Title

HYDROLOGIC AND ECONOMIC MODELS FOR SUBSURFACE DRAINAGE

Bhattacharya

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by

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ABSTRACT

Ph.D. Agricultural Engineering

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HYDROLOGIC AND ECONOMIC MODELS FOR SUBSURFACE DRAINAGE

Field and laboratory experiments were conducted on some soils of the St. Lawrence Lowlands region, with a view to evaluating some pertinent soil water properties, relating actual evapotranspiration to potential evapotranspiration and available water, and relating drainable porosity to water table depth.

A water balance model was developed and used with the results of the above investigations to predict daily water table depths.

Based on information from literature, a crop loss model was developed, and the two models were used with 76 years of weather data from the Ottawa station to compute yearly losses and the associated probabilities, and the average annual loss for corn.

The average annual revenue increase from a subsurface drainage system was computed by subtracting the average annual crop loss and the equivalent annual investment cost from the no-loss crop value. This analysis was done for various combinations of soil, economic and design parameters.

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Ph.D ASHIM K. BHATTACHARYA Genie rural

MODELES HYDROLOGIQUES ET ECONOMIQUES DE DRAINAGE SOUTERRAIN

On a procédé à des expériences au champ et en laboratoire sur quelques sols des basses terres de la plaine du St-Laurent en vue: (1) de detérminer quelques caractéristiques hydrologiques des sols; (2) d'établir un rapport entre l'évapotranspiration réelle, l'évapotranspiration potentielle et l'eau disponible; (3) et d'obtenir une relation entre la porosité utile au drainage et la profondeur de la nappe phreatique.

Les résultats de ces expériences ont permis de construire un modele du bilon d'eau utilisable pour prédire les profondeurs quotidiennes de la nappe phréatique.

Se basant sur les études d'autres chercheurs on a construit un modele pour calculer les pertes de récolte causées par une nappe d'eau élevée tout en tenant compte de la durée de cette élévation de la nappe.

Les deux modèles mentionnés plus haut ainsi que les données météorologiques des 76 dernières années a Ottawa ont été utilisés en vue d'estimer les pertes annuelles de récolte, degré de probabilité de ces pertes ainsi que les pertes annuelles moyennes pour la recolte de maïs.

On a calculé l'augmentation du revenu annuel retiré d'un système de drainage souterrain en retranchant la perte annuelle moyenne de récolte et la coût d'investissement annuel s'équivalent de la valeur d'une récolte intacte.

On a effectué cette analyse pour diverses combinaisons de types de sol, de paramètres économiques et de dimensionnement de systemès de drainage souterrain.

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CONTRIBUTION TO KNOWLEDGE

Although much research work has been done in allied fields on matters dealing with subsurface drainage design, to date only a few of the results have found general applicability. This is due to: failure of some of the assumptions of the theory to be satisfied under field conditions; assuming some pertinent soil water properties to be constant, which, in reality, are variable; and the deterministic nature of most of the analyses, while important causative factors for drainage problems, are stochastic in nature.

In this work an attempt has been made to alleviate some of the above-mentioned shortcomings. This thesis contributes to the knowledge of subsurface drainage design theory and methodology in the following respects:

1. The variability of drainable porosity with respect to water table depth has been taken into account - Table 7.

2. The transient water content in a soil column has been used in the prediction of water table depth - equation (5.3), Figures 9 (a) and (b).

3. It accounts for the variation of actual evapotranspiration with respect to both available water in the soil and potential evapotranspiration, and uses a multiple regression equation between these to predict actual evapotranspiration - equation (5.6).

4. It uses a simple water balance model for water table prediction that incorporates the results of 1, 2 and 3 above.

5. A considerable saving in computer cost has been achieved by reducing the required number of zones of the soil profile to only two.

6. The applicability of the water balance approaches for subsurface drainage design work in the St. Lawrence Lowlands region has been justified by comparing some statistical properties of the distributions of the observed and the predicted water table depths, and by an appropriate statistical test - Table 11.

7. It proposes a method to compute the crop losses due to various depths and durations of water table and uses the distributions of predicted water table depths to compute the probabilities of different annual crop losses for various combinations of saturated hydraulic conductivity and drain spacing - Section 3.2 and 3.3, Table 1 and Figure 14.

8. It has been found, using the limited crop loss data available, that under the conditions of no subsurface drainage, the average annual loss from a corn crop would be a little more than 50 percent of the full production - Figure 15.

9. A method for revenue increase computations from the information obtained from 8 above, has been proposed for selected values of installation cost, interest rate, amortization period and no-loss crop value - equation (5.7) and Table 14.

10. It uses the procedure of 9 above corresponding to different drain lateral spacing - hydraulic conductivity combinations and other pertinent information to calculate average annual revenue increases for different drainage design alternatives - Figures 16 and 17.

11. The results obtained above indicate that the location of the maximum average annual revenue increase with respect to the spacing is insensitive to installation cost, interest rate, amortization period, and no-loss crop value, but is sensitive to soil hydraulic conductivity - Table 15 and subsequent discussion.

The predicted water table depths obtained from the water balance model were found to be close to the observed values, and a statistical test indicated no significant difference between the two at the 99 per confidence level. The accounting for the variability of drainable porosity with water table depth, and of actual evapotranspiration with available water and potential evapotranspiration, is considered to be the main reason for this close agreement.

The results provide quantitative information - hitherto scanty - about the effects of various parameters on the average annual revenue increases from subsurface drainage systems.

The analysis has been done for selected hydraulic conductivities and economic parameters to cover a wide range of possible combinations. From the graphs and tables presented, results corresponding to some intermediate conditions of various parameters can be obtained by interpolation. A constant value for drain depth of 1200 mm has been assumed for the economic analysis. Although, a change in this parameter will alter the predicted water table depths, it is recognized that the flexibility in this parameter is limited, and the selected drain depths in most cases in Quebec are close to the chosen value. In locations where the drains are placed shallow, e.g. 0.8 m, narrower spacings between drain laterals will need to be used.

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LIST OF SYMBOLS AND ABBREVIATIONS

All except the symbols of some standard statistical parameters are listed below. The latter have been defined where they appear. Symbols used in the computer program have been defined in the comment cards at the beginning of the program in Appendix D.1.

AAW	Available water in a soil column
ACI	Uniform annual investment cost
ACL	Cost of average annual crop loss
ACL_0	Cost of average annual crop loss on a field with no subsurface drainage
ADW	Allowable water table depth from soil surface
AE	Actual evapotranspiration
AP	Amortization period
AW	Available water in a soil column (differs in unit from AAW)
AWL	Lower limit of available water
AWU	Upper limit of available water
a	Coefficient
b, β_1, β_2, β_3, β_4	Coefficients or constants
C	Slope of R vs. (K) (HW) curve
c	Constant or coefficient
cc	Cubic centimeter
cm	Centimeter
D	Diameter of drain
DD	Depth of drain from soil surface
d	Height of drain above impermeable layer
f	Drainable porosity
g(h)	A function of h
H_o, H_a	Null and alternate hypothesis
HW	Height of water table from drain level at mid-way between drains

h	Suction or depth of water table from soil surface
ha	Hectare
I	Interest rate
IC	Total cost of subsurface drainage per meter length of installation
i	Subscript
K	Saturated hydraulic conductivity
k	Constant
L	Length of drain
l	Cost of crop loss
m	Meter
mm	Millimeter
NLCV	No-loss crop value
n	Manning's roughness coefficient or subscript
PE	Potential evapotranspiration
p	Fraction to compute crop loss
Q	Maximum drain capacity to flow
R	Steady drainage rate
RCV	Remaining crop value
RE	Average annual revenue increase due to the installation of subsurface drainage system
R_H	Steady drainage rate obtained from Hooghoudt's equation
RMAX	Maximum or design drainage rate
S	Spacing between drain laterals
s	Slope of drain laterals
TR	Remaining transient water in soil column
TRMAX	Maximum transient water content
V	Volume of transient water drained from a soil column of unit cross sectional area, due to a certain water table drawdown
Z	Depth of upper zone of the soil column

I. INTRODUCTION

1.1 Statement and Nature of the Problem

Subsurface drainage is a recognised and increasingly practised method of removing excess soil water. The design of subsurface drainage systems and the physics of flow of water through the soil towards the drains have been extensively studied in the last quarter of the century. With the present state of the knowledge, it is possible to determine appropriate spacing and depth of drains, for either a steady rainfall or a desired rate of water table drawdown. The success of a design is, however, dependent upon how closely the assumptions of the theory are satisfied in a given region.

Because of large spatial variability of many soil water properties and failure to justify the assumptions of the theory under field conditions, the results of exhaustive theoretical research have scarcely been used in practice. Instead, subsurface drainage design has long been based on some simple relationships between the most pertinent variables and in many cases based merely on experience. This has prohibited the designers from attaching a prior economic justification to a certain design alternative.

Various theoretical procedures for design, irrespective of their rigor, are deterministic in nature. This means that they provide solutions for specific values of variables such as a steady rainfall rate or a desired rate of water table fall. However, in reality, the factors such as rainfall and evapotranspiration, which are directly responsible for causing water table fluctuations, are stochastic hydrologic events. Conventional theoretical procedures are at present not well equipped to incorporate this feature of the causative factors. The present theories are also not well equipped to incorporate the continuously changing property of drainable porosity with respect to water table depth changes. These difficulties, and the advent and availability of high-speed computers, have given rise to the water balance approach for subsurface drainage design.

The water balance approach is sufficiently flexible to enable one to consider the stochastic nature of the pertinent variables and to provide solutions to drainage problems on a regional basis by incorporating appropriate values of soil-water-plant parameters.

When the weather data are available for a large number of years, the water balance approach is particularly useful as it enables the computation of probabilities of various depths and durations of water table. This, coupled with the information on crop physiology and its susceptibility to damage due to various depths and durations of water table, makes possible the estimation of probabilities associated with various magnitudes of crop losses and average annual loss of income due to crop damage. This type of information is required when an analysis is aimed at providing an economic justification for a certain design alternative.

In Canada, with a growing awareness among farmers of the benefits of subsurface drainage, substantial government subsidies for drainage installations in some provinces, and a cool moist climate in many regions, the drainage installation rate is increasing annually. A recent study (Broughton, 1976), shows that during the decade from 1964 to 1973, the total land under subsurface drainage in Canada has increased from 0.824 million hectares to about 1.26 million hectares. The present rate of installation is approximately one per cent of the land, needing subsurface drainage, per year. This rate is expected to increase with the availability of more efficient drainage machinery. It has also been estimated, on the basis of prevailing installation costs in 1972 (Broughton, 1972), that approximately $544 million would be needed to install the subsurface drains required in Québec alone.

This situation, therefore, demands that rational guidelines be established with a view to obtaining technically sound, and at the same time economically attractive, design alternatives for subsurface drainage work.

The two major controllable design variables which have significant influence in subsurface drainage performance are the spacing and depth of drains. Since drain depth is often governed by the capacity of the drain-laying machinery and the terrain slopes, spacing becomes the only major controllable variable.

In the St. Lawrence Lowlands region in Canada, relatively impervious layers are found at shallow depths and therefore, according to Hooghoudt's steady state drainage equation (Luthin, 1973), the single soil property of hydraulic conductivity is most important in determining an appropriate drain spacing for a certain design drainage rate. Alternatively, designs with different drain spacing-hydraulic conductivity combinations will result in different subsurface drainage rates, and each combination will give a different pattern of water table fluctuations.

For a given hydraulic conductivity, closer spacing will reduce the chances of high water table conditions, thereby reducing the chances of crop damage. This will yield more income to the farmer. But a closer spacing will also result in a higher installation cost. A wider spacing, on the other hand, will have just the opposite effect on the chances of crop loss and installation cost. The revenue increase from a certain system may be obtained by subtracting the cost of crop loss and the cost of drainage installation from the no-loss crop value. The no-loss crop value is here defined as the market price of the crop, harvested per unit area, under normal growth conditions.

A water balance model operated with appropriate values of pertinent soil-water-plant parameters and economic information such as installation cost, rate of interest and amortization period of the invested money, is expected to result in a series of additional revenue values associated with different drain spacings. From these, it should be possible to select an appropriate spacing for known values of other parameters in a region, that would give a maximum average annual revenue increase to the investor.

It is the purpose of the present study to: determine the pertinent soil-water properties of some soils of the St. Lawrence Lowlans region; develop and test a water balance model; and use an integrated water balance and crop loss model for 76 years of weather data from the Ottawa weather station to analyse the revenue increases from different design alternatives for subsurface drainage systems for corn fields.

1.2 Objectives

The objectives of the present study are:

1. To conduct field experiments to study the influence of potential evapotranspiration and available water on actual evapotranspiration from corn fields and to develop a functional relationship between these variables.

2. To conduct laboratory and field experiments to study the effec of water table depth on drainable porosity and to develop a functional relationship between the two.

3. To develop a simple water balance model, which will use the results of 1 and 2 above, in addition to other pertinent soil-water parameters and climatic data, to predict daily water table depths.

4. To test the above model by comparing its results with the recorded data of water table depths and perform appropriate statistical tests for the comparison.

5. To establish a corn crop loss pattern with respect to various depths and durations of water table, based on available experimental results reported in the literature.

6. To develop a crop loss computation model, which will use the output of predicted daily water table depths from the water balance model, to compute yearly loss of crop.

7. To integrate and run the water balance and crop loss model for 76 years (1900 to 1975) of weather data from the Ottawa weather station for computing yearly losses, probability distribution of yearly losses, and average annual loss of corn corresponding to various hydraulic conductivity - drain spacing combinations.

8. To use several combinations of installation cost, interest rate and amortization period and the result of 7 above for computing average annual revenue increase from subsurface drainage systems in corn fields for different hydraulic conductivity - drain spacing combinations.

1.3 Scope of the Work

The results of the investigations of this dissertation, with respect to actual evapotranspiration, drainable porosity and water balance model are expected to be applicable to the clay and the sand soils of the St. Lawrence Lowlands region having flat topography. The results of the cost and revenue increase analyses are only for corn crops grown in the clay soils of the region to which the precipitation and evapotranspiration patterns of Ottawa apply. The methods should apply to other localities, but detailed results could be different when the weather and the soil data pertinent to the locality are used.

II. REVIEW OF LITERATURE

A considerable amount of research on subsurface drainage design problems has been done during the last 25 years. A substantial part of this work has been done since the advent of high speed electronic computers. It is generally recognised that a unique analytical formulation of the various processes involved in subsurface drainage is very complex, and even if it could be achieved, the improvement over the results obtained by simpler methods, may be insignificant. The recent trend is to use a water balance approach, due mainly to its flexibility to accommodate a wide range of input data and relationships.

The author has therefore elected to review and comment on a few of the major contributions in the field of subsurface drainage design based on an analytical approach, and on the water balance approach.

2.1 The Analytical Approach to Subsurface Drainage Design

Numerous solutions for subsurface drainage design problems are available in the literature, for both steady state and non steady state flow conditions. One of these, which has probably been used most extensively, is Hooghoudt's steady state formula for drain spacing computation (van Schilfgaarde et al., 1956; Luthin, 1973). Hooghoudt's approach combines both radial and horizontal flow theories and considers the effect of convergence of streamlines near the drains.

Van Beers (1965) had given a nomographic solution of Hooghoudt's equation, and more recently, the nomographic solutions have been further generalized by Sakkas (1975). Subsequent to Hooghoudt's formula, which was published in 1940, various theories and interpretations of the subsurface drainage flow processes have been forwarded by a number of research workers.

Aronovici et al. (1946) mentioned four basic parameters to be considered for subsurface drainage. These were, volume of water to be drained, hydraulic conductivity of the medium, hydraulic gradient, and cross-sectional area through which flow occurred. Based on Darcy's law, they obtained the spacing equation:

$$S^2 = 4K \frac{(HW + d)^2 - d^2}{R} \quad \ldots (2.1)$$

where

- S = spacing between the drains
- K = hydraulic conductivity of the medium
- HW = height of the water table above the plane of the drain center lines mid-way between the drains
- d = height of drain center above the impermeable layer
- R = amount of water to be removed from the soil profile per unit time.

The above relation considers horizontal flow only and design values of the spacing S are calculated when the water table is at or very near the soil surface. The convergence effect of flow lines near the drain had not been considered. Therefore, Hooghoudt's equation, which takes into consideration the convergence effect by introducing a term DE (equivalent depth) is considered superior to Aronovici's approach.

A logical step-by-step derivation of equation (2.1) and a discussion on the selection of appropriate value of d and HW for design spacing calculations have also been given by Slater (1950). In spite of the simplicity of the assumptions, however, equation (2.1) was found to give computed values of spacings that were close to the actual spacings in some fields with subsurface drains.

A more rigorous approach was later proposed by Kirkham (1949, 1951). He had used the potential theory of flow and had evaluated the potentials under different boundary conditions. His analysis was done for steady flow under ponded water conditions. No comparison between the results from the theory and field observations was given.

Walker (1952) presented another approach for drain spacing computations under falling water table conditions. Soil porosity was also taken into consideration for the first time. In his approach, the water table recession for an assumed velocity of flow was expressed in terms of the velocity and position of a point on the phreatic line mid-way between the drains. The velocity was then expressed as the ratio of hydraulic conductivity (K) to porosity (f). Spacing was calculated from the required value of water table recession in a given time.

The analysis is simple but the substitution of resultant velocity of flow in terms of the ratio K/f is questionable. In fact, this implies a constant hydraulic gradient in the entire flow region, which is seldom, if ever, realized.

Van Schilfgaarde et al. (1956) have shown that, of the various steady and nonsteady state drainage flow equations, Hooghoudt's equation for spacing computation gives the least percentage difference between the actual and computed spacings.

The falling water table or the nonsteady case of drainage was later considered by Luthin (1959). His spacing formula was expressed as:

$$S = \frac{CK(t_2 - t_1)}{f.\ln(\frac{HW_1}{HW_2})} \qquad . . . (2.2)$$

where

S = drain spacing

C = slope of the curve obtained by plotting flow rate per unit length of drain against the product [(K)(HW)] where HW is the height of the water table above the plane of the drain center lines mid-way between the drains

K = hydraulic conductivity

t_1, t_2 = two consecutive times, and

HW_1, HW_2 = heights of the water table above the plane of the drain center lines mid-way between the drains at times t_1 and t_2 respectively.

The value of C is obtainable from field experiments.

This method does not take into account the convergence of the flow lines near the drains, and it assumes a flat water table. No comparisons with the field data were given to check the validity of the derived equation.

Kirkham (1958), in one of his works on steady drainage under constant rainfall, has given a solution to the flow problem based on rigorous mathematical analysis, but subsequently, Wessling (as reported by Sakkas, 1975) has shown that Kirkham's results did not differ by more than five per cent from those obtained by applying Hooghoudt's equation under similar conditions.

Another evaluation of the various drainage flow equations under nonsteady state conditions was made by Johnston et al. (1965). They have computed the time taken by the water table to fall through a given distance, using the integrated Hooghoudt's equation, the integrated Toksoz-Kirkham equation, the Luthin-Worstell equation and the Van Schilfgaarde equation. The observed data for the water table were taken in fields where spacings between subsurface drains varied from 57 to 365 meters, drain depth varied from 1.65 to 2.13 meters, and drainable porosity varied from 0.0064 to 0.17. The observed times for water table drop were either five days or ten days. The computed values, using the above-mentioned equations, ranged from 2.4 to 17.7 days as against five days of observed time and from 2.5 to 35.3 days as against ten days of observed time. Of the four, Van Schilfgaarde's equation was found to give least deviation from actual observations.

Glover (1966) has applied the theory of heat conduction in soils for an idealised case where the ground water flow follows Dupuit-Forchheimer assumptions. The usual objections to these assumptions, i.e., negligible variation of hydraulic gradient with depth, and horizontal flow, are also applicable to Glover's approach. He, however, has restricted the use of his equation to cases where the height of the water table above the drain is smaller than the depth of the impermeable layer below the water table. This is objectionable because the effect of streamline convergence will be significant under this situation.

More recently, a generalized theory, formulation and nomographic solution of flow to subsurface drains have been proposed by Toksoz et al. (1971, 1971a) for layered soils. Their two papers appear to be a very comprehensive document on the subject. One of the main assumptions in their work is that the head loss under the arched water table is negligible. This leads to the situation of horizontal flow. In the actual field case, since the flow lines are arched, there is a significant head loss, and a smaller quantity of water flows into the drains. For a given water table configuration above the drains, this permits the installation to be done at a wider spacing between the drains. In this respect, a comparison with Hooghoudt's equation indicated that under similar field conditions, the nomographs proposed by Toksoz et al., resulted in 50 per cent less spacing than that obtained using Hooghoudt's equation.

From the above discussion it appears that no single theory can be relied upon to give reasonable results under all field situations. None of the theories discussed above considered the stochastic nature of the input data, which can be taken into account by the water balance approach.

2.2 The Water Balance Approach to Subsurface Drainage Design

The basic idea in the water balance approach is to compute the changes in soil moisture in response to input of precipitation and outputs of evapotranspiration, drain outflow and deep seepage. Mathematical book-keeping equations are used to balance the input and output for some specific time intervals and the process is repeated for a large number of years for which the weather data are available.

Van Schilfgaarde (1965) has mentioned that the design criteria for drainage systems would be more meaningful if they take into account the probability distribution of water table heights rather than considering only a constant water table level (steady state), or an assigned rate of water table fall (nonsteady state). He had calculated the number of times a prescribed water table height was exceeded for a given length of days during a 25-year period. He concluded that the concept of developing frequency distributions of water table heights using a water balance model was important and feasible.

Taylor et al. (1967) have used a water table model to compute water table frequencies corresponding to various arbitrarily selected spacings, varying between 18 and 46 meters, and drainage coefficients of 12.7 and 25.4 millimeters per day. They did not use any particular spacing equation, but had developed regression equations between water table depth and time. Although the method would be limited in application, an important conclusion was that under many situations, an increase in spacing after a certain limit, did not appreciably influence the performance of a drainage system in controlling the water table.

Young et al. (1972) have used a water balance model to compute both soil moisture and water table probabilities. They modified the method earlier proposed by Van Schilfgaarde, by incorporating a feature to compute surface runoff. Also, the soil moisture capacity was divided into two parts, one for an unidentified upper layer and the remaining for the lower layer.

Dividing the soil column into several zones enables adjustments for variable soil-water properties at different depths. Foroud (1974) and Chieng (1975) have used water balance models by dividing the soil profile from surface to drain level, respectively into three and four zones. The former had used the model to obtain design drainage coefficients and the latter had used it to obtain frequency distributions of water table depths for different drainage coefficients and drain depths, in the St. Lawrence Lowlands region in Canada.

In all the studies mentioned above, the predicted water table depths showed reasonable agreement, visually, with the observed data. No tests of significance were done to statistically evaluate the extent of agreement. The comparisons with observed values were done with limited data, sometimes even for a very limited period of a few weeks. It is, therefore, difficult to judge the relative merit of one over the other.

In water balance work, the soil moisture is generally divided into two parts. The portion between saturation and field capacity is called the transient water and the portion between field capacity and wilting point is called the available water. The water table fluctuation is related to the changes in the transient water content. Also, the volume of water drained from the transient storage due to a certain water table drop, or the rise of the water table due to the replenishment of transient water by rainfall, is dependent upon drainable porosity of the medium. Evapotranspiration is assumed to deplete the available water storage. In the models discussed above, drainable porosity has been assumed to be constant while, in reality, it varies with water table depth. Similarly, actual evapotranspiration has either been taken as a constant fraction of potential evapotranspiration (PE), or a variable fraction of PE for different zones of the soil profile. In reality, however, AE is a function of several variables, the most important of which are probably the incoming energy (characterized by PE), water available for evapotranspiration, and stage of crop growth. In the next two subsections, brief reviews of the work done on these two components of the water balance model are presented.

2.2.1 Drainable porosity

A saturated soil block will release different quantities of water as a result of different suctions. This property is generally described by the moisture desorption curve of the soil. In the context of field drainage, without evaporative water loss the suction applied to a soil layer can be taken as the mean depth of water table from the layer. In this study, the two terms, suction and water table depth, have sometimes been used interchangeably.

Taylor et al. (1957) reported that the quantity of water released from the A-horizon (0 to 0.2 m), was five times as large as that from the B-horizon (0.2 to 1.27 m), when the water table was lowered from the surface to the drain level (0.76 m). Based on this result, they have questioned the utility of installing subsurface drains any deeper than is necessary to protect them from the weight of heavy field machines. However, on the basis of a subsequent work by Taylor (1960), the above observation loses its justification. In the latter work he had considered the dependence of water release on the applied suction and had attempted its evaluation by field experiments. Although several researchers, such as Luthin and Worstell (1957) and Luthin and Miller (1957), have studied the dependence of drainable porosity, f, on suction, h, Taylor (1960) appears to be the first to give a simple functional relation between them:

$$f_n(h) = \frac{TR_{n-1} - TR_n}{A(h_n - h_{n-1})} \qquad \ldots (2.3)$$

where

$f_n(h)$ = drainable porosity of a layer of soil situated at a distance of h [= $(h_n + h_{n-1})/2$] from water table,

TR_n, TR_{n-1} = volumes of transient water in the soil corresponding to water table depths of h_n and h_{n-1} from the soil surface, A = cross-sectional area of the soil column,

n = number of arbitrarily chosen intervals.

There are several objections to using this relationship for field situations. Firstly, the equation was developed by conducting experiments on a sand-glycerine system. Secondly, the equation does not take into account the curvature of the moisture desorption curve, and hence will not be applicable for the entire range of suctions which can possibly occur in the field. Also, the nature of the equation does not lend itself to continuous evaluation of f with respect to a variable h.

A more useful concept, presented by Luthin (1973), also permits the computation of the volume of water drained from an initially saturated soil column due to a certain water table drop. According to this, if f can be expressed as a function of h as, $f = g(h)$, then the volume of water drained from a soil column of unit cross-sectional area, when the water table is lowered from h_1 to h_2, can be given by:

$$V = \int_{h_1}^{h_2} f \, dh = \int_{h_1}^{h_2} g(h) \, dh \qquad \ldots (2.4)$$

The above integral is the area under the f-h curve between $h = h_1$ and $h = h_2$. Since f is dimensionless and h has the dimension of length, V will also have the dimension of length. As most drainage design computations are based on a unit area, the length dimension of V is very convenient and is also in a useful form to directly include it in the water balance computations.

Luthin (1973), assuming a linear relation between f and h, has expressed the volume of water drained from a soil column of unit area by the type of relation given in equation (2.4), as:

$$V = c\,(h_2^2 - h_1^2) \qquad \ldots (2.5)$$

where

V = volume of water drained in terms of depth from a soil column of unit area,

c = a constant, and,

h_1, h_2 = initial and final water table depths from soil surface, respectively.

In reality, however, the f – h relations for most soils are far from being linear and the assumption of linearity will give incorrect estimates of the value of V.

In spite of the dependence of f on h and the influence of f on drainage design, most steady state drainage relations do not take f into account in the analysis. For example, Aronovici (1946), Van Schilfgaarde et al. (1956), Jenab et al. (1969) and Toksoz et al. (1971, 1971a). An exception to these is Hooghoudt's steady state relation which was modified by Bouwer et al. (1963). On the other hand, all nonsteady state relations include f in the analysis; for example, Walker (1952), Dumm (1954), Dylla (1966), and Jenab et al. (1969). Researchers such as Vaigneur et al. (1966), Young et al. (1972), Foroud (1974) and Chieng (1975), who have used a water balance approach for drainage design, have mostly included f as a parameter in the water balance model. Exceptions to these are the researchers who have used drain outflow as a predictor of water table heights, such as Bird et al. (1971).

So far, both analytical and water balance procedures for drainage design have used a constant value of f for a particular soil. Some of the more recent workers such as Foroud (1974) and Chieng (1975), have made attempts to consider its variability by assigning to it different values for the successive zones of the soil profile. However, the functional dependence of f on h was ignored.

Attempts have been made to incorporate f in the drainage design by using K/f ratio. Skaggs et al. (1975) have mentioned f to be one of the basic parameters for drainage design, and subsequently Skaggs

(1976) attempted to evaluate K/f ratios by solving differential equations of flow of water in porous media, based on Dupuit-Forchheimer assumptions. He mentioned that the ratio K/f tended to include an overall effect of soil heterogeneity, and since most of the nonsteady state drainage relations contained a K/f term, a field evaluated value of the ratio could be expected to give a better result when spacing calculations are made using such formulas.

An important point which had not been considered in the analytical or water balance approaches for drainage design until now is that a subsurface drainage system is expected to function under a variety of weather conditions for a long period of time. Since drainable porosity is a function of suction, the more the water table recedes, the larger the volume of water which is drained. Therefore, a soil column with a deep water table will have more capacity to store water from a subsequent rainfall event than if the water table were shallow. In other words, a certain amount of rainfall will cause a varying extent of water table rise, depending on the volume of water drained prior to the rainfall event. In nature, the process is further modified by evapotranspiration and due to this, it is common to have a negligible water table rise from a heavy rain occurring after a long dry period. Evapotranspiration does not remain constant at all times of the year. Since it is one of the important components of a water balance model, considerable efforts have been made in the past and are still being made to study the behaviour of this component. In the next subsection, some of the more relevant studies on this subject will be discussed.

2.2.2 Evapotranspiration

The usual practice has been to evaluate potential evapotranspiration, PE, from climatic data, and modify the PE by factors representing the effects of soil and crop, to obtain actual evapotranspiration, AE. Formulas presented by Penman, Thornthwaite, Lowery-Johnson and Blaney-Criddle (as reported by Israelsen et al., 1962), use a different number of climatic parameters. Penman's formula has been considered to be theoretically the most sound, but too complicated for practical use. Often the input data necessary to use Penman's equation are not available. The other three methods are much simpler but are unsuitable for computing daily values of PE.

Smith (1959) has collected data of soil moisture depletion under permanent grass cover and has found Thornthwaite's formula to fit most closely the observed data. He has also concluded that PE alone would be a poor estimator for AE.

Various studies for direct measurement of AE have been done since as early as the beginning of this century. These were lysimetric studies, soil moisture studies and inflow-outflow studies for large areas. Detailed description of the various methods, their uses and limitations, have been given by Jensen (1973). The main limitation is that very few of the methods are suitable for computing AE or PE on a daily basis.

Researchers have also used the water balance approach for estimating AE. For example, Baier et al. (1966) have developed a versatile soil moisture budget model that takes into account PE and available moisture in the soil in computing daily AE. They found that, depending on the type of soil, AE/PE ratios may be independent, linearly dependent, or nonlinearly dependent on available water. Consequently, they proposed the use of a group of curves for AE computation, each of which would be applicable for a different soil type.

Baier's model shows a significant correlation between the observed and the predicted soil moistures. The procedure, however, is quite involved. In water balance studies for drainage, in which AE is only a component of the overall model, using Baier's approach for AE computation may be impractical. Baier's model does not take into account the crop cover, which was found to have a linear effect on AE, by Stuart et al. (1969).

Influence of PE as well as available moisture on AE has been studied by Bouchet (as stated by Solomon, 1967). Solomon gave an expression for AE in terms of solar radiation, AE/PE ratio and available soil moisture. Solomon's approach was basically derived for large areas for time periods of one year. The approach was claimed to be applicable for shorter time periods as well, but no attempt was made to justify this claim using actual data.

The procedure of computing AE from AE/PE ratio has also been used by Saxton et al. (1974, 1974a), with varying degrees of modelling details, and by Ligon et al. (1965), in a much simpler way. Some variation to the latter approach has been made by Jensen et al (1971), who had developed a user oriented water budget model. In this model, AE is computed as a fraction of PE, but the factor used to modulate PE is derived from crop cover, available soil moisture and an exponential function of time.

The importance of available water in determining AE was further confirmed by Ritchi et al. (1972). They considered evapotranspiration as a two-stage process. Stage 1 refers to the condition when the soil is wet and incoming energy limits AE. Stage 2 refers to the condition when soil moisture limits AE. A formula was proposed by Ritchi et al. (1972a) to compute daily AE, taking into account PE and time from beginning of the stage 2 condition. Subsequently, Ritchi (1972) proposed a water budget model to compute daily AE. Although the percentage difference between computed and measured daily AE was large, the cumulative values for the study period of 37 days showed much less difference.

Expressing AE as a function of time or AE/PE ratio, or a combination of the two, has notable disadvantages. Use of AE/PE ratio, in effect, amounts to losing some information, in the statistical sense. Use of time after drying as an indicator of AE is not unique, since the time rate of depletion of soil moisture does not remain constant for all soil types or at all times for the same soil type.

More recently, Rowse (1975) has used an isothermal flow equation incorporating soil water diffusivity to calculate soil water flux. His model is also capable of utilising rainfall input and its redistribution in the soil. The model study was done on a column of sand and the observed and predicted volumetric water contents at different depths were close. Field experimental results were reported for a 2.5 cm layer of surface soil only.

In contrast to using AE/PE ratio, time after start of drying or diffusivity type equations, Shimshi <u>et al</u>. (1975) and Strateener <u>et al</u>. (1975), in their studies on wheat fields, found that for a 90 cm soil layer, the actual evapotranspiration could be expressed as a simple linear regression equation with soil moisture as the independent variable or, as a multiple regression equation with soil moisture and pan-evaporation as independent variables. They also found that, statistically, the equation incorporating soil moisture and pan evaporation, did not give any significant difference in the results over the equation which considered AE as a function of soil moisture alone.

It appears from the above reported work that the general procedure for evaluating AE on a daily basis, is to relate it with climatic as well as soil-water-plant parameters. Climatic factors are reasonably represented by PE and the available soil moisture may be used as an overall index to represent the influence of the soil-water-plant phase of the system.

2.3 Economic Considerations in Subsurface Drainage Design

Broughton (1974), based on his experiment on variable drain spacing, and its effect on controlling water table, has concluded that much of the area in the St. Lawrence Lowlands region could be drained at spacings wider than previously practised and up to about 50 m, giving a substantial reduction in installation cost per unit area.

Higgins et al. (1974) have analysed the existing drainage systems in Nova Scotia and have concluded that the drain spacings in most of the areas were unjustifiably low. They found that the spacings for most of the cases were based on tradition rather than on some scientific approach.

Much research has been done in the past to study the performance of various crops under different water table conditions. Notable among these are those by Hiler et al. (1971), Williamson et al. (1970) and Ritter et al. (1969). These studies indicate that different crops have different degrees of tolerance to high water table conditions. Detailed information on the above and other studies has been compiled by Van Schilfgaarde (1974).

Based on the concept of interaction between spacing-cost and spacing-water table relations, Wiser et al. (1974) have presented an approach for an optimized design of a subsurface drainage system. The design has been considered optimum in the sense that it maximized the amount by which the system benefit exceeded the system cost. The model used for obtaining water table fluctuations was the same as had previously been proposed and used by Van Schilfgaarde (1965). A model to consider the effect of water table depth on crop growth was formulated based on the work of Tovey (as reported by Van Schilfgaarde, 1974). The results of the two models were combined to find the relation between the net benefit and the spacing. The net benefit was found to increase with spacing, to a maximum value, and then drop.

Kraft et al. (1972) have presented a different approach for optimized design of a subsurface drainage system, based on the probability density function of head gradient in soil required to maintain outflow. Methods were presented to compute the statistical parameters of the distribution and these were used to compute the probabilities associated with certain critical values of the head gradient. A critical value of the head gradient was defined as that which must be maintained such that the water table does not come closer to the surface than a predetermined level. This predetermined level was fixed from the information on crop root zone depth. The system cost was expressed in relation to the depth and spacing of drains.

There could be two basic objections to this approach. Firstly, the author's assumption of head gradient to be a function of mean annual rainfall is difficult to justify. The head gradient required for some individual rainfall events may be entirely different than that required for others. Mean annual rainfall may be a poor indicator of individual rainfall events. Secondly, and more important, treating K as a random variable and using its probability distribution may not have much significance in the light of large spatial variability of this soil property. A safer way would be to evaluate K by field measurements, in areas where drainage systems are to be installed.

More recently, a water balance approach for subsurface drainage design has been proposed by Bhattacharya _et al_. (1976). In this approach, the system installation cost and the market value of the harvested crop were compared for drainage systems designed with different drainage rates. Each drainage rate corresponded to a certain spacing, and the water balance model run with a certain drainage rate and spacing, resulted in a particular frequency distribution of water table depths. The frequency distribution of water table depths was used to find the crop loss. A drainage system was considered to be inadequate, and crop loss occurred, if the water table remained closer to 40 cm from the surface for more than two successive days. The above study indicated that a drainage system could be designed with a risk of being inadequate for some years in the stipulated amortization period of the invested money, and yet could yield maximum net benefit to the investor.

The probability distribution and the derived risk concept associated with natural events, and their use in investment decisions have been studied by Angell (1960). He mentioned that probability calculus was based on a form of knowledge derived from the outcomes of a long series of identical past trials, and since no two acts of investments were, or could be identical, investment decisions should not be based on any such form of knowledge or prediction.

Mathematically, Angell's explanation is correct. However, in investment decisions, particularly when there are several alternatives, a rational investor is expected to invest money in an alternative that has a possibility of yielding a maximum weighted average return. It is well known that if a certain alternative has been chosen, only the actual outcome will show the quality of the predictions. This does not make the probability concept invalid, but makes the investor aware of the uncertainties of natural events and provides a tool to partially cope with them.

Installation of a subsurface drainage system involves a large capital expenditure. The farmer, naturally, is anxious to have some idea of the extent of benefit he may eventually expect to realize from such a system. The search, therefore, continues to find better and economically more attractive design alternatives.

In the present work, the author has endeavoured to present a comprehensive procedure which will use available information on weather, soil-water-plant properties and relevant cost parameters, to establish rational guidelines to enable the investor to select an appropriate design alternative which will result in maximum average annual increase in revenue.

III. THEORETICAL CONSIDERATIONS

3.1 Prediction of Water Table Depths using a Water Balance Model

In extensive areas of flat land with cool, moist climate and poor natural drainage, a subsurface drainage system is installed primarily to lower the water table to such depths that the root zone of the crops remains free of excess water. The maximum depth of drain tube installation is limited in most cases by the drain laying machinery, the terrain elevations, and the outlet elevation. The only major variable which can be controlled by the designer is therefore the spacing of the drains. In addition, where the soil is permeable and has relatively large capacity to transmit water for a specific gradient, and rapid drainage is desired, the drain tube diameter and slope may limit the drainage rate. However, in practice, a single drain size is selected for laterals and larger drain tube sizes are selected for collectors, based on some assumed maximum drainage rate, called the drainage coefficient. Once the drain size is fixed, spacing becomes the only major variable to be considered in the design.

The cost of installation increases with narrow spacing and the chances of the water table remaining closer to the surface increases with wider spacing. The designer, therefore, is required to make a compromise between these two interacting factors and provide a design that will produce a most economic system for the investor.

Since one is dealing with hydrologic events of rainfall and evapotranspiration in the design of a subsurface drainage system, no confidence can be attached to any design based on single valued deterministic inputs of rainfall, evapotranspiration and drain outflow. This deficiency can be overcome by studying the actual occurrences of various water table depths in the growing season of a crop. Since recorded water table depth data are very limited, a water balance model can be developed to work with the more abundantly available data on rainfall and evapotranspiration for the purpose of predicting the water table depths. The model is first to be tested with some observed data, and if it is found to represent adequately the few years of recorded data of water table depths, the model is then run for long periods of weather data. From the results thus obtained, frequency distributions or recurrence intervals of various water tables or crop losses associated with various water tables, can be calculated.

The water balance model developed for this study is assumed to be applicable for flat areas such as the St. Lawrence and Ottawa Lowlands region in Canada, with a moist climate and low surface runoff potential.

In modelling the physical system, the soil column is divided into two zones. The depth of the first zone is taken as 400 mm. The depth of the second zone is from 400 mm to the depth of the drains. This division is based on two important criteria, namely,

1. The direct effect of field operations seldom exceeds 400 mm.
2. For most field crops, more than 50 per cent of the moisture is extracted from the top 400 mm of the soil and the rest from the lower depths (Schwab <u>et al</u>., 1970).

Thus, it is assumed that separate moisture balancing for the two zones will adequately describe the physical processes of the entire column from the surface to the drain level.

Each zone is assumed to have two distinct soil moisture storage capacities. These are the available soil moisture storage capacity, AAW, and the transient soil moisture storage capacity, TR. AAW is the moisture held between field capacity and wilting point, and TR is the moisture held between saturation and field capacity. From an initially saturated soil column, moisture depletion takes place due to actual evapotranspiration, AE, and drain outflow, R. AE continues to deplete the TR until the latter becomes zero, after which AE starts depleting the AAW. Drain outflow continues as long as there is TR, after which it becomes zero. In the event of rainfall, any excess for the day, of precipitation over PE, is added to the soil moisture storage. The excess is assigned, firstly, to the AW until the latter is filled, then to the TR until its capacity is reached. Any water remaining after the TR is filled is considered to be surface runoff. When rainfall is zero or less than PE, depletion is in the form of AE and drain outflow.

AE is considered to be related to both PE and per cent of available water, as:

$$AE = a.PE + b.AW + c \qquad \ldots (3.1)$$

where

AE = actual evapotranspiration, mm,

PE = potential evapotranspiration, mm,

AW = volume percentage of available water,

a,b = regression coefficients, and

c = intercept.

Further discussion about this equation will be given in Chapter V.

It was realised that since drainable porosity f is dependent upon water table depth or suction (i.e., volume of water drained from transient storage is dependent upon change and depth of water table) a constant f value cannot be used in predicting water table heights. Let

$$f = g_1(h) \qquad \ldots (3.2)$$

represent the relation between drainable porosity and water table depth. In equation (3.2), $g_1(h)$ is some function of water table depth, h. The actual nature of this relationship is a characteristic of the medium and is generally nonlinear. The total volume of water drained from a soil column, when the water table drops from h_1 to h_2, can be calculated by integrating equation (3.2), as:

$$V = \int_{h_1}^{h_2} f\,dh = \int_{h_1}^{h_2} g_1(h)\,dh \qquad \ldots (3.3)$$

If the functional relationship of equation (3.2) is known or experimentally determined, one can calculate the volume of water drained from a soil column of unit area, for any water table fall from the surface (i.e., h = 0) using equation (3.3). This volume can be subtracted from the total transient water content of the column (i.e., transient water content at saturation) to obtain the remaining transient water at a certain position of the water table. This will yield a relation as:

$$h = g_2(TR) \qquad \ldots (3.4)$$

where h is the suction or water table depth and $g_2(TR)$ is some function of the remaining transient water TR.

The water balance model proposed in this study uses an equation of the type (3.4) to compute the water table depth with respect to the transient water content of the soil column. The problem associated with layered soils can also be handled in the same way by developing these types of relationships for the heterogeneous soil layers separately. As will be discussed later in Chapter IV, this was done for model verification for one of the locations of the study area where the soil encountered had two distinct layers.

The drainage was assumed to follow Hooghoudt's steady state drainage equation written as:

$$R = \frac{4K}{S^2}(2.DE.\ HW + HW^2) \qquad \ldots (3.5)$$

where

R = drain flow rate, m/day,

K = hydraulic conductivity of the medium, m/day

S = spacing between laterals, m,

DE = equivalent depth, a function of d and S, (Luthin, 1973)

HW = height of water table above the plane of drain mid-way between two adjacent drain tubes, m.

These symbols are depicted in Figure 1. Noting from this figure that HW = (DD-ADW), equation (3.5) may be written as,

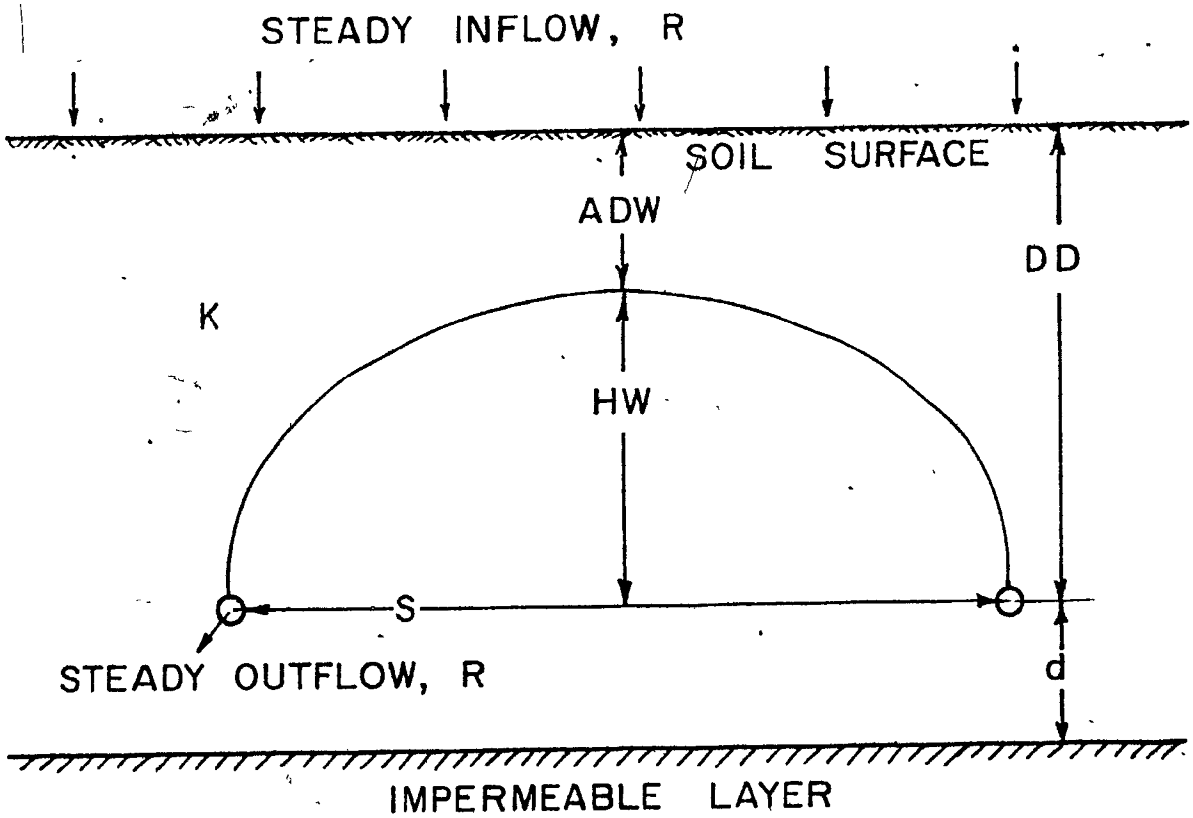


Fig. 1. Schematic representation of steady drainage in homogeneous soil

$$R = \frac{4K}{S^2} (2.DE.\ (DD-ADW) + (DD-ADW)^2 \qquad \ldots (3.6)$$

where

DD = depth of drain, m,

ADW = allowable water table depth from soil surface, m.

For a given region and given spacing, K, S, DE and DD of equation (3.6) are fixed and hence R will be a maximum when ADW = 0. The drain will flow at this maximum rate unless its capacity restricts the flow. Drain capacity can be found from Manning's equation (Chow, 1959) as:

$$Q = (\frac{\Pi}{4}\ D^2)\ [\frac{1}{n}\ (\frac{D}{4})^{2/3}\ (s)^{1/2}]\ \frac{1}{SL} \cdot 86400 \qquad \ldots (3.7)$$

where

Q = maximum drain tube capacity, m^3 per day per unit drained area,

D = diameter of the drain, m,

n = Manning's roughness coefficient,

s = slope of the drain tubes,

S = spacing between adjacent laterals, m,

L = length of a drain tube, m,

86400 = conversion factor from flow rate per second to flow rate per day.

If drain tube capacity is not to be exceeded, the flow rate obtained from equation (3.6) must not exceed the flow rate given by equation (3.7) or $R \leq Q$

$$\text{or } L \leq \frac{86400\ \frac{\Pi}{4}\ D^2\ \frac{1}{n}\ (\frac{D}{4})^{2/3}\ (s)^{1/2}}{4K[2\ DE\ (DD-ADW) + (DD-ADW)^2]}\ S \qquad \ldots (3.8)$$

For a given region, the coefficient of S in the above equation consists of constant parameters. However, since the variability of K is high, the numerical value of this coefficient will vary to a large extent. Some calculations using assumed appropriate values of parameters indicate that the numerical value of the coefficient may vary from 5 for K = 2 m/day, to 100 for K = 0.1 m/day. This range of K values is not uncommon for the soils in the St. Lawrence Lowlands.

It can be seen from equations (3.6) and (3.8) that including ADW will lead to a conservative design in the sense that, for a given drainage rate, either the drain spacing or the length of drain tube for a given spacing will be reduced if ADW > 0.

ADW in the model can be assigned suitable values depending on the types of crops grown and their root zone depths. It appears from the literature (Chieng, 1975; Foroud, 1974; Luthin, 1973) that 400 to 500 mm is a widely used range for ADW and in the present study a value of 400 mm will be used. If the water table depth from the soil surface is closer than ADW, drainage takes place at the rate equal to the capacity of drain tubes, otherwise the drain flow rate is assumed to follow Hooghoudt's equation. If the drain capacity is not reached when the water table is at or closer than ADW, drainage is assumed to take place at a rate corresponding to a water table at ADW. This again will lead to a somewhat conservative design as the drainage flow rate will be taken as being slightly less than actual. However, past experience and the present analysis have shown that the number of occurrences per year for which the water table depths are closer than 400 mm (i.e., ADW) from the soil surface, are only a few and this assumption will therefore not materially affect the results of the analysis.

In the proposed model, the concepts discussed above have been used to obtain the output of predicted daily water table depths.

3.2 Yield Reduction due to High Water Table

A shallow depth of water table from the soil surface as well as the duration for which the water table remains at that depth are important factors in reducing the crop yield. Some quantitative information about the former is available for various crops from the works of Williamson et al. (1970) and Van Schilfgaarde (1974).

According to these, static water levels between 300 and 900 mm from the soil surface may cause various degrees of damage to different crops. The data for corn, as reported by Van Schilfgaarde (1974) are plotted in Figure 2a. The average of the curves of Figure 2a has been plotted in Figure 2b with a modification to give no loss when the water table depth is 750 mm. The lower broken portion of the line in this figure indicates an extrapolation made for the purpose of loss computation at various water table levels starting at the surface. Also, to simplify the computation, the losses for a one-hundred-millimeter interval of water table depths have been assumed to be constant. This is exemplified by the step curve of Figure 2b.

Information on the effect of duration of certain water table depths on yield reduction is primarily qualitative. There is very little quantitative data available. The effects of flooding periods of one to eight days have been studied by various authors for different crops, and it has been generally found that the longer the duration of flooding, the more harmful it is for the crops (Joshi, Dastane, as reported by Williamson _et al._, 1970). Ritter _et al._ (1969) report that flooding at the early stages of growth for four or five days could completely kill corn plants. This implies that the time of occurrence of excessively high water table depth is also important in determining the crop loss. In the absence of more definitive data on the effect of duration of flooding on yield reduction, it was decided to use the step curve of Figure 2b with the following modifications. The reduction of yield indicated for a certain range of water table depths will be taken at 100 per cent if the water table stays in that range for more than seven consecutive days. The losses will be taken as 75, 50, 25 and 0 per cent of the loss indicated by the step curve of Figure 3.2, if the consecutive durations of water table in a given range are respectively six or seven days, four or five days, two or three days, and less than two days. The actual losses to be considered for various water table depths and durations are summarized in Table 1.

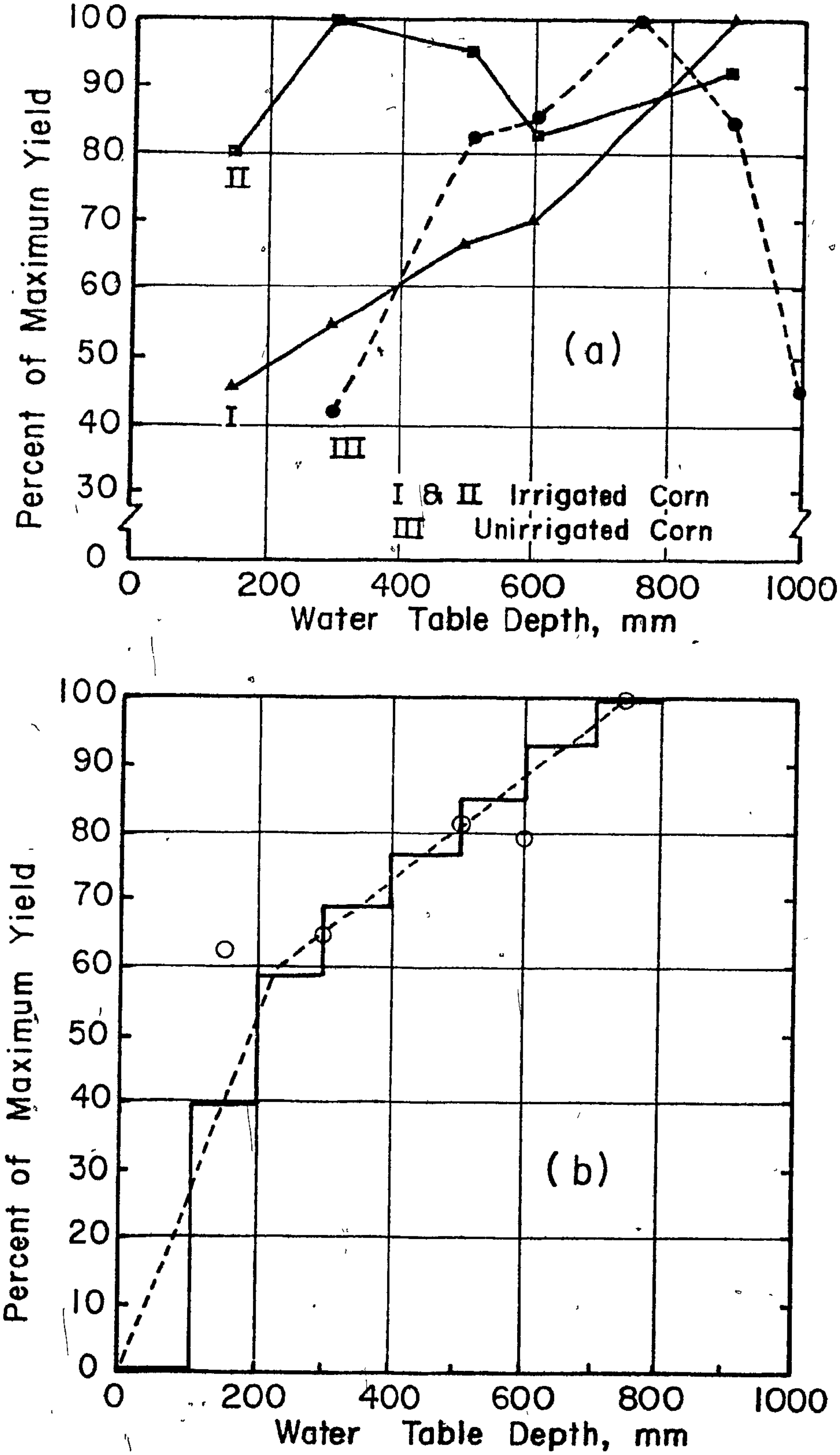


Fig. 2. Corn yield - static water table relation. (a) Individual studies. (b) Average of (a) and proposed step function. (Data for figure 2 (a) are obtained from Van Schilfgaarde, 1974)

TABLE 1. Crop loss for various water table depths and durations

Range of water table depth from soil surface mm	Crop loss in per cent of maximum yield %	Crop loss in per cent of maximum yield for water table at depths indicated in Column 1, and for the following durations				
		Consecutive duration, days				
		< 2	2-3	4-5	6-7	> 7
(1)	(2)	(3)	(4)	(5)	(6)	(7)
100 > h ≥ 0	100	0	25	50	75	100
200 > h ≥ 100	60	0	15	30	45	60
300 > h ≥ 200	41	0	10	21	31	41
400 > h ≥ 300	31	0	8	16	23	31
500 > h ≥ 400	23	0	6	12	17	23
600 > h ≥ 500	15	0	4	8	11	15
700 > h ≥ 600	7	0	2	4	5	7
h ≥ 700	0	0	0	0	0	0

Note: 1. h = water table depth from soil surface in millimeters.

2. Values of Column 2 are the assumed losses for static water table depths as indicated in Column 1.

3. Values in Columns 3, 4, 5, 6 and 7 are respectively, 0, 25, 50, 75 and 100 per cent of the values in Column 2.

3.3 Probabilities of Crop Losses and the Average Annual Revenue Increase Computation

As a result of working the water balance model with weather data, predicted daily water table depths from the soil surface can be obtained for the period under consideration. The general nature of the predicted water table depths has been shown schematically in Figure 3.

From the crop physiological point of view the growing season for corn may be reckoned from 1st May to 31st August. Therefore, any high water table before 1st May and after 31st August, is considered to have no influence on the yield of corn. From Figure 3, it is possible to determine the number of consecutive days the water table had been above a specific depth. Since, from Table 1, it is seen that there is no crop loss when the water table is beyond 700 mm from the soil surface, one needs to consider only those events when the water table had been at or closer than 700 mm from the soil surface.

For example, referring to Figure 3, it can be seen that during some period in the growing season from B to C, the water table was $\leq$ 900 mm from the soil surface for 33 consecutive days and $\leq$ 800 mm from the soil surface for 22 consecutive days. According to the information in Table 1, there will be no loss of crop for these two events. However, there were 18 consecutive days when the water table was $\leq$ 700 mm, 13 consecutive days of water table at $\leq$ 600 mm, and so on, up to one day of water table at $\leq$ 100 mm from the soil surface. For the last event, although the water table came as close to and even closer than 100 mm from the soil surface, according to Table 1, there will be no loss for this event as the water table did not stay long enough at this level to cause any damage to the crop. The losses corresponding to all other events will be computed as follows:

Let us assume that prior to B in Figure 3, some crop has already been lost due to high water tables and the remaining value of the crop, in dollars per hectare is RCV_{i-1}. The loss corresponding to the event of 18 consecutive days of water table being $\leq$ 700 mm from the soil surface is given by:

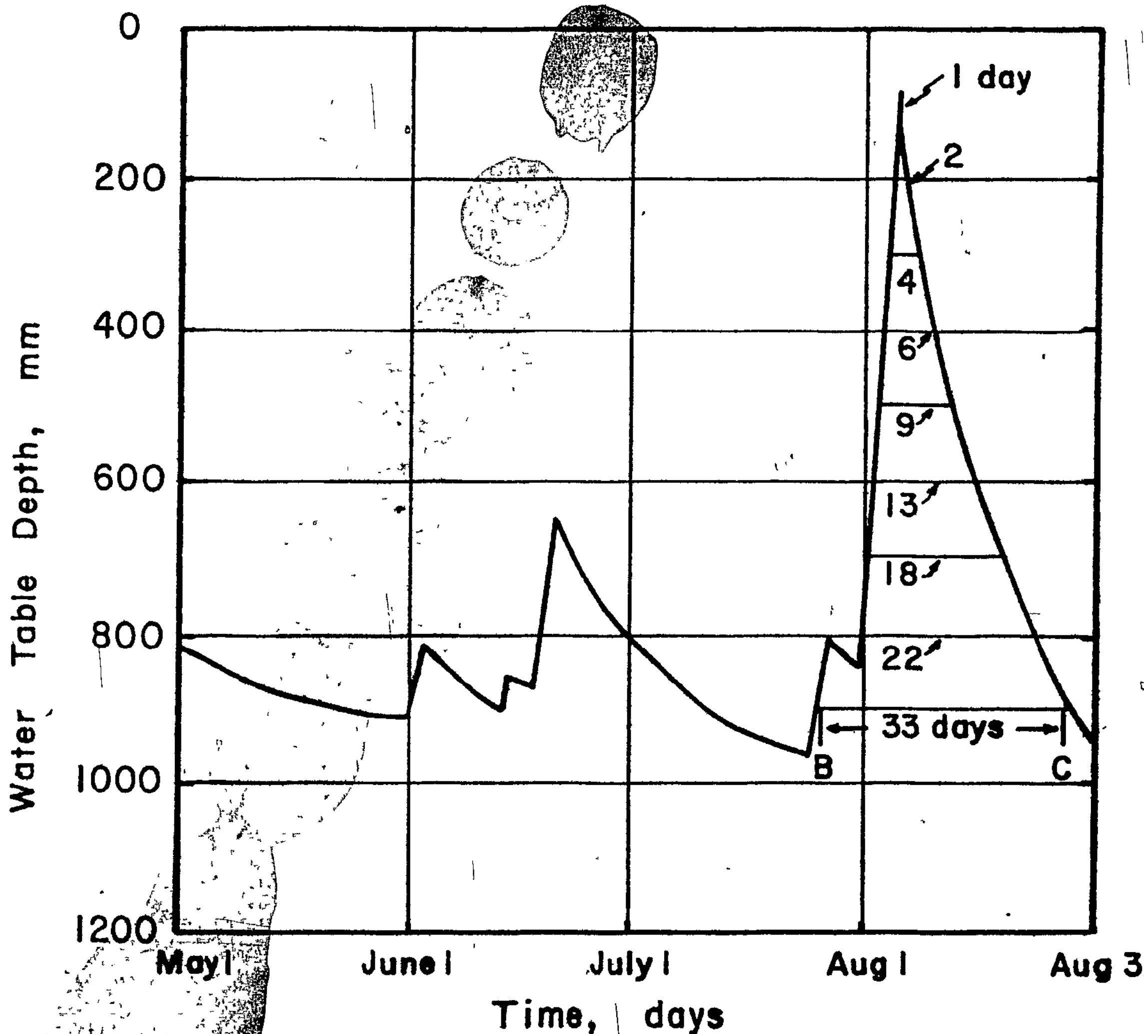


Fig. 3. Sketch of a hypothetical distribution of water table depths in the growing season of corn

$$l_i = (7/100)(100/100)\ RCV_{i-1}$$

$$= 0.07\ RCV_{i-1} \quad \ldots (3.9)$$

where

l_i = loss corresponding to the ith event of the water table being at or above a specific depth for a specific duration,

and RCV_{i-1} is as discussed earlier.

The new remaining crop value is obtained as:

$$RCV_i = RCV_{i-1} - l_i \quad \ldots (3.10)$$

Again from Figure 3, there were 13 consecutive days of water table at ≤ 600 mm from the soil surface, and the loss for this event will be given by:

$l_{i+1} = 0.15\ RCV_i$, and the remaining crop value will be written as,

$$RCV_{i+1} = RCV_i - l_{i+1}$$

In general, the loss and the remaining crop value accounting can be represented by the set of two equations as:

$$l_i = p_i\ RCV_{i-1}$$
$$RCV_i = RCV_{i-1} - l_i \quad \ldots (3.11)$$

where p_i is the appropriate fraction to be obtained from the percentage values given in Table 1, for various depths and durations of water table.

The computations using the set of two equations (3.11) are continued for the growing season to account for all possible losses associated with various high water table events. If this scheme is repeated for a large number of years and total yearly losses are arranged in order of magnitude, the recurrence intervals or probabilities associated with various magnitude of losses can be computed. Since different losses will have different associated probabilities, the average annual loss will be obtained by computing the area under the probability curve (see Figure 14 for example). As has been mentioned earlier, since drain spacing is the most important parameter from the point of view of controlling the water table and in determining the cost of the system, a set of probability curves may be drawn for several spacings, and the area under the probability curve for each spacing will give the corresponding average annual loss. Also associated with a certain spacing will be one installation cost, which can be broken down into an annual cost for an assumed amortization period and a suitable interest rate. The average annual revenue increase due to the installation of a subsurface drainage system at a given spacing will then be obtained from:

$$RE = (NLCV - ACL - ACI) - (NLCV - ACL_0)$$

or,

$$RE = ACL_0 - ACL - ACI \quad \ldots (3.12)$$

where

RE = average annual revenue increase due to the installation of subsurface drainage system,

NLCV = no-loss crop value,

ACL = cost of average annual crop loss,

ACL_0 = cost of average annual crop loss on a field with no subsurface drainage,

and ACI = uniform annual installation cost.

All of the above terms are expressed in dollars per hectare. A plot of revenue increase RE and spacing S, is expected to reveal the spacing at which the average annual revenue increase will be maximum.

The actual sequence of crop loss computation in the model will be slightly different. The model will first select a number of reference levels, starting from the soil surface down to a depth of 700 mm, at intervals of 100 mm. The program will then compare the data of predicted water table depths against the reference level. Whenever the depth of predicted water table is found to be above the reference level for a certain number of consecutive days, the losses corresponding to this event are computed from the given information in Table 1, and the remaining crop value is updated. The computation then proceeds for the same reference level for the remaining number of days in the growing season. At any time, when the predicted water table is found to be above the selected reference level, the losses (if any), are computed.

After the end of the growing season, the whole sequence of computations is repeated for each of the pre-established reference levels. The remaining crop value after all such computations is subtracted from the no-loss crop value to obtain the total cost of crop loss in one year. This scheme will make the computation simpler without altering the results of the computation.

It may be mentioned here that in this study, no attempt has been made to include the effect of inflation on various cost items. It is assumed that inflation equally influences the cost of the system and the selling price of the crop, and therefore, has no overall significant effect. Tweeten et al. (1971) have shown, based on their studies on the impact of input price inflation on the farming industry, for the decade 1958 to 1967, that the net farm income was reduced by four per cent, considering some short periods of one to two years, and by only two per cent, considering larger periods of many years. In this respect, it can be argued that, since the average functional period of a subsurface drainage system is more than 10 years, and since the subsurface drainage installation cost may be considered to be only a part of the total investment in a farming industry, the long term effect of inflation may be relatively small.

IV. MATERIALS AND METHODS

Field and laboratory experiments were set up to investigate two of the variables used in the water balance model, namely, drainable porosity, f, and actual evapotranspiration, AE. The drainable porosity measurements in the field and in the laboratory were done on two soil types, namely, the Upland Sand and the Ste. Rosalie Clay. The particle size distributions of these two soils, sampled from three depths, are given in Appendix A. The objective of the drainable porosity experiments was to develop functional relationships between the water table depths and the transient water contents of soil columns. The field experiments to determine actual evapotranspiration were done on five plots, two of which were in St. Amable Loamy Sand, and one each in the Upland Sand, the Rideau Clay and the Ste. Rosalie Clay. The plot numbers and the corresponding soil types are given in Table 10 of Chapter V. The objective of the evapotranspiration experiment was to relate AE to PE, and AW of a soil column. Using the results of the above investigations, a water balance model was developed. A procedure for crop loss computation was proposed and used, and net benefits from subsurface drainage systems were computed for various design alternatives.

4.1 Drainable Porosity Measurements

Field investigations of drainable porosity and its relations to water table depth were made at two locations, representing somewhat, the two extremes of soil conditions in the St. Lawrence Lowlands region. One of the experimental sites was located at the Macdonald College farm and the soil here was predominantly sandy to about 1.7 m from the surface, then changed to clayey. The other site was located on the farm of Mr. J. P. Martineau, at St. Clet, about 32 km west of Macdonald College. The soil here was clayey from surface to great depths.

In each location, an area of approximately 3 m x 3 m was selected and hydraulically separated from the surrounding soil by plastic sheets, to a depth of 2 m. The necessary digging was done by a back hoe which operated from outside the experimental plot. Before backfilling the trenches, undisturbed soil samples were taken in thin aluminum rings of approximately 5 cm diameter and 2.5 cm height in the sand, and 5.5 cm diameter and 1.5 cm height in the clay, at 0, 5, 10, 15, 20, 25, 30, 40, 50, 60, 80, 100, 120, 140, 170 and 200 cm depths from the surface, with four replicates at each depth, for the determination of bulk densities. Some of the bulk density values were later used to convert soil moisture contents from a weight basis to a volume basis.

Undisturbed samples were also taken from 10, 60 and 120 cm depths, with five replicates at each depth, for laboratory determination of drainable porosity. The trenches were then backfilled. Fourteen water table observation pipes, in two parallel rows, were installed, of which four were inside the plot and ten were outside the plot. The depth of installation of these pipes was 2.2 m. Thirteen tensiometers were installed in the plot, three at each of 15, 30 and 45 cm depths, and two at each of 80 and 120 cm depths. The sketch of the installations is shown in Figure 4.

After a day of completing the installations, a set of initial observations of water table depths and soil moisture tensions was taken. The plot and the surroundings were then irrigated by three sprinklers at an average rate of 6 mm per hour to bring the water table to the surface. This took about four days for the clay soil plot and about five days for the sandy soil plot. The application rate was kept low to reduce air entrapment. After saturation, as indicated by the appearance of the water table at the surface, irrigation was discontinued. A set of five replicates of soil samples was taken at intervals of 5 cm from surface to 30 cm depth; at intervals of 10 cm from 30 to 60 cm depth; and at intervals of 20 cm from 60 to 140 cm

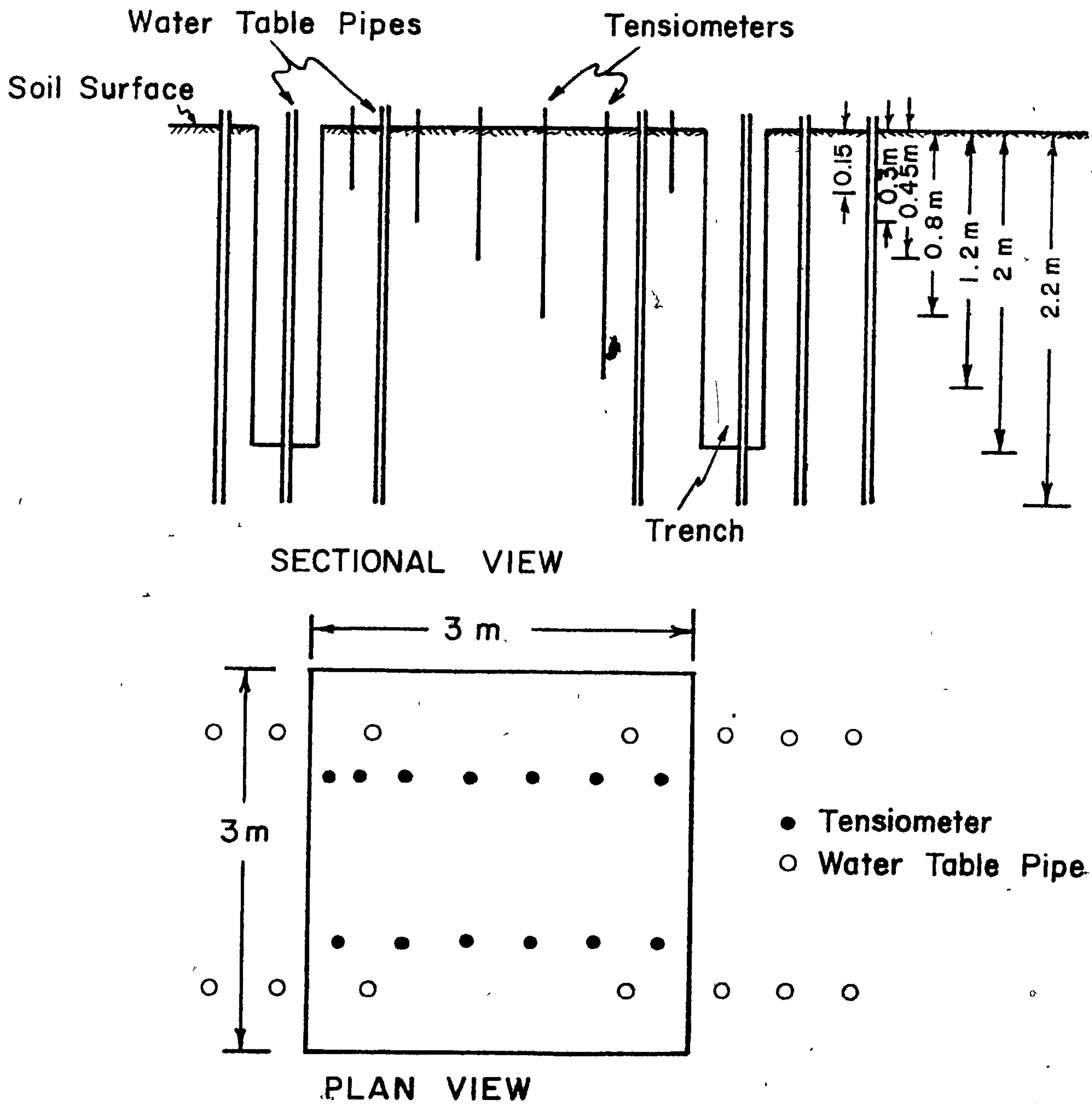


Fig. 4. Details of installations in the experimental plot

depth. These samples were taken by soil auger, and were used for saturation soil moisture content determination. The plot was then covered with two plastic sheets, then by 100 mm thick white styrofoam, and finally by plywood, to eliminate evaporation losses as completely as possible. Subsequent water table and moisture tension observations and soil sampling for moisture content determination at previously mentioned depths, were done at an approximate interval of six hours for the first day, 12 hours for the second day, and once a day until successive water table depressions became negligible. The water table was measured using a graduated blow tube. Soil moisture was measured gravimetrically on a percentage dry weight basis and was converted to percentage volume basis by multiplying the dry weight basis moisture content by the bulk density. The volume basis moisture content values were used to compute field values of drainable porosity.

The laboratory determination of drainable porosity was done on Haines' apparatus, following the procedure outlined by Luthin (1973). The purpose of the laboratory test was to investigate the extent of agreement between the field and laboratory test results. It was thought that in case of a good agreement, laboratory data would be used to empirically relate drainable porosity with water table depth because the suction in the laboratory could be adjusted at various predetermined levels to give more data points. Also, a close agreement between the two would justify future studies on drainable porosity, in other regions, by laboratory tests only, resulting in a substantial reduction in time and cost.

The previously collected samples for determination of f were placed on the saturated porous plate of Haines' apparatus, one at a time, and slowly saturated. A series of water table depths was maintained at suitable levels, up to an elevation difference of 2 m between the sample and the water level. At each water level position, the corresponding volume of water released into the attached burette, was measured after the system came to equilibrium. In the case of the sandy soil, equilibrium was reached approximately eight hours after the position of the water table was fixed. In the case of the clay soil, the time taken to reach equilibrium was about 24 hours. Evaporation from the system was minimized by covering the sample and the burette with plastic sheets with a small hole to maintain atmospheric pressure at both ends. The holes were covered with a wet sponge. The volume of water drained corresponding to a certain elevation difference was expressed as a fraction per unit volume of the soil sample.

4.2 Actual Evapotranspiration Measurements

Five sets of experimental plots, each set having one open and one covered plot side by side, and approximately 2 m x 2 m in size, were selected for AE determinations. These plots were in different soil types (see Table 10 of Chapter V). The effect due to the difference of soil type was expected to be reduced by using available soil moisture, rather than total soil moisture in the analysis. Soil moisture measurements were done gravimetrically, for samples taken at 0, 6, 15 and 25 cm depths from the soil surface. Soil moisture contents at these specific depths were assumed to represent the moisture contents for the depth ranges of 0-2 cm, 2-10 cm, 10-20 cm and 20-30 cm, respectively. Accordingly, the average moisture content for a depth of 30 cm of the soil column was computed by multiplying the moisture contents at the above mentioned depths by the assumed corresponding depth ranges, summing these products and dividing the sum by the depth of the column (i.e., 30 cm). This was done so that the average soil moisture for the whole column was not unduly influenced by the very low values of surface soil moisture as a result of rapid drying. Soil moisture computations were made on a percentage dry weight basis. These were subsequently converted to percentage volume basis by multiplying by the dry bulk density, and then to equivalent depth of soil moisture, by multiplying the volumetric moisture content, expressed as a decimal fraction, by the depth of soil column. AE, in millimeters for one day, was computed by subtracting the equivalent depth of soil moisture of a certain day from that of the previous day.

Available water content, AAW, for a day was expressed as a percentage of total available soil moisture, AW. Total available soil moisture is the difference between the moisture contents at field capacity and at wilting point. These two parameters were determined separately, for the individual plots, based on the soil moisture values observed during the experiment.

Daily values of PE were obtained from the Agrometeorological Tables (Russelo et al., 1974). A multiple regression analysis was done with AE as dependent variable, and PE and AW as independent variables. In the regression model, AE and PE were expressed in millimeters and AW was expressed as a percentage. The relatively dry summer of 1975 allowed approximately 20 days of soil moisture measurements with minor interruptions due to rain.

The determination of total available soil moisture was done separately for each of the experimental plots, for the purpose of developing a relation between AE, AW and PE. In the subsequent work with the water balance model, considerable averaging was done and modifications were made to arrive at the most appropriate values for a large area. This gave a satisfactory performance of the model. This does not invalidate the refinements made for AE determination because the developed regression equation for AE was used in the water balance model.

4.3 The Water Balance Model

The water balance model developed in this study has the primary purpose of predicting daily water table depths. Daily rainfall and PE values are inputs to the model. Other pertinent soil water parameter values are assigned, based on the experiments described. The model does the water budgeting computations to find the transient water content, from which the water table position is predicted. A flow chart, representing the basic operations in the model, is shown in Figure 5. The model, with minor modifications can also be used to compute average soil moisture content for a soil profile, but this was not done in the present study.

Various assumptions for the operation of the model have been described in Section 3.1. One of these, that the input rainfall first fills up the available water storage capacity, was modified by allowing a certain percentage of input rainfall to directly join the transient water storage. This modification resulted in a much better performance of the model in the clay soil area, and is justified from actual evidence in many cases, when the water table was found to rise without saturation of the top layers being attained. The percentage of input directly joining the transient water storage was varied in different computer runs to arrive at a most appropriate value. It was found that for the clay soil, an addition of 50 per cent of the input to transient storage, gave the best performance of the model.

This modification was not required for the sandy soil area. For this area it was found that, in general, where daily water table observations had been made in the past, the soil profile had two distinct layers. The top layer of sand was of approximately 80 cm thickness, underlain by sandy clay. Since observed water table data from this area were used for model verification, two separate f-h relations, and hence two separate relations between transient water content and water table depth were used. Similarly, appropriate values for saturation, field capacity, and wilting point moisture contents were used for the two distinct soil layers in the sandy soil area at Macdonald College farm. The criteria for judging the performance of the water balance model were the comparisons of various statistical parameters of the distributions of predicted and observed water table depths. The statistical parameters used were, the mean, the standard deviation and the sum of the squares of deviations between the observed and the predicted water table depths. It was considered that the

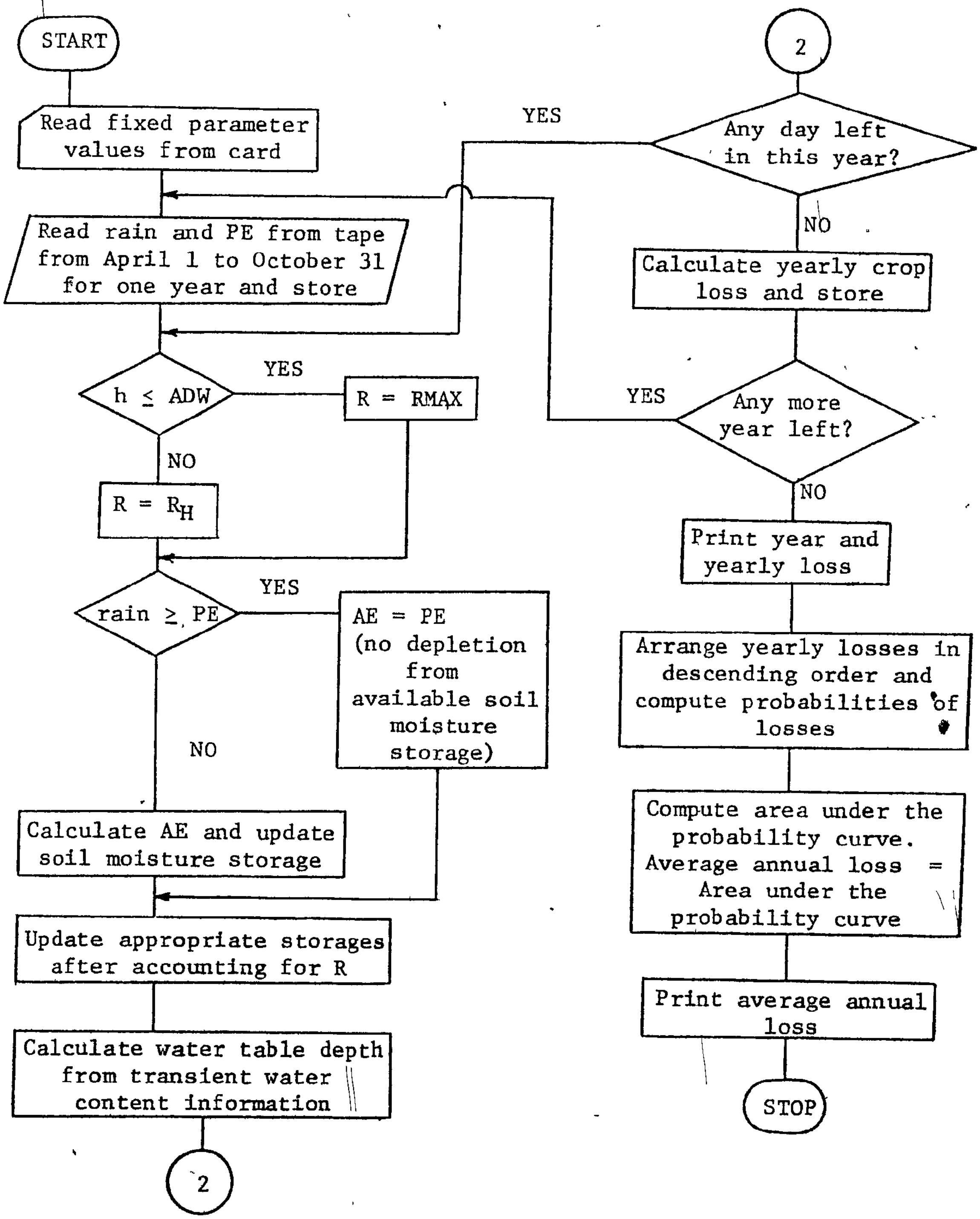


Figure 5. Flow chart for the integrated water balance and crop loss model.

recorded water table depth data were inadequate to establish the nature of their probability distributions, hence a non-parametric test was done to test whether the observed and the predicted water table depths were significantly different. Recorded water table depth data for 1974 and 1976 were used for the comparison and the test mentioned above.

4.4 Crop Loss Computation in the Model

The proposed method of crop loss computation has been explained in Section 3.3. To increase work efficiency, it was decided to include the computer program of crop loss computations at the end of the water balance computations. Accordingly, the combined flow chart has been shown in Figure 5. The crop loss program also calculated the area under the crop loss probability curve. The probability intervals for successive area computation by the average ordinate rule, were kept sufficiently small for a reasonably accurate estimation of the actual area. The area under the crop loss probability curve represented the average annual crop loss, and was expressed as a percentage of the no-loss crop value. The installation cost per hectare was calculated for various assumed values of installation rate per unit of length. Suitable interest rates and amortization periods were used to convert the initial value of the installation cost to an equivalent uniform annual cost. With these information, the average annual revenue increase due to the installation of a subsurface drainage system was computed by using equation (3.12). In this equation, an appropriate value for ACL_0 was selected based on the results of crop loss analyses, and will be discussed later.

V. RESULTS AND DISCUSSION

Broadly speaking, the study reported in this dissertation consists of two parts. In the first part, a water balance model was developed for daily water table predictions and was verified with recorded data. In the second part, the model was used with 76 years (1900 to 1975) of weather data from the Ottawa weather station, to obtain daily water table depths. These results were subsequently used to compute crop losses and the average annual revenue increase due to the installation of subsurface drainage systems designed with different hydraulic conductivities and spacings between laterals. Some experiments were done in connection with the first part of the work, to evaluate some soil water properties to be used in the model, to investigate the drainable porosity and its relationship to suction, and to study the variation of actual evapotranspiration with respect to potential evapotranspiration and available water in the soil.

In this section the results of the experiments will be reported first, followed by descriptions of further analyses done using the experimental results.

5.1 Drainable Porosity

Drainable porosity was determined by both field and laboratory experiments. Laboratory tests were done on constant volume undisturbed samples, During the field experiments, however, due to the large number of samples and the sampling depths involved, volumetric measurements could not be made. Instead, these were determined by multiplying the dry weight basis moisture contents, obtained from samples taken with a soil auger from various depths, by the appropriate dry bulk densities. It was, therefore, necessary to determine the dry bulk densities of the soils at different depths.

TABLE 2. Bulk densities vs. depths in the experimental plots

Depth from soil surface m	Dry bulk density, g/cm^3			
	Upland sand		Ste. Rosalie clay	
	Mean	Standard deviation	Mean	Standard deviation
(1)	(2)	(3)	(4)	(5)
0.00	1.09	0.06	1.04	0.11
0.05	1.10	0.10	1.20	0.15
0.10	1.19	0.08	1.21	0.11
0.15	1.27	0.02	1.38	0.23
0.20	1.27	0.03	1.36	0.21
0.25	1.32	0.12	1.38	0.22
0.30	1.25	0.10	1.46	0.11
0.40	1.18	0.07	1.41	0.13
0.50	1.05	0.14	1.37	0.07
0.60	1.12	0.11	1.41	0.09
0.80	1.34	0.07	1.45	0.05
1.00	1.42	0.04	1.28	0.17
1.20	1.49	0.12	1.33	0.12
1.40	1.57	0.04	1.16	0.10

Note: each entry in columns 2 and 4 is an average of four replications.

Table 2 presents the averages of the observed values of bulk densities at different depths and their standard deviations in the two plots selected for drainable porosity measurements. The standard deviation values indicate the variability of bulk densities within the respective experimental plots. To investigate further the variability of bulk density with respect to depths and locations, an analysis of variance was done using the average values of the columns 2 and 4 of Table 2. The results of the analysis are given in Table B-1 of Appendix B.

From the analysis of variance it was found that the average bulk density did not significantly vary with respect to depth or location, at the 99 per cent confidence level. This may lead one to assume, incorrectly, that the soil is homogeneous with respect to depth and location, in so far as the bulk density and other related properties are concerned. From Table 2, one can calculate the largest difference between the bulk densities of two soil layers of the same plot to be 33.1 per cent (see values of column 2 at 0.5 and 1.4 m depths). Also, one can calculate the largest difference between the bulk densities of two locations at the same depth to be 26.1 per cent (see values of columns 2 and 4 at 1.4 m depth). Since bulk density values are used as multipliers for the weight basis moisture contents to obtain the volume basis moisture contents, the latter values, for different soil layers or locations, will give a larger magnitude of total difference. This will result in incorrect estimates of drainable porosities for soils at different layers and locations, if such estimates are made using a single average value of the bulk density.

One alternative to tackle this problem is to carry out the water balance computations, simultaneously for a large number of zones, where each zone is homogeneous in bulk density. But this is not generally done because of very high computer costs involved in water balance computations for a large number of zones, and also because the bulk density variations with respect to depth are not unique for all locations. A certain amount of grouping and averaging is often done, and this sometimes is a reasonable alternative to the use of a large number of zones and is superior to using an average value of bulk density for the whole column. Also, operating a water balance model for a large number of zones is impractical and often unnecessary, whereas some of its components may be more correctly evaluated by taking the zonal variation of some of the soil water properties into account. It was therefore decided to use the information in Table 2 for the drainable porosity computation, rather than using these directly in the water balance model.

The soil moisture and water table depth variation data obtained during the field experiments are shown in Tables 3 and 4, for the Upland sand and the Ste. Rosalie clay plots respectively. Results for down to 40 cm depth have been reported, as these will be later used for f calculations. Calculation of f for lower layers was not done because of the limited suction obtainable under field conditions. The data in Tables 3 and 4 were obtained by multiplying the observed per cent dry weight basis moisture contents by the appropriate average bulk densities for each of the layers considered.

The data in Tables 3 and 4 are not in a convenient form to compute drainable porosity for different soil layers because the water table depths of column 2 were measured from the soil surface. The average suctions applicable to the soil layers will be different to those reported in the tables. These data were, therefore, plotted in Figures 6 and 7 respectively, to enable the determination of actual water table depths with respect to the center of the soil layers. To emphasize the effects of water table drawdown on soil moisture changes, the data for the former have been plotted directly under the latter, over the same time scale. The averages of the tensiometer readings, expressed as suctions in meters of water at the soil surface, have also been plotted along with the water table drawdown curves. A very close agreement is observed between the two in the case of the sand.

TABLE 3. Variation of water table depth and soil moisture in the drainable porosity experimental plot in the Upland sand

Time since start of drainage days	Average water table depth m	Average soil moisture content in per cent by volume for 10 cm soil layers at the following depths			
		Depth of soil layers from the surface, cm			
		0-10	10-20	20-30	30-40
(1)	(2)	(3)	(4)	(5)	(6)
0.00	0.00	30.60	39.10	43.70	39.80
0.35	0.60	22.98	29.16	33.75	31.86
0.57	0.68	18.77	22.71	26.33	25.88
0.97	0.73	19.08	21.09	19.47	19.81
1.31	0.80	18.09	20.70	16.14	15.67
2.00	0.89	17.68	20.29	17.32	11.54
3.21	1.00	16.67	16.10	10.80	9.11
4.25	1.07	16.42	17.56	12.51	9.36
5.29	1.08	15.95	16.89	13.66	9.54
8.00	1.20	16.01	16.45	10.74	7.41
11.04	1.29	15.00	17.56	13.17	7.90

TABLE 4. Variation of water table depth and soil moisture in the drainable porosity experimental plot in the Ste. Rosalie clay

Time since start of drainage days	Average water table depth m	Average soil moisture content in per cent by volume for 10 cm soil layers at the following depths			
		Depth of soil layers from the surface, cm			
		0-10	10-20	20-30	30-40
(1)	(2)	(3)	(4)	(5)	(6)
0.00	0.00	49.25	52.11	48.22	43.43
0.27	0.09	48.30	51.36	45.08	41.73
0.52	0.23	47.35	46.45	39.86	41.10
0.77	0.39	46.15	45.70	40.28	40.18
1.04	0.53	46.35	42.90	36.96	39.67
2.02	0.89	46.49	45.15	37.55	38.51
3.05	1.12	42.78	40.17	34.72	36.79
4.07	1.19	43.51	38.35	36.05	37.51
5.02	1.24	43.18	40.38	34.80	40.64
8.01	1.30	43.93	42.38	36.11	37.89

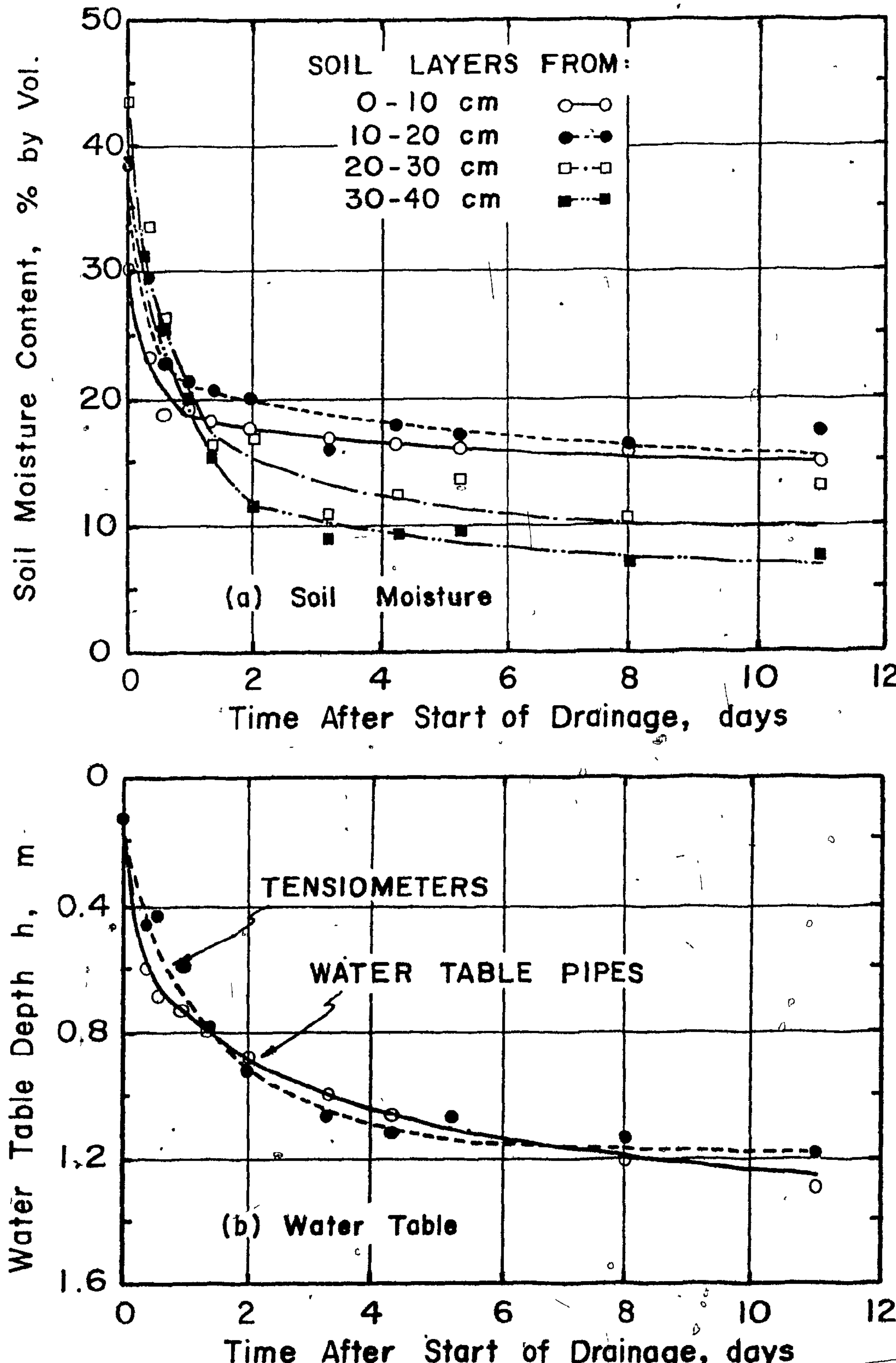


Fig. 6. Water table and moisture content variation with time in the experimental plot in Upland sand

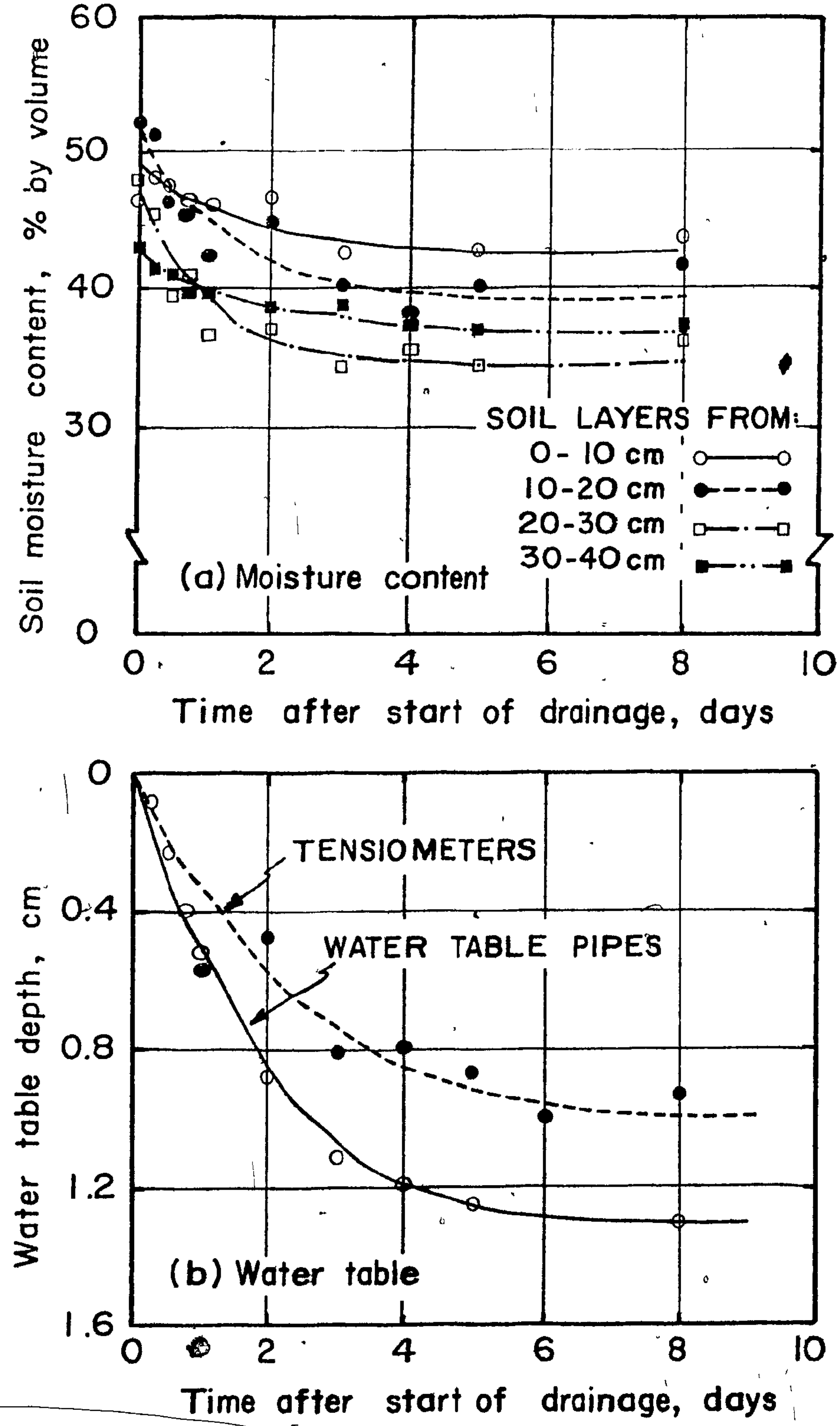


Fig. 7. Water table and moisture content variation with time in the experimental plot in Ste. Rosalie clay

In the case of the clay the average suctions indicated by the tensiometers were lower than the water table depths. This is not unusual because in clay, during tensiometer installation, the soil gets smeared and becomes compacted around the tensiometer cup. This results in slow depletion of water in the immediate vicinity of the tensiometer cup compared with the surroundings. The apparent suction at the tensiometer cup will, therefore, be lower than the actual suction in the soil at the same level but away from the tensiometer. For this reason, suction for the soil layer at a certain depth was taken as the mean distance of the water table from the soil layer. The mean distances of the water table for various soil layers at different times from the start of drainage were obtained from Figures 6 and 7, for the sand and the clay respectively.

Referring to these figures, it is seen that both the water table drawdown and the transient soil moisture depletion are very rapid in the first 24-hour period of drainage. There is very little relative change in the transient storage after the second day of drainage. This indicates that the time taken to attain field capacity for the soils studied may be roughly taken as 48 hours. This also indicates that the movement of a large part of the transient water is fast and corresponds well to the water table drop. A unit time period of one day, chosen as the time interval for water balance computation seems, therefore, to be reasonable. This was further corroborated by an examination of rainfall and water table fluctuation data from previous years, which showed that the time difference between rainfall and water table rise (if there was a water table rise), was at most 12 hours.

Using Figures 6 and 7, the drainable porosities of soil layers of indicated depths, for a certain water table position, were computed by subtracting the volumetric water contents at the specified water table depth from the volumetric water contents at saturation. These were then expressed as a decimal fraction. The results of these computations for the four layers and at various water table depths are

TABLE 5. Field and laboratory values of drainable porosity for the Upland sand

Mean water table depth m	Field values of drainable porosity f in fraction, for 10 cm soil layers at the following depths				Average field value of f fraction	Average laboratory value of f fraction
	Depth of soil layers from the surface, cm					
	0-10	10-20	20-30	30-40		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.20	0.026	0.016	0.037	0.073	0.038	0.019
0.40	0.051	0.071	0.167	0.198	0.122	0.094
0.60	0.101	0.179	0.272	0.291	0.211	0.218
0.80	0.126	0.201	0.317	0.318	0.241	0.269
1.00	0.141	0.221	0.337	-	0.233	0.284
1.20	0.156	-	-	-	-	0.292
1.50	-	-	-	-	-	0.297
2.00	-	-	-	-	-	0.302

Note: The blanks indicate that the corresponding mean water table depths were not realized in the field.

TABLE 6. Field and laboratory values of drainable porosity for the Ste. Rosalie clay

Mean water table depth m	Field values of drainable porosity f in fraction, for 10 cm soil layers at the following depths				Average field value of f fraction	Average laboratory value of f fraction
	Depth of soil layers from the surface, cm					
	0-10	10-20	20-30	30-40		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.20	0.023	0.061	0.090	0.037	0.053	0.029
0.40	0.030	0.073	0.105	0.044	0.063	0.065
0.60	0.040	0.095	0.111	0.049	0.074	0.093
0.80	0.048	0.109	0.127	0.058	0.086	0.106
1.00	0.056	0.121	0.132	-	0.103	0.111
1.20	0.063	-	-	-	-	0.115
1.50	-	-	-	-	-	0.117
2.00	-	-	-	-	-	0.117

Note: The blanks indicate that the corresponding mean water table depths were not realized in the field.

shown in Tables 5 and 6, for the sand and the clay, respectively. The last column of these tables shows the drainable porosities obtained from laboratory experiments for various water table depths. Each of the laboratory experiment values is the average of 15 samples, made up of five replications for samples at each of the three depths of 10 cm, 60 cm, and 120 cm, from the soil surface.

It is to be noted here that, strictly speaking, the average values of f from the field and laboratory experiments are comparable when the relative change of water table depth in the field for successive days became negligible. Intermediate values from the field experiments do not represent the drainable porosity under equilibrium water table conditions, since the water table could not be treated as static for any extended period of time during the field experiment. Consequently, the field values of drainable porosity, should normally be lower than the corresponding laboratory values. This is seen to be violated in three cases in the data presented in Tables 5 and 6. This could be partially attributed to the averaging effect in the field experiments. While the field values are the averages over a depth of 10 cm of soil, the laboratory values are the averages over only 2.5 cm of soil for the sand, and 1.5 cm of soil for the clay. The general agreement between the field and the laboratory results is quite close, particularly at larger water table depth values.

5.1.1 Drainable porosity - water table depth relationships

Since a large number of data points were available from the laboratory tests (only partial results have been shown in column 7 of Tables 5 and 6), these were used to obtain a functional relationship between drainable porosity f, and water table depth h. A fortran computer program for non linear least squares curve fitting (Daniel et.al 1971) was used for this purpose. For the clay, a combination of exponential terms was found to result in a close agreement between the predicted and the observed data. This, however, did not work for the sand, as the plotted data for the sand indicated substantial change of curvature between the initial and final values of water table depths. From the literature (Daniel et.al 1971), a relation of the type,

$$f = 1/(a + b^{-ch}) \qquad \ldots (5.1)$$

where a, b, and c are constants, seemed to represent the plotted data of f and h most closely. This form was, however, modified to:

$$f = [1/(a + b^{-\beta_1(h-k)}) - 1/(a + b^{\beta_1 k})]/\beta_2 \qquad \ldots (5.2)$$

where a, b, c, β_1, β_2 and k are constants.

This modification for the sand was necessary for two reasons. Firstly, it is seen from the laboratory data of Figure 8a that the curvature changes distinctly between h = 0.4 m and h = 0.6 m, making it necessary to change the exponent of b in equation (5.2) from positive to negative in this region of h. This was achieved by changing the exponent of b from $-ch$ to $-\beta_1(h-k)$. The parameter k will have the dimension of h, and will indicate the water table depth at which the f - h curve changes its curvature. Secondly, an additional term was introduced to equation (5.2), to force f to be zero at h = 0. The value of k was taken as 0.5 m for the present case and a and b were fixed after some initial trials as a = 1 and b = 2. The parameters β_1 and β_2 were found by the curve fitting program. The f - h equations, obtained for the two soil types, are shown in Table 7.

The field and laboratory experimental data of drainable porosity and the curves fitted by non linear regression are shown in Figures 8 (a) and (b), for the sand and the clay respectively. The variability of f has been emphasized by drawing the mean ±1 standard deviation curves for the laboratory data. These figures help us to conclude that for the soils studied, the field values of f can be reasonably approximated by laboratory data, the sand and the clay soils have distinct f - h relations, these relations are curvilinear,

TABLE 7. Non linear regression equations between dependent variable f and independent variable h

Soil type	Type of equation	Values of fitted parameters
(1)	(2)	(3)
Upland sand	$f = \left[\frac{1}{1 + 2^{-\beta_1 (h-0.5)}} - \frac{1}{1 + 2^{0.5\beta_1}} \right] / \beta_2$	β_1 = -11.6466 β_2 = - 3.2993
Ste. Rosalie clay	$f = h(e^{-\beta_1 h} - e^{-\beta_2 h}) + \beta_3 (1 - e^{-\beta_4 h})$	β_1 = 0.4154 β_2 = 0.3484 β_3 = 0.3305 β_4 = 0.6480

Note: f is in fraction and h is in meter.

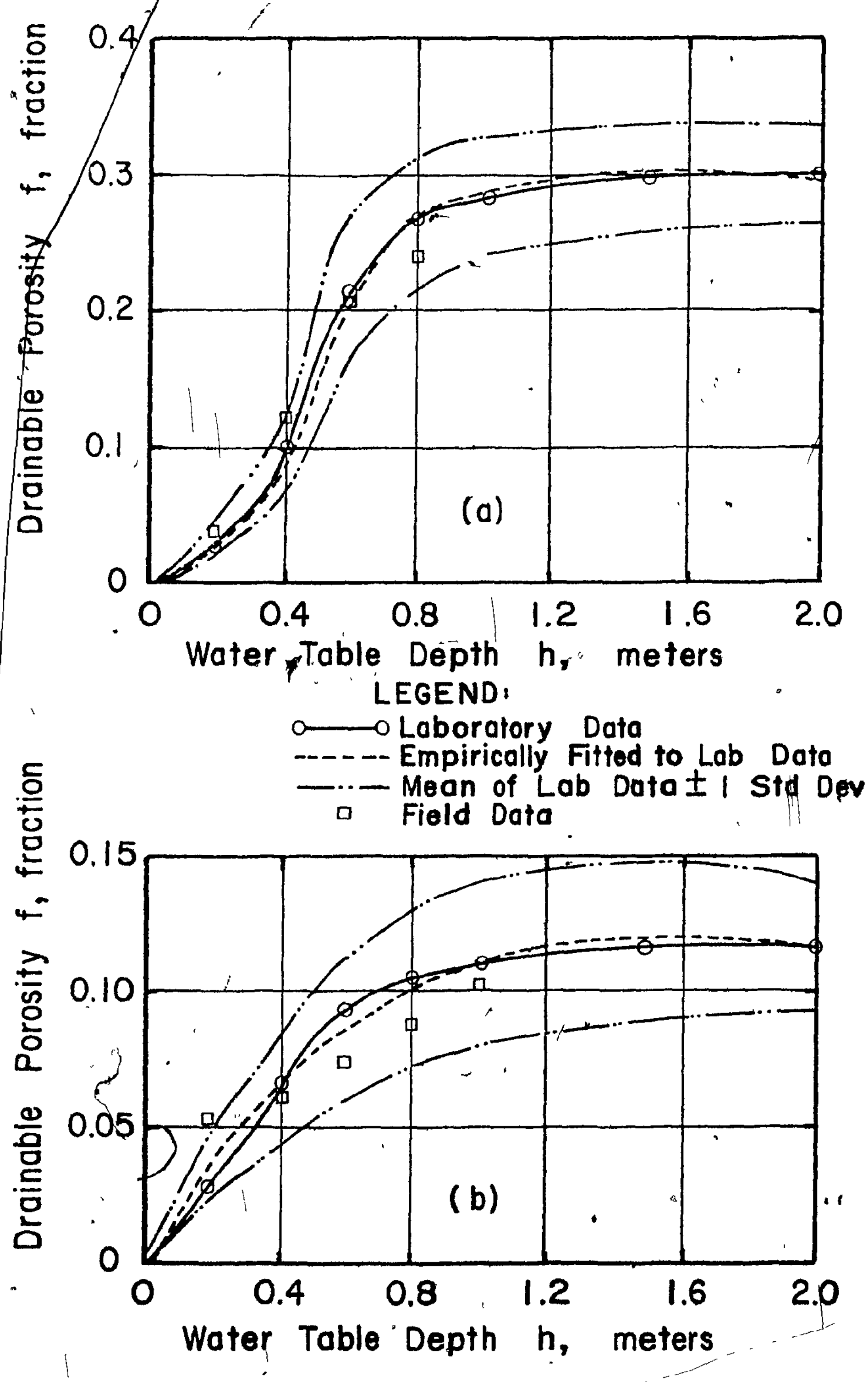


Fig. 8. Water table depth - drainable porosity relations. (a) Upland sand (b) Ste. Rosalie clay

almost all transient water is drained within a suction of 2 m of water, and the empirical f - h relations of Table 7, adequately describe the variation of drainable porosity with water table depth.

It must be mentioned here that the f - h relations developed in this study are applicable for a maximum suction of 2 m of water. However, the water table in the St. Lawrence Lowlands region in general, and the study area in particular, seldom drops below 2 m from the soil surface. In addition, the actual depths of subsurface drain installation are generally less than 1.5 m. Thus the f - h relations developed here can be applied with reasonable confidence in this region for the soil types mentioned in Table 7.

5.1.2 Water table depth - transient water content relation

The f - h relations discussed in the previous section were proposed to be used for computing the volume of water drained due to a certain water table drop, and to relate the remaining transient water content in a soil column of known depth to the position of the water table. For these reasons, the selections of the functional forms of the relationships between drainable porosity and water table depth were so made that they are analytically integrable. Integration of the relation for the clay is direct and simple, while the integration of the relation for the sand can be done by appropriate substitution. The results of the integrations are given in Appendix C. These results were used to compute the volumes of water drained from a saturated soil column due to the various extents of a drop in the water table from the soil surface. These volumes were subtracted from the maximum transient water capacity for a soil column of known depth to obtain the remaining transient water content in the soil column. Results of these calculations are shown in Tables 8 (a) and (b) for the sand and the clay respectively. In order to arrive at the values shown in these tables, appropriate saturation and field capacity moisture contents for the whole column had to be established. This was done by referring to the

TABLE 8. Water table depth and remaining transient water in soil columns of different depths

Water table depth m	Remaining transient water depth TR, mm, within soil columns of indicated depths						
	Soil column depth, m						
	0.4	0.6	0.8	1.0	1.2	1.4	1.6
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(a) Upland sand							
0.0	121	181	242	302	362	423	483
0.2	119	179	240	300	360	421	481
0.4	110	170	231	291	351	412	472
0.6	82	140	201	261	321	382	442
0.8	42	92	151	211	271	332	392
1.0	14	44	95	154	214	275	335
1.2	3	14	45	95	154	215	275
1.4	1	3	15	45	95	155	215
1.6	0	1	4	14	45	96	154
(b) Ste. Rosalie clay							
0.0	48	72	95	118	142	165	189
0.2	44	68	91	114	138	161	185
0.4	34	58	81	104	128	151	175
0.6	23	43	66	89	113	136	160
0.8	14	28	47	70	94	117	141
1.0	8	17	29	49	73	96	120
1.2	4	9	17	30	50	73	97
1.4	1	4	8	16	30	49	73
1.6	0	2	3	7	16	29	49

Note: 1. First rows of Tables 8 (a) and (b) represent the maximum transient water storage in millimeters per square centimeter of soil columns of the indicated depths.

2. Maximum transient water content in millimeters is equal to [(saturation moisture content, per cent by volume - field capacity moisture content, per cent by volume)/100] x (depth of soil column in millimeters).

soil moisture depletion values from the field, and saturation soil moisture content values from both field and laboratory experiments, and by consulting the related work done on similar soils by Lake (1968) and Shrivastava (1968). The final average values selected, and considered applicable for the whole column are given in Table 9 below. This table also shows the average wilting point moisture content values used in the water balance model computations.

TABLE 9. Some average soil water properties used in the present study

Soil type	Saturation moisture content % by volume	Field capacity moisture content % by volume	Wilting point moisture content % by volume
(1)	(2)	(3)	(4)
Upland sand	46.1	15.9	12.1
Ste. Rosalie clay	53.0	41.2	35.0

In the water budgeting computations for drainage design in humid areas, it is generally assumed that the lower limit of the water table position is the drain depth (Foroud, 1974; Chieng, 1975). Values of appropriate columns in Tables 8 (a) or (b) can be used, depending on the proposed depth of the subsurface drains. It will, however, be necessary to use the values selected, to derive a suitable functional form to enable one to predict water table depth from a known value of transient water content. Plots of these values, given in Figures 9 a and b, indicate that for a soil column of given length, a relation of the type:

$$h = a(TRMAX - TR)^b \qquad (5.3)$$

where

h = water table depth,

TRMAX = maximum transient water storage capacity of a column,

TR = remaining transient water content in the soil for a water table depth of h, and

a, b = constants,

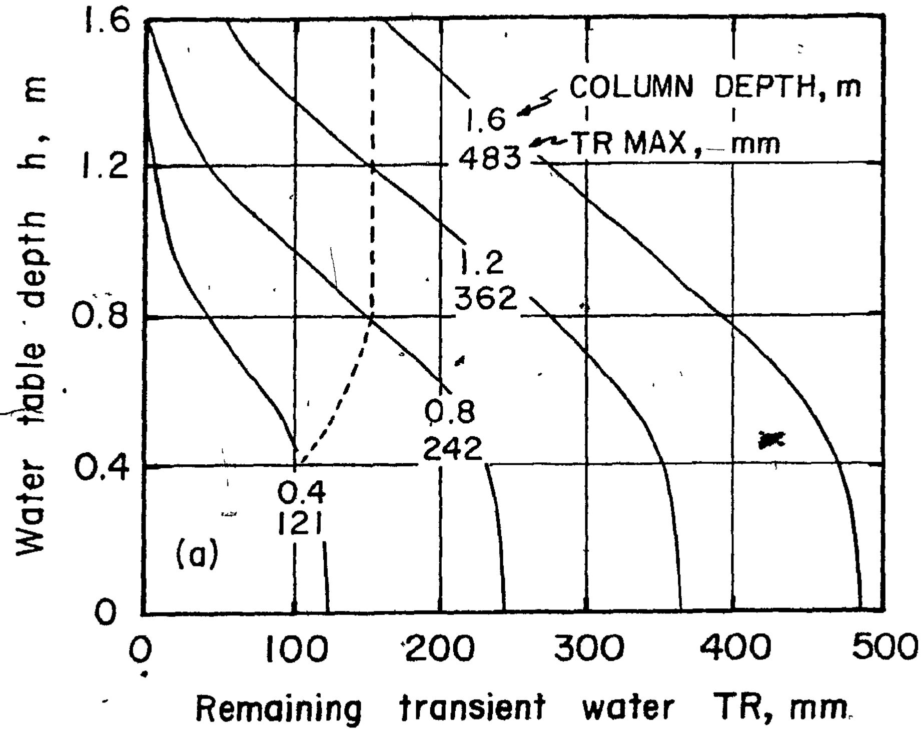
will adequately represent the water table depth - transient water content relation, as long as the water table remains within the soil column. For example, for a clay soil column of 1.2 m depth, the relation will be,

$$h = 0.083(143-TR)^{0.5896} \quad \ldots (5.4)$$

In equation (5.4), h is in meters and TR is in millimeters. The fit for this type of equation starts deviating from the actual points when the water table depth exceeds the depth of soil column. It is clear that this situation will not arise for a homogeneous soil, because the depth of the water table is not allowed to go beyond the drain depth in the water balance model. In the case of layered soils, the best approach is to use two equations for the two distinct sections, above and below the dashed lines of the plotted curves of Figures 9 (a) and (b). A relationship such as equation (5.4) is only applicable to the portion of the curves below the dashed lines in the above mentioned figures.

The results discussed in this section draw our attention to the fact that the drainable porosity - water table depth relations can be advantageously used to compute the remaining transient water content in a soil column for a certain water table depth. In addition, it has been demonstrated that simple expressions may be used to describe the water table depth - remaining transient water content relationship. For heterogeneous soils, more than one function may be necessary.

The main assumption, implicit in the derivation of the various relations, is that the water table depth and the transient water content are in a form of dynamic equilibrium with each other. This assumption is reasonable for the sand and for the well structured clay with interconnected cracks. For compact or massive clay, the movement of water will be slow and the assumption will be unjustified. Also, the tables



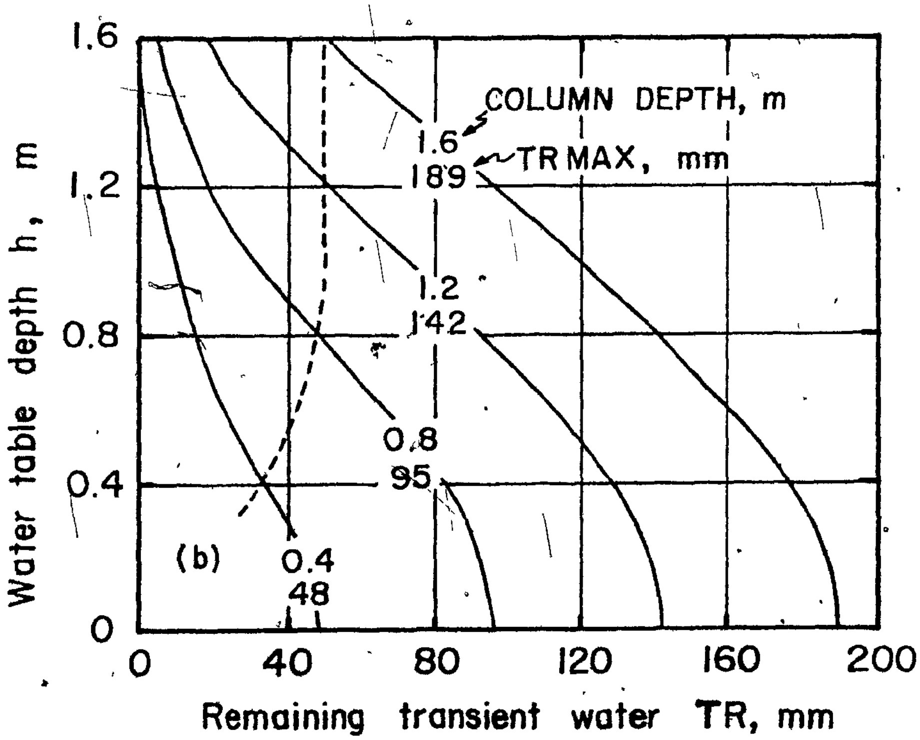


Fig. 9. Water table depth - transient water content relations. (a) Upland sand. (b) Ste. Rosalie clay

and graphs given in this section are for the average soil water properties for the study area. However, these properties can be determined for particular soils, and hence modification of the numerical values in the tables and the orientation of the graphs can be accomplished for some other soils.

5.2 Actual Evapotranspiration

Experiments for AE determination were conducted on two sets of five plots, each approximately 2 m x 2 m in size. After a heavy rainfall in early June in 1975, one set of five plots was covered. These plots attained constant moisture contents within two to three days after the rainfall.

These soil moisture content values were taken as field capacities, or the upper limits of available soil moisture for the soils of respective plots. The other set of five plots was kept open, and they were found to reach almost constant low soil moisture contents in about 20 days time. These low values were taken as the wilting points, or the lower limits of available soil moisture. The upper and lower limits, and the maximum available water for a 300 mm soil column of 1 sq.cm area, are given in Table 10.

The variations of average soil moisture content for a 30 cm soil column in all of the open plots are shown in Figure 10. The period reported was mainly dry, except for a few days with small amounts of rainfall, during which time soil moisture values are seen to increase slightly. The plotted points seem to suggest some type of curvilinear relationship between the moisture content and elapsed time since wetting. But this was not investigated as it would obscure the effects of two distinctly influencing factors, the available water content and the potential evapotranspiration. Instead, the values in Figure 10, for individual plots, were used to compute the reduction in water content between two consecutive days. This reduction, expressed in millimeters, was taken as the AE for one day. The corresponding PE

TABLE 10. Some soil water properties for the experimental plots for AE determination

Plot no.	Type of soil	Soil moisture content, % by volume, at: Upper limit of available water	Lower limit of available water	Maximum available water, mm for a 300 mm soil column
(1)	(2)	(3)	(4)	(5)
1	St. Amable Loamy Sand	34.70	20.21	43
2	St. Amable Loamy Sand	41.62	29.12	38
3	Upland Sand	24.72	15.37	28
4	Ste. Rosalie Clay	55.82	49.00	21
5	Ste. Rosalie Clay	48.71	37.66	33

Note:

Col.(5) = [Col.(3) - Col.(4)] (Depth of soil column in mm)/100

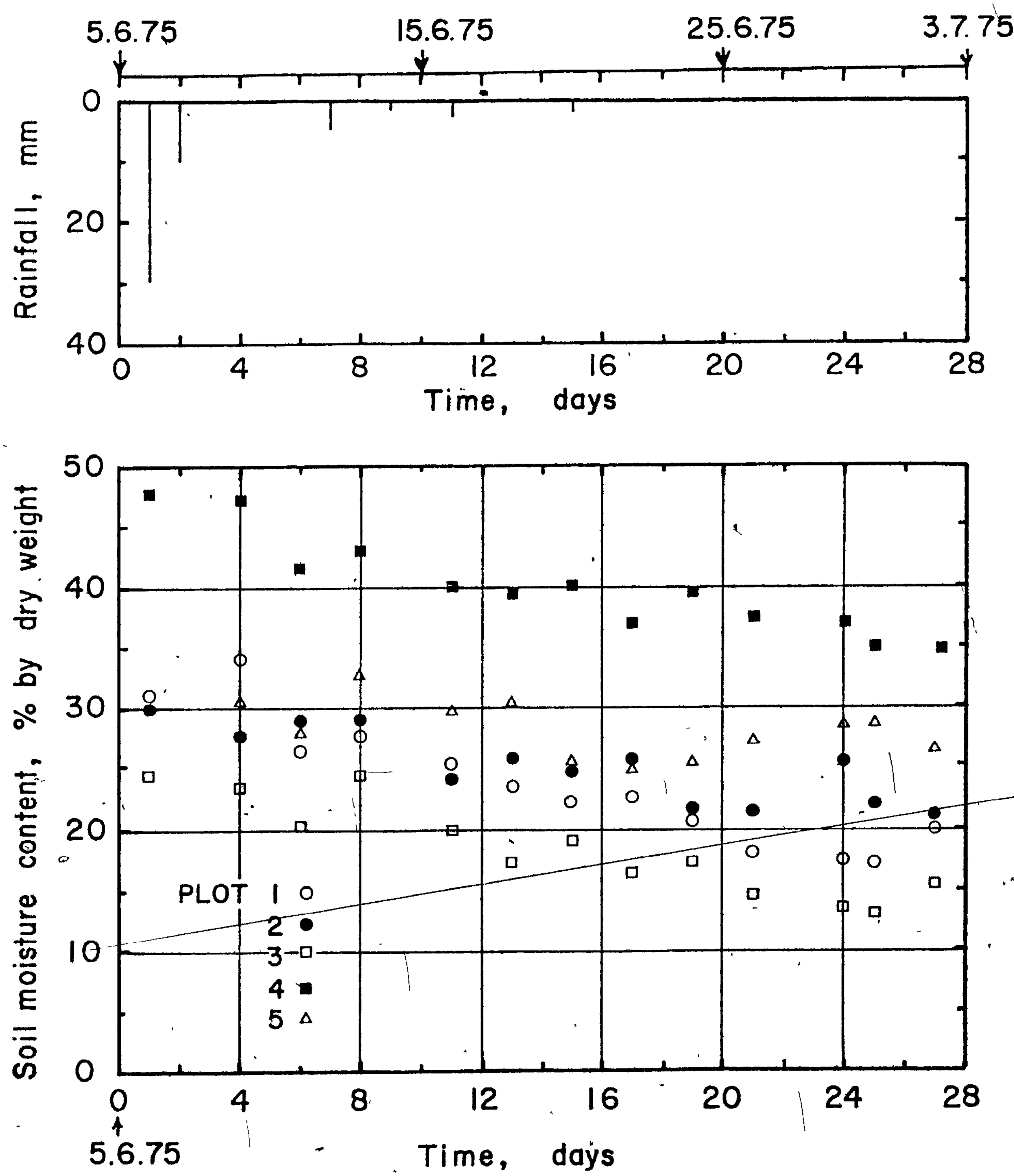


Fig. 10. Soil moisture variation and rainfall in the plots for actual evapotranspiration measurement

values, in mm, were obtained from published tables (Russelo et al., 1974), from recorded values of maximum and minimum temperatures for the appropriate days, and from known values of solar radiation for the study area. The percentage of available water before evapotranspiration was computed by the formula,

$$AW = \frac{AAW - AWL}{AWU - AWL} \times 100.0\% \qquad \ldots (5.5)$$

where

AW = per cent of available water before evapotranspiration,

AAW = actual available water, in millimeters, before evapotranspiration,

AWL = lower limit of available water in millimeters, and

AWU = upper limit of available water in millimeters.

The values of AE, AW and PE, obtained as discussed above, were used in developing a multiple regression equation between these, and the equation was obtained as,

$$AE = -0.2285 + 0.4753\ PE + 0.019\ AW \qquad \ldots (5.6)$$

The analysis of variance for multiple regression and some other statistical information about the association between the three variables are given in Tables B.2 and B.3 of Appendix B.

Since the soil profile had been divided into two zones for water budgeting, AE was calculated separately for the two zones. It was assumed that 50 per cent of the available water is depleted from the first 400 mm of the soil profile and the rest from the lower layers. Thus, in equation (5.6), daily PE was replaced by (PE - rain)/2 (when rain $\leq$ PE), and the computed value of AE was multiplied by 2, to obtain the total AE from the whole soil column for one day. The whole soil column referred to here is restricted in depth from the soil surface to the drain level.

Equation (5.6) has two limitations. Firstly, the computed AE becomes negative when both PE and AW are either zero or have very low magnitude; and secondly, it will estimate AE in excess of PE, when the latter is less than 3.5167 mm at AW of 100 per cent. These two problems arise due to the relatively small amount of data used for the multiple regression. More data could not be collected during a one-year experiment. However, the estimates obtained from the equation for most of the days during the growing season for corn, were found to be reasonable, and the problems mentioned above were subsequently taken care of in the water balance computations by slight modifications in the operation of the model. Also, the above equation was justified by appropriate statistical tests, and is considered superior to computing AE as an arbitrary fraction of PE. In fact, the value of the partial regression coefficient of AE on AW was approximately 1.5 times greater than that of AE on PE (Table B.3 of Appendix B). This indicates that AW is relatively more important than PE in estimating AE. Use of the proposed equation is also considered superior to the practice of computing AE from AE/PE ratios corresponding to different soil moisture availability curves because, in equation (5.6) the continuous nature of the variables is retained, and being in a functional form, can be readily used in computer water balance computations.

The coefficient of variability of AE was obtained as 28 per cent. Although this is high, in order to know if this is unusual, one has to have experience with similar data. The author could not find similar information from other sources for comparison. Some data on the variability of moisture content in a sandy loam soil were available from the works of Carvallo et al. (1976), and working with their data, the coefficients of variability of soil moisture content, under various suctions, were found to lie between 1.6 and 44.8 per cent. Since AE is dependent on soil moisture content, it is reasonable to expect AE to exhibit a large variability.

In general, it may be said that equation (5.6) has taken two important factors, namely, PE and AW, into consideration in evaluating AE; it gives reasonable estimates for a large part of the growing season; it is simple to use and is easily adaptable to computer application.

5.3 Water Balance Model

5.3.1 Comparison of observed and predicted water table depths

The basic flow chart of the water balance model for computing daily water table depths has been shown in Figure 5. The detailed program was made flexible to include the following modifications:

1. To assign to AE a constant low value (= PE/4, subject to a minimum of 1 millimeter) when AE computed by equation (5.6) becomes negative.

2. To assign to AE the value of PE when either rainfall is greater than or equal to PE, or AE computed by equation (5.6) becomes greater than PE.

3. To allow (if warranted by the evidence of observed water table rise), a certain percentage of input rainfall to join directly the transient water storage, without first satisfying the entire available water storage capacity.

4. To use more than one relationship between water table depth and transient water content when the soil profile indicates distinct soil layers.

The model verification was carried out using 1974 input data for one area (St. Amable Loamy Sand, Beef farm, Macdonald College), with subsurface drains installed; 1976 input data for one area (Ste. Rosalie Clay, Seed farm, Macdonald College), with negligible artificial subsurface drainage; and 1976 input data for another area (Ste. Rosalie Clay, farm of Mr. J. P. Martineau, St. Clet), with subsurface drains installed. The results of the model verification for the above three cases are shown in Figures 11, 12 and 13 respectively.

From these figures, the general agreements between the observed and the predicted water table depths are seen to be close with respect to: the trend of rise and fall of water table, number of rises, and the maximum extent of rise. The water table rise for the poorly drained

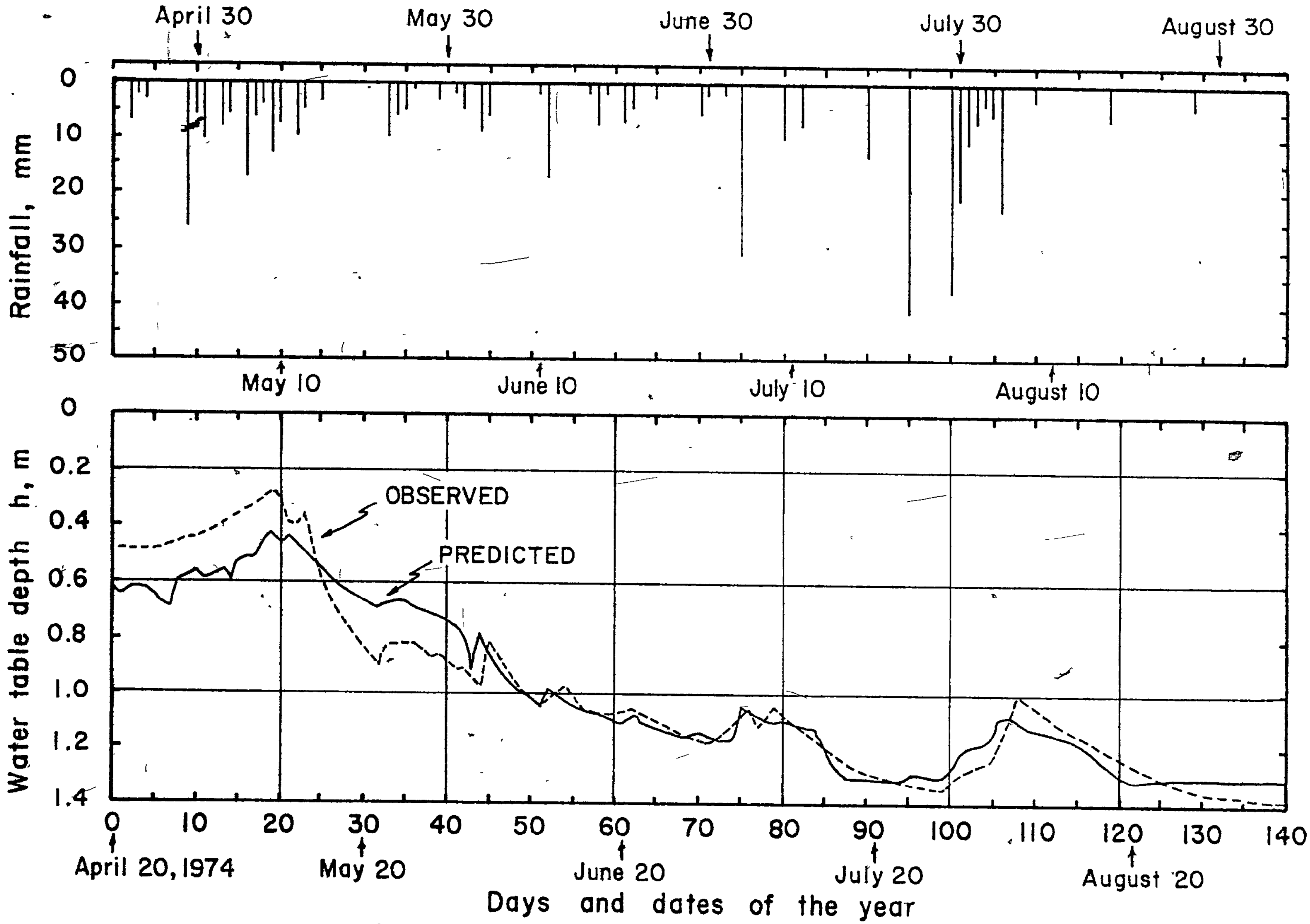


Fig. 11. Water balance model verification - St. Amable Loamy Sand (Beef farm)

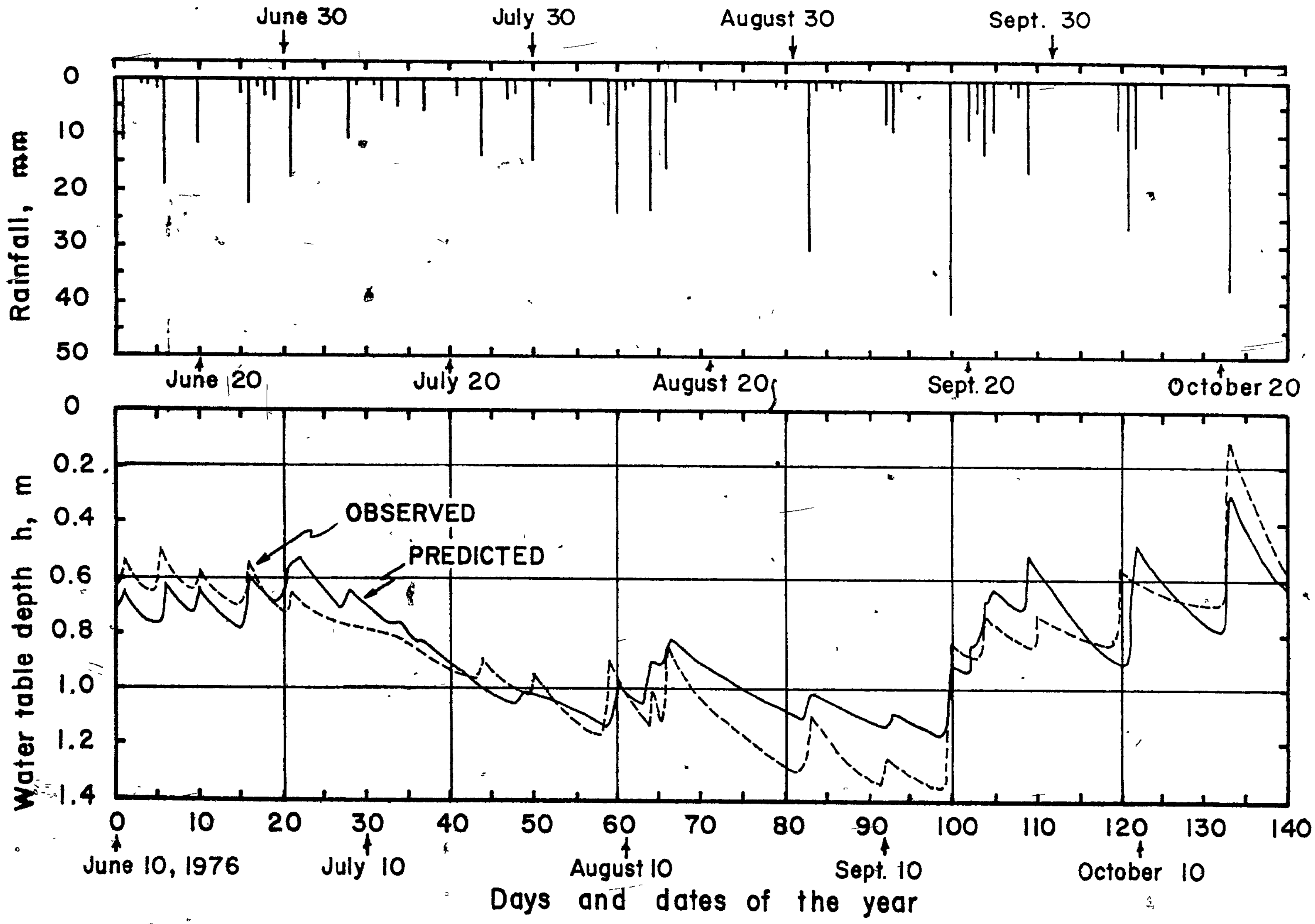


Fig. 12. Water balance model verification - Ste. Rosalie Clay (Seed farm)

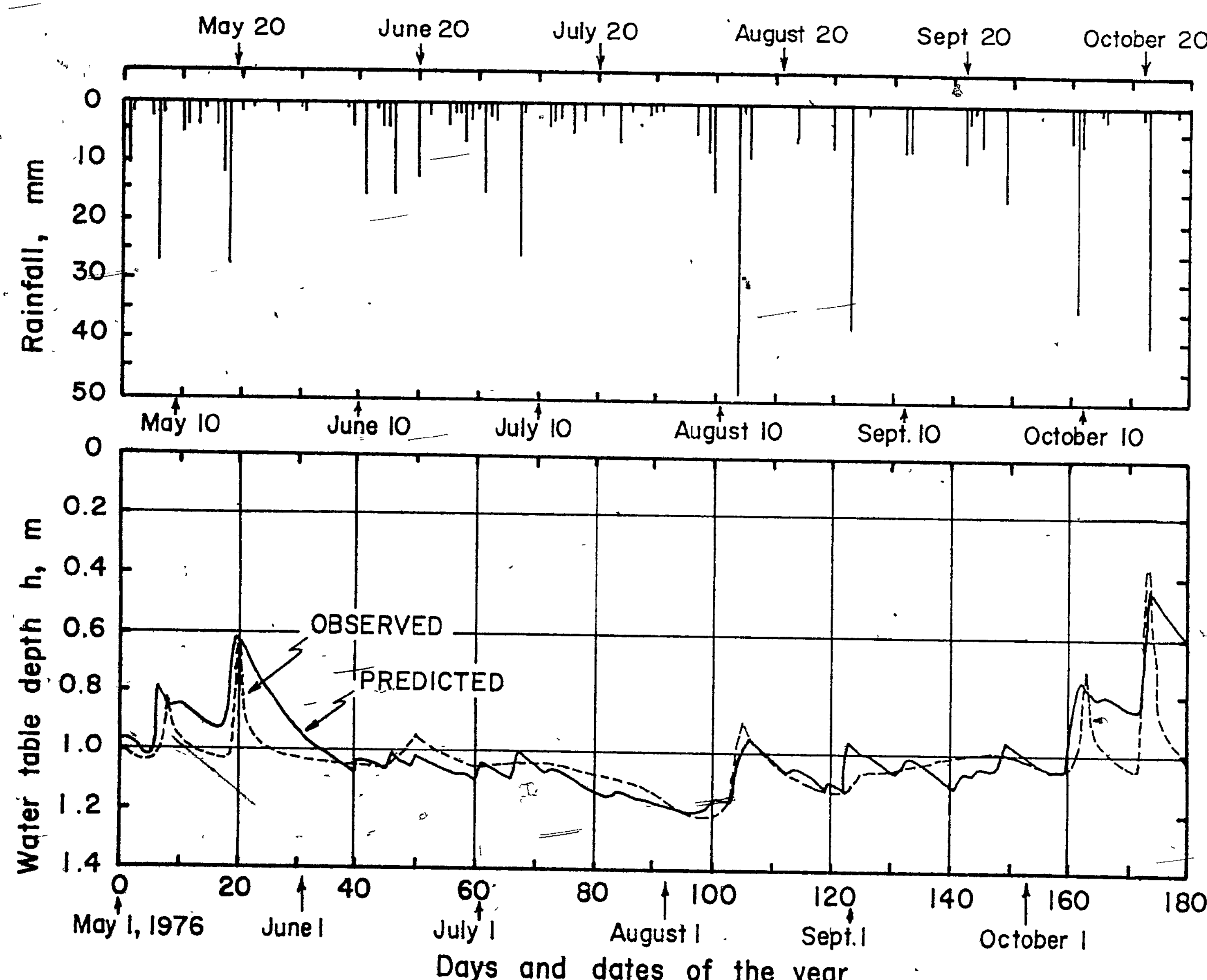


Fig. 13. Water balance model verification - Ste. Rosalie Clay (Mr. J. P. Martineau's farm at St. Clet)

clay, in Figure 12, is seen to be more numerous than for the others. This is to be expected because in a poorly drained soil, the moisture depletion is slow, and at any stage, there is a greater chance that the water table will respond to the rainfall, because of the initially high moisture content of the soil medium.

The fact that the drainable porosity varies with the water table depth under field conditions, can be made evident by some approximate calculations based on the observed water table depths in Figures 11, 12, and 13. Thus, with respect to the loamy sand in Figure 11, in a ten-day period (July 28 to August 6), a 113 mm rainfall caused the water table to rise from 1340 mm to 1000 mm from the soil surface, or by an amount of 340 mm. The total AE (obtained from the output of the model operation and not shown here), for the ten-day period was 53.6 mm. Hence, since the water table was near the drains, thus allowing us to neglect the effects of drainage, the actual depth of rainfall responsible for water table rise is obtained as (113.0 - 53.6) or 59.4 mm. The average drainable porosity is therefore 59.4/340 = 0.175. Similar calculations for the period June 29 to July 4, when the initial position of the water table was higher than that for the previously considered period July 28 to August 6, indicate that a net rainfall of 13.6 mm caused a water table rise of 130 mm (from 1180 mm to 1040 mm from the soil surface), yielding an average drainable porosity value of 13.6/130 = 0.105.

Computations similar to those above were done for some events presented in Figures 12 and 13. With respect to the undrained clay in Figure 12, for the period September 17 to September 20, when the initial water table depth was 1360 mm, the drainable porosity was found to be 0.11, compared with 0.09 for the period August 7 to August 8, when the water table was at a depth of 1170 mm. For the drained clay in Figure 13, during the period August 9 to August 13, and with the initial water table at 1210 mm, the drainable porosity was obtained as 0.13, as against 0.08 for the period May 18 to May 19, when the initial water table depth was 1030 mm.

One of the reasons for obtaining a closer agreement between the observed and predicted water table depths in the present model, as compared with a previously used model (Chieng, 1975), is that, in the present model the variability of drainable porosity with respect to water table depth has been considered. The previous model had used constant values of drainable porosity for different zones.

In each of the Figures, 11, 12, and 13, during certain periods, the predicted water tables are seen to deviate from the observed values. Particular mention may be made of the periods April to May in Figure 11, mid-August to mid-September in Figure 12 and, the end of October in Figure 13. The deviations for the sand in Figure 11 during April and May are probably due to the melting of ice lenses in the soil or the melting of some above-ground snow, causing the water table to be higher than predicted, as the water balance model does not take into account the snow factor. Broughton (1972) has mentioned that immediately after winter months there are times when the soil water may refreeze on cold nights, reducing the capacity of the soil to transmit water. This could also be a reason for measured water table to be closer to the surface during this time of the year. A steep drop of the water table in the following few weeks is probably due to very high evaporation rates (equal to PE), since seedbed preparation in this period causes enough soil disturbance to bring wet soil to the surface, in which case available water does not restrict evaporation. Another reason for the steep drop may be a large subsurface drainage rate because the soil has been temporarily rendered more porous as a result of the melting of ice lenses. Also, during some periods of heavy rainfall, some of the rain directly joins the water table through large cracks, causing the water table to rise temporarily; but in the subsequent period, the water redistributes itself in the surrounding soil, causing a sharp drop in the water table. This particular phenomenon is seen to occur for the undrained clay in Figure 12 during August and September, and for the drained clay in Figure 13 during mid-May and at the end of October.

The overall agreement between the observed and the predicted water table depths during the main growing season of corn (May 1 to August 31), was found to be reasonable. The predicted water tables in several cases were found to be a little closer to the surface than the observed values, and this may be considered to be due to a conservative performance of the water balance model. Also, realizing the large spatial variability in many of the used soil water properties, no attempt was made to further refine the model to obtain a more close fit between the observed and the predicted water table depths.

One likely reason why the predicted water tables may be higher than the actual water tables is that the predictions were based on the assumption that there was no surface runoff until the profile was saturated. In reality, a part of some intense rainfalls may have been lost as surface runoff before the profile was saturated. The large discrepancies between the observed and the predicted water table depths during late August to late September in the case of the poorly drained clay in Figure 12, however, remains unexplained.

5.3.2 Statistical analyses using observed and predicted water table depths

To gain more insight into the performance of the developed water balance model, it was decided to evaluate some statistical parameters of the distributions of the observed and predicted water table depths for comparison, and to perform an appropriate statistical test to determine if there is any significant difference between the two. One year's data from a previous model, referred to hereafter as the fixed drainable porosity model or the fixed f model, were also available for the St. Amable Loamy Sand area. This model had used some constant values of f for various soil zones, and had computed AE from total available soil moisture and AE/PE ratios for different zones. The results from this model and from the present model were used for a comparative evaluation of the two. For the clay soils no other data, except the recorded water table depths, were available for comparison.

TABLE 11. Statistical properties of the distributions of observed and predicted water table depths

Criteria of comparison and/or test and hypothesis	1974 St. Amable loamy sand Beef Farm 64 data points			1976 Ste. Rosalie clay Seed Farm 36 data points		1976 Ste. Rosalie clay St. Clet 35 data points	
	Observed	Predicted by		Observed	Predicted by variable f model	Observed	Predicted by variable f model
		fixed f model	variable f model				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Mean	0.9402	0.9961	0.9241	0.8242	0.7900	0.9605	0.9098
Standard deviation, m	0.2795	0.4175	0.2754	0.2718	0.2291	0.1638	0.2127
Coefficient of correlation	-	0.7935	0.9409	-	0.9215	-	0.8068
Sum of the squared deviation, sq.m	-	2.5399	0.5930	-	0.4482	-	0.6290
Coefficient of skewness	-0.7594	-	-	-0.0451	-	-1.0486	-
Wilcoxon's Matched Pairs Signed Ranks test: H_o: No difference between observed and predicted water table H_a: Observed and predicted water tables are different		Do not reject H_o at 0.01 α level. Reject H_o at 0.05 α level	Do not reject H_o at 0.05 α level		Do not reject H_o at 0.05 α level		Do not reject H_o at 0.05 α level

Note: α = size of Type I error

The variable f model in the above table refers to the model developed in the present study.

The statistical parameters of the distribution and the results of the test are given in Table 11.

From the limited data from these three areas, the coefficients of skewness were found to be -0.0451, -0.7594 and -1.0486. According to Yevjevich (1972), the two latter values do not satisfy the requirements of the preliminary test criterion for symmetry and, hence, for normality of the distribution of the data. Thus, tests on various statistical parameters could not be performed. Instead, a nonparametric test, namely, Wilcoxon's Matched Pairs Signed Ranks Test for related samples (Siegel, 1956) was performed to discover whether the samples of observed and predicted water table depths had any significant difference in any respect. In this regard, Aitken (1973) had suggested the use of a simple Sign Test, but a Sign Test only considers the direction of difference between two series of data, without considering the magnitude of the differences. Wilcoxon's Matched Pairs Signed Ranks Test is considered superior to a simple Sign Test as the former takes into account both the direction and the magnitude of difference between two series of data. The pair of observed and predicted water table depths for the same day is considered to be related, because in the model, as well as in nature, the same main causative factors are responsible for bringing about the fluctuation in the water table depth.

From Table 11, the performance of the present model as compared with the fixed f model appears to be satisfactory. For 1974, a substantial reduction in the sum of the squares of deviations between observed and predicted water table depths, and an improvement of the correlation coefficient between these were achieved in the present model. By using the Wilcoxon's Matched Pairs Signed Ranks Test, the distributions of the observed and predicted water table depths were found to be identical at the 95 per cent confidence level.

At this stage, because of the limited availability of data on recorded water table depths in other areas and also because of the large spatial variability of the different soil water parameters involved, it is felt unnecessary to further refine the model. Also, the present model is quite simple to operate and takes only 34 seconds of computing time for the analysis of 76 years of weather data. The previous fixed f model took 57 seconds of computing time to analyse 27 years of weather data. This saving in computer time in the present model is achieved mainly by reducing the number of zones of the soil profile from four to two, and by replacing various step functions by continuous functions.

5.3.3 Application of the integrated water balance and crop loss model in average annual crop loss calculations

In the previous subsection, the performance of the water balance model in predicting water table depths was discussed, and it was seen that the developed model was able to predict the daily water table depths with reasonable accuracy. The results of the present subsection have been obtained by using the integrated water balance and crop loss model for 76 years (1900 to 1975) of weather data from the Ottawa weather station. The crop loss computation was based on the theory presented in Section 3.2. The model was run for the average soil water properties of the clay soil region. These properties, along with the values used for depth of drain, equivalent depth DE, and allowable water table depth are given in Table 12.

Figure 14 shows a sample result of computed crop loss and associated probabilities for a specific set of values of design parameters, namely, spacing between laterals $S = 35$ m and hydraulic conductivity $K = 0.1$ m/day. The crop losses have been expressed as per cent of no-loss crop value, NLCV.

For a specific crop, the actual losses can be obtained by multiplying the values of the ordinate of Figure 14 by the factor which, when multiplied by 100, yields the maximum value of the crop under normal growth conditions, in dollars per hectare. In this discussion, the maximum value of the crop and the no-loss crop value are synonymous.

TABLE 12. Values of some fixed soil water and design parameters used in the water balance and crop loss model

Number	Parameter	Numerical value mm
(1)	(2)	(3)
1	Lower limit of available soil moisture in zone 1	140.0
2	Upper limit of available soil moisture in zone 1	165.0
3	Lower limit of transient soil moisture in zone 1	0.0
4	Upper limit of transient soil moisture in zone 1	48.0
5	Lower limit of available soil moisture in zone 2	280.0
6	Upper limit of available soil moisture in zone 2	329.0
7	Lower limit of transient soil moisture in zone 2	0.0
8	Upper limit of transient soil moisture in zone 2	95.0
9	Depth of zone 1 (or allowable depth of water table in Hooghoudt's equation)	400.0
10	Depth of zone 2	800.0
11	Depth of drain	1200.0
12	Equivalent depth DE in Hooghoudt's equation	1000.0

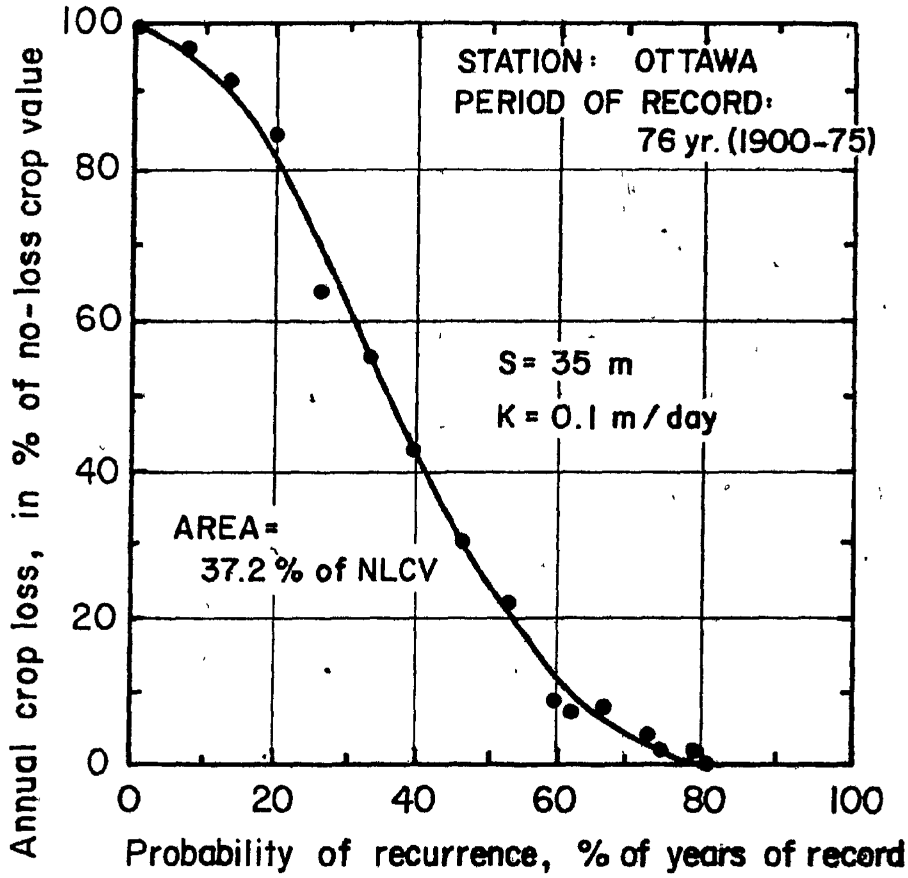


Fig. 14. An example of crop loss probability curve and computation of average annual crop loss (Assumed DD = 1200 mm)

The information of Figure 14 may be useful in obtaining the probabilities, on the average, associated with a certain crop loss. Thus, we may say that there is expected a crop loss of at least 50 per cent in 35 per cent of the years, or, roughly in 7 out of 20 years, if the design parameters are as indicated above. Similarly, we may also say that in 40 per cent of the years or in 4 out of 10 years, on the average, the loss would be expected to be 10 per cent or less.

It must be mentioned at this stage that the above assertions about the probabilities of recurrence are not in any way rigorous, and simply mean that if a long period of years is subdivided in blocks of 20 or 10 years, and the losses in each block are computed, then, on the average, one may expect a loss of 50 per cent or more in 7 out of 20 years, or a loss of 10 per cent or less in 4 out of 10 years. It is for this reason that the reliability of this type of prediction is increased when one has a large number of years of data. This, nevertheless, is a standard procedure for loss computation in engineering design (Metropolitan Toronto Region and Conservation Authority, 1959).

From Figure 14, it is seen that different losses have different associated probabilities. This makes the comparison between various design alternatives a tedious task. A very useful parameter for such comparisons may, therefore, be selected as the average annual loss, which is the area under the probability curve of Figure 14. This parameter has been computed and shown to be 37.2 per cent of the no-loss crop value, in dollars per hectare. Obviously, different sets of design parameters will yield different values of average annual loss.

The integrated water balance and crop loss model was used to compute average annual losses, as explained above, for different design parameters. Rather than computing the area under the probability curve manually, a subroutine was used to approximate the area by the average ordinate rule. The intervals were kept sufficiently small to obtain a close approximation of the area. The result of this analysis is shown

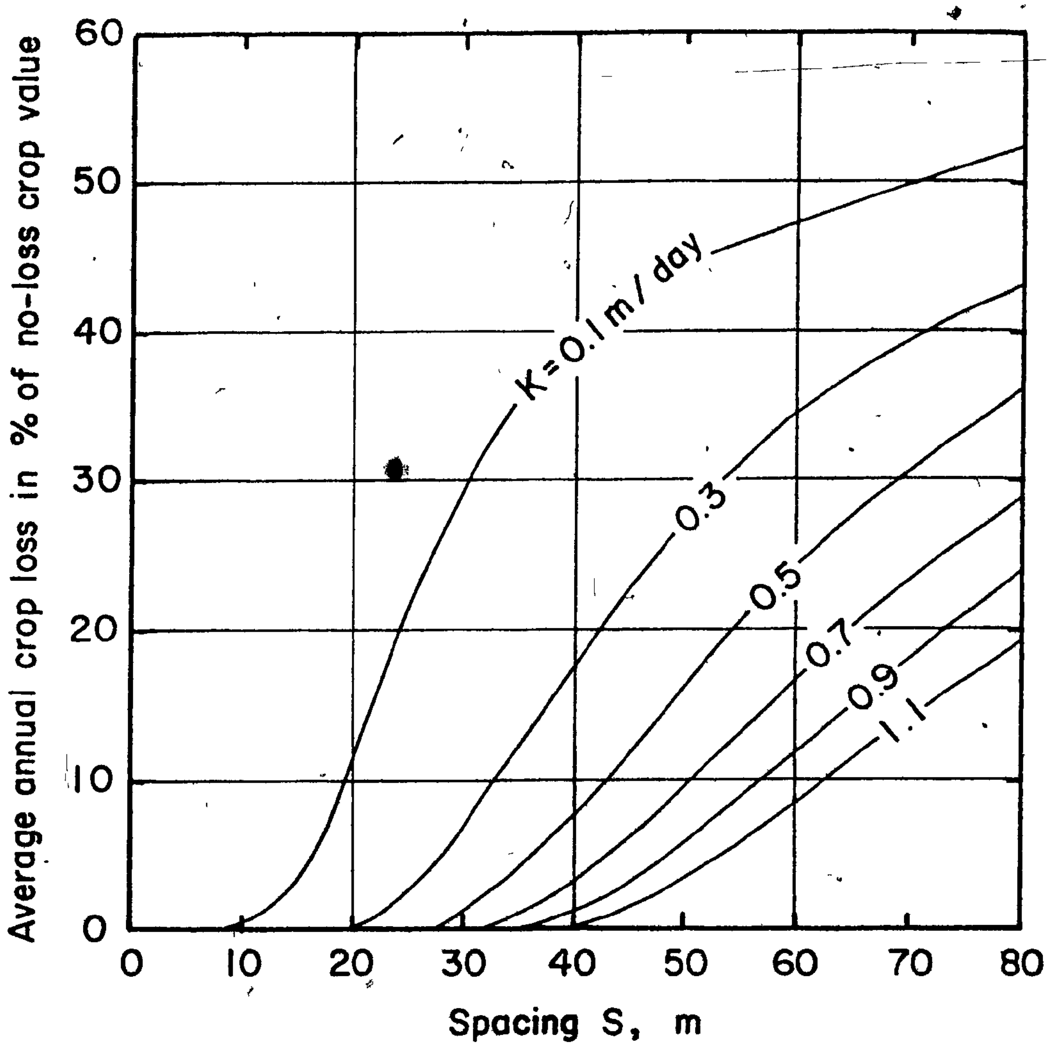


Fig. 15. Average annual crop loss for various spacing - hydraulic conductivity combinations (Assumed DD = 1200 mm)

in Figure 15. In this analysis, six values of hydraulic conductivity, from 0.1 m/day to 1.1 m/day, in steps of 0.2, were taken and the model was run for spacings varying between 5 and 80 meters. This analysis was done with soil water properties for the clay soil region (Table 8 b, equation (5.4) and Table 12), and for an assumed crop loss pattern for corn (Table 1). Since drain depth and other soil water parameters, such as available and transient water capacities, were fixed, each spacing - hydraulic conductivity combination gave one value of design drainage rate or RMAX. All values of RMAX for all combinations of S and K are given in Table 13. These values are also the same as drainage coefficients, which is the design rate of water removal by the subsurface drains. This rate is used in the model when the water table is closer than or equal to the allowable water table depth, ADW. The values of hydraulic conductivity used in the calculation constitute the commonly encountered range in the St. Lawrence Lowlands region.

From Figure 15, it is also seen that, for the lowest hydraulic conductivity, the average annual crop loss will be 51.9 per cent of the no-loss crop value, at a spacing of 80 meters. This condition is almost equivalent to no subsurface drainage, as the RMAX for this case will be 0.14 mm/day. The minimum value of AE ever attained in the model is about seven times as large as this value of RMAX. It can be concluded from this that, in the extreme case, if a farm has no subsurface drains, the average annual loss will be a little more than 50 per cent of the no-loss crop value. The benefit from drainage will be due to the salvaging of part of this loss. We are, however, ignoring the loss of crop that may occur due to delay in timely field operations as a result of high water table conditions in the undrained soil during the early growing period. The information in Figure 15 may be useful as a tool in comparing the crop losses for a wide range of values of design parameters. It is to be noted here that the crop loss computation has been done for a corn crop considering the main growing period to be from May 1 to August 31, in the climatic conditions of St. Lawrence

TABLE 13. Values of design drainage rate for various combinations of S and K

Drain spacing m	Values of design drainage rate RMAX in mm/day for indicated K values					
	Hydraulic conductivity K m/day					
	0.1	0.3	0.5	0.7	0.9	1.1
(1)	(2)	(3)	(4)	(5)	(6)	(7)
5	35.840	-	-	-	-	-
10	8.960	-	-	-	-	-
15	3.982	-	-	-	-	-
20	2.240	-	-	-	-	-
25	1.434	4.301	-	-	-	-
30	0.996	2.987	4.987	-	-	-
35	0.731	2.194	3.657	5.120	6.583	-
40	0.560	1.680	2.800	3.920	5.040	6.160
45	0.443	1.327	2.212	3.097	3.982	4.867
50	0.358	1.075	1.792	2.509	3.226	3.942
55	-	-	1.481	2.073	2.666	3.258
60	-	0.747	-	1.742	2.240	2.738
70	-	-	0.914	1.280	1.646	2.011
80	0.140	0.420	0.700	0.980	1.260	1.540

Note: 1. Blanks in the above table indicate that the model was not run for these combinations since they were unnecessary for the economic considerations.

2. Values in the above table are obtained using Hooghoudt's equation (equation 3.6).

3. In equation (3.6), DE, DD, and ADW were taken as 1.0 m, 1.2 m and 0.4 m, respectively, for all S-K combinations.

Lowlands. Any high water table due to inadequate drainage before or after this period has been assumed to have no influence on corn yield.

5.3.4 Average annual revenue increase due to subsurface drainage for different design alternatives

Earlier, in section 3.1, it was mentioned that the selection of a design alternative for a subsurface drainage system depends upon the interaction of two requirements, namely, minimizing crop loss by closely spacing laterals and minimizing installation cost by spacing laterals as widely as possible. The average annual revenue increase due to drainage, RE, was also expressed by equation (3.12) of section 3.2, and is rewritten here for convenience :

$$RE = ACL_0 - ACL - ACI \qquad \ldots (5.7)$$

In the above equation, the value of ACL_0 can be estimated for an assumed no-loss crop value NLCV, and an estimated value of production at negligible drainage. In view of the results shown in Figure 15 and the discussion on page 86, and for an assumed NLCV = \$500.00/ha, ACL_0 may be estimated as; ACL_0 = \$500.00 x (51.9/100.0) = \$259.50 per hectare. The quantity ACL of equation (5.7) is obtainable from the water balance model, as explained earlier in this section, for a specific set of design parameters, and the equivalent series of installation cost ACI, per year, may be obtained for different amortization periods AP, rates of interest I, and installation cost per unit area. Information on the last item has been worked out and published by Conseil des Productions Végétales du Québec (1976), for various drain spacings and installation rates per unit length, along with a group of curves to convert initial installation cost to a uniform annual series of costs for different interest rates and different amortization periods. This information has been used to work out the average annual revenue increase due to subsurface drainage for different design alternatives. A sample result is shown in Table 14. The information in this table has been obtained by assuming: NLCV = \$500.00/ha, AP = 20 years, I = 8%, cost of installation and material IC = 98¢/m, and the value of average annual crop loss at negligible drainage = 51.9% of the no-loss crop value = \$259.50/ha, and using the crop loss information

TABLE 14. Sample table of the economic analysis results

Spacing between drains m	Average annual crop loss $/ha	Uniform annual series of installation cost $/ha	Average annual revenue increase due to subsurface drainage $/ha	Modified benefit/cost ratio
(1)	(2)	(3)	(4)	(5)
5	0.00	116.20	143.30	1.23
10	0.35	102.50	156.65	1.53
15	15.90	68.67	174.93	2.55
20	59.35	51.87	148.28	2.86
25	105.05	42.23	112.22	2.66
30	142.40	35.07	82.03	2.34
35	175.35	31.12	53.03	1.70
40	197.60	27.66	34.24	1.24
50	223.10	23.71	12.69	0.54
80	259.50	15.06	-15.06	-

Note: 1. Values of column 2 are obtained from the integrated water balance and crop loss model, for K = 0.1 m/day.

2. Values of column 3 are obtained from Conseil des Productions Vegetales du Quebec (1976), for an amortization period of 20 years, an interest rate of 8 per cent per annum, and an installation cost of 98 cents per meter which includes the cost of material.

3. The no-loss crop value is assumed to be $500.00 per hectare

4. Column 4 = 500.00 x (51.9/100.0) - (column 2 + column 3), and column 5 = column 4/column 3.

5. Assumed drain depth = 1200 mm.

from the curve corresponding to K = 0.1 m/day in Figure 15. The last column of this table shows the modified benefit/cost ratios, the significance of which will be discussed later.

Repeating the procedure used for computation of Table 14, the average annual revenue increase due to subsurface drainage, RE, was computed for all the combinations of S and K, and for two values of interest rate (8 and 12 per cent), two values of amortization period (10 and 20 years), and two values of installation rate per meter (98 ¢/m and $1.64/m). These are considered reasonable ranges. RE for intermediate cases may be interpolated by approximation. The no-loss crop value, or the value of the harvested crop under normal growth conditions has been assumed to be $500.00/ha for all the combinations of the other parameters in the economic analysis. In this regard, studies by Fisher (1976) indicate thatduring the period 1945 to 1964 the gross return of grain corn in Ontario varied from $109.17 to $257.87 per hectare, with an average of $187.12 per hectare. And a subsequent study by Fisher et al. (1976), indicates that the average gross return for corn in the same region for the two years 1973 and 1974 was $717.00 per hectare. Thus, the selected no-loss crop value of $500.00 per hectare represents an intermediate approximate figure.

The results of the economic analyses are shown in Figures 16 and 17. These figures clearly indicate the interactions between the saving from reduced crop loss due to closer spacing of drains and the saving in installation cost at wider spacing of drains. The most desirable condition of spacing is that for which the average annual revenue increase RE is a maximum. The numerical value of RE is different under different combinations of hydraulic conductivity, interest rate, amortization period and installation cost. Of the various parameters considered in the economic analyses, only hydraulic conductivity is seen to influence the spacing at which a maximum RE is obtained. This is further clarified in Table 15. This table is prepared to indicate the spacings needed to achieve maximum average annual revenue increase due to subsurface drainage under various levels of other parameters.

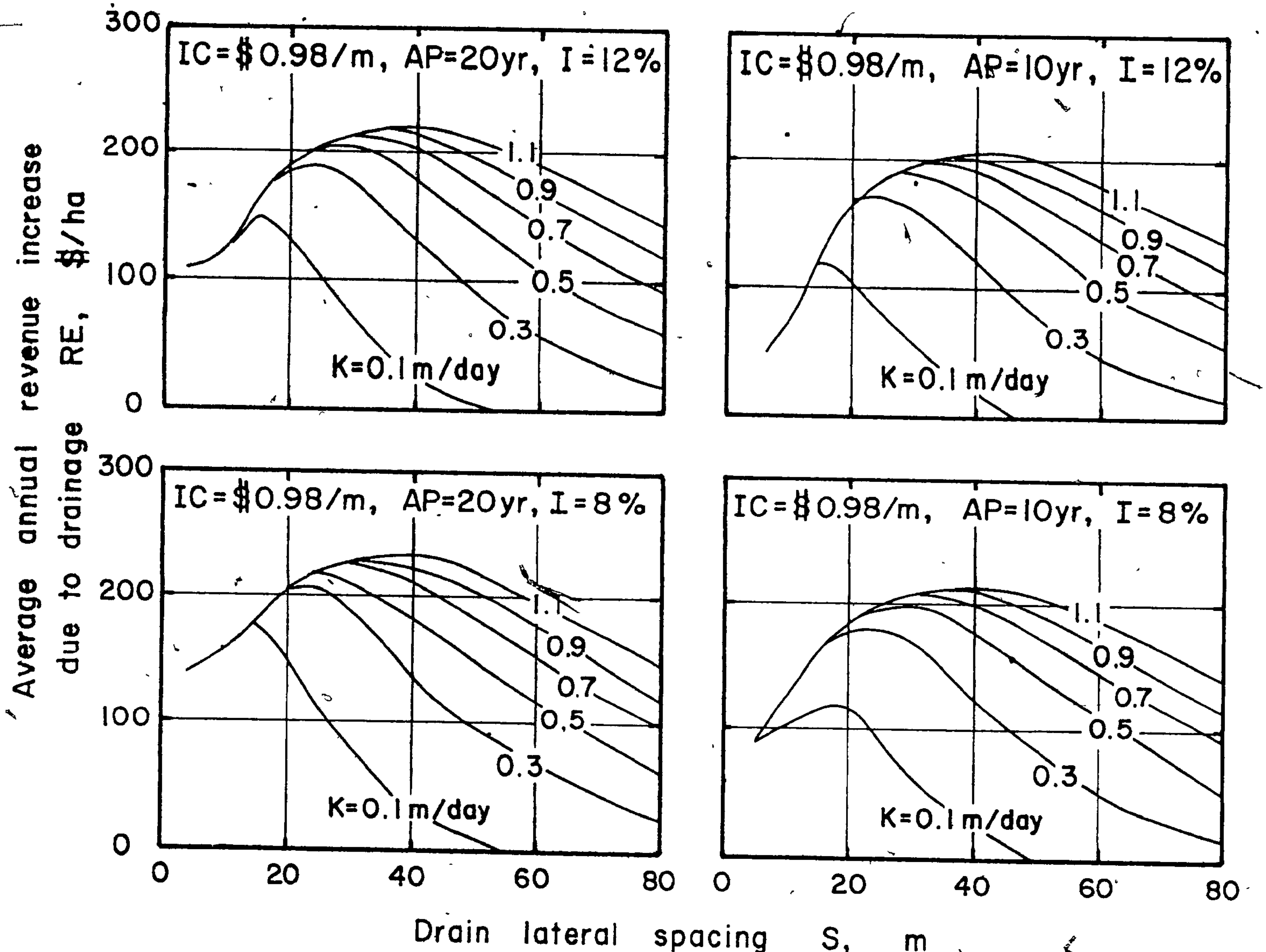


Fig. 16. Average annual revenue increase due to subsurface drainage for various S - K combinations at I = 8%, AP = 20 years, IC =$0.98/meter, NLCV = $500.00/ha and ACL_0 = $259.50/ha. (Assumed depth of drain = 1200 mm)

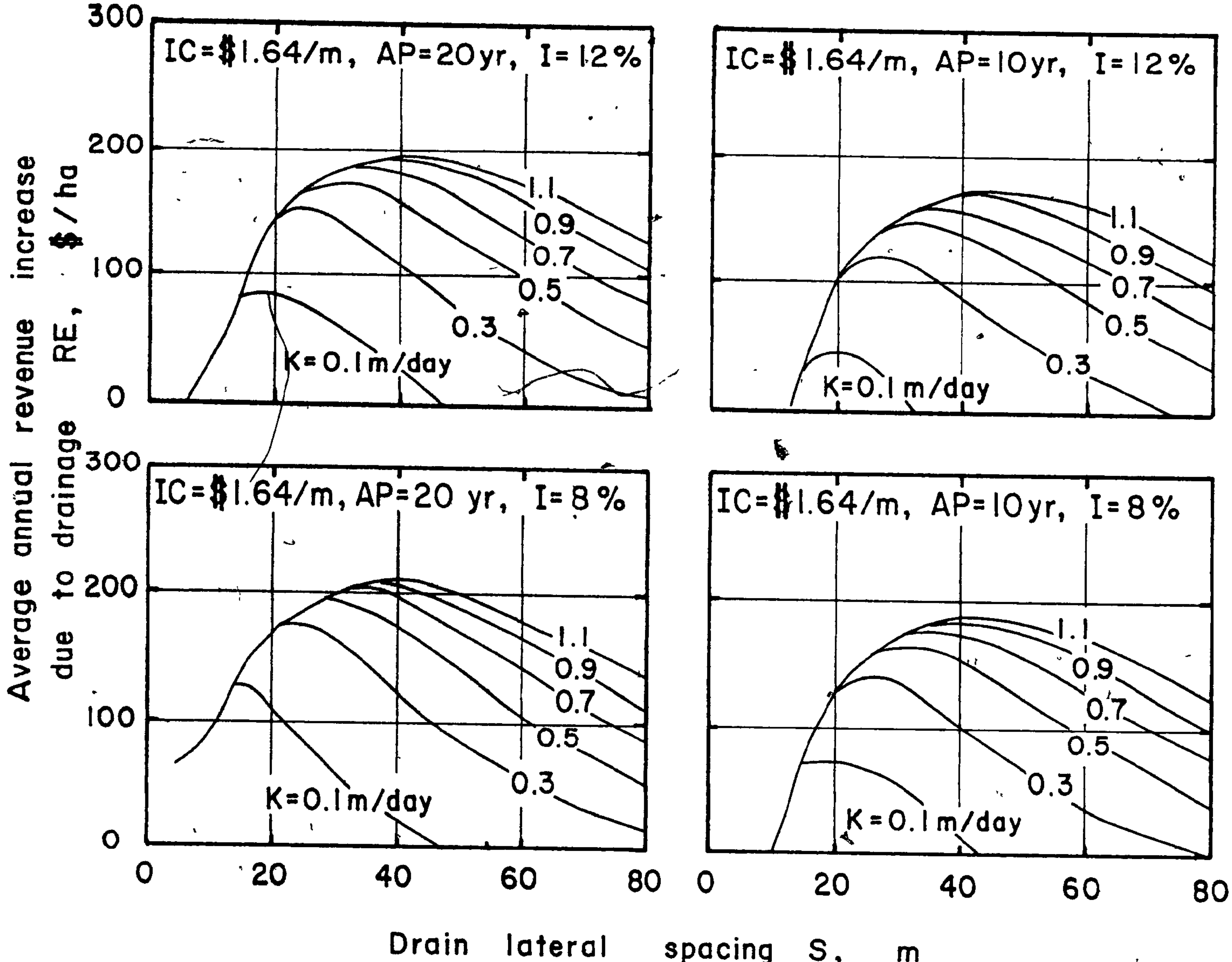


Fig. 17. Average annual revenue increase due to subsurface drainage for various S - K combinations at I = 8%, AP = 20 years, IC = $1.64/meter, NLCV = $500.00/ha and ACL_0 = $259.50/ha. (Assumed depth of drain = 1200 mm)

TABLE 15. Drain spacings for obtaining maximum average annual revenue increase due to subsurface drainage

Interest rate	Amortization period	Installation cost	Spacing, m for maximum average annual revenue increase due to subsurface drainage for indicated K values					
			Hydraulic conductivity K, m/day					
%	years	$/m	0.1	0.3	0.5	0.7	0.9	1.1
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
8	20	0.98	15	20	30	30	35	40
8	20	1.64	15	25	30	35	40	40
8	10	0.98	15	20	30	30	35	40
8	10	1.64	15	25	30	35	40	40
12	20	0.98	15	25	30	30	35	40
12	20	1.64	15	25	30	35	40	40
12	10	0.98	15	25	30	35	40	40
12	10	1.64	15	25	30	35	40	40

In the above table, under columns 5, 7 and 8 for K values of 0.3, 0.7 and 0.9 m/day, respectively, the variations in spacings for maximum revenue increase are minor. These variations may have been obtained because the intermediate values of spacings were not used in the water balance model computations. Thus, in column 5 of Table 15, the actual spacing corresponding to maximum revenue increase may have been between 20 and 25 meters, in column 7, between 30 and 35 meters, and in column 8, between 35 and 40 meters. The information from figures 16 and 17 and from Table 15 is important as it helps one to select the most appropriate spacing of drains under a wide range of pertinent parameters. Although the no-loss crop value has been taken as constant at $500.00/ha, it is obvious from equation (5.6) that, since ACL_0 is a certain percentage of NLCV, a change in the numerical value of the latter would shift the curves of figures 16 and 17 vertically, and the spacing required for obtaining a maximum average annual revenue increase would remain unchanged.

The modified benefit/cost ratios given in Table 14 are obtained by dividing the average annual revenue increase by the uniform annual cost of drainage installations. The decision as to which drain spacing is most appropriate will be different if one uses the criterion of the greatest average annual revenue increase or the greatest modified benefit/cost ratio. The modified benefit/cost ratio criterion may be more appropriate if the benefit values were obtained by considering all the production and other costs, better data on the effect of drainage on yield, and in situations of restricted capital when the efficiency of the capital use becomes a more sound basis for comparing alternatives. The RE criterion chosen for comparison in this thesis signifies the amount of additional return that a farmer may expect to get as a result of subsurface drainage installations.

It must be mentioned here that the results of this analysis are dependent upon the assumed crop loss pattern given in Table 1 and discussed in section 3.2. The assumptions about the crop loss were based on the evidence of a considerable amount of research work on yield reductions due to static water tables, and the relatively little information which was available on the effect of fluctuating water table on reduction of crop yield. The results of the cost analysis presented here strongly suggest the need for conducting further research work on the effect of fluctuating water tables, particularly the effect of varying duration of different water table depths on the growth and yield of crops. Also, this analysis is done for corn only and the loss pattern will have to be modified if it is desired to use this type of analysis for other crops.

In general, it can be said that the water balance approach is a very useful tool, not only for moisture budgeting, but the results of the model can have diverse uses. One such use is the computation of the probabilities of recurrences of water table depths and durations. Such probability information can then be used to compute losses associated with various degrees of inadequacy of drainage, and appropriate design alternatives which yield maximum revenue increase can be selected under different soil conditions.

VI. SUMMARY AND CONCLUSIONS

The design of subsurface drainage systems has been based generally on previous experience and on some simplified formulae. A deterministic approach to the design is, at best, incomplete because of the stochastic nature of some of the factors causing drainage problems.

Drainage system installation involves large sums of money, and the investor is, naturally, anxious to recoup his expenditure by being able to increase the productivity of his land. The conventional methods of design most often ignore this important economic implication associated with the subsurface drainage installations. An economically attractive design will be one which is expected to yield maximum revenue increase to the investor over the amortization period of the investment. Several factors can be thought to have an influence on the determination of a most economic design alternative. Some of these are, the type of crop grown, its market value and susceptibility to damage due to inadequate drainage, soil physical properties, installation cost and opportunity cost of the invested money. Design methods hitherto used are not well equipped to handle the diverse nature of the influencing factors.

The principal objective of this study was to set out a rational procedure which could be used under different soil, crop and economic conditions to arrive at a most economic alternative for subsurface drainage design.

A computer water balance model was developed for predicting daily water table depths. The inputs to the model were the daily values of rainfall and potential evapotranspiration, and some fixed average values of soil water properties applicable for the soils of the St. Lawrence Lowlands region. Actual evapotranspiration was expressed as a linear function of potential evapotranspiration and available soil moisture content. Drainable porosity was expressed as a non linear function of suction or water table depth. Field and laboratory investigations were carried out to evaluate some soil water properties and to develop the above-mentioned relationships. The output of the model - the predicted daily water table depths - was compared with some observed data from the study area. Statistical parameters of the distributions of observed and predicted water table depths were computed and compared, and a nonparametric test was used to test the hypothesis that the two distributions were identical. Performance of model was compared with that of a previously developed model which was based on some constant soil-water properties. The model was then operated with 76 years of weather data from the Ottawa weather station for the period from April 1 to October 31 of each year, and frequencies of various depths, and durations of water tables were worked out. Several combinations of spacing and hydraulic conductivity were used for this purpose. Based on available information, a crop loss pattern with respect to different depths and durations of water tables, was established. Yearly crop losses were computed for corn considering the main growing period to be from May 1 to August 31 of each year. From the probability distribution of yearly losses, average annual losses were computed. Initial investment cost per unit area was determined for known spacings and installation cost per unit length. This cost was broken down into uniform annual series using suitable interest rates and amortization periods. A constant no-loss crop value was assumed. The average annual revenue increase per unit area was computed by subtracting annual installation cost and the average annual value of crop loss from the no-loss crop value. This analysis was done for six values of hydraulic conductivity, two values of interest rate, two values of amortization period and two values of installation cost per unit length. Based on the results of this research the following conclusions are drawn:

1. Actual evapotranspiration from a corn field can be expressed as a linear function of available soil moisture and potential evapotranspiration. From the relation developed from experimental data, the multiple correlation coefficient was found to be significant at the 99 per cent confidence level. The partial regression coefficient of AE on PE was significant at the 95 per cent confidence level, and partial regression coefficient of AE on AW was significant at the 99 per cent confidence level. AE was found to be influenced more by AW than by PE.

2. Drainable porosity was found to be curvilinearly related to water table depth. Field experimental values of drainable porosity were found to be within the range of mean ±1 standard deviation of the values obtained under controlled laboratory measurements. The types of relations were found to be different for the sand and the clay soil. The empirical relations very closely approximated the results obtained in the laboratory measurements. The above relations can be integrated between given limits to yield volumes of transient water drained due to a certain water table drawdown. The result may subsequently be used to derive empirical relationships between the transient water content and the water table depth of a soil column.

3. It was found that a water balance model could satisfactorily simulate the water table fluctuations in the field, when the interdependences of some of the soil-water-plant properties are taken into consideration. In this respect, it was found that, accounting for the dependence of drainable porosity on water table depth, and of actual evapotranspiration on available water content and potential evapotranspiration, substantially improved the performance of the water balance model. Also, the reduction in the number of zones of the soil profile, and the replacement of the step functions of soil water properties with continuous functions helped achieve a considerable saving in computer costs.

4. By using the water balance approach for predicting daily water table depths and for an assumed crop loss pattern with respect to various depths and durations of water tables, it was possible to find

the probabilities associated with various magnitudes of crop loss, under different combinations of drain spacing and soil hydraulic conductivity. The average annual loss was then obtained from the area of the probability curve. The results indicated that in the extreme combination case of very low K and very wide S, for which the water depletion rate by the subsurface drains is negligible, the average annual loss was a little more than 50% of the no-loss crop value. This means that the installation of the subsurface drainage system will salvage a part of this loss.

5. From the information on average annual crop loss for a certain combination of S and K, installation cost, interest rate, and amortization period, it is possible to calculate the average annual revenue increase resulting from the installation of a subsurface drainage system. This can be done for a wide range of S and K values, and the results will allow one to select the most appropriate spacing that will be expected to give a maximum increase in revenue in a region having approximately constant K values.

6. The magnitude of the maximum average revenue increase is influenced by all the parameters considered for its computation, namely, drain spacing, soil hydraulic conductivity, interest rate, installation cost, amortization period and no-loss crop value. However, the most important feature was that the location of the maximum average annual revenue increase with respect to spacing was found to be greatly affected by hydraulic conductivity, but practically independent of the other parameters. This suggests that the fluctuating market conditions may not have a significance influence on the selection of a design alternative. This also stresses the importance of field evaluated values of K for subsurface drainage design.

7. The effect of drain depth on the predicted water table depths was not investigated. While it is recognized that the drain depth may have an influence on the economic decision for selecting an appropriate design alternative, the value of DD, 1.2 m, chosen for the present work is close to the commonly used values in the St. Lawrence Lowlands. In locations where the drains are placed shallower e.g. 0.8 m, narrower spacings between drain laterals will need to be used.

SUGGESTIONS FOR FUTURE RESEARCH

As a result of the study conducted for this dissertation, the following topics are considered important for further investigation:

1. Field experiments should be conducted in several areas to study the actual evapotranspiration in the growing season of the crop and tests should be done to find if a generally applicable relation between some pertinent variables exists.

2. Laboratory measurements should be made to determine drainable porosity vs. water table depth relations for the soils at various depths and in different areas, to establish the extent of variability of drainable porosity with depth of soil profile and location. Attempts should be made to develop relations which can be generally used with a particular soil type if possible.

3. More observations of water table depths should be made in different areas, particularly for several continuous days after rainfall events, and the results of the developed water balance model should be tested further against the observed data for other areas and other years.

4. Detailed experiments should be planned and conducted to investigate crop loss with respect to time, depths and durations of water table levels. This is very important for increasing the reliability of the economic analysis for the subsurface drainage systems. In this respect, it may be beneficial to induce some farmers to maintain crop yield and water table depth records from subsurface drained, and undrained fields.

5. The effects of different crop loss patterns on the average annual losses and hence on average annual revenue increases from subsurface drainage should be studied.

6. A survey should be carried out to classify the poorly drained areas of the St. Lawrence Lowlands region, with respect to the saturated hydraulic conductivity. It may be more useful if such hydraulic conductivity measurements are made *in situ* rather than in the laboratory. The results of the survey will help in identifying regions with similar subsurface drainage potential. This will facilitate large scale planning for subsurface drainage design works.

7. In the regions where drain depth is considered to be a major variable, its effect on the predicted water table depths and hence on the crop losses and revenue increases should be studied.

8. The applicability of the water balance model for obtaining the daily or the seasonal soil moisture depletions, irrigation requirements and drought frequencies should also be investigated.

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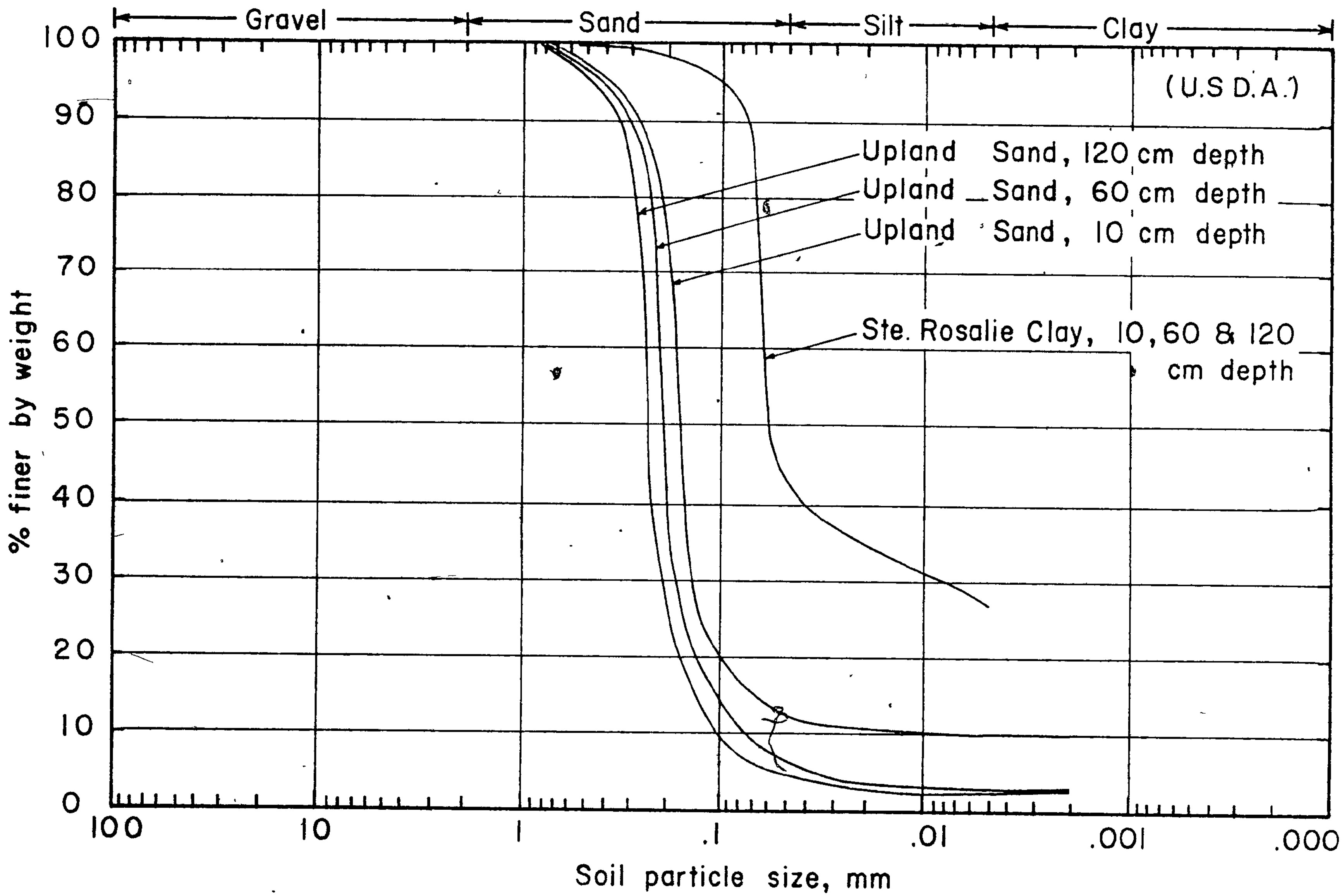
APPENDICES

APPENDIX 3.

Particle size distribution for Uplard sand and Ste. Rosalie clay.



APPENDIX B

TABLE B.1. Analysis of variance for bulk densities of Upland sand and Ste. Rosalie clay

Source	Degrees of freedom	Sum of squares	Mean square	$F_{calculated}$	$F_{tabulated}$	
					0.05	0.01
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Total	27	0.5322				
Depth	13	0.2594	0.0200	1.0363	2.58	3.91
Location	1	0.0217	0.0217	1.1244	4.67	9.07
Error	13	0.2511	0.0193			

TABLE B.2. Analysis of variance for multiple regression between actual evapotranspiration (AE) as dependent variable, and potential evapotranspiration (PE) and per cent of maximum available water (AW) as independent variables

Source	Degrees of freedom	Sum of squares	Mean square	$F_{calculated}$	$F_{tabulated}$	
					0.05	0.01
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Regression	2	10.2389	5.1194	6.2140**	3.35	5.49
Error	26	21.4211	0.8239			
Total	28	31.6600				

** Significant at 0.01 α level

α = size of Type I error

TABLE B.3. Some pertinent statistical parameters associated with multiple regression between AE, AW and PE

Number	Description of the statistical parameter	Value of the parameter
(1)	(2)	(3)
1	Standard partial regression coefficient of AE on PE, b_1	0.3367*
2	Standard partial regression coefficient of AE on AW, b_2	0.4581**
3	Partial regression coefficient of AE on PE, b_1	0.4753*
4	Partial regression coefficient of AE on AW, b_2	0.0190**
5	Partial correlation coefficient between AE and PE, $r_{AE\ PE.AW}$	0.3790*
6	Partial correlation coefficient between AE and AW, $r_{AE\ AW.PE}$	0.4866**
7	Multiple correlation coefficient between AE, PE and AW, $R_{AE.AW\ PE}$	0.5687**
8	Simple correlation coefficient between PE and AW, $r_{PE\ AW}$	-0.00004315
9	Coefficient of variability, CV	28%

Note: 1. Notations used in the above table are taken from Steel and Torie (1960).

2. $r_{PE\ AW}$ has been included to stress the independence between the two.

3. *Significant at 0.05 α level, **significant at 0.01 α level, where α is size of Type I error.

4. The significance tests for 1, 3 and 5 are identical and also the significance tests for 2, 4 and 6 are identical. They are included in the table merely to report the numerical values.

APPENDIX C

C.1. Integration of f – h relation for sandy soil and an example of the use of this integrated function to compute volume of water drained from a saturated soil column due to a certain water table drawdown

The relation between drainable porosity f (in fraction) and water table depth or suction h (in meter), was obtained as,

$$f = \left[\frac{1}{1 + 2^{-11.6466(h-0.5)}} - \frac{1}{1 + 2^{(0.5)(11.6466)}}\right] \Big/ 3.2993 \qquad \ldots (C.1)$$

On simplification, this yields:

$$f = \frac{0.0054}{0.0177 + 2^{-11.6466h}} - 0.0053 \qquad \ldots (C.2)$$

Therefore, total water drained from a soil column, when water table drops from h_1 to h_2 is given by:

$$V = \int_{h_1}^{h_2} f dh = \int_{h_1}^{h_2} \frac{0.0054}{0.0177 + 2^{-11.6466h}} dh - \int_{h_1}^{h_2} 0.0053\ dh$$

This is of the form,

$$V = \int_{h_1}^{h_2} \frac{A}{B + C^{-Dh}} dh - \int_{h_1}^{h_2} E\ dh,$$

$$= \quad I_1 \quad - \quad I_2 \quad \text{, where}$$

A = 0.0054, B = 0.0177, C = 2.0, D = -11.6466, and E = 0.0053.

(continued)

Integration of I_1

Substituting, $B + C^{-Dh} = z$, we get, $C^{-Dh}.\ln C.(-D).dh = dz$

$$\text{or,}\quad dh = \frac{dz}{-DC^{-Dh}.\ln C}$$

$$\text{and,}\quad C^{-Dh} = z-B$$

$$\therefore\quad I_1 = \int_{h_1}^{h_2} \frac{Adz}{-D.\ln C.z(z-B)} = \frac{A}{-D.\ln C}\int_{h_1}^{h_2} \frac{dz}{z(z-B)}$$

$$= \frac{A}{BD\ln C}\left[\ln\frac{z}{z-B}\right]_{h_1}^{h_2} = \frac{A}{BD\ln C}\left[\ln(B + C^{-Dh}) - \ln C^{-Dh}\right]_{h_1}^{h_2}$$

$$= \frac{A}{BD\ln C}\left[\ln(B + C^{-Dh}) + (D\ln C)h\right]_{h_1}^{h_2}$$

Also, from I_2, we get, $I_2 = E\left[h\right]_{h_1}^{h_2}$

Subtracting I_2 from I_1, substituting upper and lower limits, substituting values of A, B, C, D, E, and simplifying, we obtain,

$$V = 0.038\ln(0.0177 + 2^{-11.6466h_2}) + 0.3017h_2 -$$

$$0.038\ln(0.0177 + 2^{-11.6466h_1}) - 0.3017h_1 \qquad \ldots \text{(C.3)}$$

(continued)

Example

To calculate volume of water drained from a saturated soil column when water table drops from surface to a depth of 1.2 meter from the surface.

Solution

Substituting $h_1 = 0.0$, and $h_2 = 1.2$ in equation (C.3), we get: $V = 0.2081$ m. Thus, a volume of water, equivalent to 0.2081 meter over an area of 1 sq.cm. of a saturated soil column will be released. This is also equal to 20.81 cc of water.

C.2. Integration of f - h relation for clay soil

The relation between drainable porosity f (in fraction) and water table depth or suction h (in meter) was obtained as,

$$f = he^{-0.4154h} - he^{-0.3484h} - 0.3305e^{-0.648h} + 0.3305 \quad \ldots \text{(C.4)}$$

Integration of equation (C.4) is direct and simple. The final result of volume of water drained obtained by integrating the above equation is given as:

$$\begin{aligned} V = & -2.4073h_2e^{-0.4154h_2} - 5.795e^{-0.4154h_2} + 2.87h_2e^{-0.3484h_2} + \\ & 8.238e^{-0.3484h_2} + 0.51e^{-0.648h_2} + 0.3305h_2 + 2.4073h_1e^{-0.4154h_1} + \\ & 5.795e^{-0.4154h_1} - 2.87h_1e^{-0.3483h_1} - 8.238e^{-0.3484h_1} - \\ & 0.51e^{-0.648h_1} - 0.3305h_1. \end{aligned}$$

As before, the volume of water drained can be calculated from known values of water table drawdown, i.e., h_1 and h_2.

Note: e = base of natural logarithm.

APPENDIX D.

D.1 Computer program of the integrated water balance and crop loss model.

```
      DIMENSION RAIN(215),PE(215),AW1(215),AW2(215),TR1(215),TR2(215),
     1SR(215),DW(215),AE(215),HW(215),XLOSS(76),P(8,5),S(10),RMAX(10)
C     THE FOLLOWING ARE THE EXPLANATIONS OF SOME BASIC ABBREVIATIONS
C     USED IN THE COMPUTER PROGRAM AND OUTPUT.
C
C     AE = DAILY ACTUAL EVAPOTRANSPIRATION,MM., AWB = AW2 = AVAILABLE
C     WATER IN THE BOTTOM ZONE,MM.,AWT = AW1 = AVAILABLE WATER IN THE
C     TOP ZONE,MM.,AW1L & AW1U - LOWER AND UPPER LIMITS OF AVAILABLE
C     WATER IN THE BOTTOM ZONE,MM.,CV = UPDATED CROP VALUE AFTER ACC-
C     -OUNTING FOR PREVIOUS LOSSES,$/HA.,CVNL = NO-LOSS CROP VALUE
C     (ASSUMED TO BE 100),$/HA.,DD = DRAIN DEPTH,MM.,DE = HOOGHOUDT'S
C     EQUIVALENT DEPTH,M.,DW = PREDICTED DAILY WATER TABLE DEPTH,MM.,
C     HW - DAILY WATER TABLE HEIGHT FROM DRAIN LEVEL,MM.,P = CROP LOSS
C     IN % OF FULL PRODUCTION.,PE = DAILY POTENTIAL EVAPOTRANSPIRATION,
C     MM.,SR = DAILY SURFACE RUNOFF,MM.,TRB = TR2 - DAILY TRANSIENT
C     WATER IN THE BOTTOM ZONE,MM.,TRT = TR1 = DAILY TRANSIENT WATER
C     IN THE TOP ZONE,MM.,TR1L & TR1U = LOWER AND UPPER LIMITS OF
C     TRANSIENT WATER IN THE TOP ZONE,MM.,TR2L & TR2U = LOWER AND UPPER
C     LIMITS OF TRANSIENT WATER IN THE BOTTOM ZONE,MM.,XLOSS = ANNUAL
C     CROP LOSS IN % OF NO-LOSS CROP VALUE,$/HA.,Z1 = DEPTH OF TOP
C     ZONE,MM.
C
C     READ PARAMETER VALUES FROM CARD
C
      READ(5,1)DD,DE,Z1,TR1L,TR1U,TR2L,TR2U,AW1L,AW1U,AW2L,AW2U,CVNL
    1 FORMAT(12F5.1)
      DO 10 IC=1,5
      READ(5,2)(P(IR,IC),IR=1,8)
    2 FORMAT(8F5.1)
   10 CONTINUE
      READ(5,3)(S(I),I=1,10)
    3 FORMAT(10F3.1)
      READ(5,4)(RMAX(I),I=1,10)
    4 FORMAT(10F5.3)
      XK=0.1
      WRITE(6,5) DD,DE,Z1,TR1L,TR1U,TR2L,TR2U,AW1L,AW1U,AW2L,AW2U,CVNL
    5 FORMAT('1','FOLLOWING FIXED PARAMETER VALUES HAVE BEEN USED IN',
     1' WATER BALANCE COMPUTATION'/1X,76('-')//1X,10X,'DD    =',F8.2,
     2' MM',8X,'DE    =',F8.2,' M',8X,'Z1    =',F8.2,' MM',8X,'TR1L =',
     3F8.2,' MM'/1X,10X,'TR1U =',F8.2,' MM',8X,'TR2L =',F8.2,' MM',
     48X,'TR2U =',F8.2,' MM',8X,'AW1L =',F8.2,' MM'/1X,10X,'AW1U =',
     5F8.2,' MM',8X,'AW2L =',F8.2,' MM',8X,'AW2U =',F8.2,' MM',8X,
     6'CVNL =',F8.2,' $/HA'/1X,10X,104('-')/)
```

[redacted]

```
      WRITE(6,6)
6     FORMAT('0',10X,'THE FOLLOWING IS THE PERCENTAGE CROP LOSS',
     1' MATRIX FOR'/1X,10X,'VARIOUS DEPTHS AND DURATIONS OF WATER',
     2' TABLE       '/1X,10X,52('-')//1X,10X,'WATER',18X,'DURATION IN',
     3' CONSECUTIVE DAYS'/1X,10X,'TABLE'/1X,10X,'DEPTH',3X,'<1 DAY ',
     43X,'2 OR 3 DAYS',3X,'4 OR 5 DAYS',3X,'6 OR 7 DAYS',3X,'>7 DAYS'/
     51X,10X,'  MM '/1X,10X,67('-')/)
      IK=-100
      JL=0
      DO 20 IR=1,8
      IK=IK+100
      JL=JL+100
      IF(IR.EQ.1) GO TO 8
      IF(IR.EQ.8) GO TO 11
      WRITE(6,7)IK,JL,(P(IR,IC),IC=1,5)
7     FORMAT(' ',9X,I3,'-',I3,2X,F7.1,3F14.1,3X,F7.1)
      GO TO 20
8     WRITE(6,9)JL,(P(IR,IC),IC=1,5)
9     FORMAT(' ',12X,'<',I3,2X,F7.1,3F14.1,3X,F7.1)
      GO TO 20
11    WRITE(6,12)IK,(P(IR,IC),IC=1,5)
12    FORMAT(' ',12X,'>',I3,2X,F7.1,3F14.1,3X,F7.1)
20    CONTINUE
      DO 30 ISP=1,10
      WRITE(6,13)XK,S(ISP),RMAX(ISP)
13    FORMAT('1','K =',F5.2,' M/DAY',6X,'S =',F5.2,' M',6X,
     1'AND RMAX =',F7.3,' MM/DAY  FOR THIS RUN'/1X,75('-')/)
C
C     INITIALISE ANNUAL CROP LOSS VECTOR
C
      DO 40 I=1,76
      XLOSS(I)=0.0
40    CONTINUE
C
C     REWIND TAPE AND SET UP YEAR LOOP
C
      REWIND9
      MAD=1900
      DO 50 IYEAR=1,76
      LY=IYEAR
C
C     INITIALISE ARRAYS AND ASSIGN VALUES FOR THE FIRST DAY
C
      DO 60 I=1,215
      SR(I)=0.0
       AW1(I)=0.0
      AW2(I)=0.0
      TR1(I)=0.0
      TR2(I)=0.0
      DW(I)=0.0
```

```
      HW(I)=0.0
      AE(I)=0.0
60    CONTINUE
      AW1(1)=AW1U
      AW2(1)=AW2U
      TR1(1)=TR1U
      TR2(1)=TR2U
      DW(1)=0.0
      HW(1)=DD
      R1=(4.0*XK)/(S(ISP)*S(ISP))
      R2=2.0*DE
C
C     READ DATA FROM TAPE,SKIPPING JAN TO MAR AND NOV - DEC.
C
      DO 70 JT=1,90
      READ(9,14)
70    CONTINUE
      READ(9,14)NSTN,NYR,MONTH
14    FORMAT(I7,I2,I2,50X)
      IF(MONTH.EQ.4) BACKSPACE9
      DO 80 KT=1,214
      READ(9,15)RAIN(KT),PE(KT)
15    FORMAT(44X,F4.2,11X,F2.2)
80    CONTINUE
      DO 90 JT=1,61
      READ(9,14)
90    CONTINUE
      NY=NYR+NAD
      DO 100 IM=1,214
      RAIN(IM)=RAIN(IM)*25.4
      PE(IM)=PE(IM)*25.4
100   CONTINUE
C
C     START OF DAILY COMPUTATION
C
      DO 110 I=1,214
      J=I+1
C
C     CHECK FOR MISSING DATA
C
      IF(RAIN(I).GT.100.0.OR.PE(I).GT.15.0) GO TO 44
      TEMP1=RAIN(I)
      TEMP2=PE(I)
      IF(DW(I).LE.Z1) GO TO 16
      R=R1*(R2*HW(I)+HW(I)*HW(I)/1000.0)
      GO TO 17
16    R=RMAX(ISP)
17    IF(RAIN(I).GE.PE(I)) GO TO 34
      TEMP2=0.0
      APE=PE(I)-RAIN(I)
```

```
      PAW=((AW1(I)-AW1L)/(AW1U-AW1L))*100.0
      AEV=-0.2285+0.4753*APE+0.019*PAW
      IF(AEV.LE.0.0) GO TO 18
      IF(AEV.GT.APE) GO TO 19
      GO TO 21
18    AEV=0.25*APE
      GO TO 21
19    AEV=APE
21    AE1=0.5*AEV
      AE2=0.5*AEV
      IF((TR1(I)-AE1).LE.TR1L) GO TO 25
      TR1(I)=TR1(I)-AE1
      IF((TR1(I)-AE2).LE.TR1L) GO TO 24
      TR1(I)=TR1(I)-AE2
22    IF((TR1(I)-R).LE.TR1L) GO TO 23
      TR1(I)=TR1(I)-R
      TR=TR1(I)+TR2(I)
      IF(TR.GE.143.0) GO TO 42
      DW(I)=((0.01472*(143.0-TR))**0.5896)*1000.0
      HW(I)=DD-DW(I)
      GO TO 42
23    R=R-(TR1(I)-TR1L)
      TR1(I)=TR1L
      GO TO 28
24    RAE2=AE2-(TR1(I)-TR1L)
      TR1(I)=TR1L
      GO TO 27
25    RAE1=AE1-(TR1(I)-TR1L)
      TR1(I)=TR1L
      IF((AW1(I)-RAE1).LE.AW1L) GO TO 26
      AW1(I)=AW1(I)-RAE1
      GO TO 27
26    RAE1=0.0
      AE1=0.0
      AW1(I)=AW1L
27    IF((TR2(I)-AE2).LE.TR2L) GO TO 31
      TR2(I)=TR2(I)-AE2
28    IF((TR2(I)-R).LE.TR2L) GO TO 29
      TR2(I)=TR2(I)-R
      TR=TR1(I)+TR2(I)
      IF(TR.GE.143.0) GO TO 42
      DW(I)=((0.01472*(143.0-TR))**0.5896)*1000.0
      IF(DW(I).GE.DD) DW(I)=DD
      HW(I)=DD-DW(I)
      GO TO 42
29    TR2(I)=TR2L
      GO TO 33
31    RAE2=AE2-(TR2(I)-TR2L)
      TR2(I)=TR2L
      IF((AW2(I)-RAE2).LE.AW2L) GO TO 32
```

```
      AW2(I)=AW2(I)-RAE2
      GO TO 33
32    AE2=0.0
      RAE2=0.0
      AW2(I)=AW2L
33    DW(I)=DD
      HW(I)=DD-DW(I)
      GO TO 42
34    XW=RAIN(I)-PE(I)
      XWU=0.5*XW
      XWL=0.5*XW
      AE1=0.0
      AE2=0.0
      TEMP1=0.0
      IF((XWU+AW1(I)).GE.AW1U) GO TO 35
      AW1(I)=AW1(I)+XWU
      XW=XWL
      GO TO 36
35    XWU=XWU-(AW1U-AW1(I))
      AW1(I)=AW1U
      XW=XWU+XWL
36    IF((XW+AW2(I)).GE.AW2U) GO TO 37
      AW2(I)=AW2(I)+XW
      XW=0.0
      XWU=0.0
      XWL=0.0
      GO TO 42
37    XW=XW-(AW2U-AW2(I))
      AW2(I)=AW2U
      IF((XW+TR2(I)).GE.TR2U) GO TO 38
      TR2(I)=TR2(I)+XW
      GO TO 21
38    XW=XW-(TR2U-TR2(I))
      TR2(I)=TR2U
      IF((XW+TR1(I)).GE.TR1U) GO TO 39
      TR1(I)=TR1(I)+XW
      GO TO 22
39    XW=XW-(TR1U-TR1(I))
      TR1(I)=TR1U
      IF((XW-R).GE.0.0) GO TO 41
      R=R-XW
      GO TO 22
41    SR(I)=XW-R
      DW(I)=0.0
      HW(I)=DD-DW(I)
42    AE(I)=AE1+AE2+TEMP1+TEMP2
43    AW1(J)=AW1(I)
      AW2(J)=AW2(I)
      TR1(J)=TR1(I)
      TR2(J)=TR2(I)
      DW(J)=DW(I)
```

```
      HW(J)=HW(I)
      GO TO 45
44    RAIN(I)=-1.0
      PE(I)=-1.0
      AE(I)=-1.0
      GO TO 43
45    AW1(I)=AW1(I)-AW1L
      AW2(I)=AW2(I)-AW2L
      TR1(I)=TR1(I)-TR1L
      TR2(I)=TR2(I)-TR2L
110   CONTINUE
      CV=CVNL
      CALL FREQ(CV,DW,P,XLOSS,LY,NY,CVNL)
50    CONTINUE
      WRITE(6,46)LY
46    FORMAT('1','YEARLY LOSSES ARRANGED IN ORDER OF DECREASING',
     1' MAGNITUDE :     NUMBER OF YEARS =',I4/1X,82('*')/)
      CALL SORT(XLOSS,LY)
30    CONTINUE
9000  STOP
      END
C
C     SUBROUTINE FOR FINDING ANNUAL CROP LOSS
C
      SUBROUTINE FREQ(CV,DW,P,XLOSS,LY,NY,CVNL)
      DIMENSION DW(214),X(214),P(8,5),XLOSS(76)
      DO 10 I=1,214
      X(I)=0.0
10    CONTINUE
      WLEVEL=0.0
      DO 50 I=1,7
      WLEVEL=WLEVEL+100.0
      DO 20 J=1,214
      X(J)=WLEVEL-DW(J)
20    CONTINUE
      NUM=0
      N0=0
      N1=0
      N2=0
      N4=0
      N6=0
      N8=0
      DO 30 L=31,123
      IF(X(L).LT.0.0) GO TO 1
      NUM=NUM+1
      GO TO 30
1     N0=N0+1
      IF(NUM.GT.7) NUM=8
      IF(NUM.LT.1) GO TO 30
      GO TO (2,3,3,4,4,5,5,6),NUM
2     N1=N1+1
```

```
      CV=CV-P(I,1)*CV/100.0
      GO TO 7
3     N2=N2+1
      CV=CV-P(I,2)*CV/100.0
      GO TO 7
4     N4=N4+1
      CV=CV-P(I,3)*CV/100.0
      GO TO 7
5     N6=N6+1
      CV=CV-P(I,4)*CV/100.0
      GO TO 7
6     N8=N8+1
      CV=CV-P(I,5)*CV/100.0
7     NUM=0
30    CONTINUE
      DO 40 IJ=1,214
      X(IJ)=0.0
40    CONTINUE
      IF(NUM.GT.7) ND=5
      IF(NUM.EQ.6.OR.NUM.EQ.7) ND=4
      IF(NUM.EQ.4.OR.NUM.EQ.5) ND=3
      IF(NUM.EQ.2.OR.NUM.EQ.3) ND=2
      IF(NUM.LT.2) ND=1
      CV=CV-P(I,ND)*CV/100.0
50    CONTINUE
      CLOSS=CVNL-CV
      WRITE(6,8) NY,CV,CLOSS
8     FORMAT(' ',10X,'YEAR:',I6,6X,'REMAINING CROP VALUE:',F8.2,
     1' $/HA',6X,'CROP LOSS:',F8.2,' $/HA')
      XLOSS(LY)=CLOSS
      RETURN
      END
C
C     SUBROUTINE SORT ARRANGES ANNUAL LOSSES IN DECREASING ORDER
C
      SUBROUTINE SORT(Y,LY)
      DIMENSION Y(LY),B(76),PROB(76)
      DO 10 I=1,LY
      PROB(I)=0.0
10    CONTINUE
      DO 20 I=1,LY
      B(I)=Y(I)
20    CONTINUE
      KY=LY-1
      DO 40 I=1,LY
      DO 30 J=1,KY
      JJ=J+1
      IF(B(JJ).LE.B(J)) GO TO 30
      TEMP=B(J)
      B(J)=B(JJ)
```

```
      B(I,J)=TEMP
20    CONTINUE
40    CONTINUE
      WRITE(6,1)(B(I),I=1,LY)
1     FORMAT('0',14F9.2)
      WRITE(6,2)
2     FORMAT(' ',132('*')//1X,'THE PROBABILITY OF RECURRENCE IN',
     1' PERCENT OF YEARS ARE GIVEN BELOW'/1X,65('-')/)
      PPN=0.0
      DO 50 I=1,LY
      PPN=PPN+1.0
      PROB(I)=PPN*100.0/76.0
50    CONTINUE
      WRITE(6,3) (PROB(I),I=1,LY)
3     FORMAT(' ',14F9.2)
      AREA=(B(1)*PROB(1))/100.0
      LY1=LY-1
      DO 60 I=1,LY1
      J=I+1
      AREA=AREA+(((B(I)+B(J))/2.0)*(PROB(J)-PROB(I)))/100.0
60    CONTINUE
      WRITE(6,4) AREA
4     FORMAT(' ',132('-')/1X,'AREA UNDER THE PROBABILITY CURVE',
     1' (=AVERAGE ANNUAL LOSS)-',F10.2,' $/HA  (IN PERCENT OF NO',
     2' LOSS CROP VALUE)')
      RETURN
      END
```

Note: 1. The above model has been operated using the weather data from Ottawa, and the soil water properties of the Ste. Rosalie Clay.

2. The 'WRITE' option for the output tables as shown in D.3 has been removed from the program as the tabular output was not required for further analysis. The values under the column 'DW' in the above mentioned tables were stored in the computer memory and were subsequently used in the crop loss computations.

D.2 Basic input values into the model.

FOLLOWING FIXED PARAMETER VALUES HAVE BEEN USED
IN WATER BALANCE COMPUTATION

DD =	1200.00 MM	DE =	1.00 M
TR1U =	48.00 MM	TR2L =	0.0 MM
AW1U =	165.00 MM	AW2L =	280.00 MM
Z1 =	400.00 MM	TR1L =	0.0 MM
TR2U =	95.00 MM	AW1L =	140.00 MM
AW2U =	329.00 MM	CVNL =	100.00 $/HA

THE FOLLOWING IS THE PERCENTAGE CROP LOSS MATRIX FOR
VARIOUS DEPTHS AND DURATIONS OF WATER TABLE

WATER TABLE DEPTH MM	DURATION IN CONSECUTIVE DAYS				
	≤1 DAY	2 OR 3 DAYS	4 OR 5 DAYS	6 OR 7 DAYS	>7 DAYS
<100	0.0	25.0	50.0	75.0	100.0
100-200	0.0	15.0	30.0	45.0	60.0
200-300	0.0	10.0	21.0	31.0	41.0
300-400	0.0	8.0	16.0	23.0	31.0
400-500	0.0	6.0	12.0	17.0	23.0
500-600	0.0	4.0	8.0	11.0	15.0
600-700	0.0	2.0	4.0	5.0	7.0
>700	0.0	0.0	0.0	0.0	0.0

The values in the preceding table are the crop losses, in percent of full production, for the indicated depths and durations of water table.

D.3 A sample of output from the model showing recorded daily RAIN and PE and predicted values of other variables.

* STATION NUMBER: 6105976 * * STATION NAME: OTTAWA * * YEAR: 1958 * JULY

DAY	RAIN MM	PE MM	AE MM	AWT MM	AWB MM	TRT MM	TRB MM	DW MM	HW MM	SR MM
1	1.0	6.1	3.4	1.9	49.0	0.0	27.6	1200.0	0.0	0.0
2	0.0	4.8	2.2	0.8	49.0	0.0	26.5	1200.0	0.0	0.0
3	0.0	4.8	1.1	0.0	49.0	0.0	25.5	1200.0	0.0	0.0
4	0.0	4.3	0.9	0.0	49.0	0.0	24.5	1200.0	0.0	0.0
5	14.0	1.8	1.8	6.1	49.0	0.0	30.6	1200.0	0.0	0.0
6	0.0	3.0	1.7	5.3	49.0	0.0	29.8	1200.0	0.0	0.0
7	0.0	4.1	2.1	4.2	49.0	0.0	28.7	1200.0	0.0	0.0
8	17.0	3.8	3.8	10.8	49.0	0.0	35.4	1200.0	0.0	0.0
9	0.0	4.6	2.8	9.4	49.0	0.0	34.0	1200.0	0.0	0.0
10	0.0	5.1	2.9	8.0	49.0	0.0	32.5	1200.0	0.0	0.0
11	0.0	3.3	1.9	7.0	49.0	0.0	31.5	1200.0	0.0	0.0
12	9.9	4.3	4.3	9.8	49.0	0.0	34.3	1200.0	0.0	0.0
13	0.0	4.3	2.6	8.5	49.0	0.0	33.1	1200.0	0.0	0.0
14	4.6	4.8	4.8	8.4	49.0	0.0	32.9	1200.0	0.0	0.0
15	20.8	4.8	4.8	16.4	49.0	0.0	40.9	1200.0	0.0	0.0
16	0.0	3.6	2.7	15.0	49.0	0.0	39.6	1200.0	0.0	0.0
17	0.0	4.1	2.8	13.6	49.0	0.0	38.2	1200.0	0.0	0.0
18	26.9	5.1	5.1	24.5	49.0	0.0	49.1	1200.0	0.0	0.0
19	3.3	3.0	3.0	24.7	49.0	0.0	49.2	1200.0	0.0	0.0
20	0.0	4.1	3.6	22.9	49.0	0.0	47.4	1200.0	0.0	0.0
21	0.0	5.1	3.9	20.9	49.0	0.0	45.5	1200.0	0.0	0.0
22	0.0	5.3	3.9	19.0	49.0	0.0	43.5	1200.0	0.0	0.0
23	0.0	4.3	3.3	17.3	49.0	0.0	41.9	1200.0	0.0	0.0
24	0.0	5.6	3.7	15.5	49.0	0.0	40.0	1200.0	0.0	0.0
25	0.0	5.6	3.6	13.7	49.0	0.0	38.2	1200.0	0.0	0.0
26	6.3	4.3	4.3	14.7	49.0	0.0	39.2	1200.0	0.0	0.0
27	0.0	3.8	2.7	13.3	49.0	0.0	37.9	1200.0	0.0	0.0
28	43.4	3.6	3.6	25.0	49.0	0.0	66.1	1067.8	132.2	0.0
29	8.6	4.1	4.1	25.0	49.0	0.0	67.8	1053.2	146.8	0.0
30	0.0	4.8	4.0	23.0	49.0	0.0	62.7	1095.6	104.4	0.0
31	0.0	4.1	3.5	21.3	49.0	0.0	58.8	1127.2	72.8	0.0

D.4 A sample output from the model showing yearly crop losses and the remaining values of crop.

K = 0.10 %/DAY S = 20.00 MM AND RMAX = 2.240 MM/DAY FOR THIS RUN

YEAR	REMAINING CROP VALUE $/HA	CROP LOSS $/HA	YEAR	REMAINING CROP VALUE $/HA	CROP LOSS $/HA
1900	100.00	0.0	1939	96.04	3.96
1901	100.00	0.0	1940	96.00	4.00
1902	100.00	0.0	1941	100.00	0.0
1903	98.00	2.00	1942	100.00	0.0
1904	98.00	2.00	1943	68.10	31.90
1905	100.00	0.0	1944	100.00	0.0
1906	100.00	0.0	1945	23.66	76.34
1907	54.14	45.86	1946	98.00	2.00
1908	21.77	78.23	1947	7.96	92.04
1909	25.57	74.43	1948	100.00	0.0
1910	100.00	0.0	1949	100.00	0.0
1911	100.00	0.0	1950	100.00	0.0
1912	100.00	0.0	1951	98.00	2.00
1913	100.00	0.0	1952	100.00	0.0
1914	100.00	0.0	1953	92.16	7.84
1915	100.00	0.0	1954	88.47	11.53
1916	34.82	65.18	1955	100.00	0.0
1917	100.00	0.0	1956	55.16	44.84
1918	100.00	0.0	1957	100.00	0.0
1919	74.76	25.24	1958	100.00	0.0
1920	98.00	2.00	1959	100.00	0.0
1921	100.00	0.0	1960	100.00	0.0
1922	100.00	0.0	1961	92.16	7.84
1923	100.00	0.0	1962	75.29	24.71
1924	58.37	41.63	1963	100.00	0.0
1925	100.00	0.0	1964	100.00	0.0
1925	100.00	0.0	1965	100.00	0.0
1927	100.00	0.0	1966	100.00	0.0
1928	60.88	39.12	1967	100.00	0.0
1929	48.50	51.50	1968	100.00	0.0
1930	100.00	0.0	1969	55.28	44.72
1931	100.00	0.0	1970	100.00	0.0
1932	100.00	0.0	1971	100.00	0.0
1933	75.58	24.42	1972	96.04	3.96
1934	100.00	0.0	1973	86.70	13.30
1935	100.00	0.0	1974	74.93	25.07
1936	91.20	8.80	1975	100.00	0.0
1937	100.00	0.0			
1933	100.00	0.0			

D.5 A sample of output from the model showing the yearly losses, associated probabilities and the average annual loss.

YEARLY LOSSES ARRANGED IN ORDER OF DECREASING MAGNITUDE : NUMBER OF YEARS = 76

****

92.04	78.23	76.34	74.43	65.18	51.50	45.86	44.34	44.72	41.63	39.12	31.90	25.24	25.07
24.71	24.42	13.30	11.53	8.80	7.84	7.84	4.00	3.96	3.96	2.00	2.00	2.00	2.00
2.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0								

****

THE PROBABILITY OF RECURRENCE IN PERCENT OF YEARS ARE GIVEN BELOW

1.32	2.63	3.95	5.26	6.58	7.89	9.21	10.53	11.84	13.16	14.47	15.79	17.11	18.42
19.74	21.05	22.37	23.68	25.00	26.32	27.63	28.95	30.26	31.58	32.89	34.21	35.53	36.84
38.16	39.47	40.79	42.11	43.42	44.74	46.05	47.37	48.68	50.00	51.32	52.63	53.95	55.26
56.58	57.89	59.21	60.53	61.84	63.16	64.47	65.79	67.11	68.42	69.74	71.05	72.37	73.68
75.00	76.32	77.63	78.95	80.26	81.58	82.89	84.21	85.53	86.84	88.16	89.47	90.79	92.11
93.42	94.74	96.05	97.37	98.68	100.00								

AREA UNDER THE PROBABILITY CURVE (=AVERAGE ANNUAL LOSS)

= 11.87 $/HA (IN PERCENT OF NO LOSS CROP VALUE)