Topics in Early Universe Cosmology

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Contributions of Author

For the paper: Y. -F. Cai, E. McDonough, F. Duplessis and R. H. Brandenberger, *Two Field Matter Bounce Cosmology*, JCAP **1310**, 024 (2013) [arXiv:1305.5259 [hep-th]].

I initiated this project as a general investigation of perturbation theory for multi-field bounce models, and Robert Brandenberger suggested it be focused on a specific setup he developed with Yifu Cai. I performed, in tandem with the authors Yifu Cai and Francis Duplessis, all calculations contained in the paper, both analytical and numerical. I made significant contributions to the writing of the paper, especially in discussing the background cosmology and perturbations. Robert Brandenberger wrote the introduction to the paper, and provided guidance throughout the project. Yifu Cai also contributed to the writing of the paper.

For the paper: K. Dasgupta, R. Gwyn, E. McDonough, M. Mia and R. Tatar, *de Sitter Vacua in Type IIB String Theory: Classical Solutions and Quantum Corrections*, arXiv:1402.5112 [hep-th].

I performed (often in tandem with the other authors) all calculations contained in the paper. I contributed the key insight that conditions on curvature corrections could be connected to general constraints on the sign of the stress energy tensor, and Rhiannon Gwyn, Keshav Dasgupta, and myself did the neccessary calculations. Mohammed Mia was responsible for realizing that equation 5.8 could be a powerful constraint, and all members of the collaboration worked towards applying this to various setups. Keshav Dasupta provided his expertise to make the ideas more precise, and guided the project from its initiation to completion. Mohammed Mia wrote much of sections 5.2-5.3, while I wrote section 5.1, 5.4, much of 5.5-5.7, and sections 5.8-5.9.

ABSTRACT

This thesis concerns two questions in Early Universe Cosmology, as addressed in the papers arXiv:1305.5259 and arXiv:1402.5112.

In the first part of this thesis, we examine the evolution of cosmological perturbations in a two-field non-singular bouncing cosmology. The field content of this model consists of a canonical massive (“matter") scalar field and a galileon-like (“bounce") field. The matter field leads to an effective cold, pressureless, matter dominated contracting universe at early times, while the bounce field leads to a violation of the Null Energy Condition and a non-singular bounce at a sub-Planckian energy scale. We study the evolution of both curvature and entropy fluctuations through the bounce, and show that both have a scale-invariant spectrum. However, we find that the entropy fluctuations have an amplitude that is much smaller than that of the curvature perturbations, due to gravitational amplification of curvature perturbations during the bounce phase.

In the second part of this thesis, we study the supergravity limit of Type IIB string theory coupled to fluxes, scalar fields, D-branes, anti D-branes and Orientifold-planes. We show that this theory does not admit solutions with a four-dimensional de Sitter space. We extend the analysis to include higher-order curvature corrections, and find that a de Sitter solution in type IIB theory may be achieved if the higher-order curvature corrections are carefully controlled.

ABRÉGÉ

Cette thèse concerne deux questions de la cosmologie de l'univers primordial, telles qu'adressées dans les articles arXiv:1305.5259 et arXiv:1402.5112.

Dans la première partie de cette thèse, nous examinons l'évolution des perturbations cosmologiques dans un univers rebondissant sans singularité à deux champs. Les champs que contiennent ce modèle consistent en un champ scalaire massif (champ de matière) et un champ de type "galileon" (champ de rebondissement). Le champ de matière mène à un univers en contraction dominé par de la matière froide et sans pression, tandis que le champ de rebondissement mène à une violation de la condition d'énergie de genre lumière et à un rebondissement sans singularité à une énergie sous l'échelle de Planck. Nous étudions l'évolution des fluctuations de courbure et d'entropie à travers le rebondissement, et nous montrons qu'elles ont tous les deux un spectre indépendant de l'échelle. Par contre, nous trouvons que les fluctuations d'entropie ont une amplitude qui est beaucoup plus petite que les perturbations de courbure lors du rebondissement.

Dans la deuxième partie de cette thèse, nous étudions la limite de la supergravité de la théorie des cordes de type IIB couplée à des flux, des champs scalaires, des D-branes, des anti D-branes, et des plans orientifold. Nous montrons que cette théorie n'admet pas de solutions avec un espace de Sitter à quatre dimensions. Nous étendons l'analyse pour inclure des corrections de courbure d'ordres supérieurs, et nous trouvons qu'une solution de Sitter dans la théorie de type IIB peut Ãłtre obtenue si les corrections de courbure d'ordres supérieurs sont soigneusement contrôlées.

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Chapter 1
Introduction

This thesis is a study of two topics at the interface of high energy theoretical physics and cosmology, using two fundamentally different approaches. The first approach is to work in the context of an effective field theory, specifically chosen to remove the initial singularity and replace it with a 'bounce': a smooth transition from a contracting universe to an expanding universe. This allows for a consistent semi-classical analysis of cosmology at all times, including during the bounce, which in turn allows for detailed observable predictions, while remaining agnostic on the precise details of quantum gravity.

The second part of this thesis will be much more ambitious in some ways, and much less ambitious in others. We will abandon the 4d effective field theory approach, and instead work within a UV complete theory: Type IIB superstring theory. This allows for a much clearer understanding of the possible physics, however we will not attempt to understand the initial singularity or do any precise phenomenology. Instead we will be focused on a much simpler task: the search for four-dimensional de Sitter solutions.

To place both these approaches into context, let us recall the earlier frameworks. Standard Big Bang cosmology [1] is a description of the universe as being homogeneous, isotropic, and with matter content described by classical perfect fluids. The initial state of this universe is hot and dense, with subsequent evolution governed by General Relativity. This picture has had many successes, for example Hubble's law, the prediction of the Cosmic Microwave Background (CMB), and the prediction of the primordial abundances of the light elements.

Standard Big Bang cosmology can be supplemented by introducing a period of accelerated expansion, called inflation, that occurs before the radiation and matter dominated periods [1]. Provided the inflationary phase lasts for long enough, inflation solves many problems of the Standard Big Bang. For example, an inflationary universe evolves towards homogeneity, isotropy, and flatness, and dilutes the density of defects such as monopoles. In addition to this, inflation provides the origin of primordial perturbations that seed large scale structure, and a causal mechanism for generating large-scale correlations in the CMB.

However, there *are* aspects of early universe physics for which inflation does not provide an understanding [3]. The example relevant to this thesis is the *Singularity Problem*: inflation does not provide a way to understand what came *before* inflation, and hence inflationary cosmology has the same initial singularity as Standard Big Bang cosmology. Furthermore, in the context of inflation, the primordial perturbations that correspond to observable scales today were generated at energy scales where quantum gravity cannot be ignored, and hence any semi-classical analysis is inherently inconsistent. This forms the basis of the *Trans-Planckian Problem* of inflationary cosmology.

There are two paths to making circumventing these problems: (1) construct an addition, or alternative, to inflationary cosmology that avoids both the singularity and high energy scales altogether, or (2) work in a theory, such as string theory, that is in valid at all energy scales, such that physical quantities can (in principle) be reliably computed in the quantum gravity regime.

This thesis will focus on answering two questions, as presented in the papers [4] [5], each motivated by one of the two paths mentioned above. In Chapter 4, we will study the evolution of cosmological perturbations, and in particular entropy perturbations, as they pass through a specific realization of a sub-Planckian non-singular bounce. The background material for this will be reviewed in Chapter 2. In Chapter 5 we will change course and take path (2): we will study the construction of a 3+1 dimensional accelerating universe in the context of Type IIB string theory. The background material for this is reviewed in Chapter 3.

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Chapter 2
Review of Cosmological Perturbation Theory

2.1 Overview of Cosmological Perturbations

This chapter will function as a review of the tools that will be immediately applicable to the analysis done in this thesis. Namely, we will, to a large degree, avoid any discussion of perturbation theory of perfect fluids, and instead focus on cosmological perturbation theory of scalar fields. The starting point is a single scalar field with minimal coupling to gravity:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right). \tag{2.1}$$

The equations of motion that follow from varying this action with respect to the metric are the Einstein equations, which determine the background evolution of spacetime. A simple case is a homogenous and isotropic universe described by the Freidman-Robertson-Walker metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}x_i\mathrm{d}x^i, \tag{2.2}$$

which leads to the equations of motion:

$$\ddot{\varphi} + 3H\dot{\varphi} + V(\varphi)_{,\varphi} = 0, \tag{2.3}$$

$$H^2 = \frac{1}{3M_{Pl}^2}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right], \tag{2.4}$$

where $H = \dot{a}/a$ is the Hubble parameter.

We now seek to understand perturbations about this background solution. It is not enough to merely look at fluctuations in the field value, $\varphi \to \varphi + \delta\varphi$, since matter is coupled to gravity via Einstein's general relativity. We are forced to consider the most general form of perturbations that satisfy the perturbed Einstein equation:

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}, \tag{2.5}$$

where the left hand side is perturbed Einstein tensor, built out of metric fluctuations, and the right hand side is the perturbed stress-energy tensor. The stress energy tensor of a canonical scalar field has the same form as that for a perfect fluid, $T_{\mu\nu} = \mathrm{diag}(-\rho, p, p, p)$, with pressure and energy density given by [1]

$$\rho = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \;\;,\;\; p = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi), \tag{2.6}$$

which allows perturbations to $T_{\mu\nu}$ to be built out of field fluctuations $\delta\phi$ and metric fluctuations $\delta g_{\mu\nu}$.

The metric is a rank-2 symmetric tensor, and thus has 10 independent degrees of freedom. Given our ansatz for the metric, we can decompose this into into irreducible representations of the group of spatial rotations, which classifies the perturbations as scalar, vector, and tensor components. The first order scalar metric fluctuations can be written as [8]:

$$\delta g^S_{\mu\nu} = a^2 \begin{pmatrix} 2\phi & -B_{,i} \\ -B_{,i} & 2(\psi \delta_{ij} - E_{,ij}) \end{pmatrix}, \tag{2.7}$$

where ϕ, ψ, E, and B are scalars, and $_{,i} = \partial/\partial x^i$. Vector perturbations take the form:

$$\delta g^V_{\mu\nu} = a^2 \begin{pmatrix} 0 & -S_i \\ -S_i & F_{i,j} + F_{j,i} \end{pmatrix}, \tag{2.8}$$

where S and F are divergence-less vectors. Finally, tensor perturbations take the form:

$$\delta g^T_{\mu\nu} = a^2 \begin{pmatrix} 0 & 0 \\ 0 & \gamma_{ij} \end{pmatrix}, \tag{2.9}$$

where γ_{ij} is traceless and divergence-less.

The scalar, vector, and tensor perturbations completely decouple from one another at the linear level, end hence can be treated independently. The scalar, vector, and tensor modes are be mixed by the non-linearities of gravity, but this is precisely the effect that is removed by considering only first order perturbations. An excellent proof of this can be found in [3]. Furthermore, vector modes decay in amplitude in an expanding universe, and hence in many cases it suffices to study only the scalar and tensor fluctuations.

It is important to note that not all 10 metric degrees of freedom carry physical significance. In fact, for a universe consisting of a single fluctuating scalar field, there is only *one* physical scalar degree of freedom, as opposed to the 5 that one would naively expect (4 from the metric and 1 from the field). Diffeomorphism invariance of the action (2.1) requires invariance under the two possible scalar coordinate transformations: $t \to t + \alpha$ and $x_i \to x_i + \partial_i \beta$, where α and β are scalars, and hence two scalar degrees of freedom can removed by fixing the choice of coordinates [3]. The Einstein constraint equations remove two more scalar degrees of freedom, such that the physics of all scalar fluctuations can be written in terms of the equation of motion of a single scalar.

An easy way to deal with this issue is to introduce the concept of *gauge-fixing*. A standard example is longitudinal gauge: $B = E = 0$, which leaves only ϕ and ψ as the metric scalar degrees of freedom. This gauge is particularly useful since ϕ and ψ are equal in the absence of anisotropic stress (that is, off-diagonal spatial elements in $\delta T_{\mu\nu}$), which indeed vanishes for a scalar field. The Einstein equations can then easily be reduced to a single scalar degree of freedom.

2.2 Scalar Fluctuations

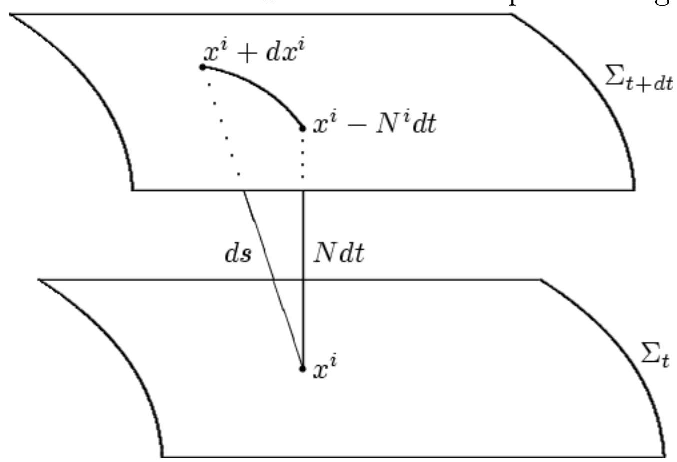
2.2.1 The ADM formalism

We will study scalar perturbations using a technique called the ADM formalism [4]. This method is a choice of foliation of spacetime into spacelike hypersurfaces Σ_t, labelled by the time t. The natural perturbation variable in this setup is the curvature perturbation on uniform density hypersurfaces, denoted ζ, which can be isolated by working in the uniform field gauge $\delta\varphi = 0$. The variable ζ is related to the *comoving* curvature perturbation $\mathcal{R}$ by [5]

$$-\zeta = \mathcal{R} + \frac{2\rho}{2(p+\rho)}\left(\frac{k}{aH}\right)^2 \Psi, \tag{2.10}$$

where p and ρ are the pressure and energy density of the universe, and Ψ is a gauge-invariant combination of the gravitational potentials. The potential Ψ vanishes on super-Hubble scales, and hence on these scales ζ and $\mathcal{R}$ are equivalent perturbation variables.

Figure 2–1: The ADM formalism. See text for description. Image taken from [6].



The time evolution of perturbations is given by the mapping of perturbations from one spacelike hypersurface to another, specified by a *lapse function* N and *shift vector* N^i, as shown in Figure 2.2.1. The metric is written in the ADM formalism as

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t). \tag{2.11}$$

Using this, the 4-dimensional Ricci scalar R_4 can be decomposed as

$$R_4 = R_3 + \kappa_{ij}\kappa^{ij} - \kappa^2, \tag{2.12}$$

where R_3 and κ_{ij} are the Ricci scalar and extrinsic curvature of the spatial slices, with κ_{ij} given by

$$\kappa_{ij} = \frac{1}{2N}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i) \quad , \quad \kappa = \kappa_i^i, \tag{2.13}$$

and ∇_i is the covariant derivative on spatial slices. Similarly, the canonical kinetic term of a scalar field takes the form

$$X \equiv \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi = \frac{1}{2N^2}\left(\dot{\varphi} - h^{ij}N_i\partial_j\varphi\right)^2 - h^{ij}\partial_i\varphi\partial_j\varphi, \tag{2.14}$$

where we have defined X as the canonical kinetic term. Finally, the action (2.1) is written in the ADM formalism as:

$$S = \int \mathrm{d}^3x\mathrm{d}t\sqrt{h}\frac{N}{2}\left[R_3 + \kappa_{ij}\kappa^{ij} - \kappa^2 + X - V(\varphi)\right]. \tag{2.15}$$

We can now begin studying this action perturbatively. To simplify the analysis, we fix the gauge to uniform field gauge $\delta\varphi = 0$, and choose our physical perturbation variable to the ζ. This gauge is fully specified by:

$$h_{ij} = a^2\left[(1-2\zeta)\delta_{ij} + \gamma_{ij}\right] \ ; \ \partial_i\gamma_{ij} = 0 \ , \ \gamma_{ii} = 0 \ , \ \delta\varphi = 0. \tag{2.16}$$

The lapse function and shift vector act as Lagrange multipliers, and can be redefined as

$$N_i = \partial_i\sigma + \partial\tilde{N}_i \ , \ N \equiv 1 + \alpha, \tag{2.17}$$

where $\tilde{N}_i$ is a divergence-free, $\partial_i\tilde{N}_i$. These can then be expanded perturbatively as:

$$\begin{aligned} \alpha &= \alpha_1 + \alpha_2 + ... \\ \sigma &= \sigma_1 + \sigma_2 + ... \\ \tilde{N}_i &= \tilde{N}_{i1} + \tilde{N}_{i2} + ... \end{aligned}$$

where the subsripts $n = 1, 2...$ denote terms at order ζ^n. The variation of the action at first order gives the background dynamics of the field ϕ, and also gives first-order equations for the variables α, σ, and $\tilde{N}_i$. These are given by [7]

$$\alpha_1 = \frac{\dot{\zeta}}{H} \;\;,\;\; \partial^2 \tilde{N}_{i1} = 0 \;\;,\;\; \partial^2 \sigma_1 = -\frac{\partial^2 \zeta}{H} + \frac{1}{2}\left(\frac{\dot{\phi}}{M_{Pl} H}\right)^2 \dot{\zeta}, \tag{2.18}$$

where $\partial^2 = \partial_i \partial^i$. Upon expanding the action to second order, and using the first order equations of motion, we obtain the second order action for ζ:

$$S = \frac{1}{2}\int \mathrm{d}^3x \; a\mathrm{d}t \, \frac{z^2}{2}\left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i \zeta)^2\right], \tag{2.19}$$

where z is defined as

$$z^2 = a^2 \frac{\dot{\phi}^2}{H^2}. \tag{2.20}$$

After a changing to the Mukhanov-Sasaki variable $v \equiv z\zeta$, and writing the action in terms of conformal time $\mathrm{d}\tau = a\mathrm{d}t$, denoting $\mathrm{d}/\mathrm{d}\tau = {}'$, the action becomes

$$S = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3x \left[(v')^2 - (\partial_i v)^2 + \frac{z''}{z} v^2\right]. \tag{2.21}$$

The equation of motion that follows from this is the famous Mukhanov-Sasaki equation:

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0 \tag{2.22}$$

where we have decomposed v into Fourier modes,

$$v_k(\tau) = \int d^3x \; e^{-ik\cdot x} v(\tau, x). \tag{2.23}$$

2.2.2 Quantization of Scalar Cosmological Perturbations

We now seek to quantize the scalar fluctuations, which will be used in the next subsection to solve the Mukhanov-Sasaki equation for the mode functions v_k. The classic text on this is Birrell and Davies [8].

Quantum field theory (QFT) in an arbitrary curved spacetime is *difficult*. For starters, an important step in QFT in Minkowski space is the splitting of the field into positive and negative frequency modes. This splitting requires a global time-like

Killing vector, an object with which an arbitrary spacetime is not generally endowed. In practice, one may still define a set of positive and negative frequency modes, but the non-uniqueness of the time variable means there is no inherent reason to prefer this set over any other.

We will illustrate this with the most common example, and also the most physically relevant, the FRW metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}x_i\mathrm{d}x^i. \tag{2.24}$$

To quantize v, we promote fields to operators, and express the variable v as:

$$\hat{v}(\tau, x) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} \left[\hat{a}_k v_k(\tau) e^{ik\cdot x} + \hat{a}^\dagger v_k^*(\tau) e^{-ik\cdot x}\right], \tag{2.25}$$

where $\hat{}$ denotes an operator, and the operators $\hat{a}_k, \hat{a}_k^\dagger$ are identified as the usual annihilation/creation operators. The mode functions $v_k(\tau)$ in the above decomposition define an *inertial observer*, and the $\hat{a}_k, \hat{a}_k^\dagger$ are defined with respect to this observer. The annihilation/creation operators obey the commutation relations:

$$[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta(k - k') \ \ , \ \ [\hat{a}_k, \hat{a}_{k'}] = [\hat{a}_k^\dagger, \hat{a}_{k'}^\dagger] = 0. \tag{2.26}$$

The mode functions must also satisfy the normalization condition

$$v_k' v_k^* - v_k v_k^* = -i. \tag{2.27}$$

The Hilbert space of quantum states can then be constructed by acting creation operators on the vacuum state $|0\rangle_a$, defined as the state that is annihilated by a_k:

$$a_k|0\rangle_a = 0, \tag{2.28}$$

which defines the minimum energy state.

Physically, the vacuum $|0\rangle_a$ is the state which observed to be empty of particles by the inertial observer defined by the set of mode functions $v_k(\tau)$. However, a *different* choice of inertial observer $u_k(\tau)$ would define a different set of annhilation/creation operators b_k, with vacuum $|0\rangle_b$, and in general the b_k will *not* annihilate the a_k vacuum: $b_k|0\rangle_a \neq 0$. It follows that different observers disagree on how many particles are observed, which is quite different from QFT in Minkowski space, where all inertial observers agree on the choice of vacuum state.

However, there exists a limiting case where the physics simplifies. Consider a massless canonical scalar field in a matter dominated contracting universe with $a(t) \sim t^{2/3}$, which is precisely the initial state of the cosmology described in Chapter 4. The Mukhanov-Sasaki equation, written in conformal time $\mathrm{d}\tau = a(t)^{-1}\mathrm{d}t$, is given by

$$v_k'' + (k^2 - \frac{2}{\tau^2})v_k = 0. \tag{2.29}$$

Modes that are on relevant cosmological scales today were deep inside the horizon in the far past, where $\tau \to -\infty$. It follows that the k^2 term dominates in the equation of motion for fluctuations at early times times:

$$v_k'' + k^2 v_k = 0 \quad \text{for } \tau \to -\infty, \tag{2.30}$$

which is the Klein-Gordon equation in Minkowski space, and hence we have a vacuum state that is agreed upon by all inertial observers. Geometrically, this result follows from the fact that any manifold is locally flat. A mode with fixed comoving wavenumber k that corresponds to a large physical scale today was a very small physical scale in the far past, and hence the mode function at early times is effectively living, and can be quantized, in Minkowski space.

Provided the frequency $\omega_k = k^2 - 2/\tau^2$ is varying adiabatically, the vacuum state at any later time can then be built using the WKB approximation. Hence, the vacuum state is fully specified by a choice of initial conditions for quantum mode functions, which can be applied by solving the Mukhanov-Sasaki equation (2.29) with said initial conditions. One example is

$$v_k(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau} \text{ at } \tau \to -\infty, \tag{2.31}$$

where we have identified the positive frequency mode as the minimal-energy excited state, as in QFT in flat space. This is called the *Bunch-Davies initial condition.* It is important to note the above condition can be imposed at a finite $\tau = \tau_0$, in which case this is only as an approximate solution for the mode function. This becomes exact in the UV limit of k, which corresponds to imposing the initial condition at $\tau_0 = -\infty$.

In our present example, the solution to equation (2.29) is given by

$$v_k(\tau) = C_1 \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) + C_2 \frac{e^{ik\tau}}{\sqrt{2k}} \left(1 + \frac{i}{k\tau}\right), \tag{2.32}$$

where $C_{1,2}$ are constants. Applying the Bunch-Davies initial condition determines the mode function to be

$$v_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right), \tag{2.33}$$

and the full solution for the quantized perturbation $\hat{v}(\tau, x)$ is given by equation (2.25).

2.2.3 The Power Spectrum of Scalar Fluctuations

The power spectrum of long-wavelength fluctuations can be calculated by evaluating the vacuum expectation value of quantum fluctuations at the moment that fluctuations cross the horizon. Let's consider the quantum expectation value of the fluctuation $\hat{v}$ (or more specifically, a single Fourier mode):

$$\langle 0|\hat{v}(k)\hat{v}(k')|0\rangle = \langle 0| \left(a_k v_k + a^\dagger v_k^*\right) \left(a_k v_k + a^\dagger v_k^*\right) |0\rangle = |v_k|^2 \delta(k + k'). \tag{2.34}$$

This defines the (dimensionful) power spectrum of v fluctuations:

$$P_v^{dim} = |v_k|^2_{k=aH}. \tag{2.35}$$

The convention that will be used here is to work with a power spectrum defined as

$$P_v \equiv \frac{k^3}{2\pi^2} |v_k|^2_{k=aH}. \tag{2.36}$$

which will allow the spectrum of curvature perturbations to be written in a dimensionless form. The power spectrum of curvature perturbations is then given using the definition $v = a\zeta$,

$$P_\zeta \equiv \frac{k^3}{2\pi^2} |\frac{v_k}{z}|^2_{k=aH}. \tag{2.37}$$

For the example of a massless scalar field in a matter-dominated contracting universe, the power spectrum has a simple form:

$$P_\zeta = \frac{H^2}{16\pi^2 M_{Pl}^2} \tag{2.38}$$

This is often parametrized as a power law,

$$P_\zeta \sim A_\zeta k^{n_s - 1} \tag{2.39}$$

where n_s is called the *spectral tilt* of scalar fluctuations, and A_ζ is the amplitude of the power spectrum. The example given above has $n_s = 1$, which is known as a *scale-invariant* spectrum. We will now proceed to consider tensor fluctuations.

2.3 Tensor Perturbations

Let's now consider a tensor fluctuation to the spatial metric. This takes the form $\delta g_{ij} = a^2 h_{ij}$, where h_{ij} is symmetric and traceless. Remarkably, the expansion of the action (2.1) in terms for h_{ij} gives the action for a massless scalar field in a curved background:

$$S = \frac{M_{Pl}^2}{2} \int \mathrm{d}\tau \mathrm{d}^3 x a^2 \left[(h')_{ij} - (\nabla h_{ij})^2 \right], \tag{2.40}$$

where h_{ij} is dimensionless, and ∇ is the covariant derivative on spatial slices. This is not a miracle, but follows from the fact that tensor fluctuations decouple from scalar fluctuations, and hence do not back-react on the background cosmology. The tensor field can decomposed into Fourier components,

$$h_{ij}(\tau, x) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \sum_\lambda \epsilon(k, \lambda)_{ij} h_k(\tau, \lambda) e^{ik\cdot x}. \tag{2.41}$$

where the sum is over polarizations λ, which can be $+$ or $\times$. The action can then be written as

$$S = \sum_\lambda \int \mathrm{d}\tau \mathrm{d}^3 k \frac{a^2}{4} M_{Pl}^2 \left[(h_k'(\lambda))^2 - k^2 h_k^2(\lambda) \right]. \tag{2.42}$$

In analogy with our scalar perturbation analysis, we can define the Mukhanov-Sasaki variable for tensor fluctuations

$$u_k = \frac{a}{2} M_{Pl} h_k, \tag{2.43}$$

such that our action becomes

$$S = \sum_{\lambda} \frac{1}{2} \int \mathrm{d}\tau \mathrm{d}^3 k \left[{u_k'}^2 - \left(k^2 - \frac{a''}{a} \right) u_k \right]. \tag{2.44}$$

The power spectrum of tensor fluctuations is then given by

$$P_h = \frac{k^3}{2\pi^2} |\frac{u_k}{a}|^2. \tag{2.45}$$

This is often parametrized as

$$P_h \sim A_T k^{n_T}, \tag{2.46}$$

where A_T is the amplitude to tensor fluctuations, and n_T is the spectral index of tensor fluctuations. An important quantity is the tensor to scalar ratio, defined as

$$r = \frac{A_h}{A_\zeta}, \tag{2.47}$$

where A_h is the amplitude of P_h, and A_ζ is the amplitude of P_ζ.

We will now take a detour and review the essential elements of string theory, before we return to cosmological perturbation theory in our analysis of a bouncing universe in Chapter 4.

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Chapter 3
Short Review of String Theory

As a disclaimer, let us mention we will only cover those ideas that are crucial to understanding the details of Chapter 5, as well as a bare minimum of background. There will be *many* crucial elements of string theory which we will *not* cover.

3.1 Type IIB Supergravity

Type IIB string theory is a supersymmetric, chiral theory of closed and open strings propogating in a 10 dimensional spacetime. The low energy field content of this theory corresponds to massless excitations of strings, and is split into two sectors according to the boundary conditions applied to the string. The closed string sector, where Neveu-Schwarz boundary conditions are applied to both left and right moving oscillations of the string, gives rise to a scalar field, an antisymmetric 2-form, and a symmetric two form. These are known as the dilaton ϕ, the Kalb-Ramond two-form B_2, and the metric $g_{\mu\nu}$.

The open string sector, where Ramond boundary conditions are imposed on both left and right movers, gives rise to form-fields of odd-dimension, and can be thought of as a generalization of electrodynamics to p-form electrodynamics. The form fields, analogous to the vector potential A_μ in electromagnetism, are denoted C_0, C_2, and C_4. The corresponding field strengths are obtained by taking exterior derivatives.

At a more formal level level, the spectrum of massless fields in type IIB can be written in Light-Cone gauge as a tensor product of SO(8) supermultiplets:

$$(\mathbf{8}_v + \mathbf{8}_c) \otimes (\mathbf{8}_v + \mathbf{8}_c), \tag{3.1}$$

where v and c denote the vector and conjugate sprinor representations of SO(8), respectively. The bosonic part of the NS-NS sector is given by

$$\mathbf{8}_v \otimes \mathbf{8}_v = \mathbf{1} + \mathbf{28} + \mathbf{35}, \tag{3.2}$$

which correspond to the dilaton, antisymmetric two-form, and metric respectively. Similarly, the bosonic part of the RR sector is given by

$$\mathbf{8}_c \otimes \mathbf{8}_c = \mathbf{1} + \mathbf{28} + \mathbf{35}_+, \tag{3.3}$$

which corresponds to the RR gauge fields: p-form potentials with p=0,2,4, and with the added condition that the four-form potential have a self-dual field strength.

The action for this theory is given, at low energies, by the supergravity action:

$$\begin{aligned} S_{\rm SUGRA} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G_{10}}\left(R - \frac{\partial_M\tau\partial^M\bar{\tau}}{2|{\rm Im}\tau|^2} - \frac{|\widetilde{F}_5|^2}{4\cdot 5!} - \frac{G_3\cdot\bar{G}_3}{12{\rm Im}\tau}\right) \\ &+ \frac{1}{8i\kappa_{10}^2}\int\frac{C_4\wedge G_3\wedge\bar{G}_3}{{\rm Im}\tau}. \end{aligned} \tag{3.4}$$

Here $\tau = C_0 + ie^{-\phi}$; $G_{10} = \det g_{MN}, M, N = 0,..,9$; g_{MN} is the metric in Einstein frame; $G_3 = F_3 - \tau H_3$; $F_3 = {\rm d}C_2$ is the three-form RR (Ramond-Ramond) feld strength, $\tilde{F}_5$ is the 5-form self-dual RR field strength, and $H_3 = {\rm d}B_2$ is the three-form NS-NS (Neveu-Schwarz Neveu-Schwarz) field strength.

Note that this action is only a *perturbative description* of string theory. Moreover, there are in principle *two* perturbative expansions, one in the expectation value of the string coupling $g_s = e^{-\phi}$, and another in the string length scale $l_s = \frac{1}{\sqrt{4\pi\alpha'}}$. The string coupling, which has been absorbed into the fields in the above action (this is the 'Einstein frame'), generates loop (i.e. quantum) corrections from string interactions, or equivalently, counts the genus of the string worldsheet. The α' expansion is a *tree-level effect*, which comes from the classical description of the string worldsheet in terms of the non-linear sigma model.

Even at the level of the supergravity action above, we are presented with a striking deviation of String Theory from 'standard physics': This theory is defined in

10 dimensions! The choice D=10 (so called 'critical string-theory') is made to cancel the Weyl anomaly of the string worldsheet in flat space. In terms of Conformal Field Theory (CFT), the trace of the stress energy tensor is proportional to the central charge of the CFT. The type IIB string worldsheet is described by a $\mathcal{N} = (1, 1)$ Super-Conformal Field Theory, with central charge given by $c = D + (1/2)D - 26 + 11$, where we have seperated the contributions from bosons, fermions, and their corresponding ghosts. Hence by setting $D = 10$ we can cancel the anomaly, a feature which is discussed in more detail in many textbooks, for example [1].

In order to describe our 3+1 dimensional universe, string theorists are thus forced to consider splitting the 10 dimensional spacetime into our 3+1 dimensions, and a 6-dimensional internal manifold. The simplest topology is a direct product space:

$$\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6. \tag{3.5}$$

This splitting lends itself to an easy derivation the 4d (low-energy) physics, which can be obtained via Kaluza-Klein reduction, such that massless modes for each supergravity field correspond to harmonic forms on the internal manifold [2]. In this way, the low energy field content is encoded in the topology of $\mathcal{M}_6$. To see how this arises, let's consider what happens to an RR p-form field upon compactification on a product space, following Section 9.5 of [3]. Assuming no other background fields, the equation of motion for $F_p = \mathrm{d}C_{p-1}$ is given by

$$\mathrm{d} \star F_p = 0. \tag{3.6}$$

In terms of the potential, this is

$$\Delta C_{p-1} = 0, \tag{3.7}$$

where Δ is the 10d Laplacian. This can be decomposed as

$$\Delta = \Delta_4 + \Delta_6. \tag{3.8}$$

Fluctuations of F_p in the internal space will lead to slight violations $\Delta_6 C_{p-1} \neq 0$, which can be treated as a source term in the 4d equation of motion. These fluctuations, and hence the 4d fields, can be written in a basis given by solutions to the equation

$$\Delta_6 C_{p-1} = 0. \tag{3.9}$$

The solutions of this equation can be studied via the Atiyah-Singer index theorem, which (in loose terms) gives a one-to-one correspondence between zero modes of a differential operator and elements of the cohomology on the manifold defined by the operator. More formally, the Atiyah-Singer index theorem equates the the analytical index of an elliptic operator to the topological index. This allows the low-energy fields to be read off directly from the Betti numbers of the internal manifold: compactification of F_p on $\mathcal{M}_4 \times \mathcal{M}_6$ will lead to a 4d theory with have b_n number of n-form fields, where b_n are the Betti numbers of $\mathcal{M}_6$.

A similar procedure applies to the metric. For the case of a Ricci-flat internal manifold, the analogous 6d equation is the Lichnerowicz equation, given by

$$\nabla_k \nabla^k \delta g_{mn} + 2R_m{}^p{}_n{}^q \delta g_{pq} = 0, \tag{3.10}$$

where δg_{ab} is a fluctuation of the metric component g_{ab}. In complex coordinates, this leads to the 'Kahler Moduli', which describe the purely holomorphic or antiholomorphic deformations, and 'Complex Structure Moduli' which describe the mixed deformations.

3.2 Branes and Planes

There are two different approaches to introducing D-branes. The first is develop the worldsheet theory of a p-dimensional RR-charged object, and to discover that these branes exist as solitons of string theory. An alternative method is to consider the background fields induced by open strings with Neumann boundary conditions along p-directions, and Dirichlet boundary conditions along $10 - p$ directions, such that the open strings can be said to 'end' on Dp-branes. The realization that branes could be treated as both classical solutions and as boundary conditions for the fundamental string was discovered in [4], a paper in what is now referred to as the 'Second Superstring Revolution'.

Recall that the action for a point particle is given by

$$S_0 = -\alpha \int \mathrm{d}s, \tag{3.11}$$

where α is a constant, and $\mathrm{d}s$ is the invariant length element along the worldline of the particle. The generalization of this to a p-dimensional object is then given by

$$S_p = -T_p \int \mathrm{d}V_p, \tag{3.12}$$

where T_p is the mass per unit volume, called the tension, and V_p is the p-dimensional volume element:

$$\mathrm{d}V_p = \sqrt{-\det G_{\alpha\beta}}\mathrm{d}^{p+1}\sigma. \tag{3.13}$$

The coordinates σ_α, $\alpha = 1...p$, are the coordinates on the brane worldvolume, and $G_{\alpha\beta}$ is the induced metric defined as

$$G_{\alpha\beta} = g_{\mu\nu}(X)\frac{\partial X}{\partial\sigma^\alpha}\frac{\partial X}{\partial\sigma^\alpha}, \tag{3.14}$$

where $g_{\mu\nu}$ is the spacetime metric, and X^μ are the spacetime coordinates.

We can generalize the above action to the case with a more general background. Specifically, we need to account for the gauge theory of open strings that end on the brane, as well as the contribution to the closed-string background coming from the Kalb-Ramond two-form. This leads to the DBI action, given by

$$S_{DBI} = -T_{Dp} \int \mathrm{d}^{p+1}\sigma e^{-\phi}\sqrt{-\det(G_{\alpha\beta} + B_{\alpha\beta} + F_{\alpha\beta})} \tag{3.15}$$

where we have set $\alpha' = 1$, F is the field strength of the $U(1)$ gauge field that lives on the brane, and $B_{\alpha\beta}$ is the pull back of the Kalb-Ramond two-form. In a setup with N coincident branes, the U(1) gauge group is enhanced to the non-abelian group U(N). This can be seen by a simple counting argument: the two ends of an open string have N choices of which brane to end on, which gives $N \times N = N^2$ degrees of freedom. This forms the N^2 gauge bosons of a $U(N)$ Yang-Mills theory on the brane world-volume.

The coupling of Dp-branes to the RR fields is given by a Chern-Simons term,

$$S_{CS} = \mu_p \int C \wedge e^{B+F} \tag{3.16}$$

where μ_p is proportional to the p-form charge density. It follows that the case $\mu_p > 0$ describes a positive charge object, while $\mu_p < 0$ is a negatively charge object which we call an 'anti-brane'. Putting these two contributions together, and changing to the Einstein frame, we arrive at the action of a Dp-brane:

$$\begin{aligned} S_{Dp} &= -\int d^{p+1}\sigma \, T_p \, e^{\frac{\phi(p+1)}{4}} \sqrt{-\det(G_{\alpha\beta} + B_{\alpha\beta} + F_{\alpha\beta})} \\ &\quad + \mu_p \int \left(C \wedge e^{B+F}\right)_{p+1}, \end{aligned} \tag{3.17}$$

where C_{p+1} is the RR flux. Note that the sign of μ_p determines whether we have a brane or an anti-brane. However both branes and anti-branes have positive tension $T_p > 0$. In this convention, μ_p and T_p are related by $|\mu_p| = e^{\phi}|T_p|$.

A seemingly similar object is the 'orientifold plane', or Op-plane. While a Dp-brane enforces boundary conditions on the open strings, the Op-plane enforces boundary conditions on the closed string spectrum. In terms of geometry: Op-planes arise as the fixed points of an orientifold projection, which combines a spacetime reflection (an orbifold) with a worldsheet parity reversal and worldsheet orientation reversal. For example, consider an orientifold $\mathcal{M}/\Gamma$, $\Gamma = \mathbb{Z}_2 \times \Omega \times (-1)^{F_L}$, where Ω and $(-1)^{F_L}$ are worldsheet parity and orientation reversal respectively. This orientifold will generically have a set of fixed points. The closed string partition function can built directly on $\mathcal{M}/\Gamma$, or else defined on $\mathcal{M}$ with boundary conditions enforced by the Op-planes that sit at the points in $\mathcal{M}$ that would be fixed under $\mathcal{M}/\Gamma$. It follows that Op-planes are not free to move around, but rather have a fixed position. This highlights a key difference from Dp-branes: Op-planes are topological in nature, and thus are *non-dynamical*.

The action for an Op-plane is given by

$$S_{Op} = -\int d^{p+1}\sigma\ T_{Op} e^{\frac{\phi(p+1)}{4}}\ \sqrt{-\text{det}G} + \mu_{Op}\int C_{p+1}, \tag{3.18}$$

where the orientifold has negative tension, i.e. $T_{Op} < 0$. Here μ_p is the charge of the Op-plane and we have the relation $|T_{Op}| = e^{-\phi}|\mu_{Op}|$. Also note that since the Op plane has negative charge, we have $\mu_p = e^{\phi}T_{Op} = -e^{\phi}|T_{Op}|$.

A key poiny is the relation between the tensions of a Dp-brane and an Op-plane: $T_{Op} = 2^{p-4}T_p$ (see Chapter 8 of [5]). This allows Dp-branes and Op-planes to be used in combination to achieve many different types of compactification.

3.3 From IIB to M-theory, and back again!

Type IIB string theory can be connected to an 11-dimensional theory known as $M-theory$. The field content of M-theory is considerably simpler: the only bosonic fields are the metric, the dilaton, and a 3-form potential. There is no Kalb-Ramond two-form, and hence M-theory is a 'string theory' theory without strings! Moreover, there are no Dp-branes or Op-planes in M-theory. There are only membranes: the M2 brane, and the M5 brane.

The bulk supergravity action for M-theory is given by

$$S_{bulk} = \frac{1}{2\kappa^2}\int \text{d}^{11}x\ \sqrt{-g}\left[R - \frac{1}{48}G^2\right] - \frac{1}{12\kappa^2}\int C\wedge G\wedge G + \frac{1}{2\kappa^2}\int C\wedge X_8, \tag{3.19}$$

where the $C\wedge X_8$ term is a Chern-Simons term required for anomaly cancellation, and X_8 is a rank-8 tensor constructed out of curvatures. M-theory can be related to type II string theories by dimensional reduction, the easiest case being M-theory that is a T^2 fibration of a 9-dimensional manifold $\mathcal{M}_9$. Dimensional reduction along one of the torus directions leads to IIA on a circle, which can then be T-dualized to IIB.

Let's follow a simple example, given in [6]. Take M-theory on $\mathcal{M}_9 \times T_2$, with metric given by

$$\text{d}s_M^2 = \text{d}s_9^2 + \frac{v}{\tau_2}\left((\text{d}x + \tau_1\text{d}y)^2 + i\tau_2^2\text{d}y^2\right), \tag{3.20}$$

where (x, y) are the coordinates on the T^2, $\tau_{1,2}$ are the complex structure moduli of the torus, $\tau = \tau_1 + i\tau_2$, and $\mathrm{d}s_9^2$ is the metric on $\mathcal{M}_9$. After dimensional reduction along x and T-duality along y, this leads to the IIB metric (in the Einstein frame):

$$\mathrm{d}s_{IIB}^2 = \frac{\sqrt{v}}{L}\left(\mathrm{d}s_9^2 + \frac{l_s^2 L^2}{v^2}\mathrm{d}y^2\right), \tag{3.21}$$

where L is the radius of the y-direction, and l_s is the IIB string coupling (which is also given by the M-theory geometry). For the case of $\mathcal{M}_9 = \mathbb{R}^{1,2} \times B_6$, and using $L \equiv \sqrt{v}$, the metric is

$$\mathrm{d}s_{IIB}^2 = -(\mathrm{d}x^0)^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2 + \frac{l_s^4}{v}\mathrm{d}y^2 + \mathrm{d}s_6^2. \tag{3.22}$$

In the limit that $v \to 0$, with l_s held fixed, the y-coordinate decompactifies. The metric becomes

$$\mathrm{d}s_{IIB}^2 = -\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}s_6^2, \tag{3.23}$$

which is 3+1 dimensional Minkowski space and a 6d internal manifold.

This will be the most relevant result for the current work, although for completeness we will note a few other results, which are given in Section 3.3 [6] as well as Table 5 of [7]. The bulk IIB RR-fields are determined by the M-theory 3-form potential, which can be seen by (locally) decomposing the M-theory 3-form as

$$C_3 = C_3' + B_2 \wedge L\mathrm{d}x + C_2 \wedge L\mathrm{d}y + B_1 \wedge L\mathrm{d}x \wedge L\mathrm{d}y. \tag{3.24}$$

Upon taking M-theory to IIB as described above, B_2 and C_2 become the NSNS and RR two-forms, and C_3' becomes the 4-form $C_4 = C_3' \wedge \mathrm{d}y$. The 1-form B_1 becomes the IIA 1-form, which mixes with the IIA metric under T-duality to give the IIB metric a leg along on the y-direction.

Localised sources in IIB can be inferred by tracking the localised contributions to the flux in M-theory. Specifically, space-filling M2 branes lead to space-filling D3-branes, while singularities in the M-theory elliptic fibration lead to D7 branes, and M2 branes wrapping vanishing cycles of the elliptic fibration lead to a *stack* of D7 branes. In each case, the gauge fields on the branes can be determined by the decomposition of the M-theory flux at that point. For the $D7$, the decomposition can be written in terms of the M-theory 4-form field strength $G = dC$, which takes the following form at singular points of the torus:

$$\frac{G}{2\pi} = \sum_{i=1}^{k} F_i \wedge \Omega_i \tag{3.25}$$

where $i = 1..k$ are the singular points, the Ω_i are a basis harmonic forms localized at the singularities and the F_i are the gauge fields on the worldvolume of the D7-branes.

This concludes the review components of this thesis. We will now proceed to present original research.

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Chapter 4
Two Field Matter Bounce Cosmology

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Abstract

We re-examine the non-singular Matter Bounce scenario first developed in [20], which starts with a matter-dominated period of contraction and transitions into an Ekpyrotic phase of contraction. We consider both matter fields, the first of which plays the role of regular matter, and the second of which is responsible for the non-singular bounce. Since the dominant matter field is massive, the induced curvature fluctuations are initially not scale-invariant, whereas the fluctuations of the second scalar field (which are initially entropy fluctuations) are scale-invariant. We study the transfer of the initial entropy perturbations into curvature fluctuations in the matter-dominated phase of contraction and show that the latter become nearly scale invariant on large scales but are blue tilted on small scales. We study the evolution of both curvature and entropy fluctuations through the bounce, and show that both have a scale-invariant spectrum which is blue-tilted on small scales. However, we find that the entropy fluctuations have an amplitude that is much smaller than that of the curvature perturbations, due to gravitational amplification of curvature perturbations during the bounce phase.

4.1 Introduction

Current high precision data from ground-based [1,2] and space-based [3,4] cosmic microwave background (CMB) telescopes indicate that the origin of structure in the universe is due to a primordial spectrum of nearly adiabatic and nearly scale-invariant cosmological fluctuations. As realized long before these observations [5] (see also [6–8]), a phase of cosmological inflation during the very early universe will generate such a spectrum. On the other hand, inflation is not the only way to generate such a spectrum. As realized in [9, 10], a scale-invariant spectrum of curvature fluctuations on super-Hubble scales is also generated during a phase of matter-dominated contraction. In order to make contact with the present expanding universe, new physics is required to allow for the transition between the contracting and expanding phases. Such a transition can in principle either be singular and from the point of view of the low-energy effective theory (as in the case of the original Ekpyrotic scenario [11]), or non-singular. There are various ways of obtaining a non-singular bounce, e.g. by modifying the gravitational action as in Horava-Lifshitz gravity [12], torsion gravity [13], or by adding Null Energy Condition violating matter such as a ghost condensate [14] or Galileon [15] field [1] . A cosmological model with an initial phase of matter-dominated contraction and a non-singular bounce is called the *Matter Bounce* scenario and it provides an alternative to cosmological inflation for generating the observed spectrum of cosmological fluctuations (see e.g. [17] for review articles on the matter bounce scenario) [2] .

A problem for most bouncing cosmologies is the instability against anisotropic stress, the BKL instability [18]. An intuitive way of understanding this problem is to note that the effective energy density in anisotropies evolves with the cosmological

[1] See also [16] for a review of bouncing cosmologies.

[2] Note that there are other alternatives to inflation for generating a scale-invariant spectrum of cosmological perturbations which, however, will not be discussed in this article.

scale factor $a(t)$ as $\rho_{anis} \sim a^{-6}$, and thus increases much faster in a contracting universe than the energy densities in matter and radiation. Hence, unless the initial anisotropies are not tuned to zero to a very high precision, no homogeneous bounce will occur.

The solution to this problem, first implemented in the context of the Ekpyrotic scenario [11], is to introduce a new matter field ϕ during the contracting phase whose energy density scales with a higher power of a^{-1} than that of the anisotropy term and which hence dominates the total energy density during the later phases of contraction. With such a field, the BKL instability can be avoided [19]. In [20], a concrete model was proposed in which the new field ϕ generates both the Ekpyrotic contraction phase and the non-singular bounce. This is obtained by giving ϕ a Galileon-type non-standard kinetic action (which yields the non-singular bounce), and by providing it with a negative exponential potential which then yields the Ekpyrotic contraction. If we assume that the contracting period starts with a phase of matter-domination, we obtain a realization of the "matter bounce" scenario. In [20] the evolution of the spectrum of cosmological fluctuations across the bounce phase was studied in detail. In particular, it was shown that the two problems for a certain class of non-singular bounce models discussed in [21] do not arise [3] The stability of this model against anisotropic stress was then confirmed in [22] by following the cosmological evolution in the context of an anisotropic Bianchi ansatz.

In the model of [20] (and in many other implementations of the "matter bounce") there are two matter fields, the field ϕ and a field ψ representing the matter which initially dominates the phase of contraction, and which has an equation of state $p = 0$, p denoting the pressure density. Thus, in general there will not only be adiabatic cosmological fluctuations, but also entropic ones. In this paper we give a careful analysis of the evolution of both background and cosmological perturbations in the two field scenario in which a first field ψ generates a matter phase of contraction,

[3] The anisotropy remains small during the bounce phase, and there is no dangerous non-scale-invariant fluctuation mode which emerges in the bounce phase.

and a second field ϕ which has a negative exponential potential and hence yields a later phase of Ekpyrotic contraction, and which has a non-trivial kinetic action which generates a non-singular bounce.

We begin in the matter-dominated period of contraction with vacuum fluctuations of both scalar fields. For ψ, the resulting power spectrum is blue, since the field has a mass. For ϕ, the resulting power spectrum on super-Hubble scales is scale-invariant. In the far past, the spectrum of ϕ corresponds to the entropy mode, while ψ corresponds to the adiabatic mode. However, at the transition between the matter phase and the Ekpyrotic phase, ϕ becomes the adiabatic field, and thus a scale-invariant spectrum of curvature fluctuations results. Due to the gravitational mixing between the two modes during the matter phase of contraction, the ϕ fluctuations induce a scale-invariant component to the spectrum of ψ fluctuations at the end of the matter phase of contraction (this is the analog of the "curvaton" scenario of structure formation [23] - see also [24]). Hence, the mode which becomes the entropy mode during the later phases of evolution also inherits a scale-invariant contribution in addition to the original contribution which has a steep blue spectrum.

The outline of the paper is as follows: in Section II we discuss the model for a non-singular bounce proposed in [20], and how this is affected by the addition of an additional scalar field of K-essence form. In Section III we describe the background cosmological evolution by splitting the time history of the universe into phases: matter contraction, Ekpyrotic contraction, non-singular bounce, and fast roll expansion. To justify this phase structure, and to serve as a evidence that this model is feasible, we study the background numerically. In Section IV, we consider the evolution of perturbations our model, which we then use in Section V to calculate the power spectra at late times. We finish with some concluding remarks in Section VI.

A word on notation: We define the reduced Planck mass by $M_p = 1/\sqrt{8\pi G_{\rm N}}$ where $G_{\rm N}$ is Newton's gravitational constant. The sign of the metric is taken to be $(+,-,-,-)$. Note that we take the value of the scale factor at the bounce point to be $a_B = 1$ throughout the paper.

4.2 Cosmology of a Non-Singular Bounce

As discussed in the introduction, the model of interest for the present work is that of two scalar fields: a matter field which dominates at very early times, and a bounce field which violates the Null Energy Condition for a brief period, inducing the bounce. We begin with the most general Lagrangian for this class of models, given by

$$\mathcal{L} = K(\phi, X) + G(\phi, X)\Box\phi + P(\psi, Y) \ , \tag{4.1}$$

where ϕ is the bounce field of Galilean type, ψ is a K-essence scalar of general form, and have defined

$$X \equiv \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \ , \quad Y \equiv \frac{1}{2}\partial_\mu\psi\partial^\mu\psi \ , \tag{4.2}$$

as well as the d'Alembertian operator

$$\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu. \tag{4.3}$$

The Lagrangian terms for the bounce field are defined as

$$K(\phi, X) = M_p^2 \left[1 - g(\phi)\right] X + \beta X^2 - V(\phi), \tag{4.4}$$

$$G(\phi, X) = \gamma X, \tag{4.5}$$

where we have parametrized the model via the positive-definite constants[4] β and γ, as well as the functions $g(\phi)$ and $V(\phi)$. The term $G(\phi, X)$ is a Galileon-type operator which we have introduced to stabilize the gradient term of cosmological

[4] The positive-definiteness of β ensures that the kinetic term is bounded from below at high energy scales

perturbations, and leads to a sound speed which is positive-definite at all time except for during the bounce. Note that we have adopted the convention that ϕ is dimensionless, and so we include a factor of M_{pl}^2 in $K(\phi, X)$.

The bounce is triggered when $g(\phi) < 1$, which causes ϕ to form a ghost condensate and hence violate the Null Energy Condition. The function is negligible far from the bounce, such that the bounce field ϕ will have canonical kinetic terms at early and late times, given suitable behaviour for X. We can build this function by setting the bounce to occur at $\phi = 0$, and requiring that $g < 1$ when $|\phi| \gg 1$ but $g > 1$ when $\phi \sim 0$. We choose its form to be

$$g(\phi) = \frac{2g_0}{\mathrm{e}^{-\sqrt{\frac{2}{p}}\phi} + \mathrm{e}^{b_g\sqrt{\frac{2}{p}}\phi}}, \tag{4.6}$$

where $g_0 \equiv g(0)$ and p are positive constants, with g_0 larger than unity, $g_0 > 1$ and p smaller than unity, $p < 1$.

The bounce field potential $V(\phi)$ is chosen to ensure that the bounce is preceded by a phase of Ekpyrotic contraction, which is necessary to dilute anisotropy and avoid the BKL instability. The potential can also be chosen to give an attractor solution in both the expanding and contracting branches of the cosmological evolution, by making use of exponential functions. We take the form of the potential to be

$$V(\phi) = -\frac{2V_0}{\mathrm{e}^{-\sqrt{\frac{2}{q}}\phi} + \mathrm{e}^{b_V\sqrt{\frac{2}{q}}\phi}}, \tag{4.7}$$

where V_0 is a positive constant with dimension of $(\text{mass})^4$, q is a positive constant that must be smaller than 1/3 in order to obtain Ekpyrotic contraction, and the constant b_V is an asymmetry parameter for the potential. The attractor solution is induced during expansion by the positive-valued exponential, while the negative exponential leads to an attractor solution in the contracting phase.

We now turn to the second field ψ, which we introduce to play the role of an arbitrary matter field satisfying the Null Energy Condition. Initially, we take its Lagrangian to be of K-essence form, $P(\psi, Y)$, but eventually we will consider a canonical massive free scalar field. It has pressure and energy density given by

$$p_\psi = P \ , \tag{4.8}$$

$$\rho_\psi = 2YP_{,Y} - P \ . \tag{4.9}$$

There are two important quantities for this system: the equation of state w_ψ and the sound speed square c_ψ^2. These are given by

$$w_\psi \equiv \frac{p_\psi}{\rho_\psi} = -1 + \frac{2YP_{,Y}}{2YP_{,Y} - P} \ , \tag{4.10}$$

$$c_\psi^2 \equiv \frac{p_{\psi,Y}}{\rho_{\psi,Y}} = \frac{P_{,Y}}{2YP_{,YY} + P_{,Y}} \ . \tag{4.11}$$

We now consider the spatially flat FRW universe whose metric is given by

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2 \ , \tag{4.12}$$

where t is cosmic time, x are the comoving spatial coordinates and $a(t)$ is the scale factor. The evolution of the scale factor can be characterized by the Hubble rate:

$$H \equiv \frac{\dot{a}}{a} \ , \tag{4.13}$$

where the dot denotes the derivative with respect to cosmic time t.

At the background level the universe is homogenous, and thus both the bounce field ϕ and the matter field ψ are only functions of cosmic time. Thus, the kinetic terms of these two fields become

$$X = \dot{\phi}^2/2 \ , \quad \Box\phi = \ddot{\phi} + 3H\dot{\phi} \ , \quad Y = \dot{\psi}^2/2 \ . \tag{4.14}$$

The pressure and energy density of the bounce field are given by

$$p_\phi = \frac{1}{2}M_p^2(1-g)\dot{\phi}^2 + \frac{1}{4}\beta\dot{\phi}^4 - \gamma\dot{\phi}^2\ddot{\phi} - V(\phi) \ , \tag{4.15}$$

$$\rho_\phi = \frac{1}{2}M_p^2(1-g)\dot{\phi}^2 + \frac{3}{4}\beta\dot{\phi}^4 + 3\gamma H\dot{\phi}^3 + V(\phi) \ , \tag{4.16}$$

where dynamics of ϕ are governed by the equation of motion

$$\mathcal{P}\ddot{\phi} + \mathcal{D}\dot{\phi} + V_{,\phi} = 0 \ , \tag{4.17}$$

and we have introduced

$$\begin{aligned} \mathcal{P} &= (1-g)M_p^2 + 6\gamma H\dot{\phi} + 3\beta\dot{\phi}^2 + \frac{3\gamma^2}{2M_p^2}\dot{\phi}^4, & (4.18) \\ \mathcal{D} &= 3(1-g)M_p^2 H + \left(9\gamma H^2 - \frac{1}{2}M_p^2 g_{,\phi}\right)\dot{\phi} + 3\beta H\dot{\phi}^2 \\ &\quad -\frac{3}{2}(1-g)\gamma\dot{\phi}^3 - \frac{9\gamma^2 H\dot{\phi}^4}{2M_p^2} - \frac{3\beta\gamma\dot{\phi}^5}{2M_p^2} - \frac{3G_{,X}}{2M_p^2}(\rho_\psi + p_\psi)\dot{\phi} \ . & (4.19) \end{aligned}$$

From Eq. (4.17), it is clear that the function $\mathcal{P}$ determines the positivity of the kinetic term of the scalar field and thus can be used to determine whether the model contains a ghost or not at the perturbative level; the function $\mathcal{D}$ on the other hand, represents an effective damping term. By keeping the first terms of the expressions for $\mathcal{P}$ and $\mathcal{D}$ and setting $g = 0$, which is a good approximation far from the bounce where $\dot{\phi} \ll M_{Pl}$, one can recover the standard Klein-Gordon equation in the FRW background. Note that the friction term $\mathcal{D}$ contains the contributions from the matter fluid, which can be suppressed for small values of $\dot{\phi}$. However, these terms will become important during the bounce phase where $\dot{\phi}$ reaches a maximal value.

For completeness, we can write down the Einstein equations in this background,

$$M_p^2\left(R_{\mu\nu} - \frac{R}{2}g_{\mu\nu}\right) = T^{\phi}_{\mu\nu} + T^{\psi}_{\mu\nu}, \tag{4.20}$$

and the corresponding Friedmann equations,

$$\begin{aligned} H^2 &= \frac{\rho_{\rm T}}{3M_p^2} \ , & (4.21) \\ \dot{H} &= -\frac{\rho_{\rm T} + p_{\rm T}}{2M_p^2} \ , & (4.22) \end{aligned}$$

where $\rho_{\rm T}$ and $p_{\rm T}$ represent the total energy density and pressure in the FRW universe, e.g. the sum of the contributions of the bounce field and the matter field.

4.3 Background evolution

The initial conditions of the background are chosen such that the universe is initially dominated by regular matter in the contracting phase, which in our model is mimicked by the matter field ψ. Since the potential of the bounce field $V(\phi)$ has an Ekpyrotic potential for $\phi \ll 1$, the corresponding energy density grows faster than that of regular matter. As a consequence, ϕ eventually becomes dominant, signaling the end of matter contraction. After that, the Ekpyrotic phase of contraction begins, and lasts until the non-singular bounce interval begins (this is the phase where the effects coming from new physics dominate), followed by a period of fast-roll expansion, which in turn ends at a transition to the expansion of Standard Big Bang cosmology. We choose the initial conditions for the density of regular matter and for the value of ϕ such that the temperature at which the Ekpyrotic phase begins is higher than that at the time of equal matter and radiation in the Standard Big Bang expanding phase.

4.3.1 Analytic estimates

In the following we briefly investigate the evolution of the universe in each of the periods mentioned above, and refer to [22] for a more generic analysis in which the anisotropy was taken into account as well.

Matter contraction

We start by considering the period when the universe is dominated by the matter field ψ. We take the Lagrangian of ψ to be that of a free canonically normalized massive scalar field:

$$P(\psi, Y) = Y - \frac{1}{2}m^2\psi^2 \ . \tag{4.23}$$

Thus the matter field oscillates around its vacuum state $\psi = 0$ and the time-averaged background equation of state parameter is roughly $w = 0$. In this phase, the scale factor evolves as

$$a(t) \simeq a_E \left(\frac{t - \tilde{t}_E}{t_E - \tilde{t}_E}\right)^{2/3} , \tag{4.24}$$

where t_E denotes the final moment of matter contraction and the beginning of the Ekpyrotic phase, and a_E is the value of the scale factor at the time t_E. In the above, $\tilde{t}_E$ is an integration constant which is introduced to match the Hubble parameter continuously at the time t_E,

$$\tilde{t}_E \simeq t_E - \frac{2}{3H_E}. \tag{4.25}$$

Hence the Hubble parameter can be approximated by

$$\langle H(t) \rangle = \frac{2}{3(t - \tilde{t}_E)} \ . \tag{4.26}$$

where the angular brackets stand for averaging over time. The solution for the scalar field ψ can be asymptotically expressed (modulo a phase) as

$$\psi(t) \simeq \tilde{\psi}(t) \sin(m(t - \tilde{t}_E)) \ , \tag{4.27}$$

with a time dependent amplitude

$$\tilde{\psi}(t) = \frac{1}{\sqrt{3\pi G} m(t - \tilde{t}_E)} \ , \tag{4.28}$$

which yields an equation of state with has vanishing pressure when averaged over an oscillation period of the field.

Ekpyrotic contraction

We assume a homogeneous scalar field ϕ which is initially placed in the region $\phi \ll -1$ in the phase of matter contraction. In this case, the Lagrangian for ϕ approaches the conventional canonical form. Once ϕ begins to dominate the energy-momentum tensor of matter, it then approaches an attractor solution which is given by

$$\phi(t) \simeq -\sqrt{\frac{q}{2}} \ln \left[\frac{2V_0 (t - \tilde{t}_{B-})^2}{q(1-3q)M_p^2} \right] \ , \tag{4.29}$$

where $\tilde{t}_{B-}$ is an integration constant which chosen such that the Hubble parameter at the end of the phase of Ekpyrotic contraction matches with the one at the beginning of the bounce phase. This attractor solution corresponds to an effective equation of state

$$w \simeq -1 + \frac{2}{3q} \ . \tag{4.30}$$

During the phase of Ekpyrotic contraction, the scale factor evolves as

$$a(t) \simeq a_{B-} \left(\frac{t - \tilde{t}_{B-}}{t_{B-} - \tilde{t}_{B-}} \right)^q \ , \tag{4.31}$$

where a_{B-} is the value of scale factor at the time t_{B-} which corresponds to the end of Ekpyrotic contraction and the beginning of the bounce phase. Therefore, the Hubble parameter is given by

$$H(t) \simeq \frac{q}{t - \tilde{t}_{B-}} \ , \tag{4.32}$$

where, in order to make $H(t)$ continuous at the time t_{B-}, one must set

$$\tilde{t}_{B-} = t_{B-} - \frac{q}{H_{B-}} \ . \tag{4.33}$$

Additionally, we require the scale factor to evolve smoothly and continuously at the time t_E. This leads to the relation

$$a_E \simeq a_{B-} \left(\frac{H_{B-}}{H_E} \right)^q \ . \tag{4.34}$$

Bounce phase

In our model the scalar field evolves monotonically from $\phi \ll -1$ to $\phi \gg 1$. For values of ϕ between $\phi_- \sim -\sqrt{p/2}\ln(2g_0)$ and $\phi_+ \sim \sqrt{p/2}\ln(2g_0)/b_g$ (assuming one term in the denominator of $g(\phi)$ dominates over the other at each transition time), the value of the function $g(\phi)$ becomes larger than unity and thus the universe enters a ghost condensate state. The occurrence of the ghost condensate naturally yields a short period of Null Energy Condition violation and this in turn gives rise to a non-singular bounce [14].

As shown in Ref. [20], we have two useful parameterizations to describe the evolution of the scale factor in the bounce phase. One is the linear parametrization of the Hubble parameter

$$H(t) \simeq \Upsilon t \ , \tag{4.35}$$

and the other is the evolution of the background scalar

$$\dot{\phi}(t) \simeq \dot{\phi}_B \mathrm{e}^{-t^2/T^2} \ , \tag{4.36}$$

where the coefficient Υ is set by the detailed microphysics of the bounce. The coefficient T can be determined by matching the detailed evolution of the scalar field at the beginning or the end of the bounce phase, which will be addressed in next subsection. Thus, during the bounce the scale factor evolves as

$$a(t) \simeq a_B \mathrm{e}^{\frac{1}{2}\Upsilon t^2} \ . \tag{4.37}$$

Note that a non-singular bounce requires that the total energy density vanishes at the bounce point. The total energy density includes the contributions from the matter fields and the anisotropy factors. This leads to the following result for the value of $\dot{\phi}_B$

$$\begin{aligned} \dot{\phi}_B^2 &\simeq \frac{(g_0-1)M_p^2}{3\beta}\left[1+\sqrt{1+\frac{12\beta(V_0+\rho_\mathrm{m}+\rho_\theta)}{(g_0-1)^2M_p^4}}\right] \\ &\simeq \frac{2(g_0-1)}{3\beta}M_p^2 \ , \end{aligned} \tag{4.38}$$

where we have made use of approximations that ρ_m and ρ_θ are much less than V_0 and $V_0 \ll M_p^4$ in the second line. These approximations must be valid for the model to hold since both ρ_m and ρ_θ are greatly diluted in the Ekpyrotic phase and V_0 is the maximal absolute value of the potential of ϕ which, according to the observational constraint from the amplitude of cosmological perturbations, must be far below the Planck scale.

Fast-roll expansion

After the bounce, the universe enters the expanding phase, where the universe is still dominated by the scalar field ϕ. During this stage, the motion of ϕ is dominated by its kinetic term while the potential is negligible. Thus, the background equation of state parameter is $w \simeq 1$. This corresponds to a period of fast-roll expansion, where the scale factor evolves as

$$a(t) \simeq a_{B+} \left(\frac{t - \tilde{t}_{B+}}{t_{B+} - \tilde{t}_{B+}} \right)^{1/3} , \tag{4.39}$$

where t_{B+} represents the end of the bounce phase and the beginning of the fast-roll period, and a_{B+} is the value of the scale factor at that moment. Then one can write down the Hubble parameter in the fast-roll phase

$$H(t) \simeq \frac{1}{3(t - \tilde{t}_{B+})} , \tag{4.40}$$

and the continuity of the Hubble parameter at t_{B+} yields

$$\tilde{t}_{B+} = t_{B+} - \frac{1}{3H_{B+}} . \tag{4.41}$$

Recall that, in Eq. (4.36), we made use of a Gaussian parametrization of the scalar field evolution in the bounce phase, with characteristic timescale T. In the fast roll phase we find the following approximate solution for the evolution of ϕ:

$$\dot{\phi}(t) \simeq \dot{\phi}_{B+} \frac{a_{B+}^3}{a^3(t)} \simeq \dot{\phi}_B \mathrm{e}^{-t_{B+}^2/T^2} \frac{H(t)}{H_{B+}} , \tag{4.42}$$

where we have applied (4.36) in the second equality. This implies that

$$\rho_\phi \simeq \frac{M_p^2}{2} \dot{\phi}^2 \simeq \frac{M_p^2 \dot{\phi}_B^2}{2\mathrm{e}^{2t_{B+}^2/T^2}} \frac{H^2}{H_{B+}^2} . \tag{4.43}$$

Moreover, the Friedmann equation requires that $\rho_\phi \simeq 3M_p^2 H^2$ in the fast-roll phase, so that T^2 is given by

$$T^2 \simeq \frac{2H_{B+}^2}{\Upsilon^2 \ln\left[\dfrac{M_p^2(g_0-1)}{9\beta H_{B+}^2}\right]} \ . \tag{4.44}$$

4.3.2 A (Numerical) Proof of Principle

To justify our claims that the background does exhibit this phase structure, we numerically solve the background equations of motion. We present this solely as a 'Proof of Principle', in order to illustrate the occurrence of a non-singular bounce in the model under consideration. By this we mean that the parameters are chosen to make the effect of the matter field ψ manifest during the bounce, but this parameter choice does not necessarily satisfy the bounds imposed by observations. Assuming parameter values taking into account the experimental constraints would lead to an Ekpyrotic phase which is long enough to dilute all the matter fields, which would decrease the significance of entropy perturbations. Similarly, in the limit that the Ekpyrotic phase stretches to the infinite past, the evolution of the background approaches that obtained in a regular isotropic bounce model realized by a single field as studied in [20].

In the numerical calculation we work in units of the Planck mass M_p for all variables. We specifically set a group of model parameters as,

$$\begin{gathered} V_0 = 10^{-10} \ , \quad g_0 = 1.1 \ , \quad \beta = 5 \ , \quad \gamma = 10^{-3} \ , \\ b_V = 5 \ , \quad b_g = 0.5 \ , \quad p = 0.01 \ , \quad q = 0.1 \ , \quad m = 5 \times 10^{-6} \ . \end{gathered} \tag{4.45}$$

Moreover we choose the initial conditions for the bounce field and matter field as follows,

$$\begin{gathered} \phi_{\rm ini} = -2.11 \ , \quad \dot{\phi}_{\rm ini} = -8.87 \times 10^{-8} \ , \\ \psi_{\rm ini} = -0.025 \ , \quad \dot{\psi}_{\rm ini} = -3.57 \times 10^{-8} \ . \end{gathered} \tag{4.46}$$

Our numerical results are presented in Figs. 4–1 and 4–2. In order to enlarge the details of the cosmic evolution, we introduced a parameter

$$N_a \equiv \begin{cases} -\ln \frac{a}{a_0} & t < t_B \\ \\ \ln \frac{a}{a_0} & t \geq t_B \end{cases} \tag{4.47}$$

(where a_0 is a normalization constant) as the horizontal axis in Fig. 4–1. The vertical axis shows the dynamics of the Hubble parameter and the equations of state of scalar fields as well as the overall one.

From the upper panel of Fig. 4–1, one can see that the Hubble parameter evolves smoothly through the bounce point with an approximately linear dependence on cosmic time. However, the bounce phase is not symmetric with respect to the bounce point in this model. The lower panel of Fig. 4–1 shows that the background equation of state initially takes an average value $w = 0$ since the universe is dominated by the oscillating matter field ψ. During the matter contraction, the bounce field slowly becomes dominant over and triggers a period of Ekpyrotic contraction, where for our parametrization the equation of state is approximately equal to $w = 5.67$. When the universe enters the bounce phase, the background equation of state experiences a sudden decrease to negative infinity and then evolves back to a value $w = 1$ which signals a fast-roll expanding phase.

In order to better characterize the transitions between different phases, we plot the evolution of the energy densities and density parameters in Fig. 4–2. The density parameters are defined as

$$\Omega_i \equiv \frac{\rho_i}{\rho_{\rm T}} \ , \tag{4.48}$$

where the subscript "i" represents ϕ and ψ, respectively. This figure explicitly shows that the universe in this model experiences four phases: Matter contraction, Ekpyrotic contraction, the bounce, and fast-roll expansion.

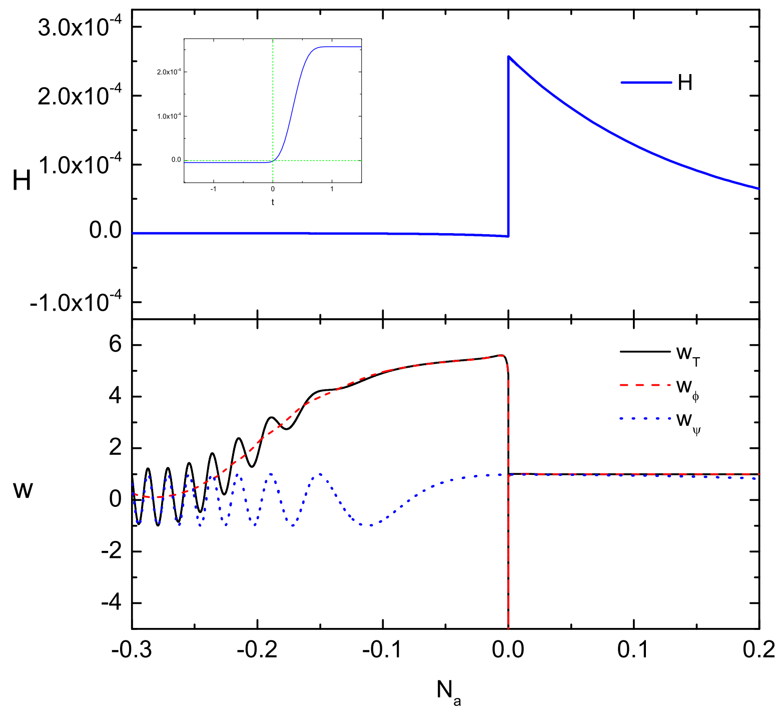


Figure 4–1: Cosmic evolution of the Hubble parameter H (blue line in the upper panel) and the equations of state (black solid, red dashed, and blue dotted lines in lower panel for the total background $w_{\rm T}$, the bounce field w_ϕ and the matter field w_ψ, respectively), in units of the reduced Planck mass M_p, with background parameters given by (4.45) and initial conditions as in (4.46). The main plot shows that a non-singular bounce occurs, and that the time scale of the bounce is short (it is a "fast bounce" model). The inner insert shows a zoomed-in view of the smooth Hubble parameter during the bounce phase as a function of cosmic time.

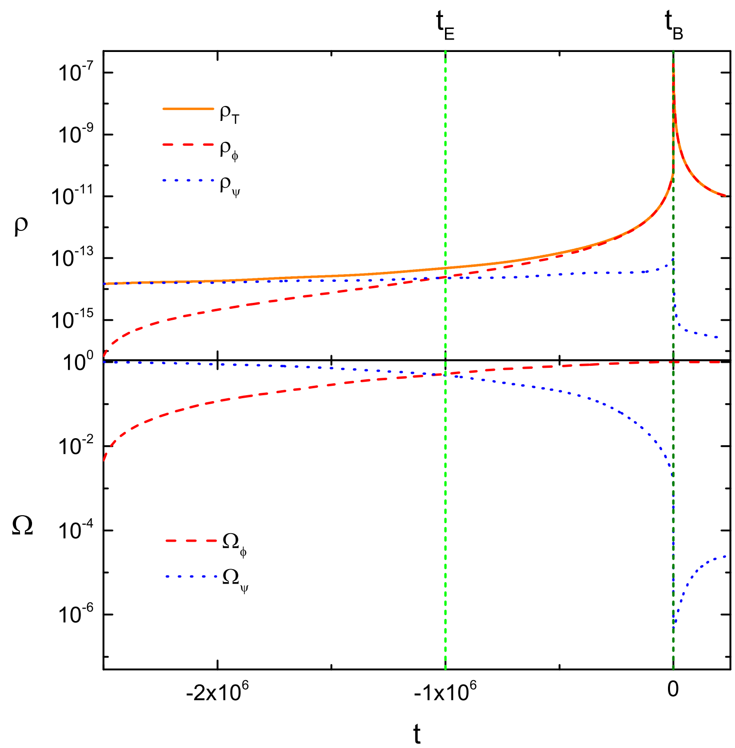


Figure 4–2: Cosmic evolution of the energy densities ρ and density parameters Ω of the background universe (orange solid line in upper panel), the bounce field (red dashed line) and the matter field (blue dotted line), respectively. The horizontal axis is the cosmic time. The initial conditions and model parameters are the same as in Fig. 4–1.

4.4 Cosmological Perturbations

4.4.1 Overview

In this section we study the dynamics of linear cosmological perturbations in the Two Field Matter Bounce. One attractive property of a non-singular bounce cosmology is that perturbation modes can be evolved smoothly through the bounce phase. In linear theory, perturbations of scalar type evolve independently from those of vector and tensor type. This reduces the number of degrees of freedom which must be analyzed. In addition, as a consequence of linearity one can track each Fourier mode independently (see e.g. [25] for a survey of the theory of cosmological perturbations and [26] for an introductory overview). The evolution of the Fourier modes depends on the background cosmology.

As per the analysis presented in the previous section, our cosmological background will first undergo matter contraction, then a period of Ekpyrotic contraction, followed by a non-singular bounce, and then a phase of fast roll expansion. We begin with vacuum fluctuations on sub-Hubble scales in contracting phase. During the phase of contraction, wavelengths exit the Hubble radius (which is shrinking in co-moving coordinates). Once they are on super-Hubble scales, the modes are squeezed. Both the exiting of the Hubble radius and the squeezing on super-Hubble scales is similar to what happens during the phase of accelerated expansion in inflationary cosmology. However, in the case of inflation the Hubble length has constant physical size while the physical wavelength of fluctuations increases exponentially. Hence, if the period of inflation was long, the physical wavelength of the fluctuations was initially smaller than the Planck length, leading to the 'trans-Planckian problem' for fluctuations [27]. This problem does not arise in a bouncing cosmology as long as the energy scale of the bounce is smaller than the Planck scale, as is required for the self-consistency of any effective field treatment such as what we are presenting, since then the physical wavelength of the fluctuation modes which we measure today were always much larger than the Planck length scale.

As was initially realized in [9, 10], a consequence of the super-Hubble growth of fluctuations in a contracting universe is that the initial vacuum fluctuations of a massless scalar field (and consequently also the curvature fluctuations in a model in which the only matter component is this massless field) are converted to a scale-invariant spectrum. It is in this sense that the matter bounce can provide an alternative to inflationary cosmology as a mechanism to form the cosmological fluctuations we observe today.

However, the model under consideration involves two scalar fields, ϕ and ψ, with ψ leading to a phase of a matter contraction at early times, and ϕ being responsible for the Ekpyrotic phase and the bounce. The dominant field in the initial matter phase of contraction is massive and hence its vacuum spectrum does not evolve into a scale-invariant form in isolation. The field ϕ, on the other hand, is effectively massless at early times and hence evolves to a scale-invariant spectrum on super-Hubble scales. The field ϕ acts an entropy field during the phase of matter contraction. However, once the Ekpyrotic phase begins, ϕ becomes dominant and becomes the curvature mode, while the ψ fluctuations become the entropy modes.

As is well known, entropy modes source a growing curvature perturbation on super-Hubble scales [5] Thus, to determine the final spectrum of curvature and entropy fluctuations in our model we must carefully study the interaction of the two fluctuation modes in each cosmological phase. As we will show, in the matter phase of contraction, the scale-invariant ϕ mode (which acts as an entropy fluctuation) seeds a curvature fluctuation (the ψ mode in the initial phase) of comparable magnitude. Thus, at the end of the matter-dominated phase of contraction, both modes are scale-invariant and have comparable amplitude. After that time, it is no longer important to consider the sourcing of the adiabatic mode by the entropy mode since the adiabatic mode is already larger in amplitude (and the effect of the sourcing cannot induce a larger amplitude than that of the source)

In many non-singular bounce models it has been shown that the scale-invariance of curvature fluctuations is preserved during the bounce phase (see, however, the exceptions discussed in [21]). We will show that this is also the case in our model.

[5] See e.g. [28] for an early discussion in the context of an axion dominated inflationary universe.

We will also evolve the entropy fluctuations on super-Hubble scales and will show that they preserve their scale-invariance on large scales. Moreover the curvature mode are amplified compared to the the entropy mode during the bounce phase, and thus the final spectrum of fluctuations is almost completely adiabatic.

In both the matter contraction phase and the Ekpyrotic phase, the Lagrangian of the bounce scalar recovers the canonical form, since the higher derivative terms are suppressed by the small value of $\dot{\phi}$. In the matter contraction phase, it is convenient to study the evolution of perturbation modes in the spatially flat gauge ($\zeta = 0$) and the initial conditions for two field fluctuations can be imposed inside the Hubble radius. However once the initial conditions have been set, we can switch into the uniform ϕ gauge ($\delta\phi = 0$) for the Ekpyrotic and subsequent phases. In this way, the curvature perturbation becomes manifest.

To perform this perturbation analysis we use three sets of perturbation variables. For the initial conditions, we consider the field fluctuations in the spatially flat gauge

$$Q_\phi = M_p(\delta\phi + \frac{\dot{\phi}}{H}\Phi) \quad , \quad Q_\psi = \delta\psi + \frac{\dot{\psi}}{H}\Phi \ . \tag{4.49}$$

where Φ is the Bardeen potential (see Appendix A). We can change to the uniform ϕ gauge, where the perturbation variables become $\delta\psi$ and

$$\zeta = H\frac{(M_p\dot{\phi}Q_\phi + \dot{\psi}Q_\psi)}{M_p^2\dot{\phi}^2 + \dot{\psi}^2} \qquad \delta\psi \to Q_\psi \ . \tag{4.50}$$

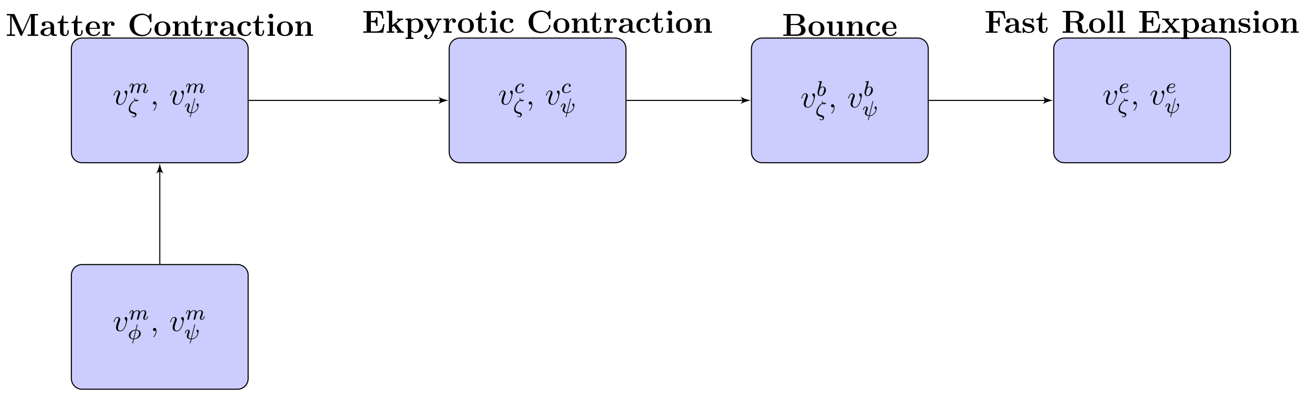
We lay out the general recipe for the perturbation analysis in Figure 4.4.1

4.4.2 Field fluctuations during matter contraction

At the beginning of matter contraction, the universe is dominated by the matter field ψ which is oscillating around its vacuum point; this yields a time averaged value of the background equation of state $w \simeq 0$ and thus the universe is in a matter dominated phase. During this phase, the bounce field ϕ is subdominant and fast rolling down along its potential with an effective equation of state $w_\phi \simeq 1$.

One can perturb the metric and the two scalar field to linear order, which includes three scalar type perturbation modes, ζ, $\delta\phi$ and $\delta\psi$, respectively. However,

Figure 4–3: Stages of the Perturbation analysis. We begin in the matter dominated phase by using the field fluctuations, then change gauge to study the entropy and curvature perturbations. We match the curvature perturbations through the bounce to solve for the behaviour in the fast roll expansion phase. We denote the phases of the bounce by indices on the perturbation variables: m , c , b, and e, for **m**atter domination, Ekpyrotic **c**ontraction, **b**ounce, and fast roll **e**xpansion.



one of these three variable can be eliminated by making a gauge choice. We start by considering the evolution of cosmological perturbations using the gauge invariant field fluctuations Q_ϕ and Q_ψ defined in Eq. (4.49), which are the Mukhanov-Sasaki variables [29, 30].

One can introduce the gauge invariant curvature perturbation as in Eq. (4.50), as well as the entropy perturbation

$$\mathcal{S} = \frac{\left(M_p \dot{\phi} Q_\psi - \dot{\psi} Q_\phi\right)}{\sqrt{M_p^2 \dot{\phi}^2 + \dot{\psi}^2}} \ . \tag{4.51}$$

At early times in the matter dominated phase, $\dot{\psi} \gg \dot{\phi} M_p$, which implies that

$$\zeta \simeq H \frac{Q_\psi}{\dot{\psi}} \ , \quad \mathcal{S} \simeq -Q_\phi \quad \text{ for } \ t \to -\infty. \tag{4.52}$$

Therefore, one can immediately observe that at very early times the main contribution to the curvature perturbation is from the matter field fluctuation, and the entropy perturbation is dominated by the fluctuation of the bounce field. However, one can see from Eq. 4.50 that by the end of the matter contraction phase, the contribution to the curvature perturbation from each field will become equally important. With this in mind, we follow the evolution of v_ϕ and v_ψ during the matter contraction in order to determine the resulting spectrum of v_ζ at t_E. We will see that the result of this is that ζ acquires a scale invariant spectrum from the bounce field (which was initially the entropy perturbation). This is an explicit realization of the Matter Bounce Curvaton scenario proposed in [24].

The field fluctuations evolve following the general equations of motion provided in (4.113) as analyzed in Appendix A. The perturbation equations can be written in terms of canonical variables

$$v_\phi = aQ_\phi \ , \quad v_\psi = aQ_\psi \ . \tag{4.53}$$

The equations of motion can then be written in Fourier space as

$$v_\phi'' + (k^2 - \frac{a''}{a})v_\phi = J_{\phi\psi}v_\psi + J_{\phi\phi}v_\phi \ , \tag{4.54}$$

$$v_\psi'' + (k^2 + m^2a^2 - \frac{a''}{a})v_\psi = J_{\phi\psi}v_\phi + J_{\psi\psi}v_\psi \ , \tag{4.55}$$

where the prime denotes the derivative with respect to conformal time, and we define the source (interaction) terms:

$$J_{\phi\phi} = -\frac{9}{2}\mathcal{H}_E^2\Big(\frac{a_E}{a}\Big)^4 \ , \tag{4.56}$$

$$J_{\phi\psi} = \frac{3}{2}m\mathcal{H}_E\frac{a_E}{a}\cos\left[ma(\tau - \tilde{\tau}_E)\right] \ , \tag{4.57}$$

$$J_{\psi\psi} = \frac{9}{2}\mathcal{H}_E^2\frac{a_E}{a} \ . \tag{4.58}$$

We can treat this system perturbatively, using the first order Born approximation to estimate the effect of the source terms. We begin by analyzing the source-free ('homogeneous') system:

$$v_\phi^{(0)\prime\prime} + (k^2 - \frac{a''}{a})v_\phi^{(0)} = 0 \ , \tag{4.59}$$

$$v_\psi^{(0)\prime\prime} + (k^2 + a^2 m^2 - \frac{a''}{a}) v_\psi^{(0)} = 0 \ . \tag{4.60}$$

One can see from Eq. (4.59) that the k^2 term will initially dominate, and so the squeezing factor $a^{''}/a$ can be neglected. Thus the dynamics for v_ϕ corresponds to a free scalar propagating in a flat space-time, and the initial conditions take the form of the Bunch-Davies vacuum:

$$v_\phi^{ini}(\tau, k) \simeq \frac{e^{-ik\tau}}{\sqrt{2k}} \ . \tag{4.61}$$

However, the situation for v_ψ is different, due to the presence of a non-zero mass. Specifically, in Eq. (4.60) when we neglect the last term a''/a, the mass term becomes important in addition to the k^2 term at the initial moment. Thus one can introduce an effective frequency for v_ψ as

$$\omega_k^2 = k^2 + a^2 m^2 \ , \tag{4.62}$$

and Eq. (4.60) has an asymptotic solution which oscillates rapidly with this time dependent frequency on sub-Hubble scales. This is what is expected since the adiabaticity condition $|\omega_k'/\omega_k^2| \ll 1$ is satisfied which corresponds to a situation in which the effective physical wavelength is much smaller than the Hubble radius. Therefore, the modes can be regarded as adiabatic when they are in the sub-Hubble regime with $|\omega_k \tau| \gg 1$, and we can impose suitable vacuum initial conditions by virtue of a Wentzel-Kramers-Brillouin (WKB) approximation

$$\sqrt{2\epsilon} \ v_\psi^{ini}(\tau, k) \simeq \frac{1}{\sqrt{2\omega_k}} e^{-i\int^\tau \omega_k(\tilde{\tau}) \mathrm{d}\tilde{\tau}} \ , \tag{4.63}$$

where $\epsilon \equiv -\dot{H}/H^2 = 3/2$ in the phase of matter contraction.

During both the matter and Ekpyrotic phases of contraction, the fluctuations modes on scales of cosmological interest today exit the Hubble radius and become classical perturbations [6] . For matter dominated contraction, one has

$$a \propto (\tau - \tilde{\tau}_E)^2 \ , \quad \tilde{\tau}_E = \tau_E - \frac{2}{\mathcal{H}_E} \ , \tag{4.64}$$

where $\mathcal{H}_E$ is the conformal Hubble parameter at the moment t_E. The gravitational term a''/a leads to the squeezing of field fluctuations. Making use of the vacuum initial condition, we obtain an exact solution to (4.59):

$$v_\phi^{(0)}(\tau, k) \simeq \frac{e^{-ik(\tau-\tilde{\tau}_E)}}{\sqrt{2k}} \left[1 - \frac{i}{k(\tau - \tilde{\tau}_E)}\right] \ , \tag{4.65}$$

in the phase of matter contraction. For the v_ψ mode, there exists a mass term in the expression for the dispersion relation, and thus the field fluctuations do not get squeezed on super-Hubble scales. Instead, one can neglect the k^2 term and derive an asymptotical solution as follows,

$$v_\psi{}^{(0)}(\tau, k) \simeq \frac{e^{-iam(\tau-\tilde{\tau}_E)}}{\sqrt{6am}} \ . \tag{4.66}$$

These homogenous solutions correspond to a scale invariant spectrum of the entropy mode ϕ, and a spectrum of the initial curvature mode ψ that is deeply blue:

$$P_\phi^{(0)} \equiv \frac{k^3}{2\pi^2} |\frac{v_\phi^{(0)}}{a}|^2 = \frac{H^2}{16\pi^2} \ , \tag{4.67}$$

$$P_\psi^{(0)} \equiv \frac{k^3}{2\pi^2} |\frac{v_\psi^{(0)}}{a\sqrt{3}}|^2 = \frac{k^3}{12\pi^2 m a^3} \ . \tag{4.68}$$

As we now show, the entropy mode sources a growing contribution to the curvature mode which then inherits the scale-invariant spectrum of the entropy mode. To compute this effect, we use the 1st order Born approximation in which we evaluate the form of the source terms using the zero'th order solutions. This means that the

[6] The classicalization is a consequence of squeezing and decoherence via nonlinear interactions, as discussed in [31, 32].

1^{st} order corrections are determined using the equation of motion with the following background-dependent source terms:

$$v_\phi^{(1)\prime\prime} + (k^2 - \frac{a''}{a})v_\phi^{(1)} = J_{\phi\psi}v_\psi^{(0)} + J_{\phi\phi}v_\phi^{(0)}, \tag{4.69}$$

$$v_\psi^{(1)\prime\prime} + (k^2 + a^2m^2 - \frac{a''}{a})v_\psi^{(1)} = J_{\phi\psi}v_\phi^{(0)} + J_{\psi\psi}v_\psi^{(0)} \ . \tag{4.70}$$

We solve these for modes on super-Hubble scale and obtain the homogeneous solution plus first order correction,

$$v_\phi \simeq v_\phi^{(0)}\left[1+\frac{1}{3}\left(\frac{a_E}{a}\right)^3\right] + \frac{ma_E}{3\mathcal{H}_E}\left[1+\mathrm{Log}|\frac{2k}{\mathcal{H}_E}|\right]v_\psi^{(0)} \ , \tag{4.71}$$

$$v_\psi \simeq v_\psi^{(0)}\left[1+\frac{9\mathcal{H}_E}{4a_Em}\left(\frac{a_E}{a}\right)^{\frac{1}{2}}\right] + \frac{3}{2}e^{iam(\tau-\tilde{\tau}_E)}\left(\frac{a_E}{a}\right)^{\frac{1}{2}}v_\phi^{(0)} \ . \tag{4.72}$$

Correspondingly, the power spectra for two field fluctuations near the end of matter contraction are given by [7]

$$\begin{aligned} P_\phi &\simeq \frac{16}{9}P_\phi^{(0)} + \frac{1}{9}\left(\frac{m}{H_E}\right)^2\left[1+\mathrm{Log}|\frac{2k}{\mathcal{H}_E}|\right]^2 P_\psi^{(0)} \ , \\ P_\psi &\simeq \left[1+\frac{9}{4}\frac{H_E}{m}\right]^2 P_\psi^{(0)} + \frac{9}{4}P_\phi^{(0)} \ . \end{aligned} \tag{4.73}$$

We can see from the above expression that the gravitational interaction mixes the spectra of the two fields, such that both fields have a scale invariant piece which is the one which dominates in the infrared.

4.4.3 Perturbations in the phase of Ekpyrotic contraction

During the matter contraction, the energy density of the ϕ field becomes more and more important since it is fast rolling along its tachyonic potential. At some

[7] Note that the precise form of the mode functions actually includes an arbitrary phase, each of which is drawn from an independent gaussian distribution. The result of this is that the cross term of ϕ and ψ vanishes vanishes when averaged over both distributions to compute the power spectrum.

moment t_E, its contribution to the background energy density starts to dominate over that of the ψ field. We still have $|\phi| \gg 1$ and $\dot{\phi} \ll M_p$ and thus the Lagrangian of ϕ is of canonical form with an Ekpyrotic potential. This model then yields an attractor solution of Ekpyrotic contraction

$$a \propto (\tilde{\tau}_{B-} - \tau)^{\frac{q}{1-q}} \ , \quad \tilde{\tau}_{B-} = \tau_{B-} - \frac{q}{(1-q)\mathcal{H}_{B-}} \ . \tag{4.74}$$

We have introduced the instant of time $\tilde{\tau}_{B-}$ when the scale factor would meet the big crunch singularity if there was no non-singular bounce. If we were not interested in the bounce phase, it would make sense to normalize the time axis such that $\tilde{\tau}_{B-} = 0$, and in this case we would find that the function g would become unity slightly earlier, namely at a time $\frac{q}{(1-q)\mathcal{H}_{B-}}$ (keeping in mind that $\mathcal{H}_{B-}$ is negative). This signals the beginning moment of the bounce phase τ_{B-}.

Note that, when the universe has not yet arrived at the non-singular bounce phase, the Lagrangian has canonical form and thus the analysis based on gauge invariant field fluctuations (shown in the previous subsection) is still valid. However, one can see that the main contribution to the curvature perturbation has changed from $\delta\psi$ to $\delta\phi$. To render the analysis of cosmological perturbations through the non-singular bounce easier, we switch to the uniform ϕ gauge in the Ekpyrotic phase. The detailed analysis of the second order action for perturbations is performed in Appendix B. The simplified quadratic action in this phase is given by:

$$S_2 = \int d\tau dk^3 \frac{1}{2} \sum_i \left[v_i'^2 - \left(k^2 - \frac{q(2q-1)}{(1-q)^2(\tau - \tilde{\tau}_{B-})^2} \right) v_i^2 \right], \tag{4.75}$$

the subscript 'i' runs over $\{\zeta, \psi\}$. In Appendix B we introduce two new perturbation variables $\{v_\sigma, v_s\}$ which are linear combinations of v_ζ and v_ψ. This rotation decouples the kinetic terms of v_ζ and v_ψ in the general evolution. However, in the model under consider, we can find quadratic actions for v_ζ and v_ϕ which allows for an easy analysis without resorting to a field rotation.

The quadratic action (4.75) yields the following equations of motion for perturbation variables

$$v_i'' + \Big(k^2 - \frac{q(2q-1)}{(1-q)^2(\tau - \tilde{\tau}_{B-})^2}\Big)v_i = 0 \ , \tag{4.76}$$

One can solve for the general solutions to the above equations of motion as follows,

$$\begin{aligned} v_i^c = C_{i,1}\sqrt{\tau - \tilde{\tau}_{B-}} J_{\nu_c}(k(\tau - \tilde{\tau}_{B-})) \ &+ \ C_{i,2}\sqrt{\tau - \tilde{\tau}_{B-}} Y_{\nu_c}(k(\tau - \tilde{\tau}_{B-})) \ , \\ i \ &= \ \zeta, \ \psi \end{aligned} \tag{4.77}$$

where $\nu_c = \frac{(1-3q)}{2(1-q)}$ and the subscript "c" denotes the Ekpyrotic contracting phase. In addition, J_{υ_c} and Y_{υ_c} are the two linearly independent Bessel functions with indices ν_c. The coefficients $C_{i,1}$ and $C_{i,2}$ are functions of comoving wave number k, and are determined by matching the perturbations at the surface of t_E, as we will address in Section 4.5. For the moment we keep the coefficients general.

Recall that the expression of curvature perturbation ζ is given by Eq. (4.50). When the universe evolves into the Ekpyrotic phase, the trajectory of the background evolution becomes dominated by the bounce field and thus the curvature perturbation is mainly contributed by Q_ϕ, or equivalently v_ϕ. Since the matter field ψ no longer dominates over in the background evolution, its field fluctuation Q_ψ plays the role of entropy perturbation.

4.4.4 Perturbations through the bounce

When the bounce field ϕ evolves into the range of the ghost condensation, the kinetic term in its Lagrangian is no longer approximately canonical. This triggers a violation of the Null Energy Condition. This causes the universe to exit from the Ekpyrotic phase at some moment t_{B-} and to enter the bounce phase. In this period the bounce field yields a negative contribution to the energy density which will eventually cancel all the other positive contributions, including that of the matter field ψ, at a time we denote by t_B. We normalize the time axis of the background evolution such that $t_B = 0$. At this moment, the Hubble parameter transits from negative to positive values, crossing $H = 0$. As a result, a non-singular bounce takes place.

During the bounce phase, it is a good approximation to model the evolution of the Hubble parameter near the bounce as a linear function of cosmic time:

$$H(t) = \Upsilon t \,, \tag{4.78}$$

where Υ is a constant. Such a parametrization is applicable to a wide class of fast bounce models, and the value of Υ depends on the detailed microphysics of the bounce as shown in (4.35). In addition, the evolution of $\dot{\phi}$ during the bounce is given by (4.36). Making use of the parameterizations for $\dot{\phi}$ and the Hubble parameter H, we can keep the dominant terms of the quadratic action which then simplifies to

$$S_2 = \int d\tau dk^3 \frac{1}{2}\left[v_\zeta'^2 - \left(c_\zeta^2 k^2 - \frac{z''}{z} \right) v_\zeta^2 + v_\psi'^2 - \left(c_\psi^2 k^2 + a^2 m^2 - \frac{a''}{a} \right) v_\psi^2 \right] \,, \tag{4.79}$$

where we discuss the role of each term below.

First, we study the gradient terms of the two perturbation modes. The stability of the gradient terms is characterized by the sound speed square parameters, c_ψ^2 and c_ζ^2, which are defined in (4.126). In our explicit model, the matter field ψ takes canonical form and thus simply leads to $c_\psi^2 = 1$. Moreover, if we make use of the parameter choice (4.45) used in the numerical estimates in the previous section and insert the value of $\dot{\phi}_B^2$ from (4.38) as well as the parametrization of the Hubble rate (4.35) into the definition of c_ζ^2, then it takes the following approximate form:

$$c_\zeta^2 \simeq \frac{1}{3} - \frac{2}{3\sqrt{1 + \frac{12\beta V_0}{M_p^4 (g_0 - 1)^2}}} \,, \tag{4.80}$$

in the bounce phase. If we make use of the parameter choice (4.45), we immediately get $c_\zeta^2 \simeq -1/3$ which implies that the perturbation ζ suffers from an gradient instability during the bounce. However, as the duration of the bounce is extremely short, such a exponential growth does not spoil the perturbative control of the analyses [8] .

We have also introduced two quantities to characterize the effective squeezing rates of the perturbation variables

$$\frac{a''}{a} \simeq a_B^2(\Upsilon + 2\Upsilon^2 t^2) \ , \qquad \frac{z''}{z} \simeq a_B^2 \left[\Upsilon + \frac{2}{T^2} + \Big(2\Upsilon^2 + \frac{6\Upsilon}{T^2} + \frac{4}{T^4} \Big) t^2 \right] \ . \qquad (4.81)$$

The coefficient T is approximately one quarter of the duration of the bounce phase, and was initially introduced in Eq. (4.36) to better understand the dynamics of $\dot{\phi}$ during the bounce. In the limit of a slow bounce, one finds that both squeezing rates are equal which implies that there is no differential growth of the curvature fluctuations relative to the entropy mode across the bounce. In contrast, if we consider a fast bounce model, the gravitational terms a''/a and z''/z differ and lead to enhanced growth of v_ζ relative to v_ψ. However, the overall growth during the bounce phase is bounded from above since the duration of a fast bounce cannot be smaller than the Planck time if the effective field theory description is to be self-consistent. The bottom line is that given the validity of the effective field theory analysis we can obtain a controllable amplification effect of cosmological perturbations when they evolve through the bounce phase.

The equations of motion for cosmological perturbations during the bounce phase are given by,

$$v_\psi'' + \Big(a^2 m^2 + k^2 - \frac{a''}{a} \Big) v_\psi = 0 \ , \qquad v_\zeta'' + \Big(c_\zeta^2 k^2 - \frac{z''}{z} \Big) v_\zeta = 0 \ . \qquad (4.82)$$

[8] It does in the bouncing model discussed in [21] in which there is a long bounce phase.

The general solutions to these equations of motion are given by

$$v^b_\psi(k,\tau) = D_{\psi,1}(k) e^{-\int_{B-}\omega_\psi d\tau} + D_{\psi,2}(k) e^{\int_{B-}\omega_\psi d\tau} , \tag{4.83}$$

$$v^b_\zeta(k,\tau) = D_{\zeta,1}(k) e^{-\int_{B-}\omega_\zeta d\tau} + D_{\zeta,2}(k) e^{\int_{B-}\omega_\zeta d\tau} , \tag{4.84}$$

with the frequencies ω_ψ and ω_ζ being

$$\omega^2_\psi \simeq -k^2 - a_B^2 m^2 + a_B^2(\Upsilon + 2\Upsilon^2 t^2) , \tag{4.85}$$

$$\omega^2_\zeta \simeq -c_\zeta^2 k^2 + a_B^2 \left[\Upsilon + \frac{2}{T^2} + \Big(2\Upsilon^2 + \frac{6\Upsilon}{T^2} + \frac{4}{T^4}\Big) t^2\right] , \tag{4.86}$$

respectively. The subscript "b" indicates that we are discussing the solutions in the bounce phase.

Note that we are mainly interested in the infrared modes of cosmological perturbations which are expected to be responsible for the large scale structure of the universe at late times. Therefore, we neglect the k^2 terms in the expression for the frequencies and then easily find that v_ψ and v_ζ are amplified during the bounce phase. Specifically, the amplification factor $\mathcal{F}_\psi$ for the entropy perturbation v_ψ takes the form:

$$\mathcal{F}_\psi \equiv e^{\int_{B-}^{B+}\omega_\psi d\tau} \simeq \exp\left[\Upsilon^{\frac{1}{2}} t + \frac{1}{3}\Upsilon^{\frac{3}{2}} t^3\right]\Bigg|_{B-}^{B+} , \tag{4.87}$$

where $B+$ and $B-$ stand for the end and beginning of the bounce phase, respectively. A reasonable bounce model requires Υ to be a very small quantity (which is equivalent to taking the 'fast bounce' limit), so that the amplitude of perturbations is in agreement with observations. In this case, the amplification of the entropy mode is in general very small. As a consequence, it is safe to approximately take $\mathcal{F}_\psi \simeq 1$.

On the other hand, the curvature perturbation experiences an exponential growth through the bounce phase, which can be described by the amplification factor

$$\mathcal{F}_\zeta \equiv e^{\int_{B-}^{B+}\omega_\zeta d\tau} \simeq \exp\left[\sqrt{2+\Upsilon T^2}\frac{t}{T} + \frac{2+3\Upsilon T^2 + \Upsilon^2 T^4}{3\sqrt{2+\Upsilon T^2}}\frac{t^3}{T^3}\right]\Bigg|_{B-}^{B+} . \tag{4.88}$$

This result is exactly the same as the growth factor obtained in the model of single field bounce [20], and thus shows that the amplification effect brought by the effective tachyonic mass term during the bounce is generic. In the limit of a fast bounce scenario, this amplification factor can be as large as of order $O(10^5)$ as shown in [20]. This effect is very important to non-singular bounce cosmologies since such a controllable growth suppresses the tensor-to-scalar ratio, which was originally found to be too large in matter bounce models [33].

4.4.5 Perturbations in Fast Roll Expansion

After the bounce, the potential for ϕ tends to zero very rapidly. Since the energy density in ϕ dominates over the density in ψ, this causes us to enter a phase of fast roll expansion, where the quadratic action is given by

$$S_2 = \int d\tau \, d^3k \frac{1}{2} \sum_i \left[v_i'^2 - (\frac{1}{4(\tau - \tilde{\tau}_{B+})^2} + k^2) v_i^2 \right] \ , \tag{4.89}$$

where the subscript "i" denotes ζ and ψ, respectively. This gives the equations of motion

$$v_i'' + (k^2 + \frac{1}{4(\tau - \tilde{\tau}_{B+})^2}) v_i = 0 \ , \tag{4.90}$$

which yield the solutions

$$v_i^e = E_{i,1}(k) \sqrt{\tau - \tilde{\tau}_{B+}} J_0 \left(k(\tau - \tilde{\tau}_{B+}) \right) + E_{i,2}(k) \sqrt{\tau - \tilde{\tau}_{B+}} Y_0 \left(k(\tau - \tilde{\tau}_{B+}) \right) \ , \tag{4.91}$$

with

$$a \propto (\tau - \tilde{\tau}_{B+})^{\frac{1}{2}} \ , \quad \tilde{\tau}_{B+} \equiv \tau_{B+} - \frac{1}{2\mathcal{H}_{B+}} \ . \tag{4.92}$$

The subscript "e" indicates that we are discussing the solutions in the fast-roll expanding phase. The coefficients $E_{i,1}(k)$ and $E_{i,2}(k)$ can be determined by matching the perturbations at the moment τ_{B+}. Modulo the square root term, the first mode is constant on super-Hubble scales but the second is growing as a logarithmic function of conformal time. As a consequence, one can see the second term Y_0 finally dominates and form the power spectra of cosmological perturbations at late times.

4.5 Power spectra of cosmological perturbations

Having solved equations of motion for cosmological perturbations phase by phase, now we are able to study how the solutions can be transferred from initial states to the final ones. We leave the detailed matching processes to Appendix C and here merely provide a rough description of the analysis.

Our first matching surface is chosen at the moment τ_E where the Ekpyrotic contraction starts and thus is defined by $\rho_\psi = \rho_\phi$. The matching conditions simply require

$$v^m_{\zeta,\psi}(\tau_E) = v^{\zeta,\psi}_c(\tau_E) \text{ and } \frac{d}{d\tau} v^{\zeta,\psi}_m(\tau_E) = \frac{d}{d\tau} v^{\zeta,\psi}_m(\tau_E)\,. \tag{4.93}$$

In the Ekpyrotic phase, the growing modes are characterized by the coefficients $C_{\zeta,2}$ and $C_{\psi,2}$ as shown in (4.77), and we focus on super-Hubble scales as it is the long wavelength fluctuations that we are interested in. As a consequence, we can obtain the dominant modes of cosmological perturbations during the Ekpyrotic phase.

Similarly, we match the perturbation modes in the Ekpyrotic contracting phase with those in bounce phase at the moment τ_{B-}. Then we can solve for the coefficients of the growing modes in the bounce phase which are characterized by the coefficients $D_{\zeta,2}$ and $D_{\psi,2}$, respectively. The last matching surface is chosen at the moment τ_{B+} where primordial cosmological perturbations just pass through the bounce phase and enter the fast-roll expansion. In this case, we are able to determine the forms of $E_{\zeta,2}$ and $E_{\psi,2}$ which are the coefficients of the dominant modes after the bounce.

Substituting the coefficients $E_{\zeta,2}$ and $E_{\psi,2}$ back into the solutions (4.91), we can solve for the asymptotic solutions of the cosmological perturbations in the final stage. On super Hubble scales, these become

$$v^e_\psi \simeq \frac{\mathcal{F}_\psi H_E}{2} \gamma_\psi e^{-2m/H_E} \left[U^{(0)}_\psi \frac{1}{\sqrt{6 a_E m}} + U^{(k)}_\psi \frac{a_E m}{k^{3/2}} \right] \frac{a_{B-}}{a_{B+}} a(t) \ , \tag{4.94}$$

$$v^e_\zeta \simeq \frac{\mathcal{F}_\zeta H_E}{2} \gamma_\zeta \left[U^{(0)}_\zeta \frac{1}{\sqrt{6 a_E m}} + U^{(log)}_\zeta \frac{\mathrm{Log}(\frac{-2k}{a_E H_E})}{\sqrt{6 a_E m}} + U^{(k)}_\zeta \frac{a_E m}{k^{3/2}} \right] \frac{a_{B-}}{a_{B+}} a(t) \ , \tag{4.95}$$

where we have defined,

$$\gamma_\zeta = \frac{1}{2(1-3q)}\left[1+\left(1-\frac{\sqrt{2}}{H_{B+}T}(1+\frac{t_{B+}^2}{T^2})\right)\ln\frac{a_{B+}}{a(t)}\right] , \tag{4.96}$$

$$\gamma_\psi = \frac{1}{2(1-3q)}\left[1+\left[1-\frac{\sqrt{\Upsilon}}{H_{B+}}(1+\Upsilon t_{B+}^2)\right]\ln\frac{a_{B+}}{a(t)}\right] . \tag{4.97}$$

and the U's are dimensionless coefficient whose detailed form are given in Appendix C.

As a result, we can easily calculate the primordial power spectra of curvature perturbations in the fast roll phase. Up to leading order in k, the result is scale invariant,

$$P_\zeta(k) \simeq \frac{k^3}{2\pi^2}\left|\frac{v_\zeta^e}{a}\right|^2 \simeq \frac{\mathcal{F}_\zeta^2 H_E^2 a_E^2}{8\pi^2}\gamma_\zeta^2\frac{a_{B-}^2}{a_{B+}^2}(m|U_\zeta^{(k)}|)^2\Big[1+\mathcal{O}(k^{3/2})\Big] . \tag{4.98}$$

From the above expression, we can see that the curvature perturbation is dominated by a scale invariant component while there are other terms which can lead to a scale dependence at small length scales. In our model the maximal value of H_E is of the order of the mass parameter m, and thus for the perturbation modes which exit the Hubble radius during matter contracting phase the primordial power spectrum is nearly scale-invariant. However, if we consider the perturbation modes on small length scales, the spectrum becomes blue which may lead to interesting observational signals for experiments. The absence of a red tilt on large scales indicates that the mechanism for a bounce studied here is not the full story, and other ingredients are necessary to have a complete description of cosmology. We discuss this issue in more detail in the discussion.

To provide a check of our analytic calculation of the power spectrum of curvature perturbations, we numerically track its amplitude on super-Hubble scales through the bounce. From the analytical calculation, we expect the amplitude of curvature perturbation to be conserved before the bounce and to undergo an amplification during the bouncing phase. Specifically, we take the same model parameters as in the background numerics introduced in Section III, and numerically compute the

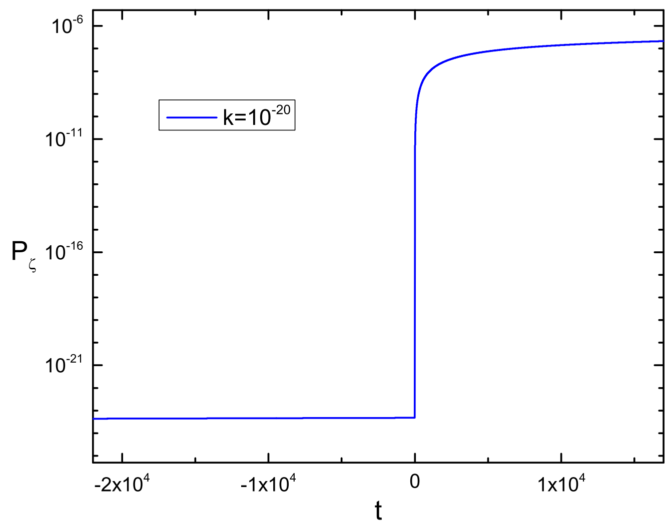


Figure 4–4: Evolution of the power spectrum of curvature perturbation P_ζ at super-Hubble scale (with a fixed comoving wave number $k = 10^{-20}$) as a function of cosmic time. The background parameters are the same as in Fig. 4–1, and the initial condition for the perturbation is chosen as vacuum fluctuation.

curvature perturbation for a fixed comoving wave number. We show the result in Fig. 4–4, in which one can see that the amplitude of curvature perturbations is nearly constant during the contracting phases. During the bounce, the curvature perturbation obtains a dramatic amplification of order $O(10^10)$, corresponding to an amplification factor $\mathcal{F}_\zeta$ of order $O(10^5)$, in exact agreement with the analytical analysis performed in previous subsections.

The power spectrum of the entropy modes, which is carried (except in the initial matter phase of contraction) by the matter field ψ, is also scale-invariant on large scales. It inherits this spectrum from the ϕ mode during the matter phase of contraction. However, the amplitude of the entropy mode is negligible in the case of a fast bounce, as the adiabatic mode undergoes a much larger amplification during the bounce.

Thus, this result shows that the universe after the bounce is isotropic and homogeneous and has a nearly scale-invariant spectrum which is almost purely adiabatic.

In this sense, our model provides a alternative to inflationary cosmology for explaining the observed spectrum of cosmological perturbations.

We expect the perturbation modes forming the above power spectrum of curvature perturbations will eventually be responsible for the CMB anisotropies. It is therefore interesting to check that the modes relevant to the CMB will exit the Hubble radius during the matter contraction phase. Requiring that the modes do exit the Hubble radius imposes a condition on the bounce. We now perform an estimate of this condition. First, we can write down the wavelengths of the modes exiting the Hubble radius at the beginning and the end of the Ekpyrotic phase associated with today's wavelength λ_0 as follows,

$$\lambda(t_E) = \frac{a(t_E)}{a(t_0)}\lambda_0 \simeq \frac{a(t_E)}{a(t_{B-})}\frac{a(t_{B+})}{a(t_F)}\frac{a(t_F)}{a(t_0)}\lambda_0 \ , \tag{4.99}$$

$$\lambda(t_{B-}) = \frac{a(t_{B-})}{a(t_0)}\lambda_0 \simeq \frac{a(t_{B+})}{a(t_F)}\frac{a(t_F)}{a(t_0)}\lambda_0 \ , \tag{4.100}$$

respectively, where t_F denotes the end of the Fast-roll expansion with $\rho_\psi(t_F) = \rho_\phi(t_F)$. Recall that the background energy density scales as $\rho \sim a^{-3(1+w)}$, which depends on the background equation of state w. Making use of this relation, we derive:

$$\lambda_E \simeq \left(\frac{\rho_{B-}}{\rho_E}\right)^{\frac{2}{q}} \left(\frac{\rho_F}{\rho_{B+}}\right)^{\frac{1}{6}} \left(\frac{\rho_0}{\rho_F}\right)^{\frac{1}{3}} \lambda_0 \ , \tag{4.101}$$

$$\lambda_{B-} \simeq \left(\frac{\rho_F}{\rho_{B+}}\right)^{\frac{1}{6}} \left(\frac{\rho_0}{\rho_F}\right)^{\frac{1}{3}} \lambda_0 \ . \tag{4.102}$$

Since the wavelength of today's observable mode scales as $\lambda_0 \sim t_{eq}$ with t_{eq} being the moment of equality, we require the wavelength of the mode exiting the Hubble radius during the beginning of the Ekpyrotic phase to satisfy

$$\lambda_E \geq |t_E| \ , \tag{4.103}$$

so that the observable modes were generated during matter contraction.

Specifically, we take the density of the universe at present to be $\rho_0 \sim (10^{-12}\text{GeV})^4$, while at the end of the Fast-roll it is $\rho_F \sim (10^3\text{GeV})^4$, and around the bounce $\rho_B \lesssim (10^{15}\text{GeV})^4$. From this, we get $\lambda_E \gtrsim 10^{-28}\lambda_0$, which has to be larger than $|t_E|$. This requires $\frac{|t_E|}{t_{eq}} \lesssim 10^{-28}$. Recall that $\rho_m \sim a^{-3} \sim t^{-2}$ during matter contraction, which yields

$$\frac{\rho_m(t_E)}{\rho_m(t_{eq})} \simeq \left(\frac{t_{eq}}{t_E}\right)^2 \gtrsim 10^{56} \ , \tag{4.104}$$

in the specific case considered above. By inserting the value of the density at matter-radiation equality $\rho_m(t_{eq}) \sim (10^{-9}\text{GeV})^4$, one can obtain the lower bound on the density of the universe at the beginning of the Ekpyrotic phase,

$$\rho_m(t_E) \gtrsim (10^5\text{GeV})^4 \ . \tag{4.105}$$

This condition needs to be satisfied in order for the modes exiting the Hubble radius during matter contraction to be responsible for the CMB anisotropies. Note that if we consider a bounce at lower energy scales then the above condition to be satisfied for a large portion of parameter space .

4.6 Tensor perturbation

Similar to scalar modes, tensor perturbations are generated from vacuum fluctuations on sub-Hubble scales in the matter-dominated contracting phase. As the universe contracts, the tensor modes exit the Hubble radius. As is well known, the equation of motion for the tensor fluctuations is the same as that of a massless scalar field. Hence, vacuum initial conditions lead to the same amplitude of the tensor modes and the curvature fluctuations on sub-Hubble scales. Once on super-Hubble scales, the tensor modes are squeezed. During the phases of Ekpyrotic contraction, bounce and fast-roll expansion the equation of motion for the tensor modes is the same as that for the entropy mode (in the absence of mass for the latter). In particular, the squeezing factor of the modes is a''/a. As we showed above, the amplitude of the entropy mode at the beginning of the Ekpyrotic phase is of the same order as that of the curvature modes, which in turn is the same order as that of the tensor modes. After the beginning of the Ekpyrotic phase the tensor and entropy modes evolve the same way. Therefore, it is easy to derive the power spectrum of primordial tensor modes. Making use of the expression (4.140), one obtains the following expression for the power spectrum of primordial tensor perturbations:

$$P_T \equiv \frac{k^3}{2\pi^2}|\frac{u_h}{a}|^2 \simeq \frac{\mathcal{F}_\psi^2 \gamma_\psi^2 H_E^2}{64\pi^2 M_p^2 (2q-3)^2} \frac{a_{B-}^2}{a_{B+}^2} \ . \tag{4.106}$$

One can see the power spectrum of primordial tensor modes in our model is scale-invariant. This spectrum is inherited from the power spectrum of primordial curvature perturbations on large scales. The evolution and the amplification during the bounce phase, however, follow the behavior of entropy perturbations.

One can define a tensor-to-scalar ratio,

$$r_T \equiv \frac{P_T}{P_\zeta} \simeq \frac{\mathcal{F}_\psi^2 \gamma_\psi^2}{\mathcal{F}_\zeta^2 \gamma_\zeta^2} \ . \tag{4.107}$$

This ratio is given by the ratio of amplification factors of curvature and entropy modes during the bounce phase. Thus, this ratio can be greatly suppressed by a large value of the factor $\mathcal{F}_\zeta$. Considering the group of canonical values for model parameters as given in the previous section discussing the background analysis, we find that this ratio can be as low as of order $O(10^{-8})$.

4.7 Conclusion and Discussion

We have studied the evolution of the background and of the linear cosmological fluctuations in a two field matter bounce model in which one field (ψ) represents the regular matter which has a time-averaged equation of state $p = 0$, and the second field (ϕ) is responsible for both an Ekpyrotic phase of contraction which follows the initial matter-dominated period, and which yields a non-singular bounce. As a consequence of the Ekpyrotic phase of contraction, there is no BKL instability in this model [9] . Thus, as long as the initial conditions are chosen such that the Ekpyrotic

[9] The BKL instability to the growth of anisotropies is a problem which afflicts most bouncing cosmological models.

period of contraction begins before the anisotropies dominate, the background will evolve towards a homogeneous and isotropic state.

Since there are two matter fields present, it is important to study not only the adiabatic fluctuations (as was done in [20]), but also the entropy mode. We have shown that in the matter phase of contraction the adiabatic mode (which is seeded by the massive field ψ) starts out with a deep blue spectrum, and it is only the entropy mode (which is seeded by the effectively massless field ϕ) which acquires a scale-invariant spectrum via squeezing on super-Hubble scales during the phase of matter contraction. However, the entropy mode continuously seeds a contribution to the curvature fluctuation. This contribution is scale-invariant, and we have shown that its amplitude at the end of the matter phase of contraction is of the same order of magnitude as the initial entropy fluctuation. Once the Ekpyrotic phase of contraction begins, the roles of the adiabatic and entropy modes change: it is now the dominant field ϕ which determines the adiabatic mode, and ψ becomes the entropy mode. Since the fluctuations in ϕ have a scale-invariant spectrum, the curvature perturbations inherit a scale-invariant spectrum at the beginning of the Ekpyrotic phase, whereas the fluctuations associated with ψ which have developed a scale-invariant form (due to the seeding mentioned above) become the entropy mode.

We followed the evolution of both the adiabatic and the entropy modes from the beginning of the Ekpyrotic phase of contraction through the non-singular bounce phase and into the following fast-roll phase of expansion. Both modes preserve their scale-invariant spectrum. The curvature fluctuations are amplified during the bounce phase, but for a fast bounce the amplification of the entropy mode is negligible. Hence, the entropic contribution to the late time fluctuations is suppressed. It is, in fact, suppressed by the same factor as the tensor perturbations to the scalar ones, since the tensor modes have the same squeezing factor as the entropy field.

One serious shortcoming of the model under consideration is the lack of a prediction for the spectral tilt of perturbations in agreement with CMB observations, which require a red tilt. At best, this model is capable of producing a scale invariant spectrum for the CMB, however scale invariance has been ruled out by Planck at the 5σ level [5]. Given this, we emphasize that the focus of our work is the study of perturbations through a non-singular bounce, and hence we are primarily concerned with the evolution inside of the 'black-box' that separates the contracting and expanding branches of the cosmological evolution. The tilt is due to the choice of contracting branch, and in this study we have chosen matter contraction as our toy model, purely for the sake of simplicity.

However, there do exist mechanisms which could induce a red tilted spectrum in this model. The simplest possibility is to generate the red tilt via a tachyonic coupling to a curvaton field. The effect of curvatons in a matter bounce was originally investigated in Ref. [24], where one can quickly see that a tachyonic coupling $g^2 < 0$ will cause the the spectral index in eq. (23) of [24] to be red, without generating any instability. Another mechanism is to change the matter field to a fluid with slightly negative pressure, as was mentioned in [43]. We plan to investigate these mechanisms in future work.

Finally, we would like to comment on the reheating process. In this specific model under consideration, we assume the two fields are only coupled through gravitational interactions. Therefore it is straightforward to track the evolution of both the background and perturbation modes. In a more generic case, the universe described by our model can be reheated by several different methods, e.g. the usual treatment of reheating in the fast roll phase, perturbative decay of the bounce field in Ekpyrotic phase, and gravitational particle production during a phase transition such as the bouncing phase [44]. Another mechanism of reheating the universe it to introduce a kinetic coupling such as was done for the defrosting process in an emergent galileon cosmology [45]. This aspect, as well as the comparison with the CMB data, provides us with quite a few interesting topics which we will explore in future work.

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Appendix A: Cosmological perturbations in a double field model of canonical form

In this Appendix we shall review the equations of motion for the coupled curvature and entropy modes in a model with two canonical scalar fields. Particular focus is on the curvature modes induced by an initial mode. We will apply this theory to the matter-dominated phase of contraction during which both of the scalar fields in our model have kinetic terms in the action which are approximately canonical. Note that the matter fields ϕ_i considered below have mass dimension one, and hence to apply these formulas our Galileon field ϕ must be multiplied by M_p.

We shall work in longitudinal gauge in which the linearized scalar metric fluctuations appear in the metric in the following way (see e.g. [25, 26]):

$$ds^2 \;=\; (1+2\Phi)dt^2 - a^2(t)(1-2\Psi)d\vec{x}^2 \;, \tag{4.108}$$

where t is cosmic time and x^i are the comoving spatial coordinates. The scalar metric fluctuations are characterized by two functions Φ and Ψ which depend both on space and time. We take matter to consist of a set of scalar fields ϕ_i, which in our explicit model are the bounce field ϕ and the matter field ψ. If the gravitational action is the usual one, then the matter sector does not admit linearized anisotropic stress the off-diagonal components of the perturbed Einstein equations imply $\Psi = \Phi$. By expanding the Einstein and matter equations to first order, we obtain the following perturbation equations:

$$\delta\ddot{\phi}_i + 3H\delta\dot{\phi}_i + [-\frac{\nabla^2}{a^2}\delta\phi_i + \sum_j V_{,ij}\delta\phi_j] \quad = \quad 4\dot{\phi}_i\dot{\Phi} - 2V_{,i}\Phi \ , \tag{4.109}$$

$$-3H\dot{\Phi} + (\frac{\nabla^2}{a^2} - 3H^2)\Phi = 4\pi G \quad \sum_i \quad [\dot{\phi}_i\delta\dot{\phi}_i - \dot{\phi}_i^2\Phi + V_{,i}\delta\phi_i] \ , \tag{4.110}$$

$$\dot{\Phi} + H\Phi \quad = \quad 4\pi G\sum_i \dot{\phi}_i\delta\phi_i \ , \tag{4.111}$$

where $V_{,i}$ denotes the derivative of the scalar field potential with respect to ϕ_i.

We can recast the above equations in terms of the Sasaki-Mukhanov variables [29, 30] which are defined as

$$Q_i \equiv \delta\phi_i + \frac{\dot{\phi}_i}{H}\Phi \ , \tag{4.112}$$

and in terms of which the equations of motion are given by [10, 36, 37]

$$\ddot{Q}_i + 3H\dot{Q}_i - \frac{\nabla^2}{a^2}Q_i + \sum_j[V_{,ij} - \frac{1}{a^3M_p^2}\frac{\mathrm{d}}{\mathrm{d}t}(\frac{a^3}{H}\dot{\phi}_i\dot{\phi}_j)]Q_j = 0 \ . \tag{4.113}$$

To combine the above equations, one can define the quantity ζ which is the curvature perturbation on the uniform density slice,

$$\zeta = H\frac{\sum_i \dot{\phi}_i Q_i}{\sum_j \dot{\phi}_j^2} \ . \tag{4.114}$$

This quantity is conserved on super-Hubble scales in an expanding universe if there are only adiabatic fluctuations [38,39]. However, the presence of entropy fluctuations on large scales will lead to a growth of ζ which corresponds to the seeding of an adiabatic fluctuation mode by the entropy mode.

At linear order, the equation for the time derivative of ζ in the case of two matter fields ϕ and ψ (both with mass dimension one) is given by [36, 37]

$$\dot{\zeta} = -\frac{H}{\dot{H}}\frac{\nabla^2}{a^2}\Phi - \frac{H}{2}\Big(\frac{\delta\phi}{\dot{\phi}} - \frac{\delta\psi}{\dot{\psi}}\Big)\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{\dot{\phi}^2 - \dot{\psi}^2}{\dot{\phi}^2 + \dot{\psi}^2}\Big) \ . \tag{4.115}$$

On large scales, the first term of the r.h.s of Eq. (4.115) is negligible. The second term describes the transfer of entropy to adiabatic fluctuations, the term we are interested in.

Appendix B: General second order action for cosmological perturbations in uniform ϕ gauge

It is useful to study perturbation theory by making use of the ADM metric. Particularly, we focus on the part of the action involving the scalar metric perturbation ζ and the matter field fluctuations $\delta\phi$ and $\delta\psi$. It is well known that one scalar degree of freedom can be fixed by a gauge choice. We choose the following uniform field gauge:

$$\delta\phi = 0\ , \quad h_{ij} = a^2 e^{2\zeta}\delta_{ij}\ . \tag{4.116}$$

After a lengthy calculation, the Lagrangian (4.1) expanded to quadratic order in the fluctuations becomes

$$\begin{aligned}
S_2 = \int dtdx^3 a(t)^3 \Big[& (2M_p^2\dot{\zeta} - 2M_p^2 H\alpha + \dot{\phi}^3 G_{,X}\alpha + \dot{\psi}P_{,Y}\delta\psi)\frac{\partial_i^2\sigma}{M_p^2 a^2} \\
& - 3M_p^2\dot{\zeta}^2 - 2M_p^2\alpha\frac{\partial_i^2\zeta}{a^2} + 6M_p^2 H\alpha\dot{\zeta} - 3\dot{\phi}^3 G_{,X}\alpha\dot{\zeta} \\
& + M_p^2\frac{(\partial_i\zeta)^2}{a^2} - 3M_p^2 H^2\alpha^2 + \frac{\dot{\phi}^2}{2}K_{,X}\alpha^2 \\
& + \frac{\dot{\phi}^4}{2}K_{,XX}\alpha^2 + 6H\dot{\phi}^3 G_{,X}\alpha^2 + \frac{3}{2}H\dot{\phi}^5 G_{,XX}\alpha^2 \\
& - \dot{\phi}^2(G_{,\phi} + \frac{\dot{\phi}^2}{2}G_{,X\phi})\alpha^2 + 3\zeta(\delta\psi P_{,\psi} + \dot{\psi}P_{,Y}\dot{\delta\psi}) \\
& + \frac{\dot{\psi}^2}{2}P_{,Y}\alpha^2 + \frac{\dot{\psi}^4}{2}P_{,YY}\alpha^2 \\
& + \alpha\Big(\delta\psi(P_{,\psi} - \dot{\psi}^2 P_{,Y\psi}) - \dot{\delta\psi}\dot{\psi}(P_{,Y} + \dot{\psi}^2 P_{,YY})\Big)\Big]\ ,
\end{aligned} \tag{4.117}$$

where α and $\partial_i\sigma$ are the lapse function and shift vector, respectively. Varying the quadratic action (4.117) with respect to α and σ yields

$$\alpha = \frac{2M_p^2\dot{\zeta} + \dot{\psi}P_{,Y}\delta\psi}{2M_p^2 H - \dot{\phi}^3 G_{,X}} \ , \tag{4.118}$$

as well as the expression for σ.

Substituting α and σ back into the action, we then obtain a much simplified form

$$\begin{aligned} S_2 = \int dtdx^3 \Big\{ & \frac{a}{2}z^2\Big[\dot{\zeta}^2 - \frac{{c_\zeta}^2}{a^2}(\partial_i\zeta)^2\Big] \\ & + \frac{a}{2}y^2\Big[\dot{\delta\psi}^2 - \frac{{c_\psi}^2}{a^2}(\partial_i\delta\psi)^2 + \frac{2}{ay^2}M_{\delta\psi}^2\delta\psi^2\Big] \\ & + C_1\delta\psi\zeta + C_2\delta\psi\dot{\zeta} + C_3\dot{\delta\psi}\dot{\zeta} + C_4\partial^i\delta\psi\partial_i\zeta\Big\} \ , \end{aligned} \tag{4.119}$$

where $C_{1,2,3,4}$ are the coefficients in front of the interaction terms

$$C_1 = 3a^3\dot{\psi}^2\Big(\ddot{\psi}P_{,YY} - P_{,Y\psi}\Big) \ , \tag{4.120}$$

$$\begin{aligned} C_2 = & \frac{2M_p^2a^3}{(2M_p^2H - \dot{\phi}^3G_{,X})^2}\Big(12\dot{\phi}^3\dot{\psi}HG_{,X}P_{,Y} - 6M_p^2\dot{\psi}H^2P_{,Y} + \dot{\phi}^2\dot{\psi}K_{,X}P_{,Y} \\ & + \dot{\psi}^3{P_{,Y}}^2 + 3\dot{\phi}^5\dot{\psi}HG_{,XX}P_{,Y} + \dot{\phi}^4\dot{\psi}K_{,XX}P_{,Y} + \dot{\psi}^5P_{,Y}P_{,YY} \\ & - 2\dot{\phi}^2\dot{\psi}G_{,\phi}P_{,Y} + 2M_p^2HP_{,\psi} - \dot{\phi}^3G_{,X}P_{,\psi} - \dot{\phi}^4\dot{\psi}G_{,X\phi}P_{,Y} \\ & - 2M_p^2\dot{\psi}HP_{,Y\psi} + \dot{\phi}^3\dot{\psi}^2G_{,X}P_{,Y\psi}\Big) \ , \end{aligned} \tag{4.121}$$

$$C_3 = -\frac{2M_p^2a^3\dot{\psi}}{2M_p^2H - \dot{\phi}^3G_{,X}}\Big(P_{,Y} + \dot{\psi}2P_{,YY}\Big) \ , \tag{4.122}$$

$$C_4 = \frac{2M_p^2a\dot{\psi}P_{,Y}}{2M_p^2H - \dot{\phi}^3G_{,X}} \ . \tag{4.123}$$

The parameters z^2 and y^2 are defined as the coefficients of $\dot{\phi}^2$ and $\dot{\psi}^2$ respectively, and are given by

$$z^2 = \frac{4M_p^4 a^2}{(2M_p^2 H - \dot{\phi}^3 G_{,X})^2}\Big(6H\dot{\phi}^2 G_{,X} + \frac{3}{2M_p^2}\dot{\phi}^2 G_{,X} + \dot{\phi}^2 K_{,X} + \dot{\phi}^4 K_{,XX}$$
$$+ \dot{\psi}^2 P_{,Y} + \dot{\psi}^4 P_{,YY} + 3H\dot{\phi}^5 G_{,XX} - 2\dot{\phi}^2 G_{,\phi} - \dot{\phi}^4 G_{,\phi X}\Big) \tag{4.124}$$
$$y^2 = a^2\Big(P_{,Y} + \dot{\psi}^2 P_{,YY}\Big) \ . \tag{4.125}$$

The sound speeds of ζ and ψ are denoted $c_{\psi,\zeta}$, and are given by

$$c_\psi{}^2 = \frac{P_{,Y}}{P_{,Y} + \dot{\psi}^2 P_{,YY}} \ , \tag{4.126}$$
$$c_\zeta{}^2 = \frac{2a^2}{z^2}\,[\ M_p^2 - \frac{3M_p^4 H}{2M_p^2 H - \dot{\phi}^3 G_{,X}}$$
$$- \frac{3M_p^4(3\dot{\phi}^2\ddot{\phi}G_{,X} + \dot{\phi}^4 G_{,\phi X} + \dot{\phi}^4\ddot{\phi}G_{,XX} - 2M^2\dot{H})}{(2M_p^2 H - \dot{\phi}^3 G_{,X})^2}\ ] \ . \tag{4.127}$$

One can define canonical variables for perturbation modes ζ and ψ as follows,

$$v_\zeta \equiv z\zeta \ , \quad v_\psi \equiv y\delta\psi \ , \tag{4.128}$$

and then the time derivative terms in the quadratic action become of canonical form in conformal coordinates.

Appendix C: Matching Coefficients

The matching conditions for cosmological perturbations were discussed in [40, 41]. The idea was to match two solutions of General Relavitity across some space-like matching surface which is endowed with the localized stress-energy to enable the transition between the two space-times. The matching conditions state that the induced metric on the matching surface must be the same when calculated from either side, and that the extrinsic curvature jumps by an amount given by the localized stress-energy on the surface.

In our case, the background is continuous across the various matching surfaces (this would not have been the case had we cut out the bouncing phase and tried to match directly between the contracting Ekpyrotic phase and the expanding fast-roll period). Hence, there is no jump in the extrinsic curvature across the matching surface. Matter fields must also evolve continuously across the bounce. Hence, the matching conditions are

$$v^1_\zeta(\tau_m) = v^2_\zeta(\tau_m) \;\text{ and }\; \frac{d}{d\tau}v^1_\zeta(\tau_m) = \frac{d}{d\tau}v^2_\zeta(\tau_m)\ , \tag{4.129}$$

at each matching time τ_m, where the superscripts 1 and 2 indicate the values of the variables computed in the phases after and before the matching surface, respectively, and

$$v^1_\psi(\tau_m) = v^2_\psi(\tau_m) \;\text{ and }\; \frac{d}{d\tau}v^1_\psi(\tau_m) = \frac{d}{d\tau}v^2_\psi(\tau_m)\ . \tag{4.130}$$

We first match the cosmological perturbation v_ζ and v_ψ at the beginning moment of the Ekpyrotic phase τ_E. The matching conditions allow us to determine the dominant coefficients $C_{i,2}$ in the Ekpyrotic phase, with the result

$$C_{i,2} \simeq \frac{\pi\sqrt{\tau_E - \tilde{\tau_{B-}}}}{2v_c\Gamma_{v_c}}\Big(\frac{k(\tau_E - \tilde{\tau}_{B-})}{2}\Big)^{v_c}\Big[\frac{d}{dt}v_i^m - v_i^m\frac{1+2v_c}{2(\tau_E - \tilde{\tau}_{B-})}\Big] . \quad (4.131)$$

The coefficients $D_{i,1}$ and $D_{i,2}$ for the solutions in the bounce phase are also derived by matching v_ζ and v_ψ at the end moment of Ekpyrotic phase τ_{B-}. Picking out the dominant terms yields

$$D_{\zeta,2} \simeq \frac{-C_{\zeta,2}\Gamma_{\nu_c}e^{\int_{\tau_B}^{\tau_{B+}} w_\zeta d\tau}}{2^{2-\nu_c}\pi\omega_\zeta k^{\nu_c}(\tau_{B-} - \tilde{\tau}_{B-})^{\frac{1}{2}+\nu_c}}\Bigg[1 - 2\nu_c + 2\omega_\zeta(\tau_{B-} - \tilde{\tau}_{B-})\Bigg] , \quad (4.132)$$

$$D_{\psi,2} \simeq \frac{-C_{\psi,2}\Gamma_{\nu_c}e^{\int_{\tau_B}^{\tau_{B+}} w_\psi d\tau}}{2^{2-\nu_c}\pi\omega_\psi k^{\nu_c}(\tau_{B-} - \tilde{\tau}_{B-})^{\frac{1}{2}+\nu_c}}\Bigg[1 - 2\nu_c + 2\omega_\psi(\tau_{B-} - \tilde{\tau}_{B-})\Bigg] . \quad (4.133)$$

After the bounce, we match the cosmological perturbations at the moment τ_{B+} and then determine the coefficients $E_{\zeta,i}$ and $E_{\psi,i}$. Both are important so we write them all,

$$E_{1,i} = -D_{i,2}\frac{e^{\int_{\tau_B}^{\tau_{B+}} w_i d\tau}}{2\sqrt{\tau_{B+} - \tilde{\tau}_{B+}}}\Bigg[-2 + \big(2(\tau_{B+} - \tilde{\tau}_{B+})w_i - 1\big)\Big(\ln\Big[\frac{k(\tau_{B+} - \tilde{\tau}_{B+})}{2}\Big] + \gamma_E\Big)\Bigg] , \quad (4.134)$$

$$E_{2,i} = D_{i,2}\frac{\pi e^{\int_{\tau_B}^{\tau_{B+}} w_i d\tau}}{4\sqrt{\tau_{B+} - \tilde{\tau}_{B+}}}\Bigg[1 - 2(\tau_{B+} - \tilde{\tau}_{B+})\Bigg] . \quad (4.135)$$

By making use of these coefficients, we can extract the dominant mode of cosmological perturbations in the fast-roll expanding phase, namely

$$v_\psi^e \simeq \frac{\mathcal{F}_\psi H_E}{2}\gamma_\psi e^{-2m/H_E}\Big[U_\psi^{(0)}\frac{1}{\sqrt{6a_E m}} + U_\psi^{(k)}\frac{a_E m}{k^{3/2}}\Big]\frac{a_{B-}}{a_{B+}}a(t) , \quad (4.136)$$

$$v^e_\zeta \simeq \frac{\mathcal{F}_\zeta H_E}{2}\gamma_\zeta\Big[U^{(0)}_\zeta\frac{1}{\sqrt{6a_E m}}+U^{(log)}_\zeta\frac{\mathrm{Log}(\frac{-2k}{a_E H_E})}{\sqrt{6a_E m}}+U^{(k)}_\zeta\frac{a_E m}{k^{3/2}}\Big]\frac{a_{B-}}{a_{B+}}a(t)\ , \qquad (4.137)$$

with

$$U^{(k)}_\zeta = -(25+49q)i\frac{H_E}{24m}-\frac{27}{24}q, \quad U^{(log)}_\zeta = \sqrt{2}\frac{m}{H_E}\Big(1-\frac{5}{2}q\Big)$$
$$U^{(0)}_\zeta = \sqrt{2}\Big(1-\frac{3}{2}q-\frac{27}{8}iq+\frac{9H_E}{8m}(1-q)+\frac{m}{3H_E}(1-q-9iq)\Big)$$
$$U^{(k)}_\psi = -\frac{3}{8}\Big(\sqrt{3}(1-q)\frac{H_E}{m}-3q\Big), \quad U^{(0)}_\psi = 1+\frac{9}{2}\frac{H_E}{m}-(\frac{3}{2}-\frac{27}{8}i)q-\frac{9}{8}\frac{H_E}{m}q-3i\frac{mq}{H_E}.$$

We have also defined the constants γ_ζ and γ_ψ as the coefficients who comes from the asymptotic form of the Bessel function Y_0 on large length scales,

$$\gamma_\zeta = \frac{1}{2(1-3q)}\left[1+\left(1-\frac{\sqrt{2}}{H_{B+}T}(1+\frac{t^2_{B+}}{T^2})\right)\ln\frac{a_{B+}}{a(t)}\right]\ , \qquad (4.138)$$

$$\gamma_\psi = \frac{1}{2(1-3q)}\left[1+\left[1-\frac{\sqrt{\Upsilon}}{H_{B+}}(1+\Upsilon t^2_{B+})\right]\ln\frac{a_{B+}}{a(t)}\right]\ . \qquad (4.139)$$

Similarly, one can track the evolution of primordial tensor modes and determine the matching relations. Comparing with the evolution of entropy perturbation, the tensor fluctuations differ only in the mass term and the choice of initial conditions which only affects the evolution before the Ekpyrotic phase. During and after the phase of Ekpyrotic contraction, the evolution of entropy perturbations and tensor fluctuations are described by the same equation of motion. Working at the level of homogeneous solutions, the tensor fluctuations will have the same amplitude as v_ϕ at the end of the matter contraction phase (both come from a massless field that has the same vacuum amplitude). Hence we conclude that the final amplitude of the tensor modes will be of the form,

$$u_h^e(k,\tau) \simeq -\frac{i\mathcal{F}_\psi \gamma_\psi \mathcal{H}_E}{4a_E\sqrt{2k^3}(2q-3)}\frac{a_{B-}}{a_{B+}}a(\tau) \ , \tag{4.140}$$

in the fast-roll expanding phase.

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Chapter 5
de Sitter Vacua in Type IIB String Theory: Classical Solutions and Quantum Corrections

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Abstract

We revisit the classical theory of ten-dimensional two-derivative gravity coupled to fluxes, scalar fields, D-branes, anti D-branes and Orientifold-planes. We show that such set-ups do not give rise to a four-dimensional positive curvature spacetime with the isometries of de Sitter spacetime. We further argue that a de Sitter solution in type IIB theory may still be achieved if the higher-order curvature corrections are carefully controlled. Our analysis relies on the derivation of the de Sitter condition from an explicit background solution by going beyond the supergravity limit of type IIB theory. As such this also tells us how the background supersymmetry should be broken and under what conditions D-term uplifting can be realized with non self-dual fluxes.

5.1 Introduction

The hot, dense state of the early universe and its subsequent evolution offer a unique testing ground for theories of high-energy physics; if string theory is the correct theory of the earliest universe, it should be possible to embed all the known results from cosmology in a consistent string theory description. Our best observational data of the early universe, from the cosmic microwave background (CMB) [1–6], and late time acceleration [7], point to a universe that is very close to spatially flat, in which large-scale structure was generated from an almost scale-invariant spectrum of primordial density perturbations with a nearly Gaussian distribution. This is consistent with a large class of inflationary models [5], which we will have in mind here, as well as a variety of alternatives to inflation [8–10].

However, the dynamics of the early universe is necessarily studied via an effective field theory (EFT) approach. Although one might expect a decoupling of energy scales, leading to suppression of higher-order terms in the Lagrangian by increasing powers of the cut-off, the predictions of inflation can be highly sensitive to corrections of both the potential or inflaton mass [11] and the kinetic terms [12,13]. This forces one to consider the UV sensitivity of inflation, which has been addressed from many perspectives: see [11,14] for reviews, [15] for a recent take, and [16] for a completely different approach. The dependence of cosmological observables on the detailed embedding of inflation into string theory offers a unique window into the high-energy physics of the early universe, and may provide evidence that string theory could be the correct description of physics at these scales.

A consistent string compactification with a de Sitter (or quasi de Sitter) vacuum in the 3+1 non-compact directions is crucial to such an embedding. Achieving such a compactification has proved to be an extremely difficult endeavour. No-go theorems exist for supergravity [17,18] and for string theory (without time-dependent fields or higher-curvature corrections), the well-known Maldacena-Nunez result [19]. This was extended to the heterotic case with higher-order corrections (but without non-perturbative effects) included [20,21].

In Type II string theory, dS solutions have been studied in many works, for example [22–32]. In addition, many models of inflation in string theory have been proposed (see the reviews by [11,14]), together with 'uplift' mechanisms for obtaining dS [33–35] by lifting an AdS minimum of the scalar potential to a metastable dS minimum.

In this paper, we revisit the question from the full ten-dimensional setup of Type IIB string theory, generalizing the analysis of Maldacena-Nunez [19] by including extended localized sources in the gravity action. In particular we consider the traced-over Einstein equations, identifying the conditions for achieving de Sitter space in the non-compact dimensions for the cases of fluxes, scalar fields and different localized sources, e.g. D-branes, anti D-branes and orientifold planes, in Type IIB with two-derivative gravity. We find that none of these ingredients satisfy the required condition, suggesting that one must consider additional terms in the gravity action.

One example of such additional terms is the set of higher-order curvature corrections. We perform an explicit calculation using an M-theory uplift, so as to simplify the form of the available fluxes. To study the effect of curvature corrections, we are forced to take an indirect route and instead consider a generalized correction to the action. We make an ansatz for the stress-energy tensor of the perturbative corrections, noting that the correction terms are built from curvatures. We explicitly find that positive curvature in the non-compact directions is only possible if curvature corrections are present and satisfy a certain inequality.

We further find that the fluxes in any dS solution must be non self-dual, as is consistent with broken supersymmetry. These fluxes, combined with D-brane instantons, are enough to fix both the complex structure and Kahler moduli, including the volume modulus. In addition to this, the instantons are one possible source for the curvature correction terms required to give positive curvature to the non-compact space. We do not propose a specific form for these corrections, and as the complete set of supported corrections is not yet known, further conclusions cannot be made at this point.

The structure of this paper is as follows: Sections 2 and 3 rederive the Gibbons-Maldacena-Nunez No-Go theorem, and apply it to bulk fields (fluxes and scalar fields) and localized sources. In Section 4, we set up our M-theory calculation, which we perform in Section 5. We then examine the resulting equations of motion in Section 6 and 7, and discuss the origin of higher order curvature corrections in Section 8. We conclude our work with a short discussion of our results in Section 9.

5.2 Einstein gravity in D dimensions

Consider the following Einstein-Hilbert action coupled to matter in D spacetime dimensions:

$$S_{\text{total}} = \frac{1}{\mathcal{K}_D}\int d^Dx\sqrt{-G_D}R_D + \int d^Dx\mathcal{L}_{\text{int}}, \tag{5.1}$$

where $\mathcal{K}_D$ is the D-dimensional Newton constant, R_D is the Ricci scalar in D dimensions, G_D is the determinant of the D-dimensional metric $g_{MN}, M, N = 0, .., D-1$, and $\mathcal{L}_{\text{int}}$ is the Lagrangian for the local or global fields that couple to gravity. It can contain global fluxes, scalar fields, local sources and terms that describe graviton self coupling. In the Einstein equations, $\mathcal{L}_{\text{int}}$ enters through the stress-energy tensor

$$T_{MN} = -\frac{2}{\sqrt{-G_D}}\frac{\delta \mathcal{L}_{\text{int}}}{\delta g^{MN}}. \tag{5.2}$$

Variation of (5.1) with respect to g^{MN} gives the following Einstein equation:

$$R_{MN} = \frac{\mathcal{K}_D}{2}\left(T_{MN} - \frac{1}{D-2}g_{MN}T\right), \tag{5.3}$$

where T is defined in the usual way, i.e.

$$T = g^{MN}T_{MN}. \tag{5.4}$$

Now we will split the geometry into two manifolds: M_4, spanned by coordinates $x^\mu, \mu = 0,..,3$ and a transverse space $\mathcal{M}^{D-4}$, spanned by coordinates $x^m, m = 4,..,D-1$. We want M_4 to describe our four dimensional non-compact space-time geometry and thus choose $(x^0, x^1, x^2, x^3) = (t, x, y, z)$, where t is timelike. $\mathcal{M}^{D-4}$ can be either a compact or non-compact $D-4$ dimensional manifold, described by spacelike coordinates x^m. We will often refer to x^m and x^μ as describing internal and external directions respectively. The line element is

$$ds_D^2 = ds_4^2 + ds_{D-4}^2 \equiv g_{\mu\nu}dx^\mu dx^\nu + g_{mn}dx^m dx^n. \tag{5.5}$$

Now if the D-dimensional manifold has a direct product topology $M_4 \times \mathcal{M}^{D-4}$, then the Ricci scalar for M_4 is:

$$R_4 \equiv g^{\mu\nu}R_{\mu\nu}. \tag{5.6}$$

If $R_4 > 0$ we obtain a positive curvature spacetime, of which de Sitter space is one example, as is consistent with our universe. Alternatively, if $R_4 < 0$, we have Anti-de Sitter type geometry, which is not consistent with the current universe.

Taking the trace of (5.3) in the μ, ν directions, we get

$$R_4 = -\frac{\mathcal{K}_D}{2(D-2)}\left[T^\mu_\mu(6-D) + 4T^m_m\right]. \tag{5.7}$$

Thus for a positively curved spacetime, i.e. $R_4 > 0$, we must satisfy the condition:

$$(D-6)\,T^\mu_\mu > 4T^m_m. \tag{5.8}$$

Whatever the content of the Lagrangian, we must satisfy (5.8) if we are to obtain a positively curved four-dimensional universe. If we do not have a direct product space, but rather a warped product space, then the manifold cannot be nicely separated: $M_D \neq M_4 \times \mathcal{M}^{D-4}$. However, we can still try to obtain an effective four-dimensional space at low energies. In this case, the transverse dimensions are not accessible, which is possible if the size of $\mathcal{M}^{D-4}$ is small compared to the typical distance scale of interactions in M_4. We will separately address the case of a warped product space in the context of type IIB string theory in Section 3.2, where we will again see that the condition (5.8) plays a crucial role.

We can now proceed to analyse different choices for the Lagrangian.

5.2.1 Fluxes and scalar fields coupled to gravity

We can reproduce the No-Go theorem of Gibbons [17,18] and Maldacena-Nunez [19] by including fluxes in the Lagrangian. We consider the flux Lagrangian

$$\mathcal{L}^F_{\text{int}} = -\sqrt{-G_D}F_{a_1\ldots a_q}F^{a_1\ldots a_q}, \tag{5.9}$$

where F is a q-form. The above Lagrangian leads to the following stress-energy tensor:

$$T^F_{MN} \;=\; -g_{MN}F^2 + 2qF_{Ma_2..a_q}F_N^{\;a_2\ldots a_q}. \tag{5.10}$$

One can readily check that with the above form of the tensor, condition (5.8) will be satisfied if

$$4(1-q)F^2 > -F_{\mu a_2..a_q}F^{\mu a_2..a_q}q\,(D-2). \tag{5.11}$$

We will consider two types of fluxes: the first type with legs only along the internal directions and the second type with legs in M_4. Also note that the overall minus sign in the Lagrangian is chosen to give positive energy, i.e. $T_{00} > 0$. For the first type of flux $a_i = m, n$ for all i and $F^2 \geq 0$ with $F_{\mu a_2..a_q} F^{\mu a_2..a_q} = 0$. Thus we find that condition (5.8) is *not* satisfied for $q > 1$.

If $q < 4$ then all the legs will be along $\mathcal{M}^{D-4}$ since otherwise the isometries of $d = 4$ Minkowski or de Sitter like space will be broken. Thus when we consider the second type of flux which has legs in M_4, we will restrict to the case $q \geq 4$. For $q \geq 4$ we will consider 4 out of q legs along M_4 i.e. flux with legs in all the directions of M_4 and the rest of its legs along the internal directions. With this condition on the fluxes, one obtains the following identities:

$$F^2 = F_{a_1 a_2..a_q} F^{a_1 a_2..a_q} = C(q,4) F_{\mu_1..\mu_4 a_5..a_q} F^{\mu_1..\mu_4 a_5..a_q}$$
$$F_{\mu a_2..a_q} F^{\mu a_2..a_q} = C(q-1,3) F_{\mu_1..\mu_4 a_5..a_q} F^{\mu_1..\mu_4 a_5..a_q}, \tag{5.12}$$

where the coefficient $C(q,k)$ is defined by

$$C(q,k) \equiv \frac{q!}{(q-k)!k!}. \tag{5.13}$$

This in turn gives us

$$F_{\mu a_2..a_q} F^{\mu a_2..a_q} = \frac{4}{q} F^2. \tag{5.14}$$

Using the above relation and the fact that $F^2 < 0$, condition (5.8) will be satisfied if and only if

$$D < q + 1. \tag{5.15}$$

Thus for $D > q + 1$, we find that a q-form flux with legs in M_4 *does not* give rise to positive curvature for M_4. Any flux that preserves the desired isometries of M_4 can be written as a combination of the two types of fluxes described above. Thus, whatever the form of the flux, q-form flux for $D > q + 1$ *does not* give rise to positive curvature for M_4, as was first demonstrated by Maldacena and Nunez [19].

Next we consider scalar fields. The most general interaction Lagrangian for a scalar field interacting with gravity is given by

$$\mathcal{L}_{\text{int}}^{\phi} = -\sqrt{-G_D}\left(\partial_M \phi \partial^M \phi + V(\phi)\right). \tag{5.16}$$

Note that the overall minus sign is chosen so that when $V(\phi) = 0$ (for example massless fields with only kinetic energy), we get positive energy, i.e. $T_{00} > 0$. The stress-energy tensor is given by

$$T_{MN}^{\phi} = -g_{MN}\left(\partial_K \phi \partial^K \phi + V(\phi)\right) + 2\partial_M \phi \partial_N \phi. \tag{5.17}$$

Then with the stress-energy tensor given above, the only way (5.8) is satisfied is if and only if

$$\partial_\mu \phi \partial^\mu \phi + V(\phi) > 0. \tag{5.18}$$

Now if we demand that the M_4 is isotropic in space but dependent on time, we readily find $\partial_\mu \phi \partial^\mu = g^{tt}\partial_t \phi \partial_t \phi < 0$ since $g^{tt} < 0$. Thus *if* $V(\phi) < 0$, M_4 *will not have positive curvature*. In type IIB string theory, which will be the focus of our study, the scalar axio-dilaton field τ has no potential and thus will not aid in constructing positive curvature.

5.2.2 Localized matter coupled to gravity

Another possibility for the interaction Lagrangian is that of localized matter. For a p-dimensional object embedded in D-dimensional geometry, the most general Lagrangian that couples to the metric is the worldvolume Born-Infeld Lagrangian:

$$\mathcal{L}_{\text{int}}^{\text{BI}} = -T_p\sqrt{-\tilde{f}}\sqrt{g_{D-p-1}}\,\delta^{D-p-1}(x-\bar{x}), \tag{5.19}$$

where $\tilde{f}$ is the determinant of the metric $\tilde{f}_{ab}$, defined in the following way:

$$\tilde{f}_{ab} = f_{ab} + \widetilde{F}_{ab}, \qquad f_{ab} = g_{MN}\frac{\partial X^M}{\partial \sigma^a}\frac{\partial X^N}{\partial \sigma^b} \quad \text{and} \quad \widetilde{F}_{ab} = F_{ab} + B_{ab}. \tag{5.20}$$

Here T_p is the tension, F_{ab} is the worldvolume flux, B_{ab} is the pullback of the background magnetic flux, $a, b = 1, .., p+1$, and $\widetilde{F}_{ab}$ is raised or lowered with the pullback metric f_{ab}. Also note that $\delta^{D-p-1}(x-\bar{x})$ is the $(D-p-1)$-dimensional delta function, $x = \bar{x}$ is the location of the p-dimensional object, and g_{D-p-1} is the determinant of the $(D-p-1)$-dimensional metric such that we have the normalization

$$\int d^{D-p-1}x\sqrt{g_{D-p-1}}\delta^{D-p-1}(x-\bar{x}) = 1. \tag{5.21}$$

We have picked worldsheet parameters $\sigma^a = x^a, a = 0, .., p-1$. T_p can be considered as mass per unit length and thus it is typically positive.

If the Lagrangian is of the form (5.19) with positive mass term, i.e. $T_p > 0$, one obtains:

$$\begin{aligned} T_\mu{}^{\mu\,(\mathrm{BI})} &= -T_p\frac{1}{\sqrt{-G_D}}\sqrt{-\tilde{f}}\sqrt{g_{D-p-1}}\tilde{f}_{ab}g^{\mu'\nu'}\frac{\delta\tilde{f}^{ab}}{\delta g^{\mu'\nu'}}\delta^{D-p-1}(x-\bar{x}) < 0 \\ T_m{}^{m\,(\mathrm{BI})} &= -T_p\frac{1}{\sqrt{-G_D}}\sqrt{-\tilde{f}}\sqrt{g_{D-p-1}}\tilde{f}_{ab}g^{m'n'}\frac{\delta\tilde{f}^{ab}}{\delta g^{m'n'}}\delta^{D-p-1}(x-\bar{x}) < 0 \end{aligned} \tag{5.22}$$

Using (5.22) in (5.7) one readily sees that (5.8) is satisfied if $D < 6$. For $D > 6$ (5.8) is not automatically satisfied. In particular string theory gives $D = 10$ or 11 and thus we must have T_m^m non-vanishing to obtain our four-dimensional positive curvature universe.

String theory also allows negative tension objects, i.e. $T_p < 0$, and higher-derivative terms in the low-energy effective action for gravity. Then, using the form of the *localized* stress-energy tensor (5.22) and adding the contributions from the fluxes, scalar fields and higher derivative terms, it may be possible to satisfy the condition (5.8). We will discuss this possibility in Sections 4 to 8.

5.3 dS in Type IIB String Theory with Branes and Planes

With a general understanding of gravitational coupling to fluxes and localized matter fields in D dimensions, we will now consider the specific case of low-energy type IIB superstring theory with the following action in Einstein frame:

$$S_{\mathrm{total}} = S_{\mathrm{SUGRA}} + S_{\mathrm{loc}}, \tag{5.23}$$

where

$$\begin{aligned} S_{\rm SUGRA} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G_{10}}\left(R-\frac{\partial_M\tau\partial^M\bar{\tau}}{2|{\rm Im}\tau|^2}-\frac{|\hat{F}_5|^2}{4\cdot 5!}-\frac{G_3\cdot\bar{G}_3}{12{\rm Im}\tau}\right) \\ &+ \frac{1}{8i\kappa_{10}^2}\int\frac{C_4\wedge G_3\wedge\bar{G}_3}{{\rm Im}\tau}. \end{aligned} \tag{5.24}$$

Here $\tau = C_0 + ie^{-\phi}$; $G_{10} = \det\, g_{MN}, M, N = 0, .., 9$; g_{MN} is the metric in Einstein frame; $G_3 = F_3 - \tau H_3$; F_3 is the three-form RR flux, H_3 is the three-form NS-NS flux, and $\hat{F}_5$ is defined by

$$\hat{F}_5 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3. \tag{5.25}$$

For the localized action we will consider Dp-branes and orientifold planes in various dimensions. The action for a Dp-brane is given by

$$S_{Dp} = -\int d^{p+1}\sigma\; T_p\, e^{\frac{\phi(p+1)}{4}}\,\sqrt{-\tilde{f}} + \mu_p\int\left(C\wedge e^{\hat{F}}\right)_{p+1}. \tag{5.26}$$

Here $\tilde{f}$ is the same as in (5.19) and C_{p+1} is the RR flux. As above, $\widetilde{F}_{ab}$ is raised or lowered with the pullback metric f_{ab}. Note that the sign of μ_p determines whether we have a brane or an anti-brane. However both branes and anti-branes have positive tension $T_p > 0$.

On the other hand, for an orientifold, we have the action

$$S_{Op} = -\int d^{p+1}\sigma\; T_{Op}e^{\frac{\phi(p+1)}{4}}\,\sqrt{-f} + \mu_{Op}\int C_{p+1}, \tag{5.27}$$

where the orientifold has negative tension, i.e. $T_{Op} < 0$. Here μ_p is the charge of the Op-plane and we have the relation $|T_{Op}| = e^{-\phi}|\mu_{Op}|$. Also note that since the Op plane has negative charge, we have $\mu_p = e^{\phi}T_{Op} = -e^{\phi}|T_{Op}|$.

With the above localized action and the bulk supergravity action, we can write (5.23) in the form (5.1) with the interaction Lagrangian being[1]

$$\begin{aligned}
\mathcal{L}_{\rm int} &= \mathcal{L}_{\rm bulk} + \mathcal{L}_{\rm Dp} + \mathcal{L}_{\rm Op} \\
\mathcal{L}_{\rm bulk} &= \sqrt{-G_{10}}\left(-\frac{\partial_M\tau\partial^M\bar{\tau}}{2|{\rm Im}\tau|^2} - \frac{|\hat{F}_5|^2}{4\cdot 5!} - \frac{G_3\cdot\bar{G}_3}{12{\rm Im}\tau}\right) \\
\mathcal{L}_{\rm Dp} &= -T_p e^{\frac{\phi(p+1)}{4}}\sqrt{-\tilde{f}}\sqrt{g_{D-p-1}}\delta^{10-p-1}(x-\bar{x}) \\
\mathcal{L}_{\rm Op} &= |T_{Op}|e^{\frac{\phi(p+1)}{4}}\sqrt{-f}\sqrt{g_{D-p-1}}\delta^{10-p-1}(x-\bar{x}).
\end{aligned} \tag{5.28}$$

In the above $\mathcal{K}_{10}$ has been replaced by $2\kappa_{10}^2$. Using the above form of the Lagrangian we can readily obtain the stress-energy tensor (5.2) and check whether the constraint (5.8) is satisfied or not.

To evaluate the trace of the stress-energy tensor, we will restrict the form of the fields to ensure Poincaré invariance in the non-compact spacetime. This way even without solving for the on-shell values of the fluxes and metric, we can check whether the inequality (5.8) is satisfied. These conditions are the following:

•The fluxes H_3 and F_3 only have legs along $\mathcal{M}^6$, and τ depends only on x^m, the coordinates of $\mathcal{M}^6$.

• $\hat{F}_5$ will have legs in the x^μ directions. Then by imposing self duality and Poincaré invariance, one obtains the general form

$$\hat{F}_5 = (1 + *_{10})\, d\alpha\wedge dt\wedge dx\wedge dy\wedge dz, \tag{5.29}$$

where $\alpha(x^M)$ is a scalar field which is a function of all coordinates $x^M, M = 0, .., 9$.

[1] The topological term cannot enter the stress-energy tensor since $\frac{\delta S_{\rm CS}}{\delta g^{MN}} = 0$ where $S_{\rm CS} = \mu_p\int\left(C\wedge e^{\widetilde{F}}\right)_{p+1}$ is the Chern-Simons action. Therefore we omit it in the Lagrangians here. For Dp-branes $\hat{F}$ is not generally zero but Op-planes do not carry gauge fields, and have $\hat{F}$=0.

Having laid down the required conditions, we will now analyze the individual cases with branes, anti-branes and orientifold planes.

5.3.1 Direct product space with Branes and Planes

We will first consider product spaces $M_{10} = M_4 \times \mathcal{M}^6$ with branes and planes, where the transverse space $\mathcal{M}^6$ can be either compact or non-compact. For $p = 3$, we have D3 or anti-D3 branes which fill up M_4. Thus the induced metric is

$$\begin{aligned} f_{ab} &= g_{ab}, \quad \text{for} \quad a, b = \mu, \nu \\ f_{ab} &= 0 \quad\quad \text{for} \quad a, b \neq \mu, \nu. \end{aligned} \tag{5.30}$$

Then we find [2]

$$\begin{aligned} &T^{\mu}_{\ \mu\ (D3/\bar{D3})} = -T_3 e^{\phi} \frac{\sqrt{-\tilde{f}}\sqrt{g_6}}{\sqrt{-G_{10}}} \left(4 + \hat{F}^{\mu}_{\mu}\right) \delta^6(x - \bar{x}) \\ &T^{m}_{\ m\ (D3/\bar{D3})} = 0. \end{aligned} \tag{5.31}$$

However, since the flux $\hat{F}$ is anti-symmetric while the metric is symmetric, $\hat{F}^{\mu}_{\mu} = 0$. Thus *neither* the D3 nor the anti-D3 brane tensor satisfies the constraint (5.8).

The results for D3 and anti-D3 branes can easily be generalized to Dp and anti-Dp branes with $p = 5, 7$. For Poincaré invariance in the noncompact dimensions, we will fill up M_4 with the Dp or anti-Dp branes and the remaining worldvolume will fill up some S^{p-3} cycle inside the transverse space $\mathcal{M}^{D-p-1}$. If x^m, x^n denote coordinates of the cycle S^{p-3}, then we have

$$\begin{aligned} f_{ab} &= g_{ab}, \quad \text{for} \quad a, b = \mu, \nu, m, n \\ f_{ab} &= 0 \quad\quad \text{for} \quad a, b \neq \mu, \nu, m, n. \end{aligned} \tag{5.32}$$

[2] Note that the upper indices here and elsewhere in this section have been raised with the metric g^{MN}, which is free of any warping in the case of a direct product space . For the warped compactifications studied in later sections, we will make the distinction between the warped metric and unwarped metric, where we introduce ‘tilded’ quantities, $\tilde{A}^m$, that are defined with respect to the unwarped metric.

And we obtain

$$T^{\mu}_{\ \mu\ (Dp/\bar{D}p)} = -T_p e^{\frac{\phi(p+1)}{4}} \frac{\sqrt{-\tilde{f}}\sqrt{g_{D-p-1}}}{\sqrt{-G_{10}}} \left(4 + \hat{F}^{\mu}_{\mu}\right) \delta^{D-p-1}(x-\bar{x})$$
$$T^{m}_{\ m\ (Dp/\bar{D}p)} = -T_p e^{\frac{\phi(p+1)}{4}} \frac{\sqrt{-\tilde{f}}\sqrt{g_{D-p-1}}}{\sqrt{-G_{10}}} \left(p - 3 + \hat{F}^{u}_{u}\right) \delta^{D-p-1}(x-\bar{x}). \quad (5.33)$$

Again, the worldvolume flux $\hat{F}$ is anti-symmetric while the metric is symmetric. Hence $\hat{F}^{\mu}_{\mu} = 0$. Using the form above, we can readily see that *neither* the Dp nor anti Dp-brane stress-energy tensor satisfies the constraint (5.8) for $p = 5, 7$.

Now for the five-form flux: using self-duality, i.e. $|\hat{F}_5|^2 = 0$, one finds that the constraint (5.8) for the stress-energy tensor of the $\hat{F}_5$ will be satisfied if and only if

$$\hat{F}_{\mu abcd}\hat{F}^{\mu abcd} > 0. \quad (5.34)$$

However, using the form of the flux (5.29), it is straightforward to see that $\hat{F}_{\mu abcd}\hat{F}^{\mu abcd} < 0$ and thus the constraint (5.8) is not satisfied by the five-form flux. Alternatively, $\hat{F}_5$ can be written as a sum of two types of fluxes as described in section (5.2.1), and again we arrive at the same conclusion.

Finally, using the condition that G_3 has legs along $\mathcal{M}^6$ and τ only depends on x^m, one finds that the stress-energy tensors for G_3 and τ do not satisfy the constraint (5.8). Since stress-energy tensors arising from fluxes, scalar fields or localized Dp or anti-Dp branes individually do not satisfy the constraint (5.8), the total stress-energy tensor for the entire system consisting of all these ingredients will also not satisfy the constraint.

We can generalize the case for the localized Dp or anti-Dp branes to include smeared Dp or anti-Dp branes along the compact directions.[3] The only difference in the smeared case is that the delta function in the stress-energy tensor (5.22) will

[3] A discussion of smeared sources can be found in [37–39]. This procedure is a way to incorporate the global nature of charge cancellation into the 10d equations

be replaced by some distribution i.e. $\delta(x-\bar{x}) \to \Gamma(x^m) > 0$. Smearing the branes in this fashion will allow one to compute the Ricci curvature *on the brane*, which will be a finite quantity. Again, since $\Gamma(x^m) > 0$, the stress-energy tensors will not obey the constraint (5.8). In summary, we conclude that local or non-local branes or anti-branes in the presence of global fields do not satisfy the condition (5.8) .

The only remaining case is the Op-planes. Orientifold planes are the loci of fixed points of some discrete symmetry group, arising from a Z_2 quotient of the theory combining worldsheet orientation reversal with an involution on the spacetime manifold [36]. The number of fixed points of this orientifolding then gives the number of orientifold planes, which fill all the noncompact dimensions. They have no gauge fields on their worldvolume, and have negative fractional charge and tension. As the planes are fixed points of a symmetry group, their location in the internal space is fixed and cannot be arbitrarily chosen. Thus the planes are essentially localized and cannot be thought of as smeared objects.

To construct an explicit gravity solution, we consider the localized action for the plane coupled with the bulk action. The tension of O3-planes taken to lie in M_4 is given by

$$\begin{aligned} T^{\mu}_{\ \mu\ (O3)} &= 4|T_{O3}|e^{\phi}\frac{\sqrt{-f}\sqrt{g_6}}{\sqrt{-G_{10}}}\delta^6(x-\bar{x}) \\ T^{m}_{\ m\ (O3)} &= 0, \end{aligned} \tag{5.35}$$

while for Op-planes with $p = 5, 7$, assuming as above that the spacetime directions M_4 are filled, we find

$$\begin{aligned} T^{\mu}_{\ \mu\ (Op)} &= 4|T_{Op}|e^{\frac{\phi(p+1)}{4}}\frac{\sqrt{-f}\sqrt{g_{D-p-1}}}{\sqrt{-G_{10}}}\delta^{D-p-1}(x-\bar{x}) \\ T^{m}_{\ m\ (Op)} &= |T_{Op}|e^{\frac{\phi(p+1)}{4}}\frac{\sqrt{-f}\sqrt{g_{D-p-1}}}{\sqrt{-G_{10}}}\,(p-3)\,\delta^{D-p-1}(x-\bar{x}). \end{aligned} \tag{5.36}$$

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of motion, which are inherently local. Not all 'smeared' solutions correspond to solutions of the full 10d equations.

Orientifolds have negative tension, $T^{\mu}_{\mu\ (Op)} > 0$, so there is a possibility that the constraint (5.8) might be satisfied when O-planes are included. However we will see that this does not lead to positive curvature in four dimensions. To see this first consider the Einstein equations arising from variation of the action (5.23) with respect to the metric:

$$R_{\mu\nu} = -g_{\mu\nu}\left[\frac{G_3\cdot\bar{G}_3}{48\ \mathrm{Im}\tau} + \frac{\hat{F}_5^2}{8\cdot 5!}\right] + \frac{\hat{F}_{\mu abcd}\hat{F}_{\nu}^{\ abcd}}{4\cdot 4!} + \kappa_{10}^2 N_f\left(T^{\mathrm{loc}}_{\mu\nu} - \frac{1}{8}g_{\mu\nu}T^{\mathrm{loc}}\right),$$

$$R_{mn} = -g_{mn}\left[\frac{G_3\cdot\bar{G}_3}{48\ \mathrm{Im}\tau} + \frac{\hat{F}_5^2}{8\cdot 5!}\right] + \frac{\hat{F}_{mabcd}\hat{F}_{n}^{\ abcd}}{4\cdot 4!} + \frac{G_m^{\ bc}\bar{G}_{nbc}}{4\ \mathrm{Im}\tau} + \frac{\partial_m\tau\partial_n\tau}{2\ |\mathrm{Im}\tau|^2}$$
$$+\kappa_{10}^2\left(T^{\mathrm{loc}}_{mn} - \frac{1}{8}g_{mn}T^{\mathrm{loc}}\right), \tag{5.37}$$

where N_f is the number of localized objects contributing to S_{loc}. Since we are considering manifolds which have the product form $M_{10} = M_4 \times \mathcal{M}^6$, we have the following form for the metric:

$$ds^2 = g_{\mu\nu}(x^\mu)dx^\mu dx^\nu + g_{mn}(x^m)dx^m dx^n. \tag{5.38}$$

With this metric ansatz, taking the trace of the first equation in (5.37) gives

$$R_4(x^\mu) = -\frac{G_3\cdot\bar{G}_3}{12\ \mathrm{Im}\tau} + \frac{\hat{F}_{\mu abcd}\hat{F}^{\mu abcd}}{4\cdot 4!} + \frac{\kappa_{10}^2 N_f}{2}\left(T^{\mu\ \mathrm{loc}}_{\mu} - T^{m\ \mathrm{loc}}_{m}\right). \tag{5.39}$$

The left-hand side is independent of x^m, and hence the right-hand side should be as well. It follows that we can evaluate the right-hand side at *any* value of x^m, and so we are free to consider x^m away from the localized Op-planes, where the local O-plane stress-energy tensor gives zero. As we have already studied, the flux and local or smeared Dp or anti-Dp brane contributions to R_4 are negative definite. Thus we obtain

$$R_4 \leq 0. \tag{5.40}$$

Since we have a product space $M_{10} = M_4 \times \mathcal{M}^6$, R_4 is the Ricci scalar of M_4. Thus we conclude that neither Dp-branes, anti-Dp branes, nor Op-planes, in the presence of type IIB fluxes and scalar fields, give rise to positive curvature for M_4.

5.3.2 Warped Product Manifold with Branes and Planes

Now we consider the more general case where the ten-dimensional manifold is not a direct product space, but rather a warped product. We look for solutions to (5.37) which take the following warped form:

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu + g_{mn}dx^m dx^n \\ &= e^{2A}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + e^{-2A}\tilde{g}_{mn}dx^m dx^n, \end{aligned} \tag{5.41}$$

where $A(x^m)$ is a scalar function, $\tilde{g}_{\mu\nu}(x^\mu)$ is independent of internal coordinates x^m while $\tilde{g}_{mn}(x^m)$ depends on x^m . Now, using the ansatz (5.41) for the metric, we get

$$R_{\mu\nu} = \tilde{R}_{\mu\nu} - \tilde{g}_{\mu\nu}e^{4A}\widetilde{\nabla}^2 A, \tag{5.42}$$

where the Laplacian is defined as

$$\widetilde{\nabla}^2 = \widetilde{g}^{mn}\partial_m\partial_n + \partial_m\widetilde{g}^{mn}\partial_n + \frac{1}{2}\widetilde{g}^{mn}\widetilde{g}^{pq}\partial_n\widetilde{g}_{pq}\partial_m, \tag{5.43}$$

and $\tilde{R}_{\mu\nu}$ is the Ricci tensor for the metric $\tilde{g}_{\mu\nu}$. Since the geometry is not a direct product, there is no notion of a separate four-dimensional space at all energies. If the internal space is compact and small, then at low energies we effectively have a four-dimensional non-compact space $\tilde{M}_4$ with metric $\tilde{g}_{\mu\nu}$. Then the condition $\tilde{R}_4 = \tilde{g}^{\mu\nu}\tilde{R}_{\mu\nu} > 0$ states that $\tilde{M}_4$ has positive curvature. Thus, for a warped product geometry with metric of the form (5.41), we will restrict to the case where $\mathcal{M}^6$ is compact and look for local and global fields in ten-dimensional type IIB theory that can give rise to $\tilde{M}_4$ with positive curvature.

We take the trace of the first equation in (5.37) and use the relation (5.42) to get

$$\begin{aligned}\widetilde{\nabla}^2 e^{4A} &= \tilde{R}_4 + \frac{e^{2A} G_3 \cdot \bar{G}_3}{12\ \mathrm{Im}\tau} - \frac{e^{2A} \hat{F}_{\mu abcd} \hat{F}^{\mu abcd}}{4 \cdot 4!} + e^{-6A} \partial_m e^{4A} \partial^m e^{4A} \\ &+ \frac{\kappa_{10}^2}{2} e^{2A} \Big(\sum_i \Big[T^m_{\ m\ (Op/\bar{O}p)i} - T^\mu_{\ \mu\ (Op/\bar{O}p)i} \Big] + \sum_j \Big[T^m_{\ m\ (Dp/\bar{D}p)j} - T^\mu_{\ \mu\ (Dp/\bar{D}p)j} \Big] \Big). \end{aligned} \tag{5.44}$$

Here $T^a_{\ a\ (Op/\bar{O}p)i}$ denotes the trace of the stress-energy tensor of the Op or anti-Op planes localized at $\bar{x}_i$, and similarly $T^a_{\ a\ (Dp/\bar{D}p)j}$ denotes the trace of the stress-energy tensor of the Dp or anti-Dp branes at $\bar{y}_j$. The fluxes, branes, and planes, are related globally by charge cancellation, although we will not discuss the precise details here. We can integrate (5.44) over the compact internal manifold $\tilde{M}^6$ (which has the metric $\tilde{g}_{mn}$) to get

$$\begin{aligned} 0 \ = \ & \tilde{V}_6 \hat{R}_4 + \int d^6x \ \sqrt{\tilde{g}_6} \mathcal{I}_{\mathrm{global}} + \int d^6x \ \sqrt{\tilde{g}_6} \Big[\frac{\kappa_{10}^2}{2} e^{2A} \Big(\sum_i \Big[T^m_{\ m\ (Op/\bar{O}p)i} - T^\mu_{\ \mu\ (Op/\bar{O}p)i} \Big] \\ & + \sum_j \Big[T^m_{\ m\ (Dp/\bar{D}p)j} - T^\mu_{\ \mu\ (Dp/\bar{D}p)j} \Big] \Big) \Big], \end{aligned} \tag{5.45}$$

where we have defined $\mathcal{I}_{\mathrm{global}}$ and $\tilde{V}_6$ as

$$\begin{aligned} \mathcal{I}_{\mathrm{global}} &\equiv \frac{e^{2A} G_3 \cdot \bar{G}_3}{12\ \mathrm{Im}\tau} - \frac{e^{2A} \hat{F}_{\mu abcd} \hat{F}^{\mu abcd}}{4 \cdot 4!} + e^{-6A} \partial_m e^{4A} \partial^m e^{4A} \geq 0, \\ \tilde{V}_6 &\equiv \int d^6x \sqrt{\tilde{g}_6} > 0. \end{aligned} \tag{5.46}$$

Since Op and anti-Op planes are localized objects, they give rise to physical singularities in the manifold. The metric is not well defined at these singular points and thus when we integrate over the manifold, we exclude these singular points. The local stress-energy tensor for both Op and anti-Op planes is zero away from these singular points, and hence they do not contribute to the volume integral:

$$\int d^6x \sqrt{\tilde{g}_6}\frac{\kappa_{10}^2}{2}e^{2A}\left[T^m_{\ m\ (Op/\bar{O}p)i} - T^\mu_{\ \mu\ (Op/\bar{O}p)i}\right] = 0. \tag{5.47}$$

Similarly, localized Dp and anti-Dp branes will also not contribute to the integral. However, one can smear the Dp or anti-Dp branes along $\mathcal{M}^6$ and then the integral will not be zero. As discussed in the previous section, $T^m_{\ m\ (Dp/\bar{D}p)} - T^\mu_{\ \mu\ (Dp/\bar{D}p)} \geq 0$, and thus we get

$$\tilde{R}_4 \leq 0. \tag{5.48}$$

In summary, *neither Dp nor anti-Dp branes with arbitrary worldvolume fluxes in the presence of type IIB fluxes and scalar fields result in positive curvature in four dimensions. Moreover, even Op or anti-Op planes with negative tension do not give rise to positive curvature.* Hence we look for higher-derivative gravity terms which also arise in string theory.

5.4 Curvature Corrections and Background Solutions from M-theory

In the above sections we have argued that it is impossible to get a four-dimensional de Sitter spacetime in a ten-dimensional two-derivative gravity coupled to fluxes, scalar fields, D-branes, anti D-branes and Orientifold-planes. However string theory can have higher-curvature corrections which, as we show below, could indeed help us to overcome the no-go theorem.

The analysis thus far has been done solely in the context of Type IIB string theory. However, the full set of quantum corrections in IIB is not known, and in addition there are many fields present which can complicate the analysis. To make the computations easier, we work in M-theory, where the bosonic field content is just the metric, g_{MN}, and the three-form, C_{MNP}, and make an ansatz for the form of the stress-energy tensor arising from any curvature corrections, given in (5.70). A T^2

reduction of M-theory in the limit when the torus size goes to zero, will reproduce the answer for Type IIB theory.[4]

We begin by setting up the M-theory uplift of the IIB system we are interested in. The action for M-theory is given by

$$S = S_{bulk} + S_{brane} + S_{corr}, \tag{5.49}$$

where S_{bulk} is the standard supergravity action for M-theory with a 3-form flux C and corresponding field strength G_4, S_{brane} is the contribution from $M2$-branes, and S_{corr} is a curvature correction to the action. The supergravity and brane actions are given by

$$S_{bulk} = \frac{1}{2\kappa^2}\int \mathrm{d}^{11}x\,\sqrt{-g}\left[R - \frac{1}{48}G^2\right] - \frac{1}{12\kappa^2}\int C\wedge G\wedge G, \tag{5.50}$$

$$S_{brane} = -\frac{T_2}{2}\int \mathrm{d}^3\sigma\sqrt{-\gamma}\left[\gamma^{\mu\nu}\partial_\mu X^M\partial_\nu X^N g_{MN} - 1 + \frac{1}{3!}\tilde{\epsilon}^{\mu\nu\rho}\partial_\mu X^M\partial_\nu X^N\partial_\rho X^P C_{MNP}\right], \tag{5.51}$$

where T_2 is the tension of the $M2$-brane, X^M denotes the worldsheet coordinates of the brane, $\gamma^{\mu\nu}$ is the induced metric on the brane, and we have assumed a minimal coupling of the brane to the fluxes.

The corrections to the action are of the form R^n or G^n (or a combination thereof)[5] and can come from several sources: instanton corrections, tree level α' corrections, and loop corrections. We delay a proper discussion of the R^n terms to Section 5.8. To study the effect of these corrections, we first assume that S_{corr} has two types of contributions: those that depend on the metric and are therefore non-topological, which we denote $\hat{S}_{ntop}$, and those that are topological and do not

[4] Earlier studies using EOMs but without invoking quantum corrections may be found in [41].

[5] See for example [42] for more detail, up to four-point amplitudes, on this.

depend explicitly on the metric, $\hat{S}_{top}$. In other words we have

$$S_{corr} \;=\; \hat{S}_{ntop} + \hat{S}_{top}, \tag{5.52}$$

where $\hat{S}_{top}$ can depend on the topological classes constructed out of the curvature form R.

Both sets of corrections depend on the curvatures R_{MNPQ} and G_{MNPQ} of the metric g_{MN} and the three-form field C_{MNP} respectively, and we brand them curvature corrections. The contributions to $\hat{S}_{ntop}$ and $\hat{S}_{top}$ at lowest order in α' are known (see [44] for example, as well as Section 8) and using these we can express $\hat{S}_{ntop}$ and $\hat{S}_{top}$ as

$$\begin{aligned} \hat{S}_{top} &= -T_2 \int C \wedge X_8 + \mathcal{S}_{top}(R,G) \\ \hat{S}_{ntop} &= \frac{T_2}{9.2^{13}\cdot(2\pi)^4}\int d^{11}x\sqrt{-g}\left(J_0 - \frac{1}{2}E_8\right) + \mathcal{S}_{ntop}(R,G), \end{aligned} \tag{5.53}$$

where X_8 is the curvature correction eight-form built completely with curvature two-form, such that $C \wedge X_8$ is a gravitational Chern-Simons term required to cancel the anomaly on the fivebrane worldvolume [46]; and J_0 and E_8 are given in [44]. The additional contributions $\mathcal{S}_{ntop}$ and $\mathcal{S}_{top}$ are functions of both the curvatures (R, G). Some details of $\mathcal{S}_{ntop}$ and $\mathcal{S}_{top}$ have been worked out and they are given in [42] and [43] respectively. We will give a more complete discussion in Section 8.

In Section 5.5 we will make an ansatz for the variation of the correction terms with respect to the metric, which acts as an effective stress-energy tensor T^{MN}_{corr}, rather than deal with the action of the correction terms directly. In other words, we will make an ansatz for

$$T^{MN}_{corr} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_{corr}}{\delta g_{MN}}\Big|_{g,C} \;=\; -\frac{2}{\sqrt{-g}}\frac{\delta \hat{S}_{ntop}}{\delta g_{MN}}\Big|_{g,C} \quad , \tag{5.54}$$

where the subscript denotes a given choice of the metric and the three-form flux.

From the action (5.49), we obtain three key equations which govern the evolution of the system. The first is the Einstein equation,[6]

$$R^{MN} - \frac{1}{2} g^{MN} R = T^{MN}, \tag{5.55}$$

where T^{MN} is the total stress-energy tensor coming from fluxes, brane sources and quantum or curvature corrections, and which we compute in Section 5.5. The second is the flux equation [44],

$$d *_{11} G = \frac{1}{2} G \wedge G + 2\kappa^2 \left(T_2 X_8 + *_{11} J\right) + S_G, \tag{5.56}$$

where J is the source term coming from n_3 M2-branes, $*_{11}$ is the Hodge star with respect to the warped metric unless mentioned otherwise, and S_G is the contribution from $\mathcal{S}_{ntop}$ and $\mathcal{S}_{top}$ in (5.53) that we will discuss later.

The third equation is the $M2$-brane equation,

$$\Box X^P + \gamma^{\mu\nu} \partial_\mu X^M \partial_\nu X^N \Gamma^{P_{MN}} = \frac{1}{3!} \epsilon^{\mu\nu\rho} \partial_\mu X^M \partial_\nu X^N \partial_\rho X^Q G^P{}_{MNQ}, \tag{5.57}$$

where $\epsilon_{\mu\nu\rho} = \sqrt{-\gamma} \tilde{\epsilon}_{\mu\nu\rho}$. The source term at a spacetime position x is related to the spacetime position X of the brane, and is given by

$$J^{PQR}(x) = \frac{2\kappa^2 n_3 T_2}{\sqrt{-g}} \int \mathrm{d}^3\sigma \sqrt{-\gamma} \tilde{\epsilon}^{\mu\nu\rho} \partial_\mu X^P \partial_\nu X^Q \partial_\rho X^R \delta^{11}(x - X). \tag{5.58}$$

We would like to find a solution to these equations that is conformally de Sitter when brought to IIB, such that the IIB metric can schematically be written as

$$ds^2 = \frac{1}{t_c^2} \eta_{\mu\nu} dx^\mu dx^\nu + ds^2_{internal}, \tag{5.59}$$

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[6] We are assuming that the volume of the internal fourfold is large so that an equation like (5.82) can be used to describe the metric there. This brings us to the issue of moduli stabilization, which will be discussed towards the end of Section 7.

where the time coordinate t_c is conformal time, usually denoted τ or η, which in the de Sitter space is related to physical time by

$$t_c \sim e^{-t_{phys}}. \tag{5.60}$$

It follows that the infinite future ($t_{phys} \to \infty$) is given by the limit $t_c \to 0$, as is the case during inflation. From this point onward we will drop the subscript c, and denote conformal time as t.

We make the following ansatz for the metric in M theory:

$$\begin{aligned} ds^2 &= \frac{1}{(\Lambda(t)\sqrt{h})^{4/3}}(-dt^2 + \eta^{ij}dz_i dz_j) + h^{1/3}\left[\frac{\tilde{g}_{mn}dy^m dy^n}{(\Lambda(t))^{1/3}} + (\Lambda(t))^{2/3}|dz|^2\right] \\ &\equiv e^{2A(y,t)}(-dt^2 + \eta^{ij}dz_i dz_j) + e^{2B(y,t)}\tilde{g}_{mn}dy^m dy^n + e^{2C(y,t)}|dz|^2, \end{aligned} \tag{5.61}$$

where $i, j = 1, 2$, $\tilde{g}_{mn}$ is the *unwarped* metric, A, B and C are warp factors that can be written in terms of $\Lambda(t)$ and $h(y^m)$, which we leave unspecified for the moment, and

$$dz \equiv dx_3 + idx_{11}, \tag{5.62}$$

so that the only time dependence in the system comes from $\Lambda(t)$. Specifically, the internal eight-dimensional manifold only depends on time via the warp factor $\Lambda(t)$ as we saw earlier, i.e.

$$ds_8^2 = \frac{\tilde{g}_{mn}dy^m dy^n}{\Lambda^{1/3}(t)} + \Lambda^{2/3}(t)|dz|^2. \tag{5.63}$$

This ansatz is chosen as the M-theory uplift for the solution we want to obtain in Type IIB, i.e. by shrinking the torus specified by coordinates $(z, \bar{z})$ or (x_3, x_{11}) to zero size one may recover type IIB theory. It is a generalization of the ansatz considered in [44], and describes a system of M2-branes moving towards orbifold singularities of the torus fibration of the fourfold (where the D7 fluxes are localized). This was developed as a first step towards an M theory uplift of D3/D7 [45].

The IIB metric that follows from dimensional reduction of the M theory metric (5.61) is given by

$$ds^2 = \frac{1}{\Lambda(t)\sqrt{h}}(-dt^2 + \eta^{ij}dz_i dz_j + dx_3^2) + \sqrt{h}\tilde{g}_{mn}dy^m dy^n, \tag{5.64}$$

so that, taking $\Lambda(t) = \Lambda|t|^2$ (taking the absolute value to avoid any imaginary warping in the M-theory metric), we obtain

$$ds^2 = \frac{1}{\Lambda t^2\sqrt{h}}(-dt^2 + \eta^{ij}dz_i dz_j + dx_3^2) + \sqrt{h}\tilde{g}_{mn}dy^m dy^n. \tag{5.65}$$

For this to be a dS solution, we demand that Λ be strictly positive. We also require a suitably well-behaved functional form for $h(y)$, to avoid any pathology. However, for our purposes, we will leave its functional form to be completely general.

Turning now to the flux equations, the equation for the G-fluxes can be rewritten as:

$$\begin{aligned} D_M\left(G^{MPQR}\right) &= \frac{1}{\sqrt{-g}}\widetilde{\epsilon}^{PQRM_1....M_8}\left[\frac{1}{2\cdot(4!)^2}G_{M_1....M_4}G_{M_5....M_8} + \frac{2\kappa^2 T_2}{8!}(X_8)_{M_1....M_8}\right] \\ &+ \frac{2\kappa^2 T_2 n_3}{\sqrt{-g}}\int d^3\sigma\widetilde{\epsilon}^{\mu\nu\rho}\partial_\mu X^P\partial_\nu X^Q\partial_\rho X^R\delta^{11}(x-X) + \frac{1}{\sqrt{-g}}\left(\frac{\delta\mathcal{S}_{ntop}}{\delta C_{PQR}} + \frac{\delta\mathcal{S}_{top}}{\delta C_{PQR}}\right). \end{aligned} \tag{5.66}$$

The above equation is in general hard to deal with because of the quantum corrections etc. However the the G-fluxes are related to the membrane motion via the membrane EOM. In the limit where the membrane motion is very slow, $\gamma_{\mu\nu}$, which is the pull-back metric, is simply equal to the spacetime metric given in (5.61). This implies

$$G_{m\mu\nu\rho} \;=\; \partial_m\left(\frac{\widetilde{\epsilon}_{\mu\nu\rho}}{h\Lambda(t)^2}\right), \tag{5.67}$$

which shows that the spacetime part of the three-form field $C_{\mu\nu\rho}$ should be time-dependent to maintain a metric of the form (5.61) with a membrane fixed at a point on the eight-dimensional internal space. However to solve all the background equations we need more flux components. Let us then switch on the following three additional G-fluxes:

$$G_{mnpq} \equiv 4\partial_{[m}C_{npq]}, \qquad G_{mnpa} \equiv 3\partial_{[m}C_{npa]}, \qquad G_{mnab} \equiv 2\partial_{[m}C_{nab]}. \tag{5.68}$$

To add some flexibility to the equations we seek to solve, and since we generically expect a mix of time-dependent and time-independent fluxes, we assume that the components G_{mnpa} are time independent, whereas all other fluxes depend on the internal coordinates y^m, as well as on (a, b) – i.e. on (x_3, x_{11}) – and the time t.

5.5 The Einstein Equations

In what follows we solve the Einstein equations (5.82) by including the general form of the stress-energy tensor T_{MN} in Section 5.5. This way we will be able to tabulate all the equations for the metric components satisfying (5.61), in Section 5.6. Subsequently, in Section 5.7, we study the flux equations (5.56) and resulting consistency conditions.

5.5.1 General Form of The Stress-Energy Tensor

Like the action, the stress-energy tensor has 3 contributions:

$$T^{MN} = T_G^{MN} + T_{corr}^{MN} + T_B^{MN}, \tag{5.69}$$

where G is for G-flux, *corr* is for correction, and B is for brane. As discussed in Section 5.4, we will study the effect of higher-order curvature corrections to the action by making an ansatz for the resulting T^{corr}_{MN}. Since our goal is to study solutions that are de Sitter in the non-compact dimensions, we are primarily concerned with tracking the time dependence of each component of the action and resulting Einstein equation. In line with this, we choose an ansatz for T^{corr}_{MN} that allows us to keep track of the time dependence. The stress-energy contributions are then given by

$$T_G^{MN} = \frac{1}{12}\left[G^{MPQR}G^N_{PQR} - \frac{1}{8}g^{MN}G^{PQRS}G_{PQRS}\right] \tag{5.70}$$

$$T_B^{MN}(x) = -\frac{\kappa^2 T_2 n_3}{\sqrt{-g}}\int \mathrm{d}^3\sigma\sqrt{-\gamma}\gamma^{\mu\nu}\partial_\mu X^M\partial_\nu X^N\delta^{11}(x - x_b) \tag{5.71}$$

$$T_{corr}^{MN} = \frac{-2}{\sqrt{-g}}\frac{\delta\hat{S}_{ntop}}{\delta g_{MN}}\Big|_{g,C} \equiv \sum_i [\Lambda(t)]^{\alpha_i+1/3}\mathcal{C}^{MN,\,i}, \tag{5.72}$$

where again x_b is the spacetime position of the brane (which is generically time dependent), and we have defined

$$\mathcal{C}^i_{MN} = g_{MN}\tilde{\mathcal{C}}_i - 2\frac{\delta\tilde{\mathcal{C}}_i}{\delta g^{MN}}. \tag{5.73}$$

In the following sections we will attempt to search for solutions, by separately examining the mn, ab, and $\mu\nu$ components of the Einstein equation. Note that the scalars $\tilde{\mathcal{C}}_i$ are defined in terms of the *unwarped metric*, such that the only dependence on warp factors in $\mathcal{C}_{MN}$ comes from the explicit factors of the warped metric g_{MN}.

5.5.2 Internal (m, n) components

We will start with the internal (m, n) components along the six-dimensional base. Two set of equations need to be solved now: the Einstein equation and the flux equation. For the Einstein equation we need the Einstein tensor from the M-theory metric (5.61). The Ricci tensor R_{mn} is given by

$$\begin{aligned} R_{mn} &= \tilde{R}_{mn} + 3\left[2\partial_{(m}A\partial_{n)}B - \partial_m A\partial_n A - \tilde{g}_{mn}\partial_k A\partial^k B\right] + 4\left[\partial_m B\partial_n B - \tilde{g}_{mn}\partial_k B\partial^k B\right] \\ &\quad -3D_{(m}\partial_{n)}A - 2D_{(m}\partial_{n)}C + 2\left[2\partial_{(m}C\partial_{n)}B - \partial_m C\partial_n C - \tilde{g}_{mn}\partial_k C\partial^k B\right] \\ &\quad -4D_{(m}\partial_{n)}B - \tilde{g}_{mn}\Box B + e^{2(B-A)}\left[\ddot{B} + \dot{A}\dot{B} + 6\dot{B}^2 + 2\dot{C}\dot{B}\right]\tilde{g}_{mn}, \end{aligned} \tag{5.74}$$

and the warped curvature scalar R is given by

$$\begin{aligned} R &= -e^{-2B}\left[10\Box B + 6\Box A + 4\Box C + 20\partial_m B\partial^m B\right] - 3e^{-2B}\left[4\partial_m A\partial^m A + 8\partial_m A\partial^m B\right] \\ &\quad -2e^{-2B}\left[3\partial_m C\partial^m C + 8\partial_m B\partial^m C + 6\partial_m A\partial^m C\right] + e^{-2B}\ \tilde{R} \\ &\quad +2e^{-2A}\left[6\ddot{B} + 2\ddot{A} + 2\ddot{C} + 21\dot{B}^2 + 6\dot{A}\dot{B} + 12\dot{C}\dot{B} + 2\dot{A}\dot{C} + \dot{A}^2 + 3\dot{C}^2\right], \end{aligned} \tag{5.75}$$

where remaining raising and lowering operations are done by the unwarped internal metric $\tilde{g}_{mn}$. The Einstein tensor G_{mn} is found to be

$$G_{mn} = \tilde{G}_{mn} - \frac{\partial_m h\partial_n h}{2h^2} + \tilde{g}_{mn}\left[\frac{\partial_k h\partial^k h}{4h^2} - 6\Lambda h\right], \tag{5.76}$$

where Λ is the coefficient of t^2 in $\Lambda(t)$, and hence the above expression is independent of time.

To study the stress-energy tensor from the G-fluxes we have to first express the various components of the G-fluxes G_{MNPQ} in terms of their *unwarped* components $\widetilde{G}_{MNPQ}$ as:

$$\begin{aligned} G^{012m} &= \widetilde{G}^{012m}[\Lambda(t)]^{13/3}h^{5/3}, & G^{012a} &= \widetilde{G}^{012a}[\Lambda(t)]^{10/3}h^{5/3} \\ G^{0mna} &= \widetilde{G}^{0mna}[\Lambda(t)]^{4/3}h^{-1/3}, & G^{0mab} &= \widetilde{G}^{0mab}[\Lambda(t)]^{1/3}h^{-1/3} \\ G^{mnpa} &= \widetilde{G}^{mnpa}[\Lambda(t)]^{1/3}h^{-4/3}, & G^{mnab} &= \widetilde{G}^{mnab}[\Lambda(t)]^{-2/3}h^{-4/3} \\ G^{0mnp} &= \widetilde{G}^{0mnp}[\Lambda(t)]^{7/3}h^{-1/3}, & G^{mnpq} &= \widetilde{G}^{mnpq}[\Lambda(t)]^{4/3}h^{-4/3} \end{aligned} \tag{5.77}$$

where what we have done here is to simply isolate the warp factor dependences of G^{MNPQ} and express its components in terms of $\widetilde{G}^{MNPQ}$. This also means that $G_{MNPQ} \equiv \widetilde{G}_{MNPQ}$ by definition. We can also isolate the warp factor from the metric and write the determinant as

$$\det\, g = -[\Lambda(t)]^{-14/3}h^{2/3}\det\, \tilde{g}. \tag{5.78}$$

The stress-energy tensor is easily expressed in the language of the unwarped G-fluxes (5.77) and the determinant (5.78):

$$\begin{aligned} \mathcal{T}^{(G)}_{mn} &= \tilde{g}_{mn}\frac{\partial_k h\partial^k h}{4h^2} - \frac{\partial_m h\partial_n h}{2h^2} + \frac{1}{4h}\left[\widetilde{G}_{mlka}\widetilde{G}_n{}^{lka} - \frac{1}{6}\tilde{g}_{mn}\widetilde{G}_{pkla}\widetilde{G}^{pkla}\right] \\ &+\frac{\Lambda(t)}{12h}\left[\widetilde{G}_{mlkr}\widetilde{G}_n{}^{lkr} - \frac{1}{8}\tilde{g}_{mn}\widetilde{G}_{pklr}\widetilde{G}^{pklr}\right] + \frac{1}{4h\Lambda(t)}\left[\widetilde{G}_{mlab}\widetilde{G}_n{}^{lab} - \frac{1}{4}\tilde{g}_{mn}\widetilde{G}_{pkab}\widetilde{G}^{pkab}\right]. \end{aligned} \tag{5.79}$$

The stress-energy tensor from the membrane (M2 brane) will not contribute however. This is because the stress-energy tensor, given by [44],

$$\mathcal{T}^{(B)}_{mn} = -\kappa^2 T_2 n_3 \tilde{g}_{pm}\tilde{g}_{qn}\frac{h^{1/3}[\Lambda(t)]^{5/3}}{\sqrt{\tilde{g}}}\int d^3\sigma\sqrt{-\gamma}\gamma^{\mu\nu}\partial_\mu X^p\partial_\nu X^q\delta^{11}(x-X), \tag{5.80}$$

where $\tilde{g}$ is the determinant of the metric in the m, n directions, vanishes in the limit where the membrane motion is very slow. The only other contribution will be from the correction terms, which, using $g_{mn} = e^{2B}\tilde{g}_{mn}$, gives

$$\mathcal{T}^{corr}_{mn} = h^{1/3}\sum_i [\Lambda(t)]^{\alpha_i}\tilde{\mathcal{C}}^i_{mn}. \tag{5.81}$$

The equation that we need to solve now is

$$G_{mn} = \mathcal{T}^{(G)}_{mn} + \mathcal{T}^{corr}_{mn}. \tag{5.82}$$

This can be split into a time-independent piece,

$$\tilde{G}_{mn} - \tilde{g}_{mn}6\Lambda h = \frac{1}{4h}\left[\widetilde{G}_{mlka}\widetilde{G}_n{}^{lka} - \frac{1}{6}\tilde{g}_{mn}\widetilde{G}_{pkla}\widetilde{G}^{pkla}\right] + h^{1/3}\sum_{\alpha_i=0}\tilde{\mathcal{C}}^i_{mn}, \tag{5.83}$$

where we made use of our assumption that the G_{mnpa} are time independent, and a time-dependent piece given by

$$\frac{\Lambda(t)}{12h}\left[\widetilde{G}_{mpqr}\widetilde{G}_n{}^{pqr} - \frac{1}{8}\tilde{g}_{mn}\widetilde{G}_{pqrs}\widetilde{G}^{pqrs}\right] + \frac{1}{4h\Lambda(t)}\left[\widetilde{G}_{mpab}\widetilde{G}_n{}^{pab} - \frac{1}{4}\tilde{g}_{mn}\widetilde{G}_{pqab}\widetilde{G}^{pqab}\right]$$
$$+h^{1/3}\sum_{\alpha_i\neq 0}[\Lambda(t)]^{\alpha_i}\tilde{\mathcal{C}}^i_{mn} = 0. \tag{5.84}$$

Note that at this stage the only possible way G_{mnpr} and G_{mnab} can also be time independent and yet still satisfy (5.116) is if the α_i are allowed to take the values

$$\alpha_i = (1, -1, 0, 0, ...0). \tag{5.85}$$

It is not clear we can have this condition for our case, and so we will assume that the only time-independent components of the G-fluxes are G_{mnpa}.

5.5.3 Internal (a, b) components

The Ricci tensor for the (a, b), i.e. the x^3 and x^{11} components, is given by

$$R_{ab} = -\delta_{ab}e^{2(C-B)}\left[\Box C + 3\partial_m C\partial^m A + 4\partial_m C\partial^m B + 2\partial_m C\partial^m C\right]$$
$$+\delta_{ab}e^{2(C-A)}\left[\ddot{C} + \dot{A}\dot{C} + 6\dot{C}\dot{B} + 2\dot{C}^2\right], \tag{5.86}$$

which can be used to compute the Einstein tensor G_{ab}. For the M-theory metric (5.61), G_{ab} is given by

$$G_{ab} \;=\; \delta_{ab}\Lambda(t)\left[-\frac{\tilde{R}}{2} - 9h\Lambda + \frac{\tilde{g}^{pk}\partial_p h \partial_k h}{4h^2}\right], \tag{5.87}$$

where we note that there is an overall time dependence given by $\Lambda(t)$. The stress-energy tensor due to the fluxes is given by

$$\begin{aligned}\mathcal{T}_{ab}^{(G)} \quad &= \frac{\Lambda(t)}{12h}\left[\widetilde{G}_{amnp}\widetilde{G}_b{}^{mnp} - \delta_{ab}\frac{\widetilde{G}_{mnpc}\widetilde{G}^{mnpc}}{2} + \delta_{ab}\frac{3\tilde{g}^{mp}\partial_m h\partial_p h}{h}\right] \\ &+\frac{1}{4h}\left[\widetilde{G}_{acmn}\widetilde{G}_b{}^{cmn} - \frac{1}{4}\delta_{ab}\widetilde{G}_{mncd}\widetilde{G}^{mncd}\right] - \delta_{ab}\frac{[\Lambda(t)]^2}{4\cdot 4!h}\widetilde{G}_{mnpq}\widetilde{G}^{mnpq}.\end{aligned} \tag{5.88}$$

The interesting thing about the above formula is that the time dependence of the first term (involving $\widetilde{G}_{mnpa}$) is exactly the same as the time dependence of the G_{ab}. This means that the $\widetilde{G}_{mnpa}$ components can remain time independent, as we had earlier. The correction term contribution to the stress-energy tensor for the (a,b) directions is

$$\mathcal{T}_{ab}^{corr} = h^{1/3}\sum_i [\Lambda(t)]^{\alpha_i+1}\tilde{\mathcal{C}}_{ab}^i. \tag{5.89}$$

As before, we can write the resulting Einstein equation as a time-independent expression (where we collect the terms linear in $\Lambda(t)$):

$$\begin{aligned}\left(\frac{\tilde{R}}{2} + 9h\Lambda\right)\delta_{ab} + \frac{1}{12h}\left[\widetilde{G}_{amnp}\widetilde{G}_b{}^{mnp} - \delta_{ab}\frac{\widetilde{G}_{mnpc}\widetilde{G}^{mnpc}}{2}\right]& \\ +h^{1/3}\sum_{\alpha_i=0}\tilde{\mathcal{C}}_{ab}^i \;=\; 0,&\end{aligned} \tag{5.90}$$

and a time-dependent expression:

$$\frac{1}{4h}\left[\widetilde{G}_{acmn}\widetilde{G}_b{}^{cmn} - \frac{1}{4}\delta_{ab}\widetilde{G}_{mncd}\widetilde{G}^{mncd}\right] - \delta_{ab}\frac{[\Lambda(t)]^2}{4\cdot 4!h}\tilde{G}_{mnpq}\tilde{G}^{mnpq}$$

$$+h^{1/3}\sum_{\alpha_i\neq 0}[\Lambda(t)]^{\alpha_i+1}\,\tilde{\mathcal{C}}^i_{ab} \;=\; 0. \tag{5.91}$$

Once again, we must assume G_{mnpq} and G_{mnab} are time dependent in such a way as to solve (5.91). Thus the conclusion of this section is perfectly consistent with the conclusions of the previous section.

5.5.4 Spacetime (t, z_1, z_2) components

We now study the spacetime components. The curvature tensors R_{00} and R_{ij} are given by

$$\begin{aligned} R_{ij} &= -\eta_{ij}e^{2A-2B}\left[\Box A + 3\partial_m A\partial^m A + 4\partial_m A\partial^m B + 2\partial_m A\partial^m C\right] \quad (5.92)\\ &\quad + \left(\ddot{A} + 6\dot{A}\dot{B} + \dot{A}^2 + 2\dot{A}\dot{C}\right)\eta_{ij} \\ R_{00} &= e^{2A-2B}\left[\Box A + 3\partial_m A\partial^m A + 4\partial_m A\partial^m B + 2\partial_m A\partial^m C\right] \quad (5.93)\\ &\quad - \left[2\ddot{A} + 6(\ddot{B} + \dot{B}^2 - \dot{A}\dot{B}) + 2(\ddot{C} + \dot{C}^2 - \dot{A}\dot{C})\right], \quad (5.94) \end{aligned}$$

using which the Einstein tensor $G_{\mu\nu}$ is found to be

$$G_{\mu\nu} \;=\; -\frac{\eta_{\mu\nu}}{\Lambda(t)}\left[\frac{\tilde{R}}{2h} + \frac{\tilde{g}^{mk}\partial_k h\partial_m h}{4h^3} - \frac{\Box h}{2h^2} + 3\Lambda\right], \tag{5.95}$$

where we see that the overall time dependence is provided by $1/\Lambda(t)$. The above equation should be balanced by the stress-energy tensor from the G-flux and corrections, as well as from the membrane. The latter term is there because the almost static membrane *does* contribute to the stress-energy tensor along the spacetime directions.

The stress-energy tensor from the G-flux is given by

$$\mathcal{T}^{(G)}_{\mu\nu} \;=\; -\eta_{\mu\nu}\left[\frac{(\partial h)^2}{4\Lambda(t)h^3} + \frac{\widetilde{G}_{mnpa}\widetilde{G}^{mnpa}}{4!\Lambda(t)h^2} + \frac{\widetilde{G}_{mnpq}\widetilde{G}^{mnpq}}{4\cdot 4!h^2} + \frac{\widetilde{G}_{mnab}\widetilde{G}^{mnab}}{16h^2[\Lambda(t)]^2}\right] \tag{5.96}$$

As expected, $\mathcal{T}^{(G)}_{\mu\nu}$ has a piece that scales as $1/\Lambda(t)$, so we should be able to maintain the time independence of the G_{mnpa} components.

The stress-energy tensor coming from the correction terms can be found to be

$$T^{corr}_{\mu\nu} = h^{-2/3} \sum_i [\Lambda(t)]^{\alpha_i - 1} \tilde{\mathcal{C}}^i_{\mu\nu}. \tag{5.97}$$

Finally we will need the stress-energy tensor for the static membrane. The EOM of the worldvolume metric gives us, in the case where the brane is moving very slowly,

$$\gamma_{\mu\nu} \;=\; \partial_\mu X^M \partial_\nu X^N g_{MN} \;\approx g_{\mu\nu} \;=\; \frac{\eta_{\mu\nu}}{[\Lambda(t)\sqrt{h}]^{4/3}}. \tag{5.98}$$

Using this we can show that the stress-energy tensor is given by

$$\mathcal{T}^{(B)}_{\mu\nu} = -\frac{\kappa^2 T_2 n_3}{h^2 \Lambda(t)\sqrt{\tilde{g}}} \delta^8(x - X)\eta_{\mu\nu}, \tag{5.99}$$

which is again suppressed by $1/\Lambda(t)$, confirming the time independence of the components G_{mnpa}.

Again, we can split the full Einstein equation into a time-independent part:

$$\left(\frac{\tilde{R}}{2h} - \frac{\Box h}{2h^2} + 3\Lambda\right) = \frac{\widetilde{G}_{mnpa}\widetilde{G}^{mnpa}}{4!h^2} + \frac{\kappa^2 T_2 n_3}{h^2\sqrt{\tilde{g}}}\delta^8(x - X) - \frac{1}{3h^{2/3}} \sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}^{\mu,i}_\mu \tag{5.100}$$

where we have traced over the μ, ν components using $\eta_{\mu\nu}$, and a time-dependent part:

$$\eta_{\mu\nu}\left[\frac{\widetilde{G}_{mnpq}\widetilde{G}^{mnpq}}{4 \cdot 4!h^2} + \frac{\widetilde{G}_{mnab}\widetilde{G}^{mnab}}{4!h^2\Lambda(t)^2}\right] - \frac{1}{h^{2/3}} \sum_{\{\alpha_i\}\neq 0} [\Lambda(t)]^{\alpha_i - 1}\tilde{\mathcal{C}}^i_{\mu\nu} = 0. \tag{5.101}$$

5.6 Analysis of the EOMs and Consistency Conditions

We have now split the Einstein equations into 6 equations, 3 of which are time dependent, and 3 of which are time independent. To deduce the properties of these equations, it suffices to look at the traced over form of each. The traced-over time independent equation for the spacetime (μ, ν) components is

$$\left(\frac{\tilde{R}}{2h} - \frac{\Box h}{2h^2} + 3\Lambda\right) = \frac{\widetilde{G}_{mnpa}\widetilde{G}^{mnpa}}{4!h^2} + \frac{\kappa^2 n_3 T_2 \delta^8(x-X)}{h^2\sqrt{\tilde{g}}} - \frac{1}{3h^{2/3}} \sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_\mu^{\mu,\, i}, \tag{5.102}$$

whereas for the internal (m, n) components, it is

$$36h\Lambda + h^{1/3} \sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_m^{m,\, i} = \tilde{G}_m^m. \tag{5.103}$$

Note that the flux contribution in (5.83) is traceless, so it doesn't appear in the above equation. Finally, for the internal (a, b) components the trace equation is

$$\frac{\tilde{R}}{2} + 9h\Lambda + \frac{h^{1/3}}{2} \sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_a^{a,\, i} = 0, \tag{5.104}$$

where again the flux contributions from (5.90) do not enter. The last two equations, (5.103) and (5.104), are quite similar and can be rewritten as

$$\sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_m^{m,\, i} = -\frac{2}{h^{1/3}}(\tilde{R} + 18h\Lambda), \tag{5.105}$$

$$\sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_a^{a,\, i} = -\frac{1}{h^{1/3}}(\tilde{R} + 18h\Lambda), \tag{5.106}$$

from which we can read off that

$$\sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_m^{m,\, i} = 2 \sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_a^{a,\, i}. \tag{5.107}$$

Using (5.103) and (5.104) we can also write

$$\tilde{R} = -18h\Lambda - h^{1/3}\left(\frac{1}{2} \sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_a^{a,\, i} + \frac{1}{4} \sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_m^{m,\, i}\right), \tag{5.108}$$

which allows us to rewrite the constraint (5.102) as

$$
\begin{aligned}
-\Box h &= \frac{\widetilde{G}_{mnpa}\widetilde{G}^{mnpa}}{12} + 12h^2\Lambda + \frac{2\kappa^2 n_3 T_2 \delta^8(x-X)}{\sqrt{\tilde{g}}} \\
&\quad + h^{4/3}\left(\frac{1}{2}\sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_a^{\ a,\ i} + \frac{1}{4}\sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_m^{\ m,\ i} - \frac{2}{3}\sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}_\mu^{\ \mu,\ i}\right). \quad (5.109)
\end{aligned}
$$

There are three further equations that arise from (5.90) in the limit when $a \neq b$, $a = b = 3$ and $a = b = 11$ respectively. These are

$$
\begin{aligned}
&\widetilde{G}_{amnp}\widetilde{G}_b^{mnp} + 12h^{4/3}\sum_{\{\alpha_i\}=0} \tilde{\mathcal{C}}^i_{ab} = 0, \\
&\widetilde{G}_{3mnp}\widetilde{G}_3^{mnp} - \widetilde{G}_{11,mnp}\widetilde{G}_{11}^{mnp} = 24h^{4/3}\sum_{\{\alpha_i\}=0}\left(\frac{1}{2}\tilde{\mathcal{C}}_a^{a,\ i} - \tilde{\mathcal{C}}^i_{33}\right), \\
&\widetilde{G}_{3mnp}\widetilde{G}_3^{mnp} - \widetilde{G}_{11,mnp}\widetilde{G}_{11}^{mnp} = -24h^{4/3}\sum_{\{\alpha_i\}=0}\left(\frac{1}{2}\tilde{\mathcal{C}}_a^{a,\ i} - \tilde{\mathcal{C}}^i_{11,11}\right). \quad (5.110)
\end{aligned}
$$

If we now consider integrating equation (5.109) over the compact eight-dimensional manifold, we see that the LHS integrates to zero, and we get

$$
\begin{aligned}
0 &= \frac{1}{12}\int d^8x\sqrt{\tilde{g}}\ \widetilde{G}_{mnpa}\widetilde{G}^{mnpa} + 12\Lambda\int d^8x\sqrt{\tilde{g}}\ h^2 + 2\kappa^2 T_2 n_3 \\
&+ \int d^8x\sqrt{\tilde{g}}h^{4/3}\left(\frac{1}{2}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}_a^{a,\ i} + \frac{1}{4}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}_m^{m,\ i} - \frac{2}{3}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}_\mu^{\mu,\ i}\right). \quad (5.111)
\end{aligned}
$$

In the absence of fluxes and higher-curvature corrections the above equation implies that the simplest solution will be $\Lambda = 0$, i.e. a four-dimensional Minkowski space. In the presence of fluxes, and in the presence or absence of the higher-curvature corrections, it is not difficult to see that the $\Lambda < 0$ solution is favored. However to allow a $\Lambda > 0$ solution from (5.111), it is *at least* necessary to have the higher curvature corrections, because the first three terms in (5.111) are positive definite. Moreover, if all the curvature corrections in (5.111) add up to some positive value, a $\Lambda > 0$ solution will again be impossible.

This means that for a $\Lambda > 0$ solution to exist, the curvature terms in (5.111) should integrate to a negative definite value. This conclusion should be valid for all possible choices of the warp factor h and the internal metric $\tilde{g}_{mn}$. In particular, for certain choices of the fluxes the warp factor may be localized over a small patch on the internal manifold (for example like a M2-brane solution). Then the integral condition on the higher-curvature terms will have to be realized at every such patch on the internal manifold. On a small patch, since there is no local transformation that can make the metric flat everywhere, $\tilde{\mathcal{C}}_M^{M,\,i}$ can be viewed as the expectation or the average value on the patch, or more explicitly:

$$\langle \tilde{\mathcal{C}}_M^{M,\,i} \rangle \equiv \int d^8x \sqrt{\tilde{g}}\; h^{4/3} \tilde{\mathcal{C}}_M^{M,\,i}. \tag{5.112}$$

In other words, for a solution to exist we must have the following condition

$$\frac{1}{2} \sum_{\{\alpha_i\}=0} \langle \tilde{\mathcal{C}}_a^{a,\,i} \rangle + \frac{1}{4} \sum_{\{\alpha_i\}=0} \langle \tilde{\mathcal{C}}_m^{m,\,i} \rangle - \frac{2}{3} \sum_{\{\alpha_i\}=0} \langle \tilde{\mathcal{C}}_\mu^{\mu,\,i} \rangle \;\; < \;\; 0. \tag{5.113}$$

Since $T_{mn}^{corr} \sim \tilde{C}_{mn}^{i}$, this equation is almost analogous to (5.8) but expressed in the language of curvature corrections.[7] This makes sense because only these corrections will allow us to overcome the Gibbons-Maldacena-Nunez [17–19] no-go theorem. Under this assumption, (5.113) gives non-trivial constraints on the curvature corrections required to have a four-dimensional de Sitter solution in Type IIB theory.

[7] One subtlety however is that this constraint arises from the Einstein equations of an 11-dimensional M theory, in which μ runs from 0 to 2, while in (5.8) it runs from 0 to 3, so the numerical factors are not expected to be the same in both expressions. We would have to redo the calculation in IIB to get the same expression. However in both cases the condition is that the four-dimensional curvature upon compactification be positive.

The curvature terms may be further constrained if we look at the time-dependent equations. These equations are

$$\frac{\widetilde{G}_{mnpq}\widetilde{G}^{mnpq}}{4} + \frac{\widetilde{G}_{mnab}\widetilde{G}^{mnab}}{\Lambda(t)^2} = 8h^{4/3} \sum_{\{\alpha_i\}\neq 0} [\Lambda(t)]^{\alpha_i - 1}\tilde{\mathcal{C}}^{\mu,\, i}_{\mu}, \tag{5.114}$$

$$\widetilde{G}_{acmn}\widetilde{G}^{acmn} - \frac{[\Lambda(t)]^2}{6}\widetilde{G}_{mnpq}\widetilde{G}^{mnpq} = -8h^{4/3} \sum_{\alpha_i\neq 0} [\Lambda(t)]^{\alpha_i + 1}\,\tilde{\mathcal{C}}^{a,\, i}_{a}, \tag{5.115}$$

$$\frac{\Lambda(t)}{6}\widetilde{G}_{pqrs}\widetilde{G}^{pqrs} - \frac{1}{\Lambda(t)}\widetilde{G}_{mpab}\widetilde{G}^{mpab} = -8h^{4/3} \sum_{\alpha_i\neq 0} [\Lambda(t)]^{\alpha_i}\tilde{\mathcal{C}}^{m,\, i}_{m}. \tag{5.116}$$

From the first equation above, and noting that both the terms on the LHS are positive definite, we deduce one new condition on the corrections by integrating over the eight-dimensional manifold:

$$\sum_{\{\alpha_i\}\neq 0} a^{\alpha_i}\langle\tilde{\mathcal{C}}^{\mu,\, i}_{\mu}\rangle \;>\; 0, \tag{5.117}$$

where $a \equiv \Lambda(t_a)$ for a fixed t_a. In fact (5.117) will be an infinite set of constraints because, due to its time dependence, a^{α_i} can take any (positive) values including arbitrary fractional numbers. Note that

$$\langle\tilde{\mathcal{C}}^{\mu,\, i}_{\mu}\rangle > 0 \tag{5.118}$$

will always solve (5.117) if the α_i appearing in (5.117) are not equal to each other. However a generic statement cannot be made unless we actually solve all the EOMs. In view of that we will only demand (5.117) as our constraint equation. The other two equations involve relative signs and therefore tell us nothing about the signs of $\sum_{\{\alpha_i\}\neq 0}\tilde{\mathcal{C}}^{a,\, i}_{a}$ or $\sum_{\{\alpha_i\}\neq 0}\tilde{\mathcal{C}}^{m,\, i}_{m}$.

In total we have the following conditions on the form of the corrections:

$$\frac{1}{2}\sum_{\{\alpha_i\}=0}\langle\tilde{\mathcal{C}}_a^{a,\,i}\rangle + \frac{1}{4}\sum_{\{\alpha_i\}=0}\langle\tilde{\mathcal{C}}_m^{m,\,i}\rangle < \frac{2}{3}\sum_{\{\alpha_i\}=0}\langle\tilde{\mathcal{C}}_\mu^{\mu,\,i}\rangle, \tag{5.119}$$

$$\sum_{\{\alpha_i\}\neq 0} a^{\alpha_i}\langle\tilde{\mathcal{C}}_\mu^{\mu,\,i}\rangle \;>\; 0. \tag{5.120}$$

5.7 Analysis of the background fluxes and additional consistency checks

The above set of conclusions was derived by analyzing the Einstein's equations alone. The next question is whether any conclusions are altered when the equations of motion for the G-fluxes are taken into account. Before moving ahead with the exact flux equations, we will do a more careful analysis of the background fluxes to see how the type IIB fluxes should be viewed from our choices of the M-theory fluxes. Imagine we rewrite the flux components in M-theory as [45]:

$$\widetilde{G} = G_{\mu\nu\rho m}dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^m + \widetilde{\mathcal{G}}_{mnqa}dx^m \wedge dx^n \wedge dx^q \wedge dx^a + \sum_{i=1}^{N} F^i \wedge \Omega^i, \tag{5.121}$$

where we have taken the time-dependent components $\widetilde{G}_{mnpq}$ and $\widetilde{G}_{mnab}$ to be localized around certain singular points on the eight-dimensional internal space and we have decomposed $\widetilde{G}_{mnpa}$ into a *delocalized* and a *localized* piece as

$$\widetilde{G}_{mnpa} = \widetilde{\mathcal{G}}_{mnpa} + \widetilde{G}^{loc}_{mnpa}. \tag{5.122}$$

In (5.121), the localized pieces are contained in the last term, where the sum is over the points at which the F-theory torus degenerates, the Ω^i are the normalizable harmonic forms located at these points, and the F^i represent the gauge fields on the resulting D7-branes at these points in IIB, such that only the F^i are functions of time.[8] Then it turns out that the delocalized piece $\widetilde{\mathcal{G}}_{mnpa}$ gives rise to the type IIB three-forms in the following way:

$$\begin{aligned}\widetilde{\mathcal{G}}_{mnpa} dx^m \wedge dx^n \wedge dx^p \wedge dx^a \;&\equiv\; 2(H_3)_{mnp} dx^m \wedge dx^n \wedge dx^p \wedge dx^3 \\ &\quad +2(F_3)_{mnp} dx^m \wedge dx^n \wedge dx^p \wedge dx^{11}, \quad (5.123)\end{aligned}$$

where H_3 and F_3 are the NS and RR three-forms of type IIB theory respectively, while the localized fluxes should appear as gauge-fields on the type IIB seven-branes. A straightforward decomposition immediately gives us:

$$\begin{aligned}\int G \wedge *_{11} G \;&\to\; \int d^{10}x \sqrt{g_{10}} \left[\frac{1}{g_B^2}\left(|H_3|^2 + |F_5|^2\right) + |F_3|^2\right] \\ &\quad + \sum_{i=1}^{N} \int d^8\sigma \; F^i \wedge *_B F^i, \\ \int C \wedge G \wedge G \;&\to\; \int C_4 \wedge H_3 \wedge F_3 \;+\; \sum_{i=1}^{N} \int d^8\sigma \; C_4 \wedge F^i \wedge F^i, \quad (5.124)\end{aligned}$$

where for the first relation, the first three terms appear in the type IIB bulk and the last term collects the interactions on the D7-brane worldvolume. We have also assumed that the self-duality of F_5 is imposed via the EOM, so that the action is explicitly non-selfdual. The five-form piece comes from the spacetime part of the G-flux and the three-form fluxes come from the components G_{mnqa}. For the second relation, the first term is the bulk term and the second one is the seven-brane Chern-Simons term. The $C \wedge X_8$ term gives rise to the couplings on the D7-branes and O7-planes and possibly some contributions to the bulk interactions. For example we expect some parts of $C \wedge X_8$ to reproduce

$$a_1 \int_{\mathrm{D7}} C_{\mathrm{RR}} \wedge \sqrt{\hat{A}(R)} + a_2 \int_{\mathrm{O7}} C_{\mathrm{RR}} \wedge \sqrt{H(R/4)}, \quad (5.125)$$

[8] A discussion of these issues is also given in [48] and [49]. Note that the existence of these points do not mean that the eight-dimensional manifold is singular.

where $\hat{A}(R)$ and $H(R)$ are the corresponding A-roof genus and Hirzebrusch polynomial respectively. We have also used the orthogonality condition for the components of Ω^i to get the interactions of the seven-brane worldvolume gauge fields. Note that this analysis only gives the abelian part of the gauge group (i.e the Cartan subalgebra), which could be extended to include a non-abelian gauge group by including M2-branes wrapping vanishing 2-cycles of the fourfold.

Once the structure of the fluxes is laid out, the physics away from the singular points will be captured by the delocalized fluxes only. The G-flux EOM (5.66) then gives us the following equation for the warp factor h:[9]

$$\begin{aligned}-\square h = \quad & \frac{1}{12}\widetilde{\mathcal{G}}_{mnpa}(*_8\widetilde{\mathcal{G}})^{mnpa} + \frac{2\kappa^2 T_2}{8!\sqrt{g}}(X_8)_{M_1\dots M_8}\widetilde{\epsilon}^{M_1\dots.M_8} \\ & + \frac{2\kappa^2 T_2 n_3}{\sqrt{g}}\delta^8(x-X) - \frac{2\kappa^2 T_2 \bar{n}_3}{\sqrt{g}}\delta^8(x-Y) + \alpha_1\frac{\delta\mathcal{S}_{ntop}}{\delta\widetilde{C}_{012}} + \alpha_2\frac{\delta\mathcal{S}_{top}}{\delta\widetilde{C}_{012}},\end{aligned} \tag{5.126}$$

where $*_8$ is the Hodge star with respect to the unwarped metric unless mentioned otherwise, α_i are coefficients that can be derived from (5.66), and we take only the delocalized flux components. Equation (5.126) can be compared to the Einstein equation:

$$\begin{aligned}-\square h \quad = \quad & \frac{\widetilde{\mathcal{G}}_{mnpa}\widetilde{\mathcal{G}}^{mnpa}}{12} + 12h^2\Lambda + \frac{2\kappa^2 n_3 T_2\delta^8(x-X)}{\sqrt{g}} + \frac{2\kappa^2 \bar{n}_3 T_2\delta^8(x-Y)}{\sqrt{g}} \\ & + h^{4/3}\left(\frac{1}{2}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}_a^{a,\,i} + \frac{1}{4}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}_m^{m,\,i} - \frac{2}{3}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}_\mu^{\mu,\,i}\right),\end{aligned} \tag{5.127}$$

where we have re-expressed (5.109) in terms of the delocalized fluxes instead of the total fluxes. The factors $(n_3, \bar{n}_3)$ denote the number of M2 and anti-M2 branes

[9] We have defined the covariant derivative D_q in the following way: $D_q G^{qmnp} \equiv \frac{1}{\sqrt{-g}}\partial_q\left(\sqrt{-g}G^{qmnp}\right)$.

located at (X, Y) respectively and X_8 is defined in the usual way [50] such that

$$\int X_8 \;=\; -\frac{1}{4!(2\pi)^4}\chi_4, \tag{5.128}$$

where the integral is over the eight-dimensional manifold with Euler characteristic χ_4, which could in general take any sign.

Comparing (5.127) and (5.126) we get the following consistency relation which should be compared with the consistency condition that we had from (5.111):

$$\frac{1}{12}\widetilde{\mathcal{G}}_{mnpa}\left[\widetilde{\mathcal{G}}^{mnpa} - (*_8\widetilde{\mathcal{G}})^{mnpa}\right] + 12\Lambda h^2 + \frac{4\kappa^2 T_2 \bar{n}_3}{\sqrt{\tilde{g}}}\delta^8(x-Y) - \alpha_1\frac{\delta\mathcal{S}_{ntop}}{\delta\widetilde{C}_{012}} - \alpha_2\frac{\delta\mathcal{S}_{top}}{\delta\widetilde{C}_{012}}$$
$$+h^{4/3}\left(\frac{1}{2}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}_a^{a,\,i} + \frac{1}{4}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}_m^{m,\,i} - \frac{2}{3}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}_\mu^{\mu,\,i}\right) - \frac{2\kappa^2 T_2}{8!\sqrt{\tilde{g}}}(X_8)_{M_1\ldots M_8}\widetilde{\epsilon}^{M_1\ldots.M_8} = 0. \tag{5.129}$$

Firstly note that in the presence of curvature corrections and positive cosmological constant Λ it is in general *not* possible to maintain the self-duality of the G-fluxes. This may be more obvious if we re-express (5.110) using (5.123) as

$$|H_3|^2 - |F_3|^2 = \frac{h^{4/3}}{12}\sum_{\{\alpha_i\}=0}\left(\tilde{\mathcal{C}}_{11}^i - \tilde{\mathcal{C}}_{33}^i\right), \tag{5.130}$$

which may not be consistent with $H_3 = -*_6 F_3$ and $F_3 = *_6 H_3$, where $*_6$ is the six-dimensional Hodge star measured with respect to the unwarped metric. In other words:

$$\widetilde{\mathcal{G}}^{mnpa} - (*_8\widetilde{\mathcal{G}})^{mnpa} \neq 0, \tag{5.131}$$

meaning that supersymmetry should be broken to allow for a positive cosmological constant. One may also note that the contribution from the anti-M2 branes in (5.129) allows the self-duality of the G-fluxes to be broken even for vanishing cosmological constant Λ and vanishing higher-order corrections. This means supersymmetry can be broken in *flat* space by the anti-M2 branes.

The above relation can in fact be extended to the *full* G-fluxes, i.e. including both the localized and the delocalized pieces. To show this we make use of another component of the G-flux equation, finding

$$\begin{aligned}\Lambda(t) D_q \widetilde{G}^{qmnp} + D_a \widetilde{G}^{amnp} &= \frac{\partial_q h}{h}\left[\Lambda(t)\widetilde{G}^{qmnp} - \frac{1}{12}\left(*_8 \widetilde{G}\right)^{qmnp}\right] \\ &+ \frac{\partial_a h}{h}\left[\widetilde{G}^{amnp} - \frac{1}{12}\left(*_8 \widetilde{G}\right)^{amnp}\right] + \beta_1 \frac{\delta \mathcal{S}_{ntop}}{\delta \widetilde{C}_{mnp}} + \beta_2 \frac{\delta \mathcal{S}_{top}}{\delta \widetilde{C}_{mnp}},\end{aligned} \tag{5.132}$$

which is expressed in terms of the total fluxes and is again consistent with (5.131). In deriving the above equation we have assumed

$$(X_8)_{012M_1....M_5} \approx 0. \tag{5.133}$$

Note that for the delocalized flux components $\widetilde{\mathcal{G}}_{mnpa}$, away from the singular points, (5.132) simplifies to

$$D_a \widetilde{\mathcal{G}}^{amnp} = \frac{\partial_a h}{h}\left[\widetilde{\mathcal{G}}^{amnp} - \frac{1}{12}\left(*_8 \widetilde{\mathcal{G}}\right)^{amnp}\right] + \left[\beta_1 \frac{\delta \mathcal{S}_{ntop}}{\delta \widetilde{C}_{mnp}} + \beta_2 \frac{\delta \mathcal{S}_{top}}{\delta \widetilde{C}_{mnp}}\right]_{\widetilde{G}^{loc}_{mnpa}=0}^{\widetilde{G}_{mnpq}=0} \tag{5.134}$$

meaning that the delocalized flux components are not covariantly constant. Another consequence of the above equation is that the $\widetilde{\mathcal{G}}_{mnpa}$ components will continue to remain time independent provided

$$\frac{\partial}{\partial t}\left[\beta_1 \frac{\delta \mathcal{S}_{ntop}}{\delta \widetilde{C}_{mnp}} + \beta_2 \frac{\delta \mathcal{S}_{top}}{\delta \widetilde{C}_{mnp}}\right]_{\widetilde{G}^{loc}_{mnpa}=0}^{\widetilde{G}_{mnpq}=0} = 0, \tag{5.135}$$

giving us another constraint on the curvature corrections in the theory, although solutions should also exist for cases which violate this constraint and hence require a more general analysis that includes a time dependence for $\widetilde{\mathcal{G}}_{mnpa}$.

Now looking at (5.131) and (5.126) we conclude that a four-fold with *negative* Euler characteristic χ_4 may easily accommodate fluxes of the kind (5.131) and simultaneously account for the supersymmetry breaking, although this is not a necessary condition for a solution to exist. In other words, without loss of generality, we can demand

$$\frac{1}{4}\int \sqrt{\tilde{g}}\ \widetilde{\mathcal{G}}_{mnpa}\left(*_8\widetilde{\mathcal{G}}\right)^{mnpa} \ = \ \int H_3 \wedge F_3 \ < 0, \tag{5.136}$$

which in turn can be made consistent with the first equation in (5.110), namely

$$\int d^6x\sqrt{\tilde{g}}\ (H_3)_{mnp}(F_3)^{mnp} = -3\sum_{\alpha_i=0}\langle\mathcal{C}^i_{3,11}\rangle, \tag{5.137}$$

provided $\sum_{\alpha_i=0}\langle\mathcal{C}^i_{3,11}\rangle > 0$. This could be taken as another constraint on the curvature corrections, which applies in the case that $\chi_4 < 0$. A similar constraint would apply for the case $\chi_4 > 0$.

Yet another possible class of solutions are those with vanishing Euler characteristic $\chi_4 = 0$. These solutions could correspond to an internal M-theory eight manifold that is an elliptical fibration of a Calabi-Yau threefold, since the Euler characteristic of the eight manifold is related to the Chern classes of the base by [71]:

$$\chi_4 = 12\int_B c_1(c_2 + 30c_1^2). \tag{5.138}$$

If the base manifold is Calabi-Yau, then $c_1 = 0$, and hence χ_4 vanishes. This, in conjunction with the condition $\tilde{R} = 0$, leads to its own set of solutions, with the modified conditions:

$$\sum_{\{\alpha_i\}=0}\langle\tilde{\mathcal{C}}^{m,\ i}_m\rangle_{\chi=0} \ < \ 0, \tag{5.139}$$

$$\sum_{\{\alpha_i\}=0}\langle\tilde{\mathcal{C}}^{a,\ i}_a\rangle_{\chi=0} \ < \ 0. \tag{5.140}$$

As an interesting corollary, in the *absence* of any curvature corrections and due to (5.103), (5.104) or (5.105), it is *impossible* to get a four-dimensional de Sitter spacetime if the internal six-dimensional base of the M-theory eight-fold is a Calabi-Yau manifold because

$$\tilde{R} = -18h\Lambda. \tag{5.141}$$

We now make a few observations. Note that to stabilize all the complex structure moduli, we will have to switch on G-fluxes in the internal manifold. The $\widetilde{\mathcal{G}}_{mnqa}$ components are the ones that will do the required job for us. However due to the background constraint (5.131) we cannot allow supersymmetric fluxes. In fact we can extend (5.131), by incorporating the localized fluxes in (5.126) and (5.127), to *full* G-fluxes G_{mnpa}, G_{mnpq} and G_{mnab}. This means, in addition to (5.131) we will have another relation

$$G^{loc} - *_4 G^{loc} \neq 0, \tag{5.142}$$

where $*_4$ is the Hodge star on a four-dimensional surface Σ_4 inside the six-dimensional base of our eight-manifold. Since the localized fluxes are related to the gauge fields on the seven-branes wrapping Σ_4 in type IIB theory, this immediately implies that the gauge fluxes (both the abelian and the non-abelian pieces) will create a D-term potential satisfying the background constraint relations (5.129) and (5.111).

In addition to that, the decomposition (5.122) switches on an FI term from the $H_3 = dB_2$ of $\widetilde{\mathcal{G}}_{mnqa}$ and from the $F_2 = dA$ of $\widetilde{G}^{loc}_{mnqa}$, proportional to

$$\int_{\Sigma_4} \mathcal{F}^- \wedge \mathcal{F}^- \tag{5.143}$$

where $\mathcal{F}^- \equiv \mathcal{F} - *_4\mathcal{F}$ and we have defined $\mathcal{F} \equiv F_2 - B_2$.

Since the background supersymmetry is broken by the G-fluxes, the F-term is explicitly non-zero allowing us to switch on a non-zero D-term in the presence of higher-curvature quantum corrections. The fact that the F-term and D-term are related to each other can be inferred from the decomposition (5.122) where *both* three-form and gauge fluxes in type IIB are sourced by M-theory G-fluxes. This way we take care of the issues raised by [69].[10] Note that in the *absence* of the quantum corrections, this wouldn't have been possible.

Finally, we need to switch on D-brane instantons that would help us stabilize all the Kähler structure moduli, including the volume moduli. As mentioned earlier, we have to make sure that the internal manifold is stabilized at large volume so that the dynamics can be captured by the set of EOMs described above. In the presence of the D-brane instantons higher-curvature terms are automatically generated (some aspect of this will be discussed in Section 5.8). These curvature terms are the last pieces of the link required to satisfy the consistency relations (5.111) or (5.129).

Thus both the fluxes and the curvature corrections are therefore *necessary* consequences of stabilized moduli in this set-up. As such they could lead to a positive cosmological constant solution, and a natural realization of D-term uplifting [34].

5.8 A discussion on the curvature corrections

In this section we discuss in more detail the possible origins for the higher-order curvature corrections[11] we have argued might allow for construction of de Sitter vacua in IIB compactifications. While our calculations were done in M-theory, it is interesting to first look at the corrections that can appear in type IIB string theory. These terms can be sourced by tree- and loop-level n-graviton scattering amplitudes, or equivalently loop corrections to the underlying σ-model, and are also induced by D-instanton corrections. The general form of these corrections is given by (adopting the notation of [72], combined with [63] but with the substitution $s = (m+6)/4$):

$$(\alpha')^{n-m+1} t_{m,n} Z_m^{(w,w')} D^{2m} R^n \tag{5.144}$$

[10] It will be interesting to compare our results with the ones in [70] regarding D-term uplifting.

[11] We will restrict ourselves to R^n corrections as these have been studied in more detail than the G^n corrections. For an analysis of G^n corrections, the readers may refer to [42, 43].

where $t_{m,n}D^{2m}R^n$ is the contraction of $2m$ covariant derivatives and n Riemann tensors with a tensor $t_{m,n}$. The coefficient $Z_m^{w,w'}$ is an eigenfunction of the Laplace operator on the fundamental domain of $SL(2,\mathbb{Z})$, with modular weight (w,w'). This coefficient can be written as an Eisenstein series [63], and is necessary for $SL(2,\mathbb{Z})$ invariance of the corrections to the action.

The lowest-order correction can be calculated from 4-graviton scattering; see for example [64] in type II and [65] in Heterotic, which induces a $D^0\ R^4$ correction at both tree level (at order $(\alpha')^3$) and at the one-loop level. In the calculation by Gross and Witten [65], this led to a gaussian path integral that can alternatively be written as a contraction of four copies of the Riemann tensor with two copies of a rank-8 tensor denoted t_8. This allows one to write the correction as (equations 10 and 11 of Gross and Witten):

$$\int \mathrm{d}\psi_L^\alpha \mathrm{d}\psi_R^\beta \exp\left[\bar{\psi}_L^\alpha \Gamma_{\alpha\beta}^{\mu\nu}\psi_L^\beta \bar{\psi}_R^{\alpha'}\Gamma_{\alpha'\beta'}^{\sigma\tau}\psi_R^{\beta'} R_{\mu\nu\sigma\tau}\right], \tag{5.145}$$

or in terms of the t_8 tensor:

$$t^{\mu_1\mu_2\ldots\mu_8}t^{\nu_1\nu_2\ldots\nu_8}R_{\mu_1\mu_2\nu_1\nu_2}R_{\mu_3\mu_4\nu_3\nu_4}R_{\mu_5\mu_6\nu_5\nu_6}R_{\mu_7\mu_8\nu_7\nu_8}, \tag{5.146}$$

with the t_8 tensor defined by

$$\sqrt{\det \Gamma^{\mu\nu}F_{\mu\nu}} = t^{\mu_1\mu_2\ldots\mu_8}F_{\mu_1\mu_2}F_{\mu_3\mu_4}\ldots F_{\mu_7\mu_8}. \tag{5.147}$$

The above correction is often written in the literature as simply $t_8t_8R^4$. Another approach to calculating this correction is to consider loop corrections in the sigma model (see for example [66]), where an n-loop effect will lead to an R^n correction that is order $(\alpha')^n$ in the corresponding string theory. Collecting all the terms at order R^4 yields a correction of the form:

$$\left(\frac{1}{8}\epsilon_{10}\epsilon_{10} - t_8t_8\right)R^4, \tag{5.148}$$

where ϵ_{10} is the rank-10 totally anti-symmetric tensor .

One might also wonder if there are R^2 or R^3 terms. The sigma model analysis does not produce these terms, which would indicate that type II theories are protected from α'^2 and α^3 corrections, as shown in the sigma model in [67]. This was also done in the context of type I, II and heterotic string theory in [68], which confirmed the result that R^2 and R^3 corrections do not appear. One can also check that R^5 terms do not arise, and in fact the next corrections coming from the tree-level graviton scattering are D^2R^4, D^2R^5, and R^6, all at order $(\alpha')^5$ (see table I of [72]). At the loop level, there has been recent work [73–75] showing that perhaps string loop corrections at order $g_s^2(\alpha')^2$ can become important in a certain class of compactifications (dubbed the Large Volume Scenario).

Another contribution comes from calculating the graviton scattering amplitude in a D-instanton background, as was done by Green and Guterperle [76], which gives an extra contribution to $Z_m^{(w,w')}$ that is neccessary for the correction to be $SL(2,\mathbb{Z})$ invariant. The coefficient for the D^0R^4 correction has modular weight $(w,w') = (0,0)$, and is given by (equation 1.15 of [63] with $s = 3/2$, or in our notation, $m = 0$):

$$\begin{aligned} Z_0 = \quad & 2\zeta(3){C^{(0)}}^{3/2} + 8\zeta(3){C^{(0)}}^{-1/2} \\ & +4\pi \sum_{k\neq 0} \mu(k,3/2) \exp\left[-2\pi(|k|e^{-\phi} - ike^{-\phi})\right] \sqrt{|k|} \left(1 + \frac{3}{16\pi|k|C^{(0)}} + ...\right), \end{aligned} \tag{5.149}$$

where $C^{(0)}$ and ϕ are the axion and dilaton. The first term on the RHS is the tree-level correction, while the second term is the 1-loop correction. The set of terms on the second line is an infinite set of D-instanton corrections, with the function $\mu(k,3/2)$ defined as in Appendix A of [63].

The picture in M-theory is slightly simpler, as there is only one curvature superinvariant. A review of the corrections to M-theory supergravity, as well as the supersymmetrization, can be found in [77], while the detailed derivations can be found in [78] and [79]. A feature of the M-theory picture that is fairly well understood is the necessity of an additional Chern-Simons term to cancel the 5-brane anomaly, via anomaly inflow. This term takes the form

$$C \wedge X_8, \tag{5.150}$$

where X_8 is built out of R^4. As this term includes a factor of the M-theory 3-form flux, it will contribute to the equation of motion of the fluxes.

A key feature of these corrections is that the form of the contraction conspires to choose only the Weyl part of the Riemann tensor, such that the corrections vanish on manifolds with vanishing Weyl tensor. This was shown explicitly by Banks and Green in [80], where they considered $AdS_5 \times S^5$. This is great news for AdS/CFT, since the correspondence is protected from loop corrections. However, it makes the search for scenarios where corrections may be important a non-trivial exercise. One possibility for finding non-negligible corrections is to consider Calabi-Yau manifolds, and indeed this is the internal manifold used in the 4D effective picture of these corrections in Kahler Uplifting [35, 40]. However, this introduces a new difficulty: many Calabi-Yau manifolds can not be given an explicit metric – for example the explicit realization of Kahler uplifting in [40] is done on $\mathbb{CP}^{11169}$.

5.9 Conclusion

This paper has been a close examination of de Sitter solutions in Type IIB string theory, from the perspective of the 10-dimensional equations of motion (and the corresponding 11-dimensional M-theory equations). We have reached two key conclusions:

1. By applying the Gibbons-Maldacena-Nunez No-Go Theorem [17–19] to localized static sources we have found that the inclusion in IIB supergravity of Dp-branes, anti Dp-branes, Op-planes, and by extension any linear combination thereof, does not lead to positive curvature in the 3+1 non-compact directions.

2. The addition of curvature corrections, sourced by D-instantons as well as tree and loop-level graviton scattering, may lead to a de Sitter solution in the 3+1 non-compact directions, although an explicit construction of this would require specifying a metric on the internal manifold as well as a subset of correction

terms to consider. Furthermore, this solution naturally leads to compactification with broken supersymmetry, all moduli stabilized, and the generation of a D-term in the scalar potential of the 4d effective field theory.

The first result is a fairly simple extension of the analysis performed by Maldacena-Nunez [19], and Giddings, Kachru, Polchinski [58], among others. Our assumptions in deriving this were limited to demanding (i) maximal symmetry in the 3+1 dimensions, as well as (ii) positive curvature in the 3+1 dimensions. Since we only consider time-independent matter configurations, the 3+1 dimensional non-compact spacetime we are looking for is 'pure' de Sitter, as opposed to quasi-de Sitter as is usually considered in cosmology. However, to construct any 3+1 dimensional positive curvature geometry, the stress-energy tensor must satisfy the condition (2.8) regardless of the symmetry, and in particular regardless of time dependence.

Note that there are many existing proposals which we have not considered, for example IIA on nilmanifolds [81], IIA on solvmanifolds [82], and non-geometric fluxes [83]. These proposals should also be subject to condition (2.8).

The second result is a non-trivial check that curvature corrections do indeed evade the No-Go theorems. In this calculation we have used an ansatz for the effective stress-energy tensor induced by the curvature corrections, which we view as an appropriate way to proceed given the freedom to set the internal manifold as well as the complicated (and not completely known) form of the curvature corrections.

A worthy question at this point would be the sensitivity of our second result to the form of the ansatz, as it is entirely possible that some choices of internal manifold do not lead to curvature corrections that can be parametrized in this way. Thus a conservative restatement of our second result would be as follows: given a class of internal manifolds that allow the time dependence of the curvature correction to be isolated from other contributions, there do exist de Sitter solutions provided a set of consistency conditions (5.119) - (5.120) is satisfied. This hints at interesting further work, to clarify the consistency of our claims with the work of Sethi et al. [20] which found that such corrections in Heterotic theory do *not* lead to dS solutions.

Upon studying the dS solution obtained via curvature corrections, we uncovered a number of interesting features. Solutions exist for any choice of the Euler characteristic of the internal manifold, including an elliptic fibration of a Calabi-Yau threefold. Furthermore, this setup generically leads to non self-dual fluxes, which break supersymmetry, and induce a D-term in the scalar potential, suggesting that this construction may be a realization of D-term uplifting [34]. The moduli of this setup can be fully stabilized: the complex structure moduli are fixed by the fluxes, while the Kähler moduli are stabilized by the D-instantons, which in turn source the curvature corrections. Hence our analysis indicates that curvature corrections *can* do the job at hand.

This work has opened up several directions for future research. One option, motivated by the desire for a deeper understanding of string theory, is to continue the investigation of de Sitter solutions, using dualities to relate the solutions in different string theories. This has the potential to clarify subtleties of dualizing non-BPS states, and to allow one to 'map out' the space of dS vacua in string theory.

An alternative way forward is to push this work closer to cosmology, and in particular, inflationary cosmology. While the full 10d equations do not lend themselves to model building, this approach does provide a clear path to studying compactifications with a time-dependent scalar curvature ('quasi-dS'). The appeal of this option lies in building self-consistent embeddings of inflationary cosmology in string theory, with the (albeit ambitious) goal of teasing out distinctive signatures of string theory in the sky. As has happened before, it may be that effects from a full 10-dimensional construction result in observational signatures which do not arise in the effective field theory approach.

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