

PARETO OPTIMALITY AND BEYOND

by

Li, Cheuk Ming

Department of Economics

McGill University, Montreal

April, 1985

A thesis submitted to the Faculty of
Graduate Studies and Research in partial
fulfillment of the requirement for
the degree of Doctor of Philosophy.

© Li, Cheuk Ming, 1985

to my family

ABSTRACT

The problem of social choice is the central theme of this study. Our main objective is to prove the existence of a social welfare function in order to put to rest the doctrine of 'natural liberty.' We reject most of the recently suggested solutions to the problem on the basis that they are either incomplete or inconsistent. Our proposed social welfare function is along the utilitarian line. Ratio-scale interpersonal comparisons of cardinal utilities are used to prove its existence. If we are allowed to define utilitarianism more broadly, then our social welfare function will also be unique. Finally, the study argues strongly for more positive action on the part of the government to rectify social injustice.

RESUME

Le thème central de la présente étude est le problème du choix social. Notre but principal est de prouver l'existence d'une fonction de bien-être social à fin de mettre un terme à la doctrine de 'Laissez-faire.' La plupart des récentes solutions à ce problème sont ou incomplètes ou déçousues, et sont rejetées par conséquent. La présente étude propose une fonction de bien-être social axée sur l'approche utilitariste. Des comparaisons 'ratio-scale' interpersonnelle des utilités cardinales sont utilisées à fin de prouver l'existence de cette fonction. Si on défini plus largement l'utilitarisme, la fonction de bien-être social sera unique. En conclusion, l'étude soutient fermement que le gouvernement doit intervenir, avec des actions positives dans l'économie, à fin de combattre l'injustice sociale.

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT.....	iii
RESUME.....	iv
ACKNOWLEDGEMENTS.....	viii
CHAPTER I Introduction.....	1
CHAPTER II Positive and Normative Economics.....	3
Some Reflections on their Distinction.....	3
The View of the Protagonist.....	5
The Response of the Heretic.....	7
CHAPTER III Social Decisions without Interpersonal Comparisons.....	21
The Competitive Solution and Interpersonal Comparison.....	21
The Choice of Space for the Compensation Test.....	33
The Hicks-Kaldor Criterion.....	42
The Scitovsky Paradox.....	50
Samuelson's 'Objective' Solution.....	59
Little's Value-Loaded Criterion.....	64
CHAPTER IV The Possibility of Social Decisions.....	79
The Bergson-Samuelson Social Welfare Function.....	79
Arrow's Impossibility Theorem.....	86

	What Causes the Arrow Paradox.....	98
	Why We can Ignore Samuelson.....	99
CHAPTER V	Cardinal Utility Functions under Stochastic Situations.....	102
	Some Historic Remarks.....	102
	Some Reasons for Preferring the Original von Neumann-Morgenstern Formulation.....	110
	The Allais Paradox.....	113
	The Independence Axiom.....	115
	The Ellsberg Paradox.....	125
CHAPTER VI	Cardinal Utility Functions under Non-Stochastic Situations.....	140
	The Jevonsian Method.....	140
	Answering Some of the Criticism.....	147
	Attempts to Axiomatize the Jevonsian Method.....	161
	Alt's Proof.....	165
CHAPTER VII	Interpersonal Comparison.....	171
	Some Preliminary Remarks on Taxonomy....	171
	Partial Comparability.....	177
	Ordinal Comparability.....	187
	A Special Case: Rawls's Maximin Principle.....	190
	Ordinal Non-Comparability.....	201
	Two More Proposals on Fairness.....	212
	A Second Best Solution.....	215
	Unit Comparability.....	224
	Cardinal Non-Comparability.....	243

	Ratio-Scale Comparability.....	247
	The Empty State.....	264
	The Other Touch.....	272
	The Possibility of Consensus.....	284
CHAPTER VIII	Configuring a Social Decision Rule.....	289
	Difficulties in Hedonism.....	289
	Intuitionism.....	292
	Rule-Utilitarianism.....	295
	Act-Utilitarianism.....	296
	Pinpointing one Particular Function.....	301
	Should the Will of the People Always Prevail?.....	310
CHAPTER IX	Conclusion.....	314
	FOOTNOTES.....	319
	SELECTED BIBLIOGRAPHY.....	347

ACKNOWLEDGEMENTS

I am indebted to many people for their input into this thesis. In particular, I would like to thank my supervisor Professor J.C. Weldon for his guidance and inspiration. Also, I am grateful to Professor A.L. Levine for his very useful comments and suggestions without which I could not have dealt with some of the issues discussed. Professor J. Kurien read parts of an earlier draft of this thesis and offered many helpful comments and I am much in debt to him for this. I also wish to express my sincere thanks to my fellow students at McGill, V. Nallainayagam, S. Narainen, K. Wampah and A. Nigam for their invaluable discussions and barbers.

Introduction

Economics, like other disciplines, has different schools of thought. However, unlike many other disciplines, the different schools in economics are very much influenced by the political convictions of those who belong to them. A rock is always a rock to a geologist, and his political convictions usually would have nothing to do with the way he identifies the rock. However, the same cannot be said of economists. Have you ever seen a good monetarist belonging to a socialist party? Economics and politics are closely related and in our opinion, the purpose of studying economics is to help governments implement their policies. This was perhaps the accepted view until ordinalism and positivism swept through the profession early in this century. Economic positivism insists that we should leave value judgements out of economic analyses. Ordinalism, on the other hand, has successfully prevented the construction of any consistent group decision rule.

The purpose of this study is two fold. Firstly, we would like to restore the importance of the government in the economic sphere. Secondly, we would also like to argue the existence of a unique function which is capable of maximizing social welfare. To arrive at the latter, interpersonal comparisons of cardinal utilities are required.

We will begin our investigation with a discussion on the

distinction between positive and normative economics. Upon refuting the existence of such a distinction, we will move on to examine the existence of a social welfare function. Unsuccessful attempts like the compensation tests and Arrow's theorem will be looked at. Finally, we will argue for full interpersonal comparison of cardinal utilities and the existence of a unique social welfare function. All in all, we feel that the government has an important role to play in the economic welfare of a nation, and that the society would be made better if the government can represent the will of its people as reflected by the social welfare function.

Positive and Normative Economics

Some Reflections on the Distinction

It is a common practice to divide economics into positive and normative economics: the former is supposed to deal with factual problems, i.e., problems concerned with 'what is,' while the latter is left to handle ethical problems that answer the question, 'what ought to be?' It is the opinion of most economists that if economics is to be regarded as a science, it must concern itself with positive economics exclusively. Such sentiments can be traced back to the views as propounded by the logical positivists of the Vienna Circle.<1>

As a philosophical doctrine, logical positivism has already passed its prime. The same, however, cannot be said of positive economics. It yet enjoys a commanding influence over economists and it is a fact that countless of value-free economists were manufactured in the past few decades. One cannot deny that there is a marked improvement in the deductive techniques used by these new professionals; this is especially true of their mathematical skills. On the other hand, there seems to be a general lack of purpose in most of their exercises. To put it bluntly, they simply do not know what they are trying to get at. In a way, this is quite natural. When the positivists decided to purge value

judgements from economics, they have in the process banished all the issues concerning distribution. This is so because, according to their demarcation, problems pertaining to distribution are not value free. As strict adherents to the scientific method, they would refrain from commenting on whether the society will be better off if one individual rather than another is allowed to consume certain outputs of the society. Given this position, there is no justification whatsoever to produce more since one may not know beforehand (a) who will get the extra outputs and (b) whether anyone is affected as a result. Even if everyone gets more, one would still not be able to consider the increase in output as a blessing to society unless one accepts some form of value judgement like the Pareto criterion.<2> The problem of efficiency will become meaningless if we do not have any value judgement to back it up. That is why we say that the positivists do not know what they are trying to get at. Production will be for the sake of production, and economists will play the role of computers. They will try to enlist all the possibilities available and, considering the speed at which the economists are working, it is not difficult to envisage the day when a single computer will replace the entire economic profession.

To avoid this calamity, we propose to go back and examine the fundamental arguments of the positive economists and see if their methodology really merits our unconditional acceptance. In particular, our main concern is to find out whether the line drawn between positive and normative

statements at present is appropriate. It is our belief that certain problems of distribution related to interpersonal comparisons should be included in economics. If we want to view economics as a positive science, then we would have to regard statements concerning interpersonal comparisons as positive statements. If, on the other hand, we want to consider statements of interpersonal comparisons as normative statements, we would also have to treat economics as a normative subject. In essence, we do not see any intrinsic difference between statements which involve interpersonal comparisons and other fundamental statements in economics, and we will make an attempt to show that the alleged difference is unfounded. Before we can make our view clear, we would like to allude to briefly but more explicitly on the position taken up by the positivists. Towards this end, we will follow the exposition put forward by Friedman,^{<3>} for the simple reason that it is a clear cut case which makes an unpretentious defense of the positivist dogma.

The View of the Protagonist

According to Friedman, "the ultimate goal of a positive science is the development of a 'theory' or 'hypothesis' that yields valid and meaningful (i.e. not truistic) predictions about phenomena not yet observed."^{<4>} To develop that hypothesis, two elements are required. Firstly, we need a language to serve as a 'filing system' which we can usually

identify as the rules of deductive logic.' Secondly, we also require a set of substantive hypotheses 'to abstract essential features of complex reality.' Actually the function of these substantive hypotheses is much more important than merely abstracting the reality. As far as we can see, they also serve as the base on which we build the other hypothesis.

A point to note at this juncture is that a recursive procedure has been introduced here. Friedman is using hypotheses to construct hypotheses, and we will soon find out that this procedure will later manifest itself as containing a crucial internal inconsistency.

After adopting a hypothesis, the next step for a positivist is to test it against evidence. Here Friedman's approach follows closely Popper's Law of Falsification.<5> A hypothesis is to be rejected if its predictions are contradicted by evidence, and tentatively accepted if the contrary is true. Direct verification is never possible, and "factual evidence can never 'prove' a hypothesis. It can only disprove it."<6>

In general, there are always more than one hypothesis which is consistent with the available evidence, and it is up to the positivists to decide which of these alternative hypotheses should be accepted as the 'true' hypothesis. To make that decision, he will have to look at the differences among these hypotheses. Since the positivists are willing to work within deductive logic, there should be, in principle, no difference with respect to the 'filing system' as employed

by the different hypotheses. As a result, if these are different at all, then the dissimilarities must come from the basic hypotheses (assumptions) on which they are based.

In view of the vast number of assumptions we might have to choose from, Friedman has this rule of thumb to offer us:

In so far as a theory can be said to have 'assumptions' at all, and in so far as their 'realism' can be judged independently of the validity of predictions, the relation between the significance of a theory and the 'realism' of its 'assumptions' is almost opposite of that suggested by the view under criticism.... in general, the more significant the theory, the more unrealistic the assumptions.<7>

The Response of the Heretic

There are a few points we want to raise against positivism as outlined above. Some are concerned with the distinction between positive and normative statements, but we will begin with a critical look at Friedman's controversial assertion first. Presumably, the original intention of Friedman's remark on assumptions was to eliminate unnecessary details. Examples of this sort are plentiful. In consumer theories, we often assume that there are only two commodities in an economy, and in production theories we likewise assume only two inputs. However, whenever we make these assumptions, we are not saying that the realism of assumptions is not important. On the contrary, it is of utmost importance, and the only reason we allow ourselves to 'abstract' from reality is that there⁹ is no loss of realism at all through this

process of abstraction. When we assume that there are only two commodities in an economy, we are fully aware of the fact that the results of our analysis can be generalized to include any number of commodities. In other words, the simpler assumption is logically equivalent to the more complex assumption, and we have not left any part of reality out of the picture. In the event that such an equivalence is not substantiated, as in the case of Ricardo's trade model, we would have to reject the simplification involved although it may be of the same order as some other similar but accepted assumptions. Hence what we are doing is not compromising on realism but using concise notations.

While we have interpreted Friedman in this way, the latter obviously has something else in mind. As one reads his essay further, one will immediately discern that his paradoxical assertion is intended to justify the assumptions of perfect competition in contrast to those of monopolistic competition, and the differences between these definitely do not pertain to the kind of notational economy we have been considering. In that respect, the two sets of assumptions are not logically equivalent, and neither are their consequences. One wonders why Friedman would insist on applying his rule of thumb in this case.

As we know, assumptions are hypotheses in their own right, and in so far as two sets of assumptions are not logically equivalent, a good positivist will have to judge them according to the fundamental rule of factual evidence. We do not share the view of Cyret and March, who maintain

that "laws of higher order (which include the kinds of assumptions we are examining) used to explain lower-order laws need not contain terms referring directly to observables." <8> A possible case which Cyret and March have in mind would likely be the Euclidean points and lines as mentioned by Friedman.

It was argued by Friedman that assumptions in geometry, like the concepts of points and lines, are not only unrealistic but non-existent in the real world, and since these concepts are functioning perfectly well in geometry, there does not seem to be any reason why we should insist on having assumptions which are realistic. If this line of reasoning is accepted, then it will imply that there exist some higher order laws which are above the crucial test of factual evidence. However, if these laws are not based on empirical evidence, what else could they be based on? A natural way is to regard them as self-evident truths <9> and yet, that would still leave us at the mercy of some form of evidence all the same. When empirical evidence has been ruled out, the only possible alternative will have to be mental evidence which has no basis in fact. Or equivalently, positivism will be turned into idealism or Platonism. Understandably, this is not what Friedman had in mind when he invoked geometry to state his case. Perhaps a revisit to the Euclidean space will clarify some of our perplexities on this point.

It is granted that what Euclid had axiomatized is not exactly what we can find in the real world. Yet, we also have

to admit that Euclid must have had his eyes on the real world when he propounded his axioms. Evidently, there are disparities between the real world and Euclid's postulates. Nevertheless, we can attribute those disparities to the imperfect measuring tools we have. With more sophisticated tools, we can find out that the area which a point occupies will diminish. To assert that Euclid had taken the limit of these observations will be really taking the matter to the limit. However, it is hard not to believe that Euclid had done his inductive exercise and constructed his postulates thereon. As a result, we can confidently conclude that Euclidean geometry is, on the one hand, not wholly based on some 'idealistic' concepts but also on observations recorded by our senses; and on the other hand, that it is realistic as its axioms are based on the trends of their counterparts in the real world and are not arbitrarily constructed to accommodate the cleavages between its hypotheses and the real phenomena concerning the hypotheses.

Hence, if we want to stay within the boundaries of positive economics, we will have to abandon Friedman's rule of thumb, or else contradictions and chaos will prevail. We will use an example to demonstrate why this is the case.

From elementary logic we know that both true and false premises can imply a true conclusion. Hence, even if the evidence proves that the conclusion of the hypothesis is correct, we may still have a considerable number of wrong premises. These premises are wrong not in the sense that they have simplified reality for we know that simplification of

this kind does not amount to any wrong choice. Rather they are outright counter-factual. Take the case where we are trying to predict the output of an economy. Usually, we will assume that firms try to maximize profits, and individuals their utilities. Let us suppose that this set of assumptions will yield the same result as the set of assumptions which expects firms to maximize their losses, and individuals their disutilities. Moreover, we will assume their predictions to be correct. If we take Friedman's advice, we would have to choose the most unrealistic set of assumptions which, in this case, presumably would be the second set. If there is nothing else in this world beside these two sets of assumptions, it would probably make no difference whether we choose the first or the second. Unfortunately, our world is not so simplistic, and there do exist other things that we have to consider. More specifically, there is a body of accepted knowledge with which our newly postulated hypothesis has to align. Since the latter is unrealistic by choice, it will most likely conflict with some elements in this body of accepted knowledge. In our example, past observations and our introspection will tell us that, in general, firms do not try to maximize their losses, and consumers their disutilities. In the same way, we also know that firms operate more or less in monopolistic markets rather than in perfectly competitive markets. By following Friedman in these two cases, we will inevitably be contradicted by this body of generally accepted knowledge. If we follow Friedman in all cases, there will be no accepted knowledge. Contradiction will be the rule of the

day, and chaos will rule the system.

Consequently, we must reject Friedman's recommendation and start looking for some other rule(s) that may help us determine a set of 'objective' assumptions. We will argue that such a rule does not exist, and will make use of the distinction between positive and normative statements to establish our point.

To reiterate, positive statements are those statements that are concerned with 'what is' and normative statements with 'what ought to be.' If there is only one set of assumptions, or if all sets of assumptions are identical, then we will be able to say that this set of assumptions is the set of assumptions which is appropriate for the economic problem on hand. In mathematics, examples of 'what is' statements are plentiful. The initial segment of ω is $\{1,2,3,4,5,6\}$; the power set of $\{1,2\}$ is $\{\{1\}, \{2\}, \{1,2\}\}$ etc. Even though we may follow Zermelo-Fraenkel or von Neumann to axiomatize set theory, the examples above would not change in the two schemes. The two ways of axiomatization are both consistent with each other and can both be formalized into first-order theories with the binary relation ϵ .^{<10>} Hence, the choice of assumptions will not constitute a problem in this case.

The blessings of mathematics are by no means extendable to economics. There is a major difference between the two disciplines. In mathematics, one is dealing with arithmomorphic concepts ^{<11>} while in economics we are not. In our field, we are always confronted with aggregates. To

put it in simple words, the problem of aggregation, of which interpersonal comparison is one, is a problem of ordering n-tuples.<12> From an elementary theory in set theory, we know that every set can be well-ordered, and the well-ordering can be conducted in many ways. To decide which way is appropriate is a delicate matter. There are certainly competent economists who would insist that development is equivalent to a per capita income of U.S.\$1,000; that unemployment below 4% are due only to frictional forces. Nonetheless, most economists who possess a little sophistication would shy away from taking a firm view on such matters without qualifications. It seems that there does not exist an unequivocal answer to questions like: What is the rate of inflation? Or what is the productivity of the economy?

Refuge may be sought by some in the realm of individual utility theories. They may try to quibble on the apparent fact that utility theories -- the core of modern economics -- do not deal with aggregate concepts. If this argument is valid, they may have a chance to turn things around, or at least in consumer theories. What they can do is to postulate on each individual without involving aggregation first, and once the hypotheses are made, all they have to do is to add up the predictions and pray that the fallacy of composition would not arise.

Once again, this reasoning is deceptive. The fact that utility is a personal attribute does not prevent it from being an aggregate concept. An individual derives his

satisfaction from a vector of commodities. To compare this vector with another vector is just another routine of ordering n-tuples. Though this aggregation problem does not directly pertain to our economist, he will however be indirectly affected by it. It has been documented that preference intransitivities may arise when individuals are being asked to rank order commodity vectors.^{<13>} In fact, intransitivity may also arise when individuals are only rank ordering scalars.^{<14>} When such incidents happen, there is no general rule as to how to determine whether our individual prefers A to B or B to A. Our economist may, as threshold theorists have done, choose a particular percentage and insist that if our individual's preference for A over B is above that threshold, then, he really does prefer A to B. Needless to say, this threshold is arbitrarily set by our economist, and it can only be the result of his own value judgement.

Another seemingly innocuous and value-free postulate in consumer theory is the non-satiation axiom,^{<15>} which says that an individual will prefer more to less of a good. This seems so true to us that beginners in economics will hardly discern its normative implication, since, to most of us, this is a daily fact, and almost a tautology. We may choose to attack this axiom from the viewpoint that there will always exist some irrational individuals for whom this law would not hold. However, this issue will easily be resolved by simply restricting the eligible consumers to the set of rational consumers. As this is a circular explanation, objections

along the previous line will no longer hold.

Undoubtly, the decision to exclude the irrational consumers¹⁶ is a normative one, and we can show our disapproval to the positivists at this point. But we will attempt to disprove the positivists' of this axiom by means of the aggregation analysis introduced earlier.

Though for the novice it may not be so obvious, the Pareto criterion is a value judgement. It is so because, through this, one is imposing one's opinion of the society on the society. One is evaluating the consequences of situations that concern not only oneself but also others. The fact that there may be consensus about this criterion will not change it from a value judgement to a positive statement. Pareto's criterion is 'a' criterion about aggregation, but not 'the' criterion. There are other ways to aggregate welfare also. So, granted that we are dealing with rational people, and that everyone feels that his share has been increased, the Pareto criterion will still be normative.

To reveal the relationship between the non-satiation condition and the Pareto criterion, it will be more convenient, though not necessary, to introduce some symbols. We will denote the weak social preference relation by R , and R_i as the corresponding relation of the i th individual. Let X, Y be situations; by the Pareto condition, $XRY \Leftarrow \forall i x_i R_i y_i$, where x_i and y_i are the commodity vectors for the i th individual in X and Y respectively. By the non-satiation condition, we have $x_i R_i y_i \Leftarrow x_i \geq y_i$. By universal generalization, non-satiation will give us

$\forall_i (x_i R_i y_i) \Leftarrow \forall_i (x_i \succeq y_i)$, but $\forall_i (x_i R_i y_i) \Rightarrow XRY$. So, non-satiation implies the Pareto criterion.<17> Since we have only included the rational members of the society in our analysis and since non-satiation has been considered as one of the behaviours we would expect from a rational being, there is really no need to prove the other part of the equivalence. Consequently, when we assume both axioms, the acceptance of one will lead automatically to the acceptance of the other. However, we must not forget that the Pareto criterion is a normative concept by consensus.

Other fundamental assumptions of utility theories, like the Archimedean axiom and the postulate of the monotonic increase of the marginal rates of substitution among commodities, are all disputable. To comment on their non-positive characteristics would indeed be superfluous.

Leaving the assumption issue aside, the next phase in the process of economic inference will be the logical deduction of the theory. Unless there are flaws in the logic of the researcher, this would probably be the only part of the whole process that can claim to be truly positive.<18> Even if there are flaws, sooner or later, they will be detected either by the researcher himself, or by some of his fellow economists.

After establishing a hypothesis through deduction, it is time to test its predictions against evidence. Here our positivist will again run into trouble. He will be confronted with a series of questions like: What should my data include? Where to find them? Who are the relevant people to observe?

To whom should I direct my results? When should I start and end my observations? etc. The decisions to make are numerous, and all of them are to be decided by our positivist himself, if not arbitrarily then by reference to his personal convictions. Hence there is no such thing as the existence of the objectively correct data, and subjectivity is a necessary ingredient of data collection.

Upon completion of data collection, he will have to face the critical task of approving or disapproving the hypothesis. Yet, the criterion to be used is still in doubt. Should he accept a hypothesis only when it is perfectly in accordance with the evidence? This seems to be too demanding, as most phenomena in the real world are of stochastic nature. However, to settle for anything less would require justification. Calling the level of acceptance a confident one is not a positive vindication. One has to show why the choice of ninety nine out of a hundred is better than ninety five out of a hundred in one case and vice versa in another.

Such discretions abound in economic literature, and they are in fact indispensable. Rigidity is an exception rather than a rule in economics, and it is advisable to treat each problem as a distinct case. Since there does not exist a unique format in economics binding the profession to treat a particular problem in a particular way, the economists are, to a great extent, free to accept, reject or make reservation about a hypothesis. In order to do this, he must inevitably use his own judgement.

Lastly, let us put one more straw on the camel's back by pursuing a meticulous point which is often overlooked by more profound minds. We will follow Hare^{<19>} and classify words into three kinds, viz., descriptive, prescriptive and evaluative. Evaluative words is a subset of prescriptive words which also possess descriptive meanings. Since prescriptive words include the evaluative ones, they can either have purely prescriptive meanings, or both prescriptive and descriptive meanings simultaneously. Descriptive words must have descriptive meanings only. Evaluative words are also referred to as moral words. If we adhere to the positivists' construction, we should only employ descriptive words in economic analyses, as the other two kinds of words both involve value judgements. Descriptive words are universalizable, and it is this feature that makes them suitable for generalizing theories. But descriptive words are not the only ones universalizable. Moral words are also universalizable because they also have a descriptive component. Nevertheless, they are different from descriptive words as they also prescribe. Examples of descriptive words are 'open,' 'close' whereas 'good' and 'right' belong to the category of moral words. To further illustrate the difference, we will quote Hare at length:

For in saying that it is proper to call a certain kind of man good we are not just explaining the meaning of a word, it is not mere verbal instruction that we are giving, but something more: Moral instruction. In learning that, of all kinds of men, this kind can be called good, our learner will be learning something synthetic because of the added prescriptiveness of the word 'good'; in learning it, not merely to use a word in a certain

way, but to comment, or prescribe for imitation, a certain kind of man. A man who wholeheartedly accepts such a rule is likely to live, not merely talk, differently from one who does not. Our descriptive meaning rule has thus turned into a synthetic moral principle. <20>

Needless to stress, moral words abound in literature, positive and normative alike. To mention only a few, we have words like 'perfect,' 'efficient,' 'free,' 'favourable,' 'optimal,' 'backward,' 'rational,' 'golden path' etc.... And as people's awareness of our field continues to increase, the very word that denotes our subject has more or less acquired a moral overtone. If we do not object to Hare's interpretation of moral words, then it seems that the only alternative available is to admit that economics is by its very nature a moral discipline. More often than not, those who claim objectivity are the very same people who slip in most of the moral words in economics. By sustaining their plea and acting conscientiously to the contrary, these people are unquestionably inviting us to cast doubts on their sincerity.

In closing, let us restate that the sharp distinction between positive and normative economic statements which Friedman insists on was never there in the first place. Value judgements are present in all economic analysis, and subjective opinions are unavoidable. In fact, the idea that there exists some methodology or epistemology which stands above and governs a scientific discipline is somewhat out-dated.<21> As a result, we must consider problems of distribution as an integral part of our investigations, and

in subsequent chapters, we will examine how such problems can be re-instated.

Social Decisions without Interpersonal Comparisons

The Competitive Solution and Interpersonal Comparison

We have seen that a certain amount of arbitrariness is unavoidable in any economic investigation and it will be shown in later chapters that analyses which involve interpersonal comparisons are no more or less arbitrary than any other analysis in economics. Admittedly this view does not represent the opinion of most economists. Amongst them is Robbins, who has explicitly made the following claim:

The assumptions of the proposition which did not involve interpersonal comparison of utility were assumptions which had been verified by observation or introspection, or at least, were capable of such verification. The assumptions involving interpersonal comparison were certainly not of their order.<1>

Robbins has given some justifications for his position though none of them seems to be acceptable to us. We will defer the discussions on them until chapter 7. In the meantime, what we want to do is to bring out the implication hidden in the above quotation. It occurs to us that despite Robbins self-proclaimed positive attitude, his assertion is very much value-loaded in the favour of the market solution. To see why this is the case, we must turn our attention to the relationship among the competitive equilibrium, the Paretian equilibrium, and the welfare maximum.

The first step people take to hail the market solution

is usually by asserting that every competitive equilibrium is also a Paretian equilibrium. This is being done in spite of the fact that it has already been demonstrated that "perfect competition is neither a necessary nor sufficient condition for meeting the allocation rule, or the (Paretian) optimum condition."<2> In recent years, that belief has obviously been strengthened by the numerous mathematical proofs which claim to have established the equivalence of the two kinds of equilibrium.<3> Taking this belief as given, i.e., a competitive equilibrium is necessarily a Paretian equilibrium, we will illustrate why, if we prohibit interpersonal comparisons, we will be doing the competitive equilibrium a favour. To do that, we will have to take a further look at the Paretian condition.

There are two features which we can identify with the condition. Firstly, this condition describes a monotonic increasing transformation of individual utilities into social welfare; secondly it does not specify how the gains of a certain party can override the losses of another party. Monotonicity is really not at issue. Every now and then there may be a handful of economists who would like to question its appropriateness by invoking the fact that people's utilities may be interdependent, i.e., we may have a situation where everyone's commodity bundle has been increased, but some people's utility level might have gone down as a result of jealousy or other psychological factors. However, these objections do not constitute a major threat to the Paretian condition, and it seems that most economists are willing to

accept at least the first part of the condition. The major problem of the condition comes from its second feature. By not specifying how the gains of one party can override the losses of another party, the Paretian condition has left the economist silent on a lot of occasions. Naturally, attempts have been made to break this deadlock, and they usually take the form of comparing different people's utilities. However, if we follow Robbins' reasoning, we would have to consider these attempts as value judgements and should refrain from making them as long as we want to stay on the positive track. Without interpersonal comparisons, we will not be able to judge the merits among Paretian equilibria, and without such judgements, there will be no justification for the government to prefer any one of the Paretian equilibria. Consequently, the government will be prevented from playing an active role in formulating the decisions of the society. Such a state of passiveness, however, does not mean that there is no policy recommendation. Where there is no active action on the part of the government, laissez-faire will take hold, and that will in turn lead to the free market competitive solution. We must realize that inaction is also a policy option, and that the *status quo* is also an element of the set of possible alternative states. The competitive equilibrium is also a Paretian equilibrium, and in the absence of any interpersonal comparisons, no other alternative state will be able to challenge its chosen position once the society has arrived at it through laissez-faire policies. Hence by adopting Robbins' definition of value judgement, we are in effect giving our

endorsement to the free market solution, and we do not see how such a recommendation could be regarded as value-free if interpersonal comparisons of utilities are considered to be value judgements.

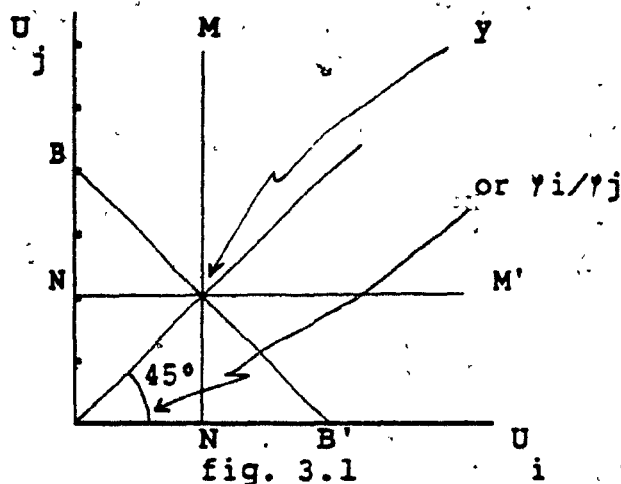
The gist of the problem here lies in the Paretian criterion. In positive economics, this criterion has been used indiscriminately to denote allocative efficiency, and it has been treated as if it does not involve any interpersonal comparison of utility. In chapter one we have briefly asserted that this is not the case. In the following pages we will make an attempt to back up that assertion. It will be shown that an extreme form of interpersonal comparison of utility is implicit in the criterion. If we are successful in our effort, the positivists will be faced with two choices. Either they will have to accept interpersonal comparison of utility, or give up the Paretian criterion. By giving up the Paretian criterion, the competitive equilibrium will lose its edge against other alternative situations. Allocative efficiency will cease to matter, and there is no reason why the government cannot interfere with the free market solution. If the positivists choose to accept interpersonal comparison of utilities, then the government will again be able to interfere, since it is unlikely that the market solution will still be able to retain its chosen position once other forms of interpersonal comparison of utility are allowed.

When we deal with production or preference functions in orthodox theories, we often characterize them with perfect

substitutabilities. Such characterization, however, does not prevent us from recognizing the existence of other forms of preference or production functions which display less than perfect substitutabilities. The Leontif production function is an extreme case in point; exhibiting no substitutability at all. Yet, few economists today would refuse to consider it as a production function. It will be argued that the place the Paretian criterion occupies among interpersonal comparison criteria is analogous to the place the Leontif production function occupies among production functions. If we are willing to regard the latter as a production function, then there is no reason why we could not treat the Paretian criterion as an interpersonal comparison scheme.

In a recent discussion of the Rawlsian maximin principle,^{<4>} Alexander^{<5>} has tried to subsume that principle under the general utilitarian rule. By picking a typical utilitarian welfare function $W = U_1^\alpha + U_2^\alpha$, Alexander argues that among the many, there are three notable forms of conflation which can be represented by the utilitarian function given. The three are the Benthamite, the Rawlsian, and the Nietzschean. When the parameter α is equal to one, the function will give us the Benthamite utilitarian rule. When α approaches $-\infty$, W will be the Rawlsian maximin principle and when α approaches ∞ , we will have the Nietzschean maximax principle. Since all three forms of conflations are representable by the utilitarian function, Alexander concludes that all three are just special cases of utilitarianism. In figure 3.1 the three rules are

represented by ByB' , MyM' and NyN' respectively, with y as the hypothetical status quo. Realise that the shape of MyM' is very much similar to that of a Leontif isoquant, and we hope to be able to complete our analogy by establishing a certain relationship between the Paretian criterion and the maximin and/or the maximax principle.



Before we set out to show the proof, we want to remind the readers that interpersonal comparisons of utilities are implicit in all three of the ethical principles mentioned above. No one should have doubted this in the case of the Benthamite rule, where different individuals' utilities are treated as equals. As for the maximin principle, the evidence is sometimes not as obvious, and this is mostly attributable to the veil of ignorance introduced by Rawls. When the agents in the Rawlsian system started out from the original position to find the worst-off position in the society, it

seems that they had already arrived at a consensus among themselves with regard to that position without even making any interpersonal comparison. It is widely believed that behind the veil of ignorance, people's self-seeking motives will be neutralized and impartiality will make each of them identify the same worst-off social position. However if one makes a closer examination of the matter, one will easily discern that the consensus in Rawls is arrived at through the introduction of a primary good index and the construction of the latter is not possible without interpersonal comparison of utility.<6> Since the maximax principle is symmetric to the maximin principle, the same argument concerning interpersonal comparison of utility must apply.

In the meantime, we will follow Alexander's interpretation of Rawls and start with a situation where there is absolute equality. According to Alexander, such absolute equality would mean the same amount of primary good for each agent, and what we think could safely be translated as the same amount of utility for all concerned. Since each agent's utility function is independent of those of the others, u_i may be quite different from u_j numerically though the society may want to consider them to be equal. Luckily, both the primary good and the utility indexes are real variables, and, by the Archimedean axiom, there will always exist a vector γ such that $\gamma_i u_i = \gamma_j u_j$. This means that a simple transformation will enable us to bring different people's utilities into line. Finally, it is conceivable that we may need different vectors of γ as the level of well being

in the society changes. Nonetheless, we will at the moment ignore the issue with the understanding that it is nothing more than a minor complication.

Having laid down the preliminary groundwork, we will now set out to prove our assertion, i.e., the Paretian criterion also involves interpersonal comparison of utility. Referring back to figure 3.1, we will take y as the initial position of absolute equality with $u_i/y_j = 1$. Let MN denote the relation that is at least as good as under the maximin principle; then $\forall x, y \in X, \forall i, j, l \in N$,

$$x(MN)y \Leftrightarrow \exists j \forall i [u_i(x, i) \geq u_i(y, j)]$$

where X is the set of all situations, N the set of all individuals, and (x, i) the i th role in situation x . Similarly, for the maximax principle MX ,

$$x(MX)y \Leftrightarrow \exists l \forall i [u_i(x, l) \geq u_i(y, i)]$$

Furthermore, by inverting (MX) we will form NB such that

$$x(NB)y \Leftrightarrow \exists l \forall i [u_i(y, l) \geq u_i(x, i)]$$

where NB is some kind of equity principle. This principle ensures that the welfare of the best-off person will be reduced. Let us call it the 'non-believer' principle for the moment; the reasons for this will become evident later on. If we join (MN) and (NB) together, we will have

$$x(MN \cap NB)y \Leftrightarrow \exists j \exists l \forall i [u_i(x, i) \geq u_i(y, j) \wedge u_i(y, l) \geq u_i(x, i)]$$

Since y is a situation of absolute equality,

$$u_i(y, j) = u_i(y, l) = u_i(y, i) \text{ and}$$

$$x(MN \cap NB)y \Leftrightarrow \forall i [u_i(x, i) \geq u_i(y, i) \wedge u_i(y, i) \geq u_i(x, i)]$$

Hence x must be identical to y if the relation $(MN \cap NB)$

is to hold between them. Since both (MN) and (NB) are utilitarian rules, their intersection will have to be utilitarian also. The seemingly contradictory (or trivial) result of $(MN \cap NB)$ should come as no surprise. The relation requires that in situation y there must be a position which, in everyone's judgement, is inferior to or, at most, as good as every position in x . At the same time, it also requires that there must exist a position in y such that, in everyone's judgement, is at least as good as every position in x . But y is a situation of absolute equality. Every position in it is regarded by everyone as yielding the same amount of satisfaction. Then, every position in y must be (1) at least as well off, and (2) at most as well off as every position in x . This is only possible when x and y coincide.

In the above discussion, we have simplified the case by assuming $\gamma_1 = 1 \quad \forall_1$. In figure 3.1, γ_1/γ_2 is the slope of the norm of y . If $\gamma_1 = \gamma_2$ then the slope is 1 and the analysis goes through easily. But if the slope is not 1, all we have to do is to multiply γ_1/γ_2 to the corresponding individuals' utility, and the conclusion of the analysis will not change. This reasoning is not only true when y is an absolute equality position, but for any position. All we have to do is to make an appropriate rotation of γ_1/γ_2 , and any point on the utility space will give us similar results.

The Paretian equilibrium status quo is one of these points. As we have mentioned before, we will not (at least for the moment) dispute the monotonic part of the Pareto

criterion. What is left therefore is the second part of the Pareto criterion which deals with the problem of social choice when we are at one of the Pareto equilibria. What the Pareto criterion says is that we should not move to any other situation unless no one is hurt by such a process. Since we are at one of the equilibria, that condition cannot be fulfilled, and the choice of the Pareto criterion when we are already at one of the Paretian equilibria is the existing status quo y .

This result is equivalent to what we have derived from $(MN \cap NB)$, which implies that the two criteria, $(MN \cap NB)$ and the Pareto criterion, are equivalent when we are dealing with the Paretian equilibrium set. Since the $(MN \cap NB)$ criterion is under the utilitarian rule, this must also be true of the Pareto criterion by virtue of the tautological axiom.

Before investigating what kind of interpersonal comparison is involved with $(MN \cap NB)$, it may be advisable to dispel certain confusion between the maximin principle and the Pareto criterion. Apart from their initial position, which can be transformed by a rotation of γ_1/γ_2 , the maximin is quite similar to the Pareto criterion. As originally formulated by Rawls, "social and economic inequalities are to be arranged so that they are both reasonably expected and to be to everyone's advantage." If in a situation, inequality is not to the benefit of some people, it should be regarded as worse than the original position. This condition effectively rules out all those weak Pareto improvements under which at least one party will not receive any benefit.

So, the maximin principle is not the same as the strong Pareto condition. The maximin is a reflexive condition, but the weak Pareto condition is not; so by definition the two cannot be equivalent. To be exact, maximin is a subset of the strong Pareto condition, and includes the weak Pareto condition.

Maximin is a lexicographical ordering. Under it, social welfare will be increased only if the welfare of the worst-off person improves. Any possible improvement of the worst-off person must come from one of these two sources: (1) from other members of the society, and (2) not from other members of the society, e.g., manna from heaven. By augmenting the non-believer relation to (MN), we are in effect affirming that there is no heaven and therefore no manna from heaven. In other words, 'free' output is no longer possible. So, the improvement must come from other members in the society. Since we have started with absolute equality, these other members, once deprived of some goods in their possession, will become the worst-off group in the society. By maximin, the society will have to consider their welfare as primal, and that will surely mean a halt to the original transfer.

The same kind of intuition applies equally well to the Pareto criterion, as the absolute equality position can be transformed to suit the Paretian status quo. As before, improvement of one individual can only be brought about either by efficient allocation or by redistribution. The first is no longer possible because of the definition of a

Pareto equilibrium. Redistribution will necessarily enable the losers to claim the same kind of privileges possessed by the worst-off person in the maximin case, and this will automatically nullify any argument for change. As a result of the γ transformation, the Paretian status quo now corresponds to a situation of absolute equality. Everyone is the worst-off person and the maximin principle becomes anonymous. The infinite weight is therefore applicable indiscriminately to everyone and that will stifle any hope for redistribution, and the reactionary bias is thus ensured.

By this time, we should be able to recognize that interpersonal comparison is not only important, but also indispensable in any economic analysis. Proposals for changes or no change, all involve interpersonal comparisons. Hitherto, we have been misled into performing an extremely reactionary form of interpersonal comparison, because we have put all our trust on the Paretian criterion. As a result, the problem of distribution has almost been completely ignored in economics. In what follows we will try to examine some other forms of interpersonal comparison which are supposed to supplement the Paretian criterion. Naturally, all of them are Pareto-inclusive comparisons, i.e., satisfying the monotonicity part of the Pareto criterion.

The Choice of Space for the Compensation Test

In contrast to the Pareto criterion, the compensation test is a principle that aims for changes. Its prescription was meant to be a response to the quiescent attitude of the profession. Nonetheless, the people who propounded the principle were in no way revolutionary, and this was made explicit right from the beginning by Kaldor:

It is not the purpose of this note to question Professor Robbins' view regarding the scientific status of such (interpersonal) comparison; with this the present writer is in entire agreement.<8>

The first impression one gets from the Kaldor-Hicks version of the compensation test is that it is dealing with a Pareto sub-optimal situation in which production efficiency can be increased.<9> Since both Hicks and Kaldor had avowed to abide by the positive rule, production efficiency can only be increased if there exists a feasible situation such that it has more of every commodity than in the original situation. However, if that is what was intended, we would find it hard to explain why Samuelson would want to put out his own compensation test which is an obvious repetition of the above interpretation. Besides, it is extremely difficult to find any support for this position in the entire 'compensation test' literature since few, if any, had ever refused to consider the case where two or more utility possibility frontiers intersect. Kaldor was apparently not an exception. His example of the Corn Law is a case in point.<10> And given the state of opinion in the profession,

we think we are quite justified to say that, in general, the compensation test also deals with pairs of situations where neither one can totally dominate the other with respect to physical efficiency. As a matter of fact, these are the cases which the compensation test addresses itself to most of the time.

There is another observation concerning methodology which we would have to clarify before we can actually proceed to analyse the logical aspect of the compensation test. In modern economics, individuals are postulated to maximize their utilities, and it is definitely correct for us to conduct the compensation test in the utility space of the individuals. This procedure, however, is not universally observed. Many have chosen to carry out their analyses in the commodity space, and it seems, at least to these people, that the two spaces are interchangeable and their subsequent deductions are equivalent. We, however, wish to show that this assumed equivalence does not exist, and that it is conceptually indefensible to conduct the compensation test in the commodity space.

The most important idea in the compensation test is that of the redistribution set. Let us denote the redistribution set of situation x by $D(x)$. In the utility space, there can be one and only one meaning to any element in this set, and that is the resultant utility achievements by different individuals of an economy after a redistribution of situation x . In the commodity space, $D(x)$ is subject to several different interpretations, <11> namely, the production

possibility frontier, the Scitovsky indifference surface and the iso market value set. All three have been employed in the discussion of the compensation test, and we will examine them one by one.

The production possibility set is related to the utility possibility set in that an element in the former set when transformed into the utility space will give us a utility possibility set. Therefore when we transform all the points on the production possibility frontier $P(x)$ into the utility space, we get a sequence of utility possibility sets. Take the union of all these sets, and the maximal elements in that union will give us the welfare frontier.^{<12>} The production possibility frontier will correspond to this welfare frontier but not to a utility possibility frontier. When we are dealing with $D(x)$ in the utility space, it is the utility possibility frontiers that we are looking at. $P(x)$ and $D(x)$ differ from each other in many aspects besides being in different spaces. First of all, they are of different cardinal orders. The number of elements in $P(x)$ is infinite, but each element in $P(x)$ can be transformed into a $D(x)$ which in turn has infinitely many elements. So the cardinality of $D(x)$ is greater than that of $P(x)$. According to the Schroder-Bernstein theorem,^{<13>} $D(x)$ will dominate $P(x)$, and it is not possible for the two to be equivalent.

The second thing to notice between $D(x)$ and $P(x)$ is that an element in $P(x)$ will necessarily, through the welfare frontier, correspond to some notion of global maximum in the utility space while an element in $D(x)$ in general will

represent no more than a local maximum. Moreover, a situation y may dominate x in the commodity space, but not in the utility space and vice versa.

Lastly, we must point out that the concept of $P(x)$ is not suitable for the application of compensation tests. In view of Scitovsky's objection, we need not only $D(x)$, but also $D(y)$, to conduct the compensation test, $D(y)$ being the status quo post. In the utility space this does not cause any trouble. There are infinitely many such frontiers, and there will exist $D(y)$ for any situation y . The same reasoning, however, cannot be extended to the commodity space, i.e., we may not be able to find $P(y)$ for situations y such that $P(y) \neq P(x)$. Admittedly, our view is not shared by all economists, and Samuelson is one of those who would like to identify more than one production possibility frontier.<14>

In order to make our point clear, we will have to go back to the definition of the production possibility frontier. At any point of time, a society will have at its disposal an array of roundaboutness of production. Ex ante, all these methods are available, so that any situation which is within the feasible set of any one of these methods must also be considered as feasible for the society. Thus the possibility set for the society includes any situation that is within the feasible set of any one of these methods. Ex-post, we may be confined to a particular way of roundaboutness, but this is of no concern to our production possibility set ex ante. When making decisions, we are not limited by situational limitations. All situations are

available to us, provided the situation is attainable with our state of art. If x and y are feasible, then they must both be bounded by the same frontier. If they are at the same time efficient, then they will be on the frontier. In both cases, $P(x)$ will be identical to $P(y)$.

One may be tempted to stretch this argument to the utility space and conclude that there exists only one $D(x)$. However, there is a major difference between the construction of $D(x)$ and $P(x)$. $D(x)$ is constructed on the condition that x is being produced. If instead of x , y is being produced, we will have $D(y)$. The whole concept of $D(.)$ is a conditional one. It is based on the condition that production decisions precede consumption decisions. With this conditional clause, we will be able to identify distinctly $D(x)$ from $D(y)$. Realize that the conditional clause is a subjunctive clause; it does not require that x or y is actually produced. What corresponds to the unique $P(x)$ is not $D(x)$, but the welfare frontier; and since $P(x)$ is the same as $P(y)$, the compensation test will become inoperative in the commodity space.

The Scitovsky indifference curves drawn on the commodity space are often regarded by many as perfect substitutes for the utility possibility frontiers. Some have gone even further and insisted that it is more appropriate to use the Scitovsky indifference curves to conduct the compensation test. According to Mishan, for example, "the community indifference curves technique has always seemed (to him) a superior expositional device than the use of utility

possibility curves"<15> when it comes to the discussions of the compensation test. We will soon find out why Mishan would want to make such a comment.

As in the case of $P(x)$, the Scitovsky indifference curves, $S(x)$ of x , are of a different cardinal order than $D(x)$. Unlike $P(x)$, which is dominated by $D(x)$, $S(x)$ dominates over $D(x)$. For each element y in $P(x)$, there corresponds only one $D(y)$, but we find an infinite number of Scitovsky indifference curves, and $S(y)$ is only one among the infinite curves which pass through y . Applying the Schroder-Bernstein theorem again, we conclude that $D(x)$ and $S(x)$ cannot be equivalent. Obviously, our conclusion does not coincide with what Mishan had in mind.

The Scitovsky indifference curves are constructed by setting $(U_1, U_2, \dots, U_n) = K$, <16> where K is some ordinal scaler. If we know how different combinations of $(U_1, U_2, \dots, U_n)$ would constitute the same K for all values of K , we would have the knowledge of a complete social welfare function, and the Scitovsky indifference curves would be a redundant intermediate step. Consequently, the usual way to ensure that the society will achieve the same K while we change its output is to set each (U_i) constant. This can be done by varying the commodity composition of each individual so as to keep it equal to the same U_i . The usual exposition of this technique is to slide one individual's indifference surface against another individual's indifference surface. This technique gives us the impression that every point on the community indifference surface necessarily implies

allocative efficiency in consumption. This is a false impression. It is true that for each point on a social indifference curve of number K there will exist a situation such that the output of that situation will be allocated efficiently; but it is also true that there may exist many other situations such that these situations will also attain that K th level of social satisfaction, but in which their output is being inefficiently allocated among the members of the society. Any situation x which satisfies the property $(U_1, U_2, \dots, U_n) = K$ will have itself placed on the K th social indifference curve. Two situations x and y will be on the same indifference curve K if $(U_{1x}, U_{2x}, \dots, U_{nx}) = (U_{1y}, U_{2y}, \dots, U_{ny}) = K$ even though x may have more of every output than y . In that case, we simply would have to misallocate the output of x . The indifference curve concept, be it for an individual or a community, only displaces the iso-achievement of various combinations of arrays of commodities. It does not require these combinations to be allocated efficiently. Thus the point $(U_{1x}, U_{2x}, \dots, U_{nx})$ may well represent such an inefficient consumption allocation, and it will correspond to an interior point of the set bounded above by $D(x)$ in the utility space. Since x is on $S(x)$ does not necessarily imply that $(U_{1x}, U_{2x}, \dots, U_{nx})$ is on $D(x)$, it is not at all justifiable for us to shift from $S(x)$ to $D(x)$ as if they were essentially equivalent concepts in different spaces.

Given that $D(x)$ and $S(x)$ are not equivalent, we think it best to disregard $S(x)$ as much as possible, with perhaps the exception of those cases where the original endowment does

play a role. The reason for this is obvious. Since for every efficient $S(x) = K$ that passes through x , there will co-exist an infinite number of y such that the allocation in anyone of these y is inefficient, and $S(y) = S(x) = K$, and since there corresponds an infinite number of efficient $S(x)$ for every $D(x)$, the correspondence between $D(x)$ and those inefficient, $S(y)$ must be close to, if not engulfed in outright chaos. In such a situation, it is unimaginable that one will be able to deduce anything significant therefrom without running into inconsistency. On the other hand, it is utterly simple to take advantage of the situation, and get almost anything we want with the appropriate manipulation. It seems to us that Mishan's result is one such manipulation, and that perhaps explains why he would rather work with $S(x)$ than $D(x)$.

For any individual in a large society, we can assume that the price vector is given to him as long as the impact of his decisions on market prices are negligible. The revealed preference theory for an individual consumer is founded on such an assumption. Despite its ex-post flavour, the theory is valid so far as this assumption is concerned. Any point that lies on or inside the budget constraint is feasible for the individual through the market. Let $M(x)$ be the budget constraint of x . Assuming that there is no price discrimination, $M(x)$ will be a hyperplane separating the feasible set from the infeasible set; it is therefore the set of maximal elements of all the elements bounded by $M(x)$. $D(x)$ shares this last feature with $M(x)$; mainly because of this similarity many economists hold the view that they can

also conduct the compensation tests on $M(x)$.^{<17>} Yet by definition, this test is an aggregate criterion; if we want it to be conducted on $M(x)$, we must first transform the individual $M(x)$ into an aggregate $M(x)$. In the economic literature, the aggregate $M(x)$ problem is known as the real national income or the index number problem. In the aggregate, as in the individual case, $M(x)$ is depicted as a linear function, and the compensation criteria are transformed in a straight forward manner as: $\sum P_2 Q_2 \geq \sum P_2 Q_1$, $\sum P_1 Q_2 \geq \sum P_1 Q_1$. Is this a legitimate procedure?

The answer to the above question depends on whether the linearity of the aggregate $M(x)$ is feasible. For an individual, we have shown that a linear $M(x)$ is feasible, as the price vector will not be affected by the individual's choice. For the society as a whole, this cannot be true. The relative prices of the commodities will change as the society moves from one commodity bundle to another. A bundle y which is in $M(x)$ does not imply that x is in $M(y)$. Unlike D , M is not always symmetric. Lump sum non-disturbing transfers only apply to the same commodity bundle and it cannot be used as an argument for fixed prices in the $M(x)$ case which involves different commodity bundles.

Apart from asymmetry, a further difficulty in $M(x)$ is its infeasibility. If we assume, as do most real income analyses, that x is an efficient point, then $M(x)$ will not be feasible under any kind of return to scale. We are confronted with the same difficulty as with the Scitovsky curves, namely, what is the purpose of comparing $M(x)$ and $M(y)$ if

these represent just some imaginary bundles.

The Hicks-Kaldor Criterion

We have so far attempted to demonstrate the irrelevance of the commodity space in the compensation test; we will from now on confine our analysis to the utility space unless otherwise stated. In so doing, we shall bear in mind that we have dismissed simultaneously the theory of consumer surplus as well as the real national income investigations, and perhaps some others....

After a lengthy preliminary distinction, we now have a chance to engage in a brief discussion of the compensation principle. Let K be the binary relation of strict preference in terms of Kaldor's compensation principle, then $\forall x, x', y \in X$,

$$(x, y) \in K_s \iff \{\exists x' \in D(x) : (x', y) \in P\}$$

where P is the Pareto strict preference relation.

The Kaldor criterion is a piecemeal approach. Its purpose is to supply a link between two points in the utility space. The link, so established, will supposedly enable us to make comparisons that go beyond the dogmatic Paretian condition. It is a piecemeal approach because it is only contemplating two situations, at a time, and because two situations will not always necessarily be related by K_s . In other words, K_s may not be complete.

Many have overlooked what Hicks was trying to do when he

supplemented Kaldor's compensation criterion with his own criterion.<18> At first glance, his criterion seems rather redundant, and few have taken the effort to distinguish between the two. But underneath, Hicks was aiming at something more ambitious. According to him, a situation II is better than situation I if (f) "it is impossible to make everyone as well off as he is in situation II by any redistribution of the actual quantities acquired in situation I."<19> It will be helpful if we again put these into notations that we have been using. Let H_B denote the binary relation of strict preference according to Hicks, then $\forall x, x', y, y' \in X$:

$$(x, y) \in H_B \iff \{ \neg [\exists y' \in D(y) : (y', x) \in P] \}$$

Defining H_W as the worse off counterpart of H_B , we have:

$$(x, y) \in H_W \iff \{ \neg [\exists x' \in D(x) : (x', y) \in P] \}$$

If we take the union of K_B and H_W , we will arrive at the following definition:

$$(x, y) \in (K_B \cup H_W) \iff \{ (\exists x' \in D(x) : (x', y) \in P) \vee \neg (\exists x' \in D(x) : (x', y) \in P) \}$$

The result is obvious. K_B and H_W are complements, and together they always define a relation between any two situations x, y in X . By the time Hicks proposed his criterion, the asymmetric and transitive properties of the compensation test had not been disproved by Scitovsky. Hence $(K_B \cup H_W)$ is no longer piecemeal; it is complete, asymmetric and transitive, and thus a linear order in the fullest sense. Who else would need Bergson and Samuelson? We already have a social welfare function without resorting to interpersonal comparison!

We want to insist that the compensation test does involve interpersonal comparison. Not only that, the kind of comparison involved is a very special one, a crude one, so crude that we may as well call it *ad hoc*.

There are two possibilities after the compensation test has been fulfilled: (1) compensation is paid, and (2) compensation is not paid. If the compensation is paid, then the compensation test would be redundant. The change is justified by the Pareto criterion, and our previous analysis will apply here. Thus the only interesting and in a sense non-trivial, case of the compensation test is the second possibility where compensation is not paid. Does the test involve any interpersonal comparison in this case?

Little's answer is both yes and no. In his opinion, "since the criterion under consideration only provides hypothetical compensation for the losers; it follows that any one who accepts it must either make interpersonal comparisons, and decide that money has the same utility for every one, or he must abandon the view that 'the happiness of the group' is a name for a collection of individual satisfactions." <20> Little chooses the second alternative to explain the compensation criterion, and arrives at the conclusion that the criterion is value-loaded after all. We do not want to deny that this is a viable route. Nevertheless, in accepting this explanation we are, in effect, forcing the compensationists to abandon the Pareto criterion altogether. Since 'the happiness of the society' has a substantive identity, its magnitude, if there is any,

need bear no functional relationship to the level of utilities of the individuals in the society. This is surely not what the compensationists have intended.

At this point, we will restrict ourselves to a more modest attempt and put aside Little's suggestion, at least for the time being. The choice is not to downplay the gestalt theories^{<21>} and all that. Rather it is our belief that in so doing we will be more loyal to Kaldor and Hick's original formulation.

Since we have denounced the commodity space, we can no longer talk of money compensation. As a result, the notion of money utility that Little mentioned is irrelevant. This is in no sense a loss to us. By discarding money we have just got rid of an unnecessary intermediary of interpersonal comparison, and this will enable us to conduct the comparison in a more direct fashion in the utility space.

During a move from x to y with $x' \in D(x)$, $y' \in D(y)$, $(u, x') \in P$ or $(u', x) \in P$, certain parties' utilities will be diminished while those of others will be increased. To recommend such a move within the individualistic framework, we are in fact saying that the gainers' gains are sufficient to outweigh the losers' losses. If this is not interpersonal comparison, what else can we call this? In the compensation test, this conclusion is being 'rationalized' by the physical fact that the gainers can over compensate the losers if they choose to. However, the compensation is not paid, it is not supposed to be paid. The losers are left with the comfort which is analogous to seeing the cake and not being able to

eat it.

We will try to elucidate some latent characteristics of the compensation test. For the sake of exposition, let there be two individuals in the society. Individual 1 being the loser, and individual 2 the gainer in a move from x to y . Let their respective gains and losses be ΔU_2 and ΔU_1 . All these are shown in figure 3.2. In this two-person economy, let us call the social evaluation at situation x under the Kaldor-Hicks criterion the social rate of substitution (SRS) between U_1 and U_2 (SRS) at x in a two-person economy. Furthermore, we want to identify three other rates as follows:

- (1) The rate of transformation between U_1 and U_2 at x made available by y (RTY)
- (2) The rate of transformation between U_1 and U_2 at x made available by x' (RTX')
- (3) The rate of transformation between U_1 and U_2 of $D(x)$ at x (RTX)

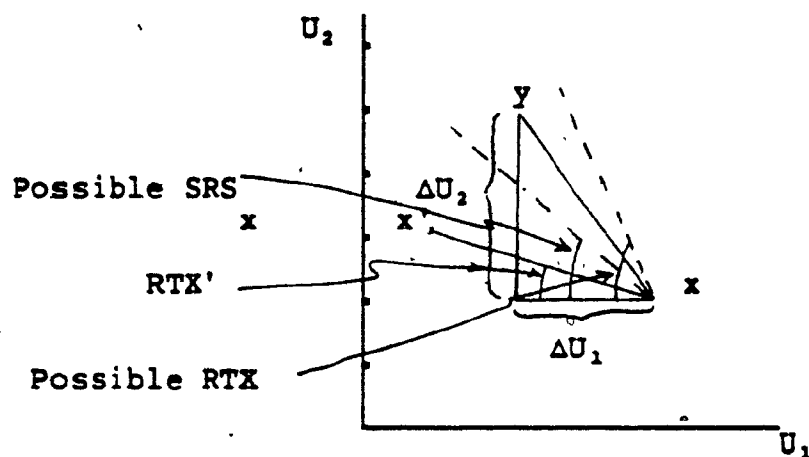


fig. 3.2

Since $(x, y) \in H_w$ (the case of $(x, y) \in K$, can be similarly dealt with) the society through the compensation test will express its preference of ΔU_2 against ΔU_1 . Such a declaration is an explicit indication of the existence of a social interpersonal comparison scheme. At this point, we still do not have sufficient information to claim full knowledge of such a scheme. However, the information given to us by $(x, y) \in H_w$ enables us to postulate that the SRS at x is less than $\Delta U_2 / \Delta U_1$. The latter, as we know, is RTY . In explaining the compensation test in this way, we have achieved one enormous task, and that is to circumscribe the compensation test within the traditional optimization framework. Like all other optimization problems, the only reason a society would want to move away from some situation x through the compensation test will have to be the existence of certain disparity between the rate of substitution and the rate of transformation at x .

As $SRS < RTY$, the society will desire to move away

from x to some other situation which will promise more utilities to individual 2. There are three ways to fulfill this objective. (1) The society can make an indivisible move from x to y ; (2) it can move along $D(x)$ in the direction of x' , and (3) it can move to x' directly.

By definition of the compensation test, $RTY > RTX'$, which means that for the same ΔU_1 , y will give us more U_2 in return. This puts y in a preferred position with respect to x' according to the Pareto criterion. So, what is left to compete with y is the movement along $D(x)$. RTX can be greater or less than RTY . If $RTX < RTY$, it seems that we can repeat the reasoning as in the case of x' , and conclude that y should be preferred to movements along $D(x)$ in the direction of x' . This conclusion is fallacious. When dealing with x' , both movements, to x' and y are indivisible, involving a common loss of ΔU_1 . The two movements are comparable without further information. The situation is different with RTX . Even if RTX is smaller than RTY , it does not necessarily mean that the movement along $D(x)$ is worse than the move from x to y , the latter being an indivisible move while the former is a continuous movement. Suppose the social evaluation takes a sudden turn to favour individual 1 in the neighbourhood of x , we may easily have some elements in $D(x)$ between x and x' which are socially preferred to y . Figure 3.3 demonstrates such a possibility. The case of $RTY < RTX$ is similarly inconclusive.

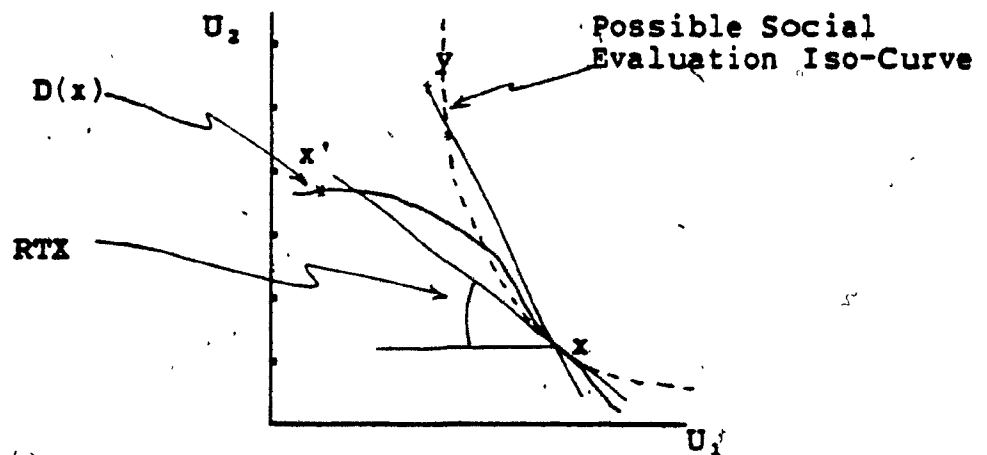


fig. 3.3

The crux of the problem lies in the *ad hoc* structure of the interpersonal comparison mentioned earlier. Instead of constructing a comprehensive scheme of social evaluation, the compensation test only limits the scope of SRS to one point at a time. It is based on this one point SRS that the compensation test makes its recommendation of the indivisible move from x to y . The way they have chosen to deal with the RTX issue is to write it off completely. When conducting the compensation test, we are allowed only to compare y with one and only one element in $D(x)$, and that is x' . If x' is inferior to y , the move is justified although superior alternatives exist.

The Scitovsky Paradox

Obviously, the conclusion so formed cannot be carried through without logical errors. Since we have only SRS at x , we do not know what will be the SRS at y , or at any other point, in the utility space. There is always a chance that the SRS at y is greater than RTY , and the society is urged to move from y to x . This, of course, is the source of the familiar Scitovsky's paradox.<22>

The Scitovsky's paradox can be looked at from two different perspectives. First, we can regard it as a final blow to the compensation test as it reveals the intrinsically, inconsistent, structure of the social judgement of the test. Secondly, we can treat the paradox as a warning, signalling the inadequacies of the test. If we choose to adopt the first view, and if we care about consistency, we should abandon the classical compensation test altogether. If we do not care for consistency, then the whole paradox should be of no concern to us, even if we are faced with overlapping, intersecting social preference curves. On the other hand, if we choose to treat the paradox as only a proof of inadequacy and not an outright rejection of the compensation test, we will have to consider the possibility that we may some day come up with certain appropriate modifications to the test such that Scitovsky's objection can be met. In other words, we should not exclude the possibility that we may in the future be able to trace out a social welfare function with

the Hicks-Raldor kind of piecemeal information, even though at the moment the social preference curves intersect and overlap.

If we accept that social preference curves do not intersect, and if SRS at x is less than RTY and SRS at y is greater than RTY, we can deduce that the social preference curves are not linear. Most likely, they will display certain diminishing rates of substitutability between U_1 and U_2 . This is a major result. It explains the whole paradox as the result of over substituting U_2 for U_1 . But how does this over substitution come about? Basically we can assign it to the indivisibility from x to y .

Earlier we have asserted that the main reason which gives rise to the Scitovsky's Paradox is that people have only considered the move from x to y , and not anything else in between. Now, if we do not make any other specifications, the distance from x to y may be considerable, and it is certainly plausible that the distribution under the two situations are so different that those who were once favoured by society for their poverty have turned into much hated millionaires. If we are allowed to move gradually on xy , it is very probable that we may be able to locate a point or an interval at which further advances to y or retreats back to x are not recommended. If this point or interval is found, the Scitovsky's paradox will be solved. However, in order to be able to cruise along xy , we need much more than the

availability of y and x' . We need to consider all the $D(\cdot)$'s since all of them are liable to intersect xy . Our informational and feasibility bases have to be much enlarged. Such requirements are apparently not promised by the compensation test. Actually, if we have all the $D(\cdot)$'s on hand, why should we limit ourselves to xy ? Once the optimal point or interval on xy is located, it will always be a Pareto move to the north east direction if it is possible. Such possibility is shown in figure 3.4.

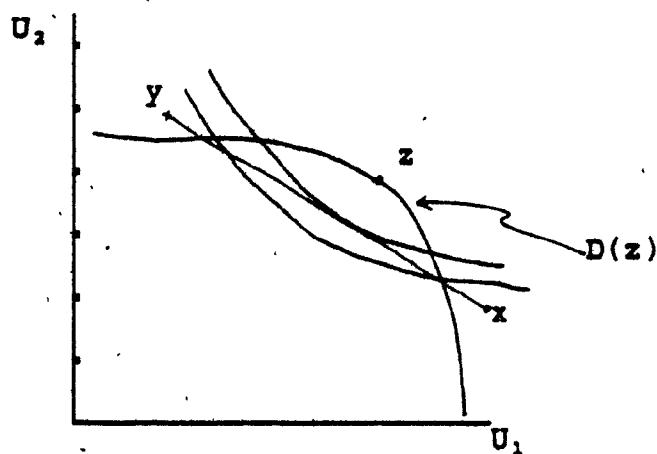


fig. 3.4

From the above discussion, we have arrived at a very interesting result. If we take the social judgement of the compensation test seriously, then what seemed once to be a liability has now turned into a blessing. The Scitovsky's Paradox is no longer something to be feared of. As a matter of fact, it should be something that we are looking for. Let us start with x ; through the compensation

judgement (which always exists) we move to another situation y . If at y , the Scitovsky's Paradox arises with respect to x , we know that we have over-substituted the two individuals' utilities. Somewhere between x and y there must be a situation preferable to both. By conducting the test for points in between x and y , the local optimal situation can easily be located. To achieve this global optimum, we will move to feasible situations lying in the north east of the local optimum and re-conduct the compensation test thereof. By repeating such a process *ad infinitum*, there is little doubt that we will find the global social optimum. If we do not encounter a paradox by moving from x to y , we should proceed further in the same direction until a paradox is found. Once it is found, we can apply the same procedure as before and locate the global optimum.

Obviously, this is an optimism that would not last. Before we go on to dismiss it, we will endeavour to examine an alternative interpretation of the Scitovsky's Paradox by Mishan. Working with community indifference curves, Mishan also attributed the Paradox to the difference in the rate of social substitution between the utilities of the individuals. However, his construction is quite different from ours. While we are considering the differences of the SRS in the *status quo ante* and *post*, Mishan is only concerned with the possible differences in the SRS in the *status quo ante* x . This is so because in the commodity space, all that we know are the commodity

bundles. How these bundles are divided between the individuals is not known. Hence, we cannot postulate that the society has over-substituted U_1 for U_2 , or vice versa. Barred from this option, we have no other choice but to explain the paradox by differences in the distribution of the initial commodity bundle.

At any initial bundle x , there corresponds an Edgeworth box and a contract curve. Assume that we are at some point on the contract curve. At this point, the marginal rate of substitution of good A for good B must be the same for the two individuals and for the society as whole. Let this rate be greater than the one at the point we are considering. Let y be another bundle with more of good B and less of good A. Then a "movement from x to y is a movement to a better welfare position for the community, since for every individual in the community the unit of B added is valued more highly than the unit of A which is subtracted." <23>

So it is said, when two sets of community indifference curves constructed at x are based on two different distributions of x : one with $MRS_{A,B} > 1$ and one with $MRS_{A,B} < 1$; then the two sets of community indifference curves will intersect between x, y .

There are two flaws in Mishan's argument. (1) He does not know the distribution of y . Not everyone will necessarily get more of B for the amount of A they have given up. To assume the contrary will be equivalent to imposing restrictions on the distribution of y which is

essentially not a part of the compensation test. Since not everyone will be better off at y , we cannot guarantee that y will be on a higher community indifference than x , even if $MRS_{A,B} > 1$ for the society at x . (2) Even if we assume that at y everyone will be getting more of B for the A they have given up, we still cannot be sure that y is on a higher community indifference curve than x . Mishan's assertion will be true only if y is in the immediate neighbourhood of x . He probably did realize this point, as we can see from the quotation above. He was only talking about per unit change of A and B .

A preference for an infinitesimal substitution of A for B at x does not imply that the same preference will remain valid for a change of bigger magnitude. Since the distance between x and y is unspecified, it may well transcend the vicinity of x , and thus the resulting preference relation between x and y will no longer be governed by the $MRS_{A,B}$ at x . When this happens, the community indifference curve from x with an initial $MRS_{A,B} > 1$ may pass above or below y . This is also true of the other indifference curve which is constructed at x with $MRS_{A,B} < 1$.

Mishan's concern about distribution is correct, but his effort is misdirected. By working on the commodity space with the indifference curve concept, he is practically trying to canvass a distribution problem without distribution. Since the Scitovsky's Paradox arises as a result of the differences in distribution before and

after the recommended move, we certainly cannot find an answer in the community indifference curves. His attempt to manipulate MRS_{AB} at x as an indicator for distribution is an effort in vain. By insisting on reading more than what the MRS at x supplies, we have to specify what the distribution is every time we are conducting the test. Even with this specification, we still would not know whether we have, during the move, over substituted U_2 for U_1 . In order to know this, we must also have knowledge of the distribution of y (i.e., MRS_{AB} at y) and the exact details of the contract curves of x and y . If MRS_{AB} at y is also greater than unity, and if there is no preference reversal, then we may perhaps say that the indifference curve so drawn at x will pass above y . This is still a supposition which we have no intention to verify. We do not not think it is worthwhile to go through all such fuss when we can accomplish a better result much more easily by simply choosing to conduct our test in the utility space.

With our digression behind us, we will now return to the results of the compensation test. Is it really as potent as what we have demonstrated? Can it actually locate the social optimum singlehanded? The answer seems to be in the affirmative as we have solved the problem of symmetry in the test. Upon further examination, this does not appear to be the case. There is yet another stumbling block preventing us from accepting the compensation test as a 'rational' choice function.

Take any two situations x and y . By (K_B, U, H_W) they

must be related, and therefore there exists a restriction on the SRS at x with respect to y . Take another situation z , it must be again related to x , and it provides us with another restriction on the SRS at x . As there is no pre-arranged agreement with the two restrictions, they can easily conflict with each other. Such a case is shown in figure 3.5.

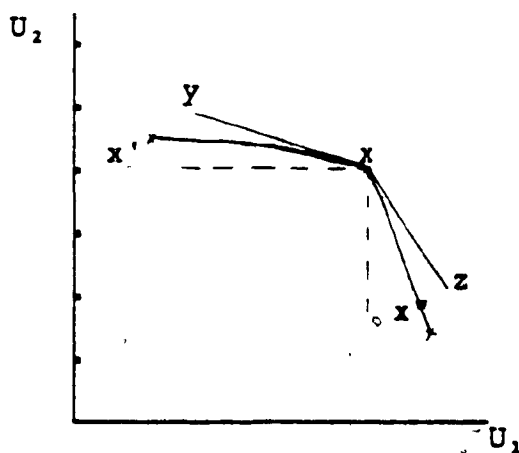


fig. 3.5

The two restrictions do not signify that there is an abrupt change in the preference of the social judgement at x , but rather two different social evaluation schemes, one independent of another. In fact, we can have as many as $(m-1)$ social evaluation schemes at x , where m is the numbers of situations available. Each evaluation scheme will have to be internally consistent, but there is no guarantee that any two of them will be consistent with each other. The reason for the non-uniqueness of our

social evaluation scheme is fundamentally a result of the binary nature of the compensation test.

In earlier discussions, when we were trying to locate the global social optimum between x and y , we had to reconduct the compensation test for situations lying between x and y . However, in so doing, it is possible that we might have switched to a different evaluation scheme. When that happens, the original social optimum will lose its relevance, and we will have to redefine our target or we will be pursuing the previous optimum, which is in a sense 'obsolete'. A reader who is familiar with utility theories will immediately realize that the problem we are facing is that of integrability. Or if one chooses to use the more recent terminology -- path independence. Both integrability and path independence are implied in transitivity, and it is exactly the lack of transitivity that leads to the ultimate failure of the compensation test. Scitovsky's double criterion resurrects asymmetry at the expense of completeness. It has not however dealt with transitivity, <24> and that leaves us still haunted by the same old problem.

Samuelson's 'Objective' Solution

All along, we have been trying to rank the elements in the utility space, and we have argued at length that this is the proper way if our ultimate aim is the welfare of the people. Samuelson has missed this point in his effort to rectify the compensation test, but because of his authority, his misconception is still followed by some in the profession.<25> Usually those who share this misconception will identify Samuelson's criterion with asymmetry, transitivity though not completeness.

These properties would have been a correct description of Samuelson's criterion if this criterion is being used to rank order elements in the commodity space or equivalently the $D(.)$'s, the two being isomorphic to each other. If X, Y represent commodity bundles and X has more of every commodity than Y ,<26> $D(X)$ will be lying totally above $D(Y)$. This last sentence satisfies the Samuelson criterion which states that:

$$X(Sa)Y \Leftrightarrow [\forall y' \in D(Y), \exists x' \in D(X) : (x', y') \in P]$$

Interpreted in this way, the Samuelson criterion is indeed transitive, asymmetric and incomplete, but it is no more and no less than the Pareto ranking of the commodity space. Since we have exhaustively discussed the Pareto criterion, we shall not repeat ourselves again.

The second alternative is to employ the Samuelson criterion to rank, not the $D(.)$, but the elements in the utility space. This time let x, y represent two

situations. From each of the corresponding x and y , we will be able to derive numerous utility possibility curves. If this interpretation is accepted, the Samuelson criterion will be complete. Nevertheless, the inconsistencies, viz., intransitivity and symmetry, of the compensation test that it sought to solve will re-appear.

Let us first re-examine what kind of social evaluation scheme is offered by the Samuelson criterion when it is conducted in the utility space. Let $x(Sa)y$ read as ' x is better than y ' according to the Samuelson criterion. Then

$$x(Sa)y \Leftrightarrow \exists D(X) \exists D(Y) [\forall y' \in D(Y), \exists x' \in D(X): (x', y') \in P]$$

$D(X) \in \lambda(X)$ and $D(Y) \in \lambda(Y)$ where

$$\lambda(X) = \{D(X): x \in D(X)\}$$

$$\lambda(Y) = \{D(Y): y \in D(Y)\}.$$

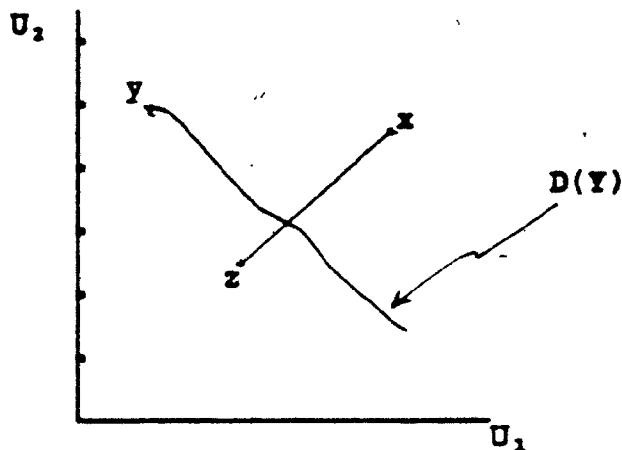


fig. 3.6

Let $x(Sa)y$ and $y(Sa)z$ as shown in figure 3.6. Furthermore, let $A = \{w: w = \alpha x + (1 - \alpha)z\}$ and $0 \leq \alpha \leq 1$.

A is closed. It is also continuous by virtue of the continuity of the commodity space. Let $F = \{ w: \neg y(Sa)w \}$, $G = \{ w: \neg w(Sa)y \}$. Both F and G are closed, and non-empty, $x \in F$ and $z \in G$ and $(F \cup G) = A$. Since F and G are closed and by the connectedness of A, $(F \cap G) \neq \emptyset$. Hence there must exist some α^* such that $w^* = \{ \alpha^*x + (1 - \alpha^*)z \} \in (F \cap G)$. We will say that this w^* is indifferent to y under the Samuelson criterion.

Next, we will try to prove the uniqueness of α^* . Assume that there are α_1 and α_2 such that $\alpha_1 < \alpha_2$. $w_1 = \alpha_1x + (1 - \alpha_1)z$, $w_2 = \alpha_2x + (1 - \alpha_2)z$ and both $w_1, w_2 \in (F \cap G)$. Since the Samuelson criterion is also Pareto inclusive, $\alpha_1 < \alpha_2 \Rightarrow (w_2, w_1) \in P \Rightarrow (w_2, w_1) \in Sa$, a contradiction, if α_1 and α_2 are distinct variants of α^* .

w^* can be on $D(Y)$ or not on $D(Y)$. If it is not on $D(Y)$, it must be on some $D(\cdot)$. Let this $D(\cdot)$ be $D(T)$. Let the respective commodity bundle associated with $D(Y)$ be Y and $D(T)$ be T. Given Y and T distinct, T either has more of every commodity than Y, or has less of every commodity than Y, or more of some and less of other commodities than Y. If T has more of every commodity than Y, $D(Y)$ will lie totally above $D(T)$ and $t(Sa)y$. So T cannot have more of everything than Y, and similarly not less of everything than Y for distinct T and Y. We are then left with only one alternative, and that is T will have more or some and less of other commodities than Y. Let there be r commodities and $T_i > Y_i, \forall i = 1..j$; $T_i < Y_i, \forall i = j+1...r$. Moreover let $c_i = 0, \forall i = 1...j$

and $c_i > 0$, $\forall i = j+1 \dots r$. Adding c to Y we get $S = c + Y$. Since S has more of some and not less of any commodities than Y , S is preferred to Y by the Pareto criterion. Since the Samuelson criterion is Pareto inclusive, for any $s \in D(S)$, $y \in D(Y)$, $s(Sa)y$. But as S does not have more of everything than T , for the same $D(T)$ and $D(S)$ as before, we have $\neg[s(Sa)t]$ and this contradicts the Pareto criterion, and hence $w^* \in D(Y)$.

Since this proof can be repeated for every situation in $D(Y)$, we have to conclude that $D(Y)$ is the set whose elements, given Y , are indifferent to y . With reference to Y , and with the continuity of the commodity space, any situation x in the utility space will lie on a $D(\cdot)$ such that either $x(Sa)y$ or $y(Sa)x$ or $x \in D(Y)$. Thus the Samuelson criterion is complete, and even transitive and asymmetric, if Y is properly specified. The social preference surface is the same as the utility possibility surface. Distribution is totally disregarded. A situation is better than another one if, and only if, it is derived from a larger bundle than the other. Since the SRS at any x is the same as the RTX, interpersonal comparison is given by RTX. Any situation is optimal by SRS, = RTX unless we can move to a larger bundle.

Though in most people's opinion, this would represent an extremely oppressive scheme of value judgement, it would still be a consistent scheme if all the properties mentioned above can be secured. Our next task is therefore to prove that even this last hope of the compensationists

is not founded on solid logical ground.

We have maintained earlier that for any situation x in the utility space, there exist numerous commodity bundles whose utility possibility frontier will pass through x . Let X be such a bundle and $D(X)$ the corresponding utility frontier as shown in figure 3.7.

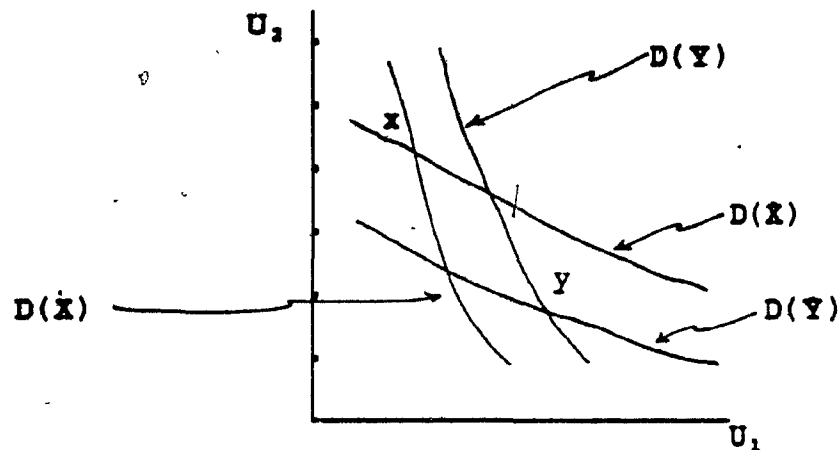


fig. 3.7

For another situation y , $D(Y)$ is the utility frontier that corresponds to y which has more of every commodity than x . Hence we have $y(Sa)x$. Let X be another commodity bundle whose utility frontier $D(X)$ also passes through x . Similarly, $D(Y)$ passes through y . From the diagram, we can see that X and Y can be so constructed that $D(X)$ totally dominates $D(Y)$, and hence $x(Sa)y$. By $x(Sa)y$ and $y(Sa)x$, we have proved that the Samuelson criterion is not asymmetric. Intransitivity can be proved along the same line. To avoid needless repetition, we have substituted

figure 3.8 for proof.

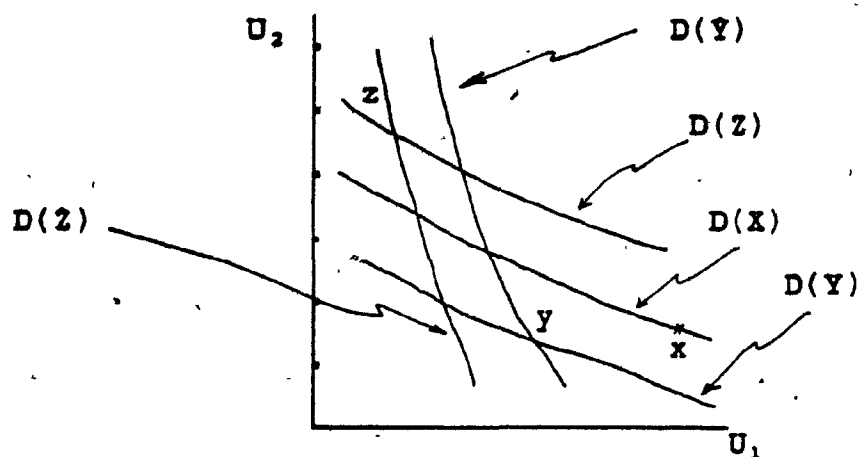


fig. 3.8

Little's Value-Loaded Criterion

It appears that the last effort to save the 'objective' and 'rational' compensation test has also failed. Like the Hicks-Kaldor or the Scitovsky criterion before, Samuelson criterion's failure is due mainly to the plurality of its social judgements. The reason why we cannot derive a unique social evaluation scheme from all these criteria is the way they are formulated. The social judgements in all cases do not have any substantive identity. They are 'derived' judgements which owe their existence to the physical feasibilities of the society.<27> When the latter changes, so will the former, and social judgements at different situations often

conflict with each other.

Little realizes that interpersonal comparison is inevitable in Welfare economics, and pledges to make it explicit in the compensation criterion (such judgement, as we understood, is additional to the interpersonal comparison provided by the Pareto criterion.) Instead of being a puppet on the string, the social value judgement under Little's criterion will stand on its own. It will be able to evaluate two specified distributions and declare a preference.

Before we can judge two distributions, we should be able to say what we mean exactly when we are talking of distribution. Little supposedly is talking about distribution of real income. Then we will have to know what real income is. There are several ways to define real income. The first is to equate it with money income, or with the money income adjusted by a 'suitable' scalar. We have argued against the legitimacy of such an approach, and have no intention to engage at length in the same discussion again.

The second way, which is also the way we would advocate, is to follow the lead of Pigou and treat real income as the same as welfare. When we speak of real income in such a context, we are referring to a commodity-cum-distribution situation, and to the fact that every situation represents a distinctive real income level. There is no way of separating real income from distribution since the latter is a part of a whole, and

the whole requires this part to accomplish its completeness. Needless to say, Little's distinction is not applicable to this definition.

Maybe it is due to their tangibilities that commodity bundles are often taken as equivalent to real income. Problems with this approach to real income are well documented by Graaff. He argues that the commodity bundle to be produced by the economy is not independent of its distribution. But how can we know the distribution of real income if we do not have real income?<28> The argument is circular. Hence we must conclude here, as in the previous case, that real income and distribution cannot be distinguished.<29>

So far, all the definitions of real income we have considered are incompatible with Little's dichotomy of real income and distribution. The next, and also the last, definition of real income that we are going to discuss is probably the only hope for Little -- naturally it was propounded by Little. The stage was set for Little when Robertson defends his thesis by not requiring the same commodity bundle as a necessary condition for the same real income. Nevertheless, Robertson is still demanding that for two real incomes to be equal, there is not "any shrinkage, or the reverse, in the total supply of such factor (work, saving, risk-taking)."<30> Little promptly seizes this opportunity and pushes it one step further. He questions why Sir Dennis thinks that he "should not have permitted any change in the amounts of

work, risk-taking and saving"<31> when there is a change in distribution of the same real income. Definitely, real income is not the same as commodity bundle anymore. Yet, what is the difference between them under Little's new definition? Very intuitively, we can say that the commodity bundle depends on distribution whereas real income does not.

Like the physical output, the real income of an economy depends on an array of factors and to a large extent these factors are more or less the same for both. If we put them into broad categories, these factors will include the resource availability; the technical know-how; the allocation method and many others, that are beyond the scope of this study to list.

Given that the allocation method, resource availability and technical know-how all remain the same, the economy will be able to produce certain commodity bundles with certain distributions. The latter may be defined as a vector whose components are respective utility levels of the members of the economy. A reshuffling of this vector will lead to a change in the commodities produced in the economy as a result of work-leisure substitution. When we try to give someone more of one and less of the other, the commodity vector changes, and naturally the original intention may not be fulfilled. Nonetheless, after maybe a few rounds of re-adjustments, the economy will eventually settle at a new commodity bundle with a new distribution vector.

However, this new bundle cannot represent a new level of real income for the economy without contradicting Little's claim for the dichotomy between real income and distribution. In order to maintain that dichotomy, Little will have to come up with some factor which will only affect either real income or distribution, but not both. Little is nowhere explicit about this, and all we can do is to make a conjecture of what he has in mind.

The name of the factor which we want to add to Little's theory can be tentatively termed 'economic consciousness'. At any point of economic development, the population of an economy is always characterized by a particular kind of attitude towards economic activities. At times, they might want to engage in more economic activities, and at other times less. This is not to be confused with the work and leisure trade off from labour economics. Our economic agents may be well aware of the fact that they can increase their economic well-being by working more, and they might actually not mind working more. However, their economic initiatives may be suppressed by other social, cultural or even habitual reasons.

With the introduction of this 'economic consciousness' factor, real income will no longer stand for an array of output potentialities. Rather it will be more appropriate to regard it as a vector of input potentialities. Included in this vector are all tangible and intangible inputs as well as the state of mind of the

people, i.e., the level of their economic consciousness. We will say that real income has remained the same if none of these inputs has changed. Under this definition, real income will be independent of real output. A redistribution of the society's resources will not have any effect on real income since it will not change the value of any input. On the other hand, such a move will likely change the real physical output of the society as people may want to make the appropriate adjustments with respect to their work-leisure decisions.

Admittedly, this definition of real income is still far from satisfactory. First of all, there is the problem of recognizing what is 'economic consciousness'. Even if there can be agreement on its nature, there will still be the problem of quantification. How can we know that it has or has not changed? Though all these can be put aside as practical complications, without them we will have no way of monitoring changes in real income, and the concept of 'economic consciousness' will more or less become a tautology.

Having pointed out the deficiencies in our interpretation of real income, we are still of the opinion that it is the closest we could have come to what Little had in mind during his exchange of views with Robertson and others, and we will temporarily adopt it to facilitate our analysis of Little's compensation criterion. Later, it will be revealed that our objection to that criterion is independent of the above mentioned shortcomings in our

interpretation.

We will now proceed to examine Little's version of the compensation test. Annoyed by the Scitovsky Paradox, Little has used his criterion in another effort to resolve that paradox. In Little's criterion, a desirable economic change constitutes two parts: (a) it would result in a good distribution of wealth, and (b) the potential losers could not profitably bribe the potential gainers to oppose the change.<32> The two parts taken together are sufficient, but part one alone is necessary for a desirable economic change.<33> The second part of the criterion amounts to what has been known as the Scitovsky criterion (the reverse of the Kaldor criterion, and not to be confused with the double criterion).

There has been a debate between Little and some leading economists over the exclusion of the Kaldor criterion in part two of the Little criterion. Little's defence is that when the Scitovsky criterion is fulfilled, the preferred situation is always a Paretian optimum with respect to all the distributions available in the two real income levels while the Kaldor criterion is not. We do not wish to enter into this debate at this late stage. However, we must say that when Little chooses to discard the Kaldor criterion, he is also giving up completeness. Little is well aware of this fact, and overtly proclaims that his approach is of a piece-meal nature.

In the literature on the subject, the main distinction between the Little criterion and the rest of

the compensation criteria is that the former involves value judgement, and the latter is free from it. After we have shown that value judgements are implicit in all compensation criteria, the distinction vanishes. The only identifiable difference one may still cling to is that the value judgement in Little's criterion is explicit, and the other criteria are implicit. But explicit or implicit, value judgements are value judgements, and this difference could not possibly constitute a resolution of the Scitovsky Paradox.

The crucial contribution of Little's criterion is to separate the value judgement in the compensation test into two parts. The first part is the traditional Paretian value judgement, and the second part is some kind of 'distributional' judgement. Still haunted by the profession's abhorance of the dichotomy between production and distribution -- obvious in Kennedy and Meade's criticisms, <34> Little is reluctant to call the second part of his value judgement as 'distributional' judgement. So, when Sen offers him a way out of the difficulty, he is more than happy to accept it. <35> Sen argues that all that is involved in Little's criterion is no more than the simple assumption of transitivity. <36> Using our symbols, Sen's interpretation of the Little criterion amounts to $(y \succ x') \wedge (x' \succ x) \Rightarrow (y, x) \in L$, where L denotes the Little criterion, $\succ$ being the Paretian strict preference relation and $\succ$ can be a strict preference relation of 'anything'. <37> Transitivity of L requires that

$\forall a, a', b, b', c, \text{ and } c' \text{ in the set of situations,}$
 $[(a > b') \wedge (b' > b)] \wedge [(b > c') \wedge (c' > c)] \Rightarrow (a, c) \in L$ and Sen "do(es)
 not see how it (transitivity) can be avoided." <38>

By agreeing with Sen, Little has to shift from the previous judgement of $>$ on x and y to $>$ on x and x' . This obviously is an improved argument against Kennedy and Meade's criticism. They both question why Little does not compare x and y directly if the value judgement is on x and y . By introducing $>$ on x and x' , Little can now claim that x' is a necessary intermediary for the test. Such a move however is not without costs, and we will be dealing with some of them in later discussions. To say that the $>$ relation is not a relation on distribution is in a certain sense to be engaging in semantics. There is little doubt that x and x' differ from each other in their relative distribution. Actually, their construction is based on this difference. Even if we admit that there exist other factors determining $>$, the concrete evidence of the distributional difference between x and x' certainly demands us to embrace distribution into the $>$ relation. As we have at our disposal a ready, though unsatisfactory, distinction between real income and distribution, the detour seems grandiose.

Hence we will assume that the second value judgement in Little's criterion pertains to distributional judgements. In what way does the introduction of two distinct value judgements help solve our paradox? In previous tests, the two judgements are mingled into one,

and are revealed in the final recommendation. When the two judgements are blended together, the properties of one may be overridden by those of the other, or vice versa. In the Little criterion, we give them separate identities. Especially we demand that the distributional judgement not be overruled. Implicit in Little's analysis, the distributional judgement is asymmetric and transitive. By the asymmetry of $\succ$, the asymmetry of the Little criterion is automatically guaranteed. If $(x \succ x')$, and y is distributional similar to x' , we cannot have $y \succ x'$; since the distributional judgement is necessary in the Little criterion, y cannot be better than x according to the Little criterion. The distributional judgement alone cannot assure transitivity of the Little criterion, but it does manage to deter cyclicity. By transitivity of $\succ$, $(x \succ y) \wedge (y \succ z) \Rightarrow (x \succ z)$. So we cannot have $(z \succ x)$ and therefore z cannot be better than x by Little's criterion.

Despite the inability of the Little criterion to fulfill transitivity, acyclicity does serve as a consolation prize. It ensures that any chosen situation from a set of alternatives according to the Little criterion is in the maximal set of the set of feasible alternatives, and this characteristic satisfies one of the major requirements of a consistent decision function. The asymmetric property requires that the choice set of any pair of comparable alternatives consist of one and only one element. If the two properties can be sustained, the

Little criterion is definitely an improvement on its predecessors, and the improvement is the result of substituting a well-structured (asymmetric and transitive) interpersonal comparison for the *ad hoc* interpersonal comparison of the Kaldor-Hicks criterion.

Since the Little criterion is nothing more than an assumption of transitivity, its fate will naturally be related to the credibility of this transitivity assumption. The transitivity of the Paretian criterion by itself is taken for granted by most people, and the transitivity of $\succ$ for the same level of real income also seems indisputable. Originally, Little was conducting the distributional judgement on x and y , and he was right in pointing out that it requires nothing more than an order relation to compare x and y .^{<39>} However if we can compare x and y directly, why would we need x' ? It must be the case that we do believe that the compensation test is meaningful. To find x' , we are no longer involved in a simple order relation only. Something more complicated is required. We must find an x' such that it is distributionally similar to y , and this requires that the relative distributions of different real income levels be comparable. Even if we adopt Sen's advice and conduct the distributional judgement on x and x' first, we still have to find an x' which is distributionally similar to y . So the method suggested by Sen will indeed avoid the embarrassment of comparing the distribution of different real income levels at the beginning but only to find out

that the same difficulty surfaces at the end anyway.

There are two major problems when we are dealing with distributional judgement at different income levels. The first is the problem of defining what is the same relative distribution at two income levels. If we admit that it is meaningful to talk about similar distributions, the second problem will then be concerned with the reasonableness of postulating transitivity over different income levels.

We will deal with the second problem first. It is a popular belief that at different income levels the distributional judgement may run in opposite directions. To take an extreme example, most people would say that at an income level, on the average, below physical subsistence, an uneven distribution of income which gives surviving chances to some is better than an even distribution which condemns the whole society to extinction. The distributional judgement may change and favour even income distribution as the income level rises. Let the relative distribution of x be similar to y' and x' similar to y . From the above example, we may have x' distributionally preferred to x and y' to y . Such conclusion certainly violates the transitivity of the distributional judgement assumed earlier. Without the transitivity of the distributional judgement, all the properties of consistency of the Little criterion will have to go, and we are back to square one.<40>

We have been defining the distribution of a certain level of income as a vector whose components are the

utility levels of the members in a society. Since vector comparisons are usually incomplete, it is necessary to normalize these distributional vectors into scalars of relative distribution so that any two situations are comparable. Naturally, the normalization vector has to be the same for all income levels if consistency is to be observed. Moreover, the components in the distributional vectors cannot be ordinal utilities. If that is not the case, then each component of the distributional vector will be equivalent up to any monotonic increasing transformation. Given an identical normalization vector for all income levels, the relation between the relative distribution of two income levels will vary with a change in the choice of transformation. Apparently it does not make any sense to talk about similarity of distribution under such circumstances.

Now that we have seen what similar income distribution means, it should be clear that there is no gain whatsoever by shifting from direct interpersonal comparisons of utilities to relative income comparisons. Both methods will involve vector normalization, and technically they are of the same order of difficulty. On the other hand most of us will probably feel more at home with interpersonal comparisons of utilities. After all, we do make them almost everyday. As for the concept of similar income distribution, we think that only a few will actually attach any substance to it even if they do not reject it outright. Sometimes one really wonders why we

have to go through all these troubles to arrive at something that is inferior to what we already have.

After a detailed exposition of Little's proposition,^{<41>} we have arrived at a conclusion that is similar to all the compensation criteria: the Little criterion is inherently inconsistent even as a piecemeal criterion. Unlike the tests before it, the rejection of the Little criterion is not due to the existence of many schemes of ad hoc social judgements. There seems to be only one explicit scheme of distributional judgement in Little's criterion. Little does not specify how this scheme of judgement is derived. In the absence of such specification, one is free to attribute the judgement to a teleological or deontological explanation. But deontological ethics does not always guarantee coherence. *Prima facie* reasons under intuitionism are often liable to conflict with each other, and when they do, the distributional judgement will display intransitivity even though there is only one scheme of judgement. The extreme example we gave above may perhaps be classified as an expression of such conflicts.

It should be clear by now that if we want the decisions of the society to be consistent, it is not sufficient to deduce these decisions solely from the physical feasibilities of the society as in the case of Kaldor-Hicks criterion, nor is it possible to include all possible value judgements as in Little's criterion.

This suggests that consistency is already a rather

demanding condition by itself. The willingness to make interpersonal comparisons is neither necessary nor sufficient to guarantee the fulfillment of this condition in the social decision function. It is not necessary because we can always rank the social alternatives by a mapping with the set of natural numbers, and it is not sufficient because the use of some criteria which do involve interpersonal comparisons cannot produce consistent choice decisions.

IV

The Possibility of Social Decisions

The Bergson-Samuelson Social Welfare Function

We have shown that the New Welfare Economics runs into a theoretical deadend if it observes the consistency conditions. Its failure to meet such conditions stems mostly from its deontological structure. Hence, if we want to devise a consistent social evaluation scheme, the most obvious thing we must do is to start with something that is more teleological in nature.

A safe way to arrive at such a scheme would be to take into consideration all conceivable situations and ensure that they form an ordering according to the judgement of the society. By the representation theorem proved by Debreu,^{<1>} we know that there exists a function which transforms each of these situations into a numerical value and which is unique up to any increasing monotonic transformation. This function has been known as the social welfare function. The very word function implies an incorporation of the demanding and deep-rooted consistency conditions.

As it stands, this social welfare function is hardly defined; it can be any function, so long as this function can well-order the set of alternative situations. As indicated earlier, the one-one onto mapping between the social states and any subset of the set of natural numbers will produce one

such ordering. This kind of arbitrariness definitely tells us how much weight we should attach to the recommendations of an unconstrained social welfare function, and that Samuelson was certainly entitled to regard such treatment of social choice as banalities.<2>

The constraints that a society imposes on its own social welfare function are likely to depend heavily on the prevailing morality of that particular society; obviously, they will tend to differ from those of other societies, or from those of the 'same' society at a different time. It is therefore impractical for theorists to specify to the last detail how the social welfare function of any one society is derived. Instead, what we should do is to exercise our value judgements and extract some common ethical beliefs which we think are dominant in all relevant societies we are contemplating. Many economists have tried such an approach in contemporary western societies. Prominent among them are Lerner, Marshall, Pigou, Pareto and Barone. But it was Bergson<3> who clarified and synthesized the views of these authors. He grouped them into three main categories: (i) the Lerner condition; (ii) the Pareto-Barone-Cambridge conditions, and (iii) the Cambridge conditions.

Bergson approves the first two conditions and is rather negative toward the third. In his opinion, "the utility calculus introduced by the Cambridge economists is not a useful tool for welfare economics....these value propositions, which imply the complete measurability of the Economic Welfare Functions aside from an arbitrary origin and

a scalar constant, are not necessary for the derivation of the Maximum conditions, and accordingly are not essential to the analysis."<4>

Samuelson, who popularized Bergson's conditions -- later known as the Bergson-Samuelson social welfare function -- also shares Bergson's view that there is no need for cardinal utility. The latter is introduced by Samuelson as the last of eight suggested constraints on the social welfare function; it is also the one which he regards as "essentially unnecessary."<5> Since then economists have followed the Bergson-Samuelson's formulation of the social welfare function and completely ignored cardinal utility.

Basically, the Bergson-Samuelson (B-S) social welfare function can be represented as follows:

$$E = E(U_1(x_{11}), U_2(x_{12}), \dots, U_n(x_{1n}))$$

for all $i=1\dots m$

where E is the economic welfare level of the society; it depends on the ordinal utility levels of the members of the society and these, in turn, depend on their consumption levels of the m commodities.

The social welfare function presented above is in its most general form. It has the individuals' utility functions as arguments because of the Paretian condition in both Bergson and Samuelson's formulations. The Paretian condition requires that the individuals' preferences be counted, and since an individual's preference can be represented by a utility function, we can substitute the individuals' utility functions for their preferences on the right hand side of the

social welfare function.

With this general formulation, many economists often consider the use of the B-S social welfare function as a substantial improvement on the earlier utilitarian formulation. This improvement, in their view, comes from avoiding the restrictive utilitarian summation principle. With the B-S social welfare function, it is now possible to maximize not only the sum, but also the product and any other arithmetic operand of people's utilities whereas, with the utilitarian framework, this was considered impossible.

Such reasoning is certainly true when we consider the early utilitarians, like the Benthamites, who rigidly summed up people's utilities without any transformation. However, utilitarianism has undergone quite a drastic change in recent years. Instead of attaching a constant weight to each individual's utility, a method which will subsequently produce a linear social welfare function, people are now arguing for variable weights for the same individual at different utility levels, and this may yield a non-linear social welfare function.<6>

The reason for the adoption of this new procedure is mainly the surging egalitarianism of the twentieth century. Benthamite utilitarianism was probably regarded as a radical egalitarian approach in its time, for it declared that different individuals' utilities should be given the same weight in the social welfare function. Such superficial treatment of equality inevitably overlooked the possibility of conspicuous disparities in incomes among the members of a

society. The thought that one unit of my utility is to be counted exactly as one unit of utility of a monarch would be gladdening only if I have sufficient utilities to be counted.

An improved version of utilitarianism consists in attaching different, but still constant, weights to different individuals' utilities. Supposedly, a rich man's utility will receive a smaller weight than that of a poor man. This rule is no longer anonymous, and it seems particularly amoral when there is a dramatic change in the distribution of income in the society. So what we have in mind as egalitarianism is probably a form of social welfare function under which the relative weights of an individual's utilities depend on the total amount of utilities he is going to enjoy. When the prospect of satisfaction changes for the individual, so will the relative weights of his utilities. A notable opponent of this form of formulation of the social welfare function is Harsanyi.^{<7>} His objection is understandable. Non-linear social welfare functions are incompatible with the kind of Bayesian social welfare function he has been advocating in the past few decades. Harsanyi agrees that we should discount the money income of rich people, and this is put into effect by the postulate of diminishing marginal utility of money income. But he disputes the legitimacy of discounting people's utilities, which he considers to as "unnecessary and inadmissible double-counting."^{<8>}

In our opinion, Harsanyi's allegation of double-counting is unfounded. If we agree that we should have a more egalitarian society, then it is up to us to incorporate this

conviction into the social welfare function. The diminishing marginal utility of money income is not a conviction; it is simply a fact of life that is revealed by experiments. If we simply insert this fact into the social welfare function without any modification, we will not have made our wills known. At the most, all we can say is that we have joined the band-wagon favouring the 'status quo.' Therefore, in order to translate a society's ethical belief into its social welfare function, one must be able to inflate or deflate each individual's utility at will to accommodate for natural imbalances. With this version of utilitarianism, the social welfare function will have the following form:

$$E = f(U_1)U_1(x_1) + \dots + f(U_n)U_n(x_1)$$

which is equivalent to

$$E = g(U_1(x_{11})) + \dots + g(U_n(x_{1n}))$$

By transforming the linear utilitarian social welfare function into a non-linear one, we have gained one degree of freedom, namely the g function, which supposedly reflects the ethics of the society. With this extra degree of freedom we can easily turn the utilitarian function into a monotonic transformation of the B-S social welfare function. Since the B-S function is an ordinal construct, any of its monotonic transformations is equivalent to its original form. This amounts to saying that the B-S function, despite its apparent generalization, is still within the confines of the widely defined utilitarianism. The reasoning behind this is simple, and we can explain it in two parts. Firstly, summation, from which all other operations are derived, is a primitive

operation in arithmetic. No matter how complicated a function may be, it can always be arrived at by summation. Secondly, in its unconstrained form, the arguments in the B-S function need not be the utilities of the individuals in the society. They can be anything as remote as the stars in the sky. But once we follow the Bergson-Samuelson formulation and admit the Pareto conditions or its weaker cousin, the individualistic postulate, into the B-S social welfare function, it will no longer be possible to retain its previous flexibility. Individual preferences must be taken into account. Since there exists a utility function corresponding to every individual preference, we have no other choice but to insist that individual utility functions are on the right hand side of the B-S social welfare function. Because of the first part of our explanation, the value of this function is always attainable by the utilitarian function up to a monotonic transformation.

In a recent article, Maskin has proved a set of necessary and sufficient conditions for the Benthamite utilitarianism. They consist of six conditions altogether: (1) independence, (2) the strong Pareto condition, (3) anonymity, (4) elimination of indifferent individuals, (5) continuity, and (6) full comparability.<9> He further noted that "with continuity, independence and the strong Pareto property alone, a 'generalized utilitarianism'"<10> of the Bergson-Samuelson form can be inferred. In fact, Maskin is requiring too much. It is not at all necessary for the B-S function to be continuous,<11> and we can easily drop the

continuity axiom without affecting the possibility of 'generalized utilitarianism.' In addition, the independence axiom in Maskin's formulation is essentially a multiprofile axiom, it is not relevant when we are dealing with the B-S social welfare function. The latter, by construction, is nothing but a single profile welfare function.<12> Finally, it is not even imperative for us to postulate the strong Pareto condition to arrive at 'generalized utilitarianism.' In its place, it is sufficient to require that the social welfare function be responsive to individual preferences. One way to postulate this weaker axiom is to establish a subset of the members of the society whose preferences are sufficient to determine social preference over two alternative situations. On the other hand, a utilitarian social welfare function with only individual utility functions as arguments would of course entail the individualistic postulate. Hence, given the other axioms, this individualistic postulate is both necessary and sufficient for 'generalized utilitarianism.'

Arrow's Impossibility Theorem

Having intuitively arrived at the equivalence between the B-S social welfare function and the 'generalized utilitarian' social welfare function, and having subsequently found out the necessary and sufficient conditions for their existence, we will now digress and examine Arrow's

impossibility theorem. This theorem will prove to be of much relevance to our discussion of social welfare functions. Despite Samuelson's denial,<13> common sense tells us that there is much resemblance between the B-S social welfare function and Arrow's constitution.<14> The major difference between the two, as Samuelson admits, is in the domain of the aggregating function. The B-S function will only aggregate one and only one set of individual preferences to produce a corresponding social preference. It can be any set, but only one set should be chosen. For another set of social preferences, there will exist another B-S social welfare function for the purpose of aggregation. If two or more such functions are not incompatible with each other, that incompatibility can only be the result of chance rather than necessity. On the other hand, Arrow demands that the same constitution should accommodate any possible configuration of the preferences of the members of the society, that is to say, all the B-S functions in different situations must be the same.<15> Obviously, Arrow's condition is stricter than that of the B-S function. The question is: would this difference be significant enough to constitute a difference between the possible and the impossible? Samuelson's answer is in the affirmative.<16> Yet, his authoritative view has recently been challenged by other economists.<17> To settle this controversy, we will have to find out what exactly is the Arrow Paradox.

We will not attempt to prove the paradox as this formidable task has been undertaken so many times before by

others.. What we will do instead is to try to make some sense out of the paradox and single out the main fallacy, if there is any, and discuss, whether on moral and logical grounds that particular condition should be retained or not. This is in contrast to the mood prevailing in the profession, which is to try to find a way out of the paradox.<18> We want to find the way out.

Most economists are familiar with the paradox, and some even can prove it off-hand. However, when confronted with the question how the paradox comes about, most of the time they will only repeat the few conditions verbally, and at most offer a literary version of the mathematical proof. Misconceptions are not at all rare, and they can be entertained by non-specialists as well as experts in the field. To take a less remote example, Mackay, a contributor to notable journals in the field, has recently made the following remark about the paradoxical nature of the Arrow conditions: All the conditions are logically independent; no one bears any logical relation to any other, neither contradicting nor implying it.<19> The readers are supposed to be in a state of shock after they have read the above statement. But why should they be? If one condition implies the other, the latter must be redundant. If one contradicts the other, the two together will be sufficient, and the rest can be ignored. We can partly attribute this state of ignorance to Arrow's failure to supply an intuitive explanation of the paradox, and partly to the neglect of scholars who intentionally or unintentionally overlooked the

raison d'être of the whole exercise.

Following Wilson,<20> we will modify Arrow's Pareto condition into our individualistic axiom. The other three conditions are: unrestricted domain, independence of irrelevant alternatives, and non-dictatorship. The four conditions together are sufficient to produce the paradox. In recent years, numerous scholars of diverse disciplines have rigorously attempted to resolve the paradox by relaxing or partially relaxing one of the four conditions. However, their methodologies are mostly mechanistic, and very few of them have taken the trouble to explain why they had chosen to relax one particular condition, instead of another.

We will argue in favour of dispensing with the independence of infeasible alternatives condition.<21> This view is by no means novel. It has been propounded by Asimakopulos and Kemp, as well as by Weldon<22> immediately after the first publication of Arrow's book, and subsequently by many others, notably Hasson,<23> over the past three decades. Nevertheless, we will try to substantiate our position with arguments which we believe to be more convincing than those supplied earlier.

At first sight, the independence condition seems extremely reasonable, at least to most laymen. Why should our decision process be determined by our concern for some hypothetical creatures on Mars, or whether we want Napoleon I to be the present Premier of Quebec? There is however, another feature of the condition which is not yet unveiled by its literal name. As in the case of taking a sugar-coated

pill, most people often swallow the whole condition because the name of the condition is agreeable to them and they do not know that they have swallowed much more than what they are willing to accept.

It was quickly pointed out by Asimakopoulus and Kemp, and later elaborated by Sen, <24> that the independence condition in Arrow's theorem not only prohibits the use of information concerning infeasible alternatives but also suppresses the expression of individuals' utilities in cardinal units.

What have we left if we decide to discard cardinal utility? Recently Basu <25> has put forward a concept called the degree of cardinality. Based on the idea of utility difference, Basu argues that one will not be able to arrive at full cardinality if one only postulates "second order utility differences. Rather one must keep on postulating the third order, fourth order, and so on, up to the nth order. Basu pointed out that only when one approaches infinity will one have full cardinality. Anything less than that will only produce something between full ordinality and full cardinality. Such cases are rank-ordered by Basu according to the order of utility difference each postulates or, in Basu's terminology, their degree of cardinality. It will be shown in chapter 6 that it is sufficient for us to postulate only second order utility to arrive at full cardinality, and that all that Basu has come up with is nothing more than an overkill. If we put aside Basu's objection and accept Arrow's independence axiom, we can either arrive at a social decision by employing the concept of ordinal utility alone,

or by other means, which will be necessarily outside the realm of utilitarianism as we have defined it. The latter approach would imply the relaxation of the rather weak individualistic axiom. However, if we are willing to take such a course, there will in fact be no issue at all. The welfare of individuals will lose its significance, and any arbitrary rule which leads to a decision is a possible candidate for our social welfare function, even if this may mean an extremely oppressive and deteriorating environment for all the members in the society.

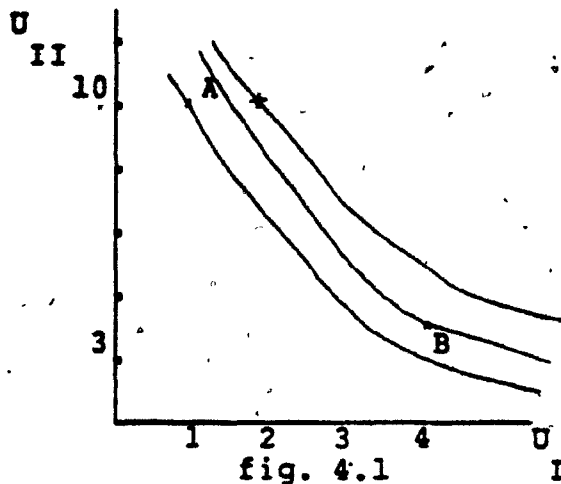
With regard to the prevailing ethical beliefs in the world, the mere thought of not responding to the welfare of the entire population is unthinkable. This applies not only to western capitalism but also to socialist and communist states. And is this not the name of the game and the whole purpose of the study of welfare economics? So, if we wish to stay within some workable framework in the course of our investigation, orthodox and radical alike, we would have little choice but to retain the individualistic axiom.

The only option available to us now is to construct an ordinal social welfare function from the ordinal utility functions of individuals. Is this possible? Intuition tells us that it may be so. After all, it has been proved by Debreu that we can construct ordinal utility functions for individuals from their preferences, and a social welfare function would appear to be a simple extension of Debreu's representation theorem. However, intuition can seldom carry us far, and this is one of those many cases in which

intuition fails us.

There is one major difference between constructing an individual's ordinal utility function and an ordinal social welfare function with ordinal individual utility functions as arguments. In the former case, variables like apples and pears, are supposed to be cardinal variables whose magnitudes do denote significance. In the latter case, both the dependent and independent variables are of ordinal order. It will be easily shown that there may not exist a functional relationship between two ordinal variables in general. We shall prove our assertion by supplying a counter-example.

Let there be two individuals in our society and let $f:O^2 \rightarrow O^1$ be a social choice function of the Arrow type, mapping elements in the two dimensional ordinal space onto elements in the one dimensional ordinal space. In particular, let $f:[1, 10] \rightarrow [1]$, $f:[4, 3] \rightarrow [2]$ and $f:[2, 10] \rightarrow [3]$. This is a perfectly legitimate hypothetical social choice function; in fact, it is a rather traditional one, as we can easily represent it by the usual indifference techniques in figure 4.1. Suppose we are now contemplating two situations, A and B.



The ordinal utility levels of the two individuals in situation A are given by the ordered pair $[1, 10]$, and in situation B by $[4, 3]$. According to our social choice function, situation B should be ranked above situation A. However, if we perform a monotonic increasing transformation on I's utility function as follows: $T:[1, 4] \rightarrow [2, 4]$, then situation A will be ranked above situation B, which is a contradiction. It may be argued that, after performing the transformation on I's utility function, it is always possible to perform another transformation on the social choice function such that the choice of the society remains unchanged. Such a transformation is definitely available, and in our example it will be $R:[3, 2] \rightarrow [1, 2]$. Nevertheless, we cannot rely on such a transformation to ensure consistency in our choice function because, in doing so, we will no longer be working within the boundaries of ordinal functions. The transformation R is not a monotonic transformation. And since we have only conducted an ordinal transformation on

individual I 's utility function, the most we are allowed to do is to perform a similar transformation on f , and if f is equivalent up to any monotonic increasing transformation as assumed, this latter transformation will in effect be superfluous.

This is obviously not the case in our example. In this example we were faced with a dilemma. Either we can work within ordinality and later find ourselves confronted with a contradiction, or we can avoid the contradiction by totally abandoning ordinality. Given the choice, most people would probably abandon pure ordinality and avoid the impending contradiction. However, this is not the route economists have taken. To our surprise, most of them have chosen to adhere to their ordinal aspirations, and have totally ignored the inherent inconsistencies in their analyses.

Thus far, we have shown that if we are willing to accept the individualistic axiom, the independence axiom will be sufficient to nullify all the possibilities of constructing a social welfare function. This nevertheless is not exactly true because we have not yet eliminated the trivial case where the social preference is identical to a particular individual's preference. If that happens, the social choice problem will degenerate into an individual's choice problem, and the standard individual's choice theorems will apply. Yet, we should not delude ourselves by resorting to this last possibility as the solution to the ordinal spell because that would be equivalent to evading the issue of social choice rather than solving it.

One way to guard ourselves against such triviality is to adopt Arrow's non-dictatorship axiom. The name of the axiom can be misleading. However, it should be clear from the start that it is not the dictator *per se* we are opposed to. One can easily conceive of an all-out benevolent dictator whose preferences and social preferences are perfectly matched. Instead, it is the uninteresting case of transforming a uni-dimensional preference into another uni-dimensional preference that we are trying to avoid.⁶ Our acceptance of the non-dictatorship axiom implies that we do recognize the existence of benevolent dictators. However, since these cases do not actually constitute a problem of social choice, we will, for the sake of convenience, overlook them while bearing in mind that their exclusion is not the real reason for our inability to configurate a social choice function.

The example given above is fictitious. Our ability to conclude that this fictitious example is sufficient to dismiss any chance of conflating ordinal personal preferences into ordinal social preferences rests on: (i) the implicit assumption that it is possible to have such a preference profile in a society that is composed of only two members, and (ii) we do not want to disregard or suppress the expression of such a profile. To do otherwise will necessarily restrict our social choice to a sub-class of all the alternatives available, and this begs the question of predetermining the social choice before the act of choosing has actually begun. Such theoretical contradiction applies to the works of Duncan Black²⁶ and others. To avoid this

contradiction and maintain a level of theoretical generality, we would have to formally postulate the axiom of unrestricted domain. This axiom is essentially a non-regularity axiom whose purpose is to admit, in spite of its credibility, any profile of individual preferences as an eligible profile to represent the state of actual tastes of the members of the society concerned. The denial of such profile would reduce the supposedly cosmic structure of our social choice function to the kind of piece-meal scheme that we had before. This time the piece-meal character is not pertinent to social situations as in compensation tests, but to profiles of preferences. We would have to devise rules for discriminating against certain patterns of profiles, which are to be considered inadmissible, and it is in this sense that our social choice function loses its universal character and degenerates into a piece-meal choice function. The dismissal of the impossibility result through this channel can hardly be regarded as a solution at all. The core of the problem still persists, and what we have done is simply to assume it away. If this is the procedure we allow ourselves, we would not have had to engage in the foregoing discussions about social welfare function in the first place. The compensation test can fulfil the task just as well. All we have to do is to eliminate all those situations that cause inconsistency. Since we do not feel that this is the proper procedure to undertake a theoretical investigation then, it would be rather awkward to justify our change of mind now. Consequently, we are left with but one alternative, and that

is to declare that the independence axiom alone is responsible for the impossibility result.

The above is naturally hard for the ordinalists to swallow, and predictably, something has to be done to rectify the situation. Not wanting to disappoint friends and foes, Samuelson^{<27>} has come to the rescue and affirmed that the reign of ordinalism would not be disrupted by Arrow's findings. Politely, he cast off the paradox as something that does not belong to economics proper, but to his newly invented subject -- mathematical politics. Hence, whether rational social choice is possible or not is of no relevance to the welfare economists. According to him, the difference in results between the two approaches is due to the difference in allowable profile structures that we have discussed earlier. To recapitulate, the B-S social welfare function is basically a single profile function, while the Arrow choice function is a multiprofile function. Moreover, we also know that the real cause of impossibility is the exclusion of cardinal utility. Hence, in order to determine whether the difference in profile structure does constitute a significance to the impossibility result, we must first find out if this difference lead to a difference between ordinality and cardinality.

What Causes the Arrow Paradox

Ordinality is implied by cardinality, and therefore, we cannot exclude cardinality merely by insisting on an ordinal framework. It has to be rejected distinctly. Arrow's independence condition is sufficient to eradicate cardinality. How about the B-S social welfare function? In The Foundations, Samuelson introduces ordinalism by requiring that "individual's preferences are to 'count'".^{<28>} Correspondingly, in the mathematical supplement of the same book, he expressed a similar requirement by demanding that the social welfare function W be of the form $W = W(U_1(x_{11}, \dots, x_{n1}; v_{11}, \dots, v_{m1}), \dots, U_s(x_{1s}, \dots, x_{ns}; v_{1s}, \dots, v_{ms}))$.^{<29>} However, both the literary and the mathematical conditions are only endorsements of ordinality. They do not amount to the prohibition of cardinality. The two are of course fundamentally different. Samuelson did not slip only once. He showed the same confusion again on another occasion. While defending the ordinal B-S social welfare function against Arrow's view, he propounded the following axiom to contrast Arrow's axiom:

The social ordering over the states of the world, $[a_{AB}, \dots]$ must put A over B, i.e., $a_{AB}^0 = -1$ if every individual ordering does so, $a_{AB}^1 = -1$ for all n ; more generally, if no $a_{AB}^1 = +1$, $a_{AB}^0 \leq 0$, the strong inequality being required if with no $a_{AB}^0 = +1$, $\sum_i a_{AB}^i < 0$.^{<30>}

He referred to the above axiom as the "only Axiom restricting a Bergson Social Welfare Function".^{<31>} However, the axiom just stated is nothing more than the Pareto Axiom

which to the best of our knowledge, does not forbid cardinality. If this is the only condition imposed upon the B-S social welfare function, no one would be surprised that the B-S welfare function is immune to Arrow's result. The immunization comes about not because of the difference in profile structures as claimed by Samuelson, but because cardinality is readmitted through the back door. This is most vividly revealed in the example<32> Samuelson recently gave, in a last effort to reaffirm the existence of the welfare function which bears his name. The possibility given in his example is immediately proved to be fallacious by Kemp and Ng's rejoinder.<33>

Why We Can Ignore Samuelson

If we consider Samuelson's plea against cardinality as a bona fide one, and moreover if we wish to verify his allegation with respect to the B-S social welfare function, we must devise a method for expelling cardinality from the single profile B-S social welfare function and observe if the impossibility still holds. The obvious way to achieve this is to introduce Arrow's independence axiom to the B-S welfare function. In recent years, many economists<34> have tried to include the B-S social welfare function into the Arrowian paradigm by devising a single profile analogue of the independence axiom. Unfortunately, such an analogue does not exist because the independence axiom is basically a

mutiprofile axiom. At present, the closest we can do is to postulate an Independence-Neutrality Axiom. To be precise, we will quote the version as given by Kemp and Ng:

The social ordering of any two social alternatives depends only on the m individual orderings of the alternatives. Formally, for any (not necessarily distinct) x', x'', x''', x'''' in X , if $(x'Rx'' \Leftrightarrow x'''Rx'')$ for all i then $(x'Rx''' \Leftrightarrow x''Rx'')$, where R stands for one (but not necessarily the same for all i) of P, I and contra P .^{<35>}

The above axiom is a little stronger than its counterpart in the multiprofile case as it requires not only the exclusion of cardinality but also the neutrality of all alternatives. The latter characteristic is furiously and yet rightfully condemned by Samuelson.^{<36>} From the logical point of view, it is blatantly wrong to say that the Arrowian result will encompass the B-S function when we actually impose a different, and even stronger axiom, on the latter. On the other hand, if our objective is only to extend Arrow's result to the B-S function, then there is no need for us to postulate the Independence-Neutrality Axiom. Actually even the independence axiom will be too strong. What we really need is not independence, but that part of independence which excludes cardinal utility. The same end can easily be achieved by allowing not only linear, but also monotonic transformations on utility functions.

When one looks back to the proof we have supplied earlier, one should observe that we have only utilized that part of the independence axiom which prohibits cardinal utility. Moreover, we have never employed more than one

profile in our analysis. Hence, what we have proved is not Arrow's impossibility theorem but the impossibility of the B-S social welfare function. Since the Arrow's independence axiom is more restrictive than what we have assumed, our result will apply equally to the Arrovian framework as well.

It should be clear by now that the crux of the problem lies in the issue of cardinality. It does not matter whether we are working with single or multi-profile preferences. As soon as we try to exclude cardinal utility from our agents' utility functions, impossibility will result. Having arrived at this conclusion, it should now be apparent that if we are serious about making the kind of social decisions as outlined in welfare economics, we will have to bring cardinal utility back to the centre of our analysis.

V

Cardinal Utility Functions under Stochastic Situations

Some Historic Remarks

At the end of last chapter we came to the conclusion that both single and multi-profile social decision functions are subject to Arrow's impossibility result. In addition, we also argued for the admission of cardinal utility as the only sensible way to get us out of the paradox. Notwithstanding the amount of emphasis given to cardinal utility, we still have to prove its existence. On this issue, the wisdom of the profession is evidently not on our side. Most of our fellow economists are of the opinion that either cardinal utilities do not exist, or even if they did, they are redundant. Now that we know that cardinal utilities are by no means redundant, the next thing we have to do is to show that they exist. Basically, there are two ways to axiomatize cardinal utility. The first makes use of stochastic events, and the second of non-stochastic events. In this chapter, we will deal with the first method; the second method will be dealt with in chapter 6.

It was well known in the thirties and the forties that if an individual can rank order adjacent first-order differences in his preferences as he rank-orders pairs of commodity bundles, then a measurable utility scale unique up to any linear transformation can be constructed.

Unfortunately, unlike the ranking of commodity bundles, one cannot possibly reveal an individual's ranking of preference differences through behaviouristic procedures. This is so because we cannot display in front of the subject physical entities called "preference-differences," and ask him to choose. Consequently, if we want to construct a cardinal utility function through preference-differences, we would have to appeal to the individual's introspection by direct questioning.

After the Hicks-Allen revolution, it is generally held that if we do not want to find out what the consumer is actually striving for when he engages in economic activities, then the concept of marginal utility can be totally dispensed with. Claims are constantly made that the indifference analysis, together with the newly introduced concept of marginal rate of substitution, is sufficient to accommodate consumer choice theories in non stochastic situations. As is well known, only ordinality is required in the indifference curve approach.<1>

A battle has been won, but the war is still on. The fundamental drawback of the indifference analysis is that, ultimately, it will also have to rely on the consumers' introspection. The reason for this is that under the indifference approach the commodity space is also rank-ordered by directly questioning the consumer's preference on hypothetical situations, and none of the situations are supposed to have been actualized at the time of the interrogation. But if the indifference analysis has to

rely on introspection all the same, what superiority can it claim to have over the preference-differences theory? After all, most of us would agree that we can rank-order preference-differences besides rank-ordering commodity bundles. Then would it not be more consistent for us to grant the full fledged introspectionists' demand rather than to stop short in between?

Realizing their vulnerability, the ordinalists were forced to defend their position by appealing to Samuelson's revealed preference approach for help. As indicated before, this approach is characterized to a large extent by an ex-post overtone, and it will have predictive power only to the extent that events can repeat themselves. Despite such shortcomings, it does seem that this newly developed approach may perhaps enable the ordinalists to investigate consumer choices in a strictly behaviouristic environment. Predictably, the ordinalists were flamboyant. According to them, with the revealed preference analysis, "a chapter in the history of utility theory has now been brought to a close...",^{<2>} and it was taken for granted that there would be no place for cardinal utility in the upcoming chapter of consumer choice theory.^{<3>}

While the above conclusion may perhaps be true of utility theories that deal with non-stochastic events, it is in no way clear that it will also apply to those that deal with stochastic events. Samuelson was of the opinion that it will.^{<4>} His verdict was arrived at by augmenting probabilities^{<5>} to commodity bundles. Accordingly, each

commodity bundle will have a certain probability of realization. So, if on top of the commodity bundle $[x]$ the consumer can also take into account its probability p_x , then it would perhaps be legitimate to require that the consumer rank-orders commodity vectors $[x, p_x]$ which have their corresponding probabilities of realization as new components. Given this extra requirement, we will again be able to construct indifference surfaces for the same pattern as in the previous non-stochastic case. Analogous to the non-satiation axiom, most theorists in the field have imposed the absolute preference axiom^{<6>} on the consumer. Essentially this axiom requires the consumer to prefer that lottery ticket $[x, p_x]$ to $[y, p_y]$ if $x > y$, $p_x > p_y$; or $x > y$, $p_x = p_y$; or $x = y$, $p_x > p_y$.^{<7>} If the above exposition is correct, there should not be any reason for the ordinalists to worry about the stochastic case, given the fact that they are content with their achievements in the non-stochastic case. Yet, the majority of them still feel much unease with the situation, and even hardliners like Friedman have defected to the opposite side on the issue. An answer is definitely called for.

When we are dealing with nonstochastic events, the commodity bundle $[x]$ is a complete description of the consequence of a particular action, and if we take another course of action we will have another consequence. Such clear cut separation among consequences is however restricted to non-stochastic events. As for stochastic events, the kind of description of the state $[x, p_x]$ which we have outlined

above is incomplete. When p_x is less than unity, $[x, p_x]$ can only tell us that the likelihood of $[x]$'s occurrence is p_x . But what happens if $[x]$ does not occur? Some other consequence $[y]$ with p_y , where $p_y \leq (1 - p_x)$, will occur. If we are allowed to simplify things by assuming $p_y = (1 - p_x)$, a complete description of the state will now be $[x, y; p_x]$ in contrast to the previous description $[x, p_x]$. Hence, when we ask people to indicate their preferences in stochastic situations, we should not only ask them to rank-order states in the form $[x, p_x]$, but instead $[x, y; p_x]$. Yet, this is equivalent to asking our consumer to rank order probability distributions! We must remind ourselves that probability distributions are much more complicated than usual commodity bundles, and it is questionable whether our consumer will have the ability to satisfy such demanding requirements. Even if we are willing to stretch the meaning of rational economic being to enable our individual to evaluate probability distributions, we would still be at a loss as to how to analyse such behaviour. Should we postulate all standard axioms as in the case of non-stochastic events or should we just postulate the innocuous absolute preference axiom? Putting aside all outstanding questions, it seems that the self-evident absolute preference axiom is not so obvious after all. Why should we regard one lottery ticket as better than another just because its pay-off or its probability of winning is higher? We must bear in mind that when our individual compares probability distributions, he does something very different from comparing non-stochastic

consequences resulting from different courses of actions. In the latter, our individual has only to concentrate on the consequences, and there is little doubt that to him a bigger bundle in the Paretian sense would be the better bundle. However, this line of reasoning cannot be applied to stochastic events. As is well documented, decision making under risk often involves consideration of the whole probability distribution. Partial approximations by concentrating emphasis on the first moment of the distribution may at times serve as a crude indicator of action. Yet, it is not uncommon to find cases where these approximations fail to provide the predictive power necessary for constructive theorising. Since it is not possible to take into account the infinitely many moments of the distribution, any cutting point seems as arbitrary as any other. By postulating only the 'absolute preference axiom, we are in effect displaying a favourable bias towards the maximization of the first moment of the distribution, for the absolute preference axiom is entailed by the expected hypothesis even though the converse may not be true. Nonetheless, when either μ or p , changes, it will necessarily alter other moments of the distribution as well. An increase in the expected value of the distribution may be accompanied by an increase or decrease of the various moments of the distribution. As under our basic postulate, our individual's reaction over such changes is not restricted, and his response to them may either be favourable or unfavourable. In the event that the ~~expected~~ value effect

does not operate in the same direction as the effects of some other moments, it is conceivable that the absolute preference axiom may have to yield.<8>

So even if we act as cautiously as the most conservative ordinalists, we may still frequently find exceptions to our basic axiom. Should we, after noticing such exceptions, abandon the absolute preference axiom? What will be left in our investigation if we decide to carry out this last defeatists' step? Most likely, we will degenerate into a state of lawlessness and nothing can be said before the actual happening has taken place. In fact, this is exactly the kind of state that Samuelson termed 'banalities,' as we pointed out when we were dealing with the social welfare function. It must be somewhat surprising that the very same word describes so vividly something that is of the ordinalists' own making.

What would happen if one disregards the apparent contradictions between the absolute preference axiom and other maximizing behaviours, and let the former over-rule all other considerations? The utility function thus derived will belong to what Samuelson has called as the Ysidro family. Ysidro utility functions are ordinal functions which satisfy the absolute preference axiom but not the strong independence axiom, or any of its related axioms. However, it was shown to Samuelson by Savage that "if he behaves like an Ysidro, he can, so to speak, make book against himself and end up making -- or shall we say losing? -- money!"<9> This inherent intransitive feature has exposed a major inadequacy of the

Ysidro family, and finally made Samuelson decide to cross the line and become a 'cardinalist' in the Paris conference of 1952. Samuelson was convinced by Savage that the strong independence axiom is a reasonable postulate in the case of stochastic events because the outcomes in stochastic events are mutually exclusive, and therefore there should not be any contemporaneous correlation between them.

By accepting the strong independence axiom, it can be readily shown that a cardinal utility function can be constructed for our individual. The above expositions of the absolute preference axiom are reminiscent of the comment made by von Neumann and Morgenstern concerning the indifference curve analysis. They wrote: "at any rate we hope we have shown that the treatment by indifference curves implies either too much or too little...."<10> If we add the word stochastic in front of indifference curve analysis, we will arrive at what Samuelson had realized in 1950. If we only postulate the absolute preference axiom, the indifference curves are not integrable; yet if we add a seemingly mild strong independence axiom to the absolute preference axiom, our individual's utility function will be of the cardinal type.

Some Reasons for Preferring the Original von Neumann-Morgenstern Formulation

At this point, we would like to digress and allude to two counts of technicalities. Firstly, there are several ways to axiomatize cardinal utility through stochastic events, and basically, all of them come to more or less the same result. In his survey of the subject, Edwards^{<11>} has indicated his preference for the original von Neumann and Morgenstern's formulation, and he attributed this preference to a paper by Georgescu-Roegen.^{<12>} There are many good reasons for us to side with von Neumann and Morgenstern, and one of them is that their axioms are less demanding than those of the others. In the Marschak, or the Savage, or the Samuelson^{<13>} formulation, the subject is required to order lottery tickets. In contrast, the von-Neumann-Morgenstern version only requires the individual to have a complete ordering of the commodity bundles. Since the former requirement implies the latter, and since they achieve similar results,^{<14>} we must by Occam's Razor favour the latter.

The other point we want to bring out is a response to the hard-line ordinalists like Friedman and Savage. After conceding that the unmodified indifference curve analysis is insufficient to handle stochastic choices, these hardliners are still unwilling to admit cardinal utility to the domain of non-stochastic choices. In their opinion "the ordinal properties of utility functions can be used to rationalize choices not involving risk."^{<15>} On a later occasion, the

same two authors again confirmed their position when they remarked on the expected utility hypothesis. According to them, "the meaning, relevance, and role of the hypothesis has been greatly obstructed by a twofold error in its earlier use: the belief, first, that a numerical utility, unique except for origin and unit of scale, was necessary for the analysis of choices among alternatives involving risk."<16> It was not until 1965 that Samuelson admitted that a 'logical paradox' may have been involved between the measurable utilities under risky choices and the ordinal utilities under non-stochastic choices. In the last paragraph of his postscript, he confessed that even "the actual world under view involves only $(p_1, p_2, \dots) = (1, 0, \dots)$, the possibility that the p 's could be fractional -- even though they are not -- puts restrictions on the $(1, 0, \dots)$ case."<17> Basically, Samuelson is saying that the ordinal utility scale derived from non-stochastic events has to be compatible with the measurable utility scale obtained from stochastic events. Since the measurable scale is the more restrictive of the two, compatibility between them would mean that we will have a measurable utility scale in either case. In other words, if we are willing to accept cardinal utility for risky prospects, it will no longer be logically consistent to retain ordinal utility for sure prospects. The above argument will be valid even if we take into account the Bridgeman thesis, which was drawn to our attention by an outstanding undergraduate thesis by Ellsberg. Bridgeman states:

We must demand that the set of operations equivalent to any concept be a unique set, for otherwise there are possibilities of ambiguity in practical application which we cannot admit....

If we have more than one set of operations we have more than one concept, and strictly there should be a separate name to correspond to each different set of operations.<18>

The issue on hand now is to decide whether there is sufficient evidence to regard the operations from which non-stochastic ordinal utilities and stochastic cardinal utilities are derived as distinct and deserving different nomenclatures. The question looks complicated. It seems that the two operations are quite similar as they both require the subject to rank-order a set of alternatives, and yet the alternative sets, in the two cases are not quite identical. Upon what kind of Criterion or criteria can we base our judgement one way or the other? Once again, we will find the von Neumann-Morgenstern formulation more useful in this connection. Instead of starting with stochastic events, under this formulation we can start the analysis with non-stochastic events. Two of the sure prospects are assigned with two arbitrary ordinal numbers. From these two ordinal numbers an interval of cardinal utility scale will be constructed. Once we have constructed such an interval, the two numbers which we had originally assigned to the two sure prospects will lose their ordinal significance and take on cardinal characteristics. Given this formulation, is it still possible for one to maintain the simultaneous existence of a cardinal utility scale for risky choices and an ordinal utility scale for riskless choices? It is obviously

contradictory for certain numbers to have cardinal and ordinal significance at the same time, and we can easily prove our assertion by performing a monotonic and a linear transformation on any two numbers at the same time. Therefore, if we are to follow the von Neumann-Morgenstern procedure of using ordinal utility function under riskless choices to derive cardinal utility functions, we would have to give up ordinal utility altogether. This is so because under that procedure, the cardinal utility function thus derived will also assign cardinal numbers to sure prospects. Furthermore, it will assign exactly the same numbers to the two sure prospects from which our whole process originated. Since there is only one operation for both risky and riskless prospects, it does not seem appropriate for us to use different names for the two kinds of prospects. Consequently it is not only questionable, but outright improper, for Friedman and Savage to advocate a new name to replace 'utility' to deal with choices under uncertainty.<19>

The Allais Paradox

We will now proceed to analyse the expected utility hypothesis and explain why we do not feel comfortable in constructing cardinal utility scales with this hypothesis. We will cite two types of recalcitrant behaviours to substantiate our position. The first is attributable to the works of Allais.

The way Allais goes about to prove his thesis is to ask his subjects to answer a series of questionnaires. One part of these questionnaires is particularly famous, and has come to be known as the Allais paradox.

In the unmodified version of the Allais paradox, the subject is asked to reveal his or her preference on two pairs of situations. The first pair consists of situations A and B. Situation A guarantees the certainty of \$100 million to our subject, whereas situation B offers him a 10% chance of winning \$500 million; a 89% chance of winning \$150 million, and a 1% chance of winning nothing. Immediately following the above choice, the subject is given the other pair of situation, C and D, both lottery tickets. Situation C offers the subject a 11% chance of winning \$100 million and a 89% chance of winning nothing. In situation D, the subject faces a 10% chance of winning \$500 million and a 90% chance of winning nothing. For those of us who do not very often have the fortune to fondle even \$100 million, the choice in the first pair is most likely A. However, between C and D, the choice is not that obvious. Among those who were questioned, some chose D over C, but the bulk of them preferred C over D.

The choice of A over B is intuitively clear. It is not that we cannot discern the difference between \$100 million and \$500 million, nor do we want to revoke the law of non-satiation. Nevertheless, the certainty that A guarantees, together with the comfortable thought of living happily thereafter easily annul the appeal of the additional \$400 million. The certainty which dominates our choice of A over B

is lacking in the second pair of situations. Here both situations are lottery tickets and the difference in probability is minimal. On the other hand the difference in payoffs is significant this time. With little hesitation, greed reassures itself and urges us to choose D over C.

The seemingly rational behaviour outlined above is incompatible with the expected utility hypothesis. By the latter we mean an individual is trying to maximize the sum of the utilities from a prospect, viz., $\max \sum p_i u(g_i)$. It is easy to show the contradiction. Let $u(0) = 0$, $u(\$100m) = a$ and $u(\$500) = b$. By choosing A over B, our subject has revealed that $a > 0.1b + 0.89a$ and by preferring D over C, he has also confirmed that $0.1b > 0.11a$. Since $0.1b + 0.89a > 0.11a + 0.89 = a$, we have the desired contradiction.

The Independence Axiom

When an axiomatic theory is violated, it follows logically that at least one of the axioms which makes up the theory must be incompatible with people's behaviour. Allais has drawn our attention to the independence axiom in the expected utility hypothesis. In his view, this is the axiom which gives rise to his paradox.

The independence axiom takes on different forms under different authors though the essential feature remains more or less the same. At first glance, all of them appear harmless. Indeed Savage had once commented that "except

possibly for the assumption of simple ordering" he knew "of no other extralogical principle governing decisions that finds such ready acceptance." <20> Before we can critically evaluate whether these independence axioms are acceptable, it seems appropriate to know what exactly they mean.

We will examine Savage's version first. Let $P_1, P_2, P_3,$ and P_4 be four events and $0 \leq \alpha \leq 1$. If it happens that an individual is indifferent between the lottery tickets $[\alpha p_1 + (1 - \alpha)p_3]$ and $[\alpha p_2 + (1 - \alpha)p_3]$, then he should also be indifferent between $[\alpha p_1 + (1 - \alpha)p_4]$ and $[\alpha p_2 + (1 - \alpha)p_4]$. Savage tries to justify his postulate by giving an example of a businessman who is contemplating buying a certain piece of property. Besides taking into consideration the property alone, he also attempts to base his decision on the next federal election. In the end, he realizes that he will purchase the property no matter whether the Republicans or the Democrats win. Savage argues that there is sufficient ground to conclude that the businessman should purchase the property even though the election has not been held, and he named his axiom the surething principle. 'Surething' because the individual would prefer one prospect over the other regardless of the outcome of the election, and it seems that we could safely consider that preference as a 'surething' free from extraneous uncertainties.

No doubt, the example given by Savage is very convincing. Nonetheless, if we allow ourselves to accept the surething principle based solely on this example, we would have committed a major logical error, i.e., the confusion

between the existential and the universal quantifiers. The implication of the surething principle is much more than what this single incident is purported to show. It actually prescribes that in all cases where two prospects have certain payoffs in common, the preference of an individual on the prospects should remain the same if it happens that those common payoffs change values while maintaining their equality between the two prospects.

According to the surething principle, if an individual prefers situation A to situation B in the Allais paradox, he would have to prefer C over D. The logic is simple to perceive. We can separate the payoffs of A into three parts. The first part being a 10% chance of winning \$100 million; the second and the third parts being an 89% chance and a 1% chance of winning \$100 million respectively. By performing similar operations on situations C and D, the four prospects will appear as follow:

Situation A:

- a 10% chance of winning \$100 m
- an 89% chance of winning \$100 m
- a 1% chance of winning \$100 m

Situation B:

- a 10% chance of winning \$500 m
- an 89% chance of winning \$100 m
- a 1% chance of not winning anything

Situation C:

- a 10% chance of winning \$100 m
- an 89% chance of not winning anything
- a 1% chance of winning \$100 m

Situation D:

- a 10% chance of winning \$500 m
- an 89% chance of not winning anything
- a 1% chance of not winning anything

Situations A and B have in common a 89% chance of getting

\$100m and suppose that our individual prefers A to B. Now if we compare A with C, we will notice that the two are the same except in their second payoffs. A similar situation prevails with B and D. Furthermore, the second payoff in C is the same as that in D. Hence when we compare C and D, we are in effect comparing situations A and B with altered second rows. As the second row payoff in A is the same as that in B before and after the alternation, it should not affect our individual's decision if he observes Savage's surething principle. We have assumed that our individual prefers A to B, and by the surething principle he must also prefer C to D. It should be clear by now that if people's choices are similar to those reported by Allais, then the surething principle will be sufficient to give rise to the Allais paradox.

Samuelson's version of the independence axiom is even more restrictive, and he has appropriately called it the Strong Independence axiom. Under this axiom, an individual would be indifferent between two prospects P_1 and P_2 if and only if $\langle 21 \rangle$ he also exhibits an indifference between $[\alpha P_1 + (1 - \alpha)P_3]$ and $[\alpha P_2 + (1 - \alpha)P_3]$ for any P_3 and α with $0 \leq \alpha \leq 1$. Since the choice of P_3 is arbitrary, we can easily substitute P_4 for P_3 and arrive at the indifference between $[\alpha P_1 + (1 - \alpha)P_4]$ and $[\alpha P_2 + (1 - \alpha)P_4]$. With the equivalence relation provided by the axiom, the following three way relation will hold:

$$\begin{aligned} [P_1 = P_2] &\Leftrightarrow [[\alpha P_1 + (1 - \alpha)P_3] = [\alpha P_2 + (1 - \alpha)P_3]] \\ &\Leftrightarrow [[\alpha P_1 + (1 - \alpha)P_4] = [\alpha P_2 + (1 - \alpha)P_4]] \end{aligned}$$

which will in turn give us Savage's surething principle. In other words, Samuelson's strong Independence axiom is even stronger than Savage's surething principle. Since we have shown that the surething principle is sufficient to induce the Allais Paradox, we are definitely entitled to make a similar conclusion on Samuelson's more demanding independence axiom.

When the expected utility hypothesis<22> was first introduced in its modern axiomatic form by von Neumann and Morgenstern, it did not explicitly contain an independence axiom. However, as experts in the field tried to reformulate the hypothesis in different ways, they realized that an independence axiom of one form or another is somewhat indispensable. All these frustrations led Samuelson and others to suspect that an implicit form of independence axiom was implanted in the von Neumann-Morgenstern system. Without solid confirmation, Samuelson has tentatively named this unborn child the Zeroth Axiom. It was not until 1952, at the Paris Conference, that Samuelson received the necessary formal proof from Malinvaud.<23>

In the Theory of Games, an operation on the equivalence classes of utility was being inserted before the authors brought in their formal axioms. The operation is as follows:<24>

$$\alpha u + (1 - \alpha)v \in w$$

Where u , v and w are equivalence classes in the utility space and $0 < \alpha < 1$. Letting $P_1, P_2 \in u$, $P_3 \in v$ and substituting them in the above operation, we will readily obtain:

$$\alpha P_1 + (1 - \alpha)P_2 \in w$$

$$\alpha P_2 + (1 - \alpha)P_1 \in w$$

Since both $[\alpha P_1 + (1 - \alpha)P_2]$ and $[\alpha P_2 + (1 - \alpha)P_1]$ are in w , our individual must be indifferent to both of them; Samuelson's strong independence axiom is thus entailed.

The first author who successfully remoulded the von Neumann-Morgenstern system was Marschak.^{<25>} Without exception he had also included an independence clause in his re-formulation. It is in his fourth postulate. Marschak states that for any pair of events P_1 and P_2 such that P_1 is indifferent to P_2 , the convex linear combination of P_1 and P_2 , P will also be indifferent to P_1 and P_2 . Obviously, this is a special case of Samuelson's Strong Independence Axiom. Let P_1, P_2 be indifferent to one another and P , be identical to P_1 . By the strong independence axiom, $[\alpha P_1 + (1 - \alpha)P_2]$ is indifferent to $[\alpha P_2 + (1 - \alpha)P_1]$. Substituting P_2 by P_1 , we have P_1 indifferent to $[\alpha P_2 + (1 - \alpha)P_1]$ and since P_1 is indifferent to P_2 , we have established Marschak's axiom.

We have demonstrated that the several independence axioms are inter-related, and anyone of them is sufficient to cause the Allais paradox; that is, if we recognize the paradox.

Reactions to the Allais paradox are mixed. Some have welcomed it as a cogent proof of the inadequate nature of the expected utility hypothesis. Others have dismissed it as a wily trap whose success depends solely on the astronomic figures devised by Allais. There are still others who would on the one hand acknowledge the significance of Allais's

findings, while on the other try to uphold the expected utility hypothesis. In order to have it both ways, these people have proposed to treat the expected utility hypothesis as a normative theory rather than a positive theory. With such a twist, they maintained that the expected utility hypothesis will no longer be bound by the relentless demands of the positive methodology. In particular, predictions can now deviate from observable facts. This is so because a normative theory "is not a descriptive or predictive theory of behaviour. It is a theory which purports to advise any one of its believers how he should behave in complicated situations,"^{<26>} and "should be regarded as rules of behaviour to be followed."^{<27>} So, instead of feeling embarrassed, Raiffa's reaction was a sentiment of triumph. His inconsistency, according to him, has given him "renewed respect for the importance of the Savage theory."^{<28>}

The way in which Raiffa and Savage solicited people to act according to the expected utility hypothesis is to decompose Allais's questionnaire into several parts. They then analyse our individual's preference in each part separately. Such a procedure has definitely the advantage of bringing out the essence of independence in the surething principle. However, by so doing, they have already denied Allais's major contention -- the complementarity effects among different payoffs on the subject. By complementarity effect, we mean that the consideration of one payoff is conditioned by the values of other payoffs.

There is little dispute that Allais has chosen an

extreme example to illustrate his point. Nonetheless, in our view it is not sufficient to belittle his central thesis. What Allais has tried to do is to demonstrate to us that independence among mutually exclusive prospects is possible, and that the independence axiom as postulated by the expected utility school is unacceptable. In his paradox, Allais has illustrated that the independence axiom will be violated in the neighbourhood of certainty. However, that need not always be the case, and we will later explain why independence may fail on other occasions. In the meantime, let us find out how the supporters of the expected utility school defend their independence axiom.

In the theory of non-stochastic choices (to be discussed in chapter 6) there is also a strong independence axiom similar to the one in stochastic choices. Together with the provisions of the other non-crucial axioms, that strong independence axiom is sufficient, though not necessary, for the construction of cardinal utility functions. However, Samuelson rejects that axiom, and rightly so. The explanation is elementary. With that strong independence axiom, we would have implied non-complementarity for all goods and an additive utility function for our individual.<29> Since this is in absolute contrast to our introspection and observable facts, we must decline to accept it.

The question then remains: on what ground does Samuelson reject one and accept another strong independence axiom? Here is his own defense:

Within the stochastic realm, independence has a legitimacy that it does not have in the nonstochastic realm. Why? Because either heads or tails must come up: if one comes up, the other cannot; so there is no reason why the choice between P_1 and P_2 should be 'contaminated' by the choice between P_3 and P_4 .<30>

Samuelson's opinion is actually preceded by von Neumann and Morgenstern's and this is what they wrote:

It is important to realize that we are doing this (assuming additive utility functions) solely in a situation where there can indeed be no complementarity. As pointed out in the first part of 3.3.2., our u , v are the utilities not of definite -- events. The u , v of (3:1:b) in particular refer to alternatively conceived events u , v , of which only one can and will become real. I.e. (3:1:b) deals with either having u (with the probability α) or v (with the remaining probability $1 - \alpha$) -- but since the two are in no case conceived as taking place together, they can never complement each other in the ordinary sense.<31>

The argument sounds convincing enough. If we are not consuming two commodities together, how can they complement each other? This is just like eating bread without butter. Although it might be universally agreed that the two complement each other, one would definitely be reluctant to induct on their complementarity in this situation. To most people, the concept of complementarity can only be invoked with the actual act of consumption. When we are eating bread without butter, it does not make sense to speak of butter complementing bread.

However, like most concepts in economics, the notion of complementarity is not concerned with *ex-post* descriptions. It is an *ex ante* concept. Whether one good should be regarded as a complement to another will be decided by

whether our individual's desire to consume one will be affected by the hypothetical presence or absence of the other. It is important to remind ourselves that the actual consumption has not taken place. Hence even if our individual eats bread without butter *ex post*, we cannot, and in fact should not, conclude that there is no sign of complementarity between the two. This remains true even if there is further restrictions on the feasible set such that bread and butter are not allowed to be consumed together.

If we follow the logic given above, we will find it quite easy to expose the fallacy in the argument of the expected utility school. Basically, this school asserts that two events are exempted from complementing each other if they are mutually exclusive, i.e., the two will not happen simultaneously *ex-post*. Yet, from our previous discussion, we understand that this is not the case. Even though the payoffs of a lottery ticket may be mutually exclusive, their presence in the same lottery ticket *ex ante* will suffice to make them complement or substitute of each other.

The concept of complementarity in the stochastic case is however a little different from that in the non-stochastic case. In the latter, a hamburger and a hot dog will most likely be regarded as substitutes. As one can easily observe, this is not exactly true of the Allais paradox, or for that matter, of any other 'mutually exclusive' choice events. What used to be substitutes in the ordinary sense is more prone to become complements. This is so because in non-stochastic events, a hamburger and a hot dog will both satisfy the same

need, and as their occurrences are both certain, the acquisition of one is sufficient to satisfy that need. Hence the two will tend to be substitutes for each other. In stochastic events, the situation is completely different; the major difference being that the occurrence of the events is no longer certain. If the want that needs to be satisfied is somewhat non-negotiable as in the case of subsistence, then it is always better to increase the probability of satisfying that want by one commodity or another. In such situations, competing commodities will not oust each other anymore. They will each occupy their own probability space and welcome others to join rank. Because the more competing commodities there are, the higher will be the chance of their fulfilling that want. The case is particularly obvious as one approaches the neighbourhood of certainty. Nevertheless, the complementary effect is still there in other regions, and there is no reason whatsoever to expect independence to exist among payoffs in stochastic choices.

The Ellsberg Paradox

When von Neumann and Morgenstern first constructed their cardinal utility index, they were adhering to the frequency interpretation of probability.<32> This approach is patently inadequate when one is trying to apply the procedure to a non-recurring event, or to an event that does not repeat itself *ad infinitum*. In order to rectify this inadequacy and

make the expected utility hypothesis more general, people like Ramsey and Savage have turned to Bayes for help. With the concept of subjective personal probability, one will be able to assign probabilities to events which may not recur again. The procedure is simple.

Let us assume that we are interested in knowing someone's subjective probability on Reagan's re-election. Since this is not an event that will happen too often, the frequentists will not give us a legitimate clue to the problem. The Bayesians, on the other hand, will provide us with an answer. Firstly, they will suggest to us to find an event in some gamble whose objective probabilities of success and failure are known and can be varied, e.g., the drawing of a red ball from a probability urn. They will then ask our individual to choose between the two events, viz., Reagan's re-election and the successful draw of a red ball from the urn. Initially, we will set the probability of success of drawing a red ball at 0.5. If at this point our individual chooses to place his bet on the urn rather than Reagan's re-election, the Bayesians will say that the individual's subjective probability of Reagan's re-election is less than 0.5. By adjusting the probability of success through trial and error, we will easily arrive at a probability in which our individual will be indifferent between the urn and the re-election. That probability, according to the Bayesians, is the subjective or personal probability the individual attaches to Reagan's re-election. Since this process is universally applicable, we have eliminated the frequentists'

difficulty, and made it appear that the expected utility hypothesis has leaped over another hurdle.

Unfortunately, this optimism proved to be short-lived, as it soon became clear that subjective probability is entangled with another kind of difficulty known as 'non-additivity.' (It would perhaps be easier if we approach the problem by means of Ellsberg's example. The subject in Ellsberg's experiment is informed that there are two urns, the first containing 100 red and black balls in a ratio entirely unknown to the subject, and the second one containing 50 red and 50 black balls. The subject is then presented with the following series of questions:

1. "Which do you prefer to bet on, Red₁ or Black₁; or are you indifferent?" That is drawing a ball from Urn 1, on which 'event' do you prefer the \$100 stake, red or black; or do you care?
2. "Which would you prefer to bet on Red₂ or Black₂?"
3. "Which do you prefer to bet on, Red₁ or Red₂?"
4. "Which do you prefer to bet on, Black₁ or Black₂?"<33>

If there is no colour bias, most people would most likely show indifference between the alternatives in the first two questions. However, the answers to the last two questions will vary among the respondents. According to the majority, Red₂ is preferred to Red₁ and Black₂ to Black₁.

People's indifference between Red₂ and Black₂ is probably due to the fact that they have the same objective probability. As for the indifference between Red₁ and Black₁, some kind of reasoning pertaining to the principle of insufficient reason must be at work. With that principle, Red₁ is considered to have the same likelihood as Black₁. As

the two exhaust all possibilities, we have no other choice but to conclude that our individual confers on each of Red₁ and Black₁ a 'subjective' probability of 0.5. Since each of Red₂ and Black₂ has an objective probability of 0.5, the four should be considered as equivalent. If that should be the case, then how can our subject show a preference in the last two questions of the experiment? By indicating such a preference, he has again violated the surething principle.<34>

The question that now arises is why would our subject attach unequal weights to the same probability of 0.5? Are these weights somehow different from each other? According to Ellsberg and Fellner, the answer is in the affirmative. In the Knightian fashion, they argue that we must distinguish clearly between risk and uncertainty. There is more to stochastic choices than just the probability measures. In our example, the probability distribution of the second urn, but not that of the first urn, is known to us. Even though, we can assign a probability of 0.5 to occurrences of both Red₁ and Black₁, by applying the principle of insufficient reason, these probabilities are in a sense less secure, or in Fellner's term, more 'ambiguous' than those of the second urn. This ambiguity constitutes, so to speak, another dimension of the stochastic choices. A prudent person who wants to see less cloud over his head will, no doubt, prefer less ambiguity to more. In other words, he would like to stake his wager on the second urn than on the first which in our case will put subjective probability in an inferior

position vis-a-vis objective probability.

At this point, we must point out that the 'subjective' probability in the above paragraph is derived in a different way as compared with the subjective probability we first introduced. In our earlier method, the subjective probability of Red_1 will be less than 0.5. This is due to the fact that when asked to bet on Red_2 and Red_1 , our individual chose Red_2 . Let us assume it to be 0.3. The same reasoning is also applicable to $Black_1$ and $Black_2$, and by the principle of insufficient reason, $Black_1$ and Red_1 should have the same probability, which is 0.3. However $Black_1$ and Red_1 exhaust all possibilities in the first urn, and yet 0.3 and 0.3 do not add up to unity. Obviously there is an acute problem of non-additivity here.

When faced with this non-additivity problem, the proponents of the expected utility hypothesis are in a no win position. They can either acknowledge the problem and try to restore additivity, or they can leave it as it is. If they ignore the problem they will fail to construct cardinal utility scales because additivity is indispensable for the construction of quality probability^{<35>} which, in turn, is necessary for the axiomatization of cardinal utility scales. On the other hand, if they choose to acknowledge the problem, they will have to make certain adjustments to their model. The usual way to do this is to try to normalize the subjective probabilities so that their sum add up to unity. Fellner calls the normalized subjective probability the 'corrected' subjective probability, in contrast to the

uncorrected ones. Even if such corrections are made, there are still difficulties. The first of this is the identification of the subjective probabilities that are to be normalized, i.e., the problem of whether we should normalize the sum of certain events' occurrence or their non-occurrence. Naturally the two need not come to the same thing when there are personal distortions. The second point to note is that our subject may confer subjective probability on an event through different processes. He may, for example, use the probabilities urn, or a deck of playing cards etc.... The subjective probabilities thus derived from each process may be different as our individual may have certain biases against some processes. So there is the question of choosing which objective game to determine the subjective probabilities. The same is true when one uses the same objective game to derive the subjective probabilities of different non-recurring events. To illustrate the point, let us say that we are interested in finding the subjective probability of drawing a black card from a deck of cards of whose proportion of black cards is unknown, and the chance of drawing a black ball from the first urn. Theoretically, they should have the same corrected subjective probability. Yet their uncorrected subjective probabilities may well be 0.4 and 0.3 respectively. If that is the case, we would require different normalization factors for the two events and "the differences between the correction factors applicable to these (events) will lead to the establishment of different operational utility functions...."<36> The embarrassment then

consists in deciding which utility function we should adopt. Even if we can identify one such utility function, we will still be haunted by the Ellsberg Paradox. This is because as long as our individual can discern a difference between equal values of objective probabilities and corrected subjective probabilities, he will be trapped by that paradox and will violate the 'surething' principle.

At this juncture, one is tempted to retreat to the von Neumann-Morgenstern approach and to ignore the whole issue of non-recurring events. However, this is not quite possible after we have been exposed to the concept of subjective probability. Experiments have confirmed that personal distortions of probabilities are not restricted to events<37> whose objective probabilities are not available but also to those events whose objective probabilities are taken for granted.

We are still very much in the dark as to why people would want to distort objective probabilities. A couple of tentative explanations may be given. Firstly, people may tend to distort objective probabilities as a result of past inculcations. Imprinted in the minds of many people is the myth that '7' will bring good luck while '13' bring bad luck. So when these people engage in a dice game, they may consistently avoid '13' and prefer '7' even though it may be known to them that '13' has a higher objective probability of realization than '7' in that particular game.

The other cause that we want to mention about distortion is more or less an objection to the frequentists' approach to

probability. According to this approach, the objective probability of an event is defined as the relative frequency of its occurrence when the number of trials approaches infinity. But even with regard to recurring events, no one has yet witnessed an infinite number of trials. The objective probabilities so far available are based on finite trials. If that is the case, then how can the frequentists claim that their probability measures are the correct ones? It may be true that their figures are based on more, or even much more observations. However, the very fact that they are not based on an infinite number of observations has deprived them of the right to assume any superiority over other figures observed. One must realize that closeness to infinity is irrelevant as a theoretical argument.

For any event it is not unusual for us to observe a relative frequency that deviates from its objective probability. When that happens, what we usually do is to discredit our own observations as unrepresentative and console ourselves with the excuse that as in a single trial, in a sequence of trials we are perfectly entitled to accept such deviations. More importantly, it is possible for the relative frequency of a shorter sequence, to resemble more of the objective probability of an event x than a longer sequence. To argue that a longer sequence may have a higher probability to do better than a shorter sequence is of no help. Upon reflection, one should realize that this is not a resolution of the problem because that is equivalent to using probability to explain probability! However, if that is the

case, then how much weight should we attach to the objective probability of any event? Is it not arrived at through a sequence of trials all the same? Though that sequence may be longer, we believe that it is still a finite sequence after all? Hence, when it is compared with an infinite sequence, it will have no theoretical advantage over other finite sequences. The law of large numbers, which fits so perfectly into the frequentists' framework, can then be easily dismissed as a happy coincidence, or may be a prolonged coincidence whose shorter counterparts we often find in our daily encounters of random events.

Given our past personal experience with certain stochastic events, and given that they may not exactly concur with the frequentists' experience, we may, with the aid of the above reasoning, assign subjective probabilities to events whose objective probabilities are well documented. If this much is granted, the propounders of expected utility hypothesis will find their tenets even more difficult to defend. In no instance will they be entitled to use objective probabilities to construct utility functions. The issue of non-additivity will come back to torture them once again, and perhaps with added vengeance. If they do not normalize people's subjective probabilities, they will naturally run into the same problem of not fulfilling one of the conditions of qualitative probability. If on the other hand, they try to normalize people's subjective probabilities, they will find themselves in the predicament that Handa once found himself in, i.e., they will not be able to deal meaningfully with any

stochastic situation that has more than two events.<38>

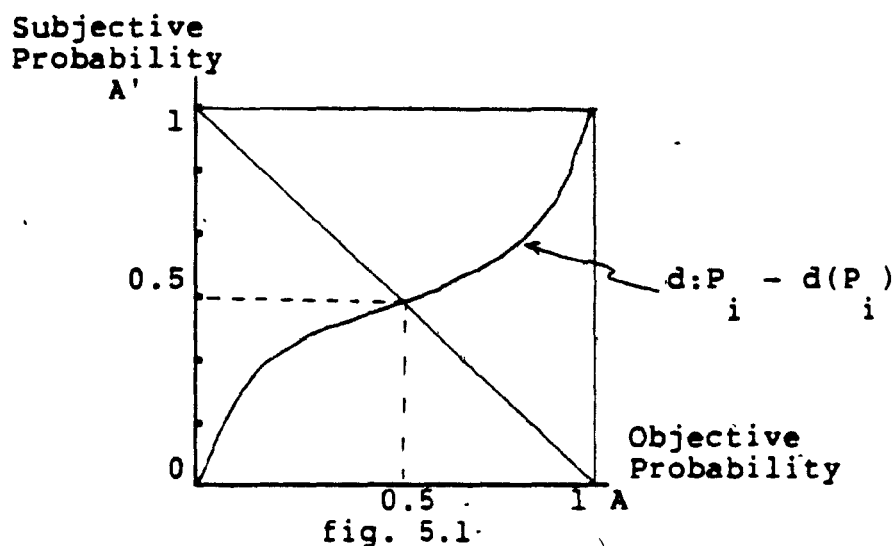
The documentation of personal distortion of objective probabilities is not a recent revelation. Yet, all along, people have tended to ignore such phenomena when they theorize on the expected utility hypothesis, i.e., for the term $\sum p_i u(x_i)$, they will leave p_i intact. Noticing the deficiency in these formulations, Handa tries to rectify it by moving to the other extreme. He postulates that for small variations in the commodity bundle, marginal utility will remain constant so that the mapping between commodity and utility will be linear in that neighbourhood. Having done so, he goes on to axiomatize a theory on the distortion of objective probabilities. Basically, what he managed to do is to construct a function d such that $d: p_i \rightarrow d(p_i)$ and $\sum d(p_i) = \sum p_i = 1$. The summation constraint has been imposed so as to assume that the analysis can stay within the purview of qualitative probability and observed additivity. However, by imposing such a constraint, Handa has unconsciously invalidated his whole thesis.

When one investigates personal distortions of objective probabilities, it is always preferable to incorporate in the analysis a symmetric clause so that the results will be generally applicable. The proposed symmetry is on events with the same objective probabilities. For example, an individual may have a tendency to attach to certain event a subjective probability. When that happens, the symmetric clause will require the individual to assign the same subjective probability to other events which have the same

objective probability as the preceding event. Without such a mild regularity provision, the individual's behaviour will be unpredictable, as the function d will be liable to changes at all times, thus denying any workable specification. However, in adopting such a symmetric clause, one will find himself bewildered by another kind of difficulty. The function d , which is supposed to be an unspecified permutation of p_i , is now constrained. Except in the uninteresting case of two events, d has become the identical mapping which makes $p_i = d(p_i)$.

We will now try to illustrate, in an intuitive manner, how this comes about.<39> In figure 5.1 a hypothetical d function for the case with only two events is depicted. Notice that this function passes through the point $(0.5, 0.5)$ and is symmetric over the line AA' . It has to pass through $(0.5, 0.5)$ because when one event has an objective probability of 0.5, the other event will also possess the same objective probability. By our symmetric clause, the two events should be transformed into the same subjective probability, and $(0.5, 0.5)$ is the only candidate that meets this requirement. It has to be symmetric over AA' because when there are only two events, one must complement the other with respect to probability, i.e., when the objective probability of the first event is α , that of the second event must be $(1 - \alpha)$ and vice versa. So, when the function d transforms an event with an objective probability α to $d(\alpha)$, it simultaneously transforms the second event with an objective probability $(1 - \alpha)$ to $d(1 - \alpha)$. By moving along

the domain of d , the objective probabilities of the two events will change, and there exists a point such that the objective probability of the first event will be $(1 - \alpha)$ and that of the second event α . But unlike before, our individual will no longer be allowed to express his personal distortion freely. This time, the subjective probabilities of the two events are both pre-determined by his dispositions and particularly by the symmetric clause. They will swap the values of their subjective probabilities in the same fashion as they had swapped their objective probabilities. It is exactly due to this double swap that d is symmetric over AA' .



When we have three or more events, the symmetric requirement is even more irreconcilable. To make things simpler, we will bring out the essence of this impairment with the case of three events. As in the case of two events, we will again start with the point $(1/3, 1/3, 1/3)$, where the

three events are all equally probable. With analogous reasoning from previous analysis, it should be obvious that $d(1/3)$ is equal to $1/3$. Since $2/3$ is the probabilistic complement of $1/3$ and since $d(1/3)$ is determined, $d(2/3)$ will have to be equal to $2/3$. And when one event takes on an objective probability of $2/3$, the other two events must split the remaining probability. Let us assume that they split them equally so that we are now at the point $(2/3, 1/6, 1/6)$. As we are bound by the symmetric clause and as $d(2/3)$ is $2/3$, we have no other choice but to assign $d(1/6)$ to $1/6$ for each of the remaining events and thus proving inductively that $d(p_i)$ is the same as p_i .

The implication of the equality between $d(p_i)$ and p_i is profound, for it virtually undermines all attempts to accommodate personal distortions of objective probabilities. Yet such distortions are as real as any other phenomena that we have observed; to disregard them is to ignore reality. Actually this attitude may not be possible at all.<40> In Handa's formulation of probability preference, the marginal utility function is held constant; and in the usual expected utility formulation the probability preferences are ignored. Yet these two cases are only theoretical extremes. In real life, the effects of the two kinds of preferences will most likely intermingle with each other and make it impossible for the experimenter to tell whether a certain result is caused by one preference or the other. The situation will be further aggravated if the two kinds of preferences are not independent. An example: "the subjective probability function

is influenced by the utilities involved in the bets." When that happens "there is no hope of predicting risky decision unless their law of combination is known."<41>

So far we have brought forth quite a few arguments and evidences all of which have indicated that the expected utility hypothesis is not a tenable hypothesis. It appears that the best thing for us to do is to abandon the idea of constructing cardinal utility functions through stochastic events entirely, for apart from being disproved by observations, they are also unnecessary. We have in our command another logically consistent method of constructing utility functions for non-stochastic events. It starts by assigning cardinal utilities to non-stochastic events first; with these utilities known, the choices under stochastic events can easily be handled. To foresake this method and opt for the expected utility hypothesis seems unconvincing. It is just like putting the cart before the horse; no wonder the whole analysis backfires.

As always, there are those who are reluctant to yield to reason. To cite just one such case, it is often observed that people buy both lottery tickets and insurance at the same time, and as it stands, such behaviour cannot be accommodated by the expected utility hypothesis. Nevertheless with a little twist, the problem is solved. The twist was performed by Friedman and Savage on the utility function. Rather than finding explanations in other dimensions of risk-taking, the individual's utility function is twisted upwards when he is buying lottery tickets and downwards when

he is buying insurance. So the individual's utility function will now display both a convex and a concave segment. Unorthodox as this function may seem, it is not the end of the story. With much honesty, the two authors also admitted that they have found that many people refuse to take long shots when they are gambling. To reconcile these behaviours, the utility function will have to make another twist so that it will have three segments. In fact, the absurdity does not stop here. Friedman and Savage are willing to go all the way and do not intend to exclude the possibility of adding even more segments to the utility function. For "it may well be, however, that it will be necessary to add such segments to account for behaviours revealed by further empirical evidence."<42> Unmistakably, the whole manipulation is an ex-post exercise which cannot be refuted empirically. It offers no explanation for the behaviour observed, and it never will. Finally, it is only sad to learn that our minds have to be bent over so many times before we can be qualified as utility maximizing homo sapiens. The saddest part, however, lies not so much on the bending, but rather in the fact it is something that we have to learn to know.

VI

Cardinal Utility Functions under Non-Stochastic Situations

The Jevonsian Method

After expending a considerable effort to repudiate the logical foundations of the expected utility school, it seems now opportune to redirect our attention to the concept of constructing a cardinal utility function through the 'preference-difference' method. To avoid introducing a new terminology, we will from now on follow the precedent set by others and call this method the 'Jevonsian' method.<1>

The modern rehabilitation of the Jevonsian method is associated with Pareto.<2> In his Manuel, he alluded to the possibility of arriving at a cardinal utility function for an individual if that individual is able to compare not only commodity bundles, but also the first-order preference-differences of any two pairs of commodity bundles. What it means is that for any two pairs of commodity bundles A, B and C, D, if our individual can tell whether A is preferable to B and C to D, and if he can also judge whether the excess in satisfaction of A over B is greater or less than or equal to that of C over D, then we will be able to construct for that individual a utility function which is equivalent up to any linear transformation.

The logic of the above conclusion is clear, intuitively. For ordinal utility functions, the commodity bundles are only

well ordered. The differences in utilities among bundles denote no significance at all. Now, if we turn around and say that the first-order differences in utilities are not a vacuous concept, and if we also agree that the two kinds of preferences belong to the same utility function, then we obviously have a contradiction.

Beside being non-rigorous, the above explanation is also incomplete. A few outstanding issues have to be settled before this Jevonsian method can be used to convert people to the cardinalists' persuasion. The foremost question that raises immediate concern is obviously the ability of the individual to discern such first order differences in his or her own preferences. Can we, as in the case of consumer choice theories, resort to observable facts to verify the validity of the Jevonsian method? The answer appears to be no. Unlike commodity bundles, first-order 'preference-differences' are not tangible entities that can be displayed before our individual; his choices among these differences are therefore not directly observable. This difficulty is well recognized by the propounders of the 'difference' school.^{<3>} An attempt has been made by Suppes and Winet^{<4>} to design an experiment so as to capture even the choices among 'preference-difference', thus taking us back to the behaviouristic scene. Their experiment consists in presenting a housewife with a pair of household appliances of more or less the same monetary value, e.g. a toaster and a waxer. Then, she will be given an opportunity to trade. She can either trade the toaster for a waffle iron, or the waxer

for a blender, but she cannot engage in both trade. According to Suppes and Winet, if the housewife chooses the first rather than the second trading opportunity, that would be regarded as sufficient evidence that her utility-difference between the toaster and the waffle iron is greater than the utility difference between the waxer and the blender. Since these trades are observable, we may again fall back to behaviourism. Nonetheless, if we really want to adhere to behaviourism to that extent, we will have to make certain sacrifices. In this particular case, we must be willing to accept that the consumption of different commodities are independent. For if there exists complementary effects of consumption, the housewife's decision to engage in any trade will also reflect those effects; we thus cannot attribute the choice solely to utility differences.

That is of course a very high price to pay, and since information concerning utility differences is also obtainable by directly questioning the individual, it is unjustifiable why we should hang so firmly onto behaviourism. Moreover, behaviourism cannot claim any superiority over the direct questioning method. As a methodology, behaviourism appeals to our visual and tactile experiences, and it is on an equal footing with direct questioning as the latter appeals to our mental faculties through introspection. As long as we do not lie to ourselves and to the experimenter, direct questioning is just as good, if not better, a method of discovery through our experiences.

The best way to find out whether people can rank-order

first order differences of utility is to ask ourselves whether we can rank-order such differences. The issue has been handled by Weldon^{<5>} in a lucid manner. He offers to his subjects three monetary values: \$1, \$2 and \$10,000. With little hesitation, most people will prefer them in the reverse order. After revealing their usual preferences, the subjects will then be asked if they have a higher preference for passing from \$1 to \$2 or from \$2 to \$10,000. To nobody's surprise, most respondents do not find the second question nonsensical. In fact, the majority of them are confident that they prefer passing from \$2 to \$10,000 more than from \$1 to \$2.

As for economists, we know of no one amongst them who would challenge the above hypothesis. Pareto perhaps has come closest to that position. Yet, he does not deny the existence of preference-differences. His doubts are over whether people have the ability to discriminate such preferences when changes in utilities are refined.^{<6>} However, that skepticism is directed at the practicality rather than at the validity of the Jevonsian method. And if that is an issue at all, it is not restricted to the Jevonsian method. The usual indifference curve analysis is also plagued by that problem. As pointed out by Armstrong,^{<7>} the indifference relation is not transitive. When we add small amounts of a commodity to a commodity bundle A, changing it to B, C, D.... and if we make the amounts sufficiently small, it is highly probable that our individual will not be able to tell the difference between A and B; B and C; C and D and so on. A good example

is adding a grain of sugar to a cup of coffee each time. Let A be the initial situation where there is no sugar in our individual's cup of coffee, and B be the situation after we have added one grain of sugar to the same cup of coffee. Naturally, our individual will not find any difference between A and B and will show an indifference between the two. Nevertheless, if we keep on adding grains of sugar to the same cup of coffee, and changing situation B to C, D and so on, there will certainly come a situation Z such that our individual will feel the difference between A and Z. If that is the case, indifference will fail to be transitive, and that is no less a revelation of the imprecision of human perceptibility than is the example cited by Pareto.

If the above analogy still cannot satisfy the ordinalists, one would only have to remind them that in adopting such a stance, their whole indifference analysis may be jeopardized by Georgescu-Roegen's thesis. As suggested by its name, indifference analysis requires the existence of indifference classes, and the latter is by no means self-evident. Georgescu-Roegen argues that "it is not at all necessary that between two states there should always be an intermediary one." <8> According to him, an individual's preference pattern is more analogous to real time than the real line. Between two real intervals there is always a real point, but between two real time periods like life and death, there may not exist anything in between. As a gesture of good faith, we have all along granted the ordinalists the existence of indifference classes. In so doing, we did not

imply that Georgescu-Roegen's objection is meticulous or invalid. Rather, we are confident that even equipped with indifference classes, the ordinalists' argument is doomed to founder. But if it is their wish to articulate on painstaking details, we are always ready to play their game. In that case we would demand a precise explanation of the concept of indifference in the behaviouristic sense. Such explanation cannot be provided because the revealed preferences approach does not make any allowance for indifference. Sometimes doubts are even cast on the validity of the asymmetric strong preference relation when differences among commodity bundles are small and subtle.<9>

We do not want to deny that precision may be an inherent problem. Yet, as we have pointed out, it is not a problem restricted to the Jevonsian method, and we do not regard ourselves in any sense negligent in ignoring it. In an absolute sense, there does not exist any exact science, and economics is no exception. No one is prepared to construct for all the members of a society utility functions that are accurate to the last detail. If any utility function is to be determined at all, it must be a gross approximation of what theory would dictate. Under such circumstances, one would require the individual to possess no more than the ability to rank order utility differences that are of considerable magnitude. We will not concern ourselves at this stage with how big these differences should be. The important thing is to find out whether such awareness exists among individuals through their own introspection. On this last point, the best

judge is ourselves. In case we need any confirmation, the whole economic profession is behind us. Supportive statements are readily available. To cite just a few:

That such a feeling of contrast exists, if we have conscious of different degrees of intensity seems to us proved by the simplest self observation.<10>

Introspection seems to show beyond doubt that if C^1PC^1 , C^2PC^1 , one can always tell whether C^2 is 'nearer' to C^1 or to C^3 .<11>

Subjective service(utility) is not physically measurable, but its quantitative variation is factually indubitable; and even if it were possible to explain the facts of demand without it, the procedure would be interesting only as an intellectual 'stunt'.<12>

Apart from not being able to come up with solutions to welfare problems, ordinal analysis as formulated by Hicks and Allen is also deficient in another aspect. In the old classical world, the complementary effect between two goods is indicated by the sign of their cross derivatives, and it will exist irrespective of the availability of other goods. The same cannot be said of the Hicks-Allen approach. Cross derivatives are devoid of any meaning in their analysis. As a result, two goods can be complements only when there are three goods or more. Otherwise, they must be substitutes for each other even though they may be undisputed complements in everyday language as bread and butter. This difficulty can of course be avoided by drawing only butter and margarine on the board, and in fact this is a usual practice among economic professors. However, we do not think such an evasive attitude should be encouraged, and the "fact that the new theory of complementarity breaks down where the relationship of two

commodities only is concerned, it is therefore axiomatic proof of the unavailability and validity of the second postulate (utility-difference)."<13>

Answering Some of the Criticism

Now that we have accepted the relevance of the concept of utility differences, we will move on to examine some of the more technical aspects of the Jevonsian method. The first item on the agenda pertains to the question of sufficiency; to a great extent this question is related to the problem of precision that we have discussed earlier. Despite their willingness to admit that they can discern utility differences, many economists are still holding on to their dogmatic belief that cardinal utility functions are not conceivable. They maintain that the postulate concerning utility differences is not sufficient to prove the existence of cardinal utility functions. In their opinion, the difference postulate is inadequate in the sense that the individual is only required to compare first-order utility differences. As pointed out by Basu,<14> full cardinal utility function will exist only if utility differences can be discerned up to the n th order where n approaches infinity. Anything that falls short of n will not give us a full cardinal utility function. However, the utility function formed with $\infty > n > 0$ will obviously give us more information than the ordinal utility function. Clearly, it should be a

function that lies between the isotonic and the linear group.<15> That description fits exactly into the group of functions which Coombs has called the metric ordered transformation.<16>

The metric-ordered scale differs from the linear scale in that the former cannot order internal differences above the nth order for some $n < \infty$, which is exactly Basu's objection against the Jevonsian method. An excellent example has been provided by Stevens to help us to visualize how this could happen.<17> Imagine that we have a beam balance with which we can rank-order a set of objects A, B, C, D,.... according to their weights. Moreover, we have also at our disposal plenty of fine sand so that we can make up the difference in weight between say A and B with an amount of sand a, and the difference between B and C with b and so on. Naturally a, b, c, d can also be rank-ordered according to their weights, and so can their differences, i.e. (a-b), (b-c), etc. However, we cannot carry on the induction indefinitely. There will soon come a round when we will actually be weighing one grain of sand against another, and if the balance is sensitive enough, the weights between the two grains of sand must differ. Yet, with the apparatus on hand, it is no longer possible for us to capture this difference as we did before, and our measurement will fall short of full cardinality.

We must agree that the above example is perfectly credible and the analogy is close. But is it close enough for us to accept Basu's argument? To answer these questions, let

us go back to our earlier discussions. We have dismissed Pareto's doubts on people's ability to discern some of the first order utility differences on the ground that imperfect perceptions are common in both ordinary preferences and in first order preferences. It seems to us that if we are willing to concede completeness to one set of preferences, we should also allow the same to the other unless there is sufficient evidence to the contrary.

Taking that much for granted, we will now be able to apply the Jevonsian method to any pair of utility differences of the first order. We do not however count on our individual to compare preference differences of any higher order. It will be shown that even without comparability of higher orders, the utility function derived by the Jevonsian method is of the interval scale.

Let there be several commodity bundles A, B, C, D preferred by our individual in that order. Furthermore, the utility difference between A and B is smaller than that of C and D. From the usual Archimedean axiom and the continuity axiom, we will be able to find a bundle E such that the utility differences between E and D are the same as between A and B. For convenience, let us make the utility difference between B and E greater than that of A and B. As before, we can again find a bundle F such that the difference between F and E is the same as the between A and B. Note that we have not invoked any utility difference higher than the first order. As represented in figure 6.1, the difference between A and B is now greater than the difference between B and F.

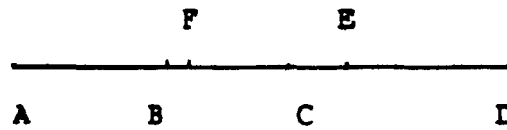


fig. 6.1

By repeating the same process, we will soon arrive at bundles G, H, I,.... The differences between these subsequent bundles will become smaller and smaller. Depending on the degree of refinement we desire, we can pick out any one of these differences as the yardstick and construct a full cardinal utility function upon it. Needless to say, there is an apparent contradiction between this conclusion and the one indicated by Steven's example; this contradiction is mainly attributable to the continuity postulate to which we have already referred.

As we have learned in elementary mathematics, distance in a uni-dimensional space can be fully represented by pairs of points. If these points are everywhere dense, we will be able to give significance to any distance in such a space. The latter is of course a metric space property and its presence entails measurability. The reason why Steven's example does not give us full measurability is that both the objects to be measured and the sand by which the objects are measured are discrete. If full continuity is granted, Basu's objection will not constitute an issue. In a metric space, the difference of a distance is again a distance, and since all distances are accountable as a result of continuity, it will suffice to postulate only first order differences to

obtain measurability. The ordered metric scale is therefore a possibility only when we do not have continuity -- an idea which will undermine even the usual preference analysis. As we have all spent enough time on this problem and arrived at a verdict when we first embarked into economics, it would seem somewhat repetitive if we are to engage in the same old arguments all over again. It would therefore be advisable to close Basu's case and proceed to other topics of more urgent concern.

The first formal proof of the Jevonsian method was supplied by Lange in two papers.<18> The proof in the first paper was both 'defective and unsatisfactory' even by Lange's own standard.<19> The proof in the second paper was relatively more complete. In it, Lange took for granted all the usual postulates of the ordinal analysis. In addition he also introduced his 'Postulate(2),' which is equivalent to recognizing first order utility differences:

$$\phi(x_2) - \phi(x_1) \cong \phi(x_3) - \phi(x_2)$$

depending on as to whether a transition from the combination (x_1) to the combination (x_2) involves a gain in utility greater than, equal to, or less than a transition from combination (x_2) to combination (x_3) ,<20> where ϕ denotes an ordinal utility function.

The proof is elementary. By the mean value theorem, we have, for any two combinations x_1 and x_2 , a combination x_0 such that $x_1 < x_0 < x_2$ and $\phi(x_2) - \phi(x_1) = d\phi(x_0)$, $G(x_2) - G(x_1) = dG(x_0)$ where ϕ and G are any two utility functions. If both of them belong to the same individual, we

would demand $G = F(\phi)$ and $F'(\phi) > 0$. By the same token, we will have for some x^* where $x_2 < x^* < x_1$, and $\phi(x_1) - \phi(x_2) = d\phi(x^*)$, $G(x_1) - G(x_2) = dG(x^*)$. By augmenting Postulate (2) to the above, we are in effect requiring:

$$\begin{aligned} (\phi(x_1) - \phi(x_2)) - (\phi(x_2) - \phi(x_1)) &= (G(x_1) - G(x_2)) - (G(x_2) - G(x_1)) \\ d\phi(x^*) - d\phi(x_0) &= dG(x^*) - dG(x_0) \\ &= F'(x^*)d\phi(x^*) - F'(x_0)d\phi(x_0) \end{aligned}$$

provided, of course, that we agree to the fact that Postulate (2) refers to the same utility function as the other postulates. In the special case where two utility differences are the same, $(\phi(x_2) - \phi(x_1))$ will equal $(\phi(x_1) - \phi(x_2))$, thus making $F'(x^*)d\phi(x^*) - F'(x_0)d\phi(x_0) = 0$. Since x^* and x_0 are arbitrary, the value of F 's first order derivative must be the same throughout, which is to say, by definition, F is a linear transformation. Furthermore, for any linear transformation, the value of its first order derivative must be a constant; hence Lange has proved both the necessary and sufficient condition of the Jevonsian method.

Though the ordinalists were unquestionably disenchanted, there was little that they could do. Given the assumptions, Lange's result is manifestly valid, and since the proof proper was accomplished with the state of the art of mathematical techniques of the day -- the one which the ordinalists' endorsed, it is inconceivable that it could be repudiated on logical grounds. Consequently, the majority of the criticisms that have been directed

against Lange is mainly attacks on his postulates and amongst them, there are certainly one or two that deserve a passing rebuttal.

The first of these criticisms is closely related to an age old problem that has long baffled economists. John Stuart Mill was one of those who have explicitly dealt with this problem. When he was arguing for the utilitarian principle, he belittled his opponents' claim that the principle is only good for swines by maintaining that utilities from different sources may differ in their qualities even for the same individual. Swines are naturally deprived of the capacity to appreciate utilities of higher qualities. Whether this last assertion is true or not is irrelevant to us at a time when we are still preoccupied with the immense miseries of mankind. Rather, we are more interested in learning how to differentiate utilities of different qualities and on that Mill has this to say:

If one of the two (pleasures) is, by those who are competently acquainted with both, placed so far above the other that they prefer it, even though knowing it to be extended with a greater amount of discontent, and would not resign it for any quantity of the other pleasure which their nature is capable of, we are justified in ascribing to the preferred enjoyment a superiority in quality, so far outweighing quantity as to render it, in comparison, of small account.<21>

Theoretically, there may be two possibilities for an individual to show non substitutability between two kinds of consumptions beyond the level of minimum subsistence. The first possibility is that the amount of utilities derived

from one consumption takes on an infinite value so that there is no need to consider what trade may bring forth. Mill evidently was not anticipating this as it would render his utilitarian principle meaningless.<22>

The second possibility is that the two kinds of utilities are not comparable, and in this case, the range of the individual's utility function will be partitioned into two disjoint sets, and the elements in each set will be of a different order. It is normally not possible for a single utility function to handle the two different orders of utilities. Even if it can, it is still preferable and methodologically correct to separate the transformation into two distinct functions and allow each of them to deal with one and only one of the two sets of consumptions. Since the mapping between a preference and a utility function is unique after accounting for allowable transformation, we in effect admit that an individual may possess more than one preference when we let him differentiate qualities in his utilities. In addition, utilitarianism will become a lexicographical rule as there will exist priorities among utility functions. Mill claimed that utilities that are derived from higher aspirations like art should always be ranked before those derived from vulgar indulgences. This assertion is readily disproved by facts. Man cannot live by bread alone, nor can he live on books alone. Even lofty economists may at times spare an hour or two to attend to their tomatoes. Substitutions do take place, and when that happens, our individual is in the fullest sense of the word, trading one

pleasure for another. In order for such trade to occur, the different kinds of pleasures must be able to agree with each other, and that is possible only if these pleasures are of the same order. As a result, our individual will have one and only one preference.

Our digression on Mill is not without purpose. Basically, it serves to clarify some conceptual difficulties with regard to individual preferences, and thus makes us better equipped to handle one line of criticism that has been launched against the Jevonsian method.

Phelps Brown^{<23>} was the first to voice his uneasiness with the proof supplied by Lange. In his counterargument, he held that even if we accept Lange's second postulate, we will not be able to construct a cardinal utility function. According to him, the major error committed by Lange was to conflate the consequences of the two postulates. Recalling from the proof, we were able to construct an ordinal utility function ϕ with the first postulate alone and this function is equivalent to G for any $G = F(\phi)$ with $F'(\phi) > 0$. On top of that we also required in the name of the second postulate that $(\phi(x_1) - \phi(x_2)) - (\phi(x_2) - \phi(x_1)) = (G(x_1) - G(x_2)) - (G(x_2) - G(x_1))$. But that is more than what the second postulate gives us. For if we treat the second postulate the same way as we treated the first postulate, then the ability to discern utility differences will give rise to a utility function with utility differences of ϕ as arguments and this function is equivalent to J for any $J = H(\downarrow)$ with $H'(\downarrow) > 0$. There is no explicit requirement from the postulates that ϕ and $\downarrow$ should

be the same function. As long as the individual is entitled to two distinct preferences, it is perfectly legitimate, at least theoretically, for δ and $\downarrow$ not to be identical.

Taking the example of the dictionary to illustrate his point, Phelps Brown explained that, just like the number system, the dictionary is a form of rank ordering, and we certainly can map an individual's preference into a dictionary. However, should we want to rank order under both the first and the second postulate, we must employ two dictionaries, and if "for the first assumption we have appropriated the words of an English dictionary, we may for the second take the words of, say, a French dictionary."<24> There is obviously the advantage of clarity by using two dictionaries instead of one single number system to represent two preferences if these preferences are genuinely unconnected. For firmly inculcated in people's mind is the misconception that the number system is inseparable from some form of metric space properties. So if we represent two preferences by the same number system, there is always the tendency for people to presume that the two are somehow conjoined even though there is no logical necessity for such relationship to exist.

Phelps Brown's case is even stronger if we consider the fact that the ordering of utility differences is not, in the strictest sense of the term, a preference. Instead, it is just an inner feeling within ourselves. For by preferring one thing to another, we normally imply that this act, when realized, will lead to a betterment of our well-being. That

is definitely not the case with regard to utility differences. For whatever we stand to gain from any act, we acquire it through the preference as governed by the first postulate. We do not benefit anything in real term by passing from one utility difference to another, and if there is anything at all, it would have been captured by the preference under the first postulate. Therefore, it would be paradoxical to call the ordering of utility differences a preference as the individual is quite 'indifferent' to all these differences.

If this plea for disparity is accepted, the implications of the two postulates would then differ not only in degree but also in kind, and it would render Lange's proof of existence incomplete. As an ardent ordinalist, Samuelson was delighted by the force of Phelps Brown's argument and he readily lent his support to the latter. In an article which is still being regarded as the one that has dealt the final blow to the Jevonsian method, Samuelson reinstated that "Dr. Lange has shown that if two utility indexes differ by more than a linear transformation, their differences cannot both be used as a numerical representation of the ordered terms of his second postulate."<25>

The whole matter might have drawn to a close and a verdict arrived at had Samuelson chosen to leave it there. However, he decided to carry on with his crusade further; in doing so he unknowingly planted the seed of his own defeat. This happened when he gave the concluding remark to his attack on the single representation method used by Lange.

Observing the rules of sportsmanship, Samuelson confessed, despite his strong commitment to the contrary, that the respective representation of the two postulates may not be totally unrelated after all. For "if we fix the initial point of a movement, then our index of the second kind regarded as a function of the end point alone as a variable must itself be capable of serving as an index of the first kind."<26>

The above quotation definitely calls for some explanation. Essentially, what Samuelson tried to say is that if we share the objection raised previously, then we should make an effort to distinguish clearly the implications of the two postulates. One might, as suggested by Phelps Brown, use two dictionaries to mark the contrast. But, it is also possible to use two number systems, or as Samuelson put it, two numerical indexes to serve the same purpose as long as we bear in mind that the two sets of numbers belong to two distinct systems and are not convertible into one another.

There is one exception. When we construct a utility index of the first kind, we assign numerical values to commodity bundles. These values take on significance only if they have some common reference point. Usually this common point is taken to be zero. So the numerical value 5 of bundle A is actually the short form for writing that there is a five unit difference in utility between bundle A and that bundle which has been used as the reference bundle. Take another bundle B and the same line of reasoning will equally apply. We should realize that A and B are automatically related to each other because the ordering under the first postulate is

complete. And this is the point where the exception comes in. In fact, it is not that much of an exception. As observed by Samuelson, when two members from the index of the second kind, in the happiest coincidence, can be expressed in terms of the index of the first kind, then the two indexes would have to be consistent with each other with respect to these two members. This will happen only if the two utility differences we are comparing share a common end point and that common end point is the reference bundle as adopted in the index of the first kind. Admittedly, if we perceive the matter this way, the chance of such occurrences is slim, as the reference point is only a point in a continuum and there is a strong case to write off this possibility as insignificant. Luckily, we are not restricted to this view. For if instead of taking selective members of the index of the second kind and make them into members of the index of the first kind we reverse the whole procedure, i.e., taking members of the first index and transform them into members of the second index, then it is possible that we might have substantially enhanced the linkage between the two indexes.

As we have seen earlier, every commodity bundle can be expressed in the form of utility difference; it is therefore perfectly admissible to compare all commodity bundles in terms of the second index. To cite an example, let there be two bundles A and B with 5 and 6 units of utility respectively under the first postulate. This, however, would also mean that there are 5 units of utility difference between A and the reference bundle, say X, and 6 units

between B and X. Since these two differences fit exactly into the description of the second postulate, it is inconceivable that anyone would object to comparing A and B under that postulate. Furthermore, the numerical values of these utility differences will be the same under the two postulates since we have not made any transformation on them when applying the second postulate.

In the same manner, this requirement would also apply when we use two dictionaries instead of two numerical indexes. If both dictionaries are of the same language, the second one must totally contain the first one and the arrangement should be in such a way that the positions of the words of the first dictionary are exactly the same whether that dictionary is taken independently or as a part of the second dictionary, and the extraction of the words of the first dictionary from the second dictionary would not have the slightest effect on the positions of the words remaining in that dictionary. In the case where we use two dictionaries of two different languages, there must be, in addition to what has been mentioned above, a one to one correspondence between the words in the first dictionary and their counterparts in the second. With this correspondence, we shall obtain through the two-language representation the same results as the single language representation, and it serves no purpose other than complicating the problem.

By performing a twist on Samuelson's reasoning, we seem to have revealed the existence of a relationship between the two indexes that is much stronger than what he had in mind.

When clearly defined, this relationship will enable us to dismiss Phelps Brown's charge of disparity. For any relation $f: Y \rightarrow Z$, and $X \subset Y$, the relation g is said to be the restriction of f to X if $g: X \rightarrow Z$ such that $g(x) = f(x)$ for all $x \in X$. Sometimes g is also known as a sub-relation of f . In mathematics, we certainly regard f and g to be identical relations with respect to all $x \in X$. The relation g is a useful concept only if we are solely interested in $(Y-X)$ or if the information concerning the complement of $(Y-X)$ is not available. Other than these possibilities, the total domination of f over g makes the latter redundant. In any case, we should never entertain the thought that the two might have come from two different families of relations.

With little effort, we should be able to identify the index of the first and the second kind as g and f respectively. Since they are just particular cases of the general definition, our discussion concerning the restriction of a relation will apply, and that should be sufficient to convince the ordinalists that there is no purpose in pursuing the matter any further in this direction.

Attempts to Axiomatize the Jevonsian Method

After Lange had put forward his original proof, there were several attempts to reproduce his results by means of an axiomatic method. Despite their rigour, they all failed to prove the existence of a general cardinal utility function u .

What they showed instead is that there exists a function w such that $w(x+y) = w(x) + w(y)$ and w is equivalent up to any linear transformation.<28> This function w is of course very restrictive. Not only does it prohibit any complementary effects between any x and y , but it also excludes the possibility of increasing or decreasing the marginal utility of any good. To see this, let $x=y$, then $w(x) + w(x) = w(x+x)$ which is equivalent to saying that the marginal utility of x is constant throughout.

There is little doubt that w is a cardinal utility function. In fact, it is a proper subset of u . As a result, it is easier to prove the existence of w than u . However, we do not think this contributes significantly to one's understanding as to why people have chosen to prove w instead of u . More likely, that favouritism is the result of a massive misconstruction of Lange's second postulate. That postulate requires our individual to compare $u(x) - u(y)$ and $u(w) - u(z)$. Let us assume that these two differences are the same so that we have $u(x) - u(y) = u(w) - u(z)$. Since we are all familiar with simple arithmetical operations, it is extremely tempting to rearrange the equation into the following form;

$$u(x) + u(z) = u(w) + u(y); \quad (1)$$

as we are always contemplating an atemporal situation, our individual can never be able to consume x and z separately. The possession of x and z together requires that we conceive of x and z as forming a single bundle with a utility equals to $u(x+z)$. Together with the acceptance of (1), we would

have $u(x) + u(z) = u(x+z)$ and that more or less lays bare why people have been proving cardinal utility functions with additive properties.

The major error of these people lies in assuming (1) to be equivalent to the second postulate, or more appropriately, in confusing an equivalence relation with the equality sign. Under the second postulate we have $u(x) - u(y) = u(w) - u(z)$, and the "=" sign here is an equivalence relation denoting the internal feeling of the individual. In order to derive (1) from the second postulate, we would have to treat "=" as an equality which is basically illegitimate. Hence (1) is not equivalent to the second postulate. The fundamental difference between them is rooted in the "+" and "-" signs in the two equivalence relations. As we have noted before, we are not entitled to write in the form $(u(x)+u(y))$ unless the consumption of x is independent of the consumption of y , and this is exactly what is implied by (1).

Lange's second postulate, on the other hand, does not imply such independence. Under that postulate, our individual is asked to evaluate one bundle of commodities after another and then evaluate the utility differences between them. However, he is never asked to evaluate two bundles simultaneously because he has in his possession only one bundle of goods at any one time. The "-" sign here should be interpreted as a relation of removal so that as soon as one bundle is made available, the existing bundle will be removed, thus leaving our individual with only one bundle to assess. Interpreted in this way the second postulate is able

to avoid the difficulty which is encountered by (1). In particular, it enables our individual to express his opinion on different bundles of goods without losing track of the complementary effects and satiation effects.

It is evident that we only need utility differences and not additivity to derive cardinal utility functions and there seems to be no justification at all to overstate our case. If there is no objection to our line of argument, then we will have to dismiss all the independent functions put forward by Debreu and others.

We have spent a lot of our efforts in Lange's defense, and we are justifiably interested in knowing whether our efforts are well spent, i.e., whether Lange's proof is worth defending. The answer to this question is somewhat ambivalent. With respect to all the arguments that have been advanced, we feel that everyone of them is justifiable and that there is no reason to retrack from anyone of them. Nevertheless, there is one major failing in Lange's proof -- a logical incoherence that almost completely nullifies the validity of that proof right from the start.

Like most economists, Lange has taken for granted in his two essays many assumptions, and among them is the assumption of the differentiability of the function ϕ . But ϕ is an ordinal utility function until proven otherwise, and "it is meaningless to speak of the slope, or marginal utility, of an ordinal utility function; such a function cannot be differentiated." <29> A utility function is differentiable only if it is a cardinal utility function. By assuming

differentiability to prove cardinality, Lange has in effect assumed away what he has set out to prove. Yet Lange was not alone in this respect. Many ordinalists still commit the same error today as Lange did in the thirties. Usually they will dismiss cardinal utility functions on the one hand and then plead for a well behaved utility function.<30> Samuelson was likewise trapped when he criticized Lange.<31> Finally it was Debreu, the mathematician who, quoting another mathematician of higher calibre, restored meaningfulness to the debate between the two camps.<32> Nonetheless, Debreu did not supply us with a complete proof of the existence of a general cardinal utility function, and the entire difference approach was left in limbo after we disclosed the inadequacy in Lange's methodology.

Alt's Proof

This, however, is actually an unnecessary state of affairs. Had the English speaking writers been a little more opened minded, they would have noticed that there had already existed a cogent and valid proof for the Jevonsian method in German as far back as 1936. This brilliant work was by Alt<33> who, to our dismay, is again not from our discipline. To the best of our knowledge, it was Debreu<34> who first mentioned Alt's article in his bibliography in 1958, and it took no less than thirteen years before it was finally translated into English. Alt openly declared that his work

was inspired by Lange, whose proof he regarded as incomplete.

To a certain extent, Alt's reasoning is much similar to that of Weldon's^{<35>} as they both have made use of the induction method to arrive at their results. However Alt's proof is more rigorous and complete. Not only did he utilize the axiomatic approach to handle the problem, he also successfully proved the necessary and sufficient conditions for measurability corresponding to a real line. Since his proof is almost^{<36>} logically correct, our evaluation on it will depend solely on whether we accept his axioms or not.

There are altogether seven of them, or five more than what Lange has postulated. Most of the additional axioms are just formal statements of what we have taken for granted in previous discussions. Among them we can find the Archimedian axiom(VII); the ordering axiom(III); and the continuity axiom(VI). Only axioms IV and V are indeed novel. The former is there to serve as a bridge between the first and the second postulate in Lange's system. Its introduction is a recognition of Phelps Brown's objection by Alt. As we have already indentified g as the restriction of f to X , there is no need for us to elaborate further on this axiom as it posits a condition which is weaker than what we have conceded.

The one that really deserves some justification is axiom V. We will first reproduce the axiom:

If $(x) - (y) > (x') - (y')$ and $(y) - (z) \equiv (y') - (z')$,
then $(x) - (z) > (x') - (z')$; the same holds for c
and $\equiv$ in place of $>$.^{<37>}

At first glance, this axiom can be easily confused with Frisch's axiom IIc<38> which is essentially an axiom of addition. But if we peruse it more closely, we will realize that axiom V does not involve any property of addition. It is true that this axiom is crucial for the construction of cardinal utility functions, and it probably attributed more to measurability than any other axioms in Alt's system. Nevertheless, unlike Frisch's axiom, axiom V does not imply independent utilities.

Assume that we are dealing with the ordinary preference of an individual over three bundles x , y and z . Hypothetically, let us say that he prefers x to y and y to z . From the information given, are we entitled to make any speculation on his preference between x and z . Certainly we can invoke transitivity and conclude that he prefers x to z . But why would we allow for transitivity? The reason is simple enough. We postulate transitivity to enable the individual's preference to be compatible with another important axiom, viz., the axiom of non-satiation. When the individual prefers x to y and y to z , we assign to each of them a $u(x)$, $u(y)$ and $u(z)$ respectively with $u(x) > u(y)$ and $u(y) > u(z)$ and $u(x) > u(z)$. Hence, non-satiation will dictate our individual to prefer x to z . We wish to stress that we have not performed any addition of utilities, and as a matter of fact, we have only told the story the way the ordinalists would like to have it told.

Eventhough, strictly speaking, we cannot call the relation under the second postulate a preference function

because the arguments in that function are not exactly something that our individual would be able to consume, it is nevertheless an ordering relation all the same. The difference between this function and the previous one is in their arguments. We have seen that it is not sufficient to accept only the first postulate to arrive at a utility function. The non-satiation axiom is equally necessary. It is only reasonable that if we made a certain allowance for the first postulate, we should give similar treatment to the second postulate under similar circumstances, or otherwise, it will not be able to produce what it has set out to produce. In other words, we should also permit a non-satiation clause for the second postulate. Here we run into difficulties. It is rather ridiculous to tell someone to want more of something which does not really add anything to his level of satisfaction. So, non-satiation has to be interpreted figuratively. Or, if we like, we could call it the extended non-satiation axiom. As one would expect, this axiom should have more or less the same characteristics as the same axiom under the first postulate, i.e., having more of something should not reverse an established relationship. In this case that something is the feeling of intensity.

Having committed ourselves to that egalitarian principle, let us take a further look at what exactly axiom V does amount to. Loosely it says that if an individual feels the utility difference between x and y is greater than the difference between x' and y' , and if he also feels the difference between y and z is greater than the difference

between y' and z' , then the simultaneous feeling of the two greater differences will still maintain their domination over the simultaneous feeling of the two smaller differences.

The word 'simultaneously' does not involve any addition of utility differences. It only requires that our individual is capable of simulating two feelings into one. It is analogous to his ability to consume two bundles of goods together and then express a unique satisfaction level, this should be well within the ordinalists' field of analysis.

As far as we are concerned, there is nothing objectionable about axiom V. It is there because we have postulated an additional ordering and that ordering will not be consistent even in the ordinal sense if we do not have this extended non-satiation axiom. Since the acceptance of Lange's second postulate is almost universal, and since it is impossible to make any sense out of that postulate without the extended non-satiation axiom, we will take it that there is a general agreement on its acceptance.

In closing this chapter, we will allude to a minor point which in our opinion is illegitimate in Alts' proof. After successfully proving the existence of a real valued function with the properties:

$$(i) f(x) \geq f(y) \Leftrightarrow x \supseteq y \text{ and}$$

$$(ii) f(x) - f(y) \geq f(z) - f(w) \Leftrightarrow x - y \supseteq z - w,$$

he tried to prove that for all the functions g such that g satisfies (i) and (ii), $g(x) = a + bf(x)$ where a and b are constants.

In Alt's deduction, he lets $g(x)=b$, $g(x_1)=c$ and by one

of his lemmas he was able to get $g(x_0) - g(x_{1/2}) = g(x_{1/2}) - g(x_1)$. Then he proceeded to move $g(x_1)$ to the left hand side, and $g(x_{1/2})$ to the right hand side and here is where he slipped. As we have mentioned before, the equality sign here is not the usual equality sign in arithmetic. It is just an equivalence relation as Alt had clearly identified in his lemma on page 427; to confuse the two is tantamount to postulating independent utilities.

In fact, Alt did not have to make such a concession. His proof can well be carried on without moving anything from one side of the relation to the other. Obvious as it may be, we will make an untiring effort to produce it in the following:

$$\text{let } g(x_0) = b; g(x_1) = c$$

$$g(x_0) - g(x_{1/2}) = g(x_{1/2}) - g(x_1)$$

$$b - 1/2(b+c) = g(x_{1/2}) - c$$

$$1/2(b) - 1/2(c) = g(x_{1/2}) - c$$

$$1/2(b-c) = g(x_{1/2}) - c$$

let $(b-c) = a$ and substituting $1/2$ by r which stand for any real number

$$ra = g(x_r) - c$$

and since $f(x_r) = r$ has been proven by Alt in his article,

$$af(x_r) = g(x_r) - c$$

which is the desired result.

Readers who are interested in empirical findings concerning the Jevonsian method can consult the work published by Allais recently.<39>

VII

Interpersonal Comparison

Some Preliminary Remarks on Taxonomy

In the earlier chapters we have successfully proven the logical necessity for the measurability of utility; it now remains for us to demonstrate how we can take advantage of cardinal utility functions to construct acceptable orderings of social alternatives. When taken on its own, cardinal utility function will produce results that can hardly claim any superiority over its ordinal counterparts, as it is easy to conceive that any social decision thus arrived at can be readily reversed by an appropriate linear transformation of certain individual's utility function. So, if any ground is to be gained, it would not be gained by annexing cardinal utility alone. Some extraneous information must be made available to complement cardinal utility. For that we must turn to interpersonal comparisons. In order to be fair, we will try to enlist and discuss all possible cases of interpersonal comparisons; in doing so we hope no party will be left with the feeling that they had been victims of prejudice. As we proceed, we will realize that most of these possibilities are not actual possibilities at all. Sometimes their results are inapplicable, but more often they are logically inconsistent. By eliminating them one by one, we will eventually find ourselves confronted with one last

possibility, and that is the one we opt for.

There was a time when interpersonal comparison was a rather vague concept covering an array of evaluating schemes. Very often, debates on the topic were misguided as participants did not exactly know what kind of comparison or non-comparison their opponents were propounding. The situation has much improved now. Even though the core of the problem apparently still persists, there appears to be a more precise classification of the different kinds of interpersonal comparisons due mostly to the mathematical treatments of the subject. In what follows we will first briefly review the different forms of comparisons that are most commonly discussed in the literature;^{<1>} thereafter, we will attempt to comment on their credibilities.

The taxonomy will be performed by a two-way partition. Recalling that by an ordinal utility function for individual i , we mean a function u_i such that $u_i(x) = u_i(y)$, if and only if $u_i'(x) = u_i'(y)$, where $u_i'(\cdot) = \phi_i(u_i(\cdot))$ and ϕ_i is any strictly increasing function. A cardinal utility function for the same individual will be u_i , if and only if u_i' is also a utility function of the individual, where $u_i' = a_i + b_i u_i(\cdot)$ and a_i, b_i are any constants with $b_i > 0$. It is certainly possible for us to impose on an individual's utility function restrictions that are even weaker than the positive monotonic transformation. However, judging from the present circumstance, this extra effort would serve no purpose. The controversy in utility theories is centred around only two major contenders, viz., ordinal and cardinal

utility, and there is hardly a single economist who would urge us to go beyond ordinal representations. Given this boundary, it is just natural to make our first partition along the familiar ordinal-cardinal line.

The second partition comes in even more naturally. Since there are advocates and opponents of comparability, what we have to do is simply to split them according to their convictions. By the law of combination, we can now at least identify four kinds of interpersonal comparison: (i) ordinal non-comparable; (ii) cardinal non-comparable; (iii) ordinal comparable and (iv) cardinal comparable. There may exist other cases which lie in between the four listed, just as there may exist ordered-metric transformations embedded in between monotonic and linear transformations. We will turn our attention to them once we have properly defined the forementioned classes.

By ordinal non-comparability we mean that each individual's preference in the society is represented by an ordinal utility function as defined above, and this function is fully independent of other's utility functions. Furthermore, each of these functions is free to undergo any monotonic transformation on its own. In other words, each individual has an independent and full-fledged ordinal utility function. The reason why it is not possible to make interpersonal comparison in this case is because there does not exist any rigid correspondence between people's utility functions. When we want to compare one person's utility with another's, we need a certain kind of correspondence between

their utilities; when these utility functions can transform at will and are independent of each other, it is simply impossible to conduct such a correspondence.

The explanation in the ordinal non-comparable case is equally applicable to the cardinal non-comparable case. Only this time the various individuals' preferences are represented by different independent cardinal utility functions. These functions are still allowed to undergo transformations, but these transformations will be restricted to the form $u_i'(\cdot) = a_i + b_i u_i(\cdot)$, with $b_i > 0$. Each individual i 's utility function is independent in the sense that the constants a_i and b_i are allowed to take on any value without regard to a_j and b_j , for all $j \neq i$. Hence, although it may be true that, as non-linear transformations are excluded, the set of possible transformations is smaller than in the previous case, the task of establishing correspondence among people's utilities has not been made any easier. As long as the a_i 's and the b_i 's are permitted to vary without bounds, any individual i can always choose suitable a_i and b_i such that his preference can never be overruled by others. Since each and every individual is entitled to act likewise, social decisions under cardinal non-comparability are potentially as indeterminable as in the case of ordinal non-comparability.

It should be evident that if we want to make interpersonal comparisons, we must restore a certain kind of correspondence among people's utility functions. One professed way to accomplish this is to further restrict the

allowable transformations that are opened to our individual's utility function. Some economists have maintained that even if these additional restrictions are imposed, it would still be possible for people's utility functions to retain their ordinal significance. They feel that comparability and ordinal utility functions are perfectly compatible with each other. According to them, we can still perform any monotonic transformation on an individual's ordinal utility function as long as an identical² transformation is also performed on the utility functions of the other members in the society. Comparability comes about as a result of the identical transformation. If parity exists before the transformation between two individuals' ordinal utility levels, parity will be retained after it. There is a notion of invariance among utility functions of different individuals -- an invariance that ignores which transformation is being undertaken.

Since cardinal utility entails ordinal utility, if we are able to make interpersonal comparisons of ordinal utilities, we must also be able to make similar comparisons of cardinal utilities. Here again, the comparison is made possible by identical transformations. By an identical transformation, in the cardinal case, we mean that $a_i = a_j$ and $b_i = b_j$, for all i and j in the society. A special case of cardinal comparability is the ratio-scale comparability, which requires $a_i = a_j = 0$, for all i, j . Ratio-scale comparability is stronger than ordinary cardinal comparability in that it fixes an absolute zero for every utility function. In essence, it is no more than a natural

normalized form of cardinal comparability, since, by the latter, a_i can take on all values, as long as these values are matched by corresponding adjustments in the a_j 's. However, all the a_i 's are real numbers and they can take on significance only if there pre-exists an absolute zero.<3>

According to some economists,<4> both the ordinary and ratio-scale cardinal comparability schemes are unduly restrictive. They claim that full cardinal comparability can still be achieved with weaker conditions, and they have advanced a version of cardinal comparability known as unit comparability. Under this kind of comparability, full interpersonal comparison is attained if the b_i 's are the same for all individuals. There is no restriction imposed on the a_i 's, and these can vary freely among individuals. It is called unit comparability because b_i is the coefficient responsible for the choice of unit in individual i 's utility function. The a_i 's, which denote the initial levels of satisfaction people enjoy, are left out. They are inconsequential, in the sense that when we compare two situations, the status quo will be common in both; hence, the utilities so derived from it for any individual in the two situations can be regarded as cancelling each other out. If we can compare the gains and losses in utilities among individuals through the equality of the b_i 's, we will be able to make a social evaluation of the situations based on that information.

Besides the forementioned, there are other cases of comparability. On the whole, what we have defined represent

the extreme cases. Among them, we can either find full comparability or a total lack of it. There is nothing in between. Such an attitude naturally will leave some moderates in the cold and before we know it they have already come up with something called partial comparability.

Partial Comparability

The idea behind the development of partial comparability is mostly attributable to the rigidity of full comparability. It occurs to many that given the vast differences among people, the requirement confining each individual's marginal utility to the same identical transformation coefficient b_i is simply too restrictive. Before we go on to analyse how one can relax this rigidity, we would like to point out that the differences in individual tastes and capacities are already well expressed by their own utility functions, and the weights and the functional form of the social welfare function have also taken care of the needs for social priorities and preferences. So the appeal for the modification of the uniform b_i appears to us no more than a redundant re-iteration of what has already been accounted for.

The term 'partial comparability' was first introduced by Sen, who illustrated it with the story of King Nero playing the fiddle when Rome was burning. In his discussion, he argued that in order to say that "the sum total of welfare

went down as a consequence" of Nero's action, we need not be "assuming that every Roman's welfare units can be put into one-to-one correspondence with the welfare units of every other Roman."<5>

Assume that the decision to burn Rome is in the hands of two groups of people. The first group being Nero himself and the second group being his subjects. As self-centred individuals, it is likely that both of these groups will try to influence the social decision to serve their own interests. This can be done in three ways: (i) they can try to change the rules of the decision process; (ii) they can give false statements about their utilities by exaggerating their satisfaction levels; (iii) they can insist that the linear transformations of which their cardinal utilities are subject to should be different from those of the other group. Specifically, the b_i for his group should be higher than that for the other group. We will ignore the first two possibilities by assuming that all members of the society are willing to observe the fixed rules of the game.

If each group wants to set its own b_i equal to infinity, then we will be back to non-comparability again. Hence, what partial comparability begs for is a certain sense of restraint and altruistic feeling. It sets both an upper bound of infinity and a lower bound of zero to the value of each b_i . Each group has to value its own b_i below infinity and those of others' above zero. Taking the previous example, let Nero be the j th person in the community, and assume that he sets $b_j = 10$ and $b_i = 1$ for all $i \neq j$.

Furthermore, assume that there are twelve Romans altogether, and that Nero would gain one unit of utility by seeing Rome burn, and the rest would lose one unit of utility each by witnessing their homes vanish in fire. If the social welfare function is of the simple utilitarian type, then the total utilities gained if Rome were set on fire would be 10 units, the total utilities lost would be 11 units, and the society should not burn Rome even according to Nero's judgement. With little doubt, the result will again be confirmed if we conduct the evaluation with the set of b_i 's that belongs to the rest of the Romans.

To a great extent, partial comparability is just a modified version of the Paretian condition as it also relies on the consensus rule <6> to conduct social decisions. But, unlike the Paretian condition, it may sometimes be able to preside over situations where at least one party will suffer. As it is well known, the Paretian condition is totally inoperative in such situations. The logic behind this improved cutting power is simple. It may be true that some parties will be hurt, as the society moves from one situation to another, but if those who are going to be hurt agree that the social gains so derived will be worth what they are giving up, then it is probable that they may follow their conscience and approve such a social move even though they are fully aware of the fact that they are the ones who will bear the costs.

One may tend to think that because of the lack of disparity in opinion, partial comparability will have to be

replaced by full comparability when the number of social decision makers is reduced to one. Yet this is not quite the case. On the contrary, partial comparability was first propounded by Sen with the single ethical observer in mind. Should such a case arise, partial comparability will be sustained on the excuse of the fuzziness in the observer's judgement.<7> Very often, such fuzziness will take the form of a band, i.e., with a supremum and an infimum, and each b_i can lie anywhere between its own bounds. One can justify this kind of inexactness in many ways, e.g., informational constraints; human indecisiveness and so on. Regardless of what the excuse may be, whenever fuzziness occurs we may have to settle for something less than full comparability even though there may be only one social decision maker. If one does not object to this line of reasoning, the whole analysis will follow in exactly the same fashion as in the previous case, where the differences in the b_i 's are caused by the differences in individuals' opinion.

The choice of Nero's story by Sen is by no means a random one. As an example, it offers us a clear cut illustration where one situation is unequivocally preferred to another. However, this need not always be the case, and it is not hard to construct examples where unanimity is absent. Whenever such cases should present themselves, which is often, there are two options open to us to deal with them. Either we can regard any two situations, which do not call for strict preference of one over the other under partial comparability, as belonging to the same equivalent class,<8>

or we can consider them as non-comparable.<9> Neither is satisfactory.

If we follow the first option, we will have to treat any two situations a and b as indifferent from each other whenever $\neg(aPb)$ and $\neg(bPa)$. However, this need not, and usually does not, have the same amount of social welfare as b . Given an unrestricted domain, it is always possible to construct situations $c, d, e, \dots$ such that $aIcIdI\dots Pb$. Worse still for the logic, cyclic sequences like $aIbIcI\dots PxIyIz\dots Pa$ are also conceivable.<10> Since consistency is non-negotiable, it seems we have no other choice but to discard the hope of resolving the indecisiveness in partial comparability through this first option. Hence we are left with the only other alternative, and that is to declare a and b as non-comparable. This is manifestly the defeatists' route, and it may render the social ordering so derived incomplete. However, as the name might suggest, this probably is the route that the forerunners of partial comparability had intended for us. To no one's surprise, the porosity of the social ordering depends critically on the values of the b_i 's each decision maker assigned to each individual utility function. As a rule, the smaller the variations, the more complete will be the social ordering. Full comparability will be achieved when all the b_i 's are identical for all decision makers. But is that at all necessary? The answer is no, and a proof has been given by Fine<11> in a short technical note. The idea behind his argument is intuitively simple. Given that each

decision maker has a set of b_i 's, each of them will be able to perform pairwise comparison of all the situations. For any pair of x, y , some may rank x over y , while others may do the opposite. As a result of vested interests, there must exist two decision makers such that the differences in their valuations over some pair of situations x and y will be the greatest among all the decision makers and situations. If under such unfavourable circumstances, there is still agreement between the two individuals over the ranking of x and y , then it would be safe to conclude that there is a complete social ordering of all the possible situations notwithstanding the fact that we do not have an identical b_i for all utility functions.<12> Besides the values of the b_i 's, the possibility of a complete, or near complete, social ordering also depends critically upon the number of decision makers involved. In an actual society where the number of people is almost uncountable and conflict of interests is taken for granted, it is hard to imagine how a complete ordering of the social states could ever be achieved if we allow every individual to have some fuzzy input into the social decision process. Since we have no *a priori* reason to dismiss universal participation, the chance of getting an incomplete social ordering partial comparability does not appear to us to be just a remote possibility. We must presume that this is the rule rather than the exception.

The above comment pertains only to the practicality of partial comparability and has no bearing on the theoretical level of the concept at all. In what follows, we will

attempt to construct an example which will reveal some of the logical difficulties in the concept of partial comparability. To do that, we must re-examine the fundamental idea of partial comparability. Let us define the degree of partial comparability d as $\inf.(b_i/b_j)$ for all $i, j < 13$ and recall that the social ordering, derived from partial comparability, may be complete and that it all depends on the maximum difference between any b_i and b_j or equivalently on d .

Assume that we are dealing with a society comprising two persons and that we are trying to rank order two social states x and y . Since there are only two individuals, $d = b_i^*/b_j^*$, and we will take it to be $1/2$. Furthermore, let $a_i = a_j = 0$, $u_i(x) = 8$; $u_i(y) = 1$; $u_j(x) = 1$; and $u_j(y) = 2$. To make things simple, we will again conduct our illustration with the Benthamite rule. From the figures given, it is obvious that $u_i(x) + u_j(x) = 9$ is greater than $u_i(y) + u_j(y) = 3$, and x should be socially preferred to y . However this evaluation is carried out under the implicit assumption that $b_i = b_j = 1$, i.e., before we have performed any allowable transformation on the two utility functions. What would happen to the social ordering of x and y , if we actually apply the allowable transformations? With $d = 1/2$, one can easily be deluded to think that the social ordering of x and y will not change even after all the allowable transformations have taken place. If that is the case, then partial comparability will prove to be a useful concept, as it completely orders x and y whereas the Paretian condition could not do so. Letting $u_i' = b_i^*(u_i)$ and $u_j' = b_j^*(u_j)$;

the total utility of x with the most unfavourable transformation will be $u_1'(x) + u_2'(x) = 10$ while with its most favourable transformation y will have a total utility of $u_1'(y) + u_2'(y) = 5$. Needless to say, x is still preferred to y . But that may not last long! By definition u_1' and u_2' are also authentic utility functions, as they are derived by performing linear transformations on two utility functions u_1 and u_2 . In all respects, they are as qualified as any other pair of utility functions from the same equivalence class. If that much is accepted, then u_1' and u_2' should be entitled to whatever treatments are available to u_1 and u_2 ; in particular, the transformations prescribed by d .

By repeating the same operation, we will have $u_1'' = b_1 \cdot (u_1')$ and $u_2'' = b_2 \cdot (u_2')$. In our example, $u_1''(x) + u_2''(x)$ is still greater than $u_1''(y) + u_2''(y)$, and situation x will again edge out situation y . But as one keeps on transforming the newly formed utility functions with d , one will eventually come to a pair of utility functions, in our case to u_1''' , u_2''' such that the ordering relationship between x and y will be reversed. This conclusion is true for any pair of situations as long as the b_i 's are not identical. With this preference reversal phenomenon, partial comparability is evidently inherently inconsistent, and this will render it unsatisfactory even as a theoretical construct.

One naturally wonders why would such a promising and intuitively appealing concept fail to hold. In our opinion, the flaw of partial comparability lies in misidentifying the

issue. There is no dispute that, at times, our ability to make interpersonal comparison is limited and fuzzy. The fuzziness, however, does not seem to come from our inability to decide on an identical b for all the individuals. That kind of reasoning, after all, is extremely technical, and very often, beyond the comprehension of most people. When people say that they cannot exactly compare two persons' satisfactions, what they likely mean is that they cannot uniquely identify one of individual B's satisfaction level to correspond to a particular satisfaction level of individual A's. For example, they may not know how many units of B's utility would exactly correspond to 5 units of A's utility, yet they do believe that it should fall between 2 and 8 units.

If we recall our basic transformation $u_1' = b_1(u_1)$, we will notice that fuzziness can come in two ways, namely from b_1 and u_1 . Using either one of these fuzziness will achieve the idea behind partial comparability. As it stands now, we have one extra degree of freedom. People have chosen to conduct the exercise by varying b_1 because it is mathematically elegant and precise. Just imagine how unmanageable that might have been if they had chosen the other alternative, as that would mean finding a set of fuzzy sets to account for each and every satisfaction level of all the individuals. To illustrate, 5 units of A's utility may be considered as equivalent to some value of B's utility that lies between 2 to 8 units, and of C's utility between 21 to 40 units, and so on until all the individuals in the society

have been likewise accounted for. Furthermore, similar equivalence relationships must be constructed for other levels of A 's utilities as well. Since the formulation of these fuzzy sets are independent of each other, there is little hope that they can be consistent with each other simultaneously.

Unmanageable as it may seem, the choice of the u_i 's as the source of our fuzziness does, nonetheless, enable us to avoid the logical embarrassment that we have encountered before. Besides, if we look back to our earlier example, we will readily realize that we were concerned with the fuzziness of the u_i 's rather than the b_i 's. When one carefully rereads Sen's discussion on partial comparability, one will certainly share the feeling that Sen was talking about the same kind of indetermination though he has mis-directed it under the banner of the b_i 's.

We have proved that partial comparability via the variation of b_i is inherently inconsistent. Later on we will argue that the existence of some fuzziness on the u_i 's may not deter us from getting full comparability, and to that extent partial comparability is redundant. Consequently we will hereby discard partial comparability as a viable method in making interpersonal comparisons.

Theoretically, partial comparability can take on a third form, and that is through the fuzziness of the a_i 's. Since all the criticism of the b_i 's apply equally to the a_i 's, we will try not to repeat ourselves here, but will move on to discuss the extreme cases which we have outlined earlier.

The first is that of ordinal comparability.

Ordinal Comparability

We will start by giving a more precise definition of ordinal comparability. Two ordinal utility functions, u_1 and u_2 , from two different individuals are considered comparable if, and only if, there exists a list of identical, strictly monotonic transformations ϕ such that $\phi', \phi'', \dots, \phi_n$, and for all situations x, y , $\phi'(u_1(x)) R \phi'(u_2(y))$ if, and only if, $\phi''(u_1(x)) R \phi''(u_2(y))$, R being the social ordering, and u_1 and u_2 not being identical. Some people have taken R as the moral judgement of some single ethical observer. That interpretation may be convenient but it cannot be regarded as an answer to our question, for there is simply no interpersonal comparison involved if we were to invoke the ethical observer scenario. What we would have instead is some kind of intrapersonal comparison pertaining only to the observer himself on various social positions. For example, $u_1(x) R u_2(y)$ would mean that the observer considers being individual 1 in situation x is at least as well off as being individual 2 in situation y . Hence R must be treated as the configuration of the opinions of a group of people, and preferably the same group of people which makes up the society under scrutiny. In the most general form, R will be a function of $(R_1, R_2, \dots, R_i)$. We should not confuse u_i and R_i : the former being the i th utility function while the

latter is some kind of ethical observation by individual i.<14>

Once we have agreed to this frame of investigation, there are several problems awaiting us. Firstly, there is the difficulty of construction. Unlike cardinal comparability where, when you know one, you will know all of them, ordinal comparability demands the comparison of every possible social position. To illustrate, let $u_1(x) = 1$; $u_1(s) = 9$; $u_1(w) = 10$; $u_2(y) = 1$; $u_2(t) = 2$; $u_2(z) = 10$ and $u_1(x) R u_2(y)$; $u_1(w) R u_2(z)$. With all these information, can we say something about the relation between $u_1(s)$ and $u_2(t)$? There is little doubt that intuitively, most of us are inclined to speculate on $u_1(s) R u_2(t)$. However this is by no means certain. The social decision can easily go the opposite way. In fact, there is no a priori reason for favouring either outcome. When we have only ordinal comparability, interpolation and extrapolation of any sort on R over unranked alternatives will have no logical basis. This difficulty, on the other hand, is not totally unsurmountable as we can always exhaust all possible comparisons through unyielding patience and dexterity. It will just take more time!

The second problem of ordinal comparability comes from the definition. What does it mean when we say that there exists an identical monotonic transformation among the utility functions of the individuals? Does it mean that when we transform 5 units of A's utility into 10 units, we would have to do likewise for 5 units of B's utility? If that is

the case, does it somehow suggest to us that there is a certain kind of cardinal parity between the two utility functions? Since any other possibility would only lead to more complication and chaos, there is no other recourse for the ordinalists but to accept this construction. Nor does the problem end there, for, even with such an identical transformation, one could still easily envisage a case where the values of the newly formed utilities would reverse the previous social ordering. A possible way out of that difficulty is to perform a simultaneous transformation on the social ordering R such that it will restore the old order. But that procedure is questionable on two counts.

Firstly, the procedure mentioned was not adopted by any of the exponents of ordinal comparability. We have come to this conclusion through observation. In all the writings on the subject, there is not a single piece which has made the explicit but necessary notational change. For if one agrees to the transformation of R , one would have to change R to something like R' after the transformation has taken place. Secondly, if we are allowed to adjust the social decision rule in this way to accommodate ordinal comparability, there is little left to be disputed. However, that may not be a good approach, for what we have done is to turn ordinal comparability into a tautology. Since R' is adjustable, $u_1(X) R' u_2(Y)$ is equivalent to $\phi'(u_1(X)) R' \phi'(u_2(Y))$ always. It is just like saying X is better than Y if and only if X is better than Y . It certainly does not add anything to our knowledge, and still less, to the construction of a decision

rule.

A Special Case: Rawls's Maximin Principle

As one may recall, an Arrowian social welfare function will be dictatorial if it satisfies (i) the individualistic axiom, (ii) the Paretian axiom, and (iii) the independence axiom. With the acceptance of the independence axiom, our individuals will be allowed only to represent their satisfaction levels with ordinal utility functions. Since we have shown that the concept of an identical monotonic transformation for two ordinal utility functions does not really make sense, we would therefore expect that Arrow's impossibility result will also apply to those social welfare functions which only allow interpersonal comparisons of people's ordinal utility levels.

Our whole discussion of ordinal comparability would have ended at this point had it not been for Rawls,^{<15>} for it seems that the difference principle, as devised by him, has enabled us to avoid Arrow's omen with some sleight of hand. Even Arrow^{<16>} himself has conceded that Rawls's maximin principle is not subject to his impossibility theorem. He attributed this to the fact that ordinal comparability is allowed in the Rawlsian maximin; and more specifically, it is the identical monotonic transformation that does the trick. We know that this cannot possibly be the case because there simply does not exist something called an identical monotonic

transformation for two individuals' ordinal utility functions. Hence, if the maximin rule is really an exception at all, we must seek some other more convincing reasons to back up such a claim. In order to discover whether such reasons exist, we will have to go back and examine some of the fundamental issues in Rawls' difference principle.

Despite the fact that some people may still want to argue that by "adopting 'maximin'," Rawls has presupposed "not merely a cardinal measure of utility invariant up to a linear transformation, but a ratio scale with fixed zero-point,"^{<17>} Rawls has insisted that his analysis is ordinal in nature. In his own words:

The difference principle meets some of the difficulties in making interpersonal comparison....as long as we can identify the least advantaged representative man, only ordinal judgements of well-being are required from then on....The further difficulties of cardinal measurement do not arise since no other interpersonal comparisons are necessary. And, of course, in maximizing with respect to the least favoured representative man, we need not go beyond ordinal judgements....We never have to calculate a sum of advantages involving a cardinal measure. While qualitative interpersonal comparison are made in finding the bottom position....^{<18>}

Rawls suggested that we should use something that is commonly desired by all individuals as the standard to locate this bottom position. A commodity with such attribute will be called a primary good. The common primary goods are liberty, rights, income, wealth and self-respect, and so on. It was claimed by Rawls that this position can be found with some kind of consensus in the society, and if that is true, one could certainly not regard the difference principle as a

dictatorial rule. Arrow's theorem would, therefore, not apply. But unless there is a position such that it has less of all primary goods than any other position -- a position which we should not count on, we would still have to confront the basic aggregation problem. Only this time, we aggregate not to find the best situation for the society but to search for the worst-off social position that is recognized by all. Since the two aggregating problems are of the same order, there is nothing gained by switching from one to the other. Rawls pleaded that it is possible to construct an index of primary goods which allows us to rank the social positions according to that index. With this index, the least favoured social position will be easily found. There is no clear indication as to how this index is to be constructed. It seems that Rawls was counting on the existence of a group of people whose share of every primary good is less than that of any other group in the society. This supposition is of course questionable. The question however, does not stop there, for what Rawls needed is the identification of a single individual and not a whole group of people. To do that, Rawls has reverted to the concept of the representative individual. This is how he put himself through the ordeal:

We try to do this by taking up the standpoint of the representative individual from this (the worst-off) group and asking which combination of primary social goods it would be rational for him to prefer. In doing this we admittedly rely upon our intuitive capacities. This cannot be avoided entirely, however.<19>

By not avoiding one, Rawls has successfully avoided the other, for the concept of the representative individual completely bypasses the question of aggregation. If we choose to regard this representative individual as some single individual chosen from the group, we would have to supply reasons why this, and not some other, individual is being chosen. On the other hand, if this individual is chosen just through some mathematical operation, e.g., averaging, we would still have to explain why a certain operation is used and not others. Rawls's appeal to intuition is not acceptable as a solution. All it has done is to shy away from the problem rather than furnishing it with an answer.

On another occasion Rawls has indicated that he might want to solve the problem with a representative primary good rather than with a representative man. Instead of taking some kind of an average, the representative primary good is supposed to be chosen according to its importance. Among all the primary goods he has indicated, 'self-respect' is the one which deserves that honour, and he thinks that it should rank lexicographically before any other primary goods. To procure his point, he claimed that "without (self-respect) nothing may seem worth doing, or if some things have value for us, we lack the will to strive for them. All desire and activity become empty and vain, and we sink into apathy and cynicism. Therefore the parties in the original position would wish at almost any cost to avoid the social conditions that undermine self-respect."<20>

Most of us will find this Rawlsian idea hard to accept. Casual observation tells us that people very often are prepared to exchange self-respect for other goods in life, and when such substitution takes place, self-respect will cease to be the representative good and will lose its lexicographical priority. Even if we assume that each and every individual regards self-respect as sacred and would not let go any part of it for some other blessings in life, we would still not have identified the worst-off person in the society. This is because self-respect is not a homogeneous good. Its units are vaguely defined, if they are defined at all. Different people may have different ideas as to what self-respect is, and depending on those ideas, they may have very divergent opinions as to which is the worst-off position in the society. Rawls has tried to get rid of this problem by introducing what he called the 'bases' of self-respect, <21> which are supposed to be common to all individuals. Whether these bases of self-respect really exist is now immaterial because as soon as Rawls starts to invoke the word 'bases' in its plural form, the same old aggregation problem will confront us anew. Obviously, Rawls is trapped in a vicious circle, and he will never be able to get out of it if he keeps on insisting not to go beyond ordinal comparability.

Recently there has been a rejuvenation of Rawls's difference principle. With much mathematical sophistication, that principle has been successfully axiomatized by several economists. <22> It turned out that the difference principle is closely related to the set of axioms originally set out by

Arrow. In fact, with the addition of an extra axiom, the Arrovian system will entail the difference principle. This proof was first propounded by Strasnick, and it soon attracted much attention from the profession.

The extra axiom is usually in the form of an equity axiom, which states that the will of the worst-off person should not be blocked in every social decision. Having read Arrow's impossibility theorem, one should not be shocked by the newly established result. Actually, we can say that Arrow's theorem is bound to lead to Strasnick, and one can say that the whole profession has really overreacted, considering the amount of homage paid to the latter.

We have seen that the Rawlsian maximin satisfies (i) and (iii) of Arrow's Axioms.^{<23>} Moreover, it also satisfies a weak version of the Paretian axiom. Hence all signs are indicating that there is a dictator in the Rawlsian paradigm. In fact there is one. Rawls's maximin principle clearly identifies the worst-off person as the dictator. The contribution of Strasnick and others rests on supplying the missing link between the Arrovian and the Rawlsian system, that is to turn Arrow's anonymous dictator into Rawls's proletariat with the equity axiom. The idea behind the transformation is mundane; what it amounts to is that if there is a dictator, and if we let the worst-off person be the dictator, then of course the worst-off person will be the dictator. Naturally, the actual proof of the theorem is more difficult than this. The complication, however, comes mainly from people trying to reiterate Arrow's result. To see how

this works, we will assume Arrow's three axioms. With the equity axiom, the worst-off person will be decisive for at least a pair of alternatives. As it is well known, under the axiom of independence of irrelevant alternatives, an individual who is decisive for one pair of alternatives will be decisive for all pairs of alternatives. Hence the worst-off person will be the dictator. Rawls's difference principle is of no exception to Arrow's impossibility result.

It is possible that we might have missed something somewhere down the line because our view is being contradicted by Arrow. Arrow has overtly professed in his article, cited above, <24> that there is no dictator in the Rawlsian system. His whole line of argument was carried through by the introduction of a new axiom called 'anonymity'. <25> This anonymity axiom is supposed to be stronger than the non-dictatorship axiom and hence entails this axiom. Consequently, any social welfare function which is dictatorial will, by 'definition', <26> not be anonymous. Since it has been proven by Strassnick and others that the maximin principle satisfies the anonymity axiom, it therefore follows that there will not be a dictator in the Rawlsian system.

The crux of the whole exercise now rests on what we mean by a dictator. If a dictator is identified by his name, e.g., Nero, then Rawls' maximin principle will in fact be anonymous as well as non-dictatorial. Assume Nero is the worst-off individual in the society and his preference over-rides the preferences of all others in the society. Now, if we perform

a permutation on the preferences of the individuals as well as on their positions, <27> a person i will pick up Nero's preference and his position, and become the worst-off individual in the society. Since he has the same preference as Nero before, and occupies the worst-off position, he will replace Nero and become the new decision maker in the society even though social decision will be exactly the same as before. Hence a permutation on people's preferences cum positions will not change social decision, and Rawls's maximin principle is anonymous in this limited sense. There is no dictator in the system because by dictator we mean a person whom we can only identify by name. Nero is not a dictator because once we have performed the permutation, the person whose name is Nero will cease to be the decision maker in the society.

It appears that given this restrictive definition of dictatorship, the second part of Arrow's assertion is correct. Nonetheless, the two parts of that assertion will rise and fall as a whole, and there is no need for us to concede Arrow unless there is sufficient evidence to believe that his original formulation of the social choice problem is inconsistent with the anonymity axiom.

When we look back to the general possibility theorem, there does not appear to be any ground for supporting Arrow's assertion. What he has proved is the possibility of the existence of an individual, who can be any individual, whose preference will always be the same as the social preference. Since this person can be any one, a permutation on

individuals' preferences will leave the original social ordering of the alternative states unchanged. The so-called dictator in the general possibility theorem is an extremely weak dictator. It can very well happen by chance that his preference is the same as the social preference. In that case he will be a dictator not by choice but by coincidence.

We do not understand why Arrow could claim that his theorem is incompatible with anonymity. A possible explanation may be traced back to the concept of the smallest decisive set.^{<28>} Very briefly, V is a subset of the set of all the members in the society, and it will be decisive over x and y if, and only if, for all $i \in V$, $xR_i y \Rightarrow xRy$. V will be the smallest decisive set over x and y if there does not exist any other decisive set over x and y such that it has fewer members than V .

Until now, this concept of decisive set is still perfectly general. Members of a decisive set need not be replaceable, i.e., it is not only the number of elements in a decisive set that counts. We may have both A and B being decisive over x and y where $A = \{\text{John, Mary}\}$, $B = \{\text{Joe, John, Sam}\}$ and Mary is irreplaceable by either Joe or Sam. In this case, A would be the smallest decisive set. If we can prove that Mary is the dictator, then a permutation on everyone's preference will alter social preference because Mary is irreplaceable in A , and since Mary will have a different preference after the permutation, she would naturally want to adjust the social preference according to her newly acquired tastes.

We are not sure whether this is what Arrow had in mind when he claimed that his theorem violates the anonymity axiom, but it is the closest we can think of to explain why he could have made such a claim. However, when we look back to his proof, we definitely could not find the kind of strong dictator that Mary represents. It seems to us that the members of the smallest decisive set are interchangeable among themselves, and the only reason why one particular individual and not another could become the dictator is his preference and not his name. Take any member of the decisive set and exchange his preference with that of the dictator, and this member will become the new dictator of the society. Names certainly do not matter within the decisive set.

In order to generalize this result to include the other members of the society, we must point to the fact that Arrow was a bit hasty in his proof. After he had illustrated that there is a person whose preference will be decisive for the pair of alternatives x, z ,^{<29>} he should not have concluded that this individual is the desired dictator. This is because Arrow's proof is a proof by contradiction, and what he had contradicted is what he had assumed in the beginning, i.e., V is the smallest decisive set. To prove that there exists a dictator in the society, we must invoke induction and prove that always the set $(V - 1)$ is not the smallest decisive set unless V contains only one element.

We are not being meticulous. The reason why we want to take this route is that we want to make sure that anonymity applies not only within V . If we follow Arrow, all we know is

that names do not matter within V , and yet we are not sure if we can switch the preferences between the dictator and someone outside V and still leave social preference unchanged. On the other hand, the proof by induction will confirm that anonymity holds for all members of S . When we are conducting the induction, member after member of V will be expelled from V , and all of these members are eligible to be the dictator because they were in the set V before. After being expelled, these ex-members of V will have exactly the same characteristics as any one who is not in V , and if we switch the preference of one of these ex-members of V with that of someone in the complement of V , social preference should remain unchanged. In other words, anyone in the society could have been in V or equivalently, anonymity holds for all members in S . The problem can be put in another way. We could have started with $V = S$ and then arrived at a dictator through induction. In this way, anonymity will be more clearly perceived since each and every member of S will be a direct candidate for the role of dictator.

We must admit that we are still somewhat sceptical about the above conclusion because it just does not seem possible that an authority as respectable as Arrow could have overlooked such an obvious point. However, in the absence of any reasoning to the contrary, we will tentatively hold onto our deduction until someone can convince us that we are mistaken. That is to say, we will reject Arrow's claim that Rawls' maximin principle is not subject to his general possibility theorem. Moreover, we will consider ordinal

comparability as a non-starter for the problem of social choice.

Ordinal Non-Comparability

If traditional wisdom can be of any guide, we would probably want to spare ourselves the trouble going into ordinal non-comparability altogether. This is because the informational basis of ordinal comparability entails that of non-comparability, and since we have not been able to come up with anything substantial in the former, investigations into the latter will most likely prove to be, to say the least, equally futile.

However, this is not the experience of recent years. There has been a growing interest in the possibility of using ordinal non-comparability to render the Paretian ordering more complete. In particular, the concept of 'fairness' has been widely discussed, or shall we say, abused. The basic idea behind this so-called 'fairness' approach dates back to thousands of years and yet we owe its present day inspiration to Foley.<30> Essentially, it is a way of apprehending other's feeling by imagining oneself to be in another person's position, or as some would prefer to put it -- stepping into another person's shoes. Due to this characteristic of sharing other people's feeling, the approach is also known as the extended sympathy or the empathy approach.

As documented in the literature, there are at least two ways of stepping into another person's shoes. The first is analogous to total involvement; it has won the endorsement from Harsanyi^{<31>} amongst others. By total involvement, we mean that empathy does not stop at just imagining oneself taking up another person's social position, or his properties etc., but to acquire the other person's taste and character as well, or in other words, to be the other person himself. If such ability exists, and if it is common to every individual,^{<32>} then the problem of interpersonal comparison would have been solved. There will be no need to make interpersonal comparison any more since there is only one person or more correctly, one character. The differences in people's preferences will only be the results of differences in pre-conditions. The other person is really not the other person but the same person with a different name and situation. Ranking of social positions will be handled by this abstract human through intrapersonal comparison. Of course, we do not want to dismiss this deterministic approach off-hand, and to a great extent we do believe that human beings are intrinsically equal. However, we would be deluding ourselves if we profess that total empathy exists. Its absence is caused by many factors; among them are, selfishness and physical differences.

The second kind of extended sympathy is by far the more typical.^{<33>} It is self-centred. Instead of imagining himself to be another person, our individual will try to evaluate other individuals' consumption bundles from the

standpoint of his own preference. The evaluation will generally take the form of $x_i R_i y_j$, i.e., individual i would prefer his bundle in x to individual j 's bundle in y and R_i is just the ordinary preference relation over commodity bundles for individual i . Obviously, there is no interpersonal comparison. R_i is just a personal preference relation. The only difference between R_i and the ordinary preference relation is that the former has a wider domain, and it enables our individual to compare not only his own prospective bundles but the prospective bundles of other individuals as well. The enhanced ability is allegedly important. For one thing, it is supposed to supplement the Paretian ordering and make the resulting social ordering less porous. The trick to attain that end is to establish a 'non-envy' criterion.

To begin with, we will introduce some definitions. Individual i is said to be envying individual j in situation x if $x_j R_i x_i$. A situation that is freed from envious individuals is defined as 'equitable,' and an 'equitable situation which is also efficient in the Paretian sense will be called 'fair.' The equity relation so formulated will be independent of the Paretian relation, and it is capable of rank ordering alternative social states. But just as the Paretian relation, this ordering may also be incomplete. The relation says that situation A should be preferred to situation B if there exists envious individuals in B and none in A , and yet the definition has no indication at all as to how to deal with cases where there are two or more equitable

or fair allocations. Nor does it tell us what to do when we are confronted with two or more inequitable allocations. When confronted with the latter, should we rank them according to the number of envious individuals as some people have suggested?<34> To do that would imply that we are treating one envious person as the same as another envious person. Naturally, there is no a priori reason to justify such a scheme. For those who are willing to endorse this view, it is questionable why they would not accept the idea of giving each individual's preference an equal priority in the social welfare function in the first place.

Besides being incomplete, there is another intolerable drawback in the fairness approach -- intolerable even to the ones who originally proposed the idea. For "it is now known that even under classical convexity and selfishness assumptions on the economic environment, allocations that are both (equitable) and Pareto efficient will not always exist in economies with production."<35>

The reason for this is intuitively simple. Let there be two individuals i and j in the economy. We will assume that j is less productive than i , while i desires more leisure than j . The consumption bundles of the two individuals will include their commodity bundles as well as their leisure time. To be efficient, society must ask i to work more and let him enjoy the lion's share of the total product. However, that might cause j to envy i . On the other hand, i might also envy j for j 's leisure. Swapping position between the two will not solve the problem because that will decrease

total output -- a violation of the efficiency requirement.<36>

Since the Paretian axiom is the corner stone of the ordinalist's study, anything in its way will have to go. There is no exception even for equity. It did not however take our ordinalist long to find a replacement, and in fact there are quite a few of them. We will selectively discuss some of these proposals.

Firstly, there is the wealth-fair criterion suggested by Varian.<37> An allocation is said to be 'wealth-fair' if there does not exist in the economy any individual who envies the consumption *cum* production bundle of another individual. It has been proved that wealth-fair allocations are consistent with the Paretian condition and that they always exist.<38> Yet existence is not everything, and we cannot approve an ethical judgement on its existence and on its compatibility with the Paretian condition alone. We need something more, something that is at least ethically palatable to most of us.

Does wealth-fair allocation in general possess such agreeable characteristics? We believe that they do not, and it would be a sham if we pretend that they do, just because they fulfill some vacuous symmetric requirement that one may feel compelled to accept at first sight.

What the wealth-fair definition says is something like this: if you like to consume what another person is consuming, you have to produce what he is producing, and if you cannot produce what he is producing, you'd better stop

envying him because your envy will not be legitimate under the definition.

One does not need much insight to notice that wealth-fair allocations are there to justify the market solution. This is because, theoretically at least, free movements of goods and resources are guaranteed in a market economy. Let us at first set aside the issue of innate talents and bequests. Under such condition, if a person envies another person's social position, i.e., his composite bundle of consumption cum production, he is always free to move to that position. The fact that he has not done so implies that he prefers his own position to the other position. Of course, this does not exclude the possibility that our individual lacks the required productive ability demanded by the other position, and for that he has only himself to blame as he was supposedly free to acquire that ability earlier on in the game. Apparently, the symmetry in this case is the symmetry of opportunities. However, if we allow innate talents and bequests to play their parts, the symmetry will degenerate even further into the symmetric right to compete -- something that is trivially fulfilled in a competitive system.

Prior to the introduction of this wealth-fair criterion, the Paretian condition was about the only ethical condition that the competitive equilibrium could satisfy, and even that honour it has to share with innumerable other alternative situations. If it could claim any superiority over the others, it would have to come passively from it being the

status quo. Now the perspective has changed; instead of being the reluctant champion of the non-chosen, the competitive solution can come out of the closet and boost itself to be the righteous heir to fairness as well as the defender of equity. It seems that once a man has acquired wealth, fame will prove to be of equal importance.

Apart from being ethically repugnant, there is also a possibility that the wealth-fair criterion may be inconsistent. As we know, the most important contribution of the fairness approach is the extra ranking power made possible by the 'non-envy' judgements. Situations that are without envy are considered superior to those with envy. If there were mutual envy, a permutation of the social positions will be recommended to eliminate them, and that supposedly will bring the society to a preferred situation. However, that may not always work and the only case we are sure that it will work is the one where the preferences of all the individuals are identical.

When a person envies another person's position, of course he would like to move to that position. However, such desires are often formed under the assumption that someone like himself will occupy his original position. Naturally, that cannot be the case as our individual is unique in his own right. He has his own consumption pattern and his own work-leisure tradeoffs. The newcomer might have taken up our individual's position on the condition that he will accept the same commodity bundle and put up the same hours of work. But that does not exclude him from making substitutions. For

the same hours of work, it is very likely that he will produce something that is very different from what our individual used to produce. The problem cannot be solved by demanding that the newcomer duplicates the production pattern of our individuals as that is equivalent to treating the two individual as the same person. Since no one in this world could exactly duplicate the work of another person, if one insists in doing so, he would only turn the already vacuous definition of fairness into something meaningless.

As for consumption, the substitution will be even more obvious. In an exchange economy, our new comer can always trade his consumption bundle for another bundle, and in fact he will do so because, by elementary trade theory, he will always be better off trading his bundle if his preference is different from that of the person who previously occupied that position.

Coming back to our individual, he will most likely make the same kind of substitutions when he acquires his new position and that will in turn ruin someone else's expectation. One who believes in rational expectation may want to suggest that before swapping their positions, the participants of the swap should have taken fully into account the possible substitutions that might arise and their decision to move on to another position should also depend on these expected substitutions. This is nothing but a fairly tale. In an n -person economy, one may know which position one wants to move to, but one may not have the slightest clue who is moving in to his position. To speculate that he might

know what kind of substitutions this unknown person will make is hilarious. Just imagine how large each individual's informational basis would have to be before he can make such predictions. With that kind of information, one would be better off making interpersonal comparisons. Hence there is a potential inconsistency here; people who try to eliminate their envy under the wealth-fair definition may find themselves in the same agonizing state when all the swappings have been conducted.

When one considers the inconsistency, one just cannot help realizing the similarity between this cyclic phenomenon of the wealth-fair judgement and the Scitovsky Paradox. We will not attempt to set up a 'double-criterion' to salvage the wealth-fair approach, but those who are sympathetic to the idea are most welcome to do so, though it is our fear that their efforts will some how be in vain as a revisit to the compensation muddle will most vividly prove. The core of our argument is this: like the relationship between distribution and production, there simply does not exist something called constant social environment. As the social actors exchange their roles, the social environment will change with them. To theorize upon a fixed social environment in the face of all these swapping activities is just like maintaining absolute elements in a field that is governed by the theory of relativity. The contradiction is inevitable.

The second Pareto compatible fairness judgement that we are going to consider is the 'income-fair' criterion.<39> To

be income-fair, an allocation must satisfy the condition that there does not exist any individual in the society who envies the income of another individual. Here income is defined as the monetary value of his consumption bundle plus the implicit value of his leisure evaluated at his particular wage rate. Since both the price vector for commodities and the wage rates are competitively determined, the resulting allocation has to be efficient in the Paretian sense. If we allot each member of the society an equal share of this income, the resulting allocation will be free of envy and thus income-fair. Furthermore, this will be the only income-fair allocation available because if we choose some other allocation there will always be some getting more, and others getting less, than the equal share. Since the market is functioning, more of this 'income' will automatically mean more of both goods and leisure, and there are all the reasons in the world for those who are getting less to envy those who are getting more.

This income-fair criterion is definitely much more 'egalitarian' than the previous wealth-fair criterion. Nevertheless, there are still inequalities, and those who are productive are in some sense penalized. Due to their abilities, the wage rates of the productive members are higher than those of the others, and this would lead to higher price tags on their leisure. Since no one would be interested in other people's leisure, the same income for everyone would therefore mean less leisure time for the able.

Most people, and the conservatives in particular, would

think that this scheme is inoperative. There is no incentive for people to work. Anyone with average intelligence will realize the fact that by understating his productivity, he will be rewarded with more of his own leisure time. But would such kind of behaviour imply that the productive workers are actually envying the positions of the less productive workers despite the equality of their 'income?' And if that is the case, why should it happen? The reason is simple enough. It is certainly true that our present conception of fairness involves the notion of symmetry and 'non-envy' is probably the most conspicuous form of its manifestation. However, that does not mean that any kind of symmetric distribution will necessarily lead to 'non-envy' situations. There are things that we regard as ends and others as means to those ends. 'Income,' being an abstract artifact invented by the human mind, is necessarily a means to an end. By equally distributing income, there is no guarantee that its corresponding end will also be evenly distributed. In our case, there are still people envying others for their leisure time. Nor is the latter an end in itself, at least not in economics. Leisure is useful only if it enables our individual to achieve a higher level of satisfaction. The picture is finally clear. If we want to use 'non-envy' as an ethical criterion, we must, to say the least, take into consideration all the factors that contribute to the well being of an individual. But would that include also consumption and production? Apparently, the vicious circle is still with us, and the problem would

not be solved by considering 'income' alone.

Two More Proposals on Fairness

There is a common feature in both of these definitions and that is the abusive use of the notion of symmetry. Firstly, there is the idea of 'fair trade' developed by Schmeidler and Vind.^{<40>} A net trade is 'fair' "if, for any agent, his net trade is at least as good as the net trade of any other agent."^{<41>} The symmetry is set on the opportunity to trade, or to put it simply -- the price vector facing the individuals. That means that as long as there is a uniform price vector in the market, trading in that market will be fair. Evidently, that would include the case where there exists one or more parties who want to take advantage of the trading opportunities but lack the resources to carry out their intentions. In the market, only demands which are backed up by abilities to engage in actual transactions are considered effective; all others are irrelevant.

The trouble with this criterion is its total reliance on something which is often asymmetrically determined. For it does not matter what kind of economy you are in, competitive or otherwise, the price vector is determined partly by the initial endowments of the participants. When those initial endowments are asymmetric, so will the resulting trading opportunities be, notwithstanding the existence of a uniform price vector across the agents. A bias is established when,

at the existing prices, an individual who wants to be a buyer (seller) is deterred because of the limitations in his initial endowment. Fairness is construed as equal dollar power for everyone, while it ignores the critical fact that some people may not have any dollars at all.

The second proposal is even more absurd. It was introduced by Pazner and Schmeidler.^{<42>} As defined by them, "an allocation is said to be egalitarian-equivalent if there exists a fixed commodity bundle (the same for each agent) that is considered by each agent to be indifferent to the bundle that he actually gets in the allocation under consideration."^{<43>} An egalitarian-equivalent allocation may not be as good nor as commendable as an egalitarian allocation, but the intention to associate the two is undeniably there.

The egalitarian bundle which is being 'used as the yardstick' can be any hypothetical bundle, and its actual existence is of no consequence. The major problem with this egalitarian-equivalent relation is that it does not provide us with a non-trivial choice set. We will use a two by two exchange economy to illustrate our point. Furthermore, we will ignore the uninteresting case of identical preference. Let the two individuals be i, j ; the two goods be A and B , and the economy initially at situation X . Without any loss of generality, we will assume that when compared with j , i has a relative preference for A over B . Imagine another hypothetical situation Y where the total amount of goods are distributed equally between i and j . From Y we will have

$u_i(Y)$ and $u_j(Y)$. If $u_i(Y) = u_i(X)$ and $u_j(Y) = u_j(X)$, X will be an egalitarian-equivalent allocation. If not, there are four possibilities: (i) $u_i(Y) > u_i(X)$ and $u_j(Y) > u_j(X)$; (ii) $u_i(Y) < u_i(X)$ and $u_j(Y) < u_j(X)$; (iii) $u_i(Y) > u_i(X)$ and $u_j(Y) < u_j(X)$ and (iv) $u_i(Y) < u_i(X)$ and $u_j(Y) > u_j(X)$.

In case (i), X is obviously not egalitarian-equivalent to Y . However, that would not prevent X from being egalitarian-equivalent to some other allocation, and that allocation should have less of both goods than Y . To find that allocation, we would have to adjust Y downwards, and with the assumption that the commodity space is continuous, it is certain that we will eventually reach an allocation that belongs to either one of (ii), (iii) and (iv).

Since (ii) is the converse of (i), it cannot possibly have included any allocation that is egalitarian-equivalent to X . So let us adjust Y to another allocation Z as described under (iii). By definition, Z is still not egalitarian-equivalent to X , as individual i prefers Z to X and individual j , X to Z . If we can still recall, among the two commodities, individual i prefers A to B when compared with individual j . Now we have a situation where we want to lower the level of satisfaction for i and raise that for j , and the most appropriate thing to do is to change the proportion of A and B such that there will be more of B and less of A in Z . Since we have only two inequalities to deal with in (iii) and we have at hand two instruments, viz., the overall size of Z and the proportion of A and B in Z , by Tinbergen's rule, we can always turn the two inequalities

into equalities. Hence there will always exist an egalitarian-equivalent allocation Z for X in (iii). As (iv) is similar to (iii), we will not bother to go over the whole exercise again. Thus, under all circumstances, every allocation will also be egalitarian-equivalent at the same time.

A Second Best Solution

It should be obvious by now that we are not particularly keen on using the idea of 'non-envy' as a guide to social decisions.<44> In our opinion, the most severe drawback of this approach is its inability to account for the differences in felicity capacities in people due to its non-commitment to interpersonal comparison. Nevertheless, we do realize that this line of ethical thinking is deeply rooted in people's mind, and that it may take a while before it can be completely eradicated. In the meantime, we will endeavour to supply a more egalitarian oriented version of 'non-envy' judgement.

We must admit that our proposal is in no way original; with little difficulty, one will be able to discern its resemblance to Ricardo's concept of rent. To avoid repetition and to ensure that our proposal is Pareto-compatible, we will follow the lead of the 'wealth-fair' criterion and define a social position as the composite bundle of production *cum* consumption. Moreover, we will also

adopt their 'non-envy' definition until we find it apt to discard it. As always, for every situation, there are three possibilities: (i) there exists mutual envy among some individuals; (ii) there exists one way envy among some individuals and (iii) there does not exist any envy. The idea behind our proposal is to extract the surpluses from those who do not envy others and then distribute them among the members of the society. Our first task is to make the society 'envy-free,' and that is easily accomplished in case (i). For if we are permitted to ignore our own objection on all the possible cyclic behaviours that might arise, then all we have to do is to let those people who envy each other switch their positions, and envy will be eliminated.

When it comes to case (ii), there is a little complication, but it is nonetheless manageable. We will assume that there are only two individuals A and B in the society with A envying B's position. Since we do not know whether A will envy his own position if he were in B's position, the best way to find out is to let them exchange their position as in case (i). If after the exchange, envy ceases from both sides, then the newly created situation will be the envy-free situation we are looking for. If not, individual A's original position must be the better position of the two. With this information in hand, we will now make an attempt to 'equalize' the two positions. One can do this in different ways, and we will choose to lower in equal proportion the overall composite bundle of consumption goods cum leisure of the better position. To make things less

clumsy, we will from now on simply call this composite bundle 'income.' Since the preferences of A and B are not identical, and 'income' is a continuous variable, we can always find a level of 'income' for A's position such that one and only one of the two individuals will prefer B's position to that of A's. Let this individual be A himself, and we will now assign him back to his old position. Only this time a certain amount of 'income' will be taken away from him. Since A prefers his old position even after the reduction in 'income,' we have therefore no reason to worry that he will envy B. On the other hand, A is the only one who prefers his old position, so we are assured that B will not envy A. Consequently, we have again eliminated envy in the society. For the time being, we will act as trustees to oversee the 'income' we have extracted from position A.

Case (iii) is even easier than case (i). There is simply no envy by definition. Hence we have demonstrated that under all circumstances, it is always possible to turn a society into an envy-free society. However, this kind of envy-free society may be extremely oppressive at times. For as productivity differs among members of the society, one individual is not entitled to envy another individual if he cannot produce what the other is producing even though there may exist a great disparity between their 'income.' To rectify this potential injustice, we suggest that we should redefine 'non-envy' so that it can approximate to what people have in mind when they invoke that term. From now on, when we say that a person A is not envying another person B, we

mean that if we assume A to be able to produce what B is producing, A would still not want to exchange his position with B.

To arrive at a 'non-envy' society we will start with a 'wealth-fair' allocation. For any individual, we will assume that he is multi-talented and capable of performing all jobs equally well. In addition, we will ask him to identify a position in the society which he regards as the worst position. To guard against false representation, we can always reveal this worst position in an indirect manner. What we have to do is to lower, in a continuous fashion, our individual's 'income' level from his original position. On top of that, we will ask our individual if there is any position in the society he would like to be in rather than his own depleted position. As the process continues, it is likely that our individual will reveal position after position which he would like to be in rather than his own position. Since R defines a well-ordered set, there must exist at least an element among all the social positions such that the amount of 'income' reduced from our individual's original position is the greatest before he would consider that position to be indifferent to his own position. For our purpose, that position is the worst position in the opinion of our individual and that amount of 'income' is the surplus he is enjoying by holding his original position instead of the position which he considers as the worst.

Due to differences in tastes, people may have in mind very different ideas of what the worst position is, and as a

result, each of them must be enjoying a certain amount of extra 'income' in excess of what one would have enjoyed if one were relocated to the position which one regards as the worst. In the meantime, we will extract all these extra 'income' or surpluses from the individuals and pool them into a fund together with those 'income' which is under our custody. This fund of 'income' is to be distributed back to the individuals later.

After being deprived of their surpluses, it is certain that all members of the society will feel worse-off than before. Many and especially those free enterprise economists have argued that this feeling of deprivation may some how work its way through people's incentives and affect productivity. We think that this line of argument is nonsensical. For productivity will only be affected if there are changes in (i) social positions and/or (ii) work efforts. But in our redistribution scheme, neither one will be affected.

Social positions will not change because all that we have extracted from the agents are their surpluses -- the amount of 'income' they could do without. An agent will not fare any better if he were to demand reallocation. In the new position, his surplus will be extracted likewise. Only in that position which he considers as the worst would he be exempted from such extraction. In any case, he will not do any better than his present position. To ensure that the *status quo* is maintained, it is always possible to keep an individual in his original position with an infinitesimal

premium. If our scheme works, and there is no reason why it should not, we will be able to dismiss the usual outcry against redistribution, for they will no longer be able to justify the excessive income and profits paid to certain social positions as incentives to induce those who are holding those positions not to leave their jobs, and to lure those who have potentials to exercise all their potentials to acquire those positions. For as long as an infinitesimal premium exists, there will be sufficient motivation for people to get into what they think will be the best for them. Anything on top of that will be superfluous.

The question with regard to work effort is even more easy to handle. As we have learned from introductory sociology, there are positive and negative sanctions. Throughout the history of capitalism, the two kinds of sanctions are distinctly applied to different positions. Entrepreneurs, professionals, etc., are often encouraged with positive sanctions, while the mass of the productive force is deterred from slackness by negative sanctions. Just imagine what is there to motivate those who are stuck in deadend jobs to work hard besides the fear of losing their jobs. One may argue that there were positive sanctions at the time when they were still in school. That, however, is besides the point. As a channel of inducement, sanctions must not be a once and for all measure, and as long as someone is still in the productive force, there is a need for sanctions to continue. Of course, the society does continue to apply sanctions on our individuals even though most of them come in

the negative form and if our observations have not deceived us, these sanctions surely do work.

There is no doubt that by extracting all the surpluses from every position, we have taken away most, if not all, the positive sanctions from the society. Nevertheless, that does not mean that less effort will be exerted by our individuals when they are carrying out their jobs; appropriate negative sanctions such as job security, further 'income' reduction and reallocations have all been proven effective in their fight against job apathy. Hence we should not be deceived by the fallacy that redistribution will only lead to inefficiency.

Having set aside the question of productivity, we will now go back to examine what kind of a mess we have turned our society into! If we compare it with a 'wealth-fair' society, it is certainly the poorer of the two. However, it is also more egalitarian since a big chunk of the surplus income of the latter society has now been eliminated. The result is a more egalitarian society that satisfies our new 'non-envy' definition. People are treated, irrespective of their talents, skills and aspirations, equally in the sense that they are all given what they themselves regard as the worst outcome. Even though they have been assumed to have the abilities to perform the tasks of other positions, people would still prefer to stick with their own lots. It is certainly true that inequality will still persist, but that is something we cannot eliminate under the existing structure of the model. Considerations with regard to differences in

tastes and preferences can only be taken into account if we are willing to make interpersonal comparisons. In the absence of that, we do sincerely believe that our proposition is a rather accurate reflection of what people have in mind when they talk about 'non-envy' situations.

Having arrived at a 'non-envy' situation, our next question is how to deal with the surplus that we have appropriated from the people? Naturally we have to return it to the people, but in what proportion? We cannot make our decision based on people's preferences because we have no information concerning differences among preferences. We cannot distribute according to some redress scheme due to the fact that everyone in the society is equally deprived. In the face of inadequate information and indiscernible differences, it seems that the best and the only thing for us to do is to distribute the loot in such a way that the new situation will not violate our 'non-envy' condition. Given this objective, we have no other choice but to distribute the loot equally among all members of the society. Since if we try any other reallocations, some social positions will end up getting more than other positions, and since we have to begin with a situation where everyone in the society is indifferent to all social positions, an uneven distribution of this kind will for certain re-introduce envy into the society.

Some may feel that our argument is circular and question begging. Why would we want to go through all these troubles if we are willing to adopt equal distribution? Why would we

not do it in the first place? To these sceptics we must say that equal distribution was not what we have originally intended, and it has come in only as a convenient by-product of our quest to fulfill our 'non-envy' definition within the empathy framework. If we accept the latter and ignore all possible complementarity effects, equal distribution of the loot will come as the only logical consequence.

Like all criteria in this 'non-envy' category, our proposed ordering is also incomplete for the same reasons that we have discussed before. However, the existence of a solution is guaranteed under all circumstances as we can always trace out a local maximum by the method outlined above. Furthermore, the solution, being a 'wealth-fair' solution in the first place, will also be Pareto compatible. And if desired, we can still allow the agents to engage in recontracting activities even after the redistribution so that a trade equilibrium can be established to satisfy the free traders. All in all, we think that our criterion is the more justifiable and more equitable criterion among the whole bunch of 'non-envy' criteria and we would certainly stand behind its policy recommendations as long as we are prohibited from making interpersonal comparisons. However, why should we opt for a second best solution when we have not found any valid reason to rule out the best yet? And it is to that best we will now give our attention.

Unit Comparability

By unit comparability we mean that each individual i 's utility function u_i is equivalent up to the following form of transformation: $u_i' = a_i + b(u_i)$, where a_i is independent of j for all $j \neq i$, and b is the same for all i and j . Given this explicit definition, our approval or disapproval of unit comparability will depend solely on whether we are willing to accept the restrictions it has imposed on the coefficients a_i and b . Note that apart from the defunct notion of partial comparability, unit comparability is the ranking scheme that demands the least information and yet provides us with a complete ordering. Hence, from the point of view of economics, we should look no further if we are satisfied with the way unit comparability is ordering the social situations.

Of the two restrictions, we have little argument with the uniformity of the b_i 's, and we will postpone its discussion until we have come to ratio-scale comparability. In our opinion, this delay will not in any way prejudice unit comparability. Our eventual rejection of this form of comparability will be based on our uneasiness with the other restriction.

To be consistent and complete, it is mandatory for a social welfare function not to have any undefined values in its domain. Specifically, none of the utility functions that go into the social welfare function should in any case take on an infinite value. However, as there are no restrictions imposed on the a_i 's, it is perfectly possible for some or all

of them to take on undefined values. If we insist on having a complete ordering of all the social states under unit comparability, we must somehow prevent the a_i 's from entering the domain of the social welfare function. The easiest, and also the most popular, way is to make the social ordering a function of utility differences. For any individual i , a factor a_i will be subtracted from the amount of utility that he is going to derive from any situation. This procedure is equivalent to moving the origin of the utility space along the y -axis. The resulting utility index will then be rid of its incomparable component a_i , and it will render interpersonal comparison a straightforward business. Very often a_i is taken to be individual i 's utility level with respect to the *status quo*, and if that is the case, the social welfare function will be a marginal evaluating scheme with the *status quo* being the origin.

How could we on the one hand not allow ourselves to compare the utility levels of different individuals and then on the other casually accept to appraise the marginal utilities of the same group of people? According to our earlier analysis on Lange and others, the identification of marginal utilities necessarily implies the recognition of utility levels, and one would, if there is any difference between the two at all, expect it to be easier to make interpersonal comparison of utility levels than utility differences.

That, however, is not the opinion of those who support unit comparability, and amongst them, Little had probably

given the most pictorial and explicit justification for their position. In order not to do him any injustice, we will let Little speak for himself:

the mind is like a well of unknown depth, partly filled with water.... and also it is impossible to say how much water there is in the well. One cannot therefore significantly ask how much economic welfare someone has; but one can say that the level of water has risen or fallen as the result of turning the economic tap....one cannot add up the total amount of water in a number of different wells if one cannot say how much water there is in any well. What, however, is not always noticed in that the utilitarians had no need to add up different people's happiness in order to prove their thesis....(the theory) requires only that one should be able to compare the difference in happiness which an extra pound would make to different people. If one, then, moves money about until a pound more or less makes the same difference to everyone, one has maximized happiness. The addition of happiness is never required. The addition of differences in happiness may, however, be necessary.<45>

The refusal to make interpersonal comparison of utility levels is extremely costly. To say the least, it will deprive us of the ability to make ethical judgements apart from the kind of unethical ethical judgement that is made by the primitive utilitarians. For if we could compare only utility differences, we would have no idea whether individual A or individual B is enjoying a higher satisfaction level than the other. Lacking this information, we will not be in a position to apply any differential treatment to the marginal utilities of the two individuals, and equal treatment is the only alternative and that suggests the method of simple summation.

Apparently, this gloomy prospect did little to dampen Little's enthusiasm for equity as he was still trying

prodigiously to justify egalitarianism even after he had revealed his endorsement for unit comparability. In his opinion, egalitarianism will come through if the hypothesis of diminishing marginal utility is observed by everyone in the society, for if that is the case, the marginal utility of an extra dollar for those who are getting more than the equal share must be less than the marginal utility of the same dollar for those who are getting less than the equal share. To maximize total utility in a society like this will call for equal share of income for each individual.

There are two fallacies in the above rationalization. Little recognized one, but not the other. He recognized that the marginal utility may not fall monotonically for everyone. Little belittled that possibility as an exception and maintained that on the whole the hypothesis of diminishing marginal utility is a good approximation of observed facts.<46> There is no need to argue with Little on this point since his conclusion hinges on both of the fallacies. The successful refutation of one is sufficient to reject his overall assertion.

The second fallacy is the one which eluded Little's attention, and in our view, it is the more critical of the two. To put it succinctly, this fallacy is nothing more than the supposition that people's preferences are identical, for if that is not true, the maximization of total utility in the society will not necessarily call for an equal distribution of income even if the hypothesis of diminishing marginal utility is accepted. This is so because we cannot rule out

the possibility that different individuals may have very diverging absorption capacities. If the privileged class happens to be a very sensitive group of individuals, it is conceivable that they may be more able to appreciate the benefit of an extra dollar of income than the underprivileged despite the monotonic decreasing nature of both groups' utility functions. Since we do not intend to concede this supposition (and if we do we would not have come this far), it seems proper to conclude that unit comparability is incapable of accommodating any ethical judgement and it is ethically immoral for us to use it as a tool for such purposes.

After tackling the moral issue, we will now return to the logical side of Little's example. Without much scrutiny, one will notice that there is an inconsistency in Little's simile. How can one claim that there is no way to find out how much water is there in the well or equivalently, how deep is the well while maintaining that we can measure the amount of water we pour in or fetch out of the well? One will never find out how much water is in the well if one keeps on adding water into that well. However, Little's allegation is intended to go both ways, i.e., we also can measure how much water each time we draw out from the well. If we are persistent enough, there seems to be no reason why we could not exhaust all the water of a well. Even if one argues that some parts of the well may not be accessible to us, it is still possible for us to get an approximation, and very often a close approximation, of the depth of the well as well as

the actual amount of water in it. In any case, both of these quantities will be finitely measurable due to the fact that both the amount of water on earth and the diameter of the earth are finite magnitudes. However, we must admit that our argument is not conclusive in the absolute sense, and there is still ground for a sceptic to come around and make a final challenge. Predictably, he would condemn our choice of origin. The earth after all, is only an inconsequential planet in the universe from the cosmological standpoint, and there appears to be no particular logical nor a priori reason to consider our well to have its origin here. We could actually have insisted that the bottom of our well is somewhere remote in the universe, and what we are seeing is just the surface of the well. Since our present conception of the universe is still that it is infinite in size and since the well can start anywhere in the universe, it is of course legitimate for someone to claim that we cannot measure the depth of the well. However, we hope most of us would agree that it is not this kind of general nihilism that we are looking for. One can argue as much as one wants as how far away the origin of the well may be, and yet it is of no concern to us. As earthlings, we are only interested in the depth of the well as it is measured on earth. For all our purposes, that measure has fared sufficiently well. On the other hand, we do not need to negate the possibility of an absolute measurement with a different origin other than the one we are using. But until we discover that, we will hold on to what we have at present. This approach to measurement

is not only methodologically correct, it also happens to be the same approach on which all our knowledge rests. As for people as knowledgeable as Little we are sure that they have not the slightest intention to give up all their knowledge in exchange for a bottomless well. Even if they do, their sceptical position will prevent them from telling whether the object they are seeing is a well or not since later revelations may well prove that to be a mirage.

Our prime interest naturally does not rest in Little's well per se. Rather we are concerned with what it stands for, and if the simile between the economic agent and the well carries through, i.e., if the economic agent cannot claim that he is deriving an infinite amount of utility from any of the situations, then the use of unit comparability will prove to be inappropriate. Some people may have the impression that we are being too hawkish in our rebuttal of Little's analogy. According to them, there does not seem to be any apparent need for us to venture into outer space to make our point across. That view is deceptive because the simile between the well and the economic agent will be incomplete if we do not take ourselves beyond this planet. To see why this is the case, we must draw attention to those economists who argue that the amount of utility which an individual is getting is always above infinity. In their view, life is extremely valuable to an individual, and the mere fact that he is surviving will confer on him an infinite amount of utility. Those who are unconvinced have only to ask themselves what they are willing to give up to avoid

certain death. Since we will no longer be able to appreciate anything else once our lives are taken away from us, the only rational answer for a self-centred economic agent to the above question must then be 'anything,' or even anything that he can imagine to have. By the non-satiation axiom, the latter will give an infinite amount of utility to our individual, and since he prefers life to everything else he can imagine to have, he must be getting at least an infinite amount of utility from being barely alive.<47> If one accepts the above reasoning, one will also have to approve unit comparability. Every individual's utility level will then start with infinity, and there is no way we can compare their utility levels as we have learned that depth on earth is finite, it seems that our trip to outer space is after all not conducted for its sheer extravagance.

There are now two issues awaiting to be resolved. Firstly, there is the question of whether we should endorse the infinity thesis, and secondly, if we have chosen to endorse it, how much weight should we attach to it. We will take up the second issue first, and here is where our expedition to the universe will fit in.

Essentially, our conclusion on the well will apply equally to the individual, that is to say, we will regard the infinity thesis as irrelevant. When dealing with the well, we did not dismiss outright the legitimacy of measuring it from another reference point. Nonetheless, when that reference point is so far out that it defies all forms of measurement and at the same time it is common to all other

wells, we think it would be best if we leave it out of our picture. Apparently, those who advocate the infinity thesis also share the same feeling. As summed up by Georgescu-Roegen, these "economists felt, however faintly, that man's economic life is like that of a navigator on high seas, for whom the depth of the ocean, whether infinite or finite, is immaterial." And "to circumvent the ensuing analytical difficulties (of the infinity issue); it has been suggested that we 'take life for granted' and measure utility above the level of subsistence."<48>

One should not however, construe this remark as an approval of unit comparability. Two distinctions must be made. In the first place, we must recognize that the *status quo* is not the same as the level of subsistence, and secondly, we must distinguish between what cannot be known from what we do not care to know. By accepting the infinity thesis, we have conceded that there may be levels of utility which we cannot know. However, the emphasis is not on our not knowing. In effect, we do not care if we know it or not. Moreover, this unknown factor is definitely not attributable to the *status quo* and it would be incorrect to treat it as such. If we take life as given and adjust our measuring rod accordingly, levels of utility will become measurable. There is nothing marginal about this analysis once we have disregarded the infinity issue. Of course, one can still insist on the marginal tenet as Lange's second postulate always implies his first postulate. But then, every analysis of utility will be a marginal one and the argument will

become meaningless.

We will now try to make an effort to tackle our first question -- the question of whether we should accept the infinity thesis. To our mind, this is by far the more important issue of the two. Before we come to any conclusions, we want to forewarn the readers that these conclusions are not going to be decisive. It is not that we do not have faith in our own reasoning. Instead, it is the controversial nature of the concept of infinity that is at stake. As long as the philosophers and mathematicians are still disagreeing on the existence of infinity, there is simply no way we can give a definite verdict on the matter. In what follows, we will try to make the best use of what we have along the orthodox line and deduce some sensible speculations therefrom.

The first point we want to make is a refutation of the argument that the mere fact of existence will give a human being an infinite amount of utility. It may be provocative to some, but from the social point of view, we must say that we have never seriously entertained this idea. Social decisions are made everyday concerning human lives. In many cases, simplistic cost-benefit analyses are applied, and lives are attached certain weights by planners. However, it has never occurred to anyone that we should regard life as giving rise to an infinite amount of utility to an individual. If we do, we will no longer be in a position to recommend any major policy, as almost all of them involve the inevitable loss of lives at the macro level. Take for

example the building of a highway. Assume that if we do not build the highway, people would have to keep on travelling slowly but safely on donkey backs. From statistics, we know that some people will surely get killed on the highway. So the policy maker is confronted with a decision of lives versus conveniences. History has proven which of the two society values more. One can say whatever he wants, but it is simply not true that social decision makers treat human lives, or at least other people's lives, as invaluable entities.

Convincing as it may be, the above argument is insufficient to disprove the infinity thesis. This is so because economic agents are non-altruistic animals and the fact that we do not treat other people's lives as invaluable entities does not prevent us from deriving an infinite amount of utility from our own lives. After all, social decision makers are entitled to make mistakes. To dismiss even that possibility, we will have to go back and re-examine the truism of the assertion on bare existence.

It is undoubtedly true that a rational economic agent will give up anything in exchange for his life. However, it would be logically erroneous if we construe this kind of behaviour as a sufficient proof of the infinity thesis. There are two possible explanations why an individual would behave in this way. Firstly, he may feel there is nothing worth appreciating in life, and in which case one really does not care to give away everything. Secondly, he may feel that there is something worth appreciating in life, and in order

to be able to appreciate that something, one must stay alive. If one loses one's life, one will automatically lose everything, and that is why one is willing to give up everything for one's life. In that sense, life is a necessary condition, a prerequisite to any enjoyment. Nevertheless, it does not follow that life will always give rise to an infinite amount of utility though it does not exclude that possibility either. In what follows, an attempt will be made to give a more direct refutation of the infinity thesis.

Consider the case where an individual is making a decision concerning future events. He will be confronted with a probability distribution of outcomes one of which may mean the loss of his life. If our agent really considers life as giving rise to an infinite amount of utility, he should always avoid choosing the action that may lead to his eternal rest. This reasoning is valid irrespective of how small the probability of death may be since any mathematical operation such as the expected utility rule on infinity will again produce infinity. Those who still have reservations on the legitimacy of our agreement have only to observe the identical structure of our logic and that of the St. Petersburg's Paradox. Since the operation on infinity has been widely accepted as valid in the latter, there appears to be no ground to bar us from using the same line of reasoning in the utility context.

We will now move on to see whether this implication of the infinity thesis fits human behaviour. We will again use

the example of the highway. As we have mentioned before, travelling on the highway will, for certain, involve some fatalities when the society is taken as a whole. For each individual traveller, that would be translated into some positive probability of getting killed. If the utility of survival is infinite as claimed, it would be nonsensical for anyone to risk his life by travelling on the highway. Yet, it seems that few of us are willing to go to work on donkey backs or stay home just because we risk premature death. The kind of people we have in mind here includes even the most risk adverse conservatives. As for the more spirited minds, active actions are often undertaken to incur risks. It is not uncommon that situations are created such that the chance of survival may be even smaller than its complement. Nonetheless, we cannot consider these people as fanatics. In most cases, they are just as consistent as we are, and their decisions are by no means arrived at without premeditation. We are thus left with only two alternatives. Either we recant our commitment to the orthodox view with regard to the concept of infinity, or we will have to admit that the utility of life is always finitely measurable. As indicated before, we have chosen to adhere to the second option. Finally, we want to point out that the decision to choose the first alternative will still not enable us to say that life gives rise to an infinite amount of utility. This is because by so doing we will have to explain to people what we mean by infinity in the first place!

What we have established so far is the idea that mere

existence by itself will not bring forth an infinite amount of utility. That conclusion, however, is insufficient to dismiss the infinity thesis altogether, as it still leaves open the possibility for our economic agent to derive an infinite amount of utility through consumption.<49> Following the traditional terminology, we will call such a state of mind 'bliss.'⁵⁰ Due to differences in absorption capacities, it is in general not necessary for the agent who experiences bliss to be the richest person in the society. However, he will most likely be a sorry man after that experience since from thereafter he can only go downhill on his utility scale. In fact, we really wonder if there is any meaning in life after bliss. In the event when bliss can be sustained by means of constant stimuli, the individual concerned will probably be so much uplifted that he will cease to share the mundane feelings of the rest of the society. In either case, we are extremely sceptical about the relevance of the whole idea of bliss as it stands in the context of social decisions.

Our scepticism will of course be superfluous if we can show that there does not exist a state of mind called 'bliss.' In the following, we will try to produce an exceedingly fragile argument to substantiate that view. Before we do that, we want to point out to our readers that we are not the only ones who do not believe that utility can reach the level of infinity. Among those who share our belief is Ramsey, and this is how he expresses his opinion on the issue:

On the other hand, the rate of enjoyment may even stop increasing as capital increases. There are then two logical possibilities: either the rate of enjoyment will increase to infinity, or it will approach asymptotically to a certain finite limit. The first of these we shall dismiss on the ground that economic causes alone could never give us more than a certain finite rate of enjoyment.<51>

Those of the readers who are religiously inclined should have no problem at all in comprehending our assertion against the infinity thesis. For them, the ultimate and eternal enjoyment is supposed to come after life. Whether they know what heaven is, or whether they will like what they see up there, is immaterial to our discussion. As long as they claim that there is something more fulfilling than every experience in life, it will be contradictory for them to consider the latter as giving rise to an infinite amount of utility. This is so because in order for any experience to give rise to an infinite amount of utility, the foremost necessary condition is that it must not be surpassed by any other actual or conceivable experience in utility level. Since the acceptance of heaven runs counter to that condition, our readers with a religious bent will have to be contented with finite rates of enjoyment in this world.

Having dealt with the faithfuls, we will now turn our consideration to the infidels among us. We will have to use another way to find out whether they have experienced or are capable of experiencing utility levels that are infinite in magnitude. It occurs to us that the method of direct investigation of people's own introspection is most appropriate for that purpose.

One can easily make up a few questions that will bring forth answers which are capable of provoking approval or rejection of the infinity thesis. For example, we can ask the subject which occasion he would judge as the happiest moment of his life. After he answered that, we can then go on to ask him if he would consider that experience as unsurpassable by any later experience. If not, that experience could not possibly be the bliss we are looking for. Hence, we will assume the answer to be yes and we will call that experience x . Whether this x is really the bliss we are after will now depend on the correctness of our individual's response. To check that, all we have to do is to see if there is any situation that our subject would prefer to x . By the conventional non-satiation axiom, our individual will prefer y to x if materialistically y has more of everything than x . Most subjects will likely yield to this axiom. However there are exceptions. The ability to appreciate materialistic improvements is often constrained by the time factor, and it is not unconceivable that people may be indifferent to increases in material wealth. For these people, we will have to device some other questions in order to test if x is truly their blissful experience.

In consumer theory, when the budget is the constraint, the consumer will usually move onto a preferred position if the constraint is somewhat relaxed. Essentially we face the same problem here, only this time the constraint is the time factor. It should therefore follow that our individual will feel happier if we can relax the time constraint on him.<52>

Needless to say, it is not possible for us to give more actual time to our individual. However, that is not at all necessary. The whole exercise can be conducted on a hypothethical level. What we will do is to create a situation y where in y our individual will have everything he has in x , and on top of that, ten more youthful and non-repetitive years to live. One must realize that this y is structurally no different from the fictitious apples and oranges in an introductory economic textbook. If we are willing to say that the utility of two apples is greater than that of three oranges simply because the consumer hypothetically chooses the former, we should also be willing to admit that the utility of y is greater than that of x if our individual prefers y to x . Since life is finite, there will always exist a situation y such that for any individual who desires to live longer, y is unattainable during his life time but is preferred to x by our individual, and this contradicts our assumption that x is the bliss point for our individual. In addition to that, if each isolated experience in one's life time only gives rise to an finite amount of utility, it is extremely likely that utility derived from the whole of this person's life will also be finite. Our speculation is based on the fact that the summation, the multiplication or any other mathematical manipulations of a set of finite quantities will again produce a finite quantity. Undoubtly, it is also possible for there to exist a mysterious kind of complementary effect, so that the prospective utility of the experiences of a life time may be

infinite while the utility of each isolated experience is finite. We do not intend to go into that question any further, though it appears that the same argument against the existence of bliss given above will again apply. Moreover, we do not need to prove that to reject unit comparability. So long as we can show that a_1 is finitely measurable, our task will have been accomplished, and that means for people to concede that there does not exist any single situation which gives them an infinite amount of utility.

Although it is our belief that the above argument is cogent, we must also admit there is a catch in it which makes it not quite as decisive as we would like it to be. Since we are basically dealing with static situations, all consumptions are meant to be instantaneous. Such a restriction does not constitute much difficulty in the case of apples and oranges; our individual simply has to eat faster. Any difference between theory and practice can be attributed to statistical errors. Time, on the other hand, is in a different category. One simply cannot be in one time period while living in another time period. So even if our individual does prefer ten more years to live, the amount of utility which he is expecting to get from that ten more years will not, by temporal order, be realized in the same period. Given this argument, it is therefore conceivable that an individual may be experiencing 'bliss' and at the same time would like to lead a longer life.

In order to solve this difficulty we will have to find another way to put that ten extra years into our individual's

present utility function. Consider for a moment a man who is trying to match the number on his lottery ticket with the winners' numbers on the newspaper, and to his surprise, his number matches one of the winner's numbers and he is the winner of a million dollars. At this moment, he has not yet received the million dollars, and still less has he consumed any of the prospective goods which he could buy with that million dollars. However, the man is jumping up and down with joy and with tears in his eyes and is shouting, "I am very, very happy." If you ask him whether he is now happier than a moment ago, we can assure you that the answer will be in the affirmative. Then, with all these evidence, what can we say about the man's utility level before and after he has matched his lottery ticket? If external behaviours can be of any guide and if the man's answer is to be taken seriously, then there is no alternative but to conclude that the man is at a higher utility level after he has learned about his sudden fortune.

Coming back to our original question, what will happen to our individual if he were told that he will have ten more years to live? Since it is his desire to live longer, he must be delighted with the prospect, in a similar fashion as the man who won the lottery, though may be with much less intensity. However, intensity does count in this case, and as long as there is an increase in the utility level of our individual after he has learned about his longer life span, we will be able to say that he was not enjoying 'bliss' in his original position. Of course we are fully aware of what

we are postulating. One can call these behaviours as 'illusions' if one wants to, but that is definitely the way how people react. Notice that the policy implications here are immense. All those debates on the neutrality of money will now be unnecessary. Not only are monetary policies non-neutral, but all those policies which subject people to the impression that their well beings will be affected in the future will also be non-neutral. There is definitely a more active role for the government in such an environment.

Turning to the empirical side, we must say that we are quite encouraged by our findings. Among the several people we have sampled, none has indicated any support for the infinity thesis. They either said outright that they have never had such an experience, or came up with a similar answer once we started questioning them. In the main, we feel that there is not enough evidence for us to regard unit comparability as the interpersonal comparison scheme that accurately represents the true preference structures of our individuals.

Cardinal Non-Comparability

Our arguments against unit comparability are equally applicable to the case of cardinal non-comparability, as the latter is fully entailed by the former. In order for a social choice function to be cardinally non-comparable on the one hand and yield a complete set of decisions on the other,

there are two requirements that it must fulfill. Firstly, the two constants a_i and b_i in each individual's cardinal utility function must be free to vary, and secondly, the rulings of the social choice function must be independent of the choice of these constants.

The bargaining solution proposed by Nash^{<53>} is one of such social choice functions. Basically, that solution suggests that we should maximize the product of each individual's marginal utility with respect to some *status quo* as the reference point. As in the case of unit comparability, the *status quo* is there to ensure that the a_i 's are excluded from exerting any influence on what is eventually decided by the society. The ingenuity of Nash's solution lies, however, in maximizing specifically the product of each individual's marginal utility. For only by the maximization of that product could we have possibly made the choice of unit, or equivalently the b_i 's, irrelevant in the social decision making process. This is so because when we are maximizing the product, a change of unit for any individual's utility function will have the same proportional effect on all situations, and that will leave the original ordering among them intact.

There is, however, a price to be paid when we use the product instead of the sum as the criterion for social decisions. When we were discussing unit comparability, an additive social welfare function was used. As a result, the a_i 's were allowed to take on any value. Furthermore, the choice of the *status quo* is inconsequential. For

convenience's sake, we have chosen $[a_1, a_2, \dots a_n]$ as the *status quo* but it could have been any other situation as long as we can eliminate the alleged unknown factors in the a_i 's. In Nash's bargaining solution, the *status quo* is most crucial. Due to the multiplicative form of that solution, a change in the choice of the *status quo* is capable of producing a completely different set of results on the social orderings.<54>

This may sound a little confusing to some, since by definition there can be one and only one *status quo* and it is absolutely absurd to say that we can choose the *status quo* at will. Obviously, we could not have taken the phrase in its literal meaning. Instead what we mean by the *status quo* is just a reference situation. It can be any situation, provided that the social decision so derived is devoid of any influence from the a_i 's.

In our opinion, this interpretation of the *status quo* is more appropriate when look at from the ethical point of view. There is no doubt that social decisions are not made in a vacuum, and it is perfectly normal to expect a state of the world to be affected by the state which immediately precedes it. In that sense, there seems to be nothing wrong with the dependence of Nash's solution on the *status quo*, and there is apparently one and only one *status quo* for any situation. On the other hand, we must not forget that the *status quo* is nothing but one of the possible situations, and it should not receive any special treatment when we are making ethical judgements for a society. Perhaps an example will clarify our

point.

Consider the dispute between the Arabs and the Jews. Both parties are claiming a legitimate right to the same piece of land, and each party has a different opinion as to which situation is the *status quo*. For the Jews the *status quo* can be either the present time or the time of the old testament. As for the Arabs, the *status quo* may mean the era before the Six Days War or the time prior to the Second World War, or even the period which preceded British rule. Depending on which *status quo* one accepts, the eventual settlement will be different.

By now one should realize that sometimes it is not a matter of fact that some state is the *bona fide status quo*, and secondly, a solution which hinges on the choice of the *status quo* is not in general ethically agreeable. In our example, there is no doubt that the choice of any one of the suggested *status quo* will not only infuriate the other party, but it will also not gain the general approval of the present world opinion. We do not want to deny that our ethical judgements are conditioned and constantly changing. Nonetheless, that is not a valid reason for social decisions to change with the choice of the *status quo*. At any particular time, there can be only one set of opinion, and in order for that set of opinion to become an ethical judgement, it must possess certain ingredients of durability and impartiality. To our mind, one such ingredient is the need to make every participant feel that he or she is not being prejudiced against. That does not mean that no one will be

hurt in the process, nor does it require a general consensus to approve every social action. Rather it will serve the purpose if we can make sure that the decisions arrived at by the society are the product of the sincere representation of people's opinions when they act according to their moral consciousness. Since there can be one and only one set of sincere representation of people's moral consciousness at any one time, there can be one and only one set of social decisions that is compatible with people's ethical judgements. Evidently Nash's solution, and for that matter, all other cardinal non-comparability solutions, have failed to provide us with this ingredient of impartiality, and it would be a mistake to treat them as anything more than technical possibilities for rank ordering social alternatives.<55>

Ratio-Scale Comparability

We have, hitherto, practically dismissed every possible form of interpersonal comparison, and yet our attitude is not completely negative. We do want to lend our support to full comparability and it is our belief that this is the only form of interpersonal comparison that will enable us to construct an acceptable social welfare function which does not disregard relevant information. In particular, we think that there is sufficient reason to go further and settle for the even stronger form of ratio-scale comparability.

We will have full comparability whenever $a_i = a_j$ and $b_i = b_j$ for all i and j in the society. On top of that, ratio-scale comparability requires $a_i = a_j = 0$. At first glance, there does not seem to be much difference between the two forms of comparability besides the fact that the latter being more restrictive. On the theoretical level, both will perform the same function and produce the same social ordering. However, our preference for ratio-scale comparability, rests on its feasibility.

When making full cardinal comparability between two individuals, there must be at least two common points on each individual's utility function. If we can capture one of these points, our problem will be half solved. It will be discussed later that in our opinion, there is one such point common to all individual's utility functions which deserves to be called the zero point.

Of course we can transform this zero to some other value and still obtain the same social ordering although that is obviously a redundant procedure. Why should we bother to perform an extra transformation if the original representation is sufficient to accomplish the task. Some may want to argue that there are other common points in people's utility functions, and that any one of these points may as well serve as the origin and thus the need for ratio-scale comparability will be nullified. We must admit that we have contemplated that possibility for a long time, but up till now, these other points are still eluding us. And even if we can find them, it is still our belief that they cannot

replace the zero point which we are going to introduce.

There have been many suggestions for ratio-scale comparability. Among them is the threshold school initiated by Armstrong, who in turn was inspired by the method of psychoanalysis in psychology.<56> The essence of this school's position is its emphasis on the inability of human perceptions to discern infinitesimal differences in external stimuli; its application to welfare economics is most succinctly expressed by an axiom propounded by Armstrong:

Any individual welfare variation, sufficient to give rise to a preference, causes a greater variation of group-welfare than any individual welfare variation, sufficient only to give to indifference.<57>

When we were dealing with the issue of the existence of cardinal utility, we had rejected the threshold approach on the ground that different bare preferences of the same person need not amount to the same basic unit on that person's cardinal utility scale. Without a doubt, the above axiom is stronger than anything that Armstrong had proposed so far. This is because in addition to implying what we had rejected before, the new axiom also asserts that bare preferences of different individuals should be regarded as equal in the social welfare function. Obviously, this assertion cannot be factually based. Variations in people's perceptive abilities are well documented, and it would be a gross error to insist that they are similar. As a result, if we ever decide to accept this axiom, it would be because of its ethical content. In that respect, we are again not at all optimistic. It may be true that there is a certain kind of anonymity

implied in the axiom as bare preferences of different individuals are equally treated. Nonetheless, that is nothing more than the simple minded Benthamite solution, and it is certainly not self-evident that our society would go along with it. Moreover, we also do not see why an individual's welfare variation which gives rise to preference will always cause a greater variation in group welfare than an individual's welfare variation which can only give rise to indifference. If that is the case, how would we compare two or more variations of indifferences with that of a single variation that leads to preference? It is highly suggestive from the axiom that we should always consider the latter before the former. Instead of leaving us to ponder in vain, Armstrong was kind enough to confirm our speculation with another of his axiom:

If, in a group, there is a variation of the situation of one individual, within the limits of his indifference, then the variation of group-welfare is trivial.<58>

In approving the above axiom, Armstrong has put himself in a rather awkward position. The first thing he should realize is that he can no longer claim shelter under the Benthamite school. The latter, being a simple summation rule, will definitely not allow the kind of non-cancellation property inherent in the axiom. Secondly, the axiom also contradicts the main feat of Armstrong's whole argument, i.e., the intransitivity of the indifference relation. If for an individual, bits of indifference variations can eventually lead to a preference variation, then there is no reason why

we cannot apply the same logic to social preferences. After all, if the contrary is adopted, it would be a severe setback to the egalitarian movement. In a mass society, the benefits of a few are often built on the miseries of the many. According to the axiom, such exploitations will increase social welfare as long as those who are being exploited are unaware of their own deprivation while those who take advantage of others can prove that they are in a preferred position. Of course, the whole process can be repeated *ad infinitum*, and acute social injustices will easily become justifiable.

What Armstrong proposed was to extend the bare preference theory from the level of the individual preference to that of social preference. He did not, as we might have misled the reader to believe, go so far as to suggest ratio-scale comparability. That idea in threshold theories came only recently.

In a rather interesting paper, Waldner recommended that we should fix an interpersonal zero point by the following condition:

$$\begin{aligned} u_i(x) = u_i(-x) &\Rightarrow d_i(x) = d_i(-x) = 0 \\ u_j(x) = u_j(-x) &\Rightarrow d_j(x) = d_j(-x) = 0 \quad <59> \end{aligned}$$

As usual, u_i and u_j are the utility functions of individuals i and j and d_i , d_j are just their normalized counterparts. What needs explaining here is the meaning of the negation of x . A critic of Waldner^{<60>} pointed out that $(-x)$ can mean nothing other than the non-occurrence of x . And if that is accepted, Waldner's proposition would be

equivalent to identifying the status quo as the zero point for all individuals. Needless to say, this cannot be a very satisfactory solution as far as social welfare is concerned. Since by assuming "all individuals to be at the same zero point (in the status quo) denies the fact that some people are happier than others, or that anyone changes his satisfaction level."<61>

It appears that Waldner's attempt to revive the bare preference approach is something less than a success. His failures are two-fold. In the first place, he should not have chosen to work within a framework with so much proven conceptual difficulties as the bare preference analysis, and secondly, his idea of the zero point is exceedingly unconvincing. On the other hand, we must confess that we do believe that his effort with regard to trying to find a common zero point for everyone is a step in the right direction.

Waldner is not alone in this respect. To our knowledge, there are at least two other suggestions of the same nature. The first one was advanced by von Wright.<62> After realizing the underlying difficulties in the explication of $(-x)$, he was quick to turn to Moore for inspiration. According to his interpretation of Moore, "an 'empty' world, 'the existence of nothing at all', was necessarily a state of affairs of zero-value."<63> However, this definition is incomplete as it stands since it still has to specify what exactly an 'empty' world is.

Being a logician, it is predictable that von Wright

would seek a logical solution to the problem. In his own words:

Among the truth-functions of the members of S (a finite set of n logically independent states of affairs), one is the tautology, another the contradiction. The first is a disjunction of all the possible total states, the second a disjunction of none of them. For this zero-termed disjunction we could introduce a special symbol 0 . It represents a logically impossible world, a world of which nothing is true and nothing is false.<64>

The first thing that comes to our mind is the difficulty in perceiving such a contradicting state. Presumably, this zero level of utility is present in everyone's utility function, and yet we simply cannot see how people who have not been exposed to elementary logic can comprehend such a fictitious construct. We cannot, on the other hand, deny these people the right to determine their own utility functions. Unlike the English law system which states that ignorance of the Law is no excuse for anyone who breaks it, we do think that ignorance of the contradicting state is a reason valid enough to grant immunity to anyone from possessing the alleged zero level of utility, and it would be an imposition if we insist otherwise.

For those who have some exposure to logic, we must remind them that von Wright has not given us a direct definition of this contradicting state. What we were given instead is just the negation of a state that is always true. To derive the contradicting state from this tautological state, we must apply the law of the excluded middle. If one rejects this law, the negation of the tautological state will not necessarily lead to contradiction and emptiness. Without

that feeling of emptiness, the whole of von Wright's argument for the zero point will collapse. Since that zero point is supposed to be common in everyone's utility function, it will be sufficient to have it refuted if there exists one person in the society who fails to recognize the law of the excluded middle. In other words, we need only find one intuitionist!<65>

Von Wright's original intention in introducing the zero utility level was to differentiate between what is good and what is bad. Accordingly, a possible world is good if, and only if, it gives rise to a positive level of utility, and something is bad if, and only if, from it emanates a negative level of utility. The contradicting state is there to serve as a watershed. In our opinion, this way of differentiation is extremely counter intuitive. For those of us who dare pretend to have some idea of what a contradicting state might be, we are sure that we would still prefer many things in this world which we would consider as bad to the arrival of this contradicting state. For example, an egalitarian would certainly condemn as bad a state where the distribution of income is extremely uneven, though if you ask him to choose between this inegalitarian state and the contradicting state, it is almost certain that he will opt for the former. Good and bad are ethical terms, and many things that are not ethically appealing and thus bad are still tolerable both physically and mentally. In contrast, the contradicting state is totally foreign to us. If there is anything we have learnt about this state, it must be the feeling of annihilation. To

most of us, this is just too radical an attitude, and that is probably why we would prefer a bad state to this contradicting state.

It was claimed by von Wright that there "would be no logical hindrance to ranking 0 higher than any of the possible worlds. This would correspond to an attitude of extreme pessimism or gloom: every possible world is a bad one. To the reverse ranking would correspond an extremely cheerful and optimistic attitude which regards all possible worlds as good ones. An attitude of complete apathy and indifference would correspond to a ranking of all possible worlds equal with 0."<66>

However, if we agree that people do rank some bad states before the contradicting state, the above allegation will have no practical relevance. At the same time, we would have some vague idea concerning the range of an individual's utility function. For one thing, the utilities of some states must be positive.

Even though we do not agree with von Wright completely, we must say that we have learned a lot from him. His idea of associating zero utility with emptiness is most refreshing, and because of his construction, we are also able to make certain deduction of the whereabouts of this zero utility point. All these will prove to be very useful when we try to put the puzzle together. In the meantime, let us take a brief look at another one of such suggestions.

This one was propounded by Wagner<67> in defense of Nash's solution. Wagner realized that the *status quo* is

crucial in determining the solution to the Nash problem, and if it can be manipulated by the players in the game, the whole of Nash's exercise will lose its significance. As a result, Wagner was eager to propose a *status quo* that will withstand such manipulation. The state below which life cannot be sustained was chosen for this purpose, and it was conveniently identified as the state with zero utility. It was claimed that due to the low level of economic activity, a society in such a state will collapse and everyone in it will perish if any one of its member refuses to co-operate. Hence unanimous consent is necessary for any social action, and players' manipulation of the *status quo* will become impossible.

Wagner's proposal is different from that of von Wright's on two counts. First, it is more humane. Unlike the contradicting state, the state at which life cannot be sustained is not a logical artifact. It is and will always be an immediate concern to our human existence. Even though most of us may not have a first hand experience of this state, we do have a fairly general idea of the kind of predicaments that are involved in it. In the perspective of choice theory, the decision makers are supposed to have knowledge of the consequences of their own actions. While that condition is satisfied by the subsistence state, most of us do not have a clue about the contradicting state. It is just too foreign to us. As a result, if we have to choose between the two according to choice theory criteria, we would prefer Wagner's suggestion.

The second difference between the two proposals pertains to the preciseness of the definition of emptiness. As we have noted, von Wright was quite correct in recognizing the close relationship between emptiness and the level of zero utility, and the way he defined this empty state is exact and precise. We may object to the use of the contradicting state as the zero point for some other reasons, but there is no doubt that there can be only one such state. In contrast, Wagner's state at which life cannot be sustained is only vaguely defined. We do not know what exactly was in his mind when he introduced the concept of bare existence. Was it supposed to mean subsistence? Since we cannot think of a better answer, we will assume this to be the case. Although subsistence is a familiar concept, it nevertheless does not clearly identify any particular state to us. As we have learned in economics, the meaning of subsistence can vary from individual to individual, as well as over time, and given the state of confusion over the term, it can actually mean anything. Even if we try to be more sympathetic and put a further restriction on bare existence to make it to mean physical subsistence, the problem of identifying a unique state will remain all the same. Different people may subsist physically under quite different environments, and it is obvious that Wagner's definition as it now stands is incomplete, and it will fail to pick out uniquely one single state and assign that state as the absolute zero on every individual's utility scale.

Although we have rejected both von Wright and Wagner's

arguments for the zero point, we must acknowledge that they have provided us with invaluable insights into the matter and have narrowed down the scope of our search for the zero point considerably. Basically, we would agree with von Wright that the state to which we could assign zero units of utility should be unique and possess the notion of general emptiness. On the other hand, we do share Wagner's view that we should seek such a state in something that is closer to human experience than in some logical construct. Hence, what we are looking for is a unique state of the world that is available to each and every individual in their alternative set, and it will offer them the emptiness and only the emptiness as demanded by Moore. Before we go on to identify this particular state of the world, we would like to add some justifications for the relation between emptiness and the 'zero' on an utility scale. We feel that Moore's argument is inadequate and amounts to nothing more than a pure assertion.

The first reason for advocating emptiness as the common zero point is a pragmatic one. Recall that if we want to make meaningful interpersonal comparisons, full cardinal comparability with $a_i = a_j$; $b_i = b_j$ for all i, j is sufficient, and there is no need to fix the origin of people's utility functions at any particular point. However, there is a drawback if we allow people to change the origin of their utility functions at will as it will be much harder, at least from the practical point of view, to find a common interval among individuals' utility functions to make the necessary crossover from one person's utility functions to

another person's utility function. The matter will be made much easier if there is a common zero among all individuals. Since each individual is entitled to his own preference, it does not follow logically that there will always exist some commodity to which each individual will assign the same amount of utility. In the absence of such a commodity, it is hard to imagine how anyone could say the utility A gets from an apple is the same as the utility B gets from an orange.

Theoretically speaking therefore, we need only $a_i = a_j$ and not $a_i = a_j = 0$, but it seems to be much easier to find $a_i = 0$ and then equate $a_i = 0 = a_j$ than to find $a_i = a_j$ directly, and it is partly because of our inability to produce the direct equality between a_i and a_j that we have in the past been vulnerable to the attack of the non-comparison school, thus failing to convert the profession *en masse* to our conviction.

The more fundamental reason for our choice of 'nothingness' as the absolute zero is a conceptual one; it is closely related to the process of counting in mathematics. Supposedly, if our individual's utility functions are of the cardinal type, they will be equivalent up to any linear transformation, and the choice of origin is arbitrary. In other words; we can denote any commodity bundle as zero, and it will work equally well as any other bundle! The idea is that zero is just like any other number, and it will make no difference to replace it by another number. What is involved is nothing more than a simple mapping.

Under certain circumstances, this rule may not strictly

apply. Take the example of counting pencils. We can count them by each unit so that we have one pencil, two pencils, and so on. Or if we like, we can also call twelve pencils a dozen pencils. However, we usually would not call twelve pencils no pencils. The renaming of twelve pencils a dozen pencils is pertained to a change of unit whereas it will need a change of origin before we can call twelve pencils no pencils. The same can be said in the case of speed. A man can be walking at one mile per hour or two miles per hour, and so on. Similarly, we can also use kilometers or any other units to denote how fast he is traveling. Nonetheless, most of us would be reluctant to say that a man is motionless when he is obviously walking.

Admittedly, there are exceptions to our rationalization, and the most well known one to an economist is perhaps the concept of the natural rate of unemployment. Depending on the definition, one can easily call 3 or 4 percent of unemployment zero percent of natural unemployment. Naturally, Friedman had a much higher figure in mind. But then, he is perfectly entitled to his opinion. What makes the natural rate of unemployment different from our two previous examples is that, despite its claim, there simply does not exist a natural zero for the former; that is why we can almost choose any rate of unemployment and call it the natural rate.

With all these elementary counting lessons, what we are trying to get at is the existence of a zero in an individual's utility function. In our opinion, the representation of a person's preference by a utility function

is more akin to the case of counting pencils than to the natural rate of unemployment. Utility is a measure of satisfaction level and satisfaction (or dissatisfaction) can only be the result of experiences, physical or otherwise. When an individual encounters an agreeable experience, his satisfaction level will rise, and we will be obliged to represent it by a higher utility level. Likewise, when the same individual encounters an unpleasant experience, we will have to adjust his utility level downwards accordingly. In either case, however, our individual will be experiencing something, and as in the case of counting pencils, we think it would be inappropriate to represent any of these substantial experiences by the value zero on his utility scale.

The problem is a fundamental one. Essentially, what we are asking is the meaning of zero. In order to find that out we must trace our way back to the number system, since it is there where all other zero units derive their common property from. We know that in order to construct the number system, we need to assert the existence of zero. However, to assert something is not the same as explaining what something is, and it was not until recently that an attempt was made by von-Neumann to explain what zero is.<68>

We do not claim that von Neumann's explanation is completely satisfactory. After all, there is so much we can reduce and something is bound to be taken as basic. The point is, however, if what has been taken as basic in the explanation of zero by von Neumann is generally accepted as

reasonable and self-evident, then we would consider him to have scored. Apparently, von Neumann's number system has fulfilled those requirements as it has been recognized as one of the most reputable number systems ever axiomatized.

The whole of von Neumann's system is based upon the definition of zero. Taking the empty set as irreducible, he defines zero as the set that contains only the empty set, and then he builds the rest of the numbers inductively upon zero and the empty set. One of the reasons why he has chosen to define zero in this way is probably that there is nothing to start with. But more likely, his decision is affected by our general conception of the number zero. Without begging the question too much, we think that whenever we utilize the number zero, we are trying to denote the non-existence of something. Since in mathematics we are dealing with universally applicable abstracts, the zero there must denote none other than the non-existence of everything. To fill that vancancy, it seems to be exceedingly difficult to find a more qualified candidate than the set which contains only the empty set.

When we say one pencil, two men and so on, the units used have their own significance. Together with the numbers which precede them, each one of them will convey something different to us. However, that is not the case with zero. When you have zero unit of something, it will make no difference whether that something is pencil or banana. In all cases, zero will convey the same sense of nothingness. The unit is significant to the extent that it will suggest what

one might have. Hence when one has no pencil, one might still have some bananas and or some apples, and so on. However, as far as what one does not have is concerned, i.e., the complement of what one has, zero pencil is the same as zero utility and the same as the zero in the von Neumann number system. The latter is special in the sense that it has no unit and therefore its complement can take on any units. As a result, it is entitled to claim total emptiness. In contrast, those zeroes which have units attached to them will inherit the emptiness property from the one without unit and can only claim partial emptiness. In either case, zero will have the same meaning, and that is emptiness. We hope most of us are satisfied with the arguments given above, and hence accept the relationship between zero and emptiness. For those who are still unconvinced, we must remind them that what they are questioning is not our proposition in particular, but the whole of the number theory in general, and we think they would agree that this is an extremely costly route to take.

Wagner's definition of bare existence has come close to locating this empty state. However, it has not gone far enough. If we push the concept of bare existence to the limit, we may perhaps interpret it as purporting death or the state that just precedes death. Since death does confer on us a sense of nought in this world, it may seem possible, at least tentatively, that we may use Wagner's state of bare existence as the common zero.

The matter, however, is not quite as simple as it might have appeared to be because the idea of death is not that

simple. There is not one state by which we can unambiguously recognized as death. After all, people do die in different ways, and we would certainly agree that the agony which Bobby Sand had gone through must be drastically different from what Charles II had felt during his execution. Hence, if we were given two ways to die and if we assume we have decided to represent one of the ways by zero on the utility scale, we would then be excluded from representing the other by the same zero, given the fact that we are not indifferent between the two. It is therefore not true that death will always result in emptiness. To our minds, that is an *ex post* argument. There is another objection which we may raise against Wagner's definition. Since we are basically concerned with individual choice theory here, the decision of our individual on any state should therefore be the result of his own 'free' choice. In most societies, the state of bare existence will not satisfy this condition. People do not usually choose bare existence, rather it is something that is forced onto them.

The Empty State

If we agree that death does offer us that emptiness property, then the next step is to find among different death states a state that will offer us nothing other than emptiness.

Among all the alternative states, there seems to be one

that fits exactly our description, and that is the state of painless suicide. Very often, society has regarded suicide as a kind of sickness, and people who try to commit it are considered irrational and abnormal. If this view is correct, it will certainly be a mistake to let suicide be our common zero point. This is so because (i) if suicide is abnormal, it will not be something that is common among all individuals and (ii) if it is irrational, it will be irrelevant to us since we are only dealing with rational choices.

However, we must admit that many rational people do commit suicide, and in fact there are those who insist that suicide be included among the solutions that may be proposed to a person concerning the conduct of his life.<69> Actually, we are about the only living creature who can commit suicide, and that is all because of our rationality and intellegence. Animals are prevented from committing suicide as a result of their ignorance. There is no dispute that the lives of many animals are much more miserable than many of those people who had committed suicide, and if the option of suicide is available to these animals, we are sure that most of them would be delighted to join rank with us at least in this respect. Unfortunately, they are deprived of that privilege. They could not commit suicide beause they do not know that such an option exists, and their ignorance can be accounted for in two ways. Firstly, they do not always know the consequences of their actions. A dog standing at the edge of a cliff would not know that if he jumps off the cliff, he will die. However, he does get a message from his own

reflexes telling him that he should not take the jump. These reflexes are provided by nature to protect the lives of all living creatures, and they have created a barrier preventing all animals from killing themselves. Human beings are the only exception because we know that if we jump off a cliff and if the cliff is high enough, we will surely die.

The second way in which animals are prevented from committing suicide is their ignorance of death.<70> We do not want to dispute that animals have witnessed death. Nevertheless we do want to speculate that they do not know what death is and especially the emptiness that is associated with it. Death to them is often extremely agonizing, and it is just unimaginable how they could possibly conceive it as an exodus from misery. If our speculation is accepted, it will follow logically that animals cannot commit suicide since it is impossible for them to choose something which they do not know to exist. Consequently we must reject the claim that suicide is an irrational act committed by morons, and start to recognize it as an alternative state available to an individual under free choice conditions.

It may be against the law, but it is always possible for a free individual to kill himself, and we have never any doubt about the availability of suicide as an alternative state. In other words, if there are people who refuse to commit or even consider suicide under all circumstances, then suicide will be excluded from these people's alternative sets. Most of the time, the negative opinion held by these people are influenced by the way in which they identify

suicide. As in the case of death, there are also many forms of suicide, and some of the ways by which people killed themselves are admittedly uninviting. Examples of this kind are not difficult to find. We think few of us would ever seriously entertain the thought of burning ourselves to death or of cutting our own throats. If an individual's idea of suicide is somehow associated with one or more of these unmerciful self-killing, it would be easy to understand why he would refuse to include suicide in his alternative set. When that happens, our proposed common zero point will not apply to this individual. Since our zero point is meant to be universally applicable, a counter-example of this nature will prove to be detrimental.

Fortunately, the above argument is fallacious, and the fallacy lies in allowing people's misconceptions to prevail. We have suggested that some forms of suicide might deter people from considering killing themselves; however, that does not imply that all forms of suicide are equally atrocious. For those who can only identify suicide as some brutal way of ending one's life, we must regard their view as restrictive if not prejudiced. Knowledge of other forms of suicide which are more agreeable is readily available to civilized societies. Painless suicide has been around for quite some time. Hemlock has been in use since the Platonian time, and today even prisoners on death penalty are given the right to die in peace with lethal injections. With little reservation, we think we can say that anyone who wants to kill himself can do so without much pain inflicted upon

himself, and there is no reason to deny suicide a place in the alternative set just because some misinformed people have indentified some subset of suicide as suicide. Our proposal is to use painless suicide as the common zero point and to make all other forms of suicide irrelevant to our discussion.

Besides being able to dismiss some of the prejudices that are clouding over self killing, our choice of painless suicide as the zero point will also satisfy our original requirement for 'nothingness.' Since we are unaware of any form of death that will produce pleasure, painless suicide is apparently the supremum among all death states, and the closest we can come to total 'nothingless.' And even if we may in the future discover some enjoyable way to die, there will still be no reason to abandon painless suicide. Afterall, pleasure and pain are just like two sides of a coin, and if one fails to convey a sense of nothingness to our individual, so will the other.

In real life, it is conceivable that some people may be denied the opportunity to kill themselves painlessly. These can well be the result of the lack of information on the part of our individuals, or of the deliberate legislations by the establishment. But whatever the cause may be, they will not in any way constitute a threat to our previous argument. As in the example given by Allais, our proposal to use painless suicide as the common absolute zero in people's utility functions does not rest on the actual existence of such an alternative in everyone's alternative set. Its hypothetical existence will suffice. That is, as long as our individuals

are able to envisage what is involved when someone commits painless suicide, we will be able to establish our case. After all, we know of no one who has successfully committed painless suicide and still survives. Hence, experience will not be a factor, and whether someone is able, actually or hypothetically, to commit such suicide is inconsequential.

Finally, we must not forget there might exist some righteous beings in this world who will vow never to consider suicide as an option opened up to them. If we allow these people to uphold their opinion, our common zero will again be in jeopardy. Luckily, the words of these people do not carry much weight, as this can be exemplified by the group which has waged the strongest campaign against suicide. In an emotional way, Szasz condemns the psychiatrists as those who "ceaselessly agitate against suicide, and lock up those of their unfortunate fellow citizens who they can accuse of suicidal proclivities", and at the same time, "commit suicide, in greater numbers than the members of any other professional groups in the United States."<71>

When a preacher cannot practise what he preaches, it is apparently not unreasonable for the unconverted like us to question the sincerity of his intention. Introspectively, there is a strong indication that we will yield to certain circumstance, and life will cease to be meaningful below that point. That circumstance can take the form of physical or mental torture or even some other distasteful event which may be unheard of to us. The important point is, however, that such a yielding point will exist for everyone. We are

certainly not alone in holding this view. As pointed out by Baechler, it "was George Orwell who, in his 1984 declared that for each individual there is at least one torture even the most courageous person cannot resist."<72>

Hence, if we believe in our own introspection and feel that George Orwell's assertion is not unreasonable, we would then be obliged to assert that suicide is in the alternative set of every individual. As for those who insist that they will never contemplate suicide, we have no other choice but to treat their insistence as a conscious or unconscious effort to mislead us.

Knowing that each one of us will choose to end our life voluntarily under certain circumstances can be much more informative than we expect. It will, above all, confirm the existence of a common absolute zero in everyone's utility function. In addition to that, we will also be able to speculate on the range of people's utility functions. Since life usually has something better to offer us than suicide, it is not unjustifiable to say that our utilities will in general take on positive values.

Under free choice conditions, a rational individual should never have actually experienced negative utilities, as he could always choose to kill himself and prevent himself from going down to the negative values on his utility scale. On the other hand, it is not hard to imagine prospective utilities to take on negative values. For a person who would rather commit suicide than be tortured continuously, the prospective utility of the latter must be below that of zero.

All these are admittedly pedestrian at first glance and yet their implications can be immense. As an illustration, let us look at the ongoing debate on population growth. Should we or should we not let the population of a society grow indefinitely? Since when there is one more person in the society, there will be one more to be counted, the simple utilitarian rule which advocates for 'the greatest good of the greatest number' would therefore suggest that we should multiply population until it reaches infinity, in which case the sum of utility of the society will also be maximized. This conclusion will be true despite the existence of diminishing marginal utility which, presumably, is capable of lowering the sum total of society's utility. This is because when the number of people in the society approaches infinity, the total utility of the society will also approach infinity provided that the utility of each individual is positive. On the other hand, if our hypothesis is correct, i.e., if it is possible for a person's total utility to become negative, then the number of people in a society will cease to be the only dominant criterion to the utilitarian solution of the population problem. Other utilitarian rules like the 'Millian' and the 'Harsanyian' average rules will be open up to us, and we may after all not have to procreate at maximum capacity.<73>

From the previous discussions, we also know that people's utility functions are bounded from above with a supremum less than infinity. As of now, we still do not have any idea whether there is a corresponding lower bound and we

have no intention, at least for the time being, of pursuing the question any further. Our inability to verify the existence of that lower bound will in no way obstruct our investigation. Since our main concern is redistribution, and since people can always choose to kill themselves, the lowest point in an individual's utility function we need to consider is the zero point, and what comes after zero will serve nothing but pedagogic interest.

The Other Touch

We have up till now discovered the common zero point in people's utility functions. Yet this is insufficient to allow us to make full interpersonal comparisons. This is so because even with the common zero point, people are still entitled to say what they like and what they dislike and set whatever utility to any commodity as long as their preferences are consistent. And it is due to this prerogative which we have conferred on our individuals, people have in the past been able to cite outrageous examples to deny us the possibility of making interpersonal comparisons.

One such example was given by Robbins whose caricatures of his colleagues must be reproduced in full. He was trying to find out which of the two professors x and y would derive more enjoyment from the possession of the first edition of Ricardo and he was expecting the following responses from them:

"What, compare my refined delight in this superb item with the vulgar complacency of that fellow," sniffs Professor x. "That effet snob may say what he likes," bellows Professor y, "but in fact my poor treasures mean more to me than he has ever felt about anything."<74>

It is comforting to know that intellectual animosity is a universal phenomenon. Nonetheless, we must also realize that the above example is a grossly exaggerated one. Being economic students, we know that Ricardo's treatise is crucial to all streams of economic thought, and one can hardly believe, if not for past grievances, two learned professors would make such irresponsible comments on each other. It would of course, be a different story if the stress of the example is on the first edition of Ricardo's book rather than its content. In that case, we must consider the two professors as rare book collectors instead of economists. And like all self-claimers of 'refined' tastes, their will to possess the book would most likely be the result of some painstaking efforts whose purpose is to articulate certain cultivated differences among individuals. Apparently, there is no way to deny these differences. Wine lovers who clean their mouths with processed alcohol are sometimes capable of telling the year of a bottle of wine down to the last digit, and we must also admit that some people do take the 'Pepsi challenge' with the utmost seriousness. However, we should not be discouraged by the existence of such differences. In fact, we have always been expecting them. Contrary to what has been alleged by Robbins, these differences among us are far from being sufficient to prohibit us from making

interpersonal comparisons. In our opinion, it does not matter if the tastes of the two individuals are worlds apart for some or even most commodities, and as long as we can identify one commodity which for the two have equal regards, we will be able to conduct interpersonal comparisons between them.

What makes Robbins's example, and for that matter, all other examples of similar nature so convincing is a trick which they have employed in their illustrations. There is essentially nothing new to it. All one has to do is to emphasize what is to one's advantage. Since Robbins is for non-comparability, it is of course to his advantage to dramatize the difference in tastes among individuals. To no one's surprise, there is a great deal of irreconcilable differences in tastes between a king and a pauper. However, if we are to make interpersonal comparisons, should it be the similarities or the differences among us that are to be stressed? The answer is unequivocally clear.

It seems inconceivable that as human beings we do not share something in common, and this would apply equally well to those whose deviant behaviours are well publicized. If one can still recall, Sen used the story of Nero to demonstrate how different individuals can have completely opposite reactions to the same event. Supposedly, Nero was fiddling while Rome burnt, and the rest of his people lost their homes. That, however, is not the opinion of some columnist of the Montreal Gazette. Though the source may not be something we would usually associate with lexical excellence, we must

admit that we do share its view. This is how the story was retold:

Then there is the story about Nero fiddling while Rome burned. I'm no public relations man for Nero, who was a genuinely nasty bit of work. But Nero gets a bum rap. It was a translator's error.

In the original Latin source for that story, Nero was 'fretting' while Rome burned. And fretting and fiddling can be confused in Latin.

So Nero was actually fretting, as in wringing his hands, and not fiddling while Rome burned.<75>

Hence Nero did not react differently. He was worried as most of us would be under like circumstances, and there is no ground to believe that he was totally different from us and did not share one single trace of mutual human feeling.

It is exactly that single trace of mutual human feeling which is needed to make the necessary crossover to complete our interpersonal comparison scheme. In other words, if we are comparing the satisfactions of two individuals, we must be able to tell how happy the two individuals are with at least one consumption bundle. Of course, there are still those who would say that we can never tell how happy another individual can be. That position, however, is no longer tenable since our problem is neither more, nor less, complex than the idea of telling that another person is feeling hot.<76>

We think there is a general consensus that we know when we are hot. Staying in an closed greenhouse under a bright summer sun without any air-conditioning will definitely make us feel hot. The knowledge of our hotness is a first hand experience. But how can we know that the person who stands next to us also feels hot? Due to the laws of physics, we can

never be the person himself, and therefore will never be able to have a first hand experience of his experience. However, there is no doubt that we do make statements like "he is hot," and these statements are seldom intended to be meaningless strings of words.

Our judgement on whether someone is hot is often based on induction and belief. In the above example, the first way to find out whether the person is hot is to ask him directly. When he tells us that he is hot, we can accept or reject his answer. If we choose to accept his answer, what we are actually doing is to believe in what he said. There can be many reasons why we would like to believe him, but all of them would probably come back to the same theme, i.e., he is honest and what he has said in the past was often true. Naturally, beliefs made in this way are also inductions, since what we believe in is inferred from past experiences.

Another way in which we can tell that the person is hot is to extrapolate our own experiences. We know that we are hot in that greenhouse, and we have always felt hot under similar conditions. Furthermore, when we feel hot, there will be certain physiological changes in our bodies. We will perspire more and our hearts will beat faster, etc., Now that we see this person standing next to us in the greenhouse, sweating and behaving more or less like us, can we not infer that he is also feeling hot?

We understand that due to our unique individualities, each one of us will perspire differently, and our hearts will beat differently even if we are placed under exactly the same

conditions. However, we think the case will be overstated if we try to use these differences to proclaim that we can never tell whether the person next to us is feeling hot or not! For if it is not possible for us to tell that he is hot, it will be equally impossible for us to tell that he is cold, or angry, or for that matter, any kind of feeling that he might have. In other words, we will be practising solipsism^{<77>} if we refuse to recognize that the person next to us is hot just because he is not built exactly like ourselves!

There is a high price to be paid when we choose to deny the existence of other minds. First of all, the value of all cultures will become questionable as the purpose of any form of art hinges on arousing the inner feelings of others so that mind and mind can echo. When there is no other mind, there is obviously no place for culture. The second problem of solipsism is related to epistemology. When we deny that a person possesses a mind, we will never be able to know what that person is thinking, and therefore it will be impossible to understand anything he says unless it can be translated into some statements about our immediate experience.^{<78>} So we will not know that he is hot, but we may find out the rod he claimed to be one meter long is true or not. Such an attitude towards knowledge is that of the most primitive form of empiricism and it is known as radical reductionism.

We may not be aware of it, but by endorsing radical reductionism we are in effect endorsing strict behaviourism at the same time as the latter is only the former applied to human behaviour. To be a hard headed behaviourist, one must

know that introspection has no place in his theories. Everything that can be known of himself are supposed to be observable through his external behaviour, and that makes the existence of his mind otiose. In fact, a behaviourist does not only deny the existence of other minds, he also denies the existence of his own mind.

It seems something must have gone wrong! We had started the exercise by denying the existence of other minds, and yet we end up losing our own minds also. Intuitively, the denial of one's own mind is obviously much stronger; it implies the denial of other minds, and it is natural for one not to expect to derive a stronger conclusion from a weaker premise.<79> However, that is not the opinion of the behaviourists as they maintain "that 'our' knowledge of ourselves is no different in kind from our knowledge of other people. We may see more, because our own body is easier to observe than those of other people;" but we do not see anything radically unlike what we see of others."<80>

The idea of losing one's own mind is not a particularly entertaining thought, and even then, one must not forget that he cannot think without a mind! Fortunately, the method of radical reductionism has almost been abandoned by everyone. Even self-proclaimed empiricists are no exception. While behaviourists think that knowledge of ourselves is no different in kind from our knowledge of other people, Quine argues that the same relationship is true of physical objects and other myths:

Physical objects are conceptually imported into the situation as convenient intermediaries -- not by definitions in terms of experience, but simply as irreducible posits comparable, epistemologically, to the gods of Homer....in point of epistemological footing the physical objects and the gods differ only in degree and not in kind.<81>

Reductionism runs into trouble because reduction can only be carried so far, and there is always something which is irreducible. The problem is first seen in mathematics and logic where the axiomatic method has proved most successful. Immediate experience will never confirm points and lines to us, nor will it assure us of the law of the excluded middle. By definition, the axiomatic method recognizes that something is irreducible, and it requires us to appeal to our minds as well as to other people's minds for consensus.

If others do not have minds, how can we know that the point we are talking about is the same point they have in mind. By the same token, we are certainly entitled to say that the person next to us is feeling hot. We will make mistakes at times, but that will by no means deprive us of the right to make such statements. Mistakes of this nature are inevitable. When two people measure the same rod, variations will always occur. It may be true that the precision problem may be more acute in one case than in the other. However, inaccuracies are inaccuracies and they can only differ in form but not in substance.

We understand that people from the north pole will have less tolerance to heat than people from the tropics, and the same applies to those people who have a higher fat composition than those who have a lower one. Taking all these

relevant factors into account, it seems that the more a person resembles us in these respects, the more accurate will we be able to detect his feeling of hottness.

At the limit, we know that we are as hot as ourselves. In addition, we should also know that we are almost as hot as A who resembles us in every relevant aspect. We may have very sensitive ears while A may be deaf. Yet, that is of no concern to us as long as we are only interested in telling whether A is hot. Admittedly, a certain degree of arbitrariness is unavoidable, but that is also universal to all forms of knowledge, and despite these shortcomings, we think that most of us would agree in principle that we could tell (i) another person is hot and (ii) someone is almost feeling as hot as ourselves.

The analogy between hottness and felicity is easy to discern. Sometimes, the difference in our cultures, upbringings and temperaments is much greater than the difference between the man from the north pole and the man from the tropics, and it is not unreasonable to say that interpersonal comparisons in these cases are difficult though not impossible to make.

However, as we have stressed before, it is not our intention to make comparisons of this sort. Instead, what we should do is to compare individuals who are similar and who share the same taste in a particular area. For example, let there be two amateur baseball players A and B who live in the same neighbourhood. They are about the same age and earn comparable incomes. Moreover, they are both married and have

a family to attend to. Yet, every week they both voluntarily decide to devote the same amount of time to play baseball and they play with equal vigor. Then can we speculate that the two players might have equal preference for baseball? Naturally, the criteria are arbitrary, and it is preferable to have them decided by the people involved rather than imposed by some outsider. The point to note, however, is that such comparison can be made, and unlike the examples cited by people who argued for non-comparability, comparisons reached in this way will be voided of instances where each individual regards his own taste as refined and holds an uncompromising attitude. Since the criteria are decided by the individuals involved and participation is voluntary, there is definitely a sense of consensus among those who claim to be comparable to each other.

On the other hand, individuals need not be identical to be comparable, and it is conceivable that they might have diverging tastes in areas other than the one that they have in common. To illustrate, A may be allergic to seafood while B is crazy about it. So when you ask them about their opinions on seafood, they will be miles apart. In such circumstances, direct comparison of the two players's preferences on seafood will be extremely difficult. However, if we know that their preferences for baseball are the same, then comparison of their preferences on seafood will become a routine exercise. Assume that A assigns 10 units of utility to baseball and 5 units of utility to seafood, while the corresponding figures for B are 20 and 100. Since the two

players have the same preference for baseball, 10 units of A's utility must be equivalent to 20 units of B's utility. By simple ratio, 5 units of A's utility must then be transformed into 10 units of B's utility, and we have already obtained a rough idea where the two players stand on seafood.

Although we have emphasized that people have to be similar before they can be compared, it does not mean that each and every one of us will have to be a middle-class amateur baseball player. Nor does it mean that our proposal can only apply to baseball players, and leave out the rest of the society out in the cold. In fact, there are endless ways in which people can be similar to each other, and there is no need to restrict ourselves to conduct our comparison on the basis of some particular taste or interest.

With this idea in mind, let us group all the baseball players who are similar enough to warrant comparison into a set and call it P. In a similar fashion, we will have a set Q for soccer players, R for musicians and so on. By construction, it is possible to make interpersonal comparisons among members of the same set. But can we say the same for individuals belonging to different sets? Normally we cannot, because a baseball player is seldom a soccer player, and musicians are usually less enthusiastic about sports. Nevertheless, we must not forget about B who is a baseball player as well as a seafood lover. If there is a set T for seafood lovers, there is little doubt that B will be in it. In that case, sets P and T will no longer be disjoint as they have at least one common element B.

When two sets are not disjoint, interpersonal comparisons among their members will be made possible with the help of the elements they have in common, and this will be true even if their intersection is a singleton. To see how this works, let there be a C in T who assigns 50 units of utility to seafood and 100 units of utility to playing the piano. We want to compare A 's preference for baseball with C 's preference for playing the piano. Taken alone, this is certainly a formidable task, but not so with the existence of B , since seafood means 100 units of A 's utility. Hence, transformation is actually not necessary for comparing A and C 's utilities, and C 's preference for playing the piano is exactly five times that of A 's for baseball.

Moreover, the comparison between A and C is also guaranteed by consensus. Though it is not a direct consensus between A and C , it is nevertheless a consensus that is equally strong. In the absence of a more appropriate name, we will tentatively call this kind of mutual agreement the 'extended consensus,' and represent it by the symbol E . As defined in this way, E is a ternary relation, and it can hold for any subset of the set of all the members of the society as long as the number of elements in that subset is equal to three. There is no requirement for the three elements to be distinct. $(B)E(A)(C)$ would mean A is in agreement with C through B . When A and B or C , and B are identical, E will be reduced to the ordinary consensus relation between two individuals. E is an equivalence relation. It is reflexive since it is always true that $(A)E(A)(A)$; it is symmetric

because $(B)E(A)(C) \Leftrightarrow (B)E(C)(A)$; and finally it is also transitive as $(B)E(A)(C) \wedge (D)E(C)(F) \Rightarrow ((B)(D))E(A)(F)$. If in addition E is also complete, then all the members in our society will be in agreement as to how each one's utility should correspond to those of the rest of the society.

The Possibility of Consensus

We will now examine what are the necessary conditions that will give rise to the completeness of E . Let us call sets P, Q, R etc. comparable sets. Since a comparable set which contains only one element is trivial, we will from now on assume that a comparable set will at least have two elements that are not identical. In order to be comparable with other members in the society, an individual must belong to at least one of these comparable sets. This is indeed not a demanding condition, and we really doubt that an individual who does not share certain feeling with someone else should be considered as part of a society.

Two comparable sets with a non-null intersection are said to be chained. For any n number of comparable sets $A, B, C, \dots, N$; if A is chained to B , B to $C, \dots, M$ to N and N to A , then we say sets A to N form a loop. A self-contained loop is a loop which has none of its member chained to any comparable set outside the loop.

A society will fulfill the extended consensus condition only if all comparable sets are chained, and every individual

in the society must be a member of one of the comparable sets which make up the smallest self contained loop. Again these are fairly reasonable conditions. Like the condition posed before, they are there to prevent alienation. Only this time we are aiming at groups of individuals, and groups of groups of individuals.

If a group of people is to be regarded as a society, we see no other way but to assume that there exists some link among them. For any subgroup of a society which can form its own self-contained loop, that sub-group is no longer dependent upon the existence of the mother society. Very often, aristocrats in the past had approached that limit as they lost touch with their subjects, and that partly explains the efforts that had been exerted to eliminate them from their original societies.

For those who cannot imagine how all the members of a society can be linked together, we advise them to probe into the relation of 'friendship.' As a speculation, we think that all of us may perhaps be linked together by friendship. This is obviously a testable hypothesis, and we strongly encourage the non-believers to verify it. Since 'friendship' is irreflexive at times, we certainly cannot solely rely on it to achieve our extended consensus. However the relation does give us an idea how easily we can be linked together, and makes us recognize the immediate possibility of the existence of an extended consensus.

Not only is it possible that we are linked together, it is also probable that we may be linked in more than one way.

For example, a set of comparable sets will be chained in more than one way when they form a loop. In our previous illustration, we have come to the conclusion that C's preference for playing the piano is five times that of A's preference for baseball, and that conclusion was arrived at from the data we were given. A certain set of data will produce a certain set of results, and there is no way we can check whether the results are accurate.

The story, however, will be told differently if A and C are linked in more than one way. Since in that case we would have two sets of results that are arrived at through different paths and because they are still in the same metric space, path independence must be observed, and these results must be consistent with each other.

When that happens, we will be able to perform checks on our results, and our hypothesis will cease to be a mere tautology. Apparently, the more checks there are, the more reliable our results will be. But as checks increase, inconsistencies caused by measuring errors and other reasons are bound to occur. In such circumstances, it will be up to the investigator's discretion to accept or reject our hypothesis, and that is true of other instances when we try to apply Popper's law of falsification.

Finally, we must point out that our extended consensus may not be as difficult to obtain as it seems. Depending on the degree of refinement one desires, comparable sets can be easily enlarged to encompass more people. In fact, societies are often subdivided in such a way that comparable sets are

almost always ready at hand. In our analysis, we have started with the individual for conceptual clarity, but for practical purposes, we may well embark upon social classes. The important point to note is that we have now on hand a theoretical apparatus which is intuitively acceptable as well as logically consistent. There are lots of things we can do with such an apparatus and the construction of a social welfare function is definitely among one of them. Throughout our analysis, we have never specified a particular form of social welfare function for our society to follow, and upon the eventual specification of that form, we will have a deterministic solution to our social choice problem. We will not be surprised if it turns out that the solution offered by our extended consensus is not much different from, or even identical to, what governments have been doing all along. After all, a government is supposed to be a means to express social opinion. However, that does not mean that our concept of extended consensus is totally superfluous. Without a theoretical foundation, government policies will always be vulnerable to the sceptisms of the 'natural liberty' school. What we have done is to provide certain credible and consistent justifications for public actions, and perhaps some insights into how such actions can be modified. There is not the slightest intention on our part to claim that the extended consensus rule is the only rule to arrive at social decisions. People can reject any one of the axioms which we have postulated, and we will end up with a completely different rule. For example, one can bypass the individual

and take the society as the basic unit. This will be equivalent to dropping the individualistic axiom and subsequently the elimination of the aggregation problem altogether. In principle, we feel that there is no prima facie reason to reject any particular solution to the social choice problem except the solution which says there exists no solution.

VIII

Configurating a Social Decision Rule

Difficulties in Hedonism

We have hitherto proved (i) the existence of cardinal utility functions, and (ii) the possibility of full comparability. Although (i) and (ii) together will be sufficient for the construction of social welfare functions, they still leave us with the problem of deciding which particular function is appropriate for the society in question. In what follows, we will try to show that it is possible to pinpoint that particular function if our individuals are all trying to maximize social welfare. Before we can come up with that solution, however, it is necessary to examine some of the recent debates on utilitarianism.

Early utilitarians, like Bentham, were mostly hedonists.<1> Their epicurean leanings made them believe that pleasure is the only thing that matters. Consequently, utility was taken to mean pleasure, and utilitarianism as the doctrine which dictates the maximization of people's pleasure. It should not be difficult to see that this position of the hedonists' is hard to defend. People simply do not always opt for the most pleasant alternative available to them. A good example is charity.<2> Whether at the personal or the national level, charity is definitely ill-qualified for the hedonists' quest. An individual who

gives away part of his wealth in return for nothing will no doubt be deprived of certain niceties in life. The same is also true of a nation. If their only goal is pleasure, then why would they engage in charitable acts?

In response to the difficulties of hedonism, a modified version of utilitarianism known as ideal utilitarianism was put forward by people like Moore.<3> Under this brand of utilitarianism, utility is no longer treated as another word for pleasure. Rather, it is asserted that many other things can contribute to utility. Among the commonly accepted ones one may include knowledge, fraternity, and perhaps even aesthetic qualities like beauty. Since pleasure ceases to be the only contributor to utility, it is not hard to conceive of situations in which pleasure may have to give way to knowledge or fraternity. With these modifications, it seems that the objections from the critics of hedonism are met, and utilitarianism is again in safe water.

This attitude of optimism was soon proved to be premature, as another wave of attacks was launched by the non-utilitarians. This time, the latter argued that utilitarianism is inconsistent with even the inclusion of the other non-hedonistic characteristics. Usually, their claims were supported by some counter-examples. A typical one may run as follows: at his final moments, a dying man may, through certain means, make another person promise him to destroy a considerable amount of grain. It is understood that if this grain was not destroyed, it could be used to feed many of those hungry people in the society. Since the

dying man could never know whether the promise would be kept after his death, and since he was the only party who might possibly be injured by the breaking of that promise, it seems that utilitarianism would suggest that our promisor should ignore the promise, and put that grain to some better use. This solution however, is not intuitively acceptable -- at least not to the intuitionists. They argue that most people probably would feel that there is a strong moral obligation to honour a promise irrespective of whether or not that promise will produce the best result for all. Perhaps another example will further clarify their thesis.

Consider a community with intense racial unrest. Suppose one day a prominent member of one race is assassinated, and no one knows who the murderer is. It is generally believed, however, that he is a member of the other race. Discontentment is on the increase, and an all-out riot is imminent if the guilty party is not apprehended soon. Faced with this situation, the sheriff of the community has two alternatives: either he could frame an innocent person as a scapegoat and avoid the riot, or he could just let nature take its course and await the inevitable. The decision is a tough one. As moral beings, most of us would probably refrain from framing an innocent person. On the other hand, if we try to balance out the sufferings of different individuals, as utilitarians do, it does appear that there is a strong case for the sacrifice of one individual in exchange for the peace of the society as a whole. So, we again have another conflict between moral intuitionism and utilitarianism.<4>

Intuitionism

For the intuitionists, <5> this kind of conflict is to be resolved by resorting to our own intuitions. They claim that it is within our capacity to identify a set of *prima facie* moral obligations, <6> and also that it is to this set of obligations that our moral behaviour should conform.

Insofar as there is only one *prima facie* obligation applicable to each different situation, the intuitionists are saved from inconsistency. For example, the *prima facie* obligation to keep promises would require us to honour our pledge to a dying man, and the *prima facie* obligation to be honest would prohibit us from framing an innocent person.

The problem, however, is not as simple as it appears to be. Putting aside the issue of whether we do in fact have moral intuition, there is still a couple of questions waiting to be resolved. The first one is concerned with how to draw up the list of *prima facie* obligations. Since different people might well have very different ideas as to what is morally obligatory, the list of *prima facie* obligations may vary from individual to individual. When there is a significant disparity with regard to the sense of moral obligation among people, expectations will often be dashed, and that will jeopardize the stability as well as the continual existence of the society.

The second difficulty with intuitionism arises when there are more than one *prima facie* obligations in any single situation. To illustrate, we may have a case where a man is

being ordered by a judge to reveal a secret which he had promised to a friend that he would keep. Two *prima facie* obligations are at work here: (1) the obligation to keep a promise, and (2) the obligation to tell the truth in court. How and upon what judgements can one decide which of these two *prima facie* obligations to follow? Many have suggested that we should again rely on intuition to make that decision. Needless to say, this will inevitably involve some form of the theory of types. At the lowest level, we will have the ordinary *prima facie* obligations. When there is a conflict among these obligations, we will have to go to a meta level of intuition to decide which of the lower level obligations should prevail. It has been recognized that, depending on the situation, one obligation may come before another in some situations, and the opposite may be true in others. That is to say, there is no fixed rules prescribing which *prima facie* obligation shall come first, and what will prevail, will prevail.<7>

Such modifications have undoubtedly stripped intuitionism most of its explanatory power. Moreover, they have also raised the question of intrinsic goodness. Can there be things which we can regard as intrinsically good? Something is said to be intrinsically good if, and only if, the goodness in it does not depend on circumstantial relationship.<8> If we were to follow Ross<9> and consider *prima facie* obligations to be intrinsically good, then we would have to ask why the relationship between two such obligations may reverse under different situations. By

definition, if A is intrinsically better than B, it will always be intrinsically better than B. The requirement for consistency will not allow us to have different opinions from situation to situation, and this will contradict the intuitionists' claim that 'relative stringencies' of *prima facie* obligations may vary.

There is another way out, and that is to break away from Ross and give up intrinsic goodness as an undefined concept. We will come back to this point when we discuss act-utilitarianism.

When the utilitarians are confronted with the kind of objections raised in the two examples above, their reactions are mixed. Some are undisturbed. Seeing those objections as irrelevant, these utilitarians are quite content to cling on to their beliefs. Others, however, are more receptive to the oncoming criticism. They admit that there are defects in utilitarianism as it stands. Nonetheless, they do not think that these defects are serious enough to make them desert utilitarianism. They have instead introduced some minor changes into their doctrine to accommodate the growing opposition.<10> With this move, they have put forth yet another brand of utilitarianism which is known as rule-utilitarianism.

Rule-Utilitarianism

Unlike the usual kind of utilitarianism (i.e., act-utilitarianism) which allegedly judges actions on their immediate consequences, rule-utilitarianism is supposed to evaluate actions according to their long run benefits. It explains that moral people sometimes do not maximize the overall utility of the society because such actions may be detrimental to the subsequent well being of the society.

Take the case of promise keeping. It may be true that at times the society will be better off if we break certain promises. However, if this type of behaviour is permitted, people cannot know at all whether what others had promised them will be kept or not. The whole notion of promise will lose its meaning. Understandably, the end result of such a system will be far worse than the one which demands that people keep their promises at the costs of some occasional mistakes.

There are other rules besides those governing promise keeping, but the general idea is more or less the same. Seemingly irrational and non-maximizing behaviours of individuals are often attributed to these people's conformity to a set of rules. By observing these rules, we will supposedly maximize social welfare in the long run. Yet, who made these rules? Or do we know them by intuition? If yes, what is the difference between rule-utilitarianism and intuitionism?

It is obvious that our behaviours cannot always be

guided by rules. There will come a time when we will have to break away from certain rules. When are we allowed to do that? Or are we allowed at all under rule-utilitarianism? What should we do when two or more rules clash?

Surely, we have met all these questions before when dealing with intuitionism. In fact, it is our opinion that, for all practical purposes, rule-utilitarianism is just another name for intuitionism. As we have learned from theories of profit maximization, there will be little left to dispute once we have brought long run scenarios into the picture. In the long run, anything can happen! The final outcomes of rule-utilitarianism are just as true by necessity as those of intuitionism.

Act-Utilitarianism

It should not be misunderstood that we are strongly against the tautological implications underlying the two theories discussed. On the contrary, our view on the matter is, to say the least, fatalistically inclined. We do not endorse intuitionism or rule-utilitarianism because we do not think that their detour is necessary. In our opinion, act-utilitarianism is fully capable of making the same kind of defense as offered by its rivals -- perhaps in a more consistent way.

In order to substantiate our assertion, we must first find out what is it that our moral individuals are trying to

maximize. We know that they are trying to maximize the welfare (or utility) of the society. But what is this utility? It cannot be people's hedonistic pleasure nor can it be the fulfillment of *prima facie* obligations. The former, at the most, can only be part of the social welfare proper, while the latter is but a means to an end. Of course, we can always say that something will increase social welfare if it is good for the society. However, that will in turn leave us with the word 'good' to explain. By 'goodness' we could not have meant relational goodness, since that does not enable us to construct a consistent welfare index. So, we are stuck with only one alternative, and that is to regard something as giving rise to social welfare if, and only if, it is being considered as intrinsically good by the members of the society.

We must admit that we do not know much about this concept of intrinsic good. However, given the fact that we are working within an individualistic framework, we will have to postulate that what is intrinsically good for the society will depend on what is intrinsically good for each morally responsible individual. It will be argued that we cannot define in an exact manner what is intrinsically good for the individual. As a result, we will have to leave the concept of intrinsic goodness for the society as undefined.

An individual's moral convictions are necessarily influenced and molded by those prevailing in his society. On the other hand, he still possesses the discretion to accept or reject moral principles, and we must consider him as an

independent moral unit, making his own moral decisions, and responsible for them.

Having identified the individual as the basic decision unit, we will ask ourselves if there are any rules governing his decision which may lead us eventually to discover uniquely the thing which he would call intrinsic good.

There are many things which we know will be considered by our individual as intrinsically good at one time or another. To name only a few: knowledge, benevolence and personal pleasure. Each of these is not intrinsically good per se because all of them are capable of being overridden by other things which our individual might value more highly under certain circumstances.

Since none of the above can be considered as intrinsic good itself, we have little choice but to go to our individual and examine more carefully what he has in mind when he proclaims that he is experiencing something which he considers to be intrinsically good.

Putting aside the mind-body problem, modern neurology and biochemistry have revealed that our human brain is responsible for all our actions and feelings. When certain stimulus activates our nerve system, a signal will be sent to our brain. Depending on the kind of stimulus, different fibres in the brain will be excited.

It should be noted that our brain responds to both physical and non-physical stimuli. If our individual devours an apple, he will experience certain satisfaction as a result of the stimulation of some set of fibres in his brain. On the

other hand, the same individual may also feel satisfied for some benevolent act he has done. Again, the satisfaction so derived must be the outcome of the agitation of a set of brain fibres.

The distinction between physical and non-physical stimulations is only apparent. It has generally be conceded that, given sufficient advancement in neurology, it may be possible to create the sensations of physical objects in the human brain without the actual existence of such objects.

To adopt the common analogy, our brain is just like a sophisticated experience machine,¹¹ deciphering informations that are fed to it to determine our feelings and sensations.¹² The same brain will then rank-order these feelings and sensations. Again, this applies to both physical as well as non-physical objects. A similar ranking which is restricted to consumer goods has already given us the consumer utility index. In that ranking, the consumer's preference is being treated as undefined. One bundle is better than another if it is so proclaimed by our consumer. No explanation is necessary, and there is no ground for argument. The whole exercise is tautologically true had there not been the possibility of judgement errors. The situation is not much different when it comes to ordering social alternatives by our moral individual. It may be true that things are a little more complicated. Even local non-satiation of consumer goods may not always apply, as one's will to suppress greed may prevail. However, the underlying logical construction is the same in both cases.

With the consumer utility index, the consumer's preference is taken as undefined. The same is true of the social utility index. Here, we have left intrinsic goodness undefined. Something is intrinsically better than another else if, and only if, it is preferred by our moral agent.<13>

Prima facie obligations as well as moral rules will cease to be primary objectives of our moral agents. Rather, they will be treated like everything else and will only be observed on certain occasions if our agent finds them agreeable. Acts will be evaluated according to the amount of intrinsic goodness or social welfare they can bring forth. Our brain will be the judge to that effect. All these procedures are undoubtably well within the description of act-utilitarianism. Every now and then the problem of time may come up, as the consequences of an act may spread out through time. However, we should not be at all bothered by that. Problem of the same nature are everyday encounters in consumer theories. The practice of discounting is a notable example. In fact, it is hardly conceivable for the consequences of an act not to span any duration. All in all, the whole exercise in act-utilitarianism is of the same nature as the ones they have been performing on consumer preferences.

There are a few things we want to recapitulate before going on. First, we have not assumed an identical social utility function for any two members in the society. Each moral agent is allowed to have his own moral standards. What we demand from him is that he tries to act according to that

standard. Secondly, we have seen that their behaviours are all compatible with act-utilitarianism. In other words, they are all trying to maximize social welfare. Thirdly, the social utility function for each individual is not tautologically true. Specifically, it is also possible to persuade people to change their conviction by pointing out their inconsistencies or by convincing them with more agreeable solutions. Lastly, even if one considers each individual's social welfare function to be tautological, the social welfare function for the society is not. We will now make an attempt to show why this is the case.

Pinpointing One Particular Function

In previous chapters we have attempted to show how it is possible to make interpersonal comparisons of utility. The type of utility functions considered were strictly private in the sense that moral contingencies were excluded, as were their effects on the level of satisfaction of our individual. This state of affair is admittedly unsatisfactory, since such an utility function is only a special case of the more encompassing utility functions we have been examining in this chapter.

This latter function, which takes into account everything that our individual appreciates, will produce an index of intrinsic good for our individual. Every situation, irrespective of whether morality is involved, will be ordered

by this same function. In other words, our individual will be able to compare situations that are morally implicated with those that are not. To give an example, let us suppose that Robinson Crusoe catches two fish on some day. From introspection, he knows that he will derive 10 units of intrinsic goodness by consuming both fish. There is no moral question involved. Suddenly, Crusoe notices there is something on the beach, and to his surprise, it is the hunger-stricken body of a living person (Friday). A moral decision now faces Crusoe. Should he split the catch with Friday? Crusoe figures that he will retain 8 units of intrinsic good by consuming only one fish. Moreover, he thinks that he will also get an extra 5 units by knowing the fact that he saves the life of another person. To make things simple, we will assume that Crusoe's intrinsic goodness is additive. As a result, the morally implicated situation, with its 15 units of intrinsic goodness, is equivocally better than the first situation.

The above conclusion is too simplistic to be true, as we have left many considerations out of the picture. To maximize social utility, Crusoe has not only to pay attention to his own intrinsic good, but also to Friday's intrinsic good. Perhaps another look at the example will clarify the matter.

We will assume that Friday can derive 10 units of intrinsic good from consuming one fish, and 12 units from consuming both. In addition, these units will have exact correspondence to those of Crusoe's.

As always, there is a limit to altruism. Crusoe's

empathy for Friday decreases as Friday's well being improves. In fact, he will be a bit annoyed if he has to give both fish to Friday. We will deduct 2 units of intrinsic good from Crusoe for these irritations.

Taking inventory, we find out that if Crusoe gives one fish to Friday, he will enjoy 13 units of intrinsic good, whereas Friday will enjoy 10 units. On the other hand, if Crusoe gives both fish to Friday, he will only achieve 3 units of intrinsic good while Friday will enjoy 12 units.

At this point, the two individuals will differ in their opinions even though they are both bound by their morals. What we have to do is to conflate each people's social welfare function into a social welfare function for the society. It should be noted that we have only taken into account the first round repercussions of Friday's consumption on Crusoe. It is obvious that Crusoe's consumption will also affect Friday. Furthermore, second or higher round repercussions are also possible, and as long as such sequences of repercussions are convergent, our individuals' social welfare functions will be defined.

Given the fact that people's social welfare functions are interpersonally comparable, one way, and also the easiest way, to conflate them into a social welfare function for the society is to simply add them up and choose the situation that maximizes the sum of people's intrinsic good. This method will favour the egalitarian solution in our example. The important thing here is that we are trying to do something with which every morally responsible person would

agree, i.e., to maximize social welfare. For those who do not agree to this objective, there was never a problem of social choice in the first place. Our solution is 'objectively' arrived at through interpersonal comparison of intrinsic good. Naturally, there will remain differences in the moral beliefs among people, and in this sense, the chosen situation may fall short of unanimous consent. In another sense, however, unanimous consent is ever present. People are trying to maximize social welfare, and our method will always guarantee an objective solution in that regard.

There are two kinds of reaction one can have when one finds out that one's moral beliefs are different from what the society would dictate if social welfare is to be maximized. One can either adapt one's moral beliefs to that of the society so that conformity will eventually come about. Many of us who have lived in a foreign land are no stranger to such practices. Of course, such convergent behaviours will only help to uphold the hegemony of the society's original choice as social welfare will be re-evaluated in its favour.

The other kind of reaction one can have is to ignore the objective facts and hold on to one's own moral beliefs. Even by acting this way, however, our individual will fail to revert the social decision. In fact, the social welfare vector for all situations will remain as it were before since no one, including our individual, has changed his opinion on any of the social alternatives.

Although non-conforming behaviours of the above mentioned type are unable to change the society's opinion on

the set of social alternatives, they are fully capable of preventing the society from realizing its social optimum. When there are people who disagree with the society as to what actions are the best for the society, and if these people are allowed to act, freely and without restrictions, according to their own moral convictions, then it is almost certain that the collective will of the society will be undermined. At this point, the government will have to come in and play its role. Its function is to enforce that social consensus which was derived from people's social welfare functions. This might involve telling people what to do, and what not to do; explaining to them that egalitarianism and capitalism cannot go together. It will ensure that the society will reach that optimum which everyone of its morally responsible members has consented to, though some of them might have been misguided as not to recognize it.

In simplifying our earlier example, we have assumed that the society will try to maximize the unweighted sum of intrinsic good of its members. Obviously, this assumption is unduly restrictive as well as unnecessary. Theoretically speaking, there can exist numerous other conflation rules which are equally eligible for the purpose of deciding a social optimum. However, it will be argued that even if we allowed for these other rules, our conclusion with respect to the social consensus will still remain valid.

As in previous discussions, we will again rely on our moral agents to decide on what is optimal for the society. Social welfare functions, like many things which we feel

strongly about, are non-physical objects. However, as we must have recognized by now, non-physical objects are also capable of giving rise to intrinsic good in us. By adopting different social welfare functions, one's level of fulfillment will vary depending on one's attitude toward that particular function. That is to say, besides the usual amount of intrinsic good, there will be an additional (possibly different) amount that comes with every conceivable social welfare function. To illustrate, let A and B be the only two members in the society, and each has 10 units of intrinsic good from standard sources. There are two rules under consideration: (i) the simple summation rule, and (ii) the weighted average rule with weights being 2 and 1 in favour of A. Being egalitarians, both A and B detest the weighted average rule, and their feelings are well reflected by their preferences. After taking into account all repercussions, we find out that A will get 10 and 5 more units of intrinsic good for the society's choices of (i) and (ii) respectively. The corresponding figures for B are 20 and 1. As a result, rule (i) will produce 50 units of intrinsic good and rule (ii) will produce only 36 units. If the society is to maximize social welfare, it will have to choose rule (i).

The above example is unsatisfactory as it stands. What if we have a rule which assigns weights 10 and 1 to the two individuals' intrinsic good respectively? Since with the multiple of 10, the resulting social welfare from this rule will necessarily be greater than that from a simple summation rule. In fact, given the properties of the number system, we

will always be able to find a rule such that it will reverse the decision of any previous rule. A decision system of this nature certainly warrants no sympathy. Apparently, something has to be done to resume consistency. It appears that some scalings will have to be performed on people's social welfare functions in order to re-establish consistency. Following our individualistic approach, we will let the individuals themselves to do the actual scalings. Presumably, those rules which have been inflated will be scaled down. The opposite will be true for those which have been deflated. Ideally, these scalings are not supposed to totally offset the original impact of a particular decision rule, or else there is no point in having different rules in the first place. In our example, we have assumed that A will derive a total of 15 units of intrinsic good from the application of rule (ii). This figure, however, has not taken into consideration A's scaling factor for rule (ii). Let us suppose that factor to be 0.6. In other words, A will actually get 9 units of intrinsic good with the application of rule (ii). Of course, this re-evaluation will not affect the outcome in our example, although it is sufficient to give us an idea how the problem can be solved.

Finally, it should be noted that, due to interdependence of people's social welfare functions, an individual's intrinsic good will not be determined unless he also knows other people's level of intrinsic good. Hence, if we are to find a solution to our problem, we will either have to solve everyone's social welfare function simultaneously, or ask

each of them to make conjectures of the other's level of intrinsic good. It should be understood that these conjectures are liable to revisions as our moral agent sees fit. The situation here is similar to the recontracting process, and a solution will be guaranteed as long as one's feelings for others do not exceed those he has for himself.

The social welfare function for the society portrayed above does appear to have satisfied almost every requirement that western economists demand. It is, first of all, oriented towards the individual. Group decisions are derived from aggregating decisions of every single member in the society. Secondly, universality in the sense that everyone's well being must be accounted for, is guaranteed. Lastly, fundamental consensus with respect to the constitution is achieved by the very definition of the whole exercise -- maximizing social welfare. This result obviously represents a major improvement over other approaches, which often run into infinite regress when it comes to the problem of constitution. On the other hand, we have not incorporated the usual anonymity and neutrality axioms into our analysis. In our opinion, these axioms are too restrictive. A crippled person certainly will have very different needs than a blind person. To insist on equal treatment for all is still a little premature. The same can, of course, be said of different social situations.

From our discussions, one may have the impression that our social welfare function for the society is an all encompassing totalitarian monster, and probably wonders if

there is any escape from the 'tyranny' of the group consensus. To answer this question, it is perhaps advisable to re-iterate our discontent with the competitive system and its solution. We do not think that the policy of 'natural liberty' is the one that a government should follow in order to enhance the well-beings of its members. Given this premise, one can either treat social welfare with a substantive identity, and make it on par with individual preference. There should not be any hierarchic structure between the two, as one is not derived from the other. There will be two undefined concepts: social welfare and individual preference. Government policies will no longer be responsible for individual preferences. They are made simply because it is the government's desire to make them. We have never down played this interpretation of the social choice problem. In fact, most social decisions are made in this way. Since there is no group consensus in this interpretation, the 'tyranny' of the group consensus is thus dismissed.

Alternatively, one can follow the competitive economists, and treat social welfare as a secondary concept to be derived from individual preference. This interpretation has the advantage of appealing to the modern concept of 'democracy'. However, the choice of this second interpretation in our analysis is not because of this particular appeal. Rather, it is our conviction that is at stake. If we can prove that even under this individualistic approach, active government policies are necessary to ensure

that society attains its social optimum, then government policies will be required in all cases.

Should the Will of the People Always Prevail?

We believe that we have successfully proved that government actions are indeed necessary to procure the will of the people in the individualistic approach. The question now remains: Can that will of the people be somehow overridden? We will produce two examples where it is necessary to ignore that will. Both of these examples are mainly the results of judgement errors on the part of the people.

Remember the time when it was believed that the earth was flat? And then came Columbus telling people that the earth was in fact round. Had Columbus yielded to public opinion, the earth would, probably, have been thought to be flat until today. Due to insufficient information, individual decision makers are often liable to make decisions which they will certainly regret later on. At this time, the government, with its expertise and information, will have the obligation to make sure that the society does not go astray.

The second example which we want to bring up is concerned with the shape of the utility function of the individual. In economics, utility functions are usually treated as continuous, unique (up to some transformation) and everywhere differentiable. Marginal utility at any point of the functions is supposed to be well defined. This means

that, if upon receiving $\$x$ our individual gets an extra y units of utility, then he will lose y units of utility if he is to give up $\$x$. All too often, our own introspection will tell us this is not the case. The situation is especially vivid with gamblers. The feeling of winning $\$x$ is definitely different from losing $\$x$, and this difference does not only reside in one being a win and the other a loss. There is a difference in intensity, and most people will likely feel more acute about the loss.

Some economists have attributed this type of behaviour to the shape of the utility function. As a result, attempts have been made to bend it to suit what had been observed. However, it was soon found out that in order to accommodate all, or even most, observations, the origin of the co-ordinates of the utility function would have to move with the *status quo* every time.<14>

In our opinion, the problem on hand is not one that can be resolved, by using a traditional utility function. More drastic changes would have to be made. Specifically, continuity and uniqueness would have to go. It is not the utility function that bends. Rather, there is a tendency for people to attach a greater significance to something which is in their possession, and which they have to give up. Such psychology is in no way restricted to stochastic events alone. In the realm of certainty, people are overtly displaying similar behaviours. To prove our point, one has only to look at the way people are willing to fight for things which are already in their possession. In contrast,

few would dare challenge others for something which they do not own in the first place. Note that law and order is not at issue here, although behaviours of this sort would certainly enhance it.

It appears to us that all those apparently erratic behaviours can be more appropriately explained by re-assessing the concept of marginal utility. Essentially, for any good x , why should the marginal utility of x be the same in either direction? Can we have marginal utility of x less than the marginal utility of $-x$? And if that is the case, then for any status quo S , our individual's utility function would be discontinuous at S .

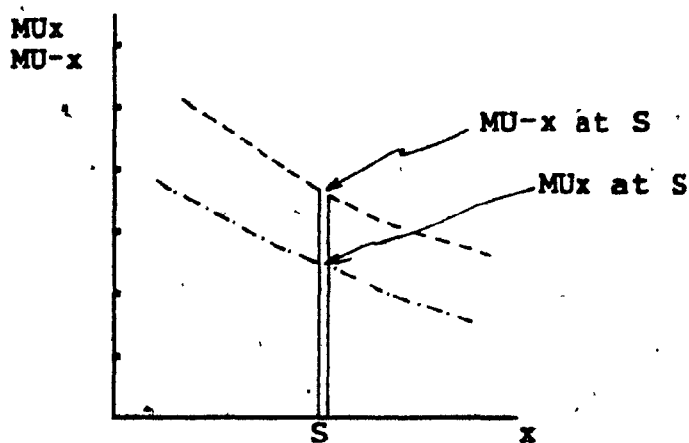


fig. 8.1

As a result, our individual's preference would have to be represented by two or even more utility functions. There is no doubt that the problem of social choice will be made much more difficult with these complications. Nonetheless, we

still believe that a solution is possible although we have no intention to pursue it any further. Our main concern in the whole issue is with the possible reactionary bias that may ensue. When there is a tendency for everyone to value more highly what they have to give up than what they are about to receive, one can hardly argue that there is no bias against impending adjustments. Under such circumstances, any *status quo* will have an edge over all other situations. Understandably, social re-distributions will be more difficult to justify. Even if people act according to their *bona fide* moral conscience, the obstacles created by their unconscious 'exaggeration' of the *status quo* will almost surely make change improbable. However, once people are freed from their former *status quo*, they will start to acquire a different attitude. The new situation will now assume the position of the old one. And when people look back from the stand-point of the present *status quo*, they will readily recognize the improvement in social welfare. The abolition of monarchies, the entrenchment of minority rights, and the liberation of women are among the prominent examples in this respect. Since these reactionary biases, when left alone, will often prevent the society from attaining its welfare optimum, the government must therefore exert active efforts to ensure the choices made by the society are in the best interest of the people even if it may mean temporary departures from the consensus of the society.

IX

Conclusion

The time is now opportune for us to recount what we have accomplished, or have attempted to accomplish in the previous chapters. We stated at the very outset that our primary objective is to justify, from the perspective of individualism and within the confines of economics, public policies that are directed towards enhancing social welfare. Such an objective, however, is incompatible with economic positivism. The latter, being renowned for its suppression of value judgements in economic analyses, has virtually made it impossible for anyone to make anything out of the notion of social welfare. Hence, the first step in fulfilling our objective would necessarily involve the dismissal of economic positivism and this is exactly what we sought to do at the beginning of our study.

The majority of economists in the past few decades are positivists and quite a number of them had tried to overcome the dilemma of value neutrality by devising social choice rules that are free of value judgements. More specifically, what they have tried to do is to bypass interpersonal comparisons used by early utilitarians. The different versions of compensation tests are notable examples of these efforts. For a while, these tests paved the way for a sense of optimism in the profession. Some economists even thought that these tests could actually generate a complete ordering on the set of alternatives states. Fortunately, this state of

affairs did not last long. In 1941, Scitovsky pointed out that it is possible for the Hicks-Kaldor test to be inconsistent if one reverses the order of evaluation on the two alternative states. In the same article, Scitovsky also proposed replacing the Hicks-Kaldor criterion by a 'double-criterion.' However, it was later found out that even this Scitovsky criterion is insufficient to deliver a consistent ordering.

Two other major attempts were made by Samuelson and Little to restore consistency to the compensation principle. Nonetheless, neither of them was successful. We have shown that their failures were due to the piecemeal nature of the compensation tests. Consistency is a condition that requires the consideration of all alternative states simultaneously. However, the compensation test takes into account only two situations at one time. In the absence of a unified evaluation scheme, we have argued that, the piecemeal judgements like the compensation tests are inherently inconsistent, and one should not be surprised to learn that none of them is capable of constructing a complete ordering on the set of alternative states.

Since piecemeal principles cannot guarantee a consistent ordering of the alternative states, the next natural thing for us to do is to seek a solution from the all-encompassing social decision rules. We noted, in this connection, that a stumbling block that needs to be removed was Arrow's Impossibility theorem. With four extremely appealing postulates, this theorem rules out any possibility of

constructing a general group decision rule for any society.

We pointed out that the postulate of irrelevance of infeasible alternatives is the one that does the trick in Arrow's Theorem. Moreover, we have also demonstrated how the exclusion of cardinal utility by this postulate leads to the impossibility result. Recent studies have shown that even the traditional Bergson-Samuelson social welfare function has not escaped this theorem. To invoke Arrow's result on the B-S social welfare function, all one has to do is to postulate a similar axiom for the latter to assure that no utility function of the cardinal type is allowed. These discussions made it clear to us that cardinal utility is one of the key issues, and in our opinion, an indispensable factor, in the construction of a group decision rule.

In the literature, cardinal utility is axiomatized under stochastic or non-stochastic events. We rejected the stochastic approach because we do not think that its Independence axiom is reasonable. In addition, we also feel that we are actually putting the cart before the horse by axiomatizing cardinal utilities under stochastic events and then applying these utilities to non-stochastic events. To substantiate our claims, we have provided a detailed discussion for both cases. In particular, we have explained why we favour cardinal utility over ordinal utility and how cardinal utility functions can be constructed.

Although cardinal utility is necessary, it alone is not sufficient to generate a group decision rule. Some form of interpersonal comparison is also necessary. In chapter 7, we

have tried to enlist and examine every kind of interpersonal comparisons in use. In the end, we decided to go all the way to settle for the ratio-scale comparison. Moreover, we also proposed the use of painless suicide to provide a common zero point for everyone's utility function. As for the other point in the utility function which is necessary for interpersonal comparison, we suggested the method of 'extended consensus.' It was argued that with the zero point and the 'extended consensus,' one will be able to conduct all relevant interpersonal comparisons.

In fact, we even went one step further than just making interpersonal comparisons. By redefining utilitarianism more broadly, we were able to include almost everything, which an individual feels, into his utility function. As a result, we argued that if a society is willing to maximize its social welfare, then there will exist a unique social function preferred by the society as a whole, and this function will deliver a solution to our problem of social choice.

Understandably, an enormous amount of work would still be needed if one really wants to put our proposed solution to test. It is our wish that this study of ours might initiate some subsequent research in the same direction that could eventually solve the problem of social choice. In the meantime, let us be content with the possibility of interpersonal comparisons and its profound impact on government policies. Lastly, it is our belief that one day, with our collective will and co-operation, we might be able to rectify as well as reverse all the injustices that Nature

has done to the human race. This is certainly an ambitious prophecy and its fulfilment will depend solely on the support of all mankind. Hence, let us not indulge ourselves anymore in those trifle non-altruistic doctrines of 'natural liberty.' Rather, what we should do is to act swiftly to bring back some significance to words like 'fairness,' 'equality,' and 'justice.'

FOOTNOTES

CHAPTER II

1. For a recent critique of this school as applied in economics, see D.N. McCloskey, "The Rhetoric of Economics," *Journal of Economic Literature* vol. 21 (1983): 481-517.
2. It will be discussed later in this chapter why the Paretian criterion is a value judgement.
3. M. Friedman, *Essays in Positive Economics*, ch. 1.
4. *Ibid.*, p. 7.
5. K. Popper, *The Logic of Scientific Discovery*. Eng. trans. N.Y: Harper, 1959.
6. M. Friedman, *op. cit.*, p. 9.
7. M. Friedman, *ibid.*, p. 14.
8. R.M. Cyert and J.G. March, *A Behavioral Theory of the Firm*, p. 299.
9. R. Shone, *Microeconomics: A Modern Treatment*, 1975, pp. 4-6.
10. R. Stoll, *Set Theory and Logic*, 1963, p. 452.
11. This terminology was introduced by N. Georgescu-Roegen in *The Entrophy Law and the Economic Process*, 1971, pp. 43-44.
12. An ordering is a relation that is reflexive, transitive and complete.
13. K.O. May, "Intransitivity, Utility and the Aggregation of Preference Patterns," *Econometrica* vol. 22 (1954): 1-13. See also N. Georgescu-Roegen, "Choice, Expectations and Measurability," *Quarterly Journal of Economics* vol. 68 (1954): 503-534.
14. W.E. Armstrong, "The Determinateness of the Utility Function," *Economic Journal* vol. 49 (1939): 453-67.

15. What we are here referring to is 'local' non-satiation. In contrast to 'local' there is 'global' non-satiation. However, it was never seriously considered by anyone.
16. Gary Becker had made an effort to incorporate irrational behaviours into the system. However, his model was based on the assumption that irrational behaviours are distributed normally among the population and the average behaviour of the population as a whole is rational. When one makes these two assumptions, one is invoking the law of large numbers and it is no different from the representative man in Marshall. Becker insisted that his model is not the same as Marshall's but we do not understand why. See G. Becker, "Irrational Behaviour and Economic Theory," *Journal of Political Economy* vol. 70 (1962): 1-13.
17. Let $V_1(x_1, y_1)$, $V_1(x_1, R_1, y_1)$ and XRY be a , b , c respectively. Given $a \Rightarrow b \Rightarrow c$, we will have $(a \Rightarrow b \Leftrightarrow b \Rightarrow c)$ and therefore $(a \Rightarrow b) \Leftrightarrow (b \Rightarrow c)$. The equivalence of the non-satiation condition and the Pareto criterion will hold only if we assume both conditions in the first place.
18. Of course, one can argue that the same kind of reasoning we have applied to economic assumptions can also apply to the axioms in logic. However, that does not mean that we cannot put more faith in the logical system. According to Quine, there may be only one general body of knowledge, but we surely can consider some knowledge as occupying the centre of this body while others have barely made it to the periphery. When recalcitrant experiences occur, the system will usually make adjustments at the periphery and leaving the centre intact. There is little doubt that logical laws are in that centre and should be regarded as positive statements if we ever decide to recognize the existence of such statements. For a more detailed discussion, see W.V.O. Quine, "Two Dogmas of Empiricism", in *From a Logical Point of View*, 1961.
19. R.M. Hare, *Freedom and Reason*, Oxford, Clarendon Press, 1963.
20. *Ibid.*, p. 23.
21. See for example R. Rorty, *Philosophy and the Mirror of Nature*. Princeton: Princeton University Press, 1979.

CHAPTER III

1. L. Robbins, "Interpersonal Comparisons of Utility: A Comment," *Economic Journal* vol. 48 (1938): 637.
2. E.J. Mishan, "A Survey of Welfare Economics," in *Surveys of Economic Theory* Vol. 1, p.167.
3. There are plenty of examples. G. Debreu in his "Valuation Equilibrium and Pareto Equilibrium," (*AER Reading in Welfare Economics*, ed. by Arrow and Scitovsky, pp.39-45) has established the equivalence of valuation equilibrium and Pareto equilibrium; Duncan Foley even pushed this result to include the public sector in "Resource, Allocation and The Public Sector," *Yale Economic Essays* vol. 7 (1967): 45-98. See also Arrow and Hahn, *General Competitive Analysis*, for an exposition, of how the core of the competitive equilibrium approaches the set of Pareto equilibrium when the number of individuals approaches infinity.
4. The idea is propounded by John Rawls, and it can be found in his book *A Theory of Justice*, 1970.
5. S. Alexander, "Social Evaluation through Notional Choice," *Quarterly Journal of Economics* vol. 88 (1974): 611.
6. This point about primary good index will be discussed in more detail in chapter 7.
7. J. Rawls, *op. cit.*, p. 6.
8. N. Kaldor, "Welfare Propositions of Economics and Interpersonal Comparisons of utility," *Economic Journal* vol.49 (1939): 549.
9. The phrase 'an increase in physical productivity' was stressed by Kaldor though he immediately confused it with an increase in aggregate real income. *Ibid.*, p.550.
10. We will be examining Samuelson's criterion later on. As a preview, let us note that Samuelson has recommended that a situation x should be preferred to another situation y if, and only if, the utility possibility frontier of y is totally dominated by that of x. For that to happen, x must have more of every commodity when compared to y. One should realize that this is exactly what is demanded in the previous footnote and if this is what Hicks and Kaldor had in mind, we would have no other choice but, to consider Samuelson's effort as repetitive.

11. See the exchange of notes in *Economic Journal* vol. 72 (1962) among Robertson, Meade, Little, Mishan, and in the subsequent year Sen. They have interpreted $D(x)$ in different ways and we will be discussing later on in this chapter.
12. J. de V. Graaff, *Theoretical Welfare Economics*, p.59.
13. "Speaking vaguely we may say that the cardinal number of a set is the property that the set has in common with all sets equivalent to it," from P.R. Halmos, *Naive Set Theory*, 1960. Interested readers may take a look at section 22, 23, 24 and 25.
14. Examples of these economists include Samuelson with his "Evaluation of Real National Income," *Oxford Economic Papers* vol. 2 (1950): 1-29; R. Baldwin, "A Comparison of Welfare Criterion," in *The Review of Economic Studies* vol. 21 (1954):154-167, and M.J. Bailey, "The Interpretation and Application of the Compensation Principle," in *Economic Journal* vol. 64 (1954): 39-52. See also E. Mishan, "The Resolution of a Paradox," *Economic Journal* vol. 83 (1973): 745.
15. E. Mishan, "Welfare Criteria: An Exchange of Note, A Comment," *Economic Journal* vol. 72 (1962): 238.
16. J. de V. Graaff, *op. cit.*, p. 49.
17. Hicks, Samuelson, and Little all use this method to conduct their real national income analyses.
18. J.R. Hicks, "The Foundation of Welfare Economics," *Economic Journal* vol. 49 (1939): 700.
19. J.R. Hick, "The Valuation of The Social Income," *Economica* vol. 46 (1980): 111.
20. See J. Rothenberg, *The Measurement of Social Welfare*, 1961, chapters 3 and 4.
21. I.M.D. Little, "The Foundation of Welfare Economics," *Oxford Economic Papers* vol. 2 (1950): 228, or Rothenberg, chapter 3. By 'gestalt theories' we mean nothing more than those theories which advocate a substantive identity for social welfare.
22. T. Scitovsky, "A Note on Welfare Propositions in Economics," *The Review of Economic Studies* vol. 9 (1941): 77-88.
23. E. Mishan, "The Compensation Principle' Reconsidered," *Journal of Political Economy* vol. 60 (1952): 318.

24. See W.M. Gorman, "The Intransitivity of Certain Criteria Used in Welfare Economics," *Oxford Economic Papers* vol. 7 (1955): 25-35 for a diagrammatic exposition of the intransitivity problems in Hicks-Kaldor and Scitovsky's double criterion.
25. For example, see K. Suzumura, "On Distributional Value Judgements and Piecemeal Welfare Criteria," *Economica* vol. 48 (1980): 125-139.
26. A few words on notation: In this section, we are using lower case x, y to denote elements in the utility space and upper case X, Y to stand for corresponding elements in the commodity space.
27. One should distinguish between the Samuelson criterion, as discussed here, and the Social Welfare Function as propounded by Samuelson. The latter, of course, embraces much more value judgements than just 'physical feasibilities.'
28. J. de V. Graaff, *op. cit.*, p. 92.
29. See Kenneth Arrow, "Little's Critique of Welfare Economics," *American Economic Review* vol. 41 (1951): 927.
30. D. Robertson, "Welfare criteria: An exchange of Notes," *Economic Journal* vol. 72 (1962): 227.
31. I.M.D. Little, "Welfare Criteria: An Exchange of Notes," *Economics Journal* vol. 72 (1962): 230.
32. I.M.D. Little, *A Critique of Welfare Economics*, p.275.
33. This is because of Little's assumption B, a move to H is also available and if H is distributionally better than Q_2 , the move will by itself sufficient for an improvement.
34. J. Meade's review of Little's Critique in *Economic Journal* vol. 69 (1959): 125. C.M. Kennedy, "The Economic Welfare Function and Dr. Little's Criterion," *The Review of Economic Studies* vol. 21 (1953): 137-42.
35. I.M.D. Little, "Two comments," *Economic Journal* vol. 73 (1963): 778.
36. A.K. Sen, "Distribution, Transitivity and Little's Welfare Criteria," *Economic Journal* vol. 73 (1963): 772-3.
37. *Ibid.*, p. 773.

38. *Ibid.*, p. 772.
39. See A.K. Sen, **Collective Choice and Social Welfare**, 1970, ch.1*.
40. I.M.D. Little, "Exchange of Notes," **Economic Journal** vol. 72 (1962): 231.
41. Based on cardinality and similar distribution, Mishan and Ng. have proved that Little's criterion is inconsistent and even violates the Pareto condition. See E. Mishan, "An Exchange of Note," **Economic Journal** vol. 72 (1962): 234-244, and Y.K. Ng, "Little's Welfare Criterion Under the Equality Assumptions," **The Economic Record** vol. 47 (1971): 579-83. For an attack of Little's position see Arrow, "Little's Critique...", **American Economic Review** vol. 41 (1951): 928, and D. Robertson, "Exchange of Notes," **Economic Journal** vol. 72 (1962): 229. Little's defence appears also in **Economic Journal** vol. 72 (1962): 231. In passing, we want to mention that there are efforts to establish a trade-off between income and distribution. The major proponent of this thesis is F.M. Fisher, "Income Distribution, Value Judgements and Welfare," **Quarterly Journal of Economics** vol. 70 (1956): 380-24. Disregarding his use of money income as real income, the approach misses the point which Little tries to make, i.e. to separate income from distribution. Mishan's idea to resort everything to distributional judgement in his articles, "A Resolution of a Paradox," **Economic Journal** vol. 83 (1973) is simply ridiculous. With his manipulation of the Scitovsky curve, any two situations will have the same real income level.

CHAPTER IV

1. G. Debreu, "Representation of a Preference Ordering by a Numerical function," in Thrall, Coombs and Davis (eds.), *Decision Process* (New York: Wiley, 1954). The Representation theorem is not undisputed. Using a theorem in lattice theory that is similar in substance to the Schroder-Bernstein theorem in set theory which we have introduced earlier, Georgescu-Roegen argued that the density of our wants is higher than that of the real number system so that the former cannot be represented by a real-valued utility function. This argument appears in "Choice, Expectations, and Measurability," *Quarterly Journal of Economics* vol. 68 (1954) 503-34.
2. P.A. Samuelson, *Foundations of Economic Analysis*, p. 222.
3. A. Bergson, "A Reformulation of Certain Aspects of Welfare Economics," *Quarterly Journal of Economics* vol. 52 (1938) 310-34. Reprinted in *AEA Readings in Welfare Economics*, pp. 7-25.
4. *Ibid.*, pp. 20-1.
5. P.A. Samuelson, *op. cit.*, p. 226.
6. Simple examples can be found in A.K. Sen, *On Economic Equity*. See especially his Weak Equity Axiom. For a short argument for non-linear social welfare function, one can see P. Diamond, "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility: A Comment," *Journal of Political Economy* vol. 75 (1967): 765-6.
7. J.C. Harsanyi, "Bayesian Decision Theory and Utilitarian Ethics," *American Economic Review* vol. 68 (1978): 223-8; "Non linear Social Welfare Functions," *Theory and Decision* vol. 6 (1975): 311-32; "Cardinal Welfare, Individualistic Ethics and Interpersonal Comparisons of utility," *Journal of Political Economy* vol. 63 (1955): 309-21.
8. J.C. Harsanyi (1975), p. 320.
9. E. Maskin, "A Theorem on Utilitarianism," *The Review of Economic Studies* vol. 43 (1976): 93-6. A more detailed discussion on full and partial comparability will be given in Chapter 7.
10. *Ibid.*, p. 94.

11. Although the B-S function has usually been identified by most as a real-value function, it is absolutely legitimate for it to take up other forms and still stay within the original formulation of Bergson and Samuelson. Hence the continuity requirement is basically a matter of convenience rather than necessity.
12. It will soon be clear why the B-S social welfare function is a single profile function.
13. P.A. Samuelson, "Arrow's Mathematical Politics," in *Human Value and Economic Policy*, ed. by S. Hook, 1966; "Reaffirming the Existence of 'Reasonable' Bergson-Samuelson Social Welfare Functions," *Economica* vol. 44 (1977): 81-8.
14. This name was suggested by M. Kemp and A. Asimakopulos in "A Note on 'Social Welfare Functions' and Cardinal Utility," *Canadian Journal of Economics and Political Science* vol. 18 (1952): 195-200, and adopted by Arrow in the second edition of his book.
15. In his book, *The Theory of Social Choice*, 1973, P. Fishburn has suggested to use intra and inter profiles to denote single profile and multiprofile comparisons and it has been widely adopted.
16. P.A. Samuelson, (1977), *op. cit.*
17. M. Kemp and Y.K. Ng, "On the Existence of Social Decision Functions," *Economica* vol. 43 (1976): 59-66.
18. For a simple overview, see C.R. Plott, "Axiomatic Social Choice Theory: An Overview and Interpretation," *American Journal of Political Science* 1976, pp. 511-96.
19. A.F. Mackay, *Arrow's Theorem, the Paradox of Social Choice: A Case Study in the Philosophy of Economics*, 1980, p. 121.
20. R. Wilson, "Social Choice Theory Without the Pareto Principle," *Journal of Economic Theory* vol. 5 (1972): pp. 478-86.
21. Again this is Fishburn's terminology. In a way, it is more general than the use of the word 'irrelevant.'
22. M. Kemp and A. Asimakopulos, *op. cit.*, and J.C. Weldon, "On the Problem of Social Welfare Functions," *Canadian Journal of Economics and Political Science* vol. 18 (1952): 452-463.
23. B. Hannson, "The Independence Condition in the Theory of Social Choice," *Theory and Decision* vol. 4 (1973): 25-49.

24. A.K. Sen, *Social Welfare and Collective Choice*, 1970.
25. K. Basu, *Revealed Preference of Government*, ch. 6, 1980.
26. D. Black, "On the Rationale of Group Decision-Making," *Journal of Political Economy* vol. 56 (1948): 23-34.
27. P.A. Samuelson (1966), *op. cit.*
28. P.A. Samuelson, *Foundations of Economics Analysis*, p. 223.
29. *Ibid.*, p. 229.
30. P.A. Samuelson (1966), *op. cit.*, p. 49.
31. *Ibid.*, p. 49.
32. P.A. Samuelson (1977), *op. cit.*, p. 85.
33. M. Kemp and Y.K. Ng, "More on Social Welfare Functions: The Incompatibility of Individualism and Ordinalism," *Economica* vol. 44 (1977): 89-90.
34. P. Parks, "An Impossibility Theorem for Fixed Preferences: A Dictatorial Bergson-Samuelson Welfare Function," *The Review of Economic Studies* vol. 44 (1976): 447-50; M. Kemp and Y.K. Ng (1976), *op. cit.*; R.A. Pollak, "Bergson-Samuelson Social Welfare Functions and The Theory of Social Choice," *Quarterly Journal of Economics* vol. 93 (1979): 73-90.
35. M. Kemp and Y.K. Ng (1976), *op. cit.*, p. 60.
36. P.A. Samuelson (1977), *op. cit.* He argues that neutrality is not acceptable because it is amoral. For if we favour a particular individual in a certain situation, we will have to favour him again in other situations if the preference profile is similar to the first situation even though the characteristics of the individual may have undergone drastic changes.

CHAPTER V

1. For a lucid exposition of the underlying arguments, see D.H. Robertson, *Utility and All That and Other Essays*, George Allan & Unwin, 1952, pp. 13-41; "Utility and All What," in his *Economic Commentaries*, pp. 42-58, Staples Press Ltd., London, 1956.
2. P.A. Samuelson, "The Problem of Integrability in Utility Theory," *Economica* vol. 17 (1950): 355. The comment was made by Samuelson right after Houthakka had come up his 'strong axiom' to deal with the problem of integrability in revealed preference; see also H.S. Houthakka, "Revealed Preference and the Utility Function," *Economica* vol. 17 (1950): 165.
3. For a critical review of the approach, see A.K. Sen, "Behaviour and the Concept of Preference," *Economica* vol. 40 (May, 1973): 241-259.
4. P.A. Samuelson, "Probability and the Attempts to Measure Utility," *The Economic Review*, Tokyo (1950). Reprinted in *The Collected Scientific Works of P.A. Samuelson*, pp. 117-26.
5. Samuelson was probably talking about objective probability. A distinction between objective and subjective probability will be given later in this chapter.
6. M. Allais, *The Foundations of a Positive Theory of Choice Involving Risk*. English translation in *Expected Utility Hypothesis and the Allais Paradox*, ed. by M. Allais and O. Hagen, p. 39.
7. *Ibid.*, p. 40 and P.A. Samuelson, *op. cit.*, p. 119.
8. In another direction, the Absolute Preference Axiom is regarded by many as too weak. Essentially, this axiom is equivalent to what has been known as Stochastic Dominance of the First Degree. See K. Borch, "Utility and Stochastic Dominance," in *Expected Utility Hypothesis and The Allais Paradox*, ed. by M. Allais and O. Hagen. One can also postulate stochastic dominance of higher degrees, e.g., third degree stochastic dominance as advocated by G.A. Whitmore in "Third-Degree Stochastic Dominance," *American Economic Review* vol. 60 (1970): 457-459. As a general rule, stochastic dominance of a higher degree is entailed by that of lower degrees.
9. P.A. Samuelson, *op. cit.*, (1950) postscript, p. 124.

10. von Neumann and O. Morgenstern, *Theory of Games and Economic Behaviours*, p. 19.
11. W. Edwards, "The Theory of Decision Making," *Psychological Bulletin* vol. 51 (1954): 392.
12. N. Georgescu-Roegen, "Utility, Expectation, Measurability, Prediction," paper read at Econometric Society, Kingston, September 1953.
13. J. Marschak, "Rational Behaviour, Uncertain Prospects, and Measurable Utility," *Econometrica* vol. 18 (1950): 111-141; L.J. Savage, *The Foundation of Statistics*, 1954; P.A. Samuelson, "Probability, Utility, and The Independence Axiom," *Econometrica* vol. 20 (1952): 670-78.
14. Actually, according to Friedman and Savage in "The Expected Utility Hypothesis and The Measurability of Utility," *Journal of Political Economy* vol. 60 (1952): 463-474, the von Neumann-Morgenstern formulation achieves even more than the others.
15. M. Friedman and L.J. Savage, "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy* vol. 56 (1948): 279-304. Reprinted in *AEA Readings in Price Theory*, pp. 61-2.
16. M. Friedman and L.J. Savage, *op. cit.*, (1952), p. 464.
17. P.A. Samuelson, *op. cit.*, (1950), p. 126.
18. D. Ellsberg, "Classic and Current Notions of 'Measurable' Utility," *Economic Journal* vol. 64 (1954): 528-556. Reprinted in *Utility Theory: A Book of Readings*, ed. by A.N. Page, p. 285.
19. M. Friedman and L.J. Savage (1952), p. 464.
- 120 L.J. Savage, *op. cit.*, p. 21.
21. P.A. Samuelson, "Utility, Preference, and Probability," in *The Collected Scientific Works of P.A. Samuelson*, p. 133.
22. The modern expected utility school was inspired by a paper written by D. Bernoulli in the sixteenth century. That paper was translated into English by L. Sommer as "Exposition of a New Theory on the Measurement of Risk," *Econometrica* vol. 22 (1954): 23-36. Due to this fact, many have referred to the expected utility school as the neo-Bernoullian school.
23. E. Malinvaud, "Note on von Neumann-Morgenstern's Strong Independence Axiom," *Econometrica* vol. 20 (1952): 679.

24. von Neumann and O. Morgenstern, *op. cit.*, p. 26.
25. J. Marschak, *op. cit.*
26. H. Raiffa, "Risk, Ambiguity, and the Savage Axioms: Comment" *Quarterly Journal of Economics* vol. 75 (1961): 690.
27. J. Marschak, *op. cit.*, p. 112.
28. H. Raiffa, *op. cit.*, p. 691.
29. Again this will be discussed in chapter 7.
30. P.A. Samuelson (1952), p. 672.
31. von Neumann and O. Morgenstern, *op. cit.*, p. 682.
32. von Neumann and Morgenstern think that "the simplest procedure is, therefore, to insist among the alternatives, perfectly will founded interpretation of probability as frequency in long run." *Ibid.*, p. 19.
33. D Ellsberg, "Risk, Ambiguity, and The Savage Axioms," *Quarterly Journal of Economics* vol. 75 (1961): 651-653.
34. *Ibid.*, p. 652.
35. L.J. Savage, *op. cit.*, chapter 3.
36. W. Fellner, "Distortion of Subjective Probabilities as a Reaction to Uncertainty," *Quarterly Journal of Economics* vol. 75 (1961): 676.
37. W. Edwards. *op. cit.*, pp. 395-6.
38. J. Handa, "Risk, Probabilities, and a New Theory of Cardinal Utility," *Journal of Political Economy* vol. 85 (1977), pp. 97-122 and pp. 99-108 in particular.
39. For a formal proof, see P. Fishburn, "On Handa's 'New Theory of Cardinal Utility' and the Maximization of Expected Return," *Journal of Political Economy* vol. 86 (1978): 321-24.
40. Fellner, in the above cited article, points out that "the operational utility functions shows some combination of the properties of the 'pure' utility-of-wealth function with impurities caused by the utility or disutility of gambling....there exists no dependable operational method for separating the pure utility-of-wealth function from the impurity." (p. 682)
41. W. Edwards, *op. cit.*, p. 400.

42. M. Friedman and L.J. Savage, *op. cit.*, (1948), p. 88.

CHAPTER VI

1. This name was first adopted by J.C. Weldon in "A Note on Measures of Utility," *Canadian Journal of Economics and Political Science* vol. 16 (1950): 230. It was later picked up by D. Ellsberg in "Classical and Current Notions of 'Measurable Utility'," *Economic Journal* vol. 64 (1954): 528-56. As Weldon puts it, the term "is intended to signify no more than that Jevons assumed that preferences could be directly compared...."
2. Pareto admits that if an individual can compare first-order preference differences, it "would be enough to permit us to consider the pleasure or ophelimity as a quantity," *Manuel of Political Economy*, p. 191, translated by Ann S. Schwier, and ed. by Ann S. Schwier and Alfred N. Page.
3. As O. Lange puts it in "The Determinateness of the Utility Function," *The Review of Economic Studies* 1934, p.224, "the assumptions underlying postulate (2) is not capable of expression in terms of acts of choice"
4. P. Suppes and M. Winet, "An Axiomatization of Utility based on the Notion of Utility Differences," *Management Science* vol. 1 (1955): 259-70.
5. J.C. Weldon, *op. cit.*, p. 228.
6. Immediately after eliciting the Jevonsian method, Pareto wrote, "But it is not possible for us to attain that precision...he can in no way tell what quantity of wine he must drink after the second glass in order to get pleasure equal to that which the second glass of wine provided him," *Manuel of Political Economy*, p. 191.
7. W.E. Armstrong, "The Determinateness of the utility Function," *Economic Journal* vol. 49 (1939): 453-67.
8. N. Georgescu-Roegen, "Choice, Expectation and Measurability," in *Analytic Economics*, p. 201.
9. We have discussed on this point in earlier exposition. Interested readers are advised to see K.O. May, "Intransitivity, Utility, and aggregation in preference patterns," *Econometrica* vol. 22 (1954): 1-13.
10. H. Bernardelli, "Notes on the Determinateness of the Utility function," *The Review of Economic Studies* vol. 2 (1935): 74.
11. N. Georgescu-Roegen, *op. cit.*, p. 202. C^1 , C^2 and C^3 can be viewed as commodity bundles.

12. F. Knight, "Realism and Relevānce in the theory of Demand," *Journal of Political Economy* vol. 52 (1944): 298.
13. H. Bernerdelli, op. cit., p. 72.
14. K. Basu, *Revealed Preference of Government*, 1980, ch. 6.
15. Isotonic group includes all those functions that are equivalent up to any monotonic transformation and linear group includes all those functions that are equivalent up to any linear transformation. See S.S. Stevens, "Measurement, Psychophysics and Utility," in *Measurement: Decisions and Theories*, ed. by Churchman and P. Ratoosh, 1959, p. 25.
16. C.H. Coombs's works are cited by S.S. Stevens in his article in footnote 15 and by W. Edwards in "Theory of Decision Making," *Psychological Bulletin*, vol. 51 (1954): 380-417. As interpreted by Stevens, "the ordered metric scale is said to lie between the ordinal and the cardinal scales. It assumes that we have an operation for ordering objects on a continuum and also for ordering the intervals between the objects," p. 34.
17. S.S. Stevens, *ibid.*; pp. 35-36.
18. O. Lange, "The Determinateness of the Utility Function," *The Review of Economic Studies* vol. 1 (1934): 218-225; "Notes on the Determinateness of the Utility Function," *The Review of Economic Studies* vol. 2 (1935): 75-77.
19. "It (the proof) is deficient for it gives only a necessary but not sufficient condition that G be a utility function if $\emptyset$ is one this proof is unsatisfactory. For....it is based on some of the material properties of the utility function, i.e., the property that its first partial derivatives are positive and the property that the second partial derivatives of the utility function cannot be expected to be all of the same sign," *ibid.*, 1935, p. 75.
20. O. Lange, *ibid.*, p. 75.
21. J.S. Mill, *Utilitarianism*, reprinted in *John Stuart Mill: A Selection of His Works*, ed. by J.M. Robson, p. 159.
22. The issue of 'infinity' in utility will be dealt with in more detail in chapter 7.
23. E.H. Phelps Brown, "Notes on the Determinateness of the Utility Function," *Review of Economic Studies* vol. 2 (1935): 66-9.

24. E.H. Phelps Brown, *ibid.*, p. 67.
25. P.A. Samuelson, "The Numerical Representation of Ordered Classifications and the Concept of Utility," *The Review of Economic Studies* vol. 6 (1938): 66.
26. P.A. Samuelson, *ibid.*, p. 68.
27. The minus sign here is the infix operator for the difference between two sets.
28. See for example, P. Suppes and M. Winet, *op. cit.*, p. 261; G. Debreu, "Topological Methods in Cardinal Utility Theory," in *Mathematical Methods in Social Sciences*, ed. by K. Arrow et. al., 1959, pp. 16-26; A. Camacho, "Societies and Social Decision Function," in *Developments in the Methodology of Social Sciences*, ed. by Leinfellner and Kohler, 1974, pp. 217-53 and "On Cardinal Utility," in *Theory and Decision* vol. 10 (1979): 131-45.
29. W. Edwards, *op. cit.*, p. 384.
30. A well behaved utility function is generally taken as one that is everywhere differentiable with $u' > 0$ and $u'' < 0$. Examples are too numerous to cite and the readers are urged to pick up any neo-classical journal to verify our allegation themselves. To see why an ordinal utility function may not be differentiable one has only to realize that differentiability implies continuity and an ordinal function needs not be continuous.
31. P.A. Samuelson, *op. cit.*, p. 66. In fact differentiability is just like the foundation of *The Foundations*. Could one imagine what would *The Foundations of Economic Analysis* be liked without all its first and second order derivatives!
32. G. Debreu, *op. cit.*, p. 16. The work quoted by Debreu is by W. Blaschke, "Topologische Fragen der Differentialgeometrie I," *Mathematische Zeitschrift* 1928, pp. 150-57. In fact the first topological proof was achieved by P. Suppes and M. Winet. The credit given to Debreu is mainly for his explicit indication of the shortcomings of differentiability.
33. F. Alt, "Über die Messbarkeit des Nutzens," *Zeitschrift für Nationalökonomie*, 7, 2. Heft 1936, pp. 161-69. Translated into English by Siegfried Schach as "On the Measurability of Utility," in *Preferences, Utility, and Demand*, ed. by J.S. Chipman et. al., 1971, pp. 424-31.
34. G. Debreu, "Stochastic Choice and Cardinal utility," *Econometrica* vol. 26 (1958): 440-44.

35. J.C. Weldon, *op. cit.*, pp. 228-229.
36. There is one minor point that will be discussed later.
37. F. Alt, *op. cit.*, p. 427.
38. R. Frisch, "Sur un Probleme d'economie Pure," *Norsk Matematisk Forenings Skrifter, Serie 1, Nr.16, Oslo: Grøndahl & Sons Boktrykkeri 1926*, pp.1-40. Translated by J.S. Chipman in *Preferences, Utility, and Demand*, p. 389.
39. M. Allais, "The 'Allais Paradox' and Rational Decisions," in *Expected Utility Hypotheses and the Allais Paradox*, ed. by M. Allais and o. Hagen, pp. 438-439.

CHAPTER VII

1. What we are presenting is nothing spectacular. In fact, it is a mere reiteration of something that has already been standardized in the literature. For a more thorough picture of these definitions, the reader is advised to consult A.k. Sen, "On Weights and Measures: Informational Constraints in Social Welfare Analysis," *Econometrica* vol. 45 (1977): 1539-1572; P.J. Hammond, "Equity, Arrow's Conditions, and Rawls's Difference Principle," *Econometrica* vol. 44 (1976): 793-804; L. Gevers, "On Interpersonal Comparability and Social Welfare Orderings," *Econometrica* vol. 47 (1979): 75-89.
2. The notion of an identical transformation will be explained later.
3. One can say an apple gives him 3 or 4 units of utility as he likes, but what can 3 and 4 units mean if they are not backed up by an absolute zero. One cannot argue that they are derived from different zeroes because all of the a_i 's are from the same set of linear transformations which presupposes that they share the same zero.
4. I.M.D. Little, *Critique of Welfare Economics*, ch. 4.
5. A.K. Sen, *Collective Choice and Social Welfare*, pp. 99-102. The idea is very likely the result of the development of fuzzy reasoning in mathematics. For a simple exposition, see Hans-Werner Gottinger, "Toward Fuzzy Reasoning in the Behavioural Sciences," in *Developments in the Methodology of Social Science*, ed. by W. Leinfellner and E. Kohler, pp. 287-308. There Gottinger defines "a fuzzy set A in X is characterized by the assignment $X \rightarrow f_A(X)$, $f_A: X \rightarrow M = [0, 1]$ where f_A denotes a generalized characteristic function which assigns to every object $x \in X$ its grade of membership $f_A(x)$," p. 290.
6. Pareto was not the first one to come up with the idea of the consensus requirement, though he surely is responsible for its present popularity in economics. Tracing a little further back there is K. Wicksell who argued for the 'benefit principle' with universal consent. See K. Wicksell, "A New Principle of Just Taxation," translated by J. Buchanan in *Classics in The Theory of Public Finance*, ed. by R. Musgrave and A.T. Peacock, 1958.
7. As defined by Gottinger, $f_A(x)$ is not exact equivalence class. This can be indicated by the existence of 'grades of membership.' On the other hand, Sen did not make such a distinction in his exposition, and presumably anything

that falls within the band would still be considered equivalent.

8. This approach is suggested by C. Blackorby in "Degrees of Cardinality and Aggregate Partial Ordering," *Econometrica* vol. 43 (1975): 848.
9. A.K. Sen, *Collective Choice and Social Welfare*, ch. 7*, pp.108-110.
10. We usually identify the first type of intransitivity as indifference intransitivity, and Sen has termed the second cyclic relation as IP intransitivity. See A.K. Sen, *Collective Choice and Social Welfare*, ch. 1*.
11. B. Fine, "A Note on 'Interpersonal Aggregation and Partial Comparability,'" *Econometrica* vol. 43 (1975): 169-72.
12. Similarly, we do not need to allow the b_i 's to vary without limit to achieve full non-comparability. Since the reasoning is just the converse of what has been discussed, we will leave it as an exercise to the readers. The explanations given here is somewhat heuristic and it is stronger than what is necessary to prove the theorem. Readers who are interested in rigorous proofs are advised to refer to those materials on partial comparability by Sen and Fine cited above.
13. Again, this definition is different from the ones given by Sen and Fine. Nevertheless, it is sufficient to illustrate our point.
14. Writing the Social Welfare function as $R = f(R_1, R_2, \dots, R_n)$ will not contradict our earlier specification which takes the form $R = g(u_1, u_2, \dots, u_n)$. Since $R_i = f_i(u_1, u_2, \dots, u_n)$, $R = f(f_1, f_2, \dots, f_n) = g(u_1, u_2, \dots, u_n)$.
15. The readers are assumed to have general knowledge of the two principles of justice devised by Rawls. For revision, see J. Rawls, *A Theory of Justice*, Part I, or "Justice as Fairness," *Philosophical Review* vol 67 (1958): 164-94 and "Distributive Justice" in *Philosophy, Politics and Society*, third series, ed. by P. Laslett and W.G. Runciman, pp. 58-82.
16. R.J. Arrow, "Extended Sympathy and the Possibility of Social Choice," *AER Papers and Proceedings* vol. 67 (1977): 219-225.
17. R.P. Wolff, "On Strasnick's 'Derivation' of Rawls's 'Difference Principle'," *The Journal of Philosophy* vol. 73 (1976): 850.

18. J. Rawls, *A Theory of Justice*, pp. 91-92.
19. *Ibid.*, p. 94.
20. *Ibid.*, p.440.
21. *Ibid.*, p.303; for a more in depth discussion on 'self-respect, see A.L. Levine, "Rawls and The Utilitarian Decision Rule: A Critique of Arrow's Case for 'Convergence'," paper presented to the Canadian Economics Association's annual meeting, May, 1978.
22. S. Strasnick, "Social Choice and The Derivation of Rawls's Difference Principle," *The Journal of Philosophy*, vol. 73 (1976): 85-99; "The Problem of Social Choice: Arrow to Rawls," *Philosophy and Public Affairs* vol. 6 (1976): 241-273; "Extended Sympathy Comparisons and the Basis of Social Choice," *Theory and Decision* vol. 10 (1979): 311-28; P. Hammond, "Equity, Arrow's Conditions, and Rawls' Difference Principle," *Econometrica* vol. 44 (1976): 793-804; "Why Ethical Measures of Inequality Need Interpersonal Comparisons," *Theory and Decision* vol. 7 (1976): 263-74; Claude D'Aspremont and Louis Gevers, "Equity and the Informational Basis of Collective Choice," *The Review of Economic Studies* vol. 45 (1977): 199-210; R. Deschamps and L. Gevers, "Leximin and Utilitarian Rules: A Joint Characterization," *Journal of Economic Theory* vol. 17 (1978): 143-63; L. Gevers, "On Interpersonal Comparability and Social Welfare Orderings," *Econometrica* vol. 47 (1979): 75-89; K.W.S. Roberts, "Interpersonal Comparability and Social Choice Theory," *The Review of Economic Studies* vol. 48 (1980): 421-39.
23. Some people have abandoned the maximin in favour of Leximin in order to compile the strong Paretian axiom. By leximin we mean that a social decision will be based on the preference of the least well-off non-indifferent individual. See R. Deschamps and L. Gevers, "Leximin and 'Utilitarian Rules: A Joint Characterization," *Journal of Economic Theory* vol. 17 (1978): 143-63.
24. K.J. Arrow., *op. cit.*, 1977.
25. Anonymity requires that if $u(x, i) = u'[x, \lambda(i)]$ for all x and i , then $f(u) = f(u')$, where λ is a permutation of N .
26. Arrow has not really defined the relationship between dictatorship and anonymity. He has only asserted that an implicative relationship exists between the two.
27. If we do not permute their positions, the social ordering of alternatives will change because we have changed the worst-off person's preference and that will

certainly affect social choice.

28. K.J. Arrow, ***Social Choice and Individual Values***, second edition 1963, p. 57.
29. *Ibid.*, p. 58.
30. D.K. Foley, "Resources allocation and the Public Sector," ***Yale Economic Essays*** vol. 7 (1967): 45-98.
31. J.C. Harsanyi, "Cardinal Welfare, Individualistic Ethics and Interpersonal Comparisons of Utility," ***Journal of Political Economy*** vol. 63 (1955): 309-321, reprinted in ***Economic Justices***, ed. by E.S. Phelps, pp.266-285; "Non-Linear Social Welfare Functions," ***Theory and Decision*** vol. 6 (1975): 311-332; "Bayesian Decision Theory and Utilitarian Ethics," ***American Economic Review*** vol. 68 (1978): 223-228.
32. Harsanyi (1975, p. 313 and 1955, p. 283) believes that as information increases, the social welfare functions (extended sympathy) of different individuals will become identical, and therefore the ability will be common to everyone.
33. H.R. Varian, "Equity, Envy, and Efficiency," ***Journal of Economic Theory*** vol. 9 (1974): 63-91.
34. A. Feldman and A. Kirman, "Fairness and Envy," ***American Economic Review*** vol. 64 (1974): 995-1005.
35. E.A. Pazer, "Pitfalls in the Theory of Fairness," ***Journal of Economic Theory*** vol. 14 (1977): 458.
36. E.A. Pazner and D. Schmeidler, "A Difficulty in the Concept of Fairness," ***The Review of Economic Studies*** vol. 42 (1974): 441-433; H.R. Varian, *op. cit.*, p. 72.
37. H.R. Varian, *ibid.*, pp. 72-74; E.A. Pazer, *op. cit.*, 1977, pp. 459-60.
38. H.R. Varian, *ibid.*, theorem 3.2.
39. This criterion was first proved by E. Pazner and D. Schmeidler in "Decentralization, Income Distribution and the Role of Money in Socialist Economies," Technical Report No.8, The Foerder Institute for Economic Research, Tel-Aviv University, 1972. Another proof was supplied by H.R. Varian, *ibid.*, theorem 3.4.
40. D. Schmeidler and K. Vind, "Fair Net Trades," ***Econometrica*** vol. 40 (1972): 637-642.
41. *Ibid.*, p. 637.

42. E.A. Pazner and D. Schmeidler, "Egalitarian Equivalent Allocations: A New Concept of Economic Equity," *Quarterly Journal of Economics* vol. 92 (1978): 671-687.
43. *Ibid.*, p. 672.
44. There are some more suggested definitions and for those who are interested, see T.E. Daniel, "A Revised Concept of Distributional Equity," *Journal of Economic Theory* vol. 11 (1975): 94-109; S.M. Goldman and C. Sussangkarn, "On the Concept of Fairness," *Journal of Economic Theory* vol. 14 (1978): 210-216.
45. I.M.D. Little, *Critique of Welfare Economics*, ch. 4, pp. 51-2.
46. *Ibid.*, p. 61.
47. Here is the same view put in a different way by one of the leading economists: Up to the minimum of subsistence, say, x , the utility of a basic food item is that of life itself and hence, must be infinite; the utility of any amount greater than that minimum also must be infinite. N. Georgescu-Roegen, "Utility," *International Encyclopedia of the Social Sciences*, 1968, p. 241.
48. *Ibid.*, pp. 241-2.
49. If we want to be exact, consumption here can be interpreted as net consumption, and our 'utility' will be equivalent to Ramsey's rate of enjoyment, i.e., utility minus disutility. Since units are preserved under subtraction, Ramsey's rate of enjoyment and our 'utility' must have the same units. See F.P. Ramsey, "A Mathematical Theory of Saving," *Economic Journal* vol. 38 (1928), reprinted in *AEA Readings in Welfare Economics*, ed. by K. Arrow and T. Scitovsky, pp. 620-1.
50. Ramsey was examining an issue different from ours. He was looking at the optimal rate of saving for a community (actually it is an eternal individual) over time. When the number of periods of time approaches infinity, the rate of enjoyment will also approach infinity. On the other hand, we are dealing with a single individual whose life span is finite and it is not necessary for the amount of utility he enjoys during his entire life to be anywhere near infinity. However, what Ramsey refers to as 'Bliss' is not the integral of the rates of enjoyment over time. Rather it is the upper bound of the rate of enjoyment function and that is in agreement with our own idea of 'Bliss.'
51. F.P. Ramsey, op. cit., pp. 620-1. The definition of bliss in Ramsey's article is somewhat different from

ours. According to him, both the finite and infinite rate of enjoyment are referred to as bliss whereas we have restricted the name only to the kind of ultimate fulfillment in the latter category.

52. The idea may sound ridiculous at first but this is exactly what medical science has been contributing to mankind all the time.
53. J.F. Nash, "The Bargaining Problem," *Econometrica* vol. 18 (1950): 155-162.
54. Sen has proved an impossibility result with regard to this point. See A.K. Sen, *Collective Choice and Social Welfare*, ch.8*, pp. 128-30.
55. For a more elaborate discussion on the same point, see A.K. Sen, *ibid.*, ch. 8.
56. W.E. Armstrong, "The Determinateness of The Utility Function," *Economic Journal* vol. 58 (1948): 1-9; "Utility and the Theory of Welfare," *Oxford Economic Papers* vol. 3 (1951): 259-271.
57. W.E. Armstrong (1951), p. 264.
58. *Ibid.*, p. 267.
59. I. Waldner, "Bare Preference and Interpersonal Utility Comparisons," *Theory and Decision* vol. 5 (1974): 319.
60. G. Becker, "Difficulties with Bare Preferences," *Theory and Decision* vol. 70 (1974): 329-331. Becker has pointed out there are three possible interpretations for $(-x)$. Firstly, it can mean the reverse of x . However, he excluded this possibility because not every state will have an exact opposite. "For example, suppose x is a 'ten day, all-expenses-paid trip to New York', or 'a ten day, pay-your-own-way trip to New York, or 'a ten day, all-expenses-paid trip to California'?" (p. 330) Another interpretation of $-x$ is to treat it as "the disjunction of all possible outcomes excluded by x ." Nevertheless, Becker was not satisfied with this explanation either since it would "imply that the individual is totally indifferent between a world where x is realized and any of the innumerable worlds where x is not realized." (p. 331) Subsequently, we were forced to take up the third possibility and that is to regard $(-x)$ as the non-occurrence of x .
61. *Ibid.*, p. 330.
62. G.H. von Wright, "The Logic of Preference Reconsidered," *Theory and Decision* vol. 3 (1972): 140-169.

63. *Ibid.*, p. 163.
64. *Ibid.*, pp. 163-164.
65. Even though the basic argument against the contradicting state may apply to our earlier discussion on infinity, the nature of the problem is totally different. When we were dealing with the infinity issue, we were asking for your approval of our methodology. And if there is no objection on your part, our argument will go through between us. There is no third party involved, and we do not care if there is someone in some corner of this world who finds our idea repellent. In contrast, von Wright is making a hypothesis on every individual's utility function in a society, and whether we accept or reject that hypothesis will of course depend on these people's opinions on the hypothesis.
66. *op. cit.*, p. 164.
67. R.H. Wagner, "Impartiality and Equity," *Theory and Decision* vol. 12 (1980): 61-74.
68. See K. Kuratowski and A. Mostowski, *Set Theory* 1968, pp. 269-273.
69. J. Baechler, *Suicide*, translated by B. Cooper 1979, p. 14.
70. It may be true that we are also quite ignorant of death and especially what will come about after life. Nonetheless as we have discussed before, we know that when we are dead, our brains will cease to function, and we will no longer be able to experience anything in this world. This is the knowledge of death which we have acquired and which is not available to animals, and most of the time when people commit suicide, it is this kind of annihilation they are looking for rather than their desires to meet their Gods earlier.
71. T.S. Szasz, "The Ethics of Suicide," in *Between Survival and Suicide*, ed. by B.B. Wolman and H.H. Krauss, 1976, pp. 183.
72. J. Baechler, *op. cit.*, p. 45.
- See, J.S. Mill, "Utilitarianism," reprinted in *The Utilitarians*, pp. 401-472; J.C. Harsanyi, "Cardinal Welfare, Individualistic Ethics and Interpersonal Comparison of Utility," *Journal of Political Economy* vol. 63 (August, 1955): 309-321.
74. L. Robbins, "Roberson on Utility and Scope," *Economica* vol. 19 (1953):, pp. 107-108.

75. **The Montreal Gazette**, December 12, 1980, p. 3.
76. We are basically reiterating one of B. Russell's example in **An Inquiry into Meaning and Truth**, 1940. The example appears frequently throughout the book, but it will be sufficient to read only chapter XVI to follow our argument.
77. The point has been raised and agreed by several authors. Notably there are, B. Russell, *op. cit.*, p.292; I.M.D. Little, **A Critique of Welfare Economics**, ch. 4, pp. 54-7; J. Harsanyi, *op. cit.*, (1955), p. 279; J.Rawls, *op. cit.*, p. 91.
78. W.V.O. Quine, "Two Dogmas of Empiricism," in **From a Logical Point of View**, 1961, p. 38.
79. There is a similar case in the theory of social choice. A person i is called a semi-dictator if and only if xRy whenever $xR_i y$ and $yR_j x$ for all $j \neq i$ and he will be called a dictator if and only if xRy whenever $xR_i y$. Apparently, a dictator is the more totalitarian of the two, and yet it turns out that the two are equivalent. For a formal proof, see P. Hammond, "Why Ethical Measures of Inequality Need Interpersonal Comparisons," **Theory and Decision** vol. 7 (1976): 263-74.
80. B. Russell, **The Analysis of Mind**, 1951, ch. 1, p. 29.
81. W.V.O. Quine, *op. cit.*, p. 44.

CHAPTER VIII

1. Hedonists believe that the only objective of human existence is the procurement of happiness, or equivalently, the avoidance of pain.
2. Interdependence of utility functions, altruism and some related topics have been under close examination in recent years. For an overview of the literature in this area, see E.S. Phelps, (ed.), *Altruism, Morality, and Economic Theory*, 1975; H.M. Hochman and J.D. Rogers, "Pareto Optimal Redistribution," *American Economic Review*, vol. 59 (1969), pp.543-57; E.J. Mishan, "The Futility of Pareto-Efficient Distributions," *American Economic Review*, vol. 62 (1972), pp.971-76 and D.A. Kennett, "Altruism and Economic Behaviour," *American Journal of Economics and Sociology*, vol. 39 (1980), pp.181-98.
3. G.E. Moore, *Principia Ethica*, 1903 and *Ethics*, 1947.
4. Both of these are standard examples in the literature. The first one may be found in J.J.C. Smart, "Utilitarianism," in *Encyclopedia of Philosophy*, vol. 8, p. 208, and the second one may also be located in H.J. McCloskey, "A Non-Utilitarian Approach to Punishment," *Inquiry*, vol. 8 (1965): 249-263.
5. The intuitionists are also known as formalists, and they are the ones who advocate *prima facie* ethical obligations. See R. Brandt, *Ethical Theory*, 1959. For a brief and systematic explanation of the nomenclature in question, see also the introduction in M.D. Bayles, *Contemporary Utilitarianism*, 1968.
6. A representative view of the twentieth century intuitionism can be found in W.D. Ross, *The Right and The Good*, 1930, and *Foundations of Ethics*, 1963. Here is how Ross sees it: That an act...is *prima facie* right, is self-evident;...in the sense that when we have reached sufficient mental maturity and have given sufficient attention to the proposition it is evident without the need of proof or of evidence beyond itself (*The Right and The Good*, p.29).
7. Brandt has also asked a similar question: But which is the 'more stringent' (*prima facie*) obligation? He concludes that "Ross' list of *prima facie* obligations tells us nothing of relative stringencies; and indeed Ross thinks that we cannot frame any simple generalizations on this point, for the stringency of a *prima facie* obligation varies with the circumstances". See R.B. Brandt, *Ethical Theory*, *op. cit.*, p. 393.

8. W.D. Ross, *The Right and the Good*, op. cit., pp. 115-40.
9. W.D. Ross, *ibid.*, p. 140.
10. R.F. Harrod, "Utilitarianism Revised," *Mind* vol. 45 (1936):137-156; J. Rawls, "Two Concepts of Rules," *Philosophical Review* vol. 64 (1955): 3-32; S. Toulmin, *The Place of Reason in Ethics*, 1951.

11. R. Nozick, *Anarchy, State, and Utopia*, ch. 3. Nozick has argued that there is something more to us than an experience machine. To support his argument, he supposes that there is such a machine which, when plugged to the human brain, would produce any experience one desired. Nozick believes that any sensible human being would prefer to be himself rather than being plugged to that machine. In his opinion, we would, "in addition to our experiences....want to do certain things, and not just have the experience of doing them....to be a certain sort of person....(not) to limit us to a man-made reality...."(p.43)

Nozick's argument is objectionable at least for two reasons. Firstly, our abhorrence of the experience machine is basically due to the information given to us, i.e., we will be plugged to a machine remotely different from ourselves. Supposedly, one is not able to make out the difference between being oneself and being plugged to the machine. Nonetheless, this precondition is broken once Nozick poses his question to us, and naturally our answer to the question must be biased. On the other hand, we must recognize that this is an intrinsic difficulty with the problem itself. Had we been plugged to the machine, our answer might well be the opposite. Yet, it can be even more farfetched. We might have already been plugged to that machine and without knowing it. In any case, there will always be a problem whenever we are using the brain to analyse the brain.

12. There is no doubt that we are adhering to a deterministic view on the subject. In his book, *Foundations of Ethics*, ch. 10, Ross has presented a very forceful defence of determinism. For a less remote account of the controversy, see K. Lehrer, *Freedom and Determinism*, 1966.

The case has often been made that if everything is pre-determined, then it would be possible to have them all written in a book which will give perfect predictions of future events. Suppose an adherent of free-will has somehow been able to get hold of such a book, and to no one's surprise, he tries to do things contrary to those predicted in the book. Will determinism fall apart in the face of his actions?

We have to look at the problem in this way: if determinism is true, can our indeterminist ever do anything that is not in the book? or can the book not

have recorded all the acts our indeterminist will perform?

It is obvious that the reason our indeterminist's will to contradict the book is that he wants to refute determinism. However, if determinism is true, it will be able to predict our indeterminist's rebellious actions as well as the consequences it will have on subsequent events. Critics may say that the indeterminist may also be able to get hold of this 'modified' book (to be accurate, there can only be one and only one book, and modifications are not allowed) and again try to act contrary to it. The whole of the previous argument has been pushed one level 'higher,' and the same logic will apply. Eventually, when the book is written, it will take into account all the intentions of our indeterminist, and his actions will be as predictable as everything else.

13. Actually, the social utility index entails the consumer index. To be a moral being is not a part-time undertaking. If one chooses to be a moral being, one will have to be consistent in one's behaviour throughout. Hence, even when our moral agent is dealing with consumer goods, he will still have to evaluate them from the moral point of view.
14. M. Friedman and L.J. Savage, "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy* vol. 56 (1948): 279-304; H. Markowitz, "The Utility of Wealth," *Journal of Political Economy* vol. 60 (1952): 151-158.

SELECTED BIBLIOGRAPHY

- Akerlof, George A. "The Case Against Conservative Macroeconomics: An Inaugural Lecture." *Economica* vol. 46 (1979): 219-237.
- Alexander, S. "Social Evaluation Through Notional Choice." *Quarterly Journal of Economics* vol. 88 (1974): 597-624.
- Allais, Maurice and Hagen, Ole. *Expected Utility Hypothesis and the Allais Paradox*. Dordrecht, Holland, D. Reidel Publishing Company, 1978.
- Allen, R.G.D. "A Note on the Determinateness of the Utility Function." *Review of Economic Studies* vol. 2 (1935): 155-158.
- Alt, Franz. "On the Measurability of Utility." In *Preference, Utility and Demand*, pp. 424-31. Edited by J.S. Chipman et. al. Translated by Siegfried Schach. New York, Harcourt Brace Yovanovich, 1971.
- Armstrong, W.E. "The Determinateness of the Utility Function." *Economic Journal* vol. 49 (September, 1939): 453-467.
- . "Uncertainty and the Utility Function." *Economic Journal* vol. 58 (March, 1948): 1-10.
- . "Utility and the Theory of Welfare." *Oxford Economic Papers* vol. 3 (October, 1951): 259-271.
- Arrow, Kenneth J. "Alternative Approaches to the Theory of Choice in Risk-Taking Situations." *Econometrica* vol. 19 (1951): 404-437.
- . "Little's Critique of Welfare Economics." *American Economic Review* vol. 41 (1951): 923-934.
- . *Social Choice and Individual Value*. 2nd ed. New haven: Yale University Press, 1963.
- . "A Theory of Justice by John Rawls: Two Review Articles." *Journal of Philosophy* vol. 70, no. 9 (May, 1973): 245-263.
- . "Extended Sympathy and the Possibility of Social Choice." *American Economic Review* vol. 67 (February, 1977): 219-225.

Arrow, Kenneth J. and Hahn, Frank. **General Competitive Analysis**. San Francisco, Holden-Day, 1971.

Baechler, J. **Suicide**. Translated by B. Cooper. New York: Basic Books, 1979.

Bailey, Martin J. "The interpretation and Application of the Compensation Principle." **Economic Journal** vol. 64 (March, 1954): 39-52.

———. "The Possibility of Rational Social Choice in an Economy." **Journal of Political Economy** vol. 87 (February, 1979): 37-56.

Baldwin, R. "A Comparison of Welfare Criteria." **Review of Economic Studies** vol. 21 (1954): 154-167.

Basu, Kaushik. **Revealed Preference of Government**. New York: Cambridge University Press, 1980.

———. "Determinateness of the Utility Function: Revisiting a Controversy of the Thirties." **Review of Economic Studies** vol. 49 (1982): 307-311.

Batra, Raveendra N. and Pattanaik, Prasanta K. "On Some Suggestions for Having Non-Binary Social Choice Functions." **Theory and Decision** vol. 3 (1972): 1-11.

Baumol, William J. "The Neumann-Morgenstern Utility Index -- An Ordinalist View." **Journal of Political Economy** vol. 59 (February, 1951): 61-68.

———. "Economics of Athens Drama: Its Relevance for the Arts in a Small City Today." **Quarterly Journal of Economics** vol. 85 (August, 1971): 365-376.

Bayles, Michael D. **Contemporary Utilitarianism**. Garden City, N.Y., Anchor Books, 1968.

Becker, Gary S. "Irrational Behavior and Economic." **Journal of Political Economy** vol. 70 (February, 1962): 1-13.

Bergson, Abram. "A Reformulation of Certain aspects of Welfare Economics." **Quarterly Journal of Economics** vol. 52 (February, 1938): 310-334.

Bernardelli, Harro. "Notes on the Determinateness of the Utility Function." **Review of Economic Studies** vol. 2 (1935): 69-75.

Bernoulli, Daniel. "Specimen theoriae novae de mensura sortis." **Commentarii academiae scientiarum imperiales Petropolitanae** vol. 5 (1738): 175-192. Translated from Latin into English by Dr. Louise Sommer under the title, "Exposition of A New Theory

- on the Measurement of Risk." *Econometrica* vol. 22 (1954): 23-36.
- Bishop, Robert L. "Professor Knight and the Theory of Demand." *Journal of Political Economy* vol. 54 (1946): 141-169.
- Black, D. "On the Rationale of Group Decision Making." *Journal of Political Economy* vol. 56 (1948): 23-34.
- Blackorby, C. "Degree of Cardinality and Aggregate Partial Ordering." *Econometrica* vol. 43 (September, 1975): 845-852.
- Blau, Julian H. "The Existence of Social Welfare Functions." *Econometrica* vol. 25 (1957): 302-313.
- Blau, Julian H. and Deb, Rajat. "Social Decision Functions and the Veto." *Econometrica* vol. 45 (May, 1977): 871-879
- Borch, Karl. "Utility and Stochastic Dominance." In *Expected Utility Hypothesis and the Allais Paradox*, pp. 193-201. Edited by M. Allais and O. Hagen. Dordrecht, Holland, D. Reidel publishing Company, 1978.
- Brandt, R.B. *Ethical Theory*. Prentice-Hall, Englewood Cliffs, N.J., 1959.
- Brown, Phelps E.H. "Notes on the Determinateness of the Utility Function." *Review of Economics Studies* vol. 2 (1935): 66-69.
- Buchanan, James M. "Social Choice, Democracy, and Free Markets." *Journal of Political Economy* vol. 62 (1954): 114-123.
- . "Individual Choice in Voting and the Market." *Journal of Political Economy* vol. 62 (1954): 334-343.
- Camacho, A. "Societies and Social Decision Functions." In *Developments in the Methodology of Social Science*, pp. 217-253. Edited by W. Leinfellner and E. Kohler. Dordrecht: D. Reidel Pub. Co., 1974.
- . "On Cardinal Utility." *Theory and Decision* vol. 10 (1979): 131-145.
- Campbell, Donald E. "Social Choice and Intensity of Preference." *Journal of Political Economy* vol. 81 (1973): 211-218.
- Campbell, Donald E. and Fishburn, Peter C. "Anonymity Conditions in Social Choice Theory." *Theory and*

Decision vol. 12 (1980): 21-39.

Chipman, John S. "The Welfare Ranking of Pareto Distribution." **Journal of Economic Theory** vol. 9 (1974): 275-282.

Crawford, Vincent P. "A Game of Fair Division." **Review of Economic Studies** vol. 44 (1977): 235-247.

Cyert, R. M. and Grunberg, E. "Assumption, Prediction, and Explanation in Economics." In **A Behavioral Theory of the Firm**, pp. 289-311. Edited by R.M. Cyert and J.G. March. Englewood Cliffs, N.J., Prentice-Hall, 1963.

Daniel, Terrence E. "A Revised Concept of Distributional Equity." **Journal of Economic Theory** vol. 11 (1975): 94-109.

_____. "Pitfalls in the Theory of Fairness -- Comment." **Journal of Economic Theory** vol. 19 (1978): 561-564.

D'Aspremont, Claude and Gevers, Louis. "Equity and the Informational Basis of Collective Choice." **The Review of Economic Studies** vol. 45 (1977): 199-210.

Davidson, Donald and Suppes, Patrick. "A Finitistic Axiomatization of Subjective Probability and Utility." **Econometrica** vol. 24 (1956): 264-275.

Davidson, Donald and Marschak, Jacob. "Experimental Tests of a Stochastic Decision Theory." In **Measurement: Definitions and Theories**, pp. 233-269. Edited by C.W. Churchman and P. Ratoosh. N.Y., Wiley, 1962.

Debreu, Gerald. "Valuation Equilibrium and Pareto Optimum." **Proceedings of the National Academy of Sciences** vol. 40 (1954): 588-592.

_____. "Representation of a Preference Ordering by a Numerical Function." In **Decision Processes**, pp. 159-165. Edited by R.M. Thrall, C.H. Coombs, and R.L. Davis. New York: Wiley, 1954.

_____. "Stochastic Choice and Cardinal Utility." **Econometrica** vol. 26 (1958): 440-444.

_____. "Topological Methods in Cardinal Utility Theory." In **Mathematical Methods in the Social Science**, pp. 16-26. Edited by K. Arrow et. al. Stanford, Calif., Stanford University Press, 1960.

Deschamps, Robert and Gevers, Louis. "Leximin and Utilitarian Rules: A Joint Characterization." **Journal of Economic Theory** vol. 17 (1978): 143-163.

Diamond, P. "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility: A Comment." *Journal of Political Economy* vol. 75 (1967): 765-766.

Dictionary of Political Economy, 2nd ed. S.v. "Deductive Method," by J.S. Mill.

Dobb, Maurice. "A Further Comment on the Discussion of Welfare Criteria." *Economic Journal* vol. 73 (December, 1963): 765-771.

Dreze, Jacques H. "Axiomatic Theories of Choice, Cardinal Utility and Subjective Probability: A Review." In *Uncertainty in Economics: Readings and Exercise*, pp.3-23. Edited by P. Diamond and M. Rothschild. New York: Academic Press Inc., 1978.

Edwards, Ward. "The Theory of Decision Making." *Psychological Bulletin* vol. 51 (1954): 380-417.

_____. "Behavioral Decision Theory." *Annual Review of Psychology* vol. (1961): 473-498.

Ellsberg, D. "Classic and Current Notions of 'Measurable Utility'." *Economic Journal* vol. 64 (September, 1954): 528-556.

_____. "Risk, Ambiguity, and the Savage Axioms." *Quarterly Journal of Economics* vol. 75 (1961): 643-669.

Encyclopaedia of Philosophy, 1967. S.v. "Utilitarianism," by J.J.C. Smart.

Encyclopaedia of Social Science, 1968. S.v. "Utility" by N. Georgescu-Roegen.

Feldman, Allan and Kirman, Alan. "Fairness and Envy." *American Economic Review* vol. 64 (1974): 995-1005.

Felner, William. "Distortion of Subjective Probabilities as a Reaction to Uncertainty." *Quarterly Journal of Economics* vol. 75 (1961): 670-689.

Fine, Ben. "A Note on 'Interpersonal Aggregation and Partial Comparability.'" *Econometrica* vol. 43 (January, 1975): 169-172.

Fine, B. and Fine, K. "Social Choice and Individual Ranking -- I." *Review of Economic Studies* vol. 41 (1974): 303-322.

Fishburn, Peter C. *Theory of Social Choice*, Princeton, N.J., Princeton University Press, 1973.

_____. "On Harsanyi's 'New Theory of Cardinal Utility' and

- the Maximization of Expected Return." *Journal of Political Economy* vol. 86 (1978): 321-324.
- Fisher, Franklin M. "Income Distribution, Value Judgements, and Welfare." *Quarterly Journal of Economics* vol. 70 (August, 1956): 380-424.
- Flemming, M. "A Cardinal Concept of Welfare." *Quarterly Journal of Economics* vol. 66 (1952): 366-384.
- Foley, Duncan K. "Resource Allocation and the Public Sector." *Yale Economic Essays* vol. 7 (1967): 45-98.
- Frazer, W. and Boland, Lawrence A. "An Essay on the Foundations of Friedman's Methodology." *American Economic Review* vol. 73 (March, 1983): 129-144.
- Friedman, M. *Essays in Positive Economics*. Chicago, University of Chicago Press, 1953.
- Friedman, Milton and Savage, L.J. "The Utility Analysis of Choices Involving Risk." *Journal of Political Economy* vol. 56 (1948): 279-304.
- _____. "The Expected-Utility Hypothesis and the Measurability of Utility." *Journal of Political Economy* vol. 60 (December, 1952): 463-474.
- Gardenfors, Peter. "Positionalist Voting Functions." *Theory and Decision* vol. 4 (1973): 1-24.
- _____. "Manipulation of Social Choice Functions." *Journal of Economic Theory* vol. 13 (1976): 217-228.
- Georgescu-Roegen, N. "Choice, Expectations and Measurability." *Quarterly Journal of Economics* vol. 68 (1954): 503-534.
- _____. *The Entrophy Law and the Economic Process*. Cambridge, Mass., Harvard University Press, 1971.
- Gevers, Louis. "On Interpersonal Comparability and Social Welfare Orderings." *Econometrica* vol. 47 (January, 1979): 75-89.
- Gibbard, Allan. "Disparate Goods and Rawls' Difference Principle: A Social Choice Theoretic Treatment." *Theory and Decision* vol. 11 (1979): 267-288.
- Goldman, A.H. "Rawls's Original Position and the Difference Principle." *Journal of Philosophy* vol. 73 (1976): 845-849.
- Goldman, S.M. and Sussangkarn, C. "On the Concept of Fairness." *Journal of Economic Theory* vol. 14

(1978): 210-216.

Goodman, Leo A. and Markowitz, Harry. "Social Welfare Functions Based on Individual Rankings." *American Journal of Sociology* vol. 58 (1952):257-262.

Gorman, W.M. "The Intransitivity of Certain Criteria Used in Welfare Economics." *Oxford Economic Papers* vol. 7 (1955):25-35.

Gottinger, Hans-Werner. "Towards Fuzzy Reasonings in the Behavioral Sciences." In *Developments in the Methodology of Social Science*, pp. 287-308. Edited by W. Leinfellner and E. Kohler. D. Reidel Pub. Co., Dordrecht, Holland, 1974.

Graaff, J. de V. *Theoretical Welfare Economics*. Cambridge, University Press, 1967.

Hammond, Peter J. "Why Ethical Measure of Inequality Need Interpersonal Comparisons." *Theory and Decision* vol. 7 (1976): 263-274.

———. "Equity, Arrows Conditions, and Rawls' Difference Principle." *Econometrica* vol. 44 (July, 1976): 793-804.

Handa, Jagdish. "Risk, Probabilities, and a New Theory of Cardinal Utility." *Journal of Political Economy* vol. 85 (1977): 97-122.

Halmos, P.R. *Naive Set theory*. Princeton, N.J., Van Nostrand, 1960.

Hansson, Bengt. "The Independence Condition in the Theory of Social Choice." *Theory and Decision* vol. 4 (1973): 25-49.

———. "A Proof Technique for Social Choice with variable Electorate." *Journal of Economic Theory* vol. 13 (1976): 193-200.

Hare, R.M. *Freedom and Reason*. Oxford, Clarendon Press, 1963.

Harrod, R.F. "Utilitarianism Revised." *Mind* vol. 45 (1936): 137-156.

Harsanyi, J.C. "Cardinal Welfare, Individualistic Ethics and Interpersonal Comparisons of Utility." *Journal of Political Economy* vol. 63 (August, 1955): 309-321.

———. "Non-Linear Social Welfare Functions." *Theory and Decision* vol. 6 (1975): 311-332.

- _____. "Bayesian Decision Theory and Utilitarian Ethics." **American Economic Review** vol. 68 (1978): 223-228.
- _____. "Rule Utilitarianism, Rights, Obligations and the Theory of Rational Behavior." **Theory and Decision** vol. 12 (1980): 115-133.
- Hicks, J.R. "The Foundations of Welfare Economics." **Economic Journal** vol. 49 (1939): 696-712.
- _____. "The Valuation of the Social Income." **Economica** vol. 7 (1940): 105-124.
- _____. "The Rehabilitation of Consumers' Surplus." **The Review of Economic Studies** vol. 9 (February, 1941): 108-110.
- _____. "The Four Consumer's Surpluses." **The Review of Economic Studies** vol. 11 (1943): 31-41.
- _____. "The Generalized Theory of Consumer's Surplus." **Review of Economic Studies** vol. 13 (1946): 68-74.
- _____. "Preference and welfare." In **Economic Theory and Planning -- Essays in Honour of A.K. Das Gupta**, pp. 3-16. Edited by Ashok Mitra. Calcutta: Oxford University Press, 1974.
- Hicks, J.R. and Allen, R.G.D. "A Reconsideration of the Theory of Value -- I." **Economica** n.s. vol. 1 (February, 1934): 52-73.
- _____. "A Reconsideration of the Theory of Value -- II." **Economica** n.s. vol. 1 (May, 1934): 196-219.
- Hildreth, Clifford. "Alternative Conditions for Social Orderings." **Econometrica** vol. 21 (1953): 81-94.
- Hochman, H.M. and Rodgers, J.D. "Pareto Optimal Redistribtuion." **American Economic Review** vol. 59 (1969): 542-557.
- Houthakka, H.S. "Revealed Preference and the Utility Function." **Economica** vol. 17 (1950): 159-174.
- Kaldor, N. "Welfare Propositions of Economics and Interpersonal Comparison of Utility." **Economic Journal** vol. 49 (September, 1939): 549-553.
- Kemp, Murray and Asimakopulos, A. "A Note on 'Social Welfare Functions' and Cardinal Utility." **Canadian Journal of Economics and Political Science** vol. 18 (1952): 195-200.
- Kemp, Murray and Ng, Yew-Kwang. "On the Existence of Social

Welfare Functions, Social Orderings and Social Decision Functions." *Economica* vol. 43 (February, 1976): 59-66.

_____. "More on Social Welfare Functions: The Incompatibility of Individualism and Ordinalism." *Economica* vol. 44 (1977): 89-90.

Kennedy, Charles. "The Economic Welfare Function and Dr. Little's Criterion." *Review of Economic Studies* vol. 21 (1953): 137-142.

Kennett, D.A. "Altruism and Economic Behaviour." *American Journal of Economics and Sociology* vol. 39 (1980): 181-198.

Klappholz, K. and Agassi, J. "Methodological Prescriptions in Economics." *Economica* n.s. vol. 26 (February, 1959): 61-74.

Knight, Frank H. "Realism and Relevance in the Theory of Demand." *Journal of Political Economy* vol. 52 (December, 1944): 289-319.

_____. "Comment on Mr. Bishop's Article." *Journal of Political Economy* vol. 54 (1946): 170-176.

Kuratowski, K. and Mostowski, A. *Set Theory*. Translated from Polish by M. Maczynski. Amsterdam, North-Holland, Warszawa, Polish Scientific Publishers, 1967.

Lange, O. "The Determinateness of the Utility Function." *Review of Economic Studies* vol. 1 (1934): 218-225.

_____. "Notes on the Determinateness of the Utility Function." *Review of Economic Studies* vol. 2 (1935): 75-77.

Lekker, K. *Freedom and Determinism*. New York, Random House, 1966.

Levine, A.L. "Rawls and the Utilitarian Decision Rule: A Critique of Arrow's Case for 'Convergence'." Paper presented at the Annual Meeting of the Canadian Economic Association, May, 1978.

Little, I.M.D. "The Foundations of Welfare Economics." *Oxford Economic Papers* vol. 2 (1950): 227-246.

_____. "Social Choice and Individual Values." *Journal of Political Economy* vol. 60 (1952): 422-432.

_____. *Critique of Welfare Economics*. Oxford Clarendon Press, 1957.

- _____. "Welfare Criterion: An Exchange of Notes." *Economic Journal* vol. 72 (March, 1962): 229-231 and 233-234.
- _____. "Two Comments." *Economic Journal* vol. 73 (December, 1963): 778-779.
- Luce, D. and Raiffa, H. *Games and Decisions: Introduction and Critical Survey*. John Wiley & Sons, New York, N.Y., 1957.
- Luce, D. "How We May Have Been Misled into Believing in the Interpersonal Comparability of Utility." *Theory and Decision* vol. 15 (1983): 247-260.
- McCloskey, H.J. "A Non-Utilitarian Approach to Punishment." *Inquiry* vol. 8 (1965): 249-263.
- McCloskey, N. "The Rhetoric of Economics." *Journal of Economic Literature* vol. 21 (June, 1983): 481-517.
- Mackay, Alfred F. *Arrow's Theorem, the Paradox of Social Choice: A Case Study in the Philosophy of Economics*. New Haven: Yale University Press, 1980.
- McKenzie, G.W. and Pearce, I.F. "Welfare Measurement -- A Synthesis." *American Economic Review* vol. 72 (September, 1982): 669-682.
- Malinvaud, E. "Note on von Neumann-Morgenstern's Strong Independence Axiom." *Econometrica* vol. 20 (1950): 679.
- Manne, Alan S. "The Strong Independence Assumption -- Gasoline Blends and Probability Mixtures." *Econometrica* vol. 20 (1952): 665-678.
- Margolis, J. "'That All Men Are Created Equal'." *Journal of Philosophy* vol. 52 (June, 1955): 337-346.
- Markowitz, Harry. "The Utility of Wealth." *Journal of Political Economy* vol. 60 (1952): 151-158.
- Marshak, Jacob. "Rational Behavior, Uncertainty Prospects, and Measurable Utility." *Econometrica* vol. 18 (1950): 111-141.
- Maskin, Eric. "A Theorem on Utilitarianism." *Review of Economic Studies* vol. 43 (1976): 93-96.
- May, Kenneth O. "A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision." *Econometrica* vol. 20 (1952): 680-684.
- _____. "Intransitivity, Utility, and the Aggregation of

- Preference Patterns." *Econometrica* vol. 22 (January, 1954): 1-13.
- Meade, J.E. "Little: A Critique of Welfare Economics." *Economic Journal* vol. 69 (March, 1959): 124-129.
- _____. "Welfare Criteria: An Exchange of Notes." *Economic Journal* vol. 72 (March, 1962): 231-233.
- Mill, John Stuart. "Utilitarianism." Reprinted in *The Utilitarians*, pp. 401-472. Anchor Press, Garden City, New York, 1973.
- _____. "Other Minds." In *Reason and Responsibility*, 3rd ed., pp. 212-213. Edited by J. Feinberg. Encino, Calif.: Dickenson Pub. Co., 1974.
- Mishan, E.J. "The Principle of Compensation Reconsidered." *Journal of Political Economy* vol. 60 (1952): 312-322.
- _____. "Welfare Criteria: An Exchange of Notes." *Economic Journal* vol. 72 (March, 1962): 234-244.
- _____. "A Survey of Welfare Economics." In *AEA Survey of Economic Theory* vol. 1, pp. 154-222. London, Macmillan; New York, St. Martin's Press, 1965.
- _____. "The Futility of Pareto-Efficient Distributions." *American Economic Review* vol. 62 (1972): 971-976.
- _____. "Welfare Criteria: Resolution of a Paradox." *Economic Journal* vol. 83 (September, 1973): 747-767.
- Moore, G.E. *Principia Ethica*. Cambridge University Press, 1903.
- _____. *Ethics*. London, Oxford University Press, 1947.
- Mosteller, Frederick and Nogee, Philip. "An Experimental Measurement of Utility." *Journal of Political Economy* vol. 59 (October, 1951): 371-404.
- Mueller, Dennis C. "Public Choice: A Survey." *Journal of Economic Literature* vol. 14 (1976): 395-433.
- Nash, John F. "The Bargaining Problem." *Econometrica* vol. 18 (April, 1950): 155-162.
- Neumann, von John and Morgenstern, Oscar. *Theory of Games and Economic Behavior*. Princeton University Press, 1947.
- Ng, Y.K. "Little's Welfare Criterion Under the Equality Assumption." *Economic Record* vol. 47 (December, 1971): 579-583.

- _____. "Value Judgements and Economists' Role in Policy Recommendation." *Economic Journal* vol. 82 (1972): 1014-1018.
- _____. "Bentham or Bergson? Finite Sensibility, Utility Functions and Social Welfare Functions." *Review of Economic Studies* vol. 43 (1975): 545-569.
- Nozick, R. *Anarchy, State, and Utopia*. Oxford, Blackwell, 1974.
- Pareto, Vilfredo. "Manuel D'Economie Politique (Ophelinite)." In *Utility Theory: A Book of Reading*, pp. 168-383. Edited by Alfred N. Page. New York, Wiley, 1968.
- Parks, P. "An Impossibility Theorem for Fixed Preferences: A Dictatorial Bergson-Samuelson Welfare Function." *Review of Economic Studies* vol. 43 (1976): 447-450.
- Pattanaik, P.K. "Risk, Impersonality and the Social Welfare Function." *Journal of Political Economy* vol. 76 (December, 1968): 1152-1169.
- Pazner, Elisha A. "Pitfalls in Theory of Fairness." *Journal of Economic Theory* vol. 14 (1977): 458-466.
- Pazner, Elisha A. and Schmeidler, David. "A difficulty in the Concept of fairness." *Review of Economic Studies* vol. 42 (1974): 441-443.
- _____. "Egalitarian Equivalent Allocations: A New Concept of Economic Equity." *Quarterly Journal of Economics* vol. 92 (November, 1978): 671-687. Phelps, E.S. *Altruism, Morality, and Economic Theory*. Russell Sage Foundation, New York, 1975.
- Plott, Charles R. "Path Independence, Rationality, and Social Choice." *Econometrica* vol. 41 (November, 1973): 1075-1091.
- _____. "Bergson-Samuelson Social Welfare Functions and The Theory of Social Choice." *Quarterly Journal of Economics* vol. 93 (1979): 73-90.
- Pollak, R.A. "Bergson-Samuelson Social Welfare Functions and the Theory of Social Choice." *Quarterly Journal of Economics* vol. 93 (1979): 73-90.
- Popper, K. *The Logic of Scientific Discovery*. New York, Harper & Row, 1968.
- Quine, W.V.O. "Two Dogmas of Empiricism." In *From a Logical Point of View*. Cambridge, Harvard University Press, 1961.

- Raiffa, Howard. "Risk, Ambiguity, and the Savage Axioms: Comment." *Quarterly Journal of Economics* vol. 75 (1961): 690-694.
- . "Bayesian Decision Theory." In *Recent Developments in Information and Decision Processes*, pp. 92-101. Edited by R.E. Machol and R. Gray. New York, Macmillan, 1962.
- Ramsey, Frank P. "A Mathematical Theory of Saving." *Economic Journal* vol. 38 (1928): 543-559.
- Rawls, John. "Two Concepts of Rules." *Philosophical Review* vol. 64 (1955): 3-32.
- . "Justice as Fairness." *Philosophical Review* vol. 67 (1958): 164-194.
- . "Distributive Justice." In *Philosophy, Politics and Society*, pp. 58-82. Edited by P. Laslett and W.F. Runciman. Blackwell, 1967, third series.
- . *A Theory of Justice*. Harvard University Press, 1971.
- Ray, Paramesh. "Independence of Irrelevant Alternatives." *Econometrica* vol. 41 (September, 1973): 987-991.
- Robbins, Lionel. "Interpersonal Comparisons of Utility: A Comment." *Economic Journal* vol. 48 (1938): 635-641.
- . "Robertson on Utility and Scope." *Economica* vol. 19 (February, 1953): 99-111.
- . *An Essay on the Nature and Significance of economic Science*. London, Macmillan, 1962.
- Roberts, Kevin W.S. "Interpersonal Comparability and Social Choice Theory." *Review of Economic Studies* vol. 48 (1980): 421-439.
- Robertson, Dennis. *Utility and All That and Other Essays*. George Allen & Unwin, 1952.
- . "Utility and All What." In *Economic Commentaries*, pp. 42-58. Staples Press Ltd., London, 1956.
- . "Welfare Criteria: An Exchange of Notes." *Economic Journal* vol. 72 (March, 1962): 226-229.
- Rorty, Richard. *Philosophy and the Mirror of Nature*. Princeton: Princeton Press, 1979.
- Ross, W.D. *The Right and the Good*. Oxford, Clarendon Press, 1930.

- _____. **The Foundation of Ethics.** Oxford, Clarendon Press, 1963.
- Rothenberg, J. **The Measurement of Social Welfare.** Englewood Cliffs, N.J., Prentice-Hall, 1961.
- Russell, Bertrand. **The Analysis of Mind.** London, G. Allen & Unwin Ltd., 1921.
- _____. **An Inquiry into Meaning and Truth.** London, G. Allen and Unwin Ltd., 1940.
- Samuelson, Paul A. "The Numerical Representation of Ordered Classifications and the Concept of Utility." **Review of Economic Studies** vol. 6 (1938): 65-70.
- _____. "Evaluation of Real National Income." **Oxford Economic Papers** vol. 2 (1950): 1-29.
- _____. "Probability and the Attempts to Measure Utility." **The Economic Review**, Tokyo (1950). Keizai Denkyu, Hitotsubashi University. Reprinted in **Collected Scientific Papers of Paul A. Samuelson**, vol. I, pp. 117-123. Edited by J.E. Stiglitz. M.I.T. Press, 1966.
- _____. "The Problem of Integrability in Utility Theory." **Economica** n.s. vol. 17 (1950): 355-385.
- _____. "Probability, Utility, and the Independence Axiom." **Econometrica** vol. 20 (1952): 670-678.
- _____. **Foundations of Economic Analysis.** New York, Atheneum, 1965.
- _____. "Utility, Preference, and Probability." Colloque International d'Econometrie 'Les Fondements et Applications de la theorie du Risque en Econometrie', Centre National de la Recherche Scientifique, Paris. Also in **Collected Scientific Papers of Paul A. Samuelson**, vol. I, pp. 127-137. Edited by J.E. Stiglitz. M.I.T. Press, 1966.
- _____. "Arrow's Mathematical Politics." In **Human Values and Economic Policy**, pp. 41-51. Edited by S. Hook. New York: New York University Press, 1967.
- _____. "Reaffirming The Existence of 'Reasonable' Bergson-Samuelson Social Welfare Functions." **Economica** vol. 44 (1977): 81-88.
- Savage, L.J. **The Foundations of Statistics.** John Wiley & Sons, New York, N.Y., 1954.
- Schmeidler, David and Vind, Karl. "Fair Net Trade."

Econometrica vol. 40 (July, 1972): 637-642.

Schmeidler, David and Sonnenschein, Hugo. "Two Proofs of the Gibbard-Satterthwaite Theorem on the Possibility of a Strategy Proof Social Choice Function." In **Decision Theory and Social Ethics, Issues in Social Choice**, pp. 227-234. Edited by Hans W. Gottinger and Werner Leinfellner. D. Reidel Pub. Co., Dordrecht, Holland, 1978.

Schutt, Klaus-P. "A Model to Support the Assessment of Subjective Probabilities." **Theory and Decision** vol. 12 (1980): 173-183.

Scitovsky, T. "A Note on Welfare Propositions in Economics." **Review of Economic Studies** vol. 9 (1941): 77-88.

Sen, A.K. "Distribution, Transitivity and Little's Welfare Criteria." **Economic Journal** vol. 73 (December, 1963): 771-778.

_____. **Collective Choice and Social Welfare**. San Francisco: Holden-Day; Edindurgh: Oliver & Boyd, 1970.

_____. **On Economic Inequality**. Oxford: Clarendon Press; New York: Norton, 1973.

_____. "Real National Income." **Review of Economic Studies** vol. 43 (1976): 19-39.

_____. "Social Choice Theory: A Re-examination." **Econometrica** vol. 45 (January, 1977): 53-89.

_____. "On Weights and Measures: Informational Constraints in Social Welfare Analysis." **Econometrica** vol. 45 (October, 1977): 1539-1572.

_____. "personal Utilities and Public Judgements: Or What's Wrong with Welfare Economics." **Economic Journal** vol. 89 (september, 1979): 537-558.

_____. "Development: Which Way Now?" **Economic Journal** vol. 93 (December, 1983): 745-762.

Shone, R. **Microeconomics: A Modern Treatment**. New York: Academic Press, 1975.

Spiegelberg, Herbert. "A Defense of Human Equality." **Philosophical Review** vol. 53 (March, 1944): 101-124.

Stefanik, Richard J. "Harsanyi's Critical Rule Utilitarianism." **Theory and Decision** vol. 13 (1981): 71-80.

Stevens, S.S. "Measurement, Psychophysics and Utility." In **Measurement: Decisions and Theories**, pp. 18-64. Edited by C.W. Churchman and P. Ratoosh. N.Y., Wiley, 1962.

Stoll, R. **Set Theory and Logic**. San Francisco, W.H. Freeman, 1963.

Strasnick, Steven. "The Problem of Social Choice: Arrow to Rawls." **Philosophy and Public Affairs** vol. 6 (1976): 241-273.

_____. "Social Choice and the Derivation of Rawls's Difference Principle." **Journal of Philosophy** vol. 73 (February, 1976): 85-99.

_____. "Extended Sympathy Comparisons and the Basis of Social Choice." **Theory and Decision** vol. 10 (1979): 311-328.

Suppes, Patrick. "Some Formal Models of Grading Principles." **Synthese** vol. 16 (1966): 270-306.

Suppes, Patrick and Winet, Muriel. "An Axiomatization of Utility Based on the Notion of Utility Differences." **Management Science** vol. 1 (1955): 259-271.

Suzumura, Kotaro. "On Distributional Value Judgements and Piecemeal Welfare Criteria." **Economica** vol. 47 (1980): 125-139.

Szasz, T.S. "The Ethics of Suicide." In **Between Survival and Suicide**, pp. 163-183. Edited by B.B. Wolman and H.H. Krauss. New York, Gardner Press, Inc. 1976.

Toulmin, S. **The Place of Reason in Ethics**. Cambridge, 1951.

Varian, Hal R. "Equity, Envy, and Efficiency." **Journal of Economic Theory** vol. 9 (1974): 63-91.

Wagner, R. Harrison. "Impartiality and Equity." **Theory and Decision** vol. 12 (1980): 61-74.

Waldner, Ilmar. "Bare Preference and Interpersonal Utility Comparisons." **Theory and Decision** vol. 5 (1973): 313-328.

Weldon, J.C. "A Note on Measures of Utility." **Canadian Journal of Economics and Political Science** vol. 16 (1950): 227-233.

_____. "On the Problem of Welfare Economics." **Canadian Journal of Economics and Political Science** vol. 18 (1952): 452-463.

- Wicksell, K. "A New Principle of Just Taxation." In **Classics in the Theory of Public Finance**, pp. 72-118. Translated by J. Buchanan and edited by R. Musgrave and A.T. Peacock. London, Macmillan, 1958.
- Whitmore, G.A. "Third-Degree Stochastic Dominance." **American Economic Review** vol. 60 (1970): 457-459.
- Wilson, Robert. "Social Choice Theory Without the Pareto Principle." **Journal of Economic Theory** vol. 5 (1972): 478-486.
- . "On the Theory of Aggregation." **Journal of Economy Theory** vol. 10 (1975): 89-99.
- Wittman, Donald. "A Diagramatic Exposition of Justic." **Theory and Decision** vol. 11 (1979): 207-237.
- Wold, H. "Ordinal Preferences or Cardinal Utility?" **Econometrica** vol. 20 (1952): 661-663.
- Wolff, R.P. "On Strasnick's 'Derivation' of Rawls's 'Difference Principle.'" **Journal of Philosophy** vol. 73 (1976): 849-858.
- Wright, G.H. von. **The Logic of Preference**, University Press, Endinburgh, 1963.
- . "The Logic of Preference Reconsidered." **Theory and Decision** vol. 3 (1972): 140-169.
- Young, J.P. "An Axiomatization of Borda's Rule." **Journal of Economic Theory** vol. 9 (1974): 43-52.
- Zeuthen, F. "A Note on the Determinateness of the Utility Function." **Review of Economic Studies** vol. 2 (1935): 236-239.