part I maximizes the revenues generated by the products produced. Parts II, III, IV and VIII aim to minimize the cost of processing, hauling, reclaiming and mining, respectively. Parts V, VI and VII minimize the deviations from mining, mineral processing, and SAG mill capacities, while Parts IX and X minimize deviations from smoothing and sink rate constraints. The cashflows are subject to an established economic discount rate, d, while the deviation penalty costs are subject to a geological risk discount rate, r (Dimitrakopoulos and Ramazan, 2004). The geological risk discount rate reduces the magnitude of the penalty cost over time, deferring the risk of failing to meet production requirements to later periods, when more information will become available.

2.2.4. Constraints

The objective function is subject to constraints, including reserve, slope, destination policy, and so on. Only select constraints of particular interest to the case study presented in this paper are presented; comprehensive definitions and explanations for the remaining constraints can be found in Goodfellow and Dimitrakopoulos (2016). The different processing streams of the mineral value chain can only accept certain material types based on their geometallurgical attributes. Each processing stream is subject to a capacity constraint; deviations from the capacity constraints are calculated using equations (2) and (3) and are penalized in the objective function (1). Similarly, the material extracted from the mines is subject to the mining capacity (4).

$$v_{hits} - d_{hits}^{process} \leq ProcessingCap \qquad \forall t \in T, s \in \mathcal{S}, i \in \mathbb{P}, h \in H \tag{2}$$

$$v_{hits} - d_{hits}^{SAG} \leq SAGAvail \qquad \forall t \in T, s \in \mathcal{S}, i \in \mathbb{P}, h \in H \tag{3}$$

$$\sum_{m \in \mathbb{M}} \sum_{b \in \mathbb{B}_m} x_{bt} \times tonnage_b - d_{ts}^{mine} \leq MiningCap \qquad \forall t \in T, s \in \mathcal{S} \tag{4}$$

In order to ensure a mineable schedule, a smoothing constraint (5) is applied. Based on the methodology presented in Dimitrakopoulos and Ramazan (2004), a smoothing window, $\mathbb{W}_b$, is defined centered around block b. The number of blocks which make up this window is defined according to a smoothing radius: all blocks whose centers reside within a certain distance of block b's center are considered to be in block b's smoothing window. Constraint (5) counts the number of blocks which are scheduled to be mined in a different period than that of block b; this number (d_{bt}^{smooth}) is then penalised in part VIII the objective function (1). In addition, a sink rate constraint (6) also ensures a mineable schedule by limiting the mine's vertical advance rate in any period. If a block b is mined in the same period as the overlying block $\bar{b} \in \mathbb{Q}_b$, located at a distance equivalent to the sink rate plus the block's length in the vertical direction, the deviation variable, d_{bt}^{sink}, takes on a value of one and is penalized in part IX of the objective function (1).

$$|\mathbb{W}_b| x_{bt} - \sum_{\bar{b} \in \mathbb{W}_b} x_{\bar{b}t} \leq d_{bt}^{smooth} \qquad \forall m \in \mathbb{M}, b \in \mathbb{B}_m, t \in \mathbb{T} \tag{5}$$

$$x_{bt} + \sum_{\bar{b} \in \mathbb{Q}_b} x_{\bar{b}t} - d_{bt}^{sink} \leq 1 \qquad \forall m \in \mathbb{M}, b \in \mathbb{B}_m, t \in \mathbb{T} \tag{6}$$

2.2.5. Solution Approach

The simultaneous stochastic optimization of mining complexes requires a metaheuristic solution approach due to the large number of decision variables that must be considered. The metaheuristic approach used in this work is the simulated annealing (Kirkpatrick et al, 1983) extended to consider multiple perturbation neighbourhoods and adaptive neighbourhood search (Goodfellow and Dimitrakopoulos, 2016, 2017; Montiel and Dimitrakopoulos, 2017).

2.3. Case Study at a Gold Mining Complex

2.3.1. Overview

The SSO mathematical programming formulation described previously is applied to the Rosebel Gold Mines (RGM) mining complex in Suriname. The case study considers three deposits: Rosebel Mine, Pay Caro Mine, and Royal Hill Mine, as shown in Figure 3. Each deposit has four material types: waste, laterite/saprolite, transition, and hard rock. The extracted material can be sent to the processor, related stockpile or waste dump. At the processor, constraints are considered on the material throughput at the SAG mill and on the total tonnage of material sent to the processor.

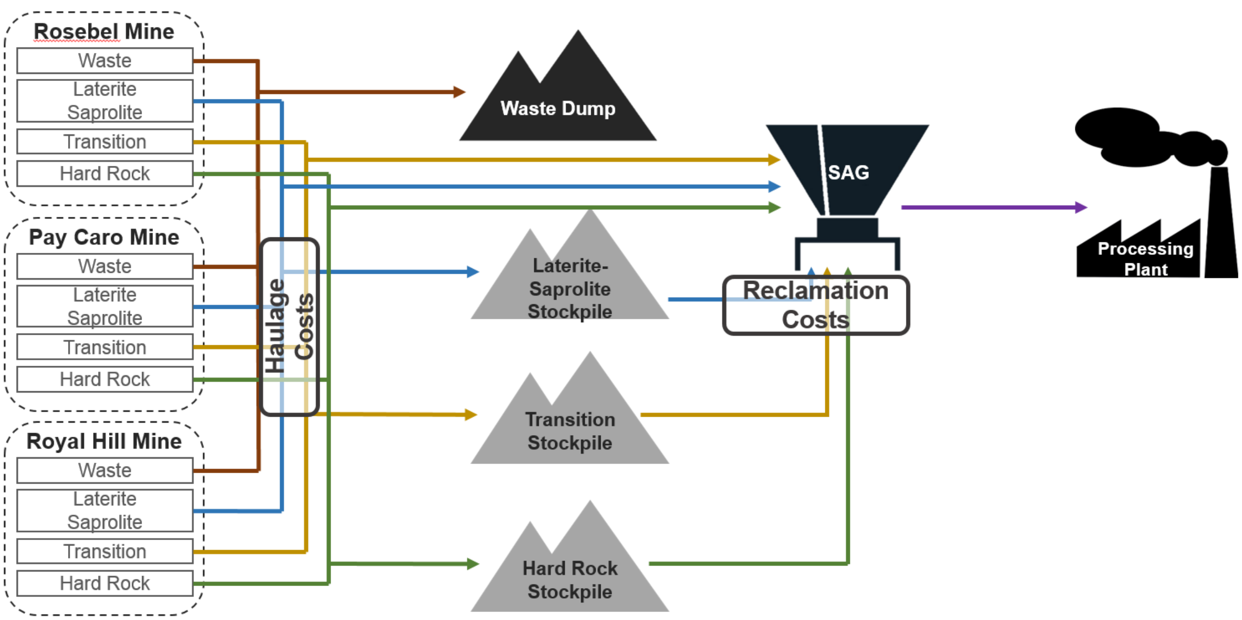


Figure 3 Material flow diagram at the gold mining complex considered in this case study

Each deposit is made up of blocks of 16x12x9 m^3, for a combined total of 1.07 million mining blocks: 0.26 million at Rosebel Mine, 0.38 million at Pay Caro Mine, and 0.43 million at Royal

Hill Mine. The material uncertainty is quantified using 10 stochastic simulations of the gold grades per mine, for a total of 1,000 uncertainty scenarios. Note that Albor and Dimitrakopoulos (2009) determined that 10 to 12 simulations are sufficient to obtain a stable solution for stochastic optimization of mining complexes. The simulated realizations of the three deposits are generated using the direct block simulation (DBSIM) method (Godoy, 2003; Boucher and Dimitrakopoulos, 2009). Each deposit is separated into different geological domains which were simulated separately.

The economic parameters used in the optimization process are listed in Table 1. The mining cost is separated into the drill and blast cost, loading cost, dump maintenance cost and closure cost while the total processing cost is separated into processing cost, administration cost and sustaining capital cost, to account for the different costs associated with the different material types. In addition, the haulage costs are separated from the mining costs to account for differences in the haulage distance from the different mineral deposits to the different processing stream destinations. For example, the Rosebel Mine is the furthest from the processor and therefore has the highest transportation cost, while the Pay Caro Mine is the closest and therefore has the lower transportation cost. Furthermore, an incremental mining cost is included to account for the increase cost of mining deeper into each pit. Table 2 summarizes the targets for each component of the mineral value chain including the stockpiles and processors. Table 3 denotes the mineability constraints applied to create smooth schedules.

Table 1 Economic parameters

General	Material-Dependent	Mine-Dependent
Economic Discount Rate	Gold Recovery Rate	Reclamation Cost
Geological Risk Discount Rate	Drill and Blast Cost	Haulage Costs
Gold Price	Processing Cost	Incremental Mining Cost
Selling Cost	Administration Cost	
Royalties	Sustaining Capital Cost	
Loading Cost		
Dump Maintenance Cost		
Closure Cost		

Table 2 Capacity constraints

Constraints	Capacity
Mining Capacity (years 1-5)	67.3 Mt/year
Mining Capacity (years 6-18)	74.0 Mt/year
SAG Mill Capacity	876 hours/year
Processing Capacity	8.83 Mt/year

Table 3 Scheduling constraints

Constraint	Distance
Smoothness	48 m
Max sink rate	63 m

2.3.2. Results

In the following figures, the results of the simultaneous stochastic optimization are represented, where applicable, by the P10, P50 and P90. These represent the 10%, 50% and 90% probability, respectively, of obtaining values below the corresponding forecast. The results of the case study are scaled for confidentiality reasons. The mining complex has an 18-year life, as shown in Figure 4 alongside the NPV results. Figure 5 presents the ore mined and recovered gold over the mining complex life and Figure 6 presents the tonnage mined throughout the long-term plan of the mining complex. Figure 7 shows the production schedules generated which, as noted in previous sections, comply with smoothing and sink rate constraints. Figure 8 to Figure 10 present the ore tonnage mined and recovered gold over the life-of-mine for Rosebel Mine, Pay Caro Mine, and Royal Hill Mine, respectively.

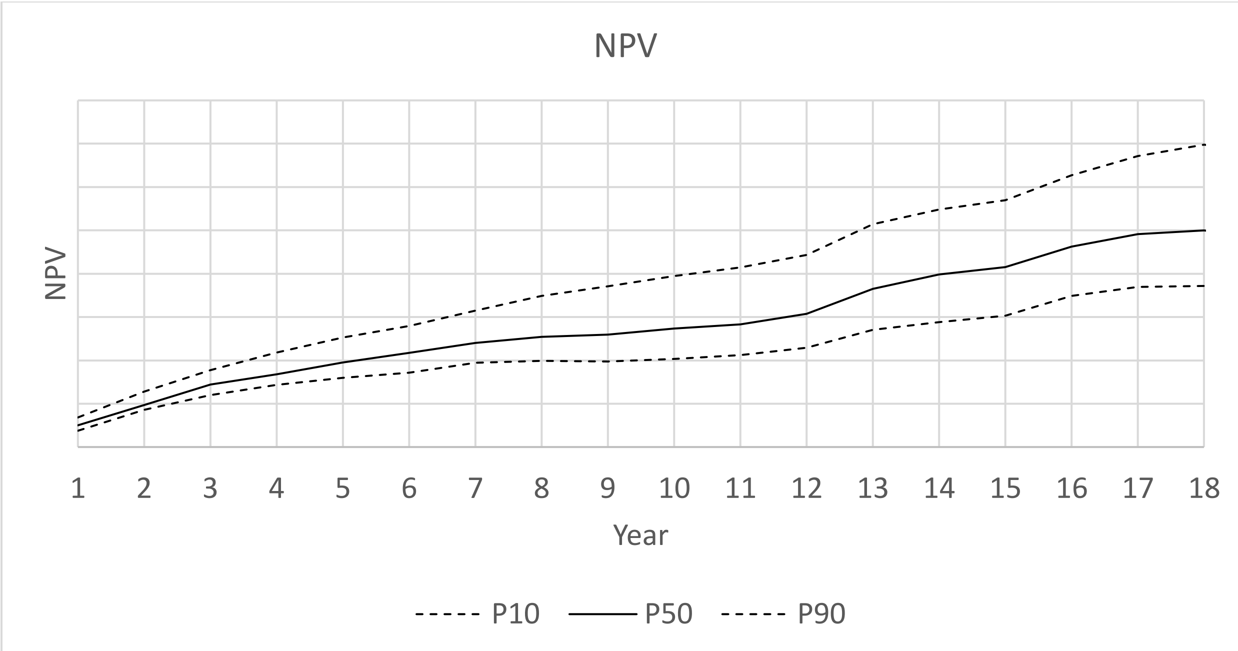


Figure 4 Net present value of the RGM mining complex

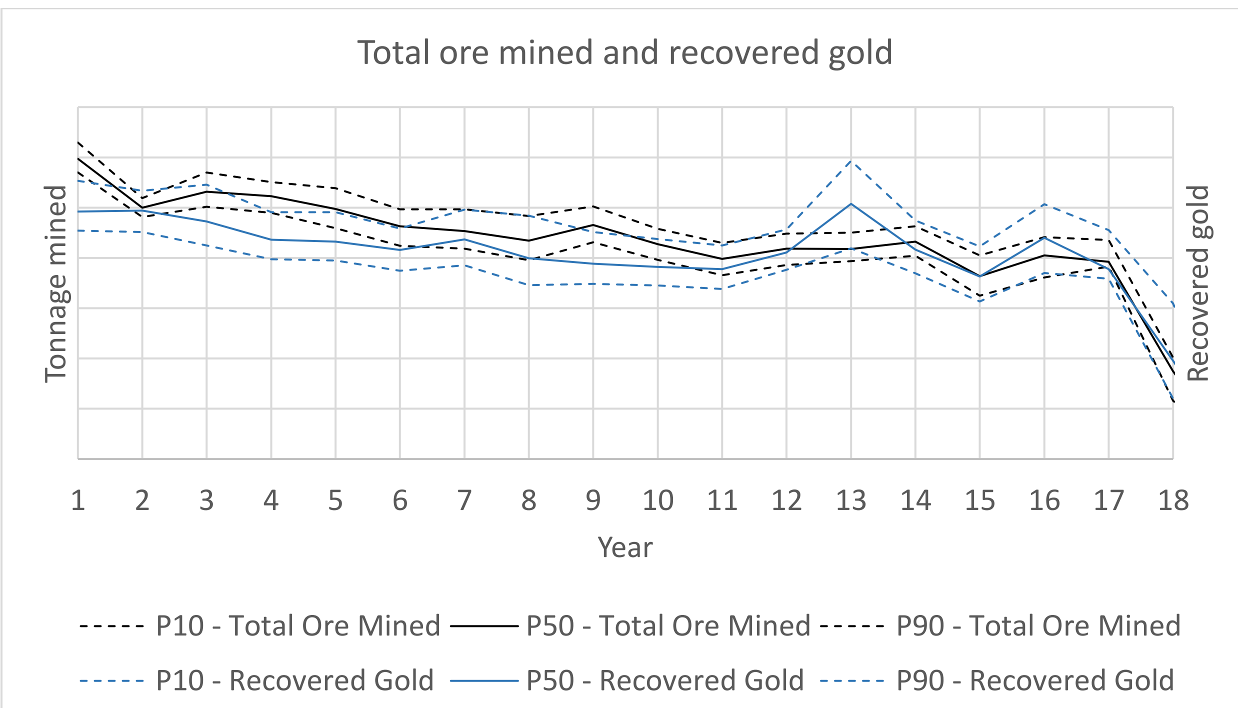


Figure 5 Total ore tonnage mined and recovered gold from the three mines at the RGM mining complex

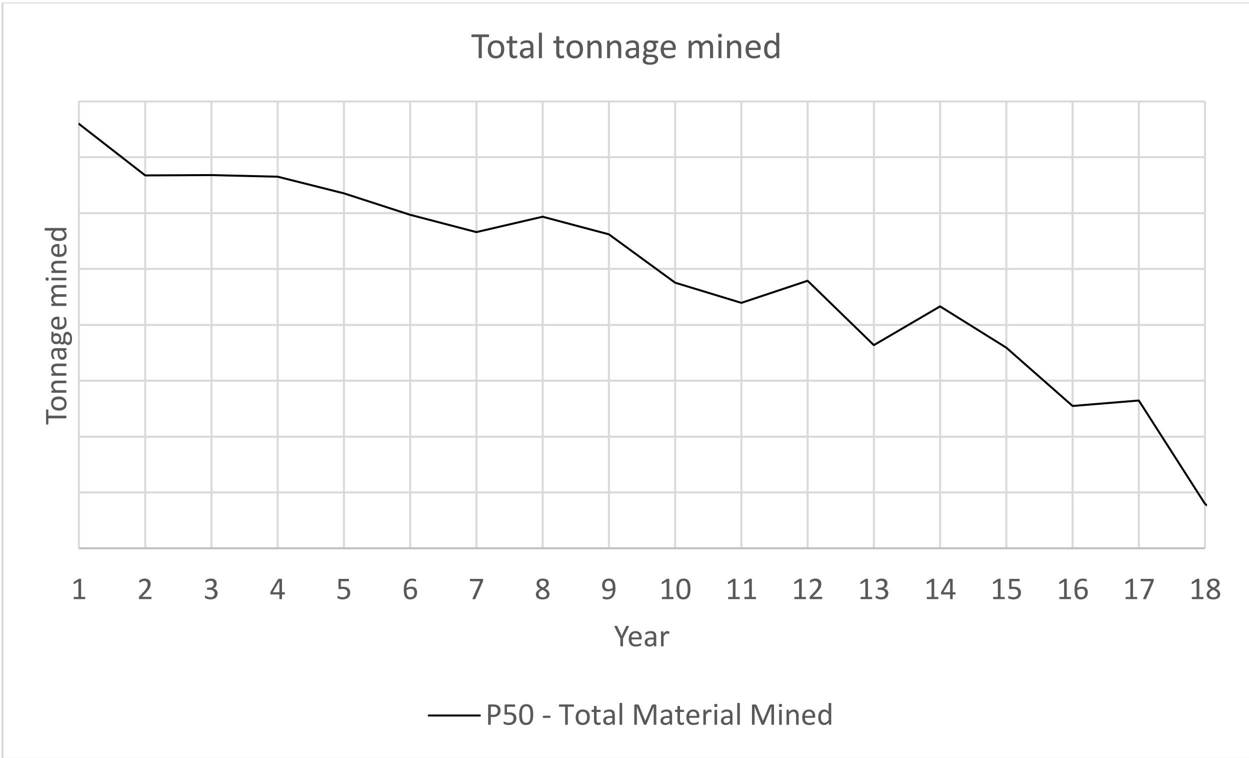


Figure 6 Total tonnage mined from the three mines at the RGM mining complex

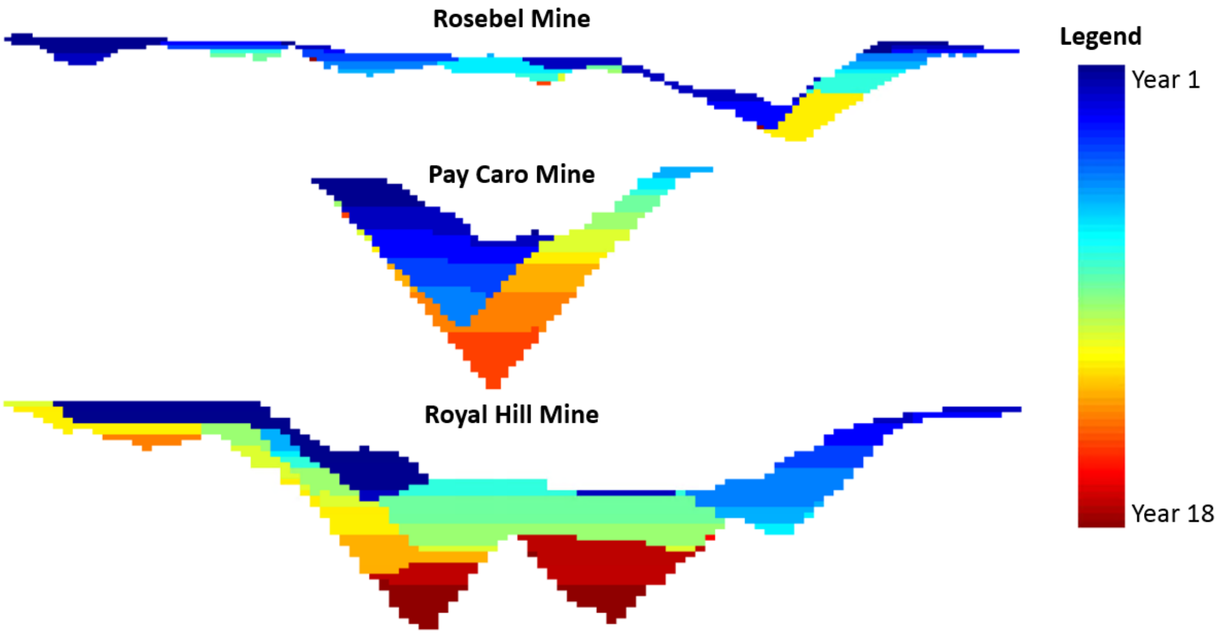


Figure 7 Stochastic life-of-asset production schedules at the three mines at the RGM mining complex

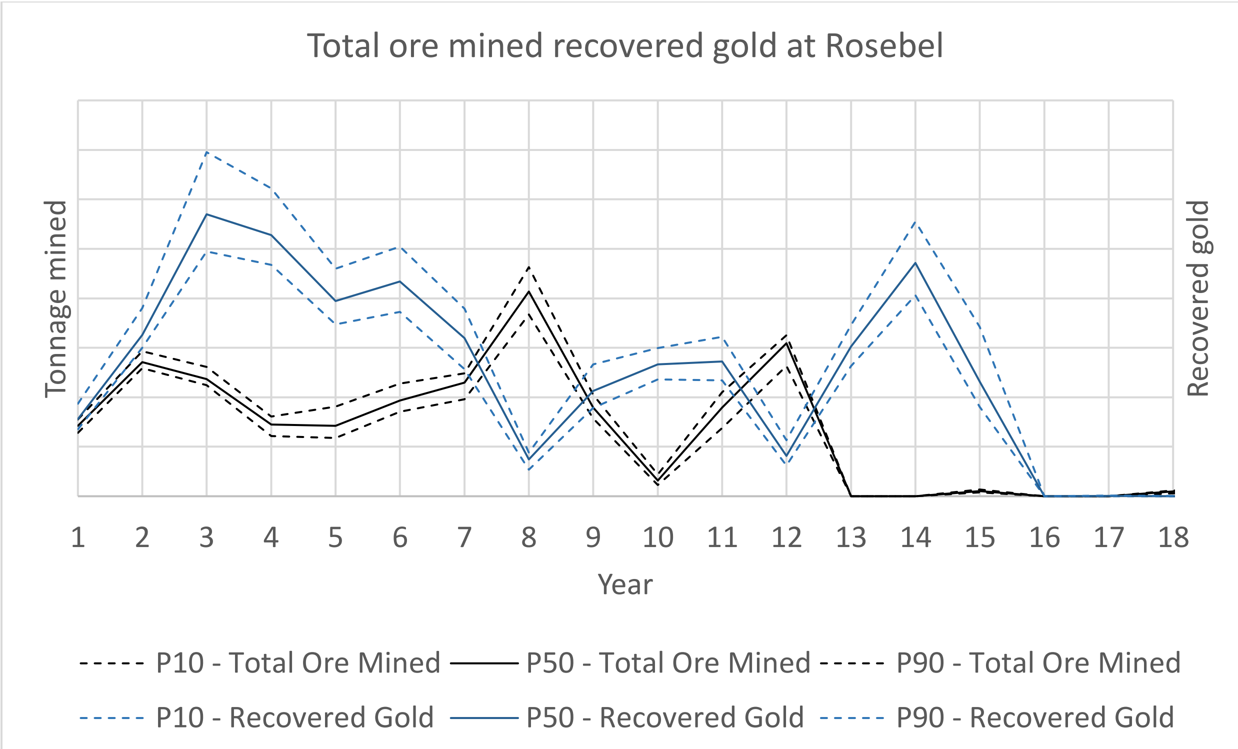


Figure 8 Ore tonnage mined and recovered gold at the Rosebel Mine

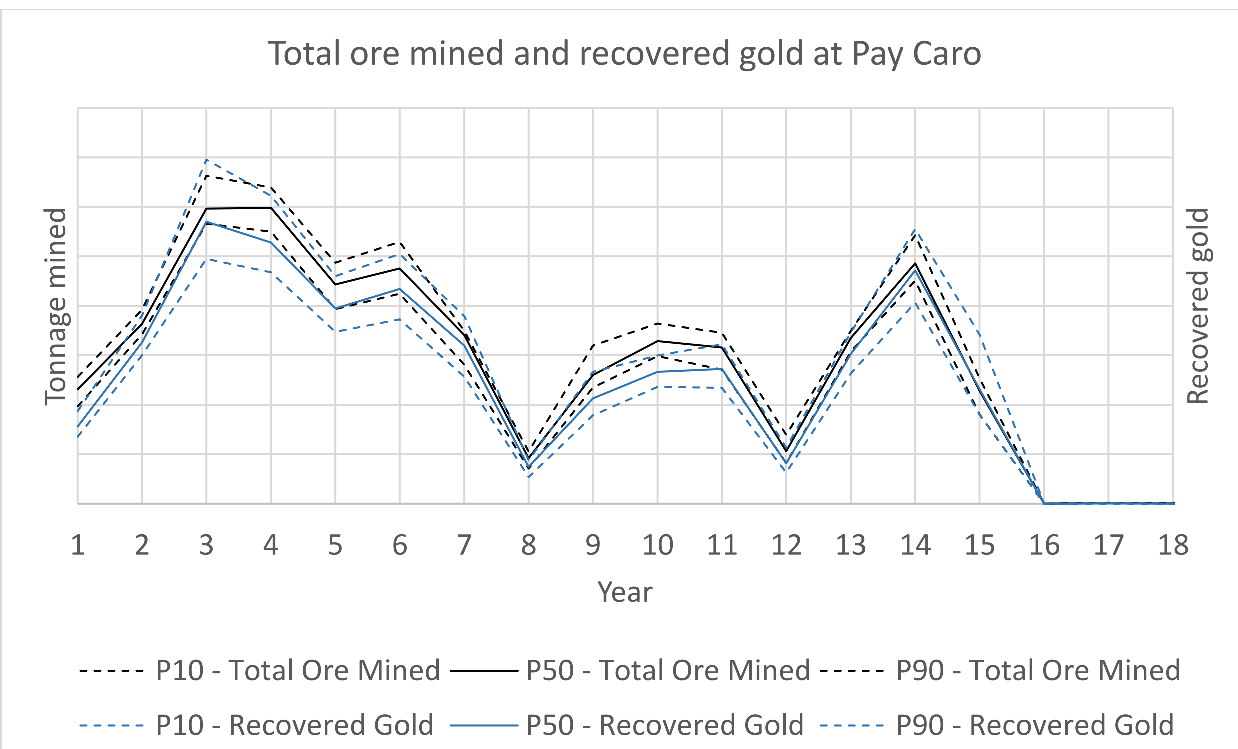


Figure 9 Ore tonnage mined and recovered gold at the Pay Caro Mine

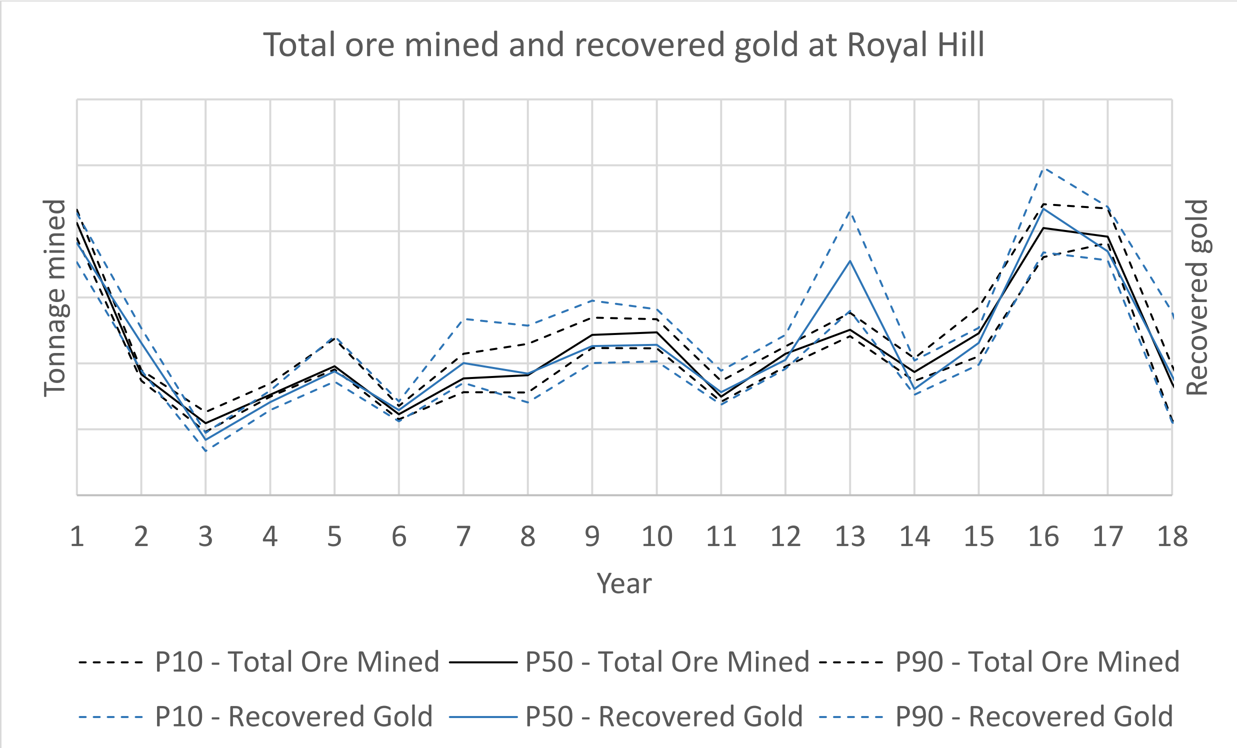


Figure 10 Ore tonnage mined and recovered gold at the Royal Hill Mine

The extracted ore material can be sent to either the appropriate stockpile or directly to the processor. Figure 11 displays the amount of material stockpiled throughout the life-of-mine. The laterite-saprolite material is stockpiled to a greater degree in the earlier years and is reclaimed over time. Towards the end of the mining complex life, the hard rock material is stockpiled to a greater extent, while the transition material is rarely stockpiled. The ore reclaimed from the stockpiles or sent directly to the processor is crushed at the SAG mill. Each material type has a different throughput rate at the SAG mill, based on the material hardness. As such, a constraint is placed on the SAG mill availability rather than a tonnage capacity (Figure 12). The SAG mill is used to capacity however the processor throughput capacity (Figure 13) and the mining capacity (Figure 6) are not reached throughout the long-term plan of the mining complex, documenting that the SAG mill is a bottleneck for the mining complex.

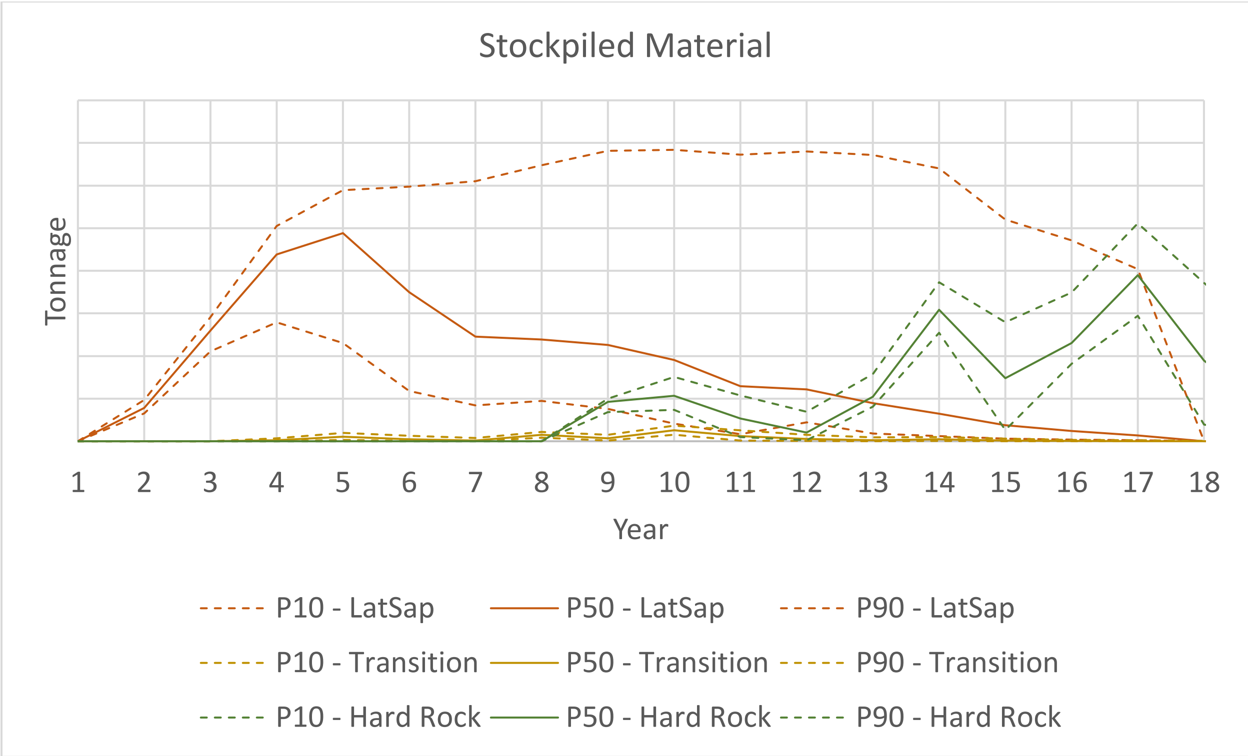


Figure 11 Stockpiled material at the RGM mining complex

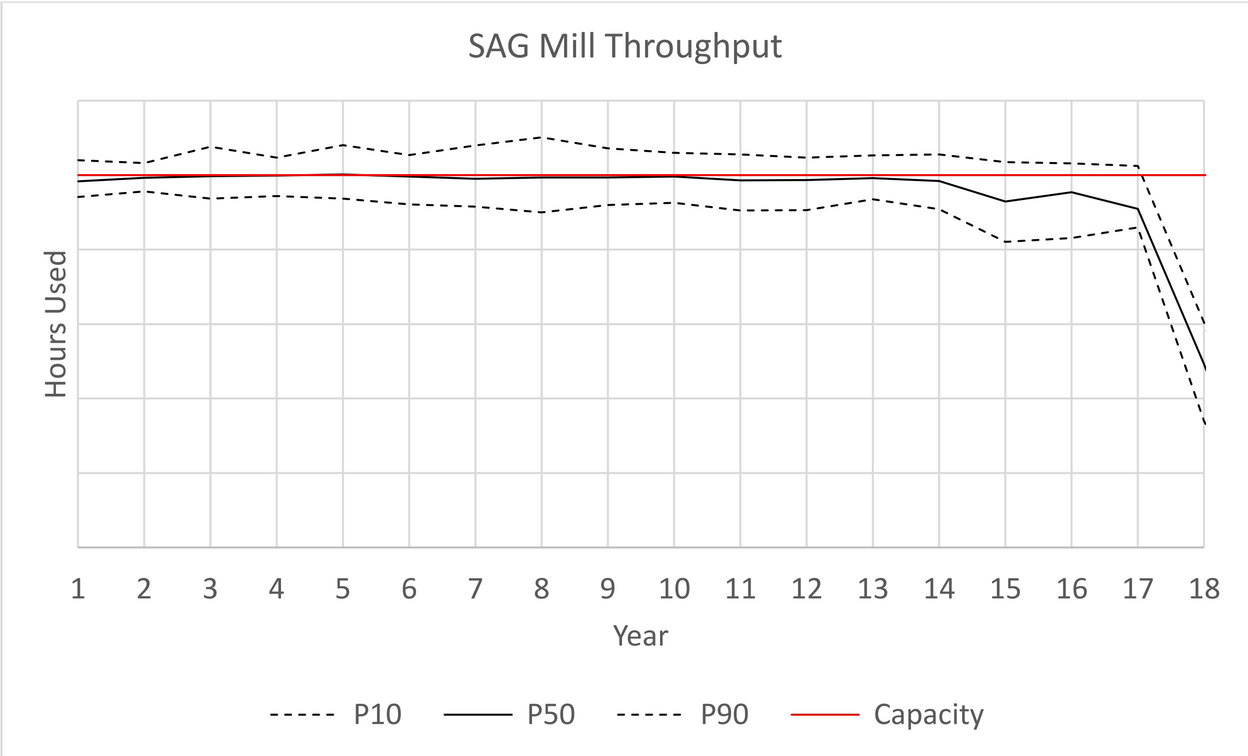


Figure 12 SAG mill utilization

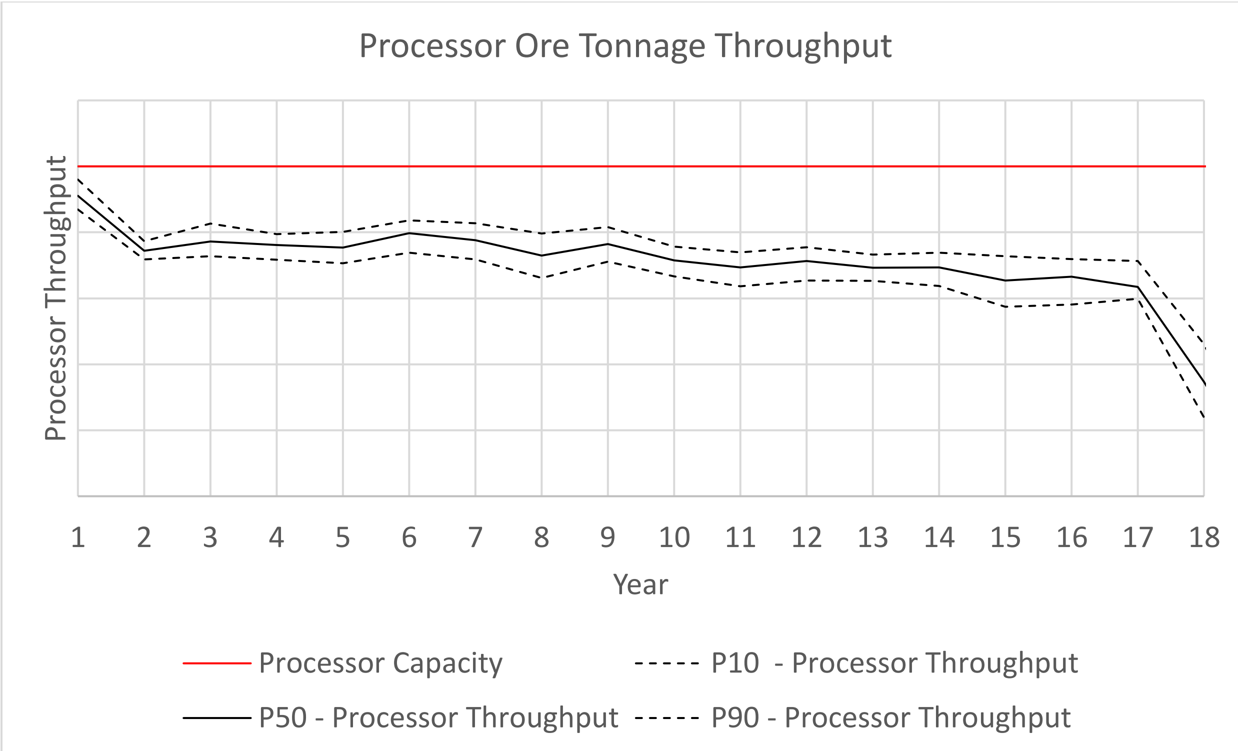
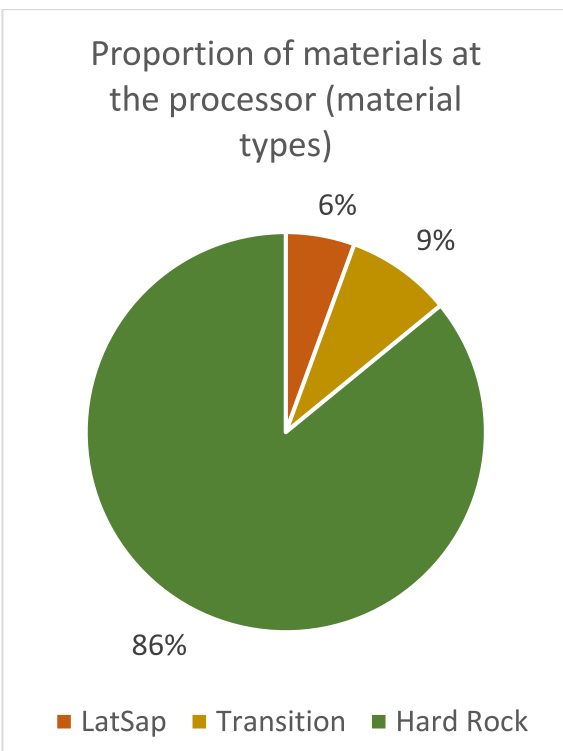


Figure 13 Processor ore tonnage throughput



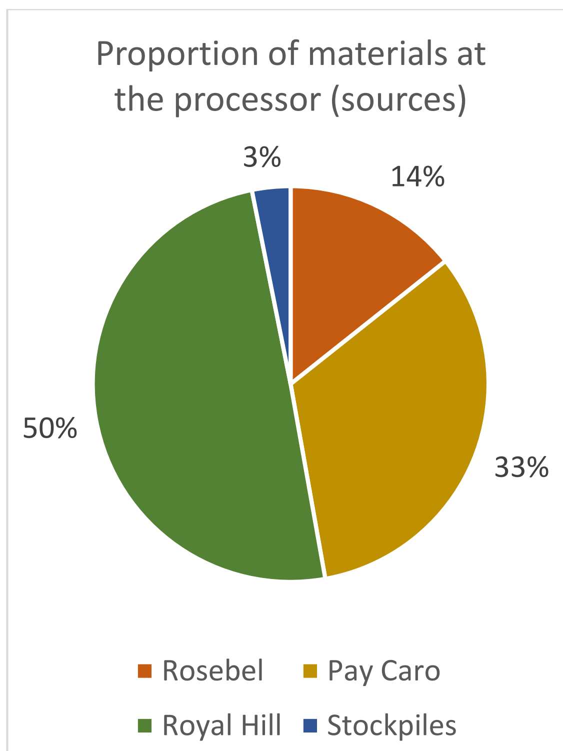


Figure 14 Proportion of different material types (left) and material from different sources (right) at the processor

Figure 14 shows the proportion of material at the processor by material type and by source. Most of the ore sent to the processor is hard rock, which has a significantly lower SAG mill throughput than the laterite-saprolite and transition materials. Indeed, each RGM deposit has a mined-out portion which has depleted the softer material types, leaving only harder material and contributing to the SAG mill utilization. The early stockpiling and reclamation of the laterite-saprolite material (Figure 11) will assist the SAG mill utilization. Regarding the source of the ore sent to the processor, the Royal Hill Mine provides most of the material, followed by the Pay Caro Mine, the Rosebel Mine, and stockpile reclamation. Figure 15 shows the proportion of recovered gold by material type and by source. Though the trends are similar to those shown in Figure 14, it can be noted that 92% of the gold is recovered from the hard rock material while that material only constitutes 86% of the processor feed. The hard rock material has the lowest recovery rate of the three material types as well as the highest mining and processing costs. Similarly, 55% of the gold is recovered from Royal Hill ore, whereas Royal Hill ore makes up only 50% of the processor feed. This reflects that Royal Hill's material is richer in gold than the other deposits. Finally, Figure 16 presents the cut-off grades obtained from the simultaneous stochastic optimization process, using Royal Hill's hard rock material as an example. The cut-off grades of the simultaneous stochastic optimization study are outputs of the optimization process.

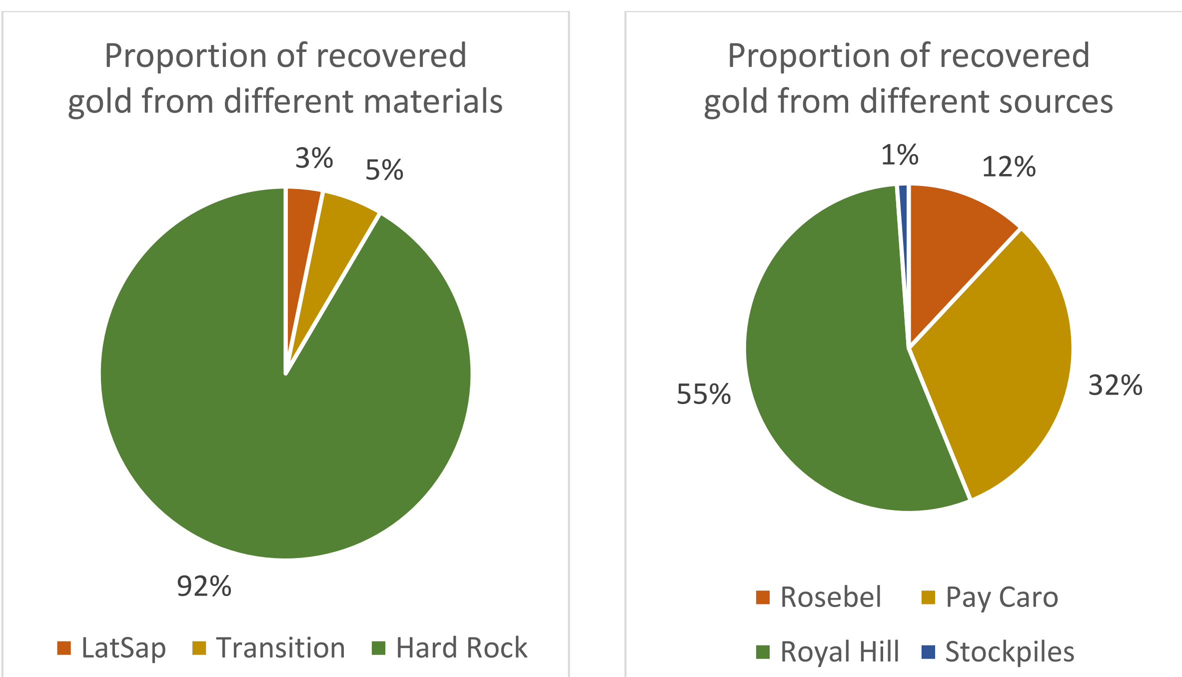


Figure 15 Proportion of recovered gold from different material types (left) and sources (right)

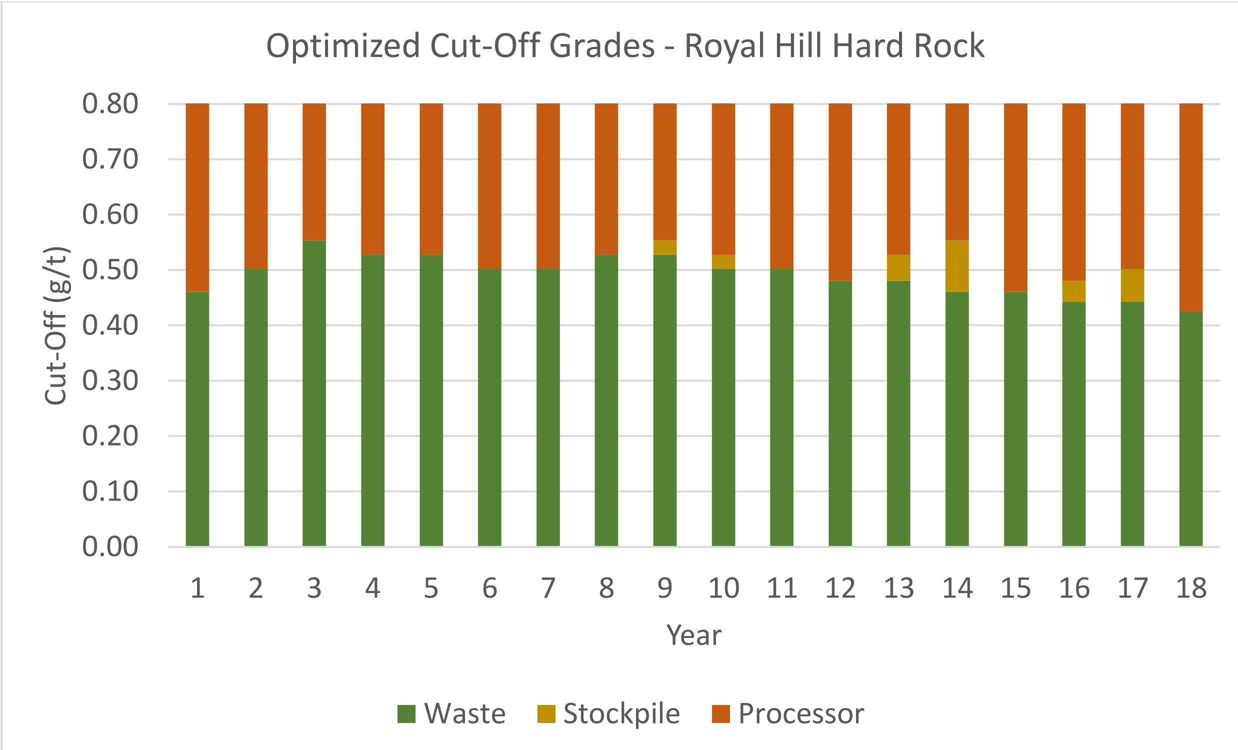


Figure 16 Royal Hill Mine cut-off grade policy

2.4. Conclusions

An application of simultaneous stochastic optimization at the RGM mining complex is presented herein. The mining complex includes three deposits, three stockpiles, one processor and one waste dump. The case study maximized NPV and gold production over the life of the mining complex. It generated production schedules for the three deposits, indicating that Royal Hill would be mined to a greater extent than Pay Caro and Rosebel, as well as stockpile management plans for the three stockpiles considered. The SAG mill was determined to be the bottleneck of the operation, with a 100% utilization rate throughout the life-of-mine. Future work could consider the incorporation of more components of the RGM mining complex, such as additional deposits and mine-to-mill transportation scheduling, as well as the incorporation of capital investment options to reduce the effect of the identified bottleneck. Future work could also consider stochastic simulations of the orezone boundaries as well as densities and hardness to better represent the deposits and the utilization of the SAG mill.

3. Simultaneous Production Scheduling and Transportation Optimization from Mines to Port under Uncertain Material Supply

3.1. Introduction

Industrial mining complexes or mineral value chains consist of various components such as mines, stockpiles, waste dumps, transportation systems, and processing facilities, among others. The simultaneous stochastic optimization (SSO) approach for long-term scheduling capitalizes on the synergies of these components to generate an optimal production schedule while managing technical risks, using several material supply (material deposit) simulations (Goodfellow and Dimitrakopoulos 2016, 2017; Montiel and Dimitrakopoulos 2015, 2018). The SSO approach is an extension of previous stochastic integer programming (SIP) approaches, optimizing a single mine under material supply uncertainty (Ramazan and Dimitrakopoulos, 2007, 2013; Dimitrakopoulos and Ramazan, 2008). However, mine-to-port transportation might be important for the extraction of certain commodities, yet it is not included in the SSO approaches. Indeed, in some situations, such as in iron ore mining complexes, the mine-to-port transportation is a key element ensuring that the products extracted at the mines reach their respective clients. These types of complexes can include several mines, stockpiles, and ports connected by complex railway systems, accounting for material supply uncertainty, a critical source of technical (geological) risk (Baker and Giacomo, 1998; Vallée, 2000). The interactions between the locations and the mine-to-port transportation system can be included in the optimization process to ensure that the value of the operation is maximized (Everett, 2001) while managing technical risk (Gomes Leite et al., 2019).

Developments in mine-to-port transportation scheduling optimization have been limited to short-term production planning where the mine extraction schedules are optimized beforehand and exclude sources of uncertainty. Liu and Kozan (2011) propose a model to schedule trains on a single-track railway connecting two mines to a port using a job-shop problem representation. Singh et al. (2014) present a model optimizing the mine-to-port transportation scheduling over the medium term for a large iron ore mining complex. These two models require a pre-determined extraction schedule at the mines and, therefore, a fixed tonnage and product quality to be transported by the mine-to-port transportation system. This approach ignores the interdependencies of the two components of the mining complex: a change in the mine-to-port transportation system's schedule affects the amount and quality of material that can be delivered, in turn affecting which material should be extracted at a given time, and vice versa. Additionally, neither study considers uncertainty in the amount or quality of the extracted material, further limiting the reliability of the optimization forecasts. Bodon et al. (2011) propose a combination of optimization and a discrete event simulation method to optimize the extraction sequence, the mine-to-port transportation scheduling, and the port operations simultaneously. The approach is applied to a case study to analyze different capital expenditure options and operation modes, evaluating their impact on the quantity and quality of delivered products. The approach successfully combines mine production and mine-to-port transportation scheduling in a single model.

Belov et al. (2020) develop a method for the short-term scheduling of trains and vessels as well as port stockpiles management in order to guide long-term infrastructure capacity planning. This approach allows the model to maintain the level of detail of a short-term optimization while covering a longer scheduling period. The approach does not include extraction scheduling at the mines, nor does it directly incorporate sources of uncertainty. Montiel and Dimitrakopoulos (2015) incorporate transportation alternatives for material output from processing facilities in long-term production scheduling within a stochastic optimization framework. These alternatives allow some flexibility in managing the transportation equipment by determining the proportion of material types being transported by the different methods. However, these proportions are predetermined.

There is no attempt in the technical literature to integrate mine-to-port transportation scheduling into the overall long-term production optimization framework for mining complexes under supply uncertainty. As such, this work presents a general long-term stochastic integer programming model incorporating mine-to-port transportation constraints into the mine production scheduling optimization under material supply uncertainty. This uncertainty is represented by stochastic simulations of the pertinent attributes (e.g. grade and material types) of the related mineral deposits (Goovaerts, 1997; Boucher and Dimitrakopoulos, 2009). The proposed SIP model includes multiple mines, stockpiles, waste dumps, loading facilities, different transportation system layouts, and a single port. Extracted materials can be sent to stockpiles, waste dumps, or to the port via the mine-to-port transportation system. At the port, fixed yearly demand (quantity and quality) for the multiple products is considered for the material extracted from the related mines. The overall aim of the model is to minimize the costs associated with meeting the product demand at the port as well as to manage the risks associated with meeting these targets. The model presented herein produces long-term extraction schedules for the related mines, as well as a schedule for the mine-to-port transportation equipment utilization. This schedule can be used to guide overall strategic mine planning decisions. In the following sections, the proposed mathematical programming model is presented, as well as an overview of the solution method. Then, a case study for a two-mine, single-port iron ore mining complex is presented. Finally, conclusions and directions for future work are given.

3.2. Method

This section outlines the model developed to simultaneously optimize the long-term mine scheduling and the mine-to-port transportation for an iron ore mining complexes accounting for uncertain material supply. The proposed model is developed for mining complexes with a single port at which a yearly demand is specified for several products.

3.2.1. Definitions and Notations

This section includes the definitions and the notation used to specify the proposed SIP model.

3.2.1.1. Indices and Sets

s: Stochastic orebody simulation, $s \in S$

t: Time period, $t \in T$

i, j: Nodes in the graph representing the mining complex, $i, j \in N$

m: Mine, $m \in \mathcal{M}$

ℓ: Loading area at a mine m, $\ell \in \mathcal{L}_m \subset N$

h: Stockpile at a mine m, $h \in \mathcal{H}_m \subset N$

w: Waste dump at a mine m, $w \in \mathcal{W}_m \subset N$

d: Destination at a mine m, $d \in D_m = \mathcal{L}_m \cup \mathcal{H}_m \cup \mathcal{W}_m$

b: Mining block at a mine m, $b \in B_m$

r: Final product, $r \in R$

e: Element making up the final products, $e \in E$

w: Mine-to-port transportation equipment, $w \in W$

θ: Path, starting and ending at the port, followed by mine-to-port transportation equipment, $\theta \in \Theta$

$\mathbb{P}_b$: Set of extraction predecessors of block b

$\mathbb{W}_b^{smooth}$: Set of blocks within block b smoothing window

$\mathcal{S}_h$: Randomized order in which the blocks sent to stockpile h can be reclaimed

3.2.1.2. Parameters

M: big M (scalar with large value)

d: Economic discount rate

$\mathcal{D}$: Geological risk discount rate

c_m^{mine}: Cost of extracting a block at mine m

c_d^{trans}: Cost of transporting material from mine m to a mine destination d

c_h^{rec}: Material reclamation cost in stockpile h

c_w^{fixed}: Fixed cost associated with using equipment w

c_θ^{path}: Travel cost of equipment on path θ

c_i^{load}: Loading cost at node i

c_0^{load}: Unloading cost at port

c^{cap}: Cost of using equipment under capacity

c_r^{O-}, c_r^{O+}: Penalty cost of deviating below or above, respectively, from the ore tonnage demand for product r

c_r^{e-}, c_r^{e+}: Penalty cost of deviating below or above, respectively, from the element e grade demand for product r

c^{smooth}: Penalty cost associated with mining blocks within block b's smoothing radius in different periods t

Q: Tonnage of a block

Q_d^{dest}: Capacity of mine destination d

Q_{mt}^{min}: Minimum mining rate at mine m in period t

Q_{mt}^{max}: Maximum mining rate at mine m in period t

Q_w: Single-trip capacity of equipment w

H_{wt}^{time}: Maximum time available to equipment w in period t

H_{rt}^{ore}: Ore tonnage demand of product r in period t

H_{rt}^{e-}, H_{rt}^{e+}: Lower and upper bound, respectively for element e content in product r in period t

ψ_{bs}^{e}: Grade of element e in block b in geological scenario s

o_{ij}^{θ}: Indicates whether or not arc (i, j) is included in path θ

α_{ij}: Indicates whether or not nodes i and j are connected in the graph

τ_{ij}: Time required to travel from node i to node j

Q_{ij}: Maximum number of equipment which can travel on arc (i, j) in a period

T: Time required to load a tonne of material onto a piece of equipment

n^{trips}: Minimum number of trips to be completed by a piece of equipment per period

3.2.1.3. Decision Variables

3.2.1.3.1. Discrete Decision Variables

x_b^{td}: Indicates whether or not block b is extracted and sent to destination d in period t

$\xi_b^{t\hbar}$: Indicates whether or not block b is reclaimed from stockpile $\hbar$ in period t

ρ_b^{tr}: Indicates whether or not block b is assigned to product r in period t

$\mathcal{O}_{wt}$: Indicates whether or not equipment w is used in period t

z_θ^{wt}: Number of times equipment w travels on path θ in period t

3.2.1.3.2. Continuous Decision Variables

$y_{w\theta b}^t$: Proportion of block b loaded onto equipment w travelling on path θ in period t

$d_{t\theta w}^-$: Unused capacity of equipment w travelling on path θ in period t

d_{rt}^{O-}, d_{rt}^{O+}: Deviations below or above, respectively, from the ore demand target of product r in period t

$d_{rst}^{e-}, d_{rst}^{e+}$: Deviations below or above, respectively, from the element e grade target of product r in period t and scenario s

d_{bt}^{smooth}: Number of blocks in block b smoothing radius which are mined in a different period t

3.2.2. Optimization Model

3.2.2.1. Objective Function

$$
\min\left(\overbrace{\underbrace{\sum_{t\in T}\sum_{m\in\mathcal{M}}\sum_{d\in D_m}\sum_{b\in B_m}\frac{c_m^{mine}Qx_b^{td}}{(1+d)^t}}_{Part\ I}+\underbrace{\sum_{t\in T}\sum_{m\in\mathcal{M}}\sum_{d\in D_m}\sum_{b\in B_m}\frac{c_d^{trans}Qx_b^{td}}{(1+d)^t}}_{Part\ II}+\underbrace{\sum_{t\in T}\sum_{m\in\mathcal{M}}\sum_{\hbar\in\mathcal{H}_m}\sum_{b\in B_m}\frac{c_\hbar^{rec}Q\xi_b^{t\hbar}}{(1+d)^t}}_{Part\ III}}^{Section\ I}\right.
$$

$$
+\overbrace{\underbrace{\sum_{t\in T}\sum_{w\in W}\frac{c_w^{fixed}\mathcal{O}_{wt}}{(1+d)^t}}_{Part IV}+\underbrace{\sum_{t\in T}\sum_{w\in W}\sum_{\theta\in\Theta}\frac{c_\theta^{path}z_\theta^{wt}}{(1+d)^t}}_{Part\ V}}^{Section\ II}
$$

$$
+\overbrace{\underbrace{\sum_{t\in T}\sum_{m\in\mathcal{M}}\sum_{\ell\in\mathcal{L}_m}\sum_{b\in B_m}\frac{(c_\ell^{load}+c_0^{load})Qx_b^{t\ell}}{(1+d)^t}+\sum_{t\in T}\sum_{m\in\mathcal{M}}\sum_{\hbar\in\mathcal{H}_m}\sum_{b\in B_m}\frac{(c_\hbar^{load}+c_0^{load})Q\xi_b^{t\hbar}}{(1+d)^t}}_{Part\ VI}+\underbrace{\sum_{t\in T}\sum_{w\in W}\sum_{\theta\in\Theta}\frac{c^{cap}d_{t\theta w}^-}{(1+\mathcal{D})^t}}_{Part\ VII}}^{Section\ II} \tag{7}
$$

$$
\left.+\overbrace{\underbrace{\sum_{t\in T}\sum_{r\in R}\frac{c_r^{O-}d_{rt}^{O-}+c_r^{O+}d_{rt}^{O+}}{(1+\mathcal{D})^t}}_{Part\ VIII}+\underbrace{\frac{1}{S}\sum_{s\in S}\sum_{t\in T}\sum_{r\in R}\sum_{e\in E}\frac{c_r^{e-}d_{rst}^{e-}+c_r^{e+}d_{rst}^{e+}}{(1+\mathcal{D})^t}}_{Part\ IX}}^{Section\ III}+\overbrace{\underbrace{\sum_{t\in T}\sum_{m\in\mathcal{M}}\sum_{b\in B_m}\frac{c^{smooth}d_{bt}^{smooth}}{(1+\mathcal{D})^t}}_{Part\ X}}^{Section\ IV}\right)
$$

The proposed model is a two-stage stochastic integer program (SIP); its objective function (7) is a minimization function aiming to reduce mining and mine-to-port transportation costs as well as to reduce the risks associated with meeting product demand at the port. The objective function has four main sections. Section I minimizes the overall production scheduling costs at the mines: Part I involves the extraction costs of blocks at the mines; Part II involves the transportation costs to mine destinations; and Part III involves the stockpile reclamation costs. Section II includes the mine-to-port transportation costs: Part IV involves the equipment fixed cost; Part V involves the equipment's path-dependent travels costs; Part VI involves the equipment's loading and unloading costs at the different locations; and Part VII involves the cost of underutilizing equipment. Section III is related to the risk of deviating from product demand targets at the port: Part VIII involves the ore tonnage product demand target deviation penalty costs; and Part IX involves the ore product quality target deviation penalty costs. Section IV aims to generate a mineable schedule by ensuring a minimum mining width. The model aims to mine block b in the same period as the blocks within a smoothing window; a penalty is applied to the blocks within this window not mined out (Dimitrakopoulos and Ramazan, 2004; Ramazan and Dimitrakopoulos, 2007, 2013). Parts VII to X include a geological discount rate (GDR), $\mathcal{D}$. Like the economic discount rate ($\mathcal{d}$) which reduces the value of costs over time, the GDR aims to reduce the cost of deviating over time. The inclusion of the GDR makes it more costly to deviate from targets in earlier periods than later periods, hence deferring the risk of not meeting production targets at the port (Dimitrakopoulos and Ramazan, 2004).

3.2.2.2. Constraints

3.2.2.2.1. Production Scheduling Constraints

$$\sum_{t\in T}\sum_{d\in D_m} x_b^{td} \leq 1 \qquad \forall m \in \mathcal{M}, b \in B_m \qquad (8)$$

$$\sum_{d\in D_m} x_b^{td} \leq \sum_{\tau\leq t}\sum_{d\in D_m} x_{\bar{b}}^{\tau d} \qquad \forall t \in T, m \in \mathcal{M}, b \in B_m, \bar{b} \in \mathbb{P}_b \qquad (9)$$

$$\sum_{b\in B_m} Q x_b^{td} \leq Q_d^{dest} \qquad \forall t \in T, m \in \mathcal{M}, d \in D_m \qquad (10)$$

Constraint (8) ensures that a block cannot be extracted more than once, and that it can only be sent to a single destination. Constraint (9) ensures that the slope constraints and that the block precedence are satisfied. Constraint (10) ensures that the amount of material sent to a destination does not exceed its capacity in any period.

$$\sum_{b \in B_m} \sum_{d \in D_m} Q x_b^{td} \leq Q_{mt}^{max} \qquad \forall t \in T, \mathcal{m} \in \mathcal{M} \tag{11}$$

$$\sum_{b \in B_m} \sum_{d \in D_m} Q x_b^{td} \geq Q_{mt}^{min} \qquad \forall t \in T, \mathcal{m} \in \mathcal{M} \tag{12}$$

Constraints (11) and (12) ensure that the maximum and minimum mining rates, respectively, at each mine are respected throughout the life of the operation.

$$\left|\mathbb{W}_b^{smooth}\right| \sum_{d \in D_m} x_b^{td} - \sum_{d \in D_m} \sum_{\bar{b} \in \mathbb{W}_b^{smooth}} x_{\bar{b}}^{td} \leq d_{bt}^{smooth} \qquad \forall t \in T, \mathcal{m} \in \mathcal{M}, b \in B_m \tag{13}$$

Constraint (13) counts the number of surrounding blocks which are mined in a different period; these blocks incur a cost in Section IV of the objective function (7). This ensures a certain connectivity between the mined blocks, producing a more mineable schedule.

3.2.2.2.2. Stockpile Constraints

$$\xi_b^{t\hbar} \leq \sum_{\tau < t} x_b^{\tau\hbar} \qquad \forall t \in T, \mathcal{m} \in \mathcal{M}, \hbar \in \mathcal{H}_m, b \in B_m \tag{14}$$

$$\sum_{\tau \leq t} \xi_b^{\tau\hbar} \geq \xi_{(b+1)}^{t\hbar} \qquad \forall t \in T, \mathcal{m} \in \mathcal{M}, \hbar \in \mathcal{H}_m, b \in \mathcal{S}_\hbar \tag{15}$$

Constraint (14) ensures that blocks are sent to a stockpile before they can be reclaimed. Constraint (15) implements a random block removal order policy. Indeed, when blocks are sent to a stockpile, they are randomly placed in a list indicating the order in which they are reclaimed. Accordingly, a first-in-first-out rule is applied in order to remove all the blocks introduced in previous periods before those introduced in the current one.

It should be noted that the above policy avoids the disadvantages associated with the assumptions of standard stockpile modelling approaches. The perfect blending approach assumes that all material in a stockpile is homogenous, while the perfect selection approach assumes that the material's location within a stockpile is well know. Typically, stockpiles are heterogeneous and highly variable, therefore neither assumption is realistic (Dirkx and Dimitrakopoulos, 2018). Moreover, the perfect blending approach requires non-linear constraints, adding significant complexity to the model which cannot be solved with linear programming commercial solvers. The random block removal order strategy applied to the stockpiles overcomes the previously listed disadvantages because it does not make assumptions about a stockpile's material, and it also ensures that the model remains linear.

3.2.2.2.3. Linking Constraints

$$\sum_{w\in W}\sum_{\theta\in\Theta} y_{w\theta b}^{t} = \sum_{\ell\in\mathcal{L}_m} x_b^{t\ell} + \sum_{\hbar\in\mathcal{H}_m} \xi_b^{t\hbar} \qquad \forall t\in T, m\in\mathcal{M}, b\in B_m \qquad (16)$$

$$x_b^{t\ell} \le \sum_{\hbar\in\mathcal{H}_m}\sum_{w\in W}\sum_{\theta\in\Theta} y_{w\theta b}^{t} o_{\ell\hbar}^{\theta} \qquad \forall t\in T, m\in\mathcal{M}, \ell\in\mathcal{L}_m, b\in B_m \qquad (17)$$

$$\xi_b^{t\hbar} \le \sum_{\ell\in\mathcal{L}_m}\sum_{w\in W}\sum_{\theta\in\Theta} y_{w\theta b}^{t} o_{\ell\hbar}^{\theta} \qquad \forall t\in T, m\in\mathcal{M}, \hbar\in\mathcal{H}_m, b\in B_m \qquad (18)$$

Constraint (16) ensures that the blocks sent directly to a loading area or that are reclaimed from a stockpile are loaded onto mine-to-port transportation equipment in the same period. Constraints (17) and (18) ensure that the blocks sent to a loading area or reclaimed from a stockpile are loaded onto equipment travelling on a path including that destination.

3.2.2.2.4. Mine-to-Port Transportation Constraints

$$\sum_{w\in W}\sum_{\theta\in\Theta} z_{\theta}^{wt}\left(o_{ij}^{\theta} + o_{ji}^{\theta}\right) \le Q_{ij}\alpha_{ij} \qquad \forall t\in T, i<j\in N \qquad (19)$$

Constraint (19) ensures that a path segment's capacity is respected. A path is defined as the route taken by mine-to-port transportation equipment traveling to the different locations within the mining complex. For the purpose of this model, each path starts and ends at the port. A path segment is defined as a portion of the path, connecting two different locations. Each segment has a maximum number of equipment traveling on it within a period.

$$\sum_{\theta\in\Theta}\sum_{i<j\in N}\tau_{ij}z_{\theta}^{wt}\left(o_{ij}^{\theta}+o_{ji}^{\theta}\right)+\sum_{m\in\mathcal{M}}\sum_{b\in B_m}\sum_{\theta\in\Theta}\mathrm{T}Qy_{w\theta b}^{t}\leq H_{wt}^{time}\qquad\forall t\in T,w\in W\tag{20}$$

Constraint (20) ensures that each piece of equipment has a limited availability time in each period, and that the resulting transportation schedule is operationally feasible. This constraint allows the inclusion of the planned equipment maintenance in the long-term schedule.

$$\sum_{m\in\mathcal{M}}\sum_{b\in B_m}Qy_{w\theta b}^{t}+d_{t\theta w}^{-}=Q_w z_{\theta}^{wt}\qquad\forall t\in T,w\in W,\theta\in\Theta\tag{21}$$

$$\sum_{t\in T}\sum_{w\in W}\sum_{\theta\in\Theta}y_{w\theta b}^{t}\leq 1\qquad\forall m\in\mathcal{M},b\in B_m\tag{22}$$

Constraint (21) ensures that the equipment's capacity is never exceeded. In addition, the unused capacity of the equipment in each period is penalized in the objective function (see Section 3.2.2.1). Note that the capacity constraint is specified over the total number of times the equipment is used. Constraint (22) ensures that a mining block cannot be transported more than once.

$$z_{\theta}^{wt}\leq M\mathcal{O}_{wt}\qquad\forall t\in T,w\in W,\theta\in\Theta\tag{23}$$

$$\mathcal{O}_{wt}n^{trips}\leq\sum_{\theta\in\Theta}z_{\theta}^{wt}\qquad\forall t\in T,w\in W\tag{24}$$

$$\mathcal{O}_{w(t+1)} \geq \mathcal{O}_{wt} \qquad \forall t \leq T-1, w \in W \qquad (25)$$

Constraint (23) ensures that the equipment fixed costs are paid by activating the binary variable $\mathcal{O}_{wt}$. Once it is activated, the equipment's use is subject to constraints (24) and (25). Constraint (24) ensures that used equipment will complete a minimum number of trips while constraint (25) ensures that once a piece of equipment is used in one period, it will continue to be used in the following periods. Together, these constraints reduce the number of equipment in use at any time.

3.2.2.2.5. Demand and Blending Constraints

$$\sum_{w\in W}\sum_{\theta\in\Theta} y_{w\theta b}^{t} = \sum_{r\in R}\rho_b^{tr} \qquad \forall t \in T, m \in \mathcal{M}, b \in B_m \qquad (26)$$

$$\sum_{m\in\mathcal{M}}\sum_{b\in B_m} Q\rho_b^{tr} + d_{rt}^{O-} - d_{rt}^{O+} = H_{rt}^{ore} \qquad \forall t \in T, r \in R \qquad (27)$$

$$\sum_{m\in\mathcal{M}}\sum_{b\in B_m} Q\rho_b^{tr}(\psi_{bs}^{e} - H_{rt}^{e+}) - d_{rst}^{e+} \leq 0 \qquad \forall s \in S, t \in T, r \in R, e \in E \qquad (28)$$

$$\sum_{m\in\mathcal{M}}\sum_{b\in B_m} Q\rho_b^{tr}(\psi_{bs}^{e} - H_{rt}^{e-}) + d_{rst}^{e-} \geq 0 \qquad \forall s \in S, t \in T, r \in R, e \in E \qquad (29)$$

Constraint (26) ensures that every block delivered to the port is assigned to a final product. Constraint (27) sets the deviations from the ore tonnage target for each product. Moreover, for each material uncertainty scenario considered, constraints (28) and (29) set the deviations from the upper and lower bound targets of the different elements considered. These constraints allow the optimization process to make the best decisions to reduce the overall risk of missing demand targets.

3.2.2.2.6. Integrality and Non-Negativity Constraints

$$x_b^{td} \in \{0,1\} \qquad \forall t \in T, m \in \mathcal{M}, d \in D_m, b \in B_m \tag{30}$$

$$\xi_b^{t\hbar} \in \{0,1\} \qquad \forall t \in T, m \in \mathcal{M}, \hbar \in \mathcal{H}_m, b \in B_m \tag{31}$$

$$\rho_b^{tr} \in \{0,1\} \qquad \forall t \in T, r \in R, m \in \mathcal{M}, b \in B_m \tag{32}$$

$$\mathcal{O}_{wt} \in \{0,1\} \qquad \forall t \in T, w \in W \tag{33}$$

$$z_\theta^{wt} \geq 0, integer \qquad \forall t \in T, w \in W, \theta \in \Theta \tag{34}$$

$$y_{w\theta b}^{t} \geq 0 \qquad \forall t \in T, w \in W, \theta \in \Theta, m \in \mathcal{M}, b \in B_m \tag{35}$$

$$d_{t\theta w}^{-} \geq 0 \qquad \forall t \in T, \theta \in \Theta, w \in W \tag{36}$$

$$d_{rt}^{O-}, d_{rt}^{O+} \geq 0 \qquad \forall t \in T, r \in R \tag{37}$$

$$d_{rst}^{e-}, d_{rst}^{e+} \geq 0 \qquad \forall s \in S, t \in T, r \in R, e \in E \tag{38}$$

$$d_{bt}^{smooth} \geq 0 \qquad \forall t \in T, m \in \mathcal{M}, b \in B_m \tag{39}$$

Constraints (30) to (34) enforce integrality on the variables while constraints (35) to (39) enforce non-negativity.

3.3. Case Study

The formulation presented in Section 3.2 is applied to study an iron ore mining complex where components of a mine-to-port transportation system consist of a railway system with a fleet of trains. An overview of the operation as well as key parameters are first introduced, and the results obtained are then presented.

3.3.1. Overview

This case study considers an iron ore mining complex with two mines and a single port; each mine has a waste dump, a loading area, and a stockpile, as shown in Figure 17. In the figure, the arrows depict the flow of the extracted material, and the railway tracks exhibit the existing railway system connecting the mines to the port. At the mines, a total of approximately 2,000 mining blocks are available, having dimensions of 25 m by 25 m by 12 m and a mass of 22,500 tonnes. Material supply uncertainty is included using fifteen geostatistically simulated scenarios (Boucher and Dimitrakopoulos, 2009, 2012) to quantify the uncertainty and variability of the five different elements considered: iron, silica, aluminum oxide, phosphorus, and loss-on-ignition (LOI). At the port, the demand for two products is considered. Each product is characterized by a fixed yearly tonnage target as well as product quality constraints for the elements considered, as shown in Table 4.

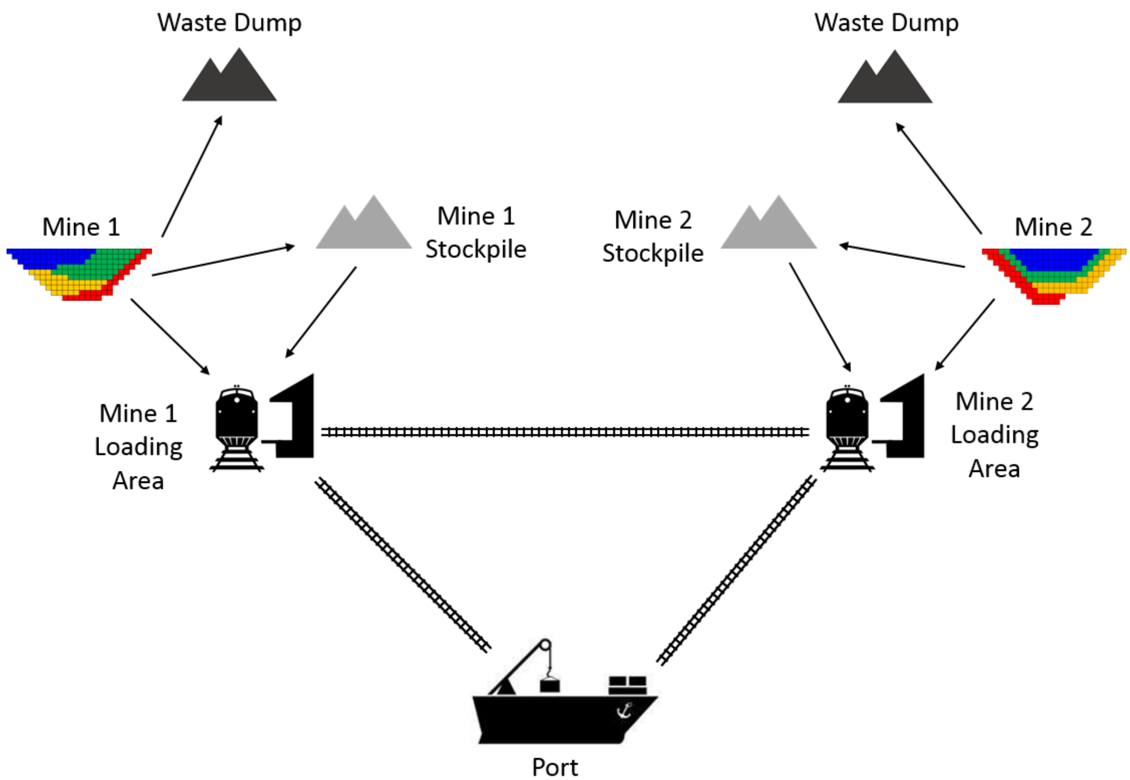


Figure 17 Components and layout of the mining complex

Table 4 Ore and grade targets for each product

	Year	Ore Tonnage	Fe (%)	SiO_2 (%)	Al_2O_3 (%)	P (%)	LOI (%)
Product 1	1	5,000,000	57.9-59.4	4.6-5.2	1-1.05	0.033-0.04	8.8-11
	2	5,000,000					
	3	4,000,000					
	4	4,000,000					
	5	4,000,000					
Product 2	1	4,000,000	57.1-58.5	4.9-5.5	0.9-1.05	0.031-0.038	9.5-13
	2	4,000,000					
	3	4,000,000					
	4	3,000,000					
	5	2,000,000					

The transportation system of the mining complex includes a fleet of two trains as described in Table 5. Each train is available for 6,300 hours per year. Three paths are available for the trains to follow when transporting material, as shown in Figure 18 and described in Table 6. The mine-to-port transportation costs depend on the travel distance between the locations on each path. The economic parameters used in the optimization model are listed in Table 7. For this case study, the transportation costs within the mine (therefore from a mining face to a loading area, a stockpile, or a waste dump) are the same for all destinations and for both mines.

Table 5 Fleet characteristics

Type	Number Available	Capacity per Trip (Tonnes)
I	1	23,000
II	1	32,000

Table 6 Train path definitions

Path	Definition	Cost per Trip ($)
I	Port – Mine 1 – Port	800
II	Port – Mine 2 – Port	640
III	Port – Mine 1 – Mine 2 – Port or Port – Mine 2 – Mine 1 – Port	960

Table 7 Economic parameters

Parameter	**Value**
Mining cost ($/t)	3
Transportation costs within the mine ($/t)	2
Reclamation costs ($/t)	0.1
Economic discount rate (%)	10
Geological risk discount rate (%)	12

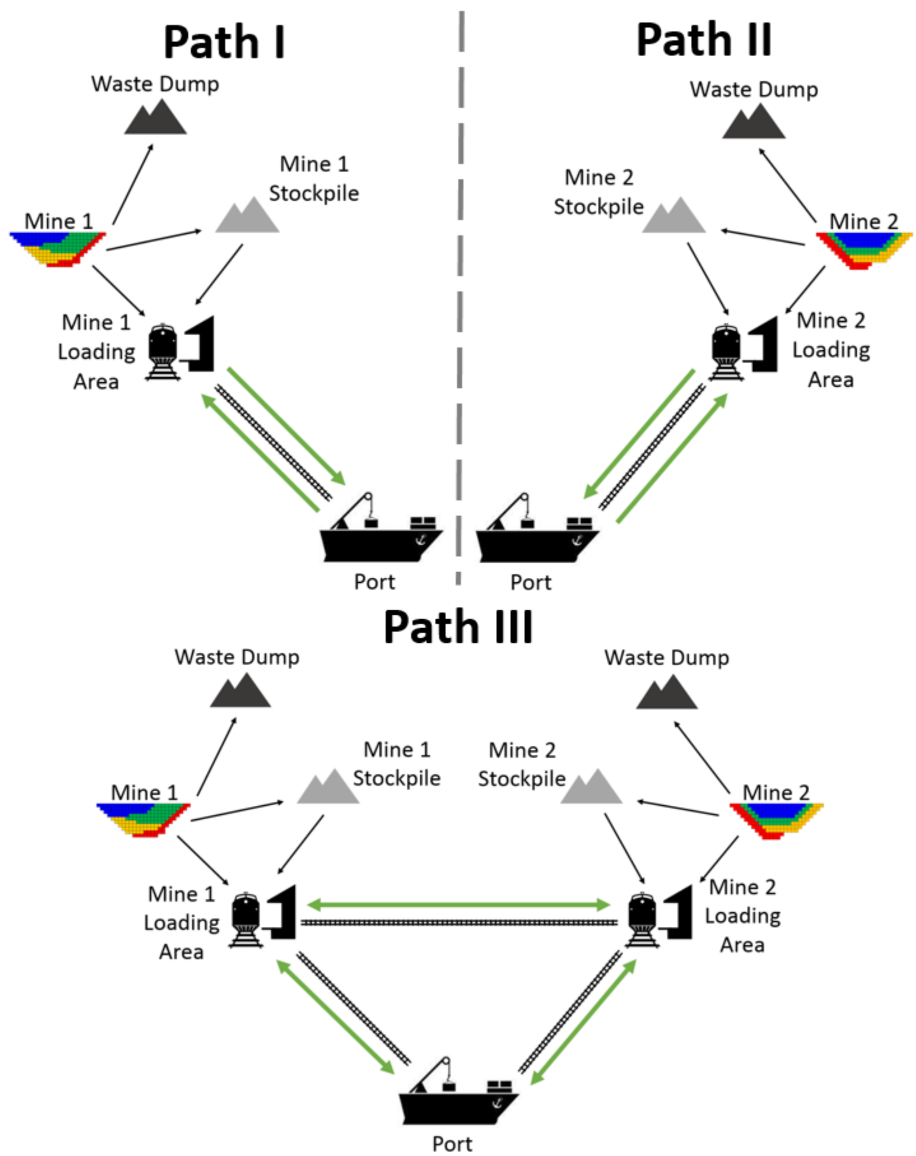


Figure 18 Representation of the possible paths taken by the mines-to-port trains

3.3.2. Results

The model in the case study described previously is solved using the branch and cut algorithm implemented in CPLEX v.12.6.1.0 in a Visual Studio 15 (C++) environment. The number of binary and integer variables (in the order of 70,000) and the number of constraints (in the order of 175,000) in the model are too large to obtain results in reasonable time and the rolling time horizon approach is applied (Dimitrakopoulos and Ramazan, 2008; Ramazan and Dimitrakopoulos, 2013). The time horizon chosen is two years, with a one-year overlap; each horizon is solved to an optimality gap less than 1%.

3.3.2.1. Production Schedules

Production schedules are generated for both mines, as shown in Figure 19. The cross-sections show that, during the first two years, Mine 2 is mined more extensively than Mine 1. Since the extraction and mine transportation costs are identical for both mines, the results indicate that either Mine 2 provides better supply to meet the product demand at the port, or that Path II's smaller cost (Table 6) induces less expensive extraction for Mine 2 in earlier years, or for both reasons.

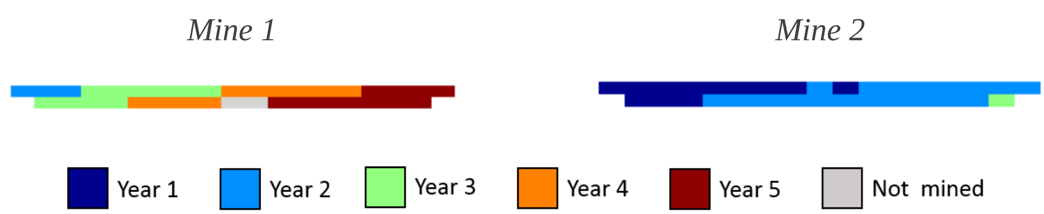


Figure 19 Cross-sections of the mining schedules for the two mines

Figure 20 compares the tonnage delivered at the port for each product relative to its demand. Note that the decisions relative to the distribution of the material between the products at the port are scenario-independent, therefore there are no risk profiles. For both products, the demand is well met, with deviations of less than 0.5%. Figure 21 to Figure 23 present the yearly forecasts (P50 of the results) for meeting each quality constraint for both products, along with the associated risk profiles (P10 and P90 of the related forecasts). The P10, P50 and P90 represent the 10%, 50% and 90%, respectively, probability of obtaining values below the corresponding amount. Note that the risk profiles shown are created using a set of simulations different than those used in the optimization process. Figure 21 presents the yearly iron grade of the final products along with the related risk profiles. Product 1's forecasted iron grades are well within the given bounds for years 1 through 4, however, a small deviation can be seen in year 5; the P10 value is slightly lower than the required lower bound. For Product 2, the forecasted iron grades are within the given bounds for years 2 to 5. In year 1, there is a slight deviation from the upper bound; the P90 value exceeds the upper bound limit marginally. Overall, the iron demand is expected to be well met for both products.

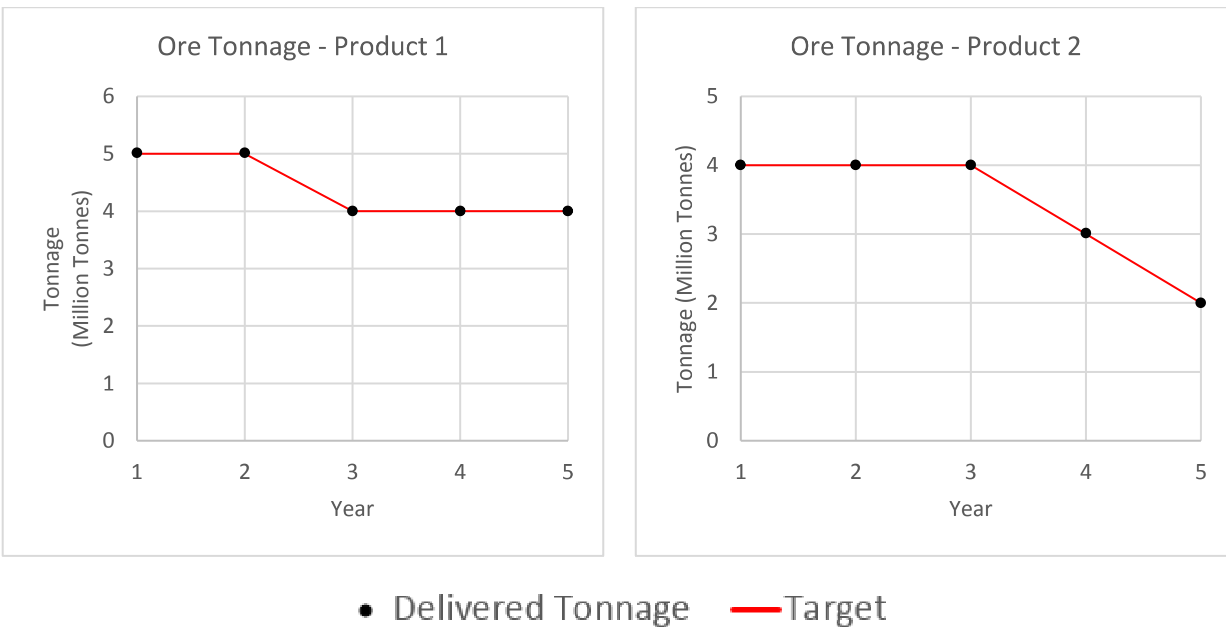


Figure 20 Yearly tonnage of products delivered to the port

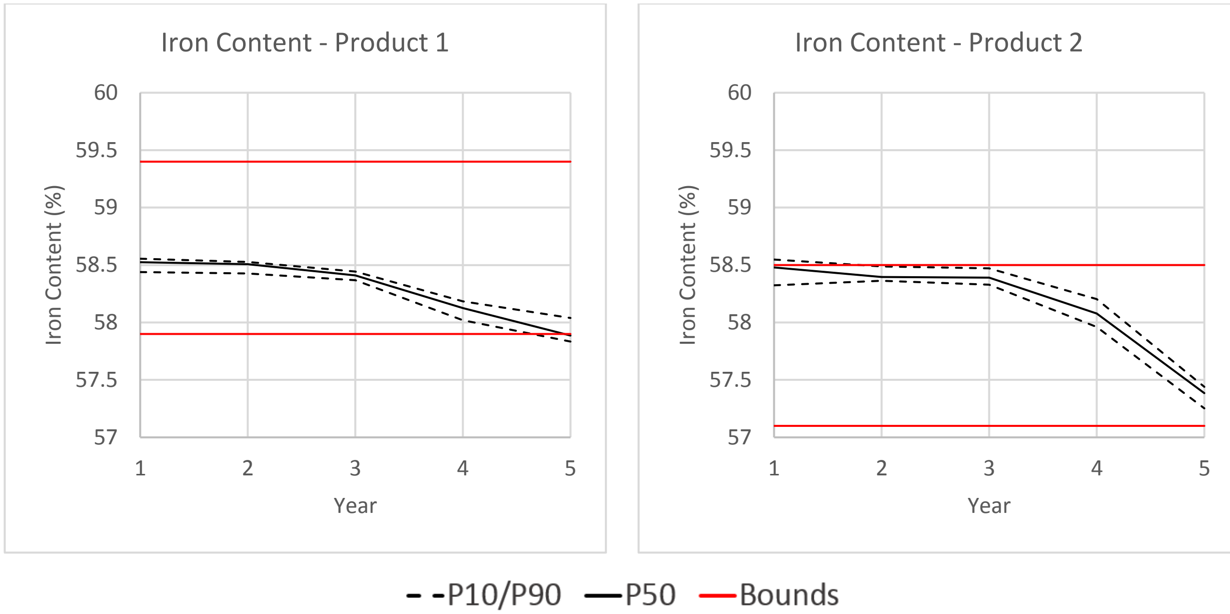


Figure 21 Yearly iron grade of products at the port

Figure 22 illustrates the yearly forecasted silica content of the final products along with the related risk profiles. Both products exhibit significant deviations from the upper bound. For Product 1, there are deviations during all years. In years 1 to 3, the deviations are relatively small (the P90 deviates less than 3%) before increasing in year 4 (the P90 deviates almost 10%) and reaching a maximum in year 5, where the P10, P50 and P90 deviate by approximately 20%. There are also deviations for Product 2 during years 3 to 5, and they are small (less than 1%) but increasing over time reaching a maximum in year 5 where they reach over 20%. These results indicate that, for the years in which there are larger deviations, the material that can be extracted from the deposit may not have the silica properties required to meet the demand for these products. Hence, blending the ore extracted from the deposits with ore from other sources may be necessary to meet demand. Additionally, Figure 22 illustrates the effects of the geologic discount rate (Section 2.2.1): the deviations, and thus the risk, are generally smaller in earlier years than in later years. Deferring risk to later years may allow an operation to consider other sources of material allowing to meet demand with higher certainty. Figure 23 shows the yearly forecasts and the related risk profiles of the alumina content of the final products. Product 1 exhibits minor forecasted deviations in all years except for year 2; the P10 deviates from the lower bound by less than 2% in those four years, and the P50 deviates by less than 1% in year 4. As for Product 2, minor forecasted deviations occur in periods 4 and 5, where the P10 deviates by less than 3%.

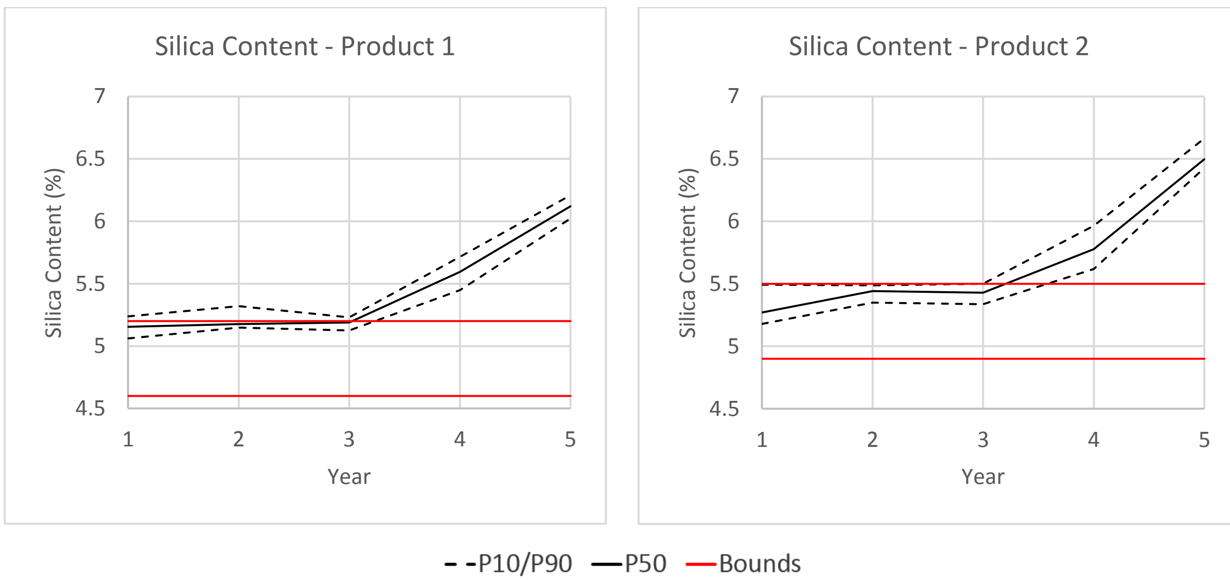


Figure 22 Yearly silica grade of products at the port

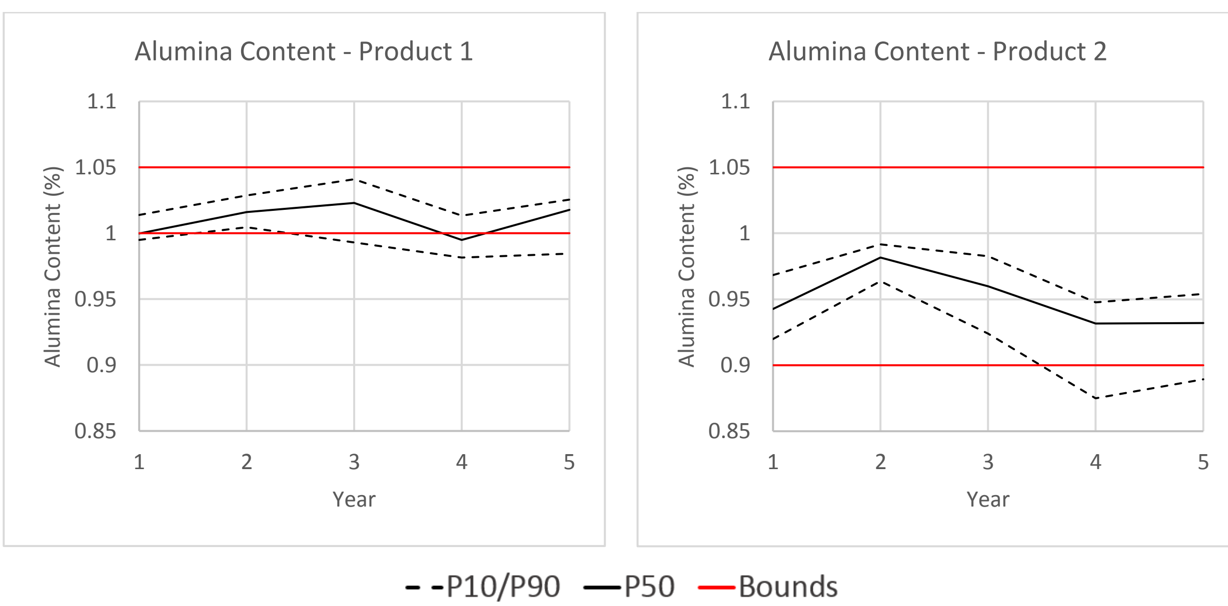


Figure 23 Yearly alumina grade of products at the port

Figure 24 illustrates the yearly forecasts and the related risk profiles of the phosphorus grade of the final products. For Product 1, the phosphorus grades are within the bounds in all years at the exception of year 4, where the P10 deviates marginally from the lower bound. Also, for Product 2, the phosphorus grades are well within the bounds during all years, with no forecasted deviations. The risk profiles for the phosphorus grades of each product are very tight; i.e., there is very little difference between the P10, P50, and P90 values. Figure 25 shows the yearly forecasts and the risk profiles of the loss on ignition (LOI) of the final products. There are no forecasted deviations during any years for either product, and therefore the demand is expected to be met.

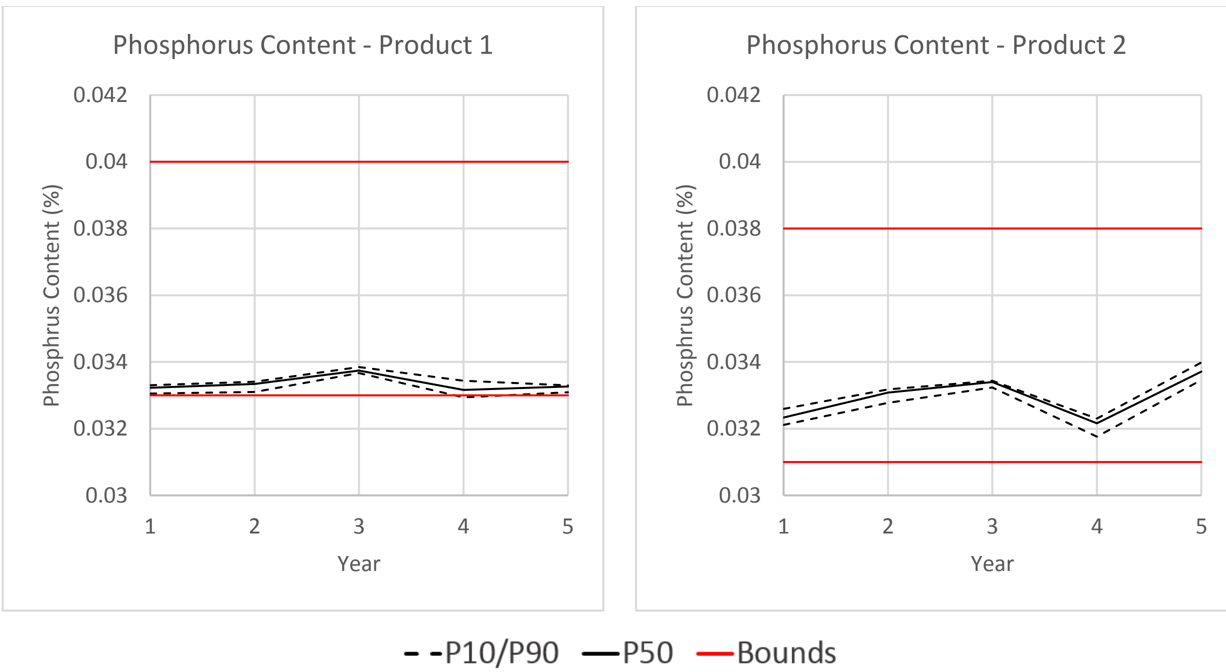


Figure 24 Yearly phosphorus grade of products at the port

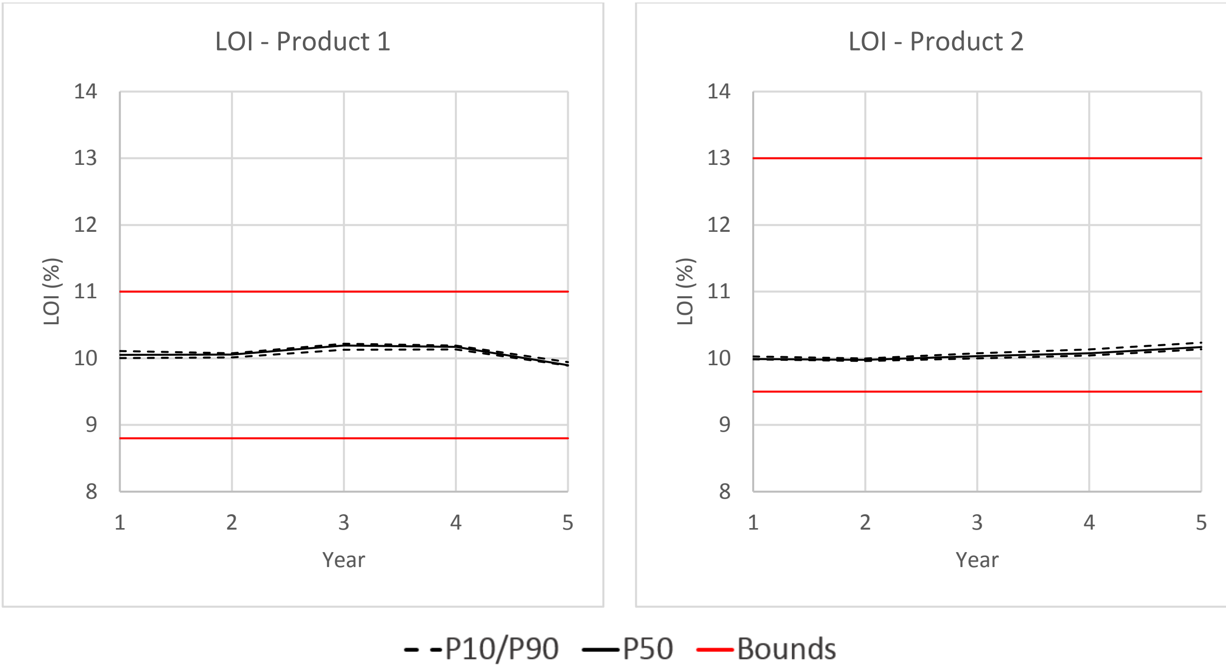


Figure 25 Yearly LOI of products at the port

The origin of the material included in the final products is presented in Figure 26. Most of the material delivered to the port is taken directly from the mines, with limited contribution from the stockpiles. Thus, the stockpile material left could be used in the future to deal with future demand. Figure 26 highlights the need to optimize all components of a mineral value chain simultaneously. Indeed, both products require blending material from both mines; the demand would not have been met as well if the mines had been optimized individually.

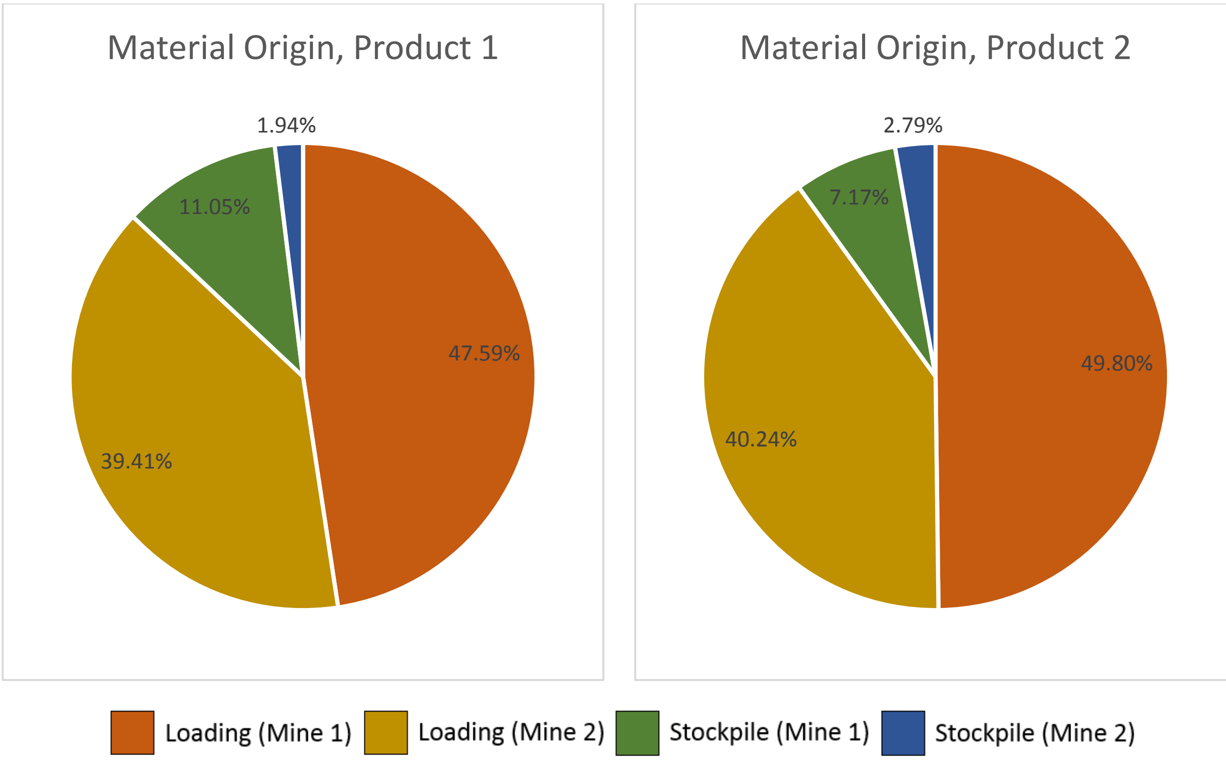


Figure 26 Provenance of material making up the products at the port

3.3.2.2. Transportation Schedules

The optimization process provides a schedule for the mine-to-port transportation fleet indicating the forecasted use of each train each year. Figure 27 shows the number of trips for each path completed by each train yearly. Path III is not shown as it was used only once by Train 2 in year 3. Moreover, only the largest train (Table 5) is used during the first 3 years, and it is used to a greater extent than Train 1 in periods 4 and 5. These results indicate that the smaller train can be allocated to other operations during this time, or, in the case of a future project, that it should not be purchased before year 4. This result highlights the need to incorporate mine-to-port transportation scheduling into the optimization of the mining complex. Indeed, traditionally if only the fixed yearly capacity of the mine-to-port transportation system is specified, then only a percentage use of the system is obtained rather than the expected utilization of each equipment considered. In addition, Figure 27 indicates that Train 1 is scheduled to use Path II to a higher extent in years 1 and 2 since, as mentioned in the previous section, Mine 2 is scheduled to produce more material than Mine 1 in this period. When the production shifts over to Mine 1 during years

3 to 5, the number of trips on Path I increases accordingly. Figure 28 summarizes the yearly number of trips on all paths completed by each train. It also indicates that the number of trips decreases over time, accordingly to the decrease in total demand and the delivered tonnage for the products at the port over time (Table 4 and Figure 20).

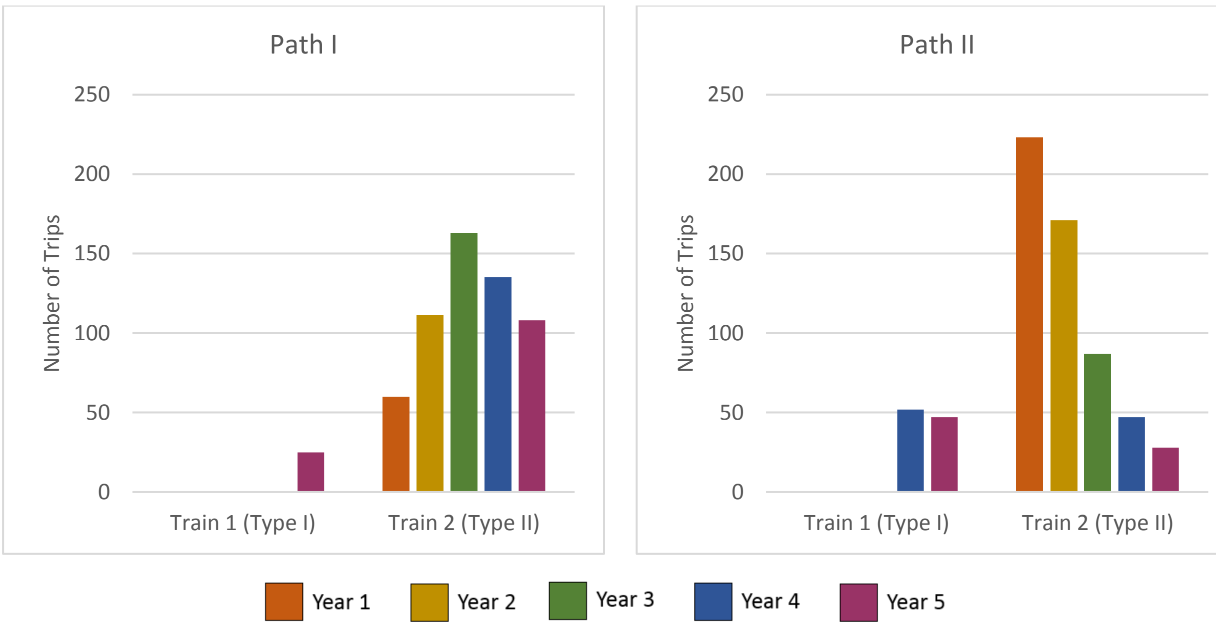


Figure 27 Train use per path and year

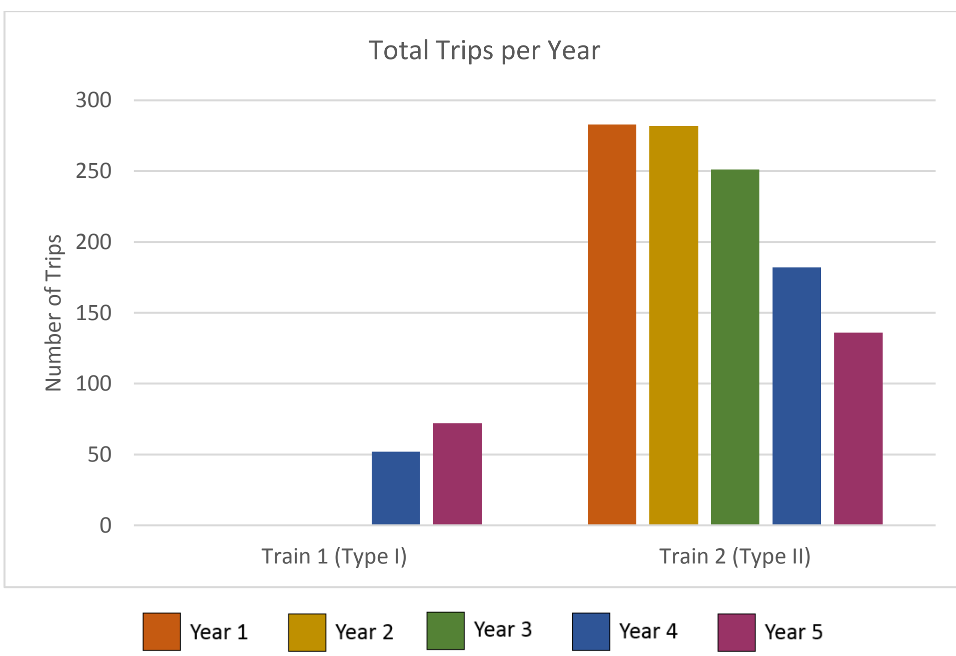


Figure 28 Overall number of trips completed by each train

Finally, Figure 29 allows the comparison of the total capacity of the trains available (obtained by the product of the number of trips completed and the single-trip capacity (Table 5) and the yearly amount of material transported. As shown in the figure, the trains are being used near capacity during each year. This follows from penalizing the unused capacity in the objective function (7) as well as from reducing the overall cost. Simultaneously optimizing the mine-to-port transportation schedule and the production schedules allows the adjustment of the transportation schedule according to the tonnage of material extracted at each mine in order to reduce train travel only as required to maximize their use.

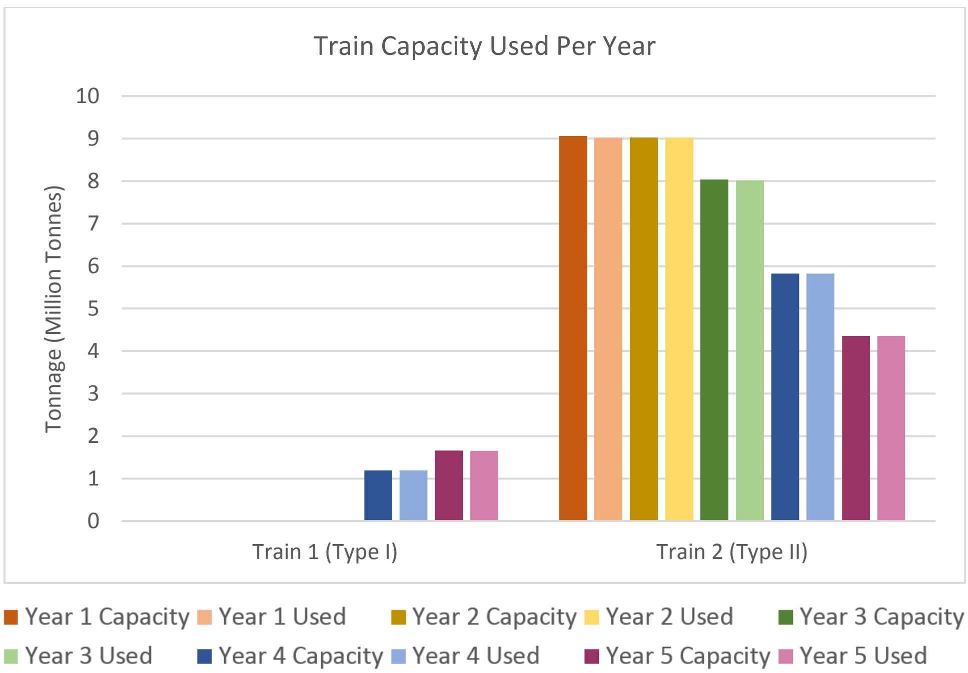


Figure 29 Summary of the capacity of each train used

3.4. Conclusions

In this paper, a new stochastic mixed integer program is formulated to simultaneously optimize the schedule of the production and the mines-to-port transportation of mining complexes under uncertain material supply. This formulation can include different numbers of mines, stockpiles, waste dumps, loading areas, trains, and railway layouts; however, only a single port is allowed. The mining complex is subject to multiple operating constraints related to capacities, transportation, and to the geochemical blending of uncertain material supply to meet product targets at the port. A case study is also introduced to apply the proposed model to an iron mining complex composed of two mines, each having a stockpile, a loading area and a waste dump, and a single port. The results indicate the model's ability to meet product demand and quality constraints while minimizing the risk of not meeting the targets. The inclusion of the mine-to-port transportation scheduling in the long-term optimization of a mining complex allows for the analysis of how the fleet is used over time. The current formulation could be extended to mining complexes with multiple ports, each having a stockpile available. Furthermore, the current formulation can be extended to integrate port-to-client transportation optimization.

4. Conclusions and Future Work

4.1. General Conclusions

This thesis presents research advancing the use of the simultaneous stochastic optimization method for the optimization of a mineral value chain. A mineral value chain or mining complex is composed of all components of a mining operation, from mines to customers or the spot market, for the extraction, transportation and transformation of materials. Simultaneous stochastic optimization aims to optimize all components of a mining complex jointly to maximize the value of the operation while managing the technical risks associated with material supply or geological uncertainty. This thesis focuses on the simultaneous stochastic optimization framework presented by Goodfellow and Dimitrakopoulos (2016, 2017). First, the framework is applied to a real-world three-mine gold mining complex, maximizing NPV and metal production and managing technical risk by generating the optimal long-term production schedule. The application highlights the framework's ability to incorporate multiple components of a mineral value chain into a single mathematical model. Then, a new mathematical programming formulation for the joint optimization of long-term production scheduling and mine-to-port transportation is presented. The model extends the simultaneous stochastic optimization framework to incorporate the mine-to-port transportation component. A case study application highlights the proposed model's ability to meet product quantity and quality constraints while minimizing technical risks.

The case study applies long-term simultaneous stochastic optimization to a large gold mining complex, the Rosebel Gold Mining (RGM) Complex, composed of three open pit mines, three stockpiles, a waste dump and a processing facility. This is the first attempt in technical literature to apply simultaneous stochastic optimization to a three-mine complex. Stochastic orebody simulations of the gold grades represent the deposits, quantifying the material supply uncertainty. The case study generated extraction schedules and a stockpile management plan which maximize NPV while managing the risks of not meeting production forecasts. The optimization process resulted in an 18-year life for the mining complex and indicated that the Royal Hill deposit would be mined to a greater extent than the other two deposits. The extraction schedules, in addition to the stockpile management plan, optimize the use of the SAG mill through material hardness management. The SAG mill was identified as the operation's bottleneck due to its near-capacity utilization throughout the life of the operation while other components of the mining complex had unused capacity remaining. In addition, cut-off grades were generated reflecting the production schedules and defining the destination policies.

A proposed stochastic integer programming (SIP) model for the joint long-term optimization of extraction schedules, stockpile management and mine-to-port transportation scheduling is presented. Given a fixed product demand at the port, the two-stage SIP aims to minimize the costs associated with meeting the demand and manage the technical risks associated with meeting the quantity and quality constraints for the products. The inclusion of mine-to-port transportation scheduling, consisting of trains and railways, in the long-term optimization of the mining complex extends previous optimization models, which do not include this component. In addition, the model includes a new stockpile representation method: the random block removal order. This approach overcomes limitations associated with the perfect blending and perfect selection representations while maintaining linearity. The proposed model was applied to a case study at an iron ore mining complex consisting of two mines, each with a stockpile, waste dump and train loading area, as well as a single port. The results illustrate the model's ability to minimize the risks of deviating from product demand and quality constraints while managing costs. In addition, an analysis of the fleet use over the mine life determined that only one train is necessary in the first three years of the mine life. Therefore, the other train could be allocated to another operation for better use.

4.2. Future Work

Future work for the case study at the three-mine gold mining complex should consider the inclusion of a larger number of mineral deposits. The RGM complex is composed of 8 deposits, of which the three largest were utilized for the case study. Further, simulations of other geological attributes and sources of uncertainty can be included. Density, hardness, geometallurgical attributes and material-type boundaries can be considered in order to better represent the orebodies and material interactions at different locations within the value chain. The SAG throughput rate as well as the recovery rate and cost for each material type can also be simulated the reflect the uncertainty at the processing facility; market uncertainty can be incorporated using commodity price simulations. Capital investment decisions can also be incorporated into the optimization process. This can allow the optimizer to consider the fleet replacement costs over the life-of-mine as well as consider SAG mill expansion options, to alleviate the operation's bottleneck. Other components of the mineral value chain, such as transportation systems, tailings dams and existing stockpiles, can also be included in the optimization. However, the inclusion of additional deposits, sources of uncertainty, capital expenditure options and other mineral value chain components will significantly expand the size of the optimization problem, increasing the difficulty of finding a reasonable solution. Additional mines and sources of uncertainty will increase the number of uncertainty scenarios considered, and therefore the size of the problem, exponentially. Therefore, a strategy for the reduction of the number of scenarios while ensuring all simulations are still included should be considered. Moreover, the metaheuristic solution approach used presents limitations when applied to large problems. A method exploring the solution space more efficiently, using combinatorial optimization and machine learning, can be applied to ensure a good solution is found in a reasonable time.

Future work for the proposed model jointly optimizing extraction scheduling and mine-to-port transportation should consider the incorporation of additional components of a mineral value chain. For example, crushers at the mines and port stockpiles can be included, incorporating important components of iron ore mineral value chains. In addition, the proposed model can only accommodate a single port; however, many large iron ore mining complexes include multiple ports. Therefore, the inclusion of multiple ports will allow the model to be applicable to a larger number of cases. Additionally, the proposed model can only accommodate a fixed demand at the port and does not include the delivery of products to clients. The model could be modified to include more complex contractual agreements for the sale and delivery of extracted products, and it could be extended to include port-to-client logistics. Moreover, additional sources of uncertainty can be included. Economic and market uncertainty can be considered, as well as uncertain mine-to-port availability and travel times. In addition, capital investment alternatives can be considered, such as the installation of railways, the doubling of existing single-track railways and the purchase of trains. Finally, a metaheuristic solution method can be developed in order to solve larger, more complex instances.

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