

SHORT TITLE

TURBULENT WALL JETS OVER CURVED
SURFACES

**Some Contributions to the Study of
Equilibrium and Non-Equilibrium
Turbulent Wall Jets over Curved Surfaces**

by

D.E. Guitton

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**Department of Mechanical Engineering
McGill University
Montreal, P.Q.**

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A B S T R A C T

The flow of two-dimensional turbulent wall jets in still air over two logarithmic spiral surfaces and over a circular cylinder is studied both theoretically and experimentally. The former surfaces permit the development of equilibrium flows while the latter is an example of a simple non-equilibrium case.

The analysis of the equilibrium flows is based on Townsend's latest theoretical model and uses experimental information about the turbulence structure - principally $\overline{u'v'}/\overline{q'^2}$ - in conjunction with the requirement of overall kinetic energy balance.

The flow on the circular cylinder is analysed using a momentum integral technique; momentum integral equations are written for the inner and outer layers respectively and solved together with information about the rate of growth of the jet. The unknown physical quantities appearing in the equations are obtained or inferred from data gathered in the experiments on the equilibrium flows. Separation is satisfactorily predicted by assuming rapid retardation of the boundary layer, i.e. that pressure forces are much larger than shear forces.

The curved jets are characterized by highly intense turbulence and the accuracy of the shear measurements in these flows was suspect. A measure of the possible error was indispensable because a satisfactory theoretical prediction of the flow is hampered by the pervading influence of three-dimensional effects, - to which curved convex flows are particularly susceptible - and

a measure of these effects can be obtained by comparing the calculated and measured shear stress distributions. Consequently special attention is given in this work to the influence of high intensity turbulence on hot wire measurements.

The outer flow of a plane wall jet influences strongly the inner boundary layer and the experiments revealed that this effect increases with curvature. This represents a rather unique physical situation and some attention is given to this part of the flow. In particular there is developed a novel technique for measuring skin friction with a hot film probe.

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Table of Contents

	Page
Summary	1
Acknowledgements	11
Notation	111
Chapter 1 Introduction	1
1.1 Curved shear layers, occurrence in practice and flow of present thickness	1
1.2 History and some comments on previous investigations	5
1.2.1 Plane wall jet in still air	6
1.2.2 Measurements of a wall jet over a cylinder of constant radius	8
1.2.3 Measurements of the self-preserving curved wall jet	11
1.2.4 Theoretical attempts to generalize curved flow data	14
1.3 Purpose and Scope of Work	15
Chapter 2 Basic Equations and the Conditions for Self-Preservation	20
2.1 Equations of motion	20
2.2 Self-preservation	22
Chapter 3 Theoretical Analysis and Required Measurements	28
3.1 The linearized integral method - applied to the spirals and to both the concave and convex cylinders	29
3.2 The structural similarity theory of Townsend (1969)	37
3.2.1 Theoretical considerations	38
3.3 The strip integral method - applied only to the cylinder	43
3.3.1 The integral equations	47

Chapter 4	Separation on the Coanda Cylinder	52
4.1	Introduction	52
4.2	Conditions for validity of both analyses	53
4.3	Theoretical development	56
4.3.1	Extension of the Reshotko-Tucker analysis to the Coanda jet boundary layer	56
4.3.2	The position of the threshold	58
4.3.3	The Stratford-Townsend separation criterion	59
4.3.4	Application of the criterion to the Coanda cylinder	63
Chapter 5	Experimental Apparatus, Measuring Equipment and Procedure	65
5.1	Apparatus	65
5.1.1	Equilibrium curved flows	65
5.1.2	Circular cylinder-convex	70
5.1.3	The plane wall jet in still air	72
5.1.4	Calibration rig	73
5.2	Measuring equipment : hot wire and associated equipment	74
5.3	Experimental procedure	75
5.4	Concave cylinder	82
Chapter 6	Two-Dimensionality : 'Establishment' and Verification	84
6.1	Slot lip effect	88
6.2	Secondary flow	91
Chapter 7	The Effect of High Intensity Turbulence on the Response of a Hot Wire	94
7.1	Introduction	94
7.2	Review of analytical methods used to correct signals	98

7.3	Calculation of wire response	100
7.3.1	General	100
7.3.2	Determination of effective normal component	101
7.3.3	Voltage response of linearized system	102
7.3.4	Response equations for specific wire orientations (single wire)	103
7.3.5	Equations for mean velocity and Reynolds stresses	108
7.3.5.1	Mean velocity	108
7.3.5.2	Reynolds stress equations for single slanted wires having $k \sim 0$ [c]	109
7.4	Approximations to the quadruple correlations	111
7.5	Calculation of high intensity correction factors E, F and G ($k = 0$ equations (7.25) and (7.26))	113
7.5.1	Cases where $V = 0$	113
7.5.1.1	The circular pipe - experiments by Laufer (1953)	114
7.5.1.2	The free two-dimensional jet in still air measured in a streamline coordinate by Heskestad; corrections to Reynolds stresses	115
7.5.2	The equilibrium wall jets in still air	118
7.6	Conclusions	122
Chapter 8	A Transient Technique for Measuring Skin Friction using a Flush Mounted Heated Film and its Application to a Wall Jet in Still Air	124
8.1	Introduction	124
8.2	Theoretical approach to the heat loss relation	127

8.2.1	General	127
8.2.2	Dimensional analysis	128
8.2.3	Other theoretical approaches	131
8.3	Experimental equipment	133
8.3.1	The flush probe and mounting	133
8.3.2	Pipe	134
8.3.3	Boundary layer and wall jet	135
8.3.4	Preston and Pitot tubes	135
8.4	Experimental procedure and calibration results	136
8.4.1	General	136
8.4.2	Effective length of surface temperature profile	139
8.4.3	Pressure gradient effect in flush probe measurement in a pipe and on the logarithmic spirals	139
8.4.4	Limit of film length	140
8.4.5	The effect of voltage accuracy on the measurement of skin friction	142
8.5	Measurements in the flat plate boundary layer	142
8.5.1	General	142
8.5.2	Initial unsuccessful measurement of C_f with hot film	143
8.6	The transient technique for measuring τ_w	144
8.7	Further measurements in the turbulent boundary layer	146
8.8	Measurements of skin friction in a plane wall jet in still surroundings	147
8.8.1	Previous work	147
8.8.2	The experimental conditions and procedure	150
8.8.3	Results and discussion	151
8.9	Measurements of skin friction in the wall jet over the logarithmic spiral	151
8.9.1	Results and discussion	155
8.10	Conclusions	158

Chapter 9	Experimental Results on the Logarithmic Spirals and Comparison with both Plane Wall Jet and Theory	161
9.1	Quality of the curved flows - verification of two-dimensionality	161
9.2	Experimental results	164
9.2.1	Mean velocity profiles	164
9.2.2	Growth of jets	166
9.2.3	Maximum velocity decay	168
9.2.4	Surface pressure and static pressure variations throughout the jet	169
9.2.5	Reynolds stresses	170
9.2.6	Intermittency	177
9.3	Experimental Results in Inner Layer	183
9.4	Application of the Frictionless, First Order Theory of Section 3.1	186
9.5	Townsend's Structural Similarity Theory	189
Chapter 10	Experimental Results and Theoretical Prediction of the Wall Jet over the Cylinder of Constant Radius	194
10.1	Quality of the flow on the cylinder $\therefore$ verification of two-dimensionality and universality of results	194
10.2	Experimental results	197
10.2.1	Bulk parameters and comparison with the results of Fekete	197
10.2.2	Reynolds stresses	199
10.3	The linearized integral method and a discussion of the rate of growth equation	200
10.4	The strip integral method : (application of)	208
10.4.1	The closure relations	208
10.4.2	Predicted results	213
Chapter 11	Summary and Conclusions	216

Appendix I

Tables

Figures

Notation

A	Angle between hot wire and instantaneous velocity vector (Chapter 7)
A	Constant in the log-law (Chapters 4 and 8)
A _c	Angle between hot wire and calibration stream (Chapter 7)
a	Exponent for the velocity decay law $U_m \propto x^a$
a	Constant in equation 8.10 (Chapter 8)
B	Constant in the log-law (Chapters 4 and 8)
b	Slot width
b	Constant in equation 8.10 (Chapter 8)
$\left. \begin{matrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{matrix} \right\}$	Constants defining hot-wire non-linear response
C	Constant in equation 4.20 (Chapter 4)
C	Calibration constant for hot-wire (Chapter 7)
C _f	Skin friction coefficient
C _p	Specific heat of air at constant pressure
d	Wire diameter
dq _e	Fluctuation in cooling component
E	Voltage
E	Turbulence intensity correction coefficient defined in equation 7.24 (Chapter 7)
E _f	Operating wire voltage
E ₁	Heskestad's correction coefficient
e'	Fluctuating voltage
e [•] _{A+B}	Instantaneous sum signal from X-wire
e [•] _{A-B}	Instantaneous difference signal from X-wire

F	Turbulence intensity correction coefficient defined in equation 7.25
F_1	Heskestad's correction coefficient
$f()$	General function
G	Turbulence intensity correction coefficient defined in equation 7.26
G_1	Heskestad's correction coefficient
$g()$	General function
$g_{12}(\eta)$	Shear stress profile
H	Boundary layer form factor (Chapter 4)
H	Turbulence intensity correction coefficient defined in equation 7.27 (Chapter 7)
H	Heat dimension (Chapter 8)
H_s	Form factor at separation
H_t	Form factor at threshold
$h()$	General function
h	Height above wall (Chapter 8)
I_p	$\int_0^\infty f \int_0^\infty f^2 d\eta d\eta$
I_1	$\int_0^\infty f d\eta$ or as in Appendix I
I_2	$\int_0^\infty f^2 d\eta$
I_{2a}	$\int_0^\infty \eta f^2 d\eta$
I_3	$\int_0^\infty f^3 d\eta$ or as in Appendix I
I_5	Integrals defined in Appendix I
I_7	
I_8	
I_9	
I_{10}	
i	Current

J	Momentum
K } K ₁ }	Constants
k	Thermal conductivity (Chapters 5 and 8)
k	Longitudinal cooling correction factor (Chapter 7)
L ₁	(Streamwise) convection length scale (Chapter 8)
L ₂	(Transverse) diffusion length scale (Chapter 8)
L _e	Dissipation length scale
l	Length of hot-wire
ℓ	Length of film (Chapter 8)
l _o	Velocity profile length scale
l _e	Length of rectangular temperature profile
ℓ _l	Mixing length
M	Mass scale
N	Exponent for C _f
Nu	Nusselt number
n	Exponent in the boundary layer power profile
n _t	Exponent for the threshold power profile
P _f	Power dissipated in the film
P _o	Reservoir total pressure
P _r	Prandtl number
p	Mean static pressure
p _e	Static pressure at the edge of the flow
p _∞	Ambient pressure
p _w	Static pressure on the wall
p'	Pressure fluctuation

Q	Mean total velocity
Q_e	Mean effective cooling velocity
q_a	Heat flux due to either forced or free convection (Chapter 8)
q_e	Instantaneous effective cooling component (Chapter 7)
q_N	Instantaneous velocity component normal to wire (Chapter 7)
q_s	Heat flux to substrate (Chapter 8)
q_t	Total heat input to hot film (Chapter 8)
q_o^2	Turbulence energy scale (Chapter 3)
$\overline{q'^2}$	Mean turbulence kinetic energy
R	Radius of curvature
R	Correlation coefficient, as defined by subscripts (Chapter 7)
R_c	Cold resistance (hot-wire or film)
R_e	Slot Reynolds number
R_f	Hot resistance (hot-wire or film)
R_f	Measured correlation coefficient (Chapter 7)
R_1	Series resistance
r	Radial co-ordinate
T_b	Temperature of brass mount
T_f	Operating temperature (hot-wire or film)
T_1	Air temperature
t	Time scale
t_D	Large eddy life time
U	Mean velocity in the x-direction (Chapter 7 only)
U_c	Calibration velocity

U_{ce}	Effective cooling velocity at calibration (includes longitudinal cooling effect)
U_{cN}	Normal cooling velocity at calibration
U_e	Effective cooling velocity
U_j	Slot velocity
U_m	Maximum velocity in wall jet velocity profile
U_{m_r}	Maximum velocity at reference station
U_N	Normal velocity with respect to hot wire
U_p	Velocity parallel to wire
U_τ	Skin friction velocity
u	Mean velocity in the x-direction (except in Chapter 7)
$u -$	Time dependent velocity in x-direction (Chapter 7 only)
u'	Velocity fluctuation in the x-direction
v	Mean velocity in the y-direction (Chapter 7 only)
v_e	Value of v at edge of flow (entrainment velocity)
v_m	Value of v at $u = U_m$
v	Mean velocity in the y-direction (except in Chapter 7)
v	Time dependent velocity in the y-direction (Chapter 7 only)
v'	Velocity fluctuation in the y-direction
w	Mean velocity in the z-direction (Chapter 7 only)
w	Mean velocity in the z-direction (except in Chapter 7)
w	Time dependent velocity in the z-direction (Chapter 7 only)
w	Width of film (Chapter 8)
x	Distance from the origin of the spiral or hypothetical origin

x_r	Value of x at reference station
x_o	Value of x at the slot
x_1	Distance from slot ($x = x_o + x_1$)
y	Distance normal to surface
y_e	Value of y at half intermittency point
y_m	Value of y at $U = U_m$
$y_{m/2}$	Value of y at $U = \frac{1}{2} U_m$ ($y_{m/2} > y_m$)
z	coordinate orthogonal to x and y

Greek Symbols

α	Angle defined in figure 7.1
α_c	Value of α at calibration conditions
α_1	Thermal coefficient of resistivity (hot-wire or film)
β	Growth rate
β_o	Growth rate for plane wall jet in still air
β_1 to β_5	Constants in equation 3.3a and 3.23
ϵ	Dissipation (Chapter 3)
ϵ	Order of magnitude of turbulence intensity (Chapter 7)
ξ	$0.881 (y - y_m)/l_o$
η	$y/y_{m/2}$
η_o	$(y - y_m)/l_o$
θ	Polar coordinate
θ	Angle defined in figure 7.1 (Chapter 7 only)
θ_s	Value of θ at separation
θ_t	Value of θ at threshold
Λ	Free convection allowance
μ	Viscosity of air
ν	Kinematic viscosity of air

ν_T	Eddy viscosity
ν_{T_1}	Eddy viscosity acting on large eddies
ρ	Density of air
σ	Standard deviation of superlayer from mean position
τ	Shear stress
τ_m	Value of τ at $u = U_m$
τ_{MAX}	Maximum value of τ
τ_w	Value of τ at the wall
ψ	Dividing streamline (Chapter 4)
ψ	Inclination of slanted wire (Chapter 7)

Subscripts

c	Calibration conditions
e	Conditions at edge of jet
e	Effective value (Chapter 7, only)
m	Conditions at maximum velocity point
m	Measured values (Chapter 7 only, where indicated)
N	Normal component
r	Conditions at reference station
s	Conditions at separation
t	Conditions at threshold of pressure rise
l	Parameter referred to slot (e.g. x_1)
∞	Ambient conditions

Superscripts

d	Conditions on dividing streamline
.	Differentiation with respect to η or x where appropriate (except where identifying turbulence quantities)

—
o[]

11

(Over bar) Time average

of the order of

Absolute value

PART A

INTRODUCTION AND THEORETICAL ANALYSIS

Chapter 1

Introduction

1.1 Curved shear layers, occurrence in practice and flow of present interest

In recent years much of the research on the nature of turbulent motion has been devoted to understanding and predicting the streamwise development of plane shear flows. Methods of calculation which are increasingly general in their applicability have evolved principally as a result of the continual exploratory and pathfinding work of Townsend; and his models of the basic turbulence mechanisms underlying the development of shear flows have been largely responsible for catalyzing today's research activities.

But, while plane flows do occur in practice it transpires that many situations of present engineering interest involve shear flows that are curved. Examples of particular importance are boundary layer control by blowing whereby a jet is blown over an aerofoil surface (or the knee of a flap) to energise the boundary layer; and air cushion vehicles which use peripheral jets to maintain the high base pressure.

Curvature has a marked influence on the turbulent mixing but the mechanisms underlying this effect are uncertain. It is well known (Townsend, 1956) that the stability of small disturbances in a curved laminar flow depends on the sign of the angular momentum gradient, $\frac{\partial}{\partial r}(ur)$ and it has been postulated

(Taylor, 1932) that the same mechanism acts on curved turbulent flows to influence the scale and intensity of the turbulence. A fluid particle is considered to be displaced with a constant angular momentum from one fluid stratum to another of larger radius and the pressure gradient necessary for its equilibrium is noted. It is easily shown (Stratford, 1962) that the particle will be displaced further outwards in a negative, and hence unstable angular momentum gradient. In a similar way the positive gradient leads to stability. Such arguments can be extended to suggest that either the "mixing length" or the large eddy length scale will be increased in the unstable flow (see also Sawyer, 1962; Margolis, 1963; Giles et al., 1966; and Newman, 1967 respectively). Margolis has measured the turbulent energy balance (using the isotropic relation to calculate dissipation) in both convex and concave mixing layers and has found that in the stable case (concave) the transport of energy away from the region of high production is inhibited and that the converse is true for the unstable case. In the latter, the overall production of energy was much increased due to the availability, in zones which would normally be energy-starved in a plane flow, of Reynolds stresses against which the mean flow could work to produce more turbulent energy. Townsend (1969) in a very recent paper, presents evidence which implies that the dominant structure of the turbulence in a wall jet is not altered by curvature and this is discussed more fully in section 1.2.4.

The influence of curvature on the turbulent mixing has

suggested that convex curved jets (increased mixing) be applied to augment the heat transfer from a surface or to increase the entrainment in an ejector.

Of particular interest in the study of turbulent shear flows are cases in which the motion is self-preserving or self-similar. Such situations imply a state of moving equilibrium wherein the terms of the energy balance are always constant fractions of each other. Parameters specifying the turbulence structure or mean flow at any station can be expressed non-dimensionally by using either a length or velocity scale which is a function of a single variable (usually the downstream distance). In these flows the position downstream is no longer relevant (providing the flow is fully developed) and measurements can be made at only one station. It is interesting that whenever self-preservation is predicted theoretically by using the non-dimensional self-similar forms, it almost invariably occurs in the real flow. (There are three known exceptions to this and they are discussed by Townsend (1969).) A number of authors have suggested that this indicates a tendency in all flows towards self-preservation.

Much of the advance in understanding plane shear flows has come from studies of the self-similar cases and information derived from these has been used with some success to predict the development of many relaxing flows. The work described in this thesis continues this tradition. For obvious reasons

very simple curved shear flows are chosen. A number of two-dimensional and self-preserving convex curved wall jets in still air and a plane wall jet in still air (which has $R = \infty$ and represents the limiting case of the preceding flows), are studied experimentally and an attempt is made to extend the results to predict the development of a simple non-equilibrium case; a wall jet in still air on the outside (convex) of a circular cylinder.

The flow on the convex surface is usually termed the Coanda effect after Henri Coanda who first exploited the possibility that a jet could stay attached to a nearby surface. Figure 1.1 illustrates the flow around the outside of the cylinder. The jet emanates from the slot and flows around the surface and there is formed a boundary layer at the surface - called the 'inner layer' - and an 'outer layer' which resembles a half-jet. The effects of mixing with surrounding fluid at rest and, of lesser importance, skin friction diffuse the jet's momentum with the result that the jet becomes thicker and the maximum velocity decreases. Due to centrifugal effects the pressure at the surface is lower than ambient, but, in the case illustrated, slowly rises and the jet eventually separates.

The curved self-preserving cases are those for which the thickness of the jet ($y_{m/2}$) - conveniently defined as the point where the velocity is one-half the maximum - and the radius of curvature (R) are both proportional to the downstream distance (x). (See Chapter 2.) This is almost exactly possible

at high Reynolds numbers, on surfaces shaped like logarithmic spirals. (Figure 1.2 illustrates two log spirals for which $R = \frac{3}{2}x$ and $R = x$ respectively.) At low Reynolds numbers the effect of viscosity at the wall becomes important and the skin friction coefficient is not invariant. For similar reasons the plane wall jet in still air is also very nearly self-preserving at high Reynolds numbers.

Previous investigations of these flows will now be reviewed.

1.2 History of and some comments on previous investigations

A concise summary of the work on turbulent jets to May 1969 may be found in Newman (1969). An earlier summary relating to Coanda effect is given by Wille and Fernholz (1965).

A number of experiments on the Coanda effect have been made to study (i) the effect of geometric changes near the slot such as gaps and offsets, Korbacher (1962); (ii) the effect of small aspect ratio, Fernholz (1966); (iii) the influence of sound wave acting normal to the flow at the slot, Chang (1966); and (iv) the large $\frac{b}{R}$ case in which there exists a potential core between the boundary layer forming at the wall and the external shear layer, Kadosch (1957, 1967); Fernholz (1967). There exist also a number of experiments dealing with the deflection of a jet by an adjacent boundary such as a flat plate and cases where there is an external stream above the curved shear layer. Within the limits and

goals imposed on this thesis the flows cited in this paragraph are considered hybrid and will not be discussed.

1.2.1 Plane wall jet in still air

The earliest known work on plane wall jets is an experimental investigation by Förlthmann (1934). Glauert (1956) revived interest in the flow and attempted a theoretical prediction of its development. Although his analysis used the eddy viscosity concept which implied erroneously that the shear at y_m is zero it nevertheless described correctly the basic behaviour of the flow. One particularly relevant finding was that viscosity hardly affects the jet's development providing one is far enough downstream of the slot.*

The most recent extensive study of the plane wall jet in still air is by Kohan (1968) in which he gives a resume of the work to that date and no further historical account is judged useful here. Kohan has concentrated on the measurement of wall friction and intermittency and these topics are discussed in subsequent chapters. Certain of his results are noted here because they are of general interest to the proposed experiments on curved flow. Firstly Kohan finds that the scales U_m and $y_m/2$, defining the flow, change their behaviour

* When the slot width is small the position of the jet's hypothetical origin corresponds nearly to that of the slot. The maximum velocity U_m , varies approximately as $x^{-1/2}$ and inner layer thickness, y_m , like x . Thus the skin friction coefficient $C_f = \frac{U_m y_m}{\nu} N_m = x^{N/2}$ and is very large at small x .

for approximately $\frac{x}{b} > 300$: the slope of $y_{m/2}$ vs x changes and the exponent 'a' in $U_m \sim x^a$ changes to -1 from approximately $-\frac{1}{2}$. Kohan also claims that the turbulence structure, characterized by $\frac{u'^2}{U_m^2}$, does not become self-preserving until $\frac{x}{b} \approx 300$. This is surprising because wall effects are known not to play a significant role in the jet-development (Guitton, 1969) and that for a plane free jet the results of Heskestad show that self-preservation of $\frac{u'^2}{U_m^2}$ occurs beyond $\frac{x}{b} \approx 65$ and that the energy balance reaches equilibrium for $\frac{x}{b} \approx 100$.

A considerable amount of experimental data on the plane wall jet has now been accumulated and indeed at the onset of the present investigation it had not been planned to measure this flow. However, some special experimental techniques were required to correct Reynolds stress measurements (Guitton, 1968) and to measure skin friction (Guitton, 1967), and intermittency, and it was found useful to verify these techniques in the plane flow before trying them in the more unknown curved cases. As a corollary to these special measurements in the plane flow, most terms in the Reynolds stress tensor were obtained (see Guitton, 1968), and when compared to the available data they showed, much to the author's surprise, that the present measurements were probably of superior quality to those of earlier workers. This is emphasized by figures 1.3 and 1.4 and some specific information will be given here because the plane wall jet is not considered in detail elsewhere in this work. Figure 1.3 shows the normal stress $\frac{v'^2}{U_m^2}$ measured by various other experimenters. The present results are judged favourably for three reasons: (a) the measured

shear stress distribution agrees very well with the calculated one (figure 1.4); (b) the measurements were made using techniques more sophisticated than those of the other experimenters (linearized constant temperature equipment, single slanted and normal wires, and long integration times - the method is discussed fully in Chapter 5); and (c) the growth of the flow, $y_{m/2}' = .071$ at $Re = 3.0 \times 10^4$, agrees very well with the value calculated from the mean line

$$y_{m/2}' = -4.05 \times 10^{-7} Re + 0.0805$$

plotted by Kohan through all the experimental data available to him. (Points (a) and (c) will be elaborated on in the discussion of results.) In view of the quality of the results, and to provide consistency in the measurement technique and conditions (all measurements were made at $Re = 3 \times 10^4$), the author's own wall jet measurements will be used as the plane flow standard against which the curved flow results will be compared.

1.2.2 Measurements of a wall jet over a cylinder of constant radius

Nakaguchi (1961) and Newman (1961) were the first to attempt detailed studies of the fully developed two-dimensional jet flowing over the outside of a circular cylinder of constant radius. Newman's work is more basic in that he was the first to provide a general physical understanding of the flow. He defined the parameters describing a universal flow and verified

their usefulness by using them to correlate his experimental data. Fekete (1963) later undertook more accurate measurements on a much improved piece of apparatus, and further demonstrated the usefulness of Newman's original concepts. When a flow is universal the flow development can be referred to a hypothetical origin and the measurements taken on a given cylinder of radius R are applicable to any other. Newman reasoned that for this to be possible the conditions downstream of the slot must be independent of the particular slot geometry (e.g. the slot width) but should depend only on the jet's initial momentum. Universality is, moreover, only possible if the flow is independent of Reynolds number (viscosity). Embodied in these two statements are requirements that the slot width, b , be not too large - or else the jet will retain the slot characteristics until it separates - and that it be not too small in order to avoid large skin-friction effects.* The experiments of Fekete give the limits as $.01 < \frac{b}{R} < .05$ for $Re > 1.3 \times 10^4$. In a recent publication Sridhar and Tu (1969) present measurements made on a number of cylinders of different radii but they were unable to collapse their results on the basis of a universal flow. Examination of their data reveals that the slot width to radius ratio is $.055 < \frac{b}{R} < .167$ and consequently is beyond the range specified above. (A study by the author (Guitton 1964, 1967) of a wall jet on a concave surface of constant radius showed that the same universal

* See footnote on page 6.

growth law is applicable to both the convex and concave flows and some discussion of this work is included in this thesis.)

Newman and Nakaguchi also attempted simplified theoretical analyses. The latter obtained a relation between the velocity and length scales by substituting into the first order momentum equation the similarity form, $\frac{u}{U_m} = f(\frac{y}{y_{m/2}})$, of the velocity profile and then assumed, as is usually done to obtain the conditions for self-preservation (Townsend, 1956) that the coefficients of the resulting equation are proportional to each other. If this is done for a plane free jet the result is

$$\frac{U_m'}{U_m} = a \frac{y_{m/2}'}{y_{m/2}}$$

where 'a' is the exponent in $U_m \propto x^a$ and Nakaguchi assumed the same relation holds for the Coanda cylinder. The arguments leading to this are strictly incorrect - the flow on the Coanda cylinder is not self-preserving - but the result is equivalent to assuming the conservation of linear momentum and it so happens that this is true to first order if skin friction is neglected (this will be shown in section 3.1). Nakaguchi obtained a relation for U_m from the above by using an empirical relation for $y_{m/2}$. His prediction of the velocity decay is good but the result is somewhat deceiving since the theoretical curve was forced through a reference value chosen near the centre of the experimental range. The surface pressure was also predicted by using the first order y-momentum equation.

Newman's analysis assumes zero wall friction and is based on a simplified integral method which replaces the actual velocity profiles by rectangular profiles having the same mass flow and momentum. The maximum velocity decay and the surface pressure are predicted moderately well if an empirical growth law is supplied.

Separation is not predicted in either of the above theories.

The experiments on the convex cylinder have established with some confidence the bulk behaviour of the flow. These experiments were associated with exploratory theories and were principally concerned with recording the flow's mean velocity and surface pressure. Consequently the skin friction and the turbulence structure in both the inner and outer layers are largely unknown.

The available theory is limited to first order, does not consider separation and requires empirical knowledge about the flow to predict the flow and generality is thus severely limited. A more sophisticated approach must bridge these gaps and this is attempted in this thesis.

1.2.3 Measurements of the self-preserving curved wall jet

Recall that a self-preserving flow exists when $y_{m/2} \propto R \propto x$ and that this is possible on logarithmic spirals. There exists only one detailed experimental study, made by Giles et al (1966),

of the flow over such surfaces. Another investigation by Sawyer (1962) preceded, and is included in, the above work. A preliminary experiment on one spiral by the author (Guitton, 1965) yielded far different results than Giles et al. but the conditions in that test were such that it was impossible to measure far enough downstream of the slot. Giles et al. measured mean velocity and surface pressure on three convex logarithmic spirals and two concave ones. They also present measurements of the Reynolds stresses $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u'v'}$, and intermittency taken on their most highly curved spiral ($\frac{x}{R} = 1$). These results are then compared to similar experiments on the plane wall jet. The convex flows spread much more rapidly than the plane case; for example the jet on the $\frac{x}{R} = \frac{2}{3}$ spiral had a growth rate over twice that of the plane wall jet. On the other hand, in the concave flow the spreading was inhibited; the $\frac{x}{R} = -\frac{2}{3}$ spiral had a growth rate about $\frac{2}{3}$ that of the plane wall jet.

Giles et al. compared their growth rates with that obtained by Fekete on his cylinder ($\frac{dy_{m/2}}{dx}$ vs. $\frac{y_{m/2}}{2}$) and concluded, from the substantial disagreement, that the cylinder flow necessarily has a large history effect and is not solely defined by local parameters. Newman (1969) subsequently reduced the discrepancy by using the universal growth law proposed by the author and concluded that the history effect is not large. This point will be further discussed in sections 5.3.1 and 10.3.

A comparison of the maximum shear stress measured by Giles et al. on the $\frac{x}{R} = 1$ spiral, with that calculated from

the equations of motion reveals a discrepancy of over 45% and the following principal reasons suggest why the accuracy of their measurements is suspect: (a) the two-dimensionality of their flows is not adequately proven. It is shown in this thesis (Chapter 6) that slight irregularities in the slot lip can lead to large amplitude and small wave length variations of the jet thickness in the lateral direction. (b) The Reynolds stresses $\overline{v'^2}$ and $\overline{u'v'}$ were measured with a standard Disa X-wire (Type 55A38) probe and for this instrument Jerome et al. (1970) have shown that thermal wake interference may cause significant errors in measurements. (c) The anemometer signal was not linearized.

Giles et al. present a theory similar to that first used by Sawyer (1962) and later corrected and extended by the author (Guitton, 1964). The analysis uses the well known eddy viscosity approach in which the shear stress is related to the local mean velocity field (see Chapter 3). Using three empirical constants (one of which is obtained from the plane wall jet) they obtain a good prediction of the mean velocity profiles but a less adequate one of the shear stress gradient. They do not calculate either the variation of the velocity and length scales, nor the surface pressure distribution and in this respect the work lacks completeness. Furthermore, they predict, for all spirals, that the velocity scale varies as $x^{-1/2}$ and this is incorrect (Chapters 2 and 3) even if skin friction is neglected.

The above discussion has shown that even the basic properties of the self-preserving curved wall jets are still uncertain. Furthermore, such unknowns as the shear stress at the maximum velocity point, the skin friction and the turbulence structure parameter $\frac{\overline{u'v'}}{\overline{q'}^2}$ should be determined if the equilibrium flow data is to be more useful for predicting other curved flows. These reasons amply justify a more comprehensive investigation of the equilibrium flows and this is also attempted in this thesis.

1.2.4 Theoretical attempts to generalize curved flow data

The theoretical analyses by Fekete, Newman, Nakaguchi and Giles et al. (in the reference cited above) have dealt solely with the particular flows under study and do not attempt to generalize and verify the applicability of the results to a wider class of flows. Their value, as data correlations lie in their ability to shed some light on both the validity of assumptions and the quality of the flow. Nevertheless, some attempts have been made to predict a specific curved flow using results taken in a different situation and these are discussed below.

Sawyer (1962) attempted to correlate experimental measurements of curved free jets (formed by the attachment of a jet to a nearby surface), with measurements made on a logarithmic spiral, by using a single empirical constant based on an eddy viscosity hypothesis. The local entrainment was related to both this constant and the curvature ratio ($\frac{y_m/2}{R}$) by using the equations of motion, and was compared in the two classes of

flows. The results could not be correlated on the basis of a single value of the constant but the conclusion is not firm since there were inaccuracies in the analysis and experiments.

Newman (1967) attempted to predict the development of a two-dimensional free jet with small curvature, by using the data of Fekete (1963) to relate the local curvature ratio ($\frac{y_m/2}{R}$) and the increase of entrainment over the plane flow. Using this entrainment relation in conjunction with an integral technique he attempted to calculate the flow of interest but found that more precise knowledge was needed about the magnitude of the shearing stress at the velocity maximum.

The boldest generalization has been attempted by Townsend (1969) in which he used integrals of the momentum and total energy equations in conjunction with the principal assumption that the turbulence structure, depicted by $\frac{\overline{u'v'}}{q'^2}$ is nearly invariant among most shear flows. Using data from the plane wall jet he predicts with reasonable accuracy the rate of growth of the jet on Fekete's cylinder. But there are some order of magnitude inconsistencies in Townsend's calculations and a parallel analysis, presented in this work (section 9.5) is shown to be particularly sensitive to slight variations in $\frac{\overline{u'v'}}{q'^2}$.

1.3 Purpose and Scope of Work

The particular aim of the present investigation is to predict the development up to separation, of a fully developed two-

dimensional turbulent wall jet in still air flowing over the outside of a cylinder of constant radius. (The analysis and experiments are not concerned with that region influenced by slot conditions and all theoretical predictions use as starting conditions the parameters at the first measured station. Note however, that the momentum at the slot is often used to make certain parameters non-dimensional and this implicitly assumes that no momentum is lost in the region preceding fully developed flow.) This flow, as mentioned in previous sections, constitutes the simplest non-equilibrium (or relaxing) convexly curved wall jet flow and its prediction implies the resolution of three important difficulties:

- (a) the failure of the thin jet assumption because the jet soon becomes thick with respect to the cylinder radius and
- (b) the strong influence of the outer layer on the inner layer and
- (c) the occurrence of separation.

This particular Coanda flow has been the subject of a number of experimental investigations and its principal features have been satisfactorily established. Little is known, however, about its turbulence structure (see section 1.2.2).

The essence of knowledge is generalization; consequently an attempt is made to predict the above non-equilibrium flow,

using data obtained from equilibrium flows. This, in a sense, defines a general aim of this work which is to provide more knowledge about the basic mechanism determining the development of curved wall jets and, by extension, all curved shear layers.

The backbone of the work is provided by some experimental data gathered on a plane wall jet and on two self-preserving jets flowing over logarithmic spirals (discussed in Chapter 9). To facilitate comparison the curved surfaces are identical to two of the spirals used by Giles et al. Some repetition of the work of these authors was judged essential because of and the need for additional data possible uncertainties in their data (see section 1.2.2). In all the experiments the velocities are low enough that the flow can be treated as incompressible. All measurements of the equilibrium flows are taken as far from the slot as possible to eliminate "history" effects. The distance downstream was limited by two-dimensionality problems and it was never possible to attain that range of distance where the scales describing the flow change their behaviour (Kohan, 1968).

Three theories are presented (Chapter 3). In the first and simplest approach it is assumed that the effects of skin friction are small and that the wall jet profile can be replaced by a half jet. A linearized integral technique is used to

calculate the change, due to curvature, of the growth and mean velocity of the plane half-jet and the change is added to the known behaviour of the plane wall jet. The maximum velocity decay and the surface pressure coefficient are expressed in terms of an empirical constant related to the growth (or entrainment) of the jet. Using the equilibrium flow data the theory is applied to the flow on the convex cylinder.

The second analysis is based entirely on Townsend (1969) and is somewhat more sophisticated in its assumptions regarding the turbulence. Momentum and total energy equations are combined together using specific information about the turbulence structure which is taken from the plane wall jet; and a prediction of the growth is made.

In the third analysis aimed principally at predicting the convex flow, an integral approach is used. Two momentum integrals are taken in the inner and outer layers respectively and solved together with an entrainment relation. The unknown physical quantities appearing in the equations are obtained or inferred from the self-preserving data. Possible failure of the cylinder flow to be predicted with the self-preserving quantities is therefore a measure of the influence of history effects in its development and this knowledge is indispensable to any more advanced theory. Separation is predicted (Chapter 4) by assuming rapid retardation of the boundary layer and

frozen flow for the outer half-jet.

In the course of the experimental work it transpired that certain difficulties arose in conjunction with the realization of two-dimensionality and there existed large differences between the measured and calculated shear stress distributions. The curved jets are characterized by highly intense turbulence and the accuracy of shear measurements in these flows was suspect. Consequently special attention was given to the influence of high intensity turbulence on hot-wire measurements and this is discussed in Chapter 4. The corrections to the Reynolds stresses, developed in that chapter were also useful in establishing a more accurate evaluation of the parameter $\frac{u'v'}{q'^2}$ which is used in Townsend's analysis.

The experiments revealed that the inner layer ($y < y_m$) of the curved wall jets is strongly influenced by the outer layer and since this represents a rather unique physical situation it was decided to investigate this part of the flow in some detail (Chapter 8). In particular there was developed a technique for measuring skin friction when there is no universal logarithmic law of the wall. The knowledge so gained was particularly useful when predicting separation on the circular cylinder because an analysis of the data of Fekete revealed that small changes in ^{initial} skin friction influence strongly the predicted position of separation.

Chapter 2

Basic Equations and the Conditions for Self-Preservation

2.1 Equations of Motion

The full equations of motion ^(neglecting viscous terms) and continuity for steady, incompressible, two-dimensional, turbulent mean flow over a curved wall of radius R , are (see for example Goldstein (1938) and Townsend (1956))

$$\frac{R}{R+y} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{uv}{R+y} = -\frac{R}{R+y} \frac{1}{f} \frac{\partial}{\partial x} (p + f \overline{u'^2}) + \frac{1}{f} \frac{\partial \tau}{\partial y} + \frac{\tau}{f(R+y)} \quad 2.1$$

$$\frac{R}{R+y} u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} - \frac{u^2}{R+y} = -\frac{1}{f} \frac{\partial}{\partial y} (p + f \overline{v'^2}) + \frac{R}{R+y} \frac{\partial \eta}{\partial x} + \frac{\overline{u'^2} - \overline{v'^2}}{R+y} \quad 2.2$$

$$\frac{R}{R+y} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{v}{R+y} = 0 \quad 2.3$$

where

$$\frac{\tau}{f} = -\overline{u'v'} \quad 2.4$$

In the flows considered the viscous contribution to τ is negligible except near the wall. (R is defined to be positive for flow on a convex surface and negative for flow on a concave surface.)

Multiplying all equations by $(\frac{R+y}{R}) = (1 + \frac{y}{R})$ and adding $\frac{\partial}{\partial x} (p \overline{v'^2})$ to both sides of equation (2.1) yields:

$$u \frac{\partial u}{\partial x} + (1 + \frac{y}{R}) v \frac{\partial u}{\partial y} + \frac{uv}{R} + \frac{\partial}{\partial x} (\overline{u'^2} - \overline{v'^2}) = -\frac{1}{f} \frac{\partial}{\partial x} (p + f \overline{v'^2}) + \frac{1}{f} (1 + \frac{y}{R}) \frac{\partial \tau}{\partial y} + \frac{\tau}{fR} \quad 2.5$$

$$u \frac{\partial v}{\partial x} + (1 + \frac{y}{R}) v \frac{\partial v}{\partial y} - \frac{u^2}{R} = (1 + \frac{y}{R}) \frac{1}{f} \frac{\partial}{\partial y} (p + f \overline{v'^2}) + \frac{1}{f} \frac{\partial \tau}{\partial x} + \frac{\overline{u'^2} - \overline{v'^2}}{R} \quad 2.6$$

$$\frac{\partial u}{\partial x} + (1 + \frac{y}{R}) \frac{\partial v}{\partial y} + \frac{v}{R} = 0 \quad 2.7$$

For a given surface the flow of a jet is self-determining and not influenced, as in the case of the boundary layer by an arbitrary external pressure gradient. If orders of magnitude are considered and it is assumed that

$$u \sim O[1]$$

$$|R| \sim O[1]$$

$$\frac{y}{|R|} \sim O[\frac{y}{R}] \ll 1$$

and x and R are of the same order, it is then observed that, unlike the Prandtl boundary layer equations, the term:

$$\frac{\partial}{\partial y} (p + \rho \bar{v}^2) \sim O[1]$$

hence

$$(p - p_\infty) \sim O[\frac{y}{R}]$$

and it transpires that

$$\frac{\partial}{\partial x} (p + \rho \bar{v}^2) \sim O[\frac{y}{R}]$$

Following the order of magnitude analysis the equations (2.5), (2.6) and (2.7) are written to $O[\frac{y}{R}]$, $O[1]$, $O[\frac{y}{R}]$ respectively:

$$u \frac{\partial u}{\partial x} + (1 + \frac{y}{R}) v \frac{\partial u}{\partial y} + \frac{uv}{R} = -\frac{1}{\rho} \frac{\partial}{\partial x} (p + \rho \bar{v}^2) + \frac{1}{\rho} (1 + \frac{y}{R}) \frac{\partial \tau}{\partial y} + \frac{1}{\rho} \frac{\tau}{R} - \frac{\partial}{\partial x} (u^2 - \bar{v}^2) \quad 2.8$$

$$\frac{\partial}{\partial y} (p + \rho \bar{v}^2) = \frac{\rho u^2}{R} \quad 2.9$$

$$\frac{\partial u}{\partial x} + (1 + \frac{y}{R}) \frac{\partial v}{\partial y} + \frac{v}{R} = 0 \quad 2.10$$

If the jet is thick, i.e. if

$$\frac{y}{R} \sim O[1]$$

then the simplification embodied in equation (2.9) is not valid, the full y-momentum equation must be used, and this presents a problem of considerably increased complexity. The equilibrium flows considered in this work satisfy the thin jet assumption ($O[\frac{y}{R}] \ll 1$) and consequently, the self-preserving requirements sought in the following paragraphs are based on equation (2.9).

2.2 Self-Preservation

To investigate the possibility of a similarity solution of equation (2.8) for the outer portion of the wall jet ($y > y_m$) assume the following non-dimensional parameters

$$u = U_0 f'(\eta) \quad 2.11$$

$$-\overline{uv'} = U_0^2 g_1(\eta) \quad 2.12$$

$$(\overline{u^2} - \overline{v^2}) = U_0^2 g_2(\eta) \quad 2.13$$

where

$$\eta = \frac{y - y_m}{L_0} \quad 2.14$$

and

$$l_0 = y_{m0} - y_m$$

2.15

Note that in the region considered the flow is fully turbulent the viscous stresses are negligibly small, and

$$\frac{\tau}{\rho} = -\overline{u'v'}$$

2.16

Also at y_m the mean lateral velocity V_m is not zero.

Using continuity

$$v\left(\frac{R+y}{R}\right) = -l_0 \left[U_0' f - \frac{U_0}{L} \frac{dl_0}{d\alpha} (\eta f' - f) - \frac{U_0 y_m'}{L} (f' - 1) \right] + V_m \left(\frac{R+y_m}{R} \right) \quad 2.17$$

where $f(0) = 0$ and $f'(0) = 1$ by definition.

Integrating equation (2.9) and substituting equation (2.11) yields

$$\left[\frac{p}{\rho} + \overline{v'^2} \right]_{\eta}^{\infty} = \frac{l_0}{R} U_0^2 \int_{\eta}^{\infty} f'^2 d\eta \quad 2.18$$

Substituting equations (2.11)-(2.18) into equation (8) and rearranging yields (see also Guitton (1964))

$$\begin{aligned} & \frac{l_0}{U_0} \frac{dU_0}{d\alpha} \left[f'^2 - ff'' + 2g \right] - \frac{dl_0}{d\alpha} \left[ff' + \eta g' \right] - \frac{dy_m}{d\alpha} \left[f' - g' \right] \\ & + \frac{V_m}{U_0} \left(1 + \frac{y_m}{R} \right) f'' - g'' + \frac{l_0}{R} \left[\frac{l_0}{U_0} \frac{dU_0}{d\alpha} (-ff' - 2 \int_{\eta}^{\infty} f'^2 d\eta) \right. \\ & \left. - l_0' (ff' + \int_{\eta}^{\infty} f'^2 d\eta) - y_m' f - \eta g' - 2g \right. \\ & \left. + \frac{l_0}{R} \frac{dR}{d\alpha} \int_{\eta}^{\infty} f'^2 d\eta \right] = 0 \end{aligned} \quad 2.19$$

For self-preservation equation (2.19) must be independent of x and it follows that the coefficients must be constants. This condition results in

$$\begin{aligned} R &\propto l_0 \propto y_m \propto x \\ V_m &\propto U_0 \propto x^a \end{aligned} \quad 2.20$$

The shape of a surface having its radius of curvature proportional to the distance along the surface is that of a logarithmic spiral. (If $x \propto R$ then in polar co-ordinates $r \propto e^{R\theta/x}$)

These conditions should also be compatible with the behaviour of the inner layer ($y < y_m$). In the fully turbulent part of this layer the similarity forms can be written (e.g. Townsend, 1956).

$$\begin{aligned} u &= U_m + U_c f' \left(\frac{y}{y_m} \right) \\ -\overline{u'v'} &= U_c^2 g_{12} \left(\frac{y}{y_m} \right) \end{aligned} \quad 2.21$$

For the flow considered the momentum equation can be simplified by neglecting curvature terms (since $\frac{y_m}{R} \ll \frac{1}{R} \ll 1$) and since the profiles are very full (a typical profile with $\frac{y}{y_m} = \left(\frac{u}{U_m}\right)^{\frac{1}{n}}$ would have $n = 13$) it is permissible to adopt a profile form applicable to the small deficit wake where $\frac{U_m - U}{U_m} \ll 1$. Substituting equations (2.20) yields

$$\frac{U_m}{U_c} \left[\frac{y_m}{U_c} \frac{dU_c}{dx} f - \frac{dy_m}{dx} \frac{y}{y_m} f' \right] + g_{12}' = 0$$

and again a self preserving flow will require

$$U_m \propto U_e \propto x^a$$

$$y_m \propto x$$

and continuity shows that

$$V_m \propto U_m$$

Compatibility of inner and outer conditions requires that

$$R \propto l_0 \propto y_m \propto x$$

$$V_m \propto U_m \propto U_e \propto x^a \quad 2.22$$

and

$$U_m \propto U_e$$

The last requirement is never exactly met because

$$\frac{U_r}{U_m} \left(\equiv \sqrt{\frac{C_f}{2}} \right) = f(R_e) \quad 2.23$$

but the Reynolds number dependence is weak, especially at high Reynolds numbers, and the experimental evidence shown later verifies that, within experimental accuracy, the flow on a logarithmic spiral is self-preserving. Consequently it is permissible to write

$$\begin{aligned} \frac{u}{U_m} &= f(\eta) & \eta &= \frac{y}{l_{m/2}} \\ -\frac{\overline{u'v'}}{U_m^2} &= g_{12}(\eta) \end{aligned} \quad 2.24$$

and

$$\overline{u'^2} - \overline{v'^2} = U_m^2 g(\eta)$$

The exponent 'a' defining the decay of the maximum velo-

city may be found from the integral of the momentum equation (2.8). Multiplying that equation by $(R+y)$, integrating from $y = 0$ to ∞ , and substituting the pressure obtained from equation (2.9), ^{and neglecting the normal stress term} yields after some manipulation

$$\frac{2}{\partial x} \int_0^{\infty} u^2 dy + \frac{1}{R^2} \frac{dR}{dx} \int_0^{\infty} y u^2 dy = - \frac{c_w}{\rho} \quad 2.25$$

where terms of $O\left[\frac{1}{R^3}\right]$ have been dropped and $\frac{\tau_w}{\rho}$ is the skin friction. Substituting the similarity forms equation (2.24) yields

$$\int_0^{\infty} f'^2 d\eta \left[\gamma'_{m/2} + 2 \frac{\gamma_m u'_m}{u_m} \right] + \int_0^{\infty} \eta f'^2 d\eta \left[R' \left(\frac{\gamma_{m/2}}{R} \right)^2 \right] = - \frac{c_w}{\rho u_m^2} \quad 2.26$$

Writing

$$\frac{\gamma_{m/2}}{x} = \beta = \frac{d\gamma_{m/2}}{dx} \text{ in equilibrium flows.}$$

and using

$$u_m \propto x^a$$

Equation (2.26) gives

$$a = -\frac{1}{2} - \frac{C_f}{4\beta \int_0^{\infty} f'^2 d\eta} - \frac{\int_0^{\infty} \eta f'^2 d\eta}{2 \int_0^{\infty} f'^2 d\eta} \frac{\gamma_{m/2}}{R} \quad 2.27$$

This result is in disagreement with Giles, Hays and Sawyer who give $a = -\frac{1}{2}$ for all values of the curvature parameter, $\frac{\gamma_{m/2}}{R}$.

Measurements, which will be described in a later chapter, show that the skin friction coefficient does not vary substantially between different logarithmic spirals, and equation (2.27) shows that the dependence of 'a' on skin friction

decreases as the growth rate ($dy_{m/2}/dx = \beta$) increases. The neglect of the shear at the wall forms the basis of one of the theoretical methods.

Chapter 3

Theoretical Analyses and Required Measurements

Three different, and complementary theoretical approaches are undertaken and they will be discussed in detail in the sections which follow. The first and least sophisticated is essentially a preliminary method whose primary aim is to yield information on the order of magnitude of terms and the applicability of equilibrium data to non-equilibrium cases. It is designed for flows in which the jet is assumed to be thin in comparison to the local radius of curvature and it is applied to both the equilibrium and non-equilibrium flows. The second theory based on Townsend (1969), uses the total energy equation and hence incorporates assumptions about the turbulence structure that are unnecessary in the preceding analysis. It uses the order of magnitude approximations set forth by the first but its treatment of the turbulence yields information on the principal parameters which influence the local growth of a jet ($y_{m/2}$). This is useful to the third approach which uses the growth ($y_{m/2}$) as an auxiliary equation. This approach is aimed at predicting the development of the flow on the Coanda cylinder and consequently comes to grips with the three primary difficulties mentioned in section 1.3 of the Introduction: failure of the thin jet assumption, the occurrence of separation, and the strong interaction of the outer with the inner layer.

The theory uses a 'strip' integral method which, considering how little is known of this flow, appears to be the only reasonable approach at this stage.

3.1 The Linearized Integral Method - applied to the spirals and to both the concave and convex cylinders

The first and least sophisticated in its scope is essentially an extension of ideas first used by Newman (1961). It is noted from experimental work on plane wall jets, that the behaviour of the jet depends largely on the outer layer; the downstream variation being chiefly caused by entrainment of ambient fluid into the mixing layer rather than by a loss of momentum due to the wall friction. This will be especially true when there exists at the slot the combination of large momentum and thick boundary layer. (Some effects of friction on the jet on the cylinder have been discussed in section 1.2.) Figure 3.1 shows the effect of friction on the maximum velocity coefficient of the plane wall jet (the plane free jet analysis predicts that $\frac{0.054 \rho U_m^2 x}{J} = 1$). It is observed that it is possible to apply a frictionless analysis up to approximately 100 slot widths and still be within 10% accuracy. The curve through the data of Gartshore and Hawaleshka (1964) shows a greater wall friction effect which is probably due to the care taken by these experimenters to assure the existence of a rectangular profile at the slot (small boundary layer).

Skin friction does not substantially affect the development of the plane wall jet for a limited number of $\frac{x}{b}$ and it is convenient to assume that this is also true when there exists

surface curvature). In the method to be described below, skin friction is neglected, and the wall jet profile is replaced by a half-jet (i.e. there is admitted a finite slip velocity at the wall). A linearized integral technique is used to calculate the change, due to curvature, of the growth and mean velocity of the plane half-jet and the change is added to the known behaviour of the plane wall jet.

Some empirical knowledge is required and, to complement the strip integral analysis of section 3.3, it is convenient to use the local jet growth ($\frac{dy_{m/2}}{dx} = y'_{m/2}$) as the auxiliary equation. The theory of Townsend - to be discussed in the next section - shows, in conjunction with the experiments, that this parameter can be written generally as (see Chapter 10)

$$y'_{m/2} = f\left(\frac{y_{m/2}}{R}, R'\left(\frac{y_{m/2}}{R}\right)^2\right) \quad (3.1)$$

For the equilibrium flows on the logarithmic spirals

$$R'\left(\frac{y_{m/2}}{R}\right)^2 = y'_{m/2} \left(\frac{y_{m/2}}{R}\right)$$

and equation 3.1 is written

$$y'_{m/2} = f_1\left(\frac{y_{m/2}}{R}\right) \quad (3.2a)$$

but this is not the same function as given for the circular cylinder by equation 3.1 when $R' = 0$

$$y'_{m/2} = f_2\left(\frac{y_{m/2}}{R}\right) \quad (3.2b)$$

The result of Giles et al. and that of Fekete indicate that linearized forms for functions 'f₁' and 'f₂' may be acceptable and thus

$$\text{cylinder} \quad y'_{m/2} = \beta_0 + \beta_1 \frac{y_{m/2}}{R} \quad (3.3a)$$

$$\text{spirals} \quad y'_{m/2} = \beta_0 + \beta_3 \frac{y_{m/2}}{R} \quad (3.3b)$$

If the flow on the cylinder is in equilibrium, then there is a relation between β_0 , β_1 and β_3 and this will be discussed in Chapter 10.

Note that in the flows considered $y_{m/2}'$ is directly proportional to the entrainment rate. Thus

$$\frac{V_e}{U_m} = \frac{1}{U_m} \frac{d}{dx} \int_0^\infty u \, dy \quad (3.4)$$

and using equation (2.25) it is easily shown that

$$\frac{V_e}{U_m} \propto y'_{m/2} \quad (3.5)$$

Newman (1969) related the local entrainment rate to $\frac{y_{m/2}}{R}$ and obtained a similar result to equations (3.3), but his theory suggested, contrary to the above, that $\beta_1 = \beta_3$. Giles et al. (1966) also suggested that β_1 and β_3 are equal for all curved flows in equilibrium.

It transpires that the assumption of a linearized growth, though valid on the log spirals tested, does not hold for the whole of the flow on the Coanda cylinder and consequently in that case the above analysis is valid only for small x (in the present flow for about $\theta_1 < 145^\circ$). However, it is applied with success to the flow on the inside (concave) of a circular cylinder where the jet remains thin (Guitton, 1964).

An analogous method which uses the eddy viscosity concept rather than an integral approach is described by Guitton (1964, 1967) but because of its limited usefulness and mathematical complexity, it will not be discussed here.

A. The Self-preserving cases

Neglecting skin friction equation (2.25) is written

$$\frac{\partial}{\partial x} \int_0^\infty u^2 dy + \frac{1}{R^2} \frac{dR}{dx} \int_0^\infty y u^2 dy = 0 \quad (3.6)$$

and substituting the similarity form

$$\frac{u}{U_m} = f(\eta) \quad \eta = y/y_{m/2} \quad (3.7)$$

yields

$$I_2 \left[\frac{y'_{m/2}}{y_{m/2}} + \frac{2 y_{m/2} U'_m}{U_m} \right] + I_{2a} R' \left(\frac{y_{m/2}}{R} \right)^2 = 0 \quad (3.8)$$

where for a half-jet $I_2 = \int_0^\infty f^2 d\eta = 0.76$

and $I_{2a} = \int_0^{\infty} \eta f^2 d\eta = 0.29$

Using the assumed universal growth rate

$$Y'_{m/2} = \beta_0 + \beta_3 \left(\frac{Y_{m/2}}{R} \right) \quad (3.3b)$$

discussed above and recalling that for the self-preserving flow

$$U_m \propto x^a \quad (2.22)$$

(x is measured from the origin of the surface)

then equation (3.6) yields to $0 \left[\frac{Y_{m/2}}{R} \right]$

$$a = -\frac{1}{2} - 0.2 \frac{Y_{m/2}}{R} \quad (3.9)$$

where $\frac{Y_{m/2}}{R} = \frac{\beta_0 \frac{x}{R}}{(1-\beta_3 \frac{x}{R})}$ (3.10)

As noted in the Introduction relation (3.9) contradicts the result of Giles et al. who gave $a = -\frac{1}{2}$ for all $\frac{Y_{m/2}}{R}$.

To determine the constant of proportionality in (2.22), it is common, in self-preserving flows, to find a parameter which remains constant downstream. For the plane free turbulent jet the invariant is the jet momentum and it is found that

$$U_v = \left(\frac{J}{\rho I_2 \gamma'_{m/2}} \right)^{\frac{1}{2}} x^{-\frac{1}{2}} \quad (3.11)$$

and this relation also almost holds for the plane wall jet. For the plane laminar wall jet the invariant is Glauert's "flux of exterior momentum flux". For the logarithmic spirals an invariant was found (Guitton, 1968) but its validity appears to be limited to only the self-preserving region of the flow and not to the flow at or in the region of the slot (Glauert's invariant is valid at the slot). As an alternative, it was found convenient, when analysing the equilibrium flow data, to refer the maximum velocity to a known reference value - usually the first measurement station (Gartshore, 1965) - and this gives

$$\frac{U_m}{U_{m_v}} = \left(\frac{x}{x_v} \right)^{-\frac{1}{2}} - \frac{0.2 \beta_0 x}{R(1 - \beta_0 \frac{x}{R})} \quad (3.12)$$

The surface pressure is obtained by integrating, across the flow, the transverse equation of motion (2.2).

$$\begin{aligned} \frac{p_s - p_\infty}{\rho} = & \int_0^{\infty} \frac{u^2}{R+y} dy - \frac{V_\infty^2}{2} - \int_0^{\infty} \frac{R}{R+y} u \frac{\partial v}{\partial x} dy - \int_0^{\infty} \frac{R}{R+y} \frac{\partial u v}{\partial x} dy \\ & + \int_0^{\infty} \frac{u^2 - v^2}{R+y} dy \end{aligned} \quad (3.13)$$

Noting that at the edge of the jet the Bernoulli equation gives

$$\frac{p_s}{\rho} = \frac{p_\infty}{\rho} + \frac{V_\infty^2}{2} \quad (3.14)$$

and writing equation (3.13) to $O\left[\left(\frac{y}{R}\right)^2\right]$ yields

$$\frac{p_0 - p_\infty}{\rho} = \int_0^\infty \frac{u^2}{R} dy - \int_0^\infty \frac{yu^2}{R} dy - \int_0^\infty u \frac{\partial v}{\partial x} dy - \int_0^\infty \frac{\partial u v}{\partial x} dy + \int_0^\infty \frac{u^2 - v^2}{R+y} dy \quad (3.15)$$

It was shown by the author, Gaulton (1964), that with the thin jet assumption the third and fourth terms on the right hand side are negligible and it will be assumed here that the fifth is also negligible. The surface pressure can therefore be calculated from

$$\frac{p_0 - p_\infty}{\rho} = \int_0^\infty \frac{u^2}{R} dy - \int_0^\infty \frac{yu^2}{R} dy \quad (3.16)$$

providing $\frac{y}{R} \ll 1$. Substituting the velocity profile (3.7) and using the calculated integrals I_2 and I_{2n} yields

$$\frac{p_0 - p_\infty}{\rho U_m^2} = 0.76 \frac{y_{m/2}}{R} \left[1 - 0.38 \frac{y_{m/2}}{R} \right] \quad (3.17)$$

In terms of a reference value at some upstream station, the surface pressure is written (see equation 3.12)

$$\frac{p_0 - p_\infty}{(p_0 - p_\infty)_\infty} = \left(\frac{x}{x_\infty} \right)^{-(1 + 0.4 \frac{y_{m/2}}{R})} \quad (3.18)$$

and in equation (3.17) and (3.18) the value of $\frac{y_{m/2}}{R}$ is obtained from equation (3.10).

B. The Constant Radius Case

For this case $R' = 0$ and equation (3.6) reduces to

$$\frac{\partial}{\partial x} \int_0^{\infty} u^2 dy = 0 \quad (3.19)$$

and to the order of magnitude being considered the linear momentum (J) is conserved. Combining equations (3.19) and (3.7) yields

$$\int_0^{\infty} u^2 dy = \frac{J}{\rho} \quad (3.20)$$

and

$$(U_m^2 y_{m/2})' = 0 \quad (3.21)$$

From the growth rate equation (3.3a) it is easily shown that

$$\frac{y_{m/2}}{\alpha} = \beta_0 + \frac{1}{2} \beta_1 \frac{y_{m/2}}{R} \quad (3.22)$$

and assuming an analogous expansion for $U_m x^{\frac{1}{2}}$ (Guitton, 1964) yields

$$U_m x^{\frac{1}{2}} = \beta_4 + \beta_5 \frac{y_{m/2}}{R} \quad (3.23)$$

Combining equation (3.20), (3.21), (3.22), (3.23) and (3.3a) yields

$$\frac{\rho U_m^2 \alpha}{J} = \frac{1.81}{\beta_0} \left(1 - \frac{\beta_1}{2\beta_0} \frac{y_{m/2}}{R} \right) \quad (3.24)$$

The surface pressure is given by (see the self-preserving analysis)

$$\frac{p_s - p_\infty}{\int U_m^2} = 0.76 \frac{Y_{m0}}{R} \left(1 - 0.38 \frac{Y_{m0}}{R} \right) \quad (3.25)$$

and substituting the velocity decay (3.24) and the growth (3.19) yields

$$\frac{(p_s - p_\infty) |R|}{(P_s - p_\infty) b} = 2 \frac{|R|}{R} - 0.76 \frac{Y_{m0}}{R} \quad (3.26)$$

In the above equation, β_0 , is the plane wall jet growth and β_1 , will be determined from the equilibrium flow experiments.

3.2 The Structural Similarity Theory of Townsend (1969)

In a recent paper Townsend (1969) demonstrates that the dominant turbulence structure (both shape and intensity of large eddies) in a shear flow seems to arise from the rapid distortion of originally near-isotropic turbulence which is created in the vicinity of the interface (superlayer), and this structure is very similar to that produced by plane shearing of isotropic turbulence. From experiments and theoretical calculations he notes that after a given strain the structure, defined by $\frac{\overline{u'v'}}{q^2}$, does not change substantially; and using this fact plus the requirement of overall kinetic energy balance he is able to predict the development of a wide variety of flows.

Townsend applies these ideas to the prediction of curved wall jets and in particular the experiments of Fekete on the Coanda cylinder, and he shows that the assumption of a constant structure parameter predicts this flow with reasonable accuracy. However, there is an order of magnitude error in the early portion of the analysis as well as the liberal use of "ball park" approximations, and it is the purpose of the following analysis to present a more rigorous and correct calculation.

For reasons cited in the linearized theory (section 3.1) the inner boundary layer is not explicitly considered, and the wall jet profile is replaced by one of a free jet. The analysis furnishes information about the spread (or entrainment or eddy viscosity Reynolds number) of the jets and consequently is particularly complementary to the "strip integral" theory which requires the entrainment rate and its possible variation from equilibrium to non-equilibrium flows.

3.2.1 Theoretical considerations

The overall kinetic energy balance in a curved flow is written in the usual curvilinear coordinates as

$$\frac{d}{dx} \int_0^{\infty} u \left[\left(\frac{p}{\rho} + \bar{v}^2 \right) + \frac{\bar{u}^2 + \bar{q}^2}{2} \right] dy = - \int_0^{\infty} \left(\frac{R+y}{R} \right) \epsilon dy \quad (3.53)$$

where no curvature approximations are made but where the term

$$\frac{d}{dx} \int_0^{\infty} \left[u (\bar{u}^2 - \bar{v}^2) + v \bar{u} \bar{v} + \overline{u' \left(p' + \frac{q'^2}{2} \right)} \right] dy$$

is assumed much smaller than the left hand side of the above equation and is neglected.

Using

$$\frac{p}{\rho} + \bar{v}^2 = - \int_y^{\infty} \frac{u^2}{R} dy \quad (3.54)$$

written to suit an $O(\frac{Y}{R})$, $\frac{Y}{R} \ll 1$ analysis and assuming the similarity forms

$$\begin{aligned} \frac{u}{U_m} &= f(\eta) & \eta &= y/y_{m/2} \\ \frac{p}{\rho U_m^2} &= g(\eta) \end{aligned} \quad (3.55)$$

and

$$\int_0^{\infty} E dy = \frac{q_0^2 y_e}{L_e}$$

equation (3.53) is written

$$\begin{aligned} \frac{d}{dx} \left[\frac{U_m^3}{2} y_{m/2} \int_0^{\infty} f^3 d\eta + \frac{U_m}{2} q_0^2 y_{m/2} \int_0^{\infty} f d\eta - \frac{U_m}{R} y_{m/2}^2 \int_0^{\infty} f \int_0^{\infty} f^3 d\eta d\eta \right] \\ = - \frac{q_0^2}{L_e} y_e \left(1 + 0.5 \frac{y_e}{R} \right) \end{aligned} \quad (3.56)$$

where q_0^2 is now defined from

$$q_0^2 = \frac{\int_0^{\infty} u q_1^2 dy}{\int_0^{\infty} u dy}$$

The equation (3.56) is rewritten

$$\begin{aligned} \frac{d}{dx} \left[U_m^3 y_{m/2} \left(\frac{I_3}{2} - I_p \frac{y_{m/2}}{R} \right) + U_m q_0^2 y_{m/2} \frac{I_1}{2} \right] \\ = - \left(\frac{q_0^2}{L_e} \right) y_e \left(1 + 0.5 \frac{y_e}{R} \right) \end{aligned} \quad (3.57)$$

where

$$\begin{aligned} I_n &= \int f^n d\eta \\ I_p &= \int f \int f^2 d\eta d\eta \end{aligned} \quad (3.58)$$

The momentum equation is obtained by integrating equation (3.) in section 3.1 to yield equation (3.8) or

$$I_n \frac{d}{dx} (U_m^2 \gamma_{m/2}) + I_{2n} \left(\frac{\gamma_{m/2}}{R} \right)^2 \frac{dR}{dx} U_m^2 = - \frac{\tau_w}{\rho} \quad (3.59)$$

where

$$I_{2n} = \int \eta f^n d\eta$$

Combining equations (3.57) and (3.59) yields for the self-preserving case (γ/R and q_0^2/U_m^2 are constant)

$$\begin{aligned} \left(\frac{I_n}{2} - I_p \frac{\gamma_{m/2}}{R} + \frac{q_0^2}{U_m^2} \frac{I_1}{2} \right) \left[\frac{\gamma'_{m/2}}{2} + \frac{3}{2} \frac{I_{2n}}{I_n} R' \left(\frac{\gamma_{m/2}}{R} \right)^2 + \frac{3\tau_w}{2I_n U_m^2 \rho} \right] \\ = \frac{\gamma_{m/2}}{L_n} \frac{\gamma_0}{\gamma_{m/2}} \frac{q_0^2}{U_m^2} \left(1 + 0.5 \frac{\gamma_0}{R} \right) \end{aligned} \quad (3.60)$$

In the cases considered C_τ is very small (Chapter 9, section 9.4) and consequently may be dropped from the preceding equation.

In order to introduce a structure parameter, chosen for convenience to be $\frac{\tau_{MAX}}{\rho q_0^2}$, it is necessary to find an expression for τ_{MAX} . Still following Townsend, if $\frac{\gamma_m}{R} \sim \frac{\gamma_m/2}{R}$ the equation of motion at γ_m is nearly

$$\frac{\partial}{\partial x} \left(\frac{p}{\rho} + \bar{v}^2 + \frac{u^2}{2} \right) = \frac{\partial}{\partial y} \left[\left(1 + \frac{\gamma}{R} \right) \frac{\tau}{\rho} \right] \quad (3.61)$$

and using the y-momentum equation (1.9) and the similarity profiles yields

$$\frac{\partial}{\partial x} \left[\frac{U_m^2}{2} \left(1 - 2I_2 \frac{y_{m/2}}{R} \right) \right]_{y=y_m} = \frac{\partial}{\partial y} \left[\left(1 + \frac{y}{R} \right)^2 \tau \right]_{y=y_m} \quad (3.62)$$

Assuming similarity of shear stress profiles

$$\frac{\tau}{\tau_{max}} = g_{12} \left(\frac{y}{y_{m/2}} \right) \quad (3.63)$$

where g_{12} is assumed to be the same function in both plane and curved jets (figure 9.17a), yields

$$\frac{\partial}{\partial y} \left[\left(1 + \frac{y}{R} \right)^2 \tau \right]_{y_m} = \frac{\tau_{max}}{y_{m/2}} \left[g'_{12} \left(\frac{y_m}{y_{m/2}} \right) + 2g_{12} \left(\frac{y_m}{y_{m/2}} \right) \frac{y_{m/2}}{R} \right] \quad (3.64)$$

For reasons discussed in the introduction the author's plane wall jet measurements are used. (These results are fully discussed in section 9.5). The calculated shear stress distribution corresponding to a growth $\frac{y_{m/2}}{x} = .071$, agrees well with the measured, and subsequently corrected distribution, and yields

$$\begin{aligned} \frac{\tau_m}{\tau_{max}} &= g_{12} \left(\frac{y_m}{y_{m/2}} \right) = 0.2 \\ \left[\frac{\partial(\tau/\tau_{max})}{\partial(y/y_{m/2})} \right]_{y_m} &= 2.74 \end{aligned} \quad (3.65)$$

Substituting equations (3.59), (3.64) and (3.65) into (3.62) and neglecting skin friction and the term $R' \left(\frac{y_{m/2}}{R} \right)^2$ because $\frac{\tau_{MAX}}{\rho U_m^2}$ is already of 0 $\left[\left(\frac{y_{m/2}}{R} \right)^2 \right]$ yields for a self preserving

flow

$$\frac{\tau_{max}}{\rho U_m^2} = -0.183 \left[1 - (2I_2 + 0.146) \frac{y_{m/2}}{R} \right] y'_{m/2} \quad (3.66)$$

For the plane wall jet the above equation gives $\frac{\tau_{MAX}}{\rho U_m^2} = .013$ which is in excellent agreement with the measured value (see figure 1.4). Combining equation (3.60) and (3.66) yields

$$\left\{ \frac{I_3}{2} - I_p \frac{y_{m/2}}{R} + \frac{I_1}{2} \frac{q_o^2}{|\tau_{max}|} \left[1 - (2I_2 + 0.146) \frac{y_{m/2}}{R} \right] 0.183 y'_{m/2} \right\} \left[\frac{y'_{m/2}}{2} + \frac{3}{2} \frac{I_{2a}}{I_2} R' \left(\frac{y_{m/2}}{R} \right)^2 \right] = \frac{y_{m/2}}{L_e} \frac{y_e}{y_{m/2}} \left(\frac{q_o^2 \rho}{|\tau_{max}|} \right)^{1/2} (0.183 y'_{m/2})^{1/2} \quad (3.67)$$

$$\left[1 - (2I_2 + 0.146) \frac{y_{m/2}}{R} \right]^{1/2} \left(1 + 0.5 \frac{y_e}{y_{m/2}} \frac{y_{m/2}}{R} \right)$$

Replacing the plane wall jet profile by a half-jet results in

$$\begin{aligned} I_1 &= 1.12 & I_3 &= 0.605 & I_p &= 0.32 \\ I_2 &= 0.76 & I_{2a} &= 0.29 \end{aligned} \quad (3.68)$$

and it is of interest to examine the possibility of predicting the spread of the wall jet on the logarithmic spirals by assuming that the parameters $y_{m/2}/L_e$, $y_e/y_{m/2}$ and $\rho q_o^2/|\tau_{MAX}|$ are identical to those of the plane wall jet. There is available little data which can shed light on the validity of this assumption. Giles et al. did not measure all the Reynolds stresses and for reasons mentioned in the introduction there is some doubt regarding the accuracy of their hot-

wire work and the two-dimensionality of their flow. A purpose of the experimental investigation is to supply some of the missing experimental data and the measurements and results are discussed in Chapter 9.

The reduction of equation (3.67) to a form similar to the proposed universal growth relation (3.3), will also be discussed in Chapter 10.

3.3 The Strip Integral Method - applied only to the cylinder

This analysis is concerned with the prediction of the flow on the Coanda cylinder; but it is limited to a region bounded upstream by an ill-defined zone corresponding to the onset of fully developed flow (or flow that has 'lost all significant memory of the slot conditions) and downstream by the more precisely defined point at which separation occurs. If separation is to be predicted, the behaviour of the inner layer must be considered and consequently the assumptions of the previous theory are too naive and the following must be accounted for: skin friction, shear stress at the velocity maximum (which is far from zero), the possible variation in shape of the inner layer velocity profile, and the impossibility of simplifying the y -momentum equation when $\frac{y_m/2}{R}$ becomes large as it does when the jet tends towards separation. A strip integral method is chosen because the velocity profile in fully developed jets is almost invariant and because information about the turbulent shear stress is required only at the strip boundaries.

Guidelines for the theory are provided by the experiments of Fekete (1963) (Section 1.2) and it is of interest to present a sample of his data. Because the results are universal it is only necessary to consider one test. Figure (4.1) shows the pressure distribution, the maximum velocity decay and the growth of the inner layer for the case $\frac{b}{R} = .0531$ and $Re = 1.31 \times 10^4$.

In figure (4.1) it is noted that the surface pressure is almost constant up to say 120 or 130° , after which it increases rapidly to ambient and the jet separates.

Beyond about 130° , where $\frac{y_m/2}{R} = .5$, the jet becomes very thick and the 'thin jet' assumption necessary to simplify the y-momentum equation (section 2.1) is untenable. This was shown by Newman (1961) who attempted to calculate the surface pressure using

$$\frac{p_\infty - p_w}{\rho} = \int_0^\infty \frac{u^2}{R+y} dy$$

and obtained progressively worse agreement with experiment as $\frac{y_m/2}{R}$ increased. But it is very tedious to solve the x-momentum integral equation when the pressure gradient term is calculated from the full y-momentum equation. Furthermore, as discussed below, the stability of the numerical solution is uncertain when $\frac{y_m/2}{R} \ll 1$. Therefore another approach was sought.

A simple system of equations valid for all $\frac{y_m/2}{R}$ is one written in streamline co-ordinates, and this was adopted in a preliminary attempt to solve the flow development. This

method gave rise to a third order differential equation in which all derivatives of order higher than unity had coefficients of $O\left(\frac{y_{m/2}}{R}\right)$, but unfortunately, at small θ where $\frac{y_{m/2}}{R} \ll 1$, the numerical solution was unstable. (The nature of the instability, though not fully understood, indicated that a method employing the curvilinear co-ordinates and the full y-momentum equation would also be unstable when $\frac{y_{m/2}}{R}$ is small.) Two other approaches seemed feasible: (a) the use of simplified set of equations ((2.8) and (2.9)) which are valid for $\frac{y_{m/2}}{R} \ll 1$ but nevertheless are applied right up to the threshold of rapid pressure rise ($\theta = 130^\circ$) and from there the use of an approximate technique to predict separation; or (b) the use of the small $\frac{y_{m/2}}{R}$ solution up to an arbitrary point where say $\frac{y_{m/2}}{R} \approx .3$ or $.4$ at which point the solution is used as the starting condition for a method based on the full y-momentum equation. The latter method is tedious and might still be plagued by instability problems and thus the former technique was chosen as a reasonable introductory approach.

To begin the threshold point is approximately defined using a criterion inspired by the Reshotko-Tucker (1955) theory of shock induced boundary layer separation. The essence of that theory is the assumption that shear forces are small when compared to the pressure forces, and that they can be neglected. A preliminary version of this theory was published by Newman (1969).

Consider the flow development between a point which is downstream of the slot and at which the flow is fully developed, and the threshold point. The surface pressure between these is almost constant and hence it is permissible to assume a constant shape of the inner profile and experience indicates that the outer layer is well represented by a half-jet profile. Three unknowns exist - the maximum velocity and the outer and inner layer thicknesses - and three equations are required to solve the flow. These are taken to be: (a) two momentum integral equations over the outer and (b) inner portions respectively, and (c) the growth equation. Two boundary conditions on these integrals are the shear at y_m and the skin friction and these parameters are introduced as additional unknowns. The first, τ_m , is related to the maximum shear which in turn is found by a relation similar to equation (3.6) in the discussion of Townsend's theory. The empirical information in this equation plus information on skin friction is obtained from equilibrium flows.

The flow beyond the threshold point is treated using the Stratford-Townsend analysis of rapidly retarded boundary layers (Stratford, 1959; Townsend, 1960) in conjunction with an assumed linear pressure rise.

The full method is described in the following sections. To begin with the integral equations are presented and the required additional equations or unknown parameters are noted. These are entrainment, shear at y_m and τ_w and are discussed

individually in subsequent sections. Finally in Chapter 4 the method used for determining separation and establishing the threshold point is described. (It was decided to include the separation analysis in a different chapter because of its length and large number of subsections.)

3.3.1 The integral equations

As described in the previous section the wall jet is broken up into an inner layer extending from the wall to the maximum velocity point, y_m , and an outer layer extending from y_m to the ambient surroundings. (Figure 1.1.)

The integral of equation (2.8) between the limits y_m and ∞ is

$$\begin{aligned} \int_{y_m}^{\infty} (R+y) \frac{\partial u^2}{\partial x} dy - \frac{(R+y_m)^2}{R} U_m y_m + \int_{y_m}^{\infty} (R+y) \frac{\partial (u^2 + v^2)}{\partial x} dy \\ + \int_{y_m}^{\infty} (R+y) \frac{\partial (P/\rho)}{\partial x} = - \frac{(R+y_m)^2}{R} \frac{\tau_m}{\rho} \end{aligned} \quad (3.71)$$

where

$$\frac{P}{\rho} = \frac{p}{\rho} + \frac{v^2}{2}$$

In section 2.1 the y -momentum for thin jets was written

$$\frac{\partial (P/\rho)}{\partial y} = \frac{u^2}{R} \quad (1.9)$$

but there may be an intermediate point between the values $\frac{y_m/2}{R} \ll 1$ and $\frac{y_m/2}{R} \sim O[1]$, where the lateral velocity v is still small with respect to u such that the streamlines can

still be approximated as concentric circles. In this zone a better representation for (1.9) is

$$\frac{\partial(p/p)}{\partial y} = \frac{u^2}{R+y} \quad (3.72)$$

and this was used by Newman (1961) and Fekete (1963) and will be adopted in these calculations.

With equation (3.72), (3.71) becomes

$$\begin{aligned} \frac{1}{2} \int_{y_m}^{\infty} (R+y) \frac{\partial u^2}{\partial x} dy + \frac{(R+y_m)^2}{2} \int_{y_m}^{\infty} \frac{1}{R+y} \frac{\partial u^2}{\partial x} dy + \frac{1}{2} \frac{dR}{dx} \int_{y_m}^{\infty} u^2 dy - \frac{dR}{dx} \frac{(R+y_m)^2}{2} \int_{y_m}^{\infty} \frac{u^2 dy}{(R+y)^2} \\ + \int_{y_m}^{\infty} (R+y) \frac{\partial (u^2 - v^2)}{\partial x} dy - \frac{(R+y_m)^2}{R} u_m v_m = - \frac{(R+y_m)^2}{R} \frac{\tau_m}{\rho} \end{aligned} \quad (3.73)$$

(Note that $R = R(x)$ for completeness.)

To obtain v_m the continuity equation is used in the inner layer

$$\begin{aligned} \frac{(R+y_m)}{R} v_m &= - \int_0^{y_m} \frac{\partial u}{\partial x} dy \\ &= - \frac{\partial}{\partial x} \int_0^{y_m} u dy + u_m y_m' \end{aligned} \quad (3.74)$$

The inner layer integral equation, written to $O\left[\frac{y_m}{R}\right]$ is

$$\begin{aligned} \left(1 + \frac{y_m}{R}\right) \int_0^{y_m} (2u - u_m) \frac{\partial u}{\partial x} dy + \frac{R'}{R} \int_0^{y_m} \left(\frac{y - y_m}{R}\right) u^2 dy + \int_0^{y_m} \frac{\partial (u^2 - v^2)}{\partial x} dy \\ = - \frac{p_w'}{\rho} \left[y_m \left(1 + 0.5 \frac{y_m}{R}\right) \right] + \left(1 + \frac{2y_m}{R}\right) \frac{\tau_m}{\rho} - \frac{\tau_w}{\rho} \end{aligned} \quad (3.75)$$

where the variation of pressure across the layer has been included and p_w' is the surface pressure gradient. Orders higher than $O\left[\frac{y_m}{R}\right]$ are negligible in the region of interest (at the threshold of rapid pressure rise $\frac{y_m}{R} = .1$).

The outer velocity profile is well represented (figure 9.6 & 9.7) by the relation

$$u = u_m \operatorname{sech}^2 \xi \quad (3.76)$$

where

$$\xi = 0.881 \frac{y - y_m}{l_0}$$

and

$$l_0 = y_{m/2} - y_m \quad (3.77)$$

and the inner profile is adequately described by the power law

$$\frac{u}{u_m} = \left(\frac{y}{y_m}\right)^{\frac{1}{n}} \quad (3.78)$$

Recalling again that in the region to be analysed the surface pressure is almost constant, it is permissible to assume that the shape of the inner layer profile is invariant. Consequently

$$\frac{1}{n} = \text{constant}$$

Substituting equations (3.74), (3.76) or (3.78) into either (3.73) or (3.75), putting $R' = 0$, and calculating $\frac{p_w'}{\rho}$ from (3.72) yields:

OUTER INTEGRAL

$$\begin{aligned}
 & (U_m^2)' l_o \left[0.378 + 0.645 I_1 \frac{l_o}{R+y_m} + 0.568 I_2 \left(\frac{R+y_m}{R} \right) + \frac{0.5n}{n+1} \frac{y_m}{l_o} \right] \\
 & + y_m' U_m^2 \left[2 I_3 + 2.27 I_3 \frac{l_o}{R+y_m} + 2 I_4 \left(\frac{R+y_m}{R} \right) - \frac{1}{n+1} \right] \\
 & + l_o' U_m^2 \left[2.27 I_3 + 2.57 I_4 \frac{l_o}{R+y_m} + 2.27 I_5 \left(\frac{R+y_m}{R} \right) \right] \\
 & = - \left(\frac{R+y_m}{R} \right) \frac{\tau_m}{f} - \frac{l_o}{0.88(R+y_m)} \int_0^{\infty} (R+y) \frac{\partial(u^2 - v^2)}{\partial x} dz
 \end{aligned}
 \tag{3.79}$$

INNER INTEGRAL

$$\begin{aligned}
 & (U_m^2)' y_m \left[\frac{n^2}{(n+2)(2n+2)} - \frac{n}{2n+2} \frac{y_m}{R} - \frac{I_3}{0.88} \frac{l_o}{R} \left(1 + 0.5 \frac{y_m}{R} \right) \right] \\
 & + y_m' U_m^2 \left[- \frac{n}{(n+1)(n+2)} - \frac{n}{n+1} \frac{y_m}{R} \right] \\
 & = \left(1 + \frac{2y_m}{R} \right) \frac{\tau_m}{f} - \frac{\tau_o}{f} - y_m \int_0^1 \frac{\partial(u^2 - v^2)}{\partial x} d\left(\frac{y}{y_m}\right)
 \end{aligned}
 \tag{3.80}$$

The integrals are listed in Appendix I, and for convenience, there follows a summary of the important assumptions:

SUMMARY OF ASSUMPTIONS LEADING TO EQUATIONS (3.79) AND (3.80)

- (a) The region analysed is one of near constant pressure and the shape of the inner profile is invariant.
- (b) The outer and inner layer velocity profiles are represented by similarity profiles defined by the scales U_m , $l_o = (y_m/2 - y_m)$, and y_m .
- (c) The simplified y-momentum equation (2.9) is used to calculate all pressure terms.
- (d) The inner integral equation includes curvature terms but only to $O\left[\frac{y_m}{R}\right]$.

The two equations (3.79) and (3.80) introduce the following unknowns

U_m , l_o , y_m , τ_m , τ_w , η , normal stress terms

and the information necessary to close the problem is discussed in Chapter 10.

Chapter 4

Separation on the Coanda Cylinder

4.1 Introduction

Newman (1961) has predicted, by dimensional arguments that for a flow assumed to emanate from a virtual origin, the angle for Coanda separation from a circular cylinder is fixed at high enough Reynolds numbers. This prediction is subject to the limitation that $\frac{b}{R}$ should remain large enough that friction forces are unimportant. If $\frac{b}{R}$ does not become too large the slot position can be considered coincident with the virtual origin.

The measurements of Fekete (1963) suggest that the angle is $210^\circ \pm 10^\circ$ for jet momentum Reynolds numbers $Re > 1.3 \times 10^4$ and for $.0074 < \frac{b}{R} < .0531$. The separation position has not yet been predicted analytically.

For a Coanda flow of this type two independent criteria for separation can be developed by observing that the pressure distribution over the cylinder remains essentially constant to near separation and then rises quickly to ambient conditions as the jet separates. If this pressure gradient is severe enough it becomes possible to divide the boundary layer* into two

* In the discussions that follow the term boundary layer refers to the inner layer of the wall jet, i.e. $y < y_m$. No direct reference is made to the flow in the outer portion but its presence is implied through the use of the surface pressure distribution.

zones: (i) an outer zone where the pressure forces are much larger than the shear forces and where the total head along each streamline is essentially conserved; and (ii) an inner zone where the inertia forces are small so that the pressure forces are balanced by the shear forces.

The criterion to be discussed in the following pages is based on separate analyses of each of these zones. The first, suggested by Tyler and Shapiro (1953) and later modified by Reshotko and Tucker (1955) is the discontinuity analysis of shock induced turbulent boundary layer separation and is essentially an outer zone criterion. It is used to define the position of the threshold from which the pressure rapidly increases. A preliminary version of this analysis was published by Newman (1969). The second stems from an analysis by Stratford (1959), later modified by Townsend (1960) and considers the inner region of the boundary layer. The usefulness of the method is assessed by comparing with Fekete's measurements.

4.2 Conditions for Validity of Both Analyses

The discontinuity and the Stratford analysis are based on the assumption that the acceleration of the outer part of the boundary layer due to pressure gradient should be much larger than that due to shear. Neglecting curvature effects (assuming $\frac{y_m}{R} \ll 1$) and assuming the pressure constant across the boundary layer the integral of the x-momentum equation is written

$$\int_0^{y_m} \frac{\partial u^2}{\partial x} dy + U_m V_m = y_m \frac{d(b/\rho)}{dx} + \frac{\tau_m - \tau_w}{f}$$

The analyses will therefore be valid if

$$\left| \frac{y_m}{\tau_m - \tau_w} \frac{dp}{dx} \right| \gg 1 \quad (4.1)$$

Fekete's data for $.01 < \frac{b}{R} < .053$ satisfies the conditions for universal flow and consequently it is only necessary to evaluate the separation analysis for one particular case. Figure 4.1 shows the inner layer thickness, the surface pressure and the maximum velocity data of Fekete for $\frac{b}{R} = .0531$ and $Re = \left[\frac{2(P_o - p)b^2}{\rho v^2} \right]^{1/2} = 1.3 \times 10^4$. It is observed that the pressure distribution consists of a plateau region followed by a steep pressure rise. For convenience in evaluating relation 4.1, the start of this pressure rise or threshold, is defined to be at the intersection of the dotted lines in figure 4.1 which represent the asymptotes to the plateau and constant gradient regions.

A conservative estimate of 4.1 can be made by evaluating y_m and the shear stresses at the threshold point where they are most likely to be small and large respectively. Considering the idealized linear pressure rise, and defining subscripts 't' and 's' to imply 'threshold' and 'separation' respectively then expression 4.1 can be rewritten

$$\left| \frac{y_{mt}}{R} \left[\frac{\frac{p_s - p_t}{\frac{1}{2} \rho U_{mt}^2 \theta_t}}{\frac{(\tau_m - \tau_{wt})}{\frac{1}{2} \rho U_{mt}^2}} \right] \left(\frac{\theta_t}{\theta_s - \theta_t} \right) \right| \gg 1 \quad (4.2)$$

which can be easily evaluated from the data of Fekete providing an expression for τ_m is found.

From fig 10.17, at mid-point of rapid pressure rise ($\theta \approx 190^\circ$ in the present measurements)

$$\frac{\tau_m}{\rho U_{mt}^2} = 0.011 \quad (4.3)$$

The wall shear stress at θ_t may be obtained to a satisfactory accuracy by using the drag law proposed by Escudier et al. (1966) for the two dimensional turbulent wall jet in still surroundings.

$$\frac{\tau_w}{\rho U_{mt}^2} = \frac{0.4}{\left[\ln \left(7.62 \frac{y_m U_{mt}}{\nu} \right) \right]^2} \quad (4.4)$$

and this relation evaluated at θ_t yields

$$\left(\frac{\tau_w}{\frac{1}{2} \rho U_{mt}^2} \right)_t = 0.0062 \quad (4.5)$$

The measured pressure distribution gives

$$\frac{p_s - p_t}{\frac{1}{2} \rho U_{mt}^2 \theta_t} = 0.25 \quad (4.6)$$

Using the above values (4.1) becomes

$$\left| \frac{y_m}{(\tau_w - \tau_m)_t} \frac{dp}{dx} \right| \approx 6 \quad (4.7)$$

The pressure forces are therefore approximately an order of magnitude greater than the shear forces and an analysis based on the neglect of the latter will be attempted.

4.3 Theoretical Development

4.3.1 Extension of the Reshotka-Tucker Analysis to the Coanda Jet Boundary Layer

If a one-parameter family of profiles is assumed for the inner boundary layer (equation 3.1), two unknowns appear y_m and n (note that U_m and p are considered known boundary conditions) and two equations are necessary to specify the flow. These are taken to be the momentum and energy integral relations which yield simpler results than the momentum and momentum equations used by Reshotko and Tucker.

Since $\frac{y_m}{R} \ll 1$ all curvature terms are neglected, and it follows that the pressure is essentially constant across the layer (the contribution to the surface pressure comes almost entirely from the outer flow: $y > y_m$). Dropping all turbulent stress terms and the skin friction, the momentum and energy integral equations for a power profile $\frac{u}{U_m} = \left(\frac{y}{y_m}\right)^{1/n}$ are respectively:

$$\frac{2H(H+1)}{(H-1)U_m^2} \frac{d(\frac{p}{\rho})}{dx} + \left(\frac{2}{H-1}\right) \frac{d}{dx} \ln U_m^2 - \frac{2d \ln y_m}{dx} + \frac{2(H^2-2H-1)}{H(H+1)(H-1)} \frac{dH}{dx} = 0 \quad (4.8)$$

and

$$\left(\frac{3H-1}{H-1}\right) \frac{1}{U_m^2} \frac{dU_m^2}{dx} + \left(\frac{1}{H-1}\right) \frac{d \ln U_m^2}{dx} - \frac{d \ln y_m}{dx} + \frac{3H^2-6H-1}{(3H-1)(H^2-1)} \frac{dH}{dx} = 0 \quad (4.9)$$

where H is the usual shape parameter defined as the ratio of displacement to momentum thicknesses.

Equations (4.7) and (4.9) may be combined to eliminate the terms containing $\ln y_m$ and $\ln U_m^2$ to yield

$$-\frac{1}{\rho U_m^2} \frac{d(p_o - p_w)}{dx} = \left[\frac{1}{(3H-1)(H-1)H} \right] \frac{dH}{dx} \quad (4.10)$$

Integrating between the start of the pressure rise (subscript t) and separation (subscript s) yields

$$\int_{(p_o - p_w)_t}^{(p_o - p_w)_s} \frac{1}{\rho U_m^2} d(p_o - p_w) = - \int_{H_t}^{H_s} \frac{dH}{H(3H-1)(H-1)} \quad (4.11)$$

The left-hand integral prescribes the overall influence of the boundary conditions on the inner layer and is a fixed quantity, defined by the right-hand integral, for particular values of the shape parameter before and after separation. The use of H_s to define separation is usually considered a vague criterion since values between 1.8 and 2.7 have been known to occur, but fortunately the integral is quite insensitive to this terminal quantity. The difficulty lies rather in its sensitivity to the initial value of H_t . This is illustrated in the following table where n_t is the power profile exponent corresponding to H_t

n_t	H_t	H_s	$\int_{H_t}^{H_s} \frac{dH}{H(3H-1)(H-1)}$
12	1.166	2.2	.409
12	1.166	2.8	.44
14	1.143	2.2	.463
14	1.143	2.8	.49

Particular values of n_t are not available from the data of Fekete; but the range of n_t and H_s quoted above seem physically plausible and thus, for

$$12 < n_t < 14$$

$$2.2 < H_s < 2.8$$

the separation criterion is

$$-0.41 \geq \int_t^s \frac{d(p_{\infty} - p_w)}{\rho U_m^2} \geq -0.49 \quad (4.12)$$

The following section describes how this relation is used to determine the threshold point 't'.

4.3.2 The position of the threshold

Within the approximations of the analysis the total pressure is constant along the streamlines of the boundary layer, and in particular at $y = y_m$ the Bernoulli equation gives

$$\left(p + \frac{1}{2} \rho U_m^2 \right)_t = \left(p + \frac{1}{2} \rho U_m^2 \right)_s \quad (4.13)$$

and therefore

$$\frac{d}{dx} \left(\frac{p_0 - p}{\rho} \right) = \frac{1}{2} \frac{dU_m^2}{dx} \quad (4.14)$$

Combining equation (4.14) with equation (4.12) yields

$$0.82 \leq \int_0^t \frac{dU_m^2}{U_m^2} \leq 0.98$$

or

$$0.43 \geq \frac{U_m^2}{U_{m,0}^2} \geq 0.375 \quad (4.15)$$

and using equation (4.13), noting that at separation $p = p_m$, yields

$$0.43 \geq \left[1 - \frac{2(p_0 - p_t)}{\rho U_{m,t}^2} \right] \geq 0.375 \quad (4.16)$$

Using the measured data of figure 4.1 only the left-hand limit is met, and this at

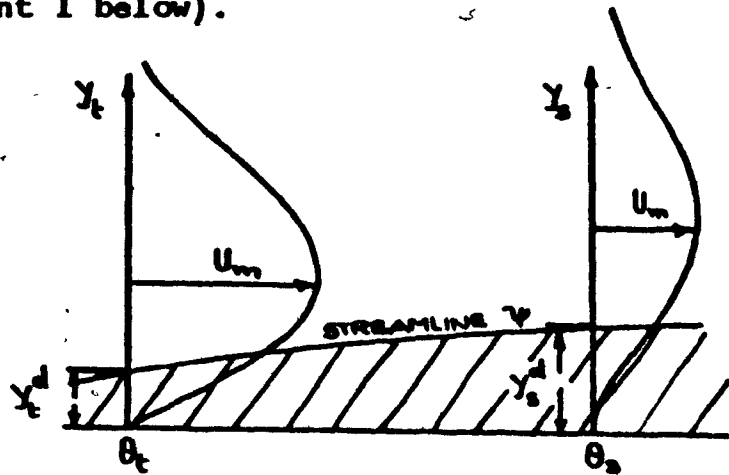
$$\theta_t = 155^\circ$$

which corresponds exactly to point defined by the idealized pressure distribution given by the asymptotes. The following analysis is concerned with predicting θ_s once θ_t is known.

4.3.3 The Stratford-Townsend separation criterion

In the sketch below, streamline ψ is defined as the approximate division between the outer region of the boundary layer, uninfluenced by shear forces, and the inner region where inertia

effects are small (a more precise definition is given in statement 1 below).



As before subscript 't' defines the point where the pressure gradient begins and subscript 's' where separation takes place. Superscript (d) refers to conditions along streamline ψ . The analysis of Stratford modified by Townsend proposes a relation between $(\frac{dp}{dx})_s$, $p_s - p_t$, and τ_{w_t} obtained from the following equations and associated assumptions.

(1) It is assumed that changes are so rapid between θ_t and θ_s that the shear stress distribution in the upper layer is essentially "frozen" at the θ_t conditions. In particular along ψ

$$\tau_t^d = \tau_s^d$$

At θ_t the pressure gradient is small and it will be assumed that a near constant stress layer exists* and that ψ is its

* In a curved wall jet the influence of the outer flow imposes a stress gradient and the assumption must be verified experimentally. It is not necessary to have a constant stress region to have a log-law of the form 4.21. A linear stress region would have different constants A and B.

approximate outer boundary. Hence

$$\tau_s^d = \tau_{wt} \quad (4.17)$$

and the total pressure is therefore conserved along ψ such that

$$\left(p + \frac{1}{2} \int u^{2d} \right)_t = \left(p + \frac{1}{2} \int u^{2d} \right)_s \quad (4.18)$$

(2) The flow in the inner region (below ψ) is in a state of local equilibrium defined by the local parameters

$$\gamma, \tau_w, \left(\frac{\partial \tau}{\partial y} \right)_w, \left(\frac{\partial^2 \tau}{\partial y^2} \right)_w, \dots, \rho \quad (4.19)$$

This is possible if advection of energy from the external flow is small and if near the wall the rate of turbulent energy dissipation is so large that any decrease in production due to a local decrease in velocity will lead to an immediate decrease in turbulence intensity. Where $\tau_w = 0$, and close enough to the wall where a linear variation of shear is representative, dimensional reasoning yields

$$u_s = c \left(\frac{\gamma}{\rho} \left(\frac{\partial \tau}{\partial y} \right)_s \right)^{1/2} \quad (4.20)$$

In the assumed near constant stress region* at θ_t

$$\left(\frac{u}{u_\tau} \right)_t = A \ln \left(\frac{\gamma u_\tau}{\nu} \right)_t + B \quad (4.21)$$

* See footnote on previous page.

and the values of the constants A, B and C will be discussed later (Chapter 10).

(3). The first compatibility condition

$$\left(\frac{\partial \tau}{\partial y}\right)_s = \left(\frac{dp}{dx}\right)_s \quad (4.22)$$

is applicable at separation. This assumption is known to be inexact (see Newman, 1951; Spangenberg et al., 1967) due to the importance of the gradient $\frac{\partial u^2}{\partial x}$ in the momentum balance. Bradshaw and Galca (1967) conclude that it would be preferable to express the separation criterion in terms of $\frac{\partial \tau}{\partial y}$ rather than $\frac{dp}{dx}$ and comparison with their experiments justify this conclusion. However, it is not practical to use the shear gradient in the analysis and it will not be considered.

The separation criterion is determined by substituting equations (4.20) and (4.21) into the condition that y_t^d and y_s^d lie on the streamline ψ

$$\int_0^{y_t^d} u_t dy = \int_0^{y_s^d} u_s dy \quad (4.23)$$

and then using relations (4.22), (4.17) and (4.18) to yield

$$\left(\frac{1}{\tau_{wt}} \left(\frac{dp}{dx}\right)_s e^{\left\{ \left[\frac{2(p_s - p_t) + C^2 \tau_{wt}^2}{A^2 \tau_{wt}} \right]^{1/2} - \frac{B}{A} \right\}} \right) \left\{ \left[\frac{2(p_s - p_t) + C^2 \tau_{wt}^2}{\tau_{wt}} \right]^{1/2} - A \right\} = \frac{2CU_\tau}{3A} \quad (4.24)$$

4.3.4 Application of the criterion to the Coanda cylinder

Equation (4.24) may be used to predict the point of separation on the Coanda cylinder providing the constants A, B and C are known and that a linear pressure rise is assumed between θ_t and θ_s .

Townsend determines from the experiments of Stratford that $c = 4$. The constants of the log-law present some difficulty as it is not clear that the values $A = 2.30$ and $B = 5.5$ shown to be applicable to pipes, boundary layers and wall jets (Guitton, 1969) are valid for the curved wall jet. Nevertheless, at this stage it is of interest to use the data available and the values quoted above will be used for the time being. The experiments presented and discussed in Chapter 10 will verify the assumptions.

Using the skin friction coefficient given by equation (4.5), the above constants, and the experimental conditions, equation (4.25) yields

$$\left(\frac{d(pR)}{d\theta} e^{\left\{ \left[22.8 \left(\frac{p_s - p_t}{J} \right) R + 2.82 \right]^{\frac{1}{2}} \right\}} \right) \left\{ \left[129 \left(\frac{p_s - p_t}{J} \right) R + 16 \right]^{\frac{1}{2}} - 2.38 \right\} = 502 \quad (4.25)$$

Taking the actual surface pressure

$$\left(\frac{p_s - p_t}{J} \right) R = 1.4$$

and recalling that a linear pressure gradient is assumed,

then the above equation predicts separation at 200° which is in excellent agreement with experiment. If the skin friction coefficient is increased by 10% to .0068 then separation is predicted at 181° which constitutes a considerable change in view of the 67° angle between threshold and separation. In curved wall jets it will be seen that the outer layer exerts a strong influence on the inner and the validity of both the logarithmic law of the wall (equation (4.21)) and the skin friction law given by Escudier et al. (equation (4.4)) is uncertain. Thus a special investigation into the measurement of skin friction was undertaken and is described in Chapter 8.

PART B

EXPERIMENTAL APPARATUS AND METHOD

Chapter 5

Experimental Apparatus, Measuring Equipment and Procedure

5.1 Apparatus

5.1.1 Equilibrium curved flows

Photographs of the apparatus, including the traversing gear and the end-plate 'ladder' arrangement used for containing secondary flows, are shown in figures 5.1a and 5.1b, and a schematic is presented in figure 5.2. The experimental arrangement consisted essentially of a centrifugal blower (Sturtevant type 20062, 20 HP at 3500 rpm; operated well away from the flat region of its characteristic curve) which supplied air via a flexible rubber hose to an expansion chamber followed by a contraction leading to a slot whose width could be varied.

The intake to the blower was fitted with a filter box to minimize the quantity of dust carried by the jet (necessary for hot-wire work). The flow was made variable by a bleed valve, which was followed downstream by a heat exchanger to control the air temperature.

The expansion chamber consisted of a 40° expansion fitted with two perforated plates to prevent separation and two turning vanes upstream of the first plate to ensure equal distribution of the entry flow. The diffuser was terminated by two gauze type screens fitted into a frame which could be easily

removed to facilitate access. Following this a section of honeycomb was installed ($\frac{1}{4}$ inch cell size and 2 inches deep).

The lower portion (the term 'lower' will be used to refer to the side of the contraction abutting the spiral surface) of the contraction could be moved and was terminated by a soft flexible rubber strip which rested against a contoured fairing attached to the flat plate beginning the spiral (see below). With this arrangement it was possible to vary both the slot width (0 to $1\frac{1}{2}$ ") and its position relative to the spiral. The latter adjustment was always necessary when adjusting the virtual origin of the flow to make it coincide with the origin of the spiral.

The upper wall and lip, shown in figure 5.2, are the end product of numerous alterations caused by the considerable trouble encountered in realizing two-dimensionality. (The principal difficulty was associated with slight irregularities in the slot lip and this is discussed fully in Chapter 6). The upper wall, as shown, finally consisted of a very flat steel plate attached to the wooden backing and abutting the honeycomb at one end and resting on the slot lip, to form a small step, at the other. The slot lip was a massive mild steel block carefully heat treated to relieve internal stresses, then ground with great care and finally rubbed with gun bluing compound to especially prevent corrosion of the sharp edge. The accuracy of the slot was not checked for fear of damaging

the sharp edge but experience with similarly ground stock suggests that variation was less than .0005".

Two convex logarithmic spirals were constructed and, to facilitate the comparison of results, their shapes were purposely chosen identical to two of the spirals used by Giles et al. A logarithmic spiral is a curve whose local radius of curvature is proportional to the distance from the origin and the spirals chosen had equations

$$\frac{r}{R} = \frac{2}{3}$$

and

$$\frac{r}{R} = 1$$

In polar coordinates these curves can be written

$$r = e^{\frac{2}{3}\theta}$$

and

$$r = e^{\theta}$$

respectively.

Each surface was 24" wide. The first ($\frac{x}{R} = \frac{2}{3}$), built in the McGill Low Speed Aerodynamics Workshop, was constructed from four 1" thick plywood ribs, strengthened by a number of crosswise stringers and covered with a $\frac{1}{8}$ " sheet of plywood. The other surface was constructed at the Von Karman Institute for Fluid Dynamics, Bruxelles (Guitton, 1965 and see acknow-

ledgements) using a quite different technique. It was built up of thick mahogany strips glued together and carefully shaped to conform to the specified contour. Both surfaces were repeatedly sanded and painted until they were mirror-smooth.

Static pressure taps were provided by placing lengths of .015" ID brass tubing through the surfaces, gluing them with epoxy, and then carefully sanding them flush. The position of the static holes are given by the experimental points in figures 9.13 and 9.14.

The floor of the slot was made from $\frac{1}{2}$ " thick, ground mild steel plate bolted and glued on one side to the wooden spiral and prolonged on the other by the wooden contoured fairing mentioned above (see also figure 5.16). The location of the plate on the spiral was chosen such that when the spiral was positioned, with the plate parallel to the 'upper' slot lip, the furthest downstream end of the spiral surface would leave about a one foot clearance with the 'lower' part of the diffuser. This precaution facilitated access to both the static taps protruding inside and the surface mountings required for holding the hot film probe used in measuring skin-friction. The length of the metal plate was determined by the requirement that the virtual origin of the jet should coincide with the origin of the surface and after numerous tests (see section 5.3) the following dimensions were adopted.

Spiral $\frac{\Delta}{R}$	Distance between Downstream end of Plate and Origin of Surface (inches)	Length of plate (inches)
$\frac{2}{3}$	.85	3
1	2.26	9

The sides of the plenum chamber were prolonged outwards to form large end plates (figures 5.1 and 5.2) held apart by steel rods placed at the furthest extremes. The spirals could be just glided between the plates and firmly bolted in position. A fairing was fitted around the end plates to prevent possible separation of the entrained air flow. Some control over the effects of secondary flows was provided by some additional end plates shown in figures 5.1a and 5.2 and discussed in Chapter 6.

The flow was probed using the traversing gear shown in figure 5.1a, which consisted of a Baty dial gauge, with a finely controllable travel of $2\frac{1}{2}$ " (smallest division - .001"), attached to a sliding bar permitting an additional rough travel of 8". The whole unit could be moved vertically to permit flow traverses at any spanwise position. On the centre-line, where the final traverses were made, all probing devices - hot wires or pitot tubes - were fixed to a long $\frac{3}{16}$ " diameter rod attached to the dial gauge at one end and passing through a guide bushing, installed in the surface, at the other end. This technique effectively prevented flow induced vibrations of the measuring probe. The guide rod was positioned perpendicular to the surface tangent at the measuring station and was sufficiently far down-

stream of the station ($2\frac{1}{2}$ ") to effectively eliminate interference. (This was checked by comparing the measurements to some made using a $\frac{1}{2}$ " rod.)

The stagnation pressure within the plenum chamber was measured by a pilot tube placed at the exit of the jet. The air temperature could be periodically monitored by a telethermometer, the sensing element of which was inserted through a hole in a side wall of the plenum chamber.

The experimental procedure and the measuring equipment are discussed in subsequent sections.

5.1.2 Circular cylinder - convex

This apparatus was designed, first used, and fully described by Fekete (1963) and its details will not be discussed here. Some changes, however, were necessary to improve two-dimensionality and these will be described below. Schematics of the apparatus are shown in figures 5.3 and 5.4.

The surface consisted of a very accurately machined, cadmium plated, steel cylinder having an O.D. of 8". Air was supplied to an adjustable slot via a plenum chamber situated within the cylinder. (The air supply and control were identical to that used in the equilibrium flow experiments.) This configuration, as evidenced in figure 5.4, eliminated almost completely, any interference with the curved jet.

The slot width could be varied, using a shim (figure 5.4), within the range .03 inches to .212 inch, which corresponded to

an aspect ratio variation of approximately 1200 to 120 respectively.

The modifications made by the author to the original apparatus consisted of (see figure 5.4): the vinyl sheet installed to block a major portion of the perforations within the plenum chamber; the addition of an extra perforated plate and a honeycomb; and two sheet metal fairings forming a new contraction section. These changes, forced by poor two-dimensionality obtained with the original apparatus (see next section), were suggested by solutions to similar difficulties experienced on the logarithmic spirals (Chapter 6). Indeed, a small variation in the thickness of the boundary layer on the upper slot lip was known to influence the downstream flow and the object of the changes was to eliminate any possible interference with the flow as it emerged from the plenum chamber. To further assure a high-quality flow, the slot lip was also reground.

The traversing mechanism was identical in principal to that described in the previous section save that no guide rod transfixing the surface was used. Probe vibrations were no problem, however, because for most of the measurements the jet was thin and the cantilevered probe holder could be kept short. In this apparatus the traversing gear remained fixed and a change of station was effected by rotating the cylinder.

The experimental procedure and the measuring equipment are discussed in subsequent sections.

5.1.3 The Plane Wall Jet in Still Air

The apparatus and experimental conditions were identical to those described by Gartshore and Hawaleshka (1964). The slot Reynolds number was set at $Re = 3 \times 10^4$ for a nominal slot width of .300 inches. The experimental procedure and instrumentation were identical to that used for measuring the curved flows and this is described in the next section.

The slot width was not uniform but varied by 2% due to distortion caused by the later addition of hold-down screws. Nevertheless, this did not prove a hardship because the plane flow was not as sensitive as the curved flow to initial irregularities at the slot. The two-dimensionality was measured in a number of different ways; (a) figures 5.6 and 5.7 show pitot tube traverses taken at three transverse positions - the centre-line and six inches to either side - for two downstream positions. For $x = 40\frac{5}{8}$ " corresponding to $\frac{x}{b} = 136$ the total pressures are lower on the centre-line by approximately 4.4% and velocities are therefore lower by about 2%; (b) figure 5.8 shows a shear stress traverse taken at the centre-line and six inches to either side. The variation in maximum shear is 4%; (c) figure 5.9 shows Preston tube readings taken at the farthest possible downstream station ($\frac{x}{b} \approx 200$). The whole shear seems relatively constant two inches to either side of the centre-line and the variation over 12" is $\pm 5\%$ in Δp (where $\Delta p = p_o - p_w$) which is equivalent to about $\pm 2\frac{1}{2}\%$ in U_T .

The above variations in two-dimensionality were judged acceptable because the growth of the jet ($y_{m/2}' = .071$ see section 1.2.1) was close to that found by Gartshore and Hawaleshka and that it also agreed well with a linear relationship between $y_{m/2}'$ and Re obtained by Kohan (1968) from a survey of much of the experimental work to date. Further discussion is given in Chapter 9.

5.1.4 Calibration rig

Calibration of the wire was performed in a specially constructed apparatus (figure 5.1b) consisting of a large steel drum - acting as a plenum chamber - fitted internally with some screens to smooth the flow and provided with a 1 inch diameter contoured outlet nozzle. The quantity of air into the drum was varied by a hand adjusted sluice valve.

A hot wire probe could be positioned above the nozzle, by a holder which permitted a 2 degree of freedom movement in the plane of the nozzle and with this arrangement the axis of the probe could be adjusted to lie parallel to the oncoming velocity vector. This adjustment was especially important when calibrating slanted wires and for these cases the adjustment was performed by repeatedly rotating the wire 180° until the anemometer output became invariant.

The temperature of the air flow was controlled by a heat exchanger.

5.2 Measuring Equipment: Hot Wire and Associated Equipment

All mean velocity and turbulence measurements were made with either Disa type 55A25 normal wires* or type 55A29 single slanted wires* used in conjunction with a type 55A01 anemometer. The signal from the anemometer was fed to a linearizer (type 55D10). A schematic of the complete instrument arrangement is shown in figure 5.5.

The linearizer output could be connected either directly to a voltage to frequency (VF) converter or to an RMS meter having a DC output linearly related to the measured RMS signal. The output of the VF converter - whether it be representative of the mean or fluctuating signal - was fed to a digital counter which integrated the input signal over a time period determined by the setting of an attached audio oscillator. Finally, after integration, the output of the counter was printed. In order to guard against possible clipping of the linearizer signal which could be caused by an accidental overshoot of its range, the output of the linearizer was continuously visually monitored on an oscilloscope.

The pressures within all plenum chambers were measured with a mercury manometer. Surface pressures were recorded from a Lambrecht alcohol filled manometer having an adjustable inclination.

* These are platinum plated tungsten wires of diameter 5μ and length approximately 1 mm.

Other more specialized instrumentation was used in studying the effects of high intensity turbulence or in measuring skin friction and a discussion of that equipment is reserved for the particular chapters devoted to these investigations (Chapters 7 and 8 respectively).

5.3 Experimental Procedure

At the onset of each experimental investigation general flow visualization studies as well as a number of exploratory total pressure traverses were made. It was in the course of these preliminary studies that serious difficulties were encountered in obtaining two-dimensionality in the curved flows. The sheer magnitude of the possible irregularities suggested that it would be worthwhile to discuss in some detail the experience encountered in tracing their origins and in finally realizing a more acceptable flow. This is done in Chapter 6.

After satisfactory two-dimensionality was obtained it was necessary, in the equilibrium flow experiments, to match the origin of the flow to that of the spiral and for this a trial and error procedure was adopted. The slot was set at an arbitrary position relative to the flat bottom plate. Velocity traverses were obtained at three or four downstream stations by using a total head tube and assuming a linear static pressure distribution between the wall pressure and the ambient condition at the edge of the jet assumed to be located at $\frac{y}{y_m/2} = 2.0$. This technique was much simpler than using hot wires and it had

already proved useful in the concave flow experiments (Guitton, 1964) and its accuracy was judged sufficient for the adjustments being attempted (see also section 9.2.4). The jet's growth obtained from the profiles was then extrapolated to zero, the origins compared, and the procedure repeated until matching occurred. With a slot width of .125 inch on the $\frac{x}{R} = \frac{2}{3}$ spiral, a 3 inch lower plate permitted sufficient movement to match the origins. For the $\frac{x}{R} = 1$ spiral and the same slot width, it was found necessary, in order to finally achieve coincidence of origins, to extend the plate to 9 inches and this was similar to the experience reported by Giles et al. The final position of each spiral was double checked by taking hot wire traverses and ascertaining that the jet growth extrapolated to $x = 0$. The slot Reynolds number for both surfaces was maintained at $Re = 2.8 \times 10^4$ which is close to the value used in the plane wall jet experiments. On the convex cylinder the same two-dimensionality checks were used and large lateral variations in the jet thickness were found. The cause of the difficulty was suspected (on the basis of experience described in section 6.2) and accordingly the modifications described in 5.1.2 were undertaken. The difficulties encountered do not, however, justify skepticism about the quality of Fekete's original flow, because inspection of the slot lip revealed that some corrosion had taken place in the time interval (1963-1969) during which the apparatus lay idle. (Indeed, the experiments obtained in this work described in Chapter 10 are in good agreement with Fekete's results.)

Acceptable two-dimensionality on the convex cylinder was possible only for the smallest slot-width; $\frac{b}{R} = .0074$, and this was adopted although it was realized that the value was just below the lower limit ($\frac{b}{R} = .01$) suggested by the experiments of Fekete (see Introduction 1.1) for a universal flow. Recalling that universality fails at such small slot widths because of large skin friction effects attributable to a low boundary layer Reynolds number, it seemed probable that the flow could be made universal by simply increasing the slot Reynolds number beyond that used by Fekete. Accordingly, the maximum possible value was used, $Re = 7.0 \times 10^3$ but this was less than twice that given by Fekete and the experiments did not compare favourably with his universal flow although they did agree with his small $\frac{b}{R}$ results (discussed in Chapter 10).

The procedure for obtaining hot wire traverses was identical for all surfaces. To begin, it was necessary to position the wire at a known distance from the surface and this was accomplished by the electrical contact method (Dodds, 1960). One terminal of an oscilloscope was attached to a gauge block placed squarely on the surface just over the station of interest, and the other terminal was attached to the stem of the hot wire probe. By turning the dial gauge the probe was slowly moved towards the block and when it just touched, the extraneous electrical pickup on the screen immediately disappeared. This procedure was followed for both normal and slanted wires. The distance between the wire axis of the normal wire or the mid-point of the slanted wire, and the gauge block was determined by obtaining, on an optical

comparator, the dimensions of the probe.

As noted previously each voltage was averaged over a convenient time period. This was chosen so that the standard deviation of the voltages at $y = y_{m/2}$ was less than 1% of the mean quantity and at the furthest downstream station it was usually necessary to use one minute or longer integration times.

The experiments were performed at night - usually between 9 p.m. and 6 a.m. - and ambient conditions were usually found to be particularly stable. (The maximum ambient temperature variation encountered was about 1°F.) All traverses necessary to obtain mean velocity and Reynolds stresses at one station were completed before moving on to the next. In this way the error in the structure parameter $\overline{u'v'}/\overline{q'^2}$, due to a slight drift in the conditions, was minimized.

Although the main air supply was filtered, it soon mixed with ambient air and consequently, at some distance downstream, the flow contained enough dust particles to require cleaning of the wire after each traverse and this was accomplished in an ultrasonic cleaner. Before each new traverse the wire was again calibrated. A measure of the drift occurring during a particular traverse was obtained by returning to the first measured point and, if the change was greater than ^{about} 1%, the traverse was repeated.

If a slight change in average local temperature occurred it was compensated for in the following way: Koch and Gartshore

(1969) showed that the calibration of a Disa hot wire (type 55A25) over a wide range of wire temperatures and a narrow range of ambient temperatures can be correlated by the expression

$$\frac{E_f^2}{k R_f l (T_f - T_1)} \left(\frac{R_f}{R_c} \right)^{0.4} = b_1 + b_2 \left(\frac{U l}{\nu} \right)^{0.45}$$

where E_f is the voltage across a wire having a resistance R_f , an average temperature T_f and R_c is its cold resistance at temperature T_1 . On a Disa type 55A01 anemometer the relation between wire voltage and bridge voltage is nearly

$$E_f = E \left(\frac{R_f}{100 + R_f} \right)$$

For small changes in air temperature the fluid properties can be considered constant and noting also that

$$R_f = R_c [1 + \alpha_1 (T_f - T_1)]$$

then the calibration is written

$$\frac{R_f \left(\frac{R_f}{R_c} \right)^{0.4}}{\left(\frac{R_f}{R_c} - 1 \right)} E^2 = b_3 + b_4 U^{0.45}$$

and for an invariant E vs. U relation it is necessary that the parameter

$$\frac{R_f \left(\frac{R_f}{R_c} \right)^{0.4}}{\left(\frac{R_f}{R_c} - 1 \right)} = \text{constant}$$

which implies that for $\frac{R_f}{R_c} = 1.8$.

$$\Delta R_f = 3.9 \Delta R_c$$

Whenever a change in R_c occurred then ΔR_f was calculated from the above quantities and added to R_f and with this procedure

the error in V was minimized.

An idea of the possible measurement accuracy can be obtained from the work of R.P. Patel (1968) who, using some of the above techniques, obtained consistent agreement within 2-3% with the calculated shear stress distribution in a circular pipe (if longitudinal cooling corrections are included). Some slanted and normal wire traverses were also attempted in the pipe by this author and the Reynolds stresses obtained were in excellent agreement with those given by Patel.

In a curved wall jet the turbulence intensity is very high (e.g. $\frac{\sqrt{u'^2}}{U} \sim .5$ at $y = y_{m/2}$) and its effect on hot wire readings was largely unknown at the time the author began the present curved flow investigation. Furthermore, the difficulties with two-dimensionality emphasized that agreement between the calculated and measured shear stress distribution was a reliable criterion for ascertaining the flow quality. It also transpired that preliminary measurements of the shear stress were in significant disagreement ($\approx 50\%$) with the calculated values and it was therefore decided to investigate the effects of high intensity turbulence on the response of a hot wire. This work is fully discussed in Chapter 7.

It is also known (Champagne et al., 1967) that the readings of a wire, of finite aspect ratio, inclined to the velocity vector, must be corrected for an additional cooling effect (other than the normal velocity component) ascribed to

the longitudinal component of velocity. This effect is also included in the analysis of Chapter 7.

The following summarizes the precautions taken to obtain accurate hot wire measurements.

- (a) the distance from the surface was very accurately obtained
- (b) a calibration was performed prior to every traverse
- (c) the drift occurring during a traverse was verified by always returning to the first measured point.
- (d) all voltages were averaged over long time periods
- (e) local temperature changes were accounted for by changing the wire's 'hot' resistance to remain on the same calibration curve.
- (f) at a particular station all normal and slanted wire traverses were made during the same run
- (g) a number of measurements were made in a pipe flow where, in particular, the shear stress can be easily calculated for comparison purposes
- (h) corrections for longitudinal cooling and high intensity turbulence were applied.

Further discussion of the accuracy of the measurements taken on the log spirals and convex cylinder is given in Chapter 9

and 10 respectively.

Other parameters measured on the convex curved surfaces were skin friction and separation (only on the cylinder). The latter was detected with a tuft made of down and could be ascertained to within $\pm 3^\circ$. The former parameter is often measured with a Preston tube or other technique which requires the existence of a universal law of the wall. In the curved wall jet the outer region influences strongly the inner layer and the shear stress gradient is very large even in a zero pressure gradient. Consequently, it was not known to what extent Preston tube readings (whose accuracy depend on the existence of a universal law of the wall in the fully turbulent region) could be trusted, and a comparative investigation based on the uses of hot film flush elements was undertaken and is described in Chapter 8.

(no page 83)

Chapter 6

Two-Dimensionality: 'Establishment' and Verification

The difficulties met in obtaining two-dimensionality can hardly be over-emphasized and the purpose of this chapter is to describe the principal pitfalls encountered and the remedies employed to cure them.

The most common method of verifying two-dimensionality is to compare velocity profiles (and static pressure if necessary) measured at various stations situated in a plane normal to the principal flow direction and deciding, more or less arbitrarily, whether the variation is acceptable. Another approach, and one that gives more quantitative results, is to verify that the velocity and length scales that describe the flow vary downstream in a manner consistent with the 2-D momentum equation. However, evidence shows that the most sensitive technique is to compare calculated and measured shear stress distributions.

The latter point can be demonstrated for the curved wall jets in still air, by determining the effect of a small flow divergence on the exponent 'a' (of $U_m \propto x^a$) and thus on the shear stress distribution. The effect of divergence on a thin curved jet can be roughly estimated by using the integrated plane flow x-momentum equation

$$\frac{\partial}{\partial x} \int_0^{\infty} u^2 dy = - \frac{\tau_w}{\rho} - \int_0^{\infty} u \frac{\partial w}{\partial x} dy \quad (6.1)$$

where it has been assumed that

$$\omega \frac{\partial u}{\partial x} \ll u \frac{\partial \omega}{\partial z}$$

For a source type flow with origin at $x = -x_0$

$$\omega = \frac{u z}{x_1 + x_0} \quad (6.2)$$

and equation (6.1) becomes

$$\begin{aligned} \frac{\partial}{\partial x} \int_0^\infty u^2 dy &= - \frac{\tau_w}{f} - \int_0^\infty \frac{u^2}{x_1 + x_0} dy \\ &= - \frac{\tau_{w \text{ calc.}}}{f} \quad \text{say} \end{aligned} \quad (6.3)$$

Assuming similarity profiles yields

$$\frac{\tau_{w \text{ calc.}}}{f U_m^2} = \frac{\tau_w}{f U_m^2} + I_2 \frac{y_{m/2}}{x_1 + x_0} \quad (6.4)$$

and assuming that a 24 inch span of the jet has an included angle of divergence of 2° , then

$$x_1 + x_0 = (12) \left(\frac{180}{\pi} \right) \text{ inches}$$

and

$$\frac{\tau_{w \text{ calc.}}}{f U_m^2} = \frac{\tau_w}{f U_m^2} + 0.0011 y_{m/2}$$

where $I_2 = \int_0^\infty \left(\frac{u}{U_m} \right)^2 d \left(\frac{y}{y_{m/2}} \right) = .76$. On the $\frac{x}{R} = 1$ spiral, measurements gave $\frac{y_{m/2}}{x} = .272$ and at $x = 20$ inches, which is in the middle of the experimental range, equation (6.5) gives

$$\frac{\tau_{w \text{ calc.}}}{\rho U_m^2} = \frac{\tau_w}{\rho U_m^2} + 0.0066$$

The additional term is nearly $3 \frac{\tau_w}{\rho U_m^2}$ and therefore for a 2° divergence

$$\frac{\tau_{w \text{ calc.}}}{\rho U_m^2} \approx 4 \frac{\tau_w}{\rho U_m^2}$$

Using the curved flow equations (2.8) to (2.10) and the known growth rate, it is easily shown that the effective increase in $\frac{\tau_w}{\rho U_m^2}$ accounts for about a 4% change in 'a' and a 20% change in the calculated shear at $y_m/2$. Furthermore, if the velocity decay is given by

$$\frac{U_m}{U_{m_r}} = \left(\frac{x}{x_r} \right)^a$$

then at $\frac{x}{x_r} = 5$ (which corresponds to $\frac{x}{b} \sim 150$ if $\frac{x_r}{b} \sim 30$) there is an error of only 2% in $\frac{U_m}{U_{m_r}}$. Comparing calculated and measured shear stress distributions for a given growth rate, is therefore a much more sensitive technique than comparing the variation of the velocity or length scales.

Ideally, the three methods described earlier are applied successively; a very poor quality flow is first improved by comparing traverses; further refinements are then made using the momentum equation as reference; and finally, if necessary, an attempt is made to reduce the discrepancy between calculated

and measured shear.*

When verifying the flow on all surfaces an effort was made to follow the above procedure. On the least curved spiral surface ($\frac{x}{R} = \frac{2}{3}$) the value of $\frac{U_m}{U_{mr}}$, calculated via the momentum equation, was within 2% of the experimental value (see Chapter 9) but the maximum measured shear stress was still 25% lower than the respective calculated quantity. (This difficulty could not be resolved and the flow was finally measured as such.) On the $\frac{x}{R} = 1$ spiral a less satisfactory momentum balance was achieved but the discrepancy in shear was about the same as for the previous surface.

The difficulties with two-dimensionality originate from two sources: the one is related to conditions at the slot lip or within the plenum chamber leading to the slot; and the other is due to secondary flows originating in the boundary layers forming on the end-plates. These effects will now be described individually and, since they are ^{similar} ~~small~~ for all surfaces, attention is focussed on only one; the $\frac{x}{R} = \frac{2}{3}$ spiral.

The traverses, the momentum balance, and the shear stress distribution, specifying the quality of the flow on each surface, are presented with other experimental results in Chapters 9 and 10.

* This, of course, implies that the measurements of shear are accurate and, in view of the considerable discrepancies between the measurements and calculation, a considerable effort was made to investigate possible sources of error (Chapter 7)

6.1 Slot Lip Effect

Fekete (1963) in a preliminary investigation preceding his adoption of a much improved apparatus discovered that marked deviations from two-dimensionality, as measured by large lateral variations in the jet thickness, were caused by slight irregularities in both the wall leading to the slot lip and the edge of the lip itself. He noted that "even a small imperfection of the slot lip will have a detrimental effect on two-dimensionality due to Coanda effect".

The essence of the difficulty can be appreciated by considering the ratio of the radius of the edge of the slot lip to the slot boundary layer thickness. As this parameter increases there is a greater tendency for the jet emanating from the plenum chamber to curl around the lip due to the Coanda effect and this necessarily causes a locally thicker flow. On a convex surface, analysis reveals that this thick region may tend to grow compared to an adjacent thin region because the surface pressure under the former is lower and consequently creates a centre of inflow.

The apparatus used for studying the equilibrium flows was designed with this hazard in mind. The slot lip was shaped like a knife edge and carefully ground sharp and the wall leading to it was straight and free of obstructions. Nevertheless, the initial measurements made on the $\frac{x}{R} = \frac{2}{3}$ spiral revealed a flow of very poor quality (figure 6.1).

In these measurements a pitot tube was moved parallel to, and at a distance y from the surface and the total head was recorded every inch on either side of the centre-line. (It should be noted that comparing traverses taken on the centre line and 6" to either side would have revealed little discrepancy.) In figure 6.1 the readings taken three and five inches below the centre line differ significantly from each other and traverses taken at these positions, and shown in figure 6.2, confirm the discrepancy at all values of y . (Figure 6.3 shows the significantly improved results obtained after the tests and changes described below.)

The results of figure 6.1 were quite unexpected and the initial reaction was to suspect early separation within the diffuser. Consequently turning vanes and additional screens were installed (figure 5.2) but these were of no help. To further verify that irregularities in the flow upstream of the honeycomb did not cause the difficulty, lengths of tape were strung across the honeycomb to partially block the flow but again no significant changes in the jet could be detected.

The possibility that leaks or an unevenness in the $\frac{1}{32}$ in. thick rubber seal (Chapter 5) should be the cause was next investigated. A piece of corrugated cardboard was placed along the "bottom" of the plenum chamber, extending from the honeycomb, over the wooden contraction piece and covering the rubber seal. Again no change in the jet thickness was detected although there was some effect on Preston tube readings taken just downstream of the slot.

Attention was now focussed on conditions at the upper wall although, as previously stated, the care taken to assure a very sharp lip and smooth entry made this area an unlikely suspect. A length of masking tape (a few "thou." thick) was first placed over the smooth joint between the metal lip and the wooden plenum chamber and this had a remarkable effect (figure 6.1; dotted line) which reinforced Pekete's observations. Various changes were now attempted in an effort to create a thicker boundary layer on the upper wall. The metal lip was first shimmed to form a step projecting .030" beyond the wood but this was unsuccessful. So also was an attempt to increase the boundary layer thickness by using large rods placed flat on the 'upper' wall and perpendicular to the flow direction.

The last possibility, the edge of the slot, was now considered. The knife edge was honed flat using a hand-held grind stone, and the sharp 90° edge formed resulted in an immediate improvement in the flow. It appeared therefore that a ground 90° edge is more likely to form a uniformly sharp lip than a knife edge, and this could be due to difficulties in avoiding small burrs in the latter configuration. Accordingly, a completely new rectangular slot lip was designed (figure 5.2) and to improve the quality of the flow leading to it a piece of flat steel plate was placed over the wooden wall, its upstream edge abutting the honeycomb and its downstream edge resting on the lip, and forming a step to it.

The resulting flow at $\frac{x}{b} \approx 125$ is shown in figure 6.3. There is only a small variation within the central 8". Figure 6.7 shows the lateral total pressure variation at $\frac{x}{b} \approx 260$ and though the variation is still not excessive it is barely satisfactory and the comparison of figures 6.3 and 6.7 corroborates the prediction that, in convex flow, a thick region grows relative to an adjacent thin one. No further improvements in the slot region could be conceived and attention was now focussed on the influence of secondary flows.

6.2 Secondary Flow

In the course of the work described in the previous section it was observed that a marked build-up of flow was taking place along the side walls and that in this region the jet grew much more rapidly than it did along the centre line. Somewhat surprisingly this secondary flow differs from that which occurs in a curved channel. In the channel the pressure gradient imposed on the boundary layers formed on the two plane side walls is balanced by an increased curvature of the slower moving fluid, and consequently the flow along these walls has an additional component of velocity directed towards the centre of curvature of the channel.

Figure 6.4 shows schematically some flow directions detected, on the $\frac{x}{R} = \frac{2}{3}$ spiral, with smoke puffs and tufts made of down. Near the end plates the surface flow on the spiral is directed inwards (towards the centre line), indica-

ting that the static pressure is higher near the corner. Not far above the curved surface (at approximately y_m in the case studied) the secondary flow is directed outwards, and at the end plate it appears to split into two parts - one completing the vortex flow in the inner layer ($y < y_m$) and the other contributing to the rapid expansion of the boundary flow.

The principal objective was to eliminate the secondary flow or at least to render it negligible, and consequently no detailed experiments were made to determine its exact form. Nevertheless, a few observations are shown in figures 6.5 and 6.6. In the former figure there is shown a comparison of a total pressure traverse taken at the centre line, with a number of traverses made very near the end-plate.

Two static pressure traverses^{were} made at the centre line and at $\frac{1}{2}$ " from the end plate respectively. In these measurements an attempt was made to align the static tube with the mean flow direction as indicated by a tuft, and the error is probably identical for the two traverses. The static tube also agreed well with a surface static hole. The relative magnitude of the static pressure readings is consistent with the flow directions shown in figure 6.4; below y_m the surface pressure near the end plate is higher than at the centre and the secondary flow is inwards; and the opposite is true for $y > y_m$. Figure 6.6 shows various slices, made at constant y , through the traverses of figure 6.5. (Although the measurements are not taken at the same value of $\frac{x}{b}$ figure 6.5 can be

imagined as a qualitative extension to figure 6.3.)

The rapid expansion of the side flow soon created a U-shaped wall jet (figure 6.4) in which the interference between the side flow and the centre portion could no longer be disregarded. Consequently, a number of techniques were attempted to eliminate this 'runaway' expansion. The method finally adopted consisted of a number of end plates arranged in a ladder formation so that each succeeding plate cut off the boundary layer formed immediately upstream of it (figure 5.1a). An attempt was made to limit the number of plates in order to minimize blockage and brief trials indicated that a 'ladder' consisting of three end plates would be satisfactory. The position of each plate was determined by trial and error. With this method no U-shaped formation could be detected. Figure 6.7 shows the effect of the end-plate arrangement on the lateral total pressure variation at $\frac{x}{b} \approx 260$ for the $\frac{x}{R} = \frac{2}{3}$ spiral. The effect at the centre line is negligible and it must be concluded that for this surface the secondary flows did not influence the centre region even at this extreme downstream distance. However, on the $\frac{x}{R} = 1$ spiral (shown in figure 5.1a) the 'ladder' end plates were necessary and caused a 7% change in $y_{m/2}$ at $\frac{x}{b} = 284$.

PART C

DEVELOPMENT OF SPECIAL EXPERIMENTAL TECHNIQUES

(see Introduction)

Chapter 7

The Effect of High Intensity Turbulence on the Response of a Hot Wire

7.1 Introduction

The usual approach to the calculation of a hot wire response is to assume small intensity with respect to the local mean velocity and thus to linearize the heat balance equation. While this may be an acceptable procedure for certain flows such as turbulence pipe flow, in which the obtainable accuracy is better than 2% (Patel, 1968), the perturbation approach becomes suspect when applied to measurements made in free shear flows such as jets where the intensity level is high. This fact was noted by Corrsin and Uberoi (1949) in their experiments with heated turbulent free jets. The table below gives some typical longitudinal turbulent intensities found in various jet flows at the point, $y_{\pi/2}$, where the velocity is half the maximum. None can be considered small. (In the table and what follows (except for section 4.1.2), the coordinate x is parallel to either the wall or the line of symmetry of the flow, depending on the case and $U + u'$, $V + v'$ and $W + w'$ are instantaneous velocity components in the x , y and z direction respectively.)

Flow (Experimenter)	$\frac{\sqrt{u'^2}}{u}$ at $y = y_m/2$
Plane free jet in still air (Heskestad, 1965)	51%
Swirling axi-symmetric free jet (Corrsin and Uberoi, 1949)	48%(linearized)
Plane Wall Jet (Patel, 1962)	40%
Wall Jet on a Convex Cylinder (Fekete, 1963)	43%
(Present results)	46%(linearized)

Before discussing more fully the effects of high intensity turbulence it is of interest to consider two other sources of measurement error; the effects of prong interference and of longitudinal cooling.

The former phenomenon is associated with blockage of the flow due to the presence of the stem and prongs and can arise when a wire is being used in an orientation other than the one for which it was originally calibrated. Such might be the case in high intensity turbulence where the instantaneous flow angles are large. It transpires, however, that modern probe design has largely eliminated this error. (Some measurements reported in this text were made with older type probes - the commercial Disa 55A25 and 55A29 miniature normal and slanted wire probes - which do suffer from interference effects (Gilmore, 1967). But more extensive calculations reported in a lengthier version of this work (Guitton, 1968) revealed that, for these sensors, the interference effect can be neglected when calculating high

intensity turbulence corrections.)

The longitudinal cooling effect is not peculiar to high intensity turbulence but is associated with additional heat convection effects when a wire is yawed to the mean flow.* The experiments of Champagne (1965), (also Champagne et al., 1967) for yaw angle $< 55^\circ$, indicate that the deviation from normal component cooling is not due to altered end losses but is primarily due to additional heat convection attributable to the tangential velocity component. He confirms an empirical relation, identical to an earlier equation suggested by Hinze (1959) for an effective cooling velocity:

$$\begin{aligned} U_e^2 &= U^2 (\sin^2 A + k^2 \cos^2 A) \\ &= U_N^2 + k^2 U_p^2 \end{aligned} \quad (7.1)$$

which is the velocity indicated by a wire whose axis is inclined at an angle A to a velocity U . Champagne concludes that the factor k is primarily a function of the aspect ratio of the wire and is equal to 0.2 for $\frac{l}{d} = 200$ and tends to zero for

* In practice this difficulty may be surmounted without determining a quantitative relationship for the added heat loss, nor indeed the wire angle, by experimentally determining the directional sensitivity of the wire. This procedure requires very precise knowledge of differences between yaw angles, and graphical differentiation, and is therefore subject to error. In this work the other approach is used whereby the fluctuating response of the wire is calculated from a known response equation which incorporates longitudinal cooling effects, blockage effects, and a measured wire angle.

$\frac{L}{d} > 600$. The value $k = .2$ for Disa miniature probes (Catalogue No. 55A29, $\frac{L}{d} = 200$) has been corroborated, for wire Reynolds number $5 < Re < 14$, by Gilmore (1967) by calibrating a normal wire at various yaw angles in a wind tunnel, and by Patel (1968) from careful shear measurements in a pipe. The neglect of longitudinal cooling effects can lead to significant errors (e.g. 0 [10%] in $\overline{u'v'}$) in slanted wire measurements and for high intensities, second order longitudinal cooling corrections may become necessary.

Using a perturbation theory to analyse hot wire signals obtained in a high intensity flow leads to two types of error:

- a) it produces an error associated with the non-linearity of the response equation
- b) it produces an error associated with large changes of flow direction with respect to the wire.

The first error arising from magnitude fluctuations can be conveniently removed by linearizing the signal to

$$E \propto U$$

and this type of response will be assumed throughout this work. (This is conveniently achieved using a Disa linearizer.)

A simple example of the second source of error, that due to directional effects, can be obtained by postulating that

the wire is cooled by a simple turbulent flow having velocity components $U + u'$ and v' . The wire will be insensitive to the direction of v' so that the same average cooling is obtained whether v' is positive or negative. Hence for this particular case, the average cooling sensed is greater than the cooling sought; viz., that due to U alone.

7.2 Review of Analytical Methods used to Correct Signals

Several analytical methods have been presented to correct the small disturbance approach. None consider the case of a wire oriented such that both the mean U and V components are present. Such is the case for measurements done in rapidly expanding shear layers such as wall jets where it is usually most convenient to keep the probe at a fixed orientation with respect to a given datum (e.g. parallel to the wall in a wall jet traverse).

Rose (1962) has considered high intensity corrections to the linearized constant temperature response but he limits the correction terms to third order correlations (e.g. $\overline{u'v'^2}$). Heskestad (1963, 1965) corrects this by including fourth order correlations (e.g. $\overline{u'^4}$), but the resulting corrections to the Reynolds stresses are not given analytically. Furthermore, by using Heskestad's experimental data, it was not possible to reproduce his plotted correction coefficients. Champagne and Sleicher (1967) have determined the instantaneous voltage response to third order correlations and to $O[k^4]$ in longitudinal cooling effects, but like Heskestad, they have not

carried the work to its logical conclusion which is to provide analytic corrections to the mean velocity and Reynolds stresses. Sandborn (1967) discusses the problem of high intensity and presents some experimental data that indicates errors of less than 2% in nonlinear normal wire measurements of $\sqrt{u'^2}$ in intensities up to $\frac{\sqrt{u'^2}}{U} = .50$. No physical reason is offered for this unexpected behaviour which, in this author's opinion, could rest on the complicated interaction of magnitude and directional effects. Sandborn does not discuss either the effect of a three-dimensional turbulence field on the normal wire response, or the behaviour of a slanted wire. Wichner and Peebles (1963) suggest a method of determining the Reynolds stresses which requires an accurate knowledge of the mean velocity.

The purpose of this chapter is to provide a means of correcting mean velocity and Reynolds stresses which are being measured in a two-dimensional mean flow of high turbulence intensity by means of a linearized constant-temperature hot wire anemometer. The wire response is taken to fourth order in the velocity fluctuations and includes corrections due to deviations from normal cooling. Equations are given which relate the mean and fluctuating voltages of a single wire to the mean and fluctuating velocity components of the flow for particular orientations of the wire. When a number of these equations are solved simultaneously it is possible to express the desired Reynolds stresses or mean velocities

in terms of the measured voltages and some correction terms. To evaluate these terms it is necessary to measure both triple and fourth order turbulence correlations. This has previously been done by Heskestad in a two-dimensional free jet, by Laufer (1953) in a pipe and for these two flows the correction terms are determined.

An important part of this chapter is the experimental portion which is concerned with the determination of the correction factors in other high intensity flows: two-plane and curved dimensional turbulent wall jets in still surroundings.

7.3 Calculation of Wire Response

7.3.1 General

The overall objective of this section is to present equations from which the mean velocities and Reynolds stresses in a specified high intensity turbulent flow may be determined. The flow considered is two-dimensional and has a single dominant flow direction. If U , V and W represent the three orthogonal mean velocities, this restriction can be stated as:

$$\begin{aligned} W &= 0 \\ \frac{V}{U} &= 0 (\epsilon) \text{ or less} \end{aligned} \quad (7.2)$$

where ϵ represents the typical order of magnitude of the turbulent intensity (e.g. $\sqrt{\frac{u'^2}{U}}$). In such a flow the unknowns of interest are the mean velocity components U and V and the turbulent Reynolds stresses $\overline{u'^2}$, $\overline{v'^2}$, $\overline{w'^2}$ and $\overline{u'v'}$ and it

follows that at least two mean and four fluctuating response equations are necessary. These equations are obtained by noting the signal from a wire placed at various orientations to the flow.

The calculation of the mean and fluctuating response equations is both lengthy and algebraically tedious and only an outline of the analysis will be presented here. For further details the reader is referred to the original report (Guitton, 1968).

7.3.2 Determination of effective normal component

A co-ordinate system is chosen coincident with the time dependent orthogonal velocity components, u , v and w as shown in figure 7.1. The wire is arbitrarily positioned as shown.

Equation (7.1) which links the measured effective velocity to the actual velocity by correcting for longitudinal cooling effects can be rewritten

$$q_e^2 = q_N^2 (1 + k^2 \cot^2 A) \quad (7.3)$$

where q_N is the velocity component normal to the wire, 'A' the instantaneous angle of yaw of the wire, and q_e the effective normal component due to longitudinal cooling. It is possible to express q_N and A in terms of the orthogonal velocity components and the wire orientation, and q_e can be written generally as

$$q_e = f(u, v, w, \theta, \alpha, k) \quad (7.4)$$

7.3.3 Voltage response of linearized system

For linearized operation the output voltage is proportional to the effective cooling velocity, U_e and at calibration conditions

$$E = \frac{U_{ce}}{C} \quad (7.5)$$

where C is a constant of proportionality obtained by calibration. This calibration is obtained with the wire positioned in a laminar flow or a turbulent flow of moderately low intensity ($\sim 10\%$ should be sufficient, R.P. Patel, 1968) with probe axis parallel to the calibration velocity U_c so that both the V and W components are equal to zero.

The voltage signal obtained in the test turbulent flow is composed of a mean fluctuating component produced by the mean and fluctuating components of q_e .

$$E + e' = \frac{\bar{q}_e}{C} + \frac{dq_e}{C}$$

The time dependent or fluctuating signal is given by

$$\frac{e'}{E} = \frac{dq_e}{\bar{q}_e} \quad (7.6)$$

and the mean signal by

$$E = \frac{\bar{q}_e}{C} = \frac{Q_e}{C} \quad (7.7)$$

The quantity dq_e can now be seen to represent the variation of the effective cooling component about an operating point whose voltage on the calibration curve corresponds to a calibration velocity U_{ce} . From equation (7.3)

$$U_{ce} = U_{cn} (1 + k^2 \cot^2 A_c)^{1/2} \quad (7.8)$$

where $\alpha_c = 0$ and A_c is obtained from figure 7.2 when $V = W = 0$. For these conditions

$$U_{cn} = U_c \sin \psi$$

Equation (7.6) is now written as

$$\frac{e'}{E} = \frac{dq_e}{U_{ce}} = \frac{dq_e}{U_{cn} (1 + k^2 \cot^2 A_c)^{1/2}} \quad (7.9)$$

It is possible to evaluate dq_e by carrying out a series expansion of equation (7.4) about the operating point and in terms of the velocity components u , v and w . This is done such that the equation for $\frac{e'}{E}$ will contain terms of $O[\epsilon^3]$. This will yield a mean square response, valid to $O[\epsilon^4]$; i.e. to terms typically like $\frac{u^4}{U^4}$. The results of this calculation are given, in the following section, for some specific wire positions and the reader is referred to the original work for details.

7.3.4 Response equations for specific wire orientations (single wire)

It was pointed out in section 7.3.1 that four response

equations are required to obtain the Reynolds stresses and two to obtain the mean velocities and the positions given below are chosen for convenience.

Position 1 (see figure 7.2) $\theta = 0$, $\alpha = 90^\circ$

$$\text{MEAN: } \frac{U}{U_{c1}} = 1 - \frac{1}{2} \frac{V^2}{U_{c1}^2} - \frac{1}{2} \frac{\overline{v'^2}}{U_{c1}^2} \quad (7.10)$$

$$\text{where } U_{c1} = C_1 E_1$$

TURBULENT:

$$\begin{aligned} \frac{\overline{e'^2}}{E_1^2} = & \left(1 - \frac{V^2}{U_{c1}^2} \right) \frac{\overline{u'^2}}{U_{c1}^2} + \frac{V^2}{U_{c1}^2} \frac{\overline{v'^2}}{U_{c1}^2} + \frac{2V}{U_{c1}} \frac{\overline{u'v'}}{U_{c1}^2} \\ & + \frac{V}{U_{c1}} \frac{\overline{v'^3}}{U_{c1}^3} + \frac{\overline{u'v'^2}}{U_{c1}^3} - \frac{2V}{U_{c1}} \frac{\overline{v'u'^2}}{U_{c1}^3} \\ & + \frac{1}{4} \left[\frac{\overline{v'^4}}{U_{c1}^4} - \frac{(\overline{v'^2})^2}{U_{c1}^4} \right] - \frac{\overline{u'^2 v'^2}}{U_{c1}^4} \end{aligned} \quad (7.11)$$

Position 2 (see figure 7.2) $\theta = \psi$, $\alpha = 0$

$$\text{MEAN: } \frac{U}{U_{c2}} = 1 + \cot \psi \frac{V}{U_{c2}} - \frac{1}{2 \sin^2 \psi} \frac{\overline{\omega'^2}}{U_{c2}^2} \quad (7.12)$$

$$\text{where } U_{c2} = C_2 E_2 / \sin \psi (1 + k^2 \cot^2 \psi)^{1/2} \quad (7.13)$$

TURBULENT:

$$\begin{aligned} \frac{\overline{e_2^2}}{E_2^2} &= \frac{\overline{u^2}}{U_{c2}^2} + \cot^2 \psi \left(1 - \frac{2k^2}{\sin^2 \psi}\right) \frac{\overline{v^2}}{U_{c2}^2} - 2\cot \psi \left(1 - \frac{k^2}{\sin^2 \psi}\right) \frac{\overline{uv}}{U_{c2}^2} \\ &+ \frac{1}{\sin^2 \psi} \frac{\overline{u\omega^2}}{U_{c2}^2} - \frac{\cot \psi}{\sin^2 \psi} \frac{\overline{v\omega^2}}{U_{c2}^2} + \frac{2\cot \psi}{\sin^2 \psi} \frac{\overline{uv\omega^2}}{U_{c2}^4} \\ &- \frac{\cot^2 \psi}{\sin^2 \psi} \frac{\overline{v^2\omega^2}}{U_{c2}^4} - \frac{1}{\sin^2 \psi} \frac{\overline{u^2\omega^2}}{U_{c2}^4} \\ &+ \frac{1}{4\sin^4 \psi} \left(\frac{\overline{\omega^4}}{U_{c2}^4} - \frac{(\overline{\omega^2})^2}{U_{c2}^4} \right) \end{aligned} \quad (7.14)$$

Position 3 (see figure 7.2) $\theta = 180^\circ - \psi$, $\alpha = 0$

$$\text{MEAN:} \quad \frac{\overline{u}}{U_{c3}} = 1 - \cot \psi \frac{V}{U_{c3}} - \frac{1}{2\sin^2 \psi} \frac{\overline{\omega^2}}{U_{c3}^2} \quad (7.15)$$

$$\text{where} \quad U_{c3} = C_3 E_3 / \sin \psi (1 + k^2 \cot^2 \psi)^{1/2} \quad (7.16)$$

TURBULENT:

$$\begin{aligned} \frac{\overline{e_3^2}}{E_3^2} &= \frac{\overline{u^2}}{U_{c3}^2} + \cot^2 \psi \left(1 - \frac{2k^2}{\sin^2 \psi}\right) \frac{\overline{v^2}}{U_{c3}^2} + 2\cot \psi \left(1 - \frac{k^2}{\sin^2 \psi}\right) \frac{\overline{uv}}{U_{c3}^2} \\ &+ \frac{1}{\sin^2 \psi} \frac{\overline{u\omega^2}}{U_{c3}^2} + \frac{\cot \psi}{\sin^2 \psi} \frac{\overline{v\omega^2}}{U_{c3}^2} - \frac{2\cot \psi}{\sin^2 \psi} \frac{\overline{uv\omega^2}}{U_{c3}^4} \\ &- \frac{\cot^2 \psi}{\sin^2 \psi} \frac{\overline{v^2\omega^2}}{U_{c3}^4} - \frac{1}{\sin^2 \psi} \frac{\overline{u^2\omega^2}}{U_{c3}^4} \\ &+ \frac{1}{4\sin^4 \psi} \left(\frac{\overline{\omega^4}}{U_{c3}^4} - \frac{(\overline{\omega^2})^2}{U_{c3}^4} \right) \end{aligned} \quad (7.17)$$

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Position 4 (see figure 7.2) $\theta = 0$, $\alpha = \psi$

$$\text{MEAN: } \frac{U}{U_{c4}} = 1 - \frac{1}{2\sin^2\psi} \frac{V^2}{U_{c4}^2} - \frac{1}{2\sin^2\psi} \frac{\overline{v'^2}}{U_{c4}^2} \quad (7.18)$$

$$\text{where } U_{c4} = C_4 E_4 / \sin\psi (1 + k^2 \cot^2\psi)^{1/2}$$

TURBULENT:

$$\begin{aligned} \frac{\overline{e_4'^2}}{E_4^2} = & \left(1 - \frac{1}{\sin^2\psi} \frac{V^2}{U_{c4}^2} \right) \frac{\overline{u'^2}}{U_{c4}^2} + \frac{1}{\sin^4\psi} \frac{V^2}{U_{c4}^2} \frac{\overline{v'^2}}{U_{c4}^2} \\ & + \left[\cot^2\psi \left(1 - \frac{2k^2}{\sin^2\psi} \right) - \frac{\cot^2\psi}{\sin^2\psi} \frac{V^2}{U_{c4}^2} \right] \frac{\overline{\omega'^2}}{U_{c4}^2} \\ & + \frac{1}{\sin^4\psi} \frac{V}{U_{c4}} \frac{\overline{v'^3}}{U_{c4}^3} + \frac{1}{\sin^2\psi} \frac{\overline{u'v'^2}}{U_{c4}^3} - \frac{2}{\sin^2\psi} \frac{V}{U_{c4}} \frac{\overline{v'u'^2}}{U_{c4}^3} \\ & + \frac{2\cot^2\psi}{\sin^2\psi} \frac{V}{U_{c4}} \frac{\overline{v'\omega'^2}}{U_{c4}^3} - \frac{\cot^2\psi}{\sin^2\psi} \frac{\overline{\omega'v'^2}}{U_{c4}^3} \\ & - \frac{1}{\sin^2\psi} \frac{\overline{u'^2v'^2}}{U_{c4}^4} - \frac{\cot^2\psi}{\sin^2\psi} \frac{\overline{v'^2\omega'^2}}{U_{c4}^4} \\ & + \frac{1}{4\sin^4\psi} \left(\frac{\overline{v'^4}}{U_{c4}^4} - \frac{(\overline{v'^2})^2}{U_{c4}^4} \right) \end{aligned} \quad (7.19)$$

7.3.5 Equations for Mean Velocity and Reynolds Stresses

7.3.5.1 Mean Velocity

The four mean velocity equations permit a number of independent evaluations of the two unknown components U and V. The equations given below, which do not represent all the possible solutions, are sufficient to obtain a check on the measurements. They are all written to O (ϵ^2).

From the two slanted wire equations (7.12) and (7.15) (UV plane)

$$V = \frac{U_{c3} - U_{c2}}{2 \cot \psi} \quad (7.20)$$

$$U = \frac{U_{c3} + U_{c2}}{2} - \frac{1}{2 \sin^2 \psi} \frac{\overline{w_m^2}}{U_{c2}} \quad (7.21)$$

From equations (7.10) and (7.18)

$$U = \frac{U_{c1} U_{c4} \cos^2 \psi}{U_{c4} - U_{c1} \sin^2 \psi} \quad (7.22)$$

and substituting equation (7.20) into (7.10)

$$U = U_{c1} - \frac{1}{8} \left(\frac{U_{c3} - U_{c2}}{\cot \psi} \right)^2 \frac{1}{U_{c1}} - \frac{\overline{u_m^2}}{2 U_{c1}} \quad (7.23)$$

The last equation, from the normal wire response, is probably less subject to error from wire misalignment and inadequate knowledge of the wire angle ψ and is therefore

probably the most accurate evaluation of U . As the correction terms are of $O[\epsilon^2]$ it is not necessary that they should be in "corrected" form given by the following section and hence they are subscripted ' m ' for 'measured' value or value calculated by neglecting third and fourth order terms in the 'turbulent' equations of section 3.4.

7.3.5.2 Reynolds Stress equations for single slanted wires having $k \sim O[\epsilon]$

It is convenient to express the corrected Reynolds stresses in terms of coefficients multiplying the measured quantities.

Defining

$$\overline{u_c'^2} = E \overline{u_m'^2}$$

$$\overline{v_c'^2} = F \overline{v_m'^2}$$

$$\overline{u_c v_c'} = G \overline{u_m v_m'}$$

$$\overline{\omega_c'^2} = H \overline{\omega_m'^2}$$

the values of E , F , G and H are:

$$E = 1 + \frac{V^2}{U_{c1}^2} - \frac{V^2 \overline{v'^2}}{\overline{u_m'^2} U^2} - \frac{2V \overline{u v'}}{U_{c1} \overline{u_m'^2}} - \frac{\overline{u' v'^2}}{\overline{u_m'^2} U_{c1}} + \frac{\overline{u'^2 v'^2}}{\overline{u_m'^2} U^2} - \frac{1}{4} \left(\frac{\overline{v'^4} - (\overline{v'^2})^2}{U^2 \overline{u_m'^2}} \right)$$

(7.24)

$$F = \frac{1}{1 - \frac{2k^2}{\sin^2 \psi}} - \frac{\overline{u\omega^2}}{\cos^2 \psi \overline{v_m^2} U_{c2}} \left(1 + \cot \psi \frac{V}{U_{c2}} \right) - \frac{(E-1)}{\cot^2 \psi} \frac{\overline{u_m^2}}{\overline{v_m^2}} + \frac{1}{\sin^2 \psi} \frac{\overline{v\omega^2} V}{\overline{v_m^2} U^2} + \frac{1}{\cos^2 \psi} \frac{\overline{u^2 \omega^2}}{\overline{v_m^2} U^2} + \frac{\overline{v^2 \omega^2}}{\sin^2 \psi \overline{v_m^2} U^2} - \frac{1}{\sin^2 2\psi} \left(\frac{\overline{v^4} - (\overline{v^2})^2}{\overline{v_m^2} U^2} \right) \quad (7.25)$$

$$G = \frac{1}{1 - \frac{k^2}{\sin^2 \psi}} - \frac{1}{2 \sin^2 \psi} \left(1 + \cot \psi \frac{V}{U} \right) \frac{\overline{v\omega^2}}{U_{c2} \overline{u'v_m}} + \frac{1}{2 \sin^2 \psi} \frac{\overline{u\omega^2}}{\overline{u'v_m}} \frac{V}{U^2} + \frac{1}{\sin^2 \psi} \frac{\overline{u'v\omega^2}}{\overline{u'v_m} U^2} \quad (7.26)$$

$$H = \frac{1}{1 - \frac{2k^2}{\sin^2 \psi}} + \frac{1}{\sin^2 \psi} \frac{V^2}{U_{c4}} - \frac{(E-1)}{\cot^2 \psi} \frac{\overline{u^2}}{\overline{\omega_m^2}} + \frac{1}{\cos^2 \psi} \frac{V^2}{U^2} \frac{\overline{u^2}}{\overline{\omega_m^2}} - \frac{1}{\sin^2 \psi \cos^2 \psi} \frac{V^2}{U^2} \frac{\overline{v^2}}{\overline{\omega_m^2}} - \frac{1}{\sin^2 \psi \cos^2 \psi} \frac{V}{U} \frac{\overline{v^3}}{U \overline{\omega_m^2}} - \frac{1}{\cos^2 \psi} \frac{\overline{u'v^2}}{U \overline{\omega_m^2}} + \frac{2}{\cos^2 \psi} \frac{V}{U} \frac{\overline{v\omega^2}}{U \overline{\omega_m^2}} - \frac{2}{\sin^2 \psi} \frac{V}{U} \frac{\overline{v\omega^2}}{U \overline{\omega_m^2}} + \frac{1}{\cos^2 \psi} \frac{\overline{u'v^2}}{U^2 \overline{\omega_m^2}} + \frac{1}{\sin^2 \psi} \frac{\overline{v^2 \omega^2}}{U^2 \overline{\omega_m^2}} - \frac{1}{4 \sin^2 \psi \cos^2 \psi} \left(\frac{\overline{v^4} - (\overline{v^2})^2}{U^2 \overline{\omega_m^2}} \right) \quad (7.27)$$

and H
where the correction factor E appears in F_A due to the iteration required for consistency in order of magnitude. (Details regarding the above equation are again available in Guilton, 1968). Note that to the order of magnitude being considered the longitudinal cooling corrections appear only in the first term and consequently in what follows these effects will no

longer be considered and the measurements will be concerned solely with the high intensity turbulence effects ($k = 0$ in equation (7.25) or (7.26)).

7.4 Approximations to the quadruple correlations

The complicated correction factors presented in the previous section can be reduced to forms more easily measurable by introducing certain plausible approximations which are justified later. A survey of experimental data (see Laufer, Comte-Bellot (1965) and Heskestad) for the circular pipe, the two-dimensional channel and the free jet and wall jet in still air indicates that the flatness factors of the fluctuating velocity components are approximately constant in the flow region of interest, and are given by

$$\frac{\overline{u'^4}}{(\overline{u'^2})^2} \approx \frac{\overline{v'^4}}{(\overline{v'^2})^2} \approx \frac{\overline{w'^4}}{(\overline{w'^2})^2} \approx 3 \quad (7.28)$$

the value associated with a Gaussian distribution of intensity. A comparison of the correction coefficients obtained by using the actual measured flatness factors, with those calculated from the above assumption, is made later and shows a completely negligible difference. Relation (7.28) will therefore be assumed in the following discussion.

The fourth order correlations such as $\overline{u'^2 w'^2}$ and $\overline{v'^2 w'^2}$ can be reduced by noting that the instantaneous signals v'^2 and w'^2 are composed of mean and fluctuating components

$$u'^2 = \bar{u}^2 + f_1(t)$$

$$v'^2 = \bar{v}^2 + f_2(t)$$

$$w'^2 = \bar{w}^2 + f_3(t)$$

(7.29)

where

$$\overline{f_1(t)} = \overline{f_2(t)} = \overline{f_3(t)} = 0$$

Taking the product u'^2 and v'^2 yields

$$\overline{u'^2 v'^2} = \bar{u}^2 \bar{v}^2 + \overline{f_1 f_2}$$

(7.30)

and using equations (7.20) and (7.30), it can easily be shown that

$$1 \leq \frac{\overline{u'^2 v'^2}}{\bar{u}^2 \bar{v}^2} \leq 3$$

(7.31)

A similar relation holds for $\overline{v'^2 w'^2}$, $\overline{u'^2 w'^2}$ and $\overline{u' v' w'^2}$.

The condition for zero $\overline{f_1 f_2}$ in (7.30), corresponding to the lower limit in (7.31), is that there be no correlation between the magnitudes of u' and v' * but it is evident from even simple mixing length arguments that u' and v' are definitely correlated (note that Heskestad assumed the lower limit). The quadruple correlation $\overline{u'^2 v'^2}$ appears in the correction factors E and H. The former is small and for its calculation it is judged acceptable to take an arithmetic mean

* Whereas for $\overline{u' v'} = 0$ it is necessary and sufficient that the signs of u' and v' should be uncorrelated.

for equation (7.31) and to assume

$$\frac{\overline{u'^2 v'^2}}{\overline{u'^2} \overline{v'^2}} \approx 2.0 \quad (7.32)$$

For the latter correction, the correlation may contribute significantly to the result and for this reason the factor H is less certain.

In contrast to the above, mixing length arguments would predict no correlation between u' and w' or between v' and w' such that

$$\begin{aligned} \overline{v'^2 w'^2} &= \overline{v'^2} \overline{w'^2} \\ \overline{u'^2 w'^2} &= \overline{u'^2} \overline{w'^2} \\ \overline{u'v'w'^2} &= \overline{u'v'} \overline{w'^2} \end{aligned} \quad (7.33)$$

If correlations do exist between the magnitudes of u' and w' , and v' and w' , then equation (7.33) represents conservative estimates and should make the correction coefficients lower than actual.

7.5 Calculation of High Intensity Correction Factors E, F and G ($k = 0$ in equation (7.25) and (7.26))

7.5.1 Cases where $V = 0$

It can be seen that the high intensity correction factors reduce to their simplest possible form in a flow where $V = 0$, and an appreciation of their importance can be obtained by investigating two flows which differ widely in their intensity

levels. Experimental data is available for two such flows; fully developed turbulent flow in a circular pipe (Laufer), and the free turbulent jet in still air (measured by Heskestad along mean flow streamlines). These flows and their associated correction coefficients, as well as some new measurements in the pipe, are discussed in sections 7.5.1.1 and 7.5.1.2.

7.5.1.1 The Circular Pipe - Experiments by Laufer (1953)

Laufer has measured certain triple and quadruple correlations relevant to the correction coefficients cited in the previous section, by using an x-wire and processing the individual sum ($e'_{A+B} \approx u'$) and difference ($e'_{A-B} \approx v'$) signals as shown in figure 7.3. The ratio meter determines R from the relation

$$R = \frac{e'^2_{A-B}}{e'^2_{A+B}}$$

$$\frac{(K_1 e'^2_{A-B} + K e'^2_{A+B})^2}{(K_1 e'^2_{A-B} - K e'^2_{A+B})^2} = \frac{1+R}{1-R}$$

providing the signals are equalized by adjusting K_1 , before the mixing circuit. Laufer has verified the accuracy of the technique by measuring the correlation coefficient

$$R_{\overline{u'v'}} = \frac{\overline{u'v'}}{\sqrt{\overline{u'^2}} \sqrt{\overline{v'^2}}}$$

and comparing this with measurements obtained by the conventional method using an x-wire and with theory. In his higher Reynolds experiment he found excellent agreement between these techniques and therefore only results from this experimental condition will be considered here. Laufer measured triple correlations to obtain the convection term in the turbulent

energy equation and it transpires that only the correction factor G can be assessed from his measurements. This is not a major inconvenience, however, because it will become evident that the intensity corrections in the pipe are extremely small and can be effectively neglected. The factor has been calculated and is shown in figure 7.4. Figure 7.5 shows the effect of G on the $\overline{u'v'}$ measurements made by Patel (1968) using a Dism type 5A29 single slanted wire probe having $\frac{L}{d} \approx 200$ in fully developed turbulent pipe flow. The intensity correction improves slightly the agreement between the theory and experiment (after the initial measurements have been corrected for longitudinal cooling effects) and the remaining discrepancy could be removed by choosing a bigger k .

7.5.1.2 The free two-dimensional jet in still air measured in a streamline co-ordinate by Heskstad: Corrections to Reynolds stresses

N.B. In this section the velocity components u' and v' are tangential and Normal to the streamlines.

Measurements of the free jet in still air have been made by Miller and Comings (1957), van der Heyge Zijnen (1958) and Heskstad (1963). Of the three experiments, Heskstad's results seem preferable because his measurements were carried out to $\frac{x}{b} = 100$ compared to $\frac{x}{b} = 40$ for Miller and Comings (x is the distance from the slot and b is the slot width) and the turbulence measuring equipment and techniques were more sophisticated.

Heskestad has also corrected his Reynolds stresses by using correction coefficients which (see later paragraph for discussion) require triple and fourth order correlation. The $\overline{u'v'^2}$ correlation was calculated by taking the time average of the cubed instantaneous fluctuating voltage response of a single slanted wire. In the UV plane this procedure yields, to first order

$$\frac{e'}{E} = \frac{u'}{U_c} - \cot \psi \frac{v'}{U_c} \quad (7.34)$$

and

$$\frac{\overline{e'^3}}{E^3} = \frac{\overline{u'^3}}{U_c^3} - 3 \cot \psi \frac{\overline{v' u'^2}}{U_c^3} + 3 \cot^2 \psi \frac{\overline{u' v'^2}}{U_c^3} - \cot^3 \psi \frac{\overline{v'^3}}{U_c^3} \quad (7.35)$$

The correlations were found by measuring $\overline{e'^3}$ at four different wire angles, 45°, 60°, 90° and 135° and solving algebraically. This method, as indicated by Comte-Bellot, tends to be inaccurate because the possible errors introduced by slight wire misalignment and the neglected high intensity terms, may be a significant portion of the small signal $\overline{e'^3}$. This probably explains the large scatter in Heskestad's results (Figs. 21-24 of Heskestad).

The $\overline{u'w'^2}$ correlation was obtained by forming the ratio of the mean square response of a normal wire in the UV plane (equation (7.17) $\psi = 90^\circ$). Finally Heskestad obtained the term $\overline{v'w'^2}$ from the assumption

$$\frac{\overline{v'w'^2}}{\sqrt{\overline{v'^2}} \sqrt{\overline{w'^2}}} = \frac{\overline{u'w'^2}}{\sqrt{\overline{u'^2}} \sqrt{\overline{w'^2}}}$$

The resulting intensity correction coefficients obtained using Heskestad's measurements, are shown in figure (7.6). The intensity corrections to $\overline{u'^2}$ and $\overline{u'v'}$ are nearly constant and of order 5% and 10% respectively. The correction to $\overline{v'^2}$ is large at $y/y_{m/2} = 1$. Beyond this point the corrections are suspect because the flow is intermittent and the intensity becomes so high that it is insufficient to limit the corrections to $O(\epsilon^4)$.

The coefficients F and G are shown in broken down form in figures (7.7) and (7.8) to illustrate the weight of each contributing term. It is observed that no particular term dominates so that it is not possible to further simplify the analytical correction factors.

A comparison between the present calculations and Heskestad's results is shown in figures (7.9) to (7.11). Heskestad's coefficients are given as (note that the subscript to U_m does not imply 'measured value')

$$\left(\frac{\overline{u'^2}}{U_m^2} \right)_c = E_1 \left(\frac{\overline{u'^2}}{U_m^2} \right)_m$$

$$\left(\frac{\overline{v'^2}}{U_m^2} \right)_c = F_1 \left(\frac{\overline{v'^2}}{U_m^2} \right)_m$$

$$\left(\frac{\overline{u'v'}}{U_m^2} \right)_c = G_1 \left(\frac{\overline{u'v'}}{U_m^2} \right)_m$$

and the present results are accordingly modified to account for the intensity correction to U_m - the maximum jet velocity. Hence

$$\overline{u_c^2} = E_1 \left(\frac{U_m^2}{U^2} \right) \overline{u_m^2}$$

which using equation (7.10) yields

$$\overline{u_c^2} = E_1 \left(1 - \frac{\overline{v^2}}{U^2} \right)_{y=0} \overline{u_m^2}$$

or

$$E = E_1 \left(1 - \frac{\overline{v^2}}{U^2} \right)_{y=0}$$

Similarly

$$F = F_1 \left(1 - \frac{\overline{v^2}}{U^2} \right)_{y=0}$$

$$G = G_1 \left(1 - \frac{\overline{v^2}}{U^2} \right)_{y=0}$$

Figures (7.9) to (7.11) show the discrepancy between the coefficients which is especially significant for F . The difference is not due to assumptions about the magnitude of the flatness factors, nor is it due to the additional correction to U_m (which Heskestad may not have included). Heskestad unfortunately does not present the coefficients in analytical form and it is therefore not possible to make a more direct comparison.

7.5.2 The Equilibrium Wall Jets in Still Air

The apparatus and experimental conditions were identical

to those described in Chapter 5.

The method used to obtain the triple and quadruple correlations differed from Laufer's and Heskestad's. In the present experiments Disa equipment was used, in conjunction with a precision full wave rectifier, and the schematic is shown in figure (7.3b).

A Disa miniature x-wire was mounted with its stem parallel to the plate. The linearized signals from the two anemometers controlling each wire were fed into the first correlator. This instrument was used simply as a summing and difference device such that the signals e_{A-B} and e_{A+B} were respectively proportional to v' and u' . This is, of course, only possible if the wire calibration curves are identical and this was assured by adjusting the operating resistance of each wire to yield equal DC voltages at the maximum calibration velocity. It transpired that the voltages remained equal for all subsequent lower velocities, thus yielding a single calibration curve valid for both wires. The validity of this technique was checked by measuring $\overline{v'^2}$ in the pipe and excellent agreement was obtained with R.P. Patel's results.

If, for example, the correlation $\overline{v'u'^2}$ was desired the signal e'_{A+B} was passed through a full wave rectifier and then squared by the Disa Linearizer. The rectifier - the only new piece of electronic equipment required - was necessary because the linearizer does not accept negative signals.

Finally the signals e'_{A-B} and e'_{A+B} were fed into the second correlator and the coefficient

$$R_f = \frac{e'_{A-B} e'^2_{A+B}}{\sqrt{e'^2_{A-B}} \sqrt{e'^2_{A+B}}}$$

where

$$\overline{e'^4_{A+B}} = \left(\overline{e'^2_{A+B}} \right)^2 + \overline{e'^4_{A+B}}$$

was read directly on the ratio meter. It should be noted that R_f is not the actual correlation coefficient because the Disa correlator can only accept the AC component of a signal and $\overline{e'^2_{A+B}}$ has a DC component. The manner in which this affects R can be shown by finding the correlation of the two signals.

$$e_1 = E_2 + e'_1$$

$$e_2 = E_2 + e'_2$$

The actual correlation coefficient is

$$\begin{aligned} R &= \frac{\overline{e_1 e_2}}{\sqrt{\overline{e_1^2}} \sqrt{\overline{e_2^2}}} \\ &= \frac{R_f + \frac{E_1 E_2}{\sqrt{\overline{e'^2_1}} \sqrt{\overline{e'^2_2}}}}{\left[1 + \frac{E_1^2}{\overline{e'^2_1}} + \frac{E_2^2}{\overline{e'^2_2}} + \frac{E_1 E_2}{\sqrt{\overline{e'^2_1}} \sqrt{\overline{e'^2_2}}} \right]^{1/2}} \end{aligned}$$

where

$$R_f = \frac{\overline{e'_1 e'_2}}{\sqrt{\overline{e'^2_1}} \sqrt{\overline{e'^2_2}}}$$

A number of tests, including the measurements of skewness factors (e.g. $\overline{u'^3}/(\overline{u'^2})^{3/2}$) and flatness factors (e.g. $\overline{u'^4}/(\overline{u'^2})^2$), were made to assure the proper functioning of the

system and these are described in the original work (Guitton, 1968). It was not possible to measure the correlation and this parameter was obtained by noting that the experiments give

$$R_{\overline{v}} \approx R_{\overline{u}} \approx R_{\overline{u'v'}} \approx R_{\overline{u'w'}}$$

If it is assumed that

$$R_{\overline{v}} \approx R_{\overline{u'w'}}$$

then

$$\frac{\overline{u'w'^2}}{U_m^2} = \left(\frac{\overline{u'w'^2}}{U_m^2} \right) \left(\frac{\overline{v'^2}}{u'^2} \right)^{1/2}$$

The correction factors E, F, G and H obtained from equations (7.24) to (7.26) are shown in figures (7.12) to (7.15). The corrections to $\overline{u'^2}$ are, like in the free jet, small and do not exceed 5% except for $y > y_{m/2}$ where the corrections are suspect (see discussion of plane free jet). The transverse stress $\overline{v'^2}$ is again shown susceptible to large errors which, unexpectedly in the plane wall jet (which is a lower intensity flow), exceed those of the free jet and the spirals. The following reasons suggest why this may only be an apparent inconsistency: (a) the correction factors E, G and H vary in a perfectly consistent manner; (b) in figure 7.13 it is observed that at small $y/y_{m/2}$ the value of F for the $\frac{x}{R} = \frac{2}{3}$ spiral exceeds that of the $\frac{x}{R} = 1$ case, and if this trend were continued the plane wall jet results would be plausible. The correction factor to $\overline{w'^2}$, H, is very large and, of all the terms contributing

to it, the correlation $\overline{u'^2 v'^2}$ is by far the most important. In calculating H, equation (7.32) was used, and in view of the uncertainty of this assumption the value of H should be regarded as uncertain.

7.6 Conclusions

(i) The intensity corrections to $\overline{u'^2}$ are generally small and of the order of 5% in the free jet and plane and curved wall jets.

(ii) The shear stress intensity correction in all the flows is of order 8% over most of the region below $y_{m/2}$.

(iii) The correction to $\overline{v'^2}$ becomes large for $y < y_{m/2}$ in the free and wall jets. At $y/y_{m/2} = .32$ in the wall jet, the correction is

$$\overline{v_c'^2} = 1.18 \overline{v_m'^2}$$

and at $y/y_{m/2} = 1$ the correction is unity. In the free jet the correction at this point is

$$\overline{v_c'^2} = 0.8 \overline{v_m'^2}$$

and is also a minimum. Below $y/y_{m/2} \sim .5$ it is surprising that the correction to $\overline{v'^2}$ in the plane wall jet is larger than the correction in the more intensely turbulent flows but close examination of the data reveals a trend which intimates that the results are consistent.

(iv) The correction to $\overline{w'^2}$ is generally very large but

is suspect due to uncertainty in the correlation $\overline{u'^2 v'^2}$ which is the major contributing term.

(v) In the pipe the intensity corrections, as expected are small.

Chapter 8

A Transient Technique for Measuring Skin Friction using a Flush-Mounted Heated Film and its Application to a Wall Jet in Still Air

8.1 Introduction

A device used for measuring skin friction in a turbulent boundary layer should, ideally, meet four basic requirements. It should be:

- (a) simple to manufacture (or readily available commercially)
- (b) easily calibrated
- (c) insensitive to the level and scale of turbulence near the wall
- (d) readily corrected for pressure gradient effects.

Table 8.1 lists advantages and disadvantages of the most common techniques used for measuring wall shear. Devices which remain in the sublayer are to be preferred because they do not invoke universality of the law of the wall for the turbulent part of the flow. Among these the Stanton tube and the boundary layer fence are the most common. But they have an uncertain response to velocity fluctuations (Bradshaw and Gregory, 1959); their calibrations are dependent on $\frac{U_d}{v}$ (d = height above surface); and they are affected by dust and other particles in the stream. The flush mounted heated

film, on the other hand, does not interfere physically with the flow; it can be easily cleaned and is relatively simple to construct and mount. Furthermore, there is strong evidence (Brown, 1967) that with a properly designed probe, the calibrations in laminar and turbulent boundary layers can be made identical. This is an impressive quality in view of the high turbulence level ($\frac{\sqrt{u'^2}}{U} \approx 32\%$, e.g. Mitchell and Hanratty (1966)) in the viscous sublayer of the turbulent boundary layer in zero pressure gradient. The hot film therefore is useful for measurements in a wide variety of experimental configurations and was chosen for these measurements in the curved wall jets in still air. In the plane wall jet the turbulence intensity $\frac{\sqrt{u'^2}}{U}$ within the viscous sublayer is, surprisingly, about the same as that for a turbulent boundary layer (Kohan, 1968; Tailland and Mathieu, 1967) and therefore it should not vary appreciably if the jet is curved.

The surface heated element has been studied by a number of authors and its history is well recorded. (Bellhouse and Schultz, 1966). To permit highly localized measurements and ensure mobility it has been developed to consist of a very narrow strip of metallic film (usually platinum) deposited on the smooth end of a small cylindrical rod (usually glass) and connected to external terminals with conducting paint. (e.g. the commercial Disa flush mounted hot film probe - see Section 8.3.1.) The above assembly is encased in a sleeve of low thermal conductivity so as to form a rugged probe which can be readily moved from one position to another. A calibra-

tion curve can be obtained by placing the probe in a flow in which the skin friction is known and relating the heat input to the local skin friction. Bellhouse and Schultz have shown that the calibration can be repeated with little scatter so that τ_w can be measured to $\pm 3\%$, but they obtained different calibrations in laminar and turbulent boundary layers. (Liepmann and Skinner (1954) had previously claimed identical calibrations in laminar and turbulent flow but Bellhouse points out that the scatter in their results does not warrant this conclusion.) Brown (1967) has recently shed light on the controversy by obtaining identical calibrations with a probe especially designed to limit its heating effect to the viscous sublayer. The design suggested by Brown attempts to reduce the heat lost to the material around the film (and hence reduce the effective streamwise length of the element) by shielding it with a circumferential air gap which is in turn surrounded by copper. Reducing the element's streamwise length thins the thermal boundary layer and with this method it may be possible to obtain accurate evaluations of the skin friction. The experiments described in this report were done with a Disa hot film probe which has neither air gap nor copper heat conductor. However, on the basis of a criterion developed by Brown (section 8.4.4) the results indicated that the heat transfer was always limited to the sublayer.

An element is most useful when it can be calibrated in a known simple flow and then moved to the flow under investigation. But it is often the case that the calibrating apparatus

will be constructed with a different material of different physical form. For example, in the McGill University low speed aerodynamics laboratory it was found most convenient to use, as a calibrating flow, a long brass pipe which had been carefully aligned and thoroughly checked by R.P. Patel (1968). The film was then used to measure the skin friction in a turbulent boundary layer, a plane wall jet boundary layer formed on a plexiglas surface and in the boundary layer of a curved wall jet formed on a wooden surface. These measurements were complicated by the fact that there was a different heat loss to the material surrounding the film in the calibration and the subsequent test runs. Since a significant proportion of the total heat input leaks to this material a slight difference can lead to significant errors in measured skin friction. This problem, first identified by Liepmann and Skinner but not yet investigated by any author, is resolved in this chapter. A simple technique is developed and it is used to determine the skin friction in the plane and curved wall jets in still air. Information on the validity of the law of the wall in these flows is therefore obtained. The investigation is limited to air flow at low Mach numbers in which the temperature is uniform far away from the heat element.

8.2 Theoretical approach to the heat loss relation

8.2.1 General

For the purpose of the discussion it is useful to sub-

divide the total input of heat flux to the film as follows:

$$q_t = q_a + q_s \quad (8.1)$$

where q_a represents the heat flux transferred by either forced or free convection from the film and surrounding material to the flow and q_s is the remainder. (The maximum film temperature considered is 185°C and the heat transferred by radiation can be neglected. Also, the experiments are at sufficiently high Reynolds numbers (turbulent flow) that the natural convection is very small compared with the forced convection.)

The requirement of any theoretical approach is a relation between q_a and τ_w valid in any pressure gradient. This can be done by: (i) dimensional analysis (Liepmann and Skinner, 1954); (ii) obtaining an exact solution of the energy and momentum equations (Myers et al., 1961; Spence and Brown, 1968); (iii) integral methods (Curle, 1962; Bellhouse and Schultz, 1966; Brown, 1967). The next section presents a dimensional analysis which is more formal than that used by Liepmann and Skinner. The other theoretical methods are briefly discussed in section 8.2.3.

8.2.2 Dimensional analysis

The heat transferred, q_a , from a heated surface due to forced convection within the viscous sublayer when the pressure gradient is zero will depend on: the skin friction, the temperature difference between the film and the fluid,

the fluid temperature, the length and breadth of the film, the fluid properties and a number of parameters describing the geometry and composition of the material surrounding the element (substrate)

$$q_a : \tau_w, (T_f - T_i), T_i, l, w, \rho, \nu, k, C_p, \text{substrate parameters} \quad (8.2)$$

There are five fundamental units: mass (M), length (L), time (t), temperature (T), heat (H). The fundamental dimension H is introduced because there is no significant interchange between mechanical and thermal quantities; e.g. the heat produced by fluid friction is insignificant compared to that transferred by the external source. Dimensional reasoning yields

$$\frac{q_a}{kl(T_f - T_i)} = F \left(\frac{\tau_w l^2}{\rho \nu^2}, R_s, \frac{T_f - T_i}{T_f}, \frac{l}{w}, \text{non-dimensional substrate parameters} \right) \quad (8.3)$$

This very basic functional relationship can be refined by implicitly introducing the boundary layer assumptions (Newman, 1967) into the dimensional analysis. To do this, it is noted that in a boundary layer two approximately independent transport modes exist: convection of vorticity (and heat) in the x direction (not significant in the y direction) and diffusion of these quantities in the y direction (not significant in the x direction). Thus two length scales may be defined: L_1 associated with the streamwise direction x and L_2 associated with y. The dimensions of each parameter in equation (8.2)

can be obtained by the following reasoning. Since

$$\mu = \frac{\tau}{2 \frac{\partial u}{\partial y}}$$

therefore

$$\mu = \frac{ML_2}{L_1^2 t} \quad (8.4)$$

and because ρ has dimensions $\frac{M}{L_1^2 L_2}$

$$\nu = \frac{L_2^2}{t} \quad (8.5)$$

Since vorticity is predominantly diffused in the y direction, the above result is perhaps obvious and could have been written by inspection. Using similar arguments

$$k = \frac{HL_2}{\tau L_1^2 t} \quad (8.6)$$

The other parameters have dimensions which are self-evident.

In relation (8.2) there are now six fundamental units and dimensional analysis yields

$$\frac{q_a}{h l (\tau_f - \tau_i)} \left(\frac{\rho \nu^2}{\tau_0 l^2} \right)^{1/3} = f \left(R_0, \frac{\tau_f - \tau_i}{\tau_f}, \frac{l}{\omega}, \text{non-dimensional substrate parameters} \right) \quad (8.7)$$

and the heat lost by forced convection from the surface element is proportional to the cube root of the surface shear

$$q_a \propto \tau_w^{1/3} \quad (8.8)$$

if the substrate parameters do not involve the length scale L_2 which seems likely. Liepmann and Skinner derived the same result by studying the Rayleigh problem.

Substituting equation (8.8) into the heat balance, equation (8.1), and noting that

$$q_t = \frac{E^2}{R_f}$$

yields

$$\frac{E^2}{R_f k l (T_f - T_i)} = \left(\frac{\tau_w l^2}{\rho \nu^2} \right)^{1/3} f + \frac{q_a}{k l (T_f - T_i)} \quad (8.9)$$

which is the working relation for analysing hot film results. This equation is compared in the following section with the results obtained by the integral and exact solutions.

8.2.3 Other theoretical approaches

For: (i) small $\frac{dp}{dx}$; (ii) a "top hat" or rectangular temperature profile of length l_e ; (iii) constant fluid properties; and (iv) an infinite aspect ratio. Both the integral and exact analyses give a solution of the following form:

$$\frac{q_a}{k l_e (T_f - T_i)} = a \omega \left(\frac{R_e}{\rho \nu^2 l_e} \right)^{1/3} \tau_w^{1/3} + b \frac{\omega}{\tau_w} \frac{dp}{dx} \quad (8.10)$$

(see for example Brown's equations (5) and (11)). In the integral analysis l_e , the length of the equivalent rectangular profile, is defined by the relation

$$q_a l_e = \int q_a(x) dx$$

where the integral is evaluated over the heated portion of the surface. For the "top-hat" temperature distribution at temperature T_f , which may be particularly applicable to Brown's specially designed probe, Spence and Brown obtained $a = .807$ and $b = .10$ by an exact solution of the boundary layer energy equation. In zero $\frac{dp}{dx}$, substituting 'a' and combining equation (8.10) with (8.1) yields

$$\frac{E^2}{R_f k l_e (T_f - T_i)} = \omega \left(\frac{P_r}{1.9 f \nu^2 l_e} \right)^{1/3} \tau_w^{1/2} + \frac{q_a}{k l_e (T_f - T_i)} \quad (8.11)$$

Brown calculated l_e from equation (8.11) using the slopes of calibration curves obtained in a laminar boundary layer with a probe of nominal size twice the present Disa model, and in a turbulent boundary layer with the air gap probe previously discussed. He found l_e to be respectively two and three times the actual physical length of the film. Although this difference may be due in part to the theoretical assumptions which permit the determination of the constant ($a = 1.9$) in equation (8.11), it is primarily due to substrate heating. This discrepancy between the calculated and physical length of film illustrates the difficulty of keeping l_e small which

is desirable to limit the thickness of the thermal layer and hence keep the heat transfer within the viscous sublayer. Furthermore, it casts doubt on the assumption of a top-hat temperature distribution.

The above evidence, combined with the knowledge that a very precise heat transfer relation is required (see section 8.4.5), indicates that a completely theoretical approach to the problem does not eliminate the necessity for calibrating a probe. The usefulness of the theory is that it provides corrections for the effect of pressure gradient and furnishes design criteria (e.g. how to maintain the thermal layer within the viscous sub-layer).

8.3 Experimental Equipment

8.3.1 The flush probe and mounting

The hot film probe used in the experiments was the standard Disa sub-miniature model type 55A92*. The platinum film had an aspect ratio of 5 with smaller length, $l = .008"$ in the streamwise direction. In all the tests the probe was held firmly in place, flush with the surface of interest, by means of the special mounting shown in figure 8.1a. A number of these mountings were glued into the surface with epoxy cement

* Disa Electronic A/S, Herlev Denmark supply two flush, hot-film, probes type 55A92 and 55A93. The latter has a quartz covered film to permit measurements in conducting fluids.

and were sanded and polished so as to be perfectly smooth and flush. The probe, fitted on the standard miniature hot-wire holder, slid through the hole in the threaded plug and was held flush (see experimental procedure) with the surface of the mounting by means of a squashed 'O' ring. The mounting was made of brass to assure good conductivity and compatibility with the brass pipe used for calibration.

The hot-film probe was heated and maintained at constant temperature by the standard Disa 55A01 anemometer. The Disa linearizer was not used. In the pipe and boundary layer experiments the bridge output was fed to a Hewlett-Packard Dymec 2010B system which read and printed the bridge voltage at desired time intervals. For the wall jet experiments (plane and curved) the anemometer output was measured by the VF converter and counter arrangement described in Chapter 5.

8.3.2 Pipe

A very carefully aligned, long, circular pipe with air blown down its length was used for calibrating the hot film probe. The pipe was made of precision brass tubing 3 inches in diameter with a length to diameter ratio of 142, and was thoroughly checked by R.P. Patel (1968) to assure symmetry, lack of swirl and fully developed flow. A $\frac{1}{2}$ " hole was drilled into the wall of the pipe, 1" from the exit and the brass mounting mentioned in the previous section was inserted and epoxied in place. When firmly held the mounting was filed and

buffed until it conformed perfectly to the inside contour of the pipe. The air temperature was controlled by a heat exchanger placed downstream of the compressor.

8.3.3 Boundary layer and wall jet

The boundary layer and wall jet measurements were made in the McGill blower cascade wind tunnel, a general description of which has been given by Wygnanski and Gartshore (1963). The exit of the tunnel was fitted with a working section described by Gartshore and Hawaleshka (1964) which was designed primarily to study wall jets.

In the boundary layer study the adjustable blowing slot was closed tightly and a smooth fairing 6 in. long and .2 in. high was installed downstream of the protruding slot lip. At the measuring station, which was $55\frac{5}{8}$ in. downstream of the fairing, a hot film probe mounting was installed flush with the surface, at the centre-line of the plate.

For the plane wall jet experiments, four more brass mountings were installed at distances of $11\frac{11}{32}$, $16\frac{5}{8}$, $26\frac{5}{8}$ and $41\frac{5}{8}$ ins. from the slot.

In the curved flows the brass mountings were installed at each measuring station. (See Chapters 5 and 9.)

8.3.4 Preston and Pitot tubes

It was desired to compare the heated element results with predictions obtained from Preston tubes and for this

purpose six different tubes were made. Their characteristics are listed in table 8.2. It is observed that all have the ratio of internal diameter to external diameter well over .2, the value above which, according to Head and Rechenberg (1962), the calibration becomes independent of the internal diameter. The readings of the tubes were interpreted using V.C. Patel (1965)'s calibration.

Boundary layer velocity traverses were made using the flattened tube shown in figure 8.1b.

8.4 Experimental Procedure and Calibration results

8.4.1 General

The Disa hot film probe was inserted into the pipe mounting and adjusted until visual examination, through a powerful magnifying glass, revealed that it was flush with the pipe wall and that the long dimension of the platinum film was perpendicular to the flow direction. Tests with a surface gauge indicated that this visual technique satisfactorily positioned the film normal to the surface to an accuracy of about .001 in. and subsequent experiments showed excellent repeatability. The boundary layer and plane wall jet were both on horizontal surfaces whereas curved jets were on vertical surfaces and accordingly the pipe was rotated until the film was in a plane corresponding to the appropriate experimental surface. The air temperature, controlled by the heat exchanger, was held to room temperature with an accuracy of better than .5°F.

Calibration of the probe was done by varying the airflow while noting a reference static pressure reading, taken from a pressure tap on the pipe wall, and recording the resultant bridge voltage for a one second integration time. Figure 8.2 shows a number of calibration lines obtained at constant ambient temperature for different film temperatures (expressed as overheat ratios) and these confirm the straight line relationship of equation (8.10). (The parameters are expressed in dimensional form for convenience.) It is emphasized that the ordinate of the figure is calculated from the bridge voltage of the anemometer and is proportional to the power supplied to both the film and the resistance, R_1 say, in series with it in the bridge circuit. R_1 is composed of a 100 Ω resistance in one bridge arm plus the resistance of the cable and hot wire holder but the latter two amount to less than 1 Ω and are negligible. The true power to the film is, on the other hand given as

$$P_f = i^2 R_f \quad (8.12)$$

with

$$i = \frac{E}{R_1 + R_f} \quad (8.13)$$

Hence

$$\begin{aligned} P_f &= \frac{E^2}{R_f \left[\left(\frac{R_1}{R_f} \right)^2 + 2 \left(\frac{R_1}{R_f} \right) + 1 \right]} \\ &= \frac{E^2}{R_f \left[\left(\frac{100}{R_f} \right)^2 + \frac{200}{R_f} + 1 \right]} \end{aligned} \quad (8.14)$$

Figure 8.3 gives the relation between the true power and $\tau_w^{1/3}$. The lack in uniformity between the spacing of the lines for different overheat ratios seemed improbable (the prevailing conditions were the same for all curves) and the experiments were therefore repeated together with an additional calibration (shown in figure 8.3) for the overheat ratio $\frac{R_f}{R_c} = 1.4$. The new results were in excellent agreement with the previous set. The irregular distribution can be plausibly explained by noting that the stated overheat ratios are, in reality, only nominal values which could not be achieved exactly because, for a given cold resistance, the value of the relevant operating resistance must be rounded off to the nearest one hundredth of an ohm due to the limitations of the three decade adjustable resistance in the Disa control unit. Indeed, the shift in position required to bring the lines, corresponding to the overheat ratios 1.35 and 1.30, into a more plausible formation would amount to approximately .005 ohms in .800 (.800 is the difference between the operating resistance at these two overheat ratios) or a .6% change.

The results from the two extreme overheat ratios, 1.2 and 1.4, are plotted non-dimensionally, according to equation (8.9), in figure 8.4. The collapse is extremely good and is not entirely due to the larger scale, and substantiates well the calibration results. This figure also shows that a large proportion of the total heat input is lost to the surrounding material and is not transferred to the air flow (see discussion in sections 8.5.5 and 8.5.6).

8.4.2 Effective length of surface temperature profile

An indication of the extent of substrate heating may be obtained by calculating l_e from the theoretical relation (8.14). The slope of the line in figure 8.3 corresponding to an overheat ratio of 1.25 gives

$$\frac{\omega k}{\alpha_1} \left(\frac{P_e}{1.9 P v^2} \right)^{1/3} l_e^{2/3} = 0.22 \quad (8.15)$$

and noting that the temperature coefficient of resistance of the film (α) is approximately $2.5 \times 10^{-3}/^\circ\text{C}$ then

$$\omega l_e^{2/3} = 0.0025 \text{ (in.)}^{5/3}$$

The width of the film is approximately .0472 in. (1.2 mm.) and therefore

$$l_e = 0.012 \text{ in.} \quad (8.16)$$

which is 1.5 times the actual physical length of the film and compares very favourably with Brown's results. It will be seen in subsequent sections that, at a given overheat ratio, the slope of the line representing the probe response ($E^2 \propto \tau_w^{1/3}$) does not alter when the hot film is installed in the other surfaces of interest: the relation $l_e = 1.5l$ is also valid for these surfaces.

8.4.3 Pressure gradient effect in flush probe measurement in a pipe and on the logarithmic spirals

In a small pressure gradient the theoretical solution

given by Spence and Brown is:

$$\tau_w = 1.9 \frac{\rho U^2}{R} \frac{1}{l_e^2} Nu^3 - \frac{5}{18} l_e \frac{dp}{dx} \frac{1}{Nu} \quad (8.17)$$

where

$$Nu = \frac{E^2}{R_f k_w (T_f - T_i)} - \frac{q_w}{k_w (T_f - T_i)} \quad (8.18)$$

Using the calibration curve corresponding to an overheat ratio $\frac{R_f}{R_c} = 1.25$ in figure 8.3 and the length $l_e = 1.5 l$, the effect of $\frac{dp}{dx}$ on the calculated value of τ_w was determined for the pipe and the $\frac{x}{R} = 1$ spiral (which represented the worst case). The error was very small and $\Delta\tau/\tau_w$ amounted to less than 10^{-4} in both cases. (The surface pressure gradient on the spiral will be discussed in Chapter 9.)

8.4.4 Limit of film length

As discussed previously, the film length should be limited to maintain the thermal boundary layer within the viscous sublayer. It is usually assumed that the parameter

$$\frac{y U_\tau}{\nu} = 12 \quad (8.19)$$

denotes the sublayer limit. Spence and Brown, solving the energy equation using an exponential temperature profile, show that less than 10% of the heat supplied to the air at $y = 0$ will cross a plane parallel to, and situated at distance $y = h$ from the wall, providing the criterion

$$\left(\frac{R_e \tau_w l_o^2}{\rho \nu^2} \right)^{1/3} \frac{h}{l_o} > 3 \quad (8.20)$$

is met.

Combining the above equations yields a design criterion given by Brown as

$$\frac{U_r l_o}{\nu} < 64 R_e \quad (8.21)$$

and for air with $P_r = .7$

$$\frac{U_r l_o}{\nu} < 45$$

In the pipe, using $l_o = 1.5l$ and $\tau_w^{1/3} = .4 \left(\frac{1 \text{ bf}}{\text{ft}^2} \right)^{1/3}$

$$\frac{U_r l_o}{\nu} \approx 32 \quad (8.22)$$

The maximum value of U_r measured in the experiments was $U_r = 7.5 \text{ ft./s}$ (Table 8.4) obtained in the wall jet and in this case

$$\frac{U_r l_o}{\nu} \approx 46$$

and the heat transfer is satisfactorily limited to the viscous sublayer in all cases to be studied.

8.4.5 The effect of voltage accuracy on the measurement of skin friction

The shaded band surrounding the $\frac{R_f}{R_c} = 1.25$ line in figure 8.2 indicates the accuracy in voltage required if an arbitrarily selected $\pm 6\%$ error in shear is tolerated. At $\tau_w^{1/3} = .3$ this corresponds to a variation

$$\frac{dE^2}{E^2} = \frac{0.04}{17.34} \quad (8.23)$$

or

$$\frac{dE}{E} = 0.00115 \quad (8.24)$$

Hence to remain within $\pm 6\%$ shear tolerance requires $\pm 0.1\%$ accuracy in voltage and great care must be taken in obtaining the measurements. The recording system easily met this required accuracy.

8.5 Measurement in the flat plate boundary layer

8.5.1 General

The boundary layer measurements were made to verify the generality of the pipe calibration. Before proceeding with the experiment, the two-dimensionality of the boundary layer was checked by noting the reading of Preston tube no. 3 at 1 in. intervals up to 6 in. on either side of the centre-line. A 10% random variation in C_f was discovered and, following the work of R.P. Patel (1964) this was attributed to screens.

placed downstream of the honeycomb in the settling chamber. Accordingly, the variation in C_f was reduced to 2% by removing all downstream screens and leaving upstream only a single screen of pressure drop coefficient K , equal to two ($K = \frac{\Delta p}{\frac{1}{2}\rho U_1^2}$). This procedure increased the tunnel's turbulence intensity level to about 1% at $U_1 = 135$ ft/sec., but existing evidence shows that such free stream turbulence should not affect the wall law region (Kline et al., 1960; Junkhan and Serovy, 1967).

8.5.2 Initial unsuccessful measurement of C_f with hot film

With the flush probe set in the plate the bridge DC voltage (and therefore the heat input to the element) did not remain steady, as it did in the pipe calibration, but slowly dropped and this was attributed to a gradual heating of the brass mounting which in this case did not behave as an infinite heat sink due to its being now surrounded by plexiglas rather than brass. Figure 8.5 shows the resulting value of $\frac{E^2}{R_f(\frac{R_f}{R_e} - 1)}$ obtained after a 1.5 hour waiting period and the resulting prediction of shear using the pipe calibration is seen to be highly inaccurate.

One obvious solution to this difficulty is to measure the voltage when the brass mounting has not had the time to heat up and when, by definition, conditions on the plate and in the pipe are identical. Accordingly, it was decided to obtain a plot of voltage vs time and to extrapolate the curve

back to $t = 0$.

The Dymec digital voltmeter was set at its maximum sampling rate of 3.5 samples/sec. for a sample time of 0.1 sec. and the output was recorded by an automatic printer. The resulting time variation of film voltage was a damped oscillation and is shown in Figure 8.6. Also shown is the value $E = 11.12$ volts; the voltage corresponding to

$$\frac{E^2}{R_f \left(\frac{R_f}{R_c} - 1 \right)} = 17.45$$
 which, using the pipe calibration of figure 8.5 is required to correctly predict the shear. No obvious portion of the $E(t)$ curve could, a priori, be extrapolated with any accuracy to this point and the method had to be abandoned.

In figure 8.6 it takes less than 4 seconds for the film voltage to drop below that specified by the pipe calibration. This illustrates how rapidly the brass mounting heats to a level sufficient to invalidate, for the required experimental accuracy, the heat balance, and therefore the calibration curve, which was found in the pipe experiments. The technique proposed to counter this difficulty involves the construction of a hypothetical calibration line which is compatible with a small change in the brass mounting temperature. This is described in the following section.

8.6 The transient technique for measuring τ_w

The transient technique makes it possible to construct an instantaneous calibration curve using the 'wind-off' voltage

read in the test and the slope of the original calibration obtained when the probe is surrounded by an infinite heat sink. The argument behind the procedure is based on two assumptions:

- (i) that the difference, Λ , between the wind-off reading and the point given by the intersection of the calibration line with the $\tau = 0$ axis, is due to free convection. (See figure 8.2.)
- (ii) that Λ , measured in the calibration flow, will not change for the test flow providing the brass mounting temperature is effectively the same for the two cases.

The general expression for Λ is

$$\Lambda = \left(\frac{q_a}{T_f - T_i} + \frac{q_b}{T_f - T_i} \right)_{\text{wind off}} - \left(\frac{q_b}{T_f - T_i} \right)_{\text{calibration}} \quad (8.25)$$

and the first assumption is equivalent to assuming that the second and third terms cancel so that

$$\Lambda = \left(\frac{q_a}{T_f - T_i} \right)_{\text{free convection}} \quad (8.26)$$

Some justification for this result is obtained from the analysis of laminar free convection from an upward facing horizontal plate of length w and width l_e . This is given approximately by (McAdams, 1954)

$$\left(\frac{q_a}{T_f - T_1} \right)_{\text{free convection}} \propto \omega l_e^{3/4} (T_f - T_1)^{1/4} \quad (8.27)$$

The slopes of the calibration lines in figure 8.3 are almost identical and it follows from equation (8.15) that $\omega l_e^{3/4}$ is nearly constant. Hence $\omega l_e^{3/4}$ will also be nearly constant and equation (8.27) can be written

$$\Lambda \propto (T_f - T_1)^{1/4} \quad (8.28)$$

In the range of film temperatures used in the experiments, the parameter $(T_f - T_1)^{1/4}$ hardly varies and this is corroborated in figure 8.7 where it is observed that the experimental scatter masks any trend in the results. Thus for the calibration results it is permissible to consider Λ - constant.

In the test flow, if the temperature rise of the brass is restricted such that

$$\frac{T_b - T_1}{T_f - T_1} \ll 1$$

(in the calibration $T_b - T_1 = 0$) then the change in free convection from that which occurred in the pipe calibration will be small and Λ may be assumed equal to the calibration value and constant. Assuming Λ - constant, for small changes in the brass mounting temperature, is the key to the construction of the hypothetical calibration line. It is used in the following way.

The electric power to the film is again initiated at $t = 0$ and, as discussed before, the recording system adjusted to sample the voltage. At a time 't' close to $t = 0$, but far enough away that the transients will have sufficiently subsided (usually $15 < t < 20$ sec. e.g. figure 8.6)*, a cap is suddenly placed over and around the film so as to cut off all forced convection. An example of this method applied to a boundary layer in zero pressure gradient is shown in figure 8.8. The resulting $E(t)$ locus which clearly indicates the sharply reduced heat loss that occurs. The time varying 'no flow' voltage is then extrapolated back in time to a point just preceding the air cut-off, and the two voltages, corresponding to forced and free convection are noted. These values, are plotted in figure 8.9 for an $\frac{R_f}{R_c} = 1.2$. The point marking the intersection of the proposed calibration line with the ordinate is found by subtracting Λ from the flow 'off' reading. For this purpose the value $\Lambda = 2.3$ watts, an average obtained from figure 8.2, is chosen. The hypothetical calibration line is now drawn with the slope given by the pipe calibration line corresponding to an overheat ratio $\frac{R_f}{R_c} = 1.2$ (figure 8.1) and this is shown in figure 8.9. The agreement between predicted shear and the one given by the average of five Preston tubes is very good.

* The actual value is not critical since the slope of the lines is small.

8.7 Further Measurements in the Turbulent Boundary Layer

The validity of the method was further established by measuring a number of other wall shears produced by varying the tunnel's free stream velocity. Figure 8.10 compares the flush probe results - each with its own particular calibration line - with the values determined by Preston tubes. The results are also tabulated in Table 8.3. The maximum discrepancy between the hot film results and the average Preston tube predictions is +9% in τ_w , and occurs in only one of the six cases - of the other 5 cases, two give +6%, 1 gives +2% and 2 give 0% (see table 8.3) - and this is no more than the maximum scatter in the Preston tube readings.

Although the agreement is good, the flush probe results do seem to be very slightly but consistently higher and this may be attributed to three possible causes: (a) an incorrect free convection allowance Λ ; (b) the existence of a different law of the wall than that used by Patel to give the Preston tube calibration; (c) an incorrect slope which could be due to a different effective length of film than that which existed in the pipe calibration. Point (a) is the most probable but it was not possible to pinpoint the exact reason for the small discrepancy and the present measurements in the boundary layer have determined τ_w to within 6% (neglecting the one high discrepancy case). The agreement between the hot film and Preston tubes, having V.C. Patel's calibration, also implies a verification of Patel's universal law of the wall. But the maximum extent of the tube was $\frac{U_d}{v} \approx 100$,

which is well below the approximate outer limit, $\frac{U_\tau d}{\nu} = 300$, and the experiments only imply a verification of the constants in the logarithmic law

$$\frac{U}{U_\tau} = A \log \frac{y U_\tau}{\nu} + B \quad \text{for } 30 < \frac{y U_\tau}{\nu} < 100 \text{ approximately}$$

and where V.C. Patel gives $A = 5.5$, $B = 5.45$.

Table 8.3 sums up the boundary layer measurements. For one case, case A, the experimental C_f was compared with that given by the Ludwig-Tillman equation and the agreement is excellent: $C_f = .00301$ and $.00305$ respectively.

8.8 Measurements of Skin Friction in a Plane Wall Jet in Still Surroundings

8.8.1 Previous work

The skin friction in a wall jet in still air has been frequently measured (Sigalla (1958) Myers et al. (1961), Schwarz and Cosart (1961), R.P. Patel (1962), Bradshaw and Gee (1962), Kruka and Eskinazi (1964), Parthasarathy (1964), Tailland and Mathieu (1967), Alcaraz et al. (1968) and Kohan (1968)) but the uniqueness of the relationship linking the skin friction coefficient with the Reynolds number $\frac{y_m U_m}{\nu}$ and especially the universality of the law of the wall are still in doubt. Newman (1968) has given a critical review of most of the work to date and some of the comments below are based on that review.

A number of experimenters have used either Preston tubes

or Clauser plots but these methods imply or require 'a priori' particular values of A and B in the log-law of the wall (V.C. Patel, Dickinson 1964) and therefore cannot be used with confidence to establish other values if such exist. Schwarz and Cosart obtained C_f from the integral momentum equation but this is known to be extremely sensitive to three-dimensional effects (see Chapter 6). Tailland and Mathieu, and Kohan used mean velocity measurements obtained with a hot wire in the sublayer, to infer C_f but this technique suffers from uncertainty in assessing the additional heat loss to the wall and none of the authors accounted for the effect. Kohan's measurements are in good agreement, while Tailland and Mathieu's results are 15 - 20% lower than those given by the floating element balance of Alcaraz et al.

Figure 8.11 shows the results of C_f vs $\frac{y_m U_m}{\nu}$ given by those experimenters who used either Stanton tubes (R.P. Patel, 1962), the hot film probe (Myers et al.) or the floating element balance (Alcaraz et al., Parthasarathy). The results of Myers et al. are uncertain due to a lack of two-dimensionality (a variation of 10% in velocity in the central 30 inches).

R.P. Patel used Stanton tubes - with $\frac{du}{v} < 10$ - calibrated in a boundary layer, using boundary layer traverses and Clauser plots drawn for log constants A = 5.6 of B = 4.9 to give τ_w . The tubes were placed 1 in. to the side of the measuring station and hence, in addition to introducing an error due to the uncertainty of A and B, Patel's calibration may also have been subject

to a calibration error caused by a lateral variation in shear stress produced by upstream screens (R.P. Patel, 1964).

The floating element results are shown in figure 8.11 by the straight line which is Parthasarathy's mean line and by the shaded band which delineates the boundaries of the scatter in Alcaraz et al.'s data. Parthasarathy obtained good agreement (maximum 10% discrepancy) between his balance data and that given by a Preston tube (extending to $\frac{yU}{\nu} = 100$) calibrated in a pipe and found to have the same calibration as the one originally proposed by Preston ($A = 5.8$, $B = 5.5$). V.C. Patel has shown that, at $\frac{Ud}{\nu} = 100$, the original Preston calibration gives a skin friction approximately 5% too low and, if this is accepted, Parthasarathy's line in figure 8.11 should be shifted to the right. Moreover, the readings given by the balance were corrected for gap effects using empirical formulae, and thus the agreement is not fully conclusive.

Alcaraz et al.'s balance predicts well the skin friction of the boundary layer in zero pressure gradient (see Mathieu and Alcaraz, 1967) and it is therefore probable that their measurements in the wall jet are quite reliable. Their results are also in good agreement ($\pm 7\%$ in skin friction) with measurements using five different diameter Preston tubes which were calibrated in a pipe and found to have a calibration nearly identical to the one obtained by V.C. Patel. What is surprising is their conclusion that in the wall jet, a logarithmic law having $A = 4.15$ and $B = 7.5$ exists. This result is

clearly inconsistent with the close agreement they obtained between the balance and Preston tube measurements because the latter, which extended to $\frac{yU}{\nu} = 200$, were calibrated in a different wall law region ($A = 5.6$, $B = 5.4$). This point is discussed further in section 8.3.

8.8.2 The experimental conditions and procedure

The apparatus and experimental conditions were discussed in Chapter 5.

In the present work all measurements, at a given $\frac{x}{D}$, were made at the same point and should consequently be free of errors due to spanwise variations of C_f such as those caused by upstream screens. However, it should be noted that the effect of three-dimensionality on the law of the wall is unknown.

The skin friction was measured using techniques identical to those described earlier for the turbulent boundary layer, save that in this case an overheat ratio $\frac{R_f}{R_c} = 1.25$ was used because experience indicated less scatter in the results than for $\frac{R_f}{R_c} = 1.2$. Five different Preston tubes were used at the farthest downstream station where the boundary layer was thickest. Nearest to the slot at $\frac{x}{b} = 38$, where y_m is only .13 inches, the two smallest tubes were tried. Table 8.4 lists the non-dimensional distance $\frac{U_d}{\nu}$ occupied by the probe orifice. In addition to Preston tubes two estimates of wall shear were obtained from Clauser plots drawn using the log-law constants $A = 5.5$, $B = 5.45$ and $A = 4.15$, $B = 7.5$ (see Discussion in next section). To

do this mean velocity profiles were obtained with the flattened tube described in section 8.3.4 (also see figure 8.1b). Care was taken to determine accurately the position of the geometric centre of the probe with respect to the surface, by using an electrical contact method to set the zero. Thereafter the probe was moved with a dial gauge adjustable to .0005".

8.8.3 Results and discussion

Table 8.4 summarizes the results. It is seen that in three of five cases there is excellent agreement between the Preston tube and the heated element. In the other two ($\frac{x}{D} = 38$ and 139) the Preston tube predicts a 10% lower τ_w . This trend, whereby the hot film results are, if anything, higher, is similar to what was observed in the turbulent boundary layer measurements of section 8.7, except that now the maximum discrepancy is slightly larger. (The larger discrepancy is not explainable on the grounds of an increased turbulence intensity in the sublayer because this would cause the Preston tube to read higher. The effect on the hot film is thought to be negligible since identical calibrations have been reported in laminar and turbulent boundary layers.)

Table 8.4 lists $\frac{U_d}{v}$ - the extent to which the Preston tube penetrated into the wall law region - and the maximum is approximately 100. The agreement between the flush probe and the tubes therefore implies that the law of the wall, given by V.C. Patel, is at least valid in this zone ($0 < \frac{yU_d}{v} < 100$) and in particular that of the log law

$$\frac{U}{U_\tau} = 5.5 \log \frac{yU_\tau}{\nu} + 5.45$$

holds in the region $30 < \frac{yU_\tau}{\nu} < 100$.

Alcaraz et al., however, show that in a zone starting further from the wall - $40 < \frac{yU_\tau}{\nu} < 300$ - it is possible to fit the curve

$$\frac{U}{U_\tau} = 4.15 \log \frac{yU_\tau}{\nu} + 7.5 \quad (8.29)$$

But by definition of the law of the wall it is the region closest to the wall that should least be affected by changes in the outer portion of the boundary layer and, from the point of view of the establishment of a logarithmic law, this suggests that $\frac{yU_\tau}{\nu} = 30$ should be used as an 'anchor point'. If this is done then it can be shown that the relation

$$\frac{U}{U_\tau} = 5.5 \log \frac{yU_\tau}{\nu} + 5.3$$

fits quite well the data of Alcaraz et al. in the approximate region $30 < \frac{yU_\tau}{\nu} < 100$ and substantiates the results obtained by this author.

The Clauser plots for V.C. Patel's and Alcaraz et al.'s log-law are shown in figures 8.12 - 8.13, and 8.14 - 8.15 respectively. The points shown are uncorrected but the two

extreme cases - smallest and largest $\frac{x}{b}$ - are marked by a horizontal line indicating the possible shift in position introduced by a variation of $\pm .001$ in. in y . The possible error is very significant at small $\frac{yU}{\nu}$. Although some care was taken to accurately position the tube, by using the electrical contact method to establish the 'zero' point, a positioning error of .001 in. was possible. In addition a displacement correction may be significant. Some recent results due to Ozarapoglu and Dickinson (1969) indicate that the displacement correction to flat pitot tubes in a turbulent boundary layer, varies from a large negative correction near and on, the wall ($\Delta y \approx - 0.3 \times$ height of tube) to an identical but positive one further out and that the maximum corrections are larger in magnitude than any suggested for round tubes. Hence, the points near the wall - on the plots approximately $\frac{yU_m}{\nu} = 1500$ or roughly $\frac{yU}{\nu} < 100$ are not considered reliable. Beyond about $\frac{yU}{\nu} = 100$ the results are generally more consistent with the Clauser plot based on Alcaraz et al's log law (figures 8.14 and 8.15) than with that given by V.C. Patel's and the values of C_f obtained from figures 8.14 and 8.15 are given in Table 8.4. The discrepancy between these and the hot film results is, in three out of four cases, bigger than the discrepancy between the film and the Preston tube.

The above results support the existence of a universal log-law in the zone $30 < \frac{yU}{\nu} < 100$ and show that another logarithmic relation can be fitted through the data if a different lower

value of $\frac{yU_I}{\nu}$ is chosen as an 'anchor point' through which the line is forced to pass. This is also true in pipe flow; for example, taking the results of Alcaraz et al. it is possible to fit the line

$$\frac{U}{U_\tau} = 6.35 \log \frac{yU_\tau}{\nu} + 3.65$$

in the range $100 < \frac{yU_I}{\nu} < 1000$

whereas, in the range $30 < \frac{yU_I}{\nu} < 300$

these authors show that

$$\frac{U}{U_\tau} = 5.6 \log \frac{yU_\tau}{\nu} + 5.4$$

fits the data well. The existence of a number of possible logarithmic regions, with the universal one being closest to the wall has also been observed in the wall jet by Kohan.

The data is plotted as C_f vs $\frac{y_m U_m}{\nu}$ in figure 8.11 and falls approximately within the shaded strip representing the scatter in Alcaraz et al's data. The results are also in good agreement with Parthasarathy's mean line and the experimental range of $\frac{y_m U_m}{\nu}$ is not large enough to establish which mean line is correct.

8.9 Measurements of Skin Friction in the Wall Jet over the Logarithmic Spiral

See Chapter 5 and section 8.8.2 for the experimental conditions and procedure respectively. Note that for these surfaces the velocity profiles used in the Clauser plots were obtained with hot wires (Chapter 5) and they should not be subject to errors caused by displacement effects. The wire readings would

be in error due to additional heat loss to the surface but the points of interest for the Clauser plots were always well over 100 wire diameters from the wall. In this section, only the results relevant to the discussion of skin friction measurements will be treated. Other results will be considered in Chapter 9.

8.9.1 Results and discussion

The results for the $\frac{x}{R} = \frac{2}{3}$ and 1 spirals are summarized in tables 8.5 and 8.6 respectively and for these surfaces, unlike the previous, there was no tendency for the hot film measurements of τ_w to be slightly higher than those given by the Preston tube. For the $\frac{x}{R} = \frac{2}{3}$ surface, the values of τ_w given by the hot film were 0%, 18% and 21% of the Preston tube predictions (1 indicates the hot film result is higher), whereas for the $\frac{x}{R} = 1$ surface the hot film results are 21% and 23% higher.

The considerable scatter on the $\frac{x}{R} = \frac{2}{3}$ spiral gives an average deviation which is nearly zero and this suggests that the universal wall law is valid in the range covered by the Preston tube which in this case is $\frac{yU_\tau}{\nu} = 150$ (the reading of tube no. 1 at $\frac{x_1}{b} = 116$ is considerably higher than the rest and indicates the wall law may fail for this size tube).

Further evidence to justify this is provided by the excellent agreement between the prediction of the Clauser plot, for the log-law constants $A = B = 5.5$ (figure 8.17), and that of the hot film. (The data for $\frac{x_1}{b} = 239$ is felt to be more reliable

because the inner layer thickness, y_m is twice that at $\frac{x_1}{b} = 116$.) But the conclusion can only be tentative because the prediction of the Clauser plot and Preston tubes disagree by 10% even though the two methods are based on the same values of A and B and this will be discussed in more detail below. For interest the Clauser plot, derived using the constants $A = 4.15$ and $B = 7.5$ given by Alcaraz et al. for the plane wall jet (see previous section), is shown in figure 8.18 and the agreement with the hot film is not so satisfactory even though the points beyond about $\frac{yu_1}{v} = 150$ seem to agree better with this relation than with the log-law given in figure 8.17.

The results on the $\frac{x}{R} = 1$ spiral are few, but they do intimate that the law of the wall fails for this surface. The Clauser plot results are 10% lower than those given by the Preston tubes and this discrepancy is identical to that found for $\frac{x_1}{b} = 239$ on the $\frac{x}{R} = \frac{2}{3}$ surface. In view of this discrepancy between the Clauser plots and the Preston tubes and between the latter and the hot film, it is of interest to examine whether either the existing pressure gradient or the shear stress gradient may be responsible for the failure of the logarithmic law.

It was shown by Townsend (1961) that the condition for the log law to be valid is that

$$\frac{y}{\tau_w} \frac{\partial \tau}{\partial y} \ll 1$$

and for the log-law to be valid at $\frac{yu_1}{v} = 200$ say, requires

$$\frac{\nu}{\rho U_{\tau}^3} \frac{\partial \tau}{\partial y} < 0.005 \quad (8.30)$$

Consider the $\frac{x}{R} = 1$ spiral at $\frac{x_1}{b} = 284$. The point $\frac{yU_1}{\nu} = 200$ occurs at $\frac{y}{y_m} = .071$ and the shear and surface pressure measurements reveal (Chapter 9) that a conservative estimate for $\frac{\partial \tau}{\partial y}$ at this point can be calculated from

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x} \quad \text{at } y=0 \quad (8.31)$$

and the results give

$$\frac{\nu}{\rho U_{\tau}^3} \frac{\partial \tau}{\partial y} = 0.0014$$

This value is not an order of magnitude less than the limit imposed in equation 8.30 and the validity of the log-law on the $\frac{x}{R} = 1$ spiral is therefore doubtful. Thus the disagreement between the hot film and Preston tubes, on this surface, may be due to the affect of pressure gradient rather than the influence of the outer layer.

On the $\frac{x}{R} = \frac{2}{3}$ spiral at $\frac{x_1}{b} = 239$ the value $\frac{yU_1}{\nu} = 200$ occurs at $\frac{y}{y_m} = .145$ and again using 8.31 and the results described in Chapter 9 yields

$$\frac{\nu}{\rho U_{\tau}^3} \frac{\partial \tau}{\partial y} = 0.00058$$

The value meets the criterion given by equation (8.30) and supports earlier evidence ascertaining the validity of the logarithmic law having constants $A = B = 5.5$

Figure 8.16 shows the hot film measurements plotted as C_f vs $\frac{y_m U_m}{\nu}$. It is noted that at a given $\frac{U_m y_m}{\nu}$ the skin friction coefficient is higher on the $\frac{x}{R} = 1$ spiral than for the other surfaces and it follows that a given U_τ is associated with a smaller U_m .

8.10 Conclusions

(1) A successful method has been developed by which a hot film probe (Disa type 55A92) could be calibrated in a standard calibration rig (in this case a circular pipe) and then moved to the surface of interest. (This has a great advantage when the device cannot be calibrated locally such as when it is used on a wing.) Shifting the probe to another apparatus altered the heat transfer and therefore changed the temperature in the material surrounding the film which caused a shift in the calibration curve. This was overcome with a transient measuring technique which gave the heat loss by forced and free convection a very short time after the film was first heated.

(2) When the flush probe was calibrated in the pipe the results agreed well with the well known linear relation between heat output and the cube root of the shear stress. Using this calibration, in conjunction with the technique mentioned above,

the flush probe was used to measure the shear stress in a plane turbulent boundary layer in zero pressure gradient. The results were in good agreement - an average overall discrepancy of 5% (the maximum was 9% in one reading) - with Preston tube results obtained from calibrations given by V.C. Patel (1965).

(3) In the wall jet in still air, good agreement was obtained between predictions of the hot film and of numerous Preston tubes. (At five downstream stations, discrepancies of +3%, 2%, -11%, 0, -11% respectively, were obtained where + sign implies a higher average Preston tube reading.) The Preston tubes extended to $\frac{yU_\tau}{\nu} \approx 100$ and this confirmed the validity in the range $30 \leq \frac{yU_\tau}{\nu} \leq 100$ - to within roughly $\pm 5\%$ in the constants - of the logarithmic law of the wall proposed by V.C. Patel

$$\frac{U}{U_\tau} = 5.5 \log \frac{yU_\tau}{\nu} + 5.45$$

and is in substantial agreement with results given by Parthasarathy (1964), and Kohan (1969). Beyond $\frac{yU_\tau}{\nu} \approx 100$, Clauser plots indicated the logarithmic relation proposed by Alcaraz et al.

$$\frac{U}{U_\tau} = 4.15 \log \frac{yU_\tau}{\nu} + 7.5$$

fits the data and this again is in agreement with the observations of Kohan. When plotted as C_f vs $\frac{y_m U_m}{\nu}$ the points agree satisfactorily with the floating balance measurements of

Alcaraz et al. (figure 8.11) and of Parthasarathy.

(4) On the logarithmic spirals the results were not numerous enough to permit definitive conclusions. Calculations based on the derivation of the log-law indicated that it should be valid on the $\frac{x}{R} = \frac{2}{3}$ spiral and an average of the hot film data tended to substantiate the existence of the constants $A = B = 5.5$ in the range $30 < \frac{yU}{\nu} < 150$. On the $\frac{x}{R} = 1$ surface the calculations indicated that the log-law might fail and this was substantiated by the hot film results which gave a shear stress about 20% higher than that predicted by the Preston tubes.

PART D

EXPERIMENTAL RESULTS AND COMPARISON WITH THEORY

adjacent to the end plates. Three convenient methods for verifying two-dimensionality were suggested: (a) a comparison of velocity profiles measured at various lateral positions, (b) a verification that the velocity and length scales describing the flow vary downstream in a manner consistent with the two-dimensional momentum equation, and (c) a comparison between measured and calculated shear stress. The results of the first approach were considered in Chapter 6 and the results of the last two methods are discussed below.

Line 1 of figure 9.10 shows the experimental variation of maximum velocity on both the $\frac{x}{R} = \frac{2}{3}$ and 1 spirals, (for convenience the data is referred to the first measured station called $x - x_r$) and lines 1 and 3 represent the respective calculated variations obtained by solving, with a Runge Kutta technique, the integral x-momentum equation using the known growth, skin friction, Reynolds stress distribution and shape of velocity profile. (All these quantities are individually described in the ensuing paragraphs.) It is observed that the calculated behaviour for the $\frac{x}{R} = \frac{2}{3}$ spiral gives excellent agreement with results measured by either pitot tubes (assuming a linear static pressure variation - see Chapter 5) or hot wires and the adequate two-dimensionality of the flow appears confirmed. The agreement for the $\frac{x}{R} = 1$ surface is not as good and at $\frac{x}{x_r} = 2$, which corresponds roughly to 150 slot widths downstream, the difference between calculated and measured mean velocity is 3%.

For comparison the same calculation is applied to the results of Giles et al. for the $\frac{x}{R} = 1$ surface, and this is

shown in figure 9.11. The skin friction coefficient was not measured by these authors and was therefore taken from the present hot film measurements, but this is considered acceptable in light of the relative insensitivity of the flow to the precise value of this parameter (to be discussed below). The two-dimensionality of their flow seems quite satisfactory up to approximately $\frac{x}{x_r} = 2$ but it is evident from figure 9.12 that their velocity decay does not follow the self-preserving form: $U_m \sim x^a$.

The momentum check performed above appears to confirm that both of the present equilibrium flows were satisfactorily two-dimensional. But a comparison between the measured (and corrected for both high intensity turbulence and longitudinal cooling - Chapter 7) and calculated shear stress distributions shown in figures 9.22 and 9.27 was not as encouraging and the discrepancy at the point of maximum shear stress was 13% and 19% of the calculated value for the $\frac{x}{R} = \frac{2}{3}$ and 1 spirals respectively.

It is difficult to attribute the discrepancy in shear to errors in measurement. Much effort was made to assure accurate data and the excellent non-dimensional collapse of the parameters (see discussion below) as well as very satisfactory measurements of shear stress performed in a pipe confirmed the validity of the measurement technique. As an additional check on the measurements the shear stress profile was measured on different days and with different wires of type 55A29 and the excellent collapse of the results is shown in figure 9.23. (See section

5.3 for a detailed description of the procedure.)

It was shown in Chapter 6 that a slight divergence of the flow ($0[2^\circ]$), while giving rise to only a small discrepancy between the calculated and measured maximum velocity decays, can lead to a large difference in the corresponding shear stress distributions. The calculated shear stress is therefore not representative of the value in a two-dimensional flow, and is in error because the loss of momentum due to flow divergence has been incorrectly attributed to the action of shear forces. On the other hand for such a small flow divergence the measured shear is unlikely to be largely in error and it will be assumed most representative of conditions in an absolutely two-dimensional flow.

It was seen above that good agreement exists up to $\frac{x}{x_r} = 2$ between the measured and calculated maximum velocity decay on the $\frac{x}{R} = 1$ surface of Giles et al. Nevertheless, the discrepancy between their measured and calculated shear is extremely large; at $y = y_m/2$ the calculated value is $\tau_w/\rho U_m^2 = .046$ while the measured quantity is .016.

9.2 Experimental Results

9.2.1 Mean velocity profiles

The mean velocity, Q , profiles, measured with normal hot wires, are shown plotted for both spirals in figures 9.1 to 9.3 and in these and subsequent figures the measurement stations are defined in terms of their distances from both the slot (x_1)

and the origin of the spiral (x). The latter represents the distance to the virtual origin of the flow while the former includes a length of flat plate required to permit this virtual origin to match the physical origin of the spiral (Chapter 5). The table below shows the relative length of the flat plate in relation to the total distance from the slot to the 1st measuring station (slot width $b = .125$ inches).

Spiral	Approximate number of slot widths between slot and end of flat plate	Approximate number of slot widths between slot and first measuring station
2/3	16	55
1	64	160

The data of figures 9.1 to 9.3, reduced non-dimensionally by using the scales $y_m/2$ and U_m , is shown plotted in figures 9.4 and 9.5. It is observed that the collapse of the data is good although at the furthest measurement station there is some tendency to stray from the self-similar form but in each case this takes place at well over 200 slot widths. In these figures there is also shown both the total velocity, Q , and the x component, U , which was obtained by calculating the y -component of velocity V using the continuity equation. The only significant corrections are near the outer edge of the jet.

The shape of a wall jet mean velocity profile suggests a division between an inner layer, which resembles the usual boundary layer and an outer layer which resembles a half-jet profile, and this observation has been exploited in a number of analyses of jet flows (e.g. Gartshore and Newman, 1969) and

is used in the present work to analyze the flow on the circular cylinder. Comparing figures 9.4 and 9.5 it is seen that the relative thickness of the inner layer increases with surface curvature and results presented below will show that this is due to an increase in the influence of the outer flow on the inner.

The outer layer profiles, plotted non-dimensionally in figures 9.6 and 9.7, are compared with some analytical expressions for the profiles, and with the present measurements of the plane wall jet. The 'sech²' profile is that given by a constant eddy viscosity theory (e.g. Glauert, 1956) and agrees most closely with the experiments especially in the region $y < y_{m/2}$ which contains about 95% of the total jet momentum. Figure 9.7 shows that in that region the profile on the $\frac{x}{R} = 1$ surface is fuller and agrees more closely with the 'sech²' profile than does the velocity distribution on the $\frac{x}{R} = \frac{2}{3}$ spiral and this intimates the existence of a more vigorous mixing process which tends to smooth out the velocity variations. The mechanisms involved will become more evident in the discussion of the turbulence parameters and the intermittency. Figure 9.5 also shows the mean velocity distribution given by Giles et al. and the discrepancy with the present results is noteworthy.

9.2.2 Growth of jets

The thickness of the wall jets is characterized by the parameters $y_{m/2}$ and its downstream development is shown in

figure 9.8. The results are (note that $y_{m/2}' = \frac{dy_{m/2}}{dx}$):

$\frac{x}{R} = 1$ spiral :	Present results	$y_{m/2}' = .277$	Difference = 8% of mean
	Giles et al.	$y_{m/2}' = .300$	
$\frac{x}{R} = \frac{2}{3}$ spiral :	Present results	$y_{m/2}' = .155$	Difference = 15% of mean
	Giles et al.	$y_{m/2}' = .180$	
plane wall jet	Present results	$y_{m/2}' = .071$	Difference = 8% of mean
	Giles et al.	$y_{m/2}' = .0765$	

It is interesting that the present measurements are consistently lower than those of Giles et al. It may be that three dimensional effects, attributable to irregularities at the slot lip are responsible (Chapter 6) but it is curious that these should cause a difference having consistently the same sign, especially since the results of Giles et al. are composed of two independent investigations: that due to Sawyer (1962) on a $\frac{x}{R} = \frac{2}{3}$ spiral with an 18" span and that due to Giles and Hays (1965) on the plane wall jet and on the $\frac{x}{R} = 1$ spiral with 12" span. (The present spirals had 24" spans.) A consistent error could be caused by secondary flows near the end plates but this explanation is not convincing because the importance of secondary flows should decrease as the span width increases but the largest discrepancy between the present results and those of Giles et al. occurred between the spirals $\frac{x}{R} = \frac{2}{3}$ which had comparable spans: 18" for Sawyer vs. 24" in the present case. Photographs shown by Sawyer (1962) of the traversing gear of Giles et al. reveal that it is particularly bulky and it may possibly have caused some outward displacement of the flow.

Although no satisfactory explanation can be offered for the differences encountered it is emphasized that the discrepancies are not excessive and are well within the scatter shown by published plane wall jet results. Indeed, from the survey of Kohan (1969), the range of growths of plane wall jets at $Re > 1.2 \times 10^4$ is approximately $.085 < y_{m/2}' < .065$ which represents a variation of $\pm 13\%$ of the mean value.

The development of the inner layer is shown in figure 9.9 and the linear growths, which give constant value of the ratio $y_m/y_{m/2}$, further corroborate the existence of self-preservation. The results are:

$\frac{x}{R} = 1$ spiral	$y_m/y_{m/2} = .202$
$\frac{x}{R} = \frac{2}{3}$ spiral	" = .159
plane wall jet	" = .150

9.2.3 Maximum velocity decay

These results are contained in figures 9.10, 9.11 and 9.12. The first two were discussed at length in section 9.1 and attention is now focussed on figure 9.12 which shows the data of figure 9.10 replotted using logarithmic scales and from which the value of the exponent 'a' in $U_m \propto x^a$ can be calculated. Again it is seen that the experimental results fall approximately on one straight line and there is no clear variation in 'a' between the two spirals. Lines 2 and 4 are based on values of 'a' calculated from the equation 2.27. They coincide with ~~points taken from~~ lines 1 and 3 of figure 9.10 and this provides

a check on the computer analysis.

The discrepancy between the calculated and experimental lines is attributed to three dimensional effects. The difference in the value of 'a' on the $\frac{x}{R} = 1$ spiral is 8% of the measured value and as seen in section 9.1 this results in a 3% discrepancy in (U_m/U_{mr}) at $(x/x_r) = 2$.

9.2.4 Surface pressure and static pressure variations throughout the jet

The full y momentum equation 2.6 or the simplified version

$$p_{\infty} - p_w = \int_0^{\infty} \frac{u^2}{R+y} dy \quad (3.72)$$

show that $\frac{p_{\infty} - p_w}{\rho U_m^2} = \text{constant}$

and the computed results for the two spirals are shown in figure 9.13. The difference between the two calculations is very small and, in the case of the $\frac{x}{R} = \frac{2}{3}$ spiral cannot be discerned on the graph. The discrepancy with the measurements for the $\frac{x}{R} = 1$ spiral is expected from the difference between the measured and calculated maximum velocities: the measured velocity is higher and the surface pressure is consequently lower.

Figure 9.14 will be discussed later (see section 9.4).

Figure 9.15 shows the distribution $\frac{p_{\infty} - (p + \rho v^2)}{\rho U_m^2}$ calculated using both the full Y-momentum and the relation

$$p_0 - (p + \rho \overline{v'^2}) = \int_y^\infty \frac{u^2}{R+y} dy$$

The higher order terms become important for about $y/y_{m/2} > 0.6$. The static pressure across the jet can be obtained from figure 9.15 by using the measured distribution of $\frac{\overline{v'^2}}{U_m^2}$ and this is shown in figure 9.16. Comparison of the two figures shows that $\overline{pv'^2}$ is comparable in magnitude to p and this agrees with the observations of Newman (1961). The preliminary velocity traverses required to establish the equilibrium flows (section 5.3) were obtained by assuming a linear static pressure distribution between the wall pressure and ambient conditions at $y/y_{m/2} = 2.0$, and figure 9.16 shows that the procedure gives a reasonable mean line through the actual distribution, especially in the range $0 < y/y_{m/2} < 1$.

9.2.5 Reynolds stresses

The mean velocity profile of a wall jet can be divided into inner and outer layers corresponding to $y < y_m$ and $y > y_m$ respectively, and this observation was found mathematically useful when deriving the integral theory for predicting the behaviour of the flow on the Coanda cylinder. However, it is now common knowledge that in the plane wall jet the shear stress at the velocity maximum is not zero; the inner and outer layers interact and it is not possible to assume that each layer possesses the properties of either the usual boundary layer or of the plane free jet.

Indeed, in the flows being considered here, the outer layer dominates the inner and the turbulence structure in the outer portion is hardly affected by the relative thickness of the inner layer. This is shown in figures 9.17a and 9.18 where the shear stress distribution - which will be discussed in detail later - is shown plotted as $\overline{u'v'}/\overline{u'v'}_{MAX}$ versus either the coordinates $y/y_{m/2}$ or $y-y_m/y_{m/2}-y_m$ respectively. It is evident that using the distance from the wall provides a much better collapse of the data than using the distance from the maximum velocity point, and in figure 9.17 it is seen that the point of maximum shear stress is almost invariant for the different flows shown. The above evidence suggests that the parameter $y/y_{m/2}$ should be used to correlate all the turbulence data. In fig. 9.17b it is seen that the collapse is further improved if the shear stress is referred to the wall value.

The Reynolds stress data is presented in the following way: the raw data is first shown and the necessary corrections for high intensity turbulence and longitudinal cooling are applied; the Reynolds stresses are then compared for each surface and their variation with surface curvature examined.

The measured Reynolds stresses are shown plotted in figures 9.19 to 9.23 and 9.24 to 9.27 for the $\frac{x}{R} = \frac{2}{3}$ and 1 spirals respectively. Attention is given primarily to the outer region of the flow, and details of the inner layer, such as the possible existence of a law of the wall, are reserved for section 9.3.

A. $\frac{x}{R} = \frac{2}{3}$ spiral

The distribution of $\frac{\overline{u'^2}}{U_m^2}$ is shown in figure 9.19 and it is observed that self-preservation occurs first in the outer portion of the jet and then gradually encompasses the entire flow. This phenomenon can also be observed in the work of other experimenters (e.g. Wygnanski and Fiedler, 1968) and is not surprising since after the slot, a shear layer first begins to form at the outer edge of the flow. The jet appears completely self-preserving by about $\frac{x_1}{b} \approx 116$. Further downstream, it appears that deviations from self-preservation first begin in the inner layer at about $\frac{x_1}{b} = 197$ and then move progressively outwards towards the edge of the jet. These deviations can be attributed to three-dimensional effects and the increase in $\frac{\overline{u'^2}}{U_m^2}$ suggests a divergence of the flow in the inner layer whereby vortex lines aligned in the z-direction are stretched so as to increase both u' and v' . However, measurements of $\overline{v'^2}$ shown in figure 9.20 contradict this hypothesis and show that the data at $\frac{x_1}{b} = 197$ is in excellent agreement with upstream measurements. The measurements of $\overline{w'^2}/U_m^2$ are shown in figure 9.21 and again the deviation from self-preservation begins between $197 \sim \frac{x_1}{b} 239$. The measured shear stress distribution is shown in figure 9.22 and was discussed in some detail in section 9.1. It is observed that it is narrower than the normal stress distributions and this may be because shear is zero and normal stresses are finite in the irrotational fluid outside the turbulent front. The dotted lines in the above figures represent the mean Reynolds

stress distribution corrected for both longitudinal cooling and high intensity turbulence effects. The latter corrections were discussed in Chapter 4 and are given in Figures 7.12 to 7.15. The former are calculated from the first term in equations 7.24 to 7.27 using the longitudinal cooling correction factor $k = .2$ which is appropriate to the type of wire used (see section 7.1).

B. $\frac{x}{R} = 1$ spiral

These results are shown in figure 9.24 to 9.27. The scatter is more pronounced than for the previous spiral but there is no obvious trend away from self-preservation as was seen at large $\frac{x_1}{b}$ for that case. Also shown are the results of Giles et al. who obtained their data using non-linearized normal and X-wires. Their measurements of $\overline{u'^2}/U_m^2$ were obtained with the normal wire and the agreement with the present data is quite good below $y/y_{m/2} = 1$, and the increasing discrepancy as y increases may be attributed to errors introduced by the non-linearity of their anemometer response. The agreement between the $\overline{v'^2}/U_m^2$ measurements is less satisfactory but nevertheless is surprisingly good in view of the rather large scatter that exists between the various published measurements in the plane wall jet (see introduction and figure 1.3). Indeed Giles et al. used a X-wire to measure $\overline{v'^2}$ and it is known (Jerome et al., 1970) that this probe is susceptible to errors introduced by thermal wake interference. If to this error there is added the possible errors due to non-linearity of the signal

and divergence effects due to three dimensional flow then the agreement shown in figure 9.25 is indeed good. Giles et al. did not measure $\overline{w'^2}/U_m^2$.

The shear stress distribution is shown in figure 9.27. The comments made for the $\frac{x}{R} = \frac{2}{3}$ spiral are applicable and the relation between measured and calculated shear is discussed in section 9.1. The result of Giles et al. again agree reasonably well with the present measurements up to say $y/y_{m/2} \approx .8$ but although their maximum shear stress is higher than the present it does not agree more closely with the calculated value; for the present results the ratio of calculated maximum shear to the measured and corrected value is 1.25 whereas for Giles et al. the ratio is 1.35 (the growth of their jet is $y_m/2/x = .30$).

Again the dotted line in the preceding figures represent the mean Reynolds stress distribution corrected for both longitudinal cooling and high intensity turbulence effects.

C. Comparison of stresses and structure

Figures 9.28 to 9.30 compare, for each surface, the measured and corrected Reynolds stresses. An immediate observation is that on the curved surfaces $\overline{v'^2}$ has increased relative to $\overline{u'^2}$ and this is brought out more clearly in figure 9.35 which shows the variation of the parameter $\overline{u'^2}/\overline{v'^2}$. In plane flows the equation describing the rate of change of $\overline{u'^2}$ ($D\overline{u'^2}/Dt$) is the only Reynolds stress equation to contain a production term related to the mean flow. In curved flow,

however, it transpires that the equation for $\overline{v'^2}$ also contains such a term and it is possible to draw a parallel between production due to curvature and production due to buoyant effects which occurs in flows having a density stratification. Indeed, Bradshaw (1969) has shown that the ratio of v' production to u' production in curved flow is

$$R_f = \frac{2\overline{u'v'} \frac{u}{R+y}}{\frac{\overline{u'v'}}{R+y} \frac{\partial [(R+y)u]}{\partial y}}$$

which to $O(\frac{y_m/2}{R})$, and for the self-preserving velocity profile, can be written at $y = y_m/2$ as

$$R_f = 1.25 \frac{y_m/2}{R}$$

The value of R_f for the $\frac{x}{R} = 1$ spiral is about three times that for the $\frac{x}{R} = \frac{2}{3}$ spiral. The experimental results do give a slightly smaller value of $\overline{u'^2}/\overline{v'^2}$ for the former case than for the latter and this is consistent with the above reasoning. There is also a substantial change from the plane flow. It should be noted that the above observations remain unchanged if the uncorrected stresses are used.

The effect of curvature as seen between the plane flow and the $\frac{x}{R} = \frac{2}{3}$ spiral is to increase w' relative to u' (figure 9.36) and it is curious that this trend should be reversed between the $\frac{x}{R} = 1$ and $\frac{2}{3}$ spirals. It may be that divergence of the flow, which tends to stretch favourably oriented vortex lines (see discussion of $\frac{x}{R} = \frac{2}{3}$ spiral) and thus increase u' relative to w' , is responsible but there is no direct evidence

to confirm this.

Figures 9.31 to 9.33 show the variation of each normal Reynolds stress with surface curvature and as observed above it is only $\overline{w'^2}$ that does not continuously increase. An interesting feature of these plots is that, in the outer part of the jets, the measured distributions of $\overline{u'^2}$ and $\overline{v'^2}$ tend towards a unique and isotropic relation. This data is in agreement with Townsend (1969) who suggests that at the interface between turbulent and non-turbulent fluid the newly created turbulence is isotropic and has not yet acquired the characteristics of the particular flow.

The variation of turbulent energy, $\overline{q^2}/U_m^2$, on the surfaces is given in figure 9.34 and confirms the overall increase due to the additional 'buoyant' production term. The structure parameter $\overline{u'v'}/\overline{q'^2}$ has been calculated using the measured (and corrected) stresses and is shown in figure 9.37. No obvious trend can be detected; the maximum scatter about an approximate mean line is $\pm 6\%$ and it appears that, within the experimental accuracy, the structure is invariant. This is in agreement with the hypothesis made by Townsend (1969) (see section 3.2) and will be discussed further in section 9.5.

In order to correct the Reynolds stresses for the effects of high intensity turbulence it was necessary to measure a number of correlations between the fluctuating velocity components (see Chapter 7) and these are shown in figures 9.38 to 9.40.

Figure 9.38 shows the shear stress correlation coefficient measured directly, with an electronic correlator, on the two spirals and in the plane wall jet; the measurements fall convincingly on a single curve and thus provide further evidence of the structural similarity between these flows and indeed between most shear flows since the measurements do not differ substantially from those taken in a plane free jet by Bradbury (1965) or in the round jet by Gibson (1963). The shear stress calculated from $\overline{u'v'}/\sqrt{\overline{u'^2}}\sqrt{\overline{v'^2}}$ using the measured values of $\overline{u'^2}$ and $\overline{v'^2}$ is in good agreement with the one measured directly using a slanted wire and this provides an additional check on the measurements.

The flatness factor $\overline{v'^4}/(\overline{v'^2})^2$ is shown in figure 9.39 and, over most of the jet, is close to the value (= 3.0) given by a Gaussian distribution.

The triple correlations $\overline{u'v'^2}$ and $\overline{u'w'^2}$ are shown in figure 9.40 and 9.41. In the former figure $\overline{u'v'^2}$ becomes more positive with the increasing curvature and mixing length arguments (which postulate that a positive u' is associated with a positive v') suggest that convex curvature inhibits the existence of negative v' .

9.2.6 Intermittency

Figure 9.42 compares the present measurements of intermittency in the plane wall jet in still air, with measurements by Gartshore (1966) and Giles et al. (1966), and the agreement

is indeed poor. No obvious explanation can be given for the discrepancy. The growth and velocity decay in the flows were similar and both Gartshore and Giles et al. obtained their data with electronic circuits which were apparently checked by using a visual analysis identical to the present method. To be consistent, the present wall jet measurements will be used for comparison with the measurements on the logarithmic spirals.

Figures 9.43 and 9.44 show the measurements taken on the $\frac{x}{R} = \frac{2}{3}$ and 1 spirals respectively. In the former figure the data collapses onto a single curve. In the latter, there is a downstream variation which, at such a large distance from the slot ($\frac{x_1}{b} > 200$), may be attributed to three-dimensional effects but this cannot be claimed with certainty since such a variation did not appear in the measurements of Reynolds stress. Nevertheless, it will be assumed that the more reliable data is at $\frac{x_1}{b} = 205$. The dotted line in figure 9.44 represents the data of Giles et al. They did not measure beyond $y/y_{m/2} = 1.7$ but their results nevertheless indicate a much wider intermittent zone. For small $y/y_{m/2}$ they agree with the present results measured at $\frac{x_1}{b} = 284$, but they disagree at large values of $y/y_{m/2}$ where the intermittency is more easily and accurately measured.

The present intermittency results are gathered in figure 9.45 and two important observations can be made: (a) the relative mean position of the superlayer moves inward as the curvature increases, and (b) the relative width of the inter-

mittent zone - as specified by the standard deviation $(\sigma/y_{m/2})$ - does not vary in a consistent manner with increasing curvature, but it is nevertheless smaller for the two spirals than for the plane wall jet. These results are particularly interesting in light of some recent ideas on the behaviour of turbulent shear flows.

Gartshore (1966) has suggested that the standard deviation of the superlayer from its mean position, is a measure of the length scale of the large eddies but Bradshaw (1964)

suggests that the size of these eddies is more likely related to the average position of the interface. The present results, in conjunction with either of the above hypotheses, show that the ratio of the large eddy length scale to the width of the flow decreases with increasing curvature.

By considering the large eddy as a vortex, Gartshore (1967) and Gartshore and Newman (1969) argued that its scale, relative to that of the mean flow, is primarily influenced by the mean rates of strain $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ and that consequently

$$\frac{y_{m/2}}{\sigma} = h \left(\frac{\partial u / \partial x}{\partial u / \partial y} \right)$$

The function h is shown to be even and Gartshore uses

$$\left(\frac{y_{m/2}}{\sigma} \right)^2 \propto 1 + \text{constant} \cdot \left(\frac{\partial u / \partial x}{\partial u / \partial y} \right)^2_{y_{m/2}}$$

The present results are insufficient and too scattered to corroborate for curved flows, the form of the above relation (see table at end of section) but they nevertheless do indicate

that the decrease in size of the large eddies with respect to $y_{m/2}$ in the convex flows is qualitatively in accord with the effect of rate of strain on the eddy.

It has been suggested by Gartshore (1966), on the basis of the large eddy equilibrium hypothesis (Townsend, 1956) that the mixing length ' l_2 ' in plane flows

$$l_2^2 \propto \frac{\overline{u'v'}}{(\frac{\partial u}{\partial y})^2} \quad (9.1)$$

can be taken as the length scale of the large eddies and thus for identical profile shapes that

$$\frac{\overline{u'v'}}{U_m^2} \propto \left(\frac{l_2}{y_{m/2}} \right)^2 \propto \left(\frac{\sigma}{y_{m/2}} \right)^2 \quad (9.2)$$

Newman (1967) has used equation (9.2) in order to predict the development of curved free jets but this particular part of his analysis appears incorrect for the following reasons:

The form of equation (9.1) changes for curved flow. The correct equation is obtained by writing the energy equation - in curvilinear coordinates - for the turbulent motion of the large eddy (Townsend, 1956) and then following the analysis of Gartshore (1966): the production and dissipation terms are equated and the kinematic viscosity ν is replaced by the 'eddy viscosity' ν_{T_1} which is assumed to represent the action of the smaller eddies on the large eddies. Furthermore, the large eddy is assumed to have little energy and therefore $\nu_{T_1} = \nu_T$. This

procedure yields

$$(u''^2) \left(\frac{\partial u}{\partial y} - \frac{u}{R+y} \right) \propto N_T \left(\frac{u''^2}{l_l^2} \right) \quad (9.3)$$

where u'' is the characteristic velocity scale of the large eddy.

~~(b) The above equation is incompatible with the present experimental evidence. Rewriting (9.3) yields~~

$$\frac{\tau}{\rho} \propto l_l^2 \left(\frac{\partial u}{\partial y} - \frac{u}{R+y} \right)^2 \quad (9.4)$$

Note that this equation is not the same as the usual mixing length formulation which is written (Guitton, 1964; Bradshaw, 1969)

$$\frac{\tau}{\rho} \propto l \left(\frac{\partial u}{\partial y} + \frac{u}{R+y} \right)^2 \quad (9.5)$$

Consequently l_l cannot be identified as the mixing length as was done in plane flow equation (9.2). Equation (9.4) can also be rewritten as

$$\frac{\overline{u'v'}}{U_m^2} \propto \left(\frac{\sigma}{Y_{m/2}} \right)^2 \left(1 - 0.68 \frac{Y_{m/2}}{R} \right) \quad (9.6)$$

where, following Gartshore (1966) l_l has been replaced by the standard deviation. The experimental data gives $\overline{u'v'}/U_m^2$ increasing with $\frac{Y_{m/2}}{R}$ while the right-hand side of equation

(9.6), in conjunction with experiments, decreases. Thus if the experiments have correctly identified, and measured, the large eddy scale, then equations (9.6) and therefore equation (9.4) are invalid in curved flow.

γ_{flow}	$\left(\frac{\partial u}{\partial x} / \frac{\partial u}{\partial y}\right)^2_{y_{m/2}} \times 10^4$	$\left[\frac{R_T}{y_{m/2}}\right]$	$\left(\frac{y_{m/2}}{0}\right)^2$	$\frac{y_e}{y_{m/2}}$	$y_{m/2}$
Plane free jet in still air (Gartshore, 1966)	39.2	31	6.25	1.81	.115
Plane wall jet in still air (Guitton)	22.1	69	10.9	1.82	.071
$\frac{x}{R} = \frac{2}{3}$ spiral (Guitton)	102	43	15.15	1.62	.155
$\frac{x}{R} = 1$ spiral (Guitton)	344	30	13	1.50	.277

9.3 Experimental Results in Inner Layer

The inner layer of the equilibrium wall jets is a rather unique example of a 'boundary layer type' flow which is submitted to the influence of an intensely turbulent external flow and upon which this outer flow imposes a strong shear stress gradient (figure 9.27).

Figure 9.46 shows the mean velocity distribution in the inner layer. The profile on the $\frac{x}{R} = 1$ spiral is slightly less full and this may be attributed to the increased pressure gradient in this flow: e.g.

$$\frac{x}{R} = \frac{2}{3} \quad \frac{y_m}{U_m^2} \frac{\partial p}{\partial x} = 0.08$$

$$\frac{x}{R} = 1 \quad \frac{y_m}{U_m^2} \frac{\partial p}{\partial x} = 0.14$$

Figure 9.47 shows the distribution of the normal stresses $\overline{u'^2}$ and $\overline{v'^2}$ and the data from each surface collapses quite well. The shear stress measurements are shown in figure 9.48 and they extrapolate convincingly (considering the difficulty of this measurement) to the wall value. The large shear at y_m has already been discussed in the previous section.

The velocity defect plots are shown in figure 9.49. In deriving this relation for 'classic' boundary layers it is assumed that ^{the} velocity difference $(U_m - u)$ in the outer part of the layer is independent of viscosity and depends only on y , y_m and U_τ and thus that:

$$\frac{U_m - u}{U_\tau} = f\left(\frac{y}{y_m}\right)$$

For wall jets where there is a strong shear stress at y_m and it has been suggested by Tailland and Mathieu (1967) that the previous argument can be extended to include $\overline{u'v'}_m$ such that

$$\frac{U_m - u}{U_\tau} = f\left(\frac{y}{y_m}, \frac{\overline{u'v'}_m}{U_\tau^2}\right)$$

The data in figure 9.49 does suggest a slight streamwise variation in the shape of the defect plot. The logarithmic law of the wall can be derived by assuming that the defect law given above and the inner law

$$\frac{u}{U_\tau} = f\left(\frac{yU_\tau}{\nu}\right)$$

overlap. Thus a failure of the velocity defect law implies that the 'log-law' fails but the scatter in the results is important and no definitive conclusions can be drawn. (Note that in Chapter 8 it was suggested that the log-law holds in the $\frac{x}{R} = \frac{2}{3}$ spiral but fails due to pressure gradient on the $\frac{x}{R} = 1$ surface.)

The variation of the Reynolds stress in the wall region is given in figure 9.50 and the results indicate clearly that there is no tendency for $\overline{u'^2}/U_\tau^2$ to collapse when plotted against yU_τ/ν (cf. fig. 9.47) and this corroborates the idea of 'inactive'

or swirling motion parallel to the wall and imposed by the outer flow (Townsend, 1961). Only a few measurements of $\overline{v'^2}$ were made but they suggest a collapse of the data for both surfaces as far out as $yU_\tau/\nu = 450$. The hot film measurements intimated that the classical logarithmic law (with constants $A = B = 5.5$) fails on the $\frac{x}{R} = 1$ spiral but some results in Chapter 8 suggested (in agreement with the observations of Alcaraz et al., 1968) that a different wall law may still be valid, and the plot of $\overline{v'^2}/U_\tau^2$ substantiates this.

9.4 Application of the Frictionless, First Order Theory of Section 3.1

It is recalled that this simple and preliminary theory was introduced to gain more insight into the order of magnitude of the terms in the relevant equations, and to investigate the applicability of equilibrium data to non-equilibrium flows. The empirical information for the theory was provided by assuming equilibrium conditions whereby the growth of the jet is a function of only local conditions and given by

$$\frac{dy_{m/2}}{dx} = \beta_0 + \beta_3 \frac{y_{m/2}}{R} \quad (3.3b)$$

where β_0 is the plane wall jet growth rate (.071 in this case) and β_3 is assumed universal for flow on the spirals.

The present experiments on the logarithmic spirals give (see section 9.2.2):

$$\frac{x}{R} = \frac{2}{3} \text{ spiral} : \frac{dy_{m/2}}{dx} = 0.071 + 0.806 \frac{y_{m/2}}{R} \quad (9.6)$$

$$\frac{x}{R} = 1 \text{ spiral} : \frac{dy_{m/2}}{dx} = 0.071 + 0.742 \frac{y_{m/2}}{R} \quad (9.7)$$

The values of β_3 differ by only about 6% which appears to confirm the universality of equation (3.3b).

The experiments of Giles et al. give:

$$\frac{x}{R} = \frac{1}{3} \text{ spiral}$$

$$\frac{dy_{m/2}}{dx} = .077 + 1.038 \frac{y_{m/2}}{R}$$

$$\frac{x}{R} = \frac{2}{3} \text{ spiral}$$

$$\frac{dy_{m/2}}{dx} = .077 + .857 \frac{y_{m/2}}{R}$$

$$\frac{x}{R} = 1 \text{ spiral}$$

$$\frac{dy_{m/2}}{dx} = .077 + .743 \frac{y_{m/2}}{R}$$

For reasons discussed in Chapter 6 and in section 9.1 the present flows are presumed to be of better quality and the average of the equations (9.6) and (9.7)

$$\frac{dy_{m/2}}{dx} = 0.071 + 0.775 \frac{y_{m/2}}{R} \quad \begin{matrix} (3.3b) \\ (9.8) \end{matrix}$$

will be taken as the required universal relation. Equation (9.8) can be rewritten as (see equation 3.10)

$$\frac{y_{m/2}}{R} = \frac{0.071 \frac{x}{R}}{(1 - 0.775 \frac{x}{R})} \quad \begin{matrix} (3.10) \\ (9.9) \end{matrix}$$

Using the above relation, the maximum velocity decay (equations 3.12) on the spirals is given as:

$$\begin{aligned} \frac{x}{R} = \frac{2}{3} & : \quad \frac{U_m}{U_{m0}} = \left(\frac{x}{x_0} \right)^{-0.52} \\ \frac{x}{R} = 1 & : \quad \frac{U_m}{U_{m0}} = \left(\frac{x}{x_0} \right)^{-0.56} \end{aligned}$$

and this agrees extremely well with a numerical solution which includes skin friction and the y-momentum equation to first order. (See figure 9.12.)

The surface pressure distribution is given by equation (3.17) and in conjunction with equation (9.9) yields

$$\frac{p}{R} = \frac{2}{3} \text{ spiral} : \frac{p_{\infty} - p_w}{\rho U_m^2} = 0.072$$

$$\frac{p}{R} = 1 \text{ spiral} : \frac{p_{\infty} - p_w}{\rho U_m^2} = 0.2105$$

and these predictions give good agreement with the experiments shown in figures 9.13 and 9.14.

The 'frictionless first order' theory has confirmed, for the equilibrium flows, the ideas of Newman (1961) and Nakaguchi (1961) and it has been shown that

(a) a solution limited to $O\left(\frac{Y}{R}\right)$ gives excellent agreement with experiment even up to $\frac{Y_m/2}{R} \approx .3$

(b) the bulk features of the equilibrium flows can be predicted quite well by neglecting the influence of skin friction.

In addition it has been shown that the linear universal growth law

$$\frac{dy_{m/2}}{dx} = 0.071 + 0.775 \frac{Y_{m/2}}{R} \quad (9.8)$$

gives excellent prediction of the velocity decay and the surface pressure. It should be noted that this relation does not represent a perturbation away from the plane flow growth. Thus rewriting equation (9.8) yields

$$\frac{dy_{m/2}}{dx} = 0.071 \left(1 + 10.7 \frac{Y_{m/2}}{R} \right)$$

and, for a perturbation type analysis to hold it is necessary that $y_{m/2}/R \ll 1$ and this condition is not met in any of the flows studied in this work. More insight into the nature of this relation (9.8) is given in the next section in which Townsend's theory is discussed.

9.5 Townsend's Structural Similarity Theory (refer to section 3.2)

In this theory a prediction of the development of the curved wall jets is attempted by assuming that in equation (3.67), the turbulence parameters $q_o^2 / |\tau_{max}|$, $y_o / y_{m/2}$ and $y_{m/2} / L_e$ are invariant between the plane and curved flows. Some basic simplifications to the working equations were made by calculating pressure to 0 ($\frac{y_m/2}{R}$) and neglecting skin friction and these assumptions were shown valid in the previous section.

The plane wall jet measurements discussed earlier give

$$\frac{y_o}{y_{m/2}} = 1.8$$

$$\frac{y_{m/2}}{L_e} = 0.071$$

$$\frac{q_o^2}{|\tau_{max}|} = 4.13 \quad (9.10)$$

and substituting these values into equation (3.67) when $\frac{y_m/2}{R} = 0$ yields

$$\frac{y_{m/2}}{L_s} = 0.521 \quad (9.11)$$

The equation (3.67) for the spread of the curved jet is therefore written

$$\left[0.3 - 0.32 \frac{y_{m/2}}{R} + 0.423 y'_{m/2} \left(1 - 1.67 \frac{y_{m/2}}{R} \right) \right] \left[\frac{y'_{m/2}}{2} + 0.572 R' \left(\frac{y_{m/2}}{R} \right)^2 \right] \\ = 0.617 \left(1 + 0.9 \frac{y_{m/2}}{R} \right) \left(1 - 1.67 \frac{y_{m/2}}{R} \right)^{3/2} y'^{3/2}_{m/2} \quad (9.12)$$

and solving for the spirals $\frac{x}{R} = \frac{2}{3}$ and 1 yields

$$x/R = 2/3 \text{ spiral} : y'_{m/2} = 0.100$$

$$x/R = 1 \text{ spiral} : y'_{m/2} = 0.280$$

These predictions are substantially lower than the result of Giles et al (they obtained $y_{m/2}' = .18$ and $.3$ respectively) and are closer to the present results ($y_{m/2}' = .155$ and $.277$) but the discrepancy is still large. The solution is extremely sensitive to errors in the coefficients ($\approx .617$) of the right-hand side of equation (9.12). For example, if this number is changed by 10% to equal $.556$, then for the $\frac{x}{R} = \frac{2}{3}$ spiral the predicted growth is $y_{m/2}' = .171$ which represents a 70% difference with the previously calculated results. This feature unfortunately limits the usefulness of the method since the parameters $y_e/y_{m/2}$ alone changes by more than 10% between the plane wall jet and the jet on the $\frac{x}{R} = 1$ spiral.

It is of interest to consider equation (3.60) which represents equation (3.67) before the equation for the maximum shear

stress is introduced. The jet growths can be obtained from equation (3.60) by using

$$\frac{y_{m/2}}{L_c} = 0.321$$

and taking the present experimental values of q_o^2/u_m^2 and $\frac{y_e}{y_{m/2}}$ which are shown in figure 9.5. This procedure yields

$$\frac{x}{R} = \frac{2}{3} \text{ spiral} : y'_{m/2} = 0.183 \quad (9.14)$$

$$\frac{x}{R} = 1 \text{ spiral} : y'_{m/2} = 0.254$$

The growth, predicted for the $\frac{x}{R} = \frac{2}{3}$ spiral, is more rapid than that found in the present experiments but agrees well with the flow of Giles et al. The growth on the $\frac{x}{R} = 1$ spiral agrees most closely with, and is less than, the present data. More insight into the influence of errors in the parameters q_o^2/τ_{max} , $y_e/y_{m/2}$ and $y_{m/2}/L_c$ on the growth calculated from equation (3.60) can be obtained by noting that the equation can be considerably simplified, by neglecting small terms, to yield

$$y'_{m/2} \propto \left(\frac{y_{m/2}}{L_c} \right) \left(\frac{y_e}{y_{m/2}} \right) \left(\frac{q_o^2}{u_m^2} \right)^{1/2} \quad (9.15)$$

In this form the equation is explicit in $y_{m/2}$ and a 10% error in the right-hand side will cause a similar error in $y_{m/2}$. The possible error in the growth calculated from equation (3.60) is smaller than that for a calculation using equation (9.12) and it is therefore surprising that in relations (9.14) the better agreement with the present experiments is obtained for the $\frac{x}{R} = 1$ spiral. (The two-dimensionality tests described

in Chapter 6 and in section 9.1 indicated that the flow on the present $\frac{x}{R} = \frac{2}{3}$ spiral is of better quality than that obtained by Giles et al. on their spiral or by the author on the $\frac{x}{R} = 1$ spiral.)

In figure 9.5 there is shown the variation with curvature of the quantities $y_e/y_{m/2}$, $q_o^2/(\tau_{MAX})$ and q_o^2/U_m^2 . It is seen that $y_e/y_{m/2}$ changes quite consistently with curvature and it appears doubtful that it is much in error. It is clear from equation (9.14) and (9.15) that no matter how $y_{m/2}/L_e$ varies with curvature (providing it is a consistent variation) it is not possible to have both of the predictions in equations (9.14) agree with the present experiments. This suggests that experimental scatter in q_o^2/U_m^2 is responsible for the unexpected predictions in equations (9.14). The modifications to q_o^2/U_m^2 that are required to make the calculated growths agree with the present results ($y_{m/2} = .155$ and $.277$ respectively) are quite plausible and are represented by the full line in the lower plot of figure 9.51. Thus errors encountered when calculating y_e' from equation 9.12 must be due to incorrect estimate of τ_{MAX} .

Figure 9.52 compares the variation, with curvature of the measured (and corrected) $\tau_{MAX}/\rho U_m^2$ with various predictions which are similar, except in details to the method used by Townsend and which was followed in developing equation (3.66) (given by line No. 1 in the figure). Line No. 3 is given by

$$\frac{\tau_{max}}{\rho U_m^2} = \frac{\tau_w}{\rho U_m^2} - 0.25 y_{m/2} \left(1 - 1.54 \frac{y_{m/2}}{R} \right) \quad (9.16)$$

which represents a best straight line fit through a polynomial

of $O\left(\left(\frac{y_m/2}{R}\right)^2\right)$, and was obtained by noting that: (a) the collapse of the almost self-similar shear stress profiles in figure 9.17a can be improved by plotting as in figure 9.17b; (b) some of the coefficients to $\frac{y_m/2}{R}$ are large and limiting the calculation to $O\left(\frac{y_m/2}{R}\right)$ is invalid (see last paragraph in preceding section); and (c) terms of $O\left(\frac{y_m}{y_m/2}\right)$ are significant. Line No. 2 is identical to No. 3 save that the coefficient of $y_m/2$, ($= .25$), in equation (9.16) was replaced by .215 in order to force the line through the plane wall jet results. This relation will be used in the analysis of the Coanda flow over the circular cylinder.

Chapter 10

Experimental Results and Theoretical Prediction of the Wall
Jet over the Cylinder of Constant Radius

10.1 Quality of the Flow on the Cylinder: verification of
two-dimensionality and universality of results

Figure 10.1 and 10.2 show the lateral variation of total pressure at $y = y_m$ and $y_{m/2}$ for $\theta_1 = 60^\circ$ and 150° respectively and it is observed that, even with the care taken, it was impossible to prevent the occurrence of large and very rapid variations in the pressure. (Indeed, the variations take place in such short distances, that lateral traverses taken every 2 inches (Fekete, 1963) may not reveal the difficulty.) The variations are more pronounced at $y_{m/2}$ than at y_m and this is consistent with suggestions (Chapter 6) that the variations are due to slight irregularities of the slot lip. As mentioned in Chapter 5, it is quite possible that corrosion and slight flaking of the cadmium plating is responsible for the irregularities and this may not have occurred in Fekete's original experiments made just after the apparatus was constructed.

The variations are especially pronounced near the centre-line and consequently no traverses were attempted at this location. The irregularities were a minimum for all angles, θ_1 , at $\frac{1}{52}$ inches to the left of the centre-line and the measurements were performed at this position. This, of course, subjected the flow to possibly more influence from secondary flows near the end plates but a perusal of figures 10.1 and 10.2 and a

consideration of the time spent in achieving even this degree of quality (Chapters 5 and 6) revealed that there was little further choice in the matter.

The verification of momentum balance is given in figures 10.3 which shows the measured decay of maximum velocity compared with the results of two calculation methods. The first, line 1, is calculated using a Runge-Kutta numerical solution of equations (3.79) and (3.80) in conjunction with: (a) the known growth; (b) the measured shear stress at y_m (τ_m); and (c) the measured skin friction. The second, line 2, is calculated like the previous except that τ_m is not the measured value but is calculated from the inner layer equation (3.80) using the known growth of the inner layer.

If it is accepted that a small divergence of the flow does not significantly change $\overline{u'v'}$ (Chapter 9) then the measured shear stress distribution is most representative of conditions in a perfectly two-dimensional flow and the difference between lines 1 and the measured points is a measure of the existing three-dimensional effects. On the other hand for line 2 the three-dimensional effects are artificially compensated by incorporating them into a fictitious shear at y_m and hence the perfect agreement with experiment. These arguments also apply to figure 10.4 which shows the above numerical procedures applied to the calculation of y_m . The excellent agreement between experiment and line 2 of figure 10.3 or line 2 of figure 10.4

provides a verification of the computer program.

A further attempt to investigate two-dimensionality was made by measuring the stress $\overline{u'w'}$ at three streamwise stations and this is shown in figure 10.5. Note that the points where $\overline{u'w'}$ is a maximum occur roughly at the same value of $y/y_m/2$ and this is also close to where $\overline{u'v'} = \overline{u'v'}_{MAX}$ (figure 10.24). The ratio of the maximum shear stresses is

$\theta_1 = 110^\circ$

 $\theta_1 = 170^\circ$

$$\frac{\overline{u'w'}_{MAX}}{\overline{u'v'}_{MAX}} \approx 0.14$$

$$\frac{\overline{u'w'}_{MAX}}{\overline{u'v'}_{MAX}} \approx 0.06$$

To verify the quality of these results the measurement at $\theta_1 = 145^\circ$ was repeated three months later with a different wire, anemometer, linearizer, and RMS meter and the result is given by the dark points in figure 10.5. The two measurements are in good agreement considering the difficulty of the measurement and the extremely small magnitude of the parameter being measured. The negative value of $\overline{u'w'}$ at $\theta_1 = 145^\circ$ and $y = y_m/2$ suggest a velocity which is decreasing with increasing distance from the centre-line and the lateral total pressure variation shown in figure 10.2 tends to corroborate this deduction.

The most accurate measure of three dimensional effects is, of course, a comparison between measured and calculated shear stress. For these experiments the calculation of shear is more tedious than in the self-preserving flows and it must be repeated for every measurement station. It was found just as useful to

calculate one point, and the shear at y_m was chosen (it was used in the computer program related to the results of figures 10.3 and 10.4) and is shown, in figure 10.26,^(m) compared to the measured and corrected values. The discrepancies are very large and are in the range 30 - 70% but they are responsible for only a 5% discrepancy in the maximum velocity (figure 10.3).

To verify universality it is of interest to compare the measurements with those of Fekete (1963) but this will be reserved for the discussion of the individual results.

10.2 Experimental Results

10.2.1 Bulk parameters and comparison with the results of Fekete

The measured mean velocity profiles are shown in figures 10.6 and 10.7 and it is observed that the corrections due to the transverse v component, are negligibly small except at the edge of the jet and over the whole of station $\theta_1 = 200^\circ$. (The v component was calculated from the continuity equation and the ' u ' profiles are shown by the dotted lines.)

The non-dimensional profiles Q/Q_m or u/U_m versus $y/y_{m/2}$ are shown in figures 10.8. There is an excellent collapse of the data even up to $\theta_1 = 170^\circ$ which is in the region where the pressure gradient quickly steepens; and only at $\theta_1 = 200^\circ$, which is near the separation point, does the profile shape undergo a marked change. These observations are generally in agreement

with Fekete's results but the present measurements give rise to a better collapse of the data. It is interesting to note that the changing profile shape is mainly a result of a change in shape of the inner layer accompanied by a change in the ratio $y_m/y_{m/2}$. This is clearly seen in figure 10.9 where the dimensionless plot of the outer layer shows an excellent collapse of all data including that of $\theta_1 = 200^\circ$. This fact, plus the excellent agreement between the measurements and 'sech²' profile, provides excellent justification for the 'strip' integral analysis described in Chapter 3 and used in section 10.4.

The variation of both $y_{m/2}$ and y_m is shown in figure 10.10; the surface pressure variation is also shown and it is observed that the rapid thickening of the inner layer is associated with the sharp pressure rise. This phenomenon was discussed in detail in Chapter 4. It is observed that in this case the plateau region seems not as well defined as for the results of Fekete and a comparison with these results will be shown later.

The surface pressure, referred to the local maximum velocity is plotted in figures 10.11 and 10.12. In the former it is seen that the pressure calculated from the full y-momentum equation agrees well with that calculated from the linearized integral theory (equation 3.17) and this is in accord with calculations made for the equilibrium flows. The discrepancy with the measurements is considerable and no satisfactory explanation can be suggested. A comparison with the equilibrium flow results is given in figure 10.12.

The position of the threshold* is easily obtained by using equation (4.16) in conjunction with figure 10.11. The results give

$$0.25 \geq \frac{p_{\infty} - p_t}{\rho U_{m_t}^2} \geq 0.23$$

and the threshold is predicted to lie somewhere between $\theta_1 = 145^\circ$ and 170° . This is in good agreement with the surface pressure distribution shown in figure 10.10.

The results are compared with Fekete's data in figures 10.13 to 10.15 and the agreement is good. In particular the decay of maximum velocity agrees very well with Fekete's large $\frac{b}{R}$ results and this suggests that the present data is close to the universal conditions discussed in the introduction. Thus universality of the flow will be assumed for the present results and no attempt will be made to apply the theoretical analyses to Fekete's data (except, of course, for the separation analysis discussed in Chapter 4).

10.2.2 Reynolds stresses

The theory of Townsend will not be applied explicitly to the flow on the circular cylinder (see discussion at beginning of Chapter 3) and consequently the Reynolds stress measurements are of less theoretical interest in the present case and they will not be discussed in detail. They are shown in figures 10.16

* Recall that the 'threshold' point is where the surface pressure is assumed to begin the cascade type increase towards separation (Chapter 4).

to 10.19 for the whole jet layer and in 10.20 and 10.21 for the inner boundary layer.

Of present interest is the observation that the flow does not appear close to equilibrium conditions. The values of $\frac{y_{m/2}}{R}$ obtained on the two spirals were $y_{m/2}/R = .103$ and $.277$ for $\frac{x}{R} = \frac{2}{3}$ and 1 respectively. The two stations on the cylinder which have values of $y_{m/2}/R$ close to these are

$$\theta_1 = 62^\circ \quad \frac{y_{m/2}}{R} = 0.103$$

$$\theta_1 = 110^\circ \quad \frac{y_{m/2}}{R} = 0.28$$

It is seen in figure 10.16 that there is a significant history effect and that the local $\overline{u'^2}/U_m^2$ on the cylinder is smaller than that of on equilibrium flow with an equivalent $y_{m/2}/R$. This property will be discussed in section 10.3.

10.3 The Linearized Integral Method and a Discussion of the Rate of Growth Equation

Recall that the principal objective of the linearized integral method was to examine the applicability of a first order - i.e. $O(\frac{y_{m/2}}{R})$ - theory in conjunction with the assumption that the jet is in local equilibrium and only these two aspects will be considered in this section.

The integral of the x-momentum equation in conjunction with the y-momentum written to $O(\frac{y}{R})$ shows that, to this order, the linear momentum is conserved over the cylinder (equations (3.19) to (3.21)). The table below gives the relevant measured data

θ_1	40°	75°	110°	145°	170°	200°
$\frac{\rho U_m^2 y_{m/2}}{(p_o - p_m)^{1/2}}$	1.99	1.96	1.94	2.00	1.97	1.614
$\frac{y_{m/2}}{R}$	.07	.16	.28	.45	.605	.96

and it is seen that the linearized analysis is quite valid even up to very large values of $y_{m/2}/R$ where a 'perturbation' type analysis is evidently inapplicable. This observation is in agreement with the behaviour of the equilibrium flows (section 9.4).

Figures 10.11 and 10.12 show the surface pressure distribution. In the former figure there is shown a comparison between the pressure calculated from equation (3.17) and from the full y-momentum equation and the agreement is excellent up to about the maximum point in the curve. Beyond this, only the full y-momentum gives reasonable agreement with the measured pressure. The latter figure shows a comparison between the log-spiral and the cylinder data. The point $\frac{y_{m/2}}{R} = .45$ corresponds to $\theta_1 = 145^\circ$ and figure 10.10 shows that the flow at this location is ^{just} beyond the 'plateau' region of pressure, and in the region where the surface pressure is rapidly increasing (Chapter 4) and this explains the increasing difference between the calculations based on the full y-momentum and the simplified relation, equation (3.17).

It follows from the above table that if the streamwise variation of $y_{m/2}$ is given experimentally (as was done in the analysis of Newman, 1960) then the predicted velocity decay should be in excellent agreement with experiment up to $\theta_1 = 170^\circ$.

Of primary interest is the development of a growth law. It was suggested in Chapter 3 (section 3.1) on the basis of Townsend's analysis, that an equilibrium growth law should be written

$$y'_{m/2} = f \left(\frac{y_{m/2}}{R}, R' \left(\frac{y_{m/2}}{R} \right)^2 \right) \quad (3.1)$$

This formulation follows directly from equation (3.67) of section 3.2 where the growth, $y_{m/2}'$, is a function of the rate of change of surface curvature ($\frac{dR}{dx} = R'$), if it is assumed that the turbulence structure - characterized by the parameter $\frac{y_{m/2}}{L_\epsilon}$, $\frac{y_e}{y_{m/2}}$ and $\frac{q_0^2}{\tau_{MAX}}$ - is a function only of local conditions, i.e. of $\frac{y_{m/2}}{R}$. A form of function 'f' which suits observation and can be reduced to the linear relations 3.3a and 3.3b is

$$y'_{m/2} = \beta_0 + \beta_1 \frac{y_{m/2}}{R} + \beta_2 R' \left(\frac{y_{m/2}}{R} \right)^2 \quad (10.1)$$

Thus for equilibrium flows the above is written:

$$y'_{m/2} = \beta_0 \left[\frac{1 + \frac{\beta_1}{\beta_0} \frac{y_{m/2}}{R}}{1 + \beta_2 \frac{y_{m/2}}{R}} \right] \quad (10.2)$$

For very small $\frac{y_{m/2}}{R}$ (section 9.4), and using the binomial expansion to $O\left(\frac{y_{m/2}}{R}\right)$ the above becomes

$$y'_{m/2} = \beta_0 + \beta_1 \frac{y_{m/2}}{R} \quad (3.36)$$

where

$$\beta_1 = \beta_1 - \beta_2 \beta_0 \quad (10.3)$$

but for the range of $y_{m/2}$ encountered in this work this procedure is invalid. Nevertheless, an average linear relationship - equation (9.8) - can be used.

The measurements on the logarithmic spirals ^{in conjunction with Townsend's theory} give

$$y'_{m/2} = 0.071 + 0.91 \frac{y_{m/2}}{R} - 0.6 R' \left(\frac{y_{m/2}}{R} \right)^2 \quad (10.4)$$

and therefore the hypothetical linear equilibrium growth law, applicable to the circular cylinder is

$$y'_{m/2} = 0.071 + 0.91 \frac{y_{m/2}}{R} \quad (10.5)$$

This is shown in figure 10.22. Compared to the relation

$$y'_{m/2} = 0.071 + 0.69 \frac{y_{m/2}}{R} \quad (10.6a)$$

which is a linear fit through the measured data at small $\frac{y_{m/2}}{R}$ on the cylinder, the agreement is very poor.

It is of interest to compare this equation with that derived by the author (Guitton, 1964) from experiments on the concave cylinder and from the experiments of Fekete (1963). The suggested growth was

$$\frac{y_{m/2}}{x} = 0.067 + 0.3 \frac{y_{m/2}}{R} - 0.08 \left(\frac{y_{m/2}}{R} \right)^2$$

This equation was obtained from the large $\frac{b}{R}$ results of Fekete to assure a more universal flow but it included points measured at large $\frac{y_{m/2}}{R}$ (or θ) which are beyond the threshold point discussed in Chapter 4. The overall growth of the jet, $y_{m/2}$, beyond this threshold is clearly not due to the same process as that in the constant pressure region, and consequently the second order term in the above equation is doubtful. Neglecting this term and taking the x-derivative yields

$$y'_{m/2} = 0.067 + 0.6 \frac{y_{m/2}}{R}$$

The average growth rate obtained on the concave cylinder alone was

$$y'_{m/2} = 0.069 + 0.64 \frac{y_{m/2}}{R}$$

and both of the above results are in good agreement with the equation 10.6a.

A more plausible fit through the cylinder data should account for the fact that the point at smallest $\frac{y_{m/2}}{R}$ appears to be close to equilibrium and the following polynomial fit through the data is suggested:

$$\dot{\gamma}_{m/2} = 0.071 + 0.911 \frac{\gamma_{m/2}}{R} - \left(\frac{\gamma_{m/2}}{R} \right)^2 \quad (10.6b)$$

and is shown by line 4 in figure 10.22.

The difference between the suggested equilibrium rate of growth -- obtained from the log-spiral data -- and the measured rate on the cylinder shows that the flow on the latter is not in equilibrium and that there is a history effect. This is in agreement with measurements of Reynolds stresses discussed earlier. It is therefore impossible to use the equilibrium growth law for predicting the development of the flow on the cylinder and it is of interest to examine how this law may be modified to account for history effects.

The local rate of growth on the cylinder can be considered identical to that which would exist in an equilibrium flow but at a smaller value of $\frac{\gamma_{m/2}}{R}$. Local conditions on the cylinder are governed by eddies created upstream and it is plausible to assume that they move downstream with essentially 'frozen' or unchanging length and velocity scales and that their effect on the non-equilibrium flow is described by the particular properties of the corresponding equilibrium flow at their place of 'birth'. Thus if in the non-equilibrium flow a large eddy is

* The author is indebted to Dr. B.G. Newman for suggesting the ensuing analysis.

'born' at the value $(\frac{y_{m/2}}{R})_n$ corresponding to the station $x-\Delta x$ say, then its effect on the flow at x will be described by the equilibrium flow: $(\frac{y_{m/2}}{R})_n = \text{constant}$. Now the average distance a large eddy, which is centered at $y_{m/2}$ say, travels downstream is given somewhat crudely as

$$\Delta x \approx \frac{U'_{m \text{ ave.}} t_D}{2 \left(1 + \frac{(\frac{y_{m/2}}{R})_n + \frac{y_{m/2}}{R}}{2} \right)} \quad (10.7)$$

where t_D is the 'life' time of the large eddy. Townsend (1956) suggested that

$$t_D \approx \frac{\chi}{\left| \frac{\partial u}{\partial y} \right|_{\text{ave.}}} \quad (10.8a)$$

and in an equilibrium flow χ is indeterminate. Newman (1970) has shown by considering the rotation and stretching of a vortex line, assumed to be originally in a position normal to the surface, that approximately

$$1 \leq \chi \leq 4 \quad (10.8b)$$

depending on where the large eddy spends most of its life; the smaller limit corresponds to a lifetime and decay spent mostly at an orientation of about 45° to the surface; the larger limit corresponds to a decay near the x -axis.

Combining equations (10.7) and (10.8) for self-similar velocity profiles yields

$$\frac{\Delta x}{R} \approx \frac{0.7 \chi \frac{y_{m/2}}{R}}{\left[1 + \frac{\frac{y_{m/2}}{R} + \left(\frac{y_{m/2}}{R} \right)_n}{2} \right]} \quad (10.9)$$

It is now of interest to: (a) find the value of $\left(\frac{y_{m/2}}{R} \right)_n$ at the new station $x - \Delta x$; and (b) to examine the consequence of assuming that the conditions at x are determined by an equilibrium structure or growth related to the value $\left(\frac{y_{m/2}}{R} \right)_n$.

(a) Using the growth rate given by equation (10.6b) yields

$$\left\{ \begin{array}{l} \left(\frac{y_{m/2}}{R} \right) \\ \left(\frac{y_{m/2}}{R} \right)_n \end{array} \right\} \frac{d \left(\frac{y_{m/2}}{R} \right)}{1 - 12.8 \frac{y_{m/2}}{R} - 14.1 \left(\frac{y_{m/2}}{R} \right)^2} = 0.071 \frac{\Delta x}{R} \quad (10.10)$$

Combining (10.10) with (10.9), and integrating yields

$$\begin{aligned} \tan^{-1} \left[0.868 - 1.9 \left(\frac{y_{m/2}}{R} \right)_n \right] - \tan^{-1} \left[0.868 - 1.9 \left(\frac{y_{m/2}}{R} \right) \right] \\ = 0.34 \chi \frac{\frac{y_{m/2}}{R}}{1 + \frac{\left(\frac{y_{m/2}}{R} \right)_n + \frac{y_{m/2}}{R}}{2}} \end{aligned} \quad (10.11)$$

(b) The equilibrium growth rate at $\left(\frac{y_{m/2}}{R} \right)_n$, (or at $x - \Delta x$)

is

$$y'_{m/2} = 0.071 + 0.91 \left(\frac{y_{m/2}}{R} \right)_n$$

and setting this equal to the growth rate at $\frac{y_{m/2}}{R}$, (or at x), yields from equation (10.6b)

$$0.91 \left(\frac{y_{m/2}}{R} \right)_n = 0.91 \left(\frac{y_{m/2}}{R} \right) - \left(\frac{y_{m/2}}{R} \right)^2 \quad (10.12)$$

Combining equations (10.12) and (10.11) and writing $\frac{y_m/2}{R} \equiv \epsilon$ then

$$\chi = \left(\frac{1 + \epsilon - 0.85\epsilon^2}{0.34\epsilon} \right) \left[\begin{array}{c} \tan^{-1}(0.863 - 1.9\epsilon + 2.2\epsilon^2) \\ - \tan^{-1}(0.863 - 1.9\epsilon) \end{array} \right] \quad (10.13)$$

This equation represents the variation of χ , which determines the eddy life that is necessary for the flow at $\frac{y_m/2}{R}$ on the cylinder to be governed by the equilibrium relation, equation (10.5) and χ is shown plotted versus ϵ (or $\frac{y_m/2}{R}$) in figure 10.23. At $\frac{y_m/2}{R} = 0$ the flow is near equilibrium and the development is determined by local conditions and the plot indicates that in an equilibrium flow $\chi = 0$. It is also seen that over the range of $\frac{y_m/2}{R}$ of interest, the value of χ is generally between the limits suggested by Newman (see equation 10.8b).

For the purpose of developing the integral technique to be discussed in the next section, the plausible variation of χ with $\frac{y_m/2}{R}$ derived above will be used, and this, of course, implies the use of the actual rate of growth law measured in the cylinder (equation 10.6b).

10.4 The Strip Integral Method: (application of)

10.4.1 The Closure Relations

This analysis was formulated in section 3.3 and left with two equations and the seven unknowns

$u_m, l_0, y_m, \tau_m, \tau_w, n$, normal stress terms

and a purpose of the investigation of the equilibrium flow on the log-spirals, and of the other theoretical studies in section 3.1 and 3.2, was to supply the missing information. The suggested 'closure' relations are discussed below:

(a) The rate of growth will be taken as the one actually measured experimentally and approximated by equation (10.6b) (see detailed discussion in preceding section).

$$\dot{\gamma}_{m/2} \left(\equiv \dot{\ell}_0 + \dot{\gamma}_m \right) = 0.071 + 0.911 \frac{\dot{\gamma}_{m/2}}{R} - \left(\frac{\dot{\gamma}_{m/2}}{R} \right)^2 \quad (10.6b)$$

Although this is not a completely satisfactory approach, in that the required generality has not been achieved, it is a plausible preliminary attempt in view of the difficulty in handling non-equilibrium flows.

(b) A relation for τ_m can be deduced from the observation (section 9.5) that it is possible to collapse all the measured shear profiles by plotting

$$\frac{\tau - \tau_w}{\tau_{max} - \tau_w} = g_{12} \left(\frac{\dot{\gamma}}{\dot{\gamma}_{m/2}} \right)$$

From figure 9.17b it is possible to write

equilibrium
flow data

$$\frac{\tau - \tau_w}{\tau_{max} - \tau_w} = 1.95 \left(\frac{\dot{\gamma}}{\dot{\gamma}_{m/2}} \right) \quad \text{for } \frac{\dot{\gamma}}{\dot{\gamma}_{m/2}} < 0.4 \quad (10.14)$$

or

$$\frac{\tau_m}{\rho U_m^2} = \frac{\tau_w}{\rho U_m^2} + 1.95 \left(\frac{\tau_{max} - \tau_w}{\rho U_m^2} \right) \frac{\dot{\gamma}_m}{\dot{\gamma}_{m/2}} \quad (10.15)$$

Using the shear profiles of figure 9.17b and the procedure used in developing equation (3.66) (and subsequently improved upon for equation (9.16)), and noting that, in equation (3.59), the term $R' = 0$, yields

$$\frac{\tau_{max} - \tau_w}{\rho U_m^2} = -0.215 y'_{m/2} \left(1 - 0.89 \frac{y_{m/2}}{R}\right) \quad (10.16)$$

where the linear relation $\frac{y_{m/2}}{R}$ represents a very good fit through a polynomial of 0 ($\frac{y_{m/2}}{R}$). Using the measured rate of growth given by equation (10.6b) and the measured skin-friction, the streamwise variation of τ_{MAX} can be calculated and is compared, in figure 10.25, to the measured and corrected values (line 2 in the figure). The agreement with measurement is very poor. In order to verify that a failure of the shear stress profiles to be self-similar was not responsible for the discrepancy, the shear stress measured on the cylinder was plotted in the manner of figure 9.17b and this is shown in figure 10.24. There appears to be a slight change in shape of the profiles and accordingly new estimates of relations (10.14) and (10.16) can be made from an average between line 1 and line 2 of the figure. These are now

Circular
Cylinder data

$$\frac{\tau - \tau_w}{\tau_{max} - \tau_w} = 2.3 \left(\frac{y}{y_{m/2}} \right) \quad \text{for } \frac{y}{y_{m/2}} < 0.4 \quad (10.17)$$

or

$$\frac{\tau_m}{\rho U_m^2} = \frac{\tau_w}{\rho U_m^2} + 2.3 \left(\frac{\tau_{max} - \tau_w}{\rho U_m^2} \right) \frac{y_m}{y_{m/2}} \quad (10.18)$$

and

$$\frac{\tau_{max} - \tau_w}{\rho U_m^2} = -0.215 y'_{m/2} \left(1 - 0.65 \frac{y_{m/2}}{R}\right) \quad (10.19)$$

The value of τ_{MAX} obtained from the preceding equation is shown by line 3 in figure 10.25 and the agreement with measurement is no better than before. For interest there is also shown on this figure (line 4) the variation of τ_{MAX} predicted by assuming the equilibrium growth law equation (10.5) and the discrepancy, as expected, increases.

The shear stress at y_m predicted from the combinations of: (a) equations (10.15) and (10.16) or (b) equations (10.18) and (10.19), is shown in figure 10.26. The dotted line uses the measured growth rate in conjunction with the first combination, and is given by

$$\frac{\text{equilibrium data}}{\rho U_m^2} = \frac{\tau_m}{\rho U_m^2} = 0.419 \gamma'_{m/2} \left(1 - 0.89 \frac{\gamma_{m/2}}{R}\right) \frac{\gamma_m}{\gamma_{m/2}} \quad (10.20)$$

The solid line in the figure also uses the measured growth rate and represents the second combination

$$\frac{\text{circular cylinder data}}{\rho U_m^2} = \frac{\tau_m}{\rho U_m^2} = 0.495 \gamma'_{m/2} \left(1 - 0.65 \frac{\gamma_{m/2}}{R}\right) \frac{\gamma_m}{\gamma_{m/2}} \quad (10.21)$$

The 'dot-dash' line on the figure represents equation (10.20) used in conjunction with the equilibrium growth rate. The agreement between these equations and measurement is poor but it is nevertheless of interest to use them in the strip integral analysis.

The remaining 'closure' relations are:

(c) The skin friction does not influence too strongly the development of the scales ℓ_0 (or $y_{m/2}$) and U_m (e.g. see Chapter 9) but it does influence significantly the predicted position of separation (see end of Chapter 4) and to a lesser extent the rate of growth of y_m . The hot film measurements in Chapter 8 showed that the skin friction in the plane wall jet and on the $\frac{x}{R} = \frac{2}{3}$ spiral is satisfactorily correlated by the relation

$$C_f = 0.0315 \left(\frac{y_m U_m}{\nu} \right)^{-0.182} \quad (10.22)$$

The data on the $\frac{x}{R} = 1$ surface was not correlated by this relation and it was judged that this was due to pressure gradient rather than the increased influence of the outer flow on the inner. In the zone of interest on the cylinder - the plateau region - the pressure is nearly constant and equation (10.22) will be adopted.

(d) The value of the power exponent n did not vary significantly between the log spiral inner layers; and since the region of interest on the cylinder is one of essentially zero pressure gradient it is permissible to take a constant n and the value

$$n = 13.0$$

is chosen on the basis of the equilibrium data.

(e) The normal Reynolds stresses will be taken as the

equilibrium flow values. This is known from previous studies to be incorrect but this term is small and unimportant in view of the difficulties associated with three dimensional effects.

The above information constitutes sufficient knowledge to attempt a prediction of the development of the jet up to the threshold point. The solution is given by a Runge-Kutta numerical procedure and is too standard to be discussed here.

10.4.2 Predicted results

The predictions of U_m , ℓ_o , and y_m are shown in figures 10.27, 10.28 and 10.29 respectively. The calculations are stopped at $\theta_1 = 170^\circ$ because this defines the limiting threshold position and the solution is not valid beyond this point. Line 1 represents the results calculated using the growth law, (10.6b), in conjunction with a shear at y_m , (τ_m), taken from the equilibrium data (equation 10.20). Line 2 uses a growth rate identical to the previous calculation but incorporates a shear profile representing the average similarity shear profile on the cylinder between 75° and 170° (equation 10.21). The following observation can be made:

(a) Spectacular results should not be expected because the quality of the flow is uncertain. Indeed, in section 10.1 a calculation of y_m , made using the measured shear stress at y_m , was in poor agreement with observations and three-dimensional effects were thought to be influential.

(b) The prediction of U_m is satisfactory and is hardly influenced by the variation in l_0 or y_m and consequently:

(c) The difference in the two predicted growths seems due to differences in τ_m .

(d) The prediction of y_m made using the equilibrium shear stress profiles goes completely askew and considerable improvement is realized when an average cylinder shear profile is used. In the region of y_m the shear stress distribution on the cylinder (figure 10.24) moves progressively away from the plane flow profile and the above results suggest that further improvement in the prediction of y_m could be obtained by incorporating this variation. But this would amount to a method of calculation that 'uses the flow to predict the flow' and is not very satisfactory.

It is interesting to determine why the calculated y_m begins to decrease (see dotted line). It is observed that the equation specifying τ_m gives a shear which is proportional to $y_m/y_{m/2}$. Thus if $y_m/y_{m/2}$ decreases, then τ_m decreases; also the retarding force on the inner layer is reduced, and $y_m/y_{m/2}$ further decreases and the unstable process continues. Thus it is of interest to attempt a calculation when $y_m/y_{m/2}$ is fixed and the equilibrium flows suggest the value

$$\frac{y_m}{y_{m/2}} = 0.16$$

and lines 3 and 4 in this figure represent the results of the calculation for a growth rate given by equation (10.6b) and

values of τ_m given by the same relations as used in obtaining lines 1 and 2 respectively. The prediction of all the quantities is much more satisfactory and, in particular, y_m now diverges near $\theta_1 \approx 145^\circ$ and its magnitude is quite unrealistic at $\theta_1 = 170^\circ$. It is of little interest to attempt the separation analysis for such grossly inaccurate starting conditions; it is known that the methods used to predict the threshold and separation point are satisfactory (section 10.2.1 and Chapter 4 respectively) if y_m , U_m and p_w are given accurately.

It was seen in section 10.1 that the growth of y_m is significantly underestimated if it is predicted using the measured shear, τ_m , and this was tentatively attributed to three-dimensional effects. Consequently, in the preceding analysis, even the satisfactory prediction of y_m up to $\theta_1 = 145^\circ$ should be viewed with caution.

It is seen therefore that further progress towards a more accurate prediction of the flow hinges on:

- (a) more quantitative and definitive knowledge of three-dimensional effects or their elimination;
- (b) a further improvement of the relation defining the shear at y_m
- (c) a more satisfactory account of history effects so that the equilibrium flow data can be applied with more generality to other non-equilibrium flows.

Chapter 11

Summary and Conclusions

The general aim of the work in this thesis has been to contribute to the study of the streamwise development of two-dimensional turbulent wall jets in still air flowing over curved surfaces. A particular aim of the work has been to develop a calculation method capable of predicting the development up to separation of the flow over a circular cylinder. These two objectives have been wed by studying both theoretically and experimentally some equilibrium curved wall jets, developing on logarithmic spiral surfaces, and applying the information so gained to predict the non-equilibrium flow over the cylinder.

The theoretical method for predicting the flow on the cylinder has been based on the observation that there are two distinct zones characterizing this flow: (a) the upstream 'plateau' region for which the surface pressure is essentially constant, followed by (b) a region where the inner boundary layer develops rapidly towards separation. The equilibrium data is applied to the first. The second is treated by using the theory of rapidly retarded boundary layers in which shear forces are negligible with respect to pressure forces (Chapter 4).

The upstream zone is analysed by using an integral method in which momentum integral equations are written for both the

outer (above the point of maximum velocity) and the inner regions of the flow (section 3.3). These equations introduce additional unknowns: principally the shear (τ_m) at the velocity maximum (y_m) and the skin friction at the wall. Furthermore, an additional auxiliary equation is required and is taken (see below) to be the local rate of growth of the jet ($d y_m/2/dx$). Information on these parameters is obtained from measurements made in two equilibrium jets flowing over logarithmic spirals having $\frac{x}{R} = \frac{2}{3}$ and 1 respectively.

Some ideas of Townsend (1969) are used to obtain formulations for the rate of growth and the shear, τ_m . His theory based on the total energy equation is used (sections 3.2 and 9.5) and his basic assumption that the structure parameter $\overline{u'v'}/\overline{q'^2}$ is essentially unchanged by curvature, is verified for the equilibrium flows. It is assumed that the shear stress profiles are similar in all the flows considered and the fact permits τ_m to be related to the maximum shear stress. The latter is obtained by using the x-momentum equation written at y_m in conjunction with the assumption of similar shear profiles. A Runge-Kutta numerical technique is used to solve simultaneously the differential equations.

In the course of the experimental work it transpired that certain difficulties arose in conjunction with the realization of two-dimensionality and there existed large differences between the measured and calculated shear stress distributions. The curved jets are characterized by highly intense turbulence

and the accuracy of shear measurements in these flows was suspect. Consequently special attention was given to the influence of high intensity turbulence on hot-wire measurements and this is discussed in Chapter 4. The corrections to the Reynolds stresses, developed in that chapter were also useful in establishing a more accurate evaluation of the parameter $\overline{u'v'}/\overline{q'^2}$ which is used in Townsend's analysis.

The experiments revealed that the inner layer ($y < y_m$) of the curved wall jets is strongly influenced by the outer layer (e.g. τ_m is large) and since this represents a rather unique physical situation it was decided to investigate this part of the flow in some detail (Chapter 8). In particular there was developed a technique for measuring skin friction when there is no universal logarithmic law of the wall. The knowledge so gained was particularly useful when predicting separation on the circular cylinder because an analysis of the data of Fekete revealed that small changes in skin friction influence strongly the predicted position of separation.

The principal theoretical and experimental contributions to knowledge are:

A. General

(a) The curved flows are remarkably sensitive to three-dimensional effects (Chapters 6, 9 and 10).

(b) Limiting the governing equations to terms of $O(\frac{y}{R})$ gives an excellent description of the flow even up to $y_m/2/R \approx .4$

(c) The shear at y_m is large (of order $\left[\frac{\tau_{MAX}}{5}\right]$ say) and increases as $y_{m/2}/R$ increases.

(d) The effect of high intensity turbulence causes an error of about 10% to the measurement of shear stress. Conclusions regarding the effect of high intensity are given at the end of Chapter 7.

(e) The law of the wall with universal constants ($A = B = 5.5$) seems valid in the inner layer of the curved flows up to $\frac{yU}{\nu} \sim 150$, but is not applicable when the pressure gradient is sufficiently large (e.g. the $\frac{x}{R} = 1$ surface.) The relation linking C_f to $y_m U_m / \nu$ is identical to that confirmed for plane wall jets by Alcaraz et al (1960) (Chapter 8).

B. Equilibrium Flows on Log-Spirals

(a) The development of the flow on the spirals is in reasonable agreement with the original experiments of Giles et al. but the present turbulence data is much more detailed and is of better quality.

(b) The structure parameter $\overline{u'v'}/q'^2$ is invariant with curvature (Chapter 9).

(c) The large eddy scale seems to decrease with $y_{m/2}/R$.

C. Cylinder Flow

(a) The flow on the circular cylinder is conveniently analysed by subdividing it into two streamwise zones (see above and Chapter 4): zone 1 of constant pressure and zone 2 of rapidly increasing pressure.

(b) Separation can be predicted using rapid retardation theory.

(c) The flow in the upstream zone (zone 1) is not in or near equilibrium and its deviation from equilibrium can be explained on the basis of large eddy life time which is a function of $y_{m/2}/R$.

(d) The original universal growth law suggested by the author (Guitton, 1964) should be modified to apply only to zone 2 of the flow.

(e) A satisfactory theoretical prediction of the flow was hampered by three-dimensional effects and the remarks made at the end of chapter 10 suggest future directions of study.

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General Description and Calibration
of the McGill 17 in. x 30 in.
Blower Cascade Wind Tunnel
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Appendix I

Integrals in equation (3.79)

$$I_1 = \int_0^{\infty} \eta \operatorname{sech}^4 \eta \, d\eta$$

$$I_2 = \int_0^{\infty} \eta \operatorname{sech}^4 \eta \tanh \eta \, d\eta$$

$$I_5 = \int_0^{\infty} \operatorname{sech}^4 \eta \tanh \eta \, d\eta$$

$$I_7 = \int_0^{\infty} \eta^2 \operatorname{sech}^4 \eta \tanh \eta \, d\eta$$

$$I_8 = \int_0^{\infty} \frac{\operatorname{sech}^4 \eta \, d\eta}{\left[1 + \frac{l_0}{0.882} \eta + \frac{y_m}{R}\right]^2}$$

$$I_9 = \int_0^{\infty} \frac{\operatorname{sech}^4 \eta \tanh \eta \, d\eta}{\left[1 + \frac{l_0}{0.882} \eta + \frac{y_m}{R}\right]}$$

$$I_{10} = \int_0^{\infty} \frac{\eta \operatorname{sech}^4 \eta \tanh \eta \, d\eta}{\left[1 + \frac{l_0}{0.882} \eta + \frac{y_m}{R}\right]}$$

TWO-DIMENSIONAL WALL JET ROUND A CIRCULAR CYLINDER

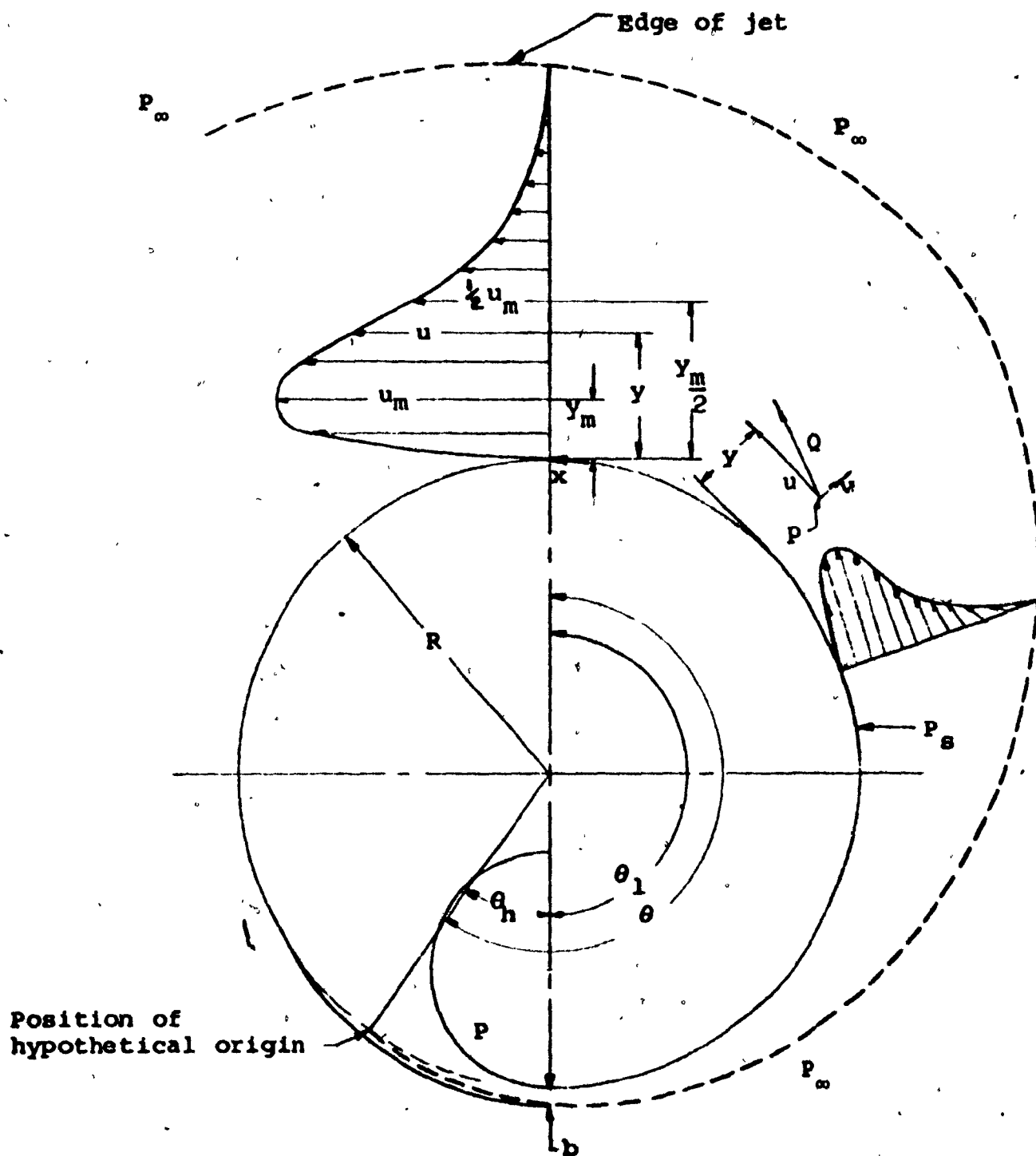


Fig. 1.2

SCHEMATIC OF FLOW OVER LOGARITHMIC SPIRALS

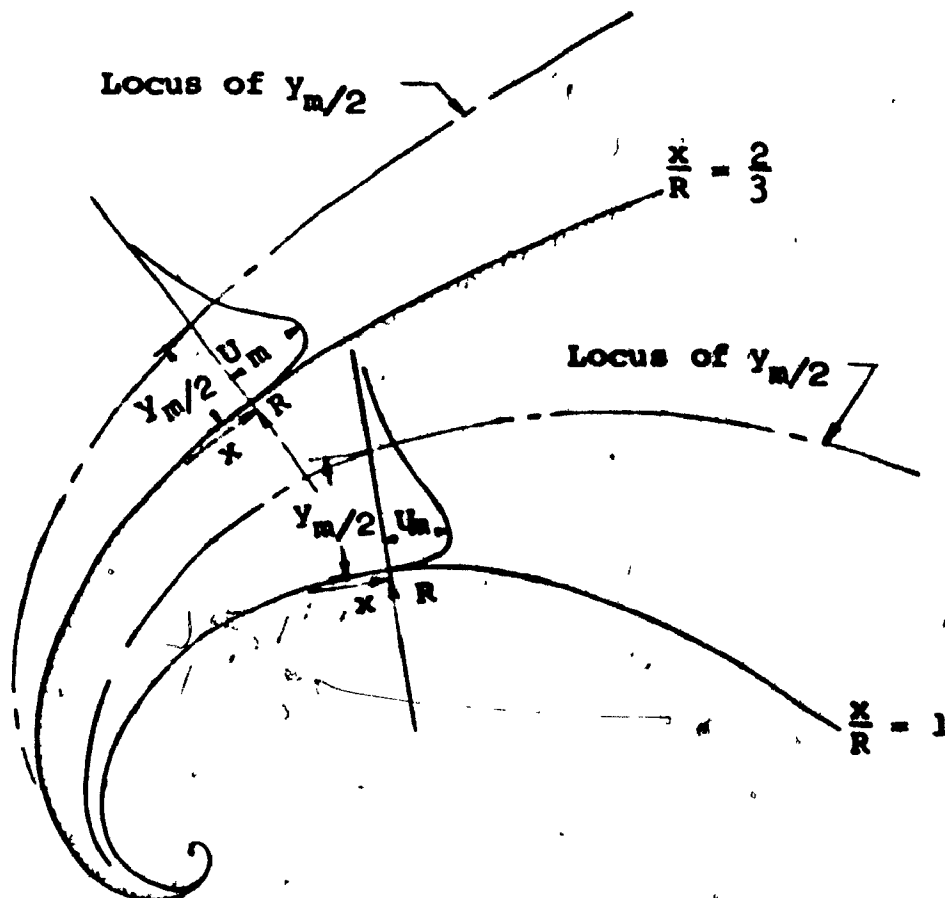
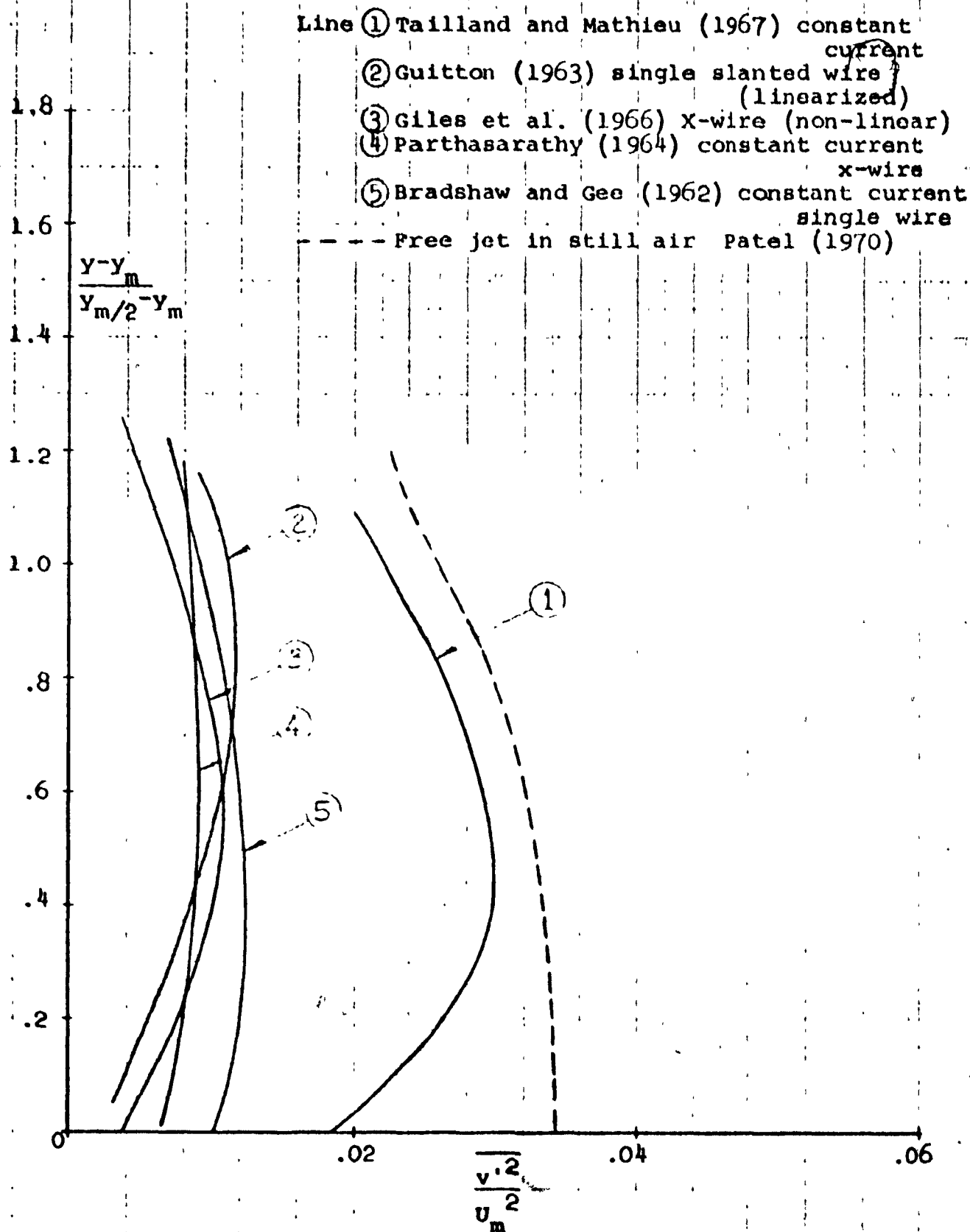


Fig. 1.3

VARIOUS MEASUREMENTS OF $\frac{\sqrt{v \cdot 2}}{u_m^2}$ IN A PLANE WALL JET IN STILL AIR



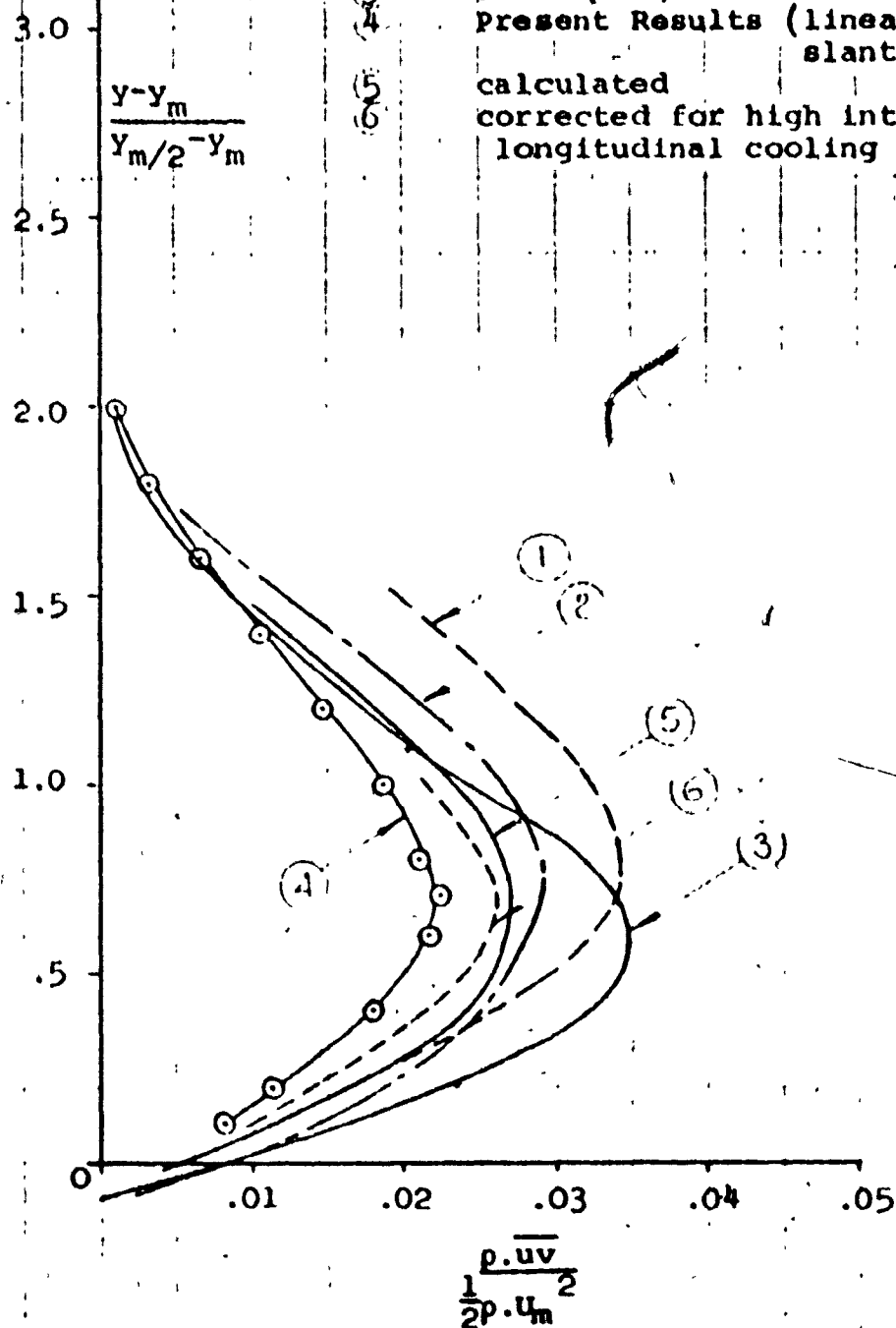
COMPARISON BETWEEN MEASUREMENTS OF THE SHEARING

STRESS DISTRIBUTION (UNCORRECTED)

2-D WALL JET IN STILL AIR

$$b = .300" \quad \frac{x}{b} = 88.4 \quad Re = 3.0 \times 10^4$$

- Line (1) Bradshaw and Gee (1952)
 (2) Tailland and Mathieu (1961)
 (3) Patel (1962)
 (4) Present Results (linearized single
 slanted wire)
 (5) calculated
 (6) corrected for high intensity and
 longitudinal cooling



EFFECT OF FRICTION ON THE MAXIMUM VELOCITY DECAY COEFFICIENT OF
THE PLANE WALL JET BASED ON A COMPARISON WITH THE FREE JET VALUE

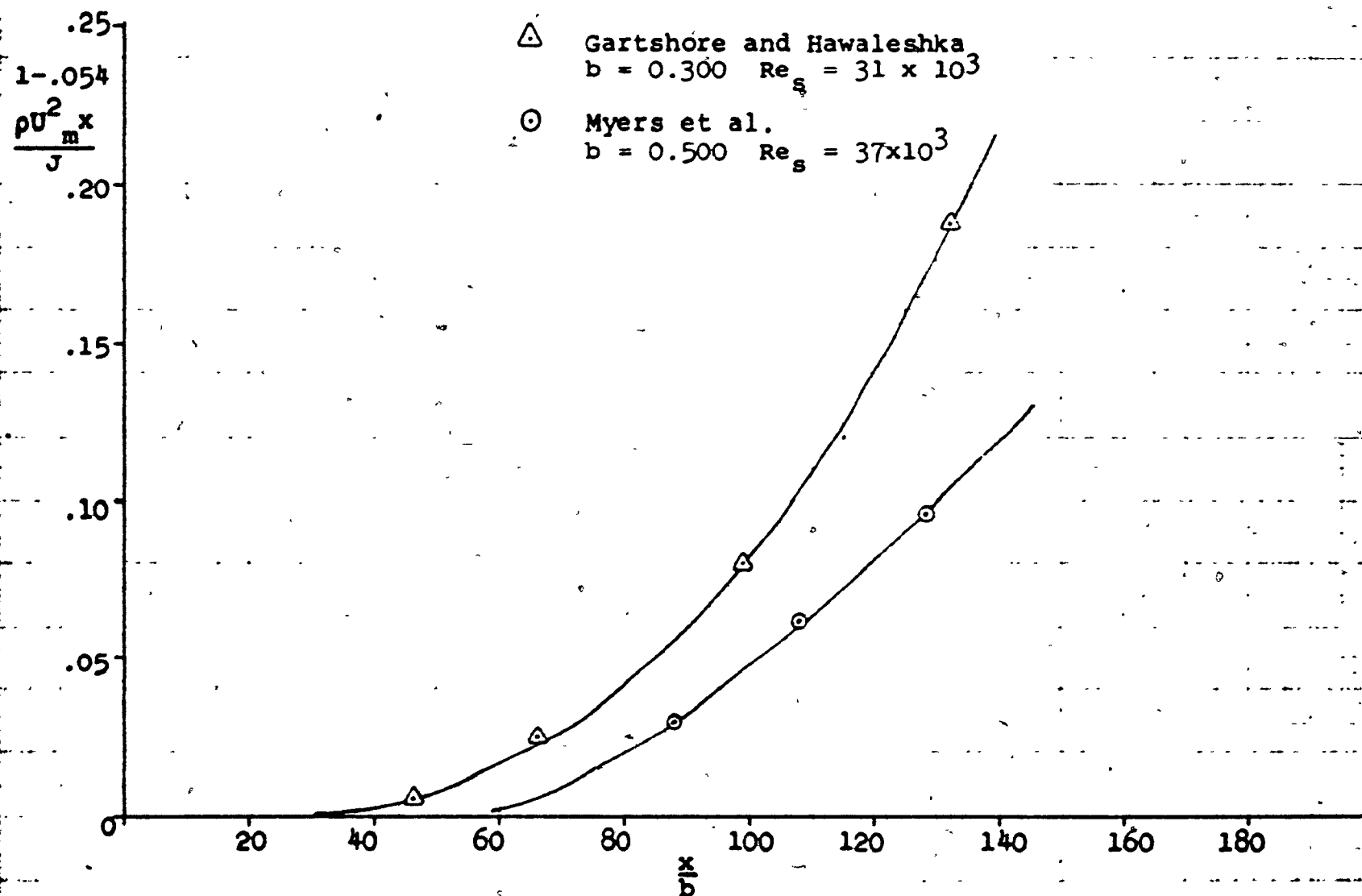


Fig. 3.1

CONVEX CIRCULAR CYLINDER - FEKETE (1963) RAW DATA FOR CASE $\frac{b}{R} = 0.531$ and $Re = 1.26 \times 10^4$

Lines ① $\frac{\rho U_\infty^2 R \theta}{J}$ ② $\frac{(p - p_w)R}{J}$ ③ $\frac{Y_w}{b}$

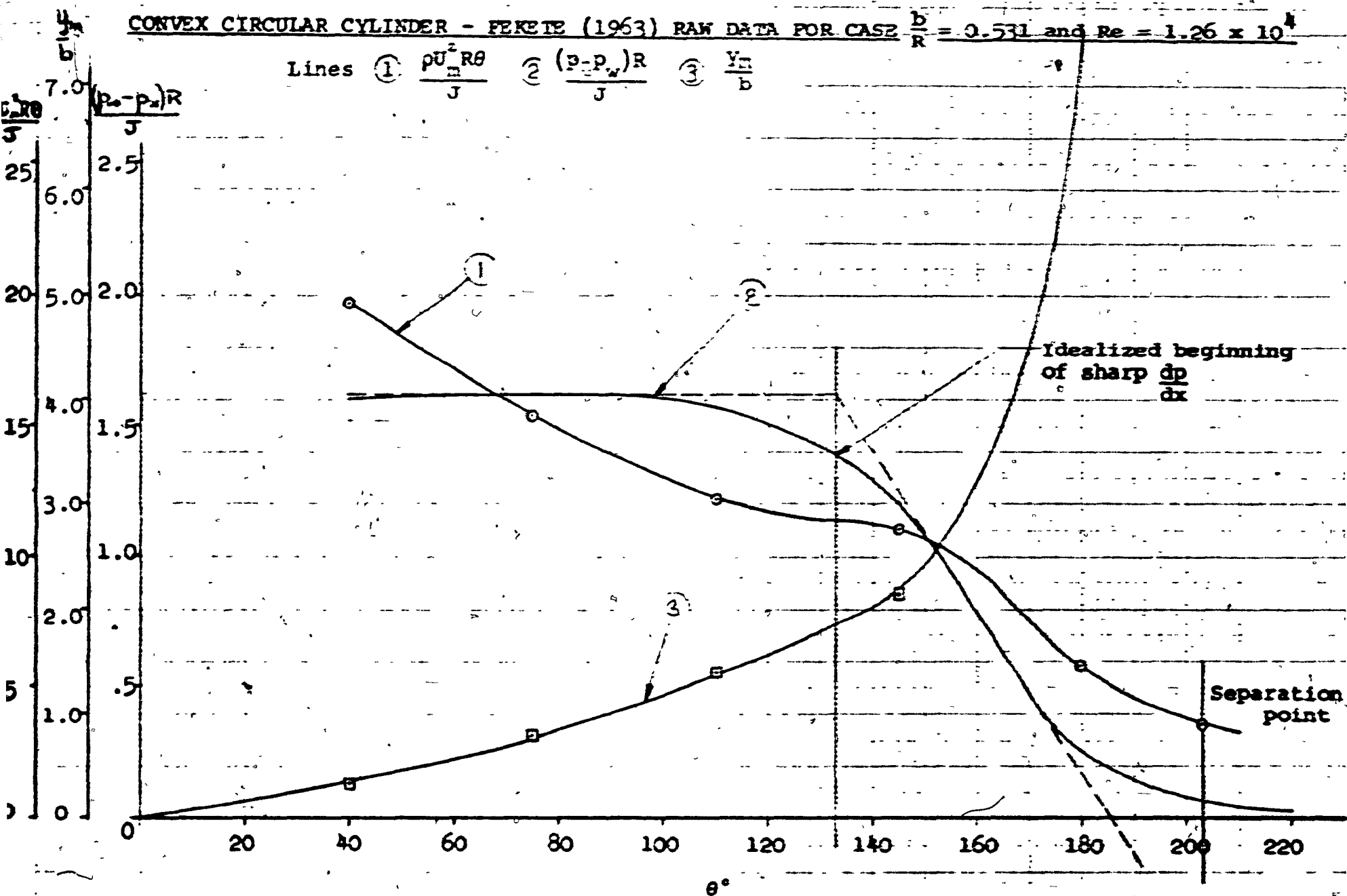


Fig. 5.1a

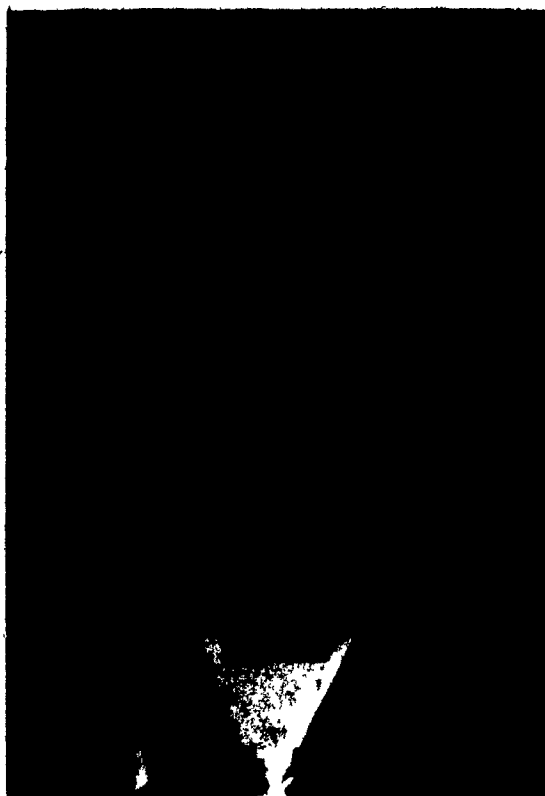


Fig. 5.1a Apparatus used in studying
the curved equilibrium flows
(shown installed is the
 $\frac{X}{R} = 1$ spiral)

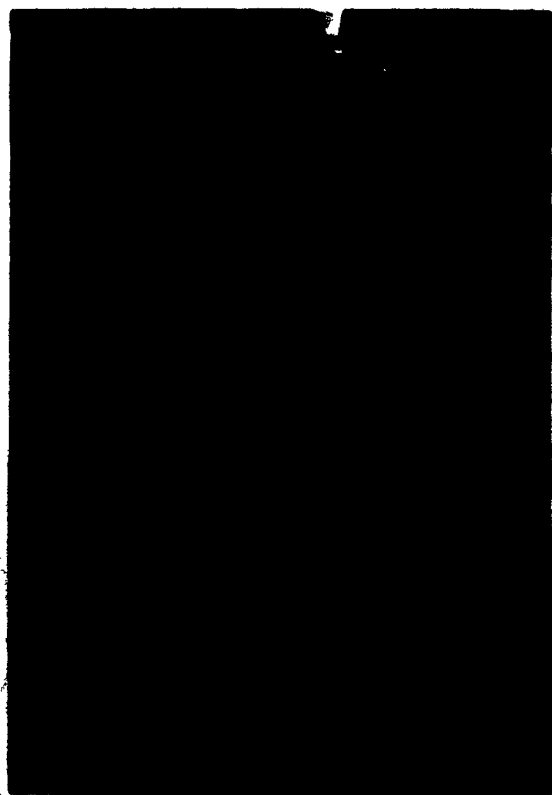
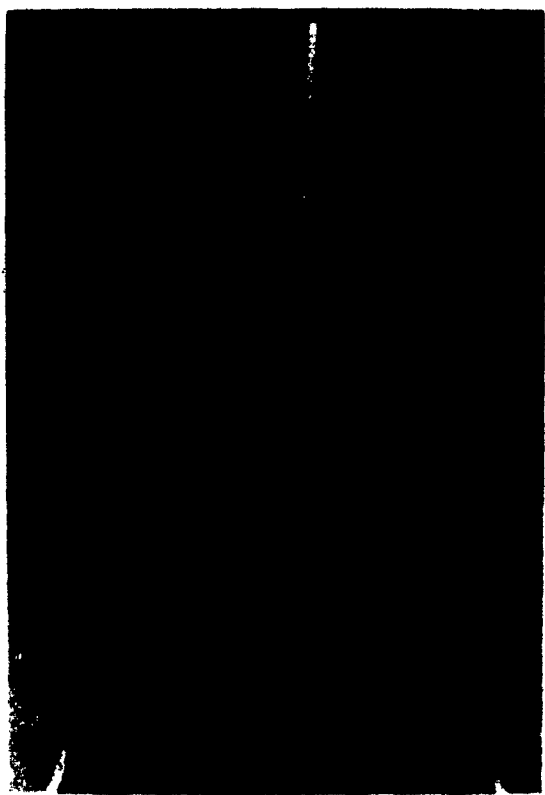
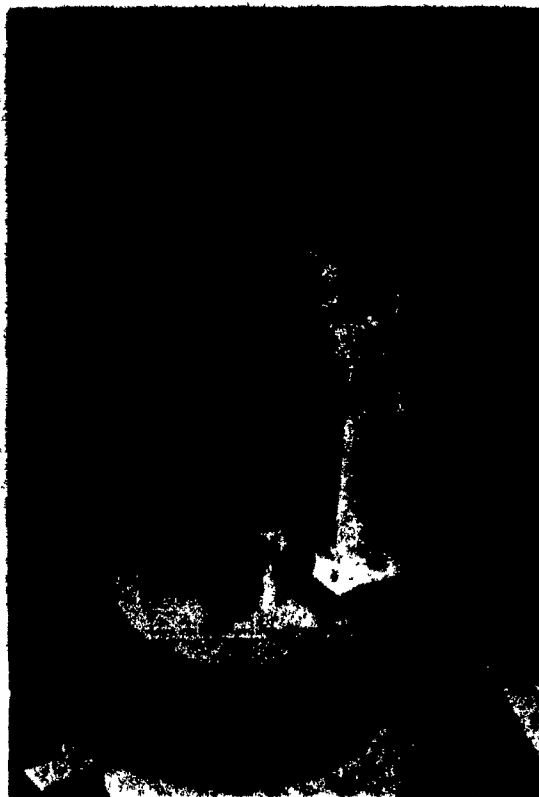
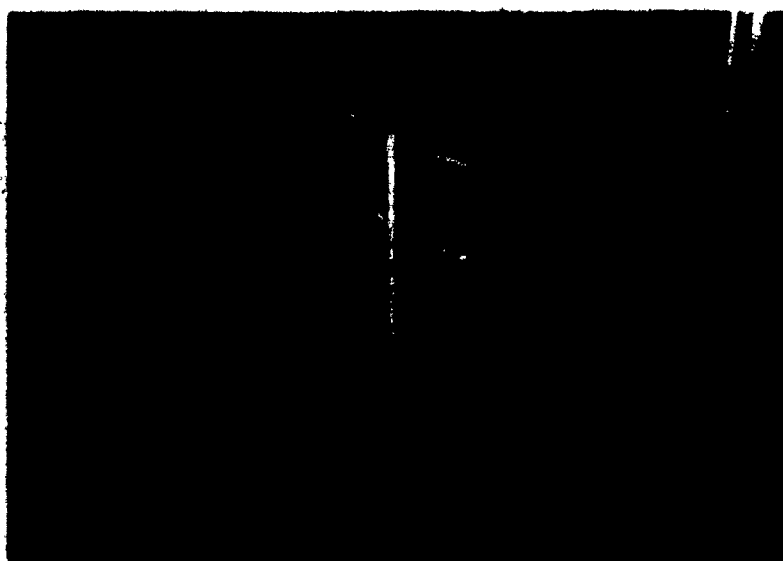


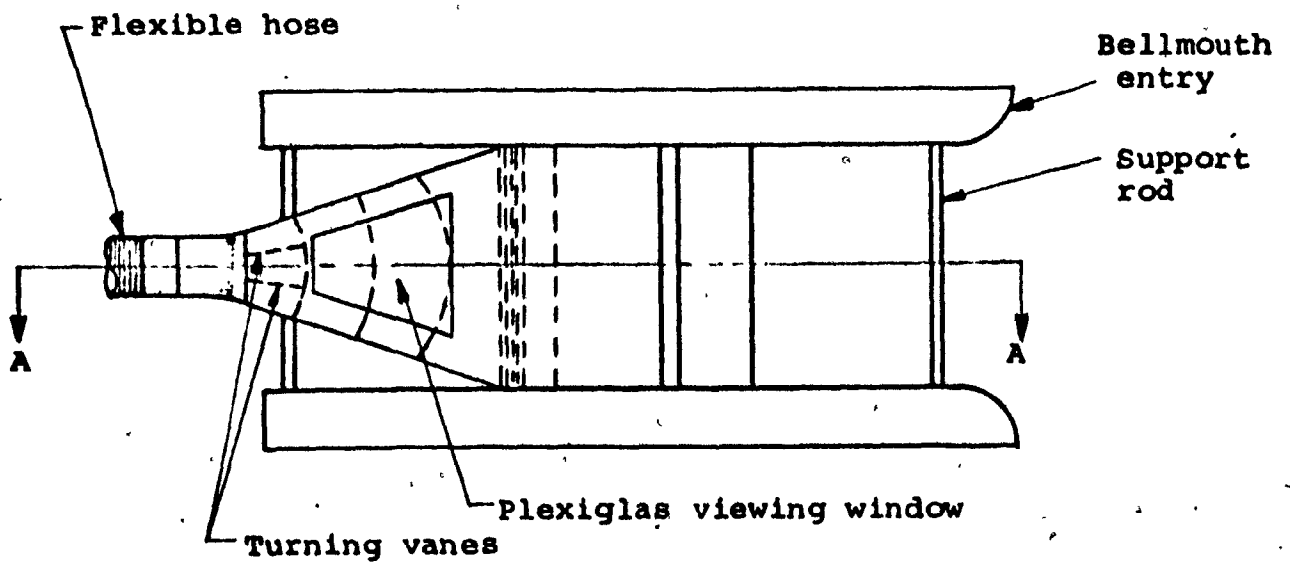
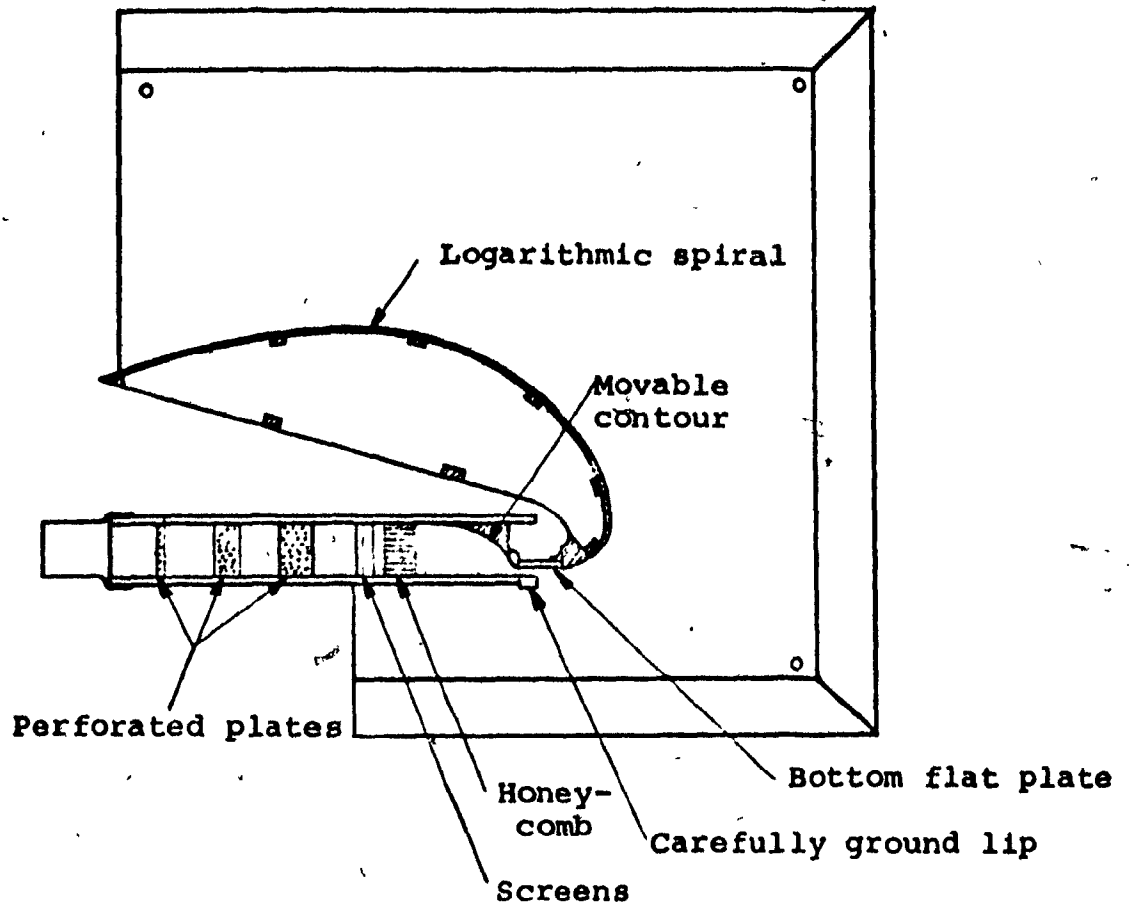
Fig. 5.1b



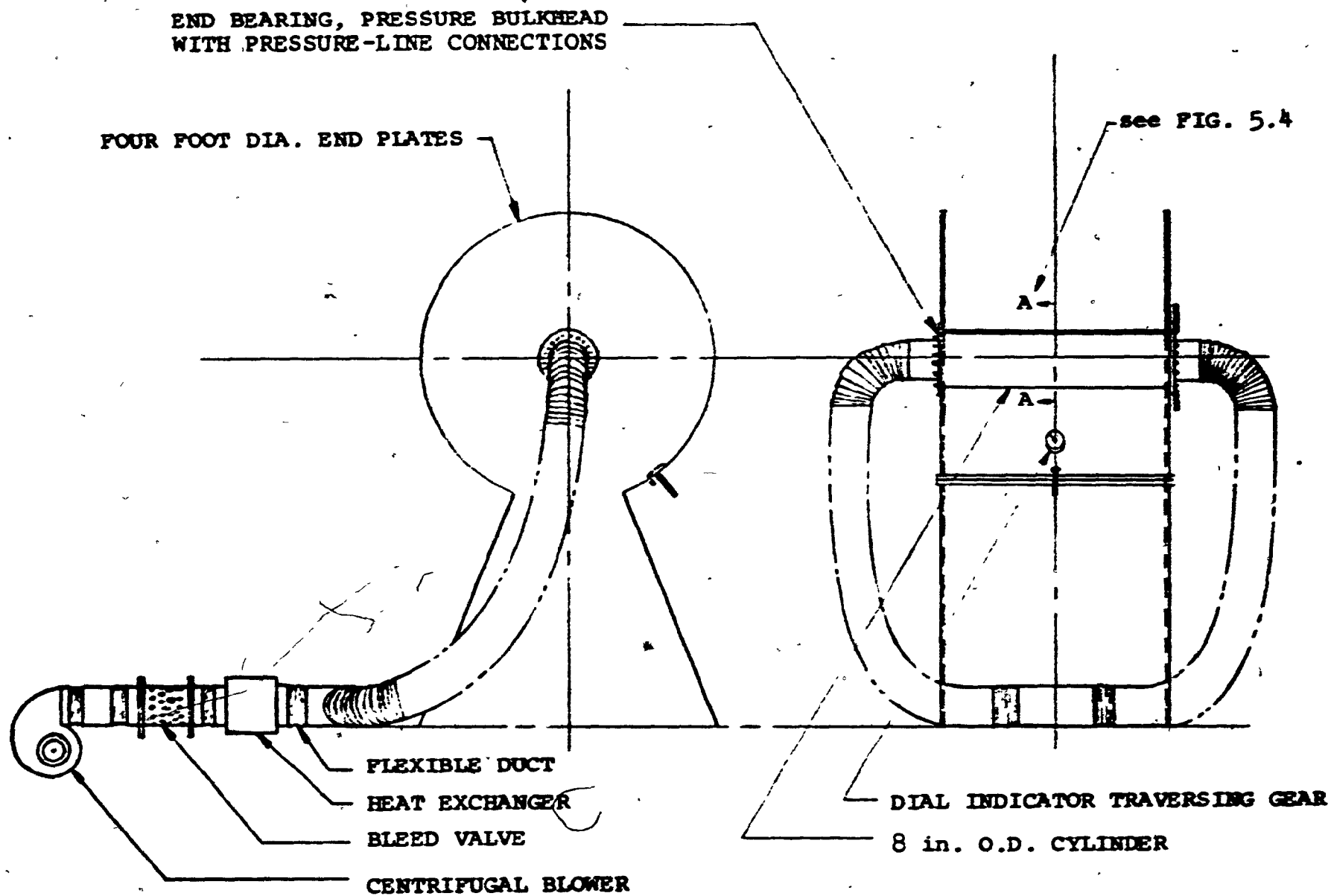
Calibrating nozzle and mechanism
for adjusting wire orientation



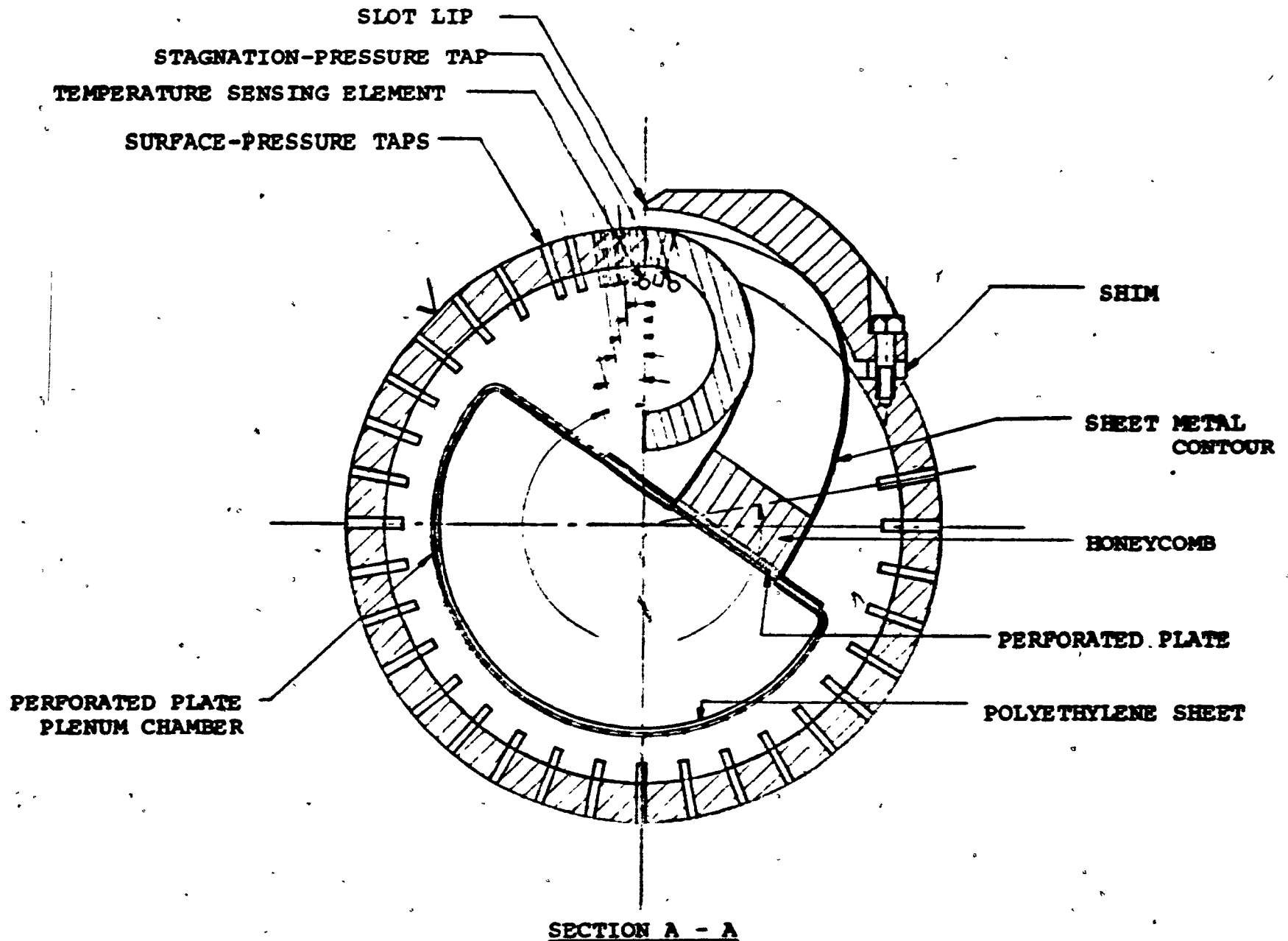
View of $\frac{X}{R} = \frac{2}{3}$ spiral showing
construction details

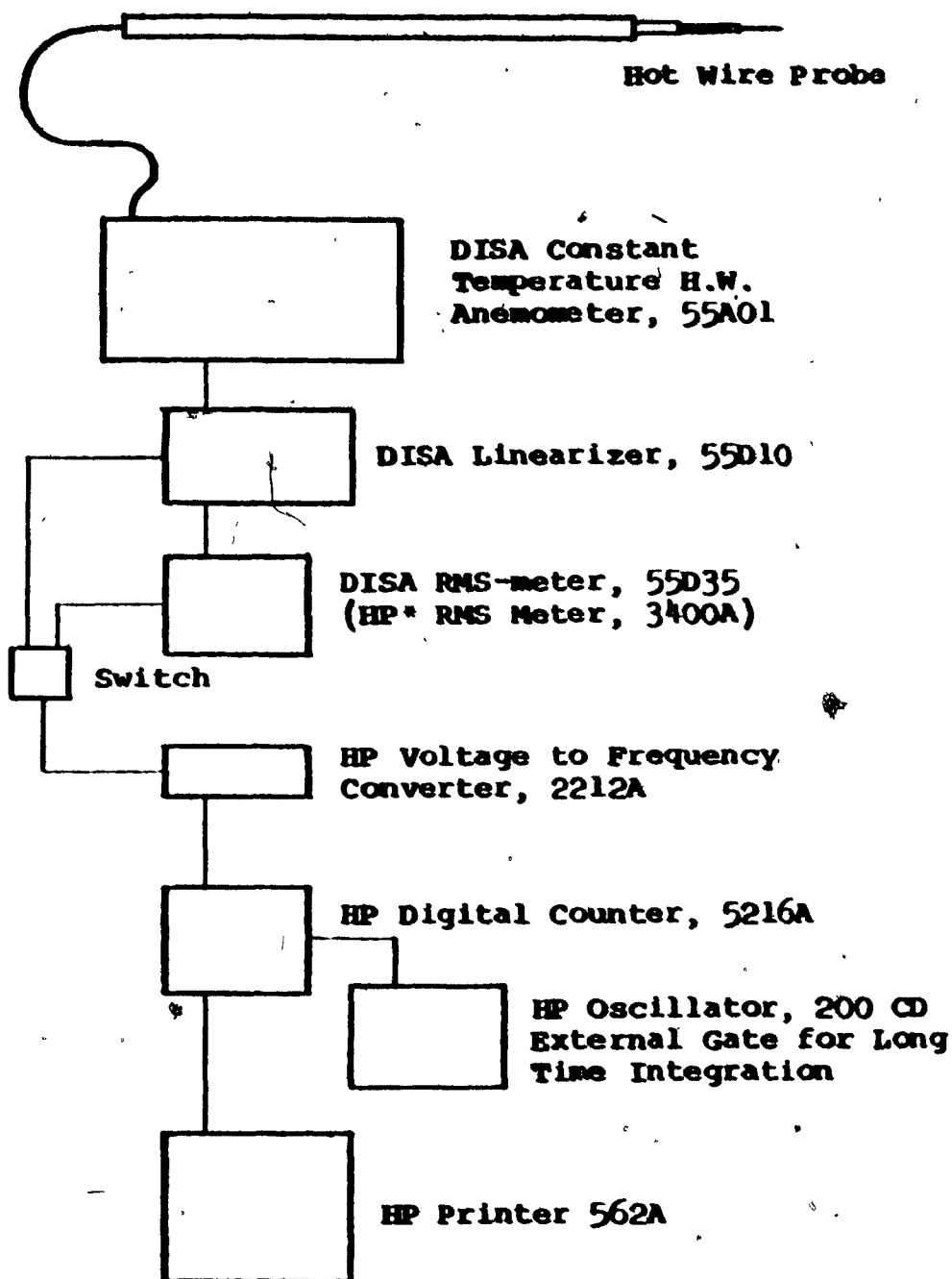


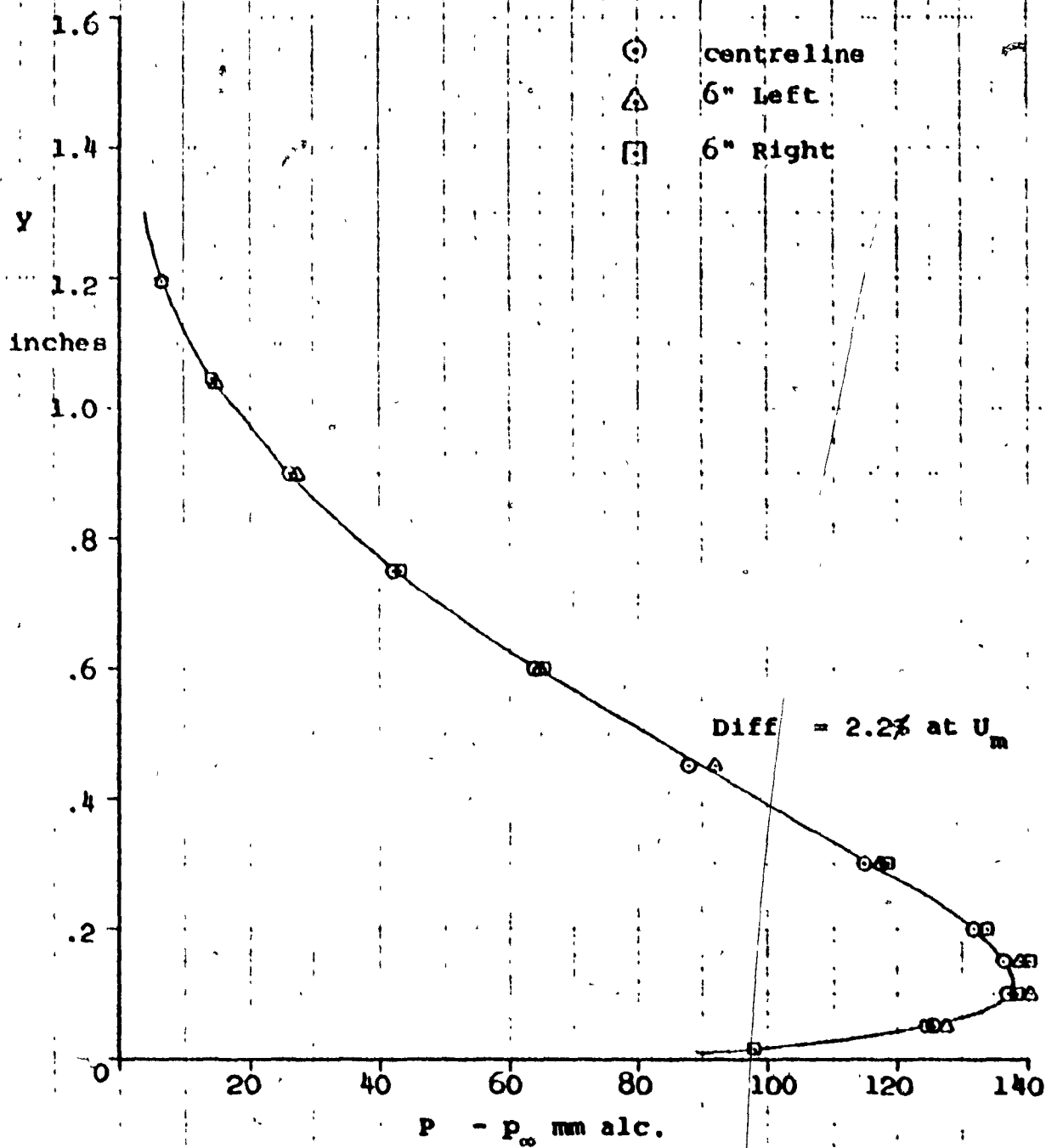
COANDA FLOW OVER CIRCULAR CYLINDER-APPARATUS



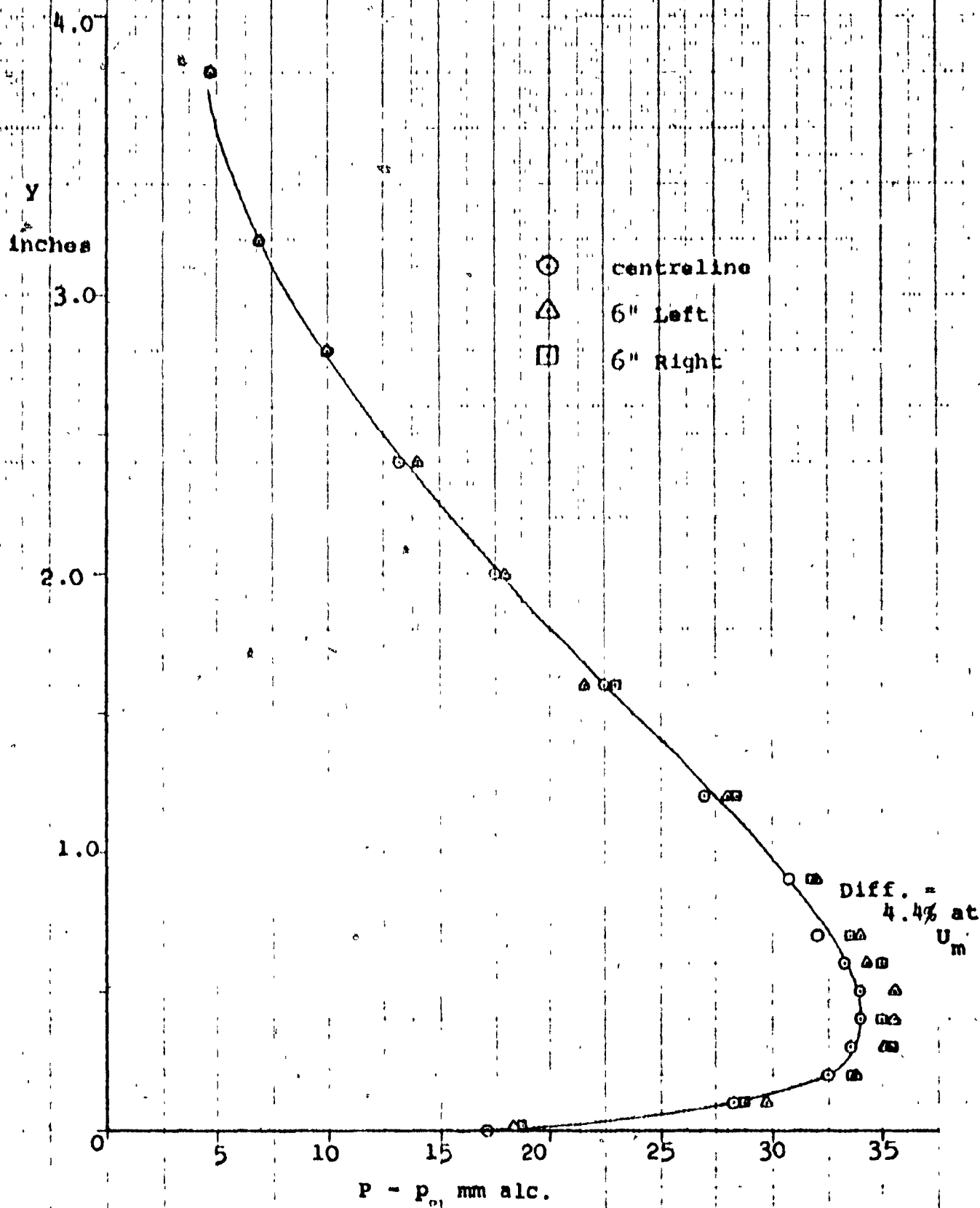
SECTION THROUGH CYLINDER OF FIGURE 5.3

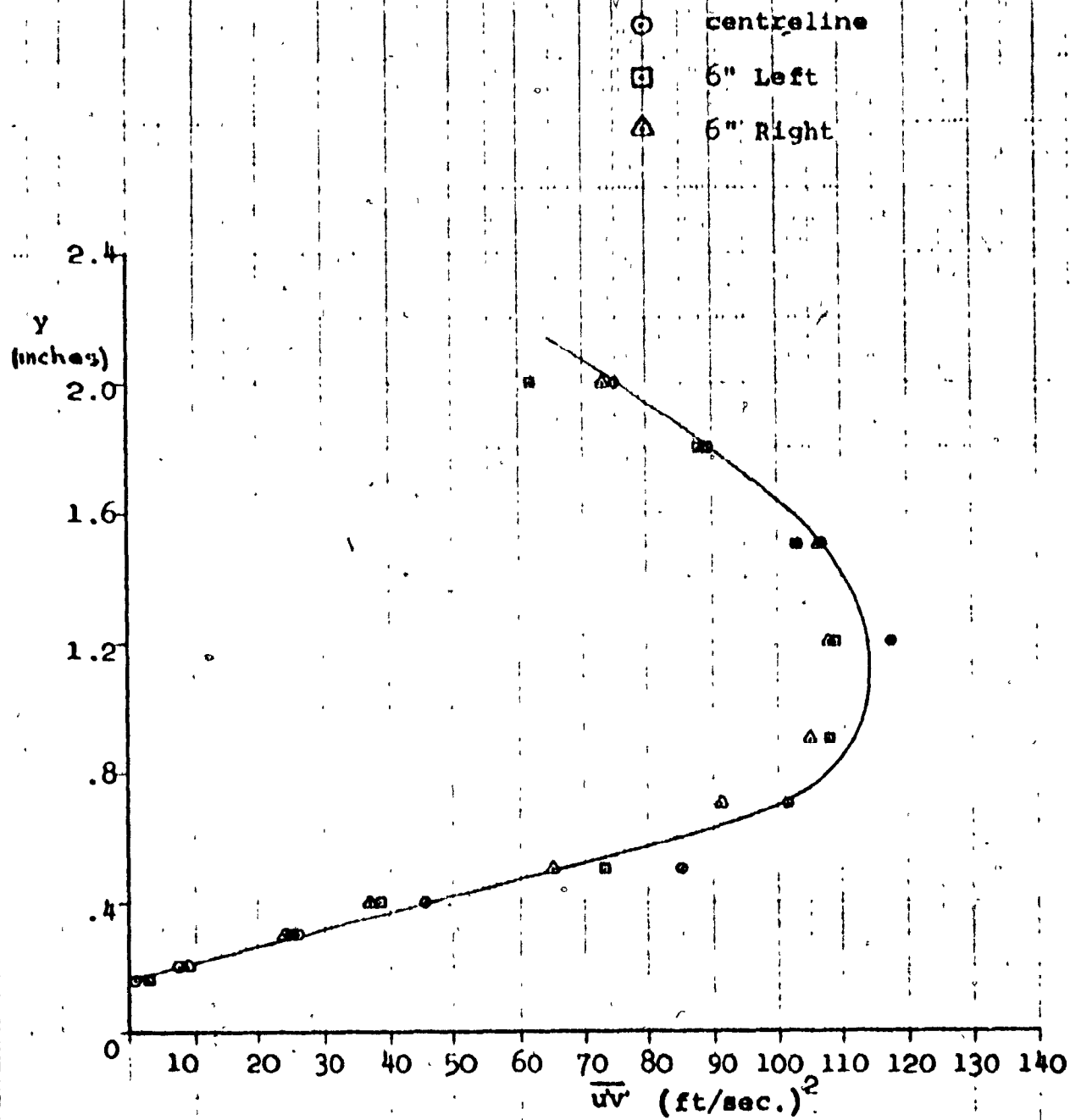


INSTRUMENTATION

WALL JET IN STILL AIR - TWO-DIMENSIONALITYCHECK AT $\frac{x}{b} = 31$, $Re = 3 \times 10^4$ 

WALL JET IN STILL AIR - TWO-DIMENSIONALITY

CHECK AT $\frac{x}{b} = 125$, $Re = 3 \times 10^4$ 

WALL JET IN STILL AIR - TWO-DIMENSIONALITYCHECK AT $\frac{x}{b} \approx 70$, $Re = 3 \times 10^4$ 

PLANE WALL JET IN STILL AIR

WALL SHEAR TWO-DIMENSIONALITY - PRESTON TUBE

$\Delta p = (p_o - p_s)$ = Preston Tube Reading
relative to local static pressure

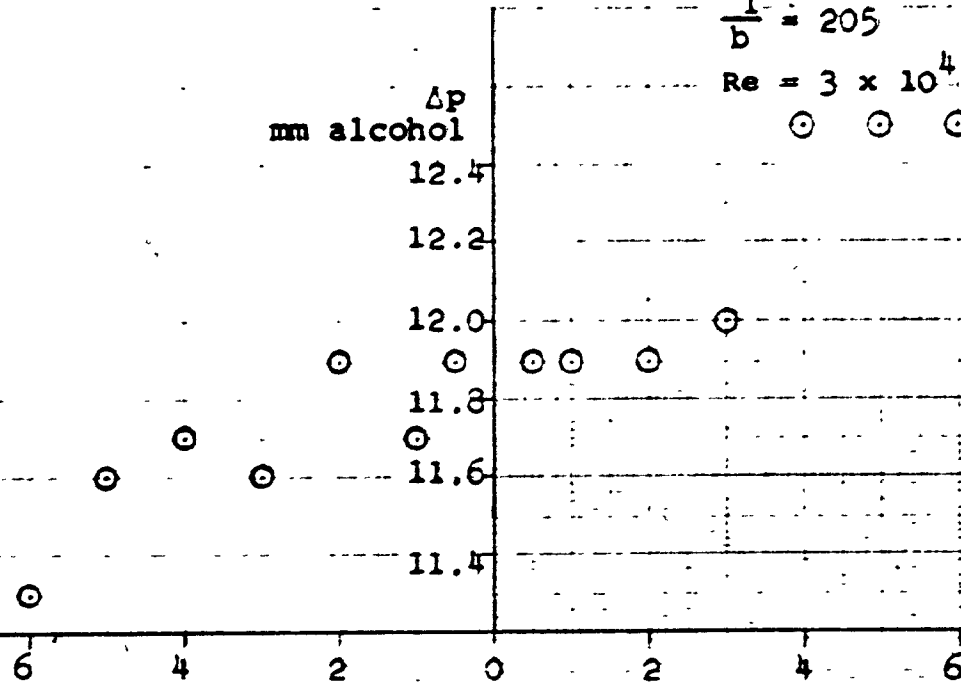
$b = .300"$

$x_1 = 61.6"$

$\frac{x_1}{b} = 205$

$Re = 3 \times 10^4$

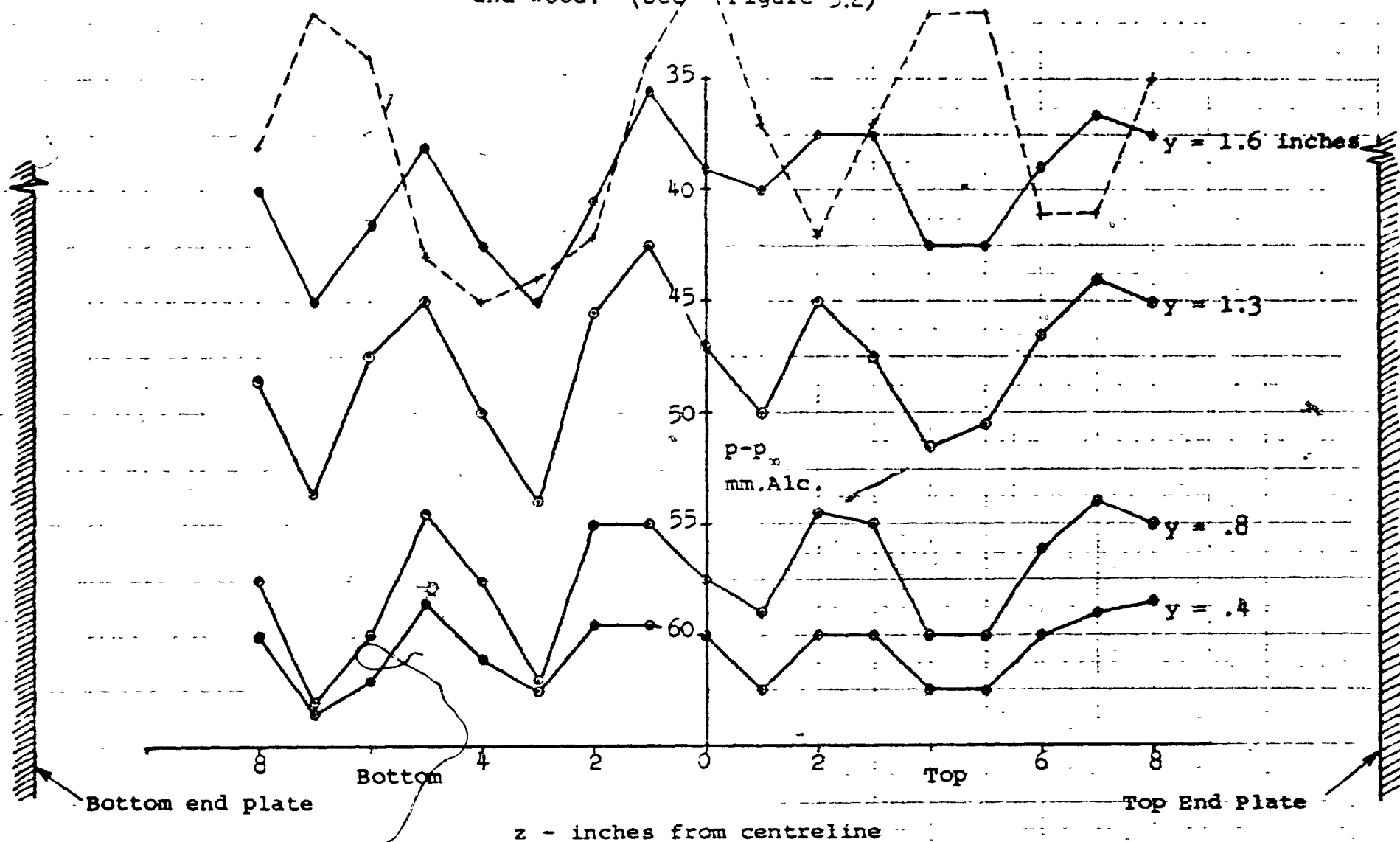
Δp
mm alcohol



DISTANCE FROM CENTRE-LINE (INCHES)

LATERAL TOTAL PRESSURE VARIATION AT CONSTANT DISTANCES FROM SURFACE ($\gamma R = 2/3$)

— Initial Trial
 - - - Change caused by masking tape covering smooth joint between lip and wood. (See Figure 5.2)

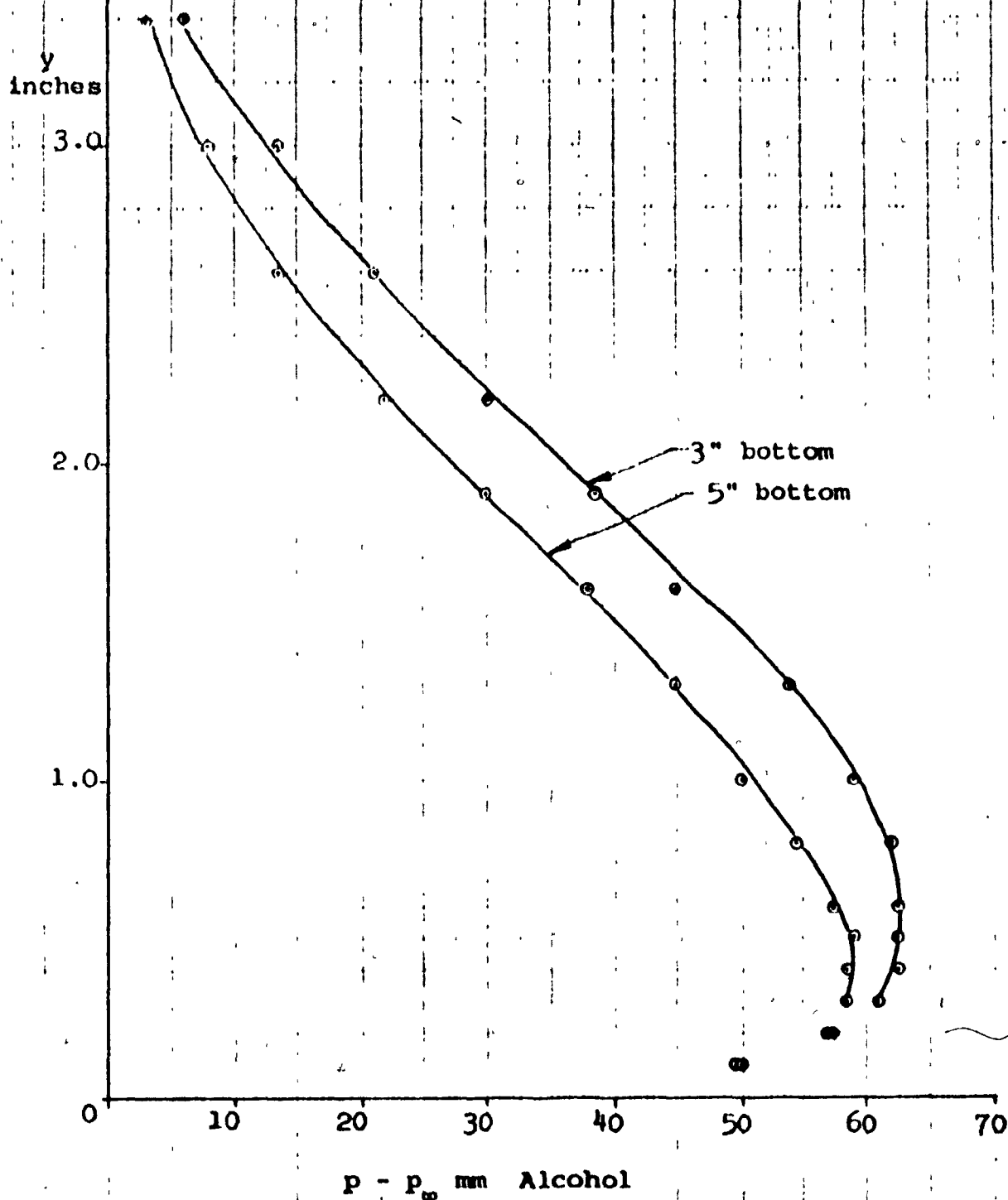


TRAVERSES TAKEN 3" AND 5" BELOW CENTRE-LINE (SEE FIG. 6.1)

INITIAL TRIAL ON $\frac{x}{R} = \frac{2}{3}$ SPIRAL

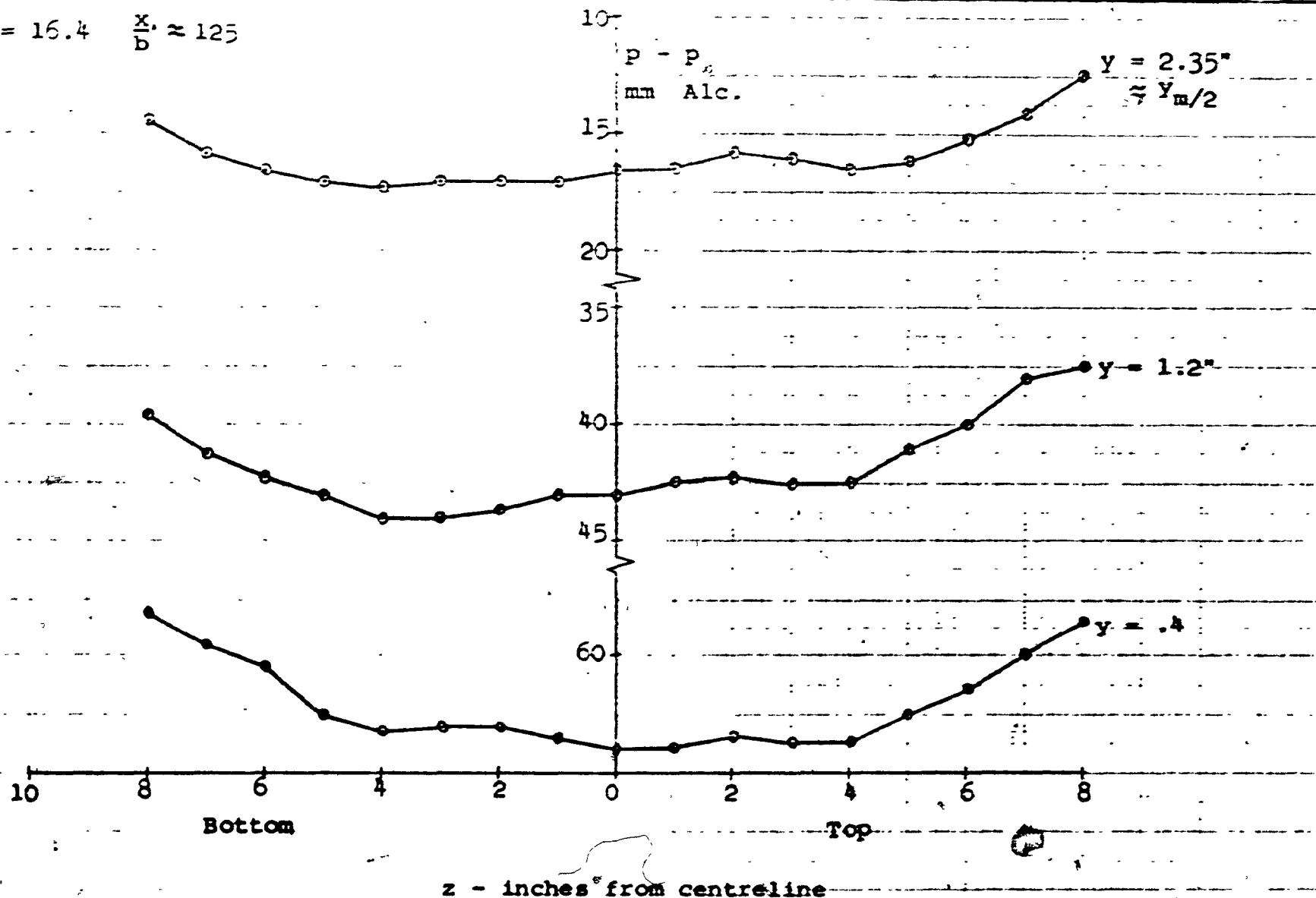
$x = 16.4$ inch

$\frac{x}{b} \approx 125$

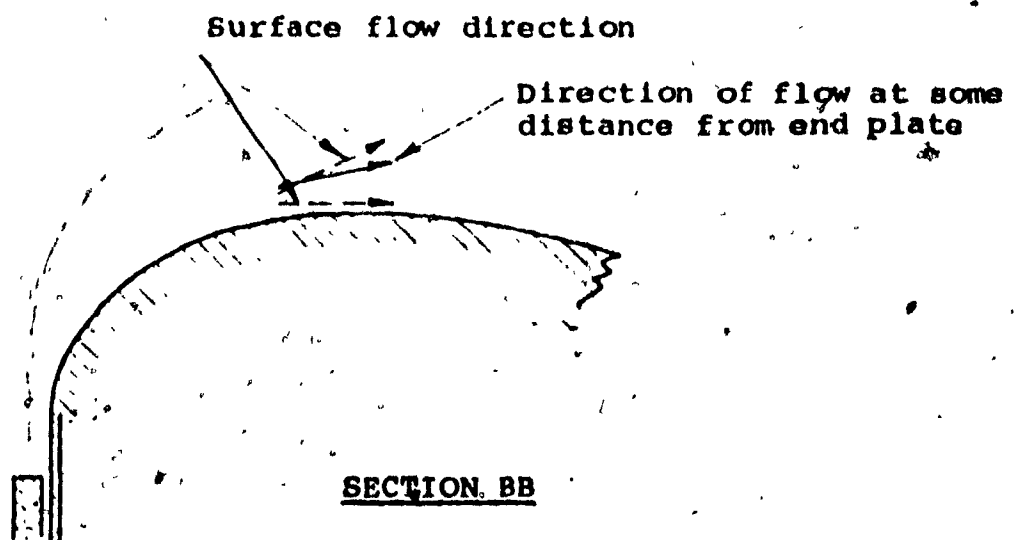
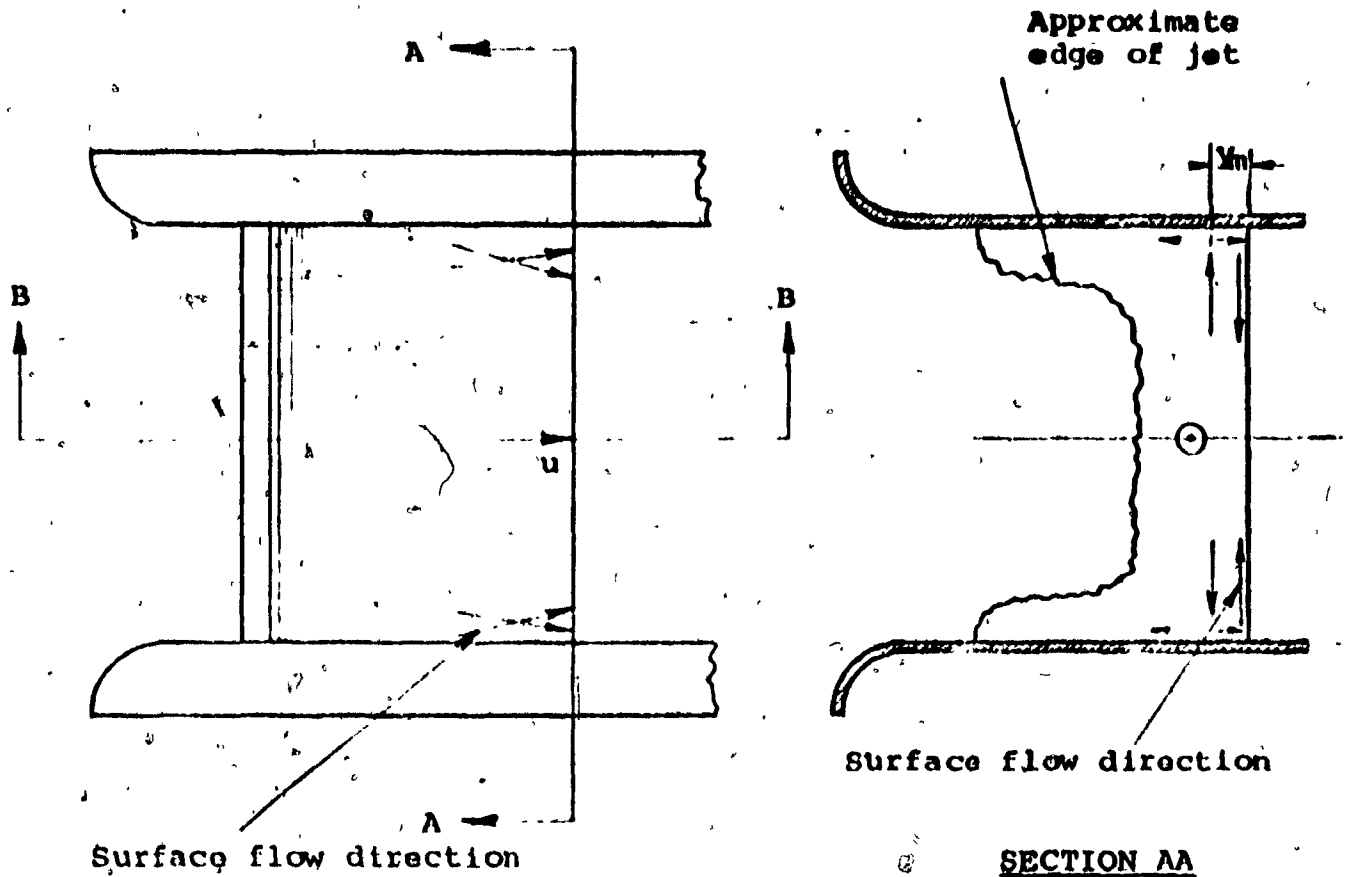


LATERAL TOTAL PRESSURE VARIATION ON $\frac{x}{R} = \frac{2}{3}$ SPIRAL AFTER NEW PRECISION GROUND 90° SLOT LIP
INSTALLED AND PLENUM CHAMBER CONDITION IMPROVED - COMPARE WITH FIGURE 6.1 SHOWING INITIAL CONDITIONS

$x = 16.4$ $\frac{x}{D} \approx 125$



SECONDARY FLOWS: SCHEMATIC OF OBSERVED FLOW DIRECTIONS ON SPIRALS. OBSERVATIONS MADE WITH DOWN TUFTS



SECONDARY FLOWS - TYPICAL TOTAL STATIC PRESSURE PROFILES MADE AT THE CENTRE LINE AND

VERY NEAR TO THE END PLATE, $\frac{x}{R} = \frac{2}{3}$ SPIRAL, $x = 8.7$ INCHES, $\frac{x}{D} \approx 115$

•• 9.5" B (2.5" BOTTOM, SEE FIGURE 6.1)
 ▷ 10.5" B
 † 11" B
 + 11.25" B
 + 11.5" B

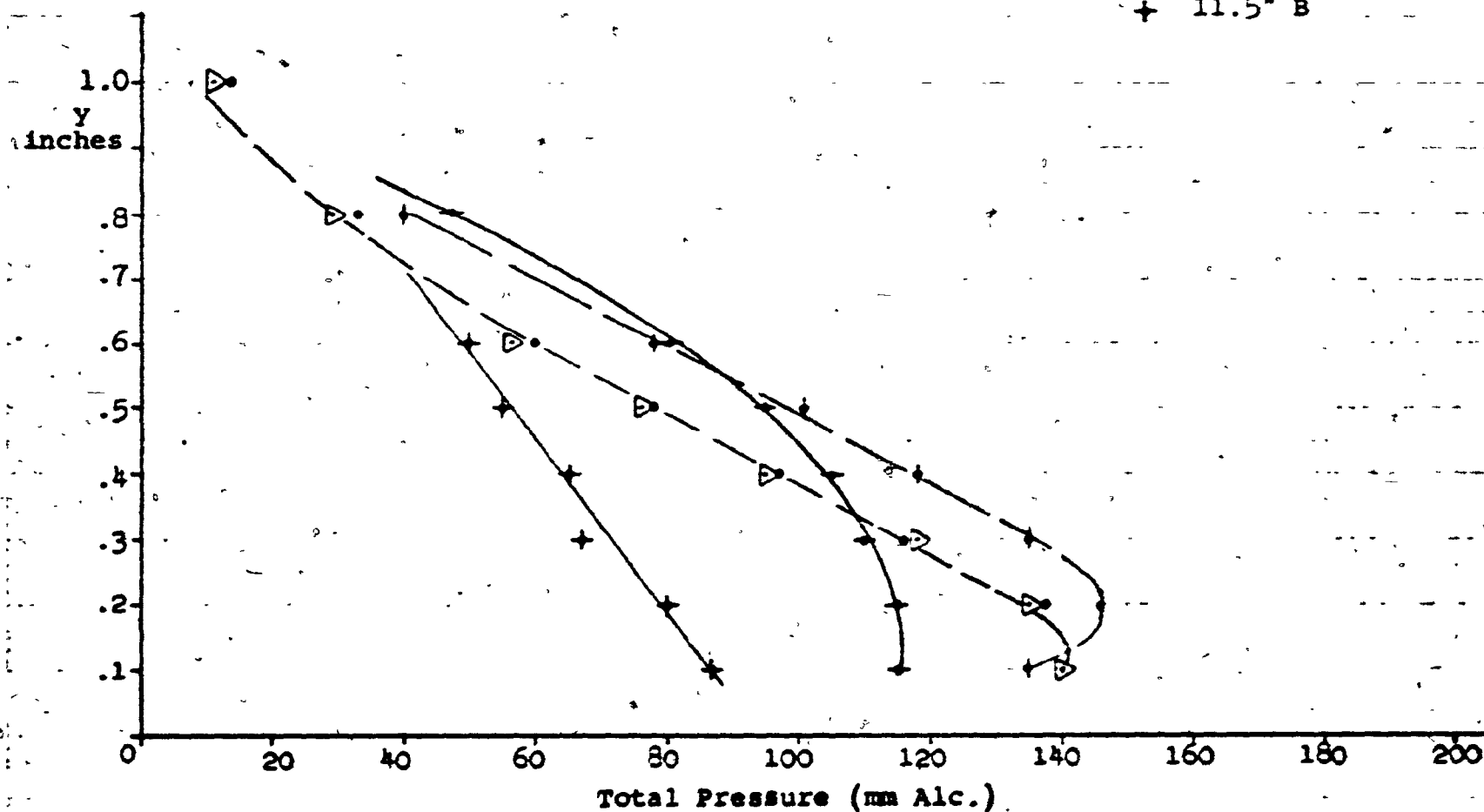
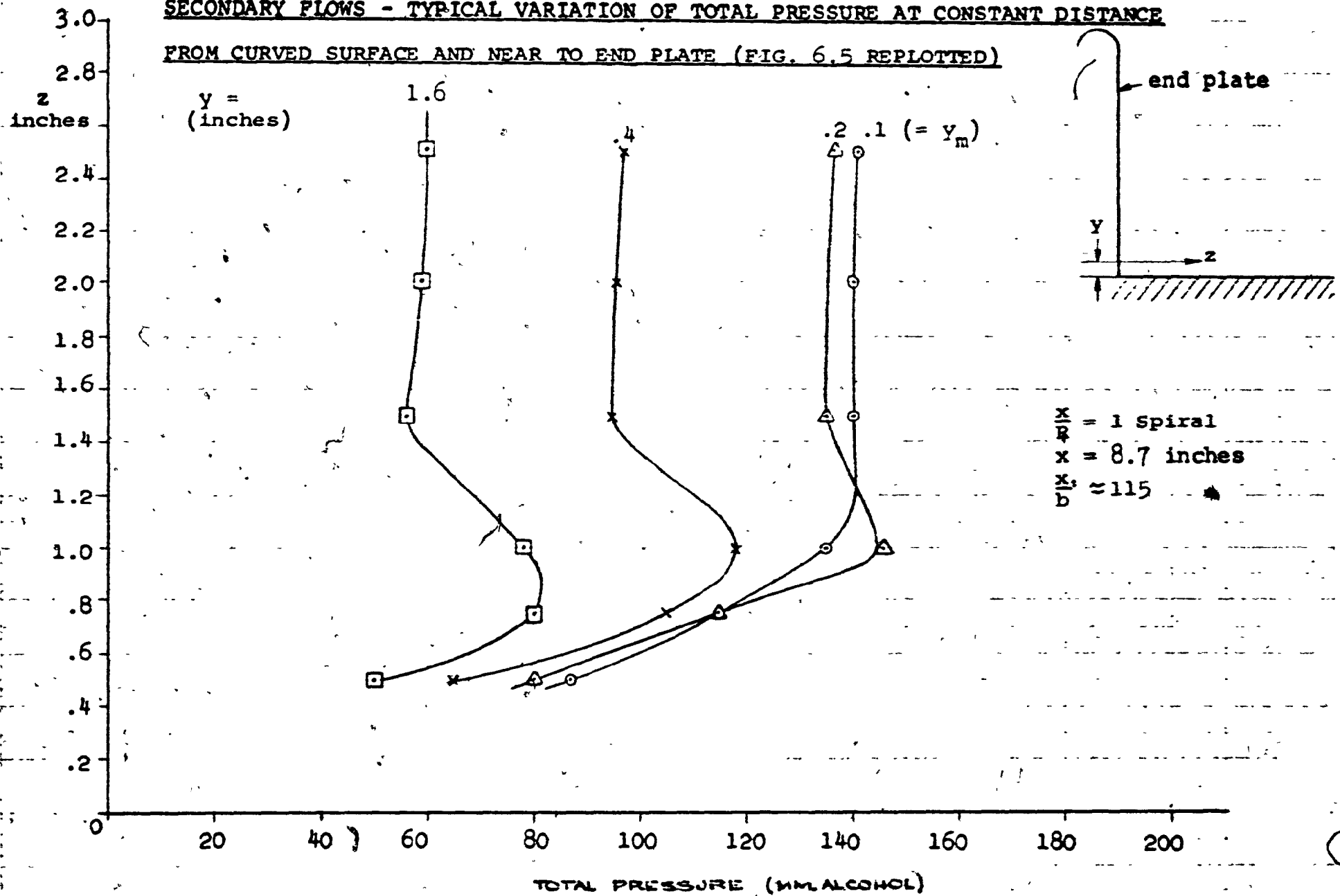


Fig. 6.5

SECONDARY FLOWS - TYPICAL VARIATION OF TOTAL PRESSURE AT CONSTANT DISTANCE
FROM CURVED SURFACE AND NEAR TO END PLATE (FIG. 6.5 REPLOTTED)



SECONDARY FLOW - EFFECT OF "LADDER TYPE" END PLATES ON LATERAL TOTAL PRESSURE VARIATION

$y = .8$ inches
 $\approx y_m$

Total Pressure
 (mm Alcohol $\times 5$)

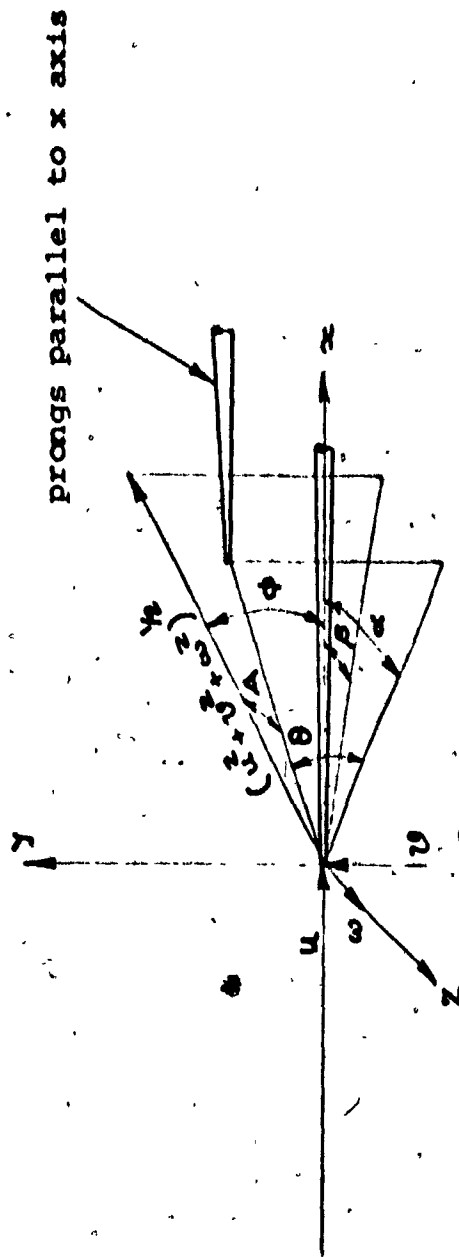
$\frac{x}{R} = \frac{2}{3}$
 $U_{max} \approx 260$
 $x = 32.3$ inches

○ end plate in
 x no end plate

$y = 2.8$ inches

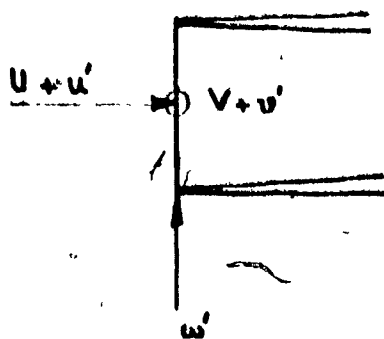
$y = 4.85$ inches
 $\approx y_m/2$

8 6 4 2 2 4 6 8

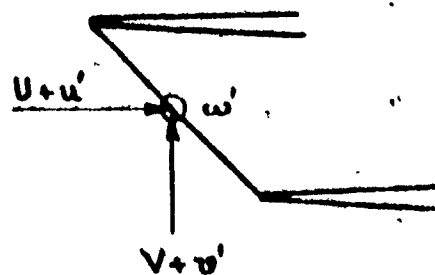


COORDINATE SYSTEM AND NOTATION FOR DERIVING THE GENERAL RESPONSE OF A HOT WIRE

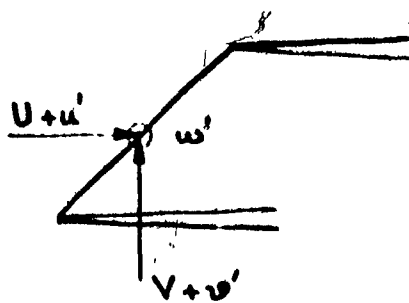
Fig. 7.2



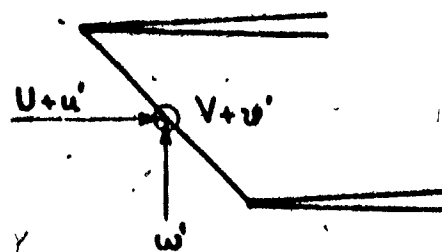
position 1



position 3

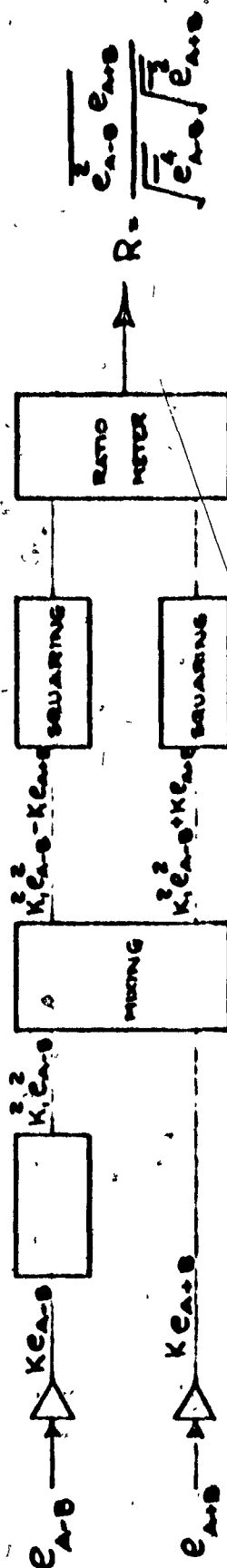


position 2

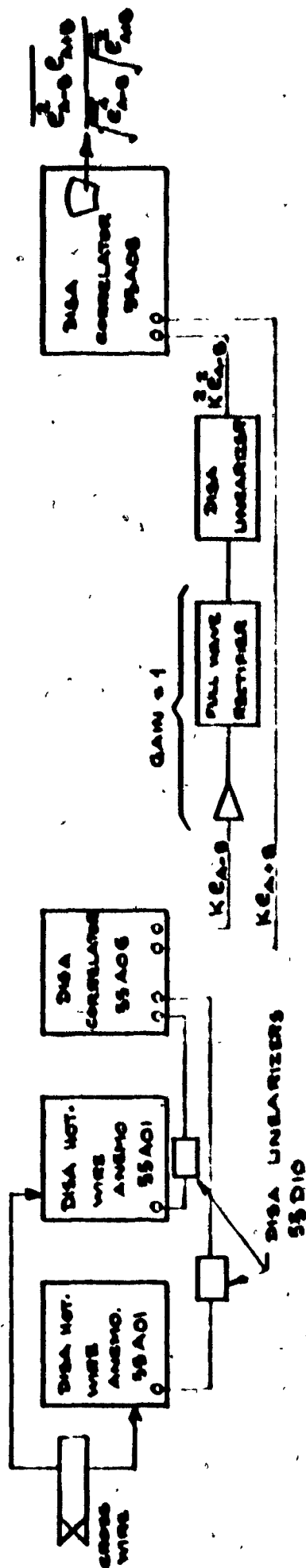


position 4

WIRE POSITIONS TO DETERMINE REYNOLDS
STRESSES



(a)



(b)

BLOCK DIAGRAMS OF CIRCUITS USED TO OBTAIN TRIPLE AND QUADRUPL2 CORRELATIONS

(a) Lafer (b) Present experiment

INTENSITY CORRECTION TO SHEAR STRESS IN PIPE

$$\overline{u'v'}_c = G \overline{u'v'}_m$$

Note: Calculations based on
triple correlations
measured by Laufer
 $Re = 5 \times 10^5$

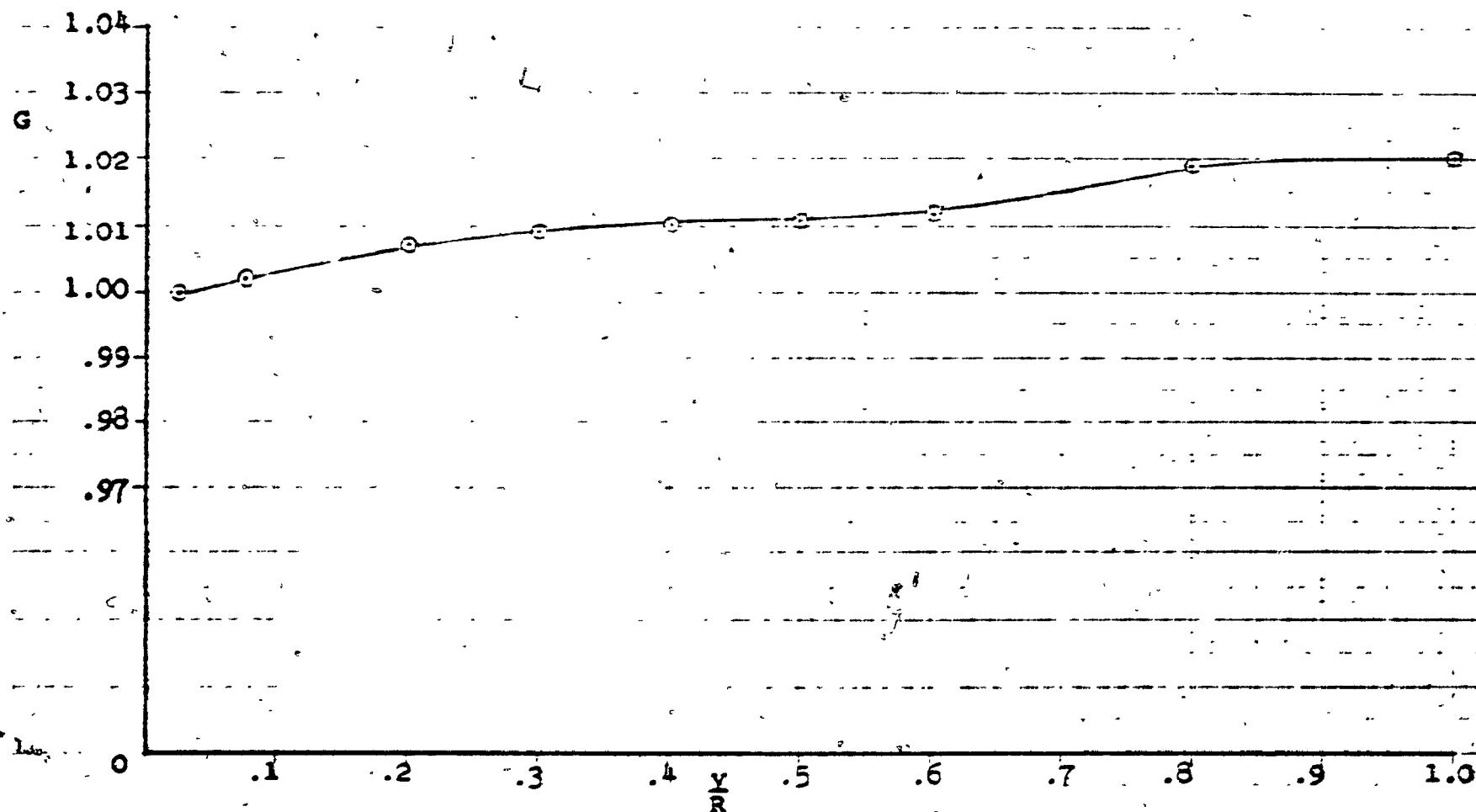
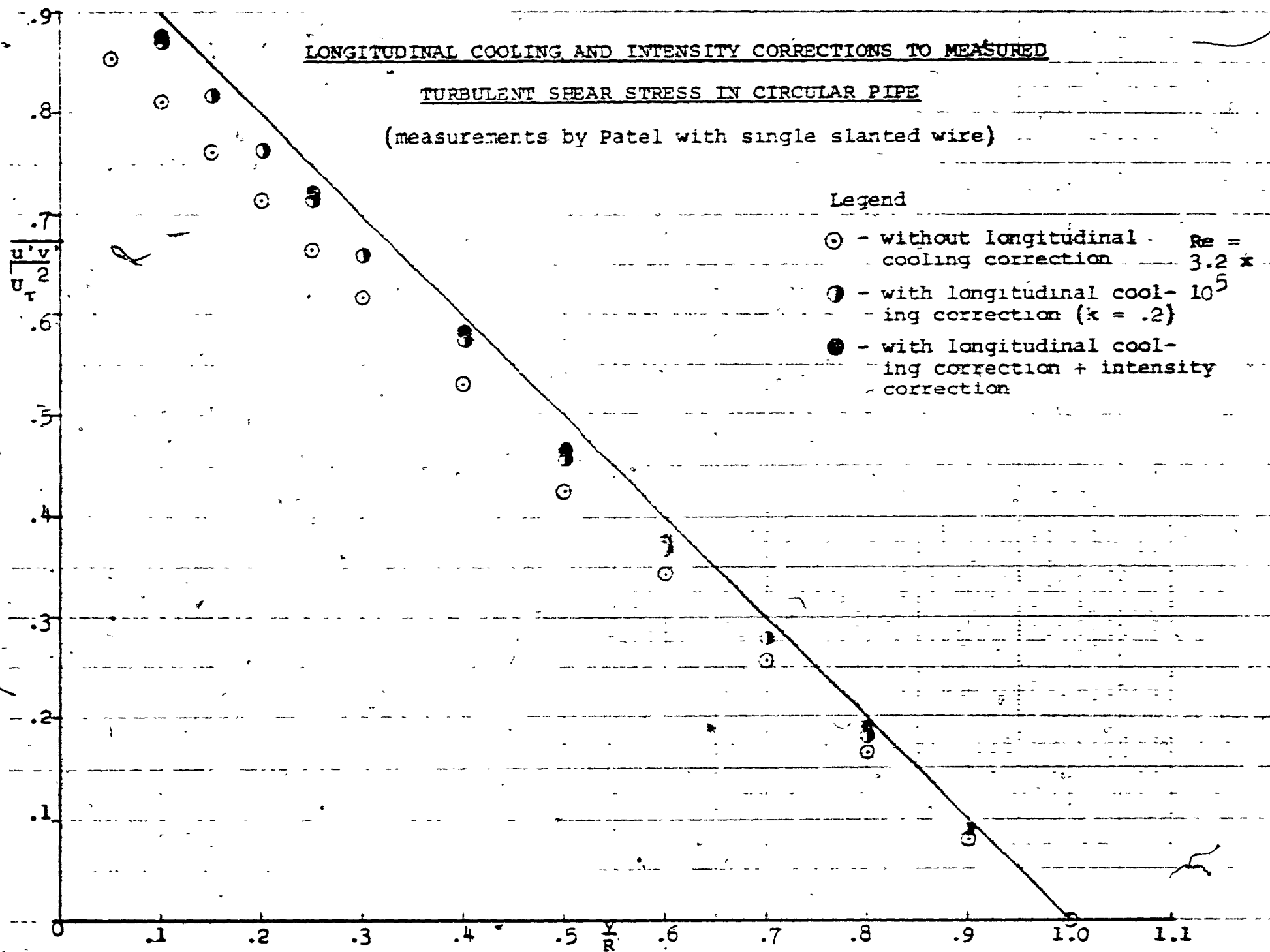


FIG. 7.4



TURBULENCE INTENSITY CORRECTION TO $\overline{u'^2}$, $\overline{v'^2}$ AND $\overline{u'v'}$

IN A FREE TWO-DIMENSIONAL JET

(Based on measurements by Heskstad)

Σ, F, G

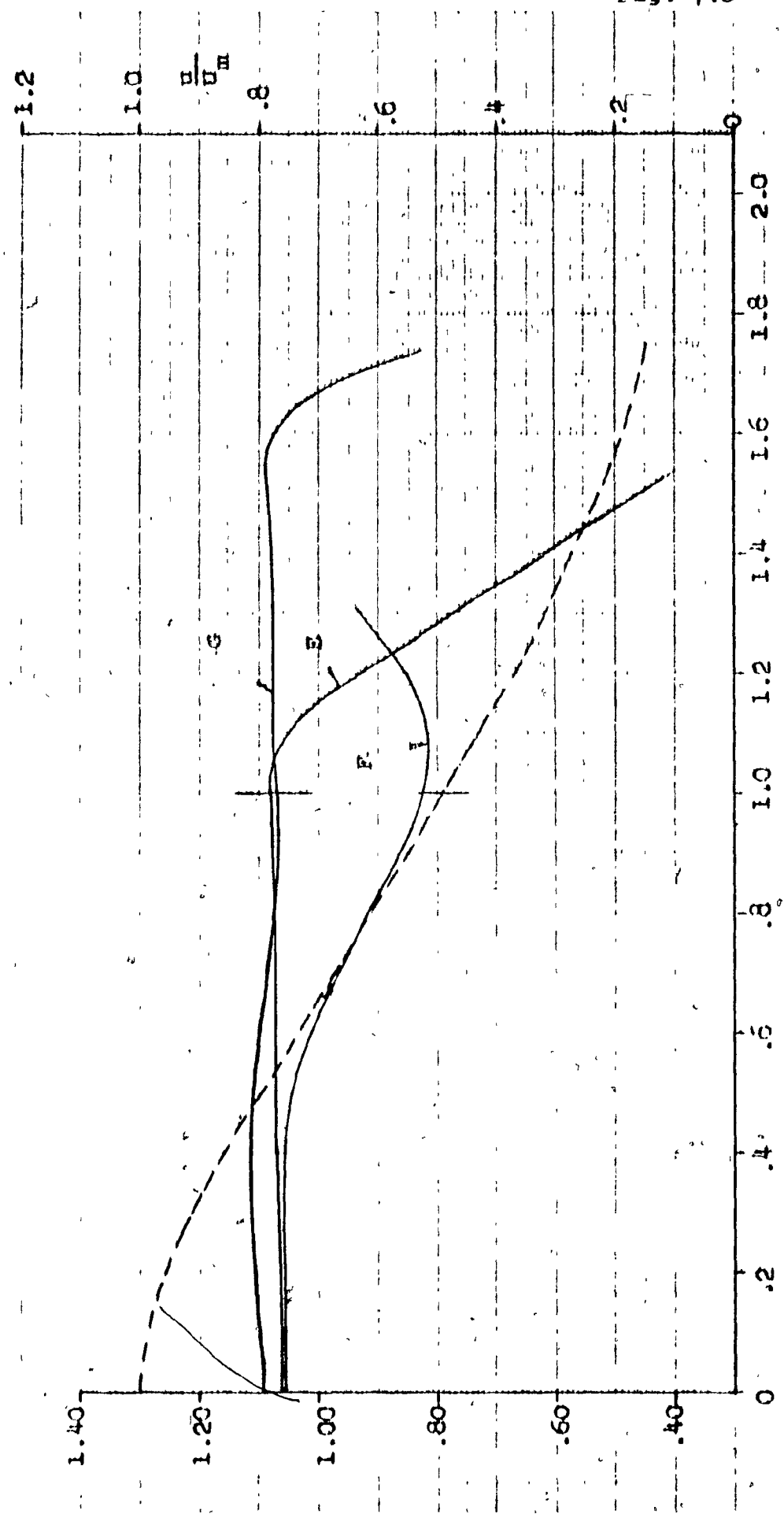


Fig. 7.6

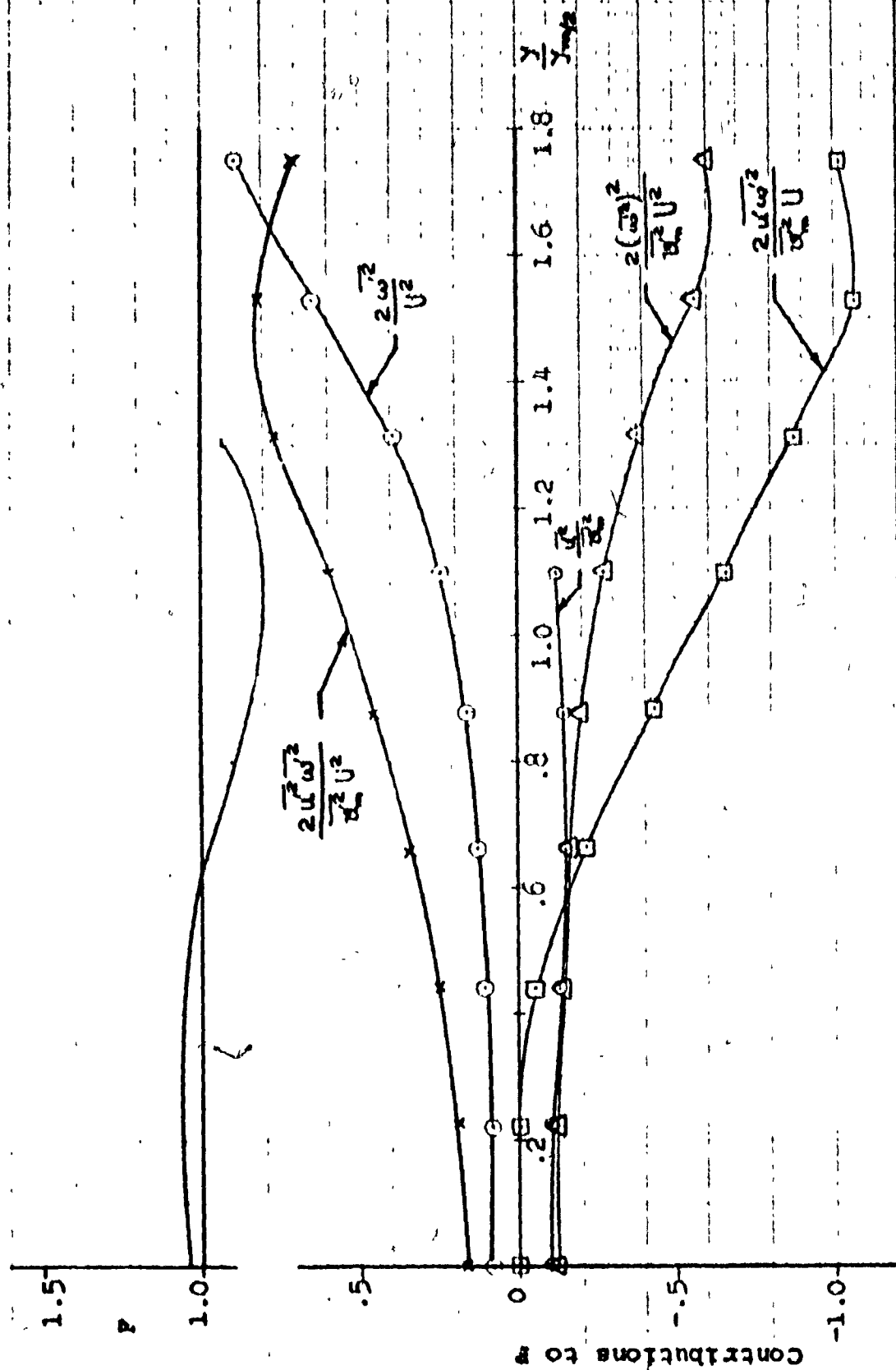
x/gz

CONTRIBUTION OF EACH TERM IN THE γ'^2 CORRECTION FACTOR F

FREE JET IN STILL AIR

(Measurements by Heskstad)

$$\frac{x_1}{D} = 100$$



CONTRIBUTION OF EACH TERM IN THE $u \cdot v$ CORRECTION FACTOR G

FREE JET IN STILL AIR

(measurements by Beskestad)

$$\frac{x_1}{D} = 100$$

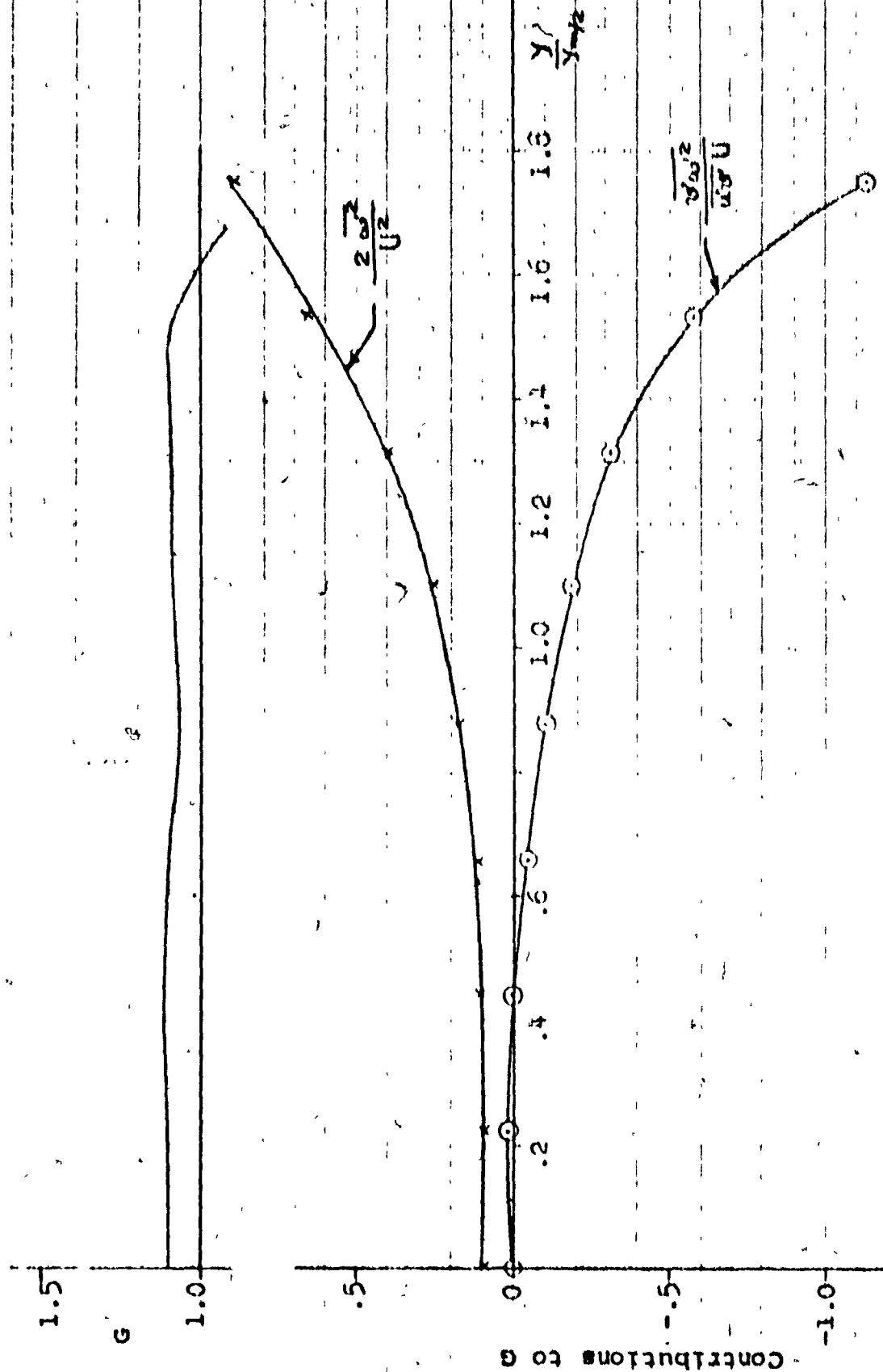
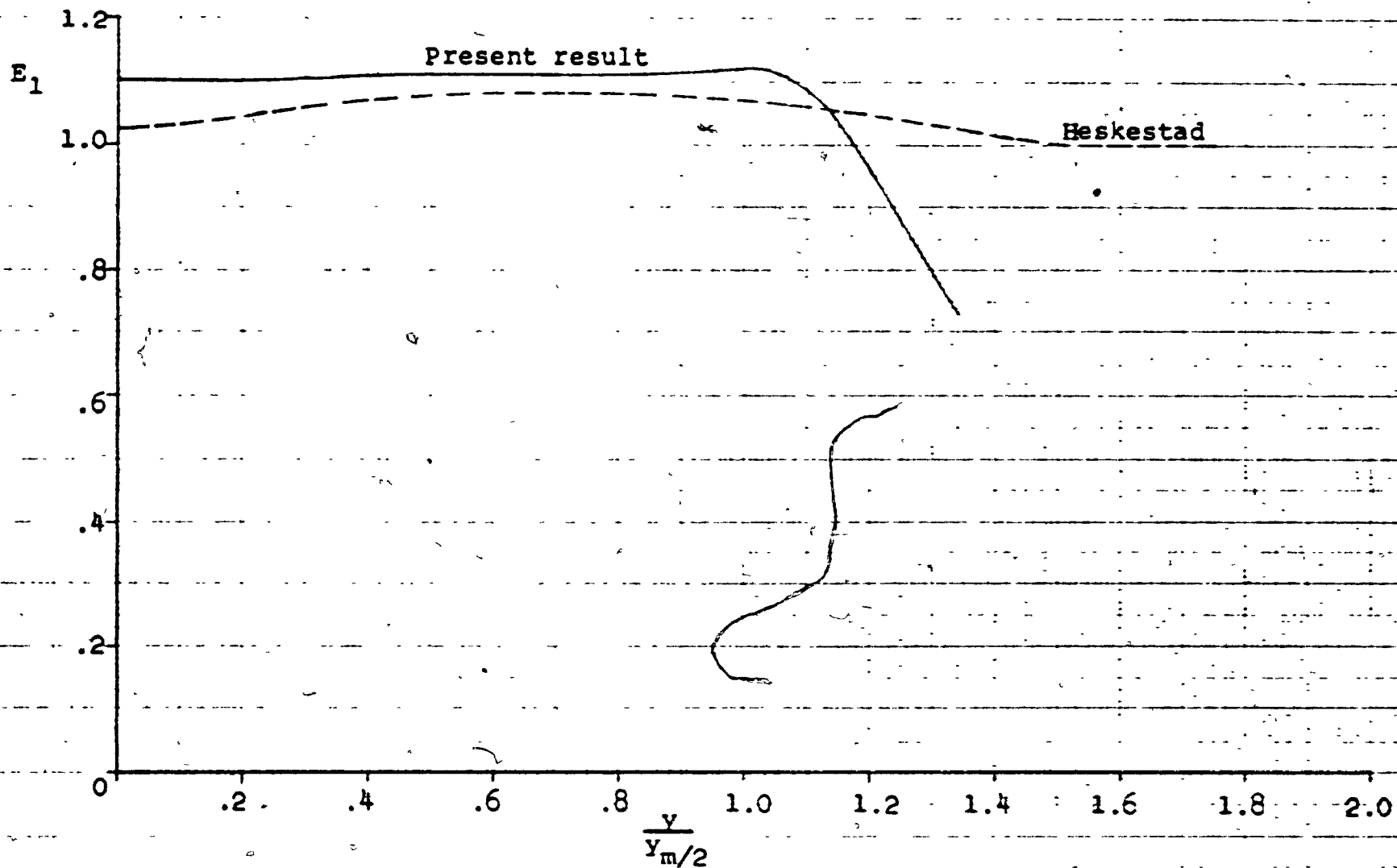


Fig. 7.8

COMPARISON OF PRESENT HIGH INTENSITY CORRECTION TO $\overline{u'^2}$ WITH THAT PRESENTED BY

HESKESTAD - FREE 2-D JET IN STILL AIR

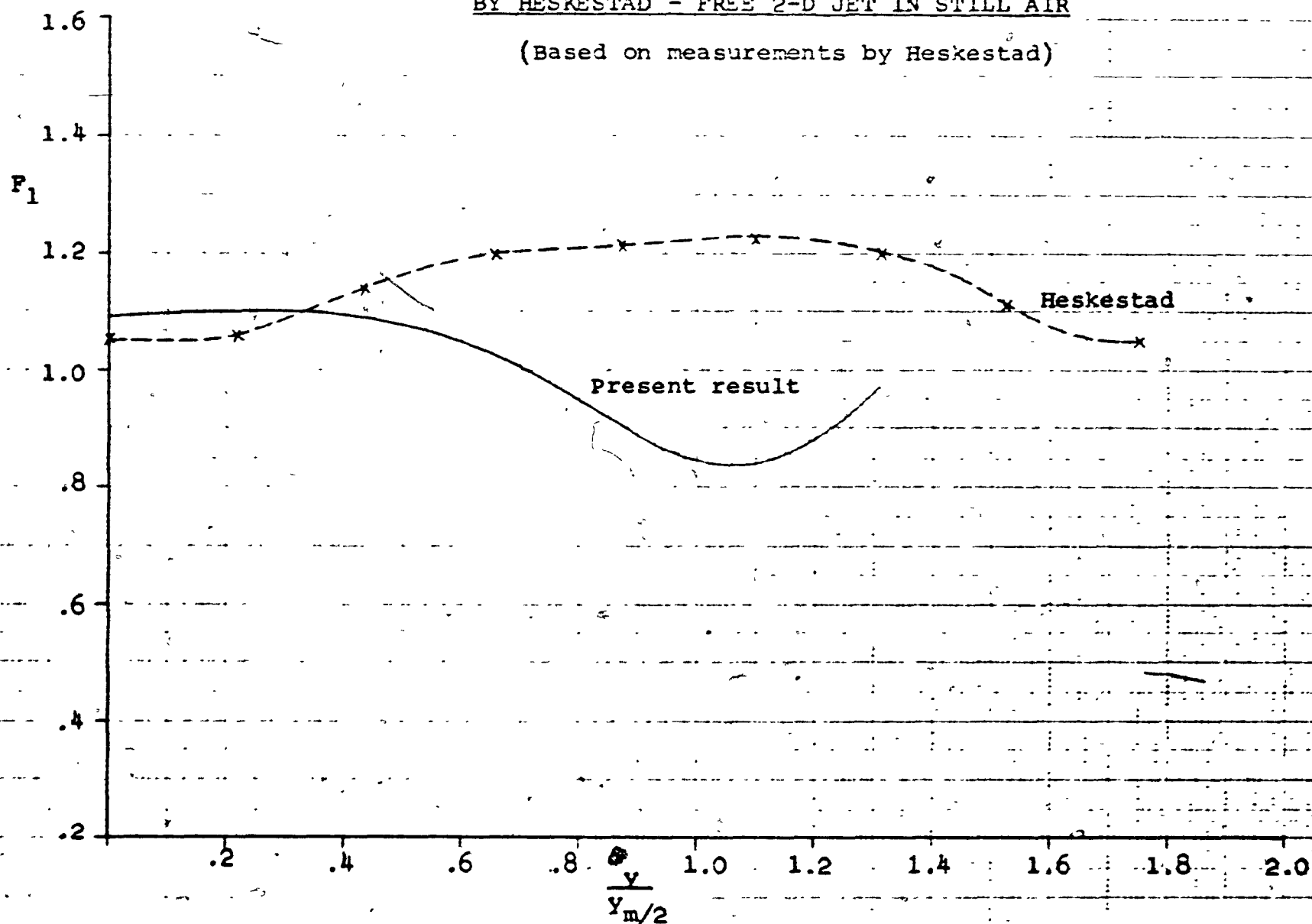
(Based on measurements by Heskestad)



COMPARISON OF PRESENT HIGH INTENSITY CORRECTION TO $\overline{v'^2}$ WITH THAT PRESENTED

BY HESKESTAD - FREE 2-D JET IN STILL AIR

(Based on measurements by Heskestad)



COMPARISON OF PRESENT HIGH INTENSITY CORRECTION TO $\overline{u'v'}$ WITH THAT PRESENTED

BY HESKESTAD - FREE 2-D JET IN STILL AIR

(Based on measurements by Heskestad)

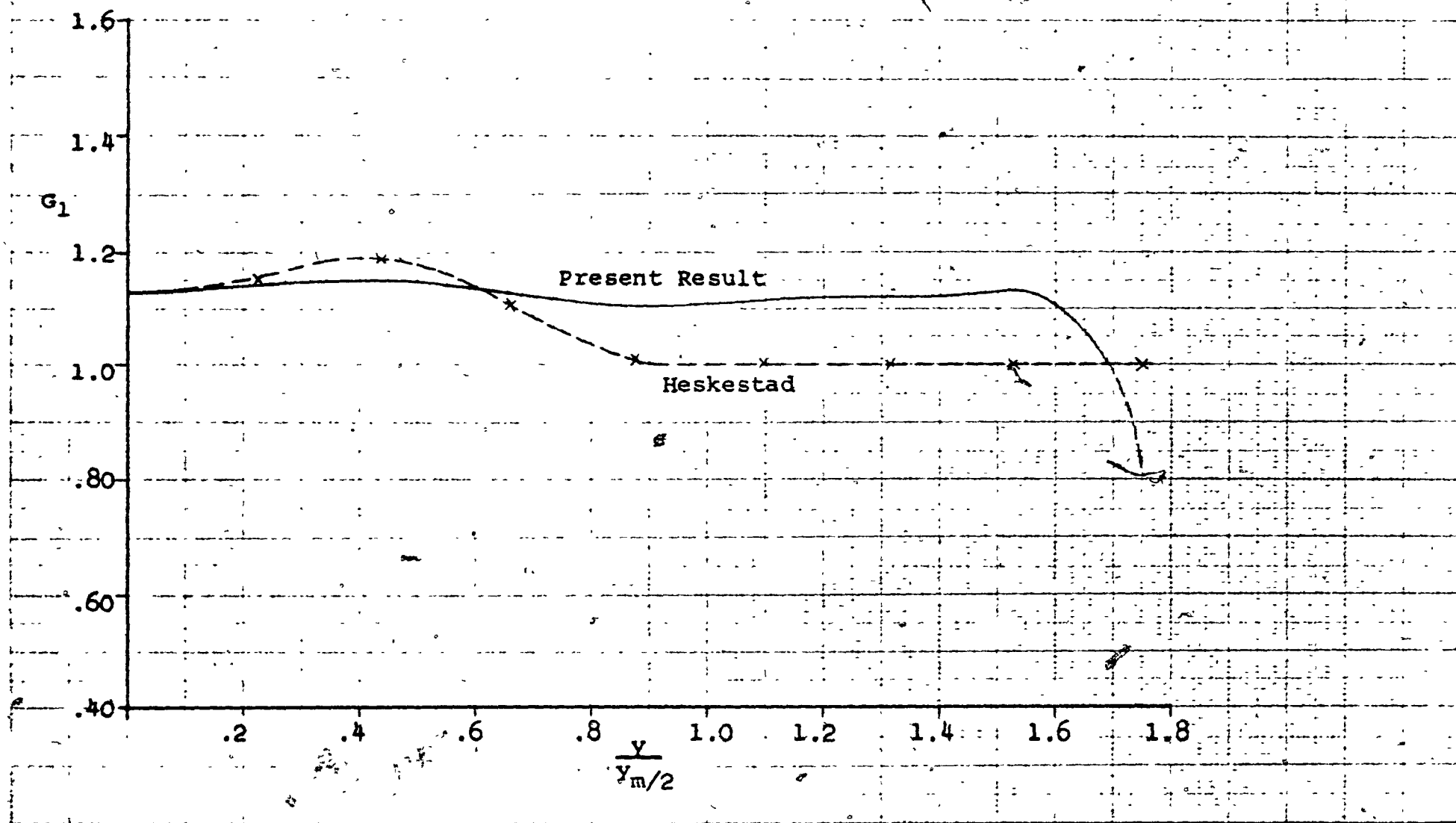


Fig. 7.11

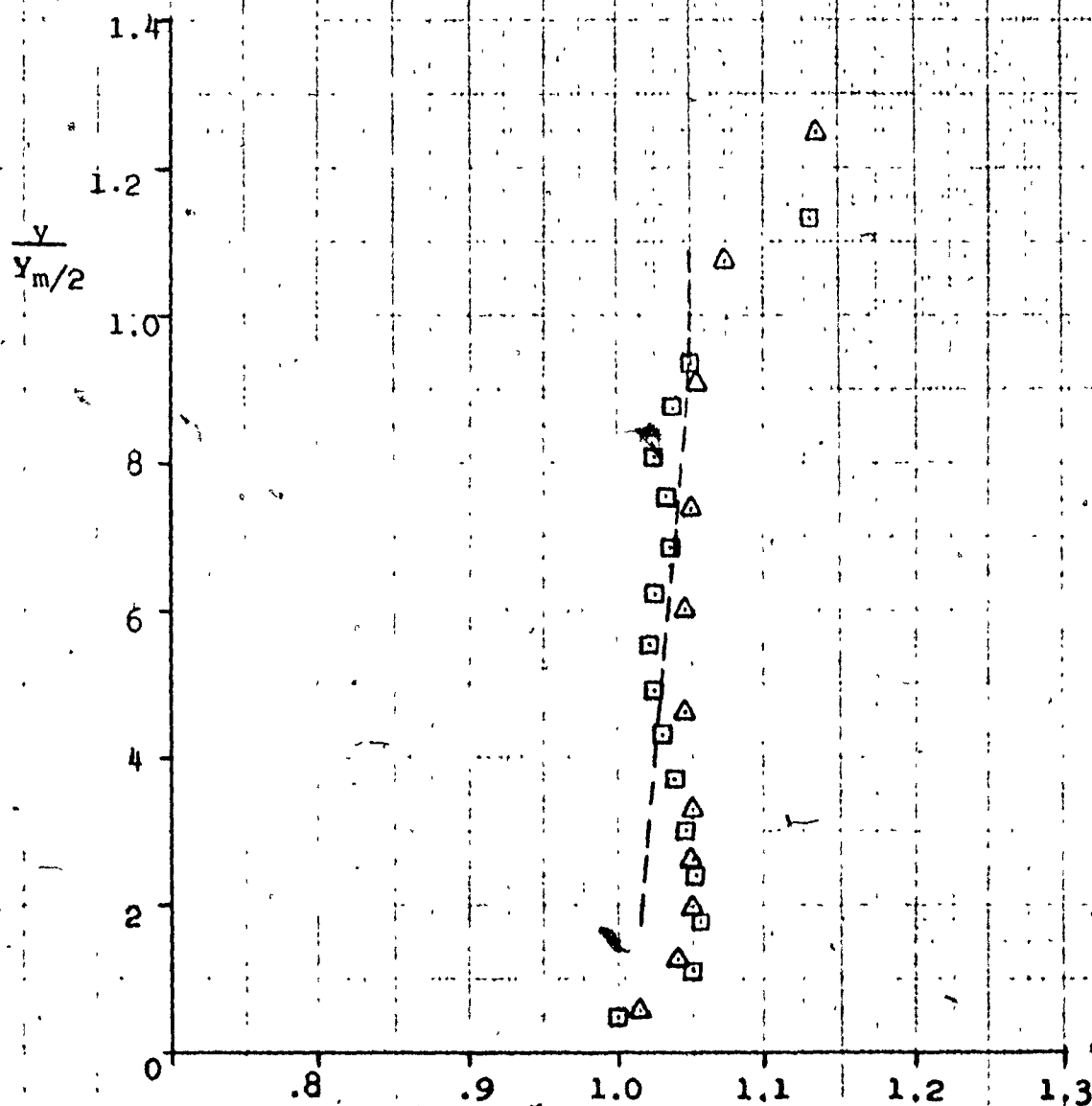
NORMAL STRESS, $\overline{u'^2}$, CORRECTION FACTOR DUE TO
HIGH INTENSITY TURBULENCE

$$\triangle \quad \frac{x}{R} = \frac{2}{3}$$

$$\square \quad \frac{x}{R} = 1$$

Plane wall jet in still air

$$\overline{u'^2}_c = E \overline{u'^2}_{meas.}$$

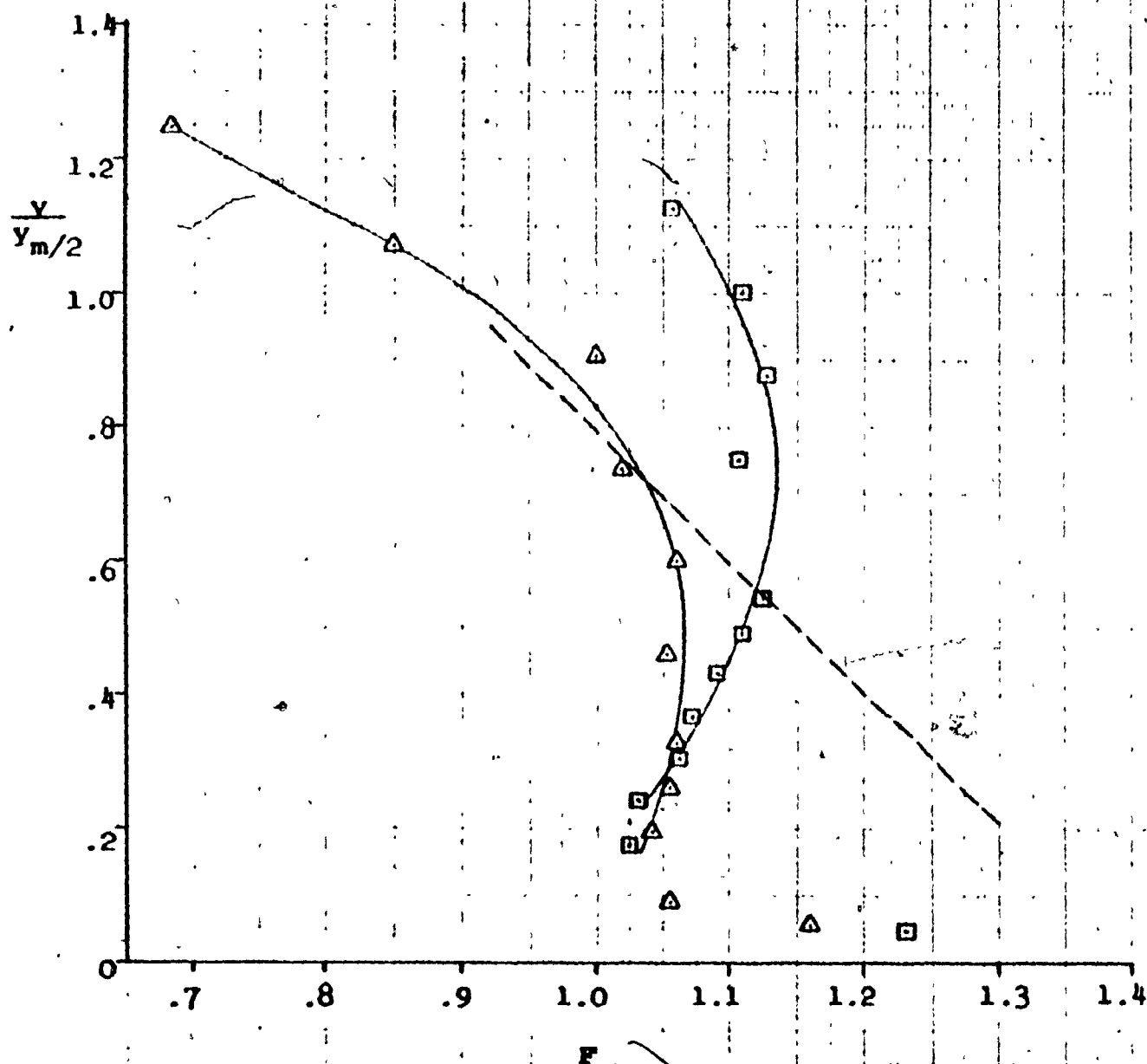


NORMAL STRESS, $\sqrt{v'^2}$, CORRECTION FACTOR DUE TO
HIGH INTENSITY TURBULENCE

$\Delta \quad \frac{x}{R} = \frac{2}{3}$

$\square \quad \frac{x}{R} = 1$

----- Plane wall jet in still air



SHEAR STRESS CORRECTION FACTOR DUE TO
HIGH INTENSITY TURBULENCE

$$\Delta \quad \frac{x}{R} = \frac{2}{3}$$

$$\square \quad \frac{x}{R} = 1$$

----- Plane wall jet in still air

$$\overline{u'v'}_c = G \overline{u'v'}_{\text{meas.}}$$

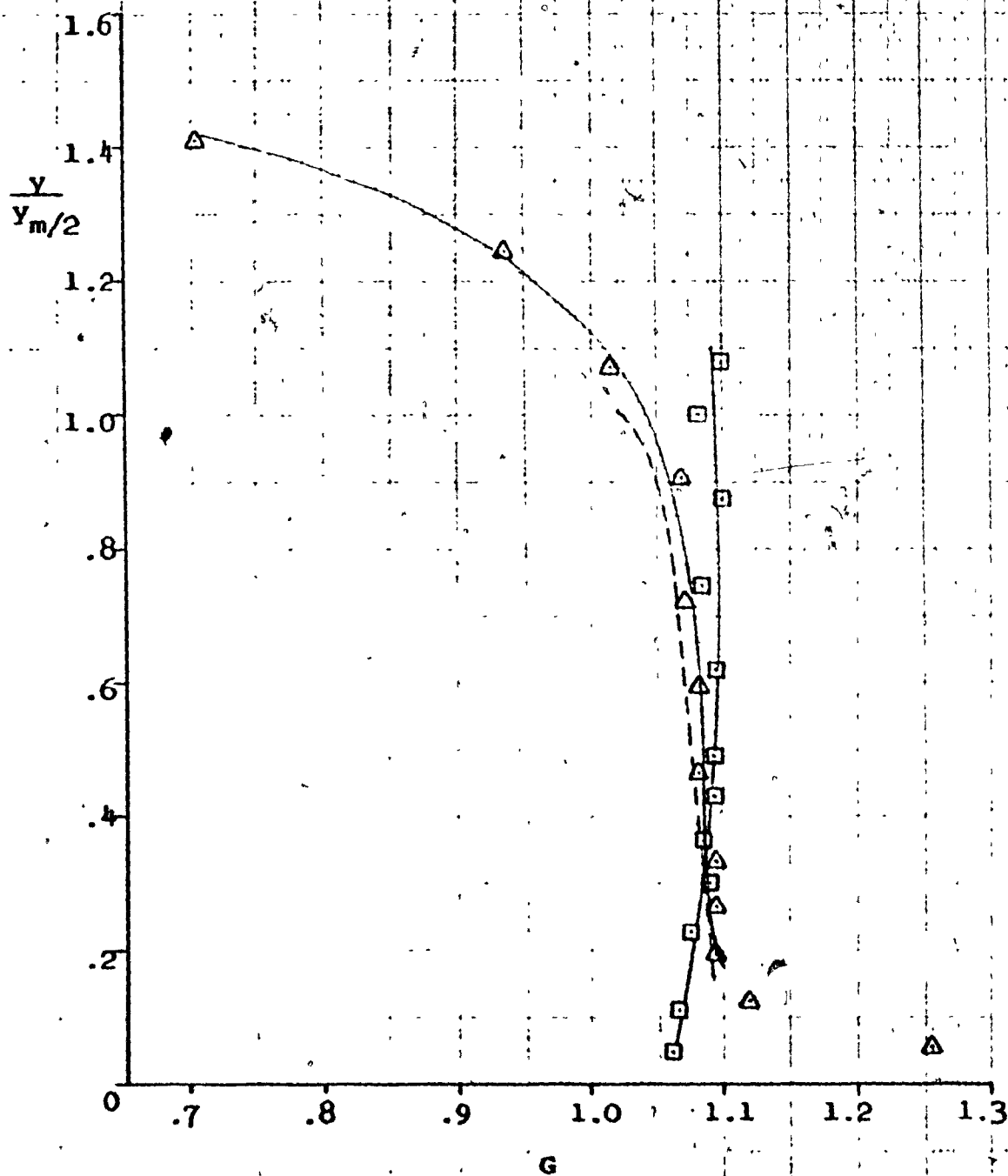


Fig. 7.15

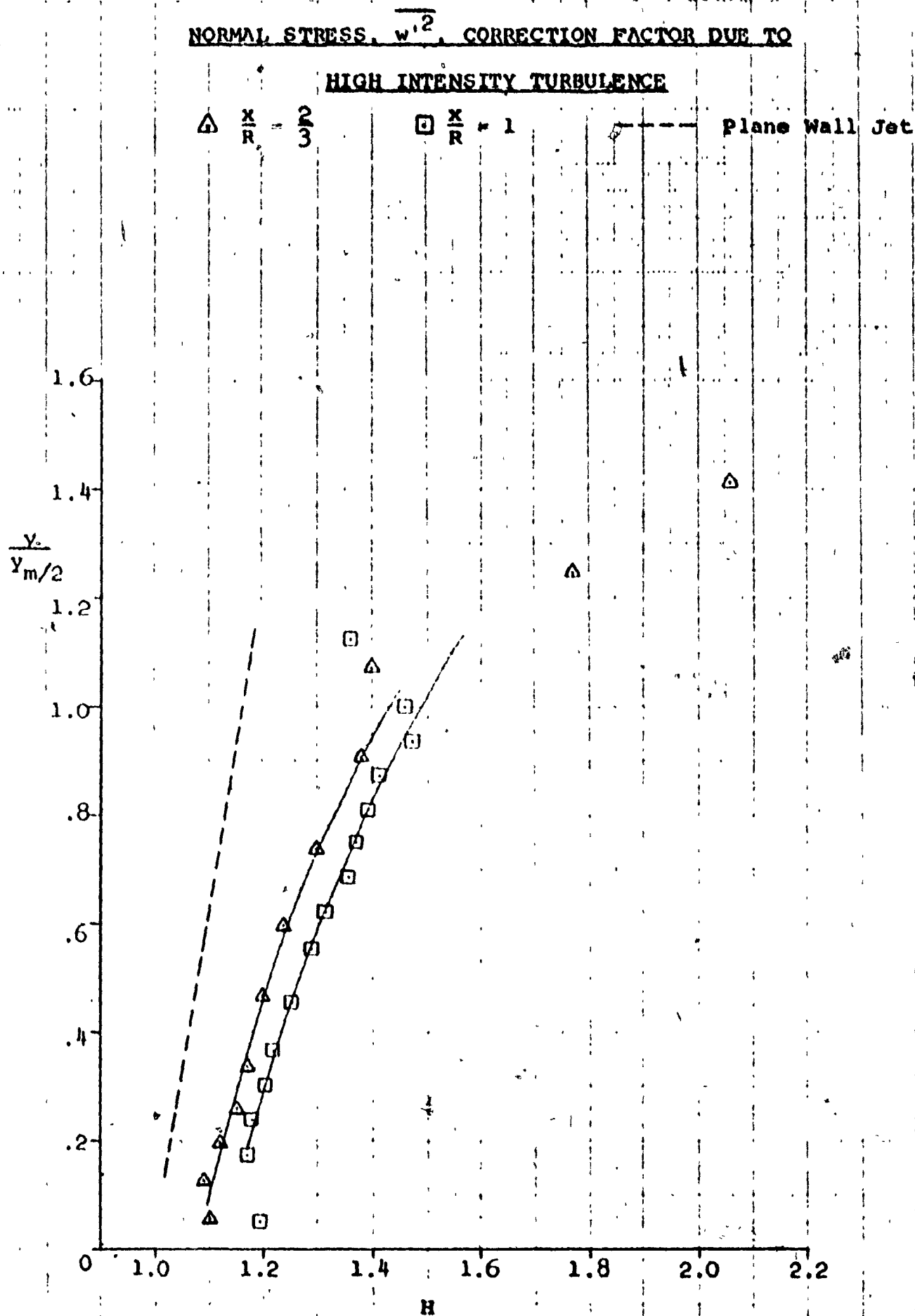


Table 8.1

Type	Method	Advantage	Disadvantage
Direct Measurement	Wall balance	(i) Measures τ_w directly	(i) difficult in pressure gradient (gap and frequency effect) (ii) difficult on incline due to gravity (iii) difficult to manufacture and very costly (iv) cannot measure at a point
Indirect: requiring calibration in known flow	Preston tube	(i) easily manufactured (ii) very mobile (iii) much data about behaviour is available (Patel, V.C.)	(i) requires wall region of local equilibrium - a feature uncertain in high intensity turbulent boundary layers (ii) response uncertain in high turbulence (iii) pressure gradient limited (Patel, V.C.)
	Stanton tube	(i) in sublayer and hence no requirement for universal region	(i) response uncertain in high intensity turbulence - requires knowledge of total and static pressures both of which are strongly influenced by large velocity fluctuations. Result is different calibrations in laminar and turbulent boundary layer (Bradshaw, Gadd) (ii) very tiny and therefore difficult to manufacture - it must be made small enough to limit its field of influence to the sublayer (Bradshaw, Gadd) (iii) pressure gradient limited though approximate corrections available (Patel, V.C.) (iv) requires measurement of small pressures

Type	Method	Advantage	Disadvantage
	B.L. fence	same as Stanton tube	
	Hqt wire near surface	(i) same as above (ii) readily available commercially.	(i) heat loss to wall which may be a function of wall velocity fluctuations - result is different calibration in turbulent and laminar boundary layers. (Bradshaw)
	Heat from surface element	(i) same as above (ii) apparent independence on <u>low</u> turbulence intensity - same calibration in laminar and turbulent layers (Brown) (iii) presents no obstruction to the flow (iv) quick response time (Bellhouse) (v) readily available commercially (Disa Electronic)	(i) calibration extremely sensitive on heat lost to substrate (present report) (ii) requires extreme care in measuring heat input (present report) (iii) non linear response to fluctuations. (iv) pressure gradient dependent though corrections available (Brown)
Indirect - no calibration in known flow is required	Extrapolation of measured shear	(i) good in any pressure gradient (ii) no special instrumentation required other than usual hot-wire equipment	(i) requires thick boundary layer (ii) depends on hot-wire measurements and their related difficulties (iii) procedure questionable in pressure gradients
	Momentum balance	(i) can be done with simple pitot tube	(i) extremely sensitive to three dimensional flow effects (Bradshaw) (ii) effect of turbulence intensity must be included

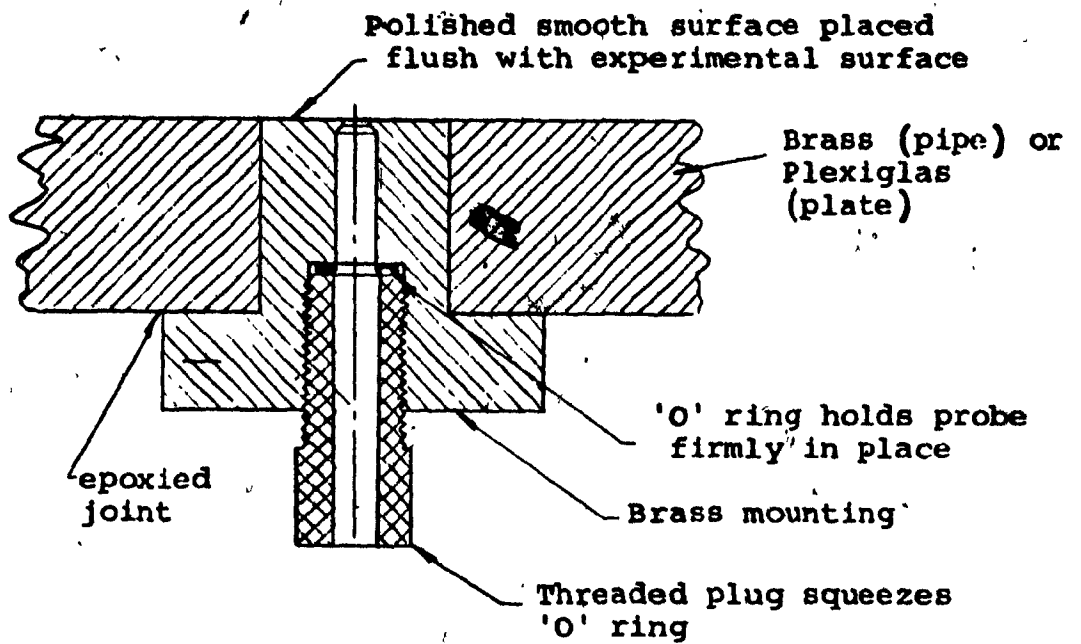


Fig. 1a

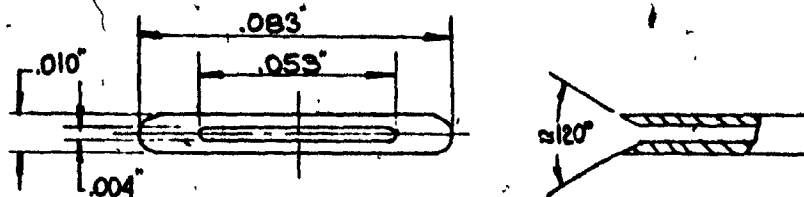


Fig. 1b

Fig. 8.1a Mounting to hold hot film probe

Fig. 8.1b Flat pitot used to traverse boundary layers

Table 8.2 Details of Preston Tubes

Preston Tube Number	External Diameter d (inches)	Approx. Ratio of Internal to External Diameter
1	.062	.5
2	.043	.52
3	.035	.65
4	.0255	.5
5	.0225	.52
6	.0284	.55

Table 8.3 Boundary Layer on Flat Plate - Comparison of different Measuring and Prediction Methods to obtain the Shear Wall Stress

Case	Measuring technique	Preston tube $\frac{U_d}{\tau}$ $\frac{1}{\nu}$	U_τ ft/sec.
A $U_1 = 132.0$ ft/sec $R_\theta = 9.6 \times 10^3$ $H = 1.25$	Preston tube no. 2	105	4.91
	3	855	4.91
	4	62.2	5.01
	5	54.9	5.01
	Average		4.96
	Flush Probe		5.10
B $U_1 = 113.8$ ft/sec	Preston tube no. 2	90.1	4.215
	Flush probe		4.37
C $U_1 = 101.0$ ft/sec	Preston tube no. 1	120	3.87
	2	83.4	3.90
	3	68.4	3.825
	4	49.1	3.95
	5	43.5	3.965
	Average		3.90
D $U_1 = 84.6$ ft/sec)	Preston tube no. 1	98.4	3.17
	2	67.4	3.146
	3	55.4	3.181
	4	40.7	3.28
	5	34.25	3.12
	Average		3.18
E $U_1 = 66.0$ ft/sec	Preston tube no. 2	55	2.57
	Flush probe		2.57
F $U_1 = 46.6$ ft/sec	Preston tube no. 2	38.9	1.82
	Flush probe		1.82

Table 8.4 Wall Jet in Still Air - Skin Friction Results Using Different Techniques

Experimental Conditions: (same as Görtshore's and Hawaleshka's)

slot width : $b = .300$; $Re = 3.1 \times 10^4$; $T = 80^\circ F$; $\frac{y_m/2}{x} = .071$

heated probe : Disa miniature (uncoated) operated at $R_f/R_c \approx 1.25$

Distance from Slot	Measuring Technique	$\frac{U_d}{v}$	U_τ ft/sec	$C_f = \frac{2U_\tau^2}{U_m^2}$
$\frac{x}{b} = 206$ $U_m = 58.1$ fs $y_m = .65$ in.	Clauser plot (using $A=4.15$ and $B=7.5$)	-	3.05	.0055
	flush heated element		2.824	.00473
	Preston tube no. 2	61.6	2.874	
	3	50.1	2.876	-
	4	36.6	2.906	
	5	32.2	2.862	
	6	40.7	2.844	
	Average (excluding No. 4)	-	2.864	.00486
$\frac{x}{b} = 139$ $U_m = 70.4$ fs $y_m = .45$ in.	heated element	-	3.60	.00524
	Clauser plot	-	3.86	.0060
	Preston tube no. 6	50.6	3.57	.00514
$\frac{x}{b} = 89$ $U_m = 89.6$ fs $y_m = .3$ in.	heated element (two tests)	-	4.97	.00615
	Clauser plot	-	4.86	.0059
	Preston tube no. 6	66.8	4.71	.00552
$\frac{x}{b} = 55.5$ $U_m = 115.8$ fs $y_m = .164$ in.	heated element	-	6.01	.00541
	" "	-	5.846	.00511
	Average	-	5.93	.00526
	Preston tube no. 6	84.1	5.944	.00527
$\frac{x}{b} = 38$ $U_m = 138$ fs $y_m = .13$ in.	heated element (two tests)	-	7.56	.0060
	Preston tube no. 4	93.2	7.33	.00564
	5	82.2	7.21	.00546
	Clauser plot		7.81	.0064

Table 8.5 $\frac{x}{R} = \frac{2}{3}$ Spiral : Skin Friction Measurements Using Various Techniques

$$Re = 2.8 \times 10^4 \quad b = .125 \quad T = 71.5^\circ F$$

Hot Film Probe Operated at $R_f/R_c = 1.25$

Position on Spiral	Measuring Technique	$\frac{U_d}{v}$	U_τ ft/sec	$C_f = \frac{2U_\tau^2}{U_m^2}$
$x = 13.3$ in. $\frac{x_1}{b} = 116$ $U_m = 117$ fs $y_m = .35$ in. $\frac{U_m y_m}{v} = 2.11 \times 10^4$	heated element: test a	-	5.905	.00509
	b	-	5.94	.00515
	c	-	5.97	.00521
	average	-	5.94	.00515
	Preston tube no. 1	194.3	6.09	.00542
	2	131.6	5.95	.00517
	3	107.2	5.95	.00518
	4	76.7	5.84	.00499
	5	67.8	5.86	.00501
	average	-	5.94	.00515
$x = 19.6$ in. $\frac{x_1}{b} = 166$ $U_m = 98.9$ fs $y_m = .5$ in. $\frac{U_m y_m}{v} = 2.54 \times 10^4$	heated element: test a	-	5.21	.00531
	b	-	5.17	.00524
	c	-	5.19	.00526
	average	-	5.19	.00526
	Preston tube no. 1	158.2	4.96	.00503
	2	108.0	4.88	.00487
	3	87.6	4.87	.00484
	4	63.7	4.85	.00482
	5	56.4	4.87	.00486
	average	-	4.89	.00488
$x = 28.7$ in. $\frac{x_1}{b} = 239$ $U_m = 74.8$ $y_m = .71$ in. $\frac{U_m y_m}{v} = 2.73 \times 10^4$	heated element: test a	-	3.57	.00455
	b	-	3.47	.00432
	average	-	3.52	.00444
	Preston tube no. 1	121.0	3.79	.00514
	2	83.2	3.76	.00505
	3	69.0	3.83	.00525
	4	48.8	3.72	.00495
	average	-	3.78	.00510
	Clauser Plot (log law constants $A=B=5.5$)	-	3.54	.0045

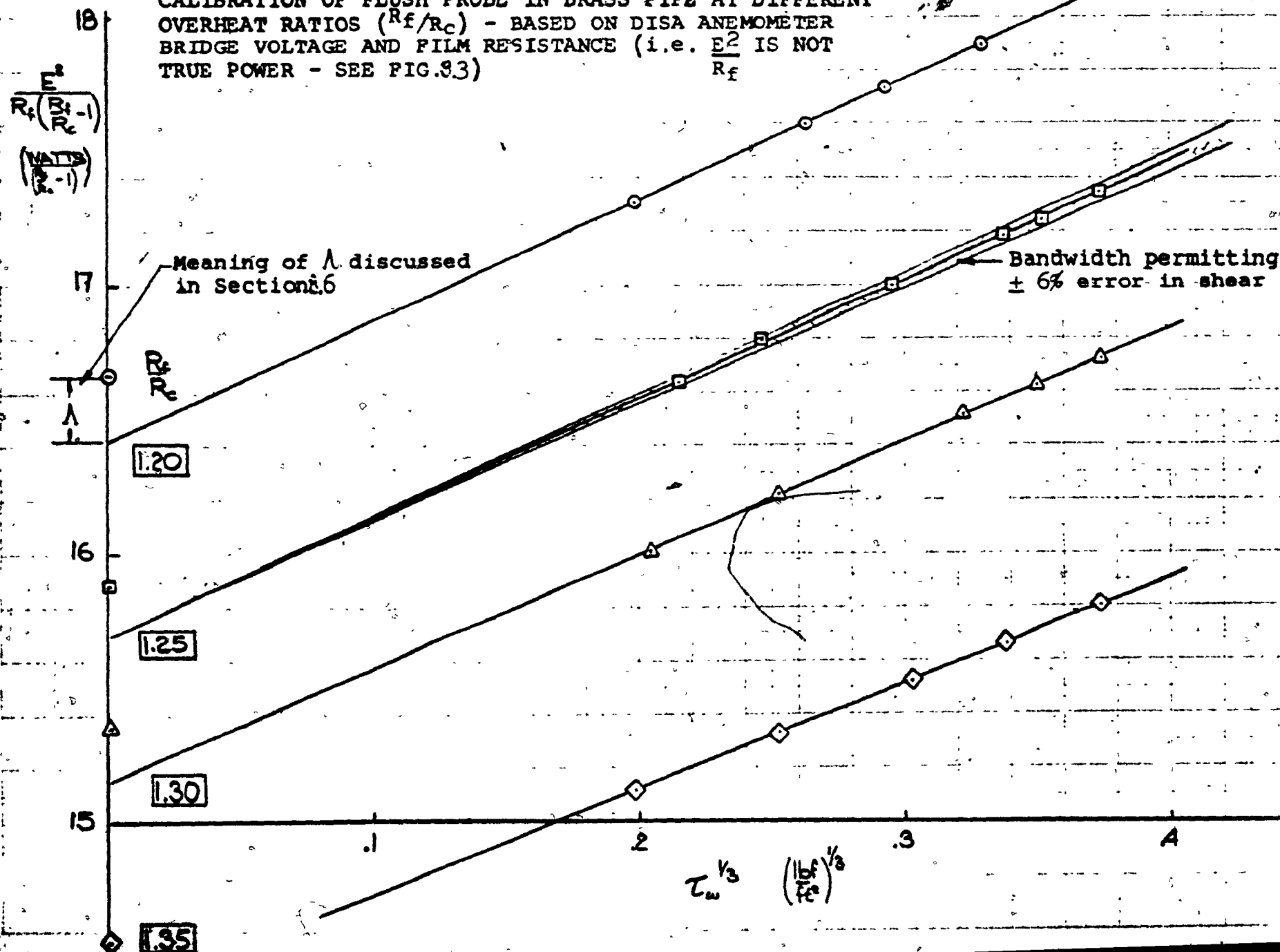
Table 8.6 $\frac{x}{R} = 1$ Spiral : Skin Friction Measurements Using Different Techniques

$Re = 2.8 \times 10^4$ $b = .125$ inch $T = 80^\circ F$

Hot Film Probe Operated at $R_f/R_c = 1.25$

Distance from Slot	Measuring Technique	$\frac{U_\tau d}{\nu}$	U_τ ft/sec	$C_f = \frac{2U_\tau^2}{U_m^2}$
$x = 14.26$ in. $\frac{x_1}{b} = 160$ $U_m = 101.1$ fs $y_m = .8$ in. $\frac{U_m y_m}{\nu} = 4.04 \times 10^4$	heated element: test a	-	5.41	.00574
	b	-	5.21	.00531
	c	-	5.13	.00515
	average	-	5.25	.00540
	Preston tube No. 1	155.6	4.88	.00466
	2	106.2	4.80	.00451
	3	86.4	4.80	.00450
	4	61.0	4.65	.00423
	5	55.4	4.79	.00449
	average	-	4.78	.00448
$x = 29.75$ in. $\frac{x_1}{b} = 284$ $U_m = 69.4$ fs $y_m = 1.75$ inch $\frac{U_m y_m}{\nu} = 6.06 \times 10^4$	Clauser (log law constants $A=B=5.5$)	-	4.56	.00407
	heated element: test a	-	3.54	.00519
	b	-	3.48	.00501
	c	-	3.54	.00519
	average	-	3.52	.00513
	Preston tube no. 1	100.7	3.16	.00414
	2	70.3	3.18	.00419
	3	57.3	3.18	.00421
	4	41.5	3.16	.00415
	5	36.7	3.17	.00416
	average	-	3.17	.00417
	Clauser	-	3.03	.00380

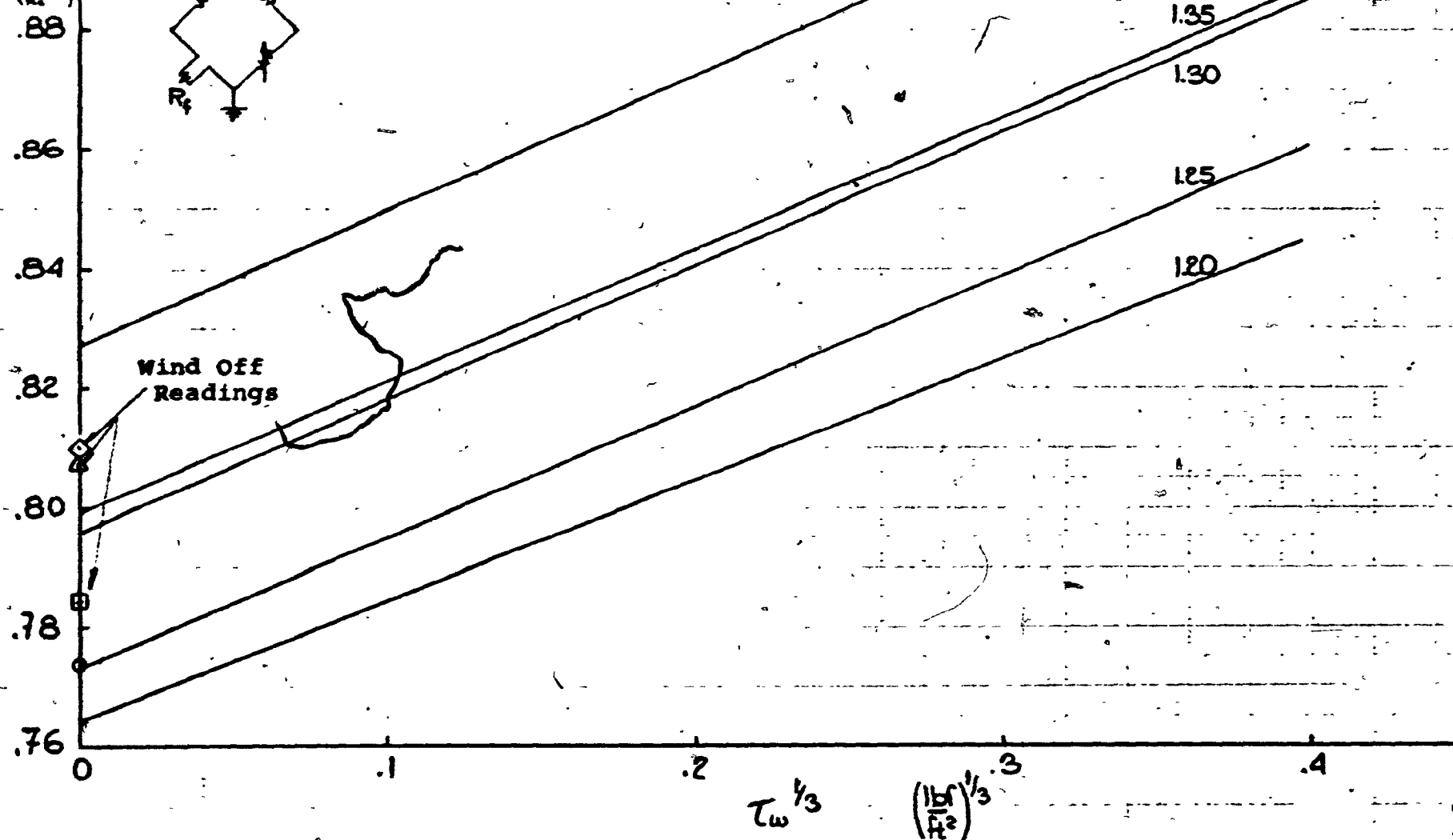
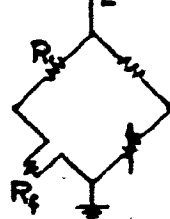
CALIBRATION OF FLUSH PROBE IN BRASS PIPE AT DIFFERENT
OVERHEAT RATIOS (R_f/R_c) - BASED ON DISA ANEMOMETER
BRIDGE VOLTAGE AND FILM RESISTANCE (i.e. $\frac{E^2}{R_f}$ IS NOT
TRUE POWER - SEE FIG.33)



CALIBRATION RESULTS OF FIG. 8.2 PLOTTED USING THE
TRUE POWER TO THE FILM

$$\text{FILM POWER} = \frac{E^2}{R_f \left(\left(\frac{R_f}{R_e} \right)^2 + 2 \frac{R_f}{R_e} + 1 \right)}$$

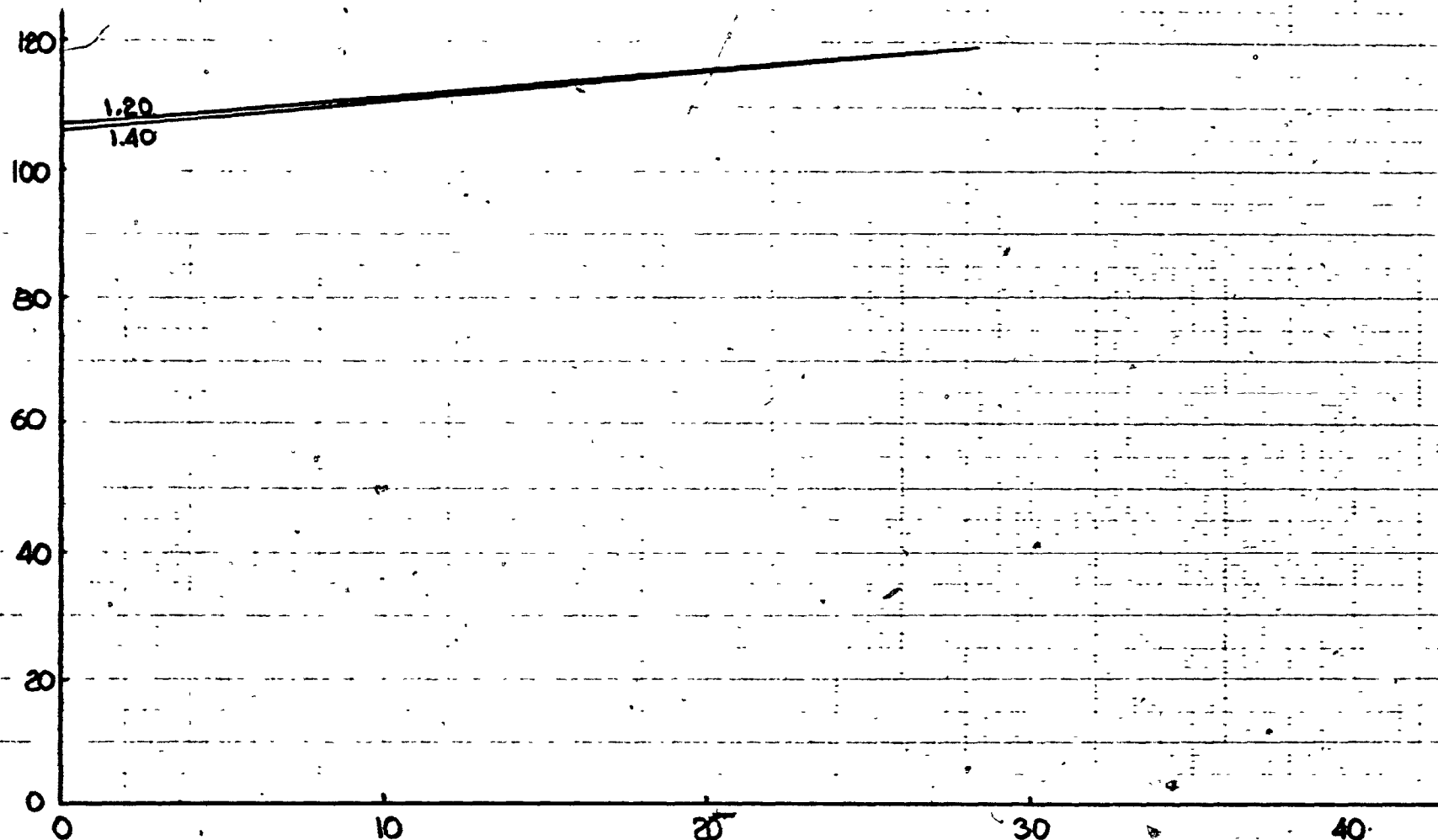
(WATTS)
 $\left(\frac{R_f}{R_e} - 1 \right)$



$Nu =$

NON-DIMENSIONAL EXPRESSION OF CALIBRATION RESULTS FOR THE TWO
EXTREME OVERHEAT RATIOS PLOTTED IN FIG. 8.2 (HOT FILM IN PIPE)
(see Eq'n 8.11)

$$\frac{E^2}{R_f k l_o (T_f - T_i)}$$



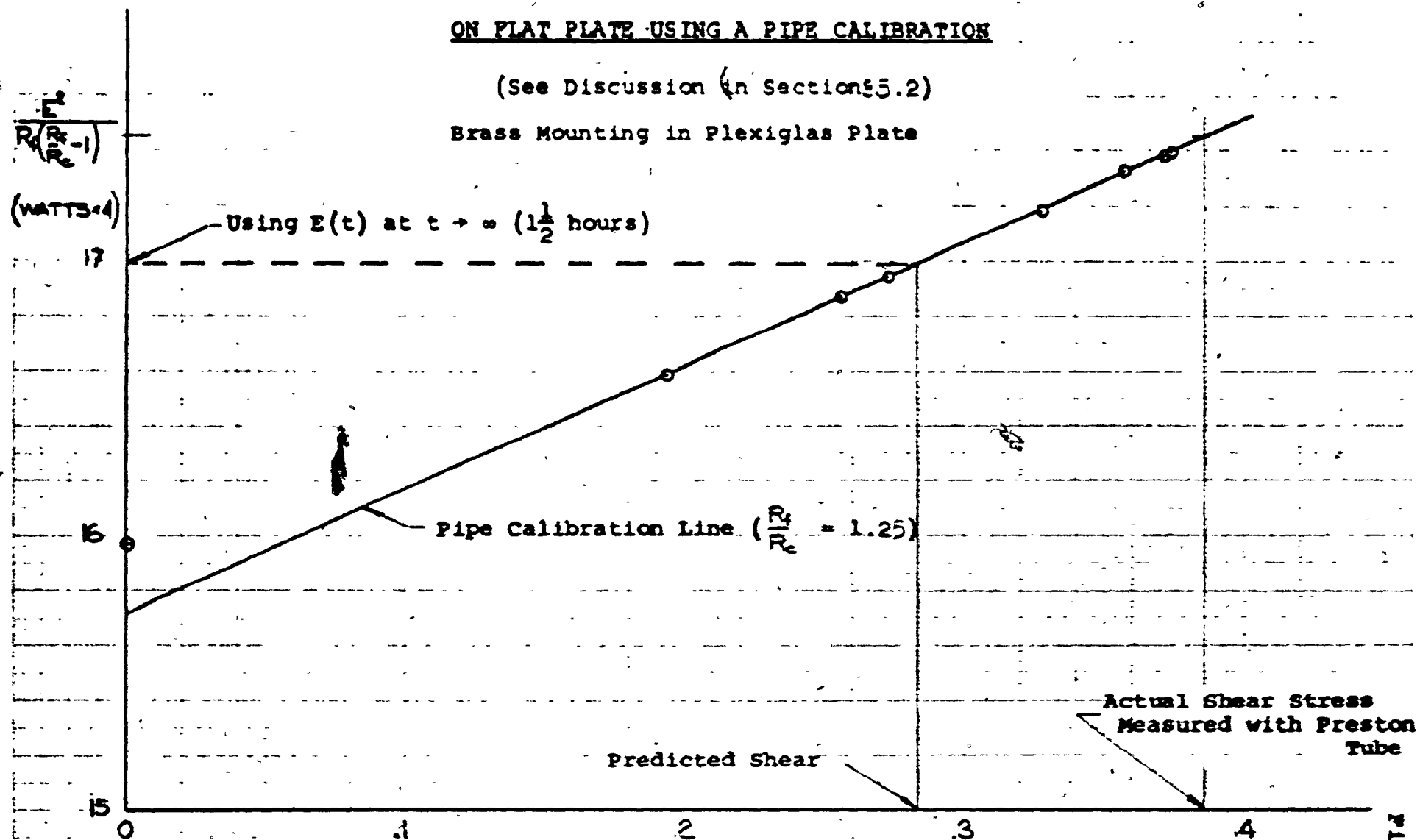
$$P_r \left(\frac{W}{l_o} \right) \left(\frac{T_w l_o^2}{1.9 \rho v^2} \right)^{1/3}$$

PRELIMINARY ATTEMPT TO MEASURE SHEAR STRESS IN TURBULENT BOUNDARY LAYER

ON FLAT PLATE USING A PIPE CALIBRATION

(See Discussion in Section 5.2)

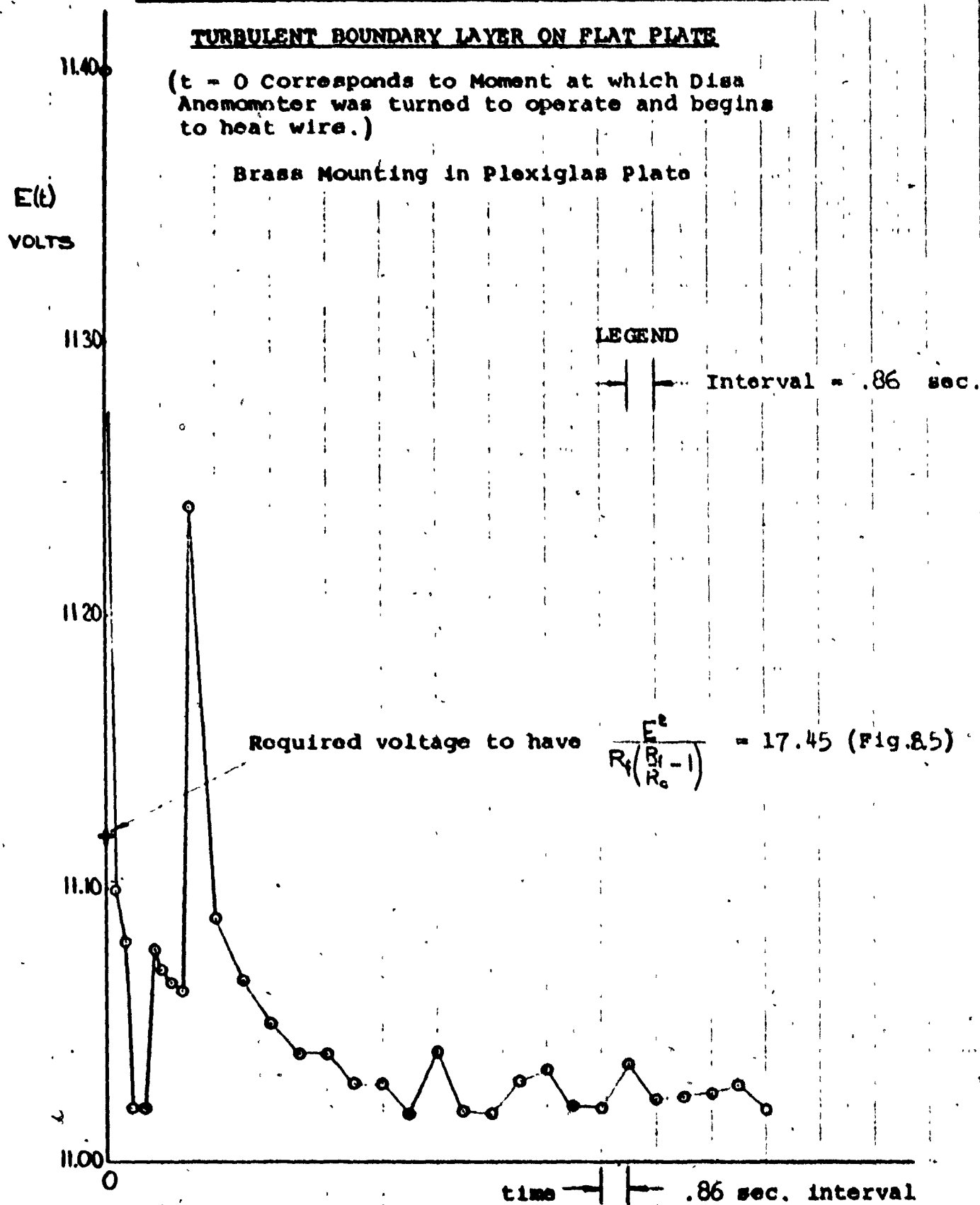
Brass Mounting in Plexiglas Plate



TYPICAL VARIATION OF VOLTAGE ACROSS FILM WITH TIMETURBULENT BOUNDARY LAYER ON FLAT PLATE

($t = 0$ Corresponds to Moment at which Disa Anemometer was turned to operate and begins to heat wire.)

Brass Mounting in Plexiglas Plate



VARIATION OF FREE CONVECTION HEAT LOSS, Λ (See section 8.6)

WITH FILM TEMPERATURE - PIPE CALIBRATION

$$\frac{R_f}{R_e} = \alpha (\bar{T}_f - T_i) + 1$$

LEGEND

- □ △ ◇ - From Fig. 8.2
- ▽ - From Fig. 8.5
- ■ ▲ ◆ - Other experiments

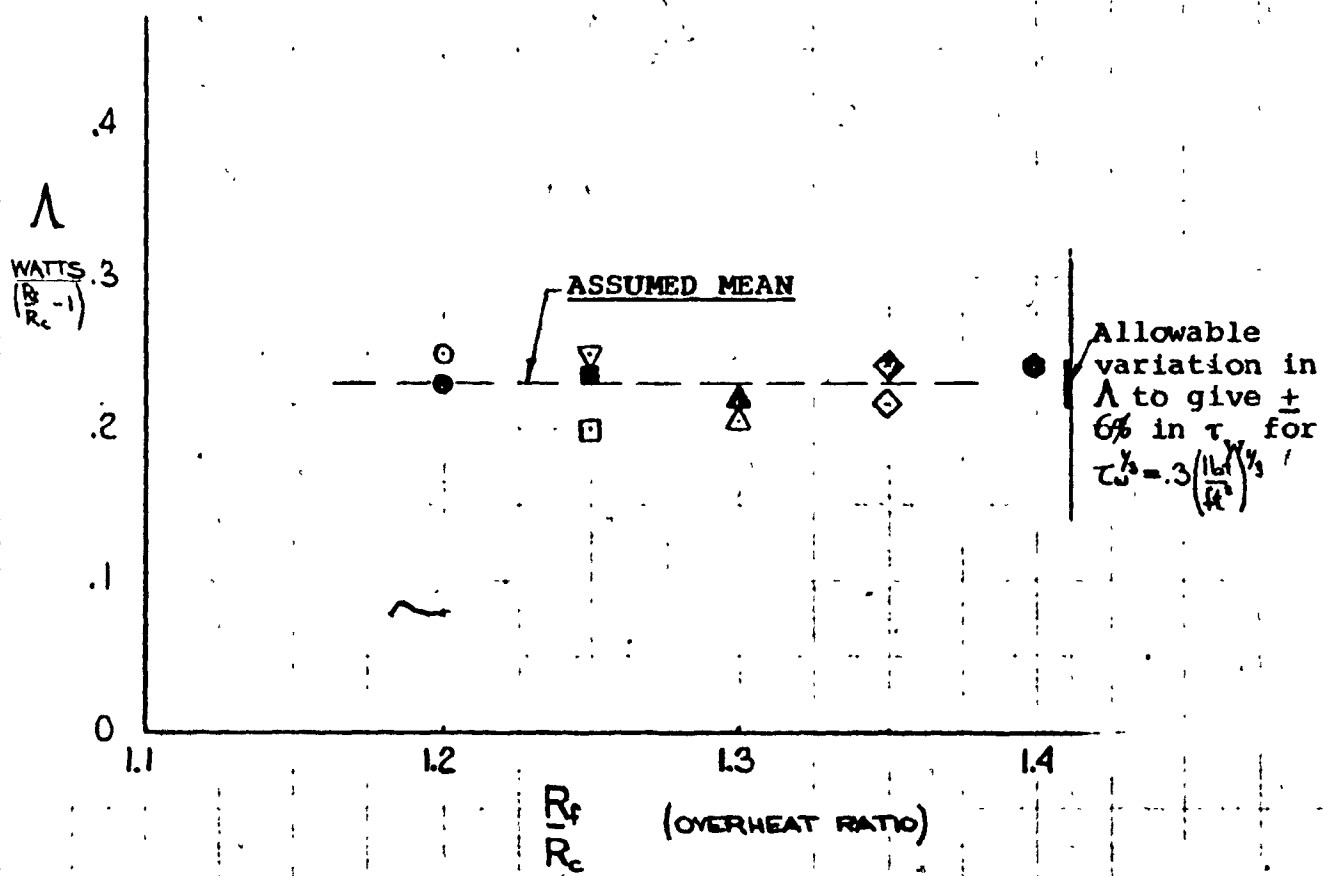


Fig. 8.8

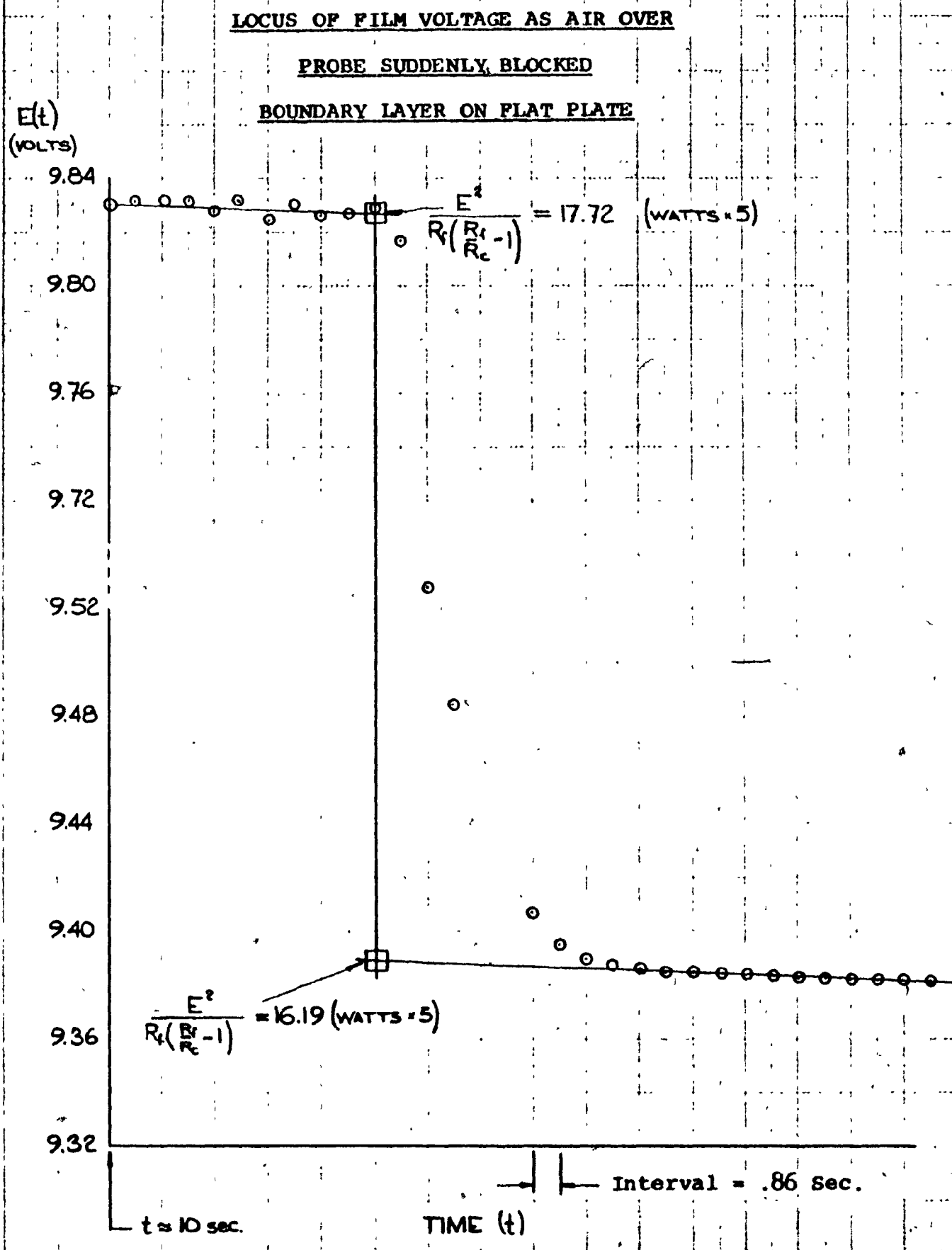


Fig. 8.9

SKIN FRICTION MEASUREMENT IN TURBULENT BOUNDARY LAYER

USING OVERHEAT $\frac{R_f}{R_c} = 1.2$

DEVELOPMENT OF HYPOTHETICAL CALIBRATION LINE

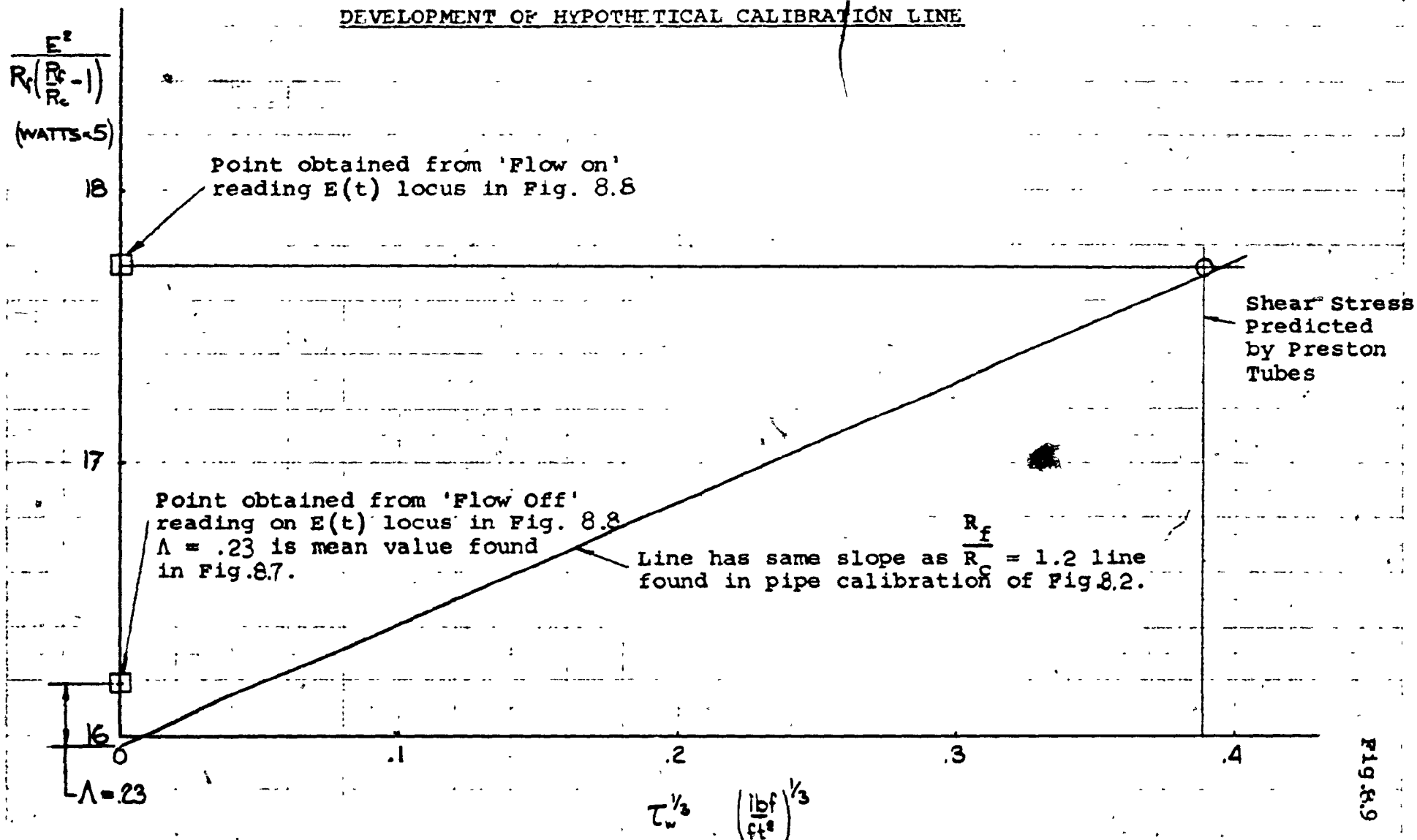


Fig. 8.9

SKIN FRICTION MEASUREMENTS IN TURBULENT

$$\text{BOUNDARY LAYER} - \frac{R_f}{R_c} = 1.2$$

E^2
 $R_f(R_c - 1)$
(WATTS-5)

LEGEND

○ Value of shear obtained from Preston Tube

— Calibration lines based on "no flow" reading and pipe calibration at specified overheat ratio

18

A

B

C

D

17

E

F

Case	'Flow Off' Point
A	16.19
B	16.21
C	16.15
D	16.20
E	16.19
F	16.20

16

0

.1

.2

.3

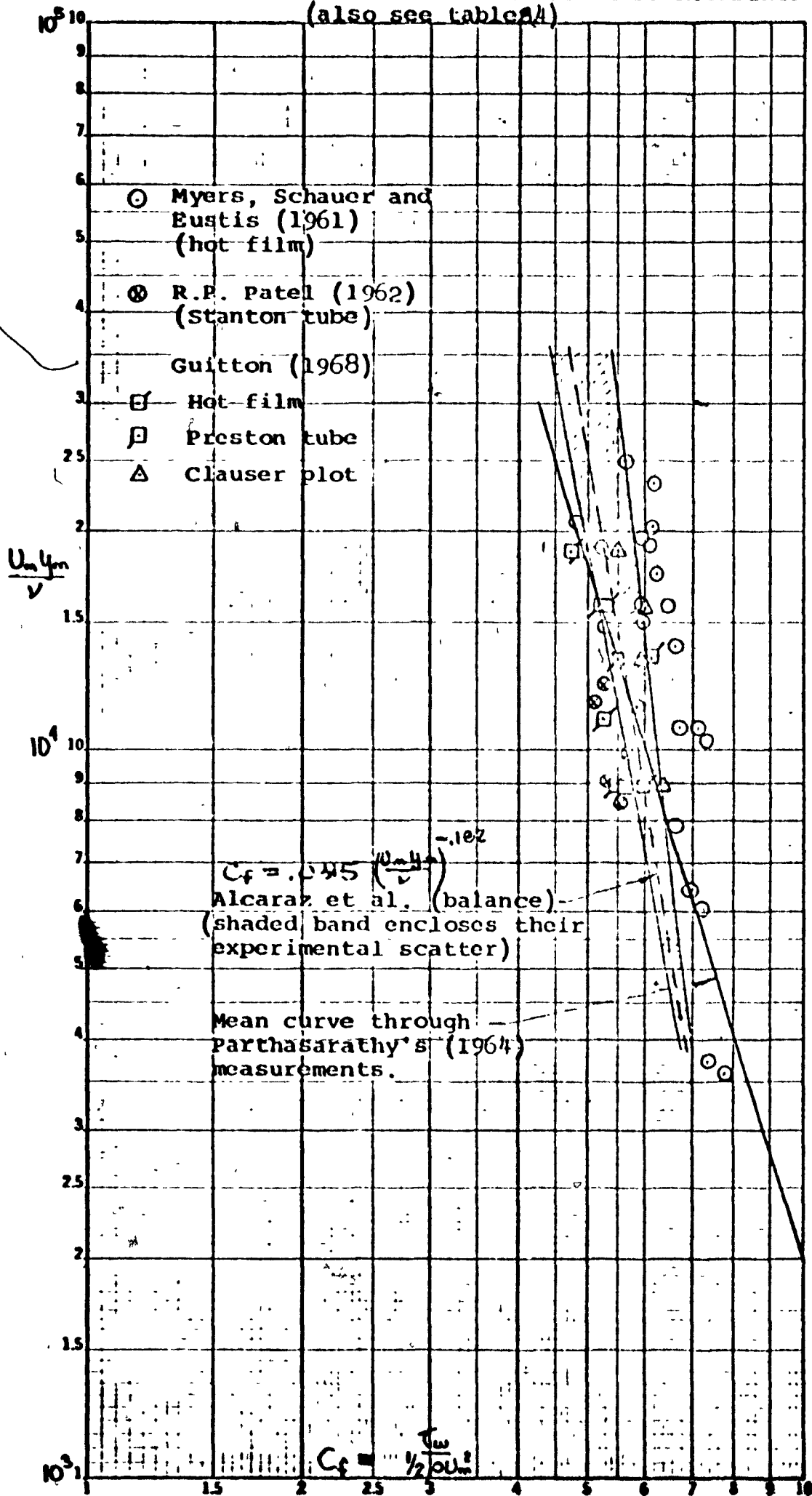
.4

$\tau_w^{1/3}$ $\left(\frac{\text{lb}_f}{\text{ft}^2}\right)^{1/3}$

Fig. 8.10

SKIN FRICTION IN WALL JET WITH STILL SURROUNDINGS (also see table A4)

Fig.8.11



CLAUSER PLOT FOR LOG-LAW: $\frac{y}{\delta} = 5.5 \log_{10} \frac{yU_c}{\nu}$ (RE: Y.C. PATEL JFM 1963)

WALL JET IN STILL AIR

$b = .2 \ln .4$
slot $Re = 3 \times 10^3$

$\frac{x}{b}$ shown below

LEGEND

$\odot \frac{x}{b} = 206$

$\nabla \frac{x}{b} = 89$

— Denotes region of uncertainty of point

$C_f = \frac{\tau_w}{\frac{1}{2} \rho U_c^2}$

value of $\frac{yU_m}{\nu}$ at $\frac{x}{b} = 206$ for which:

$\frac{yU_m}{\nu} = 30$

$\frac{yU_m}{\nu} = 300$

CLAUSER PLOT FOR LOG-LAW: $\frac{U}{U_c} = 5.5 \log_{10} \frac{y U_c}{\nu} + 5.45$ (REF: Y.C. PATEL JFM 1965)

WALL JET IN STILL AIR

$$b = .3 l_n$$

$$Re = 3 \times 10^5$$

$\frac{x}{b}$ shown below

LEGEND

$$\triangle \frac{x}{b} = 38$$

$$\square \frac{x}{b} = 139$$

$\overline{\triangle}$ denotes region of uncertainty of point

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho U_c^2}$$

value of $\frac{y_m U_m}{\nu}$ at $\frac{x}{b} = 38$ for which:

$$\frac{y U}{\nu} = 30$$

$$\frac{y U}{\nu} = 300$$

CLAUSER PLOT FOR LOG-LAW: $\frac{yU}{b} = 4.15 \log_{10} \frac{yU}{b} + 7.5$

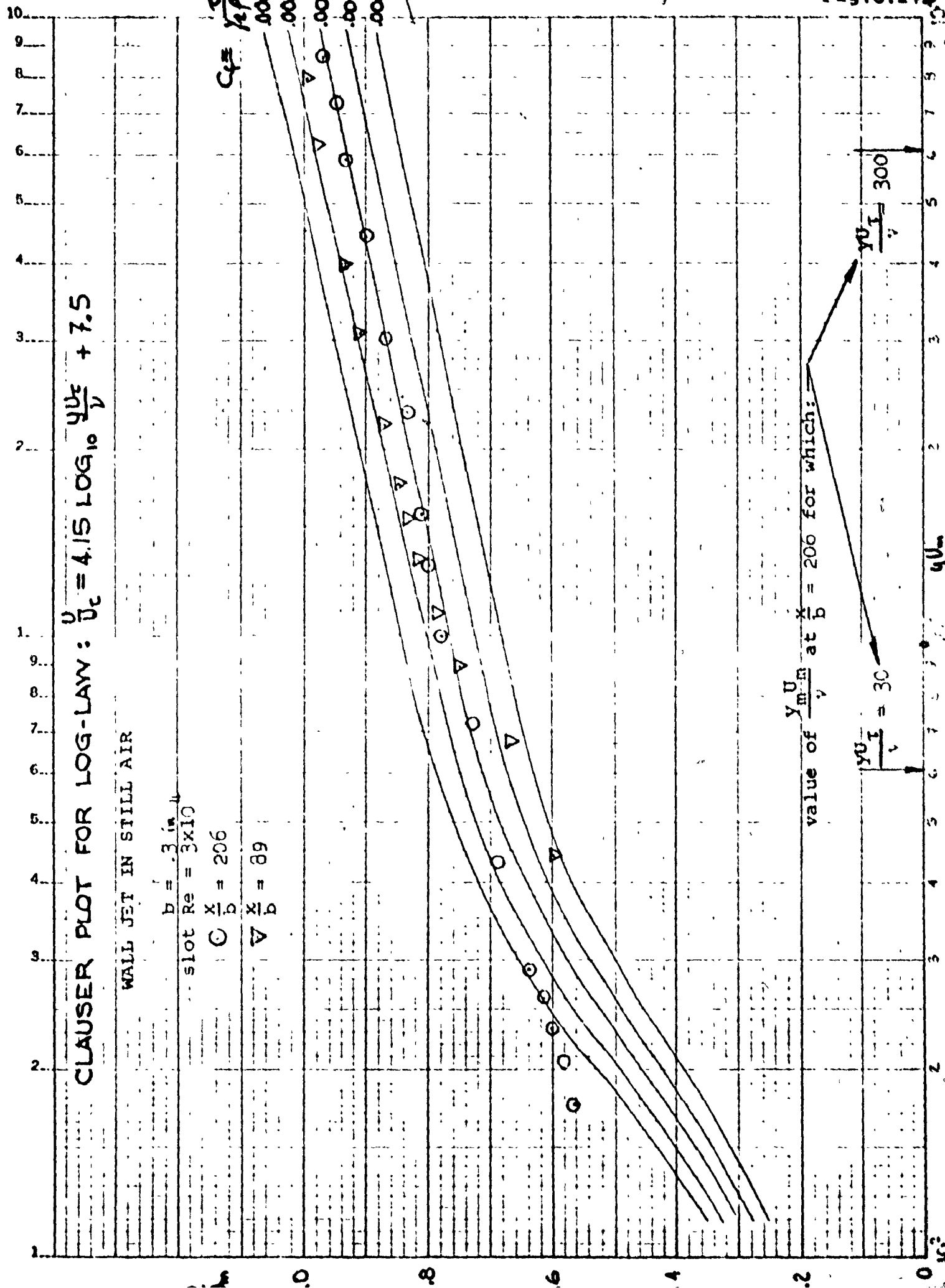
WALL JET IN STILL AIR

$b = .3 \text{ in.}$
slot $Re = 3 \times 10^4$

$\bigcirc \frac{x}{b} = 206$

$\nabla \frac{x}{b} = 89$

$C_f = \frac{\tau_w}{\frac{1}{2} \rho U^2}$
 .0045
 .0050
 .0055
 .0060



value of $\frac{yU}{b}$ at $\frac{x}{b} = 206$ for which:

$\frac{yU}{b} = 30$

$\frac{yU}{b} = 300$

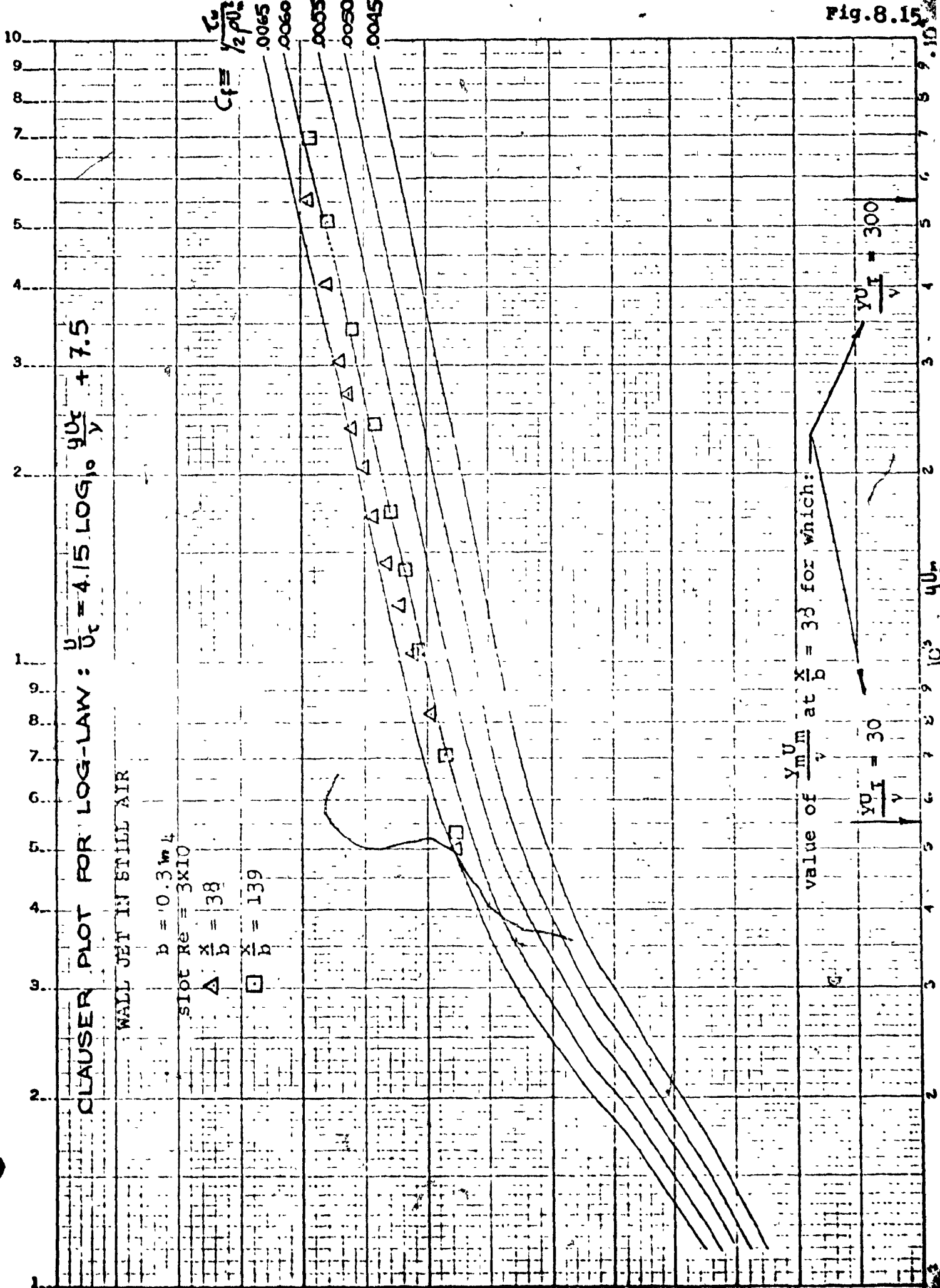
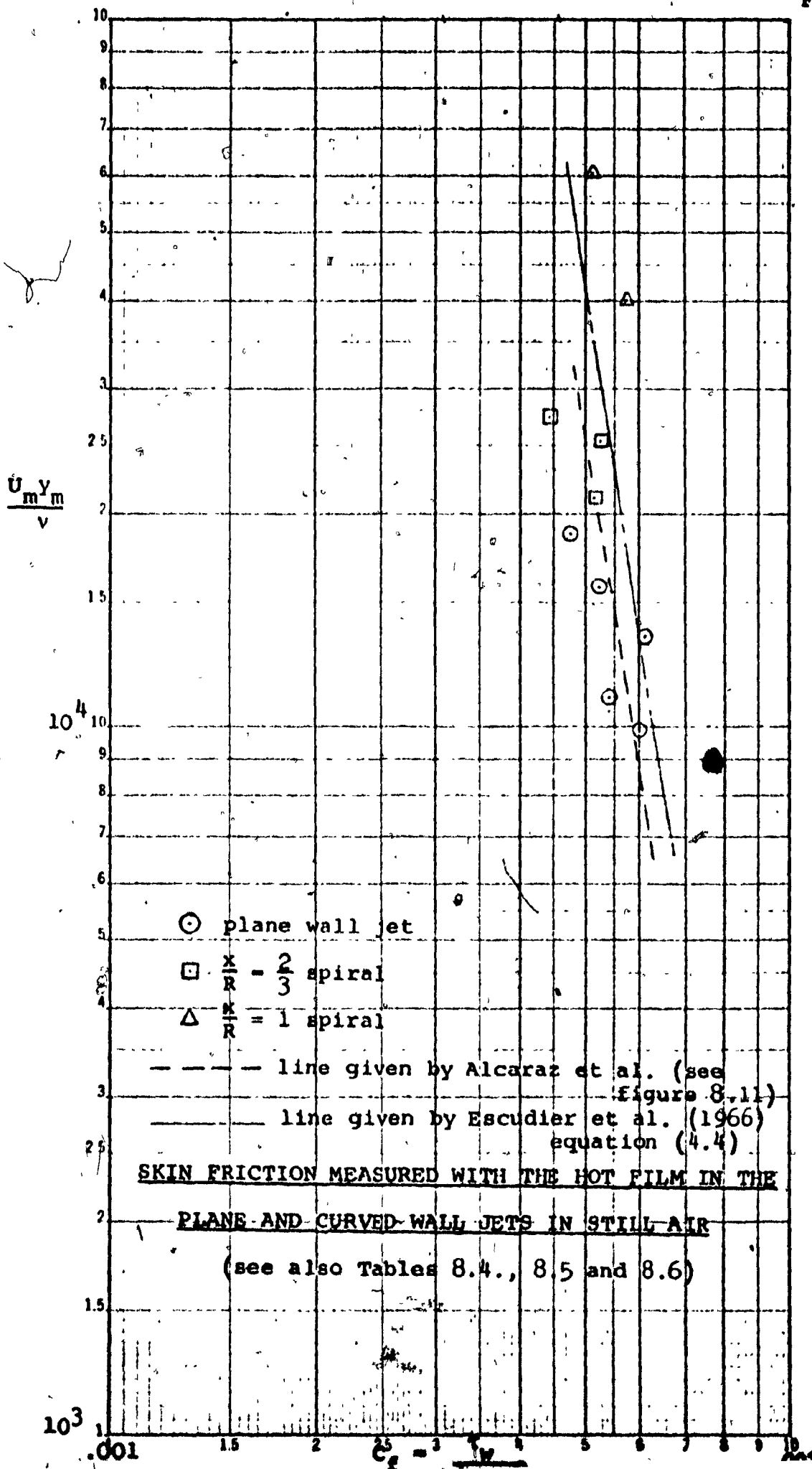
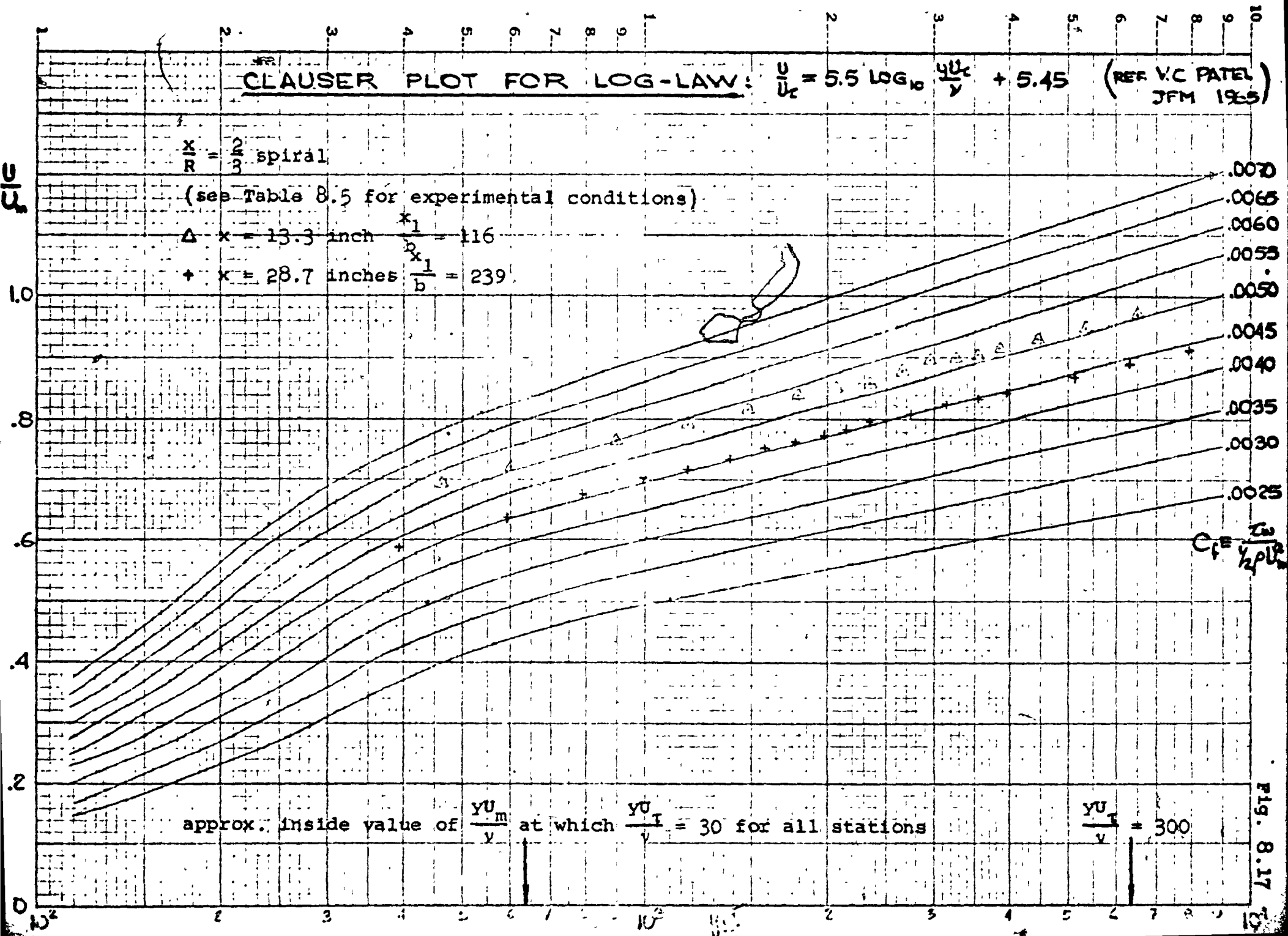


Fig. 8.16





CLAUSER PLOT FOR LOG-LAW:

$$\frac{u}{u_c} = 4.15 \log_{10} \frac{yu}{\nu} + 7.5 \quad (\text{ALCÁRAZ et al.})$$

$x = 2$ spiral
 $R = 3$

(see Table 8.5 for experimental conditions)

$\Delta x = 13.3$ inches $\frac{x_1}{b} = 116$

$+ x = 28.7$ inches $\frac{x_1}{b} = 239$

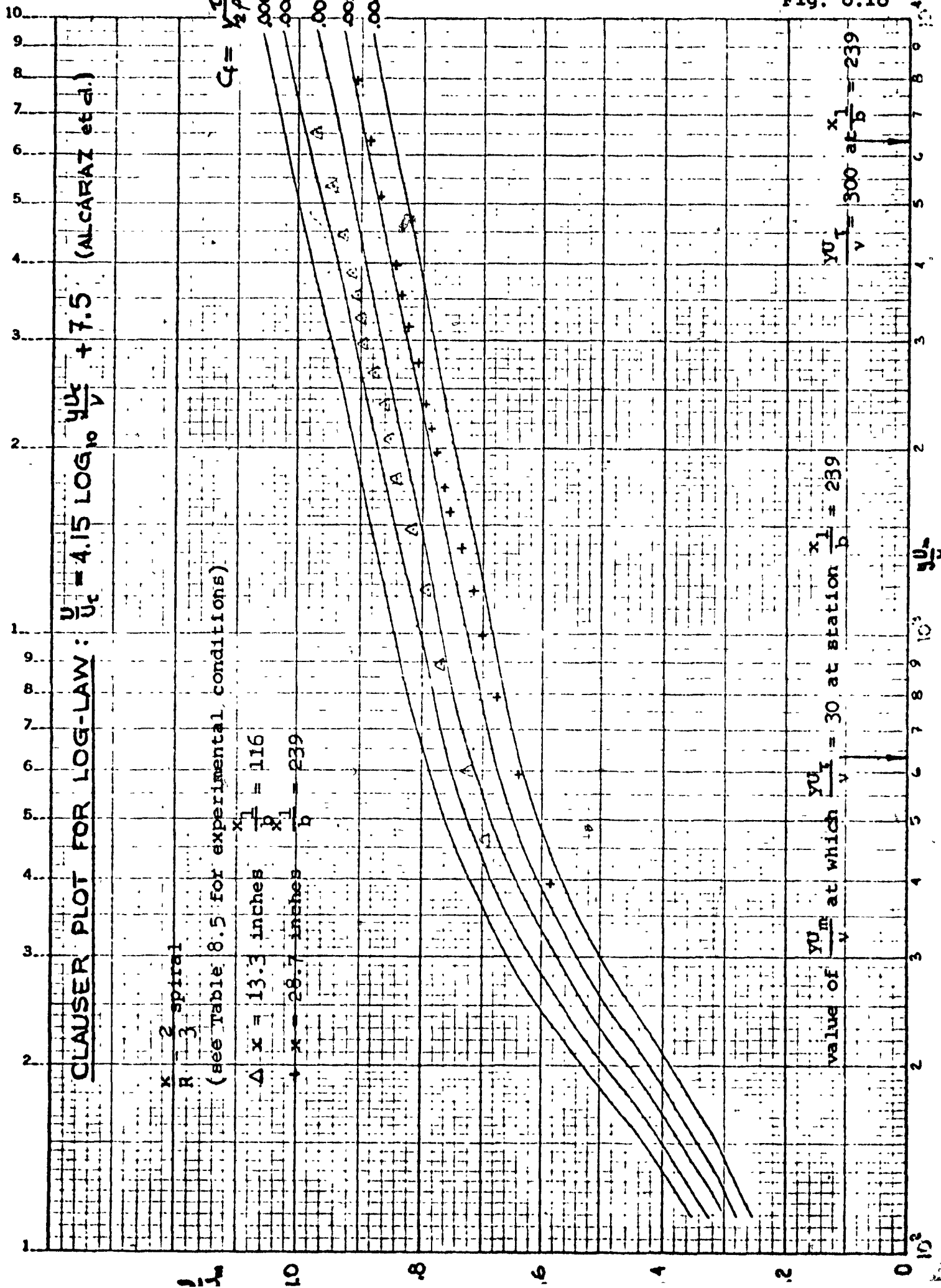
$$C_f = \frac{\tau_w}{\frac{1}{2} \rho U^2}$$

.0065
.0060
.0055
.0050
.0045

value of $\frac{yu_m}{\nu}$ at which $\frac{yu}{\nu} = 30$ at station $\frac{x_1}{b} = 239$

$$\frac{yu}{\nu} = 300 \text{ at } \frac{x_1}{b} = 239$$

Fig. 8.18



CLAUSER PLOT FOR LOG-LAW: $\frac{U}{U_c} = 5.5 \log_{10} \frac{yU_c}{\nu} + 5.45$ (RE: V.C. PATEL JPM 1965)

$\frac{x_1}{b} = 1$ spiral

(see Table 8.6 for experimental conditions)

○ $x = 14.26$ inches $\frac{x_1}{b} = 160$

+ $x = 29.75$ inches $\frac{x_1}{b} = 284$

$C_f = \frac{\tau_w}{\frac{1}{2} \rho U_m^2}$

approx. value of $\frac{yU_m}{\nu}$ at which $\frac{yU}{\nu} = 30$ for both stations

$\frac{yU}{\nu} = 300$

Fig. 9.1

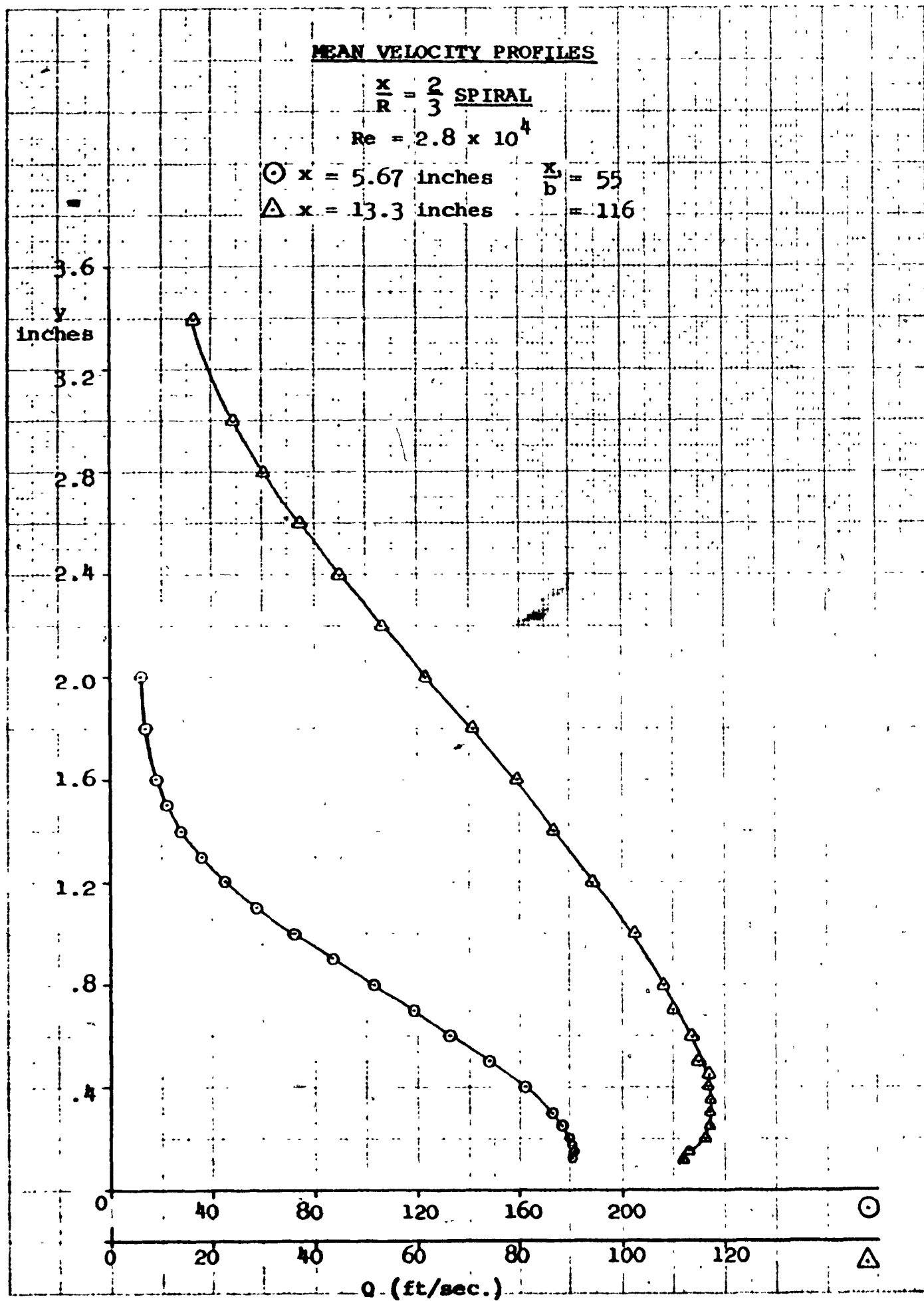


Fig. 9.2

MEAN VELOCITY PROFILES

$$\frac{x}{R} = \frac{2}{3} \text{ SPIRAL}$$

$$Re = 2.8 \times 10^4$$

$$\odot \quad x = 19.6 \text{ inches} \quad \frac{x}{b} = 166$$

$$\square \quad x = 23.5 \quad \frac{x}{b} = 197$$

$$+ \quad x = 28.7 \quad \frac{x}{b} = 239$$

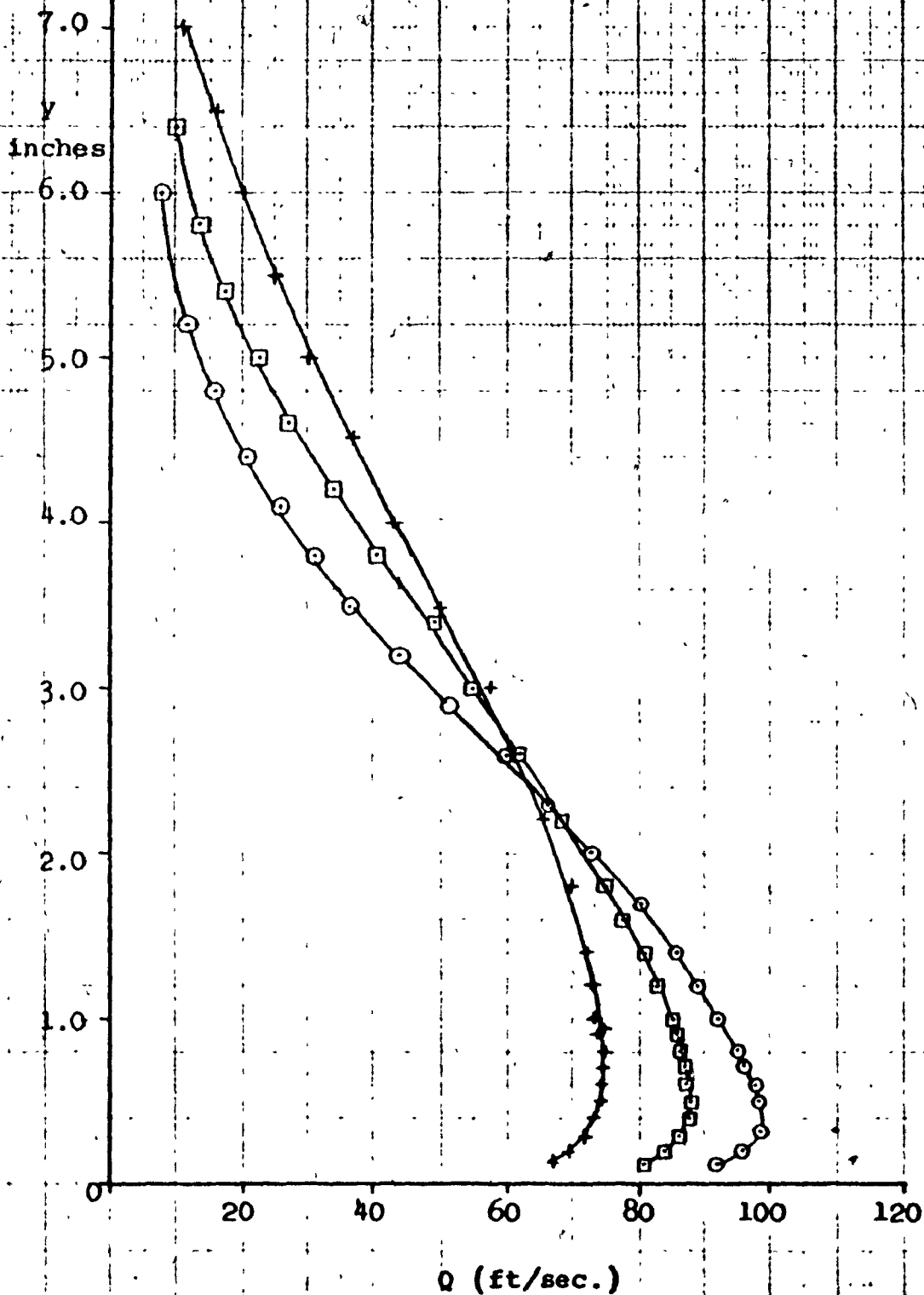


Fig. 9.3

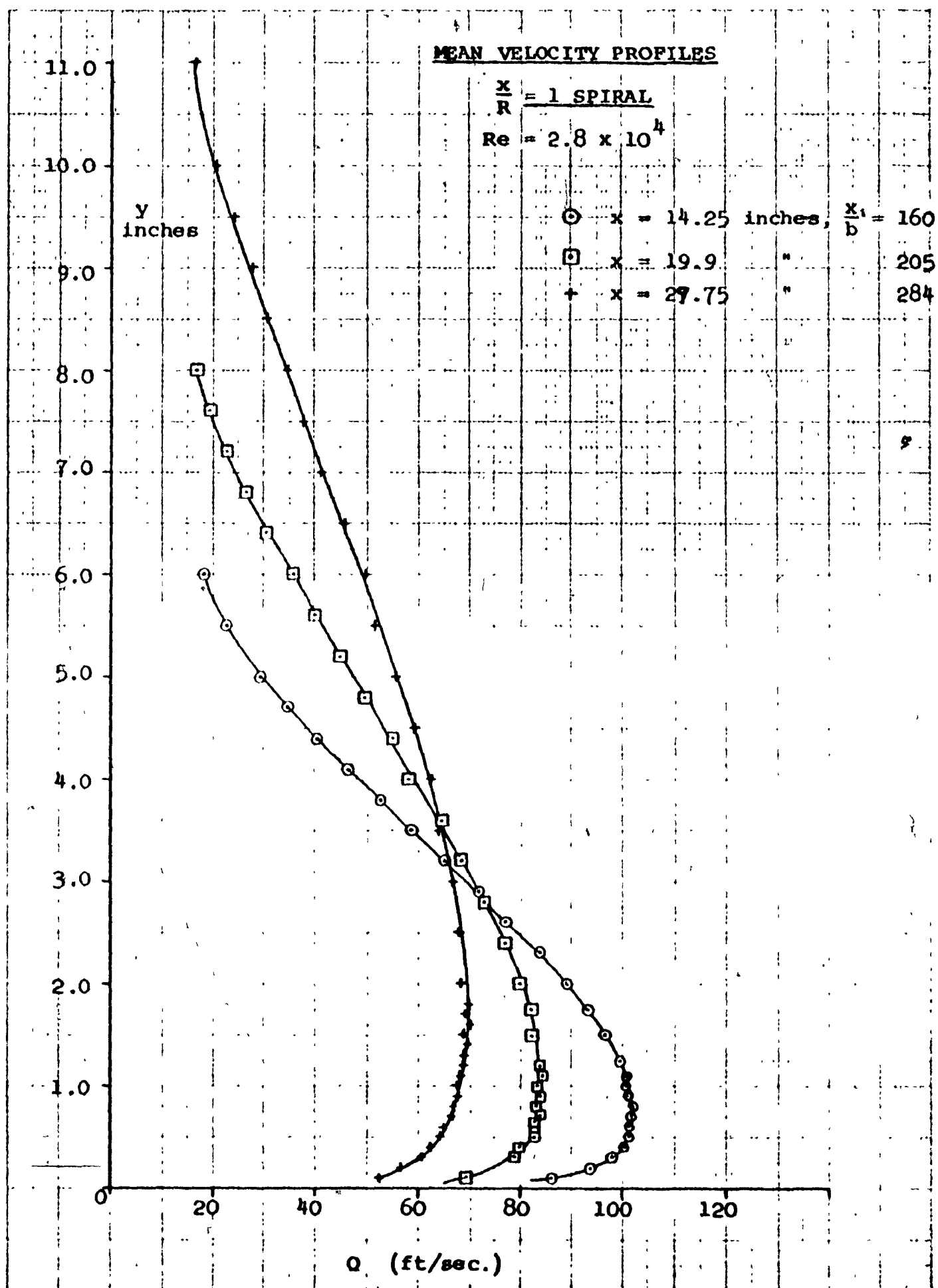


Fig. 9.4

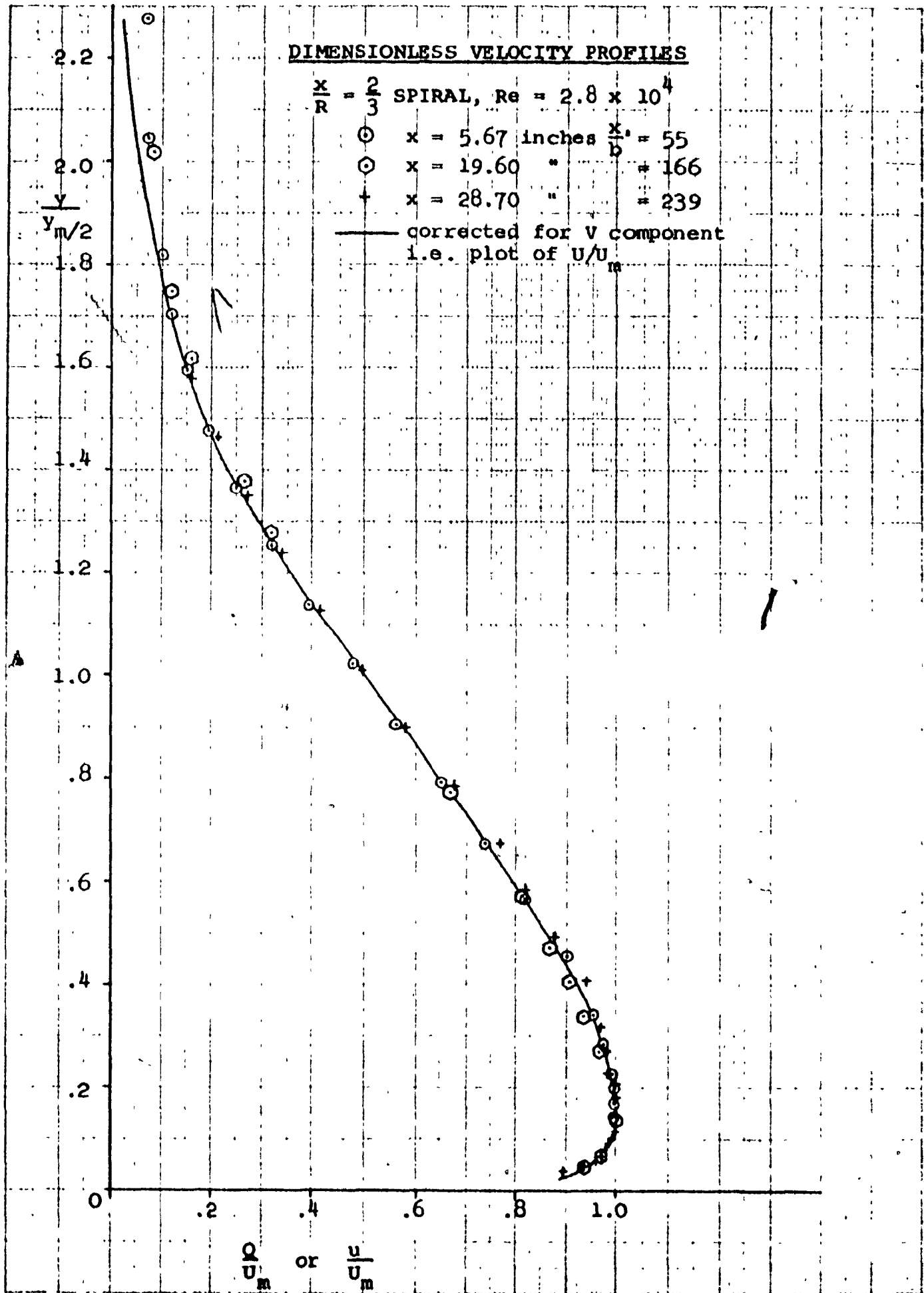
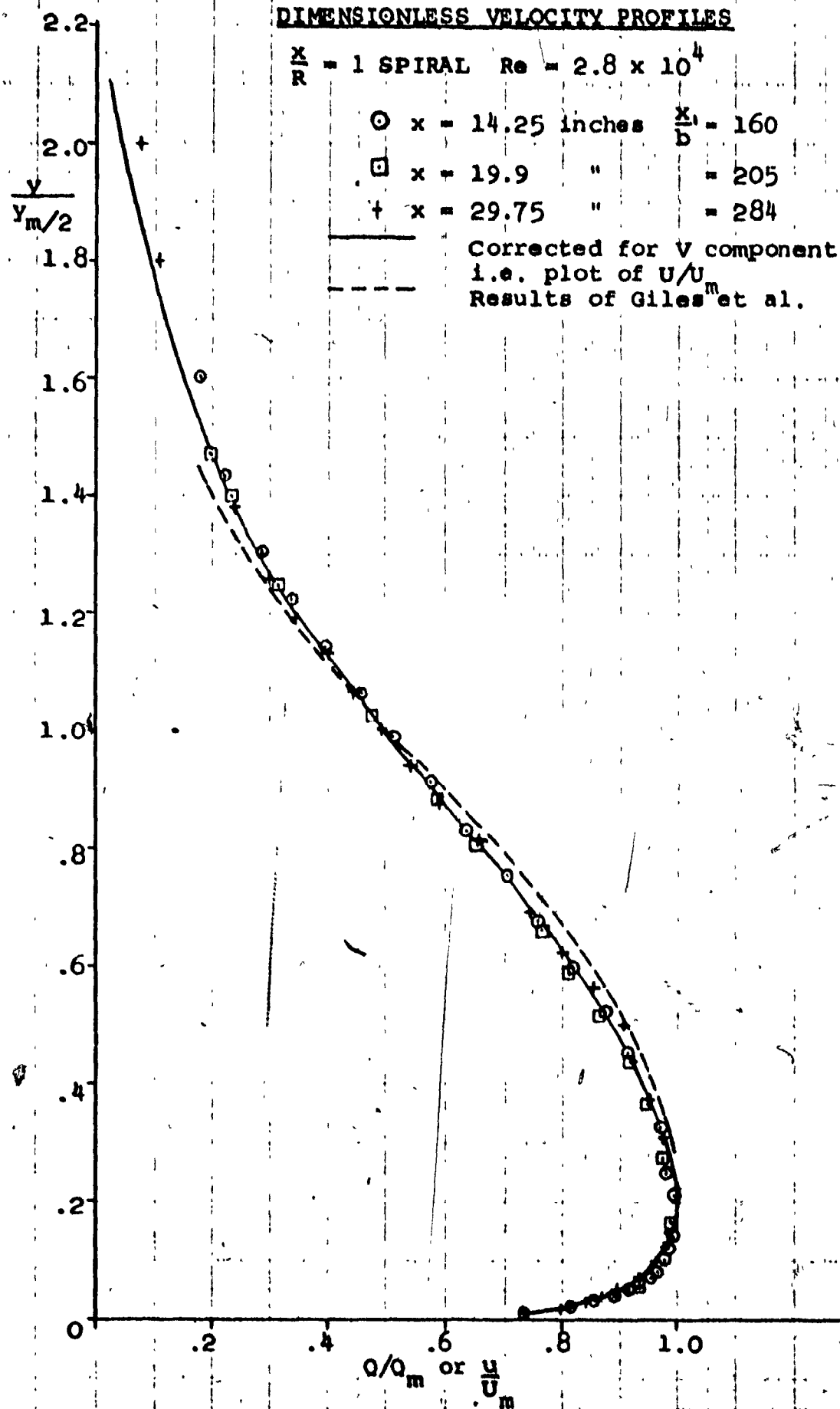


Fig. 9.5



DIMENSIONLESS VELOCITY PROFILES FOR OUTER LAYER

$$\frac{x}{R} = \frac{2}{3} \text{ SPIRAL } Re = 2.8 \times 10^4$$

$$\odot x = 5.67 \text{ inches } \frac{x}{b} = 55$$

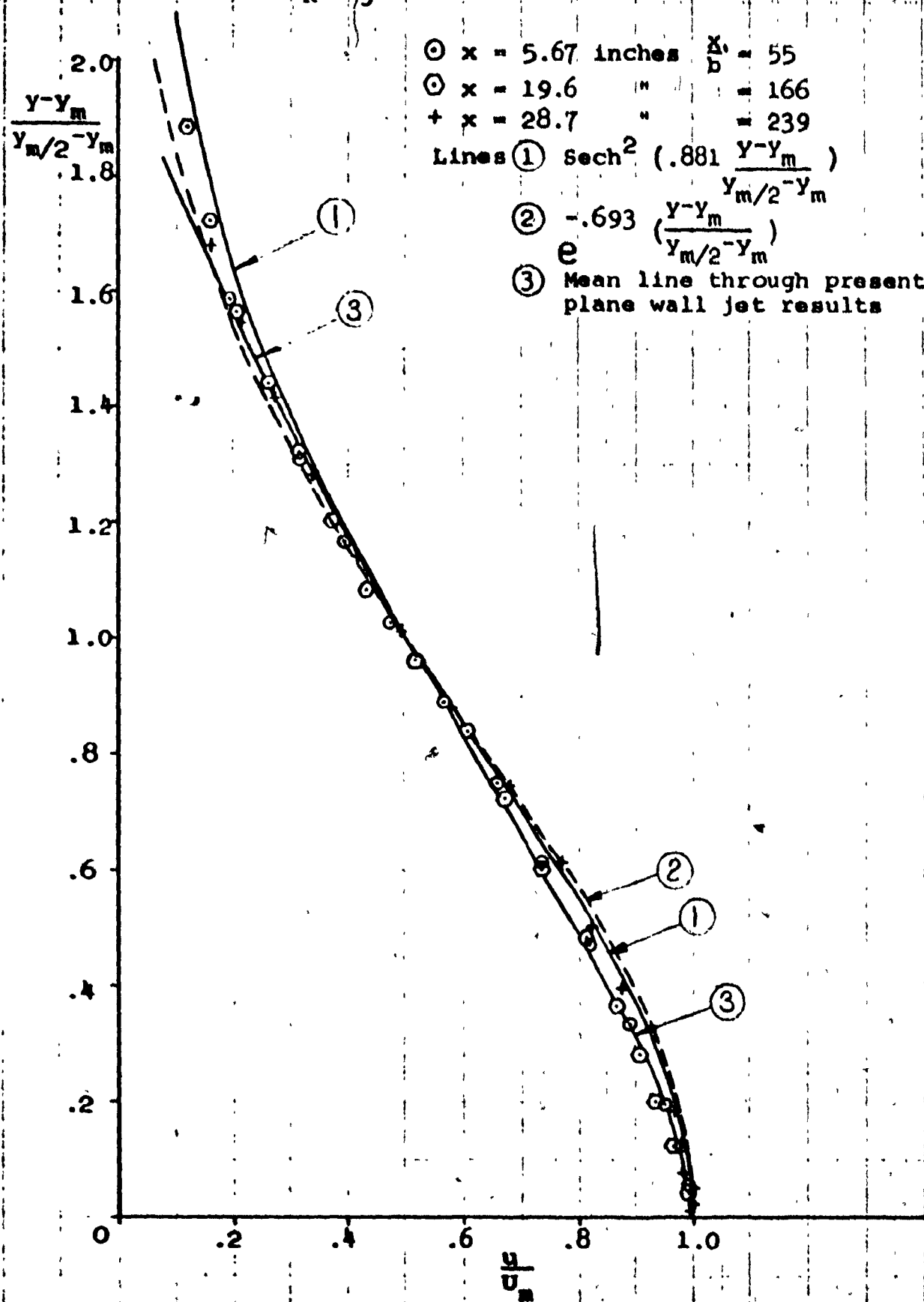
$$\odot x = 19.6 \text{ " } = 166$$

$$+ x = 28.7 \text{ " } = 239$$

$$\text{Lines } \textcircled{1} \text{ sech}^2 \left(.881 \frac{y-y_m}{y_m/2-y_m} \right)$$

$$\textcircled{2} -.693 \left(\frac{y-y_m}{y_m/2-y_m} \right) e$$

$\textcircled{3}$ Mean line through present plane wall jet results



DIMENSIONLESS VELOCITY PROFILE FOR OUTER LAYER

$\frac{x}{R} = 1$ SPIRAL $Re = 2.8 \times 10^4$

○ $x = 14.25$ inches $\frac{x}{b} = 160$

□ $x = 19.9$ " $\frac{x}{b} = 205$

+ $x = 29.75$ " $\frac{x}{b} = 284$

$$\text{sech}^2 \left(.881 \frac{y - y_m}{y_m/2 - y_m} \right)$$

$\frac{y - y_m}{y_m/2 - y_m}$

1.8

1.6

1.4

1.2

1.0

.8

.6

.4

.2

0

.2

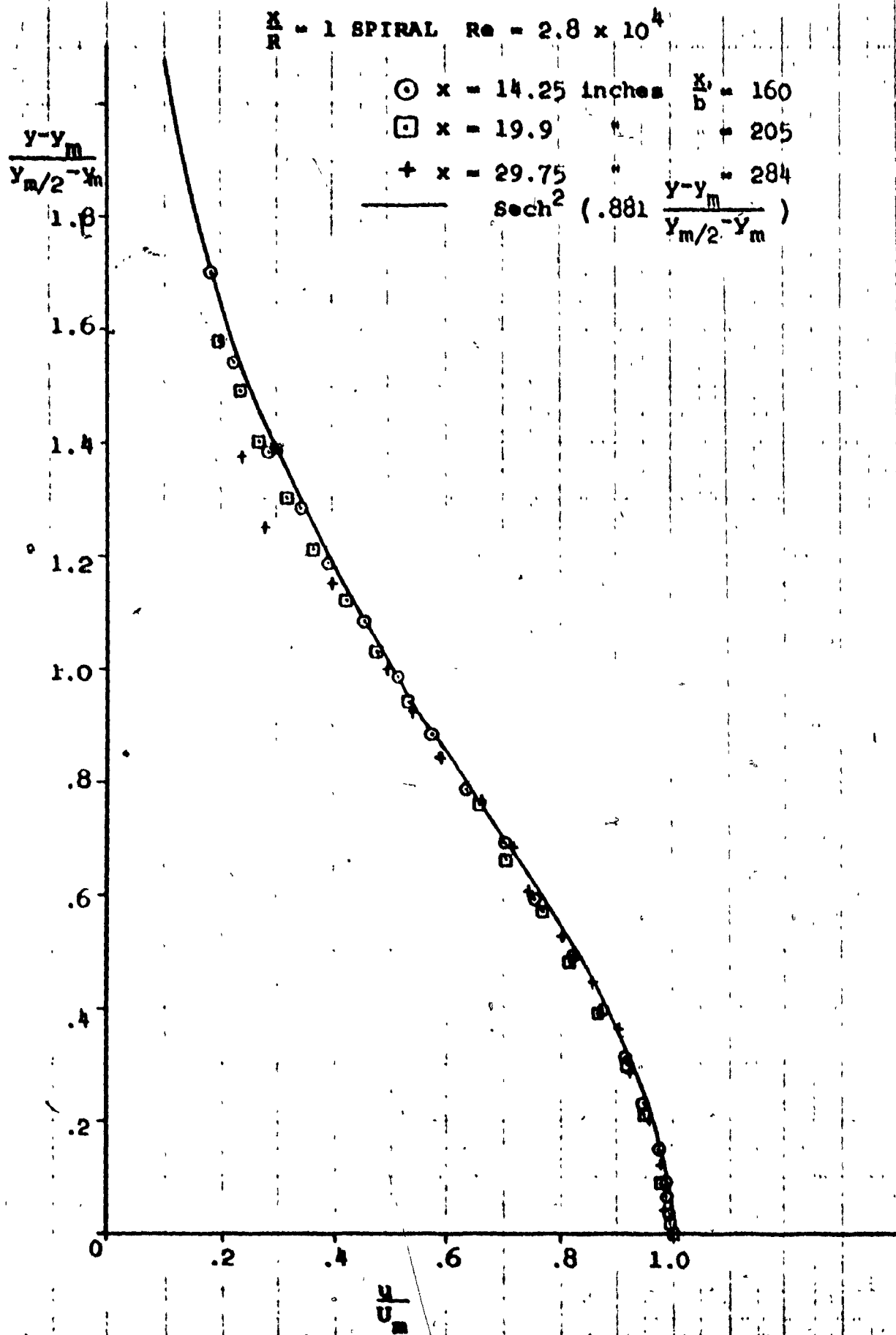
.4

.6

.8

1.0

$\frac{u}{u_m}$



GROWTH OF JETS ON LOGARITHMIC SPIRALS

AND ON A PLANE WALL JET

$$Re = 2.8 \times 10^4$$

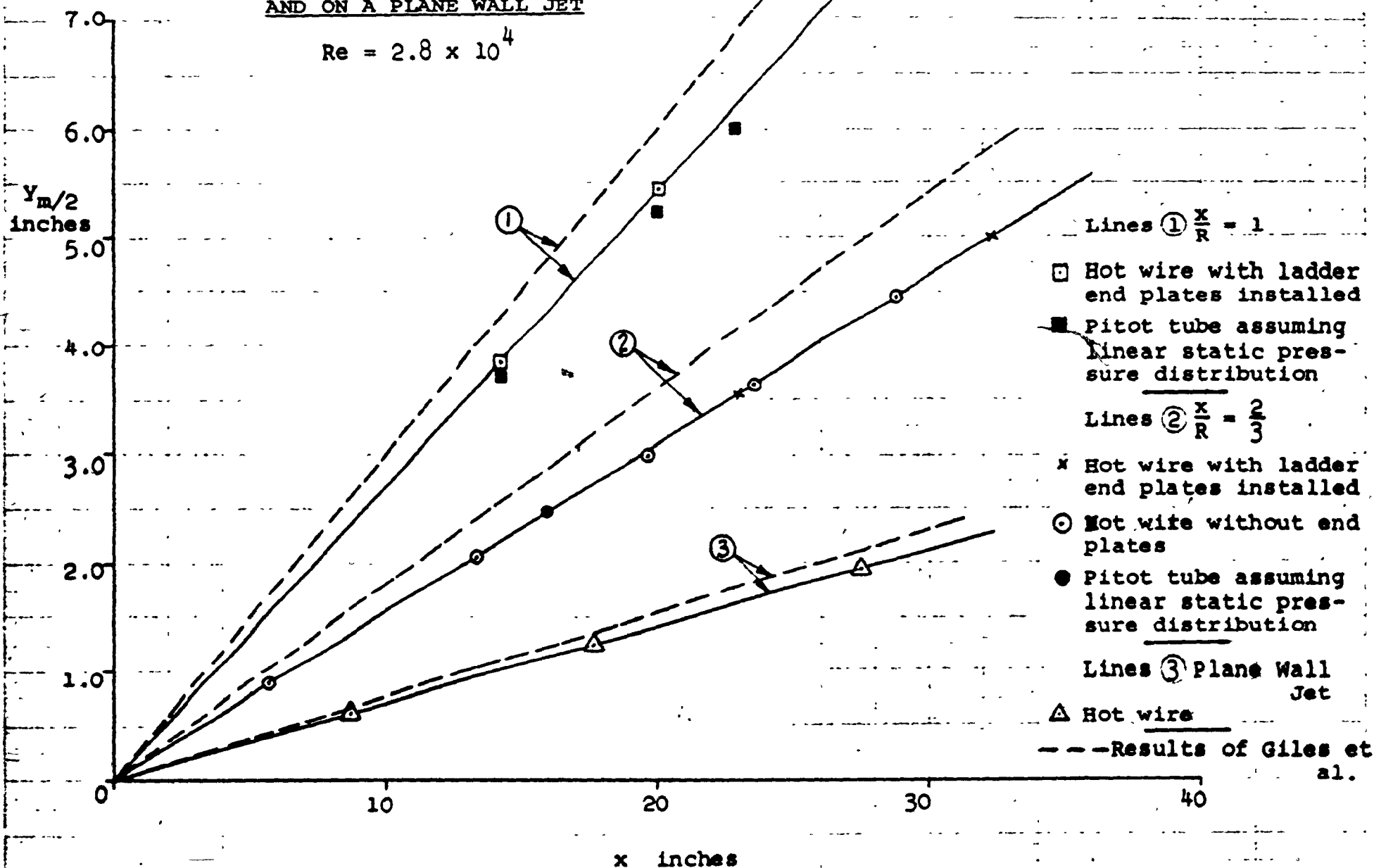


Fig. 9.8

GROWTH OF INNER LAYER (y_m) ON SPIRAL SURFACES

$Re = 2.8 \times 10^4$

$\square x/R = 1$

$\circ x/R = 2/3$

$\triangle$ Plane wall jet (Guitton)

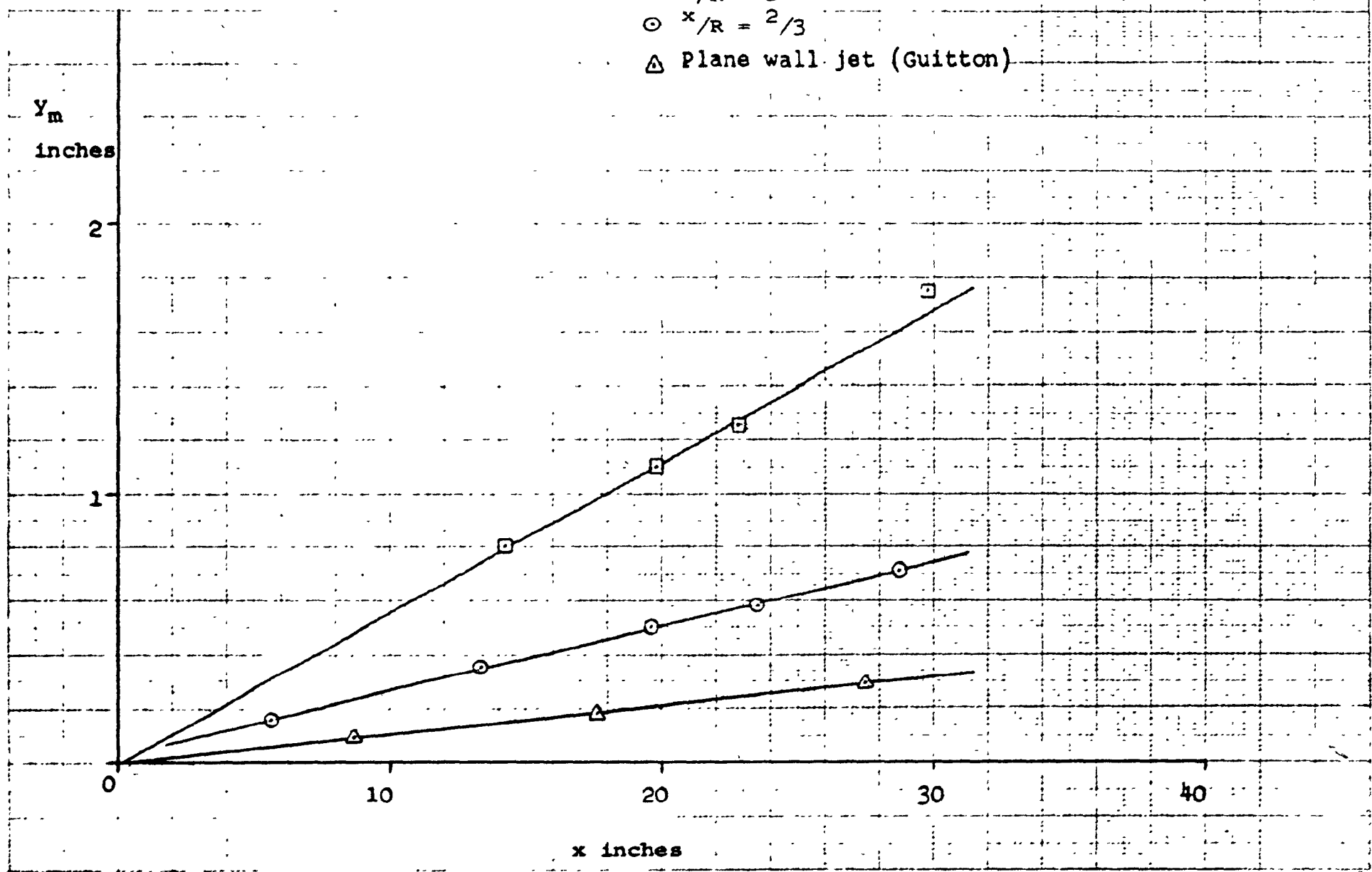


Fig. 9.9

DECAY OF MAXIMUM VELOCITY ON LOGARITHMIC SPIRALS

- VERIFICATION OF TWO-DIMENSIONALITY USING MOMENTUM EQUATION
- COMPARISON WITH RESULTS OF GILES ET AL.

LEGEND $Re = 2.8 \times 10^4$

$\frac{x}{R} = \frac{2}{3}, x_r = 5.67$ inches, $U_r = 182.2$ ft/sec.

○ Hot Wire Measurements

● Pitot Tube Measurements assuming linear static pressure distribution

Line ① Calculated from momentum equation using measured skin friction and growth ($y_m/2'$)

$\frac{x}{R} = 1, x_r = 14.26$ inches, $U_r = 101.1$ ft/sec.

□ Hot Wire Measurements

Line ② Mean line through results of Giles et al. $x_r = 5$ inches

Line ③ See Line ① above

Line ④ $U_m \propto x^{-1/2}$

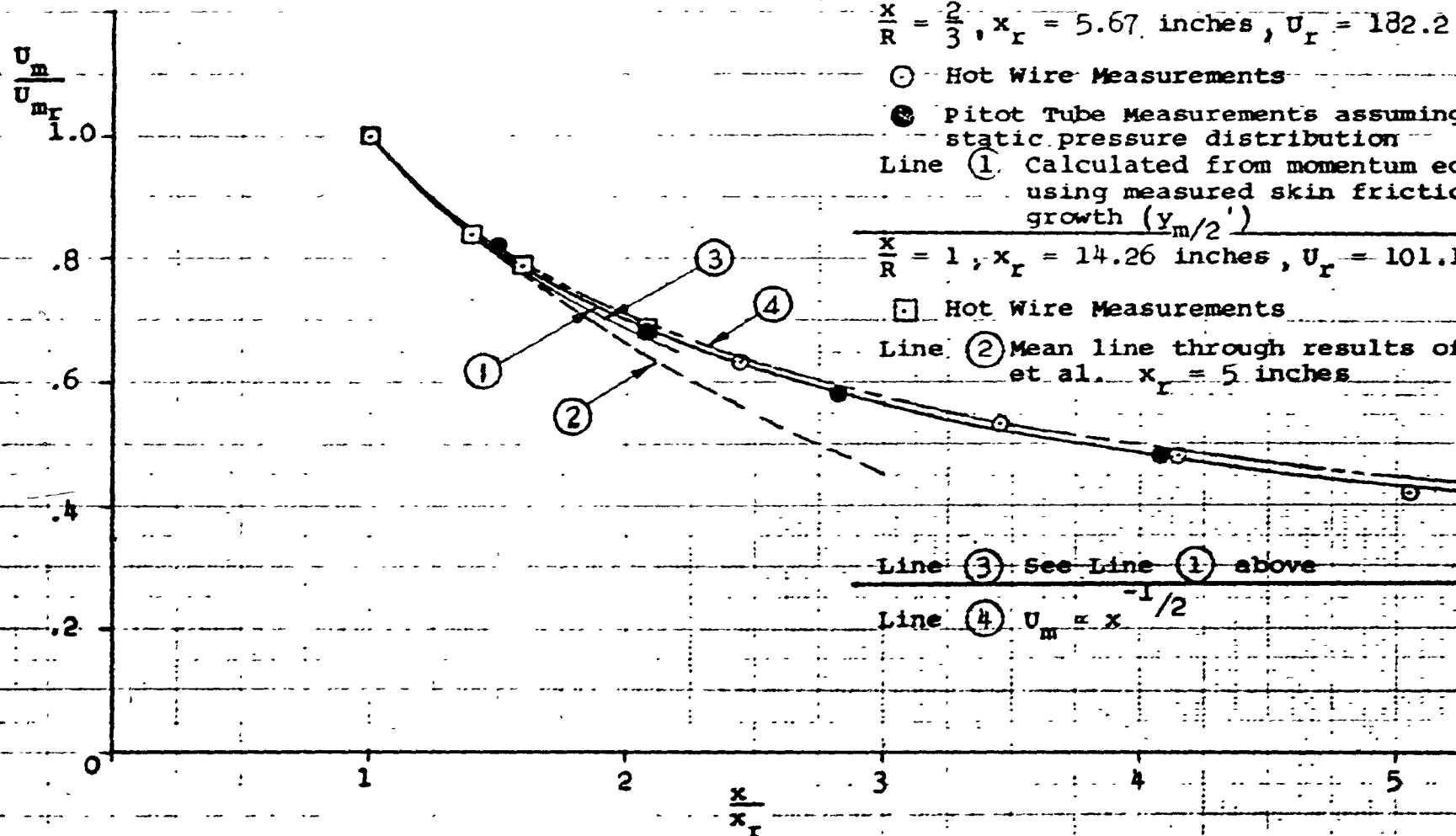


Fig. 9.10

RESULTS OF GILES ET AL : DECAY OF MAXIMUM VELOCITY

$\frac{x}{R} = 1$ SPIRAL (LINE ①) AND VERIFICATION OF TWO-DIMENSIONALITY (LINE ②)

USING MOMENTUM EQUATION IN CONJUNCTION WITH KNOWN PROFILE SHAPE AND
PRESENT MEASUREMENTS OF SKIN FRICTION

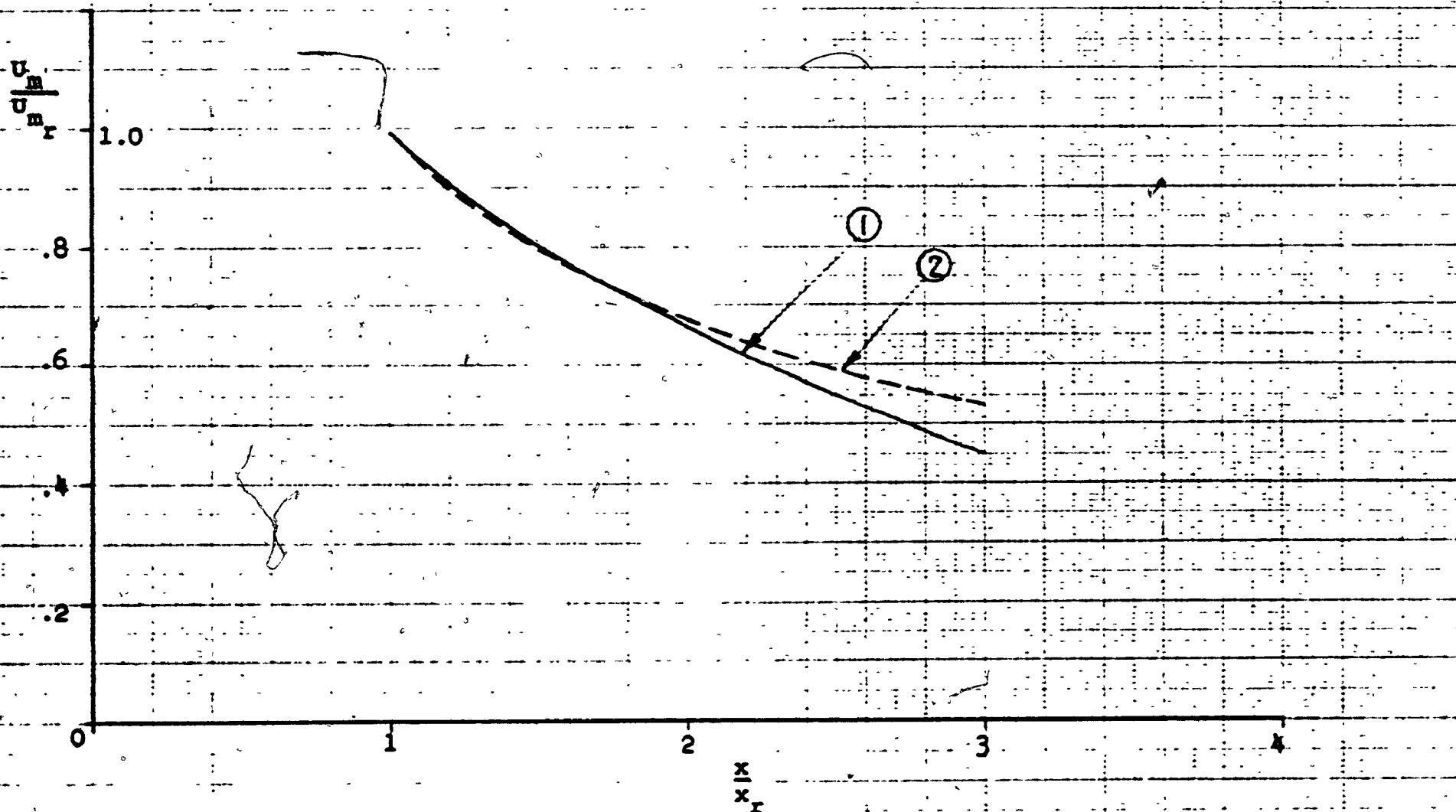
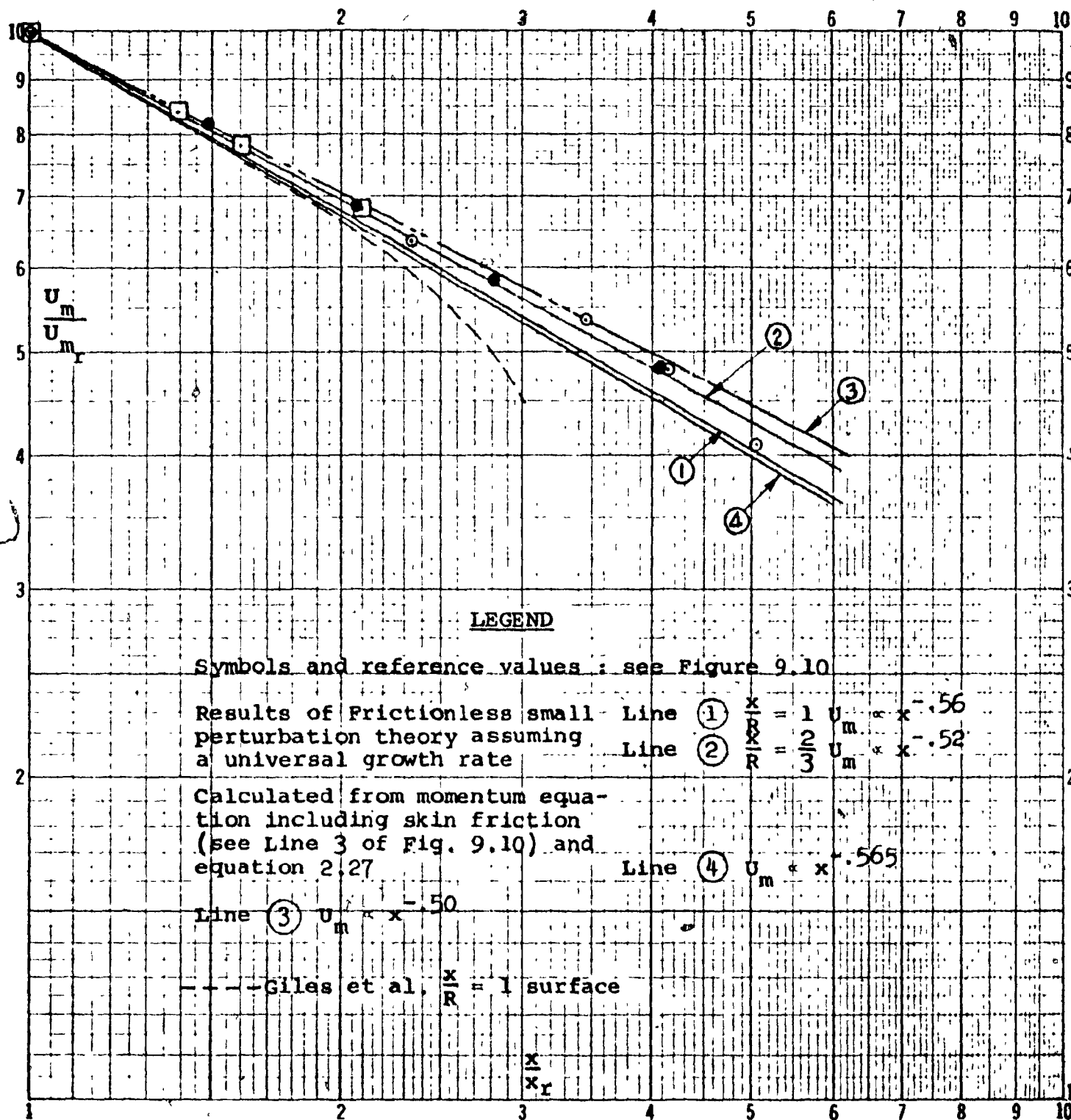


Fig. 9.11

DECAY OF MAXIMUM VELOCITY : COMPARISON BETWEEN
EXPERIMENTS AND SMALL PERTURBATION THEORY



SURFACE PRESSURE DISTRIBUTION ON SPIRALS

$$\bigcirc \quad \frac{x}{R} = \frac{2}{3} \quad \square \quad \frac{x}{R} = 1$$

$$\frac{p_{\infty} - p_w}{\rho U_m^2}$$

— Calculated from full y-momentum equation

- - - Calculated from $\int_0^x u^2/R + y \, dy$ and $u/U_m = f(y/y_{m/2})$
using measured $y_{m/2}/R$. for $x/R = 2/3$
the result coincides with the solid line.

.2

.1

0

10

20

30

40

x inches

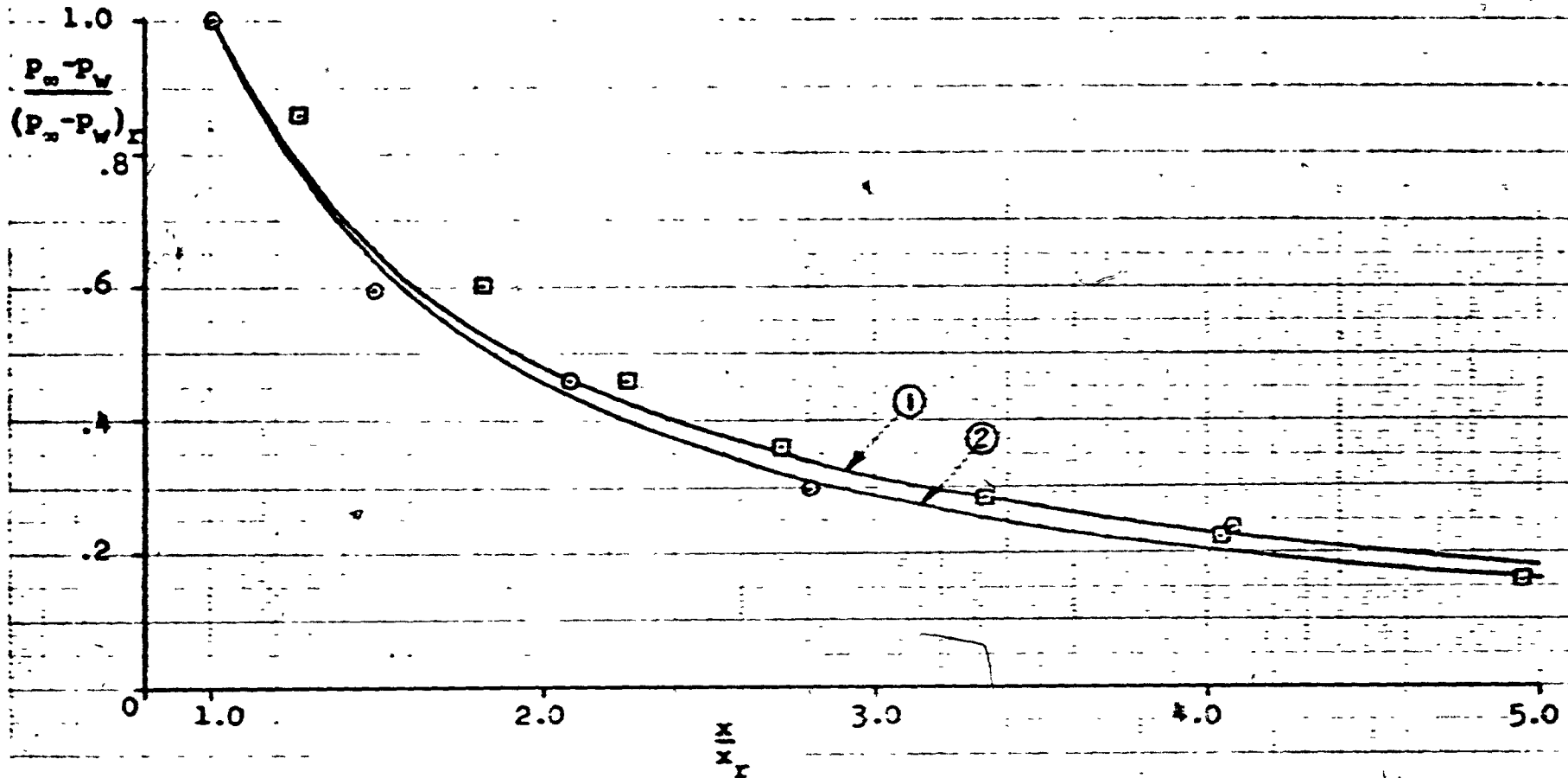
SURFACE PRESSURE DISTRIBUTION ON SPIRALS

$$\bigcirc \frac{x}{R} = \frac{2}{3} \quad \square \frac{x}{R} = 1$$

Prediction of frictionless
small perturbation theory
(equation 3.31)

Line ① $\frac{x}{R} = \frac{2}{3}$

Line ② $\frac{x}{R} = 1$



CALCULATED DISTRIBUTION OF $p + \rho v^2$ ACROSS JETS ON SPIRALS

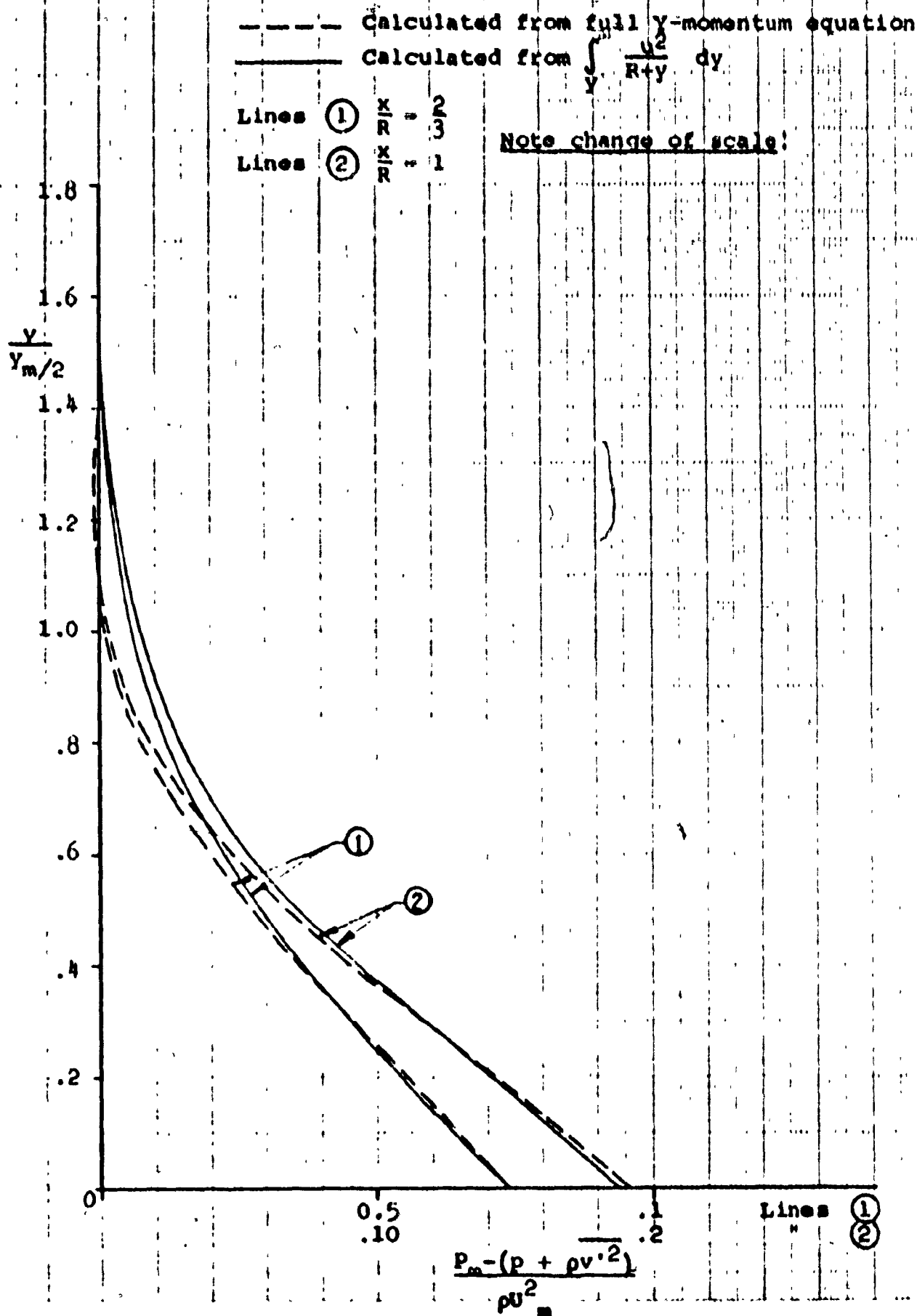


Fig. 9.16

STATIC PRESSURE DISTRIBUTION ACROSS JETS ON LOGARITHMIC SPIRALS

CALCULATED FROM FULL Y-MOMENTUM EQUATION (FIG. 9.15)

AND MEASURED

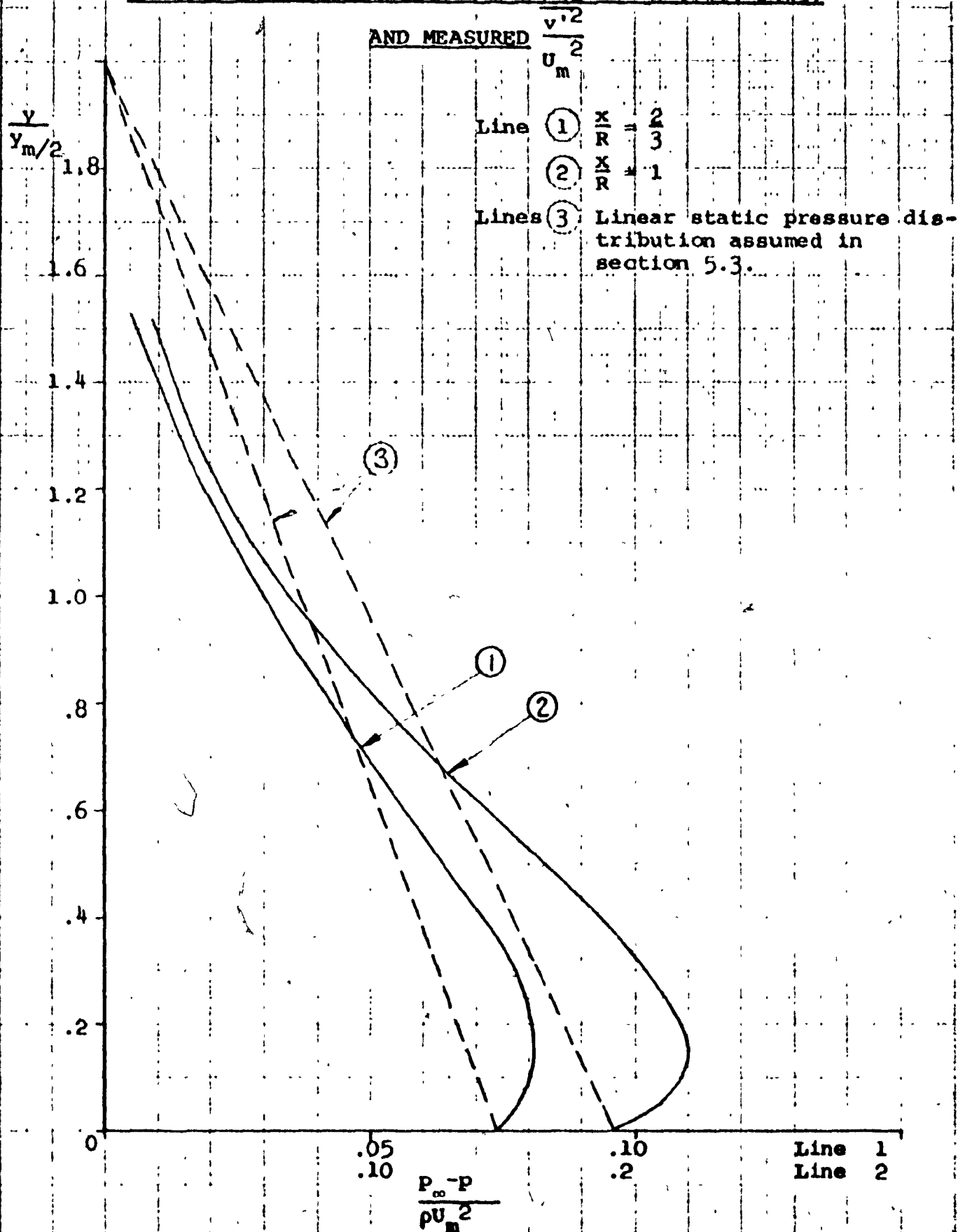


Fig. 9.17a

CALCULATED SHEAR STRESS DISTRIBUTION

$$\frac{\overline{u'v'}}{\overline{u'v'}_{\text{MAX}}}$$

- Lines (1) $\frac{x}{R} = \frac{2}{3}$ (2) Plane wall jet in still air
 (3) $\frac{x}{R} = 1$ (4) Plane free jet in still air
 (Patel, 1970)

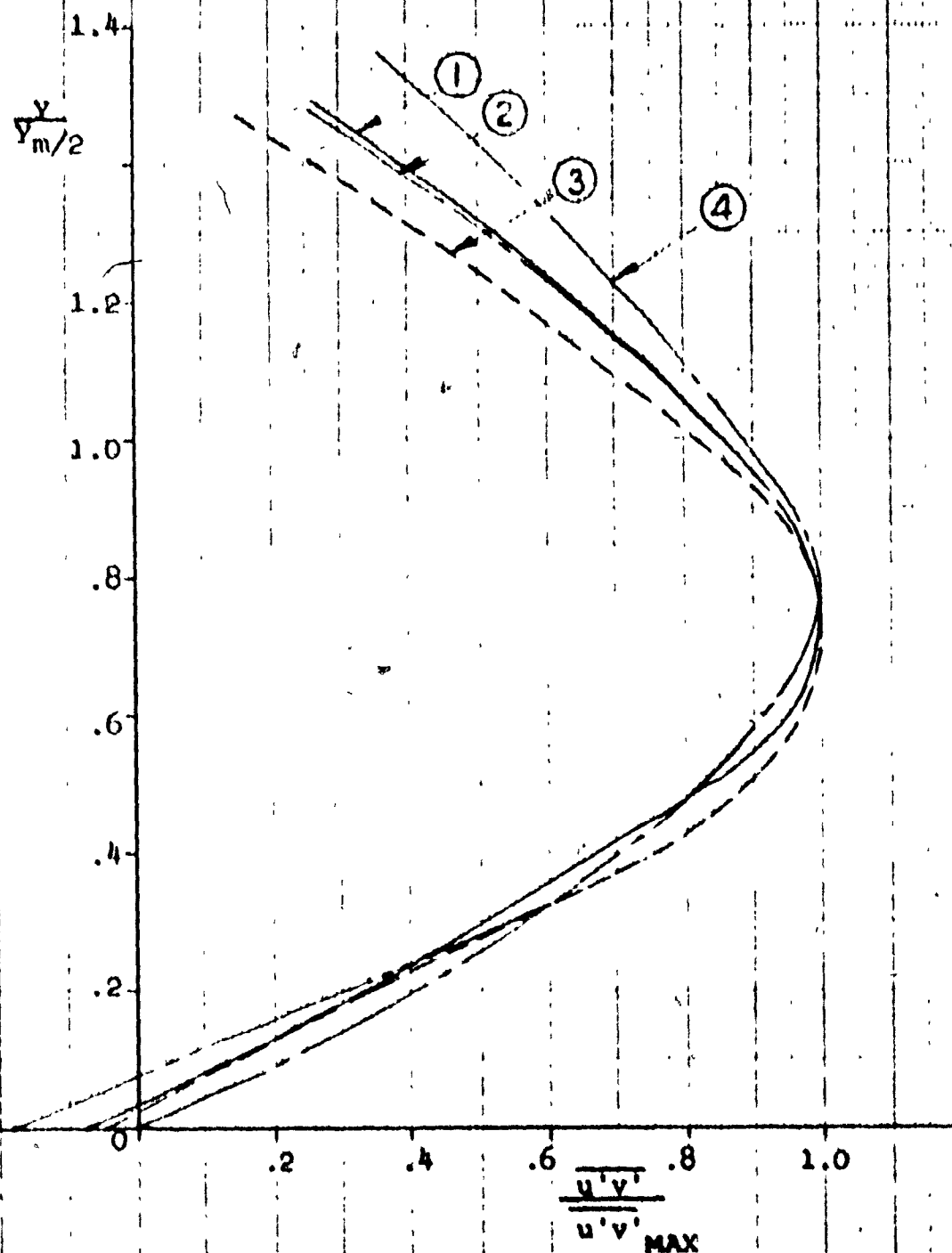
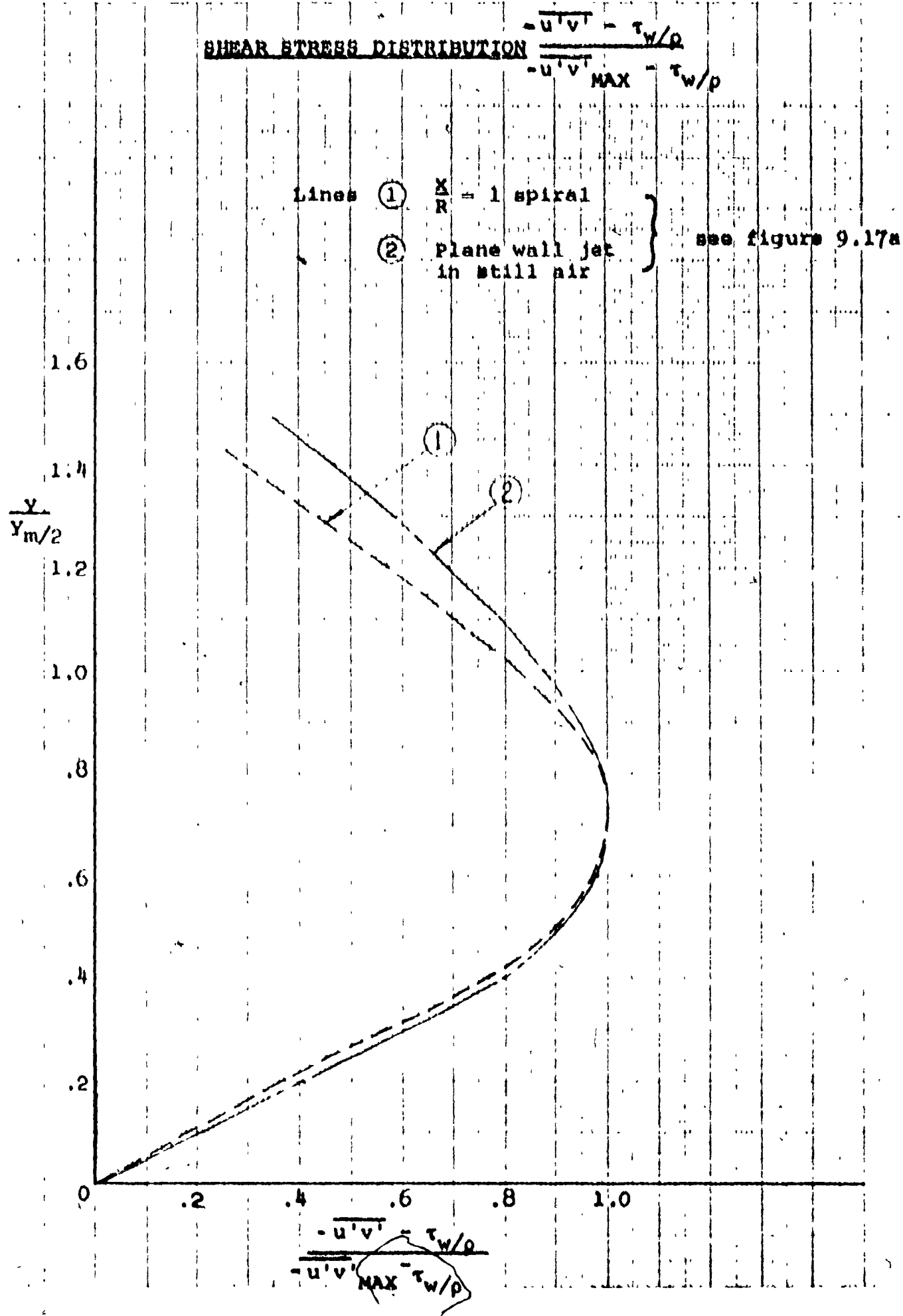


Fig. 9.17b



CALCULATED SHEAR STRESS DISTRIBUTION

$$\frac{u'v'}{u'v'_{\text{MAX}}}$$

- Lines ① $\frac{x}{R} = \frac{2}{3}$ ② Plane wall jet in still air
 ③ $\frac{x}{R} = 1$ ④ Plane free jet in still air (Patel, 1970)

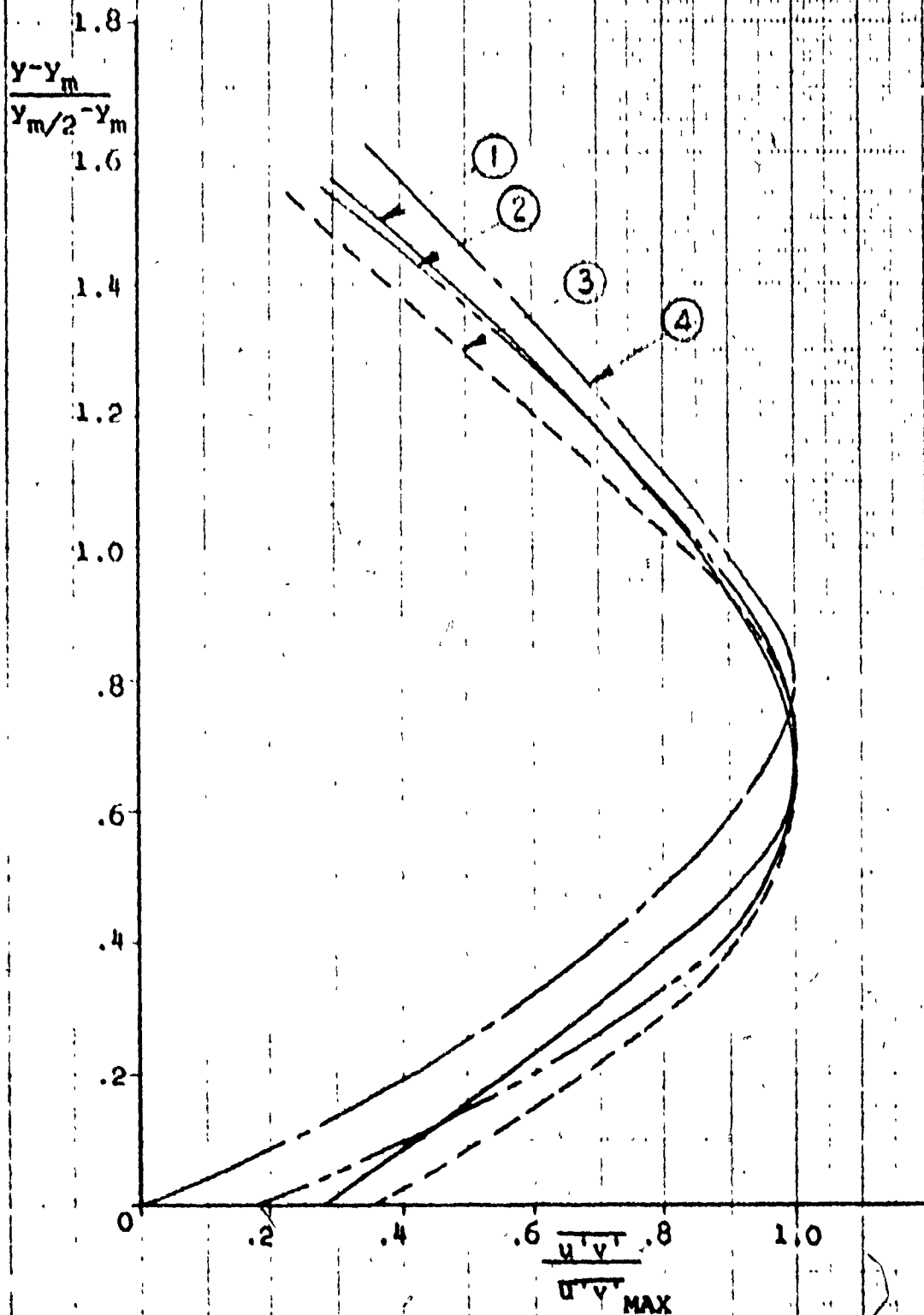


Fig. 9.19 -

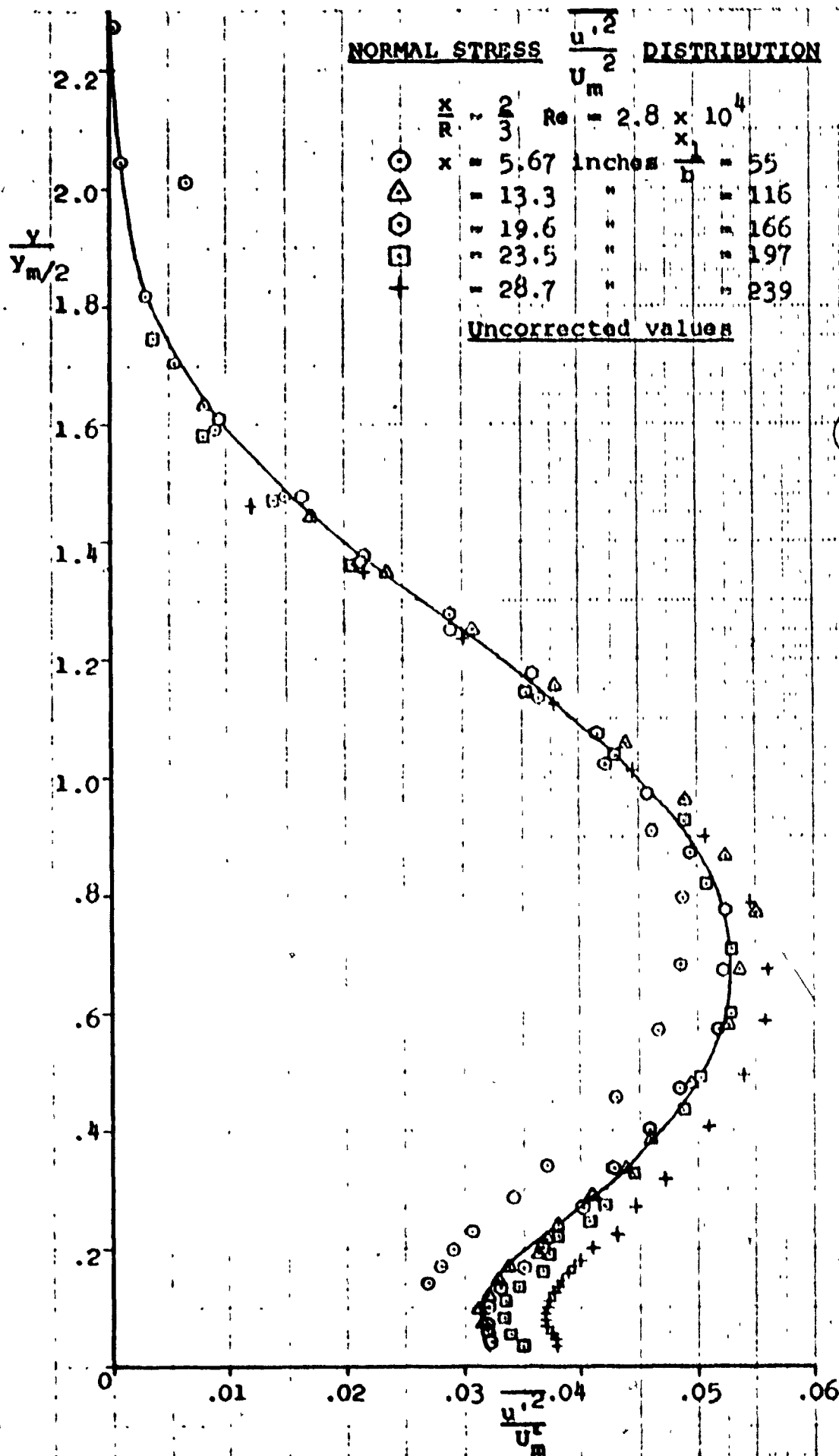


Fig. 9.20

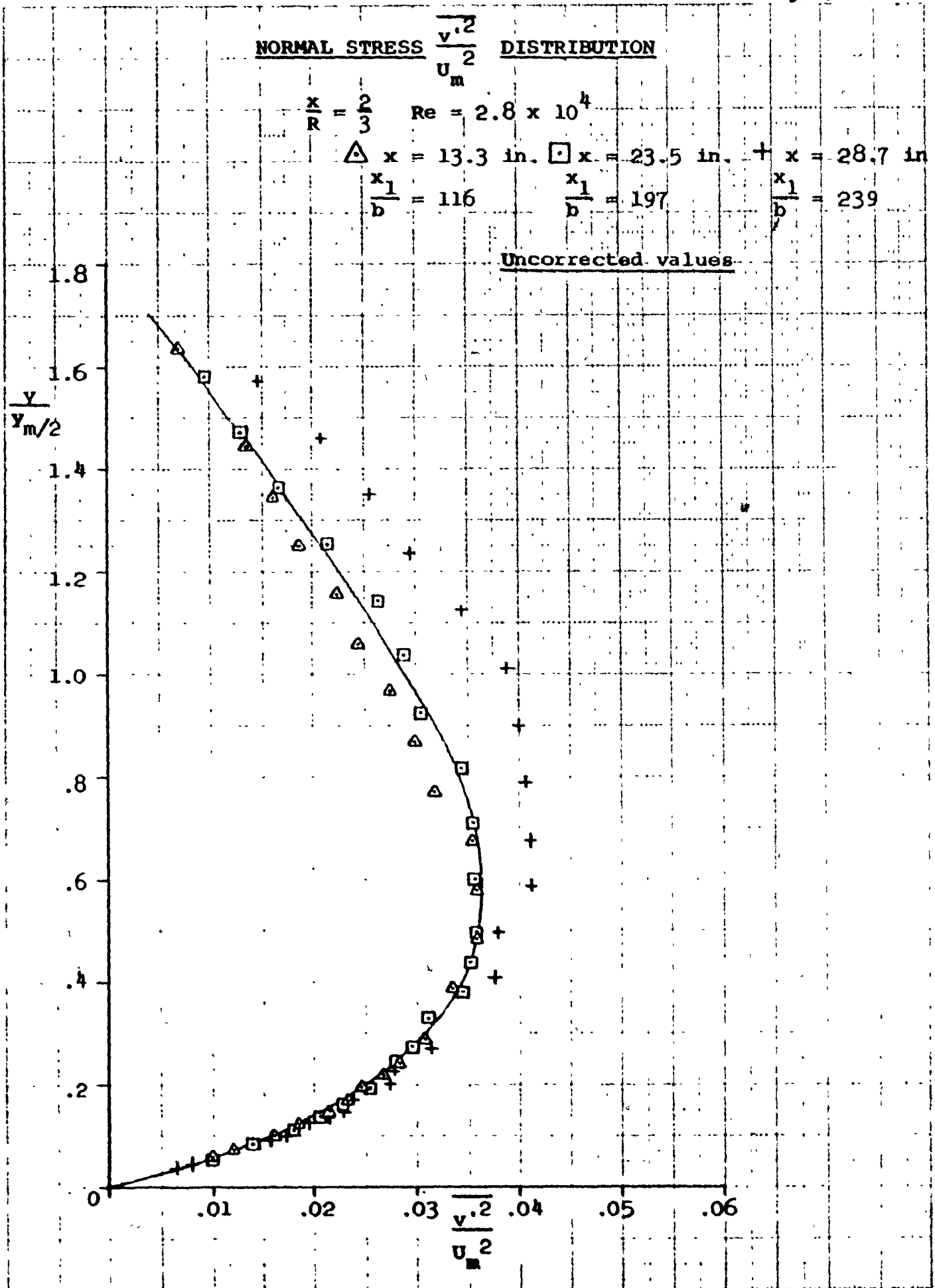


Fig. 9.21

NORMAL STRESS $\frac{w'^2}{2}$ DISTRIBUTION

$$\frac{x}{R} = \frac{2}{3} \quad \text{Re} \quad U_m = 2.8 \times 10^4$$

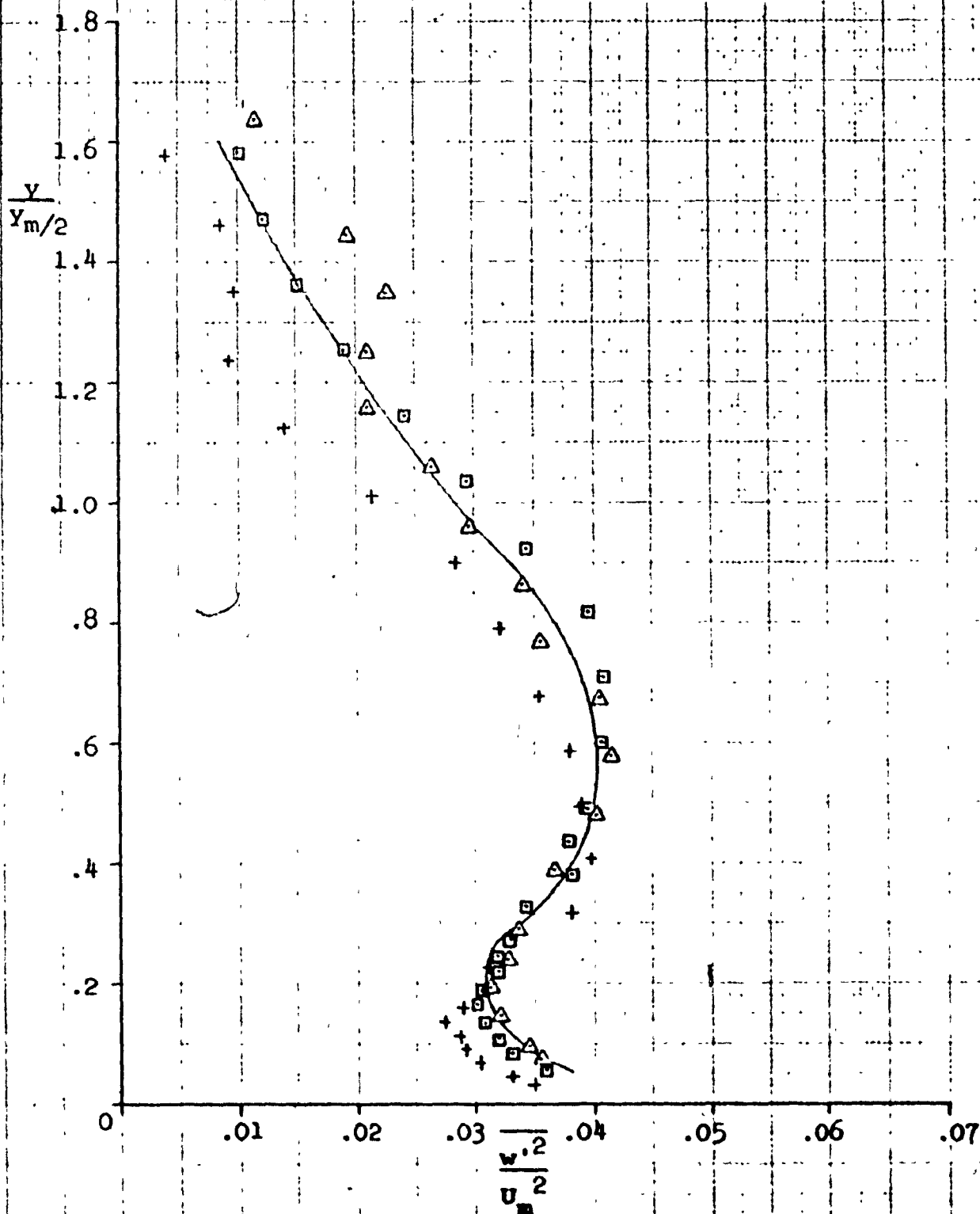
$$\Delta x = 13.3 \text{ in.} \quad \square x = 23.5 \text{ in.} \quad + x = 28.7 \text{ in.}$$

$$\frac{x_1}{b} = 116$$

$$\frac{x_1}{b} = 197$$

$$\frac{x_1}{b} = 239$$

Uncorrected values



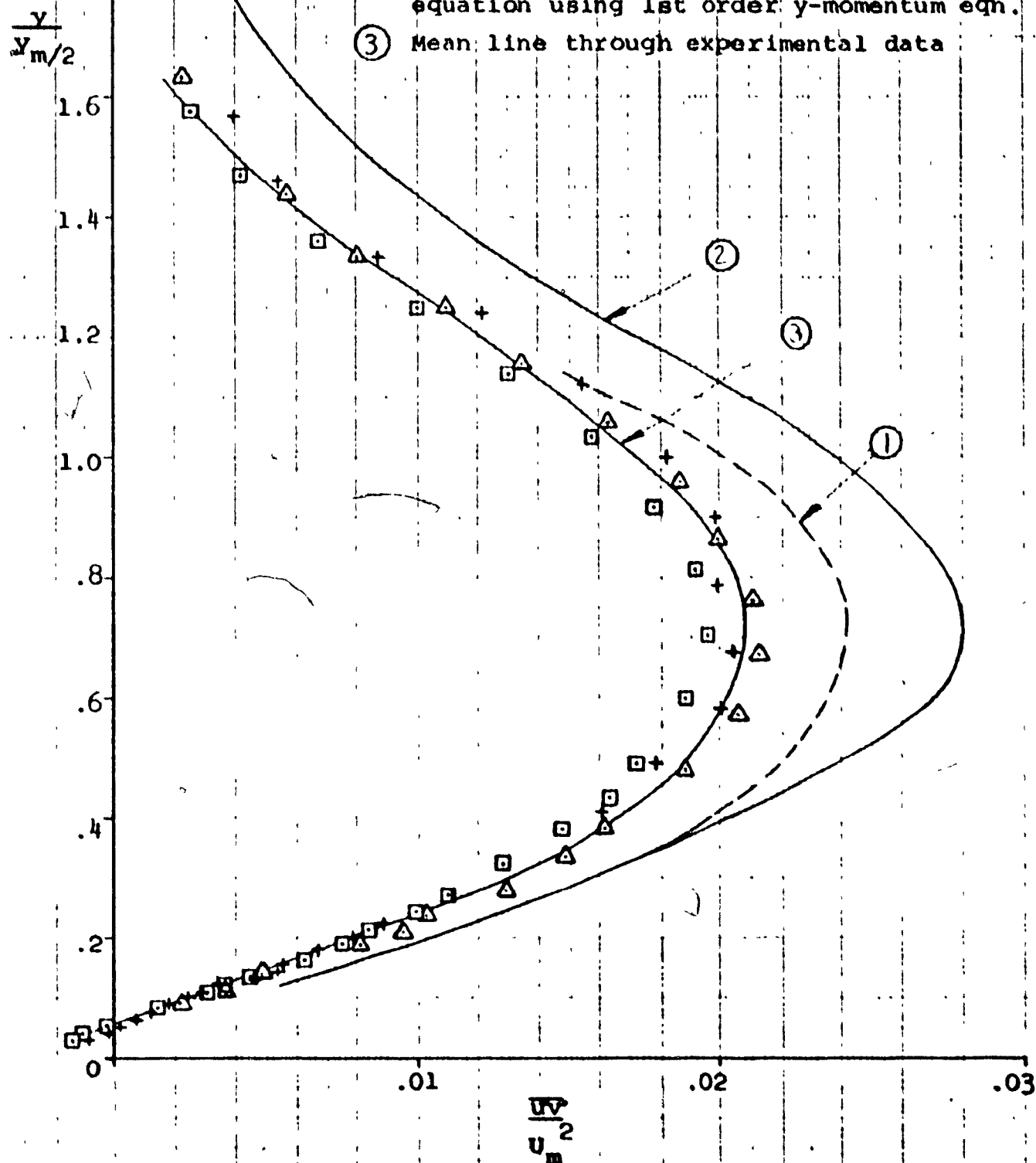
SHEAR STRESS DISTRIBUTION

$$\frac{x}{b} = \frac{2}{3} \quad Re = 2.8 \times 10^4$$

$$\Delta \quad x = 13.3 \quad \square \quad x = 23.5 \quad + \quad x = 28.7$$

$$\cdot \quad \frac{x_1}{b} = 116 \quad \frac{x_1}{b} = 197 \quad \frac{x_1}{b} = 239$$

- Line ① Shear stress corrected for longitudinal cooling and high intensity turbulence
 ② Shear stress calculated from momentum equation using 1st order y-momentum eqn.
 ③ Mean line through experimental data



SHEAR STRESS DISTRIBUTION MEASURED ON
DIFFERENT DAYS AND WITH DIFFERENT
HOT WIRE PROBES

$$\frac{x}{b} = \frac{2}{3}, x = 13.3 \text{ inches}, \frac{x_1}{b} = 116$$

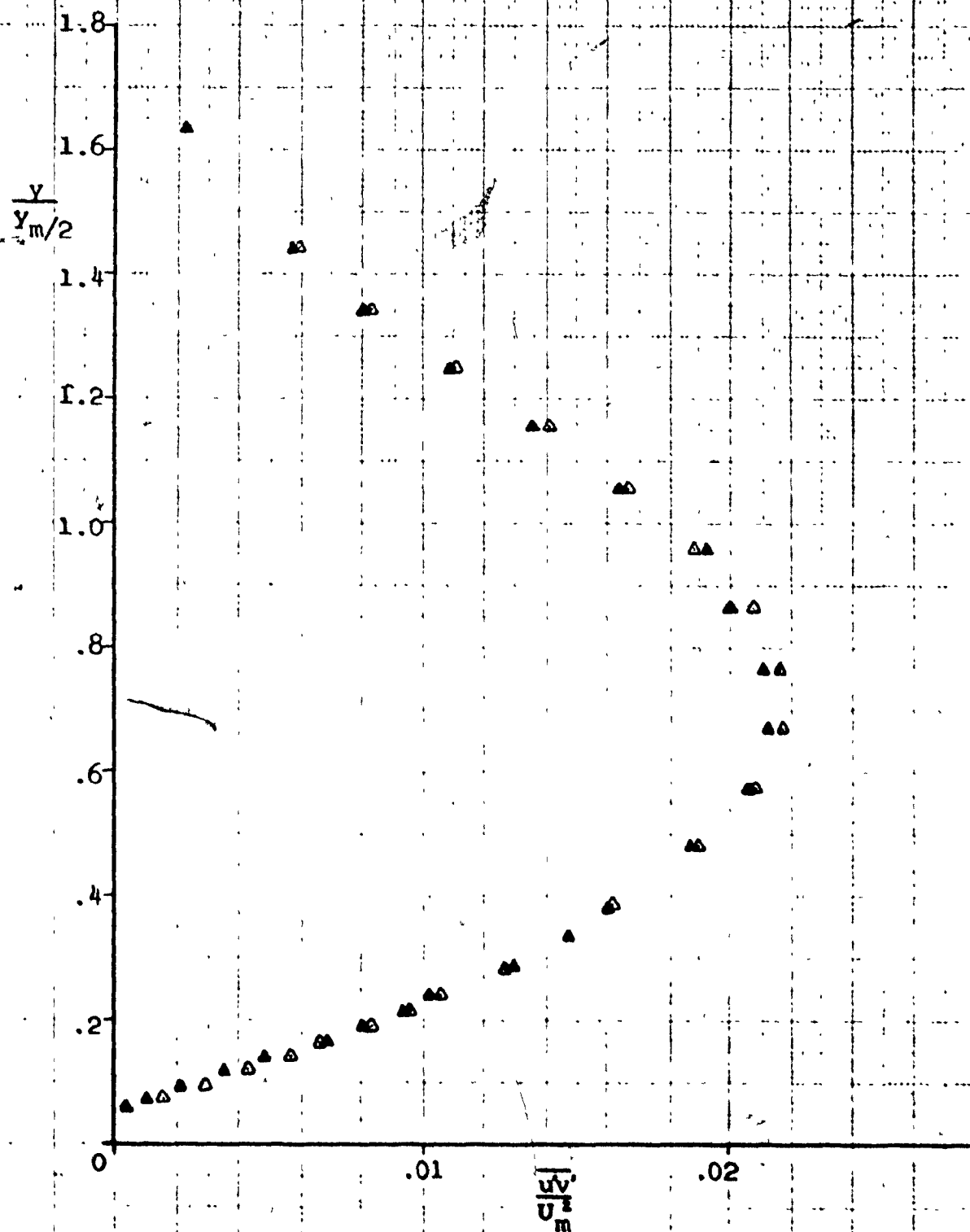


Fig. 9.24

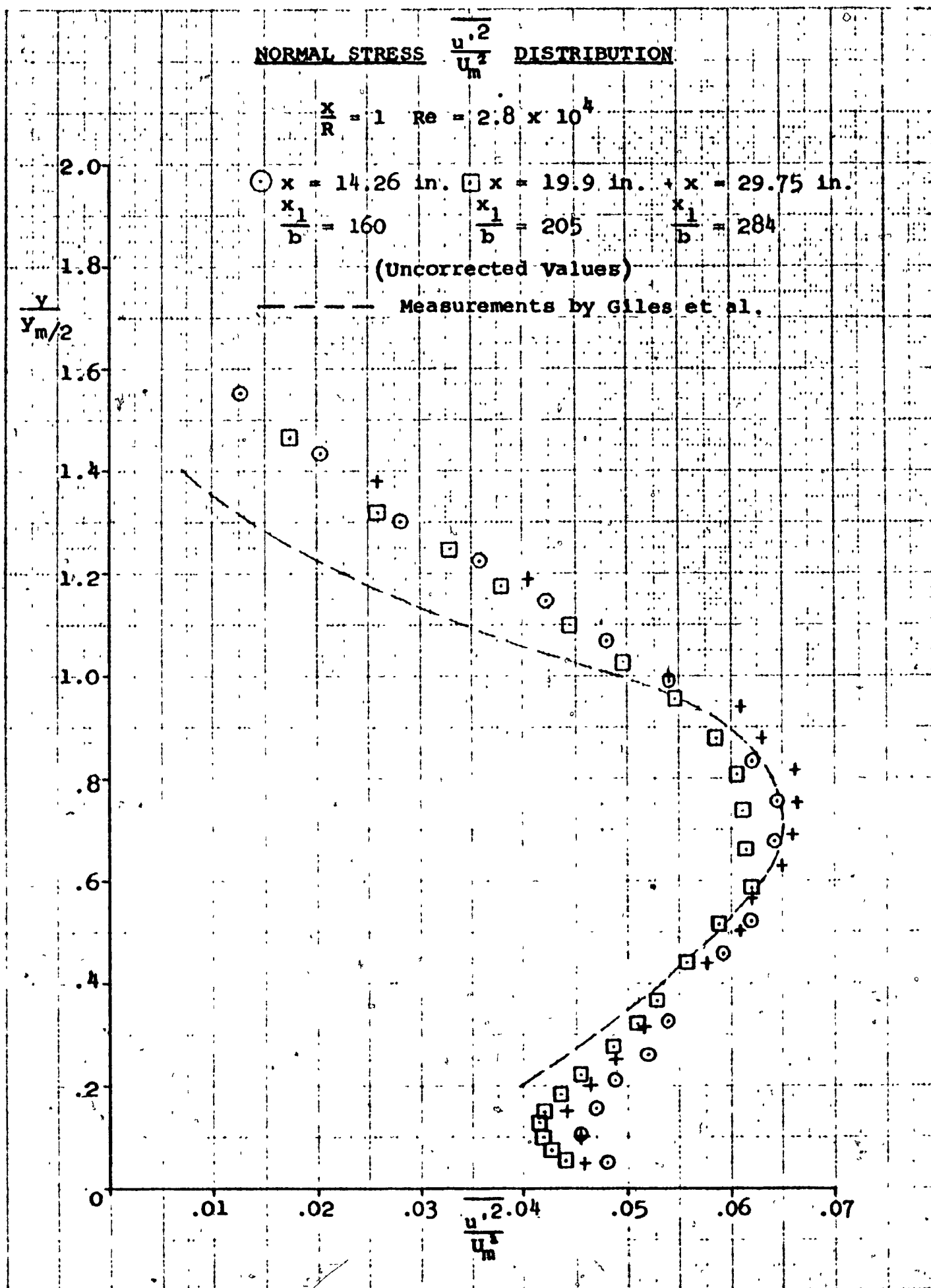
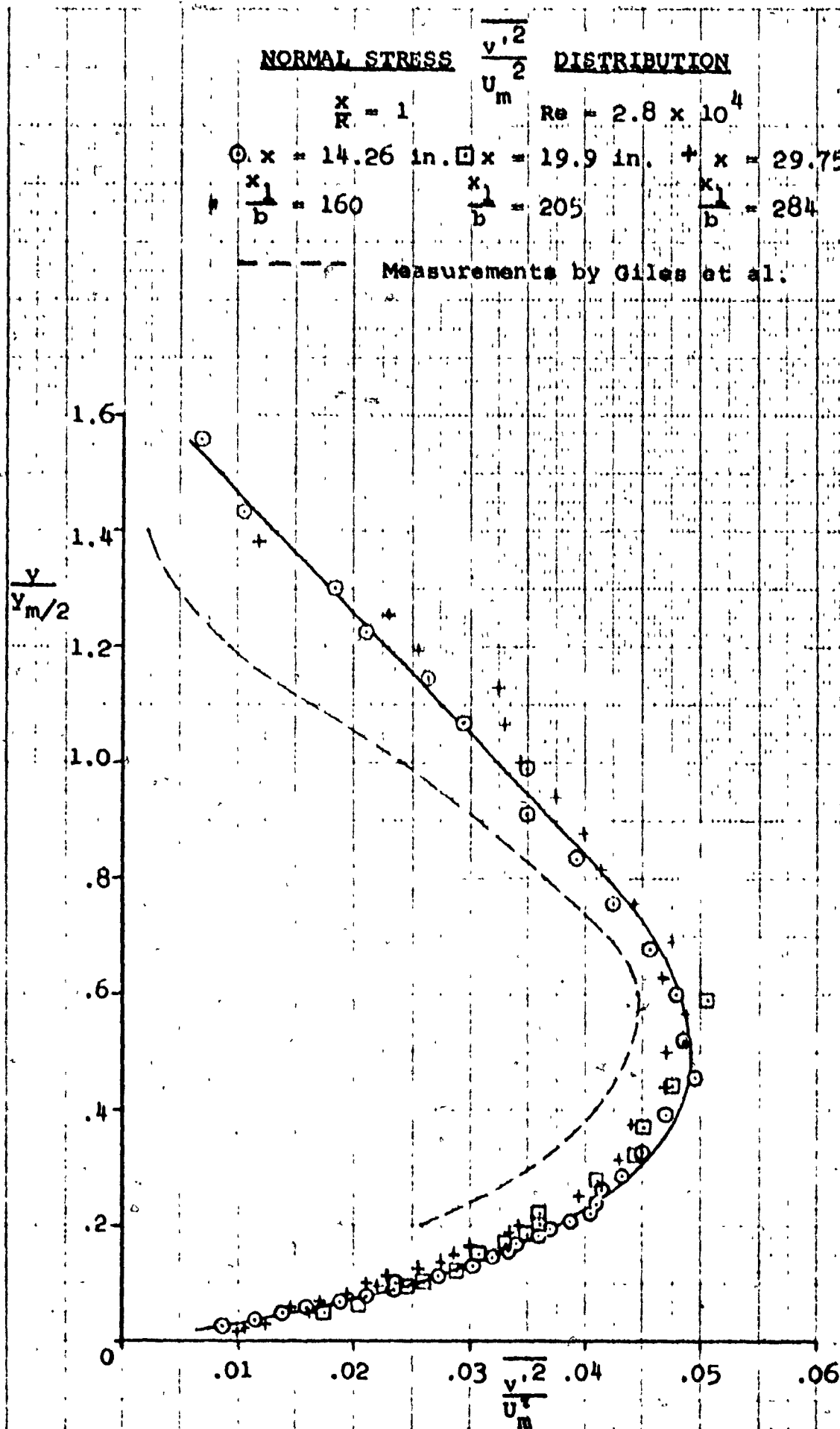


Fig. 9.25

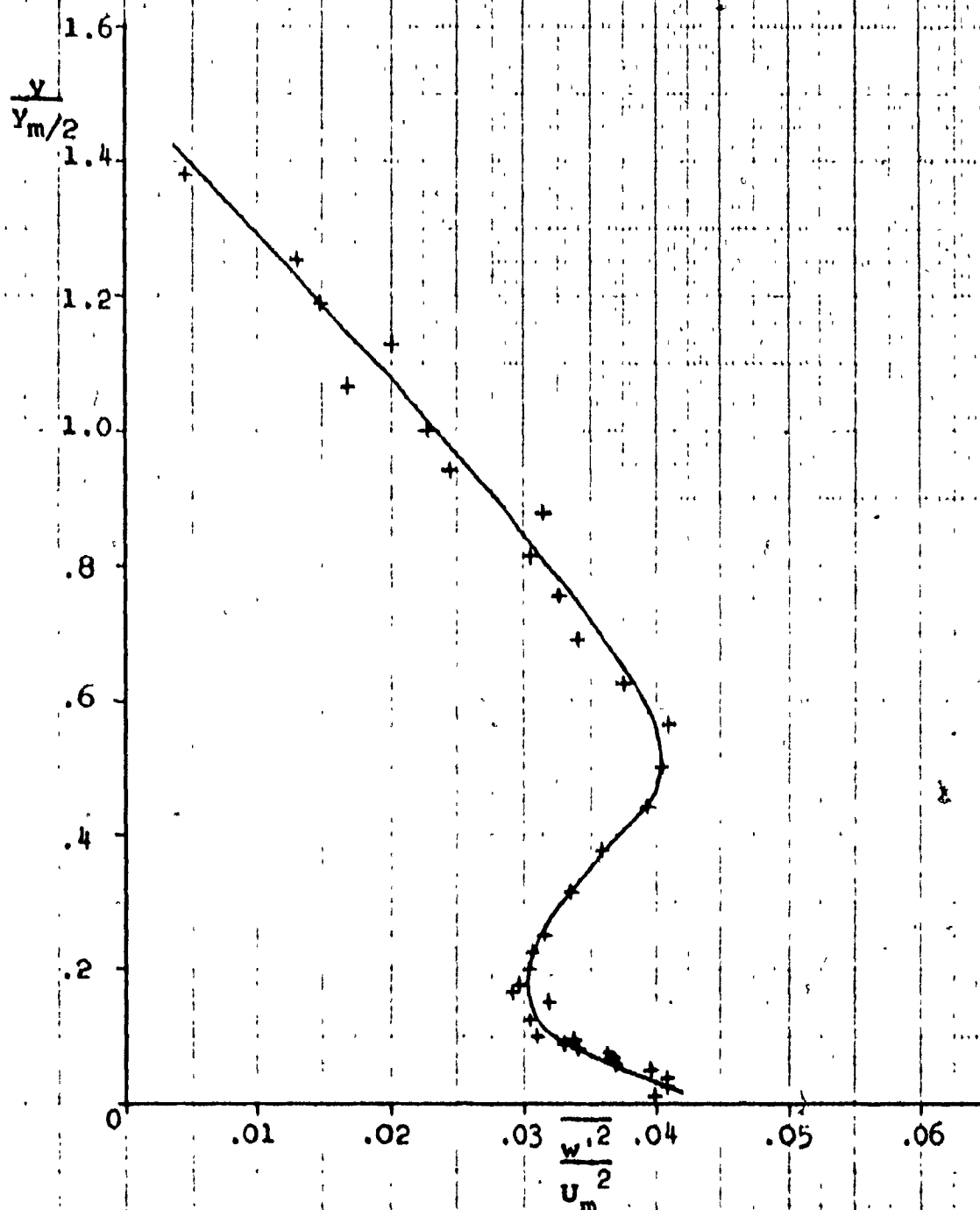


NORMAL STRESS $\frac{w^2}{u_m^2}$ DISTRIBUTION

$$\frac{x}{R} = 1 \quad Re = 2.8 \times 10^4$$

$$+ x = 29.75 \text{ in. } \frac{x_1}{b} = 284$$

(Uncorrected)



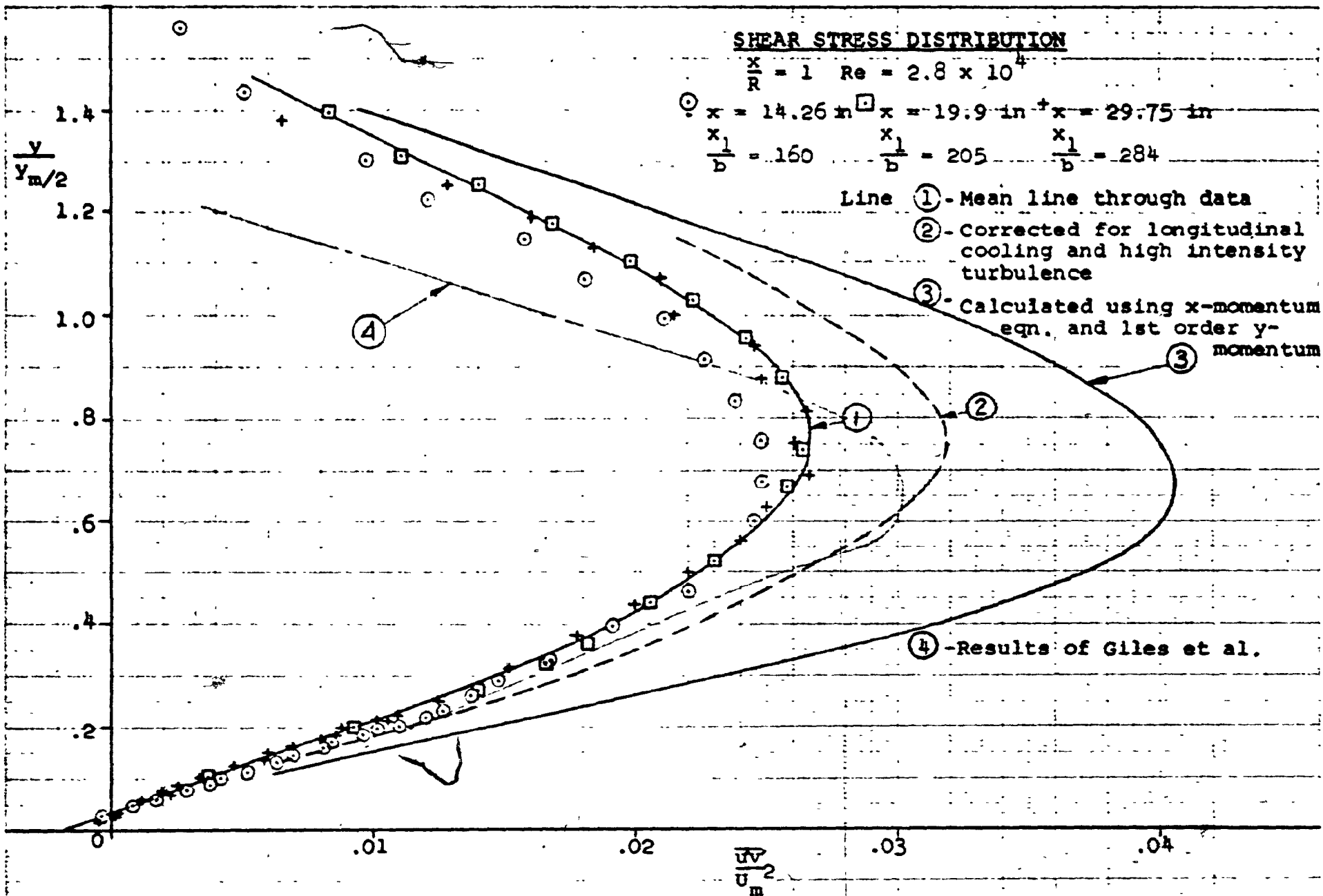
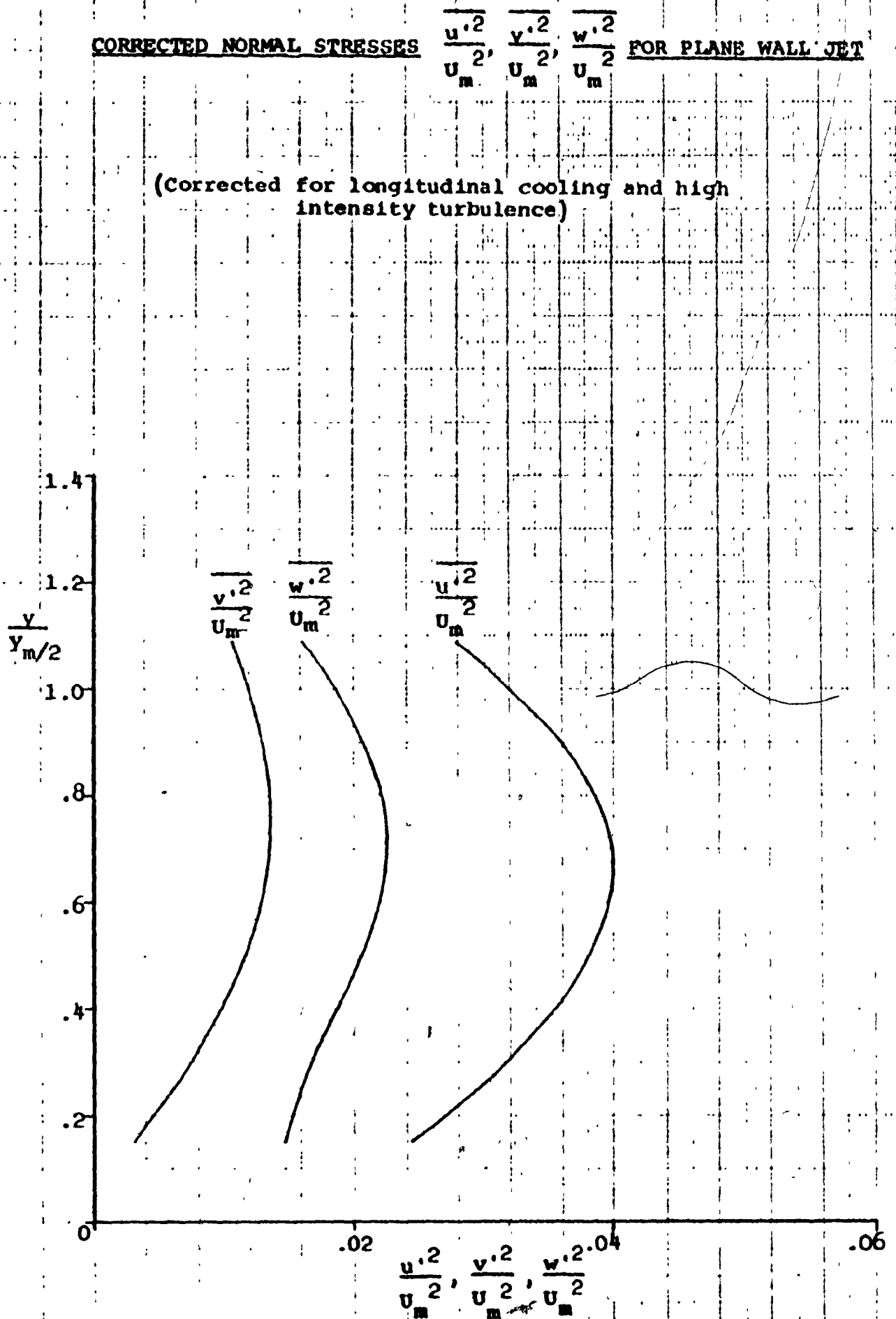


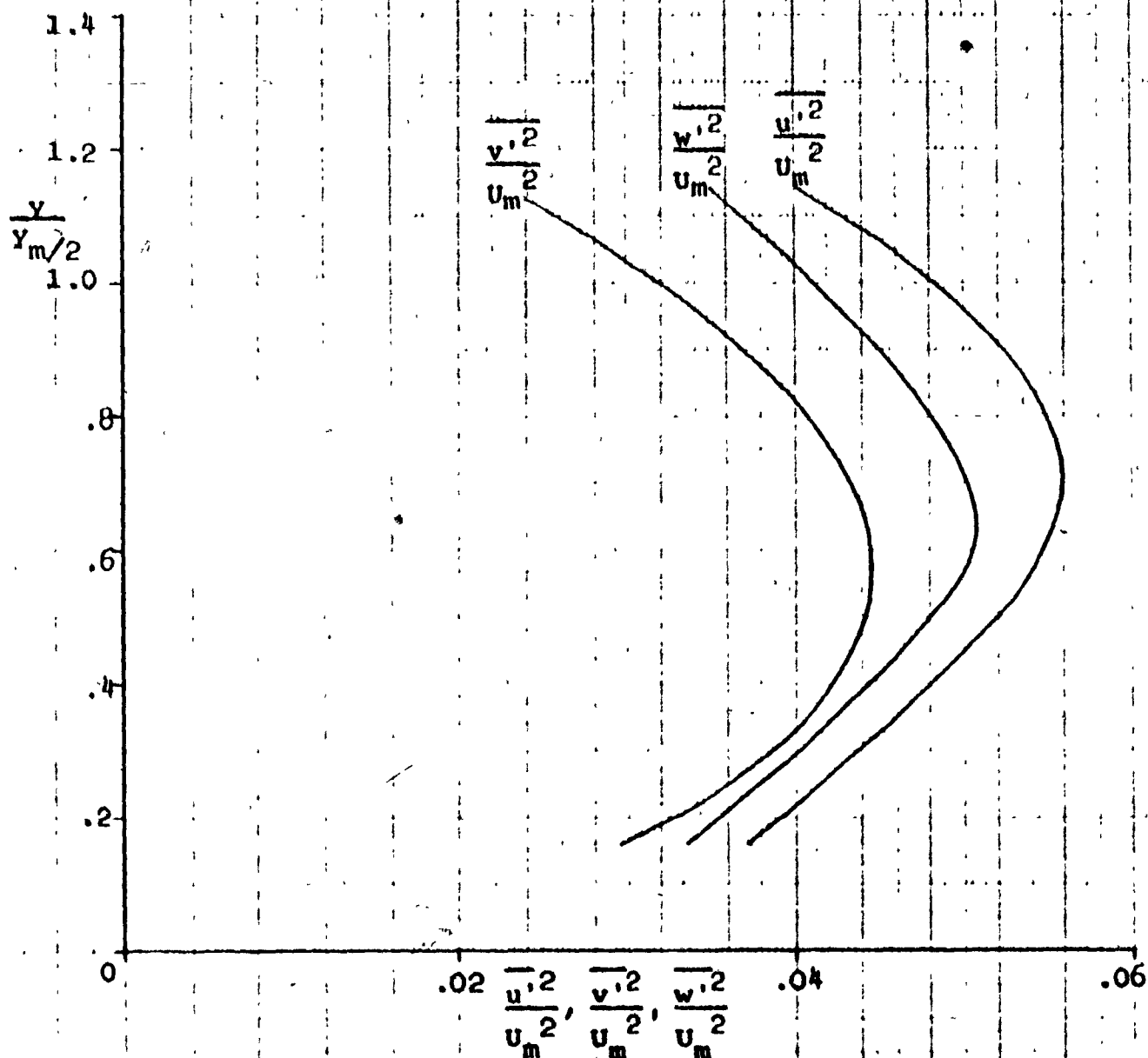
Fig. 9.27

Fig. 9.28



CORRECTED NORMAL STRESSES $\frac{\overline{u'^2}}{U_m^2}, \frac{\overline{v'^2}}{U_m^2}, \frac{\overline{w'^2}}{U_m^2}$ ON $\frac{x}{R} = \frac{2}{3}$ SPIRAL

(Corrected for longitudinal cooling
and high intensity turbulence)



CORRECTED NORMAL STRESSES $\frac{\overline{u'^2}}{U_m^2}, \frac{\overline{v'^2}}{U_m^2}, \frac{\overline{w'^2}}{U_m^2}$ ON $\frac{x}{R} = 1$ SPIRAL

(Corrected for longitudinal cooling and high intensity turbulence)

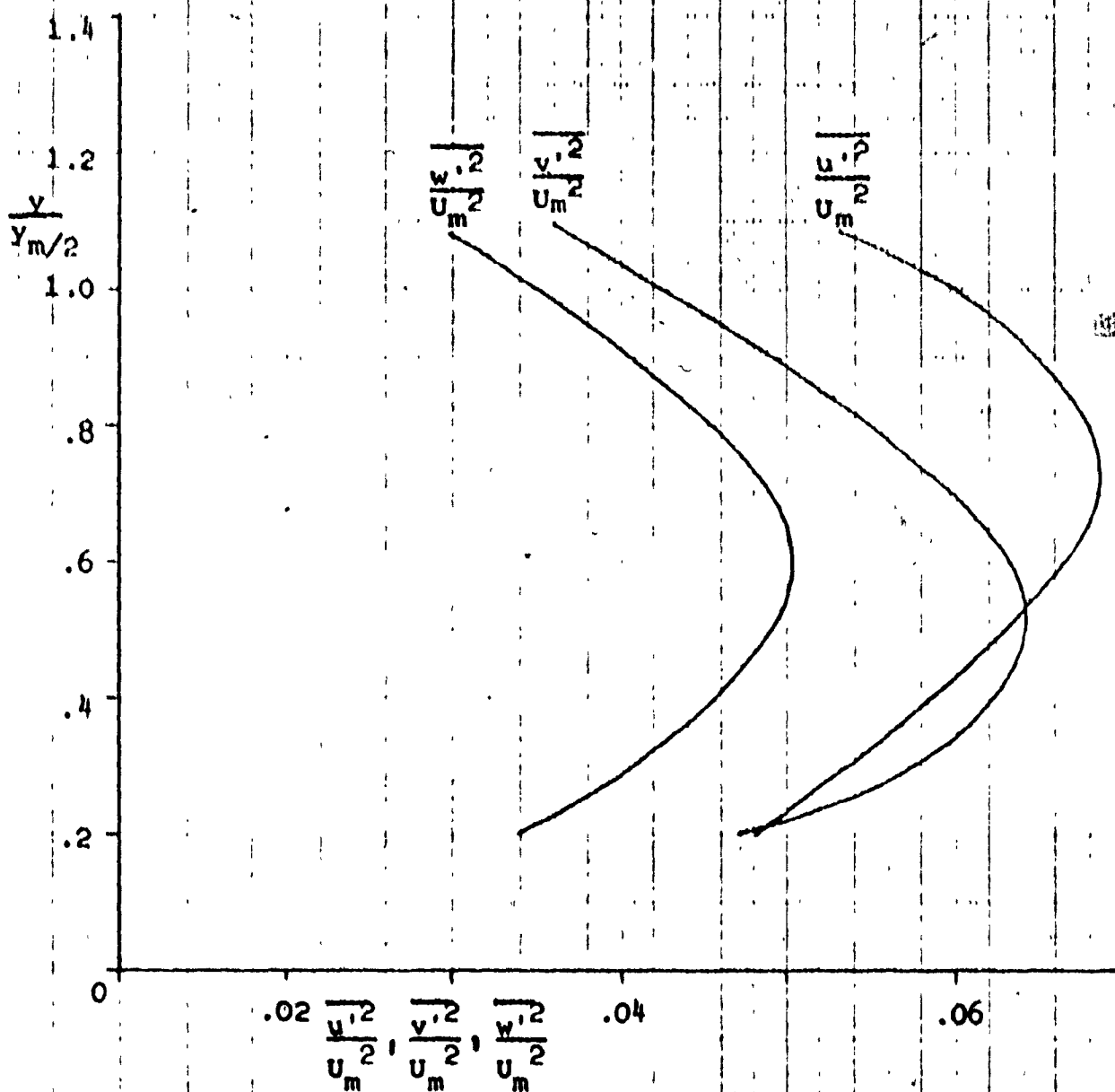
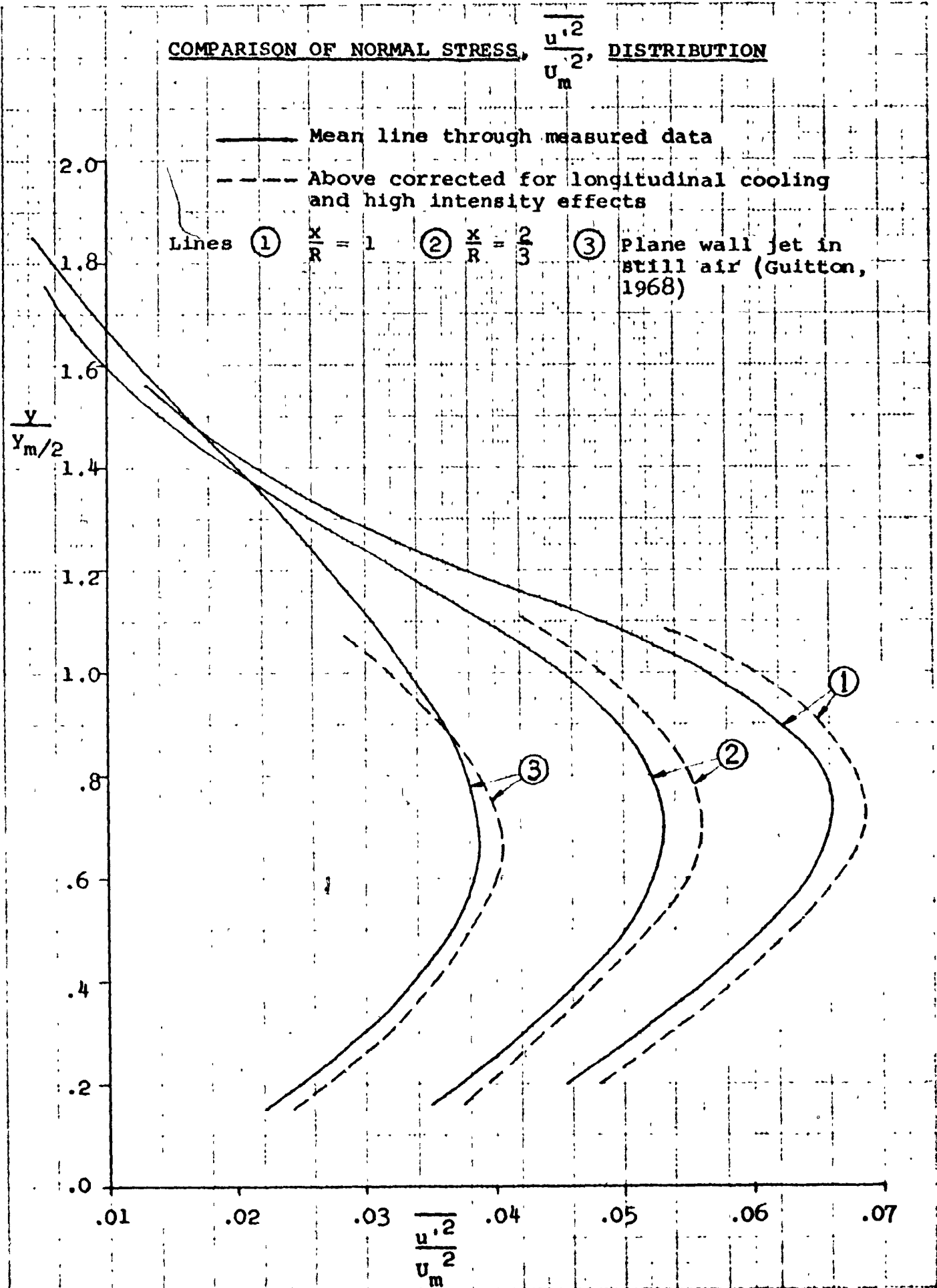
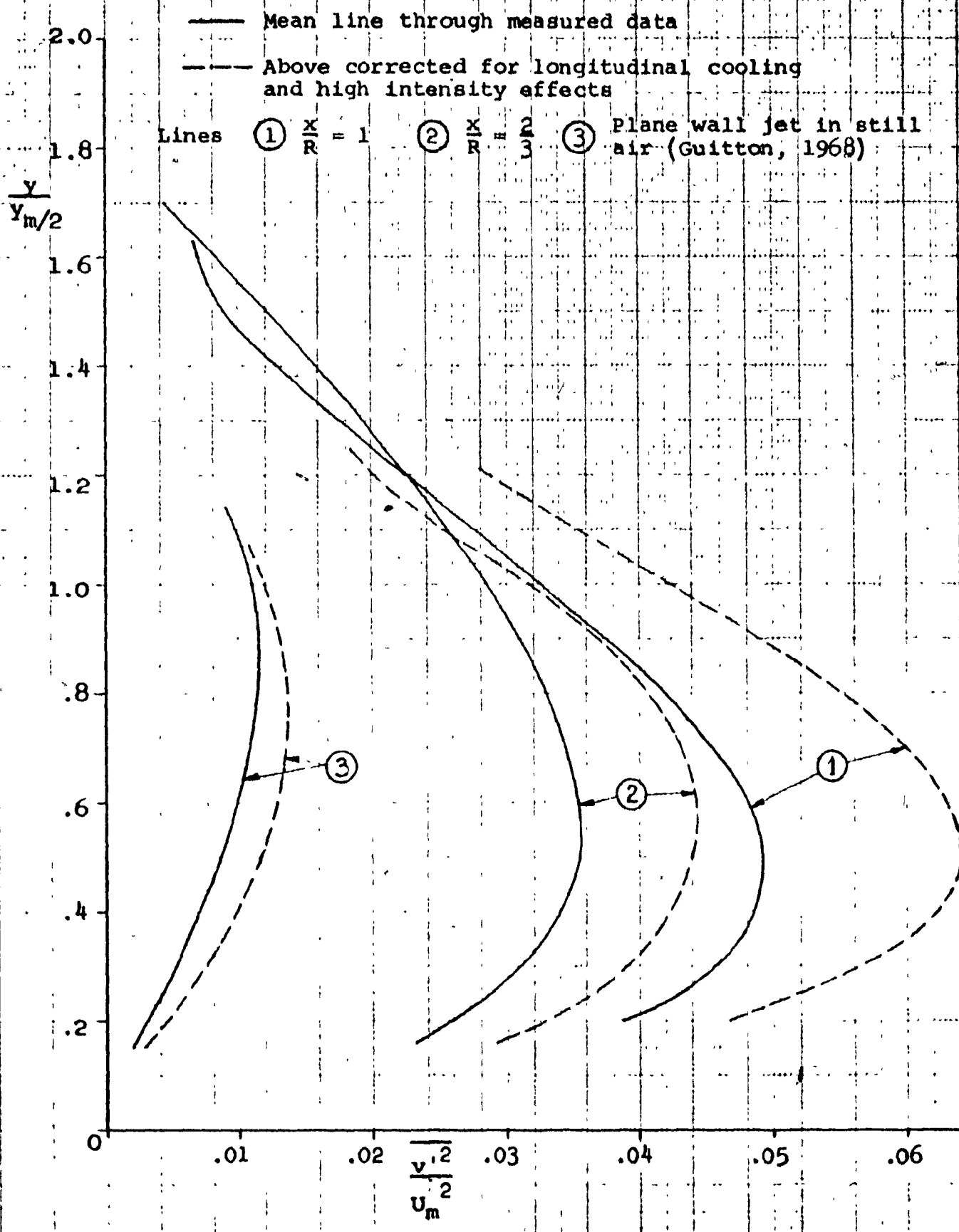


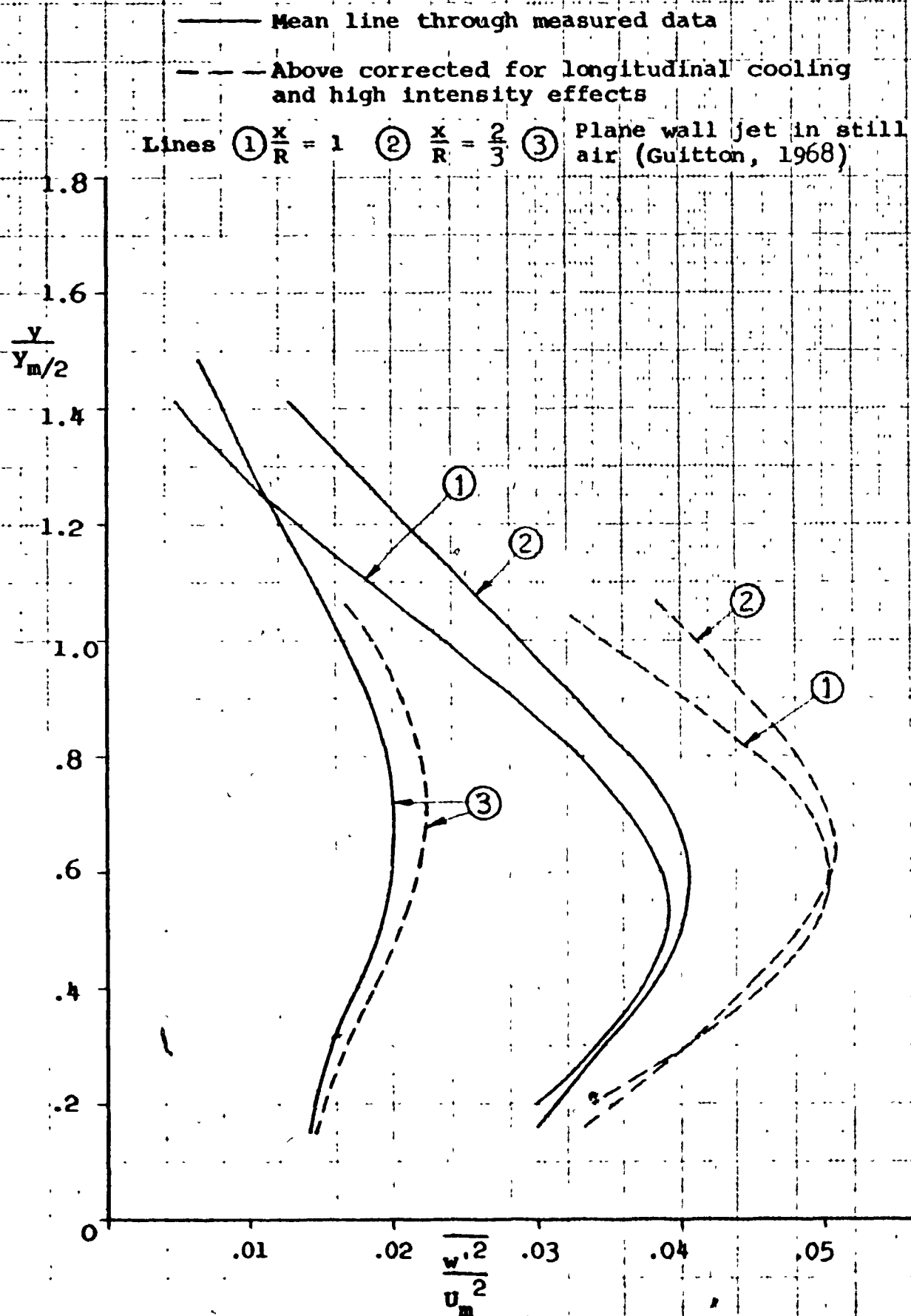
Fig. 9.31



COMPARISON OF NORMAL STRESS, $\frac{v \cdot 2}{u_m^2}$, DISTRIBUTIONS



COMPARISON OF NORMAL STRESS, $\frac{w^2}{u_m^2}$, DISTRIBUTION

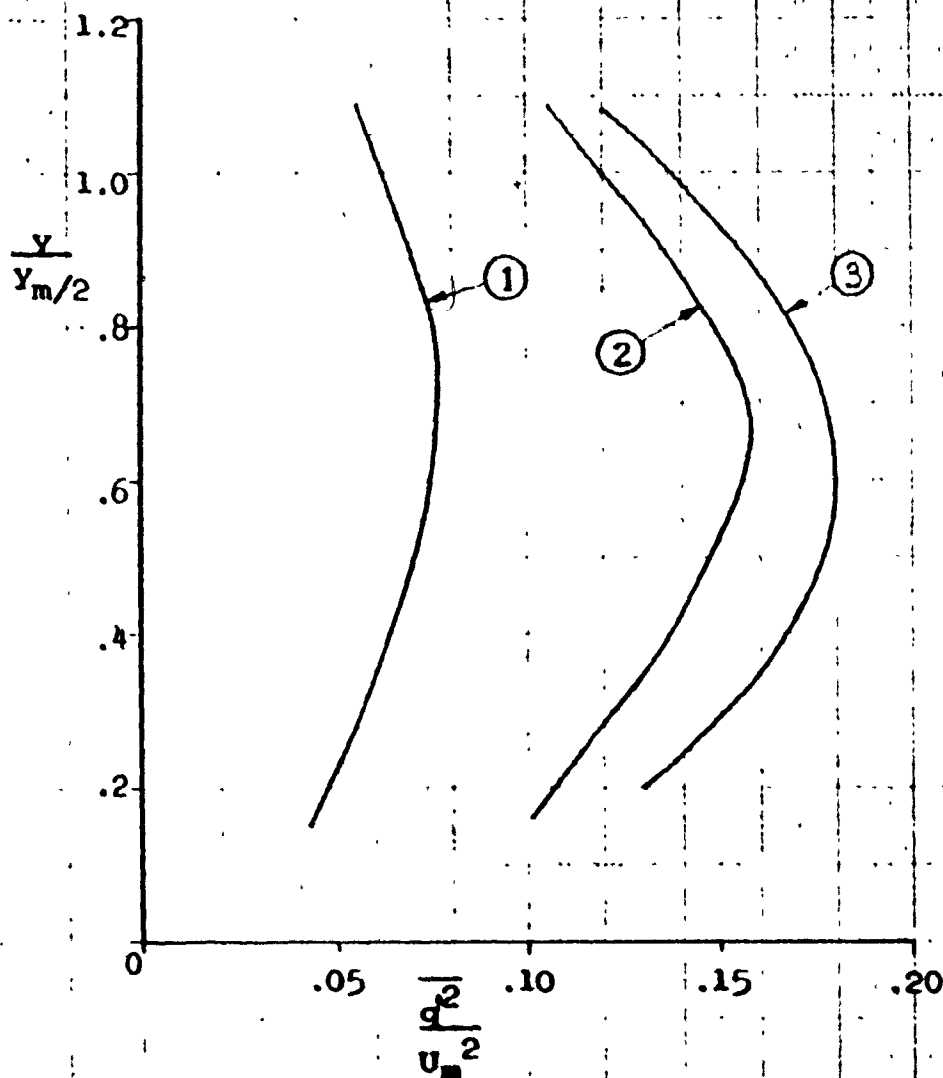


TURBULENT ENERGY, $\frac{q^2}{2}$, DISTRIBUTION BASED ON
CORRECTED REYNOLDS STRESSES

Lines ① Plane wall jet in still
air (Guitton, 1968)

② $\frac{x}{R} = \frac{2}{3}$

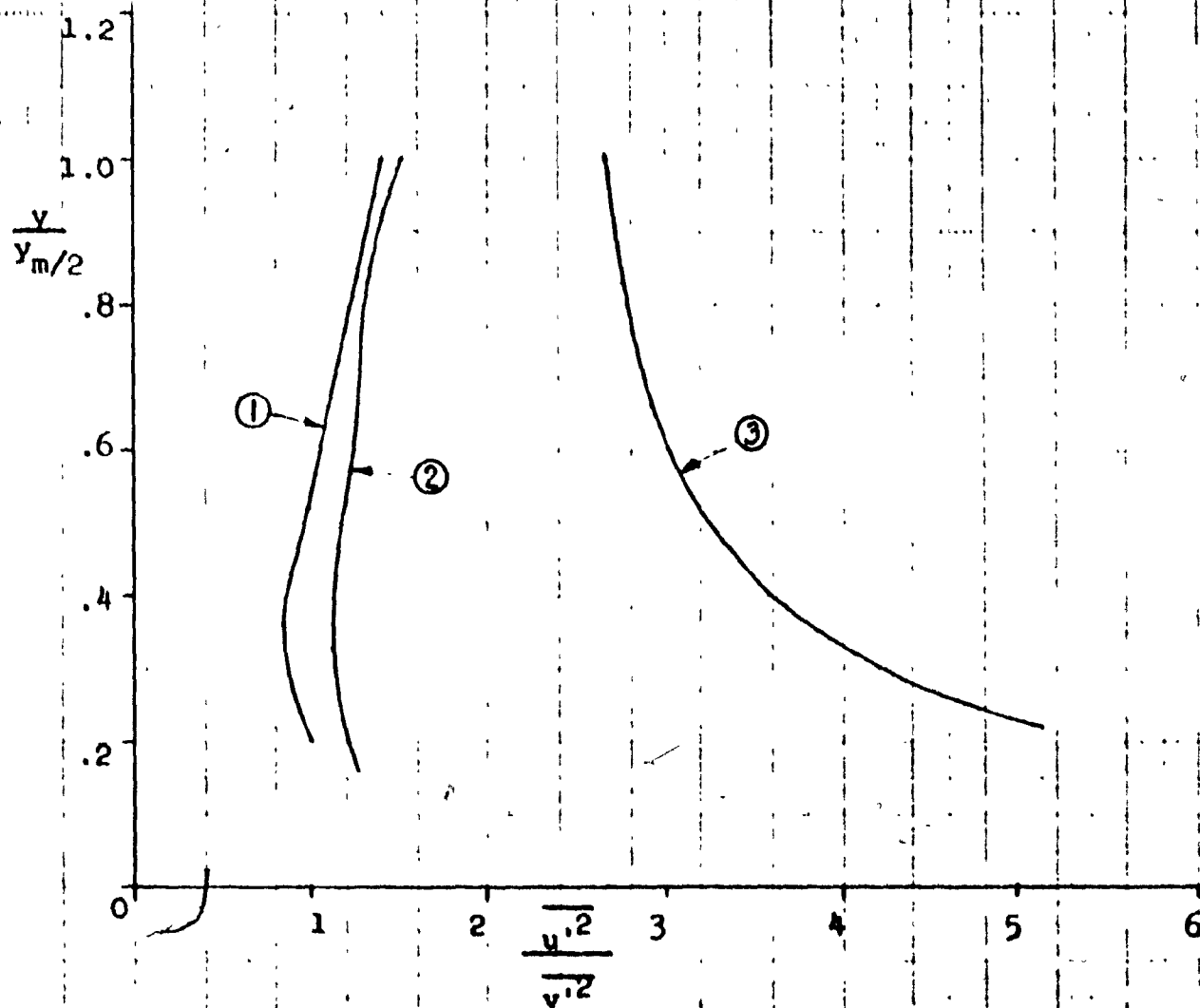
③ $\frac{x}{R} = 1$



VARIATIONS OF $\frac{u^2}{v^2}$ ACROSS JETS

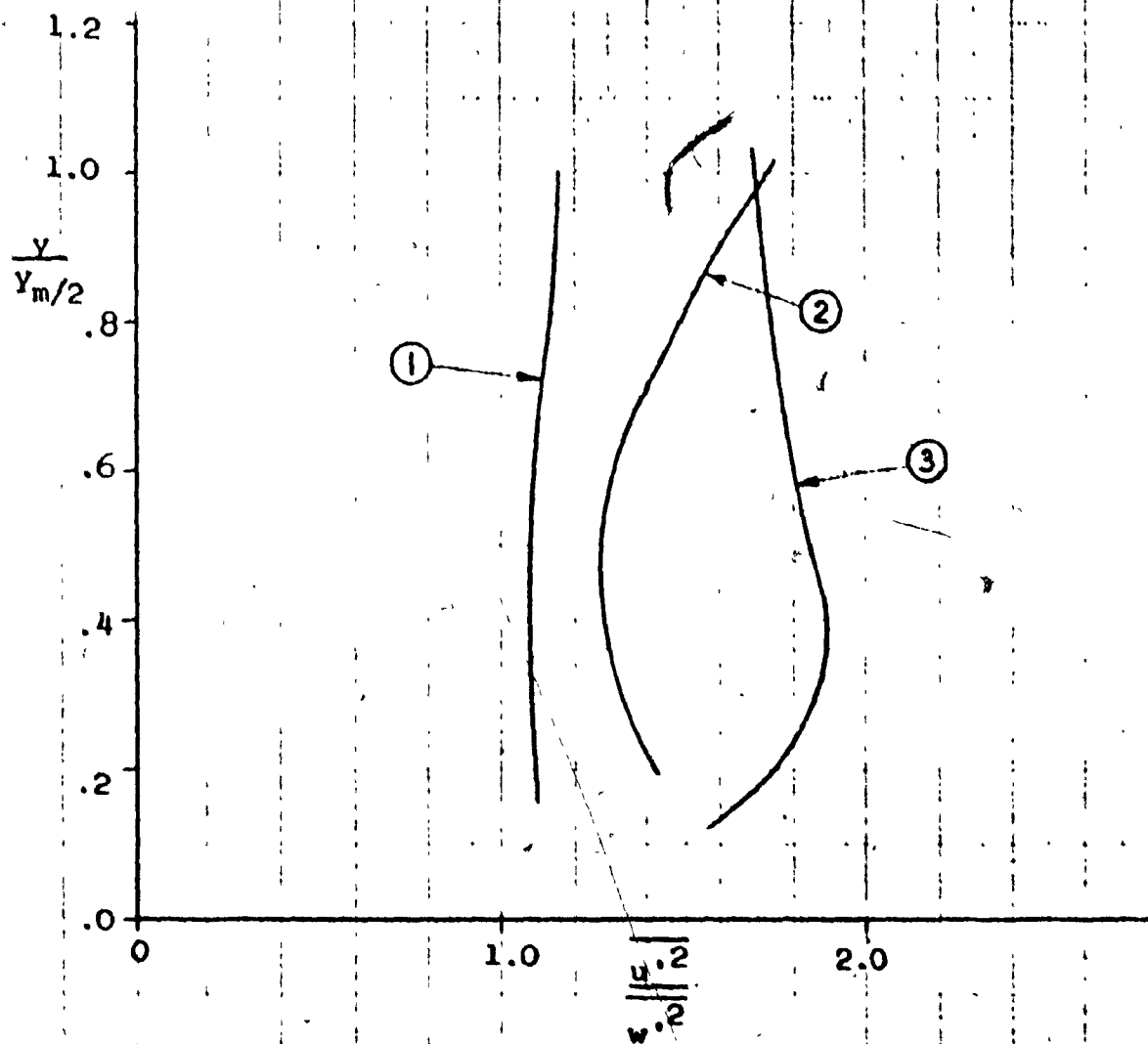
(Corrected Reynolds stresses used)

Lines ① $\frac{x}{R} = 1$ ② $\frac{x}{R} = \frac{2}{3}$ ③ Plane wall jet in still air (Guitton, 1968)



VARIATION OF $\frac{u^2}{w^2}$ (USING CORRECTED VALUES)

Lines ① $\frac{x}{R} = \frac{2}{3}$ ② $\frac{x}{R} = 1$ ③ plane wall jet in still air (Guitton)



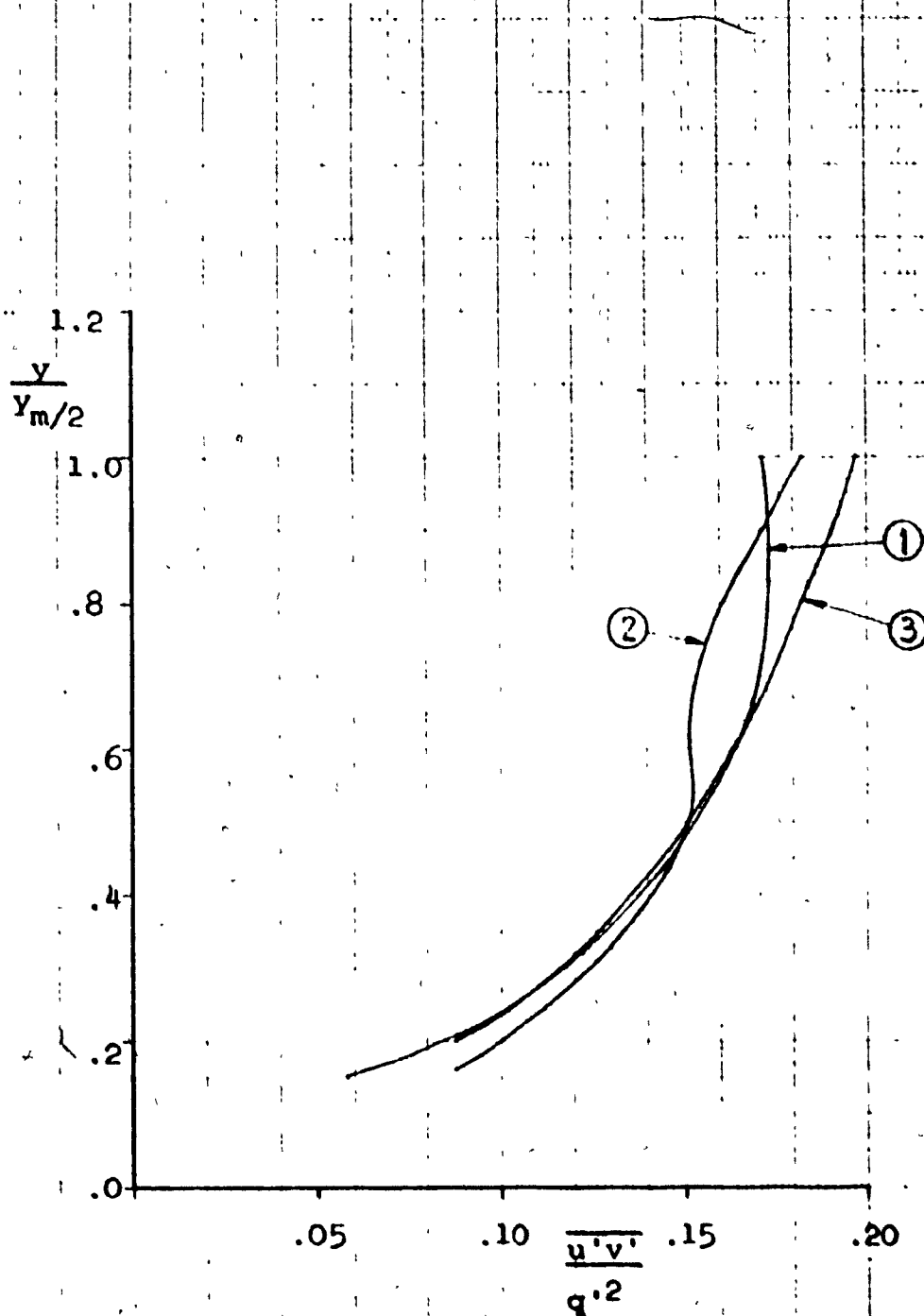
STRUCTURE PARAMETER, $\frac{\overline{u'v'}}{q'^2}$, DISTRIBUTION

Lines ① Plane wall jet in still air

② $\frac{x}{R} = \frac{2}{3}$

③ $\frac{x}{R} = 1$

(USING CORRECTED MEASURED VALUES)



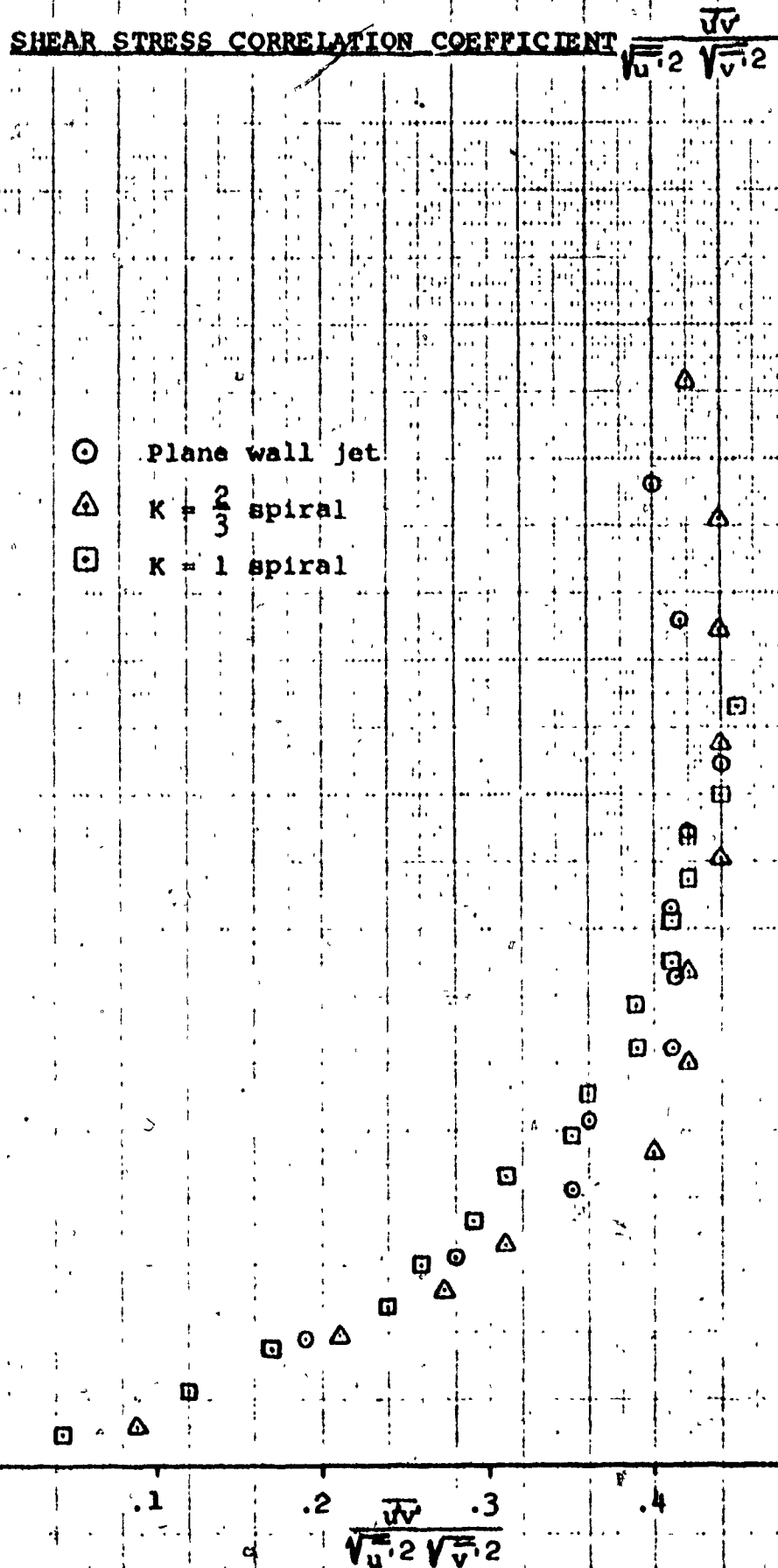
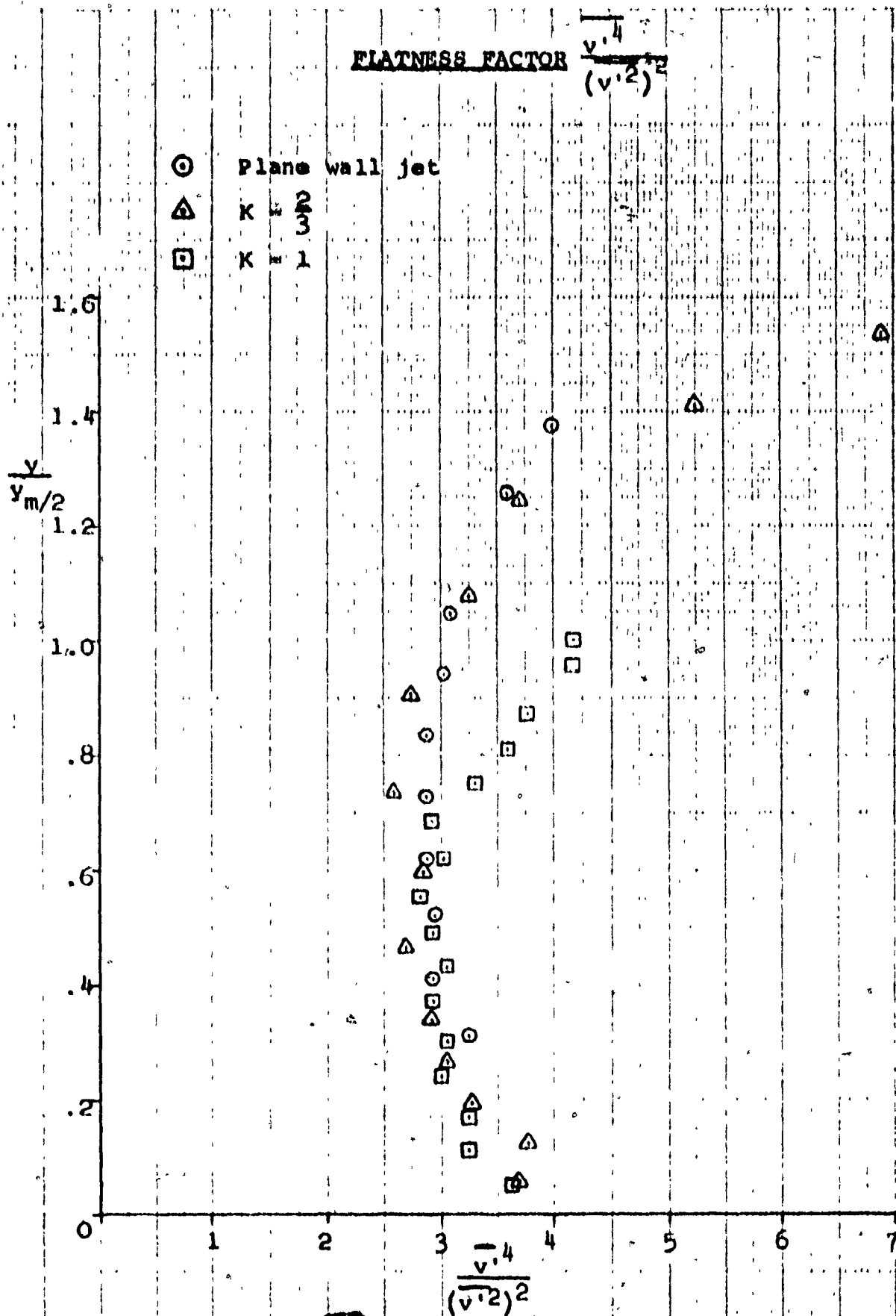


Fig. 9.39

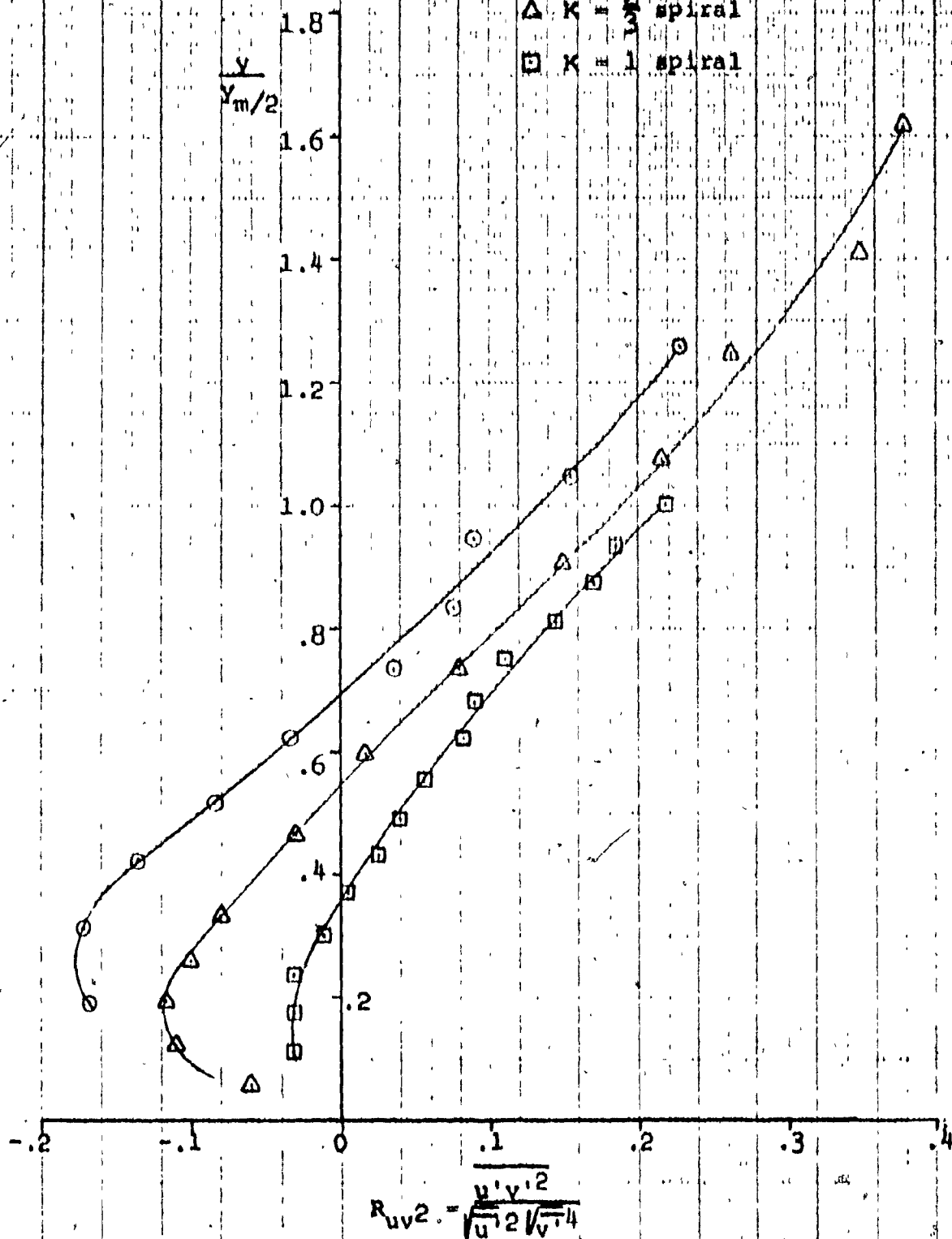


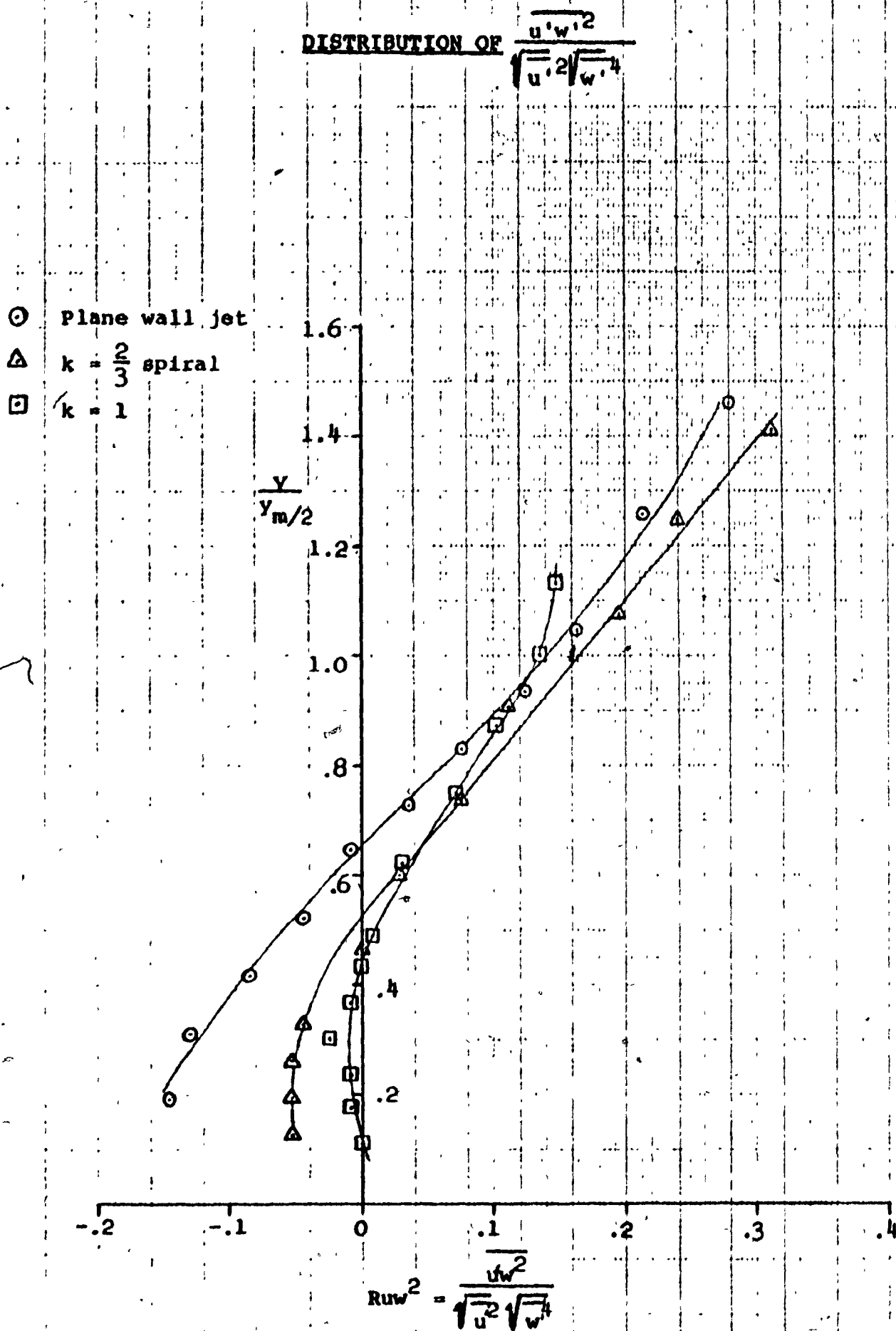
DISTRIBUTION OF $\frac{u'v'^2}{\sqrt{u'^2}\sqrt{v'^4}}$

○ plane wall jet

△ $\kappa = 1.5$ spiral

□ $\kappa = 1$ spiral

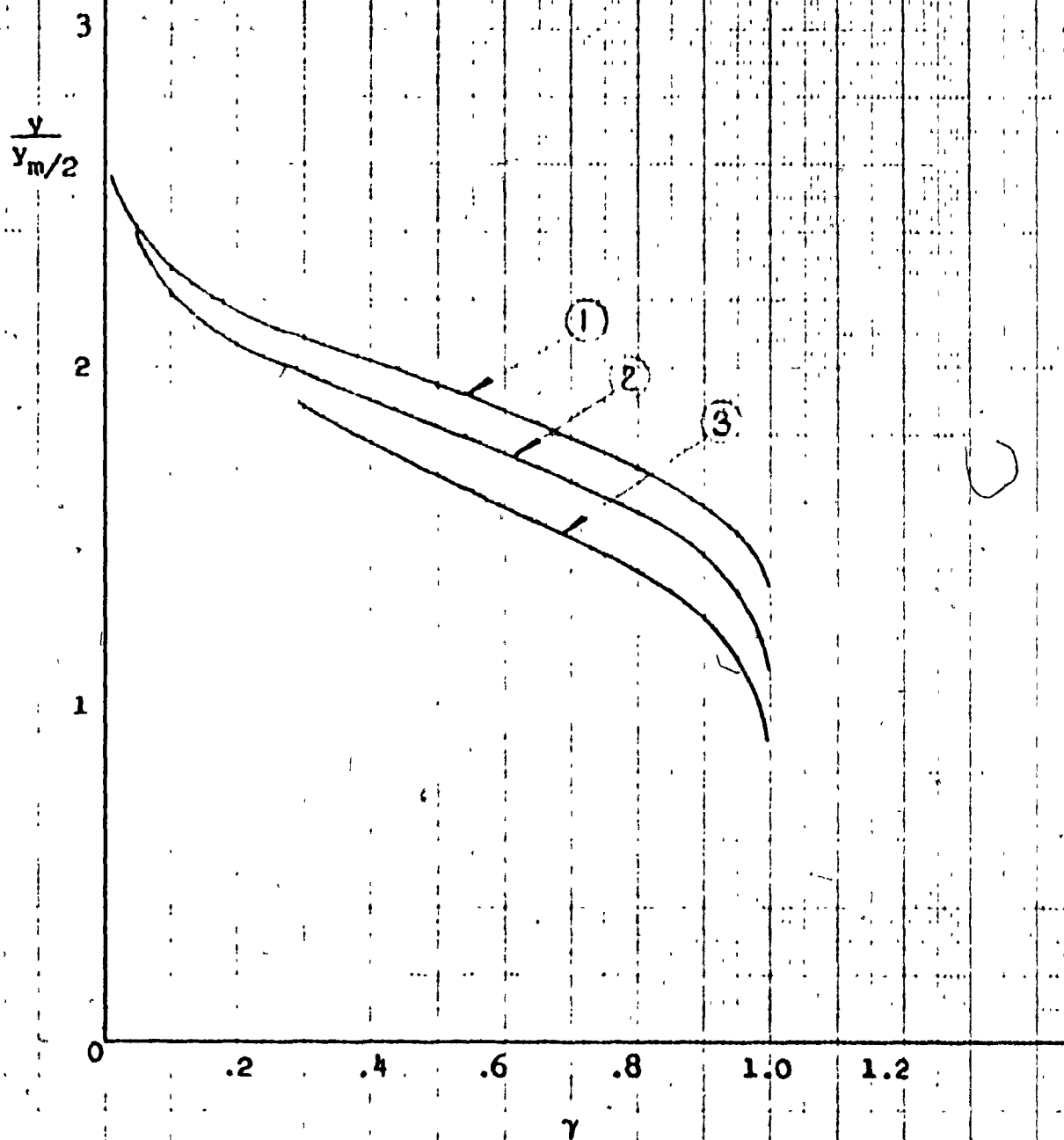




COMPARISON OF VARIOUS INTERMITTENCY MEASUREMENTS IN THE PLANE WALL

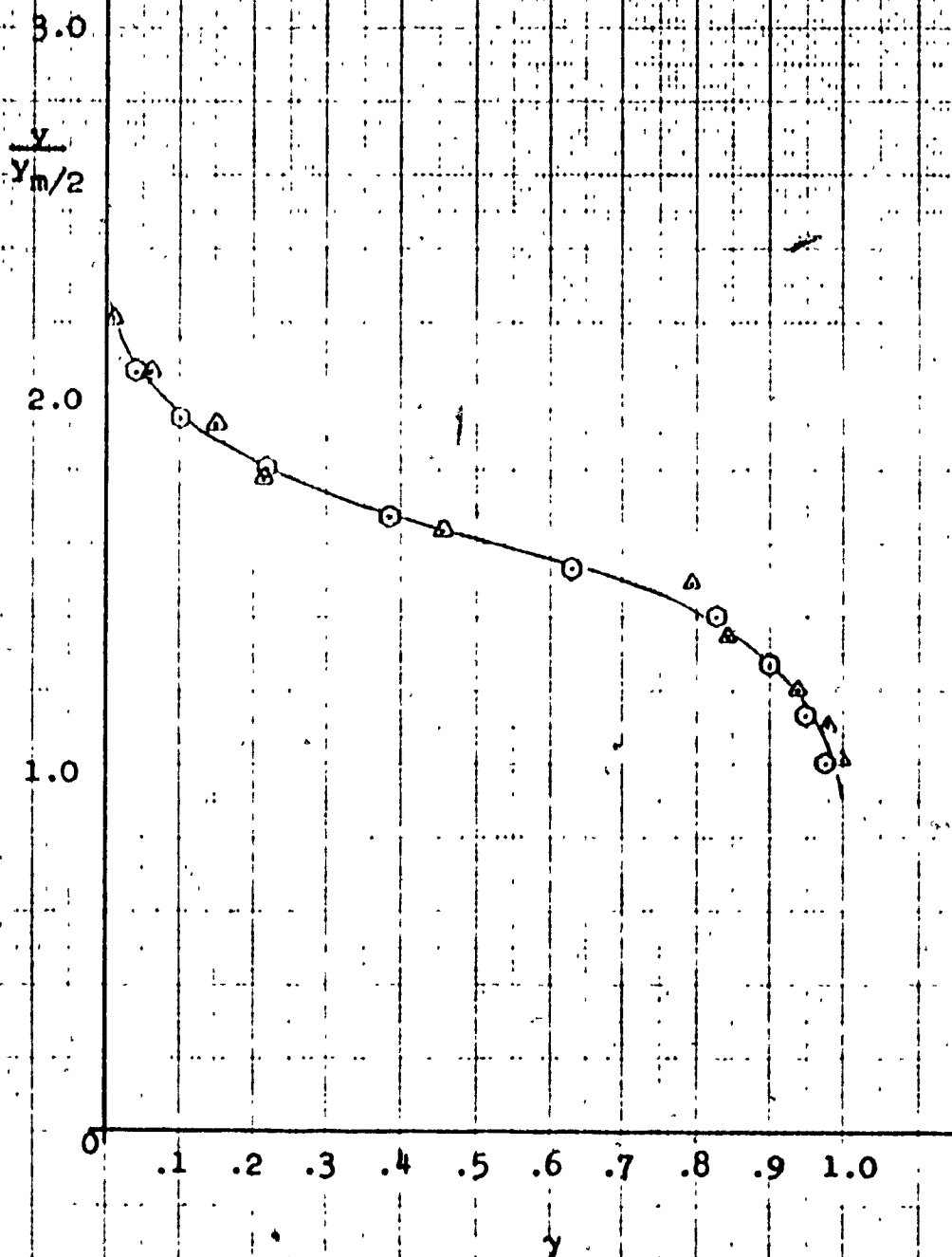
JET

Lines ① Gartshore (1966) ② Present Measurements ③ Giles et al. (1966)



INTERMITTENCY ON $\frac{x}{R} = \frac{2}{3}$ SPIRAL

Δ $x = 13.3$ inches $\frac{x_1}{b} = 116$
 $\circ$ $x = 19.6$ inches $\frac{x_1}{b} = 166$

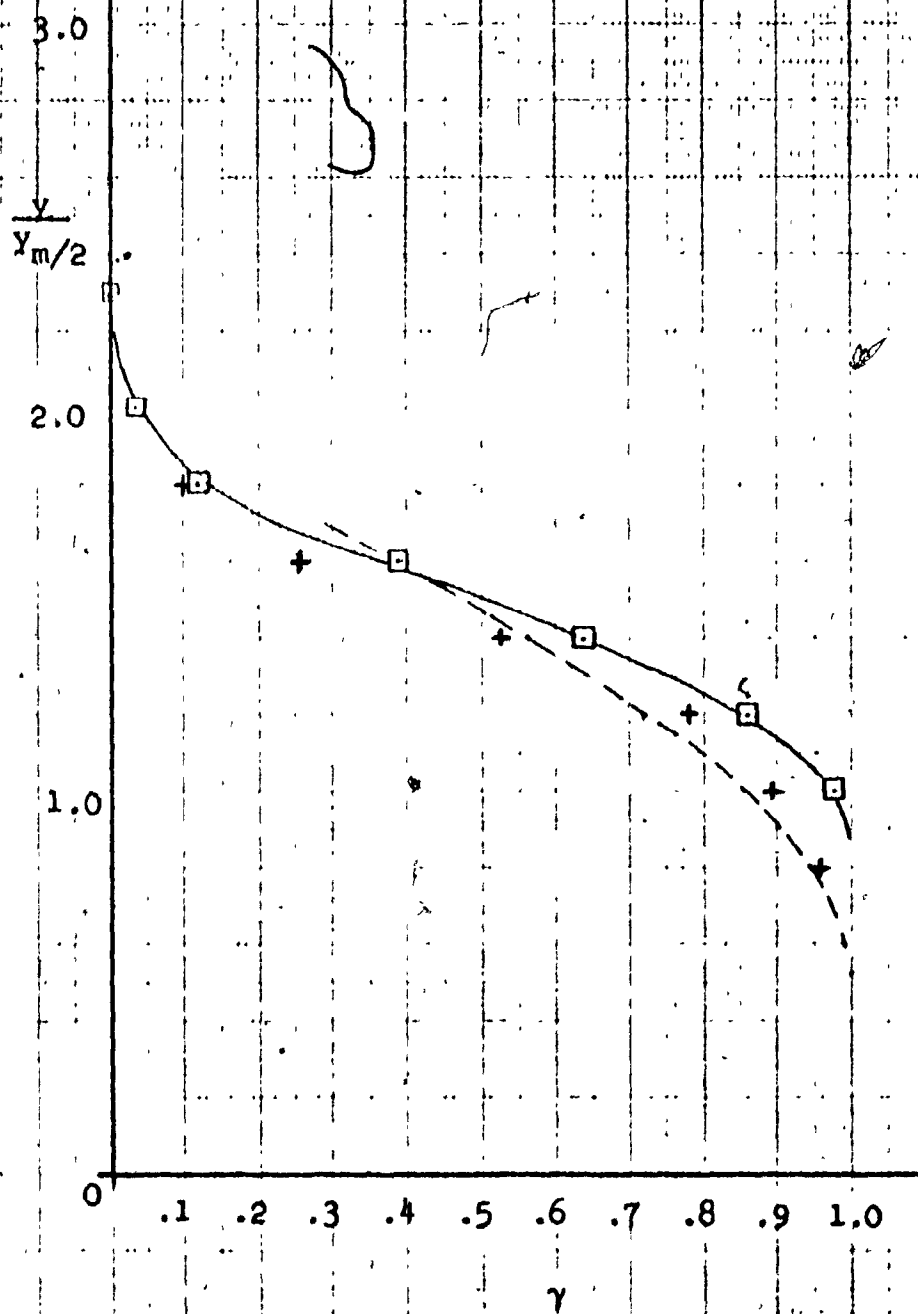


INTERMITTENCY ON THE $\frac{x}{R} = 1$ SPIRAL

□ $x = 19.9$ inches $\frac{x_1}{b} = 205$

+ $x = 29.75$ inches $\frac{x_1}{b} = 284$

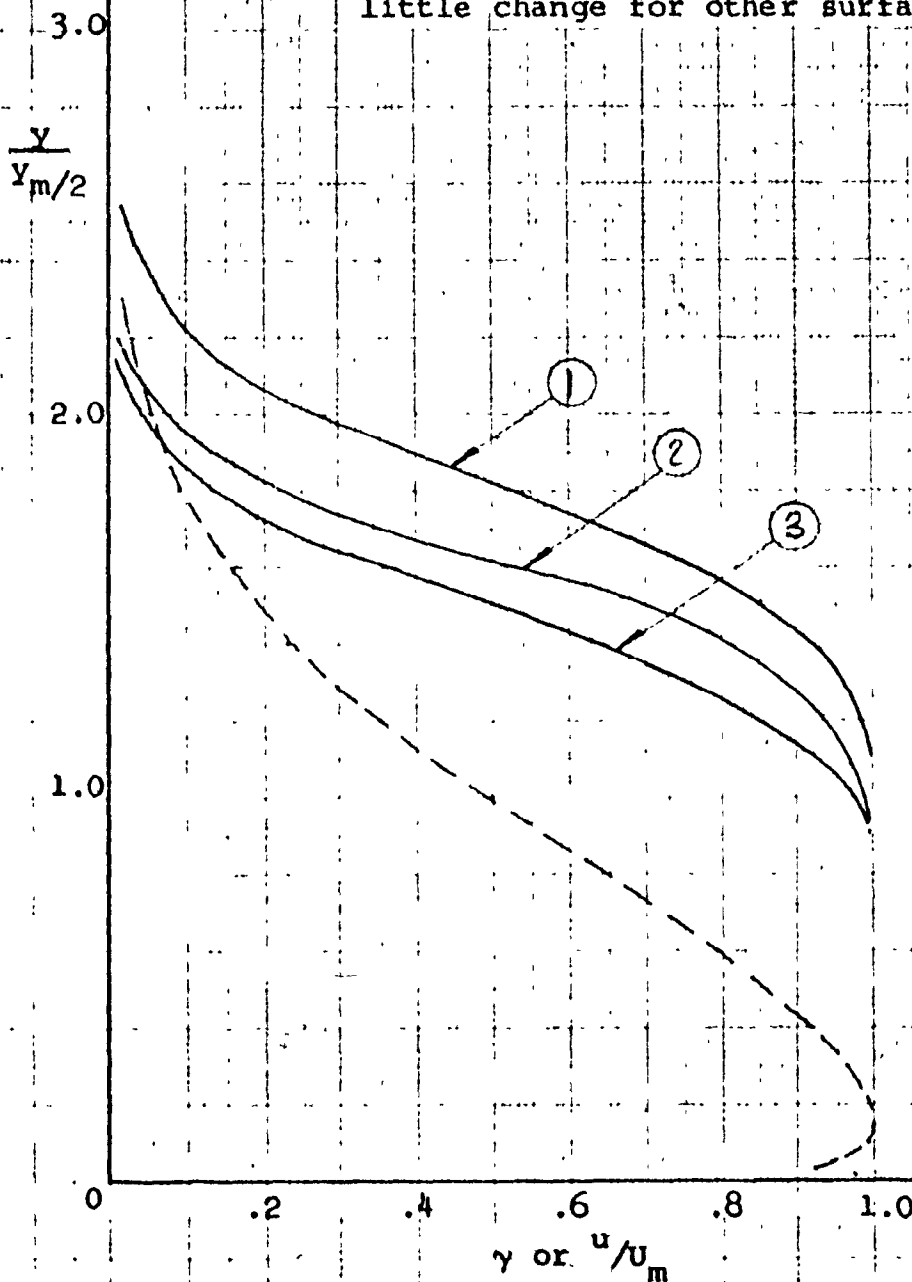
----- measurements of Giles et al. (1966)



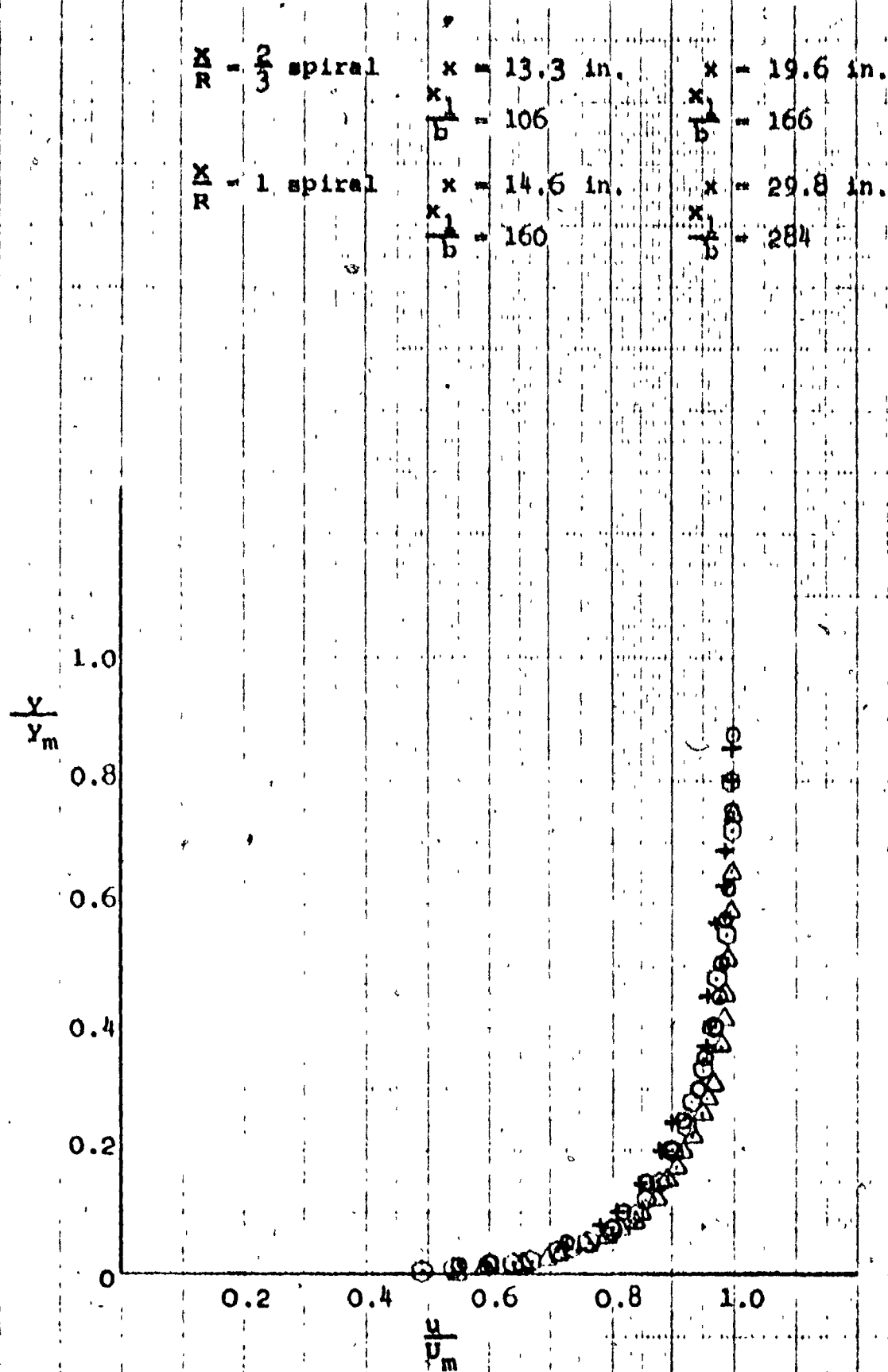
VARIATION OF INTERMITTENCY DISTRIBUTIONS WITH SURFACE
CURVATURE (MEAN LINE THROUGH DATA OF FIGURES 9.43 and 9.44)

Lines ①	Plane wall jet	② $\frac{x}{R} = \frac{2}{3}$ spiral	③ $\frac{x}{R} = 1$ spiral
	$\frac{\sigma}{y_{m/2}} = .302$	$\frac{\sigma}{y_{m/2}} = .257$	$\frac{\sigma}{y_{m/2}} = .278$
	$\frac{y_e}{y_{m/2}} = 1.82$	$\frac{y_e}{y_{m/2}} = 1.62$	$\frac{y_e}{y_{m/2}} = 1.50$

--- mean velocity profile for $\frac{x}{R} = \frac{2}{3}$ spiral but
 little change for other surfaces



MEAN VELOCITY DISTRIBUTION IN INNER LAYER



DISTRIBUTION OF REYNOLDS STRESSES $\overline{u'^2}$ AND $\overline{v'^2}$
IN INNER LAYER

$\frac{x}{R} = \frac{2}{3}$ spiral Δ $x = 13.3$ inch $\odot$ $x = 19.6$ inches

$$\frac{x_1}{B} = 106$$

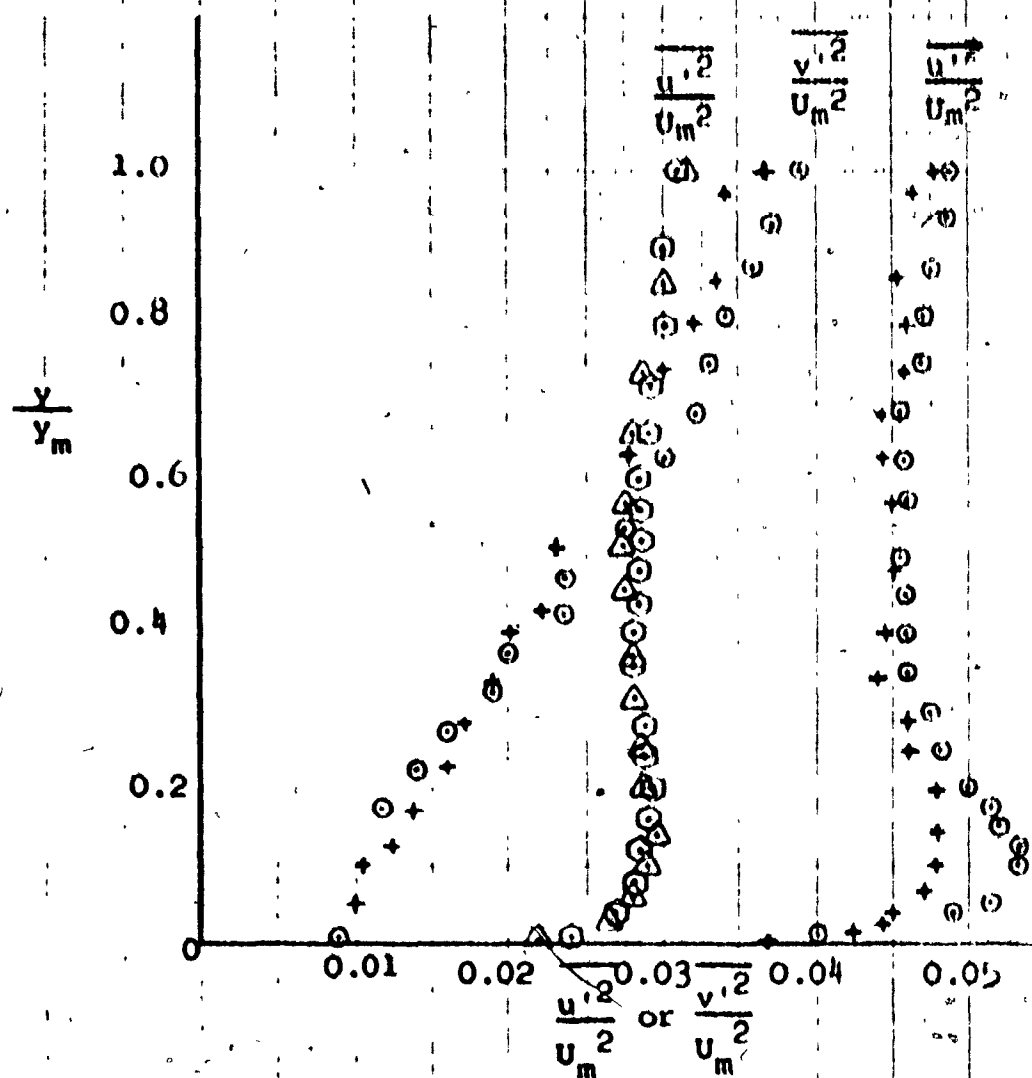
$$\frac{x_1}{B} = 166$$

$\frac{x}{R} = 1$ spiral $\odot$ $x = 14.6$ inches $+$ $x = 29.8$ inches

$$\frac{x_1}{B} = 160$$

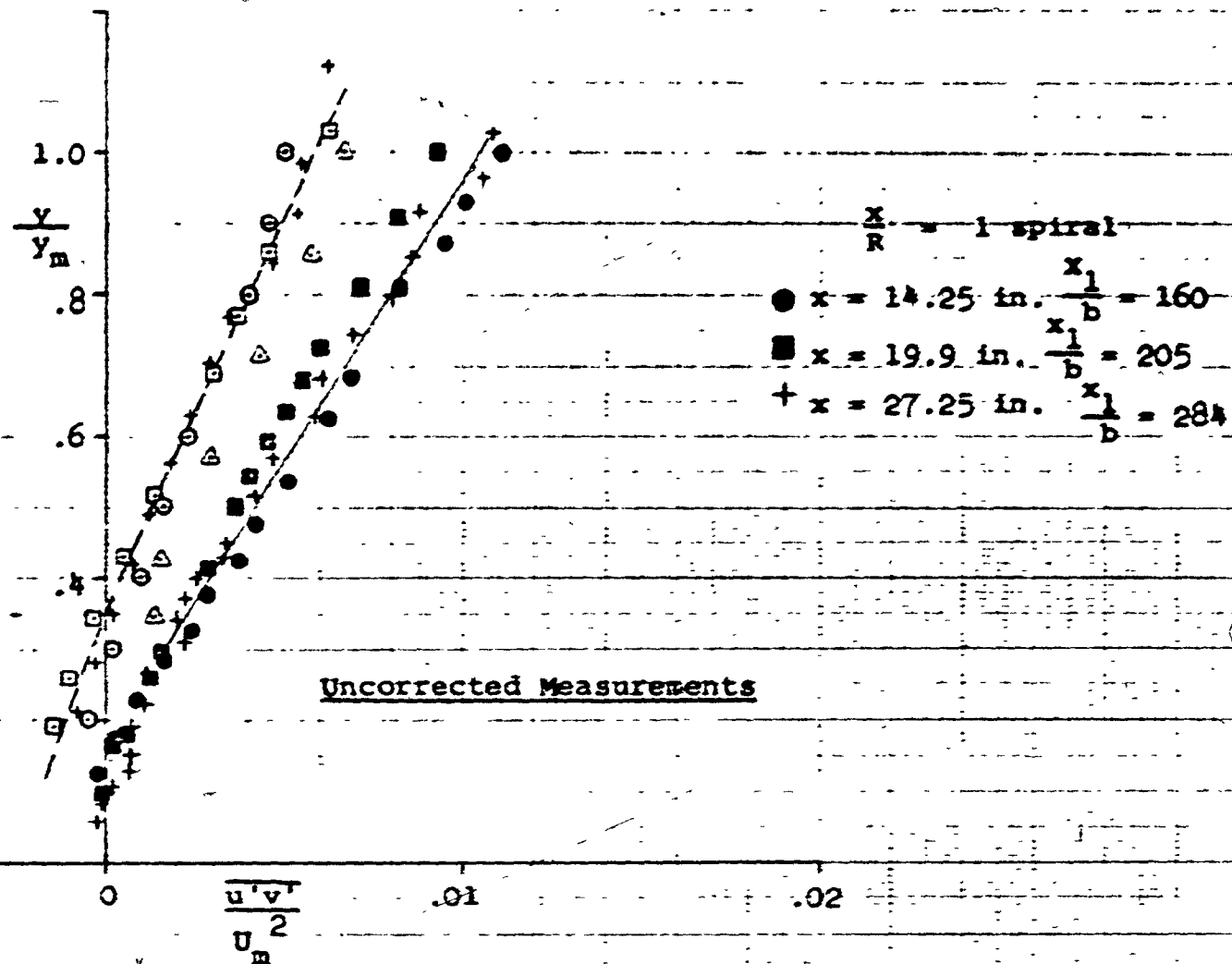
$$\frac{x_1}{B} = 284$$

(uncorrected measurements)

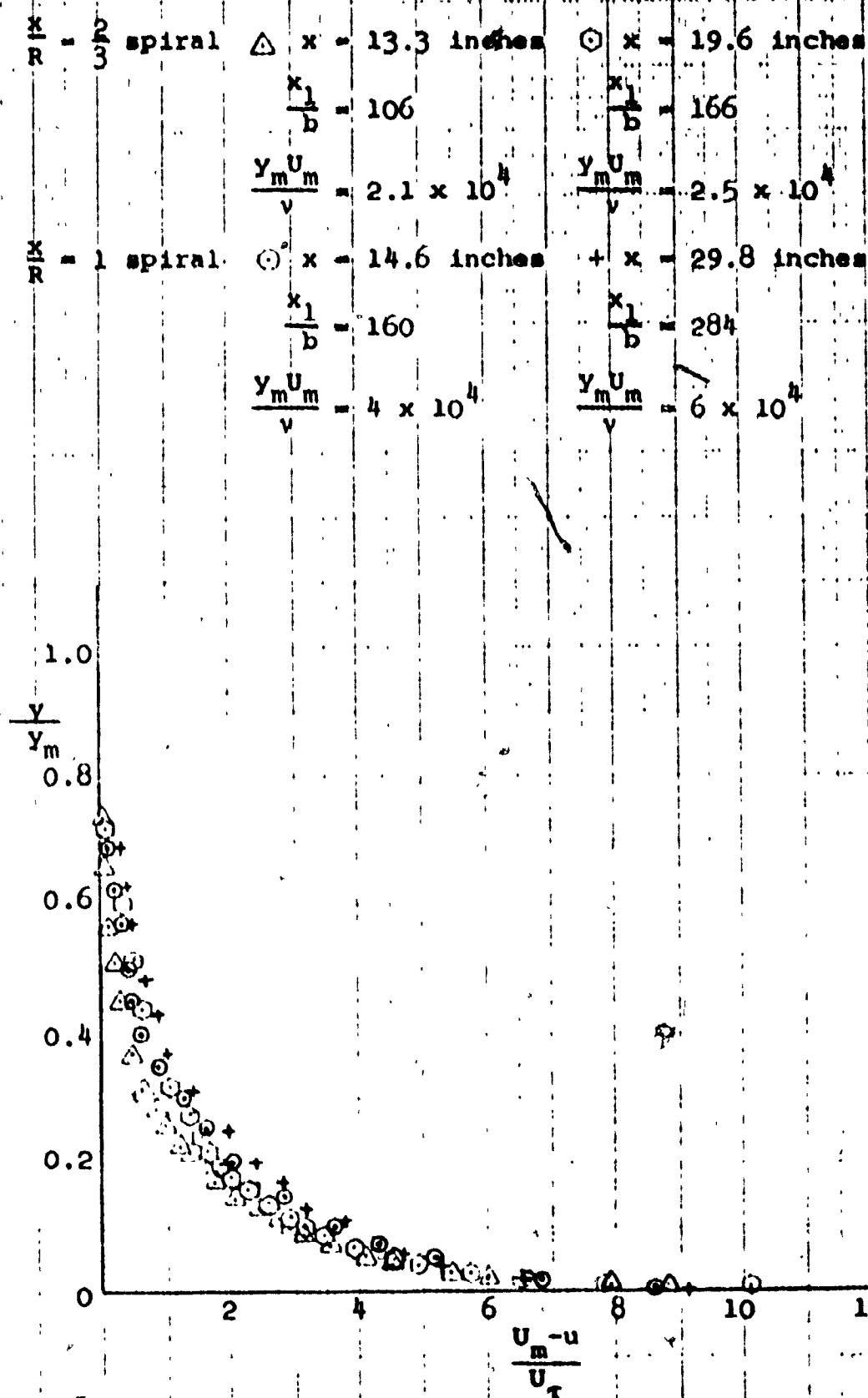


SHEAR STRESS DISTRIBUTION IN INNER LAYER

$\frac{x}{R} = \frac{2}{3}$ spiral Δ $x = 13.3$ in. $\odot$ $x = 19.6$ in. $\square$ $x = 23.5$ in. $+$ $x = 28.7$ in.
 $\frac{x_1}{b} = 116$ $\frac{x_1}{b} = 166$ $\frac{x_1}{b} = 197$ $\frac{x_1}{b} = 239$



VELOCITY DEFECT PLOT FOR INNER LAYER



DISTRIBUTION OF REYNOLDS STRESSES, $\overline{u'^2}$ AND $\overline{v'^2}$, IN WALL REGION

$\frac{x}{R} = \frac{2}{3}$ spiral

$\Delta x = 13.3 \text{ in.}$; $\frac{x_1}{b} = 106$ $\odot x = -19.6 \text{ in.}$; $\frac{x_1}{b} = 166$

$\frac{x}{R} = 1$ spiral

$\odot x = 14.6 \text{ in.}$; $\frac{x_1}{b} = 160$ $+ x = 29.8 \text{ in.}$; $\frac{x_1}{b} = 284$

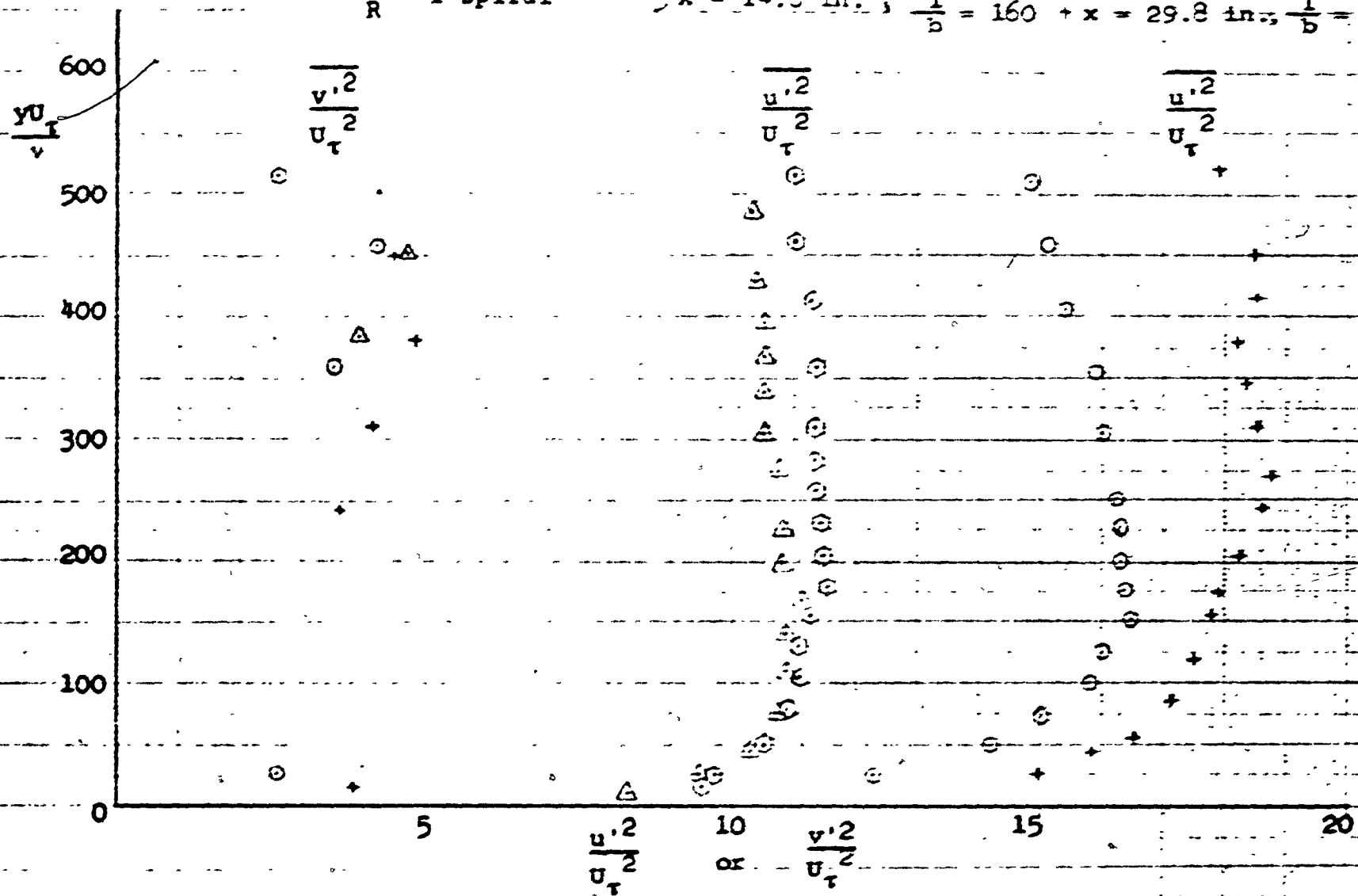
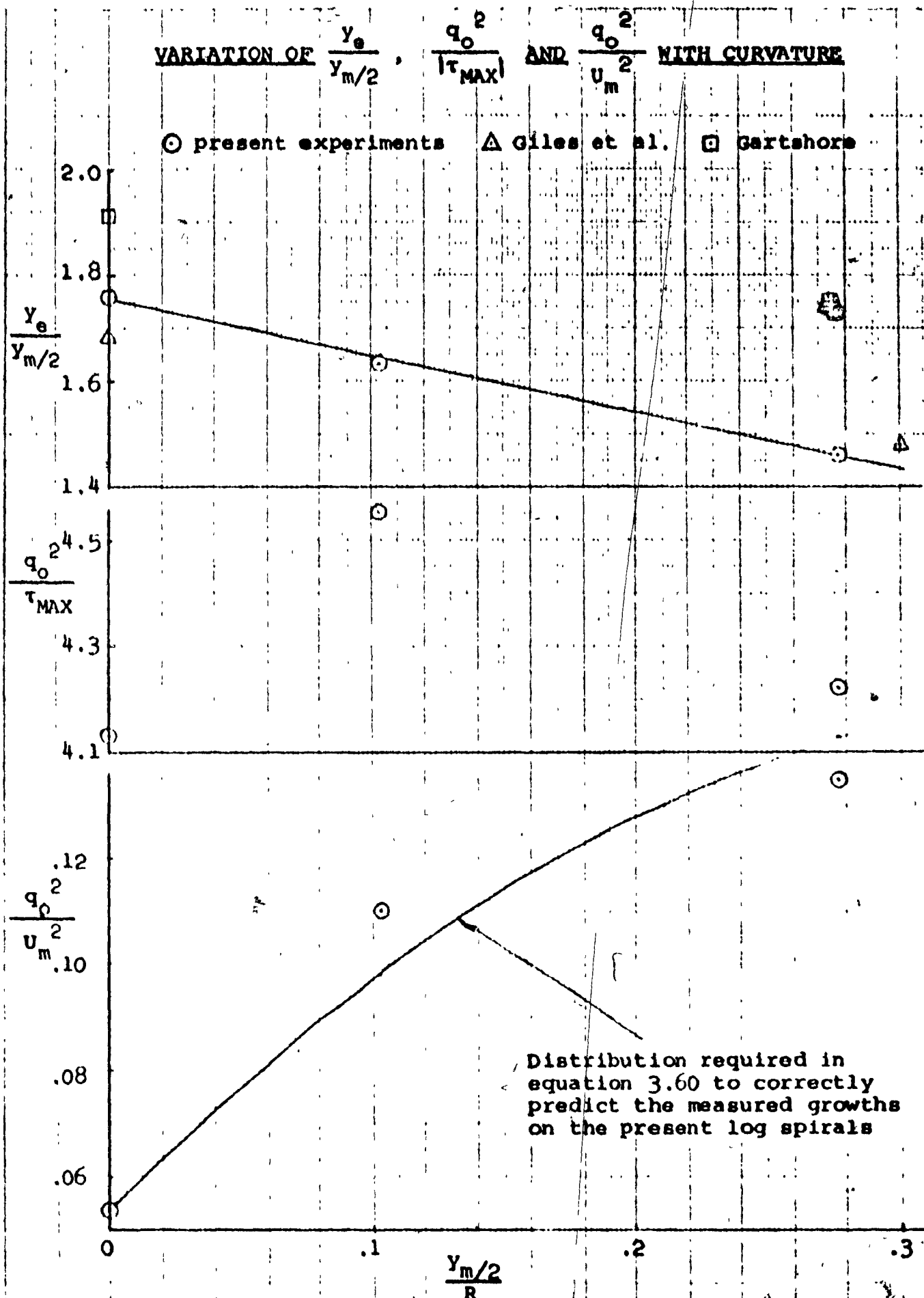
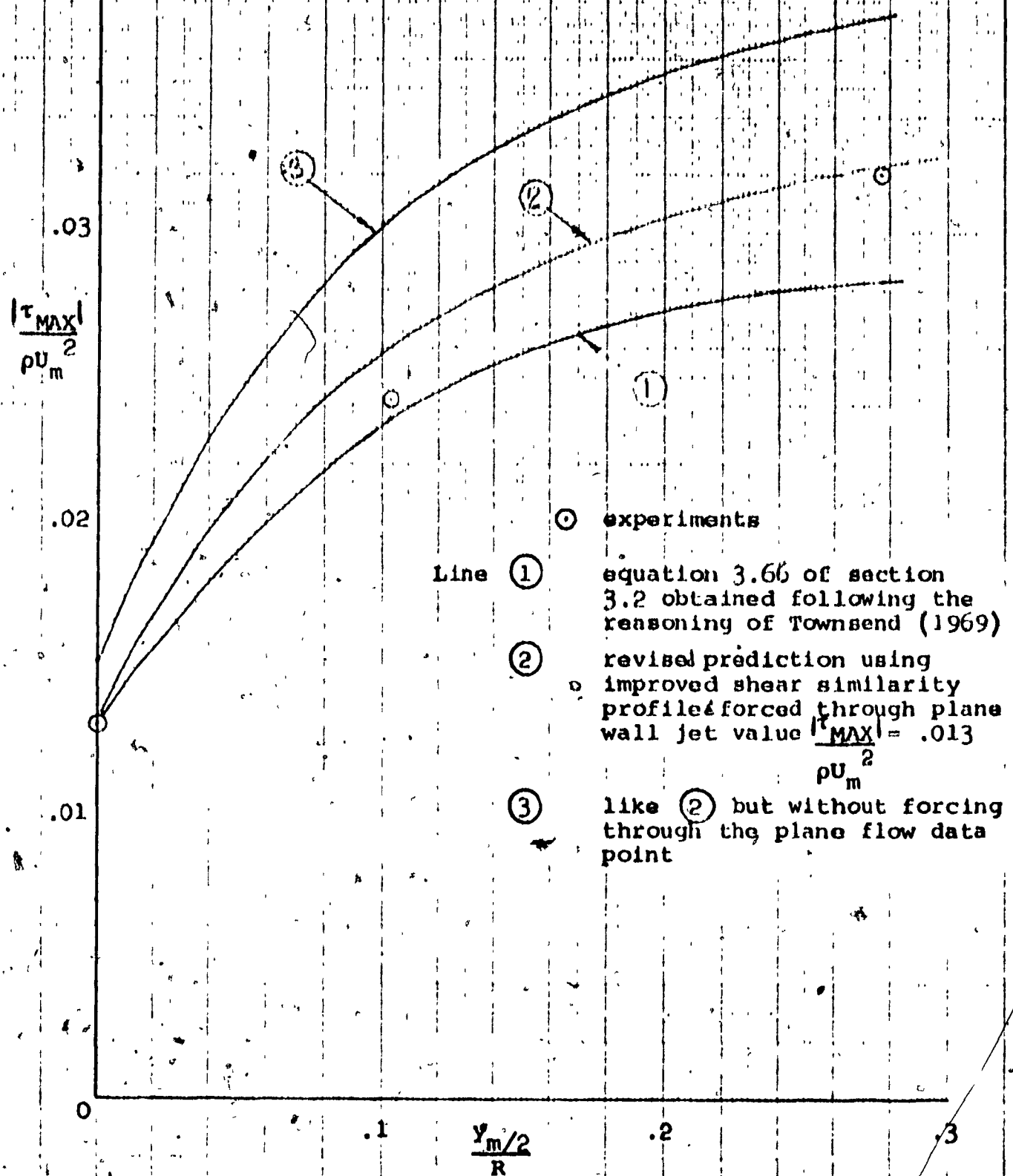


Fig. 9.51



VARIATION OF $\frac{|\tau_{MAX}|}{\rho u_m^2}$ WITH CONVEX CURVATURE



LATERAL TOTAL PRESSURE VARIATION AT CONSTANT DISTANCE

FROM SURFACE OF CIRCULAR CYLINDER

$$\frac{b}{R} = .0074 \quad Re = 7.0 \times 10^4 \quad R = 4.0 \text{ inch} \quad \theta_1 = 60^\circ$$

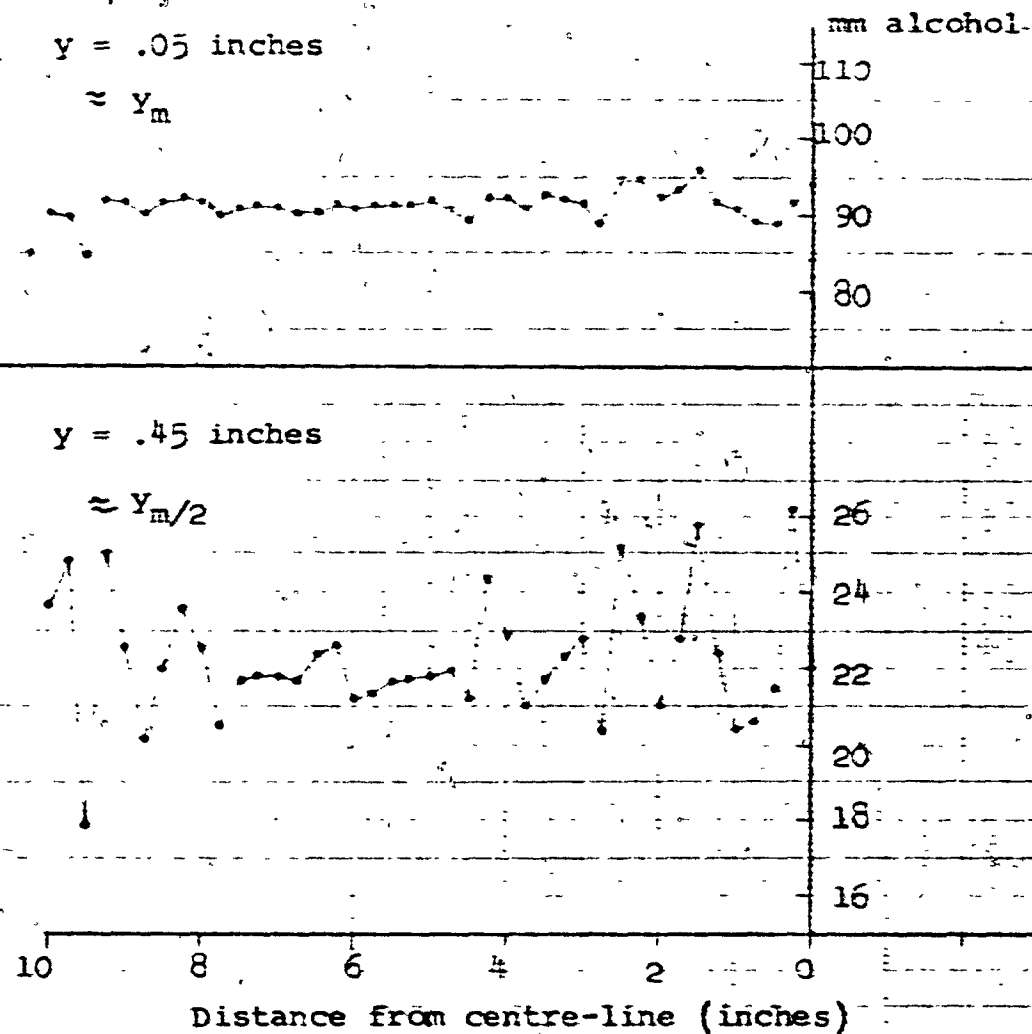
Final Result after
Modifications Described
in Chapter 5.

$y = .05 \text{ inches}$

$\approx y_m$

$y = .45 \text{ inches}$

$\approx y_m/2$



LATERAL TOTAL PRESSURE VARIATION AT CONSTANT DISTANCE

FROM SURFACE OF CIRCULAR CYLINDER

$$\frac{b}{R} = .0074 \quad Re = 7.0 \times 10^2 \quad R = 4.0 \text{ inches}$$

$$\theta_1 = 150^\circ$$

Final Result After Modifications described in Chapter 5.

$y = .45 \text{ inches}$

$\approx Y_m$

mm alcohol

13

14

$y = 1.8 \text{ inches}$

$\approx Y_m/2$

5.2

4.8

4.4

3.6

3.2

12

10

8

6

4

2

0

2

4

6

8

10

Distance on either side of centre-line (inches)

DECAY OF MAXIMUM VELOCITY ON CIRCULAR CYLINDER AND VERIFICATION OF
TWO-DIMENSIONALITY USING MOMENTUM EQUATION

$R = 4.0$ inches $\frac{b}{R} = .0074$ $Re = 7.0 \times 10^3$ $U_{mR} = 149$ ft/sec

$x_r (= \theta = 40^\circ) = 2.8$ inches

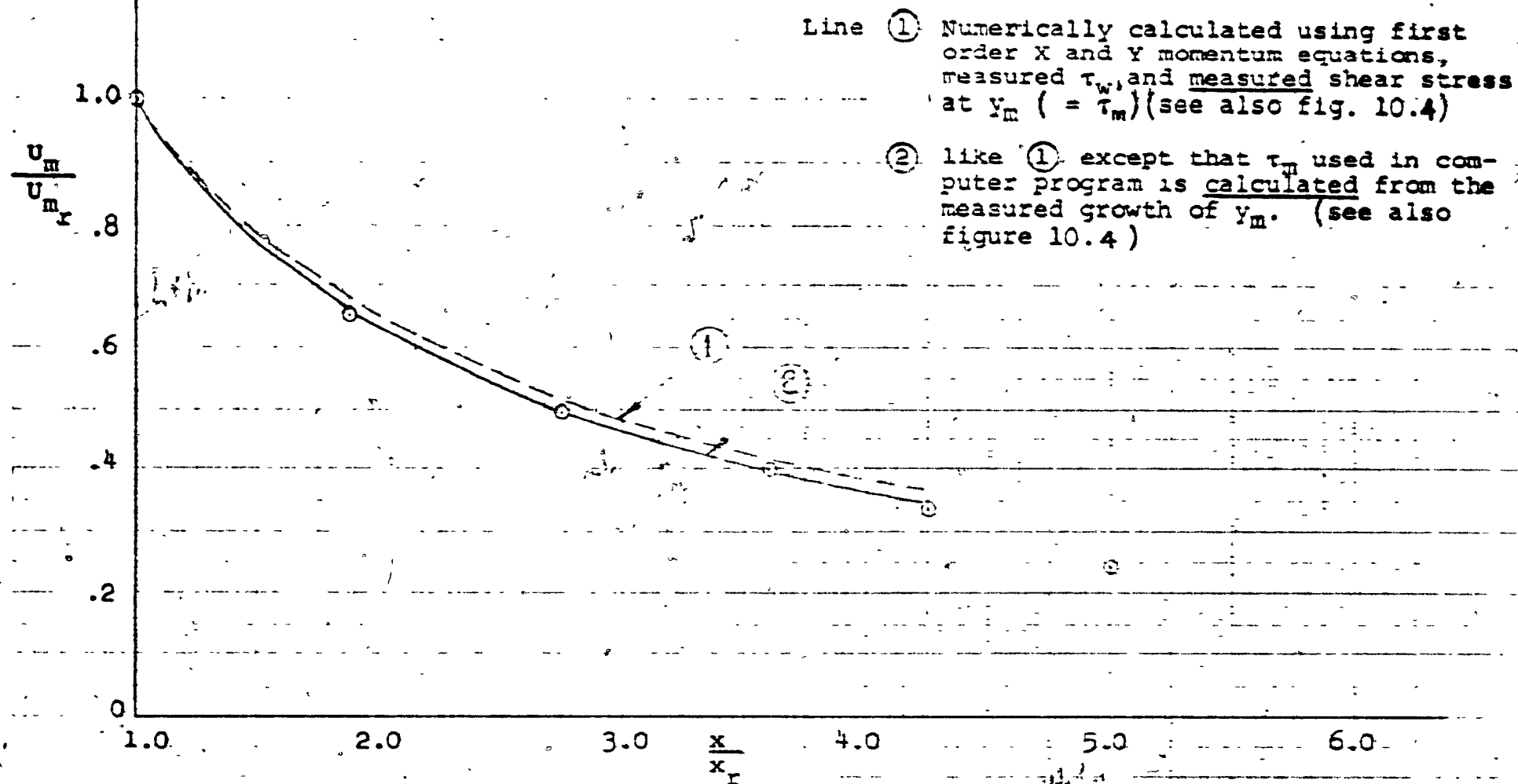
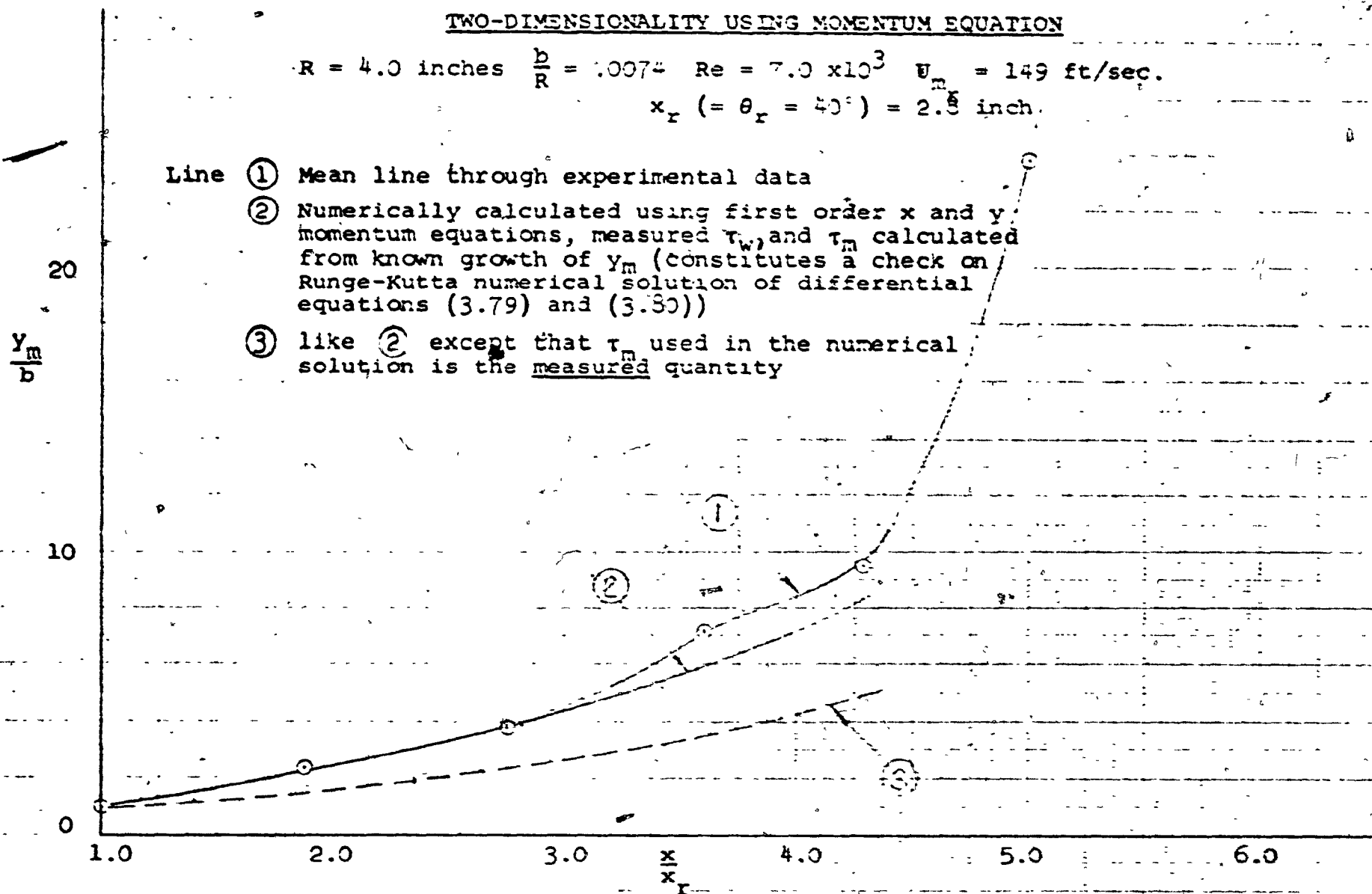


Fig. 10.3

GROWTH OF INNER LAYER ON CIRCULAR CYLINDER AND VERIFICATION OF
TWO-DIMENSIONALITY USING MOMENTUM EQUATION

$R = 4.0$ inches $\frac{b}{R} = 1.0074$ $Re = 7.0 \times 10^3$ $U_m = 149$ ft/sec.
 $x_r (= \theta_r = 40^\circ) = 2.8$ inch.

- Line ① Mean line through experimental data
 ② Numerically calculated using first order x and y momentum equations, measured τ_w , and τ_m calculated from known growth of y_m (constitutes a check on Runge-Kutta numerical solution of differential equations (3.79) and (3.80))
 ③ like ② except that τ_m used in the numerical solution is the measured quantity



VERIFICATION OF TWO-DIMENSIONALITY $\frac{u'w}{U_m^2}$ vs $\frac{y}{y_m/2}$ ON

CIRCULAR CYLINDER

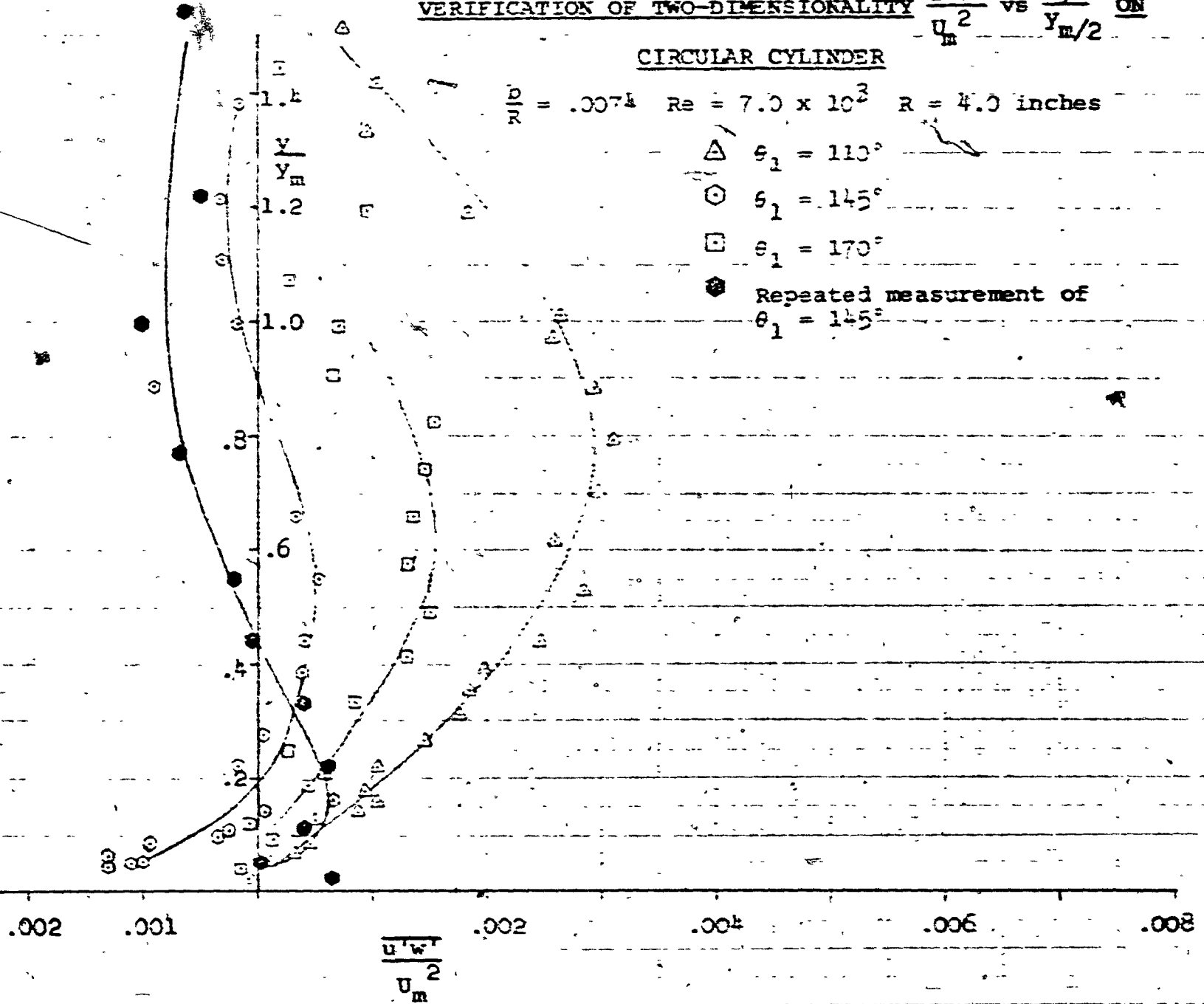
$\frac{b}{R} = .0074$ $Re = 7.0 \times 10^2$ $R = 4.0$ inches

$\Delta \theta_1 = 110^\circ$

$\odot \theta_1 = 145^\circ$

$\square \theta_1 = 170^\circ$

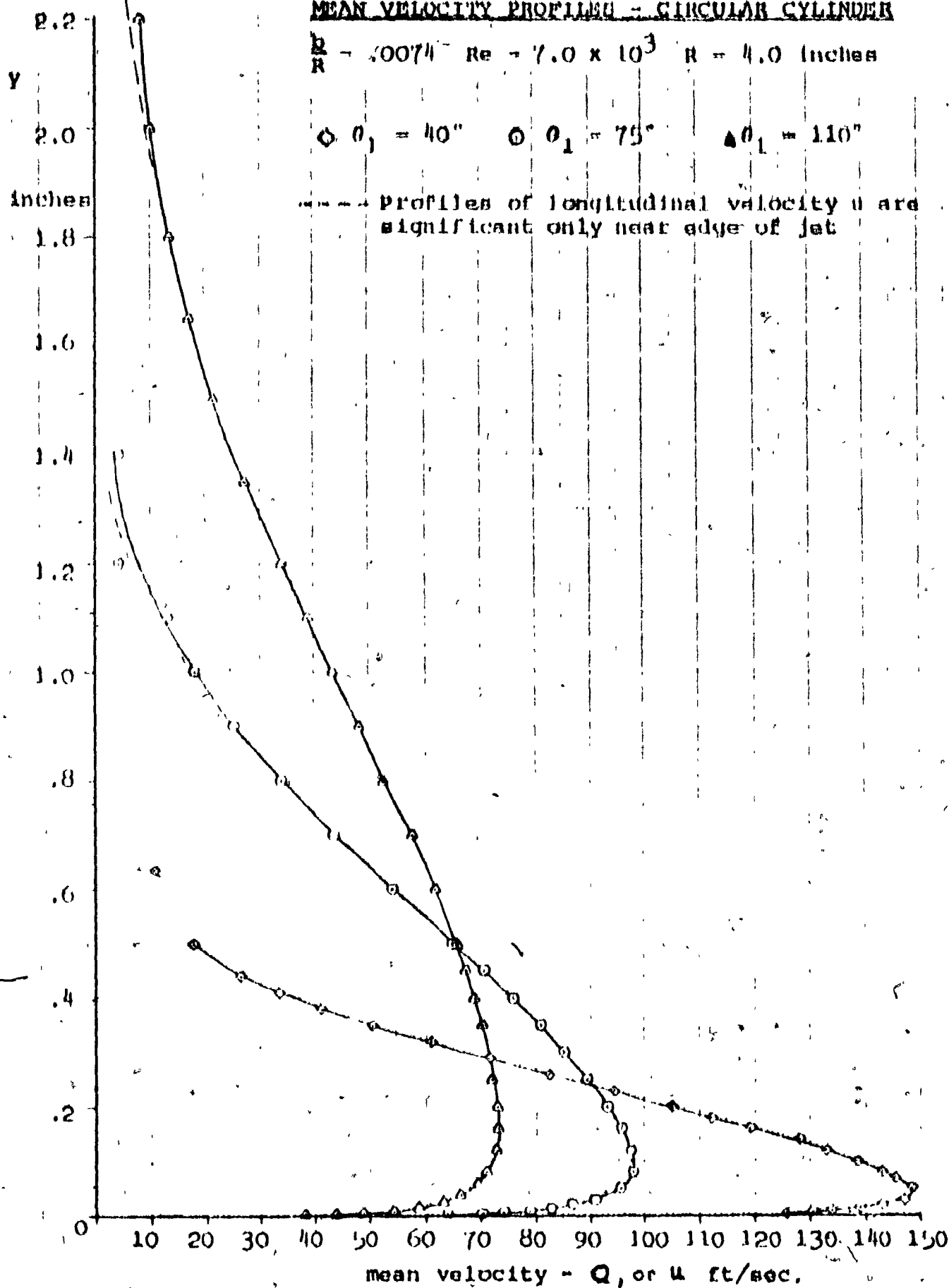
$\bullet$ Repeated measurement of $\theta_1 = 145^\circ$



MEAN VELOCITY PROFILE - CIRCULAR CYLINDER

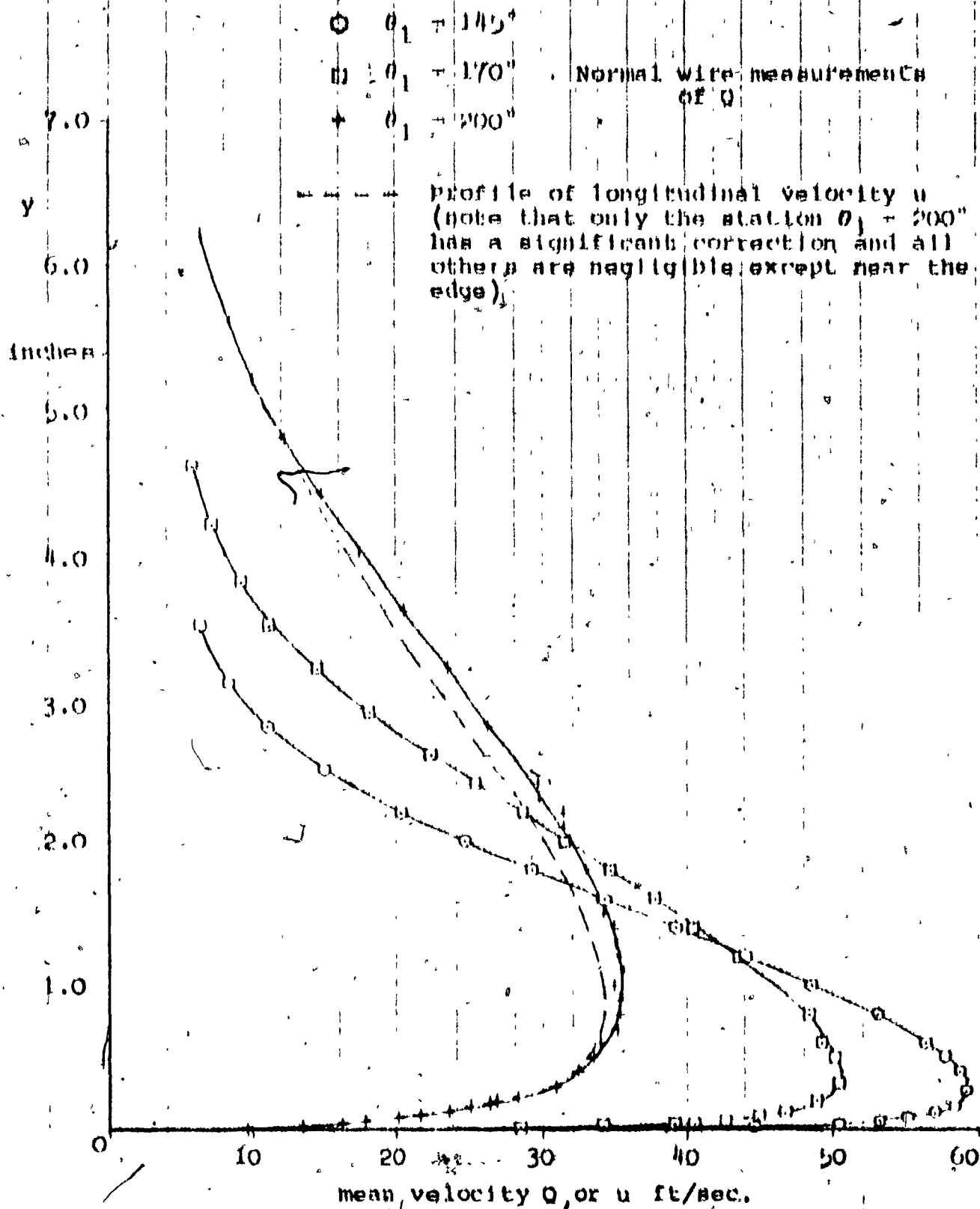
$$\frac{b}{R} = .0074 \quad Re = 7.0 \times 10^3 \quad R = 4.0 \text{ inches}$$

$$\diamond \theta_1 = 40^\circ \quad \circ \theta_1 = 75^\circ \quad \triangle \theta_1 = 110^\circ$$



MEAN VELOCITY PROFILES - CIRCULAR CYLINDER

$$\frac{b}{R} = .0074 \quad Re = 7.0 \times 10^3 \quad R = 4.0 \text{ inch}$$



DIMENSIONLESS MEAN VELOCITY PROFILE ON CIRCULAR CYLINDER

$$\frac{b}{R} = .0074$$

$$Re = 7.0 \times 10^3$$

$$R = 4.0 \text{ inches}$$

$$\frac{y}{y_m/2}$$

2.2

2.0

1.8

1.6

1.4

1.2

1.0

.8

.6

.4

.2

0

.2

.4

.6

.8

1.0

1.2

○

 $\theta_1 = 75^\circ$

△

 $\theta_1 = 110^\circ$

□

 $\theta_1 = 170^\circ$

+

 $\theta_1 = 200^\circ$

— mean line through $\frac{Q}{Q_m}$ data

--- $\frac{U}{U_m}$ (above results corrected for lateral component v)

 $\frac{Q}{Q_m}$

DIMENSIONLESS VELOCITY PROFILE FOR OUTER LAYERCIRCULAR CYLINDER

$$\frac{b}{R} = .0074 \quad Re = 7.0 \times 10^3 \quad R = 4.0 \text{ inches}$$

$$(A) \quad \theta_1 = 75^\circ \quad \Delta \theta_1 = 110^\circ \quad \theta_1 = 200^\circ$$

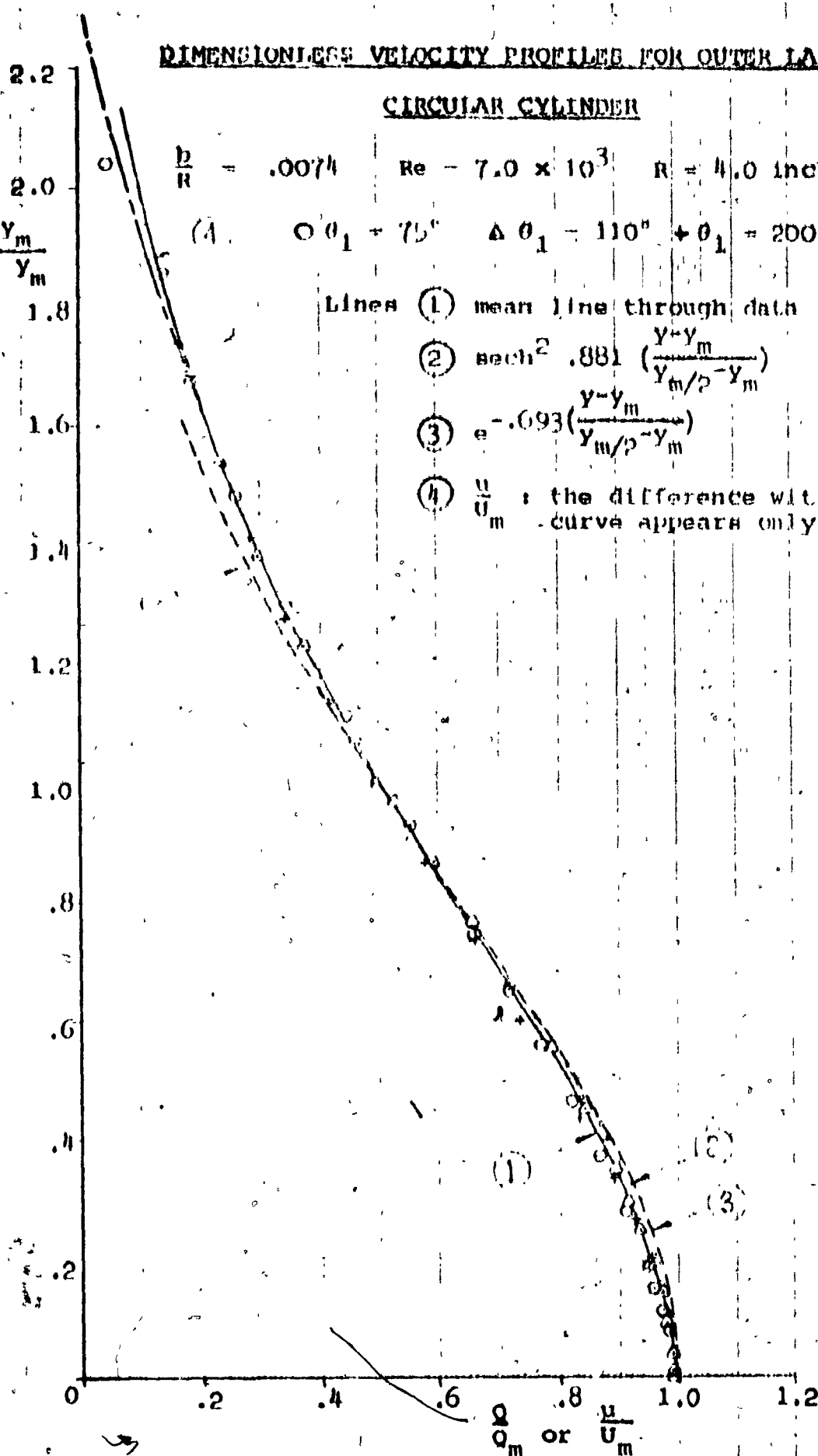
$$\frac{y-y_m}{y_m/2 - y_m}$$

Lines (1) mean line through data

$$(2) \quad \text{sech}^2 .881 \left(\frac{y-y_m}{y_m/2 - y_m} \right)$$

$$(3) \quad e^{-.693 \left(\frac{y-y_m}{y_m/2 - y_m} \right)}$$

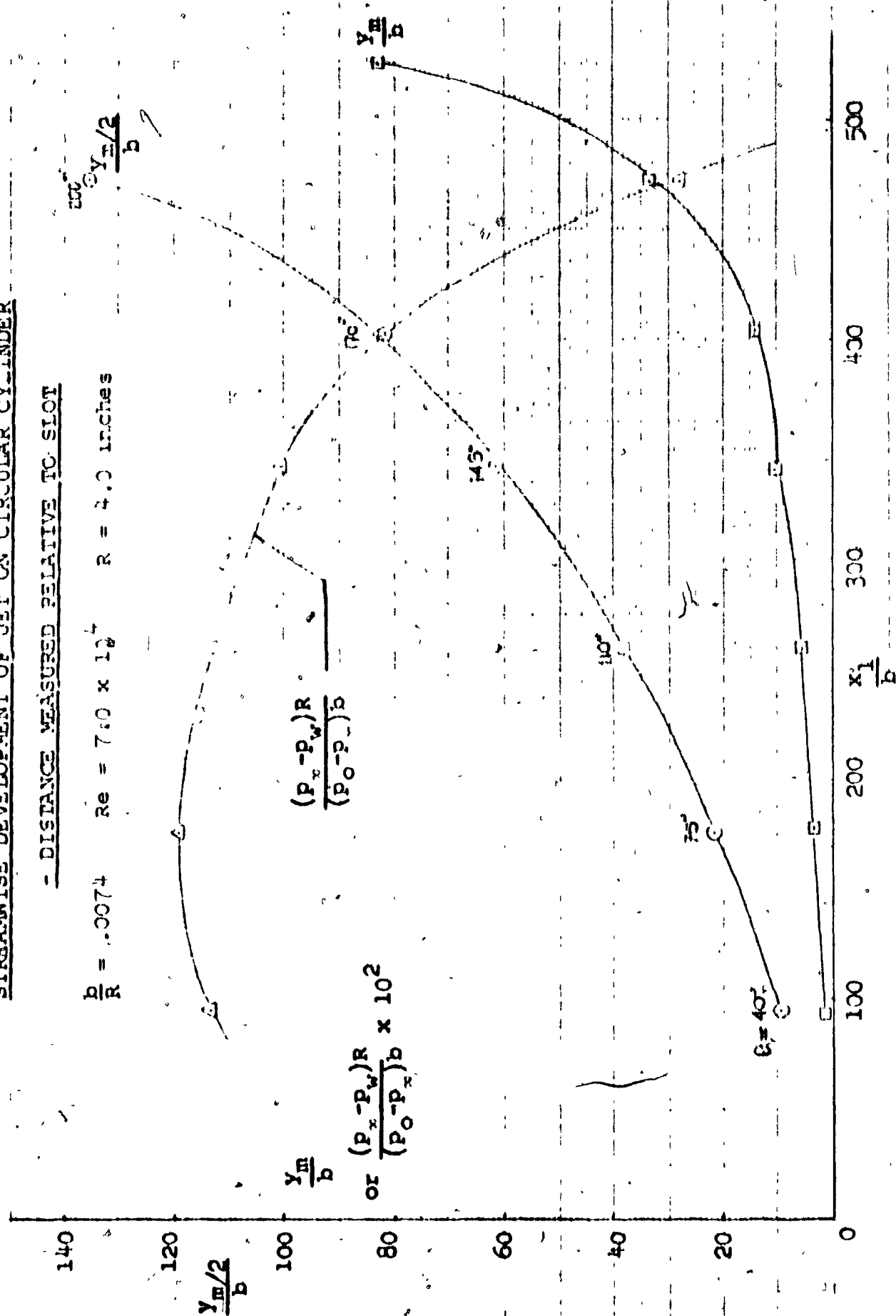
(4) $\frac{u}{u_m}$: the difference with the Q/Q_m curve appears only near edge



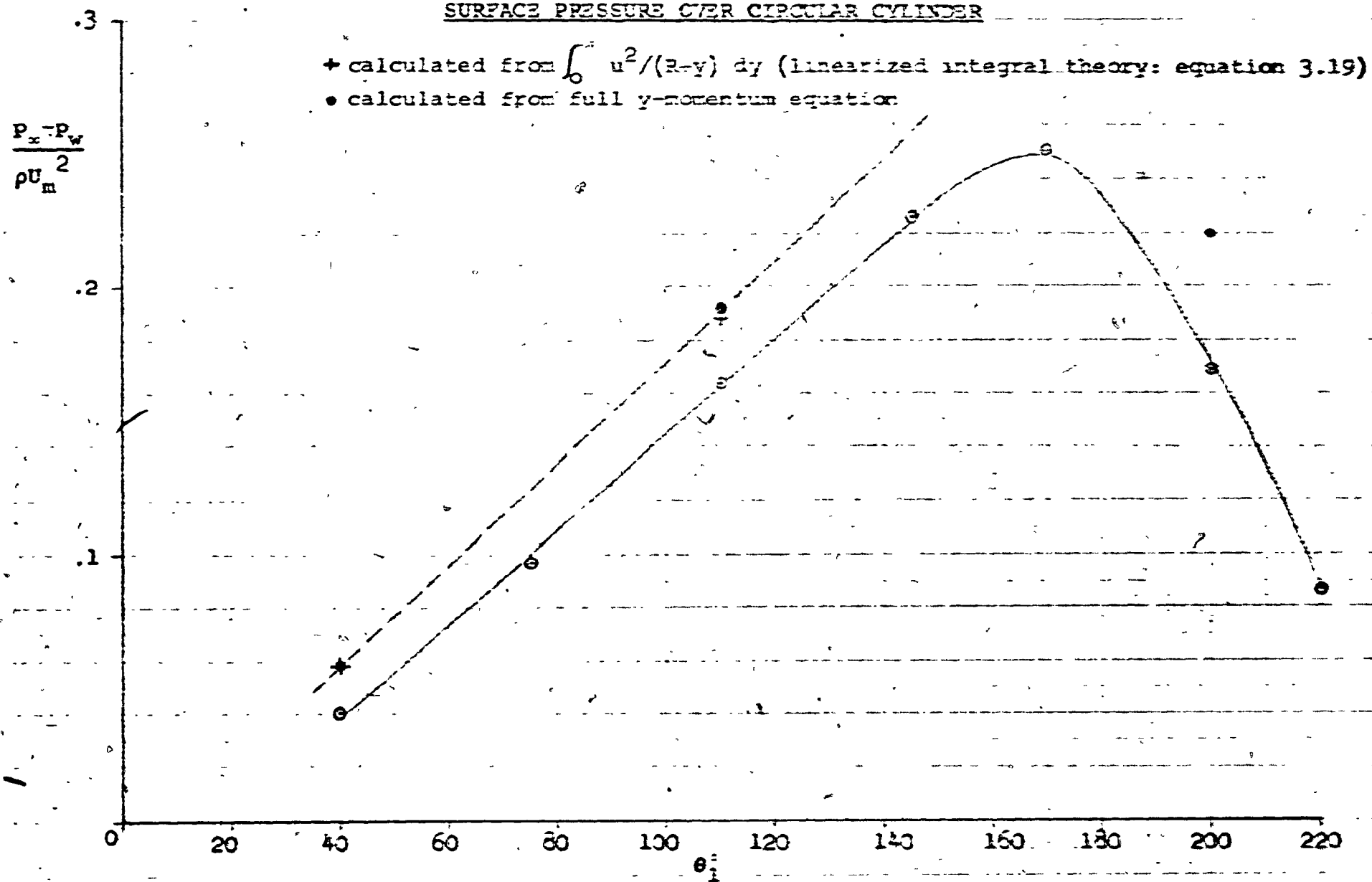
STREAMWISE DEVELOPMENT OF JET ON CIRCULAR CYLINDER

- DISTANCE MEASURED RELATIVE TO SLOT

$$\frac{b}{R} = .0074 \quad Re = 7.0 \times 10^4 \quad R = 4.0 \text{ inches}$$

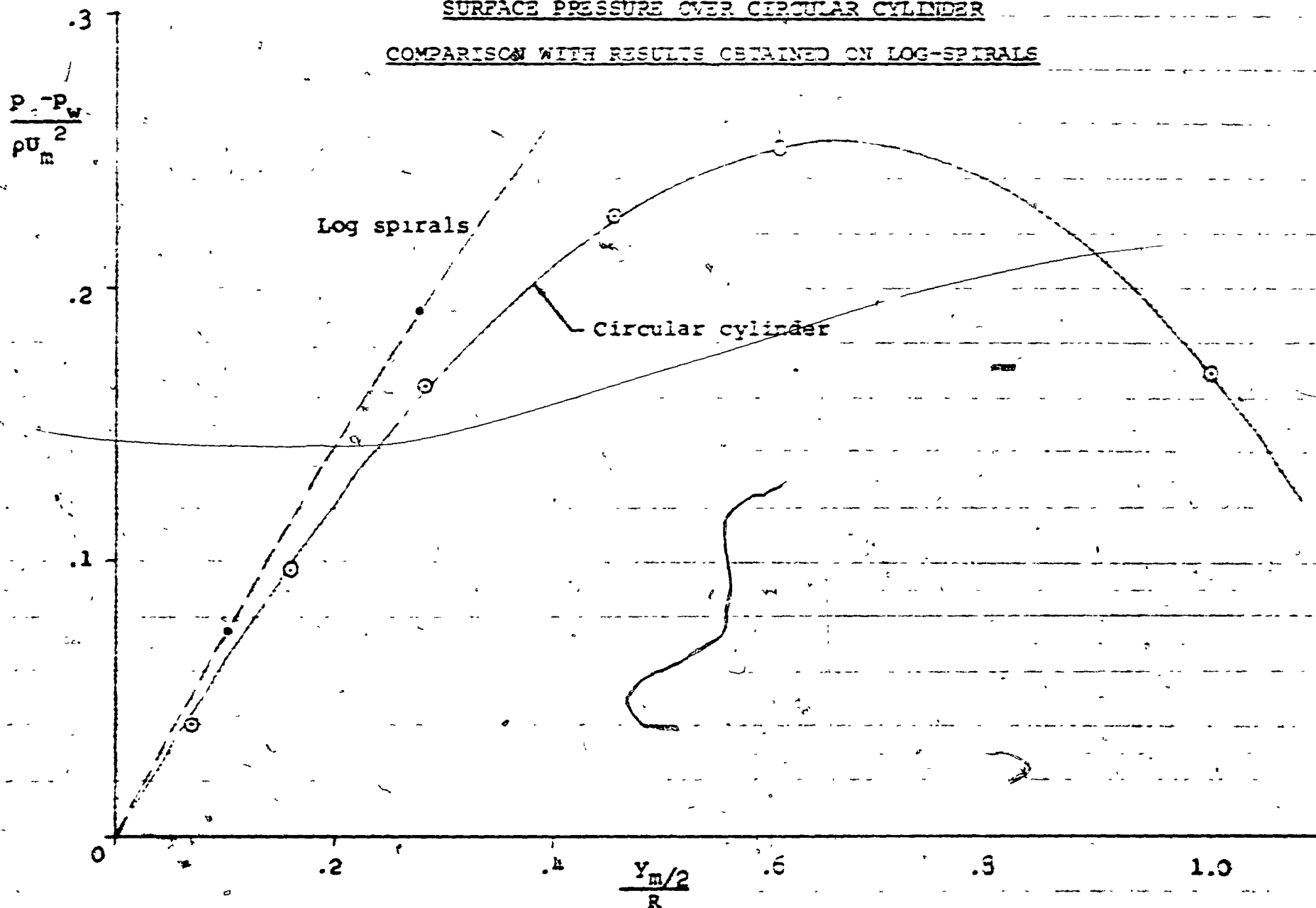


SURFACE PRESSURE C/ER CIRCULAR CYLINDER



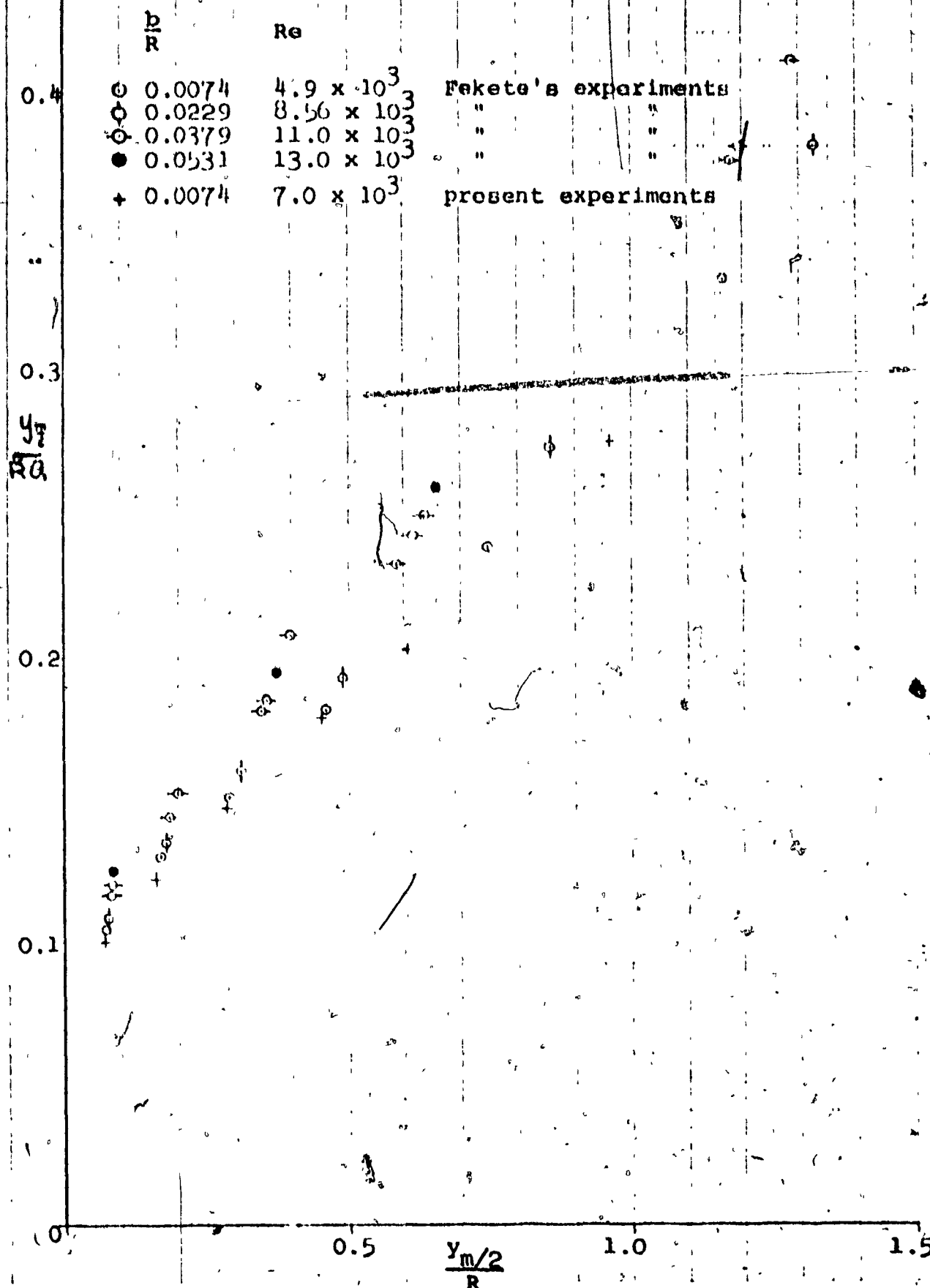
SURFACE PRESSURE OVER CIRCULAR CYLINDER

COMPARISON WITH RESULTS OBTAINED ON LOG-SPIRALS



STREAM VARIATION OF $\frac{y_m/2}{R}$ WITH $\frac{y_m/2}{R}$

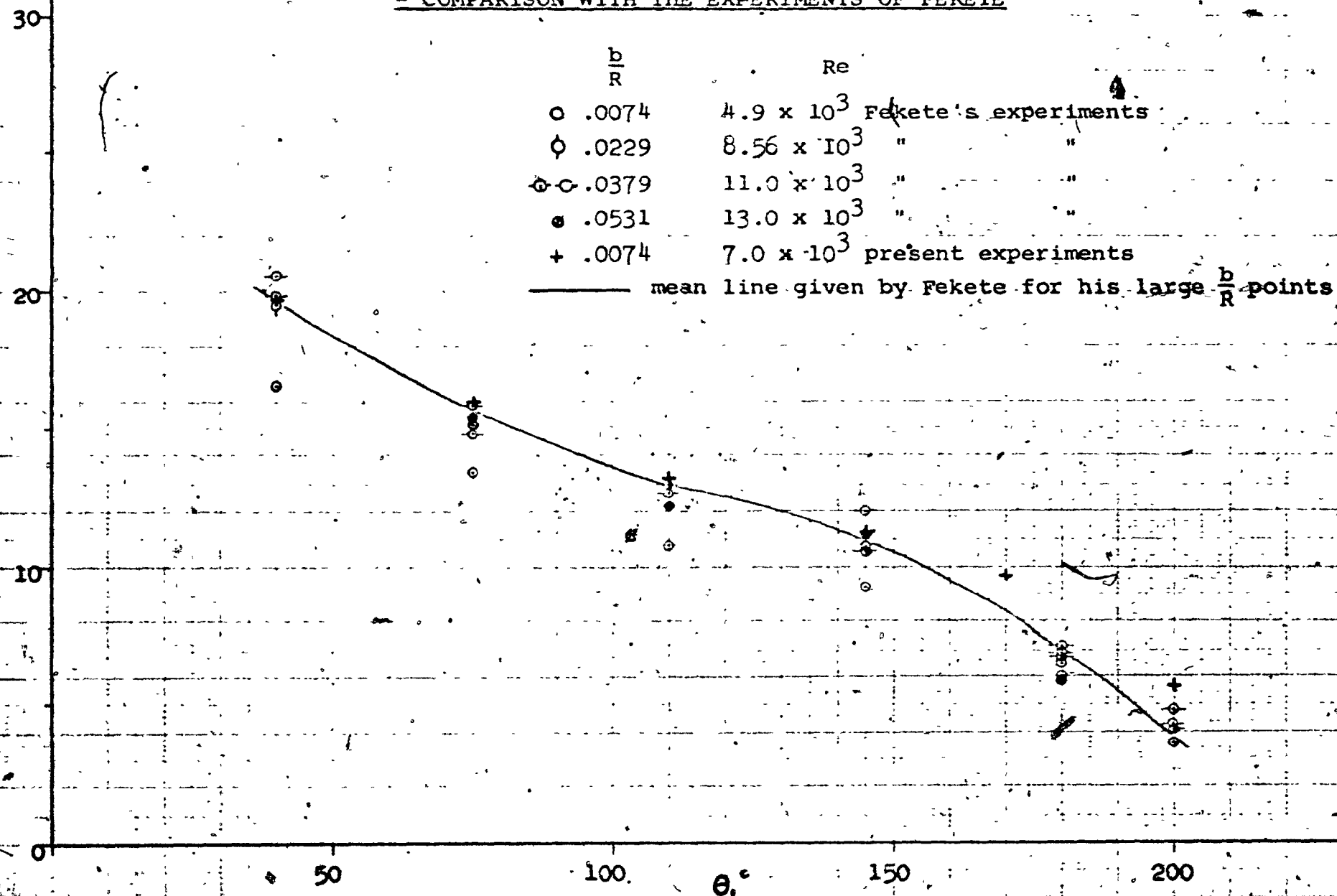
- COMPARISON WITH THE EXPERIMENTS OF FEKETE



$$\frac{\rho U_{\infty}^2 R \theta}{(P-P_{\infty})b}$$

DECAY OF MAXIMUM VELOCITY ON CIRCULAR CYLINDER

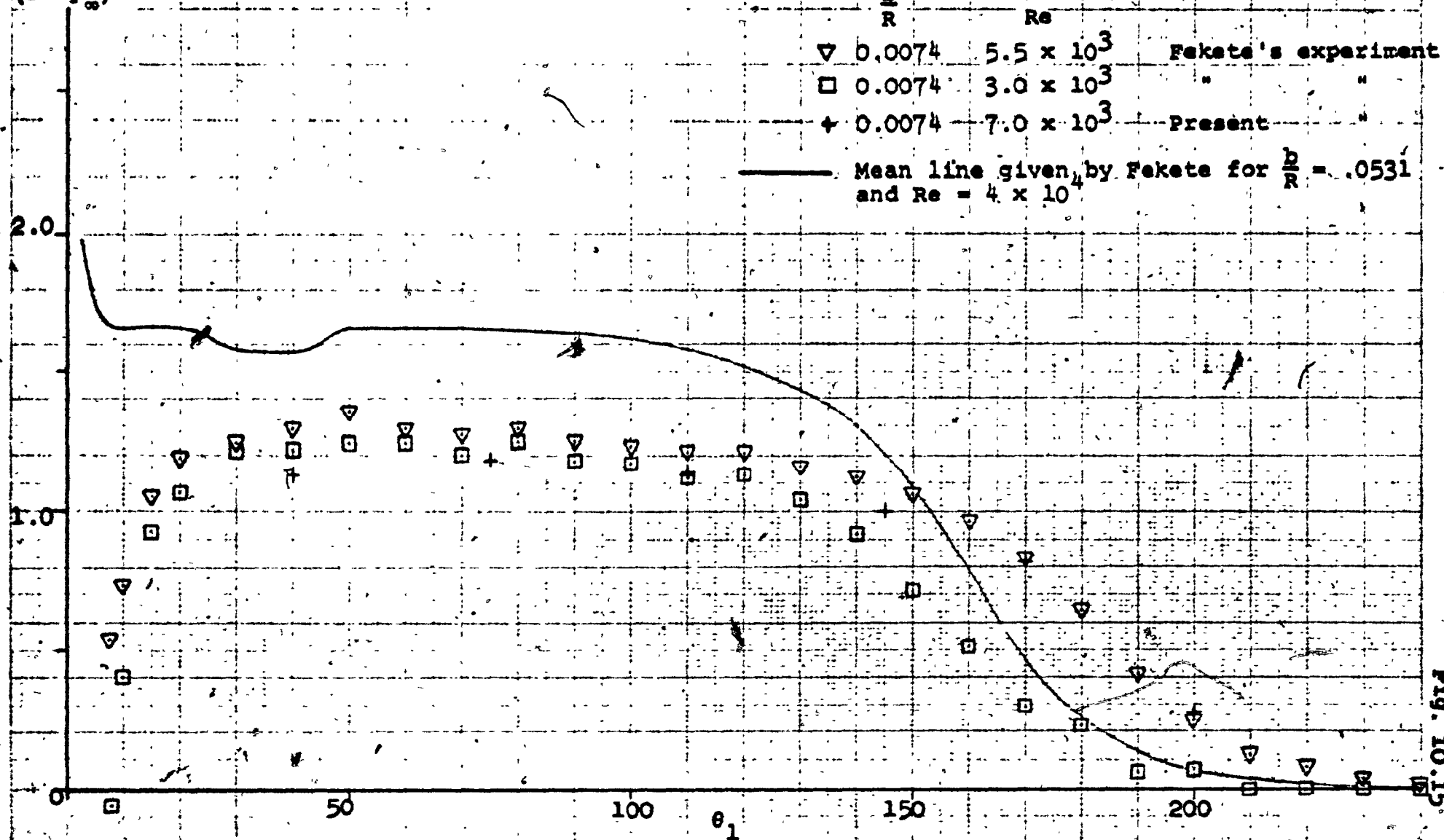
- COMPARISON WITH THE EXPERIMENTS OF FEKETE

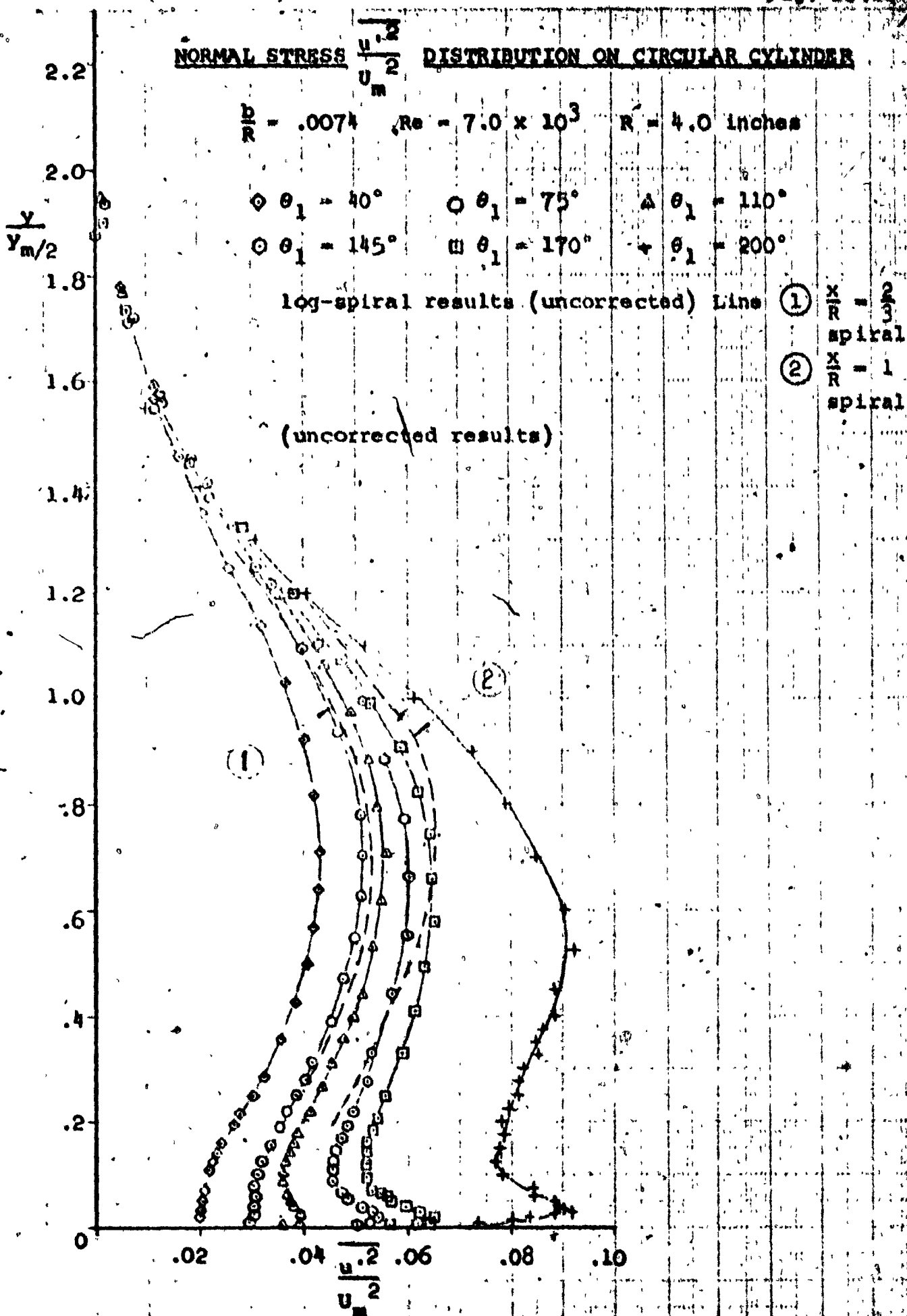


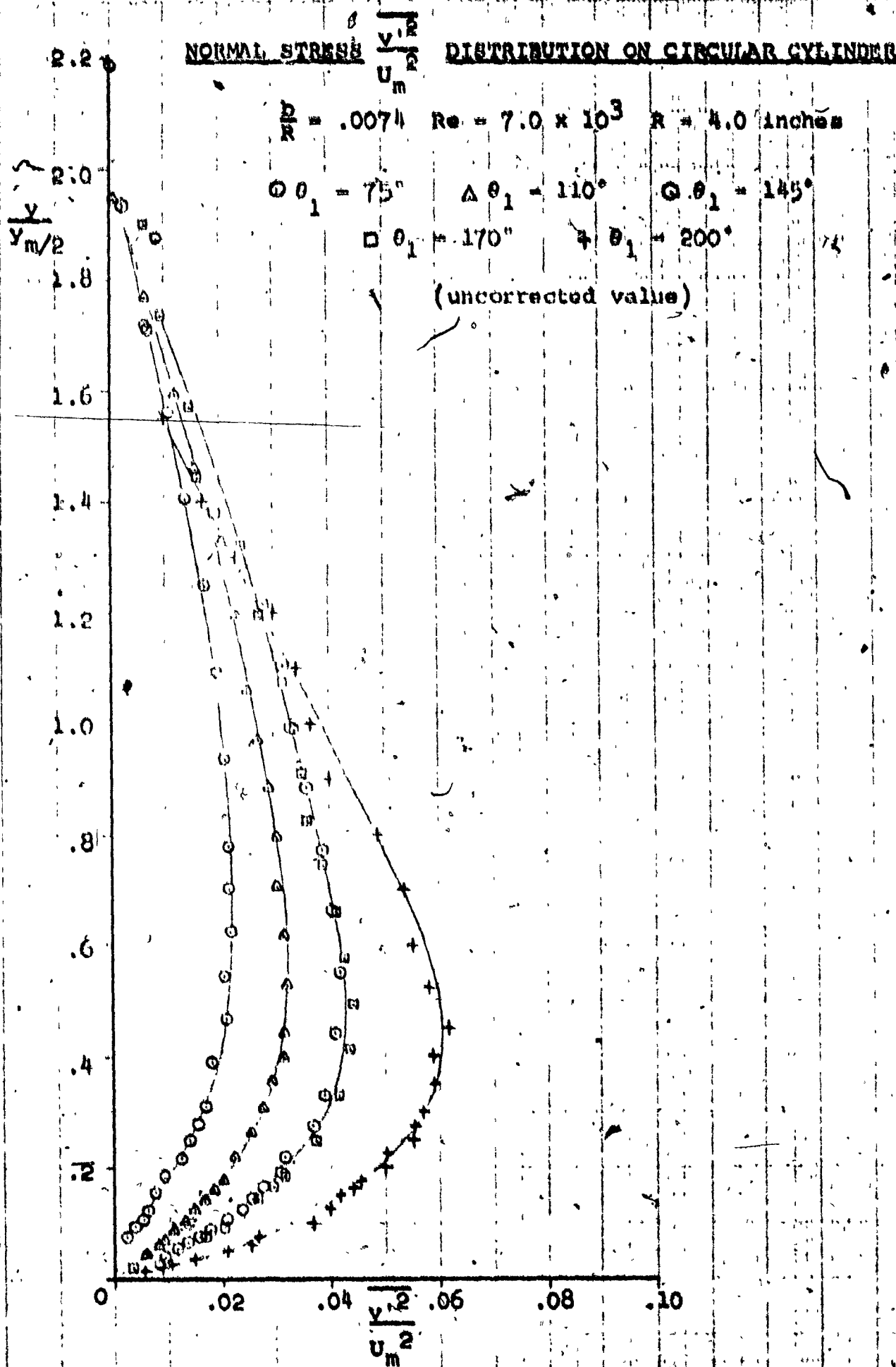
SURFACE PRESSURE DISTRIBUTION - COMPARISON WITH THE
EXPERIMENTS OF FEKETE

$R = 4.0$ inches

$$\frac{(P_{\infty} - P_w)R}{(P_{\infty} - P_w)b}$$

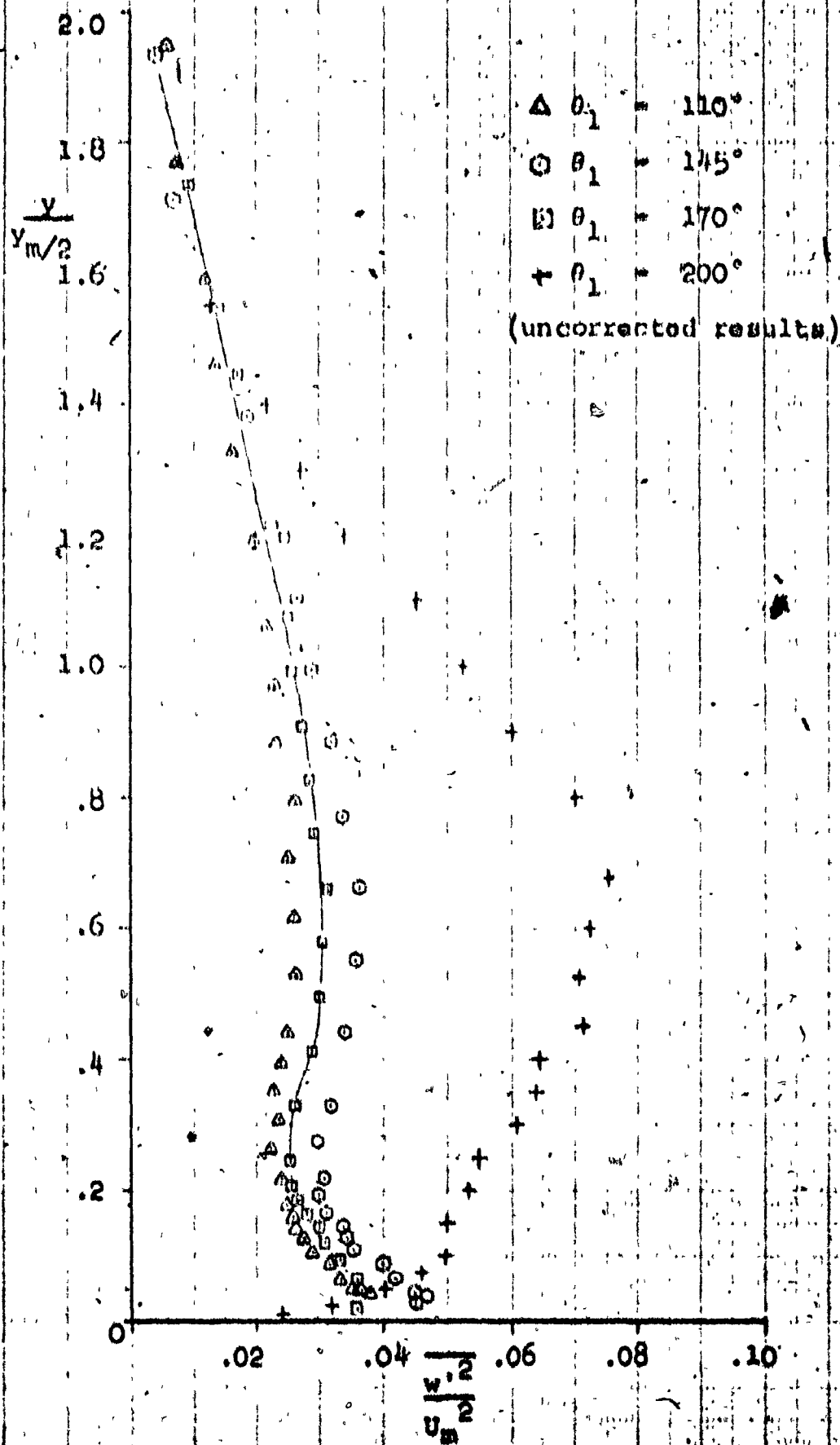






NORMAL STRESS $\frac{w}{U_m^2}$ DISTRIBUTION ON CIRCULAR CYLINDER

$$\frac{D}{R} = .0074 \quad Re = 7.0 \times 10^3 \quad D = 4.0 \text{ inches}$$



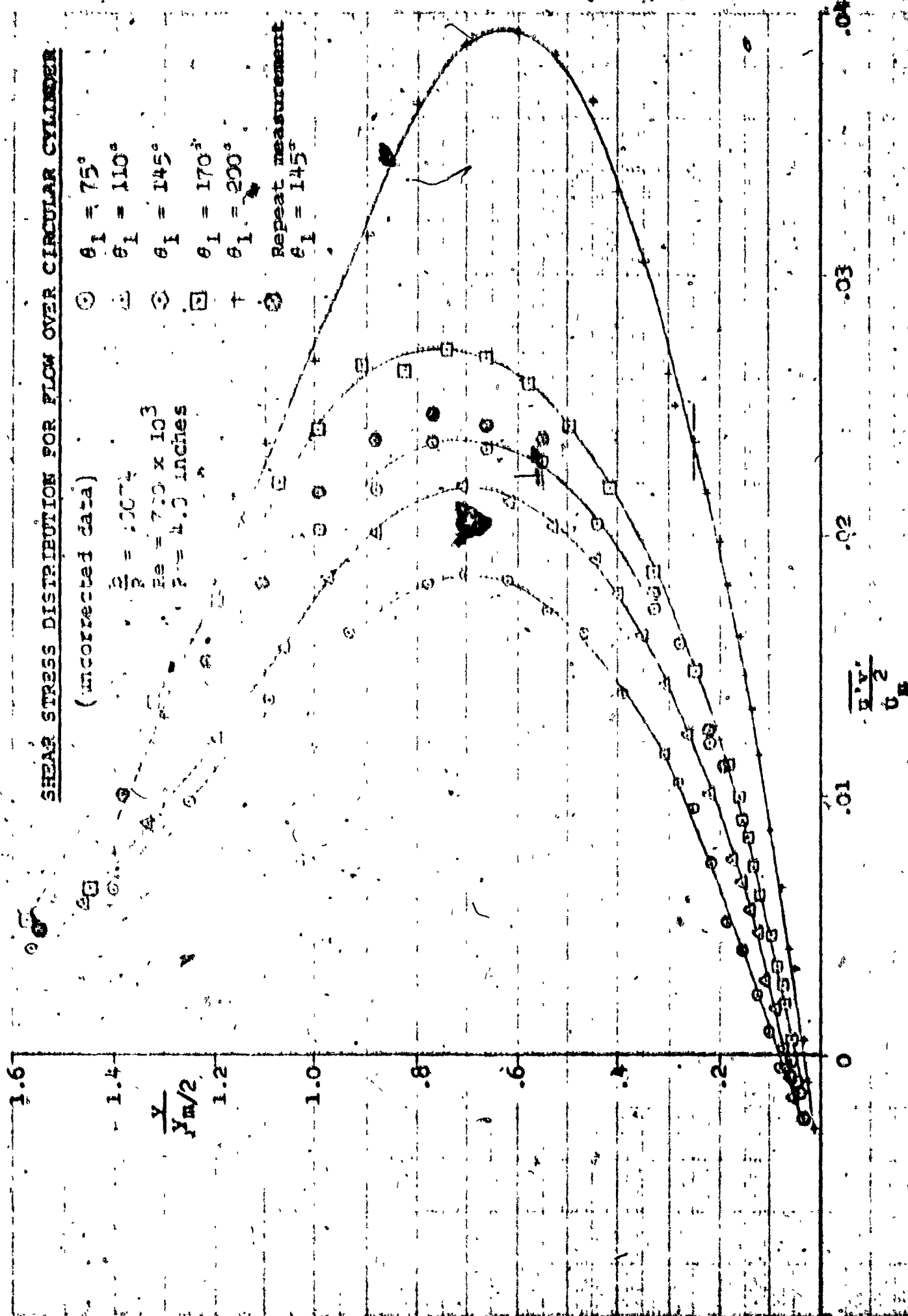
SHEAR STRESS DISTRIBUTION FOR FLOW OVER CIRCULAR CYLINDER

(uncorrected data)

$\frac{y}{y_m/2} = 10^{-4}$
 $Re = 7.0 \times 10^3$
 $\mu = 4.0$ inches

- $\theta_1 = 75^\circ$
- △ $\theta_1 = 110^\circ$
- ◇ $\theta_1 = 145^\circ$
- $\theta_1 = 170^\circ$
- + $\theta_1 = 200^\circ$

Repeat measurement
 $\theta_1 = 145^\circ$



NORMAL STRESS u'^2 DISTRIBUTION IN INNER LAYER OF

FLOW ON CIRCULAR CYLINDER (Uncorrected measurements)

- $\theta =$
- $\diamond$ 40°
 - $\circ$ 75°
 - $\triangle$ 110°
 - $\square$ 145°
 - $\blacksquare$ 170°
 - $+$ 200°

PLANE WALL TEST
(TULLAUX & MATHIEU)

$C_D = 1.4$

1.0

.8

.6

.4

.2

0

0

.02

.04

.06

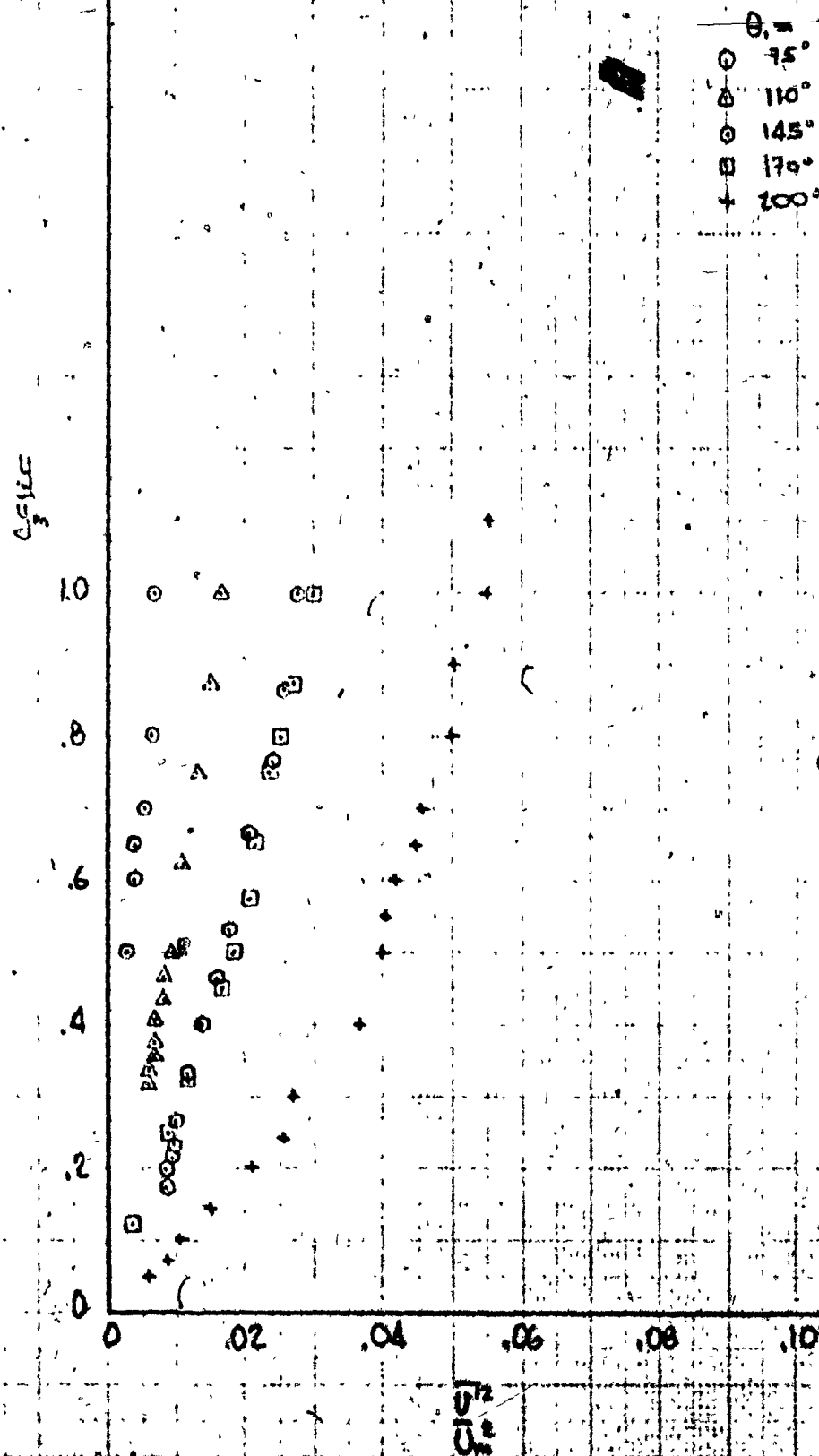
.08

.10

u'^2

NORMAL STRESS $\sqrt{\sigma^2}$ DISTRIBUTION IN INNER LAYER

(Uncorrected measurements)

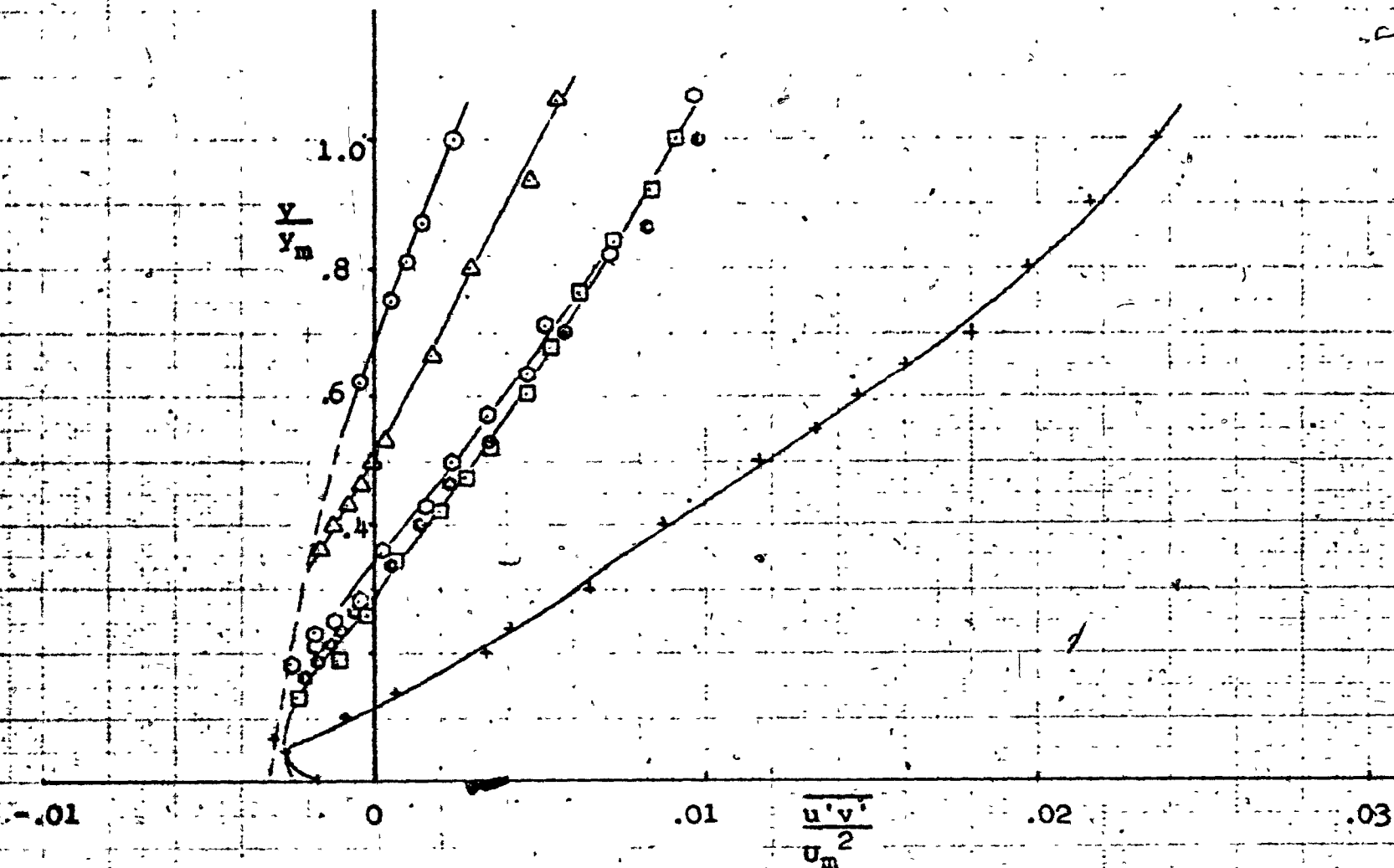


SHEAR STRESS DISTRIBUTION IN INNER LAYER OF FLOW ON CIRCULAR CYLINDER

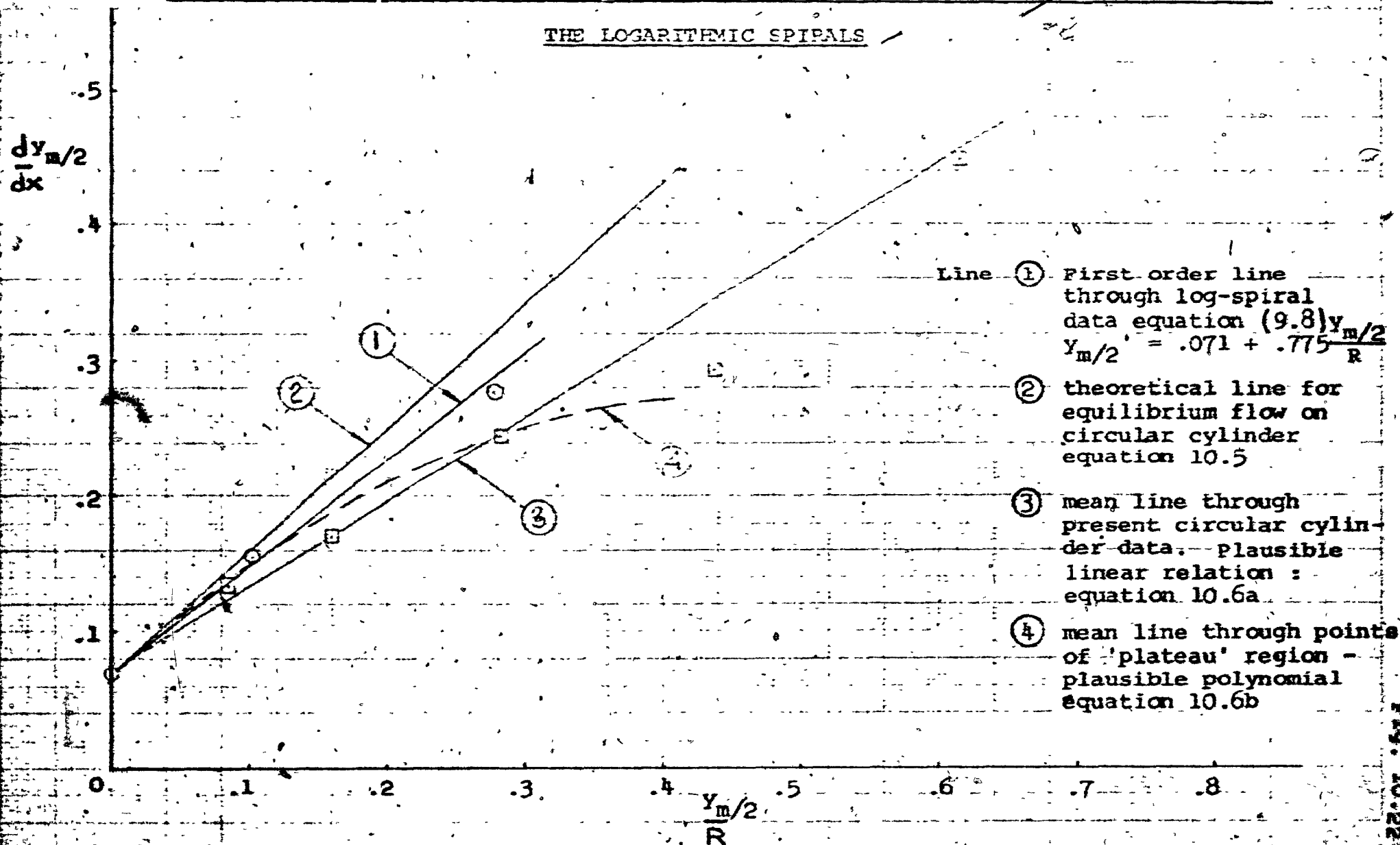
$\odot \theta_1 = 75^\circ$
 $\triangle \theta_1 = 110^\circ$
 $\circ \theta_1 = 145^\circ$

$\square \theta_1 = 170^\circ$
 $+ \theta_1 = 200^\circ$

(Uncorrected measurements)



THE PROXIMITY TO EQUILIBRIUM OF THE FLOW ON THE CIRCULAR CYLINDER - A COMPARISON
OF THE RATE OF GROWTH OF THE JET ON THE CIRCULAR CYLINDER WITH THAT OBTAINED ON
THE LOGARITHMIC SPIRALS



STREAMWISE VARIATION OF PARAMETER DEFINING LIFE OF
LARGE EDDY IN THE FLOW ON THE CIRCULAR CYLINDER

life of large eddy = $t_D = \frac{\lambda}{\left| \frac{\partial u}{\partial y} \right|}$

4.0

X

3.0

2.0

1.0

0

.1

.2

.3

.4

.5

.6

 $\frac{y_{R/2}}{R}$

0

SHEAR STRESS DISTRIBUTION

$$\frac{-\overline{u'v'}}{\overline{u'v'}_{\text{MAX}}} = \frac{\tau_w/\rho}{\tau_w/\rho}$$

ON CYLINDER

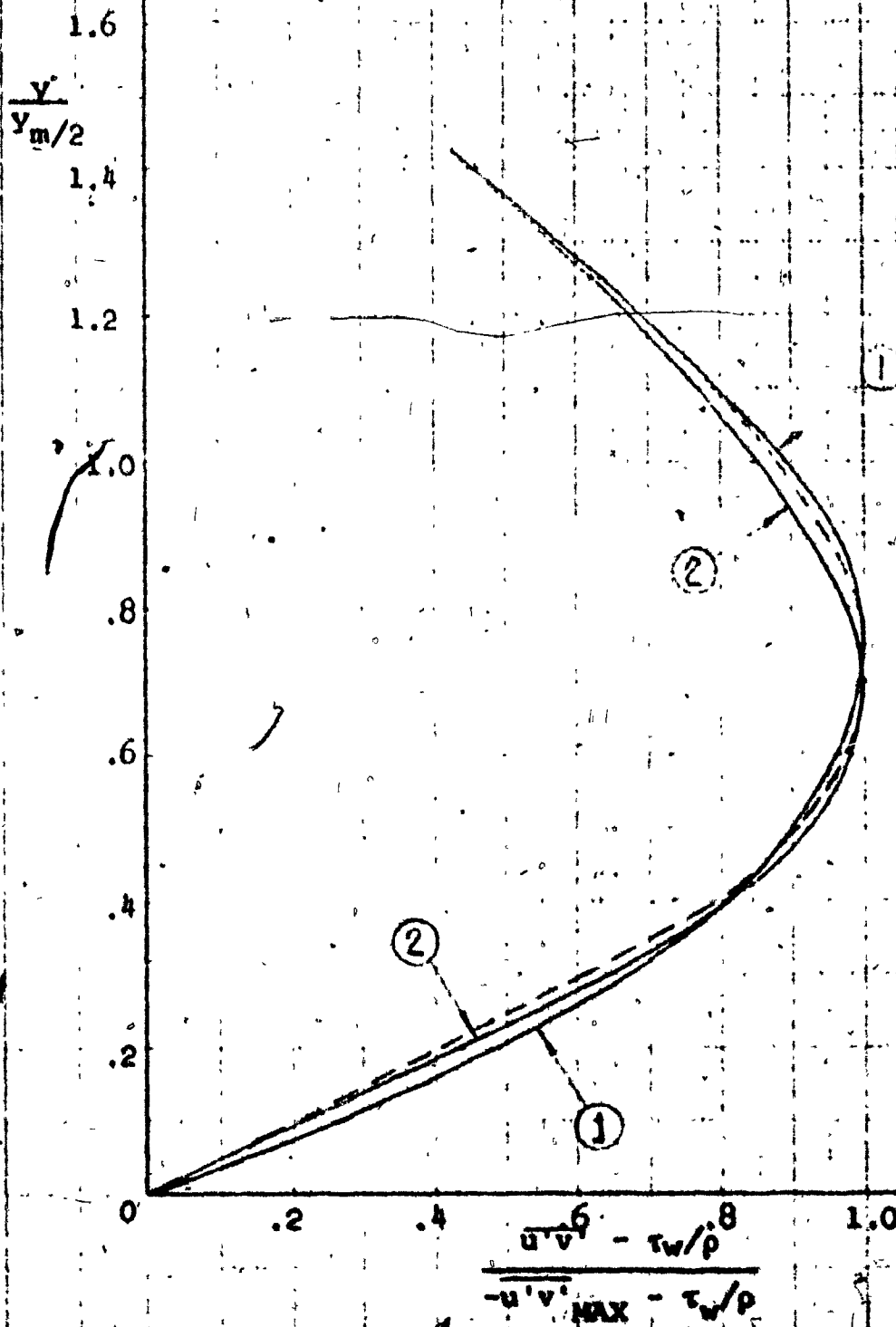
$$\frac{-\overline{u'v'}}{\overline{u'v'}_{\text{MAX}}} = \frac{\tau_w/\rho}{\tau_w/\rho}$$

(see also Fig. 9.17b)

Line ① $\theta_1 = 170^\circ$

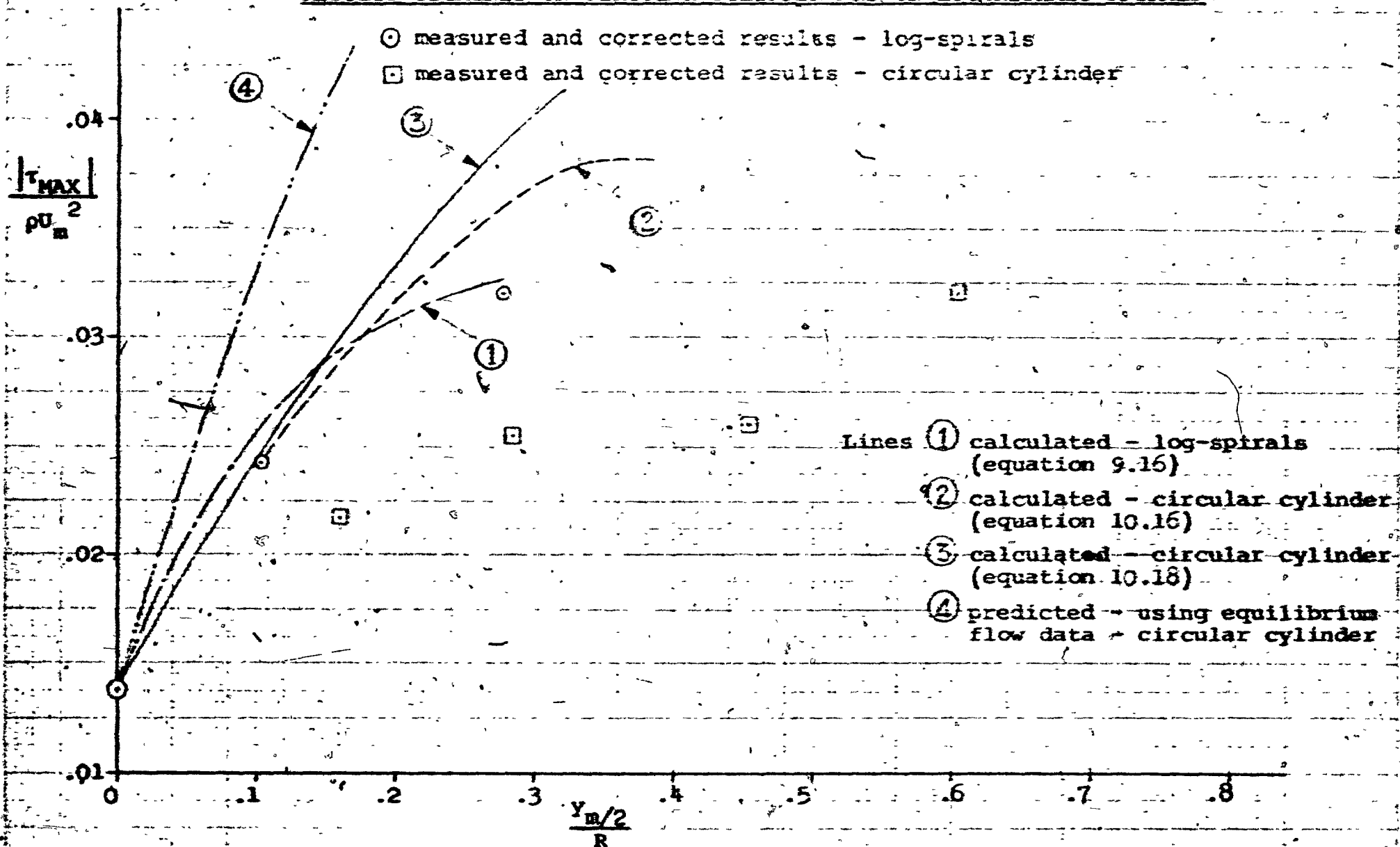
② $\theta_1 = 75^\circ$

--- Plane wall jet in still air

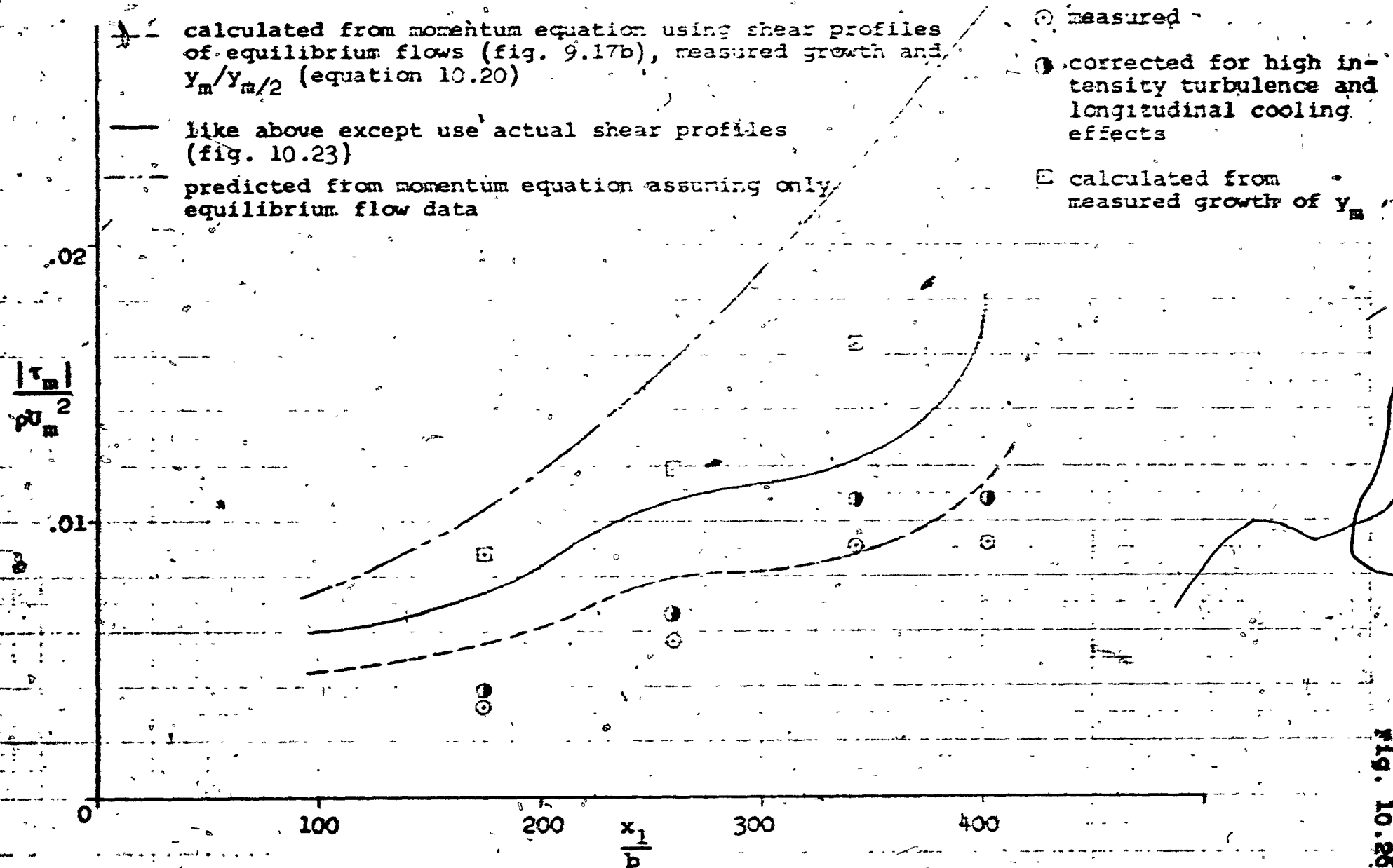


STREAMWISE VARIATION OF MAXIMUM SHEAR STRESS : COMPARISON BETWEEN EXPERIMENTAL

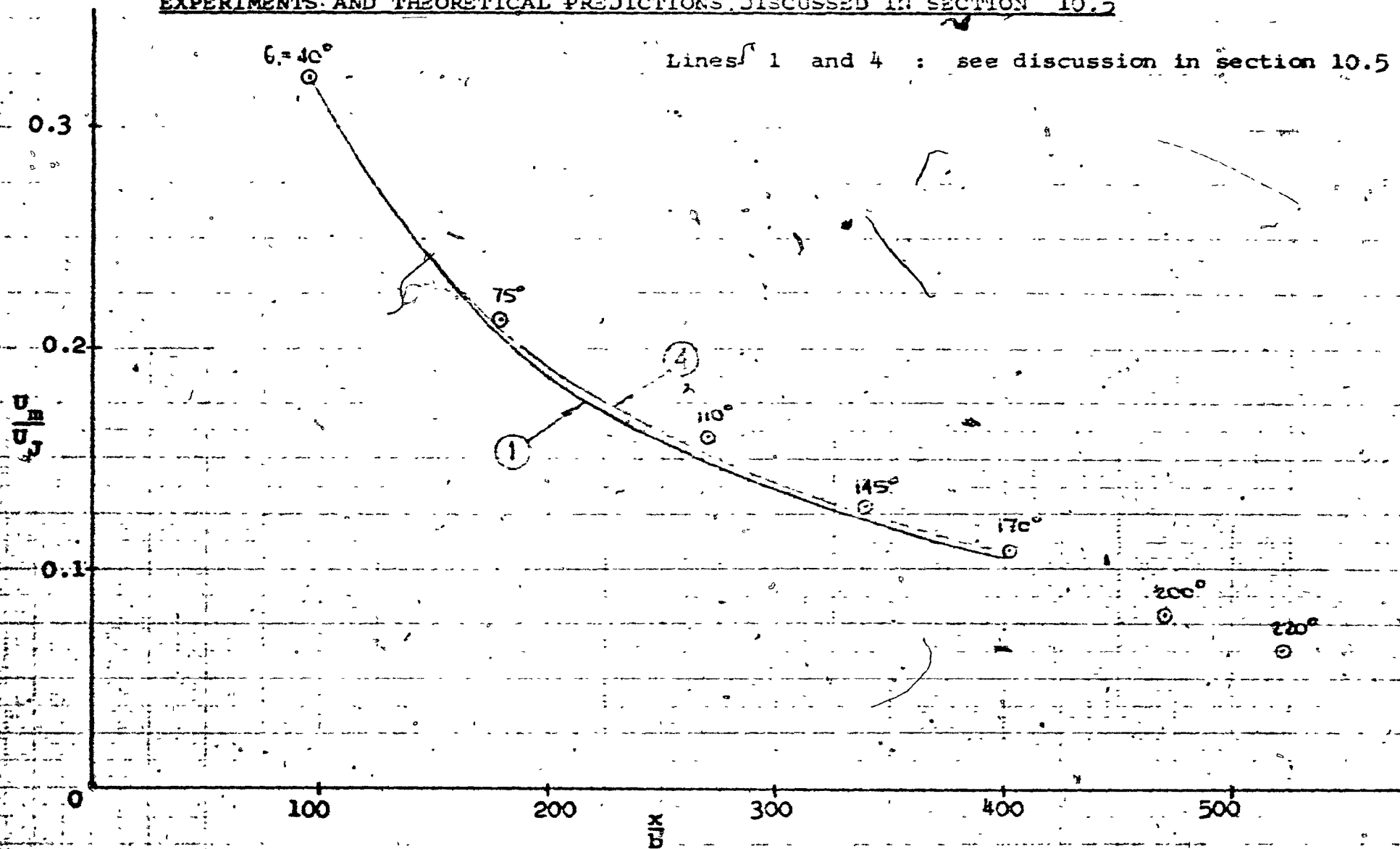
RESULTS OBTAINED ON CIRCULAR CYLINDER AND ON LOGARITHMIC SPIRALS



VARIATION OF SHEAR STRESS AT y_m WITH STREAMWISE DISTANCE ON CIRCULAR CYLINDER

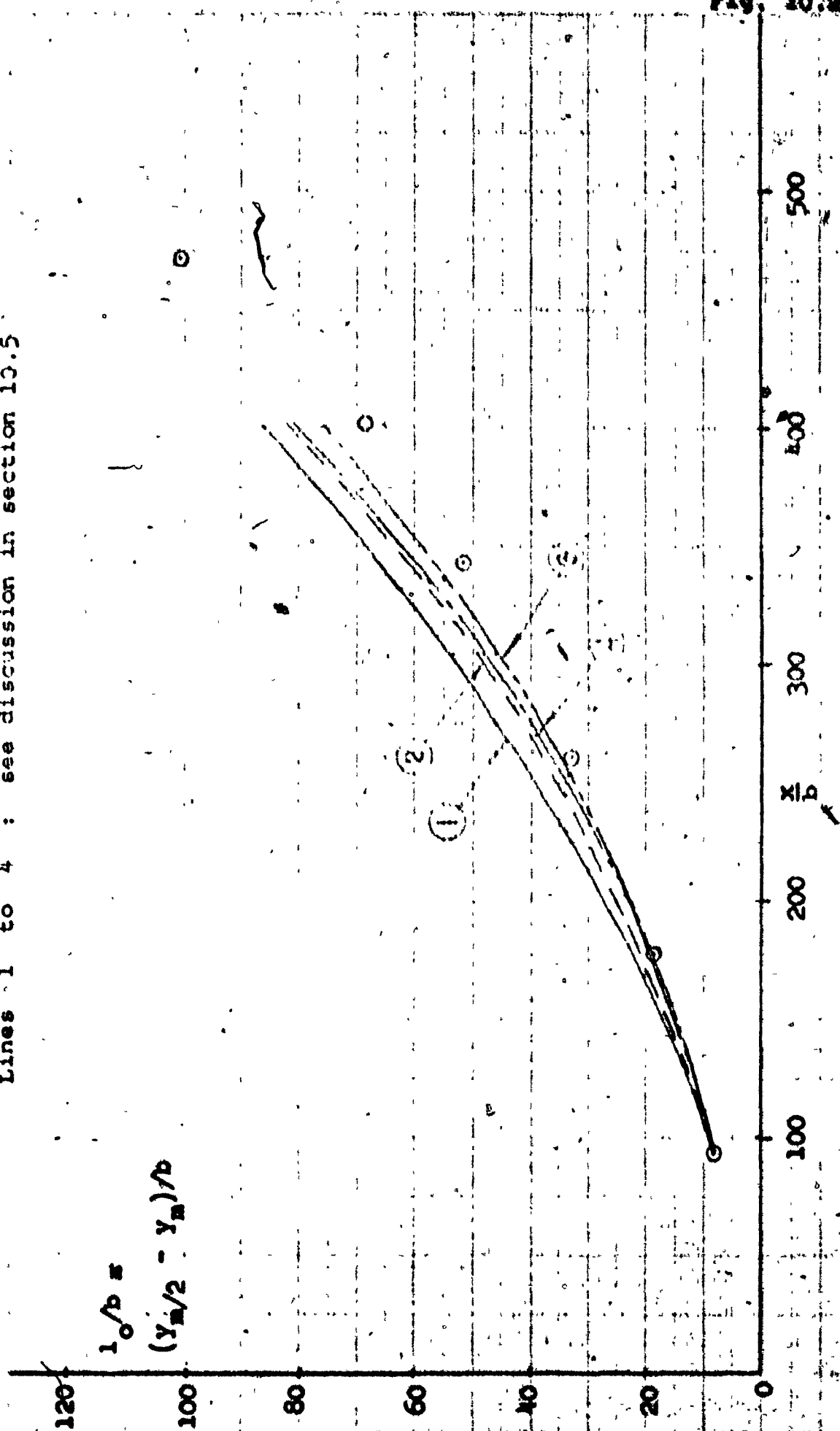


MAXIMUM VELOCITY DECAY ON CIRCULAR CYLINDER ; COMPARISON BETWEEN
EXPERIMENTS AND THEORETICAL PREDICTIONS DISCUSSED IN SECTION 10.5



GROWTH OF $(y_m/2 - y_m)$ ON CIRCULAR CYLINDER : COMPARISON BETWEEN
EXPERIMENTS AND THEORETICAL PREDICTIONS DISCUSSED IN SECTION 10.5

Lines 1 to 4 : see discussion in section 10.5



GROWTH OF INNER LAYER, y_m , ON CIRCULAR CYLINDER : COMPARISON
BETWEEN EXPERIMENTS AND THEORETICAL PREDICTIONS DISCUSSED IN SECTION 10.5

Lines 1 to 4 : see discussion in section 10.5

