AUTOMATIC CONTROL IN FREQUENCY SHIFT TRANSMISSION

Thesis submitted to the Faculty of Graduate Studies and Research, McGill University, in partial fulfilment of the requirements for the degree of Master of Engineering.

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INTRODUCTION:

The "Frequency Shift" principle has come into wide use in the past decade as a means of transmitting automatic signals over a radio link. It has been demonstrated (1) that a signal-to-noise improvement of 11 db may be obtained over an "on-off" system. The spurious sideband transmission is reduced, (2) and the problem of keying is greatly simplified as the carrier is, essentially, frequency modulated rather than being switched on and off.

This method of transmission is at present being used extensively by military and civil communication services working between fixed points. The keying units employed are not, however, readily adapted to mobile use where flexibility and compactness are required in addition to the stability and reliability which is provided.

The keyer unit in most common use (2) contains a local oscillator as part of the frequency determining circuit. This requires oven control of critical components and consequently great bulk. The mixer principle which is employed calls for crystals whose frequency differs from the transmitted frequency by 200 Kc/s, which complicates the crystal procurement problem.

Various systems have been devised to give crystal control to normal F.M. transmitters. These include the Armstrong system, discriminator control systems and frequency comparison systems. These methods have not been adapted to frequency shift transmission for various reasons as will be outlined in the following pages.

The object of this thesis is to investigate various methods of frequency control and to determine whether a system may be developed which meets the particular requirements of mobile operation of frequency shift equipment.

1. PRELIMINARY INVESTIGATION

1.1 CHARACTERISTICS OF FREQUENCY SHIFT SIGNALS

In frequency shift, as in most other keying systems, a binary code is employed to convey the information. The two conditions designated "mark" and "space" consist, in this instance, of two radio frequencies separated by a small deviation, usually 1,000 cycles / sec. or less. It is actually a special case of frequency modulation. It does, however, exhibit some very important differences from normal frequency modulation, as here listed, and consequently it requires special treatment:-

(a) Non-harmonic modulation is employed, i.e. the modulation does not necessarily repeat at equal time intervals.

(b) The modulation is not symmetrical about the arithmetic mean frequency. One of the alternate condition may obtain for long periods of time. It is to be noted that there is no "carrier" frequency, in the strict sense, in this keying system.

(c) A low modulating frequency of approximately rectangular form is employed. The teletype terminal equipment (3) utilizes a signal with a top keying rate of 23 c/s. To transmit these signals with sufficient fidelity it is considered that the fundamental, 3rd, and 5th harmonic must be passed. The modulation network must therefore be capable of responding to frequencies up to 45 c/s, and of attenuating higher frequencies which would increase the bandwidth unduly.

The above conditions make the control of the transmitted frequency quite difficult. A study was made of existing methods of controlling F.M. and frequency shift transmitters to determine whether a simple and effective method could be worked out. The various systems investigated are discussed in the following paragraphs, with particular reference to their adaptability to this type of service.

1.2 METHODS OF STABILIZING FREQUENCY SHIFT KEYERS

Within the past few years various methods have been employed for the stabilization of frequency shift keyers. The author has had some personal experience with the first three mentioned. A brief discussion and comparison of the side band distribution for these systems is given by Hatfield (2). The basic theory of operation, and a note on the suitability of each method, in view of the particular requirements, is here given.

1.2.1 "PULLED CRYSTAL" SYSTEM

The oscillator frequency is changed by shunting a capacitor across the crystal. The switching operation may be performed mechanically by a relay or electronically by a keying valve. A keyer unit employing this principle was constructed by the Canadian Signals Research and Development Establishment, Ottawa, and was used successfully on trans-Canada and trans-ocean links.

Crystal control is obtained with an extremely simple circuit. However, not all crystals are capable of being "pulled" by the required amount, and the two signals are not usually equidistant from the nominal crystal frequency. An even more serious fault is that the transmitted signal does not have a good wave form and spurious sideband frequencies are generated. This is because of the sudden transition between the "mark" and "space" frequencies, as the keying valve is driven beyond the conducting and non-conducting points. If a varying capacity were applied across the frequency determining circuit the instantaneous frequency during the transition period could be made to pass through all values between "mark" and "space" and to follow a "frequency vs. time" curve determined by the input keying wave form.

1.2.2 MASTER OSCILLATOR PLUS CRYSTAL

The operation of this keyer is illustrated by the block diagram. The frequency, f_o, of the master oscillator is maintained at a mean value of 200 Kc/s. This is mixed in a balanced modulator with the output of a crystal oscillator, f_x, to produce the carrier frequency. The crystal must therefore oscillate at a frequency 200 Kc/s above or below the required transmitted value.

The master oscillator is frequency shifted by means of a reactance tube. If a suitable low-pass filter is employed ahead of the reactance tube the side band can be made to drop off very rapidly on either side of the transmitted frequencies. This unit is the most satisfactory of the existing frequency shift keyers and is used very extensively.

The frequency stability depends upon the stability of the industance-capacity tuned master oscillator. To maintain sufficient constancy requires that this circuit and the crystals be mounted in a temperature regulated oven. This contributes considerable bulk to the unit and makes it unsuitable for mobile operation. The balanced modulator contributes some spurious radiations which are troublesome at close range. Consequently other systems have been investigated.

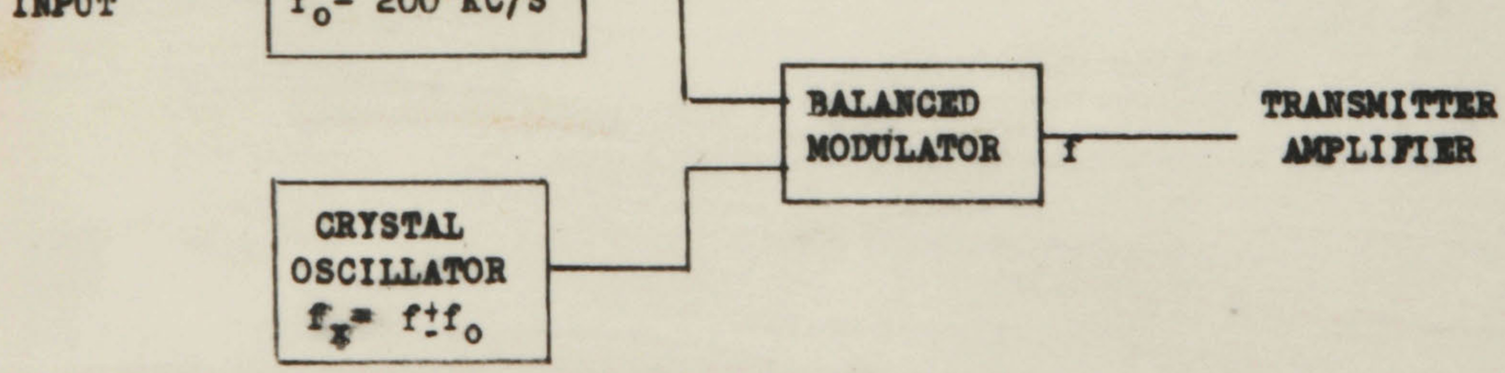

FIG. 1 FREQUENCY SHIFT KEYER

1.2.3 SWITCHED OSCILLATORS

Two oscillators differing slightly in frequency are used. Keying is applied at the buffer amplifier stages to switch between one oscillator and the other, thus giving the required shift to the transmitted signal.

In one such system two crystals are employed. They are especially ground to have frequencies which differ by the required shift value, and are equally spaced on either side of the nominal carrier values.

This system gives a sudden transition from mark to space. Furthermore there may be a time overlap or a gap between the two frequencies.

Another similar system (4) employs a crystal oscillator, and a reactance tube controlled oscillator which is held at the required frequency separation from the crystal by an AFC circuit. An audio beat between these two oscillators is passed to a discriminator which acts to maintain the required audio frequency. The mode of operation, and disadvantages are the same as for the previous case.

Either of the above circuits may be employed to actuate a locked oscillator. It would appear that this should make the system act as a single oscillator and remove the spurious conditions. This was attempted by a commercial firm but was dropped when it was found that the expected improvement was not obtained.

1.2.4 BALANCED MODULATOR

This system uses special phase shifting networks and a balanced modulator to produce a single side band from a carrier and a modulating tone. The shifting operation consists of changing the transmitted side band from one side of the carrier to the other. A special phase shifting network as described by Dome (5) gives the same phase shift to the modulation regardless of its frequency and makes this operation possible.

This scheme is also subject to a sudden transition, and hence is undesirable for the reasons given above.

1.3 METHODS OF STABILIZATION OF F.M. OSCILLATORS

1.3.1 INDIRECT F.M.

This is a method due to Armstrong (6) in which the primary frequency source is a crystal oscillator. The oscillator signal is phase modulated by an audio signal which has been integrated. This causes the instantaneous frequency, rather than the instantaneous phase to be a direct function of the audio amplitude. Hence frequency modulation is obtained with direct crystal stabilization.

In order to keep the distortion below 2% the total phase deviation must not be allowed to exceed 30 degrees. For this reason the modulation index must be kept low, in fact low enough that essentially only the 1st. order side band is produced for each modulation frequency. This requirement is specially limiting for low audio frequencies, and the lowest frequency to be passed is an important design consideration.

In order to increase the modulation index to a value suitable for transmission a considerable amount of frequency multiplication must be employed. This widens the band by bringing more side bands into prominence, as may be observed from a consideration of the Bessel functions for increasing values of modulation index. Frequency conversion is also employed so that the bandwidth may be maintained as the carrier frequency is lowered. The bandwidth is then further increased by additional frequency multiplication.

It can be seen that quite an elaborate system of multipliers and converters is required with this system. For a 23 c/s signal and shift of 1,000 c/s it can be shown (7) that a multiplication of 100 would be required.

However, the main objection to this circuit is that it produces an integration upon the audio signal. As F.S. signals are asymmetrical about the "carrier" the instantaneous frequency may be on one side of the carrier for much more than half the time. This would give rise to a net phase shift, which may be of a very high order, for a given period of keying. However it is known that the phase modulator can give only a very small maximum phase deviation, and therefore this system is unsuited to this use.

1.3.2 DIRECT MODULATION - DISCRIMINATOR CONTROL

A master oscillator is used as the source of the R.F. signal. Modulation is applied directly to the M.O., usually by means of a reactance tube. The oscillator is maintained at correct mid-frequency by a direct current feedback from a discriminator circuit to the reactance tube. Transmitted frequency stability is largely dependent upon the stability of the discriminator.

In one such system (8) the oscillator and the discriminator circuit both operate at the transmitted frequency. The discriminator consists of two crystal filters working into differentially connected detectors. The difficulty of obtaining closely matched crystals makes this system suitable only to the case of a large transmitter operating on a single frequency.

In another system (9) the M.O. (which is at the transmitted frequency) is heterodyned with a crystal to produce a low intermediate frequency. A variation of a given percentage in the discriminator circuit elements will therefore contribute a much lower percentage change in the output frequency, and a fairly high degree of stability is achieved. A crystal filter may likewise be used in this system. The restriction of the preceding paragraph does not hold as the I.F. can be maintained at a fixed value for all values of transmitted frequency.

These systems are both used commercially (9) for the control of F.M. transmitters. However, they are used to maintain the mean carrier frequency and hence are not directly applicable to this asymmetrical case.

If this system could be modified to hold a given instantaneous frequency it would then be suitable for the control of frequency shift keying. This possibility is discussed at length in the body of this thesis.

1.3.3 DIRECT MODULATION - FREQUENCY COMPARISON CIRCUITS

Various control circuits are in use which compare the mean transmitted frequency with a crystal at the required frequency, or a sub-multiple thereof. A device is required which will indicate the error both in magnitude and direction. A correction is applied to the master oscillator by means of a variable reactance with electronic or motor control. The advantage of motor control is that the error may be brought very close to zero at which point the motor will rest in equilibrium. Special compact motors have been developed for this purpose. In the normal electronic circuit a continuous correcting force must be applied in order to reduce the error to a small fraction. To produce this force requires that a small error be maintained, so that the error cannot be brought to zero. However electronic "memory" or "integrating" circuits (8) can be made to simulate the action of motors and bring the frequency as close to zero as is consistent with the sensitivity of the control system.

Some of the presently employed frequency comparison systems are described briefly in the following sections.

1.3.4 PHASE DISCRIMINATOR

By utilizing a phase comparison control circuit it is possible to prevent the mean frequency from drifting from the crystal standard by a single cycle over the entire period of operation. However the carrier must first be stripped of its modulation, as this has the effect of reducing the carrier to as low as zero and of introducing a considerable number of sidebands. This can be done by reducing the carrier frequency by means of divider circuits. This reduces the deviation, ΔF, while the modulating frequency, f, remains unchanged. Hence the deviation ratio ($M_f = \frac{\Delta F}{f}$) is reduced in direct proportion to the amount of division. For a 40 mc/s F.M. transmitter with a deviation ratio of 4,000, a division by 8,000 gives a deviation ratio of 0.5. This corresponds to a carrier reduction of 1:093, first order side bands of relative amplitude 0.24 and negligible higher order side bands. This condition is considered to be satisfactory and gives a frequency at the discriminator of $\frac{40,000}{8,000} =$ 5 Kc/s, which is typical.

The operation of the phase discriminator may be seen by reference to figure 2. This circuit is seen to be analogous to the Foster-Seeley (10) phase discriminator when the two input signals are of the same frequency. Hence the output will be a function of the phase of E_3 with reference to E_1 and E_2 (which are in anti-phase). The output from the diode V_1 is proportional to E_2 and E_3. Hence zero d.c. output is obtained when E_3 is at 90^o to both E_1 and E_2. A change in the relative phase of E_3 will cause a net positive or negative voltage at the detector output depending on the direction in which the phase shifts. This output may be used to control a reactance tube bias. By utilizing phase variation as the source of correcting voltage the controlled oscillation can be made to follow the reference frequency cycle for cycle.

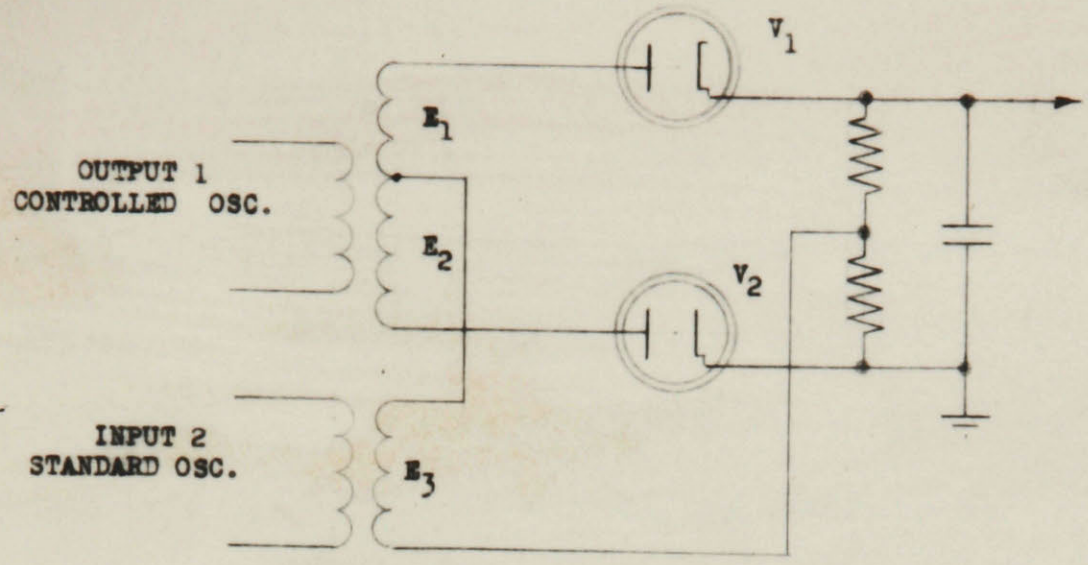

FIG. 2 PHASE DISCRIMINATOR

1.3.5 PHASE DISCRIMINATOR UTILIZING PULSES

By a slight modification of the above circuit a more flexible system is derived which has several applications. A pair of triodes is employed, and the circuit is biased beyond cut-off. The voltage E_3 is made sufficiently large to drive the tubes into the conducting condition for a very small part of the cycle, thereby giving rise to pulses of current. The magnitude of the pulse current in the tubes V_1 and V_2 depends upon the instantaneous value of E_1 and E_2 respectively. These will be equal only when E_3 is at 90° to both E_1 and E_2, as illustrated. A lack of balance, indicating a shift in the phase relationship, may be used to actuate a control system. Two applications of this principle are given in paragraphs 1.3.6 and 1.3.7.

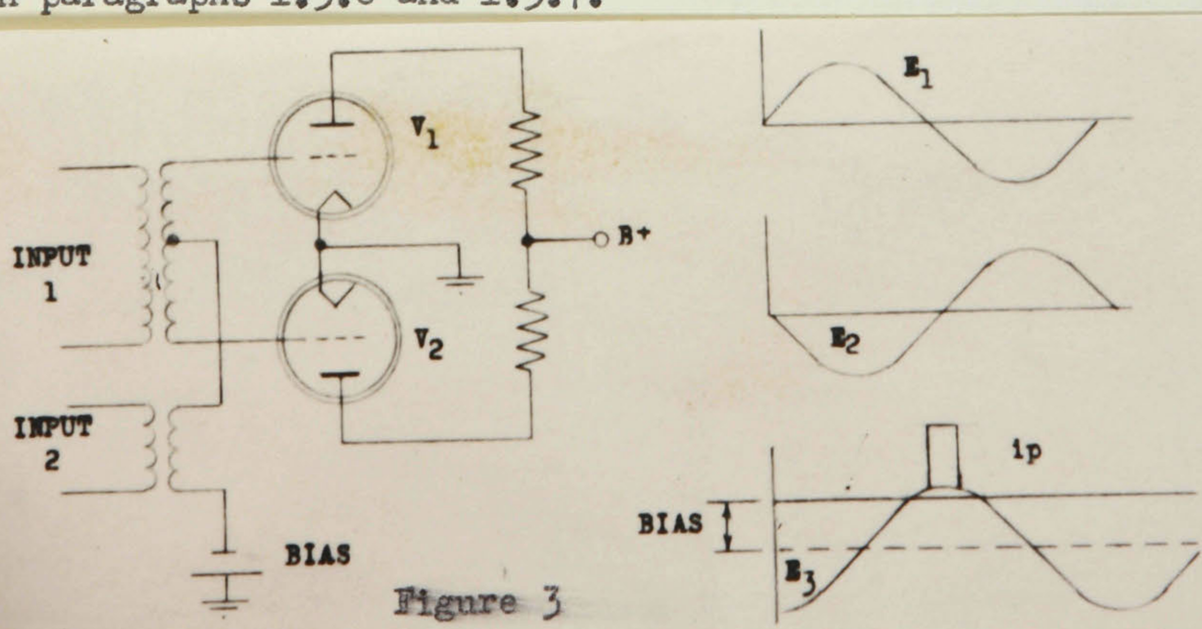

Figure 3

1.3.6 FREQUENCY SYNCHRONIZER

A method has been described ("Waveform" - M.I.T. Radiation Series No.19, page 554) which employs short duration pulses of a sub-harmonic frequency to control an oscillator. The basic principle is illustrated in the accompanying sketch. The pulse causes the tube to conduct for a very short period of time at a recurrence rate determined by the standard oscillator. The pulse amplitude at the output will be a function of the instantaneous amplitude of E_1 at the time of the pulse. A mean value is obtained at the phase condition represented by "b". If a phase shift occurs there will be an increase or decrease in the tube pulse current corresponding to the sampling occurring at "c" and "a" respectively.

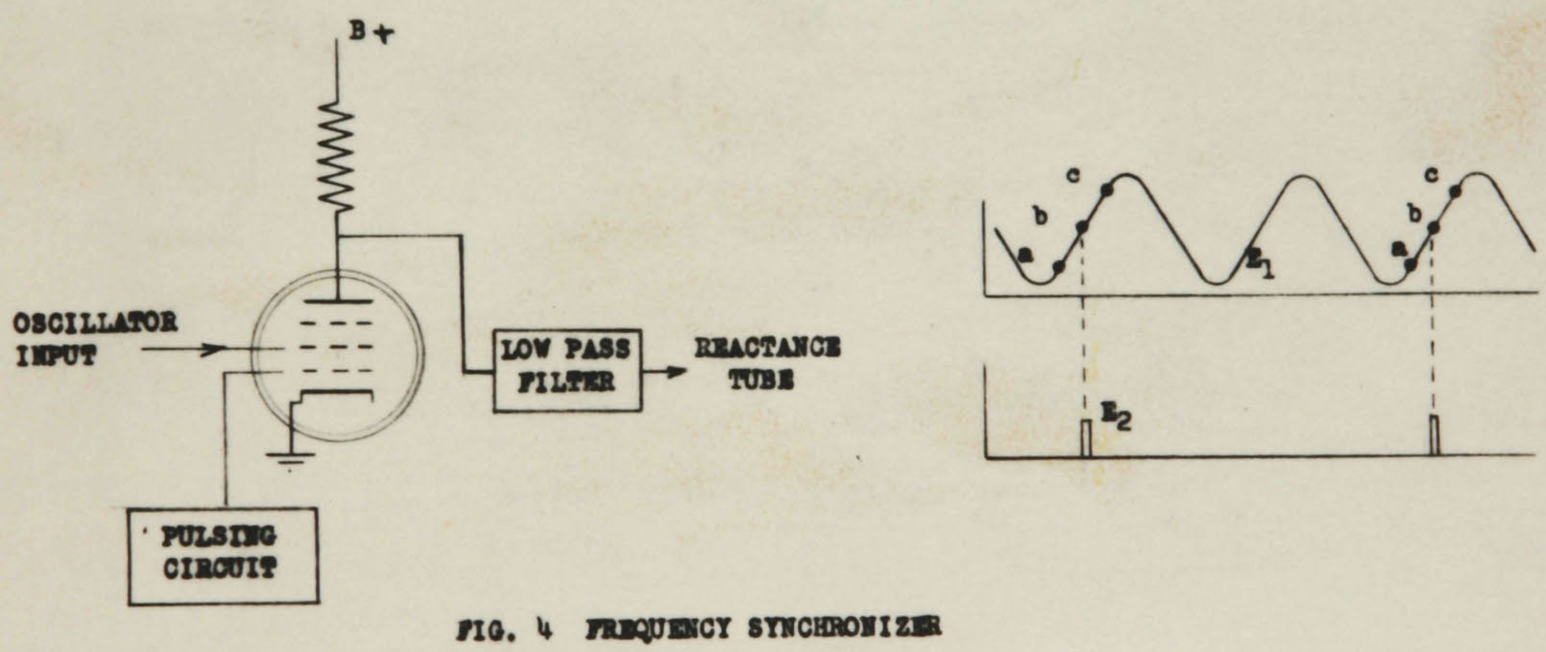

FIG. 4 FREQUENCY SYNCHRONIZER

Sampling need not take place for every cycle of the oscillator provided sufficient filtering is provided at the output. Hence a sub-harmonic control frequency may be employed.

1.3.7 PHASE DISCRIMINATOR WITH MOTOR CONTROL

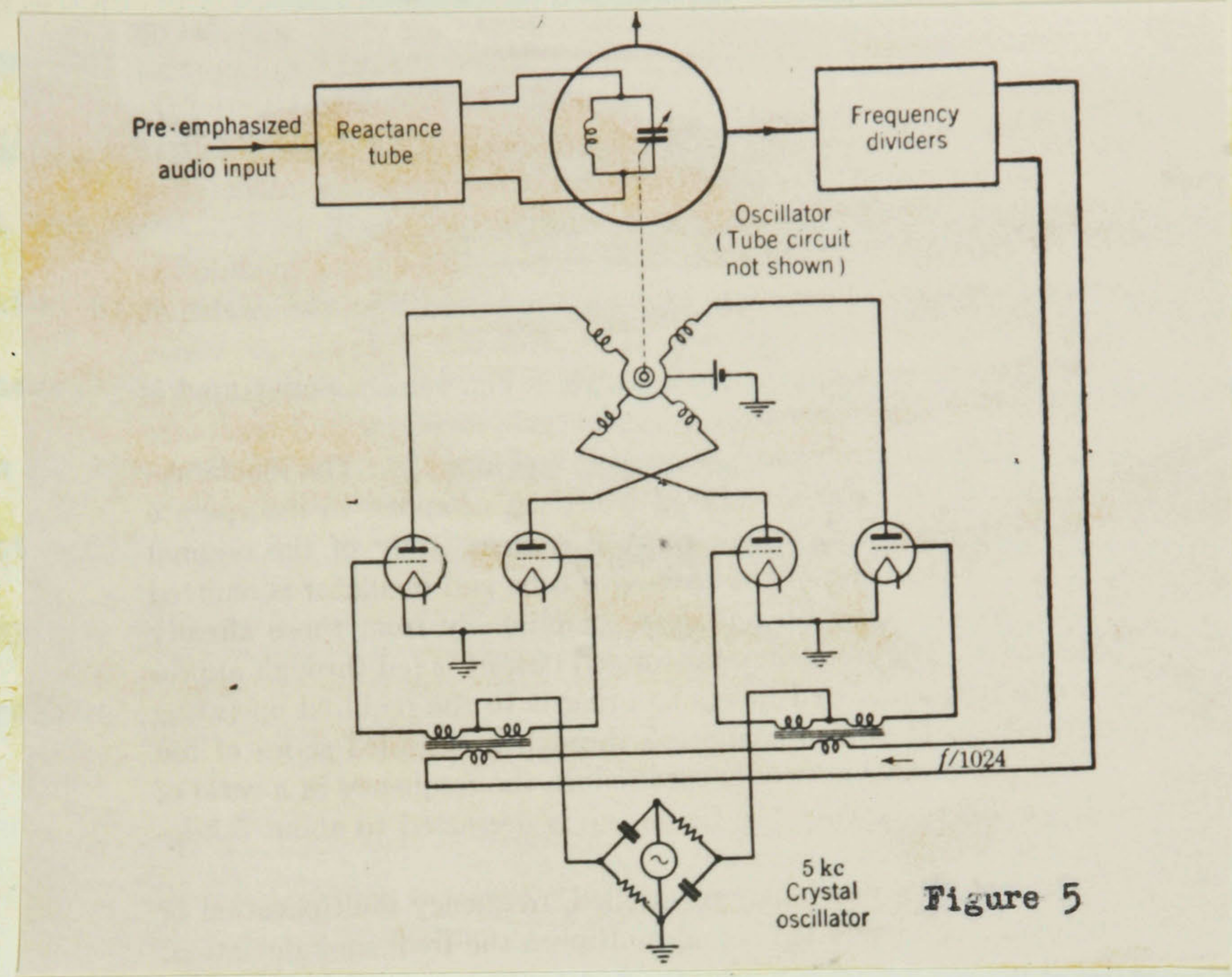

Figure 5

This circuit was developed by Morrison (11) for use in Western Electric F.M. transmitters. Two phase discriminators are employed. Tubes V_1 and V_2 supply alternating current to a pair of differentially wound field coils. If the current magnitudes are equal in both windings zero net field is produced. The same is true of the pair of windings supplied from the tubes V_3 and V_4.

The crystal standard generator produces two voltages which differ in phase by 90°, due to the phase shifting network employed. Hence if a balance is achieved in one set of windings it will not be achieved in the other set. There will consequently be a resultant field set up by the two sets of field windings whose direction is dependent upon the phase relationship between the standard and the controlled frequency. This field may vary by the complete 360° and, in fact, if the frequencies are different a revolving field is set up.

The motor shaft is connected to the oscillator variable condenser through a reduction gearing.

1.3.8 PHASE DISCRIMINATOR WITH ELECTRIC CONTROL

It has been shown that the phase discriminator comparison circuit must be operated at a very low frequency because of the modulation on the carrier. This necessitates a great amount of frequency division and consequent complication of the modulator. A system which is used in the Brown Boveri F.M. transmitters and described by Guanella (12) operates at the transmitted frequency and employs electronic rather than mechanical coupling, to the control circuits.

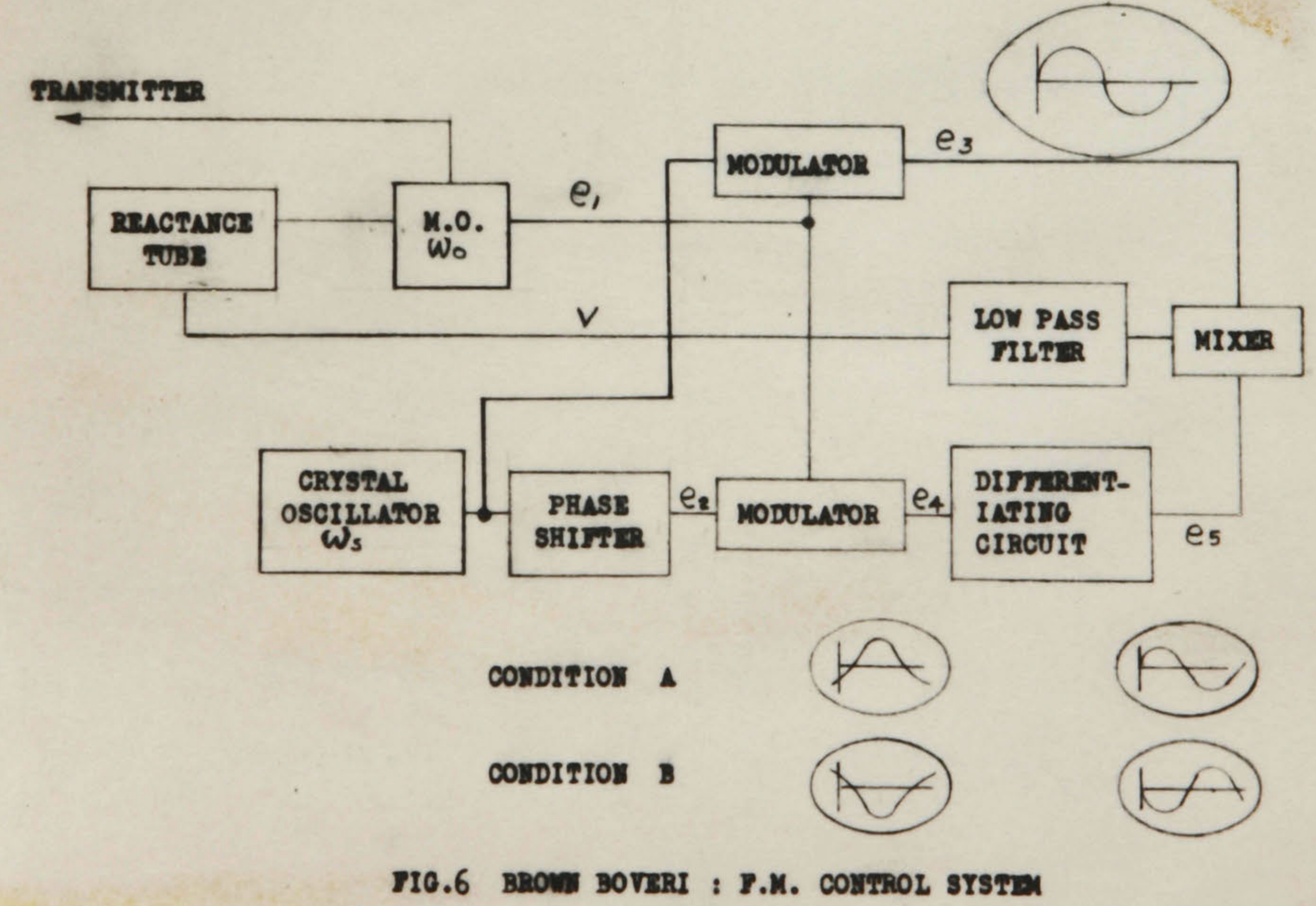

FIG.6 BROWN BOVERI : F.M. CONTROL SYSTEM

The motor shaft is connected to the oscillator variable condenser through a reduction gearing.

1.3.8 PHASE DISCRIMINATOR WITH ELECTRIC CONTROL

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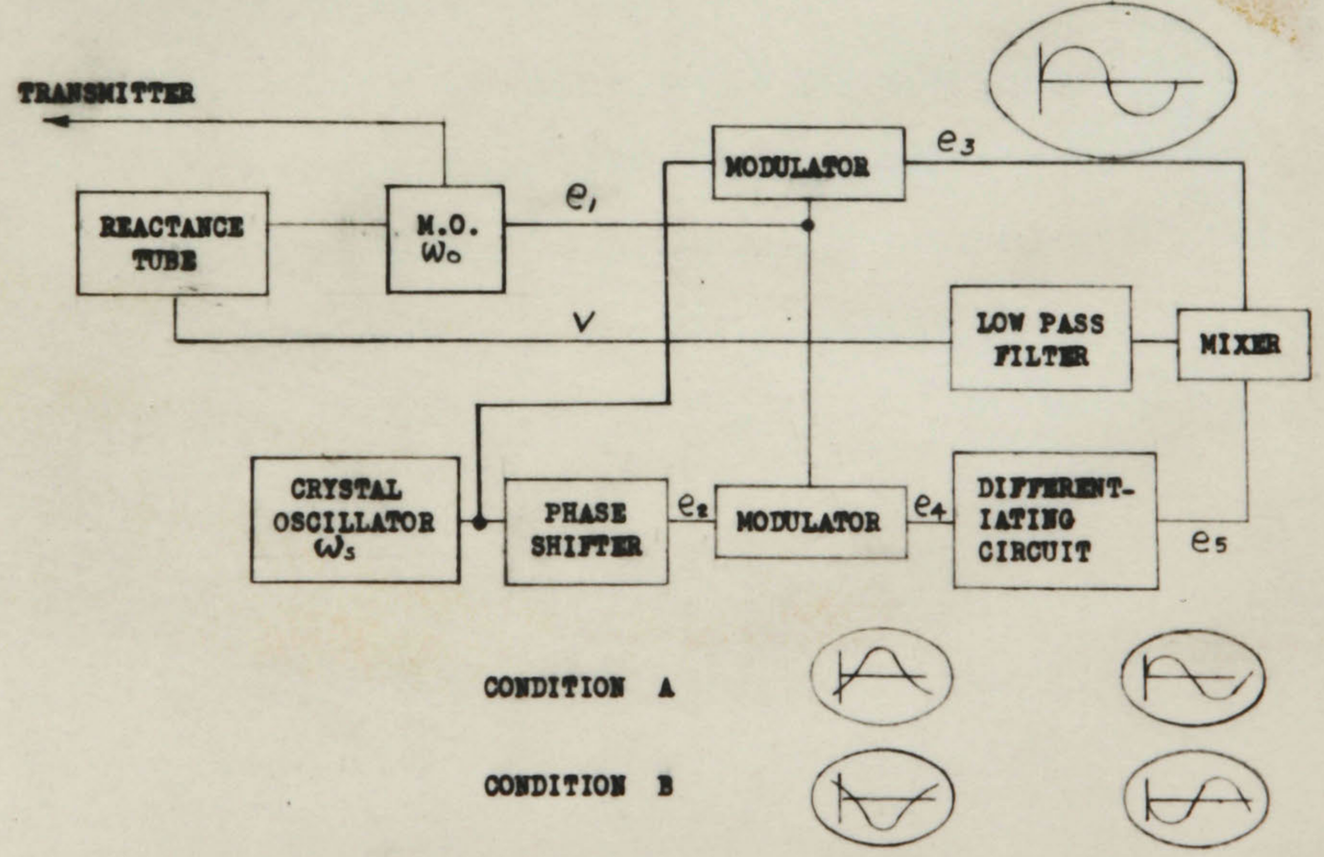


FIG.6 BROWN BOVERI : F.M. CONTROL SYSTEM

By referring to the figure 6 it can be seen that the system is the same as the Morrison circuit up to the differentiating circuit. Voltage e_4 is 90° out of phase with e_3, as before, and may lag (case A) or lead (case B) depending upon whether ω_o lags or leads ω_s. The differentiating circuit shifts e_4 by 90° so that e_5 is in phase or phase opposition with respect to e_3. These later two signals are then applied to a mixer, which produces a d.c. component of the modulation product which is proportional to the frequency error.

The method of operation may best be illustrated by an analysis of the voltages throughout the circuit using the symbols here given:

ω_o = M.O. frequency

ω_s = Standard crystal frequency

ω_n = Modulating frequency

ω = Change in oscillator frequency due to the modulation

ϕ = a sine function indicating phase variation of

$(\omega_o - \omega_s + \omega)$ = instantaneous frequency error.

The circuit voltages are:

$e = E \sin(\omega_o t + \phi)$

$e_1 = E_1 \sin(\omega_s t)$

$e_2 = E_2 \cos(\omega_s t)$

$e_3 = h_3 e e_1 + h_3 = E_3 \cos\left\{(\omega_o - \omega_s)t + \phi\right\}$ ...(omitting h_3)

$e_4 = h_4 e e_1 + h_4 = E_4 \sin\left\{(\omega_o - \omega_s)t + \phi\right\}$ ...(omitting h_4)

Note: h_3 and h_4 are higher modulation products which are neglected here.

$$e_5 = \frac{h_5 \, de_4}{dt} = K_5 (\omega_o - \omega_s + \omega) \cos\left\{(\omega_o - \omega_s)t + \phi\right\}$$

The modulation product of e_3 and e_5 is

$$e_6 = h_6 e_3 e_5 + h_6 = K_6 (\;_o - \;_s + \;)$$

This is a constant which is proportional to the frequency drift both in magnitude and sign. The variable component is filtered out, so that the d.c. signal obtained is:

$v = K_7 (f_o - f_s) = K_7 \Delta f$ where Δf is the difference between the standard frequency and the mean transmitted frequency.

Therefore in this system the control circuit operates with any degree of phase modulation. To limit the pull-in range it is necessary to put a limiter in the circuit at e_5.

1.3.9 WIDE RANGE PULSE CONTROL SYSTEM

Another system which is used with diathermy equipment is described by Lower (13). It has a pull-in range of 10% on either side of the mid frequency. Integral control is provided. This type of control, which is described in most books on servo-mechanism theory (14) provides a correcting force which accumulates with time and hence assures that the error eventually returns to precisely zero. In the above system, however, there is a dead spot of 1 to 2 kc/s at the mid frequency. A variable oscillator, which sweeps continuously over a wide range of frequencies is employed. The master oscillator and reference crystal oscillator are both beat with the V.F.O. giving two separate sweeping intermediate frequencies. These are applied to two crystal tuned I.F. channels. When either of the frequencies comes within the range of its I.F. channel a pulse is formed. The pulse due to the M.O. will be produced sooner or later than the pulse due to the standard oscillator depending upon whether the M.O. frequency is higher or lower than that of the standard. The crystal circuit pulse reverses the polarity of an Eccles-Jordan circuit from plus to minus. The M.O.

pulse gives an output to an integrating circuit whose polarity depends on the condition of the Eccles-Jordan circuit at the time of that pulse, and hence depending upon whether the M.O. is at a higher or lower frequency than the crystal oscillator. The filtered output of the integrating circuit is applied to a motor which performs the oscillator tuning.

1.3.10 SAMPLING DISCRIMINATOR

Another scheme which occured to the author is based on some of the previous described systems but is not mentioned in the literature in the form here presented. It is noted that the weakness of the frequency discriminator control system lies in the instability of the discriminator. If the discriminator itself could be stabilized from a standard frequency source by varying one or more of its frequency determining elements this difficulty would be overcome. This requires that the standard frequency be applied directly to the discriminator circuit, and the d.c. output used to stabilize the discriminator, which would then not be available for its main purpose of controlling the master oscillator. However it is known that a discriminator may be used on a sampling basis, as is indicated in a book by MacColl (15), hence it may be shared between the two functions. Switching of the necessary circuits may be accomplished by means of a multivibrator and gating circuits.

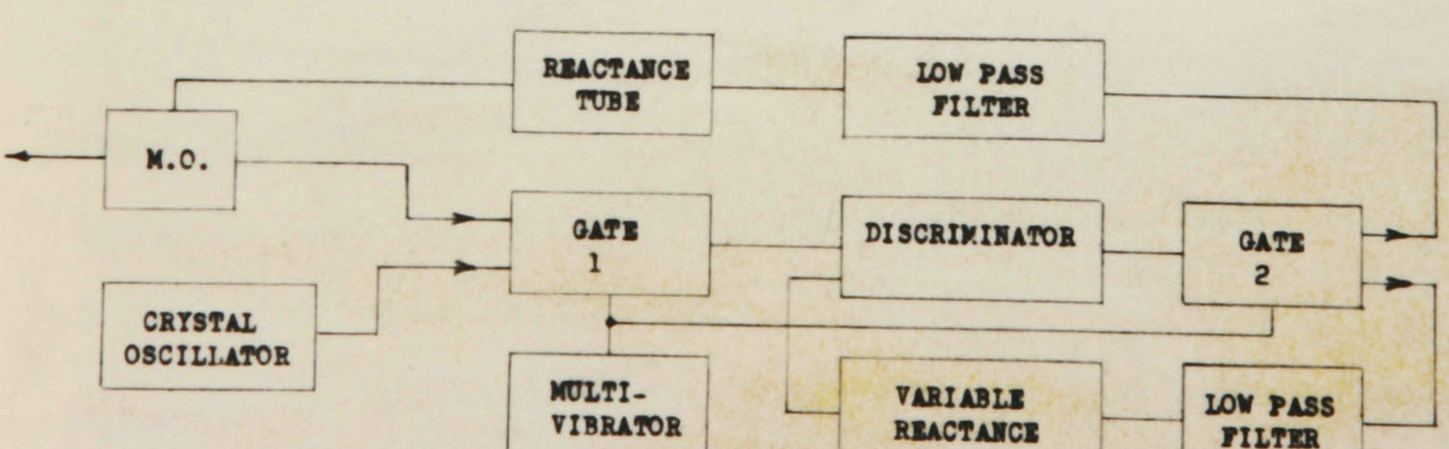


FIG.7 SAMPLING DISCRIMINATOR SYSTEM

The frequency comparison systems discussed above are normally used to control the mean carrier frequency. Those which act on a phase comparison basis must be used in this way and hence are not applicable to frequency shift. Any system with integral control is unsatisfactory as it will not allow the carrier to remain on one side of the carrier for any length of time. The sampling system may be made to hold a frequency which differs from the reference frequency by the required amount. However, it and the other systems described in paragraph 1.3.3 were thought to be far too complex for this application, and were not considered further.

1.4 PROPOSED METHOD

It is seen that none of the techniques discussed above may be applied directly to give a simple and stable frequency shift keyer with good output wave form. A modification of the method of paragraph 1.3.2 appeared to offer a solution. An investigation was carried out to determine the suitability of this system and to work out a practical circuit.

The feedback circuit is required to have a very rapid response. In addition, it must be capable of holding the signal continuously on one side of the carrier. In general the A.F.C. loop must be capable of following variations of a random keying signal with good fidelity. The system then becomes analagous to a servo-mechanism, wherein a control circuit is used to cause the output to follow faithfully the variation of an input at any velocity up to the design maximum. It can be seen that a modification to the A.F.C. feedback loop will be required.

The circuit to be investigated is of the form indicated by the accompanying block diagram. This is identical to the control system of 1.3.2 except that the filter in the feedback loop is designed to respond to the required keying wave form, and a voltage from the keying input (also suitably filtered) is added in series with the control voltage from the discriminator.

It will be shown in section 2 that the voltage produced by the feedback circuit is approximately equal to and is opposite in polarity to the keying voltage. As the feedback voltage is brought about by a change in the master oscillator frequency it follows that this frequency will follow the variations of the keying wave, regardless of the asymmetrical nature of the latter.
The operation of the control loop will be investigated from the point of view of feedback theory. A generalized theory has been developed by workers in electronic feedback and servo-mechanism fields which organizes and simplifies the problem of analysis of control system, and which lends itself to the necessary synthesis which is required in order to obtain the most satisfactory operation. This method is briefly developed in the next section. In section 3 the theory is applied to the design of the frequency shift keyer. The operational characteristics of the system, as found by actual measurement, are given in section 4.

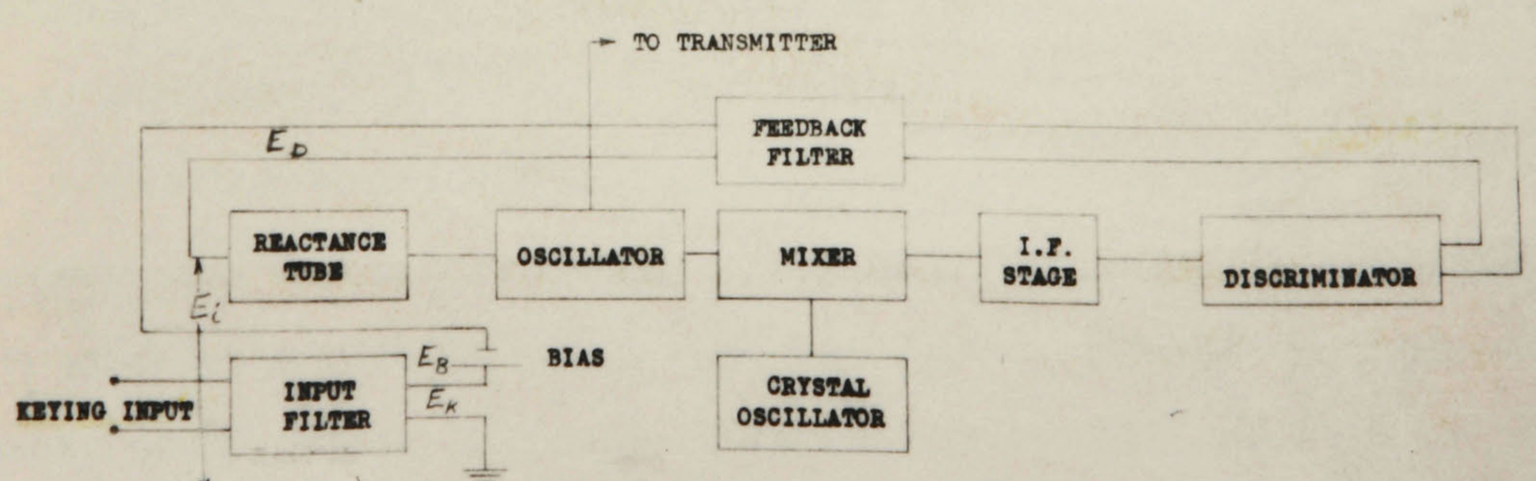


FIG. 8. PROPOSED CONTROL SYSTEM

2. CONTROL THEORY WITH APPLICATION TO CONTROL OF FREQUENCY

2.1. General

The application of closed loop control systems is by no means new. Watt's steam engine governor, developed early in the 19th. century employed such a system. Leonardo da Vinci (1452-1519) is credited with having used "servolike" devices (16). The theory of closed loop systems, and a criterion for stability in a network are discussed in a book by Routh (17) published in 1877.

Automatic control of frequency was first suggested by Round (18) of Britain who patented a very crude circuit in 1921. White (19) of U.S.A. developed a workable circuit in 1935 which brought about the first use of A.F.C. in radio receivers the following year. More practical circuits were soon developed by Travis (20), Foster & Seeley (10), and Freeman (21), which are still widely used. The discriminator circuits used in the above systems, and newer types such as the ratio detector and C.B.C. detector, are discussed in a thesis by Rioux (22).

Armstrong (6) first gave impetus to the use of frequency modulation by an article on the subject written in 1936. The special circuits required for A.F.C., i.e., the discriminator and reactance tube (or other type of frequency modulator) were soon being adapted for F.M. use. Great strides were made in their development.

Much theoretical work on feedback circuits has been carried out in recent years at Bell Telephone Laboratories by Nyquist (23), Black (24), Bode (25), and others. This theory, though first developed for feedback amplifier design, can be extended to all control problems including A.G.C. (26) and A.F.C.

During the recent war a great demand existed for the use of a wide variety of automatic control systems and much effort was expended in the development of servo-mechanisms. When security regulations were lifted a large volume of literature became available on the subject. A very comprehensive theory has been built up dealing with all aspects of control engineering. A book on servo-mechanisms by Brown and Campbell (27) forms the basis for much of the theoretical work in this thesis.

2.2 FEEDBACK THEORY

A very comprehensive discussion of feedback is given by Black (27) which indicates clearly the effect which it produces upon the fidelity and stability of an amplifier. It is shown that the effect of variations and non-linearity in the amplifier can be largely nullified. Feedback acts as a control system in that it assures that the output is a faithful reproduction of the input, or reference, signal. The input and output are compared by combining them in an additive nature and it is the difference or error voltage which is actually applied to the amplifier. If the gain is high the error will necessarily be limited to a small value.

Referring to feedback theory, which is thoroughly discussed in current literature, we have the familiar relationship:

$$\text{Amplification} = \frac{A}{1 - AB} = -\frac{1}{B}\,\frac{1}{1 - \frac{1}{AB}} = -\frac{1}{B}$$

where A = amplification without feedback

B = fraction of output feedback

AB = "feedback factor"

Hence if the feedback factor is large the amplification is practically independent of the characteristic "A", which includes non-linearities within and random disturbances imposed upon the amplifier.

It will be noted that infinite amplification obtains when AB = 1. Hence an output is obtained without any input being applied, which represents a sustained spurious oscillation. As A and B are, in general, frequency dependent complex functions, this relationship implies that

$$|AB| = 1$$

and $\phi = 2n\pi$, i.e., the signal fedback is in phase with the input

where $A = A \angle\theta$

$B = B \angle\alpha$

$\phi = \theta + \alpha$

This condition, known as the Barkhausen condition, usually occurs at only one frequency which is determined by the angle ϕ. However, it is not usual for $|AB|$ to be exactly unity at this frequency. In very simple designs the system is considered to be oscillatory if this factor is greater than unity. In more complex circuits a criterion due to Nyquist (23) must be used, which states that oscillations will occur if the complex locus of AB encloses the point (1,0). Or alternately, a method due to Routh, as discussed by Gardner and Barnes (28) page 197, which depends on a knowledge of the characteristic equation of the feedback system, may be used.

The derivation of the Nyquist condition is based upon the theory of functions of a complex variable, and will not be given here. However, it may be illustrated by a consideration of the locus of the function AB on a complex plane.

It is not possible to pass an infinite range of frequencies through electronic circuits, and an amplifier is always designed to pass a given band. All frequencies above and below this range are attenuated. There is a definite relationship between the attenuation and phase shift characteristics, as established by Bode (25) and in fact, if one is known the other can be determined directly without reference to the particular network employed. There is normally zero phase shift imposed by the network at the centre of the pass band. As the cut-off frequencies are approached both the phase and amplitude of the transmitted signal undergo variations.

In a control system it is essential to pass zero frequency. Hence the networks must form low pass filters, in which case minimum attenuation and zero phase shift is obtained at zero frequency. It must also be kept in mind that each stage of amplification contributes a shift of 180° at zero frequency. Hence an odd number of stages is required in order to obtain the necessary negative feedback.

The locus of a typical AB vector is represented in figure 9.

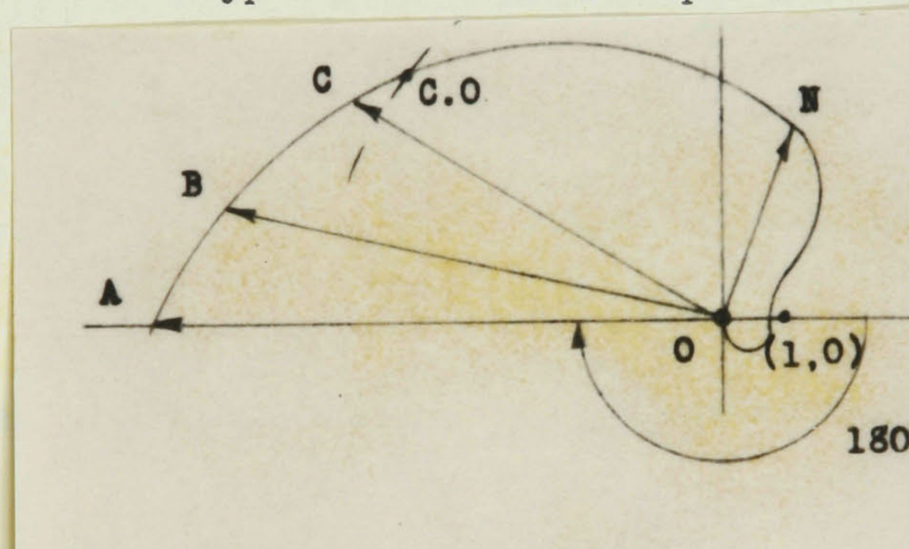


FIG.9 TYPICAL VECTOR LOCUS

For negative feedback the amplifiers give rise to a shift of 180° so that the feedback factor at zero frequency may be represented by the vector OA. For a higher frequency there will be an attenuation and a change of phase in the negative direction. This is represented by the fectors OB, OC, etc. which are arranged in order of increasing frequency. To simplify the diagram the fectors may be omitted and the locus ABC etc. shown.

If two low pass stages are used the phase shift cannot exceed 180°. If more stages are used there will be some frequency at which the network contributes 180° shift, giving a total of 360 degrees. If the length of the vector is then unity the Barkhausen condition is satisfied and oscillations result. This corresponds to the curve locus passing through the point (1,0). However the gain may be greater than unity at this frequency without giving instability. This depends upon whether the point (1,0) is enclosed by the curve. The curve in figure 10(a) represents an unstable system. However, it has been proved experimentally (29) that a circuit with the characteristics of the curve in figure 10(b) is stable, although it is only "provisionally stable" as oscillations will result if the gain is reduced.

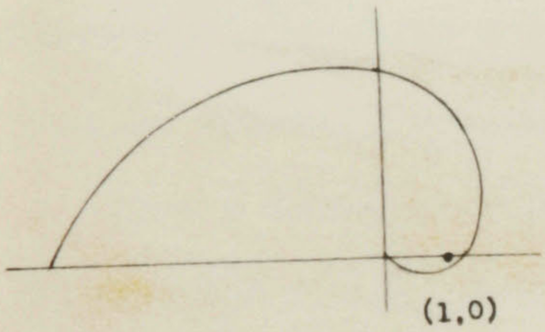


FIG 10 A
VECTOR LOCUS OF AN UNSTABLE SYSTEM

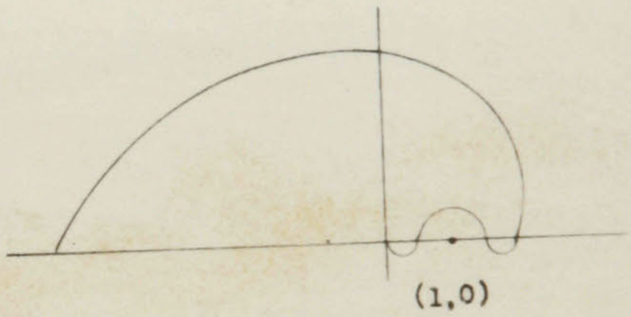


FIG 10 B
VECTOR LOCUS OF A PROVISIONALLY STABLE SYSTEM

The frequency at which the instability may occur is seen to be much higher than the cut-off frequency of the amplifier. The ratio may be greater than 100 to 1. For this reason an analysis must be carried well beyond the pass band of the system.

2.3 TRANSFER FUNCTIONS

In any practical feedback circuit a number of frequency varient elements will be present. One or more feedback paths may be employed, which loop a certain number of these elements. The problem of determining the response of the system to a given input by classical mathematics is normally very difficult. To organize and greatly simplify the work each element is assigned a transfer function based upon the Laplace transform , as discussed by Brown and Campbell. The overall frequency response for the system may be obtained from algebraic manipulation of the individual transfer functions.

The output of a circuit element is related to the input by a constant K, which represents the maximum amplification and a complex term $G_{(s)}$ which represents the effect of frequency upon the phase and amplitude of the output, or

$$\text{transfer function} = \frac{E_o}{E_i} = KG(s)$$

where $s = j\omega$ is the frequency variable of the Laplace transform . For passive networks K is normally equal to unity, unless an attenuator is employed. In an active network K represents the amplification factor.

When a number of elements are connected together, as in figure 11(a) the overall transfer function is given as:

$$KG(s) = K_1G_1(s) \times K_2G_2(s) \times K_3G_3(s) \text{ etc.}$$

For a simple feedback circuit, as illustrated in figure (b) the following relationships exist:

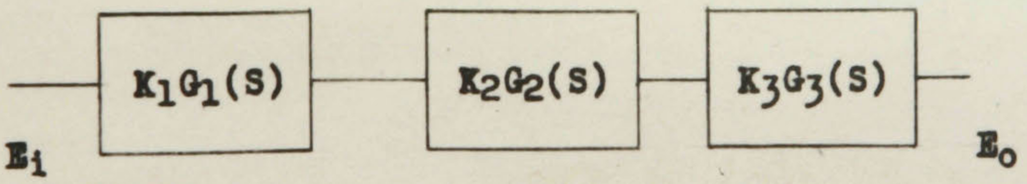


FIG. 11 A NETWORK IN CASCADE

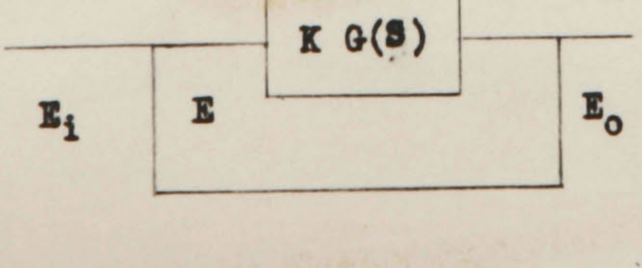


FIG. 11 B SIMPLE FEEDBACK SYSTEM

$$\frac{E_{o(s)}}{E_{(s)}} = KG(s)$$

$$\frac{E_{o(s)}}{E_{i(s)}} = \frac{KG(s)}{1 + KG(s)}$$

$$\frac{E_{(s)}}{E_{i(s)}} = \frac{1}{1 + KG(s)}$$

These formulae are seen to be identical to the feedback formulae given in section 2.2. except that transfer functions are employed in place of the symbols A and B.

As has been noted, the frequency response of a system may be obtained from the overall transfer function. At each frequency the phase and amplitude of the output signal can be determined and plotted. However the equation for the overall system will contain many terms, and this process can be very tedious. For example, if the transfer function is $K_1K_2K_3G_1(s)G_2(s)\ G_3(s) = KG_1(s)\ G_2(s)G_3(s)$ the frequency variant, i.e., G(s) functions must be analyzed to determine the amplitude and phase factor for each frequency. The amplitude factors for the three elements are then multiplied together, and the phase factors are added.

To simplify this operation a graphical method may be employed. A phase vs. log frequency curve and a log amplitude vs. log frequency curve is made for each stage. The total phase and total amplitude curves may be constructed from direct addition of the individual curves. An example of this may be seen in figure 44 .

This form of frequency response curve may also be used in place of the polar curve to investigate the degree of stability of the feedback loop. From the phase curve, the frequency at which positive feedback exists may be found. This will be the point at which the G(s) function gives a shift of 180 degrees, and will be called the "critical frequency." From the amplitude curve the degree of attenuation that takes place between zero frequency and the critical frequency may be noted. This figure represents the amount of gain that is required in order to sustain a spurious oscillation. For stable operation this must be somewhat greater than the actual gain employed. In servo systems a "gain margin" of 10 to 20 db is employed (30). However in a purely electronic system there are fewer variables and a smaller margin should be sufficient.

If a network could be made to give a rapid degree of attenuation without much phase shift it would be possible to obtain a very high control ratio. However, it is known that, regardless of circuit configuration, there is a definite relationship between phase and amplitude characteristics. For example, a single section low-pass filter gives an increase in attenuation of 6 db per octave beyond the cut-off frequency and a final phase shift of 90 degrees. A two-section filter gives 12 db attenuation per octave and a shift of 180 degrees. In order to maintain the phase shift of any network below 180 degrees the rate of increase in the attenuation must not exceed 12 db per octave.

2.4 AMPLITUDE AND PHASE CHARACTERISTICS

In order to determine the overall transfer function for the network it is necessary to analyze the individual circuits that comprise the system and to specify the amplitude and phase characteristics of each.

From the point of view of audio oscillations we may consider the A.F.C. loop, figure 8, as an audio amplifier with feedback. An audio signal at the reactance tube grid will produce frequency modulation upon the oscillator. This modulation is also present in the I.F. stages. At the detector the audio signal is recovered, and returned through the feedback path to the reactance tube. In passing around the loop the audio signal is acted upon by various frequency selective circuits. Of these the oscillator tuned circuit, I.F. stage, and the detector input circuit may be considered as carrier stages, i.e., the audio exists as side bands upon a high frequency carrier. The detector output and any filters in the feedback loop act as audio frequency networks. The effect of each of these stages will be considered in turn.

2.4.1 AUDIO STAGES

For a simple low pass filter the transfer function is derived as follows:

$$\frac{E_o}{E_i} = \frac{X_c}{R + X_c} = \frac{1}{1 + jWRC}$$

which is equivalent to

$$KG(s) = \frac{1}{1 + \tau s}$$

where τ = the time constant = RC

$s = j\omega$

hence the cut-off frequency is $\frac{1}{\tau} = \frac{1}{RC}$ radians/second

and $\frac{\omega}{\omega_{co}} = \omega RC$

then $$KG(s) = \frac{1}{1 + jx} = \frac{1}{\sqrt{1 + x^2}} \underline{/\tan^{-1} x}$$

$$= \cos \phi \, e^{-j\phi}$$

where

$x = \omega RC$

$\phi = \tan^{-1} x$

then

$\cos \phi$ = the variable amplitude component of KG(s)

$e^{-j\phi}$ = the variable phase component

The amplitude and phase curves are plotted from $\cos \phi$ and $e^{-j\phi}$ respectively. It will be noted that the general shape of either of these curves is unaltered by a shift in the cut-off frequency. For this reason a template may be used to construct the curves. It is moved along the sheet to correspond to the cut-off frequency, which is the only variable. These templates are shown in figure 43 .

A true high pass filter cannot be used in a control circuit because it would give infinite attenuation at zero frequency and the circuit would loose its holding function. The loop gain of the system is given as its zero frequency gain, which should be as high as attainable. However it is found that a modified form of high pass filter can generally give an improvement in the overall characteristics and such filters are treated in section 2.5.

2.4.2 DETECTOR CIRCUIT

It is shown by Sturley (31) that the detector circuit of figure 12(a) may be considered as a low pass filter whose time constant is determined by the effective resistance of the diode, R^1_d, and the load capacitor C. However when a low cut-off frequency is employed and the modulation is approximately in the form of a square wave there is a distinct possibility that diagonal clipping will take place. This prevents the output signal from following variations in the modulation envelope, as illustrated in figure 12(c). This effect results when the rate of decay due to the time constant of R and C in the detector load is too low to respond to the signal variations. As the diagonal clipping distorts the output waveform a large error signal will be set up at the input which may overload the system. This must be prevented if the system is to function satisfactorily.

Using the notation

R_d = diode resistance as calculated from the slope of diode $i_p e_p$ curve

R'_d = the effective resistance of the diode to the audio signal

C = diode load condenser

R = diode load resistor

m = percentage of A.M. modulation

Proof is given by Sturley that the necessary condition for no diagonal is that

$$\frac{X_c}{R} \geqq \frac{m}{\sqrt{1 - m^2}}$$

where

X_c = impedance of the capacitor at highest audio frequency used with full modulation.

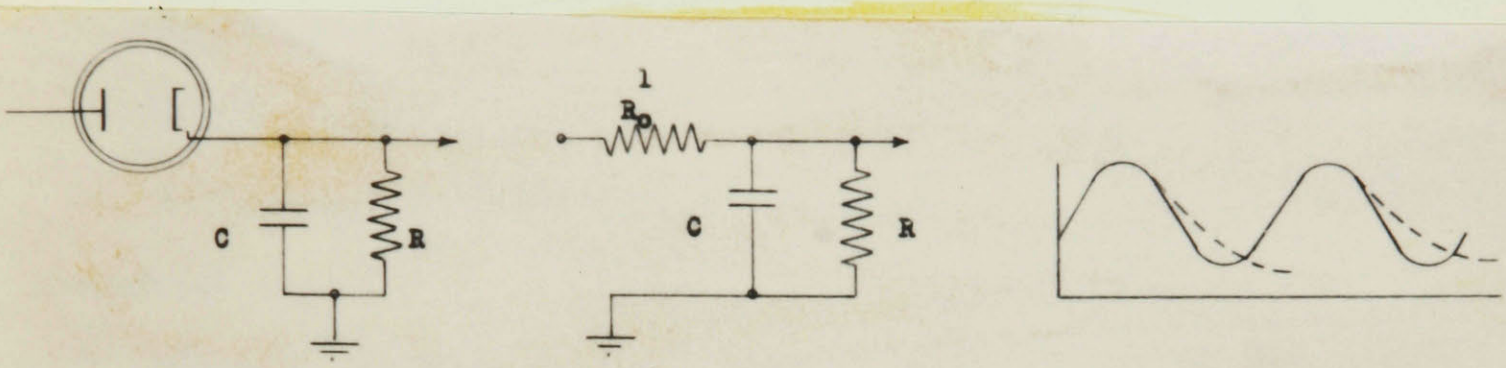


FIG. 12 A DETECTOR CIRCUIT

FIG. 12 B EQUIVALENT AUDIO CIRCUIT

FIG. 12 C DIAGONAL CLIPPING

The value of R is usually set by consideration of the loading effect of the rectifier and will be in the order of 1 megohm. The percentage of modulation for F.M. may be obtained by considering the maximum deviation in relation to the width of the detector filter. Hence the maximum value for C is fixed.

As R is large compared to R_d' the former may be disregarded in calculating the frequency response of the detector to the audio signal. The cut-off frequency is $\frac{1}{2\pi C R_d'}$, from which the amplitude and phase curves may be plotted using templates.

The value of R_d' may be determined by measuring the efficiency of rectification from the equation

$$\text{Efficiency} = \eta = \frac{R}{R + R_d'}$$

A formulae and a curve relating R_d, R and η are given by Sturley, page 351. Normally R_d need not be known. However if the desired value of cut-off frequency is not obtained it may be necessary to alter R_d'. This can only be done by altering the diode resistance R_d, and hence it is necessary to correlate the two values.

For example, assume the following values:

R_d = 600,000 ohms

C = 5,000 pfd

Efficiency = 94% (from measurement)

therefore

$$R_d' = \frac{600,000 \times (1 - .94)}{.94} = 40,000 \text{ ohms}$$

$$\frac{R_d}{R} = \frac{1}{250} \text{ (from equation, page 351 of Sturley)}$$

$$\therefore R_d = 2,400 \text{ ohms}$$

$$f_{c.o.} = \frac{1}{2\pi C R_d^1} = 650 \text{ c/s}$$

To obtain a lower cut-off frequency either C or R_d^1 must be increased. However, C is fixed by consideration of diagonal clipping. Therefore R_d' must be increased. An example calculation for a cut-off frequency of 115 c/s is given in section 3.1.5.

2.4.3 CARRIER STAGES

An I.F. filter or R.C. filter which has a given bandwidth does not pass all side bands equally. In fact, for a modulating frequency of one-half the bandwidth there is an attenuation of 3 db and a phase shift of 45° as referred to the centre frequency. It can be shown that the effect of the I.F. filter upon the audio signal is identical to that which would be produced by a low pass filter cutting off at half the I.F. filter bandwidth.

Hence if the bandwidth of the carrier filter is known, its cut-off frequency is established. The phase and amplitude characteristics may be drawn by means of the templates referred to in the previous section.

This analysis may be applied to the oscillator coil and any single tuned isolated I.F. stages. The transfer function becomes

$$KG(s) = \frac{K}{\tau s + 1}$$

where τ' is the time constant $= \dfrac{1}{\pi\,(BW)} = 2\ CR$

C = total tuned circuit capacity

R = total effective shunt resistance across the tuned circuit

BW = the tuned circuit bandwidth to half power points.

A tuned coupled circuit is more difficult to analyze. The equation of the output, and hence the transfer function, has a very complicated form. However, universal curves of the amplitude and phase characteristics for coupled circuits under various conditions of "Q" and coupling coefficient are given by Terman (32) page 160. It will be shown that these curves, taking the portion to the right of the carrier frequency, may be transferred directly to the phase and amplitude curves for the modulation transfer function. This is illustrated with reference to figure 13, which shows the effect of a filter upon the side bands. It is seen that no attenuation or phase shift is imparted to the carrier(f_c). For a side band pair consisting of an upper side band (f_u) and a lower side band (f_l) there is both an attenuation and a phase shift. The upper side band is retarded in phase by an angle ϕ and the lower side band is advanced by the same angle. The instantaneous value of the carrier envelope may be obtained by adding the vector representing the carrier and side bands where the carrier vector is considered as remaining stationary. For the time instant indicated by the diagram the carrier amplitudes are described by the formulae:

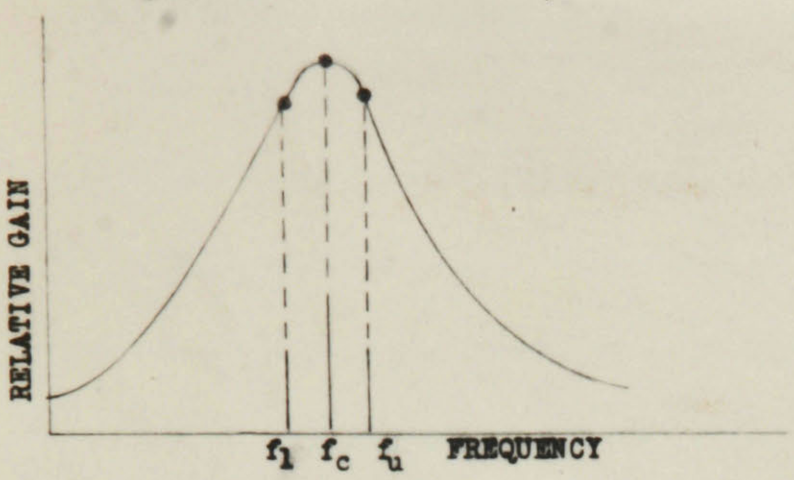


FIG. 13A NETWORK TRANSMISSION CHARACTERISTIC

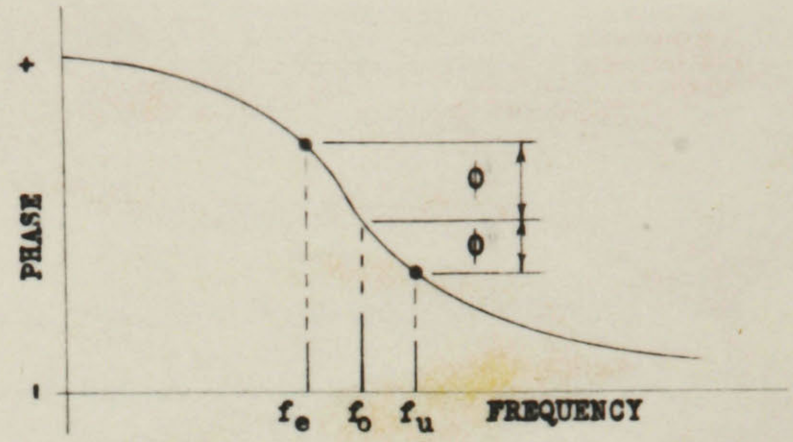


FIG. 13B NETWORK PHASE SHIFT

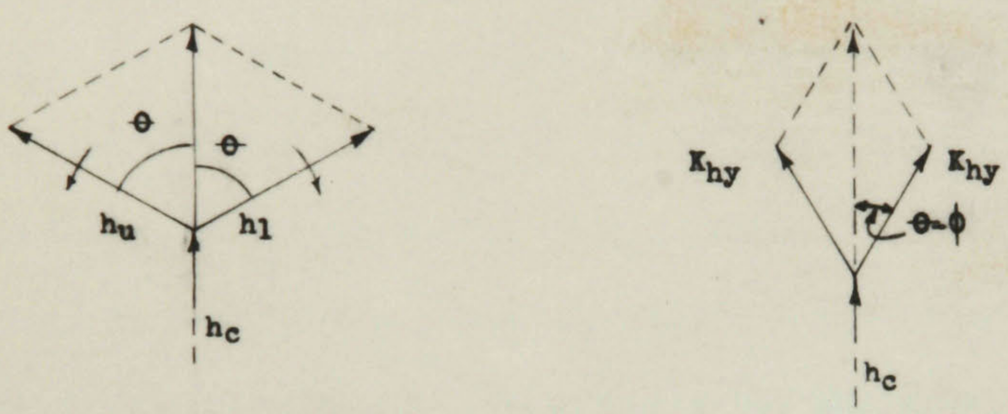


FIG. 13C VECTOR ADDITION OF CARRIER AND SIDE BAND

Input amplitude $= h_c + (h_l + h_u) \cos \Omega$

Output amplitude $= h_c + k(h_l + h_u) \cos (\theta - \phi)$

where

h_c = amplitude of the carrier vector

h_u = amplitude of the upper side band vector

h_l = amplitude of the lower side band vector

k = amplification of the side band frequencies relative to mid band amplification

ϕ = the phase shift imposed upon the side bands.

Hence the modulation envelope is retarded by the angle ϕ and attenuated by the factor "k". These values are obtained from the universal filter curve and may be applied directly to the transfer function curves.

2.4.4 NON-SYMMETRICAL FILTER

The I.F. filters considered above exhibit symmetry about the carrier both in amplitude and phase characteristics. An F.M. to A.M. converter is, of necessity, non-symmetrical. The carrier must fall upon a sloping portion of the curve so that a transition from frequency variation to amplitude variation may take place. An analysis of the effect of tuned circuits upon a frequency-modulated wave is given by Roder (33). His discussion of the converter assumes that the amplitude and phase curves are linear over a range which covers most of the important side bands. This condition holds for conventional F.M. detector circuits such as the Foster-Seeley discriminator.

Some filters, notably the crystal filter discussed in section 3, do not have linear charateristics. Furthermore the analysis must be carried to side bands far beyond the normal pass band of the filter. For this reason another method of analysis was employed, which is illustrated by the following vector diagrams.

To simplify the calculations the first side band only is considered, i.e. a low modulation index is assumed. This is justifiable for the following reasons:

(a) If oscillations are to exist they must start at a low level, and hence a study of the stability on the basis of first order F.M. side bands is sufficient.

(b) The oscillations take place at high audio frequencies, where the side bands are highly attenuated.

(c) It is the first side band which give rise to the oscillations. Higher order side bands cannot contribute to spurious oscillations at a given frequency.

The effect of a symmetrical filter upon an A.M. wave has been given in section 2.4.3. The effect of the same filter on an F.M. wave is shown in figure 14 (a) and (b). It is seen that the modulation is reduced and is made to lag as

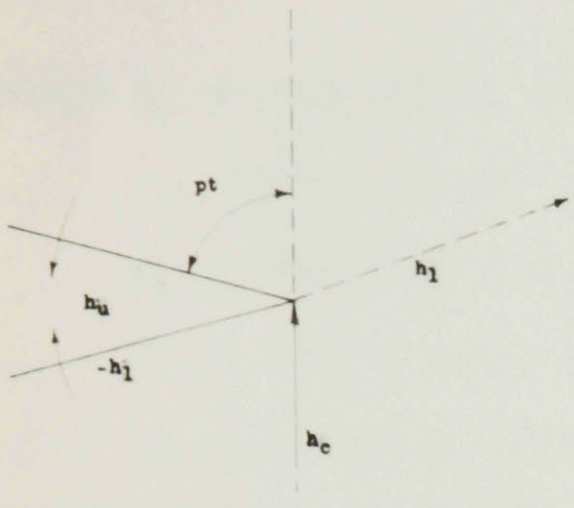


FIG. 14 A INPUT

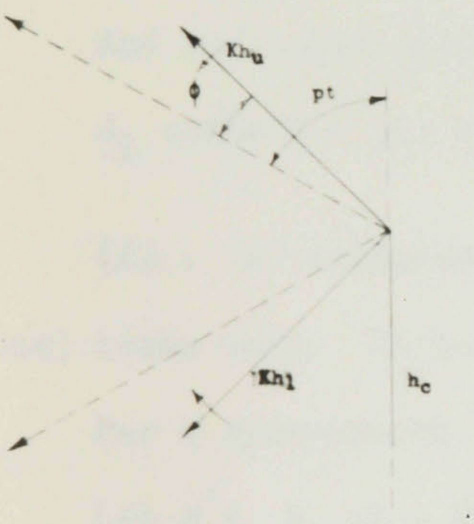


FIG. 14 B OUTPUT

FIG. 14 A & B EFFECT OF A SYMMETRICAL FILTER UPON AN FM SIGNAL

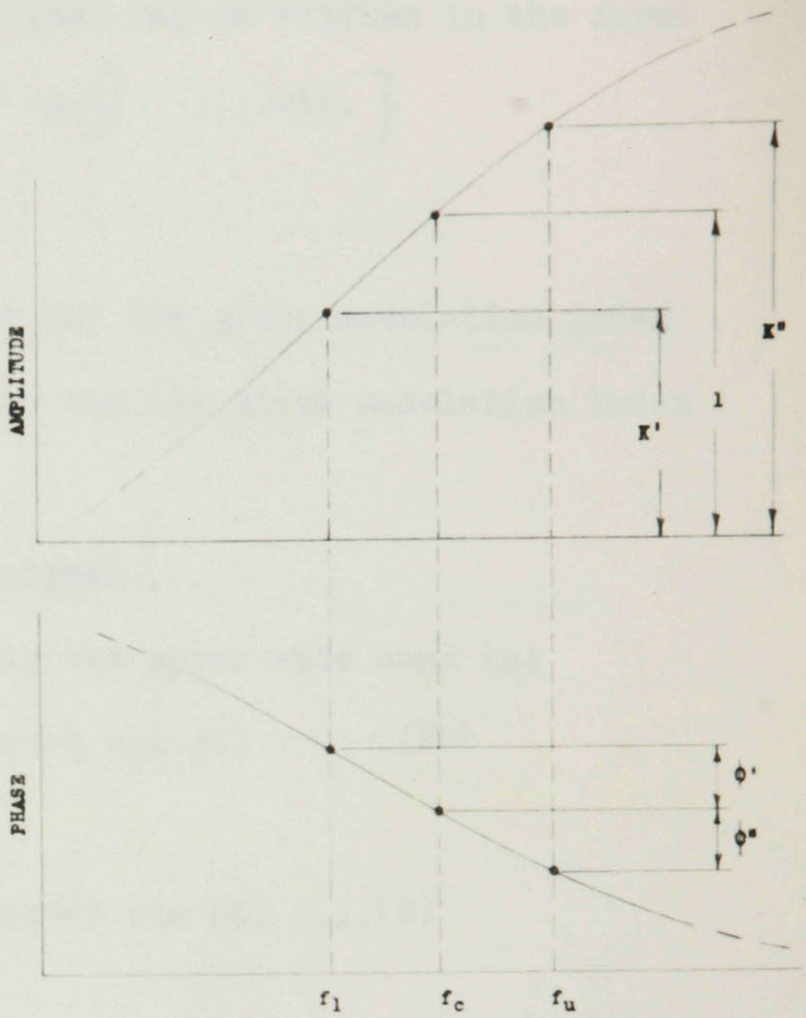


FIG 14 C NON-SYMMETRICAL FILTER CHARACTERISTICS

FIG. 14 D OUTPUT FROM NON-SYMMETRICAL FILTER

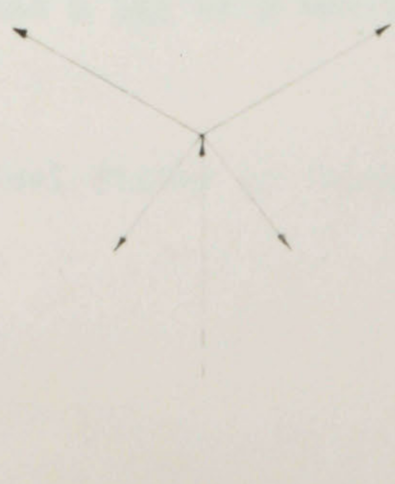


FIG. 14 E "IN PHASE" COMPONENT ONLY (PER CALCULATIONS)

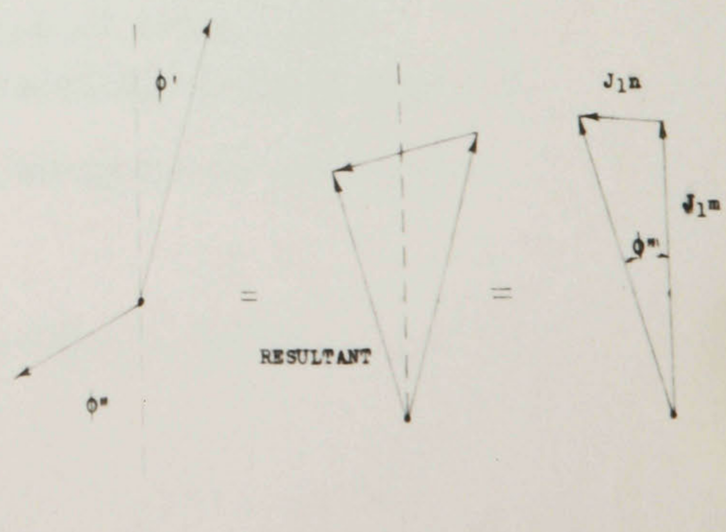


FIG. 14 F UPPER SIDE BAND ONLY, AT TIME $t = 0$

in the previous case.

The equation for a frequency-modulated signal may be written in the form:

$$E_o\left\{J_o \sin\omega t + J_1\left[\sin(\omega + p)t - \sin(\omega - p)t\right] \quad \ldots\text{etc.}\right\}$$

where E_o = peak carrier voltage

J_o = Bessel function of zero order for the given modulation index

J_1 = Bessel function of first order for the given modulation index

ω = carrier frequency

P = frequency of the modulating signal

Considering the first order side band only the upper side band is:

$$J_1 \sin(\omega t + pt) = J_1(\sin\omega t \cos pt + \cos\omega t \sin pt) \quad \cdots\text{(A)}$$

And the lower side band is:

$$J_1 \sin(\omega t - pt) = J_1(\sin\omega t \cos pt - \cos\omega t \sin pt) \quad \ldots\text{(B)}$$

(A) - (B) contains no "in phase" (sin) terms of ωt but contain quadrature (cos) terms only. It is therefore a case of F.M. with no A.M. components.

For a symmetrical filter the following equation is obtained.

Let $p't = pt - \phi$

Then for the upper side band

$$kJ_1 \sin(\omega t + p^1 t) = kJ_1\left\{\sin\omega t \cos(pt - \phi) + \cos\omega t \sin(pt - \phi)\right\}$$

for the lower

$$kJ_1 \sin(\omega t - p't) = kJ_1\left\{\sin\omega t \cos(pt - \phi) - \cos\omega t \sin(pt - \phi)\right\}$$

This gives only a quadrature component on subtraction, therefore no A.M. is introduced. The modulation has a lag of ϕ and an attenuation due to the factor "k".

The case of a non-symmetrical filter is illustrated by figure 14(c), and the equations become:

Upper side band

$$J_1K^1 \sin\left\{\omega t + (pt - \phi^1)\right\} = J_1K\left\{\sin\omega t \cos(pt - \phi^1) + \cos\omega t \sin(pt - \phi^1)\right\}$$

Lower side band

$$J_1K'' \sin\left\{\omega t - (pt - \phi'')\right\} = J_1K''\left\{\sin\omega t \cos(pt - \phi'') - \cos\omega t \sin(pt - \phi'')\right\}$$

Where K^1 = filter gain at the upper side band frequency relative to the gain at centre frequency.

K'' = relative gain for the lower side band.

ϕ^1 = phase lag imposed upon the upper side band relative to the phase shift at centre frequency.

ϕ'' = phase lead imposed upon the lower side band.

The "in phase" or amplitude modulation component is

$$J_1 \sin\omega t \left[K^1 \cos(pt - \phi^1) - K'' \cos(pt - \phi'')\right]$$

$$= J_1 \left[\frac{K^1}{2} \sin(\omega t - (pt - \phi^1)) + \frac{K^1}{2} \sin(\omega t + (pt - \phi^1))\right.$$

$$\left. - \frac{K''}{2} \sin(\omega t - (pt - \phi'')) - \frac{K''}{2} \sin(\quad t + (pt - \phi''))\right]$$

These four components are illustrated in figure 14(e). It is seen that there are two pairs of vectors. Each pair acts upon the carrier vector in such a manner as to produce amplitude modulation. Hence an FM to AM conversion has been brought about. There are still side band pairs present which give rise to frequency modulation, but these have been dropped from the analysis.

In order to obtain the AM component of the output it is necessary to consider only one side band from each pair, as the side bands are symmetrical about the carrier. The total contribution to the amplitude modulation is twice that which is obtained from one side band only. The in-phase and quadrature components must be considered together to give the amplitude and phase lag or lead of the output.

Taking the upper side band only of each pair yields

$$J_1 \left[\frac{K}{2} \sin(\omega t + (pt - \phi^1)) - \frac{K''}{2} \sin(\ t + (pt - \phi'')) \right]$$

$$= J_1 \left[\sin\omega t (\frac{K^1}{2} \cos(pt - \phi^1) - \frac{K''}{2} \cos (pt - \phi'')) \right.$$

$$\left. + \cos\omega t (\frac{K^1}{2} \sin(pt - \phi^1) - \frac{K''}{2} \sin(pt - \phi'')) \right]$$

when pt = o

Total "in-phase" component $= 2J_1 (\frac{K^1}{2} \cos \phi^1 - \frac{K''}{2} \cos \phi'') = J_1 \, m$

Total "quadrature" component $= 2J_1 (\frac{K^1}{2} \sin \phi^1 - \frac{K''}{2} \sin \phi'') = J_1 \, n$

The two upper side band vectors for the instant pt - o are shown in figure 14 (f). They are at the same frequency, $\omega + p$, and hence actually constitute but one side band, which is given in relative phase and magnitude by the resultant vector. The components m and n also define this vector so that the modulation amplitude is $J_1 \sqrt{m^2 + n^2}$ and the phase shift is

$$\phi^{111} = \tan^{-1} \frac{n}{m}$$

The output contains the term J_1, and hence it is directly dependent upon the modulation index. This causes the amplitude to vary with frequency, because for a given audio voltage at the modulator the deviation remains constant as the frequency is varied : that is,

Modulation index $= \frac{\Delta F}{f}$ where ΔF is constant.

Therefore B varies inversely with the frequency. For a low modulation index ($B < 0.2$) J_1 varies directly with B. The J_1 term therefore causes the output to decrease with an increase in modulating frequency.

If the phase vs frequency and amplitude vs frequency curves for the filter have been determined, the terms K^1, K'', ϕ^1 and ϕ'' may be read off for each side band. When these values are known the transfer function curve can be constructed from the above equations. A separate calculation is required for each modulation frequency. A sufficient number of frequencies must be chosen to plot an accurate curve over a range that extends well beyond the pass band of the filter.

2.4.5 TIME CONSTANT RELATIONSHIPS

It has been seen that all the above networks form low pass filters. The time constant of most of these circuits can be varied to some extent. It is desirable to use an available combination of time constants which gives rise to the highest control ratio.

For example, if there are three low pass networks with equal time constants, the transfer function for each may be written as $\cos \phi \, e^{-J\phi}$ where

$$\phi = \tan^{-1} x$$

$$x = \frac{f}{f_{c.o.}}$$

For a total shift of 180° each network will contribute 60°, hence ϕ at the critical frequency is 60°. Therefore $\cos \phi = \frac{1}{2}$, and $\cos^3 \phi = 1/8$ The total attenuation ratio for the three networks in series is only 8. This value can be increased if the time constant of one of these filters is changed by the factor "p".

Where f_{π} = the frequency giving a total shift of π radians.

f_1 = the cut-off frequency of the two similar networks.

then $\pi = 2 \tan^{-1} \frac{f_{\pi}}{f_1} + \tan^{-1} p \frac{f_{\pi}}{f_1}$

hence $$\frac{f_{\pi}}{f_1} = \sqrt{1 + \frac{2}{p}} \qquad \text{-------------------- 1.}$$

The amplitude at this frequency is $\cos^2\left[\tan^{-1} \frac{f_{\pi}}{f}\right] \cos\left[\tan^{-1} p \frac{f_{\pi}}{f_1}\right]$

$$= \frac{1}{\left[1 + \left(\frac{f_{\pi}}{f_1}\right)^2\right] \sqrt{1 + p^2 \left(\frac{f_{\pi}}{f_1}\right)^2}} \qquad \text{-------------- 2.}$$

Combining equations 1. and 2. yields

$$\frac{A_\pi}{A_o} = \frac{P}{2(p+1)^2}$$

where A_π = amplitude at frequency which gives 180° shift

A_o = amplitude at zero frequency.

The reciprocal, which represents the absolute value of the loss at f is

$$\frac{A_o}{A} = \frac{2(P+1)^2}{P} \cong 2p \text{ when } P \gg 1$$

Hence the less which can be obtained before the shift becomes 180° is seen to increase with an increase in the factor P. The variation for values of P greater than and less than unity is shown in figure 15.

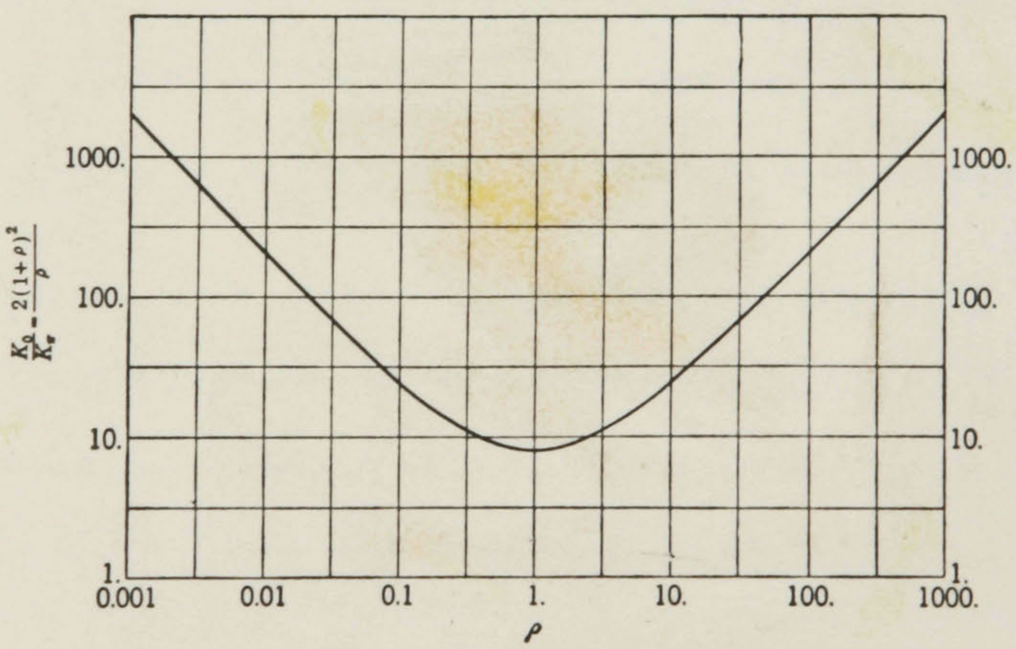


Figure 15

2.5 COMPENSATING NETWORKS

2.5.1 GENERAL

In most cases it is found that the overall transfer function of a control circuit will not have the best characteristics until some form of compensating network has been added. The transient response and the degree of stability of the system may be modified and hence improved upon. The technique of performing the necessary synthesis is described fully by Brown and Campbell.

The stability may be expressed in terms of the gain margin, and phase margin. The gain margin is the difference between the gain employed and that value which satisfies the Barkhausen condition for oscillation. The phase margin is the difference between 180° and the angle introduced by KG(s) when $|KG(s)|$ is unity. These values may be determined from a polar plot of KG(s) as in figure 47 or from the log curves of figures 44 and 45.

However, these factors are not sufficient to ensure good design. A system may be stable and yet have a response peak at some high frequency, that is, although the system is not oscillatory there may be some frequency at which $\frac{E_o}{E_i}$ may be considerably greater than the value at zero frequency. This causes a damped oscillation to be present in the transient response.

In order to ascertain the actual degree of stability the polar plot of KG(s) may be employed. A series of M circles are drawn as in figure 16. Each M curve represents the locus of a constant value of $\frac{KG(s)}{1 + KG(s)}$. Hence if a plot of KG(s) follows one of these curves, the gain $\frac{E_o}{E_i}$ will remain constant. If the system has a large resonant peak its characteristic as plotted on the KG(s) plane will pass into a high M region.

It is convenient to employ the log base curves to arrive at the over-all transfer function and to determine the gain and phase margins. This must be supplemented by a polar plot to determine whether excessive responses will be obtained.

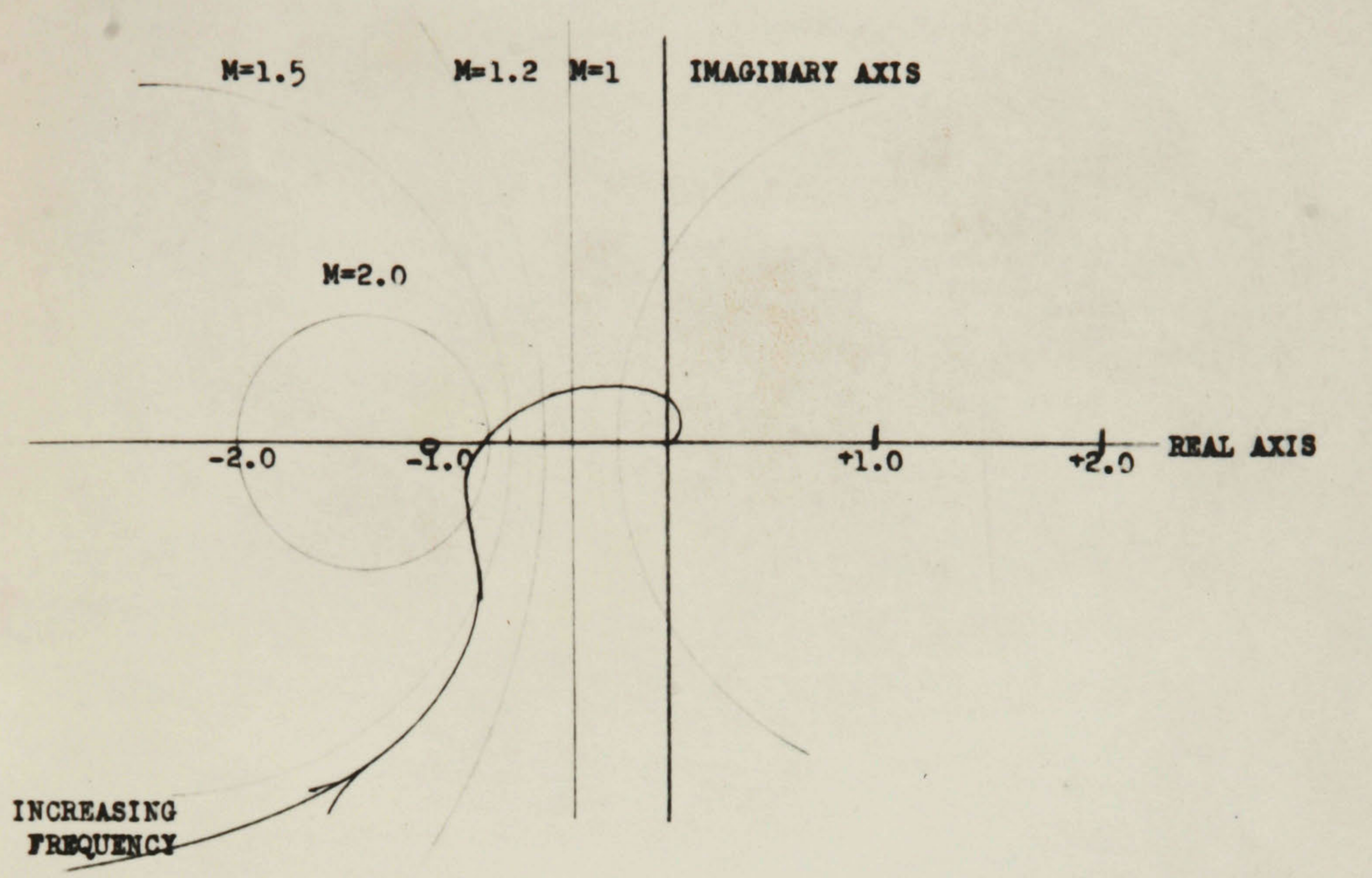


FIG. 16 M CIRCLES SUPERPOSED ON THE KG LOCUS

2.5.2 AUDIO FREQUENCY NETWORK

It was seen that the transfer functions encountered in the control system are all essentially low pass filters. The network which is employed to improve the overall characteristics will consist of one or more modified high pass filters. The purpose of this network is to modify the phase and amplitude functions in such a manner that sufficient attenuation is obtained before the phase shift becomes too close to 180°. A low pass filter causes a lag in phase and a high pass filter produces a lead. Hence a reduction in the total phase shift can be brought about.

The network shown in figure 17(a) may be used for the purpose. If C_2 and R_1 alone were employed suitable high frequency characteristics would be obtained, but there would be infinite attenuation at zero frequency. With the addition of R_1 the circuits act as an attenuator at low frequencies and the constant of the transfer function is $K = \dfrac{R_2}{R_1 + R_2}$

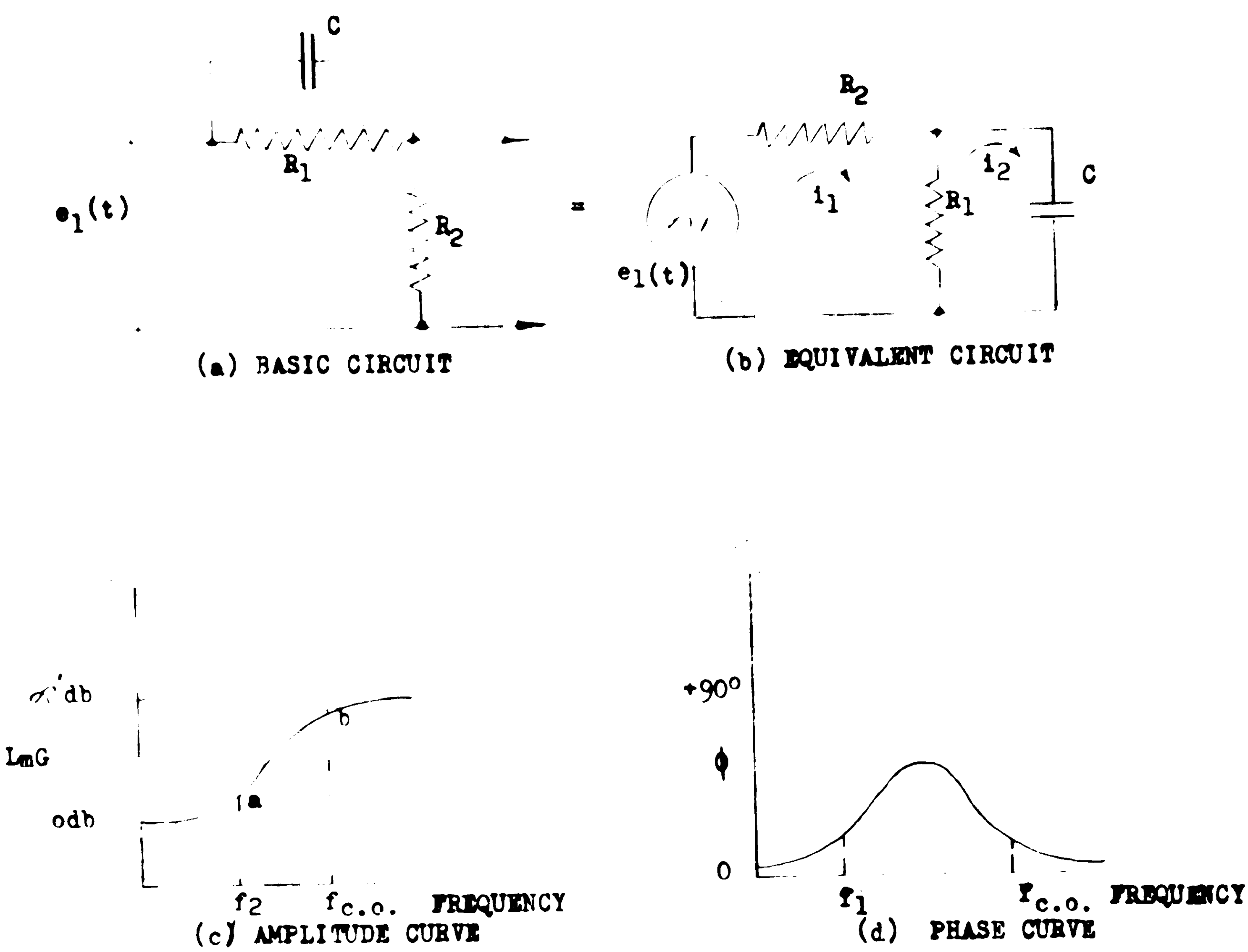


FIG. 17 COMPENSATING CIRCUIT

The complete analysis for the transfer function KG(s) is given with reference to the equivalent circuit in figure 17(b).

The mesh equations are:

$$e_1\,(t) = (R_1 + R_2)\; i_1 - R_1\, i_2$$

$$0 = R_1(i_2 - i_1) + \frac{1}{C}\int i_2\; dt$$

The Laplace transformation is

$$E_1(s) = (R1 + R2)\;\; I_1(s) - R_1 I_2\,(s)$$

$$0 = R_1(I_2\,(s) - I_1(s)) + \frac{1}{CS}\; I_2 + \frac{1}{CS}\; I_2^{-1}\;(o +)$$

$$= -R_1\, I_1\,(s) + (R_1 + \frac{1}{CS})\; I_2\,(s),$$ (assuming the initial charge to be zero)

hence

$$I_1(s) = \frac{\begin{vmatrix} E_1(s) & -Ri \\ 0 & (R_1 + \frac{1}{CS}) \end{vmatrix}}{\begin{vmatrix} R_1 + R_2 & -R_1 \\ -R_1 & R_1 + \frac{1}{CS} \end{vmatrix}}$$

$$= \frac{E_1(s)\,(R_1 + \frac{1}{CS})}{(R_1 + R_2)(R_1 + \frac{1}{CS})} = E_1(s)\,\frac{R_1CS + 1}{R_1R_2\,CS + R_1 + R2}$$

$$E(R_2) = I_1(s)R_2 = E_1(s)\,\frac{R_1CS + 1}{R_1CS + \frac{R_1 + R_2}{R_2}} = E_1(s)\,\frac{R_2}{R_1 + R2} \times \frac{\frac{R1 + R2}{R2}\,\frac{R_2R1}{R1 + R2}\,CS + 1}{\frac{R_2R1}{R_1 + R_2}\,CS + 1}$$

$$\frac{E(R_2)}{E_1(s)} = \frac{1}{\alpha}\;\frac{\alpha\tau S + 1}{\tau S + 1} = KG(s)$$

where $\alpha = \frac{R_1 + R_2}{R_2}$

$\frac{1}{\alpha}$ = the network insertion loss at zero frequency.

$\tau = \frac{R_2R_1}{R_1 + R_2}C$ = the time constant corresponding to the cut-off frequency of the high pass filter.

The time constant $\alpha\tau = R_1C$ corresponds to a frequency below the cut-off beyond which the filter gives no further attenuation. It may be termed the "flattening-off" frequency, which is descriptive of point "a" on the curve shown in figure 17(c). Hence there are two important frequencies to be considered. It is readily seen that the ratio between these two frequencies is α , hence is dependent upon the choice of resistors.

The general form of the amplitude and phase characteristics of such a network is indicated in figure 17. It is seen that the loss decreases by the factor α as the frequency is increased to a frequency higher than cut-off, so that the total attenuation of the system at high frequencies is reduced. This is undesirable from the point of view of stability, but is more than compensated for by the fact that the phase can be retarded. A much higher frequency can be reached before the total shift becomes 180°, and the other networks will thus introduce more attenuation giving a greater total before the critical frequency is reached. The effect of such a network on the phase and amplitude characteristics is shown by figure 44 in the appendix.

For a given ratio of $R_1:R_2$ the ratio α is fixed, and the shape of the log base characteristic curves is fixed. Hence templates for the phase and amplitude curves may be drawn. The position of these curves with respect to frequency is then determined by the value of C. The templates are constructed from the following equations.

$$\text{Log}\,|G(s)| = \text{Lm}\,G(s) = \text{Lm}\,(\alpha\tau S + 1) - \text{Lm}\,(\tau S + 1)$$

$$\text{Arg}\,G(s) = \tan^{-1}\alpha\tau\omega - \tan^{-1}\tau\omega \text{ ---(since } S = j\omega)$$

or by a change in frequency constant such that u = = unity at the cut-off frequency.

$$\text{Lm}\,G(ju) = \text{Lm}\,(j\alpha u + 1) - \text{Lm}(ju + 1)$$

$$\text{Arg}\,G(ju) = \tan^{-1}\alpha u - \tan^{-1}u$$

The templates for the condition $\alpha = 6$ are shown in figure 43.

2.5.3 CARRIER FREQUENCY NETWORK

The compensating network may be placed in the audio frequency portion of the control system, in which case a network such as described in section 2.5.2 would be used. Similar results may also be obtained by placing a network of a suitable type in the carrier stages. It was shown that an IF filter was analagous to a low pass filter in its action upon the modulation. An I.F. network is required which acts as a modified high pass filter. Its attenuation must increase as the carrier frequency is approached from the upper and lower side band regions, and stabilize at the "flattening-off" frequency. Such a characteristic is illustrated in figure 18.

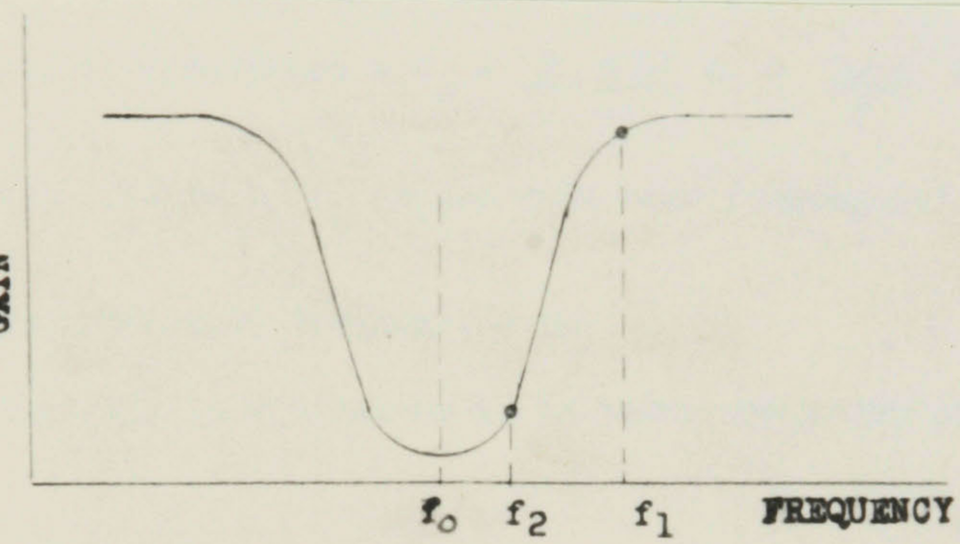
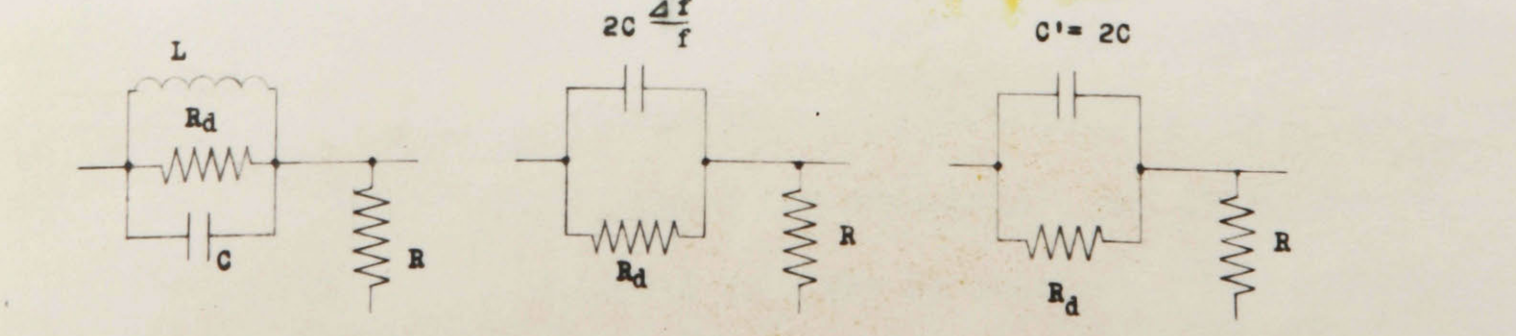


FIG. 18 REQUIRED CARRIER FILTER CURVE

It can be shown that a parallel tuned circuit in series with a load resistance will give the required characteristics. Such an arrangement is indicated in figure 19(a). It was shown in section 2.4.2 that if the response of side bands on one side of the carrier were known then the transfer function as applied to the modulating signal is of the same form.



(a) BAND REJECTION FILTER (b) EQUIVALENT CIRCUIT NEAR RESONANT FREQUENCY (c) EQUIVALENT CIRCUIT FOR THE MODULATING SIGNAL

FIG. 19 CARRIER FREQUENCY COMPENSATING NETWORK

The admittance of the parallel tuned circuit is

$$Y_f = \frac{1}{R_d} + j\omega c - \frac{1}{j\omega L} = \frac{1}{R_d} \quad \text{at resonance}$$

$$Y(f + \Delta f) = \frac{1}{R_d} + j(\omega + \Delta\omega)c - \frac{1}{j(\omega + \Delta\omega)L} \simeq \frac{1}{R_d} + 2\Delta\omega c$$

This is almost exactly true when $\omega \gg \Delta\omega$. Hence the circuit may be represented as a resistor and capacitance in parallel as in figure 19(b).

Effective shunt capacitance = $C_e = \frac{2\,\Delta\omega}{\omega} C = \frac{2\Delta f}{f} C$

The impedance offered by C_e to the side band frequencies is $X_c = \frac{1}{2\pi f C_e} = \frac{1}{4\pi\Delta fc}$

But Δf = the modulation frequency = fm

Hence this network is equivalent to an audio frequency network shown in figure 19(c).

Where $C^1 = \frac{1}{2\pi f_m X_c} = \frac{2\ fm2C}{2\ fm} = 2C$

The circuit synthesis consists in determining the required value of capacitance and dynamic resistance for the tuned circuit and the lead resistor necessary to produce the desired transfer function. As this network is equivalent to that discussed in the preceding section it may be analyzed in exactly the same manner.

A bridged T or parallel T network may also be used for this purpose. Circuits of this nature are employed in servo systems, where the frequencies involved are very low. A carrier of 60 c/s is often employed. These circuits are analyzed by Lauer Lesnick and Matson(14), and it is seen that a transfer function of the required form may be obtained. It is doubtful, however, that these T networks would be suitable at the higher frequency where the stray capacitance assumes great importance.

2.5.4 INPUT NETWORK

Attention has been directed thus far mainly to obtaining a suitable function for $\frac{E_o}{E_i}$. This is essential in order to provide stability in the closed loop. In a system which contains the controlled member at the output, the function $\frac{E_o}{E_i}$ describes how well this member will follow input variations.

However, in the present case, the important function is the frequency of the oscillator. It is not located that the output, and hence its action must be given by a different formula.

The signal which is applied to the oscillator control tube is $E = E_i - E_o$. The equation relating this to the input signal is $\frac{E}{E_i} = \frac{1}{1 + KG(s)}$

But KG(s) is large at zero frequency, and tends toward zero, at high frequency, hence $\frac{E}{E_i}$ is small for zero frequency and tends toward unity as the frequency is increased.

The amplitude versus frequency curve for this equation rises very rapidly as the frequency is increased. It is, in fact, the reciprocal of the KG(s) curve, up to the frequency region where KG(s) approaches unity. The transient response will therefore be very poor, the action being essentially that of a differentiating circuit. A square wave input would give rise to pronounced spikes at the reactance tube, causing a very wide frequency band to be transmitted.

It is desirable, not only to remove these spikes, but also to limit the rate of rise of the voltage E. It is not necessary to transmit a perfectly square wave, and the band width can be reduced if the wave is rounded somewhat.

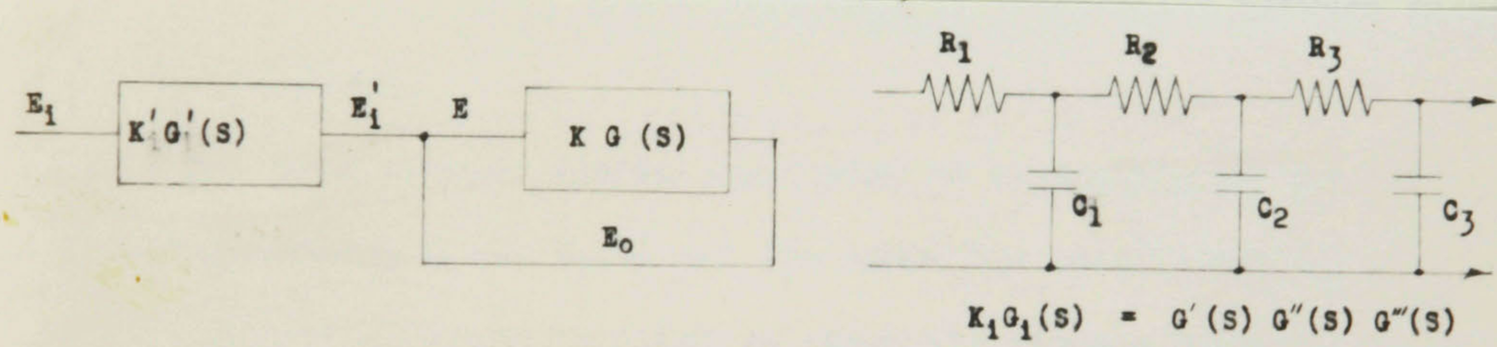
Furthermore, in order to prevent diagonal clipping at the detector, the rise time of the signal must be limited to correspond to the maximum decay rate of the detector filter as discussed in section 2.4.2.

Some form of network must be placed ahead of the voltage combining point to modify the input signal, as represented in figure 20.

$$\frac{E}{E_i^{\prime}} = \frac{1}{1 + KG(s)}$$

$$E_i^1 = E_i K^1 G^1(s)$$

$$\text{therefore } \frac{E}{E_i} = \frac{K^1 G^1(s)}{1 + KG(s)}$$



(a) OVERALL CIRCUIT (b) INPUT NETWORK

FIG. 20 CONTROL CIRCUIT WITH INPUT NETWORK ADDED

As a passive network will normally be employed, $K^1 = 1$ and $\frac{E}{E_i} = \frac{G^1(s)}{1 + KG(s)}$

For the frequency range where KG(s) 1 this becomes $\frac{G^1(s)}{KG(s)}$ and if $G^1(s)$ is made to be the same as G(s), then $\frac{E}{E_1} = \frac{1}{K}$ which will remain constant over this frequency range. As KG(s) approaches zero, the denominator approaches unity so that $\frac{G^1(s)}{1 + KG(s)}$ approaches zero.

The function $\frac{E}{E_1}$ is therefore linear (= 1/K) for a frequency range extending nearly to the region where KS(s) = 1, and falls off in amplitude for higher frequencies. Hence the variation of the oscillator frequency will be a very faithful reproduction of the keying wave form. In order to round off the oscillator response, another low-pass filter, with the required time constant must be added to the input network.

The problem of duplicating the variable portion of the transfer function, i.e. $G(s)$, in the input circuit, is rather difficult. The crystal discriminator characteristics are rather complex. The other frequency variant elements are more simple, but they are separated by vacuum tubes, and as independent filters their transfer functions may be added. It is desirable to employ only a passive network for the input, hence there will be interaction between successive filter elements.

The input network may be considered as consisting of two parts, the compensating network $G_1(s)\,G_2(s)$ and the extra low pass filter $G_3(s)$. As the highest modulation passed by $G_3(s)$ is about 115 c/s the function $\frac{G_1(s)G_2(s)}{1 + KG(s)}$ need be linear only up to this frequency. Beyond this point it is necessary for $G_1(s)G_2(s)$ to attenuate at least as rapidly as $G(s)$, and hence it does not need to be an exact duplicate of the latter.

The input network of figure 20 consists of three low pass filters in cascade. The transfer function may be analyzed following the method given in paragraph 2.5.2.

The mesh equations are

$$e_1(t) = R_1 i_1(t) + \frac{1}{C_1}\int\left[i_1(t) - i_2(t)\right] dt$$

$$0 = \frac{1}{C_1}\int\left[i_2(t) - i_1(t)\right] dt + R_2\, i_2(t) + \frac{1}{C_2}\int\left[i_2(t) - i_3(t)\right] dt$$

$$0 = \frac{1}{C_2}\int\left[i_3(t) - i_2(t)\right] dt + R_3\, i_3(t) + \frac{1}{C_3}\int i_3(t)\, dt$$

and $$e_4(t) = \frac{1}{C_3}\int i_3(t)\, dt$$

The transforms are, assuming that the charge on the capacitors is zero at t = o

$$E_1(s) = R_1 I_1(s) + \frac{1}{C_1 s}\left[I_1(s) - I_2(s)\right]$$

$$0 = \frac{1}{C_1 s}\left[I_2(s) - I_1(s)\right] + R_2 I_2(s) + \frac{1}{C_2 s}\left[I_2(s) - I_3(s)\right]$$

$$0 = \frac{1}{C_2 s}\left[I_3(s) - I_2(s)\right] + R_3 I_3(s) + \frac{1}{C_3 s} I_3(s)$$

and $E_4(s) = \frac{1}{C_3 s} I_3(s)$

The major determinant is

$$\begin{vmatrix} R_1 + \frac{1}{C_1 s} & -\frac{1}{C_1 s} & 0 \\ -\frac{1}{C_1 s} & \frac{1}{C_1 s} + R_2 + \frac{1}{C_2 s} & -\frac{1}{C_2 s} \\ 0 & -\frac{1}{C_1 s} & \frac{1}{C_2 s} + \frac{1}{C_3 s} + R_3 \end{vmatrix}$$

hence
$$D = \left(R_1 + \frac{1}{C_1 s}\right)\left\{\left(\frac{1}{C_1 s} + R_2 + \frac{1}{C_2 s}\right)\left(\frac{1}{C_2 s} + \frac{1}{C_3 s} + R_3\right) - \frac{1}{C_2^2 s^2}\right\}$$
$$- \frac{1}{C_1^2 s^2}\left(\frac{1}{C_2 s} + \frac{1}{C_3 s} + R_3\right)$$
$$= R_1 R_2 R_3 + \frac{R_1 R_3 + R_2 R_3}{C_1 s} + \frac{R_1 R_3 + R_1 R_2}{C_2 s} + \frac{R_1 R_2}{C_3 s} + \frac{R_1 + R_2 + R_3}{C_1 C_2 s^2}$$
$$+ \frac{R_1 + R_2}{C_1 C_3 s^2} + \frac{R_1}{C_2 C_3 s^2} + \frac{1}{C_1 C_2 C_3 s^3}$$

If $R_1 = R_2 = R_3 = R$ and $C_1 = C_2 = C_3 = C$

$$D = R^3 + \frac{5R^2}{Cs} + \frac{6R}{C^2 s^2} + \frac{1}{C^3 s^3}$$

$I_3(s) = \frac{E_1(s) B_{13}}{D}$ where B13, the minor determinant is $\frac{1}{C_1 C_2 s^2}$

$$\frac{E_4(s)}{E_3(s)} = \frac{1}{C_3 s} I_3(s) = \frac{\frac{1}{C_1 C_2 C_3 s^3}}{D} = \frac{\frac{1}{C^3 s^3}}{D}$$

$$= \frac{1}{R^3 C^3 s^3 + 5 R^2 C^2 s^2 + 6 RCs + 1}$$

Equating the denominator to zero and solving the cubic yields

$RCs = -1.554, -3.248$, and -0.198

or $KG(s) = \frac{E_4(s)}{E_1(s)} = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)}$

where $\tau_1 = RC/\ 0.198$

$\tau_2 = RC/\ 1.554$

$\tau_3 = RC/\ 3.248$

Hence the transfer function will be the same as for three isolated low pass filters whose time constants bear the relationship to RC given in the above equations.

By similar treatment it can be shown that the two stage cascade low pass filter, with $R_1 = R_2 = R$ and $C_1 = C_2 = C$, has a transfer function

$$KG(s) = \frac{1}{(\tau_1 s + 1)(\tau_2 + 1)}$$

where $\tau_1 = 0.382\ RC$

$\tau_2 = 2.61\ RC$

Templates for these two input networks are given in figure 49.

3. CIRCUIT CALCULATIONS

It has been seen that a stable feedback circuit may be constructed which is capable of responding to square wave signals of the type employed in frequency shift keying. In the experimental stage of this thesis, work was directed toward the construction of a suitable circuit from the theoretical foundations established in section 2, and to test its practicability. A suitable circuit must conform to the requirements indicated in section 1.1. To obtain compactness a minimum number of stages, of simple design, are indicated. At the same time, stability must not be sacrificed. The oscillator frequency stability must be of the same order as that of a crystal oscillator even though it is being shifted between two fixed values at an arbitrary rate.

3.1. GENERAL CIRCUIT DESIGN

3.1.1. BLOCK DIAGRAM

The circuit which was investigated is indicated by the block diagram of figure 8. This is a general form of AFC circuit except that provision is made to superimpose a keying signal upon the signal returned to the reactance tube. The discriminator must be operated off ground so that the keying input may be grounded as shown. The total signal applied to the reactance tube is $E_i = E_k + E_d + E_b$, and the incrimental variation due to a keying signal is

$\Delta E_i = \Delta E_k + \Delta E_d \quad = \quad e_i = e_k + e_d$

E_k = the keying signal

E_d = the signal returned from the discriminator

E_b = the bias voltage

E_i = the voltage applied to the reactance tube

As there is considerable gain in the circuit, it follows that $e_i \ll e_d$

and hence $e_d \cong -e_k$. It can be therefore seen that the discriminator must provide a voltage which is approximately equal to, but opposite in polarity to the input signal. For this to be brought about requires a change in the oscillator voltage. Hence the oscillator frequency will follow variations of the keying signal.

The frequency must be of the correct value to produce the required voltage at the discriminator. Hence the stability is a function of the discriminator and not of the oscillator circuit. If the tuning of the discriminator is altered, the frequency will be altered by the same amount. However if the oscillator components vary in such a manner that a shift would be produced in an uncontrolled oscillator, the actual shift will be reduced by a factor equal to the control ratio.

The overall circuit is similar to that described in section 1.2.2 and need not be discussed here. The calculations which pertain to the design of various portions of the circuit are given in the following paragraphs.

3.1.2 OSCILLATOR - REACTANCE TUBE CIRCUIT

A Hartley circuit with cathode feedback was employed. Separate oscillator and mixer tubes were used so that an output could be obtained at the oscillator frequency. Such an arrangement would be required if this unit were to excite a transmitter. The choice of values for the tank circuit depended upon a consideration of the stray capacity and capacity added by the reactance tube. An oscillator frequency of 4.5 Mc/s was used. The tank circuit values were:

Stray capacity (including coil)	= 25 pfd
Capacity due to reactance tube	= 4.6 pfd
Variable capacity	= 7.0 pfd
Total capacity	= 36.6 pfd
Inductance	= 33 uH

To provide good linearity and sensitivity the reactance tube must have a linear, gm vs eg curve and a large change in g_m for a given change in e_g. The 6AU6 was chosen for this purpose. Its g_m curve, for E_{c2}= 150v, is shown in figure 21(a). The linear position may be considered to extend from 0.5 to 4.5 volts bias, i.e. a change in g_m of 5,600 - 600 = 5000. The reactance tube is employed as a variable capacitance across the oscillator tank. Its sensitivity depends upon the operating frequency, the total capacitance in the tank circuit and the division ratio of the phase shifting network. For a high sensitivity a large signal is required at the grid, hence a low division ratio is indicated. On the other hand a high ratio is required if a phase shift of nearly 90° is to be developed. If the phase shift is less than 90° there will be a variable resistance in parallel with the variable capacitance and both AM and FM will be produced. A compromise circuit as indicated in an article by Ross and Sandel (34) shows that an overall deviation of 250 Kc/s may be expected with this circuit. The product RC of the phase shifting network is given by the formulae:

$$RC = \frac{2 \phi (B')^2 Co}{f}$$

where ϕ = sensitivity in cycles /mho

$B^1 = f2/f_1$

f_2 = mid oscillator frequency plus ½ deviation

f_1 = mid oscillator frequency minus ½ deviation

Co = total capacity in the tank circuit

hence $\phi = \frac{250{,}000 \text{ c/s}}{5{,}000 \text{ u mhos}} = 50 \times 10^6$ cycles/mho

$$RC = \frac{2 \times 50 \times 10^6 \times 1.08 \times 36.6 \times 10^{-12}}{4{,}500{,}000} = 900 \times 10^{-12}$$

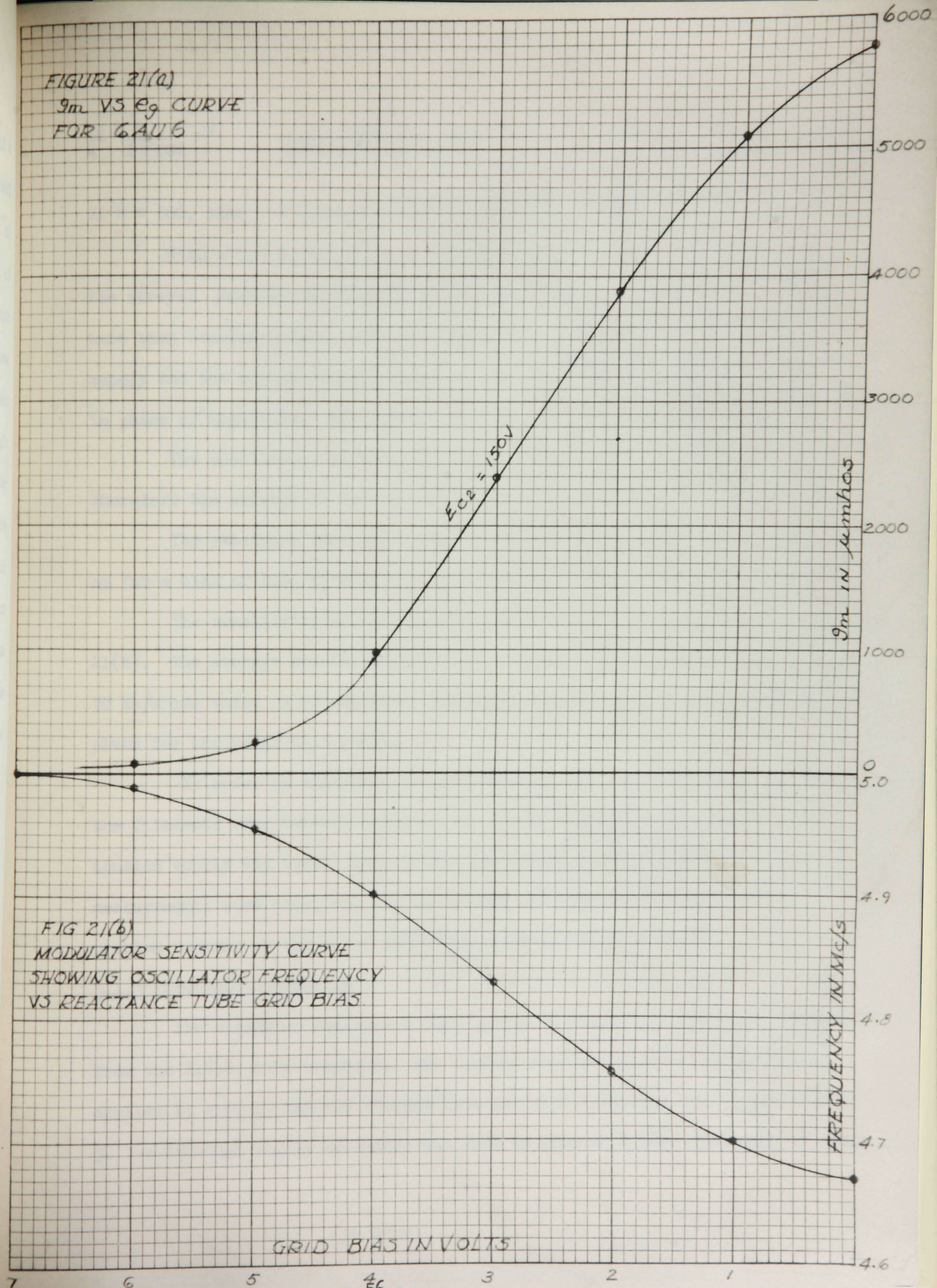


FIGURE 21(a)
gm VS eg CURVE
FOR 6AU6
Ec2 = 150V
gm IN μmhos
6000
5000
4000
3000
2000
1000
0
5.0
4.9
4.8
4.7
4.6
FIG 21(b)
MODULATOR SENSITIVITY CURVE
SHOWING OSCILLATOR FREQUENCY
VS REACTANCE TUBE GRID BIAS.
FREQUENCY IN MC/S
GRID BIAS IN VOLTS
7
6
5
4
3
2
1

$$\frac{X_c}{R} = \frac{1}{2\pi f C R} = \frac{10^{12}}{2\pi \times 4.5 \times 10^6 \times 900} = 40 = \text{the impedance ratio}$$

If R = 500, then C = 1.8 pfd., X_c = 20,000 ohms.

With a low R, X_c is low and hence C can be made somewhat larger than the strays. However the loading of the phase shift network upon the reactance tube then becomes larger, also any impedance in series with R, i.e. the bias supply and the keying circuit, must be extremely small so that R is not varied in phase or magnitude. The values 500 and 1.8 pfd proved to be obtainable.

The oscillator was designed to give 10 volts at the oscillator grid. Therefore the voltage at the reactance tube grid was 10/40 = 0.25 volts.

The reactance tube effective capacity = $C_e = g_m C R$.

or for a bias of 2.5, $C_e = 3000 \times 10^{-6} \times 900 \times 10^{-12} = 2.7$ pfd

The sensitivity of the reactance tube circuit is indicated in figure 21(b). The average sensitivity between -2 and - 4 volts bias is approximately 70 Kc/s per volt. A 2% amplitude variation was measured at the oscillator plate for the total bias range.

In order to eliminate 60 c/s frequency modulation from the circuit the filaments were battery operated and a well regulated and filtered high tension supply was used. This effect was quite pronounced when open loop measurements were made, but for the closed AFC loop this ceased to be a problem because of the noise cancellation produced by feedback.

3.1.3 FREQUENCY CONVERSION

The converter employed a 6BE6 tube with resistance values as obtained from the R.C.A. tube handbook. The voltage from the master oscillator was applied to grid 1, and the voltage from the beat frequency oscillator to grid 3. An external signal generator was used to provide a beat frequency of such value as to provide the correct intermediate frequency. Various output circuits were employed in conjunction with the converter. At first a Foster Seeley discriminator was used at the converter plate. The sensitivity was very low and no limiting was provided. Hence an I.F. filter was employed, which was followed by a limiter-buffer amplifier working into the discriminator.

For reasons of feedback stability as outlined in section 2.4.5, it was found desirable to make the I.F. filter very wide. A single tuned circuit was employed.

For the coil

$$L = 1 \text{ mh}$$

$$C_d = 7 \text{ pfd}$$

$$Q = 84$$

$$R_D = 337{,}000 \text{ at } 1 \text{ mc/s (from Q meter measurements).}$$

A bandwidth of 200 kc/s, hence a Q of 5, was required. It was therefore necessary to have a total shunt resistance of 30,000. This was obtained by shunting R_D with 33,000 .

3.1.4 DISCRIMINATOR CIRCUIT

A Foster Seeley phase discriminator was employed in the early stages of the investigation. This was a conventional circuit operated at 465 Kc/s. A sensitifity of 1.5V/kc was obtained. The loop gain was therefore 73 x 1.5 = 110. This circuit was discarded in favour of a crystal filter because a higher gain was required, and because the necessary degree of frequency stability could not be obtained with an LC circuit.

3.1.5 CRYSTAL FILTER

Good stability and sensitivity can be obtained by the use of a crystal filter as an FM demodulator. It is more difficult to obtain a linear curve than is the case with the conventional discriminator. Hollis (8) employed two crystal filters, with detector outputs connected in series opposition to obtain good linearity in a control circuit of a commercial F.M. transmitter. The problem of crystal matching was found by Hollis to be very exacting. Effort was therefore directed to an investigation of a single crystal filter.

The crystal impedance characteristic as discussed in section 3.2.1, has resonant and anti-resonant frequencies that are very close together. As a shift of 1000 c/s may be employed, a slope width in the order of 2000 c/s should be provided. To increase the separation between these two frequencies, various modifications to the crystal circuit may be made which consist essentially of adding inductance or capacitance in series or in parallel with the crystal. A parallel inductance was employed because a d.c. path was required and because stray capacitance effects can be cancelled out by a parallel circuit. The analysis of this configuration is more straightforward than for some other types as the resonant frequency of the crystal (fr) remains fixed and the anti-resonant frequency (fa) alone varies.

A sketch of the impedance vs frequency curve for the crystal-inductance combination is shown in figure 22. The d.c. output curve of the detector would be of the same form. In order to cause this curve to operate from the centre of the slope a d.c. bias must be added which gives rise to zero volts at a frequency midway between f_r and f_a, as indicated by point "b". This is not a satisfactory curve due to the complication of providing the proper bucking voltage, and because the system could also stabilize at a frequency represented by the point "d". To avoid this difficulty a bridge circuit was employed as illustrated in figure 23.

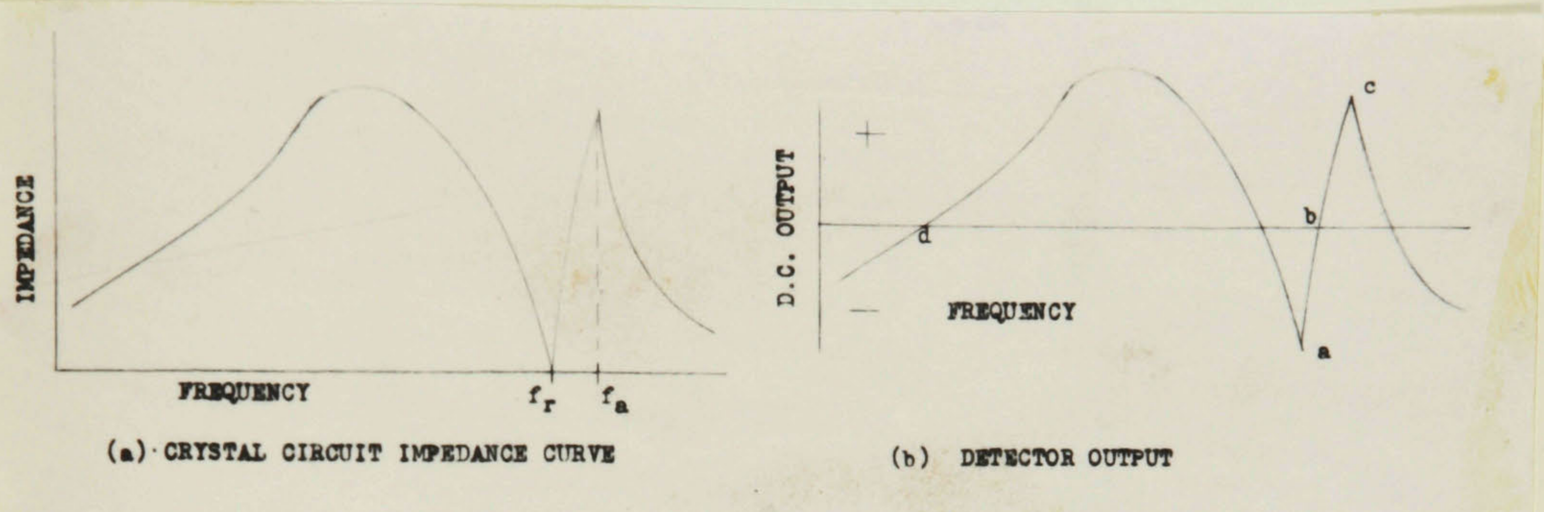


FIG. 22 CRYSTAL DETECTOR CHARACTERISTICS

The detectors are connected in such a manner that there is subtraction between the two d.c. outputs. The output obtained from arms A and B are indicated by curves a and b respectively. The difference voltage, curve c, is seen to have a desirable form.

3.1.6. DETECTOR CIRCUIT

Two valve rectifiers were employed, connected as shown in figure 23. The time constant for the detector on the L C side (A) of the bridge was not significant, and hence was made long. The crystal branch detector performed the detection operation and hence its time constant was dependent upon considerations of the frequencies to be passed and the system stability. It was seen that the fifth harmonic of the keying wave should be passed, indicating a cut-off at 115 c/s. The other cut-off frequencies in the system will be higher than this, and therefore as has been shown in section 2.5.5 the detector should cut-off at as low a frequency as possible. Hence a value of 115 c/s was chosen.

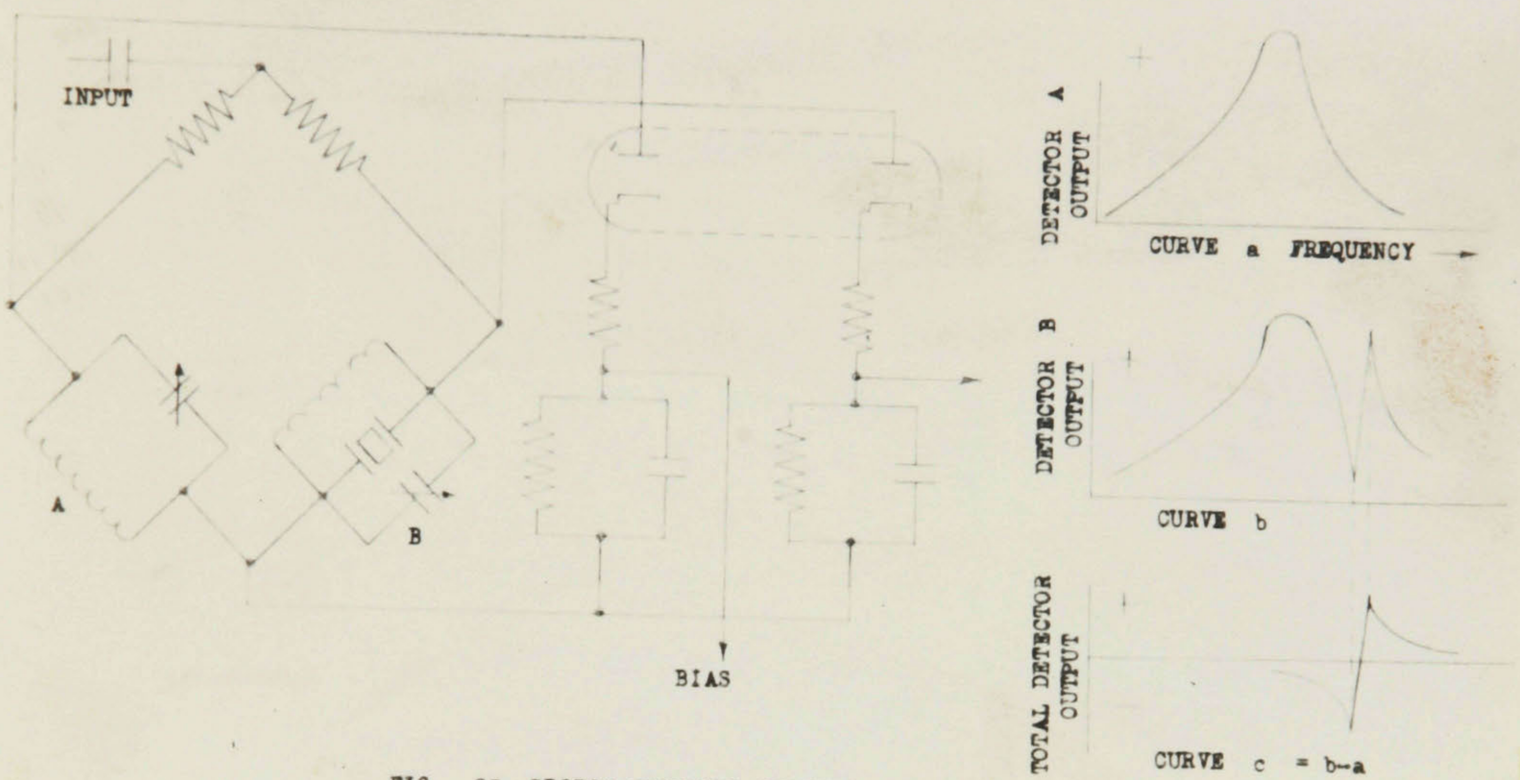


FIG. 23 BRIDGE DETECTOR CIRCUIT

Consideration of diagonal clipping as discussed in section 2.4.2 must be taken into account in the design of the detector circuit. With a total deviation of 1000 c/s and a detector band width of 2000 c/s the FM signal will be converted to AM with 50% modulation. If the upper modulation frequency is taken as 115 c/s then from the formulae given by Sturley

$$\frac{X_c}{R} = \frac{m}{1 - m^2}$$

where $m = \frac{50\%}{100\%} = .5$

therefore $\frac{X_c}{R} = \frac{.5}{1 - .25} = .575$

Taking R = 600,000 ohms

then X_c = 345,000 ohms at 115 c/s

therefore $C = .0040\mu fd$ maximum.

As 115 c/s is not present at the full assumed value of 50% modulation this figure for C allows a margin of safety. A standard J A N value, C = 4,700 was employed.

To provide cut-off at 115 c/s

$R_D' = X_c$ at 115 c/s = 345,000 ohms

$$\text{Efficiency} = \frac{R_1}{R_D' + R_1} = \frac{600}{945} = 63.0\%$$

From Sturley (P.351) it is seen that $\frac{R_1}{R_d} = 10$ for this value of efficiency.

therefore R_d = 60,000 ohms

In section 2.5.2 it was seen that R_d for the diode alone was 2400 ohms. Hence another 57,600 ohms must be added in series as shown, and J A N values of 47,000 ohms plus 10,000 ohms were used.

If the extra resistor is omitted then R_D = 2400 ohms. The efficiency as obtained by actual measurement and which checks approximately with the curve in Sturley, was 94%. The effective resistance R_D' is therefore 40,000 ohms and the cut-off frequency is $1/CR_D'$ = 650 c/s. Both forms of detector load were used in subsequent tests in order to determine the effect of varying the detector time constant.

3.1.7 CATHODE FOLLOWER

In a normal automatic frequency control circuit the d.c. output from the detector is passed through a low pass filter with a very long time constant so that there is a large value of capacity, and hence low impedance at the reactance tube input. In the present case however the detector circuit must pass some audio frequencies, and a compensating network may be required so that the impedance offered in series with the reactance tube phase shifting network may be sufficiently high to change the nature of this network considerably. Hence a cathode follower is required, which acts as an impedance transformer.

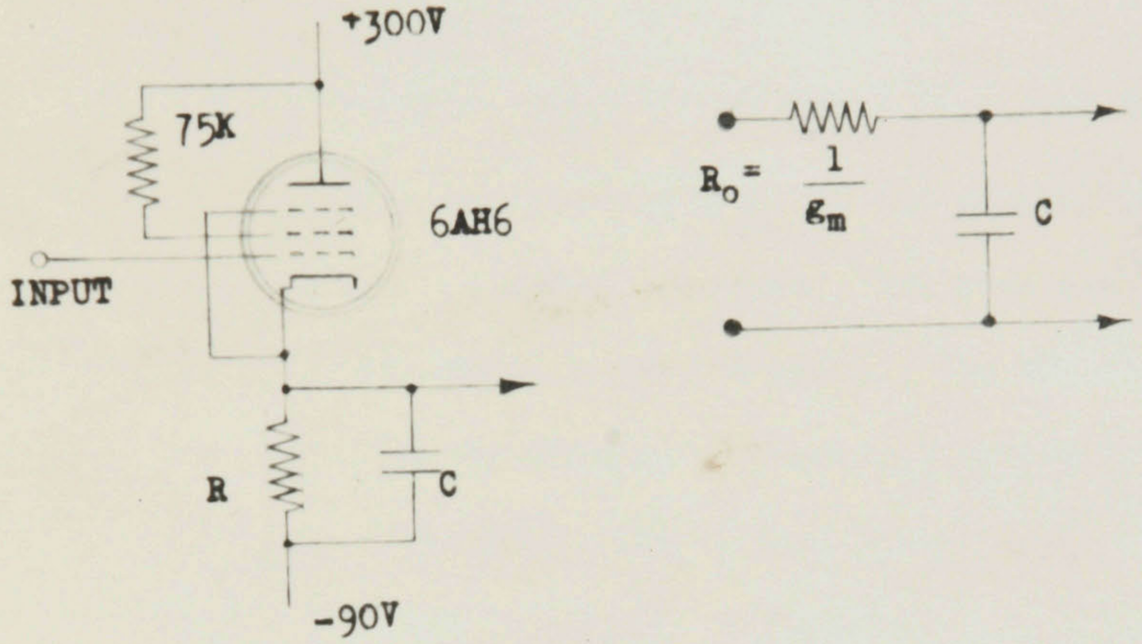


FIG. 24 CATHODE FOLLOWER CIRCUIT

It is necessary to present a low impedance, in the form of a large capacitance, to the control tube without adversely effecting the frequency response in the feedback loop. The cathode follower acts as a low impedance source whose impedance $= R_o \cong \frac{1}{g_m}$, hence it will act as an additional low pass filter as illustrated in figure 24. The cut-off frequency of this filter must be made very high, hence a tube with a high value of g_m is indicated.

A 6AH6 was used, connected as shown. For 1 volt bias $I = I_p + I_{c2} =$

$14 + 3 = 17$ mA.

and $g_m \cong 10{,}000$ mhos, from the tube curves.

therefore $R = \frac{E}{I} = \frac{90}{17 \times 10^{-3}} = 5{,}300$ ohms.

$R_o = \frac{1}{g_m} = 50$ ohms

if $C = .01\mu fd$

then $f_{c.o.} = 300$ kc/s which is well beyond the important frequency range.

The impedance offered to the control tube phase shifting network at 5 mc/s is

$$X_c = \frac{1}{2\pi \times 5 \times 10^{6} \times 10^{-8}} = 3 \text{ ohms}$$

3.2 CRYSTAL FILTER CALCULATIONS

3.2.1 CRYSTAL EQUIVALENT CIRCUIT

In order to calculate the characteristics of the crystal detector it was first necessary to find the primary constants of the equivalent circuit, figure 25. These consist of L, C, R and C_o as shown. The resonant frequency, f_r, is determined (to within a few cycles), by L C and R. The anti-resonant frequency f_a, is determined by C_o and an effective inductance formed by the series circuit. Since the crystal reactance changes rapidly as the applied frequency is varied, the separation between f_r and f_a is small.

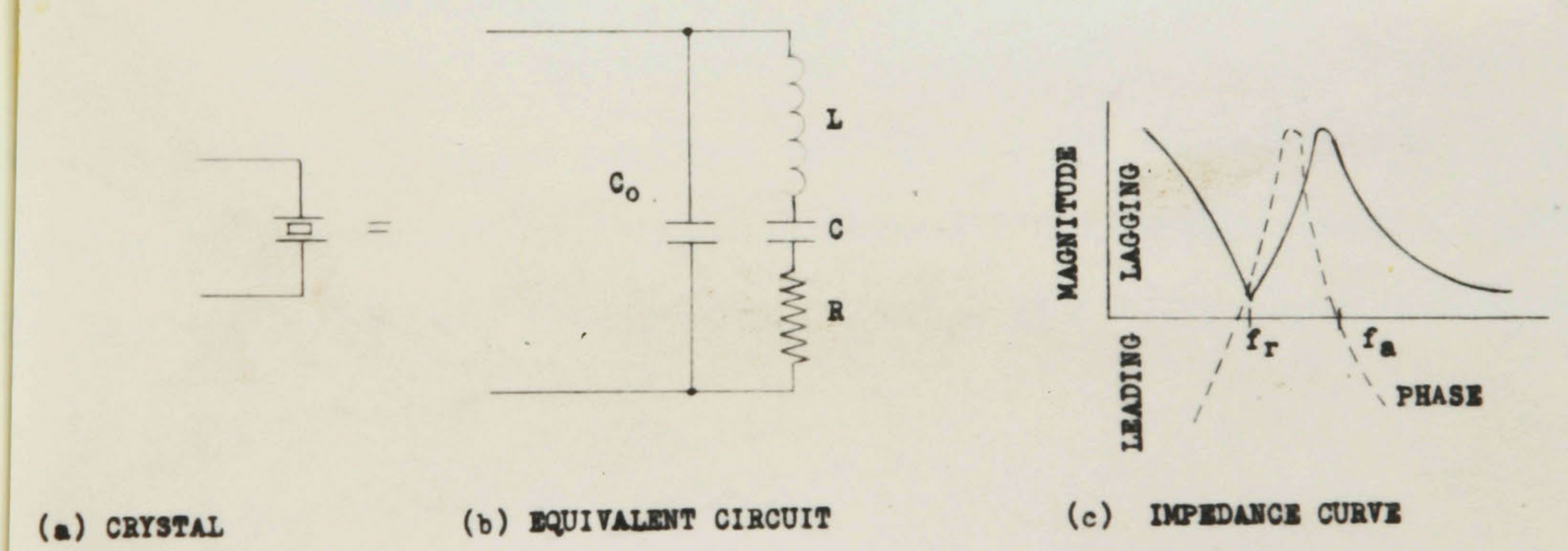


FIG. 25 EQUIVALENT ELECTRICAL CIRCUIT OF A CRYSTAL

Various methods for determining the primary constants by means of "Q" meters and bridge circuits are discussed by George, Selby, and Scolnik (35). To obtain the high degree of accuracy which is indicated in this reference would have required the use of a highly stable signal generator, with a high power output and a vernier scale, to drive the "Q" meter. As suitable equipment was not available another method was employed, which gave fairly consistent results.

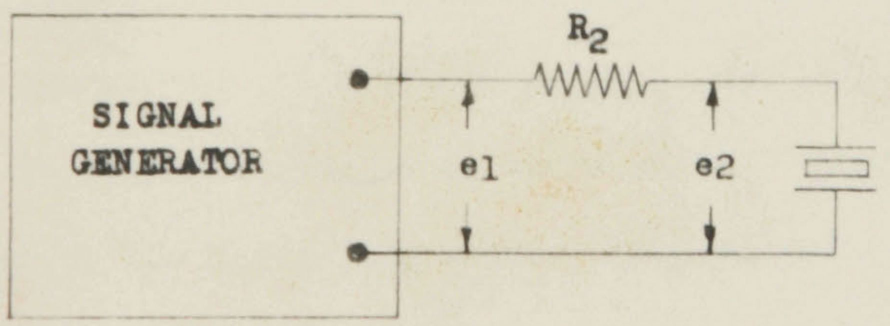
(a) Measurement of C_o

The parallel capacitance, C_o, is composed primarily of the crystal holder capacity, and is dependent upon the method of mounting the crystal. Stray capacitance across the crystal adds to the magnitude of C_o. The value of this constant may be read directly on the "Q" meter by noting the decrease in capacity required to maintain resonance when the crystal is inserted across the terminals. Care must be taken that the test frequency is sufficiently removed from the crystal resonance point that the impedance of the L C R branch will be very high.

The measured value for the crystal and holder mounted in an octal socket was 14 pfd.

(b) Measurement of R

The value of R for the crystal was determined by a voltage division network as shown. The signal generator was tuned to the resonant frequency so that the crystal was resistive. A high impedance V.T.V.M. (Hewlitt Packard 410A) was used to measure the voltage at E_2. A value of R = 470 ohms was obtained.



$$R = \frac{e_2}{e_1 - e_2} R_2$$

FIG. 26 CIRCUIT FOR MEASUREMENT OF R.

(c) Measurement of L and C

As the resonant frequency is readily determined it is necessary to obtain the value of one only of these reactances. This may be determined from a knowledge of C_o and the separation between f_r and f_o.

The anti-resonant frequency is that frequency at which the reactance of the L C R branch is inductive and equal in magnitude to the reactance of C_o.

Hence $X_L + X_c = J\left(\frac{\omega^2 L C - 1}{\omega C}\right) = -\left(-j\frac{1}{\omega C_o}\right)$

therefore $\omega_a^2 = \frac{C + C_o}{L C C_o}$

but $\omega_a^2 = \frac{1}{L C}$

therefore $\frac{\omega_a^2}{\omega_r^2} = \frac{C + C_o}{C_o}$ or $\frac{f_a}{f_r} = \sqrt{\frac{C + C_o}{C_o}}$

$1 + \frac{1}{2}\frac{C}{C_o}$

from which $\Delta f = f_r \frac{1}{2} \frac{C}{C_o}$ where $\Delta f = f_a - f_r$

and $C = \frac{2\Delta f C_o}{f_r}$

The separation, Δf, was very small(about 200 c/s) and difficult to obtain accurately. The test circuit depicted in figure 27 was at first employed. In order to obtain a small variation in frequency two signal generators, type G.R.605B, were used at the frequencies shown. With this arrangement one small division on the Vernier of the 100 kc/s generator represented 100 c/s. It was impractical to use a frequency below 100 kc/s for the variable generator as the other would have to be operated very close to 1 mc/s and would produce an error in the indicating meter. With this arrangement a frequency difference in the order of 200 c/s could not be measured accurately. Furthermore the stray capacity across the crystal is increased by the measuring equipment so that Δf is made even smaller.

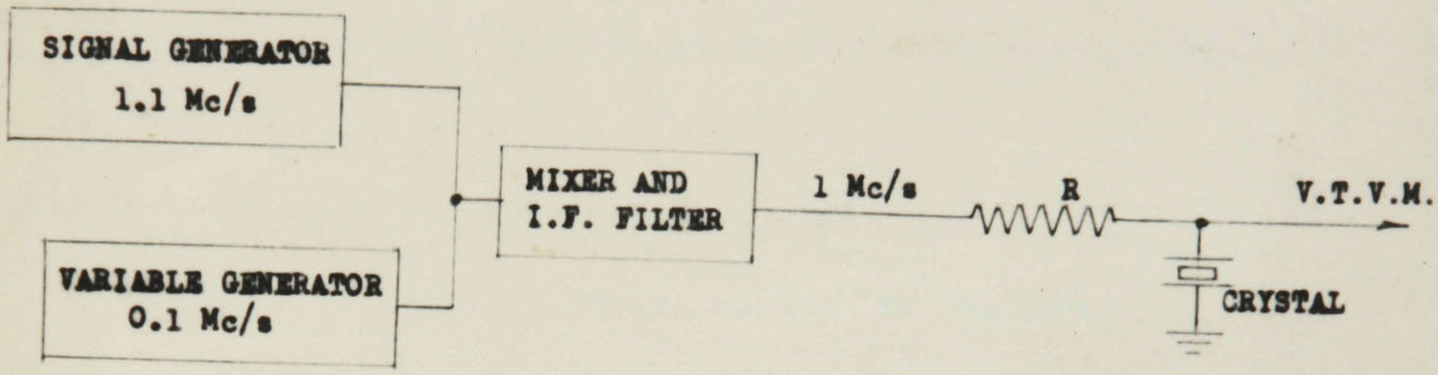


FIG. 27 CIRCUIT FOR MEASUREMENT OF L AND C

The accuracy of measurement was greatly increased by shunting the crystal with an inductance in order to spread out the interval Δf. A fixed inductance was used, and a trimmer condenser placed across it, as indicated in figure 28.

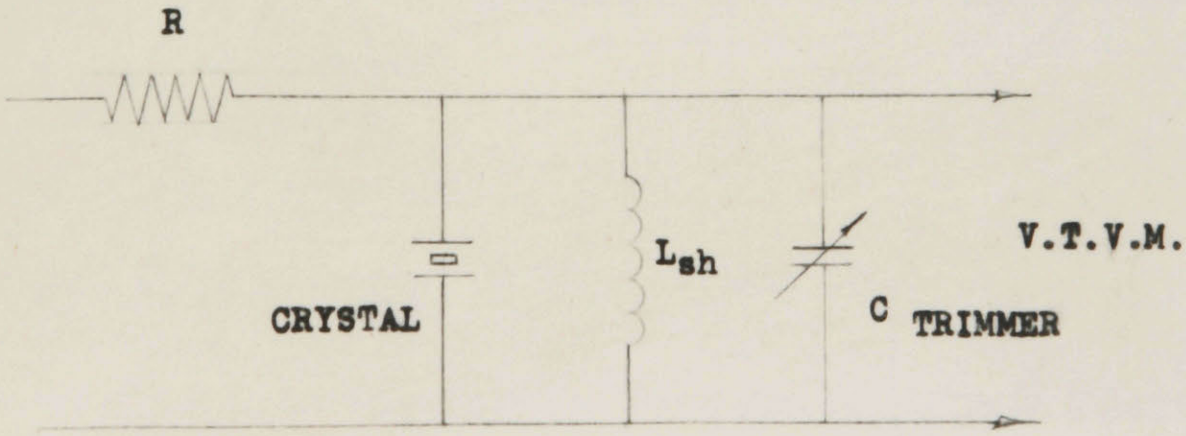


FIG. 28 MODIFICATION TO CRYSTAL CIRCUIT

The inductance was of such value that it cancelled out the crystal shunt capacity, stray capacity, and minimum trimmer capacity. The effective shunt capacity could then be set at any required value by means of the trimmer.

The actual value of total shunt capacity was determined by finding the resonant frequency due to the known inductance and capacity in parallel. This frequency is normally far enough removed from the crystal resonance that the crystal impedance is very high and hence has negligible effect.

The effective shunt capacity at crystal frequency becomes,

$$C_e = C_T - C_L$$

where C_T = total C as found above

C_L = amount of capacity cancelled by L at resonance

L = the value of inductance shunting the crystal

then $f_a - f_r = \Delta f = f_r C/2C_e$

$C = 2\Delta f C_e/f_r$ = capacity of the crystal series element.

The following example illustrates the method employed:

L_{sh} = .81 mh (by measurement)

therefore C_L (at 1 mc/s) $= \dfrac{1}{\omega^2 L} = 31.2$ pfd.

The trimmer was set to give $\Delta f = 2$ kc/s.

Resonant frequency of L_{sh} and $C_T = \omega_1 = .945$ mc/s (by measurement)

$$C_T = \frac{1}{\omega_1^2 L} = 34.9 \text{ pfd}$$

$\therefore$ $Ce = 34.9 - 31.2 = 3.7$ pfd

$$C = \frac{2\Delta f}{fr} \times C_e = \frac{4}{1000} \times 3.7 = 0.0148 \text{ pfd.}$$

The results which were obtained with various trimmer settings are given in the following chart:

f	$_1f_1$	C_T	C_e	C
1 kc/s	.905 mc/s	38.1 pfd	6.9 pfd	.0138 pfd
2	.945	34.7	3.7	.0148
3	.966	33.4	2.2	.0132
6	.983	32.35	1.15	.0138

These values are fairly consistent, and give an average of approximately .0140 pfd.

$$\therefore L = \frac{1}{\omega^2 C} = \underline{1.81 \text{ Henries}}$$

The equivalent circuit for the crystal therefore has

$R = 540$ ohms

$C = .014$ pfd

$L = 1.81$ Henries

$C_o = 14$ pfd

3.2.2 CRYSTAL IMPEDANCE CURVE

Assuming a filter band width of 2 mc/s, and crystal circuits as given in the preceding section, then

$$C_e = \frac{C\omega_r}{2\Delta\omega} = \frac{.014 \times 10^6}{4000} = 3.5 \text{ pfd}$$

$C_L = 31.2$

$C_T = 34.7$

and $f_1 = .95$ mc/s

The trimmer was set to give a resonance at 0.95 mc/s, so that the required value of C_T was obtained. The equivalent circuit assumed the form here shown, with values as given. The shunt resistance, R_1, is composed of three resistor in parallel, as follows:

(a) Dynamic coil resistance, R_d

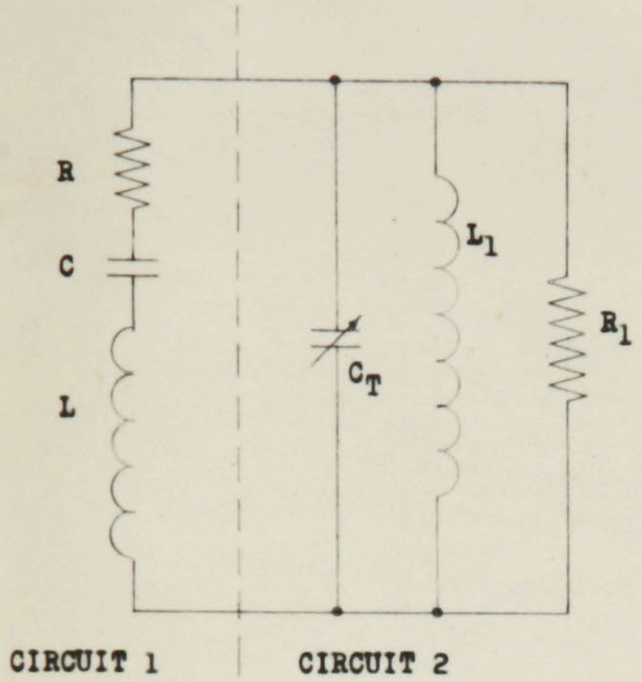
This was determined by finding the damping effect on a Q meter of the coil at the resonant frequency. A value of $R_d = 320,000$ ohms was found.

(b) Detector effective leading, $R_E = \frac{R}{2\eta}$

where $R = 600,000$

$\eta = 63.\ \%$

therefore $R_E = 390,000$ ohms.



$R = 540$ ohms
$C = .014$ pfd
$L = 1.81$ Henries
$C_T = 34.7$ pfd
$L_1 = .81$ Mh

$$R_1 = \frac{1}{\frac{1}{320K} + \frac{1}{390K} + \frac{1}{2,660K}}$$

$= 165,000$ Ohms

FIG. 29 EQUIVALENT CIRCUIT FOR CRYSTAL SHUNTED BY INDUCTANCE, CAPACITY AND RESISTIVE LOADING CIRCUIT .

(c) Crystal holder loss

This value was found by observing the loading effect of the crystal upon a Q meter coil. A frequency well removed from the crystal resonance, in this case .85 mc/s, was employed, and a value of $R_h = 2,660,000$ ohms was obtained.

The total shunt resistance is therefore

$R_1 = 165,000$ ohms.

For the purpose of analysis the circuit was broken down into two sections so indicated by the dotted line. The crystal series resonant branch is called circuit 1, and the remaining elements circuit 2.

In circuit 1

$$Z = R + \frac{1}{J(\omega_r + \Delta\omega)C} + J(\omega_r + \Delta\omega)L$$

$$= R + \frac{1 - (\omega_r + \Delta\omega)^2 LC}{J(\omega_r + \Delta\omega)C} \qquad = R + J\, 2\Delta\omega L$$

$$\therefore Y = \frac{1}{Z} = \frac{R}{R^2 + (2L\Delta\omega)^2} - J\frac{2L\Delta\omega}{R^2 + (2L\Delta\omega)^2}$$

In circuit 2

$$Y = \frac{1}{R_1} + J(\omega_r + \Delta\omega^1)C_T + \frac{1}{J(\omega^1_r + \Delta\omega^1)L}$$

$$= \frac{1}{R_1} + \frac{1 - (\omega^1_r + \Delta\omega^1)^2 LC_T}{(\omega^1_r + \Delta\omega^1)L} = \frac{1}{R} + J2\Delta\omega^1 C_T \frac{\omega^1 r}{\omega^1_r + \Delta\omega^1}$$

$$= \frac{1}{R_1} + \frac{J2(\Delta\omega + b_\omega)\,\omega_r}{\omega^1_r + \Delta\omega^1} \quad CT$$

$$= \frac{1}{R_1} + J4\pi\,(\Delta f + b_f)\,\frac{f^1_r}{f^1_r + \Delta f^1} \quad CT$$

where

$fr = \omega_r/2\pi$ = crystal frequency

$f^1r = \omega^1 r/2\pi$ = resonant frequency of L and C_T

$b_f = b_\omega/2\pi = f_r - f_r^1$

$\Delta f = \Delta\omega/2\pi$ = a variation referred to f_r

$\Delta f^1 = \Delta\omega^1/2\pi$ = a variatiom referred to f^1_r

The total admittance is

$$Y_T = Y_1 + Y_2$$

$$= \left[\frac{R}{R^2 + (2L\Delta\omega)^2} - j\frac{2L\Delta\omega}{R^2 + (2L\Delta\omega)^2}\right] + \left[\frac{1}{R_1} + j4\pi(\Delta f + bf)\frac{f^1_r}{f^1_r + \Delta f^1} \quad CT\right]$$

let $X = \frac{2L\omega}{R}$, then $X^2 = \frac{(3.62 \times 2\pi\Delta f)^2}{540}$ and $x = \frac{\Delta f}{23.75}$

Y_1 becomes $\frac{1}{R(1 + x^2)} - \frac{J x}{R(1 + X^2)}$

Substituting $C_T = 34.7$ for circuit 2 gives

$$Y_2 = \frac{1}{R_1} + J \frac{436}{10^6} \Delta f^1 \; X \; \frac{f^1{}_r}{f^1{}_r + \Delta f}$$

$$Y_T = G_1 + G_2 + J(B1 + B_2)$$

$$= \frac{1}{R_1} + \frac{1}{R(1 + X^2)} + J \frac{436}{10^6} \; \Delta f^1 \; \frac{f^1{}_r}{f^1_r + \Delta f^1} - \frac{X}{R(1 + x^2)}$$

For example, if $\Delta f = 2{,}000$ c/s, $\Delta f^1 = 52{,}000$ c/s, $X = \frac{2000}{23.75} = 84.3$

$$G_2 = \frac{1}{R_1} = \frac{1}{165{,}000} = 6.05 \; \mu\text{mhos}$$

$$G_1 = \frac{1}{R(1 + X^2)} = \frac{1}{540\ (1 + 84.3^2)} = 0.26 \mu \text{ mhos}$$

$$B_1 = \frac{-X}{R(1 + X^2)} = 084.3 \times 0.26 = -22 \mu \text{mhos}$$

$$B_2 = \frac{436}{10^6} \times \frac{950}{950 + 52} \times 52000 = + 22 \mu \text{mhos}$$

$$\therefore Y = 6.31 + J0 = 6.31 \underline{/o} \quad \mu\text{mhos}$$

$$Z = 158{,}000 \underline{/o} \quad \text{ohms}$$

A graph showing impedance vs frequency for the crystal circuit is given in figure 40.

3.2.3 LINEARIZATION OF DETECTOR IMPEDANCE CURVE

As the detector is across the crystal circuit impedance, the d.c. output at a given frequency will be largely proportional to this impedance. It is, however, also dependent upon the value of the dropping resistor R_{se} of figure 30. The following calculations were made in order to determine the proper resistance value to employ.

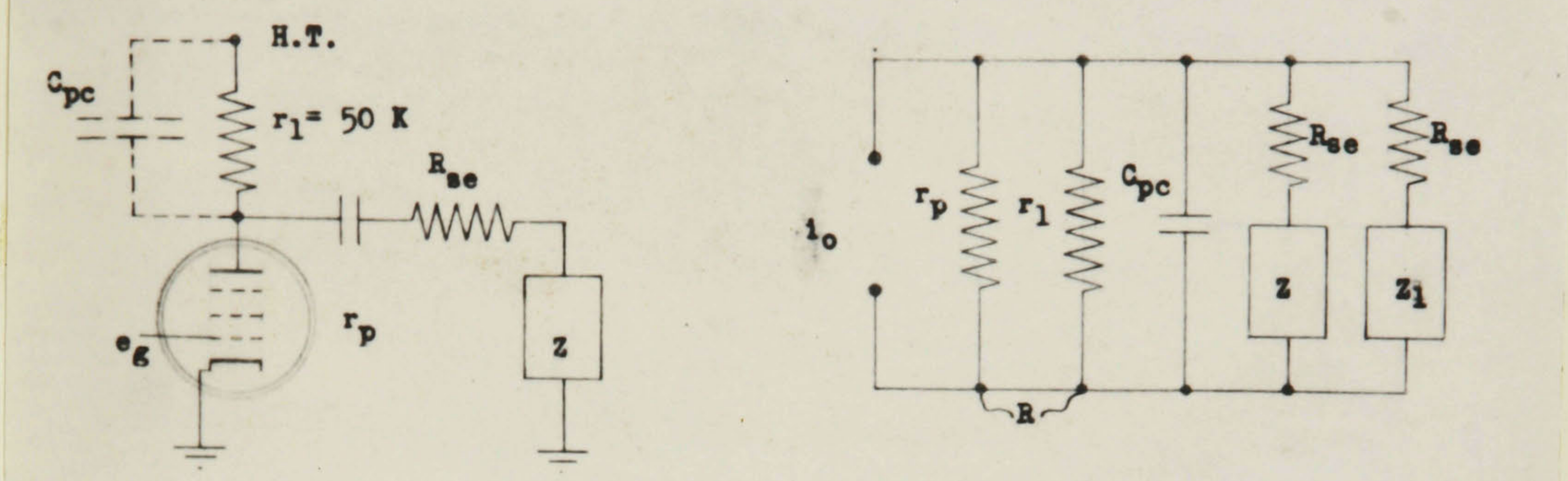


FIG. 30 EQUIVALENT CRYSTAL LOADING CIRCUIT

Due to the complexity of the circuit it was not practical to solve for a linear curve by differentiation as is normally done for discriminator circuits. The method employed was to make the output at the mid point of the curve equal to one half the maximum value. Substantially zero output was obtained at the crystal resonant frequency.

The equivalent circuit is based on the constant current generator concept. The loads are shown as being in parallel, each drawing a portion of the current. The block designated Z contains the entire circuit considered in the preceding section, including the detectors. To account for the other leg of the bridge circuit, the impedance $R_{se} + Z_1$ is added. All the shunting impedances are grouped together as Z_A, and the detector circuit is given as Z_B.

Of the terms in Z_A only R_L and X (Cgp) need be considered, as they are small compared to the other impedances.

Thus Z_A is 50,000 ohms and -J16,000 in parallel or

$Z_A = 4,650 - J14,500 = 15,240 \;\overline{\backslash 72.1^\circ}$

The other shunt impedances are large enough that this approximation introduces less error in the final result than is introduced by the tolerance of the components.

The current through the load Z is

$$i_z = (Z_A/Z_A + Z_B)\, i_T$$

and $E_z = Zi_z = \dfrac{(R_z + j\,X_z)\, Z_A\, i_T}{Z_A + R_{se} + R_z + jX_z}$

as $Z_A\, i_T$ is a constant, then

$$\frac{e_z}{k} = \frac{R_z + jX_z}{Z_A + R_{se} + R_z + jX_z} = F$$

It is therefore desired to make $|F(2000)| = 2 \times |F(1000)|$ where $|F(2000)|$ represents the absolute magnitude of F for a frequency of 2000 c/s.

at 1000 c/s

$$F = \frac{42,500}{R_{se} + 4,650 + 12,420 + J(40,600 - 14,500)}$$

at 2000 c/s

$$F = \frac{158,000}{R_{se} + 4,650 + 158,000 - J\,14,500}$$

$$\therefore |85000(r_{se} + 162,650 - j14500)| = |158000(R_{se} + 17,070 + j26,100)|$$

$$(R_{se} + 162,650)^2 + (14,500)^2 = (1.86)^2\left\{(R_{se} + 17,070)^2 + (26,100)^2\right\}$$

$$(R_{se} + A)^2 + B^2 = C^2\left\{(R_{se} + D)^2 + E^2\right\}$$

$$(C^2 - 1) R^2_{se} + 2(C^2 D - A)R_{se} + (C^2 D^2 + C^2 E^2 - A^2 - B^2) = 0$$

hence $R_{se} = 40{,}200 \pm 102{,}500 = \underline{142{,}700}$ ohms

$$e = K \frac{Z}{R_{se} + R_A + R_Z + J(X_Z - X_A)} = K \frac{Z}{R_Z + 147{,}350 + J(X_Z - 14{,}500)}$$

Hence the relative magnitude and phase angle of the detector output for various frequencies near the filter resonance may be found. These are plotted in figure 41.

3.2.4 CRYSTAL FILTER TRANSFER FUNCTION

A method of determining the effect of a non symmetrical filter upon audio modulation was developed in section 2.4.2. To illustrate the use of the equation as applied to the crystal filter characteristic figure 41 two examples are given.

(a) for a modulating frequency of 100 c/s

$$K^1 = \frac{\text{Upper side band amplitude}}{\text{carrier frequency amplitude}} = \frac{31}{27} = 1.15$$

$$K'' = \frac{\text{Lower side band amplitude}}{\text{carrier frequency amplitude}} = \frac{23}{27} = .85$$

ϕ^1 = lag introduced in the upper side band with respect to the carrier $= + 5^\circ$

ϕ'' = lead introduced in the lower side band $= \mp 5^\circ$

hence

$$m = K^1 \cos \phi^1 - K'' \cos \phi'' = .300$$

$$n = K^1 \sin \phi' - K'' \sin \phi'' = .026$$

$$\text{Amplitude} = J_1 \sqrt{M^2 + n^2} = J_1 \text{ X} .301$$

The actual value of J_1 need not be known in order to obtain relative amplitude values. It may be replaced by an arbitrary function, provided that this function varies inversely with frequency, for the reason given in section 2.4.4. For convenience the amplitude for this first example, which is for a very low frequency and hence represents a maximum, may be called 100.

$$\therefore \quad \text{Amplitude} = \frac{K}{f} \times .301 = \frac{k}{100} \times .301 = \underline{100}$$

when k = 3,320

$$\text{and phase} = \tan^{-1} \frac{.026}{.300} = \underline{5^\circ}$$

(b) for a modulation frequency of 1000 c/s

$$K^1 = 2.0$$

$$K'' = 0.018$$

$$\phi^1 = 61^\circ$$

$$\phi'' = -59^\circ$$

hence $m = .960$

$n = 1.764$

$$\text{amplitude} = \frac{3320}{f} \times 2.00 = \underline{66.4}$$

$$\text{phase} = \tan^{-1} \frac{.960}{1.764} = \underline{61.5^\circ}$$

The relative amplitude and phase values for the crystal filter transfer function are shown in figure 42 for modulating frequencies up to 100 kc/s.

3.3 OVERALL CIRCUIT CHARACTERISTICS

3.3.1 DETERMINATION OF THE OVERALL TRANSFER FUNCTION

It was shown in section 2.3 that when the phase and amplitude characteristics of each of the elements of a circuit are known, the overall transfer function may be calculated, or constructed geometrically. The following data has been determined for the frequency dependant elements.

(a) The oscillator tank for the particular operating frequency used in this investigation has a band width of 140,000 c/s, hence an effective cut-off frequency of 70,000 c/s.

(b) The tuned circuit in the mixer stage acts low pass filter which cuts off at 100,000 c/s.

(c) The crystal filter has a characteristic as defined by figure 42.

(d) A detector circuit which cuts off at 115 c/s was used for some of the tests. Another cutting off at 650 c/s was also employed. A comparison between the overall effect may be seen by observing the figures referred to below.

(e) The cathode follower provides a low pass filter cutting off at 300,000 c/s. As this frequency was a bit beyond the range investigated, this circuit was omitted from the transfer function consideration.

The curves representing each of the above listed transfer functions are plotted in figures 44 and 45 for a detector cutting off at 115 c/s. The overall characteristic is obtained by direct addition of the individual curves.

It can be seen from figure 44 (b) that a phase shift of 180° is obtained at a modulating frequency of 4,000 c/s. At this frequency the attenuation (figure 44 (a)) is 48 db. Therefore the gain of the circuit must be kept less than 48 db, ie., the amplification must be less than 250 to avoid instability. It should be considerably less than that figure to give a good transient response and to allow a margin of safety.

Similarly for a filter with a cut-off frequency of 650 c/s the critical frequency is 5400 c/s (figure 45 (b)). At this frequency the attenuation (figure 45 (a)) is 37.2 db, hence the gain must be less than 72.5. It is therefore advantageous from the viewpoint of stability, to have a low cut-off frequency at the detector. This gives confirmation to the reasoning of section 2.4.5, that any variable time constant should be far removed from the fixed time constants. The crystal detector, in this case, may be considered approximately as a second order low pass filter with cut-off frequencies in the range of 1000 to 2000 c/s.

3.3.2 IMPROVING THE TRANSFER FUNCTION

One of the compensating networks described in section 2.5 may be used to render the circuit more stable and increase the useable gain. A filter of the audio frequency type was connected across the detector output condenser, as indicated in figure 31. The network resistors R_1 and R_2 serve a dual purpose in that they also form the resistance load of the detector. The impedance of the network is considerably higher than that of the detector load capacity across which it is placed. The detector circuit and compensating network will therefore not interact to any appreciable degree.

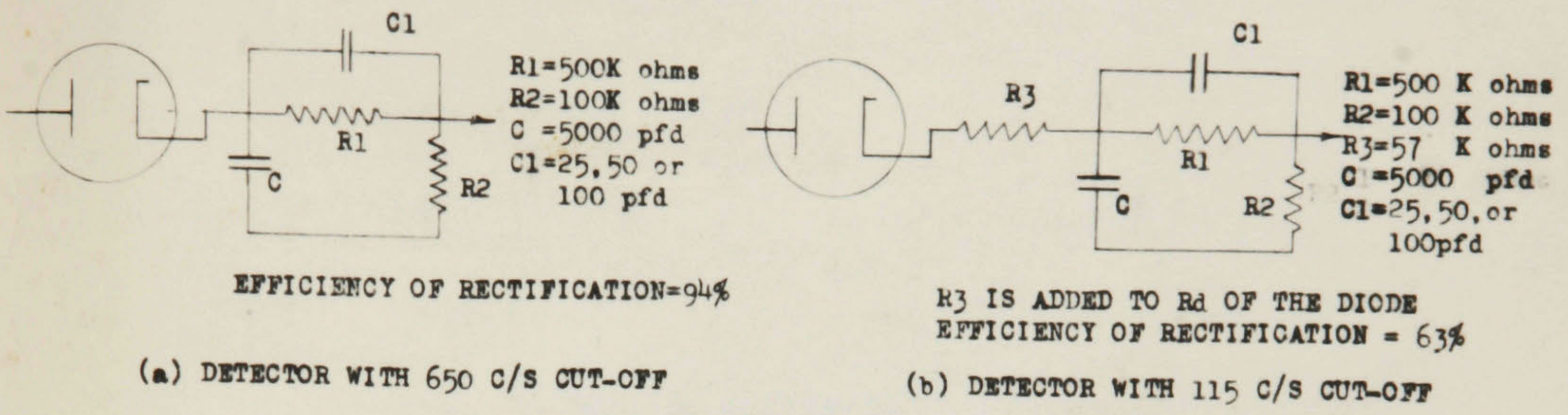


FIG. 31 AUDIO COMPENSATING NETWORK

The nature of the correcting function depends upon the choice of the components R_1, R_2, and C, and is described by the formulae in section 2.5.2. The total amount of phase lead that is produced by the network increases with an increase in the factor α. But the insertion loss also increases with α, and hence a compromise must be made.

As a trial a value $\alpha = \frac{R_1 + R_2}{R_2} = 6$ was used

where $R_1 = 500{,}000$ ohms

$R_2 = 100{,}000$ ohms

The template of figure 43 was therefore employed to represent the network transfer function. Various values of C_1 were used, corresponding to a shift of the curve along the frequency axis.

The position of the network curves was determined as follows:

(a) when $R_1 = 500{,}000$ ohms

$R_2 = 100{,}000$ ohms

$C_1 = 50$ pfd

then $\alpha = 6$

$$\tau = \frac{R_1 R_2 C}{R_1 + R_2} = 4.17 \times 10^{-6}$$

$$f_1 = f_{c.o.} = \frac{1}{2\pi T} = 38{,}200 \text{ c/s}$$

$$f_2 = f_1/6 = 6{,}350 \text{ c/s}$$

(b) when $R_1 = 500{,}000$ ohms

$R_2 = 100{,}000$ ohms

$C_1 = 100$ pfd

$$f_1 = \frac{38{,}200}{2} = 19{,}100 \text{ c/s}$$

$$f_2 = f_1/6 = 3{,}175 \text{ c/s}$$

(c) when $R_1 = 500{,}000$ ohms

$R_2 = 100{,}000$ ohms

$C_1 = 25$ pfd

$f_1 = 76{,}400$ c/s

$f_2 = 12{,}700$ c/s

The frequencies f_1 and f_2 locate the compensating curve so that it may be combined, by addition, with the other response curves. The effect of each of the three networks upon the overall characteristics is illustrated in figures 44 (a) and 44 (b) for a detector cutting off at

115 c/s, and in figures 45 (a) and 45 (b) for a 650 c/s detector cut-off frequency. The results may be summarized as follows:

(a) Detector circuit with 115 c/s cut-off.

C	f_π	loss at f_π (db)	loss ratio	f_{150}	loss at f_{150}(db)	loss ratio
0 pfd	4000 c/s	48	250	1100 c/s	24	16
25	32000	74.6	5400	1250	26	20
50	28000	70	3160	(1400 13000)	28 62	25 1260
100	23500	64.4	1660	10500	55.2	575

(b) Detector circuit with 650 c/s cut-off.

C	f_π	loss at f_π (db)	loss ratio	f_{150}	loss at f_{150}(db)	loss ratio
0 pfd	5400 c/s	37.2	72.5	1800	16	6.3
25	31000	59.6	955	2400	22	12.6
50	29000	55.2	576	15000	48	252
100	24000	49.6	302	13000	42	126

where f_π = frequency at which 180° shift is obtained.

f_{150} = frequency at which 150° shift is obtained, i.e., the condition for 30° phase margin.

Another factor to be considered was the available gain. The sensitivity of the reactance tube-oscillator circuit was found to be 70 kc/s per volts (section 3.1.2), and that of the crystal discriminator (section 3.2.2) was 14.4 volts per kc/s. As it was impossible to measure frequencies with a separation of less than 1 kc/s this last figure was based on the positive and negative peaks of the discriminator curve. The sensitivity over the linear range, judged on the basis of the impedance curve in figure 41, is 50% greater.

The loop gain is therefore 70 x 14.4 x 1.5 = 1,500. The compensating network reduces this value to $\frac{1500}{6} = \underline{250}$. These figures are based on the detector of figure 31 (b), which has an efficiency of 63%. For the other circuit the efficiency was found to be 94%, and hence the loop gain for this case is 250 x 94/63 = 374.

It is apparent that a greater degree of stability may be obtained from circuit (a). However, there is more gain available from circuit (b) due to the higher efficiency. A stable system must have both a phase margin and a gain margin. The gain margin is given by the loss ratio at f_{π}. The ratio at f_{150} indicates the rate at which the phase varies with respect to changes in attenuation. As an example, for circuit 31 (a) and C_1 = 25 pfd, the loss ratio at f_{π} is 955. If a gain of 374 is employed the gain margin will be 955/374 or 8.2 db. With 50 pfd. the gains margin is 576/374 or 3.8 db. However, in the first case, the loss ratio drops from 955 to 12.6 for a phase change of 30°. Hence the plot of the transfer function on the complex plane will pass into a high "M" range. In the latter case a more suitable trace is indicated, and this circuit was employed to make the loop measurements that follow.

A preliminary check indicated that diagonal clipping in the detector presented severe limitations upon the response of the system. The circuit was modified to increase the margin of safety by reducing C from 5000 pfd to 2000 pfd. This shortened the detector time constant from 3 to 1.2 milliseconds.

In order to maintain the same form of transfer function it was necessary to restore the detector cut-off frequency to 650 c/s.

hence $R_D^1 = 40{,}000 \times 5{,}000/2{,}000 = 100{,}000$ ohms.

$\eta = 600{,}000/700{,}000 = 86\%$

$R/R_D = 50$ (from the formula in Sturley P351)

$R_D = 12{,}000$ ohms

The diode contributed $R_D = 2{,}400$ ohms, therefore on external resistor, $R_2 = 9600$ ohms, was added. The available loop gain was then $374 \times 86/94 = 342$. Otherwise there was no alteration to the overall transfer function.

3.3.3 EFFECT OF INPUT FILTER

As a frequency control system is being considered the function of interest is the ratio between the reactance tube voltage and the input signal. It was shown in section 2.5.4 that this is given by

$$\frac{E(s)}{E_1(s)} = \frac{K^1G^1(s)}{1 + KG(s)}$$

where $\frac{E(s)}{E_i(s)} = \frac{1}{1 + KG(s)}$

and $\frac{Ei(s)}{E_1(s)} = K^1G^1(s)$ = the transfer function of the input filter

If $K^1G^1(s) = KG(s)$ then $\frac{E(s)}{E_1(s)} = \frac{E_o(s)}{E_i(s)}$ which is the system transfer function without the input filter. It has been shown that this function is stable, ie., does not have infinite response at any frequency. Linear response is obtained up to a very high frequency, since $KG(s) \approx 1 + KG(s)$ when $KG(s) \gg 1$, and a bit of a peak is obtained near f as illustrated by the M curves in figure 16. Hence for a square wave input harmonics up to a very high order will be passed, and the voltage E(s) will be very nearly square with some overshoot due to the high frequency response peak. This is not desirable for reasons given in section 1.

Furthermore since the detector has a finite time constant there will be diagonal clipping. This prevents the full value of the output signal from being fed back to give cancellation at the input for a short period following each keying transition. Hence the input signal is applied unopposed at the reactance tube grid, producing a voltage spike.

The input circuit must therefore have, in effect, an additional low pass filter to limit the build up time of the wave. The function KG(s) can be roughly approximated by a two stage filter. Hence a three stage filter is required.

The cascade network, figure 20, will perform the required function. In this case the curve $G_2(s)G_3(s)$ may be used to simulate KG(s). The template is moved right or left so that this curve falls along the KG(s) curve, figure 44 (a). It is then possible to determine the value of f_o for the nondimensionalized template curve, and hence the values for R and C in the filter section may be found. The transfer function may then be written:

$$\frac{E}{E_1} = \frac{G^1(s)}{1 + KG(s)} = G_1(s)\,\frac{G_2(s)\,G_3(s)}{1 + KG(s)}$$

$$\cong G_1(s), \text{ for } KG(s) \gg 1.$$

The position of the curve $G_1(s)$ was determined by the above process. It was seen to represent a low pass filter, cutting off at 64 c/s. This was lower than the desired value of 115 c/s. Unfortunately the relative cut off frequencies of such a cascade network cannot be altered without employing stages with unlike values of R and C. This presents a difficult problem from the point of view of synthesis and this solution was not attempted.

A three stage network was employed which had the values $R_1 = R_2 = R_3 = 500$ ohms, $C_1 = C_2 = C_3 = 0.5$ mfd. The frequency f_o of the template, figure 49 (a), was therefore:

$$\frac{1}{2\pi \times 500 \times 0.5 \times 10^{-6}} = 637 \text{ c/s}$$

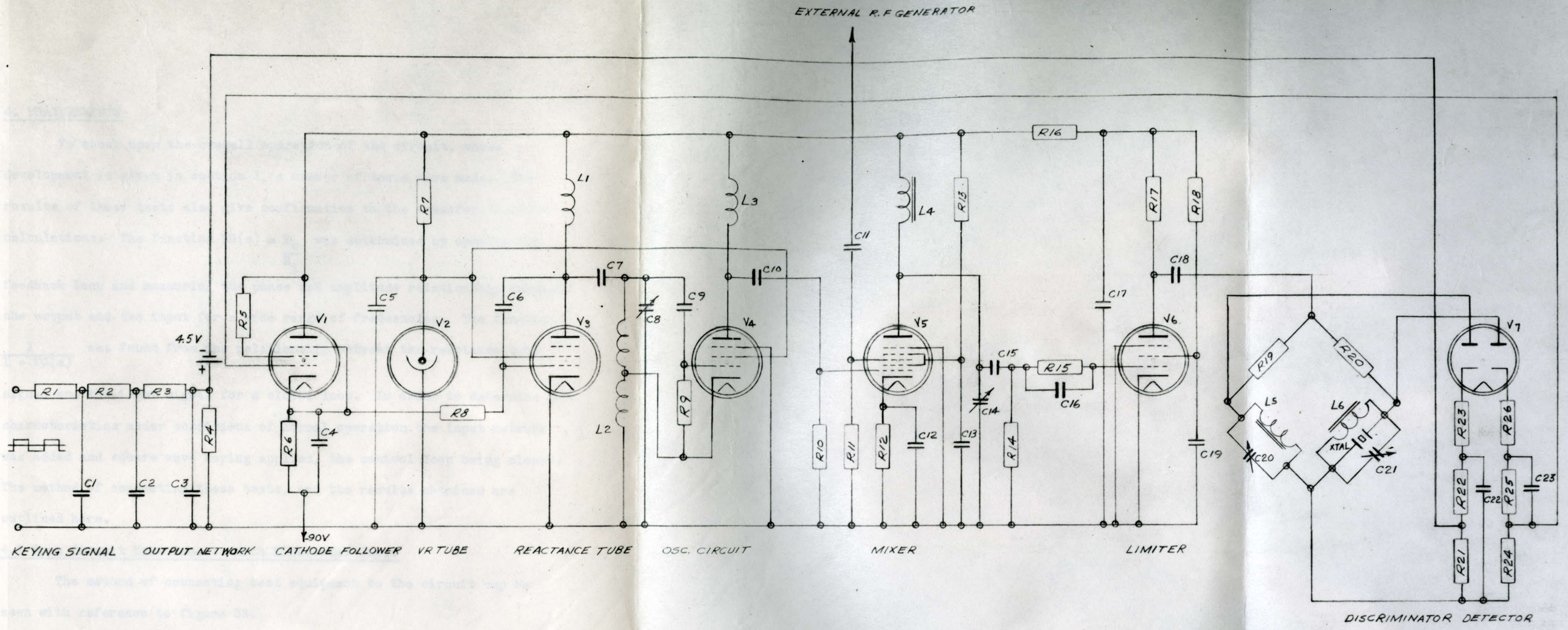
This located the curve G (s), and the function $\frac{G^1(s)}{1 + KG(s)}$ was obtained by addition of the template curve to that of the function $\frac{1}{1 + G(s)}$ in figure 48.

The function obtained has the form given in figure 50. The curve has the general shape of a low pass filter curve. An attenuation of 3 db is obtained at 130 c/s, which allows the important harmonics of the keying signal to be passed.

3.4 SUMMARY

In this section the design calculations have been given for all portions of a frequency shift control system. The characteristics of each element and of the overall circuit have been analyzed mathematically. The results indicate that the system will be stable, and that it should be possible to apply square wave keying at the input to produce a frequency shifted oscillator signal.

The complete circuit, which is illustrated in figure 32 incorporates the component arrangement and values which were shown by the calculations to give the most satisfactory results.



C1	Condenser	0.5 mfd		L1	Coil	.25 Henry R.F. Choke
C2	"	0.5 mfd		L2	"	33 mh Tapped 15 %
C3	"	0.5 mfd		L3	"	2.5 mh R.F. Choke
C4	"	.01 mfd		L4	"	1 mh
C5	"	.25 mfd		L5	"	0.8 mh
C6	"	2 pfd		L6	"	0.8 mh
C7	"	.1 mfd				
C8	"	3-18 pfd		R1	Resistor	500 ohms
C9	"	220 pfd		R2	"	500 ohms
C10	"	100 pfd		R3	"	500 ohms
C11	"	100 pfd		R4	"	120 K ohms
C12	"	.25 mfd		R5	"	75 K ohms
C13	"	.25 mfd		R6	"	5 K ohms
C14	"	3-18 pfd		R7	"	10 K ohms 5 W
C15	"	4700 pfd		R8	"	500 ohms
C16	"	220 pfd		R9	"	22 K ohms
C17	"	.25 mfd		R10	"	47 K ohms
C18	"	4700 pfd		R11	"	47 K ohms
C19	"	.25 mfd		R12	"	120 ohms
C20	"	3-18 pfd		R13	"	27 K ohms
C21	"	3-18 pfd		R14	"	33 K ohms
C22	"	.25 mfd		R15	"	330 K ohms
C23	"	47 pfd		R16	"	10 K ohms

R17	Resistor	50 K ohms
R18	"	100 K ohms
R19	"	150 K ohms
R20	"	150 K ohms
R21	"	100 K ohms
R22	"	500 K ohms
R23	"	10 K ohms
R24	"	100 K ohms
R25	"	500 K ohms
R26	"	10 K ohms
V1	Valve	6AH6
V2	"	0A2
V3	"	6AU6
V4	"	6AU6
V5	"	6BE6
V6	"	6AU6
V7	"	6AL5

Xtal Crystal G.E. Type 32C401-G18
#345598, 1 MC/S

Figure 32

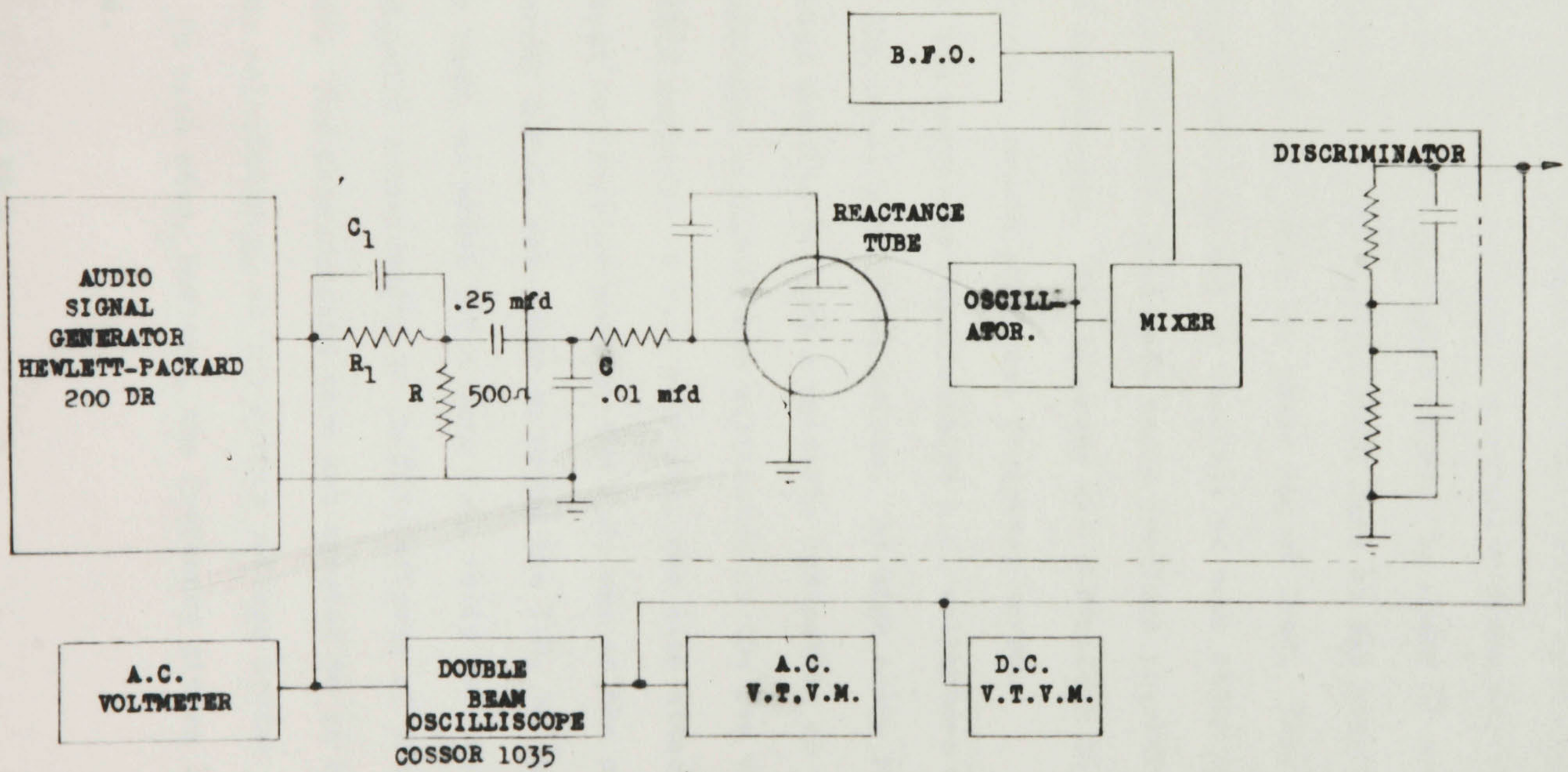
4. MEASUREMENTS

To check upon the overall operation of the circuit, whose development is given in section 3, a number of tests were made. The results of these tests also give confirmation to the transfer function calculations. The function $KG(s) = \frac{E_o}{E_i}$ was determined by opening the feedback loop and measuring the phase and amplitude relationship between the output and the input for a wide range of frequencies. The function $\frac{1}{1 + KG(s)}$ was found from the relationship between the reactance tube signal and the input signal for a closed loop. In order to determine the characteristics under conditions of normal operation the input network was added and square wave keying applied, the control loop being closed. The method of conducting these tests, and the results obtained are outlined here.

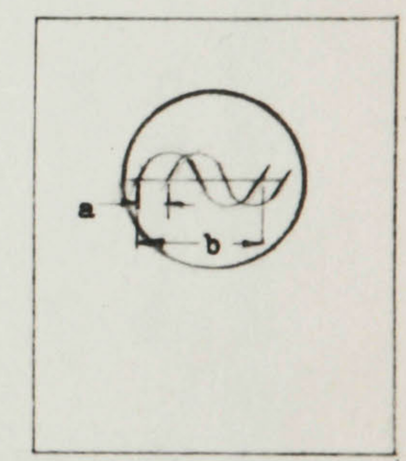
4.1 Open Circuit Measurements with Sinusoidal Input

The method of connecting test equipment to the circuit may be seen with reference to figure 33.

The control loop was opened between the discriminator output and the reactance tube. In order to keep the circuit characteristics unaltered it was necessary to have the same impedance at these two points as for a closed loop. The detector output impedance was very low, and it was not necessary to add compensation at this point. A low impedance, consisting of C and R was added to the input side to simulate the detector load. An attenuator was employed in conjunction with the signal generator in order to provide the necessary low value of input signal. This consisted of adding an impedance in series with the input load impedance to provide the required division.



(a) CIRCUIT



(b) TRACE OF INPUT AND OUTPUT WAVE ON THE DOUBLE BEAM OSCILLISCOPE

FIG. 33 ARRANGEMENT FOR OPEN LOOP MEASUREMENTS.

The input load = 500 ohms and .01 mfd in parallel,

For a ratio 1:10		R_1 = 4,500 ohms	C_1 = .0011 mfd
	1:100	R_1 = 50,000 ohms	C_1 = .0001 mfd
	1:1,000	R_1 = 500,000 ohms	C_1 = 10 pfd

In order to measure the input and output phase relationships a double beam oscilloscope was employed. A separate trace was obtained for each signal, as indicated in figure 33(b). By means of the graduated scale superimposed on the oscilloscope face it was possible to determine the ratio a:b and hence the phase lag or lead. This method of phase measurement, although not as accurate as some other systems, was the only method available that made rapid readings possible over the wide range of frequencies. The accuracy was sufficient to make it possible to construct a smooth phase vs. frequency curve.

To measure the amplitude relationships A.C. voltmeters were connected across the input and output points. At high audio frequencies the audio output was too low to give a suitable indication on the voltmeter. The oscilloscope amplifier was calibrated on the low voltage scales to serve this purpose. A D.C. voltmeter was also attached to the discriminator output to indicate whether the I.F. was at the correct frequency. The external B.F.O. was tuned to bring the I.F. to the centre of the discriminator band, at which point zero D.C. volts were developed. The B.F.O. was adjusted occasionally to nullify effects of drift in the circuit oscillator. The measurements were not carried to as high a frequency as were the calculations, as the output voltage became too small to be measured. In each case, however, the frequency giving 180° phase shift was reached.

To determine the transfer function for the crystal circuit alone it was necessary to make the cut-off frequencies of all other circuits very high. The fixed capacity was removed from the detector circuit, leaving a stray value of 14 pfd. The cut-off frequency for the detector was then:

$$\frac{1}{2 \quad CR_o} = \frac{1}{6.28 \times 14 \times 10^{-12} \times 276{,}000} = 41{,}000 \text{ c/s}$$

This accounts for the discrepancy between the measured and the calculated curve (figure 42) at the high frequency end.

This test was repeated employing a detector circuit cutting off at 115 c/s and for a detector with a 650 c/s cut-off frequency. A comparison between calculated and measured results is given in figure 42.

The effect of adding a compensating network to the latter circuit, as measured, is illustrated in figure 46. A comparison may be made with the calculated curves given in figure 45.

4.2 Closed Loop Measurements with Sinusoidal Input

For the purpose of this test the discriminator output was applied to the reactance tube grid, in series with the input signal. A cathode follower was employed to isolate the input impedance from the reactance tube circuit phase shifting network. Voltage and phase relationships were measured at the points indicated in figure 34. The voltages measured were E_i and E, and hence the function $E/E_i = \dfrac{1}{1 - KG(s)}$ was obtained.

The measurement technique employed was the same as for the previous test. The D.C. meter was used to make the initial frequency setting. With the

A.F.C. loop closed the oscillator frequency was automatically maintained at a fixed point for the remainder of the test.

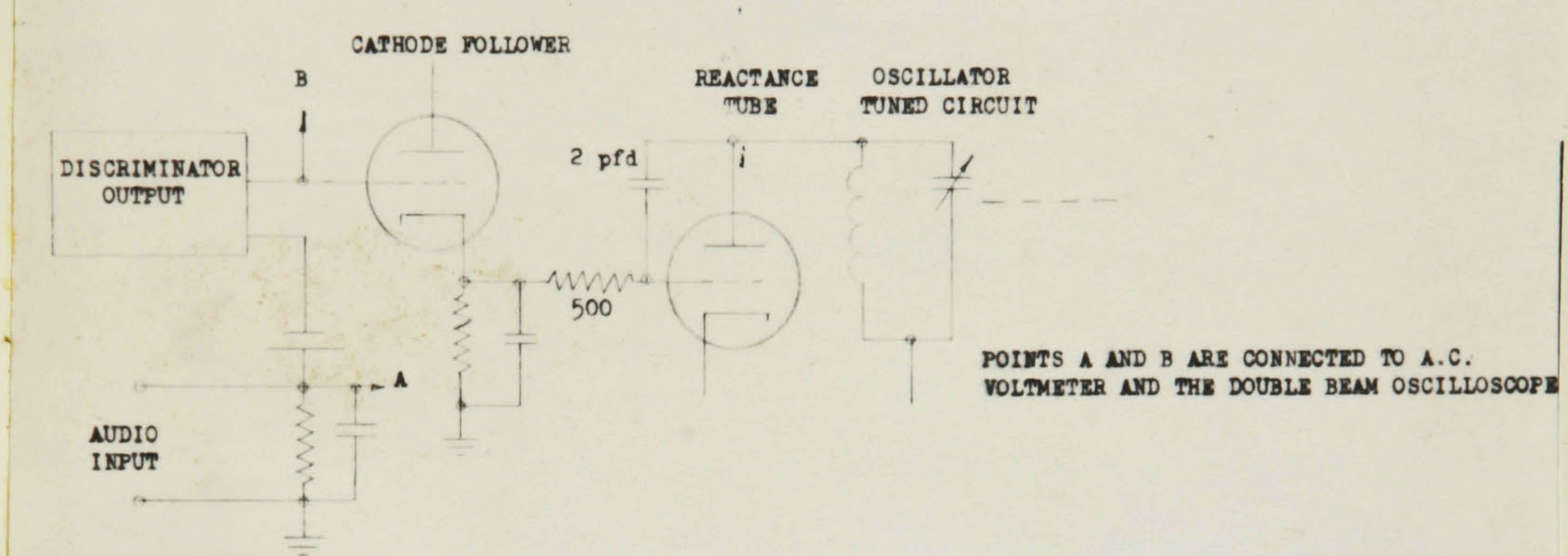


FIG. 34 ARRANGEMENT FOR CLOSED LOOP MEASUREMENTS

As the feedback loop greatly attenuated extraneous signals it was possible to carry this test to a higher frequency than was the case for the open loop. The readings for frequencies near the critical point (i.e., where the shift approaches 180°), were, for the same reason, more accurate with the loop closed. The two tests give essentially the same information, but in a different form, because when KG(s) is known $\frac{1}{1 + KG(s)}$ may be calculated.

The measured results, using a detector with 650 c/s cut-off and a compensating network with C = 50 pfd, are shown in figure 48. It may be seen that the maximum loop gain is 340 and the gain margin is $\frac{2.8 + 1}{2.8}$, or 2.7 db. At the critical frequency, $f_{\pi} = 20$ kc/s, KG(s) is negative and $\frac{1}{1 + KG(s)}$ assumes its greatest magnitude.

4.3 Closed Loop Measurements with Square Wave Input

This was the most important test as it gave the performance of the system under operating conditions. The equipment arrangement is indicated in figure 35.

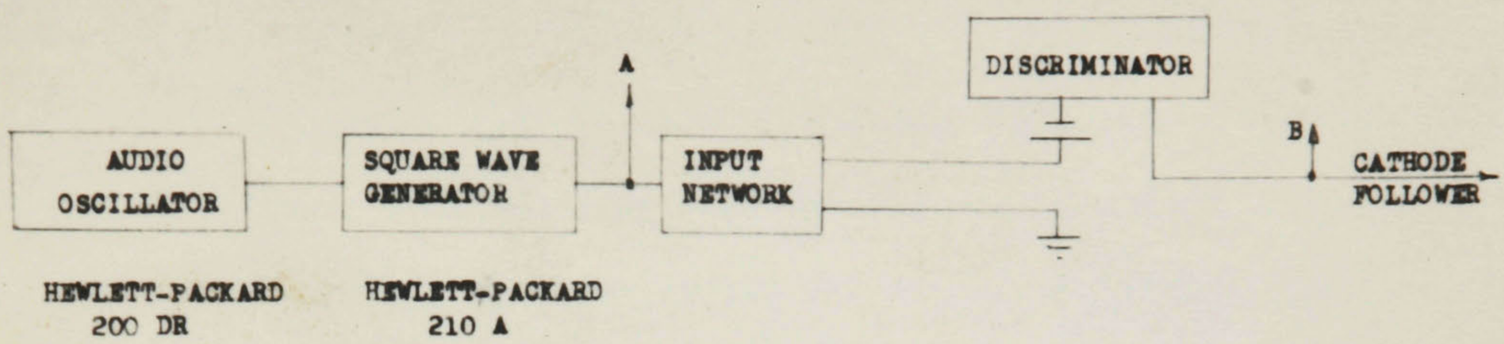


FIG. 35 ARRANGEMENT FOR SQUARE WAVE KEYING

The voltages at point A and B were observed on the oscilloscope. Voltmeters were not employed, because the form factor at point B was variable and not readily obtainable. Various forms of input network were used. A note was made of the transient response at the point B, and the maximum shift in the voltage (hence oscillator frequency shift) that was obtainable at this point. This variation was limited by the effect of diagonal clipping outlined in section 2.4.2. After a critical value of input bias was reached any further increase would give rise to a high order damped oscillation at the keying edge.

The method employed for the measurement of the frequency shift may be seen with reference to figure 36. The R.F. signal radiated from

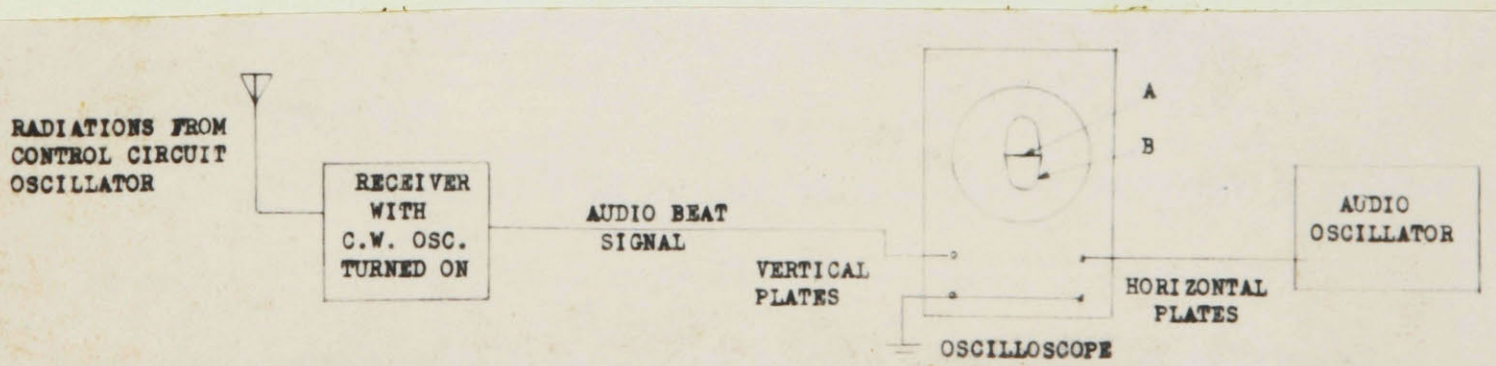


FIG 36 METHOD OF DETERMINING THE FREQUENCY SHIFT

the frequency shifted oscillator was monitored by a broadcast receiver. The C.W. oscillator was turned on so that an audio beat was produced.

When keying was applied the R.F. signal and hence the receiver audio signal, shifted between two frequencies. The C.W. oscillator was tuned to make one of the audio signals zero frequency. This gave a horizontal line on the oscilloscope, designated by A in the figure. The zero beat setting would be made accurately to a few cycles per second by setting the receiver oscillator dial midway between points where audio modulation was just discernible.

The second audio signal was therefore at a frequency equal to the value of the shift. It produced a vertical deflection to the oscilloscope pattern. With the audio oscillator set at the same frequency a Lissajous circle was produced. The shift was therefore indicated by the setting of the audio oscillator.

It was possible to carry out this test while the R.F. oscillator was being keyed. Due to the screen persistence both patterns A and B, were viewed simultaneously. The pattern contained various stray lines, due to the finite transition time between the two frequencies, but was clear enough to allow the shift to be read to an accuracy of approximately ± 20 c/s.

The results obtained with various input networks were:

(a) Direct Connection

When no input network was employed the square wave was presented to a circuit acting as a high pass filter or differentiating circuit. Voltage spikes were developed at the keying edges, and no appreciable shift was observed in the flat portion of the curve.

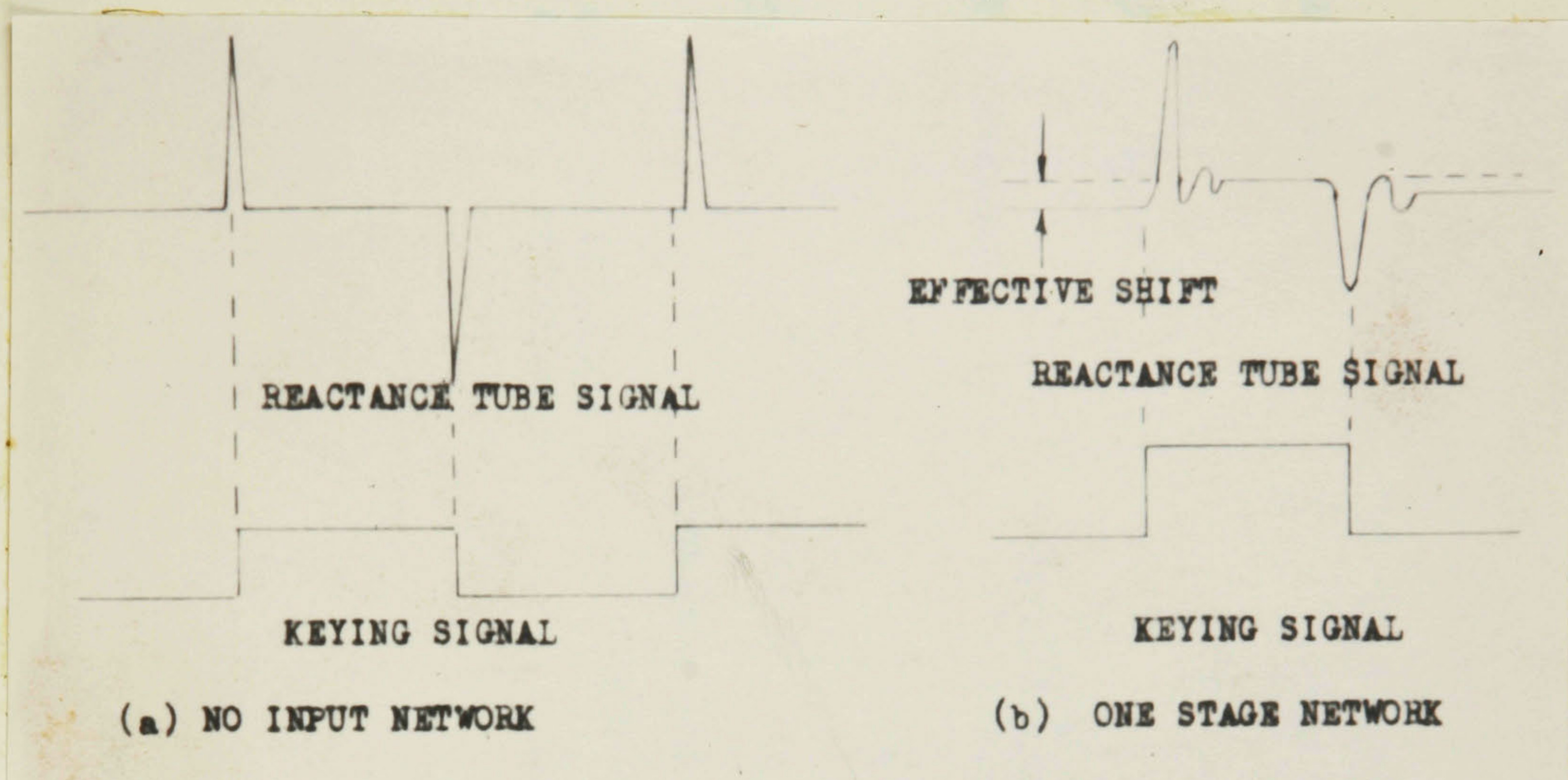


FIG .37. INPUT RESPONSE TO SQUARE WAVE KEYING

(b) One-Stage Network

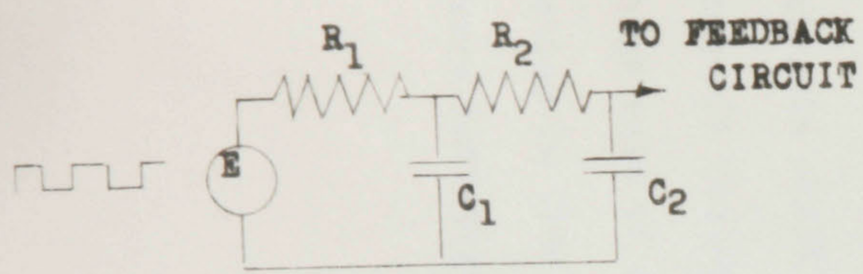
The same situation still existed, to a lesser degree. Pronounced transients were produced. A small voltage shift was observed, but its amplitude was only about 1/15 that of the pulse peak.

(c) Two-Stage Network

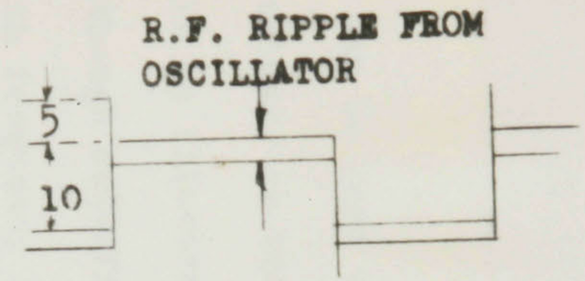
With two low pass networks it was possible to produce a transfer function which was approximately equal to the main circuit transfer function KG(s). In this case the response was quite faithfully reproduced. If the cut-off frequency was lowered the wave front of the signal at B was rounded. If the cut-off frequency was made higher then a steep front with a transient overshot resulted. The results obtained for various values of RC are shown in Figure 38.

(d) Three-Stage Network

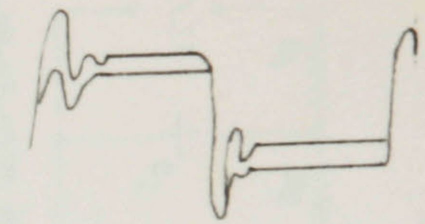
The values of R and C in this network were chosen with a view to provide a circuit giving 3 db attenuation at 115 c/s. (See section 3.3.3). The resulting wave form is illustrated in figure 39.



$R_1 = R_2 =$ 500 ohms
$C_1 = C_2 =$ .5 mfd



RESPONSE AT 100 C/S

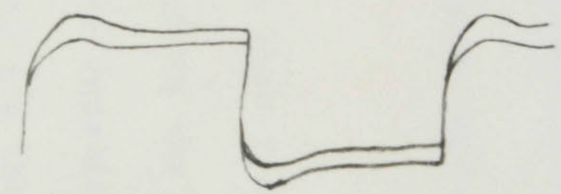


RESPONSE AT 1000 C/S
(RIPPLE IS AT 16,000 C/S)

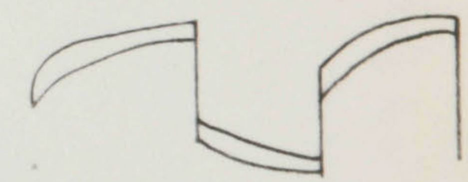
MAXIMUM FREQUENCY SHIFT =650 C/S

FIG. 38 (a) **TWO STAGE NETWORK WITH HIGH CUT-OFF FREQUENCY**

$R_1 = R_2 =$ 680
$C_1 = C_2 =$ .5 mfd



RESPONSE AT 50 C/S

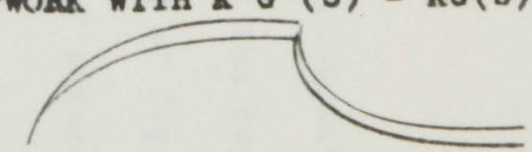


RESPONSE AT 200 C/S

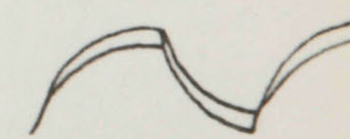
MAXIMUM FREQUENCY SHIFT =800 C/S

FIG. 38 (b) TWO STAGE NETWORK WITH K'G'(S) = KG(S)

$R_1 = R_2 =$ 1500
$C_1 = C_2 =$ .5 mfd



RESPONSE AT 50 C/S



RESPONSE AT 200 C/S

MAXIMUM FREQUENCY SHIFT = 650 C/S

FIG. 38 (c) TWO STAGE NETWORK WITH LOW CUT-OFF FREQUENCY

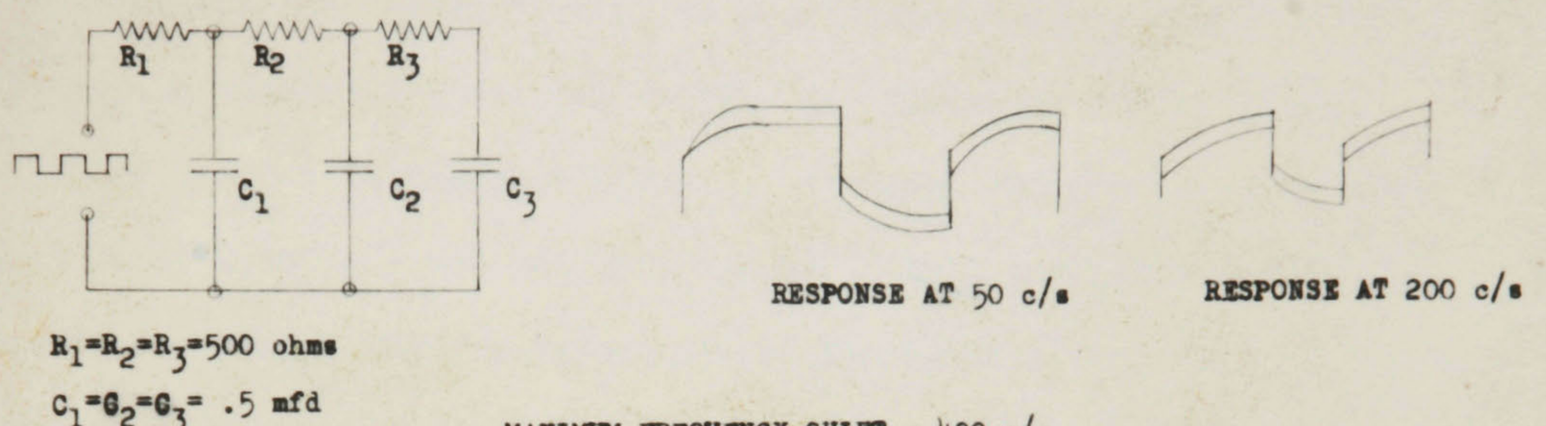


FIG. 39 THREE STAGE INPUT NETWORK

It can be seen that not sufficient high frequency attenuation is obtained, as there is a sharp front in the wave at the keying points. This is illustrated in the transfer function curve of figure 50. Although cut-off is obtained at 130 c/s the rate of attenuation with frequency is low.

With the use of isolated low pass filters whose time constants can be individually controlled, it should be possible to obtain wave forms of a more satisfactory form than those given, and to increase the available shift somewhat. The filter sections should all have an equal cut-off frequency so that the required frequencies may be passed and a high rate of attenuation be presented beyond this point. Due to restriction in time for the completion of this thesis no further networks were investigated.

DISCUSSION:

It has been shown that it is possible to construct a highly stable and compact transmitter exciter for frequency shift keying which is based on a feedback circuit. Although the goal of obtaining a shift of 1,000 c/s was not achieved it is felt that this could easily be accomplished by variations in the input network or crystal filter bandwidth, or an alteration to the detector circuit to further reduce the effect of diagonal clipping. It should be possible to extend the application of this system to the use of multiplex and other high speed keying methods. It is not restricted to binary code transmission and could be adapted for use with a variable intensity facsimile system.

The system described in the thesis would present a crystal procurement problem, as the reference crystal oscillator must differ from the transmitted frequency by the intermediate frequency, which was chosen to be 1 Mc/s. This could be solved by the addition of a subsidiary control loop. The reference oscillator would then consist of a variable oscillator controlled from a crystal which is cut for the transmission frequency. The extra control loop would operate at the same intermediate frequency, but as it carried no modulation it could be very simple in design.

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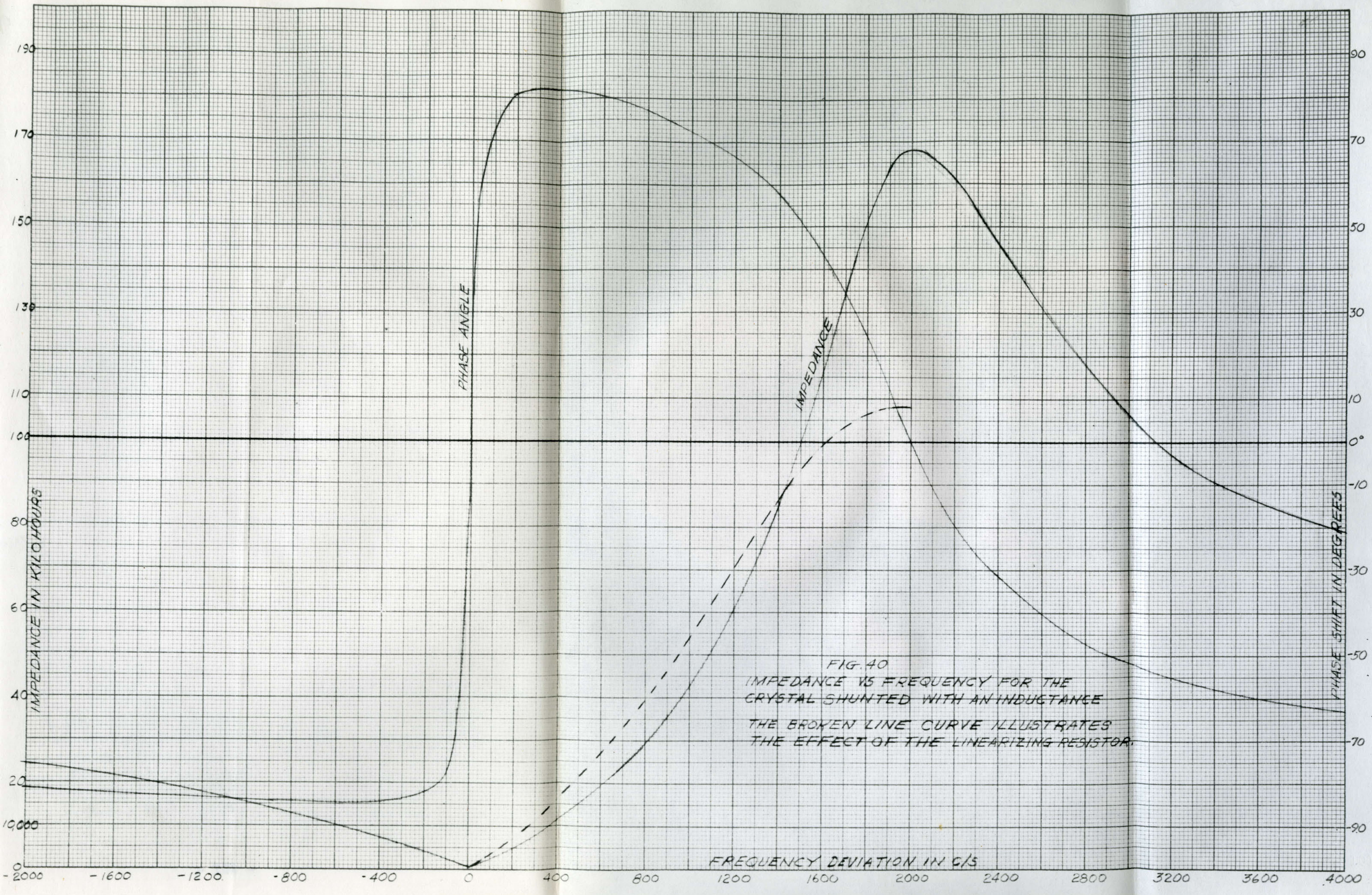


FIG. 40
IMPEDANCE VS FREQUENCY FOR THE CRYSTAL SHUNTED WITH AN INDUCTANCE

THE BROKEN LINE CURVE ILLUSTRATES THE EFFECT OF THE LINEARIZING RESISTOR.

400 C/S = 73-54 = 19°, 65-35 = 30°

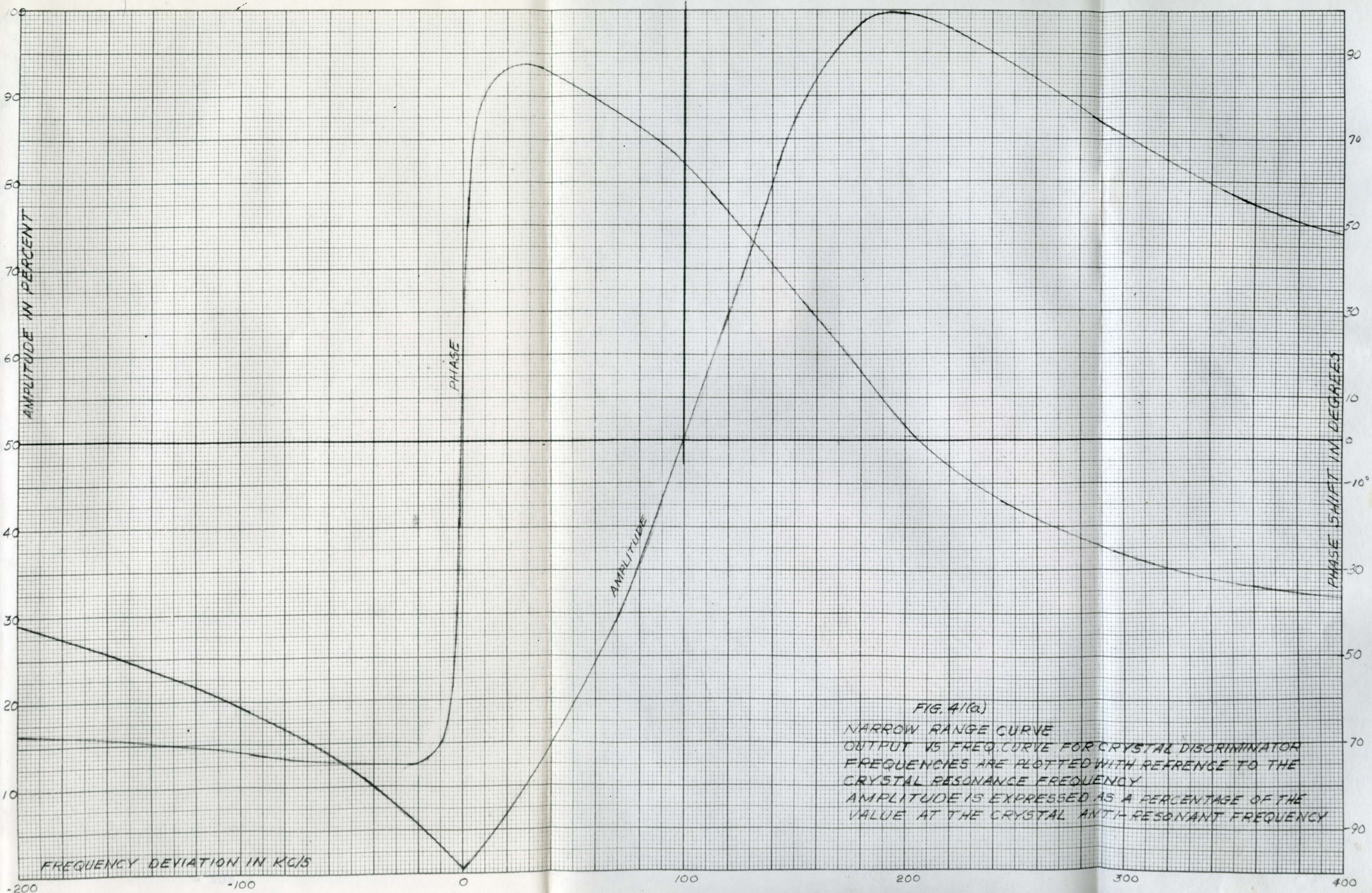


FIG. 41(a)

NARROW RANGE CURVE
OUTPUT VS FREQ. CURVE FOR CRYSTAL DISCRIMINATOR
FREQUENCIES ARE PLOTTED WITH REFRENCE TO THE CRYSTAL RESONANCE FREQUENCY
AMPLITUDE IS EXPRESSED AS A PERCENTAGE OF THE VALUE AT THE CRYSTAL ANTI-RESONANT FREQUENCY

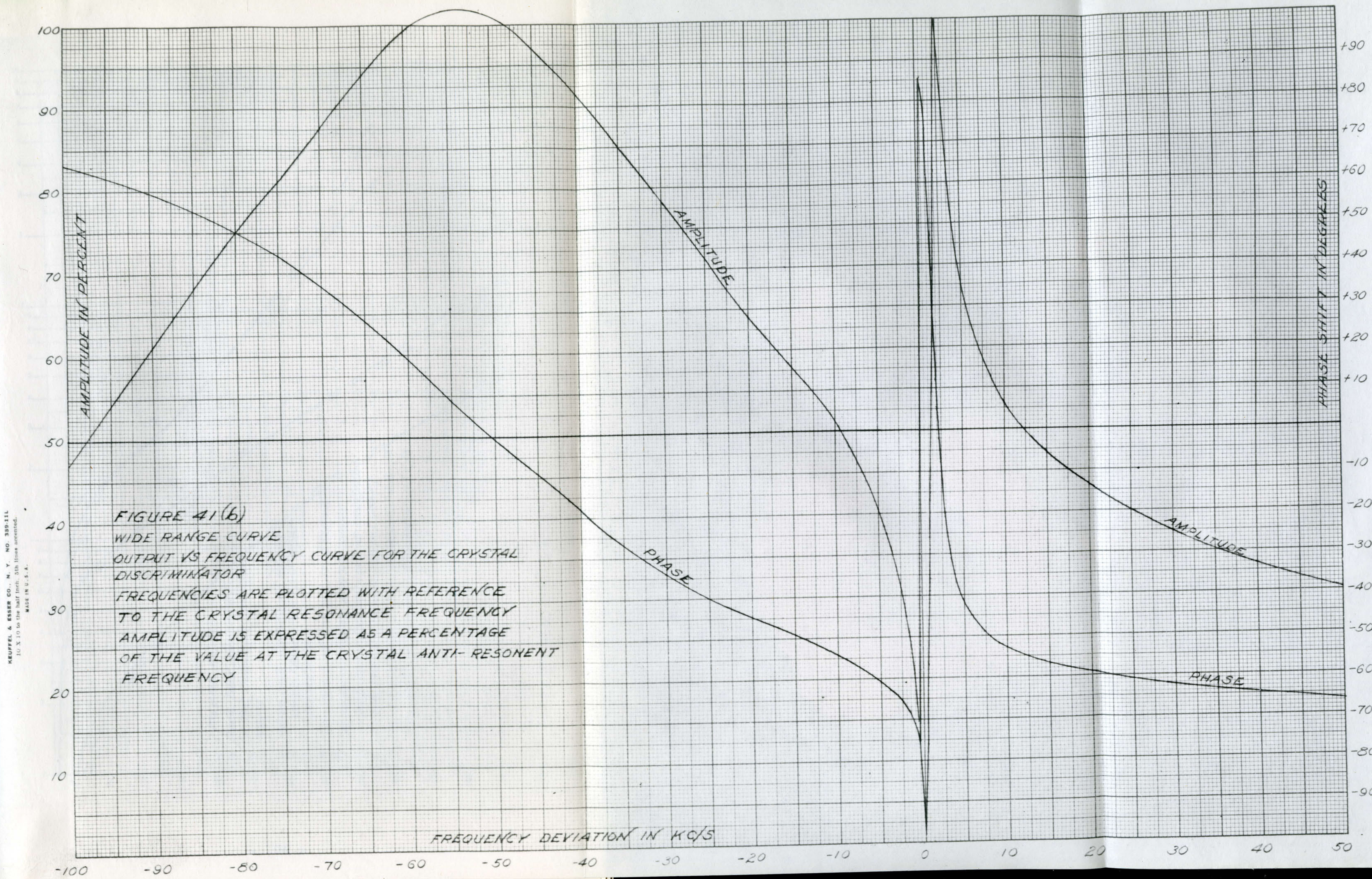


FIGURE 41(b)
WIDE RANGE CURVE
OUTPUT VS FREQUENCY CURVE FOR THE CRYSTAL DISCRIMINATOR
FREQUENCIES ARE PLOTTED WITH REFERENCE TO THE CRYSTAL RESONANCE FREQUENCY
AMPLITUDE IS EXPRESSED AS A PERCENTAGE OF THE VALUE AT THE CRYSTAL ANTI-RESONENT FREQUENCY

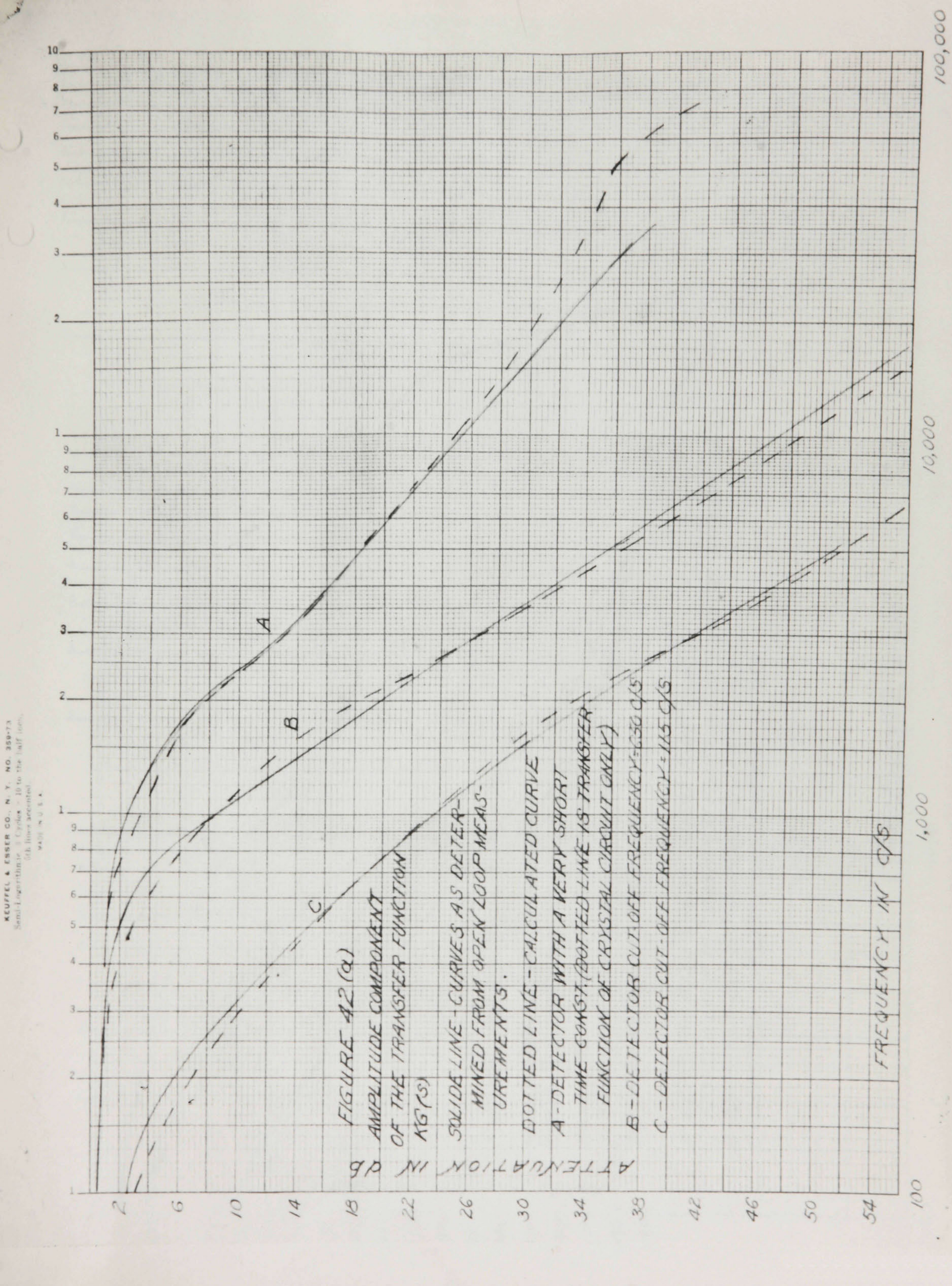


FIGURE 42(a)
AMPLITUDE COMPONENT
OF THE TRANSFER FUNCTION
KG(s)
SOLID LINE - CURVES AS DETER-
MINED FROM OPEN LOOP MEAS-
UREMENTS.
DOTTED LINE - CALCULATED CURVE
A - DETECTOR WITH A VERY SHORT
TIME CONST. (DOTTED LINE IS TRANSFER
FUNCTION OF CRYSTAL CIRCUIT ONLY)
B - DETECTOR CUT OFF FREQUENCY = 530 C/S
C - DETECTOR CUT-OFF FREQUENCY = 115 C/S
ATTENUATION IN dB
FREQUENCY IN C/S
A
B
C
2
6
10
14
18
22
26
30
34
38
42
46
50
54
100
1,000
10,000
100,000

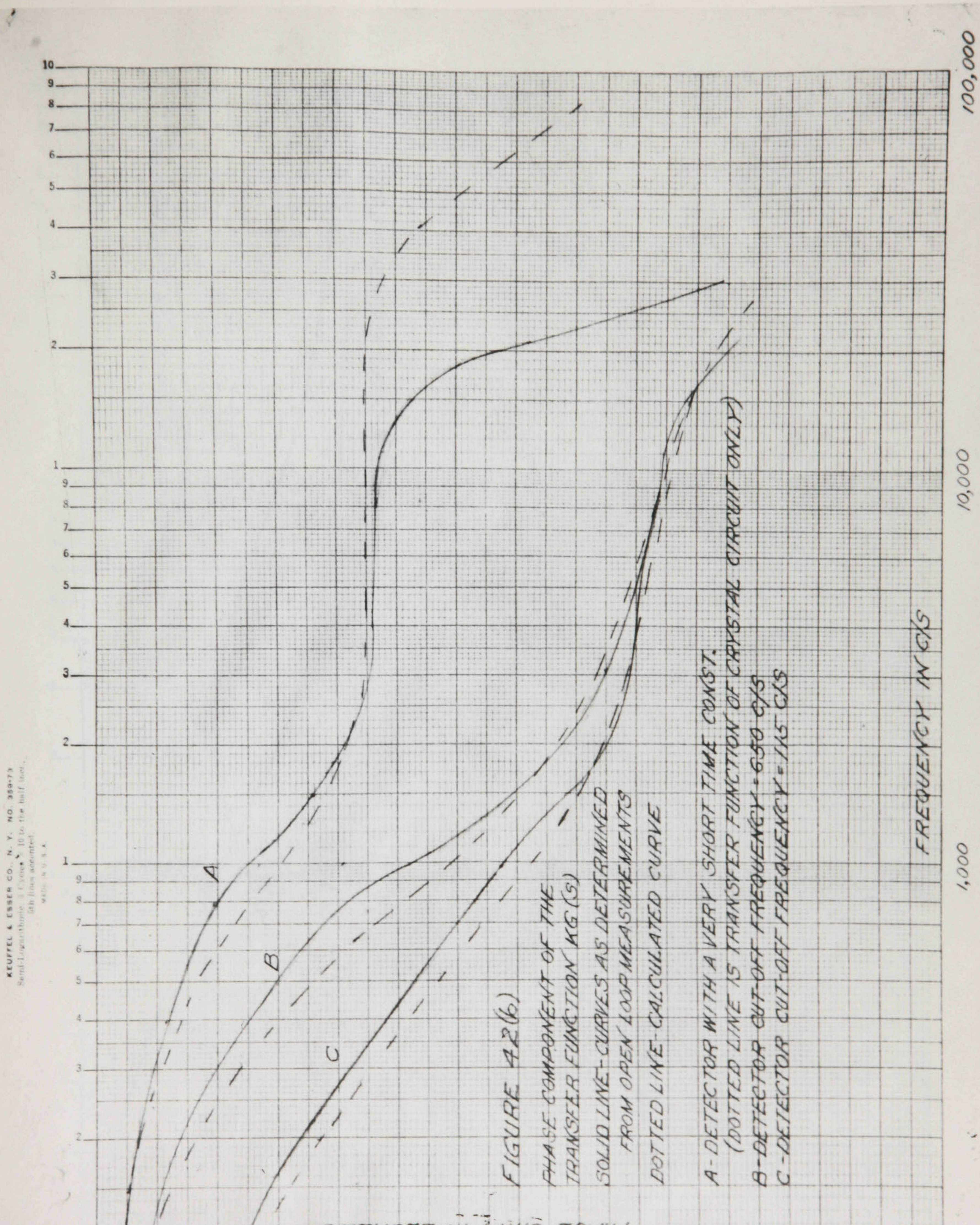


FIGURE 4.2(b)
PHASE COMPONENT OF THE
TRANSFER FUNCTION KG(S)
SOLID LINE-CURVES AS DETERMINED
FROM OPEN LOOP MEASUREMENTS
DOTTED LINE-CALCULATED CURVE
A-DETECTOR WITH A VERY SHORT TIME CONST.
(DOTTED LINE IS TRANSFER FUNCTION OF CRYSTAL CIRCUIT ONLY)
B-DETECTOR CUT-OFF FREQUENCY = 650 C/S
C-DETECTOR CUT-OFF FREQUENCY = 165 C/S
A
B
C
PHASE SHIFT IN DEGREES
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
FREQUENCY IN C/S
100
1,000
10,000
100,000
KEUFFEL & ESSER CO., N. Y. NO. 359-73

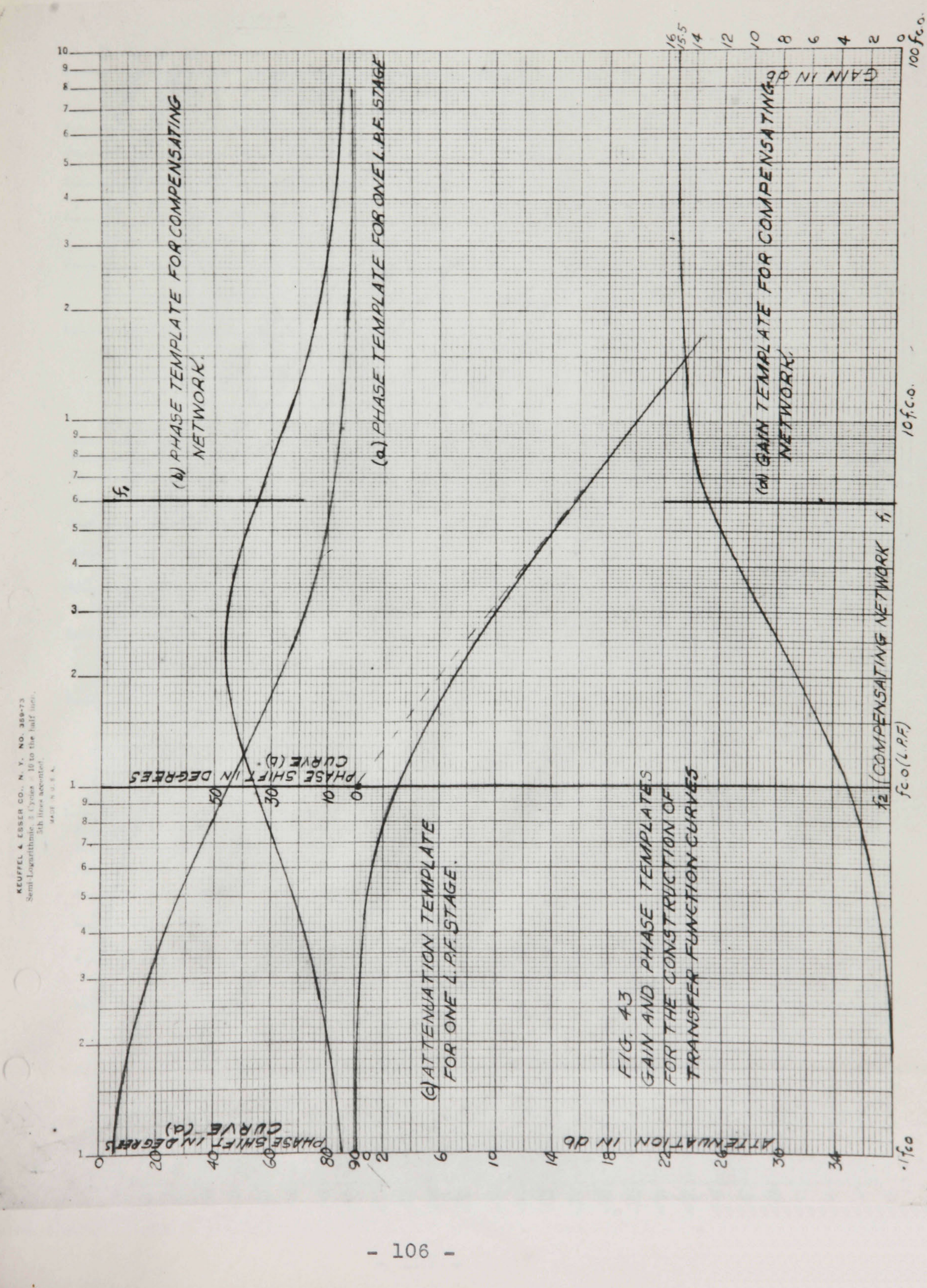


FIG. 43
GAIN AND PHASE TEMPLATES FOR THE CONSTRUCTION OF TRANSFER FUNCTION CURVES

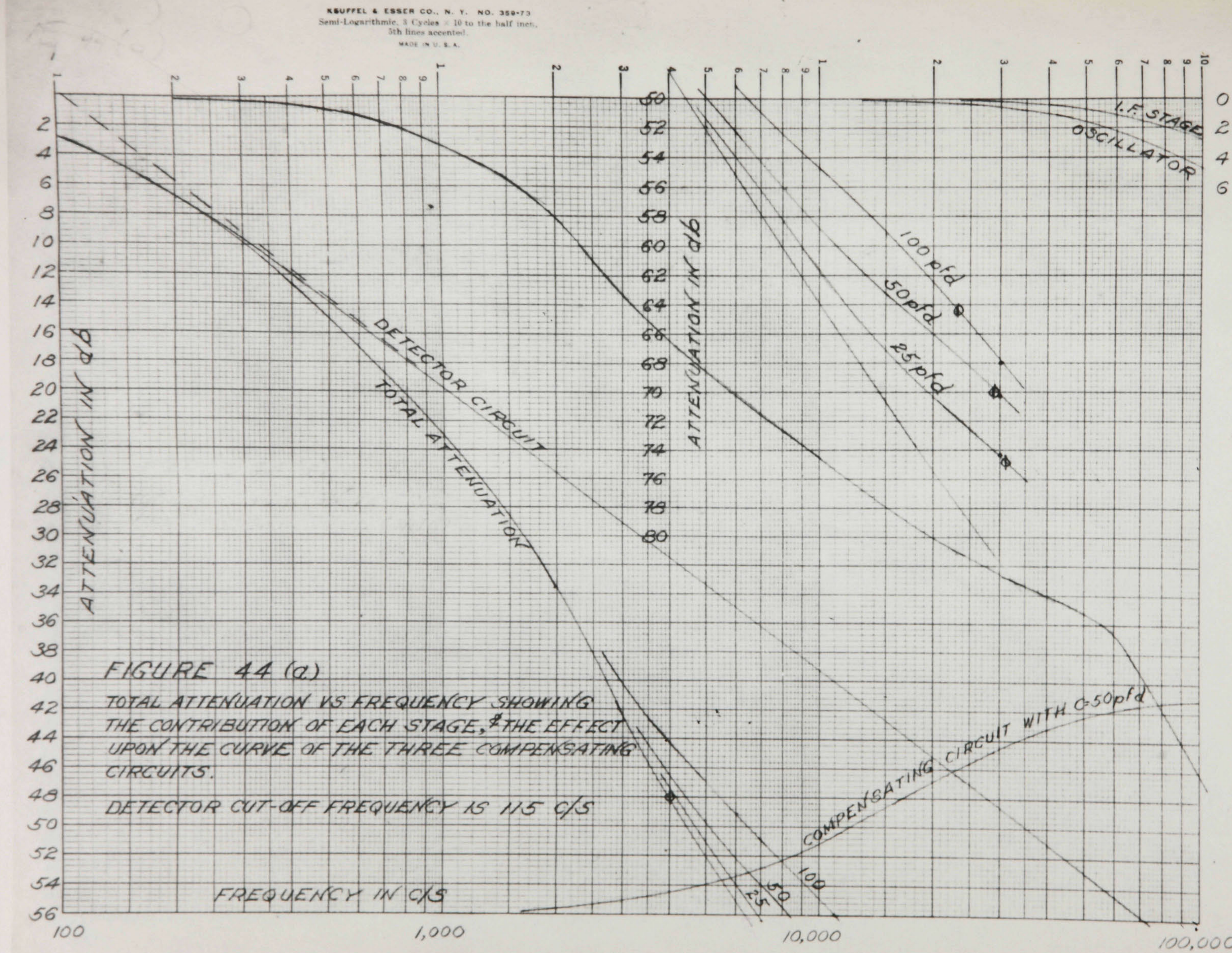
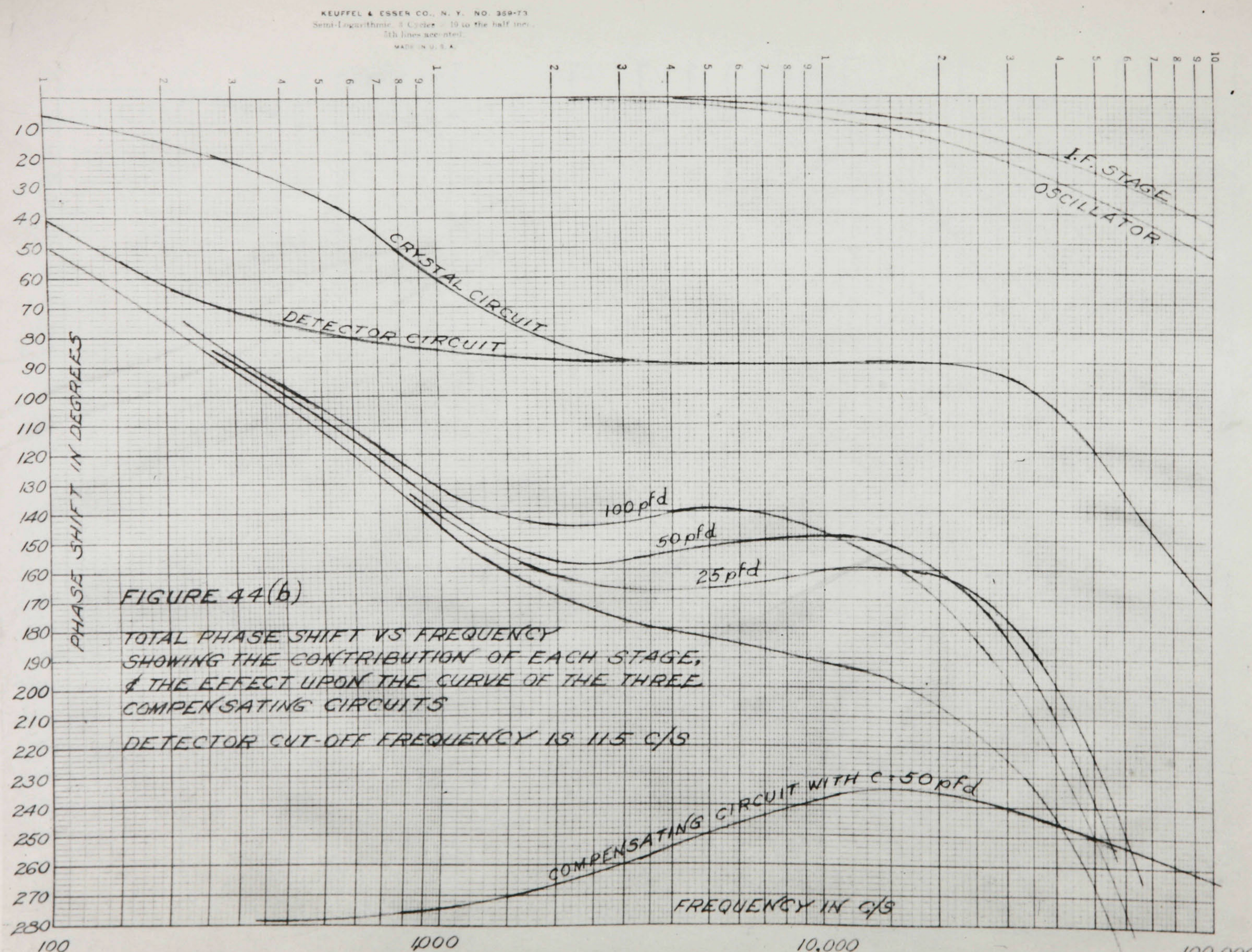


FIGURE 44 (a)

TOTAL ATTENUATION VS FREQUENCY SHOWING THE CONTRIBUTION OF EACH STAGE, & THE EFFECT UPON THE CURVE OF THE THREE COMPENSATING CIRCUITS.

DETECTOR CUT-OFF FREQUENCY IS 115 C/S


KEUFFEL & ESSER CO., N. Y. NO. 359-73
Semi-Logarithmic, 3 Cycles × 10 to the half inch, 5th lines accented.
MADE IN U. S. A.
FIGURE 44(b)
TOTAL PHASE SHIFT VS FREQUENCY
SHOWING THE CONTRIBUTION OF EACH STAGE,
& THE EFFECT UPON THE CURVE OF THE THREE
COMPENSATING CIRCUITS
DETECTOR CUT-OFF FREQUENCY IS 11.5 C/S
PHASE SHIFT IN DEGREES
FREQUENCY IN C/S
I.F. STAGE
OSCILLATOR
CRYSTAL CIRCUIT
DETECTOR CIRCUIT
100 pfd
50 pfd
25 pfd
COMPENSATING CIRCUIT WITH C = 50 pfd
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
260
270
280
100
1000
10,000
100,000

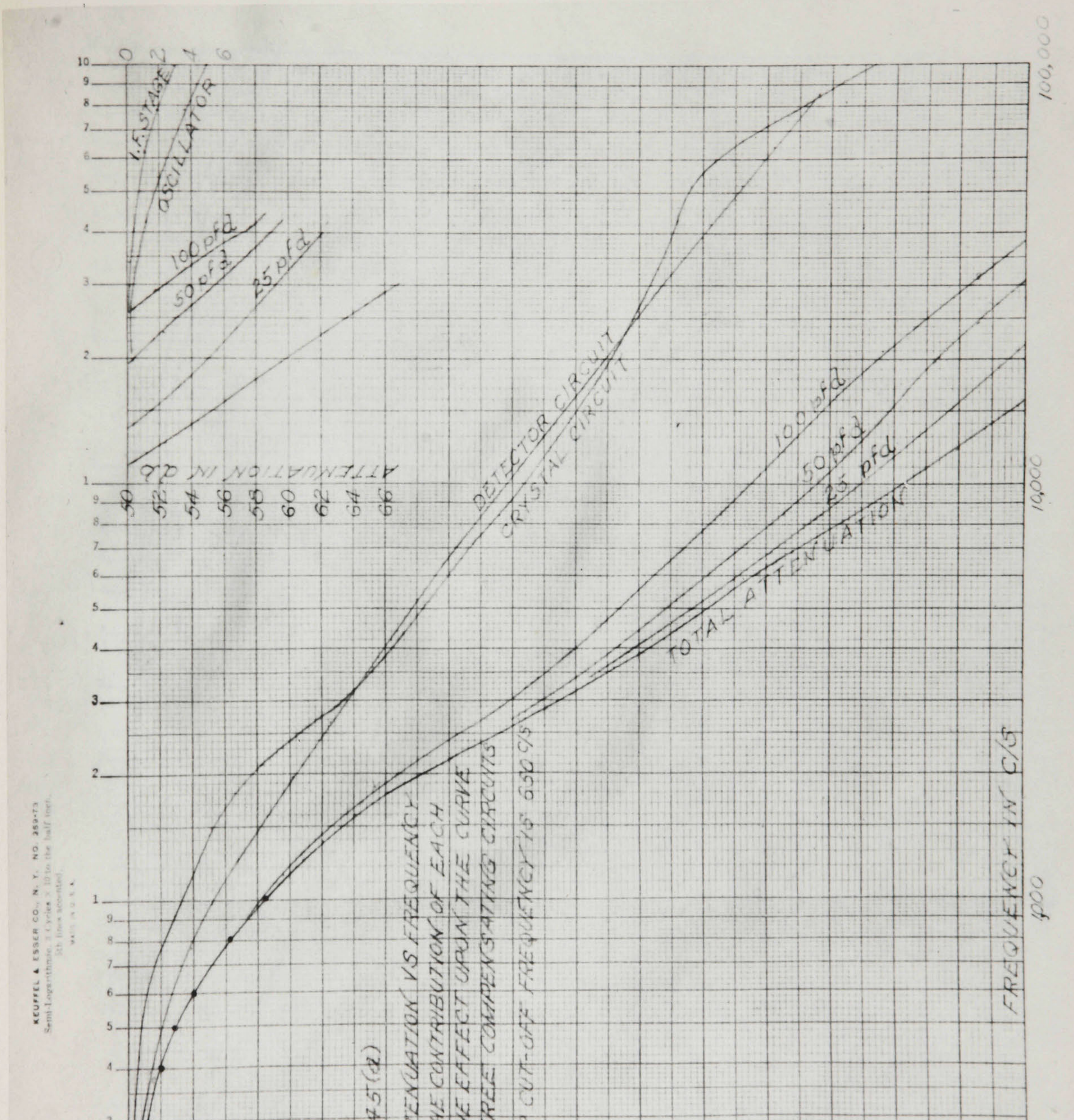


FIGURE 45(d)

TOTAL ATTENUATION VS FREQUENCY SHOWING THE CONTRIBUTION OF EACH STAGE, THE EFFECT UPON THE CURVE OF THE THREE COMPENSATING CIRCUITS

DETECTOR CUT-OFF FREQUENCY IS 650 C/S

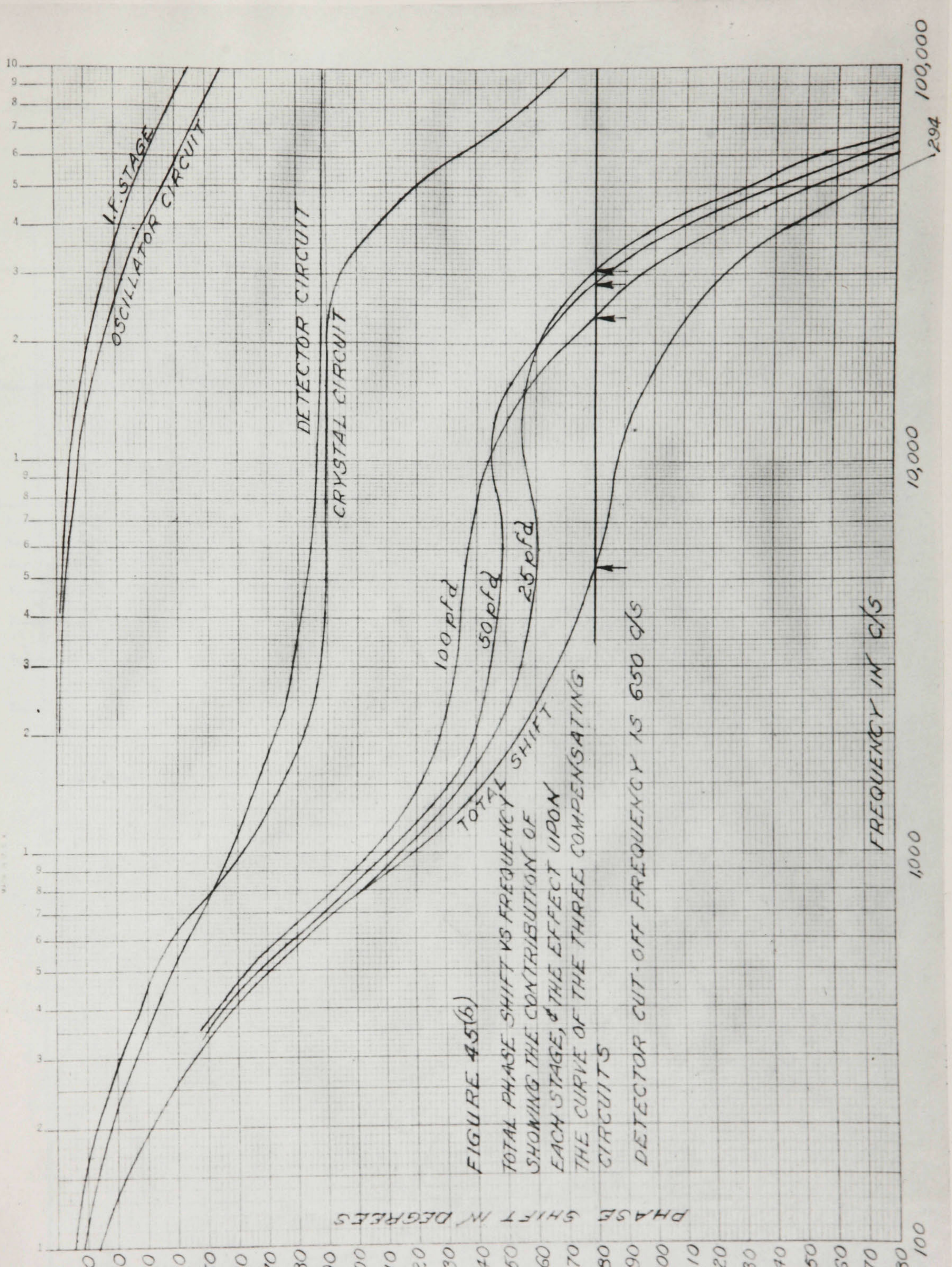
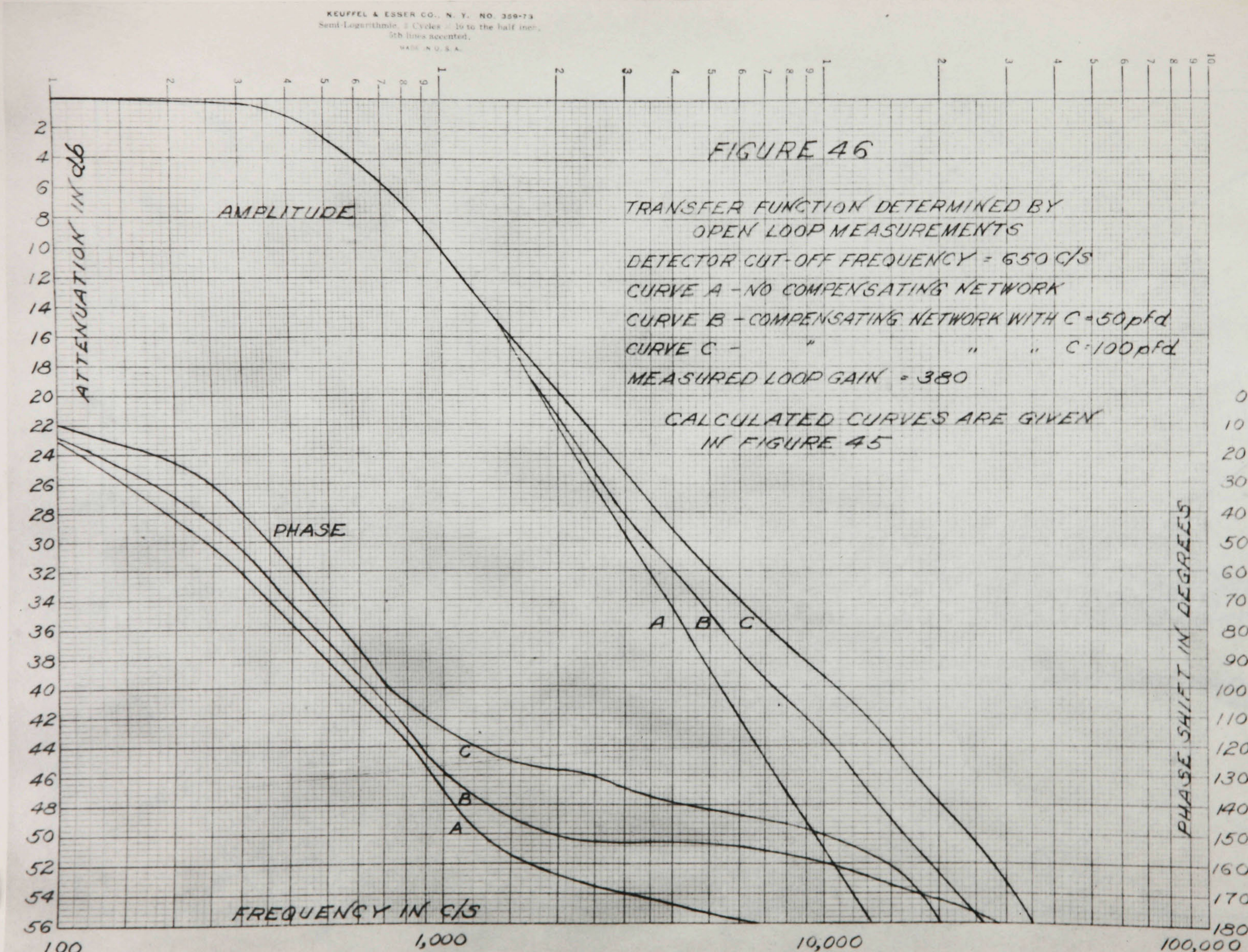


FIGURE 45(b)

TOTAL PHASE SHIFT VS FREQUENCY SHOWING THE CONTRIBUTION OF EACH STAGE, & THE EFFECT UPON THE CURVE OF THE THREE COMPENSATING CIRCUITS

DETECTOR CUT-OFF FREQUENCY IS 650 C/S


KEUFFEL & ESSER CO., N. Y. NO. 359-73
Semi-Logarithmic, 3 Cycles × 10 to the half inch, 5th lines accented.
MADE IN U.S.A.
FIGURE 46
TRANSFER FUNCTION DETERMINED BY OPEN LOOP MEASUREMENTS
DETECTOR CUT-OFF FREQUENCY = 650 C/S
CURVE A - NO COMPENSATING NETWORK
CURVE B - COMPENSATING NETWORK WITH C = 50 pfd
CURVE C - " " " C = 100 pfd
MEASURED LOOP GAIN = 380
CALCULATED CURVES ARE GIVEN IN FIGURE 45
AMPLITUDE
PHASE
A
B
C
ATTENUATION IN db
PHASE SHIFT IN DEGREES
FREQUENCY IN C/S
100
1,000
10,000
100,000

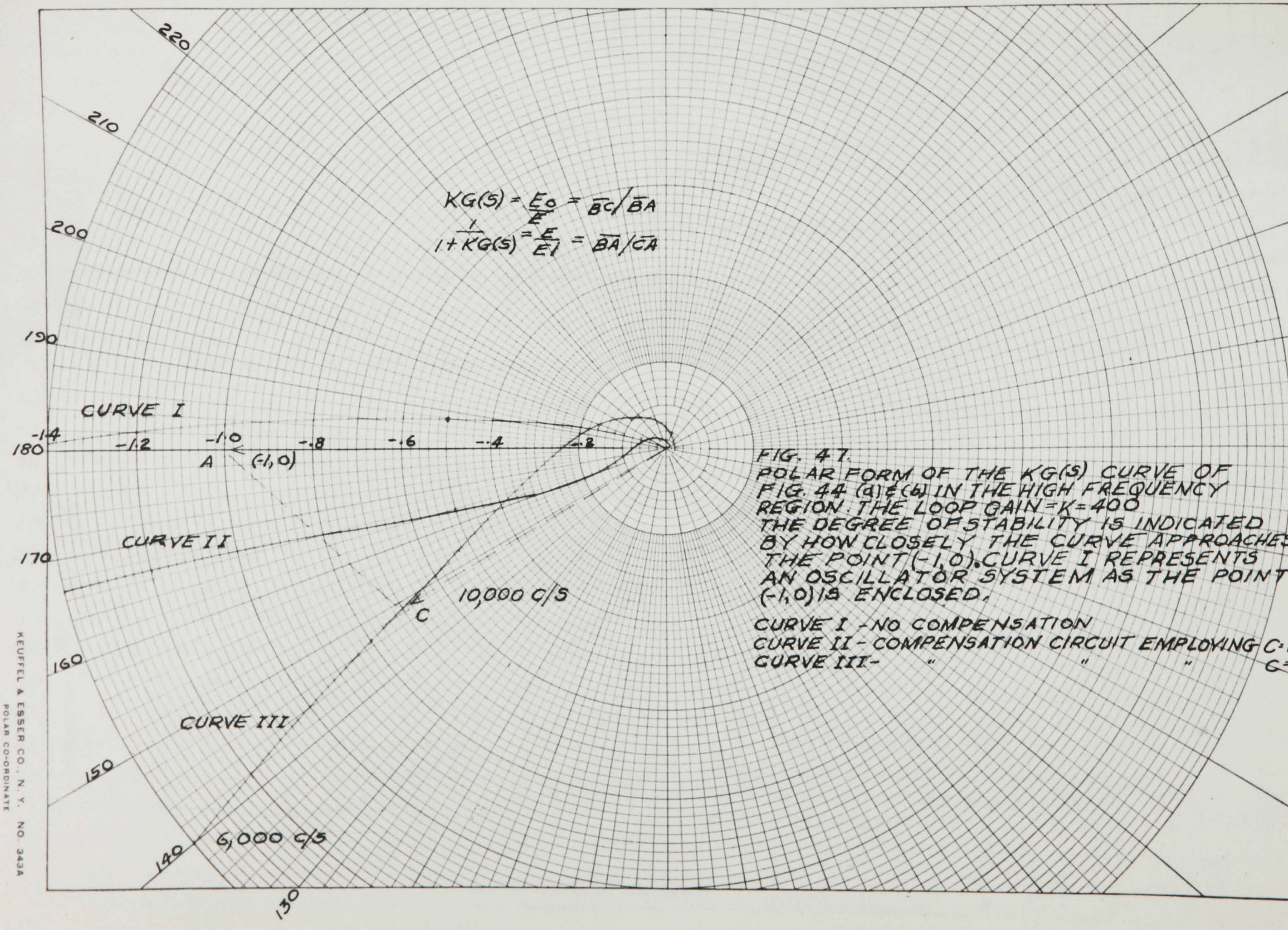


FIG. 47.
POLAR FORM OF THE KG(S) CURVE OF FIG. 44 (a) & (b) IN THE HIGH FREQUENCY REGION. THE LOOP GAIN = K = 400
THE DEGREE OF STABILITY IS INDICATED BY HOW CLOSELY THE CURVE APPROACHES THE POINT (-1,0). CURVE I REPRESENTS AN OSCILLATOR SYSTEM AS THE POINT (-1,0) IS ENCLOSED.

CURVE I - NO COMPENSATION
CURVE II - COMPENSATION CIRCUIT EMPLOYING C=25 pfd
CURVE III - " " " C=100 pfd.

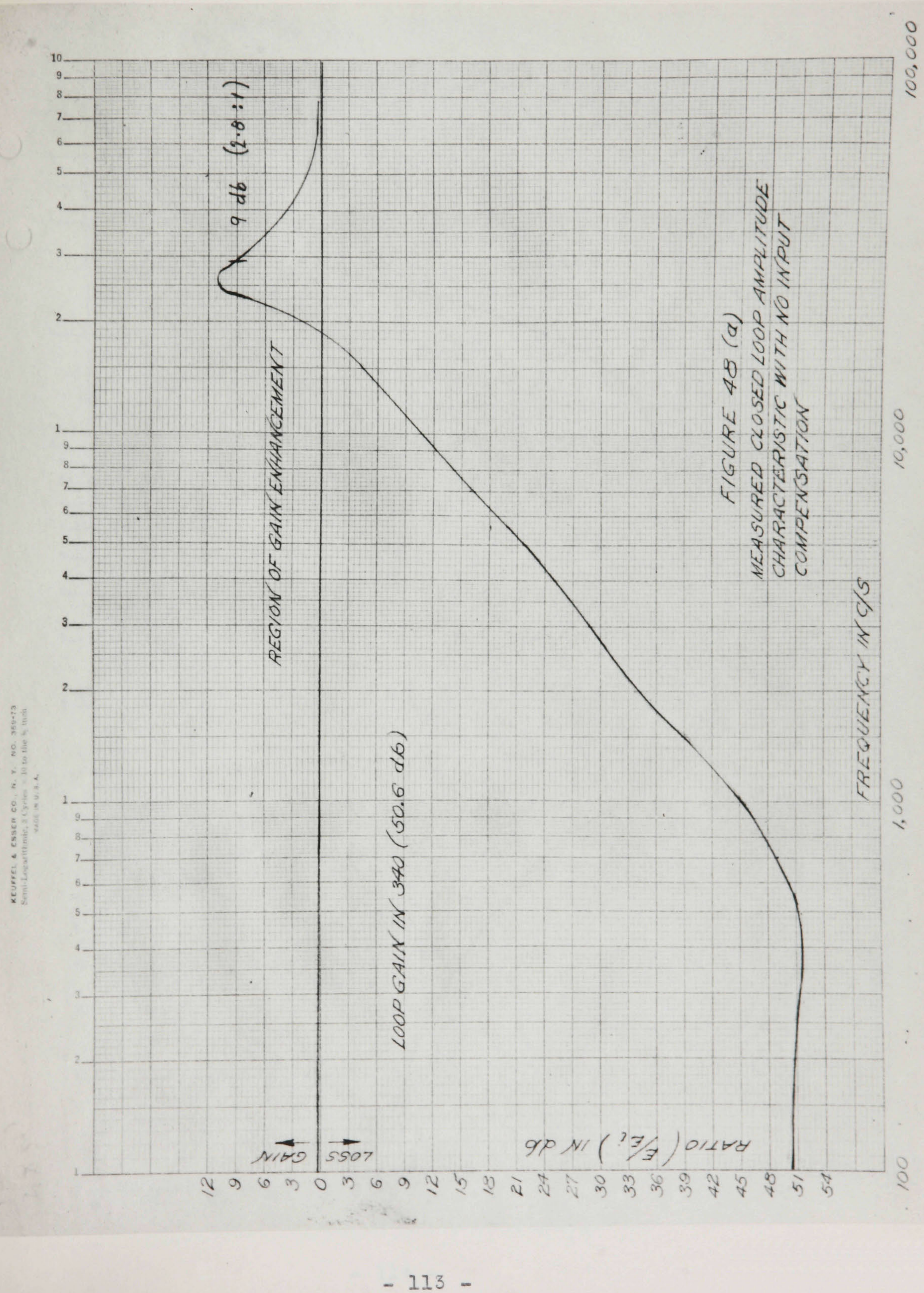


FIGURE 48 (a)

MEASURED CLOSED LOOP AMPLITUDE CHARACTERISTIC WITH NO INPUT COMPENSATION

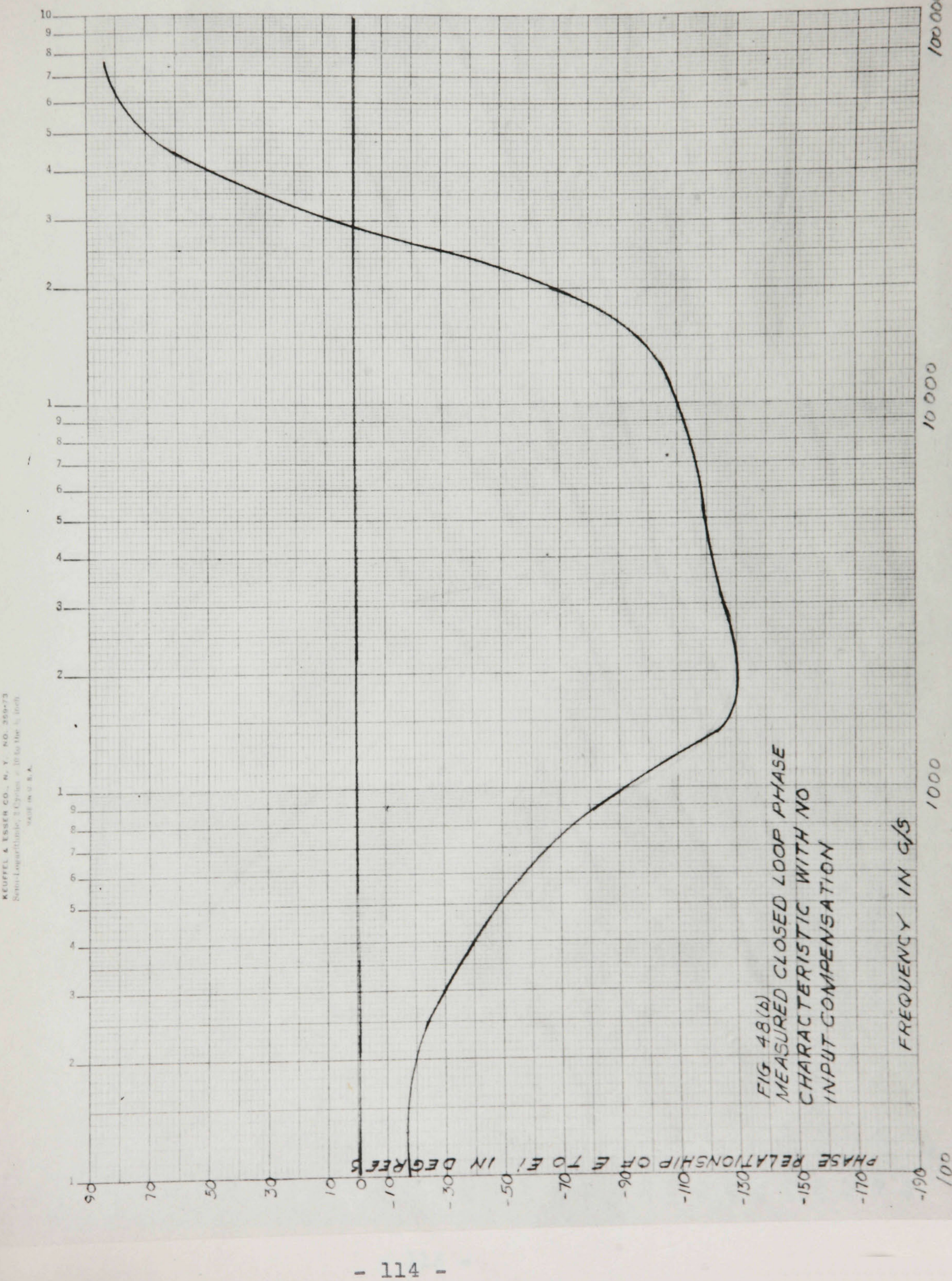


FIG. 48(b)
MEASURED CLOSED LOOP PHASE
CHARACTERISTIC WITH NO
INPUT COMPENSATION

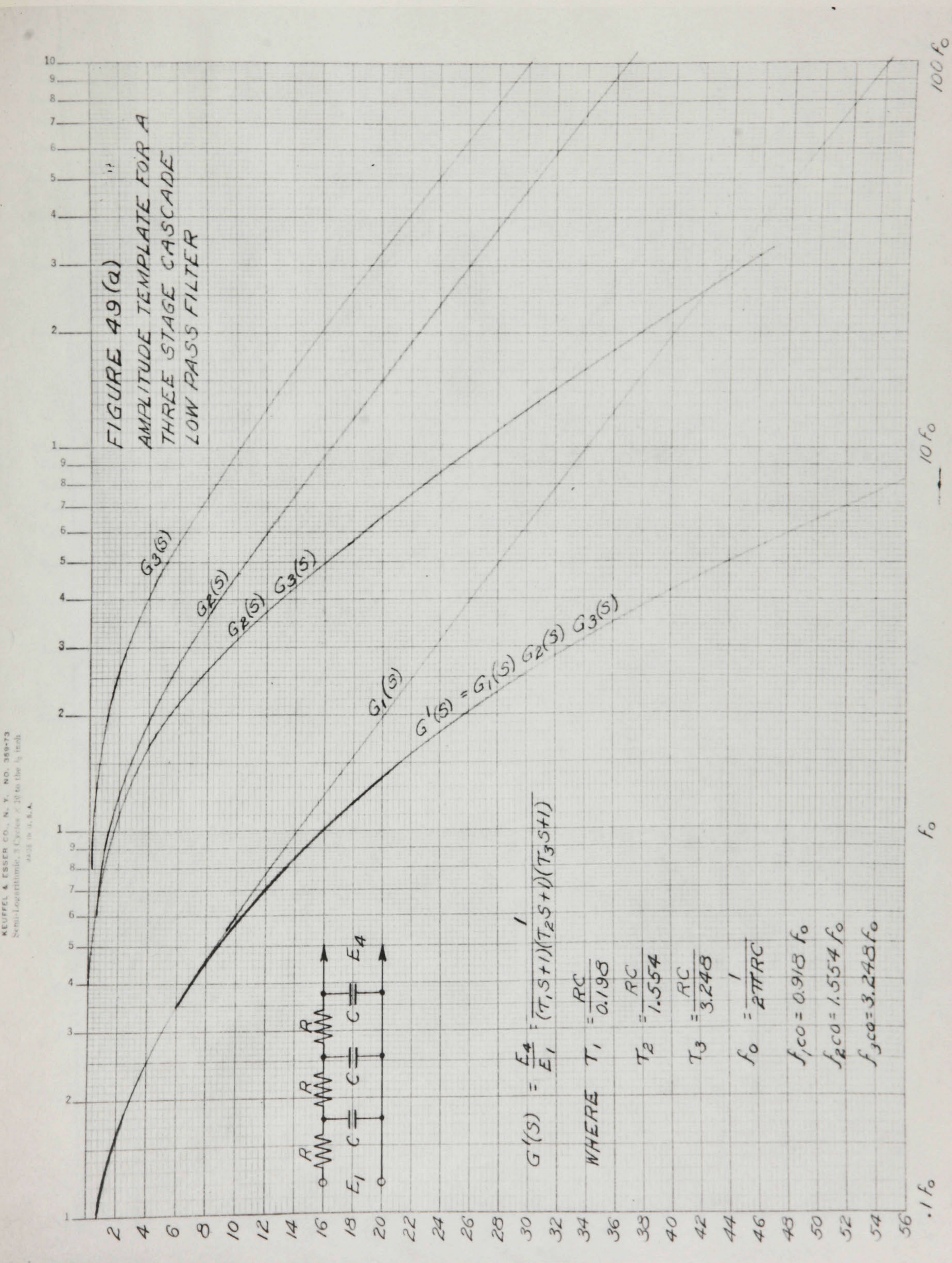


FIGURE 49(a)
AMPLITUDE TEMPLATE FOR A THREE STAGE CASCADE LOW PASS FILTER
G3(S)
G2(S) G3(S)
G1(S)
G'(S) = G1(S) G2(S) G3(S)
R R R
C C C
E1 E4
G'(S) = E4/E1 = 1/((T1S+1)(T2S+1)(T3S+1))
WHERE T1 = RC/0.198
T2 = RC/1.554
T3 = RC/3.248
f0 = 1/2πRC
f1co = 0.918 f0
f2co = 1.554 f0
f3co = 3.248 f0
.1 f0
f0
10 f0
100 f0
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56
KEUFFEL & ESSER CO., N. Y. NO. 359-73

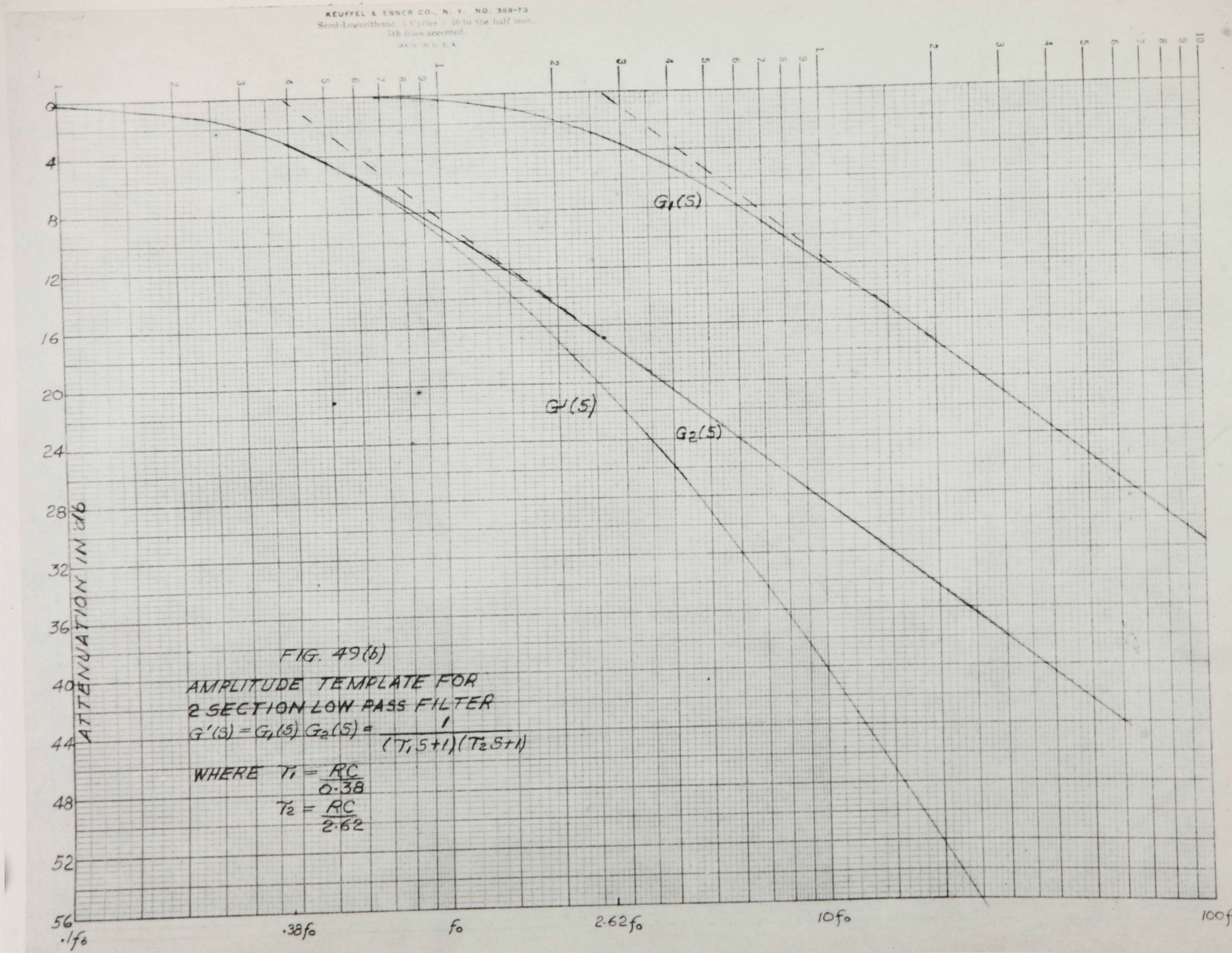


FIG. 49(b)

AMPLITUDE TEMPLATE FOR 2 SECTION LOW PASS FILTER

$G'(S) = G_1(S)\,G_2(S) = \dfrac{1}{(T_1 S+1)(T_2 S+1)}$

WHERE $T_1 = \dfrac{RC}{0.38}$

$T_2 = \dfrac{RC}{2.62}$

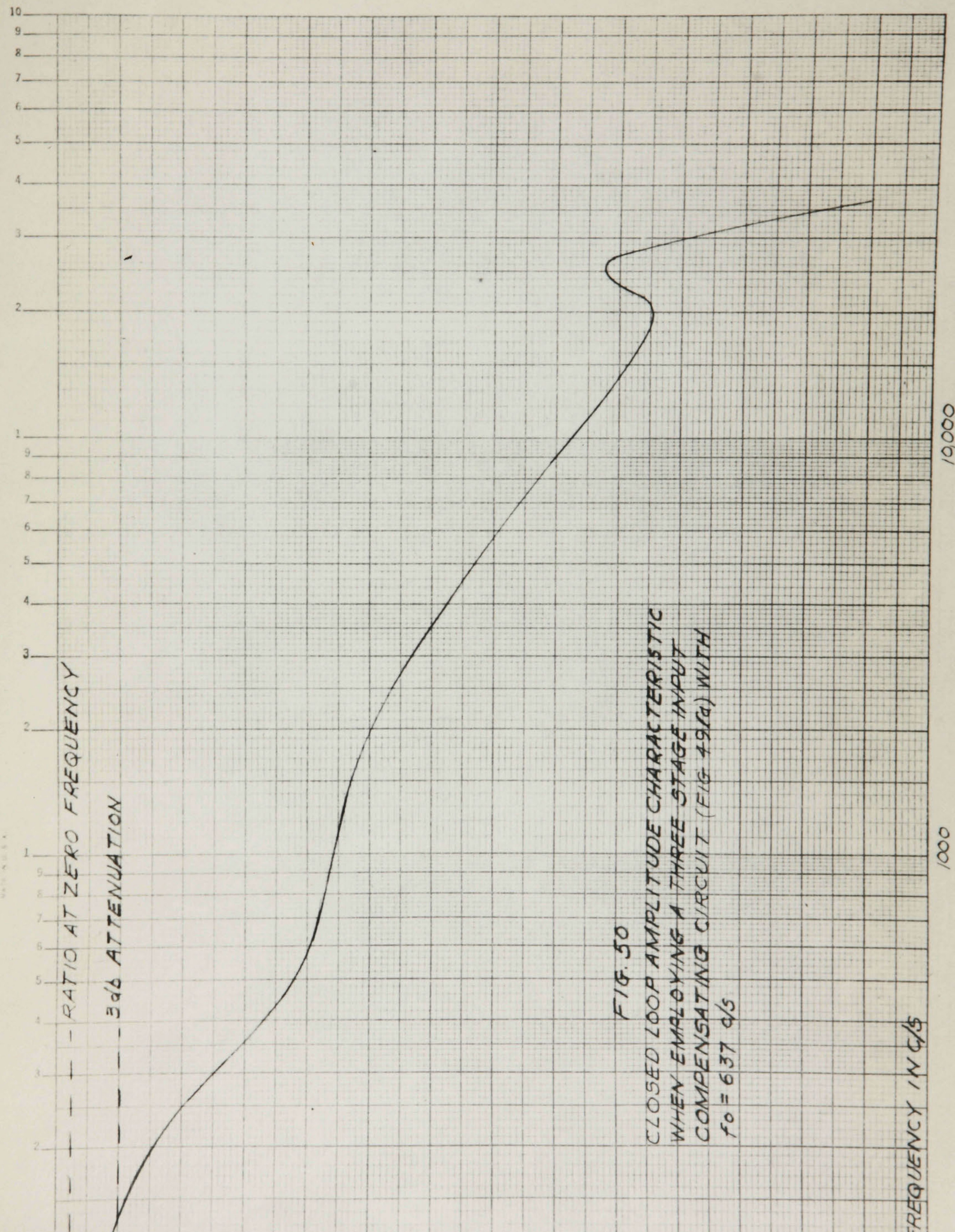


FIG. 50

CLOSED LOOP AMPLITUDE CHARACTERISTIC WHEN EMPLOYING A THREE STAGE INPUT COMPENSATING CIRCUIT (FIG 49(a)) WITH $f_0 = 637$ c/s