A STUDY OF COUNTING LOSSES DUE TO "DEAD-TIME" OF COUNTING CIRCUITS

by

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Being a thesis submitted to the Faculty of Graduate Studies and Research of McGill University, in partial fulfilment of the requirements for the degree of Master of Science.

McGill University August, 1951.

SUMMARY

High speed electrical counting may in many situations lead to large dead-time corrections. The corrections become greatly magnified when the process being studied is simultaneous with the arrival of particles from the pulsed output of a modern particle accelerator. In coincidence arrangements, even when a pulsed source is not used, losses are large and conditions are very stringent. In the calculation and experimental study of losses in the above situations, it is found convenient to insert electronically into the counting system a definitely known dead-time of specific characteristics. Using such an arrangement, an experimental study is made of three situations:

(1) Losses in single channel continuous counting.

(2) Losses in single channel pulsed counting.

(3) Losses in coincidence counting with a continuous source. In all cases, the experimental results are compared with the theoretical calculations made by Dr. C.H. Westcott.[1,2] In addition, the experimental apparatus is described in detail, with special emphasis being made on the critical dead-time circuits employed.

ACKNOWLEDGMENTS

The author is deeply indebted to Professor C.H. Westcott for the large amount of time he spent with him in discussion of the project, helping with the design and construction of the apparatus, and carrying out of experiments.

The author would also like to thank Mr. A. Densmore for help in construction of the apparatus in the early stages of the project; Mr. Dave Clark for help in performing theoretical and experimental computations; members of the McGill Radiation Laboratory for helpful hints and advice; members of the Macdonald Physics Laboratory workshop for machining the brass vacuum chamber and other parts of the apparatus.

Sincere gratitude is expressed to Dr. A. Norman Shaw, Chairman of the Physics Department, for his helpful guidance throughout the author's undergraduate and graduate years at McGill University.

This work was done under the sponsorship of a National Research Council grant awarded to Professor C.H. Westcott, and a Scientific Bureau of Quebec Scholarship awarded to the author for the years 1950-1951.

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INTRODUCTION

Due to the finite resolving time of counter action and the associated electronic circuitry, a counter-amplifier recorder circuit is limited in the number of impulses per second it can detect and record. Thus, if an impulse to the system is preceded by an interval of time t from the last count which is less than the recovery time τ of the system, then the latter will fail to register this count. In experimental arrangements where electrical counting is employed, the above situation leads to incorrect interpretation of results if not properly accounted for.

For the case of continuous counting with a uniform counting rate, consistent calculations for different experimental arrangements have been made by workers in this field. (The more important papers are listed in the bibliography.) Two simple types of situations that result in a loss of counts are usually discussed. In one type, each pulse accepted by the counting circuit excites a certain dead-time in the circuit, during which any pulse that occurs is rejected. The rejected pulse does not further extend the dead-time of the circuit. In the second situation, every pulse that occurs excites the dead-time, whether or not the pulse is recorded. The counting system is then said to have an extended dead-time or resolving time.

The latter situation is encountered in Geiger counters where the mechanism of supression is internal. The dead-time is of the order of 100 to 200 microseconds, and usually depends on the counting rate, a situation which is not always advantageous.

Of greater importance is the former situation, where the counter mechanism is fast and the dead-time of the circuit is controlled by the external circuitry. A proportional counter used in conjunction with a linear amplifier is an example of such an arrangement. In such a system where the mechanism can be regarded as not being affected at all by an impulse arriving during its recovery time, the efficiency of counting for constant τ is agreed upon by most workers to be $(1+\mu\tau)^{-1}$, where μ is the true expected counting rate. Experimental verification of this result is discussed in section (3.1).

Unfortunately, the above result is not sufficient to deal with present day situations arising in the counting of nuclear events. Out of the present trends of nuclear physics, towards the study of interaction of high energy particles, have grown complex machines for the acceleration of particles to high energies. A common and unfortunate feature of these machines is that their outputs consist of pulses of particles instead of a continuous stream. The McGill cyclotron is in this category. Its pulsed output is directly linked with the relativistic increase in mass of the protons which becomes critical when the energy of the protons exceeds approximately twenty millon electron volts.

The pulsed nature of the source is of no consequence so long as the process being studied is not simultaneous with the arrival of the particles from the accelerator. In this category lies the manufacture of radio isotopes and their study by mass spectrometer and allied techniques. On the other hand, if the process being studied is simultaneous with the arrival of the particles from the accelerator, and the pulsed phenomena appears in the nature of the particles being counted, the determination of losses due to dead-time requires new considerations. A simple application and extension of the equations of continuous counting at enhanced counting speeds is not sufficient to deal with this situation. An illustration is found in the following consideration.

In the pulsed accelerators discussed above, the pulse duration is usually only a small fraction of the total cycle. Thus it can be seen that the rate of arrival of particles during this interval will be high even though the average number arriving per cycle is comparatively low. An overall rate of 1000 per minute may mean that during the pulse the particles may be arriving at a mean rate of 10^6 per minute. The low total count makes it imperative that high speed counting is employed if large statistical errors are to be avoided. Correspondingly, dead-time corrections become very large.

This field of pulsed counting embodies a large number of important nuclear phenomena, where precise quantitative results are fundemental to the interpretation of results.

[1]In one paper, Dr. C.H. Westcott has made a careful theoretical study of the counting losses expected under pulsed conditions when one counting channel is used. A discussion of the highlights of this theory are given in Section (1.2), but it is well to point out at present that the theory specifies two conditions for the reduction of counting losses. (1) It is desirable to reduce the dead-time to a minimum so that $\tau \ll \mathcal{T}$, where $\mathcal{T}$ is the pulse length. (2) The counting rate should be as small as possible so that the parameter z, which is a product of the true counting rate and the dead-time, is kept as small as possible.

In actual practice, the first condition imposes strenuous requirements on the electronic techniques employed, since an accelerator using a pulsed deflector system gives an output pulse only a few hundreths of a micro-second long. This difficulty is overcome in the experimental arrangement used in this investigation by a method described below. It is further worth while to point out that the actual methods and techniques used in the present investigation are adaptable to machines where the beam is scattered out. Such a scattered beam could be obtained from the McGill cyclotron, where it would have a pulse length of approximately 50 microseconds.

The problem investigated and reported on in this thesis can be separated into three categories.

(1) The study of losses in continuous counting, single channel.

(2) The study of losses in pulsed counting, single channel.

(3) The study of losses in coincidence counting.

In all three instances, a known variable dead-time, which is larger than any other dead-time in the circuit, is introduced into a counting system described later. (In order to make possible a theoretical prediction of the variation of counting losses with dead-time, it is important to insert electronically into the counting system a definite dead-time greater than the largest inherent dead-time of the system, so that the variations of the latter become unimportant.) In accordance with the postulates of the theory, the mechanism of the dead-time must be such that the inoperative period only follows counts which are recorded. Such a circuit is described in the section on apparatus and its advantages and disadvantages are discussed. In the actual construction of the apparatus two such circuits are employed to provide for the study of a double channel coincidence system.

In the cases where a pulsed source is needed, it is achieved mechanically by rotating a slotted disc in front of a Polonium source. To overcome the technical difficulties in the construction of the variable dead-time, it is found convenient to scale up the pulse and dead-time durations proportionally. The circuits used in the experiment are capable of producing dead-times which are variable over the range of 10 to 5000 microseconds. The pulse lengths obtainable vary from 1 to 10 milliseconds.

The apparatus constructed is actually designed for the study of losses in experiments where pulsed coincidence counting is employed. It is very easily converted for the effective study of parts (1) and (2) by merely using a specified section of the apparatus.

In the discussion that follows, emphasis is placed on the construction of the apparatus and a thorough investigation of parts (1) and (2). Part (1), which has been investigated by many workers, is actually used as a check on the calibration of the paralysis circuits. On the other hand, calculations and experimental investigations of (2) have been sorely neglected, and it is therefore investigated very thoroughly. High statistical accuracy is at all times aimed for, since results obtained from this experiment will very likely influence later work with pulsed coincidence counting.

Due to lack of time, only preliminary experiments on continuous coincidence counting have been conducted and these are reported on. In connection with this, [2] Dr. C.H. Westcott has made calculations in an unpublished paper on the expected rate of losses. This theory is presently being revised. Wherever possible, comparison is made between this theory and experimental results. It is proposed that in the future pulsed coincidence counting will be attacked purely empirically. Results in this direction should be shortly forthcoming from Dr. Westcott's group.

SECTION I

THEORETICAL CONSIDERATIONS

No attempt is made to present any detailed proofs. Results are only quoted for the sake of completeness and easy reference. Detailed proofs may be obtained by refering to the two papers by Dr. C.H. Westcott listed in the Bibliography.

1.1 CONTINUOUS COUNTING.

As mentioned in the introduction, two situations arise. The case where the inoperative period of the dead-time follows only counts that are recorded; or the case where it follows all counts, including those suppressed by the dead period. In the former case the proportion of counts recorded is given by,

$$\frac{E'}{E} = \frac{1}{1+\mu\tau} \qquad (1)$$

where,

E' = total number of counts recorded.

E = total number of true counts.

μ = expected continuous counting rate.

τ = dead-time of circuit.

In the latter case, the proportion of counts recorded is given by,

$$\frac{E'}{E} = e^{-\mu\tau} \qquad (2)$$

In these and all other calculations below, it is assumed that the ionizing particles form a random distribution.

The above expressions neglect statistical fluctuations in the sense that E' is only an expected count. The standard deviation is usually given by $\pm\sqrt{E}$ for the true count. For the recorded count, E', it is somewhat less than the square root; in fact, to a first approximation, it can be given as $\pm\frac{\sqrt{E'}}{1+\mu\tau}$. This expression is used in calculating all standard deviations in the experimental results.

1.2 CALCULATIONS FOR PULSED SOURCES WITHOUT BACKGROUND COUNT.

These calculations may be divided into three categories according to the value assigned to the dead-time. If T is the pulse length, the three cases can be stated as follows.

(a) $\tau > T$

(b) $\tau \ll T$

(c) T greater than τ , but the ratio is not considered large.

Case (a) leads to a very simple formula for the proportion of counts recorded. Its instructive importance is diminished by the fact that the proportion of counts recorded only depends on the total number of counts occuring during the pulse. In addition condition (a) gives rise to high losses.

Case (b) is not important in the sense that in a practical situation where the pulse width is of the order of a few hundreths of a microsecond, the condition $\tau \ll T$ is difficult to achieve.

Case (c) is of considerable practical importance. It is the case we are most interested in.

In the derivations that follow, the following assumptions are made. The ionizing particles form a random distribution. The interval between pulses is much greater than the pulse length. The interval between pulses is much greater than the dead-time, so that no count recorded in one pulse can affect conditions in the next. The inoperative period of the dead-time follows only counts which are recorded. A rectangular pulse form is used throughout.

In addition to the symbols already defined, the following notation is used.

p = recurrence frequency of pulses.

$f = \mathcal{T} p$ = fractional pulse duration.

T = total duration of counting.

ν = mean counting rate during pulse.

$x = \frac{t}{\tau}$ = time in units of the dead-time.

$X = \frac{\mathcal{T}}{\tau}$ = duration of the pulse in units of the dead-time.

$z = \nu\tau$ = mean counting rate during the pulse in units of the dead-time.

(a)

The expected total count recorded is given by,

$$E' = pT(1-e^{-\nu\tau}) \qquad (3)$$

(b)

The expected total counts recorded is given by,

$$E' = pT\nu\mathcal{T}\left(\frac{1}{1+\nu\tau} + \frac{\nu\tau^2}{2\mathcal{T}.\,1+\nu\tau}\right) \qquad (4)$$

(c) The procedure to follow in these calculations is to find for different ranges of x, the probability P(x) that a count, having occured, shall be recorded, the conditions at $x = o$ and $x = \infty$ being, $P(0) = 1$ and $P(\infty) = \frac{1}{1+z}$. The expected total count recorded will then be given by,

$$E' = pT\int_0^X P(x)dx = \frac{E}{X}\int_0^X P(x)dx$$

Using the approximation that $P(x) = \frac{1}{1+z}$ for $x > 2$, it is found that the fraction of total counts recorded is given by,

$$\frac{E'}{E} = \frac{1-e^{-\nu\tau}}{\nu\tau}, \qquad \text{for } 0 < X < 1 \qquad (5)$$

$$\frac{E'}{E} = \frac{1}{Xz}\left\{2-e^{-zX}\ e^{-z(X-1)}\left[1+(X-1)z\right]\right\} \qquad \text{for } 1 < X < 2 \qquad (6)$$

$$\frac{E'}{E} = \frac{1}{Xz}\left\{2-e^{-2z}-e^{-z}(1+z)+\frac{(X-2)z}{1+z}\right\} \qquad \text{for } X > 2 \qquad (7)$$

[1]In his paper, Dr. Westcott calculates other approximations, both better and worse. He finds that the above approximation is extremely good, and only differs from an exact calculation done for values of $X < 3$ by 0.3%, when $Z = 1.0$ and $2 < X < 2.5$. For values of $z < 1$, the above approximation is all but indistinguishable from the exact calculations. In practice, one would not be likely to encounter values of z greater than 0.5, since losses would then exceed 30% of the true counting rate.

Using the above expressions for $\frac{E'}{E}$, theoretical curves for $\frac{E'}{E}$ versus X are plotted on graph 6 for values of $z = 0.2$, 0.4, 0.6 and 1.0. These are the curves that are later compared with experimentally obtained data.

Calculations for pulse shapes that are not rectangular cannot be deduced readily from the above analysis. Instead, different probability functions must be formed for each particular case and the analysis carried on from there by methods similar to those used in obtaining the above results. In any particular case, the calculations may prove to be very tedious, but at least possible. The exact procedure is outlined in [1] Dr. Westcott's paper. One of the objects of this experiment is to determine the extent to which the pulse shape affects the above theoretical results.

Up to this point, it has been assumed that no counts at all arrived between pulses. Since most practical counters have a background count, a correction for this must be introduced into the theory. In most of the practical cases, one is likely to find that the background count is small compared to the total count during the pulse. Therefore, if μ_0 designates the background rate, $\frac{\mu_0}{\nu}$ will be a small number, usually less than 1/100. Making an approximation to this effect, one obtains that the total expected count with the background present is given by,

$$E' = E_n'(\mu_o+\nu) + (1-f)E_o' - \frac{pT\mu_o\nu\tau^2}{1+\nu\tau} \qquad (8)$$

where,

$E_n'(\mu_o+\nu)$ is the total recorded count predicted by equations (5), (6) and (7), without a background count, but with a mean counting rate during the pulse of $(\mu_o+\nu)$ instead of just ν.

E_o' is the recorded count expected from the background only.

The second term on the right hand side of equation (8) is the principal correction term, while the third term is of lesser importance and is usually quite small.

1.3 COINCIDENCE COUNTING (unpulsed source)

In the case of coincidence counting, new important parameters are introduced. One of these is the coincidence resolving time. Another, which turns out to be very important, is the ratio of the coincidence counting rate to the singles rate in any channel. It is when this ratio approaches unity that the losses become difficult to compute. The results discussed below are for counting with an unpulsed source under such conditions.

We consider two similar channels, each having the same dead-time τ_0, and with true counting rates N_1=a+c and N_2=b+c, the c representing the truly coincident counts, and a and b representing the true singles rate in channels 1 & 2 respectively. In both channels, the a counts, b counts and c counts, as well as the (a+c) and (b+c) counts, form random sequences; i.e., the probability of a count being supressed is independant of whether it is a true coincidence or not. It is also assumed that the coincidence resolving time $\tau_c < \frac{1}{2}\tau_0$ so that two consecutive recorded counts in one channel can never both be within τ_c of a count in the other channel. Thus, the question never arises whether such a combination is recorded as one or two coincidences. In practice, this condition is easily achieved. Finally, it is assumed that the dead-time follows only those counts which are recorded.

Define:

$$\frac{1}{(1+N_1\tau_0)}$$, the efficiency of counting in channel 1.

$\dfrac{1}{(1+N_2\tau_0)}$, the efficiency of counting in channel 2.

$P_1(t)$, $P_2(t)$, the probability that a count occuring in channel 1, or 2 is recorded.

C, rate of recording coincidences.

The analysis proceeds by assuming that at time $t=0$, a count is recorded in channel 1, and determines the probability $P_2(t)$ that a count will be recorded in channel 2 for times $t<0$ and $t>0$. It turns out that a quantity $\gamma = \dfrac{P_2(0)}{\rho_2}$, (defined for $\tau_c = 0$), is very significant. It determines the dependency of C, the rate of recording coincidences, upon the dead-time τ_0. An approximation is made in the initial stages of the theory. (For a full explanation of this, one should refer to the original paper.[2]) The exact solution of the differential equation obtained by virtue of this approximation gives for the value of γ the following expression.

$$= e^{-(b+\frac{1}{2}c.\overline{1-\rho_1})} + (b+c)\sqrt{\frac{\pi\,\tau_0}{2c(1-\rho_1)}}\;.\,e^{\frac{b^2\tau_0}{2c(1-\rho_1)}}\;.$$

$$.\left\{\text{erf.}\left[\left(b+c.\overline{1-\rho_1}\right)\sqrt{\frac{\tau_0}{2c(1-\rho_1)}}\right]-\text{erf.}\left[b\sqrt{\frac{\tau_0}{2c(1-\rho_1)}}\right]\right\} \qquad (9)$$

where, $$\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{t^2}\,dt$$

To the first order in τ_o, γ is given by, $\gamma = 1+c\tau_o$. To the second order in τ_o,

$$\gamma = 1+c\tau_o\left[1-\tfrac{1}{2}(a+b+1/3c)\tau_o\right] \qquad (9a)$$

where the factor 1/3 is purely empirical and is obtained by comparing equation (9a) with calculations made with equation (9). $\gamma = 1+c\tau_o$ also represents the correct value for γ when a=b=o. Both equations (9) and (9a) are used later in a comparison of experimental and theoretical results.

Further analysis shows that C, the recorded coincidence counting rate, is given by,

$$C = \gamma\rho_1\rho_2\left\{c+\tau_c(bc+ca+2ab)\right\} + k\rho_1\rho_2\frac{\tau_c^2}{2} + O(\tau_c^3) + --- \qquad (10)$$

where,

$$k = c(a-b)\left\{c-b(\gamma-1)\right\} - ac(b+c)(\gamma+\rho_1) + --- \qquad (10a)$$

The term containing k should, in practical cases, be negligible. One of the objects of this investigation is to determine the dependence of C on τ_c, which should be mostly linear if the above assumption is correct.

As mentioned in the introduction, the above theory is now undergoing certain modifications. Since consideration of these modifications is at this stage not complete, experimental comparison is only made with the theory as outlined above.

To facilitate comparison of theory with experiment, it is convenient to change to the following notation.

$$z_1 = (a+c)\tau_0$$

$$z_2 = (b+c)\tau_0$$

$$z_c = c\tau_0$$

SECTION II

APPARATUS

2.1 GENERAL

The apparatus as a whole is primarily designed for the study of pulsed coincidence counting. Due allowance has been made in the construction of the electronic channels so that it is readily adaptable for the study of single channel pulsed counting, and single channel continuous counting. A block diagram of the chronological function of the apparatus is shown in fig. 1. A photograph of the laboratory set-up is shown in fig. P-1.

The ionization initiated by an alpha-particle in a counter produces a negative pulse at the anode of that counter. This pulse is then channeled into the high impedance input of a cathode follower, and subsequently, into a high gain (10000) N.R.C. amplifier. The amplifier is of low quality, but with certain alterations has been converted to serve its purpose quite well. The amplified pulse, of approximately 70 volts peak, then enters a mixer circuit where it is properly channeled and fed into

BLOCK DIAGRAM OF APPARATUS

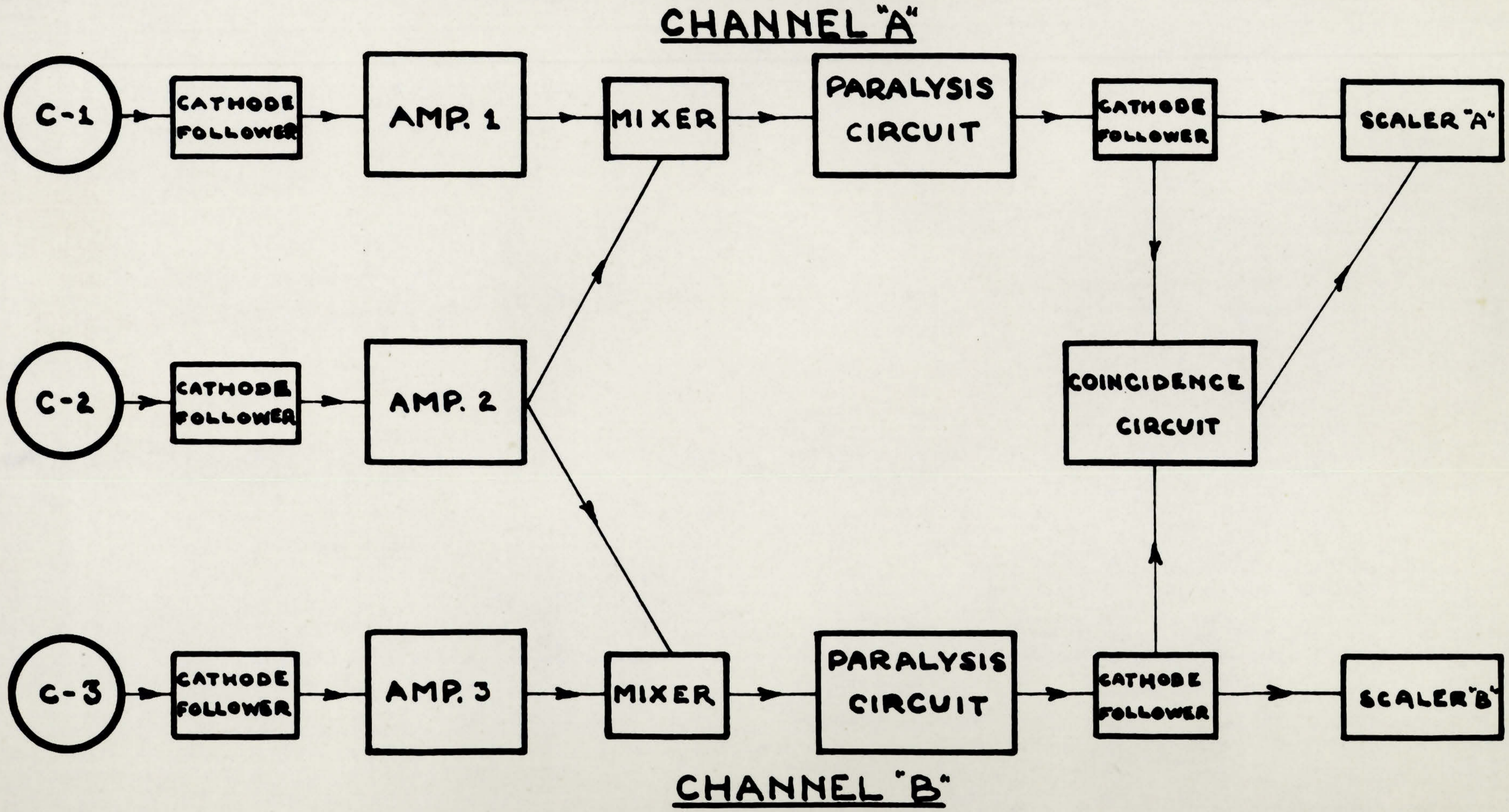


FIG. 1

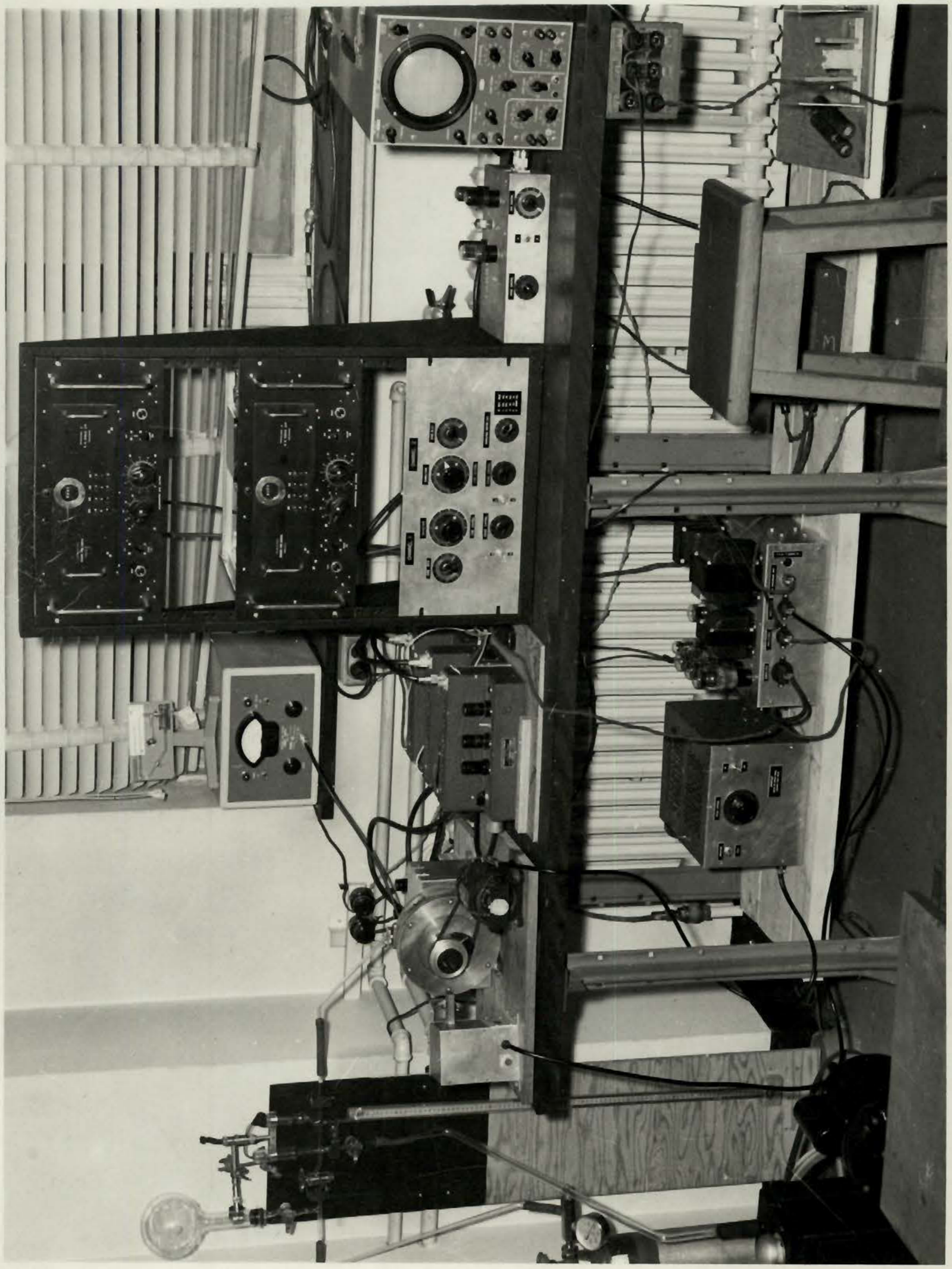


FIG. P-1

a calibrated flip-flop type of paralysis circuit. The differentiated square wave output from the paralysis circuit can then follow two paths. It may go directly into a scaler through a biased cathode follower; or into a Rossi coincidence circuit modified to produce coincidences over a specifically calibrated range, and then into a scaler where a count is registered.

When the apparatus is used to study single channel continuous counting, use can be made of either "channel A" or "channel B". For single channel pulsed counting, "channel B" must be used. "Channel A" is then occupied carrying impulses from a photo-cell arrangement which counts the recurrence frequency of the pulsed source.

When the apparatus is used in coincidence experiments, both channels are used. The sequence of events is as follows. A particle entering "counter 1" will produce a pulse which will follow "channel A" and register on "scaler A". A particle entering "counter 3" will cause a pulse to register on "scaler B" in a similar way. On the other hand, a particle entering "counter 2" will produce a pulse which will be channeled into "channels A & B" by the mixer circuit, and will then register on the scaler as a true coincidence, providing no pulse has preceded this one in both channels within a time less than the dead-time set on either of the paralysis circuits. Thus an artificial true coincidence can be simulated. Random coincidences can also be produced between particles entering "counters 1 & 3". (The number of such random coincidences will depend on the resolving time of the coincidence circuit, a parameter which in the experimental set-up is variable and is always known accurately.) In this way one can simulate conditions actually encountered in practice, such as counting proton coincidences in the presence of neutron recoils (R.R. Wilson, Phys. Rev. 71, 560, 1947). The theory can then be tested under actual experimental conditions.

As was mentioned in the introduction, it was found convenient to scale up both the pulse length and the paralysis time so that the technical difficulties encountered in designing very fast and variable paralysis times could be avoided. This also enables one to employ a mechanical method to simulate the pulsed nature of the source. A more detailed discussion is given in the next section.

2.2 VACUUM CHAMBER AND COUNTERS

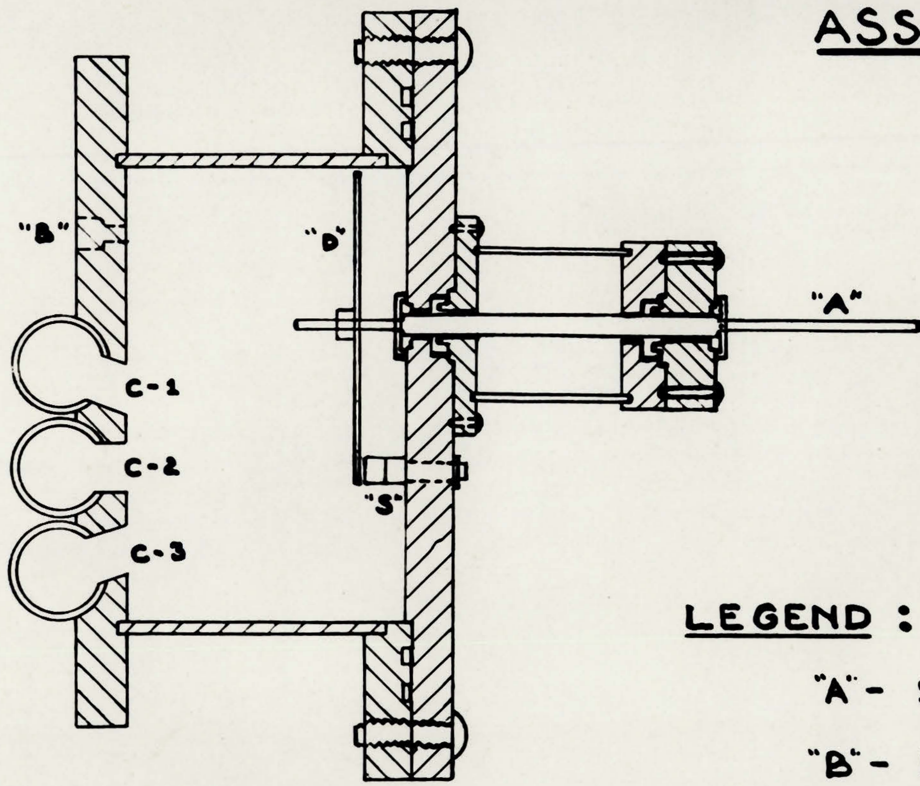
(a) VACUUM CHAMBER

The function of the vacuum chamber is to provide a housing for the source and counters. In this way, particles leaving the source always stay in an atmosphere of the desired gas under a specified pressure. One assembly and two sectional views of the vacuum chamber used are shown on figs. 2A, 2B and 2C.

The main vacuum chamber is of brass construction and is cylindrical in shape. It is composed of two sections (figs. 2B & 2C) with grooves for O-rings on the outer face of one to provide efficient vacuum sealing. In addition to the main chamber there is an auxiliary chamber (fig. 2C) which performs two functions. One is to provide a support for the rotating shaft shown in fig. 2A. The other is to diminish the leakage of air into the main chamber when the shaft is rotating. The shaft itself rests on two ball bearings. Two U-cups are used to provide vacuum sealing between the main chamber and auxiliary chamber.

A source is located on one side of the main chamber with three half inch square counter openings on the other (figs. 2A, P-2, P-3). Baffles are provided for the counter openings so that the areas exposed to the source are variable. The counters themselves are hard-soldered to the back of the main chamber, and the whole arrangement is evacuated as a whole by a megavac pump. The pressure is recorded by means of a mercury monometer

VACUUM CHAMBER ASSEMBLY



SCALE ÷ HALF SIZE

LEGEND :

"A" - SPINNING SHAFT

"B" - PUMPING OPENING

"C" - COUNTERS

"D" - SLOTTED DISC

"S" - α PARTICLE SOURCE

FIG. 2A

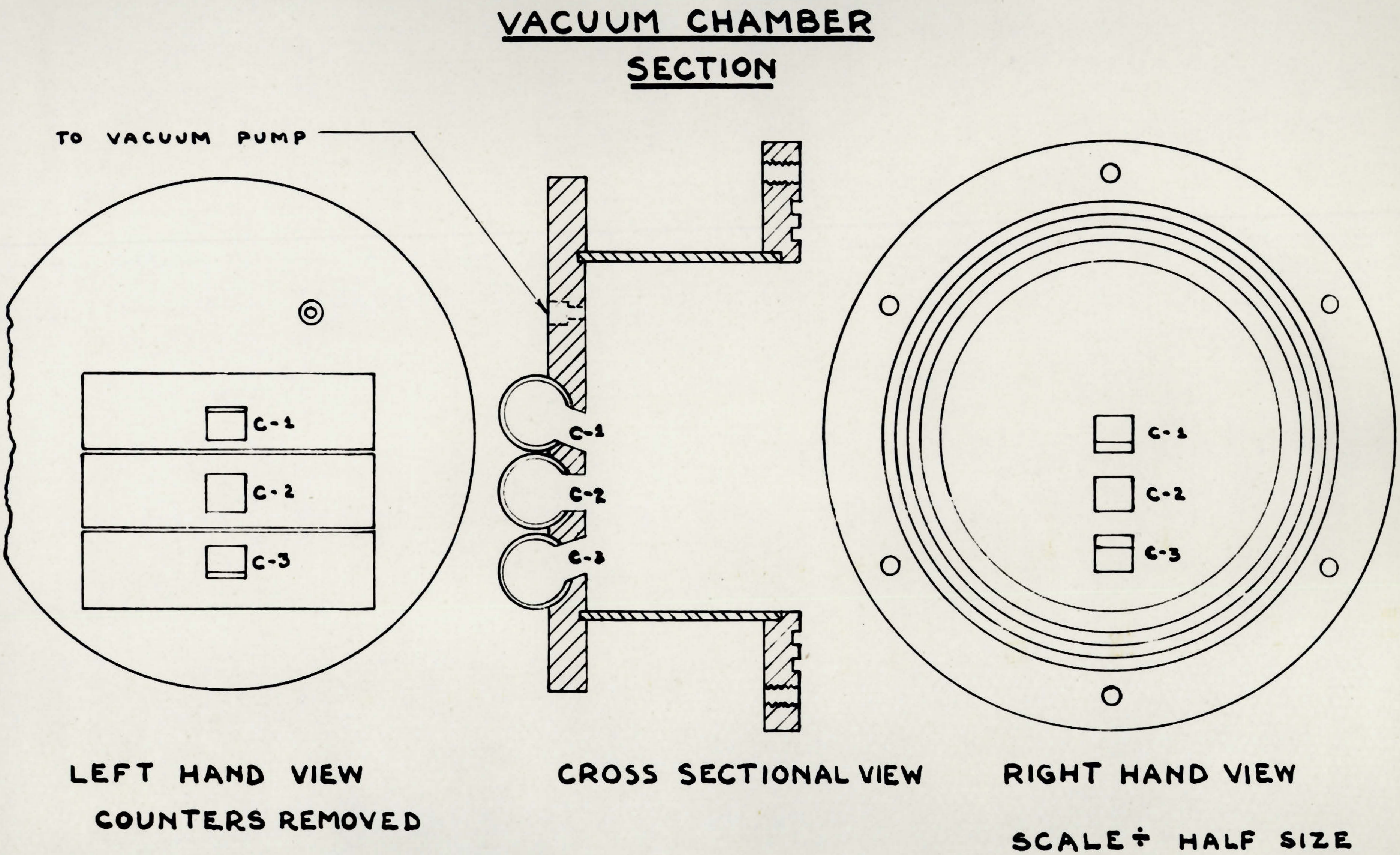

VACUUM CHAMBER
SECTION
TO VACUUM PUMP
C-1
C-2
C-3
C-1
C-2
C-3
C-1
C-2
C-3
LEFT HAND VIEW
COUNTERS REMOVED
CROSS SECTIONAL VIEW
RIGHT HAND VIEW
SCALE ÷ HALF SIZE

FIG. 2B

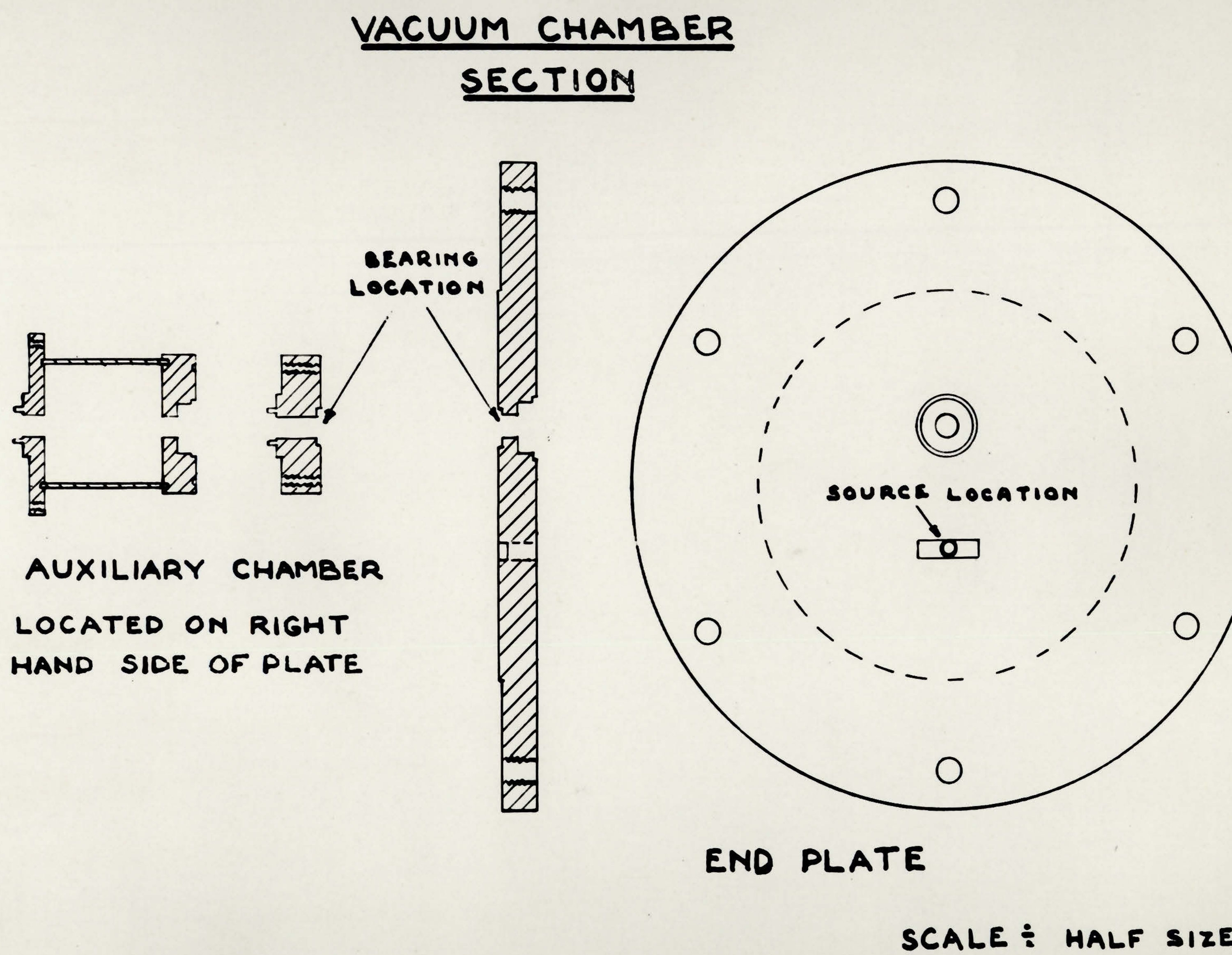

VACUUM CHAMBER
SECTION
BEARING
LOCATION
AUXILIARY CHAMBER
LOCATED ON RIGHT
HAND SIDE OF PLATE
SOURCE LOCATION
END PLATE
SCALE : HALF SIZE

FIG. 2C

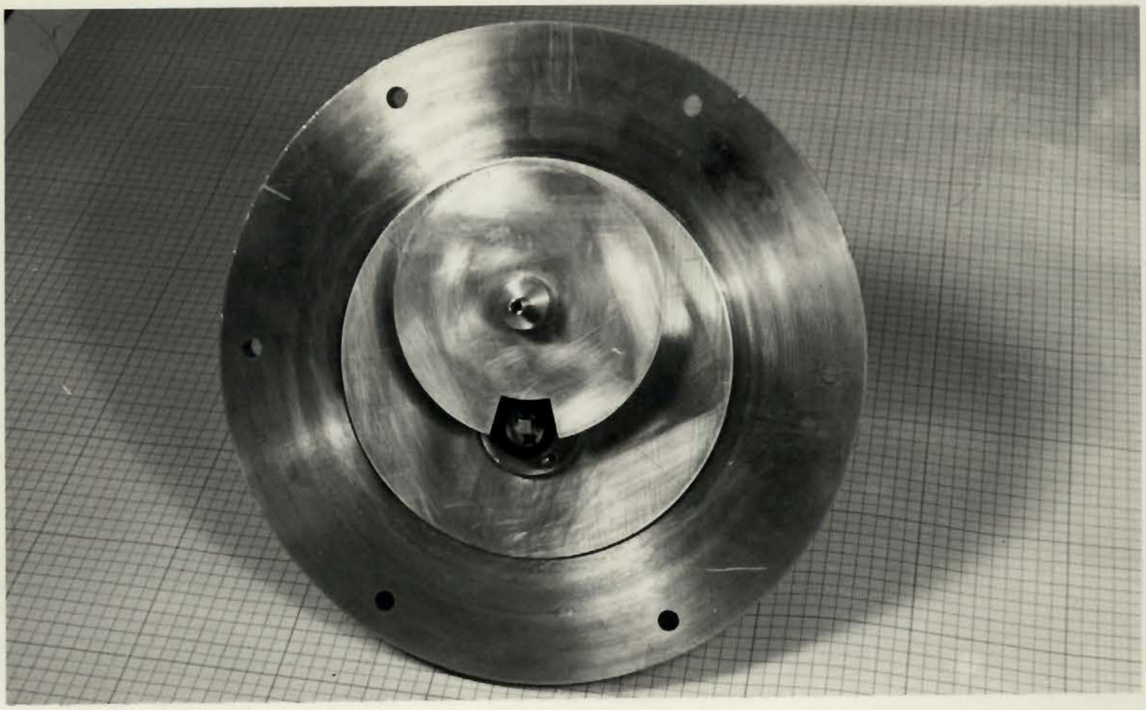


FIG. P-3

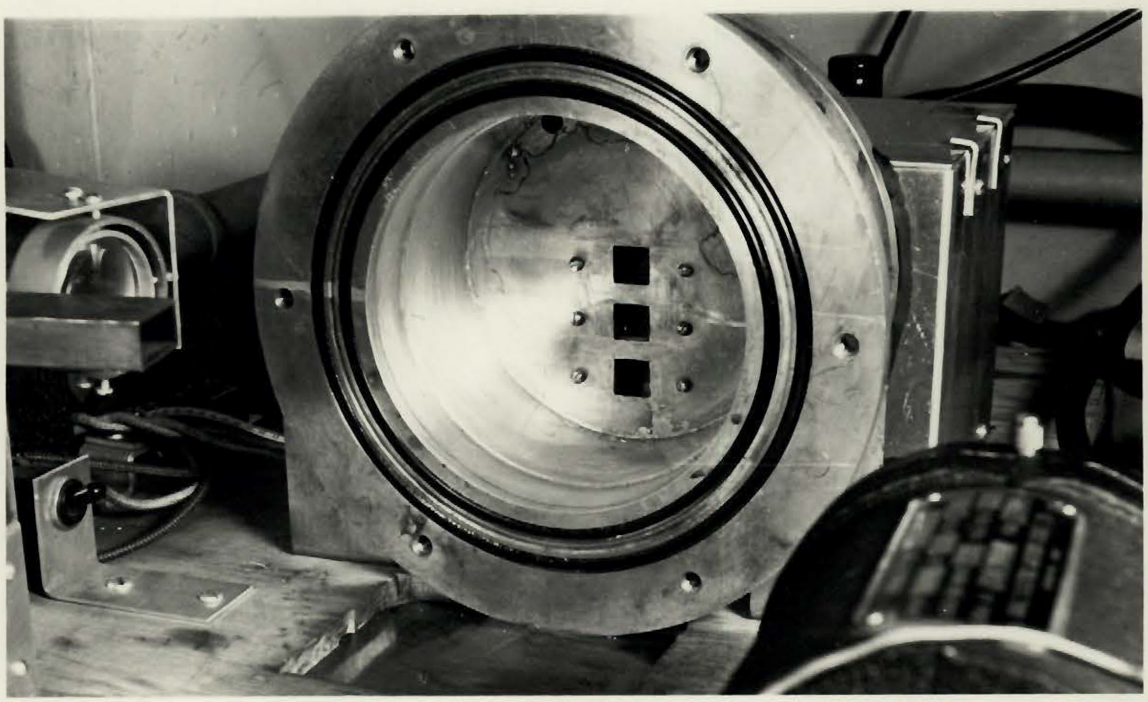


FIG. P-2

for medium and high pressures, and a Pirani gauge for low pressures.

(b) SOURCE

Alpha particles from Polonium are used as a source of ionizing particles. This gives a relatively constant source with the randomness required by theory.* The Polonium is deposited from an acid solution (equal quantities of HCl/N and $HNO_3/\frac{N}{10}$) of RaD on silver surfaces of dimensions and shapes shown on graphs 1-A & 1-B. In preparing the source, it is found necessary to clean the surface of the silver by making it the cathode in a hot bath of Roylite cleaner (mixture of NaOH, Na_3Po_4). This method is very effective in removing grease off the silver surface.

The general shape of the source holder is shown in fig. 2A. Before dipping in the RaD solution, all surfaces, except the surface to be plated, are covered with lacquer to prevent the deposition of Polonium on the sides of the source holder. When the source is placed in position in the vacuum chamber, it is covered by a mica window (fig.P-3).

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* J. Thibaud and R. Chery (C.R. Acad. Sci, Paris 230, 83-5; Jan/50) have actually proven that alpha particles from Polonium have a random distribution in time.

FIG. D-4

This is done to prevent the Polonium from migrating to all parts of the chamber due to recoils off the silver surface.

The pulsed nature of high energy accelerators is simulated by spinning a slotted disc in front of the source (fig. 2A). The disc is spun by a 1/20 H.P. d.c. variable speed motor, the speed of spinning being recorded accurately by a photo-cell arrangement which feeds directly into a scaler (fig. 5). (A photograph of the laboratory set-up is shown in fig. P-4). With a 30 degrees shutter, pulse lengths of 1 to 10 milliseconds are available, which satisfy the requirements of theory; i.e., the interval between pulses must be much greater than both the pulse length and the dead-time. Pulse shapes obtained by experimental measurement, together with their associated source shapes, are shown on graph 1-A and 1-B.

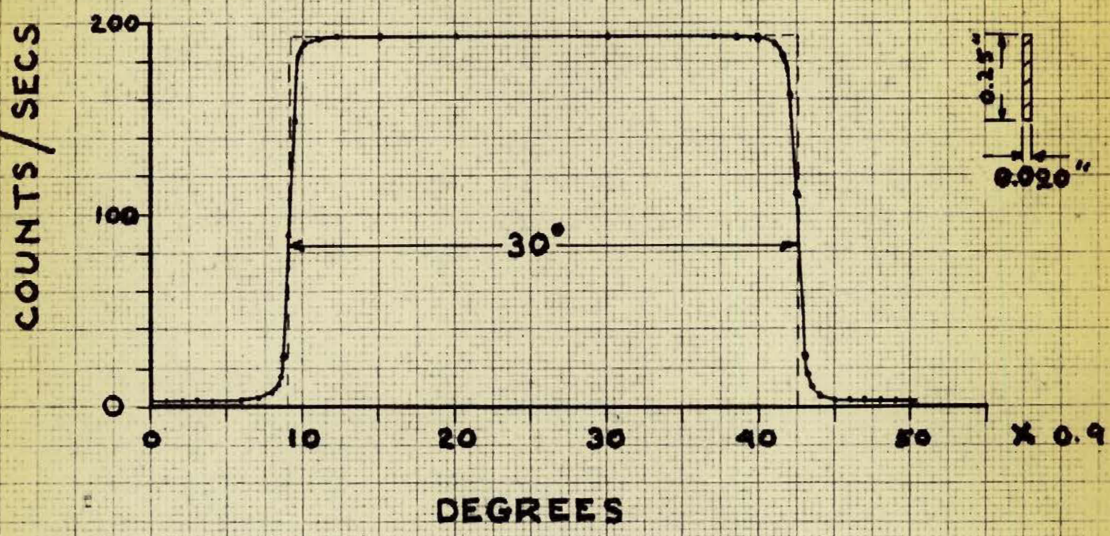
(c) COUNTERS

For purposes of this investigation, the general properties of proportional counters make them preferable to Geiger counters. Most Geiger counters have a dead-time and a resolving time of the order of 100 to 200 microseconds. The dead-time in such counters is determined

PULSE SHAPE

NARROW SOURCE

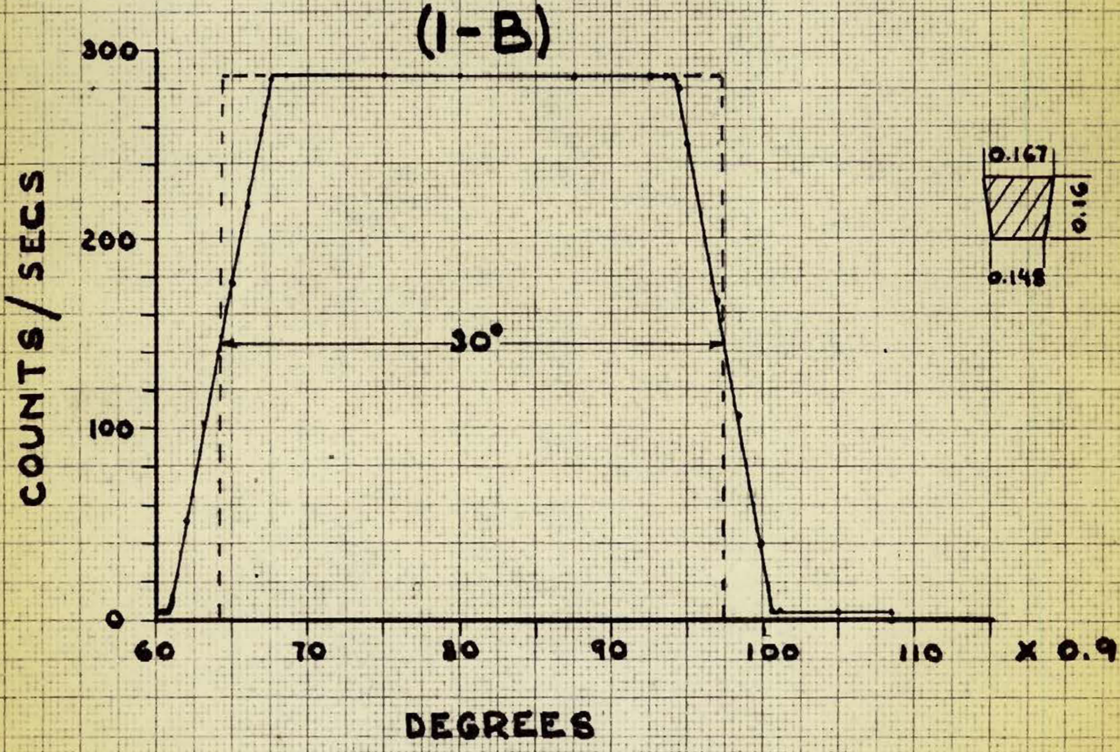
(1- A)



PULSE SHAPE

WIDE SOURCE

(1-B)



by the time taken by the positive ion sheath to move out, first to a distance such that the field near the central wire of the counter starts to recover, and finally, all the way to the outer cylinder. The mobility of these ions is such as to make the dead-time of Geiger counters relatively long. Due to this long inherent dead-time, at high counting rates, an appreciable fraction of pulses are initiated before the deionization from the previous particles is completed. At extremely high counting rates this condition becomes even normal. Thus at extremely high counting rates all pulses are much smaller than at low counting rates. Similarly, the dead-time after each reduced pulse is also smaller. Consequently, the duration of the dead-time and counter output in a Geiger counter is dependent on the counting rate, and therefore is uncontrollable in that sense.

In general, proportional counters operate much faster than ordinary Geiger counters, the increase in speed being a factor between 10 and 100, depending on such parameters as geometry, voltage, filling-gas, and where the initiating ionizing event takes place in the counter. The increase in speed of recovery is due to the fact that in a proportional counter, especially when operating at low values of gas amplification, the start of the development of the pulse on the central wire does not depend on the motion of the positive ions, but on that of the electrons. Moreover, in the proportional counter, the field near the wire is disturbed by the count only in one small region, and the rest of the wire is ready to receive additional counts at all times. Hence a second pulse can occur within the few microseconds of recovery and still be recorded as a full-sized swing in the potential of the wire. Proportional counters can be operated at rates up to 10^3 counts per second with very little loss due to statistical overlapping, and with somewhat greater tolerance up to 10^4 counts per second.

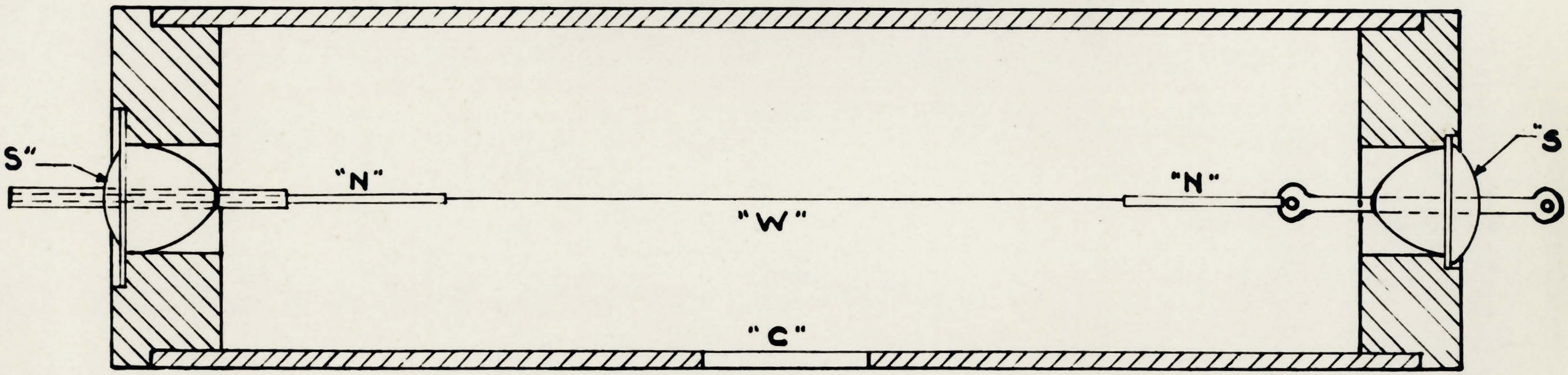
Due to the fast recovery time of a proportional counter, the dead-time of an arrangement using a proportional counter and linear amplifier is usually controlled by the resolving time of the amplifier, since two pulses coming from the counter a few microseconds apart only appear as one pulse to an amplifier whose resolving time is considerably longer than the interval between pulses. In this way, the dead period of proportional counters depends on the circuits employed in conjunction with them, a situation which is very desirable in this investigation.

In the actual construction of the counters, the traditional cylindrical geometry is used. The advantages of cylindrical design lie in the ease of construction. The wire along the axis automatically provides the high fields in its immediate vacinity which is necessary for the building up of electron avalanches. Such a field is shown in Rossi and Staub, "Ionization Chambers and Counters", page 91.

The counters (refer to fig. 3) are constructed of 7/8 inch diameter brass tubing with walls 3/64 inch thick. They are four inches long and fit snugly against one end wall of the vacuum chamber. Glass-kovar seals ("S") are used for vacuum sealing and insulating the anode wire which is a 6 mil. tungsten wire ("W"), 2 inches long. The tungsten wire is supported at each end by a 13 mil., ½ inch long, nickel wire ("N"), drilled with a 1/8 inch deep hole which provides a good connection for spot welding. This type of construction provides a field distribution which is concentrated around the centre portion of the wire. The counters are filled with 99% pure Methane.

It is found by experimentation that the optimum condition is obtained when the pressure is 24cms. of Hg, and the anode of the counter is at +1600 volts. This represents the condition that a large majority of alpha particles from the source are just getting into the counters. (Originally, a voltage of 1700 volts was used, but it was found that the background count, which was very sensitive to anode voltage, varied considerably

CROSS SECTION
OF A
PROPORTIONAL COUNTER



SCALE :- DOUBLE SIZE

LEGEND:

"S"- GLASS KOVAR SEALS.
"N"- NICKEL WIRE.
"W"- TUNGSTEN WIRE.
"C"- COUNTER WINDOW.

FIG. 3

over a small space of time, due to the instability of the bias curve for* background at that voltage.) With the above values for anode voltage and pressure, the loaded outputs of the amplifiers produce two sets of pulses. One set is about 15 to 20 volts peak, while the other is approximately 70 volts peak and quite uniform. In addition overshoots appear that are approximately 10 volts. The 15 volt pulses are of no importance, because they are considerably below the working discriminator bias level. Originally, concern was expressed over the 10 volt overshoots. But later, when the working discriminator level was chosen, it was found that the large positive peaks of the pulses completely compensated for the overshoot, and the difference between the two was safely above the discriminator level.

A typical discriminator bias curve is shown on graph 2-B. In choosing the working discriminator bias, for pulsed single channel counting, careful attention must be paid to the background count.

Fig. 4 shows the counter and cathode follower circuit. The three cathode followers are built into a separate chassis. The counter circuit is mounted on to the back

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*The background count is the count obtained when the counters are shielded from the source by the rotating disc.

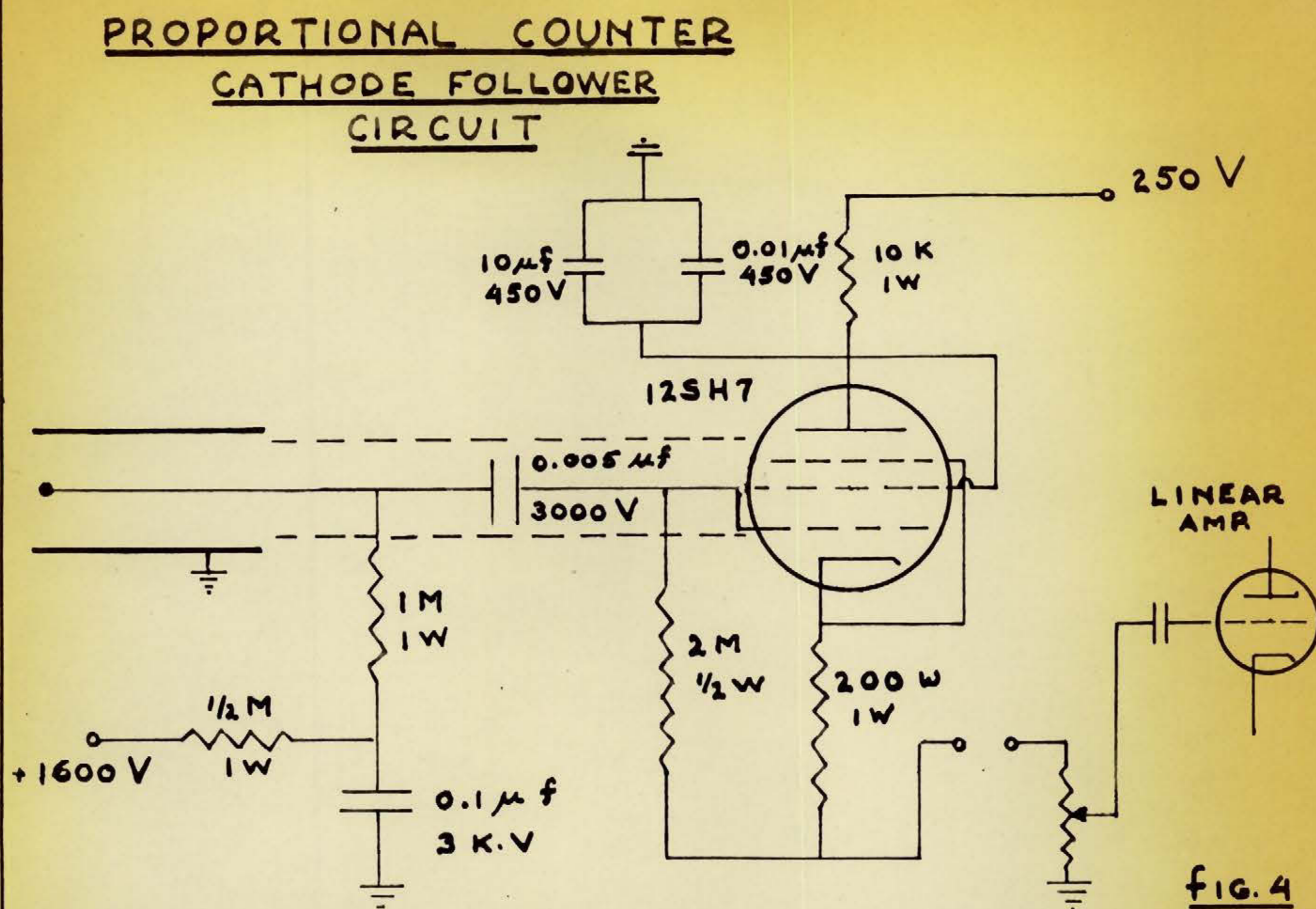

PROPORTIONAL COUNTER
CATHODE FOLLOWER
CIRCUIT
250 V
10μf
450V
0.01μf
450V
10 K
1W
12SH7
0.005 μf
3000 V
LINEAR
AMP
1M
1W
2M
½ W
200 Ω
1W
½ M
1W
+1600 V
0.1 μ f
3 K.V

FIG. 4

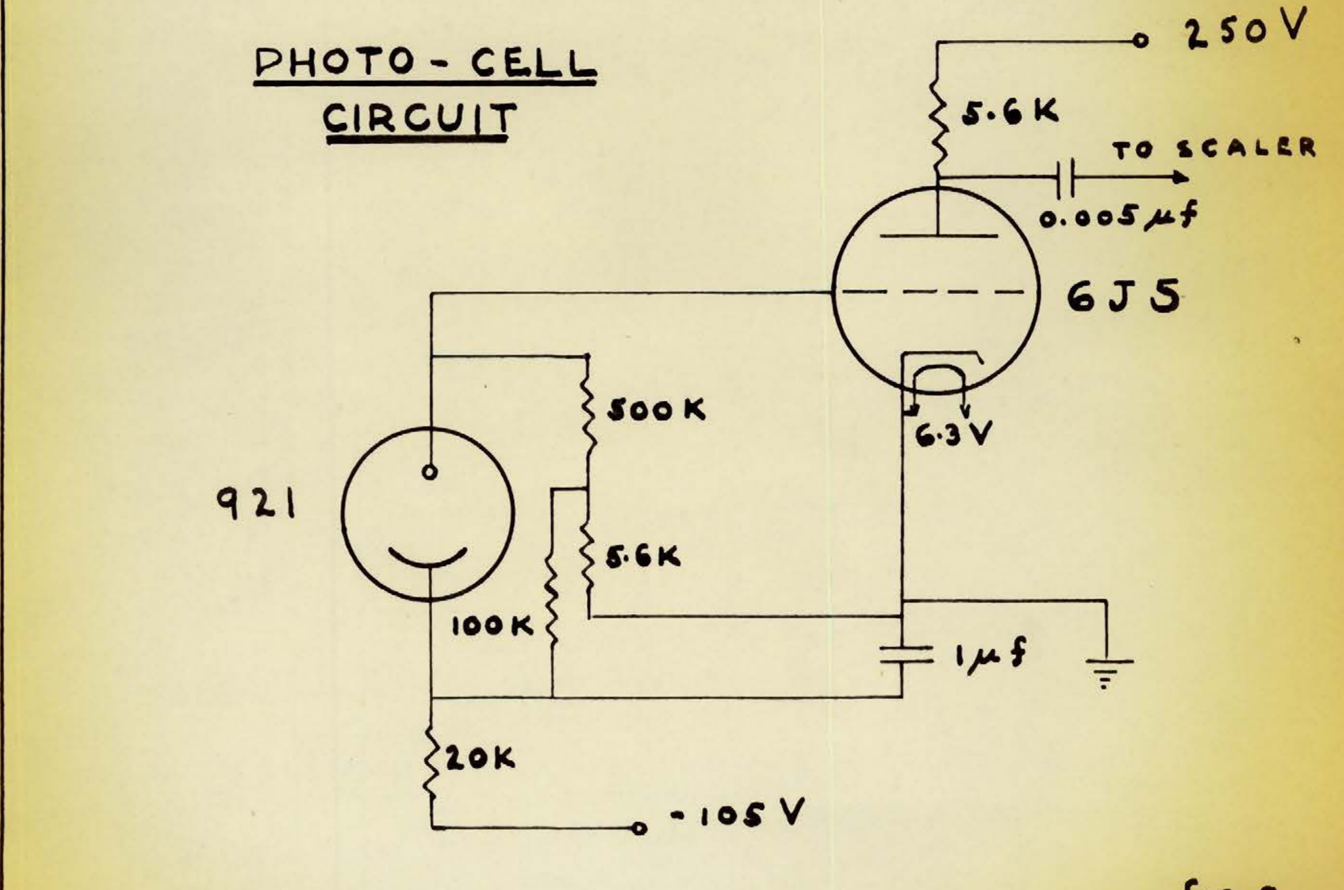


PHOTO - CELL
CIRCUIT
250V
5.6K
TO SCALER
0.005 μf
6J5
500K
6.3V
921
5.6K
100K
1μf
20K
-105 V

FIG. 5

of the main vacuum chamber next to the counters, and is shielded by an aluminum box. A tightly stretched wire surrounded by brass tube shielding carries the signal from the coupling condenser to the grid of the 12SH7. Both the cathode follower chassis and the vacuum chamber are mounted on rubber sponging to diminish the microphonics due to the vibration produced by the disc's motor.

2.3 ELECTRONIC CIRCUITS

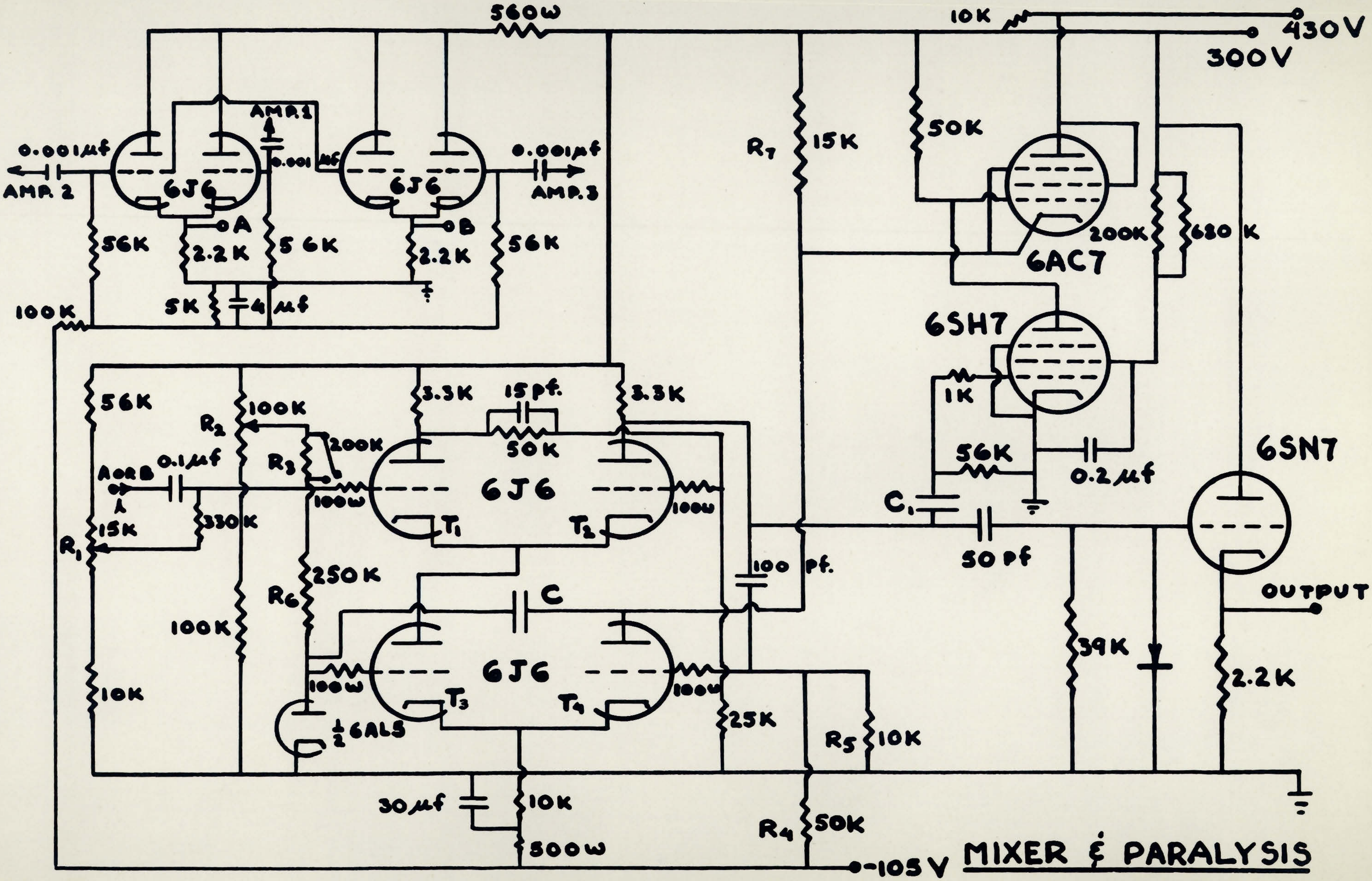
(a) AMPLIFIERS

For efficient reproduction of the pulses produced by proportional counters, amplifiers used in conjunction with these counters should preferably have a short rise time and a frequency response which lies between 10^3 to 10^4 cycles per seconds on the low frequency side, and 10^7 to 10^8 cycles per second on the high frequency side. (The lower limit is determined by the values of the coupling condensers and resistors, while the upper limit is automatically determined by the interelectrode tube capacities.) In addition, the amplifier should be able to accept pulses which are between 10^{-4} volts and 1 volt.

Though the N.R.C. amplifiers actually used in these experiments do not meet these stringent requirements, with certain alterations, they are made quite usable. The amplifiers consist of three pentode stages (12SH7), with the last stage working off a 400 volt unstabilized line for extra amplification. The other pentode stages have a plate supply of 250 volts. Originally, the high frequency response was very poor. By changing cathode bypass condensers and making other minor alterations, the frequency response was improved greatly so that the amplifiers now have a flat response to sine waves from 100 cycles to 30 kilocycles. At 50 kilocycles the response is down 5 d.b. The amplifiers have a gain of 10000 to sine waves and approximately 7000 to fast pulses. With the counters operated as described in Section (2.2c), the output of the amplifiers without load is 105 volts peak. With load, the output changes to 70 volts peak. Overshoots of 10 to 15 volts appear, but they are not serious because of the large positive peaks. The amplifiers have a resolving time of approximately 5 microseconds.

(b) MIXER, DISCRIMINATOR, PARALYSIS CIRCUIT.

The mixer circuit, (fig. 6), consists of two double



MIXER & PARALYSIS CIRCUIT

FIG. 6

triodes (6J6) wired as cathode followers. The tubes are so arranged that a pulse coming from the center counter and amplifier is channeled into both "channels A & B". Pulses coming from the two outer counters and amplifiers just follow "channels A or B", depending in which counter the ionization is initiated. In the quiescent state, each double triode is conducting 3 milliamperes. Pulses coming from the amplifiers traverse the mixer with negligible loss in gain and are fed into the discriminator paralysis circuit at points "A" or "B" (fig. 6). The mixer circuit is built into the same chassis with the discriminator paralysis circuit, and is supplied with a positive 300 volt line and a negative, -105 volt line by the principal power unit (fig.8).

The discriminator and paralysis circuit (fig. 6) is a modified version of the T.R.E. circuit used in scaler type 1009A. The circuit is arranged so that a pulse arriving during the dead-time cannot upset the circuit, a condition required by the theory of this investigation.

In its original form, the circuit consists of two double triodes (6J6) and a diode (6AL5). One double triode is arranged as a trigger circuit which operates only when the input pulse amplitude is sufficient to overcome the bias set by resistor "R_1". The second double triode and the diode form the paralysis time circuit which switches the discriminator completely out of action for the duration of the chosen paralysis time selected by varying the condenser "C" and resistances "R_2" and "R_3". At the conclusion of the paralysis time, the discriminator is brought back to its initial sensitive state.

To the above circuit have been added two modifications. (1) A recovery circuit, consisting of the 6SH7 and 6AC7, which increases the speed of recovery after the paralysis time is completed. (In its initial form, the circuit had a recovery time which was too slow to suit the requirements of this experiment.) (2) The other modification is a cathode follower output which is provided by the 6SN7.

The operation of the circuit is as follows (refer to fig. 6): In the equilibrium state, "T_3" and "T_2" are conducting. "T_1" is cut off by the bias voltage; "T_4" is cut off by the network "R_4" and "R_5". A positive pulse, (of minimum value 3 volts peak), which overcomes the bias voltage, switches current into "T_1". The negative waveform at the plate of this tube is transferred through a condenser to the grid of "T_2", cutting it off, and holding it off by means of voltage derived by the d.c. coupling of the resistors 50K and 25K. The positive waveform, which then appears at the plate of "T_2", is condenser coupled to the grid of "T_4" and switches it on. This results in a negative waveform on "C", which cuts off "T_3". The state of the circuit just after triggering is that "T_1" and "T_2" are completely cut off, "T_4" is conducting, and the grid of "T_3" is about 100 volts below ground potential. The discriminator is then paralyzed, and a pulse coming in on the grid of "T_1" is rejected.

The time during which the circuit is paralyzed depends on the time it takes for the potential on the grid of "T_3" to rise to a point where the tube will conduct. (The diode in the circuit is there to prevent the grid of "T_3" from rising above ground potential and thus drawing grid current.) The rate of rise in potential of this grid depends on a RC constant which is not readily calculable, but is calibrated experimentally. To obtain a continuous series of paralysis times, six values for "C" and variable resistances "R_2" and "R_3" are employed. "C" takes the following values:-

Range	Condenser Value
1	stray capacities
2	0.00015 microfarads
3	0.0005 microfarads
4	0.002 microfarads
5	0.006 microfarads
6	0.03 microfarads

"R_3" provides for overlapping in the ranges. In this way paralysis times which vary from 10 microseconds to 5000 microseconds are made available.

When "T_3" starts conducting, the circuit recovers to its initial state in a manner which depends on the instantaneous state of the grid of "T_1". Three situations arise.

(1) If the voltage on the grid of "T_1" is below cut off, the current drawn by "T_3" flows via "T_2", whose anode pulse cuts off "T_4". The circuit thus recovers normally.

(2) If the voltage on the grid is above cut off, (i.e., if there is a signal above the bias level on the grid of "T_1"), the current flows via "T_1", whose anode pulse holds "T_2" cut off until the signal drops below the bias level, whereupon the current is switched back to "T_2". In this way the paralysis time may be increased by a fraction of a pulse width. This situation represents a disadvantage of this circuit, though not a serious one, since the probability that this situation will arise is small.

(3) The last situation arises when two pulses come very close together so that the plate of "T_4" has not had time to recover fully from the first pulse when the second pulse initiates a paralysis. This results in an effective decrease in the second paralysis time. The

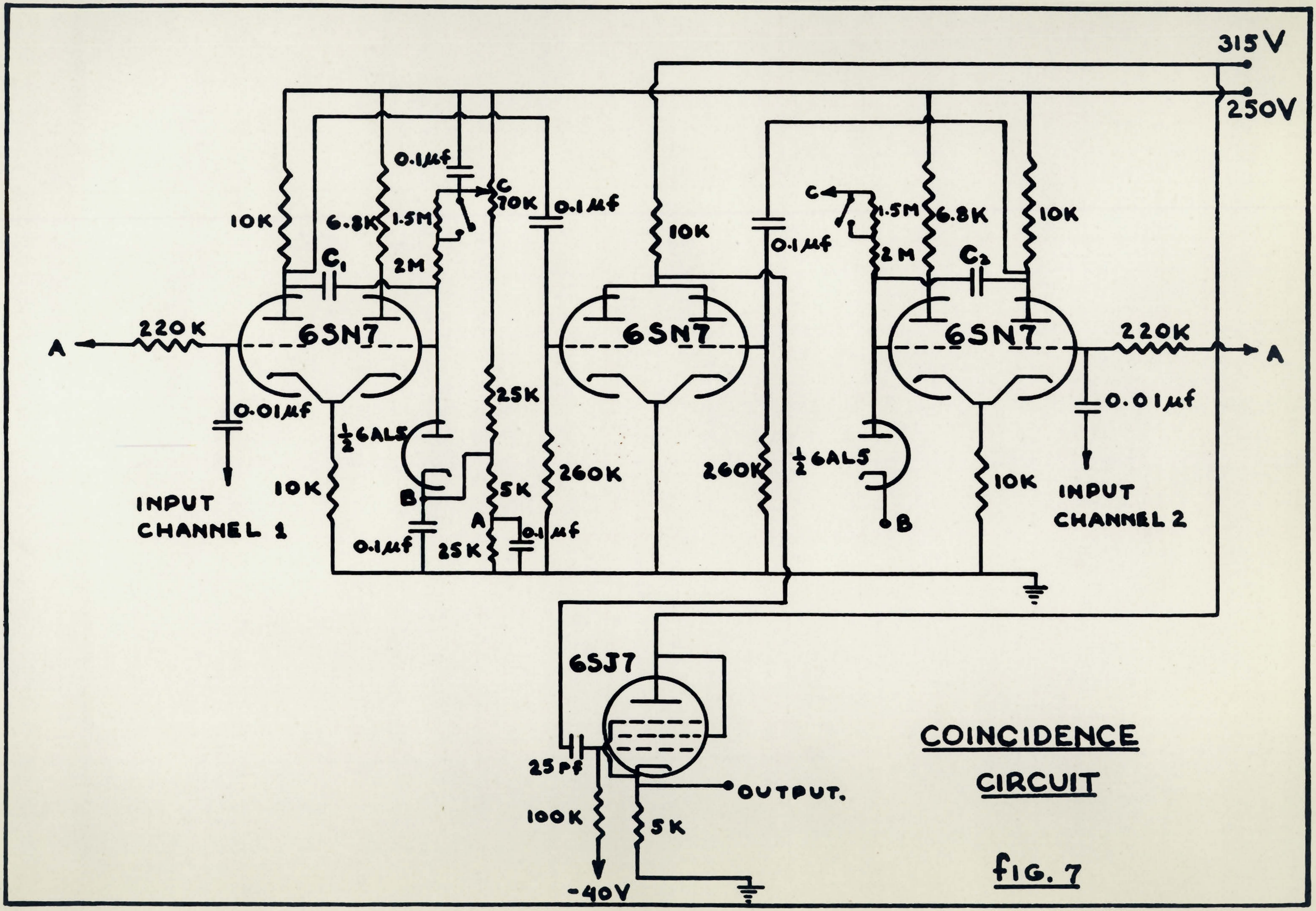
magnitude of the effect is especially noticeable when "C" is large and "R_3" is shorted. This effect was first detected when a study was being made of counting with a continuous source. The manner in which it affected the results is discussed in the following Section (3.1).

The recovery circuit, which is inserted to remedy the above situation, operates in the following manner. Normally, before the triggering of the paralysis circuit takes place, the 6SH7 is conducting and the 6AC7 is kept cut off by fixing the potential of its grid at 100 volts above ground. This is accomplished by the trial and error adjustment of the two parallel resistors 200K and 680K, and the resistor 50K. It is extremely important that the 6AC7 be kept normally non-conducting, since conduction of current through this tube at the wrong moment would completely upset the normal calibration of the paralysis circuit. The output square wave (40 volts) from the plate of "T_2" is condenser coupled to the 6SH7. The positive leading edge of this square wave does not affect the circuit except to increase the conduction of current in the 6SH7. At this instant, the lower point of "R_7" (and the cathode of the 6AC7) is at a potential of 135 volts, which is high enough above the grid potential of the 6AC7 to keep it cut off. On the other hand, the negative going trailing edge of the square wave cuts off the 6SH7 and thus drives the grid of the 6AC7 into the conducting region. In this way, a low impedance is placed in parallel with "R_7" for a duration of time equal or greater than the original recovery time of the circuit. This mechanism brings about very quick recovery of the circuit. The duration for which the 6AC7 is conducting depends on the value of the condenser "C_1". It was found by experiment that for "ranges 1 to 4", 50 microfarads is sufficient. For "range 5", 100 microfarads must be used, while for "range 6", 240 microfarads is necessary.

The output from the paralysis circuit is obtained by differentiating the square wave from the plate of "T_2" with a 2 microsecond time constant. This time constant is smaller than any paralysis time available from the circuit. A germanium diode at the grid of the cathode follower is used to cut off the negative part of the pulse, and a positive pulse of 11 volts peak is thus obtained at the output of the cathode follower. This pulse is of sufficient amplitude to drive the scaler.

(c) COINCIDENCE CIRCUIT (fig. 7)

The coincidence circuit consists of a Rossi coincidence circuit modified to produce coincidences over a



COINCIDENCE

CIRCUIT

FIG. 7

specifically calibrated range. Impulses arriving from "channels 1 and 2", (previously referred to as "A" and "B" respectively), trigger two univibrators which deliver square waves of calibrated duration to the inputs of the Rossi tube. If the square waves overlap in time, in part or in whole, a pulse of large amplitude is produced at the plate of the Rossi tube. The width of the rectangular pulse depends on the area of overlap of the two square waves coming from the univibrators. The pulse from the Rossi tube is then differentiated with a 2.5 microseconds time constant, and fed into a biased cathode follower. For two pulses from the univibrators in coincidence, the cathode follower gives an output pulse of 70 volts peak, plus an additional smaller pulse of 20 volts peak due to the differentiation of the leading part of the smaller amplitude portion of the pulse produced at the plate of the Rossi tube by the segments of the square waves coming from the univibrators which do not drive the grid of the Rossi tube negative simultaneously. For two pulses from the univibrators that are not in coincidence, only two smaller amplitude pulses are produced at the output of the cathode follower. In actual operation, the discriminator of the scaler is set so that these smaller pulses are rejected, and only the pulses of larger amplitude register as counts. The input pulse required to trigger the univibrators must be greater than 4 volts peak.

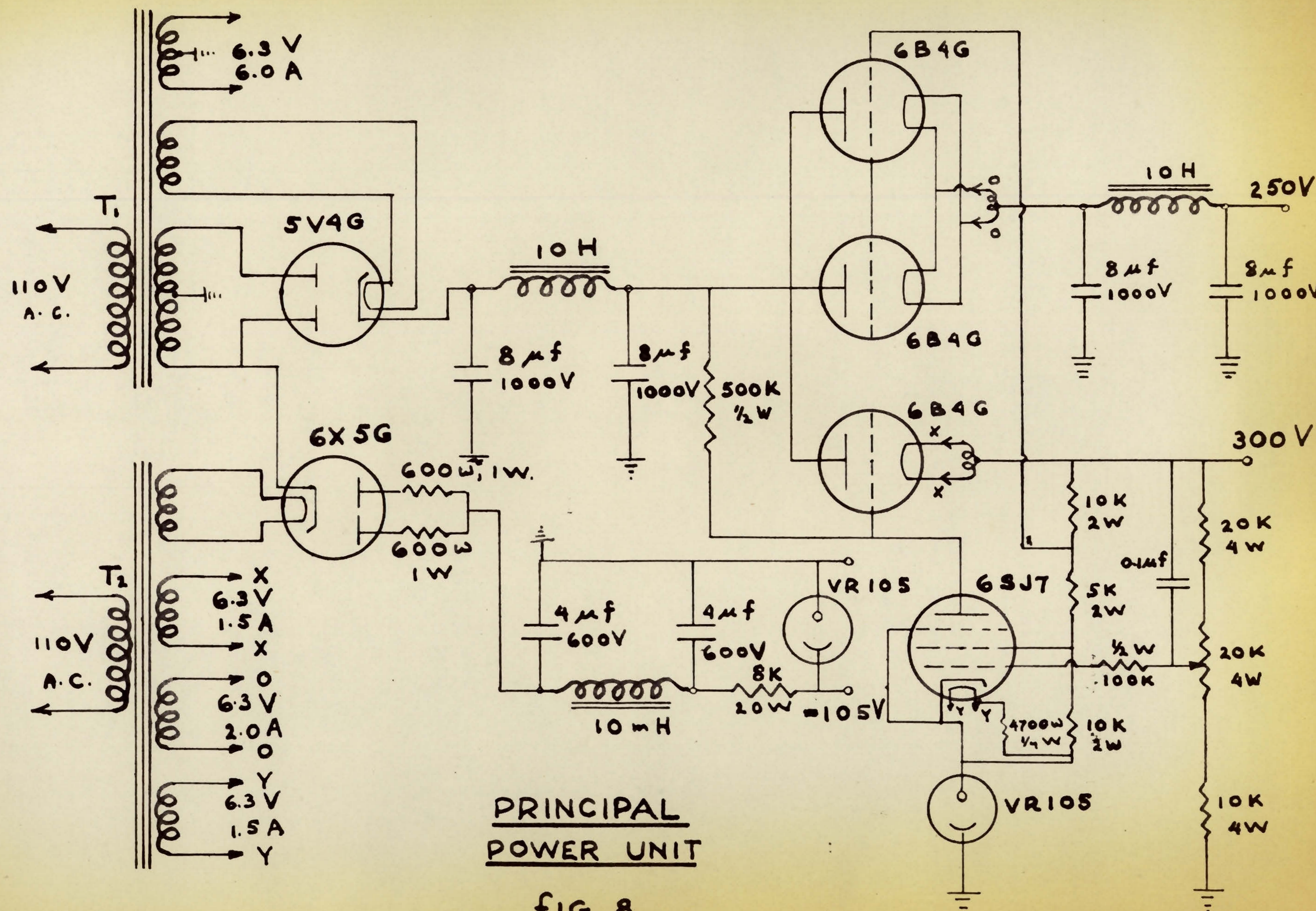
The univibrator used in this circuit is a slightly modified version of the usual univibrator where a common cathode resistor is used to complete the regenerative loop. The duration of the square wave is controlled by the condensers "C_1" and "C_2", and the chain of resistors 10K, 2M, 1.5M and the 70K potentiometer. A shorting device is provided for the 1.5M resistor to provide for overlapping of ranges. Six values for "C_1" and "C_2" are employed. They are as follows:-

Range	Condenser value $C_1 = C_2$
1	25 picofarads
2	50 picofarads
3	200 picofarads
4	700 picofarads
5	2000 picofarads
6	7000 picofarads

With this arrangement, square waves of duration 3 to 3250 microseconds are available. Both univibrators are set by the same range and fine potentiometer controls.

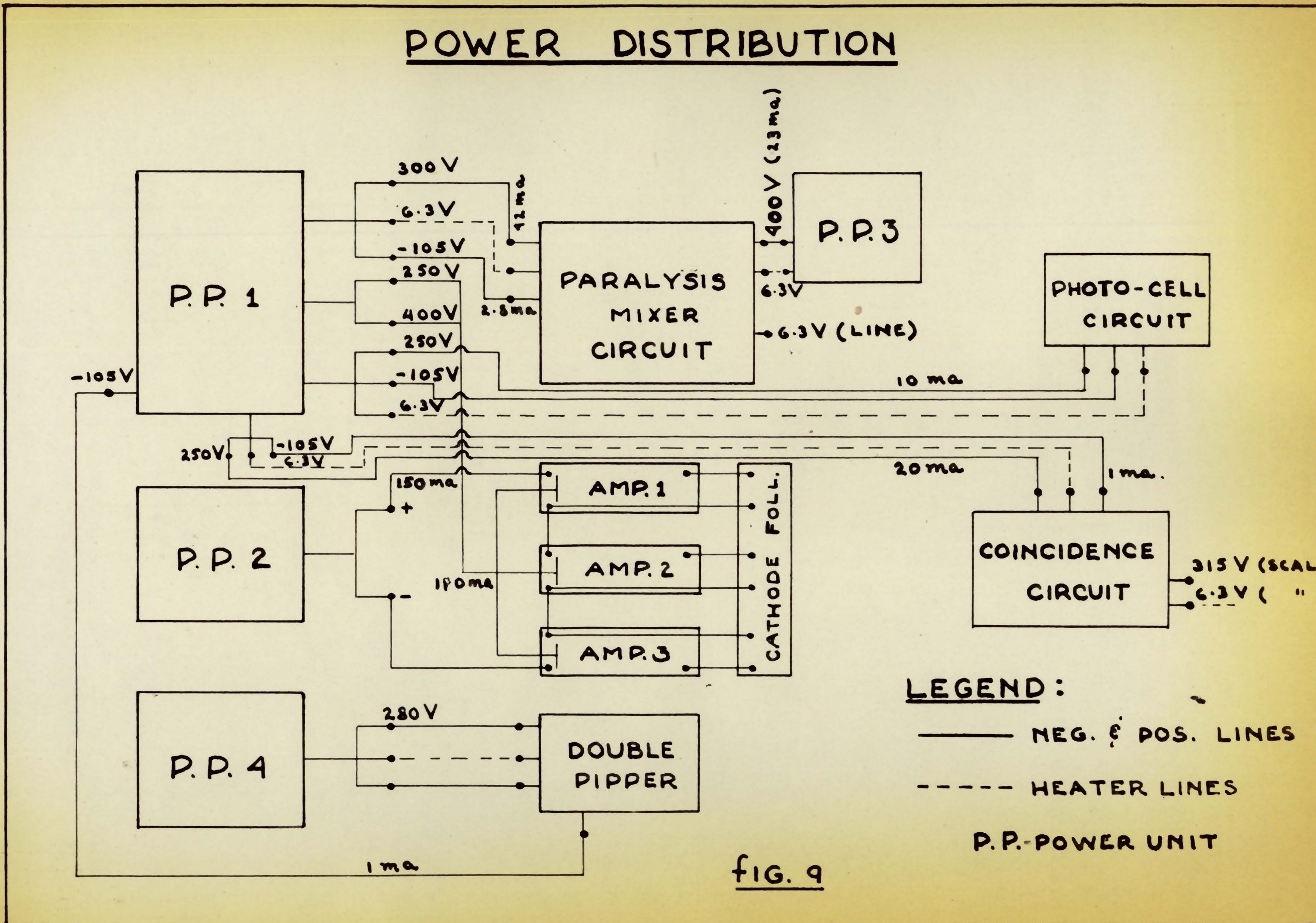
(d) POWER DISTRIBUTION

A block diagram of the power distribution is shown in fig. 9. Four power units are employed to supply



PRINCIPAL
POWER UNIT

FIG. 8


POWER DISTRIBUTION
P.P. 1
P.P. 2
P.P. 3
P.P. 4
300 V
6.3V
-105V
250V
400V
250V
-105V
6.3V
42 ma
2.8 ma
PARALYSIS MIXER CIRCUIT
400V (23 ma)
6.3V
6.3V (LINE)
PHOTO-CELL CIRCUIT
10 ma
-105V
250V
-105V
6.3V
150 ma
180 ma
+
-
AMP. 1
AMP. 2
AMP. 3
CATHODE FOLL.
20 ma
1 ma.
COINCIDENCE CIRCUIT
315 V (SCALER)
6.3 V (")
280 V
DOUBLE PIPPER
1 ma
LEGEND:
NEG. & POS. LINES
HEATER LINES
P.P.-POWER UNIT
FIG. 9

- 52 -

power to the electronic circuitry proper. In addition to these, there is a power unit to run the 1/20 H.P. motor, and a regulated "Atomic Instruments" high voltage supply for the counters.

P.P.1 (refer to fig. 9) is the principal power unit employed. A circuit diagram of it is given in fig. 8. This power supply provides a regulated 300 volt line capable of supplying 60 milliamperes to the paralysis and mixer circuit. In addition, it supplies a 250 volt, 120 milliamperes line cathode followed out through two 6B4G's. This line is shared between the amplifiers and the photo-cell circuit. The power supply also provides a negative 105 volt, 30 milliamperes, V.R. tube regulated line which is distributed to all units as shown in fig. 9.

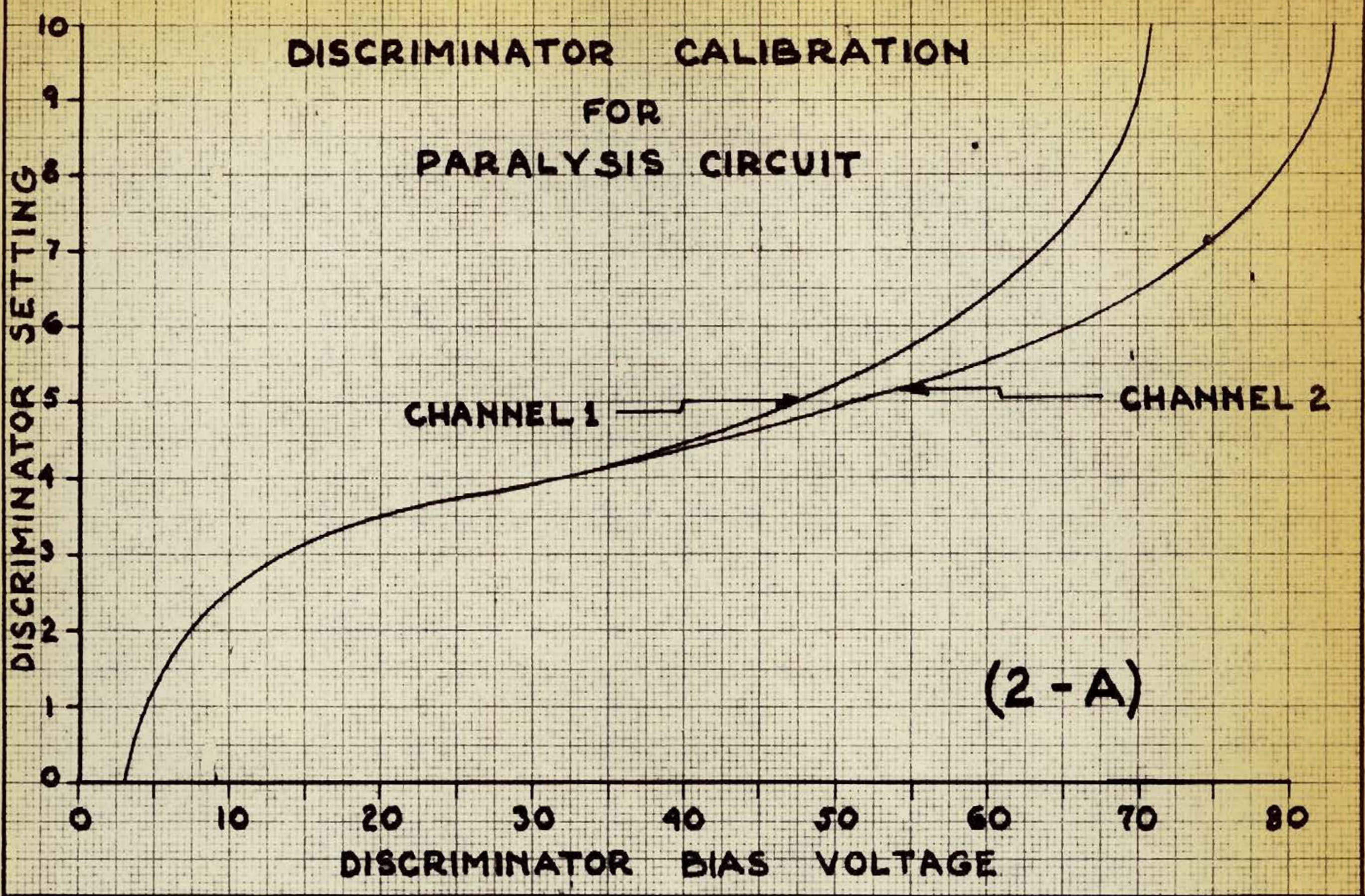
P.P.2 is a N.R.C. unit capable of supplying 180 milliamperes at 250 volts regulated. It originally came with the amplifiers, but has been modified to supply 150 milliamperes of d.c. heater current to the amplifiers and cathode followers, all connected in series. P.P.3 provides a 400 volt unregulated line and heater current for the 6AC7 of the recovery circuit in the paralysis unit. In addition, it helps to alleviate some load off the 300 volt line. P.P.4 supplies power to the double pipper constructed by Mr. A. Densmore.

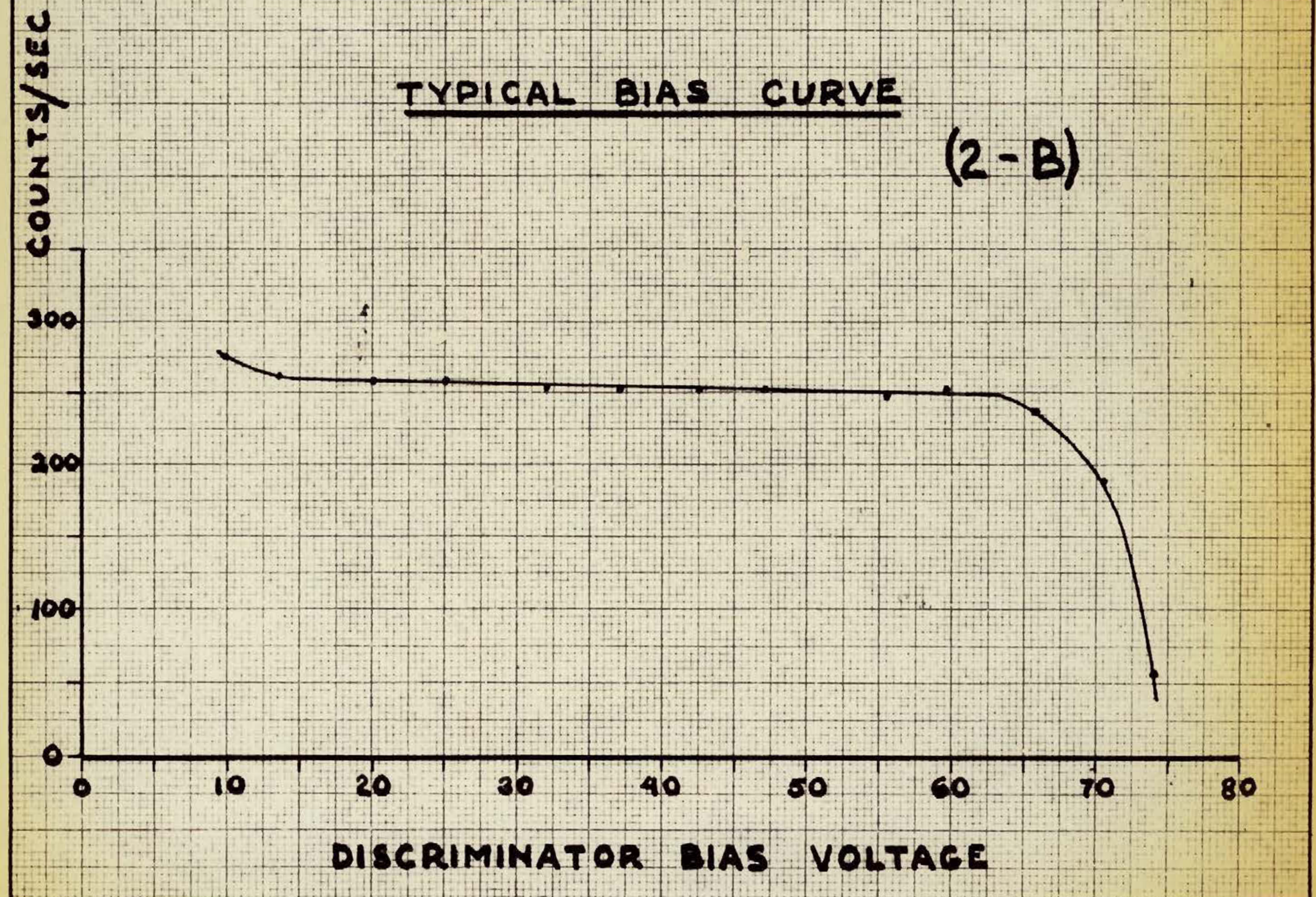
2.4 CALIBRATION OF CIRCUITS

The conditions required by theory for known, accurately reproducible paralysis times, coincidence resolving times and discriminator biases, made it essential that an accurate calibration be made of these circuits. In all such calibrations, extensive use was made of the double pipper, constructed by Mr. A. Densmore. This instrument enabled one to obtain recurring pairs of pulses of variable amplitude, 0 to 200 volts, and continuously variable time separation, 2 microseconds to 15 milliseconds. In addition, it supplied marker pips, of the type used in radar work, with recurrence frequencies of 100 kilocycles, 20 kilocycles, and 4 kilocycles. These marker pips were set against a crystal oscillator, and rechecked by direct counting into a scaler.

(1) DISCRIMINATOR

Two methods were employed in the calibration of the discriminator. The first consisted of feeding pulses from the double pipper into the discriminator circuit and checking the amplitudes of cut off points by means of a calibrated oscilloscope. The second method, which was actually used as an independent confirmation on the


DISCRIMINATOR CALIBRATION
FOR
PARALYSIS CIRCUIT
DISCRIMINATOR SETTING
0
1
2
3
4
5
6
7
8
9
10
CHANNEL 1
CHANNEL 2
(2 - A)
0
10
20
30
40
50
60
70
80
DISCRIMINATOR BIAS VOLTAGE


TYPICAL BIAS CURVE
(2 - B)
COUNTS/SEC
0
100
200
300
0
10
20
30
40
50
60
70
80
DISCRIMINATOR BIAS VOLTAGE

first, consisted of feeding the pulses into both the discriminator circuit and a T.R.E. scaler which had an accurately calibrated discriminator bias dial against which cut off points could be checked. The agreement between both methods was very good. Calibration curves obtained for "channels 1 & 2" are shown on graph 2-A.

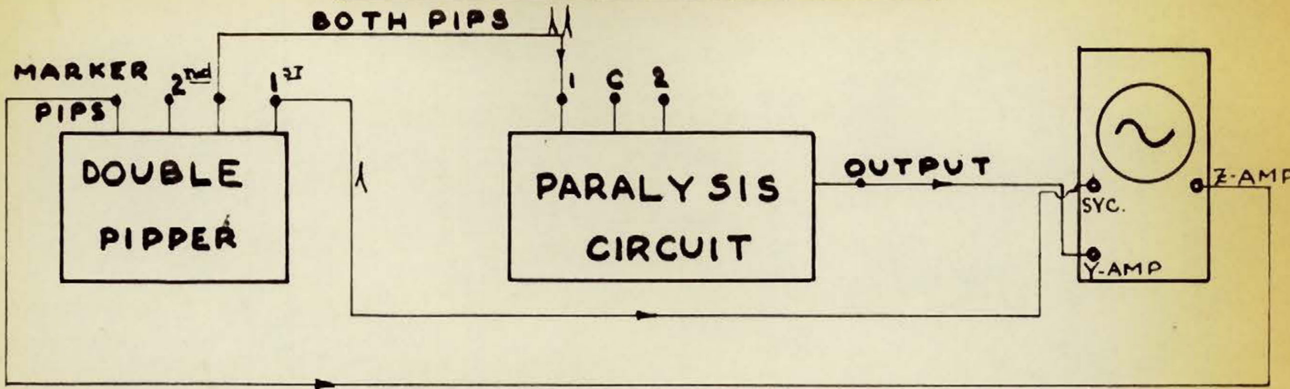
(2) PARALYSIS CIRCUIT

Three independent methods were employed in this calibration.

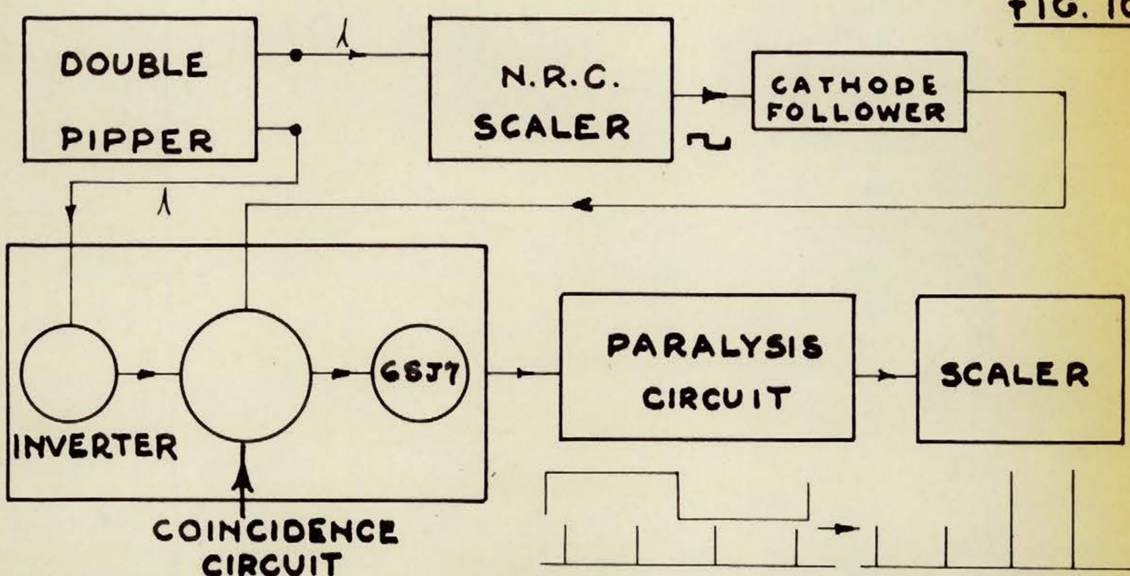
(a) (Refer to fig. 10A) Two successive pulses, the interval between them being variable, were fed from the double pipper into the channel of the paralysis circuit to be calibrated. The output from the paralysis circuit was connected to the vertical terminal of an oscilloscope. In addition, the first pulse was connected to the external synchronizing terminal of the oscilloscope. The radar type marker pips, against which the calibration was going to be made, were fed into the Z-amplifier of the oscilloscope and appeared as brightening pulses on the oscilloscope trace. Then, setting the paralysis control dials to specific values, the interval between the pulses

ARRANGEMENTS FOR CALIBRATION OF PARALYSIS CIRCUIT

A. FIG. 10

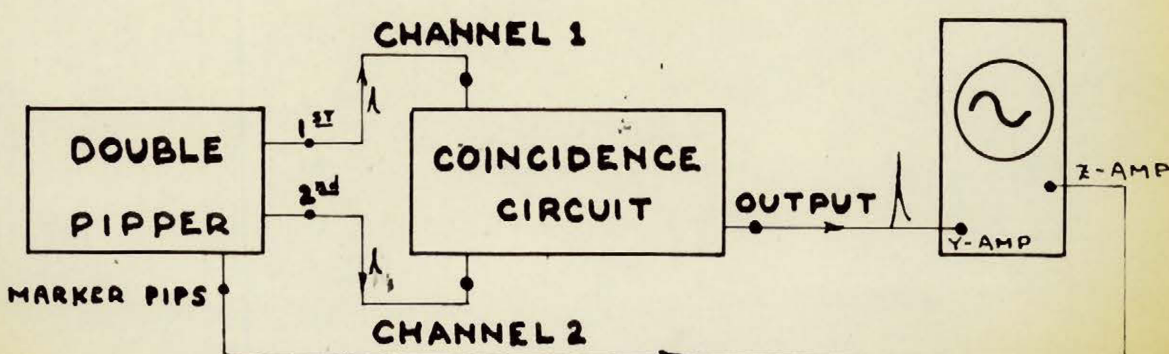


B. FIG. 10



ARRANGEMENT FOR CALIBRATION OF COINCIDENCE CIRCUIT

FIG. 11

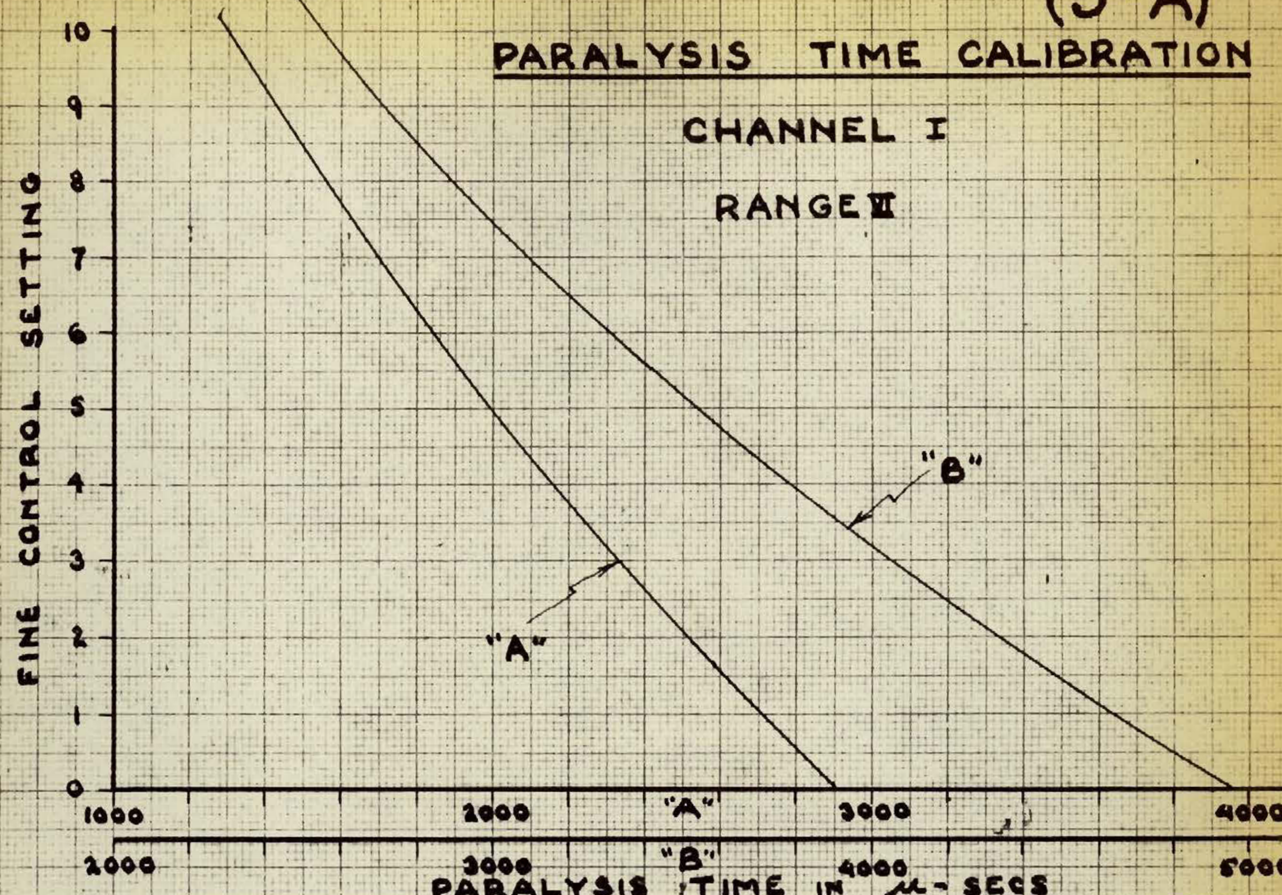


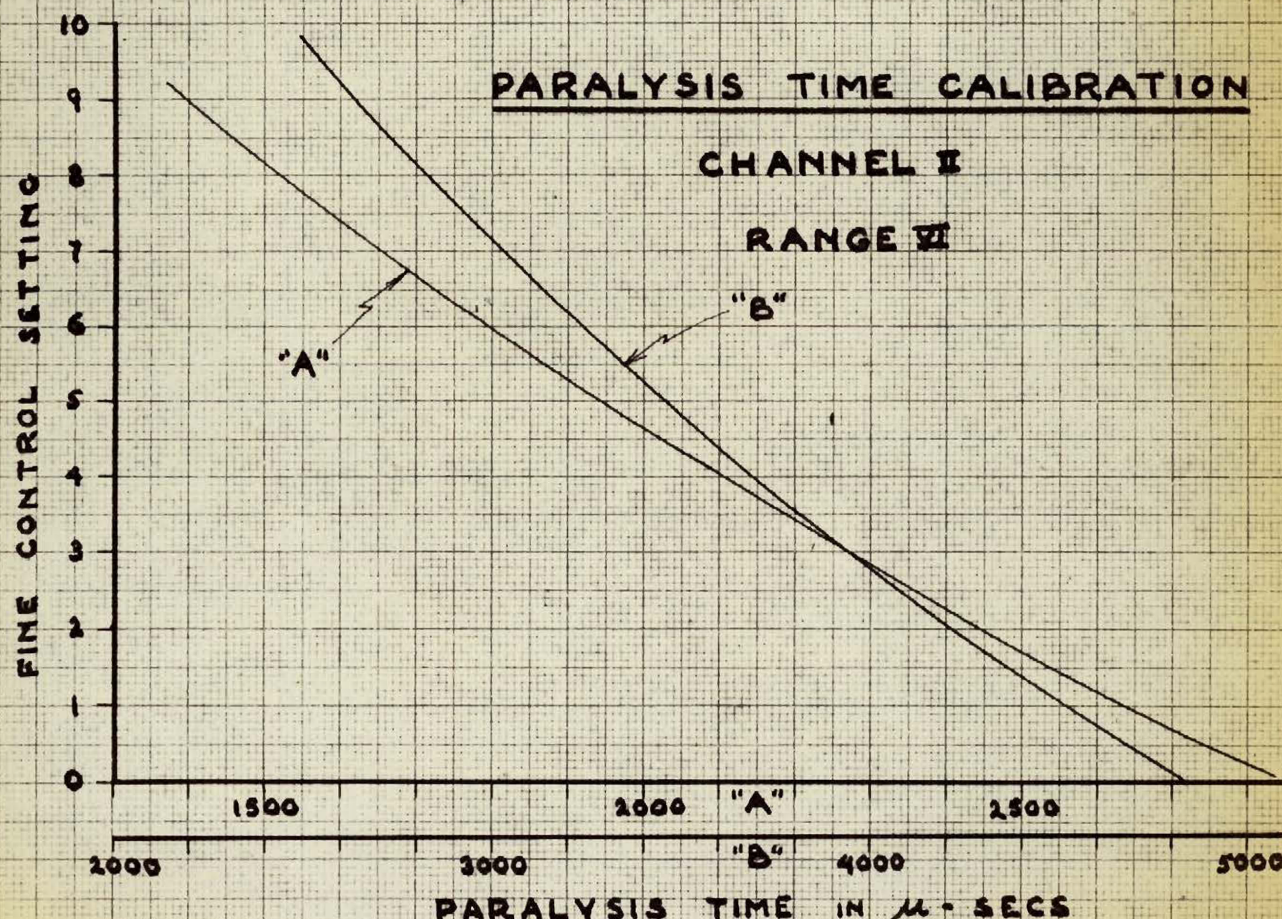
was varied until one pulse was just disappearing on the oscilloscope screen. The interval between pulses was then measured by means of the marker pips.

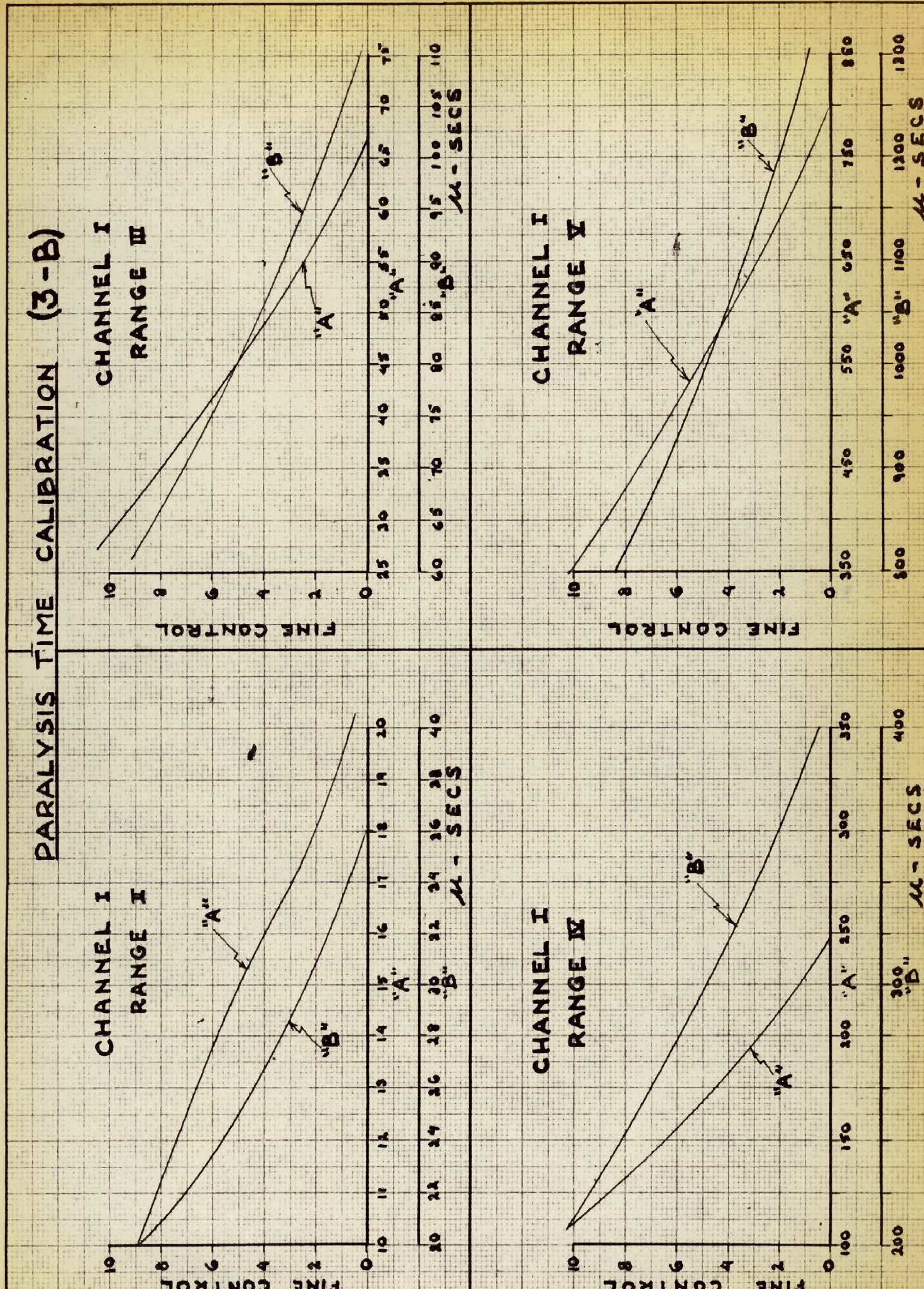
Assuming the marker pips are accurate, this calibration, except for very low paralysis times (0 to 40 microseconds), claims an accuracy of less than 1%. At 25 microseconds the accuracy is less than 2%. Calibration curves obtained in this way are shown on graphs 3-A, 3-B and 3-C.

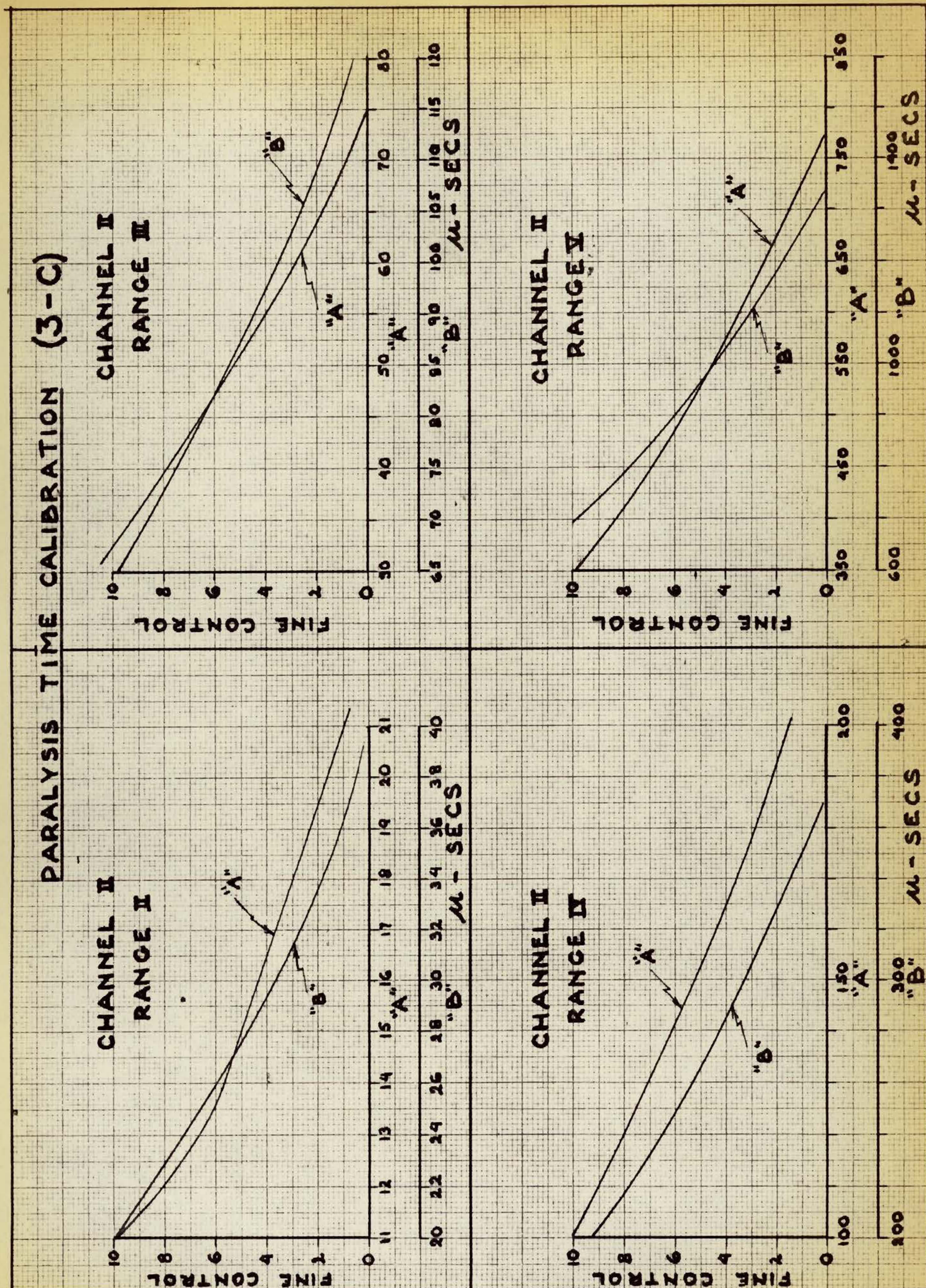
(b) This method did not employ an oscilloscope. Instead, a continuous set of equally spaced pulses were fed into the paralysis circuit whose output led to a scaler. The rate of arrival of the pulses was first noted on the scaler by straight counting, and the interval between pulses was thus computed. Subsequently, the paralysis controls were set so that the rate of arrival of pulses was diminished to three quarters of its full value. (The three-quarters rate was chosen because the transition from counting at the full rate to counting at half the rate, the latter meaning that the paralysis time was at least equal to the interval between pulses, was not sharp, but was represented by a small but finite interval on the control settings of the paralysis circuit. Thus, by using the three-quarters setting, one obtained

(3-A)


PARALYSIS TIME CALIBRATION
CHANNEL I
RANGE VI
FINE CONTROL SETTING
0
1
2
3
4
5
6
7
8
9
10
"A"
"B"
1000
2000
3000
4000
2000
3000
4000
5000
PARALYSIS TIME IN μ-SECS


PARALYSIS TIME CALIBRATION
CHANNEL II
RANGE VI
FINE CONTROL SETTING
0
1
2
3
4
5
6
7
8
9
10
"A"
"B"
1500
2000
2500
2000
3000
4000
5000
PARALYSIS TIME IN μ-SECS


PARALYSIS TIME CALIBRATION (3-B)
CHANNEL I
RANGE II
CHANNEL I
RANGE III
CHANNEL I
RANGE IV
CHANNEL I
RANGE V
"A"
"B"
FINE CONTROL
μ - SECS
10 11 12 13 14 15 16 17 18 19 20 "A"
20 22 24 26 28 30 32 34 36 38 40 "B"
25 30 35 40 45 50 55 60 65 70 75 "A"
60 65 70 75 80 85 90 95 100 105 110 "B"
100 150 200 250 300 350 "A"
200 300 400 "B"
350 450 550 650 750 850 "A"
800 900 1000 1100 1200 1300 "B"
0 2 4 6 8 10


PARALYSIS TIME CALIBRATION (3-C)
CHANNEL II
RANGE II
CHANNEL II
RANGE III
CHANNEL II
RANGE IV
CHANNEL II
RANGE V
FINE CONTROL
"A"
"B"
μ - SECS

a good mean value for the paralysis time.) The dial settings on the paralysis circuit were then compared with the calibration obtained in (a).

Doing this for a number of dial settings, it was found that the paralysis intervals obtained by this method were always approximately 4% less than those obtained in (a). This was soon explained on the basis of the recovery period of the paralysis circuit outlined in Section (2.3). The arrival of a continuous, equally spaced set of pulses was too much for a circuit whose recovery was not instantaneous. The result was that the effective average paralysis time appeared to be less than it actually was. To overcome this difficulty, the following method was suggested by Dr. C.H. Westcott.

(c) One pulse from the double pipper was fed into a N.R.C. scaler and its output from the scale of four stage was cathode followed into a Rossi coincidence circuit. The second pulse was fed into an inverter and then into the same Rossi circuit. In this way a set of pulses was obtained where alternate pairs of pulses were greatly diminished in amplitude (fig. 10B). By setting the discriminator so that the pairs of pulses with diminished amplitude were rejected, the paralysis circuit was allowed sufficient time to recover from one pair of large pulses before the next pair came along. The procedure then followed was the same as that in (b), except that to get the interval between pulses, one had to take the reciprocal of twice the full rate of arrival of the pulses of large amplitude.

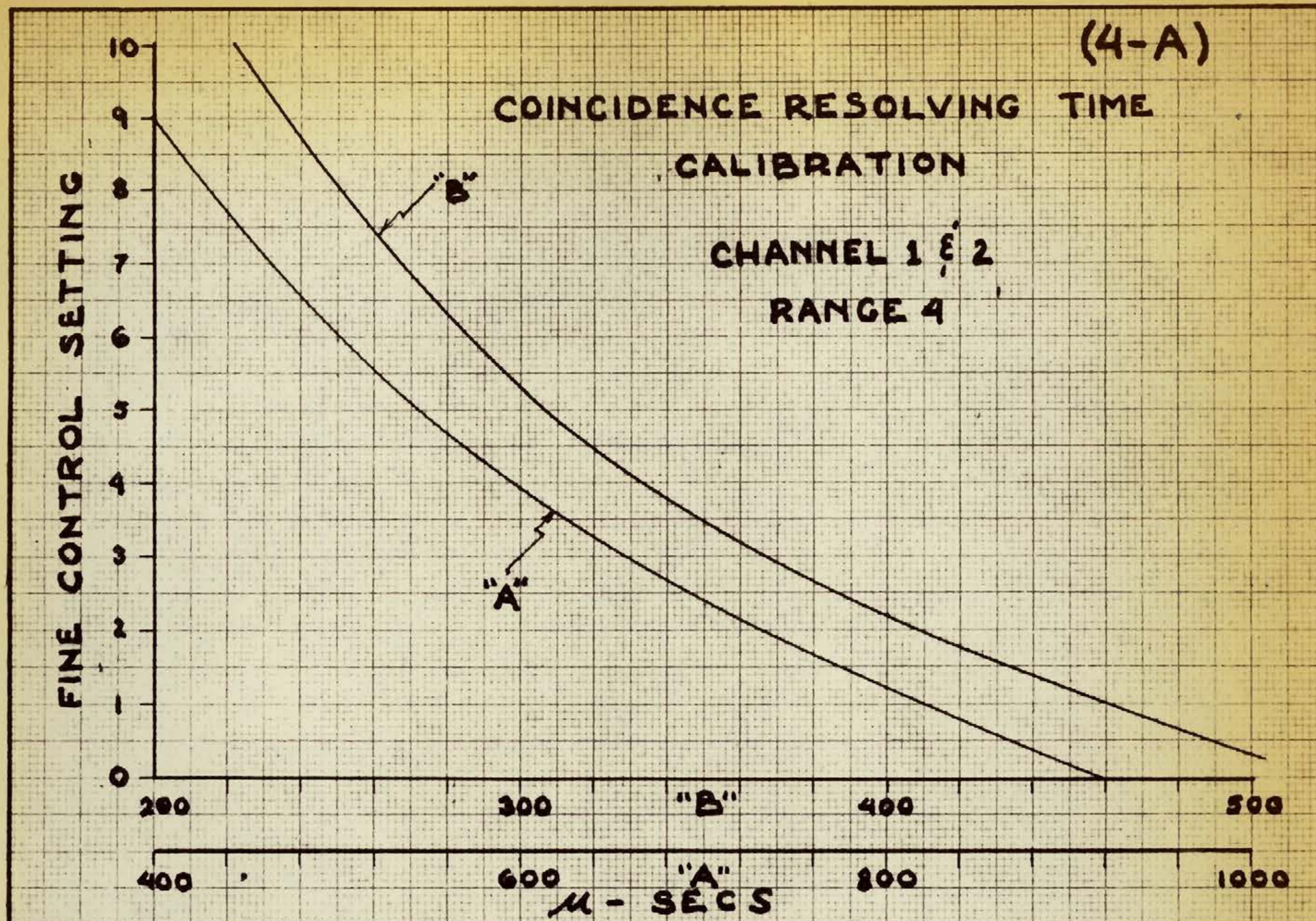
It was found that this method gave paralysis time intervals that agreed with (a) within one percent. This was as good as one hoped to get since the calibration by method (a) was only good to that accuracy. Therefore, the calibration obtained in (a) was adopted as the true calibration.

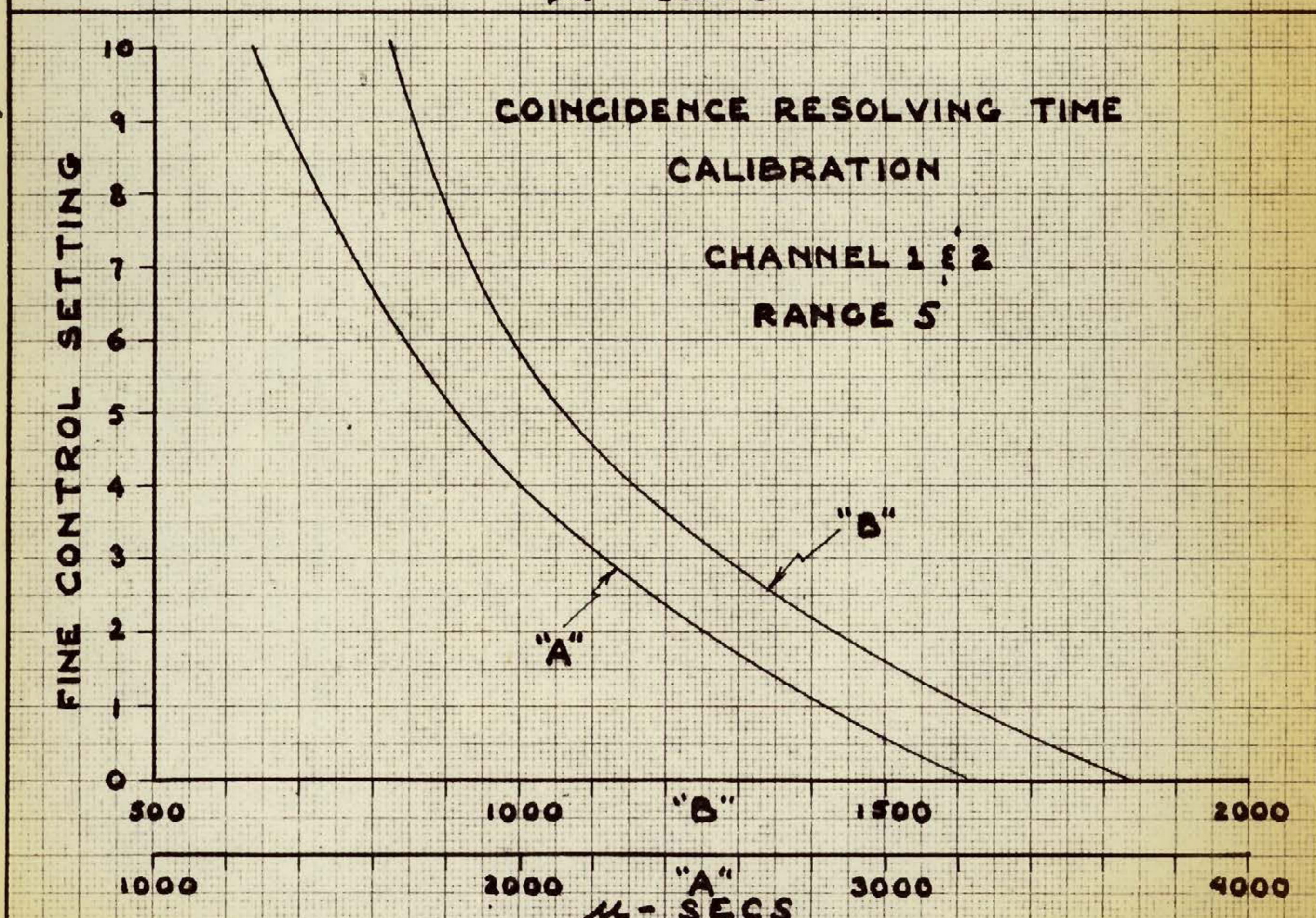
(3) COINCIDENCE CIRCUIT

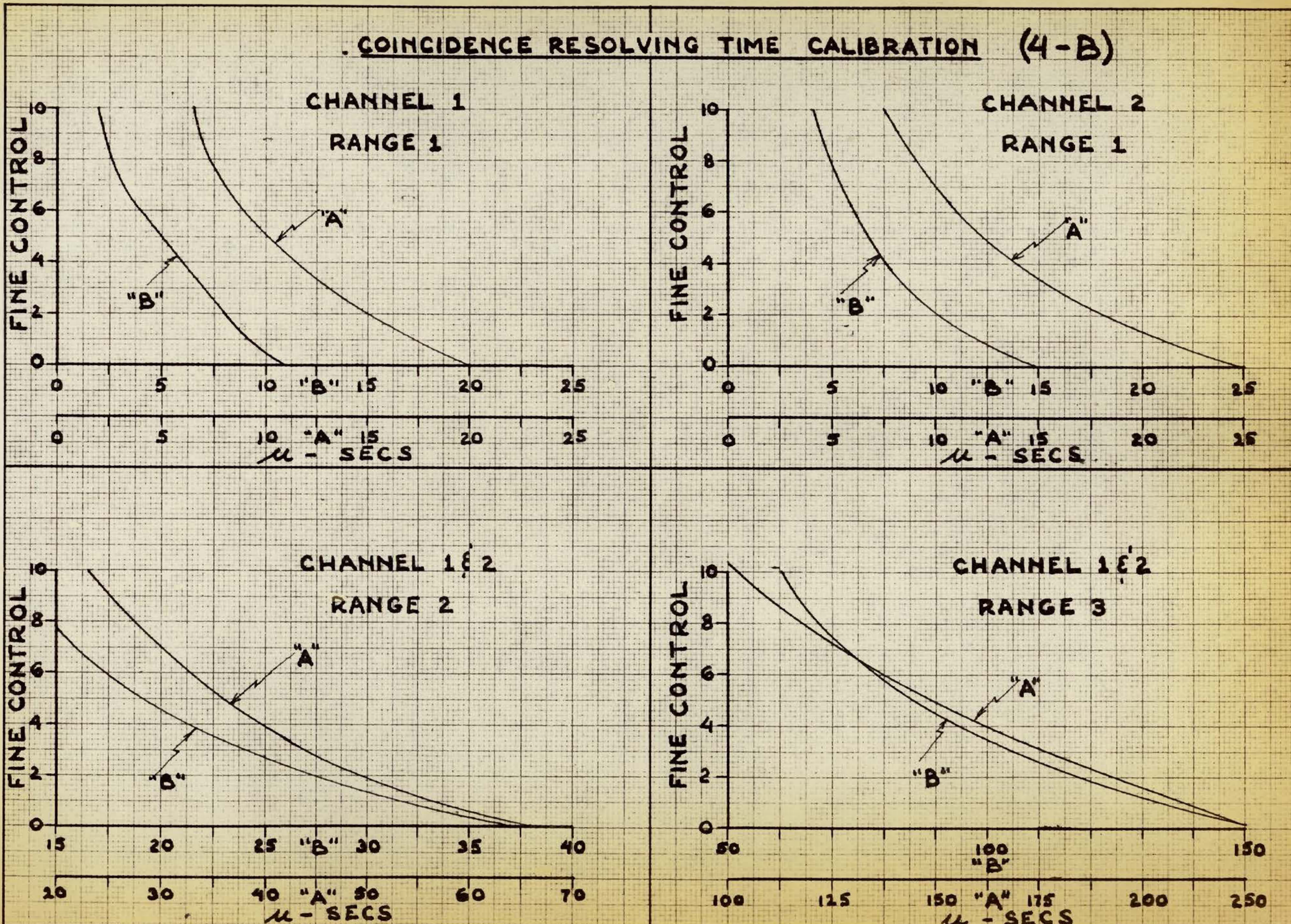
The method used in this case was the same as employed in part (2a). A block diagram of the method is shown in fig. 11. In the calibration of "channel 1", pulses 1 and 2 were fed into "channels 1 and 2" respectively. (In the calibration of "channel 2", pulse 1 was fed into "channel 2", and vica-versa.) The interval between pulses was then varied, and the square waves produced by the univbrators were observed at the plate of the Rossi tube. The width of the first square wave was measured against the marker pips by noting the point at which the large coincidence pulse first began to appear.

The interval between the leading edge of the first square wave and the point at which the large pulse first began to appear was taken as the width of the first square wave. The accuracy of measurement claimed is the same as in (2a). Calibration curves for the coincidence circuit are shown on graphs 4-A and 4-B.

Coincidence resolving time settings for both channels of the coincidence circuit are made by a common manual control. Since both univibrators circuits are not exactly identical due to deviations of circuit component values from their marked values (even though high precision components are used), the coincidence resolving times for both channels, for a given control setting, are not exactly the same. Thus an error is introduced by using a common manual control. Except for the lowest range, this error results in a deviation of the separately calibrated values of the coincidence resolving times of "channels 1 & 2" from the mean of these values at any control setting of less than 1½%. Such an error is easily tolerated for purposes of this investigation. For the lowest range the deviations from the mean are less than 3%, but at these low coincidence resolving times, this error is not important. Therefore, for all control settings of the coincidence circuit, the coincidence resolving


(4-A)
COINCIDENCE RESOLVING TIME
CALIBRATION
CHANNEL 1 & 2
RANGE 4
FINE CONTROL SETTING
0
1
2
3
4
5
6
7
8
9
10
"A"
"B"
200
300
"B"
400
500
400
600
"A"
800
1000
μ - SECS


COINCIDENCE RESOLVING TIME
CALIBRATION
CHANNEL 1 & 2
RANGE 5
FINE CONTROL SETTING
0
1
2
3
4
5
6
7
8
9
10
"A"
"B"
500
1000
"B"
1500
2000
1000
2000
"A"
3000
4000
μ - SECS


COINCIDENCE RESOLVING TIME CALIBRATION (4-B)
CHANNEL 1
RANGE 1
FINE CONTROL
"A"
"B"
"B"
"A"
μ - SECS
CHANNEL 2
RANGE 1
CHANNEL 1 & 2
RANGE 2
CHANNEL 1 & 2
RANGE 3

time used is the mean between the values obtained from calibrating "channel 1" and "channel 2" separately. For all ranges except range 1, this mean is plotted on graphs 4-A and 4-B.

SECTION III

EXPERIMENTAl PROCEDURES AND RESULTS

The procedures and results reported on below can be divided into three sections.

(1) Study of losses in single channel continuous counting.

(2) Study of losses in single channel pulsed counting.

(3) Study of losses in unpulsed coincidence counting.

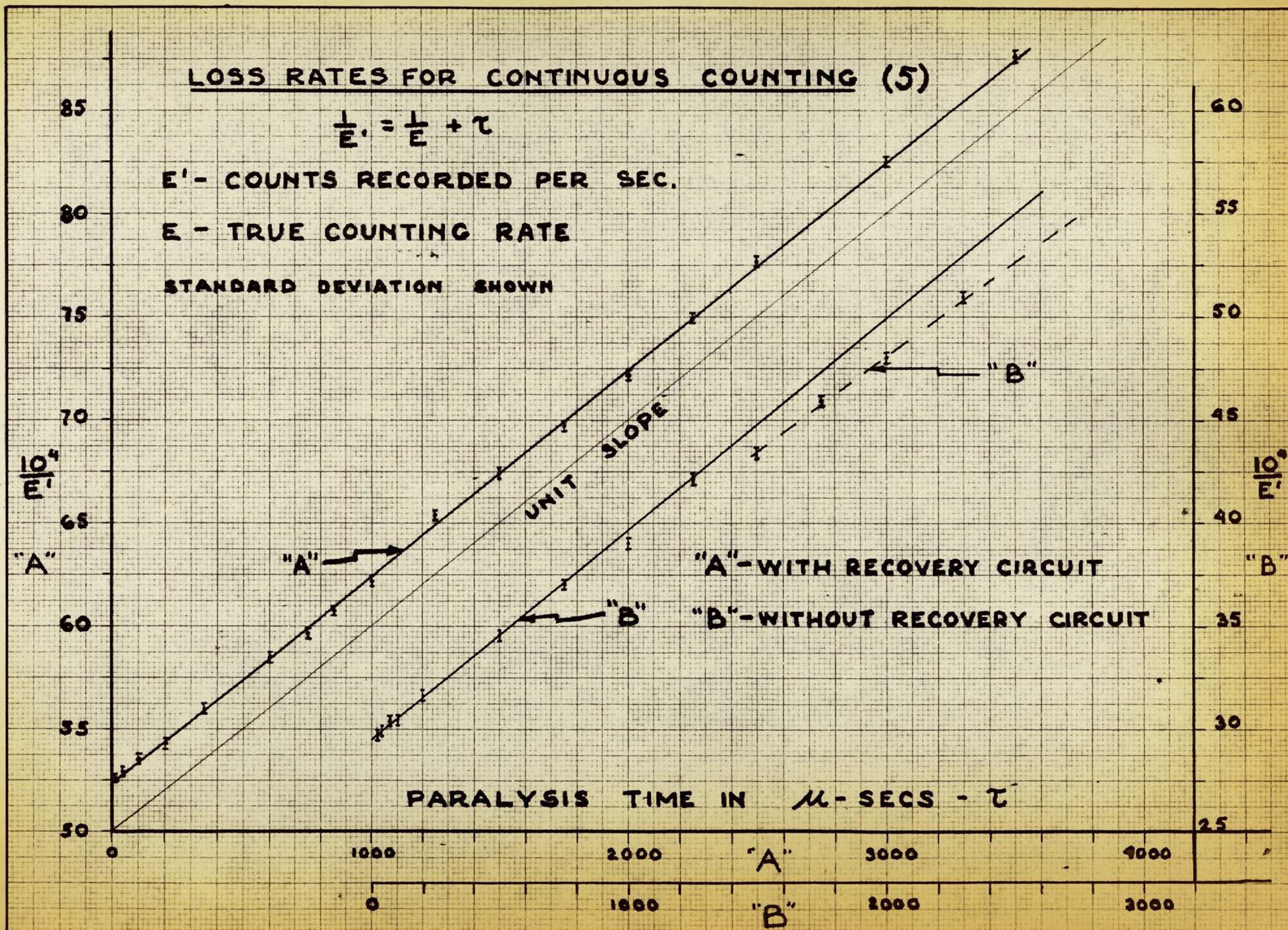
3.1 SINGLE CHANNEL CONTINUOUS COUNTING

Use was made of a source providing a true mean counting rate of approximately 200 counts per second. To obtain a continuous stream of particles, the shutter of the revolving disc was left open. Paralysis times varying from 10 microseconds to 4500 microseconds were chosen successively, and the count registered on the scaler for a period of 300 seconds was recorded in each case. Counting for 300 seconds made it possible to attain a statistical accuracy which was never in error by more than 0.5%.

It was found convenient to plot the reciprocal of the counts recorded against the paralysis time. Results agreeing with theory would then produce a straight line of unit slope.

Results obtained from experiment are shown on graph 5. Curve "A" was taken with the recovery circuit inserted in the paralysis unit. Curve "B" shows results obtained when no recovery circuit was used.

The sudden decrease in slope and increase in recorded counts of curve "B" occurs at a paralysis time of approximately 1350 microseconds. This is the point where one has to change from "range 5B" to "range 6A" on the paralysis circuit, i.e., the value of "C" changes from 0.006 microfarads to 0.03 microfarads, and "R_3", which was previously in the circuit, is now shorted. As pointed out in Section (2.3c), this represents the most undesirable situation. The value of "C" has been increased by a factor of 5, and by shorting "R_3", the impedance path "R_6", "R_2" has been decreased sufficiently to affect the current in the path of "R_7", both conditions contributing to an effective increase in the recovery time of the circuit. Approximate


LOSS RATES FOR CONTINUOUS COUNTING (5)
$\frac{1}{E'} = \frac{1}{E} + \tau$
E' - COUNTS RECORDED PER SEC.
E - TRUE COUNTING RATE
STANDARD DEVIATION SHOWN
UNIT SLOPE
"A"
"B"
"A"-WITH RECOVERY CIRCUIT
"B"-WITHOUT RECOVERY CIRCUIT
PARALYSIS TIME IN μ-SECS - τ
$\frac{10^4}{E'}$
"A"
85
80
75
70
65
60
55
50
$\frac{10^4}{E'}$
"B"
60
55
50
45
40
35
30
25
0
1000
2000
3000
4000
"A"
0
1000
2000
3000
"B"

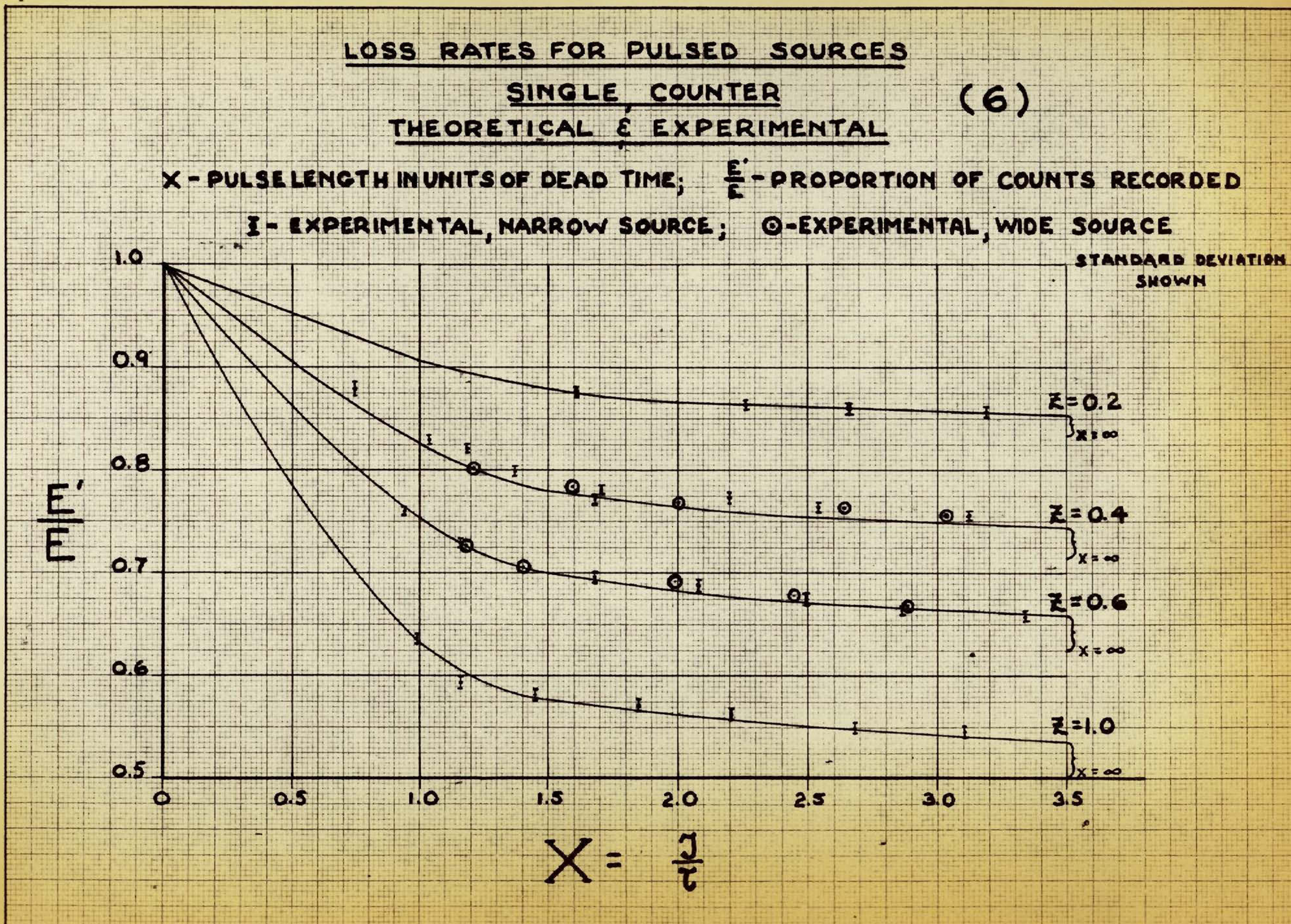
calculations show that this situation should result in a 4% increase in recorded counts and a 7% decrease in slope, due to a decrease in the effective average paralysis time. The results of the experiments bear these figures out. It was in this manner that the deficiency in the paralysis circuit was first discovered.

Curve "A", on the other hand, obtained with the recovery circuit present, has more than 2/3 of the experimental points lying on a line of slope unity. This curve again bears out the accuracy of the calibration, since a deviation of the order of a statistical error only represents a change in the calibration of 0.6%.

3.2 SINGLE CHANNEL PULSED COUNTING

An investigation was carried out as to the general agreement of experimental results with the theoretical curves plotted on graph 6. In addition, the investigation included a study of the magnitude of the effect of the pulse shape on the theory calculated with a rectangular shaped pulse.

For the former, two sources were used of dimensions


LOSS RATES FOR PULSED SOURCES
SINGLE, COUNTER
THEORETICAL & EXPERIMENTAL
(6)
X - PULSE LENGTH IN UNITS OF DEAD TIME; E'/E - PROPORTION OF COUNTS RECORDED
I - EXPERIMENTAL, NARROW SOURCE; ⊙ - EXPERIMENTAL, WIDE SOURCE
STANDARD DEVIATION SHOWN
Z = 0.2
Z = 0.4
Z = 0.6
Z = 1.0
X = ∞
E'/E
1.0
0.9
0.8
0.7
0.6
0.5
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
X = J/τ

and pulse shape shown on graph 1-A. This pulse shape approximated very closely to a rectangular pulse. The strengths of the sources were approximately 200 counts per second and 400 counts per second, so that without running the shutter at excessive speeds (which would tend to overheat the U-cups), points on the curves z=0.2, 0.4, 0.6 and 1.0 were easily obtainable.

For a curve of any given value of z, the procedure followed in obtaining an experimental point was as follows.

The true rate of counting during the pulse was first obtained by leaving the shutter open and counting for a sufficient length of time so that the statistical fluctuations were negligible. The dead-time to be used was then computed from, $\tau = \frac{z}{\nu}$. Similarly, the speed of rotation of the shutter for any given value of X was calculated from the following formula.

$$\text{Revolutions per second} = 1/12 \cdot \frac{10^6}{\tau X}$$

After obtaining a background count and setting the correct dead-time on the paralysis unit, the shutter was spun for five minute intervals (usually six such intervals) at approximately the speed calculated above.

Both the total count and shutter speed were recorded on the two scalers. From these were calculated the average counts per second and average revolutions per second. In addition, frequent checks were made of the expectation values for the true counting rate during the pulse and the background count. An average of all these values was used in the final calculations.

The total recorded count, corrected for background, was then calculated from the following formula.

$$E' = E'_n - 11/12E'_o + \frac{pT_w z\tau}{1+z}$$

where,

E'_n is the count actually recorded.

E'_o is the background count with the dead-time in the circuit.

The last correction term, which accounts for losses due to a count in the last τ of a pulse suppressing background counts, and background counts within a period τ before the pulse suppressing counts in the pulse, was only taken to the first order. This was justifiable since this correction was never greater than 1/3% of the total counts recorded, and was sometimes less than 1/6%.

Over the period of a run, there would usually be a decrease in the true counting rate during the pulse, and thus a change in the value of z. To account for this, a correction had to be made to the ratio E'/E. This correction was usually less than 0.1% and was approximated by changing E'/E by the same percentage and direction as $\frac{1}{1+z}$.

Serious difficulties were originally encountered due to the instability of the background count. This led to erroneous results that did not agree with the theoretical curves. It was found that the background count was very sensitive to counter voltage, gas purity and gas pressure. A slight change in the counter voltage usually resulted in a 5% to 10% change in background count. To overcome this difficulty the counter H.T. was changed from 1700 volts to 1600 volts where the bias curve for the background was not as steep as at 1700 volts. In addition, a long warm up period was allowed for the H.T. set, and preliminary runs were made with the motor to settle the gas in the chamber and counters.

For the case of the wider source, (graph 1-B) the same procedure was followed as outlined above. The rise time of its pulse shape was about 7½ times as long as the rise time of the two narrow sources, so that deviations from theory derived for a rectangular shaped pulse were expected to be quite noticeable if they were at all large.

Values of E'/E obtained in the above way for both the wide and narrow sources, are plotted on graph 6. In all cases the *standard deviations obtained are hardly ever greater than 0.55% and sometimes smaller than 0.4%.

The results obtained from the above experiments are as follows (refer to graph 6). The empirically obtained points for the narrow source follow the general shape of the theoretical curves very well. On the other hand, except for the curve z=0.2, the points on the average tend to fall slightly high. Excluding z=0.4, the estimated figure is something less than 1/3% high. For z=0.4, the points on the average lie 1.3% high. This discrepancy for z=0.4, is at present unexplainable. (The curve for z=0.4 was repeated with three different source strengths, the assumption being that the paralysis time used with the first source was somehow in error. All three sets of observations showed the same tendency for points to fall approximately 1.3% high. In addition, recalculation of the theoretical curve showed that no error had been made in those calculations.)

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*Standard deviations were calculated to the first order by using the formula $\pm\sqrt{N}.E'/E$.

Two explanations can be put forward to explain the slightly high lying points. One of these is contradicted by subsequently obtained experimental data, while the other is purely speculative.

(1) The tendency for the points to fall high may be due to ionization which occurs in the chamber but outside the counters. A millisecond or two after each pulse arrives from the source, this ionization may collect, enter a counter collectively, and thus produce a large enough pulse to register as a count. This process does not have to occur very often to account for the above error.

(2) The high points may be explained by the fact that the pulse shape used is not exactly rectangular. This explanation is contradicted by the results obtained with the wide source, which indicate that the finite width of the source has negligible effect, at least not large enough to account for the above discrepancy.

Intuitively, one would expect the points to fall higher for a trapezium shaped pulse than for a rectangular pulse. On the other hand, first order calculations show that the opposite effect is true. These calculations assume that $\tau \ll T$, or in other words, that the rise time of the pulse should be at least a few dead-times long. Since this condition does not exist in these experiments, no such prediction can be made. More exact and tedious calculations can be made, but the experimental results show that such calculations would not be advantageous. The only thing one can say, is that empirical indications are that a symmetric distortion from a rectangular wave has a negligible effect on the theory as calculated for a rectangular pulse shape.

Only a few points have been obtained for values of $x < 1$. In this region the proportion of counts recorded only depends on the product zx, i.e., on the total number of counts during the pulse, so that a point on one curve corresponds to points on the other curves. Therefore, in the sense that this region was not very instructive, it seemed unnecessary to spend too much time over it.

3.3 UNPULSED COINCIDENCE COUNTING

The extent to which this investigation was undertaken, was to establish both experimentally and theoretically three important points. (1) The magnitude of the losses to be expected in coincidence counting; (2) the variation of losses in coincidence counting with dead-time; (3) the variation of random coincidence counts with the resolving time of the coincidence circuit. In all cases the experimental results were to be compared with the theoretical predictions given in Section (1.3).

For the above investigations, the apparatus was set up as outlined on page 22. Since there were only two scalers available, and one was occupied counting coincidences, only the singles rate for one channel at a time could be recorded. As in Section (3.1), the nature of a continuous source was obtained by leaving the shutter of the revolving disc in an open position.

Before embarking on any of the above experiments, one first had to establish the correct discriminator working levels for both channels of the paralysis circuit. In this case, this process was a little more involved than the simple operations that had to be performed in Sections (3.1) and (3.2). The reason for this was that the three counters did not all produce pulses of exactly the same amplitude, and the three amplifiers did not all have the same amplification. Consequently, the output pulses from the amplifiers were not of a uniform amplitude, but instead depended on the counter-amplifier combination that was used. In addition, the output pulses of "amplifier 2" (fig. 1) were considerably diminished in amplitude in comparison with the two outer "amplifiers 1 & 3". "Amplifier 2" was loaded by two grids of the mixer while the latter amplifiers individually only fed pulses to one grid. The sum total effect of the above situation was to produce a discriminator bias curve which was very unlike the relatively flat curve shown on graph 2-B. Instead, for each channel, a step like curve resulted with two separate plateaus and a rapid tailing off at the end of each plateau.

In choosing the correct discriminator bias level for each channel, a compromise had to be made between points on the flat portion of the bias curve for the singles counts, and points on the flat portion of the bias curve for the true coincidence counts coming from "counter 2". This was done by taking separate bias curves with "amplifiers 1 & 3" disconnected and "amplifier 2" connected, and alternately with "amplifiers 1 & 3" connected and "amplifier 2" disconnected. In addition, due to the unequal gains of the mixer's cathode followers, the bias levels on "channels 1 & 2" had to be chosen so that in the former position equal true coincidence counts were recorded in both channels. This latter adjustment was very critical, and, in addition, very sensitive to impurities in the counter gas and counter anode voltage. Consequently, whenever the gas or anode voltage were altered, the discriminator bias levels had to be reset.

In the study of the variation of losses in coincidence counting with the dead-time τ_e, special attention was paid to two important quantities. The first of these was C/c, the proportion of true coincidence counts recorded. It gave an indication of the magnitude of the losses expected. The other quantity, γ , determined the variation of the recorded coincidence counts with dead-time. γ was found to be very sensitive to the singles count recorded in each channel (i.e., to the quantity ρ_1 & ρ_2), and to the proportion of coincidence counts recorded (C/c). Its sensitivity to the important parameters made γ an ideal quantity to use from which maximum information could be extracted. γ was calculated experimentally from the formula,

$$\gamma = \frac{C}{c} \cdot \frac{1}{\rho_1 \rho_2}$$

Since both γ and C/c are both defined for $\tau_c=0$ (refer to page 16), strictly speaking, experimental measurements had to be made with this coincidence resolving time. This, of course, was not possible, since the lowest coincidence resolving time available with the coincidence circuit was 3 microseconds. Consequently, all measurements had to be extrapolated to zero coincidence resolving time by employing two runs with different τ_c for each setting of τ_o, and then extrapolating linearly to τ_c=0. The linear extrapolation was justified by experiments performed at a later date.

The procedure followed in obtaining experimental results for the calculation of γ and C/c was as follows.

The true mean coincidence counting rate and the true singles rates in each channel were first obtained by setting τ_o in each channel equal to 11 microseconds and then extrapolating the count observed for 500 seconds to τ_o=0. (The true mean coincidence counting rate was obtained by disconnecting "amplifiers 1 & 3" and counting particles from the centre amplifier, while the true singles rate for each channel was obtained by disconnecting the centre amplifier and observing the counts recorded for each channel separately.) The above measurements were again repeated at the end of an experimental run, and an average of the two was taken. Thus the quantities a, b and c were known to a statistical accuracy of 1/5%. Successive equal* paralysis times varying from 11 to 3200 microseconds

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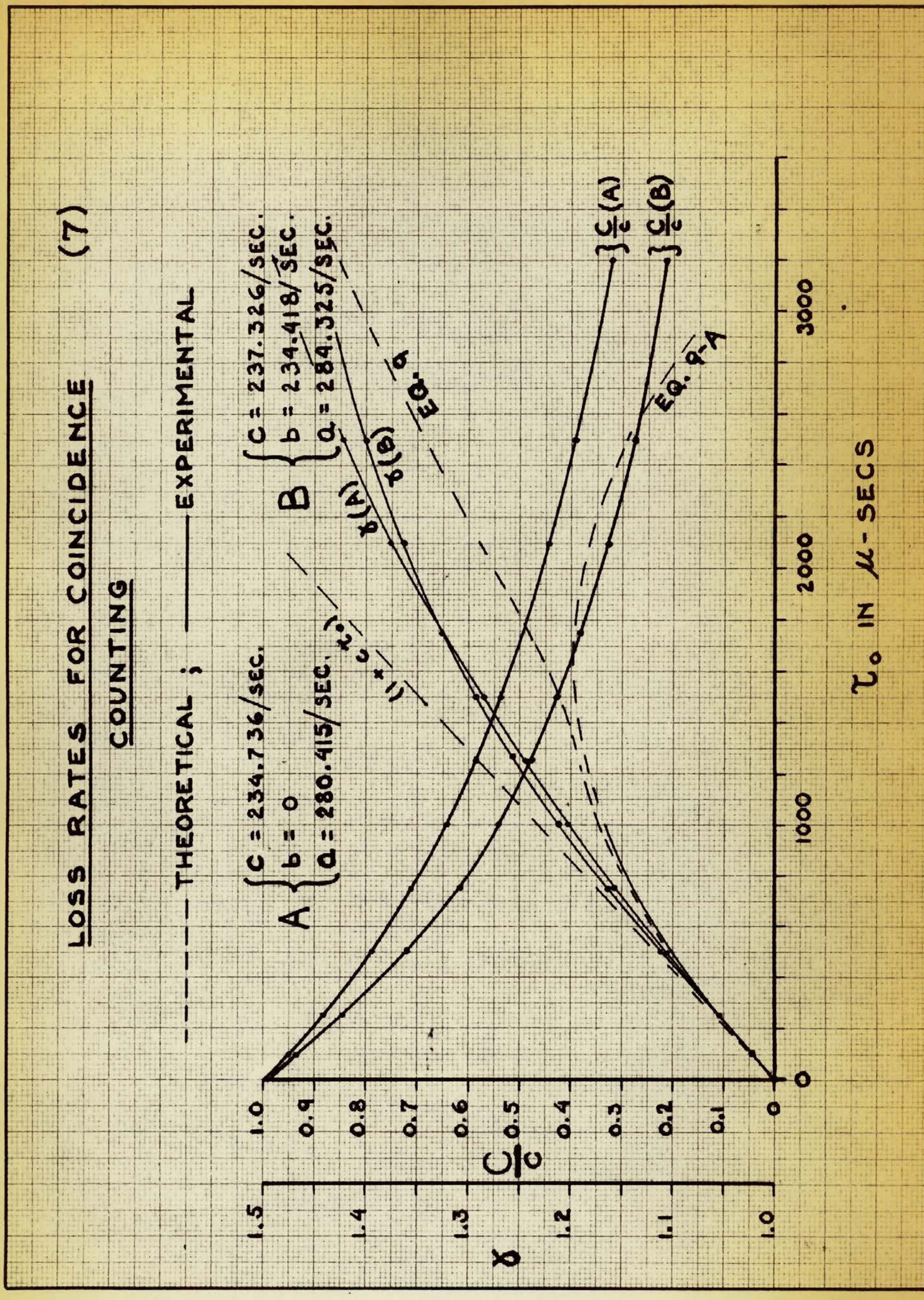
*Equal paralysis times for both channels are required by theory (page 15).

were then chosen for both channels, and the coincidence count registered on the scaler for a period of 500 seconds was recorded in each case. As mentioned in the above paragraph, two coincidence resolving times, were used for each setting of τ_0, so that an extrapolation could be made to zero coincidence resolving time.

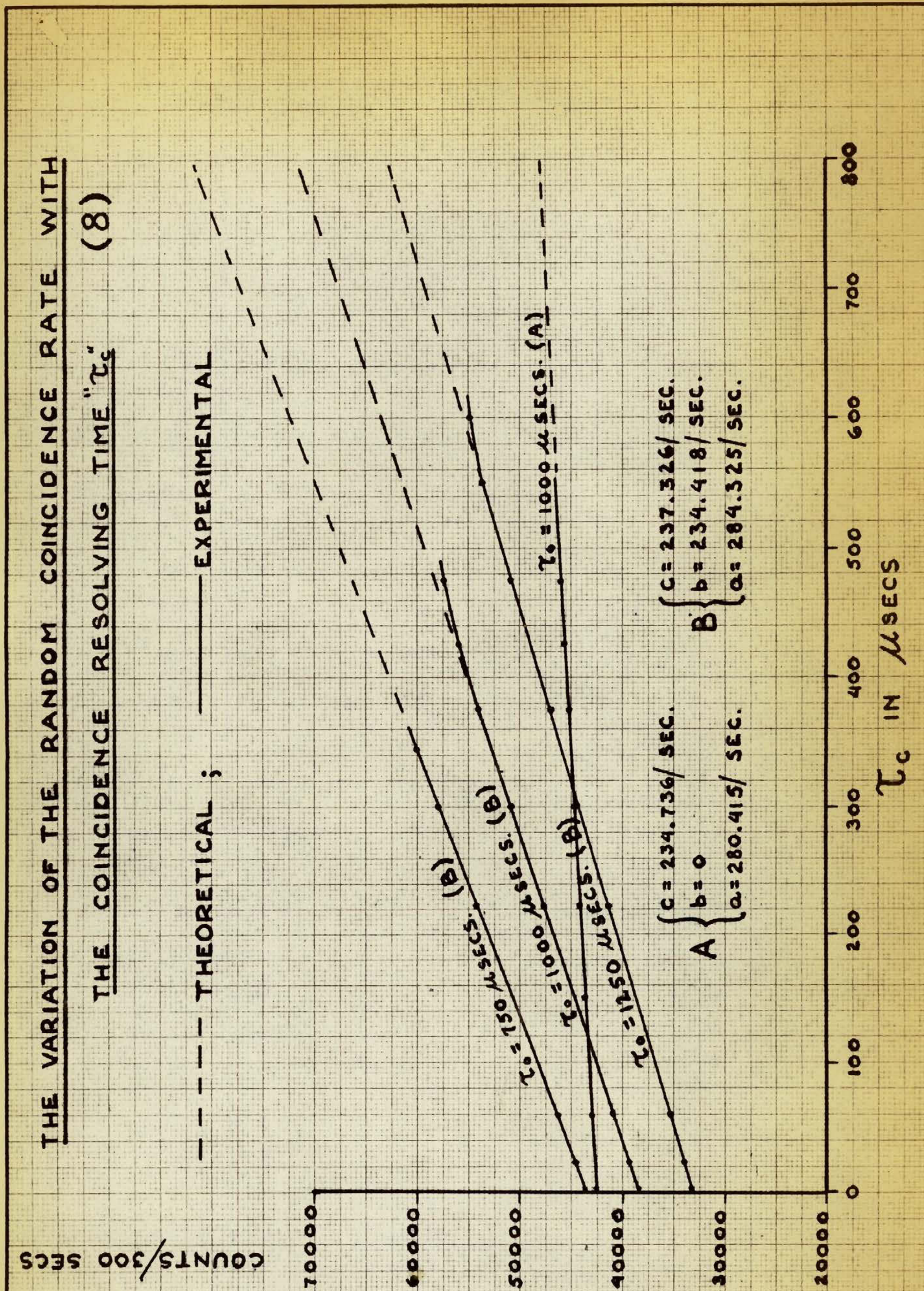
Values of C/c and γ, calculated from measurements taken in a manner outlined above, are plotted on graph 7. The graph shows two sets of such values computed from two independent sets of experimental observations in which the quantities a, b and c differ. In one case, a, b and c are approximately equal, so that at τ_0=1060 microseconds Z_1=0.55, Z_2=0.50 and Z_c=0.25. In the other case, a and c are approximately equal and b=0, so that Z_1=0.55, Z_2=0.25 and Z_c=0.25 at τ_0=1060 microseconds. For the former case, theoretical curves for γ are also plotted using both the exact equation (9) and the approximate equation (9a). In addition, the first order approximation for γ is shown (γ =1+cτ_0). This is an exact expression for γ for the case where a=b=0.

It is readily seen that theoretical predictions differ radically from the experimental observations. The meaning and practical significance of this is discussed fully under the heading of "Discussions of Results and Conclusions".


LOSS RATES FOR COINCIDENCE COUNTING
(7)
THEORETICAL ; ——— EXPERIMENTAL
A
C = 234.736/SEC.
b = 0
a = 280.415/SEC.
B
C = 237.326/SEC.
b = 234.418/SEC.
a = 284.325/SEC.
γ(A)
γ(B)
EQ. 9
EQ. 9-A
C/c (A)
C/c (B)
γ
1.5
1.4
1.3
1.2
1.1
1.0
C/c
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1000
2000
3000
τ₀ IN μ-SECS

In contrast to the above results, experimental observation as to the variation of recorded random coincidence counts with τ_c for constant τ_o, shows excellent agreement with theory. As originally predicted, the squared term in τ_c^2 containing k has negligible effect in the practical case where $\tau_c < \frac{1}{2}\tau_o$. The rate of change of recorded random coincidences with τ_c is successfully predicted by $\gamma\rho_1\rho_2(bc+ca+2ab)$, where the γ used is an experimental value.

Experimental results for the above investigation were obtained by setting a given dead-time on the paralysis circuits and observing the coincidence count as τ_c was varied from 3 microseconds up to $\frac{1}{2}\tau_o$. As in the previous experiments, the operation was performed for two sets of values of a, b and c. The dead-time, τ_o, was set to values of 750, 1000, and 1250 microseconds. Experimental curves obtained in this way, together with theoretically predicted curves are plotted on graph 8. Agreement is well within statistical accuracy.


THE VARIATION OF THE RANDOM COINCIDENCE RATE WITH
THE COINCIDENCE RESOLVING TIME "τ_c" (8)
- - - - - THEORETICAL ; ——— EXPERIMENTAL
τ_0 = 750 μSECS. (B)
τ_0 = 1000 μSECS. (B)
τ_0 = 1250 μSECS. (B)
τ_0 = 1000 μSECS. (A)
A { c = 234.736/ SEC.; b = 0; a = 280.415/ SEC.
B { c = 237.326/ SEC.; b = 234.418/ SEC.; a = 284.325/ SEC.
COUNTS/300 SECS
70000
60000
50000
40000
30000
20000
0
100
200
300
400
500
600
700
800
τ_c IN μSECS

DISCUSSION of RESULTS

and

CONCLUSIONS

The work reported on in this thesis has been categorically presented under the following headings:

(1) Design and construction of apparatus.

(2) A study of losses in single channel counting with a continuous source.

(3) A study of losses in single channel counting with a pulsed source.

(4) A study of losses in coincidence counting with a continuous source.

While (2) was used as a cross-check on the behavior of (1), (3) and (4) were studied independently.

(1) & (2). The design and construction of the apparatus was at all times guided by the assumptions of the theory. Of primary importance were the following two assumptions: (a) the source of ionizing particles had to form a random distribution; (b) the mechanism of the dead-time had to be such that the inoperative period only followed counts which were recorded. The

first was achieved by using a Polonium source. The second assumption was satisfied by using proportional counters in conjunction with circuits whose operation and calibration were fully accounted for in the text. As pointed out in Sections (2.3b) and (3.1), in their original form, these circuits showed certain deficiencies which led to erroneous results. It was in the alleviation of these deficiencies that an investigation into (1), (which otherwise would have been trivial), proved indispensable. It not only pointed out explicitely the faults of the circuits, but when these were corrected for, showed beyond doubt that the circuits functioned according to the conditions prescribed by theory. Thus, these preliminary experiments showed that in practical cases where electrical counting was employed, one could at least in principle insert electronically in the counting system a definite dead-time of the characteristics required by theory, and employ the results obtained from investigations of (3) and (4) in calculating the loss rates encountered in experiments.

(3). In Section (3.2), we see that experimental results obtained for (3) are in fairly good agreement with theory. (As pointed out before, the discrepancy in

$z=0.4$ is at present unexplainable.) How good one considers the agreement depends on how accurately one has to calculate loss rates. If the error tolerated is about 1/3% in the total count or 1% in the loss rate, the results obtained in this investigation are more than sufficient. For experiments where higher accuracy is required, further considerations have to be made into the specific nature of the experiment and the counting system employed.

As regards to the general nature of pulsed single channel counting, the following conclusions are evident from both theoretical and experimental considerations.

(a) For a mean counting rate during the pulse which is the same as for continuous counting, the loss rates in pulsed counting are less than in continuous counting. This is due to the fact that the probability that a count occuring in the first half of a pulse is recorded is relatively high.* Of course, in a practical case the above has little meaning, since one is not usually interested in the counting rate during the pulse, but in the mean overall counting rate over a period of time. The latter

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*Probability curves are shown on page 516 of reference 1.

usually has to be high to assure good statistical accuracy. This means that the counting rate during the pulse is usually very high resulting in large losses.

(b) Results show that it is desirable for the resolving time of the counting system to be much shorter than the pulse length of the pulsed source used. In practical circumstances, this is usually very difficult to achieve, since modern accelerators usually have pulse lengths of the order of a microsecond or less. On the other hand, it is undesirable for the dead-time of the counting system to be longer than the pulse length of the source. This situation results in very high losses. In the practical cases, where the pulse length is only a few times greater than the dead-time, losses are tolerable for values of z less than approximately 0.5. At this point they begin to exceed 30% of the total count.

(c) A symetrical distortion from a rectangular pulse shape has a negligible affect on the theory as calculated for a rectangular pulse. This, of course, does not mean that the above will be true for all types of pulse shapes, but it does indicate that the effect is smaller than originally expected.

In the application of the results and formulas obtained for pulsed single channel counting to actual experimental results, the procedures followed in the experiments and calculations in this thesis will have to be reversed. In all our calculations it was assumed that the true counting rate was known. By applying the correct equations, we were able to deduce the recorded one, and thus compare calculations with experiment. However, in actual practice, the problem will be the converse. One then has the recorded count and has to deduce the true count. In practice, however, this should cause little difficulty so long as the loss rate is not too great. A process of successive approximations, of assuming a true counting rate, should very rapidly lead to a correct result.

It should be mentioned that the accuracy with which the loss rates can be calculated will usually be limited by the uncertainty in our knowledge of the shape of pulse being used (i.e., the effective width of the pulse if its distortion from a rectangular pulse is not too large), and of the exact length of the dead-time inserted in the counting system. An accurate measurement of the former will usually be more difficult to obtain than an exact figure for the latter.

(4) Experimental results for loss rates for continuous coincidence counting show a general disagreement with theoretical computations. In all cases studied, theoretical values for γ computed from both equations (9) and (9a) fall short of the experimental points. Equations (9) and (9a) agree closely up to $z_c=0.25$. At about this value of z_c, equation (9) begins to behave somewhat errantically, producing a curve with two points of inflection. γ increases in value rapidly, until, at about 3200 microseconds, it approaches the experimental value of γ. Actually in the region up from 2500 microseconds, γ, as calculated from equation (9), is not reliable because of an approximation made in the theory. On the other hand, equation (9a) is in essence a parabola. Therefore γ, calculated from this equation, behaves like a parabola, increasing at first up to a maximum value of approximately 1.2 at $\tau_0=1700$ microseconds, and then decreasing rapidly to the value of unity at approximately $\tau_0=3200$ microseconds. Thus at large values of z_1, z_2 and z_c, equation (9a) is a bad approximation to equation (9). In addition, it is fundementally wrong since it does not reduce to $\gamma=1+c\tau_0$ when we put $a=b=0$.

The experimental curves for both sets of values of a, b and c show consistent behavior. The fact that they cross has no significance and can be explained on the basis that the expectation values for a and c are different in the two cases. Actually curve "A" should be above curve "B" at all times. A glance at these curves and the curve for which $a=b=0$, immediately shows that γ is not a function of a and b or of the sum $(a+b)$ in an equation of the (9a) type. If that was so, changing from a set of values of a, b and c where a and b are nearly equal (curve "B"), to a set where either a or b was zero (curve "A"), would have made the curve "A" fall about half way in between curve "B" and a curve where $a=b=0$. This did not happen. Instead the curve "A" only shifted about 1/3 of the required distance, which indicates that γ is more likely a function of the sum $(z_1+z_2+z_c)$ in a equation of type (9a). The objection to this is the same as the objection to the original form of equation (9a). When we put $a=b=0$ in an equation containing the sum $(z_1+z_2+z_c)$ in the τ_o^2 term, it will not reduce down to $\gamma = 1+c\,\tau_o$. Attempts have been made to devise empirical equations that would fit both curve "A" and curve "B". These have proved unsuccessful.

Though experimental and theoretical agreement has been lacking in these experiments, one original suspicion has been verified. A glance at the C/c curves shows that the loss rates for coincidence counting are very large, a premonition which motivated these experiments. With the counting rates indicated on graph 8, the proportion of counts recorded reached a value of 0.5 at about 1100 microseconds. This indicates that for efficient coincidence counting, conditions are very stringent, and loss rate corrections are very important.

Investigation into the variation of random coincidence counts with coincidence resolving time proved to be very successful. Experimental results showed excellent agreement with theory, indicating that an excellent approximation could be obtained for the random coincidences recorded by just using the linear term in τ_c in equation (10), and neglecting the τ_c^2 term. The validity of this approximation greatly simplifies calculations.

Considering the quantity γ again, we find that the present state of affairs is not very satisfactory. If the experiments are to be taken as a correct criterion (as would be indicated by results obtained for both pulsed single channel counting and the investigation

into variation of random coincidence counts with τ_c),
it seems necessary that some fundemental changes have to be made in the theory. Actually before one can be that definite, a large program of work must be first undertaken to determine exactly how γ would vary with many different combinations of the quantities a, b and c. A large collection of such data would probably give a good indication as to the root of the trouble. Unfortunately, such a large program of experiments is beyond the scope of this thesis, making it impossible at present time for the author to offer any definite conclusions on the inadequacies of the theory.

BIBLIOGRAPHY

1. Westcott, C.H., Proc. Roy. Soc. A 194, 508, 1948.
2. Westcott, C.H., Unpublished; read in abstract at the meeting of the Royal Society of Canada on June 7th., 1950.
3. Ruark, A.E. & Brammer, F.E., Phys. Rev. 52, 322, 1937.
4. Bateman, H., 1910, Phil. Mag. (6), 20, 704.
5. Feather, N., Proc. Camb. Phil. Soc. 45, 648, 1949.
6. Skinner, S.M., 1935, Phys. Rev. 48, 438.
7. Locher, G.L., 1933, J. Franklin Inst. 216, 553.
8. Marsden, E. & Barrett, T., 1911, Proc. Phys. Soc. London. 23,367.
9. Hole, N., 1946, Ark. Mat. Astr. Fys. 33A, No. 11.
10. Voltz, H., 1935, Z. Phys. 93, 539.
11. Wilson, R.R., Lofgren, E.J., Richardson, J.R., Wright, B.T. & Shankland, R.S., 1947, Phys. Rev. 71, 560.
12. Korff, S.A., Nucleonics 6, 5-10 (June 1950).
13. Korff, S.A., Nucleonics 7, 46-52 (Nov. 1950).
14. Korff, S.A., Nucleonics 8, 38-43 (Jan. 1951).
15. Elmore, W.C., Nucleonics 6, 26-34 (Jan. 1950).
16. Hartog, H.D. & Muller, F.A., Physica, 16, 17-29 (Jan. 1950).
17. Hartog, H.D., Nucleonics 5, 3-47 (Sept. 1949).
18. Sharpe, J. & Taylor, D., Inst. Elect. Engrs., Paper 1078.
19. Binder, D., Phys. Rev. 76, 856-8 (Sept. 15th., 1949).