Pre-composition in Two Works by Ludwig van Beethoven and Bruno Maderna

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Abstract

This thesis examines two works that use pre-compositional techniques: Op. 14, no. 1, by Ludwig van Beethoven and *Improvvisazione, no. 1*, by Bruno Maderna. The first paper compares Beethoven's piano sonata (1799) to the string quartet arrangement, which he himself composed three years after the original was published (1802). Using William Caplin's theories of formal structures, the paper concludes that the string quartet is not simply an arrangement of the piano sonata but rather a unique work in its own right. The second paper analyses Maderna's piece *Improvvisazione, no. 1* (1952). In this piece, he utilizes a new technique called "Magic Squares." By analyzing the magic squares, formal structure, elements of chance, and work by one of his pupils, it is clear how Maderna combines new and old methods, as well as freedom within constraints while simultaneously avoiding set repetition. After analyzing the two pieces, it is evident that pre-composition takes on many forms.

Cette mémoire examine deux œuvres qui utilisent des techniques pré-compositionnelles : L'opus 14, n°1, de Ludwig van Beethoven et *Improvvisazione, n°1*, de Bruno Maderna. Le premier article compare la sonate pour piano de Beethoven (1799) à l'arrangement pour quatuor à cordes qu'il a lui-même composé trois ans après la publication de l'original (1802). En s'appuyant sur les théories de William Caplin sur les structures formelles, l'article conclut que le quatuor à cordes n'est pas un simple arrangement de la sonate pour piano, mais plutôt une œuvre unique en son genre. Le deuxième article analyse l'œuvre *Improvvisazione, n° 1* (1952) de Maderna. Dans ce morceau, il utilise une nouvelle technique appelée « carrés magiques ». En analysant les carrés magiques, la structure formelle, les éléments de hasard et l'œuvre de l'un de ses élèves, il devient clair que Maderna combine des méthodes nouvelles et anciennes, ainsi que la liberté dans les contraintes tout en évitant la répétition de l'ensemble. Après avoir analysé les deux pièces, il devient évident que la pré-composition prend de nombreuses formes.

Acknowledgements

This thesis was only completed with the help of many university professors, colleagues, friends and family. I want to start by thanking my supervisor Dr. Edward Klorman for continually helping me through this complex and unique writing process. Additionally, this endeavour could not have been made possible without the area coordinator of Music Theory at McGill, Dr. Nicole Biamonte, who helped me navigate this new thesis program.

I want to give additional thanks to all the members of the faculty of Music Theory who helped with the details of each essay, including Dr. William Caplin, Dr. Robert Hasagawa, Dr. Christoph Neidhöfer, Dr. Ellen Bakulina, Dr. Peter Schubert and Dr. Jonathan Wild. Special thanks should also go out to my theory professors at Wilfrid Laurier University, Dr. Kevin Swinden, Dr. Charles Morrison, and Dr. Terry Paynter, without whom I would not be the theorist I am today.

Words cannot express my gratitude to Chloe Martin, who typeset and transcribed the entirety of Beethoven's op. 14, no. 1, i piano sonata and string quartet. Without her, this paper would not have half the comparative analysis that it does. I also could not have undertaken this venture without my colleagues Daria Michirin and Vlad Praskurnin, whose knowledge of Caplinian form is surpassed by none in the graduate music theory program.

Lastly, I would be remiss not to mention my family and friends who supported me throughout this whole journey: Fiona Owens, Isaac Bouhdana, Graham Falconer, Stephen Burzese, Sydney MacDonald, my parents, Brian and Désirée Danbrook, and my dogs Comet and Pepper Pot. Their emotional support is the reason I was able to persevere and finish this seemingly never-ending process.

List of Examples and Tables

Introduction

The idea of pre-composition is not a new concept. For centuries composers have used pre-compositional techniques, whether they are cognisant of them or not. Choosing the key of a piece, for example, is a pre-compositional choice that limits the available pitches. Selecting which instruments to write for and which timbres sound good together is also a pre-compositional decision. Some pre-compositional techniques can be so simple that a composer may not even think of them as "pre-compositional," like the ones listed above. Others can be very complicated and much more deliberate, e.g. serial music. This thesis will compare two pieces that use two very different pre-compositional devices: op. 14, no. 1 by Ludwig van Beethoven and *Improvvisazione, No. 1* by Bruna Maderna.

The first essay is entitled "Beethoven's Piano Sonata Op. 14, no. 1: The Changes Between the Piano and String Quartet Arrangements." This paper compares the Piano Sonata to Beethoven's string quartet arrangement of the piece. The methodology used is William Caplin's theory of formal structure. This paper analyzes the formal construction of the two versions and compares how Beethoven either emphasizes or de-emphasizes the sonata form.

This string quartet is unlike any other Beethoven piano sonata arrangement in that the composer himself wrote it. He despised string quartet arrangements of piano works because the two instruments are so different in timbre and style. Furthermore, he found the practice of simply placing the top voice of the piano in the first violin, the bass voice in the cello, and the middle voices in the second violin and viola too derogatory and unidiomatic. Beethoven's string quartet arrangement of op. 14, no. 1, however, does much more than simply copy and paste the original piano version. This new composition makes slight changes from the original that may seem

unimportant at first glance, but upon closer inspection, the variations are deliberate and can even lend more insight into the intent of the original piece.

This paper will explore the question *Is the string quartet arrangement of op. 14, no. 1 simply a reproduction of the piano sonata with different timbres?* The answer is no. Beethoven's string quartet arrangement of op. 14, no. 1 is not simply an arrangement of an already masterfully created work but rather a carefully crafted piece that, while sounding like the piano sonata, has its own intricacies that make it a proper string quartet in its own right.

The analysis of op. 14, no. 1 examines the formal structure of both versions of the sonata and points to some specific examples where the string quartet intensifies ambiguities found in the piano sonata and other examples where Beethoven could have loosened the structure to create a more archetypal formal structure but does not. In examining the exposition and development sections of the first movement, the validity of the string quartet's confirmed.

The second paper, entitled "Bruno Maderna's *Improvvisazione No. 1*: Magic Squares and the Constraints they Impose," is a look at Maderna's compositional device that he calls "Magic Squares." Maderna combines serial elements, concepts borrowed from the tonal era, constraints, and chance to compose his 1952 work *Improvvisazione No. 1*. The defining argument is that magic squares both constrain pitch organization and yet, paradoxically, encourage creative liberty.

This paper begins by explaining the process of creating a magic square and how the pre-compositional technique is utilized within the piece. Magic squares are much less binding than a serial matrix; Maderna creates 108 different squares, each with a unique succession of pitch classes that can be played in prime or retrograde, compared to the twenty-four rows created in a serial matrix.

Maderna is able to toy with other musical elements because the pitch class material is predetermined. This includes rhythm, timbre, dynamics, tempo, and form. The second part of the paper devotes itself to dissecting the "polka" and "can-can" sections of *Improvvisazione, no. 1*. A comparative analysis between the traditional definitions of the styles versus Maderna's use of them within the piece helps to prove Maderna's interest in moving music forward by simultaneously looking to the past.

The next section of the paper refers to outside sources, including Leonard B. Meyer's book *Style and Music: Theory, History, and Ideology,* Robert Hasegawa's chapter entitled "Creating with Constraints" from *The Oxford Handbook of the Creative Process in Music*, Alison James's chapter "A Challenge to Chance: The Poetics of the Oulipo" in *Constraining Chance*. These sources shed more light on Maderna's use of constraint and how it helps to facilitate his unique compositional ideals. A look into the ideas of the Oulipo, a 1960s French group of writers who used constraints in their works, is also helpful in understanding how Maderna's use of constraints is freeing rather than constricting. Finally, the paper will look at one of Maderna's pupils, Norma Beecroft, and how she implemented Maderna's Magic Squares into her 1962 work *Tre Pezzi Brevi*.

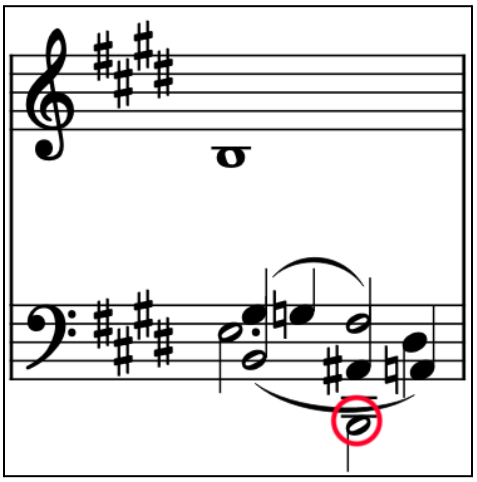
Although these two essays are about vastly different genres, they both encompass the idea of pre-composition: Beethoven in the less avant-garde but still relevant sense that he had already composed the piece he was arranging, and Maderna in the literal sense that he used pre-compositional devices. Both composers use pitch classes as the grounds for their work, not specific pitches. In this way, they can manipulate the range and timbre of the notes; in Beethoven's case, he chooses to place specific lines in the cello instead of the violin, or vice versa, as each instrument's sound and range gives a different meaning to the music. In this way,

he has much more freedom than when he originally composed op. 14, no. 1, where his only timbral option was the piano. Maderna, comparatively, has more freedom than Beethoven, as he has an entire orchestra of instruments to choose from. The piece has a nearly five-octave range of pitches from A1 in the tuba to A ♭ 6 in the flute and fourteen different timbres, including multiple woodwinds, brass, and percussion instruments. Both Beethoven and Maderna's reasons for choosing specific pitches and timbres lend insight into their works' compositional processes and show how the composers use their respective pre-compositional devices to aid in the creation of the final work.

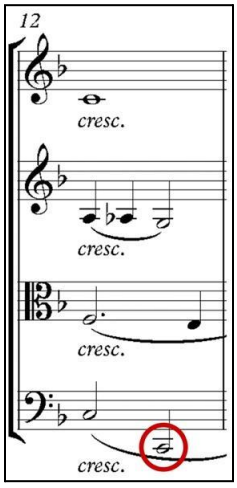
Beethoven's Piano Sonata Op. 14, no. 1:
The Changes Between the Piano and String Quartet Arrangements

Beethoven's composition Sonata in E Major, op. 14, no. 1, actually exists in two authentic transcriptions, both original to Beethoven and both published during his lifetime: the version for piano (1799) and another version for string quartet, in F major (1802). Musicians have tended to view the string quartet version as a derivative or secondary work, owing to its later release date and the tendency to dismiss arrangements as unworthy of scholarly or performance attention. This attitude was echoed by Beethoven himself. However, there are compelling reasons to analyze these two versions in tandem, partly because Beethoven appears to have conceived of the quartet version first. Regardless of which one Beethoven originally wrote, on the surface, the string quartet seems like a mechanical, almost note-for-note transcription of the piano sonata. The pitch content and formal structure are almost identical to the piano sonata, only transposed from E major to F major to accommodate the low C of the cello; had Beethoven left the string quartet in E major, the cello would have been unable to play some of the lowest notes in the piece, for example, the B2 in the piano sonata in m. 12 is too low for the cello, but in the key of F major the note becomes C2 which fits nicely within the cello's range (Example 1.1a and 1.1b). Furthermore, the key of F is much more suited to the intonation needs of a string quartet. While the harmonic, motivic, and formal functional structures of the two versions are nearly indistinguishable, the changes in instrumentation clarify structural ambiguities at some times while, at other times, muddling the already unclear analysis. By making these changes, Beethoven proves that his ability to transcribe work for other instruments

is not only sufficient but is exemplary of how a transcription can become a new piece in and of itself.



Example 1.1a. Measure 12 (piano) The lowest note is B



Example 1.1b. Measure 12 (string quartet). The lowest note in the cello is a C

This essay will cover both historical and theoretical aspects of the sonata, beginning with the context of the sonata and why it was transcribed for string quartet, despite Beethoven's dislike of string arrangements of piano works. A detailed analysis of the exposition and developments sections will follow, with a comparative look at the two versions in relation to each other, and a brief look at the recapitulation will lend insight into Beethoven's compositional process to understand how he skillfully rewrote the piano sonata for string quartet.

The Context of the String Quartet

Beethoven's arrangement of the sonata in the first place is an oddity. He rarely arranged his own works; in fact, op. 14 is the only one of his thirty-two piano sonatas that he ever arranged for another genre. What makes the arrangement even stranger is he did not like string arrangements of piano pieces. In a letter written in 1802 to the publishers Breitkopf & Härtel,

Beethoven is quoted as saying, “The *unnatural mania*, now so prevalent, for transferring even *pianoforte compositions* to string instruments, instruments which in all respects should be so utterly different from one another, should really be checked.”[1] It was common at the time for publishers to want arrangements of works so that the music could be more easily distributed and played by the masses. A translation of this letter by Watson Forbes offers insight into Beethoven's reason for arranging the sonata: “I have transcribed one of my own sonatas as a quartet for strings because people bothered me so much to do so and I know quite well that no one else could do it so satisfactorily.”[2] Micheal Steinberg further explains that it was inevitable that op. 14 would be arranged for string quartet because of the “unnatural mania” for string arrangements at the time, and Beethoven wanted to get ahead of unsavoury transcriptions: “Like any sort of translator, the musical transcriber can be suspended in excessive literalism by reverence or sheer lack of imagination; on the other hand, carelessness, irrepressible creative impulses, irreverence, or an inclination always to know better can lead to risky departures from the original.”[3] Much like how metaphors in one language cannot be literally translated into another, when arranging a piano work for a string quartet the composer understands the “metaphors” of the piece and is able to “translate” them correctly to correspond with their respective instrument. Whereas an arranger who does not have such an intimate knowledge of the language of the piece may mistranslate sections, leaving the arrangement confusing and incoherent.

Beethoven’s dislike of string arrangements stemmed not only from the publishers’ “unnatural mania” to churn them out but also because arrangers would simply tear the music

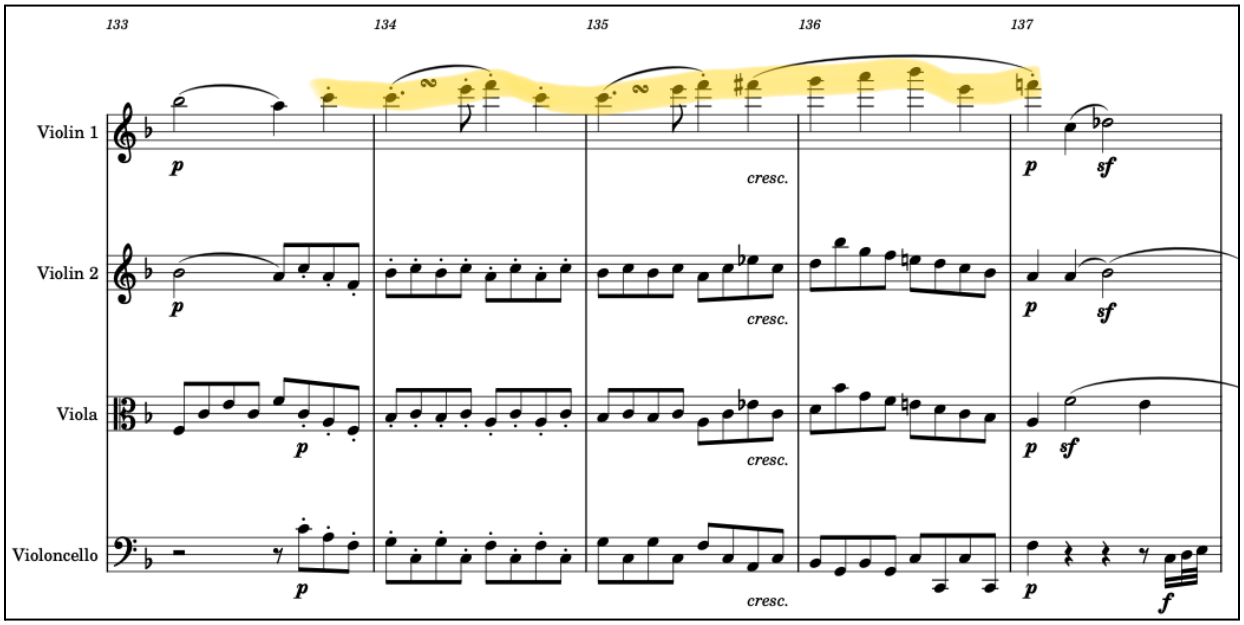
[1] Emily Anderson, *The Letters of Beethoven* (London; Toronto: Macmillan, 1961), 74 -75.
[2] Watson Forbes, “Beethoven’s Op. 14, No. 1,” *The Musical Times* 86, no. 1226 (1945): 108.
[3] Micheal Steinberg, “The Early Quartets,” in *The Beethoven Quartet Companion*, ed. Robert Winter and Robert L. Martin (Berkeley: University of California Press, 1994), 146.

directly from the piano score and drop the music into the corresponding string part: the highest notes of the piano arrangement become the first violin part, the lowest notes become the cello, and the middle voices become the second violin and the viola, with no regard to whether the arrangement sounds idiomatic or not. In Beethoven's arrangement of op. 14, no. 1, however, he constructs the string quartet as if the musical fragments from the piano arrangement were building blocks that he could arrange in any way to construct the string quartet. The placement of each line of music in the new arrangement is deliberate. The changes in the string version are well thought out and can even lend more insight into the intent of the original piece. Even differences that may seem minute, for example, more dynamic markings, are carefully calculated and help contribute to the overall aesthetic of the transcription. Lewis Lockwood said it best when he wrote: "… there can be no doubt that Beethoven's 'arrangement' is no mere transcription, but entails a recasting of the material for quartet in what he considered an appropriate degree of change."[4]

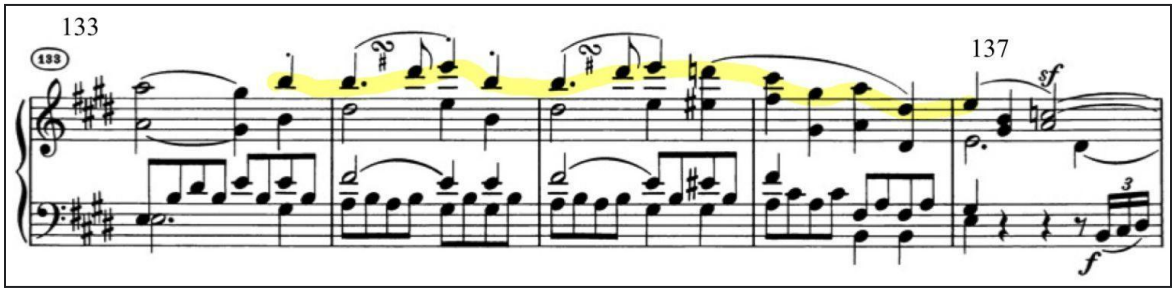
It is important to understand why the string quartet must be compared to the piano arrangement, not vice versa. The question must be "Why did Beethoven move motive *x* from the bass of the piano sonata to the second violin of the string quartet?" not "Why did Beethoven move motive *x* from the second violin of the string quartet to the bass of the piano sonata?" The difference between the two questions, although slight, can change the perspective of the analysis. The nineteenth-century musicologist Gustav Nottebohm questioned whether Beethoven conceived of the string quartet arrangement first rather than the piano arrangement. Based on Beethoven's sketches, Nottebohm concludes that they could have been drawn for either the piano or a larger ensemble: "If one considers individual passages of the sketched piece, it may seem

[4] Lewis Lockwood, "Beethoven as a Colourist: Another Look at his String Quartet Arrangement of the Piano Sonata, Op. 14 No. 1," in *Haydn, Mozart, and Beethoven: Studies in the Music of the Classical Period,* ed. Sieghard Brandenburg (Oxford: Oxford University Press, 1998), 178.

questionable whether it is intended for piano or for several instruments."[5] Micheal E. Broyles has recently examined this question, arguing that some gestures work better for a string quartet rather than for the piano.[6] For example, the melodic line in mm. 133–137 that continues up in the strings (Example 1.2a) but must be arpeggiated down in the piano part because it exceeds the piano's range (Example 1.2b). Broyles also points out the fragments in mm. 5–6 that work better moving down the four string instruments than it does in the piano (Examples 1.3a and 1.3b).



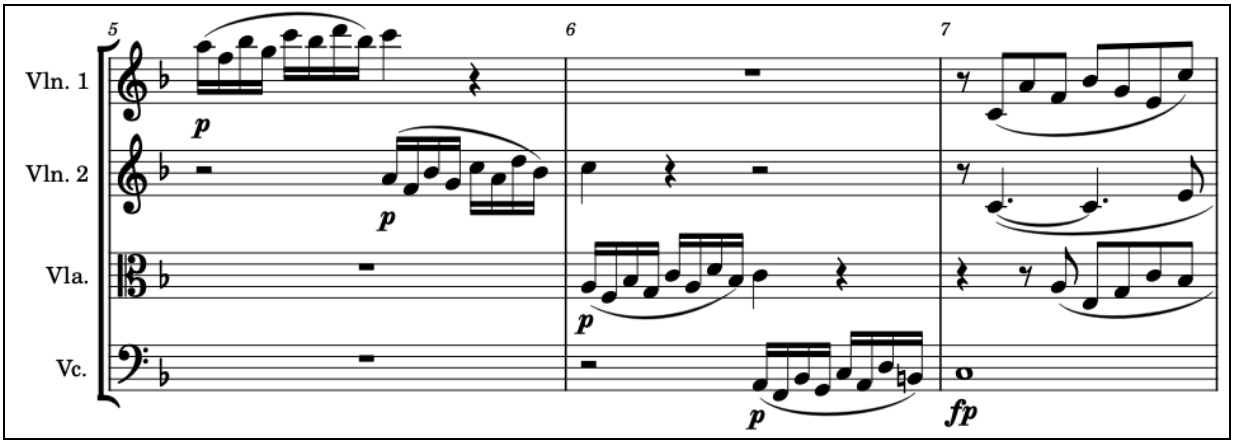
Example 1.2a. Measures 133–137 (String Quartet)



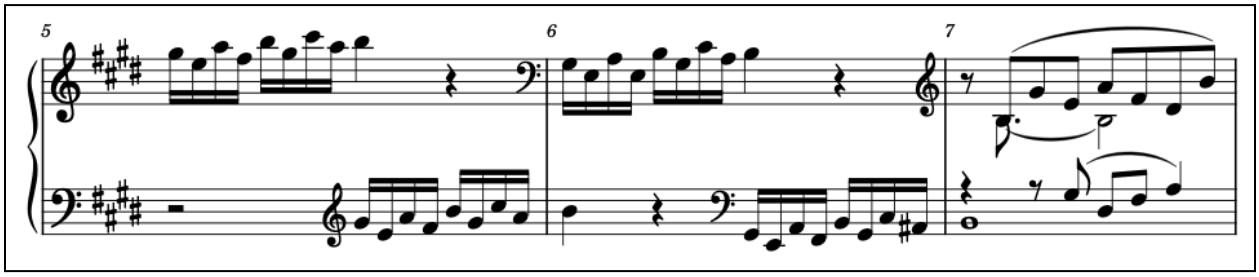
Example 1.2b. Measures 133–137 (piano)

[5] "Wenn man einzelne Stellen des skizzirten Stiickes ins Auge fasst, so kann es fraglich erscheinen, ob es für Clavier oder für mehrere Instrumente gedacht ist." Gustav Nottebohm, *Zweite Beethoveniana,* translated by Bill Danbrook (Leipzig: C. F. Peters, 1887), 47.

[6] Michael E. Broyles, "Beethoven's Sonata *Op. 14, No. 1*, Originally for Strings?" *Journal of the American Musicological Society* 23, no. 3 (1970): 409.



Example 1.3a. Measure 5–7 (String Quartet) A cycle of imitation.



Example 1.3b. Measures 5–7 (piano) The cycle of imitation in the piano sonata.

Edward Klorman explains that this line is a very idiomatic string pattern called a *cycle of imitation*:

> A cycle of imitation occurs when a motive is imitated by each instrument at a regular time-interval. Imitative entrances typically take place in order from lowest to highest (or vice versa) and often involve overlapping entries with long notes … Cycles of imitation are ubiquitous in chamber music, especially in music for strings, since they exploit the ensemble's timbral homogeneity.[7]

Additionally, Steinberg recounts that "[i]n 1795, Count Anton Apponyi … asked Beethoven for a string quartet. For whatever reason, Beethoven never delivered … but it could well be that op. 14, no. 1 was a move in the direction of fulfilling the commission."[8] This is consistent with the idea that Beethoven began conceiving op. 14 as a string piece and then changed direction.

[7] Edward Klorman, *Mozart's Music of Friends* (Cambridge: Cambridge University Press, 2016), 230–233.
[8] Steinberg, 145.

Despite all the evidence towards the contrary, Broyles concludes that it is impossible to tell which version Beethoven conceived of first by analyzing the two arrangements.[9] As such, he concludes that Beethoven's letter must be taken as fact; he wrote the piano sonata first and then the string quartet. Therefore, this analysis will compare the former to the latter, not vice versa.

Exposition

Main Theme

The sonata's exposition is constructed traditionally: a main theme that transitions to a subordinate theme group in the dominant key. The main theme is, to use Caplinian terms, looser than a typical main theme; the four-measure compound basic idea (CBI) is followed by an eight-measure continuation creating an asymmetrical grouping structure.Beethoven does not increase the harmonic rhythm in the continuation; instead, the continuation is supported solely by an implicit tonic harmony, and the medial and concluding functions are not fused together into one four-measure phrase as they typically are in a tight-knit structure.[10] The continuation is loosened further in the string quartet through its use of the cycle of imitation (see Example 1.3a). Ellen Bakulina refers to cycles of imitation by a different name: an imitative presentation. She explains they are "a polyphonic subtype of the presentation function."[11] Meaning that in the piano sonata, mm. 5–7 utilizes fragmentation — a device typically found in the medial function — while in the string quartet, mm. 5–7 exploit the initiative presentation — a device reserved more for initiatory functions. Within the first eight measures of the work, Beethoven has already shown how the arrangement can lend new insight into the original's analysis.

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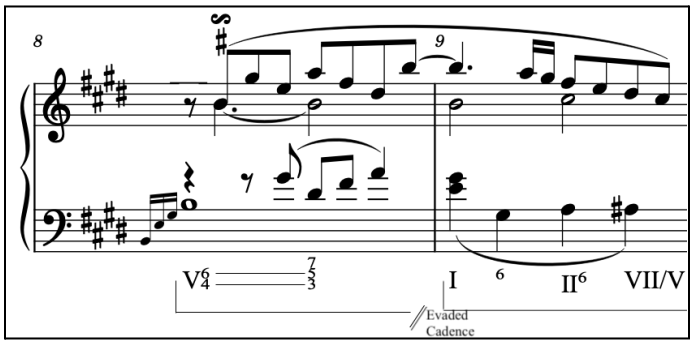
[9] Broyles, 419.

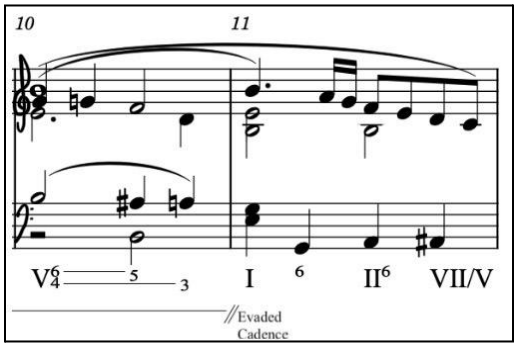
[10] William E. Caplin, *Analyzing Classical Form: An Approach for the Classroom* (Oxford: Oxford University Press, 2013), 203.

[11] Ellen Bakulina, "The Loosening Role of Polyphony: Texture and Formal Functions in Mozart's 'Haydn' Quartets," *Intersections: Canadian Journal of Music* 32, no. 1-2 (2012): 17, https://doi.org/10.7202/1018577ar.

The loosest elements of this main theme are the potential cadences from mm. 9–13. The first possible cadence appears in m. 9 with the arrival of the tonic preceded by a dominant (including its 6/4 embellishment). This harmonic progression is unequivocally not a cadence as none of the upper voices play scale degree, $\hat{1}$ and the music pushes through where the cadence would be. The downbeat of m. 9 cannot be perceived as a goal and, therefore, must be understood as an evaded cadence (Example 1.4).



Example 1.4. Measures 8–9 (piano). The first potential cadence.



Example 1.5. Measures 10–11 (piano). The second potential cadence.

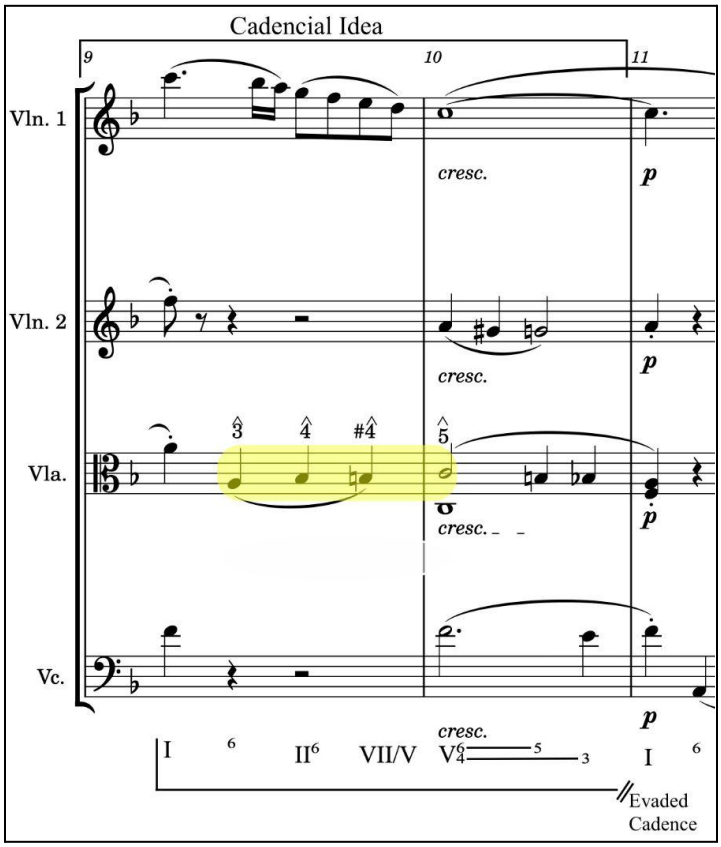
The next possible cadence is much more ambiguous. William Caplin explains that mm. 10–11 could constitute a covered perfect authentic cadence (PAC).[12] Even though the soprano holds over a B from mm. 10–11, just as it did in mm. 8–9, a clear cadential line of $\hat{3}$–♭$\hat{3}$–$\hat{2}$–$\hat{1}$ appears in the alto.[13] As Caplin points out: "If we take this alto line as the structural one, we could recognize a PAC, one that is covered by the high B."[14] The texture, however, is quite similar to the texture of the evaded cadence in mm. 8–9 (Example 1.5). Measures 10–11 cannot be analyzed as a proper cadence then when considering its similarity to mm. 8–9.

[12] William Caplin, Personal Communication, April 4, 2022

[13] When referring to different parts of the piano score the terms "soprano," "alto," "tenor," and "bass" will be used.

[14] Caplin. Personal Communications.

This sense of ambiguity in the mm. 10–11 potential cadence is different in the string quartet arrangement and is weakened by a few different factors. The $\hat{3}$–$\hat{4}$–$\hat{\#4}$–$\hat{5}$ bass line is placed in the viola in m. 9, and scale degrees $\hat{5}$–$\hat{1}$, though sounding at C3–F3 below the F4 in the cello, are in the viola, the timbre of which typically does not lend itself well to a cadential bass texture (Example 1.6).

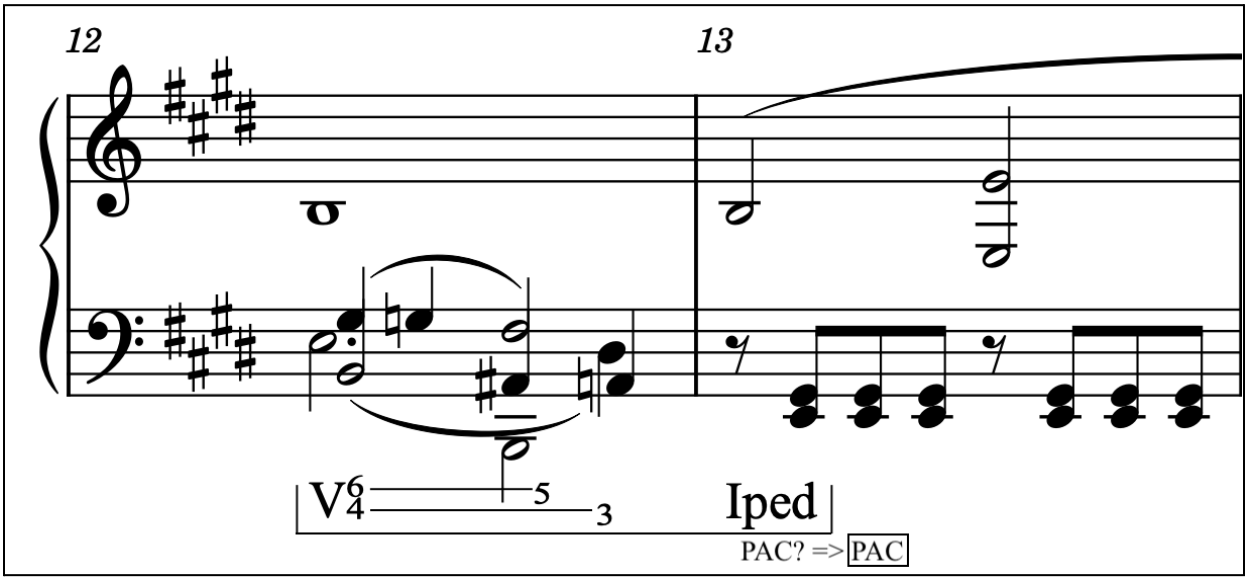


Example 1.6. Measures 9–11 (string quartet). The bass line is placed in the viola.

Additionally, Caplin points out that the $\hat{3}$–$\hat{2}$–$\hat{1}$ progression in the alto of the original becomes a $\hat{3}$–$\hat{2}$–$\hat{3}$ gesture in the second violin in the quartet, meaning the only Fs heard appear in the lowest voices.[15] The addition of new dynamic markings (*crescendo* to *piano*) also suggests that the cadence at m. 11 should be pulled back, weakening it further.

[15] Caplin, Personal Communication.

The final cadence from mm. 12 to 13 of the main theme in the piano version is equally ambiguous as the previous one (Example 1.7a). As it is the last cadence of the section, it must be retrospectively perceived as the PAC once it is clear the transition has begun, but the cadence's resolution is underwhelming, to say the least; it must, once again, be understood as a covered PAC with the B as the covering tone, the final E does not arrive until halfway through m. 13, well beyond its expected point of resolution, and the tonic arrival is delayed by an eighth rest, causing a jarring break of cadential motion.[16]

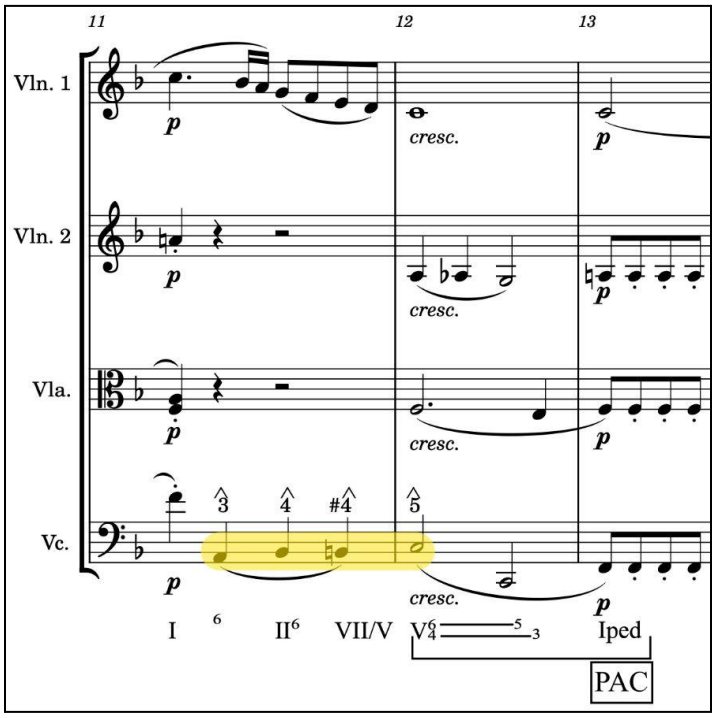


Example 1.7a. Measures 12–13 (piano). Third and final potential cadence.

The piano sonata's cadence in mm. 12–13 is much weaker than its string quartet counterpart (Example 1.7b). The string quartet's cadence is strengthened by the $\hat{3}$–$\hat{4}$–$\#\hat{4}$–$\hat{5}$ bass line in the cello in mm. 11–12 instead of in the viola as it was in mm. 9–10. By placing the bass line in the lowest voice, Beethoven is suggesting that the cadence at mm. 12–13 is stronger than the one at mm. 10–11 in the string quartet. Additionally, an octave leap in m. 12 of $\hat{5}$ from C3 to C2 in the cello creates greater anticipation of $\hat{1}$, and the arrival of the tonic chord in m. 13

[16] See Klorman, *Mozart's Music of Friends* ex. 4.8 for examples of this type of cadential break.

squarely on beat one removes the cadential halt felt in the original piano. This cadence is still, however, covered by $\hat{5}$ in the top voice. Beethoven also reuses the *crescendo–piano* dynamic from the cadence in m. 11 of the string quartet. Both arrangements tackle the ambiguity of the cadences in unique ways. Beethoven took the opportunity of the string quartet to not just copy himself but to expand upon his previous ideas.

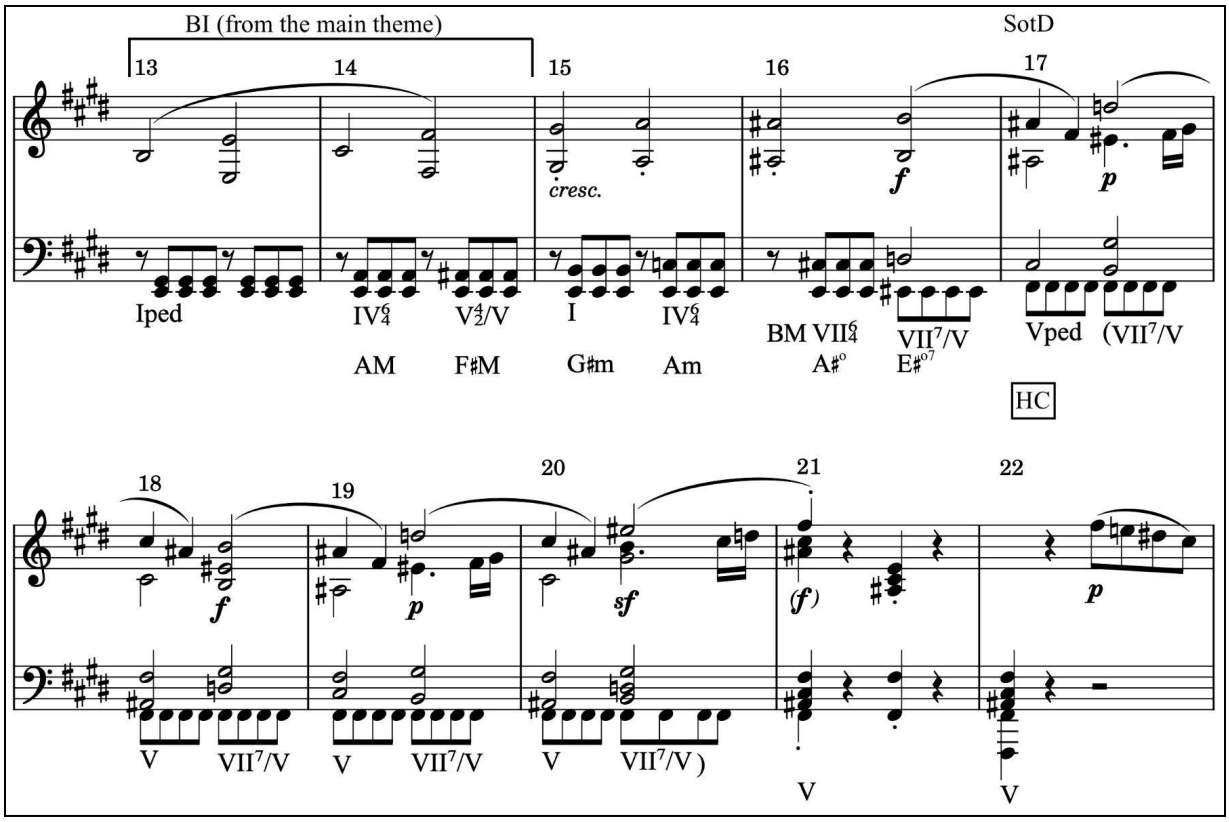


Example 1.7b. Measures 11–13 (string quartet). The bass line is in the cello this time.

Transition

The transition of op. 14, no. 1, i is a modulating transition, closing with an HC in B major in m. 17 and prolonging the section a little longer with a standing on the dominant (SotD) until m. 22 (Example 1.8). The transition begins in m. 13 by repeating the BI from the main theme, but instead of the third occurrence of the +4/-3 melodic sequence, Beethoven continues the upward motion chromatically through G#–A–A#–B–B#. The melody of mm. 14–16 is supported by an interesting progression of an ever-rising chromatic line that does not lend itself well to a

pivot chord. As such, the modulation to B major simply begins with $VII^6{}_4$ on beat one of m. 16. Were it not for the rising line, beat two of m. 15 would typically be the perfect place for a pivot between E major and B major, perhaps pivoting from I of E to IV of B or V of E to I of B. Instead, Beethoven implements an A minor triad, a chord that is not diatonic to either key. This odd use of A minor is jarring and foreshadows what is to come in the development section.



Example 1.8. Measures 13–22 (piano). The transition.

The repeated drum bass rhythm, as well as the chromatically rising melody, helps to drive the transition forward to the new key. In the string quartet version, Beethoven changes the tenor line from the drum bass to the held half notes, moving in succession with the first violin. Beethoven is able to bring out the driving bass more in the string quartet than in the piano sonata because the cello has a different, more rich texture than the higher strings. He doesn't need to

double that rhythm in another voice, as he does in the piano, because the cello stands out enough on its own.

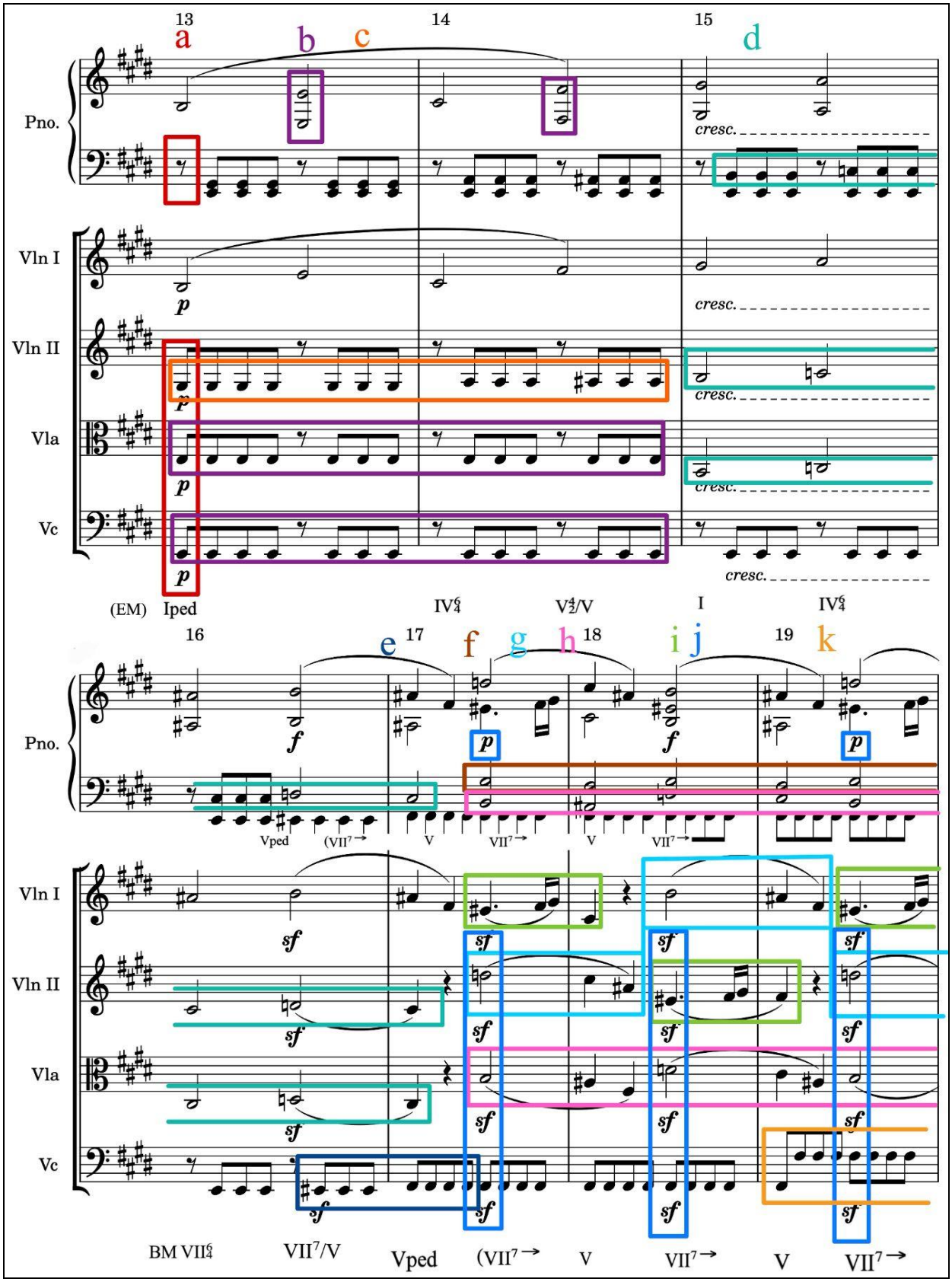
A loosening feature of the transition is its lack of discernible phrase functions. Once the HC is reached in m. 17, mm. 13–17 can be retrospectively understood as a continuation phrase, with each harmonic change being understood as a fragment, but the use of chromaticism destabilizes the continuation function.

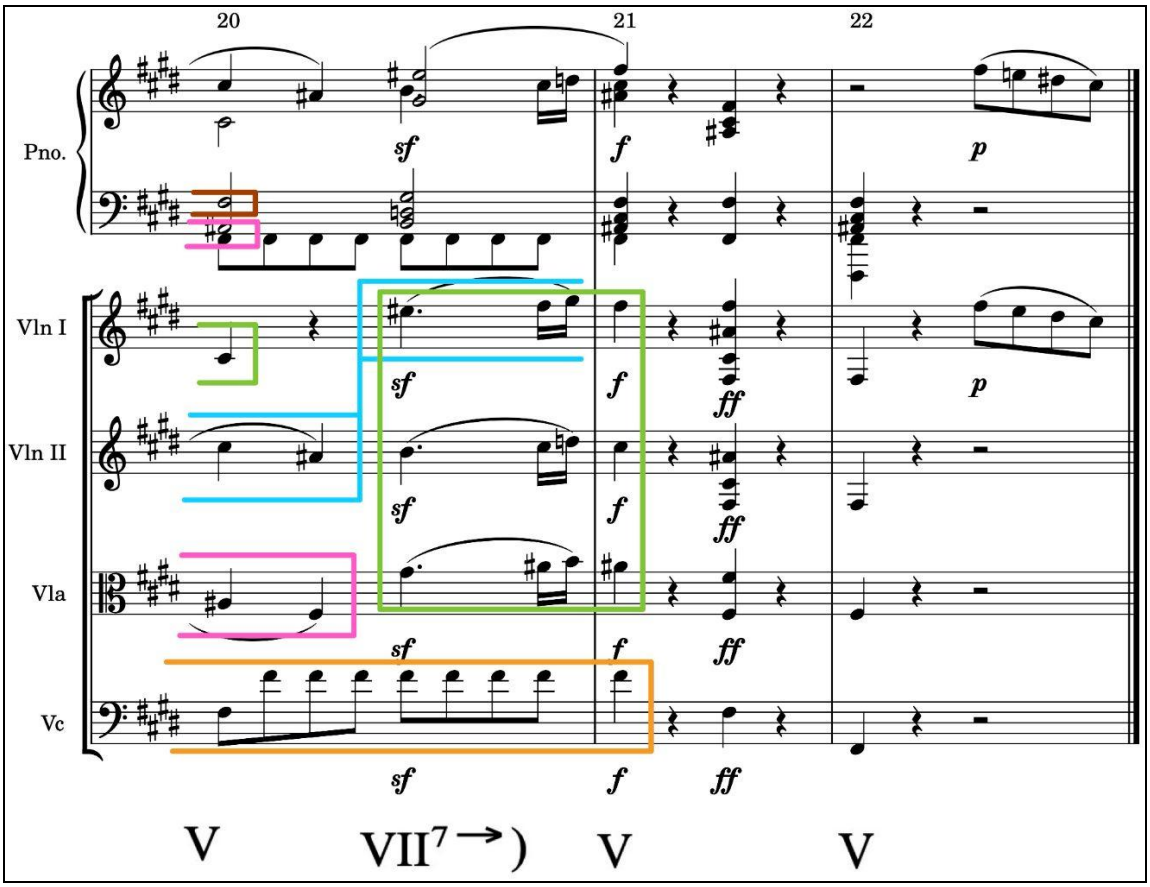
Beethoven made many changes in the string quartet version of the transition (the string quartet has been transposed to E major in this analysis for ease of comparison with the piano). Firstly, the drum bass accompaniment begins on beat one of m. 13, not the second eighth note. This is the same change made in m. 1 (Example 1.9a). The string quartet could have started on the offbeat, but beginning on the downbeat makes it easier for all the performers to come in together, a problem a solo pianist would not face. Secondly, instead of adding the alto octaves in mm. 13–14, which would be out of the violin's range, Beethoven triples the accompaniment by placing an E in two ranges, in the cello and viola (Example 1.9b), and places the tenor G in the second violin (Example 1.9c). Next, in mm. 15–16, the tenor voice is doubled instead of the bass, and the eighth note rhythm turns to half notes (Example 1.9d). The violin and viola now sound more like part of the melody instead of the accompaniment. The cello is now the only line with the eight-note rhythm, dissociating the pedal E from the rising chromatic line and creating a more jarring shift from E to E# in m. 16, allowing the half cadence to stand out more (Example 1.9e). Measures 17–20 do away with the top tenor voice, which oscillates back and forth between G# and F# (Example 1.9f). Beethoven seems to have realized that the half note G# would be unnecessary over the VII^7/V chords from mm.17–19 since the G# is heard as the last sixteenth note in the first violin.

The absence of the G#–F# motion is not the only change made in mm. 17–20. The melody passes back and forth between the two violins as if they are having a conversation (Example 1.9g), and he composes a new line in the viola that is a third below the countermelody, which itself is a variation of the lower tenor line (Example 1.9h). Beethoven also adds an additional occurrence of the dotted quartet–two sixteenth–quarter rhythm in the second violin that was not in the original in mm. 18–19. The other two occurrences of this motive are in mm. 17–18 and 19–20 in the first violin. These instances, however, play scale degrees $\hat{7}$–$\hat{1}$–$\hat{2}$–$\hat{5}$ in the key of V/V (F# major), whereas, in mm.18–19, the second violin plays $\hat{7}$–$\hat{1}$–$\hat{2}$–$\hat{1}$. This double neighbour to $\hat{1}$ once again strengthens the pull to the dominant. This motive also foreshadows mm. 20–21, in which the first violin plays the $\hat{7}$–$\hat{1}$–$\hat{2}$–$\hat{1}$ and the second violin and viola mirror it in thirds and sixths below, respectively (Example 1.9i).

The HC is treated differently in each version: in the piano sonata, Beethoven dissipates the arrival of the HC in m. 17 with a *piano* marking on VII^7/V, weakening the tonicization of the dominant, whereas (Example 1.9j), in the string quartet, he takes the opposite approach and emphasizes the arrival of the HC with a *sforzando* marking each time the VII^7/V is heard. By bringing out the tonicization of the dominant, Beethoven is reinforcing the move to the subordinate key.

The last additional moment of drama in the string quartet is added in the cello when it rises two octaves, finally arriving on the F4 at the climax of the transition in m. 21 (Example 1.9k). Beethoven takes what was a simple transition in the original piano arrangement and changes it into a dramatic ten-measure moment of anticipation and excitement in the string quartet.


13
14
15
a
b
c
d
Pno.
Vln I
Vln II
Vla
Vc
cresc.
(EM) Iped
IV6/4
V4/2/V
I
IV6/4
16
17
18
19
e
f
g
h
i
j
k
Vped
(VII7→
V
VII7→
V
VII7→
BM VII6/4
VII7/V
Vped
(VII7→
V
VII7→
V
VII7→



Examples 1.9a–k. Measures 13–22 (piano). A comparison of the two versions' transition section.

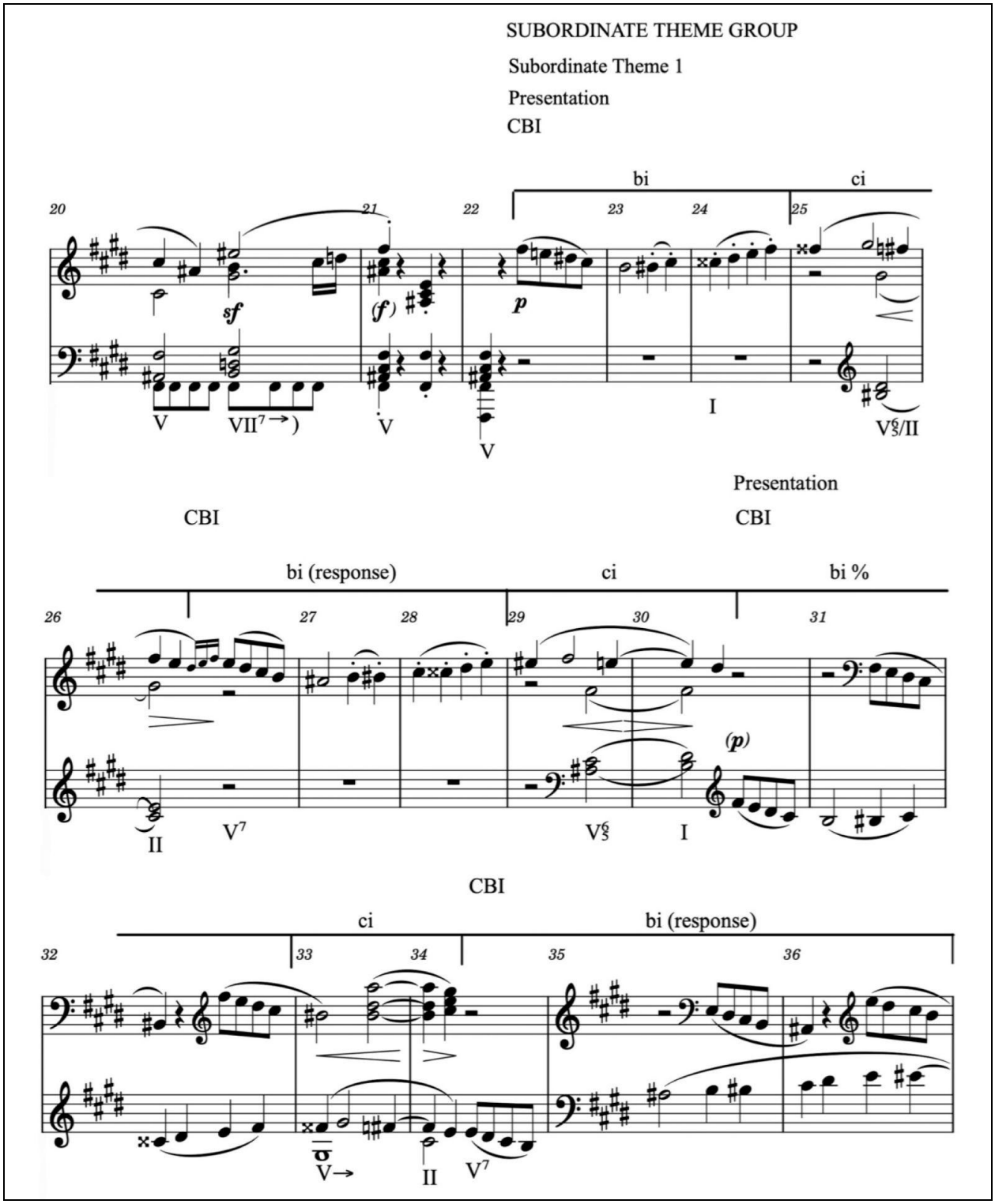
First Subordinate Theme

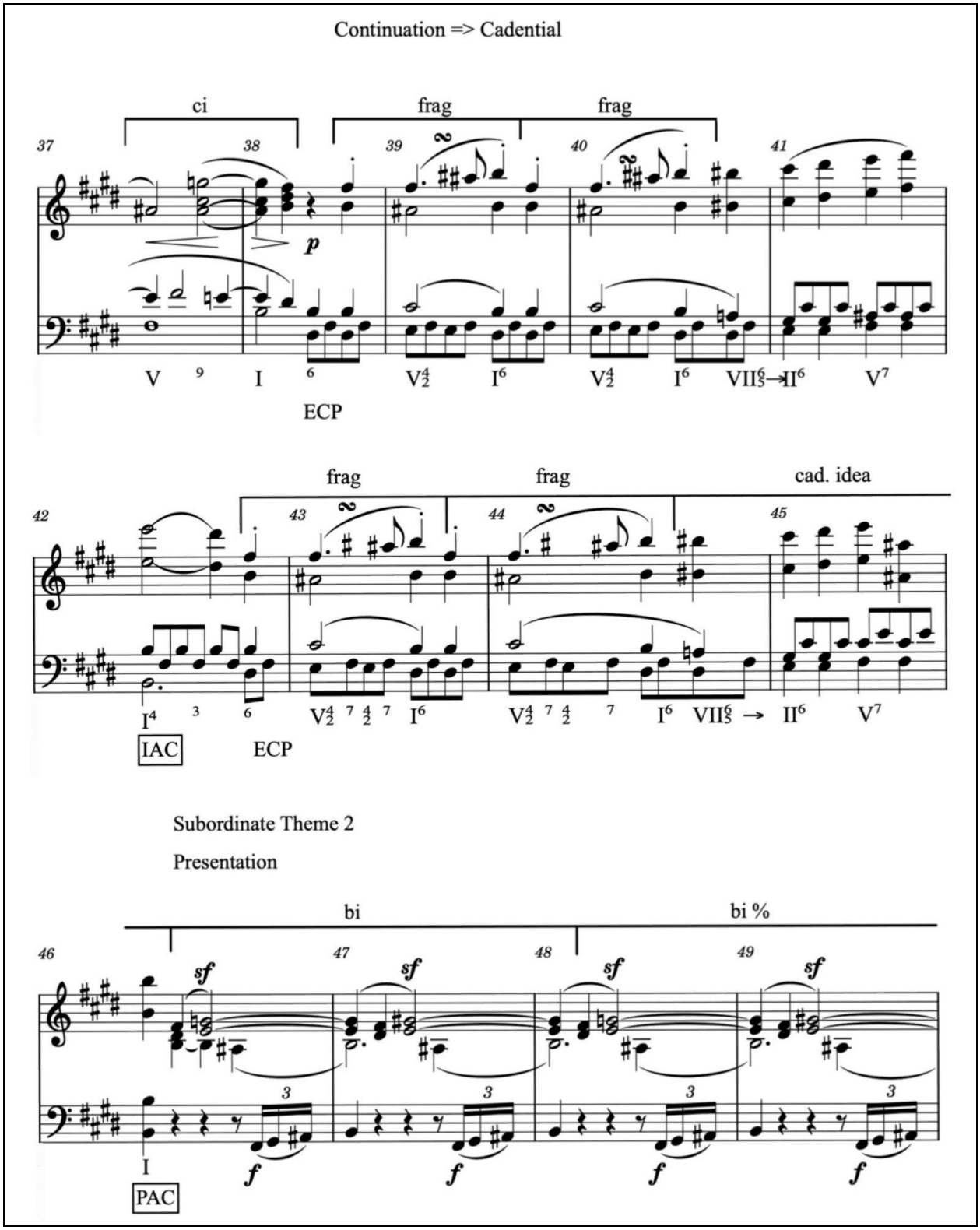
The looseness of the main theme is evident from the ambiguous cadences, but loose vs. tight-knit structures must not be compared to other sonatas but rather to other structures within the same piece. Therefore, to understand how loose or tight-knit the main theme of op. 14, no. 1, i is, it must be compared to the subordinate theme group. The first subordinate theme is the opposite of the main theme in that the initiating function is twice as long as the medial and closing functions, lasting sixteen and eight measures, respectively (see Example 1.10 for an annotated score of the first subordinate theme). The initiating function consists of two presentations, each of which consists of two CBIs, meaning that the same basic idea (BI) is heard four times, a typical characteristic of a loose structure. The loosening features continue with the

harmonic support of the sixteen measures, I–V^6_5/II–II–$V^7(^6_5)$–I, and the Fonte schemata, as theorized by Riepel in his teachings of galant music.[17] This progression of harmonies is interesting not only because it is sequential but also because it gives the impression of an expanded cadential progression (ECP) supporting an initiating phrase. This, of course, could not be an ECP since the dominant does not remain in root position, but it is of note, nonetheless, and foreshadows what is to come in the development section. Beethoven mirrors this loose nature in the string arrangement, even adding a new gesture in mm. 25 and 29 that progresses [V^6_5–V^7]/II–II and V^6_5–V^7–I, respectively (Example 1.11). This new progression creates a stronger tonicization of the supertonic.

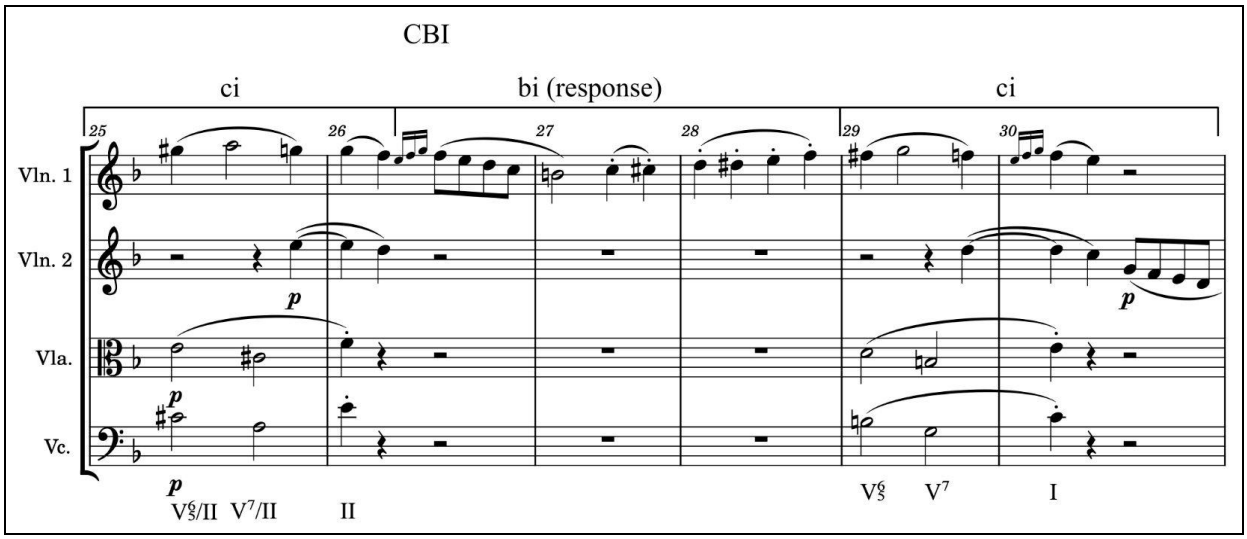
Another change in the string quartet arrangement is much more subtle and almost unnoticeable to the ear. The third occurrence of the BI begins in m. 30. In both versions, the melody moves downward in a whole-tone (WT)–semitone (ST)–WT–WT pattern just as it does in the first BI in m. 22 (Examples 1.12a and 1.12b). In m. 31, a new line appears in the bass, echoing the BI. Beethoven avoids repeating the line exactly, which would have created an augmented ninth between the B♮ and Cx on the downbeat of m. 32 (Example 1.13a). Beethoven solves this problem of the dissonant interval in different ways in each version (see Examples 1.13b and 1.13c). In the piano sonata, he leaves the original line untouched, keeping the Cx in the left hand on the downbeat m. 32. In the echo in the right hand, however, he raises the final note of the descending run from B♮ to B#, creating a major ninth between the B# and Cx in m. 32. This is an easy and elegant solution, but he composes a different solution in the string quartet, which involves switching the altered voices. The second violin begins by playing the BI in m. 30 exactly the same way the first violin does in m. 22, only an octave lower. This time,

[17] Robert Gjerdingen, "The Fonte," in *Music in the Galant Style* (New York: Oxford University Press, 2007), 61–70.

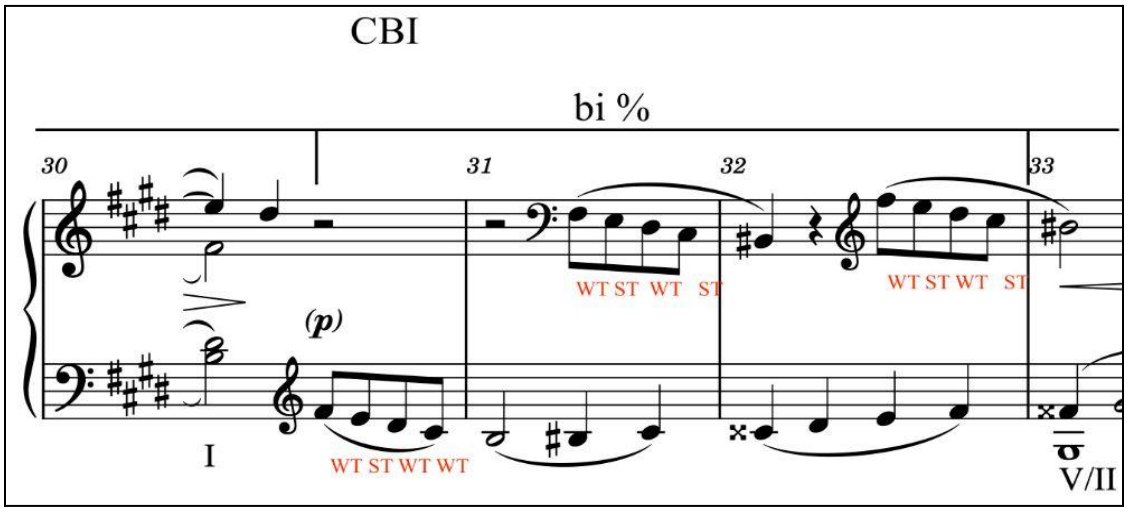

SUBORDINATE THEME GROUP
Subordinate Theme 1
Presentation
CBI
bi
ci
20
21
22
23
24
25
sf
(f)
p
V
VII7→)
V
V
I
V§/II
Presentation
CBI
CBI
bi (response)
ci
bi %
26
27
28
29
30
31
(p)
II
V7
V§
I
CBI
ci
bi (response)
32
33
34
35
36
V→
II
V7



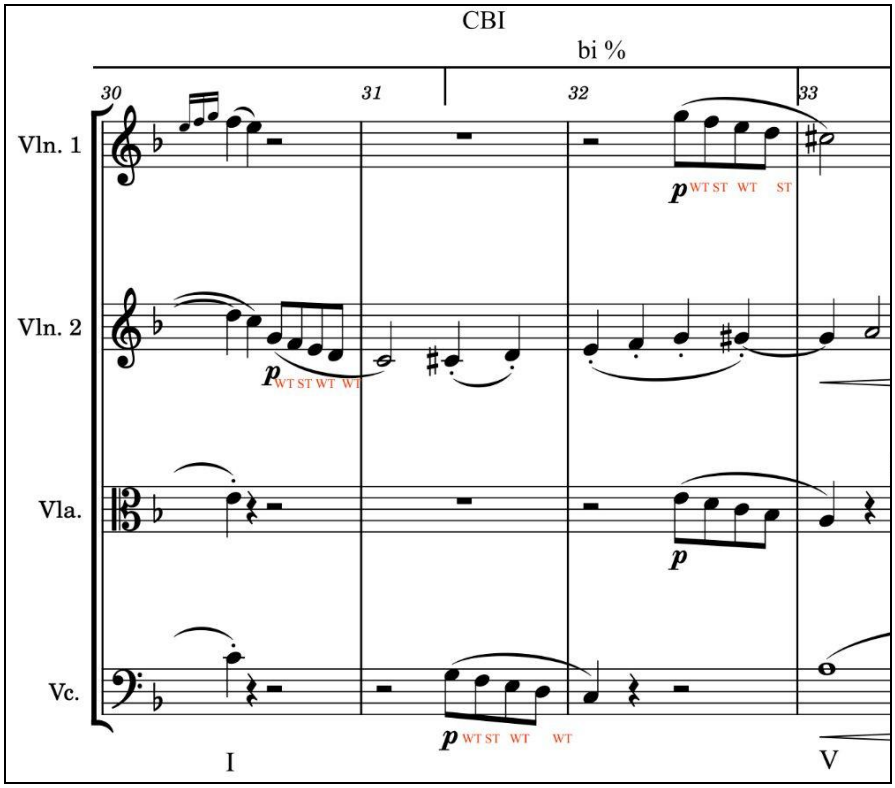
Example 1.10. Measures 20–49. The first subordinate theme.



Example 1.11. Measures 25–30 (string quartet). The new progression in the string quartet creates a stronger tonicization to II.

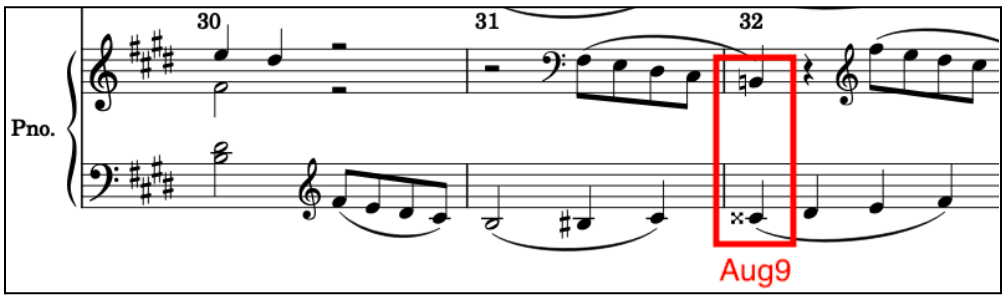


Example 1.12a. Measures 30–33 (piano). The whole-tone semi-tone patterns.

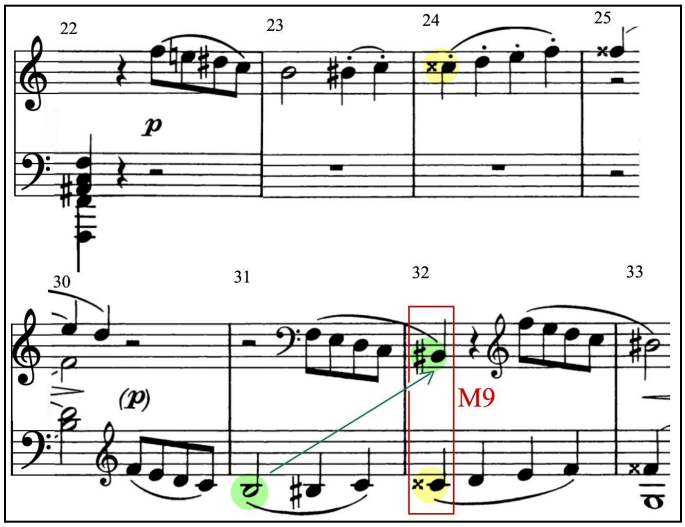


Example 1.12b. Measures 30–33 (string quartet). The whole-tone semi-tone patterns.

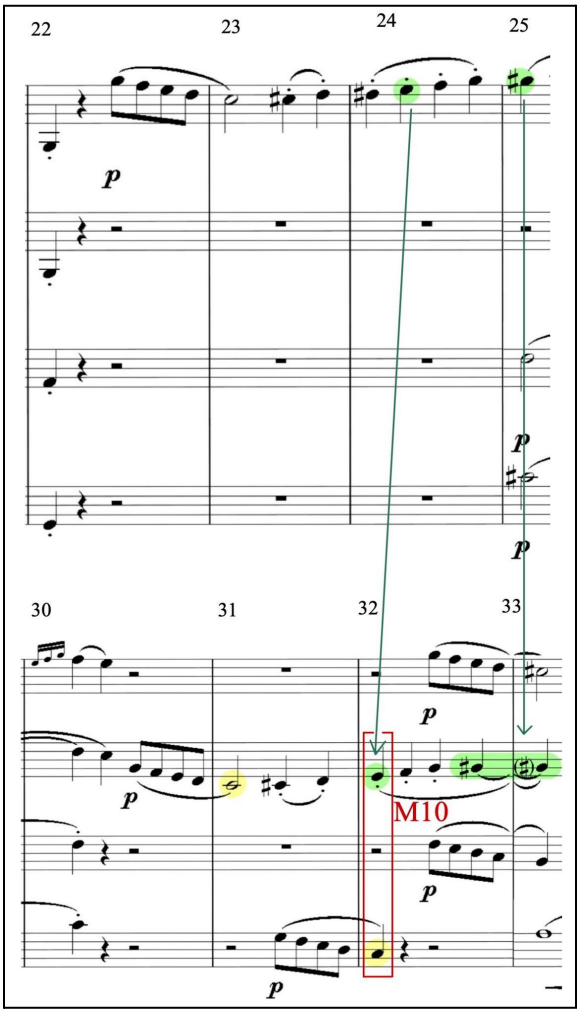
however, the second violin skips over the D# on the downbeat of m. 32 and steps directly to the E♮. The line still finishes at the appropriate time, as the G# is held for an extra beat. The cello comes in with the echo in m. 31, but this time, unlike in the piano sonata, it is unchanged, i.e. the C is not raised to a C#. This creates a major tenth between the C♮ and E♮ on the downbeat of m. 32, which is more consonant than the piano sonata's major ninth. In summary, in the piano sonata, Beethoven changes the imitating voice, whereas, in the string quartet, he adjusts the leading voice.



Example 1.13a. Measures 30–32 (piano) A hypothetical version of m. 32 had Beethoven left the pattern as WT–ST–WT–WT

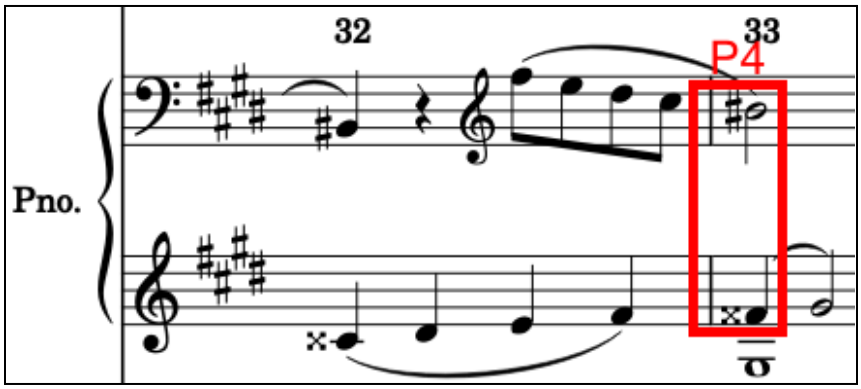


Example 1.13b. Measures 22–25 and measures 30–33 (piano). How Beethoven avoids the augmented 9th.

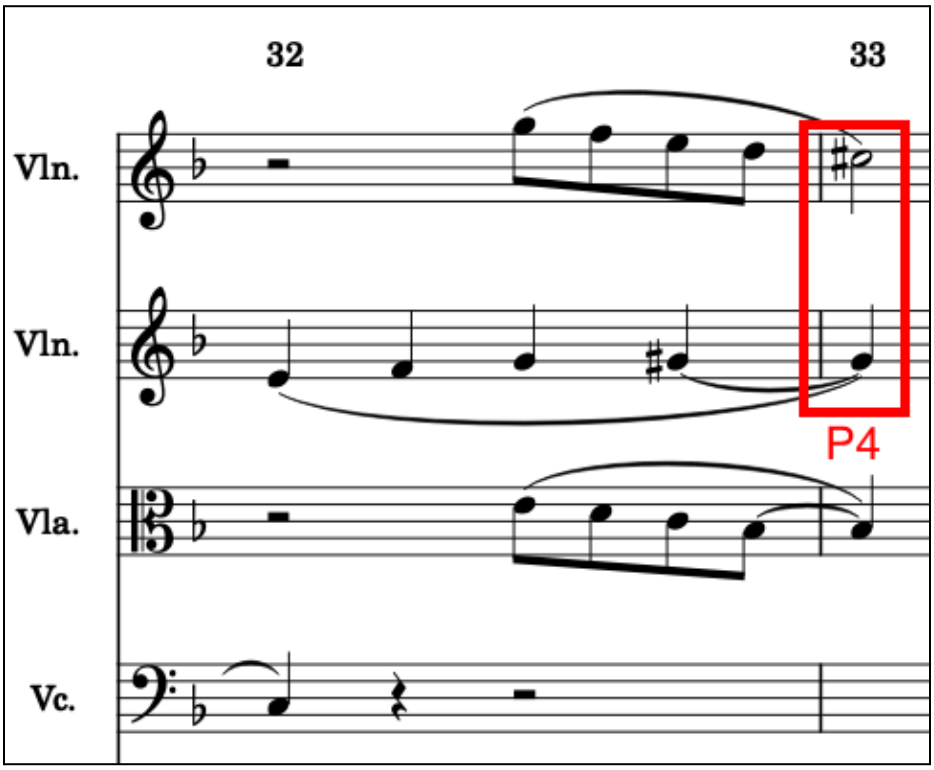


Example 1.13c. Measures 22–25 and measures 30–33 (string quartet). Note that Beethoven avoids the augmented 9th.

In both the piano and string arrangements, the first note of m. 33 is raised to B# and C#, respectively to avoid a diminished fourth (Examples 1.14a and 1.14b). In other words, Beethoven uses the same descending line in mm. 31 and 32 in the piano sonata, as opposed to in the string quartet where the descending line is the same between the second violin and cello in mm. 30 and 31 but altered in the first violin m. 32.



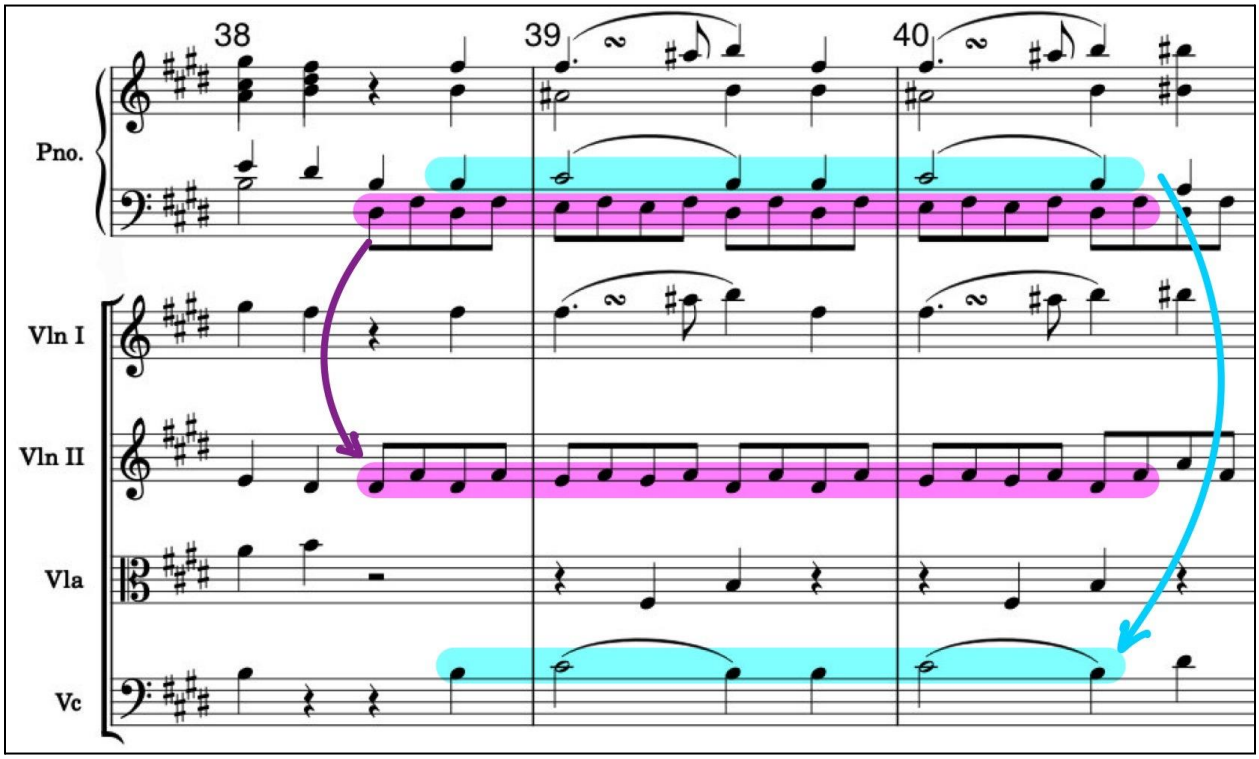
Example 1.14a. Measures 32–33 (piano). The final note of the descending melodic line is raised to B# to avoid a diminished 4th.



Example 1.14b. Measures 32–33 (string quartet). The final note of the descending melodic line is raised to C# to avoid a diminished 4th.

Beethoven does not simply copy the change from a WT to an ST and paste it into the string arrangement; he makes a calculated decision based on what will be least dissonant in each version's key.

The medial and closing functions of the first subordinate theme, much like the initiating function, also consist of a repeated structure—a continuation ⇒ cadential phrase—that has a periodic structure of an imperfect authentic cadence (IAC) followed by a PAC. The only loosening feature is the repeated phrases. Beethoven creates a looser continuation ⇒ cadential in the string arrangement by moving the bass voice in mm. 38–40 to the second violin an octave higher and placing the tenor voice in the cello, thereby creating an unstable line (Example 1.15). He also adds new dynamics that once again create the *crescendo* pulling back to *piano* at the cadence.

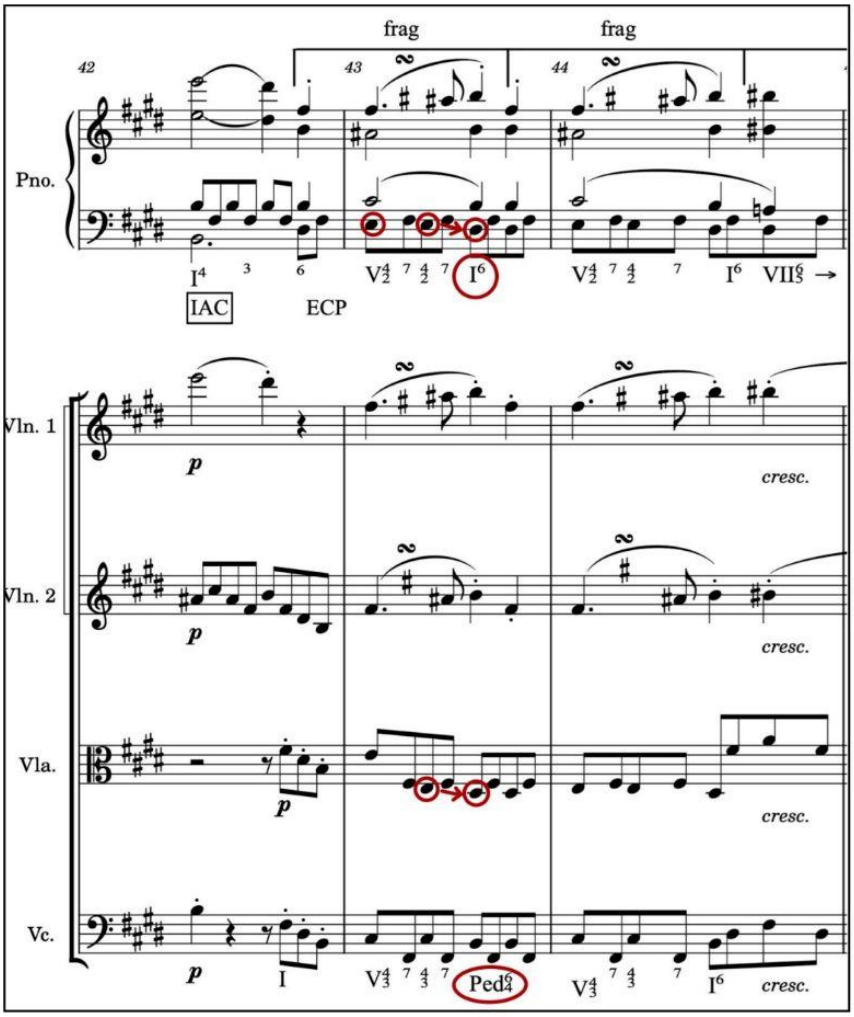


Example 1.15. Measures 38–40. The piano sonata (on top) is compared to the string quartet (below). The string quartet has been transposed into the same key as the piano sonata for ease of comparison.

The fragmentation from mm. 43 to 44 is supported by slightly different harmonies in the string quartet than in the piano sonata. Both versions support an ECP beginning with I on beat four of m. 42 and prolong tonic function using a dominant harmony. The piano sonata arpeggiates between the inversions V^7 and V^4_2, meaning that the chordal root and seventh are in the bass. Therefore, the harmony on beat three of m. 43 must be a I^6, owing to the requirement to resolve the V^4_2. Compare this to the string quartet, which prolongs the medial dominant harmony with inversions V^4_3 and V^7, meaning that the chordal seventh can resolve in another voice; Beethoven thus implements a pedal 6/4 in m. 43 instead of I^6. The chordal seventh is in the viola and is still able to resolve correctly. The difference between the two prolongational techniques is negligible but still worth taking note of, as it shows Beethoven pays close attention to voice-leading concerns and treats them individually as necessary in each version (see Example 1.16)

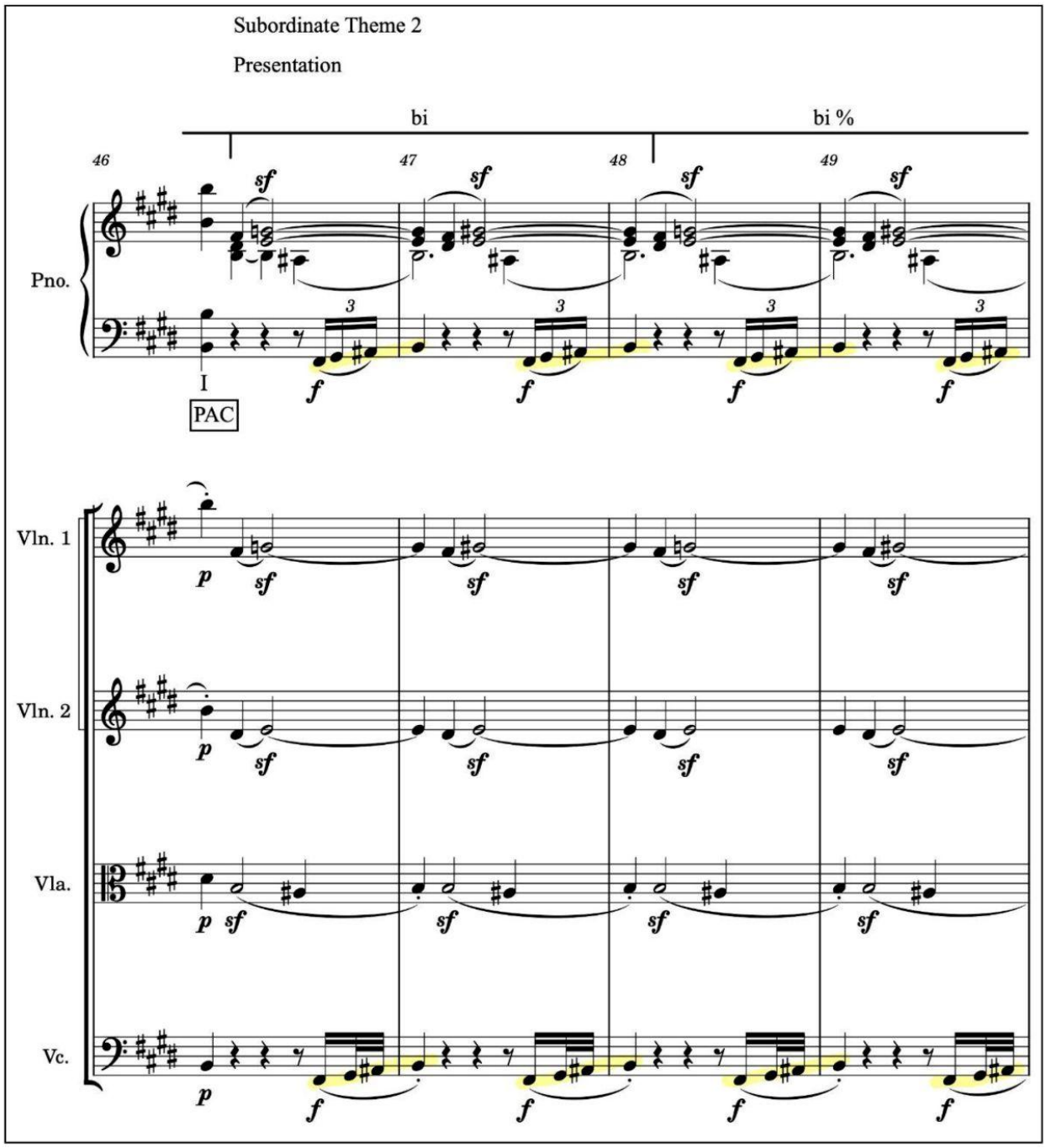
Second Subordinate Theme

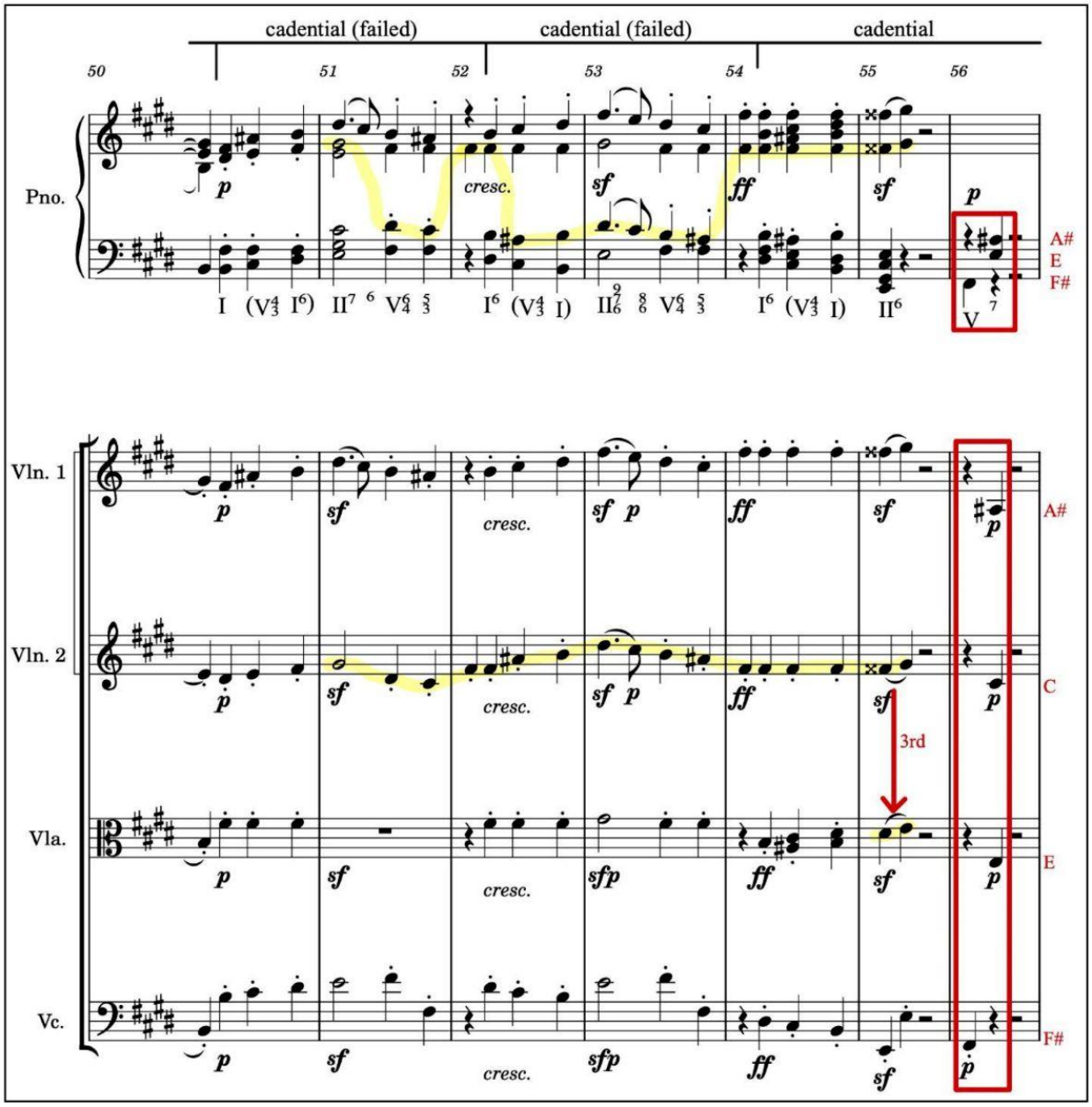
Unlike the first subordinate theme, the second subordinate theme looks remarkably like the main theme in its formal layout. It begins with a four-measure initiating function followed by an eight-measure continuation, and the cadential phrase is loosened by two consecutive evaded cadences, ending with a PAC that elides with the next section, just as the main theme closed with three consecutive ambiguous cadences; the last of which elided with the transition. The second subordinate theme appears even more tight-knit than the main theme when considering that the former closes with a clear PAC in m. 57 while the latter's cadential closure was extremely uncertain. The second subordinate theme's strong cadential closure is problematic because it does not allow for the main theme to be more tight-knit than the subordinate theme, so we see Beethoven toying with the norms of a sonata form.



Example 1.16. Measures 42–44. The piano sonata (on top) has a slightly different harmonic progression at m. 43 than the string quartet (below). The string quartet has been transposed into E major for ease of comparison.

This "too-tight-knit" problem opens the door for Beethoven to change some aspects of the string quartet to make the second subordinate theme much looser than the main theme. Strangely though, he changes very little. Some of the voices move around; for example, in mm. 51–55, the second violin takes from the tenor and alto parts: this is typical of a string quartet for the viola to play more filler material (see Example 1.17 for a comparison of the piano and string quartet versions). The triplet sixteenth rhythm in mm. 46–49 becomes a sixteenth–two thirty-second run in the cello. The viola adds a third below the melody Fx–A in m. 55, enriching the texture with more filler material. The cello part changes octaves frequently but is otherwise


Subordinate Theme 2
Presentation
bi
bi %
46
47
48
49
Pno.
I
PAC
Vln. 1
Vln. 2
Vla.
Vc.



Example 1.17. Measures 46–56. A comparison of the second subordinate theme of both the piano sonata and the string quartet. The string quartet has been transposed into E major for ease of comparison.

identical to the bass voice of the piano. Finally, m. 56 in the string quartet is more fleshed out than in the piano sonata: the cello plays the G, then the upper voices respond with the rest of the V^7 chord (B♮, D, F) a beat later. All four notes of the V^7 chord are heard, not just the root, third, and seventh, and a strong sense of call-and-response is felt in the change in timbre between the cello and the upper strings.

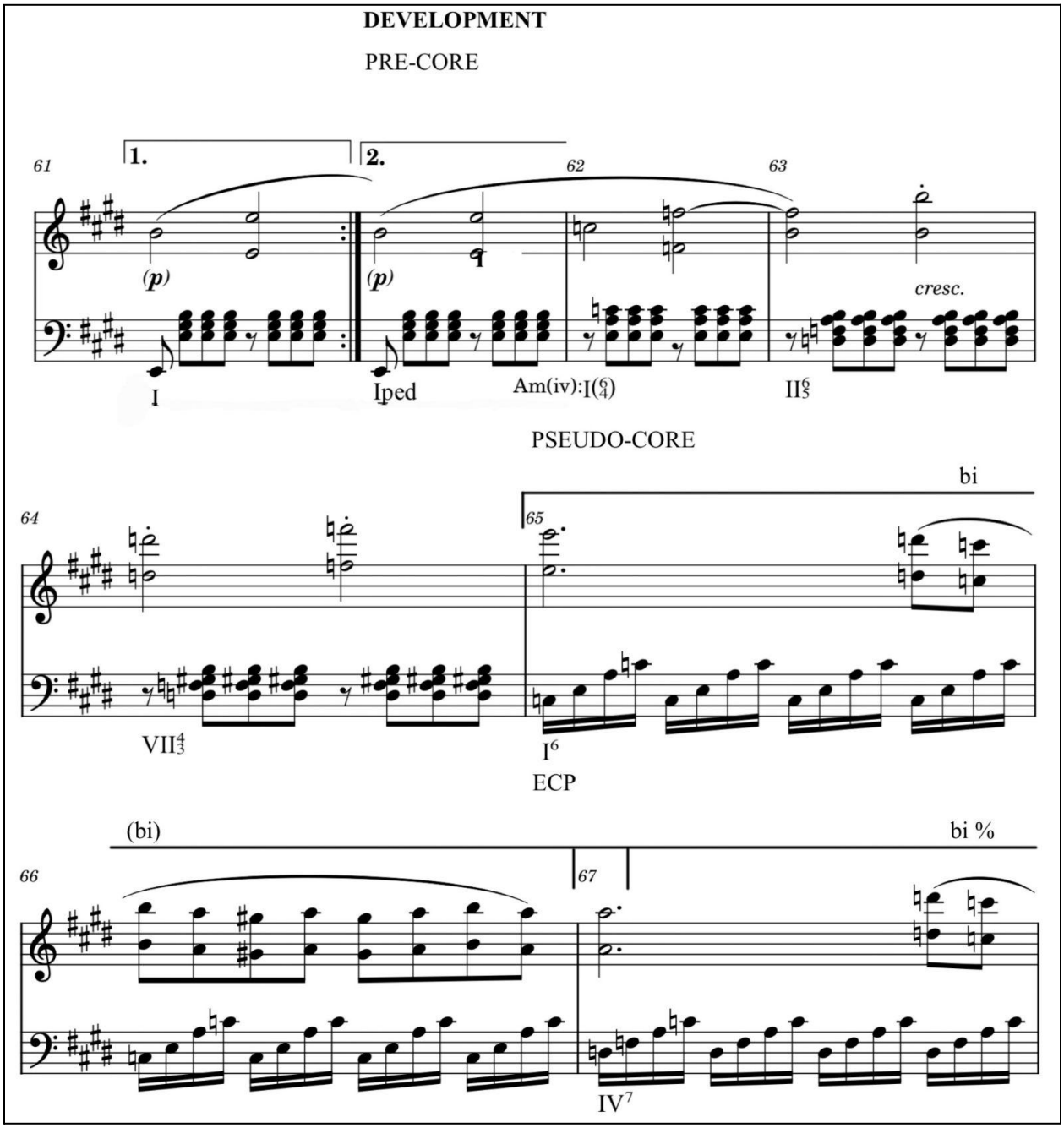
Beethoven does not loosen any of the elements that made the second subordinate theme tight-knit. Since Beethoven makes a clear, conscious effort to change elements in other sections—i.e. the main theme and the first subordinate theme—the lack of changes made to the second subordinate theme is telling. Beethoven wants to keep the second subordinate theme tight-knit, perhaps to round out the exposition with two main-theme-like phrases, or maybe the second subordinate theme is better analyzed as a large closing section following the first subordinate theme. Furthermore, this ambiguity loosens this section, meaning, paradoxically, the tight-knit nature of the second subordinate theme loosens it.

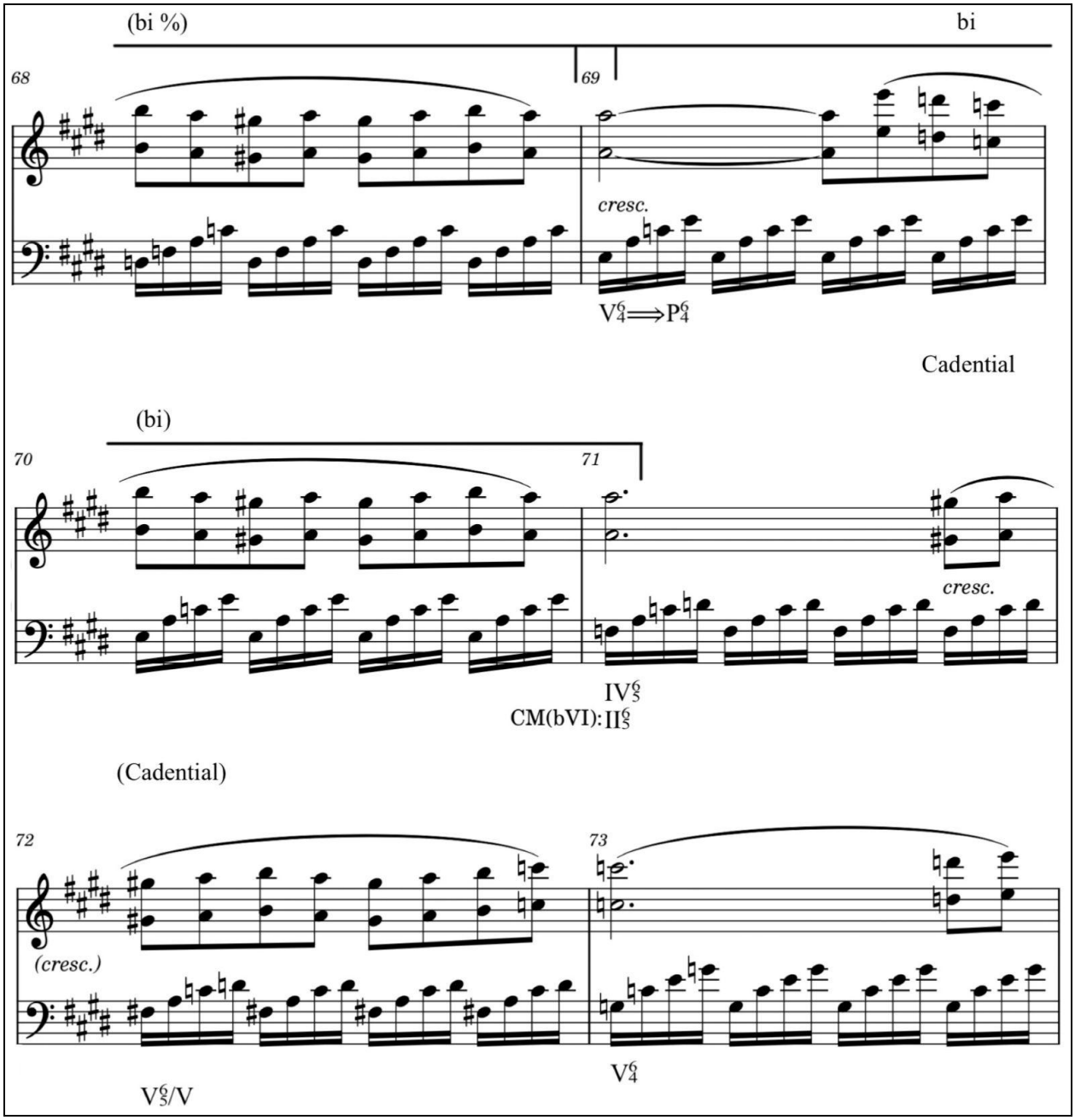
Development

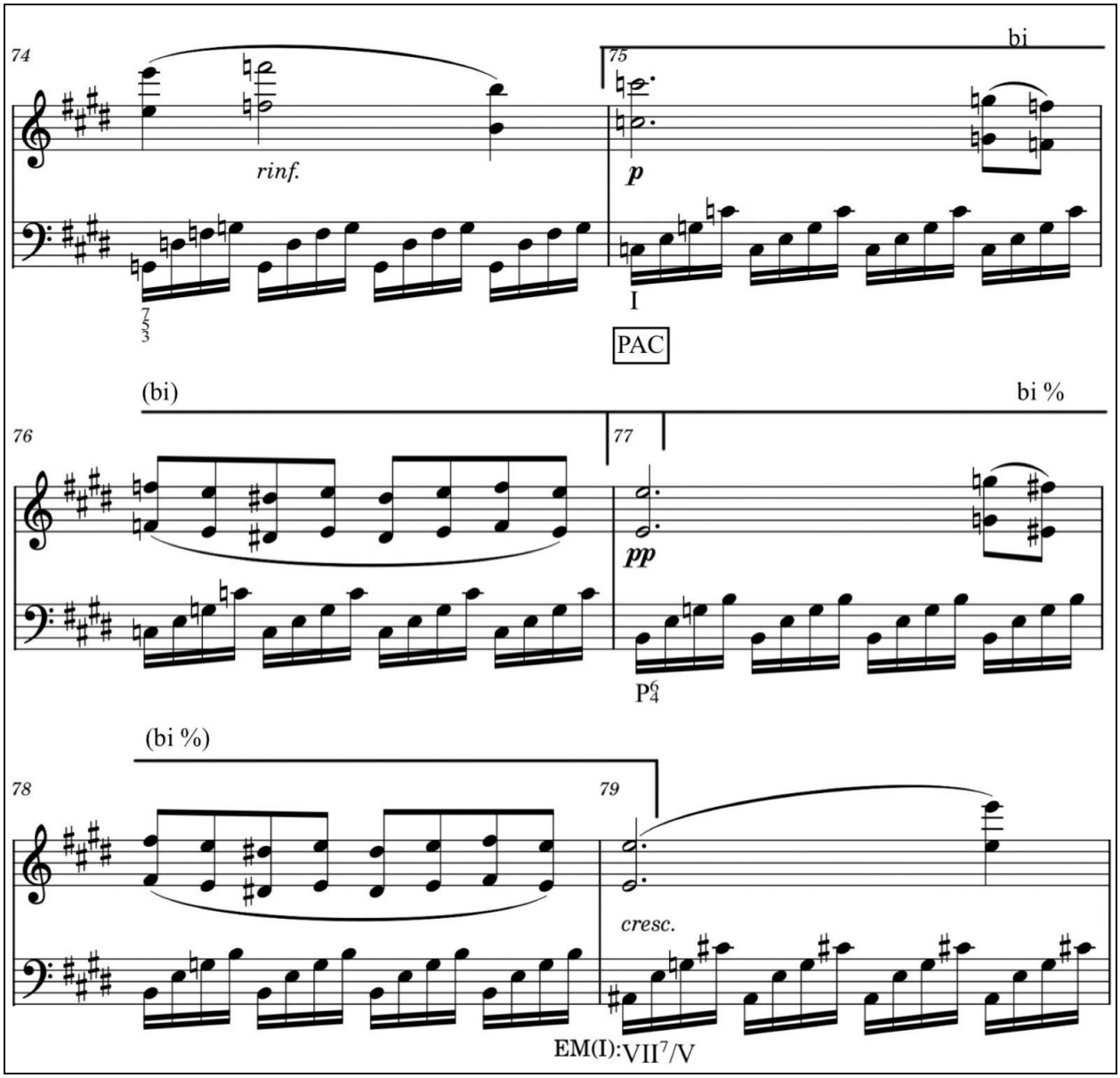
Due to the extensive formal anomalies of the development section, an understanding of the analysis of the piano sonata is necessary before comparing it to the string quartet (see Example 1.18 for an analysis of this section). The development is structured atypically. Steinberg shows that its unusual format was by design, according to annotations in Beethoven's sketches: "This was Beethoven's plan from the beginning; in fact, he had even written a reminder to himself in the sketch: 'ohne das Thema durchzuführen' (without developing the theme)."[18] As opposed to the usual modulation to the key of (♭)VI, III, or II, Beethoven opts to set this section in the key of minor IV. This is odd for a piece set in a major key, not that the development section is in a minor mode—this is typical of a development section—but that Beethoven decides to use *minor* IV, a harmony that is diatonically major in the home key. He avoids using both VI and II because he modulates to ♭VI in m. 71 (see below) and because II (F#) has already been a prominent harmony as the dominant of the subordinate key, B major. His reason for dismissing the key of III, G# minor, is less obvious. He does not modulate to G# minor anywhere else in the piece, nor is it tonicized a great deal. His choice of minor IV, however, is indicative of the

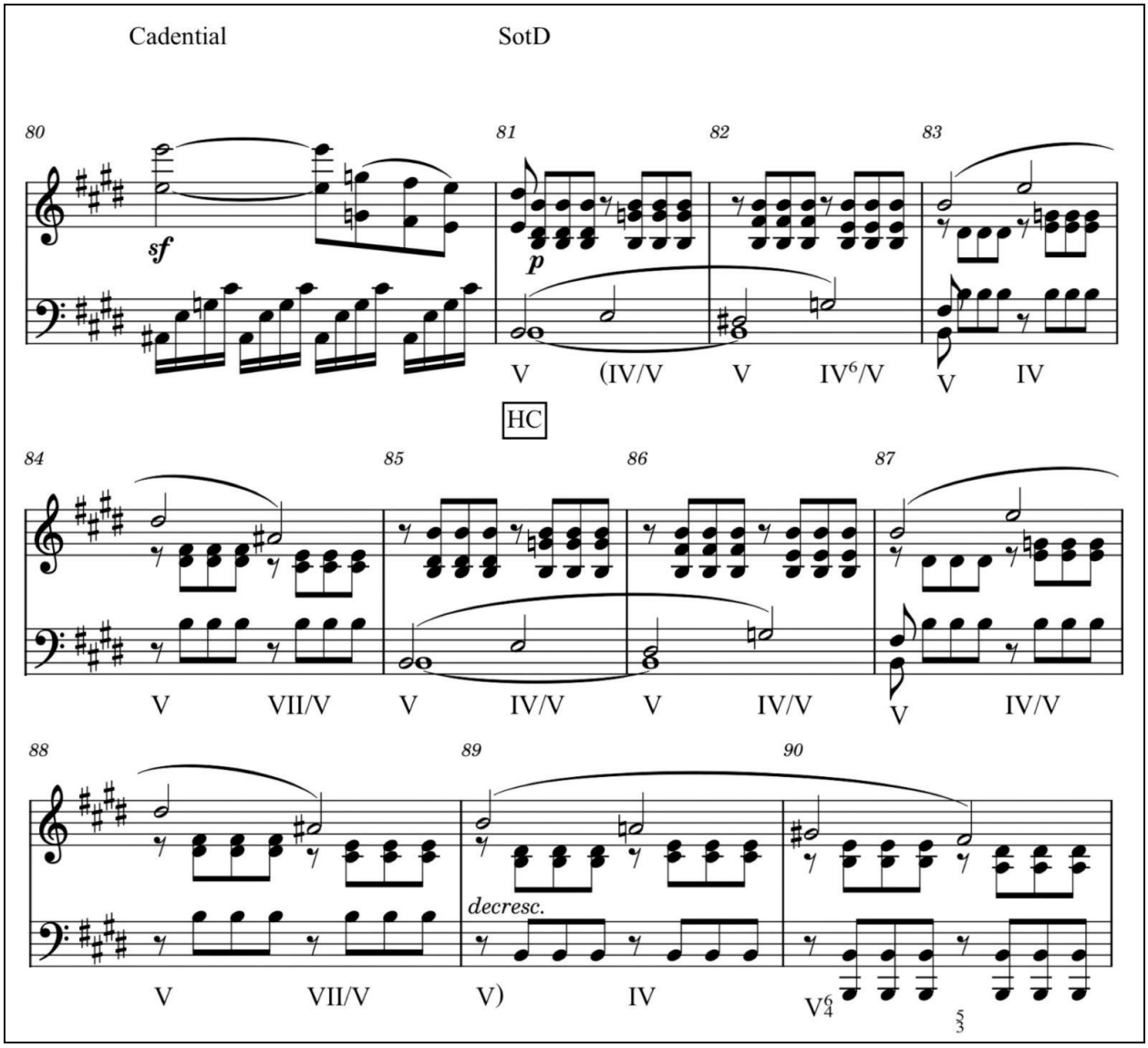
[18] Steinberg, 147.

strange nature of this development section and sets up the listener to be surprised in the coming section. Recall, also, Beethoven's strange use of A minor in the transition (m. 15). This return to the unusual A minor helps to tie the two sections of the piece together.




(bi %)
bi
68
69
cresc.
V6/4⟹P6/4
Cadential
(bi)
70
71
cresc.
IV6/5
CM(bVI): II6/5
(Cadential)
72
73
(cresc.)
V6/4
V6/5/V


74
rinf.
75
bi
p
I
PAC
(bi)
bi %
76
77
pp
P 6 4
(bi %)
78
79
cresc.
EM(I):VII7/V



Example 1.18. Measures 61–90. The development section.

While unusual for its minor mode setting, the pre-core is relatively normal in its structure; it draws material from the main theme's basic idea of the +4 /-3 melodic sequence over the drum bass and the soft dynamic. Both the piano sonata and the string quartet begin with *piano* and crescendo to m. 65, but the softness is contrasted in the latter with a quick burst of energy with a *fortepiano* to start the core. This pre-core is incomplete because of its relatively short nature and lack of a complete thematic unit. Its uncharacteristically short time frame is the second of many abnormalities in this development section, the first being the modulation to IV.

The development section cannot be said to have a strict "core" as it does not have a large-scale model that is repeated. Instead, there are two phrases that mirror each other in some ways and oppose one another in others. For this reason, among others listed below, this part of the development section is labelled "pseudo-core." Caplin describes a pseudo-core as "… a unit whose dynamics, rhythm, texture, and emotional character strongly resemble those of a core but whose material is not organized by model-sequence technique."[19] This description is true of op. 14, no. 1, i.

The first phrase begins at m. 65 and ends at m. 75 with a PAC. The phrase comprises a presentation + consequent, meaning the BI is heard three times. This label, however, is misleading as the melody of the "BI" is the same each time, but the harmonies differ. The first time the "BI" appears in mm. 65–66, it is supported by an A minor triad in first inversion; the second time, in mm. 67–68, by a D minor seventh chord in root position; and finally, in mm. 69–70, by an A minor triad in second inversion. As such, instead of labelling these two-measure segments as basic ideas, a better analysis would be the more generic title, "new idea." The new ideas are more clearly outlined in the string quartet with the *fortepianos* at the beginning of each one, but these dynamic outlines are not enough to signify a clear BI. Furthermore, this entire ten-measure section does not have proper phrase functions and, therefore, cannot be called a hybrid theme, exemplifying yet another reason why the development does not have a proper core.

Measures 65–75 are structured similarly to the first subordinate theme in that the harmonies suggest an ECP under an initiating function. This phrase's improper use of an ECP is even more egregious than the first subordinate theme's in that the section begins in A minor and modulates halfway through to C major: the phrase begins with the initial tonic I^6 moving to the

[19] Caplin, *Analyzing Classical Form*, 451.

predominant IV^7 to what is initially understood as a cadential 6/4 but must be retrospectively reinterpreted as a passing 6/4 between the two predominant harmonies. The pivot chord is at m. 71, which pivots between IV in A minor and II in C major. The predominant function is prolonged by a V^6_5/V in m. 72 before arriving at the dominant cadential 6/4 in m. 73. The phrase closes with the PAC in m. 75. Were this progression part of a closing section, it could be analyzed as an ECP. By supporting an entire initiating, medial, and closing section, however, this cannot all be considered "cadential." Furthermore, ending with such a strong cadence only halfway through the development section is quite odd. The false ECP and PAC are other atypical aspects of this development section.

The second phrase elides with the first, beginning in m. 75 and landing on an HC in the home key in 81. This section begins in a similar way to the first half of the development—by beginning with a two-measure tonic prolongation—but quickly diverges in the third measure. What was a IV^7 chord in m. 67 is now a more ambiguous passing 6/4 chord in m. 77. Furthermore, the first half of the development had three full appearances of the new idea before pivoting to the new key of ♭VI, whereas in this second half, the new idea is only heard twice before the abrupt retransition back to the home key of E major with a VII^7/V chord. This sudden modulation recalls the jarring shift to B major in the transition of the exposition in m. 16, once again tying the development back to the exposition.

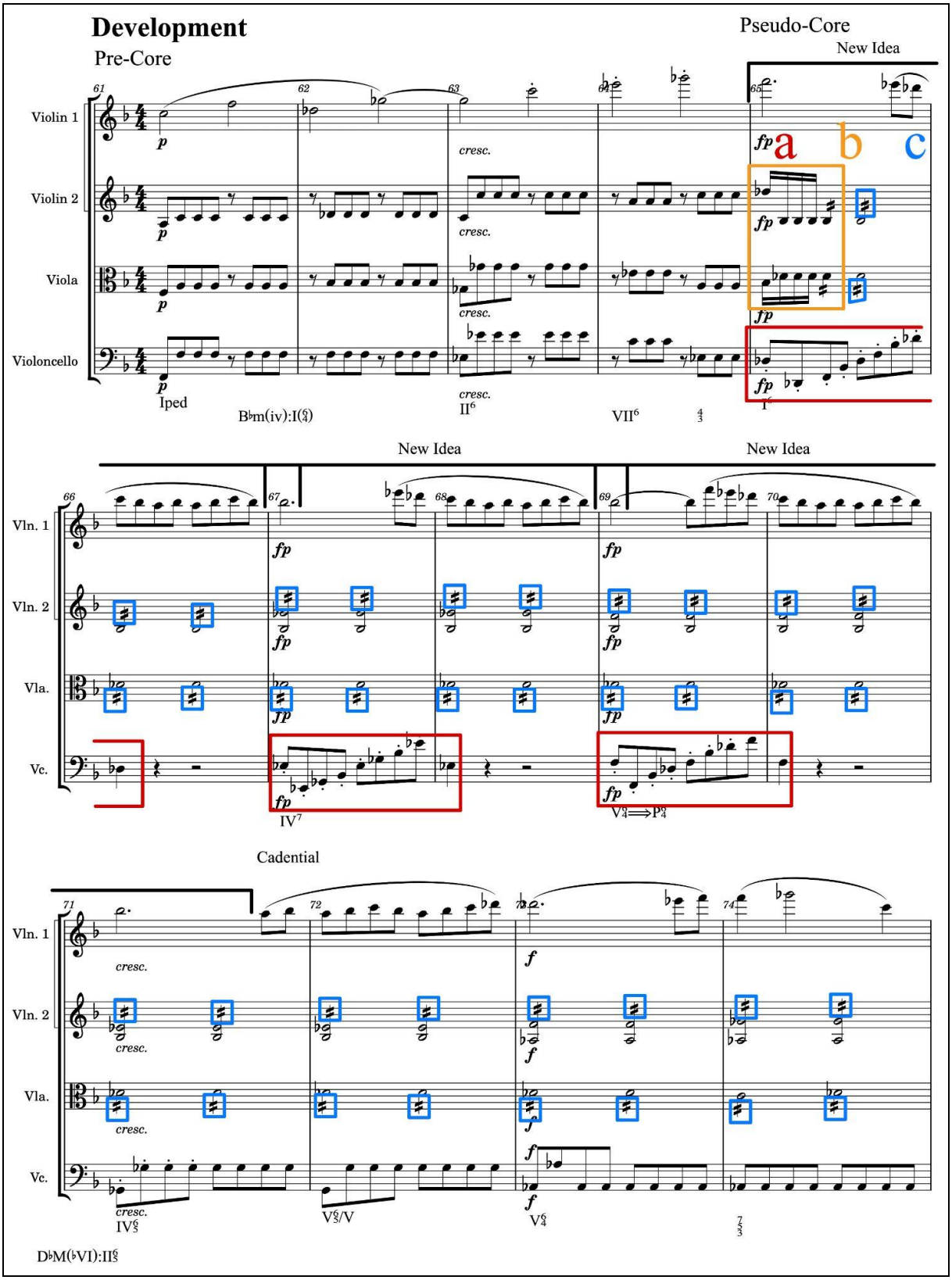
Had the first part of the development section contained more convincingly "core" material, one may be inclined to call this second phrase a second core or even a second *pseudo*-core. This idea makes sense when considering that the first phrase ends with a PAC in the development key and this section ends in the dominant of the home key—an important aspect of the definition of multiple cores: "If a core of the development ends in a development key, then a

second core may follow, this one closing with dominant harmony of the home key, in preparation for the recapitulation."[20] To have a legitimate second core, however, the second core material must be motivically different from the first core. This is not the case in op. 14, no. 1, as the motivic content between the two halves is nearly identical: a flowing legato line above an arpeggiated chord. Additionally, multiple cores are usually reserved for large-scale works, such as symphonies. It is uncommon to see one in a small-scale work like a piano sonata (or string quartet). As such, both phrases are part of the same pseudo-core, not two separate pseudo-cores.

The final section of a typical development is the SotD. Unlike the rest of the development section, the SotD is quite standard: Beethoven borrows material from the main theme, including the drum bass and the sequential melodic line, which he alters slightly so that he can continue to tonicize the key of the dominant (B major).

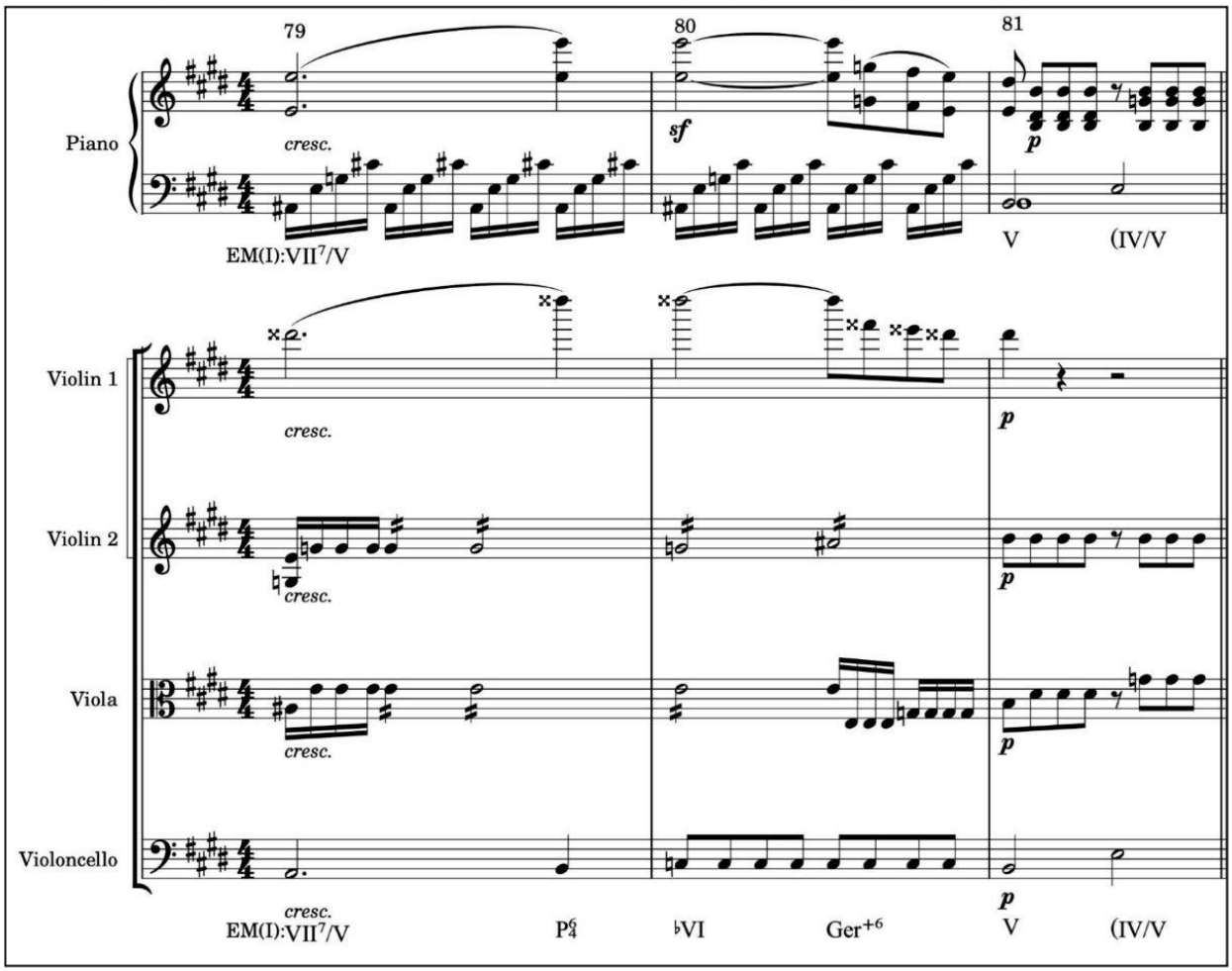
The most noticeable change between the piano sonata and the string quartet is the textural change in the bass voice. What helps to differentiate the pre-core from the pseudo-core in the original is the drastic change between the drum bass and the running sixteenth-note arpeggiation. In the string quartet, however, that running bass line is completely omitted in favour of the cello playing staccato eighth notes that arpeggiate the chords in two octaves for only half of each new idea (Example 1.19a). The sixteenth-note rhythm briefly appears in m. 65 in the second violin and viola, but quickly disappears as the two instruments fill out the harmonies in half notes from mm. 65–74 (Example 1.19b). Even though the sense of forward motion is maintained through the use of the tremolo in the second violin and viola (Example 1.19c), in removing the driving bass line, Beethoven has eliminated a significant element that allows this section to be more "core-like." As such, the pseudo-core is even more atypical in the string quartet than it is in the piano sonata.

[20] Caplin, *Analyzing Classical Form*, 422.



Example 1.19a–c. Measures 61–74. (a) The arpeggiated chords in the cello; (b) the only moment of sixteenth notes in this section; (c) how the tremolos in the second violin and viola help to push the music forward.

The modulation back to E major in m. 79 is quite different in the string quartet. Whereas in the piano sonata, the harmony continues the pattern of staying static for two full measures, in the string quartet, chromatic chords are added to build anticipation toward the HC (see Example 1.20). Beethoven loses the drive to cadence with the omission of the running sixteenth notes, but he is able to regain that momentum through increased harmonic rhythm. Moreover, the contrast of the long half notes versus rapid sixteenth notes in mm. 79–80 of the second violin and viola helps to push towards the cadence.

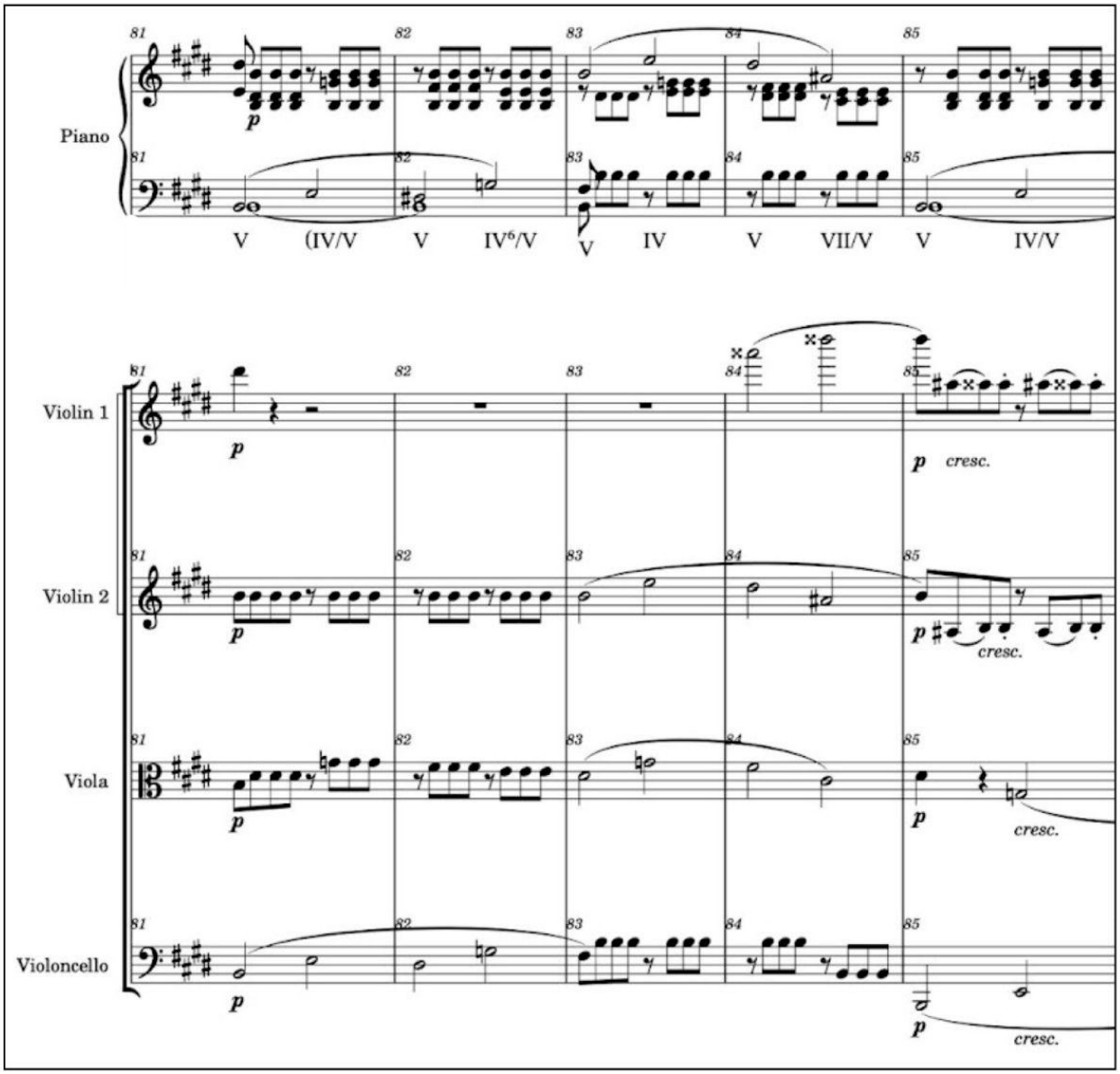


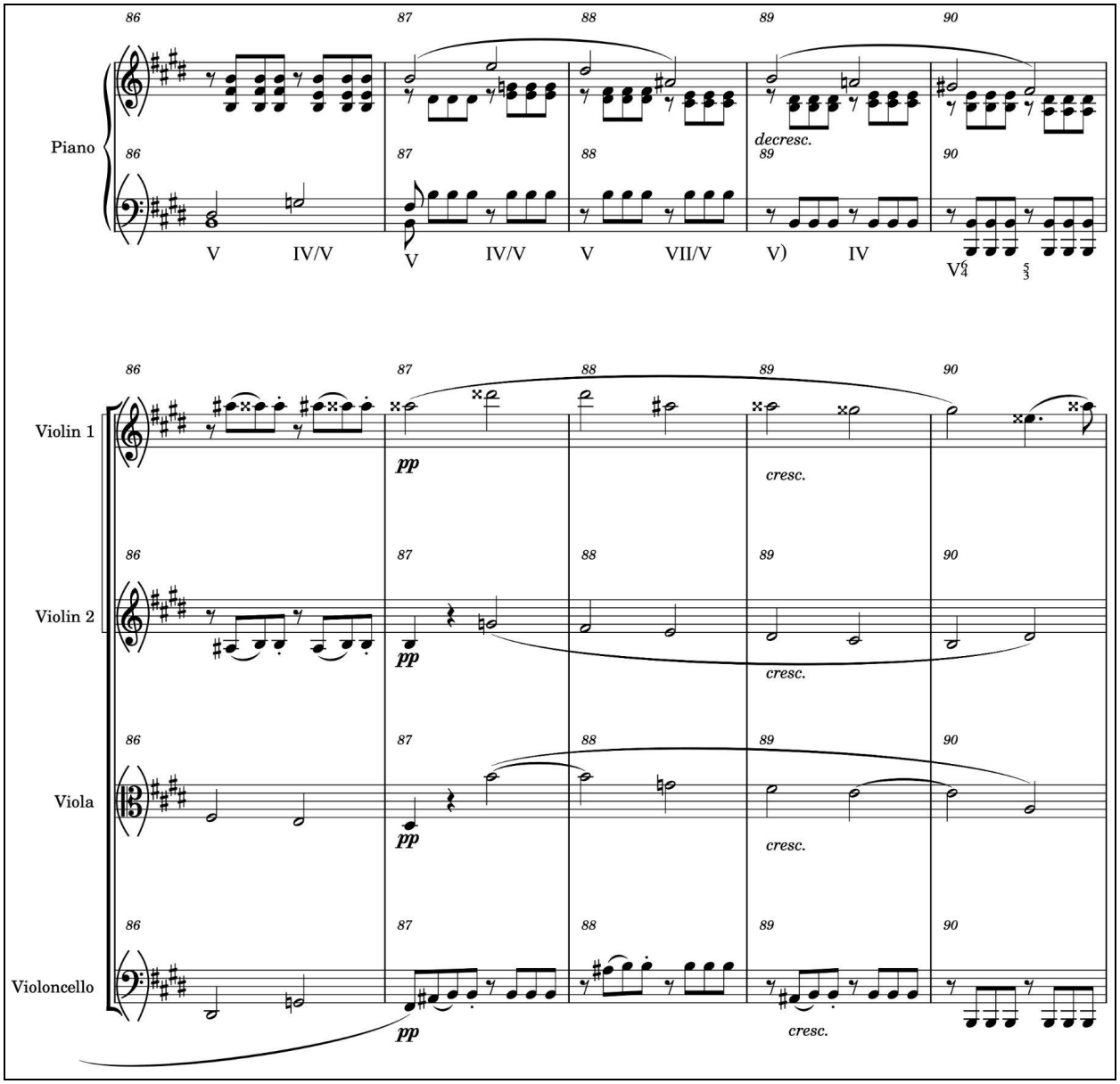
Example 1.20. Measures 79–81. The piano sonata (on top) has a different harmonic progression than the string quartet (below). The string quartet has been transposed into E major for ease of comparison.

Even though the SotD could remain untouched in the string quartet, Beethoven makes many slight changes (see Example 1.21). Beginning in m. 81, he eliminates the pedal B from the bass voice and also removes the drum bass B from the tenor voice, instead keeping the B in the second violin and omitting the first violin completely. This abrupt loss of the highest string helps to emphasize more separation from the previous section and the SotD. Secondly, the melody line that flows back and forth between the right and left hands in the piano flows similarly between the strings, except with additional harmonization. For example, the viola adds a sixth below the melody in mm. 83–84, and the first violin responds with an echo in m. 84. Similar harmonization is implemented in mm. 87–90, where the first violin plays the melody, and the second violin and viola fill out the rest of the chord. The drum bass is maintained in the cello, but the accompaniment texture in the right hand has been swapped out with melodic texture in the accompanying second violin and viola.

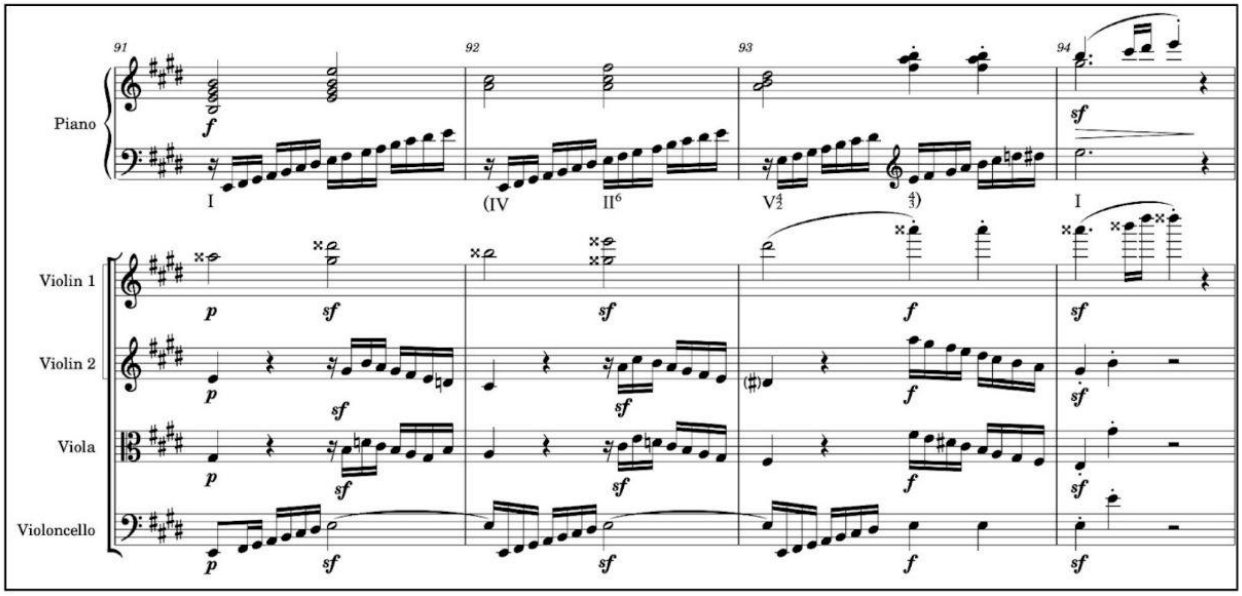
Additionally, the dynamic markings in the SotD, like many other areas of the string quartet, are much more frequent than in the piano sonata. Such an abundance of dynamic markings would be unnecessary in a piano score, as the pianist can shape the music as they see fit without consulting other performers. In a quartet, however, the musicians have to match and blend within the ensemble and must be given more guidance on how to shape the music together. Furthermore, the sustaining properties of string instruments afford certain dynamic fluidity that is not possible on keyboard instruments. Sometimes, the dynamics are changed altogether between the two versions, as in mm. 81-91, which indicates a *piano* that then decrescendos to *subito forte* in the piano sonata versus in the string quartet where a *pianissimo* at m. 87 crescendos to *piano* in m. 91. The former is an interesting dichotomy of approaching silence and then exploding into *forte,* which brings in the recapitulation with strength and bombastic energy.

In contrast, the latter begins the recapitulation in the same way the exposition began: with a soft introduction that grows with the rise of the melody. Furthermore, the string quartet mirrors the main theme's growth with the addition of the *sforzandi* from mm. 91–94 (Example 1.22).





Example 1.21. Measures 81–90. The SotD section. The piano sonata (on top) is compared to the string quartet (below). The string quartet has been transposed into E major for ease of comparison.



Example 1.22. Measures 91–94. The beginning of the recapitulation. The piano sonata (on top) and string quartet (below) do not always share the same dynamic markings. The string quartet has been transposed into E major for ease of comparison.

Lockwood uses this instance of dynamic inconsistency as an example of Beethoven's mastery of both instruments' colour, timbre, and general style:

> The two means of articulation through dynamics are completely opposed to one another. One suggests a means of pianistic shading that reinforces the chordal *Klang* of the arrival at bar 91; the other suggests a means of shading that uses the sustaining qualities of the stringed instruments to create a sense of sonorous growth as the cadence gains strength, only to back quickly at bar 91.[21]

The breadth of changes to the development section between the string quartet and the piano versions are evidence of Beethoven's deep understanding of not only the intricacies of the sonata but also the individual timbre, style, and colour of each instrument.

Recapitulation and Closing Section

The recapitulation is standard for Beethoven's sonatas; it is nearly a carbon copy of the exposition with additional embellishments and slight adjustments that allow it to remain in the home key of E major. Caplin explains that: "… the start of the recapitulation is often given an

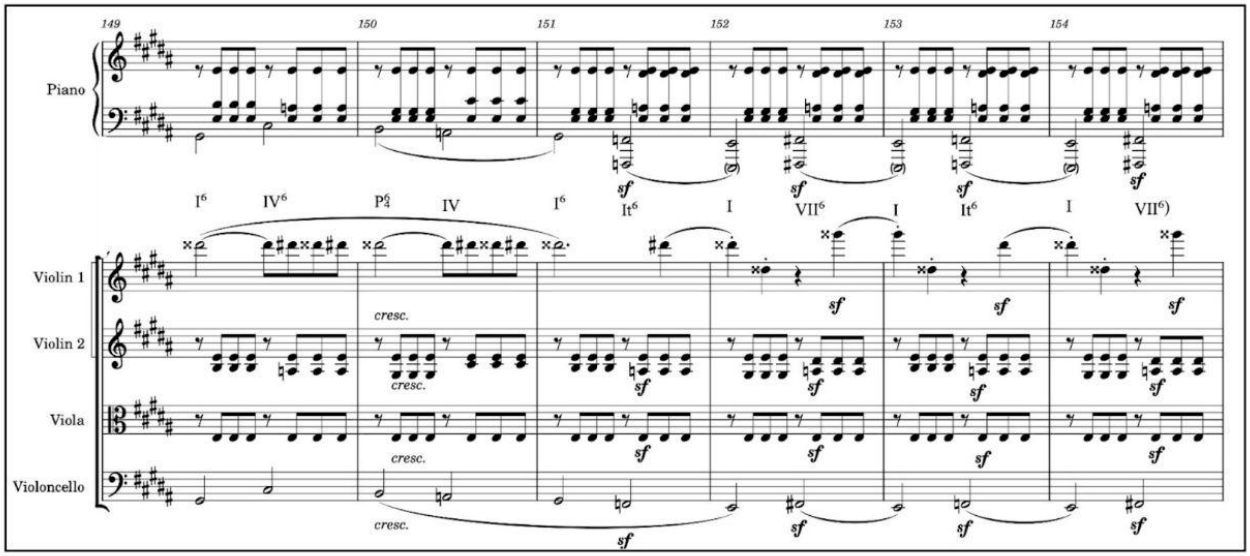
[21] Lockwood, 176.

entirely different dynamic, usually one that is louder, more forceful, and more dramatically charged than how the exposition opened."[22] Op. 14 is no exception; the quiet *piano* dynamic of the opening is replaced with a bombastic *forte* in the piano sonata, and explosive *sforzandi* scattered throughout the first four measures, from mm. 91 to 94 (see Example 1.22 above).

The differences between the piano sonata and the string quartet of the recapitulation are quite similar to those (already discussed) in the exposition. As such, an in-depth comparison of the two versions is unnecessary. However, in the closing section, Beethoven does implement a change in the string quartet that merits further investigation. A new line is added to the first violin in mm. 149–154 (see Example 1.23). Forbes explains the composer's reasoning behind adding this new melody: "In the [closing section] Beethoven can retain the mood and feeling only by altering the texture of the whole passage and adding new material; hence the justification of his claim that only the composer should be entrusted with such arrangements."[23] This gives further merit to the idea that it is only the composer who can authentically arrange a work. A stranger to the sonata would not have understood the "mood and feeling" of the closing section as well as Beethoven himself and would have, inevitably, lost those key elements that contribute to the piece's overall aesthetic.

Nottebohm's suggestion that the string quartet may have been conceived before the piano arrangement of op. 14, no. 1 is not ridiculous, especially when considering all the evidence to support the claim, including Count Anton Apponyi's commission for a string quartet, Beethoven's sketches, and lines that are more idiomatic to string quartets than piano. The question of its writing order, however, should not call into question the validity of either work. It is not necessary to consider one version as the *primary* or *true* version and the other as a

[22] Caplin, 477.
[23] Forbes, 110.



Example 1.23. Measures 149–154. In part of the closing section, Beethoven introduces a new line in the first violin of the string quartet (below) that was not previously in the piano sonata (on top). The string quartet has been transposed into E major for ease of comparison.

subsidiary because even, if the string quartet was composed after, it reveals things about Beethoven's thought process that lend more insight into the piano sonata. One could take the opposite approach and compare the piano sonata to the string quartet instead of vice versa (as this paper does). A similar conclusion would be reached: both versions are authentic compositions and are both important in understanding the complexities of the sonata.

Beethoven arranged the other two movements for string quartet as well, and they are equally as magnificent. The analysis of the Allegretto and Rondo movements are, however, beyond the scope of this paper. After looking at the differences in just the first movement, one can see how Beethoven’s arrangement is a carefully crafted work that, while sounding like the piano sonata, has its own intricacies that make it a true string quartet, not just an arrangement of another piece.

Bruno Maderna's *Improvvisazione No. 1*:

Magic Squares and the Constraints They Impose

While the history of the twentieth century has placed an emphasis on serial composers like Arnold Schoenberg, Pierre Boulez, Luigi Nono, and Karlheinz Stockhausen, another famous composer that emerged during that time was Bruno Maderna. Maderna's unique style of organizing notes into 12x12 matrices, which he called magic squares, is a complex technique that combines Oulipian chance with serial music. Maderna's 1952 piece *Improvvisazione No. 1* was the first to utilize magic squares and is an excellent example of how he combined new and old methods and freedom within constraints while simultaneously avoiding set repetition. An overview of why Maderna chose to invent his new system is critical to understand how he composed *Improvvvisazione No. 1* using magic squares. Next, the formal structures within the piece are presented, which will be followed by the criticism involved with chance composition and a glimpse into the 1960s French group of writers, the Oulipo, to see how they relate to Maderna's ideals. Finally, I will observe how Maderna's ideas influenced his pupils, specifically Norma Beecroft and her composition *Tre Pezzi Brevi*.

Maderna was born in 1920 in Italy. He, along with Boulez, Nono, and Stockhausen, taught at Darmstadt: a school where young composers could take lectures taught by the foremost serial composers of the day.[24] Maderna liked the music of the past but also felt that it was important to keep moving forward; he felt music was much like life itself in this way. In a lecture he gave at Darmstadt, he said, "Music is a continuous movement like human life."[25] As such, he

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[24] Christopher Fox, "Darmstadt School," *Grove Music Online,* January 20, 2001. https://doi.org/10.1093/gmo/9781561592630.article.49725.

[25] "La musica è un movimento continuo come l'uomo."; Bruno Maderna, [lecture without title], quoted in Christoph Neidhöfer, "Bruno Maderna's Serial Arrays," *MTO a journal of the Society for Music Theory* 13, no. 1 (2007): [1], https://mtosmt.org/issues/mto.07.13.1/mto.07.13.1.neidhofer.html.

wanted to move away from tonality, a symbol of the past. He still loved harmony, though, and the element of verticality. To accomplish this difficult task of combining atonality and harmony, he turned to different systems of composition. He composed pieces using post-tonal, neoclassical, and 12-tone serial techniques.[26] The problem he faced, mostly with serial music, was he did not like the idea of a theme and the 12-tone row was much too thematic. The technique was also too linear and did not allow enough variation within the row. Feeling confined by all the other compositional tools, Maderna created his own: magic squares.

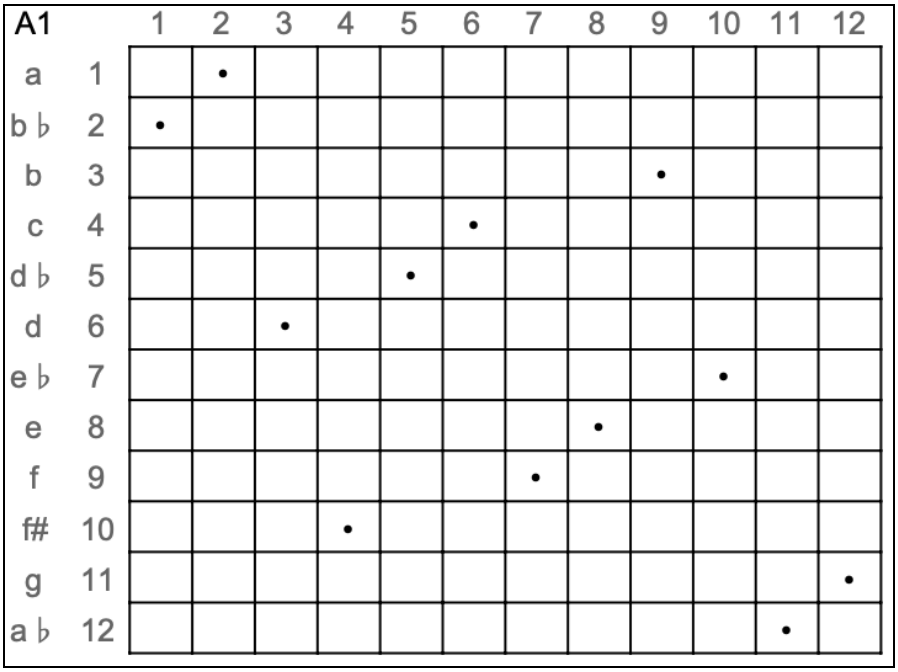
How Magic Squares Work Within *Improvvisazione No. 1*

Improvvisazione No. 1 is an orchestral work consisting of one, roughly ten minute long, movement. While the piece does touch on many different styles, Maderna does not separate it into different movements. To understand the analysis of *Improvvisazione No. 1,* an understanding of magic squares is necessary. A lot of pre-compositional work goes into a piece composed using magic squares. The first step is plotting a 12-tone row on a 12x12 matrix. The row used in *Improvvisazione No. 1* is B ♭ –A–D–F#–C#–C–F–E–B–E ♭ –A ♭ –G.[27] Example 2.1 shows the row on a matrix. The *x*-axis is time points, and the *y*-axis is pitch-classes. With the addition of a vertical axis, the "row" already looks more harmonic.

Maderna then transforms this matrix by referring to a magic square. The magic square used for *Improvvisazione No. 1* is shown in Example 2.2. Maderna calls this device a magic square because every row, column, and diagonal adds up to 132. Each line also contains two 5s, one 6, 8, 9, 10, 11, 12, 13, two 14s, and one 25. The numbers do not represent pitch classes,

[26] Veniero Rizzardi, "The Tone Row, Squared: Bruno Maderna and the Birth of Serial Music in Italy," in *Rewriting Recent Music History: the Development of Early Serialism 1947-1957*, ed. Mark Delaere (Leuven-Walpole: Peeters, 2011), 45-47.

[27] Neidhöfer, 6.



Example 2.1. The row of *Improvvisazione No. 1* is plotted on a matrix.

14	5	25	10	8	5	12	13	6	9	11	14
25	14	6	12	5	11	9	5	10	8	14	13
8	25	5	9	12	6	10	14	14	11	13	5
12	5	14	11	14	25	13	8	5	6	9	10
9	6	11	5	25	12	5	10	13	14	14	8
11	8	13	14	9	10	25	6	5	12	5	14
13	11	14	8	10	9	6	14	12	5	25	5
14	14	12	5	6	13	8	9	11	5	10	25
10	9	5	13	14	5	14	12	8	25	6	11
6	10	8	25	5	14	5	11	14	13	12	9
5	12	10	14	13	8	11	25	9	14	5	6
5	13	9	6	11	14	14	5	25	10	8	12

Example 2.2. The magic square Maderna devised for *Improvvisazione No. 1*.

intervals, or anything else musically related. They will be used in the next step of the compositional process.

While some may consider magic squares a technique of "random chance," one must understand that the matrices themselves are not random; the magic square creates them by generating numbers. Musicians include this technique in the category of "chance music" because the composer removes themselves from the process of choosing pitches. As explained below, the magic square and matrices make all the pitch choices. However, the composer creates all other elements of the composition, including rhythm, tempo, form, etc.

Maderna's magic square is, ironically, not the correct mathematical definition of a "magic square." Eric W. Weisstein defines magic squares as: "… a square array of numbers consisting of the distinct positive integers 1, 2, ..., n^2 arranged such that the sum of the n numbers in any horizontal, vertical, or *main* diagonal line is always the same number … known as the magic constant."[28] Maderna's square is not magic because he reuses numbers. He would have to use every number from 1 to 144 only once. The point of his square is to create a computer by which he will produce blank spaces on his matrices, as we will see below. Having any large number defeats the purpose of the square, as a note could be missing for a significantly prolonged time. While this is an interesting concept, it is not Maderna's intention. Christoph Neidhöfer discusses Latin squares and how they are similar to magic squares in that every row, column, and two main diagonals add to the same number.[29] Latin squares are different from magic squares, however, in that they use the same set of numbers in each row, column and main diagonal. A common example of a Latin square is a sudoku puzzle. The square Maderna uses in *Improvvisazione No. 1* is not a Latin square because he repeats the numbers 14 and 5. Robert D. Morris would refer to

[28] Eric W. Weisstein, "Magic Square," MathWorld, Wolfram Web Resource, February 14, 2023, https://mathworld.wolfram.com/MagicSquare.html.

[29] Neidhöfer, 6.

what Maderna has created as an array: "A (two-dimensional) array in an *n* by *m* rectangle of positions … The exact duration of any position is left undefined."[30] For the purposes of this essay, the technique will be referred to as *magic squares* because that is how Maderna himself refers to them.

Maderna creates eleven additional matrices, for a total of twelve, by selecting a line from the magic square. For example, Maderna chooses the top left to the bottom right diagonal for pitch-class B ♭ (Example 2.3a). He then counts the number of columns as indicated in the diagonal and places another dot. He continues this pattern until he has worked through the entire diagonal, as shown in Example 2.3b.[31] Maderna continues this process for the other eleven pitch-classes until he has plotted all the dots. The reason matrix 1 and matrix 13 are the same is because, in every row, there are 132 skipped columns plus twelve plotted dots which equals 144. His process of choosing which line of the magic square to use for each pitch-class on the matrices has a clear pattern to it, as shown in Example 2.4. The line chosen for B ♭ , as mentioned before, is read from the top left to the bottom right. The line chosen for the next pitch-class in the first matrix, A, is read from the top right to the bottom left. D from the bottom right to the top left, and F# from the bottom left to the top right. Therefore, the lines of the first four pitch-classes create a criss-cross pattern. The next four pitch-classes, C#, F, E, and C, begin at the top left and circle around the edge of the magic square in a clockwise fashion. The last four pitch-classes, B, E ♭ , A ♭ , and G start in the top left corner and circle the edge of the magic square in a counter-clockwise fashion. By laying out the path of the lines, a clear pattern emerges. Even though the final result, as seen in Example 2.3, looks random, Maderna implements a deliberate element of orderliness.

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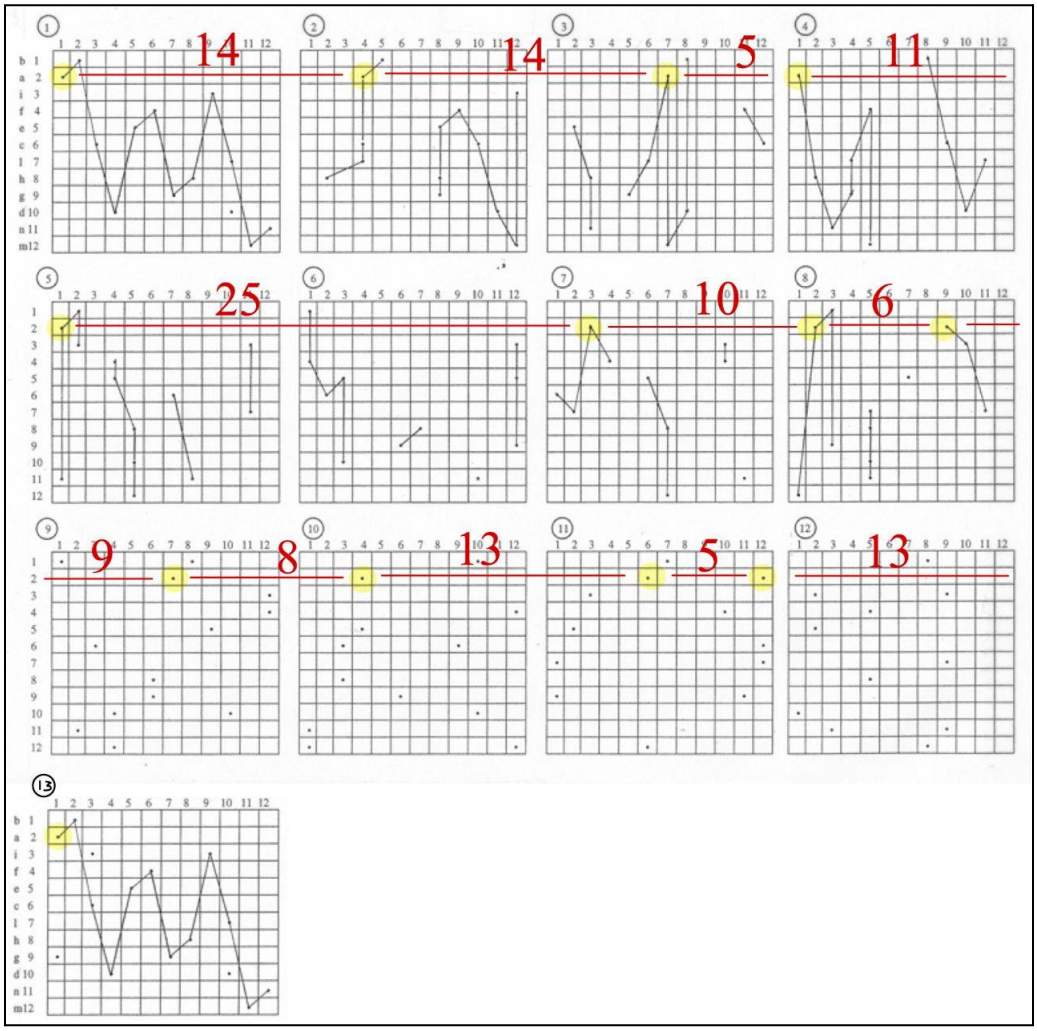
[30] Robert D. Morris, *Composition with Pitch-Classes: A Theory of Compositional Design* (New Haven: Yale University Press, 1987), 184–185.

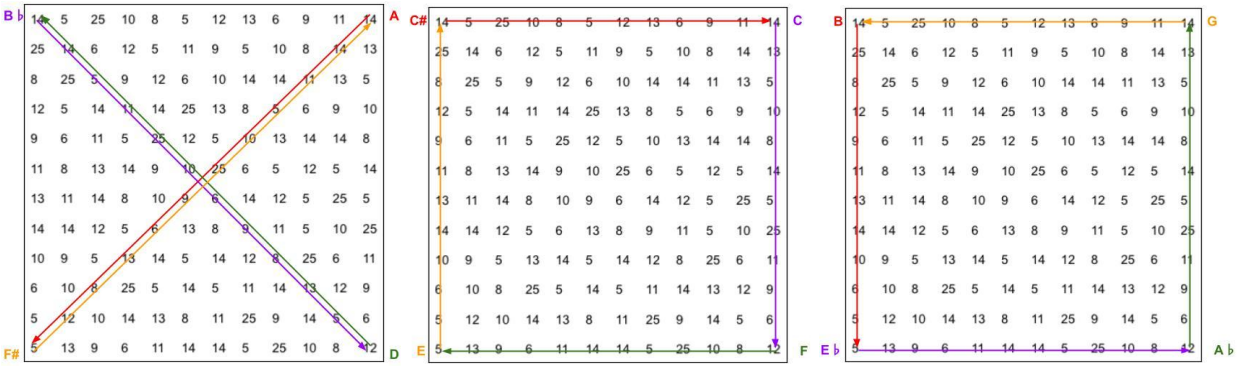
[31] Neidhöfer, example 4 (recreation of Maderna's sketch) with annotations; matrices 1 and 13 are the same.

14	5	25	10	8	5	12	13	6	9	11	14
25	14	6	12	5	11	9	5	10	8	14	13
8	25	5	9	12	6	10	14	14	11	13	5
12	5	14	11	14	25	13	8	5	6	9	10
9	6	11	5	25	12	5	10	13	14	14	8
11	8	13	14	9	10	25	6	5	12	5	14
13	11	14	8	10	9	6	14	12	5	25	5
14	14	12	5	6	13	8	9	11	5	10	25
10	9	5	13	14	5	14	12	8	25	6	11
6	10	8	25	5	14	5	11	14	13	12	9
5	12	10	14	13	8	11	25	9	14	5	6
5	13	9	6	11	14	14	5	25	10	8	12

Example 2.3a The diagonal chosen from the magic square to plot pitch class B ♭ on the matrices.

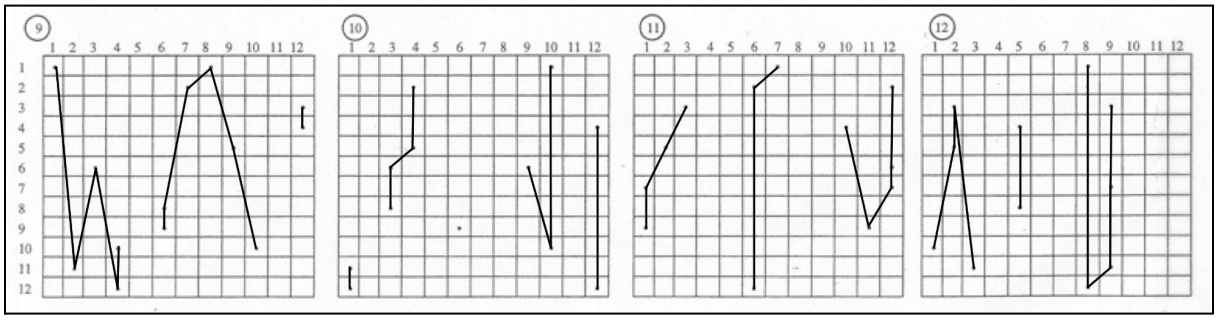


Example 2.3b How Maderna plotted pitch class B ♭ on the matrices.



Example 2.4. The pattern of how Maderna chose which line to use for each pitch-class.

Maderna draws some lines connecting dots as a way to make sense of the "randomness." The line in matrix 1 connects the pitches of the tone row. The two lines in matrix 2 connect the dots in the first half of the matrix and then the dots in the second half. He continues this pattern of connecting dots that are not separated by a void until matrix 8. The lines become more sporadic as he moves through the matrices. No lines connect dots in matrices 9–12. Example 2.5 shows how they would have been connected.



Example 2.5. How the dots in matrices 9–12 would have been connected.

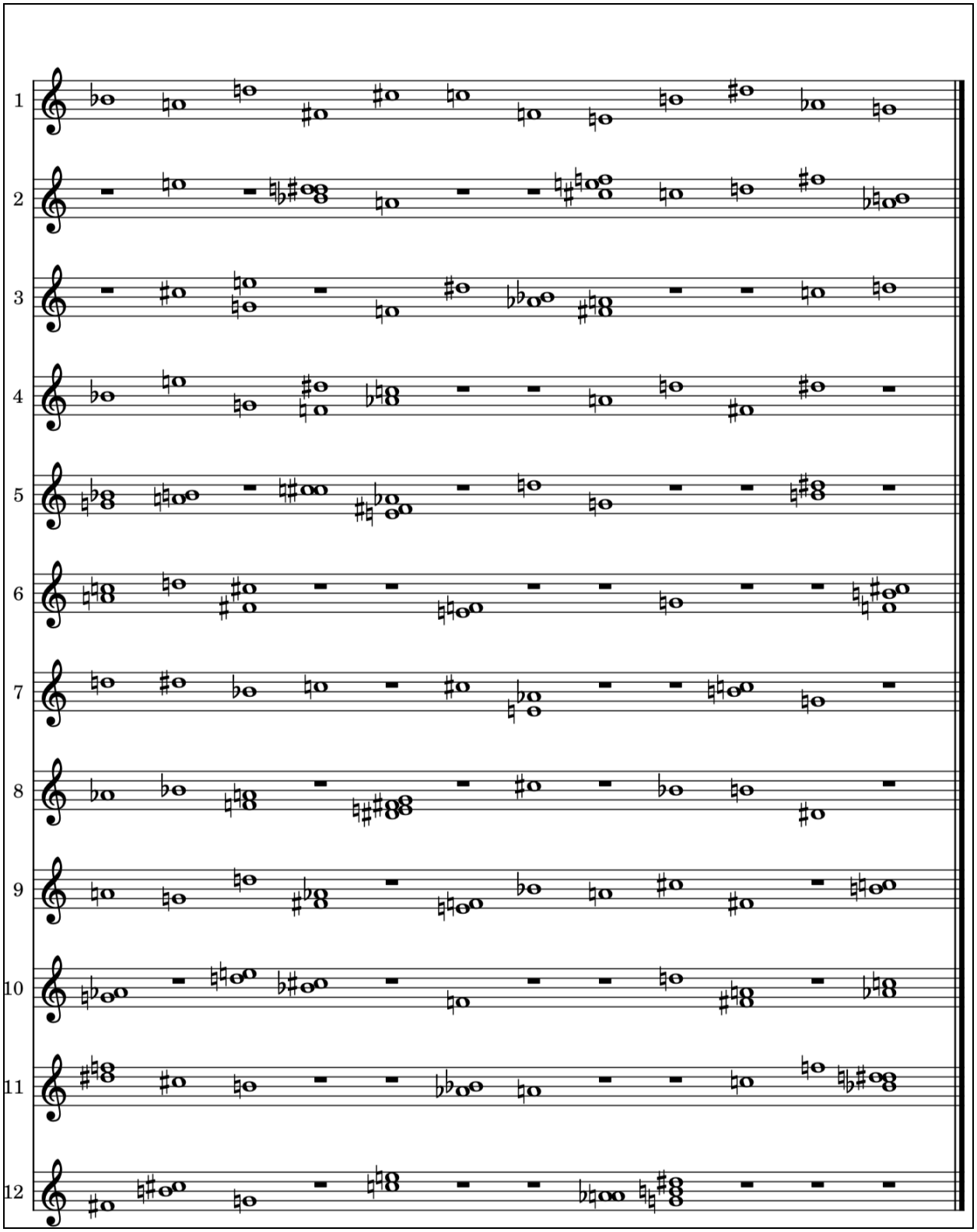
Everything in the magic square and the matrices are meant to aid in the composition of the actual work. Perhaps Maderna realized that connecting the dots in this way did not help him in the end, so he stopped doing it. Or perhaps he was able to visualize the lines without drawing them, so there was no need to physically write them out.

Take note of how different each matrix is. Every resulting “row” is compared on a staff in Example 2.6. A whole note represents each dot on a matrix. The pitches represent a whole pitch class; when Maderna composes, he does not restrain himself to a specific pitch. In laying out the matrices on a staff, some interesting discoveries come to our attention. First, one can see how far the new series stray from the original row and the 12-tone technique as a whole. Even though each series spans twelve time points, many of them have repeated notes, including number 4, which repeats D# and number 7, which repeats C♮. Others have multiple repeated notes, including number 2, which repeats the notes E♮and D♮, 5 repeats B♮and G♮, 6 repeats C# and F♮, 8 repeats B ♭ and D#, 9 repeats A♮ and F#, 10 repeats D♮ and A ♭ , 11 repeats B ♭ , D#, and F♮, and 12 repeats B♮ and G♮. Repeating a note within a 12-tone row is strictly forbidden; however, Maderna’s goal is to stray from serial techniques.

Magic squares take a further step away from serial music in that the series can be incomplete, i.e. missing some of the twelve tones. Each one of the resulting series is missing at least one pitch class. Some series are missing multiple pitch classes, including number 4, which omits B♮and C#, and number 11, which does not have E♮, F#, or G. Every pitch class is omitted at least once throughout the eleven resulting series.

The third unique element of the magic squares technique is that some pitch classes sound within the same time space. The first example is in series 2 when the B ♭ , D♮, and D# all sound together. The most shocking example is in series 8 when D#, E♮, F#, and G♮all sound simultaneously.

While its creation may have been a reaction to serial methods, the magic square technique is entirely its own structure. It follows different rules than 12-tone music and is also vastly more complicated and nearly impossible to reverse engineer as there are many possible



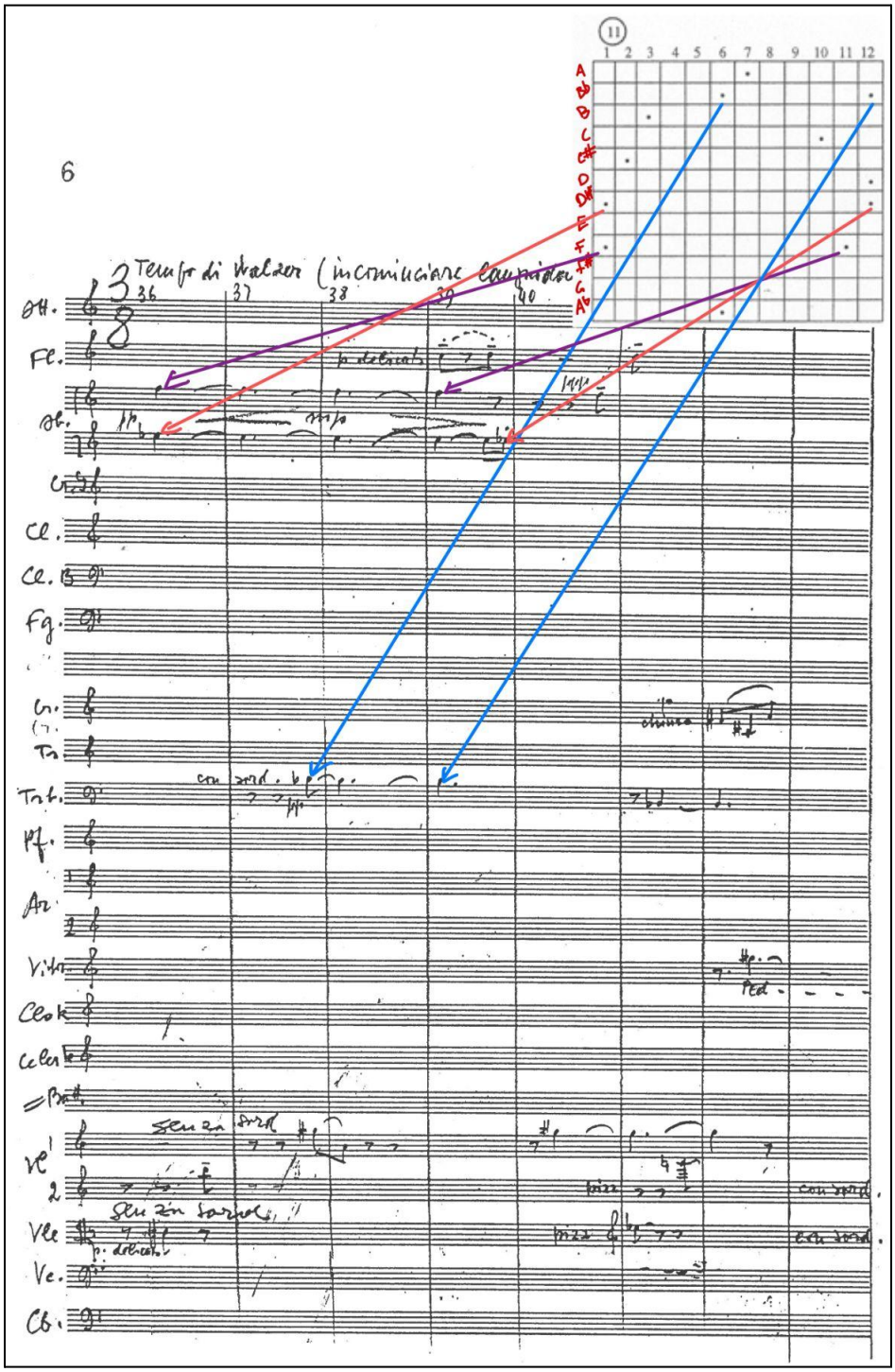
Example 2.6. Matrices 1–12 written on a staff. The rests represent blank columns.

ways to utilize the blank columns and simultaneously sounded pitches. The changes involved in reverse engineering will be illustrated later. Without Maderna's original sketches, scholars could never begin to understand how the composer constructed the work.

Magic squares allow Maderna to see how often each pitch-class is played and when pitch-classes sound together. The most unique element of the matrices is the spaces. The composer is also able to derive rhythmic content from the empty columns. The voids could represent rests, holding a pitch until the next dot, or non-pitch sounds. Neidhöfer shows how Maderna uses the voids to tie pitches together in *Improvvisazione No. 1*, as represented in Example 2.7.[32] Maderna uses matrix A11 to compose mm. 36–39. As shown in Example 2.8, pitch-classes E ♭ /D#, F, and B ♭ are all repeated in the matrix. Maderna uses this opportunity to hold the pitches for twelve, eleven, and seven beats, respectively. Conversely, when composing mm. 282–284, Maderna opts to use rests during the blank columns (see Examples 2.9 and 2.10). Even though the matrices are set, and Maderna does not change them, he still has some creative reign over the final composition. He is not simply copying and pasting the matrices onto a score.

As previously noted, Maderna's critique of serial music is that the row is too much like a theme.[33] Additionally, the composer is only able to retrograde, invert, and retrograde invert transpositions of the row. Eventually, the same intervals appear in similar positions, creating too much of a theme-like sound. By manipulating the row to such an extreme degree, as Maderna does by using the magic square, all sense of a theme is lost. However, there are only twelve possible manipulations of the row. This is not enough for Maderna; each matrix would have to be used at least twice in a single piece which is unacceptable. To avoid this repetition, Maderna creates ninety-six more matrices through a process he calls "'first proportion' (*prima*

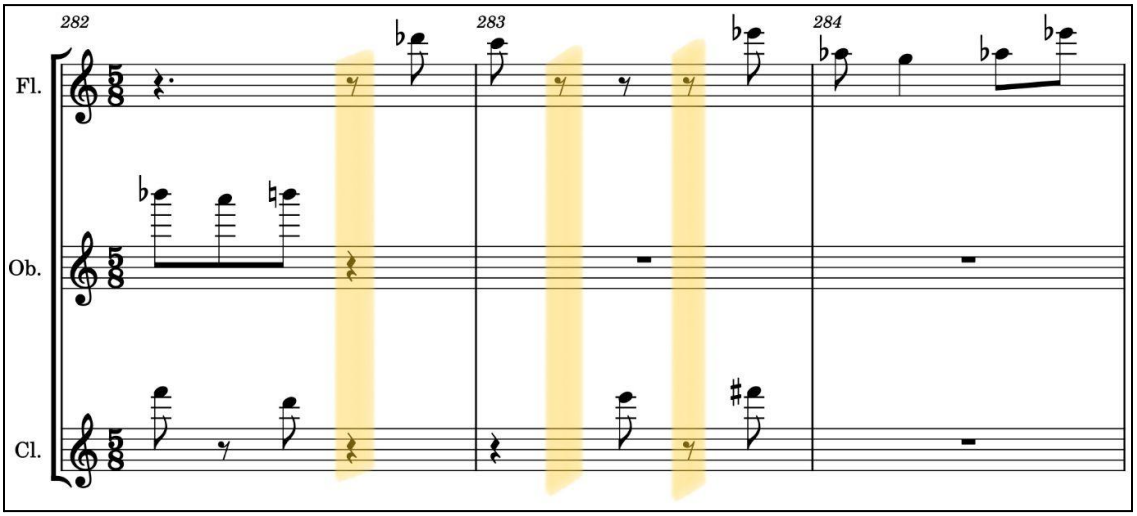
[32] Neidhöfer, 15-16.
[33] Rizzardi, 47.



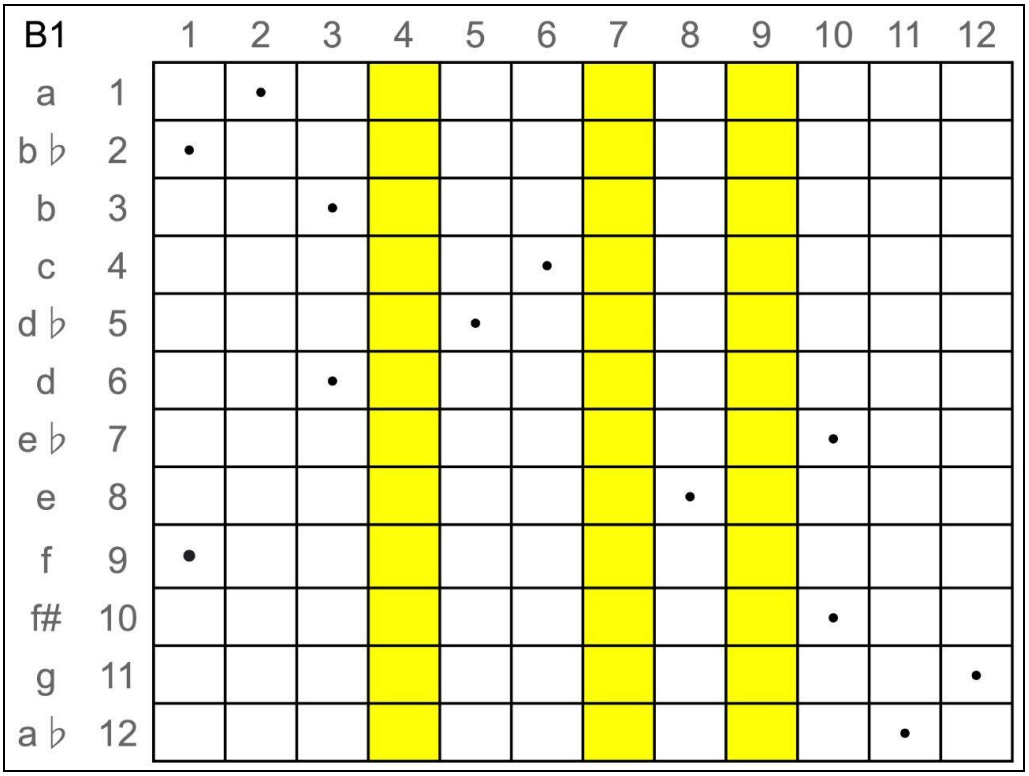
Example 2.7. How Maderna utilizes matrix A11 to compose mm. 36–39.

A11		1	2	3	4	5	6	7	8	9	10	11	12
a	1							•					
b ♭	2						•						•
b	3			•									
c	4										•		
d ♭	5		•										
d	6												•
e ♭	7	•											•
e	8												
f	9	•										•	
f#	10												
g	11												
a ♭	12						•						

Example 2.8. Matrix A11. Pitch-classes B ♭ , E ♭ /D#, and F are repeated.



Example 2.9. Matrix B1 used in mm. 282–285.



Example 2.10. Matrix B1.

proporzione)."[34] Looking back on Example 2.3, you will notice that there are two dots in matrix 13 that do not appear in matrix 1: B in column 3 and F in column 1. Veniero Rizzardi explains how Maderna deals with these extra dots:

> Since the penultimate shift generates some points on the last square (A12) that interfere with the restored A1 configuration, a new pattern of twelve points/notes is created, called by Maderna 'first proportion' (*prima proporzione*) … It is a [sic] ordinary twelve-note row and constitutes the new B1 square, starting from which a new sequence of shifts generates B2, B3, … B12. In a similar way a new proporzione will generate C1 and so on until the eighth sequence of squares, I1 … I12, reaching a total of 9 x 12 = 108 squares, that is, sets of twelve notes/voids-aggregates.[35]

By continuing this process of *prima proporzione*, Maderna is able to create nine times the number of matrices he originally had, virtually eliminating the possibility of having to repeat a matrix within *Improvvisazione No. 1*. Rizzardi creates a reproduction of Maderna's sketch,

[34] Rizzardi, 59.
[35] Rizzardi, 59.

shown in Table 2.1.[36] Maderna categorizes each matrix by the number of pitches on the top of the chart, and the number of repeated pitches (*suoni ripetuti*) along the left side of the chart.[37]

Although it is unknowable how Maderna constructed each subsequent series group, B1 through I12, it could be interesting to recreate matrices B1–B12 using A1–A12 as a starting point. Maderna uses B1 in mm. 282–284, moving the line throughout the flutes (see Examples 2.9 and 2.10).[38] The clarinet doubles some parts, but this analysis will focus on the flute part for ease of understanding. Notice that B1 and A1 are almost identical; only B1 follows the path of the additional dots from A13.

This knowledge allows for the reconstruction of the remaining dots on the other eleven B matrices. Unfortunately, the resulting matrices do not align with Rizzardi's classification table (which is itself a reproduction of Maderna's original sketch). Therefore, Maderna must not have used the same lines from the magic square to create the subsequent matrix groups. Perhaps instead of moving around the outside of the square, as he does in constructing matrix group A, he uses lines from the middle of the magic square. Alternatively, maybe he follows the same lines as matrix group A but uses them in retrograde. Unlike 12-tone music, it is impossible to reverse engineer the pre-compositional process.

Maderna's compositional process for *Improvvisazone No. 1* used the table of matrices shown in Table 2.1. Here is the first time in the compositional process in which he has some freedom, i.e. not bound by precompositional constraints; he is able to choose which matrices he wants to use and when. Starting from the top right of the table, the composition follows the order of matrices reading down and up, alternating forward and backward.[39] Therefore, the first

[36] Rizzardi, 60.

[37] Rizzardi calls the top of the chart the number of *notes*, but I believe the number of *pitches* is a better explanation.

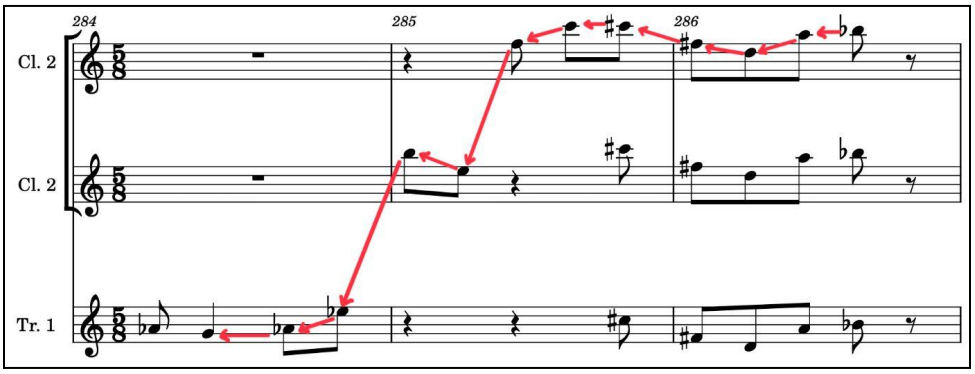
[38] In Example 2.9, only the flutes and piccolo are shown as to not overcrowd the example. See page 38–39 of the score for full instrumentation.

[39] Rizzardi, 61.

	12 note	11	10	9	8	7
	A1	A3				
	B1		B11		B3	
	C1	C5	C8			
	D1	D6				
0	E1		E3			
SR	F1	F11		F3.5.9		
	G1					
	H1				H10	
	I1		I5.10.12	I2		
			A4	A7		
	B4	B9	B5	B8		
			C3	C4.9	C12	
		D3.7.11	D5	D2.9	D12	
1		E10	E5.11.12		E2	
SR		F2	F12			
		G6	G2.10		G8	
	H9			H4		
	I8	I9	I11	I3		
		A.2.5.9	A8.10	A6.9		
				B12		
	C6		C2	C11		
				D10		
2			E8.9	E7		
SR		F4.6.10				
		G5.9	G4.11		G12	
	H3.11		H5.12	H2		H6
				A11		
	B6		B2.7			
	D4.8					
3			E6			
SR			F8			
		G7			G3	
			H7.8			
			I4.7			
			B10			
		C10	C7			
4			E4			
SR			F7			
					I6	

Table 2.1. Every matrix is laid out on a chart: the number of pitches along the *x*-axis and the number of repeated pitches along the *y*-axis.

matrix in the piece is H6 prime (P), then H6 retrograde (R), the next is I6(P)(R), then G3(P)(R), G12(P)(R), etc. By reading the table in this way, Maderna begins the piece with matrices containing more voids. As the piece continues, the matrices consist of fewer and fewer voids until near the end when matrices I1–A1, which involve all twelve pitch-classes, are used. This means that from mm. 258–286, every pitch-class is heard at least once in a twelve-beat time span. This increased amount of pitches per time ensures growth toward the climax of the piece. The first time matrix A1 is seen, the matrix that all the other matrices grew from, is at almost the very end of the piece. Measures 282–284 present B1, which, as discussed above, is very similar to A1 but not quite identical (Example 2.9).[40] We finally hear the proper A1 matrix in retrograde in mm. 284–286 (Example 2.11).[41]



Example 2.11. Maderna uses matrix A1 in retrograde to compose mm. 284–286.

Formal Construction

Whereas Maderna's use of magic squares explains his organization of musical material on the local scale, it remains to consider how *Improvvisazione No. 1* is organized in terms of large-scale form. Whereas pitch-class and rhythmic structures are predetermined by the

[40] Maderna made a numbering mistake; he jumped from m. 219 to m. 230, skipping mm.220-229. I will refer to bar numbers as he notates them (incorrectly).

[41] In Example 11, only clarinets 2 and 3 and trumpet 1 are shown as not to overcrowd the example. See page 39 of the score for full instrumentation.

precompositional process, Maderna enjoyed greater freedom to determine formal structures. Eighteen different sections are outlined by Maderna with double bar lines in the score; they are represented in Table 2.2.[42]

Polka

First, let us focus on the polka section, which is split into three parts: polka, trio polka, and coda polka. Curiously, the section labelled "polka," mm. 137–153, does not display many characteristic polka qualities. Here is Gracian Černušák's definition of the polka from the *Grove Music Online* article "Polka":

> A lively couple-dance in 2/4 time … the tempo of the polka was that of a military march played rather slowly, at 52 bars (104 crotchets [quarter notes]) per minute. The music was usually in ternary form with eight-bar sections, sometimes with a brief introduction and coda. Early characteristic rhythmic patterns are made of quavers [eighth notes] and semiquavers [sixteenth notes], generally without an upbeat.[43]

Example 2.12 (reproduced from Černušák) shows typical polka rhythms. The "polka" section of *Improvvisazione no.1* does follow a typical polka meter of 2/4, and the tempo is marked as *poco sostenuto*. The only known recording of the piece, by the Frankfurt Radio Symphony (FRS), performs this section at 128 bpm, which is somewhat faster than the typical polka tempo discussed by Černušák.[44] This polka is in a ternary form, polka - trio - coda; however, it does not contain an introduction, as Černušák suggests it should. The rhythmic part of the definition is where the "polka" section begins to lose its polka-ness. Maderna orchestrates the pitches so sporadically that any sense of flow is lost, and it is hard to tell where the phrases begin and end.[45]

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[42] If Maderna has given the section name, ex Polka, Can Can, Andante Recitativo, I have labeled that; the Waltz label is Neidhöfer's, not Maderna's, but Waltz is a fitting title and that is why it has been included.

[43] Gracian Černušák, "Polka," *Grove Music Online*, rev. Andrew Lamb and John Tyrrell January 20, 2001, https://doi.org/10.1093/gmo/9781561592630.article.22020.

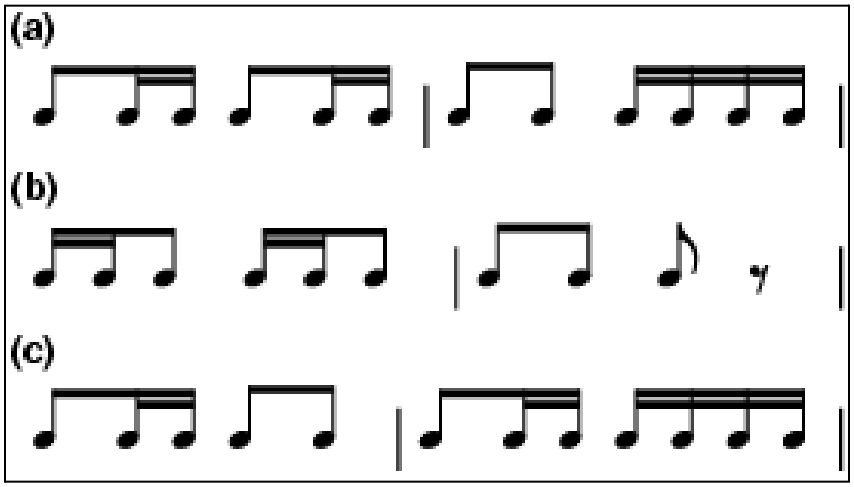
[44] Frankfurt Radio Symphony Orchestra, "Improvvisazione No.1 (1952)," The Wellesz Company, February 19, 2011, YouTube video, 4:30-4:47, https://youtu.be/NIipwlKojUE?t=270.

[45] See the score attached.

Finally, the typical polka rhythms are scarce. Not until the end of this section, leading up to the "trio polka," does a clearer beat begin to be heard, aided by the snare drum.

1 - 11	Andante
12 - 35	
36 - 106	Waltz
107 - 122	
123 - 136	
137 - 153	Polka
154 - 184	Trio Polka
185 - 190	Coda Polka
1) 191 - 195 2) 196 - 200	Two-part transition (Transizione Bipartita)
201 - 209	Can Can
210 - 218	
219 - 236	Trio
237 - 244	Trio 2
245 - 257	Trio 3
1) 258 - 269 2) 270 - 281 3) 282 - 286	Three-part transition (Transizione Tripartita)
287 - 299	Andante Recitativo
300 - 307	Andante Espressivo
308 - 312	Adagio

Table 2.2. The eighteen different sections of the piece.



Example 2.12. Traditional polka rhythms (a reproduction from Černušák's article).

The "trio polka" section displays some clearer polka figures with a much stronger foundation in the strings, playing a typical offbeat pattern throughout the section. The rhythms in the horn, trumpet, and trombone hint toward common polka sounds. The woodwinds are also more present, clearly playing rhythms seen in Example 2.12.

Finally, the coda, though very short, is the epitome of a polka. The horn and trumpet alternate, playing different combinations of eighth and sixteenth notes; the piccolo, flute, and xylophone play a clear polka melody line while the other instruments accompany them with a countermelody. Even though the pitches have been predetermined, Maderna is still able to play with a polka form. The voids in the matrices determine rest, but they do not determine the length of each rest, meaning that Maderna can play with offbeats of rests of any length he chooses.

Can-can

The can-can section is composed in a similar way to the polka section. The can-can was a wild dance during the height of its popularity in the second half of the nineteenth century. The dance was brought over to Paris from Algeria by soldiers.[46] Jacques Offenbach, who helped

[46] Harold C. Schonberg. *The Lives of the Great Composers* (New York: W.W. Norton & Company Inc., 1970), 305.

immortalize the can-can in France, was one of the most prolific composers at the Opéra-Comique:

> Offenbach worked on two levels. He was a skillful composer with a knack for creating lively melodies. But more: he had a streak in him that satirized and parodied everything within sight: Meyerbeer and Wagner, the court, the Emperor himself, the army and politicians, the entire Establishment.[47]

Offenbach's can-can—from his operetta *Orphée aux Enfers*, written in 1858—is one of the most famous of the nineteenth century. In the same way that he thumbed his nose at the bourgeoisie and at the serious musical establishment, Maderna invokes the same dance to do just that in a twentieth-century context. Maderna's parody, however, is not of the bourgeoisie but rather of 12-tone composers.

Adam Lamb describes the can-can as "… a lively 2/4 time, being derived from the quadrille or galop."[48] This definition is consistent with Offenbach's work, as he used the label "galop" for the section now known as the can-can. Despite Offenbach's tempo marking of Allegro moderato for the galop, modern recordings typically perform it much faster, at 170–200 bpm.[49] Strangely enough, Offenbach instructs a much slower tempo, writing "All^{tto} mod^{tto}" (allegro moderato), which is, at most, 120 bmps.[50] Over the years, as the piece became widely known, it became fashionable to perform it very quickly, even if that was not the composer's original intent. The FRSO plays the "can-can" section of *Improvvisazione No. 1* at a *Vivace,*

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[47] Schonberg, 306.

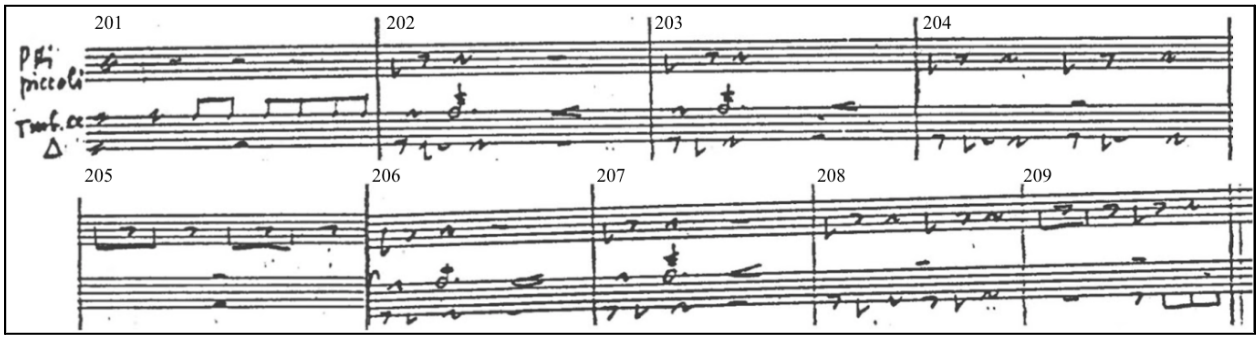
[48] Adam Lamb, "Cancan," Grove Music Online, January 20, 2001, https://doi.org/10.1093/gmo/9781561592630.article.04717.

[49] Jacques Offenbach, "Act III: Maintenant, je veux (Jupiter, Pluton)," Toulouse Capitol Orchestra, Warner Classics, *Orphee aux Enfers*, March 3, 2003, compact disc, https://mcgill.nml3.naxosmusiclibrary.com/catalogue/item.asp?cid=0077774964753; Jacques Offenbach, "Act II: Maintenant, je veux (Jupiter, Pluton)," Lyon National Opera Chorus and Lyon National Opera Orchestra, Warner Classics, *Orphee aux Enfers*, November 28, 2005, compact disc, https://mcgill.nml3.naxosmusiclibrary.com/catalogue/item.asp?cid=0724355672551.

[50] Jacques Offenbach, *Orphee aux Enfers*, 1885, https://hdl.handle.net/2027/uc1.31822013447826.

roughly 155 bmps. This is right between Offenbach's intended slower tempo and the more widely performed faster tempo. In this way, Maderna has found a suitable middle ground.

In terms of pitch, the can-cans by Offenbach and Maderna could hardly be more different. Yet, the compositions mirror each other in other ways. Maderna's Can-Can has stronger dance allusions than other areas of *Improvvisazione No. 1*. The beat is made clear by the percussion, which slowly builds rhythmically and helps to outline the first and second phrases (Example 2.13); the snare, which crescendos on beat two, also helps to build intensity through the measure.



Example 2.13. The percussion from mm. 201 to 209.

Each measure also ends with an eighth note that acts as an anacrusis to the next, giving the section that typical bouncy dance groove.

The section comprises two four-measure phrases with a pick-up measure in the snare. This form mirrors Offenbach's can-can, which also contains two phrases beginning with a pick-up. Even though the two can-cans are eight and sixteen measures, respectively, they both occupy the same amount of musical space: Maderna's is in 4/4, and Offenbach's is in 2/4.

The formal arrangements of the two versions are similar in some ways and vastly different in others. Numerous scholars have devised a great many theories of post-tonal formal analysis, including Patricia Howland's theory of "integrated parametric structures" (IPS), which

suggests that factors other than harmonic analysis can define phrases.[51] She posits five types of IPS: tension/release, departure/return, symmetric IPS, directional IPS, and, finally, steady-state IPS.[52] While Howland's research has merit, as well as others who have suggested different theories of post-tonal formal analysis, their work is beyond the scope of this paper. As such, to discuss the phrase structure of Maderna's can-can, I will adapt William Caplin's form-functional definitions.[53] Since his theory relies heavily on conventional harmonies that do not apply to *Improvvisazione No. 1*, I will focus more on rhythmic and phrasing aspects. There are no formal cadences either; the motivic acceleration simply stops at the end of phrases. This analysis is based purely on rhythmic structure, which is a vital component of Caplin's system. Specifically, the excerpt can be identified as a period; the antecedent and consequent phrases are each sentence themselves, outlining a BI, BI, continuation, cadential form on a smaller scale (see Example 2.14 for formal analysis).

Offenbach's can-can does have clear harmonies, but paradoxically, they make the formal analysis more ambiguous and difficult to impose Caplinian form structures (see Example 2.15). The harmonies only alternate between V and I, and the rhythms are consistent throughout, which does not allow for any fragmentation or acceleration to the cadence. Like Maderna's can-can, Offenbach's phrases end with a cadence on the downbeat of the fourth R=2N measure (the seventh notated measure) followed by post-cadential material. *Orphée's* two measures units, however, are not as easily defined; when read as R=2N the melody seems to suggest a periodic structure, similar to Maderna's can-can. However, this cannot be as an antecedent can never end with a PAC, as m. 7 does. So while both Maderna's and Offenbach's can-cans follow a pattern of

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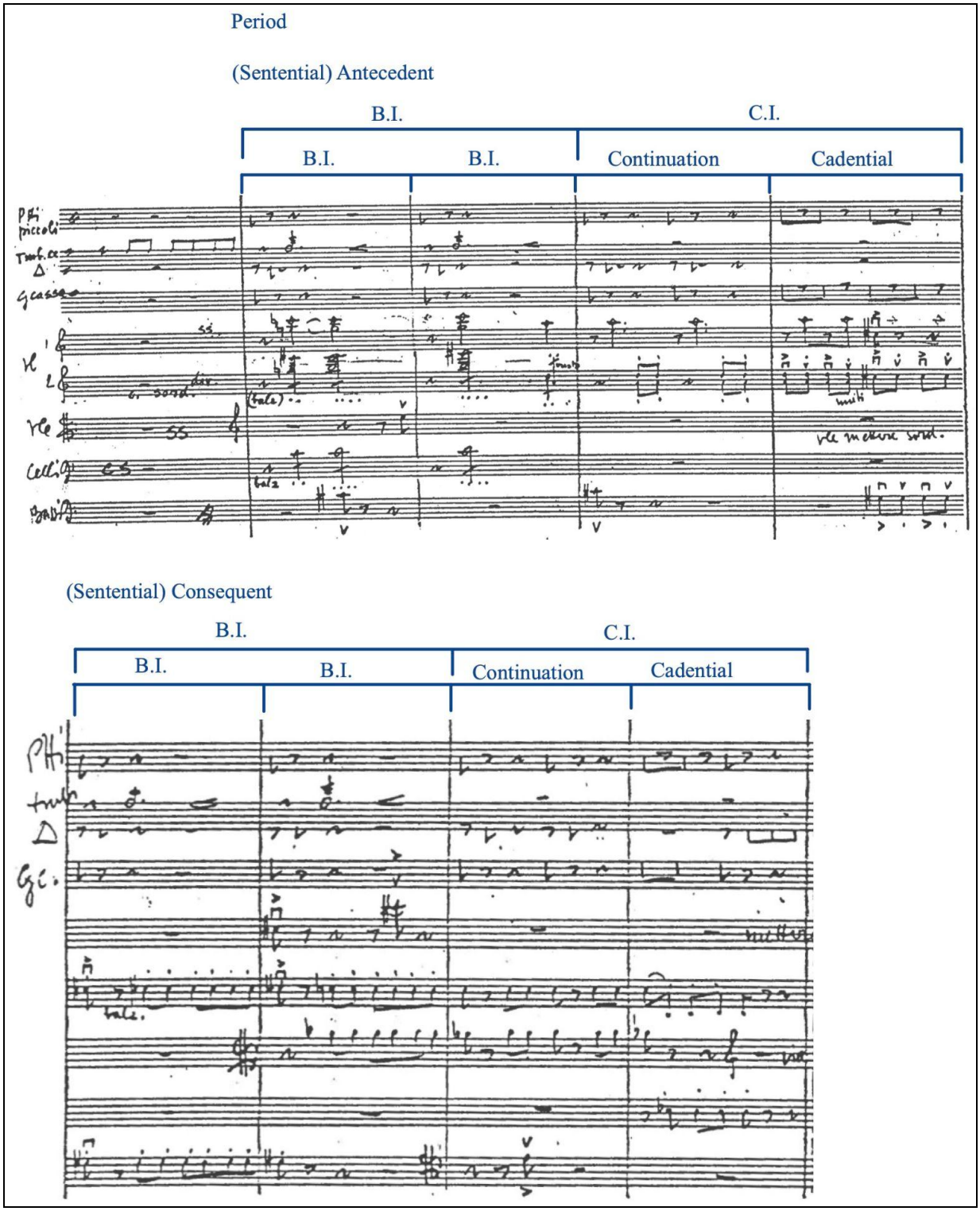
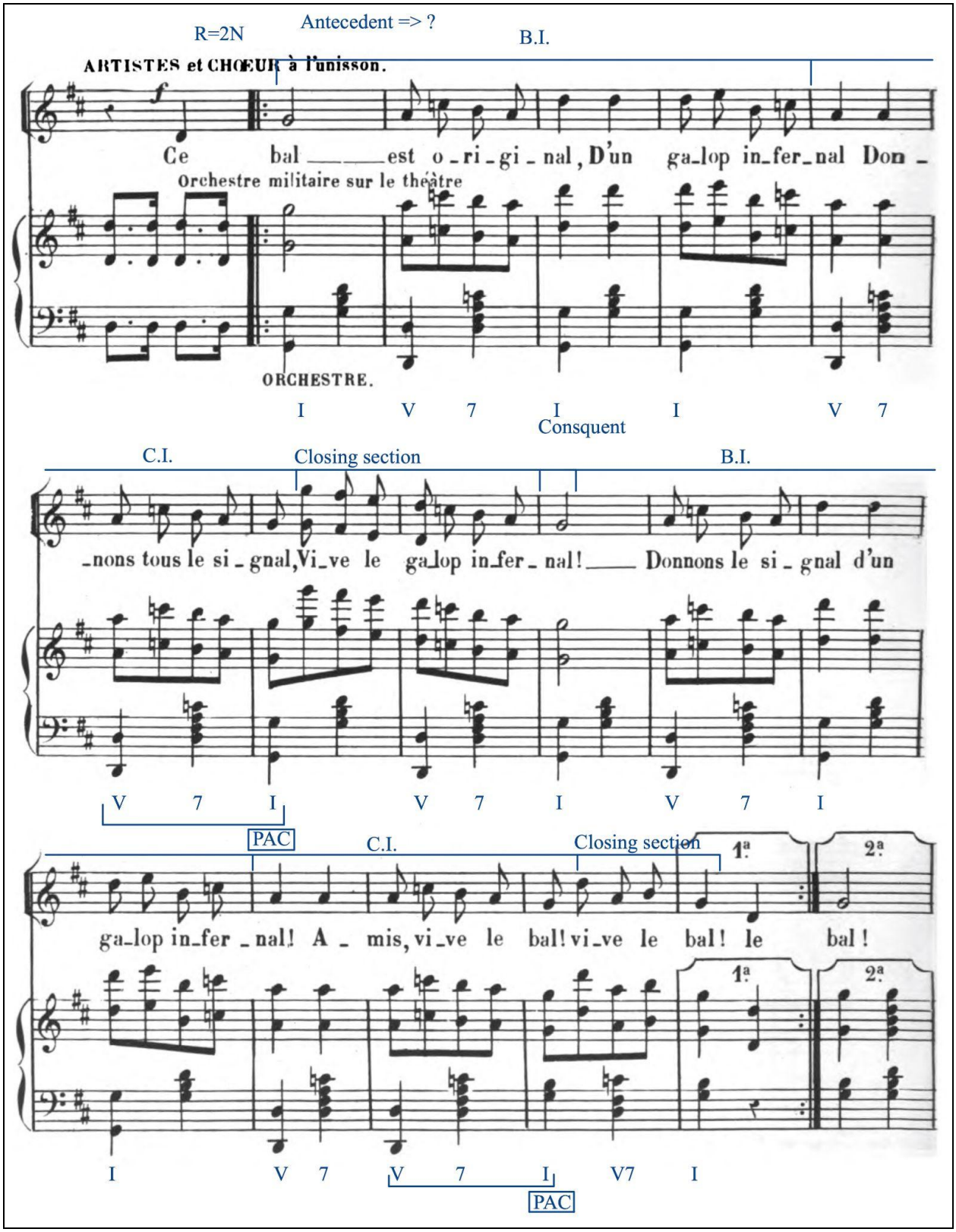
[51] Patricia Howland, "Formal Structures in Post-Tonal Music," *Music Theory Spectrum* 37, no. 1 (2015): 71, https://doi-org.proxy3.library.mcgill.ca/10.1093/mts/mtv011.

[52] Ibid.

[53] Willian Caplin, *Analyzing Classical Form: An Approach for the Classroom* (New York: Oxford University Press, 2013).



Example 2.14. *Improvvisazione No. 1*, mm. 201–209, with formal analysis.



Example 2.15. The Galop section of *Orphée aux Enfers* with Caplinian form structures.

two measure fragments, they exhibit different formal interpretations. Much like the polka section, Maderna takes some ideas from the original style and implements them relatively unchanged into *Improvvisazione No. 1*; other elements are warped until they are nearly unrecognizable. In this way, he is able to nod to the past while still moving forward.

Constraints and Chance

Without labelling the sections the way he did, one could misinterpret Maderna's compositional process and criticize him for leaving too much up to chance. By creating the constraint of the magic squares, Maderna effectively removes himself from the creative choice of pitches and, to some extent, rhythms. As Robert Hasegawa explains, however, composers need constraints in order to be more creative.[54] Hasegawa begins by explaining the importance of limitations. He explains that any artist, musical or otherwise, needs constraints in order to be more creative. Hasegawa summarizes Leonard Meyer's hierarchy of constraints.[55] Meyer has three large classes of constraints: laws, rules, and strategies. He explains that: "Laws are transcultural constraints—universal, if you will."[56] Laws transcend all music, e.g. standard five-line staves and standard rhythmic notation. They also generally involve repetition, which produces stability, for example, a return to the main theme or a sequence.

Meyer's next level in the hierarchy is *rules*:

> Rules are intracultural, not universal. They constitute the highest, most encompassing level of stylistic constraints. Differences in rules are what distinguish large periods such as Medieval, Renaissance, and Baroque from one another; and it is the commonality of rules that links Classic and Romantic music together.[57]

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[54] Robert Hasegawa, "Creating with Constraints." in *The Oxford Handbook of the Creative Process in Music*, ed. Nicolas Donin. (Oxford University Press, 2020), 2, doi: 10.1093/oxfordhb/9780190636197.013.17.

[55] Meyer's work is in relation to Western are music. It does not include all world music.

[56] Leonard B Meyer, *Style and Music: Theory, History, and Ideology* (Pennsylvania: University of Pennsylvania Press, 1989), 13.

[57] Meyer, 17.

Rules are unique to a specific style, e.g. serial music. While there may be different variations of rules—for example, some composers allow repeated notes in 12-tone music, while others don't—they are generally used throughout the entirety of the style.

The final level of Meyer's hierarchy is strategies: "Strategies are compositional choices made within the possibilities established by the rules. For any specific style, there is a finite number of rules, but there is an indefinite number of possible strategies for realizing or instantiating such rules."[58] An example of a strategy within the larger rule of twentieth-century atonal music is John Cage's use of I Ching in *Music of Changes*.

When Maderna initially wrote *Improvvisazione No. 1,* the constraint of the magic square would have been considered a strategy. When he began to use the technique to compose other pieces, and when he taught students the technique who then proceeded to compose using it, like Norma Beecroft, it became a rule. In both cases, magic squares is a chosen constraint, described by Hasegawa as a "self-imposed limitation," meaning the composer chose to compose using limitation.[59] The opposite is imposed constraints which are limitations put upon by outside sources, an example being that Western composers write using twelve pitch-classes and using a five-lined staff to represent the pitches (this is similar to Meyer's *laws*). The idea of an imposed constraint is important as it shows that even when composers write "without constraints," they are still limited by cultural norms.

Hasegawa outlines four genres of constraint: material, formal, style/genre, and process. Material constraints are, as they sound, limitations imposed by the materials at hand. This constraint somewhat maps onto *Improvvisazione No. 1*. Hasewaga explains that some material constraints "restrict the available material to a specific, limited palette: we might call the 'only

[58] Meyer, 20.
[59] Hasegawa, 2.

use *x*' constraints."[60] Maderna does not use this type of material constraint; however, he is bound by the material of the matrices developed from the magic square. Formal constraints "… work from the top down …" and govern the layout of the work.[61] In the same way that the form of a sonnet does not constraint the words within it, the formal structure of a piece does not constraint the pitches, rhythms, timbres, etc.[62] Maderna's use of polka, can-can, and the waltz are all formal constraints. Style/genre constraints are constraints that are not imposed by the composer but by the broad "style" in which they generally compose. For example, Maderna composes *Improvvissazione No. 1* using traditional Italian notation, unsurprising since he is Italian and wrote this piece in Italy. Process constraints are those that are invisible in the final product. They can include time constraints in which to create the work and precompositional devices such as magic squares. These devices can be mechanical and may either be slightly changed or rigidly accepted by the composer. In Maderna's case, he rigidly accepts the matrices that have been mechanically created. By considering *Improvvisazione No. 1* in light of Hasegawa's categories of constraints, one can see that even though Maderna added the extra constraints of the matrices, he is by no means unique in using constraints to compose. By limiting the pitch and the rhythms, Maderna is able to be freer in form and orchestration.

The Oulipo

Maderna's use of constraints brings to mind the 1960s French group of writers called the Oulipo. Even though *Improvvisazione No. 1* was written in 1952, eight years before the group was formed, understanding Oulipian ideas can help in understanding Maderna's interest in constraints. Alison James explains that the Oulipo was not avant-garde. Since the Oulipo pays a

[60] Hasegawa, 10.
[61] Hasegawa, 15.
[62] Hasegawa, 15.

"debt to tradition," they cannot be considered avant-garde; their goal is not novelty.[63] Oulipo is similar to 20th-century music styles and techniques, such as serial music and Maderna's magic squares. Unlike Hasegawa's neat description of four types of constraints, the Oulipian "constraints" are hard to define as they encompass many limitations: "mathematical, synoptical, metrical, lexical, 'lettric,' semantic …"[64] While the Oulipian writers do constrain themselves, they do not consider themselves to be "chance" writers. In fact, the Oulipo is decidedly *anti*-chance. Many critics of the Oulipo criticize its use of chance and the way that chance takes away the author's volition, but their criticism is based on a false definition of chance. The Oulipo does not use chance in the sense of "absence of intention" but rather that the process of creating the work will be indeterminate.[65] This is much like how Maderna creates all of his matrices based on a magic square that he creates; even though the matrices are "random," he determined the process by which they were created. The Oulipo also liked to point out that there are "natural rules" in all writings (and music). Natural rules are those that are "an acceptable part of literary technique"; they are very similar to Hasegawa's "imposed constraints."[66] Musical examples could be that a melody must not stay static for too long in order to maintain listener interest, or a sonata form has an exposition, development and recapitulation. These constraints are *not* Oulipian as they do not specify a set number or a length, but they are still constraints nonetheless. Artificial constraints are those that are created by the author/composer for a specific work or set of works, equivalent to Hasegawa's "self-imposed constraints."[67] Artificial constraints *are* Oulipian. An example is the S + 7 constraint, a process whereby the writer replaces every noun in their text with the noun seven letters away in the dictionary. Magic

[63] Alison James, "A Challenge to Chance: The Poetics of the Oulipo." In *Constraining Chance*, (Illinois: Northwestern University Press, 2009), 108.
[64] James, 109.
[65] James, 111.
[66] James, 114; Hasegawa, 2.
[67] Hasegawa, 2.

squares would, therefore, be considered Oulipian because they contain constraints but are not arbitrary.

James dedicates a small section of her chapter specifically to the arts and music. She refers to John Cage and explains that the Oulipo would be against his methods since "[t]he chance of the dice throw, coin flipping, or random-number generators" is too random and leaves nothing up to the composer.[68] They would prefer that the compositional process involve some sort of mechanical automatism, like that of Schoenberg's 12-tone system. "The Oulipian approach," James explains, "while it does not reject traditions, foregrounds and accentuates the arbitrary dimension of constraint."[69] Maderna has the same ideals; he likes the past but feels it is important to keep moving forward; he doesn't reject harmony but moves away from tonality.

James explains that absolute chance—that is, throwing dice, spinning a roulette wheel, etc.—is too much chance for the Oulipo.[70] The composer withdraws too much when using absolute chance. For this reason, Cage's *4'33"* is not Oulipian. Cage has completely withdrawn himself from his own composition, leaving almost 100% of the piece up to chance. The aspect over which he has control is the time length. "The paradox and strength of the Oulipian constraint," says James, "is that it is at once an organizing principle and—in its arbitrariness and in the complexity that it may generate—an embodiment of chance."[71] For all of these reasons, Maderna's approach to compositional constraints closely resembles the aesthetic philosophy of the Oulipo.

[68] James, 117.
[69] James 114.
[70] James, 127.
[71] James, 121.

Maderna's Influence on His Pupils

Maderna's ideas not only belong in France but also in Canada, as his pupils continued to use his techniques. The next part of this essay will focus on one such pupil and how she implemented his method. Norma Beecroft—born in 1934 in Canada—studied with many composers from all around the world, including Bruno Maderna. She composed the flute and piano duet *Tre Pezzi Brevi* in 1962 after studying with him at Darmstadt. Beecroft explains that while studying with Maderna, she learned his "… system of organizing musical materials, which freed [her] from the rigidity of working with 12 tone rows …"[72] That system is, of course, magic squares.

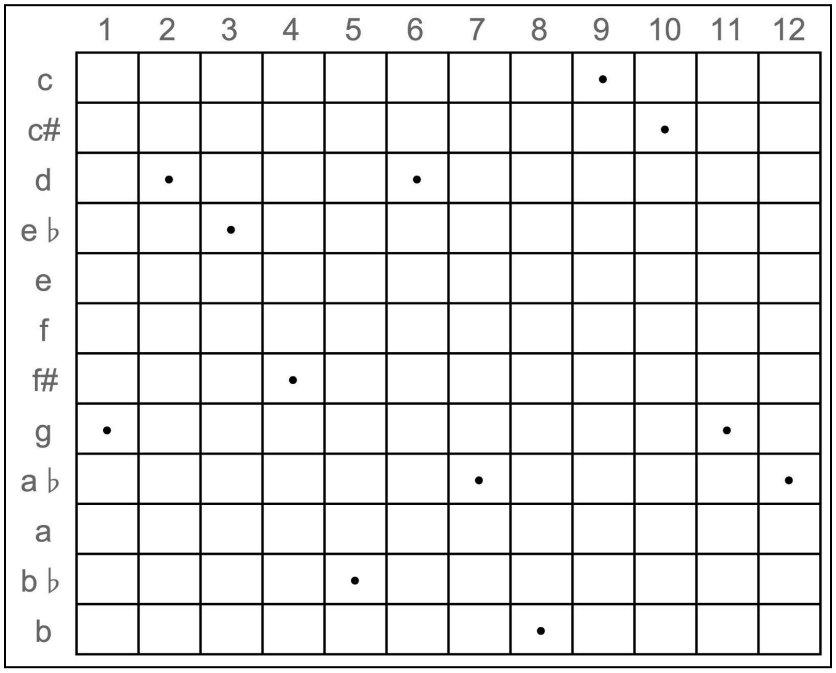
To understand Beecroft's pitch and rhythm choices in Tre Pezzi Brevi, it is instructive to attempt to reverse engineer the matrices and magic square of the piece. As stated above, it is nearly impossible to reproduce a magic square from the final product. As such, the aim of this experiment is not to discover the magic square and matrices that Beecroft used to compose *Tre Pezzi Brevi* but rather to understand how she takes Maderna's technique and uses it to suit her own musical needs.

I began by plotting pitches on a matrix similar to that of Example 2.1. I disregarded the rests in the hopes that the first twelve notes would be the initial row and, therefore, would not have any blank columns (see A1 of Maderna's matrices). The matrix formed from the first measure of movement I is shown in Example 2.16.[73] This could have been the twelve-tone row were it not for the missing E, F, and A and repeated G, D, and A ♭ pitch classes. Even though this is not the row, however, that does not mean this information is useless; it shows that Beecroft is not concerned with highlighting the row or pointing out its importance. This intentional

[72] Norma Beecroft and Canadian Music Centre, *Conversations with Post World War II Pioneers of Electronic Music* [Revised with additions 2018] (Oshawa: Canadian Music Centre, 2018), 337.

[73] See Appendix for the score of *Tre Pezzi Brevi*.

downplaying of the row is similar to Maderna's outlook in that he, too, avoided treating the original row as a theme.



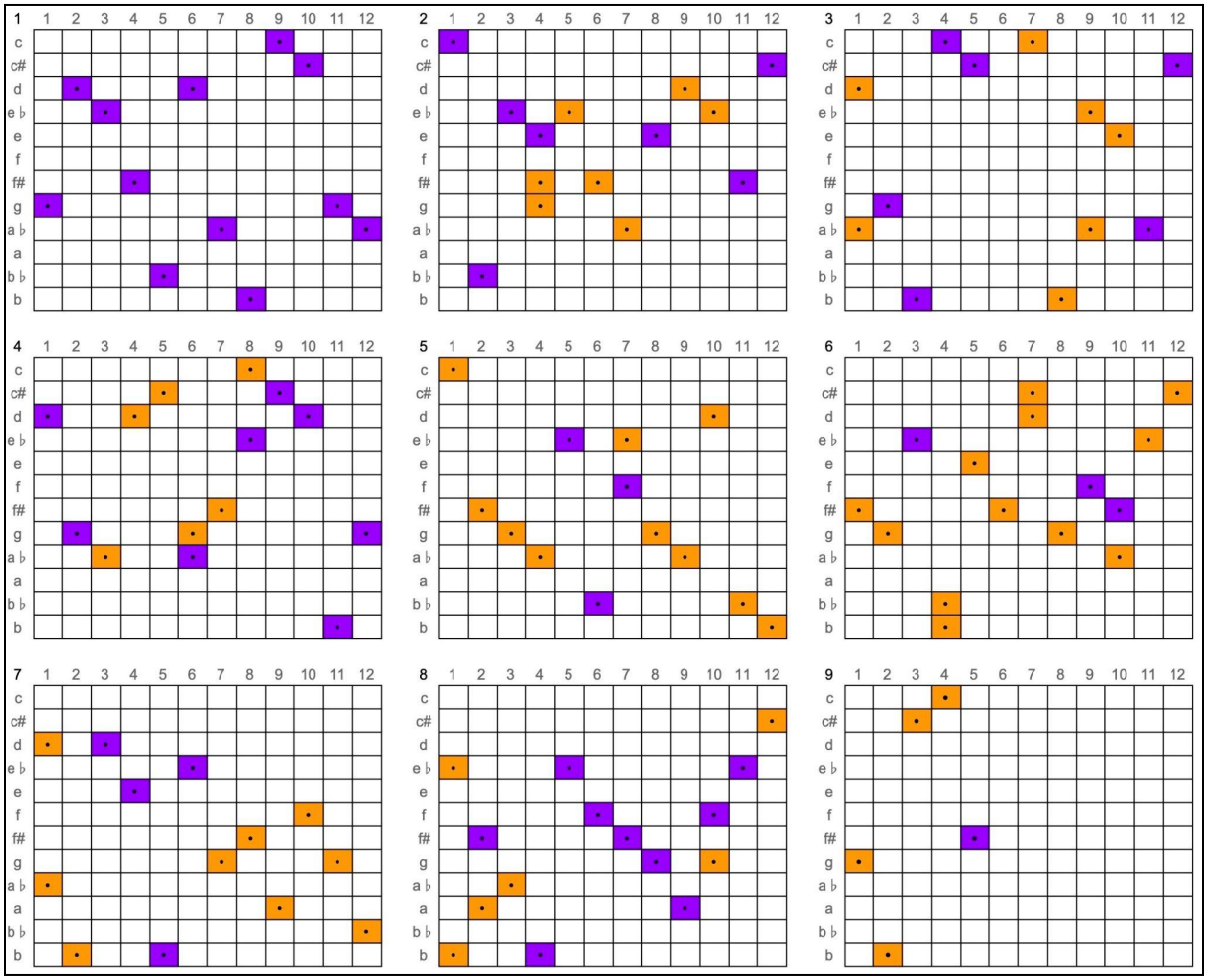
Example 2.16. The matrix formed from the first measure of *Tre Pezzi Brevi*, i.

Beecroft begins *Tre Pezzi Brevi* with a solo flute line that shows off her ability to write beautiful, flowing, Debussy-like gestures. In this way, she is able to harken back to her childhood love of Debussy, which she spoke about in an interview with Eitan Cornfield in 2003.[74] This is where Beecroft's ideals stray slightly from Maderna's since her goal was to write beautiful music, not to strictly follow a mechanical technique. She uses the matrix as a starting point and may deviate from it should her compositional ideas compel her to do so.

The task of plotting the pitches becomes difficult in m. 2 owing to a number of ambiguities: is the E6 in the flute tied over for three beats considered two separate dots on the matrix or just one? How do the rhythms line up in the piano at the end of the measure; do the triplets apply to both the right and left hands? F#, D, and G are played in the left hand and then

[74] Beecroft Documentary, "She was the first woman composer," Eitan Cornfield/Norma Beecroft, 2003, https://youtu.be/lgkVia1a8PY?t=344.

immediately in the right. Do I plot both on the matrix? Many of these types of questions arose while trying to recreate the matrices. This analysis navigates these ambiguities to create nine matrices, shown in Example 2.17, but it may not match the matrices from which Beecroft created *Tre Pezzi Brevi*. The flute and piano notes have been highlighted in purple and orange, respectively.



Example 2.17. Matrices created by attempting to reverse engineer *Tre Pezzi Brevi*.

There are a few problems in this result, by which I mean instances that are uncommon to typical magic square matrices. First, some pitches take a very long time to occur or reoccur: the first F doesn't appear until matrix 5 in the fifty-fourth spot, the first A doesn't appear until matrix 7 in the eightieth spot, and the last C appears fifty spots after the penultimate C. This is similar to

Weisstein's mathematical definition of a magic square in that a pitch is not heard for a prolonged period of time. The second and perhaps most curious element is that the last 12x12 matrix only contains five dots. Once again, these unusual instances show that Beecroft is not interested in using strictly the magic square technique. Instead, she takes more creative liberties. Unlike Maderna and the Oulipo, she does not constrain herself to mechanical automatism.

The final step in my quest to reverse engineer the magic square for *Tre Pezzi Brevi* was writing out the square itself, or at least figuring out what the numbers within the magic square might be. My results are in Example 2.18.

C	3	14	2	12	4	50									
C#	13	4	6	4	3	21	4	23	2						
D	3	9	4	3	11	2	5	11	8	5	1				
Eb	11	1	4	7	2	10	8	1	7	7	6	6	3	5	
E	3	13	30	10											
F	13	12	7	3											
F#	11	1	4	19	6	10	4	3	9	5	4	9			
G	9	4	9	11	3	5	2	4	5	5	10	3	8	1	2
Ab	4	6	5	7	1	3	2	9	4	12	2	13			
A	4	6													
Bb	8	39	4	4	19										
B	18	4	14	12	3	9	2	7	2	9					

Example 2.18. Resulting "square" of reverse engineering *Tre Pezzi Brevi*.

While this "square" does not yield results that inform what the actual magic square may have looked like, it does tell us some interesting information about how Beecroft thought about the pitch content of *Tre Pezzi Brevi*. Neidhöfer's response to seeing the "square" is "Beecroft is not interested in the statistically more even distribution of pitch classes …"[75] Her interests lie in the aesthetically pleasing sounds that music can make, not the mathematics that lie behind their construction. Therefore, the "square" is a great example of pitch-class distribution rather than a

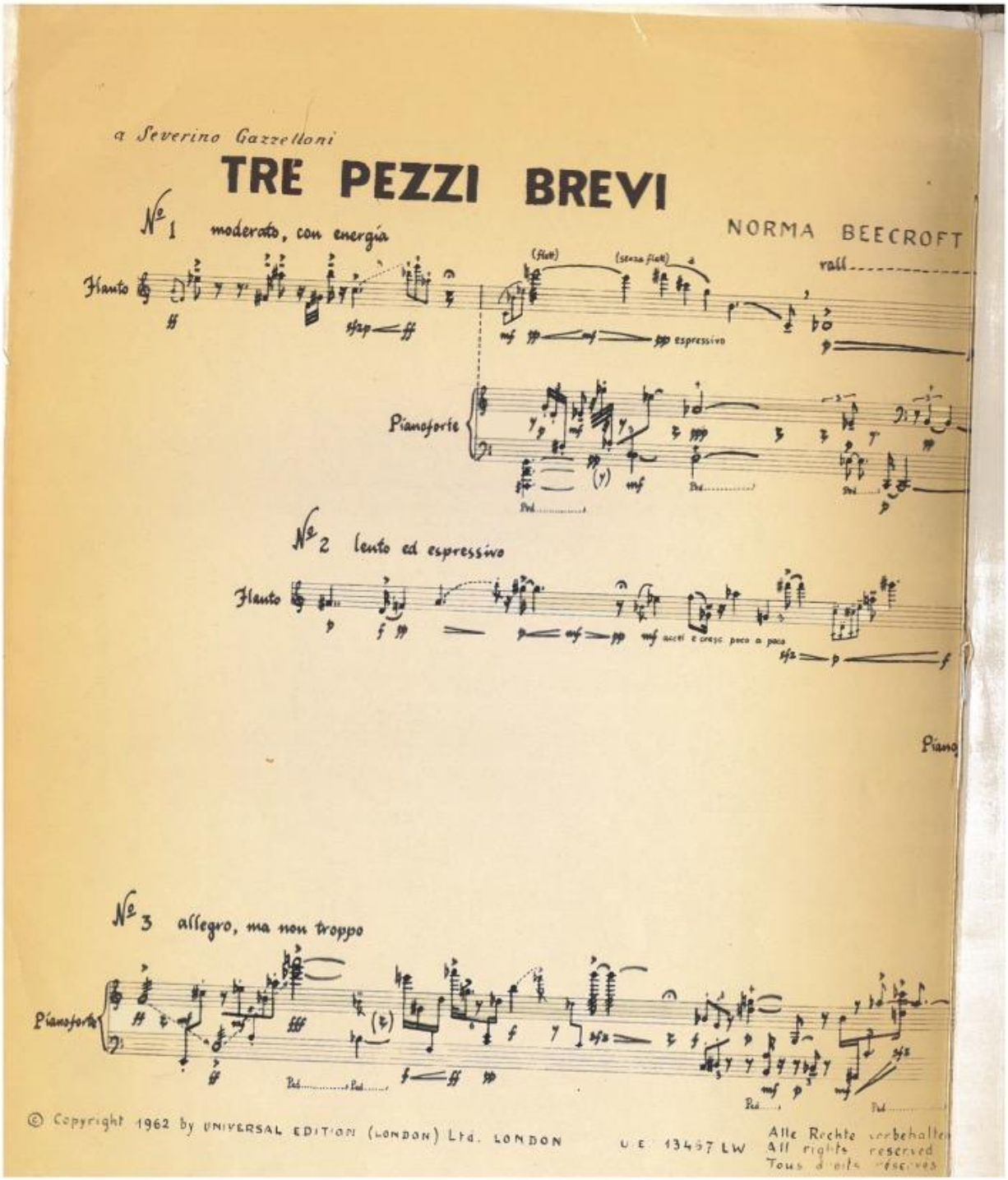
[75] Christoph Neidhöfer, Personal communication, Nov 14, 2021.

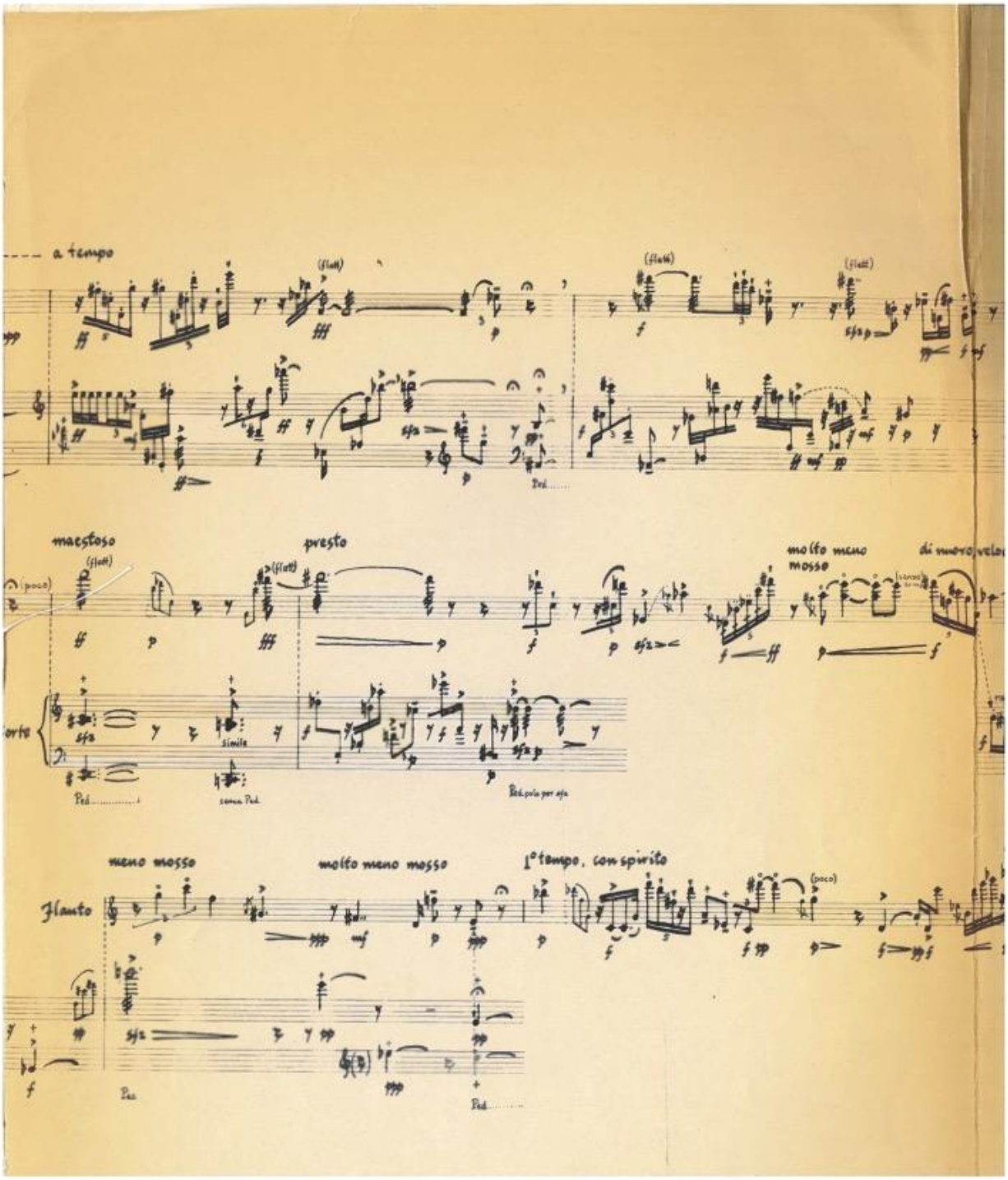
magic square. It tells us that G is the most commonly used pitch-class, appearing at least every twelve notes. E ♭ is the next most common, appearing in the same way. F# and A ♭ follow, then D, B, C#, and C. The last four pitch-classes appear very infrequently; B ♭ occurs six times with thirty-nine other pitches separating the second and third occurrence, E and F each only occur five times in the movement, and finally, A occurs only thrice. This statistical analysis of the pitch-class distribution in movement I of *Tre Pezzi Brevi* furthers the argument that Beecroft is unlike Maderna in that she is uninterested in maintaining the integrity of the magic square and also further proves that task of trying to reverse engineer a piece composed using magic squares is an impossible feat. However, she is similar to Maderna in that she is interested in moving music forward, using new methods of composition.

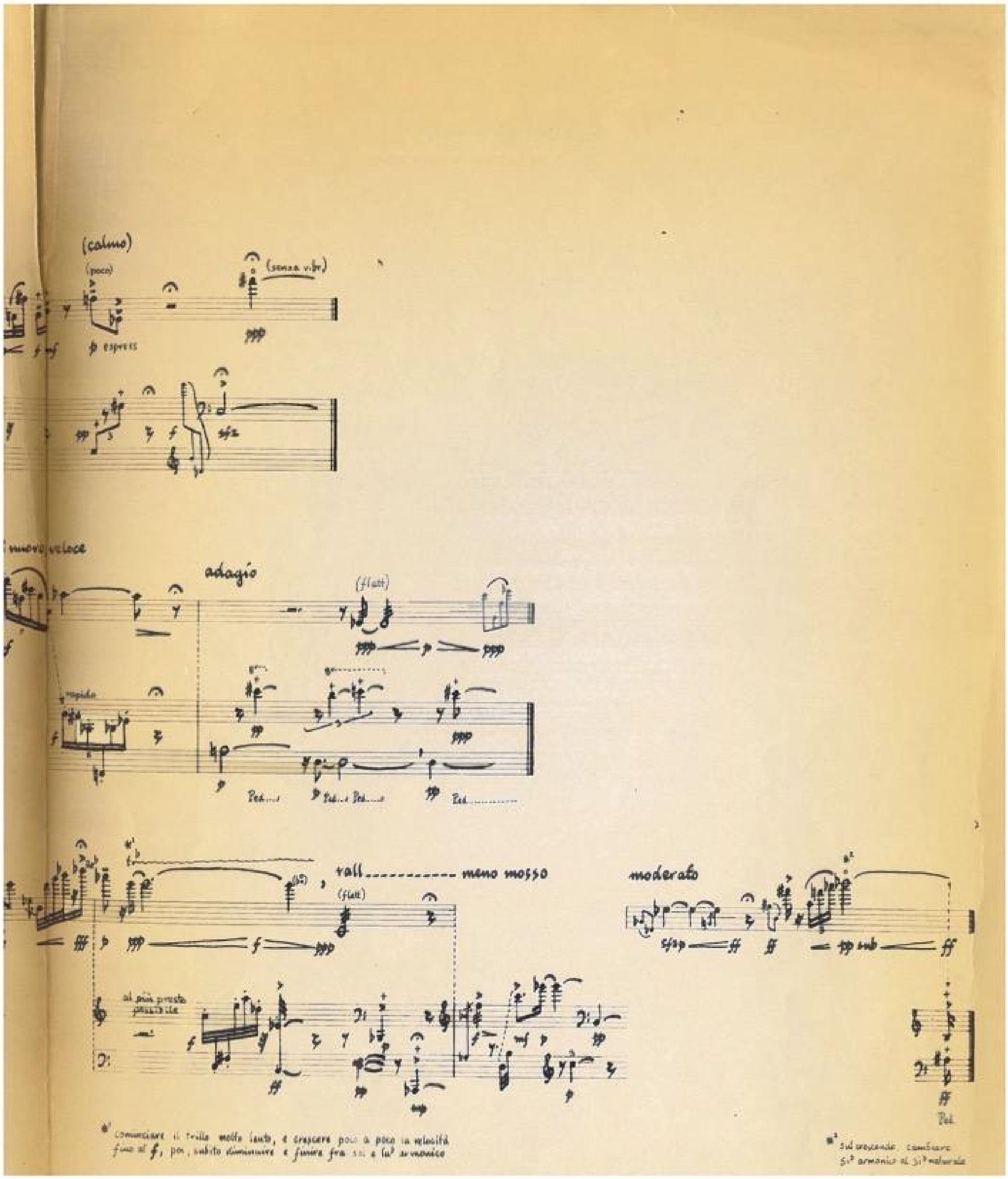
After exploring Maderna's unique system of organizing musical material, magic squares, it is clear to see how Maderna combines serial elements, old concepts, constraints and chance to compose his 1952 work *Improvvisazione, No. 1*. His work is methodical and follows a mechanical formula. However, it is not without choice; he still has many compositional freedoms, including rhythm and formal structures. In this way, his ideals line up with those of the 1960s French group of writers, the Oulipo. His time teaching at Darmstadt also gave him an opportunity to pass along his ideals to students, including Norma Beecroft, who took his ideas and used them to suit her own musical interests. Other composers, such as Sir Peter Maxwell, have used the mathematical definition of magic squares to aid in their compositional process, but Maderna's dedication to the study of moving forward by looking to the past is unrivalled by any other composer, and none have attempted the extremely complicated and convoluted system quite like he has.

Appendix

Tre Pezzi Brevi




a tempo
(flauti)
maestoso
presto
molto meno mosso
meno mosso
molto meno mosso
1° tempo, con spirito
Flauto
simile
senza Ped.


(calmo)
(poco)
(senza vibr)
p espress
ppp
sfz
nuovo veloce
adagio
(flatt)
rapido
Ped.
tall
meno mosso
moderato
al più presto possibile
Ped
cominciare il trillo molto lento, e crescere poco a poco in velocità fino al f, poi, subito diminuire e finire fra si e la armonico
sul crescendo, cambiare si armonico al si naturale

Conclusion

After analyzing op. 4, no. 1 by Beethoven and *Improvvisazione, no. 1* by Maderna, it will now be clear to see that pre-composition can take on many forms. Beethoven's work was "pre-composed" in the sense that he had already composed op. 14, no. 1 for piano. He then took that work and made many adjustments to create the string quartet version. Maderna's piece uses pre-composition in a very different sense. He used many new devices that would not directly appear in the final work but would create outputs that he would then implement into *Improvvisazione, no. 1*.

Both compositions use pitch-class as their groundwork; the op. 14 piano sonata defined Beethoven's string quartet's pitches and Maderna's matrices—created through the Magic Square—outlined in the pitches in *Improvvisazione, no 1*. Each composer is then free to manipulate timbre, range, and dynamics. In op. 14, the piano sonata already predetermined the formal structure of the string quartet. In contrast, Maderna had much more formal freedom when composing *Improvvisazione, no. 1*, as the formal layout was not part of his pre-compositional device.

In the past, pre-composition has been a term strictly associated with twentieth and twenty-first-century music. This paper proves that, although modern composers may use the term more frequently, the technique has actually been used for centuries. As music continues to grow and evolve, pre-compositional techniques will grow too. With it, a new generation of composers will be willing to experiment and see where pre-composition can take them.

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