Unearthing Challenges in a Mathematics Learning Activity Through Video-Mediated Lesson Study: A Pragmatic Grounded Analysis of Collective Reflections

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À la mémoire de Rahim Baradaran Rahmani et Shahin Oveisi

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Résumé

Comment–et par qui–une leçon est-elle réifiée? Cette thèse rassemble des citations provenant de trois rencontres de réflexions collectives transsectorielles qui décrivent, questionnent, et critiquent une activité d'apprentissage mise en œuvre dans quatre salles de classe de sixième année et captée sur vidéo. L'étude fait partie d'un projet de plusieurs années qui a adopté le modèle d'enquête de la leçon-sous-étude (“lesson study”), avec trois phases distinctes: la conception conjointe, la mise en œuvre, et la réflexion collective (Takahashi, 2014). Pour réfléchir aux défis rencontrés pendant l'enseignement et l'apprentissage des mathématiques, neuf vignettes ont été analysées à l'aide de théories qualitative combinées pour comprendre les réussites et les échecs de l'activité d'apprentissage, en se concentrant particulièrement sur les conversations qui en découlent concernant la conversion, l'estimation, et le volume par rapport à l’aire. L'étude met en évidence les différents propos de trois types d'acteurs directement engagés dans la réflexion: les enseignants, les conseillers pédagogiques, et les chercheurs. La structure de la thèse est inspirée de Borton (1970): *quoi*, *et alors*, *et maintenant?* Je décris, *quoi*, ce dont les enseignants ont parlé lors de la réflexion en trois groupes (secondaire, primaire et mixte). Ensuite, *et alors,* quelles implications peut-on en tirer? Enfin, et *maintenant,* explore les actions à mettre en place pour améliorer la tâche mathématique et les protocoles de réflexion en groupe pour la leçon-sous-étude.

Mots-clés: réflexion collective médiée par vidéo, leçon-sous-étude en mathématiques

Abstract

What/Who reifies a lesson? This thesis curates educators' quotes from three cross-sector collective reflection meetings that describe, question, and criticise a learning activity enacted in four grade six classrooms and captured on video. The study is embedded in a multi-year project that adopted the lesson study, an inquiry-based model, with three phases: codesign, enactment, and collective reflection (Takahashi, 2014). Nine vignettes were analyzed to think through unearthed math challenges during teaching and learning math, a mix of grounded qualitative techniques and content analysis were used to understand the successes and failures of the activity, specifically focusing on the educators' reflections on conversion, estimation, and volume vs. area. Following Borton (1970) the reflective model: *what*, *so what*, and *now what* will structure this dissertation to highlight the importance of collective reflection in improving learning activities and suggests that video-mediated lesson study meetings can be a valuable procedure for teachers' professional learning and development provided its prompts and outcomes are closely analyzed.

Keywords: video-mediated collective reflection, math lesson study

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List of Abbreviations

PLD	Professional Learning & Development
RPP	Research-Practice Partnership
I	Invite Elaboration
R	Make reasoning explicit
P	Positioning and Coordination
B	Build on ideas
RD	Reflect on Conversation or Activity
C	Connect
E	Express Ideas
G	Guide
TP	Teacher Personality
A	Interpersonal Affective
LP	Lesson Presentation
SP	Student Participation
M	Motivation
Ē	Physical Classroom Ē
CM	Classroom Management
LC	Lesson Content
LT	Lesson Tools
LS	Lesson Structure
SU	Student Understanding
TQ	Teacher Questions
TK	Teacher Knowledge
GD	General Description

Unearthing Challenges in a Mathematics Learning Activity Through Video-Mediated Lesson Study: A Pragmatic Grounded Analysis of Collective Reflections

In the winter of 2017, four learning sciences researchers and five consultants collaborated with two groups of math teachers, comprising four secondary math teachers and six elementary teachers, at the Sunny Side School Board (pseudonym). This collaboration was part of a multi-year (2013-2017) Research-Practice Partnership (RPP). The primary objective of this partnership was to address key issues in math teaching and learning during the transition from elementary to secondary education. Over time, the team gradually implemented a sociotechnical infrastructure inspired by the lesson study approach (Takahashi, 2014). This involved cycles of video-recorded codesign, classroom enactment, and collective reflection meetings.

On a breezy Wednesday morning in March, six elementary teachers from four different schools gathered in a dimly lit conference room at the main office of the board to reflect on their recent implementation of a classroom activity referred to as the "Donut activity." This activity required grade 5-6 students to estimate the number of donuts in a large box. Early in the meeting, four teachers from two different schools expressed contrasting views on rigor:

> *Kiera: See, I find for cycle two, I* ***don't know if it's the applications, but rigorous...in teaching grade four, rigorous it seems quite easy.*** *They get it, they understand, they give an... explanation. Maybe it's because they find, the…*
> *PI: hmm*
> ***Kiera: the material…, the information is not as challenging as***
> *Melody: Mmm*
> ***Kiera: grade six. So... in cycle two I can focus a lot on rigorous, and then when they get to***
> *Melody: hmm*
> ***Kiera: they're going to understand [pointing to Melody]***
>
> *vs.*
>
> ***Kaci: And see, with me this time, I didn't focus on the rigorous. I focused/***
> ***Dona: /me too***
> *Kaci: /more on,* ***let's stay relevant to math, "staying relevant to this problem. Let's not kill ourselves trying to explain too well. Let's go and search."***

Interestingly, during the collective reflection meeting two days prior, the secondary math teachers expressed their confusion regarding certain responses provided by the elementary teachers to students. The secondary math teachers perceived that the activity failed to address specific math concepts, including estimation, conversion, and measurement. However, two days later, the elementary teachers praised their colleagues and commended them for their good work. The elementary teachers viewed the activity as a fun way to introduce relevant questions with rigorous explanations. While some teachers expressed ease with teaching rigorous explanations, others expressed fear of overwhelming students, choosing to stay "relevant to math" instead of rigor to avoid "over-explanation." Given the discrepancies in perspectives, I sought to better understand and improve the Donut activity and researchers' lesson study procedures.

Two years later, I began the task of reexamining the video recordings of these reflections, by juxtaposing the perspectives of educators, consultants, and researchers to understand how the activity impacted student learning and how it could be improved for future use. However, reactions to the activity varied, even among group members despite using the same prompts and the same activity plan. The aim of this second glance analysis is to determine what adjustments are needed in the Donut activity's prompts, timing, and level of difficulty for it to become a Donut lesson, where math-operations are matched to math-talk and curriculum. The differing perspectives of the teachers raise the question of how to reconcile these views to improve the Donut activity. The stark differences observed in educators' reactions to the Donut activity led me to experience a Rashomon effect, a term coined by French journalist Jean Rouch in 1950 to describe Akira Kurosawa's phenomenon where people have different perspectives on the same event (Jacobs, 2022). In thriller movies, directors often use the Rashomon effect to demonstrate that reality is filtered by multiple vantage points leading to contradictions in the storyline. For instance, in the movie *The Usual Suspects*, Singer (1995) kept audiences in a twist loop chasing after the truth: who was the culprit? Thus, the Donut activity went under several loops to argue for collective redesign of its lesson plan: Who/What reifies a lesson? Why was rigor sacrificed, how can we reconcile the diverging views and balance math-talk with math-calculation in one learning activity? When did rigor become associated with over-explanation rather than well explained? To what extent elementary teachers must focus on rigorous explanations? The curiosity to understand divergences and convergences in reactions prompted me to investigate the activity that sparked conflicts among educators. As Henry Miller (1941) argued, "the moment one gives close attention to anything, even a blade of grass, it becomes a mysterious, awesome, indescribably magnificent world in itself" (p. 9). This dissertation explores these and other questions related to collective reflection in Lesson Study. The findings emphasize the importance of considering different perspectives when examining activity demands, selecting appropriate high-quality teaching practices, and maintaining rigor to facilitate student understanding.

To orient the reader, in *The Vulnerable Observer*, Behar (1996) invited the reader to join her on an unmapped journey, stating, "if you don't mind going places without a map, follow me" (p. 33). Similarly, I adopt an exploratory approach by presenting a scrapbook of data that is peeled like an onion that could to some extent describe "how things are and how they got that way" (Wolcott, 1984, p. 180). This report may appear lengthy as I transition from raw conversations to coded transcripts, interpretations, and conceptual models of reflection to provide context for key themes. Epigraphs, or quotations from the data, set the stage for different sections, which progress from the past to the future and from the singular to the collective. This combination may require an adjustment in reading style (Borton, 1970). This report consists of six main sections, each with descriptive headers and subsections. After this introduction, we begin with the Brief Dive: Body of Knowledge literature review section, which includes a Selection Criteria subsection that describes how the articles were curated to describe salient literature related to Lesson Study, Reflection, and Collective reflection. As well, the Centipede's Story: Gap section notes a gap in the literature that this research aims to illustrate. Then, the colloquial three-stage reflection model outlined by Borton (1970), is used to formulate the "Research Questions: What?, So what?, and Now what?" and to organize the process of reporting the outcomes. The Methods section details the Research Context, Research Partners, Data Sources, and Video as a Mediation Tool. Subsequently, the Lesson Study Procedures section explains the procedures for conducting collective reflection in lesson study with each teacher group. The Donut Activity: Chasing the Lesson, provides a description of the activity, its inception, and a discussion of the original activity. Furthermore, the transcripts related to three Critical Incidents from classrooms that were utilized as a mediation tool are also discussed in this section.

The Pragmatic Grounded Analysis section briefly discusses Reflexivity and Methodological Integrity, followed by an outline of the Data Categorization Procedures, Transcription, and Analysis. The What section presents nine vignettes of reflective conversations across groups identifying differences between reactions, actual effects, and intended effects. In the So What? section, the transformative implications of reflective conversations are discussed, particularly in uncovering four themes: a Wide Range of Reactions to Math Talk, challenges with Estimation, Conversion, and distinguishing between Volume and Area. However, there are

Limitations: Clifthangers associated with video-mediated lesson study procedures and the use of a pragmatic grounded analysis, therefore, I acknowledge Alternative Interpretations. The Now What section proposes directions to address the Rashomon effect by synthesizing practitioners' suggestions with established pedagogical frameworks: Understanding by Design (Wiggins & McTighe, 2011), which focuses on planning learning experiences with the end in mind, and Takahashi's (2014) Lesson Study ethos, emphasizing collective examination of *lessons*. The assumption is that by focusing on lessons we may reduce the Rashomon effect. Pathways to Future Orientation and Work-in-Progress Remarks conclude this study, together with a reference list and appendices to provide supplementary information.

Effective strategies for overcoming the challenges are identified by drawing on the perspectives of educators, consultants, and researchers. By contributing to the ongoing effort to improve math learning, this research aims to support the development of more effective lesson planning strategies, ensuring that students are exposed to learning lessons, not merely activities during math class. In short, this dissertation explores challenges in math learning identified through Lesson Study, focusing on the Donut activity.

Brief Dive: Body of Knowledge

The literature review in this dissertation is organized into seven sections. First, the selection criteria are outlined, followed by a discussion of the literature that is perceived as salient for unpacking the concepts of Lesson Study, Reflection, and Collective Reflection. Then, the types of gaps found in the literature are briefly discussed. It is important to note that this investigation was not propelled by a gap in the literature. Finally, the research questions that guide the format of the dissertation are presented.

Selection Criteria

Articles were obtained through open-access sources such as ResearchGate and McGill's institutional library, which offers privileged access via virtual private networks. The truncated keywords ‘Reflect*’ and ‘Collective, cooperative, collaborative’ were used in various databases (Google Scholar, PsycINFO Ovid, & Elsevier) to find articles related to reflection in education. Mainly English articles were included, with a few French articles. Lesson Study, lesson-study, and lesson study are used interchangeably. In Spring 2021, about 20,200 results were obtained: 90 articles were screened, and 38 articles were retained, including 19 discussions on the definition of reflection. By Fall 2022, a search using the keywords 'Lesson Study' AND 'Collective Reflection' AND 'Video' yielded 207 articles, of which only 12 reviewed articles focused on teacher learning were retained.

In short, the following review will draw upon the writings of various scholars in the fields of education, anthropology, and medicine, spanning from 1933 to 2023. While semantics are important, no standardized definitions of the concepts were taken into consideration prior to data collection in the context of this study. At the onset of the investigation, the concept of lesson study and the responsibility that came with facilitating collective reflection were unfamiliar, including selecting parts from codesign and implementations to meditate discussions.

Lesson Study: What is it?

> *Sabrina: I thought that was... I mean, it was probably one of the most valuable activities that we've done so far, in terms of the videotaping, because* ***it was much easier to compare, and contrast too, you know, the students' reactions, the students' results, the students' success at everything. So…***

This section provides a review of literature pertaining to Lesson Study to explain the definition and origins of the procedures that produced data for this study. Takahashi (2014) described a dynamic cycle for lesson development that involves codesign, implementation, and collective reflection. Lesson study is a collaborative professional development model that originated in Japan in the 1870s as "Jugyō kenkyū" (Vrikki et al., 2017, p. 213). Lewis (2010) described lesson study as a "quality circle" that enables teachers to collectively reflect on their teaching practices. Expanding on this concept, Lewis et al. (2019) introduced five integrated sets of professional learning routines aimed at enhancing teachers' knowledge (e.g., instruction, content, curriculum), beliefs and dispositions supporting instructional improvement (e.g., the belief that student thinking is essential and guides instruction), professional learning norms (e.g., collegial observation, inquiry stance), and curriculum materials (C. Lewis, personal communication, June 2023). According to Lewis and colleagues (Lesson Study Group at Mills College, 2022), the primary objective of lesson study is to move beyond a single "polished lesson." As stated by the group, "brilliant educational visions are just splotches of ink on paper until a teacher somewhere brings them to life in a classroom" (Lesson Study Group at Mills College, 2022). While it is important to plan for a big picture application of lesson study, in this dissertation, I adopt a reductionist approach, focusing on a specific sub-element of lesson study. This "over-elevation of the task" aligned with the need to start small with a pragmatic view to select the path of least resistance (C. Lewis, personal communication, June 2023).

During the coreflection stage, teachers gather with external facilitators to examine the effectiveness of their lesson and identify areas for improvement, revealing "discrepancies between expectations and observations" (Vrikki et al., 2017, p. 213). This stage is crucial for generating the basis of practice adjustment in the classroom exchange (Schön, 1983). To understand the necessary conditions for scaling up lesson study, Lewis et al. (2006) developed a five-step model: defining a problem of teaching practice collectively, codesigning a lesson that addresses the problem, enacting the lesson, reflectively examining the lesson with other teachers, and collaboratively revising the lesson before teaching it again. According to Takahashi (2006), lesson study differs from traditional professional learning and development in that it begins with questions rather than answers, is typically participant-driven, and emphasizes horizontal, reciprocal communication among teachers, rather than hierarchical communication. Due to its versatility, lesson study has multicultural applicability (Huang et al., 2023). In Lesson Study, “practice *is* research”, as opposed to being merely informed *by* research (Liptak, cited in Lewis, 2002, p. 12, italics added). Lesson study is a “contemporary teacher-led model” for professional learning and development (Kihara et al., 2020, p. 1). Its focus on educators “collaboratively studying live classroom lessons” is what gives it value (van den Boom-Muilenburg, 2022, p. 295).

Lesson study is characterized as a cultural activity due to its collaborative nature, (Takahashi, 2006). It is a process that can be school-based, district-wide, and cross-district. Although, lesson study lacks “clear definition” and specified criteria in Japan (Takahashi, 2006, p. 24), in generally high-context[1] societies (e.g., Japan) teachers have a strong sense of community, therefore, group visits to a teacher's class are a normative practice (Horn & Little, 2010). This practice reflects a process of “deprivatization of practice” (Horn & Little, 2010, p. 182). However, in somewhat low-context societies like Québec, in Canada, the default approach tends to involve teachers viewing their classrooms as private spaces, which can make it

[1] See Hofstede's culture dimensions: power, individualism-collectivism, gender, uncertainty avoidance, and long-term orientation (Hofstede & Bond, 1984).

challenging to encourage collaboration (Schoenfeld, 2014). Furthermore, to avoid the naive political position that assumes all participants have equal participatory status (Wooffitt, 2005), it is essential to acknowledge the invisible forces that maintain social harmony, as cautioned by Billings (1999). According to Schoenfeld (2014), there are clear differences in the cultural context of lesson study between Japan and North America. In Japan, collaboration time is built into the defined work of teachers, and the lesson is taught by one member and *refined* by the entire team, with an open-door policy for classroom visits by other teachers or educators. In contrast, opportunities for observation, discussion, and practice are typically more limited in North American settings (Schoenfeld, 2014). In the Québec context, the CCCM project required about two years (2012–2015) to build mutual trust and encourage classroom visits by other teachers.

Huang et al. (2023) acknowledged that lesson study is an "effective approach to teacher professional development" (p. 1). Through lesson study, teachers explore how to "bring to life standards, frameworks," and "best" teaching practices to achieve high-quality instruction (Lesson Study Group, 2022). The richness of lesson study in terms of possible outcomes is illustrated in a case study by Lewis (2009) revealing three types of knowledge that teachers can develop: "knowledge of the subject matter and its teaching-learning; development of the interpersonal relationships among teachers; and development of teachers' personal qualities and dispositions" (p. 95). As Lewis remarks, the development of professional knowledge through lesson study "entails development of the knowers as well as the knowledge, and development of communities of practice as well as individual teachers" (p. 104). Regardless of how it is practiced, lesson study promotes teacher collaboration in order to shift ineffective teaching practices to student-centered teaching practices (Takahashi, 2006). While lesson study researchers have hailed its transformative powers because of its collaborative nature, the concept of post-lesson coreflection is "notoriously undertheorized" in the field of education (Kager et al., 2021, p. 1).

To muddy the efficacy matter, variability in how lesson study research is conceptualized, leads to variability in how they are mise-en-place, further complicating interpretation thus their implications (Seleznyov, 2018). Huang et al. (2023) noted in their editorial that while researchers have documented how to connect theory and practice (Ponte, 2017), in videodisc-based cases for pre-service teachers (Risko, 1991), video clubs (van Es & Sherin, 2008), or video analysis (Tripp & Rich, 2012), video data as filters for the objectification of teaching (Roth, 2007), however, exploration into which theories best strengthen lesson study instruments is still in development (e.g., Borko & Potari, 2019; Clivas, 2018; Huang et al., 2019; Quaresma et al., 2018). Video-based lesson study solves the need for coordinating release time to watch a lesson being enacted live, but teachers still need to watch the recording of the lesson (Suh et al., 2020).

As findings from Amador and Galindo (2020) suggest, student teachers who participated in "lesson study had lesson plans that provided for sense-making, encouraged collaboration, and included investigative mathematics to a greater degree than those in the conventional group" (p. 132). For this reason, video clubs have become a robust mediating tool in lesson study for the purpose of evaluating lessons. Gaudin and Chaliès (2015) noted that video viewing has become the golden standard in professional development, in "all subject areas, at all grade levels, and all over the world" (p. 42). Miller and Zhou (2007) argued similarly that cases mediated by video are "unusually persuasive because they can function as a form of anecdote" though they are edited (p. 322). These findings correspond to Tripp and Rich (2012) research, which shed light on teachers' preference to participate in professional development mediated by videos of their practice. This development can be viewed in light of Fuller's (1970) three-stage model of teacher development: self-image (e.g., "I am so loud!"), classroom management, and learning outcomes (e.g., did the lesson come through the activity?). Often in the second stage, teachers are confronted with the challenging daily reality of most classrooms which may overshadow student learning goals. By the third stage, teachers are confident to articulate their teaching philosophy and are primarily concerned with whether the learning targets were attained (Fuller, 1970).

Although there has been extensive research on the role of video-mediated coreflection in lesson development, there is still much to learn about capturing the richness of video, reflection, and teachers' experiences in sharing quality practices. To combat silo teaching practices and enable teachers to transform their experiences into a professional knowledge base for teaching, Hiebert et al. (2002) identified three key features of knowledge: it must be *publi*c, *shared* with other professionals, and "continually *verified* and *improved*" (p. 4, emphasis added).

In their 2006 study, Lewis et al. distinguished two forms of lesson study: unguided (unpromoted gaze) and guided (prompted gaze). In 2001, Lewis et al. provided no specific guidance on which aspects of student thinking the audience should focus on, resulting in comments that were generally concerned with whether students were "on-task" or successful (p. 200). However, in their 2003 study, Lewis et al. prompted the audience to focus on specific student understanding cues such as "strategies, organization style, and analysis of errors" (p. 200). While advocating for the guided form, Lewis et al. argued that "data collection itself [is] more intentional and is planned in advance with a particular focus" (p. 275), but they did not specify who should provide guidance and for whom (A. Breuleux, personal communication, January, 2023). Nowadays, formulating guiding questions is often used to direct viewers' attention, but guidance may limit the unprompted reactions that viewers would generate. Therefore, it is argued that lesson study can focus on student understanding if teacher groups collaborate “with content specialists,” allowing them to target “particular aspects of student thinking” for observation (Lewis et al., 2006, p. 275). The procedures involved in lesson study are unique and do not prescribe a specific pedagogical approach, however, they do need to strike a balance between being prompted (directed) and exploratory (exploratory). Instead, they create space through sequenced cycles that allow for the concrete objectification of a pedagogical approach through selected lessons and their subsequent analysis. In my opinion, Lesson Study offers “scope for imagination” to think of key lesson plans in a way that strikes a balance between practitioner autonomy and curriculum mandate (Montgomery, 1908/2004, p. 62).

Reflection[2]: What is Salient?

This section provides a review of literature related to the concept of reflection. According to Merriam-Webster's dictionary, the noun 'reflection' (re·flec·tion | \ ri-ˈflek-shən) has at least nine meanings. We will explore how scholars in the field have approached the construct of reflection. The definition of reflection has evolved over centuries and currently includes a mirror, bending back, an indirect critique in the form of reproach, as well as an opinion formed after meditation. Many guides circulate on how to engage in reflection, each with their own approaches and levels of detail. For example, TeachThought (2018) recommends eight reflective questions (e.g., when were you at your best today? Where should you start tomorrow?)

[2] Reflection literature goes beyond discussed dimensions and is far too varied to describe in detail in this report.

Reflection is characterized in different ways, depending on the perspective of the researcher or teacher educator. The concept of reflection is so widespread in education settings, that some educators assume it is natural to perform it (Fendler, 2003). However, Fendler (2003) used Foucault's (1971/1998) genealogical approach to break such naturalization of the concept of reflection to avoid oversimplification and uncovered four interrelated roots of reflection: the Cartesian rationality perspective by Descartes, Dewey's action orientation, Schön's emphasis on the individual's inner workings for growth, and "anti-establishment critiques" to balance the power (Ono et al., 2013, p. 54). For Dewey (1933) reflection is a rational and analytical inquiry. In contrast, Schön's professional perspective views reflection as spontaneous knowledge. Brown (2019) defined reflective professional inquiry as "the conversations teachers have about serious educational issues or problems" (p. 8), but the meaning of 'serious' is not well-defined.

A perspective, adopted by Mezirow (1998), characterizes reflection as critical thinking and an inquiry process to identify and formulate evidence-based problems. According to Shandomo (2010), critical reflection "blends learning through experiences with theoretical and technical learning to form new knowledge constructions and new behaviors or insights" (p. 101). The Cartesian rationality perspective views self-knowledge as a "valid means of knowledge generation" (Akbari, 2007, p. 194). This perspective echoes that of Socrates, who considered any type of reflection to be a positive activity as it leads to further self-understanding (Akbari, 2007). The main assumption of the Cartesian view is that "self-awareness will provide knowledge" (Fendler, 2003, p. 17). This type of awareness promotes the emergence of a 'self-actualized practitioner' that Inchausti (1991) refers to as a "second self" that requires growth by facing bitter mirror data and deeply examining rationale for practice (p. 126). Such practitioners develop interpersonal skills and thus live fuller lives (Akbari, 2007). This idealized view of reflection is what attracts teacher educators to the Cartesian rationality perspective. However, Akbari (2007) cautioned that relying solely on the Cartesian perspective is not sufficient to handle inevitable social dilemmas, such as the need for harmonizing teachers' practice that balance fidelity and flexibility while preserving autonomous teacher practices.While researchers recognize that some form of reflection exists in all teaching settings, with different functions, what constitutes a critically valid definition of reflection is open to debate.

Discussions of reflective practice are greatly influenced by the action orientation as outlined by philosopher John Dewey (1933) book *How We Think: A Re-Statement of The Relation of Reflective Thinking of the Educational Process*. Dewey (1933) defined reflection as "action based on the active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it" (p. 9). For Dewey, reflective practice leads to professionalism, triumph over impulsivity, and the elimination of blind action (Akbari, 2007). In this sense, Dewey contrasts reflective action with impulsive and routine action. For Dewey (1933) three attributes describe a reflective person: "open-mindedness, responsibility, and wholeheartedness" (Yost et al., 2000, p. 40). Akbari (2007) clarified that instinctive trial and error principles guide impulsive action. While routine action is "based largely on authority and tradition undertaken in a passive, largely unthinking way" (Griffiths, 2000, p. 540). Reflective teachers critically examine their practices, generate ideas to improve lessons, and inject those ideas into their practice, contingent on future action (Akbari, 2007).

Half a century after Dewey's work, Schön (1983) offered a targeted perspective on reflective practice in his book *The Reflective Practitioner: How Professionals Think in Action*.

Schön argued that reflection is a creative and intuitive process, rather than a purely rational one. In other words, for Schön practitioners intuitively know how to reflect, and it is okay for the reflection to remain on the abstract level without requiring action, thus rejecting Dewey's concreteness premise. Schön defined reflection as a "cycle of appreciation, action, and re-appreciation" that involves practitioners becoming aware of their implicit knowledge and learning from their experiences (Akbari, 2007, p. 194). Schön (1983) believed that practitioners are competent and have a deep understanding of their field, usually knowing "more than" they say (p. 8). Schön distinguished between two types of reflection: reflection-in-action (ad-hoc, thinking on feet, intuitive vs, deliberate adapting), and reflection-on-action, which occur at different moments during teaching (Akbari, 2007; Fendler, 2003).

Reflection-in-action happens in the midst of actual teaching when teachers encounter a surprising element in the classroom and need to adjust their approach in real-time (Schön, 1983). Reflection-on-action happens after the fact (our lesson study adopted this approach, but with a camera, sometimes we hear the practitioner shifting gears as events went unplanned, "took a life of their own"). It occurs when teachers look back on their experiences and analyze what they could have done differently. Schön's work suggests that reflection can occur at different levels of abstraction. Mirror-data, which are representations of the activity or practice, can be helpful for certain practitioners. Practitioners may have varying opportunities to step out of the action (by distancing themselves and looking away from the object) and step in (by closely examining the tools that uphold the system). Some argue that video capture can make practice visible, allowing practitioners to watch themselves in action and gain new insights into their intuitive practice. As McVee et al. (2017) observed, practitioners may be surprised by what they see in their own videos: “I never knew I did that until I watched this video!” (p. 41). This suggests that whether practitioners are reflecting in the abstract or actively, they may need mirror representations of data to reflect effectively. Another example of reflection-in-action is “contemporaneous reflection” to refer to situations that allow the actor to “stop and think” (van Manen, 1995, p. 34). This view of reflection emerges during enactment and is typically carried out in an ad-hoc manner in the classroom by the lone teacher who needs to make decisions quickly. Positioning reflection as a deliberate response, an inner mental process, however, this perspective becomes restrictive when it comes to studying coreflection cases such as the data from the CCCM project. If we remain on the inner mental plane, then “operating signs” become essential in thinking (Wittgenstein, 1969, p. 10). By contrast, reflection-on-action occurs after enacting the lesson when teachers can distance themselves from the object of action to focus on the structure of action and have access to authentic mirror representations of the system (Visser, 2010). Wartofsky's (1979) description of representation is apt: “representations [are], in the first place, the actual praxis of creating concrete objects-in-the world, as representations” are representational objects (p. xxii). Reflection is seen as a rational and analytical inquiry (Cao, 2000, p. 3), and its cognitive element helps teachers make decisions and judge their rationality (Sparks-Langer & Colton, 1991).

Since the 1990s, research has consistently shown that reflective practice can improve teaching and better meet students' needs (Wojcik, 2020). To develop reflective skills, Wiltz (2000) has tallied several strategies, such as journaling (Calderhead, 1991; Colton & Sparks-Langer, 1993), interviewing (Trumbull & Slack, 1991), conferencing with peers (Zeichner & Liston, 1987), and video recording (Bryan, 1999). According to Rodgers (2002a), reflective inquiry requires purposeful, focused, and goal-directed thought that are "complex, rigorous, intellectual, and emotional enterprise that takes time to do well" (p. 844). Rodgers (2002b) reframed Dewey's work in modern terms, emphasizing that reflective practice involves making connections between experiences, disciplined and systematic thinking, community interactions, and valuing intellectual growth. Dewey (1933) proposed that reflection involves four essential elements: the process of making meaning, which moves learners from one experience to another with a greater understanding of the connections between events; a disciplined, rigorous, and systematic way of thinking; reflection must happen within interactions between members of the community; and intellectual growth is valued.

Reflection's "seductive allure" stems from its claim to mediate improving practice (Loughran, 2002, p. 33); hence the teaching, hence the learning. Effective teaching practice, as Wang and Paine (2010) posited, can be achieved by examining "teachers' abilities to reason about teaching and apply their thoughts to specific teaching contexts" (p. 370). For instance, Richert (1992) merged three "empowerment concepts"—reflection, agency, and voice—to construct the concept of reflection (p. 187), allowing one to access their "authentic inner self" without interference by gendered ideologies (Fendler, 2003, p. 19). Similarly, Smyth (1992) denounced non-emancipatory forms of reflective practices as they evoke the term "radical interventionism" in *The New Right ideologies* (p. 267). While these perspectives offer valuable insights into the potential limitations of hierarchical reflective teaching programs, they go beyond my escape velocity at this time.

Over the years, many have attempted to define the concept of reflective practice by describing its components. Moon (1999, 2013) tallied how reflection could be applied towards learning and improving its related materials by; representing learning and its process; building theories; developing the self; critical review; make decisions during uncertainty; and empower. Brown et al. (1989) categorized reflection under the topic of situated cognition, while Loughran (2002) captured reflection in the relationship between three important elements: time, experience, and expectations of learning (p. 33). Reflection is associated with self-consciousness, intuitive understanding of practice, inner voice, professionalism, psychological secondary-tool for reflective practitioners, empirical approaches to decision-making, and a means to denounce social injustices (Akbari, 2007). The wide range of definitions leads to a lack of consensus which then hinders how we can foster reflection among practitioners (Hatton & Smith, 1995). According to Akbari (2007), reflection involves “organized, rational, language-based decision-making processes that include gestalt (non-rational) type operations” (p. 31). Reflection can be instrumentalized as a way to gain “insights into teacher development and the nature of the relationship between teacher cognition and teacher behavior” (Korthagen & Kessels, 1999, p. 4). Tay and Johnson (2002) proposed a “typology for investigating reflective teaching”, which includes three steps: description, comparison, and criticism (p. 195). Marcos et al. (2011) argued that reflection is “essentially conceived as a cyclical and recursive process” but highlighted differences between what is promoted in practice and research evidence on reflection (p. 22). This is consistent with Borton (1970), since reflection was conceptualized to provide “an organized way” to increase awareness, evaluate information, and experiment with new behavior–referred to as the What? So What? Now What? approach (Rofle, 2014, p. 489). These are also consistent with the model of Pedagogical Reasoning and Action that defined reflection as “reviewing, reconstructing, reenacting and critically analyzing one’s own and the class’s performance, and grounding explanations in evidence” (Shulman, 1987, p. 4).

Other researchers also shape the discourse on reflection. For van Manen (1991), reflection is “just another word for *thinking*” and carries the connotation of deliberation, making choices, and coming to decisions about alternative courses of action (p. 98, emphasis added). For van Manen (1977) to reflect *is* to think as he described the three elements pertaining to technical rationality, practical reflection, and critical reflection that could define the nature of reflection. This aligns with OECD's (2018) characterization of “[reflective] practice [as] the ability to take a critical stance when deciding, choosing and acting, by stepping back from what is known or assumed and looking at a situation from other, different perspectives” (p. 6). This cyclical reflective critical positioning allows for cognitive growth that is seen in the pattern of socialization (Wolcott, 1984). In action research, it is important to identify the critical nature of reflection (Satariyan & Reynolds, 2016), as this appears to be due to the premise that “reflection and anticipation are precursors to responsible actions” (OECD, 2018, p. 6), which is equally important for lesson study. How can a math teacher’s action during activity implementation be simultaneously constructive and directed at a math learning outcome?

Over time, many scholars have highlighted the importance of solo-reflection in teaching. For instance, in 2008, Hall and Simeral noted that “the skill of self-reflection transcends all other skills, strategies, and teaching approaches because it can grow over the course of a teacher's career” (p. 38). Despite the varied interpretations of reflection in the literature, one constant assumption is that it is an ongoing process. As Brush (1999) noted, reflection is “a structured process that is used to guide personal and situational analysis and improvement. Reflection [...]

emphasizes awareness of one's own knowledge, past experiences and beliefs" (n.d.). Assuming that in order to learn from experience we ought to reflect upon said experience then reflection plays a critical role in "teachers' knowledge, skills and dispositions" (Shandomo, 2010, p. 101). Moreover, even though the unintended effects were not considered by Bortton (1970) he suggested that reflection not be viewed as fleeting in form. Reflection is constant experience, not a one hit wonder, it is a constant, just as Celine Dion said in a 1999 speech, "My Heart Will Go On is not just a hit…it's an experience that will remain forever." The word "reflection" is often defined as a mirrored image, for example, "what content is taught and the amount of time—Usually a reflection of the number of pages—that is devoted to the topic" (Chávez, 2003, cited in Reys, 2014, p. 42). If this type of definition is not contested, we can extrapolate that the teacher's choice of words and type of comments during coreflective meetings are a *reflection* of their content knowledge. However, we are not diving into the analysis of the content knowledge of teachers, we don't have that type of data.

These four perspectives on reflection–Cartesian rationality, Dewey's action-oriented approach, Schön's professionalism, anti-establishment perspective–offer a range of ideas for teacher reflection. Taken together, the earliest information found on teacher reflection dates back to Dewey's (1933/1997) theorization of reflection, followed by Schön's positioning of reflection within professional judgment in the 1980s, and Mezirow's critical reflection, to the latest studies published on collective reflection (e.g., Liu, 2020; Perry et al., 2020).

From a theoretical viewpoint, Akbari (2007) argued that in some models of reflection, a critical dimension is missing, and that there is "no common, agreed-upon definition of reflection since reflection has been influenced by many trends and philosophies" (p. 193). By drawing from each perspective, educators can develop a comprehensive approach to reflective practice to generate ideas for improvement to create optimal learning opportunities for students (Griffiths, 2000). At which point, if practitioners "draw on their repertoire of examples to reframe the situation and find new solutions" (Griffiths, 2000, p. 542), then they can argue that such in-action reflections rely heavily on their expertise –a type of knowledge in practice– that is "tacit and meant to cope with the conflicted situations of practice" (Schön, 1983, p. 9). This view of reflection, that Tremmel (1993) referred to as the Zen art of "paying attention," is similar to mindfulness in a broad sense, standing in contrast to the analytical and technical view of reflection (p. 434).

In sum, there is no universally accepted definition of reflection and no agreed-upon best practices for practicing it (Fendler, 2003; Akbari, 2007; Herring, 2004). According to Fendler (2003), tensions between practitioner-based intuition and rational thinking, as well as a historical reading of the term, have contributed to the term becoming a "catchall term" (p. 20). These multiple definitions have led to different interpretations (Akbari, 2007), which in turn have led to different styles of reflection fostered in teacher education programs (Valli, 1995). To better understand the nuances of reflection, it can be helpful to explore different models or definitions in a given context, without which we get a cornucopia of articulations.

Why does the proliferation of theoretical bases for the concept of reflection matter? It matters because having a clear understanding of the different theoretical perspectives on reflection can help teachers to identify the approach that aligns with high quality teaching practices. If not, the plurality of theoretical bases for the concept of reflection makes it difficult for teachers to critically evaluate their reflective practice. Therefore, discerning which approach to reflection is optimal could help educators develop a sense of students' learning outcomes earlier in their career. As Valli (1995) noted, different styles of reflection fostered in teacher education programs lead to different interpretations of the concept.

In conclusion, understanding the various perspectives on reflection is crucial for teachers looking to develop a comprehensive approach to lessons. Unfortunately the proliferation of different theories associated with reflection can make critical evaluation of reflective practice challenging. With all these different approaches, teachers may still face challenges in identifying glitches in teaching practices, such as those that arise during the Donut activity, and developing strategies to address them; coreflection would be then helpful in such situations. Next, we explore the concept of collective reflection.

Collective Reflection: What is Salient?

In the subsequent section, comprising four subsections, we examine various conceptualizations of collective reflection and underscore its significance in the context of The Donut Activity. The first subsection, Collective Reflection as Problem-Setting, examines the ways in which coreflection can be used to identify salient issues and problems that need to be addressed in the classroom. The second subsection, Collective Reflection as Problem-Solving, discusses how coreflection can be used to develop strategies to address the identified issues and problems. Lastly, the third subsection, Collective Reflection as Problem-Posing, explores how coreflection can be used to generate new ideas by perhaps breaking assumptions. I acknowledge the distinctive nature of collective reflection within this particular manifestation of the Lesson Study procedure, recognizing its "unique" characteristics and the valuable insights it offers for educators' professional development (H. Chestnut, personal communication, June 2023).

Specifically, this section unpacks the joint concept of collective AND reflection (coreflection), particularly in capturing the ways in which individuals engage in reflection within a collective. As Rantatalo and Karp (2016) argued, reflection is not solely an individual process but is also “socially anchored” (p. 708). Lesson study procedures create “dissonance” for solo-practitioners as they are invited to participate in collective sessions of design and reflection (Gutierez, 2020, p. 3). We will adopt the collective sense of reflection as a “tool-mediated collaborative activity” (Virkkunen & Ahonen, 2011, 229), for a nuanced conceptualization of coreflection. Lesson study gives space for reflection to be articulated (visible), while solo reflection is often internalized (invisible) unless captured in the think-aloud protocol (Virkkunen & Ahonen, 2011). Wartofsky (1979) noted, “human beings create the means of their own cognition” (p. xv). In other words, we create our own cognitive artifacts, which become models for representing our actions, and motives, and become representations of practice, such as lesson plans (Wartofsky, 1979). Schmittau (2004) suggested that reflection requires socially mediated artifacts that expand individuals' learning, which may include primary, secondary, and tertiary tools that create an environment for learners to progress from abstract to concrete objects-in-the-world concepts.

Virkkunen and Ahonen (2011), agree that reflection is a “culturally mediated action,” and have argued that mediational tools play a crucial role in supporting human engagement in reflection (p. 230). With a theoretical-genetic reflection is particularly effective in reconstructing practice by unmasking the system's logic in a change laboratory setting, thus supporting expansive learning. Additionally, arguing that similar to technical innovation, educational expansion occurs gradually, not by broad strokes (Virkkunen & Ahonen, 2011). Coreflection has been examined in various disciplines, including nurse education (Atkins & Murphy, 1993; Rolfe, 2014), police education, in which Rantatalo and Karp (2016) studied it in the context of criminal investigations; Air Force cadets, for whom Gustafson and Bennett (2002) used it to improve leadership training; and biology, where Chikamori et al. (2013) explored its use to enhance scientific inquiry. According to Chikamori et al. (2013), integrating “researching the topic” into codesign meetings is a key element for operationalizing lesson study (p. 15). Additionally, Chikamori et al. (2013) integrated a concrete prompt to obtain suggestions to improve the lesson. While having multiple approaches to coreflection may benefit practitioners and researchers, evaluating their effectiveness can be challenging. As Tallman (2020) notes, there are multiple “models of collaborative arrangements in educational literature and their assumed power for creating change” (p. 1). Regardless of the overarching name used to refer to the concept of *gathering practitioners* to discuss the subject matter of math, we have observed this diversity across two cycles.

The lack of a standardized understanding of coreflection among research partners limits the systematization of qualitative analysis. This makes it difficult to develop a useful model of coreflection that can realistically improve learning outcomes. Coreflective inquiry involves interaction, rigor, and a growth mindset (Rodgers, 2002a). In contrast, solo-written reflections during teacher preparation, often consisting of mere event description, may not truly reflect on the experience (Rodgers, 2002a). Rodgers (2002b) proposed a practice-oriented interpretation of Dewey's writings on reflective inquiry, which consists of four components: 'Presence,' 'Description,' 'Analysis,' and 'Experimentation.' According to Rodgers, 'Presence' and 'Description' slow down the pace between thinking and action, to reveal “rich and complex:

details […] paving the way for a considered response" (p. 232). Reflection enables learning from experience by applying "what we already know to what we have just experienced, turning it over and over to make sense of it and develop a reasonable response" (McVee et al., 2017, p. 69). The emphasis here is to locate detectable aspects of reflection that point to an understanding of how to improve the activity after designing, enacting and reflecting on the activity. Virkkunen and Ahonen (2011) argued that coreflection incorporates cultural-historical activity theory (CHAT) principles by integrating the entire teaching and learning experience into six pronged activity systems (subject, object, outcomes, roles, rules, and tools). CHAT, as proposed by Engeström (1987, 2000) and applied by Engeström and Pyörälä (2020), provides a medley of four approaches, including *historicizing*, *teamwork*, r*eflection* using mirror traces of the activity, and *illuminating* the practice in its authentic setting by considering it through different planes of time (e.g., past-present-future). This is consistent with sequence of inquiry proposed by Davydov (1990) first, question the presuppositions on which the current problematic practice relies upon, then collaboratively analyze the roots of the systemic problem, and lastly reconceptualize the practice to expansively innovate the purpose and principles of the activity system.

Interestingly, almost a decade ago, Goldsmith et al. (2014) reported on the four goals emphasized by the National Council of Teachers of Mathematics (NCTM) for quality professional learning and development: building teachers' mathematics knowledge, capacity to notice, analyze, and respond to student thinking, productive habits of mind, and fostering collegial relationships and structures for continued learning. While the first and third goals are relevant to problem-solving, and the second is relevant to problem-posing, the last two goals are crucial for understanding the context in which coreflection occurs. Achieving these outcomes requires a significant time investment, systemic support, and efficiently coordinated opportunities for active learning, along with opportunities to develop capacity for coreflective criticality.

In the upcoming sections, I will investigate the concept of coreflection by conceptualizing lesson study as a flipped combination of Mezirow's (1991) reflective framework, which includes problem-solving (Deweyan style), collective problem-setting (Schönian style), and problem-posing (Mezirowian style). This type of problem-posing mindset, instead of solely focusing on problem-setting or problem-solving, may foster criticality. It is important to note that this dynamic process is not a rigid linear sequence and can optimize lessons from codesign to enactment. In a traditional setting, integrating problem-solving and problem-setting questions from Suzuki et al. (2012) prior to data collection can be beneficial for encouraging problem-posing reflections and supporting problem-solving discourse. As Suzuki et al. (2012) explain, questions such as "what is the best way of teaching X?" and "did the students learn what the teacher intended them to?" can facilitate problem-solving discourse, while questions such as "what did the teacher learn from watching the students learn?" can encourage problem-setting discourse (p. 216). The CCCM research team also asked open-ended questions such as "would you have responded differently?" to further promote problem-posing reflections.

Collective Reflection as Problem-Setting

The problem setting conception is inspired by Schön in the 1980s, and offers a phenomenological view in which the classroom's uncertainties provide the main source of problems (Mezirow, 1990). According to this perspective, the teaching episode becomes the focal point of a problem, and the classroom is considered as the ultimate teacher-student space.

In such cases having cameras capturing the events can help understand part of the problem. For a discussion on how teachers experience lesson study phenomenologically, see Moquin (2019) for an interesting perspective.

Collective Reflection as Problem-Solving

According to Hubers and Poortman (2018), “problem-solving collective reflection” involves actively identifying and “questioning ineffective teaching routines,” as proactively finding “means to respond to them” (p. 198). The term ‘respond’ suggests a focus on finding solutions, which aligns with the action-oriented perspective of Dewey in the 1930s. Sparks-Langer and Colton (1991) proposed an approach that focuses on the cognitive element of reflection to underscore how teachers make decisions and judge their rationality along some scale. However, this approach has been criticized for adopting a positivist approach that emphasizes outcomes. Hatton and Smith (1995) questioned whether problem-solving “is an inherent characteristic of reflection” or if it is about gaining a clearer understanding of what happened, its purposes, and emerging difficulties (p. 34). Therefore, while problem-solving, coreflection can be an effective way to identify and address the root cause of math challenges, as it is important to consider why solutions are needed in the first place.

Collective Reflection as Problem-Posing

This subsection examines the different ways critical reflection has been defined in the field of teacher education, with a specific focus on the problem-posing approach. This approach emphasizes educators questioning their assumptions with a cultural-historical activity theory (CHAT) lens and stands in contrast to technical reflection (van Manen, 1991), which involves considering the best means to attain an outcome without examining it in depth (Dinkelman, 2000). Mezirow (1990) conceptualized reflection as questioning premises, which has influenced scholars such as Smith (2004) to theorize a reflective pattern that involves “challenging assumptions” (p. 371). For Calderhead (1991), critical reflection involved “constructive self-criticism of one's actions with a view to improvement” (Hatton & Smith, 1995, p. 35), while for Gore (1987), it was “the acceptance of a particular ideology, along with its accompanying assumptions” (Hatton & Smith, 1995, p. 35). This type of critical reflection is consistent with Cranton's (1994) definition, which emphasizes learners' awareness of assumptions, examination of sociocultural historical roots, and adjustment to their actions. This individual stance stands in contrast to the use of “tacit practical reasoning skills and competencies to accomplish social action” (Garfinkel, 1967 cited in Wooffitt, 2005, p. 73). Additionally, scholars such as Horn and Little (2010) suggest that developing the capacity to discuss “dilemmas and problems of practice” in formal working groups can “provoke learning” (p. 183). Incidentally, the Québec Ministère de l'Éducation, du Loisir et du Sport (2009) has emphasized the importance of exercising critical judgment and going “beyond.” According to the Ministry, deliberately forming an opinion is far more demanding and requires "a solid analysis" to achieve the Cross-Curricular Competency.

> beyond stereotypes, prejudices, preconceived ideas and intuitive assumptions, and to replace unconsidered opinion with judgement. So many of our beliefs about people or **things are adopted without reflection;** to **deliberately form an opinion is far more demanding**” as it “requires a solid analysis: exploring and comparing various viewpoints, finding arguments and applying strict criteria” (p. 9, Chapter 3).

For instance, a novice educator may make ad-hoc decisions about classroom management to increase control, without examining the effects of decisions on the desired learning outcomes. Another example is choosing an activity without considering the math knowledge required on the part of the teacher and students or necessary scaffolds to solve the problem are in place.

Mezirow's perspective on critical reflection derives from Dewey, who cautioned that “it is our belief in untested inferences” that can become misleading (Sparks-Langer & Colton, 1991, p. 39). Critical reflection can be considered implicit when:

> We ***mindlessly*** **choose between** good and evil because of our assimilated values, or explicit, when we bring the process of choice into awareness to examine and assess the reasons for making a choice (Mezirow, 1998, p. 186, italics added for emphasis).

Critical coreflection refers to instances where teachers question why they do things a certain way, asking “how come this?” They may have mindlessly chosen an activity or consciously picked it among many other math learning activities (Van Keulen, 2010, p. 109). Not considering why a certain activity is selected could lead to investing unnecessary time and effort in classroom management. Similarly, an activity enacted without considering how to evaluate its outcome or why it is beneficial may be repeated without necessary adjustments. To avoid assuming an activity is good in and of itself (thinking an activity becomes a lesson by osmosis), teachers can start by focusing on the plan for the lesson instead of their own teaching. This may promote critical reflection as they are guided to concentrate on the lesson plan. Furthermore, there is evidence that teacher engagement is greater when analysing problematic events (i.e., examining videos) of other teachers (Kleinknecht & Schneider, 2013). According to Wojcik (2020), reflection is an ongoing process that requires a continuous commitment to “growth, change, development, and improvement” (p. 12).

According to Valli (1993), dialogue with the community is essential to construct school expectations and values. To evaluate both pre-service and in-service teachers' ability to reflect on their teaching decisions, Sparks-Langer et al. (1990) developed a framework for reflective thinking and a coding scheme. Cross-cycle conversation can incorporate critical reflection, which includes ethical and moral components. Sparks-Langer and Colton (1991) argued that critical reflection occurs when “the moral and ethical aspects of social compassion and justice are considered along with the means and the ends” (p. 39). For example, choosing a seating arrangement that facilitates collaborative learning can foster a more equitable learning outcome. When teachers recognize differences in beliefs, knowledge, and practices, they become resources for learning (Grossman et al., 2001). This framework distinguishes between technical reflection, which focuses on the best way to reach a goal; practical reflection, which examines the means and the end by asking, What can we be learning?; and critical reflection, which considers the moral and ethical issues of social compassion (Sparks-Langer et al., 1990). Mezirow (1998), also categorized reflection into different types of critical reflection, starting with content reflection, followed by process reflection, and then Calleja’s (2014) premise reflection. Process reflection involves “examining existing knowledge, challenging assumptions, imagining and exploring alternatives” (Brigden & Purcell, 2004, p. 1). Educators can more effectively evaluate their teaching practices and develop their professional knowledge base by distinguishing between different types of reflection.

Many approaches to professional learning and development programs, including lesson study, can foster reflective critical awareness of lessons. In this study we have excerpts from two solo reflections (Appendix B), and there are non-video mediated coreflection (informal conversations off camera, through fieldnotes). Some authors posit that content and process types of reflection may carry the risk of reinforcing misconceptions. Mezirow (1998) argued that only premise reflection (the why type) can lead to transformational learning. Premise type of reflection sets the conditions to consider the big picture as well as the underlying factors in light of our value system. To create the optimal premise setting, we need to examine the relationship between workplace conditions, teacher quality, commitment, and their implications for teacher induction programs (Rosenholtz, 1989). All members understand their role in raising concerns for the group to explore and working together to question taken-for-granted assumptions (van Es, 2012). Mezirow (1998) measured criticality in relation to how frequently a teacher's statement broke an assumption by examining what has occurred and examining the presuppositions involved in the process. Collective reflections within CHAT provide a dynamic view of the organization's culture (Virkkunen & Ahonen, 2011). Virkkunen and Ahonen (2011) propose a shift away from non-historical methods of reflection. If we don't dig for the root cause, we may stifle growth with inadequate solutions. Thus, problem-posing in groups of educators may be necessary.

The reviewed research suggests that most programs need improvement until a critical productive coreflection method is introduced in the next iteration of the activity system. According to Brookfield (1990), support networks play a crucial role in engaging in critical reflection. In our case, lesson study has the transformative power to modify math lessons through coreflection on and during codesign. To understand change, it suggested that we examine the first event that led to the new development, starting with experiencing a contradiction (Virkkunen and Ahonen, 2011). The Rashomon effect is the raison d'être of this study.

Among these perspectives, the Deweyian tradition closely captures the essence of coreflection in the context of lesson study because it is action-oriented (Ono et al., 2013). However, practical tools for teachers and researchers to study teacher reflection are scarce (Russett, 2019; van Es & Sherine, 2006). Instead of focusing on a definition, a more logical approach is to obtain a *sense* of coreflection. Vygotsky allowed the concept of 'sign' to "develop and obtain somewhat different shades of meaning, depending on the context in which he used it" (Leiman, 2007, p. 420). For the purpose of our study, we require a hybrid sense of coreflection to operationalize it. This involves pre-lesson and post-lesson conversations in a research-practice partnership to discuss video representations of mathematics learning activities and improve the design, materials, and implementation of future lessons. This fluid sequence of procedures is appropriate for the concept of coreflection and was inspired by Ono et al. (2013), rooted in Dewey's (1933) actionability, Schön's (1983) professionalism, Virkkunen and Ahonen's (2011) transformative power, van Woerkom's (2003) facilitation, Hughes and Smith's (2004) practicality, and King and Kitchener's (1994) justifiability criteria.

For the purposes of this paper, coreflection is defined in terms of its key features on three dimensions. Drawing on the work of Schön (1983), van Manen (1991), and Ross (1990), we identify five key components of reflective thinking for teachers: recognizing problems in education, responding in situ, reframing activity problems, testing solutions, and analyzing consequences of change in activity. These align with the critical reflective inquiry type of reflection described by Hayden et al. (2013).

> more than simply [describing] events in a teaching interaction. Detailed analysis demands teachers [*and researchers*] to develop inquiry habits of mind, notice evidence of student learning, and then apply what is learned from these inquiries to develop responsive teaching actions (p. 4).

Therefore, the outcome of critical reflective inquiry is a "more responsive and dynamic teaching, or adaptive expertise" (Hayden et al., 2013, p. 4). This type of focused and purposeful reflective inquiry is a "complex, rigorous, intellectual, and emotional enterprise that takes time to do well" (Rodgers, 2002a, p. 844). If not, we cannot claim this:

> [video-mediated collective reflection], is a particular method of promoting lesson study, [that leads] to more effective post-lesson discussions, [unless we] go beyond intuitive impressions and develop a more rigorous method of describing these sessions (Ono et al., 2013, p. 53).

Rodgers (2002b) proposed a practice-oriented reflection method comprising four parts: “Presence, description, analysis, and experimentation” (p. 232). The first two aim to close the gap between thinking and action so that complex details are revealed, “paving the way for a considered response” (Rodgers, 2002b, p. 232). Such reflective practice, Russell (2013) asserts requires extensive support by teacher educators to model and explicitly link reflection to practical teaching experience (McVee et al., 2017). By the same token, Perry et al. (2020) have argued, the “essence of ‘professional vision’ lies in understanding (rather than taking for granted), relationships between teachers’ strategies and what students learn” (p. 4).

With all of these *senses* and interpretations in the air, how can we tell that the construct of coreflection positively influences future learning activities? This review indicates that coreflection requires a significant amount of time investment (Erickson, 2006; Horn & Little, 2010). The outcomes of this type of deliberate professional learning development partnerships are not immediate, and it takes time to see how benefits outweigh the drawbacks of investing in coreflection. From a practical point-of-view, Fleck and Fitzpatrick (2010) identified three main conditions that sustain coreflection: formal time allocation, accompaniment, and feedback and encouragement. Lewis et al. (2006) argued that engaging in lesson study yields improvements in three areas: resources for teaching and learning (e.g., “Bank of material to work with”), teachers' knowledge and beliefs (e.g., hearing differing views on a taken-for-granted activity), and professional learning communities (e.g., Barlow et al., 2021). According to Lewis et al. (2006), if we align our expectations with this theoretical model, we can expect lesson study to be a promising yet indirect path towards improving lessons and student learning, which is in line with the School Board's Mission Statement, but in reality we are cognizant of the anti-establishment perspective where we are reminded that when it comes to coreflections the model has to come from within, therefore certain flexibility is necessary, so each group creates its own lesson study procedures, we hope that it includes productive criticality as it focuses on lesson plans. Furthermore, interpreting practice as research is integral to lesson study to enable gradual progress (Pirkko, 2005). While some scholars have noted that individual teacher learning does not guarantee school learning, Säde Pirkko (2005) emphasizes that organizations such as schools only learn through individual practitioners, and without it, the organization does not learn. Wartofsky (1979) suggested that our techno-social structures connect action and cognition, theory, and practice, but the mediation between the correlated contexts remains to be shown.

For several decades, researchers such Liu (2020) have noted how Smyth (1992), Rogers (2001), and Zeichner and Liu (2010) have criticized the broadness of "reflective teaching" in education (p. 45). This is even more true for the meaning of coreflection, which has become quite elusive. The works of Dewey and Schön, particularly their unpacking of the features of reflection as a concept in education, may have paved the way for the development of the lesson study coreflection procedure. It could be argued that if learning *is* participation, as Sfard (1998) has posited, then participating in lesson study meetings brings us one step closer to learning. This also answers the call for collaboration made by the Mathematics Curriculum Team as they identified math challenges in the Québec Education Program (Ministère de l'Éducation, du Loisir et du Sport, 2009). The team in charge of the Mathematics curriculum at the Direction de la formation générale des Jeunes asked for "collaboration in order to identify the mathematical concepts and processes that students find the most difficult at the end of Elementary Cycle Three" (Ministère de L'Éducation, 2020, p. 1).

Taken together, these articles suggest that effective professional learning and development (PLD) is most effective when it is focused on learning, school-based, and continuous (Hiebert et al., 2002). Investing in video-mediated coreflection (Erickson, 2006; Miller & Zhou, 2007) offers professional learning opportunities to support teachers' implementation of the redesigned lesson (Horn & Little, 2010), and gaining a clearer understanding of challenges in lesson implementation (Coburn, 2003) are some strategies that can benefit teachers and schools. In sum, this study may shed light on educators' reactions while observing an activity captured on video during coreflection. To attain such lofty goals, we will briefly cover the niche topics of Productive Collective Reflection followed by Critical Productive Collective Reflection to dive into the forethought needed to facilitate the ultimate thinktank with educators.

Productive Collective Reflection. According to Revans (1983), learning is more than "solely the acquisition of freshly programmed knowledge, such as is purveyed by teaching institutions" (p. 25). To engage in productive reflection, teachers identify the lesson's object and enact it (Morris et al., 2009; Rodgers, 2002a). To make improvements, teachers collaborate to design a learning activity that can facilitate progress (Hoffman-Kipp et al., 2003). Reflection, as Russett (2019) points out, is the crucial step that connects these pieces and helps to achieve learning outcomes. Therefore, according to Cressey et al. (2006), learning starts when we respond to daily life challenges that require unfamiliar and complex problem-solving skills. By following these steps, we can learn from reflection and move towards productive coreflection.

Critical Productive Collective Reflection. Breidensjö and Huzzard (2006) diverge from Dewey (1933) and Schön (1983) by highlighting the collective level of reflection, rather than the individual level. Coreflection must be productive given our fast contemporary context where 'the pace of learning has to match or exceed the rate of change for organizations to survive' (Cressey et al., 2006, p. 16). Following Vince (2002), Breidensjö and Huzzard (2006) observed that reflection is an organizing process not just an activity of the "reflective practitioner" (p. 149). In this view, reflection is part of an ongoing organizing process within a community of practice (Lave & Wenger, 199/2003). Therefore, for Breidensjö and Huzzard (2006), reflection is not just a potential means for individual development. It is rather characterized by an inherent tension between coherence and constraint, a tension that is better managed by the act of collective questioning of the assumptions on which such coherences and constraints are built. This is where critical productive coreflection comes into play because Cressey et al. (2006) caution the need to identify occasions when groupthink could occur, which will be kept in mind when analyzing teacher talk. Janis (1972) has denounced the ongoing dangers of groupthink in any group process, especially coreflection. Janis (1972) elucidated how groupthink implies some group members may subordinate their thinking to the group's common point of view. In such a case, a suspension of critical thinking occurs which may then lead to inappropriate decisions that plateau the development of the lesson. According to King and Kitchener (1994), it is only when the practitioner engages in critical reflection that they can evaluate two dimensions: first, the degree of un-challenge-ability, and second, the degree of justifiability of their practice. If the practitioner's actions are "based on the evaluation and integration of existing data and theory into a solution about a problem at hand," then their solution "can be rationally defended as most plausible or reasonable, taking into account the sets of conditions under which the problem is being studied" (p. 8). In his historicization of reflection, Akbari (2007) casts a skeptical tone as he called out the uncritical embrace of reflective practice in teacher education. Akbari (2007) mainly focuses on language teachers, but his criticism applies to mathematics teacher education programs as well. As Fendler (2003) asserted, "some reflective practices may simply be exercises in reconfirming, justifying, or rationalizing preconceived ideas" (p. 16). For instance, Loughran (2002) has noted that "rationalization may masquerade as reflection" (p. 35) and serve as a source for normalization (A. Breuleux, personal communication, September, 2019).

Some teacher education initiatives are based on three problematic assumptions. The first assumes that teachers are not reflective, which Akbari (2007) argues is a misconception. For Akbari (2007), it is not possible to imagine that humans engage in cognitive level interactions *without* reflection. As Zeichner and Liston (1996) pointed out, every teacher engages in some level of reflection, "there is no such thing as an unreflective teacher" (p. 207). However, Fendler (2003) questions why reflection is being promoted if it is already inherent in teaching. This highlights the need for making teachers' reflection visible, which can be achieved through video recordings of classroom practices and collective reflection. The second assumption is that academic models can teach reflective practices, but as Akbari (2007) suggests, this may not be the most effective approach. To operationalize reflection and coreflection, I have compiled a list of definitions and models in Table 1, building on Nguyen et al.'s (2014) work, to encourage wider discussion within the learning sciences community.

Table 1

Select Definitions of Reflection

Author(s)	Year	Conceptualization, Definition, Process, Structure, or Model of Reflection
Dewey	1933	"Active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusion to which it tends" (p. 84).
Borton	1970	Three questions in sequence: 1) What? 2) So What? 3) Now What?
Schön	1983/ 1987	Questioning assumptions structures of knowing-in-action and thinking critically about that thinking that got us to fix this opportunity
Boyd & Fales	1983	Process of internally examining and exploring an issue of concern, triggered by an experience, and which creates and clarifies meaning in terms of self, and which results in a changed conceptual perspective
Kolb	1984	Reflection is conceptualised as one stage and pole of the four-stage cycle of Kolb's experiential learning; 1) Concrete experience: Describe what was seen/felt/thoughts. 2) Reflective observation: What worked? What failed? Why did the situation arise? 3) Abstract conceptualisation: How can I improve? Consult colleagues and literature. 4) Active experimentation: Theories back into your practice and try out the new
Boud et al.	1985	Generic term for those intellectual and affective activities in which individuals engage to explore their experiences in order to lead to a new understanding and appreciation
Shulman	1987	Reviewing, reconstructing, reenacting and critically analyzing one's own and the class's performance, and grounding explanations in evidence
Gibbs	1988	Six stages: 1) Description of the experience. 2) Understand feelings and thoughts. 3) Evaluation of the experience. 4) Make sense of the situation by analyzing it. 5) Conclude by determining what you learned and what you could have done differently. 6) Action plan for how you would deal with similar situations in the future.
Brookfield	1990	Process comprising three interrelated phases: 1. identifying the assumptions that underlie our thoughts and actions; 2. scrutinising the accuracy and validity of these in terms of how they connect to, or are discrepant with, our experience of reality; 3. reconstituting these assumptions to make them more inclusive and integrative.
Mezirow	1998	Process of critically assessing the content, process, or premise(s) of our efforts to interpret and give meaning to an experience. Premise reflection involves us becoming aware of why we perceive, think, feel or act as we do and of the reasons for and consequences of our possible habits
Atkins & Murphy	1993	1. Awareness of uncomfortable feelings and thoughts; 2. Critical analysis of feelings and knowledge; 3. New perspective awareness of the situation, action or experience.
Smith & Hatton	1993	Deliberate thinking about action with a view to its improvement [Mainly solo-reflection codes]
Moon	1999/ 2013	Form of mental processing with a purpose and/or anticipated outcome that is applied to relatively complex or unstructured ideas for which there is not an obvious solution
Kember et al.	2000	Reflection and critical reflection are viewed as two levels on a four-scale continuum of reflective thinking
Boud et al.	2006	Triangular representation; simplest model of reflection. The cyclical framework starts with the experience, moves on to the reflection and then onto learning. While it focuses on the essential belief that reflective thinking leads to further learning, it does not provide a guide to what the reflective process should consist of or how the learning can be translated back into the experience.

Author(s)	Year	Conceptualization, Definition, Process, Structure, or Model of Reflection
Kember et al.	2008	Operates through a careful r'e-examination and evaluation of experience', 'beliefs and knowledge' and 'leads to new perspectives'; critical reflection, the highest level of reflection, 'involving perspective transformation,' 'necessitates a change to deep-seated, and often unconscious, beliefs and leads to new belief structures'
Johns	2009	a) aesthetics, art of what is being done; b) self-awareness; c) ethics, moral knowledge; d) empirics, scientific knowledge; e) reflexivity, connect to past experiences
Mann et al.	2009	Purposeful critical analysis of knowledge and experience, in order to achieve deeper meaning and understanding
Sandars	2009	Metacognitive process that occurs before, during and after situations with the purpose of developing greater understanding of both the self and the situation so that future encounters with the situation are informed from previous encounters
Jordi	2011	"The rational analytical process through which human beings extract knowledge from their experience" (p. 181).
Ono et al.	2013	"Reflection, following the observation of a lesson, is an intellectual activity undertaken in a group setting by means of discussion among participants and observers to explore ways of improving the quality of future student learning, with particular reference to the design of the lesson, the materials used, and the mode of delivery" (p. 51).
The University of Edinburgh	2018	CARL model of reflection is adapted from a job interview technique: Context of past experiences (C), Actions that were taken (A), Results that occurred (R), Lessons that were learned (L).

Source: Adapted from Nguyen et al. (2014)

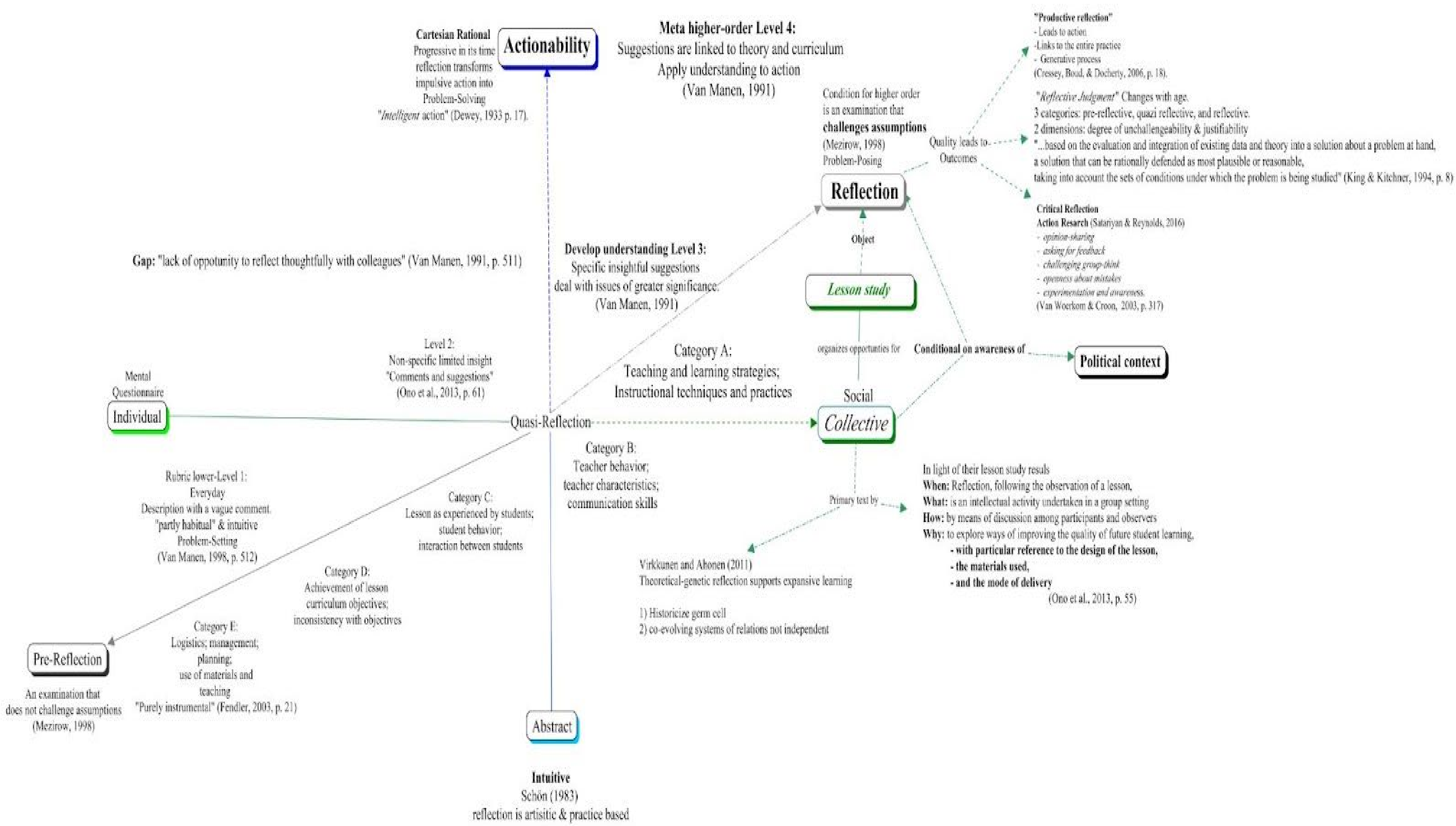
In general as exhibited by Table 1 reflection is advocated as a means to improve the lesson enactment phase (Ono et al., 2013). Jarvis (1992) stated that reflective practice is not just about thoughtful consideration, but rather a "form of practice" aiming to identify professional performance issues in order to create potential learning opportunities for practitioners (p. 174). Coreflection appeared relatively easy to operationalize at first glance; yet many researchers have grappled with defining the concepts of reflection and its collective form. As Herring (2004) argued on the "continuum of operationalizability," reflection fell on the least operationable end of the spectrum of human activities because it was an internal abstract process and was not obviously related to code categories (p. 355). Rantatalo and Karp (2016) have argued that, although "reflection has been viewed as an individual process, increased attention has been given to how reflective processes are socially anchored" (p. 708). This led us to conceptualize reflection with at least three dimensions: degree of collectivity, degree of actionability, and degree of criticality.

On page 53, Figure 1 visually presents reflection as a hypercube 3D inspired by the four historical roots identified by Akabrai (2007), Findler (2003), and Ono et al. (2013). The dimensions are considered on a spectrum: solo vs. collective (independent-x), abstractness vs. actionability (dependent-y), and pre-reflective vs. criticality (correlated-z). The independent variable on the horizontal-x axis represents solo-reflection at the negative end (far left) in contrast to coreflection at the positive end (far right), assuming that coreflection provides more opportunities for concrete change in practice. On the dependent vertical-y axis, the degree of actionability ranges from abstract reflection that leads to inertia at the bottom negative end to actionable reflection at the top positive end assuming a positive correlation between reflection and actionability when done in a group. Lastly, on the diagonal-z axis, the degree of criticality ranged from pre-reflective at the negative end (least critical, mere description of events from one's own perspective) to quasi-reflective in the middle to reflection at the positive end (constructive productive critical). For King and Kitchener (1994), when the practitioner is not thinking, then they are in the pre-reflective phase. If they reflect but not critically, they are just quasi-reflective.

This pattern moves from worst-case scenario (perfect storm: solo abstract non-critical reflection) to best-case scenario (smooth sailing: collective concrete critical reflection). This 3D view is limited, because according to Akbari (2007) we must merge "a moral dimension" (p. 197). Jay and Johnson (2002) argued that every type of reflection has to "take into consideration the social, moral, and political aspects of teaching" (p. 75). Yet, Figure 1 does not showcase the socio-politico-moral spectrums nor that of the emotional dimension. Regardless of how many dimensions are included in the concept of reflection, it serves several purposes. Perhaps its many definitions are due to its versatility as a concept, or vice versa.

Figure 1

Three Dimensional Visualization of Reflection



Fleck and Fitzpatrick (2010) proposed that when reflection was linked with learning from experience, its associated actions and behaviors were often organized into different “levels” of reflection (p. 217). These levels were utilized to recognize and evaluate reflection (Fleck & Fitzpatrick, 2010). The criticality dimension is operationalized using the criteria established by van Woerkom and Croon (2003), which include aspects such as “opinion-sharing, feedback seeking, challenging group-think, openness about mistakes, experimentation, and career awareness” (p. 317). The degree of criticality is dependent on discussion of underlying assumptions and misconceptions (both the practitioners' and students'). Based on my observations and the outcomes of this cursory literature review, it is evident that the relationships between variables are nonlinear and complex. Nonetheless, coreflection is a critical component for the development of classroom lessons. The literature on discourse analysis suggests that critical incidents may influence the spectrum of critical thinking during coreflection. However, due to time limitations, the multiplex nature of reflection cannot be fully discussed in this report. To define reflection comprehensively, it is necessary to consider at least six dimensions. For instance, Fairclough (2013) and Skovsmose (1994) recommended that critical discourse analysis must identify taken-for-granted concepts, while Cochran-Smith and Lytle (1993) suggested examining the relationships among practitioners at each grade level, their students, parents, and the provincial mathematics program. Despite Hatton and Smith's (1995) findings that critical coreflection is “ill-defined” and “loosely applied,” it can be viewed as an ongoing process rather than a fixed event (p. 33). Reflective lesson study is a process that can sustain improvement in teaching and learning mathematics through lessons extracted from activities over time.

Overall, these articles explored the concept of coreflection and its implications for research, particularly in understanding how individuals engage in it. The importance of socially mediated artifacts in expanding individuals' learning was discussed, as well as how coreflection has been examined in various disciplines. However, the lack of a standardized understanding of reflection among research partners affects the conditions required to create a useful model of coreflection that can significantly improve efforts to facilitate lesson study meetings. Multiple theoretical conceptualizations were discussed, highlighting their potential benefits and limitations (see, Moon, 1999; Boud et al., 1985). Moreover, the reviewed studies showcase how they tackled concerns related to the definitions of reflection and coreflection, their differentiation and integration, and their operationalization. This section also highlighted the need for a working sense of coreflection, which could be used to design, implement, and analyze data in research-practice partnerships for reflective lesson study. Finally, it is necessary to focus on the practical implications of coreflection to increase the success rate of lesson study procedures. This requires exploring how to incorporate the concept of coreflection into instructional design, curriculum development, teacher professional development, and assessment practices. Only then can the potential of lesson study be fully realized in improving the quality of math lessons.

Centipede’s Story: Gap

> *On a cold afternoon in 2014, my insightful neighbor advised me not to overthink my honor's thesis, lest I become like the centipede who, when asked how it walked with its thousand legs, was unable to move as it reflected on its movement (Salomon, personal communication, 2014).*
> *This oral exchange was paraphrased and is not verified for accuracy.*

In this subsection I point to a call for more empirical data using lesson study video-mediated coreflection talk to suggest directions for future improvements in teaching and learning mathematics. According to Gustafson and Bennett (2002), there is a scarcity of empirical research on reflection (Stamper, 1996). However, they note that a "substantial body of literature," such as Clift et al. (1990), does not provide conclusive evidence for the benefits of reflection (Gustafson & Bennett, 2002, p. 20). Gustafson and Bennett concluded there is room for improvement in the practice of reflection and identified ten variables that can affect its process, grouped into three categories: Reflection Task (e.g., nature of prompts, feedback quality, consequences, & report format); Learner Characteristics (e.g., experiences, skills, breadth of technological pedagogical knowledge (TPCK) (Mishra & Koehler, 2006), motivation, mental preparation, & degree of security); and Environmental Characteristics (e.g., nature of physical & interpersonal context).

While the nature of prompts and quality of feedback are important factors for improving reflective practices, the teacher's technological knowledge is also crucial for improving the learning activity. However, since we did not evaluate how teachers handled the projector and navigated the website to access the activity prompts, we do not have an actual measure of their technological knowledge. Articles on Lesson Study claim they can improve the lesson, therefore, teacher practice and student learning, however, as Hiebert et al. (2002) noted, "archived research knowledge has had little effect on the improvement of practice in the average classroom " (p. 3). Kelly and Cherkowski (2015) highlighted the importance of teachers' experiences in understanding how professional development can support professional learning. They found that teachers' descriptions provided insights into "the significance of collaboration, collegial relationships, and shifting mindsets about the work of teaching" (p. 1). In this study coreflection consists of a group of teachers, consultants and researchers discussing a math activity that was collectively designed by the elementary teachers' group following with lesson study procedures that were aligned with the change laboratory, a "formative intervention," that provides opportunities for active participation (Virkkunen & Newnham, 2013, p. xvi). A change lab is a "living toolkit" ; it can be argued that continuous lesson study can become one too (Virkkunen & Newnham, 2013, p. xvii). However, the literature related to qualitative analysis of coreflection is sparse, and data requires a custom-mix lens. While coreflection in the context of lesson study initiatives are gaining traction especially in the elementary sector, their optimal form for effectiveness is yet to be determined. Reflective practice has a long history of both direct and indirect impact on student learning outcomes (Takahashi, 2014).

Video-mediated coreflection does not necessarily produce significant results and sometimes prevents outcomes that could foster harmful outcomes, for example, groupthink and legitimizing (reifying; materializing) activities that were not vetted. We investigated the relationship between collective reflection' talk and its ability to practically improve future implementation of a learning lesson. While the many researchers provide sound scholarship, they do not, collectively, answer the question of how we can achieve lesson improvement via the implementation of coreflection. They do provide context for further study, but the idea of taking the paradigm to apply is left unanswered. Therefore, despite the amount of literature written on coreflection, there remains much unknown. For example, the literature reviewed above promotes the benefits of coreflection, there is little evidence supporting a direct correlation between this intervention and the improvement of a lesson. In fact, Hiebert et al. (2002) argued that "the teaching profession needs a knowledge base that grows and improves" to steadily improve the classroom, but research knowledge has had "little effect on the improvement of practice in the average classroom" (p. 3). Not many articles interpret data of a specific lesson with three different groups using videos from four classrooms. Reflecting on the literature, one wonders whether it is worth engaging in coreflection meetings. Akbari (2007) wondered if there is a place for the practitioner's voice in academic models of reflection to enable their empowerment. Assuming programs espouse the central premise that reflective practices assist educators in improving their practice. Hopefully in time, our data will provide evidence towards making such a claim.

The studies reviewed above differ in the specific contexts in which reflection was captured, such as teacher change, higher education, pre-service programs, nursing education, and lesson improvement. Marcos et al. (2011) noted a "lack of agreement about how to conduct reflection, as well as a wide variety of types of reflection," this persists a decade later (p. 21). Nonetheless, video-mediated coreflection is generally accepted as a great research tool in lesson study. However, few studies have assessed reflection. Gustafson and Bennett (2002) point out that the lack of studies further limits "our understanding of how to tap into this powerful strategy" (p. 3). Sumison and Fleet (1996) were unable to assess reflection, contending that the concept is so complex that they were, "unable to find a simple means of identifying reflection suitable for use" with their large group of pre-service teachers (p. 123).

Rogers (2001) explored the various forms of the concept of reflection in higher education and noted the key role of reflection in promoting deeper learning. Rogers argued that "despite the widespread adoption of reflective practices across many fields of study, a critical analysis of the concept of reflection and its application within higher education has been lacking" (p. 37). Studies that explore the nexus of in-service math teachers, video records from different classrooms, and lesson study within the context of a research-practice partnership in Québec are rare. The literature on lesson study primarily discusses its implementation in various subjects, the impact it has on teacher professional learning and development, and the methods used to analyze and report on the process. Of the 28 articles reviewed, 24 included the topic of video in their titles, while only five discussed coreflection meetings that focused on a specific lesson. Only one article by Vrikki et al. (2017) specifically addressed teacher learning through a collaborative approach, but it focused on the benefits for teachers rather than on the impact of the approach on student learning outcomes. While much of the literature emphasizes the benefits of collaboration, we acknowledge that there may be challenges associated with implementing collaborative approaches, including finding time for collaboration, balancing individual and group goals, and navigating interpersonal dynamics. Some articles explore how to effectively adapt and implement lesson study in different educational contexts. Although much of the literature on teacher collaboration is focused on formal, structured approaches, there may be value in exploring informal collaboration among teachers as well. Informal collaboration may include informal conversations or sharing of resources among colleagues and may be an important supplement to more structured approaches.

The literature on lesson study also suggests that video-mediated collective reflection can be an effective method for improving teaching practice and student learning, *depending* on its implementation and adaptation process. However, there is a need for an integrative conceptualization of coreflection, and as Hymes (2005) argued more descriptive analyses are necessary to understand video data from a “variety of communities,” of practice (p. 4). To address this gap, this study will analyze three coreflection meetings using a pragmatic grounded theory approach to contribute evidence from video-mediated coreflection in a research-practice partnership. This study will add to the descriptive knowledge on reflection in collective settings. This argument is based on the premise that to operationalize coreflection, we need more analysis that begins with descriptive participant-observation of diverse communities of practice.

Lastly, it is important to note that the procedures for analyzing the talk that occurs during collective reflection meetings are not clearly defined, particularly when comparing the range of reactions across different discussion groups. With the exception of Ono et al. (2013), who focused on creating an instrument to decipher the level of reflectivity in lesson study to improve the lesson, the literature mainly frames reflection as necessary and constructs the notion of teacher reflection as essential to learning. The majority of the literature reviewed frames reflection as necessary and constructs the notion of teacher reflection as essential to increasing student learning outcomes, but I am mindful to avoid dichotomizing the teacher from the student. A teacher can be both a learner and a teacher, as illustrated by Melody's self-reports; she often has to think about the math activity at the same time as the students. Video-mediated collective reflection may be even more beneficial than other forms of reflection, and there have been advances in harnessing co reflection by learning scientists (see Goldman et al., 2007). However, more research is necessary to understand and utilize this method effectively.

To promote teacher development, this study adopts Dewey's (1938/1997) view of researchers as interventionists. According to Dewey, research must create conditions that illuminate what teachers may have missed during an activity. Although the review of literature is not exhaustive, it indicates the need for further research on how to strengthen lesson study procedures to bring about the lesson rather than a mere activity. While Kolb (1984) argued that reflection is a process rather than an outcome of learning, this paper seeks to analyze the conversations teachers had about a single learning activity as an outcome in itself. Similar to Durkheim's (1912/1995) approach to defining religion as "to be sought from reality itself" (Field, 1995, p. 22), the conversations teachers have during reflection can provide evidence into what, why and how coreflection is necessary.

As the research reviewed above indicates, the process of engaging in reflection often begins with recognizing an educational dilemma, responding to it by considering the unique qualities of the situation in relation to other similar situations, and continuously frame and reframe the dilemma. Furthermore, reflection involves testing the implications of various solutions and closely examining the intended and "unintended consequences" of an implemented solution to determine whether they are desirable (Ross, 1990, p. 22). To accomplish this, coreflection is necessary (Høyrup & Elkjær, 2006). By this point it is not a secret that reflection is one of the most ubiquitous concepts in the field of education, yet there remain unsettled questions regarding the methodology for analyzing reflection, particularly in the context of collective reflection among elementary and secondary mathematics teachers during a learning activity. These issues may be associated with a lack of clarity regarding the concept of reflection and the nature of the relationship between reflection and lessons. Dewey (1933) has suggested that reflection hinges on action, while Schön (1983) has suggested that reflection occurs ad hoc. Once these views are established as a basis of reflection, then the collective is added to the mix and it hinges on the collective's reflective abilities to become critical of learning activities when it comes to mathematics. The edited video and transcripts from class discussions helped focus teachers' attention, with the underlying hope that we could pick up *serious* topics. Often the purpose of the act of communication is to figure out: "*Who* (says) *What* (to) *Whom* (in) *What* Channel (with) *What Effect*?" (Lasswell, 1948, p. 216).

Lasswell (1948) codes describe the "receiver" in the communication process as the group who is the intended target of the message and they may respond in various ways: accept or reject the message, be influenced by it, or modify their beliefs or actions based on it. Lasswell (1948) discussed the role of socio-cultural factors that may impact the receiver's interpretation of the message. Lasswell (1948) determined that effective communication requires an understanding of the receiver's needs, motivations, and expectations, and the ability to tailor messages accordingly. Although these types of detailed code-questions are not the focus of this study, they prove to be quite illuminating for Vignette 7.

In summary, this study aims to contribute to the growing body of evidence suggesting that activities reflected upon through lesson study with examples from live-classroom representations and minimally prompted video-mediated coreflections in the context of a RPP are promising paths to uncover challenges in learning a specific topic. While previous studies have analyzed the educational properties of coreflection and the possible impact of situational problem wording, this study will describe three video-mediated coreflections in the context of lesson study to establish where diverges occurred. We used specific research questions to describe the unfolding.

Research Questions: What, So What, Now What?

This subsection outlines the main questions that guide the focus of this dissertation. The Rashomon effect and memos from fieldwork indicate there are noticeable divergences in perspectives between secondary and elementary mathematics teachers during coreflection mediated by footage from the Donut activity. Through my experience with engaging in video-mediated lesson study and studying its underlying theories, I aim to better understand how we can harness such meetings. Inspired by Borton (1970), I consider three research questions:

I. **What?** At the heart of this thesis lies a pragmatic question; What did the educational partners talk about during collective reflection meetings mediated via video segments of the same learning activity? Conversely, what did they *not* talk about? This question has three distinct sub-questions to describe the patterns of conversations:

   A. What did the Secondary Math Teachers Collectively Reflect About?
   B. What did the Elementary Math Teachers Collectively Reflect About?
   C. What did the Mixed-Group of Math Teachers Collectively Reflect About?

II. **So What** can we make of the educators' comments made during the collective reflection meetings? So what challenges were unearthed?

III. **Now What** do we do to actually improve the lesson to improve future implementation? And **now what** can we improve in researchers' toolkit for future iterations of lesson study meetings?

By addressing these questions, I hope to shed light on the factors that contribute to the divergent perspectives of secondary and elementary mathematics teachers during coreflection-on-action The first, *What* question evokes a logical rigorous analytic mode (interested in rigorous descriptive sensing, each meeting). Borton (1970) conceded that it is challenging to foster change when too analytical, therefore, he proposed the so what and the now what questions as "contemplative mode" (p. 89). *So What* prompts us to become interested in finding patterns (making sense). While the *Now What* question prompts us towards making decisions about the lesson plan for future action. These two questions shift us into a speculative mode that could lead us to a generative state. In either case, the third question supports a critical examination of the Donut activity. In sum, adopting a critical stance towards the lesson from the start sets the tone for interactions among educational partners rather than focusing on teachers. While the sub-questions focus on displaying, explaining, and interpreting outcomes, they are key in presenting the outcomes effectively. The last two questions remind educators to check the connections between the activity's learning objectives and the codesigned objectives. A better understanding of these coreflection meetings can lead to a better understanding of how it can be facilitated. Which in turn may allow us to zoom in on the appropriateness of the Activity.

Methods

In the following five sections, I will describe the context of the RPP from which data for this study has been drawn. Followed by a discussion of the two mediational tools, namely the video and the Donut activity. Next, I will unpack how I approached the pragmatic grounded analysis, followed by a discussion of the Data Categorization procedures.

Research Context

RA1: Yeah. the à-côtés, like, the background context.

The research context section provides a brief overview of the project and is further divided into two subsections describing our partners and data sources. In 2013, educators at the Sunny Side School Board identified a performance dip between elementary and secondary cycles, with students excelling on ministry exams in elementary school but struggling in cycle 1 of secondary school. To address this challenge, the Creating, Collaborating, and Computing in Mathematics (CCCM) project was established as a research-practice partnership, bringing together teachers, consultants, and researchers to enhance math education through sustained sharing, inquiry activities, and reflection (Heo & Breuleux, 2015). The CCCM aimed to collectively address teacher practices and improve student outcomes by examining past archives on student performance and fostering a professional learning network. To achieve this goal, the partners collaboratively identified the challenges that students face during this critical transition period in their education.

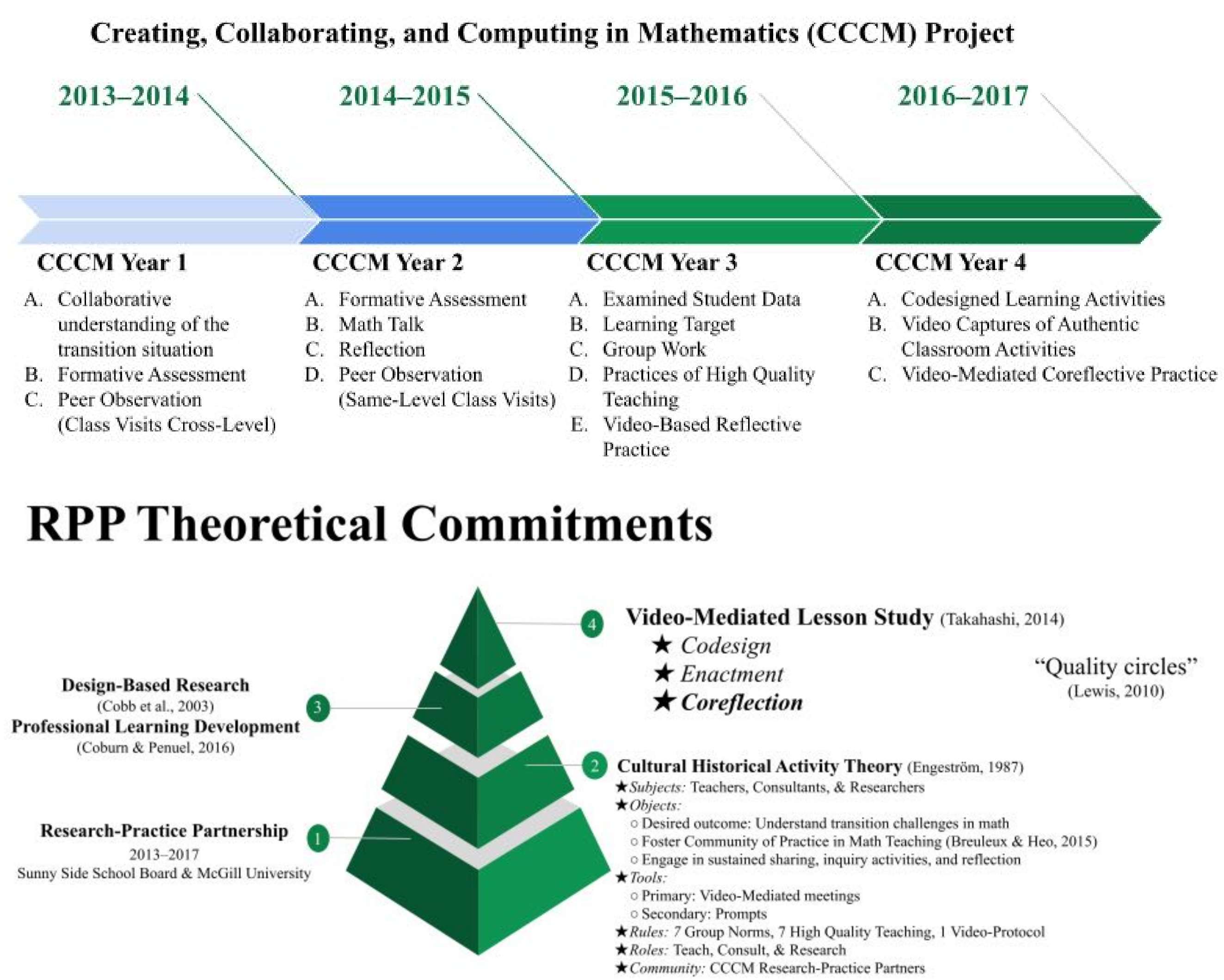
The context of the CCCM initiative is presented on page 67 in Figure 2, which consists of two panels. The first panel provides an overview of the activities that took place in the multi-year Research-Practice Partnership (RPP) between the Sunny Side School Board and McGill University (2013–2017). The first two years of the RPP, CCCM partners engaged in activities designed to build trust, such as visiting each other's classes and exchanging technological tools, leading up to the cycles of lesson study. Indeed, reciprocal moments of self-disclosure are useful for lesson study procedures to foster a community with the capacity to address challenges in mathematics transition (Bormann & Bormann, 1980).

The second panel, the green pyramid represents the research team's theoretical commitments. The base of the pyramid depicts how the team obtained a partnership development grant to engage in the RPP. The second layer of the pyramid signals that the research team adopted a socio-cultural perspective (e.g., Engeström's, 1987, 1999; Engeström & Sannino, 2010) focusing on integral activity systems, mediating tools, boundary crossing, and expansive learning (Breuleux et al, 2017). Co-design and reflective conversations are viewed as complex, dynamic mediated activity systems, with their cultural-historical context and evolving objects and tools. This perspective also provides ways of understanding the complex interactions –between participants, community, and cultural tools– and the processes of *expansive learning* taking place in a PLC and in a mathematics classroom. Collaborative design and reflection are boundary activities between teaching and inquiry/research, mediated by rich artifacts from practice (mostly–but not limited to–video records of classroom events), that are experienced mostly through discourse, and that can be scaffolded by various facilitation processes (Breuleux, 2017). The CCCM community used video-mediated representations of the activity as their primary tool, scaffolded by prompts, seven group norms (e.g. withhold all judgement), seven high-quality teaching practices, and one video-protocol. The roles of teaching, consulting, and research were central to this community, blending an "action learning" approach involving "a reflective process" to unfreeze "long-held assumptions" by subjecting them "to intense scrutiny through participative experimentation and then (temporarily) refrozen on a new level" (Dilworth, 1996, p. 17). The third part of the pyramid indicates that Design-Based Research (Cobb et al., 2003) and Professional Learning Development (Coburn & Penuel, 2016) were used to support teacher learning. Only after these structuring devices were established did the CCCM community embark on Lesson Study procedures (Takahashi, 2014), which involved codesign, enactment, and coreflection of math activities to promote sustained sharing and inquiry among teachers represented at the tip of the pyramid (see Figure 2). The CCCM study focused on fostering a sense of community, and a discussion of the differences between cycle teachers in their beliefs and practices (see Heo & Breuleux, 2015). The context for the selected data is best articulated by our technology consultant, referred to as the RÉCIT animator, who was trying to facilitate the flow of conversation by describing to the secondary math teachers the circumstances that led the elementary teachers to codesign the Donut activity. This excerpt recaps the learning target:

> RÉCIT: And this **series of [Donut & Let's Paint activities] came about when we sat down to codesign**…, I wasn't part of yours [referring to the codesign session that five secondary teachers engaged in] because I was away, when we sat down to co-planning. I just started with: "Okay, well what are some of the challenges they face?" **And a lot of them, they said the same thing, whether they're dealing with an application problem or a situational problem. And they seem to say the situational problems they're doing well in, it's the application problems that they're starting to see it. And they felt that it's because kids don't know how to find that relevant information**. So then the conversation kind of snowballed into: "Alright well maybe that's something that we need to focus on," and then they chose, you know, after talking about some of the different things we've done. **They said: "Oh, an open-ended task was an interesting approach."** So then they chose that task to lead them into... and **then they developed a series of learning targets. So what you're seeing here is one of the learning targets** [25:00] (RÉCIT animator during the secondary collective reflection Line 203 Row 302 in Sheets).

Figure 2

Overview of CCCM 2013–2017



The research team proposed a lesson study protocol that relied on capturing lessons on video and using prompts. By focusing on the activity rather than the teacher, the protocol aimed to minimize personal sentiments and facilitate more effective coreflection. By engaging in collective lesson design and collective reflective practice, the partnership was able to identify specific challenges faced by elementary students. Table 2 provides an overview of the particular activities that occurred in the fourth year of CCCM with how many partners were present at each meeting.

Table 2

Main Activities of the CCCM Project During Winter 2017

Participating Educators	6 Elementary math teacher (two new members joined in Fall 2016) 4 Secondary math teachers (two new members joined in Fall 2016) 1 member stopped participating before the end of the year. 5 Consultants (alternating at meetings + 1 ELA guest at the elementary collective reflection) 4 Researchers (+2 guests researchers at the mixed-group meeting)
Types of Activities	Four meetings plus two small group meetings per school level for codesign and coreflection

Note. Adapted from Heo & Breuleux (2015).

Research Partners

> *PI: ...I think it's a* ***very powerful method, and it's not easy to implement, because it requires all the time we built a sense of trust, and a climate of sharing and being open to each other's ideas****.*

In this subsection, I provide details on the years of teaching experience, but not on their personal experiences with math, as the focus is on the Donut activity. Although our research team employed various methods to recruit partners and collect data, I will not discuss these methods in this report. Table 3 provides an overview of the teacher partners' pseudonyms, years of experience teaching math (ranging from 4 to 23 years), and their responses to the end-of-year semi-structured interview question “How many years have you been teaching?”

Table 3

Summary of Years of Teaching Experiences

CCCM Partners 2017	Pseudonyms	Years of Experience
4 Secondary Math Teachers [6-22] years of experience teaching Cycle 1 [Sec 1–2]	Lydia Lin Ben Ellen	7 8 22 4
6 Elementary Math Teachers [8–23} years of experience teaching Cycle 3 [Grades 4-5-6]	Kate Kaci Melody Dona Kiera Sabrina	8 14 23 9 13 14
5 Consultants Referred to by their role:	RECIT, Interim Elem, ADES, Former Sec. Consult, & ELA guest [elementary meeting]	n/a
6 Researchers Referred to by their role:	PI, RA, RA1(nilou), RA2, & 2 ARIM [mixed meeting]	n/a

Data Sources

Lin: I have a question. Do they have to sign a release or something?
PI: Yes.

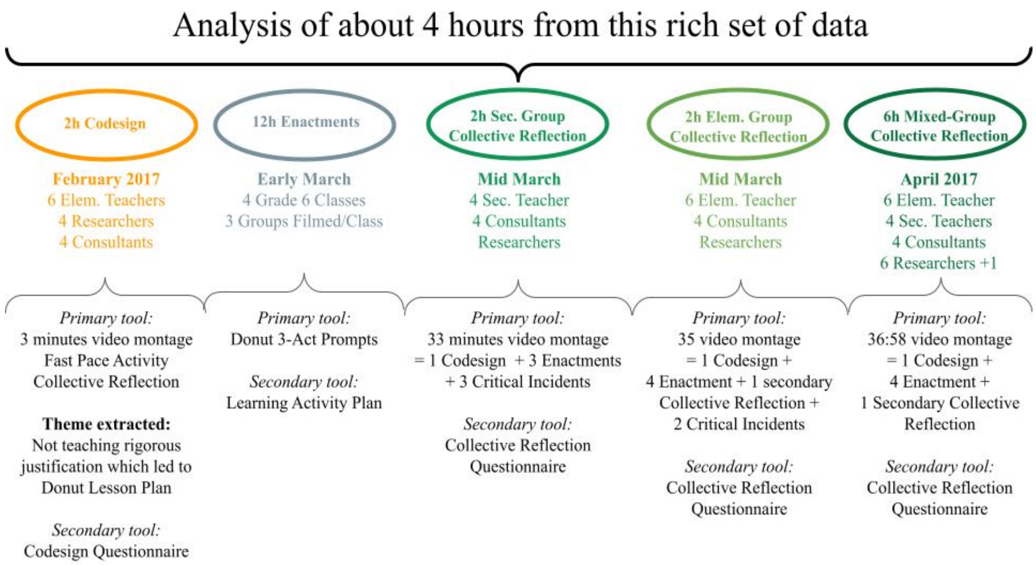
This subsection provides a brief overview of the data sources and collection methods used in the study. To capture various forms of practice representation, we employed several strategies, including two years of participant-observation (2015–2017), video recording, transcription, and analysis of teachers' codesign, implementation, and coreflective discussions. We also collected data from informal conversations, fieldnotes, and end-of-year interviews. The primary source of information for this study was video records from the last year of the CCCM project.

In this dissertation, I concentrate on a subset of data collected during the last cycle of lesson study in 2017, specifically video records of coreflections. CCCM members had eight face-to-face meetings, of which three from Winter 2017 are the focus of this study. I focused on the conversations that occurred during the enacted Donut activity, and coreflection meetings where classroom videos served as the primary “tool for fostering productive discussions about teaching and learning” (Borko et al., 2008, p. 417). At the beginning of each school year, educators gave their consent, and we obtained parental permission for classroom visits during which we recorded students' problem-solving activities. To accommodate parents who were uncomfortable with video recording, we grouped their children outside of the camera frame. Figure 3 provides an overview of the Lesson Study timeline (Feb–Apr 2017). Our data collection efforts included a 2-hour codesign session (for each of the elementary and secondary groups), during which we identified a lack of rigorous justification in teaching as a theme. We used a codesign questionnaire to create two lesson plans (this will be described in the Donut Lesson’s Inception section), and five teachers enacted the Donut Activity, of which we recorded four in early March. A school closure due to snow coincided with the recording date in the secondary sector, and we were unable to record any enactments from that group.

In March, we used multiple tools such as a reflection questionnaire, three critical incidents, and a 36-minute, 58-second video for coreflection. In April, the mixed-group briefly celebrated the year. We captured various conversations using Vialogues, an online video annotation platform that supports asynchronous discussion about classroom practices. The platform was well-liked, especially the time stamps on the Vialogues videos. Unfortunately, the website has shut down, resulting in the loss of valuable chat conversations and annotated videos. Overall, our data collection methods helped us obtain a diverse representation of teaching practices within the Donut activity.

Figure 3

Lesson Study Timeline: The Case of The Donut Activity



Typically, the meetings were scheduled early in the morning. The research team drove to the Sunny Side school board for a half-day meeting with four secondary math teachers, and two days later, we convened the six elementary mathematics teachers to discuss their experience implementing their codesigned Donut activity. A month later, we spent time in a mixed-group meeting to celebrate the milestone of having engaged in one full cycle of lesson study. Unlike typical lesson studies, we used video recordings instead of in-person visits during the enactment. Next, I describe how video was used as our main mediational tool.

Video as a Mediation Tool

> *PI: Okay, so we have the video sequence. Following the conversation we had last time, we've done quite a bit of editing. So we took [5:00] snapshots from snippets from the design meeting. Do we need to...? Is it self…?*
> *RA1: Well, bear in mind that the codesign meeting was about two hours and each class is about one hour, so this is not a hundred percent accurate representation of everything that happened in the classrooms. So in that regard, please take that into consideration when you're assessing.*
> *RECIT: And They had al**so come up with a trajectory of action**... oh, which you've got.*

> *RA2: Right. So one thing that we've made is kind of a guide as you watch the video, which will... there's a couple of* ***questions, questions about the learning targets that you can identify and the target teaching practices. So I guess I can pass these out?*** *[Referring to the collective reflection questionnaire]*

This section of this dissertation is structured into four distinct subsections to provide a comprehensive approach to describing how video is used as a mediational tool for supporting professional development. The first subsection will briefly explain the lesson procedures employed in the study. The second subsection will describe the procedures used for coreflection in the secondary math group, while the third subsection will provide a similar description for the elementary math group. The fourth subsection will outline the procedures for mixed-group coreflection. A main tool by the research team was the video montage from classrooms and data from past ministry exams to “orchestrate” CCCM meetings (Borko et al., 2014, p. 259). Heyd-Metzuyanim et al. (2019) noted that this practice promotes the “orchestrating mathematical discussions” approach among teachers, drawing on video and exam data as prompts and resources (p. 273). By “orchestrate,” we mean to facilitate and guide productive and collaborative discussions among teachers. Engeström (2015) argued that collective-level “reorchestration” is necessary to achieve expansive development of learning (p. 35). Theoretical perspectives on various aspects of video-mediated collective reflection abound, with Miller and Zhou (2007) recognizing that “video cases are perhaps the ultimate in vivid second-hand experience,” making video a “persuasive” mediational tool (p. 322).

There are several existing models for teacher learning with video. Coles (2014) investigated two primary questions: how do mathematics teachers learn from using video, and what is the role of the didactician in this process? Coles summarized four models, including the Open University technique by Jaworski (1990), Learning to Notice (van Es & Sherin, 2002), Videos as Tools (Maher, 2008), and Lesson Analysis (Santagata & Angelici, 2010), which all utilize video as a tool for teacher learning. However, Coles noted that it can be challenging for teachers to avoid making judgments and evaluations when discussing video, which may impede the learning process. Coles also highlighted a lack of understanding regarding the role of didactician in supporting teacher learning. However, empirical data suggests that didactician can model a specific type of attention when viewing classroom videos, which Coles referred to as “heightened listening” (p. 277). This approach involves focusing simultaneously on the content of teachers' contributions and the type of comment being made to establish discussion norms and support the development of new ways of seeing and acting in the classroom. For example, one didactician wondered, “but how do we bring teachers from the abstract concept of rigor to the concrete teaching of rigor in mathematics classrooms?” (C. Corriveau, personal communication, June, 2017).

This research is based on videos of collective reflection of 20 educators' conversations within the “context of the [Donut] box,” as stated by the primary investigator during a secondary meeting. Video-mediated collective reflection is one of the key methodological approaches for professional learning communities and is therefore given prominence in this dissertation. The videos served not as data, but as rich “cues designed to stimulate critical reflection” (Tobin & Hsueh, 2007, p. 77). Rouch (1995) viewed video as “a tool for feedback” (Tobin & Hsueh, 2007, p. 78), while Asch and Asch (1995) saw it as a “mnemonic device” (p. 348). For others, video clips evoke the concept of “mirror data” (Virkkunen & Newnham, 2013, p. 18). The analysis recognized that video captures are not data; rather, they are “a resource for data construction, an information source containing potential data out of which actual data must be defined and searched” (Erikson, 2006, p. 178).

In this qualitative study, a total of 4 hours, 16 minutes, 19 seconds out of the 24 hours of lesson study on the Donut Activity recorded in the last year of the CCCM RPP was analyzed. After consultation with the dissertation committee, nine segments were selected for fine-grain analysis, which involved examining discussions during and after watching the edited video representation of the Donut activity taught in four Grade 5 and 6 mathematics classrooms at three different schools within the same board. Each classroom had two cameras and two microphones, which allowed us to capture the interactions between the teacher and the students from different angles.

The video sources were captured by four researchers using the following video and audio collection equipment: Canon VIXIA HF R42 camcorder, Canon VIXIA HF R62 with two external SM57 microphones, two tripods, extension cords, and other tools to capture the meetings and classrooms with as much integrity as possible. The recording devices included a pen and notebook, MacBook Pro 2015 with 2.5GHz Intel Core i7-4870HQ PC CPU, 16GB DDR3 SDRAM 1600MHz PC Memory, 500GB storage, AMD Radeon R9 M370X Graphics abilities. In contrast to Mehan's (1979) minimally edited video, I used Final Cut Pro 2017 to semi-edit selected representations, providing educators with 'an information source within which data could be identified' (Goldman et al., 2007, p. 18). The edited video includes codesign, implementation, and coreflective conversations intended to invite constructive criticism of the Donut activity. The records for the CCCM project are saved on several Seagate Backup Plus Desktop Drives. Detailed procedures for the lesson study are provided next.

Lesson Study Procedures

PI: This conversation happened in the context of the [Donut] box.

This subsection discusses the procedures for lesson study, which are clarified separately for secondary and elementary teachers, followed by the mixed-group coreflection meeting. In this study, the approach to lesson study can be described as minimalist interventionist research. The research team remained mainly silent during the reflective conversations, with a few strategic questions in mind, while some teachers were more vocal than others, and the consultants were in a facilitative mode. The primary investigator encouraged an open interpretation of the concept of lesson study, viewing it as a social-constructivist framework that "creates opportunities for collective thinking to improve lessons" through three cycles (Breuleux, personal communication, July, 2021). At each collective reflection meeting, a sharing session lasting 20 minutes to an hour, depending on the meeting's duration, was followed by the viewing of the edited video from the Donut activity. Educators were encouraged to discuss the activity as represented in the edited video, and to use timestamps in the accompanying questionnaire when referencing specific events or thinking displayed in the video (Borko et al., 2008). The general procedure was explained by the primary investigator at the beginning of the elementary group meeting.

> PI: And **we've captured some of those moments on video, and now we can spend a bit of time this morning to look at what happened in the implementation of these activities in the classrooms. It's an opportunity to take a look at what students** do, to think about, were the learning targets actually achieved? What can be improved in that activity? And also **we're going to use a bit of video clips from the meeting last time, and the implementation in the classrooms. And one of the questions for us is, is this overall process useful to you? How can it be, maybe, done further in time and better? For example, the co-design sections, the capture of some of these conversations, hearing them back, looking at them again, trying to see what, how we can get better at that. So, so this morning I want to start with an open-ended question about anything that, in terms of thoughts you might have had after the meeting last time, and when you implemented these activities in your class.** What do you think happened? Any surprises? Any thoughts?

Additionally, I adopted an open book stance to sustain the collaborative aspect. The CHAT framework influenced the research team to adopt a more hands-on role, and the principal investigator (PI) took a semi-structured facilitative approach. The team and consultants had “lead meetings” to jointly set an agenda before meeting with the teachers, but there were still limitations to this distributed leadership approach. In some cases, teachers raised their hands to speak while the PI or a consultant moderated their turn, raising questions about the extent to which the group was truly in partnership. This highlights the need for researchers to have skills as moderators must be added to the ethnographic toolkit in education research.

In sum, we captured the educators' reactions as they drew on their math knowledge, experiences with students, pedagogical strategies, and mathematical content beyond what was captured in the video (Borko et al., 2008). For contentious scenes requiring closer examination, we prepared three critical incidents presented as skits of classroom discussions to provide opportunities for reflection on potential improvements in math teaching practices. In the following subsection, I will outline the procedures used to facilitate collective reflection among the secondary math group.

Secondary Math Group Collective Reflection

Four vignettes are drawn from the secondary math teachers' collective reflection. At the time, we had footage of three different teachers, each using the same Learning Activity Plan but with different approaches and varying outcomes. The footage was about 29 minutes long on that Monday morning in March 2017 (it was constantly evolving as I honed my editing skills). We read three critical incidents. It appears we adopted a content-focus approach. Due to various reasons, the fourth class was scheduled to be filmed a day after the secondary group's collective reflection meeting. Although the Donut activity was exogenous to the secondary math teachers, we spent 1 hour, 52 minutes, and 31 seconds discussing where the lesson fit inside the activity. Our focus remained mainly on the math concepts and vocabulary of the activity during this half-day, formally scheduled meeting.

Elementary Math Group Collective Reflection

Three vignettes were selected from the coreflection of elementary math teachers, along with an ELA guest who participated in the discussion two days after the secondary reflection meeting. With a pedagogical focus, the teachers had previously codesigned the Donut activity and were well-prepared to analyze its outcomes and implications. The video footage showed four classrooms with three groups of three to five students each, resulting in a 35-minute edited video. Two critical incidents were read and discussed during the half-day meeting, which lasted about 2 hours and 2 minutes.

Mixed-Group Collective Reflection

Two vignettes were extracted from the April 2017 mixed-group coreflection. Although the meeting was scheduled for six hours, we only spent 50 minutes discussing the Donut activity. The final video was 36 minutes and 57 seconds long and included footage from every stage of the lesson study. During the meeting, we spent 21 minutes and 27 seconds reflecting on the video and photo representations of the Donut activity. We did not read any critical incidents. At the time, we were unaware that the project was evolving into a province-wide initiative, which would later become known as the School Success Plan. This meeting marked the end of our cycle of lesson study presented as one cohesive story rather than in disjointed short segments and we celebrated our achievements in completing the year. Next, I will describe the Donut activity.

Donut Activity: Chasing the "Lesson"

This section is divided into three distinct subsections to provide a description of the math learning activity being studied. The first subsection will detail the inception of the Donut activity, which was designed to promote rigorous justification by teaching students to ask relevant questions. This section will provide a discussion of the development of the activity, including the research that informed its design and the intended outcomes. The second subsection will provide a brief description of the original activity, outlining its components and how it was implemented in classrooms. Finally, the third subsection will focus on three critical incidents from classrooms in which the activity was implemented, providing a detailed analysis of each incident and its implications for the effectiveness of the Donut activity.

As Dreyfus (1999) noted, the emphasis on explanation and argument "is consistent with the continued importance of proof in mathematics" (p. 85). The lesson plan aims to provide a model for a rigorous explanation of a mathematical solution or proof. To understand coreflection, it is critical to examine its form in specific incidents. As stated above, the team drew inspiration from the lesson study model developed by Takahashi et al. (2006), which involves three interrelated phases: codesign, implementation/enactment, and collective reflection. We fostered opportunities for teachers to identify challenges during codesign and end-of-year interviews. As we will see, the data illustrates how collectively reflecting about an activity captured on video can create dissonance; the videos break the assumption of the untraceability of reflection.

Donut Lesson's Inception

> *Kaci: Well I find, I find too that the students aren't... always great at going to pick out that information. So how could they be good at writing a rigorous*

This subsection provides context for the Krispy Kreme Me activity (renamed the Donut activity). As explained earlier in the *Donut* and *Let's Paint!* activities were codesigned in response to challenges raised by six elementary teachers during the first half of the codesign meeting in February 2017. The Donut activity was brought to the attention of the elementary groups during the second half of the codesign meeting by the interim elementary consultant, as they reached a consensus that a lack of rigor was a problem and chose to address it through two activities (see Appendix A). These challenges were based on a springboard meditational two-minute montage of pictures of student work from the 2016 *Fast Pace* activity, overlaid with collective reflection audio from five elementary teachers, one interim Consultant, one RÉCIT animator, and three researchers:

Melody: and then in their journal, they have to say why it was/
Unknown: /Yeah/
Melody: /and how it was different from their own.
Sabrina: ….I think, what we are aiming for in our journals, though is to have them explain/
Unknown: /their thinking.
Sabrina: Yeah. Exactly, yeah.
Melody: And using the math vocabulary, not just/
Sabrina: right
Melody: I did that "thing" and then I/
Kate: / I calculated it/
Melody: I calculated "it" AND Kate: I calculated it. AND Melody: "It."
Kate: How did you do it? Well I calculated it.
Melody: And I found it.
Kiera: Yeah, I did an addition.
Melody: There are a lot of "its"…
Kate: Always an addition [Chuckles]
19. **Sabrina: Because we are noticing there is a weakness in their justification too, and the use of proper math vocabulary,** so we are kind of trying to incorporate that too with the needs of our students.
20. Kate: Which is really great/
21. Sabrina: /I think we are trying to jam too much in here. I dunno [Chuckles].
22. Kate: No, but that's really just to build on those skills, s**tarting in Cycle 2, and whenever, to look at those two of the six application questions for the Grade 6 exam that are alway a *rigorous* justification is required.**
23. Sabrina: Yes.
24. Kate: We don't necessarily work on that.

The segment above is what served as springboard for the 2017 codesign session. Specifically the lack of justification raised in lines 19 to 24 as a challenge by Sabrina and Kate in 2016 propelled Melody with 23 years of experience as a teacher, to express what resonated:

> I.22. Melody: In ***the video that kind of stuck out in my head...you were talking about that whole rigorous, rigorous explanation***.

A few minutes later the RÉCIT facilitator proposed that they integrate rigour as a learning target in their codesign lesson plan:

> I.49. RÉCIT: So maybe, maybe that's essentially what you're looking at, is teaching them, ***what does rigour look like in mathematics?***

At 32:45 minutes into the codesign session, after sharing their challenges and discussing what rigour looks like, the interim elementary consultant commanded the floor by twisting her laptop to the group to show the image of a giant box of donuts. Her move to introduce the Donut activity can be construed as a possible way to bring the conversation back to the codesign aspect of the meeting. Or it could be her solution to the lack of rigorous explanations; often a faux-pas of reverting into solution mode before diving into the whys of the problem (see Change Lab procedures to first understand root causes prior to proposing solutions). Either way, the timing and the à propos manner of the introduction of the activity indicates that the consultant planned on showing it at some point in the design session. Consciously or subconsciously enacting the mantra that a consultant's role is to bring activities. The naming of a Math PLD workshop can also be interpreted as a move to legitimize the source of the activity:

> I.172. Interim Elem. Consultant: I am gonna, I did this, this week at [name of Math PLD], like: "How many [donuts] are in this box?" Right? So just open-ended, but now what questions come to mind? Right? You get the kids to like, just look at it.

The interim elementary consultant had happened to be introduced to it at a professional workshop a few weeks prior to the elementary codesign meeting. She, in turn, presented the activity to the six elementary teachers gathered on a sunny day in February 2017 to determine which student learning challenge could be mediated through a codesigned lesson. Around the table, the reaction to the giant box of donuts was such that the educators got inspired to prompt students with an open-ended question. Some would qualify prompting with an open-ended question as a "dialogic" form of interaction (see Hennessy et al., 2016). Dialogic forms of engagements invite students to share their reasoning and thinking while hearing others' perspectives (Hennessy et. al, 2016).

In all of their excitement about the Donut activity that got them involved too, no one asked *why* this donut lesson. As an example, of how some of the practitioners got involved, some volunteered their *own questions*, "how many am I going to eat?" is but one of the many replies to the interim elementary consultant's presentation of the activity. The codesign meeting was also mediated by a prompt sheet with questions that were prepared by the research team with the aim to frame the teachers' discussion.

For example, the second question guided teachers to consider: "What makes this activity worth exploring in terms of teaching (i.e., pedagogical challenges) and/or learning mathematics (i.e., student understanding)?" Sabrina filled out the questionnaire for the group: "Developing analytical thinking about problem-solving → realising that it's important to be able to identify: 1) What do I know? b) What do I need to find? and c) what information provided might be irrelevant to solving a problem?

Throughout the codesign meeting, the teachers also had access to the seven practices of high-quality teaching adapted from the nineteen practices provided by the University of Michigan (n.d.). Figure 4 indicates that the elementary teachers chose three desired practices (2nd, 3rd, and 7th are underlined): focusing on instructional goals, <u>eliciting and responding to students' thinking</u>, <u>orienting students to each other's ideas</u>, positioning students competently, establishing and maintaining expectations for student participation, representing student thinking in key ideas, and <u>using a public record of student thinking</u>. Taken together these techniques aimed to promote high-leverage teaching practices (Davis et al., 2017).

Figure 4

Seven Practices of High Quality Teaching & Four Selected Practices

- [ ] **Teaching toward an instructional goal**
- [x] **Eliciting and responding to student thinking**
- [x] **Orienting students to each other's ideas**
- [ ] **Positioning students competently**
- [ ] **Establishing and maintaining expectations for student participation**
- [ ] **Representing student thinking key ideas**
- [x] **Using a public record of student thinking**

During the codesign the elementary teachers were also seduced by the allure of a *fun* activity. To the extent that it may have guided their decisions to increase math talk time, reducing calculation time. This type of reasoning is expressed by Melody at the codesign, “we would just look at [Giant box], have a discussion”, and ask, “what were all those questions that you asked?” End of class, they walk away, that *was just something fun*.” Unfortunately, at the time the insidious “myth” of having a fun math activity was operating and I had not yet read Ayers (2015) to dissipate the need for fun activities by cautioning that fun can be “distracting” whereas learning ought to be engaging (p. 13).

Next, elementary teachers were prompted to consider what they anticipated seeing in terms of students’ work (e.g., what will the students do?). The teachers expected “students building off of each other's ideas" and "students using correct mathematical terminology (vocab)." The following prompt: "What data might be collected to ensure that students have met the learning target?" and the teachers wrote: "List of questions collected (whole-class)." Next they were asked: "How would you assess the students' work?" They indicated N/A, not applicable for this activity as many conceived of it just as a fun opening activity. This oversight may signal a design flaw of the codesign questionnaire where the assessment question was not on top of the list. Finally, those consented to being video recorded would indicate their possible video-recording dates and we recorded their implementation in the classroom.

The learning target was formulated in light of the opacity of the concept of "rigorous math solution" (Line IV.13 of the codesign transcript below). On the prompt they indicated: "Learning Topic: Rigorous Justification–Krispy Kreme Open-Ended Questions (Math Talk)."

> **IV.13. RÉCIT: So, because what became an issue for them was that idea of "what does a rigorous math solution look like?"**
> IV.14. Lydia: Hmm AND IV.15. Ellen: Hmm
> IV.16. RÉCIT: You know? And that's what you were talking about too. You look at a problem, you can't see the thinking, is that a rigorous mathematical solution?
> IV.17. Lydia: yeah
> **IV.18. RÉCIT: So those are all kinds of things that came into their discussion and their planning, when they chose this series of lessons to do [referring to Donut and Let's Paint lessons].**
> IV.19. PI: Hmm yeah that's true.
> IV.20. RÉCIT: I think that's important too.
> IV.21. PI: yeah AND IV.22. Lydia: Yes
> **IV.23. RÉCIT: I mean they did end up doing the calculations, as [inter. Elem. consultant] just said too. We…, We kind of didn't get the impression that they were necessarily going to get to that.**

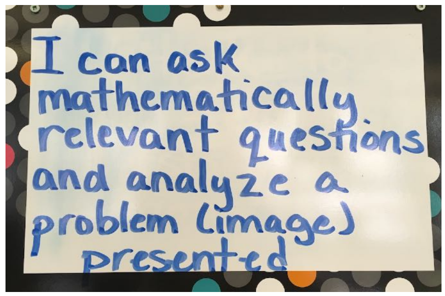
At the beginning of the coreflection meeting with secondary math teachers, the RÉCIT provided the context of the codesign session with the elementary math teacher. This is the "first aspect" towards the "overall learning goal," explained the RÉCIT:

> RÉCIT: [The Elementary teachers'] learning target because they wanted to get to **that point where the kids were not just highlighting everything.** And that's and that's what they said. And the other comment was not only highlighting but often in the elementary there's that table th**at the kids fill in What do I know? What do I need to know?** (Line III.59).

To showcase how teachers communicated their learning target Figure 5 presents a whiteboard picture where Kaci had handwritten in dark blue marker: “I can ask mathematically relevant questions and analyse a problem (image) presented.”

Figure 5

Learning Target for Donut Activity



In Part 1 of the session, the interim elementary consultant introduced the Donut activity. However, in Part 3, I asked a question about the role of knowing the diameter and area due to a personal misconception of math. The elementary teachers announced that the concept of diameter is not part of the Grade 5-6 progression of learning and may be covered in high school.

580. RA1: So, would they need to know the diameters of these donuts or the area of the box?
581. PI: Ooh.
582. RÉCIT: That might be something that comes up. Some kids might, right?
583. Kaci: Well, if you want to know how many donuts are in it.
584. Interim Elem. Consultant: Not if AND 585. [all talking at same time 27:33]
586. Kaci: well, not if you give the picture that [Donut company] answers, but if you're given no information,
587. Interim Elem. Consultant: maybe/
588. Kaci: /and are told, "Okay imagine if I had the box in here
589. PI: yeah, give them/
590. Kaci: /Maybe in high school/

In the context of elementary math, they delimited that the concept of diameter is not essential. In fact, understanding the term ‘diameter’ or its definition is not necessary to solve the problem at hand (C. Corriveau, personal communication, February, 2023).

Moreover, with the secondary group, the RÉCIT animator clarified that the Donut activity was not codesigned to challenge the students' proficiency in division, fraction multiplication, or estimation. Rather, the activity was aimed at promoting math talk by asking relevant mathematical questions:

IV.7 RÉCIT: /J**ust before you start that, sorry, just to kind of maybe bring to a little bit of a closure in terms of...like, we got kind of hung up on the actual calculations and that, and…, and you know, there are some good questions, I think, that teachers could asks the students to think: "Oh you're really confused with this because this is a large number. What would you do?" But just to kind of give you the context, also. They had a specific learning trajectory in mind, and they were doing several lessons**
IV.8 PI: Hmm
IV.9 RÉCIT: You know, over... over time, and yes, eventually some of them had decided that they may not even get to actually solving it because they had specific things, other aspects they wanted to look at. But just to give you what they produced as their learning trajectory statement, was that they said, "I can produce a rigorous mathematical argument, justification to explain my thinking and verify a solution." So that was their overall goal.
IV.10 Ellen: Hmm
IV.11 RÉCIT: And then they had broken it down into, the first aspect was identifying relevant questions, then you know working down.
IV.12 Ellen: Hmm

This exchange reads as a non-starter justification to *account for* the rationale for selecting the donut activity. This type of reasoning could be attributed to a misconception regarding the concept of math talk where it is detached from math operations. The lack of clarity about math-talk may be the second reason that led the elementary teachers to not add any clauses to assess students' understanding in the codesign plan (i.e., the lesson plan is lacking the comprehension level). For this reason, we did not collect official records of students' work.

The Original Donut Activity

In this subsection, I will describe the origins of the Donut activity, a multi-step word problem that requires students to test metacognitive abilities, mathematical operations, and interpret remainders, created by Shaw et al. (2014) and available on the Gfletchy website, which hosts a series of 3-Act tasks for the youth sector. The activity includes seven picture prompts.

Figure 6 on the next page presents the images of the Donut activity, consisting of three acts that provide students with progressively more information to help them solve the problem. Act 1 involves projecting an image of a giant box of donuts, held by four adults next to a truck, and asking students to estimate the number of donuts in the box: “How many donuts are in the box?” This is followed by two more pictures asking students to make an estimate that they “know is too high. Too low.” Act 2 provides additional information from a Donut company email, including measurements of the box, the diameter of the donut, and the number of donuts per layer, without any info about the number of donuts per row and column. Act 3 prompts students to calculate the number of donuts in the box. The final step is to watch a news report about the delivery of the giant donut box in the UK.

However, Sabrina, the self-selected note-taker, did not capture all the details of the design discussions in the handwritten lesson plan. For this reason, the photocopy of the learning activity provides a short description: 'Present students with an open-ended question (Donut image) and ask them what questions come to mind. Use the turn and talk partner strategy to share with the whole group.'

In many ways, this activity echoes Dan Meyer’s (2011) Pyramid of Pennies, as described by Corriveau (personal communication, November, 2022), which proposed four main questions: 1) How many pennies are there? 2) Guess as close as you can. 3) Give an answer you know is too high. 4) Give an answer you know is too low.

Figure 6

Activity Prompts: Act 1–Images; Act 2–Email Omitted Info; Act 3–Info

Table 4 shows an excerpt of the lesson's classification information from the three-act lesson website portal. According to its creators, Shaw et al. (2014), the lesson was designed to focus on two learning targets, Standard 1 (4.NBT.5) and Standard 2 (4.OA.3). These codes refer to specific math concepts. In the following section, I will explain these codes in more detail.

Table 4

Website Description of The Donut Activity

Date Added	Lesson Title	Standard 1	Standard 2	Big Ideas	What Do You Wonder?
10/10/2014	Krispy Kreme Me	4.NBT.5	4.OA.3	multiplication and division beyond 1000	How many Krispy Kreme Donuts are inside the box?

Source: 3-Act-Tasks

The learning targets were set by the National Governors Association Centre for Best Practices and Council of Chief State School Officers in 2010 as part of a Common Core State Standard Initiative. According to the online version of the *Common Core State Standard for Mathematic*s, the number 4 in Standard 1 (4.NBT.5) indicates that it is appropriate for Grade 4 learners and focuses on using place value understanding and properties of operations to perform multi-digit arithmetic. The acronym NBT stands for Number and Operations in Base Ten while number 5 indicates that this lesson seeks to “Use place value understanding and properties of operations to perform multi-digit arithmetic” (p. 29). Specifically, Standard 1 denoted by the code 4.NBT.B.5 tasks learners to:

> multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and **explain the calculation by using equations,** rectangular arrays, and/or area models (p. 29).

The Standard 2 (4.OA.3) is designed for Grade 4 learners and OA focuses on four operations (addition, subtraction, multiplication, and division) with whole numbers to solve problems. The number 3 indicates that this lesson seeks to nudge learners to “Use the four operations with whole numbers to solve problems” (p. 29). Specifically, Standard 2 requires student to assess reasonableness:

> solve multistep word problems posed with whole numbers and having whole-number answers using the **four operations, including problems in which remainders must be interpreted**. Represent these problems using equations with a letter standing for the unknown quantity. **Assess the reasonableness** of answers *using mental computation and estimation strategies* including rounding (p. 29).

These Standards are partially aligned with Quebec’s Education Program (QEP, 2009), which aims to promote the understanding of mathematical operations by students through teacher guidance. The intended outcome is for students to apply their knowledge by the end of the school year and be able to use this knowledge in new situations.

Our observations revealed that the teachers used various strategies to implement the activity since the original activity was not explicitly described in the lesson plan. The teachers prompted students to separate what they need to know from what they already know. We also observed that although the activity was designed for Grade 4 students, many of the 12 groups of Grade 5-6 students struggled to attain the intended learning outcomes, suggesting they require additional support to meet the standards. Next, I will detail the three critical incidents.

Three Critical Incidents From Classrooms as Mediation Tool

In this subsection, I describe and interpret the context surrounding three critical incidents (estimation, conversion, and volume/area) that were selected for further examination due to their contentious nature in classroom conversations. As classroom talk has been a focus of research for 50 years (Hennessy et al., 2020), I aim to contribute to this body of knowledge by analyzing the discussions that were read as scripts with educators to prompt coreflection. Classroom conversations are crucial for student learning as they can reveal their understanding of a topic (Hennessy et al., 2016). To analyze these discussions systematically, a structured approach was necessary. Hennessy et al. (2020) proposed an eight-category coding scheme that considers the context and interactions between students and teachers. Using this coding scheme, I determined if the discussions involved “just-in-time feedback” where teachers and students used each other's ideas to negotiate collective understanding (Davis et al., 2017, p. 277). The eight categories: Invite Elaboration (I), Make Reasoning Explicit (R), Positioning and Coordination (P), Build on Ideas (B), Reflecting on Conversation or Activity (RD), Connect (C), Express Ideas (E), and Guide direction of conversation or activity (G), were used to steer conversations toward a specific goal (Hennessy et al., 2020).

Critical Incident One. This critical incident involved estimating a number that they "know is too low" and resulted in a classroom discussion about the concept of estimation. The discussion lasted about 36 lines and took place at around 2:40 minutes into the hour-long class. Sabrina and the students discussed the definition, reasonableness, and purpose of an estimate in the context of the Donut activity, which required them to estimate the number of donuts in a box. Initially, Student 3 thought that an estimate of "1" was too low and ultimately questioned its validity. The teacher elicited further discussion by asking for explanations of what an estimate is, leading to a consensus that an estimate is an educated guess based on available information. The students then gave their opinions on what would be a reasonable estimate and what would be too high. To code this teacher's interaction with students by primarily using Hennessy et al. (2020):

Implementation (2:40 min) **Whole Class Discussion: Is 1 "too low" estimate? What is an estimate? What is "reasonable"?**	**Category**
1. Student 1: There would have to be more than one, because for sure there's one.	**I**
2. Student 2: Well you can see there's two.	**I**
3. Student 1: There's more than one.	**I**
4. Student 2: You're saying that what's there is too low.	**I**
5. Student 3: One is too low.	**RD**
6. Student 3: It has to be reasonable.	**RD**
7. Teacher: So what I'm hearing you saying is that you're agreeing with J, you're saying that one is not a reasonable estimate.	**RD**
8. Student 3: It is too low. If we look at it we know it's not...	**I**
9. Teacher: Okay. So do we want to include our one as an estimate here?	**G**
10. [Indistinguishable voices]: Yes. No.	**RD**
11. Teacher: Some of you are saying no.	**RD**
12. Student 4: Why?	**I**
13. Teacher: Persuade us.	**P**
14. Student 6: It's too low.	**I**
15. Teacher: It's too low? Isn't this your estimate?	**I**
16. Student 6: Yeah.	**RD**
17. Teacher: Okay. So now you don't like it anymore?	**RD**
18. Student 6: No.	**RD**
19. Teacher: You still like it. So why should we keep it in the "too low." Because others are saying that it's obvious that there's one. What will you have to do?	**G**
20. Student 6: Make it too low.	**RD**
21. Teacher: Make an estimate that was too low. Exactly. Is it too low?	**R**

Implementation (2:40 min) **Whole Class Discussion: Is 1 "too low" estimate? What is an estimate? What is "reasonable"?**	**Category**
22. Student 6: Yeah.	**I**
23. Teacher: Yes. Okay. Ju?	**RD**
24. Student 7: Yeah, but it's not a reasonable estimate, because, like fifty. We continue, okay there's fifty, about. But one, we know that there's more than one in the box.	**B**
25. Student 8: We said there's fifty, but there's five.	**I**
26. Teacher: One at a time. We have so many people with their hands up. C, what do you think?	**C**
27. Student 9: I think it shouldn't be there, because we're supposed to estimate, and estimate for something that's too low. But there we can know for sure that it's too low, and we're supposed to estimate.	**I**
28. Teacher: I think that maybe brings us to the question of what is an estimate? Does anybody know what an estimate is? Who can give us a definition maybe, or a reminder of what that might be? What's an estimate? Ju, do you want to answer that?	**B**
29. Student 10: Well, you know, it's approximately how much something is going to cost.	**I**
30. Teacher: So approximately, about, how much maybe something is cost, if we're talking about money.	**I**
31. Student 11: Approximation.	**B**
32. Teacher: I'm hearing a lot of similar answers, guess, approximation.	**B**
33. Student 12: An educated guess.	**B**
34. Teacher: An educated guess? What do we mean by an educated guess?	**B**
35. Student 12: It's not like we'll say there's five thousand in there, because we know there can't be five thousand.	**B**
36. Teacher: So it's not just a number that we're pulling out of a hat. We're actually basing it on something. Okay.	**B**

From the coding scheme, it can be observed that the students engaged in quantitative reasoning by discussing numbers, educated guesses, and approximations by discussing the reasonability of their estimates and the meaning of the word ‘estimate.’ By way of context of how students already knew the meaning of estimation, is that the week before they had covered the topic. Their teacher, Sabrina, asked them to remember “with our fractions and estimates that are reasonable,” as she walked around and they were writing their first estimation. In sum, Sabrina engaged students in a discussion about the concept of estimation, and they used reasoning, building on ideas and invitation to elaborate to arrive at a shared understanding of what an estimate is and what makes a reasonable estimate. At the end of the class time, the teacher asked: “What did we learn today?” Many students volunteered that they used theirs or their friend's brain, but eventually they agreed that they learned to ask questions. As the bell rang, she held the students, to complete her wrap up. Apart from initiating questions, this form of implementation of the activity did not give students time to actually calculate the number of donuts per row and column, to really apply operations and to interpret remainders. An overview of the entire class can be found in Appendix C.1 – Estimation Critical Incident 1.

Critical Incident Two. The second incident pertains to the implementation, in which a student asked a procedural question about "what do we do next?" and the teacher explained using chalkboard drawings (about 3 min 53 sec of the hour-long class). The 17-Line discussion takes place between Dona and several students in an afternoon math class, where they are discussing how to calculate the number of donuts that can fit inside a giant Donut box with length and width given in millimeters. The teacher explains how another student simplified the problem and "made it easier to work with" by converting the dimensions of the box from millimeters to meters, and then finding how many donuts fit per row and column. The teacher then demonstrates how one could determine how many donuts fit in one meter and uses that information to calculate the total number of donuts that can fit in the box. To code this teacher's interaction with students, we can see a pattern if we primarily use Hennessy et al. (2020):

Implementation (3 min 53 sec) **Whole Class Discussion: "What do we do next?" Teacher explains using chalkboard drawings.**	**Category**
1. Teacher: What numbers do you come up with when you multiply both of those? Three thousand times two thousand two hundred.	**G**
2. Student 1: Six million nine hundred thousand.	**E**
3. **Teacher: So a lot of numbers, right?** A lot of zeros. The way she did it made it smaller, made it an easier number to work with. That's always a good way to go. Making the numbers more workable. You could've done it the way you did it, but it took a little bit more time.	**B**
4. Student 2: But after doing this, what do you do? Like, after having it in millimeters?	**I**
5. Teacher: It already is in millimeters.	**R**
6. Student 2: Yes, but what do you do after?	**I**
7. Teacher: I don't understand your question.	**G**
8. Student 2: What is the next step after doing this?	**I**

Implementation (3 min 53 sec) **Whole Class Discussion: "What do we do next?" Teacher explains using chalkboard drawings.**	**Category**
9. Teacher: After doing what?	**G**
10. Student 2: Your calculation of three thousand...	**I**
11. Teacher: So. If you were to take... So we're going to say this is the box, the donut box, okay? They're saying that this length here is three thousand millimeters, which we know is three meters. Okay? This one here is millimeters, or you convert it to meters. The reason she did it is because it's easier to work with. You guys times them, right? You did three times two point three. **This would've given you the area, the inside.** What K did was instead of finding the area was, she figured out how many donuts it took... how many donuts she could place. So she knows that one donut equals eight point nine centimeters. She says, "eight point nine centimeters, too complicated." I'm going to change that to nine centimeters. She goes like this, "hmm...one meter...Hey. One meter, how many donuts do I fit in one meter? So if I was to take donut, donut, donut…" She said: "Eleven fit in here". So she said: "Well this is three times a meter stick, and if eleven donuts fit on one meter stick".	**B**
12. Student 2: You make eleven times three.	**B**
13. Teacher: Because one meter...you need three meters. Do you understand? So she found out how many donuts went here, and then she did the same thing and found out how many donuts fit here. And she said, how many donuts fit there?	**B**
14. Student 3: Twenty-five.	**B**
15. Teacher: Twenty-five. So she said, "twenty five donuts go here," and how many this way? Thirty-three. Krispy Kreme told us what? That we had...	**B**
16. Student 3: Thirty-two times twenty five.	**B**
17. Teacher: Thirty-two times twenty-five. So her way of figuring it out would have given her an answer that was pretty close. She was one off. So this was a good way of thinking. So let's actually find out exactly how many there are.	**B**

This teacher was primarily building on ideas, even revoicing, by demonstrating how another student's think-aloud calculations could be used to solve the Donut challenge. The revoicing can also be categorized as a seventh using a public record of student thinking. The students in this conversation were primarily expressing ideas and building off of each other to clarify their understanding of the challenge. How many of you found this very difficult?" A student does [the comme-si comme-ça hand gesture], the teacher asks why, and the student replies: "Well just getting confused with numbers and calculation." Dona also used positioning and coordination to point to converting the numbers to make them more manageable. To wrap up, Dona asked, "Did you know your math knowledge to answer this question?" A student replies, they had "forgotten about the diameter." Teacher reassures the student that she later remembers what the diameter was. Dona then invites others to join in, "Who was completely confused? I feel like everyone knew what to do, you had to open those brain cells. You were able to figure out math without having the exact number of donuts." I do not share her optimism. Lastly, Dona asked if there were any other questions and then advised students to put their papers in the math section of their binder. Ending the activity five minutes before the bell rings. In this enactment no time was allocated to watching the Act 3 Donut news report; many groups did not finish the task. In summary, the purpose of this section was to analyze the second critical incident. Despite the fact that Dona kept asking the students questions and revolving to invite elaboration and reasoning it seems many did not understand that area of the box is not necessary, and neither is converting. See exchanges in Appendix C.2–Conversion Critical Incident 2.

Critical Incident Three. The third critical incident pertains to the whole class discussion about Volume vs. Area (2 min 40 sec). Kate and students discuss in 31-Line the measurement of donuts in a giant box. For many groups the initial assumption was that the measurement would be in volume, but the students in this segment raise different points. They also discuss the shape and size of the donuts and the possibility of using area instead of volume. Kate guides the discussion by asking questions and summarizing the points made by the students. Applying the coding scheme by Hennessy et al. (2020) reveals the following pattern:

Implementation (2:40 min) Whole Class Discussion: Area vs. Volume	Category
1. Student 1: I think it could be volume.	**E**
2. Teacher: Why?	**I**
3. Student 1: Because it's a box of donuts and not a square of donuts. It's not flat.	**E**
4. Student 2: But there's air space between the donuts, duh.	**I**
5. Teacher: Okay. Do you have something else to say back? Why are you guys looking at area and not volume?	**I**
6. Student 3: Because, we're not filling the whole thing with donuts. It's only the bottom, because there's going to be room.	**E**
7. Student 4: Well, we don't know, if they're going on top of each other.	**I**
8. Teacher: So we have a question as to whether we're just putting them along the bottom of the	**I**

Implementation (2:40 min) Whole Class Discussion: Area vs. Volume	Category
box, or if there are layers of donuts. We don't actually know that.	
9. Student 3: Yeah.	**Dudley Agr**
10. Teacher: Okay. Boys, can you stop please? Thank you.	**Class Manage**
11. Student 5: Usually they only put one layer.	**E**
12. Teacher: So you're using your knowledge of donut boxes, that usually we have one layer, we don't usually stack them.	**I**
13. Student 4: But a donut still takes up space, it's not just flat.	**R**
14. Teacher: But this group is looking at area because they are assuming there's only one layer of donuts.	**I**
15. Student 4: Yeah, but they still take up space. They're still high.	**R**
16. Teacher: They do. Okay. j?	**I**
17. Student 6: They could stack them. There could be a piece of cardboard between the two rows of them.	**E**
18. Teacher: So you're not eliminating the possibility that they could be stacked.	**I**
19. Student 4: Yeah, because it's wide. When they showed the picture of it going in, it was really, really wide for a small donut.	**I**
20. Teacher: Hold on. When they showed it going in... let's all look at that one.	**R**
21. Student 4: Why would they make such a...	**E**
22. Teacher: So it looks quite a bit taller than one layer of donuts. Okay.	**R**
23. Student 6: But it can't be measured in volume, because even if donuts are 3D, they have a hole in the donuts, and the space between the donuts. So it can't be measured in volume, because there's a lot of air space.	**R**
24. Teacher: Very interesting observation. So you're saying we can't just use volume because we're not going to be able to fill all the spaces. So even though there's a little space in there, we're not going to cut up donut bits to fill all the empty spaces. Interesting. Jo?	**R**
25. Student 4: Well it depends what kind of donuts. Because there's the donuts that don't have the hole in the middle and it's just filled in.	**R**
26. Teacher: True. If I think of your "what if?" Do they take the same amount of space? If you've ever bought donuts before, the ones with the hole, even though they do have less volume, in the box, do they take a smaller spot in the box?	**R**
27. Multiple voices: No.	**I**
28. Teacher: No. They just have a hole in the middle.	**R**
29. Student 6: But miss, even if you take it without a hole in it, there's still the space between the donuts.	**R**
30. Teacher: Because they're round?	**R**
31. Student 6: Yeah.	**R**

The Volume vs. Area Challenge was a conversation between students and Kate about the best way to measure the amount of space taken up by donuts in a giant box. The students were considering both volume and area, with some students advocating for volume and others for area. Throughout the conversation, the students used reasoning to explain their thoughts on why volume (e.g., “Oh! because it is 3D!”, said a student in group 1) or area would be the best way to measure the space taken up by the donuts. They discussed the presence of air space between the donuts, the possibility of stacking the donuts, and the fact that donuts come in different shapes, with some having a hole in the middle. The students demonstrated an understanding of the topic being discussed and made efforts to build on each other's ideas and challenge each other's assumptions. This type of productive mathematical discourse may be positively related to students' learning of mathematical concepts. This whole class discussion brings out the third high quality teaching practice which was intended “to orient students to each other's ideas.” Kate played a key role in facilitating and coordinating the discussion, helping the students to understand each other's perspectives and encouraging them to consider new information. Kate also made sure that the conversation remained on track and helped the students to reach a consensus on the best way to measure the space taken up by the donuts

This critical conversation ends with the conclusion that the measurement cannot be made in volume due to the air space and potential to stack donuts as Kate summarized: “Very interesting observation. So you're saying we can't just use volume because we're not going to be able to fill all the spaces. So even though there's a little space in there, we're not going to cut up donut bits to fill all the empty spaces. Interesting.” This teacher recaps by saying that the important thing was that they learned to ask questions. “If we use an area,...respect the space.” that “In this case dimensions are more important than area.” Since donuts can’t be side by side, working with circles, the “triangular-ish spaces will never be filled” and the teacher concluded that students must “think about dimensions more than space.” This teacher did not show the video accompanying the activity but made comments about how each student group appeared to have full members participation (Appendix C.3–Area vs. Volume Critical Incident 3 for details).

Although we noted some students' contributions were ignored by their peers. In this classroom discussion, the implementation of area and volume concepts in relation to a box of donuts is explored. The teacher asks the students why they are considering area and not volume, to which the students provide various responses, including the assumption that there is only one layer of donuts in the box. The teacher acknowledges the possibility of stacking donuts and raises the question of how to measure the space taken up by donuts, given the air space between them and the presence of holes in some donuts. The students consider different possibilities and provide their observations on how the space is occupied in the box. The teacher manages the discussion and redirects the students when needed, maintaining a positive classroom environment. Overall, the discussion highlights the complexity of measuring space and the importance of considering various factors in determining the appropriate approach.

Taken togther, across the three critical incidents as a coder, I have problematized (i.e., identified glitches) many segments that others may not. For instance, how will the secondary teachers discuss this reply: "So her way of figuring it out would have given her an answer that was pretty close. **She was one off.** So this was a good way of thinking. So let's actually find out ***exactly*** how many there are." If this teacher had personally calculated the numbers, or had been given this detail about the Donut lesson then she would have known that no calculation would be able to reach the "exact" number 32, they all would lead to 33. In fact, this activity is about estimations (recall standard 2). I wonder who is *off*, how are the teachers' perceptions of a students' error or being "off" in math to be handled if we consider notions of building students' sense of competence in math? In the context of estimating the quantity of donuts the use of the word “exact” by the teacher influenced my perception that this teacher did not know the number of donuts per row was not going to be exactly calculable, because we have the words, approximately and gap, in the email from the donut company that signal ambiguity. To investigate whether the other educators would react the same way as I did, I explain my approach to data analysis of the educators’ talk.

Pragmatic Grounded Analysis

"This section provides an overview of the Pragmatic Grounded Analysis used in this study, including reflexivity and methodological integrity. The study employed a pragmatic grounded analysis (Charmaz, 2006; Corbin et al., 2015) that values theory based on how it informs and improves practice (Seel et al., 2017). This approach aligns with the American founders of pragmatism, Peirce and James, who argue that truth is tested by practical consequences of belief (Corbin & Strauss, 2015). For this study, analyzing coreflections on the Donut activity provided insight into classroom practices that aided in a deeper understanding of mathematics teaching and learning (Savoie-Zajc, 2010).

I prioritized interpreting three coreflections and utilized a pragmatic grounded content analysis process to qualitatively analyze video data from CCCM 2017. Since practitioners' beliefs about engaging in coreflection were not collected, I opted for a pragmatic grounded approach that focused on words, treating all observations as potentially containing meaningful themes that could impact the redesign and reenactment of the activity.

Groundedness allows for flexibility to explore the dynamic and complex nature of coreflection on the Donut activity (Groen et al., 2017). The Pragmatic Grounded Analysis aims to make sense of educators' conversation (Fuller & Goriunova, 2014) by highlighting seemingly inconsequential details. This analysis depends on my interpreter's vantage point and aims to make sense of educators' conversation by highlighting seemingly inconsequential details; this will be discussed next.

Reflexivity in This Study

In this subsection I will engage in reflexivity by detailing My Relationship With Math, Delimitations and Assumptions, to end with Methodological Integrity considerations.This is essential in qualitative research, since as a researcher, I am the instrument that needs to have the "ability to understand, describe, and interpret" the coreflective meetings (Maguire & Delahunt, 2017, p. 1). During our first lunch break in 2015, I disclosed to the CCCM partners that I was a rookie in Math Education and that my previous focus was Sex Education. They immediately began sharing their opinions, and I bonded with several of them, which made it easy for me to ask obvious math or pedagogical questions, and they patiently shared their expertise with me. From a marketing perspective, few topics are as polarizing as sex education and mathematics education. To me sex education and mathematics education are both integral to basic human rights and must be made accessible to young learners with proper vocabulary. While some students prefer sex education over math, and vice versa for some parents, teachers report that teaching either subject can be challenging from an anecdotal standpoint. As Kate would say: both require educators to: "launch back into task appropriateness."

From an educational anthropologist perspective, Behar's (1996) observations on being an insider resonate: "the lines between [being a] participant and observer, […], are no longer so easily drawn" (p. 28). For this study, the line couldn't be more demarcated yet shaded and discontinuous. On the one hand, at the beginning, I was merely an observer of math educators, on the periphery of the teaching of CCCM's activities, and I remained an outsider throughout. As a researcher, however, I engaged in handwriting copious fieldnotes. Ode to Miles and Huberman (1994), I took the 'more is better' view of notes, using a descriptive method. According to Bernard (2006) my fieldnotes were descriptive “meat and potatoes of fieldwork” (p. 397). They were attempts at writing what was going on (Spradley, 1980). By taking notes alongside pictures and recording audio-video,

I was able to trace back my thoughts on site observations and capture a “patchwork of perspectives” (Lederman, 1990, p. 90). I also tried to be socio-constructivist by sharing the link to my notes. My role evolved over time. By the second year, I participated in editing videos and intervened with questions at the secondary meeting, where my talking turns amounted to 146 out of a total of 1394. My sense was that with the secondary math teacher we could critically examine the Donut activity. At the elementary meeting, I only had 39 talking turns out of 1234, as the teachers mostly facilitated their own conversation and I aimed to maintain collegiality rather than be critical. My involvement at the mixed-group meeting was minimal, with only five talking turns out of a total of 99, and my fieldnotes from that day are not very clear. Table 5 presents an overview of my talking-turns impact, this impact interpretation.

Table 5

Number of Talking-Turns at Meetings

Video-Mediated Collective Reflection Post-viewing Discussion	Me	Rate of Intervention
Secondary Collective Reflection	RA1	146/1394
Elementary Collective Reflection	RA1	39/1234
Mixed-group Collective Reflection (Duration: 21:26:19 out of 6h meeting)	RA1	5/99

Notes: Per latest transcript count in 2020

My Relationship With Math. My academic background is in social sciences, and I used to believe that math was unnecessary for my success. This belief was rooted in my personal experience with math–I was homeschooled intermittently during the Iran-Iraq War, and my mother taught me basic math using matches. When my family fled to Montréal in 1990, I was placed in a French Accueil class. To help me catch up with my age group, my mother insisted that I focus on math and I entered high school at the age of twelve. However, I had to rely on my older brother and a French-to-Farsi dictionary to practice math. By age I7 I disliked math. Later I realized how wrong I was. In sum, my atypical academic path has enhanced my sensitivity to the student perspective during a lesson, but also limits my claims about teaching math and the recommendations that can be drawn from this study. Therefore, as a novice researcher, I present the product of analyzing for six years video data, transcripts, and fieldnotes to make sense of one activity, recognizing that I am not an expert in teaching math nor a native speaker of English.

Delimitations and Assumptions. This section will identify the boundaries and scope of interest of this study in relation to my personal position. In order to avoid appearing as an "invisible, anonymous voice of authority" (Harding, 1987, p. 9) and instead present myself as a real historical individual with specific desires and interests, I will specify the limitations and assumptions of this research. As a novice researcher in the field of Mathematics Education, I approach this study from a learning sciences perspective, aiming to improve upon the Donut activity to attain the lesson. However, I am aware that my theoretical sensitivity is influenced by my perspectives, biases, and shortcomings (those of which I am presently aware of).

My epistemological stance aligns with the empiricist perspective of Locke, Berkeley, and Hume (1910), which theorizes that knowledge results from experience and inductive reasoning, guiding this grounded analysis. Referring to Birks and Mills (2015) perspective, this dissertation is the sum of all that I have experienced. Therefore, I will be hypervigilant to avoid producing or reproducing knowledge, but rather aim to increase our understanding of how to become attuned to the math challenges raised by one activity. Learning, in this regard, is positioned as a “way of being” (Vaill, 1996, p. 66). To address my biases, I will use the anthropological concept of reflexivity to discuss them in depth in this section. This will allow me to plant my pole on the discipline of teaching and learning mathematics in the next section, where I will “make a conversational move” on how to improve lessons (A. Breuleux, personal communication, November, 2020). To begin, I must digress and recount how I arrived at this topic, which is not my area of expertise as a fifth-year PhD candidate. In the next few pages, I will use a stream of consciousness to briefly explain how I became a fly on the wall of math teaching and learning. With a vast corpus of video data, I found myself bewildered, feeling like a deer caught in the headlights. I had gone to the field like a typically naive novice researcher, without questions, just going along for the ride with a wonderful research team that welcomed me with open arms. I found myself getting stuck in the leaves of the tree and failing to see the forest or the jungle but I see opportunities in the details.

Methodological Integrity

This subsection offers an explanation for methodological integrity. As the researcher, I am responsible for any inaccuracies in “transforming” information from data to interpretation to paraphrase “into immediately relevant patterns of meaning” for the reader is my responsibility (Borton, 1970, p. 89). To compensate for my lack of expertise in math, I attended all of the meetings and listened to three years' worth of end-of-year interviews to become a genuine legitimate peripheral participant in the field of mathematics teaching and learning, as Lave and Wenger (1991) conceived of learning as situative, always involving a context. While I have never taught math, my gradual entry allowed me to understand the ropes of participating in meetings as an educational anthropologist. The field of mathematics teaching and learning can be amended with an opportunity for inquiry to acknowledge the role of research in it. Riisla et al. (2017) provided a model for thematic analysis that identified four types of consequences: individual vs. collective, and contested vs. stabilized consequences. Our data was analyzed in two overarching steps: first, open-coding of recurring patterns, and then categorizing them. The first step involved identifying common themes that emerged from the collective meetings, while the second step involved recognizing four categories of math challenges: estimation, conversions, volume vs. area, and math talk (vocabulary) as a transversal connecting thread.

To ensure the integrity of grounded analysis, potential categorical biases were considered at the initial stage of analyzing educators' conversations. Wittgenstein (1969) observed that a "language-game is, so to say, something unpredictable. This means it is not based on grounds. It is not reasonable... It is there–like our life" (p. 559). Therefore, to facilitate the conversation, we avoided fixed scripts and took a more exploratory approach, not relying on a priori coding schemes that might limit understanding of coreflection video data. Instead, we used a posteriori approach that uncovered patterns that might have been overlooked otherwise, and we missed a few opportunities to guide the discussions.

Over the years, I have read diverting theories related to coreflection and teaching math. In 2015, during my first year in the field, I met with three researchers, seven teachers, and four consultants. Over the course of a year, I became involved to the point of editing the mediational video with the research team. We developed a video protocol to mitigate tension in video research (see Baradaran, 2015). In other words, the video was prepared to act as a mediator or événement déclencheur (catalyst). At the time, there seemed to be too many segments to choose from, so we used the lesson study framework's three phases—co-design, enactment, and collective reflection—as a guiding principle to chunk the 19 hours of video captured during the three-act lesson. Most of our partners trusted the research team's perspective, which made editing somewhat easy. This level of trust is exemplified by Kaci's end-of-year interview response:

> 86. PI: Okay. Any challenges that you see in reviewing the clips?
> **87. Kaci: I'm trying to remember, when you came the first time, it was a bit on me. "Have you viewed that?" And I can't remember... Sometimes it's the time. You don't have to sit and watch them all**, but the second one we didn't have time to view. So you had just... You made us listen instead of... But I know there's no way around that. Mind you, I'm also somebody who, **I don't care which part of the clip you put, I won't be embarrassed, I trust your judgment.** But definitely having to sit and view, and communicate which parts I want included, not want included, within a timeframe, it's hard. I don't know a way around that. [0:10:01.8]

Moreover, we decided to feature video segments from each of the 12 student groups, each consisting of three to four members, during the enactment section of the video. However, this posed a challenge during the video-editing process. Specifically, I mistakenly followed the cycles of the lesson study, such as inserting codesign moments, instead of focusing solely on the clips related to the Donut activity and the mathematical glitches that occurred. For example, many of the Grade 5-6 groups we recorded were confused about the final number of donuts, and they struggled to demonstrate their understanding of the geometric shape of a round donut in a rectangular box by grossly underestimating or overestimating the quantity. If we had focused the video on these glitches, we would have had a shorter video that concentrated on the student learning outcomes.

Lastly, to interpret the data in a meaningful way, it is necessary to consider its context. I was mindful of detecting coherence across certain themes among certain roles. The consistency was not easy to measure since there are no multiple recordings of the same teacher over time. However, there are a few enactments by the same elementary math teachers, such as Kate, in the forms of Fast Pace, Donut 3-act, and Let's Paint, which can be analyzed in sequence in an upcoming post-doctoral study. Over its four-year run, some CCCM partners have already voiced their opinion that their lack of in-depth math content knowledge could be the main cause of pedagogical challenges in teaching mathematics at cycle 3–grades 5 and 6. For example, Kate explained in the summer of 2014 that "Math is the subject that I have taught the least as a matter of fact."

Data Categorization Procedures

> *Dona: My biggest thing was, I was wondering if everybody else was getting the same reactions from the kids as I was. Like, did it go the same way/*
> *PI: /we'll see*
> *Dona: in other peoples' classes, as it did in mine?*

In this section, I present how I proceeded with diverse data categorization approaches, followed by details on data transcription process and eclectic Analysis. As it will be described, the data was combed through multiple viewings of meetings and comparisons of transcripts and interviews to identify what transpired and what themes emerged using the "cyclical act" coding method (Saldaña, 2016, p. 9). As Dona's curiosity is captured here, I too wanted to see how each group reacted to the activity. The procedures for data transcription, followed by an explanation of the eclectic analysis procedures are next.

Transcription Process

All face-to-face meetings were digitally recorded, and once the research team considered the Donut activity as a crucial example of how lesson study could bring attention to transition issues as a codesigned math lesson, a third party transcribed all the discussions that pertained to the Donut activity. In mid-2017, using ExpressScribe as the transcription tool, I re-listened to the three collective reflections at a slowed speed (5–95%) to produce three sets of single-spaced transcripts (secondary, 36 pages; elementary, 38 pages; and mixed group, 6 pages) in font size 10. The purpose of these transcripts was to examine the selected nine vignettes and accompanying field notes more closely, although there were some limitations in transcribing group conversations, given their fast and fluid nature. I set out to create a Jeffersonian transcript with every utterance marked with “more details,” such as facial expressions (Jefferson, 2004, p. 22). Instead, the transcripts are filled with "inaudible" notation for instances where it was difficult to decipher/hear utterance. Indeed used many slashes (/) in the transcripts to indicate overlapping turns, which is a common feature of informal discourse. The level of formality varied across groups, with the mixed-group meeting being the most formal, having only seven overlaps recorded, the secondary group had 173, while the elementary group had 233 overlaps. And utterances were identified as not applicable if they did not fit into any category or level.

The transcripts reveal that elementary math teachers laughed significantly more than secondary math teachers, with an average of 68 and 31 times, respectively. Although the nature of the laughter is uncertain (awkward or not), it is clear that both groups used humor as a conversational strategy, consistent with Eggins and Slade’s (1997) observation that “the construction of group cohesion frequently involves using conversational strategies such as humorous banter, teasing, and joking" (p. 14). Some quotes highlight moments of laughter, providing examples of humor in the conversations. Interestingly, during the mixed-group meeting, there were only seven instances of laughter, further supporting the perception of a higher level of formality in this context. These findings suggest that humor can foster positive relationships and a supportive learning environment. In the next section, I will describe in detail the eclectic data analysis procedures used for this study.

Eclectic Analysis

To address the complexity and variability of the data in the specific context of the Donut activity, I adopted an eclectic approach to qualitative coding. Specifically, I used Saldaña's (1997) mixed and matched approach, which combines pragmatic grounded analysis (Charmaz, 2006; Corbin & Strauss, 2015) with eclectic coding (Saldaña, 1997) and contextualization techniques by Charles (1998) to merge categories and identify patterns. This approach allowed me to explore the dynamic and complex nature of coreflection and how it improves practice (Seel et al., 2017). While Kruse et al. (1995) examined educators' "conversations regarding practice, pedagogy, and student learning." I focused on how educators scrutinized the activity itself (p. 30)..

After watching the 19 hours of video data and corresponding transcripts, I adapted Charmaz's (2006) four-level grounded data analysis model. This involved open coding, category development and refinement, and thematic coding. Content data analysis (Lasswell, 1948) was used to gain insights from the semi-structured lesson study co-reflection, capturing unfolding conversation at a sentence-by-sentence and word-by-word level. An eclectic code sheet was adopted, merging categories and identifying patterns using eclectic coding (Saldaña, 1997) and contextualization techniques by Charles (1998). The eclectic approach was adopted because achieving methodological rigor in qualitative research requires new strategies (Gioia, as cited in Chandra & Shang, 2019, p. v). This approach helped avoid complexity and focus on the big picture in themes, leading to insights into the dynamic and complex nature of coreflection in the specific context of the Donut activity.

To ensure transparency and replicability, a set of general steps were followed during the conceptual content analysis phase. The videos were watched three times: once without taking notes, once while taking notes, and once while cleaning the transcripts for specific segments. To manage attention, the videos were listened to while occupied with manual tasks, such as cooking. Initially, instances where practitioners doubted their tacit knowledge and confronted biases hindering their ability to make sense of the Donut activity and its elusive lesson were identified. Three vignettes from the immediate aftermath of the video viewing were selected, and five vignettes were drawn from teachers' discussions after reading the classroom scripts, with an additional vignette from one of my comments during the mixed-group meeting. The analysis was then focused on the math concepts and themes that had emerged during the grounded analysis process.

A flexible coding scheme was used that allowed for the development of new concepts as they emerged. By using a combination of descriptive and interpretive coding, insights were gained from the data collected over a four-month period, including videos, interview transcripts, fieldnotes, and codesign questionnaires. To interpret the coreflections, a combination of ethnographic, content analysis, and discourse techniques were applied, avoiding limitations posed by a priori codes (Goulding, 2001). The data were organized chronologically within a targeted time frame, focusing on what was said after viewing the mediating video representation of the activity. A content category was defined broadly, following Phenix's (1964) tradition of "representative ideas" (p. 323), which included utterances where teachers noted math difficulties.

To achieve methodological rigor in qualitative research, I used an eclectic interpretive analysis procedure that combined different strategies, including ethnographic, content analysis, and discourse techniques, as well as a non-coding approach at the outset to gain insights from the data. This eclectic approach helped me avoid limitations posed by a priori codes and see the big picture in themes. The resulting insights provided a rich understanding of the complex and dynamic nature of coreflection in the context of the Donut activity. To explore the relationship between math concepts and challenges raised by the Donut activity in educators' conversations, I used "fuzzy sets," where multiple codes were used for the same utterances, resulting in overlap (Bazeley, 2013, p. 351). The nested model, which aligned with the study's methodological requirements of grounded pragmatism and flexibility, facilitated tracing the evolution of the educators' talk over time.

To identify idiosyncrasies across groups of teachers, I used multiple lenses to code the teachers' utterances at the collective meetings, including codes from Hennessy et al. (2016; 2020) for the whole-class conversations. However, a Classroom Observation Protocol was not used for data collection as it was not directly related to the research context at the time. I used a descriptive approach similar to van Maanen (1979) and interpreted the outcomes with Mezirow's (1998) "critical perspective on reflection" in mind (Cao, 2003, p. 122). Together, these techniques allowed me to identify four themes, providing a glimpse of patterns that help explore the dynamic and complex nature of collective reflection. I will draw mainly on the work of Ono et al. (2013), who argued that co-reflection is not just about gathering together but also about improving the lesson co-design over time. This study assumes that if the video recordings of the Donut activity are representations of teaching practice in terms of lesson planning, then it is essential to examine them carefully. According to Coburn (2003), using videos for collective reflection is an integral component of examining practice for projects like CCCM.

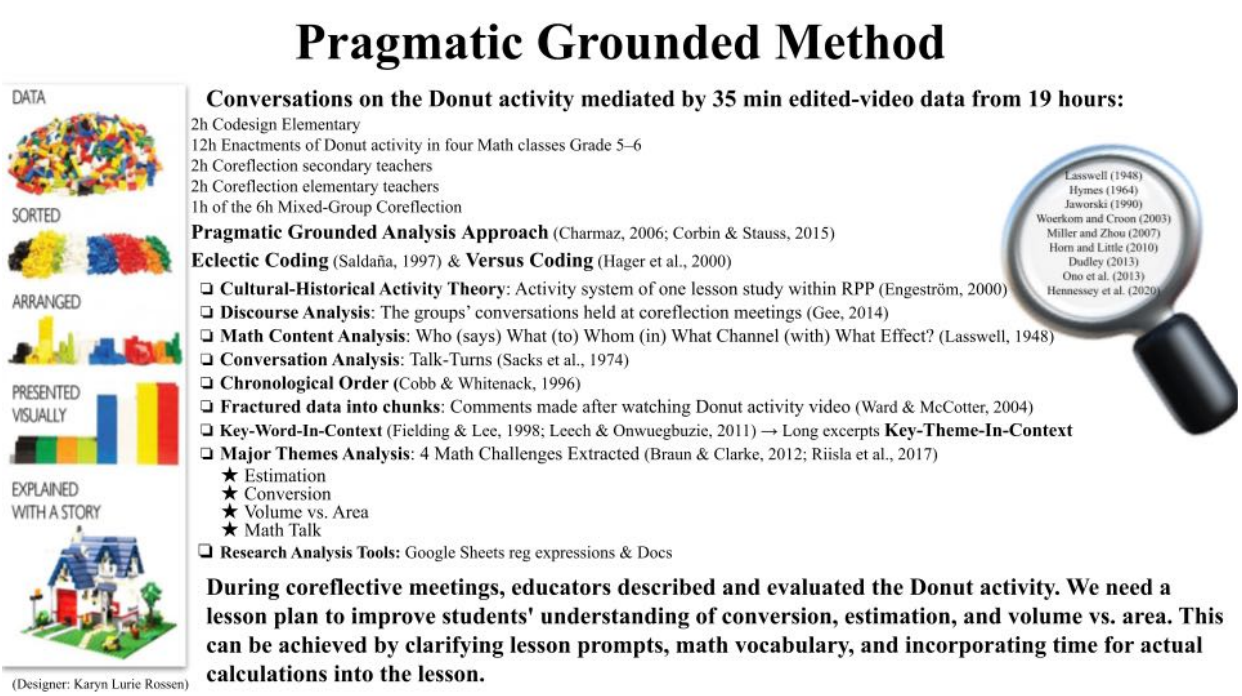
Ethnography techniques were instrumental in studying lesson study as a cultural activity with distinct territories, vernaculars, and argot. Therefore, adopting a socio-constructivist approach recognizes the 'dynamic interdependence of social and individual processes' (John-Steiner & Mahn, 1996, p. 192) and emphasizes “language as the central cultural tool” (Vrikki et al., 2017, p. 211). Because each classroom and teacher group is influenced by its own socio-cultural and historical factors (Cazden, 2001), they develop their “own language” (Green et al., 2007, p. 115), and providing “thick descriptions” can contribute to rich interpretation (Geertz, 1973, p. 3). For these reasons, I transformed significant portions of transcripts into nine vignettes to present key themes in context.

This flexible strategy allowed me to examine each utterance using the storyline technique proposed by Strauss and Corbin (1990), presenting and integrating the grounded story of the three meetings in a coherent and meaningful way. Drawing on the work of many researchers, I gained insights into various conceptual views on the process of analyzing video-mediated co-reflections. Among these researchers, there is a consensus that co-reflection mediated by videos of learning lessons involves teachers making connections between what they observed in the classroom and possible improvements to teaching practices and lesson plans in the future. To explore this process, I conducted a qualitative analysis of three coreflection meetings and identified four themes to highlight the paradoxical and ambiguous nature of video-mediated coreflection (Virkki et al., 2017).

Figure 7 depicts a sequence of LEGO bricks initially presented in a pile as raw data with emboldened key phrases, then sorted and arranged in a juxtaposed manner to visually interpret the data. This process is further explained using a storytelling approach that unearths four main themes. To explain the data section of Figure 7: Coreflective conversations on the Donut activity that were mediated by 35 minutes of edited-video data from a total of 19 hours of recordings. The study involved 2 hours of Elementary Codesign, 12 hours of enactments from four Grade 5–6 Math classes, 2 hours of Secondary Coreflection, 2 hours of Elementary Coreflection, and 1 hour of the 6-hour Mixed-Group Coreflection.

Figure 7

Procedure for Pragmatic Grounded Analysis



Moreover, Figure 7 illustrates the eclectic analysis procedures: including Discourse Analysis (Gee, 2014), Content Analysis (Lasswell, 1948), Conversation Analysis (Sacks et al., 1974), and Major Themes Analysis (Braun & Clarke, 2012; Riisla et al., 2017), among others. Using the Cultural-Historical Activity Theory (Engeström, 2000), the activity system related to one Lesson Study procedure within the RPP was identified as the unit of analysis. Talking-turns related to the Donut activity were analyzed based on their face value as projected by the edited video. Following Hiebert et al. (2002), the lesson was the unit of analysis to integrate traditionally separate components of knowledge. The use of Hager et al.'s (2000) versus coding helped to compare the perspectives of elementary and secondary math teachers, capturing conflicts between and within groups. The group level was the unit of analysis, with a focus on comments related to the Donut activity.

Mainly the five categories by Ono et al. (2013) were applied to categorize and evaluate coreflection. The first category, *Achievement of Lesson and Curriculum Objectives*, included thinking skills, conceptual understanding, and inconsistency with objectives. The second category, *Logistics*, covered management, planning, and the use of materials and teaching aids. The third category, *Teaching and Learning Strategies*, pertained to instructional techniques and practices. The fourth category, *Teacher Behavior*, referred to teacher characteristics and communication skills. The fifth category, *Lesson as Experienced by Students*, involved student learning, behavior, and interaction with one another. Then the educators’ statements are sorted into different levels of reflection, such as surface, deep, or critical reflection. Using four levels of reflection that ranged from (no reflection-level one) to comments and suggestions linked to recognized good practices (surface-level two), curriculum goals (deep-level three), or theories (critical reflection-level four). While determining the depth of reflection was a challenge, contending with these categories allowed me to gain a better understanding of the teacher's conversation and the factors that influenced the codesign and enactment of the Donut activity.

In addition, Miller and Zhou's (2007) fourteen categories were used to distill comments about teacher, lesson, and general classroom factors that could impact instruction quality and student engagement. These categories cover topics such as teacher personality, lesson presentation, student participation, classroom management, lesson content, and technological pedagogical content knowledge (TPACK). The combination of these fourteen categories provided insights into instructional practices.

Groundedness can evoke a sense of non-reliability and even non-validity. To avoid such pitfalls, I followed Campbell et al.'s (2013) method to enhance inter-coder reliability in this study. David et al. (2004) illustrated this laborious step by reporting eight rounds of coding in their study of HIV behavior. Campbell et al. (2013) reported that achieving intercoder agreement requires considerable time, with one page of text from 24 semi-structured interviews taking 11 rounds of coding to generate strong intercoder reliability, even without an initial or final codebook. The categories assigned to segments of texts underwent only three rounds of coding at different times, by me. In case of occasional disagreements during interpretation, alternative interpretations were considered while keeping the CCCM group norms in mind.

To categorize topics, I employed Charmaz's (2006) approach to create headers and detect patterns, resulting in grounded, pragmatic interpretations using Google Docs and Sheets. The resulting categories emerged from the data during the analysis process, without being preconceived. I conducted three cycles of coding (Saldaña, 2016) to generate categories, themes, and concepts by managing, filtering, and highlighting salient features of each meeting. While coding theories were relied upon midway, in the end, thematic analysis similar to Braun and Clarke (2012) emerged as the most fitting approach. The contribution of this study lies in blended methods, resulting in a "relatively unique [blend of] methodological considerations" (Prasad & Prasad, 2002, p. 6). Using Horn and Little's (2010) method to focus on macro conversational routines can uncover many challenges.

Various techniques were used to analyze the data, including Conversation Analysis to break down comments made after watching the video of the Donut activity into talk-turns. Key-Word-In-Context (KWIC) was also used to identify patterns and redundancies across codes. However, instead of the discourse-in-use approach (Bloome & Clark, 2006), Key-Themes-In-Context was found to be more useful in describing what happened during each meeting. This involved using long excerpts, for which I apologize to the reader. These research analysis methods provided a comprehensive and detailed understanding of educators' reflective conversations and their efforts to enhance student learning.

During the coding process, decisions were made about whether to code for presence or frequency of a concept. If a word was repeated in the same utterance, it was counted as many times as it was used. To logically distinguish between concepts, text was coded as it appeared, and different word forms were tracked (e.g., guide or guiding). To understand the math challenges in the activity, words related to math concepts, such conversion, estimate, and math talk, in educators' speech were focused on. Despite not having an initial codebook, this approach provided a nuanced understanding of the data. In addition to coding for specific words and phrases, concepts were inferred from surrounding text. For instance, when a string of words implied a particular concept, such as “you may have this long, empty part of your box,” those word strings were considered similar based on their implicit meaning in relation to the volume and area topic. To distill the data further, conceptual and relational content analysis techniques were applied to categorize math concepts and examine their sequence, such as the order in which math concepts and challenges appear (e.g., difficulty converting). However, content analysis has some drawbacks, such as being reductive, focusing solely on participants' words or phrases without context, and disregarding ambiguous meanings and nuances.

To check for workability, I examined whether core themes emerged with each new utterance. Constas (1992) added six sources (external, rational, referential, empirical, technical, and participative) to justify a given set of categories. This study used a grounded approach (Charmaz, 2006; Glaser, 1992; Melia, 1996), The non-coding approach allowed for the identification of overarching themes without the added complexity of coding. I triangulated findings using a mix of data (from codesign to enchantment to coreflection and end-of-year interviews) to develop a nuanced understanding of factors that contribute to effective lesson enactment. Discourse analysis of coreflection broadened conceptualizations of the challenges facing education, including math concepts, lesson objectives, lesson assessment, limited content knowledge, and the lesson's representation.

Through multiple hearings of conversations, I identified some utterances that corresponded to categories created by prominent researchers in the field. The following researchers' categories were used as main lenses for coding teachers' utterances: Lasswell's (1948) categories of communication functions, Hymes's (1964) ethnography of communication, Jaworski's (1990) communication strategies, Woerkom and Croon's (2003) interaction analysis, Miller and Zhou's (2007) categorization of communication acts, Horn and Little's (2010) communicative competence, Dudley's (2013) discourse analysis, and Ono et al.'s (2013) conversation analysis. Classroom interactions were coded using categories from Hennessy et al. (2020). Below the multi-year categorization process is listed with brief descriptions or quotes:

- **Lasswell (1948) five categories:**
  - Who: Partners involved in that specific segment
  - What: Discussing confusion about estimation in teaching and a critical incident involving students not listening to each other
  - Whom: The conversation is between the participants mentioned in the segment
  - In What Channel: The conversation takes place face to face
  - With What Effect: The partners are discussing the issue in math
- **Hymes (1964) seven categories:**
  - Setting (S): Description of context, coreflection
  - Participants (P): Who is involved
  - The (E) ends addresses the outcome
  - The Act Sequence (A) is how the educators discuss and analyze a specific aspect
  - Instrumentalities (I): The conversation is conducted verbally and is supported by visual aids
  - The conversation centered around Key (K) themes.
  - The Genre (G) of the conversation corresponds to professional learning and development
- **Jaroski Jaworski (1990) two categories:** Account Of (Descriptive), Account For (Justifications)
- **Woerkom and Croon (2003):**
  - Mathematical Thinking: Discussing the students' abilities in mathematical thinking and how their estimates are off.
  - Teacher-Student Interaction: raising questions about the student's behavior due to misunderstanding
  - Student Behavior: mentions a student's behavior is due to the student trying to be difficult.
- **Miller and Zhou's (2007) Fourteen categories in three modified umbrellas:**
  - *Teacher*
    - Teacher Personality: Passionate, Patient
    - Interpersonal Affective: agreement or affirmation (e.g. "yeah")
    - Lesson Presentation: “So that's another seven minutes or eight minutes of the class.”
    - Student Participation: “I think it's proof that students, they're not listening to each other”
    - Motivation: teachers discuss students' motivation or lack thereof (e.g. "If your low estimate, like Ben said, is a one, when it's obvious looking at the picture that it's higher."
    - Physical Classroom Environment:
    - Classroom Management: And I go around, and they see with my marker, I am like, I go: "Okay I see Don, he's listening. I see John, he's asking questions.
  - *Lesson comments*
    - Lesson Content: teachers discuss mathematical concepts such as estimation.
    - Lesson Tools: Donut box image projected
    - Lesson Structure: guided or unguided approach to revealing the information
    - Student understanding: discussion veers towards the students' understanding of estimation
    - Teacher Questions: Discussing prompts used: “Give an estimate that you know is too low”
    - Teacher TPCK: “I know” or “I don’t know” or RÉCIT: “So do we work on that rigor”
  - *General description*
- **Horn and Little (2010) six categories:**
  - Normalizing Move: indicating that differences in implementation are typical, but such reassurance overlooks potential inconsistencies in student learning

- Mathematical Content: estimation, conversion, volume vs. area
- Mathematical Representation: denotes how the conversation includes visual aids
- Mathematical Processes as educators discuss the students' abilities
- Mathematical Dialogue: is a given since the conversation on classroom dialogues
- Mathematical authority partners are experts in the field of education

- **Dudley (2013) only 10 categories out of 31**
  - Proposal: "I want to go back and talk about it in the groups too, but I think, again, that math vocabulary, and especially if the goal of the teachers was looking for rigor"
  - Echo: "It is digits."
  - Suggestions: "It's not about toute les éventualités. It's about being conscious of the possibilities of coming across obstacles"
  - Corrections: "It's digits."
  - Agrees: "Yeah."
  - Explanations: "and I'm like, "you know, honestly, this is something you're going to learn in grade nine"
  - Recounts: "She figured out how many meters. And then there was another person"
  - Reasonings: "Because I know, if I was that kid that's all I'm thinking right now"
  - Questions: "Any thoughts on what's going on here?"
  - Observations: "It's something that we're faced with all the time, kids giving us this curveball."
- **Ono et al. (2013) five categories and four levels:**
  Five categories
    - Achievement of lesson and curriculum objectives: conceptual understanding
    - Logistics: covered management, planning, and the use of materials and teaching aids
    - Teaching and Learning Strategies: instructional techniques and practices.
    - Teacher Behavior: teacher characteristics and communication skills
    - Lesson as Experienced by Students: involved student learning, behavior, and interaction with one another.
  - *Four Levels*
    - Level 1: no reflection
    - Level 2: surface descriptive comments with suggestions linked to good practices
    - Level 3: Deep: comments with suggestions linked to curriculum goals
    - Level 4: Critical: suggestions and linked to theories
- **Hennessy et al. (2020) eight categories:**
  - Invite Elaboration (I)
  - Make Reasoning Explicit (R)
  - Positioning and Coordination (P)
  - Build on Ideas (B)
  - Reflecting on Conversation or Activity (RD)
  - Connect (C)
  - Express Ideas (E)
  - Guide direction of conversation or activity (G)

The eclectic procedures used to analyze the data in this study combined descriptive non-coding and interpretive mixed-coding methods. The data included various sources such as videos, interview transcripts, field notes, and a codesign questionnaire. The outcomes are reported in nine vignettes next.

What? Coreflective Conversations on the Donut Activity

Sabrina: Yeah, I agree. And our last face-to-face meeting, too, I realized that the high school teachers had also watched the elementary school teachers, and the different versions, which I thought was really interesting too, to hear their perspectives. So, yeah.

In the following sections I will present the outcome of the coreflective conversations among math teachers regarding the Donut activity. The What section is divided into three main parts. The first part focuses on the coreflections of secondary math teachers, with four vignettes that capture their initial reactions after viewing the activity, as well as their responses to specific challenges related to estimation, conversion, and volume/area. The second part examines the coreflections of elementary math teachers, with three vignettes that explore their reactions throughout the viewing of the activity, as well as their responses to the estimation and conversion challenges. The third part explores the coeflections of the mixed-group of math teachers, with two vignettes showcasing their post-viewing reactions and my researcher's comment on reducing a large number. Together, these vignettes provide a context of the ways in which math teachers engaged with and coreflected on the Donut activity, and the implications for promoting rigorous justification in math classrooms and the themes unearthed.

The outcomes of the pragmatic grounded analysis are presented in this “What?” section, organized chronologically and grouped by meeting times. Borton's (1970) reflection three-stage model, which aims to increase “awareness” (*What?*) by evaluating information (*So What?*) and promoting experimentation (*Now What?*) with new actions (Rofle, 2014, p. 489). This approach is similar to Miller and Zhou's (2007) focus on “who notices what?” (p. 323). The educators involved in the collective reflection meetings watched, analyzed, and evaluated students' understanding of mathematical concepts using an improvisational approach guided by their initial reactions to the Donut activity. The first research question examines what was discussed in the meetings through video segments of the same educational activity, which is further divided into three sub-questions to further elaborate the teachers' coreflections:

- What did the Secondary Math Teachers Collectively Reflect About?
  - Vignette I Secondary Math Teachers Post-Viewing Initial Reactions (Theme 1)
  - Vignette II Secondary Math Teachers After Listening to Estimation Challenge (Theme 2)
  - Vignette III Secondary Math Teachers After Listening to Conversion Challenge (Theme 3)
  - Vignette IV Secondary Math Teachers After Listening to Volume/Area Challenge (Theme 4)
- What did the Elementary Math Teachers Collectively Reflect About?
  - Vignette V Elementary Math Teachers' Reactions Throughout Viewing (Theme 1)
  - Vignette VI Elementary Math Teachers After Listening to Estimation Challenge (Theme 2)
  - Vignette VII Elementary Math Teachers After Listening to Conversion Challenge (Theme 3)
- What did the Mixed-Group of Math Teachers Collectively Reflect About?
  - Vignette VIII Mixed-Group Math Teachers' Reactions Post-Viewing (Theme 4)
  - Vignette IX Mixed-Group Researcher's Comment

The focus of this section will be on reporting what was actually discussed at each meeting by the educators after they watched an edited video of the Donut activity. For this study, I interpret the educators' reflections using nine purposive vignettes from three meetings. The outcomes of this study are reported using direct quotes and describing how the Donut activity case was examined by educators. The coreflections were then inductively forged into four themes and will be discussed in the *So What* section. Lastly, the *Now What* section proposes directions for future action plans in improving the Donut activity and Lesson Study procedures.

What did the Secondary Math Teachers Collectively Reflect About?

> *It is well known that the tongue is the noblest part of the body, and what would thought be without words?… What would music be without the exécutant* (Roland, 1904).

The four vignettes in this subsection will report on what was discussed from the vantage point of the secondary math teachers. On a breezy early morning mid March 2017, we gathered with four researchers, four secondary math teachers, and four consultants gathered around a couple of rectangular desks in a small conference room in the basement of the Sunny Side school board to watch and discuss video representations of a lesson which was codesigned and enacted a month earlier by the elementary teachers. The first research question invites a description of the collective reflective talk: What was discussed during the secondary group collective reflection? Presented below is the coding outline of the secondary math teachers' coreflection, with page numbers indicating the extent to which each specific category was detected within the transcript.

Vignette I Secondary Math Teachers Post-Viewing Initial Reactions

Using the categories proposed by One et al. (2013), this vignette presents a design dilemma related to the level of guidance provided to learners, as observed by Ben and Lin. In the video Sabrina asks a guided question, "Estimate how many donuts are in the box?", while Kate and Dona take an open-ended approach and ask, “What questions come to mind when you look at this image?” Below are the categories and levels that can be extracted using Ono et al. (2013):

Utterance	Category	Level
II.2. PI: So, **any comments about those questions?** Or other thoughts that came to your mind when you were watching this?	Logistics	1
II.3. Ben: Any time you're doing anything like this, and you're asking, especially elementary kids, but young high school kids as well,	Teaching and Learning Strategies	2
II.4. PI: hmm	N/A	
II.5. Ben: Well **I guess any, but elementary for sure, you have to be very specific in terms of questions that you're asking them to think about**, unless you're... The question is: "Why would they make such a big box?" It's a perfectly legitimate question,	Achievement of lesson and curriculum objectives	3
II.6. PI: Yeah	N/A	
II.7. Ben: And I guess you don't want them to not have those ideas, those questions, **but in terms of guiding them, math questions that are relevant to the... Because if not, they could be wasting their time on things that don't really focus on what the goal is.**	Teacher Behavior; Communication skills	3
II.8. PI: hmm. Yeah.	N/A	
II.9. Lin: Yeah, I wrote the same thing, basically, that, well in particular, the **first video where they had to be guided towards the volume or area** focus, as opposed to price and... Whereas the second video, the questions **were more guided, so the kids right away went**...	Achievement of lesson and curriculum objectives	3

At the beginning of the excerpt the principal investigator (PI) facilitated the discussion with an open-ended, somewhat logistics oriented question referring to the questions that were raised in the Donut activity. During the discussion two teachers, Ben and Lin, explore scaffolding issues in relation to the openness of math prompts, and propose a goal-oriented view of guidance. Two utterances fall under the 'Achievement of lesson and curriculum objectives' category, with a couple of ‘Teaching and Learning Strategies' and one 'Logistics'. If we look for Ono et al. (2013) levels, then this conversation ranges from Level One to Level Three, and no Level Four utterances were heard in this conversation, in my opinion. Specifically, some utterances displayed no reflection (Level One), we heard a few descriptions with explanations

(Level Two), we also heard a few insightful action-oriented suggestions (Level Three), and no suggestions that were linked to overall curriculum goals (Level Four).

Among the many dilemmas illustrated by this short excerpt is to what extent elementary teachers must be "very specific in terms of questions that you're asking [especially elementary students] to think about" without stifling students' thinking about math problems. Ben continues his explanation: "in terms of guiding [students] math questions that are relevant to [...] Because if not, they could be wasting their time." What do each educator group determine as the problem in this teaching episode? What do they suggest as a way of fixing the problem?

> II. 28. Lin: **They're just two completely different approaches,**
> II. 29. PI: Hmm
> II. 30. Ellen: Hmm
> II. 31. Lin: but they bring out different skills, right?
> II. 32. RÉCIT: Absolutely**. And that was the debate they were having when they were planning, too**, you know? [15:00] In terms of..., You know**, how much do we allow that freedom, and then that fear of them going off to all different levels, and timing that too, right? How much time do we allow them to go off there, and then bring them back into focus?**

In many ways, Lin captures the essence of the conversation that took place during the secondary math teachers' coreflection meeting. The meeting involved watching three elementary teachers introduce the Donut activity in different ways, with mixed outcomes, as we will see from the incidents drawn from whole-class discussions. In this vignette, we can see that the secondary teachers were echoing Freund's (1990) investigation into whether students learn more effectively through a discovery learning approach, as proposed by Piaget in the 1930s, or through a guided learning approach that expands the zone of proximal development, as suggested by Vygotsky (1978). The secondary teachers discussed how some elementary teachers used a minimally guided approach versus a guided approach as a model for open-ended semi-guided activities. Although they noticed the difference in approach, they quickly conceded that "there isn't one right way of doing it" and acknowledged the need to adapt their teaching approach to meet their students' needs.

Vignette II Secondary Math Teachers After Listening to Estimation Challenge

This vignette features the first classroom incident presented as a skit to secondary math teachers. It has 99 lines dedicated to it. The students were debating each other's estimation of the number of donuts in a box, highlighting the confusion that can arise with the concept of Estimation. RÉCIT's request for clarification during the discussion–"We have two different opinions. Could you justify why you think that?"–prompted Ben and others to provide more detailed explanations for their comments, enriching the conversation. Below are the first 39 lines.

> Lin: thank you [laughter]
> **Ellen [whispers] to Lydia: What were they talking about?!**
> **RA1: So, how do you feel about this exchange, and the conversation? Would you have responded differently? Is there a particular part that jumps at you, in terms of student rebuttal? Or how the teacher replies?**
> **Lydia: Well, I mean, at the beginning they sounded quite confused, because I was confused.**
> Ellen: Hmm
> Lydia: At the first page I was kind of lost at what they were talking about. But then when they started asking, like: "What is an estimation?" let's, let's, Let's….And then they started throwing in their answers, "an approximation," "an educated guess," "I based it on something," then with the guiding questions, I'm like: "Oh! That's what they were talking about." But it took until like line twenty-eight
> Ellen: Hmm
> Lydia: before it all became clear to me.
> RA1: And do you agree with the definition of an approx…, an estimate? **Is this an accurate, mathematical explanation?**
> **Ben: It's... It has to be within the realm of possibility.**
> RA1: ok.
> Ellen: Hmm. Do we know how old these students are?
> RA1: I think they're/
> PI: Grade six
> RA1 /grade five and six.
> Interim Elem. Consultant: it's grade five
> Ellen: ok,
> Lydia: I/
> Ellen: /Because maybe our definition of what an estimate is with older kids will be more "theoretical" [gestures quotation],
> RA1: Hmm
> Ellen: whereas with the young kids, you have to keep it in their vocabulary.
> Lydia: Yeah.
> Ellen: It makes sense to them.
> Lydia: I think it makes sense for the age they are. That's

Ellen: Yes AND RA1: Hmm
Lydia: I don't see a problem with it.
RA1: It's because they guess, I would say. If you asked the kids: "Guess how many are in the box," then they have to consider composite numbers, you know, and non-composite numbers, and taking into account prime numbers. We can't throw, you know, something that would not be...um...
Lin: Divisible AND Ellen: Divisible.
RA1: Divisible. Whereas, when you say an estimate, you're actually being more precise, and then when you say: "Too low estimate," then you're asking them to go like, really lower, and when you're asking them for a reasonable estimate, you're asking for something that is the "best-lowest estimate". Like, so there's different words that lead to different numbers of estimations.
Ben: Personally I think the teacher was very patient.
Collective [chuckles] AND Lydia: Yeah. AND Ellen: yeah.
Ben: I wouldn't have been nearly that patient.
RA1: Hmm
Ben: Obviously, if there are…, you can *see* that there's more than one.
Interim Elem. Consultant: /then that's the wrong estimate.
Ben: So to estimate that there is one, you're wasting my time. You were not getting anything out of this discussion. Right? So right away, if it had been me, I would've either...I, I probably would've directed the other kids who are saying it's not right, to: "Okay explain, let's go. Show why it's not right."
Ellen: Hmm
Ben: The fact that you can visually see more than one, to say that there's one...
RA1: But isn't it mathematical to say an answer that: "You know is too low, estimate something that you know/
Ben: /no
RA1: is too low,
MB: /no
RA1: zero is the lowest, and one is actually the lowest estimate you can get.
Ben: No, because then you're not estimating.
RA1: Okay.
Ellen: Hmm
Lydia: no
Ben: Within, within..., within the term "estimate"
RA1: Hmm
Ben: there has to be an understanding, and if you're estimating,
Lin: /reasonable
Ben: you're going to estimate something that is in..., that is withi**n the realm of possibility**. [taps table at each point]
RA1: Okay.
RÉCIT: So that goes back to what you were just talking about before, when you said: **"Is this an accurate definition of estimating?"**
RA1: Hmm
RÉCIT: We kind of have two different opinions. So maybe if you could, like, justify why you think that...
Ben: Well, an educated guess, I, I don't necessarily agree with it being an educated guess. Um…, A reasonable guess to me would make sense.
RA1: Hmm
Ben: Um, educated, I'm not sure how that would fit in there. But I understand what, what they're saying.
Ellen: Hmm
Ben: Um…, But yeah, you know, to say that there's... Then I could say, if it's too high, I could say anything.

RA1: Yes, they said a million. They/
Ben: Well then what's/
RA1: they overestimated
Ben: Then what's the point of even doing this? Because we're not...
RA1: yeah
Ben: It's…, it's not a constructive answer.
RA1: yeah
Ben: It's not helping anybody get to where we need to get to. [10:00]
Ellen: To the point where I didn't even realize... I, I thought this was about the donut problem, but then when they started talking about the one and the two, I thought it was a whole different problem. **I didn't even know where this is going, at one point.**
Lydia: So imagine being a kid in that class. [laughs]
Ellen: yeah.
PI: So, again, it's a question of context. So
Lydia: Yeah. AND RÉCIT: yeah.
PI: This conversation happened in the context of the Krispy Kreme box.
Ellen: So then I'd have to agree with Ben.
PI: Yeah. So there are ways of improving that conversation, by bringing a definition or another point of estimation.
Lin: Right, because I remember, um, last... year here, having…, you know when you, when you're bringing up something to kids, when you're allowing everybody to contribute, it becomes confusing.
Ellen: Hmm
Lin: Whereas if you're…
PI: [inaudible 10:51]
Lin: right, and then they kid: "Is that it? Is that it?!"
Ellen: Hmm
Lin: Whereas if it were more … guided
Ellen: Hmm
Lin: you would avoid all these..
Lydia: Hmm
RA1: There was a...you know, this was a three minute exchange, and there was another one for the too high estimate.
PI: Hmm
RA1: So that's another seven minutes or eight minutes of the class. That meant fifty five minutes...
Interim Elem. Consultant: It shows where they're - sorry [RÉCIT]. It shows where they're at
RA1: Hmm AND Ellen: Hmm
Interim Elem. Consultant: in terms of mathematical thinking.
Lydia: yeah
Interim Elem. Consultant: If your low estimate, like Ben said, is a one, when it's obvious looking at the picture that it's higher.
PI: Hmm AND Ellen: Hmm AND Lydia: yeah
Interim Elem. Consultan**t: And so their idea of estimate also**
Lydia: yeah
Interim Elem. Consultant: is off,
Lydia: Hmm
Interim Elem. Consultant: and that's why when they get
PI: yeah
Interim Elem. Consultant: an application question, situational problem, when they have a budget of two hundred dollars, and then they're writing that they spent two million dollars, that they don't even make the link. Therefore, they're not even thinking
Ellen: Hmm
Interim Elem. Consultant: in terms of context.

Lydia: yeah
Interim Elem. Consultant: And so if the kids were able to estimate low or high within reason, at least…, then they could have a reasonable estimate in between those. The fact that they can't even estimate too high or too low means that their notion or /
Ben: /in all honesty
Interim Elem. Consultant: /concept of estimation/
Ben: / When you started reading this, my idea was that…, whoever that student is, was just being a pain in the ass. [collective Hmm]
Ben: No seriously. Like, to me, what are you doing?
PI: Hmm AND RA1: Hmm
PI: You're saying one. Is it just because technically you want to be right?
Interim Elem. Consultant: yeah
PI: Like,
Interim Elem. Consultant: yeah AND Lydia: yeah AND Ellen: Hmm
Ben: you'd have to know who this student is. If it was a genuine, that was the estimate, okay,
Ellen: Hmm
Ben: grade six?! That?!
Lydia: Hmm

If we apply Miller and Zhou (2007) categorization to make sense of conversations that followed the viewing of the Donut, then rapid surface coding of the excerpt above reveals some Interpersonal/Affective (A) codes (e.g., patience on the part of the teacher). Additionally, the conversation reveals some expressions of surprise or confusion (e.g. "That?!" referring to a grade six student saying "one" is an estimate) and some agreement or affirmation (e.g. "yeah", collective "hmm"). Additionally, the conversation includes some Motivation (M) as teachers discuss students' motivation or lack thereof (e.g. "If your low estimate,...is"). Lesson Content (LC) is also detected when teachers discuss mathematical concepts such as estimation. Student Understanding (SU) is also included when the discussion veers towards the students' understanding of estimation. For example, "because that student doesn't understand the true definition?" However, this excerpt does not provide any explicit examples for Lesson Structure (LS), Lesson Presentation (LP), Student Participation (SP), Physical Classroom Environment (PCE), Lesson Tools (LT), Classroom Management (CM), Teacher Questions (TQ), General Description (GD). Moreover, the same excerpt using codes from Ethnographic Communication as proposed by the SPEAKING model by Hymes (1964) can be categorized and it would reveal some redundant information named differently:

> Setting (S): A coreflection meeting or discussion among several teachers, consultants, and researchers.
> Participants (P): RA1, Interim Elem. Consultant, Ellen, Lydia, PI, and Ben.
> The (E) ends addresses the outcome to discuss and analyze the students' abilities in mathematical thinking and estimation.
> The Act Sequence (A) is how the educators discuss and analyze a specific aspect of the students' abilities in mathematical thinking and estimation, specifically their ability to estimate low or high within reason.
> The conversation is conducted verbally and is supported by visual aids such as pictures or videos pertaining to code Instrumentalities (I). The conversation is collaborative in nature and informal within a formal setting and, with participants contributing their thoughts and ideas while respecting group norms.
> The conversation centered around the students' ability to estimate low or high within reason Key (K).
> The Genre (G) of the conversation corresponds to PLD, as it involves teachers, consultants, and researchers discussing and analyzing educational content revealed by the Donut activity.

Alternatively using the codes from Horn and Little (2010) we can see this conversation can be macro coded as follows: The conversation centers around the students' abilities in *mathematical thinking* and estimation, specifically their ability to estimate low or high within reason, this is math content. The *math representation* denotes how the conversation includes visual aids such as pictures and videos. There are *math processes* as educators discuss the students' abilities in mathematical thinking and estimation. Additionally, *math dialogue* is a given since the conversation is collaborative with participants contributing their thoughts and ideas verbally. Lastly, the *math authority* is deduced by the fact that the participants are teachers, consultants, and researchers, and they can be considered experts in the field of education analyzing and discussing the students' abilities in mathematical thinking and estimation. Next, they analyzed the causes of the students' difficulty in estimating and making connections to real-world situations. Finally, they planned for future actions to improve students' understanding of estimation and evaluated the students' performance in terms of their ability to estimate.

If instead codes from Woerkom & Croon (2003) were applied to this segment of the vignette, then four codes can be accounted for. The code *math thinking* when the Interim Elem. Consultant discusses the students' abilities in mathematical thinking and how their estimates are off. The entire conversation revolves around the topic of *Estimation* and the students' understanding of *it*. The *teacher-student interaction* code emerges when Ben raises a question about the student's behavior and the possibility of it being because the student doesn't understand the definition of estimation. The RÉCIT also touches on the teacher's role in helping students understand the concept. And lastly *student behavior* code can be used when Ben also mentions that the student's behavior can be due to the student trying to be difficult. Let’s continue reading the transcript of Vignette II as the next 58 lines can be examined with Miller and Zhou (2007):

Category	Comments
Interpersonal/Affective	Chuckles and hmms
Lesson Presentation	RÉCIT: So do we work on that rigor PI: yeah RÉCIT: of the understanding of the concepts? And is that an aspect that we want to look at? And, and…,
Student Participation	**RÉCIT: or maybe it's, as well you know, logically thinking,**
Motivation	PI: We're targeting to finish at eleven, right?
Physical Classroom Environment	No specific comments noted.
Classroom Management	No specific comments noted.
Lesson Content	RÉCIT: And that to me would kind of show that that child doesn't really understand what an estimate is. RA1: You make an estimate that you know is too low. And there was a lot of fractions, and there was a lot of, Interim Elem. Consultant: Fractions? RA1: yeah, **fractions, because the activity before was a Kool-Aid activity, and they had done fractions.** So we have a lot of pictures, like one over four hundred. And there was a lot of, like we didn't know how... like they don't know how to arrange their low and high, their range form. So you see them like write they're high or they're low. Visually, it's not... It doesn't give you a range, right? Because they haven't been prompted to put brackets in the middle or high. Interim Elem. Consultant: Even with a context like that you can have a higher estimate or lower, and know that that is somewhere between, right? Lin: Right. And this is where the reasonable... Interim Elem. Consultant: The reasonable. The key word. RÉCIT: So maybe it's not educated. Maybe it's, it’s in the vocabulary. RÉCIT: Reasonable RÉCIT: estimate. Like that. So I took that down as a note. Assist. director of Ed. Services When you were talking about educated guess, that was my thing too, because it's something we hear all the time, right?
Lesson Tools	No specific comments noted.

Category	Comments
Lesson Structure	**PI: specific activity. And then using that as an opportunity to consolidate the concept of estimate.**
Student Understanding	**RÉCIT: of the understanding of the concepts? And is that an aspect that we want to look at? And, and…,** **Lin: Well, doesn't it kind of lead to, like, mental math? Like when they're doing calculations, and then they use their calculator, and shouldn't red flags go up if you don't know how to estimate an answer? Like really, f I'm going to multiply twenty-three by thirty-five, and…, I estimate one?** [This topic is raised again at the mixed group meeting]
Teacher Questions	**RA1: /you asked for lowest** **RÉCIT: /you asked for the lowest, so here's my lowest.** **Assist. director of Ed. Services: Is it that you know it's too low? It's also that there's a certainty there. So I know for sure that one is too low.** **Assist. director of Ed. Services: I would've gone for safe.** **RA1: Yes. Mathematically, it's very technically correct.** **PI: And it's a question of wording the question, the activity, more**
Teacher TPCK	RÉCIT: So do we work on that rigor

Collectively, these utterances could be categorized differently by various researchers. However, for the purposes of this dissertation, I consider them all to be part of Theme 2, which examines the Estimation Challenge. This theme will be discussed in the So What section.

Vignette III Secondary Math Teachers After Listening to Conversion Challenge

The purpose of Vignette III is to report and interpret the conversation among secondary teachers after reading the critical incident two. To provide context, I have included excerpts from the mid-conversation of part two. Specifically, Ben discusses the conversion challenge from lines 77 to 181 in the skit:

II.77. Ben: Especially in something like this, **unit conversion becomes an issue,**
II.78. Lydia: yeah
II.79. Ben: because you're dealing with the donut, and you're dealing with a giant box,
II.80. PI: Hmm
II.81. Ben: so the same unit wouldn't be useful for the whole thing, because you'd either be dealing with extremely large numbers for the box,
II.82. PI: Hmm
II.83. Ben: or extremely large decimal numbers for the donut. Right?
ll.84. Ellen: Hmm.
II.85. Ben: So this difference between decimeter and decameters, I find that's been a problem coming up from elementary school, for the longest time. And I, I…, I don't really like the King Henry Died Mother Didn't Care Much, because it doesn't differentiate
II.86. PI: Hmm

II.87. Ben: and the D.
II.88. Lin: I have one.
II.89. Ben: You might have one, but the kids that all come up from elementary school, they're all reciting the King Henry Died Mother Didn't Care Much.
II.90. Lin: Well in high school I say Kids Hate Damn Math Dumb Crazy Math, and DAM for the decameters.
II.91. Ben: Yeah. So the whole DM and,
II.92. PI: Hmm
II.93. Ben: And **DAM, I think is not really explained very well**, and **it may lead to conversion** problems.
II.94. Ellen: Hmm
II.95. RÉCIT: So it's a strategy…, But that's a strategy that the kids have learned, but they haven't really looked at the concept of what they're doing in the conversion.
II.96. Lin: right
II.97. Lydia: no
II.98. RÉCIT: They know the strategy/
II.99. Lin: /Right.
II.100. RÉCIT: /but not/
II.101. Lin: /They all know what it is.
II.102. RÉCIT: They don't have an understanding of what that represents.
II.103. Lin: Right. AND II.104. Ellen: Hmm. AND II.105. Lydia: yeah.
II.106. Interim Elem. Consultant: It's not part of the elementary school program, which is also the...
II.107. Ben: Unit conversion isn't part of the elementary program?
II.108. RÉCIT: It is, but we don't do the decameter.
II.109. Lin: But we, I mean..., in everyday life, we hardly ever use it,
II.110. Ellen: We don't
II.111. Lydia: never use it
II.112. Lin: /right? So they
II.113. RÉCIT: ok
II.114. Lin: they don't have the experience with it either.
II.115. Interim Elem. Consultant: But they are exposed to it.
II.116. Ben: That's like saying the number line. The number line is a part of the curriculum, but we don't use zero. You know? [20:00] Why would you exclude decameters?! What's the...? If you're doing unit conversion, why aren't you doing the whole thing?
II.117. RÉCIT: Well, you're exposed to... I mean, they're aware that it's there, but it's not something that they necessarily
II.118. Interim Elem. Consultant: Yeah
II.119. RÉCIT: work with. I don't know.
II.120. [brief silence]
II.121. RÉCIT: But I think, I think you just made an interesting point that goes to what we've talked about in previous meetings.
II.122. Interim Elem. Consultant: at other meetings.
II.123. RÉCIT: [inaudible]
II.124. Ben: It might not be used, but I mean/
II.125. RÉCIT: /You're aware. There's an awareness.
II.126. Ben: It's still there.
II.127. RÉCIT: But I think what you're talking about is what we've been talking about to that whole... you know, a couple years ago we talked about that whole concrete, representational, abstract.
II.128. Ellen: Hmm
II.129. RÉCIT: Like, the kids are being shown a strategy: "Here, use it". And they're doing their little jumps, and they're moving their decimal,
II.130. Lydia: Hmm
II.131. RÉCIT: but without necessarily having that.../

II.132. PI: Conceptual
II.133. RÉCIT: /Conceptual understanding
II.134. Lydia: /Conceptual understanding
II.135. PI: yeah
II.136. RÉCIT: of what that means. So I think that's again an issue that is still present.
II.137. Ben: But here, that would become important,
II.138. RÉCIT: yes
II.139. Ben: to have that understanding
II.140. Ellen: hHmm
II.141. Ben: of what those conversions mean, right?
II.142. RÉCIT: Yeah, absolutely
II.143. Lin: But wouldn't they be asked something, it seems to me, like, things that you would measure, like I'd measure this in centimetres.
II.144. Lydia: Hmm
II.145. Lin: Don't they... what unit they would use to measure
II.146. Lydia: Hmm
II.147. Lin: different things? Because you're not going to use a kilometre for being a big box.
II.148. Lydia: no
II.149. Interim Elem. Consultant: They are going to use metres.
II.150. Lin: Metres? They never, ever use decimeters?
II.151. Interim Elem. Consultant: No.
II.152. Lin: or decameters?
II.153. Interim Elem. Consultant: No. <u>It's not part of the vocabulary.</u>
II.154. Former Sec. Math Consultant: /It's not part of the culture.
II.155. Ben: /Well I just like the [inaudible]
II.156. Ellen: No.
II.157. Lin: Well, I know it isn't
II.158. Ben: It's doing them a disservice.
II.159. Former Sec. Math Consultant: In France, or Europe, they use those units
II.160. Lin: Yeah.
II.161. Former Sec. Math Consultant: everywhere. You know, food containers,
II.162. Lydia: yeah AND II.163. Ellen: Hmm
II.164. Former Sec. Math Consultant: juice boxes, or whatever.
II.165. Lydia: yeah
II.166. Former Sec. Math Consultant: It's not popular here, you know?
II.167. Lin: No, we don't. We use millilitres, litres, metres.
II.168. Former Sec. Math Consultant: So, we don't use [inaudible]
II.169. RÉCIT: They're exposed to them.
II.170. Former Sec. Math Consultant: Yeah.
II.171. **RÉCIT: They're meant to develop an understanding, it's just not entering…**
II.172. Lydia: They don't practice.
II.173. RÉCIT: don't practice
II.174. Lydia: yeah.
II.175. Ben: So then, you can't really ask them questions about that and expect them to have any genuine understanding of what/
II.176. RÉCIT: /Well I think what that shows is a conceptual... It's a misunderstanding.
II.177. Ellen: Hmm
II.178. Former Sec. Math Consultant: Yeah.
II.179. Lydia: Well that's something we can work on, right?
II.180. PI: Yeah.
II.181. Lydia: If we've discovered it. We're like: "oh!"

In the excerpt above, notice how territories are delimited quickly by the Interim Elem. Consultant on line II.106 “It's not part of the elementary.” This type of intervention is what Jaworski (1990) referred to as–an *account for–a way* to justify an event to somewhat close the conversation. However Ben immediately asked a question: “Unit conversion isn't part of the elementary program?” Line II.107 to which the RÉCIT concedes with specification: “It is, but we don't do the decameter,” Line II.108. Then we see an acknowledgement that there are conceptual misunderstandings at play (Line II.133). By juxtaposing the conversations around conversion we see how frequently the topic was top of mind at the secondary groups' coreflection but not as much in the elementary meeting, but it was pretty much the first topic of the mixed-group meeting as well. Once there is an acknowledgement that concepts are not fully taught at the cycle 3 of elementary, we can see that on Line II.175 Ben concludes: “So then, you can't really ask them questions about that and expect them to have any genuine understanding of what/” To get cut by RÉCIT: ”I think what that shows is a conceptual... It's a misunderstanding.” (Line, II.176). Two agreements (e.g., II.177. Ellen: “Hmm”; and II.178. Former Sec. Math Consultant: “Yeah”) with Lydia positively reclaiming the territory: “Well that's something we can work on, right?” (Line, II.179).

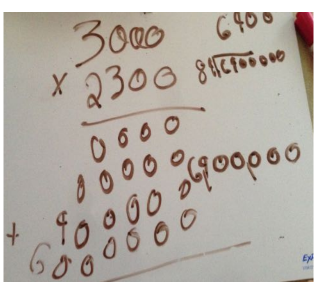
Let's fast forward to another mention of conversion during part 3 of the meeting:

> III.388. Interim Elem. Consultant: **Correct me if I'm wrong.** The kids were struggling with the millimeters, and they were given here... Like: “What if I tell you this extra piece of information? There's a row of thirty-three, of thirty-two, and a row of twenty-five.” **Now it was given more specific for those kids that had a harder time with the other information. It was just making it easier**. So it started with the challenge, with the millimeters, to allow them to struggle a bit and work on the math, and then the other whole is the extra information. “What if we tell you now, in this box, there's thirty-two donuts by twenty-five donuts? Now, can you work with that?”
> III.389. PI: Hmm
> III.390. Interim Elem. Consultant**: So the whole lesson was geared towards what information is important?** What's not? What can you work with? What can't you? What was useful? So it was more around that. **Not actually solving, but looking at relevant information.**

The original purpose of the activity was to increase students' metacognitive abilities related to estimation, but the reasoning provided by the interim elementary consultant may obscure this purpose. As noted in the Data Sources section, official records of students' calculations were not collected, but the camera angle captured instances of their work on whiteboards. Figure 8 shows a student's calculations that exemplify a common challenge faced when making sense of multiplication products: what does it mean to have the product in millimeter square (3000 mm x 2300 mm = 6900000 mm²) when we're talking about donuts? Dividing the area by donuts with a diameter of 89 mm may fall under the Progression of Learning's requirement that “students gradually learn to interpret remainder” (Ministère de l'Éducation, du Loisir et du Sport, 2009, p. 12). However, it raises the question: at what cost do we make things easier? If the problem is not solved, then how can students know if they have asked relevant math questions? The challenge is to find ways to help students understand the process of estimating and calculating the number of donuts in the box.

Figure 8

Students Puzzled Over the Product



Now that all the pre-context is set, this puzzle in Figure 8 gave rise to reactions on the second critical incident excerpted below as Vignette III as it led to the emergence of Theme 3:

IIII.241.[Mid-reding] RA1: Hold on for a second. Before we go on to the next part, think about it. **How would you have answered this?** Before we go on to what the teacher replies. [brief silence 17:31]

Lin: I think she maybe wasn't expecting to end up with something…, a value like that or

have to deal with this, and now, what do I do?
Ben: you are talking about the teacher?
Lin: Yep. AND PI: yeah AND Ellen: Hmm
Lin: Because I wasn't expecting to have to explain going, conversions and area conversions
Lydia: Hmm
Lin: versus
Ellen: Hmm
Lin: linear versus...
PI: Yeah.
Lydia: Can I continue?
PI: yes. AND RA1: Sure. AND Lydia: Ok, so…
PI: and then she goes to the black board
Ellen: That's the one with the conversion at the bottom?
AB & RA1: yes

IIII.242. [18:21-20:08] Reading Critical Incident 2–Conversion Challenge Part 2
IIII, 243. RA1: Thank you. Comments for this one?
IIII, 244. Lin: She doesn't answer her question.
IIII, 245. RA1: no
IIII, 246. Lin: Or his.
IIII, 247. Lydia: no she doesn't
Lin: And, she said: "The area inside." That's a little confusing.
RA1: Hmm.
RÉCIT: But in terms of the math talk strategies that we've talked about, what do you hear her doing?
Lin: The teacher?
RÉCIT: The teacher
Ellen: She's trying to consolidate and re-explain to the girl with her meter stick, explanation for the students to understand.
RÉCIT: So she's trying to re-state?
Ellen: Hmm
Lydia: What the kids said.
Ellen: /somebody's thinking.
PI: She's re-voicing [collective hmm]
Lin: yeah
PI: She is re-voicing a complex problem solving approach.
Lydia: Yeah. AND Ellen: Hmm
PI: Like the girl did
RÉCIT: So in terms of if we think of what we talk about with math talk, encouraging math talk, she is using one of the strategies that we've mentioned and talked about.
Lin: Could she not have addressed it though by saying: "We haven't done area conversions, and that's why you ended up with the big value"? Like, so that the kid doesn't feel like…
Ellen: mjm
Lydia: Like this question has no answer.
Lin: Yeah. Like, "I'm still lost.
PI: yeah
Lin: I don't know why I get six point nine million."
Lydia: Yeah.
PI: The girl was left with that puzzle unanswered: "So what do I do next?" [collective Hmm]
PI: What happened was a total shift
Lydia: yeah
Ellen: mjm
PI: of framing of the problem, and going straight to conversions without linking it directly to the question that the student was asking,

Lydia: Hmm
PI: "what do I do next?"
Lydia: Hmm.
Ellen: And one thing, too, is it says: "What numbers do you come up with when you multiply both of these?" So it's more... Okay, so if I was a student I would think, well I need to multiply the numbers.
RA1: Hmm
Ellen: I wouldn't even think that maybe multiplying the numbers isn't what I need to do. I would think: "Okay well I have to multiply," but I wouldn't understand what the answer gave me when... I'm thinking, even to my students.
PI: Hmm
Ellen: They'd have no clue. If there's two numbers I always multiply. But they don't know that when you multiply the dimensions, well that's giving you area.
RA1: Hmm
Lydia: Well, do we know what she was looking for when she said: "What do you come up with when you multiply both of these?" Was she looking for someone to say: "A big number". Is that what she was looking for as an answer from the kids?
Ellen: Hmm
Lydia: And then it got a little off track.
Ben: It would've been the area of the box.
Lydia: Yeah.
Ellen: But then she wouldn't have said: "What numbers do you come up with?"
Lydia: yeah
Ellen: She should've said, maybe, if she wanted the area/
Lydia: / "What does it tell us?"
RA1: "What does it tell us?"
Lydia: yeah
RA1: And she says it's a lot of zeros, and she doesn't talk about the area right now. There's a lot of zeros, and it's because it's in millimeters.
Lydia: Yeah.
Ben: I'm going to be a little picky here, but, so a lot of numbers, right?
PI: yeah
Ben: It's digits.
Lin: Mm.
Lydia: It is digits.
Ben: It makes a difference, right?
Lydia: yeah cause it's a different word
Ben: If you're talking about a ten digit number versus a ten numbered number. It doesn't... It's vocabulary. [inaudible, multiple voices 0:23:29.9]

In these excerpts, teachers discuss the second critical incident, which involves a challenge with converting measurements, which feeds to Theme 3. The participants comment on how the teacher handled the challenge and how it could have been handled differently. They also discuss the communication strategies used by the teacher to encourage math talk and problem-solving.

Vignette IV Secondary Math Teachers After Listening to Volume/Area Challenge

In this 42-line vignette, we hear secondary teachers and a former secondary consultant reflect on the third critical incident, which involved a debate among students about the use of volume or area. Applying the primary categories outlined by One et al. (2013), this excerpt provides insight into how the incident was perceived and discussed by the participants.

Utterance	Category	Level
IV.334. Lydia: There's a smart kid in that class?	Teacher Behavior	1
IV.335. Lin: Yeah, because even one row, like he said, is volume, right?	Teaching and Learning Strategies	2
IV.336. Lydia: Yes.	Dudley Agreement	n/a
IV.337. Lin: There's another dimension to the donut.	Teaching and Learning Strategies	2
IV.338. Lydia: I just found, though, the teacher was kind of like not validating him on that.	Teacher Behavior	2
IV.339. Lin: Yeah.	Dudley Agreement	n/a
IV.340. Lydia: Did you see that? I felt like she was trying to take it... I'm like, "no, but tell him or her that it is volume. He's right."	Teacher Behavior	3
IV.341. Lin: I don't know. I think there were probably a lot of things they weren't anticipating, and rightly so, right? We often think we know what they're going to throw at us and then... I tell them, "I wasn't expecting that one. You have to give me some time because..."	Teaching and Learning Strategies	2
IV.342. RÉCIT: But if we go back to what their learning target [30:00] was, ask relevant math questions, in this dialogue, did you see some relevant math questions?	Achievement of lesson and curriculum objectives	2
IV.343. Ellen: Yes.	Dudley Agreement	n/a
IV.344. Lin: Yeah.	Dudley Agreement	n/a
IV.345. Ellen: Absolutely.	Dudley Agreement	n/a
IV.346. Lydia: Yeah.	Dudley Agreement	n/a
IV.347. Lin: But then they weren't, like Lydia said, not necessarily acknowledged.	Teacher Behavior	2
IV.348. RÉCIT: That would be something to improve.	Achievement of lesson and curriculum objectives	3
IV.349. Lydia: Yeah. To acknowledge that they said something /	Teacher Behavior	3
IV.350. Interim Elem. Consultant: / Because if that was the purpose of it, then those elements should've been brought to the surface.	Achievement of lesson and curriculum objectives	3
IV.351. Lin: Right.	Dudley Agreement	n/a
IV.352. Interim Elem. Consultant: "That's a great question."	Teaching and Learning Strategies	3
IV.353. Lin: Yeah.	Dudley Agreement	n/a
IV.354. RA1: And they go from... It seems that she goes from the students referring to volume, and it goes to layers all of a sudden, right?	Achievement of lesson and curriculum objectives	2
IV.355. Lin: Yeah.	Dudley Agreement	n/a
IV.356. RA1: The conversation doesn't stay there long.	Teaching and Learning Strategies	2
IV.357. Lin: Right.	Dudley Agreement	n/a
IV.358. RA1: It switches. The teacher switches it.	Teaching and Learning Strategies	2

Utterance	Category	Level
IV.359. Lydia: Because she's trying to bring it back to what she had planned. A student brings in a layer and it's sort of like a safety boat, "okay so we'll talk about layers."	Teaching and Learning Strategies	2
IV.360. Lydia: Yeah. Yeah.	Dudley Agreement	n/a
IV.361. RA1: Anything else?	Conversational move open-end prompt seeking add	n/a
IV.362. Interim Elem. Consultant: It really comes back to the relevant questions. What was the learning target? It wasn't necessarily solving this particular problem. It was the relevant questions. We do get lost. You end up taking the long way, you forget what the/	Achievement of lesson and curriculum objectives	2
IV.363. Lydia: / The main focus was. Yep.	Dudley Agreement Teaching and Learning Strategies	1
IV.364. Ben: If you're a student in this class, and you just heard this discussion, are you walking out of that class saying, "okay, so I kind of get an idea of when volume is relevant and when volume isn't relevant," right? The fact that it's 3D doesn't necessarily mean we always have to look at volume, just because something's 3D. So I'm right in saying that it is a 3D question, it has volume involved, but the volume is not important now. I don't think anybody leaving this classroom gets that feeling. Now they're leaving and they're saying, "is volume important here or not?"	Achievement of lesson and curriculum objectives	2
IV.365. Ellen: But do they continue to address that, or no? Because we don't know what continues with the dialogue.	Lesson as Experienced by Students	2
IV.366. RÉCIT: So what Ben's bringing up is the very important last part of developing learning targets. They did it a tiny bit, recapping and saying, "did you ask questions?" But they were missing that next part. If their learning target was relevant questions, so they were missing that evaluation of, did they understand what relevant was? So that would be the part that I would work on in the next piece I think. If they were to continue this type of a strategy, keep that learning target with a different problem, I would say, "okay, so now just remember that final piece of the learning target is being able to..." Because it's "what?" the "how?" and the "how will I know?" So it's, "what are they doing? How am I going to do this with the kids?" and then "how are they going to know that they've got it?" So I think that final piece of the...	Achievement of lesson and curriculum objectives	3
IV.367. Ben: How am I going to know?	Achievement of lesson and curriculum objectives	3
IV.368. RÉCIT: Yes, as the teacher. Yeah.	Teaching and Learning Strategies	2
IV.369. Ben: They got it.	Achievement of lesson and curriculum objectives	1
IV.370. RÉCIT: I think that's probably the piece that could be tweaked, and that would bring up some interesting questions.	Teaching and Learning Strategies	3
IV.371. Former Sec. Math Consultant: And the "how will I know?" keeps you focused throughout the lesson.	Achievement of lesson and curriculum objectives	3
IV.372. RÉCIT: Yes.	Dudley Agreement	n/a
IV.371. Former Sec. Math Consultant: And the "how will I know?" keeps you focused throughout the lesson.	Achievement of lesson and curriculum objectives	2
IV.372. RÉCIT: Yes.	Dudley Agreement	n/a
IV.373. Former Sec. Math Consultant: It's not just about verifying at the end, it's about those moments, capturing those moments. Right?	Teaching and Learning Strategies	2

Utterance	Category	Level
IV.374. RA1: It also goes back to what kind of traces are left behind at the end of the lesson, what the kids would take out of it. Are they left with anything that they can put in their notebook, that they can go back to?	Lesson as Experienced by Students	2
IV.375. PI: Okay, so um [clip cuts off]	No reflection	n/a

The former math consultant reasons that exposure in daily life is important but how important is it if there is no conceptual understanding? They are “meant to develop,” clarifies RÉCIT, yet the proverbial “practice” component jumps right back at us.

III.480. Ellen: they don’t know what to do
III.481. Ben: sounds a little unfair kind of task
III.482. PI: yeah
III.483. Lydia: **yeah, kind of unfair**
III.484. PI: And so the question is, what do you do then? Because it's not clear... I think it's clear to me that the teachers have… limited options to help the kids move out of that.
III.485. Lydia: Well because.
III.486. PI: They had/
III.487. Lydia: **Well, was it an accident that they got to that? Did the teacher predict that they were going to do that and get these giant numbers, or was she kind of like: "Oh right, uh..."**
III.488. PI: Yeah. It was like: "Okay let's forget about that and
III.489. Lydia: ok
III.490. PI: and do it differently."

On the one hand, for Lin this conversation about the Donut activity brought to mind another example of situational problem where students could not solve:

III.491. Lin: **There was a year, where in grade six where they had to tile a floor**,
III.492. Ellen: yea
III.493. Lin: and one of them, the measurements of the tiles were in centimeters, and the floor in meters,
III.494. Interim Elem. Consultant: [inaudible]
III.495. Lin: and that was the year... like nobody... **because they'd end up with these huge values**,
III.496. PI: yeah
III.497. Lin: But **they had to put it on a floor that measured whatever, eight meters by five, and they could not.**

On the other hand for Ben the conversation brought him towards the common juxtaposition of the circle versus square as a task too big for grade six students to grasp. Indeed, Ben proposes a tweak to the activity by switching the round donut to a square pizza. This type of nivellement vers le bas is possibly exhibited by the exchange below:

III.500. Ben: [...] I don't know if it was just because the opportunity presented itself, or it was a [donut]

thing, but **the fact that they're using circles,**
III.501. Lin: Hmm
III.502. Ben: **probably not the best choice.**
III.503. Lin: no
III.504. Ben: If it had been squares
III.505. Lin: Hmm
III.506. Ben: right?
III.507. PI: Hmm
III.508. Ben: At least you can still find the area of the square by doing
III.509. Ellen: Hmm
III.510. Ben: side times side.
III.511. PI: yeah
III.512. Ben: The kids can understand that. Whereas pie R squared becomes something…
III.513. PI: Yeah. AND III.514. Lin: Yeah.
III.515. PI: It's a different game.
III.516. Ellen: yeah
III.517. Lydia: [inaudible] or something
III.518. Ben: **So maybe choosing a different**... Maybe pizzas instead? **Square pizzas?**
III.519. Ellen: Hmm
III.520. Ben: If an activity comes with that. Or, **understanding the fact that okay, we're going to get into a situation where there's a concept that they haven't seen yet.** So **either you're going to teach the concept, or you're going to have to give them the information that they need to be able to complete the task,** if they do need the area. Right?

Ben acknowledges that the activity may serve as a warning to students that they will encounter unfamiliar concepts. These excerpts contribute to the discussion in Theme 4 of this dissertation. Let's see how the elementary teachers' coreflect on the same activity, next.

What did the Elementary Math Teachers Collectively Reflect About?

VI.240. Melody: I find the challenge, like, if I were to re-do this question is that..., is to ***know if you've covered everything before you get to a question and there's always already so many concepts: There's the whole estimation, there's the conversion, there's the volume but not really volume.*** *Because you aren't really looking at volume. Area, the whole diameter. Did you look at parts of a circle? Like one class... because I don't teach all the classes that I teach applications to complete math. I have two homerooms that I teach math completely to, and two homerooms that it's only half an hour of strategies, application strategies. And one group, they've never even looked at parts of a circle yet.* ***They didn't even know what diameter was.***

This subsection presents the coreflective conversations of elementary math teachers regarding the challenges faced by students in math using the edited video and two critical incidents as prompts. The section includes three subsections. The first subsection, Vignette V, examines the initial reactions of elementary math teachers while watching their enactments of the Donut activity. The second subsection, Vignette VI, delves into the coreflections of teachers after reading the estimation challenge. The last subsection, Vignette VII, explores the coreflections of teachers after reading the conversion challenge. Through these vignettes, we gain insight into the perspectives of elementary math teachers on student learning and engagement with the Donut activity.

On Wednesday, March 15, 2017, we held a coreflective meeting with elementary teachers in the same small conference room to discuss the Donut activity. Despite focusing on pedagogy, the elementary teachers only superficially explored the activity in terms of time management, concept elaboration, and the scope of prompts used to stimulate relevant math questions. To avoid tension, I did not aim to determine “what would a mistake here be like? And do I have any clear idea of it?” (Wittgenstein, 1975/1969, p. 17). According to Breuleux et al. (2017), who examined the ebb and flow of the concept of rigor in the elementary group's codesign and coreflection meetings, the elusive notion of 'rigor' cannot be captured simply as a learning trajectory, but must be a meaningful modus operandi. The outline of the categorization for the coreflection of elementary teachers' transcript is presented below:

Vignette V Elementary Math Teachers' Reactions Throughout Viewing

Although this study focuses mainly on the video footage that captured the discussion *after* watching the edited video, Vignette V represents a combination of 10 lines of talks from the discussions that occurred before and during the viewing of the video edited. This inclusion was necessary as all talks at these junctions tackle the notion of problem solving strategies.

> 253. Melody: So, you know how I am in math, and actually, here's a success. **After doing this, this kind of question first or whatever, there was a question I did. It's the one about the ostrich [Fast Pace] and how many strides he would take. You know that one?**
> 254. Interim Elem. Consultant: hmm. yes.
> 255. Melody**: So it is this long of information, and in the end there's one question,** and one sentence has everything you need. So I said, **"we're going to do this question, and we're going to read the question first." I'm so excited. So I did it myself, because I never do questions before them, I always do them at the same time. So I highlighted the question and then I said: "So I'm going through the process, "oh, I need to know this and this.**" And then I answered it. And then the other teacher came, because I was giving it to her students, and she goes, "okay, I hate this question. It's really, really hard." I'm like, "what? I answered it in two minutes!" Like I was so happy because I tackled it in a way that I guess was child-like.

Around 4:47 min: teachers laugh and joke by repeating "How many?", "How many?" While viewing the video clip we paused:

> 1.1. At 5:08 min video is paused, ELA: "Did you, they created their own questions?"
> 1.2. Kate: yeah.
> 1.3. ELA: So the question that you've come up with, which is 'How many are in the box?'
> 1.4. Kate: hmm
> 1.5. ELA: Is that your question? Or is that what you, or what they gave you?
> 1.6. Kate: It was our question but it came up from them first, so the first questions there were all sorts of stuff. I had one student who asked what's the gas milage of the truck was

Taken together, comparing the excerpts in this vignette with the secondary teacher's comments, Theme 1 emerges, and it will be discussed under the So What section. Later, during the mixed-group meeting, one of the secondary teachers initiated the conversation by picking up the topic of how to read a situational-problem question, highlighting the difference in approach between arts language classes and math. Next, we will explore the coreflections of elementary teachers on the estimation challenge.

Vignette VI Elementary Math Teachers After Listening to Estimation Challenge

This section presents Vignette VI, a 32-line conversation among elementary teachers about the first critical incident. The excerpts from this vignette contribute to Theme 2, where the estimation challenge is discussed. The conversation is dominated by Dona and Kaci, possibly because neither recognized themselves or the other in the classroom exchange, creating an objective distance in their feeling dimension of reflection. The classroom exchange was complex enough for Kaci to exclaim mid-reading:

> IV.13.1. Mid script reading [1:09] Kaci: Ok, I am confused!
> IV.13.2. PI: [Laughter]
> IV.14. RA1: Thank you. So this is the end of this conversation. Anything that comes at you, or do you want to switch to the next one?
> IV.15. **Dona: It's like they don't know what an estimate is. And then they're like, the too low for them... Well, one is low but it's too low, but it could be a good estimate, but some people are thinking it's not a good estimate because it's not a reasonable estimate.**
> IV.16. **RA1: hmm**
> IV.17. **Dona: They never know what a guess is. When you say estimate, they're like: "Are we supposed to add?"**
> IV.18. **Kate: [Chuckles]**
> IV.19. **Dona: "No, you're supposed to estimate." "But are we adding?" "No, you're estimating." Like, they don't…, They feel like they always need to have the right answer, and an estimation isn't a right answer.**
> IV.20. PI: Melody?
> IV.21. Melody: Yeah. I think also, too, like, when they say estimate, it's just to kind of give you an idea of if you're on the right track or not. But now they're getting really caught up in the estimate, and now the discussion is going on the estimate, which is fine, because maybe they need to... But I think that's what happens. Is when kids start getting into a problem, they get stuck on something that in the end is not even the point. Like, it was just kind of like to guide you, and it's really taking up a lot of time now, but then you're stuck between a rock and a hard place because you don't know. Well maybe this is a perfect teachable moment.
> IV.22. [laughing]
> IV.23. Melody: We can, you know, review estimation. But that's not even the point. You're just... you know? So, I don't know.
> IV.24. PI: Kacy.

IV.25. Kaci: Well I think, too, I am seeing many things, if we're going to be talking about the talking part of it.
IV.26. PI: hmm
IV.27. Kaci: So the first time that I had to speak as teacher, I said: "So what I'm hearing you saying is that you're agreeing with Jay." I think it's proof that students, they're not listening to each other. They're just, they are stuck on... I tell my students all the time: "As I'm speaking, or as I'm asking a question and you have your hand up, once I've chosen someone to answer, put your hand down."
IV.28. PI: hmm
IV.29. Kaci: Put your hand down and listen to what they're saying, because if your hand is up, I know, in your head, you're focusing, you're repeating in your brain what you're going to say, you're sticking to your idea. You have no idea what the other person just said.
Interim Elem. Consultant: /or when someone [inaudible]/
IV.31. Kaci: /So I find the teacher here is trying to get them to maybe stop and listen to each other.
IV.32. [Collective hmm]
IV.33. Kaci: "Okay, you're both speaking as if you're arguing, but what I'm really hearing is you're saying the same thing."
IV.33. RÉCIT: Revoicing.
IV.34. Kaci: So I think that teacher is trying to - it's like what we saw at the institute this summer - is trying to prove to them, listen to each other, build on each other's ideas, can you have me repeat? So what did he just say?
Interim Elem. Consultant: hmm
IV.35. Kaci: Who else? She's definitely having to facilitate this conversation because they're just talking at each other
IV.36. RÉCIT: hmm
IV.37. Kaci: instead of listening to what the other people are saying.
IV.38. PI: Kate.
IV.39. Kate: But I think what is hard as the teacher, from having been in similar situations, is that I like... there's like wait time before we get to like... [5:00] ok now they're just having... where is that productive disagreement? And where does it stop being productive? [Collective yeah]

Dona's interpretation of the classroom conversation from the first critical incident highlights how students have difficulty understanding the concept of estimation, particularly when it comes to determining the appropriate level of precision. If we apply Lasswell's (1948) communication model, we can break down the conversation as follows

Who: Kaci, PI, RA1, Dona, Melody, Interim Elem. Consultant, Kate
What: Discussing the difficulty students have in understanding what an estimate is and the teacher's efforts to facilitate communication and understanding between students.
Whom: The conversation is between the participants mentioned above.
In What Channel: The conversation takes place in a face-to-face setting.
With What Effect: The partners are discussing the issue of estimation with the aim of improving the situation and finding a solution.

In analyzing the conversation, Kaci identifies instances of students not listening to each other. This can be challenging, especially given that at the codesign they selected the second high-quality teaching practice to orient students to each other's ideas. The conversation reveals the complexity of teaching estimation, and how facilitating productive communication between students can be a significant challenge. Next, the elementary teacher's conversion conversation.

Vignette VII Elementary Math Teachers After Listening to Conversion Challenge

> ***Kaci: Good job Miss Dona.***
> *Dona: It's me. I'm like, wait a minute, I remember this conversation. [collective laughter]*

In Vignette VII, I present the 17-Line commentary after the elementary teachers had a dynamic read-through of the second critical incident.

1. Kaci: You said that, and I was like, "okay I can picture her going donut, donut, donut."
2. Dona: That's why I did donut, donut, in my picture. Donut, donut.
3. PI: Yeah. We could show the...
4. Kaci: It's okay. Once I knew it was her I could visualize it.
5. PI: Any thoughts on what's going on here?
6. Dona: I said donut a lot.
7. [Chuckles]
8. Dona: But I was struggling, my thing was...the kid, it was the one that was talking about showing the meter stick.
9. Melody: hmm
10. **Dona: [the student] figured out how many meters. And then there was another person who had tried to calculate the area, but had made the numbers so big.**
11. Kate: hmh
12. Dona: And they couldn't see how she got it so simply,
13. PI: hmm
14. **Dona: if you can say that. Like, easily. And compared to her they have this huge number. So that's why we tried to transfer, doing it with the meter and this little…**
15. Melody: hmm
16. Kaci: Was this student the only one who was confused about what to do? Or was it...?
17. Dona: No. In that group I think the confusion was transferred among all of them, because there were actually some strong students in there, in that group, but they didn't seem to see it. They saw those numbers, and right away they said: **"But it's times means I have to multiply them."** Like, okay, but/

The conversation starting from line 14 can be interpreted as Dona initiating the conversation by noting confusion among students in a particular group, and Melody responding with a nonverbal indication of listening. Dona expands on the issue by answering Kaci's question and explaining that the confusion was not limited to just that group of students. When Dona describes her actions without evaluating their effectiveness, this is an example of surface reflection. In contrast, Kaci's question–"was this student the only one who was confused about what to do?"–represents deep reflection, as she tries to understand the underlying causes of the confusion and evaluate its extent. Our footage suggests that many students did not finish the activity before Act-3 and required calculators. See the next 59 Lines as they debate what's next:

18. PI: /What do they do next? That's the question. *[PI bringing the conversation to the concrete content, what are the math steps]*.
19. Dona: What is the next thing? Okay, you multiplied it, but what are you getting from that? They didn't know what they were getting. They were multiplying because they're like, "it says twenty-three hundred times thirty-three hundred" or whatever it was. And there's an X between them, it means you need to multiply it. But they didn't know why they were multiplying.
20. PI: Kate.
21. Kate: Well I think also in my class similar things came up, and I know I have to revisit it, but I haven't done that yet.
22. Melody: [Chuckles]
23. Kate: I have students who follow the logic of the twenty-five times [15:00] thirty-three, but they're still convinced that their area divided by area would've worked, and it doesn't. Like, it's not the right way of going about doing things.
24. Dona: [inaudible question]
25. Kate: Because they're doing the total area and then dividing it by the area of a donut, which they didn't even have, but they're convinced somehow they could have found it.
26. Dona: /the diameter
27. Kate**: But even if you had it, it doesn't consider the dimensions of the donut. So you could have empty spaces, eight point nine millimeters, nine milli…, nine centimeters wide. You could have a strip of six centimeters there. It doesn't matter. But you still can't put donuts.**
28. Melody: yeah
29. Kate: So you may have this long, empty part of your box, but, so there's still space, but if you calculate based solely on area, yes you could cut all the donuts. But that's
30. Melody: yeah
31. Kate: not what we do.
32. Melody: Yeah.
33. PI: RA2?
34. RA2: I just had a quick question. After hearing everybody talk about this big activity, I'm just wondering whether or not, in light of this particular dialogue, is there anything you would have done differently had you re-done this activity? Is there anything that you would have maybe specified or presented differently to? For example, to fit this simplification process earlier, and if so, how might…, how might that affect the way you co-design next activity, in light of what happened with this?
35. PI: Kate and then Melody.
36. Kate: I think I would've done something... I think it came up last year, where I told the students they had to produce a visual representation of the question.
37. RA2: hmh
38. Kate: I said there had to be a picture element that went with it. I don't remember what we were doing. I'm sure it was part of this group last year, where they had to draw something. I didn't want a whack of calculations**, I wanted visual evidence that we could look at and see the solution. A little bit like... I think you were talking about it maybe at our last meeting, when you said: "Just a drawing can show you a whole bunch of stuff that you understand about a question."**
39. Melody: hmm

40. Kate: But I had forced them. I had told them that even if you had the answer with numbers, your solution wasn't going to be considered, like complete.
41. Melody: hmm
42. **Kate: I think maybe by forcing them to have the visual they may have realized that this whole finding the area step was not only unnecessary, but maybe, potentially thinking about it in the wrong way.**
43. Melody**: I find the challenge, like, if I were to re-do this question is that..., is to know if you've covered everything before you get to a question and there's always already so many concepts: There's the whole estimation, there's the conversion, there's the volume but not really volume. Because you aren't really looking at volume. Area, the whole diameter. Did you look at parts of a circle? Like one class... because I don't teach all the classes that I teach applications to complete math**. I have two homerooms that I teach math completely to, and two homerooms that it's only half an hour of strategies, application strategies. And one group, they've never even looked at parts of a circle yet. They didn't even know what diameter was.
44. PI: hmh
45. Melody: Right, so I think with any of these it's always a kind of long-shot that you will have covered everything. So you always get stuck on these... Which is what happens with these applications at the end of the year, too. Like maybe you covered something way at the beginning, and they might've not... So I don't know how to solve that problem. I think something's always going to be a little bit of a glitch when it comes to that.
46. PI: Kaci?
47. Kaci: No, I mean... I'm good. I'm good. But I agree with you. It's always... even when I choose an application: "Oh but I didn't really cover that." We did it situational with the treasure chest one. "I didn't really... I know they kind of saw it last year, ten percent of" /
48. Melody: / hmm. We didn't do percent yet.
49. Kaci: Do I feel that it's fair to then have to do that part of the situation? Because really we saw it, kind of, shot in at the end of the year. And it's next. But I can't keep withholding situational problems because... So I said: "Okay, you know what? I'm going to do percentage of with them." Everybody will have...
50. Melody: yeah
51. Kaci: The insurance tax to ship the treasures is ten percent of seven thousand. Those are a minimal part of the situational.
52. Melody: hmm [inaudible]
53. Kaci: Let's do it together as kind of a teaching time, and everybody will have the same answer. But then, what happens? Snow days, nah, nah, nah. Okay, let's just start it, and then when I have time to teach you, we'll plug in the number.
54. Melody: hmm
55. Kaci: And then, "oh yeah, it's true. I have a workshop." So I left it to the sub. "Just tell them that ten percent is this, I'll teach it to them later."
56. PI: hmm
57. Kaci: Ah.
58. Sabrina: I think that that's okay, as long as it's not the key concept.
59. Kaci: hmm. But I find often that I'm putting aside applications, going, "oh we hadn't really seen that enough."
60. [collective hmm]
61. Kaci: Which I don't find you get in ELA.
62. Melody: hmm.
63. Kaci: Other than parts of... "Ooh we didn't really cover dialogues," so when they wrote their narrative, their conversations were…
64. ELA Guest: yeah
65. **Kaci: I still feel they have enough tools in terms of writing and responses no matter where they are on the curriculum, whereas math, it’s one thing always depends on the other, on the other, on the other.**
66. Melody: hmm.

67. PI: Could I ask you to... This has been a great conversation. We only have a few minutes left in regulation time.
68. [Collective laughter]
69. PI: There are a couple of questions at the bottom of the first sheet.
70. [inaudible 0:19:59.5]
71. Kaci: [inaudible] While we were watching
72. PI: What can we... [RA2], what are the questions? The inquiry. [inaudible 0:20:06.9].
73. RA2: There's just a few questions. If you want to jot down some of these things, that you mentioned... I know that one thing…, I'm curious to see on the sheet is to think about some of these difficulties we talked about today, some of these challenges, and think about the next time we have an opportunity to co-design, how might we...? We can start consolidating, I guess, by putting some of those ideas and questions that you already talked about today on this worksheet, so that we can keep some things in mind. And I think we can
74. RA1: we'll take pictures of them
75. RA2: [inaudible 0:20:47.7]. So you can take these home, but we would like to have just kind of a record, at least of the appreciative inquiry for our part, to be able to take away, before you go.
76. RA1: we are running to
77. RA2: yes. [21:05 papers shuffling, no voices, to end of clip]

Compared to the secondary teachers, Vignette VII of the elementary teachers' discussion on critical incident two had fewer comments and suggestions. This could be attributed to the fact that two teachers immediately recognized whose enactment it was, and Dona had previously expressed struggles in dealing with the heterogeneity in student knowledge, as she illustrated in her end-of-year interview in summer 2017. Overall, the three vignettes in this section provide valuable insights into the perspectives of elementary teachers. The following section will present mixed-group coreflections

What did the Mixed-Group of Math Teachers Collectively Reflect About?

In this subsection, I turn to the vantage points expressed at the mixed group coreflection by reporting two vignettes from a warm April day in 2017 with twenty educators: four secondary math teachers, six elementary teachers, six researchers, one RÉCIT, and three consultants, not quite knowing back then that the project was morphing into another School Board-wide project. We gathered in a large conference room in a horseshoe seating arrangement in the basement of the Sunny Side School Board. Among us, we had a lovely newborn baby with us, symbolizing, in my opinion, the next generation of French math learners in the province of Québec. Specifically, the two vignettes below are drawn from the 112 lines of conversation that spanned 21 minutes and 26 seconds. The data will reveal that at this meeting, we were more focused on descriptive summaries and suggestions about how to tackle situational problems rather than analysis of the donut activity. We also had a celebratory focus on having captured a full cycle of lesson study with a string of edited footage. The categorization outline provided below pertains to the mixed-group meeting after the sharing phase, following the viewing of the video:

Vignette VIII Mixed-Group Math Teachers’ Reactions Post-Viewing

Kaci: I'm going to be honest, I feel like I've dropped the ball. What have I done in math? I don't know.

In Vignette VIII, we get a glimpse into the mixed-group’s reactions to the Donut activity. However, before we even watched the video, many topics were discussed during the hour-long sharing period. These are shared first because they set the stage for the four themes generated. Kaci made a point-blank statement about the Donut activity at the beginning of the meeting, which is worth noting because it references more than just the activity and highlights a notable outcome: This elementary teacher's obvious statement about intentions of being honest somewhat shatters the implicit group norm of trust and yet somehow retains it. Trust in admitting inadequacy? Kaci's statement challenges the group's implicit trust norm but also suggests that admitting inadequacy may actually enhance trust, and or may be a result of i

> Kaci: And for me, it was really funny. I'm going to be honest, we're all about Annie, I have been away, the play, anyways. And so, "oh my god**, I'm getting videotaped tomorrow, I don't remember what the lesson is. I don't have a plan."**

Admitting to "dropping the ball" may not be comforting, but it demonstrates taking responsibility. It also suggests that the codesigned lesson plan may have been inadequate, lacking the necessary resources for Kaci to have a clé-en-main experience with the activity. Alternatively, it may signal, though unlikely, that her entire math class has been dropped for her entire cycle. Questions arise as to what will become of the cohort that went through such a class, why Kaci doesn't know what she has done, and where her learning progression tracker is. Although Kaci continues to explain, this report will not delve further into the sharing phase. In our previous work, we explored the development of meaningfulness and teacher agency in this RPP (Beck et al., 2022). Further analysis of a sharing session can be found in Beck et al. (2019), where the research team used Dudley's (2013) categories to analyze the codesign elementary meeting and discern the interaction function between consultants and teachers. We also used the same codes to track two teachers at the codesign meeting and in their respective classrooms (Zhang et al., 2019).

After the sharing period and a break, the mixed group reconvened and watched the edited video of the Donut activity together. Secondary teacher Lin's comment, which contextualized how the educators approached the activity, was particularly noteworthy as it highlighted a key strategy for situational problems: reading the question first. This echoes Melody's comments made at the elementary meeting group meeting. With this in mind, the conversation among the educators commenced, exploring various aspects of the activity and its implementation:

> **Lin: I think it was Melody who said [in the footage from the elementary group's collective reflection], trying to get [the students] to go to the question first. And I remember when I taught language arts, one of the strategies that you used in reading responses is to read the question first, before you read the text, so you know what you're looking for. And yet, we don't do it in math." Problem solving strategies typically involve, reading the problem once, then, re-reading to select important info given to solve for what's missing** (Lin, Line 1).
> [Brief silence]
> PI: I think the same idea was discussed in the secondary meeting. Okay, so what were your thoughts as you were watching this video?
> **Lin: Melody's quite animated.**
> **[Laughter]**
> **Ben: Passionate.**

Initially, the discussion among the educators centered around personality traits; however, as the meeting progressed, themes emerged that were previously identified in past coreflective meetings. It is important to note that the comments made by the educators during this meeting were not explicitly related to the critical incidents, as no skits were read. During the coreflective meeting, the educators discussed various topics, including the cancellation of a math exam that required students to engage in a "kind of abstract" task. Kate made a comment about this, noting that it was similar to the Donut activity. The comment excerpted below highlights the educators' perceptions of the activity and its relevance to their teaching practice:

> Lin: / What was the question on, essentially? That got cancelled? Like, what were they supposed to be doing?
> Kate: They had to look at the dimensions of the books, and the dimensions of the box, and determine/
> Sabrina: /how to
> Kate: /how many could fit.
> Sabrina: And which way to place them, too.
> **Kate: It was kind of abstract.** It was quite difficult, but at the same time it was to show that it wasn't about volume. Yes, there was going to be empty space in that box, that technically could've fit more books, if we could cut the books into pieces, or rip out all the pages, and re-pack it. But we don't do that.

When Kate deems a ministry math exam question about books in a box as "kind of abstract" and explains that it was cancelled, it brings to mind Davydov's plea to teach learners to ascend from abstract to concrete (Davydov, 1990). This raises questions about teaching strategies in these classes in Québec. Are these educators moving from concrete to the abstract, rather than moving from abstract to concrete? These strategies have ripple effects, and the ascension from abstract appears to provide great opportunity for the lesson to be transferable. Starting with unconventional measures is a technique that I noticed in the QEP (Ministère de l'Éducation, du Loisir et du Sport, 2009), however, when I hear pleas for manipulatives in order to be more successful at the elementary collective reflection, I wonder whether we are really moving from abstract to concrete. See, for instance, Sabrin's vocalized realization:

> Sabrina: Well, I guess it comes back to what we said earlier**, that we're trying to teach the students to talk through problems more, to use each other, you know, to bounce ideas off of each other, but then when it comes to/**
> PI: / Assessment?
> **Sabrina: Assessment, it's completely different. "You can't do this,**
> PI: yeah
> **Sabrina: no you can't. You can't use manipulatives.**" And it's hard. And I feel like if they were given those then we'd see more success.

Perhaps this Donut activity created an environment for group work where the idea of turning students to each other's ideas became a situation of the blind leading the blind. Where the student's were not able to explain their knowledge (e.g., one student performed a think-aloud four times for her peers, twice for the class and then Dona reenacted the student's process for the whole class). Perhaps the learners at this stage were not equipped to actualize the third high quality teaching practice that was meant "to orient students to each other's ideas." This contradiction in facilitating group work is best illustrated by Dona at the elementary teachers' meeting:

> Dona: /It's like we want all this great stuff, to…, it's like: "Okay, but how do we...?" Okay, I want to do centres, I want to do**... I have to make sure I do group work, group work with math talk. But I have to reinforce the math talk to make sure it's the right math talk.** And then I have to do this, and then I have to do that.

Through the sharing of reflections, the mixed-group discussion covered a wide range of topics related to mathematics teaching and learning. One notable comment came from Kate, who discussed the cancellation of a math exam that required students to engage in a “kind of abstract” task similar to the Donut activity. These discussions set the stage for an engaging and thought-provoking correflection meeting. In the following section, I will discuss how one of my comments was received by the mixed-group, providing insights into the dynamics of the group and the value of correflection in professional development.

Vignette IX Mixed-Group Researcher’s Comment

In this section, I will discuss the conversation that occurred after I shared my method for handling a challenging aspect of the activity during the mixed-group meeting. According to the transcript below, Ben, a secondary math teacher, quickly offered a clarification regarding the division of math territory between elementary and secondary teachers. Kaci expressed confusion and noted that other elementary teachers may feel similarly. The following excerpt features the discussion about dividing a large area by twice the diameter of the donut and ends with awkward laughter:

> RA1: But at the same time, I wondered. I was like, "what would I do with six million?" I sat there and I was like, okay think about the first time I [inaudible 0:04:22.6]. And I was there looking at seventy-seven points [inaudible 0:04:26.0] some digits, and I was like, "okay, well what do we do after that?" And so I divided it again by eight-nine, until my friend told me, "**well it's area divided by diameter squared**." I realised that that six million could've been reduced as soon as you divided by twice eighty-nine, eighty-nine squared. So you can still bring that big number into a, into manageable understanding if they knew the formula.
> **Ben: That's not a concept that**
> RA1: that they know
> Ben: is in elementary. You guys, you don't do...
> Kaci No, **you can look at the confusion on our faces while she's talking about divide and divide.**
> Ben: right [Laughter]

These comments raise the question of how students can transfer knowledge across different cycles. However, of note is that during the mixed-group meeting, it was revealed many practitioners did not repeat their comments to the other group. As a result, some suggestions and questions from secondary to elementary teachers were not shared in the mixed group. For example, Math Talk was mentioned more frequently in the secondary group (31 times) than in the elementary group (14 times). It was only mentioned once in the mixed group when Lin recalled, “the true meaning of the word is, and how it had to be clearer.” Other suggestions, such as Lin's way of handling unanticipated student questions, were shared during the secondary meeting but not in the mixed group. However, during the mixed-group meeting, Lin raised the concern of mental math, asking, “But isn't that all about mental math?” Kate and Kaci agreed, and Lin reiterated a challenge she observed at the secondary level: “The kids are completely... they cannot perform any mental operations.” In response, Melody offered a student's perspective, sharing their experience of anxiety due to not knowing:

> **Melody: Except if you're freaking out, right? And you're like, "I can't do this, I can't do this." And you’re just calculating and you get to a number, and now someone says, "does that make sense?" You're like, "I'm just trying to get it done.**
> Kaci yeah
> Melody: I don't know if it makes sense." As much as they see that it's reasonable, "come on it's ten dollars," or whatever, it's not reasonable when you shut down and you're like, "I don't know if it's reasonable or not. I don't know.”

To ensure that all participants heard specific feedback, I incorporate Ben's suggestion to provide clear guidance at the beginning of the activity to avoid wasting time inside the edited video for the mixed-group. However, one important recommendation from Ben–to use the term “digit” instead of “lots of numbers” when discussing the area of the box–was not shared. These examples underscore the need for more systematic communication channels to exchange ideas across different groups. Here is a last example of what was repeated at the mixed group meeting:

> RÉCIT: But I think, and **I'm just taking notes on some things that we've... I want to go back and talk about it in the groups too, but I think, again, that math vocabulary, and especially if the goal of the teachers was looking for rigor, then reminding everyone too that that math vocabulary is important.** It's not like picking on somebody and saying... you know, sometimes when you're talking.... But just

> **building that awareness again, of the importance of that math vocabulary, that then builds to rigor**. Because that was one of their initial questions, what does a rigorous math explanation look like?

Taken together, these excerpts offer important insights into the practical implications for the lesson plan and the role of lesson study in evaluating lesson quality. In the next section, I explore the “So What” aspect of these findings, discussing their implications for future practice and research.

So What? Unearthed Math Challenges

> ***Kate: I see a lot about planning there. Planning, trying, trying again and kind of working on refining processes, and I think that's kind of the ultimate goal.*** *Like going back, and/*
> *PI: / Improving*
> *Kate: I think there's also openness, different ways of doing it. I thought that was valuable, seeing different approaches,* ***both from watching clips of students in my class, so I got to see different ways.*** *They were thinking about things that I would've never thought of. I would never have thought of trying to solve a problem like that... Because I assumed it would be giving me the measurements. But some of them were saying, "but what if it gave you time?" Or, "what if it gave you sugar content?" Or something like that. [Laughs]* ***So that was interesting. I didn't see the differences between classes.***

Having laid the groundwork in the previous section, we can now delve into the practical implications. Building on the nine vignettes, we have uncovered four challenges that reveal the importance of addressing transition issues: Theme 1, which focused on the wide range of reactions to math talk mainly seen in Vignette I and Vignette V; Theme 2, which explored the estimation challenge mainly observed in Vignette II and Vignette VI; Theme 3, which highlighted the difficulties associated with converting measurements and units, mainly observed in Vignette III and Vignette VII; and Theme 4, which related to confusion between volume and area, mainly observed in Vignette IV and Vignette VIII. By examining the themes that emerged from the teachers' reflections, this section aims to provide insight into the challenges and opportunities presented by the donut activity. In the subsequent sections, the limitations, alternative interpretations, and lesson study implications of these themes will be discussed.

Theme 1 Wide Range of Reactions to Math Talk

> Secondary: She doesn't answer her question ...and, she said: "The area inside." That's a little confusing." **vs.** Elementary: "Good job ..."

This subsection presents quotes that demonstrate the range of reactions among different teachers, which is a consistent theme throughout this data set. As Montaigne (2003) argued, "each man calls barbarism whatever is not his own practice [...] for we have no other criterion of reason than the example and idea of the opinions and customs of the country we live in" (p. 191). Applying this context to the educators' divergent reactions, it is evident that the secondary group's assessment of the Donut activity as lacking in fairness, guidance, and clear vocabulary is a result of their grade-level perspective. The initial categorization of two specific passages from the reactions to the second critical incident supports the discrepancies noted in memos. Further categorization corroborated the memo findings, although over time, the differing perspectives between the secondary and elementary math teachers became more nuanced. Notably, there were more differences among elementary teachers than among secondary math teachers. Others have also explored the Rashomon effect in education; for instance, Persico et al. (2013) intentionally included contradictions in the design of one lesson to investigate this phenomenon.

There were also differences in what teachers expressed as their takeaways from our meetings. For example, Kate's takeaway from the mixed-group meeting was about [co]planning, while the RÉCIT further specified at the secondary meeting that teachers need to improve how they acknowledge and respond to students' relevant math questions.

> RECIT: But if we go back to what their learning target [30:00] was, **ask relevant math questions, in this dialogue, did you see some relevant math questions?**
> Ellen: Yes. AND Lin: Yeah. AND Ellen: Absolutely. AND Lydia: Yeah.
> **Lin: But then they weren't, like Lydia said, not necessarily acknowledged.**
> **RÉCIT: That would be something to improve.**

In this study coreflection consisted of a group of ten teachers discussing a math lesson that was collectively designed by the elementary teachers' group. However, in our case, despite the video protocol in place, we did not receive any specific requests from teachers to include a particular whole-class discussion for further analysis. As a result, our post-lesson discussion is similar to that described by Lewis et al. (2006) in the 2001 study focused on addition and subtraction without providing specific guidance about what to observe during the watching phase. Therefore, video-mediated coreflection refers to semi-prompted discussions after watching the Donut activity with a view to improve the Donut activity's lesson plan.

Our unguided approach may have led to post-lesson discussions that tended to focus on general issues such as student engagement, group work, and task guidance, rather than on specific aspects of the activity that revealed student thinking, such as their understanding, organizational style, and types of errors. Whether they detected errors or not, how teachers respond to students' errors and handle their own is the most crucial factor in addressing issues in learning math. As Lydia laughs while noting at the secondary meeting, "So imagine being a kid in that class." When reflecting on what constitutes a lesson, embodied through an activity, it is important to consider the significance of educators' comments during collective reflection meetings.

The Rashomon effect can be attributed to several reasons, but three are the most plausible. The first reason is related to the way in which the six teachers codesigned the same activity but enacted it in five different ways. Such findings suggest that the open-ended lesson plan is the culprit. At the beginning of the elementary meeting, Dona voiced her curiosity by wondering if students had a similar reaction across the classes, but it was unclear if it related to learning or feelings.

> **Dona: My biggest thing was, I was wondering if everybody else was getting the same reactions from the kids as I was. Like, did it go the same way/**
> PI: /we'll see
> **Dona: in other peoples' classes, as it did in mine?**
> PI: We'll look at what happened. Actually, it's very varied. Okay? S**o one of the things that will become very clear is, there is variety in the ways that you implemented the activity, and that's a very interesting thing. So one of the questions is, you know, to what extent these differences, what are they about? Why are they different? Is this something that we need to keep in mind for next time that we co-design an activity?** That we need to be more, maybe, specific? From the research perspective, we're not pushing for more detailed scripting of the activity. I personally don't think that that's necessarily the way that we should go. But maybe a bit more details in anticipation of what will happen when we do this in class. So we'll see a.../
> **Melody: / What I noticed is I taught it too... well I taught it to four classes, but three classes in a row**
> PI: yes
> **Melody: with Nilou, and there was, all three were very different.**
> PI: yeah,
> Melody: Like, even then, even myself. So I'm sure there was lots of differences, because you can't even control where they go with it.
> PI: yeah
> **Melody: Depending on what questions they start asking/**
> Kate: yeah
> PI: exactly
> Melody: or/
> **Sabrina: / It takes a whole different course.**
> **Melody: Yeah! Depending on: "Okay, so now that you know the question, what would be some things you would do to answer it?" Well some kids were like more using numbers, then another group we were sticking post-its all over. They were saying: "Well imagine this is about twelve donuts in there, and how many post-its could you fit on top?" And, every class was different than I was even... I don't..., I think even if I did it another time it would be different.**
> PI: yeah
> **Melody: Like you couldn't plan, you couldn't plan for the discussion or what would come out of it.** I think in the end what I got from it was that I believe, from this exercise, is that we should start teaching that earlier on, instead of reading a question from the beginning to the end. "What's the question?" And then focus on: "What would I need to know?" Like, use a bit of your brain first. What would I need to know? And then go for a treasure hunt,
> Kate: Hmm
> **Melody: and look for that information. And I started doing that at home too. Like, I just noticed that I think the biggest thing that I saw from there is, maybe we've been tackling problems backwards all this time, reading from the beginning.**
> PI: Kate? [calling her name since she had raised her hand] & Kate: I completely agree with that.

Apart from wondering why Kate raised her hand to declare her agreement, the second Rashomon effect is experienced in the different ways each elementary teacher made sense of their enactment. Melody reported each was different that by her fourth enactment, she felt depending on the “questions [students] start asking,” Although our cameras did not film the entirety of the classrooms, I observed instances of reflection-in-action, such as Melody transitioning from using a whiteboard to a blackboard with magnets to demonstrate the steps in solving the Donut problem. However, as the meeting unfolds, we begin to hear divergences even in Melody's view of the activity. Was it different or not? Melody declares, “I didn't find it very different. I thought we all kind of taught it the same way, but that was/” but Kaci cuts her off mid-observation, and we don't hear the rest of Melody's explanation. The discussion between elementary teachers revolved around some differences in outcome rather than why there were differences in the way the activity was guided.

> **Kaci: I like this, though. Because this is the first time we all do the same lesson.**
> **Melody: I didn't find it very different. I thought we all kind of taught it the same way,** but that was/
> **Kaci: /That's it. It was fun to see one video back to back, all the different ones. Like I took notes. "Maybe I should've done this, maybe I should've done that."**
> **Kate: I did the same thing.**

At the secondary meeting we were by far more focused on the way the activity was guided, as we saw in Vignette 1. The third Rashomon effect is in the difference in reactions between the four secondary teachers' views of the representations from the classrooms compared to the elementary teachers. This difference may be due to various levels of content knowledge. This, in turn, reinforces the idea that math teachers from different cycles have different cultures, which could contribute to the lack of consistency in student performance across cycles. This is consistent with May's (2012) argument that the cultural metaphor in mathematics teaching could “inhibit the process of integration and assimilation” and contribute to cultural shock as students transition from elementary to secondary math (p. 1). To address these issues, it may be helpful to initiate certain processes earlier, such as setting math notations, focusing on understanding and solving problems over time (not just during the 50-minute class time), and using correct vocabulary to discuss math operations, maybe during English hour.

To make sense of the Rashomon effect, the secondary teachers' comments were interpreted using Dudley's (2013) categorization scheme of Social, Behavioral, Cognitive, and Affective statements. During the secondary meeting, many of the statements can be classified as having a *cognitive function*. They discussed the importance of establishing learning targets and evaluating student understanding. For instance, Lin questioned, “I'm not sure if they received an answer regarding whether it was a volume or area question or if they even...” This type of concern aligns with Fuller's (1970) third stage of teacher development, where teachers focus on student learning outcomes. The secondary teachers also discussed improving the vocabulary used to explain math concepts and responding to student questions.

A combination of cognitive and *behavioral functions* can be observed in the conversation between RÉCIT and Ben at the secondary coreflection meeting. RÉCIT initiated the conversation with a declarative statement, “So what Ben is bringing up is the very important last part of developing learning targets.” Ben's response, “How am I going to know?” aligns with the behavioral function of setting assessment strategies. RÉCIT's reply, “The final piece” aligns with the affective aspect that indeed knowing if objectives were attained is the final piece of a lesson.

Additionally, Lin noted at the secondary coreflection meeting, the activity appears “open-ended, but it's really restricted”; the absence of the height dimension in the Donut activity is a major concern, not as much as the absence of clear explanations. These restrictions were overlooked during the elementary codesign meeting, but if secondary teachers had been present, they could have discussed the issue with the interim consultant and elementary teachers. If future codesign meetings involve teachers from different grade levels, then teachers like Lin would have brought up some of the limitations of the Donut activity that she would inevitably notice from her vantage point. In other words, for Lin, this activity is not as open-ended as promoted. Perhaps, she would have voiced her opinion right after the Interim Elem. Consultant talked about it at the elementary codesign meeting:

> I.183. Interim Elem. Consultant: It's huge. Now estimate how many donuts are in this box, right? That's something you could do in cycle two, **but then in high school it's, "what's the volume?** What's the weight? What's this? What's that?"

If Ben had heard this utterance (if he had been present at codesign too), he would be elaborating on Lin's comment on the limitation built into the email from the Donut company:

> III.214. Ben: So, as Lin was saying, the questions are limited. You've not provided them with a *truly open-ended* aspect that they could then further... you know. Like, **I wouldn't be able to extend that unless.., until I knew what the height of the box was.**

Lin's comment further highlights the need for closer examination of which aspects of an activity make it "truly" open-ended to allow for greater expansion in later grades. The opening epigraph demonstrates that there are different perspectives and opinions among elementary teachers, which need to be considered in future lesson planning. The exchange between Ben and Lin displays the level of agreement among secondary teachers. The study proposes clarifying practitioners' math talk during the implementation to enhance the sense-making process for students, to do so we propose to start with the lesson plan. Encouraging students to share their calculations and step-by-step processes, rather than just their thoughts, shared orally or on post-its, can bring rigor to the forefront in the Donut lesson.

There are implications for researchers facilitating lesson study meetings; this research identified differences in discourse between teacher groups, as well as variations in communication styles depending on the meeting. Using Jaworski's (1990) framework, the study found that elementary teachers provided descriptive *accounts of* the activity, while secondary teachers tended to provide evaluative comments (*accounts for*). In the secondary meeting, RÉCIT provided comments aimed at justifying the choice of activity in relation to the learning target, such explanations were not necessary at the elementary coreflection meeting. This subtle change in facilitation technique may have directed the discussion towards justifying the activity, thus stifling the need to improve it. These findings have important implications for how researchers prepare prompts to accompany lesson study groups.

The interim elementary consultant provided elaborative *accounts of* the activity at both meetings but did not participate in any of the enactments (I suspect the long distance between each school did not help). During the mixed meeting, educators tended to recount and summarize what they said, with many utterances being descriptive rather than evaluative accounts. However, a clear understanding of the purpose of the Donut activity is required to make an account justifying it (accounts of descriptive). Nevertheless, a bias was detected at the coreflective meetings, where comments stressed "the what" over "how to" (declarative over procedural) (Marcos et al., 2011, p. 32).

The first enactment allowed students more time to explore questions related to the box, but a secondary teacher perceived it as "wasting time." In contrast, the second enactment immediately guided students to the estimation question. During the first enactment, Kate allowed students to engage in hands-on calculations, both by hand and using calculators. However, in the second enactment, Sabrina's students were shown an image of the number of donuts arranged in rows and columns, giving them little chance to consider what operations to perform to calculate the number of donuts. This lack of time prevented a conversation about why the answer is 32 donuts in reality, rather than 33 donuts when calculating. The problem is that in the second enactment, a prompt intended to be used only *if necessary* by the original designers of the activity, Shaw et al. (2014), was used. Interestingly, the truncated word 'Necess*' was mentioned 16 times during the Secondary-Group meeting, six times during the Elementary-Group meeting, and once during the Mixed-Group meeting, signaling tensions not yet explored. For optimal learning with this math activity, outcomes suggest that teachers could start with a direct approach towards estimation, as in the second enactment style, and then switch to allowing actual time for calculation, as in the first enactment style. This way, students can experience the third act twist (i.e., the discrepancy between the number of calculated donuts by row and the actual number that fit in the box) and become aware of what words to take note of in a problem. Although in the third enactment, students were given time to calculate, many did not know how to proceed. In the fourth enactment, students were even less engaged in calculations, as the teacher spent most of the time talking to the students about the larger project instead of the Donut activity.

For secondary teachers, ensuring that the activity is correctly worded with appropriate math vocabulary is the main objective. To enhance the implementation of the Donut activity, practitioners closely examined how student groups handled the issue of volume versus area or how they reasoned their estimations with peers. One secondary math teacher with eight years of experience, who was also a former elementary teacher, noticed that an elementary teacher with nine years of experience did not fully explain why volume nor area were needed to solve this problem. Among the secondary teachers, there were varying reactions, with some being more critical and others more inquisitive. For example, one teacher questioned the usefulness of the king Henri conversion mnemonic. During the First International Conference on Concept Mapping in Mathematics, Schmittau (2004) criticized the curriculum for favoring calculator use over teaching algorithms. This criticism applies to the Donut activity where the teachers prioritized showing some information before students calculated their answers. Basically the teacher brought to life half of the second high quality teaching practice by eliciting students' ideas but not quite responding to students' inquiry. Perhaps an oversight in the lesson plan, after all it was “the most important segment of the lesson [..] where the teacher poses some questions to further analyse the ideas and methods shared by the students” (Takahashi, 2006, p. 200).

In short, four themes emerged from the analysis, including disparities in reactions to math talk, estimation, conversion, volume, and area. As a result of these challenges, one of the elementary teachers expressed the realization that this activity, which was intended to teach students how to find relevant information, must be presented in Grade 3 or 4 rather than in Grade 6. In the following, just as the video had reached the classroom pictures of blackboards, we paused and in this excerpt we hear the logic of the open-ended question from Melody's perspective as she explains it to the ELA guest (lines 2 to 11). And a bit later in the exchange, Melody expresses her surprise and the guest ELA agrees, suggesting starting as early as Grade 1.

> Kaci: Do you want us to pause it?
> RA1: sure
> ELA Consultant: My question doesn't really have anything to do with that. My question is, if, if the **question that they were going to pursue is, "How many donuts are in the box?" Why did they come up with questions prior to that?**
> Melody: For me, they came up with questions before, is **because my goal was for them to realize that depending on what the question is, you would need different information. So once they came up with** questions **I picked three of the best ones.** Let's say, "how much sugar do you need to make all these donuts?", and "how long will it take to deliver these donuts?", and "how many donuts are in the box?"
> **ELA Consultant: So did they solve for all three questions?**

Melody: No. I just say as a discussion, "so what if the question was
ELA Guest: ok
Melody: And then, "what if it was how far am I...?" "Well, then you would need where we're putting it, how much kilometers is it." You know, that kind of thing. And then I reveal, which one of these is the right question. "Bing, bing, bing. It's how many." So it was just to kind of let them realize that depending on the question, your..., what you need to know is different.
ELA Guest: Yeah.
Kaci: Or vice versa.
[…]
Melody: Yeah. And since we're on a pause, we were just kind of saying again that I think what's starting to frustrate me now is the whole, "I do this in grade six. Like, I teach grade six math." Grade six?***! This is when I'm showing them this, in grade six?!***
Interim Elem. Consultant: hmh
Melody: Phew. This needs to be grade three, grade four. Like, even the grade five teacher isn't... It doesn't matter if you communicate. If you're not bought into it, you're not going to do it, right? Like, that it's, it's…
ELA Guest: Even almost straight down to grade one
Melody: yeah
ELA Guest: too, right?
Kaci: Yeah, what's the question about/
Melody: Yeah. **Start reading the question from the beginning. Where is the question? Highlight it. What would I need to know?** Then go back. Like it's, I really... This is something... I think we've hit something here. I really do.

It remains to be seen who will bring such a strategy to tackle problems presented in an activity to the Grade 1 teachers and what is math talk without math calculation; two starting points for the School Board. In the next section, the second theme is discussed.

Theme 2 Estimation Challenge

Lydia: Well, I mean, at the beginning they sounded quite confused, because I was confused.	**vs.**	*Dona: It's like they don't know what an estimate is. And then they're like, the too low for them... Well, one is low but it's too low, but it could be a good estimate, but some people are thinking it's not a good estimate because it's not a reasonable estimate.*

This subsection dwells into the theme of the second critical incident, the estimation challenge drawing mainly on Vignette II and Vignette VI. According to Lehmann and Casella (2006) estimation is an "educated guess" for the unknown value (p. 4). This incident showcases how students made sense of set parameters where some information is unknown and some info available, in uncertain conditions where one impacts the other in an unknown fashion, necessitating thus conjuring hypothecation on the part of the student. Notice my clumsy opening of the discussion with the secondary teachers. Let us begin from the beginning of this particular critical incident.

> RA1: “So, how do you feel about this exchange, and the conversation? Would you have responded differently? Is there a particular part that jumps at you, in terms of student rebuttal? Or how the teacher replies? “
> Lidya: “Well, I mean, at the beginning they sounded quite confused, because I was confused.“

Apart from my compound question, in this excessively long-winded prompt, we can see that Lidya, with seven years of teaching experience, has identified ambiguity in students' understanding of estimation. This sentiment of confusion has also been voiced by elementary teachers. To analyze this critical incident on estimation, literature from Shaffer (2006) and Dreyfus et al. (2018) has been taken into account to make sense of this section. The truncated word 'Estim*' was used to generate frequency counts of 'Estimate,' 'Estimation,' and 'Estimating,' resulting in approximately 49 mentions in the Secondary group, 19 mentions in the Elementary group, and 14 mentions in the Mixed group.

The whole class discussion excerpted from the critical incident two highlights the need to clarify what is estimation in math. In this version of the activity the teacher guided her students to estimate the number of donuts and allowed the conversation to unfold amongst three students from Lines 1 to 6 with the class as “overhearing audience” (Green et al., 2007, p. 126). Next, she revoices student 3’s argument. While a bit later we can see her attempt to close the conversation (e.g, “okay. So”). However, the students’ mixed replies, “Yes” and “No” point to a possible lack of clarity in understanding why 1 belongs, i.e., is a legitimate answer. Paraphrasing a genuine question from a student, 'Is one too low?' One secondary math teacher perceived the student's response of one donut as an estimation to the teacher’s prompt “that you know is too low” as “a pain in the a**.” However, the elementary teacher reacted with less concern for the student's personality and allowed the students to debate amongst themselves perhaps because of the desire to turn students to each other’s ideas, not due to patience, not that she wasn’t. Furthermore Ben asserted that: “A reasonable guess to me would make sense” as he rejected the notion of educated guess that the students had reached consensus upon. Indeed, the student demonstrated an understanding of estimation by recognizing that there is 'at least 1 donut' regardless of what is shown on the front of the box. This is mathematically correct, regardless of the number of donuts actually inside the box. On the spectrum of guessing from zero donuts to one donut, five donuts, or a million donuts, vocabulary matters, but performing calculations to confirm estimations is even more important. In contrast, my co-supervisor, who has specialized knowledge in mathematics content, holds a different perspective:

> Not sure I have the same comprehension of “estimation.” To me, there is a confusion between estimation and maybe a process to make an appropriate estimation: which is to determine a plausible interval.

Table 6 tallies the instances of estimation or guessing discussed at the meetings, with 198 talking turns by secondary teachers and 181 talking turns by elementary teachers. Notably, the estimation topic prompted more confusion than the conversion topic.

Table 6

Turn-Talk Comparison on Critical Incident 1–Estimation Challenge

Type of F2F Meeting	Number of Utterances
Codesign Elementary Teachers	**76**
Implementation Critical Incident 1: Is 1 too low estimate? What is an estimate? What is “reasonable”? (2:40 min)	**36**
Secondary Teachers Collective Reflection	**195**
Elementary Teachers Collective Reflection	**181**
Mixed-Group Collective Reflection No Critical Incident Read	**40**

The following 40 lines are excerpted from a mixed-group conversation in which the teachers discussed estimation:

> Lin: It was about estimation. One of the kids... Do you remember that discussion that we had? **And Ben was adamant about estimating what the true meaning of the word is, and how it had to be clearer,** because, I don't know which one it was, but too low an estimate, too high of an estimate. One, and one gazillion.
> Ben: Oh, yeah. We were talking about, I don't know who it was, somebody was saying, "just give an answer." I think it might've been**... "Give an answer that is obviously wrong." And then we were talking about whether or not, just the way that was asked, and a kid who says, "one."** Or a kid who says, "a bazillion." If that... We were talking about whether that should be allowed as an answer. Seeing as how it's obvious that it is too low, or is too high. Or should we be encouraging them to try to make an estimate that would be maybe reasonable, but still too low? Or reasonable, but still too high?
> Sabrina: That's funny that you mention that, because I guess you guys didn't see the video clip of the whole debate that came from that lesson afterwards, about what a reasonable estimate was. There were some students starting to argue about how "one" shouldn't be accepted as an answer because it wasn't a reasonable estimate.
> Dona: Because there's so many in the little window, right?
> Sabrina: Exactly.
> Dona: We know there's more than one.
> Sabrina: Right. Which is kind of what happened in my lesson in my classroom when I was videotaped. The lesson took on a life of its own, and became this lesson about estimation and what's a reasonable estimate.
> Interim Elem. Consultant I forgot the conversation we had about how they often... I think it was the science one you had tried and then the dog food and they had… Their answers are so, sometimes, outrageous, and then... Remember, they were shopping for science test tubes?
> Kaci Oh, yes.
> Interim Elem. Consultant And then you were like, "but your answer is two million dollars?" And so just that lends itself well to estimation, in that if they can estimate roughly how much dog food costs are going shopping, that two million is really too high, and they should go back and double check their work.
> Dona And it comes back to Melody's conversation, and how she was saying how some kids have trouble understanding what a thousand dollars is, or what it looks like.
> Interim Elem. Consultant What it looks
> Dona: And how you said, like, my car, my house, and how much that costs.
> Interim Elem. Consultant What it means.
> Dona: And it's true, some kids are like, "how much...? That's not a lot." And I'm like, "you guys that's millions of dollars." Come on.
> Melody: [Inaudible 0:17:01.9]
> Dona: And they're like, "oh yeah?" Yeah.
> Ellen: I know that [inaudible 0:17:07.5], but I found it interesting. **I did this situational problem with my grade nines, and yesterday we finished it. It's a water reservoir, and they had to... A question with the metal. They had to replace all the metals, so surface area.** And my student comes up to me and goes, "miss, does this make sense? It's like seventeen thousand dollars." He goes, "does this make sense to you or not?" So I used the example, I said, "I know somebody," I didn't say that it was me, but when I had my car accident, I had a little piece of guardrail that my insurance needed to replace. So I just said, "I know somebody who's insurance had to pay, and just that little piece of guardrail is about ten thousand dollars." "Oh." So when I said that to him, he went back to his desk and went, "okay." But he needed to hear that seventeen thousand dollars was a lot of money. I said, "think about the millions of dollars the government invests in our roads." And then, "okay." And then it made sense. But in grade nine...
> Lin: You just don't know. But they wouldn't know. Because, you know, you're old, because you're older

than them.
Ellen: yes
Lin: So their idea of a lot... We often ask them to estimate so that we don't... In the clip you said seventeen thousand for dog food, right? But then, when they are faced with the problems that... Like I'm thinking of the sec two end of the year sit prob, where they have to desi...[corrects herself]..er, figure out the cost of coding a robotic space unit. Like, I don't know the last time you did it. How would you even know what a reasonable answer is?
Hmm
Lin: But they had to sheath it with anti-static stuff that cost a thousand dollars per meter. Like how do you even know if this is reasonable?
Kate: But I think sometimes I look at it more from the point of view of whether or not- like I would use dog food - whether or not students have ever actually bought the dog food in their house, which may or may not be relevant. Even just from a purely mathematical point of view, they know that the dog food is about ten dollars, and they've calculated it, and they're pretty reasonable. **They need two bags a month, let's say, for a year. Well, that's twenty-four bags, it's about ten dollars, and they come up with an answer that's millions of dollars. Well they should be able to mathematically realise that twenty-four bags at about ten dollars each doesn't come out to three million. And that's what I was looking at. Not so much in terms of the experience with if it's a reasonable price or not, but even just from the point of view of math and how numbers go together**.
Lin: But isn't that all about mental math?
Kate: Sure. Yeah.
Lin: The kids are completely... they cannot perform any mental operations.
Kaci yeah
Melody: Except if you're freaking out, right? And you're like, "I can't do this, I can't do this." And you're just calculating and you get to a number, and now someone says, "does that make sense?" You're like, "I'm just trying to get it done.
Kaci yeah
Melody: I don't know if it makes sense." As much as they see that it's reasonable, "come on it's ten dollars," or whatever, it's not reasonable when you shut down and you're like, "I don't know if it's reasonable or not. I don't know."
Kate: And maybe that's part of what I've been thinking about. Sometimes I find that students have a really hard time with rounding. It's always so abstract because we focus on the rules, of like, you round up, you round down. Maybe this kind of discussion is actually more beneficial than... Who cares if we round fifty-five point six to fifty-six?
Melody: right
Kate: Is that rule that important? Because depending on the situation, the highest might actually be better than the lowest.
Melody: A conversation about just life things, and the approximate value of things.
Kate: Yeah. I mean, I had a student tell me, if I'm estimating with money, I should always go higher. Even if it is fifty-five twenty, I'd be better off calculating with sixty, than fifty.
Melody: to have enough money
Kate: It was a skill that her family taught her, but that was valuable.
Melody: Give yourself enough time, also.
RECIT: Okay, so we'll take a break for lunch. Ben you can draw a little sandwich now, or an egg. [Laughs]. So we're heading over to our [inaudible 0:21:20.9], and demonstrations [clip cuts off]

Notice the realism sought by Melody: "A conversation about just life things, and the approximate value of things." With the coding scheme proposed by Hennessy et al. (2020)

identified instances that could indicate students' understanding of the concept of estimation and their ability to apply it in a real-world scenario. In the conversation the teacher and students are discussing the concept of an estimate and the idea that a given estimate of ‘one’ for the number of items in a box is considered ‘too low’. The teacher facilitates the conversation, guiding the students towards a deeper understanding of the concept of an estimate.

The first coding category, *defining estimation*, was used to identify instances where the students provided a definition of estimation. In the whole class discussion extracted from the first critical incident on estimation, this was exemplified by Student 10, who stated that “it's approximately how much something is going to cost” (line 29), and Student 11, who defined it as “approximation.” The teacher also contributed to the definition of estimation by stating that it is “approximately, about, how much maybe something is going to cost, if we're talking about money.” The second coding category, *application of estimation*, was used to identify instances where the students applied the concept of estimation in real-world situations. In the critical incident, this was demonstrated when Student 6 stated that the estimate of 'one' is too low, and when Student 7 stated that an estimate of 'one' is not reasonable because “there's more than one in the box.” Student 9 also applied the concept of estimation by stating that it should not be included because it is 'too low' and not an educated guess. The third coding category, *using evidence to support estimation*, was used to identify instances where the students used explanations to support their estimation, although not many had concrete evidence. In the conversation, this was observed when Student 7 stated that there are “fifty, about” donuts in the box and Student 8 stated that there are “five” donuts in the box.

With this theme the topic of estimation is brought forward. We get an insight into how students make sense of estimation, we know at least one of them, defined it as an “educated guess.” The student’s use of the term ‘educated’ propelled a very interesting discussion on what words must be used in an elementary classroom to optimize student learning math. For example, Ben argued that “Well, an educated guess, I, I don't necessarily agree with it being an educated guess. Um…, A reasonable guess to me would make sense.”

> **Assist. director of Ed. Services: This is not kid vocabulary. This obviously, this came from somewhere, this educated guess,**
> [collective Hmm]
> **Assist. director of Ed. Services: is misleading. They're in school, they're in the context of education, they're making a guess.**
> Lydia: Hmm
> Assist. director of Ed. Services: So I don't know exactly how revealing the use of that term...
> Interim Elem. Consultant: And it goes with the question too, the lowest estimate,
> RA1: yeah
> Interim Elem. Consultant: [15:00] versus make an estimate that is too low in this context
> [collective Hmm]
> Interim Elem. Consultant: to show your understanding.
> RA1: so your wording is important
> Interim Elem. Consultant: Your wording is very key.
> PI: yeah.
> **Ben: It's also misleading, right? You make an estimate that is obviously too low.**
> RA1: yeah
> **RECIT: Zero could work too**.

The educators in this passage are uncertain about the origin of the students' use of the expression 'educated guess.' Perhaps it was learned from an involved parent who read Lehmann and Casella's (2006) book. Using the coding scheme developed by Hennessy et al. (2020), the classroom conversation reveals the students' understanding of the concept of estimation. The results indicate that the students were able to provide a definition of estimation, which suggests they have a grasp of the concept. However, the study also highlights the importance of teachers being specific with their prompts and rigorous in their explanations of concepts like estimation:

> Or, my question is, going back to that understanding of what an estimate... If their goal is look at kids have rigorous arguments and that, is that because that student doesn't understand the true definition of what an estimate is? so that, you know, **maybe a teacher, maybe they need to be more specific and more rigorous in their explanations of concepts like that, so that a kid... It might've been the kid just trying to be difficult.**

Overall, the RÉCIT concedes some students may not have understood the concept of estimation because possibly they didn't get a true definition of an estimate and then suggests that the teachers may need to be more specific and rigorous in their explanations of concepts like estimation. A discourse analysis of this passage would likely focus on the specific language and terminology used, as well as the perspective and tone of the speaker. For example, the use of terms like 'estimate' and 'rigorous' signal a focus on precision and accuracy in the RÉCIT's approach to teaching math concepts. The use of the phrase "my question is" also indicates a personal investment in the topic, suggesting that she is engaged in the conversation and interested in finding solutions to the challenges faced by students in understanding these concepts. Yet her wonder remains at the personal "my" level not quite moving towards the "our" point of view. Conjuring that kids may have *tried* to be difficult really leaves me with questions having no classroom experience in the elementary context. Next, the conversion challenge.

Theme 3 Conversion Challenge

> *Interim Elem. Consultant: Because I think all three were given in millimeters, they wouldn't see the necessity to convert. So they're often taught to convert when there's mixed units*

In this subsection, the conversion challenge is discussed using the reflections that were reported in Vignette III and Vignette VII. When discussing donuts (an unconventional shape due to its approximate size of 89 mm) and box width and length given in millimeters, the units are technically the same, as explained by the interim elem. consultant during the secondary group's coreflection meeting. However, the students still have mixed units, as there are donuts in a 6900000 millimeter square box, which requires some conversion thinking. Ben points out that even converting to centimeters only moves the decimal two places, resulting in a number that is still too large for the box. The group discusses the need to consider what the six million represents and how to connect it to the size of a donut. As the discussion illustrates, there is still much to be explored in terms of students' understanding of measurement units.

> **Ben: Even going to centimeters, they only move the decimal two places. So you're still looking at six hundred and ninety...no, sixty nine thousand something.**
> Ellen: Hmm
> Ben: Right? It's still a big number for that box.
> PI: Yeah. But I think the kind of discussion around... it's six million of what?
> Lin: yeah. AND Ellen: Mm.
> **PI: How do you go from a box and donuts... We know the size of a donut, so how do we come up with six million? Just to get them to think about,**

According to Dreyfus (1999), students' ability to prove math solutions relies on procedural knowledge, which many students are not often exposed to. Thus, appropriate modeling in the classroom becomes a challenge, especially if some elementary math teachers cannot model rigorous proof writing. Would your interpretation of the conversion conversation change if you knew that the teacher provided candid responses during her one and only year-end interview in Summer 2017? Our PI asked, "What do you find challenging about teaching math?" Dona expressed her challenge of "not always knowing the answer" and the challenge of teaching in a heterogeneous classroom setting:

> 302. PI: And what do you find challenging about teaching math? [30:26]
> 303. Dona: Math. [inhales] **Not always knowing the answer. Like sometimes they come to me and I am, I don't see it the way they see it, and I'm trying to have them go towards trying to understand how to do that concept, yet they're seeing it completely different** and trying to... So like both thinkings.
> 304: Dona: <u>And having just so many different levels</u>. Having those that really get it, and they see it, and then those that are just struggling, and they want to, but they just don't get it.
> 305. PI: [chuckle]
> 306. Dona: And it's trying to cater to those.
> 307. PI: Right.
> 308. Dona: You know? **Because they're all in the same class. So you have the bunch that really does get it, and is already going one step further, and those that are still struggling.** And you see that those that are getting it are like...
> 309. PI: Mhm, getting bored.
> 310. Dona: Getting bored. **And those that don't get it are frustrated because they don't get it.**

During the conversion conversation, the student's question "what do we do after?" is a good case in point for pre-planning strategies that help students move forward. Starting the conversation together by acknowledging that even as a teacher, you don't know the solution right away, is what drives the need to continuously examine activities to then improve them. We have a wealth of information on what teachers consider challenging when teaching math. Aggregated data of challenges can be found here for future reports.

Table 7 displays the frequency of discussion on the topic of conversion during each meeting, including only 15 lines at codesign, 17 lines at implementation, and 13 lines at mixed meetings. The table also reveals that secondary teachers contributed a significantly higher number of talking turns (215) compared to elementary teachers (65).

Table 7

Turn-Talk Comparison on Critical Incident 2–Conversion Challenge

Type of F2F Meeting	Number of Utterances
Codesign Elementary Teachers	**15**
Implementation **Critical Incident 2: "What do we do next?" Teacher explains using chalkboard drawings. (3 min 53 sec)**	**17**
Secondary Teachers Collective Reflection	**215**
Elementary Teachers Collective Reflection	**65**
Mixed-Group Collective Reflection **No Critical Incident Read**	**13**

The excerpt below includes all 13 line pertaining to the conversion challenge uttered at the mixed-group:

> Lin: Well, we had seen the clips from before. And we had quite the discussion about area, perimeter, conversion, it was huge. And we'd seen that, or had a discussion about how much the units of measurement are really taught now in elementary**. Because in high school, the kids were struggling with decameter, decimeter, and yet we don't really use it. We expect them to know it. And they know the King Henry Died thing, but they really don't know what that means. They just know that hectometers are next to...You know? But they don't know how to convert. I know I'm seeing that now with the area. Like**

when the huge number that they had in millimetres, well had they converted properly, using area... They're still, even in sec three the whole thing is still...
Ben: The other thing that we mentioned when we were looking at that, was in terms of solving that problem, solving that problem in millimetres doesn't make sense, right?
Lin: Right.
Ben: **And for a student to look at it and see that huge number, in terms of the area of the box, or whatever they may have calculated, and it's got all these zeros in it. And conceptually, they know what the size of a donut is. You've got two very different mindsets, where you have this huge number... How does that fit with the donut? Right? So right there, by maybe not using the right unit, it creates a situation for them where now they have to figure something else out. Why... How does this go together?** So when we found out that you guys in elementary school, you don't see all of the units, or you don't use all of the units... I know, to me, and I think a couple of the other high school teachers who were sitting around the table, the question was, why not? It doesn't make sense to put them in a situation where you're going to solve the area in millimetres, when it's not an appropriate unit to use. Right?
Lin: Oftentimes activities, I remember doing the elementary math, was, what would I use to measure this table? What would I use to measure the thickness of my pencil? And yet you're asking, multiplying a huge box in millimetres, trying to encourage proper units of measure. Like, you know people who give their baby's age in months, when they're like thirty eight months, and you're sitting there trying to calculate. Like, use appropriate measures.
Kaci I think also it's not... Like in the application questions, and the situational questions we use, the measurements are accurate to what we're doing. I think this is because it was new. Wherever it comes from. Was it actually a Krispy Kreme email answer? I don't know. But that was the answer given from the Krispy Kreme company. You wouldn't see that in the MELS application, situational thing. You would use appropriate units of measure.
RA1: But at the same time, I wondered. I was like, "what would I do with six million?" I sat there and I was like, okay think about the first time I [inaudible 0:04:22.6]. And I was there looking at seventy-seven points [inaudible 0:04:26.0] some digits, and I was like, "okay, well what do we do after that?" And so I divided it again by eight-nine, until my friend told me, "well it's area divided by diameter squared." I realised that that six million could've been reduced as soon as you divided by twice eighty-nine, eighty-nine squared. So you can still bring that big number into a, into manageable understanding if they knew the formula.
Ben: That's not a concept that
RA1: that they know
Ben: is in elementary. You guys, you don't do...
Kaci No, you can look at the confusion on our faces while she's talking about divide and divide.
Ben: right [Laughing]

Table 8 presents Dudley's (2013) codes used to identify utterances related to the conversion challenge. The analysis focuses on 104 lines spoken by secondary teachers, of which four contained suggestions, while only one suggestion was made out of 65 lines spoken by elementary teachers. Although Critical Incident 2 was not specifically discussed during the mixed-group conversation, the utterances above provide valuable insights into the educators' delimited territories regarding conversion issues. The types of utterances include echo (5 secondary groups), corrections, agrees (39 secondary, 18 elementary, 2 mixed), explanations, recounts, reasonings, questions (9 secondary, 4 elementary, 1 mixed), and observations. Although Critical Incident 2 was not discussed in the mixed-group conversation, the gathered utterances provide insights into the delimited territories of the educators regarding conversion issues. The coding took place in 2019, indicating that the codes have not been challenged in some time. These findings reveal how different groups coreflected on critical incident 2.

Table 8

Dudley's (2013) Categories Critical Incident 2 Across Three Coreflections

Type of Utterances	Sec. Teachers	Elem. Teachers	Mixed-Group
Total Proposal	1	1	0
Total Echo	5	0	0
Total Suggestions	**4**	**1**	**0**
Total Corrections	**3**	**1**	**1**
Total agrees	39	18	2
Total Explanations	5	6	1
Total Recounts	0	9	1
Total Reasonings	3	2	0
Total Questions	**9**	**4**	**1**
Total Observations	**4**	**2**	**1**
Talk Turns Dudley Coded	**104/104**	**65/65**	**11/110**

The truncated word 'Conver*' was mentioned approximately 32 times at the Secondary-Group meeting, three times at the Elementary-Group meeting, and four times at the Mixed-Group meeting, referring to Conversion, Converts, Converted, and Converting. Analysis of the critical incident related to the conversion challenge revealed the use of coding categories from Hennessy et al. (2020), such as Invite elaboration (I), Make reasoning explicit (R), Positioning and coordination (P), and Build on ideas (B), by the educators to guide students through the challenge. The students demonstrated their understanding through the coding category Express ideas (E). However, the secondary teachers noted that the students were left without guidance on how to proceed with the area of the box in the conversion challenge. Meanwhile, the elementary teachers focused primarily on how a student solved the problem, rather than addressing why the teacher did not respond to the student's initial question. Theme four, pertaining to the volume and area debate, will be discussed next.

Theme 4 Volume vs. Area Challenge

Kate: It looks quite a bit taller than 1 layer of donuts

In this subsection, we discuss how students and teachers settled the debate between volume and area, drawing from Vignette IV and relevant segments of the elementary and mixed-group coreflection meetings. During the secondary coreflection meeting, after analyzing the third critical incident classroom discussions, Lydia expressed concern about the teacher's failure to validate a student, stating, “I just found, though, the teacher was kind of like not validating him on that.” Unfortunately, we could not obtain comparative data from the elementary teachers' coreflection due to time constraints. However, a frequency count of the term “volume” revealed 33 mentions in the Secondary-Group, nine in the Elementary-Group, and 10 in the Mixed-Group.

At the mixed group coreflection, Kate shared her realization that “area” was unnecessary for the Donut activity. Her class had debated the relationship between area and volume, but ultimately some students may not have realized that using the area would yield an overestimation of the number of donuts its the box, that the volume cannot be obtained with the data provided in the email, therefore, potentially it would lead to a misunderstanding of the problem. This is how Kate shared her realization in April 2017:

> Kate: I think **in my class, one of the things that came up was the whole Area, Volume debate that could generate as we went along. But I think that this came up. You know, even, even trying to use area is not necessary**. It's not even an appropriate concept because we're not talking about area, we're

> about more dimensions and spatial organization than anything. So that was interesting when it came up. The kids who were talking about volume, you know, they were stuck on this 3D thing. There was really only one student who really, really understood that volume was irrelevant in this case. And he was the one who kept bringing up, "but you're not going to fill all the space anyways, so it's not about how much stuff you can put in this box." It's about the shape and how many you can lay side by side, not about filling an entire space. So that was kind of where I went to. But i thought, that was one of the benefits that I saw to leaving the questioning quite open. **I kind of saw that this was not understood. That we had a shaky understanding at best, as to why we don't need area or volume, putting aside the fact that we don't know how to calculate the volume of a cylinder or the area of a circle. Like, I found that that was more where the kids were like, "oh well I guess we don't need area, because we don't know how to do the area of circles. So I guess we don't need that."**
> RA1: mhm
> Kate: But they didn't realize that even if you know how to do Area of a circle, you still A, don't need it, and B, shouldn't be doing that. So that was the eye-opener. That was the same question on last year's math exam**. The famous cancelled question, about putting books in the box. Kids were really, really determined to find Volume of books, and Volume of box. It had nothing to do with the volume.** There was space in that box that you just couldn't fill because of the size of the books. So I think that that was something that we/

Kate is revoicing what she had said at the elementary group meeting in March 2017 for the benefit of the secondary teachers:

> **Kate: I think maybe by forcing them to have the visual they may have realized that this whole finding the area step was not only unnecessary, but maybe, potentially thinking about it in the wrong way.**

This type of insight, “eye opener” as Kate phrased it at the mixed group meeting, underscores the importance of discussing the optimal unpacking of the concepts necessary for the Donut activity that involves two geometric shapes (a circle in a rectangle), an ambiguous task (e.g., missing information and approximated measures) during the codesign meeting. Oddly, some teachers took the time to explain that Krispy Kreme is a donut but not many took the time to unpack math concepts involved in the Donut activity.

Taken together, the different insights and experiences shared by teachers like Kate in various meetings provide a valuable foundation for improving the lesson plan and our procedures for creating them. We can begin by unpacking and discussing the concepts necessary for an ambiguous task like the Donut activity, so that teachers can provide a clearer understanding of the task and the concepts involved, thus improving students' ability to learn and retain the relevant knowledge. Table 9 shows the frequency of ten categories by Hennessy et al. (2020) across all three critical incidents.

Table 9

Dudley's (2013) Categories Critical Incident 2 Across Three Coreflections

Category	Critical Incident 1	Critical Incident 2	Critical Incident 3
Invite Elaboration (I)	13	4	11
Make Reasoning Explicit (R)	1	1	12
Positioning and Coordination (P)	1	0	0
Build on Ideas (B)	8	8	0
Reflecting on Conversation or Activity (RD)	10	0	0
Connect (C)	1	0	0
Express Ideas (E)	0	1	6
Guide direction of conversation or activity (G)	2	3	0
Dudley Agreement	0	0	1
Classroom Management	0	0	1

The table above summarizes the comparisons across three critical incidents based on the categories of Hennessy et al. (2020): Critical incident 1 had the highest number of instances of the Invite Elaboration (I) category, with a total of 13, followed by Critical Incident 3 with 11 instances, and Critical Incident 2 with only 4 instances. The Make Reasoning Explicit (R) category was the highest in Critical Incident 3, with a total of 12 instances, while Critical Incidents 1 and 2 had only one instance each. Critical Incident 1 and Critical Incident 2 had the highest number of instances of the Build on Ideas (B) category, with 8 instances each. Critical Incident 1 also had the highest number of instances of Reflecting on Conversation or Activity (RD), with a total of 10 instances. The Guide direction of conversation or activity (G) category appears in two instances in Critical Incident 1, three instances in Critical Incident 2, and no instances in Critical Incident 3. This may suggest a deliberate attempt to steer the discussion but absence of instances in Critical Incident 3 could indicate that the teachers did not feel the need to guide the conversation as part of her unguided approach.In terms of the Dudley Agreement and Classroom Management categories, both categories were present only in Critical Incident 3. These findings suggest that the patterns of conversation vary across different critical incidents, indicating the importance of examining the details of each incident separately.

Overall, classroom discussions between teachers and students revealed uneven understanding of estimation, conversion, and measurement concepts. The critical incidents related to measuring the space taken up by the donuts illuminated how reasoning was used by both teachers and students. The most common categories in the incidents were invitation and very little guide codes. However, Dudley Agreement and Classroom Management, surfaced only in the third incident. Secondary math teachers expressed concern about students making assumptions about volume being necessary because the box is in 3D. Ben questioned whether students were leaving class with a clear understanding of when volume is relevant or not.

> **If you're a student in this class, and you just heard this discussion, are you walking out of that class saying, "okay, so I kind of get an idea of when volume is relevant and when volume isn't relevant,"** right? The fact that it's 3D doesn't necessarily mean we always have to look at volume, just because something's 3D. So I'm right in saying that it is a 3D question, it has volume involved, but the volume is not important now. I **don't think anybody leaving this classroom gets that feeling. Now they're leaving and they're saying, "is volume important here or not?"**

In summary, at this stage of the discussion we may agree coreflection is probative; the coreflection meetings on the Donut activity unearthed evidence of four math challenges during implementation: math-talk, estimation, conversion, and measurement of volume versus area. Sabrina facilitated a discussion with students about basing estimates on relevant information rather than random numbers. In Dona's class, some students struggled to convert numbers into manageable units for easier calculation. While Kate and her students discussed measurement concepts and the limitations of using volume and the impact of different types of donuts in calculating the number. I echo Ben's confusion and wonder if it would affect exam results. Taken together these findings illustrate that the activity meant for students to consider all the possibilities to find relevant information to make an informed decision. However, simply picking up relevant information is not enough for metacognitive understanding and double-checking estimation calculated by providing information necessary to determine whether one's estimate is overestimated, underestimated, or closest to the actual value. As it turns out, many students were unable to complete the Donut activity on time, perhaps indicating that they did not fully grasp these concepts or ran out of time, as one student insisted.

If during math class students are not compelled to apply math knowledge and they tend not to do homework, then it raises the question of when are students given the opportunity to practice applying their math knowledge? Melody's interview answer, "they never do homework," may be a barometer of the kind of issues raised as challenges in teaching math. For a discussion on homework, see Deslandes and Barma (2018). While these four challenges offer valuable insights into the complexities of math learning, it is important to acknowledge the limitations of interpreting such rich conversations and to consider possible alternative interpretations. These issues will be further discussed in the following section

Limitations–Cliffhanger

Ellen: But do they continue to address that, or no? Because we don't know what continues…

This subsection aims to examine the limitations of the study and the intricacies involved in interpreting video data in the context of a math coreflection meeting. Interpreting video data is a complex and nuanced process, with the meaning of an activity varying depending on the viewer. To elaborate further on the limitations of this study, two subsections will be presented: one on qualitative limits and another on analysis limits. As Ellen, a secondary teacher, rightly pointed out, the video montage used in this study presents a limited view of the Donut activity and leaves many unanswered questions about what happened before and after the presented events. These qualitative limits highlight the challenges of interpreting video data in a way that captures the full richness of the context. On the other hand, the analysis limits pertain to the specific methods used to analyze the video data, which may have constrained the interpretation of the results. Overall, this section will provide a comprehensive analysis of the limitations of this study, with a focus on the intricacies involved in interpreting video data.

> PI: Ellen?
> Ellen: I just found it interesting in our responses on the elementary, because they did the whole process, and their reflection is really on the whole process. Whereas, the secondary, we just watched the clips. So our answers are really just based on the delivery of it.
> PI: Yeah

Moreover, the study's analysis is incomplete as it only examines the first of two codesigned activities. While the study primarily focused on the Donut activity and its associated challenges and learning targets, it did not include an analysis of the Let's Paint activity. Without analyzing the second activity, Let's Paint, it is not possible to fully determine whether the learning target was achieved. At best, we can only deduce that the Donut activity may have only achieved 50% of the target, or it may have had a weak plan. It's important to note that this study is not intended to provide a representative sample or generalized view of coreflection, as the groups involved were small and varied in composition, making meaningful comparisons difficult. Furthermore, as part of a unique research-practice partnership that employed lesson study procedures within the context of Québec, the interpretations of the video data are specific to this study and cannot be compared to others (Sanford, 1995; Zielinski, 2010). Qualitative and analytical limits are discussed in subsequent subsections.

Qualitative Limits. The study did not provide a way to verify the validity of the concept of collective reflection. The focus was on segments related to Donut math activity, prioritizing these over other themes in the conversations, with the understanding that this approach has limitations. Due to the nature of qualitative research, the data for this particular study cannot be replicated. While collective reflection contributed to the collegiality of the teachers' professional learning network at the Sunny Side School Board, the increase in collegiality tended towards a lack of criticality that played a key role in limiting the actual improvement of the lesson plan. In other words, the group's emphasis on collegiality in the collective reflection at Sunny Side steered the conversation towards a less critical approach, rather than addressing the root causes of the mathematical challenges that were exposed during the Donut activity enactment through students' work and unanswered questions.

Notably, discourse-in-use and content analysis, alongside key-theme-in-context, were employed in the protocol of watching and rewatching to better understand the phenomenon of coreflection. However, as Erickson (2006) warns, the researcher's “unexamined common sense notions about social interaction” (p. 180) can limit the interpretation of interaction as a learning environment. According to Erickson (2006), talk occurs within a social ecology where interaction is a simultaneous mutual influence between speakers and listeners. Therefore, talk cannot be interpreted as unproblematic evidence of teacher thinking, as it is not a straightforward exchange of words with stable meanings. Erickson warns against simplistic analysis that examines only the manifest content of words and strings without considering the hidden meanings and constant shifts. Although I acknowledge that there were many social tensions that were not diffused with the video-protocol, as it was not suitable for tensions related to the reality of video research, I was mindful of these pitfalls when categorizing the partners' utterances and words. As Spinoza (1677) wrote, “I laboured carefully” to gain “understanding” of these teachers' talk (Ch. 1, Introduction; section 4). In summary, although segmentation and interpretation are not straightforward tasks, I tried to make my process explicit.

As previously stated, this report imagined a scenario where all other factors required for an optimal learning environment were held constant during lesson study. For example, assuming that partnering in codesign of lessons would increase the feeling of agency by creating instructional resources (lesson plans) that are “more useful and usable” for teachers (Gomez et al., 2018, p. 401). The teachers did take ownership to the point that they added their own twist to the activity, causing considerable variation across enactments. The Donut activity's codesigned lesson plan was considered stable enough to serve as the constant. Since multiple regression is not a common method in education research, and our facilitation method was not scripted (i.e., guided), therefore, our focus shifted at each meeting. Another cause for variability was that the video representation of the Donut activity was adjusted as data was collected. The study's aim was to provide a sufficient description of each enactment to warrant recommendations rooted in classroom evidence. Since this study focused on the post-enactment discussion at the coreflection meetings, many insights shared during the meetings are beyond the scope of this analysis. However, one valuable insight emerged in the teachers' discussion on how to hear and respond to their students' contributions during group work. Notice all the personality and behavioral comments in the exchange below:

> **Kiera: And I was going to say that I was just asking too, were they listening to each other so cooperatively because they knew, they were being videotaped? Because in cycle two, yeah, wow! It would be great for a group discussion to be like this, but in reality it's all about listening**. Okay. Like, I **have a chart paper in my room that says, "what do you do in group work?" And I go around, and they see with my marker, I am like, I go: "Okay I see Don, he's listening. I see John, he's asking questions." Because they don't even know how to interact in a positive way in a group. So I'm [chuckles] just focusing on that.**
> **RECIT: That's something you said you've been working on for the past two years, right? Like I just noticed a big difference in your group from the last video that we watched, even of this year. The way they're talking.**
> Kaci: I mean, that group too is...
> RA1: It's a special group.
> RECIT: yeah.
> Kaci: Well, they're all very kind students.
> RA1: hmm
> RECIT: but you can tell/
> **Kaci: /There's nobody who like knows they're always right and thinks themselves better.** Which I have in other groups. And this I was lucky, because I just said... I had some kids who hadn't signed the permission slip,
> RECIT: yeah.
> Kaci: I didn't want them to miss out on the benefit of the activity. They had to sit at the back so they wouldn't be caught on camera. **So I didn't even have the forethought to place them in groups.** So I just said: "You can sit there, there, there. Get in groups."
> Interim Elem. Consultant: wow
> **Kaci: And that could be bad sometimes**

This excerpt illustrates the challenges of using video recording and grouping students for professional development. As Gomez et al. (2018) note, grouping can sometimes be difficult for teachers to manage when they do not have enough time to respond to individual learners who struggle with math concepts within their group. Additionally, with three cameras recording different groups of students, it may not be feasible for teachers to review all three hours of footage without dedicated time for lesson study. This raises questions about the practicality of using video recordings as a form of professional development, given the limited resources available to teachers (Wang & Pain, 2010). Furthermore, the interruption by the Gym teacher during the rich classroom conversation on prime factorizations raises questions about the appropriateness of impromptu announcements during math hour, which may disrupt the learning environment for some students.

Taken together, the findings suggest that elementary teachers expressed more appreciation while secondary math teachers offered more criticism during the video-mediated meetings. This raises questions about the effectiveness of the collegiality approach in this context and highlights the potential limitations of using video as a tool for facilitating collaborative knowledge-sharing sessions. While the study did not thoroughly explore the underlying reasons for these differences, it underscores the value of lesson study as a means of promoting critical reflection and replacing unadaptive teaching practices (Cribbs, 2020; Stigler & Hiebert, 2016). Given the narrow focus of the study, however, there may be other important aspects of the lesson study process that were not fully captured. Therefore, the findings and interpretations of this study should be interpreted within the limitations of the analysis.

Analysis Limits. There are several limitations, primarily methodological, that I am aware of, and others that I may not be aware of due to my limited expertise in education research. Specifically, “content analysis carries its load of human error, where the researcher's actions and decisions are open to scrutiny as they are part and parcel of the evidence that supports or contradicts the claims made in the research outcomes” (Harding, 1987, p. 9). “This evidence must also be subject” to critical examination (Harding, 1987, p. 9). Additionally, the final product of this study did not lead to a clear conceptualization of a collective reflection because according to Bryant and Charmaz (2007) “there is a set of methods essential to the research design that must be used in order for the final product to be considered a grounded theory” (cited in Birks & Mills, 2015, p. 6). The case of the data at hand is that it was not simultaneously collected, analyzed, re-collected, and re-analyzed. Therefore, every interpretation made above is subject to countless conditions. The next limitation is lack of context, pointed-out by Ellen as she observed the limit of viewing edited videos during collective reflection:

> Line 70. Ellen: It's hard, too, because we're only seeing little clips, so we don't really see what really happened
> Line 71. Lydia: Yeah
> Line 72. Ellen: step-by-step. We don't know what was said before, we don't know what was said after/
> Line 73. Lydia: /said after
> Line 74. PI: Hmm
> Line 75. Ellen: We just see one little snippet. [laughs]
> Line 76. PI: Yeah. Yeah. **It's the downside of editing, to cut it to a shortened version**, because we don't have time.

Additionally, the research team used a double-stimuli approach by combining the video watching and the questionnaire (Appendix B) as two simultaneously prompts. This method carried its own limitations, as it is excerpted here:

> Lydia: Some of **my answers** to these questions **don't make sense**.
> Ellen: I know/
> Lydia: / Because **I was writing them as I was watching, and I didn't write properly**.

The limitations of this qualitative study include the non-generalizable nature of the data, the possibility of unmeasured biases, and the particular makeup of the partners participating in the lesson study, as well as the unaccounted-for dynamics created by the facilitators. Furthermore, there are many variables that have not been considered, such as personality markers that could help evaluate a teacher's individual personality traits (Akabari, 2007).

As a novice researcher, I acknowledge the potential for trivial claims in my analysis, which has been criticized in earlier versions of conversation analysis (Wooffitt, 2005). Additionally, my limited knowledge of mathematics teaching and learning raises questions about the potential benefits of incorporating more foundational studies of communication, social interaction, language, and culture into video analysis in educational research, as suggested by Erickson (2006)

> whether video analysis in educational research would be improved by influence from more foundational studies of communication, social interaction, language, and culture. That is a presumption of this essay, but it is a wager, not a certain thing. To be fair, the subject matter guided use of data derived from videotape has made substantial positive contributions in studies of teaching and learning manifest curriculum (p. 180).

Hopefully, this study did not fall into what Vygotsky (1978) deemed as unclear methodology:

> But the relation between learning and development remains methodologically unclear because concrete research studies have embodied theoretically vague, critically undervalued, and sometimes internally contradictory postulates, premises, and peculiar solutions to the problem of this fundamental relation; and these, of course, result in a variety of errors (Vygotsky, 1978, p. 79).

In conclusion, the impact of facilitators on the practical implications of video-mediated collective reflection is often overlooked in discussions of lesson study's potential to enhance learning outcomes. Although the edited version of the classroom discussion surrounding the Donut Activity does not fully depict its logical flow, the need for improvement is undeniable. As French dramatist Rolland (1922) noted, “ne cherchez point ici de thèse ou de théorie. Voyez-y seulement l'histoire intérieure” (p. 10). In other words, readers should not expect a unifying theory within these pages, but rather an exploration of the inner workings of the research topic. Next, I will discuss alternative interpretations to expand on the implications of this study.

Alternative Interpretations

The available data suggests that elementary teachers with perhaps limited mathematics content knowledge found the activity valuable, while secondary teachers with perhaps greater math content knowledge recognized its limitations. However, as member checking sessions were not conducted, alternative interpretations are possible. Certain factors were beyond the scope of the study, but not necessarily irrelevant. For example, only my coadvisor, a math expert, referenced the plausibility realm during discussions on estimation, this was not mentioned by other educators. Due to my lack of math content knowledge, I may have missed important aspects of the discussion. Alternative methods for categorizing the dataset could yield different interpretations, but in general, the final observations were consistent across themes.

Overall, coreflection mediated by video has potential, but critical restructuring is necessary for optimal results. Qualitative coding schemes that merge overlapping codes may be useful for this purpose. This study assumed a scenario where all factors for creating an optimal coreflective environment are constant, except for the teacher's group. However, achieving these conditions was nearly impossible. Multiple interpretations exist regarding which part of coreflection leads to recodesigned lessons; this is discussed next.

Lesson Study Implications

> *Kaci: /That's it. It was fun to see one video back to back, all the different ones. Like I took notes. "Maybe I should've done this, maybe I should've done that."*

Notwithstanding the limitations enumerated above, the outcomes suggest a few practical and theoretical implications for lesson study procedures and lesson codesign considerations. These findings have important implications for teacher preparation programs and professional development initiatives. The impact of engaging in lesson study is felt when Kaci mentioned that she took notes on what she "should've done this" and not that. Unfortunately, we were not able to take a picture of her notes, therefore, we cannot decipher what she took away for future enactment of the activity. We could begin by emulating Glocal; start with one lesson in a class to many lessons in many classes to then inform the entire unit and eventually the curriculum.

The assistant director of educational services, immediately located the problem of over planning a lesson. Its design must be in a such a way as it organically reveals the lesson, because as the ADES rhetorically asked:

> **IV.311 ADES: It's not about scripting right?**
> IV.312 RÉCIT: No.
> IV.313 ADES: Like even now we're discussing it's not... **You'll never be in the exact same scenario twice**. […]
> IV.321 RA1: Which makes us wonder whether at the codesign...not scripting it necessarily but really accounting for possibilities of certain concepts that are really necessary prior to this activity so that either you define them before or you keep them in your back pockets so that you're not scrambling for words.

If we don't script, what can we do? We can anticipate. To contextualize, instead of examining each phase of the lesson study in isolation, we analyzed the entire first iteration of the lesson study cycle. As a result, we do not have data of a second iteration of lesson enactment and video-mediated collective reflection with a modified lesson plan. Our approach involved examining the codesign, followed by a review of the videos of the 12 student groups during the four enactments, and concluding with the elementary collective reflection session. Implementing a video-mediated collective reflection meeting requires significant investment and is often met with initial enthusiasm followed by skepticism about measurable improvement. Despite being difficult to achieve, and even more so to maintain practically, such interventions hold the potential to transform lessons and achieve desired outcomes indirectly.

Qualitative research on lesson study program sustainability can help lesson development interventions reach their full potential. These lead to considering: How can we organize the data in more manageable bite-sized pieces to allow for coreflection on the lesson? We have footage that keeps classroom enactments alive to an extent, awaiting review for lesson improvement purposes. Overall, videos enabled teachers' continued coreflection, creating ongoing opportunities for improved teaching practices via lessons that support learners in understanding math concepts. Next, the Now What question.

Now What? Addressing the Rashomon Effect

RÉCIT: How to improve it?

The purpose of the following sections is to propose possible directions that could enhance the effectiveness of the Donut activity and suggest revisions to the guideposts to facilitate future lesson study procedures. This section is organized into four main parts. First, a redesign of the Donut activity will be presented, considering the challenges identified in the “So What?” section. The redesign aims to provide educators with a more structured lesson plan to overcome the difficulties encountered during the implementation phase. In the second section, an idealized guidepost for lesson study procedures, incorporating critical reflection, will be elaborated upon. These guideposts aim to offer educators a framework for codesigning, implementing, and coevaluating lesson plans with a view to improve student outcomes. Additionally, potential enhancements to the toolkit for researchers facilitating lesson study meetings will be explored in this section. The third section will discuss possible avenues for future research in the field of lesson study and video-mediated coreflection. Finally, the fourth section will conclude with work-in-progress remarks.

Conceptually this last question, *now what*, steps away from the empirical side to consider the *practical* and potentially generative side of collective reflection; in a sense, such line of questioning invites us to shed light on how important it is to be able to have a pragmatic model of reflection that optimises concrete changes to the lesson using a synthesis form of the educator's talk during collective reflection. In this way positioning researchers as the potential brokers between the theory and the practice of teaching and learning mathematics.

The main purpose of coreflection during professional learning is to enhance teacher capacity to learn from peer observation and improve their practices, especially in the context of "teaching as a nuanced dance in which teachers integrate their knowledge and pedagogical content knowledge to be responsive to students' needs" (Timperley, 2011, p. 16). Hargreaves and Fullan (2012) similarly argued that professional learning communities ought not be meaningless "inconsequential talking shops nor a statistical world of scores and spreadsheets that take on a life of their own, far removed from real students" (p. 163). Our partners sought improvement opportunities, enacted a flexible lesson and employed a variety of reflective strategies which targeted different mathematics concepts. With all of these salient pieces in place, they were observed thinking together and conversing about teaching and learning math. To this end, suggestions inspired by participating in lesson study meetings will be elaborated after the Donut recodesigned plan.

Donut Activity to Donut Lesson: Possible Directions for Improvement

Kaci: …I don't remember what the lesson is. I don't have a plan.

In this subsection, possible directions for improving the Donut activity will be presented. The data suggests that there is room for adjusting the Donut activity, which aligns with the challenges identified by Horn and Little (2010) that fall under the category of "instructional triangle," focusing on the intersection of content, teachers, and students (p. 189). The approach is consistent with Biggs' (1996) advocacy for constructive alignment, emphasizing the "centrality of the learner's activities in creating meaning" through a lesson, whose vessel is an activity (p.

347). Just like a soufflé requires specific ingredients, time, and temperature to rise, the Donut activity recipe can become a lesson if we gather all the improvement suggestions gleaned from practitioners' coreflection meetings, my conversations with a math didactician (secondary to cégep), and consultations with a math teacher (cégep to university). Although such a soufflé is way more complex since humans are involved, therefore, it requires a solid codesigned lesson plan. A well-designed mathematics lesson unwraps the wisdom of codesign during classroom implementation. However, I acknowledge that given the complex dynamic of a classroom a solid codesigned plan does not guarantee that it will rise. Perhaps we can review the Donut activity to embed teacher modeling of problem posing and solving, which is an important aspect of design. Gravemeijer et al. (2000) proposed a modeling approach for designing lessons in mathematics classrooms, where an expert models how they go about lesson creation. As Montgomery (1908/2004) noted, “it'll do no harm to be thinking it over...Things like that are *all the better for lots of thinking over*” (p. 239). We too can think over the design of various aspects of the Donut activity. The Donut activity has the potential to become a wonderful lesson, making it a prime candidate for reflection and learning if it integrates: focusing on transferable skills (i.e. deep understanding).

Plan assessment first, determines expected outcome, provides feedback and opportunity to resubmit to demonstrate initial understanding and subsequent understanding.” This type of lesson plan assumes learning requires “reflection on action and the renewal that comes from adapting future actions based on that learning” (Dilworth, 1996, p. 46). Drawing on the arguments made by Artigue (2004) and Corriveau (2007, 2013) the transition from one cycle to another is mainly a transition between different math cultures. As such, viewing the activity as a tool to support learners' transition is suggested. Few scenarios exist where all these territories are so distinctly drawn as during cross sector coreflection meetings. As such the lesson is our main tool to scaffold transition across cultures (i.e., each teacher' classroom practices). With this set of data it can be argued that any math activity is not in and of itself a lesson, rather it can *become* a math lesson, if it has a vetted plan, a teacher that is a knowledge worker and keen students.

This revised lesson plan takes into account five aspects: Duration, Prompt Images, Discussion (math-talk), Calculations (math-operations) and anticipation. This structure expands Horn's (2012) two-category task design: "proof" and "what you say" (p. 25). The anticipation column predicts possible glitches (e.g., problem discerning if volume or area is needed; not knowing how to convert units). It was only after that the concept of fitting two different geometric shapes was explained to me that I realized the donut activity requires students to think about fitting circles into a rectangle. To accomplish this, they could imagine the round donut is a square using the diameter. If students chose to calculate the two sets of areas (one for the box and one for the donut), they could perform the operation of multiplying fractions with the same denominator, they learn that if they do not account for the unoccupied space of a round donut, then they overestimate the number of donuts. However, if they round up the size of the donut to 100 mm just to perform mental math they would obtain an underestimation. These would be how to bring about the metacognitive aspect of the donuts. If students are guided to perform two distinct divisions and multiply the results they would obtain the closest estimation. The Donut activity sets the stage for the concept of circle packing in higher-level math (B. Abdenbi, personal communication, March, 2017). By modeling for students that a round donut can be thought of as a square, they can overcome the limitation of not knowing how to calculate the area of a circle in a rectangular box. If the box were a circle or a square, the activity would be different altogether. Dreyfus (1999) argued that, for students to be able to explain and justify their reasoning, they need to shift their conception of mathematics from a field of structures to one of truth, instead of asking “what is the result?” to “is it true?” (p. 107). In an era where discerning truth in 'relevant info' is key, an activity that guides the learner towards capturing truth is essential.

In terms of tools to capture student thinking, I observed five classroom enactments and found that students who were given a small erasable board were quicker to erase their math process than those who wrote and traced their problem-solving steps with pen and paper. This was also true of students erasing mistakes with a crayon, but those with boards were faster. As I read the literature, I found an example from an unnamed teacher, Lewis (2000), who exemplified how observations led naturally to suggestions:

> The group that I was watching completely erased their initial plan. I thought it was a shame for them to erase all their work. **So it would have been good to give them a second worksheet to draw their new plan. To help them think about why their initial plan didn't work, it would be good for them to be able to look back at their initial plan. I felt it was a loss** (p. 12).

What do we do with such a suggestion? In police cadet training, I was taught to never erase, just strikethrough and jump to a new line. What do we want students to take with them to review?

The lesson is conceived with the worst case scenario in mind from the perspective of a grade 6 student. Imagine, if you will, a Venn Diagram where a substitute with what Gudmundsdottir and Shulman (1987) described as low pedagogical content knowledge has been asked to substitute a math heterogeneous class with mainly students that are particularly below average in math according to their term results and have low parental engagement in homework (a major component of consolidating understanding of concepts). In such a case the activity that supports the lesson ought to be maximally scripted to ease the task for the practitioner as opposed to minimally scripted scenarios where a practitioner with high technological pedagogical content knowledge (Mishra, 2019) is working with a group of students who are keen to understand math concepts and their parents are involved during practice time.

Advocating for greater *anticipation* during lesson codesign is in line with Putnam and Leinhardt (1986) call for *scripting* by assembling action segments in effective sequences. Segments are "unique goals, actions, and task environments" (Leinhardt & Greeno, 1986, cited in Leinhardt, 1989, p. 55). Leinhardt (1989) argued that for teachers dealing with unfamiliar subjects, it is necessary to "craft simple but effective scripts for teaching key topics" that can be modified over time (p. 54). Careful selection of "design and revise explanations," to orchestrate and strike a balance between "discovered, guided, or directly given" activity ensures "critical components of referent and demonstration" are included to prevent "forgetting to point out key" features of mathematical procedures (Leinhardt, 1989, p. 74). In most cases, a lesson cannot be repeated (no mulligans) for the same group of students, so it's important to ensure cohort consistency by understanding a teacher's active plans. As Leinhardt (1989) suggested, one way to do this is by "mapping specific verbal components in the agenda onto specific actions within a lesson" and annotating "with the teacher's own lesson commentary to discover how lessons are constructed and modified" (p. 73). Additionally, Leinhardt emphasized the need to compile effective procedures for explanations, which can help weave cohesive action strings. While Good et al. (1983) noted that lesson components tend to be arranged in specific sequences and develop "best" patterns based on their observations, this section presents an *ideal* pattern to consider in creating a lesson from a purely mathematical content perspective.

Though it is not explicitly stated, this activity has the potential to become a lesson that aims to develop the learners' geometric sense of circle packing, where circles are configured inside a square with a specified pattern of tangencies (Thurston, 1978; Stephenson, 2003). Assuming that the practitioners have a critical lens on during codesign, then we can shift the activity planning to lesson planning. A math lesson, in my opinion, lies at the nexus of the math curriculum that bounds a series of math operations along with a series of math-talks. Such a math lesson can serve as a mirror that diminishes blind spots for teachers, akin to a mirror that diminishes blind spots for motorists (Drexel, 2012). According to the Ministère de L'Éducation (2020), "developing a metric relation to the circle" is scheduled for secondary five students in the QEP, but it is possible to introduce this concept earlier:

> mathematical concepts do not exist as concrete entities. For example, **a circle is a mathematical object constituting an *idealized version of a shape* that is found in nature and that can be represented verbally, graphically or symbolically**. Students can use different registers of semiotic representation, either implicitly or explicitly, to become familiar with these objects, to construct a mental image of them and to give this image meaning (p. 36, emphasis added).

The Donut lesson is redesigned with the hope to help most students develop a greater sense of estimation and understand that the idealized geometry of a circle is not quite the reality of round donuts. In one of the QEP End-of-Cycle Outcomes of Cycle 2, students are expected to be able to estimate, measure, or calculate lengths and surface areas (Ministère de L'Éducation, 2020, p. 147). The Donut lesson aims to support this outcome by providing an opportunity for students to compare estimates using a variety of math operations. Through the lesson, students also learn to use "mental and written computation" to estimate (Ministère de l'Éducation, du Loisir et du Sport, 2009, p. 11). By engaging with the properties of operations, students gain a concrete understanding of math concepts that can be applied beyond the lesson itself. In this vein, the lesson is structured to develop a sense of estimation. Therefore, the lesson aims to provide an opportunity for students to make “comparisons and estimates, using a variety of conventional and unconventional units of measure” (Ministère de l'Éducation, du Loisir et du Sport, 2009, p. 17)

Furthermore, Donut lesson plan 2.0 prompts students to use their knowledge of the properties of operations, given that they are expected to have already learned how to “estimate and [measure] the dimensions of an object using unconventional units” by Grade 2 (Ministère de l'Éducation, du Loisir et du Sport, 2009, p. 17). By Grade 4 they encounter Surface Area problems that require them to estimate and measure surface area using unconventional units and by Grade 6 using conventional units. One of the reasons for which secondary math teachers deemed the Donut activity unfair is that it raises questions about whether the area of the circle is useful but in Quebec the topic of “area of figures that can be split into circles (sectors), triangles” is covered as of Sec. 1 and 2 (Ministère de l'Éducation et de l'Enseignement supérieur. 2009, p . 137). Whether the activity was unfair or not, it created tension in our math expert as well; On the one hand, the Donut activity is great for research purposes but “excerrable” for student learning. From my perspective the Donut activity simply did not realize its full potential as a lesson but it fulfilled its task at unearthing the challenges that students face at the elementary level. It can be argued that this Donut activity provides a baseline data from which we can compare future iterations of the Donut lesson to see if the same challenges resurface or not.

This Donut activity can be adopted to really dive into a discussion about the purpose of using the area of the circle if need be and how it is not the right path forward since we need full integers not decimals. However, primarily the Donut lesson is about estimation, first round up the millimeters size of donuts (to obtain the lowest estimate number of donuts) and to divide the entire area of the donut by that of the box (not considering empty spaces and smashing into bits to obtain maximum number of donuts, thus overestimation). The avenue for moving forward will be grounded in the instances where partners made suggestions to improve the lesson to transform this mere activity into a lesson. It will also take into account expert opinion on teaching mathematics by optimizing the Donut activity, Aside from removing the time allocated on the initial general open-question; "What comes to mind when looking at this image?" We can restructure the think-pair-share segment as it could be extended to two days. Here is why; letting students sit with a problem is as much a learning strategy as the content of the lesson. One day to discuss relevant concepts: estimation, conversion and then a day to actually solve the problem. The Quebec government's Ministère de l'Éducation (2020) proposed three guiding questions that led me to rethink the activity (i.e., readapting the lesson plan) by aggregating the most appropriate suggestions heard at the collective reflection:

> 1) How can these results be utilized?
> 2) How can they aid in the development of professional practices?
> 3) How can teachers and students be supported based on these findings? (p. 17).

In sum, the Donut lesson plan 2.0, "is by no means perfect" (Takhashi, 2006, p. 201), it is a proposed lesson plan primarily based on the math experts and secondary math teachers' comments on critical incidents extracted from classroom discussions (see Appendix D). This section considered: How can we improve the activity and lesson study associated prompts practically? Narrative analysis of the data suggests that a math activity may be transformed into lessons through a more guided approach during enactment and lesson study. Provided that we harness the educators' codesign and coreflective talk to improve when and how they unearth challenges that would help transform the activity into lessons. Next, I discuss a few directions for modifying the prompts that could facilitate lesson study procedures.

Idealized Guideposts: Possible Directions for Lesson Study Associated Prompts

Kaci: It was fun, it was nice to see. And not one person's way was better.

In this subsection I present an idealized guidepost for facilitating lesson study procedures. Specifically, I consider the "next steps for deciding on how to act on the best alternative and reapply it" (Borton, 1970, p. 89). My aim is to provide a brief model for lesson study that incorporates the set of reflection questions proposed by Borton (1970) to capture a type of coreflection that can lead to concrete changes in the lessons. Interpreting the data from multiple lenses helped identify areas for improvement in how lessons are created in teacher groups. As mentioned in the literature review, when glitches occur–when students are unable to solve the problem or the lone teacher's explanations are unclear–collective reflection becomes even more necessary to understand the cause of the problem. However, if all members do not perceive the same glitch or disagree, it becomes challenging to find an appropriate solution. As such to revamp our prompts, we could ask: how can we anticipate these glitches and potential areas of student weakness? Were they due to a lack of knowledge or insufficient lesson planning?

A decade ago, Meyer (2011) questioned whether "that skinny outline" was sufficient for classroom use. The ongoing dilemma about how much detail is necessary persists. For codesigning a learning activity, a consensus is needed that the default assumption should be *more* is *better* to increase the chances of attaining the optimal level of detail required for effective learning. While the primary goal of the CCCM project was not lesson study, Kaci's positive experience with the Donut activity demonstrates its effectiveness. However, when the goal is lesson study, the Rashomon effect must be addressed. To serve as research tools in the lesson study process, we propose a set of lesson study guideposts. Melody suggested.

> **Maybe we want to have some people create whole sets of questions for each other, and have someone else go through those questions to identify the ones we can use,** with others reviewing multiple choices. Then we can each have a **bank of material to work with**, but we'll work as a team dividing up and collecting it for each other.

The Interim Elementary Consultant agreed with Melody's idea. When Kate then added,

> I'd like to appeal to the kindergarten teacher, because that's you [referring to the Interim elem. Consultant]. I think that you have a good understanding of the routines and schedules that work well for kindergarteners. It's not just about content, but about the different things they do."

As the adage goes, 'hindsight is 20/20,' and reviewing my experience with multiple lenses enabled me to identify areas for improvement. To achieve the necessary level of criticality, the Donut activity can be altered through codesign in mixed groups following consultancy protocols. Techniques such as making a video montage following Brecht's (1964) approach to making theater strange can help distance the object and reveal why we selected it. Improving the activity can be the fruit of coreflection passing into recodesign, which can be viewed as a sort of bastardization of the codesigned activity (e.g., making the donut activity exogenous, basically for elementary teachers to conceive of it not as their activity).

This study identified contradictions across different sectors on the same activity while highlighting the potential of Lesson Study for math education. To enhance student engagement and knowledge development, educators could shift their focus from problem-solving to problem-finding when creating lesson plans through codesign in multiple meetings (Scardamalia, 2002). As Scardamalia (2002) argued, this approach can promote inquiry-based learning. Based on our findings, we propose directions that could potentially improve future iterations of Lesson Study while acknowledging potential biases in our reflection (Marcos et al., 2011).

This discussion presents one of the many possible versions of an 'idealized' lesson plan rather than an easily realizable one (Marcos et al., 2011, p. 33). Additionally, I try to avoid writing a prescriptive “teachers' should attitude”' without sufficient empirical support or being grounded in the realities of classrooms (Marcos et al., 2011, p. 33). As Perry et al. (2009) emphasised group norms to foster critical thinking and inquiry for improvement, perhaps we could revise to modify the withhold judgment norm to encourage constructive judgment instead. These guideposts are meant to be seen as a tool kit to alleviate possible burdens during implementation, acknowledging that the teacher is the subject knowledge broker who brings the lesson to life. In this sense, I am mindful that the donut lesson recommendations above and these guideposts are not meant to produce “teacher-proof lessons” (A. Breuleux, personal communication, April 2023).

In any case, analysis of the transcripts revealed that the term 'improve' and its variations (improved, improving) were used more frequently at the secondary (9 times) compared to the elementary (4 times) and mixed-group (2 times initiated by the PI when guiding the teachers to complete an appreciative inquiry: three things learned and how teaching practices can be improved):

> **PI: So what we distributed is the summary from the appreciative inquiry, after doing the talks, and talking about it. Take a few minutes to go through that. You have the secondary on one side and the elementary group. So this is “three things you learned today”, and the other is, “how can my teaching practices be improved?”**

Using Lasswell's (1948) framework, it became clear that the RÉCIT did not equally emphasize improvement across meetings, with a fluctuation in frequency suggesting a greater emphasis on encouraging secondary teachers to aim for improvement over elementary teachers.

Table 10 presents the frequency count for the term "improvement," although other words that signal improvement, such as “better” or “should,” were not included in this analysis.

Table 10

Frequency of the Term Improvement

Collective Reflection	improv* Freq.	Who	When
Secondary-Group	**9**	PI RA2 PI RÉCIT RA1 RÉCIT OU RÉCIT PI PI RÉCIT	Parts 3 & 4
Elementary-Group	**4**	PI RA2 RA2 PI	Part 1 & 3
Mixed-Group	**2**	PI PI	Lines 46 & 50 out of 112

Here is an excerpt summarizing one of the benefits of engaging in video-mediated coreflection from the perspective of four elementary teachers, most expressing they took notes:

> **Kate: But I really appreciate the time stamps on the Vialogues video.**
> PI: ok.
> Kaci: I can't say anything. I **was videotaped yesterday. You made all the decisions. I didn't have anything to do. So that was great. Thanks.**
> **[laughing]**
> Dona: True story.
> RÉCIT: yeah.

Although I was meant to be a fly-on-the-wall during Kaci's enactment, when she jumped to the 'if necessary' image, I inhaled audibly which led her to get flustered and show the email with the information of three layers of donuts prematurely. We discussed it briefly after. This exchange demonstrates that her entire aim was to show the students the number of donuts by row and column and then ask if that is all they need, to then say no, you still need to know the number of layers of donuts, do not jump to conclusions. As reported during Kaci's enactment, my presence in her classroom inadvertently led to a disruption in the flow of the Donut activity:

> **Kaci: …Thank god you emailed it. Got to refresh my memory. And I purposely choose to skip the first email with the blanks. And then she said: "Oh wait, you skipped that part." I thought: "Oh my god, I'm doing it wrong. Oh no."**
> [laughing]
> **Kaci: I thought: "Oh no. I wanted to skip that." Because I didn't want my kids to see the three layers. In my head I was leading them to,**
> **Interim Elem. Consultant: something**

> **Kaci: "Okay here are the number of donuts, length and width wise. Are we still able to answer how many donuts there are?" I wanted to lead them to:** "Now that we know that, what's the next piece of information we're missing?"
> RA1: hmm
> Kaci: she said: "Oh I'm sorry. I watched all the other classes, and that's how they showed it. I didn't mean to throw you off." **I was like: "You know what? We never sat down and actually decide step-by-step"**

Kaci's recount highlights that in her mind, the codesign meeting in which they codesigned two activities was not perceived as an opportunity to sit down and decide step-by-step. Even so, I am still mortified to this day that I derailed her plan. Though she had earlier declared she had no plan, thus Kaci didn't finish her sentence, but the interim completes it with "something." This excerpt sheds light on the position of the researcher in the classroom. Huang et al. (2023) argue that instruments supporting lesson study interventions are still emerging. Effective communication is one of the key factors in creating an effective small group (Bormann & Bormann, 1980). However, based on some of the teachers' end-of-year interviews, it appears that labeling participants as partners in a research-practice partnership (RPP) does not necessarily lead to a clear division of labor, which can leave some teachers uncertain about their role in meetings. Beene and Sheats (1948) have suggested that failing to establish functional group roles ahead of the discussion can have negative side effects. For example, Sabrina reflected on her experience at the end-of-year interview, stating, "for me, this year, I think that it's taken on something completely different. I think when I started, it was *uncertain* where we were going, what we were doing, and *what my role was in all of this*."

Moreover, throughout the study, it became readily noticeable that lesson study cycles are not to be viewed as separate with distinct prompts. Although the coreflection phase is often considered the most critical aspect of lesson study, it cannot be viewed in isolation from the other two cycles. During coreflection, it's essential to assess whether students were able to ask relevant math questions and receive appropriate answers to promote productive learning. Asterhan and Schwarz (2016) argued that “deliberate argumentation” and structured reasoning are essential elements of coreflection (p. 167). To facilitate the process, a pre-watching ritual involving a didactician could be implemented (Santagata & Angelici, 2010). Additionally, codesign meetings could include detailed discussions of what to anticipate so they can detect any “discrepancies between expectations and observations” during the post-lesson meeting (Virkki et al., 2017, p. 213). This requires establishing clear expectations during the codesign phase and comparing them to observations during the implementation phase. Van den Boom-Muilenburg et al. (2022) propose that integrating the seven core components of lesson study into a school's organizational routines can enhance student learning. These components include: identifying a teaching practice question related to student learning; utilizing publications, lesson material, and expertise; designing the research lesson, including observation forms; live observation of student learning during the research lesson; post-lesson discussion; adjustment and re-teaching of the research lesson based on the post-lesson discussion, followed by a final “reflection; and sharing of results with others outside of the lesson study team” (p. 299). In her work, van den Boom-Muilenburg (2022) uses the term “observation forms,” which I refer to as guidepost prompts (p. 299). To avoid a form of prescribing lesson studies, as Marcos et al. (2011) have noted as a concerning trend, I drew on Takahashi et al. (2013) to insert problem-posing in lesson study tools and justify the direction for future iterations.Effective codesigning in lesson study requires a strategic approach, similar to a master chess player anticipating their next moves. Future iterations of lesson studies should consider changing a major factor: the grouping structure. Furthermore, integrating the Understanding by Design approach (Wiggins & McTighe, 2011), which emphasizes planning learning experiences with the end in mind, and adopting Takahashi's (2014) Lesson Study ethos, which emphasize collective examination of lessons, may help reduce the Rashomon effect. However, additional research is needed to investigate the effectiveness of such an integrated approach.

In the CCCM project, we provided dedicated time for teachers to focus on grade-level codesign meetings. However, separating six elementary and four secondary teachers created a divide between math cycles and limited potential insights gained from mixing the groups. As Norman (1988) noted, “design doesn’t have to be complicated, but it does have to make things visible, exploit natural relationships that couple function and control, and make intelligent use of constraints,” with the goal of “guiding the user effortlessly to the right action on the right control at the right time” (book jacket). To achieve these goals in codesigning a lesson, we should be explicit, allow for practitioner agency, reveal required concepts, and provide a clear timeline. Mixing teachers in codesign sessions may encourage greater attention to detail and improve overall effectiveness. Thus, I recommend avoiding homogeneous grouping of teachers during codesign or any phase, especially in contexts where transitional issues are of concern.

Even though video-mediated coreflections are a superb tool for jotting memory and sparking conversation on a particular activity, guidepost prompts matter more. As far as research tools go, from a research perspective at the practical level, the data reported here suggest that there is room for improving the collective reflection prompts, which includes merging all the questions for codesign and implementation in the reflection sheet. This means creating transversal prompts and referring to them during meetings. To improve, following interpretation of the data at hand, I propose a shift in how we conceive of the three stages of lesson study:

codesign, implementation, and coreflection prompts and how they can be reiterated. These are not separate sessions with different documents; rather, they are interdependent cycles that need to be constantly reminded of each other and repeated. These cycles are meant to spill into each other, as they inform one another in an integrated fashion, akin to Biggs' (1996) constructive alignment, which merges the philosophies of constructivism (learning involved in sense-making) with instructional design (targeted assessment of learning targets). If this is read through Dewey's theory of experience (1933), it implies that the teacher's role is that of an academic course designer who aims to engage learners through participation in a curriculum that is aligned with the needs of the community. In addition to the need for repeating the lesson study procedure, there is space to incorporate a rehearsal period, between the codesign and enactment phases a simulation period that allocates ample time for practice to double-check the codesigned "Teaching-Learning Plan" (Lewis et al., 2019, p. 18).

The use of performance of understanding is how Biggs (1996) proposed to align teaching methods with assessment. Four years later, in their study of teachers' professional development, Cohen and Hill (2001) illuminated how teachers who engaged in content-focused professional development were better equipped to "reconstruct their practice to align" math standards of teaching (Penuel et al., 2012, p. 106). Alignment is the result of practitioners who "maintain continuity of thought" as "we do not learn from experience; we learn from reflecting on experience" (Dewey, 1933, p. 54). It seems plausible that the gordian knot created by the instructional triangle of content, teachers, and students can be resolved with realignment, no?

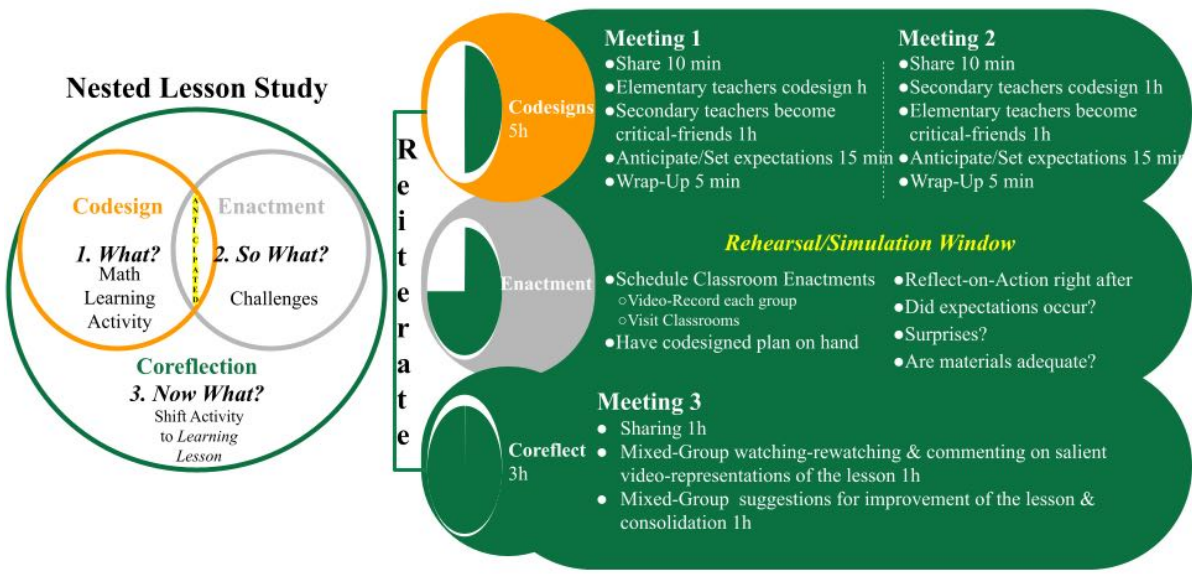
At this point in time, the most effective approach would be for lesson study coreflection conversations to move beyond the general level and examine the lesson content rigorously. This type of focus on content can help shift the starting point for codesigning lessons. Rather than viewing the cycles of lesson study as three distinct sets of meetings, it would be more productive to mix the groups twice, each time with a focus on different cycles that feed into each other.

Figure 9 presents the proposed procedures to overlap the cycles: meeting one with elementary teachers to codesign an activity while secondary teachers provide critical feedback, meeting two focusing on secondary teachers to codesign while elementary teachers become critical friends. The critical partners can adopt steps from the consultancy protocol to avoid perspectives overshadowing the conversation (Dunne et al., 2020). The specific plan involves a step-by-step process that begins with a meeting of elementary and secondary teachers. Meeting 1 begins with a 15-minute sharing session, followed by an hour of elementary teachers codesigning together and an hour of secondary teachers becoming critical friends to develop anticipation moments. Meeting 2 involves the same process, but now the elementary teachers become critical friends. The teachers then have five hours to codesign, iterate, rehearse, and enact their plans. They schedule classroom enactments and video-record each group. After the classroom enactment, the teachers reflect-on-action in Meeting 3. This meeting includes an hour of sharing, followed by mixed-group discussions for at least one hour on watching, rewatching, and commenting on salient video-representations of the lesson with the codesigned lesson plan on hand to annotate. Then, there is at least one hour for suggestions on improving the lesson and its consolidation. Such mixed codesign meetings could eventually lead to sequenced lessons designed to be enacted across two cycles, demonstrating continuity for students across sectors.

The third meeting provides an opportunity for a mixed group to examine the entire footage from one enactment. During the coreflection process, several 'Whys' must succeed each other to dig deeper into the root causes of any glitches in the learning and the lesson. Sakichi Toyoda developed the 5-Whys as a problem-solving technique to clarify the root cause of the problem and its solution (Ohno, 1989). This can be accomplished by a partner at the legitimate periphery of the activity. Once the first cycle of lesson study is completed (two codesigns, enactments, and coreflection), the same process is to be repeated. Through full cycle reiterations, the benefits of lesson study emerge, with an aim to increase the anticipation zone over the years of enactments (See Figure 9).

Figure 9

Nested Lesson Study Conceptualization & Proposed Procedure



To visualize the nested parts for a greater impact on the lesson, we can overlay the three stages of lesson study as mediated by video. This visualization demonstrates the idea of viewing coreflection as a constant dynamic backdrop for both codesign and enactment sessions, though not fully overlapped. The greater the inclusion of these elements within coreflection, the greater their overlap of anticipated events. The anticipation zone refers to the overlap between codesign and enactment. This zone is anticipated because the codesign elements are actualized during enactment without a hitch, regardless of the directions the students take with the lesson. As Kaci recalled, "it took on a life of its own," anthropomorphizing the activity. However, if this *life* becomes unique to each class, how can we guarantee lesson fidelity across classrooms?

On the one hand the Donut Lessons' great flexibility (open-endedness → vagueness) is superb to take the lesson across multiple concepts, but it also contributes to its lack of fidelity, four teachers interpreted the same plan in four distinct ways. This also generates a Rashomon effect whereby one wonders, so what was the activity's point? What is a lesson? It is an artefact and it becomes a lesson only once it is enacted; like a music score and the melody it generates depending on the musicians' rendering (A. Breuleux, personal communication, December, 2022). If comments are then aggregated at each iteration of the lesson will have a greater allotted space for anticipation to achieve optimum overlap. The space for anticipation will not be a 100% overlap because the likelihood that a lesson is codesigned and its related activities are enacted in full capacity to balance flexibility and fidelity is nearly impossible in the classroom's complex setting. As Wittgenstein (1969) already noted language-game is unpredictable, so:

> 311. Assist. director of Ed. Services: **It's not about scripting, right?**
> 312. RÉCIT: No.
> **313. Assist. director of Ed. Services: Like even now, we're discussing, it's not... You'll never be in the exact same scenario twice.**
> 314. RÉCIT: No.
> 315. Assist. director of Ed. Services: It's not about toute les eventualités. It's about being conscious of the possibilities of coming across obstacles. Even there... I think she got herself stuck in the situation. And then she wasn't able to... Like you said, it would've been okay to say, "we're stuck in this situation right now. We don't have what we need. Right now we're..." That would've been an okay answer to give.
> **316. Lin: Because I know, if I was that kid that's all I'm thinking right now. You mentioned donuts, and in my head, "how do I end up with almost seven million?" And I'd be…**

At the start of the conversation, the assistant director of education services asked, “it's not about scripting, right?” to which RÉCIT replied with a simple “no.” The assistant director then emphasized the importance of being conscious of the possibilities of facing obstacles, as it is impossible to anticipate every scenario. Lydia shared her approach of bringing down the discussion a notch when faced with a topic she can't expand on in class. Ellen added, “It's something that we're faced with all the time, kids giving us this curveball...we deal with every single day.” To address these challenges, I suggested accounting for necessary concepts and keeping them in mind during the activity. Lin noted that "the second class is always better," which Lydia and PI agreed upon, citing Melody's case as an example. PI suggested including more codesign activity to anticipate possible obstacles, and Lydia agreed. Overall, the conversation focused on the need to anticipate obstacles, handle unexpected scenarios, and try multiple approaches to improve teaching strategies without relying too much on scripting.

Considering the suggestion to use the term “anticipating” instead of “scripting,” the first step towards improving the activity is to review the codesign lesson plan and rehearse expected student actions and reactions before enacting it. During coreflection, educators can watch a non-edited video of the activity following the 3-act structure to avoid misrepresenting the implementation phase. And then the mediatonal video can be segmented into themes, each including two 30-second repeated sequences of the teacher-student or student-student conversation. As language is a psychological tool, the grounded conversation analysis revealed challenges that teachers and students encountered during the implementation of the Donut activity. These included difficulties in maintaining rigor in explanations while balancing math talk with math calculation. The study's goal was to prompt educational practitioners to reflect on the activity and determine whether it achieved the desired outcome of socio-constructivist learning. However, the findings suggest that the Donut activity had contradictory effects: although it aimed to improve students' rigor in explanation, its codesign and implementation were not aligned. To mitigate this outcome, it is essential to focus on the activity and its materials, including psychological tools such as opening prompts, actual calculation of operations, and rubrics for rigorous justification in math activities to achieve the learning target

After watching the edited-video, educators discussed whether the activity created the right conditions for learning to occur. On the spectrum of fun activity to talk activity to an activity that is the lesson, the codesign and coreflection meetings are key. Only then can the activity be deemed an adequate learning lesson. At this stage of analysis (Fall 2022), it is not evident whether the majority of educator comments point to what is not working in the activity. However, engaging in conversation analysis of the initiation, response, and evaluation technique during teacher programs could set the stage for greater collective reflection in-practice. The second possible direction for improving lesson plans is to integrate collective lesson study procedures from Takahashi (2014) with the principles of designing with the object in mind, of Understanding by Design advocated by Wiggins and McTighe (2015). Additionally, to incorporate Biggs's (1996) alignment principles into codesign sessions to create effective learning activities. Biggs proposed aligning the objectives, curriculum, and assessment methods of a unit using a constructivist “framework to guide decision-making,” which leads to improved instruction and performance (p. 347). Research has shown that instruction with aligned objectives and assessment methods produces achievement test scores that are four times higher than those of non-aligned instruction (Cohen, 1987). Comme quoi, consistency is key.

To this effect, we have compiled a Version 2.0 of the Donut lesson plan using the Backward Design as well as the protocols for conducting lesson study research. Focusing on the object of activity in order to attain the outcome of it becoming a lesson. In other words, to create “an acute sense of the learning target” (A. Breuleux, personal communication, December, 2022). Whether the new version of the lesson will be picked (high-rate uptake) remains to be seen. See Appendix E Appreciative Inquiry summary to read the teachers' answers. Where the role of the researcher is redefined as an accompagnateur[3] to liberate the practitioners from peripheral tasks (e.g., collection, analysis, & storage). This ideal is possible when the culture at School Boards set the default for research-practice partnerships (see Appendix F).

In summary, a critical-friend educator could identify flaws in the activity during the codesign meeting or reflecting on the video of the enactment, which may include various student queries, and observe how students and teachers get entangled with certain concepts (What?). The puzzlement observed would then be presented to the enacting educators, guided discussions would be held to make sense of the unearthed glitches (So What?). The codesigning educators may realize that the difference in enactment could be attributed to the lesson plan and may suggest modifications (Now What?). Through further discussions, they may discover that not all critical variables were considered during the codesign phase, and additional anticipation is necessary. This insight could lead to the development of a more comprehensive questionnaire to improve the operationalized lesson study, which could be used in future professional development, such as research-practice relationships. Furthermore, the theme of technological presence and other unenumerated variables could be incorporated. Many avenues for the future.

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[3] See LCEEQ Accompaniment Project for a discussion on the term accompaniment (Hollweck & Baradaran, 2022).

Future Orientation

Sabrina: long-term, what are we doing in cycle two? What strategies would be better to introduce earlier?

The present study has raised several questions that require further research. For example, it would be valuable to examine the effects of repeated cycles of Lesson Study with “triple-loop learning principles” to ensure that students engage in transformative learning through targeted lessons (Argyris & Schön, 1974, p. 31). Additionally, what is the potential impact of a more practical video protocol with revised group norms on the relationship between coreflection, criticality, and teacher groups?

What factors contribute to differences in conversation focus during coreflection, and how can we ensure that such conversations translate into practical improvements for school-board decision makers? Furthermore, what are the potential benefits and challenges of informal collaboration among teachers and how can it complement formal collaboration approaches? In particular, is shushing a useful classroom management strategy, or can alternative approaches be more effective? Alternatively, what can we learn from teaching and learning lessons with Tik Tok, LaTEX, and Backgammon?

Taken together, the outcomes of this study showcases three groups composed of secondary and elementary teachers in coreflection meetings focused on the Donut activity that could improve how students are encouraged to estimate with unknowns. This is where future research can begin to explore how a lesson could “support the shift from “learning math” to instead students “thinking like mathematicians?” (Resendes & Dobbie, 2017, p. 71). The findings suggest that video-mediated lesson study is a promising approach, but further research is needed to evaluate whether learning outcomes can be met by the codesigned activity before it is enacted. Such research could contribute to the integration of lesson study for professional competencies, including Competency 11, which promotes a reflective stance to "commit to own professional development and to the profession" (Ministère de l'Éducation, 2021, p. 43). However, mandatory ownership may hinder criticality by introducing ego into the process. Thus, it is important to investigate ways to facilitate critical coreflection among teacher groups. Next, I will wrap-up this dissertation with a few remarks.

Work-In-Progress Remarks

Kaci: Not one person's way didn't work. It just led to very different…

The nine Vignettes examined in this study prompted us to consider the Rashomon effects that arise during coreflective conversations of a learning activity. As an outsider to the field of math education, I will follow Abu-Lughod's (1993) guidance and refrain from offering definitive conclusions about what caused differences in activity implementation and reactions among the teachers, in order to avoid asserting an unwarranted sense of "authorial control" over their narratives (Miller et al., 2004, p. 50). Nevertheless, as an idealist critic, I acknowledge that mysteries remain. As Kaci pointed out, it can be argued that all enactments worked, but they "just" led to "very different," leaving her thought unfinished. Following Horn et al. (2010), we can classify Kaci's comment as a "normalizing move," indicating that differences in implementation are typical, but such reassurance overlooks potential inconsistencies in student learning (p. 192). Our case study of the Donut activity revealed the distinct approaches and measures of rigor that each teacher employed, which may have impeded the need for harmonization in math instruction across schools. While the interim elem. consultant asserted that the students were overwhelmed but knew what to do, Ben's reservations about the students' ability to select the appropriate unit of measurement cannot be dismissed, as it raises concerns about the effectiveness of the activity:

> Interim Elem. Consultant: "smaller numbers, or just use centimeters to work with." **Because I don't know if they didn't understand what they had to do. I think, like you said, they just got overwhelmed**
> PI: yeah
> **Interim Elem. Consultant: with the numbers. But I think they knew what they needed to do to solve the problem.**
> **Ben: Because having six million, nine hundred thousand of anything**
> Interim Elem. Consultant: is big
> **Ben: In that context, it's like, how is that even possible?**
> PI: yeah AND Lydia: Hmm
> **Ben: Six million nine hundred thousand. What have I done?**
> PI: Hmm
> **Ben: That doesn't make sense to me. I know the size of a donut. Right? Six million nine hundred thousand doesn't make sense.**
> PI: yeah
> **Ben: So not being in the right unit...**

Another mystery that remains is why secondary teachers perceived the Donut activity as unfair even when they knew the learning target, while elementary teachers did not share the same view. If content knowledge is not the cause of this difference, then what is? As my father would say, educators who engage in critical lesson study can become “sharik agahi,” or shareholders of knowledge. This stance is similar to Scardamalia's (2002) advocacy for becoming attuned to the “collective cognitive responsibility for the advancement of knowledge” (p. 67). Lesson study provides a tool for holding accountable the lesson, which can “create a new idea that preserves the value of the competing ideas while 'rising above' their incompatibilities” (Scardamalia, 2004, p. 7). Coreflections discussed here involve the Donut activity, which requires further examination to become a lesson using both guided and unguided approaches. This dissertation presents interpretations of educators' reflections, and concludes that the video montage helped unearth math challenges because we viewed one full lesson study procedure with segments from codesign, enactment, and coreflection to highlight divergences between each phase. This stance is in stark contrast to short clip viewing sessions.

In summary, repeating the lesson study procedure over time, beyond just one cycle, may help overcome challenges in codesign and contribute to more effective lesson development programs. The research outcomes support the need for educators to engage in lesson study meetings as a mode of practice to strengthen social relations and manage the tensions generated by non-harmonized lesson plans. Therefore, continued release time and funding for lesson study procedures are advocated. Moreover, this study provides a preliminary examination of the role of coreflection in the lesson study procedure within the context of Québec. Despite its limitations, the study sheds light on the potential benefits of a video-mediated coreflection approach and highlights the need for further integration of each cycle of lesson study. Educators are encouraged to critically select and enact lessons, while putting parameters in place to plan effectively, since elementary grade students deserve curated lessons where codesigners solve most glitches before enacting an activity. Overall, this dissertation sought to contribute to the ongoing discussion about the role of lesson study in professional learning and development by providing practical suggestions for educators and researchers alike.

Finally, the analysis of the educators' coreflections on the Donut activity reveal a Rashomon effect, not only for me but maybe for you too. It seems plausible that factors such as an open-ended codesigned lesson plan, an ill-defined math task with unclear definitions, and open-ended prompts for the unclear concepts of critical coreflection may have combined to create a 'perfect storm' in the reactions to the codesign and enactment phase of the Donut activity.

I acknowledge that Lesson Study procedures extend beyond the scope of this dissertation, and their complexity cannot be fully captured within the confines of the reductionist perspective focused solely on one math task. Nonetheless, this study sheds light on the crucial role of time-consuming task detailing in enhancing instructional practices. While the primary aim was to highlight how the Donut activity supported three high-quality teaching practices, the analysis indicates that the factors that led to the Rashmon effects have influenced the effectiveness of the activity. Ultimately, the effectiveness of the Donut activity hinges on educators' perspectives from their specific educational context. However, by fostering a collaborative environment where educators engage in codesigning and continuous coreflection upon lesson plans, there is a potential to actualize and reify the intended goals and outcomes of the Donut lesson.

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Appendices

Appendix A–Codesigned Donut Activity Plan (1)

Activity #1

ELEM.

(Co) Design of Learning Activities **Feb 10, 2017**

Learning Topic	Rigorous Justification — KRISPY KREME OPEN-ENDED QUES. (Math Talk)		
Name		Date and Time	
Class	Grade:	Number of students:	

1. Write a brief description of the learning activity.

Present students with an open-ended question (Krispy Kreme image) and asking them what questions come to mind. Use the turn and talk partner strategy to share with whole group.

2. What makes this activity worth exploring in terms of teaching (i.e. pedagogical challenges) and/or learning mathematics (i.e. student understanding)?

- developing analytical thinking about problem solving ~> realizing that it's important to be able to identify "What I know", "what I need to find" and what information provided might be irrelevant to solving a problem.

3. What are the Learning Targets for this activity?

I can ask mathematically relevant questions and analyze an image (problem) presented.

[Learning Trajectory (long-term) : I can produce a rigorous mathematical argument/justification to explain my thinking - and verify a solution.

4. What are your Target Teaching Practices? (Choose the teaching practice(s) that you want to focus on.)

- ☐ Teaching toward an instructional goal
- ☒ Eliciting and responding to student thinking
- ☒ Orienting students to each other's ideas
- ☐ Positioning students competently
- ☐ Establishing and maintaining expectations for student participation
- ☐ Representing student thinking key ideas
- ☒ Using a public record of student thinking

5. What do you anticipate seeing in terms of students' work (e.g., what will the students do)?

- students building off of each other's ideas
- students using correct mathematical terminology (vocab.)

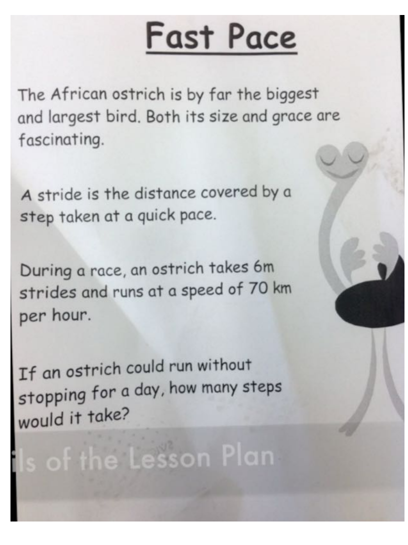
6. What data might be collected to ensure that students have met the learning target?

- list of questions collected (whole-class)

7. How would you assess the students' work?

n/a

8. Possible video recording dates (February/March)



Appendix A–Codesigned Let's Paint Activity Plan (2)

(Co) Design of Learning Activities

Feb 10, 2017

Learning Topic	RIGOUROUS THINKING – Providing App. Question w/ only diagram & final question		
Name		Date and Time	
Class	Grade:	Number of students:	

1. Write a brief description of the learning activity.

- Presenting an application question with only the diagram and final question available.
- Task is to brainstorm information that is missing in order to solve the question
  - Start individual – list on post-it
  - get into groups to decide which ideas "to keep"
  - Show final ideas as a group & reveal information
  - Separating into what I know what I am looking for

2. What makes this activity worth exploring in terms of teaching (i.e. pedagogical challenges) and/or learning mathematics (i.e. student understanding)?

- Slowing kids down
- Realizing there are several steps before finding the final answer
- Not jumping right to the final question

3. What are the Learning Targets for this activity?

- Infering / deducing the missing information in order to solve applications

4. What are your Target Teaching Practices? (Choose the teaching practice(s) that you want to focus on.)

- ☐ Teaching toward an instructional goal
- ☐ Eliciting and responding to student thinking
- ☐ Orienting students to each other's ideas
- ☑ Positioning students competently
- ☑ Establishing and maintaining expectations for student participation
- ☐ Representing student thinking key ideas
- ☑ Using a public record of student thinking

Let's Paint!

Show all your work.

According to the diagram, how many 1-litre paint cans of each colour are needed to paint the new stage decoration?

Paint for the Stage Decoration		
Colour	**Area to Be Painted (m^2)**	**Number of 1-Litre Cans**
Red		
Blue		
Yellow		

Let's Paint!
Situation Involving Applications
June 2013 3 Ministère de l'Éducation, du Loisir et du Sport
522-610 – Mathematics

Let's Paint!

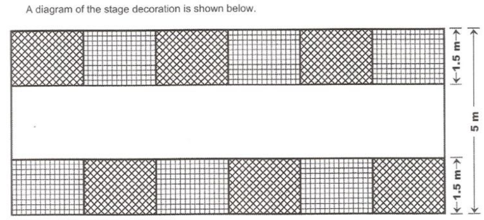
The students in the Band want to paint the new stage decoration they will be using for their next show.

This stage decoration will be made up of red, blue and yellow sections.

The red and blue sections are equal in area.

The perimeter of the stage decoration is 34 metres.

A diagram of the stage decoration is shown below.



Legend

: red : blue : yellow

A can containing 1 litre of paint will cover 9 m^2.

According to the diagram, how many 1-litre paint cans of each colour are needed to paint the new stage decoration?

Ministère de l'Éducation, du Loisir et du Sport
522-610 – Mathematics 2 *Let's Paint!*
Situation Involving Applications

Appendix B–Reflection Learning Activities v3

Reflection on Learning Activities 1

Learning Topic			
Name		Date and Time	
Class	Grade:	Number of students:	

1. After reviewing the video footage, reflect on the overall learning activity.

   a) Write a brief description of salient moments (Please provide Vialogues time stamps).

   b) Explain how and why you enacted the learning activity in this way. Refer to the practices of high quality teaching (see Table 1 below).

2. Review of the student learning data (including the video data)

   a) What do the data tell you about student thinking and learning in relation to the learning target?

   b) Were there any surprises and/or difficulties?

3. **Future planning**

   a) How would you revise the way you implemented the learning activity, the learning target and/or the target teaching practices?

Reflection on Learning Activities 2

Learning Topic			
Name		Date and Time	
Class	Grade:	Number of students:	

Table 1: Practices of high quality teaching from *"High Leverage Practices"* (TeachingWorks, University of Michigan, 2013) http://www.teachingworks.org/work-of-teaching/high-leverage-practices

❑ **Teaching toward an instructional goal**	Composed of two categories: 1) *Learning targets*, inform students what is expected of them to learn. 2) *Target teaching practices*, describe what you plan to teach (road map).
❑ **Eliciting and responding to student thinking**	Describe how you elicited students' thinking (e.g., the initial questions you asked, student responses, and your reply, etc.) Describe how your questions helped students reach the mathematical instructional goal. Why did you choose to orient students in this manner?
❑ **Orienting students to each other's ideas**	Describe how you oriented students to each other's thinking (e.g., the wording you used, etc.). Why did you do it in that way? Why was this move important for supporting students' learning of your particular mathematical instructional goal?
❑ **Positioning students competently**	Frame mathematical ideas and their use in a way that productively challenges students without going beyond their abilities. (*See 'Cognitive Demand'* http://map.mathshell.org/trumath.php#truframework)
❑ **Establishing and maintaining expectations for student participation**	Select instances where: 1) you set specific expectations for student' mathematical participation; 2) you maintained expectations for mathematical learning. Describe how you set particular expectations within each phase. How did you help students make sense of these expectations? How did you hold students accountable to these expectations? (Please move beyond classroom management expectations, such as raising hand, etc). Why were these moves important for targeting your specific mathematical instructional goal?
❑ **Representing student thinking**	Referring to your mathematical instructional goal, describe how you represented student mathematical contribution. Why did you choose to represent the students' mathematical thinking in that way? (e.g., Describe what you drew on the board to represent students' thinking.) Why was the move important for supporting students' learning?
❑ **Using a public record of student thinking**	Describe how this activity will make students' thinking and work visible. For example, photo album of projects, math journals, posters, class book, chart paper, on smart board, blackboard, white board, Padlet app. For more examples: https://onedrive.live.com/view.aspx?resid=4AC891949A1871F1!1644&ithint=onenote%2c&app=OneNote&authkey=!ABiqb8v0EZhHAJ8

Two Solo-Reflections Using the Reflection on Learning Activities in Appendix B

	Solo 1: Mediated by Fieldnotes on Four Classroom Enactments (36 pages) Boundary Object the lesson	***Solo 2: Mediated by Raw Video Data of one Full Class Enactment***
1. After reviewing the video footage, reflect on the overall learning activity. *a. Write a brief description of salient moments (Please provide Vialogues time stamps).* [Here the research team made the assumption that teachers would have video footage of the lesson. Next, iteration we could add, after reviewing your notes, recalling events, audio clips, and/or video footage]	I was very happy with the discussion and the actual math talk that was going on. Students came up with a good possible question for the picture of the donut box. I especially liked that they recognize that different information is necessary depending on the question being asked.	In general, I like how groups are spending time making sure the physical space they are using is organised. This is something I have tried to emphasise ever since the CCC-M meeting last year when we noticed that students do not necessarily spontaneously do this. 15:52 Group 1–I like how creative the solutions are. They are developing real math thinking. 16:52 Group 1–Whole group discussion about volume and area–I was pleasantly surprised that this came up from a student. I know it's something I will need to revisit with the class. I expect that they will need further discussion to understand when using volume works (measuring liquids, for example). I found groups 2 and 3 harder to understand on the video. I found myself getting distracted by comments from other groups.
b. Explain how and why you enacted the learning activity in this way. Refer to the practices of high quality teaching (see Table 1 below).	By observing the photograph and brainstorming possible questions, I was able to elicit and respond to student thinking. I wanted the students to use some of their prior knowledge and try to choose appropriate math notions that they have already acquired and apply it to the Krispy Kreme problem.	It was important for me to help the students see themselves as the true mathematicians and I think this lesson provided a good basis for this. Students could all access the problem and had something to contribute.
	Through prompting and questioning, most students participated in the conversation. I was impressed with the number of questions the students came up with and how they really stayed on task. I especially found it interesting how the students were able to think about what information they might need in order to solve the problem.	I think it was important to empower all learners by giving them the opportunity to think individually before sharing their ideas with a small group.
2. Review of the student learning data (including the video data). *a. What do the data tell you about student thinking and learning in relation to the learning target?*	The data showed that most students actually do understand the problem at hand, but are stuck at the solving stage.	Students are able to ask relevant questions about a situation and are able to determine a course of action for problem-solving. They seemed able to identify missing information needed to solve the problem. As far as the learning targets are concerned, I feel students met them fairly easily.

Two Solo-Reflections Using the Reflection on Learning Activities in Appendix B		
b. Were there any surprises and/or difficulties?	As mentioned above, once again students still had difficulty solving the problem. Transferring from discussion and understanding to applying and answering is the weak point. It took much prompting and guidance to get them to put what they knew they needed into an actual equation that would help solve the problem. I was surprised that even after such a rich discussion and the impression of true understanding, it was still difficult to come up with steps to solving the equation.	I know the big challenge to revisit centres around whether to use area, volume or dimensions to solve a problem.
3. Future planning: How would you revise the way you implemented the learning activity, the learning target and/or the target teaching practices?	The plan was to move onto a problem on paper and go through the same process. I decided to show a few more 3 act problems and go through the process a few more times before moving on. The students seemed to take this activity more lightly and participate with less stress than if I gave them a sheet with a problem on it.	I think I would spend a little less time on the preliminary questions, in the interest of time, to leave more time for solving the problem. I would have liked to have more opportunity to explore situations in which using volume would be an appropriate strategy. I definitely liked the activity though and will do it again in the future to help students see the interim steps needed to solve a problem.

Appendix C–Challenges From Classrooms

Appendix C.1–Estimation Critical Incident

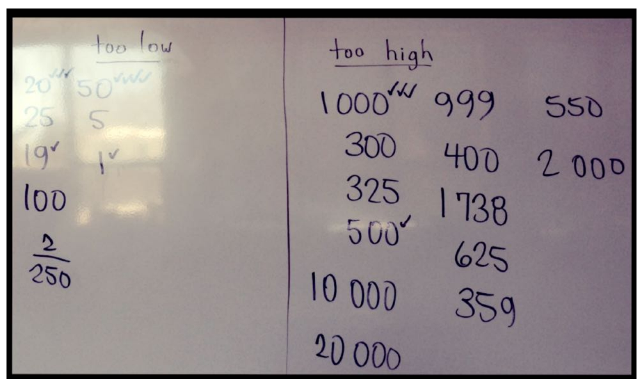
The Donut Activity Estimation Version by Sabrina (14 Year of Experience)

Act 1 Opening Images

- How many donuts are in the box? Estimate
- Make an estimate you know is too high. Too low

Source: Gfletchy 3-Act

Classroom discourse 1: Whole class discussion, is 1 too low estimate? What is an estimate? What is "reasonable"? (2:40 min)



1. Student 1: There would have to be more than one, because for sure there's one.
2. Student 2: Well you can see there's two.
3. Student 1: There's more than one.
4. Student 2: You're saying that what's there is too low.
5. Student 3: One is too low.
6. Student 3: It has to be reasonable.
7. **Teacher: So what I'm hearing you saying is that you're agreeing with J, you're saying that one is not a reasonable estimate.**
8. Student 3: It is too low. If we look at it we know it's not...
9. **Teacher: Okay. So do we want to include our one as an estimate here?**
10. [Indistinguishable voices]: Yes. No.
11. **Teacher: Some of you are saying no.**
12. Student 4: Why?
13. **Teacher: Persuade us.**
14. Student 6: It's too low.
15. **Teacher: It's too low? Isn't this your estimate?**
16. Student 6: Yeah.
17. **Teacher: Okay. So now you don't like it anymore?**
18. Student 6: No.
19. **Teacher: You still like it. So why should we keep it in the "too low." Because others are saying that it's obvious that there's one. What will you have to do?**
20. Student 6: Make it too low.

21. **Teacher: Make an estimate that was too low. Exactly. Is it too low?**
22. Student 6: Yeah.
23. **Teacher: Yes. Okay. Ju?**
24. Student 7: Yeah, but it's not a reasonable estimate, because, like fifty. We continue, okay there's fifty, about. But one, we know that there's more than one in the box.
25. Student 8: We said there's fifty, but there's five.
26. **Teacher: One at a time. We have so many people with their hands up. C, what do you think?**
27. Student 9: I think it shouldn't be there, because we're supposed to estimate, and estimate for something that's too low. But there we can know for sure that it's too low, and we're supposed to estimate.
28. **Teacher: I think that maybe brings us to the question of what is an estimate? Does anybody know what an estimate is? Who can give us a definition maybe, or a reminder of what that might be? What's an estimate? Ju, do you want to answer that?**
29. Student 10: Well, you know, it's approximately how much something is going to cost.
30. **Teacher: So approximately, about, how much maybe something is cost, if we're talking about money.**
31. Student 11: Approximation.
32. **Teacher: I'm hearing a lot of similar answers, guess, approximation.**
33. Student 12: An educated guess.
34. **Teacher: An educated guess? What do we mean by an educated guess?**
35. Student 12: It's not like we'll say there's five thousand in there, because we know there can't be five thousand.
36. **Teacher: So it's not just a number that we're pulling out of a hat. We're actually basing it on something. Okay.** [add a closure; clarify estimation, the question that was asked "too low"; acknowledge it is too low but not reasonable.]

Act 2 Email with Omitted info
Each student has their white board. Group members do not really discuss their plan for calculating.

From: **HayleyHutchison**
07/10/2014 13:21:36

Hi Graham,
We hope the information below helps:
The box was created to allow 3 x layers of doughnuts (x doughnuts on each layer)
Each doughnut is approximately 89 millimetres in diameter.
The box is 3000 mm x 2300 mm which allows a gap between each doughnut to sit comfortably.
We do not have any schematics to share as they are for internal use only but we wish you all the best with your class.
Kind regards,
Krispy Kreme

Act 3 Watched News Report
Krispy Kreme 'Double Hundred Dozen' is a Dream Come True
Wrap: What did we learn today? We learned to ask questions. ….As the bell rings, she hold the students, to complete the wrap up.

Appendix C.2–Conversion Challenge Critical Incident

The Donut Activity Conversion Version by Dona (9 Years of Experience)

Act 1 Opening Images

Before the lesson, the teacher reviewed even numbers and prime factorization. Entire footage is 01:01:04 from Group 1 perspective. When they pick the right option: “Brianiacks, I love it.” Style of encouragement. When a student proposes an answer that is incomplete, she asks for others to clarify what the other student means. She also jokes about some multiplication step they used to that was not right but that now they learned. This took 5 minutes and forty seconds before she showed the picture from the Donut company. Throughout the enactment, several students were coughing deep à la bronchitis, back in 2017 it was perceived normal to be in class despite frequent coughing symptoms.

Open-Ended Question, Think-Pair-Share: not verbatim transcription: “[5:32–We are going to do an activity today. I am going to show you a picture. I want you to come up, I want you to write down all the questions you have about the picture, and possibly all that you need to solve this picture. Any questions?…[6:32–introduces the donut company’ shows the picture and describes it: [So this is a lovely box of donuts; We have four people holding this lovely box; what questions come to mind when you see this picture? Don’t put your hand up, think about it, jog your questions down,” a student asks: “[7:00–does it have to be math related?” Teacher, “whatever questions come to mind, if you want to know the flavor, they are originals, i know my donuts. Those are originals.” Silence as the teacher goes around. Indistinctically we can hear students from other classes.

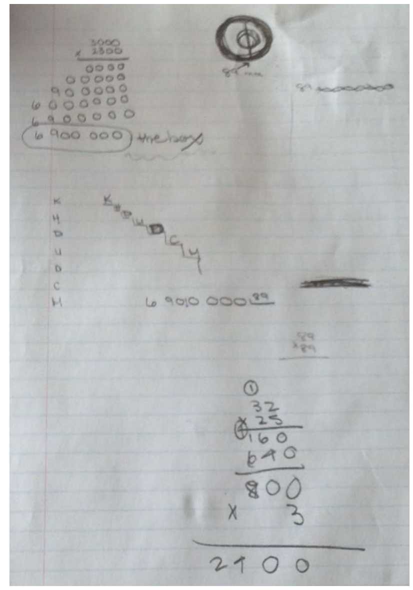
Door closed at 8:01 minutes. Stop and turn to the person beside you: “discuss what questions you came up with, what are some of the questions that came to mind?”

9 minutes–The main question: “how many donuts,” “Why are four people holding the box?” And a Discussion about optical illusion.
10:30 minutes–Summary of really good questions. How many of you thought about how many donuts …

[Shows two other pictures] 11:15 minutes–It is not an optical illusion. 11:30–[polls the main questions and checks for agreement] that’s a valid mathematical question, for a party how many can we feed?

12 minutes So if our question is [Writes on the black board with chalk]: “How many donuts are in the box?” “How would we figure that out? So, with what we have here, I want you to tell me, how many donuts do you think are in this box? In order to solve that, what information do you need? We have several images, several points of view. Think about it, what information would you need in order to find out “how many donuts are in the box?”

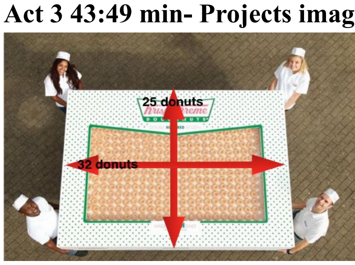
12:54 minutes [moved from pair to group]: Think about it, in your group of four. Students go back and forth in French and English, discussing the sides of the box, “What is longueur in English?” “largeur?” “length.” Comment on écrit-ça horizontally?
13:33 minutes [Research team adds a microphone in the middle of the table between five students].
[16:33 minutes–Students jump from 450 to 1000 and one adds: “approximately. It can be less”



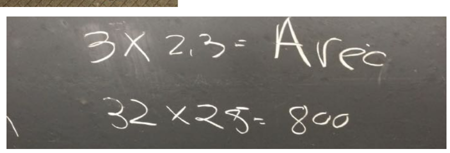
[16:46 minutes–Teacher whole class discussion: "so how many of you.. Figured out what is missing?" How many horizontally and vertically, what other words can we use? Length and width, are they the same thing, all class: "no."
[Discussion about length and height with kleenex box]
19:57 minutes–student struggles to explain, teacher refers to a sheet as flat, and a kleenex box….
22 minutes–Whole group activity estimated how many donuts. 150, 450, and: "a thousand, that's my opinion, and you can't judge me."
[24 minutes–Estimation]. Over the top, and an estimation that you know is not enough… one that's just right.
Act 2 30:58 minutes–We can't really know, there is more information. Read the email to the class.

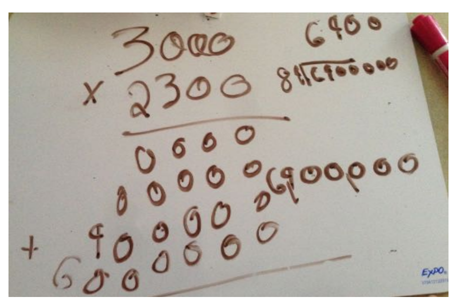
32:56 minutes–Group work. In group 1, a student says: "how big the box is; horizontally and vertically."
a student calculates: "Ok, it equals six million, 900 thousand…."
34:15 minutes: discussion about "what does the diameter mean?"
conversation with teacher: "measurement of the donut."
35:40 minutes: Student wondering, "Which one is the length between these two numbers?"
36:00 min–"King Henry died,...chocolate milk."
36:20 minutes–"which one is horizontal and which one is vertical?
38:40 min–Teacher prompt: Draw a donut, and show 89 mm diameter.
41 min–Who feels like they are at a possible answer. Who feels they need some guidance.
43:15 minute: a student exclaims unimpressed: "Si c'est ça la question du ministère la!!"
43:20 minutes–who needs some more information?

Act 3 43:49 min- Projects image of 25 donuts by 32 donuts.



46:25 min–"Before we got the added information, what were you doing? When you only had the email information what were you doing?"
What does 6900000 represent? If i multiply length and width. What is the area of the box? The endedans. [Why does the teacher discuss how the student obtained the area of the box when even the student who performed the thought aloud only divided by row and column. She did not make use of the area.]





47 minutes Conversion conversion. A student shares her math thinking out loud, converting to centimeters and the teacher asks: "what was the actual answer?" [Meanwhile you can hear group one students whispering, and seemingly disconnected from their peer's explanations. The teacher makes use of a measuring stick.
Most groups did not finish. You can hear: "we are confused."
51:40 min–I was getting somewhere but I didn't finish.
51:50 min–Student shares that she thought diameter was a unit of measurement.
52 minutes–A student asks why "put it in centimeters", the teacher says: "she put it in meters…"
Classroom Critical Incident 2: ***Student asks, "What do we do next?"***
Teacher explains using chalkboard drawings. (3 min 53 sec)

1. **Teacher: What numbers** [number] **do you come up with when you multiply both of those? [**What does that number represent when you multiply length & height]**; Three thousand times two thousand two hundred.**
2. Student 1: Six million nine hundred thousand.
3. **Teacher: So a lot of numbers** [digits]**, right? A lot of zeros. The way she did it made it smaller** [convert the units]**, made it an easier number to work with. That's always a good way to go. Making the numbers more workable. You could've done it the way you did it, but it took a little bit more time.**
4. Student 2: But after doing this, what do you do? Like, after having it in millimeters?
5. **Teacher: It already is in millimeters** [millimeter square]**.**
6. Student 2: Yes, but what do you do after?
7. **Teacher: I don't understand your question.**
8. Student 2: What is the next step after doing this?
9. **Teacher: After doing what?**
10. Student 2: Your calculation of three thousand...
11. **Teacher: So. If you were to take... So we're going to say this is the box, the donut box, okay? They're saying that this length here is three thousand millimeters, which we know is three meters. Okay? This one here is millimeters, or you convert it to meters. The reason she did it** [conversion] **is because it's easier to work with. You guys times** [multiplied] **them, right? You did three times two point three. This would've given you the area** [in meter square not millimeter square]**, the inside. What K did was instead of finding the area was, she figured out how many donuts it took... how many donuts she could place. So she knows that one donut** [has a diameter that] **equals eight point nine centimeters. She says, "eight point nine centimeters, too complicated." I'm going to change** [round-up] **that to nine centimeters. She goes like this, "hmm...one meter...Hey. One meter, how many donuts do I fit in one meter? So if I was to take donut, donut, donut…" She said: "Eleven fit in here." So she said: "Well this is three times a meter stick, and if eleven donuts fit on one meter stick."**
12. Student 2: You make eleven times three.
13. **Teacher: Because one meter...you need three meters. Do you understand? So she found out how many donuts went here, and then she did the same thing and found out how many donuts fit here. And she said, how many donuts fit there?**
14. Student 3: Twenty-five.
15. **Teacher: Twenty-five. So she said, "twenty five donuts go here," and how many this way? Thirty-three. Krispy Kreme told us what? That we had...**
16. Student 3: Thirty-two times twenty five.
17. **Teacher: Thirty-two times twenty-five. So her way of figuring it out would have given her an answer that was pretty close. She was one off. So this was a good way of thinking. So let's actually find out exactly how many there are.**

At some points she switches to French to explain something.
58:32 minutes–Teacher, "How many of you found this very difficult?" A student does [the comme si comme ça hand gesture], the teacher asks why, and the student replies: "Well just getting confused with numbers and calculation." Teacher further asked, "Did you know your math knowledge to answer this question?" Student, "I had forgotten about the diameter." Teacher: Who was completely confused? I feel like everyone knew what to do, you had to open those brain cells. You were able to figure out math without having the exact number of donuts. Asked if there are any other questions and then advised students to put this paper in their math section of their binder. Wraps up five minutes before the bell rings.

Appendix C.3–Volume vs. Area Critical Incident

The Donut Activity Area vs. Volume Version by Kate (8 Years of Experience)

Act 1 Opening Images

Write down questions about this picture, don't talk to anyone, write one question per post-it [3:28 minutes]

Source: Gfletchy 3-Act

[3:29 minutes] Project two more pictures: "Do you have any additional questions?"–*a*

[4:52 minutes] As a group compare your questions
[8:08 minutes] Whole class share your questions

[12:12 minutes] Zero in on how many donuts are in the box. Shared the Learning Target: "If we can ask relevant math questions about situations we have." Determine what information is needed to solve "how many donuts are in the box?" As a group of three, use the post-its, figure out what information you need me to provide to you, and figure out how many donuts are in the box. What information do you need?

[19:00 minutes] Whole class discussion, interested in hearing how everyone has been thinking about this question. 38 seconds with a student suggesting to buy donuts instead of travelling to Ottawa.
[19:43 minutes] What is some information that your team is looking for? Size of a donut? Why? The student mumbled an answer, 20:00 minutes, I would like you to repeat that louder so we can build off your idea.
And Area of the box. Why are we looking for Area?
Price of the box.
How much time does it take on donut, and how much time for one box
How many in a row
How much one box is [23 minutes]

Classroom Critical Incident 3: *Whole class discusses, area vs. volume* (2 min 40 seconds)

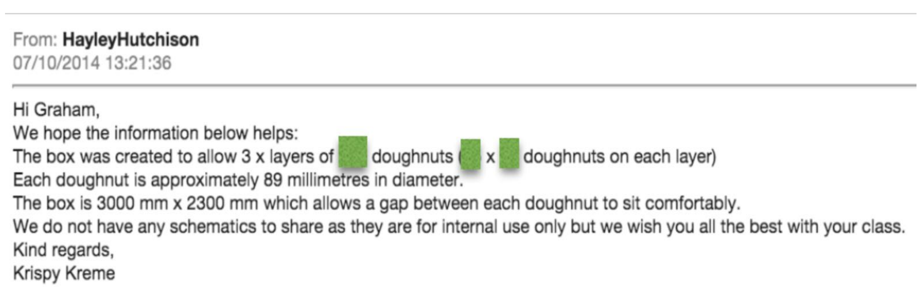
1. Student 1: I think it could be volume.
2. **Teacher: Why?**
3. Student 1: Because it's a box of donuts and not a square of donuts. It's not flat.
4. Student 2: But there's air space between the donuts, duh.
5. **Teacher: Okay. Do you have something else to say back? Why are you guys looking at area and not volume?**
6. Student 3: Because, we're not filling the whole thing with donuts. It's only the bottom, because there's going to be room.
7. Student 4: Well, we don't know if they're going on top of each other.
8. **Teacher: So we have a question as to whether we're just putting them along the bottom of the box, or if there are layers of donuts. We don't actually know that.**

9. Student 3: Yeah.
10. **Teacher: Okay. Boys, can you stop please? Thank you.**
11. Student 5: Usually they only put one layer.
12. **Teacher: So you're using your knowledge of donut boxes, that usually we have one layer, we don't usually stack them.**
13. Student 4: But a donut still takes up space, it's not just flat.
14. **Teacher: But this group is looking at the area because they are assuming there's only one layer of donuts.**
15. Student 4: Yeah, but they still take up space. They're still high.
16. **Teacher: They do. They do Okay. j?** [draws thickness donut].
17. Student 6: They could stack them. There could be a piece of cardboard between the two rows of them.
18. **Teacher: So you're not eliminating the possibility that they could be stacked.**
19. Student 4: Yeah, because it's wide. When they showed the picture of it going in, it was really, really wide for a small donut.
20. **Teacher: Hold on. When they showed it going in... let's all look at that one.**
21. Student 4: Why would they make such a...
22. **Teacher: So it looks quite a bit taller than one layer of donuts. Okay.**
23. Student 6: But it can't be measured in volume, because even if donuts are 3D, they have a hole in the donuts, and the space between the donuts. So it can't be measured in volume, because there's a lot of air space.
24. **Teacher: Very interesting observation. So you're saying we can't just use volume because we're not going to be able to fill all the spaces. So even though there's a little space in there, we're not going to cut up donut bits to fill all the empty spaces. Interesting. Jo?**
25. Student 4: Well it depends what kind of donuts. Because there's the donuts that don't have the hole in the middle and it's just filled in.
26. **Teacher: True. If I think of your "what if?" Do they take the same amount of space? If you've ever bought donuts before, the ones with the hole, even though they do have less volume, in the box, do they take a smaller spot in the box?**
27. Multiple voices: No.
28. **Teacher: No. They just have a hole in the middle.**
29. Student 6: But miss, even if you take it without a hole in it, there's still the space between the donuts.
30. **Teacher: Because they're round?**
31. Student 6: Yeah.

[25 minutes] Divide your desk, write titles: "What I know?" vs. "What do I need to find?" Put all your questions in the What I need to find because we don't know anything. [Each group has one whiteboard and marker, therefore, members have to agree on what's written]. Main question: How many donuts are in the box?

[27:40 minutes] Your ultimate objective is how many donuts are in that box, you can't go directly, you need to solve before, same thing as written math questions, steps before. See based on the information that we are given, if you find questions that become irrelevant, if you don't find info about how long it takes, then take that question off. Try to fill in what we know.

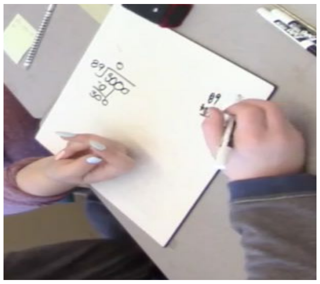
Act 2 Email With Omitted Info
[28:50 minutes] Reads email write down questions, don't talk to anyone, write
[new tape: 5 minute] group 3 students: "mrs. Kate it's a lot of zeros" "Maybe you don't need that" Group 3: "what do you need"


From: HayleyHutchison
07/10/2014 13:21:36

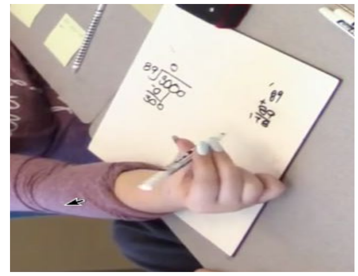
Hi Graham,
We hope the information below helps:
The box was created to allow 3 x layers of doughnuts (x doughnuts on each layer)
Each doughnut is approximately 89 millimetres in diameter.
The box is 3000 mm x 2300 mm which allows a gap between each doughnut to sit comfortably.
We do not have any schematics to share as they are for internal use only but we wish you all the best with your class.
Kind regards,
Krispy Kreme

[5:55 minute] New information, what is important? "three layers of donuts" 89 mm is the diameter, 3000 mm by 2300 mm is important, pretty much needed all. What approach are you taking? Volume. Volume? Calculate the circumference, not needed. Large calculation, area of a layer, area of a donut? No, we don't know how to calculate the area of a donut. You only have the distance across. If we know it's 89 mm across [Acting out with a tape and a book].

[9:38 minute] Let's see if this info helps, go back to your group. Group 3: began with dividing 3000 mm by 89 mm.



one students tries to perform 89 x 89 but the other erases it and does 89 + 89

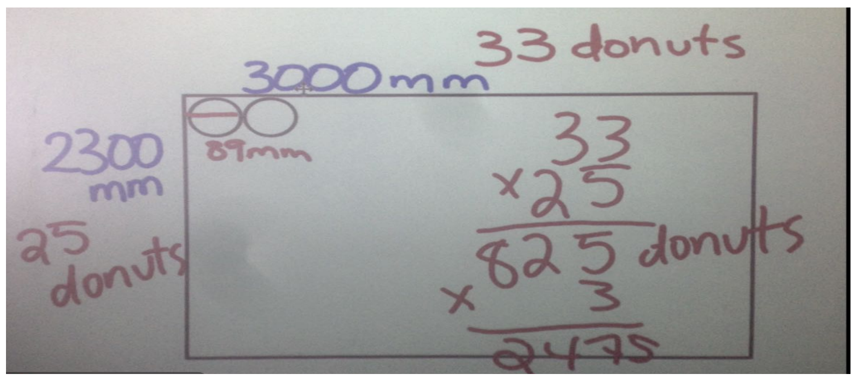


[10:52 minute] Teacher draws the diameter

[12:00 minutes] In the interest of time you may use a calculator.

Act 3 Email With Information

[14:40 minute] How many donuts fit this way? 3000/89=33, not taking decimal into account, "to breath", we can't find a 0.7 of donut, how many this way? 2300/89 = 25 donuts. What operation would I use? Times, 825 donuts, are we done? Times 3 layer. 2475 Donuts. They say "32" a miscalculation, we squeezed them in.



[17:00 minute] recap–Important thing today you learned to ask questions. If we use area, …. Respect the space… area. Not given, side by side, working with circles, the triangularish spaces will never been filled, think about dimensions than space.

From: **HayleyHutchison**
07/10/2014 13:21:36

Hi Graham,
We hope the information below helps:
The box was created to allow 3 x layers of 800 doughnuts (25 x 32 doughnuts on each layer)
Each doughnut is approximately 89 millimetres in diameter.
The box is 3000 mm x 2300 mm which allows a gap between each doughnut to sit comfortably.
We do not have any schematics to share as they are for internal use only but we wish you all the best with your class.
Kind regards,
Krispy Kreme

did not show the video accompanying the activity. Comments about group participation.

Appendix D–Donut Lesson Plan 2.0 Possible Directions for Improvement

The Donut Lesson Plan (Possible Directions for Improvement 51 minutes v.3)
Click Donut Lesson Plan 2.0 Draft

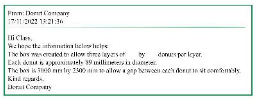
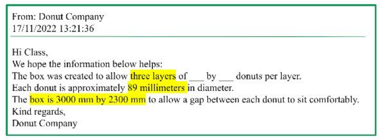
Donut activity: 3-Acts Think-Group-Share 1) Show prompt images. Estimate. 2) Read omitted info email, highlight important info, re-estimate by calculating in three ways mm, cm, and areas, with remainders. Plus one mental math example 3) Re-read email, verify results, highlight vague info and discuss approximations, test metacognitive abilities.

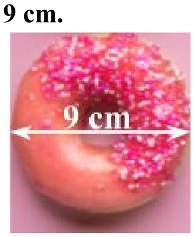
Materials: Prompt pictures with whiteboard demonstrations during the whole class discussions. A whiteboard per student and one per group. Students have their own post-its, no erasing, think alone, next in group they converge on a group whiteboard, and then share with class.

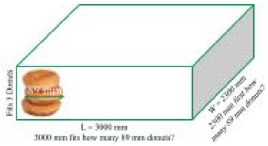
Here are the annotated classroom discussions; where the teacher's replies are made more specific with math vocabulary words such as product and multiplication words and less use of demonstrative words such as "this" & "that" CCCM Math classroom discourses Skits 1-3 annotated

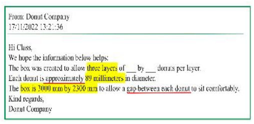
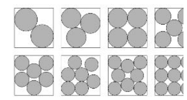
Duration	Prompt Images	Discussion	Calculations	☂Anticipate
ACT I 5 minutes Show Big Box of Donuts	**Estimate how many donuts are in this box.** **Show one pic at a time.**	The number of donuts in the box is unknown. What does it depend on? What do we need to know? Let's set our notations: Total # of Donuts in box = X Length: distance end-to-end = L Width: distance side-to-side = W Height: distance top-to-bottom = H Number of Layers = N Diameter of Donut = D Thickness of Donut = T		**What is estimation?** A rough calculation of a quantity


Duration	Prompt Images	Discussion	Calculations	☂Anticipate
10 mins	**Give an estimate that you know is too low?** **Give an estimate that you know is too high**	What is the range of the number of donuts in the box? Minimum to maximum donuts. Write guesses/approximations Differentiate between lower & upper bound vs. high & low estimations.	[0–∞] [1–5000] [200–3000] [690–2613]	Is 1 too low? Why is infinite not possible? What's reasonable?

Duration	Prompt Images	Discussion	Calculations	☂Anticipate
ACT II 5 mins Email	**First read alone in silence. Then ask a student to volunteer to read the email from the Donut company in the UK.**  From: Donut Company 17/11/2022 13:21:36 Hi Class, We hope the information below helps: The box was created to allow three layers of ___ by ___ donuts per layer. Each donut is approximately 89 millimeters in diameter. The box is 3000 mm by 2300 mm to allow a gap between each donut to sit comfortably. Kind regards, Donut Company	What do we need to know? Highlight key information in the email and match these values with information what we need to know  From: Donut Company 17/11/2022 13:21:36 Hi Class, We hope the information below helps: The box was created to allow three layers of ___ by ___ donuts per layer. Each donut is approximately 89 millimeters in diameter. The box is 3000 mm by 2300 mm to allow a gap between each donut to sit comfortably. Kind regards, Donut Company	L = 3000 mm W = 2300 mm N = 3 D = 89 mm	What is diameter? Longest chord, distance across a circle; edge to edge. Why are height & thickness not needed? We are given the number of layers.
3 mins Example	 	What if it were 100 mm in Diameter in a box that was 210 mm in length and 210 mm in width. If needed, use this image to help students make sense of what they already know. How many donuts are in this box? Therefore, there are 4 donuts in the box.	210/100 = 2.1 210mm/100 mm = 2.1 Donuts 2 x 2 = 4 ∴ Donuts X = 4	What to do with decimals? Donuts are whole, we keep the integer part.
2 mins In our Case	**In our case D = 89 mm. What if we round up just so we can perform mental math?**  	Let's start with rounding 89 mm up to 100 mm 1) If D ≈ 100 mm 3000 mm divided by 100 mm = 30 2) 2300 mm divided by 100 mm = 23 3) 30 times 23 equals 2070. Therefore, we can approximate that there are at least 2070 Donuts in the box.	If Donut mm 89 → 100 mm 2300/100 = 23 Donuts 3000/100 = 30 Donuts 23 x 30 = 690 Donuts per layer 690 x 3 = 2070 ∴ Donuts X ≈ 2070	Why "at least"? Because we rounded-up the size of the donut.

Duration	Prompt Images	Discussion	Calculations	☂Anticipate
10 mins If Area	**Think-Group-Share: What we need to find to calculate the number of donuts?**   	Let's reevaluate our estimation. Observe that each donut occupies a square of sides equal to the diameter of the donut. Let's try dividing the area of the box by the area of the square containing a donut. 1) If D ≈ 89 mm, then the space required by a donut is similar to that occupied by a square, so 89 mm times 89 mm. Then the area of the square equals 7921 mm^2 (square millimeters) 2) If L = 3000 mm and W = 2300 mm, *then* the area of the box is 3000 mm times 2300 mm which equals 69000000 mm^2 3) Each donut requires 7921 mm^2 and the area of the box is 69000000 mm^2. The area of the box divided by the area of the donut equals 871.10 just one layer. 4) We round down to 871. 5) *Since* there are 3 layers of 871 donuts, *therefore,* the total number of donuts is 2613. 6) Recall the rule of fractions multiplication with numerator (top number) and the same denominator (bottom number). 7) Therefore, we overestimate that there are 2613 donuts in the box.	89 mm (Diameter of the donut) × 89 mm (Diameter of the donut) 801 + 7120 = 7921 mm^2 ∴ 7921 mm^2 is the area of the square containing a donut 3000 mm (Length of box) × 2300 mm (Width of box) 0000 + 00000 900000 6000000 = 6900000 mm^2 ∴ Area of the box is 6900000 mm^2 $\frac{\text{area of box}}{\text{Area of square containing a donut}} = \frac{6900000\ mm^2}{7921\ mm^2} = 871.1021$ Round down to integer 871 donuts, with remainder of 809. ∴ at most 871 donuts per layer. 871 (Donuts per Layer) × 3 (Layers) = 2613 ∴ at most 2613 donuts in the box. $\frac{3000\ mm \times 2300\ mm}{89\ mm \times 89\ mm} = \frac{6900000\ \cancel{mm^2}}{7921\ \cancel{mm^2}} = 871.10$ Round down to integer 871 donuts, with remainder of 809. ∴ at most 871 donuts per layer	L x W = Area of Box D x D = Area of square In secondary school you can learn how to calculate the area of a circle which equals, R = D^2 = πR^2 Extra info: 7921 is a composite number so it can be divided by 1, 89, and 7921 itself. The prime factorization of 7921 = 89^2 The prime factor of 7921 is 89 Why is it an overestimate? Because we used all the space in the box.
3 mins Convert	**What if we convert to centimeters? Will our results change? 89 mm = 8.9 cm which we can approximate to 9 cm.**  	1) Let's simplify by rounding up 89 mm to 90 mm 2) Then convert 90 mm to 9 cm 3) 9 cm fits 11 times in a 100 cm stick (a meter), so 3 times 11 = 33 in a row. 4) Therefore, we can approximate that there are about 2475 Donuts in the box.	89 mm → 90 mm 90 mm = 9 cm 2300 mm = 230 cm 3000 mm = 300 cm 300/9 = 33.33 → 33 Donuts [3 remainder] 230/9= 25.55 → 25 Donuts [5 remainder] 825 donuts per layer x 3 = 2475 ∴ 2475 donuts	How do we convert from mm to cm? Calculate the remainder.

Duration	Prompt Images	Discussion	Calculations	☂Anticipate
4 mins Calculate	**Three layers of donuts**  	If we calculate the number of donuts by row and column and round down to 33 by 25 and then multiply by 3 we can obtain a reasonable estimate. 1) 3000 mm/89 mm = 33.70, therefore, 33 donuts by length/row (round down decimals) [63 remainder] 2) 2300 mm/89 mm = 25.84, therefore, 25 donuts by width/column (round down decimals) [75 remainder] 3) 33 x 25 = 825 Donuts per layer 4) 825 Donuts times 3 layers equals 2475. Therefore, estimate the number of donuts to be 2475.	$\frac{\text{Length of Box}}{\text{Diameter of Donut}} = \frac{3000\ \cancel{mm}}{89\ \cancel{mm}} = 33.70$ Round down to integer 871 donuts, with remainder of 63. ∴ 33 Donuts per row. $\frac{\text{Width of Box}}{\text{Diameter of Donut}} = \frac{2300\ \cancel{mm}}{89\ \cancel{mm}} = 25.84$ Round down to integer 871 donuts, with remainder of 75. ∴ 25 Donuts per column. 33 (Donuts per Column) × 25 (Donuts per Row) 165 + 660 = 825 ∴ 825 Donuts per Layer 825 (Donuts per Layer) × 3 (Layers) = 2475 ∴ 2475 Total Donuts	Notice that whether we convert or keep the same unit of measurement their results are the same. However, notice the change in remainder.
ACT III 1 min Verify	**Read Donut company email to verify which of the answers fits.** From: Donut Company 17/11/2022 13:21:36 Hi Class, We hope the information below helps: The box was created to allow three layers of 25 x 32 donuts per layer. Each donut is approximately 89 millimeters in diameter. The box is 3000 mm by 2300 mm to allow a gap between each donut to sit comfortably. Kind regards, Donut Company			Why 32 donuts not 33?! Why did we calculate 75 extra donuts?

Duration	Prompt Images	Discussion	Calculations	☂Anticipate
3rd Act Twist 2 mins Test-Meta Cognitive Abilities	**Whole class discussion to highlight in red parts of the text that indicate vagueness and explains why calculations don't match.** **Difference between circle as idealized concept and donuts as reality.**  From: Donut Company 17/11/2022 13:21:36 Hi Class, We hope the information below helps: The box was created to allow three layers of ___ by ___ donuts per layer. Each donut is approximately 89 millimeters in diameter. The box is 3000 mm by 2300 mm to allow a gap between each donut to sit comfortably. Kind regards, Donut Company	Diameter = *Approximate* Gap between donuts = Unknown Since we do not know the size of the gap between the donuts, none of these calculations can give an accurate count. Wrap-up, today we asked relevant math questions to approximate the total number of donuts in a box. Depending on how we calculate, we may obtain an underestimation if we round the diameter of the donut up to 100 mm. However, if we keep the original value of 89 mm and round down any decimals to obtain the closest estimate in donut form, we get a more reasonable estimation. Using the area of the box and the area of the square containing a donut gives us an overestimation of the number of donuts because it does not account for the gap between the donuts. In summary, rounding the diameter of the donut to 100 mm results in an underestimation, while using the area of the box and donut as a square results in an overestimation. The most reasonable estimation is obtained by keeping the original value of 89 mm and rounding down any decimals. However, we got a surplus of 75 donuts, which is likely due to the fact that donuts cannot be exactly 89 mm. Notice how they use the word "approximately 89 mm." And finally This type of calculation falls under circle packing.	If $D \approx 100$ mm then # Donuts ≈ 2070 If calculated by Area then # Donuts ≈ 2613 If $D \approx 89$ mm then # Donuts ≈ 2475 Estimation interval of the total number of donuts in the box whether one layer or three layers: [690–2613]. If donuts X = 2400 in the box then: 1) 2070 < 2400 is an underestimate since we rounded up the size of a donut (89 mm to 100 mm) 2) 2614 > 2400 is an overestimate since the entire area of the box was used. 3) 2475 ≈ 2400 closest approximation.	What does it mean to approximate? If you want to know more about circle packing;  (Musin & Nikitenko, 2016)

Google image source: GettyImages/Alexandra Fedorova

Appendix E–Summary of Appreciative Inquiry

CCC-M Collective Reflection Meeting March 2017

1) What did you notice in the video clips shown? Did anything stand out to you or surprise you?
2) What Learning Targets could you identify in the activity shown?
3) What Target Teaching Practices could you identify in the activity shown?
4) How might teaching/learning practices like the ones shown be improved?
5) How might you implement one or more of these Learning Targets and/or Target Teaching Practices in your classroom?

Appreciative Inquiry:

1) Overall, how helpful did you find the collective reflection meeting today?
   Very helpful, Helpful, Needs improvement
2) List 3 things you learned today that would help you improve your teaching/learning practices.
3) What requires follow-up or more information?
4) Other comments

CCC-M F2F Collective Reflective Meeting Compiled Survey

Question 2: List 3 things you learned today that would help you improve your teaching/learning practices.	
Secondary group (March 20, 2017): • Activities on asking relevant questions • Misconceptions with conversions • Emphasising how to label their work • Label all work on the board • Design Omit or provide more info? • Being aware of the wording we use & how we answer students questions • Work on extracting relevant information • Struggles with conversion (implementing) • Emphasise labelling of work with units (avoid gaps)	**Elementary group (March 22, 2017):** • Every lesson can be done differently so go with the flow of your students from year-to-year • Reminded of last year's lesson of 'fill-in-the-blank' with words • Talk about purposes for estimating • Wait time to encourage student-student interaction • How to continue fostering problem analysis skills-different approaches • Group work- it is very important • Taping or recording them in groups • Work on a plan or continuum between cycle 2 + cycle 3 of strategies to build problem solving skills, so that we continue not repeat. • Collaborating with colleagues in my own school • Having a clearer plan of what activities to try • Showing more video clips with students to generate further discussion • Looking at the question first, then look at the info in the problem • Different approaches that I haven't tried in order to show different concepts (ex: power hour).

Comments on Collective Reflection Questionnaire

Question 4: How might teaching/learning practices like the ones-shown be improved?	
Secondary group (March 20, 2017): • Students need more guided • Conceptual understanding of units of measure • Could provide more value (height) to elaborate the math task • Give height (box or doughnut) • * (Missing datum; respondent did not allow pictures of the top part of their survey.)	**Elementary group (March 22, 2017):** • Doing this more often in the class so students are familiar with the routine of group work and discussion. • I need to work on group work. Establishing good habits and practices. • By experience ...each time you implement one in the classroom you learn something new + reflect + improve for the next • More practice … proper behaviour.

Appendix F–Idealized Lesson Study Guideposts

Idealized: Lesson Study Guideposts v2.2 [Winter 2022]
Click here: Idealized Lesson Study Guideposts v2
Adaptation from Breuleux et al. (2017)

Once group norms have been set after being scaffolded with a few provided by the research team (e.g., assign roles, respect the person, debate ideas, resist jumping to solutions, etc.). Lesson study facilitators aim to "*bring the proce*ss of choice into awareness to examine and assess the reas*ons for making a choice*" (Mezirow, 1998, p. 186). For this reason at the codesign meetings members it would be fruitful to have partners from the other cycles present to act as critical friends. The critical partners can adopt steps from the consultancy protocol (Dunne et al., 2020) to determine conversational moves (who does what when). Alternatively discussions can be coordinated by integrating steps from Garrison and Munday (2012).

For knowledge mobilization, share beyond the group, suggestion to print a copy of this lesson plan to be shared on the school's Ideas Board under the banner "Lesson I am trying" and updated after implementation and collective reflection. The process of lesson study must be reiterated to optimize improvements.

Sparks-Langer et al. (1990) Technical reflection:consider the best way to reach an unexamined goal; practical reflection examines the means and the end (goals) by attending to "What should we be learning?"; critical reflection considers the moral and ethical issues of social compassion as well. These can be blended in with Mezirow's (1998) types of critical reflections starting with content reflection, followed by process reflection, and then premise reflection (Calleja, 2014).

Lesson's Name: **Grade:**	**Describe & Justify the Lesson Plan** [Pre-Enactment]	**Reflection-in-Action** [During Enactment]	**Reflection-on-Action** [Post-Enactment]
CoDesign *Technical Reflection: How to reach learning goals? Content.*	Why this lesson? Clarify the overall learning target. Delimit required concepts for this activity. What & How	Plan what to do. Describe the flow. First, Next. Then, and Last.	Did the lesson reveal itself? [set expectations]
Rehearsal/Simulation *[investigate solution]*	Revise solutions Anticipate [necessary concepts, possible questions, type of grouping, etc.] What, When & Where	Mark what is happening during this activity? Track surprises & challenges	Integrate as many anticipations
Implementation/ Enactment *Practical Reflection: What should we be learning? Process.*	How will you know the learning target is attained by each student? What evaluation strategies will be used to verify understanding? Delimit required concepts for this activity.	**What is happening? Track how many ad-hoc actions are needed.**	[expect-observe] Why did it happen? How has it changed?
CoReflection *Critical Reflection: Did we learn? Premise.*	What surprised you? So what? Now what?	**What happened?**	**Why that action?**

CoDesign Lesson: Describe & Justify the Lesson Plan [Pre-Enactment] Problem-Solving; *Technical Reflection: How to reach learning goals? Content.*

a. Lesson's name:
b. Why this lesson? What about this activity will lead to overall learning. Delimit required concepts for this activity.
c. How will you know the learning target is attained by each student? What evaluation strategies will be used to verify understanding?
d. What is the lesson with what activities will it come into being? Describe details for a stand alone document.
e. What format will be the classroom discussion?
   i. Think-Share (why whole class discussion? What responses do you expect to hear?
   ii. Think-Pair-Share (what kind of paring (e.g., side-by-side partners, weak-strong, random)? Why?
   iii. Think-Group-Share (what kind of grouping (e.g., homogeneous vs. heterogeneous) Why?
   iv. Group-Share (why no solo-thinking period?)
f. Calculate the answer; what is the most rigorous justification of the solution to this lesson.
g. Which of the high quality teaching will be enacted in this lesson? (Pls refer to Table 1). How will you operationalize the high quality teaching selected?

Implementation of the Lesson [Reflection-in-Action During Enactment] Problem-Setting; *Practical Reflection: What should we be learning? Process.*

h. Mark what is happening during this activity? Describe the flow. First, Next. Then, and Last.
i. What moments are key in the activity? [Descriptive lens: any surprises?]
j. What was not said but should've been said?
k. What other materials are needed to enact this lesson?
l. Were the types of pairing adequate to communicate the crux of the content to each student?
m. Does the learning target become evident as the activity unfolds?
n. How are students reacting? Are all students engaged in the activity?
o. Would you do any part of the activity differently? Mark moments that stood out in terms of concepts related to the lesson.
p. What did you learn from this lesson?
q. Any emergent questions that need further thinking?

CoReflect on the Lesson [Reflection-on-Action Post-Enactment] Problem-Posing; *Critical Reflection: Did we learn? Premise.*

r. Write a brief description of salient moments (Please provide time stamps). [note to facilitators: provide a copy of the original codesigned lesson plan to the teachers reflecting on its enactment].
s. Does the lesson plan include formative assessment as well as a plan to accommodate individual student differences during the lesson?
t. Identify which of the high quality teaching.
u. Does the lesson plan provide sufficient information about how the planning team decided to teach the lesson?
v. Review of the student learning data (including the video data):
   b) Does the lesson highlight student thinking in relation to the learning target?
      - Do the objectives of the lesson plan clearly address the curriculum?
      - Are the tasks appropriate for the students given the timing of the lesson?
   c) Were there any difficulties in discerning the lesson's impact on students' learning specific topic?
      01. Were students able to complete the task independently within the classrooms' dedicated time?
      02. Were students able to provide a rigorous justification to solve the mathematics task?
      03. **Future planning:** How would you revise the implementation phase of this learning lesson?
         a. Does the lesson plan include reasonable anticipated student responses and indicate how the teachers will help students overcome any misunderstandings?
         b. Are the key questions clear? Did they provide an opportunity for students to think mathematically?

Table 1: Practices of high quality teaching from *"High Leverage Practices"* (adapted: TeachingWorks, University of Michigan, 2013) http://www.teachingworks.org/work-of-teaching/high-leverage-practices

❑ **Focus on instructional goal**	Composed of two categories: 1) *Learning targets*, inform students what is expected of them to learn. 2) *Target teaching practices*, describe what you plan to teach (road map).
❑ **Elicit and respond to student thinking**	Describe how you elicited students' thinking (e.g., the initial questions you asked, student responses, and your reply, etc.) Describe how your questions helped students reach the mathematical instructional goal. Why did you choose to orient students in this manner?
❑ **Orient students to other's ideas**	Describe how you oriented students to each other's thinking (e.g., the wording you used, etc.). Why did you do it in that way? Why was this move important for supporting students' learning of your particular mathematical instructional goal?
❑ **Prompt students to record their thinking**	Describe how this activity will make students' thinking and work visible. For example, a photo album of projects, math journals, posters, class book, chart paper, on smart board, blackboard, white board, Padlet app. For more examples: https://onedrive.live.com/view.aspx?resid=4AC891949A1871F1!1644&ithint=onenote%2c&app=OneNote&authkey=!ABiqb8v0EZhHAJ8

** Consider the default classroom culture:

❑ **Position students competently**	Frame mathematical ideas and their use in a way that productively challenges students without going beyond their abilities. (*See 'Cognitive Demand'* http://map.mathshell.org/trumath.php#truframework)
❑ **Establish and maintain expectations for student participation**	Select instances where: 1) you set specific expectations for student' mathematical participation; 2) you maintained expectations for mathematical learning. Describe how you set particular expectations within each phase. How did you help students make sense of these expectations? How did you hold students accountable to these expectations? (Please move beyond classroom management expectations, such as one person at a time, etc). Why were these moves important for targeting your specific mathematical instructional goal?