THE CAPACITOR MOTOR

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by

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PREFACE

In spite of its simple construction and operation, the single phase capacitor motor requires rather complicated theoretical treatments such as the double-revolving-field theory and the cross-field theory.

This paper presents the circuit studies of the capacitor motor by employing the methods of three-phase symmetrical components and the two-phase right-angle components. These methods are recognized as the best tools in handling unbalanced three-phase and two-phase electric circuits. The adoption of these methods enables us to make use of the classical equivalent circuit for the three-phase induction motor in its circuit analysis and to present the studies of the exciting and rotor circuits in a clearer and more simple manner.

Part I of this paper presents the studies of the single-phase induction motor by the method of three-phase symmetrical components by considering the single-phase induction motor as a three-phase induction motor with unbalanced three-phase voltage supply. An approximate equivalent circuit and a circle diagram have been developed for calculating the slip-torque relation and the pull-out slip. Also included are the determination of machine constants from tests, the performance of a capacitor-start motor from load test and the calculation of the equivalent circuit.

In Part 2 is a complete study of the capacitor motor, with symmetrical and unsymmetrical stator windings, under unbalanced operation, which includes the starting characteristics, the construction of the positive- and negative-sequence equivalent circuits, vector diagram and the loci of vector currents. Also included is a calculation sheet suggested for the performance calculation of capacitor motor. Experiments have been done both on a capacitor-start motor operated as a capacitor-run motor with different running capacitors and on a one-value capacitor motor.

This paper concludes with Part 3 in which the design problem of the capacitor motor are discussed. Discussions are based on the circuit studies in the preceeding parts. A capacitor motor with design data was available for the purposes of experiments and in discussing the design of capacitor motor for balanced operation. The equal-volt-ampere method of capacitor motor design is also included.

The writer wishes to express his sincere thanks to Prof. W. H. Schippel of McGill University who kindly supplied the capacitor motor and its design data and gave many valuable suggestions to this work. Thanks are also given to Messrs. J. P. Grant, J. W. Clarke and D. Newman for their kind assistances in proof-reading.

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THE CAPACITOR MOTOR

PART I

Circuit Studies of Single-Phase Induction Motor by the Method of Three-Phase Symmetrical Components.

1. The Method of Three-Phase Symmetrical Components.

The basic principle of the method of three-phase symmetrical components is the resolution of any particular set of vectors, current or voltage, in the three-phases of an unbalanced electric circuit into three sets of balanced three-phase vectors, i.e., the positive-sequence, negative-sequence and zero-sequence symmetrical components. Mathematically, this can be expressed as follows:

$$C_a = C_0 + C_1 + C_2$$
$$C_b = C_0 + a^2 C_1 + a C_2$$
$$C_c = C_0 + a C_1 + a^2 C_2$$

$$C_0 = \frac{1}{3}\left(C_a + C_b + C_c \right)$$
$$C_1 = \frac{1}{3}\left(C_a + a C_b + a^2 C_c \right)$$
$$C_2 = \frac{1}{3}\left(C_a + a^2 C_b + a C_c \right)$$

in which C is the current or voltage vector of an unbalanced circuit. Subscripts a, b, and c denote the phases. Vectors with subscript 1, 2 and 0 are the positive-, negative- and zero-sequence components, respectively. The operator "a" appearing in the above expressions has the following significances:

$$a = \underline{|120^\circ} = -.5 + j.866$$

$$a^2 = \underline{|240^\circ} = -.5 - j.866$$

$$a^3 = 1$$

2. Application of Three-Phase Symmetrical Components to Single-Phase Induction Motor.

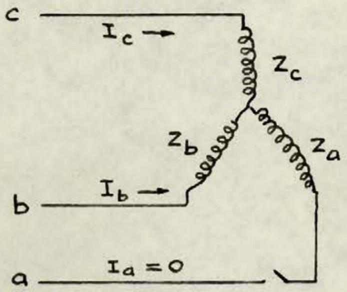


Fig. 2-1.

Consider the unbalanced three-phase circuit shown in Fig. 2-1 where Z_a, Z_b and Z_c are three identical impedances directly across a balanced supply line with lina a opened. Such a condition represents the unbalanced operation of a three-phase induction motor or a single-phase induction motor if Z_b and Z_c in series are considered as the single-phase winding of the motor.

Since there is no neutral connection in the circuit shown in Fig. 2-1, the zero-sequence current and therefore the zero-sequence phase voltage will be zero. We may write:

$$I_a = 0 \; ; \qquad I_b = I \; ; \qquad I_c = - I_b = -I;$$

$$I_0 = 0 \; ; \qquad I_1 = 1/3 \; (I_a + I_b + I_c)$$
$$= 1/3 \; (a - a^2) \; I = \frac{j\sqrt{3}}{3} I. \qquad (2-1)$$

$$I_2 = 1/3 \; (I_a + a^2 I_b + a I_c)$$
$$= 1/3 \; (a^2 - a) \; I = \frac{-j\sqrt{3}}{3} I. \qquad (2-2)$$

Thus, $$I_1 + I_2 = 0 \quad \text{or,} \quad I_1 = -I_2 \; . \qquad (2-3)$$

The phase voltages are:

$$V_a = 0 \; ; \qquad V_b = a^2 V_1 + a V_2 \; ;$$
$$V_c = a V_1 + a^2 V_2 \; .$$

The voltage across b and c is

$$V_A = V_c - V_b = (a - a^2)\cdot(V_1 - V_2)$$

$$= j\sqrt{3}\ (V_1 - V_2),$$

or,

$$V_1 - V_2 = -j\sqrt{3}\ V_A/3 \ . \tag{2-4}$$

The above mathematical expressions show how the single-phase induction motor can be treated as a three-phase induction motor with unbalanced three-phase voltage supply which consists of both positive-sequence and negative-sequence components.

Another interesting interpretation of the above equations is that the single-phase induction motor under running condition may be considered as a three-phase induction motor operated with two balanced three-phase voltage supplies of unequal magnitudes and of opposite phase rotations. Except at standstill, these two voltages produce two magnetic fields of different magnitudes revolving at synchronous speeds. As we have shown that the positive- and negative-sequence currents through the motor are equal in magnitude, an important relation can be written as follows:

$$\frac{V_1}{V_2} = \frac{-Z_1}{Z_2}$$

where Z_1 and Z_2 are the impedances to the positive-sequence and negative-sequence currents of the motor, respectively.

Before further discussing the operating theory as well as the performance of the single-phase induction motor, the basic concept of the positive-sequence and negative-sequence equivalent circuits for the single-phase induction motor must first be introduced.

3. The Positive-Sequence Equivalent Circuit.

It is understood that the positive- and negative-sequence impedances are identical for the static transformers and in some transmission circuits. For the single-phase induction motor, these sequence impedances are not constant in value but vary with the slip, and are equal when the slip is unity.

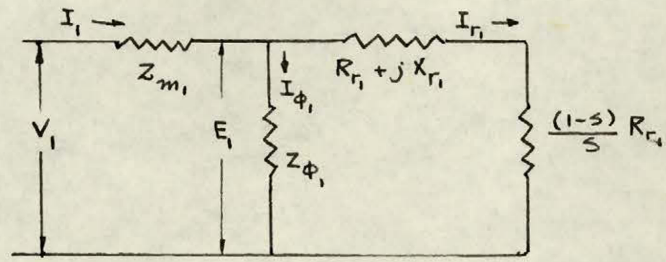


Fig. 3-1.

Fig. 3-1. shows the positive-sequence equivalent circuit of the single-phase induction motor. V_1 is the positive-sequence applied voltage. E_1 is the voltage induced in the stator winding by the air gap flux, a resultant flux produced by the stator m.m.f. and the rotor m.m.f.. Under normal conditions, the rotor rotates at a speed near to the synchronous r.p.m. , say, (1 - s)·syn. r.p.m., while the flux produced by the induced rotor current I_r has a relative speed of s times the synchronous speed. Therefore, the flux produced by I_r actually rotates at synchronous speed with respect to the stator or, in other words, the fluxes produced by I_r and I_1 are in phase. The voltage induced in the rotor will then be directly proportional to the slip and to the turn-ratio between the stator and rotor windings; and will be in phase with E_1. Referring to Fig. 3-1. and assuming a turn-ratio of unity, one can write:

$$sE_1 = I_{r_1}\,(R_{r_1} + j\,s x_{r_1})$$

$$E_1 = I_{r_1}\,\left(\frac{R_{r_1}}{s} + j\,x_{r_1}\right) = I_{r_1} Z_{r_1}$$

$$V_1 = I_1 Z_1$$

$$Z_1 = Z_{m_1} + \frac{Z_{\phi_1} Z_{r_1}}{Z_{\phi_1} + Z_{r_1}}$$

where Z_1 = positive-sequence impedance of the motor.

Z_m = positive-sequence leakage impedance of main winding.

$Z_{r_1} = R_{r_1} + j\,X_{r_1}$, the positive-sequence rotor impedance in terms of main winding.

Z_{ϕ_1} = positive-sequence exciting impedance.

It should be noticed that the resistance in the rotor circuit can further be resolved into two parts, namely: R_{r_1} and $\left(\frac{1-s}{s}\right)R_{r_1}$ which represent the rotor copper loss and the shaft load when multiplied by the square of the rotor current. At standstill, $s = 1$, the shaft load is zero, the motor is equivalent to a short circuited transformer.

4. The Negative-Sequence Equivalent Circuit.

As mentioned previously the negative-sequence impedance of static transformer is the same as the positive-sequence impedance. This would indicate that the difference between the positive- and negative-sequence impedances of the single-phase induction motor is in the rotor circuit.

Consider now that a balanced three-phase voltage of negative-sequence alone is applied to a three-phase induction motor, i.e., by interchanging any two leads of the three-phase supply that is considered as positive-sequence. The magnetic field and therefore the rotor will rotate in the direction opposite to that caused by the positive-sequence applied voltage. It can readily be seen that at synchronous speed the slip is zero when referred to the negative direction of rotation. At standstill, the slip is unity for either sequence. The negative-sequence equivalent circuit can now be constructed by replacing the slip s in the positive-sequence equivalent circuit by the quantity of(2 - s)as shown in Fig. 4-1. The negative-sequence impedance of the motor can now be written as :

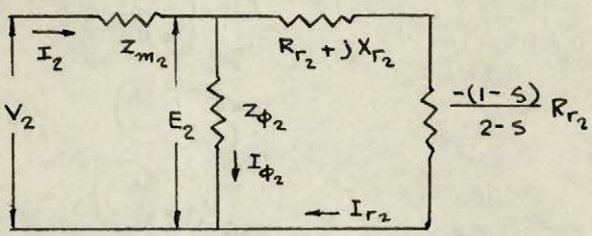


Fig. 4-1.

$$E_2 = I_{r_2} \left(\frac{R_{r_2}}{2-s} + j X_{r_2} \right)$$

$$V_2 = I_2 Z_2$$

$$Z_2 = Z_{m_2} + \frac{Z_{\phi_2} Z_{r_2}}{Z_{\phi_2} + Z_{r_2}}$$

As in the positive-sequence equivalent circuit, the resistance $R_{r_2}/2\text{-}s$ can be resolved into R_{r_2} and $-(1 - s)R_{r_2}/2\text{-}s$, which, when multiplied by the rotor current squared, represent the rotor copper loss and the shaft load, respectively. The minus sign of the latter indicates that power is being absorbed by the shaft at slips from zero to unity. This can be visualized when the positive-sequence field is taken off while the motor is operating, an equal amount of power must be supplied from an outside source to keep the motor running in the direction opposite to that of the negative-sequence revolving field. It is because of this characteristic that the single-phase induction motor can never attain synchronous speed even if there are no frictional and windage losses.

5. <u>The Performance of the Single-Phase Induction Motor.</u>

Referring to the equations given in art. 2 and the positive- and negative-sequence equivalent circuits described in art. 3 and 4, a complete equivalent circuit can now be set up as shown in Fig. 5-1., from which the complete performance of the single-phase induction motor can be calculated.

It should be noticed that all constants appearing in Fig. 5-1. are the equivalent impedances per phase of a three-phase induction motor. It would then be necessary to use one half of the actual values of the single-phase motor impedances when applied to this equivalent circuit. It is also important to point out that the sequence voltages and currents in art. 2 are the components of the phase voltage and the line current of a three-phase circuit. However, an examination on Eq. 2-3 and 2-4 will show that the factor $j\sqrt{3}/3$ can be eliminated when applied to a single-phase induction motor; the line current will be equal to the positive- or negative-sequence current in magnitude, and the applied voltage will be the difference of the positive- and negative-sequence voltages.

In what follows, the subscripts 1 and 2 will be eliminated for those impedances which are identical in both positive and negative-sequence equivalent circuits. From Fig. 5-1., we may write:

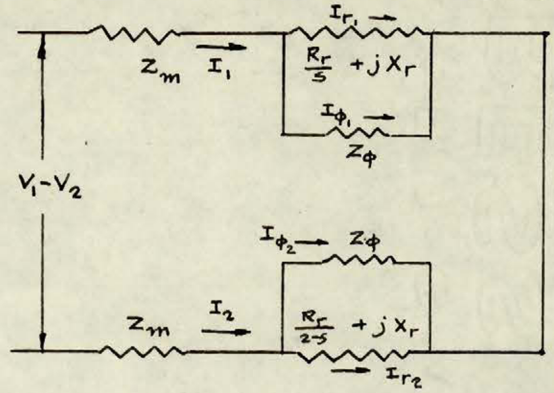


Fig. 5-1. EQUIVALENT CIRCUIT OF SINGLE-PHASE INDUCTION MOTOR.

$$I_1 = \frac{V_1 - V_2}{Z_1 + Z_2} = \frac{V}{2Z_m + \dfrac{Z_\phi Z_{r_1}}{Z_\phi + Z_{r_1}} + \dfrac{Z_\phi Z_{r_2}}{Z_\phi + Z_{r_2}}}$$

$$I_{r_1} = \frac{I_1 Z_\phi}{Z_\phi + Z_{r_1}} \quad ; \qquad I_{\phi_1} = \frac{I_1 Z_{r_1}}{Z_\phi + Z_{r_1}} \quad ;$$

$$I_{r_2} = \frac{-I_1 Z_\phi}{Z_\phi + Z_{r_2}} \quad ; \qquad I_{\phi_2} = \frac{-I_1 Z_{r_2}}{Z_\phi + Z_{r_2}} \quad .$$

The internal torque in synchronous watts of either sequence is the product of the rotor current squared and the rotor resistance of the same sequence circuit. A negative sign is attached to the negative-sequence torque to show that it is in the direction opposite to that developed by the positive-sequence rotor current. They are:

$$T_1 = I_{r_1}^2 \frac{R_r}{s}$$

$$T_2 = -I_{r_2}^2 \frac{R_r}{2-s}$$

The resultant internal torque and the resultant shaft output are respectively equal to:

$$T = T_1 + T_2 = \left(\frac{I_{r_1}^2}{s} - \frac{I_{r_2}^2}{2-s} \right) R_r \qquad \text{SYN. WATTS.}$$

$$P_{SH.} = P_{SH_1} + P_{SH_2} = \left[\left(\frac{1-s}{s} \right) I_{r_1}^2 - \left(\frac{1-s}{2-s} \right) I_{r_2}^2 \right] R_r \qquad \text{SYN. WATTS.}$$

The torque developed at the pulley in ounce-ft. can be obtained by multiplying T by 112.6/r.p.m., or P by (1 - s), less the frictional and windage losses.

The total copper loss and iron loss are:

$$P_{LOSS} = 2 I_1 R_m + (I_{r_1}^2 + I_{r_2}^2) R_r + (I_{\phi_1}^2 + I_{\phi_2}^2) R_\phi .$$

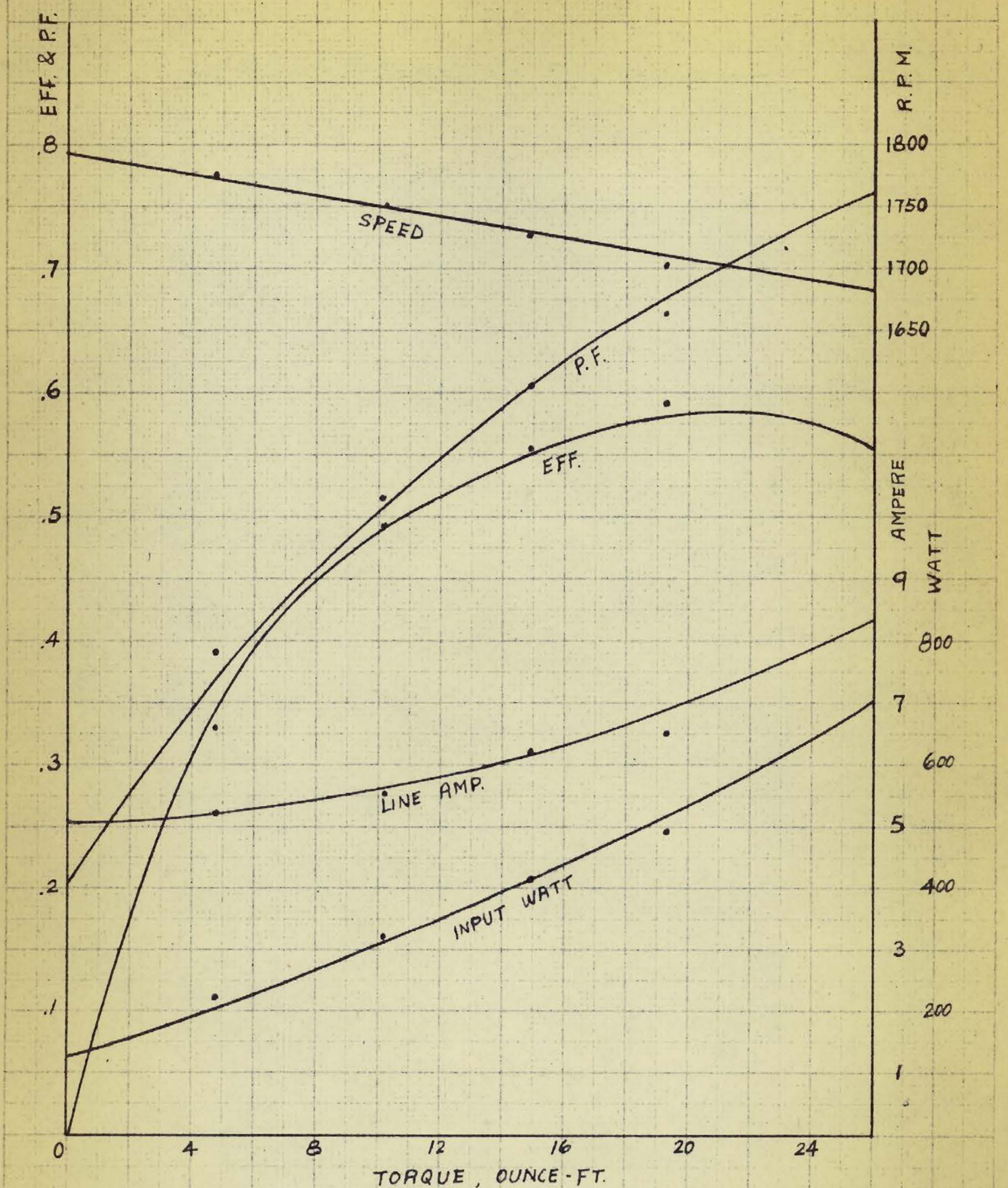


Fig. 5-2. PERFORMANCE OF A CAPACITOR-START MOTOR FROM EXPERIMENTS. 110-V 1/4 H.P. 1800 R.P.M.

The efficiency of the motor can be calculated when the output and the total losses of the motor are known. The power factor of the motor can easily be determined from the cosine of the angle by which the line current lags the applied voltage.

Complete performance characteristics of a capacitor-start motor, 110-v., 1/4 hp., 1800 r.p.m. from actual load tests are given in Fig. 5-2.

6. Determination of Machine Constants from Tests.

In order to illustrate the calculation of the performance of the single-phase induction motor from the equivalent circuit, and to check the calculated performance by experimental results, the machine constants must first be determined from tests, provided that the design data are not available.

Fig. 6-1. shows the experimental results of the no-load and locked-rotor tests made on the above mentioned motor. It should be noticed that one of the stator windings must be open-circuited during the locked-rotor tests so that the impedance measured will be the actual locked-rotor impedance of the winding under test. The capacitor should be by-passed while measuring the locked-rotor impedance of the auxiliary winding. The turn-ratio between the main and auxiliary windings may be determined from the square root of the ratio between the leakage reactances of the main and auxiliary windings. The following are the locked-rotor impedances and the effective turn-ratio determined from tests and will be used later in the performance calculation.

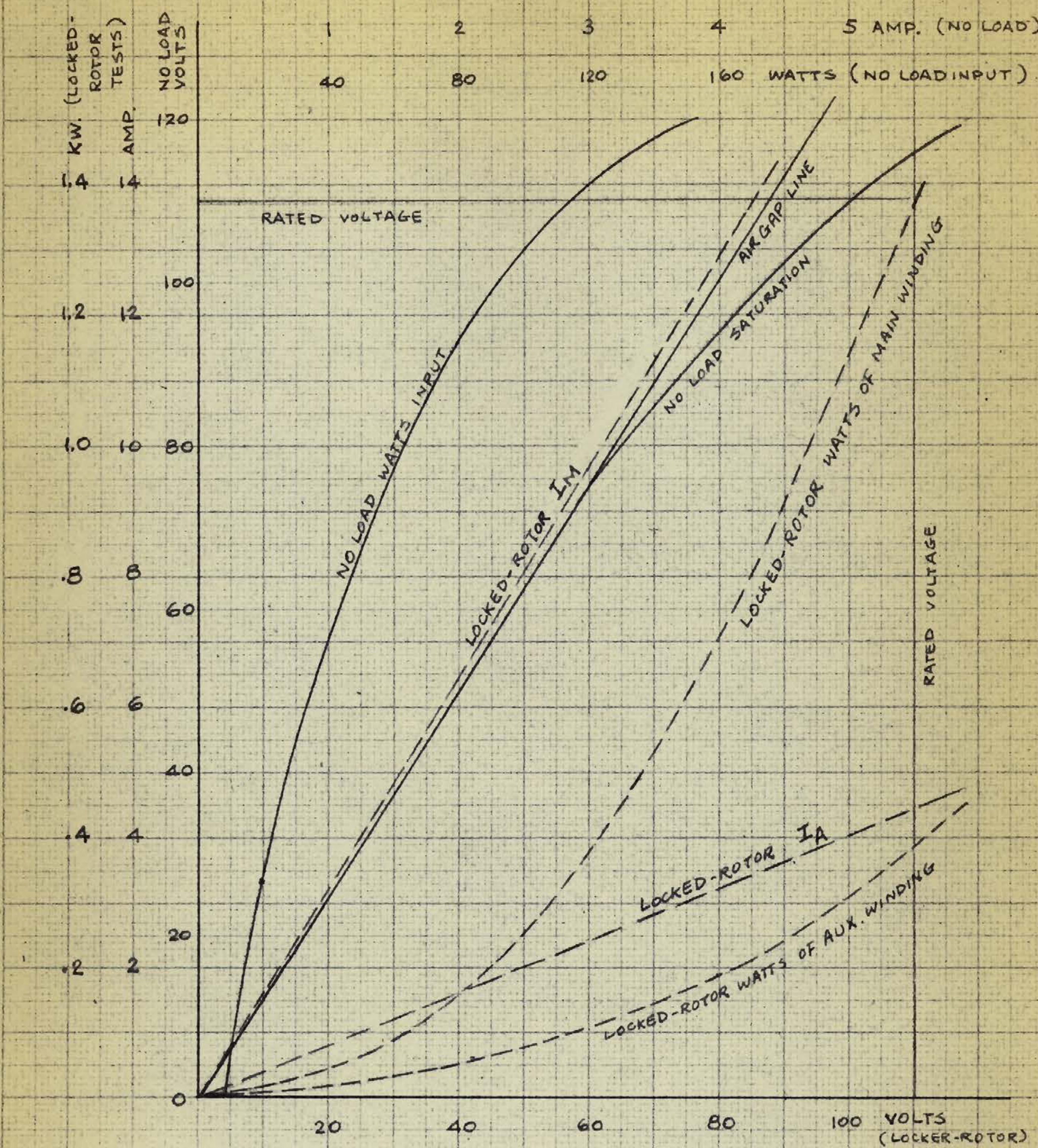


Fig. 6-1. NO LOAD AND LOCKED-ROTOR TESTS OF A CAPACITOR-START MOTOR. 110-V 1/4 HP. 60∿ 1800 R.P.M.

$$Z_M = 6.22 \angle 51.6 = 3.86 + j4.88$$

$$Z_A = 24.9 \angle 43.2 = 18.2 + j17.1$$

$$a = \sqrt{\frac{4.88}{17.1}} = 1/1.87 = .535$$

The ohmic resistance of the main winding measured immediately after a load test is found to be 1.64 ohm, The rotor resistance in term of the main winding is therefore equal to 3.86-ohm less 1.64-ohm, or 2.22 ohm.

Like in the case of three-phase induction motor, the lekage reactance of the main winding and of the rotor are assumed equal. Since the locked-rotor reactance is the sum of the locked-rotor reactance of the positive- and negative-sequence equivalent circuits of the motor, we have, therefore,

$$X_m = X_r = (1/4)(X_M) \qquad (6-1)$$

which gives 1.11 for the above motor.

The exciting impedance Z_ϕ may be calculated from the following expression:

$$Z_\phi = R_\phi + jX_\phi = \frac{\text{IRON LOSS IN WATTS}}{I_{N.L.}^2} + j\frac{E_1}{I_{N.L.}} \qquad (6-2)$$

in which the iron loss can be determined from the no-load input less the copper loss and mechanical losses. However, the induced voltage E_1 of the positive-sequence equivalent circuit has to be estimated and ranges from eighty to ninety per cent of the applied voltage V. More satisfactory results can be obtained by solving the equivalent circuit shown in Fig. 6-2. which represents the no load condition of the single-phase induction motor. At no load, the slip is practically zero so that the positive-sequence rotor current I_{r_1} and the negative-sequence exciting current I_{ϕ_2} may be considered as zero. From Fig. 6-2.,

$$E_1 \doteq V - I_{N.L.}\left(Z_m + \frac{R_r}{2} + jX_r \right) \tag{6-3}$$

$$Z_\phi = \frac{E_1}{I_{N.L.}} \tag{6-4}$$

By substituting the known constants into Eq. 6-3. and 6-4.,

$$E_1 = 110 - 5.04 \angle{-76.6} \left(1.64 + j\,2.44 + \frac{1.11}{2} + j1.22\right)$$
$$= 110 - 21.5 \angle{17.6} = 89.7 \angle{4.2^\circ}$$

$$Z_\phi = \frac{89.7}{5.04} \angle{4.2 + 76.6} = 17.84 \angle{80.8} = 2.85 + j17.6$$

The equivalent circuit is now completed as shown in Fig. 7-1.

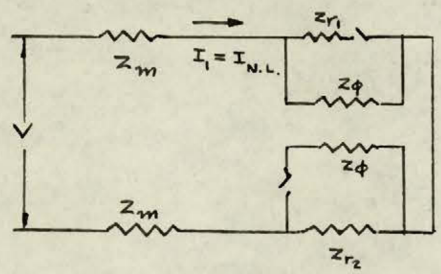


Fig. 6-2. EQUIVALENT CIRCUIT FOR SINGLE-PHASE INDUCTION MOTOR AT NO LOAD.

It is understood that the resistance R_ϕ in the exciting circuit multiplied by the exciting current squared gives the total iron losses, including the teeth surfaces and tooth pulsation losses due to the harmonics.

7. Calculation of the Equivalent Circuit and the Vector Diagram of the Single-Phase Induction Motor.

Having determined the machine constants, the performance and the speed-torque characteristics can readily be calculated. A calculation sheet giving the complete solution of the equivalent shown in Fig. 7-1. is given on P. 12-a, in which a speed of 1725 r.p.m. is calculated. The calculated results for this particular speed and for some others which are shown in solid points in Fig. 5-2. are ssufficiently close to the experimental values.

A clear picture of the equivalent circuit of the single-phase induction motor can be obtained from the studies of the vector diagram shown in Fig. 7-3.a for the no load condition and Fig. 7-3b, for the full load condition. No explaination is deemed necessary as they are clearly labelled and can readily be drawn once the calculation sheet suggested on P. 12-a is completed.

The vector diagram at no load is drawn from the solution of the a pproximate equivalent circuit at no load shown in Fig. 6-2.

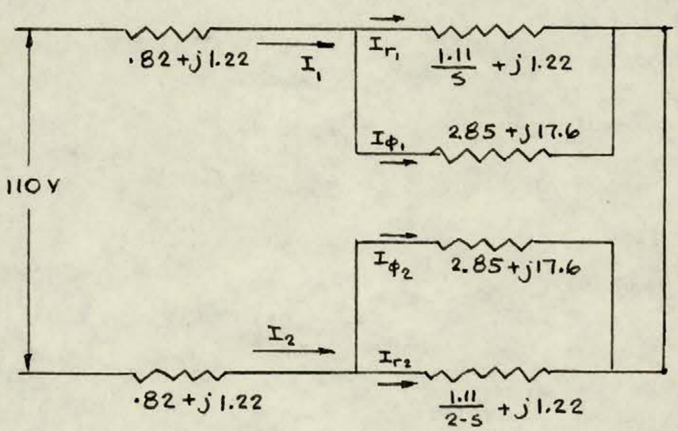
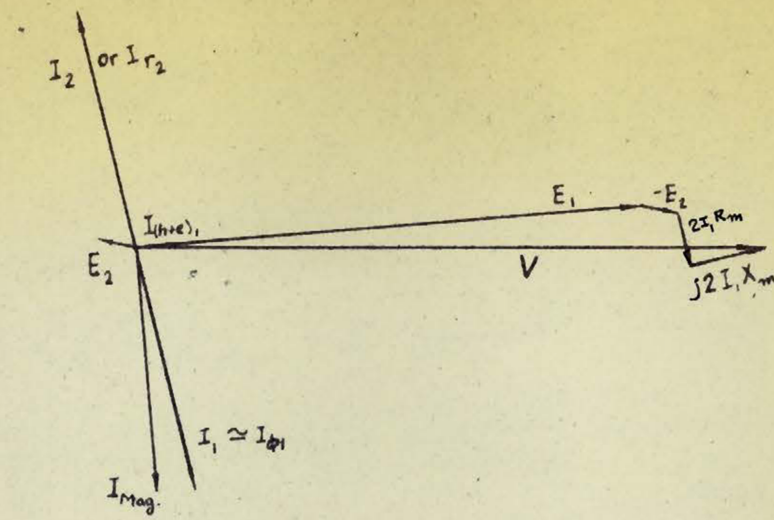


Fig. 7-1. Equivalent circuit of a capacitor-start motor, 110-v., 1/4 hp., 1800 r.p.m. with constants determined from no-load and locked-rotor tests.

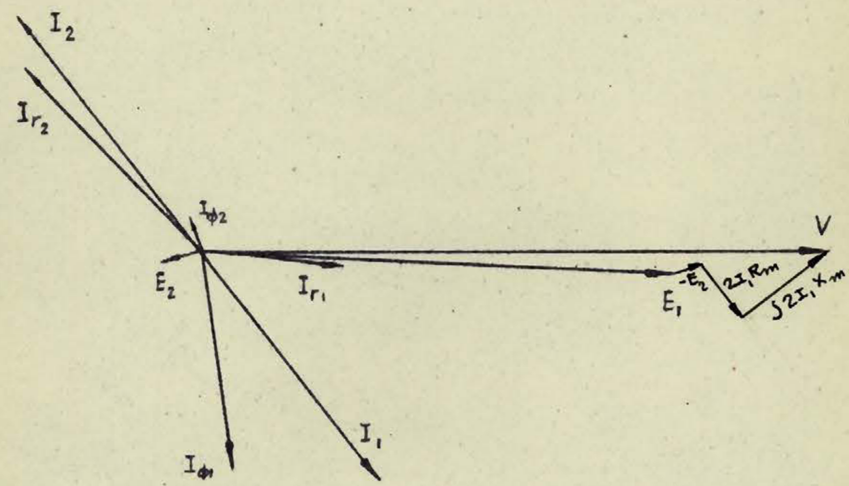
SINGLE PHASE INDUCTION MOTOR CHARACTERISTICS				MOTOR RATING: 1/4 HP. CAP. START. 1800 R.P.M.	
V	110	a	$1/1.87 = .535$	R_r	1.11 ; 2.22 FOR T_s
Z_m	$.82 + j1.22$	a^2		X_r	$j1.22$
Z_M	$3.86 + j4.88 = 6.22\angle 51.6$	Z_C	$2 - j25$	S	.0416
Z_a		Z_S	$20.2 - j7.9 = 21.7\angle -21.4$	P_{f+w}	8
Z_A	$18.2 + j17.1$	Z_ϕ	$2.85 + j17.6 = 17.84\angle 80.8$		

1	R_r/S	26.7	7	$R_r/(2-S)$	.567
2	(1) $+ jX_r$	$26.75\angle 2.62$	8	(7) $+ jX_r$	$1.35\angle 65.1$
3	(2) $+ Z_\phi$	$35.1\angle 32.5$	9	(8) $+ Z_\phi$	$19.1\angle 79.7$
4	(2) $\times Z_\phi$	$477\angle 83.42$	10	(8) $\times Z_\phi$	$24\angle 145.9$
5	(4)/(3)	$13.57\angle 50.92$	11	(10)/(9)	$1.26\angle 66.2$
6	$Z_1 = Z_m +$ (5)	$9.37 + j11.75$	12	$Z_2 = Z_m +$ (11)	$1.37 + j2.37$
13	$Z_1 + Z_2$	$10.74 + j14.12 = 17.7\angle 52.7$			
14	$I_1 = -I_2 = V/$(13)	$6.22\angle -52.7$			
15	$I_{r1} =$ (14) $\times Z_\phi/$(3)	$3.16\angle -4.7$	20	$I_{r2} =$ (14) $\times Z_\phi/$(9)	$5.8\angle -47.7$
16	$I_{\phi 1} =$ (14) $\times$ (2)/(3)	$4.73\angle 44.68$	21	$I_{\phi 2} =$ (14) $\times$ (8)/(9)	.438
17	$T_1 =$ (15)$^2 \times$ (1)	267	22	$T_2 = -$(20)$^2 \times$ (7)	-19.1
18	$E_1 =$ (14) $\times$ (5)	$84.3\angle -2$	23	$E_2 =$ (14) $\times$ (11)	7.84
19	LOSS = (14)$^2 R_m$ + (15)$^2 R_r$ + (16)$^2 R_\phi$	106.6	24	LOSS = (14)$^2 R_m$ + (20)$^2 R_r$ + (21)$^2 R_\phi$	69.6
25	$T =$ (17) + (22)				248
26	$P_{OUT} =$ (25) $\times (1 - S) - P_{f+w}$				229
27	$T = 112.6 \times$ (26)/R.P.M.				14.9
28	$P_{IN} = V \times$ (14)$_{REAL}$				413
29	P_{OUT} + LOSSES = (26) + (19) + (24) + P_{f+w}				230
30	EFF. = (26)/(29)				55.6 %
31	P.F.				.604
	STARTING CHARACTERISTICS				
32	$I_M = V/Z_M$	$17.7\angle -51.6 = 11 - j13.9$	33	$I_A = V/Z_S$	$5.08\angle 21.4 = 4.72 + j1.85$
34	$I_L =$ (32) + (33)	$15.72 - j12 = 19.8\angle 37.4$	35	P.F.	.796
36	$P_{IN} = V \times$ (34) $\times$ (35)	1730	37	$\sin\theta_{M-A}$	.956
38	$Z_\phi^2/[(R_\phi + R_r)^2 + (X_\phi + X_r)^2]$				.862
39	$T_s = 2 \times 112.6 \times$ (32) $\times$ (33) $\times$ (37) $\times$ (38) $\times R_r/[a \times$ Syn. R.P.M.$]$				38.6 OZ-FT

Fig. 7-2



(a)



(b)

Fig. 7-3 VECTOR DIAGRAM OF A CAPACITOR-START MOTOR, 110-V 1/4 HP 1800 RPM 60 ~ AT NO LOAD (a) AND AT FULL LOAD (b).

8. Approximate Equivalent Circuit for Single-Phase Induction Motor.

In the preliminary design and especially in calculating the speed-torque characteristics of the single-phase induction motor, it would be desirable if a simplified equivalent circuit could be employed requiring less calculating labour yet which would give sufficiently accurate results. In the three-phase induction motor problem, the circle diagram is used for this purpose. However, no satisfactory circle diagram has been developed from which the performance of the single-phase induction motor can be determined. The following will be given an equivalent circuit which may be found useful in some respects.

Referring to Fig. 8-1a, it is seen that except at very small slips the rotor currents in the positive- and negative-sequence equivalent circuits are practically the same in magnitude. It would be possible, therefore, to remove the exciting impedance of bothe sequence equivalent circuits and to place an equivalent exciting impedance across a and b as shown in the approximate equivalent circuit on P. 13a. Again, if a suitable voltage E, which is constant across a and b, can be predetermined, a circle diagram can be drawn from which the speed-torque characteristics can be determined.

From Fig. 8-1b, the locus of the rotor current I_r will trace a circle and it can be expressed by the following polar equation as:

$$I_r = \frac{E}{X_M} \sin\theta \qquad (8\text{-}1)$$

WHERE

$$\sin\theta = \frac{X_M}{\sqrt{\left(\frac{2R_r}{s(2-s)}\right)^2 + X_M^2}} \qquad (8\text{-}2)$$

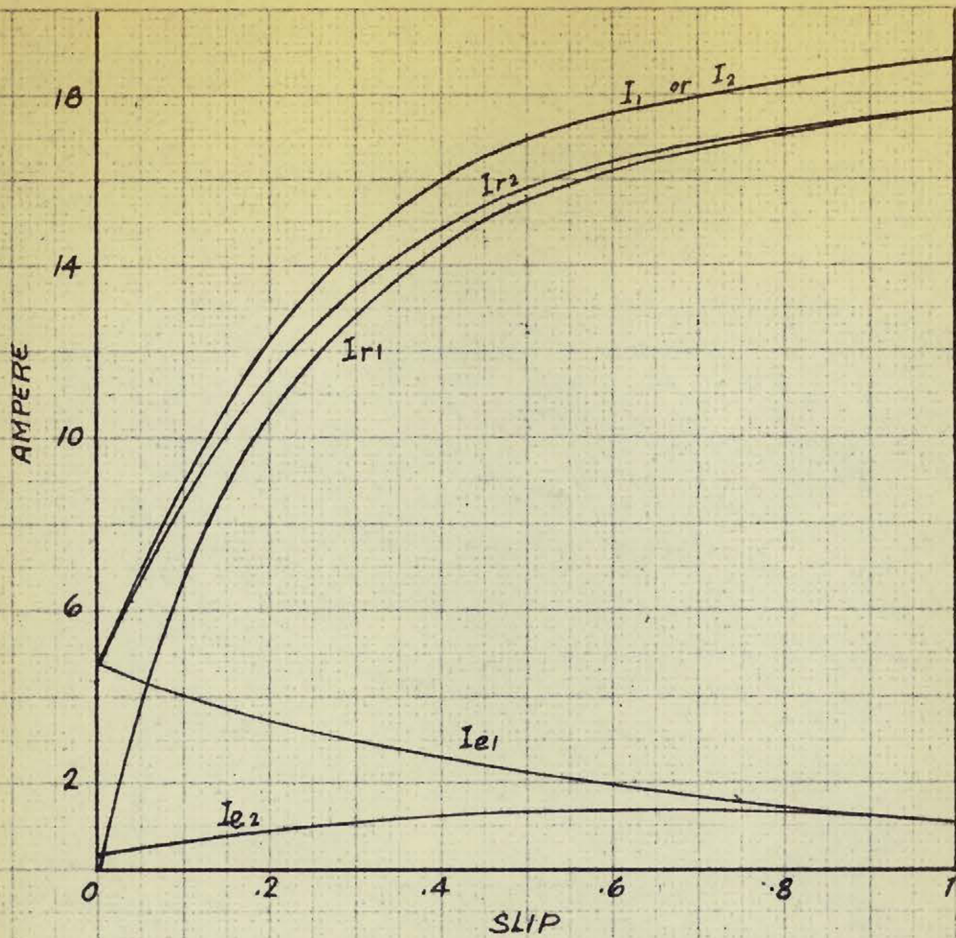


Fig. 8-1a. CALCULATED SLIP-CURRENT RELATION OF A 1/4 HP SINGLE-PHASE CAPACITOR-START MOTOR. (MAIN WINDING ONLY)

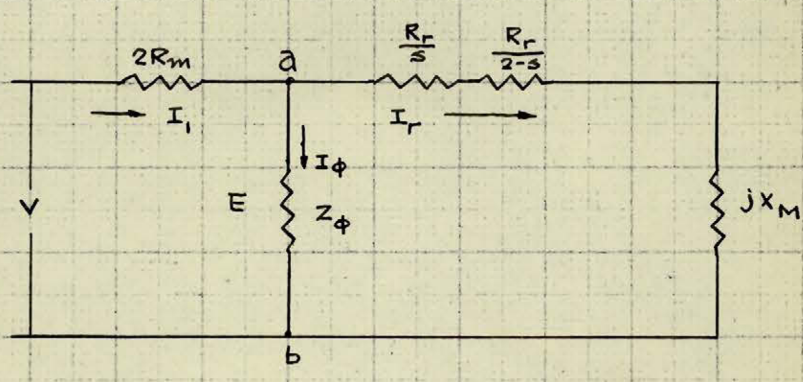


Fig. 8-1b. APPROXIMATE EQUIVALENT CIRCUIT FOR SINGLE-PHASE INDUCTION MOTOR.

The circle re presented by Eq.8-1. can be drawn if the voltage E is known. For this purpose, consider the construction shown in Fig. 8-2. where O B = V/X_M, lagging 90 degree behind the applied voltage V, is the diameter of the circle traced by the vectors of the line current. If now $I_{N.L.}$ is the line current at no load as determined from the no load test, the magnitude of the maximum rotor current I_r, the diameter A B of the circle represented by Eq. 8-1. can be determined as follows:

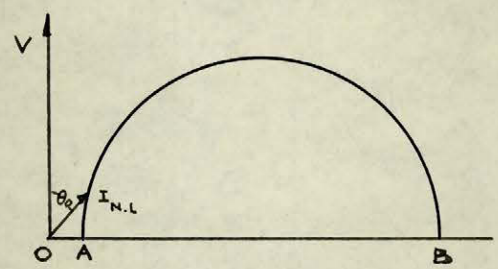


Fig. 8-2

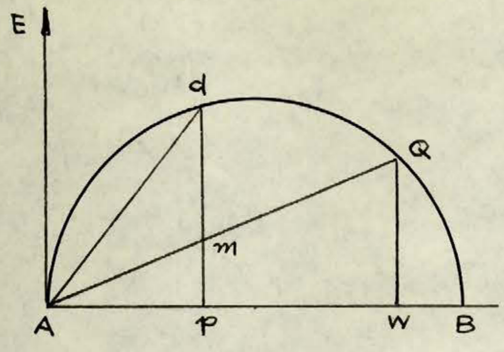
$$AB = OB - OA$$
$$= \frac{V}{X_M} - I_{N.L.} \sin\theta_o \qquad (8-3)$$

$$E = AB \cdot X_M$$
$$= V - I_{N.L.} X_M \sin\theta_o \qquad (8-4)$$

9. <u>The Circle Diagram from Approximate Equivalent Circuit</u>.

Having determined the voltage E of the approximate equivalent circuit, the circle diagram can now be constructed as in Fig. 9-1. where points A, Q and B correspond to the slip of zero, unity and infinity, respectively. At standstill, the power input to the rotor i.e., E × Q W is consumed entirely as the copper loss since there is no starting torque developed by the main winding alone of the single-phase induction motor. At a rotor current of o d, the power input to the rotor is:

$$P = E \times \overline{dp}$$
$$= I_r^2 \left[\frac{R_r}{s} + \frac{R_r}{2-s} \right]$$
$$= I_r^2 \left[2R_r + \frac{(1-s)}{s} R_r - \frac{(1-s)}{2-s} R_r \right]$$



$$= I_r^2 \left[2R_r + \frac{2(1-s)^2}{s(2-s)} R_r \right]$$

$$= E\,(mp + dm) \qquad (9\text{-}1)$$

Fig. 9-1.

in which m p and d m represent, respectively, the rotor copper loss and the shaft power in synchronous watts at the slip corresponding to the rotor current A d.

The slip for any rotor current such as o d can be determined from Eq. 9-1. as follows:

$$E \cdot dp = I_r^2 \left(\frac{R_r}{s} + \frac{R_r}{2-s} \right)$$

$$= \frac{2 I_r^2 R_r}{s(2-s)} = \frac{E \cdot mp}{s(2-s)}$$

whence,

$$s\,(2-s) = \frac{mp}{dp} \qquad (9\text{-}2)$$

By denoting mp/dp by K and solve for s in Eq. 9-2., we have:

$$s^2 - 2s + K = 0 \,,$$

$$s = 1 \pm \sqrt{1 - K} \qquad (9\text{-}3)$$

Eq. 9-3 indicates that the slip line of the circle diagram for the single-phase induction motor is not linear as in the case of three-phase induction motor. However, the locus of the variable K is a straight line as shown in Fig. 9-2.

The net torque developed at the pulley corresponding to the rotor current o d in ounce-ft. can be written as:

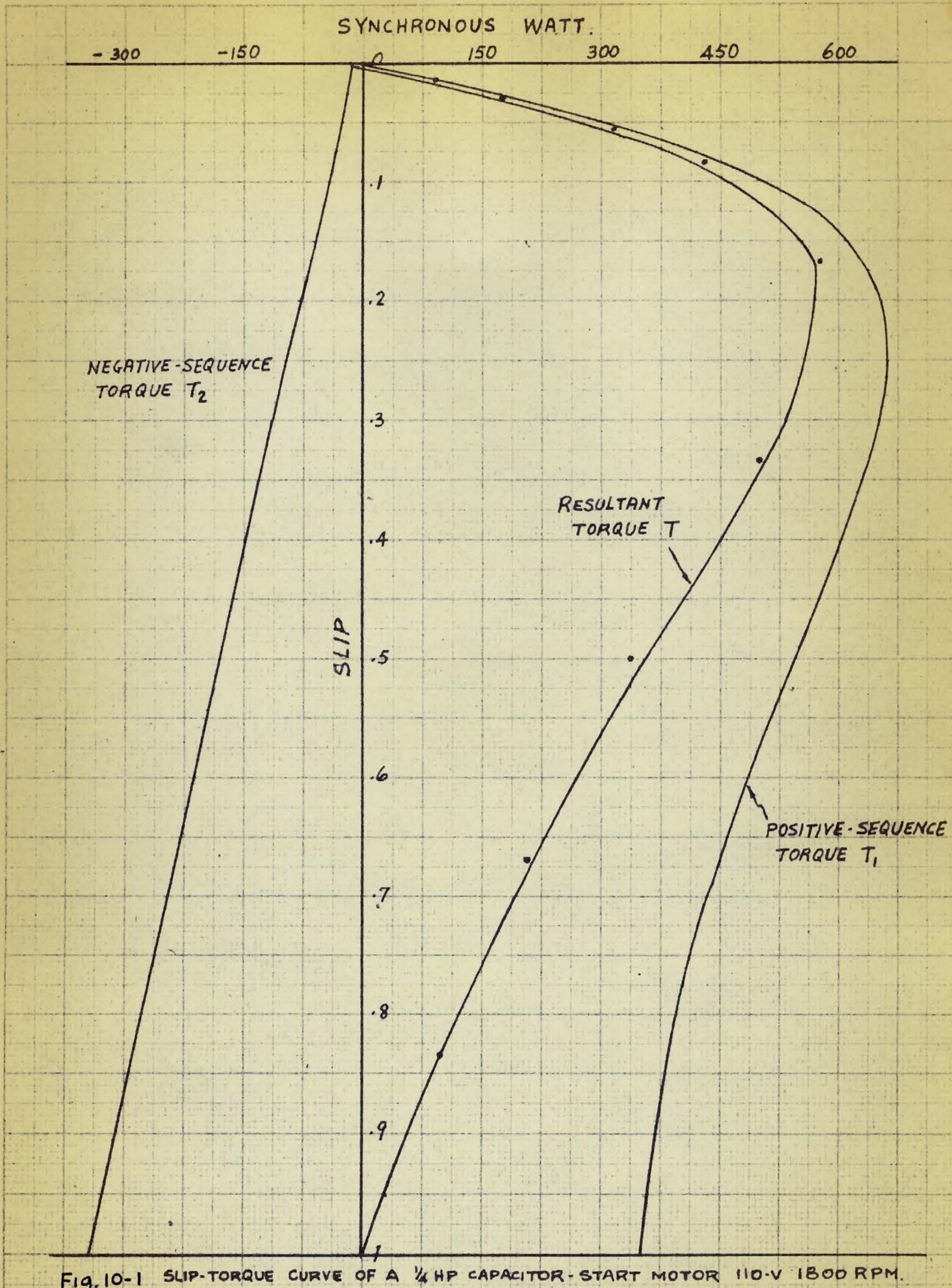


Fig. 10-1 SLIP-TORQUE CURVE OF A ¼ HP CAPACITOR-START MOTOR 110-V 1800 RPM.
SOLID POINTS CALCULATED FROM SIMPLIFIED EQUIVALENT CIRCUIT.

$$T = \frac{112.6}{R.P.M} \left[\frac{2(1-s)^2 I_r^2 R_r}{(2-s)} - P_{ftw} \right]$$

$$= \frac{112.6}{R.P.M.} \left[E \cdot \overline{dm} - P_{ftw} \right]$$

As an illustration to show the application and the construction of the circle diagram, the single-phase induction motor previously mentioned is again employed. According to the machine constants in Fig.7-1. and the no-load current $I_{n.1.}$ from the no load test, the voltage E for the circle diagram can be calculated as:

$$E = 110 - 5.05 \times 4.88 \sin 76.6$$

$$= 110 - 20 = 86 \text{ volts.}$$

The circle diagram can now be drawn with a diameter AB = 86 / 4.88 or 17.65 amperes. The rotor current at standstill AQ = 86 / 5.36 or 16.1 amperes. The locus of the variable K is along a straight line perpendicular to AB and can be constructed as follows: Draw a straight line perpendicular to AB in any position within AB. From B, where K is infinity, draw a line from B to Q, where K is unity, and produce BQ until it intersects mn at M. The length of MN is considered as unity. The value of K for any rotor current such as A d can be determined by striking from d to B a line which crosses M N at J and is read as .25, which gives a slip of $1 - \sqrt{1 - .25}$, or .135 . The construction is shown in Fig.9-2.

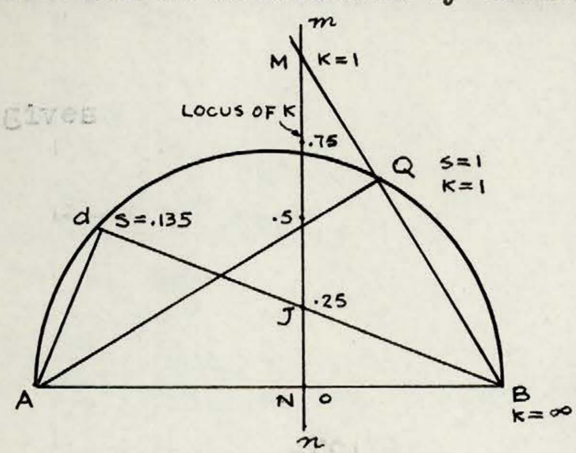


Fig.9-2. Circle diagram from approximate equivalent circuit showing how the slip is determined.

10. Slip-Torque Characteristics from Approximate Equivalent Circuit.

The slip-torque characteristics of the above mentioned single-phase induction motor calculated from the approximate equivalent circuit or the circle diagram therefrom, are shown in solid points in Fig. 10-1. The full lines in the same figure being calculated from the exact equivalent circuit shown in Fig. 7-1. Synchronous-watt is used in order to show the negative sequence torque in the figure. The torques calculated from the approximate equivalent circuit are shown slightly higher at small slips and lower at slips greater than the pull out slip when compared with those calculated from the exact equivalent circuit. The maximum discrepancy between calculated results from these two circuits is less than six per cent, but the calculating labour of the approximate equivalent circuit is considerably reduced. Unfortunately, there seem to have no satisfactory results in determing the line current and power factor from the circle diagram can be obtained without correcting the exciting current which is being assumed constant in the approximate equivalent circuit. However, this circle diagram may be found useful whenever a short cut method of calculating the slip-torque characteristics is desired.

11. Determination of Pull-Out Slip from the Circle Diagram.

The slip for maximum torque or pull-out slip, as it is called, can be determined by differentiating the torque equation given on P. 8. with respect to the slip s and equating to zero. The solution of this equation is so extremely labourous that some other means would be desired. Mr. Veinott gave a graphical solution of this problem by the Branson's circle diagram. A useful curve for the relation between the ratio of R_R/X_M and the pull-out slip was also developed. In order to show the application of the approximate equivalent circuit and the circle diagram described in Art. 9, the derivation of the pull-out slip and the curve showing the relation between R_R/X_M and the pull-out slip will be given here in a simpler manner.

From Fig. 11-1a, the maximum torque occurs at d is determined by drawing a tangent to the circle parallel to the line A Q. The pull-out slip can then be calculated from Eq. 9-3. Referring to Fig. 11-1a, we may write:

$$mp = Ap \tan\beta = Ap \frac{R_R}{X_M}$$

$$Ap = dp \tan\phi$$

$$mp = dp\, Ap \frac{R_R}{X_M}$$

WHENCE $$K = \frac{dm}{dp} = \frac{R_R}{X_M} \tan\phi$$

SINCE $$\alpha = \cot^{-1} \frac{R_R}{X_M} \quad \text{AND} \quad \phi = \frac{\alpha}{2}$$

THEREFORE $$K = \frac{R_R}{X_M} \tan\left(\frac{\cot^{-1} R_R/X_M}{2}\right)$$

Substituting the value of K into Eq. 9-3., the pull-out slip becomes:

$$S_{P.O.} = 1 - \sqrt{1 - \frac{R_R}{X_M} \tan\left(\frac{\cot^{-1} \frac{R_R}{X_M}}{2}\right)} \qquad (11\text{-}1)$$

A plot of Eq. 11-1. for various values of R_R/X_M against the pull-out slip is given in Fig. 11-1b. which will be found useful in the design of single-phase induction motor.

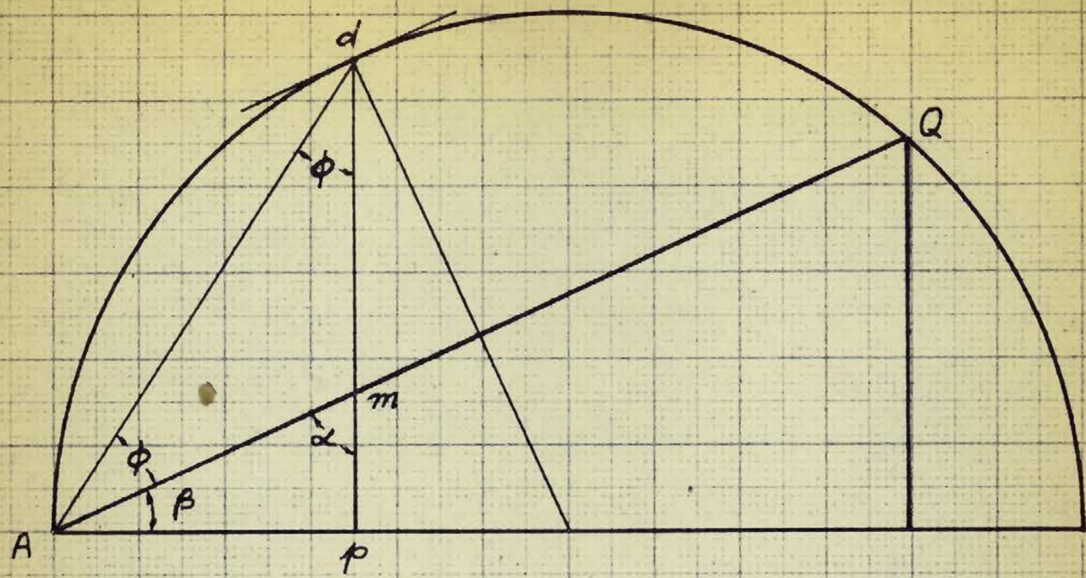


Fig. 11-1a CIRCLE DIAGRAM FOR SINGLE PHASE INDUCTION MOTOR FROM APPROXIMATE EQUIVALENT CIRCUIT SHOWING $S_{P.O.} = 1 - \sqrt{1 - \frac{mp}{dp}}$

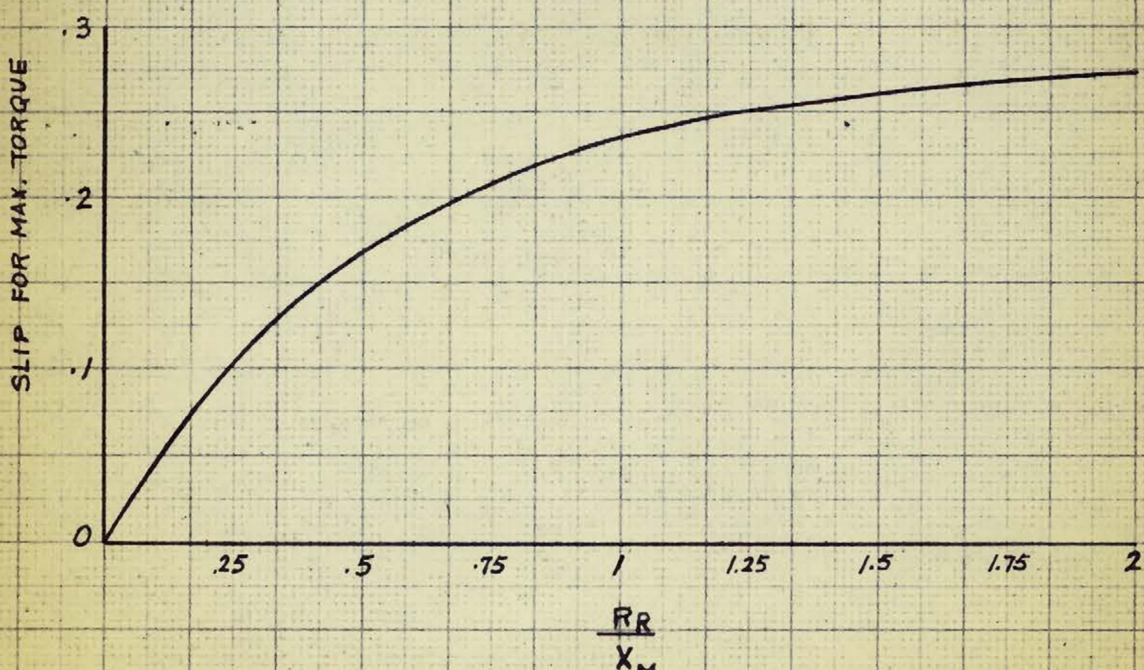


Fig. 11-1b.

PART 2

Circuit Studies of Capacitor Motor by the Method of Two-Phase Right-Angle Components.

In dealing with the circuit studies of the single-phase induction motor under starting condition and the complete performance of the capacitor motor, the method of positive-and negative-sequence right-angle components or two-phase right-angle components, as it is sometimes called, is perhaps the most suitable one as the motors under these operations act like the unbalanced two-phase machines. The advantage of this method of circuit analysis is to enable any unbalanced two-phase vectors, current or voltage, to be resolved into two sets of balanced two-phase vectors, unequal in magnitude and opposite in phase rotation. Physically, the motor may be considered as a two phase machine operated by two balanced two-phase voltage supplies, unequal in magnitude and opposite in phase rotation.

12. Basic Principle of the Method of Two-Phase Right-Angle Components

The basic principle of the positive- and negative-sequence right-angle components is to resolve any set of current or voltage vectors which are not equal nor in phase quadrature into two sets of right-angle components, unequal in magnitude and opposite in phase rotation. To illustrate this, let vectors A and B which are not equal nor in quadrature be resolved into their positive- and negative-sequence right-angle components, denoted by A_1, B_1 and A_2, B_2, respectively. We may write:

$$A = A_1 + A_2 ,$$

$$B = B_1 + B_2 .$$

Since $A_1 = jB_1$; and $A_2 = - jB_2$, therefore,

$$B = -jB_1 + jA_2 .$$

The sequence components can then be expressed as:

$$A_1 = 1/2 (A + jB), \qquad A_2 = 1/2 (A - jB),$$

$$B_1 = 1/2 (B - jA), \qquad B_2 = 1/2 (B + jA).$$

The above relations are illustrated by the vector diagrams shown in Fig. 12-1. It should be noticed that in the positive-sequence circuit the component A_1 is leading B_1 by 90 degree or in a phase sequence of A-B; while in the negative-sequence circuit the component A_2 lags 90 degree behind B_2 and the phase sequence is B-A. Any sequence component can be chosen as the reference vector and should be chosen in accordance with the phase relation between the vectors A a nd B. In both sequences, counterclockwise is considered as the positive direction of phase rotation.

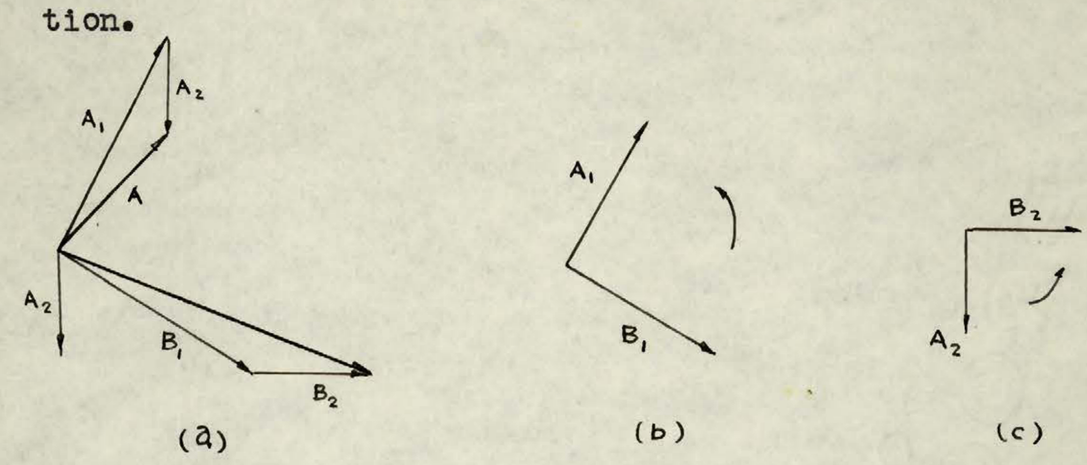


Fig. 12-1. Vector diagram in (a) showing vectors A and B being resolved into two sets of right-angle components shown in (b) and (c).

13. Equivalent Circuit of Capacitor Motor With Symmetrical Stator Windings.

To apply the method of positive- and negative-sequence right-angle components to the studies of the capacitor motor, a motor with symmetrical stator windings is first considered. The main and auxiliary windings of this motor are assumed to have similar winding distribution and to have the same weight of copper. Under these assumptions, all the constants of the auxiliary winding in terms of the main winding can be expressed in the following manner:

$$Z_a' = a^2 Z_a = Z_m \quad ; \quad R_a' = a^2 R_a = R_m \quad ;$$

$$X_a' = a^2 X_a = X_m \quad ; \quad V_a' = a V_a = V_m \quad ;$$

$$I_a' = I_a / a = I_m \quad ; \quad Z_c' = a^2 Z_c \quad .$$

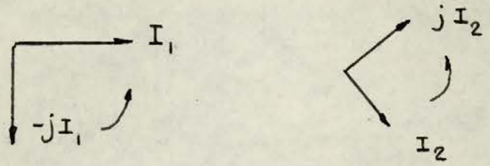
where the letter a is the effective turn-ratio between the main and auxiliary windings and the primes are used to denote the constants of the auxiliary winding in terms of the main winding.

Consider now the currents I_m and I_a in the main and auxiliary windings of a capacitor motor under unbalanced operation are to be resolved into their positive- and negative-sequence right-angle components. Thus,

$$I_a = I_{a_1} + I_{a_2}$$

$$I_m = I_{m_1} + I_{m_2}$$

As a matter of fact, the current I_a in the capacitor motor is in leading position with respect to I_m when shown in the vector diagram; and for simplicity of expression, I_{a1} and I_{a2} will be denoted by I_1 and I_2, respectively, and will be taken as the reference vectors in both sequences. Thus,



$$I_a = I_1 + I_2 ,$$
$$I_m = -jI_1 + jI_2 .$$

Fig. 13-1.

The positive-and negative-sequence current components shown in Fig. 13-1 will each produce a rotating field of different m.m.f. and rotate in opposite directions. The impedances to the positive- and negative-sequence current will again be denoted by Z_1 and Z_2, respectively. These impedances have the same expressions as those shown in the equivalent circuit for the single-phase induction motor, see Fig. 5-1. , but are twice as great in magnitude. They are:

$$Z_1 = Z_m + \frac{Z_{r_1} Z_\phi}{Z_{r_1} + Z_\phi} \quad ; \quad Z_{r_1} = \frac{R_r}{s} + jX_r \quad ;$$

$$Z_2 = Z_m + \frac{Z_{r_2} Z_\phi}{Z_{r_2} + Z_\phi} \quad ; \quad Z_{r2} = \frac{R_r}{2-s} + jX_r \quad .$$

The voltage equations can now be written while the positive- and negative-sequence component equivalent circuits can be set up as shown in Fig. 13-2.

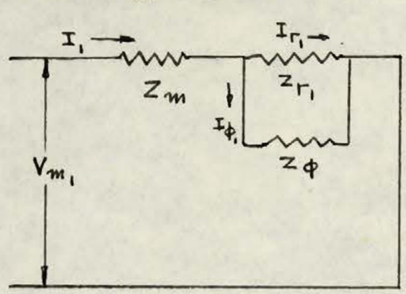


Fig. 13-2 (a) MAIN WINDING

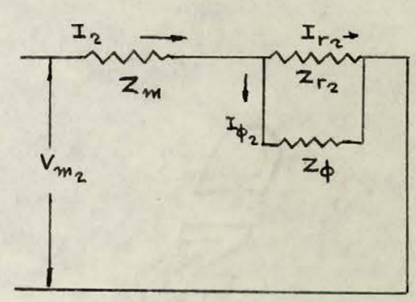


Fig. 13-2 (b) MAIN WINDING

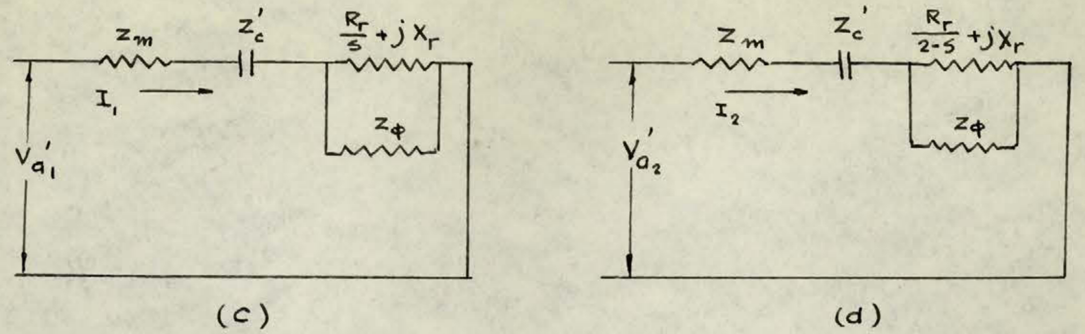


Fig. 13-2. Positive- and negative-sequence component equivalent circuits for capacitor motor: (a) (b), the main winding; (c) (d), the auxiliary winding.

$$\begin{aligned} V = V_m &= V_{m_1} + V_{m_2} \\ &= I_{m_1} Z_1 + I_{m_2} Z_2 \\ &= -jI_1 Z_1 + jI_2 Z_2 \end{aligned} \qquad (13\text{-}1)$$

$$\begin{aligned} aV_m = a^2 V_a &= a^2 (V_{a_1}' + V_{a_2}') \\ &= I_{a_1}(Z_1 + Z_c') + I_{a_2}(Z_2 + Z_c') \\ &= I_1 (Z_1 + Z_c') + I_2 (Z_2 + Z_c') \end{aligned} \qquad (13\text{-}2)$$

The positive- and negative-sequence current I_1 and I_2 can be derived by solving the simutaneous equations (13-1) and (13-2). For simplicity, use determinants.

$$\frac{\begin{vmatrix} V & jZ_2 \\ aV & (Z_2 + Z_c') \end{vmatrix}}{\begin{vmatrix} -jZ_1 & jZ_2 \\ (Z_1 + Z_c') & (Z_2 + Z_c') \end{vmatrix}} = \frac{V(Z_2 + Z_c') - jaVZ_2}{-j(Z_2 + Z_c')Z_1 - jZ_2(Z_1 + Z_c')}$$

$$= \frac{jV}{2} \frac{(1 - ja)Z_2 + Z_c'}{Z_1 Z_2 + \frac{Z_c'}{2}(Z_1 + Z_2)} \qquad (13\text{-}3)$$

$$I_2 = \frac{\begin{vmatrix} -jZ_1 & V \\ (Z_1+Z_c') & aV \end{vmatrix}}{\begin{vmatrix} -jZ_1 & jZ_2 \\ (Z_1+Z_c') & (Z_2+Z_c') \end{vmatrix}} = \frac{-jaVZ_1 - V(Z_1+Z_c')}{-j(Z_2+Z_c')Z_1 - jZ_2(Z_1+Z_c')}$$

$$= \frac{-jV}{2} \frac{(1+ja)Z_1 + Z_c'}{Z_1Z_2 + \frac{Z_c'}{2}(Z_1+Z_2)} \qquad (13\text{-}4)$$

Having determined I_1 and I_2, the currents in all branch circuits of any component equivalent circuit can be calculated as follows:

$$I_{r_1} = \frac{I_1 Z_\phi}{Z_\phi + Z_{r_1}} \qquad I_{\phi_1} = \frac{I_1 Z_{r_1}}{Z_\phi + Z_{r_1}}$$

$$I_{r_2} = \frac{I_2 Z_\phi}{Z_\phi + Z_{r_2}} \qquad I_{\phi_2} = \frac{I_2 Z_{r_2}}{Z_\phi + Z_{r_2}}$$

In calculating the torque as well as the shaft power of the motor, it must not be forgotten that the motor is equivalent to a balanced two-phase machine. This gives:

$$T_1 = 2 I_{r_1}^2 \frac{R_r}{s} \;; \qquad P_{SH_1} = 2 I_{r_1}^2 \left(\frac{1-s}{s}\right) R_r \;;$$

$$T_2 = -2 I_{r_2}^2 \frac{R_r}{2-s} \;; \qquad P_{SH_2} = -2 I_{r_2}^2 \left(\frac{1-s}{2-s}\right) R_r \,.$$

The resultant internal torque and shaft power in synchronous watts are respectively equal to:

$$T = T_1 + T_2 = 2\left[\frac{I_{r_1}^2}{s} - \frac{I_{r_2}^2}{2-s}\right] R_r \qquad (13\text{-}5)$$

$$P_{SH} = P_{SH_1} + P_{SH_2} = 2\left[\frac{I_{r_1}^2}{s} - \frac{I_{r_2}^2}{2-s}\right](1-s) R_r \qquad (13\text{-}6)$$

14. The Starting Torque T_s

At starting, both the positive- and negative-sequence impedances Z_1 and Z_2 are equal to the locked-rotor impedance of the main winding Z_M. By neglecting the resistance of the capacitor, the starting torque can be expressed in terms of Z_M, the capacitive re actance of the capacitor in terms of the main winding, the rotor resistance and the effective turn-ratio between the windings. From Eq. 13-5.

$$T_s = 2\left(I_{r_1}^2 - I_{r_2}^2\right) R_r$$

$$= 2\left[(-jI_1)^2 + (jI_2)^2\right] \frac{z_\phi^2}{(z_\phi + z_r)^2} \; R_r$$

$$= 2\,\frac{V^2}{4a}\;\frac{4R_M X_c' R_r}{z_M^2 + (X_M - X_c)^2}\;\frac{z_\phi^2}{(R_\phi + R_r)^2 + (x_\phi + x_r)^2}$$

$$= \frac{2V^2}{a\,Z_M^2}\;\frac{R_M X_c'}{R_M^2 + (X_M - X_c')^2}\; R_r \cdot K \qquad (14\text{-}1)$$

WHERE

$$K = \frac{z_\phi^2}{(R_\phi + R_r)^2 + (x_\phi + x_r)^2}$$

The starting torque in oz-ft. can be obtained by multiplying the T_s in syn.-watt by the quantity 112.6/syn. r.p.m. .

The capacitance that will give the maximum starting torque can be determined by differentiating the equation (14-1) with respect to X_c' and then equating to zero. Let the resistance of the capacitor in terms of the main winding be R_c', From Eq. 14-1,

$$\frac{d\,T_s}{d\,X_c'} = \left[\frac{2V^2\,R_r\,K\,R_M}{a\,Z_M^2}\right]\frac{d}{d\,X_c'}\left[\frac{X_c'}{(R_M \quad R_c')^2 \quad (X_M \quad X_c')^2}\right] = 0,$$

which gives

$$R_M^2 + X_M^2 + 2R_M\,R_c' - X_c^2 = 0\ ,$$

whence,

$$X_c' = \left[Z_M^2 + 2R_M\,R_c'\right]^{\frac{1}{2}}. \qquad (14\text{-}2)$$

When the resistance of the capacitor is neglected,

$$X_c' \doteq Z_M$$

whence, $$X_c = Z_M/a^2 = Z_A\ .$$

The maximum starting torque can be obtained if the capacitor is so designed as to have a reactance eequal to the blocked-rotor impedance of the auxiliary winding provided that the resistance of the capacitor is neglected.

15. Equivalent Circuits for Capacitor Motor With Unsymmetrical Stator Windings.

Single-phase induction motor and capacitor motor are described as the motors with unsymmetrical stator windings when the resistances of their stator windings do not follow the relation $R_a = R_m/a^2$.

In order to make use of the equivalent circuits described in Art. 13, it would be necessary to replace the impedance of the capacitor Z_c' by an impedance Z_e', which will include the unsymmetry of the stator windings. Let the leakage impedance of the auxiliary winding, including the capacitor, be denoted by Z_s. Then we may write:

$$Z_s' = a^2 Z_s = a^2 Z_a + a^2 Z_s - a^2 Z_a$$

$$= Z_m + Z_e' ,$$

whence,

$$Z_e' = a^2 Z_s - Z_m .$$

It is interesting to note that the impedances Z_s and Z_m shown above can be replaced respectively by the blocked-rotor impedances Z_M and Z_S as the unsymmetry of the motor is in the stator windings only. Therefore,

$$Z_e' = a^2 Z_S - Z_M \tag{15-1}$$

The equivalent circuits can now be set up as shown in Fig. 15-1. And by making use of those equations derived in Art. 13, the complete solution of these equivalent circuits may be obtained by simply replacing all the X_c's by the quantity given in Eq. 15-1. in the folowing manner:

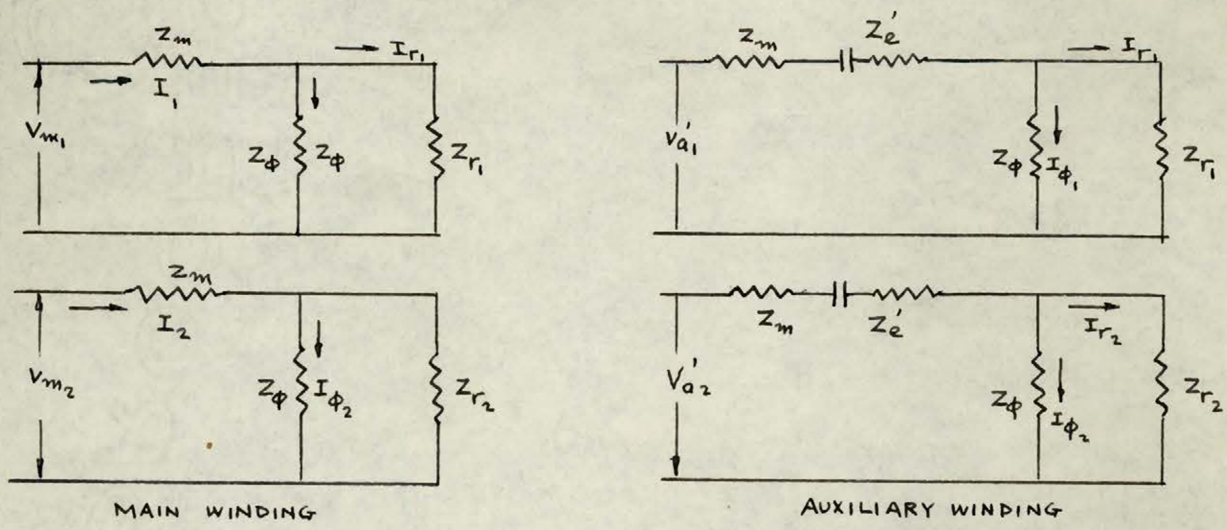


Fig. 15-1. Positive- and negative-sequence component equivalent circuits for capacitor motor with unsymmetrical stator windings.

Refer to Art. 13. and let the denominator of Eq.13-3 or 13-4 be denoted by D, thus,

$$D = Z_1 Z_2 + \frac{1}{2} (a^2 Z_S - Z_M)(Z_1 + Z_2) .$$

The positive- and negative-sequence currents are :

$$I_1 = \frac{jV}{2D} \left[(1 - ja) Z_2 + (a^2 Z_S - Z_M) \right] \qquad (15\text{-}2)$$

$$I_2 = \frac{-jV}{2D} \left[(1 + ja) Z_1 + (a^2 Z_S - Z_M) \right] \qquad (15\text{-}3)$$

The current in the main winding is:

$$\begin{aligned} I_m &= -jI_1 + jI_2 \\ &= \frac{V}{2D} \left[(1 + ja) Z_1 + (1 - ja) Z_2 + 2(a^2 Z_S - Z_M) \right] \end{aligned} \qquad (15\text{-}4)$$

The current in the auxiliary winding is:

$$\begin{aligned} I_a &= a\, I_a' = a\,(I_1 + I_2) \\ &= \frac{V}{2\,D} \left[(a^2 + ja) Z_2 + (a^2 - ja) Z_1 \right] \end{aligned} \qquad (15\text{-}5)$$

The line current is the vector sum of I_m and I_a. Thus,

$$\begin{aligned} I_\ell &= I_m + I_a \\ &= \frac{V}{2\,D} \left[(1 + a^2)(Z_1 + Z_2) + 2(a^2 Z_S - Z_M) \right] \end{aligned} \qquad (15\text{-}6)$$

The rotor current as well as the exciting current in either sequence circuit can be calculated as explained in Art.13. They are:

$$I_{r_1} = \frac{I_1 Z_\phi}{Z_\phi + Z_{r_1}} \qquad I_{\phi_1} = \frac{I_1 Z_{r_1}}{Z_\phi + Z_{r_1}}$$

$$I_{r_2} = \frac{I_1 Z_\phi}{Z_\phi + Z_{r_2}} \qquad I_{\phi_2} \quad \frac{I_2 Z_{r_2}}{Z_\phi + Z_{r_2}}$$

The resultant internal torque and shaft power are respectively:

$$T = T_1 + T_2 = 2\left[\frac{I_{r_1}^2}{s} - \frac{I_{r_2}^2}{2-s} \right] R_r \qquad (15\text{-}7)$$

$$P_{SH.} = P_{SH_1} + P_{SH_2} = T(1-s)$$

At starting, i.e., s = 1, the rotor currents become:

$$I_{r_1} = \frac{V}{2} \, \frac{Z_S - j Z_M}{a Z_S Z_M} \, \frac{Z_\phi}{Z_\phi + Z_r} \qquad (15\text{-}8)$$

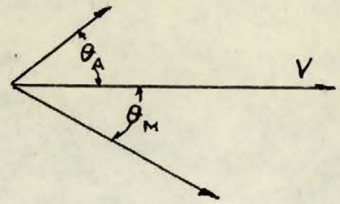
$$I_{r_2} = \frac{V}{2} \, \frac{aZ + j Z_M}{a^2 Z_S Z_M} \, \frac{Z_\phi}{Z_\phi + Z_r} \qquad (15\text{-}9)$$

Substituting Eq. 15-8 and 15-9 into Eq.15-7., the starting torque will have the following form where all constants are scalar quantities:

$$T_S = \frac{2V^2 R_r}{a(R_M^2 + X_M^2)} \cdot \frac{X_M (R_A + R_c) - R_M (X_A - X_c)}{\left[(R_A + R_c)^2 + (X_A - X_c)^2\right]} \cdot K \qquad (15\text{-}10)$$

WHERE, $$K = \frac{Z_\phi^2}{(R_\phi + R_r)^2 + (X_\phi + X_r)^2}$$

Since $I_M = V/Z_M$, and $I_A = V/Z_S$, from Fig.15-2. it can be shown that



$$\sin(\theta_M + \theta_A) = \sin\theta_M \cos\theta_A + \cos\theta_M \sin\theta_A$$
$$= \frac{X_M R_S + R_M X_S}{Z_M Z_S}$$

Fig. 15-2 SHOWING:

$$\sin\theta_A = \frac{X_S}{Z_S} \; ; \; \cos\theta_A = \frac{R_S}{Z_S} \, ,$$

$$\sin\theta_M = \frac{X_M}{Z_M} \; ; \; \cos\theta_M = \frac{R_M}{Z_M} \, .$$

Eq.15-10. can then be written as:

$$T_S = \frac{2}{a} I_M I_A K R_r \sin\theta_{M\text{-}A} \qquad \text{SYN.-WATT.} \quad (15\text{-}11)$$

Eq.15-11. is similar to the expression derived from the cross-field theory.

The capacitive reactance required to give maximum starting torque can be determined by the graphical method or by the following expression derived from differentiating the starting torque equation with respect to X_c and then equating to zero. Refer to Eq.15-10.,

$$\frac{dT_S}{dX_c} = \frac{d}{dX_c}\left[\frac{X_M R_S}{R_S^2 + (X_A - X_c)^2} - \frac{R_M (X_A - X_c)}{R_S^2 + (X_A - X_c)^2} \right] = 0$$

whence, $X_M R_S (2X_A - 2X_c) = \left[R_S^2 + (X_A - X_c)^2 \right] (-R_M) + 2R_M (X_A - X_c)^2$

Rearranged, $R_M (X_A - X_c)^2 - 2X_M R_S (X_A - X_c) - R_M R_S^2 = 0$

Solving for X_c, $X_A - X_c = \frac{R_S (X_M \pm Z_M)}{R_M}$

Thus, $X_c = X_A + \frac{R_S (Z_M - X_M)}{R_M}$ (15-12)

If the resistance of the capacitor is neglected, the capacitive reactance required to give maximum starting torque becomes:

$$X_c = X_A + \frac{R_A}{R_M}(Z_M - X_M)$$

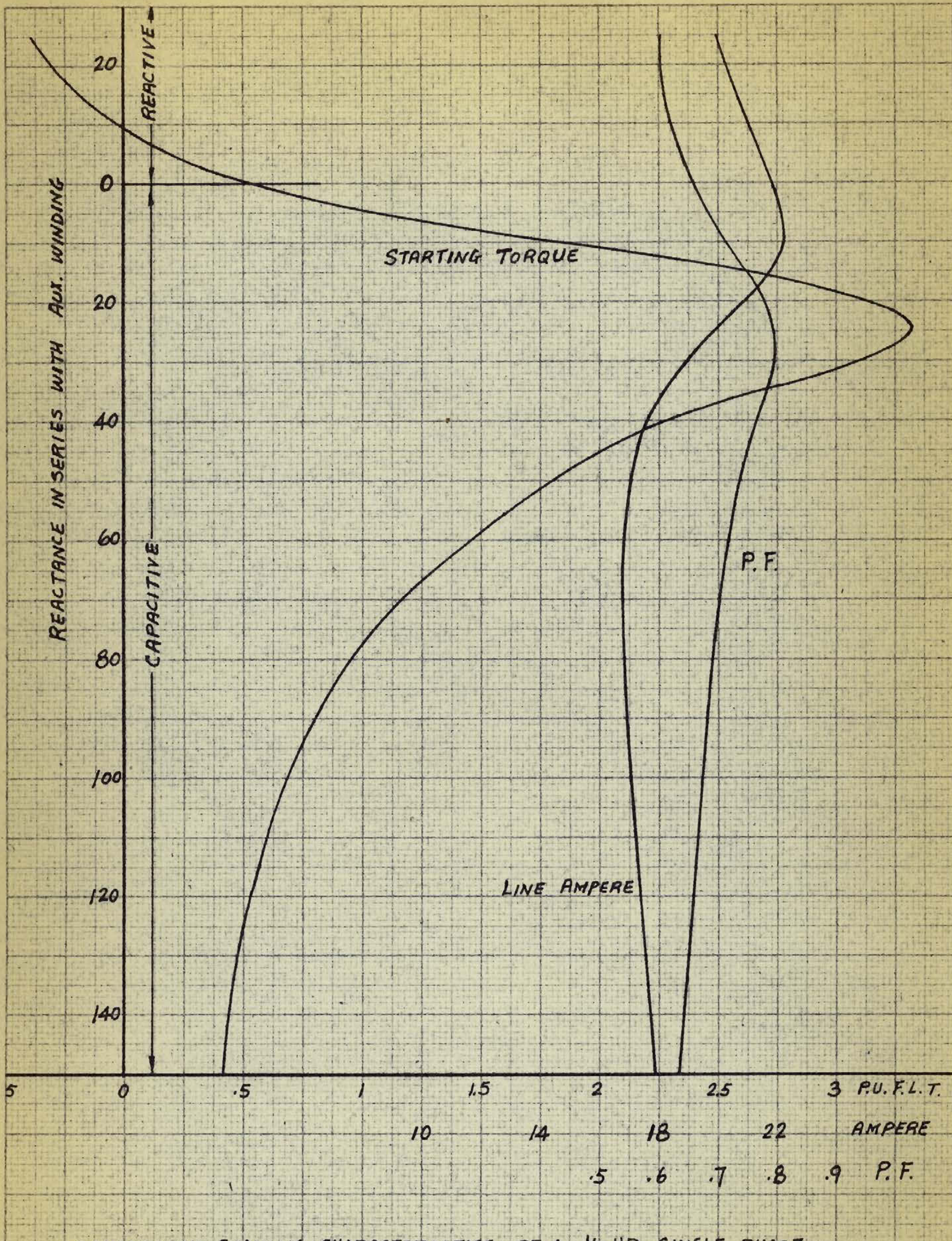
16. <u>Starting Characteristics of Capacitor-Start Induction Motor</u>.

Fig.16-1. shows the starting characteristics of the capacitor start motor mentioned in Part I of this paper, calculated by the method described in the preceeding paragraph. Starting torque, expressed in per unit full load torque, the line current and power factor versus various values of external reactance, inductive and capacitive, in series with the auxiliary winding, are plotted on the same paper. Experiments on starting torque with several different capacitors at about half the rated voltage have been made on the above mentioned motor. Reduced applied voltage will prevent the overheating of the windings during a continuous test. For reactances higher than 100 ohms, capacitive, the motor refused to start at half the rated voltage as the auxiliary winding acted practically as an open circuit. It is interesting to note that when an inductive reactance of about nine ohms is connected to the auxiliary winding, the blocked-rotor current in the auxiliary winding will be in phase with that of the main winding, and no starting torque will be developed.

The capacitor of this motor is found to have a capacitance of 106-μf or a reactance of 25 ohms at a frequency of 60 c.p.s. The resistance of the capacitor is taken as 2 ohms. As will be seen later, this capacitor has been chosen to give maximum starting torque. An example will be given below to show the calculation of the maximum starting torque of the above mentioned motor.
From Part I, the constants of the motor are:

$$X_A = 17.1 \quad ; \quad R_A = 18.2 \quad ; \quad R_c = 2 \quad ;$$

$$Z_M = 6.22 \qquad X_M = 4.88 \qquad R_M = 3.86 \quad .$$



REACTIVE
CAPACITIVE
REACTANCE IN SERIES WITH AUX. WINDING
STARTING TORQUE
P.F.
LINE AMPERE
20
0
20
40
60
80
100
120
140
.5 0 .5 1 1.5 2 2.5 3 P.U. F.L.T.
10 14 18 22 AMPERE
.5 .6 .7 .8 .9 P.F.
CALCULATED STARTING CHARACTERISTICS OF A 1/4 HP SINGLE PHASE
INDUCTION MOTOR VERSUS VARIOUS STARTING CAPACITORS

Fig. 16-1.

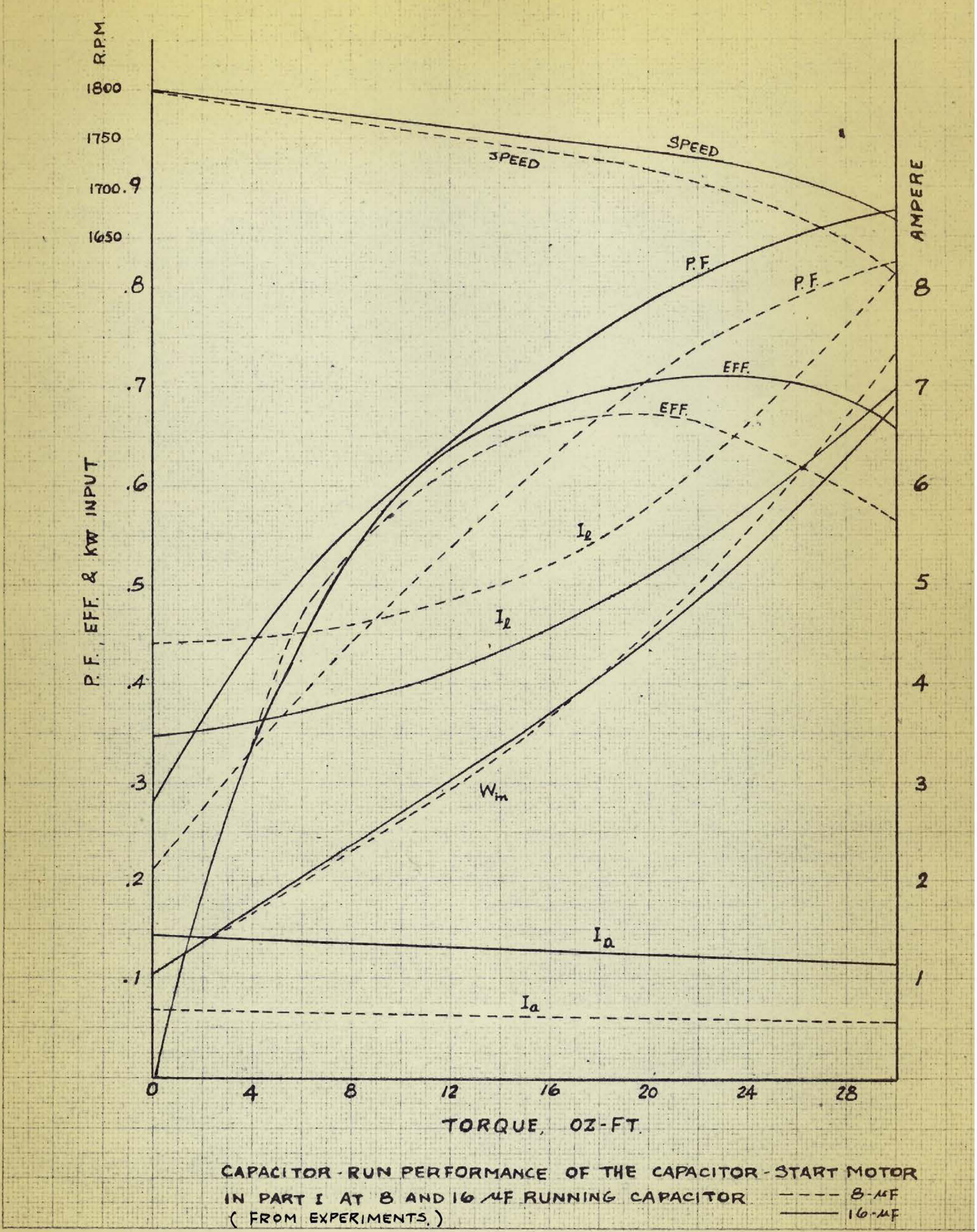
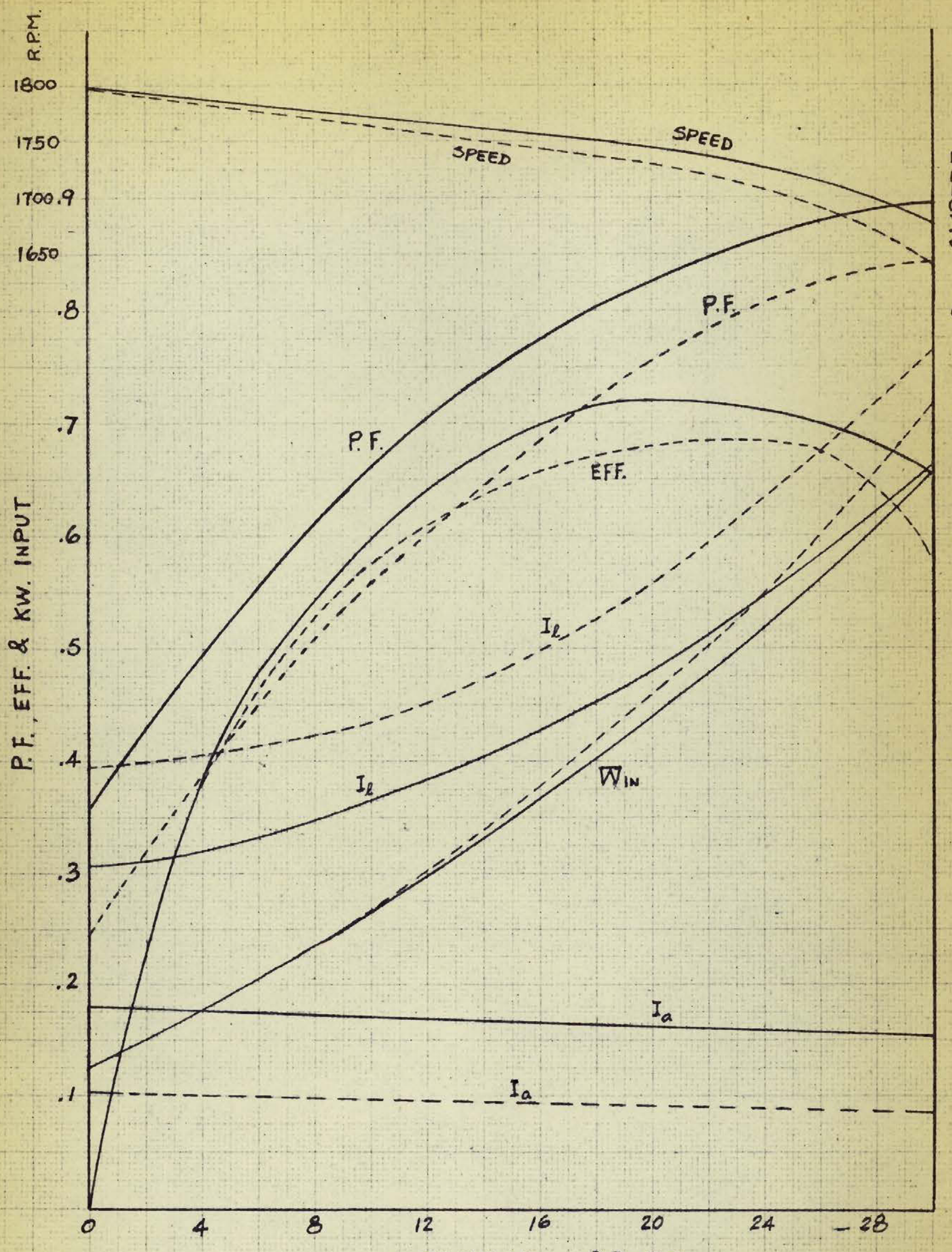

R.P.M.
1800
1750
1700
1650
P.F. EFF. & KW INPUT
.9
.8
.7
.6
.5
.4
.3
.2
.1
AMPERE
8
7
6
5
4
3
2
1
SPEED
SPEED
P.F.
P.F.
EFF.
EFF.
I_l
I_l
W_{in}
I_a
I_a
0
4
8
12
16
20
24
28
TORQUE, OZ-FT.
CAPACITOR-RUN PERFORMANCE OF THE CAPACITOR-START MOTOR
IN PART I AT 8 AND 16 μF RUNNING CAPACITOR
(FROM EXPERIMENTS.)
8-μF
16-μF

Fig. 16-2.



CAPACITOR-RUN PERFORMANCES OF THE CAPACITOR-START MOTOR IN PART I AT 12 AND 20-μF RUNNING CAPACITOR (FROM EXPERIMENT)

Fig. 16-3.

From Eq.15-12. the reactance of the capacitor to give maximum starting torque is:

$$X_c = 17.1 + \frac{(18.2 + 2)\ (6.22 - 4.88)}{3.86}$$

$$= 24.1 \text{ OHMS.} \qquad \text{OR } 110\text{-}\mu F$$

The blocked-rotor impedance of the auxiliary winding circuit will be:

$$Z_S = 20.2 - j7 = 21.36 \angle{-19.1}$$

The starting currents in the main and auxiliary windings are respectively equal to:

$$I_M = 110/6.22 \angle{51.6} = 11 - j13.85 = 17.7 \angle{-51.6}$$

$$I_A = 110/21.36 \angle{-19.1} = 4.87 + j1.69 = 5.16 \angle{19.1}$$

The starting torque:

$$T_S = \frac{2}{.535} \times 17.7 \times 5.16 \times .862 \times 2.22 \times \sin(51.6 + 19.1)$$

$$= 616 \text{ SYN. WATTS.} \qquad \text{OR } 33.6 \text{ OZ-FT.}$$

The line current is the sum of I_M and I_A and is equal to $20 \angle{-37.5}$ the power factor can readily be found as .793. These calculations have been checked by experiments with satisfactory results.

17. <u>Performance of the Capacitor Motor.</u>

The performance of the capacitor motor, whether it is originally designed as a capacitor motor, or as a capacitor-start motor operated with a proper running capacitor in series with its auxiliary winding, has decided advantages over that of the same motor when it

$Z_M = 3.86 + j\ 4.88$

$Z_A = 18.2 + j\ 17.1$

$Z_\phi = 5.7 + j\ 35.2 = 35.68 \underline{|80.8}$

$Z_c = -j\ 132.5$

$Z_S = Z_A + Z_c = 18.2 - j\ 113.9$

$Z_e' = a^2\ Z_S - Z_M = 1.34 - j\ 37.88$

$Z_m = 1.64 + j\ 2.44$

$a = 1\ /\ 1.87 = .535$

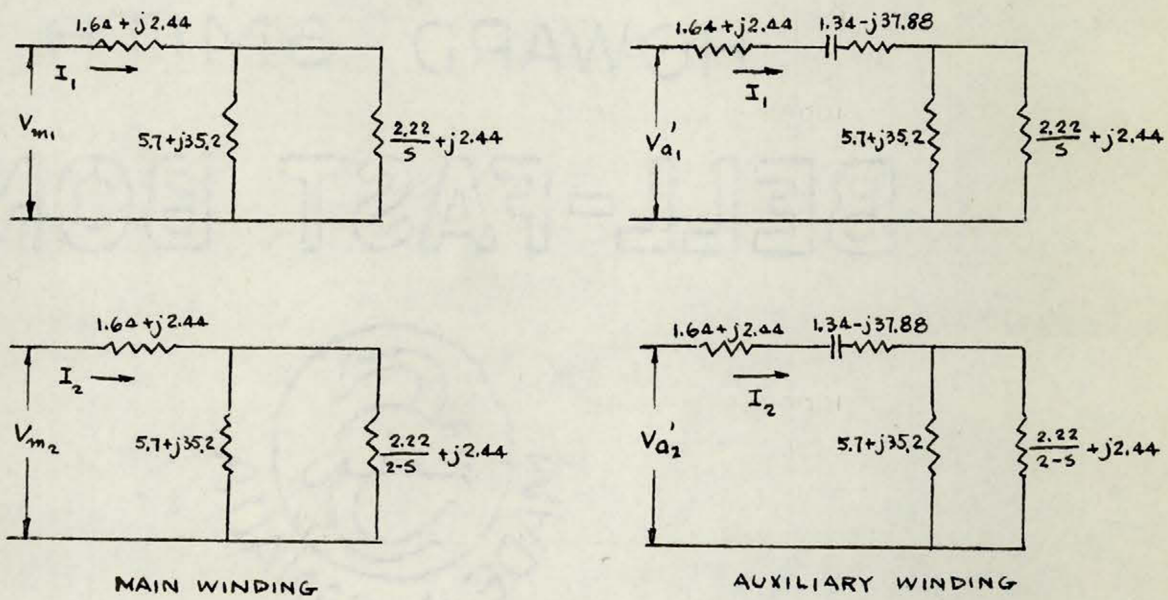


Fig.17-1. Positive- and negative-sequence component equivalent circuits of a capacitor-start motor, 110-v., 1800 r.p.m., 1/4 hp., operated as a capacitor motor with a running capacitor of 20-μf. Calculations of these equivalent circuits are given on P.33a.

is operated as a single-phase induction motor. Among these advantages are: higher efficiency and power factor, increased power output or torque at reduced line current, more stable and quiter operations. Experiments have been made on the capacitor-start motor in Part I for capacitor-run operation at various running capacitors. Experimental results are given in Fig.16-2 and 16-3. A low-starting torque one-value capacitor motor was also available for the same experiments, which will be discussed later in Part 3 of this paper.

It would be advantageous to familiarize ourselves with the equivalent circuits described in Art.13 before proceeding further with discussion of the capacitor motor. For this purpose, the complete performance calculation of the capacitor motor will first be given in the suggested calculation sheet illustrated on P. 33a.

The equivalent circuits shown on P. 32a. represent the single-phase induction motor of Part I, operated as a capacitor motor with a running capacitor of 20 micro-farad. The rotor resistances in the positive- and negative-sequence circuits have been assumed identical. However, this assumption does not hold for larger motors where skin effects in the rotor bars become important, and an increase of fifty to eighty per cent to the negative-sequence rotor resistance should be added. At near synchronous speed, the rotor is subjected to a negative-sequence rotating field which has a double frequency, i.e., two times the rated frequency, when referred to the rotor.

The calculations shown on P.33a are mostly vectorial, which might discourage those who are not interesting to deal with vectors. However, a slide rule capable of converting the vectors in polar

CAPACITOR MOTOR CHARACTERISTICS

MOTOR RATING: 1/4 HP, 1800 RPM

V	110	a	1/1.87	R_r	2.22
Z_m	$1.64 + j2.44$	a^2	.286	X_r	2.44
Z_M	$3.86 + j4.88$	Z_c	$-j132$	S	.0416
Z_a	$10.4 + j8.55$	Z_S	$18.2 - j114.9$	P_{f+w}	8
Z_A	$18.2 + j17.1$	Z_ϕ	$5.7 + j35.2 = 35.68\angle 80.8$		

1	R_r/S	53.4	8	$R_r/(2-S)$	1.13
2	(1) $+ jX_r$	$53.5\angle 2.62$	9	(8) $+ jX_r$	$2.69\angle 65.1$
3	(2) $+ Z_\phi$	$70.2\angle 32.5$	10	(9) $+ Z_\phi$	$38.2\angle 79.7$
4	(2) $\times Z_\phi$	$1908\angle 83.42$	11	(9) $\times Z_\phi$	$96\angle 145.9$
5	(4)/(3)	$27.14\angle 50.92$	12	(11)/(10)	$2.52\angle 66.2$
6	$Z_1 = Z_m +$ (5)	$18.74 + j23.5 = 30.1\angle 51.4$	13	$Z_2 = Z_m +$ (12)	$2.74 + j4.74 = 5.48\angle 60$
7	$(1+ja)Z_1$	$6.18 + j33.54 = 34.1\angle 79.6$	14	$(1-ja)Z_2$	$5.28 + j3.29 = 6.22\angle 31.9$
15	$Z_1 + Z_2$	$21.48 + j28.24 = 35.5\angle 52.73$			
16	$Z_1 \times Z_2$	$-60.25 + j153 = 164.5\angle 111.4$			
17	$a^2 Z_S - Z_M$	$1.34 - j37.88 = 37.88\angle -87.96$			
18	$D =$ (16) $+ .5 \times$ (15) $\times$ (17)	$488.7 - j235 = 542\angle -25.7$			
19	$I_1 = \frac{jV}{2D}$ [(14) + (17)]	$2.86 + j2.12 = 3.56\angle 36.6$	23	$I_2 = \frac{-jV}{2D}$ [(7) + (17)]	$-.068 - j.88 = .88\angle -94.3$
20	$I_{r_1} =$ (19) $\times Z_\phi/$(3)	$1.81\angle 84.8$	24	$I_{r_2} =$ (23) $\times Z_\phi/$(10)	$.822\angle -93.2$
21	$I_{\phi_1} =$ (19) $\times$ (2)/(3)	$2.71\angle 6.7$	25	$I_{\phi_2} =$ (23) $\times$ (9)/(10)	$.062\angle -108.9$
22	$T_1 = 2 \times$ (20)$^2 \times$ (1)	350	26	$T_2 = -2 \times$ (24)$^2 \times$ (8)	-1.52
27	$T =$ (22) + (26)	348.48 SYN. WATT.			
28	$P_{out} =$ (27) $(1-S) - P_{f+w}$	326			
29	$\tau = 112.6 \times$ (28)/R.P.M.	21.3 OZ.-FT.			
30	$I_m = -j$ (19) $+ j$ (23)	$3 - j2.93 = 4.18\angle -44.3$			
31	$I_a = a$ [(19) + (23)]	$1.49 + j.664 = 1.63\angle 24$			
32	$I_\ell =$ (30) + (31)	$4.49 - j2.27 = 5.03\angle -26.8$			
33	P.F.	.892			
34	$P_{in} = V \times$ (32) $\times$ (33)	496			
35	LOSSES = (34) − (28)	173			
36	EFF. = (28)/(34)	65.8 %			

STARTING CHARACTERISTICS

37	$I_M = V/Z_M$	$17.7\angle -51.6$	38	$I_A = V/Z_S$	$5.08\angle 21.4$
39	$I_L =$ (37) + (38)	$19.8\angle 37.4$	40	P.F.	.796
41	$P_{IN} = V \times$ (39) $\times$ (40)	1730	42	$\sin\theta_{M-A}$	.956
43	$Z_\phi^2/[(R_\phi + R_r)^2 + (X_\phi + X_r)^2]$				.862
44	$T_S = 2 \times 112.6 \times$ (37) $\times$ (38) $\times$ (42) $\times$ (43) $\times R_r/[a \times$ Syn. r.p.m.$]$				38.6 OZ.FT.

* $Z_S = 20.2 - j7.9 = 21.7\angle -21.4$ AT STARTING.

Fig. 17-2.

form into their real and j terms, or vice versa, would greatly reduce the calculating labour. For testing and checking purposes, some items in the calculation sheet given on P.33a. may be omitted, while the line current may be calculated by Eq. 15-6.

Calculated results on P.33a are ready to be compared with the experimental values plotted on P.31c. Close checks are obtained for the torques, the auxiliary winding current I_a and the line current. Calculated power factors are shown higher than the experimental values while the calculated efficiencies are lower. This is believed to be due to an overestimated exciting resistance. More satisfactory checks will be shown later in Part 3 where the machine constants calculated from design data are used for the performance calculation.

18. Vector Diagram and Loci of Current Vectors of Capacitor Motor.

The vector diagram of the capacitor motor can be conveniently drawn when the calculation sheet is completed. Readers are referred to Fig.25-4. where the vector diagram of a typical capacitor motor is given.

An interesting study of the loci of the current vectors of the capacitor-start motor has been made by the calculations of the equivalent circuits at several slips ranging from zero to unity. A capacitor of 106 μf is used for starting, and a capacitor of 20-μf for the running operation. The calculated results are shown on P. 34a and 34b.

Readers are also referred to Fig.24-2, where the loci of the positive- and negative-sequence component currents of a typical capacitor motor are given.

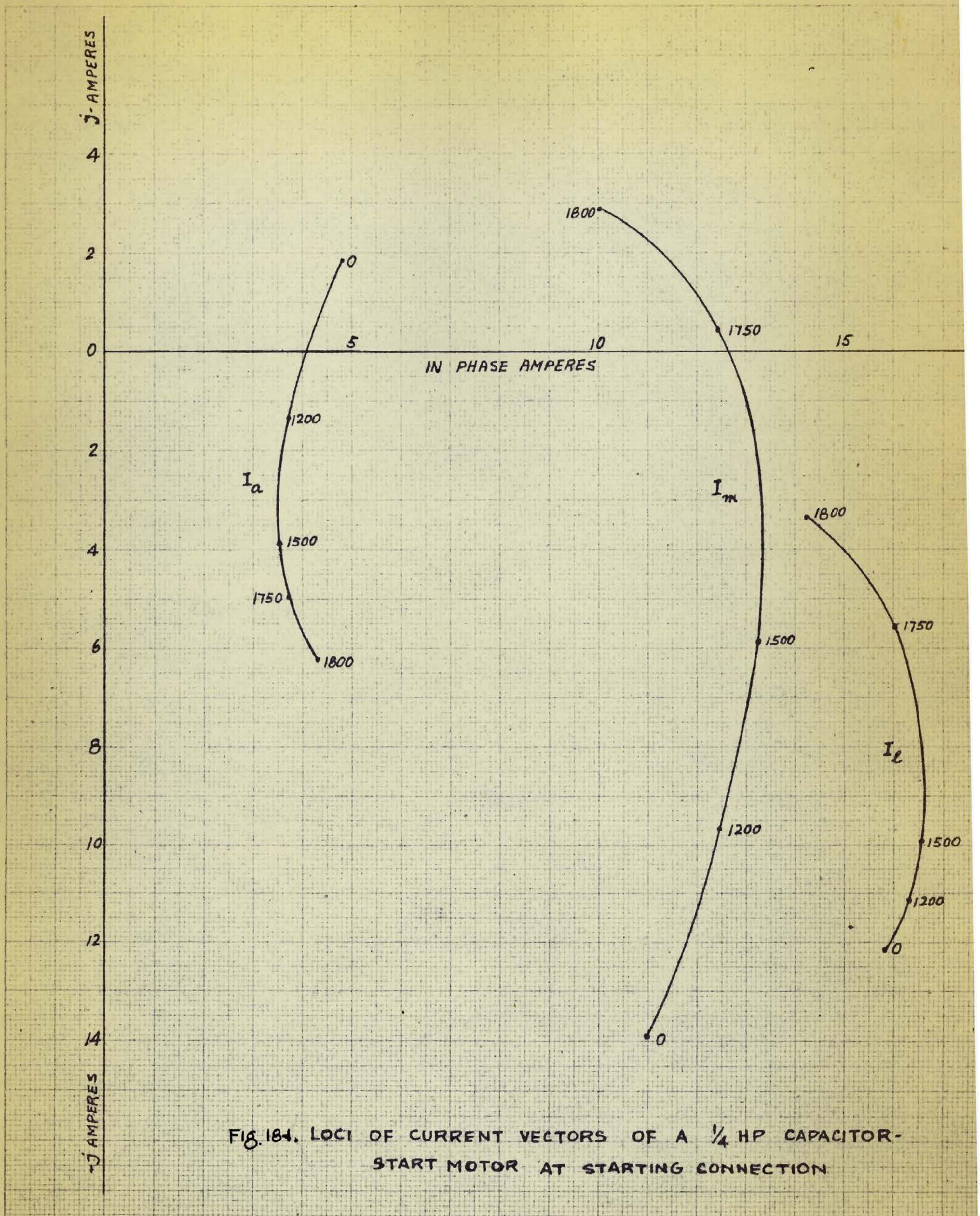
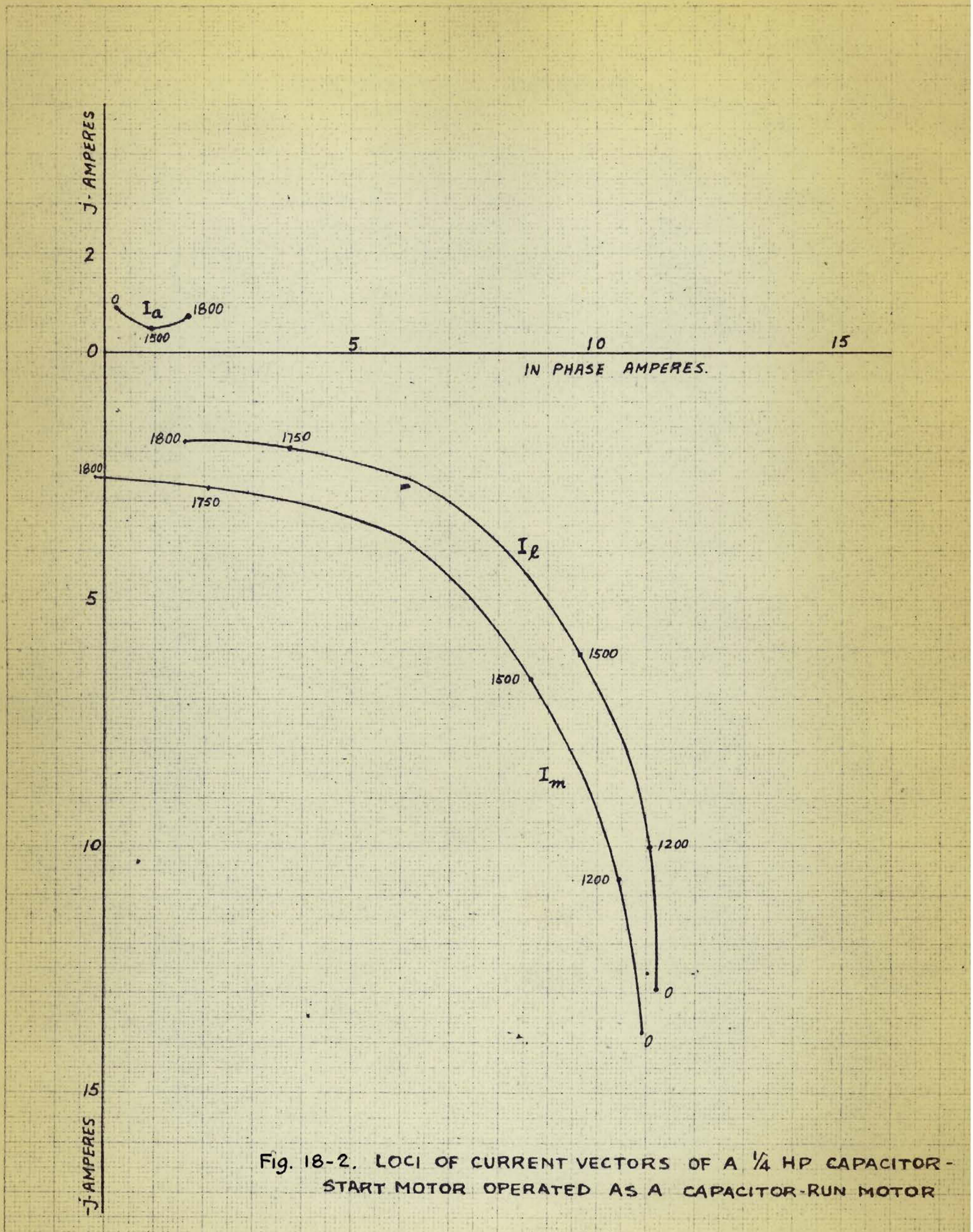


Fig. 18-1. LOCI OF CURRENT VECTORS OF A 1/4 HP CAPACITOR-START MOTOR AT STARTING CONNECTION

Fig. 18-1.



j·AMPERES
-j·AMPERES
IN PHASE AMPERES.
0
5
10
15
2
Ia
1800
1500
Iℓ
Im
1750
1200
Fig. 18-2. LOCI OF CURRENT VECTORS OF A ¼ HP CAPACITOR-START MOTOR OPERATED AS A CAPACITOR-RUN MOTOR

Fig. 18-2.

19. The Pull-Out Slip of Capacitor Motor.

There seem to have no simple method in determining the pull-out slip of the capacitor motor, and it will be extremely difficult and tedious by the method of differentiating the torque equation. It would be wise, therefore, to estimate the pull-out slip, which ranges from .1 to .3 for most machines; or by using the following expression for estimating purpose.

$$S_{P.O.} \doteq 1 - \left[1 - \frac{2R_r}{X_M + X_S} \right]^{\frac{1}{2}}$$

20. Conclusion.

During the capacitor-run operation, the current in the auxiliary winding is nearly in phase quadrature with that of the main winding, and the motor behaves as a two-phase motor. The positive-sequence and negative-sequence m.m.f.s combine vectorially into a resultant m.m.f., rotating in synchronous speed, is obviously greater than that of the main winding alone when the motor is operated as a single-phase induction motor. An experiment on the air gap flux by means of an oscilloscope picking up flux wave in the air gap by placing a few turns of fine wires along the slots when the motor is operating showed that the peak of the flux wave of the capacitor-run operation may be twice as great as that for the single-phase induction-run operation; and is entirely dependent on the capacitance of the capacitor and the condition of the load. These facts can be visualized by comparing the experimental results in Fig.16-2 aand 16-3. At a running capacitor of 20-μf, the most desirable performance is obtained. However, such a capacitor would be impracticable for this motor because of its high cost and size. Note that when a capacitor of 8-μf is used, the motor output at a speed of 1725 r.p.m. is almost 1/3 hp.

The calculation of the equivalent circuits of the capacitor motor also indicate a great reduction of the negative-sequence torque, which may be neglected when a larger capacitor is used. For a 8-μf running capacitor, the negative-sequence torque is only one half of that when the same motor is operated single-phase run and at the same speed. Except at large slips, the negative-sequence component current in the capacitor motor is much smaller than that of the same motor operated single-phase induction-run, as in the later case, the positive- and negative-sequence currents are equal. A similar statement can be made about the positive- and negative-sequence exciting currents and therefore, the corresponding rotating fields.

Discussions of design problems and the relation between machine constants and the performance of the capacitor motor will be given in Part 3. where the equivalent circuits set up with machine constants calculated from design data will be used.

PART 3

Design of Capacitor Motor.

The similarity of the equivalent circuits of the three-phase induction motor and the capacitor motor has been an advantage in discussing the operation theory and in calculating the performance of the latter. In the design of the capacitor motor, the calculations of the leakage reactance, the magnetizing reactance as well as the rotor resistance are much the same for these two types of motor because their constructions are similar in many respects. It is the writer's attempt to lay out the general procedures for the design of the capacitor motor by making use of those for the three-phase induction motor; and to discuss the sailient points regarding the design problem as observed from the studies of the equivalent circuits.

A typical low-starting torque one-value capacitor motor and a capacitor-start motor together with design data have been kindly supplied by Prof. W. H. Schippel of McGill University for the above purposes.

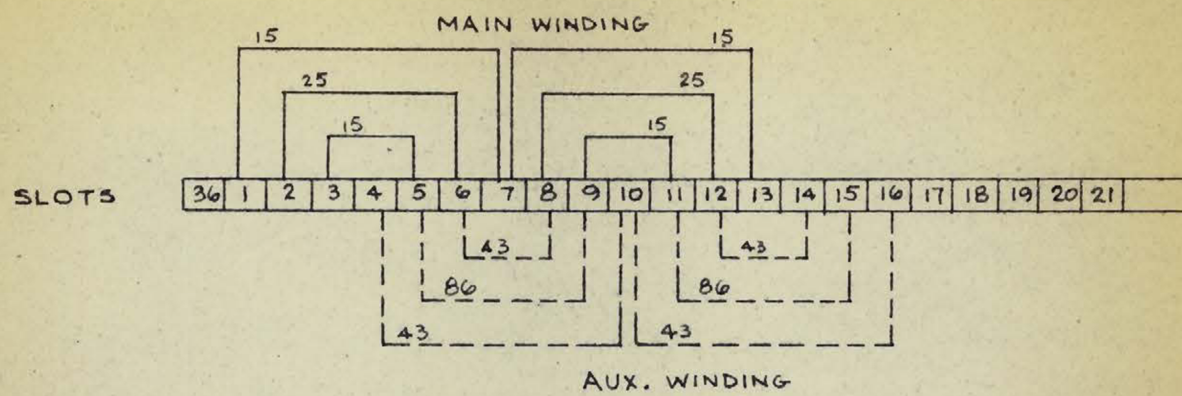
21. The Armature Windings.

Unlike the three-phase induction motor, the capacitor-start or capacitor-run motor has two windings, namely: the main winding, and the auxiliary winding, placed 90 electrical degrees apart in the armature. In the design of the capacitor-start motor, extra attention must be paid to the main winding in order to obtain good performance. The auxiliary winding, which will be switched off after the motor reaches a predetermined speed, may be designed to develope the required starting torque by use of a proper capacitor to suit the majority of the design of the main winding. The application of the capacitor in the auxiliary winding provides a wider tolerance in the design of the auxiliary winding for this type of motor and a much higher starting torque than any single-phase motor using another method of starting. In the design of the capacitor motor, especially the one-value capacitor motor, care must be given to the arrangement between the main and auxiliary windings in order to obtain good performance and high starting torque from a capacitor with lowest possible capacitance.

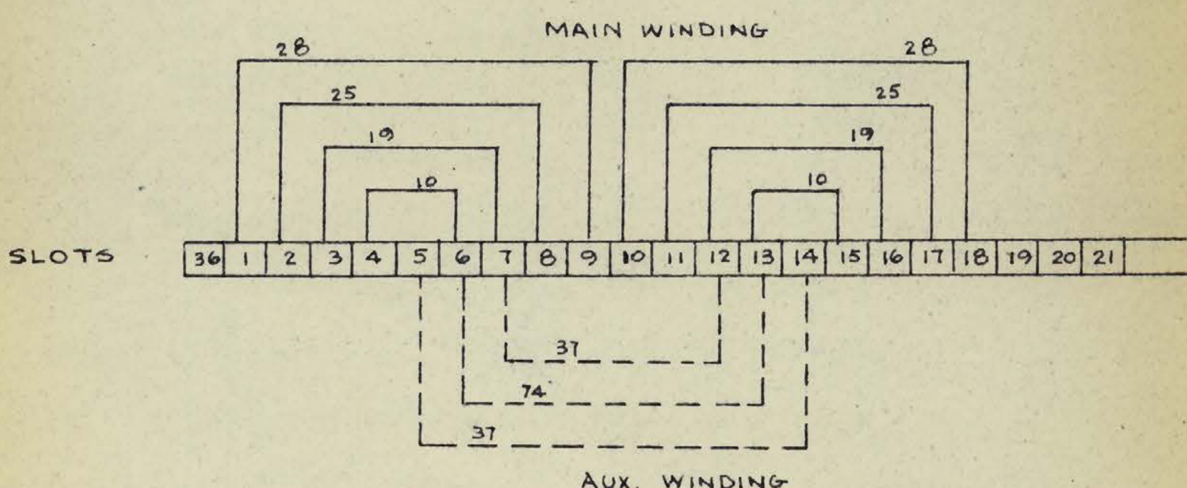
22. Type of Windings.

The stator windings of most fractional-horsepower motors are of the concentric type which have different numbers of turns per pair of slots under one pole in order to produce sinusoidal flux distribution in the air gap. Skein windings are extensively used for auxiliary windings which have comparatively more turn of smaller size. Fig.22-1 show the winding distribution of a typical capacitor-start motor, 110-v., 1/4 hp., 1800 r.p.m. with 36 slots and 82 turns per pole; and the winding distribution of a typical one-value capacitor motor, 110-v., 1/4 hp., 1200 r.p.m. with 36 stator slots and 52 turns per pole.

Note that the winding pitch of the inner pair of slots of the concentric winding is very short yet it prevents the flux wave from being flat-topped. Any harmonics resulting from non-sinusoidal flux wave would produce undesired harmonic torques which give rise to



WINDING DISTRIBUTION OF A TYPICAL CAPACITOR MOTOR
110V. 1/4 HP 60~ 1200 R.P.M. WITH 6 SLOTS PER POLE.



WINDING DISTRIBUTION OF A TYPICAL CAPACITOR-START
MOTOR. 110V. 1/4 HP 1800 R.P.M. 4 POLE. 36 SLOTS.

Fig. 22-1.

the pulsation and noise in the operation of single-phase induction motor. Factors affecting the air gap flux wave-form of single-phase induction motor include the choice of number of stator and rotor slots, the skew of the rotor bars and the shape of the teeth and slot openings. Readers are encouraged to read those technical papers prepared by experienced designers concerned with these subjects.

23. <u>The E M F Equation.</u>

The basic EMF equation for alternating machine design is:

$$E = 4.44\, f\, N\, K_p\, K_d\, \Phi\, 10^{-8}$$

in which, when applied to the single-phase induction motor:

E = Induced e.m.f. per phase; approximately one half of the applied voltage.

f = frequency of line voltage in cycles per second.

N = number of turns per phase; or one half the actual turns in series of the winding.

K_d = winding distribution factor.

K_p = winding pitch factor.

In the single-phase induction motor, the number of turns would not be the same in each coil, and the factors K_d and K_p can not be determined, as for the three-phase induction motor, from the slots per pole per phase and from the winding pitch. By assuming a sinusoidal flux wave distribution, an average winding factor can be applied to the main and auxiliary windings as given below:

$$K_d K_p = \frac{\Sigma \left\{ \text{TURNS PER COIL} \times \text{SIN} \left[\%\ \text{COIL SPAN} \times \frac{\pi}{2} \right] \right\}}{\text{NO. OF TURNS IN SERIES PER POLE.}}$$

24. The Exciting Current and Exciting Impedance.

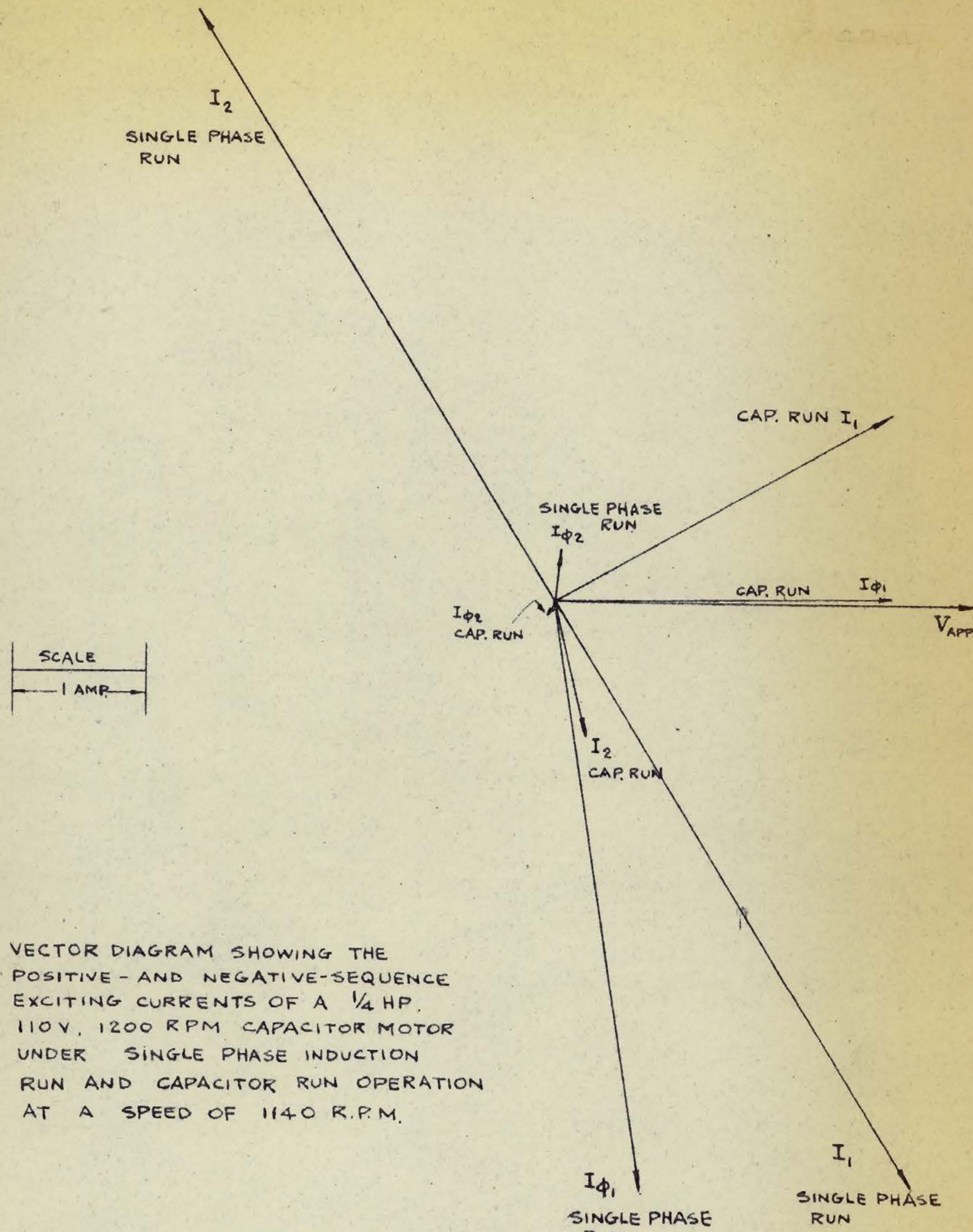
As they are restricted by size and cost, single-phase induction motors have comparatively higher no load currents than the three-phase induction motors. At no load, the resistance of the positive-sequence rotor circuit is very high so that the positive-sequence exciting current is practically equal to the no load current. The negative-sequence exciting field is weak at no load as there is a greater amount of no load current through the negative-sequence rotor circuit.

When the same motor is operated as a capacitor motor with a proper running capacitor, the required m.m.f. is provided by the exciting currents of the two windings. It had been pointed out in Part 2 that the negative-sequence current of the capacitor motor is much smaller when compared with the same motor operated single-phase induction-run, thus the negative-sequence exciting current is further reduced. Vector diagrams showing the exciting currents of a capacitor motor undersingle-phase induction-run and capacitor-run operations are shown in Fig.24-1. The loci of the vector currents of the same motor are shown in Fig.24-2.

The method of calculating the magnetizing reactance for three-phase induction motor has been found very satisfactory for the single-pha se induction motor. The formula adopted here is:

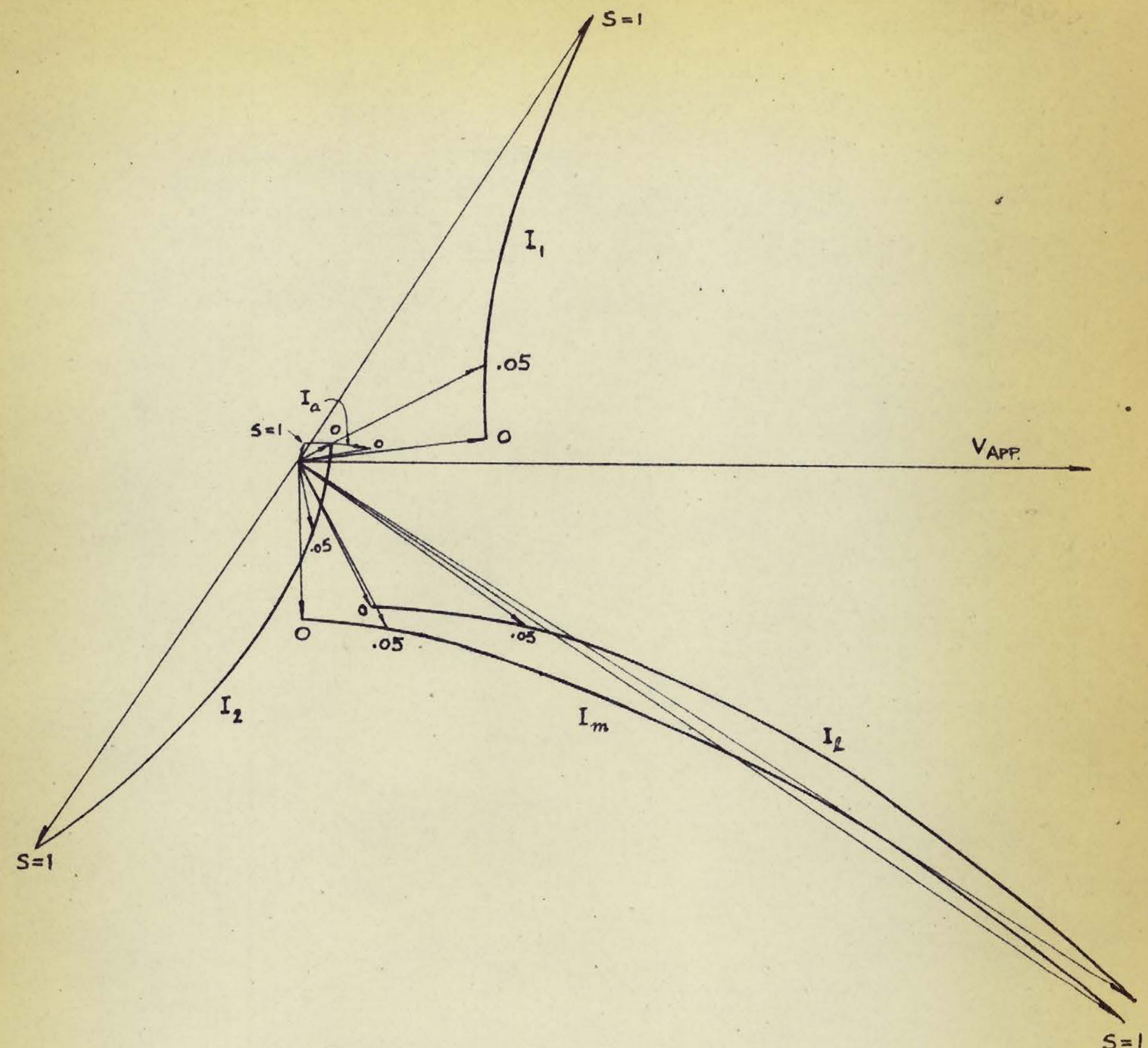
$$X_\phi = \frac{25.5\, f\, m\, D L N^2 K_d K_p}{p^2\, \delta\, C_1 C_2 \cdot 10^8}$$

where f = frequency in c.p.s.



VECTOR DIAGRAM SHOWING THE POSITIVE- AND NEGATIVE-SEQUENCE EXCITING CURRENTS OF A 1/4 HP. 110V. 1200 RPM CAPACITOR MOTOR UNDER SINGLE PHASE INDUCTION RUN AND CAPACITOR RUN OPERATION AT A SPEED OF 1140 R.P.M.

Fig. 24-1.



LOCI OF CURRENT VECTORS OF A
ONE VALUE CAPACITOR 110-V 1/4 HP 1200 R.P.M.
CONSTRUCTED FROM THE EQUIVALENT CIRCUITS
AT SLIPS OF ZERO, .05 AND UNITY.

Fig. 24-2.

m = number of phases.

D = inside diameter of the armature in inches.

L = length of armature core in inches.

N = number of turns in series per phase.

p = number of poles.

δ = air gap length in inches.

C_1 = Cartier's coefficient for stator teeth.

C_2 = Cartier's coefficient for rotor teeth.

It should be remembered that the magnetizing reactance calculated from the above equation should be multiplied by 2 when used in the equivalent circuit of the single-phase induction motor, and would be four times as great when used in the equivalent circuit of the capacitor motor.

The resistance which is in series with the magnetizing reactance has been termed as the exciting resistance, which when multiplied by the square of the exciting current would give the core loss. This resistance can be determined, when the total ampere-turns per pole required at no load and the no load core loss are known, by the following expression:

$$R_\phi = \frac{\text{CORE LOSS PER POLE AT NO LOAD}}{I_{\phi_1}^2}$$

25. Locked-Rotor Impedances.

In the design of capacitor-start motor, starting torque as high as four times the full load torque can be obtained by using high capacitance capacitor and a higher current density in the auxiliary winding. The locked-rotor impedance of the auxiliary winding should therefore be designed to suit the main winding to give the best performance. Readers are referred to Fig.16-1, where an alysis of the starting torque versus various locked-rotor impedance of the auxiliary of a capacitor-start motor is given.

In the case of the capacitor motor, high starting torque can be obtained only when a starting capacitor in parallel with the running capacitor is used. Starting torque will be a problem for the one-value capacitor motoraas the permissible capacitance is so low that a high starting torque can not be obtained by using higher rotor resistance or turn-ratio without effecting the performance of the motor under normal operation.

Fig.25-1. shows the tabulated results calculated for the one-value capacitor motor, the starting characteristics and the performances at about rated speed on different locked-rotor impedances by using various possible conductor sizes and the permissible running capacitors.

Methods of calculating the locked-rotor resistance and reactance from design data are the same as for the three-phase induction motor and will not be given here.

The design data of this motor is given on P.42-b; and the complete analysis of the equivalent circuits of this motor are given on P. 42c to 42-f, inclusive.

Fig. 25-5 shows the performance of this motor from actual load tests. The close checks between the calculated and experimental results indicate the correctness of the calculated machine constants and the equivalent circuits. The performance of the same motor operated with its main winding alone is shown in dotted

MAIN WDG	2-#20	2-#20	2-#20	2-#20	2-#21	2-#22	2-#21	2-#21
AUX. WDG	1-#26	1-#26	1-#25	1-#27	1-#25	1-#24	1-#25	1-#25
$\sqrt{R_a/R_m}$	5	5	4.43	5.6	3.96	3.15	3.96	3.96
μF	0 7 - FOR START	7	7	7	7	7	9	9
Z_m	1.11 + j1.05	2.22 + j2.1	2.22 + j2.1	2.22 + j2.1	2.8 + j2.1	3.52 + j2.1	2.8 + j2.1	2.8 + j2.1
Z_a	27.8 + j10.8	55.5 + j21.5	43.7 + j21.5	70 + j21.5	43.7 + j21.5	34.8 + j21.5	43.7 + j21.5	43.7 + j21.5
$Z_{\dot{M}}$	$7.72\angle 33$	$7.72\angle 33$	$7.72\angle 33$	$7.72\angle 33$	$8.2\angle 30.8$	$8.8\angle 28.5$	$8.2\angle 30.8$	$8.2\angle 30.8$
Z_S	$350\angle -73.6$	$350\angle -73.6$	$347\angle -75.4$	$355\angle -71.3$	$347\angle -75.4$	$345\angle -76.9$	$253\angle -69.9$	$253\angle -69.9$
I_M	14.25	14.25	14.25	14.25	13.4	12.5	13.4	13.4
I_A	.314	.314	.317	.31	.317	.32	.434	.434
I_L	14.12	14.12	14.15	14.1	13.1	12.4	13.3	13.3
$\sin\theta_{M\text{-}A}$	.958	.958	.948	.968	.958	.965	.985	.985
T_S OZ-FT.	9.55	9.55	9.54	9.54	9.08	8.53	12.8	12.8
R.P.M.	1140	1140	1140	1140	1140	1140	1140	1125
I_1	5.13	2.91	2.92	2.88	2.90	2.86	2.97	3.08
I_2	5.13	.976	.90	1.08	.878	.808	1.53	1.63
I_m	5.13	3.27	3.23	3.34	3.16	3.04	2.74	3.23
I_a	0	.894	.902	.873	.910	.912	1.21	1.17
I_ℓ	5.13	3.87	3.81	3.95	3.74	3.6	3.77	4.24
$P_{OUT.}$	141	209	211	204	210	202	215	254
TORQ. OZ-FT	13.92	20.6	20.8	19.1	20.7	18.9	21.2	25.4
$P_{IN.}$	302	350	342	356	342	333	383	433
EFF.	46.7	59.7	61.7	57.3	61.2	60.6	56.2	58.7
P.F.	.534	.82	.817	.82	.83	.84	.925	.93

PERFORMANCE AND STARTING CHARACTERISTICS OF A 1/4 HP CAPACITOR MOTOR 110/220 V. 1200 R.P.M. CALCULATED FOR VARIOUS DESIGNS OF AUXILIARY WINDING & CAPACITOR.

Fig. 25-1.

INDUCTION MOTOR — DESIGN SHEET

STATOR Lam No. E-6664

Diam. ext.	6.25	
int.	3.75	
Core stack.	3.5	
ducts	—	
Stack. factor	.95	
Iron	24 E I	
No. of slots	36	
Core depth	.469	
Slot depth	.781	
width		
Eq. area	.141	
Tooth width	.140	
Slot insul.	.020	
Wdg. type	CONC.	
connect.	2-CKT.	
Slots/ph./pole	5	
Turns/coil	15-25-15	
Cond./phase	660	
Cond. width	20 H.F.	
depth	20 E C	
Factor space	.64	
dist. / pitch	.667	
Pitch coil	1-7	
slot	.327	
pole	1.96	
Heat factor		
Cond. density		
length	6.5	
R at 75°	2.22	
volts at 75°		

ROTOR Lam No. E-9147

Diam. ext.	3.72	
int.	.75	
Core stack.	3.5	
ducts	—	
Stack. factor	.95	
Iron	24 E I	
No. of slots	45	
Core depth		
Slot depth		
width		
eq. area		
Tooth width		
Slot insul.		
Wdg. type	S.C.	
connect.		
Slots/ph./pole		
Turns/coil		
Cond./phase		
Cond. width	#8	
depth	.013 DIA	
Material	CU.	
Conductivity %		
Factor space		
dist.		
pitch		
Pitch coil		
slot		
Heat factor		
Cond. density		
length		
R at 75°		
volts at 75°		

WEIGHTS

Stator Wdg.	2.3	
aux.	1.5	
Rotor cond.		
ring.		
Stator teeth		
core		

MISCELLANEOUS DATA.

FAN		

AUX. WDG.

Capacitor	7 μF	
Wdg. type	SKEIN	
Slots per pole	5	
Turns per coil	43-86-43	
Conductor	26 EC	
Coil pitch	4-10	MAX.
Cond. length	6.45	
R. at 75°	55.5	
Rotor resist.	42.4	
Locked, X	43	
Cap. X	379	
Z	350	
I	.314	
Starting amps.	14.12	LINE

LOSSES

Cond. stator	68	
rotor	11.6	
Iron tooth / core / surface	53.4	
Wind. & frict.	8	
Brush resist.		
frict.		
Total	141	
Efficiency %	59.7	

RATING

Power	209	
Volts	110	
Phases	1	
Freq.	60 ~	
R.P.M.	1140	
Poles	6	
P.F.	.82	
Amperes	3.87	
Frame	145-2	
Enclos.	T.E.	
Temp. rise	55	
Torque start. %	50	
max. %	250	
Class		
Type	CAP. - RUN.	
Same as		
Similar to		
Supersedes		

MAGNETIC CIRCUIT

Area, gap	6.86	
S. tooth	3.38	
S. core	1.56	
R. tooth	3.06	
R. core	4.38	
Phase volts	110	
Flux/pole	204 KL	
Density, gap	46600	MAX.
S. tooth	96000	
core	65600	
R. tooth	104000	
core	23300	
Gap length	.015	
Slot open, S.	.095	
R.	.031	
Fring. stator	1.17	
rotor	1.03	
A.T. gap	264	
S. tooth	39	
core	3	
R. tooth	21	
core		
Total	327	
Eff. turns		
Amperes mag.	4.2	MAIN WINDING ALONE
" gap.	3.4	

ROTOR DESIGN

Equiv. N_1		
N_2		
Actual I_2		
Start. torq. %		
loss		
Rings ax. width	.0625	
Rad. Depth	1.48	
Material	BRASS	
Conductivity %	26	
Resist. factor		
bars		
rings		
Equiv. R_2 at 75°	4.24	
Locked, X	4.2	
Z	7.72	
I	14.25	
%		
K.W. bars		
rings.		
KW/lb. bars		
rings		
Bar length	3.6	
Slip %		
Skew angle	2 RSP	
factor		

Fig. 25-2.

Designer........................ Date........................ **DESIGN No.** A-1921

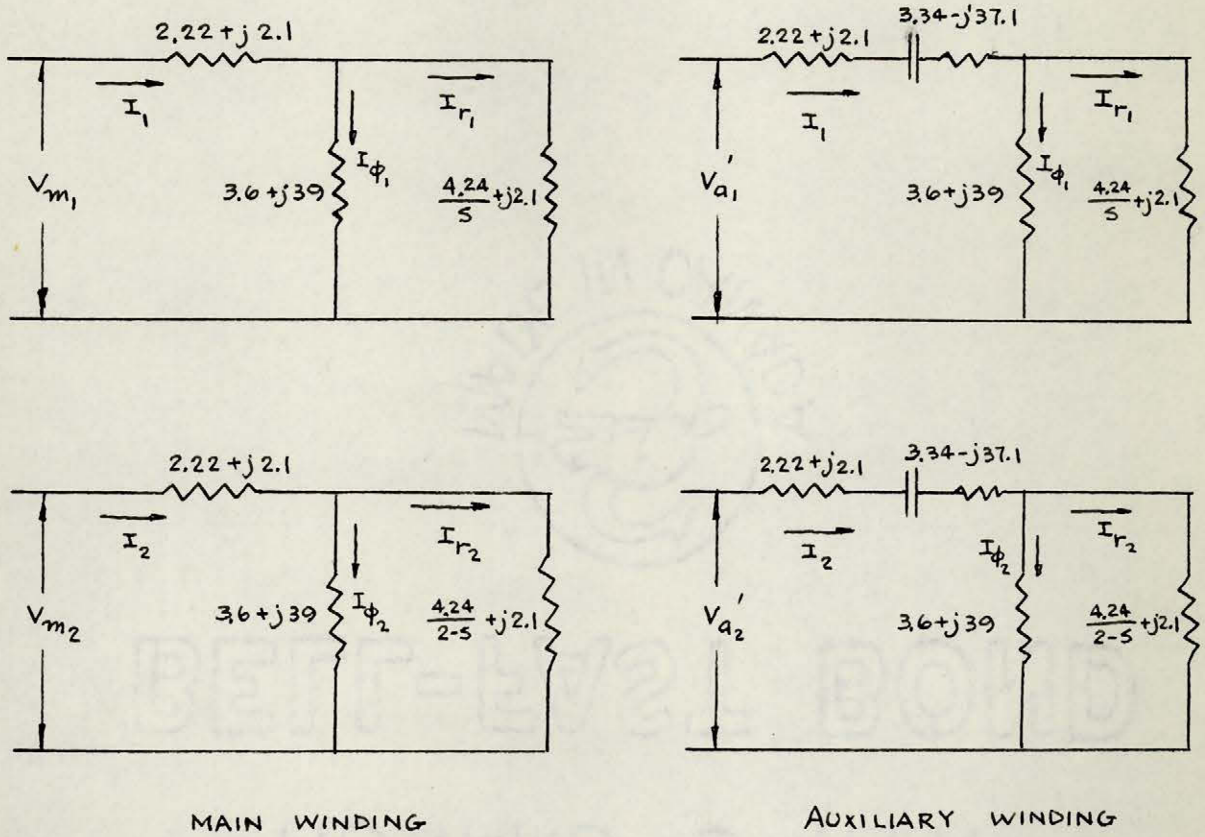


Fig. 25-3. Equivalent circuits of the capacitor motor (see P. 42-b)constructed by the method of positive- and negative-sequence right-angle components. Constants are calculated from design data given in Fig. 25-2. Calculations of these equivalent circuits are given on P. 42d. Performance characteristics of this motor from actual load tests are shown in Fig. 25-6.

CAPACITOR MOTOR CHARACTERISTICS

MOTOR RATING 1/4 H.P. 1200 RPM

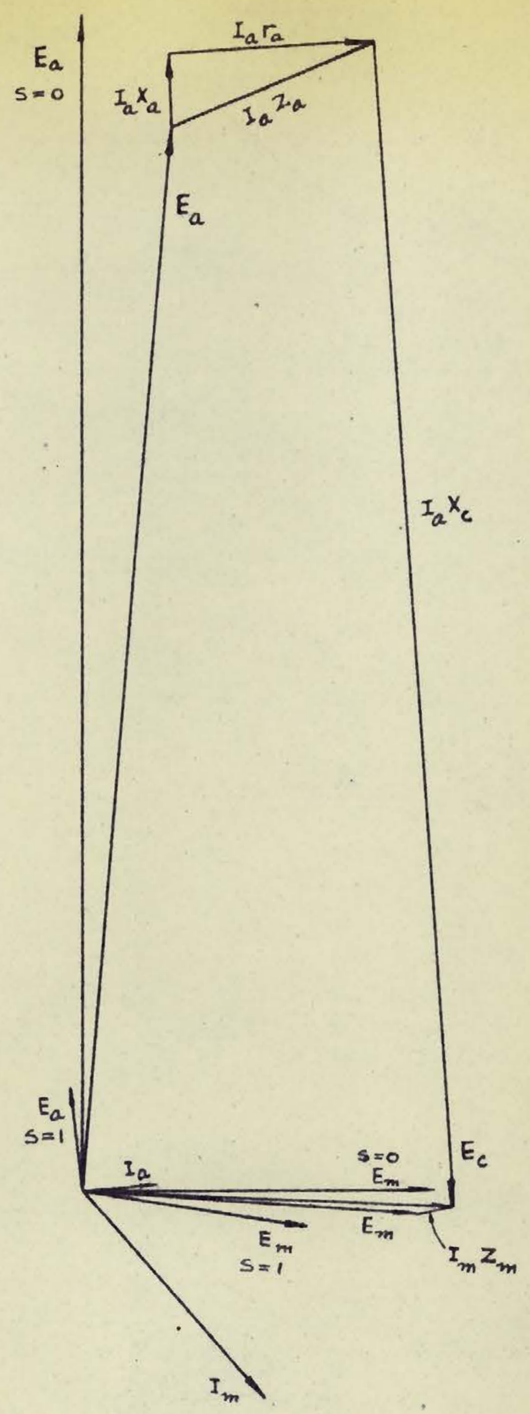
V	110	a	1/3.2	R_r	4.24
Z_m	2.22 + j 2.1	a^2	.0976	X_r	j 2.1
Z_M	6.46+j4.2 = 7.72∠33	Z_c	-j 379	S	.05
Z_a	55.5 + j 21.5	Z_s	98.9 - j 336 = 350∠-73.6	P_{f+w}	8
Z_A	98.9 + j 43.1	Z_ϕ	3.6 + j39 = 39.2∠84.7		

1	R_r/s	84.8	8	$R_r/(2-s)$	2.17
2	(1) + j X_r	84.8 + j2.1 = 84.8∠1.4	9	(8) + j X_r	2.17 + j 2.1 = 3.04∠44
3	(2) + Z_ϕ	88.4 + j41.1 = 89.4∠25	10	(9) + Z_ϕ	5.77 + j41.1 = 41.4∠82
4	(2) × Z_ϕ	3324∠86.1	11	(9) × Z_ϕ	59.6∠128.7
5	(4)/(3)	34.2∠61.1 = 16.5 + j30	12	(11)/(10)	2.88∠46.7 = 1.98 + j2.1
6	$Z_1 = Z_m$ + (5)	18.6 + j 32.1 = 37.2∠59.8	13	$Z_2 = Z_m$ + (12)	4.2 + j4.2 = 5.94∠45
7	(1+ja) Z_1	8.65 + j 37.9 = 38.9∠77.2	14	(1-ja) Z_2	5.51 + j2.88 = 6.22∠27.6
15	$Z_1 + Z_2$	22.92 + j36.3 = 42.8∠57.7			
16	$Z_1 \times Z_2$	-56.2 + j214 = 221∠104.8			
17	$a^2 Z_s - Z_M$	3.34 - j 37.1 = 37.2∠-85			
18	D = (16) + .5 × (15) × (17)	650.8 - j 151 = 668∠-13.1			
19	$I_1 = \frac{jV}{2D}$ [(14) + (17)]	2.58 + j1.34 = 2.91∠27.4	23	$I_2 = \frac{-jV}{2D}$ [(7) + (17)]	.237 - j.95 = .976∠-75.9
20	I_{r_1} = (19) × Z_ϕ/(3)	1.17∠87.1	24	I_{r_2} = (23) × Z_ϕ/(10)	.918∠-73.2
21	I_{ϕ_1} = (19) × (2)/(3)	2.53∠3.8	25	I_{ϕ_2} = (23) × (9)/(10)	.072∠-114
22	T_1 = 2 × (20)^2 × (1)	232	26	T_2 = -2 × (24)^2 × (8)	3.7
27	T = (22) + (26)	228.3 SYN. WATT.			
28	P_{out} = (27)(1-S) - P_{f+w}	209 WATT.			
29	T = 112.6 × (28)/R.P.M.	20.6 OZ-FT.			
30	I_m = -j (19) + j (23)	2.29 - j 2.34 = 3.27∠-45.6			
31	I_a = a [(19) + (23)]	.883 + j .122 = .894∠7.86			
32	I_L = (30) + (31)	3.17 - j 2.22 = 3.87∠-35			
33	P.F.	.82			
34	P_{in} = V × (32) × (33)	350			
35	LOSSES = (34) - (28)	141			
36	EFF. = (28)/(34)	59.7 %			

STARTING CHARACTERISTICS

37	$I_M = V/Z_M$	14.25∠-33 = 11.92 - j7.75	38	$I_A = V/Z_s$	.314∠73.6 = .09 + j.301
39	I_L = (37) + (38)	14.12∠-31.8	40	P.F.	.85
41	P_{in} = V × (39) × (40)		42	$\sin\theta_{M-A}$	.958
43	$Z_\phi^2/[(R_\phi+R_r)^2 + (X_\phi+X_r)^2]$				.876
44	T_s = 2 × 112.6 × (37) × (38) × (42) × (43) × R_r/(a × Syn. R.P.M.)				9.55 OZ-FT.

Fig. 25-4.



VECTOR DIAGRAM OF A TYPICAL ONE-VALUE CAPACITOR MOTOR 110-V 1/4 H.P. 1200 R.P.M. $T_a/T_m = 3.2$ AT $S = .05$

Fig. 25-5.

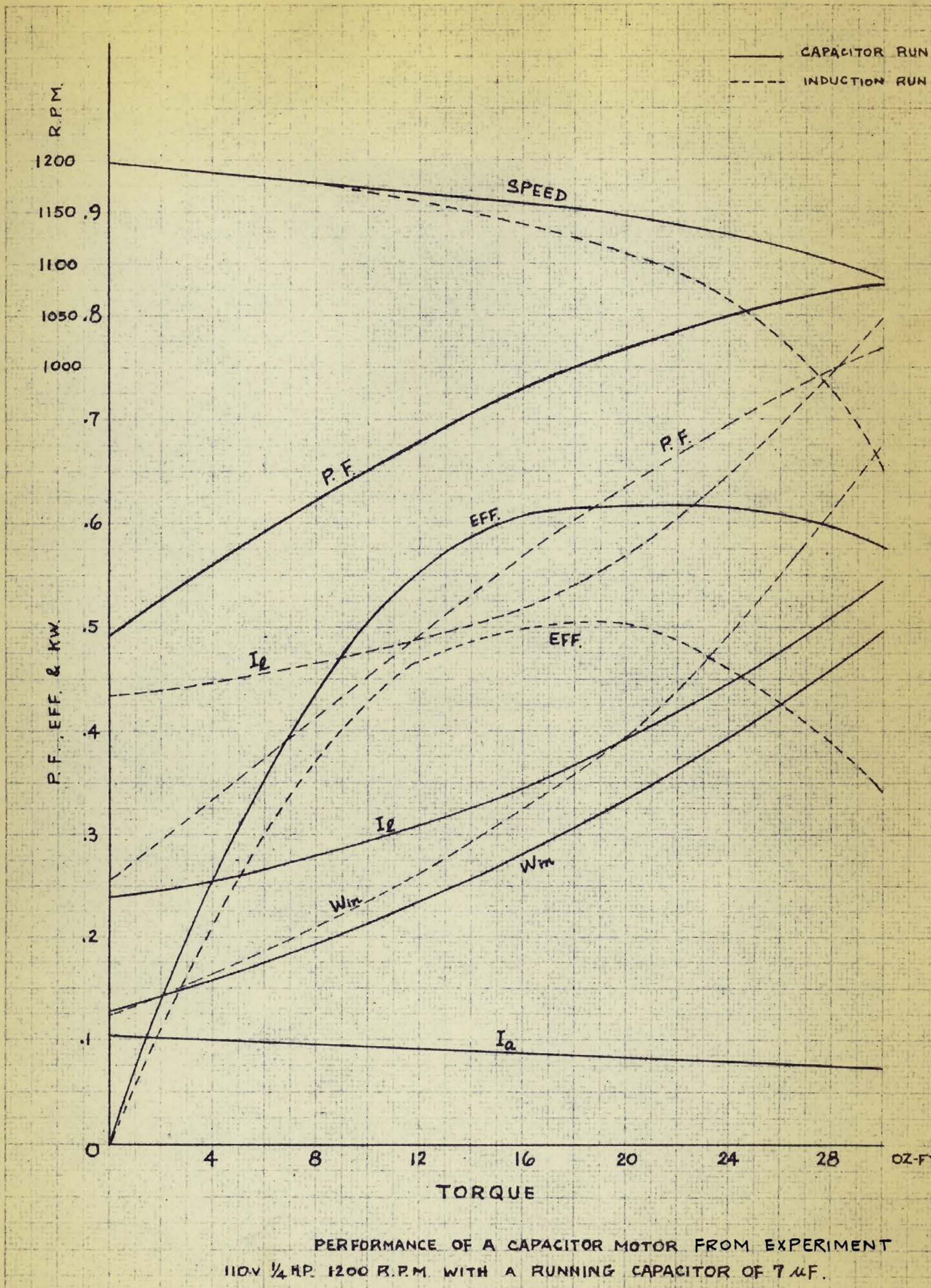

CAPACITOR RUN
INDUCTION RUN
R.P.M.
P.F., EFF. & KW.
1200
1150 .9
1100
1050 .8
1000
.7
.6
.5
.4
.3
.2
.1
0
SPEED
P.F.
P.F.
EFF.
EFF.
I_l
I_l
Wm
Wm
I_a
4
8
12
16
20
24
28
OZ-FT.
TORQUE
PERFORMANCE OF A CAPACITOR MOTOR FROM EXPERIMENT
110 V 1/4 HP 1200 R.P.M. WITH A RUNNING CAPACITOR OF 7 µF.

Fig. 25-6.

lines in the same figure. Performance calculation of such an operation can be made as illustrated in Part I of this paper.

26. Observations.:

The studies of the machine constants and the calculated results listed in Fig.25-1. enable us to sum up a few points concerning the design of the capacitor motor.

First, the turn-ratio of the motor has been chosen to give the possible maximum starting torque for the given capacitor. Calculations show that a reduction of main winding turns and an increase of auxiliary winding turns would result in a much lower starting torque; besides, the exciting current would be too high thus giving a poor performance.

An interesting fact observed is that only small change in starting torque result from changing the conductor size of both the main and auxiliary windings within the allowable slot space, the number of turns of either winding are kept unchanged. However, improvement in performance at rated speed can be observed in some cases as shown in Fig.25-1.

Secondly, the calculated results in Fig.25-1. indicate also that better performance results from more balanced current distribution between the windings, and the wider phase angle between the currents of the main and auxiliary windings. However, exact balanced operation exists only at a certain speed, and final adjustment of the winding sizes and the capacitor must be made in order to obtain balanced operation at rated output and rated speed.

For the winding arrangement given in the last coloum of Fig.25-1 and by using a 9-μf capacitor, the motor output canbe increased to one third of a horsepower at 1125 r.p.m. and a starting torque of about fifty per cent of full load torque. Note that there ism no increase in the main winding current; the increased power output resulting from an increase in positive-sequence current at a higher leading phase angle, which in turn increases the positive-sequence torque. The use of a capacitor of higher capacitance in the auxiliary winding is always accompanied by an increase of the auxiliary winding curent and a higher power factor of the motor.

In conclusion, better performance can be obtained when a capacitor motor is designed for balanced operation at rated speed. Under balanced operation, the induced e.m.f.s in the main and auxiliary windings will be in phase quadrature, with their magnitudes proportional to their turn-ratio; the same relation exists between the currents in the main and auxiliary windings, the m.m.f. in each winding will thus be equal. The symmetrical squirrel cage rotor rotates as in a symmetrical two-phase stator. In other words, the magnetic and electrical loadings will be shared equally between the main and auxiliary windings. A capacitor motor designed for balanced operation would require a smaller capacitor, and in addition, the design and performance calculations would be greatly simplified.

27. Design of Capacitor Motor for Balanced Operation.

Under balanced operation, the negative-sequence component current of the capacitor motor is zero since the main and auxiliary winding currents are in phase quadrature with their magnitudes proportional to their effective turn-ratio. As a result, the performance calculation for such condition will be greatly simplified.

By solving the equivalent circuits shown in Fig.27-1, some useful expressions concerning the design of the capacitor motor can be obtained in the following manner:

(a) The Slip for Balanced Operation.--- To determine the slip for balanced operation of a given capacitor motor, consider the equivalent circuits shown in Fig. 27-1. The voltage equations are:

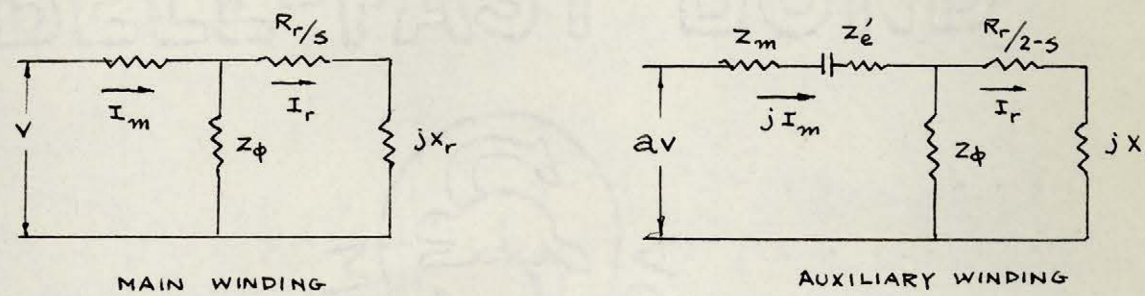


Fig.27-1. Equivalent circuits of capacitor motor under balanced operation.

$$V = I_m Z_1 \qquad (27\text{-}1)$$

$$aV = jI_m(Z_e' + Z_1)$$

OR

$$V = jaI_m\left(Z_e + \frac{Z_1}{a^2}\right) \qquad (27\text{-}2)$$

FROM (27-1) AND (27-2): $Z_1 = jaZ_e + j\dfrac{Z_1}{a}$

$$Z_1 = \frac{jaZ_e}{\left(1 - j\frac{1}{a}\right)}$$

since $$Z_1 = Z_m + \frac{Z_\phi Z_r}{Z_\phi + Z_r}$$

$$\frac{Z_\phi Z_r}{Z_\phi + Z_r} = \frac{jaZ_e}{(1-\frac{j}{a})} - Z_m = A \qquad (27\text{-}3)$$

$$Z_\phi Z_r = AZ_\phi + AZ_r$$

$$Z_r = \frac{AZ_\phi}{Z_\phi - A} = \frac{R_r}{s} + jX_r$$

therefore, $$S_{b.o.} = \frac{R_r}{\frac{AZ_\phi}{Z_\phi - A} - jX_r} \qquad (27\text{-}4)$$

From Eq.27-4, the slip for balanced operation of the capacitor motor described in Part 2 is found to be .248 or approximately 900 r.p.m..

(b) <u>Design of Auxiliary Winding for Balanced Operation.</u> For a given main winding and rotor, the leakage reactance of the auxiliary winding required for balanced operation at given speed can be worked out by a similar method as described in (a). From Eq.27-1 and 27-2, we have:

$$Z_e = \frac{(1-\frac{j}{a})Z_1}{ja} = \frac{-(1+ja)}{a^2}Z_1 = Z_s - \frac{Z_m}{a^2}$$

$$Z_s = \frac{Z_m}{a^2} - \frac{(1+ja)}{a^2}Z_1 \qquad (27\text{-}5)$$

Let Z_1' and θ_{z_1} be the scalar quantity and the argument of the vector Z_1. Eq.27-5 may be written as:

$$Z_s = \frac{Z_m}{a^2} + \frac{\dot{Z}_1}{a^2}\sqrt{1+a^2}\ \underline{\big|\tan^{-1}a + 180 + \theta_{z_1}} \qquad (27\text{-}6)$$

In Eq.27-6., the leakage impedance of the main winding Z_m and the positive-sequence impedance Z_1 at a given speed can be predetermined. The choice of the effective turn-ratio a and the size of the auxiliary winding must be well adjusted to suit the given slot and to use the capacitor with lowest possible capacitance. Note that the solution of Eq.27-6. will have the form:

$$Z_s = \pm R_s - j\,X_s\ .$$

The negative sign for R_s indicates that a balanced operation can not be obtained for the given main winding and at the given speed. However, it may occur at a larger slip. When a positive resistance R_s of reasonable value results, the auxiliary winding size can be determined. The required capacitance can readily be calculated from X_c, i.e., X_s less the leakage reactance of the auxiliary winding.

By using the constants Z_m, Z_1, and a of the capacitor motor we used before, the leakage impedance of the auxiliary winding calculated from Eq.27-6. -65 - j 364, indicates that the operation of this motor with the existing winding arrangement is not balanced at the speed of 1140 r.p.m.. Readers are referred to the calculation sheet given on P.42d in which I_m and I_a are 53 electrical degree out of phase and the ratio between their magnitudes is 1/3.67. The effective turn-ratio between the main and auxiliary windings is being 1/3.2 .

28. The Equal Volt-Ampere Method for Design of Capacitor Motor.

It can be shown from the study of Eq. 27-6. that the auxiliary winding designed for exact balanced operation would be bigger than that for the ordinary design, therefore, it would be preferable if the motor could be built cheaper though the performance at rated load would be slightly out of balance.

Mr. Trickey in his paper of Equal Volt-Ampere Method for Design of Capacitor Motor described the design of the auxiliary winding and capacitor from an apparent balanced point. This method has been found convenient in the design of the auxiliary winding and the capacitor which would give performance nearest to the balanced operation. This method will be introduced here and will be shown in use with Eq.27-6. in an sample design.

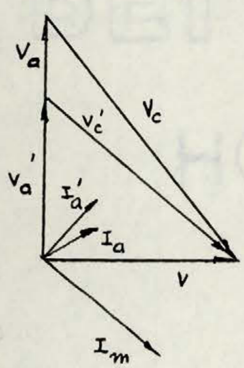


Fig.28-1.

In Fig.28-1., the primes are used for the vectors of a capacitor motor under balanced operation, which has the following particulars:

a' = effective turn-ratio,

C' = microfarad of the capacitor,

$a_r' = R_m / aR_a$,

$P_c' = V_c' \cdot I_c' = V^2/X_c'$.

With the main winding and the rotor remain unchanged, but a new capacitor and auxiliary winding are to be used, the vectors and the constants, expressed without the primes, will have the following relation to those under balanced operation. By the equal volt-ampere principle, we may write:

$$P_c' = P_c \qquad \frac{V_c'^2}{X_c'} = \frac{V_c^2}{X_c}$$

whence $$V_c = V_c'\sqrt{\frac{C'}{C}}$$

where $$V_c' = \left[1+\left(\frac{1}{a}\right)^2\right]^{\frac{1}{2}} V$$

From Fig.28-1, $$V_c = \left[V_c^2 - V^2\right]^{\frac{1}{2}}$$

whence $$a = \frac{V}{V_a}$$

and $$a_r = \frac{a\,a_r'}{a'}$$

therefore $$R_a = \frac{R_m}{a_r\,a}$$

These expressions enable us to design the auxiliary winding and the capacitor conveniently once the arbitrary balanced point from Eq.27-6. is calculated.

29. Sample Design of a Capacitor Motor Based on the Equal Volt-Ampere Method.

As an illustration, the capacitor motor described in Part 2 will be resesigned to a 1/3 hp. motor at a speed of 1140 r.p.m.

First, the main winding turns and the winding distribution will remain unchanged, as the exciting impedance of the motor has been found suitable. The main winding will have two circuits of No. 22 round wire. The leakage impedance Z_m and the positive-sequence impedance of the equivalent circuit Z_1 at a speed of 1140 r.p.m. will be, respectively,

$$Z_m = 3.52 + j\,2.1$$

$$Z_1 = 37.7\,\underline{|58.2} = 19.9 + j\,32.1$$

Under balanced operation:

$$I_m = \frac{V}{Z_1} = 2.92 \angle{-58.2}$$

$$I_r = I_m \frac{Z_\phi}{Z_\phi + Z_r} = 2.92 \times \frac{39.2}{89.4} = 1.28$$

$$T = 2 I_r^2 \frac{R_r}{s} = 2 \times 1.28 \times 84.8 = 277 \quad \text{SYN. WATTS.}$$

$$P_{OUT.} = T(1-s) - P_{f+w.}$$
$$= 277\,(1-.05) - 8 = 256 \quad \text{WATTS.}$$

which is about 1/3 hp.

Secondly, the main winding and capacitor are to be determined from Eq. 27-6. In using this equation, it would be found convenient to assume $a_r = a$, i.e., $R_a = R_m / a^2$, and to equate the argument of Eq.27-6. into 270 to avoid a negative R_s. It follows that:

$$\tan^{-1} a - 180 - 58.2 = 270$$

$$\tan^{-1} a = 31.8$$

whence $$a = .625 \text{ or } 1/1.6$$

Substituting the values of Z_m , a, $\dot{Z}_1$ and θ_{z_1} into Eq.27-6. we have:

$$Z_s = 9 - j\,114 + j\,5.4$$

which indicates that an auxiliary winding with an effective turn-ratio of 1/1.6 , a resistance of nine ohms and a capacitor of 23.3 micro-farad would give balanced operation at 1140 r.p.m. for the given main winding and rotor. Unfortunately, this auxiliary winding can not be placed in the given slot while the capacitance required is too high.

By using the equal volt-ampere method of design and to use a running capacitor of 7-μf capacitance, the auxiliary winding can be designed as follows: From the arbitrary balanced point, we have:

$$a' = .625 \qquad c' = 23.3 \ \mu F.$$

$$a'_r = \frac{3.52}{.625 \times 9} \qquad R'_a = 9 \ \text{OHMS.}$$

$$V'_c = (1 + 1.6^2)^{\frac{1}{2}} \times 110 = 207 \ \text{VOLTS.}$$

Now

$$C = 7 \ \mu F$$

$$V_c = 207 \sqrt{\frac{23.3}{7}} = 377 \ \text{VOLTS.}$$

$$V_a = \sqrt{377^2 - 110^2} = 360 \ \text{VOLTS.}$$

$$a = 110 / 360 = .305$$

$$a_r = .305 \ \ .625 / .625 = .305 = 1 / 3.28 \ .$$

The resistance of the auxiliary winding is:

$$R_a = 3.52 / (.305 \times .305) = 38 \ \text{ohms.}$$

From these calculations we see that the auxiliary winding may have the same number of turns as before, but the wire size should be changed from No. 26 to No. 24 as the latter will give a resistance of about 38 ohms as required. The performance of this re-designed motor should be calculated by the method described in Art. 15. And it is understood that the expected output of 1/3 hp. can not be developed by this motor at the speed of 1140 r.p.m. with a 7-μf capacitor. It may, however, be expected at a lower speed, or at the same speed with a capacitor of higher capacitance. The calculation of the performance of the capacitor motor has been fully described in Part 2 and will not be repeated here. Readers are referred to the performance calculations for various winding designs of this motor given in Fig.25-1. .

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a effective turn-ratio between main and auxiliary windings

a operator, $a = \angle 120° = -.5 + j.866$

R_a resistance, auxiliary winding

R_A locked-rotor resistance, auxiliary winding

R_c resistance, capacitor

R_e unsymmetry in resistance, auxiliary winding circuit

R_m resistance, main winding

R_r resistance, rotor circuit

R_R rotor resistance, locked-rotor

R_s resistance, auxiliary winding and capacitor

R_S locked-rotor resistance, auxiliary winding and capacitor

R_ϕ exciting resistance, main winding

X_a leakage reactance, auxiliary winding

X_c reactance, capacitor

X_e unsymmetry in leakage reactance, auxiliary winding circuit

X_m leakage reactance, main winding

X_M locked-rotor reactance, main winding

X_r leakage reactance, rotor circuit

X_s leakage reactance, auxiliary winding and capacitor

X_S locked-rotor reactance, auxiliary winding and capacitor

X_ϕ magnetizing reactance, main winding

Z_1 equivalent impedance, positive-sequence circuit

Z_2 equivalent impedance, negative-sequence circuit

Z_a leakage impedance, auxiliary winding

Z_A locked-rotor impedance, auxiliary winding

Z_e unsymmetry in impedance, auxiliary winding circuit

Z_m leakage impedance, main winding

Z_M locked-rotor impedance, main winding

Z_r leakage impedance, rotor circuit

Z_s leakage impedance, auxiliary winding and capacitor

Z_S locked-rotor impedance, auxiliary winding and capacitor

Z_ϕ exciting impedance, main winding

I_1 current, positive-sequence component

I_2 current, negative-sequence component

I_a current, main winding

I_m current, main winding

I_ℓ current, line

I_A starting current, auxiliary winding

I_M starting current, main winding

I_L starting current, line

I_r current, rotor circuit

I_ϕ exciting current

$P_{in.}$ power, input

$P_{out.}$ power, output

$P_{sh.}$ power, shaft

$P_{loss.}$ losses of the motor

$P_{f+w.}$ loss, frictional and windage

P. F. power factor

Eff. efficiency

S slip, (syn. speed - r.p.m.) / syn. speed

$S_{P.O.}$ slip, pull-out or maximum torque

$S_{b.o.}$ slip, balanced operation

T torque

T_s starting torque

Φ flux in lines per pole

θ_{M-A} phase angle between I_M and I_A

θ_o phase angle for no load current $I_{N.L.}$

Note that subscripts 1 and 2 are used, respectively, to indicate constants, including current, torque, power, etc., in the poeitive- and negative-sequence equivalent circuits. Primes are used for constants of the auxiliary winding when expressed in terms of the main winding, with the exception of those in Art. 28.