AN EXPERIMENTAL STUDY OF NOISE PROCESSES IN AN ELECTRON GUN

by

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ABSTRACT

An experimental investigation was made of the noise properties of a parallel-flow diode electron gun over the frequency range 200 Mc/s to 3000 Mc/s. Measurements were made at beam voltages from 100 to 1000 volts. Sufficient experimental results were obtained that by comparing them with the corresponding predictions, a critical examination of the existing high frequency noise theories was possible. The measurements are thought to be the first reported of their type to cover such a range of frequencies.

The predictions of the existing theories were shown to differ greatly and to be in serious disagreement with the experimentally measured values. The basic assumptions made in each of the theories were outlined, and a brief description of their development given.

A method of predicting electron gun noise was developed which involves the solution of a modified electronic equation by an analogue computer. The resulting predictions of the gun anode current and velocity noise still showed considerable differences from the experimental values. The correspondence between the two was found, however, to be more consistent than that obtained from the previous theories.

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1. INTRODUCTION

In all types of electrical communication the rate at which information can be transferred is proportional to the transmission bandwidth. Recent rapid extension of communication services and the introduction of high-complexity services such as television have resulted in a great demand for additional band width. To meet the demand, increasingly high radio frequencies have been exploited for transmission. Conventional electron tubes, however, prove less and less satisfactory as the signal frequency is increased because of electron transit time between the tube elements. Beam type tubes have been able to overcome this difficulty by providing a "travelling wave" interaction between an electron beam and a wave guiding structure. In addition, beam type tubes have the great advantage of being able to amplify signals over an extremely large bandwidth. A lower limit to the size of signal which can be processed by a tube at these frequencies is most often imposed by the electrical noise generated within the tube itself. The cost of setting up and maintaining a specific communications system will, therefore, be strongly dependent on the noise properties of the receiver. For the above reasons extensive research on the fundamental noise properties of the beams used in receiving tubes seems justified.

The research reported in this thesis is aimed at obtaining a better understanding of the basic phenomena related to high frequency noise in an electron flow. Interest is centered on the most fundamental situations possible, both theoretically and experimentally. The work was done as part of a group project being carried on in the Eaton Electronics Research Laboratory.

Electrical noise may be defined as the random variation with time of a current, voltage or electron velocity at some point in a circuit about its average value. For the particular case of electron flow in vacuum, current and electron velocity are the most convenient variables and will be used throughout the thesis. In all cases of practical interest the electrons are obtained by thermionic emission from a cathode at an elevated temperature. The statistical nature of thermionic electron emission both in electron velocity and in number emitted per unit time interval is well known. The statistical fluctuations form a fundamental source of noise in the electron flow.

To be useful in high frequency beam type tubes, the electrons must be accelerated to a potential of some hundreds of volts and made to form a fairly well collimated beam. This is accomplished by an electron gun which consists of a system of properly placed metal electrodes at suitable potentials relative to the cathode. The application of potentials to the gun causes a space charge potential minimum to form in front of the cathode, as described by Fry (27) and by Langmuir (7). The potential minimum modifies to some extent the fluctuations occurring at the cathode plane. The acceleration region extending from the potential minimum to the anode plane of the gun transforms the fluctuations greatly, and further transformations occur in the drift space which follows the gun.

Additional noise may be introduced into the electron beam by any process which tends to increase the disorder of the electron flow. Some of the most important such excess noise sources are:

1) gun action which is not ideal but introduces transverse velocities.

2) secondary electrons returning along the beam from the collector.

3) interception of part of the beam on various electrodes.

4) collision of beam electrons with air molecules.

It is, however, possible at least theoretically to reduce the noise from these sources to a negligibly small value. The only noise not theoretically removable is that due to the thermionic emission fluctuations which for this reason is called the fundamental noise.

Arguing from the above paragraph, the basic noise properties of an electron beam will involve only transformed quantities arising from the cathode thermionic fluctuations. A fairly well established theory of the transformations occurring in a drift space has been available for some time. The noise considerations in the interaction region are most often entirely decided by the fluctuations at the point of smallest radio-frequency signal; that is, at the signal input. Since the input is separated from the anode of the gun (usually) only by a short drift space, knowledge of the noise at the gun anode alone will allow a fairly complete description of the basic noise properties of the device.

Thus it can be seen that, as far as the fundamental theory is concerned, a successful theory need only give an adequate description of the noise which appears at the anode of an electron gun in terms of the thermionic properties of the cathode and characteristics of the gun. Considerations have almost always been restricted to the simplest possible type of electron gun which is a parallel flow diode. This is less restrictive than might be supposed, since any parallel flow gun may be considered as a diode gun followed by other spaces whose transformations can be treated separately and, in fact, by simpler theories.

In order to verify the predictions of various theories,, some measurement or series of measurements must be performed on an appropriate electron beam from which the fluctuation quantities at the anode of the gun may be derived. Measurements over a wide range of gun voltages and signal frequencies would be of particular advantage in such comparisons since trends are often equally significant with actual values.

A number of theories applicable to the prediction of anode noise have been reported in the literature. The predicted results vary widely for different theories, as do the assumed conditions. The agreement of the predictions with the few available experimental results is in general not good. There are reported no attempts to take into account the finite diameter of the beam, despite the fact that substantially different predictions would be expected. Another factor difficult to consider is the multivelocity nature of the beam near the cathode. It was thought advisable to give, in the theory section, a critical examination of the main existing theories in order to show the above mentioned differences and the assumptions involved.

Although reports of experimental noise properties of particular devices, such as travelling wave tubes, are fairly numerous, their results are not suitable for comparison with basic theory directly. The few reported fundamental measurements, by Cutler and Quate (2) and by Fried and Smullin (28) are considered to be not extensive enough to allow a sound evaluation of the theories. Most notably, no information at frequencies other than 3000 Mc/sec. is available.

In view of the situation as presented in the foregoing discussion the aims of the present research have been two fold;

1) To obtain sufficient basic experimental noise information to allow a critical examination of existing theories of noise at the anode of a simple parallel flow diode electron gun.
2) To develop an extension to existing theories which involves analogue computer solutions and makes a primary attempt to account for a finite beam diameter in prediction of the noise properties of a parallel flow diode gun.

The extent to which the research has been successful may be briefly summarized as follows:

(1) The experimental results obtained are believed to be the first available measurements of noise along the length of an electron beam which cover a substantial range of signal frequencies. The resulting fluctuation quantities computed for the gun anode plane show fairly well defined trends in spite of experimental scatter.

(2) Comparison of existing theories of noise has indicated the nature of the basic assumptions involved and discussed their validity. Numerical computations have emphasized the wide discrepancies which exist among the predictions and also the serious lack of agreement with experimental values.

(3) The solution obtained from the analogue computer,while still showing considerable differences,gives a significantly better agreement with the experimental results than do the previous theories.

2. THEORY

2-I Introductory

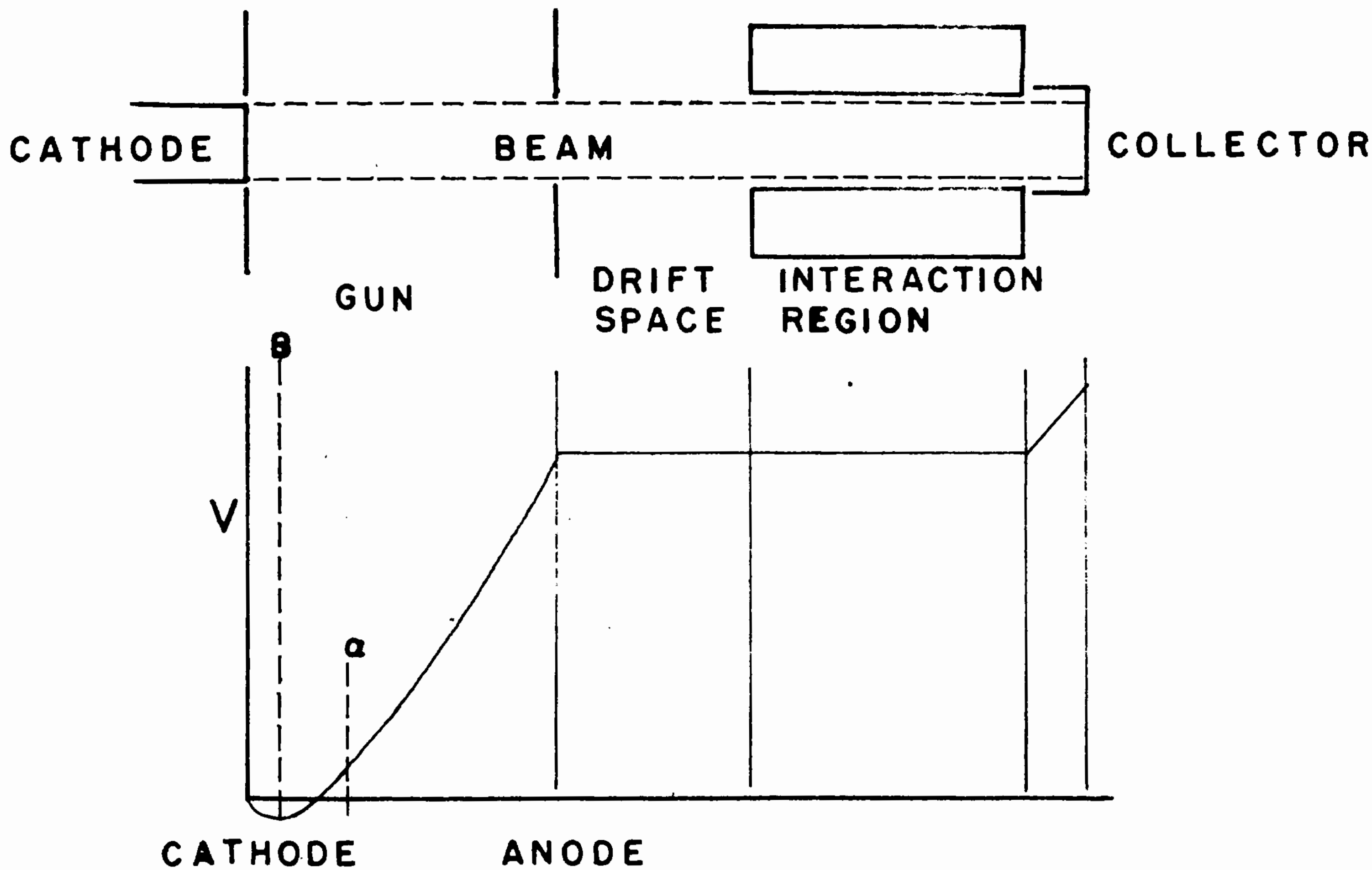
The investigation of the basic noise properties of an electron beam device reduces, as outlined in the introduction, to the more fundamental investigation of current and velocity fluctuations at the anode of a simple parallel flow diode electron gun. The fluctuations of interest must be expressible in terms of known geometric and steady state electromagnetic parameters of the gun and known thermionic properties of its cathode. Even the relatively simple case of a diode gun however, can be considered most conveniently by dividing it into a number of regions each requiring a separate treatment. A schematic diagram of the gun is given in Fig. 1.

First to be considered are the input fluctuations at the cathode surface. The magnitude of a noise fluctuation can best be described by quadratic content. By this is meant the time average value of the square of the deviation from an average value. In electron beam studies, the quadratic contents of current and of velocity are of interest. It was shown many years ago by Schottky (18) and verified experimentally by Thompson, North, and Harris (20) that the quadratic content of a thermionically emitted electron current is given by:

$$\overline{i^2_{sn}} = 2eI_o\Delta f \qquad (2.1)$$

the now familiar shot noise relation. I_o is the d.c. current and Δf the bandwidth of the measuring device. For greater generality, current densities will be considered throughout this thesis. A complete list of the symbols used, with their definitions is given in Appendix (IV). In the notation used

$$\overline{J^2_{sn}} = 2eJ_o\Delta f \qquad (2.1a)$$



THE ELECTRON BEAM

Figure 1.

It should be carefully noted that the $\overline{J_{sn}^2}$ of equation (2.1a) is <u>not</u> the square of a current density but is a quadratic content per unit beam area. From similar statistical considerations, Rack (15) has derived an expression for the quadratic content of electron velocity for randomly emitted electrons:

$$\overline{v_R^2} = \frac{(4-\pi)\,\eta\, kT_c}{J_o}\,\Delta f \tag{2.2a}$$

which, although unverified, is generally assumed to be valid. In the equation (2.2a) it should again be noted that v_R^2 is not truly a velocity squared, since it has dimensions $\frac{L^4}{T^2}$. The equations (2.1a) and (2.2a) constitute the quadratic content of current and velocity existing at the cathode surface. In order to obtain quantities whose validity is not restricted to a particular measuring system, equations (2.1a) and (2.2a) are used in their spectral density form.

$$\frac{\overline{J_{sn}^2}}{\Delta f} = 2eJ_o \tag{2.1b}$$

$$\frac{\overline{v_R^2}}{\Delta f} = \frac{(4-\pi)kT_c\eta}{J_o} \tag{2.2b}$$

At high signal frequencies, no analytic description exists for the way in which the cathode quantities are modified in proceeding to the potential minimum at plane B in Fig. 1. Until very recently all theoretical treatments chose the fluctuations at the minimum to be unchanged from those at the cathode. Late in 1956, however, a very significant paper by Tien and Moshman (22) described the modifications quite adequately for one particular set of gun conditions. Their method consisted of a "Monte Carlo" calculation of the trajectories of individual electrons randomly injected into a diode region of specified initial potential distribution. A high speed digital computer was used for the calculation. The work will be described more fully in a later section. Although the quantitative results apply only to a particular combination of gun voltage and geometry, a fair indication is given of what may be assumed at the potential minimum for any gun of a similar type.

Theories which describe the transformation of simple modulations mathematically require that the d.c. electron velocity at any cross section be single valued. Rack (15) has shown that the single velocity assumption is valid provided that the spread in the transit times of electrons across the region considered is small compared to one cycle of the radio frequency being considered. Fluctuation quantities may be treated by the same theories if it is assumed that they may be represented by equivalent single frequency modulations on a beam of the required single d.c. velocity. Proceeding from the potential minimum toward the anode, it is possible to find a plane beyond which the small transit time spread criterion holds. After Robinson (17), this is called the α-plane. It is found in practise that the α-plane need be situated at a potential of only a vote or less above the minimum for ordinary electron guns and frequencies. From the α-plane to the anode, the fluctuations are then handled by theories which were developed for considering ordinary signal modulations in acceleration regions. There remains the problem of finding a valid transformation of the fluctuations from the potential minimum to the α-plane. Robinson (17) gives a method which may be used, but there are serious difficulties which will be discussed in a later section.

It should be noted that all considerations so far have been concerned with transformations along a beam in which quantities vary only in the axial direction. This lack of edge effects implies a beam which is infinite in its lateral extent. The boundary conditions to be satisfied when a beam of finite diameter is considered results in a serious complication of the problem.

A first attempt to account for the finite size of the beam is presented as Sec. 2-V.

The accuracy with which an experimental electron gun can be made to approach the theoretical conditions is discussed in the experimental section. It is sufficient to say here that even the most carefully constructed gun will have certain deviations from the theoretical model considered.

2-II The Drift Space Relations

The relations to be discussed in this section have been used by many authors but are nowhere in the literature developed from basic physical arguments. The present development will follow that given by Woonton (25) in an unpublished report.

An electron beam in a drift space may be viewed as a moving plasma of zero net charge density. The electrons of the plasma are assumed to move with the beam velocity u_o. If an electron is displaced from its equilibrium position in the plasma it will experience restoring forces due to its immediate neighboring charges which cause it to perform simple harmonic oscillations about its equilibrium position. The frequency of oscillation will be determined by the plasma charge density of electrons (or of positive ions). In a classic paper Tonks and Langmuir (23) developed an expression for the oscillation frequency in a plasma of infinite extent. The expression is

$$\omega_p = \sqrt{\frac{e\,\rho_o}{m\,\varepsilon_o}} \tag{2.3}$$

It was found that one oscillating electron would not induce oscillations in neighboring ones, so that disturbances do not propagate through the plasma. This is equivalent to saying that the oscillating electron does not transmit energy to its neighbors.

If one imagines a pair of very closely spaced parallel grids at some position $Z = 0$ through which the plasma electrons are moving at velocity u_o, then each electron will suffer a velocity disturbance upon passing through the grids if a voltage exists between the grids. A voltage $V \sin \omega t$ applied to the grids would give the electrons a disturbance which depended on their time of passage through the grids. Assume the grids to be so closely spaced that no significant displacement occurs while the electron is between them and the voltage between the grids is constant during its passage. An electron passing through the grids at time t_1 will receive a velocity increment $\hat{v}_1 \sin \omega t_1$ where $\hat{v}_1$ is simply related to V. Thereafter, because of the plasma restoring forces, the electron will have a velocity relative to the plasma given by

$$v_1 = \hat{v}_1 \sin \omega t_1 \cos (\omega_p T) \tag{2.4}$$

where $T = \frac{z}{u_o}$ is the time elapsed since it passed through the grids. Observing the electrons at position z at time t one would find

$$v_1 = \hat{v}_1 \sin \omega (t - \frac{z}{u_o}) \cos (\frac{\omega_p z}{u_o}) \tag{2.5}$$

using trigonometrical relations, (2.5) may be written

$$v_1 = \frac{\hat{v}_1}{2} \sin (\omega t - \frac{(\omega + \omega_p) z}{u_o}) + \frac{\hat{v}_1}{2} \sin (\omega t - \frac{(\omega - \omega_p) z}{u_o} \tag{2.6}$$

showing that the behavior is equivalent to a pair of waves proceeding in the same direction as the velocity u_o, but with velocities one on either side of u_o by a small amount. The interference pattern of the two waves produces the $(\frac{\omega_p z}{u_o})$ variation noted in equation (2.5).

The electron oscillations in the plasma are accompanied by alternating electric and magnetic fields, the electric fields being by far the larger for ordinary values of u_o. Using the Lorentz force equation

$$\vec{F} = m \frac{d\vec{v_1}}{dt} = - e \quad \vec{E_1} + \frac{\vec{v_1} \times \vec{B_1}}{c} \approx -e \vec{E_1} \tag{2.7}$$

gives the relation

$$\vec{E_1} = - \frac{m}{e} \quad \frac{d\vec{v_1}}{dt}$$

or, in partial derivatives,

$$E_1 = - \frac{m}{e} \left[\frac{\partial v_1}{\partial t} + u_o \frac{\partial v_1}{\partial z} \right] \tag{2.8}$$

when equation (2.6) for velocity is used in (2.8), there results

$$E_1 = \frac{m}{e}\frac{\hat{v}_1}{2}\omega_p \cos(\omega t - \frac{(\omega+\omega_p)z}{u_o}) - \frac{m}{e}\frac{\hat{v}_1}{2}\omega_p \cos(\omega t - \frac{(\omega-\omega_p)z}{u_o}) \qquad (2.9)$$

From the Maxwell equation

$$\nabla X H_1 = J_1 + \frac{\partial D_1}{\partial t} = I_1 \qquad (2.10)$$

rearrangement gives

$$J_1 - I_1 = -\varepsilon_o \frac{\partial E_1}{\partial t} \qquad (2.11)$$

in which I_1 is called the total current. Since by mathematical identity

$$\nabla \cdot \nabla X H_1 = 0 \quad \text{or} \quad \nabla \cdot I_1 = 0$$

it is seen that I_1 does not vary with distance, but is a function of time only. Then if I_1 exists, some path external to the beam must conduct it back from the plane under consideration to the initial plane. With a beam of infinite lateral extent such a path is not possible, meaning that $\nabla \cdot I = 0$ implies $I_1 = 0$. Equation (2.11) for an infinite beam then becomes

$$J_1 = -\varepsilon_o \frac{\partial E_1}{\partial t} \qquad (2.11a)$$

Using the equivalent form of equation (2.9)

$$E_1 = \frac{m}{e}\omega_p \hat{v}_1 \sin(\frac{\omega_p z}{u_o}) \sin\omega(t - \frac{z}{u_o}) \qquad (2.9a)$$

(2.11a) gives the relation

$$J_1 = \varepsilon_o \frac{m}{e}\omega_p\omega\hat{v}_1 \sin(\frac{\omega_p z}{u_o}) \cos\omega(t - \frac{z}{u_o}) \qquad (2.12)$$

which, since $\frac{\rho_o}{\omega_p^2} = \frac{\varepsilon_o m}{e}$ from equation (3), gives:

$$J_1 = \rho_o \frac{\omega}{\omega_p}\hat{v}_1 \sin(\frac{\omega_p z}{u_o}) \cos\omega(t - \frac{z}{u_o}) \qquad (2.13)$$

Equations (2.5) and (2.13) thus give the current and velocity modulation resulting from an initial velocity modulation at $z = 0$ of the form $\hat{v}_1 \sin(\omega t)$.

In order to consider an input current as well as velocity, it is most convenient to start with the equivalent forms:

$$v_1 = \frac{\hat{v}_1}{2} \sin\omega(t - \frac{\omega+\omega_p}{\omega} \frac{z}{u_o}) + \frac{\hat{v}_1}{2} \sin \omega(t - \frac{\omega-\omega_p}{\omega} \frac{z}{u_o}) \qquad (2.6)$$

$$J_1 = \rho_o \frac{\omega}{\omega_p} \frac{\hat{v}_1}{2} \sin \omega(t - \frac{\omega+\omega_p}{\omega} \frac{z}{u_o}) - \rho_o \frac{\omega}{\omega_p} \frac{\hat{v}_1}{2} \sin\omega(t-\frac{\omega-\omega_p}{\omega} \frac{z}{u_o}) \qquad (2.14)$$

The equations (2.6) and (2.14) show that each of the two waves give a fixed ratio of J_1 to v_1. We have for the slow and fast waves respectively

$$\frac{J_s}{v_s} = \frac{\omega\rho_o}{\omega_p} \qquad \text{and} \qquad \frac{J_f}{v_f} = -\frac{\omega\rho_o}{\omega_p}$$

Then if both a current and a velocity exist at the input plane for the slow wave the input current $\hat{J}_a$ will increase the velocity magnitude by an amount $\frac{1}{\rho_o} \frac{\omega_p}{\omega} \hat{J}_a$ and for the fast wave by an amount $-\frac{1}{\rho_o} \frac{\omega_p}{\omega} \hat{J}_a$. Similarly the magnitude of the current will be increased due to the input velocity by amounts $\rho_o \frac{\omega}{\omega_p} \hat{v}_a$ and $-\rho_o \frac{\omega}{\omega_p} \hat{v}_a$ for the two waves. The resulting equations are then

$$v_1 = \frac{1}{2}(\hat{v}_a + \frac{\omega_p}{\omega} \frac{1}{\rho_o} \hat{J}_a) \sin \omega(t-\frac{\omega+\omega_p}{\omega} \frac{z}{u_o}) + \frac{1}{2}(\hat{v}_a - \frac{\omega_p}{\omega} \frac{1}{\rho_o} \hat{J}_a)\sin\omega(t-\frac{\omega-\omega_p}{\omega} \frac{z}{u_o}) \qquad (2.15)$$

$$J_1 = \frac{1}{2}(\rho_o\frac{\omega}{\omega_p} \hat{v}_a + \hat{J}_a)\sin\omega(t-\frac{\omega+\omega_p}{\omega} \frac{z}{u_o}) + \frac{1}{2}(- \rho_o \frac{\omega}{\omega_p} \hat{v}_a+\hat{J}_a)\sin\omega(t-\frac{\omega-\omega_p}{\omega} \frac{z}{u_o}) \qquad (2.16)$$

Employing exponential notation, equations (2.15) and (2.16) can be written in the more compact form:

$$v_1 = \left\{\hat{v}_a \cos (\frac{\omega_p z}{u_o}) - j \frac{\omega_p}{\omega} \frac{1}{\rho_o} \hat{J}_a \sin (\frac{\omega_p z}{u_o})\right\} e^{-j\frac{\omega z}{u_o}} \qquad (2.17)$$

$$J_1 = \left\{-j\frac{\omega}{\omega_p} \rho_o \hat{v}_a \sin (\frac{\omega_p z}{u_o}) + \hat{J}_a \cos (\frac{\omega_p z}{u_o})\right\} e^{-j\frac{\omega z}{u_o}} \qquad (2.18)$$

where the $e^{j\omega t}$ variation has not been written but is understood to exist. Equations (2.17) and (2.18) are the well known drift space relations.

Extension of the equations (2.17) and (2.18) to the case of a finite beam in a drift space involves the consideration of boundary conditions at the beam surface. A general development has been given by Birdsall and Whinnery (1) for a cylindrical beam in a cylindrical walled space which has arbitrary wall properties. For the present case, they find that the only effect on the solution of wave propagation along the beam is the modification of the plasma frequency by a real factor p, such that ω_p becomes $p\omega_p = \omega_q$. The factor p is called the plasma reduction factor. It is obtained as a function of $(\frac{\omega b}{u_o})$ by the solution of a transcendental equation which satisfies the field matching conditions and at the beam surface. For a very large diameter drift space, the required equation reduces to

$$Tb \frac{J_1(Tb)}{J_o(Tb)} = \beta_e b \frac{K_1(\beta_e b)}{K_o(\beta_e b)} \tag{2.19}$$

where p must satisfy the relation

$$p = \left[1 + \frac{T^2}{\beta_e^2}\right]^{-1/2} \tag{2.20}$$

b is the beam radius, and $\beta_e = \frac{\omega}{u_o}$

The equations (2.17) and (2.18) become, for the finite beam case

$$v_1 = \left\{ v_a \cos\left(\frac{\omega_q z}{u_o}\right) - j\frac{\omega_q}{\omega} \frac{1}{\rho_o} \hat{J}_a \sin\left(\frac{\omega_q z}{u_o}\right) \right\} e^{-j\frac{\omega z}{u_o}} \tag{2.21}$$

$$J_1 = \left\{ -j\frac{\omega}{\omega_q} \rho_o v_a \sin\left(\frac{\omega_q z}{u_o}\right) + \hat{J}_a \cos\left(\frac{\omega_q z}{u_o}\right) \right\} e^{-j\frac{\omega z}{u_o}} \tag{2.22}$$

To make equations (2.21) and (2.22) completely general, account must be taken of the total current $I_t = I_1 e^{j\omega t}$ since a return path is now possible. Using equation (2.11), equation (13) should actually be written

$$J_1 - I_1 = \rho_o \frac{\omega}{\omega_p} \hat{v}_1 \sin\left(\frac{\omega_p z}{u_o}\right) \cos \omega\left(t - \frac{z}{u_o}\right) \tag{2.13a}$$

It should be stressed again that I_t is a function of time only. Following through the arguments as before, the equations (2.21) and (2.22) become

$$v_1 = \left\{ \hat{v}_a \cos\left(\frac{\omega_q z}{u_o}\right) - j\frac{\omega_q}{j\omega} \frac{1}{\rho_o} (\hat{J}_a - I_1) \sin\left(\frac{\omega_q z}{u_o}\right) \right\} e^{-j\frac{\omega z}{u_o}} \qquad (2.23)$$

$$J_1 = \left\{ -j \frac{\omega}{\omega_q} \rho_o \hat{v}_a \sin\left(\frac{\omega_q z}{u_o}\right) + \hat{J}_a \cos\left(\frac{\omega_q z}{u_o}\right) \right\} e^{-j\frac{\omega z}{u_o}} \qquad (2.24)$$

For actual physical drift spaces, the total current need not be considered so that equations (2.21) and (2.22) are the most useful. The forms (2.23) and (2.24) are of interest only where incremental drift spaces are considered as in the development of appendix (II).

2-III. The Electronic Equation

The electronic equation is one possible general formulation of the way in which current and velocity modulations transform along a beam when the d.c. electron velocity is a function of distance in the direction of flow. In view of the equivalence mentioned in section 2-I, between signals and noise when velocity spread is small, such an equation should apply as well to noise transformations if the d.c. velocity is sufficiently large.

The equation was first developed by Smullin (19) with later similar derivations being given by Parzen (12), Tien and Field (21) and Hutter (4). A short outline of the assumptions and procedure involved in the development will most easily show the nature of the equation. The notation of Hutter is used in the outline which follows.

All the developments state or imply the following assumptions:

1) The problem is one dimensional, implying a beam of infinite lateral extent, and variation of quantities in the z direction only.

2) a.c. quantities are very small compared to the corresponding d.c. quantities, resulting in a "small signal" analysis.

3) Only d.c. velocities low enough so that a.c. magnetic forces are negligibly small are considered.

4) All a.c. quantities have a time dependence $e^{j\omega t}$.

The derivation presented by Hutter consists essentially of the following steps:

First the required basic electromagnetic relations are given:

From Maxwell's Equations

$$\nabla \times H = J + \frac{\partial D}{\partial t} = I \qquad (2.25)$$

$$\nabla \cdot D = \rho \qquad (2.26)$$

The law of conservation of charge or continuity equation

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0 \tag{2.27}$$

The Lorentz force equation

$$F = m\frac{dv}{dt} = -eE + \frac{e}{c}(v \times B) \tag{2.28}$$

and finally the equation defining beam current density

$$-J_o + J = (\rho_o + \rho)(u_o + v) \tag{2.29}$$

By considering the simplifing assumptions, restricted forms of equations (2.25) to (2.29) which apply to the present case are set up;

He uses the notation $J = J_z = J_\omega e^{j\omega t}$ with similar symbols for other variables. Using the restricted equations and the partial derivative form of $\frac{dv_z}{dt}$ as a function of z and t, relations between v_ω and J_ω are established.

$$v_\omega = -\frac{u_o}{J_o}\left[J_\omega + \frac{u_o}{j_\omega}\frac{\partial J_\omega}{\partial z}\right] \tag{2.30}$$

and

$$\left(j\omega + \frac{\partial u_o}{\partial z}\right)v_\omega + u_o\frac{\partial v_\omega}{\partial z} = -\frac{e}{m}E_\omega \tag{2.31}$$

The definitions

$$J_\omega = \frac{Y}{u_o}e^{-j\omega\tau} \qquad \tau = \int_0^z \frac{dz}{u_o} \tag{2.32}$$

then lead to Hutter's basic equations

$$v_\omega = -\frac{1}{j\omega J_o}\left[\frac{\partial Y}{\partial \tau} - \frac{Y}{u_o}\frac{\partial u_o}{\partial \tau}\right]e^{-j\omega\tau} \tag{2.33}$$

$$\frac{\partial^2 Y}{\partial \tau^2} - \frac{1}{u_o}\left[\frac{\partial^2 u_o}{\partial \tau^2} - \frac{e\,J_o}{m\,\varepsilon_o}\right]Y = \frac{e\,J_o}{m\,\varepsilon_o}I_\omega e^{j\omega\tau} \tag{2.34}$$

The manipulations involved are sometimes lengthy but are mathematically straightforward. It should be noted that J_ω and v_ω are the same quantities as the J_1 and v_1 of Sec. (2-II).

Hutter shows that for the conditions present in some particular regions the results are equivalent to those obtained by other workers in less general developments. To be more explicit, he treats a drift space, a linear acceleration space, a velocity jump and a region in which the potential distribution is decided by space charge and the end potentials (a Llewellyn(8) gap).

2-IV. Description of Existing Theories

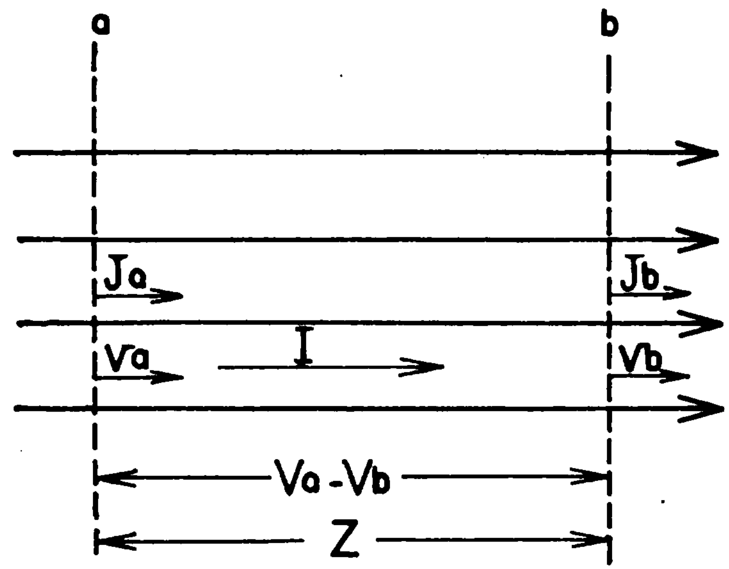
This section will include both those theories that have been used to describe the noise behavior of the complete electron gun and various analyses which considered only one particular region of the gun but which are significant in the study of noise. A relatively few reports have been published which actually make a distinct contribution to the solution of the basic problem. Those considered of the greatest importance are briefly outlined as to the assumptions made and the method of development. The theories have been entitled with the names of the authors. They appear in chronological order.

A). The Llewellyn-Peterson-Pierce Theory

The relations used by Llewellyn and Peterson (9) have their origin in the more fundamental work done earlier by Llewellyn (8) on electron inertia effects in general. Llewellyn's analysis involved very lengthy, complex manipulations leading to non-linear equations. The Llewellyn-Peterson paper employed a first order approximation to these general equations for the analysis of vacuum tube phenomena.

The situation considered in both analyses is shown schematically in Fig. 2. A pair of infinite permeable conducting parallel planes "a" and b are situated in an infinite electron beam perpendicular to the direction of the electron motion. An n alternating voltage V_b-V_a exists between the planes and a total current density I flows between them. The a.c. convection current and velocity densities J_a and v_a are known at the initial plane "a" and the corresponding J_b and v_b are to be found at the final plane "b". The following initial assumptions are made:

1) The electron beam is infinite in cross section, so that the only spatial variations are in the direction of the electron flow.



THE LLEWELLYN GAP

Figure 2.

2) All charge density in the beam is due to electrons. No positive ions exist in the beam.

3) Steady state and time dependent quantities may be expressed separately.

4) a.c. magnetic forces are negligible compared to electric forces.

Using the one dimensional form of the appropriate Maxwell Equations,

$$I = \rho u + \varepsilon_0 \frac{\partial E}{\partial t} \quad (2.35)$$

$$\rho = \varepsilon_0 \frac{\partial E}{\partial z} \quad (2.36)$$

there results the equation

$$I = \varepsilon_0 \left[\frac{\partial E}{\partial z} \frac{\partial z}{\partial t} + \frac{\partial E}{\partial t} \right] = \varepsilon_0 \frac{dE}{dt} \quad (2.37)$$

From the Lorentz force equation and equation (2.37)

$$\frac{eI}{m\,\varepsilon_0} = \frac{da}{dt} \quad (2.38)$$

If initial values of a, u, and t are now considered at the "a" plane, a breaking up of the left hand term of equation (2.38) into a steady and a time dependent part $K + \varphi'''(t)$ allows successive integrations to obtain equations for a, u and z at the final plane. Steady state equations are obtained from them by equating all time dependent parts to zero. There result the d.c. relations

$$a_b = K\,T + a_a \quad (2.39)$$

$$u_b = 1/2\ KT^2 + a_a T + u_a \quad (2.40)$$

$$z = d = 1/6\ KT^3 + 1/2\ a_a T^2 + u_a T \quad (2.41)$$

in which $T = (t - t_a)$ (2.42)

A number of other equations relating to the d.c. conditions in the region are obtained from equations (2.39) to (2.41) plus the definition of a space charge factor ζ involving the transit time T and the no-current transit time T_o.

$$\zeta = 3\left[1 - \frac{T_o}{T}\right] \quad (2.43)$$

The equations will not be reproduced here. They can be found in the Llewellyn and Peterson paper or in Pierce (13).

To study time dependent effects, the transit time when input modulations exist is taken by Llewellyn to be of the form

$$(t - t_a) = T + \delta_1 + \delta_2 + \dots \qquad (2.44)$$

the δ's being progressively smaller correction terms to the d.c. transit time T of the equations (2.39) to (2.41). A similar addition of correction terms is made to the other variables (a_a and u_a) at the initial plane, and to the $\frac{eI}{m\varepsilon_0}$ of equation (2.38). Manipulation of the equations parallel to the set (2.39), (2.40) and (2.41) when the correction terms are added becomes extremely lengthy for the general case. To make the formulation more simple, Llewellyn and Peterson include only a single correction term on each variable. In addition, they leave unconsidered all products of correction terms, making their analysis strictly a "first order" one. Thus, using the forms

$$\begin{aligned} &\text{initial acceleration} = a_a + \alpha_1(t_a) \\ &\text{initial velocity} = u_a + v_1(t_a) \\ &\text{current} = K + \varphi'''(t) \\ &\text{transit time} = T + \delta_1 \end{aligned} \qquad (2.45)$$

and assuming all the time dependent terms to vary as $e^{j\omega t}$, equations for u_b and a_b are written. Then δ_1 is eliminated from these equations by solving the "d" equation for δ_1 and substituting. The resulting equations for u_b and a_b, along with a third resulting from

$$(V_b - V_a)_1 = -\frac{m}{e} \int_a^b \alpha_1(t_a)\, u_o \, dT \qquad (2.46)$$

are then put into the form stated by Llewellyn and Peterson (9)

$$V_b - V_a = A^x I + B^x J_a + C^x v_a$$
$$J_b = D^x I + E^x J_a + F^x v_a \qquad (2.47)$$
$$v_b = G^x I + H^x J_a + I^x v_a$$

In the equations (2.47) only a.c. quantities are involved but the subscript one has been dropped and the time dependence is understood. The expressions for the nine coefficients A^x to I^x are given by Llewellyn and Peterson and also by Pierce (13) in an appendix.

The application of the equations (2.47) to noise analysis of an electron gun was reported by Pierce (13). The following assumptions are implicit in Pierce's application:

1) The Llewellyn and Peterson equations (2.47) are valid over the whole region from the potential minimum to the anode. (It should be recalled that the L-P (Llewellyn-Peterson) analysis required the d.c. electron velocity to have a single value at any cross section.)

2) The fluctuation quantities at the potential minimum are the same as those at the cathode surface.

3) The fluctuations may be treated by the equivalent modulations over the whole region: $J_a = J_{sn}$ and $v_a = v_R$

4) The d.c. velocity at the potential minimum may be taken as zero.

5) The effect of total current through the gun is small.

6) The velocity and current modulations are uncorrelated at the potential minimum. Taking the initial d.c. velocity to be the root mean square thermal velocity is thought to be an improvement over Pierce's original assumptions. Equations (2.47) then become

$$\frac{\overline{J_b^2}}{\Delta f} = \left|E^x\right|^2 \frac{\overline{J_a^2}}{\Delta f} + \left|F^x\right|^2 \frac{\overline{v_a^2}}{\Delta f} \qquad (2.48)$$

$$\frac{\overline{v_b^2}}{\Delta f} = \left|H^x\right|^2 \frac{\overline{J_a^2}}{\Delta f} + \left|I^x\right|^2 \frac{\overline{v_a^2}}{\Delta f} \qquad (2.49)$$

The method suggested by Pierce to take account of total current has been extended in Appendix (III) to apply to the equations in the modified form (2.48) and (2.49). The resulting equations are

$$\frac{\overline{J_b^2}}{\Delta f} = |E^x|^2 \left|1 - \frac{B^x D^x}{A^x E^x}\right|^2 \frac{\overline{J_a^2}}{\Delta f} + |F^x|^2 \left|1 - \frac{C^x D^x}{A^x F^x}\right|^2 \frac{\overline{v_a^2}}{\Delta f} \tag{2.50}$$

$$\frac{\overline{v_b^2}}{\Delta f} = \left|1 - \frac{G^x B^x}{H^x A^x}\right|^2 |H^x|^2 \frac{\overline{J_a^2}}{\Delta f} + |I^x|^2 \left|I - \frac{C^x G^x}{A^x F^x}\right|^2 \frac{\overline{v_a^2}}{\Delta f} \tag{2.51}$$

B). <u>Robinson's Theory</u>

From the discussion in section (2-I), it is clear that because of the single d.c. velocity assumption, a Llewellyn-Peterson type analysis is not valid in the neighborhood of the potential minimum. Far from being a single velocity stream, the flow in fact contains a Maxwellian distribution of velocities at the minimum. Robinson (17) gives a development which attempts to account for the multivelocity nature of the beam near the minimum. He suggests that an α-plane be chosen at a potential $\frac{\alpha k T_c}{e}$ far enough beyond the minimum so that the spread of electron transit times for the region between it and the anode is small compared to one radio frequency cycle. Rack (15) has shown that under these conditions the required single velocity assumption is valid. Robinson's treatment requires the following basic assumptions:

1) The Llewellyn-Peterson analysis is valid between the α-plane and the anode.

2) Those assumed by Pierce as (2) and (5) (see page 22).

3) Current fluctuations at the α-plane are given by full shot noise.

4) No correlation exists between current and velocity fluctuations at the α-plane.

Robinson applies an analysis to the region between α-plane and anode which is similar to that used by Llewellyn and Peterson except for notation. By manipulation of the Llewellyn-Peterson coefficients it is possible to obtain those given by Robinson in his paper except for the discrepancy $H^{\times} = 2A_{23}$. Robinson's actual theoretical contribution lies in his evaluation of the velocity fluctuation at the α-plane. He uses a statistical method to obtain an expression for the mean square deviations of the average electron velocity observed in a time interval, T, from the long term average velocity:

$$\overline{(\delta \tilde{v}_T)^2} = (NT)^{-1} \left[\overline{v^2} - (\bar{v})^2 \right] \tag{2.52}$$

where NT is the average number of electrons passing a given plane in a time T, and $\overline{v^2}$ and $\bar{v}$ are the mean square and mean velocities respectively, averaged over the velocity distribution at the given plane. Using a theorem given by MacDonald (10) he obtains an expression for the power spectrum of the velocity fluctuations:

$$\omega(f) = 4\pi f \int_0^\infty \frac{\partial}{\partial T} \left(T^2 \overline{(\delta \tilde{v}_T)^2}\right) \sin 2\pi f T \; dT \tag{2.53}$$

which is related to the mean square velocity fluctuation by the expression:

$$\overline{v_\alpha^{\,2}} = \omega(f) \; df \tag{2.54}$$

Evaluating the quantities $\overline{v^2}$ and $(\bar{v})^2$ at the α-plane and performing the integration of equation (2.53) leads to Robinson's expression

$$\overline{v_\alpha^2} = \frac{e}{m} \frac{4kT_c}{J_o} f(\alpha) \Delta f \tag{2.55}$$

where

$$f(\alpha) = 1 + \alpha - \left[\alpha^{1/2} + 1/2\, e^\alpha \pi^{1/2} (1 - \text{erf } \alpha^{1/2})\right]^2 \tag{2.56}$$

Robinson tabulates $f(\alpha)$ for the range $\alpha = 0$ to $\alpha = 20$ and gives the expression

$$f(\alpha) \approx \frac{1}{4\alpha} \quad \text{for } \alpha > 20 \tag{2.57}$$

A solution is now possible of the equations

$$\frac{\overline{v_b^2}}{\Delta f} = |A_{22}|^2 \cdot \frac{\overline{v_\alpha^2}}{\Delta f} + |A_{23}|^2 \frac{\overline{J_\alpha^2}}{\Delta f} \tag{2.58}$$

$$\frac{\overline{J_b^2}}{\Delta f} = |A_{32}|^2 \frac{\overline{v_\alpha^2}}{\Delta f} + |A_{33}|^2 \frac{\overline{J_\alpha^2}}{\Delta f} \tag{2.59}$$

which are equivalent to Robinson's final equations in our notation. The coefficients are given by Robinson.

C). <u>The Work of Parzen</u>

The first attempt to solve the problem of signal propagation along an accelerating beam of finite diameter was reported by Parzen (12). His contribution consisted, in essence, of giving a development to justify the replacing of the d.c. current density J_o by an effective value J_{oe} where

$$J_{oe} = \frac{J_o}{\left[1 + \frac{T^2}{\gamma_o^2}\right]} = p^2 J_o \tag{2.60}$$

in the electronic equation. To solve the resulting equation he used the WKB approximation.

The development given by Parzen begins with a formulation of the electronic equation in a form slightly different from that given by Hutter. The two forms are exactly equivalent, and the attendant assumptions are the same. His equation is

$$u_o^3 \frac{\partial^2 J}{\partial z^2} + \left[3u_o^2 \frac{\partial u_o}{\partial z} + 2j\omega u_o^2\right]\frac{\partial J}{\partial z} + \left[2j\omega u_o \frac{\partial u_o}{\partial z} - \omega^2 u_o\right] J = \frac{eJ_o}{m} j\omega E \tag{2.61}$$

To continue, he then formulates what he calls the circuit equation

$$\nabla_t^2 E + \frac{\partial^2 E}{\partial z^2} + k^2 E = j\left[k^2 J + \frac{\partial^2 J}{\partial z^2}\right]\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \frac{1}{k} \tag{2.62}$$

which is obtainable directly from Maxwell's relations. Carrying out the operations indicated in equation (2.62) with the definitions

$$E = E_1\ e^{-j\omega\tau} J_o(Tr)$$
$$J = J_1\ e^{-j\omega\tau} J_o(Tr) \tag{2.63}$$

leads to a relation between E and J. By a series of approximations this relation reduces to a simple form, which when substituted into (2.61), gives the desired reduction of the J_o term.

To obtain the relation between E and J, Parzen was forced to assume that the T in the argument of the Bessel function was not a function of z. This, however, is only the case when the beam is flowing in a conducting cylinder of the same radius as the beam, which is never true in practise. One further difficulty lies in the approximations made to simplify the relation between E and J. They include

$$\frac{\omega_p}{\omega} \ll 1 \quad \text{and} \quad \frac{1}{\omega u_o}\ \frac{\partial u_o}{\partial \tau} = \frac{2}{\omega\tau} \ll 1 \tag{2.64}$$

While these are known to be fairly good at the anode for ordinary beam conditions and high frequencies, they become much poorer in the part of the gun near the potential minimum, or if the signal frequency is lowered.

For the above reasons, the results of Parzen are thought by the author to be open to question. Certainly his development was not a rigorous one. Nevertheless, a development from another viewpoint, given in Appendix (II), has substantiated the result given by Parzen and for this reason the work has been included here as being a basic contribution.

The solution carried out by Parzen using the WKB approximation employs the equation in Hutter's notation with the appropriate modification of J_0, and total current set to zero. Parzen obtains values of the maximum to minimum ratio of the drift space noise pattern following a gun used by Cutler and Quate (2). He claims fair agreement with their experimental results, but it should be noted that the gun used by Cutler and Quate was a converging one, to which the analysis is not strictly applicable. Recent work (16) in fact has indicated that converging guns have quite different noise properties from the parallel flow type.

D). Haus's Formulation

In a fundamental paper, Haus (3) discusses the transformation of noise along a section of electron beam in terms of four terminal network theory. The following assumptions were found to be necessary:

1) The problem is one dimensional, implying an infinite beam.

2) Small signal theory only is considered.

3) At every point along the beam the spread of d.c. electron velocities is small compared to the average velocity.

Haus states that, on the basis of earlier unpublished work, the expression of noise quantities as equivalent signals through the quadratic contents is valid under the conditions of the last assumption. A plane is chosen in a similar way to Robinson's α-plane such that the section of the beam bounded by it and the gun anode obeys the assumption, which is called the single velocity assumption. Haus does not assume any specific noise quantities at the input plane, claiming that no sound information in this region is available.

He begins by writing the standard four terminal network equations for small signal theory:

$$\begin{aligned} V_1(z_2) &= A(\omega)\ V_1(z_1) + B(\omega)\ I_1(z_1) \\ I_1(z_2) &= C(\omega)\ V_1(z_1) + D(\omega)\ I_1(z_1) \end{aligned} \tag{2.65}$$

in which

$I_1(z) =$ is the convection current modulation, and

$V_1(z) = -\frac{m}{e}\ u(z)v_1(z)$ is the beam voltage modulation.

For no interaction of the beam with any external circuit, which is the case in an electron gun, the equations (2.65) may be reduced to the lossless form:

$$\begin{aligned} V_1(z_2) &= \left[A_o(\omega)\ V_1(z_1) + j\ B_o(\omega)\ I_1(z_1)\right]\ e^{j\Phi} \\ I_1(z_2) &= \left[-jC_o(\omega)\ V_1(z_1) + D_o(\omega) I_1(z_1)\right]\ e^{j\Phi} \end{aligned} \tag{2.66}$$

along with the condition

$$A_o(\omega)\ D_o(\omega) - B_o(\omega)\ C_o(\omega) = 1 \tag{2.67}$$

Equation (2.67) does not, of course, imply a reciprocity relation since the $e^{j\Phi}$ are not included.

Since noise is a stationary statistical process, it can be defined only by averages. Sufficient for the study of the phenomena involved in the present case are the auto-correlation functions of the current and velocity and the cross correlation function between the two. It is the above three quantities which will be transformed from the initial to the final plane. To carry out the transformation, Haus first uses a Fourier transform to put the equations (2.65) into the time domain. He then uses a rather complex statistical correlation operation to find the auto-correlation function of voltage at the output plane in terms of the three input quantities. A Fourier transform of the voltage auto-correlation function then gives the self power density spectrum (SPDS) of voltage modulation. By similar manipulations it is possible to get the SPDS of current modulation and the cross power density spectrum (CPDS) between them. The following notation is used:

$$\Phi(z\omega) = \text{SPDS of beam voltage modulation}$$

$$\Psi(z\omega) = \text{SPDS of convection current modulation}$$

$$\Theta_{iv}(z\omega) = \text{CPDS between voltage and current modulations.}$$

To relate the density spectra to a quantity derivable from experiments, he notes that the SPDS is simply $1/4\pi$ times the corresponding mean square fluctuation per unit frequency. Haus arrives at the general equations for noise:

$$\Phi(z_2\omega) = |A(\omega)|^2\Phi(z_1\omega) + |B(\omega)|^2\Psi(z_1\omega) + A(\omega)B^*(\omega)\Theta_{iv}(z_1\omega) + A^*(\omega)\,B(\omega)\,\Theta^*_{iv}\,(z_1\omega) \tag{2.68}$$

$$\Psi(z_2\omega) = |C(\omega)|^2\Phi(z_1\omega) + |D(\omega)|^2\Psi(z_1\omega) + C(\omega)\,D^*(\omega)\Theta_{iv}\,(z_1\omega) + C^*(\omega)\,D(\omega)\,\Theta^*_{iv}\,(z_1\omega) \tag{2.69}$$

$$\Theta_{iv}(z_2\omega) = A(\omega)\,C^*(\omega)\Phi(z_1\omega) + B(\omega)\,D^*(\omega)\Psi(z_1\omega) + A(\omega)D^*(\omega)\Theta_{iv}(z_1\omega) + B(\omega)\,C^*(\omega)\,\Theta^*_{iv}\,(z_1\omega) \tag{2.70}$$

which apply to both lossy and lossless transformations. The subscripts one refer to input quantities and two to output quantities. For the particular case of a drift space, which is a lossless transducer, Haus gives the coefficients.

$$A(\omega) = \cos\Theta\, e^{-j\omega\tau} \qquad B(\omega) = -j\, z_o \sin\Theta\, e^{-j\omega\tau}$$

$$C(\omega) = -j\, Y_o \sin\Theta\, e^{-j\omega\tau} \quad D(\omega) = \cos\Theta\, e^{-j\omega\tau} \tag{2.71}$$

where

$$z_o = \frac{1}{Y_o} = -2\,\frac{V_o}{J_o}\;\frac{\omega_p}{\omega} \qquad ; \quad \Theta = \frac{\omega_p(z_2 - z_1)}{u_o} \tag{2.72}$$

These can be shown to be equivalent to the coefficients developed in Sec. 2-II.

Using the quantities of equation (2.71) in (2.68), (2.69) and (2.70) leads to the relations for noise in a drift space:

$$\Phi(z_2\omega) = \cos^2\theta\ \Phi(z_1\omega) + z_o^2 \sin^2\theta\,\Psi(z_1\omega) - z_o\Lambda(z_1\omega) \sin 2\theta \quad (2.73)$$

$$\Psi(z_2\omega) = Y_o^2 \sin^2\theta\ \Phi(z_1\omega) + \cos^2\theta\ \Psi(z_1\omega) + Y_o\Lambda(z_1\omega) \cos 2\theta \quad (2.74)$$

$$\Lambda(z_2\omega) = \tfrac{1}{2}z_o\left[Y_o{}^2\Phi(z_1\omega) - \Psi(z_1\omega)\right] \sin 2\theta + \Lambda(z_1\omega) \cos 2\theta \quad (2.75)$$

where the definition

$$\Theta_{iv}(z\omega) = \Pi(z\omega) + j\Lambda(z\omega)$$

has been used.

Also, as a consequence of the conservation of kinetic power, $1/2\ \mathrm{Re}\left[V_1(z)\ I_1^*(z)\right]$ we must have

$$\Pi(z_2\omega) = \Pi(z_1\omega) \quad (2.76)$$

The above equations will serve as a basic foundation for the interpretation of experimental data, as is discussed in the results section.

For the gun region, let the initial plane, chosen in view of the single velocity assumption, be denoted z_o. Application of the lossless transducer restriction to the general equations (2.68), (2.69) and (2.70) results in the equation

$$\Pi(z_1\omega) = -\ B_o(\omega)C_o(\omega)\Pi(z_o\omega) + A_o(\omega)D_o(\omega)\Pi(z_o\omega) = \Pi(z_o(\omega) \quad (2.77)$$

From results valid for a coherent process and certain basic logical arguments it is possible to establish another quantity which is invariant in any lossless transformation.

$$S^2(z\omega) = \Phi(z\omega)\ \Psi(z\omega) - \Lambda^2(z\omega) \quad (2.78)$$

Haus shows that as a corollary to equation (2.78) there exists the relation

$$Z_o{}^2\,\Psi_{max}\Psi_{min} = S^2(z_1\omega) = S^2(z_o\omega) \quad (2.79)$$

where Ψ_{max} and Ψ_{min} refer to the drift space.

Haus uses equations (2.77) and (2.79) to derive expressions for the minimum noise figure obtainable in a travelling wave tube. It can be seen from the development given that the theory does not attempt to predict the quantitites on which our theory comparison is based. Comparison with the other theories and with experiments is therefore not possible in our present parameters. The formulation is included primarily because of the mathematical rigor of its development. Some evaluations of the S and Π have been done at the Research Laboratory of Electronics of the Massachusetts Institute of Technology. One and two cavity measurements in the drift space were required. Minimum noise figure computations from the above have been in fair agreement with experimental values.

E). Analysis of Tien and Moshman

A distinct contribution to the solution of high frequency noise in a parallel flow diode was recently reported by Tien and Moshman (22). The analysis was concerned solely with finding the fluctuation quantities existing at the potential minimum of a particular diode under a particular set of operating conditions. Their method of solution consisted of a "Monte Carlo" calculation of the effect of individual electrons on the conditions within an idealized model diode. A "Monte Carlo" calculation involves a numerical approximation to a physical process in which stochastic quantities are simulated by using appropriately distributed random numbers.

The following assumptions were made:

1) The motion of the electrons is one dimensional.

2) No collisions between electrons occur.

3) The behavior of the whole diode may be characterized by the behavior of a small area of it. The most useful area must be small enough so that fields may be considered uniform over it, but large enough to avoid serious interaction between electrons from neighboring surfaces. They chose an area $A_d = \frac{\pi}{4} x_m^2$

4) The diode is assumed short circuited for a.c. so that anode current fluctuations will not produce an alternating voltage across it.

According to their idealized picture, electrons are represented by infinitely thin uniform charged discs having the same area as the diode considered, $(\frac{\pi}{4} x_m^2)$.

A high speed Univac I digital computer was used to carry out the solution. The average potential distribution in the diode was first evaluated from the work of Langmuir (7), the constants of their particular diode being employed. The cathode and anode current densities also were found. The solution then proceeded as outlined below:

(1) A time interval short compared to a cycle of the highest radio frequency to be analysed is postulated. On the basis the known cathode current density, an appropriate random number is generated to represent the number of electrons emitted into the diode of area $(\frac{\pi}{4} x_m^2)$ in this interval.

(2) Additional random numbers are generated to give the exact times of emission during the interval and the emission velocity of each of the electrons.

(3) The space charge fields and electron motions inside the diode are calculated by numerical integration over the time interval. Electrons crossing cathode or anode planes are automatically erased.

(4) The electrons crossing the plane $x = 1.2\ x_m$ are tabulated by the computer along with their velocities.

(5) With the potential distribution in the diode modified by the electrons from the first time interval, a second time interval is taken and random numbers generated as before. The calculation continues for a great number of time intervals, with the numbers and velocities at the $x = 1.2\ x_m$ plane being tabulated for each one (Tien and Moshman used 3000 intervals). They use $x = 1.2\ x_m$ to avoid problems of counting electrons which may return to the cathode.

Calculations based on the variations of the quantities at the $x = 1.2\ x_m$ plane lead to the current and velocity fluctuation quantities desired. Quantitatively, of course, the results given by Tien and Moshman apply only to a gun of the particular characteristics described. Certain relations appear, however, which should be true for a fair range of conditions. The results may be summarized as follows:

The current fluctuations appearing at the minimum are a rather strong function of frequency, and may vary widely from shot noise. The actual form of the variation is shown in Fig. (3). The velocity fluctuations appear to be unaffected, having the same value at the minimum as at the cathode. There is no detectable correlation between the two fluctuations at the minimum. The last two conclusions can reasonably be assumed to apply to any diode of a type similar to that used in the analysis.

This concludes the description of existing theories of high frequency noise. In the chapter dealing with results, numerical predictions of the above theories will be given for the particular gun used in the experimental work.

A short summary of the criticisms which have been stated or implied in the descriptions may serve to clarify the present status of high frequency noise theory.

Of the theories described, only that of Parzen makes an attempt to include finite beam diameter effects. Parzen's development, however, requires the inequality $\frac{\omega_p}{\omega} \ll 1$ to hold everywhere in the gun and requires that the radial propagation number be independent of z. Neither of these requirements is actually met in the electron gun, and so Parzen's results must be viewed with suspicion.

None of the theories except that of Robinson attempt to describe the transformation of noise fluctuations through the region immediately beyond the potential minimum where the electron flow is multi-velocitied. Robinson's method applies to an infinite beam and takes no account of possible correlation at the α-plane. Further, he assumes arbitrarily that the current fluctuations are transferred without modification to the α-plane, and is concerned only with modification of the velocity fluctuations.

Known correlation of quantities at the input plane could be handled by a theory such as that given by Haus, but the determination of the existence and amount of correlation has not been accomplished analytically.

2-V. A Finite Beam Solution Using an Analogue Computer

The criticisms given in the preceding section indicate that there exists as yet no really comprehensive theory of high frequency noise in a diode electron gun.

It was thought profitable to carry out a solution of an electronic equation similar to that used by Parzen but which does not require the restriction of the radial propagation number T being independent of z (See Sec. 2-IV C for a discussion of this assumption). Allowing T to vary with z in the expected manner makes an analytic solution of the equation extremely difficult. An electrical analogue computation was therefore chosen as the most practical method of solution.

The computer work was done jointly with R. Vessot in this laboratory during the summer of 1956. The help of Prof. G.W. Farnell of Electrical Engineering and the use of the analogue computer constructed by him are gratefully acknowledged by us both. The fundamentals of operation of electrical analogue computers is readily available, for example in Korn and Korn (6), and will not be discussed in this thesis. It is sufficient to say that solution of differential equations by such computers is usually based on circuit elements which perform an integration with respect to time of the voltage appearing at their inputs. The equations must, therefore, be expressed in an integral form for solution.

The solution carried out is viewed by the author as being no more than a first attempt at solution of the actual problem. Certain of the assumptions used, as in earlier theories, are known to be unrealistic. Nevertheless, it was thought advisable to consider the simplest possible formulation of the problem. The method used should indicate a way to extension of the solutions to include more realistic conditions. It should be noted that by the computer method, solutions are obtained for any plane in the gun, not for the anode alone.

The assumptions made in the analysis are the following:

(1) The total current I_ω is taken as zero.

(2) The beam is considered to have a small d.c. velocity spread throughout the potential minimum-anode region.

(3) No correlation between velocity and current fluctuations exists at the potential minimum.

(4) The d.c. velocity of the electrons at the potential minimum is equal to the root mean square value of the thermal velocity distribution.

(5) The factor $\frac{eJ_0}{m\varepsilon_0}$ in the infinite beam electronic equation for current is replaced by $p^2 \frac{eJ_0}{m\varepsilon_0}$ for finite beam considerations.

Some comments on the above assumptions are in order.

According to the development of Appendix III, the placing of $I_\omega = 0$ appears to be very unrealistic. The assumption was made only to avoid serious complication of the computer arrangement. The possibility of resulting errors should be kept in mind.

The assumption (2) is necessary if the electronic equation is to be applicable over the whole region. While it is recognized to be unrealistic, this assumption was thought preferable to starting at an α-plane away from the minimum. At an α-plane, the input noise quantities would not be as well known, and the possibility of correlation between current and velocity fluctuations would exist. Also a considerable complication would result from the introduction of an initial electron acceleration.

Tien and Moshman, as discussed in section 2-IV (E), have shown that (3) is probably a reasonable assumption. Assumption (4) would seem reasonable from a purely logical standpoint but no rigorous justification has been attempted. Some auxiliary solutions carried out by Vessot and given in his doctorate thesis (24) indicate that the choice of initial d.c. velocity has only a minor effect on the results.

The assumption (5) concerns the modification to account for the finite beam. A derivation of the electronic equation due to Woonton (26) which starts with the finite beam drift space equation of section 2-II seems to indicate that the replacement is valid. The modification is equivalent to that used by Parzen but can take approximate account, by proper calculation of p^2, of the more general case of T varying with z.

Complete details of the method used to set up the modified electronic equation for solution by the computer are given in Appendix I, along with details of the analysis of the computer work. The basic steps of the procedure are given below:

(1) The modified electronic equation and the computer equation are put in exactly corresponding forms by appropriate normalizations of the variables.

(2) Equating of the coefficients of the parallel equations establishes the value R_x of the series resistor in the computer circuit.

(3) The method of evaluation of the function $(\frac{1 - p^2}{u_o})$ as a function of τ is outlined. It should be noted that values of p are available as a function of $\frac{\omega b}{u_o}$ (b = beam radius, ω = signal radian frequency) by solution of the transcendental equation (2.19).

(4) A treatment is given which shows the method of obtaining solutions for a range of voltages from a single computer solution.

(5) A discussion is given of the initial conditions used.

(6) Expressions for the evaluation of anode noise quantities in terms of input conditions and ratios of computer voltages are developed.

In summary, it may be said that the computer solution described makes a reasonable first attempt to solve the finite beam diode noise problem at high frequencies by applying a point by point correction to the beam plasma frequency. It is capable of extension to include more realistic initial conditions without major changes in the procedure used.

The major criticisms are the failure to take into account the multi-velocity beam properties near the minimum, the neglect of the effects of total current, and the consideration of only a single mode of propagation through the gun.

3. EXPERIMENTAL EQUIPMENT

3-I. The Electron Gun

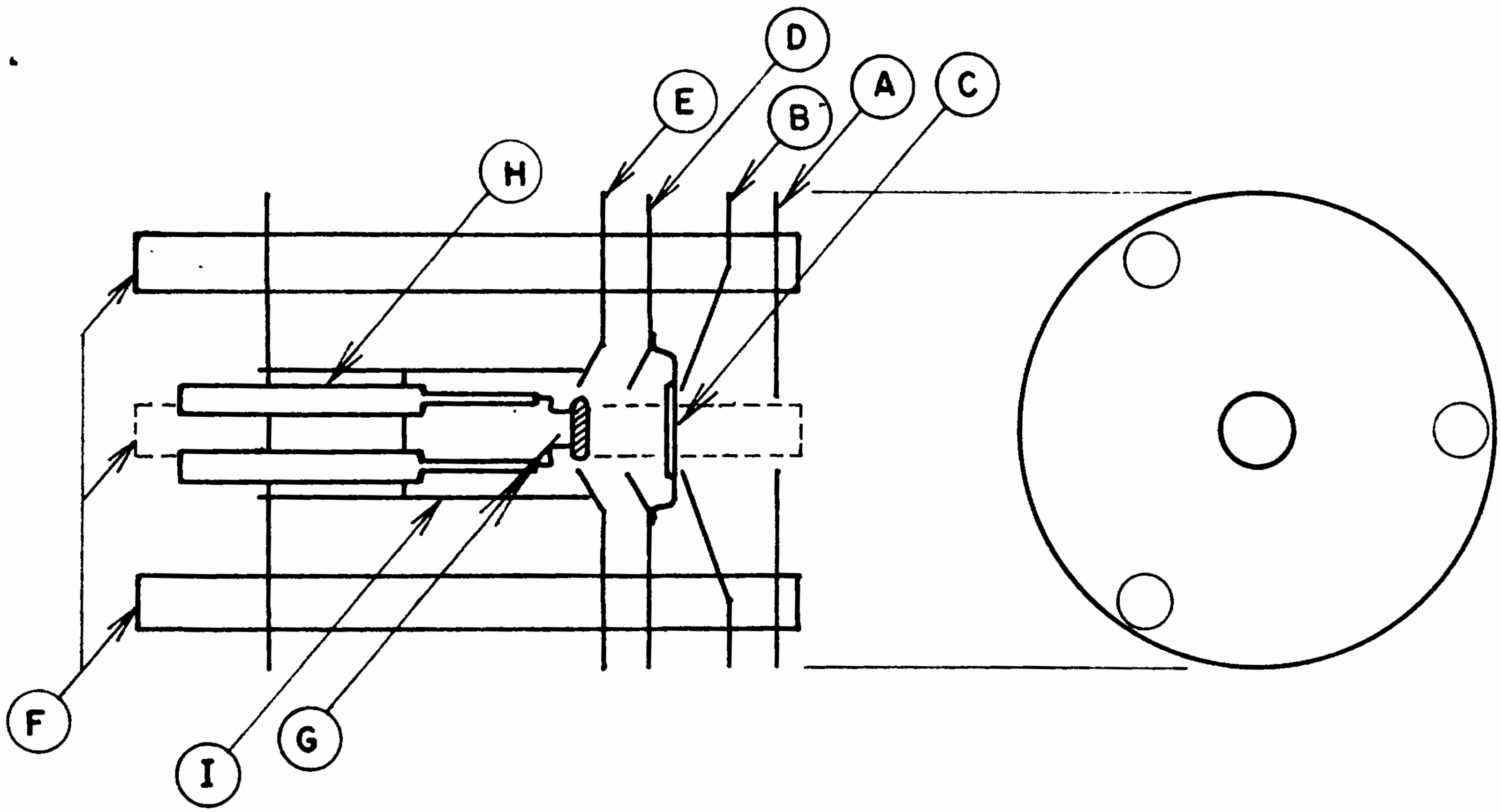
The parallel-flow, diode, electron gun used for the experimental work employed certain special features developed and built by the author. The nature of the experiments which were performed required that the vacuum system be demountable and that motion of the gun relative to the measuring cavity be possible during gun operation. The use of oxide coated emitters under these conditions is not feasible for two reasons:

(1) Severe emission poisoning would occur due to the relatively poor vacuum obtainable with available equipment, and due to the lubricating grease necessary for making sliding vacuum seals.

(2) A new oxide coating would be necessary each time the system was raised to atmospheric pressure, making precise control of gun conditions extremely difficult.

Emission was obtained, instead, from a high temperature pure metal surface, since such surfaces are not affected by exposure to air and are relatively insensitive to emission poisoning effects.

The final structural form of the gun is shown functionally in Fig. (4). The centre line indicates an axis of cylindrical symmetry, and A, B, D and E are circular metal electrodes which are mounted by cementing to three equally spaced ceramic rods, F. The diode gun on which noise studies were done consists of the anode and cathode electrodes A and B, and the cathode emitter C. All remaining parts are concerned with heating the pure metal emitter "button" C to emission temperature. The gun is a parallel flow Pierce type whose design is adequately described elsewhere (14). The anode electrode has been approximated, for further simplicity, by a plane apertured disc, and to reduce



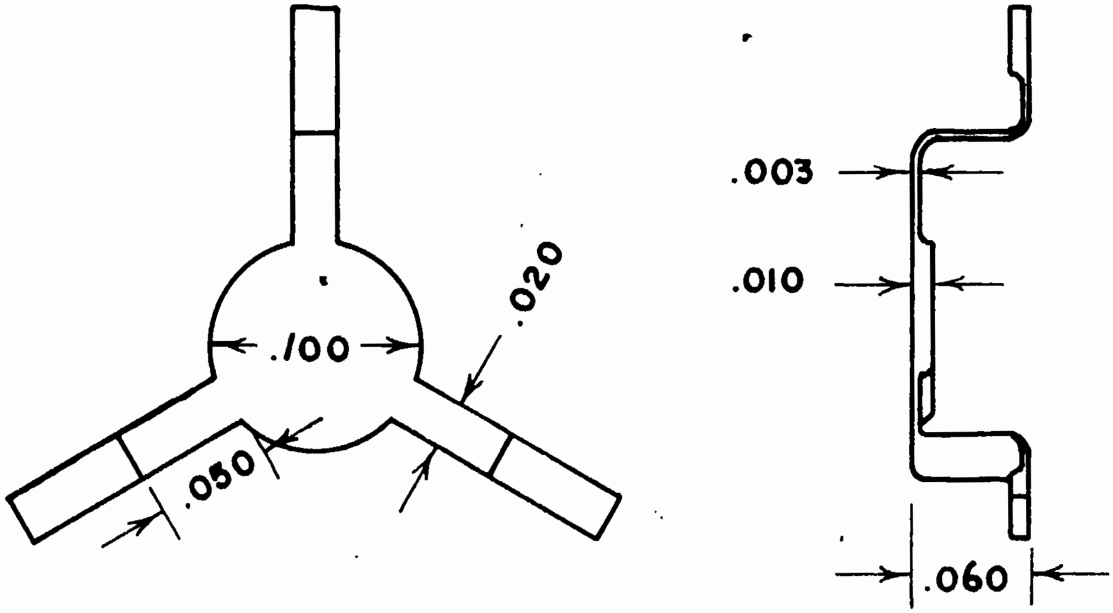
THE ELECTRON GUN

Figure 4.

required construction accuracy the emitter C has been placed a few thousandths of an inch behind ideal position. Tantalum metal was chosen for the emitter and heating was accomplished by bombarding it with a beam from a second gun formed by electrodes D and E and the primary filament G. This second gun will be called the bombarding unit.

Considerations involved in the design of the emitter C and the bombarding unit are thought to be worthy of mention. Tantalum was chosen for the emitter because of its very high melting temperature, its high emission yield, and its ease of machining. The main problem in design of the emitter button was to limit the necessary bombarding power to an obtainable value while maintaining a reasonably solid support not subject to excessive position changes during heating. Heat conduction loss from the button is proportional to the cross sectional area of the supporting arms, and inversely proportional to their length. Long arms were found to cause serious changes of position due to thermal expansion, and so reduction of the cross section to the minimum practical size was necessary. The shape of the emitter is shown in Fig. (5). It consists of a circular tantalum button with three equally spaced arms extending radially. The structure is cut from a single sheet of .010" tantalum and the arms being filed, before bending, to the thickness and width shown. The filing was done under a binocular microscope.

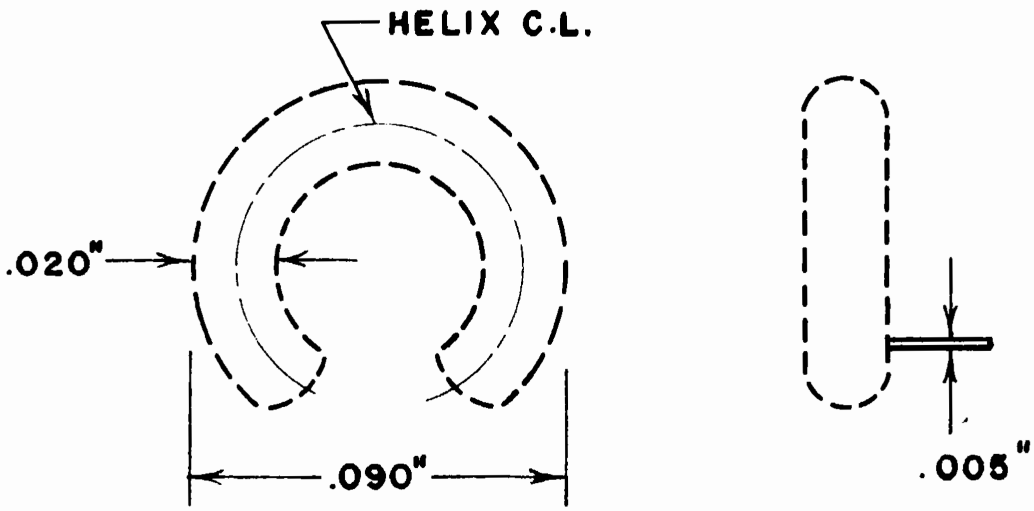
On the basis of information given by Kohl (5), the temperature required to give the desired emission current density is about 2400°K. Calculation of the radiation and conduction power losses from the emitter button indicated that about 10 watts would be radiated and 6 watts conducted away along the support arms. Leaving the arms the full sheet thickness would thus have increased the total power loss to almost double. The bombarding unit must, then, supply at least 16 watts of beam power to the



THE TANTALUM EMITTER

DIMENSIONS IN INCHES

Figure 5.



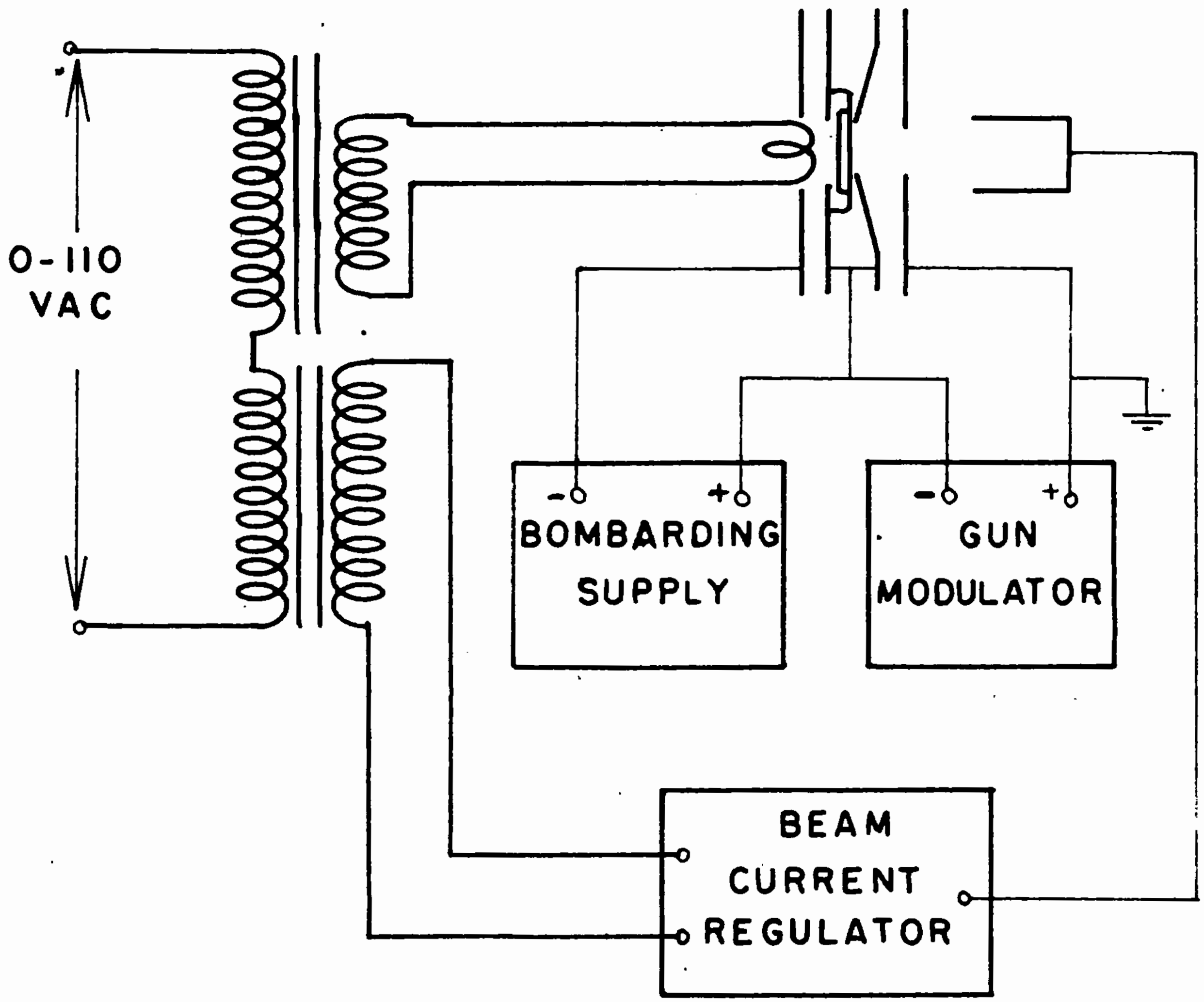
THE PRIMARY FILAMENT

Figure 6.

emitter. To avoid problems of high voltage power supplies and insulation the bombarding supply voltage was limited to 500 VDC, implying a bombarding current of 30 to 40 ma. The primary filament supplying the bombarding current is illustrated in Fig. (6). It consisted of 16 turns of a helix with .020" outside diameter made by winding .005" tungsten wire on 0.010" mandrel at 80 turns per inch. Because of the magnetic confining field used to prevent the spread of the studied beam, the primary filament was required to be smaller in diameter than the tantalum emitter since the bombarding beam was also a parallel flow. Consequently, the helix was bent into an almost complete circle with largest diameter 0.090". The filament described was able to deliver the necessary 40 ma of current under reasonable operating conditions.

The electrodes of the gun and bombarding unit were cut from 0.10" molybdenum sheet. The ceramic mounting rods and primary filament supporting block were both refractory alundum. The tantalum emitter was spot welded to the front of the bombarding gun anode D (Fig. (4)) and positioned .003" behind the cathode electrode B when cold. Electrical connections were made by spot welding molybdenum strip to the appropriate points and connecting the strip to vacuum seal-throughs.

Electrical supply circuits for the gun are shown in Fig. (7). To obtain a truly drifting beam, the gun anode was at all times at ground potential. The main gun voltage, for reasons connected with the measurement system, was a low frequency square wave supplied by the modulator. The bombarding supply was isolated from ground for the above reason, and fairly high voltage insulation was required for its power transformer, as for the one supplying the primary filament. The good beam current stability



GUN SUPPLY CIRCUITS

Figure 7.

necessary for accurate drift space measurements was achieved by the current regulator. In it the collected beam current was made to control the power being applied to the primary of the primary filament transformer, forming a feed back loop.

Significant physical dimensions of the gun are:

cathode-anode spacing	4.60 mm
cathode aperture diameter	1.50 mm
anode aperture diameter	2.40 mm

Measurements on the gun gave the following diode characteristics

V_o	$I_o (\mu a)$	$G = \frac{I_o}{V_o^{3/2}}$
300	850	1.634×10^{-7}
600	2000	1.360×10^{-7}
900	3460	1.280×10^{-7}

Best estimations of the saturation current density of the cathode under operation give a value 0.70 amp/cm^2. The corresponding temperature is 2350°K from available information on tantalum. The figure 2300°K was used in calculations as a representative figure.

In summary, the gun may be considered to produce an electron beam which is representative of the current flow in an infinite parallel plate diode of 4.6 mm cathode to anode spacing and a cathode temperature of 2300°K. The main sources of discrepancy from the above correspondence are:

(1) The potential distribution established by the electrodes along the beam edge takes no account of the fact that the potential minimum does not coincide with the cathode. Even a precisely constructed Pierce gun does not, therefore, act in the ideal fashion.

(2) The anode aperture has a diverging lens action on the beam causing undesired radial electron velocities.

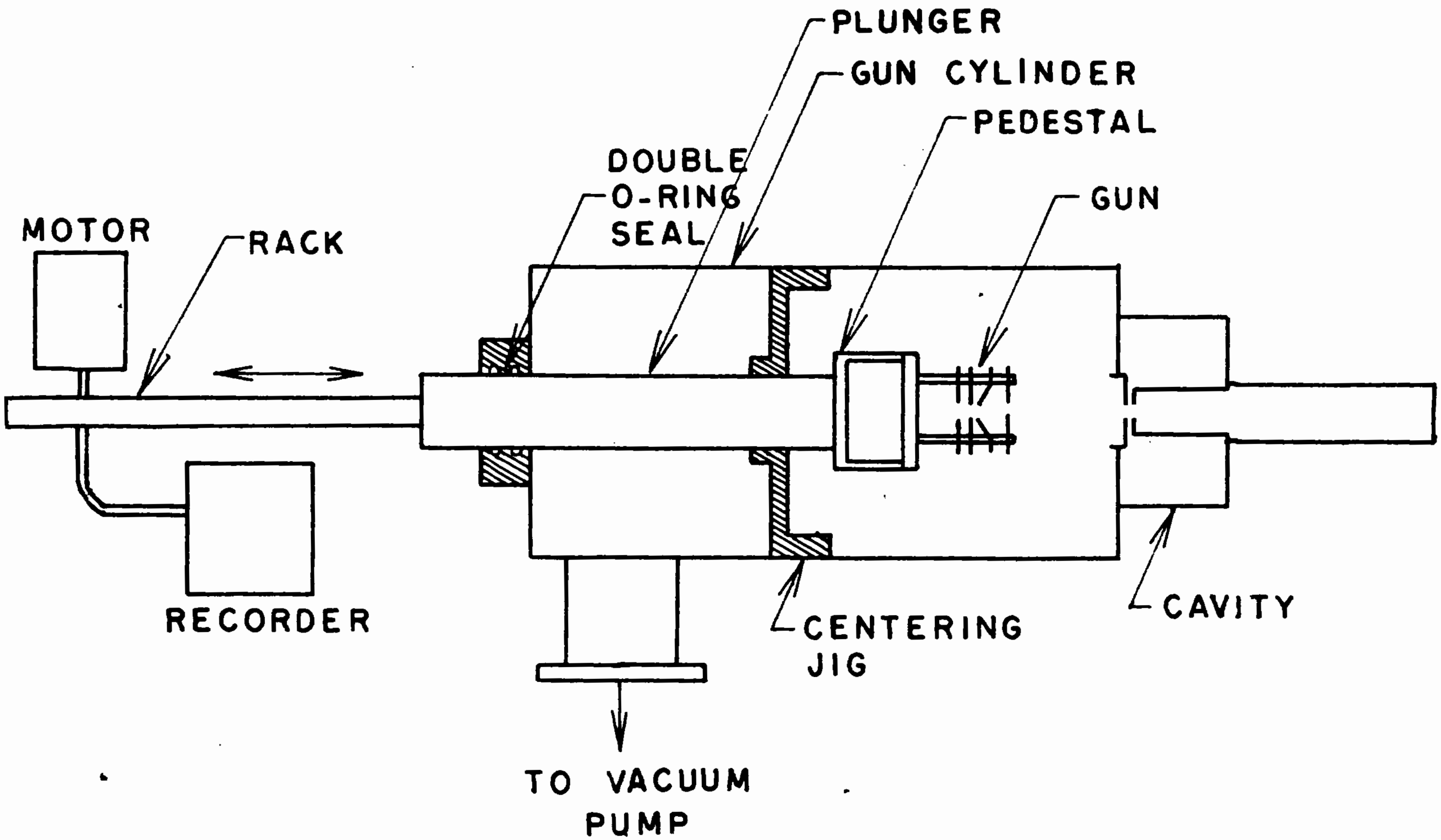
(3) A strong magnetic confining field, which may effect the statistical emission behavior of the cathode, is used to prevent spreading of the beam.

(4) The cathode surface is displaced a short distance back from its ideal position, causing further disturbance of the potential distribution.

3-II The Mechanical System

The experimental arrangement of the electron gun and measuring cavity are shown schematically in Fig. (8). The distance between gun anode and measuring gap was made variable by sliding the gun along the axis of an evacuated gun cylinder of copper plated brass. Care was taken to maintain the beam axis coincident with the cylinder axis by allowing a sliding contact between a centering assembly and the cylinder wall. The plunger by which motion was imparted to the gun entered the vacuum through a lubricated double O-ring vacuum seal in the end of the gun cylinder. Electrical leads to the gun entered the vacuum by way of seal throughs mounted in the gun pedestal which terminates the plunger. A rack attached to the plunger was engaged by a gear which was driven by a variable speed d.c. motor through a speed reducer. Gun speeds from 2 to 20 cm/min were obtainable. The paper drive of an Esterline-Angus recording milliammeter was coupled by a flexible shaft to the gun shaft mentioned above. By driving the recorded pen from the output of the noise measurement system, continuous records of the relative noise power appearing at the cavity gap as a function of gun position were obtained. The resonant cavity used to probe the beam noise was mounted concentrically on the end of the gun cylinder, a vacuum seal being made either by a ground joint or an O-ring.

The pumping system (not shown in Fig. (8)) consisted of a Welch duo-seal fore pump, a three stage oil diffusion pump of 20 l/sec capacity and a 2 litre spherical dewar flask mounted in the pumping line to allow cold trapping with liquid nitrogen. The small capacity diffusion pump and the nature of the mechanical system described limited the obtainable vacuum to a pressure of 2 to 5 x 10^{-6} mm. Hg under operating conditions.



THE MECHANICAL SYSTEM

Figure 8.

Pressure measurements done along the pumping line indicated a pressure slightly better than 10^{-6} mm. Hg which, from pumping speed considerations led to the estimate given above. The possible addition of noise to the beam by electron-air molecule collisions should be kept in mind because of the relatively poor vacuum conditions.

The axial magnetic field used to confine the beam was supplied by a 1 meter long water cooled solenoid operating from a 3 Kw a.c. motor-d.c. generator set. The entire gun cylinder up to the pumping line was mounted inside the solenoid and four micrometer type adjustment screws were used to align the gun cylinder axis along the magnetic field lines. With proper adjustment, all the beam current could be made to reach the collector beyond the cavity gap, the cavity interception current being too small to detect. (In some of the experimental runs, a slight amount of cavity interception occurred near maximum separation. These were easily detectable by the accompanying large noise increase.)

3-III. Cavity Resonators

In the course of the experimental work, three separate cavities were actually used.

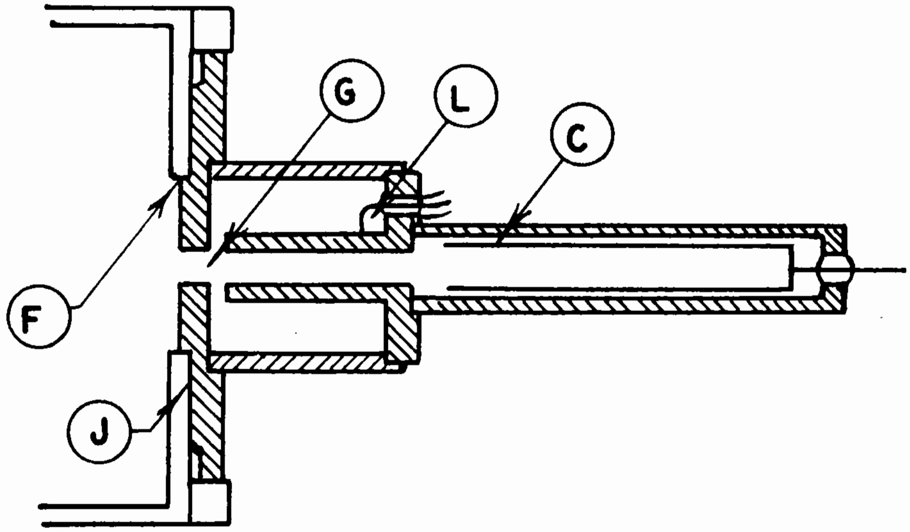
(1) a fixed frequency 480 Mc/s resonator

(2) a fixed frequency 2960 Mc/s resonator

(3) a variable frequency cavity tuneable from 200 to 1000 Mc/s.

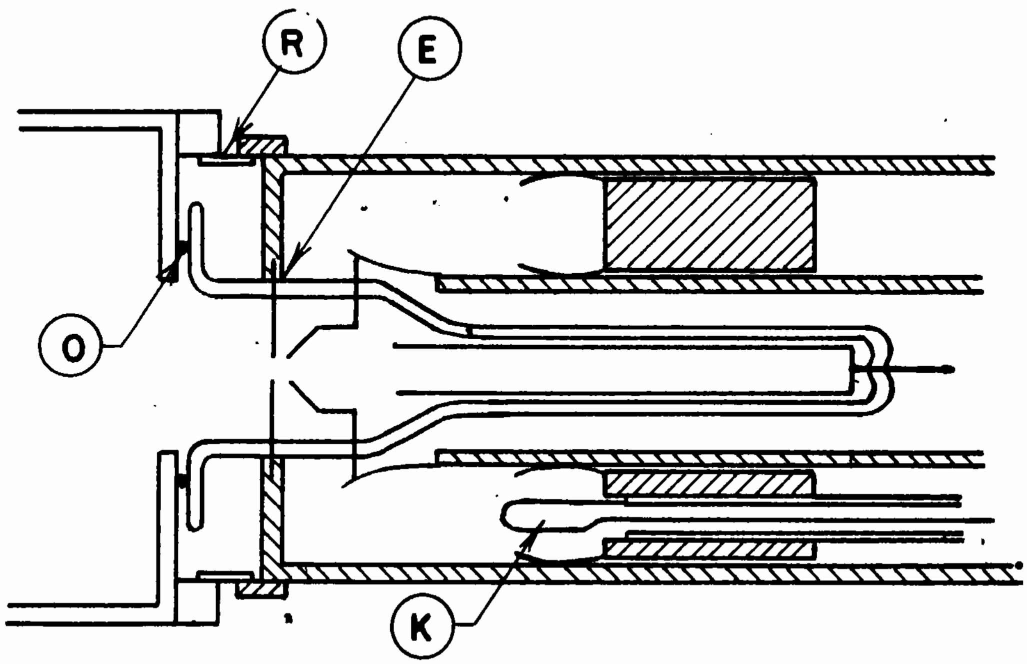
The work could have been done with the last two but the variable frequency unit was not available when measurements were started. Since the two fixed frequency cavities were different only in dimensions, only one will be described. The type of construction is shown in Fig. (9). The electron beam, passing along the symmetry axis, induces the cavity to oscillation as it passes through the narrow gap G formed by the front face and reentrant post. The resulting power in the cavity is coupled out by a loop L which is attached, by way of a vacuum seal-through, to a coaxial line leading to the amplifying system. The cavity was assembled from copper and copper plated brass parts soldered together to give vacuum tight joints. The flange F enabled accurate centering of the cavity on the gun cylinder axis. A demountable vacuum seal between the cavity and the gun cylinder was obtained by a lightly greased ground joint between the two faces at J. The long tubular collector C, turned from oxygen free copper, was found to be essential in eliminating secondary electrons from the beam. During operation a soft iron magnetic shield was placed over the collector housing to cause a strong diverging action on the beam. The secondary electron problem will be discussed more fully in a later section.

The variable frequency cavity is shown schematically in Fig. (10). It was designed by R. Vessot (24) for use with sealed off experimental



A FIXED FREQUENCY CAVITY

Figure 9.



THE TUNEABLE CAVITY

Figure 10.

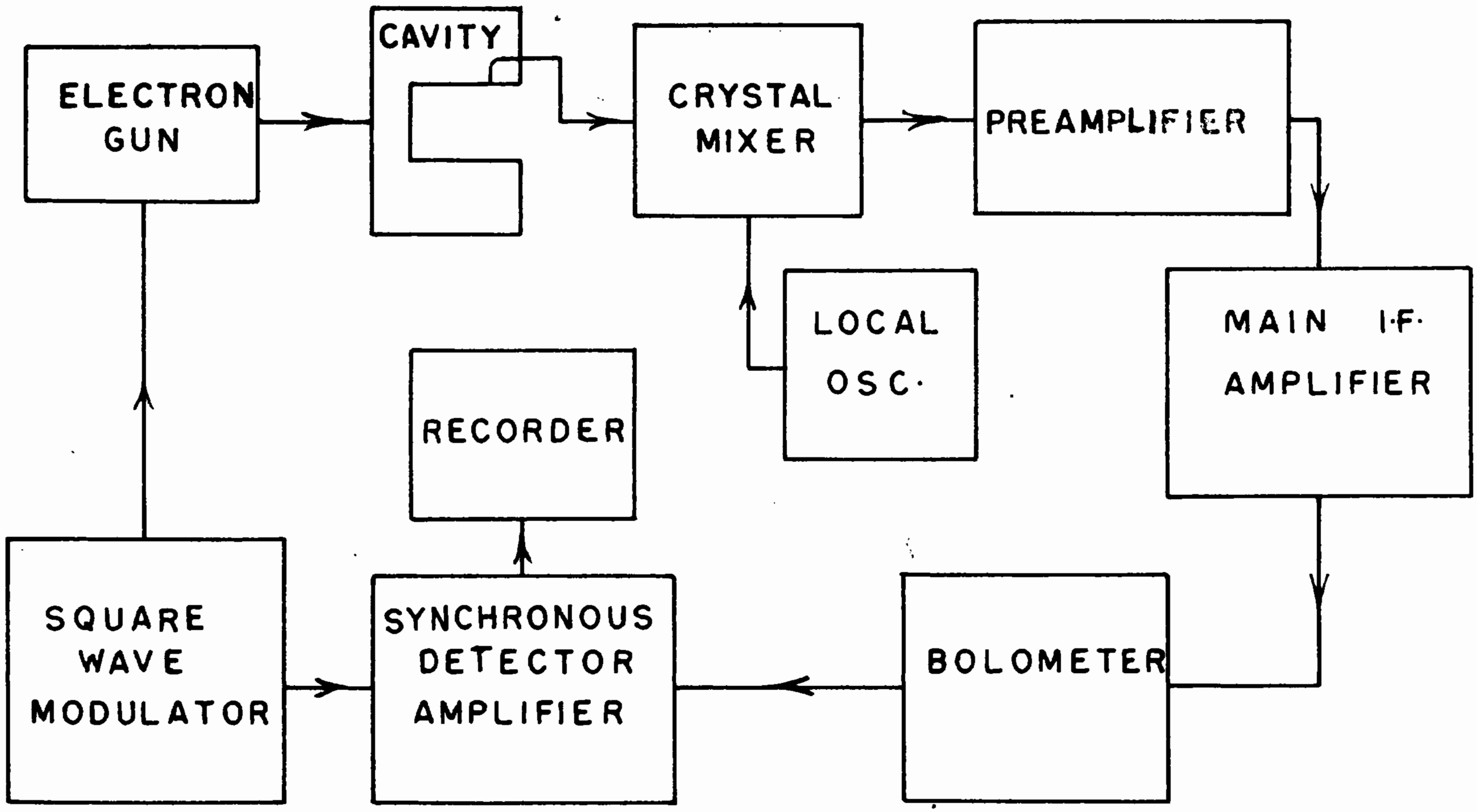
tubes and was adapted by the author for use in the present research. It does not differ functionally from the fixed cavity described above. The limits of the vacuum system are defined by the glass envelope E. The narrow gap through which the beam passes was attached to the large cavity structure outside the vacuum region by thin copper discs sealed through the glass envelope. Care was taken to assure that the gap apertures were concentric with the ring R which fits into a step on the face of the gun cylinder. An O-ring, O was used to seal the glass envelope to the gun cylinder. The collector was of the same type as in the fixed cavity, the magnetic shield in this case fitting between the glass envelope and the inner cavity tube. The tuning plunger of the cavity was moved by pushing on the rigid coaxial line leading from the coupling loop K. Sliding contacts ensured good electrical contact between the moving plunger and the cavity wall.

3-IV. The Amplify System

The amplifying equipment used to process the signal appearing at the cavity output is shown in block diagram form in Fig. (11). The noise signal from the cavity was fed to a crystal mixer along with an appropriate local oscillator signal to give a 20 Mc/s difference frequency. The 20 Mc/s signal was amplified by an intermediate frequency pre-amplifier and amplifier of 0.57 Mc bandwidth and 102 db gain. The amplifier output was detected by a littlefuse bolometer circuit resulting in an output signal proportional to the power in the input signal line. Tests of the ratio of output to input signals showed the proportionality to be good in fact. The measured noise figure of the system from cavity signal line to bolometer output was 11.2 db, which is a representative figure for such a system. For the extremely small noise signal levels to be measured, however, the above system alone proved to be quite inadequate.

To obtain maximum sensitivity, a synchronous detection system was employed. By modulating the input signal at some low frequency and using a narrow band tuned amplifier at the same frequency after the bolometer, the masking effect of crystal, local oscillator and i.f. amplifier background noise was greatly reduced. The modulation and detection equipment design is due primarily to R.A. McFarlane (11).

Modulation of the input signal was accomplished by applying a square wave rather than a continuous voltage to the electron gun. A modulation frequency of around 30 cps was used. The resulting interrupted electron beam produced a square wave modulated noise signal in the cavity output line. The amplifier signal at the bolometer output, still containing the modulated noise signal, was fed to a phase sensitive detector amplifier which employed a synchronizing signal from gun modulator. The theoretically



THE MEASURING SYSTEM

Figure 11.

expected improvement in the noise figure is given by the factor N, where

$$N = \sqrt{\frac{(B\ W)\ if}{(B\ W)\ ta}} \tag{3.1}$$

The actual improvement in the present case was close to 20 db, the final minimum detectable signal being -126 dbm, or 2.5×10^{-16} watts.

A theoretical investigation of the possibility of the square wave harmonics contributing to the cavity power at measurement frequencies was carried out. Even at 3 Mc/s, the harmonic content was found to be extremely small compared to the shot noise in the beam, and was found to decrease rapidly with rising frequency. The transient effect caused by positive ion build up in the beam should also be negligible since reports indicate that steady state ion concentration is achieved in less than 100 μsec. even at moderately good vacuum pressures (10^{-7} mm Hg). The beam "on" period is 1.67×10^4 μsec. indicating that the disturbance would be very small.

MEASUREMENT TECHNIQUE AND BASIC DATA

4-I Measuring Properties of the Cavity Gap

Analysis of the experimental results requires a knowledge of the relative response of the cavity to the current and velocity fluctuations in the beam. A development is given in this section which shows the cavity to be primarily a current measuring device under normal beam conditions.

Consider the cavity gap as a short drift region to which the Llewellyn-Peterson (9) equations apply. The total current I sets up a voltage across the gap in flowing through the cavity shunt admittance Y. Since the gap is extremely short compared to any plasma wavelength encountered experimentally, the convection current and velocity may be considered constant across it. From the first of the Llewellyn-Peterson equations

$$V_b - V_a = A^x I + B^x J + C^x v \tag{4.1}$$

If the beam area is A, the current flowing through the cavity admittance is AI. This leads to the relation

$$(V_b - V_a) Y = AI \tag{4.2}$$

which substituted into equation (4.1) gives

$$(V_b - V_a) = \frac{A}{Y} I = \left[\frac{B^x J + C^x v}{1 - \frac{A^x}{A} Y}\right] \tag{4.3}$$

The power supplied to the cavity is $P_c = \frac{1}{2} GV^2$, where G is the cavity shunt conductance and the V is obtained from equation (4.3). The resulting expression is

$$P_c = \frac{G}{2} \left| \frac{B^x J + C^x v}{1 - \frac{A^x}{A} Y} \right|^2 \tag{4.4}$$

Relative contributions to the power supplied by the current and velocity fluctuations will depend on the relative sizes of the terms $B^x J$ and $C^x v$.

Rewriting equation (4.4)

$$P_c = \frac{G}{2}\left[1 - \frac{A^x}{A} Y\right]^{-1} (B^x J)^2 \left[1 + 2\,\frac{C^x v}{B^x J} + \left(\frac{C^x v}{B^x J}\right)^2\right] \qquad (4.4a)$$

it is seen that the extent to which the cavity deviates from measuring a quantity proportional to the convection current J will depend on the size of the last two terms of the square relative to one. It should be noted that writing the expression (4.4a) has implied complete correlation between J and v. Such a situation can exist in an electron beam if the noise at the gun anode is predominantly from a single source, either velocity or current fluctuation, at the cathode. Any lack of correlation would tend to decrease the contribution of the second term until, for uncorrelated conditions, it becomes zero.

To establish the size of the quantity $2\,\frac{C^x v}{B^x J}$, consider first the ratio $\frac{C^x}{B^x}$. In terms of the more basic parameters of the gap, the expressions for the coefficients reduce to the equation

$$\frac{C^x}{B^x} = \frac{J_o}{u_o}\left[\frac{2\Upsilon}{\beta} - \frac{(1-e^{-\beta})}{(1-e^{-\beta}-\beta e^{-\beta})}\right]^{-1} = \frac{J_o}{u_o}\left[\frac{2\Upsilon}{\beta} - N\right]^{-1} \qquad (4.5)$$

in which $\beta = j\omega\tau$, τ is electron transit time across the gap, and Υ is the space charge factor. Evaluation of the magnitude of N resulted in the graph of Fig. (12). The term $\frac{2\Upsilon}{\beta}$ was found to be negligibly small for all conditions of current and voltage used in the experiments, so that (4.5) reduces to the form

$$\left|\frac{C^x}{B^x}\right| = \frac{J_o}{u_o\,|N|} \qquad (4.5a)$$

The term $2\frac{C^x v}{B^x J}$ will have its largest value at the current minimum of the space charge pattern. For this case, denote the noise quantities v_m and J_m.

More information is available about the current and velocity at the maxima of the pattern, these being very nearly the quantities appearing at the gun anode. By using the drift region space charge wave equations, $\frac{v_m}{J_m}$ can be written

$$\frac{v_m}{J_m} = \left(\frac{\omega_q u_o}{\omega\ J_o}\right)^2 \frac{J_{max}}{v_{max}} \tag{4.6}$$

the J_{max} and v_{max} referring to the fluctuations at a current maximum. There follows the relation:

$$2\left|\frac{C^x{}_v}{B^x{}_J}\right| \leq \left(\frac{\omega_q}{\omega}\right)^2 \frac{2u_o}{J_o\,|N|}\,\frac{J_{max}}{v_{max}} \tag{4.7}$$

The experimental values for J_{max} and v_{max} were obtained by assuming the left hand side of equation (4.7) to be much smaller than one. If the values thus found can be shown to result in a small value relative to one for the right hand side of the equation, the assumption may be considered justified. Calculations using an effective gap of 2 mm have shown that the maximum value of the right hand quantity for the range of experimental conditions used occurred for the 2960 Mc/s 900 V case and was 0.10. The maximum possible error introduced by neglecting the velocity contribution to the power supplied to the cavity is therefore 10%. This error would occur in measurement at a space charge minimum of the noise pattern for the above mentioned case of high frequency and high voltage if the anode velocity and current were completely correlated. All other conditions would lead to smaller error.

It was thought justified, on the basis of the above argument, to consider the cavity to measure a quantity proportional to the beam current fluctuations.

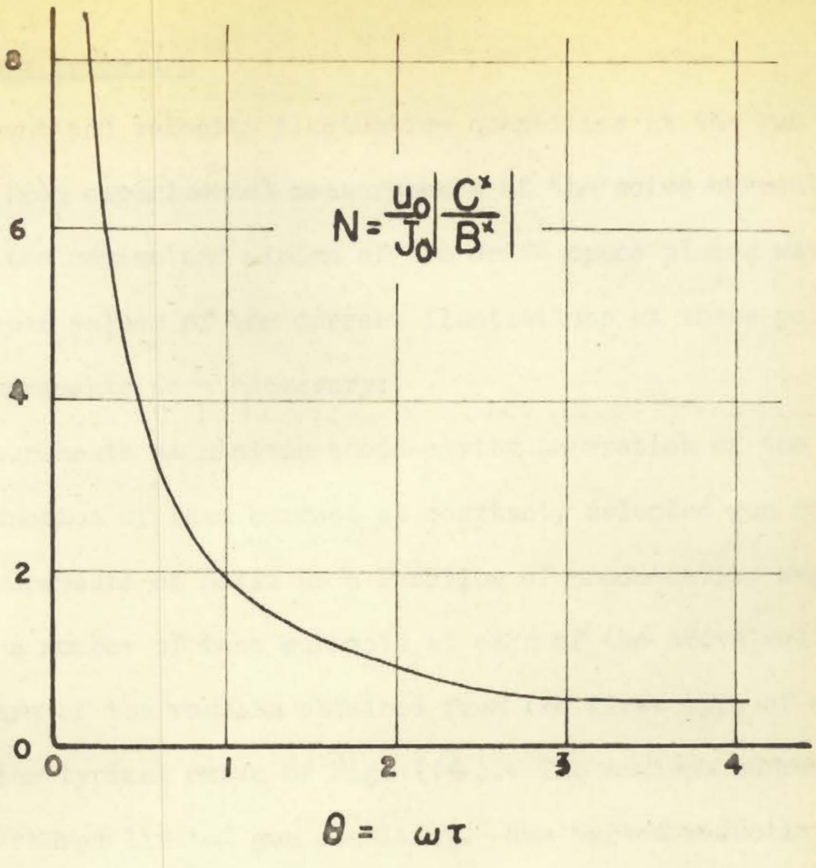


Figure 12.

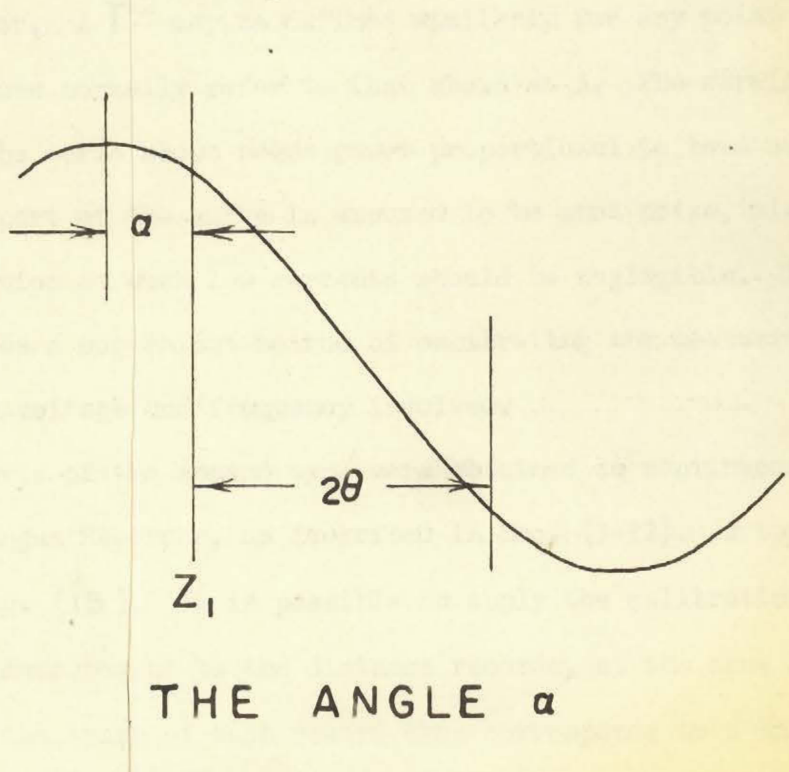


Figure 13.

4-II Measuring Procedure

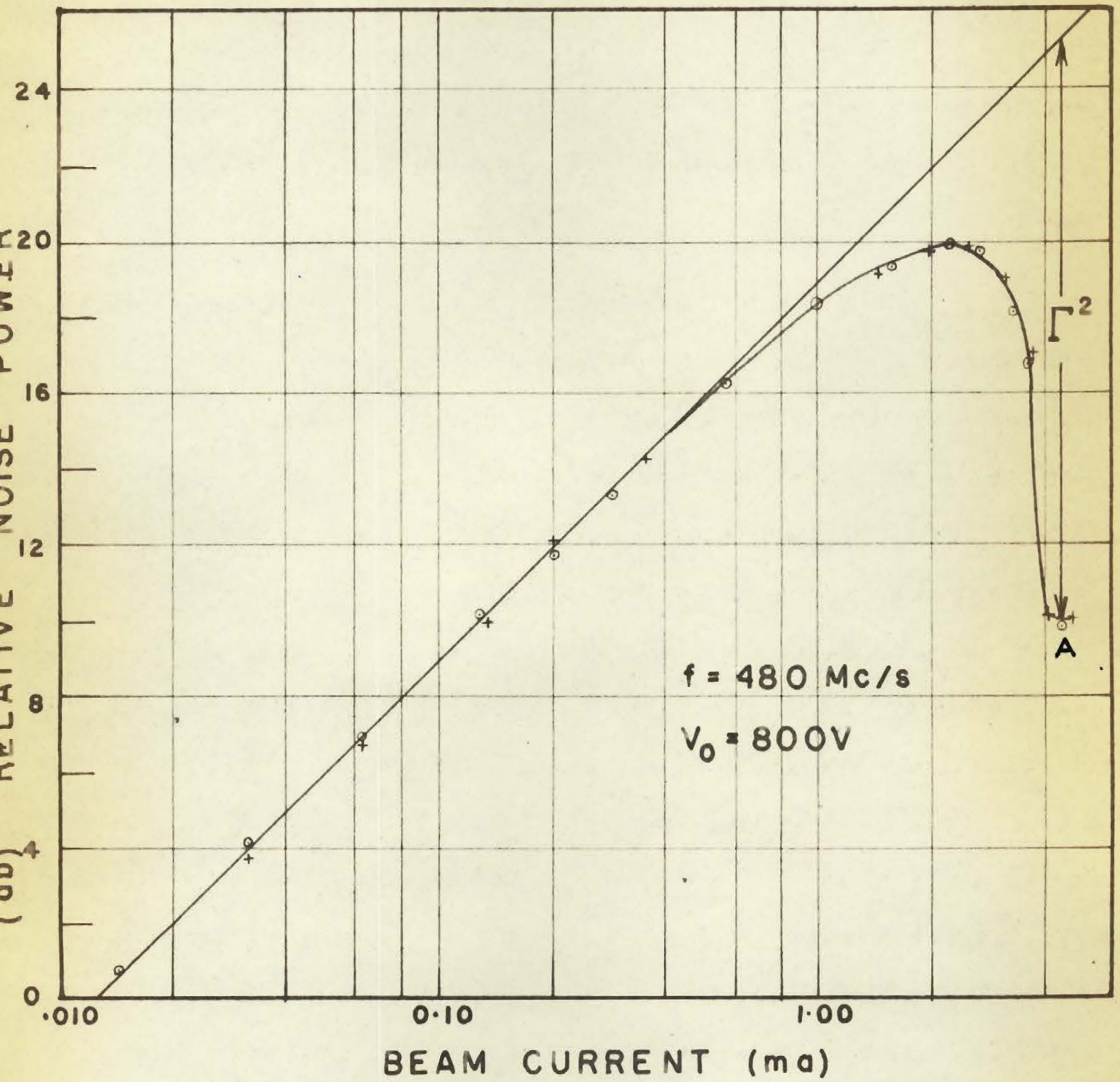
The current and velocity fluctuation quantities at the gun anode can be evaluated from experimental measurements of the noise current fluctuations appearing at the maxima and minima of the drift space plasma wave pattern. To obtain proper values of the current fluctuations at these points, two types of measurements were necessary:

(1) Measurements at minimum anode-cavity separation of the noise as a function of beam current at constant, selected gun voltages.

(2) Measurements of noise as a function of anode-cavity separation for a number of beam currents at each of the above voltages.

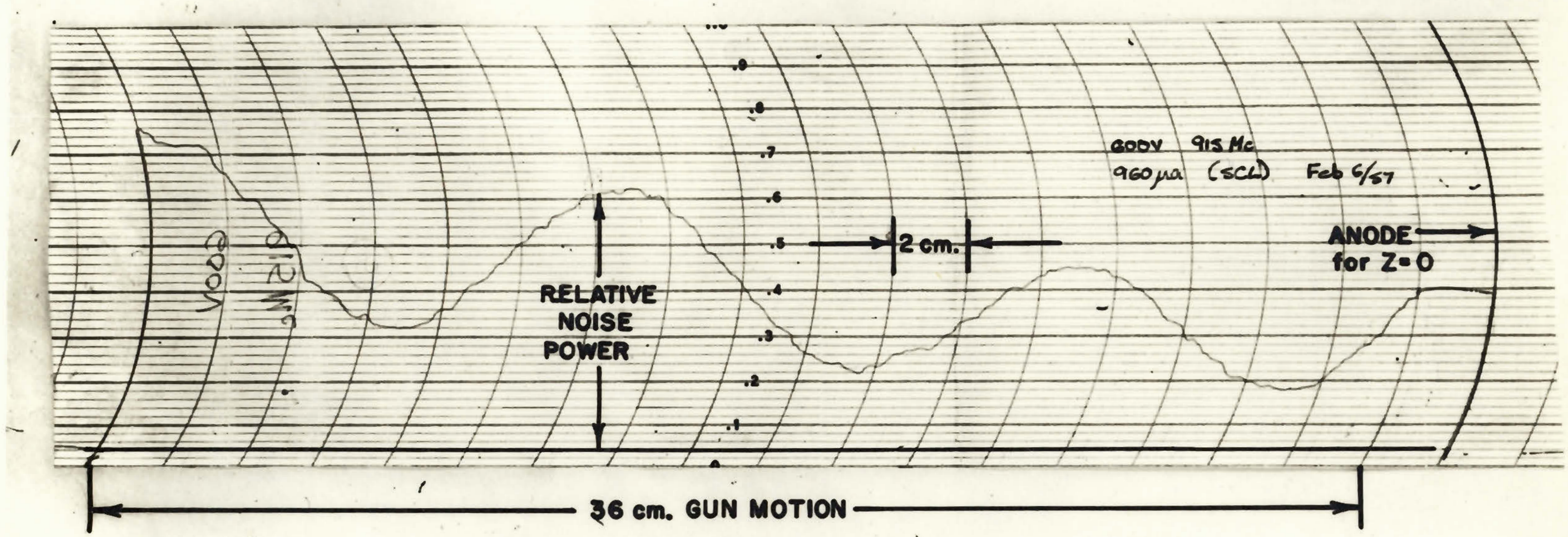
The nature of the results obtained from the first type of measurement is shown by the typical curve of Fig. (14). The maximum current point A is the space charge limited gun condition. The marked reduction of noise power at A is known as space charge smoothing and Γ^2 is known as the smoothing factor. A Γ^2 may be defined similarly for any point on the curve, but stated values normally refer to that shown at A. The straight line portion B of the curve shows noise power proportional to beam current. The noise on this part of the curve is assumed to be shot noise, since space charge interaction at such low currents should be negligible. The shot noise assumption gives a convenient method of calibrating the measuring system for the particular voltage and frequency involved.

Measurements of the second type were obtained as continuous records on an Esterline-Angus Recorder, as described in Sec. (3-II). A typical record is shown in Fig. (15). It is possible to apply the calibration from the first type of measurement to the distance records, at the same frequency and voltage since the start of each record then corresponds to a known point



ANODE CURRENT SMOOTHING

Figure 14.



A TYPICAL DRIFT SPACE NOISE PATTERN

$f = 915$ Mc/s $V_0 = 600V$ $I_0 = 1920 \mu a$ (S.C.L.)

Figure 15.

on the curve of Fig. (14). The current fluctuations existing at the space charge maxima and minima are thus obtainable as fractions Γ^2, of the shot noise $2eI_o\Delta f$ for the same current.

To maintain a consistent notation, the measurements are converted to the spectral density form. The procedure was described in Section (2-I). Thus the spectral density of beam current fluctuation is given by

$$\frac{\overline{i^2}}{\Delta f} = \Gamma^2 \; 2eI_o \tag{4.8}$$

with Γ^2 being obtainable from the experimental measurements.

The major precautions taken during the experimental measurements to ensure minimum error will be briefly mentioned. All measurements and gun supply equipment was supplied from a Sorensen line voltage regulator for maximum stability. The equipment was found to operate most satisfactorily after being on for at least 24 hours. Calibration of the phase sensitive detector amplifier showed good linearity with changing signal. The recorder patterns were therefore interpreted as having deflections proportional to the noise power at the cavity, since previous tests showed excellent square law response of the amplifier system. Current and voltage square wave values were converted to d.c. values for the gun by doubling the readings appearing on d.c. meters. Meter calibrations justified this procedure as long as the square wave modulating voltage was symmetric. The square wave was monitored on an oscilloscope and frequent checks made on the symmetry. Inaccuracies from this source should not exceed about 3%.

The measurements with the gun held fixed were done by recording the noise output signal at a slow chart speed (12 in./hr.) on a second Esterline-Angus Recorder. Each value of current, beginning with the space charge limited value was maintained for a few minutes and an average value of the recorder deflection used. The points obtained appear on Fig. (14). In some cases frequent checks of the system gain were necessary because of slight drifts in local-oscillator frequency with time.

The measurements with distance were obtained during the outward motion of the gun only. The inward motion of the plunger was found to carry small "pockets" of air into the vacuum system making operation of the gun inadvisable due to changing pressure.

The gun was driven at a speed slow enough that further decrease resulted in no change of the recorded pattern. Due to the long synchronous amplifier time constant ($\sim$ 2 sec.) the speed was limited to 6 cm/min. Lower speeds were used when the space charge minima were more than usually narrow. The motor was stopped when gain changes of the system were found necessary, also for the same reason. The smoothness of the recorded patterns indicated that the beam current stability achieved by the regulator was very good. Beam current changes of even a few microamperes were found to cause easily detectable changes in the pattern when manual control was attempted.

Measurements were taken at the following voltages and frequencies:

195 Mc/s	:	150V, 300V, 600V, 900V
336 Mc/s	:	600V
480 Mc/s	:	100V, 200V, 400V, 600V, 800V
700 Mc/s	:	600V
915 Mc/s	:	300V, 600V, 900V
2960 Mc/s	:	400V, 800V, 1200V

For each voltage a suitable number of distance patterns were recorded to show the space charge wave behavior for the non-space charge limited as well as the space charge limited cases.

Analysis and comparison with theory has, however, been restricted to the space charge limited cases in the present thesis, since no adequate theory is available for partial space charge conditions.

All useful information available from a recorder pattern is obtained by compiling a table of the noise power and position relative to the gun anode of each maximum and each minimum of the pattern. The table can then be used for all further calculations. This procedure was carried out for each of the measured patterns.

4-III Beam Radius Calculations

The first information obtained from the tabulated data was an average value of the beam radius. The method used is based on the following development:

The infinite beam plasma frequency is

$$\omega_p = \sqrt{\frac{\eta \rho_o}{\varepsilon_o}} \tag{4.9}$$

For a finite beam, a reduction factor p is used such that the plasma frequency becomes

$$\omega_q = p\omega_p = p\sqrt{\frac{\eta \rho_o}{\varepsilon_o}} \tag{4.10}$$

By using the relations $J_o = \rho_o u_o$, $\omega_q = \frac{2\pi u_o}{\lambda_q}$, and $I_o = \pi b^2 J_o$ equation (4.10) gives

$$\frac{b}{p} = \frac{\lambda_q}{2\pi u_o}\sqrt{\frac{\eta I_o}{\pi \varepsilon_o u_o}} \tag{4.11}$$

or

$$\frac{\beta_e b}{p} = \frac{\omega b}{u_o p} = \sqrt{\frac{\eta}{\pi \varepsilon_o (2\eta)^{5/2}}} \quad \frac{f \lambda_q I_o^{1/2}}{V_o^{5/4}} \tag{4.12}$$

Evaluating the numerical constants gives

$$\frac{\beta_e b}{p} = (2.936 \times 10^4) \quad \frac{f \lambda_q I_o^{1/2}}{V_o^{5/4}} \tag{4.13}$$

From the solution of the transcendental equation for p given in Sec. (2-II), a plot of the function $\frac{\beta_e b}{p}$ can be made as a function of $\beta_e b$. The beam plasma wavelength λ_q was found from the tabulated data by taking twice the average distance between the adjacent maxima or minima. Evaluation of the right hand side of equation (4.13) was then possible for the conditions indicated on any pattern. A corresponding value of $\beta_e b$ obtained from the plotted function mentioned above then led to a value of the beam radius b when divided by the known quantity

$$\beta_e = \frac{\omega}{u_o} = (1.059 \times 10^{-5}) \frac{f}{V_o^{1/2}} \qquad (4.14)$$

The beam radii computed from equations (4.13) and (4.14) showed a rather large spread for the different gun conditions and frequencies, values as large as 1.1 mm and as small as 0.25 mm being obtained. Only 82% of the 63 computations made gave values between 0.40 and 0.80 mm. The reason for the unusually large spread is best understood from the $\frac{\beta_e b}{p}$ vs $\beta_e b$ graph which is given in Fig. (16). The values of $\frac{\beta_e b}{p}$ indicated from equation (4.13) fall in the range 0.7 to 1.8. A relatively small percentage error in the measured quantities can therefore produce a considerable percentage error in the resulting and hence in b. Individual values of b should thus not be considered as significant. Since no distinct pattern was evident in the variation of b with gun conditions or frequency, the scatter was assumed to be purely random, and a mean beam radius was found by statistical analysis of the 63 computed results. The results were:

$$b_{av} = 0.612 \text{ mm} \qquad (4.15)$$

mean deviation $= 0.1265$ mm

probable accuracy $= \frac{0.1265}{\sqrt{63}} \doteq 0.0158 \text{ mm} = 2.5\%$

A calculation done by the Langmuir (7) method showed that for 900V on the gun, a temperature of 2300°K and a saturation cathode current density of 0.7 amp/cm^2, a beam current density of 0.308 amp/cm^2 would be expected. The actual measured beam current was 3.46×10^{-3} amp, which gave a calculated beam radius of 0.607 mm; in excellent agreement with the statistical value.

It should be stressed that the figures must be considered an effective beam radius only since in fact variations of current density probably occur with radius inside the beam. The statistical figure 0.612 mm was used in all conversions from beam current to current density. Using the corresponding beam area $A = 1.175 \times 10^{-6}$m^2, all the current noise magnitudes were expressed in the more general form

$$\frac{\overline{J^2}}{\Delta f} = \Gamma^2 2eJ_o = \Gamma^2 2e \frac{I_o}{A} \qquad (4.16)$$

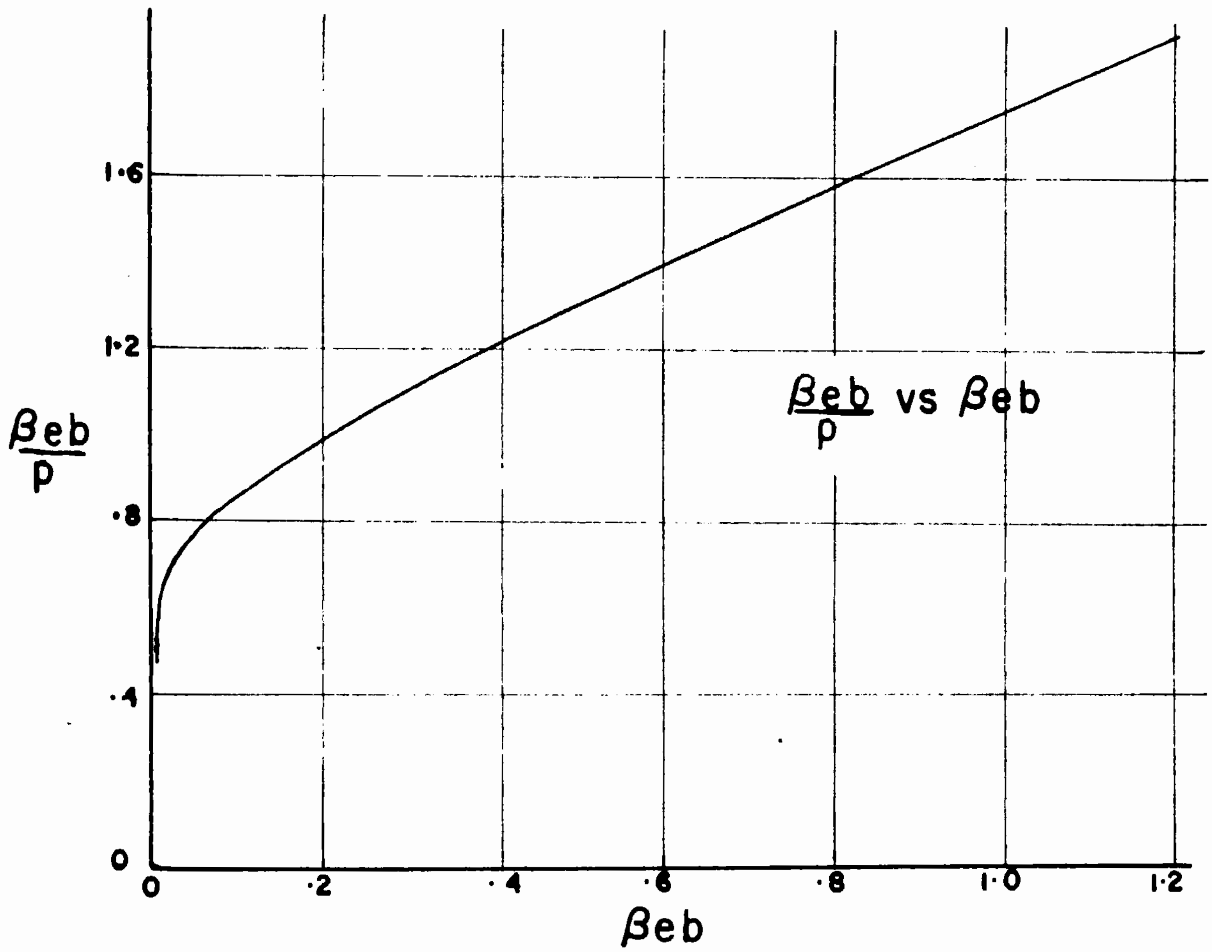


Figure 16.

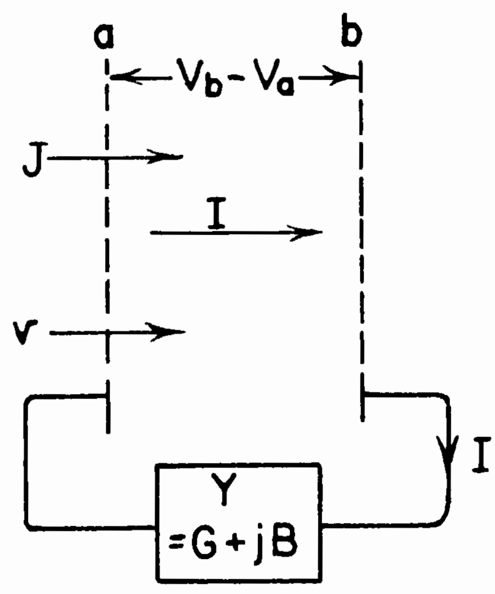


Figure 17.

4-IV. Secondary Electron Noise

At the start of the measurement program, difficulty was encountered with the effects of secondary electrons returning along the beam path from the collector. Applying a positive bias to the collector prevented the escape of low voltage secondaries but was unable to affect the reflected primary electrons since they always have an energy equal to that of incident beam. By contributing their equivalent shot noise to the cavity, the returning electrons caused serious errors in the measured noise quantities. From available information on secondary electron yields, the reflected primaries at the voltages considered could amount to between 5 and 10% of the total secondary yield. Since the total yield for copper is around one, a possible 10% of the beam current could return through the gap.

Experiments were done with oxygen-free copper tubular collectors of the type shown in Fig. (9) to find ways of eliminating the electrons. The experiments were done at 400V beam voltage, at a frequency of 480 Mc/s. The presence of secondary electrons was first definitely established by applying a reverse bias to the collector. The system could be made to oscillate when the collector was still 200V above cathode potential. It is thought that a reflex klystron action occurred, the returning electrons being secondaries. Powers more than 20 db in excess of shot noise were detected. Positive bias beyond about 10V on the collector did not have a marked effect on the measured noise indicating that low voltage secondary electrons were not affecting the measurements.

The application of a soft iron magnetic shield around the collector tube was found to give an improvement in space charge limited smoothing of between 5 and 6 db, along with a drop in the apparent gun anode interception current to a small fraction of its former value. Typical reductions were from 30 μa to 2 μa. Replacing the iron shield by a small solenoid operated to give a cancellation of the main confining field showed that both the smoothing and the interception changes took place gradually as the field within the collector was reduced. No further improvement occurred beyond the point of the zero collector field. Reflected primaries thought to be causing part of the cavity noise and impinging on the gun anode to appear as interception current appeared to be removed by the magnetic lens effect of the shield. The lens action at the start of the collector shield caused a strong divergence of the beam. A collector split into two parts, an open end cylinder followed by the usual type described above was also tested. It was found to give no further improvement. The effectiveness of the beam spreading caused by the magnetic shield was illustrated by sliding the shield over the two piece collector in small increments. Over a range of only about one inch the collected current transferred completely from one half of the collector to the other.

On the basis of the above experiments, a soft iron magnetic shield over the beam collector was assumed to remove reflected primary electrons from the beam being measured.

5. RESULTS AND DISCUSSION

5-I. Experimental Values from Basic Data

Haus (3) has given expressions, as described in Sec. (2-IV D) for the transformation of noise current and voltage along a drift space. In his equations (2.73), (2.74) and (2.75), the quantity θ is the plasma drift angle between Z_1 and Z_2

$$\theta = \frac{\omega_q}{u_o} (Z_2 - Z_1) \tag{5.1}$$

and, in our notation

$$Z_o = \frac{1}{Y_o} = -2\frac{V_o}{J_o}\left(\frac{\omega_q}{\omega}\right) \tag{5.2}$$

By trigonometric manipulation the equations given by Haus can be put in the form

$$\Phi(Z_2\omega) = \frac{1}{2}\left[\Phi(Z_1\omega) + Z_o^2\,\Psi(Z_1\omega)\right] + \frac{1}{2}\left[\Phi(Z_1\omega) - Z_o^2\,\Psi(Z_1\omega)\right]\sqrt{1+\tan^2\alpha} \times \cos(2\theta+\alpha) \tag{5.3}$$

$$\Psi(Z_2\omega) = \frac{1}{2}\left[\Psi(Z_1\omega) + Y_o^2\,\Phi(Z_1\omega)\right] + \frac{1}{2}\left[\Psi(Z_1\omega) - Y_o^2\,\Phi(Z_1\omega)\right]\sqrt{1+\tan^2\alpha} \times \cos(2\theta+\alpha) \tag{5.4}$$

in which

$$\tan\alpha = \frac{Z_o\,\Lambda(Z_1\omega)}{\frac{1}{2}\left[\Phi(Z_1\omega) - \frac{1}{2}Z_o^2\,\Psi(Z_1\omega)\right]} \tag{5.5}$$

The maximum and minimum values of equation (5.4) are:

$$\Psi_{max}(Z\omega) = \frac{1}{2}\left[\Psi(Z_1\omega) + Y_o^2\,\Phi(Z_1\omega)\right] + \frac{1}{2}\left[\Psi(Z_1\omega) - Y_o^2\,\Phi(Z_1\omega)\right]\sqrt{1+\tan^2\alpha} \tag{5.6}$$

$$\Psi_{min}(Z\omega) = \frac{1}{2}\left[\Psi(Z_1\omega) + Y_o^2\,\Phi(Z_1\omega)\right] - \frac{1}{2}\left[\Psi(Z_1\omega) - Y_o^2\,\Phi(Z_1\omega)\right]\sqrt{1+\tan^2\alpha} \tag{5.7}$$

The significance of the angle α is shown in the sketch of Fig. (13). By considering the Z_1 plane to be the gun anode, the equations (5.6) and (5.7) relate the anode noise quantities to the current maximum and minimum. Use is next made of the experimental fact that the gun anode is always close to a maximum of the current standing wave pattern. This means that the angle α will be small for all cases. Small error is thus involved by writing

$$\sqrt{1 + \tan^2\alpha} = 1 + \frac{\alpha^2}{2} \tag{5.8}$$

which results in considerable simplification. Using equation (5.8), the equations (5.6) and (5.7) can be solved for $\Psi(Z_1\omega)$ and $\Phi(Z_1\omega)$. There result the relations

$$\Psi(Z_1\omega) = \frac{(4 + \alpha^2)\,\Psi_{max} + \alpha^2\,\Psi_{min}}{2\,(2 + \alpha^2)} \tag{5.9}$$

$$Y_o^2\,\Phi(Z_1\omega) = \frac{(4 + \alpha^2)\,\Psi_{min} + \alpha^2\,\Psi_{max}}{2\,(2 + \alpha^2)} \tag{5.10}$$

The self power density spectra of current and voltage fluctuation are related to the quadratic contents by a constant factor $\frac{1}{4\pi}$

$$\Psi(Z\omega) = \frac{1}{4\pi}\,\frac{\overline{J^2}}{\Delta f} \qquad \Phi(Z\omega) = \frac{1}{4\pi}\,\frac{\overline{V^2}}{\Delta f} \tag{5.11}$$

and the voltage is related to the more commonly used velocity by the expression

$$v(Z) = -\frac{e}{m}\,\frac{1}{u_o(Z)}\,V(Z) \tag{5.12}$$

Using (5.2), (5.11) and (5.12)

$$Y_o^2\,\Phi\,(Z_1\omega) = \frac{1}{4\pi}\left(\frac{J_o\omega}{u_o\omega_q}\right)^2 \frac{\overline{v_a^2}}{\Delta f} \tag{5.13}$$

the subscript "a" indicating that anode quantities are being considered at the Z_1 plane. Then from equations (5.9), (5.10) and (5.13)

$$\frac{\overline{J_a^2}}{\Delta f} = \frac{(4 + \alpha^2)\,\frac{\overline{J^2_{max}}}{\Delta f} + \alpha^2\,\frac{\overline{J^2_{min}}}{\Delta f}}{2\,(2 + \alpha^2)} \tag{5.14}$$

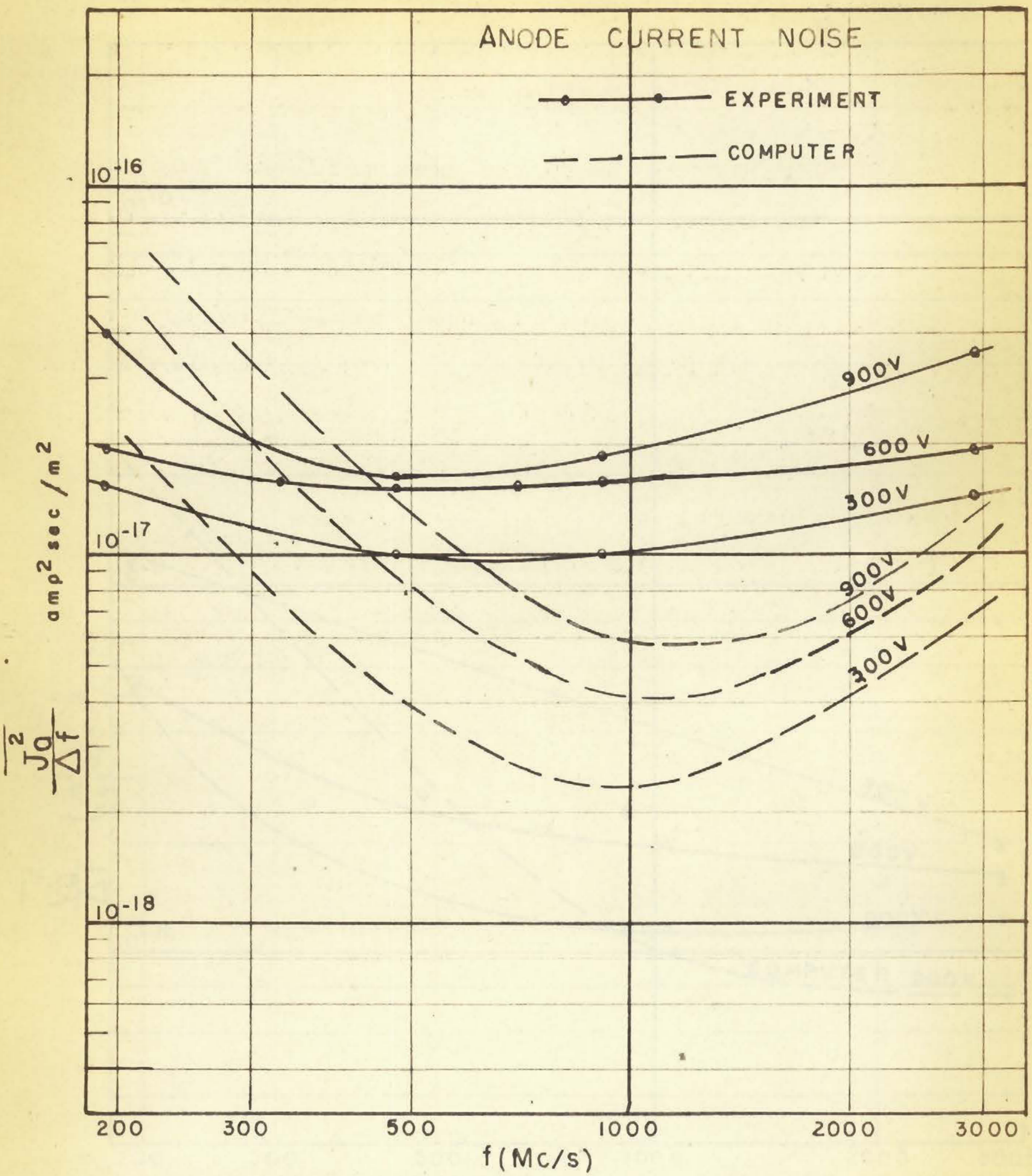
$$\frac{\overline{v_a^2}}{\Delta f} = \left(\frac{u_o\omega_q}{J_o\omega}\right)^2 \frac{(4+\alpha^2)\,\frac{\overline{J^2_{min}}}{\Delta f} + \alpha^2\,\frac{\overline{J^2_{max}}}{\Delta f}}{2(2+\alpha^2)} \tag{5.15}$$

All the quantities on the right hand sides of equations (5.14) and (5.15) are obtainable from the tabulated experimental data: The angle α is

$$\alpha = \frac{2\pi}{\lambda_q} \left[\frac{\lambda_q}{4} - d_{min} \right] \qquad (5.16)$$

d_{min} being the distance from gun anode to first minimum. The quadratic contents are found as described in Sec. (4-III). The anode noise quantities $\frac{\overline{J_a^2}}{\Delta f}$ and $\frac{\overline{v_a^2}}{\Delta f}$ have been evaluated according to (5.14) and (5.15) for all the space charge limited measurements. As representative figures, the quantities at the voltages 300, 600 and 900 V have been plotted as a function of the frequency of measurement. The current results appear in Fig. (18) and the velocity results in Fig. (19).

Only the maximum and minimum nearest the anode were used in obtaining $\frac{J^2_{max}}{\Delta f}$ and $\frac{J^2_{min}}{\Delta f}$. The angle α was found according to equation (5.10) in which λ_q was an average obtained from all maximum and minimum positions on the pattern.



COMPARISON OF COMPUTER SOLUTION WITH EXPERIMENT

Figure 18.

5-II. Relations used in Noise Calculations from Theory

For the particular gun used in the experimental work, the beam radius found in Sec. (4-III) enables calculations to be done with current densities in place of the less general gun current. The following short table of gun constants will prove useful.

V_o	I_o(amp)	$J_o\left(\frac{amp}{m^2}\right)$	$\frac{\overline{J_{sn}^2}}{\Delta f}\left(\frac{amp^2\ sec}{m^2}\right)$	$\frac{\overline{v_R^2}}{\Delta f}\left(\frac{m^2}{sec} \times m^2\right)$
300	850×10^{-6}	725	2.32×10^{-16}	6.60×10^{-12}
600	1998×10^{-6}	1700	5.43×10^{-16}	2.81×10^{-12}
900	3460×10^{-6}	2940	9.40×10^{-16}	1.626×10^{-12}

The last two columns were calculated according to equations (2.1b) and (2.2b); the cathode temperature has been assumed 2300°K for all cases. The root mean square thermal velocity is given by

$$\sqrt{\overline{v^2}} = \sqrt{\frac{2kT_c}{m}} = (2.64 \times 10^5)\ \text{m/sec} \qquad (5.16)$$

which has a voltage equivalent

$$V_t = \frac{k\,T_c}{e} = 0.198\ \text{volts} \qquad (5.17)$$

Parallel to the experimental results, theoretical values of the anode fluctuation quantities were calculated for the voltages 300, 600 and 900 volts at each of the frequencies given in Sec. (5-I). A slight change of the frequencies considered occurs for the analogue computer solution, where 230, 480, 1000, 2000 and 3000 Mc/s were used.

5-III. Theoretical Prediction of Anode Noise Quantities

The differences in the noise quantities predicted by the theories of Sec. (2-IV) is best illustrated by carrying out numerical calculations for one particular electron gun. To enable comparison with the experimental results, the gun used in the experimental work was chosen for the calculations. The way in which numerical results were obtained from the various theoretical expressions is outlined in this section.

(A) The Llewellyn-Peterson-Pierce Theory

The noise is to be calculated from the equations

$$\frac{\overline{J_a^2}}{\Delta f} = \left| E^x \right|^2 \frac{\overline{J_i^2}}{\Delta f} + \left| F^x \right|^2 \frac{\overline{v_i^2}}{\Delta f} \qquad (2.48)$$

$$\frac{\overline{v_a^2}}{\Delta f} = \left| H^x \right|^2 \frac{\overline{J_i^2}}{\Delta f} + \left| I^x \right|^2 \frac{\overline{v_i^2}}{\Delta f} \qquad (2.49)$$

The assumptions to be involved in the evaluation were fully outlined in Sec. 2-IVA. The final plane of the region being considered is the gun anode, hence the change of notation from the original equations (2.48) and (2.49). The initial plane is the potential minimum.

Although the whole diode is in the space charge limited condition, the region between the potential minimum plane and the anode is not. By requiring that a d.c. velocity exist at potential minimum and that at the same time the acceleration there be zero, the Llewellyn-Peterson relations give a value of ζ slightly less than one, as is shown in the following development.

From the d.c. velocity equation (2.40) the initial acceleration may be taken zero at the potential minimum:

$$u_a = \frac{1}{2} \frac{eJ_o}{m\epsilon_o} \tau^2 + a_i \tau + u_i \qquad (2.40)$$

$$u_a = \frac{1}{2} \frac{eJ_o}{m\epsilon_o} \tau^2 + u_i \tag{5.18}$$

From the defining expression for ζ given by Llewellyn and Peterson (9)

$$\zeta = \frac{\tau^2}{2} \frac{eJ_o}{m\epsilon_o} \frac{1}{u_a + u_i} \tag{5.19}$$

Combining equations (5.18) and (5.19) gives

$$\zeta = \frac{u_a - u_i}{u_a + u_i} \tag{5.20}$$

If equation (5.20) is substituted into the expressions for the four desired coefficients, the results are

$$E^x = \frac{u_i}{u_a} e^{-\beta}$$

$$F^x = \frac{J_o}{u_a} \beta e^{-\beta}$$

$$H^x = \frac{u_a - u_i}{u_a + u_i} \frac{2u_i}{J_o} \frac{e^{-\beta}}{\beta} \tag{5.21}$$

$$I^x = (-1 + \frac{2u_i}{u_a}) e^{-\beta}$$

in which $\beta = j\omega\tau$

If the equation (5.18) is solved for τ, and the numerical factors evaluated,

$$\tau = (.771 \times 10^{-8}) \sqrt{\frac{V_a^{1/2} - V_i^{1/2}}{J_o}} \tag{5.22}$$

where V_a and V_i are the voltage equivalents of the final and initial d.c. velocities.

Using equation (5.22) the squares of the moduli of the coefficients become:

$$|E^x|^2 = \frac{V_i}{V_a}$$

$$|F^x|^2 = \left[(6.65 \times 10^{-27})(V_a^{1/2} - V_i^{1/2}) J_o\right] \frac{f^2}{V_a} \tag{5.23}$$

$$|H^x|^2 = \left[(6.65 \times 10^{-27})(V_a^{1/2} - V_i^{1/2}) J_o\right]^{-1} \frac{V_i}{f^2} \left(\frac{V_a^{1/2} - V_i^{1/2}}{V_a^{1/2} + V_i^{1/2}}\right)$$

$$|I^x|^2 = \left[1 - 2\sqrt{\frac{V_i}{V_a}}\right]^2$$

The quantities of equation (5.23) involve only the basic gun parameters, the initial equivalent voltage V_i given by equation (5.17) and the signal frequency.

Evaluation of the coefficients according to equation (5.23) and the use of the input conditions given in Sec. 5-II for $\frac{\overline{J_i^2}}{\Delta f}$ and $\frac{\overline{v_i^2}}{\Delta f}$ then allow the anode quantities to be found for the necessary gun voltages and frequencies.

The described calculations were carried out for the standard voltages and frequencies, using the appropriate J_o's given in the table of Sec. 5-II for the experimental gun. The results are plotted for comparison with other theories and with experimental values on the Figs. (20) (21) and (22) for current fluctuation and Figs. (23) and (24) for the velocity modulations.

(B) The Robinson Theory

Again, to maintain the notation of "a" subscripts for the anode quantities, a slight change of notation is made in writing the appropriate Robinson equations

$$\frac{\overline{J_a^2}}{\Delta f} = |A_{33}|^2 \frac{\overline{J_\alpha^2}}{\Delta f} + |A_{32}|^2 \frac{\overline{v_\alpha^2}}{\Delta f} \qquad (2.59)$$

$$\frac{\overline{v_a^2}}{\Delta f} = |A_{23}|^2 \frac{\overline{J_\alpha^2}}{\Delta f} + |A_{22}|^2 \frac{\overline{v_\alpha^2}}{\Delta f} \qquad (2.58)$$

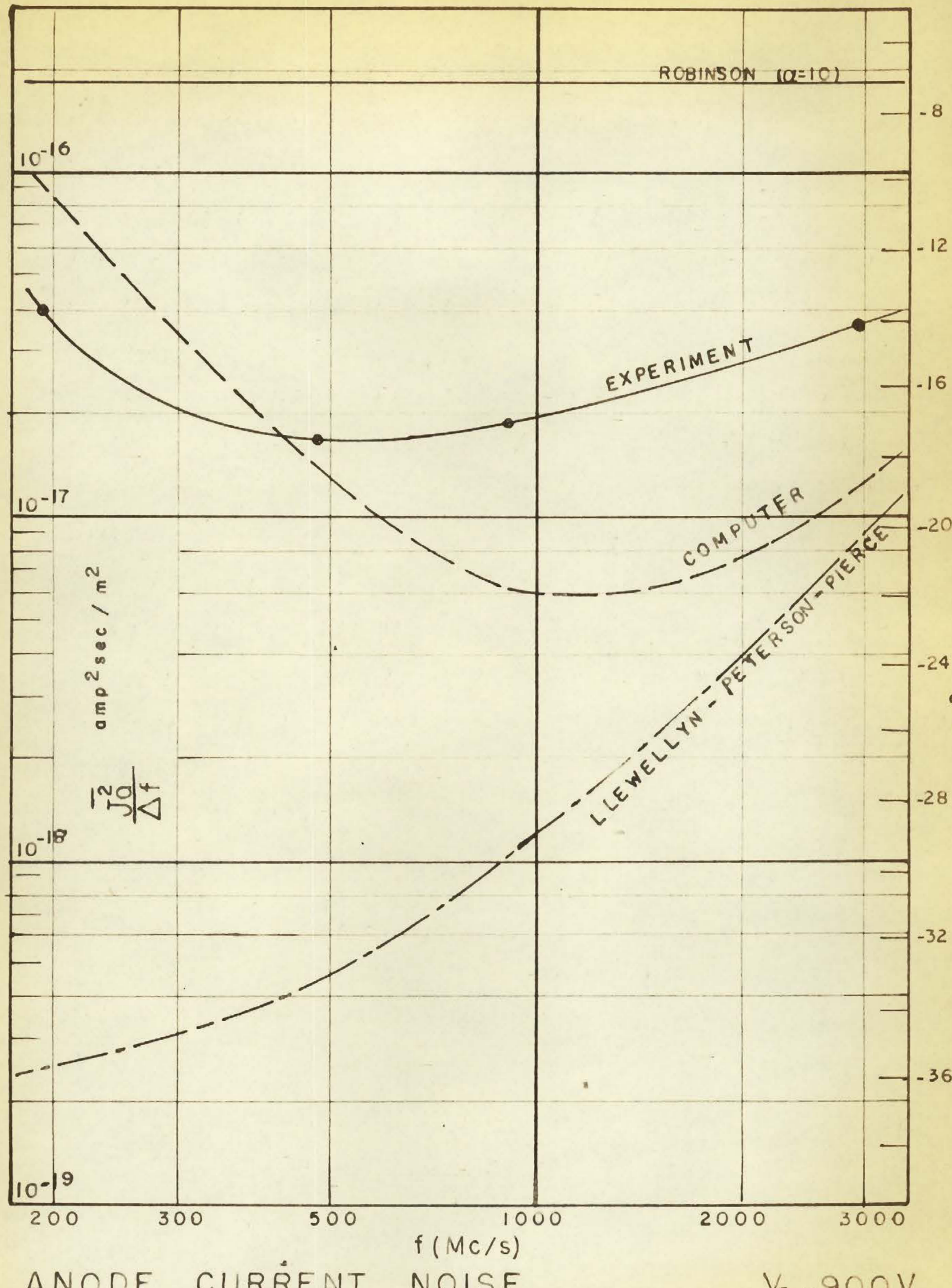
The assumptions to be used are stated in Sec. 2-IV B.

Robinson gives the coefficients in the form

$$\begin{aligned} A_{33} &= 1 - \frac{1}{(\eta+1)^{\frac{1}{2}}}\Big([\eta] - [\alpha]\Big)^2 \\ A_{32} &= K\left[\frac{1}{(\eta+1)^{3/4}}\Big([\eta] - [\alpha]\Big)\right] \\ A_{23} &= -\frac{1}{K}\left[\frac{1}{(\eta+1)^{1/4}}\Big(1 + [\eta][\alpha]\Big)\Big([\eta] - [\alpha]\Big)\right] \\ A_{22} &= 1 - \frac{2[\eta]}{(\eta+1)^{\frac{1}{2}}}\Big([\eta] - [\alpha]\Big) \end{aligned} \qquad (5.24)$$

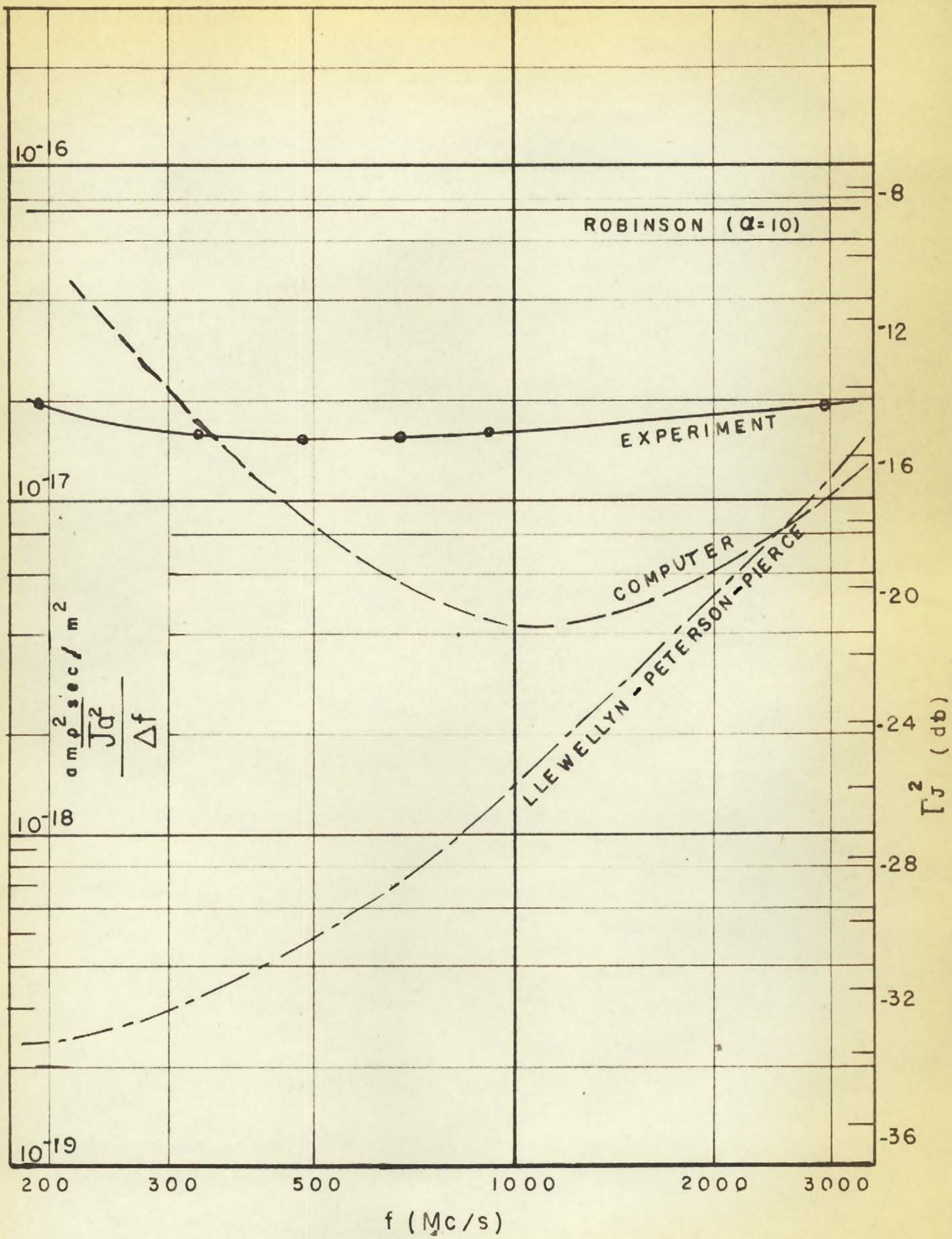
where

$$K = j\omega\, \tau_o J_o \left[\frac{2kT_c}{m}\right]^{-1/2} \qquad (5.25)$$



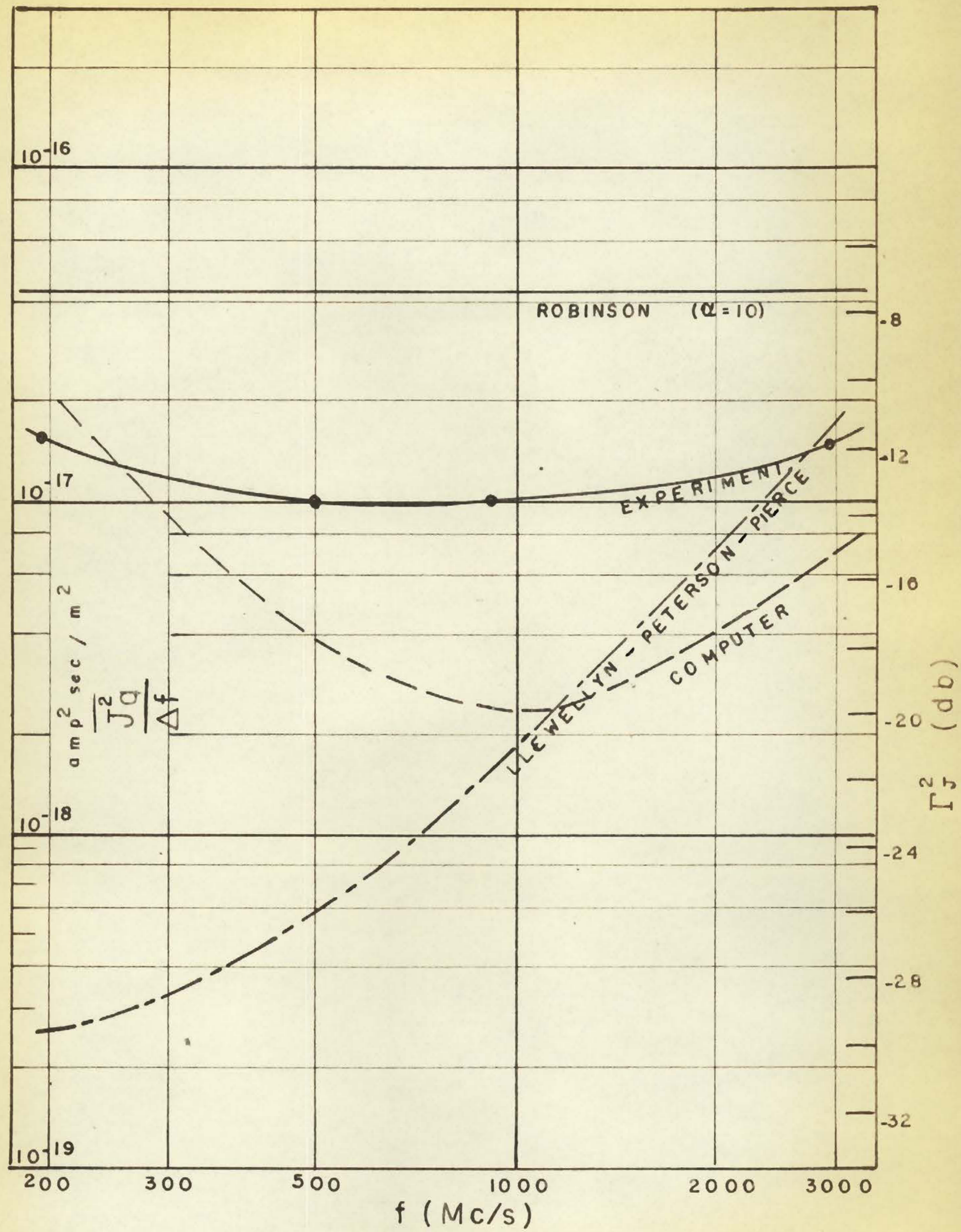
ROBINSON (α=10)
10^{-16}
10^{-17}
10^{-18}
10^{-19}
EXPERIMENT
COMPUTER
LLEWELLYN - PETERSON - PIERCE
amp^{2}sec / m^2
$\frac{\overline{J_a^2}}{\Delta f}$
Γ_J^2 · db
-8
-12
-16
-20
-24
-28
-32
-36
200
300
500
1000
2000
3000
f (Mc/s)
ANODE CURRENT NOISE
V_0 = 900V

Figure 20.



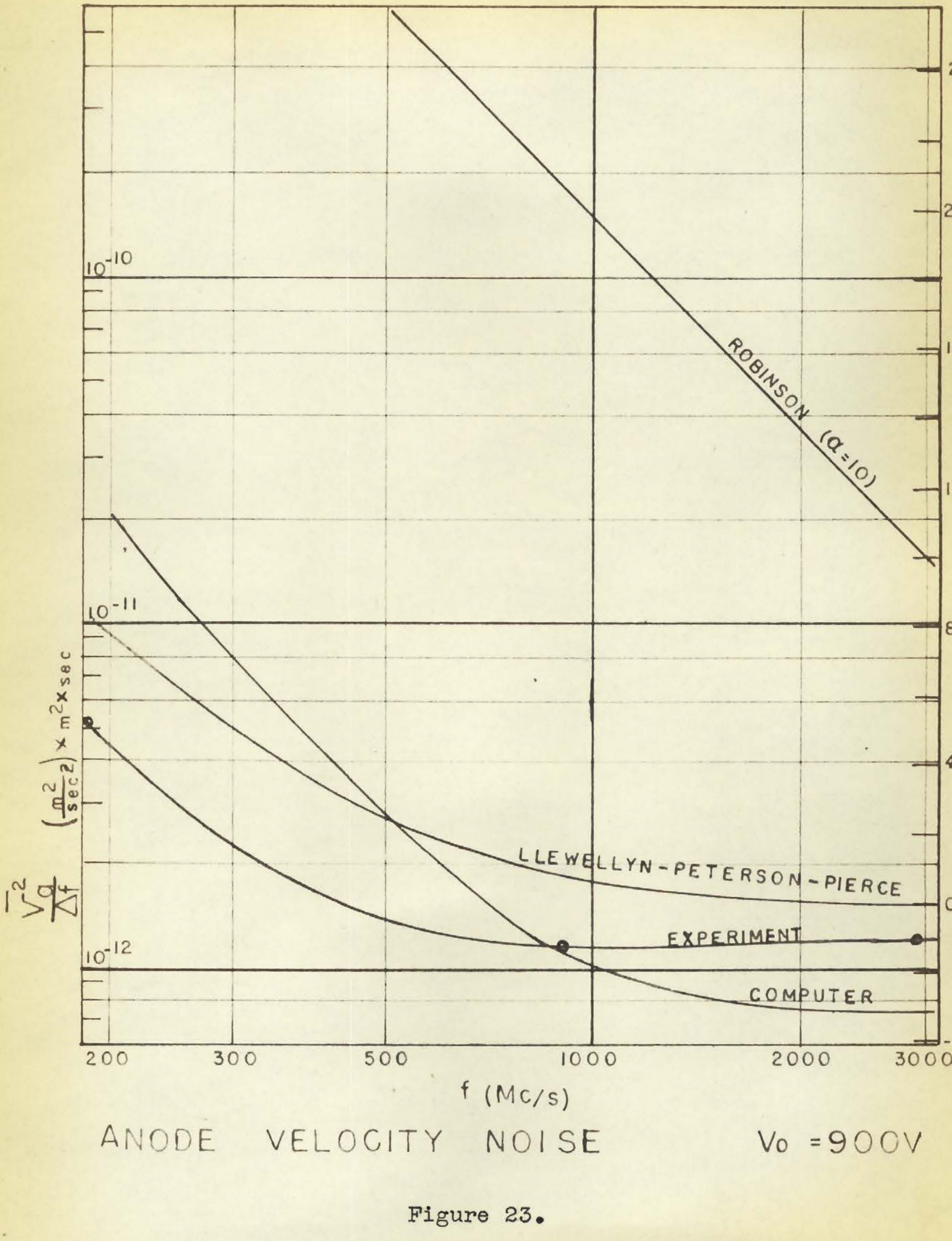
ANODE NOISE CURRENT $V_0 = 600V$

Figure 21.



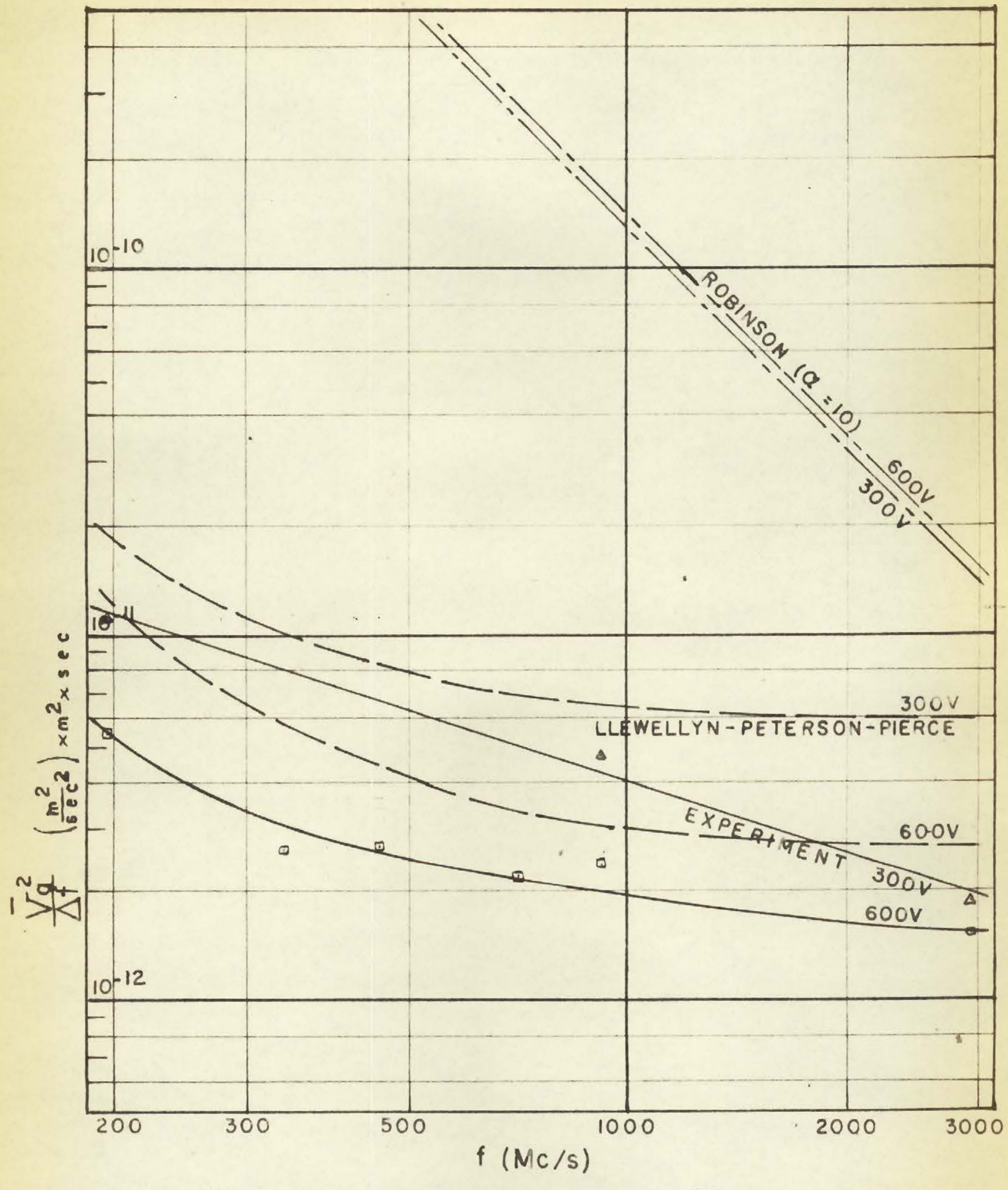
ANODE NOISE CURRENT $V_o = 300V$

Figure 22.



ROBINSON (α=10)
LLEWELLYN-PETERSON-PIERCE
EXPERIMENT
COMPUTER
10^{-10}
10^{-11}
10^{-12}
24
20
16
12
8
4
0
-4
200
300
500
1000
2000
3000
f (Mc/s)
$\frac{\overline{V_a^2}}{\Delta f}$ $\left(\frac{m^2}{sec^2}\right) \times m^2 \times sec$
Γ_v^2 db
ANODE VELOCITY NOISE
Vo =900V

Figure 23.



ANODE VELOCITY NOISE V_0 =300V, 600V

Figure 24.

and τ_o is the electron transit time from the potential minimum for zero initial energy. Using the expression (2.40) with $u_i = 0$ and $a_i = 0$ to find τ_o, equation (5.25) becomes

$$K = j\ (1.830 \times 10^{-13})\ f\ V_a^{1/4}\ J_o^{1/2} \qquad (5.26)$$

In Robinson's coefficients

$$\eta = \frac{V_a e}{kT_c} \qquad \text{and} \qquad \alpha = \frac{V_\alpha e}{kT_c} \qquad (5.27)$$

and

$$[\eta] = \left[\sqrt{\eta + 1} - 1\right]^{1/2} \qquad \alpha = \left[\sqrt{\alpha + 1} - 1\right]^{1/2} \quad (5.28)$$

The coefficients in (5.24) may thus be calculated from the gun conditions, cathode temperature, frequency and chosen value of α. $\frac{\overline{J_\alpha^2}}{\Delta f}$ is assumed to be shot noise and can be obtained from the table of Sec. (5-II). The velocity $\frac{\overline{v_\alpha^2}}{\Delta f}$ is dependent on the choice of α. It can be seen from (2.55) that $\overline{v_\alpha^2}$ is related to the Rack velocity $\overline{v_R^2}$ by the equation

$$\overline{v_\alpha^2} = \overline{v_R^2}\ \frac{4f(\alpha)}{(4-\pi)} \qquad (5.29)$$

The $\overline{v_R^2}$ from the table of Sec. (5-II) must be multiplied by the appropriate factor which depends on α.

Calculations were carried out for the standard voltages and frequencies for $\alpha = 10$. The results are plotted against frequency for comparison with experimental values and other theories on the same figures mentioned in the previous section.

(C) The Analogue Computer Solution

All details of the procedure required to calculate the anode noise fluctuations are contained in Appendix I. Because of the assumed non-correlation of input current and velocity, the squares of the contributions from the two separate solutions are added to give the total fluctuations at the anode.

The recorder potentiometer was turned by a 2 rpm motor and all but 5^{o} of the full rotation was used. The computer running time for the 900 V case was thus

$$t_a = \frac{355}{360} \times 30 = 29.62 \text{ sec} \tag{5.30}$$

giving a value of α

$$\alpha = \frac{1}{t_a} = 0.0388$$

The amplifier time constants were both 2 sec, so that

$$R_1C_1 = 2 \text{ and } R_2C_2 = 2$$

Calculations were done for the standard voltages but at the frequencies 230, 400, 1000, 2000, 3000 Mc/s. Values of the velocity fluctuations obtained were found to be seriously different from experiment for the 600 and 300 V cases. The subtraction of two terms of almost equal size in the velocity determination made the answers extremely sensitive to the exact gain of the amplifier loop. The gain is governed by the value of R_x. To justify the use of the same R_x at the lower voltages, it was necessary to assume the gun to have a constant pervance. In Sec. (3-I) this was shown to be untrue, and the subsequent error in the loop gain is thought to be the cause of the large discrepancies. An error exists for the same reason in the current calculations but it is relatively small because no difference of terms is involved.

The current results for the three voltages are shown in Fig. (18) as a function of frequency. The 900 V velocity result appears in Fig. (19) along with the experimental values and other theory predictions. The lower voltage velocity results were not plotted because of the large errors.

Proper values of the velocity fluctuations at 300 and 600 volts could be obtained by running other computer solutions with appropriate changes in R_x.

(D) The Tien-Moshman Correction

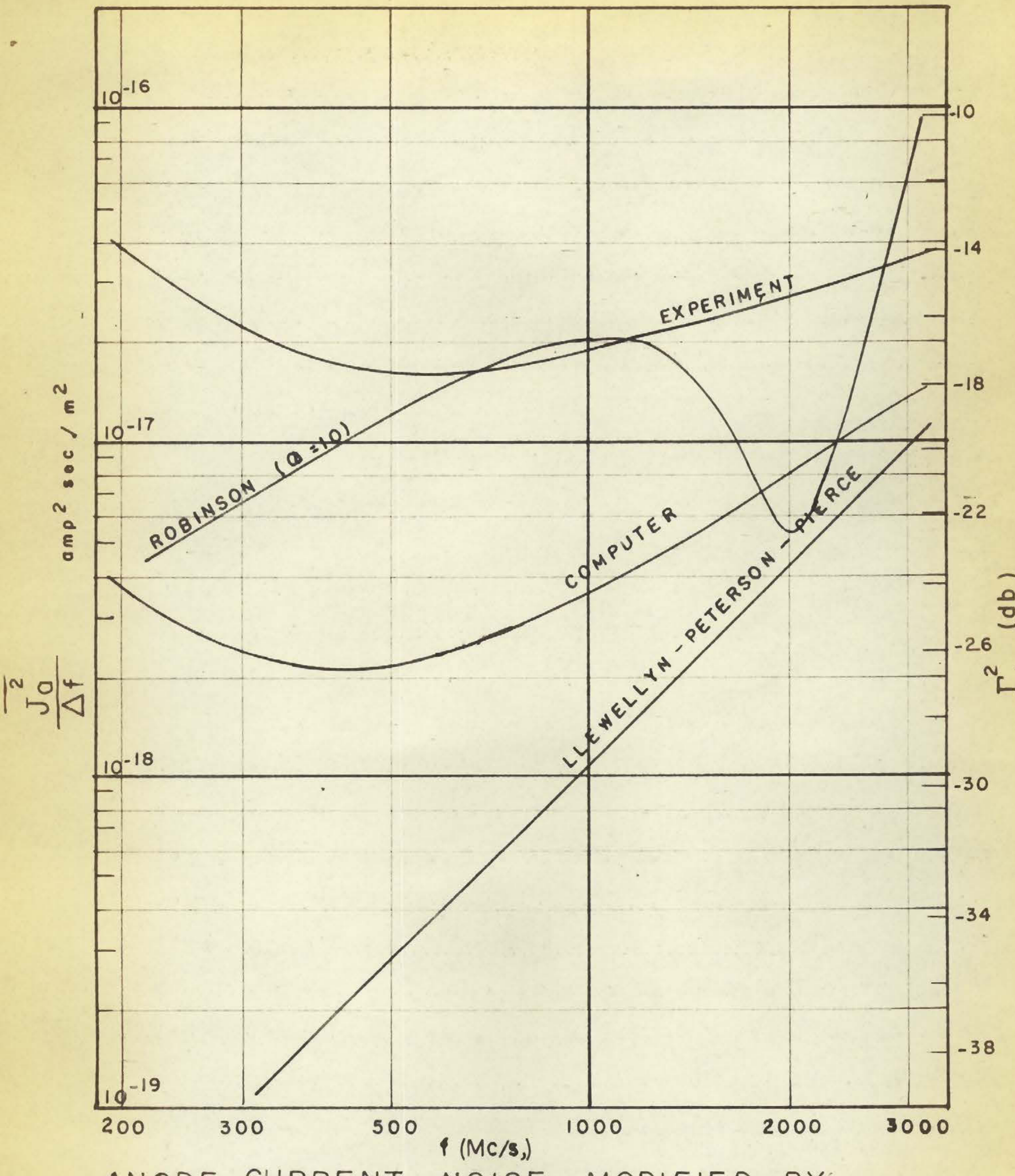
The account given in Sec. 2-IV E of the work of Tien and Moshman indicated that the current fluctuations at the potential minimum varied considerably with frequency, while the cathode velocity fluctuations were unaffected. In spite of the unusual form taken by the current smoothing, reproduced in Fig. (3), there seems no reason to doubt the validity of their results. The diode considered by Tien and Moshman is closely similar to that used in the present experimental work for an anode voltage of 900 V. A comparison is given below:

	Tien Moshman diode	Experimental Gun at 900 V	
Saturation current density	1.50	0.70	amp/cm^2
Gun current density	0.30	0.294	amp/cm^2
Potential minimum depth	0.161 V	0.181	volts
Cathode to Potential min. distance	0.75×10^{-3}	1.02×10^{-3}	cm

In view of the close similarity, the Tien Moshman smoothing was applied to the 900 V theoretical results. The shot noise input was reduced at each frequency by the amount indicated in Fig. (3), and the resulting anode quantities recalculated accordingly. The resulting values are plotted along with experimental results in Figs. (25) and (26).

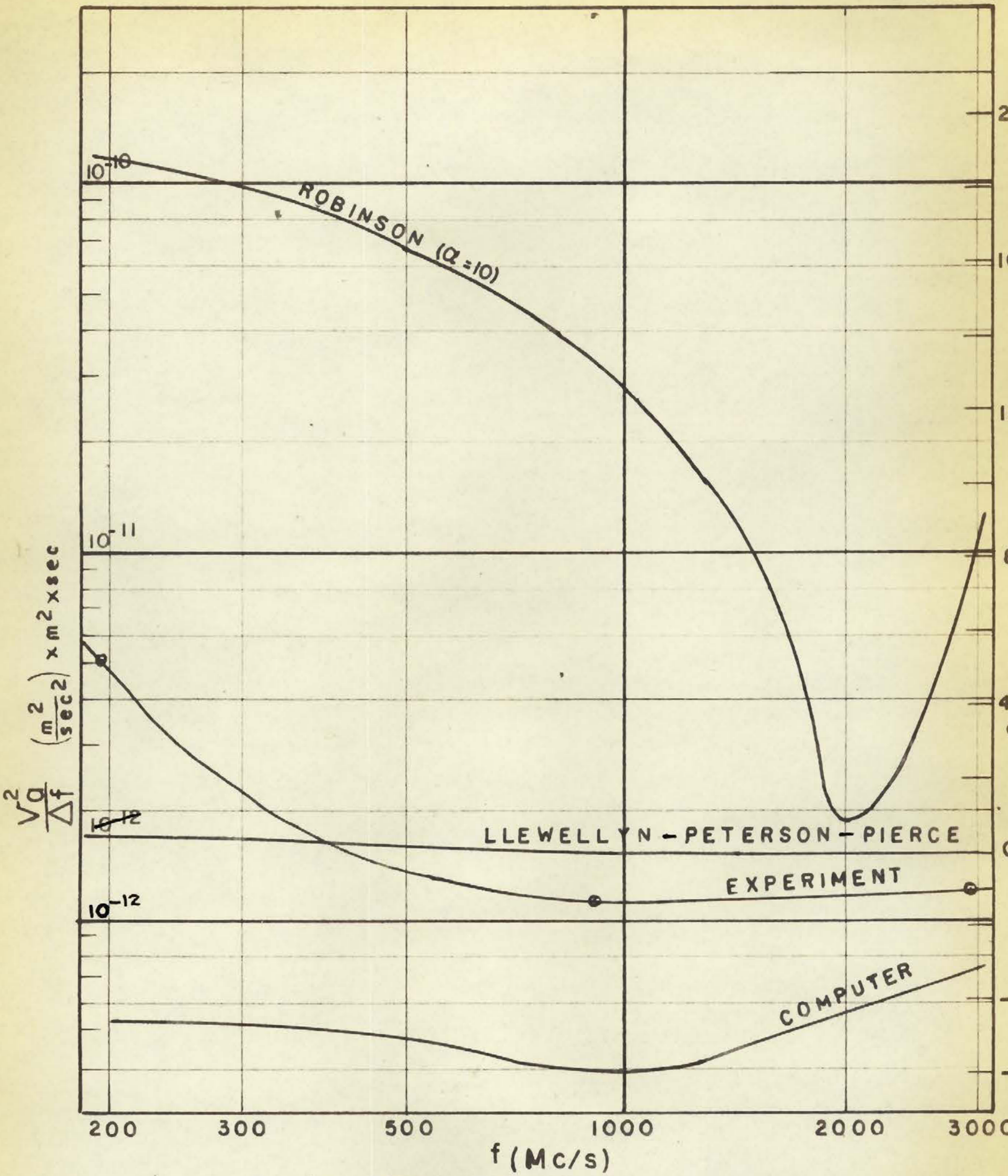
(E) The Total Current Correction to the L-L-P Theory

A method used by Pierce to account for the total gun current in the Llewellyn-Peterson equations is extended in Appendix III to the case of input current as well as velocity. The factors modifying the L-P coefficients are plotted against transit angle in Fig. (27). To show the effects on the anode noise quantities, the numerical corrections were applied to the 900 V



ANODE CURRENT NOISE MODIFIED BY TIEN-MOSHMAN SMOOTHING

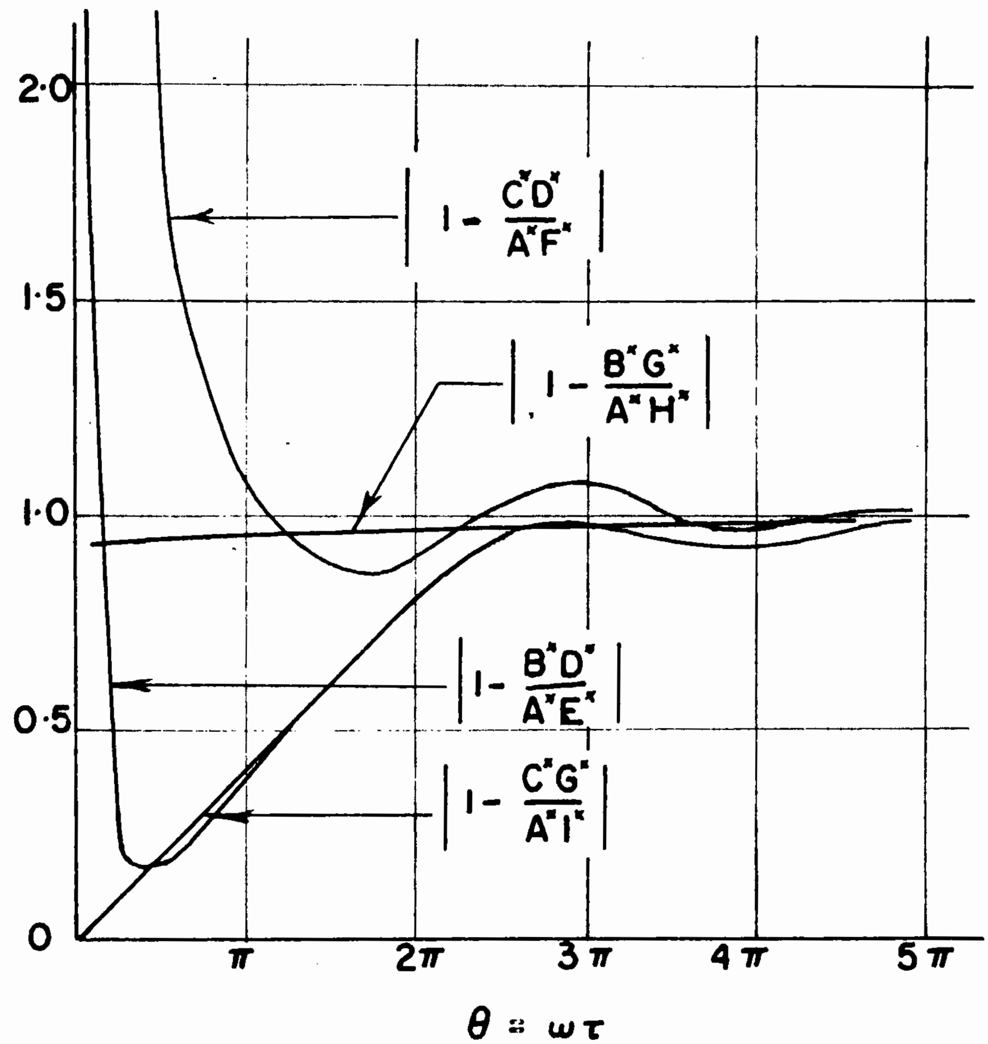
Figure 25.



ANODE VELOCITY NOISE
MODIFIED BY TIEN MOSHMAN SMOOTHING

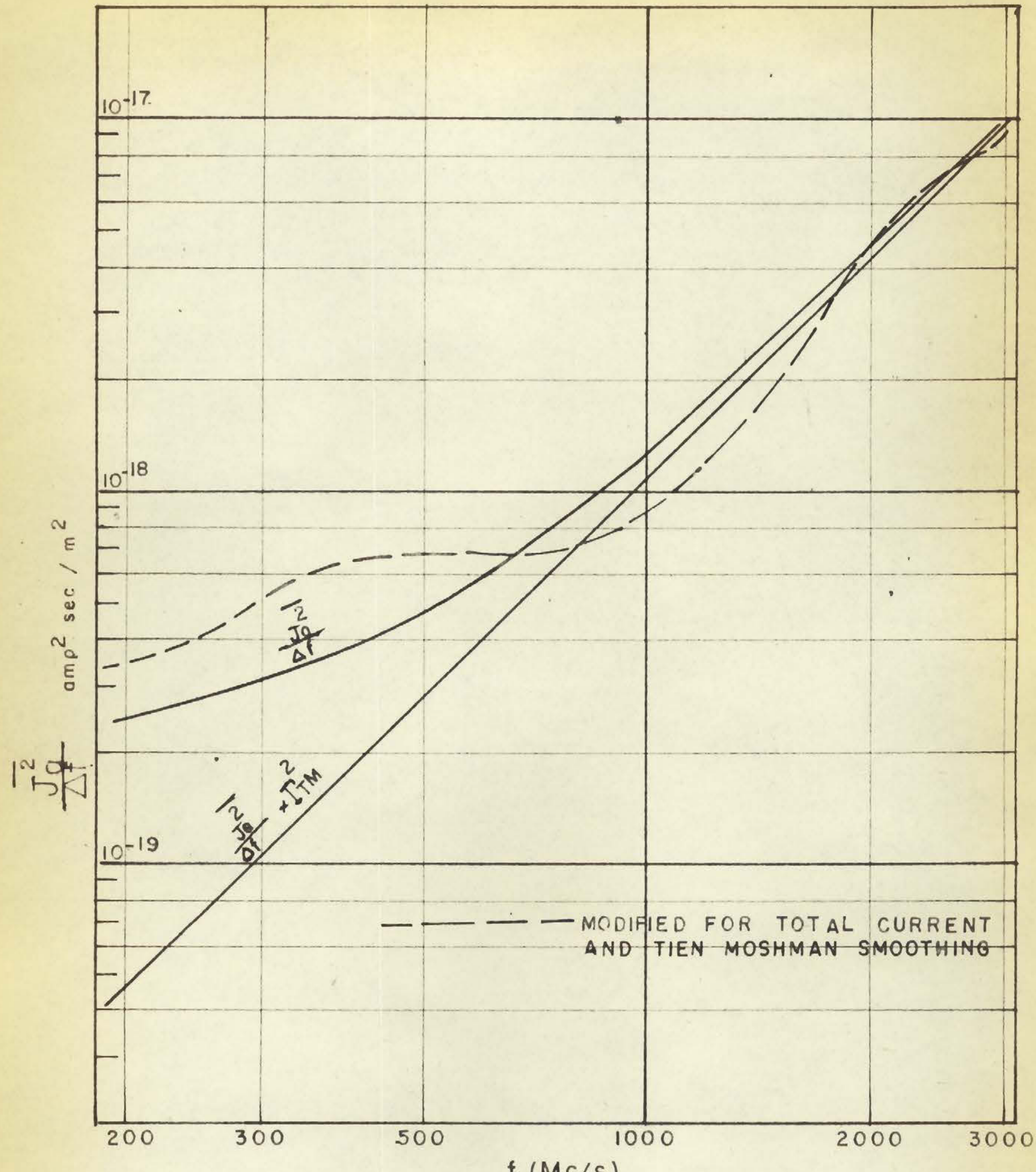
Figure 26.

case. The correspondence between transit angle and frequency was established through equation A3.13 and the necessary modifications made to the results of part A of this section. The effect on the predicted value of current fluctuations is illustrated in Figure (28) along with the correction due to Tien-Moshman smoothing. There is a tendency for the two modification effects to cancel, the final corrected curve falling fairly near the original uncorrected one. The behavior of the velocity fluctuations through the same two modifications is shown in Fig. (29). The effect when both corrections are applied is a serious one. The values at the low frequency end are reduced by a factor of about 30, while those at the high frequency end are changed only slightly.



2·0
1·5
1·0
0·5
0
$\left| 1 - \frac{C^*D^*}{A^*F^*} \right|$
$\left| 1 - \frac{B^*G^*}{A^*H^*} \right|$
$\left| 1 - \frac{B^*D^*}{A^*E^*} \right|$
$\left| 1 - \frac{C^*G^*}{A^*I^*} \right|$
π
2π
3π
4π
5π
$\theta = \omega\tau$

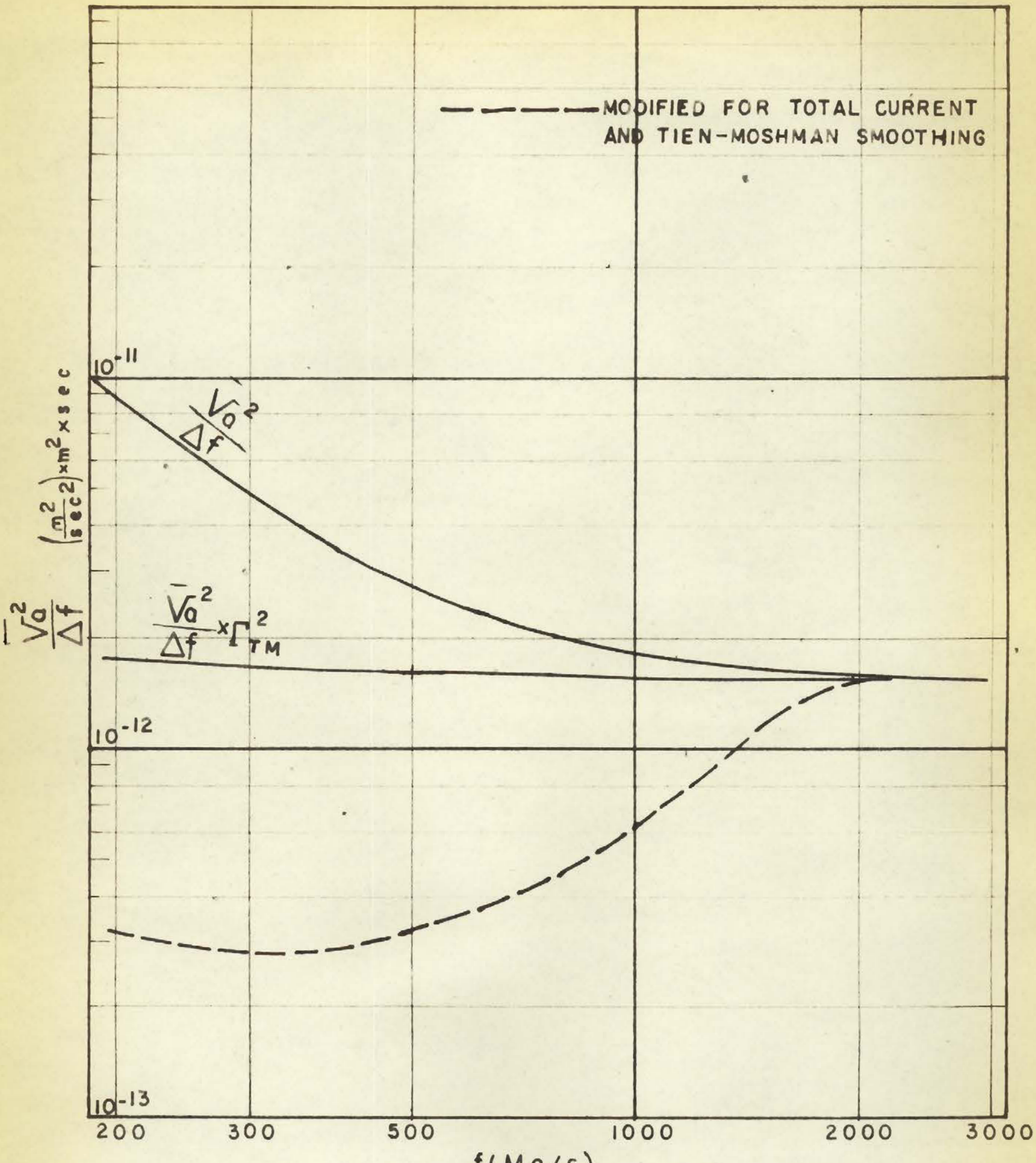
Figure 27.



ANODE CURRENT NOISE V_0 = 900V

FROM LLEWELLYN - PETERSON - PIERCE

Figure 28.



ANODE VELOCITY NOISE $V_0 = 900V$
FROM LLEWELLYN PETERSON PIERCE

Figure 29.

5-IV. Discussion of Results

(A) Reliability of Measurements

The method described in Sec. (4-II) which was used to obtain noise measurements when the gun was fixed close to the cavity is capable of good precision because of the averaging done by the recorder. Repetition of measurements showed a reproducibility of values of about 0.2 db. This is thought to be limited by the reading accuracy from the slow speed recorder charts and the short term gain stability of the amplifying system. Points obtained in two separate measurements were used, for example, in deciding the curve of Fig. (14). No significant reduction of the precision was found for the measurements done with distance except when recorder deflections were less than 0.15 of full scale. This condition was avoided whenever possible by appropriate amplifier gain changes.

Much more serious discrepancies occurred over long periods of time. Comparison of measurements under the same gun conditions and frequencies at widely separated times gave values of space charge smoothing which differed by as much as 1.2 db. Such variations could have been due to changes in vacuum or cathode conditions. The current obtained at various times for the same gun voltage was found to vary by as much as 10%. Both slight variations in gun configuration or vacuum conditions and the voltage and current measurement accuracy would contribute to this variation. The accuracy of the beam voltage and current measurements is estimated to be about 5%.

In obtaining the anode noise quantities use was made of the positions of the space charge pattern maxima and minima as well as the power levels. Spreads in the value of space charge wavelength obtained from a single pattern was usually less than 10%, although some anomalies were noted.

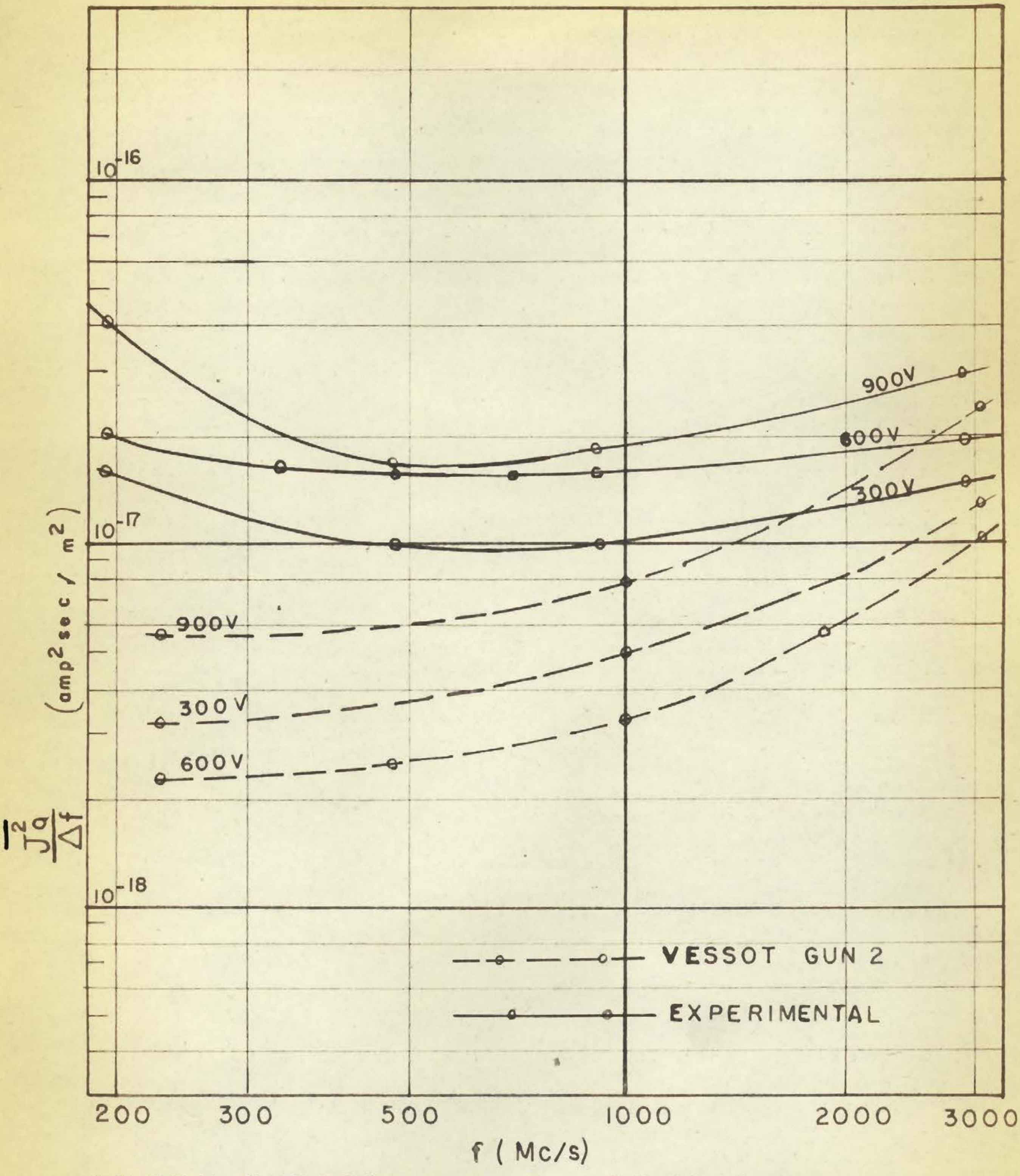
Considering the averages used, an accuracy of 5% is estimated for a representative pattern. The values obtained from patterns with few extrema would be less accurate.

On the basis of the above considerations, the overall accuracy of the resulting values is estimated to be ± 0.6 db. Relative values among the points are in many cases more accurate, being done over a single short interval of time. Differences of less than 0.8 db should not be considered significant.

(B) Comparison of Predictions with Measurements

The experimental noise values for the gun anode, obtained from the measured noise patterns and the computations of Sec. 5-I, are most easily compared with theoretical predictions by the plotted curves of Figs. (20) to (24). In this discussion, the quadratic contents of current and velocity actually considered throughout will be called simply "current" and "velocity".

The experimental current values are collected for the three basic voltages in Fig. (18). The curves exhibit a shallow minimum in the neighborhood of 500-600 Mc/s and the values increase slightly with increasing voltage. The increase is due to the fact that the input current is proportional to beam current density which rises with voltage. The current noise for the three voltages is shown in Fig. (30), along with similar current curves obtained experimentally by Vessot (24). The gun used by Vessot was of the same design as that used by the author except that an oxide cathode was used in place of the tantalum emitter. Vessot's measurements were done with sealed off tubes at very good vacuum pressure (2-5 x 10^{-9} mm Hg), and no magnetic confining field was employed. The oxide cathode temperature was 1160°K, almost exactly half that of the tantalum emitter. According to the analogue



ANODE CURRENT NOISE, COMPARISON OF EXPERIMENTAL RESULTS WITH R. VESSOT

Figure 30.

computer noise theory discussed in this thesis, current is proportional to cathode temperature, which would account for a factor two between the results given by the two cathodes. The difference in the dependence of current on frequency is significant. The measurements done by Vessot indicate a considerable drop toward lower frequencies rather than the shallow minimum found by the author. No experiments have yet been done to determine the reason for the difference. The main differences in experimental conditions between the two measurements are the pressure, the magnetic confining field, the cathode temperature and the cathode surface texture.

The experimental velocity results are shown in Fig. (19). The values are highest for the lowest voltage, attributable to the fact that the input velocity is inversely proportional to current density. The curves show a decrease with increasing frequency, with a tendency toward the formation of a broad minimum at high voltages. The measurements done at 480 Mc/s, with the exception of the 600 V case, were done with an earlier gun than that used for the rest of the measurements. Small differences were known to exist between the d.c. parameters of the two guns, but no anomalies in the current results were evident. The velocity values obtained from these measurements however, were found to show considerable deviation from those obtained by later measurement at 600 V with the second gun. In consequence, they were omitted from the plotted curves of Fig. (19) and the 600 V value used as a guide to the form of the curves in this frequency region. The current results from the first gun were retained because of reasonable agreement with the 600 V results obtained from the second gun.

The current values predicted by the Llewellyn-Peterson-Pierce (13) theory fall considerably below the experimental values and show a sharp fall with decreasing frequency. The disagreement at the lowest frequency considered is almost two decades. The results are plotted in Figs. (20), (21) and (22) for the three separate voltages along with the theory results and the experimental values for the same voltages.

The velocity predictions of the L-L-P theory are shown in Figs. (23) and (24). The agreement with the experimental values is seen to be fairly good, the curves being of roughly the same form but a factor of approximately two high. Both the velocity and the current predicted by the L-L-P theory are found to arise from initial velocity in the high frequency region of the range plotted, and from initial current in the low frequency region.

The currents values predicted by Robinson are shown in the Figs. (20) (21) and (22). There is no variation with frequency, and the values exceed the experimental ones by a factor of about 3-4 times. Robinson's predictions of velocity, shown in Figs. (23) and (24) fall extremely far above the experimental values. The values decrease sharply with increasing frequency. It should be noted that the predictions are strongly dependent on the value chosen for α, since the choice of $\alpha = 0$ would yield the Llewellyn-Peterson-Pierce results. The current and velocity are both found to arise entirely from input current.

As can be seen in Fig. (18), the values of current predicted by the analogue computer solution show a rough correspondence in shape to the experimental values. They are a factor of about 3 low at the highest frequency considered, but cross the experimental curves in the region below 500 Mc/s. The velocity predictions for the 900 V case, which are the only accurate ones available, show about the same correspondence with the experimental values as did the current. The curves cross near 1000 Mc/s, and at the lowest frequency, the values are a factor 3 higher than the experimental ones. In both the current and velocity cases, the major contributions are from input current at the lower frequencies and from input velocity at the higher frequencies.

The application of the Tien-Moshman smoothing factor to the input current quantity leads to some interesting modifications of the theories. The results for current are given in Fig. (25) and for velocity in Fig. (26). The lower frequency values have in every case been greatly reduced because of the large smoothing factor. The shape of the experimental and computer curves now agree fairly well, but the computer solution is lower by about 3 times at the high frequency and 10 times at the low for both current and velocity. The unusual shape of the Robinson prediction is due to the fact that the current and velocity were the result of input current alone with the input velocity contribution being very much smaller.

The Llewellyn-Peterson-Pierce values are entirely the result of input velocity after the modification. Agreement of the velocity case is still the best of all the theories, but the current values deviate even farther than before the modification from the experimental curve.

On the basis of the experimental measurements carried out in this research, if both current and velocity values are considered, the analogue computer solution appears to give the most reasonable predictions. Even in this case, however, the discrepancies between theory and experiment are large. The main factors which could account for the disagreement are thought by the author to be the following:

(1) Poor vacuum pressure which could, by causing electron-air molecule collisions, increase the noise content of the beam on which measurements were done.

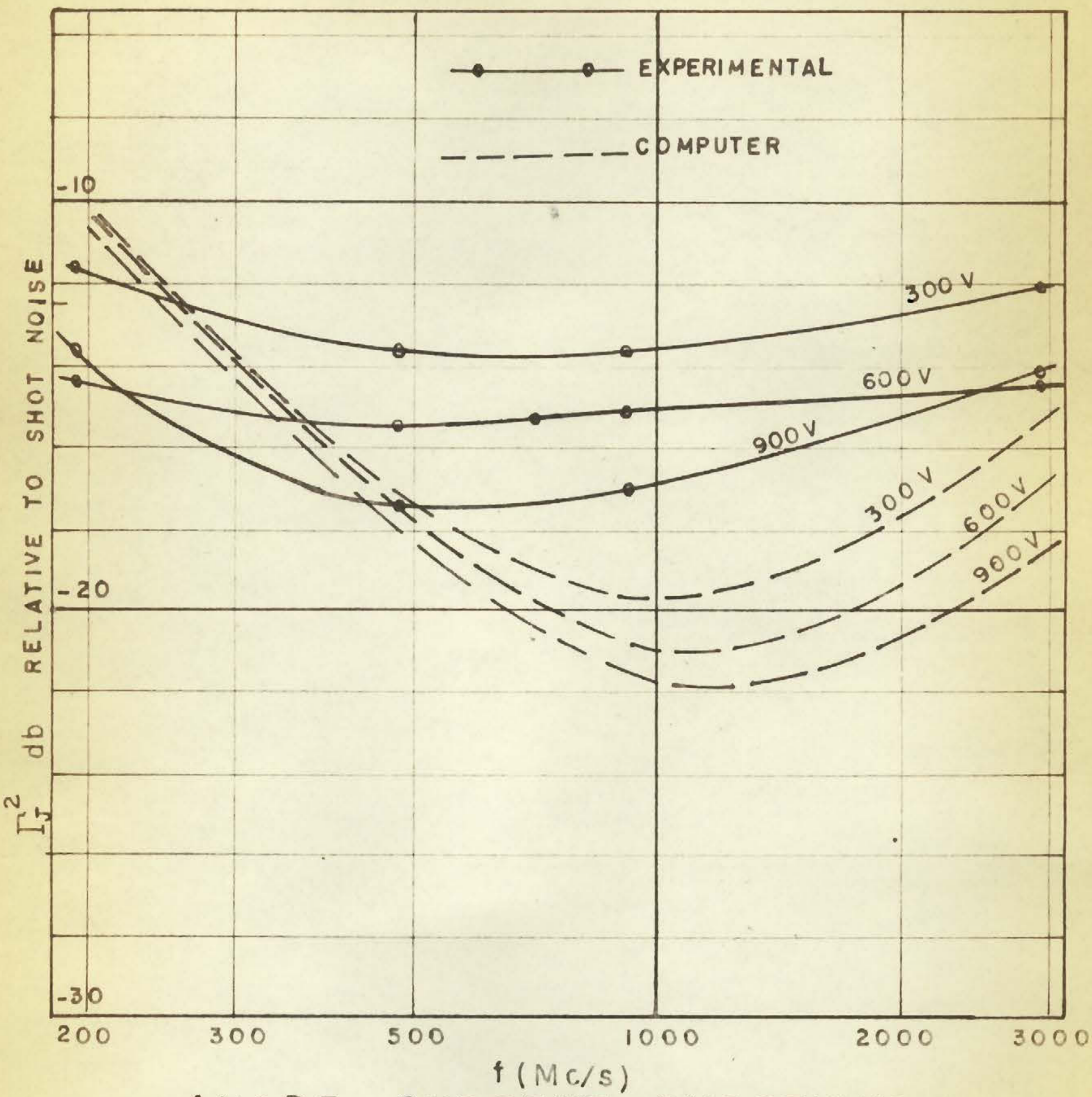
(2) The magnetic confining field, which could affect the behavior of the space

charge near the cathode, resulting in different smoothing factors than those predicted by Tien and Moshman.

(3) The assumptions made in the theoretical solution that

(a) the total current through the gun was zero.

(b) the velocity spreads at any cross section was small compared to the d.c. velocity of the electrons.



ANODE CURRENT SMOOTHING

Figure 31.

6. CONCLUSIONS

The mechanical and electrical apparatus necessary for the measurement of current noise along an electron beam was constructed. The emission behavior of oxide-coated cathodes, impregnated oxide (L-type) cathodes and indirectly heated metal cathodes in the system was examined. The oxide cathodes were found to be inoperable because of the relatively poor vacuum pressure (2 to 4 x 10^{-6} mm Hg) and because of the presence of sealing grease on the sliding vacuum seal. A bombardment heated tantalum cathode gave stable emission, however, and a parallel flow electron gun employing such a cathode was constructed.

Measurements were done of the current noise variation with distance along the beam over a 36 cm length of drift. The current noise content at four frequencies in the range 200 Mc/s to 3000 Mc/s was measured for beam voltages of 100 to 1000 volts. These are thought to be the first measurements of their type reported which cover a range of frequencies. Similar measurements reported by Rigrod (16) were concerned with noise amplification from converging electron guns. No information concerning noise transformations within a basic gun is obtainable from his results. The lowest values measured by Rigrod were in the neighborhood of the shot noise level.

Current and velocity noise at the gun anode were calculated from the current noise patterns obtained in the drift space. Only space charge limited gun conditions were analysed, although measurements were done under other space change conditions as well.

A comparison of the experimental results was made with anode current noise measurements done by Vessot (24). The electron gun used by Vessot was of the same design as that used by the author, but the experimental conditions differed widely for the two cases. Vessot's results are lower by almost tenfold in the frequency region near 200 Mc/s, but are in approximate agreement in the 3000 Mc/s region. The discrepancies are thought to be due to differences in vacuum pressure, cathode temperature, magnetic confining field and texture of the emitting surface between the two systems.

The noise values predicted by existing theories were calculated for the experimental parameters used. An examination of the assumptions used in the development of the theories was also made. Comparison of the predictions with the experimental results served to indicate their validity. The theory of Llewellyn-Peterson (9) and Pierce (13) was found to give values of current noise which were in reasonable agreement with the experimental results near 3000 Mc/s but which decreased much too rapidly with decreasing frequency, giving values nearly one hundred times too low near 200 Mc/s. The predicted velocity noise proved to be in fair agreement with the experimental values over the whole frequency range, being not more than a factor two too high. The predictions of the Robinson (17) theory for current noise gave values independent of frequency, a factor three to five times higher than the experimental ones. The velocity noise according to Robinson's theory should vary as the inverse square of the measurement frequency. The calculations showed a value ten times the experimental even at 3000 Mc/s with the discrepancy increasing rapidly toward lower frequencies. The correction to the input current noise computed by Tien and Moshman (22) was applied to the calculations for 900 V beam voltage. It was found to result in a substantial decrease of the predicted values particularly at the lower frequencies. However, no marked improvement in the agreement between the predictions and the measurements occurred. The largest modifications appeared in the predicted values of the Robinson theory. The application of a correction suggested by Pierce (13) to account for total current in the gun was applied to the Llewellyn-Peterson-Pierce theory for the 900 V cases. The resulting modification of the current noise tends to nullify that introduced by the Tien-Moshman correction. The effect of both corrections together on the velocity noise prediction, however, was a large reduction in the frequency range 200 Mc/s to 1500 Mc/s. This caused a wide disagreement with the experimental values to appear over the same range.

A method of predicting noise was developed based on the solution of an electronic equation (19) with modifications to account for the finite diameter of the beam. The solutions were achieved by the use of an electrical analogue computer for the same ranges of the variables as those considered above.

The resulting current and velocity predictions show a reasonable agreement with the experimental values. Both are somewhat low in the 3000 Mc/s region and somewhat high at the low frequency limit of the range. The Tien-Moshman smoothing factor was applied, as before, to the 900 V cases. The modified predictions for both current and velocity noise are a factor of about ten too low at the low frequency limit of the range considered, and two to three times too low at the high frequency end. The correspondence, on the basis of current and velocity predictions combined, is somewhat better than that of the other theories described.

APPENDIX I

Analogue Computer Solution of the Electronic Equation

A) Obtaining Parallel Equations

The most convenient form of the electronic equation for the particular purpose of analogue computer solution is that due to R.G. Hutter (4). As developed by Hutter, the equation describes the transformation of current and velocity modulations along an electron beam of varying d.c. velocity. The following simplifying assumptions were made:

1) Electrons move in the z direction only.
2) The beam is of infinite lateral extent.
3) Modulation quantities are in every case much smaller than the corresponding d.c. quantities (i.e. small-signal theory only).
4) d.c. velocity function of z only.

Hutter employs the variable Y defined by

$$J = \frac{Y}{u_o} e^{j\omega\tau} \qquad (A1.1)$$

Where J is the current modulation density and τ is the electron transit time $\tau = \int_0^z \frac{dz}{u_o}$. He arrives at the relations:

$$\frac{\partial^2 Y}{\partial \tau^2} - \frac{1}{u_o}\left[\frac{d^2 u_o}{d\tau^2} - \frac{e\,J_o}{m\varepsilon_o}\right] Y = \frac{e\,J_o}{m\varepsilon_o} I_\omega e^{j\omega\tau} \qquad (A1.2)$$

$$v_\omega = -\frac{1}{j\omega J_o}\left[\frac{\partial Y}{\partial \tau} - \frac{Y}{u_o}\frac{du_o}{d\tau}\right] e^{-j\omega\tau} \qquad (A1.3)$$

J_o = d.c. current density $\qquad I_\omega$ = total a.c. current.

As is shown (Appendix (III)) the effect of I_ω is negligibly small provided the transit angle exceeds approximately 2π radians. In spite of the fact that the above condition is not always experimentally satisfied, I_ω has been set to zero because retaining it would seriously complicate the computer solution. The possible error resulting at low frequencies should be kept in mind.

The development of a similar electronic equation for a finite beam starting with drift space relations has shown that it should be written: (See appendix (II))

$$\frac{\partial^2 Y}{\partial \tau^2} - K \left[\frac{1 - p^2}{u_o}\right] Y = p^2 K I_\omega e^{j\omega\tau} \qquad (A1.4)$$

in which $K = \frac{eJ_o}{m\varepsilon_o}$.

The term $\frac{d^2u_o}{d\tau^2}$ was evaluated from a velocity relation due to Llewellyn and Peterson (9).

$$u_o = 1/2 \frac{e\ J_o}{m\varepsilon_o} \tau^2 + a_a\tau + u_a \qquad (2.40)$$

$$\frac{d^2u_o}{d\tau^2} = \frac{e\ J_o}{m\varepsilon_o} = K \qquad (A1.5)$$

Thus, when the right hand side of equation (A1.4) is equated to zero we obtain:

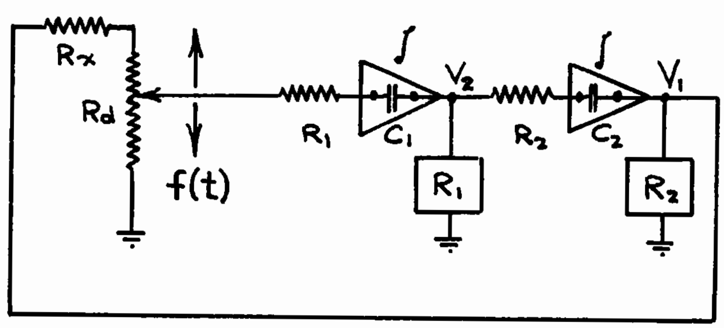
$$\frac{\partial^2 Y}{\partial \tau^2} - K \left(\frac{1 - p^2}{u_o}\right) Y = 0 \qquad (A1.6)$$

It is the solution of equation (A1.6) in the gun region, where both p^2 and u_o vary with τ, which is obtained from the computer.

A formal integration of equation (A1.6) leads to the integral form:

$$Y = K \iint f(\tau) d\tau^2 + \frac{\partial Y}{\partial \tau}\bigg|_{\tau=0} \tau + Y\bigg|_{\tau=0} \qquad (A1.7)$$

The computer connections required in the solution are shown in the sketch below:



The circuit as shown actually solves the equation

$$\frac{d^2V_1}{d\,t^2} - \left(\frac{R_d}{R_d + R_x}\right)\frac{1}{R_1C_1R_2C_2}\,f(t)\,V_1 = 0 \qquad (A1.8)$$

as can be seen by proceeding around the loop in a counter clockwise direction starting at V_1. Considering the second integrator alone there is also the relation

$$V_2 = -\,R_2C_2\,\frac{dV_1}{dt} \qquad (A1.9)$$

Equation (A1.8) may be written in the integral form:

$$V_1 = \left(\frac{R_d}{R_d + R_x}\right)\frac{1}{R_1C_1R_2C_2}\iint f(t)V_1\,dt^2 + \left.\frac{dV_1}{dt}\right|_{t=0} t + \left.V_1\right|_{t=0} \qquad (A1.10)$$

The maximum value of $f(t)$ can be made unity by proper wiring of a tapped potentiometer.

In order to compare equations (A1.7) and (A1.10) it is convenient to carry out a time normalization so that the independent variable in each equation has the range zero to unity. Let

$$T = \alpha t = \rho\tau \qquad (A1.11)$$

such that at the anode

$$T_a = \alpha t_a = \rho\tau_a = 1 \qquad (A1.11a)$$

Equation (A1.11a) indicates how the constants α and ρ may be computed. From equation (A1.11) it is seen that

$$dT^2 = \alpha^2\,dt^2 = \rho^2 d\tau^2 \qquad (A1.12)$$

A normalization of the function $f(\tau)$ of equation (A1.7) is also convenient since $f(t)$ has a maximum value of unity. Let

$$F(\tau) = F(T) = \sigma f(\tau) \text{ where } 0 \leq F(T) \leq 1 \qquad (A1.13)$$

from which it is seen that $\sigma = \dfrac{1}{f(\tau)\,\text{max}}$

The normalized forms of equations (A1.7) and (A1.10) may therefore be written:

$$Y = \frac{K}{\sigma \rho^2} \iint F(T)dT^2 + \frac{\partial Y}{\partial T}\bigg|_{T=0} T + Y\bigg|_{T=0} \qquad (A1.14)$$

the final form of the modified electronic equation, and

$$V_1 = \frac{\left(\frac{R_d}{R_d + R_x}\right)}{\alpha^2 R_1 C_1 R_2 C_2} \iint f(T)\ V_1 dT^2 + \frac{\partial V_1}{\partial T}\bigg|_{T=0} T + V_1\bigg|_{T=0} \qquad (A1.15)$$

the final form of the computer equation.

Equations (A1.14) and (A1.15) have exactly the same form provided that the function f(T) set on the computer potentiometer is made identical to the normalized $(\frac{1 - p^2}{u_o})$ function F(T). More will be said in the following section about the computation of F(T). To make the two equations identical it remains only to equate the two first term coefficients. This is most conveniently done by choosing the appropriate value of R_x. Solving the equality for R_x gives:

$$R_x = \left[\frac{\sigma \rho^2}{K\ \alpha^2 R_1 C_1 R_2 C_2} - 1\right] R_d \qquad (A1.16)$$

It is seen that Y and V_1 correspond exactly under the conditions established above so that, defining a proportionality constant β,

$$Y = \beta V_1 \qquad (A1.17)$$

B). <u>Evaluation of the Function F(T) = σ f(τ)</u>

To obtain the quantity f(τ) as a function of (τ) it is advisable first to establish the more basic relation between u_o and $(\frac{1 - p^2}{u_o})$ which will depend only on the signal frequency ω and the beam radius b. By establishing a correspondence between u_o and τ for our particular gun geometry and voltage it is then possible to express $(\frac{1 - p^2}{u_o})$ as a function of τ .

The problem of what to assume in the calculation of u_o is a serious one. If an initial u_o of zero were assumed, difficulty would be encountered with input conditions since in the equations $J = \frac{Y}{u_o}$. Again, the electronic equation is strictly applicable only to a beam in which the electrons at any cross section possess a single d.c. velocity. This condition is certainly not met in the neighborhood of the space charge potential minimum, and yet only there do we have reasonable knowledge of the input conditions. Rather than make a separate attempt to find other suitable input conditions it was decided to take the potential minimum plane as the initial plane and assume that the electronic equation still holds, at least approximately, throughout.

The root mean square thermal electron velocity was taken as the initial d.c. velocity:

$$u_{oi} = \sqrt{\overline{v^2}} = \sqrt{\frac{2kT_c}{m}} \qquad \text{(A1.18)}$$

The correspondence between u_o and τ was obtained from equation (2.40) with $a_a = 0$:

$$u_o = \frac{1}{2}\frac{e\ J_o}{m\epsilon_o}\tau^2 + u_{oi} \qquad \text{(A1.5a)}$$

The procedure for evaluating F(T) is then as follows:

1) For selected values of u_o over the range required for the highest anode voltage, reduction factor graphs are used to obtain corresponding values of $\left(\frac{1 - p^2}{u_o}\right)$.

2) For the same values of u_o, using $u_{oi} = \sqrt{\frac{2kT_c}{m}}$ and appropriate current density J_o corresponding values of τ are obtained.

3) Both $\left(\frac{1 - p^2}{u_o}\right)$ and τ are normalized to give F(T) vs T and the constants σ and ρ.

C). Solutions for Other Voltages

To obtain solutions to the equation from the computer for other anode voltages than the one for which F(T) vs T was evaluated, consider the following argument.

The function $\left(\frac{1-p^2}{u_o}\right)$ actually depends only on u_o if beam radius and frequency are held constant. A plot of the function for the highest required anode voltage V_{o1} then contains all the necessary information for lower anode voltages as well. To determine what portion T_2, of the plotted curve of F(T) vs T must be used for a lower voltage V_{o2}, note that (Al.5a) shows, for $u_o \gg u_{oi}$, the transit time τ_2 to the plane within the gun at which the voltage is V_{o2} is given by

$$\tau_2 = \sqrt{\frac{\sqrt{2\eta}\,\varepsilon_o}{\frac{1}{2}\eta J_o}}\; V_{o2}^{\frac{1}{4}} = C\, V_{o2}^{\frac{1}{4}} \qquad \text{(Al.5b)}$$

Thus

$$\frac{\tau_2}{\tau_1} = \left(\frac{V_{o2}}{V_{o1}}\right)^{\frac{1}{4}} \qquad \text{(Al.19)}$$

But from equation (Al.11)

$$\frac{T_2}{T_1} = \frac{\rho\tau_2}{\rho\tau_1} = \frac{\tau_2}{\tau_1} = \left(\frac{V_{o2}}{V_{o1}}\right)^{\frac{1}{4}}$$

And since $T_1 = 1$ by the normalization:

$$T_2 = \left(\frac{V_{o2}}{V_{o1}}\right)^{\frac{1}{4}} \qquad \text{(Al.20)}$$

For different anode voltages applied to the same gun, the d.c. current density is to a good approximation proportional to $V_o^{3/2}$. From equation (Al.5b) we then obtain

$$\frac{\tau_{a2}}{\tau_{a1}} = \left(\frac{V_{o1}}{V_{o2}}\right)^{1/2}$$

Again using equation (Al.11)

$$\frac{\rho_2}{\rho_1} = \left(\frac{V_{o2}}{V_{o1}}\right)^{1/2} \qquad \text{(Al.21)}$$

Since in the actual computer operation, the motor driving the potentiometer rotates at constant speed, if the same wiring is to be used for V_{o2} as for V_{o1} then equation (A1.20) requires

$$\frac{t_2}{t_1} = \frac{T_2}{T_1} = \left(\frac{V_{o2}}{V_{o1}}\right)^{1/4}$$

which from equation (A1.11) gives

$$\frac{\alpha_2}{\alpha_1} = \left(\frac{V_{o1}}{V_{o2}}\right)^{1/4} \qquad \text{(A1.22)}$$

The new value of R_x for the voltage V_{o2} can be obtained from equation (A1.16):

$$R_{x2} = \left[\frac{\sigma_2 \rho_2^2}{K_2 \alpha_2^2 R_1 C_1 R_2 C_2} - 1\right] R_d$$

Using equations (A1.21) and (A1.22) and noting from equation (A1.5) that $K \alpha J_o \alpha V_o^{3/2}$ so $\frac{K_2}{K_1} = \left(\frac{V_{o2}}{V_{o1}}\right)^{3/2}$, then

$$R_{x2} = \left[\frac{\sigma_2 \rho_1^2 \left(\frac{V_{o2}}{V_{o1}}\right)}{K_1 \left(\frac{V_{o2}}{V_{o1}}\right)^{3/2} \alpha_1^2 \left(\frac{V_{o1}}{V_{o2}}\right)^{1/2} R_1 C_1 R_2 C_2} - 1\right] R_d$$

$$R_{x2} = \left[\frac{\sigma_2 \rho_1^2}{K_1 \alpha_1^2 R_1 C_1 R_2 C_2} - 1\right] R_d \qquad \text{(A1.23)}$$

Comparison of equations (A1.16) and (A1.23) shows that $R_{x2} = R_{x1}$ as long as $\sigma_2 = \sigma_1$. Now equation (A1.13) requires $f(\tau)_{max}\big|_2 = f(\tau)_{max}\big|_1$ for σ_2 to equal σ_1, so that as long as the peak of the F(T) curve is included, $R_{x2} = R_{x1}$.

The important conclusion of this section is that as long as the F(T) = 1 point is included in the run, separate runs need not be done for different anode voltages. Final conditions for any voltage below V_{o1} may be obtained directly from the original computer solution simply by reading at the position indicated by equation (A1.20), as long as T does not fall below that for which F(T) = 1.

D). <u>The Initial Conditions</u>

The electronic equation is only meant to apply to transformations of uniform single frequency modulations appearing at an initial plane. For noise work, Haus (3) has justified the use of quadratic content of noise current and velocity as an equivalent modulation in a drift space, and since the equation being solved is derivable from drift space relations the same technique has been used.

Work by Tien and Moshman (22) based on statistical methods has indicated the following.

1) No detectable correlation exists between current and velocity fluctuations at the potential minimum.

2) Velocity fluctuations are unaffected by the potential minimum.

3) The effect of the minimum on current fluctuations can be described by a factor Γ_{TM}^2 which is a function of the current density and available current as well as the frequency.

The current density conditions assumed by Tien and Moshman correspond to an anode voltage of about 900 volts for the gun used here. Thus the factor Γ_{TM}^2 could be applied with reasonable accuracy to the 900 volts solution but no similar factor is available for the lower voltages.

On the basis of work due to Schottky (18) and Rack (15) the initial quantities then take the form

$$\overline{J_i^2} = \Gamma_{TM}^2 \, 2eJ_o\Delta f = \Gamma_{TM}^2 \, \overline{J_{sn}^2} \qquad (A1.24)$$

$$\overline{v_i^2} = \frac{(4-\pi)\eta kT_c}{J_o}\Delta f = \overline{v_R^2} \qquad (A1.25)$$

It should be noted that the square roots of the quantities of equation (A1.24) and (A1.25) actually do not have physical significance. The significant quantities are quadratic contents per unit area.

As a consequence of the lack of initial correlation, two separate computer solutions may be obtained; one for initial current and the other for initial velocity. The squares of the resulting fluctuations are added to give the total solution.

E). Evaluation of the Anode Fluctuations from the Computer Records

As indicated in equation (A1.24), $\overline{J^2}$ and $\overline{v^2}$ will always represent quadratic fluctuation content for a unit area. To obtain results of greater generality, spectral densities will be considered throughout; that is $\frac{\overline{J^2}}{\Delta f}$ and $\frac{\overline{v^2}}{\Delta f}$ will be evaluated.

The four quantities of interest at the anode are the current fluctuation arising from initial current fluctuation and from initial velocity fluctuation,

$$\frac{\overline{J^2_{aJ}}}{\Delta f} \qquad \text{and} \qquad \frac{\overline{J^2_{av}}}{\Delta f}$$

and the velocity fluctuations from the same two sources:

$$\frac{\overline{v^2_{aJ}}}{\Delta f} \qquad \text{and} \qquad \frac{\overline{v^2_{av}}}{\Delta f}$$

They will be evaluated in turn

1) Current due to current $\frac{\overline{J^2_{aJ}}}{\Delta f}$

By definition $J = \frac{Y}{u_o} e^{j\omega\tau}$ or, since only magnitudes are of interest

$$J_{aJ} = \frac{Y_{aJ}}{u_{oa}} \qquad \text{(the magnitude signs will be omitted)} \qquad (A1.26)$$

Since, from equation (A1.17) $Y = \beta V_1$,

$$J_{aJ} = \frac{V_{1aJ}}{u_{oa}} \beta \qquad (A1.26a)$$

As initial conditions we have

$$\frac{Y_i}{u_{oi}} = J_i = \Gamma_{TM}(2eJ_o\Delta f)^{1/2} = \frac{V_{1iJ}}{u_{oi}} \beta; \quad v_i = 0 \qquad (A1.27)$$

Solving for β,

$$\beta = \Gamma_{TM}(2eJ_o\Delta f)^{1/2} \frac{u_{oi}}{V_{1iJ}} \qquad (A1.28)$$

which, substituted into equation (A1.26), gives

$$J_{aJ} = \Gamma_{TM}(2eJ_o\Delta f)^{1/2} \frac{u_{oi}}{u_{oa}} \frac{V_{1aJ}}{V_{1iJ}}$$

or, squaring

$$\frac{\overline{J^2_{aJ}}}{\Delta f} = 2eJ_o \frac{V_{oi}}{V_{oa}} \left(\frac{V_{1aJ}}{V_{1iJ}}\right)^2 \Gamma_{TM}^2 \qquad (A1.29)$$

It should be noted that in equation (A1.27) the square root of $2eJ_o\Delta f$ was introduced only for the ease of manipulation. It appears in the final relation (A1.29) in its valid form. The squaring could have been done on equation (A1.27) instead.

2) Current due to velocity $\frac{\overline{J^2_{av}}}{\Delta f}$

Parallel to equation (A1.26a) is the relation

$$J_{av} = \frac{V_{1av}}{u_{oa}} \beta \qquad (A1.30)$$

The initial conditions are

$$J_i = 0 \qquad v_i = v_R = \left[\frac{(4-\pi)\eta kT_c}{J_o} \Delta f\right]^{1/2} \qquad (A1.31)$$

In the velocity equation (A1.3) $v_\omega = \frac{1}{\omega J_o}\left[\frac{\partial Y}{\partial \tau} - \frac{Y}{u_o} \frac{d\,u_o}{d\tau}\right]$ (A1.3)

Since only magnitude is of interest. The last term is zero at the initial plane since $\left.\frac{Y}{u_o}\right|_i = J_i = 0$. Thus we have

$$v_i = v_R = \frac{1}{\omega J_o} \left.\frac{\partial Y}{\partial \tau}\right|_{\tau=0} \qquad (A1.32)$$

Solving and using equation (A1.11)

$$\left.\frac{\partial Y}{\partial T}\right|_{T=0} = \frac{\omega J_o}{\rho} v_R \qquad (A1.33)$$

Using equation (A1.9)

$$-\alpha R_2 C_2 \left.\frac{dV_1}{dT}\right|_{T=0} = V_{2iv} \qquad (A1.34)$$

and from the relation $Y = \beta V_1$

$$V_{2iv} = -\frac{\alpha R_2 C_2}{\beta} \left. \frac{\partial Y}{\partial T} \right|_{T=0} \qquad \text{(A1.35)}$$

The resulting expression for β from equations (A1.35) and (A1.33) is:

$$\beta = -\frac{\alpha R_2 C_2}{\rho} \frac{\omega J_o}{V_{2iv}} v_R \qquad \text{(A1.36)}$$

Substituting in equation (A1.30)

$$J_{av} = -\frac{\alpha R_2 C_2 \omega J_o}{\rho u_{oa}} v_R \frac{V_{1av}}{V_{2iv}} \qquad \text{(A1.37)}$$

Squaring

$$\frac{\overline{J_{av}^2}}{\Delta f} = \left[\frac{\alpha R_2 C_2 \omega J_o}{\rho u_{oa}}\right]^2 \left(\frac{(4-\pi)\eta k T_c}{J_o}\right)\left(\frac{V_{1av}}{V_{2iv}}\right)^2$$

3). Velocity due to current, $\frac{\overline{v_{aJ}^2}}{\Delta f}$

The input conditions are

$$\overline{J_i^2} = \Gamma_{TM}^2 \, 2eJ_o\Delta f \qquad v_i = 0 \qquad \text{(A1.38)}$$

giving as in equation (A1.28)

$$\beta = \frac{u_{oi} \; \Gamma_{TM} \, (2eJ_o\Delta f)^{1/2}}{V_{1iJ}} \qquad \text{(A1.28)}$$

Equation (A1.3) for velocity, using $Y = \beta V_1$ and equation (A1.11) becomes

$$v_{aJ} = \frac{\rho\beta}{\omega J_o}\left[\frac{\partial V_{1aJ}}{\partial T} - \frac{1}{\rho u_{oa}} V_{1aJ} \frac{\partial u_{oa}}{\partial \tau}\right] \qquad \text{(A1.39)}$$

Using equation (A1.9) $v_{aJ} = -\frac{\rho\beta}{\omega J_o}\left[\frac{V_{2aJ}}{\alpha R_2 C_2} + \frac{V_{1aJ}}{\rho u_{oa}} \frac{\partial u_{oa}}{\partial \tau}\right]$ (A1.39a)

Since $u_{oa} = \frac{1}{2} K \tau_a^2 + a_i \tau_a + u_{oi}$, $\frac{du_{oa}}{d\tau} = K \tau_a + a_i$ so that for $a_i = 0$ and $u_{oa} \gg u_{oi}$

$$\frac{1}{\rho u_{oa}} \frac{\partial u_{oa}}{\partial \tau} = \frac{2}{\rho \tau_a} = 2 \qquad \text{(A1.40)}$$

Then (A1.39a) becomes

$$v_{aJ} = -\frac{\rho\beta}{\alpha R_2C_2\omega J_o}\left[V_{2aJ} + 2\alpha R_2C_2\, V_{1aJ}\right] \qquad (A1.41)$$

Substituting for β from equation (A1.28)

$$v_{aJ} = -\frac{u_{oi}\ \Gamma_{TM}(2eJ_o\Delta f)^{1/2}\rho}{\alpha\ R_2C_2\omega J_o}\left[\frac{V_{2aJ}}{V_{1iJ}} + 2\alpha R_2C_2\ \frac{V_{1aJ}}{V_{1iJ}}\right] \qquad (A1.42)$$

Squaring:

$$\frac{\overline{v^2_{aJ}}}{\Delta f} = 2eJ_o\left[\frac{u_{oi}}{\omega J_o\alpha R_2C_2}\right]^2\left[\frac{V_{2aJ}}{V_{1iJ}} + 2\alpha R_2C_2\,\frac{V_{1aJ}}{V_{1iJ}}\right]^2\Gamma^2_{TM} \qquad (A1.42)$$

4) Velocity due to velocity, $\frac{\overline{v^2_{av}}}{\Delta f}$

Input conditions

$$v_i = v_R; \qquad J_i = 0 \qquad (A1.43)$$

which leads again to equation (A1.36)

$$\beta = -\frac{\alpha R_2C_2}{\rho}\ \frac{\omega J_o}{V_{2iJ}}\, v_R \qquad (A1.36)$$

At the anode, equation (A1.3) gives

$$v_{av} = \frac{1}{\omega J_o}\left[\rho\ \frac{\partial Y_a}{\partial \tau} - \frac{Y_a}{u_{oa}}\ \frac{\partial u_{oa}}{\partial \tau}\right]$$

$$v_{av} = \frac{\rho\beta}{\omega J_o}\left[\frac{\partial V_{1av}}{\partial \tau} - \frac{V_{1av}}{\rho}\left(\frac{1}{u_{oa}}\ \frac{\partial u_{oa}}{\partial \tau}\right)\right] \qquad (A1.44)$$

which by employing equation (A1.9) and equation (A1.40) becomes

$$v_{av} = -\frac{\rho\beta}{\omega J_o}\left[\frac{V_{2av}}{\alpha R_2C_2} + 2\ V_{1av}\right] \qquad (A1.45)$$

Substituting for β

$$v_{av} = \frac{\rho}{\omega J_o\alpha R_2C_2}\left(-\frac{\alpha R_2C_2\omega J_o}{V_{2iv}}\, v_R\right)\left[V_{2av} + 2\alpha R_2C_2V_{1av}\right]$$

$$v_{av} = -v_R\left[\frac{V_{2av}}{V_{2iv}} + 2\alpha R_2C_2\ \frac{V_{1av}}{V_{2iv}}\right] \qquad (A1.46)$$

and finally, squaring

$$\frac{\overline{v^2_{av}}}{\Delta f} = \frac{(4-\pi)\varkappa kT_c}{J_o}\left[\frac{V_{2av}}{V_{2iv}} + 2\alpha R_2C_2\ \frac{V_{1av}}{V_{2iv}}\right]^2 \qquad (A1.47)$$

It should be noted that the V_2 computer voltages will always be of the opposite sign to the V_1 and so negative numbers will be required for them.

APPENDIX II

Derivation of Finite Beam Electronic Equation from Drift Space Relations

In section II of the theory, a development was given from basic physical phenomena of the widely used drift space equations

$$v(z) = \left\{ v_a \cos\left(\frac{\omega_q z}{u_o}\right) - j\,\frac{\omega_q}{\omega}\,\frac{1}{\rho_o}\,(J_a - I)\sin\left(\frac{\omega_q z}{u_o}\right) \right\} e^{-j\frac{\omega z}{u_o}} \qquad (2.23)$$

$$J(z) = \left\{ -j\frac{\omega}{\omega_q}\,\rho_o\, v_a \sin\left(\frac{\omega_q z}{u_o}\right) + J_a \cos\left(\frac{\omega_q z}{u_o}\right) \right\} e^{-j\frac{\omega z}{u_o}} \qquad (2.24)$$

The equations apply to the transformations of current and velocity modulations along a finite beam in a constant d.c. potential region or "drift space". The equations can be shown to satisfy experimentally measured quantities in an actual drift region (where the total current I_t is zero) to a fair degree of accuracy. They form, therefore, a rather firm starting point for theoretical extension.

Woonton (26) of this laboratory in unpublished work has carried out a development from the equations (2.23) and (2.24) which leads to an electronic equation for a region of varying d.c. velocity. The same lines will be followed in the development of this appendix.

Suppose that one considered an acceleration region to consist of a series of infinitesimally short drift spaces with different d.c. velocities. The discontinuity of velocity where two of the drift spaces join would act as an infinitesimal velocity jump. The theory of current and velocity modulation transfer across a velocity jump is well understood from ballistic theory. It is found that current modulation is unaffected by the jump while the velocity modulations bear an inverse relation to the d.c. velocities.

$$\frac{v_2}{v_1} = \frac{u_{o1}}{u_{o2}} \qquad (A2.1)$$

(See, for example, Field et. al. (29).

It should be pointed out that for a finite beam the theoretical relations are no longer so simple. The modulations, because of matching conditions, have a zero order Bessel function radial variation, the argument being a function of u_o. This leads to serious matching complications across the jump. For this development, since the arguments vary much more slowly than the other quantities involved, the compensating terms will be ignored in the velocity jump relations.

Consider then the situation shown in Fig.(32)

Using a small argument expansion of the sine, cosine and exponential functions, equations (2.23) and (2.24) become:

$$v_2(z + dz) = \frac{u_o}{\left(u_o + \frac{\partial u_o}{\partial z} dz\right)} \left\{ v(z) - j \frac{\omega_q}{\omega} \frac{1}{\rho_o} (J(z) - I_1) \left(\frac{\omega_q dz}{u_o}\right) \right\} \left\{ 1 - j \frac{\omega dz}{u_o} \right\} \quad (A2.2)$$

$$J(z + dz) = \left\{ -j \frac{\omega}{\omega_q} \rho_o \, v(z) \left(\frac{\omega_q dz}{u_o}\right) + J(z) \right\} \left\{ 1 - j \frac{\omega dz}{u_o} \right\} \quad (A2.3)$$

Proceeding first with equation (A2.3) if differentials of order higher than one are neglected there results

$$J(z + dz) - J(z) = -j \frac{\omega}{u_o} \rho_o \, v(z) dz - j \frac{\omega}{u_o} J(z) \, dz = dJ(z)$$

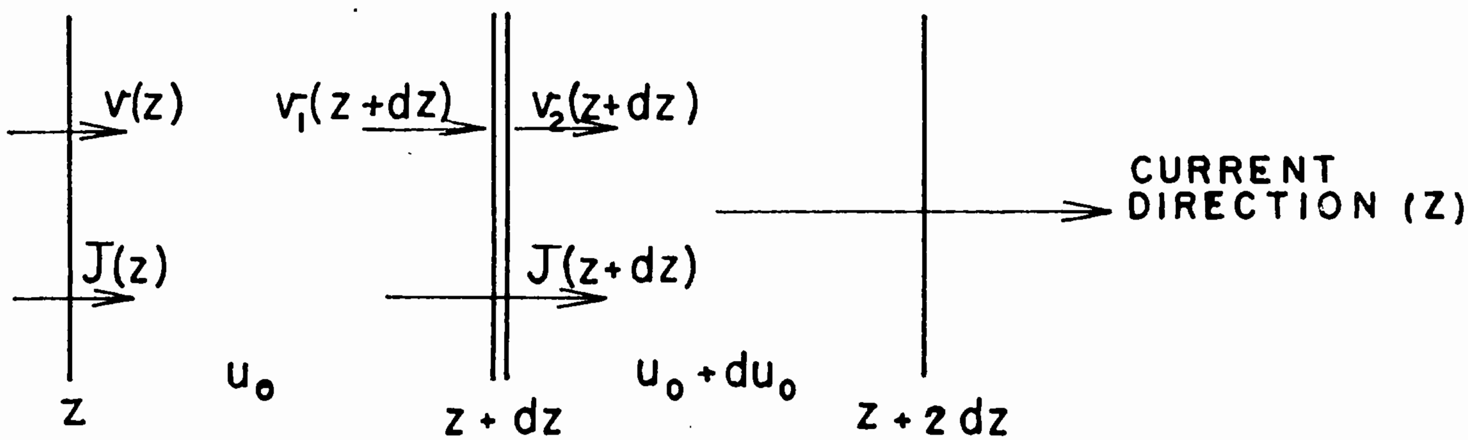
or rearranging

$$\frac{dJ(z)}{dz} = -j \frac{\omega}{u_o} \left[\rho_o \, v(z) + J(z) \right] = \frac{\partial J(z)}{\partial z} \quad (A2.4)$$

If equation (A2.4) is solved for v(z),

$$v(z) = - \frac{u_o}{j\omega \rho_o} \left[\frac{\partial J(z)}{\partial z} + \frac{j \omega}{u_o} J(z) \right] \quad (A2.5)$$

Making the substitutions $J(z) = \frac{Y(z)}{u_o} e^{-j\frac{\omega z}{u_o}}$ and using $J_o = \rho_o u_o$ the equation (A2.5) can be put in the form used by Hutter and quoted in Sec. 2-III as equation (2.33).



INCREMENTAL DRIFT SPACES

Figure 32.

Treating equation (A2.2) in a similar way, and using $(1 + \frac{1}{u_o} \frac{\partial u_o}{\partial z} dz)^{-1}$ $= 1 - \frac{1}{u_o} \frac{\partial u_o}{\partial z} dz$ we obtain

$$\frac{\partial v}{\partial z} = -v(z)\left[\frac{1}{u_o} \frac{\partial u_o}{\partial z} + j\frac{\omega}{u_o}\right] - j\frac{\omega_q{}^2}{\omega J_o}(J(z) - I_1) \qquad \text{(A2.6)}$$

Rearranging equation (A2.5)

$$v(z) = -\frac{u_o}{J_o}\left[J(z) + \frac{u_o}{j\omega} \frac{\partial J(z)}{\partial z}\right] \qquad \text{(A2.5a)}$$

and differentiating with respect to z

$$\frac{\partial v}{\partial z} = -\frac{u_o}{J_o} \frac{\partial J}{\partial z} - \frac{u_o{}^2}{j\omega J_o} \frac{\partial^2 J}{\partial z^2} - \frac{u_o}{j\omega J_o} \frac{\partial u_o}{\partial z} \frac{\partial J}{\partial z} - \frac{1}{J_o} \frac{\partial u_o}{\partial z} J - \frac{u_o}{j\omega J_o} \frac{\partial u_o}{\partial z} \frac{\partial J}{\partial z} \qquad \text{(A2.7)}$$

If equations (A2.5a) and (A2.7) are substituted in equation (A2.6) and the result multiplied through by $j\omega J_o$, the resulting differential equation for J is

$$u_o{}^2 \frac{\partial^2 J}{\partial z^2} + \left[3u_o \frac{\partial u_o}{\partial z} + 2j\omega u_o\right] \frac{\partial J}{\partial z} + \left[2j\omega \frac{\partial u_o}{\partial z} - \omega^2\right] J + \omega_q{}^2(J - I_1) = 0 \qquad \text{(A2.8)}$$

The equation (A2.8) may, again, be put in the form used by Hutter by noting

$$J = \frac{Y}{u_o} e^{-j\omega\tau} \qquad dz = u_o\, d\tau \quad \text{so} \quad \frac{\partial u_o}{\partial z} = \frac{1}{u_o} \frac{\partial u_o}{\partial \tau}$$

Since Y, u_o and $e^{-j\omega\tau}$ are all functions of τ, $\frac{\partial J}{\partial z} = \frac{1}{u_o} \frac{\partial J}{\partial \tau}$ and $\frac{\partial^2 J}{\partial z^2} = \frac{1}{u_o} \frac{\partial}{\partial \tau}\left(\frac{1}{u_o} \frac{\partial J}{\partial \tau}\right)$ while mathematically straightforward, become rather complicated. If the resulting expressions are substituted into (A2.8) and cancellation of terms carried out, one arrives at the equivalent of Hutter's current equation:

$$\frac{\partial^2 Y}{\partial \tau^2} - \frac{1}{u_o}\left[\frac{\partial^2 u_o}{\partial \tau^2} - u_o\, \omega_q{}^2\right] Y = u_o\, \omega_q{}^2\, I_1\, e^{j\omega\tau} \qquad \text{(A2.9)}$$

Since $\omega_q{}^2 = p^2\omega_p^2 = p^2 \frac{e\,\rho_o}{m\,\varepsilon_o}$, $u_o\omega_q{}^2 = p^2 \frac{eJ_o}{m\,\varepsilon_o}$ and equation (A2.9) may be written

$$\frac{\partial^2 Y}{\partial \tau^2} - \frac{1}{u_o}\left[\frac{\partial^2 u_o}{\partial \tau^2} - p^2 \frac{eJ_o}{m\,\varepsilon_o}\right] Y = p^2 \frac{eJ_o}{m\,\varepsilon_o} I_1\, e^{j\omega\tau} \qquad \text{(A2.10)}$$

which is the modified electronic equation desired for the analogue computer work.

Since the initial drift space equations are valid only if the d.c. velocity spread is small at any given cross section of the beam the same restriction is true of equations (A2.10) and (2.33). The assumptions involved in equation (A2.10) are, in fact, exactly those of the infinite beam electronic equation except for the finite beam radius.

APPENDIX III

Total Current Considerations

A development will be given which attempts to describe the effect which total gun current I_1 would have on the Llewellyn-Peterson-Pierce (9)(13) theory. The approach is the same as that used by Pierce (13) on page 150 but includes additional terms.

The complete Llewellyn-Peterson equations, discussed in section 2-III, are

$$V_b - V_a = A^x I_1 + B^x J_a + C^x v_a$$
$$J_b = D^x I_1 + E^x J_a + F^x v_a$$
$$v_b = G^x I_1 + H^x J_a + I^x v_a \qquad (2.47)$$

Recognizing that the equations actually concern a unit area of an infinite diode, it can be shown that A^x is very closely related to the capacitive impedance of the cathode and anode planes. The two are identical when the gun current is zero. Calculation has shown that the deviation from the zero current value for $\Upsilon = 1$ is not large until the transit angle has fallen to less than $\sim \frac{\pi}{4}$ where $A^x \approx 0.3\, z_c$. The voltage $V_b - V_a$ may be considered to be caused by the total current flowing through an impedance external to the beam. For an electron gun the external impedance is supplied by the electrode capacitance. The evaluation of this capacitance requires either an r.f. measurement or solution of an integral of the form

$$C = \int \frac{\varepsilon_o dA}{d} \qquad (A3.1)$$

over the electrode area. Integration has shown that for the gun used in the present research, the capacitance is very nearly that of a pair of electrodes of twice the real area separated by the cathode-anode distance. Thus for every unit area of beam we have effectively an area $2\,\frac{A_e}{A_b}$ forming an external capacitive impedance. In the first of equations (2.47) we then write

$$Z_e \; I_1 = A^x \; I_1 + B^x \; J_a + C^x \; v_a \qquad (A3.2)$$

where

$$Z_e = \frac{1}{j\omega C_e} = \frac{\varepsilon_o d}{2j\omega A_e} \qquad (A3.3)$$

However, as outlined above, $A^x \approx Z_b = \dfrac{\varepsilon_o d}{j\omega A_b}$ (A3.4)

so that

$$\frac{Z_e}{A^x} \approx \frac{A_b}{2A_e} \qquad (A3.5)$$

Since the beam radius is 0.75 mm. and the electrode radius is 10 mm

$$\frac{A_b}{A_e} = \frac{0.56}{200} = 0.0028 \text{ and so } \frac{Z_e}{A^x} = \frac{0.0028}{2}$$

From equation (A3.2)

$$0 = (A^x - Z_e) \; I_1 + B^x J_a + C^x v_a$$

$$= A^x \; (1 - \frac{Z_e}{A^x}) \; I_1 + B^x J_a + C \; v_a$$

$$0 \quad A^x \; I_1 + B^x J_a + C \; v_a \qquad (A3.6)$$

which shows that to a very good approximation, $V_b - V_a$ may be taken as zero. This is by definition taking the gun as short circuited for a.c. effects, since the total current is now entirely decided by conditions internal to the beam.

It should be noted that under particular conditions of d.c. connecting lines to the cathode and anode electrodes, it would be possible to seriously change the impedance Z_e by resonant line effects. This would imply being able to change the output noise of the gun by varying lengths of positions of the supply leads to the gun. Such possibilities will be ignored in the present discussion, since by simple adjustments, the resonance could be removed.

Solving the equation (A3.6) for I_1,

$$I_1 = \frac{B^x}{A^x} J_a - \frac{C^x}{A^x} \; v_a \qquad (A3.7)$$

substituting into the last two of equation (2.47) we obtain, for space charge limited conditions:

$$J_b = \left(1 - \frac{B^x D^x}{A^x E^x}\right) E^x J_a + \left(1 - \frac{C^x D^x}{A^x F^x}\right) F^x v_a \qquad (A3.8)$$

$$v_b = \left(1 - \frac{G^x B^x}{A^x H^x}\right) H^x J_a + \left(1 - \frac{C^x G^x}{A^x I^x}\right) I^x v_a \qquad (A3.9)$$

Since J_a and v_a are assumed uncorrelated in noise studies the last three equations take the form

$$|I|^2 = \left|\frac{B^x}{A^x}\right|^2 \overline{J_a}^2 + \left|\frac{C^x}{A^x}\right|^2 \overline{v_a}^2 \qquad (A3.10)$$

$$\overline{J_b}^2 = \left|1 - \frac{B^x D^x}{A^x E^x}\right|^2 |E^x|^2 \; \overline{J_a}^2 + \left|1 - \frac{C^x D^x}{A^x F^x}\right|^2 |F^x|^2 \; \overline{v_a}^2 \qquad (A3.11)$$

$$\overline{v_b}^2 = \left|1 - \frac{G^x B^x}{A^x H^x}\right|^2 |H^x|^2 \; \overline{J_a}^2 + \left|1 - \frac{C^x G^x}{A^x I^x}\right|^2 |I^x|^2 \; \overline{v_a}^2 \qquad (A3.12)$$

The effect of total current is thus expressed by factors which modify the four coefficients E^x, F^x, H^x and I^x. It can be shown by manipulations of the coefficients that all the modifying expressions are functions only of the transit angle $\theta = \omega\tau$

The evaluation of the modifying expressions involves considerable computation since all the coefficients are complex. The expressions modifying F^x and I^x are given by Pierce. The quantities $\left|1 - \frac{B^x D^x}{A^x E^x}\right|$ and $\left|1 - \frac{G^x B^x}{A^x H^x}\right|$ as a function of θ have been evaluated by the author and Woonton, and all four quantities are plotted in figure (27).

For a given voltage, the transit angle implies a frequency since from Llewellyn and Peterson

$$T = \sqrt{\frac{2(u_b - u_a)}{\frac{\eta J_o}{\varepsilon_o}}}$$

The resulting relation is

$$\theta = 2\pi\ (0.771 \times 10^{-3}) \sqrt{\frac{V_b^{1/2} - V_a^{1/2}}{J_o}}\ f \qquad (A3.13)$$

where V_a is the voltage equivalent of the initial velocity.

Evaluation of the total current effect on the Llewellyn-Peterson-Pierce prediction has been carried out for the gun used in this research. The modification for the 900 V anode voltage case is shown in Figs. (28) and (29).

The effect on the current predictions is to counteract the reduction resulting from the Tien-Moshman smoothing factor. The velocity predictions show a large reduction in the low frequency region.

APPENDIX IV

NOTATION

The notation employed by the original authors has in most cases been retained in the theoretical discussions in this thesis. As a result, certain quantities are denoted at different times by different symbols. In some cases also, the same symbol is used to denote different quantities. This appendix is intended to collect and define the basic symbols used in the test.

e the electronic charge (coul.)

m the electronic mass (Kg)

ρ_o time average (d.c.) volumetric charge density (Kg/m^3)

ε_o permittivity of free space (farad/m)

Z distance in direction of beam flow (m)

τ electron transit time $= \int \frac{dZ}{u_o}$ (sec)

k Boltzmann's constant (joules/deg C.)

T_c cathode temperature (^oK)

η $\frac{e}{m}$ for electrons (coul/Kg)

η $\frac{V_a e}{kT_c}$ in Robinson's theory (dimensionless)

I_o d.c. beam current (amp.)

J_o d.c. beam current density (amp/m^2)

V_o beam voltage (volts)

u_o $\sqrt{2\eta V_o}$ d.c. electron velocity (m/sec)

T normalized time (in computer theory)

T transit time in sections (2-II) and (2-IV A)

ω $2\pi f$ signal radian frequency (rad/sec)

γ space charge factor (Llewellyn and Peterson)

ω_p infinite beam plasma frequency $= \left(\frac{\eta \rho_o}{\varepsilon_o}\right)^{1/2}$ (rad/sec)

ω_q finite beam plasma frequency (rad/sec)

p reduction factor $= \frac{\omega_q}{\omega_p}$

ρ a.c. charge density (coul/m^3)

ρ transit time normalization factor (computer theory)

α radian difference of anode to current minimum distance from $\frac{\pi}{2}\lambda_q$ (in experimental section)

α computer running time normalization factor (computer theory)

$\overline{i^2_{sn}}$ shot noise current $= 2eI_o\Delta f$ (amp^2)

$\overline{J^2_{sn}}$ shot noise current per unit area (amp^2/m^2)

$\overline{v^2_R}$ Rack velocity $= \frac{(4-\pi)\eta kT_c}{J_o}$ $(\frac{m^4}{sec^2})$

$\overline{J^2}, \overline{v^2}$ quadratic content, per unit beam area, of current and velocity fluctuation

$J_\omega, v_\omega = J_1, v_1$ amplitudes of current and velocity modulation densities

J_a, v_a modulations at gun anode plane

Y $u_o J e^{j\omega\tau}$ variable used by Hutter (4) and computer theory

b beam radius (m)

A^x to I^x Llewellyn-Peterson (9) a.c. coefficients

Γ^2 space charge current smoothing factor

Γ^2_{TM} current smoothing at potential minimum predicted by Tien and Moshman

$I = I_t = I_1 e^{j\omega t} = I_\omega e^{j\omega t}$ electron gun total current (amp)

$\beta_e = \frac{\omega}{u_o}$ electron propagation constant (1/m)

$\sqrt{\overline{v^2}}$ r.m.s. thermal electron velocity $= \sqrt{\frac{2kT_c}{m}}$ $\frac{m}{sec}$

σ normalization factor for function $f(\tau)$ (computer theory)

V_{10}, V_{1a} initial and final normalized computer voltages.

θ transit angle $= \omega\tau = \omega T$.

<u>Note:</u> Llewellyn and Peterson (9) use the quantities ρ, u, I, E as the sum of a.c. and d.c. terms. (See Sec. 2-IV A). Elsewhere they indicate, ordinarily, a.c. quantities only.

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