Stochastic optimization approaches to open pit mine planning: Applications for and the value of stochastic approaches

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ABSTRACT

The mine production schedule defines the sequence of extraction of selected mine units over the life of the mine, and consequentially establishes the ore supply and total material movement. This sequence should be optimized so as to maximize the overall discounted value of the project. Conventional schedule approaches are unable to incorporate grade uncertainty into the scheduling problem formulation and may lead to serious deviations from forecasted production targets. Stochastic mine production schedulers are considered to obtain more robust mine production schedule solutions.

The application of stochastic approaches to the mine production schedule problem is recent and additional testing is required to better understand these tools and to define the value of a stochastic solution as compared to the conventional result. Two stochastic schedulers are tested in a low-grade variability copper deposit, optimization parameters are discussed and their results compared with a conventional schedule.

The first method uses a stochastic combinatorial optimization approach based on simulated annealing to address the mine production schedule problem. The method aims for maximization of the net present value (NPV) of the project and minimization of deviations from the production targets. These objectives are attained by incorporating grade uncertainty into the mine production schedule problem formulation. The second method formulates the problem as a stochastic integer programming problem, in which the objective is the maximization of the projects' NPV and the minimization of production targets deviations. The model can also manage how the risk of deviating from the targets is distributed between production periods.

Both stochastic approaches were tested in a low-grade variability copper deposit. In both case studies, the value of a stochastic solution is demonstrated to be higher than the conventional one. This fact demonstrated the misleading results that a conventional schedule may produce and shows the importance of not ignoring the presence of uncertainty when defining the mine production schedule for a project.

RESUME

Les étapes de la production minière définissent les séquences d'extraction des unités des mines pendant la durée de vie des mines et conséquemment établit la quantité de minerai produite et le mouvement de matériel requis dans sa production. Cette séquence devait être optimisée afin de maximiser la valeur du projet. Les méthodes conventionnelles sont incapables d'incorporer l'incertitude dans la formulation d'un problème de planification et peuvent mener à des déviations par rapport aux productions atteintes. Les analyses stochastiques de production de mine sont considérées comme étant des solutions plus robustes afin de déterminer le plan de production minière optimal.

L'application des approches stochastiques pour l'obtention d'un plan de production minière est récente et des tests additionnels sont requiss afin de mieux comprendre ces outils et définir la valeur d'une solution stochastique en comparaison à une méthode conventionnelle. Deux programmes stochastiques ont été testés pour un dépôt de cuivre à teneur faible variable. Les paramètres d'optimisation sont discutés et les résultats comparés à une méthode conventionnelle.

La première méthode utilise une approche combinatoire basée sur le recuit simulé pour obtenir un plan de production minière. La méthode vise la maximisation de la valeur actualisée nette du projet et la minimisation des déviations des cibles de production. Ces objectifs sont atteints en incorporant des nuances d'incertitude dans la formulation du problème de planification minière. La seconde méthode formule le problème de programmation de manière stochastique en nombres entiers, dans lequel l'objectif est la maximisation de la valeur actualisée nette et une déviation minimale des cibles de production. Le modèle peut gérer la manière dont les risques de déviations dans les cibles de production sont distribués entre les différentes périodes de production.

Les deux approches stochastiques ont été testées dans un dépôt de cuivre à faible teneur variable. Dans les deux études, la valeur de la solution stochastique a été démontrée comme étant préférable à l'étude conventionnelle. Ce fait démontre les résultats trompeurs qu'une méthode conventionnelle peut produire et montre l'importance de ne pas ignorer la présence d'incertitude lorsque le programme de production minière est défini pour un projet.

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CONTRIBUTIONS OF AUTHORS

In accordance to the format requirement of a manuscript-based thesis, this section clearly states the contribution of the co-author of the papers that comprises the preset work. All papers have Prof. Roussos Dimitrakopoulos as a co-author. It is important to emphasise that the author of this thesis has done all the work herein published, with the normal supervision, advice and orientation of his advisor Prof Roussos Dimitrakopoulos. In addition, a thorough bibliography is provided at the end of the thesis, although in order to provide a concise presentation, each paper retained its own list of references.

The papers included in the thesis are:

Chapter 3: Leite, A. and Dimitrakopoulos, R. (2007). A stochastic optimization model for open pit mine planning: Application and risk analysis at a copper deposit. IMM Transactions, Mining Technology, v. 116, no. 3, pp. 109-118.

Chapter 4: Leite, A. and Dimitrakopoulos, R. (2008). Stochastic integer programming formulation applied to the mine production schedule of a copper deposit, submitted to IMM Transactions, Mining Technology.

I herby give copyright clearance for the inclusion of the above papers, on which I am a co-author, in the MEng dissertation of Andre Nascimento Leite.

Signed by:________________________ Date: May/2008

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CHAPTER 1 INTRODUCTION

Mining activity has considerably changed over the past few decades. The previous scenario in which large, rich and superficial deposits were available has now changed to one where new deposits tend to be smaller in size, harder to mine and in some cases, only marginally economically viable. More strict environmental regulations have to be considered and a more competitive global market requires companies to be more efficient. As a result, mining companies face the challenging tasks of developing new mineral deposits and selecting from their portfolio the deposits that will generate the best return on their investment.

In this new scenario, so as to provide a solid base for investment decisions, evaluating a project should be done in the most robust and concise way possible. Mining ventures are intrinsically risky yet high capital investment decisions are often made with limited information. Project evaluation approaches should be able to account for risk when assessing the value of a project. The value of a mining project is defined by a set of activities, which are recursive by nature. These activities involve the selection of a sequence of extraction, cut-off grade strategies, and mining and processing capacities over the life-of-mine. Due to the limited amount of information available, true optimum decisions for these tasks are not possible; approximations and mathematical and statistical assumptions have to be made. The alternatives to obtaining an optimum solution for the activities mentioned above and maximizing project value involve the use of deterministic/conventional or probabilistic models.

The conventional approach for economic evaluation of a mine assumes the understanding of geological boundaries and grade distribution, mining parameters including recovery and dilution, and the economic scenario. Convention dictates the use of average responses of these parameters to define the available ore supply. In the mine-planning framework, a final economic boundary, ore and waste mining capacities and a sequence of extraction ultimately define the ore supply, which is fixed in time and space. The forecasted ore supply defines the expected revenues over the life of the project. The revenues, associated with the production costs and a discount rate, are utilized to compute the discounted cash flow (DCF) and to define the present value (PV) of the project. Final project selection is done using the computed PV. The drawbacks of the conventional approach start with the assumption of a risk-free environment. This assumption leads to misleading results obtained by the application of non-linear transfer functions, such as final pit and sequence of extraction, using average type scenarios[1]. The selection of the discount rate to be used in the DCF analysis represents the other limitation of the conventional approach. This rate is normally composed of both an opportunity cost portion and a risk related portion. In all cases, the risk portion is arbitrarly defined, since the given deterministic approach is not able to map the associated risk. DCF method also suffers from its inability to account for the managerial flexibility to react to unlikely scenarios and to take corrective actions.

Alternatives to the conventional project evaluation framework should be able to make decisions in the presence of uncertainty. Since uncertainty is an intrinsic characteristic of mining ventures, it should be considered in order to provide robust and realistic solutions. Therefore, it can be stated that a successful assessment of the economics of a mine is conditional to properly mapping the sources of uncertainty, or at least the ones that could jeopardize the economics of the project. Different sources of risk around economic or technical parameters can be considered. Common sources of uncertainty are geological boundaries, in-situ ore reserves and metal content (geological risk), dilution and recovery, geotechnical constraints, market (demand and prices), mining and processing costs and political environment. Each of the previously mentioned sources has a different impact in the economics of a project, and may or may not be predictable. One of the current alternative avenues of research involves the use of stochastic simulations to map sources of uncertainty. Since geological risk is considered to be the most frequent reason for the non-realization of the forecasted ore/metal production [2,3,4], among all potential sources of risk , there can be significant benefits when it is accounted for it in the mine planning and design stage of a project. Godoy and Dimitrakopoulos[5] present a stochastic solution based on simulated annealing that improves the expected return of a project by 28%. Menabde et al[6] describes a 4% increase in the NPV of a project when comparing a conventional scheduler with a stochastic integer programming (SIP) formulation for long-term production schedule. Ramazan and Dimitrakopoulos[7, 8] propose an approach able to directly incorporate uncertainty into the long term mine schedule framework by the use of an SIP method and obtain an increase of 10% on the project NPV when compared with the conventional approach.

It is reasonable to expect that risk should not be ignored as it substantially impacts the economics of a project and improved results should be expected by incorporating uncertainty in the context of mining planning, design and production scheduling. Further investigation is required, however, to understand and assess the possible improvements that stochastic solutions may have to offer, as compared to present conventional (deterministic) mining practices.

1.1 Goal and objectives

A modern life-of-mine production scheduling method must have the capacity to incorporate grade and, in general, geological uncertainty into the scheduling process, in order to produce more robust and realistic solutions as compared to conventional schedulers that ignore uncertainty. The use of stochastic mine schedulers to solve the mining production problem is relatively new. The goal of this thesis is to test and contribute to the understanding of two different stochastic long-term mine scheduling methods, one founded upon simulated annealing and one on stochastic integer programming, analyse their characteristics, asses their impact, as well as quantify their value and contribution. To achieve this goal the following objectives are set:

- Review the literature concerning stochastic mine production schedule formulations, and conditional simulation methods for mining related problems.
- Apply a stochastic mine production scheduling formulation based on a simulated annealing algorithm in a copper deposit and comparison with a conventional scheduler.
- Apply a stochastic integer programming formulation to solve the mine production scheduling problem at the same deposit, testing and discussing the impact of the geological discount rate and different ore selection criteria.
- Facilitate the efficient generation of inputs to the above schedulers by using the direct block simulation method.
- Analyse results, compare methods, document the value of stochastic solutions as found in this study and recommend future work.

1.2 Thesis outline

This thesis is organized into the following chapters.

Chapter 1 is an introduction to the work presented herein and briefly discusses the topic covered by this thesis.

Chapter 2 presents a literature review on fundamental topics related to mine planning optimization and in particular stochastic mine production scheduling and stresses the value of stochastic solutions in past studies and simulation methods for orebody modelling.

Chapter 3 describes a stochastic optimizer based on simulated annealing, presents its detailed application to a copper deposit and makes comparisons with the traditional approach
Chapter 4 describes a stochastic scheduler based on a stochastic integer programming formulation, presents its application to a copper deposit, discusses the major parameters involved and their impact on the stochastic schedules obtained.

Chapter 5 addresses the conclusions, revisits the contribution of stochastic solutions and suggests related future work.

CHAPTER 2 LITERATURE REVIEW

2.1 Incorporating geological uncertainty in mine production scheduling

Geological uncertainty is recognized to be the major reason for the non-realization of forecasted cash flows of mine ventures, as it has a direct impact in the ore and metal supply. The question of how to define the available ore supply is not easily answered, as it is not only a function of the metal content and spatial distribution of ore but also depends on the selected extraction sequence over time. Different sequences of extractions would produce different ore supplies from the same deposit. The definition of ore is variable over time as it is a function of economic parameters and is time dependent. Defining available ore supply is traditionally evaluated with the assumption of technical and economical constraints fixed both in time and space. In the conventional mine design and production scheduling framework, an average type of deposit is used in combination with geotechnical, economic and environmental constraints to define the extraction sequence that generates the maximum economical return. The use of a risk-free scenario has been shown to cause serious discrepancies between the forecasted and the actual economic return.

Several authors have studied the impact of geological uncertainty on the economics of a project using conditional simulations for risk-analysis of performance parameters of mine schedules[1, 2, 3, 9]. All of them conclude that the use of a smoothed image or representation of the geological reality leads to unpredictable under- or over-evaluation of the performance parameters under consideration, when non-linear transfer functions such as mine plan and design are applied. Figure 2.1 shows a risk-analysis on the PV of a mine planning pit optimization from a gold mine in Australia[1]. Each pit is evaluated using a set of simulated models and a cumulative PV probability distribution is obtained, represented by the grey lines. The black line represents the result obtained using the average type of deposit. Differences stem from the fact that an average type of deposit does not reproduce the true grade variability of the orebody and spatial correlation. One can conclude that an estimated model or average type of input does not produce an average type of response when conventional optimizers are considered. In fact, for the case presented by Dimitrakopoulos et al[1], the estimated model substantially over-estimates the PV of the pits since the risk profile shows that the estimated PV only has 2-4% possibilty of success, and a 25% difference in PV on average.

This makes clear the need to consider stochastic approaches which are able to account for risk and generate more risk-robust solutions, the natural avenue to include uncertainty into the mine design and production schedule problem formulation.

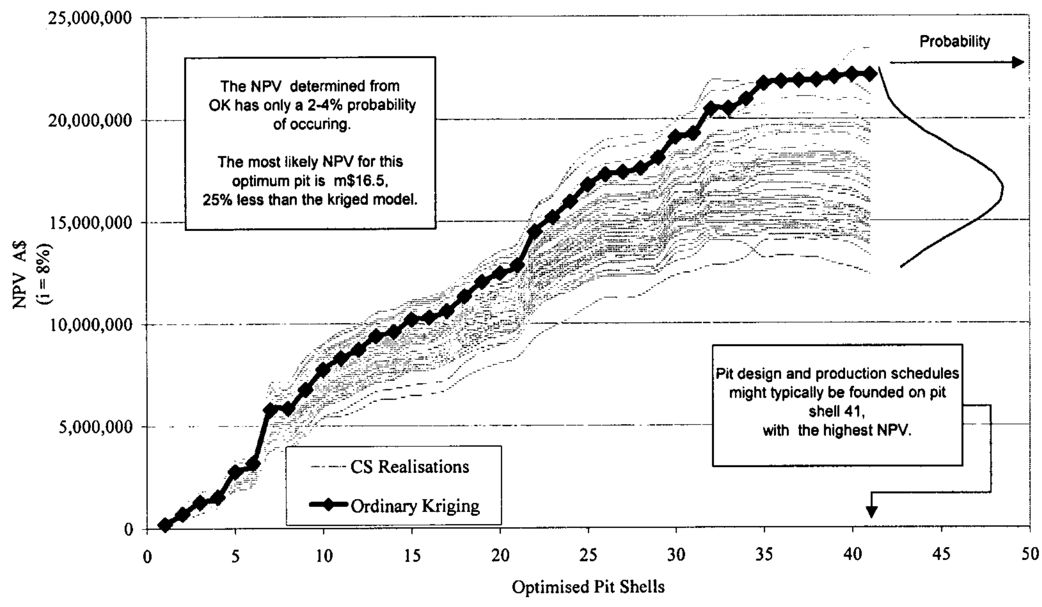

Figure 2-1 - Risk analysis of NPV distribution for a set of 41 nested pits1

The idea of incorporating geological uncertainty in the mine production schedule problem dates back to David et al[10]. In their study, the variability of the final pit boundary, generated using the Komogorov algorithm, is evaluated using a simulated orebody and the pit obtained by using the estimated orebody model. As initial observations were not fully understood at the time and due to their high computational processing and storage cost, the development of new stochastic approaches to the problem did not happen for decades. The use of a stochastic approach has been the focus of more recent studies.

A logical way to account for geological uncertainty in the mine design process would be to directly apply a conventional optimizer to a stochastic scenario. Dimitrakopoulos, Martinez and Ramazan[11] introduce the concepts of upside/downside potential to solve the long-term design and production scheduling problem in an open pit mine. Using a set of simulated orebodies, the method first defines a final pit and a set of pushbacks for each simulated orebody using a conventional pit optimizer. The set of mine designs are then evaluated using the set of simulated orebodies. A minimum mill feed requirement is first utilized to filter designs that do not meet this minimum requirement. The designs that pass the first filter are then evaluated, considering their chances of producing the minimum acceptable return (MAR). Finally, a design is selected from among those remaining by considering different price forecasts and selecting the one which has the highest upside/ lowest downside potential over the highest price scenario. The method is time consuming, as a design has to be done for each simulated model and the solution will not correspond to the optimum one, since it only considers a small and finite number of alternative mine designs.

The next logical step is to develop tools able to incorporate geological uncertainty directly into the mine production schedule problem formulation. Dimitrakopoulos and Ramazan[12] present a mathematical programming formulation to incorporate grade uncertainty into the long-term production schedule. The proposed formulation aims to minimize deviations form production targets. The probability to deviate at each production period over the life-of - mine (LOM) is considered in the objective function weight by a variable unit cost. Probabilities are defined from a set of simulation scenarios of the orebody. These costs are discounted over time using a rate defined as geological discount rate. The discounting process accounts for the fact that lower risk is preferable over the earlier periods of mining than later, therefore the costs to deviate from production targets decrease over time. This strategy to manage risk reflects the common need to ensure the payment of invested capital over the earlier periods. The probability of deviating in a given period is computed considering all blocks scheduled to be mined in that period and the probability of each one of them being in a given grade interval. The formulation also incorporates constraints to ensure equipment access and to minimize equipment movement.

The use of mixed integer programming (MIP) gives the option of solving the problem of mining partial blocks. These formulations are able to find the true optimum solution but may not be able to do it in a reasonable amount of time especially if a large number of integer variables are to be considered. Ramazan and Dimitrakopoulos[13] introduced another mixed integer programming formulation to address the mine scheduling problem. The formulation first defines the probability of each block being mined in a given period. These probabilities are obtained by applying an MIP formulation to a set of simulated models producing a set of schedules. The schedules are then used to map the probability distribution of each block being mined in all the periods of the LOM. Finally, an MIP formulation, accounting for the probability computed in the previous step and aiming to maximize the NPV of the project, is applied. The new objective function considers the probability of each block being mined in a given period. The formulation also includes constraints to ensure a feasible schedule, if equipment access is considered. Apart from successfully incorporating grade uncertainty in the scheduling process, the proposed method requires the solution of an MIP for each simulated model and one MIP to obtain the final stochastic schedule. The utilization of integer variables may render the application of the method difficult as a large number of integer variables imply an impractical solution time.

Ramazan and Dimitrakopoulos[14] further extend the use of an MIP formulation for application in complex multi-elements deposits such iron or nickel laterites. The method starts by computing the probability that each block has to be in a given grade interval or above a set of cut-off grades using simulated models of the deposit to compute such probabilities. The MIP is formulated to maximize the economic return of all blocks weighted by the respective probabilities. By stating the problem in this way, the produced probabilistic schedule mines more certain blocks to respect the quality constraints in earlier periods and postpones less certain blocks to later periods. This feature guarantees a schedule that reduces the risk of producing material which would not satisfy quality requirements. In the case study presented, a considerable difference between the probabilistic schedule and the conventional one is observed. The probabilistic schedule substantially reduced the risk of producing material outside of the quality expectations in the first years of mining, relegating the production of material with high risk of not meeting the quality expectations to later years. The conventional schedule produced a schedule with higher risk of producing material outside the quality specification in the earlier years. The method only requires one schedule to be produced but long solution times may be required as integer variables are utilized.

Godoy[5, 15] proposes a different approach to the long-term mine production schedule problem. It is an adaptation of a combinatorial optimization algorithm based on simulated annealing, and involves four stages. In the first stage, a stable solution domain for ore and waste production is defined, considering a set of simulated models. A stable solution domain is defined using concepts first introduced by Rzhenevisky[16], stating that a mine is a system with only two types of products, ore or waste, and that it is adapted to the mine schedule optimization using the concept of nested pits[17] which are used to define two possible extraction sequences. By construction, nested pits set the upper and lower limits for ore and waste production[18]. The best extraction sequence is defined by incrementally mining each nested pit bench-by-bench, which guaranties the maximum possible ore extraction with a corresponding minimum amount of waste extraction. This extraction sequence corresponds to the maximum cash flow, as it defers the waste production as long as possible. The worst-case scenario is obtained by incrementally mining each bench inside the selected final pit completely, and not considering the nested pits as separated units; this would have the highest possible stripping ratio over the LOM and therefore corresponds to the minimum possible cash-flow. These extreme scenarios are used to define the set of all possible feasible combinations of extraction rates for ore and waste. The set of all feasible combinations considering a set of orebody simulations is defined as "stable solution domain", comparing the common area of all cumulative ore with cumulative waste production considering all the simulations and defines all possible feasible combinations for ore and waste production, for a given orebody.

The second stage of Godoy's approach involves the definition of the optimum mining rates over the LOM. This problem is solved using a linear programming formulation, and the possible mining rates are constrained by the stable solution domain defined in stage one. In the third stage, a set of mining sequences are obtained, one for each simulated model, utilizing the optimum mining rates defined in stage two. In the forth and final stage, a stochastic solution is obtained by using a simulated annealing formulation, aiming to minimize the possible deviations from the optimum ore and waste production targets. The input for the algorithm is a set of schedules produced in stage three. The result is a final schedule that reduces the risk of not feeding the mill at the planned capacity and respecting the imposed mining capacity. The results of the stochastic schedule, when compared with the ones obtained by a conventional approach, produce a 28% increase in the project's NPV. There is also a lower risk of deviating form production targets, a 3% likelihood of deviating from production targets, while the conventional approach presents a 13% chance overall. The method successfully incorporates geological uncertainty into the long-term production schedule framework and the improvement in the NPV can be attributed to the better use of the available information regarding when a given block should be mined. The weakness of the method is that it does not allow control over the way risk is distributed over the LOM between mining periods. As it is a new method, it requires further testing to fully understand its capabilities and potential contribution to mining practices.

All stochastic mine production schedules described so far require that a set of schedules first be defined to feed a final optimization stage that combines them to produce a stochastic solution and that does not provide control on grade risk management. Ramazan and Dimitrakopoulos[7,8] propose an approach that directly incorporates uncertainty into the long term mine schedule framework and also provides control of grade risk management. A stochastic integer programming (SIP) formulation is proposed to solve the problem. Its objective is to maximize the PV of the project considering a set of simulated orebodies and also minimize the risk of deviating from production targets (ore tonnage, head grade, metal production). It has features that allow the mine planner to have more control over the risk of not reaching the expected targets. The user can define different penalties for deviating from each target under consideration, allowing one to not only define the priority of production targets but also the weight associated with the deviations of each of them. The method also uses the concept of a geological discounting rate. This rate is used to discount the cost of ore production deviations over time which reflects the principle that the cost to deviate from the production target earlier in time has a higher cost then it would later. Therefore, penalties for ore production deviations will decrease over time at a rate defined by the geological discounting rate. These penalties may be differentially defined for excess or under production variations, allowing more control over the type of variation which should be penalized more severely. As discussed in previous sections, IP formulations can be impractical if a large number of integer variables are considered. In the proposed formulation, to reduce the number of integer variables, only blocks classified as ore are set as integer variables, a practice that generally maintains optimal solutions[12]. By considering a probability cut-off associated with a cut-off grade, the classification of blocks is done. Both cut-off values are defined by the planner and can be variable over time. To classify a block, the probability of the block grade being greater than the grade cut-off is first computed. This probability is then compared with the probability cut-off; if greater, the block is classified as ore.

Benndorf[19] presents a SIP formulation similar to the one presented by Ramazan and Dimitrakopoulos[7,8] but that is extended to multi-element type deposits. The formulation is tested at a multi-element iron deposit in Australia. In the proposed framework, a multiple variable joint-simulation algorithm[37] is first utilized to produce simulations that respect and reproduce the spatial correlation between the variables under consideration. The simulations are then utilized as input for the multi-element SIP schedule formulation. The formulation aims for maximization of the overall economic value of the mine, but also for minimization of production targets in terms of tonnes and quality of the ore, respecting user-specified smooth mining constraints. The formulation also includes a geological risk discount rate, which is directly applied to the penalties for ore production deviation. The study shows how the proposed formulation can considerably decrease the costs associated with deviation from ore grade production targets, 150% lower than the results obtained by a conventional approach that does not include grade uncertainty in its formulation.

Jewbali[20] describes the application of a SIP formulation and the use of future data to update simulated models. The method can be divided into three components: first, a conditional Gaussian co-simulation is applied and the realizations utilized as the source of future grade control data available only at the time mining is conducted and not available at the time the production schedule is defined. Secondly, a conditional simulation based on successive residuals is developed and used to update existing representations of the orebody in order to include new information that may now be available, here represented by the future data. The third and final component is the application of the SIP formulation to the updated orebody models, which generates an optimized schedule that is consistent with the level of information available at the time mining takes place. The stochastic solution presents a higher PV, around 40%, than the handmade conventional schedule for a South African mine does.

Menabde et al[6] implements an SIP formulation for long-term production scheduling that maximizes NPV considering several possible simulated orebodies and simultaneously optimizing cut-off grades. The formulation defines a final schedule and a cut-off strategy that, combined, aim to maximize the NPV. Due to the larger number of integer variables, an aggregation procedure is done in order to reduce this number and therefore decrease the solution time.

Modeling Geological Uncertainty

As demonstrated in the previous section, all alternative solutions to the mine scheduling production problem involve the use of a set of stochastic or geostatistical simulations as the input for the geological uncertainty and local variability of orebodies. This section explains the principles of the simulation process and provides a more detailed description of common simulation methods available.

There are many simulation methods available. They can be classified into three major groups: simulation of discrete variables, continuous variables, or objects[21, 22]. The major focus of this discussion will be on simulation methods of continuous variables. The aim of simulation methods presented herein is to provide ways to generate equally probable representations of a given attribute respecting the following requirements:

1 - Simulations are conditional to the data.
2 – Simulations reproduce the frequency distribution of the data.
3 – Simulations respect the spatial correlation of the data.

To determine whether or not a simulation method is appropriate to mine-related problems under consideration, the method must be able to efficiently handle a large number of nodes to be simulated and produce conditional simulations that can map the uncertainty around the attributes of interest in order to provide the proper input for the stochastic schedules or to assess the risk associated with it. These restrictions to simulation methods are intrinsically related to their practical application on the commonly large grid of nodes to be simulated such as the ones found at Escondida in Chile, 2.5 billion, or Superpit in Australia, 500 million, (e.g.: large deposits modelled using millions/ billion of blocks), and with the need to honour and use sample points (drillhole samples) to condition the simulations.

In most simulation methods presented herein, the mining-related attribute of interest is considered as a stationary and ergotic random function (RF), eg. Kolmogorov[23]. Considering this framework, Matheron[24] introduced conditional simulation to geostatistics as a tool to map the uncertainty around a given spatially-distributed attribute. Simulations are obtained by drawing equally probable realizations of the RF model. A complete knowledge of the multi-point cumulative probability distribution function or the spatial law of the RF is not required, once the first two moments (expectation and variogram and covariance functions) of the RF are utilized to model the spatial variability. The characterization of the RF model is done considering the available information as a single realization. For that, the RF is assumed to be stationary and ergotic, meaning that its moments are invariant under translation. In general, this assumption is only extended to the first and second moments. However, non-stationary models and conditional simulation methods do exist[25], though they are rarely used due to their complexity.

Matrix decomposition or LU simulation is a method presented by Davis[26], based on the decomposition of the covariance matrix. The method is named LU decomposition since it is based on the Cholesky decomposition of the covariance matrix into a lower triangular matrix and an upper triangular matrix, and represented the first simulation method able to directly conditional simulations. By multiplying the lower triangular matrix by a vector of independent random numbers, a conditional simulation can be obtained and by construction, reproduces the original covariance matrix. The method is extremely efficient if a relatively small set of nodes has to be simulated. The method's efficiency comes from the fact that once the LU decomposition has been done, other simulations can be generated by performing a matrix multiplication and addition. Problems arise when large simulation grids (10,000 nodes or more) are considered; in this case, the covariance matrix decomposition generates computational problems related with the requirement to invert. Alabert[27] proposed the use of a local window to decrease the problem size. The simulation space is divided into overlapping windows and simulations are done inside each window, a fact that may generate artefacts when the conditioning inside each cell diverges considerably from other cells nearby. Down and Sarac[28] proposed using ring decomposition to replace the Cholesky decomposition. Neither alternative method eliminates issues related to the size of the grid to be simulated. Vargaz-Guzman and Dimitrakopoulos[29] introduce a simulation method based on column decomposition of the covariance matrix and capable of dealing with large grids of nodes. The method is based on the column-wise partitioning of the lower triangular matrix. Residual covariances of a column are calculated using the results of the previous one. By stating the problem in this way, the lower triangular matrix does not need to be inverted and the method can be applied to large simulation grids. It also has the unique capability of updating pre-existing simulations to include additional information, although to efficiently handle large grids at each iteration, memory has to be allocated to the previous residual covariances, which slows down the process. The implementation of this method is detailed by Jewbali[20] and shows that for computational reasons simulation is done using a combined row and column decomposition of the covariance matrix.

Sequential Gaussian simulation (SGS) as introduced by Isaaks[30], a variation of the Monte Carlo simulation[31], has become the most extensively used method for mining-related problems. The decomposition of a multivariate probability density function into the product of conditional distributions is the basis of the method[32,33,34]. SGS considers the decomposition of a multi-Gaussian RF into the product of its univariate Gaussian conditional distributions. Simulation is done by randomly visiting all nodes to be simulated, one at a time; the conditional mean and conditional variance are defined at each node, characterizing a conditional Gaussian distribution. From this conditional distribution a value is randomly sampled. This method is able to deal with large grid of nodes but can be time consuming, as a kriging system has to be solved for each node, a process that involves a matrix inversion operation.

Dimitrakopoulos and Luo[35] propose a generalization of sequential Gaussian simulation (GSGS). The method simulates a group of nodes at each iteration, instead of node-by-node as done by SGS. SGS can be viewed as a particular case of the LU method in which the solution is interactively implemented row-by-row. As a result, a local neighbourhood replaces the full simulation domain utilized by the LU method and each node is individually and sequentially simulated. It makes used of screen effect approximation and uses a local and common neighbourhood to define the conditional distribution that is randomly sampled to obtain the simulated node values. The method considerably reduces the time needed to produce a set of realizations, making it propitious to be used in an industrial type of problem. Benndorff and Dimitrakopoulos[36] discuss the major issues related with neighbourhood and group size and the accuracy of the simulations. It is shown that accuracy is possible but it is a function of group and neighbourhood size. The method is fast and applicable to large problems commonly found in the industrial environment but still requires the manipulation of large files as simulations are generated in point support and have to be post-processed to the appropriated selective mine unit (SMU).

Godoy[5,15] proposes a method to overcome the post-processing issue that can also speed up the simulation process. Direct block simulation (DBSIM) is a sequential Gaussian simulation method that generates simulations directly into block support. The method uses the same principle of GSGS: a group of nodes defining a block is simulated, the block value is computed by the average of the internal points, the points are discarded and only the simulated block value is added to the set of conditioning data. It assumes that the normal RF describing the attribute of interest in point and block support is joint Gaussian. (Appendix B provides a detailed description of DBSIM, as it is the selected simulation method utilized in both case studies of this thesis.)

The so-called "turning bands" (TB) method[24], a simulation method based on the simulation of independent, one-dimensional lines. The method first produces uniformly spaced lines in a plane, then a covariance function in one dimension is used to produce independent simulations in each line and the simulation in the plane is obtained by adding the values of the projections in the lines of the points to be simulated. The one-dimension covariance model is obtained by the convolution of the two-dimension covariance function. Journel[38] expanded on the method to produce three-dimensional simulations. The method maps the three-dimensional space using a maximum of 15 lines; more lines may be used if sets of 15 lines are randomly place in the space. The conditioning is done by kriging in a post-simulation stage. The method is certainly computationally efficient but once simulation is performed in lines discretizing the space, artefacts may be created. Mantouglou and Wilson[39] present a simulation method that utilizes the spectral domain approach to simulate lines in one dimension and then applies the TB algorithm to generate the simulation in two or three dimensions. Borgman et al.[40] present a spectral domain algorithm to directly simulated in two dimensions. It utilizes a discretization of the spectral density function to replace the discrete spectrum of the covariance matrix. Pardo-Iguzquiza and Chica-Olmo[41] fully expand the algorithm to three dimensions.

Spectral or frequency domain simulation methods take advantage of the computational efficiency of fast Fourier transforms (FFT) to simulate a stationary RF with an associated covariance function by simulating uncorrelated random spectral coefficients obtained by the FFT of the covariance function. To back-transform the simulations, from the spectral or frequency domain to data space, a reverse Fourier transform is applied, producing simulations with the desired variance and variogram function. The proposed method is able to account in one step for several covariance components, also allowing the presence of geometric or zonal anisotropy. First, the covariance function model is sampled at a regular grid. The discretization of the covariance function is then utilized to compute the spectral density function at all grid locations using discrete Fourier transform (DFT). By taking the square root of the density function, the spectral amplitude is obtained and if combined with a discrete phase spectrum randomly sampled in the interval 0 to 2π, defines the Fourier coefficients. An inverse Fourier transform is then applied to the coefficients to finally obtain simulation in the data space reproducing the covariance model. In general, methods based in the FFT of the covariance function do not fit the requirements of simulated mining-related problems, as a regular simulation grid is required. This is not a common situation encountered in mining. Another drawback is the requirement for post-conditioning the simulations, which increases the processing time.

Sequential indicator simulation developed by Alabert[42] is the most frequently used simulation model that requires a strictly stationary RF model and approximations of the ccdf. The method uses the same sequential approach as previously described for the Gaussian simulation methods; the difference lies in the fact that at each node an indicator kriging system is solved considering a set of thresholds. The solutions of this set of indicator systems are used to define a local probability distribution function, from which a simulated value is randomly sampled.

Multi-point simulation methods were developed essentially to deal with problems in which reproduction of two-point statistics (covariance, variogram) by the simulations is not the major concern. They aim to produce simulations that reproduce complex geological units or structures. To do so, modeling of multi-point statistics is required, therefore it incorporates much more information than the standard two-point statistics used so far by the methods previously described. Guardiano and Srivastava[43] were the first to introduce the idea of using training images to map the joint probability distribution function of multi-point geological patterns. The proposed algorithm uses the map of probability as conditional information during the simulation process. The algorithm requires the scan of the full training image at each node, therefore implying a large computational cost. Strebelle[44] proposes a solution to avoid the high computational cost, which utilizes the same principle of using training images to map probabilities of a given multi-point geological pattern, but makes use of a search tree in order to scan the training images only once. Using the search tree, the conditional probabilities for all existing patterns can be read. A conditional distribution at each location can then be defined and randomly sampled to generate the node's simulated value. This procedure considerably decreases the processing time required. The method is named single normal equation simulation (SNESIM), as only one normal variable is considered. Zhang, Switzer and Journel[45] expand on the SNESIM by filtering spatial patterns in order to avoid cutting off information as is done by SNESIM if a given pattern is not present in the training image but is present in a simulation. The algorithm defines smaller templates in a first stage, to compute reduced pattern probabilities. These probabilities are utilized if a given pattern is not present in the training image but it is obtained in a given simulation. The algorithm sets the probability of an unmapped pattern equal to one of the reduced pattern probabilities. The selection is done by considering the reduced pattern most similar to the pattern being simulated.

There are several simulation methods able to jointly simulate two or more variables; as they are not an integral part of the work presented in this thesis, no detailed description will be provided.

CHAPTER 3 Stochastic optimization model for open pit mine planning: application and risk analysis at copper deposit

3.1 Abstract

Life-of-mine (LOM) production scheduling is a critically important part of open pit mining ventures and deals with the efficient management of cash flows in the order of hundreds of millions of dollars. A LOM production schedule determines the quantity and quality of ore and waste materials to be mined over time, so as to maximize the net present value of the mine. LOM production scheduling is an intricate and complex problem to address and it is adversely affected by geological risk, which can, however, be accounted for and managed while constructing production schedules. In this study, the LOM scheduling process of a disseminated copper deposit demonstrates the intricacies of a new scheduling approach based on the technique of simulated annealing and stochastically simulated representations of the copper orebody. The study documents the benefits of incorporating geological uncertainty in the mine scheduling process through the proposed approach. The stochastic approach is found to generate a LOM schedule with a NPV 26% higher than that of the conventional schedule. Risk analysis results show that the stochastic schedule has low chances to significantly deviate from targets; the probability that the conventional schedule will deviate from production targets is high. In addition, comparisons suggest that in this specific study the conventional scheduling approach overestimates ore tonnages and underestimates the NPV of the mine design. The above suggests that LOM schedules that incorporate geological uncertainty may lead to more informed investment decisions and improved mining practices.

3.2 Introduction

A life-of-mine (LOM) production schedule aims to optimize the sequence of extraction and quantity of ore and waste mined out in each mining period throughout the life of the mine, so as to maximize its net present value (NPV). Generating such a schedule depends, among other factors, on the grade and tonnage of the ore deposit being considered. Conventional optimization techniques are typically used to generate production schedules under pre-determined technical, economic and environmental constraints using mathematical optimization algorithms. These techniques assume that the grade distribution within the mineral deposit under study is exactly as described. Orebodies, however, are only partially known through exploration drilling programs and, therefore, it is not possible to precisely define the quantity and quality of the materials available in each location within an orebody. As a result, in the presence of uncertainty, it is unlikely that conventionally constructed mine designs and production schedules are optimal and have, in fact, been shown to be misleading in some cases. For example, Dimitrakopoulos et al.[1] show the limits of conventional optimization techniques in dealing with uncertainty through the presentation of conventionally generated results in key performance indicators of a project that are shown to be misleading in the presence of geological uncertainty and grade variability. Similar concepts and risk analysis using stochastic simulation techniques have been discussed in the past[2,3] in the context of assessing the impact of uncertainty and in-situ grade variability in conventional open pit designs, production schedules and related economic evaluations.

Stochastic simulation techniques available for modelling uncertainty in orebody attributes quantify geological uncertainty by generating equally probable scenarios of the orebody under consideration and assist in enhancing mine planning. The availability of these techniques lead to the development of new scheduling techniques integrating uncertainty into the mine planning process. For example, Dimitrakopoulos and Ramazan[4,5] develop an optimization formulation introducing the new concept of production scheduling with geological risk discounting. Dimitrakopoulos et al.[6] propose an approach in which an open pit mine design is selected from a set of possible designs, using the concept of maximum upside potential and minimum downside risk for all possible designs and their key project performance indicators. Grieco and Dimitrakopoulos[8,7] propose a mixed integer programming approach for stope design in underground operations that determines optimum location, size and number of stopes based on the concept of acceptable level of risk in a design. Ramazan and Dimitrakopoulos[9,10] develop a stochastic integer programming model that uses multiple simulated orebodies to minimize deviations of ore production from LOM schedules, and show substantial monetary benefits from stochastic scheduling. A stochastic integer programming approach is also shown in Menabde et al. [11]. Godoy[12] and Godoy and Dimitrakopoulos[13] develop a new approach for mine production and scheduling optimization under uncertainty. The method integrates several new elements; these are the stable solution domain which is a characterization of all feasible combinations of ore and waste extraction rates possible from a given pit, the optimization of production rates over the life of mine for a given mine setup and mining equipment available, and a simulated annealing algorithm for scheduling optimization given multiple simulated orebody representations, given optimal production rates. The latter algorithm generates schedules that meet the optimal production rates, and minimize potential production deviations in the presence of grade uncertainty. In the same work, the associated case study shows an increase of 28% in the NPV of the mine compared to the conventional LOM schedule accompanied by substantially lower potential deviations from production.

In this study, an approach based on simulated annealing, and variant of the stochastic scheduling approach presented in Godoy[12], is first presented and then tested in a relatively low grade variability copper deposit. The objectives are several; (a) to test the simulated annealing based scheduling approach; (b) to assess the significance of incorporating geological uncertainty when scheduling a deposit with relatively low variability; (c) to ascertain if the previously reported increase in NPV when using stochastic approaches is the same in different case studies; (d) to assess the previously reported reduction in risk to deviate from production targets; and (e) to analyze the results and suggest future work. In the following sections, the stages of a stochastic scheduling approach based on simulated annealing, are stated first. Then, the case study at a copper deposit follows and results are presented. Lastly, the mine's LOM schedule from the stochastic approach is compared to a conventionally developed LOM schedule for the same copper deposit, and conclusions follow.

3.3 A stochastic production scheduling approach

The mine scheduling approach presented in this section is a multi-stage framework generating a final schedule, which considers geological uncertainty so as to minimize the risk of deviations from production targets. A basic input to this framework is a set of equally probable scenarios of the orebody, generated by the technique of conditional simulation. The stages of the approach include the (a) definition, through a conventional optimization approach, of the ultimate pit limits and mining rates to be used in subsequent stages; (b) development of a set of schedules within the predetermined pit limits that meet the ore and waste production targets defined in the previous stage; this set of schedules is developed using a conventional scheduler and simulated orebodies one at a time; and (c) generation of a single production schedule that minimizes the risk of deviation from production targets using a simulated annealing formulation. Figure 3.1 shows the stages of the stochastic production schedule framework used in this study. Each of the three stages is explained in the following subsections.

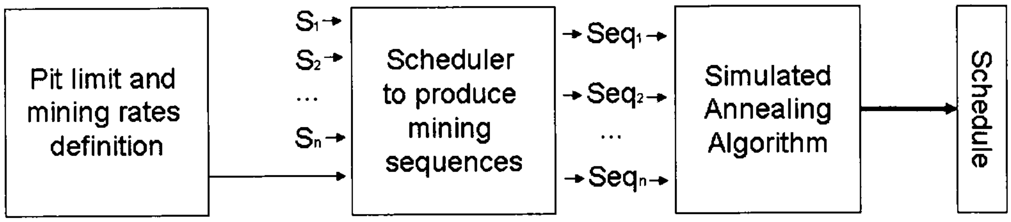

Figure 3-1- The three stages of the mine production scheduling process in this study

3.3.1. Stage 1 - Final pit and mining rates selection

In this first stage, a conventional approach is used to define the ultimate pit limits and mining rates. Without loss of generality, the approach applied in this study is based on the nested-pit implementation of the Lerchs-Grossman algorithm[14]. This procedure determines the ultimate pit limits and allows for the development of a practical sequence of extraction using a conventionally modeled obebody. Mining rates are either defined by a commonly used interactive procedure within the above framework based on the so-called Milawa scheduler[15], or are pre-selected for mine operational reasons related to mill demand and geometric constraints. Any approach to defining mining rates can be accommodated in this stage. The ultimate pit limits and mining rates are used in the subsequent stages of the scheduling approach followed in the present study.

3.3.2. Stage 2 - LOM schedules using simulated models

This second stage aims to produce a series of physical schedules describing the evolution of the working zones in the pit over the life-of-mine. Any formulation performing mining sequencing can be used for this task, provided that engineering requirements are met. These include sequencing that obeys slope constrains and satisfies mill requirements, while matching the mining rates previously derived in Stage 1. The process consists of producing multiple mining sequences, one for each simulated grade model of the orebody. These multiple alternative mining sequences are based on distinct but equally probable models of the spatial distribution of grades within the deposit. It is important to note that selecting and using one or few "representative" simulated realizations of the orebody for scheduling proposes is an erroneous practice, leading to misleading schedules.

The mining sequences generated are used next to compute the probability that a mining block belongs to a given period of the LOM schedule. The map of such probabilities defines a cumulative distribution function (cdf) for each block and this cdf forms the basic input for simulated annealing in Stage 3, where the final LOM production schedule is developed. The simulated realizations of the orebody required for Step 2 can be generated from efficient conditional simulation techniques, such as the direct block simulation used herein [16] and briefly explained next.

Direct block simulation (DBS) is a conditional simulation technique that generates realizations directly on the required block support, as detailed in Godoy[12]. The major advantages of DBS relate to substantial savings in processing time and data storage requirements. These are important issues because mining problems involve simulation of tens to hundreds of millions of nodes that need to be grouped into mining blocks. The steps of implementing the algorithm are summarized as follows:

1. Normalize the available data.
2. Select a random path to visit each block to be simulated.
3. Simulate internal nodes discretising a block in the Gaussian space using the LU simulation method, if no previously simulated blocks are involved; or otherwise using the joint LU of data points and previously simulated blocks.
4. Compute the simulated block value by averaging simulated internal nodes in both the Gaussian space and data space.

5. Discard values of internal nodes and add the average value in the Gaussian space to the conditioning dataset and the block value in the data space to the output.
6. Repeat steps 3 to 5 until all blocks are simulated.
7. Repeat steps 2 to 6 to generate additional realizations.
8. Validate the simulations generated.

Note that LU in Step 3 above stands for the simulation method based on the lower – upper decomposition of the covariance matrix.

3.3.3. Stage 3 - Simulated annealing and final schedule

Simulated annealing is a heuristic optimization approach based on the so-called stochastic relaxation[17,18]. The general principle of simulated annealing is to perturb an initial stage, for example, an initial mine sequence as in the present case, while respecting possible constraints. Perturbations are performed in order to improve an objective function which can be of any type, linear or non-linear, and include several components. For each perturbation, the relative change in the objective function is evaluated. Perturbations leading to an improvement in the objective function are readily accepted. Different rules may be defined to accept or reject unfavorable perturbations. Such a rule is the frequently used acceptance probability distribution given by

$$\text{Prob}\{\text{Accept pertubation}\} = \begin{cases} 1, & \text{if } O_{new} \leq O_{old} \\ e^{\frac{O_{old}-O_{new}}{t_i}}, & \text{otherwise} \end{cases} \qquad (3.1)$$

where O_{new} is the value of an objective function after a perturbation, O_{old} is the value of the same objective function before the perturbation, and t is the so-called annealing temperature. The idea is to start with a higher temperature, that is, with a higher probability of accepting unfavorable perturbations and gradually decrease this temperature, consequentially decreasing the chances of an unfavorable perturbation being accepted. This reduction is obtained by multiplying the temperature by a "cooling" factor. The magnitude of this factor determines how fast the probability to accept an unfavorable perturbation decreases. The perturbations mechanism continues until stopping criteria are met. Possible stopping criteria may be the maximum number of perturbations accepted without changing the objective function value, maximum number of perturbations, or reaching an upper/lower limit of the objective function value.

The simulation annealing technique used in the present study combines several mine schedules to obtain one which minimizes the risk to deviate from pre-established ore and waste production targets. The algorithm minimizes an objective function that is defined in this study as the sum of deviations from production targets for *N* mining periods

$$\text{Min } O = \sum_{n=1}^{N} \left(\sum_{s=1}^{S} \left| \theta_n^*(s) - \theta_n(s) \right| + \sum_{s=1}^{S} \left| \omega_n^*(s) - \omega_n(s) \right| \right) \tag{3.2}$$

where N is the number of mining periods, S is the number of simulated models, and n = 1, ..., N; s = 1, ...S; $\theta_n^*(s)$ and $\omega_n^*(s)$ are, respectively, ore and waste quantities of the perturbed mining sequence in simulation $\{s=1,...,S\}$ for period $\{n=1,...,N\}$; $\theta_n(s)$ and $\omega_n(s)$ are, respectively, ore and waste targets for the mining sequence in simulation $\{s=1,...,S\}$ for period $\{n=1,...,N\}$. The objective function in Eq. 3.2 measures the average deviations from ore and waste targets considering a perturbed state over all available *S* representations (simulations) of the deposit. The proposed algorithm perturbs a given state by swapping a block between possible candidate periods. Candidate periods are defined by the cdf computed in Stage 2 described above. The algorithm allows swapping either all blocks or a set of blocks defined by applying a probability threshold. A block is included in this set, if its probability to belong to any given period is smaller than the proposed threshold. Perturbed states are generated until one of the stopping criteria is met. The stopping criteria implemented in this study are the maximum number of attempted swaps, the number of acceptable swaps, the number of times the annealing temperature is reduced, the number of attempted swaps without a significant change in the value of the objective function, and if a specified lower bound on the objective function value is reached.

As noted above, the way in which a new perturbed state is obtained and the criteria that determine its acceptability are important characteristics of Eq. 3.2. Perturbations are done in such a way so as to respect slope constraints and a feasible sequence of extraction. This is achieved through the use of a connectivity test. A block is said to have connectivity, if at least one of the four surrounding blocks at the same level is scheduled in the same candidate period, the block just above it is scheduled in a previous or in the same period, and the block just below it is scheduled after or in the same period. If a block has connectivity it can be swapped to the candidate period. The objective function is then tested and the swap is accepted if the value of the objective function is improved. Otherwise, the swap is accepted or rejected by a negative exponential probability distribution such as that presented in Eq. 3.1.

The steps of implementing the simulated annealing algorithm may be summarized as follows:

1. Define all blocks that may be swapped.
2. Define the possible candidate periods with associated probabilities.
3. Loop through the steps below until a stopping criterion is met:
    - randomly draw a block to be swapped;
    - verify if the block has connectivity;

- if the block has connectivity, swap the block to the candidate period;
- update the objective function (Eq. 3.2);
- accept the swap if the objective function has improved, if not accept/reject using a negative exponential probability function (Eq. 3.1).

3.4 Case study: Risk based schedule at a low-grade disseminated copper deposit

The deposit is located in a typical archean greenstone belt. The region consists predominantly of mafic lavas with lesser amounts of intermediate to felsics volcaniclastics. Rocks are moderately deformed with a prominent cleavage sub-parallel to what is considered to be the original bedding, an E-W trend with average 64° South. The deposit itself is in a sequence of moderately to strongly foliated, sulphidic, mafic to intermediate volcanic rocks, which have been intruded by numerous sub-volcanic felsite and feldspar porphyry and/or intermediate volcanic tuff, with size ranging from lapilli to agglomerate, within a strongly chloritic and biotitic matrix. It can be traced over a strike length of 1.5km with a thickness varying from a few meters to more than 75m. Mineralization consists of about 10% sulphides, mostly chalcopiryte, pyrite and pyrrhotite, occurring as disseminations, streaks and stringers apparently controlled by the strong rock cleavage.

The geological database is compounded by 185 drillholes with 10m copper composites in a pseudo-regular grid of 50m x 50m covering an approximately rectangular area of 1600x900 m^2; the average dip is 60° North. Figure 3.2 shows the histogram and statistics for Cu% of 10m composites. Using the geological information, one mineralization domain is defined and modeled through a geostatistical study.

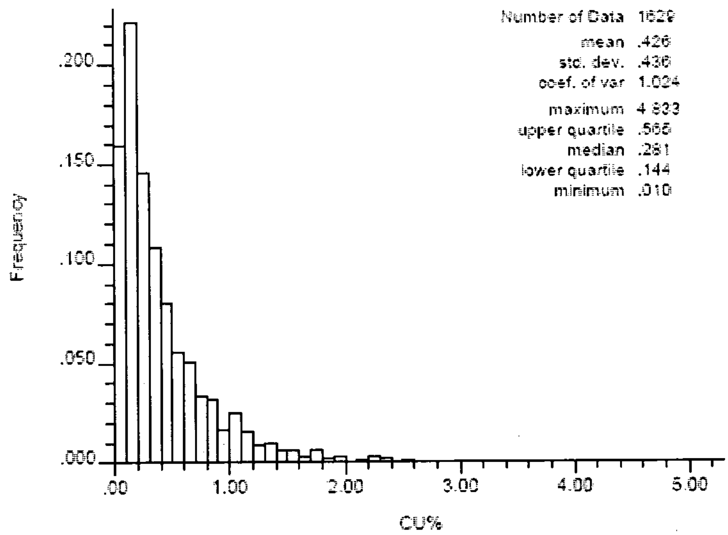

Figure 3-2 - Cu (%) histogram of 10m composites

3.4.1. Developing the mine's stochastic production schedule

Stage 1: As noted earlier, the nested Lerchs-Grossman algorithm for pit optimization[15] is used here to define a final pit. Mining, processing and selling costs, metal price, slope angles and processing recovery parameters are given in Table 3.1. In addition, this algorithm requires a single representation of the orebody as an input. In this respect, a conventionally estimated orebody model is created using ordinary kriging and $20x20x10m^3$ blocks. This model is utilized in this and subsequent sections for comparisons.

Table 3-1 - Economic and technical parameters

Copper price (US$/lb)	2.0
Selling cost (US$/lb)	0.3
Mining cost ($/tonne)	1.0
Processing cost ($/tonne)	9.0
Slope angle	45^o
Processing recovery	0.9

Using the above parameters and the conventionally estimated orebody model, a set of nested pits is generated. Pit 16 is selected as the ultimate pit limit because it corresponds to the maximum net present value. There are 14,480 ore and waste blocks inside the pit limits. After the ultimate pit definition, an interactive process is followed to define the mining rates through the life-of-mine, based on scheduling the mine with the Millawa algorithm[16] and testing different combinations of feasible mining rates given a predetermined mill demand. The mining rate is defined to be 7.5 million tonnes of ore per year with a constant striping ratio of 2.7 over the LOM. Within the limits of the conventionally constructed schedule in Stage 1, this rate appears to ensure a constant mill feed over the seven out of an expected 8 years LOM with no significant variations in the striping ratio. Note that the specifics here refer to a conventionally conduced pit optimization study.

Stage 2: This second stage produces a series of multiple mining sequences within the ultimate pit limits and with the mining rates from Stage 1. All LOM schedules are produced using the same economic parameters presented in Table 3.1, a 0.3 % Cu cut-off grade and the Milawa algorithm. Each mining sequence is generated from scheduling an equally probable realization of the copper deposit. Twenty realizations of the deposit are available and generated with the direct block simulation algorithm previously discussed. To obtain a simulated model, the orebody is divided into blocks of 20x20x10m3 within the mineralized domain defined previously. Each block is then represented by 10x10x1 nodes. This number of nodes, 100 per block, is large enough to ensure that the actual block scale variability is reproduced by the simulated orebodies.

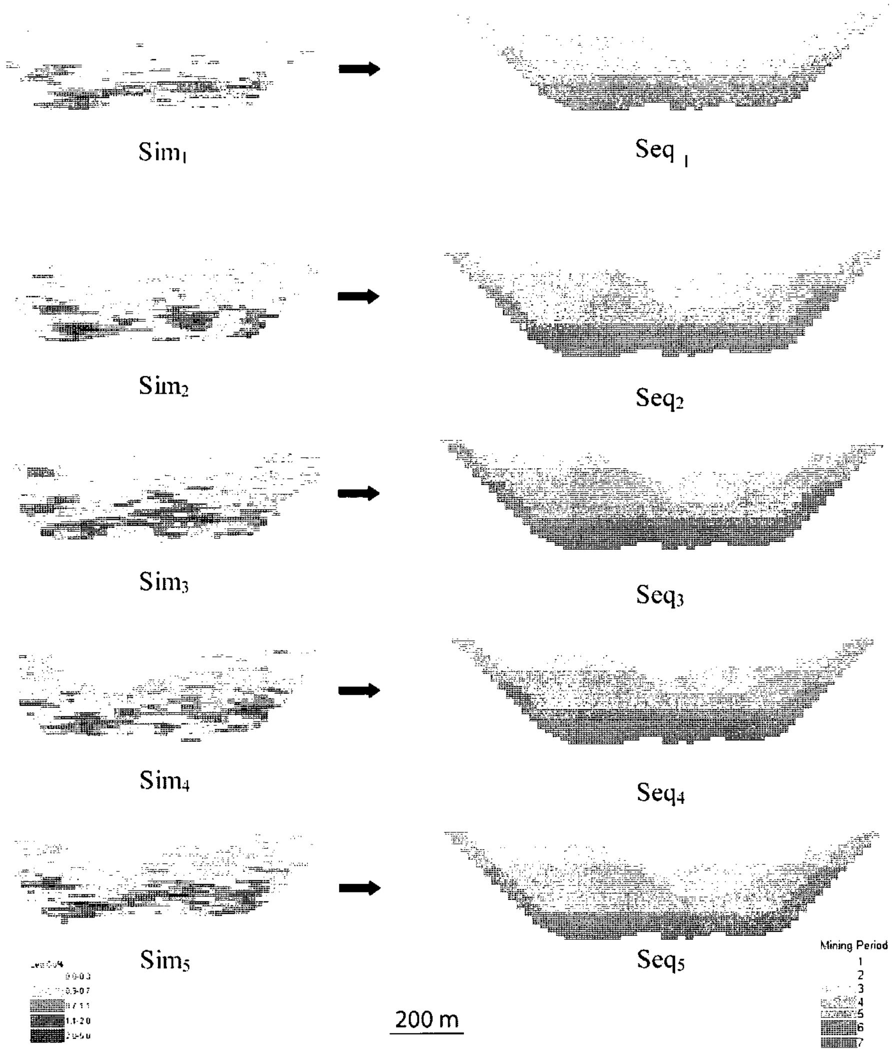

Figure 3-3 - Examples of five simulations scenarios of the copper deposit and their corresponding schedules in an East-West section

Figure 3.3 shows 5 conditionally simulated models of the copper deposit and their respective production schedules. Note that there are seven production years in all the 20 production schedules generated and the discussion of this topic is deferred for a subsequent section.

Stage 3: In Stage 3, the mining rates and the 20 mine sequences obtained in Stages 1 and 2 respectively are inputted into the simulated annealing algorithm. As defined in Stage 1, there are 14,480 ore and waste blocks inside the pit limits scheduled to 7 possible production periods. As described earlier, the algorithm works by swapping blocks among the possible periods and updates the objective function described by Eq. 3.2 after each swap. This process continues until one of the stopping criteria, described in previous section, is met. In this study, an initial annealing temperature of 10^{-5} is used, with an associated cooling factor of 0.1. The algorithm stops, in this case, after 1,581 accepted swaps since the criterion of the maximum number of perturbations without a change is met (10^7 swaps). A risk-based schedule corresponding to a 7-year mine life is finally obtained. An East-West section of the physical schedule is represented in Figure 3.4.

Risk analysis for the produced stochastic schedule is carried out using the 20 simulated orebody models. The maximum, minimum and expected (average) amounts of ore, waste, cumulative metal and cumulative net present value for each period are computed and presented in Figures 3.5, 3.6, 3.7 and 3.8, respectively. As shown in Figures 3.5 and 3.6, the differences between the expected and targeted ore and waste productions are not significant. This reflects the fact that the risk-based production scheduling approach minimizes the chances to deviate from ore and waste production targets. Figures 3.7 and 3.8 also demonstrate low potential for deviations from metal production and consistently low variations in cash flow expectations over the LOM. The evolutions of the objective functions for ore and waste components with the number of attempted swaps are shown in Figure 3.9. It is obvious that both components stabilize at approximately 2000 attempted swaps and no further significant improvements in the objective functions are achievable from additional swaps.

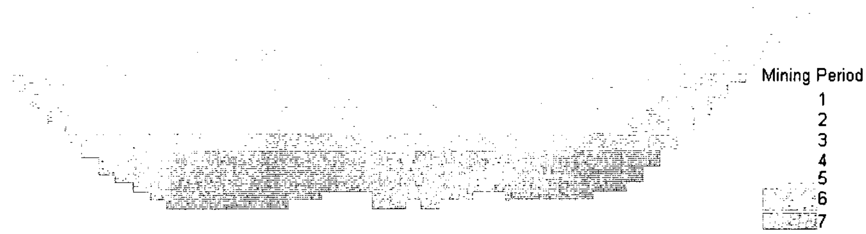

Figure 3-4 - An East-West section of the risk based mine production schedule generated with the simulated annealing algorithm

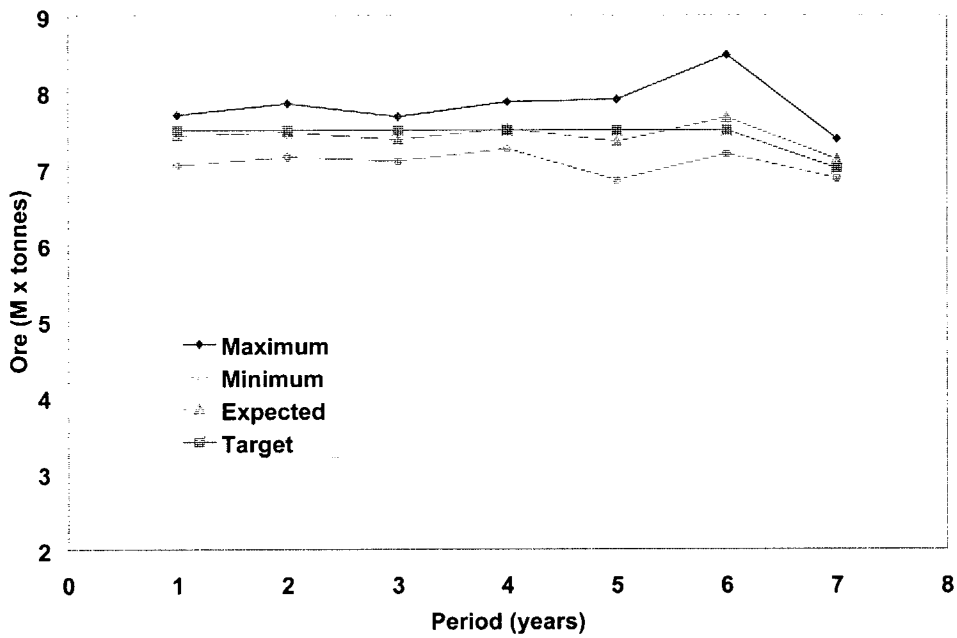

Figure 3-5 - Risk based LOM production schedule (ore risk profile)

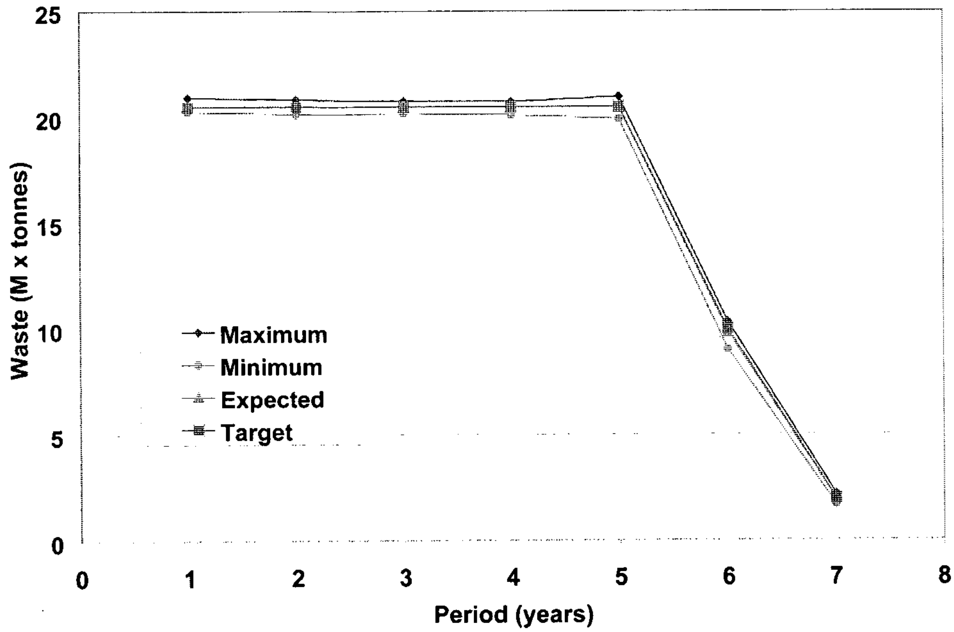

Figure 3-6 - Risk based LOM production schedule (waste risk profile)

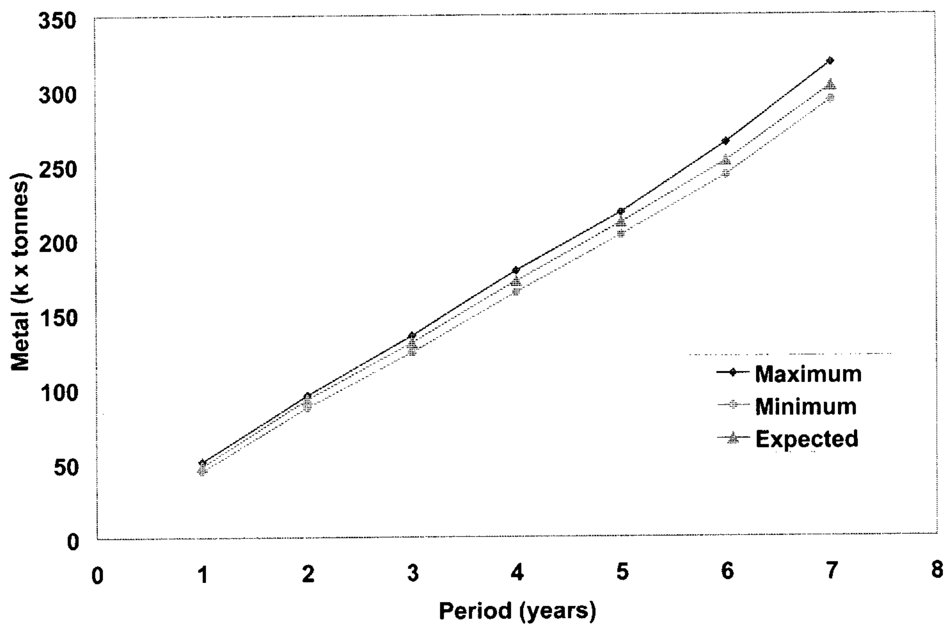

Figure 3-7 - Risk based LOM production schedule (metal risk profile)

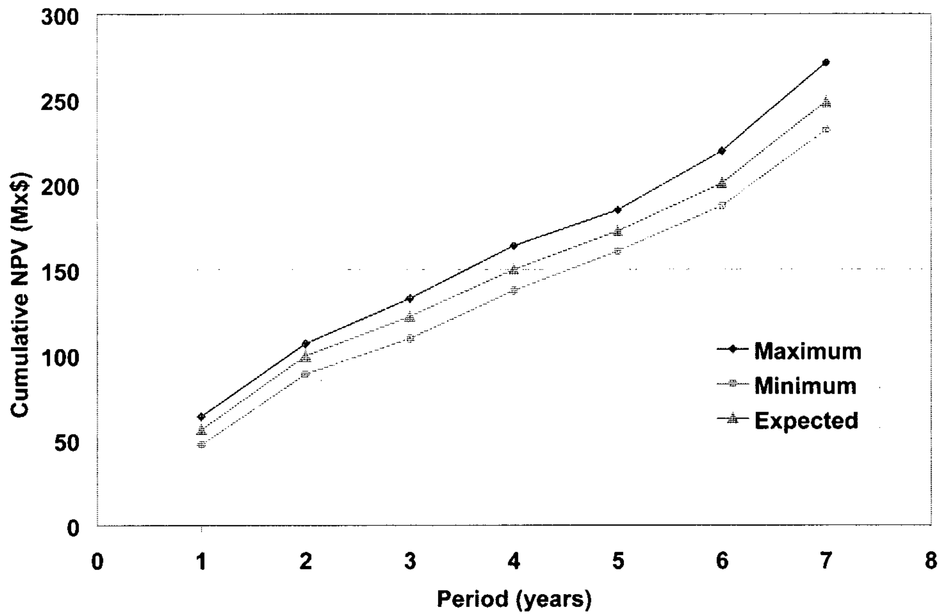

Figure 3-8 - Risk based LOM production schedule (cumulative NPV risk profile)

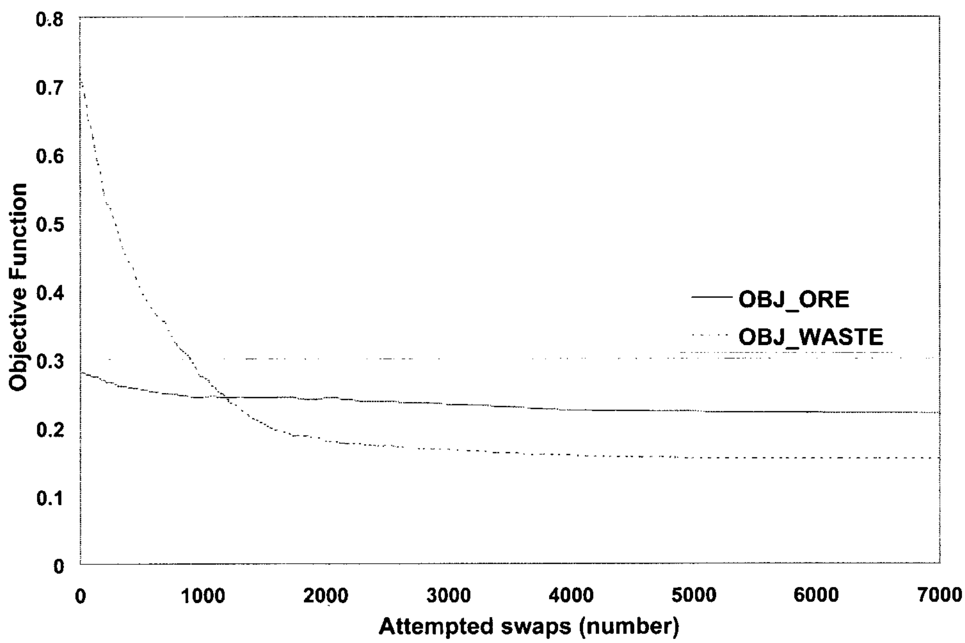

Figure 3-9 - Percentage change in the objective function components

3.4.2. Comparison with a conventional schedule

To compare the results of the stochastic LOM production schedule with those of a conventional approach, the conventional schedule produced in Stage 1 is used here. Recall that this conventional schedule is based on the same final pit, mining rates, technical and economic parameters, cut-off grade of 0.3% Cu, and uses the conventionally estimated model of the deposit. As shown in Figure 3.10, the conventional schedule forecasts an eight-year long mine life. In addition, no significant shortage or surplus of ore production, considering a target of 7.5 million tonnes of ore per year, is expected. This mine life is greater than that of the stochastic scheduling approach, which is forecasted at 7 years. The two schedules have different requirements for production capacity and associated equipment fleet. To assist with the upcoming economic comparison discussed in this section, please note that the comparison considers time costs and assumes some flexibility in the variation of production capacities and equipment fleet during the last 2 periods of production. In the last 2 periods, a decreasing stripping ratio is allowed and requires adjustment of the equipment fleet. Note that the schedules could be compared in the same timeframe, by assuming that the material mined in the last year of production of the conventional schedule is mined and stockpiled in the 7th production period. The requirement for stockpiling would lead to an even lower NPV for the conventional approach once costs for stockpiling the material are included in the economic evaluation.

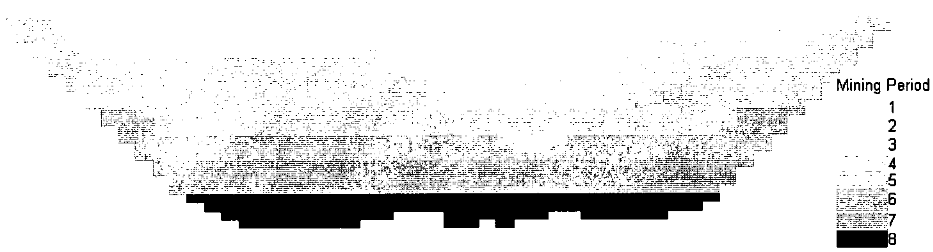

Figure 3-10 - East-west section of the conventional schedule

Risk analysis for the quantity of ore produced from this conventional schedule is carried out using the simulated orebody models. Figure 3.11 presents the ore tonnages reported by the conventional schedule as well as the maximum, minimum and expected (average) amounts obtained by evaluating the sequence using 20 simulated orebodies; it shows that the results obtained by the conventional approach are misleading. Considering that the simulations are possible representations of the actual deposit, the conventional schedule has a high probability of not meeting the ore production targets over the life of the mine. The conventional approach overestimates the ore tonnages since its estimates are higher than the expected tonnages throughout the mine's lifetime (Figure 3.11). A main contributor to this overestimation is the smoothing of the grade distribution produced by the conventional estimation techniques. It is important to emphasize that this result is specific to this case study and cannot be generalized. Conventional optimization approaches may overestimate or underestimate ore tonnages depending on the selected cut-off grade and the local grade variability in the deposit. As shown in Figure 3.12, for low cut-off grades, the conventional approach overestimates ore tonnages while for relatively high cut-offs it underestimates them.

In summary, there are significant differences between the stochastic schedule and the conventional one. First, as the conventional approach overestimates ore tonnages at the selected 0.3 % Cu cut-off, the mine life obtained by the conventional approach is longer than that suggested by the stochastic approach. Second, there are differences between the extraction sequences of the two schedules. Third, comparing Figures 3.5 and 3.11, it is clear that ore tonnages of the stochastic schedule are not significantly different from the targeted ones, while those of the conventional schedule are significantly different from the expected tonnages over the mine lifetime. The main reason for the differences between the stochastic and the conventional schedules is the dissimilar ways geological uncertainty is managed in each schedule. While the conventional scheduling approach ignores geological uncertainty, the stochastic approach integrates it into the scheduling process and manages this risk so as to minimize the risk of deviation from production targets. This management is essentially a 'blending' over a mining period of materials with more certain with less certain grade.

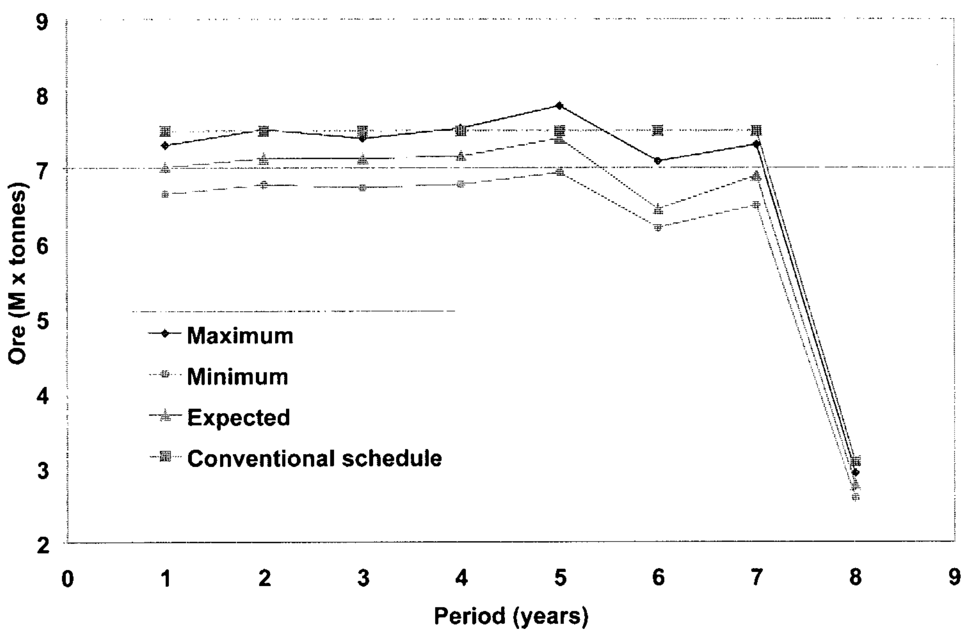

Figure 3-11 - Conventional LOM production schedule (risk profile for ore tonnes)

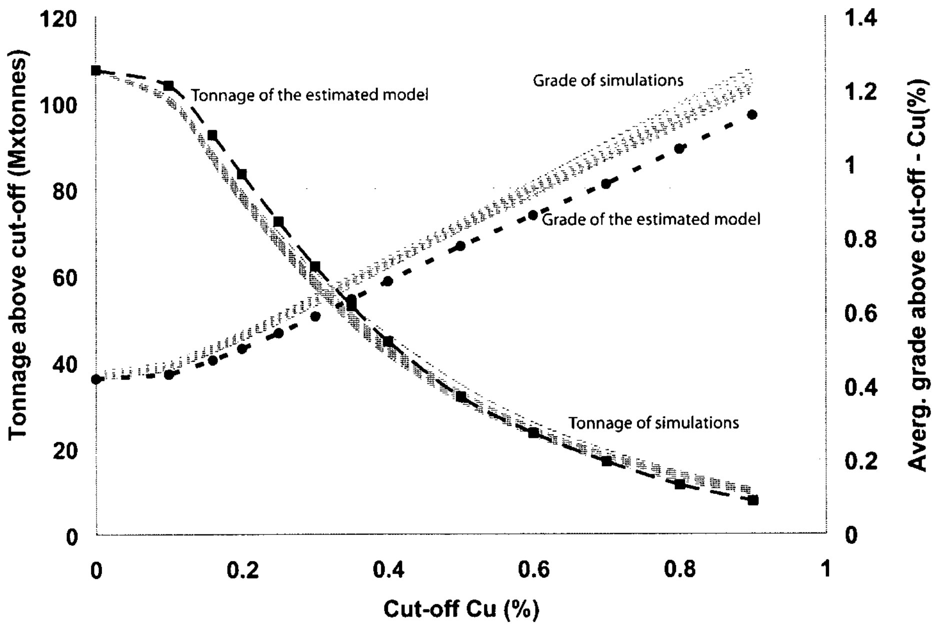

Figure 3-12 - Grade-tonnage curves for the estimated and simulated models

The economic implications of the differences between the stochastic and the conventional schedules are substantial, as shown in Figure 3.13. In the figure, the risk profile of cumulative discount cash-flows for the stochastic schedule is compared with the results obtained by the conventionally generated schedule. In addition, the risk profile for the conventional schedule is shown. A 26.2% higher NPV is obtained by the stochastic schedule when compared to the results obtained by the conventional approach. If the average NPV of the stochastic schedule is compared with the average values from the risk profile of the conventional schedule, a 15% higher NPV is obtained by the stochastic schedule. Note that the conventional scheduling practice would not be able to provide the information about the performance of such schedule. These substantial differences are mainly due to the ability of the stochastic scheduler to better assess the chances of a block to be ore or waste, and to schedule each block accordingly so as to minimize the chances of deviating from target production for ore and waste over a mining period.

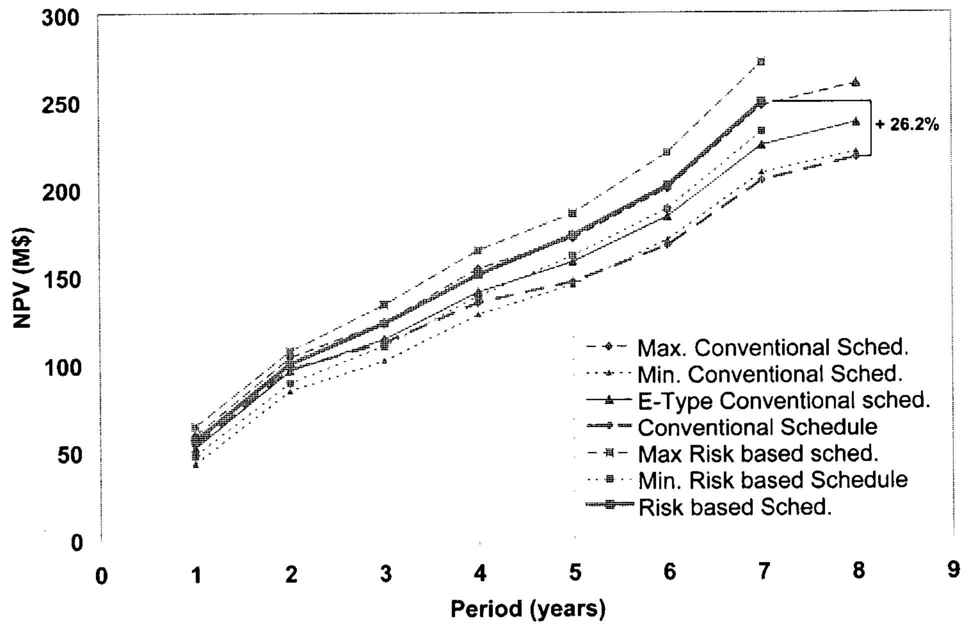

Figure 3-13 - NPV of the conventional and stochastic (risk based) schedules and corresponding risk profiles

The results of the study presented here are not sensitive to additional simulated representations of the orebody. This is documented also in the Appendix A and also expected. The reason is that the 'spaces of uncertainty' being mapped, from NPV, to metal content, waste and so on for designs discussed above are not highly variable. This also suggests that it is possible to generate comparable results to this study by using less than twenty simulated representations of the orebody. Similarly, this lack of sensitivity suggests that the use of additional simulated representations of the orebody to the 20 used herein, would not add any additional relevant information for the problem at hand.

3.5 Conclusions

This study explores the practical intricacies and performance of a stochastic scheduling approach based on simulated annealing, in an application at a copper deposit with relatively low grade variability. Despite the relatively low grade variability of the deposit, the results of the study show that there are significant differences between the stochastic and the conventional schedules. Firstly, the NPV of the stochastic schedule is found to be 26% higher than that of the conventional schedule; this is comparable to the 28% difference reported in Godoy and Dimitrakopoulos[13] for a large gold deposit. Secondly, risk analysis shows that the stochastic schedule has low chances to significantly deviate from targets; the probability that the conventional schedule will deviate from production targets is high. This is also similar to past studies mentioned above. Finally, the mine life predicted by the stochastic approach is 14% shorter than that of the conventional approach.

The results of this study suggest that the conventional approach overestimates ore tonnages and underestimates the NPV. This conclusion cannot be generalized because the conventional approach to scheduling may overestimate or underestimate ore tonnage and NPV depending on the selected cut-off grade and local grade variability within the deposit. The important point to be considered is that regardless of whether it overestimates or underestimates, a conventional approach may produce ore tonnages and NPV that are unrealistic and have low chances to be realized. This may result in misleading investment decisions where a good project is rejected or a marginal project is accepted. The results herein are not sensitive to additional stochastically simulated representations of the orebody. Future work should address the dynamics of cut-off grade, ultimate pit limit and pushback design optimization under uncertainty.

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Appendix A – Sensitivity analysis of the results

The results presented herein are found to be insensitive to the use of additional simulated representations of the orebody, and this is shown in the following test. A new set of twenty different simulations is used to evaluate the schedule generated with the stochastic scheduling approach presented in this paper. This risk profiles for NPV, ore tonnages and waste production are respectively shown in Figures 3.14, 3.15 and 3.16 respectively. The figures clearly show that there is no impact in the results when different realizations are considered. This is not surprising and, as also discussed in the main part of the paper, is the due to the fact that the 'space of uncertainty' being mapped in Figures 3.14, 3.15 and 3.16 are not highly variable. While using less than twenty realizations of the copper orebody may provide the same results, the use of additional simulations would not in this case add any useful additional relevant information for the problem at hand.

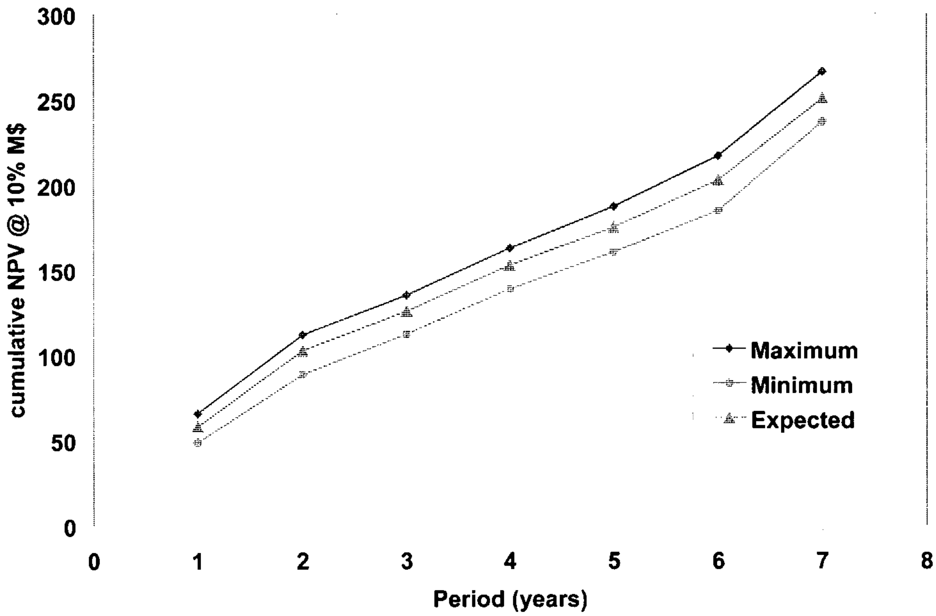

Figure 3-14 - Risk based LOM production schedule (NPV and risk profile) using a different set of simulations than those in the case study presented in the main body of this text

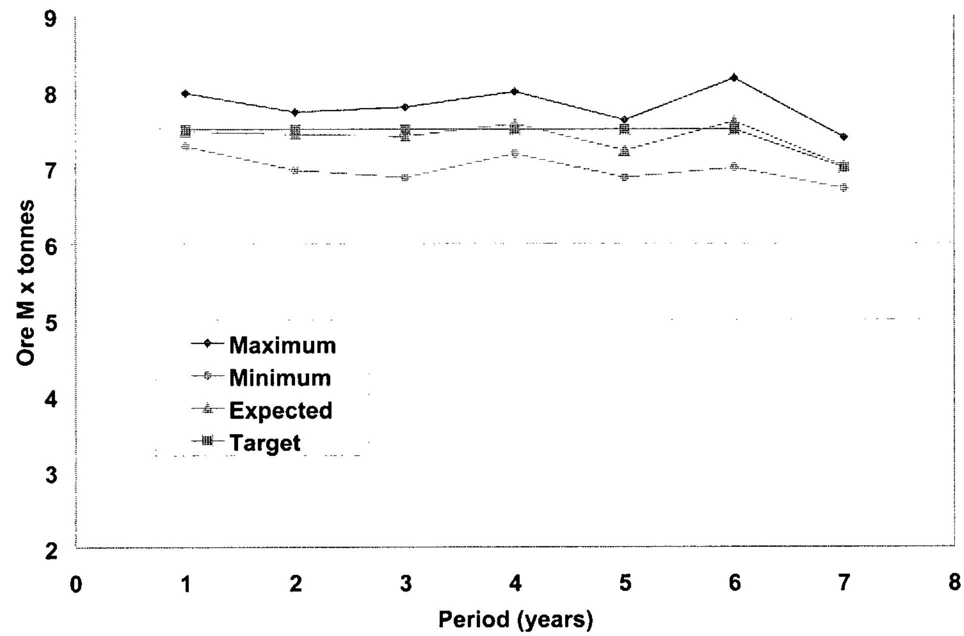

Figure 3-15 - Risk based LOM production schedule (ore production and risk profile) using a different set of simulations than those in the case study presented in the main body of this text

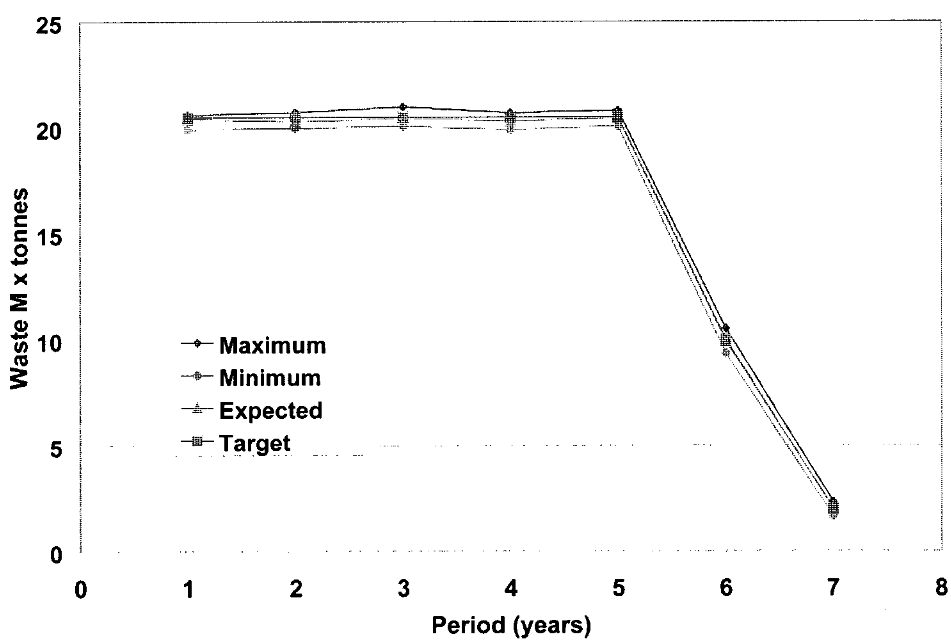

Figure 3-16 - Risk based LOM production schedule (waste production and risk profile) using a different set of simulations than those in the case study presented in the main body of this text

CHAPTER 4 Stochastic integer programming formulation applied to the mine production schedule of a copper deposit

4.1 Abstract

Determining the long-term mine production schedule of open pit mines is of major importance as it defines the ore supply that the mine can provide as well as aims to maximizes its present value. The conventional mine scheduling approach requires a single, average type representations of reality and, as a result, it ignores uncertainty around technical and economic parameters. From all sources of uncertainty affecting the planned ore supply of a given schedule, uncertainty about ore reserves is acknowledged as the most critical one. It is known that the use of a single, average type of deposit model to define reserves produces a smoothed image of the real deposit, and this smoothing may lead to a misleading forecast of ore supply, waste and related cashflow forecasts over the life of the mine. To overcome this problem, a stochastic mining production scheduling approach is considered here. The approach explicitly integrates the uncertainty of parameters or inputs to the scheduling problem aiming at generating a more risk robust solution. In this study, a stochastic integer programming (SIP) formulation for mining production scheduling is applied and tested at a low-grade variability copper deposit. The stochastic solution as implemented here aims to maximize the economic value of a project and minimise deviations from production targets in the presence of geological uncertainty. Unlike the conventional approach, the method is able to account for and manage risk. As a result, the mine production schedule obtained as a solution using the SIP formulation is shown to have a 29% higher NPV than the schedule obtained from a conventional scheduler.

4.2 Introduction

Open pit mine design and production scheduling is traditionally divided, for practical reasons, into two major tasks: first, an ultimate economic boundary beyond which mining becomes uneconomical is delineated and then, the extraction sequence of the set of selective mining units (SMU) contained inside this final boundary or pit is defined. Both problems are typically formulated such that an optimum maximum economical return for the mine is obtained. The optimum open pit mine production schedule is defined as the sequence of extraction that maximizes the present value of the project. This task is one of the most challenging and important in the mine planning framework as it defines the ore supply produced over the life-of-mine (LOM) and consequentially has an impact on the net present value (NPV) of the project. The conventional mine design and production framework defines the extraction sequence considering a single, average type of orebody model as input. As a result, it does not account for uncertainty in the related project parameters. The weaknesses of such an approach are well documented. It has been demonstrated that the use of an average model of the orebody, one that does not reproduce its actual local variability as an input for mine planning optimization algorithms, may lead to misleading results[1,2,3]. This finding makes clear that there is a need to address the mine production schedule problem using stochastic approaches. Stochastic approaches use as input a set of equally probable representations of the orebody which representations reproduce, by construction, its actual spatial variability and distribution. As a result, this set of representations is able to directly incorporate uncertainty into the formulation of the problem.

Different stochastic approaches have been considered in order to provide more robust solutions dealing with uncertainty. Dimitrakopoulos et al[4] utilize the concepts of upside/downside potential to include grade uncertainty in the long-term production schedule. Several mine designs for a set of simulated orebodies are obtained and a final one selected by considering the one with a minimum downside or maximum upside potential. Dimitrakopoulos and Ramazan[5] present a mathematical programming formulation minimizing deviations form production targets by considering a probabilistic approach. The new concept of geological discounted rate is introduced and produces a decreasing unit cost for deviation of target over the LOM. Ramazan and Dimitrakopoulos[6] introduced a mixed integer programming (MIP), which also uses a probabilistic approach. Probabilities are computed by scheduling simulated models; the final schedule uses these probabilities to maximize the NPV of the project. Godoy and Dimitrakopoulos[7] propose a different stochastic approach using a simulated annealing algorithm to obtain a stochastic mine schedule. The proposed solution is divided in three stages; in the first stage optimum mining rates are defined using a LP formulation, in the second stage the rates are utilized to schedule a set of simulated ore bodies, and the schedules are then used in a final stage in which a stochastic schedule is obtained by using a simulated annealing algorithm and then scheduled as an input. The study shows a 28% difference in NPV as compared to the conventional schedule. Leite and Dimitrakopoulos[8] apply a variant of this approach in a copper deposit and show an improvement of 25% in NPV when compared to the conventionally derived schedule. Ramazan and Dimitrakopoulos[9] extend the use of a stochastic MIP formulation to be applied in complex multi-elements type of deposits such iron or nickel laterites. Menabde et al[10] implements a stochastic integer programming (SIP) formulation for long-term production schedule that maximizes NPV considering several possible simulated orebodies and simultaneously optimizing cut-off grades. Ramazan and Dimitrakopoulos[11] propose an approach that accounts for all available realizations of the orebody simultaneously in a stochastic integer programming (SIP) formulation. Their formulation has as objectives the maximization of NPV and minimization of deviation from production targets. Different penalties may be defined for deviations of different targets. A probability cut-off, associated with a cut-off grade, is utilized for ore/waste classification proposes and shows a difference of 10% in NPV. Benndorf[12] expands and applies this formulation to a multi-element type of the deposit, also considering blending constraints. Jewbali[13] combines the use of the SIP formulation with the use of simulated future data, updating simulated models to produce an optimum stochastic mine schedule and shows a 30% inclease in NPV at a gold deposit.

Despite substantial monetary benefits, the application of stochastic schedulers is relatively recent and the value of this solution when applied to different type of deposits is still not completely understood. Therefore it is important to test the application of stochastic schedulers in different types of mineralization to assess the complexities and intrinsic characteristics of such schedulers. In the present study, the approach presented by Ramazan and Dimitrakopoulos[9,11] is applied to a low-grade variability copper deposit. The study tests the approach, quantifies the associated value of the stochastic solution, assesses the risk profile of pertinent mining parameters and, finally, analyses the results to propose future improvements. The study aims to explore the method's capability to incorporate geological uncertainty in the mine production schedule problem formulation and to manage the risk of deviating from production targets. The following sections describe the stochastic integer programming formulation in detail, present its application in a copper deposit and the results obtained, and compare the results with those obtained by a conventional scheduler. Conclusions follow.

4.3 SIP formulation for long-term open pit production scheduling

Stochastic mathematical programming approaches to the mine schedule problem are venues to directly incorporate uncertainty about ore supply in the formulation of the problem, so as to minimize risk of meeting the mine production targets. In the mine production schedule case, the decision to be made is the time period in which each block is mined, in order to maximize the overall discounted value of the project (NPV), subject to slope, reserves and processing and mining capacity constraints. The set of blocks available to be scheduled are the ones contained within the ultimate pit. The SIP formulation presented herein includes uncertainty in the formulation of the problem by considering a set of different and equally probable stochastic simulated orebody realizations in the optimization process.

4.3.1 The economic value of a block

The optimization process considers the economic value of the set of blocks to be scheduled. The expected value of a block $E\{V_i\}$ is defined herein using its expected return NR_i, which is defined as the expected gain from a set of possible stochastic simulated grades (metal content) for the given block i. The value of a given block i is therefore defined as:

$$E\{V_i\} = \begin{cases} NR_i - MC_i - PC_i, \text{if } NR_i > PC_i \\ -MC_i \qquad\qquad , \text{otherwise} \end{cases} \qquad (4.1)$$

Given that

$$NR_i = T_i \times G_i \times REC \times (price - SellingCost) \qquad (4.2)$$

where NR_i represents the expected net revenue, MC_i the mining cost, PC_i the processing cost, T_i the tonnage, G_i the grade and *REC* the recovery.

4.3.2 Objective function

The formulation aims to maximize the NPV of the mine by minimizing the risk of falling short of previously defined production targets. It includes two possible destinations for a block: processing plant or waste dump. The objective function includes two components and it is

$$\text{Max} \sum_{t=1}^{p}\left[\underbrace{\sum_{i=1}^{N} \mathrm{E}\left\{(\mathrm{NPV})_i^t\right\} * b_i^t}_{\text{Part A}} - \underbrace{\sum_{s=1}^{m}\left(c_u^{to} d_{su}^{to} + c_l^{to} d_{sl}^{to}\right)}_{\text{Part B}}\right] \quad (4.3)$$

where

i is the block identifier;

t is the time period;

to flags the ore production target type;

l stands for lower bound;

u stands for upper bound;

s stands for the simulation number

p is the maximum number of scheduling periods;

N is the total number of blocks to be scheduled;

b_i^t is a variable representing the portion of block i to be mined in period t; if defined as a binary variable, it is equal to 1 if the block i is to be mined in period t and equal to 0 otherwise.

$E\left\{(NPV)_i^t\right\}$ is the expected NPV to be generated by mining block i in period t; it is computed as the discounted value of Eq. 4.1

c_u^{to} is the unit cost for excess of ore production

d_{su}^{to} is the excess amount of ore production in period t considering simulation s;

c_l^{to} is the unit cost for the deficient ore production

d_{sl}^{to} is the deficient amount of ore production in period t considering simulation s;

The first component (Part A) in Eq. 4.3 contributes to the maximization of NPV of the project. The expected NPV of a block is computed as the expected present value if the block is mined in period t, considering all simulated values. The second part (Part B) is responsible for minimizing deviations from ore production targets, also managing the distribution of risk within and between periods over the LOM. Risk management is accomplished by the use of a geological discount rate d, which discount over time the penalties applied to the unit cost deviations as explained below. The initial penalties for excess, c_u^{0o}, or shortage production, c_l^{0o}, are user defined and should be at same order of magnitude as the first part of the objective function to ensure the second part is being properly considered. The impact of discounted penalties is a progressive decrease in the unit cost over the periods. This setup ensures that less "risky" mining blocks will be scheduled in early periods, therefore decreasing the risk of not attaining the planned targets and guarantying the minimization of production target deviations.

The unit costs in Eq. (4.3) are

for excess production $$c_u^{to} = \frac{c_u^{0o}}{(1+d)^t} \qquad (4.4)$$

where d is the discount factor mentioned earlier

or

for deficient production $$c_l^{to} = \frac{c_l^{0o}}{(1+d)^t} \qquad (4.5)$$

<u>*Constraints*</u>

Processing constraints

lower bound $$\sum_{i=1}^{N} O_{si} b_i^t + d_{sl}^{to} - a_{sl}^{to} = O_{\min} \quad \forall s,t \qquad (4.6)$$

upper bound $$\sum_{i=1}^{N} O_{si} b_i^t - d_{su}^{to} + a_{su}^{to} = O_{\max} \qquad \forall s,t \quad (4.7)$$

where

O_{si} is the ore tonnage of block i if simulation s is considered;

a_{su}^{to}, a_{su}^{to} are dummy variables to balance the equality;

$O_{\max}$ maximum ore production in a period;

$O_{\min}$ minimum ore production in a period;

Slope constraints

Two different formulations for the slope constraints are made available, though neither one is deemed better as they only differ in the final solution time which is case dependent.

The first formulation uses one constraint for all overlying blocks per period

$$y b_i^t - \sum_{l=1}^{y} \sum_{k=1}^{t} b_l^k \le 0 \qquad \forall i \qquad (4.8)$$

The second is using y constraints for each block per period

$$b_i^t - \sum_{k=1}^{t} b_l^k \le 0 \qquad \forall i \qquad (4.9)$$

where

y is the number of overlying blocks

Reserve constraints

$$\sum_{t=1}^{P} b_i^t = 1 \qquad \forall i \qquad (4.10)$$

4.3.3 Definition of ore and decision variable type

A block is classified as ore or waste on a purely economic basis. A block with positive return is classified as ore, otherwise as waste. Ideally, all blocks considered should be defined as integer (binary) variables; this setup would for most mining applications lead to impractical solution times as the number of blocks to be scheduled is normally quite large. Reducing the number of integer variables reduce the solution time, which can be accomplished by relaxing integrality constraint of some variables. The logical option is to define ore blocks as binary variables and waste blocks as linear ones. This approximation is proposed and shown to not alter the optimality of the solution obtained by Ramazan and Dimitrakopoulos[5]. Following this, the SIP formulation allows for a size reduction strategy. A probability cut-off is utilized in combination with a grade cut-off to classify blocks as ore or waste. First, the probability of the block grade to be greater than the cut-off grade is computed, which is done by counting the number of simulated grade values that are greater than the cut-off grade and dividing the total by the total number of simulated values. Then, if the computed probability is greater than the probability cut-off the block is classified as ore, if not, it is defined as a waste. Once again, variables associated to waste blocks are defined as linear variables.

4.4 Case study: Application at a copper deposit

4.4.1 The deposit

The deposit is located in a typical archean greenstone belt. The region consists predominantly of mafic lavas with lesser amounts of intermediate to felsics volcaniclastics. Rocks are moderately deformed with a prominent cleavage sub-parallel to what is considered to be the original bedding, an E-W trend with average dip 64° South. The deposit itself is in a sequence of moderately to strongly foliated, sulphidic, mafic to intermediate volcanic rocks, which have been intruded by numerous sub-volcanic felsite and feldspar porphyry and/or intermediate volcanic tuff, with size ranging from lapilli to agglomerate, within a strongly chloritic and biotitic matrix. It can be traced over a strike length of 1.5km with a thickness varying from a few meters to more than 75m. Mineralization consists of about 10% sulphides, mostly chalcopiryte, pyrite and pyrrhotite, occurring as disseminations, streaks and stringers apparently controlled by the strong rock cleavage.

The geological database consists of 185 drillholes with 10m copper composites in a pseudo-regular grid of 50m x 50m covering an approximately rectangular area of 1600x900 m2; the average dip is 60° North. Figure 4.1 shows the histogram and statistics for Cu% of 10m composites.

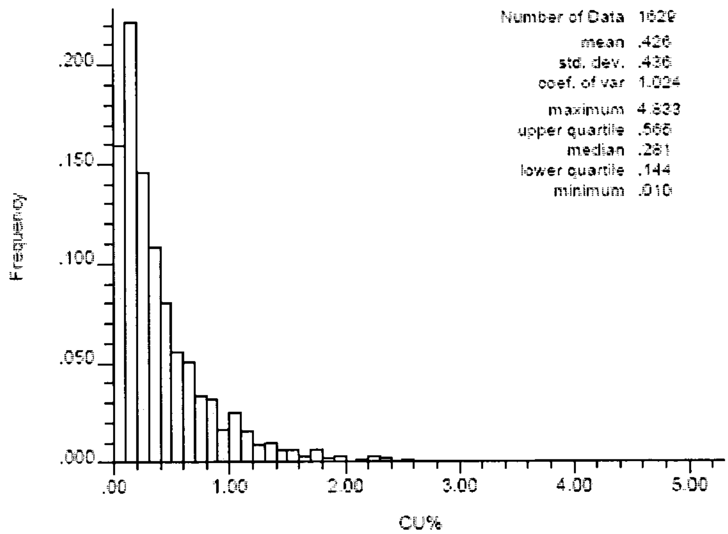

Figure 4-1 Cu (%) histogram of 10m composites

Using the geological information, one mineralization domain is defined and modeled through a geostatistical study. An estimated model, obtained using ordinary kriging, and a set of simulated models, generated using a direct block simulation method[14] are produced. All models are constrained by the ore domain previously defined. Figure 4.2 brings a section of the estimated model and three of the simulated ones. The smoothing effect on the grade, produced by the ordinary kriging model, is made clear in Figure 4.2

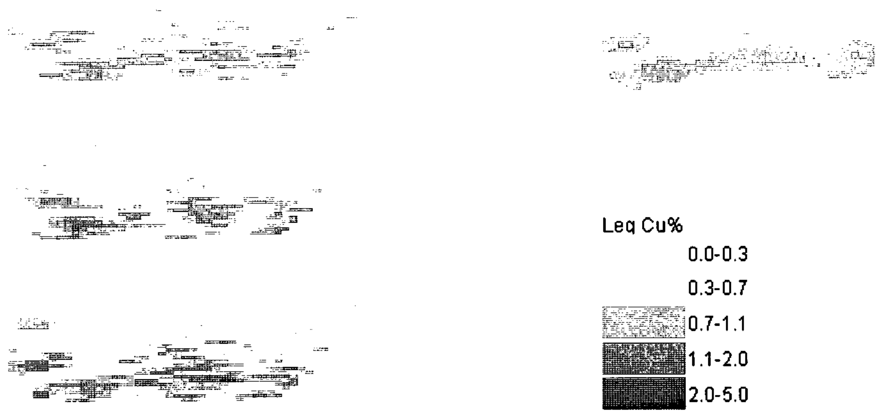

Figure 4-2 East-West sections of three simulated models (left column) and the corresponding single estimated model (right).

4.4.2 Production scheduling

The SIP formulation in Eq. 4.3 to Eq. 4.10 used to generate the mine production schedule first requires the definition of the ultimate pit and push-backs. The nested pit implementation of the Lerchs-Grossman algorithm[15] for pit optimization is used here to define a final pit and push-backs. This algorithm requires as an input a single representation of the orebody, which is created using ordinary kriging on a $20 \times 20 \times 10 m^3$ block support. The associated economic and technical parameters are given in Table 4.1. This model and parameters are utilized in the following sections for comparison with the results obtained by the SIP.

Table 4-1 Economic and technical parameters

Copper price (US$/lb)	2.0
Selling cost (US$/lb)	0.3
Mining cost ($/tonne)	1.0
Processing cost ($/tonne)	9.0
Slope angle	45^{o}
Processing recovery	0.9

Using the parameters in Table 4.1 and the conventionally estimated orebody model, a set of nested pits is generated. Pit 16 is selected as the ultimate pit limit as it corresponds to the maximum net present value. There are 14,480 ore and waste blocks inside the pit limits. In this study, a previously established ore processing capacity of 7.5 M tonnes per year is used. The yearly maximum mining capacity is set to 28 M tonnes although there is no constraint to guarantee a constant material movement over the LOM. With this setup the scheduler is free to define the optimum waste production strategy. Twenty realizations of the deposit are available and generated with the direct block simulation method[7]. To obtain a simulated model, the orebody is divided into blocks of 20x20x10m^3 within the mineralized domain. Each block is then represented by 10x10x1 nodes. This number of nodes, 100 per block, is large enough to ensure that the actual block scale variability is reproduced by the simulated orebodies. These stochastic simulated orebodies are used as an input for the SIP models and to produce risk profiles of performance parameters of the schedules generate throughout the study.

In the present study, a base case SIP schedule is first considered and then the sensitivity of pertinent key project parameters is tested and their impact on the project's NPV and performance evaluated. The stochastic schedule is generated considering a 20% geological discount rate with an associated 20% probability cut-off. All cases utilize the same parameters as specified in Table 4.2. A fixed copper grade cut-off of 0.3% is used in combination with the probability cut-off to classify ore and waste blocks. Penalties for excess and shortage production are select accordingly to the magnitude of the first part of the objective function in order to ensure that the second component, which accounts for minimization of production targets and risk management, is properly weighted in the objective function. Past work has shown[12] that it is the order of magnitude of the penalties rather that the actual values that effect the optimization process. A higher penalty for shortage production is imposed as it has a more severe impact in the project than an excess production, which could be handled by the use of stockpiles. Risk profiles for ore and waste production, cumulative NPV and production deviations are generated and presented respectively by Figure 4.3, Figure 4.4, Figure 4.5 and Figure 4.6. Two additional schedules are generated, as one of the objectives of the work is to test and better understand the effect of the geological discount rate on the schedule produced.

Table 4-2 SIP related parameters

Geologic risk discount rate	20%
Cost of shortage in ore production (unit/tonne)	10,000
Cost of excess ore production (unit/tonne)	1,000
Economic discount rate (%)	10
Cut-off (% Cu)	0.3
Number of simulated orebody models	20

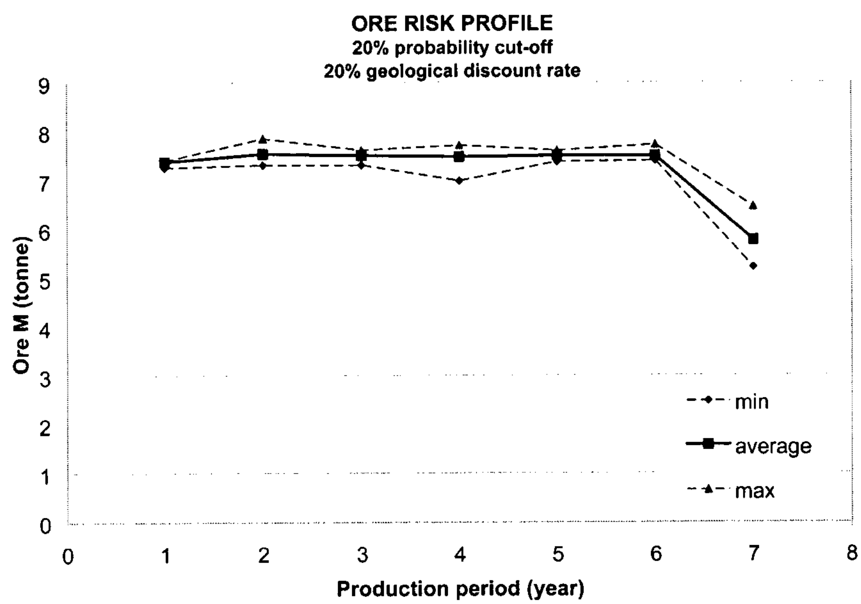

Figure 4-3 Risk profile for ore production of the base case schedule

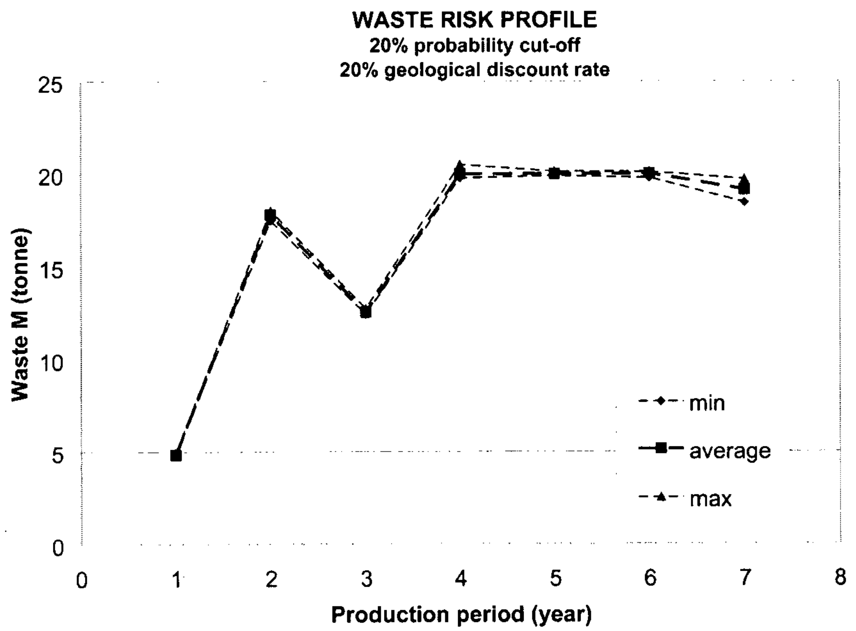

Figure 4-4 Risk profile for waste production of the base case schedule

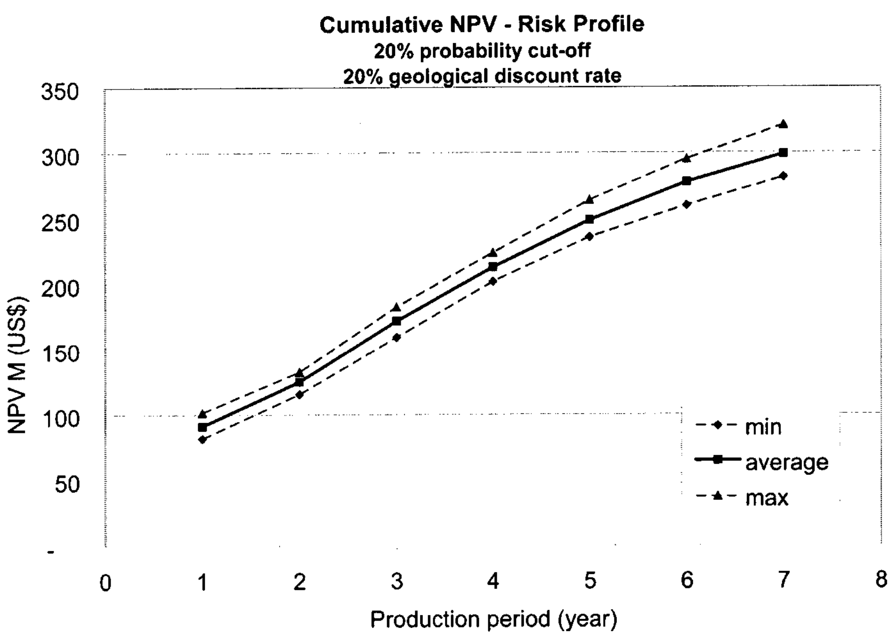

Figure 4-5 Risk profile for cumulative NPV of the base case schedule

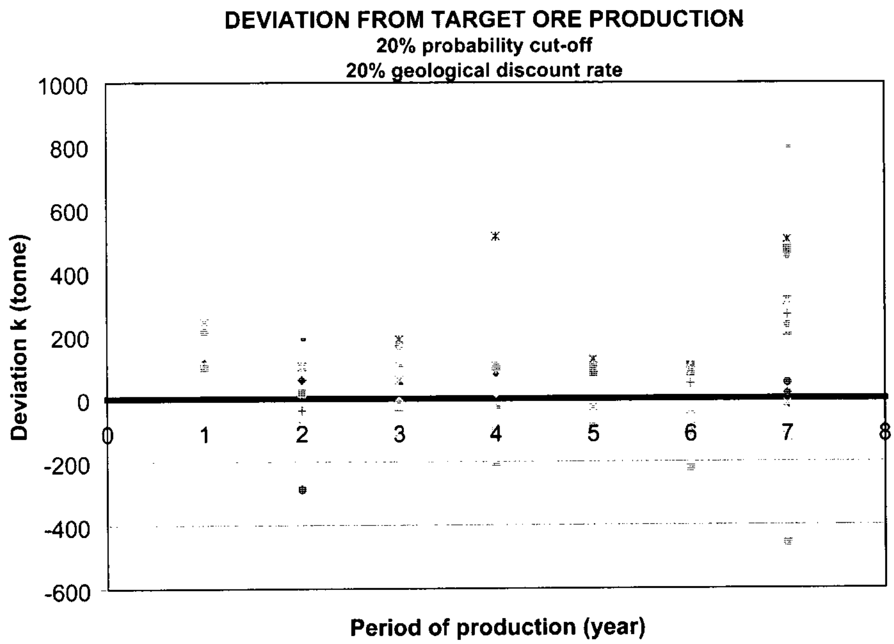

Figure 4-6 Deviations from ore production targets for the base case schedule

4.4.3 Geological discount rate and sensitivity analysis

To evaluate the impact that the geological risk discount rate has on the base case schedule, an alternative schedule is obtained using the same parameters applied to the base case schedule above, except that the geological discount rate set to 30%. With the increased geological discount rate further additional weight is given to deviations from production targets at earlier periods than later ones. Therefore would be expected a possible decrease on the variability around ore production targets in the first years of production and an increase in later ones. Figure 4.7 shows the ore production risk profile for the resulting schedule; no major difference can be observed when compared to Figure 4.3. The same situation is found when analysing Figures 4.8, 4.9 and 4.10, respectively, showing the risk profiles for waste, cumulative NPV and deviation from production targets, when compared to Figures 4.4, 4.5 and 4.6 respectively. Therefore, for is case study a 10% increases on the geological discount rate would not improve the economics of the project as it does not further decrease the deviations of production targets. The comparison of the physical schedules is presented in Figure 4.11 and Figure 4.12. There are differences in the mine's schedules produced, but in general the same areas are being scheduled to be mined in the same time periods. The most relatively significant difference is found in the two final production years. It is important to stress that the absence of a geological discount rate would have a substantive impact in the deviations from production target, thus impacting the expected NPV of the project. This is further discussed in the following section, where results obtained by a conventional scheduler is compared with the ones obtained by the base case of the SIP model.

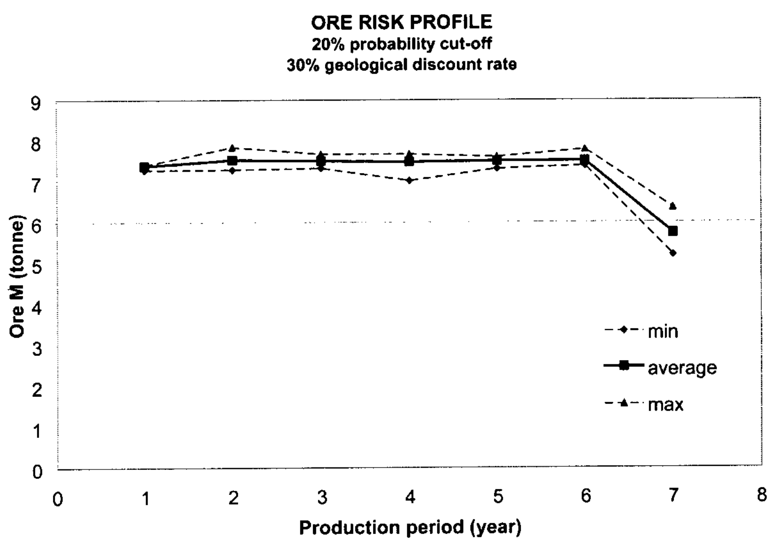

Figure 4-7 Risk profile of the mine's ore production schedule with a 30% geological discount rate

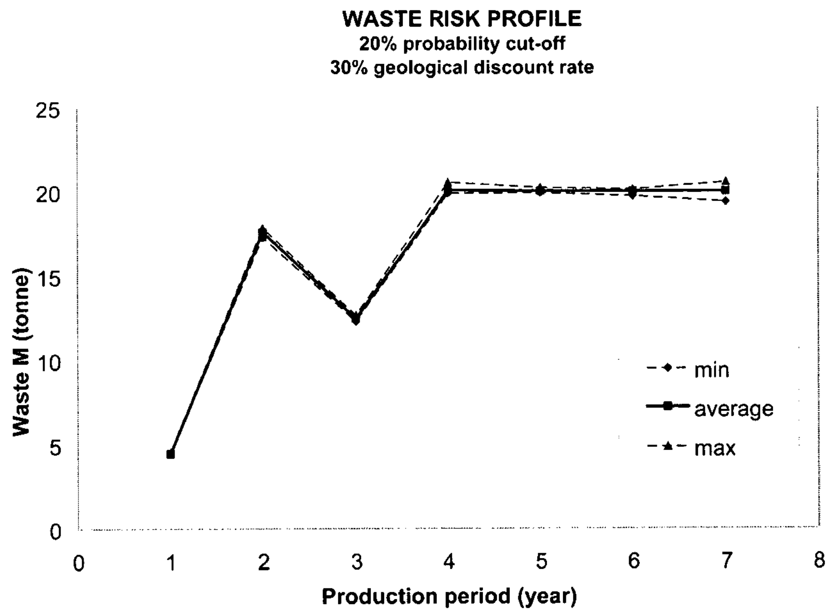

Figure 4-8 Risk profile of the mine's waste production schedule with a 30% geological discount rate

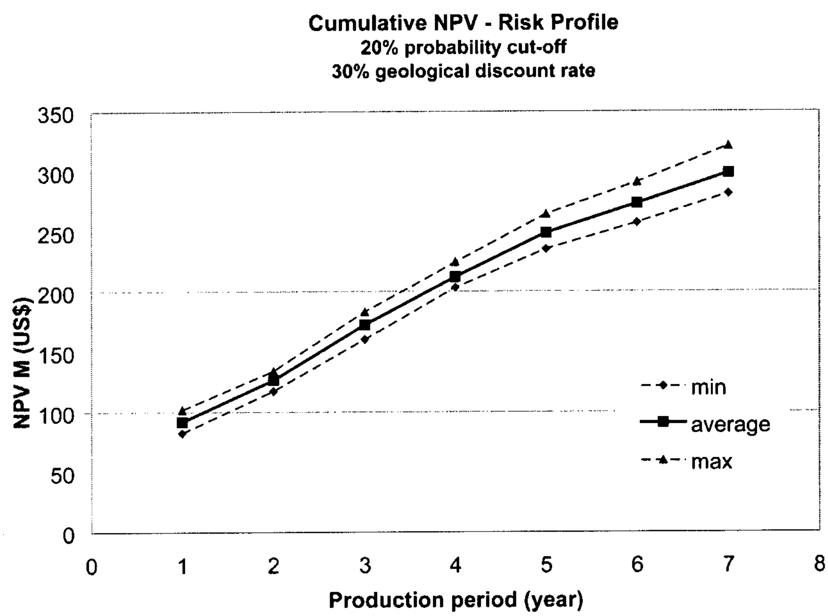

Figure 4-9 Risk profile for cumulative NPV of the mine's schedule with 30% geological discount rate

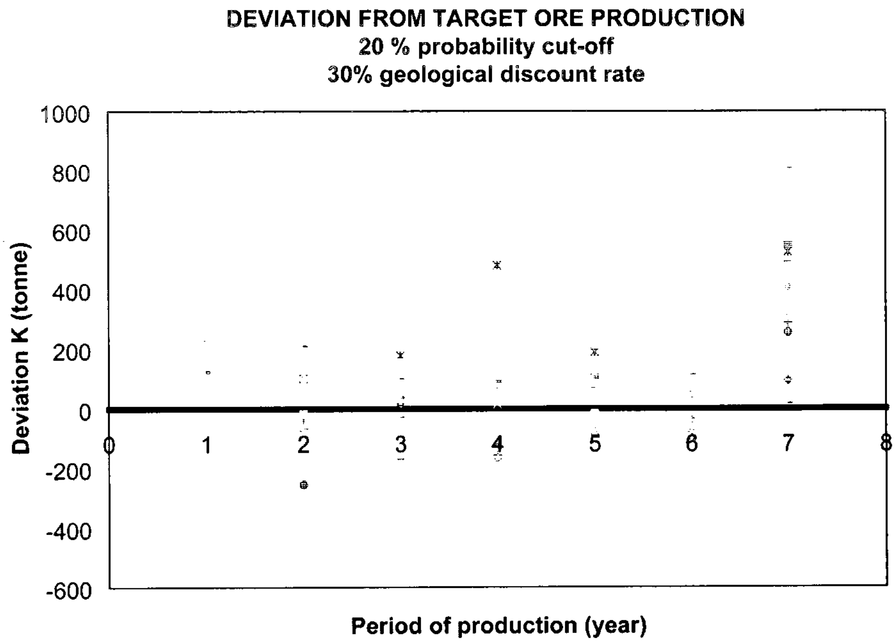

Figure 4-10 Deviations from ore production targets for the mine's schedule for the 30% geological discount rate

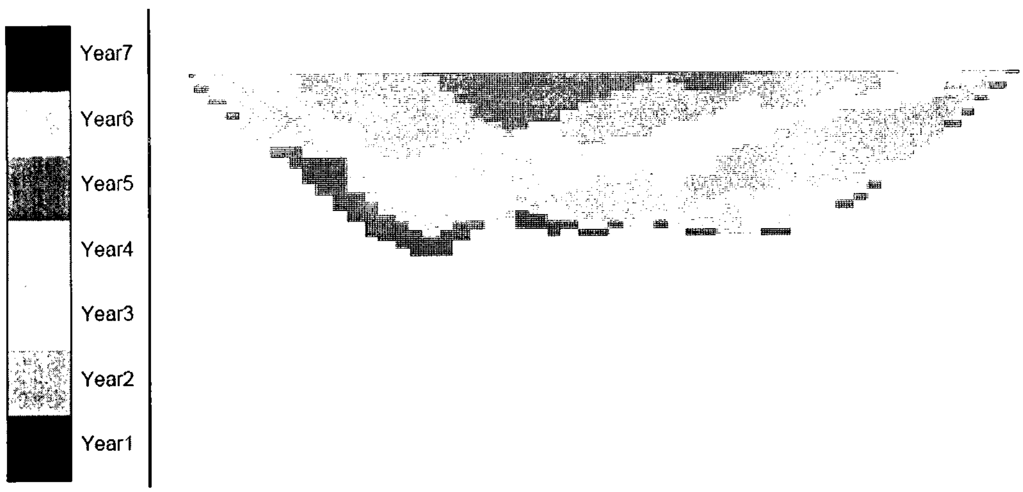

Figure 4-11 East – West section of the base case stochastic schedule (20% geological discount rate)

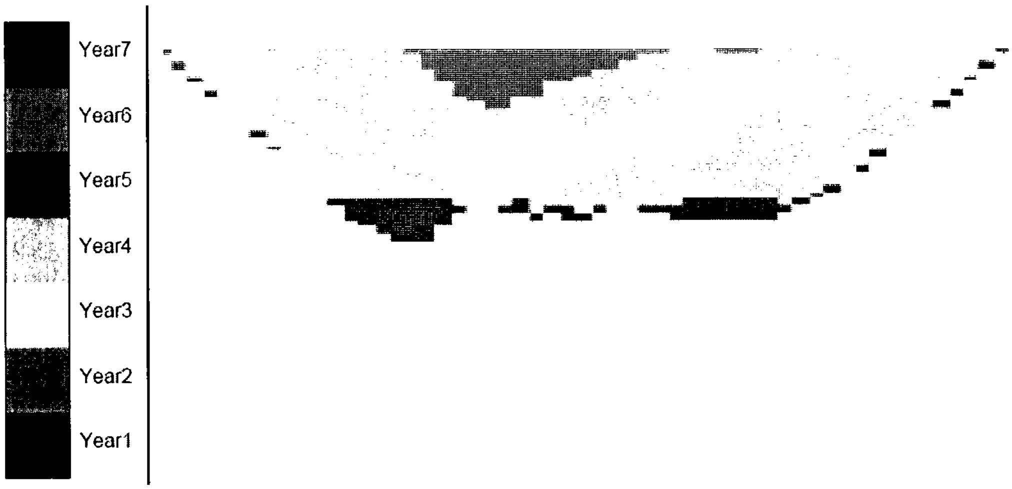

Figure 4-12 East West section of the schedule with a 30 % geological discount rate (same section as in Fig. 4.11)

To test the effects the probability cut-off has on the base case schedule, an alternative schedule is obtained using the same parameters applied to the base case except that the probability cut-off is changed to 35%. The use of a higher probability cut-off implies that a more risk-averse schedule is produced and more blocks are classified as waste. The solution time decreases as there are less integer variables in the problem formulation with more blocks being classified as waste. The impact of this new classification strategy is observed in Figure 4.13, which shows the ore production risk profile for both schedules. The alternative schedule obtained using a 35% probability cut-off has a lower ore tonnage than the base case schedule. Figure 4.14, 4.15 and 4.16, respectively show the risk profiles for waste, cumulative NPV and deviation from production targets. The major difference from the base schedule is associated with the waste material quantity and movement. The comparison of the physical schedule is presented in Figure 4.17 and Figure 4.18. There are major differences in the schedules produced with different areas are being scheduled in the related mining periods.
The schedule with a high probability cut-off presents a slightly lower NPV. The difference is associated with the lower ore tonnage produced by a more restricted ore classification. The magnitude of the difference is not significant

once the scheduler is able to mine less waste tonnes over the first years of production. The difference would be more significant if a stockpile is considered, given more flexibility to the scheduler to deal with an excess of ore production.

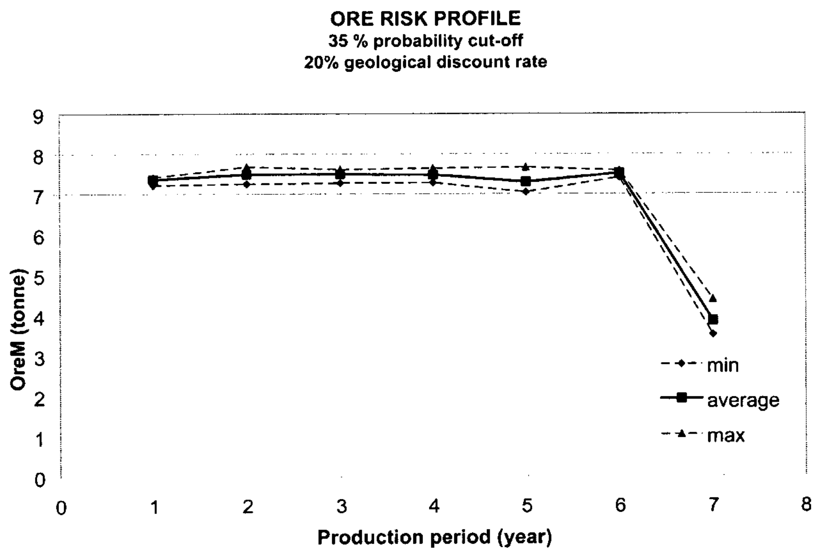

Figure 4-13 Risk profile for ore production of the schedule produced using a 35% probability cut-off

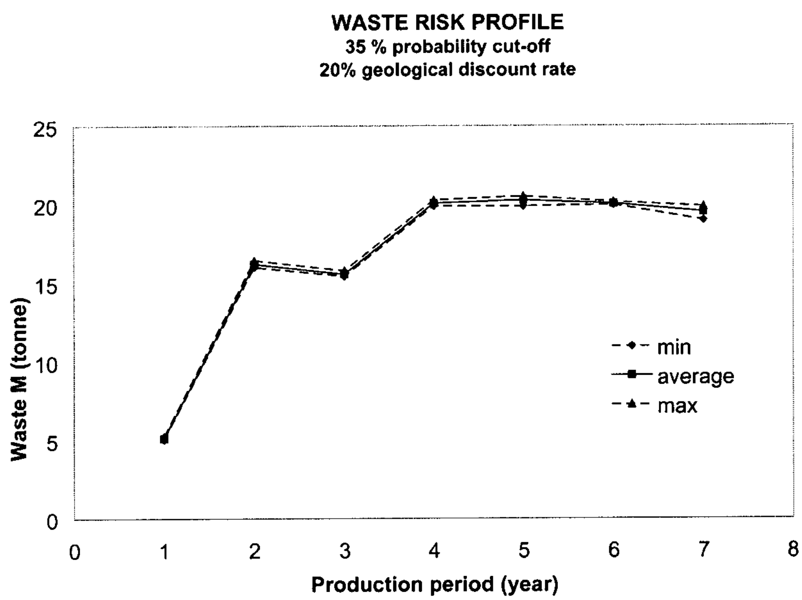

Figure 4-14 Risk profile for waste production of the schedule produced using a 35% probability cut-off

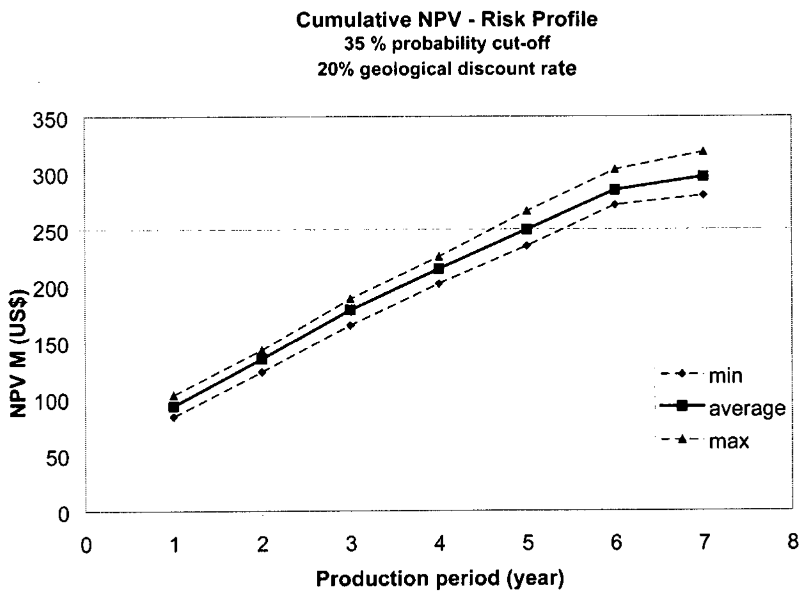

Figure 4-15 Risk profile for cumulative NPV of the schedule produced using a 35% probability cut-off

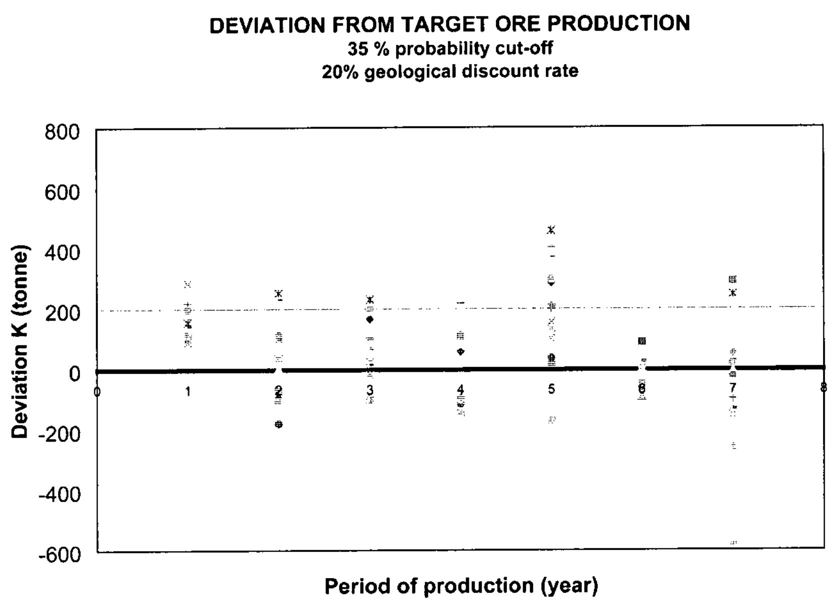

Figure 4-16 Deviations from ore production targets for the schedule produced using a 35% probability cut-off

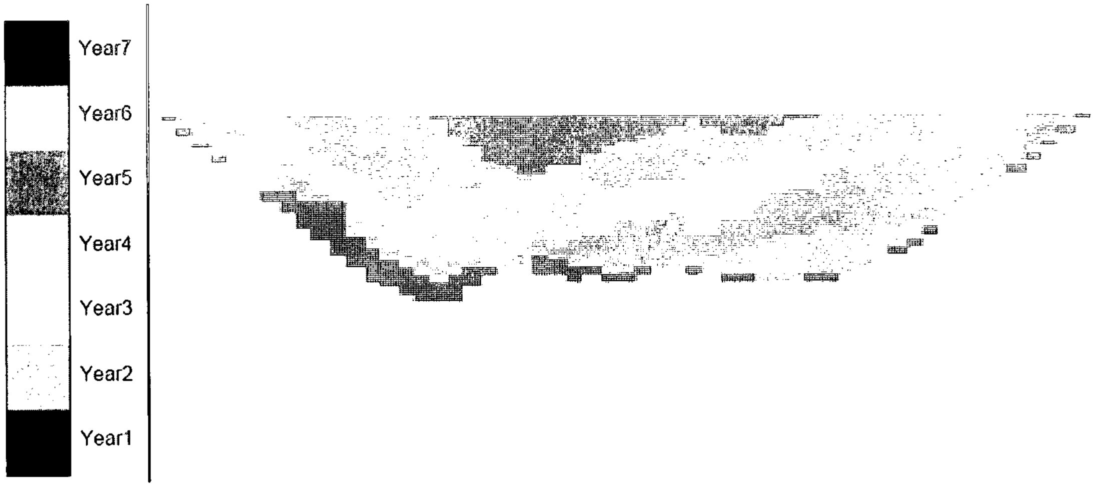

Figure 4-17 East – West section of the base case stochastic schedule

Figure 4-18 East –West section of the 35% probability cut-off schedule case

4.5 Comparison to a conventional scheduler

To assess the value of the stochastic solution in the case study presented above, a conventional LOM production schedule is developed in this section and the economic difference between the stochastic and conventional approaches evaluated. The conventional schedule is generated using the average type of deposit model previously described and obtained using the technique of ordinary kriging. The same final pit and push-backs are utilized with the associated technical and economic parameters as before (Table 4.1). The schedule is obtained using Millawa NPV algorithm[15] using a fixed 7.5 M tonnes ore production target and a maximum material movement of 28M tonnes. No constraint or limitation is imposed to the scheduler, so as to keep the total material movement constant over time. The intent is to let the optimizer decide the best waste removal strategy and to replicate the condition imposed on the SIP scheduler.

Figure 4.19 presents the ore production of the conventional schedule and its risk profile. It is clear in the figure and the related risk profile that the conventional schedule generated misleading results in the presence of grade uncertainty; the forecasted ore supply has about 5% chances (one in the twenty equally likely scenarios of the deposit will produce what is expected) to materialise for almost all years. Only in two years the forecasted production is expected to be realized, year 5 and year 7. The economic implication of such deviations can be seem in Figure 4.20 where the risk profile of cumulative NPV for both the base case stochastic schedule and the conventional schedule are shown. The average NPV difference between the conventional and the stochastic solution is approximately 29%. The higher NPV obtained by the stochastic solution is obtained by first incorporating grade uncertainty into the mine production scheduling formulation, minimizing the possible deviations of ore production target and at the same time managing the risk between mine periods. The difference between the value of the stochastic solution and the conventional one comes from the capacity of the stochastic optimizer to obtain an optimum schedule considering simultaneously several equally probable orebody models. The equally probable orebody models, obtained by conditional simulation, better than the smoothed model, utilized by the conventional scheduler, represent the spatial grade distribution of the deposit.

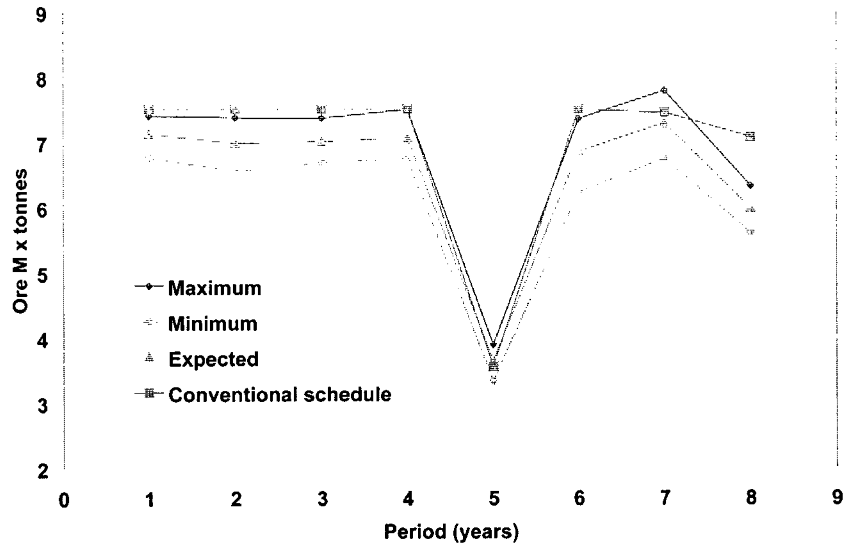

Figure 4-19 Ore production and associated risk profile for the conventional schedule

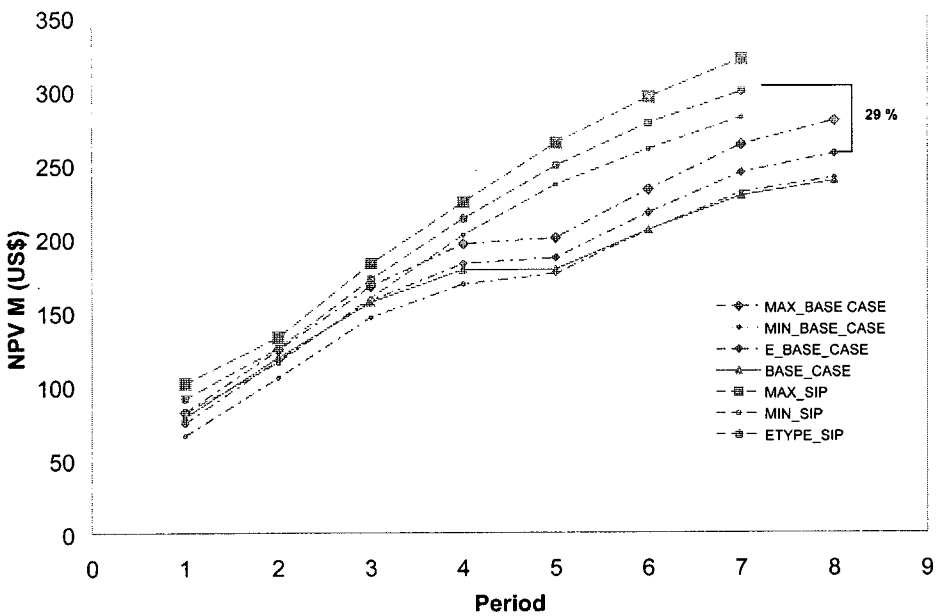

Figure 4-20 Conventional and SIP base case schedules and their respective risk profiles

The conventional scheduler forecasts a LOM of 8 years in contrast with the 7 years LOM forecasted by the stochastic scheduler. By using a smoothed scenario, the conventional scheduler overestimates the amount of ore above the 0.3% Cu cut-off therefore the extra one year of production is required to fully mine the deposit. It is interesting to emphasise that this fact can not be generalized as it is a function of the grade distribution and cut-off applied.

This study first evaluates the impact of the use of different geological discount rates and probability cut-offs in the stochastic schedule produce. The finds are that there is no major change if a higher geological discount rate is utilized, and that a higher probability cut-off produces a different schedule with different ore/waste quantity and mining strategies. The value of the stochastic solution is then computed by comparing the results obtained by the base case stochastic schedule with the one obtained by a conventional scheduler. The difference is significant and derived from the incapability of the conventional scheduler to include grade uncertainty in its formulation, in comparison with the stochastic scheduler which not only capable to account for the grade uncertainty in its formulation but also to manage the risk associate with it in the deviation of production targets.

The fact that a stochastic integer formulation solution of the mine production schedule problem generates a higher value reflects the importance of incorporating geological uncertainty into the scheduler problem formulation. Similar finding is presented by Leite and Dimitrakopoulos[16], which tested a different stochastic scheduler using the same cupper deposit. Both studies make it clear the advantages to use a stochastic approach in the mine production schedule formulation.

4.6 References

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CHAPTER 5 CONCLUSIONS

In the present work, two stochastic mining production schedule methods are tested in order to improve the understanding of the value of a stochastic solution to the mine production schedule problem. The stochastic schedules generated are analysed, their mechanisms understood and the results compared with the ones obtained by a conventional scheduler.

In Chapter 2, a comprehensive literature review of stochastic mine production schedules and methods to generate stochastic simulations is provided in order to ensure a full understanding of the subsequent sections of the thesis.

Chapter 3 proposes an application of the stochastic mine production scheduling formulation based on a simulated annealing algorithm at a low-grade variability copper deposit. Independent of the low-grade variability of the deposit, the results obtained by the stochastic schedulers are considerably more robust than the ones obtained by the conventional scheduler. The stochastic solution presents an average cumulative NPV which is approximately 26% higher than the conventional solution. The risk analyses conducted for both schedules show that the stochastic approach successfully minimizes deviations from production targets, in contrast with the conventional schedule which has a high probability of not producing the targets in any year of the life of the mine.

Chapter 4 describes the application of an SIP formulation to the mine production schedule problem, discusses the rule of the geological discount rate with examples and the probability cut-off on the stochastic schedule obtained and finally compares the stochastic results with the ones obtained by a conventional scheduler. The value of the stochastic solution is approximately 29% higher than the one obtained by the conventional schedule. As previously described, this difference is due to the ability of the stochastic scheduler to directly include grade uncertainty into the problem formulation.

Both stochastic mine production schedule approaches herein presented proved the value of a stochastic solution over a conventional one. The increase of the NPV value of the project is considerable and can be attributed mainly to the inclusion of grade uncertainty directly into the problem formulation. Further developments for the stochastic approach could involve the development of an open pit optimizer capable of defining the ultimate pit including uncertainty about the grade distribution of the orebody, as well being able to define push-backs of a fixed size. The size reduction problem also presents another issue, which requires future studies, as it is the major constraint for the application of stochastic integer programming formulations.

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Appendix B - application of direct block simulation in a copper deposit

The conventional approach to assess the value of a mine requires the assumption of a risk-free scenario and ultimately leads to single estimated valued. It is well documented and understood[1,2,3,4] that such assumption may lead to misleading results. The first step required to incorporate uncertainty into the mining planning framework involves its proper map. In the case of geological uncertainty, in this study grade uncertainty, conditional simulation methods are the logical choice for the task.

There is a large gamma of simulation methods available[21]. Simulation methods appropriate to deal with mining related problems should be able to efficiently simulate large grids in a reasonable amount of time, reproducing the spatial characteristics of the deposit and conditioning to the available data. The regular approach to the problem of simulate a given property of deposit, involves first the discretization of the deposit into a set of nodes. Simulation methods such as Sequential Gaussian Simulation[30], Turning Bands or, more recently, Generalized Sequential Gaussian Simulation[35] can produce simulations respecting all required properties but in a point support. It means that after simulation be performed an additional step has to be done in order to group nodes/points in a meaningful volume (mining blocks) in which decisions will be made. This process can be extremely timing consuming when a large number of blocks have to be simulated. To over come this problem Godoy[15], proposes a simulation method able to directly generate simulations in the selective mining unit support size. The method uses scream effect approximations to simulate group of nodes defining a block, using conditioning data with different support sizes. The next sections describe in detail direct block simulation and discuss and present the results of its application into a copper deposit.

Considering that the attribute under study can be properly modelled by a stationary and ergodic random function (RF). Direct block simulation (DBSIM) extends the principles utilized by the sequential Gaussian simulation method to generate simulations directly at the block support. As all other flavours of Gaussian simulations it requires the transformation of the RF Z(u), describing an attribute over the mineralized domain, into a normally distributed RF Y(u) = ϕ(Z(u)). As the objective is to simulate the attribute of interest directly at the block support, each block is sequentially simulated only using the original conditional data and the previously simulated blocks as conditional data. A block is simulated by simultaneously simulating N discritizing points/nodes $Y(u_i)$ i=1,...N, discritizing the given block over the its volume V. The average of the simulated nodes in the normal space is stored in memory for further conditioning. This average value can be interpreted as a RF $Y_V(v)$ (Eq. B.1), v 0 R^n discritized over the volume V. The simulated block value $Z_V(v)$ is computed and later recorded as the average of all black-transformed node values as expressed by Equation B.2. This approximation is required as no direct back-transformation in the form of $Z_V(v) = \phi^{-1}(Y_V(v))$ exits.

$$Y_V(v) = \frac{1}{N}\sum_{i=1}^{N} Y(u_i), u_i \in v \forall i \qquad \text{(B.1)}$$

$$Z_V(v) = \frac{1}{N}\sum_{i=1}^{N} \phi^{-1}(Y(u_i)), u_i \in v \forall i \qquad \text{(B.2)}$$

DBSIM algorithm randomly visits all blocks inside the simulation domain. The group of points defining a block is simulated by LU using only local conditional data in point and block support. Let C_{IIVV} be the covariance matrix of all conditional information, where II stands for point support data and VV for block support data; and C_{pIV} be the covariance matrix of discretizing points (p) and local conditional data in point (I) and block (V) support.

$$C_{pIV} = \begin{bmatrix} C_{pI} & C_{pV} \end{bmatrix} \quad \text{(B.3)}$$

$$C_{IIVV} = \begin{bmatrix} C_{II} & C_{IV} \\ C_{VI} & C_{VV} \end{bmatrix} \quad \text{(B.4)}$$

where C_{II}, C_{IV} and C_{VV} are respectively the point to point, point to blocks and block to block covariance matrixes; C_{pV} and C_{pI} are the covariance matrixes of discritizing points and conditional blocks and points. To compute the covariance between different supports, a regularized covariance is considered for point to block and block-to-block as follow:

$$\overline{C}(u_\alpha, v) \approx \frac{1}{N}\sum_{i=1}^{N} C(u_\alpha, u_i) \quad \text{(B.5)}$$

$$\overline{C}(v_1, v_2) \approx \frac{1}{N_1 N_2}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2} C(u_j, u_i) \quad \text{(B.6)}$$

The simulation of the nodes discritizing a block is done by means of Cholesky decomposition of the covariance matrix and can be defined as follow:

$$\begin{bmatrix} C_{IIVV} & C_{IVp} \\ C_{pIV} & C_{pp} \end{bmatrix} = \begin{bmatrix} L_{IIVV} & 0 \\ L_{pIV} & L_{pp} \end{bmatrix} \begin{bmatrix} L_{VVII} & L_{IVp} \\ 0 & L_{pp} \end{bmatrix} \quad \text{(B.7)}$$

$$m = Lw = \begin{bmatrix} m_{IV} \\ m_p \end{bmatrix} = \begin{bmatrix} L_{IIVV} & 0 \\ L_{pIV} & L_{pp} \end{bmatrix} \begin{bmatrix} w_{IV} \\ w_p \end{bmatrix} \quad \text{(B.8)}$$

where m_{IV} represents the matrix of conditional data, including points and blocks. The final vector containing the simulated values m_p can be described by

$$m_p = L_{pIV} L_{IIVV}^{-1} m_{IV} + L_{pp} w_p \quad \text{(B.9)}$$

The algorithm can be synthesized as follow:

- Each block to be simulated is randomly visited;
- Simultaneously simulate the internal nodes discritizing the block (Eq. B.9);
- Average the nodes values in the normal space and store for future conditioning (Eq. B.1);
- Average the back-transformed node values and record as the simulated value for the block (Eq. B.2);
- Discard the simulated internal points;
- Repeat the previous steps until all blocks have been simulated;

Application of DBSIM to simulate a copper deposit

The deposit under study is in a sequence of moderately to strongly foliated, sulphidic, mafic to intermediate volcanic rocks, which have been intruded by numerous sub-volcanic felsite and feldspar porphyry and/or intermediate volcanic tuff, with size ranging from lapilli to agglomerate, within a strongly chloritic and biotitic matrix. It can be traced over a strike length of 1.5 km with a thickness varying from a few meters to more than 75m. Mineralization consists of about 10% sulphides, mostly chalcopiryte, pyrite and pyrrohotite, occurring as disseminations, streaks and stringers apparently controlled by the strong rock cleavage. The geological database is compounded by 185 drillholes with 10 m copper composites in a pseudo-regular grid of 50 x 50 m covering all the deposit area totalizing 1629 samples with associated mean grade of 0.426 % Cu and relatively lower coefficient of variation (1) indicating the low grade variability of the orebody. Copper is the only mineral of interest and its distribution is represent below.

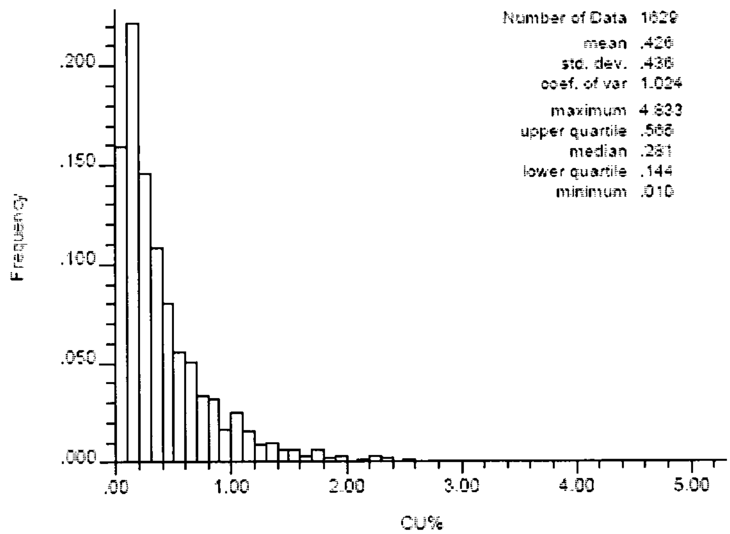

Figure B. 1 copper grade histogram for a 10m composite length

The simulation process of an orebody using DBSIM starts by the specification of a mineralized enveloped. This enveloped is discritized using a set of regular blocks, the considered selective mining unit (SMU), which represents the simulation grid. In this case study a SMU of 20 x 20 x 10 m^3 is considered. Each block is further discritized by a set of nodes defining a regular internal grid of 2 x 2 x 10 m^3. The block discritization should be such that the block grade variability of the simulated orebody reproduces the expected theoretical one obtained using regularized semi-variogram models. Other parameters involved in the simulation process are minimum and maximum number of hard sample data to be utilized, maximum number of previously simulated blocks to be considered as well the size of the search neighbourhood. In the case study herein presented, the algorithm restricts the conditional data to minimum of 3 and a maximum of 15 samples and a maximum of 7 previously simulated blocks. Using these parameters, a total of 20 simulations are generated. Sections of the simulated orebody models are presented below in Figure B.2.

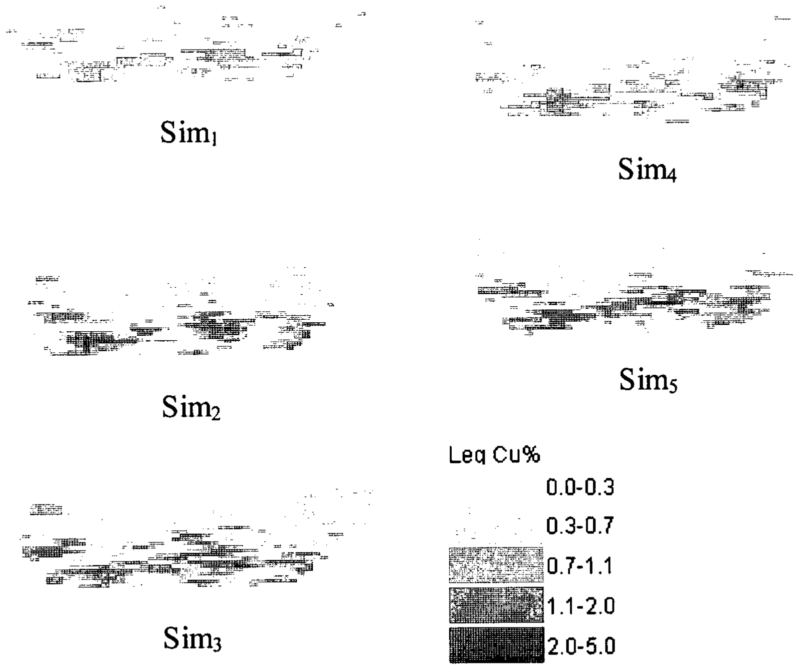

Figure B. 2 - simulations of copper grade in an east-west cross section

After simulation has been conducted a validation procedure is done to ensure the realizations adequately reproduce the spatial correlation model assumed for the deposit and also its grade distribution. The check is done comparing the variograms, computed at the block support, with the regularized variogram model utilized as simulation parameters. The results can be seem in Figure B.3, Figure B.4 and Figure B.5 which show respectively the variogram in the major, intermediate and minor directions of continuity. As expected, simulations satisfactory reproduce the desired spatial characteristics of deposit. The histogram of mean copper grades of simulations and the drillhole information utilized is also presented in order to show that simulations do reproduce the average values (Figure B.6).

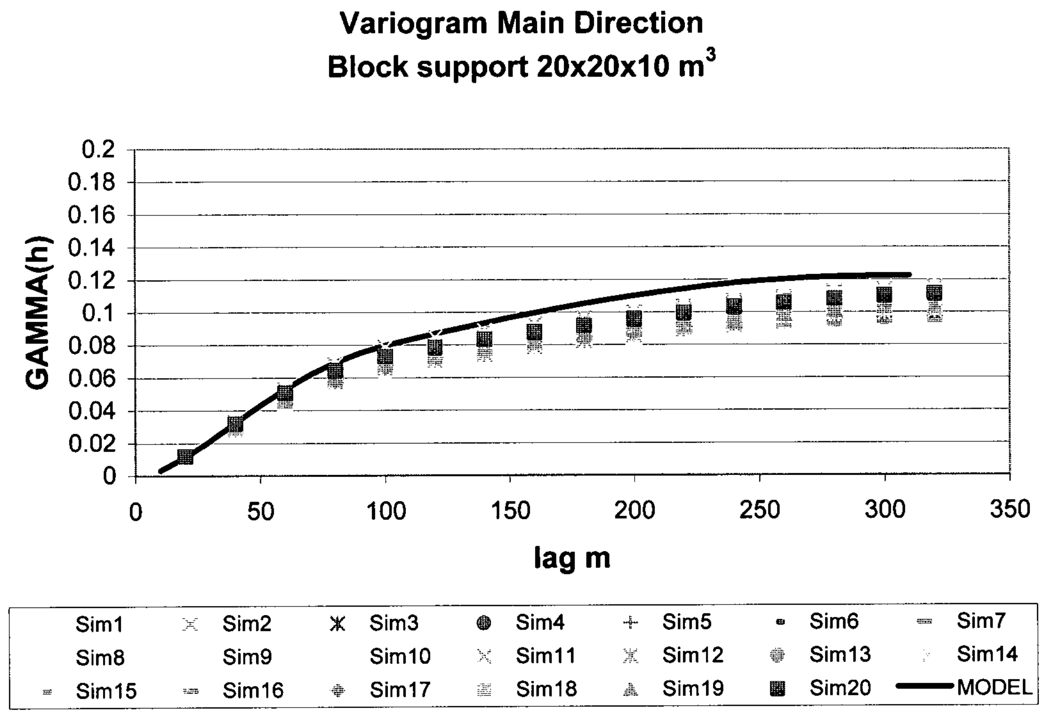

Figure B. 3 - experimental and model regularized variograms in the major direction of continuity at a block support

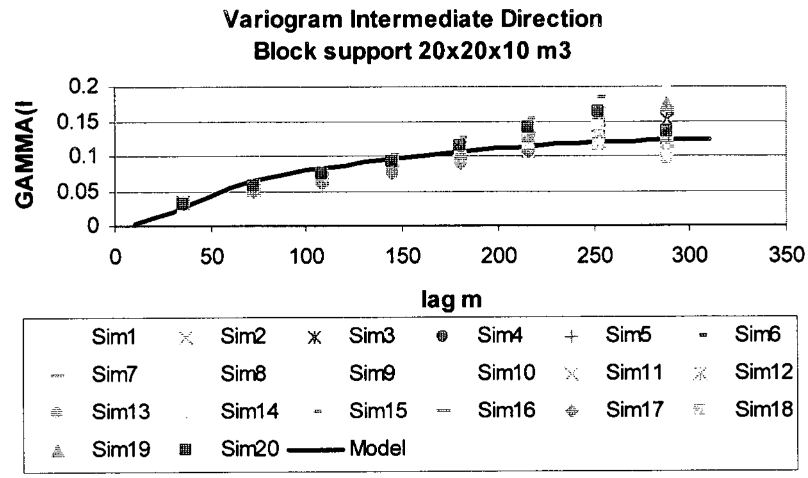

Figure B. 4 - experimental and model regularized variograms in the intermediate direction of continuity at a block support

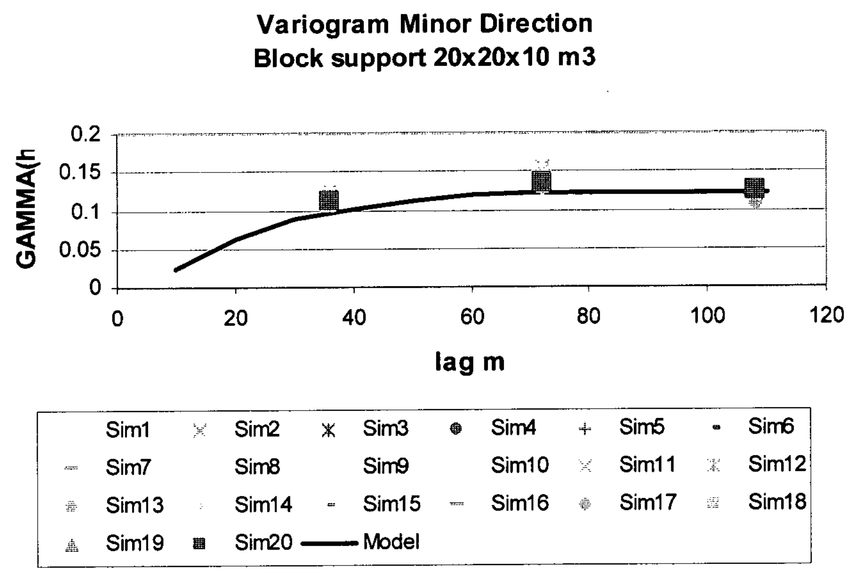

Figure B. 5 - experimental and model regularized variograms in the minor direction of continuity at a block support

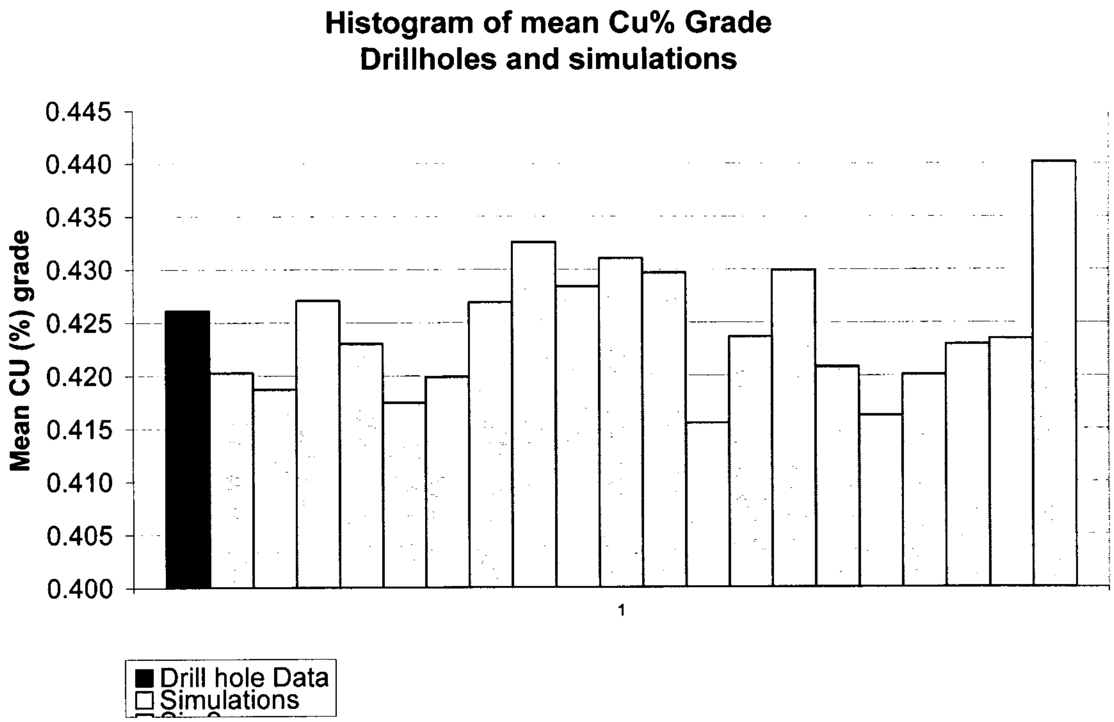

Figure B. 6 - histogram of average copper grades for drillhole and simulations

Appendix C – SIP implementation and execution: the LTSPS program

The stochastic integer program formulation present by Ramazan and Dimitrakopoulos[7] is implement in the LTSPS program. LTSPS requires input files following certain standards. These standards are here discussed as well as the sequence of execution of the program.

LTSPS requires a set of simulation models, a result final containing the push-back number of each block and a parameter file. The simulation files have to be produced using Whittle 4X and after exported reformatted to include a mineralized and waste parcel for each block as these fields are expected by the program. The result files also have to be reformatted to exclude the increment comments (lines starting with an exclamation mark) included by Whittle in the output file. The program should first be run to produce the SIP model to be feed into the optimizer (CPLEX). After a solution is obtained, it should be exported in a plan text format into a file. To accomplish that it is necessary to follow the given commands in CPLEX :

Change the problem type by typing ***change prob fixed_mip***
Re optimize by typing ***optimize***
Export the solution file by typing ***write mod.txt TXT***

The mod.txt file should then be copied inside the same folder that the model was first created. The parameter file needs to be properly modified in order to do not generate files related with slope constrains. The solution is evaluated and a result file written in the same working folder.