A SPECTRUM ANALYSER
FOR A
DIFFRACTION FIELD COMPUTER

A THESIS

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Abstract.

A computer for calculating diffraction fields is described. It has been shown that a pulse is related to its frequency spectrum in a manner which is analogous to the relation between the aperture distribution of an antenna and its radiation pattern. This analogy, in regard to typical microwave antennas, forms the basis for a computer with practical electronic constants.

The experimental work was divided into three sections, namely, a pulse source, a dispersion network and a spectrum analyser. The present discussion is primarily concerned with the analyser section of the computer. Theoretical considerations have been applied to typical microwave antennas to determine the electrical specifications and preliminary design has been tested by experimental circuits. It has been shown that a computer for distant fields is practical, design of that portion of the computer concerned with Fresnel fields requires further investigation.

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List of Abbreviations

c/s - cycles per second

kc/s - kilocycles per second

mc/s - megacycles per second

exp. - exponential operator

f_r - recurrence frequency

f_s - sweep frequency

f_{LO} - local oscillator frequency

f_{if}, I.F. - intermediate frequency

Δf_{if} - intermediate frequency bandwidth

f_o - carrier signal frequency

gm - mutual conductance

μsec - microsecond

μh - microhenry

mh - millihenry

pf - micro-microfarad

μf - microfarad

R.M.S. - root mean square

μV - microvolt

mV - millivolt

1. INTRODUCTION

1.1 Types of Computers

Computers, in general, perform mathematical operations on data with which they are provided. They are divided into two classes, digital and analogue computers. The choice between the two types for a given application depends on the nature of the problem involved. Digital computers may be exemplified by the abacus of the ancient Chinese and by the common mechanical office computing machine of today. Although the general method used in the design of the abacus has been greatly extended to solve much more complicated problems, digital computers are still characterized by the fact that they deal with discrete quantities. Analogue computers, on the other hand, are classed as such because they are the outcome of the existence of a correspondence between two entirely different systems, thus, the word "model" could be loosely applied although in the strictest sense the "model" is a mathematical one. Analogue computers, as distinct from digital computers, possess the attribute of continuity[1]. This characteristic will be clarified by a well known example.

Consider the problem involved in the determination of the potential field created by an arrangement of isolated electrically charged conductors in free space. This condition will be called system A, and it is assumed that the fundamental laws which apply with free space as the intervening medium are known. System A can therefore be described perfectly by theoretical means but the value of the field at all points in space can be evaluated only by tedious arithmetic methods and is impossible to obtain by a direct experimental method. Now let the conductors be submerged in a conducting electrolyte, this condition will be labelled system B. Since the conductors have entered an entirely different medium, the fundamental laws which apply to system B are completely divorced from those which apply to system A, however, it is found[2] that the final equations which describe the potential distribution in the two cases are identical in form. Moreover, the potential distribution can be determined experimentally under the conditions of system B, therefore, by the indirect method of the analogy the required information regarding system A can be obtained.

Although, in the above example, the potential in the electrolyte of system B happened to be the analogy of the potential in free space of system A, such is by no means the usual case. System A, for example, could comprise a phenomenon which is described in terms of mass and length whereas system B could possess the units of inductance and electric charge, this would indeed be so if an electric circuit were used to compute the path of an object falling in a gravitational field. Thus, analogies exist among the variables of the two systems as well as between the two equations which describe the overall operations.

Discussion of the individual variables in the two systems immediately brings forward the question of accuracy and range of variable. The latter, which is usually referred to as scale factor when applied to computers (systems B), must evidently be large enough to cover a range which includes operations of system A made available by direct methods, if they exist, and if possible, should exceed this value in order that new phenomena may be observed. It is clear that the analogue computer (system B) is valuable only if the results which represent system A can be achieved over a larger range of variable, more accurately, or with greater facility and speed than by use of a direct method.

1.2 Computers and the Diffraction Problem.

The identity in form of results describing two separate systems has recently been suggested as a possible method of solving problems associated with the diffraction of monochromatic radiation[3]. The diffraction pattern which results from illuminating an aperture in a plane surface by a monochromatic source comprises system A. Accordingly, system B must be a computer which will provide analogies to the source of the radiation in system A, the aperture, the measuring apparatus for the diffracted radiation and the recording device. The equation describing the final result as presented by the computer must of course agree in form perfectly, or to within approximations as laid down in the specifications of the proposed design, with the theoretical equation which describes system A. The advantage of a computer for diffraction fields is evident from the fact that the fields can be obtained, both theoretically and experimentally, only by the expenditure of great time and labour, and then for only one value of certain variables (section 2.2). An analogue computer, however, could produce recorded data for all values of variables available by direct methods in a comparatively short time and possibly data for an extended range of the variables.

Consider plane electro magnetic radiation to be normally incident on a rectangular aperture in a plane, opaque surface. If the field strength of the radiation is measured at points on the side of the surface opposite to the source, a characteristic pattern is obtained which depends on the size of the aperture, the path which the measuring device has followed and the wavelength of the radiation. If the measuring device be maintained at a constant height above the earth and the plane erected perpendicular to the earth, then the resultant pattern is no longer a function of the vertical dimension of the aperture. System A can therefore be described in terms of three variables, the aperture width, the operating wavelength and the position of the detecting device.

1.3 The Far Field Analogue.

It was observed (reference 3 and section 2.1) that the equation for the frequency spectrum of a suitably isolated rectangular pulse of radio frequency was similar in form to that of the diffraction field when the latter was measured along a line parallel to and well removed from the plane of the aperture. This field is called the Fraunhoffer or far field. The fact that a frequency spectrum exists in a pulse of voltage is probably best pointed out by recalling that an arbitrary amplitude which is of given duration and is a known function of time can be expressed in a Fourier Series which comprises many terms at successive frequencies. The pulse spectrum, referred to above, can be obtained by proceeding from the Fourier Series to the Fourier Integral thence to the Fourier Transform. The latter results in an equation which shows that the amplitude of a frequency component in the spectrum is a function of the frequency increment from the component under consideration to the frequency at the centre of the spectrum, and the time duration of the pulse. The variables in the equations of the two systems arise in such a manner that the pulse width can be associated with the aperture width and the frequency component with a point at which the field is evaluated.

The above outline has described a computer in which the light vector, or electric vector of a plane illumination has been constant across the total width of the aperture, however, two important variations from this condition can arise which do not substantially alter the theoretical reasoning. Firstly, the above theory can be shown to apply to electromagnetic horn radiators which are used in the radio microwave region of the spectrum. The only modification of the above theory required in this instance is due to a phase change which occurs across the aperture of the horn, the computer must therefore also incorporate this feature. The analogue of the phase change across the aperture can be shown[3] to be accounted for by a phase variation between the frequency components of the analogue pulses. Secondly, if a horn radiator is rotated such that the magnetic vector of the radiation is in effect distributed across the width of the gap, then the illumination can no longer be considered constant. This condition can be accounted for in the computer by using, instead of a rectangular pulse, a pulse with a trignometric envelope.

1.4 The Near Field Analogue.

Further theoretical work[3] has shown that fields measured at distances closer to the aperture than those described above, the so-called Fresnel or near fields, can be provided with suitable analogies by dispersion of the far field analogue pulses. In this case, the same correspondence between the variables of system A and system B obtains and the same analogy to the measuring apparatus of system A can be used. The dispersion network, by which the new representation is produced, must possess the characteristic that in each frequency component, a phase change takes place which is proportional to a quadratic function of the frequency increment from the component to the centre frequency.

1.5 Description of the Computer.

A schematic diagram of the computer is shown in figure 1, the function of the individual blocks will be described to ensure that the overall purpose is clear. Assuming that it is required to represent the far field diffraction pattern due to an aperture illuminated by a plane wave, both of the switches in the diagram would be closed and the envelope generator must produce a rectangular waveform of known duration. The spectrum analyser, which determines the amplitude and position in the spectrum of the various frequency components of the pulse, would present its data on a cathode ray tube. The results could be recorded by a camera as desired and thus produce a permanent record of the representation. In fact, it is possible to use a scale on the cathode ray tube which is graduated in terms of the field variables rather than those associated with the frequency spectrum. If it is required to measure the near field created by an aperture illuminated by a plane wave, the switches would be as shown in the diagram, for the representation of the far field created by a horn, S1 would be open, S2 closed.

1.6 Distribution of Experimental Work and Results

In order to provide preliminary design data on the computer, the experimental work was alloted in three sections, namely, the pulse source, the dispersion network and the spectrum analyser. The aim in all three cases was to determine whether specifications could be met rather than to construct a unified prototype computer.

Work on the pulse source, the results of which are contained in a thesis by Mr. S.A. Radley[4], was, for the greater part, successful. On the other hand, to date, in spite of research by Mr. W. E. Clarke, and others, using several avenues of approach, a complete method for dispersing the far field analogue pulses to produce the near field representation has not been found, a partial solution has been set forth by Mr. C.H. Turner. The spectrum analyser, which is discussed in this paper, has proven feasible although no great elaboration has been carried out, especially with respect to the final display of the data.

1.7 Summary.

The design data for a far field diffraction computer has been obtained and representations of various field patterns have been recorded[4]. Representations of the near field diffraction pattern which is

feasible theoretically has not as yet been produced experimentally, a further method of approaching this problem is indicated in Appendix 2 of this paper.
The design of the computer, as a whole, may therefore be described as being in a partially completed state, further development being subject to new experimental and design tactics.

2. FAR FIELD DIFFRACTION COMPUTER THEORY

2.1 The Analogous Equations.

The matching integrals which represent the diffracted field due to an aperture and the frequency spectrum of a pulsed radio frequency voltage have been developed elsewhere[3], they will therefore merely be stated directly here but will be described and related in detail. From equation (13) of reference 3, the far field intensity is given by

$$G\left(\frac{\sin\theta}{\lambda}\right) = \exp.(j2\pi ft)\int_{-a/2}^{a/2} F(x)\exp.\left(-j2\pi x\frac{\sin\theta}{\lambda}\right)dx \quad \ldots\ldots \quad (1)$$

where the symbol $G\left(\frac{\sin\theta}{\lambda}\right)$ is functional notation (G operates on the variable $\frac{\sin\theta}{\lambda}$), θ the angle between the normal to the aperture and the radius vector of the point at which the field is examined and λ the operating wavelength. G is evaluated for a constant y value along a line of constant z where the x and y axes are such that the origin coincides with the centre of the aperture. $F(x)$, consistent with the remarks in the introduction regarding phase variations, is a complex quantity representing the field distribution at the aperture, "a" is the aperture width.

Similarly from equation (14) of reference 3, the frequency spectrum of a suitably isolated pulse is

$$\overline{G}(f-f_o) = \int_{-T/2}^{T/2} \overline{F}(t)\ \exp.(-j2\pi(f-f_o)t)dt \qquad (2)$$

where $\overline{G}(f-f_o)$ is again functional, f_o the centre frequency of the spectrum and T the pulse width. $\overline{F}(t)$, as previously mentioned, is a complex quantity which includes phase differences that may exist between the various frequency components which occur at increments $(f-f_o)$ from f_o. The functions $\overline{F}(t)\ \exp.(-j2\pi(f-f_o)t)$ will be referred to as the analogue pulses.

In order to obtain a clearer relationship between the variables in the two systems, (1) and (2) must be evaluated for a given F(x) and corresponding $\overline{F}(t)$. The simplest case is that of an aperture illuminated by a plane wave and the corresponding rectangular pulse, thus $F(x) = A\ \exp.(j2\pi ft)$ and $\overline{F}(t) = B\ \exp.(j2\pi f_o t)$. The results will again be drawn from reference (3), they are

$$G\left(\frac{\sin\theta}{\lambda}\right) = \exp.(j2\pi ft)\ A\ a\ \frac{\sin\left(a\,\frac{\sin\theta}{\lambda}\right)}{a\,\frac{\sin\theta}{\lambda}} \qquad (1a)$$

and

$$\overline{G}(f-f_o) = BT \frac{\sin(\ T(f-f_o))}{T(f-f_o)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2a)}$$

It is clear that the variables must be related such that

$$T(f-f_o) = (a/\lambda)\sin\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

It must be remarked here that although (2) describes (1) perfectly, the physical facts will be represented only to within the accuracy of the assumptions made in the construction of (1). The distribution F(x) over the aperture is not a true representation of the source function since line charges and currents will exist if the aperture is cut in a metallic surface, these will therefore reradiate and thus create a distortion of the field represented by (1). Moreover, the solution of the wave equation from which (1) is derived assumes continuity of the field in the region of the aperture. This conflicts[5] with the discontinuous nature inherent in the above F(x) since the latter is assumed to fall to zero in an infintesimally short distance, that is, at the edge of the aperture.

2.2 Major Computer Specifications

The order of magnitude of the computer variables as well as their scale factor can be obtained from equation (3). The scale factors, in accordance with remarks in section 1.1, were determined by the ranges obtained by direct measurements and possible extension of these. The ranges, as governed by results of experiments in the radio microwave portion of the spectrum are $-\pi/4<\theta<\pi/4$ and $5<a/\lambda<100$. These limits are functions of the wavelength of the sources of microwave energy which are at present in common use. More especially in the case of the angular limits, little is gained by further extension. In addition to the above specifications, one degree plotting accuracy is required.

Since θ, by definition, comprises both positive and negative values, and the frequency components f, by symmetry, take on values both greater and less than f_o, the maximum angular range 2 $\sin\pi/4$ must be related by equation (3) to the maximum frequency range occupied by the spectral components, thus

$$2(f-f_o) = (a/\lambda)2\sin\pi/4 = 1.41a/\lambda \quad \ldots\ldots\ldots \quad (4)$$

Moreover, the device which analyses the spectral components must be able to discriminate between frequency differences governed by the minimum value of

$T(f_{(\theta + 1)^o} - f_{\theta^o}) = (a/\lambda)\ (\sin(\theta + 1)^o - \sin\theta^o)$.

Since $\frac{d(f-f_o)}{d\theta} = (a/\lambda T)\cos\theta$, and the cosine decreases with increasing angle, the smallest frequency difference will occur at the largest angles. Thus

$$(f_{(\theta + 1)^o} - f_{\theta^o})_{min.} = f_{46^o} - f_{45^o} = 0.012\ a/\lambda T \quad \ldots\ldots \quad (4a)$$

Equations (4) and (4a) are evaluated in Table 1 for various values of T and a/λ .

Table 1.

	$2(f-f_o)=1.41a/T\lambda$ mc/s					$f_{46^o}-f_{45^o}=0.012a/T\lambda$ kc/s				
a/λ =	5	10	20	40	80	5	10	20	40	80
T sec										
5	1.4	2.8	5.6	11.3	22.6	12	24	48	96	192
10	.71	1.4	2.8	5.6	11.3	6	12	24	48	96
20	.34	.71	1.4	2.8	5.6	3	6	12	24	48
40	.17	.35	.71	1.4	2.8	1.5	3	6	12	24
80	.08	.17	.35	.71	1.4	.8	1.5	3	6	12

2.3 Possible Relationships between the Variables

The left section of table 1 shows that if the correspondence be such that $T = a/\lambda$ and $(f-f_o) = \sin\theta$, then the range $f-f_o$ could remain constant at its maximum value and the widths of the analogue pulses be varied for various values of a/λ. This can be expressed as

$$(f-f_o)_{max.}\left(\frac{T_{max.}}{m}\right) = (a/\lambda)\sin\theta \quad \ldots\ldots\ldots \quad (5)$$

where m takes on values greater than unity which are not necessarily integral; this relationship corresponds to traversing table 1 diagonally. If it is more convenient that T be kept constant, 1/m may be associated with $(f-f_o)$, this corresponds to operation along a horizontal row of the table and is expressed as

$$T_{max.}\left(\frac{f-f_o}{m}\right) = (a/\lambda)\sin\theta \quad \ldots\ldots\ldots\ldots\ldots \quad (5a)$$

The choice between methods utilizing (5) or (5a) is subject to experimental convenience.

The right section of table 1 places a specification on the spectrum analyser which will be discussed after a general description of this instrument has been given.

2.3 Summary.

The final results which describe the operations of the two separate systems have been compared and possible relationships between the variables have been established. The order of magnitude of the variables, consistent with conventional electronic circuit practice has been indicated, detailed specifications will not be laid down until the functions of the computer units, more especially those of the spectrum analyser, have been discussed.

3. DESIGN SPECIFICATIONS

Spectrum analysers are well known as instruments used for determination of the "squareness" of pulses, more especially as related to the output of radar transmitters. This type of analyser is available at microwave frequencies since the frequency of klystrons (microwave oscillators) is readily swept in any required manner, the spectral components are then selected by a narrow band intermediate frequency amplifier and suitably displayed. Figure 2, which (with the exception of the calibrator) may be considered exemplary of the general type of spectrum analyser is actually a block diagram of the analyser in question in this paper. Several specifications, which are the outcome of the above mentioned application of spectrum analysers, are not valid in the special case under consideration, these are mentioned in this section and the reasoning behind their misapplication in this instance is indicated. Specifications with regard to display of the data will be given first, followed by the radio frequency requirements.

3.1 Computer Operations which influence the Design.

If the analogue pulses have been formed to represent the required diffraction field, dispersed or otherwise modified as the case may be, the following conditions will exist. In figure 2 the analogue pulses are shown entering the receiver of the spectrum analyser, they must recur, since the recording device can only give a continual representation of repetitive data, that is, the screen of a cathode ray tube does not possess an infinite memory. The pulses will therefore be introduced at some frequency $f_r = qf_s$ where q must evidently be integral since only a single recurring picture is desired on the display tube. Under this condition of synchronism, there will appear as vertical deflections on the cathode ray tube (neglecting the return time of the time base trace) q pulses, provided the frequency of the time base on the cathode ray tube is f_s. Moreover, the amplitude of any one of the q deflections which enters the system because it originates at a frequency characterized by

$$f = f_{LO} \pm f_{if} \quad \dots\dots\dots\dots\dots\dots \quad (6)$$

where f_{LO} is varied in the same manner and at the same frequency (f_s) as the time base, will be proportional to the magnitude of the frequency component f which corresponds to the instantaneous value of f_{LO}. For a symmetrical analogue pulse, the q deflections will be symmetrical about the centre of the trace provided

$$f_o = \bar{f}_{LO} \pm f_{if}$$

where f_o is the centre of the spectral distribution and $\bar{f}_{LO}$ is the local oscillator frequency corresponding to the midpoint of the local oscillator frequency coverage.

3.2 The Timebase and Pulse Recurrence Frequency Specifications.

The fact that the analogue pulses must be repetitive and also simulate isolated pulses immediately introduces several important specifications. It is required that $T \ll \frac{1}{f_r}$ for suitable isolation and is likewise clear, from sections 2.2 and 3.1, that $q=90$ for one degree plotting accuracy. Since $f_s = f_r/q$, f_s must evidently be chosen as low as possible to allow for practicable values of T such as those shown in table 1. f_s was set at 10 c/s, the lower limit being fixed by the visual persistence of cathode ray tube screens, therefore $f_r = 900$ c/s. Moreover, if b is the length of a time base operating at a frequency f_s on a given cathode ray tube and d is the distance separating similar points on successive pulses of frequency f_r on the trace then if $f_r/f_s=q$, $qd=b$. For a nine inch trace (which can be produced on a twelve inch cathode ray tube), $d = \frac{9''}{90} = \frac{1''}{10}$ which is a satisfactory incremental distance between the frequency components for clarity when measurements are being made.

It is evident here that a linear time base and local oscillator frequency sweep is desirable since, if they were, say, sinusoidal in nature, the analogue pulses, which are assumed generated by a periodic source such as a multivibrator, would be distributed in a sinusoidal fashion on the display tube. This would not only result in a variable value of d but also in uneven luminescence. The latter presents focussing difficulties when large cathode ray tubes are involved. For overall linear operation, by equation (3), the horizontal scale on the cathode ray tube will be linear in sin θ.

3.3 Specification of the Intermediate Frequency Amplifier Bandwidth and the Analogue Pulse Width.

The I.F. bandwidth Δf_{if} is the first radio frequency specification to be discussed because of an intimate connection with the analogue pulse width, T. Equation (2a) represents a function whose zeros occur at increments 1/T after the first zero, with an increment between the first two zeros near f_o of $2/T$. The specification laid down in most spectrum analysers, which are normally concerned with determining the "squareness" of pulses, is $\Delta f_{if} \ll 2/T$ in order that a true resolution be obtained. The problem involved with regard to bandwidth may be somewhat clarified by the obvious statement that if the bandwidth were infinitely wide, the transient $F(t)$ would be represented rather than the spectral distribution due to $F(t)$. These considerations[6] lead to a nominal specification $\Delta f_{if} \leqslant 0.1/T$.

In the present application, equations (4a) and (5a), and the right hand portion of table 1 must be consulted, for since it was decided to maintain T constant in consideration of the design of the analogue pulse generator, resolution of frequencies differing by less than 1 kc/s must be possible for $T > 60\ \mu sec$. Since bandwidths of the order 1 kc/s are practicable, the pulse width was set at $T = 50\ \mu sec$. This value, from equation (4a), specifies $\Delta f_{if} < 1.2$ kc/s for the minimum value $a/\lambda = 5$, this specification is more stringent than that previously cited where it is required $\Delta f_{if} \leqslant 0.1/T = 2$ kc/s.

3.4 Specification of Local Oscillator Requirements and the Computer Operating Frequency.

The required local oscillator frequency coverage, now that T has been fixed, can immediately be obtained from equations (5) and (6) and the specification $5 < a/\lambda < 100$. Thus for $a/\lambda = 100$ and $T = 50\ \mu sec.$, $2(f-f_o) = 2.8$ mc/s $\doteq \Delta f_{LO}$. It was decided[3] that the operating frequency of the computer should be as high as possible in view of the large percentage frequency deviation $\frac{\Delta f_{LO}}{f_o}$ which was required, in fact, the upper limit on a/λ was tentatively reduced to 50 for preliminary experimental work. The operating frequency was set at 30 mc/s, the upper limit being fixed by the use of conventional circuitry and vacuum tubes. The local oscillator must therefore provide a 1.4 mc/s frequency coverage at 30 mc/s, this coverage must be free from amplitude variations since the purpose of the analyser is amplitude discrimination between various frequency components.

3.5 Specification of the Intermediate Frequency.

The I.F. of a spectrum analyser is governed by the allowable image response. In equation (6) for example, if $f = f_{LO} + f_{if}$ is the desired response and the first order image $f = f_{LO} - f_{if}$ is to be excluded, it is evident, in view of the known maximum frequency to be swept by the local oscillator, that the I.F. is specified by $f_{if} \gg (f-f_o)_{max}$. Thus, directly from 3.4,

$f_{if} \geqslant \frac{1.4}{2} = 0.7$ mc/s. This specification supercedes those given in references (3) and (6) ($f_{if} > \frac{4}{T}$) which only accounts for amplitude discrimination of components out to the second zero of a square wave spectrum.

3.6 Summary.

A table of the design specifications is given below.

NAME	SYMBOL	SPECIFICATION
Sweep frequency	f_s	10 c/s
Analogue pulse recurrence frequency	f_r	900 c/s
I.F. Bandwidth	Δf_{if}	$<$ 1.2 kc/s
Analogue pulse width	T	50 μsec.
Computer operating frequency	f_o	30 mc/s
Local oscillator frequency sweep	Δf_{LO}	1.4 mc/s
Intermediate frequency	f_{if}	$>$ 0.7 mc/s

The above specifications have been derived by virtue of a knowledge of the operational constants in common use in the microwave region of the spectrum. All specifications are a rigid function of the pulse width, T. The fact that the method governed by equation (5a) of section 2.2 is to be used places stringent requirements on the spectrum analyser.

4. SPECTRUM ANALYSER DESIGN

A block diagram of the spectrum analyser was given in figure 2; the design of the individual blocks, as governed by the specifications of section 3 will now be discussed.

4.1 The Sawtooth Generator and Amplifier.

This unit is required to provide a 10 c/s linear sawtooth voltage of negligible recovery time which can be used to sweep the local oscillator frequency as well as operate the time base. Since it was decided to use the 60 c/s line voltage as a constant frequency synchronization source for both this unit and the analogue pulse generator to avoid possible instability due to power supply ripple and 60 c/s pickup, the sawtooth generator must be capable of scaling down by a factor of six.

T3 and T4 of figure 3 comprise an astable time base circuit[7], T3 is triggered by the positive going portion of the differentiated output of the over-driven 60 c/s amplifier T8A of figure 4. The time base circuit is convential in that C11 is periodically discharged by the thyratron, however, the exponential voltage which would appear at the output of the cathode follower is fed back by C12 to the C11-R7 circuit tending to increase the effective supply voltage and time constant of this circuit, the basic relation concerned being $e_{C11} = E_o(1-\exp.(-t/C11R7))$. An increase in C12 and R8 increase the magnitude of this effect. C11R7 was chosen as large as possible to afford maximum linearity without feedback in conjunction with a natural period which would allow 60 cycle synchronization. The output of T4 feeds the frequency modulator and a linear time base amplifier T5, both are A.C. coupled with long time constant circuits. T5 is provided with negative feedback to preserve linearity and is intended to feed a push-pull amplifier for driving a twelve inch cathode ray tube in an extended system.

4.2 The Local Oscillator and Modulator.

The local oscillator, which is probably the heart of the spectrum analyser, must produce a 30 mc/s signal which is swept over a range of 1.4 mc/s with negligible amplitude variation. Two methods available for linear frequency modulation involve the injection of either an actual or virtual reactive component into the tuned circuit of an oscillator. The former could be accomplished by mechanical rotation of a linear frequency condenser, this method was reserved to avoid possible mechanical and synchronization difficulties.

Figure 5 shows the equivalent circuit of a reactance or quadrature tube modulator. The circuit may be analysed as follows for a tube of high plate resistance:

$$i_1 = E/(R-j/wC) \quad E_g^1 = E(-j/wC)/(R-j/wC)$$

and if in addition $i_p \gg i_1$ then

$$I = gmE_g^1 = gmE(-j/wC)/(R-j/wC)$$

The effective impedance at ab is thus given by

$$Z_{ab} = E/I = +1/gm + jwCR/gm \quad \text{.............} (7)$$

A positive reactance equivalent to an inductance L_e = CR/gm therefore decreases the total inductance of the tuned circuit such that $1/L = 1/L_1 + gm/CR$.

The frequency is given by

$$f_{LO} = 1/2\pi(LC_1)^{\frac{1}{2}} = \left\{1/2\pi C_1^{\frac{1}{2}}\right\}(1/L_1 - gm/CR)^{\frac{1}{2}}$$

$$= \left\{1/2\pi(C_1L_1)^{\frac{1}{2}}\right\}(1 + gmL_1/2CR - gm^2L_1{}^2/8C^2R^2$$

$$+ \; .....) \quad \text{...............................} (8)$$

and if $gmL_1/CR \ll 1$ .... (8a)

then

$$f_{LO} = f_c\;(L_1gm/2CR + 1)$$

where $f_c = 1/2\pi\sqrt{L_1 C_1}$. If a reactance tube has a linear mutual conductance versus grid voltage characteristic such that over a given range

$$gm = kE_c + gm_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where k is a constant and E_c governs the operating point then

$$f_{LO} = f_c \left(\frac{L_1}{2CR} (kE_c + gm_o) + 1\right) \ldots\ldots\ldots\ldots (10)$$

and frequency varies linearly with grid voltage. Since the factor $f_c\left(\frac{L_1}{2CR}(k\overline{E}_c + gm_o) + 1\right)$, where $\overline{E}_c$ is the average value of E_c, can be eliminated by varying C_1, the frequency change about an operating point corresponding to any required centre frequency is

$$\Delta f_{LO} = \frac{kf_c L_1}{2CR} \Delta E_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The parameters occuring in (11) have practical limitations: k by tube design, C by input and wiring capacitances and ΔE_c by the range of linearity in (9). R has an optimum value, namely that value which for a given C will allow the radio frequency feed back voltage E_g' to cover the maximum range on the mutual inductance characteristic, E_g' for present purposes is not limited so strictly to the linear region as is E_c.

Further, from (7) and the qualitative requirement that Z be reactive,

$$wCR/gm > 1/gm \text{ or } R > 1/wC \text{ ....(12)}$$

(11) also indicates that, for a given value of C_1, f_c be as low as possible since

$$\Delta f_{LO} \propto f_c L_1 \text{ or } \Delta f_{LO} \propto \sqrt{L_1} \text{ ....(13)}$$

The upper limit on the L_1/C_1 ratio is set by oscillator stability.

The values of gm in the above derivation are assumed to be those associated with the static conditions of the tube. This reasoning is put forward since the plate load of the reactance tube, namely the oscillator tuned circuit, offers negligible impedance at the frequency f_r of E_c. Thus, if all reactance tube electrodes except the control grid are maintained at a constant potential, then the amplitude of E_c governs the operating point. A 6AG7 pentode was chosen as the modulator tube by virtue of the wide range and linear static mutual conductance characteristic. The characteristic is shown on figure 6 with the equation corresponding to (9).

In equation (11), if the optimum value of f_c, namely that value with the largest percentage frequency deviation Δf_{LO}, is less than the required operating frequency 30 mc/s, then the possibility of frequency multiplication exists. The latter occurs since a voltage $f_1 \pm \Delta f_1$ applied to a distorting amplifier whose load is tuned to pf_1 where p is integral will result in an output voltage of frequency $p(f_1 \pm \Delta f_1)$. The amplitudes of voltages at frequencies pf_1 and $p(f_1 \pm \Delta f_1)$ will be of the same order, provided the bandwidth of the load exceeds $2p\Delta f_1$. Thus, if a frequency deviation of 250 kc/s could be achieved at 5 mc/s which seems possible by evaluation of (11), then by sixfold multiplication, a 1.5 mc/s deviation should be available at 30 mc/s. The basic frequency 5 mc/s is suggested since the values C_1 and C can comprise values of the order of the stray capacitances of the circuit (see equation (13)) in conjunction with use of the smallest possible multiplication factor to obtain 30 mc/s. The small multiplication factor is consistent with power and economy considerations.

Figures 3 and 7 show the oscillator-modulator system. T1 is a conventional oscillator arranged such that the reactance tube T2 appears in parallel with the total tunable portion of L1. R_o and C_o, the latter being the input capacitance of the stage, are the R and C values of equation (11). C6 and R1 are such that their impedances are respectively low and high at the operating frequency. R2, R3 and R4 determine the range in which equation (9) is valid, the electrodes concerned are suitably decoupled at the frequencies associated with E_c and E^1_g. R6 isolates the grid circuit from any low impedances associated with the E_c source. The amplitude of the input to T2 is controlled by "Set a/λ" in consideration of equations (5a) and (11).

The 6AC7 multipliers of figure 7 were chosen by virtue of their high gain and sharp cut off characteristics, T6 and T7 are operated as a tripler and doubler with loads whose bandwidths must exceed .75 mc/s and 1.5 mc/s respectively. Both T6 and T7 must be driven below cut-off and above saturation to supply the required harmonics, the screen voltages were therefore stabilized to shorten the grid bases. The two tuned circuits in each stage are available for shaping the pass band response by staggered tuning.

4.3 The Intermediate Frequency Amplifier

The I.F. amplifier frequency specifications as given in section 3.6 are

$$\Delta f_{if} < 1.2 \text{ kc/s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

$$f_{if} \geqslant (f-f_o)_{max.} = 0.7 \text{ mc/s} \quad \ldots\ldots\ldots \quad (15)$$

The gain of the amplifier is, however, a function of the bandwidth, that is, since the energy in a pulse is spread over a wide frequency range, the total pulse power cannot be considered available at the input of a narrow band amplifier. Under the conditions of equation (2a), it can be shown[6] that the voltage loss with respect to B is $\alpha = 3T\Delta f_{if}/2$, thus for a 50 μsec pulse and $\Delta f_{if} = 1$ kc/s, $\alpha = 22$ decibels. The overall gain (G) for a one volt transient at the video detector must be

$$G = 20 \log (1/E_i) + 22 \text{ decibels} \quad \ldots \quad (16)$$

where E_i is the R.M.S. value of the analogue pulse voltage.

Moreover, the response of the amplification system must be some known function of E_i. For a linear plot the relation may be expressed as

$$h = aE_i + b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

where h is the height of a component on the cathode ray tube, a and b being constants. Preferably the system, in accordance with direct intensity measurement on microwave antennas, should respond to values E_i of (16) over a range of 30 decibels or greater.

These values, by (17), must obviously lie between the noise and saturation level of the amplification system. For a decibel plot of the component amplitudes (17) must become

$$h = a^{1} \log E_i/E_{io} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

where E_{io} is any reference value greater than the noise level. The fact that noise is present sets a limitation on the scale factor of the computer inherent solely in the spectrum analyser since observation over the full range of the input variable cannot be achieved.

Bandwidths of the order 1 kc/s are possible at 0.7 mc/s, however, due to the availability of parts, it was decided to double heterodyne. The amplifier is shown on figure 8. Condition (15) is obtained by the first mixer T9 which has a high Q load tuned to 1.8 mc/s. The first mixer has two main requirements; firstly, the grid base associated with the oscillator voltage must be short in order that the local oscillator voltage swing over the grid base for a maximum range of local oscillator frequencies, this is associated with the fact that the local oscillator voltage will fall off somewhat at the extremities of the frequency range. Secondly, the analogue pulse input circuit must be as non-frequency sensitive as possible. These requirements were taken into account by using a double injection grid tube with a fixed screen potential.

Condition (14) is met by conversion to 455 kc/s and the use of a crystal filter bridge, the conversion is accomplished by T10 which comprises a 2.255 mc/s electron coupled oscillator and the second mixer. T11 and T12 are remote cut-off 455 kc/s amplifiers operated at a high screen potential to provide a wide amplification range, this matter is related to equations (17) and (18) but the form of the amplification function was not specifically taken into account in the design. The amplifiers are followed by a conventional detector which in turn feeds a video amplifier. The latter is not critical provided its transient response does not exceed $1/f_r$, this condition actually applies to the overall amplification system. The vertical amplifier of a cathode ray oscilloscope provides suitable video amplifiers for experimental purposes.

4.4 The Time and Frequency Calibrators

It is evident that both the time base voltage and Δf_{LO} of equation (11) must be linear, in fact, since the latter is a function of the former, it is desirable that the time base be first checked for linearity in time and then in frequency, the calibrators of figure 4 were designed for this purpose.

T8B is an overdriven amplifier fed by the 120 c/s ripple voltage from a full wave rectifier. The differentiated output is fed via the time calibration switch to the video amplifier and thus to the vertical deflection plates of the display unit. Twelve calibration markers are thus provided and these correspond to a $\frac{3}{4}$ inch spacing on a 9 inch time base trace.

A general approach to frequency modulation is given in Appendix 2, however, sinusoidal frequency modulation is well known and it offers a method for checking the frequency linearity of the analyser. If a frequency w(t) is varied such that $w(t) = w_o + \Delta w \cos w_m t$ where Δw is the maximum frequency deviation and w_m the modulating frequency, then

$$\theta(t) = \int w(t)\,dt = w_o t + \Delta w/w_m \sin w_m t$$

$$e(t) = E \cos \theta(t) = E \cos (w_o t + \beta \sin w_m t)$$

where $\beta = \Delta w/w_m$ is the modulation index. e(t) can be expressed as a series[8]

$$e(t) = E\Big[J_0(\beta)\cos w_o t + J_1(\beta)\Big\{\cos(w_o + w_m)t - \cos(w_o - w_m)t\Big\} + J_2(\beta)\Big\{\cos(w_o + 2w_m)t + \cos(w_o - 2w_m)t\Big\} + \ldots\Big] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

where J_i is a Bessel function of the i^{th} order, first kind. Terms are thus symmetrically displaced in the spectrum by incriments w_m about w_o.

Since $\Delta f_{LO\ max.} = 1.41$ mc/s corresponds to a change in sin θ from -.707 to .707, it was decided to use a source for w_m of (19) at such a frequency that a suitable number of markers be available. The frequency chosen was $w_m = 100$ kc/s which, for $a/\lambda = 50$, created a correspondence between each term of (19) and sin θ = -.7, -.6 ...0, ...6, .7, where $\theta = 0$ corresponds to $EJ_o(\beta)$ sin $w_o t$. In this manner the linearity of (11) may be checked. The amplitudes J_i though eratic for various values of (β) have finite values over 1.4 mc/s for $\beta \geqslant 5$.

T14 of figure 4 is a conventional 100 kc/s crystal oscillator which feeds a 7.5 mc/s oscillator modulator circuit, T15 and T16, similar in design to that of figure 3. The oscillator output feeds a broad band quadrupler, the quadrupler output being fed to the first mixer by a matched coaxial line. The "Set β" potentiometer controls the number and form of the markers available on the display tube. The calibrator is of course switched off when analogue pulses are under investigation.

The calibration oscillator should be valuable in determining the position of the "Set a/λ " control for a required a/λ ratio since the use of a signal generator for calibration purposes is both inaccurate and lengthy especially for small a/λ values. Table 2 is a calibration chart for this purpose, it is obtained directly from equation (5a).

Table 2.

Frequency Sweep Calibration $T = 50\,\mu sec$		
a/λ	Frequency Marker	$\sin\theta$
50	5	.50
45	4	.44
40	4	.50
35	3	.43
30	3	.50
25	2	.40
20	2	.50
15	1	.33
10	1	.50

The accuracy of this calibration, neglecting measurement error on any given display unit can be obtained from equation (3), for example, for a/λ =

50, 20, 10, the calibration is valid to within 1, 2 and 1 percent respectively.

4.5 Summary.

Circuits which, theoretically, fulfill the specifications of section 3 as well as requirements inherent in the nature of a spectrum analyser have been set forth. The circuits may be termed those necessary for determining whether or not the specifications can be met. No consideration has been given to the display mechanism in its own right since the intention was to use a cathode ray oscilloscope as the display tube, the matter of an elaborate display unit offers no great difficulty. Suitable methods have been proposed for determining the performance of the analyser.

5. EXPERIMENTAL RESULTS

The outcome of the design of section 4 will now be discussed. The results will not be presented chronologically with regard to the experimental work but by units and in the same sequence as followed in section 4; any minor modifications to the design as given in section 4 are included here.

5.1 Sawtooth Generator and Amplifier.

The 60 c/s synchronization pulses from T8A, figure 4, eight volts in amplitude and of four hundred μsec duration, allowed synchronization of the generator, T3 and T4 of figure 3, from 60/8 to 60 c/s. The sawtooth output of T4 at 10 c/s had a peak magnitude of twelve volts and a recovery time of five hundred μsec. Since the analogue pulse period is 1/900, or about 1100 μsec, none of the ninety responses should be lost provided the synchronization of the analogue pulse generator is fine enough. The sweep amplifier T5 had a linear gain of ten, the cathode bias as determined by R17, R18 and R19, was fixed such that the operating point be at the midpoint of the linear portion of the transfer characteristic. The output of T5 fed directly to the horizontal plates of a Phillips five inch oscillograph, the latter being a low frequency model suitable for a 10 c/s time base frequency.

The time base condenser C11 and the feedback condenser C12 were determined experimentally for most linear operation. Overdriving or incorrect biasing of T5 was easily observed by crowding of the time markers at the extremities of the sweep. In fact, a heavy demand is placed upon T5 in driving a five inch oscilloscope; these remarks are made to ensure that sufficient amplification (preferably somewhat in excess) is provided when a twelve inch display tube is involved.

Display 1 shows a time calibration provided by T8B on a three inch time base produced by T5. Crowding of the markers is somewhat in evidence at the left end of the time base. It is to be understood that the scale in display 1 has no intimate connection with the time markers.

5.2 Local Oscillator and Radio Frequency Multipliers.

A. The local oscillator T1 and modulator T2 of figure 3 were first constructed to operate directly as a 30 mc/s system. This was attempted not only to test the theory of section 4.2 but for economic reasons (no multipliers would be necessary). L_1 was $2\mu h$ in this case and R_o was experimentally determined to require an optimum value of 50,000 ohms. For the condition $R < R_o$, the grid voltage, as viewed at the grid of T2 on a test oscilloscope would be "clipped" or flattened off at the positive portion of the wave, for $R > R_o$, the frequency deviation (keeping E_c at a constant peak to peak amplitude) would decrease. The former effect can be explained by considering the nature of the voltages at the grid of T2, namely, a 30 mc/s voltage operating about a mean value as determined by a 10 c/s sawtooth voltage, or $E_g = E_g^1 + E_c$. Thus if E_g exceeds the cathode voltage due to a high value of E_g^1, grid current will be drawn and an apparent flattening of E_c observed. The latter effect is a direct consequence of the theory of section 4.2.

The maximum overall frequency deviation under the above conditions was $\Delta f_{LO\ max.} = 450$ kc/s. From equation (11) and figure 4, using a nominal value $C_o = 15$pf, the optimum $R_o = 50{,}000$ ohms and the average value of gm, $\Delta f_{LO} = 500$ kc/s. It is apparent that the theory of section 4.2 is valid under the above conditions since $gmL_1/C_oR_o = 0.015$ and the series (8) converges rapidly. No linearity check was made on the above deviation since the maximum deviation obtained was only thirty percent of the required value.

B. The modulator oscillator circuit was modified to operate at 5 mc/s with the values as shown in figure 3. In conjunction with the multipliers of figure 7, a frequency coverage $\Delta f_{LO} = 3$ mc/s was observed. From (11), with $L_1 = 20\,\mu h$, $C_o = 15$ pf, using the multiplication factor $p = 6$,

$$\Delta f_{LO\ max.} = 6kf_c L_1 \Delta E_{c\ max}/2CR = 3 \text{ mc/s}$$

Displays 2 to 6 show frequency calibrations for $a/\lambda = 10, 20, 30, 40$ and 50, respectively. A discussion of the amplitudes of the markers is given in appendix 1, it is the horizontal increments which are of interest here. In the displays, frequency increases from left to right; since a complete sweep of the local oscillator occurs during the return time of the trace, a spectrum will appear on the flyback, this effect shows up at the left end of the time base. The displays are somewhat over exposed in an endeavour to establish contrast.

The calibrations are seen to be linear (that is, they agree with the calibration chart of section 4.4 and also have equal spacing) up to $a/\lambda = 50$. Above this value (assured always that E_c is linear) df_{LO}/dE_c began to increase at the extremities of the range. Reference to equation (11) and figure 6 indicates that since k in effect decreases at the extremities, df_{LO}/dE_c should likewise decrease. Under the above conditions $gmL_1/C_oR_o =$

0.16 and the series (8) still converges too rapidly to account for the effect. A qualitative explanation is as follows: since the reactance tube is a quadrature tube, the dynamic load lines can be shown[9] to become ellipses. Figure 9 is plot of the dynamic grid load line for T2, under the conditions of figure 3, as obtained by graphical methods from the corresponding plate load line. It is evident that if the centre of the ellipse be shifted toward the extremities of the grid base, then the principal vertices will become distorted due to cut-off and grid current. This condition will reduce E_g' in the derivation of (7) (which is invalid under the assumed conditions) and Z_{ab} and L_e will have a smaller value. Thus:

$$f_{LO} = \left\{1/2\pi\sqrt{C_1}\right\} (1/L_1 + 1/L_e)^{\frac{1}{2}}$$

$$df_{LO}/dL_e = \left\{1/4\pi\sqrt{C_1}\right\} (L_1L_e/L_1 + L_e)^{\frac{1}{2}} 1/L_e^{\ 2}$$

and as $|L_e|$ decreases df_{LO}/dL_e increases. The discrepancy between the above 3 mc/s theoretical and 1.4 mc/s ($a/\lambda = 50$) experimental value for <u>linear</u> operation is therefore due to the fact that E_g' is assumed undistorted in equations (7) to (11). The frequency linearity was highly sensitive to changes in the screen and cathode potentials of T2, for values $a/\lambda < 20$, slight bias adjustment was necessary to preserve linearity.

The R.M.S. voltages from T1, T6 and T7 were 30, 20 and 10 volts at 5, 15 and 30 mc/s respectively. The tripler and doubler bandwidths were 0.9 and 1.8 mc/s at six decibels down.

The constant amplitude requirement of the local oscillator was checked by feeding the receiver with a signal from a signal generator of constant output and varying the latter slowly over the required range. Constant response was obtained for values $a/\lambda \leq 50$, a display showing this condition was not obtained due to the long exposure time involved. The linear amplitude response was obtained by successive adjustment of the tripler and doubler tuned circuits. 60 c/s pickup throughout the system, more especially on the sawtooth voltage at the grid of T2 appeared as six departures from constant response across the trace, these were eliminated by increased decoupling.

5.3 Receiver.

The 455 kc/s I.F. section T10, T11, T12 and T13 of figure 8 was constructed prior to the double heterodyne system. The crystal filter bridge is similar to that used in the Radio Corporation of America type A.R. 88 receiver. The phasing condenser C57 was adjusted such that the response was symmetrical, this condition occurs when the crystal holder capacity is balanced out.[10]

For any other setting of C57 a "rejection slot" or region of high attenuation appeared in the response, the symmetrical response was used to avoid the possibility of double peaking. The response data of the 455 kc/s I.F. section using the filter is shown in figure 10, a further increase in C57 from this condition created a rejection slot which first appeared at approximately 455.5 kc/s, the bandwidth with the crystal shorted was 3 kc/s. Operation at 455 kc/s brought forward the requirement for a higher I.F. mentioned in section 3.

It was impossible to estimate the bandwidth of the complete circuit of figure 8, that is, including the 1.8 mc/s I.F. stage, with a General Radio type 1001A signal generator, the overall response was therefore obtained by using the analyser on a very short frequency sweep. Displays 7 and 8 show the bandpass of the receiver with and without the crystal filter, respectively; the total time base in each case corresponds to a frequency sweep of approximately 25 kc/s and the measured bandwidths are 0.7 kc/s and 4 kc/s with and without the filter respectively. Response on the return of the trace is prominent in both displays.

A ring was at all times present in the video output of the receiver. When continuous wave or frequency calibration signals were applied the ring was not as marked as when analogue pulses were under examination. In fact, in the latter case and with the filter in operation, the centre of the trace (corresponding to high amplitude signals) would lift at the base due to the increased transient response of the receiver. These observations insured that the response of the analyser to a continuous wave signal was distinct from response to repetitive pulses. In the former case, the transient is a function solely of the rate of the frequency sweep and the bandwidth of the I.F. amplifier. In the latter case, in terms of a single pulse, the pulse width is the only factor concerned provided $T \ll 1/f_r$. Both statements assume negligible transient response of the amplifier. The steeper sides of the analogue pulse therefore created a more accentuated ring, feedback to the second mixer has been suggested to correct this effect. Analogue pulses have been examined down to a/λ values of 3 and 5 at a bandwidth of 4 kc/s; the amplitudes, which are discussed in reference (4), compare favourably with theoretical values. From this result it seems probable that the specification of equation (14) is too stringent.

The amplitude characteristic of the receiver as used for all results in reference (4) and in this paper is plotted on figure 11. The response is sensibly linear over a range of 34 decibels, the equation corresponding to (17) is shown on the graph. The results in reference

(4) were obtained under the conditions $E_{1\ max.} > 5mV$, the side lobes therefore appear higher than theoretical plots due to the curved portion of the response.(display 9).

5.4 Calibrators.

Both calibrators were of great assistance in the experimental work, the frequency calibrator served in checking the analyser response for symmetry as well as frequency linearity.

A. The time markers were a function of the power supply which was being used as a source of ripple. For example, an unbalanced full wave rectifier or an improperly designed filter would result in "pairing off" in place of an equal time increment between each marker. The negative portions of the differentiated voltage (which are actually hidden by the scale in display 1) can be eliminated by a class C amplifier if necessary.

B. Further characteristics of the frequency markers are discussed in appendix 1, only their efficiency as a calibrator for the spectrum analyser is discussed here. The "Set β " control, for this purpose, would assume a fixed value corresponding to a value $\beta \doteq 5$ (see appendix 1 and display 6), the variations in amplitude in the various displays exist as a means of verifying equation (19).

It is apparent that the frequency markers do not create accurate calibration signals with the receiver in its present state of design and with the present method of displaying the calibration signals. It is clear, however, that if the markers were amplified, limited and used to grid modulate a cathode ray tube (which could also apply to the time markers) in future work, then accurate calibration would be achieved.

Several families of frequency markers may appear on the analyser due to spurious response, this can cause a certain overlapping for high a/λ values. Good shielding and decoupling was required in the frequency calibrator as well as in the local oscillator and multipliers since the spurious response was due to harmonics of the fundamental oscillators. The correct family of markers was checked by comparing the total frequency sweep width as given by a signal generator with that indicated by the markers of the calibrator.

6. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK IN AN EXTENDED SYSTEM

6.1 Spectrum Analyser.

The ability of the spectrum analyser to select frequency components, establish their intensity and position in the spectrum, and perform an overall integration of the components for display purposes, has been demonstrated by various configurations in reference (4) and this paper. Several major modifications which appear to be necessary in the spectrum analyser before the construction of a prototype computer are listed below.

A. Spectrum Analyser Modifications.

1. A single I.F. of 1 mc/s should be used. The difficulties of secondary response introduced by double heterodyning would be eliminated and the bandwidth obtainable would be suitable for values $a/\lambda \geqslant 5$.

2. Direct coupling from the 10 c/s generator to all loads is advisable to eliminate distortion of the sawtooth.

3. A certain amount of shifting of the centre frequency occurred for various positions of the "Set β" and "Set a/λ" controls since the reactance tubes assumed a slightly different operating point. Especially in the case of the local oscillator, the frequency should be

locked by a compensating voltage at the reactance tube.

4. Both calibration voltages can be used to grid modulate the display tube for greater accuracy, the frequency markers will require amplification and limiting for this purpose. Flyback elimination should be incorporated.

5. A theoretical and experimental examination of the response of narrow band radio frequency amplifiers to pulses of short duration should be carried out.

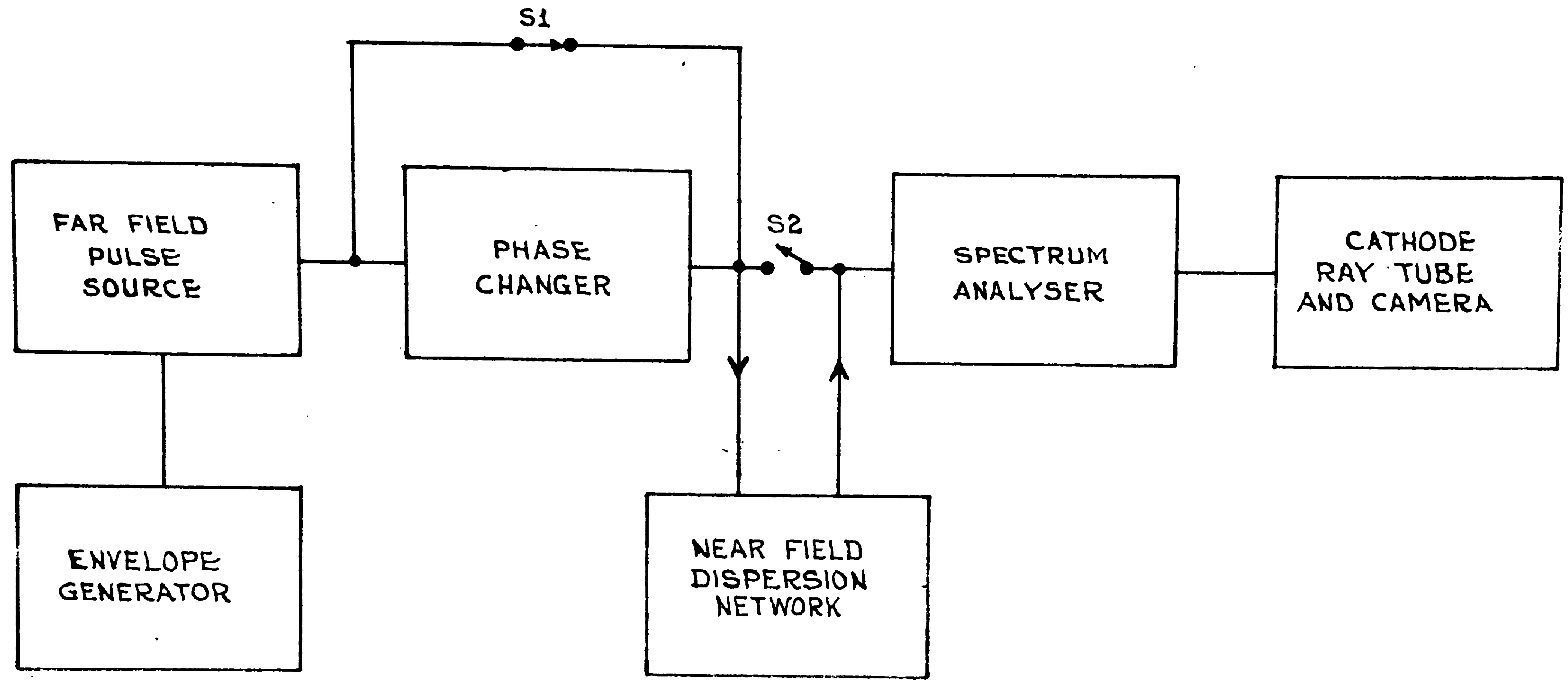
B. Accuracy of Component Frequency and Amplitude Analysis.

Since the accuracy involved in the analysis of a given pattern is a function not only of the measurement error of the given pattern on the display tube but also of the calibration accuracy, only an indication of the possible accuracy of the computer is available. This is evident from the fact that the display tube used for patterns in this paper and in reference (4) was only one third of the required size. Display (9) shows a pattern of a far field representation. Both component frequency and amplitude measurements agree to within the measurement error with theoretical values. It must be mentioned here that the patterns shown in reference (4) were recorded before the frequency linearity of the analyser had reached its present state.

Discussion of accuracy brings forward a recommendation pertinent to the overall design of the computer, namely an increase in the time base frequency. If an increase were made in the sweep frequency (f_s), less persistent display tubes could be used and more accurate representations would be obtained. This change would, of course, involve an increase in the pulse recurrence frequency (f_r) and probably a reduction in pulse width (T).

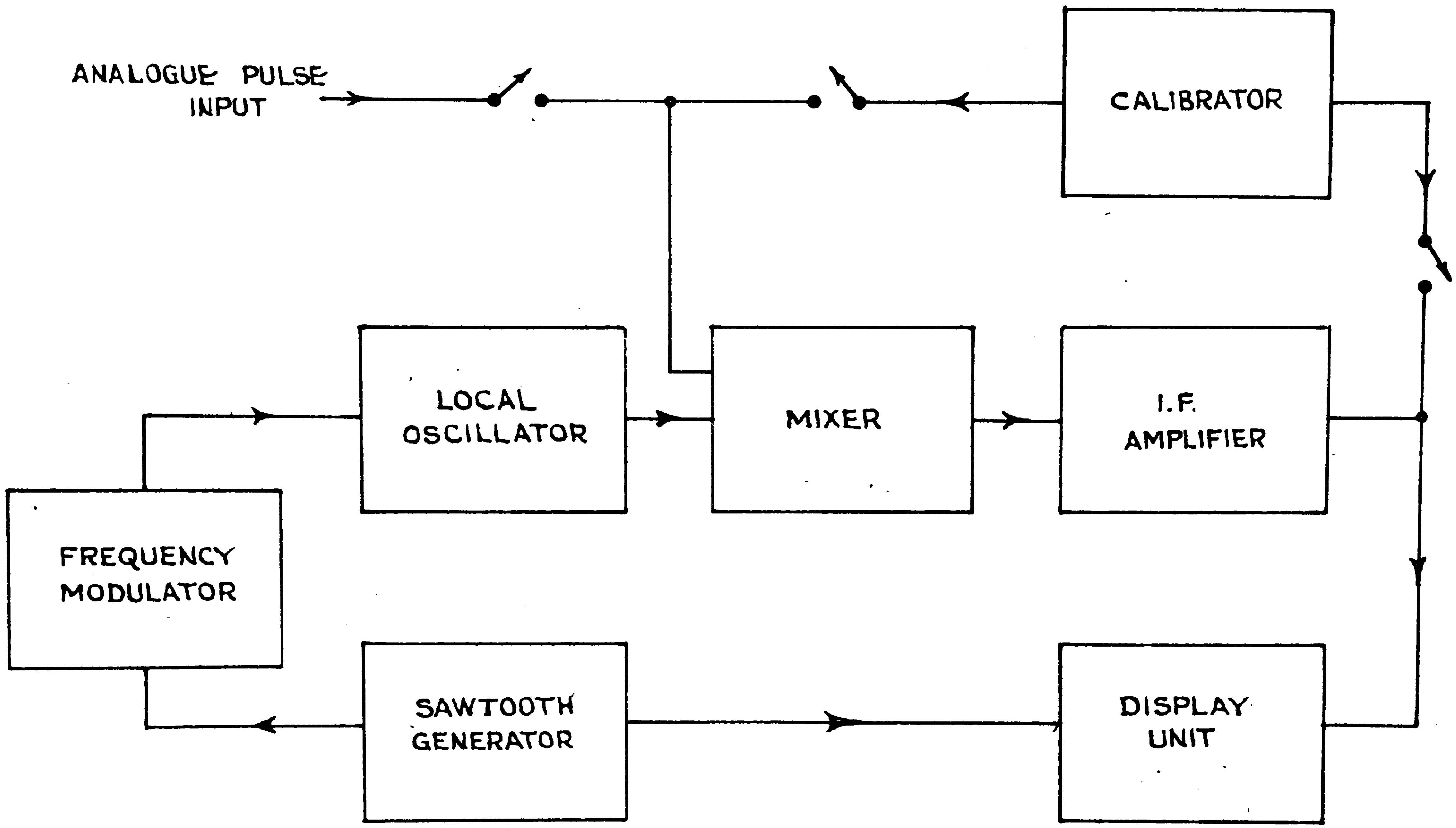
6.2 Computer.

The possibility of forming and displaying a representation of the Fraunhoffer diffraction field which is created when monochromatic electromagnetic radiation is limited by a rectangular slit has been shown to exist. Certain closely related phenomena such as the field patterns due to electromagnetic horn radiators are also feasible. On the other hand, a representation of the Fresnel, or near field has not been obtained, further investigation is necessary in this respect.



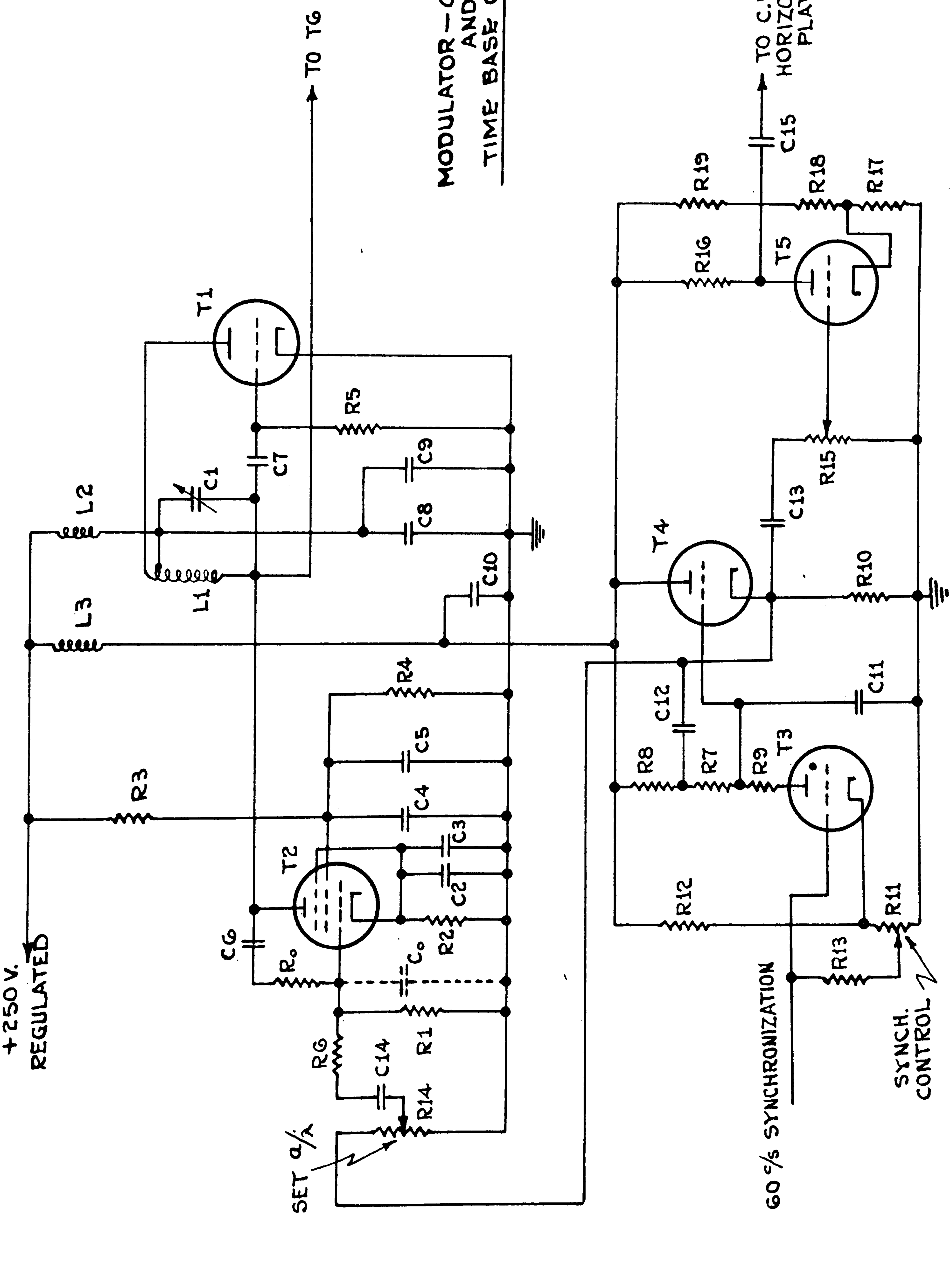
FIELD COMPUTER

FIG. 1.



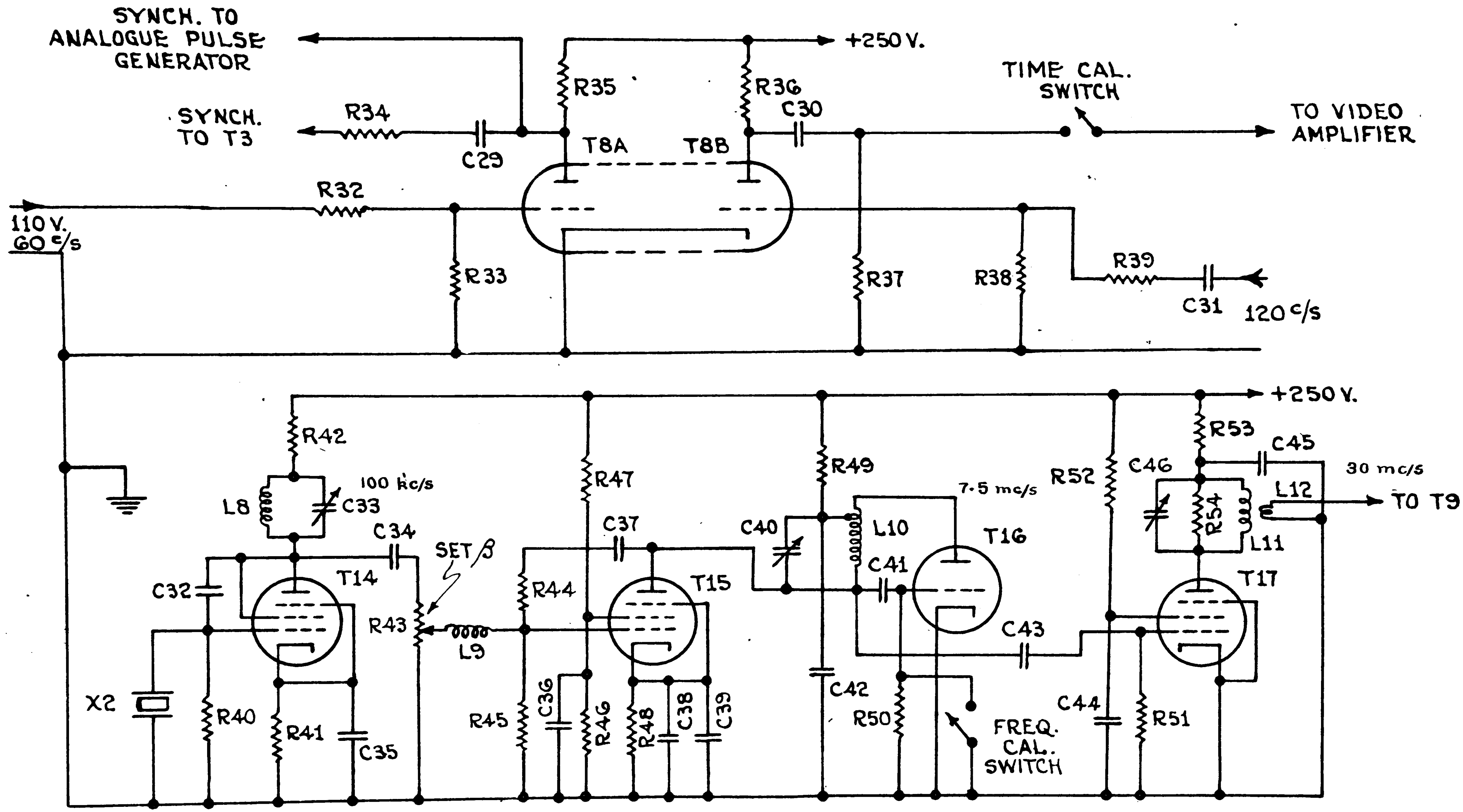
SPECTRUM ANALYSER

FIG. 2.



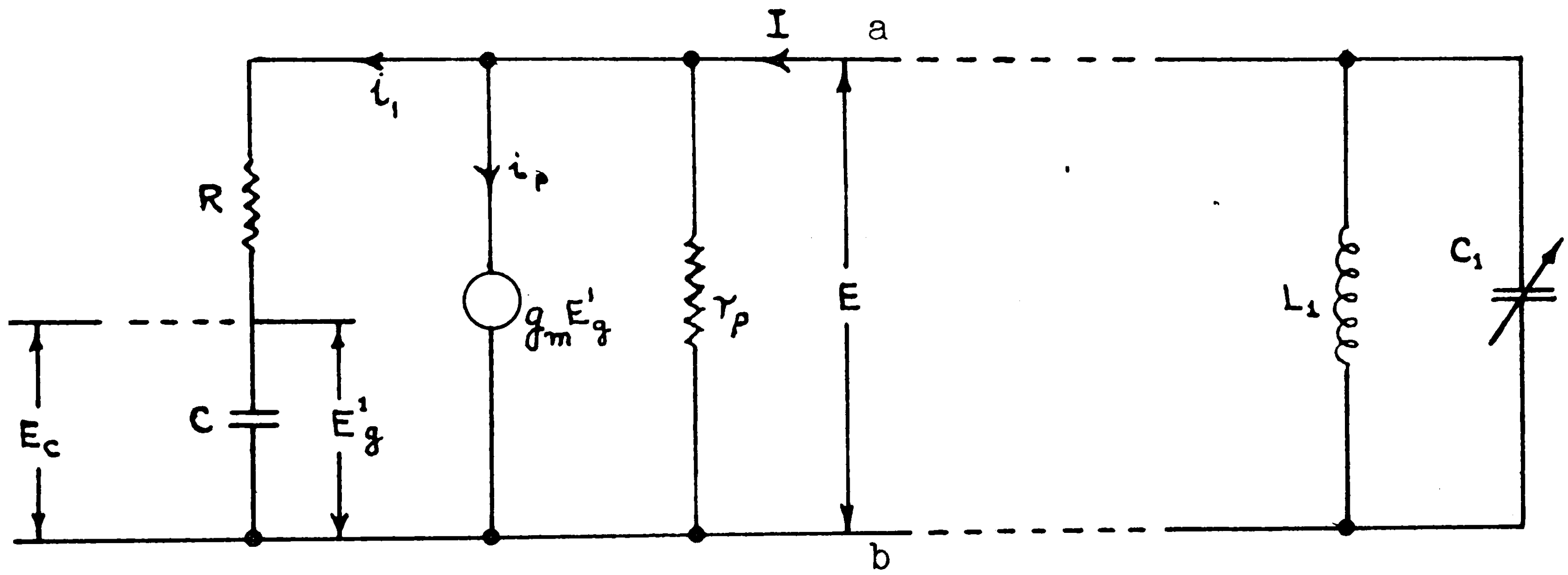
MODULATOR—OSCILLATOR AND TIME BASE CIRCUIT

FIG.3



CALIBRATION UNITS
AND
SYNCHRONIZATION SOURCE

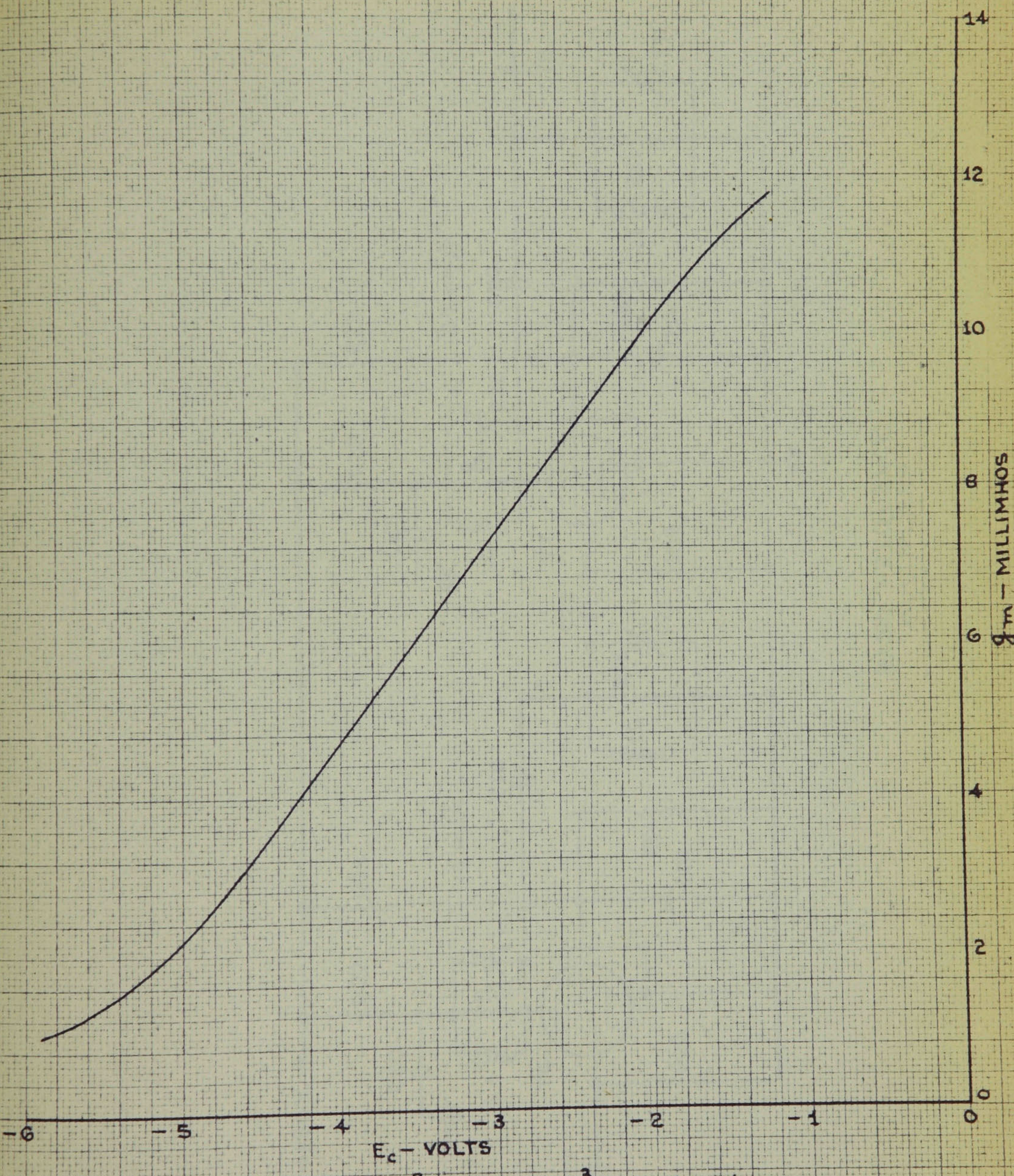
FIG. 4.



MODULATOR
EQUIVALENT CIRCUIT

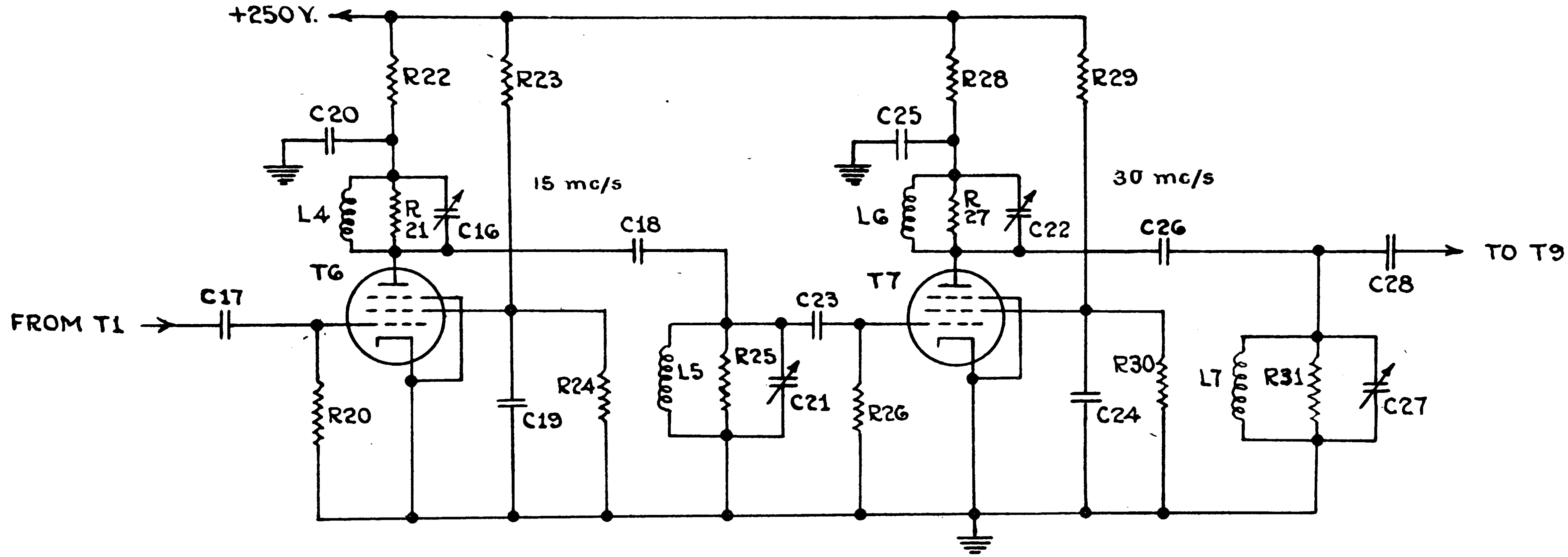
FIG.5

6AG7 STATIC CHARACTERISTIC



$$g_m = kE_c + g_{m_o} = 2.6 \times 10^{-3} E_c + 15 \times 10^{-3} \qquad -5 \leq E_c \leq -2$$

FIG. 6



RADIO FREQUENCY
MULTIPLIERS

FIG.7.

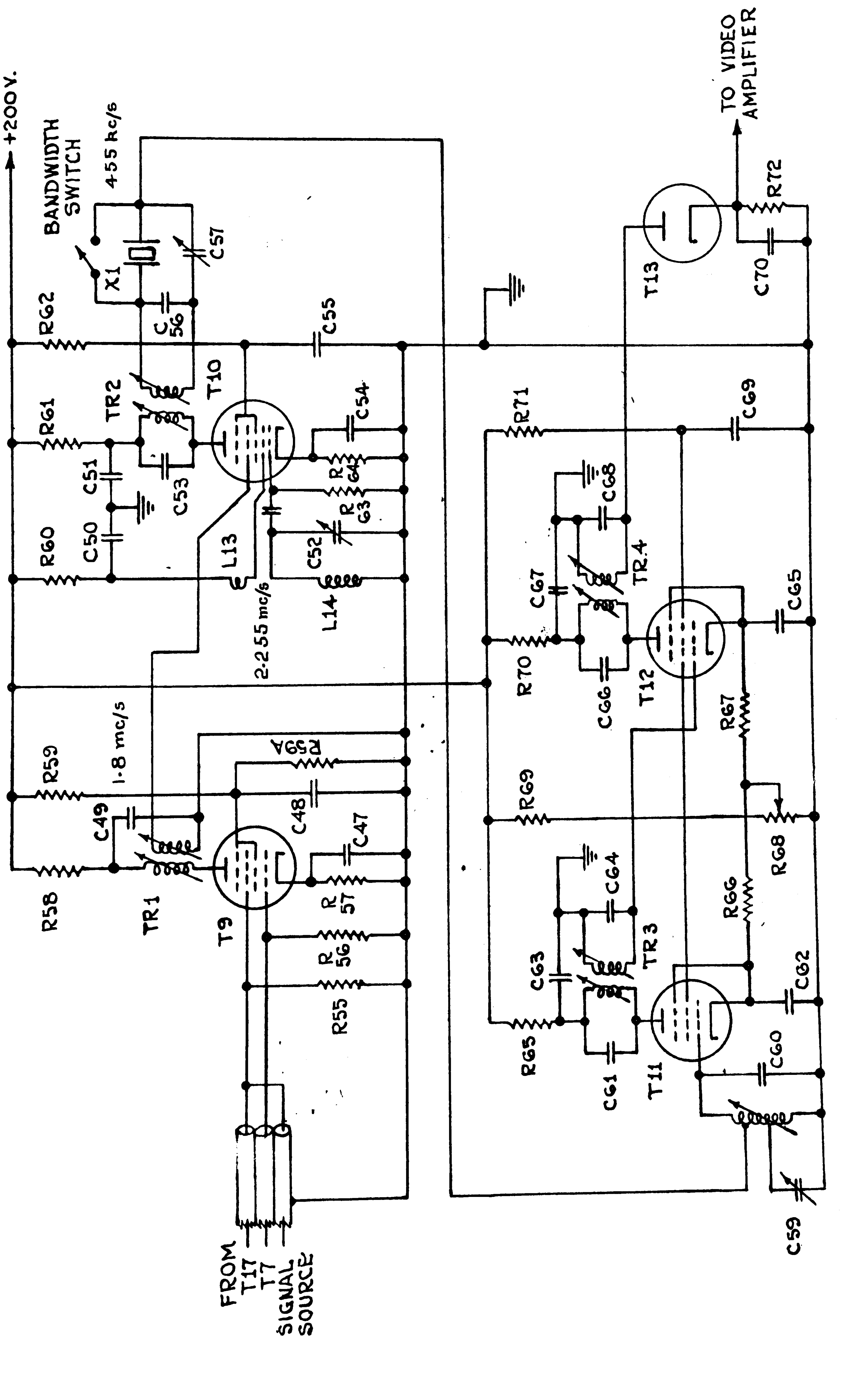
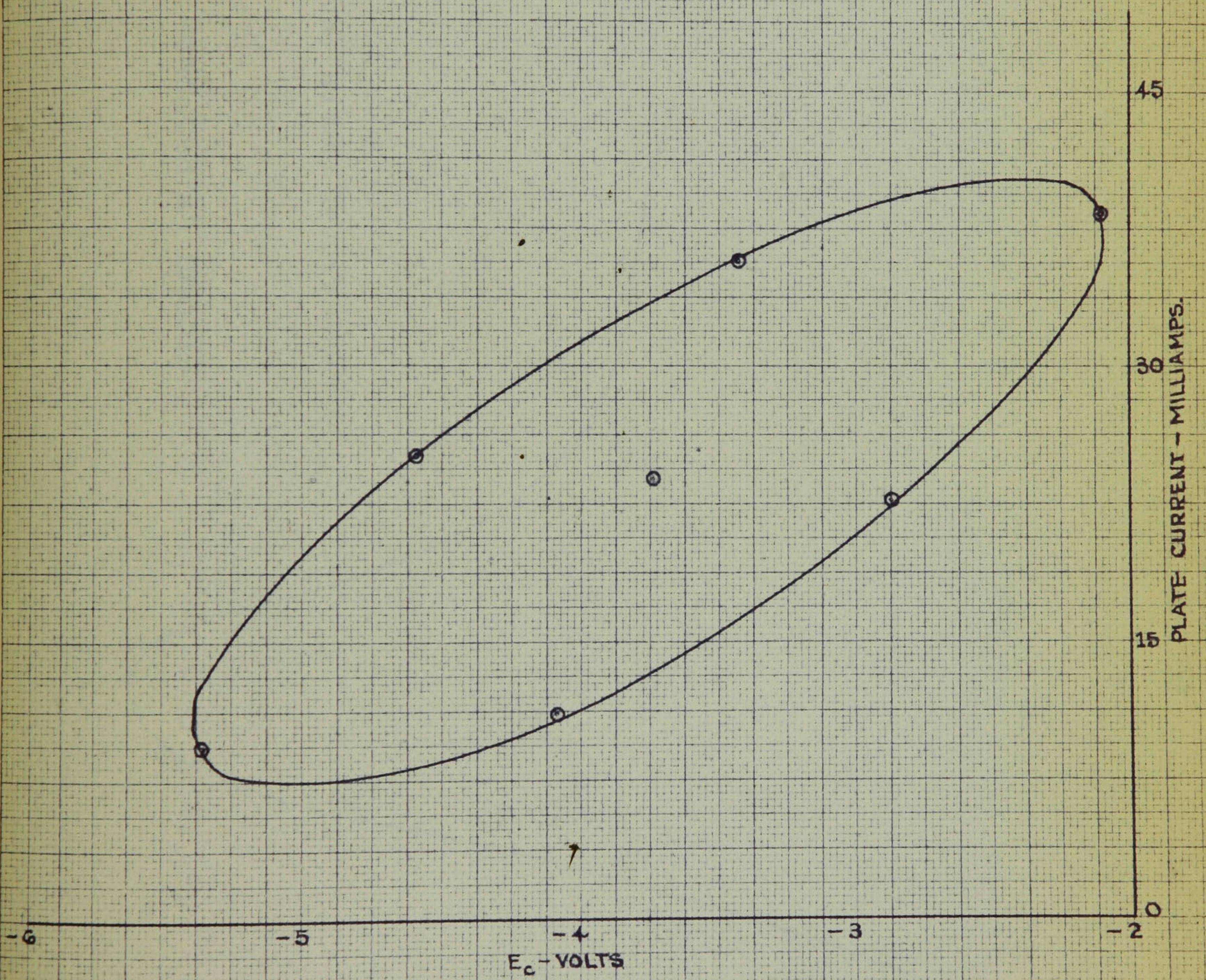


FIG. 8

I. F. AMPLIFIER



T2 DYNAMIC GRID LOAD LINE

FIG.9.

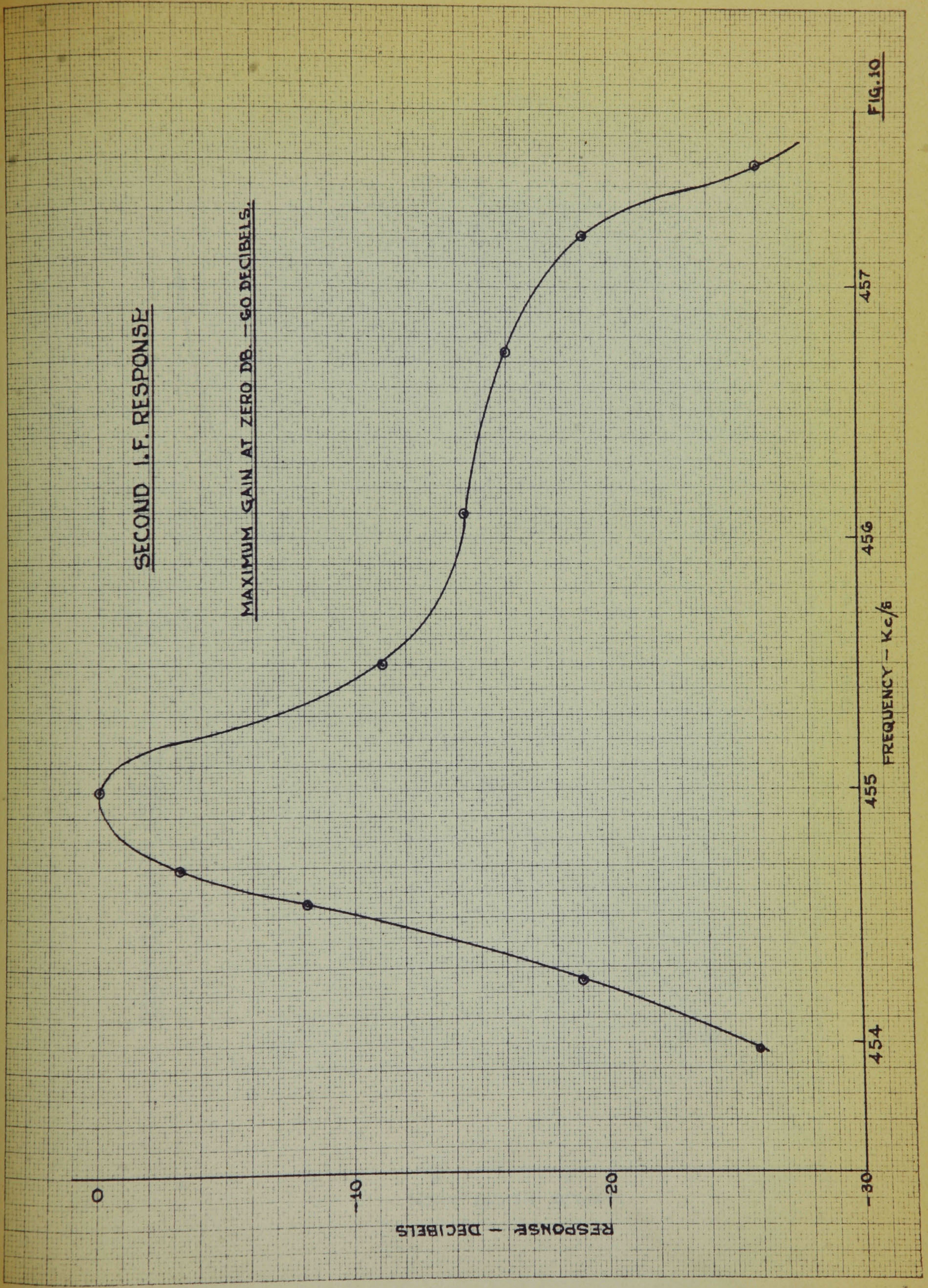

SECOND I.F. RESPONSE
MAXIMUM GAIN AT ZERO DB. — 60 DECIBELS.
FREQUENCY — Kc/s
454
455
456
457
RESPONSE — DECIBELS
0
-10
-20
-30

FIG.10

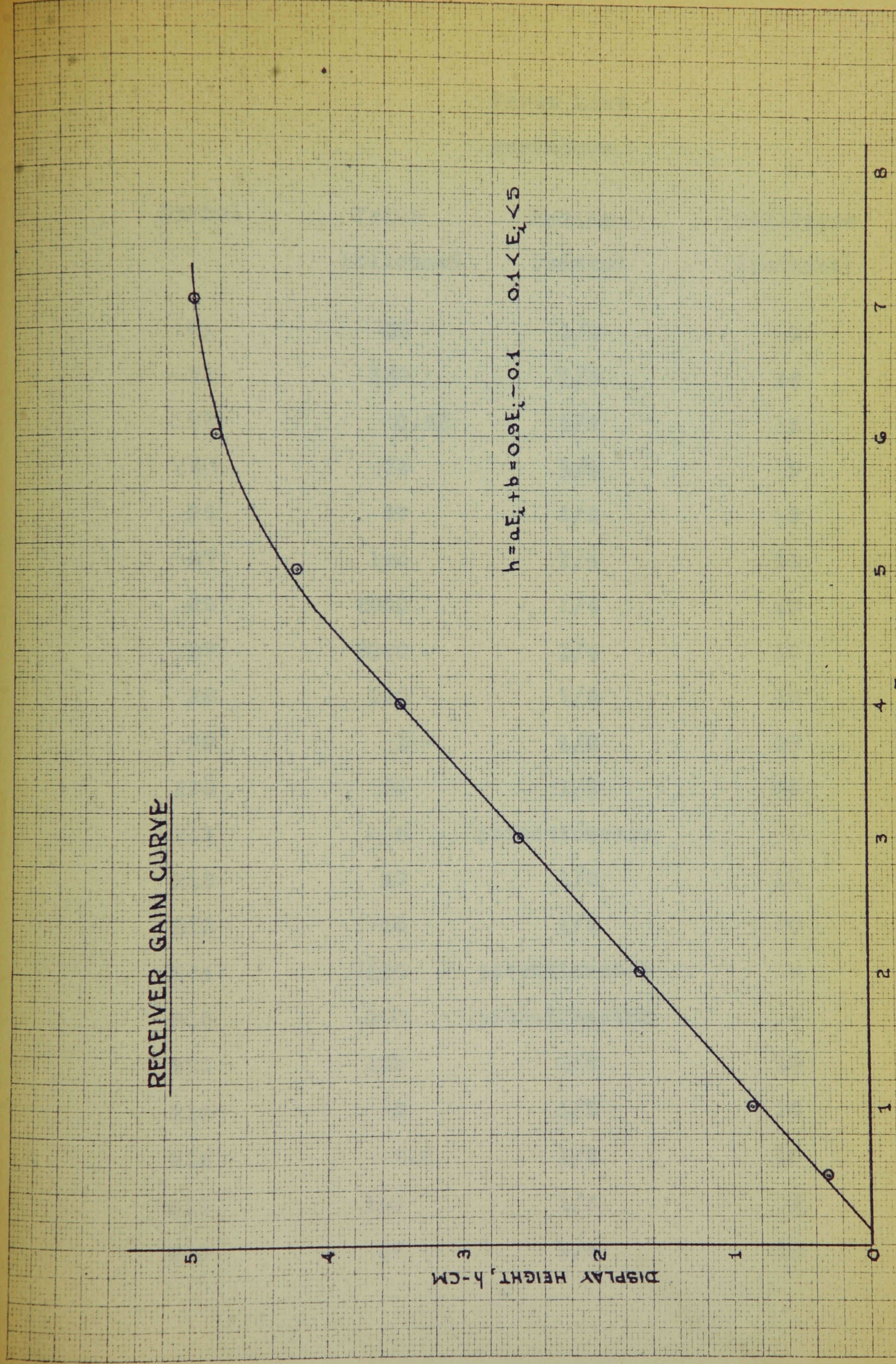

RECEIVER GAIN CURVE
h = aEᵢ + b = 0.9Eᵢ − 0.1   0.1 < Eᵢ < 5
INPUT, Eᵢ − mV
DISPLAY HEIGHT, h-CM
0
1
2
3
4
5
6
7
8

FIG.11

Parts List

Resistors.

Symbol	Value (kilohms)	Rating (watts)	Tolerance (percent)
R_o	33	1/4	5
R1	330	1/4	10
R2	0.68	1/2	5
R3	72	1/2	5
R4	39	1/2	5
R5	100	1/4	10
R6	2000	1/4	10
R7	2000	1/4	10
R8	100	1/2	10
R9	1	1/2	10
R10	50	1/2	10
R11	5	potentiometer	-
R12	47	1/2	10
R13	15	1/4	10
R14	1000	potentiometer	-
R15	100	potentiometer	-
R16	100	1/2	10
R17	2	1/2	10
R18	2.2	1/2	10
R19	100	1/2	1
R20	47	1/4	10

Parts List

Resistors (cont'd)

Symbol	Value (kilohms)	Rating (watts)	Tolerance (percent)
R21	47	1/2	10
R22	1.5	1/2	10
R23	68	1/2	10
R24	27	1/2	10
R25	47	1/2	10
R26	47	1/4	10
R27	25	1/2	10
R28	1.5	1/2	10
R29	47	1/2	10
R30	1.5	1/2	10
R31	25	1/2	10
R32	100	1/2	10
R33	30	1/2	10
R34	470	1/2	10
R35	100	1/2	10
R36	100	1/2	10
R37	470	1/2	10
R38	330	1/2	10
R39	150	1/2	10
R40	47	1/4	10
R41	.47	1/2	10

Parts List

Resistors (cont'd)

Symbol	Value (kilohms)	Rating (watts)	Tolerance (percent)
R42	1	1/2	10
R43	500	potentiometer	-
R44	33	1/4	5
R45	470	1/4	10
R46	39	1/2	5
R47	39	1/2	5
R48	.85	1/2	5
R49	1	1/2	10
R50	100	1/4	10
R51	47	1/4	10
R52	50	1/2	10
R53	2.7	1/2	10
R54	25	1/2	10
R55	.1	1/4	10
R56	.1	1/4	10
R57	.47	1/2	10
R58	4.7	1/2	10
R59	50	1/2	10
R59A	50	1/2	10
R60	50	1/2	10
R61	6.8	1/2	10

Parts List

Resistors (cont'd)

Symbol	Value (kilohms)	Rating (watts)	Tolerance (percent)
R62	39	1/2	10
R63	100	1/4	10
R64	.25	1/2	10
R65	2.2	1/2	10
R66	.25	1/2	10
R67	.25	1/2	10
R68	5	potentiometer	-
R69	68	1/2	10
R70	2.2	1/2	10
R71	7.5	2	10
R72	470	1/2	10

Capacitors

Symbol	Value	Rating (volts)	Tolerance (percent)
C_o	input capacitance	-	-
C1	0-25 pf	air dielectric	-
C2	2000 μf	6	20
C3	.01 μf	300	10

Parts List

Capacitors

Symbol	Value	Rating (volts)	Tolerance (percent)
C4	.01 μf	300	10
C5	40 μf	300	10
C6	250 pf	300	10
C7	100 pf	300	10
C8	.001 μf	300	10
C9	20 μf	300	10
C10	.01 μf	300	10
C11	.05 μf	300	10
C12	.2 μf	300	10
C13	10 μf	300	10
C14	10 μf	300	10
C15	.5 μf	300	10
C16	0-25 pf	air dielectric	-
C17	50 pf	300	10
C18	50 pf	300	10
C19	.005 μf	300	10
C20	.005 μf	300	10
C21	0-25 pf	air dielectric	-
C22	0-25 pf	air dielectric	-
C23	50 pf	300	10

Parts List

Capacitors

Symbol	Value	Rating (volts)	Tolerance (percent)
C24	.005 μf	300	10
C25	.005 μf	300	10
C26	50 pf	300	10
C27	0-25 pf	air dielectric	-
C28	50 pf	300	10
C29	100 pf	300	10
C30	100 pf	300	10
C31	.5 μf	300	10
C32	10 pf	300	10
C33	0-50 pf	air dielectric	-
C34	.02 μf	300	10
C35	.2 μf	300	10
C36	.01 μf	300	10
C37	200 pf	300	10
C38	.01 μf	300	10
C39	.5 μf	300	10
C40	0-50 pf	air dielectric	-
C41	100 pf	300	10
C42	.001 μf	300	10

Parts List

Capacitors (cont'd)

Symbol	Value	Rating (volts)	Tolerance (percent)
C43	10 pf	300	10
C44	.001 μf	300	10
C45	.001 μf	300	10
C46	0-50 pf	air dielectric	-
C47	.1 μf	300	10
C48	.1 μf	300	10
C49	.05 μf	300	10
C50	.05 μf	300	10
C51	.1 μf	300	10
C52	0-50 pf	air dielectric	-
C53	100 pf	300	10
C54	.1 μf	300	10
C55	.1 μf	300	10
C56	100 pf	300	10
C57	0-25 pf	air dielectric	-
C58	100 pf	300	10
C59	0-50 pf	air dielectric	-
C60	100 pf	300	10
C61	100 pf	300	10

Parts List

Capacitors (cont'd)

Symbol	Value	Rating (volts)	Tolerance (percent)
C62	.1 μf	300	10
C63	.1 μf	300	10
C64	100 pf	300	10
C65	.1 μf	300	10
C66	100 pf	300	10
C67	.1 μf	300	10
C68	100 pf	300	10
C69	.1 μf	300	10
C70	100 pf	300	10

Note: The filaments of T1, T2, T6, T7, T9, T15, T16 and T17 are decoupled with .005 μf condensers.

Vacuum Tubes

Symbol	Type
T1	6J5
T2	6AG7
T3	884
T4	6J5
T5	6J5
T6	6AC7

Parts List

Vacuum Tubes (cont'd)

Symbol	Type
T7	6AC7
T8	6N7
T9	6K8
T10	6K8
T11	6SK7
T12	6SK7
T13	6H6
T14	6V6
T15	6AG7
T16	6J5
T17	6AC7

Inductances

Symbol	Value	Tolerance (percent)	Winding Remarks
L1	20 μh	10	1 inch form
L2	80 mh	20	-
L3	20 mh	20	-
L4	8 μh	10	on R21
L5	8 μh	10	on R25
L6	2 μh	10	on R27
L7	2 μh	10	on R31

Parts List

Inductances (cont'd)

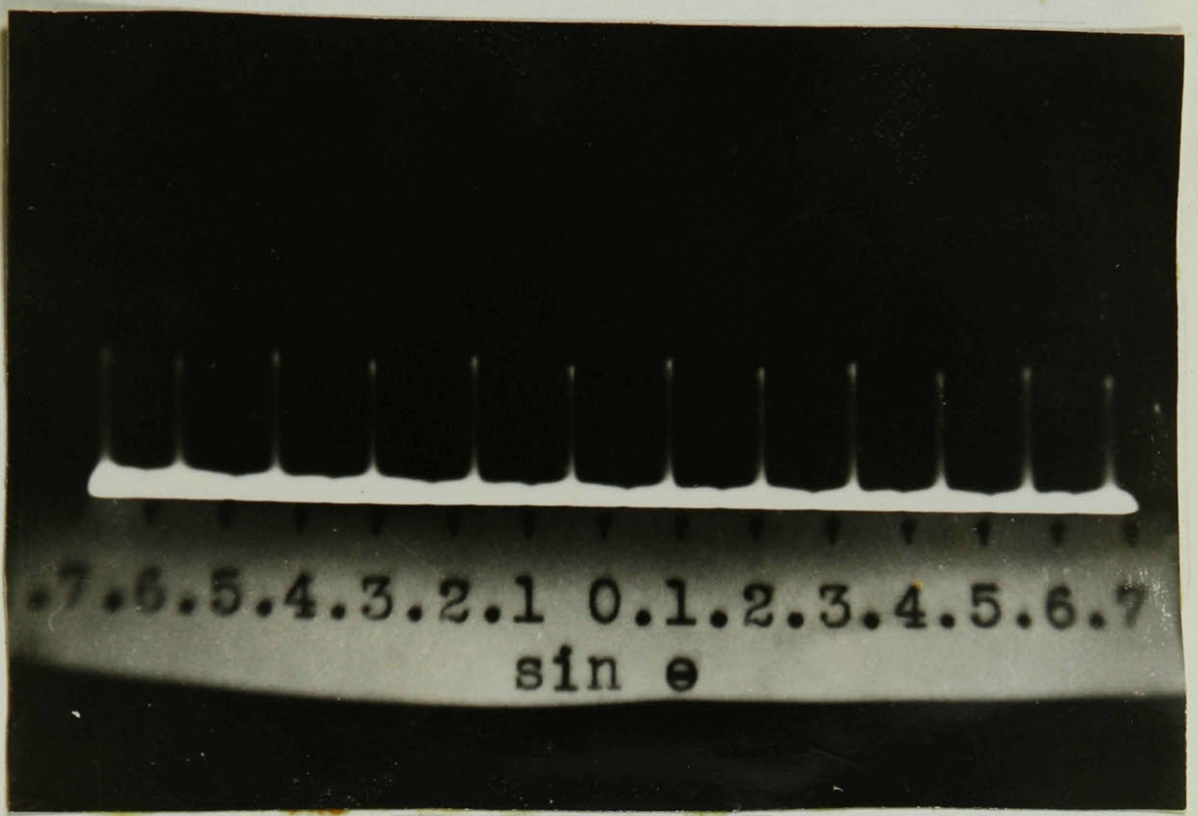
Symbol	Value	Tolerance (percent)	Winding Remarks
L8	50 mh	10	-
L9	80 mh	20	-
L10	15 μh	10	1 inch form
L11	2 μh	10	on R55
L12	-	-	3 turns
L13	-	-	8 turns
L14	100 μh	10	TR.2 modified

Transformers

Symbol	Frequency	Remarks
TR.1	1.8 mc/s	TR.2 modified
TR.2	455 kc/s	-
TR.3	455 kc/s	-
TR.4	455 kc/s	detector transformer

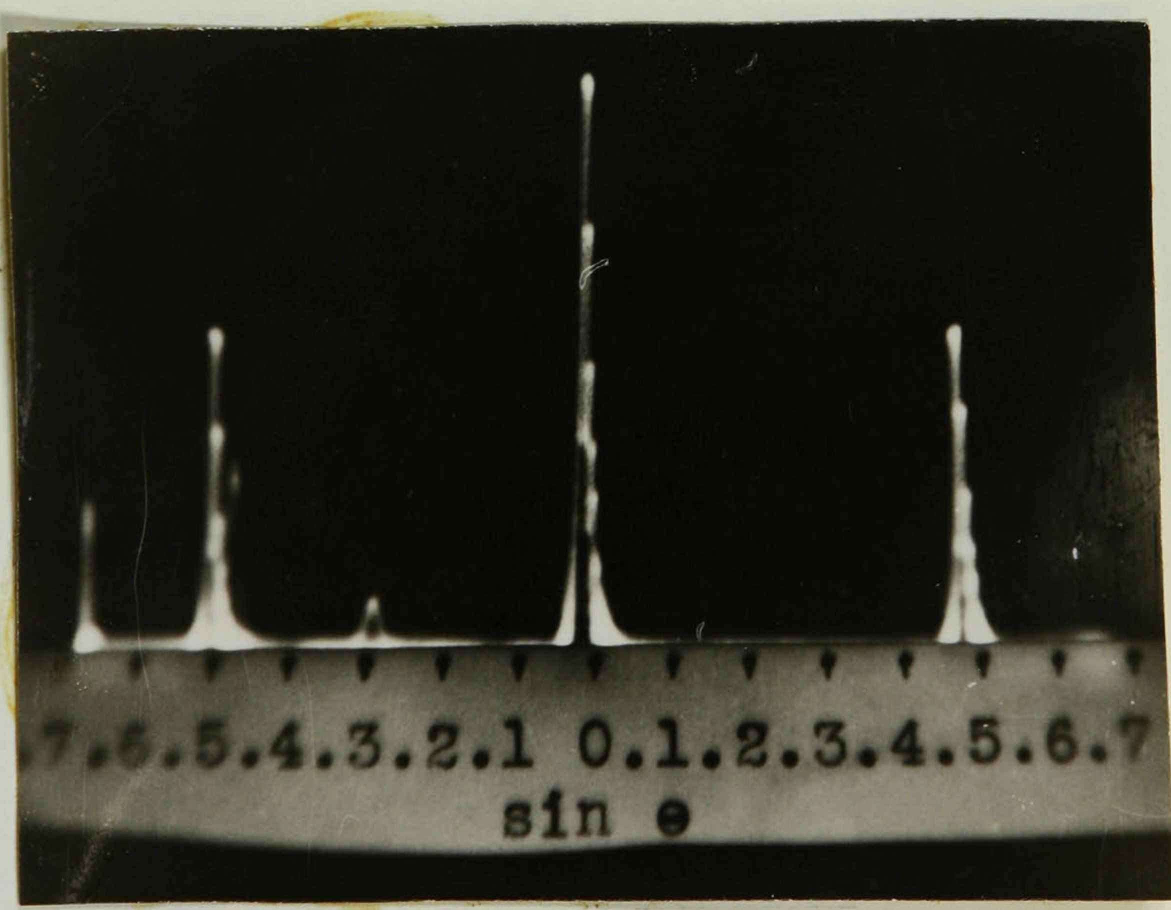
Crystals

Symbol	Frequency	Type
X1	455 kc/s	filter
X2	100 kc/s	oscillator



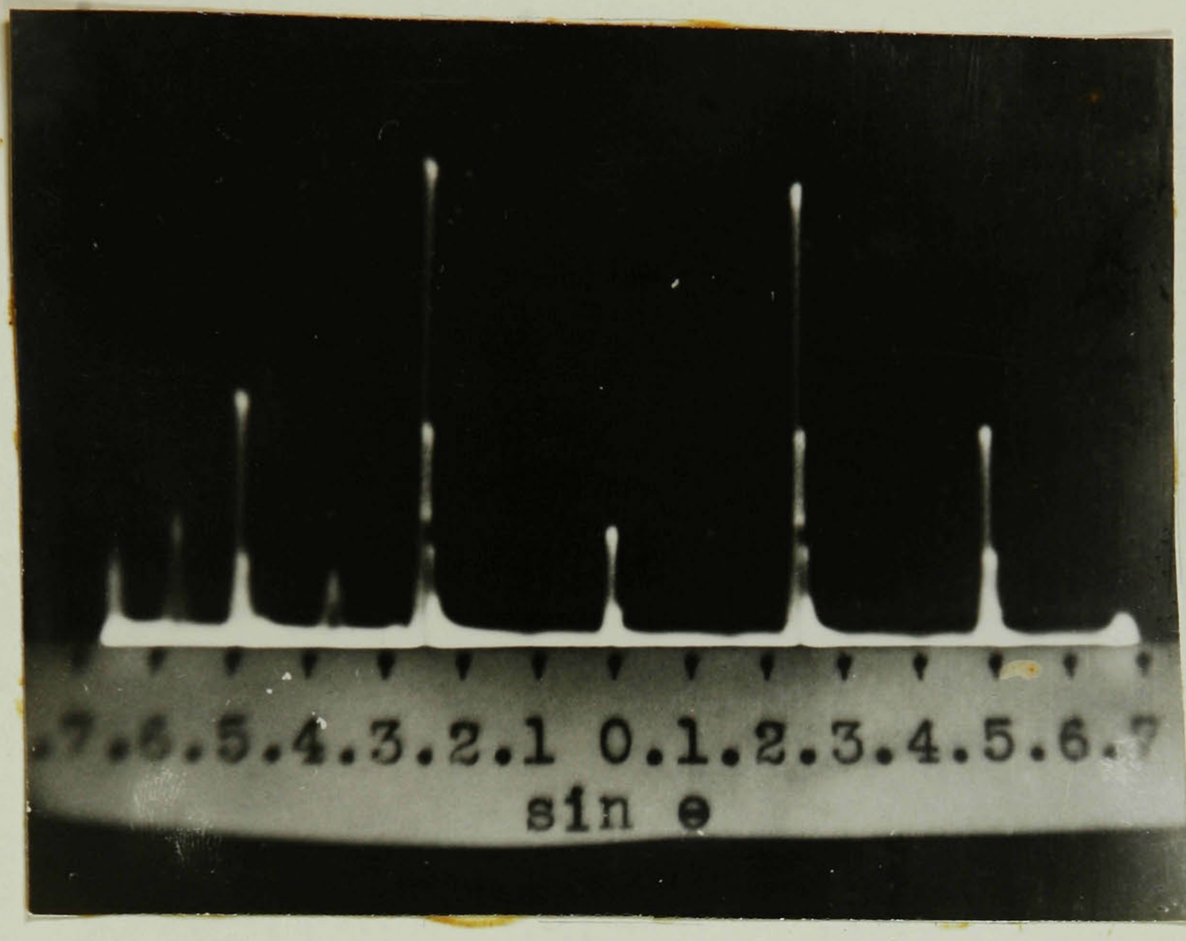
Display 1

Time Calibration



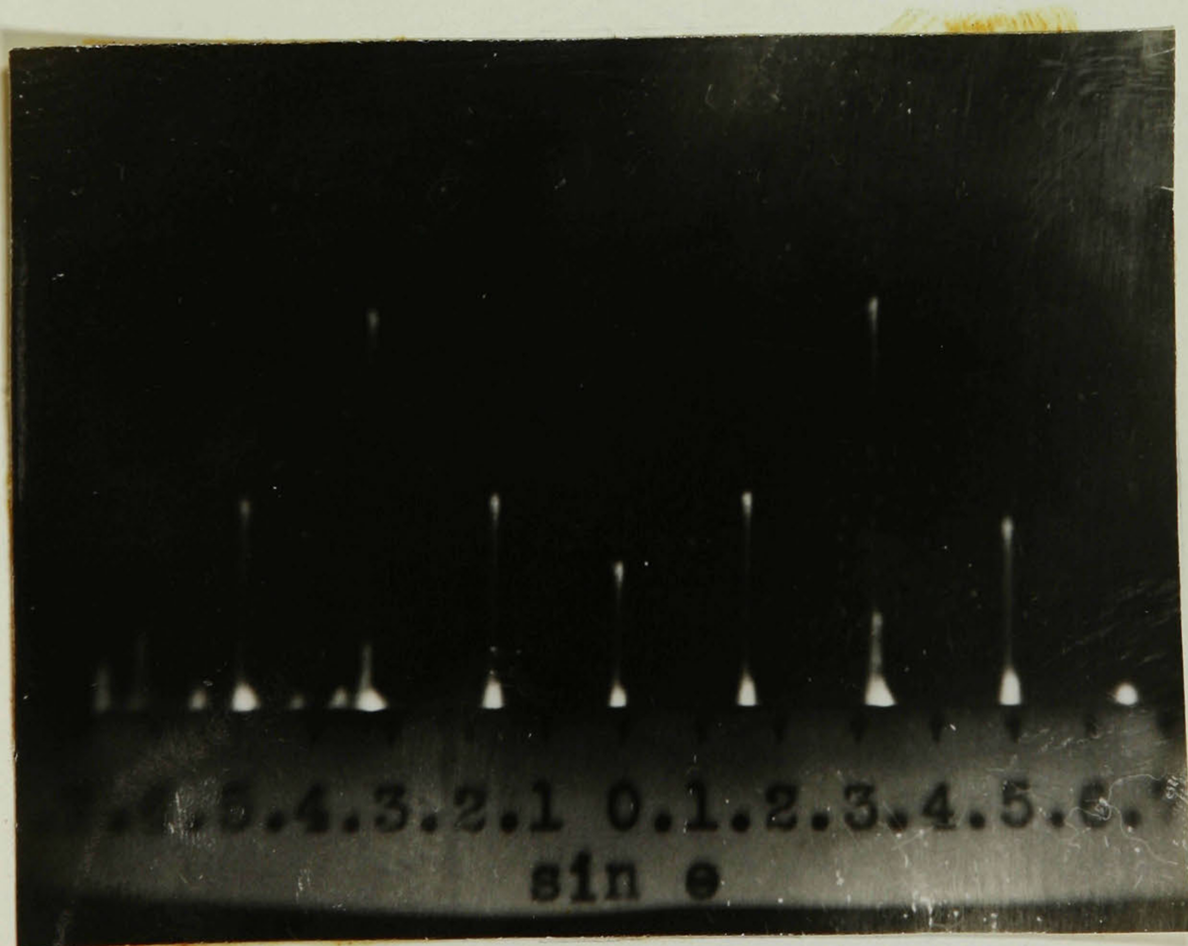
Display 2

Frequency Calibration $a/\lambda = 10$



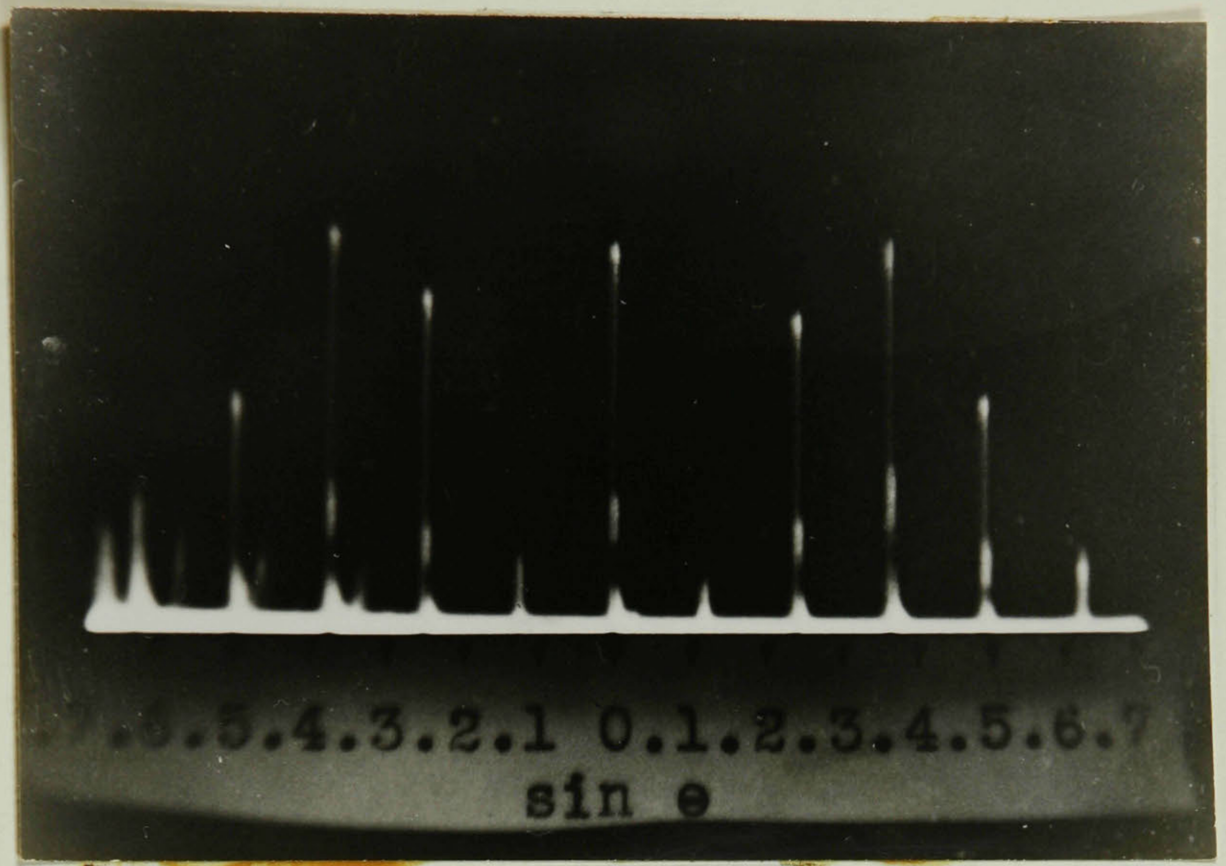
Display 3

Frequency Calibration $a/\lambda = 20$



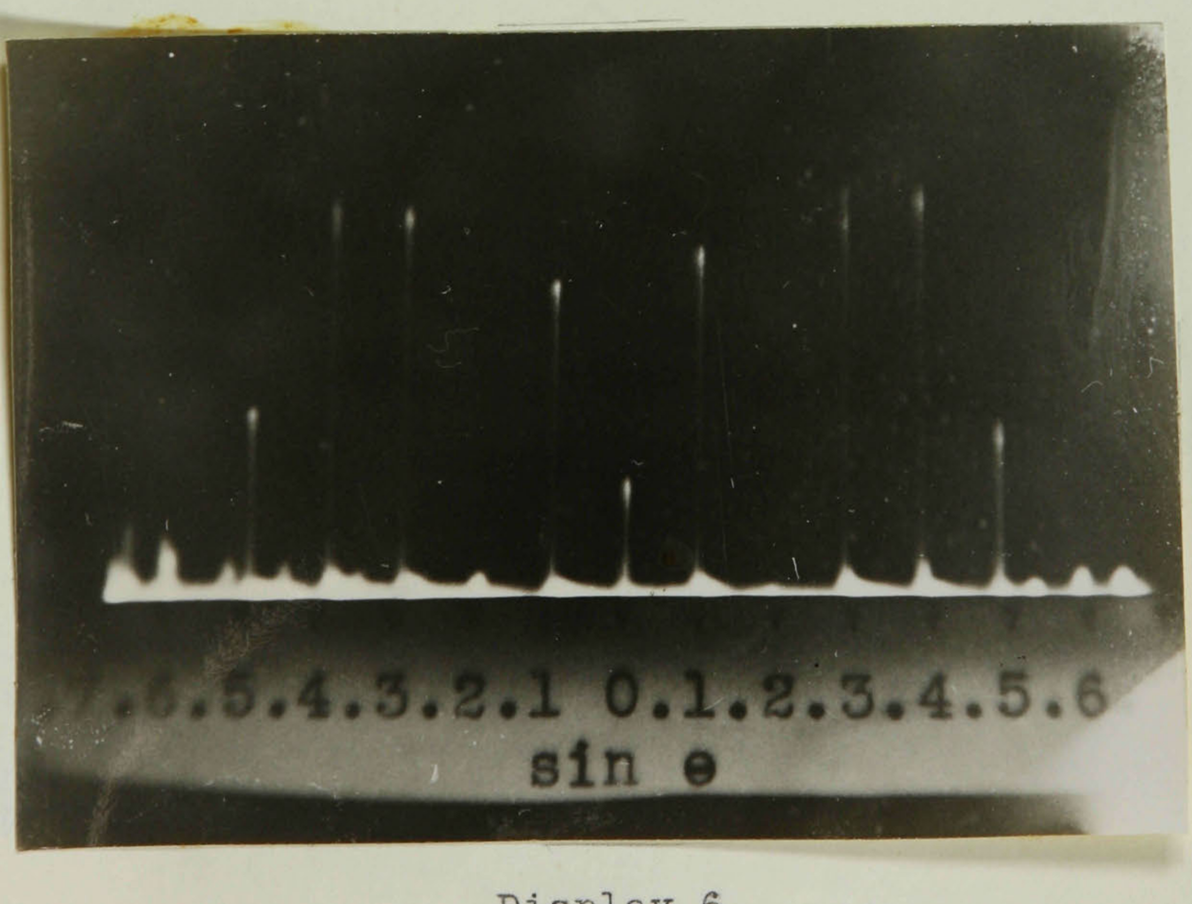
Display 4

Frequency Calibration $a/\lambda = 30$



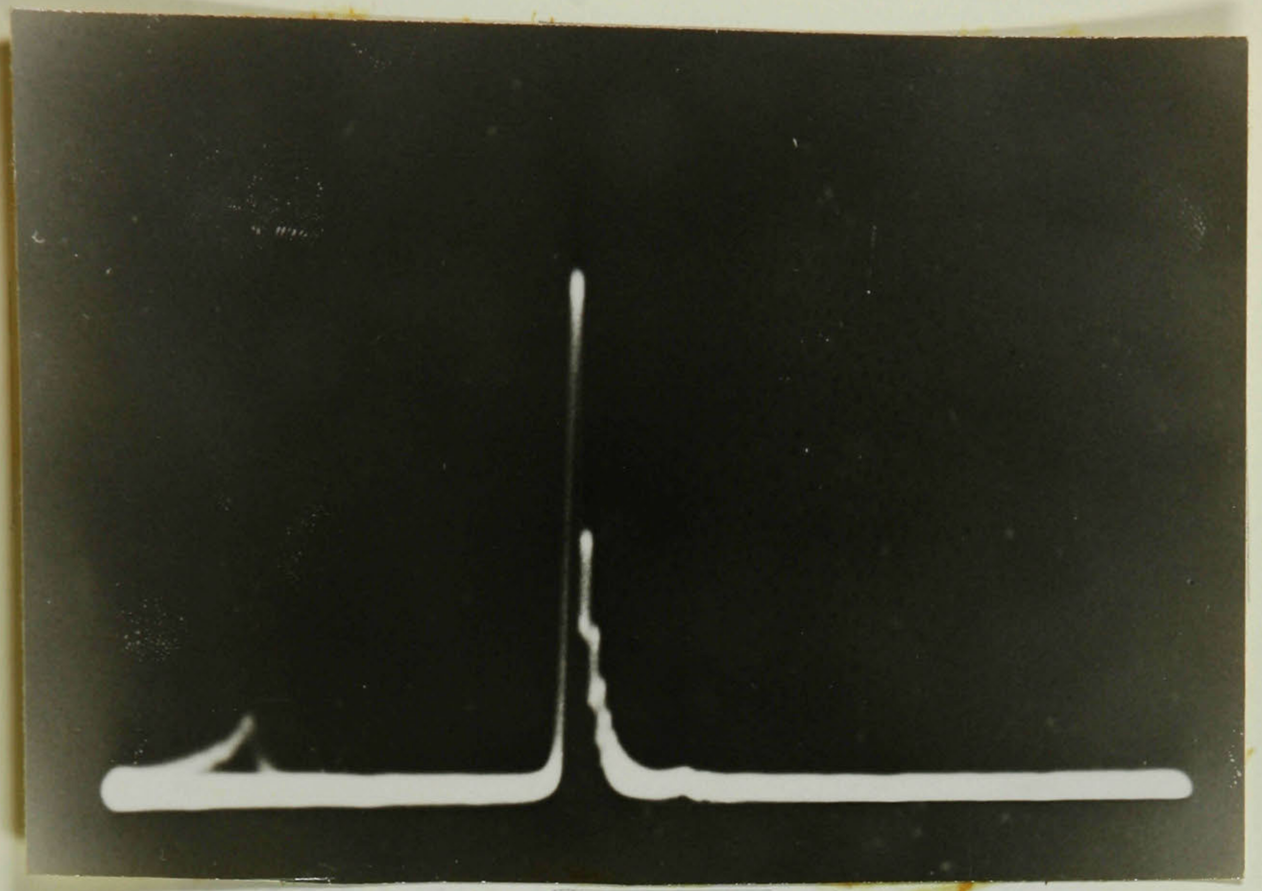
Display 5

Frequency Calibration $a/\lambda = 40$



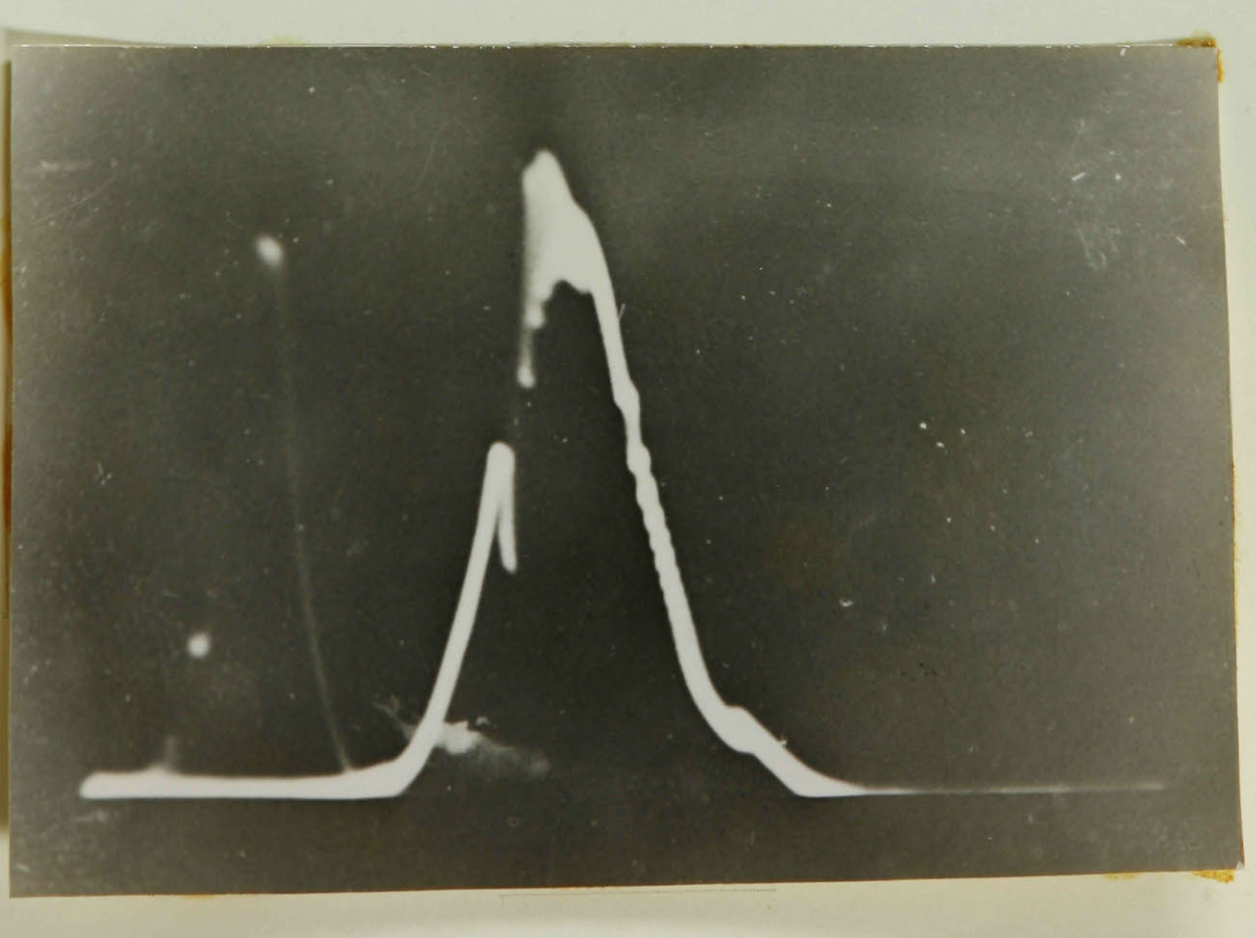
Display 6

Frequency Calibration $a/\lambda = 50$



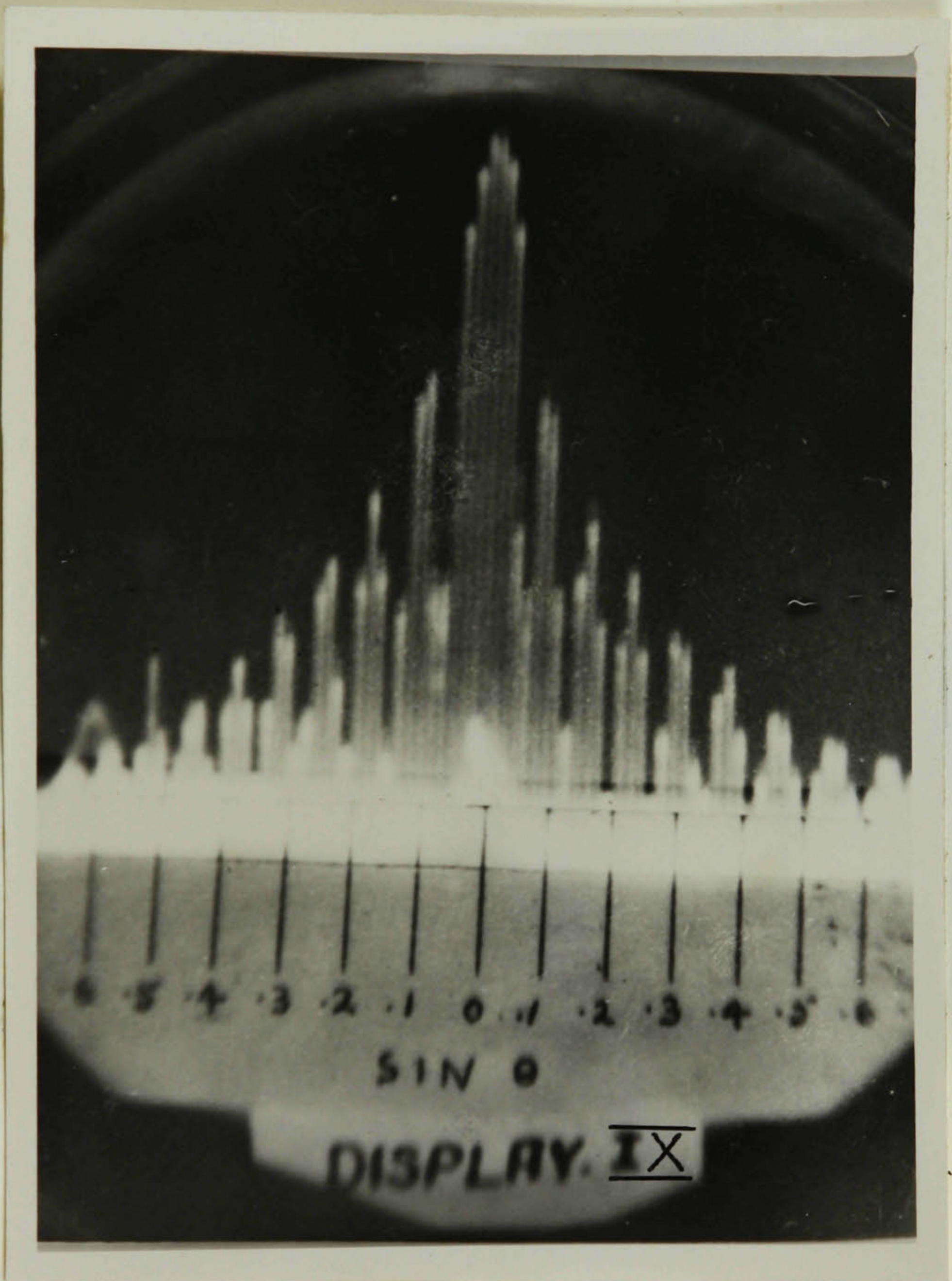
Display 7

I.F. Bandpass with Crystal Filter



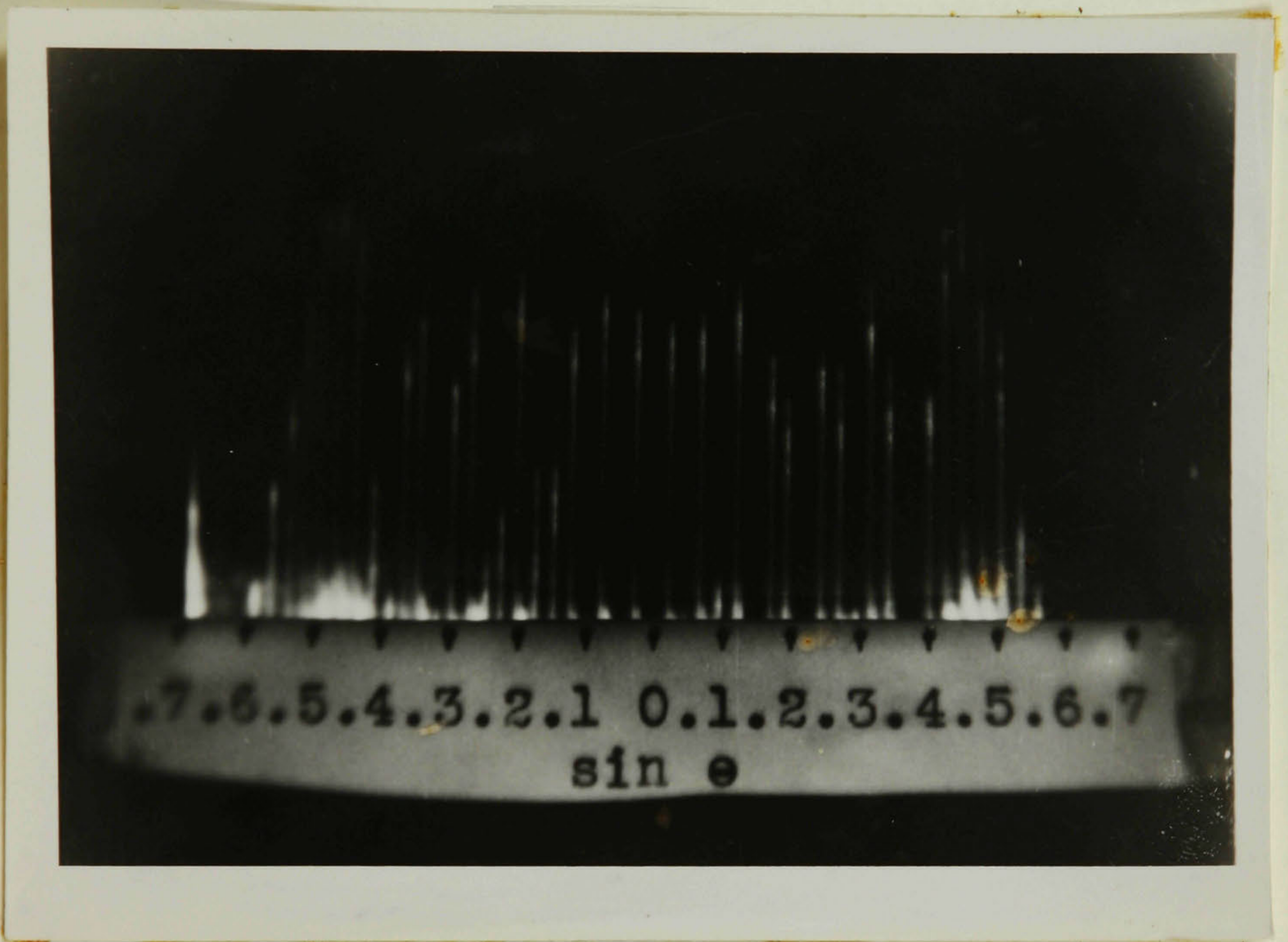
Display 8

I.F. Bandpass without Crystal Filter



Display 9

Far Field Representation $a/\lambda = 15$



Display 10

Sinusoidal Frequency Modulation Spectrum

β = 24 Sweep width = 1.4 mc/s

References

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4. S.A. Radley, "Design of a Pulse Generator for a Diffraction Field Computer", Thesis, McGill Physics Library, Montreal, P.Q., submitted April, 1950.

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11. Jahnke and Emde, "Tables of Functions", G.E. Stechert and Co., New York, 1938, pp. 171-172.

12. A. Hund, "Frequency Modulation", McGraw-Hill Co., New York, 1942, p. 34.

13. M.S. Corrington, "Variation of Bandwidth with Modulation Index in Frequency Modulation", Proc. I.R.E., Oct. 1947, Vol. 35, pp. 1013-1020.

Appendix 1.

Table 3 contains the theoretical[11] and observed values of the coefficients of equation (19) of the text. The experimental values, as read from displays 2 to 6, were obtained by adjusting R43, "Set β ", of figure 4, for the approximate relative amplitudes. The J_5 and J_6 terms are corrected for the band pass of the quadrupler T17.

Table 3.

Relative Values of the Bessel Coefficients										
β =	1		2		3		4		5	
	Th.	Obs.	Th.	Obs.	Th.	Obs.	Th.	Obs.	Th.	Obs.
J_0	.77	.77	.22	.16	.26	.17	.40	.42	.18	.10
J_1	.43	.44	.58	.58	.34	.26	.07	.06	.33	.32
J_2			.35	.29	.49	.49	.36	.37	.05	.02
J_3					.30	.23	.43	.43	.37	.37
J_4							.28	.26	.39	.40
J_5							.13	.17	.26	.20
J_6									.13	.10

Poor agreement is evident for β = 2 and 3, this is primarily due to the necessity of a more elaborate attenuator in place of R43. For example, the experimental values for β = 2 correspond more closely to

$\beta = 2.1$ as can be ascertained from the recurrence relation $J_{n+1}(x) = (2n/x)J_n(x) - J_{n-1}(x)$ and the tables of reference (11). Moreover, the theoretical values assume a perfectly sinusoidal modulation voltage, the 100 kc/s modulation voltage was not examined in great detail.

Display 10, which shows the effect of decreasing the modulation frequency, was obtained by modulating T13 with a 20 kc/s voltage from a beat frequency oscillator. In this case $\beta = 24$ with a sweep width of 1.4 mc/s. Some assymetry is due to the introduction of a low impedance source in the grid circuit of T13, however, the result compares favourably with the theoretical plot of reference (12).

Appendix 2.

Frequency modulation has been extended in a somewhat general fashion by Corrington[13] to include complex modulating voltages. Interest arose in this subject since sinusoidal[3] and complex[4] frequency modulation were suggested as methods for obtaining an analogy of the phase change which occurs across the aperture of a horn. Corrington has given the spectral distribution for rectangular and triangular modulation, both are expressed as a Fourier series and because of even symmetry in the modulation voltage, the distributions can be integrated into a closed form; the mathematical process is similar to that used in the derivation of equation (19) in section 4.4. For example, when triangular modulation is employed the amplitude of the n^{th} frequency component is given by

$$\begin{aligned} e_n = & \left\{1/(2\pi\xi)^{\frac{1}{2}}\right\} \cos \eta^2/4\xi \left\{\operatorname{sgn}\gamma\, C(\gamma^2/4\xi) - \operatorname{sgn}\eta\, C(\eta^2/4\xi)\right\} \\ & + \left\{1/(2\pi\xi)^{\frac{1}{2}}\right\} \sin \eta^2/4\xi \left\{\operatorname{sgn}\gamma\, S(\gamma^2/4\xi) - \operatorname{sgn}\eta\, S(\eta^2/4\xi)\right\} \\ & - \left\{1/\pi\gamma\right\} \sin(\pi\gamma T f_r + \epsilon) \end{aligned}$$

where

$$\xi = \beta/\pi T f_r (2 - T f_r) \qquad \gamma = (\beta T f_r / 2 - T f_r) + n$$

$$\eta = n - \beta \qquad \epsilon = -\pi\beta T f_r / 2 - T f_r$$

sgn (x) means the algebraic sign of x, C and S represent the Fresnel integrals and other symbols have the same significance as in the text.

It is felt that the possibility of determining a modulating voltage to represent the near field exists, this being so, a means would be available for a continuous transition from the far to the near field representation. Complex waveforms of odd symmetry have been examined by the general method outlined above but the result has not as yet been integrated into a closed form. The latter problem is of some importance since the type of modulation used in reference (4) to represent the above mentioned phase change is of this nature.