Two Essays on Power and Information Asymmetries in Competitive Supply Chains

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ABSTRACT

Channel parties in today's competitive supply chains are increasingly becoming asymmetric in their power and the level of information they possess regarding several important aspects of their businesses. Even though these asymmetries can be beneficial for some parties, they might create frictions towards other parties' regular practice and profitability. Studying these asymmetries can be helpful for all parties when making their strategic decisions. In particular, it helps weak and uninformed players to improve their strategies on how to respond to the additional power of their partners or competitors in the market, while the dominant players might want to reinforce these asymmetries as sources and prerequisites for their competitive advantages in the marketplace. This dissertation examines asymmetries in operations and supply chain management through two specific applications, which has resulted in two essays. The first one focuses on the impacts of market power and cost asymmetries in retail operations. More specifically, it analyzes the effects of superior cost advantage and price leadership possessed by a dominant retailer on his assortment choice. Using several game-theoretical models, this essay aims to explain the asymmetry-related causes for the retail assortment reduction, a practice taken by many big-box retailers in some product categories. The second essay is related to the information asymmetry in sourcing and supply management. More precisely, it looks at the buyers' private information regarding the quality scores of their suppliers in price-plus buyer-determined procurement (reverse) auctions. The general goal in this essay is to understand the informational and strategic implications of non-price attributes in procurement auctions as an increasingly popular sourcing mechanism. Using the Bayesian Nash Equilibrium solution concept, this essay provides normative recommendations to the buyers on when and how to share this information with the suppliers. In different ways, both essays support the idea that information and power asymmetries significantly change the motivation and action of channel partners in their operational decisions.

ABRÉGÉ

Les acteurs des chaînes d'approvisionnement concurrentielles actuelles sont de plus en plus asymétriques dans leur puissance et le niveau d'information qu'ils possèdent sur plusieurs aspects importants de leurs activités. Même si ces asymétries peuvent être bénéfiques pour certaines parties, elles peuvent créer des frictions à l'égard de la pratique régulière et de la rentabilité des autres parties. L'étude de ces asymétries peut être utile pour toutes les parties lorsqu'elles prennent leurs décisions stratégiques. En particulier, il aide les acteurs faibles et non informés à améliorer leurs stratégies sur la façon de répondre à la puissance supplémentaire de leurs partenaires ou concurrents sur le marché, alors que les acteurs dominants pourraient vouloir renforcer ces asymétries comme sources et préalables à leurs avantages concurrentiels dans le Marché. Cette dissertation examine les asymétries dans les opérations et la gestion de la chaîne d'approvisionnement à travers deux applications spécifiques, ce qui a donné lieu à deux essais. La première porte sur les impacts de la puissance de marché et les asymétries de coûts dans les opérations de détail. Plus précisément, il analyse les effets d'un avantage de coût supérieur et d'un leadership sur les prix que possède un détaillant dominant sur son choix d'assortiment. En utilisant plusieurs modèles théoriques de jeu, cet essai a pour but d'expliquer les causes liées à l'asymétrie pour la réduction de l'assortiment de détail, une pratique pratiquée par de nombreux détaillants de grandes surfaces dans certaines catégories de produits. Le deuxième essai est lié à l'asymétrie de l'information dans l'approvisionnement et la gestion de l'offre. Plus précisément, il examine les informations privées des acheteurs concernant les scores de qualité de leurs fournisseurs dans les enchères inversées déterminées par les acheteurs. L'objectif général de cet essai est de comprendre les implications informationnelles et stratégiques des attributs autres que le prix dans les enchères d'approvisionnement comme un mécanisme de sourcing de plus en plus populaire. En utilisant le concept Bayesian Nash solution d'équilibre, cet essai fournit

des recommandations normatives aux acheteurs sur quand et comment partager cette information avec les fournisseurs. D'une manière différente, les deux essais appuient l'idée que les asymétries d'information et de pouvoir modifient de façon significative la motivation et l'action des partenaires de canal dans leurs décisions opérationnelles.

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CHAPTER 1
Introduction

Channel parties in competitive supply chains are increasingly becoming asymmetric in their power and the level of information they possess regarding several important aspects of their businesses (Nahmias and Olsen 2015). These asymmetries are believed to change parties' actions significantly depending on the context of business, type and depth of asymmetry, supply chain design, etc. They might also create some frictions towards channel parties' regular practice and profitability (e.g. Özer and Wei 2006, Yang et al. 2009).

Understanding these asymmetries and their practical and strategic impacts on the incentive and actions of channel parties is crucial for the managers who adopt reactive or proactive business strategies. On the one hand, this knowledge enables weak and uninformed channel parties to act and respond more effectively in today's competitive markets. On the other hand, some dominant firms (like Walmart, Apple, Home Depot, etc.) with great informational and/or strategic powers might need to choose and work on deepening one or more of these asymmetries to help them to attain competitive advantages. A deep knowledge of these asymmetries help them to streamline their strategies and actions more successfully.

Analytically studying these asymmetries in real life situations is possible only by considering competitive settings that require the interaction among parties, necessitating the study of strategic decision making, namely *game theory*. Using mathematical and logical approaches, game theory tries to find the set of actions that decision makers (players) should take to secure the best outcomes for themselves taking into account the impacts of other players' and their own actions on their outcomes. According to Rasmusen and Blackwell [1994], a game must include the following elements: the players of the game, the information and actions available to each player at each decision point, and the payoffs for each outcome. A game theorist then uses these elements, along with a solution concept of their choice, to conclude a set of equilibrium strategies for each player such that, when these strategies are employed, no player can profit by unilaterally deviating from their equilibrium strategy.

With regard to the information assumption for the parties, a very simple form deals with *symmetric games* in which information is the same for each player. By contrast, *asymmetric games* are those games where the players do not stand on equal ground. Specifically, a game under *information asymmetry* deals with the decisions made in a situation where one player has more or better information than the other player(s). Examples of information asymmetry include: when suppliers know their own cost better than anyone else when bidding in a reverse auction, when insurance companies are unaware of insurance buyers' health condition while designing the contract clauses, when workers' potential productivity is unobservable by a hiring firm, and, when suppliers know more about their production reliability than manufacturers while signing a supply contract.

Even though supply chain asymmetries form a novel stream of research that has received increased attention from investigators recently, many types of information and power asymmetries and their potential effects have not yet been addressed. Motivated by the importance of these asymmetries, this dissertation in two essays focuses on two applications in operations and supply chain management to show how asymmetries can affect channel parties' strategic and operational decision making. In the first essay, we deal with the power asymmetries between the competitors in a retail channel and in the second essay, we focus on informational asymmetries between buyers and suppliers in two consecutive channel streams.

The first essay concentrates on the assortment reduction in certain product categories by many big-box retailers such as Walmart and attempts to explain this phenomenon by power asymmetries among competitors. Indeed, this assortment reduction is anecdotally believed to be a result of these retailers' superior market power. In order to analytically explore the possibility of this causal relationship, we focus on two sources of asymmetries among different players in retail channels: market dominance represented by market price leadership and product cost advantage. To focus on the pure impacts of cost and market power on the assortment choice of a dominant retailer, we model the games in the first essay as symmetric games with two players that are asymmetric in factors other than their information, i.e. price leadership and cost advantage. To analyze the asymmetry in pricing power, we consider two forms of competition: *simultaneous-move*, where both players make their non-variety decisions (price or quantity) independently at the same time, and *dominant-fringe* competition, where the dominant retailer sets a market price and the other retailer follows by taking the same price.

Our results show that the assortment decisions of power retailers are highly connected to their pricing power in the market, which suggests that pricing and assortment decisions are two different sides of the same coin for power retailers in a competitive market. Indeed, we contribute to the literature by providing an analytical understanding of how price leadership could lead to assortment reduction while cost advantage [if it comes from any source except assortment reduction itself] seems to play a secondary role in comparison with the market pricing leadership. For instance, in a simultaneous-move competition where the power retailer's pricing supremacy is limited, he uses his product cost advantage to increase the product varieties in order to maintain his competitiveness and profitability. But in a dominant-fringe model where he has significant pricing control over the market, he may decrease his assortment to benefit from the resulting cost savings. We also find that a larger market share in all scenarios amplifies the retailer's control over the market and can lead to his assortment reduction enabling him to carry more popular and profitable products. Moreover, we study the effects of product characteristics such as the degree of substitutability and popularity distribution of the products on the assortment decisions.

Both analytical and numerical results generated from our models provide insightful managerial implications that help explain the motivation and practice of assortment planning by power retailers. In fact, by drawing on some important aspects of retail powers in today's markets, we are enhancing our ability to theorize about the effects of advanced retail power on some critical components of retail management. Given the dynamics of this sector and the increased dominance of big-box retailers, understanding retail power impacts is arguably an issue of a particular importance to both academics and practitioners. Studying the pure impacts of each power source separately enables us to understand the relationship between retailers' operational plans and their position in the market. This is particularly important to practitioners because they increasingly need to adjust their actions as their market power and channel position evolve over time. While as a research community, we are yet to uncover the entire impacts of retail dominance, identifying the specific impact of this power on the retail assortment planning brings forth a further contribution to both theory and practice.

The second essay deals with information asymmetries with respect to non-price attributes in supply chain management. In procurement reverse auctions, non-price attributes such as product quality, supplier's reliability, and timely delivery can be sometimes more important than simply bids for the buyers, especially when it comes to highly differentiated products for which each supplier is believed to deliver a completely different product. Examples of these differentiated products include highly professional marketing and legal services. Therefore, in reverse auctions, the price-plus format, where buyers consider price and non-price attributes to evaluate their suppliers, becomes more popular than the price-only format, where the decision is made solely based on bid prices. One popular format of price-plus auctions is buyer-determined auctions in which the buyer evaluates suppliers based on the non-price attributes and assigns to each of them a unique quality score (QS). QS alters the bidding process in buyer-determined auctions in two ways. First, it makes the buyer's evaluation ambiguous from the suppliers' perspective because the suppliers may not know how the scores are eventually assigned or calculated. Second, it changes the nature of price competition among the suppliers as the winner needs to offer the lowest QS-adjusted bid price (not necessarily the lowest bid). By analyzing these informational and strategic effects of QS in a reverse auction setting, in this essay, we try to provide normative recommendations to the buyers on when and how to share suppliers' relative QS with them. In addition, we explore the impact of sharing QS information on the decisions and profits/cost of other channel parties.

To the best of our knowledge, this essay is the first study that addresses this question in an analytical framework while the majority of works in this stream use human-based experiments. While analytical and experimental studies have their own pros and cons, a special advantage of analytical studies is in their prescriptive approach while experimental studies usually help to understand current practices using descriptive analysis techniques. In addition, the main differentiator of this research from the current analytical auction literature is the reversed distribution of information between the buyer and suppliers. More specifically, the literature often considers that suppliers know their private cost information, whereas we mainly focus on the buyer's information regarding the QS he assigns to the suppliers. In addition, to increase the generalizability of the results of this research, we consider a more general setting (as an extension) in which both suppliers' marginal costs and QS are private information to the suppliers and the buyer, respectively. We identify two factors: (i) degree of homogeneity among the suppliers (as measured by relative QS) and (ii) the degree of information asymmetry (as measured by the range of uncertainty for relative QS). First of all, in both public and private cost settings, the buyer prefers not to share the relative QS with the suppliers if they are relatively similar to each other in terms of QS. This is because in this case, the suppliers engage in a more intensified price competition under information asymmetry compared to when they have access to exact value of the relative QS. However, the opposite holds true if the suppliers become more uncertain about their relative QS. Hence, in this case, the buyer finds sharing QS information with the suppliers beneficial for the sake of lowering equilibrium bid prices notwithstanding the cost of credible sharing.

Overall, this dissertation aims at deepening our understanding of power and information unevenness among channel partners in competitive supply chains. Both essays support the idea that information and power asymmetries significantly, yet differently, change the motivations and actions of players in both strategic and operational levels.

CHAPTER 2
Retail Power Impacts on Assortment Decisions

2.1. Introduction

Retail assortment planning is one of the most important decisions for retailers because product variety offered by a retailer is an essential determinant of consumers' purchasing decisions. When making this planning, retailers should take into consideration several internal and external factors. One of these critical factors is the retail competition degree and the comparative retail power of the firm. Nowadays, the retail industry has increasingly been dominated by a small circle of large power [1] retailers including supermarket chains, mass merchandisers, and wholesale clubs (Raju and Zhang 2005, Useem 2003). In the grocery industry, for example, according to Progressive Grocer Annual Report (2000) supermarket chains accounted for only 16% of the total number of stores in the US in 1999, while independent supermarkets and small stores with sales below $2 million accounted for nearly 40%. However, in terms of sales and market share, supermarket chains accounted for 61.8% of the total while independent and small stores accounted for about 27%.

With respect to product variety, retailers, on the one hand, tend to offer a broad assortment in order not to lose any potential purchase and to guarantee their required levels of profit. On the other hand, they would like to reduce the variety offered because each new product added to their shelves requires more administrative and labor costs and effort. Given these conflicting objectives, many large retailers and wholesale clubs are known to

[1] Throughout this chapter, we use generic term of 'power retailer' for any retailer that has one or more special sorts of retail power considered in this study.

limit their assortment and devote most of their shelf-space to more popular and profitable product brands. Some believe this inclination is a result of their retail power[2] . For instance, Dukes et al. [2009] considered a traditional retail setting in which the manufacturer defines the breadth of his product lines and distributes the varieties of the products through complying retailers; they showed that in this setting, if a retailer gains enough power to select his assortment while other retailers are subject to the assortment decision set by the manufacturer, the dominant retailer has an incentive to reduce the assortment depth.

In this chapter, however, in order to follow the current trend and practice, we deliberately focus on an independent retail setting in which retailers themselves determine their product variety and have no obligation to fulfill suppliers' or manufacturers' variety preferences due to their independent ownership structure. This assumption is particularly true for many large retailers whose assortment decisions are the main concern of this study. For example, in the case of Walmart, it is the retailer who tells the manufacturers which products to develop (Bianco et al. 2003).

Power retailers are known for their peculiar characteristics. Raju and Zhang [2005], for instance, identified three of these features in some current retail markets. First, they[3] usually have the ability to offer consumers a remarkable opportunity for one stop shopping and invent effective promotional services. Second, they are frequently the largest distributors for the manufacturers. For example, Walmart accounted for 17% of the $PG's$ total sales in 2002, 39% of Tandy's, and a double-digit percentage for many other large manufacturers (Useem 2003). Third, power retailers are often the price leaders, i.e. once they establish the retail price, the fringe retailers take it as the market retail price (Weinstein 2000). This feature is consistent with what has been observed earlier in the market where some small

[2] There are also other circumstances under which assortment reduction decisions may be made by a retailer. According to Aydιn and Hausman [2009], supply chain decentralization can be a common cause of offering fewer products in comparison to vertical integrated channels. It can also be a consequence of competitive cross-category management (Cachon and Kök 2007). Huang et al. [2011] reviewed some other causes of assortment reduction such as first mover advantage, assortment cost inefficiency, and increasing basket shoppers.

[3] We use masculine pronouns for all the retailers in this chapter.

retailers follow the price book of a large famous retailer (Stone et al. 1995). In addition to these factors, another important source of power in the retail sector is the unit cost advantage for the large retailers that possess a dominant market share (Samuelson and Marks 2008, Riordan 1998).

Along these lines, we model retail power in two general formats: (i) price leadership defined as the ability to control the market price of a product; (ii) cost advantage that makes the powerful retailer able to achieve lower unit costs. To assess the effect of each type of retail power on the equilibrium assortment choice by the power retailer, we consider two competition formats: simultaneous-move and dominant-fringe competition and analyze each competition format in two distinct settings: product quantities with variety, and product retail prices with variety. In the simultaneous-move setting, we assume both retailers choose their product variety and then simultaneously determine the price or quantity of the products. The powerful retailer has only cost advantage but he cannot dominate the market price by himself. But, in dominant-fringe setting, the dominant retailer sets his retail price first and the weak retailer then takes the same price. The power retailer in this model may also benefit from cost advantage on top of the market dominance.

Our analysis shows that in a competitive market, assortment provisions of a power retailer highly depends on his pricing power. For instance, in the simultaneous-move competition where the power retailer has less pricing power, he never decreases his assortment in equilibrium as his cost advantage increases. Indeed, in addition to the cost advantage by the power retailer, price-leadership is a necessary condition for assortment reduction. This result illustrates the strategic relationship between the pricing and assortment decisions of power retailers. Besides the two main types of retail power, we also analyze the effect of larger market share of the power retailer on his assortment choice. With this regard, we find that larger market share of the power retailer increases the influences of market power and product cost advantage in all scenarios so that a bigger retailer may influence the entire market strongly by offering less product varieties. We also explore the impact of product category characteristics on the assortment choice of the power retailer in different competition settings. We believe both analytical and numerical findings in this chapter pave the way for bridging the research on retail power and the research on assortment planning.

2.2. Literature Review

Our work is primarily related to two streams of research in marketing and operations: papers analyzing different sorts of retail power with emphasis on market price dominance and cost advantage, and papers dealing with determinants and impacts of retail assortment planning.

Most of the papers in the first stream can be categorized into two groups depending on the type of dominance studied: First group of the papers examine profit and contractual implications of market price leadership in a setting where a powerful retailer dominates over other competing fringe retailers by setting universal market prices (see Raju and Zhang 2005, Kolay and Shaffer 2013, Chen and Xiao 2009, Hua and Li 2008, Dukes et al. 2006, and Shi et al. 2013). The papers in the second group consider the profit and welfare consequences of channel dominance in settings where a retailer dominates a manufacturer in a Stackelberg gaming relationship[4] (Geylani et al. 2007, Dukes et al. 2009, Luo et al. 2007, Dukes et al. 2014, Chen 2003, Dobson and Waterson 1997, Chen 2003, Shi et al. 2013, and Inderst and Wey 2007). More related to our work, Geylani et al. [2007], Luo et al. [2007], and Dukes et al. [2009] analytically study the impact of shifting channel power on retail variety decisions and show that there is indeed a significant relationship between channel power distribution and the variety choices. Chen [2003] showed that an increase in the amount of countervailing power of a dominant retailer can lead to a fall in retail price for consumers. He also proves that the existence of price-taking fringe network is necessary for benefits of consumers. Dobson and Waterson [1997], Chen [2003], and Inderst and Wey [2007] study the origins and welfare consequences of retailer power. Dukes et al. [2014] study the effect of

[4] Needless to mention that channel dominance can also be exerted by participating the retailer in other non-pricing decisions such as reordering quantity and lead time and quality of products. We did not analyze these sorts of channel dominance in this chapter, but they can be addressed in future studies.

channel power on the quality choice of the dominant retailer and find the conditions under which a dominant retailer would like to reduce manufactured quality.

In line with the two groups in the first stream, there are a few papers that consider both types of dominance exerted by a power retailer (Jerath et al. 2007 and Wang et al. 2013). For instance, Jerath et al. [2007] showed that Walmart exercises both market and channel dominance. Similarly, we focus on multiple sources of retail power in this chapter. We consider market dominance as presented in the first group (e.g. Raju and Zhang 2005); however, we mainly focus on the cost implications of channel dominance referred to as the cost advantage over other retailers. This benefit can be either a result of retailer's bargaining power in the channel over suppliers (manufacturers) or due to the retailer's own operations efficiency in procurement, transportation, storage, and sale of products. Therefore, this particular definition of cost advantage as a source of retail power provides a more general framework than channel power defined by Jerath et al. [2007]. For instance, for the case of Walmart, many believe that its wholesale price benefits from its suppliers through forceful negotiation tactics plus its own superior operational capabilities and logistical efficiencies create a good margin to offer lower prices (Neff 2003, Facenda 2004, Geylani et al. 2007). Walmart is also well known for its technological innovations such as implementation of RFID, Internet-based, and scan based trading technologies, which will bring it further efficiencies compared to its competitors. Home Depot's state-of-the-art inventory system is another example of management capability that may lead to lower unit cost for the dominant retailer (Dunne and Kahn 1997).

Second vast stream that is related to this chapter is on the assortment planning in the retail operations. We refer the readers to Kök et al. [2015] for a comprehensive review of recent papers on retail assortment planning. The papers in this stream can be divided in three groups. The first group of papers focus on the perceived assortment, its difference with the real variety, and its cognitive impact on consumers (Kahn and Wansink 2004, Simonson 1999, Hoch et al. 1999, Gourville and Soman 2005, and Spassova and Isen 2013). The second group of papers concentrate on the performance impact of assortment decisions (Ton and Raman 2010, Brynjolfsson et al. 2003, Patel and Jayaram 2014, Wan et al. 2012, and Borle et al. 2005). These papers show that increased retailers' variety can generate positive economic impacts (Brynjolfsson et al. 2003) and have a U-shape effect on retailers' operational performance (Patel and Jayaram 2014 and Wan et al. 2012). Lastly, the third group study on the competitive assortment planning problem (Kök and Xu 2011, Cachon and Kök 2007, Dukes et al. 2009, Besbes and Saure 2016). Kök and Xu [2011] consider price and assortment competitions between two symmetric retailers and find that the equilibrium assortments highly depend on the consumer choice models. Cachon and Kök [2007] showed that cross-category management (as a decentralized regime for assortment management) may reduce the overall assortment to a suboptimal level. Dukes et al. [2009] showed that in a competitive retail setting, if a retailer is able to choose the product variety offered by the manufacturer, he would probably limit his variety in some situations. There are also some empirical evidences of the effect of competition on the assortment decisions(Ren et al. 2011, Olivares and Cachon 2009). Ren et al. [2011], for instance, investigate how stores select their product variety contingent on the presence of competitors and their actual distance from rivals. They show that a store's product variety increases if a rival store coexists in its market, but it starts to decrease when the rival store is collocated near the store.

Our research differs from the existing and previous literature in four ways. First, while most of the papers in marketing and OM literature focus on just a specific source of retail power, we consider retail power in a more general way that includes cost advantage, market share and price leadership or the combination of two or three in order to identify the exclusive and collective impacts of different sources of retail power on the variety choice. Second, in terms of modeling features, we assume both retailers are free to choose their desired assortment that is a condition that often happens for independent retailers (from ownership view). Third, early research assumes the manufacturer offers a particular product to both retailers at the same price and there is no room for the retailers to negotiate and change it. In contrast, we assume the retailers can get the same product at different purchase prices (from the same or different suppliers) depending on their power in the channel. Finally, in terms of managerial insights, we address the question of which type(s) of retail power can lead to its owner's assortment reduction, and find that except for cost efficiency in the absence of market price dominance, possessing any single sort of power or combinations of more than one may lead to retail assortment reduction. To the best of our knowledge, our research is the first study that examines the effect of market dominance, i.e. price leadership, on the power retailer's assortment decision via game theory models.

2.3. Model framework and assumptions

In this section, we describe the game-theoretic models that help us to study different sorts of retail power in a competitive retail market. Particularly, in order to explore the effect of price leadership on the variety decisions, we consider two forms of competition: (i) simultaneous-move, in which no retailer has the leading power to dictate the market price, and (ii) dominant-fringe, in which the dominant retailer sets the market price for his products and the fringe only follows the price. We analyze both forms of competition in two distinct settings: (i) quantity decisions with variety choices, for which the retailers make assortment decision first and then decide the quantities of the product to purchase from the suppliers (*quantity competition settings* hereinafter), and (ii) price decisions with variety choices, for which the retailers make assortment decision first and decide the product price in the market (*price competition settings* hereinafter). Although the quantity competition model, to some extent, leads to tractable results, it creates a compatibility issue: in simultaneous competition, the market shares of the two retailers have to be determined endogenously, while in the dominant-fringe competition the market shares of both retailers are assumed fixed exogenously based on the modeling specifications. Therefore, to validate the results from quantity competition settings, we also consider findings from price competition models in which market shares of both retailers will be set exogenously. Note that due to the tractability problem, however, we will only present numerical results for the price competition models.

To summarize, the four games studied in this chapter are presented in Table 2–2. In general, all the four games have three stages. In stage 1, retailers set their assortment choice decisions simultaneously. In stage 2, unit product cost of each retailer becomes known to both retailers. Any possible cost advantage for the dominant retailer can be a result of either his efficiency in administration or channel power over suppliers, or both. In the last stage, the two retailers make their decisions in either quantity or price, simultaneously or sequentially depending on their relative power and position in the market.

Before describing each game, in what follows, we provide four general assumptions necessary for our game theory models. These assumptions characterize the general market in which the two retailers compete. Note that in order to derive analytical results, some slight adjustments in assumptions might be required for some competition models.

Assumption 1 *The market is served by two asymmetric profit-maximizing retailers, $R1$ and $R2$, who can carry one category of products that is composed of two horizontally differentiated products. Within the category, the two products differ in popularity in the market.*

For simplicity, we assume Product 1 is more popular than Product 2. The asymmetry in popularity of products allows us to focus on the general interest of retailers to carry the most profitable products in their assortment. We denote the powerful or dominant retailer as $R1$ and the weak or fringe retailer as $R2$.

Assumption 2 *A retailer can decide to carry either one product (limited assortment) or both products (full assortment). Moreover, we assume a part of assortment cost which is a function of only variety- but not quantity- is equal for both retailers if they offer the same variety.*

The assortment decisions that each retailer has to make are whether to carry one or two products within a category with one more popular than the other. If a retailer decides to carry both products, we denote the decision as F (full assortment). If the retailer decides to carry only one product, we denote this decision as P (popular product) or LP (less popular product). Given Assumption 1 discussed above and that the market demand for product 2 is smaller than product 1, it is easy to observe that the power retailer's assortment decision would be either F or P, and he would never choose to carry a less popular product due to his power. Table 2–3 provides all possible scenarios of assortment decisions made by the two retailers $R1$ and $R2$ analyzed in our model.

Note that any assortment choice leads to a certain amount of assortment cost. Generally, retail assortment cost is composed of two distinct parts: variable cost that increases with the total *quantity* of products offered (e.g. sorting and moving costs), and the other is associated with the *varieties* offered (e.g. ordering and management costs). We assume both retailers are equally efficient in managing variety cost. Equality of this variety cost for both retailers enables us to focus on the capability of $R1$ to reduce the unit cost via either external channel power or internal efficiency.

Assumption 3 *Retailers may purchase the products from the same or different suppliers, but if a retailer carries both products, the unit costs of both products are the same. Nevertheless, the unit cost of the products carried by $R1$, in general, is lower than that for $R2$.*

This assumption implies that the unit costs are exogenous in our models. Although assuming a same unit cost for the products in one category seems to be restrictive, it is realistic in many circumstances in practice. For many consumable products, e.g. yogurts, toothpastes, and T-shirts, the wholesale prices from suppliers (and the final unit costs) are very similar. However, suppliers may sell their products to different retailers at different prices due to many factors such as bargaining power between suppliers and retailers, the volume of orders, the image in the market, and the market share of the retailers. As explained before, $R1$ might also benefit from efficiency in administration that may lead to his unit cost reductions compared to his competitor.

Assumption 4 *Market demand for both products is of downward linear function.*

This form of demand function is very common in economics and marketing literature and has many advantages including simplicity, robustness, and accuracy in many cases. As an example in quantity competition setting, the market price is set by $p_i = \mathbb{A} - \mathbb{B}_i Q_i^m - \mathbb{G} Q_{-i}^m$

where p_i, Q_i^m,and Q_{-i}^m are the price for product i, total quantity of product i offered in the market, and total market quantity of substitute product, respectively. Demand function in price competition setting has a similar format too. It will be discussed later in details when explaining price competition models.

Recall that we assume a retailer must make two important decisions: (i) whether to carry full assortment or only one product in the first stage; (ii) the quantity or price for the products chosen in the last stage. Different scenarios are derived based on the different sorts of decisions to be made. We model retail power in different formats as follows: (1) *Cost advantage* of the power retailer in all the games is a continuous variable that corresponds to his unit cost reduction; (2) *market power* is measured by considering two separate models: one where no retailer has special market power and the two retailers simultaneously decide the price or quantity of the products; and the other where the market dominant retailer sets the universal market price and the weak retailer follows the same price for his own products. All the notations used in this chapter are presented in Table 2–1.

In what follows, we present the game theoretical models of simultaneous-move and dominant-fringe competitions, each considering price and quantity decisions respectively, as described in Table 2–2. Note that the demand functions and product unit costs are common knowledge for both retailers in all models.

2.3.1 Simultaneous-Move Games

We first consider a situation where retailers make their decisions simultaneously[5] . In this simultaneous competition, we assume no retailer has the leading power in determining market price; therefore, the retail price of each product in the quantity competition would be determined by the total quantity of the product and its substitute in the market. In the price competition setting, however, each retailer may choose to offer a different retail price.

[5] In our analysis, we also considered a Stackelberg setting for non-variety decisions where R1 first makes his decisions and then R2 follows. However, since the assortment choices of the dominant retailer in simultaneous-move and Stackelberg games were exactly the same under the quantity setting and conceptually similar under the price setting, we only present the results of the simultaneous-move games.

Table 2–1: Notations used for model parameters and decision variables.

Indices	
i	Products, $i \in \{1,2\}$ refers to products 1 and 2, respectively (subscript).
j	Retailers, $j \in \{1,2\}$ refers to retailers $R1$ and $R2$, respectively (superscript).
m	Total market (superscript).
Model parameters	
Π_i^j	Profit of retailer j from offering product i.
Π^j	Total profit of retailer j.
w^j	Product unit cost of retailer j.
Δ	Cost advantage of $R1$ over $R2$ on any unit of product.
$V(n)$	Variety cost of offering $n \in \{1,2\}$ varieties, with $V(1) = 0$ and $V(2) = V$.
α	Intercept of demand function for quantity settings (SQ and DFQ).
β_i	Slope of demand function for quantity settings (SQ and DFQ).
γ	Degree of substitutability between products in demand function for quantity settings (SQ and DFQ).
$\mathcal{A}$	Intercept of demand function for price settings (SP and DFP).
$\mathcal{B}_i$	Slope of demand function for price settings (SP and DFP).
Γ	Degree of price competition between retailers in demand function of price settings (SP and DFP).
$\lambda; (1-\lambda)$	Market shares of $R1$ and $R2$, respectively.
Decision variables	
q_i^j	Quantity of product i offered by retailer j.
p_i^j	Price of product i offered by retailer j. †
Q_i^m	Total market quantity of product i.

† Note that we assume $p_i^1 = p_i^2 = p_i$ in quantity competition settings (SQ and DFQ) for $i = 1, 2$, and $p_1^j = p_2^j = p^j$ in price competition settings (SP and DFP) for $j = 1, 2$.

This game has three stages. In stage 1, the two retailers make their assortment (variety) decisions simultaneously by anticipating the competitor's actions in order to maximize their own profits. In stage 2, they obtain the wholesale prices from their suppliers; and in stage 3, both retailers determine their non-variety decisions (either quantity or retail price) for their selected products. Regarding stage 2, we assume retailer $R1$, the powerful retailer[6] , may have cost advantage and his unit product cost, w^1, is lower than that of retailer $R2$, w^2 by (Δ), that is $w^1 = w - \Delta$ and $w^2 = w$ where w is the wholesale price for $R2$, and $0 < \Delta < w$ to reflect the cost advantage of $R1$. As stated before, the variable cost advantage by $R1$, Δ,

[6] To be consistent throughout this chapter, in the simultaneous competition, we call the power retailer (R1) with cost advantage the *powerful retailer* and in the dominant-fringe competition *dominant retailer*. $R2$ in both scenarios is called *weak* retailer to indicate its comparative disadvantage over the other retailer either in unit cost or market power.

Table 2–2: Different forms of retail competition.

	No price leader	One retailer is price leader
Quantity setting	(1) Simultaneous Quantity competition (SQ)	(2) Dominant-fringe Quantity competition (DFQ)
Price setting	(3) Simultaneous Price competition (SP)	(4) Dominant-fringe Price competition (DFP)

can be decomposed into two parts: (i) the advantage through discounted wholesale prices charged by the suppliers due to the channel power of $R1$, and (ii) the advantage through other variable costs that imply the relative efficiency of $R1$ in ordering and managing product inventory. In this chapter, we do not differentiate between the two parts in $w^j (j = 1, 2)$, but, rather, focus on unit cost advantage, $w^1 < w^2$, possessed by the power retailer.

The total profit for retailer j (denoted by Π^j) is the sum of the profits from the products carried minus the variety cost:

$$\Pi^j = \sum_{i=1,2} \Pi_i^j - V(n),$$

where $V(n)$ is the retailers' variety cost, and n ($n = 1, 2$) is the number of products that retailer j carries. Without loss of generality, we normalize the variety cost by defining $V \equiv V(2) > V(1) \equiv 0$. This normalization indicates that carrying one more product increases the variety cost by V. We provide the payoff functions of the two retailers for each assortment decision at both simultaneous-move and dominant-fringe games in Appendix 5.2. We below discuss the simultaneous game under both quantity and price competition settings.

- **Quantity Competition (SQ)**

In this setting, we focus on a market in which the retailers compete on product quantities and derive Nash equilibria that represent the optimal quantities of the product(s) carried by the two retailers at stage 3 for each of the six assortment outcomes in Table 2–3. Then, we work backwards to identify the conditions for the six assortment decisions.

For the ease of exposition, we denote i as one product and $-i$ as the other product. Similarly, we denote j as one of the two retailers and $-j$ as the rival retailer. Let p_i^j and q_i^j be the retail price and quantity of product $i (i = 1, 2)$ offered by retailer $j (j = 1, 2)$ to the market, respectively. Since the two retailers compete for quantities, we assume both retailers offer product i at the same price p_i, i.e. $p_i^1 = p_i^2 = p_i$ for $i \in \{1, 2\}$, where p_i is the market price for product i. Then, the inverse demand function would be as follows:

$$p_i = \alpha - \beta_i \sum_{j=1}^{2} q_i^j - \gamma \sum_{j=1}^{2} q_{-i}^j \tag{1}$$

where the intercept α is the maximal market price ($\alpha > 0$) that possibly can be charged; β_i denote the sensitivity of the retail price of product i to the total quantity offered by both retailers; and γ is the degree of substitutability between the two products. In order to make sure that product 1 is more popular than product 2, we assume $\beta_1 < \beta_2$. Also, in order for the demand function to be downward, β_i should be positive and greater than γ ($0 < \gamma < \beta_1 < \beta_2$). Note that if retailer j decides not to carry product i, then $q_i^j = 0$.

For retailer j, if he decides to carry product i, he then will decide to carry quantity, q_i^j in an attempt to maximize his profit as below:

$q_i^j = \arg\max_q [\alpha - \beta_i(q_i^{-j} + q) - \gamma(q_{-i}^{-j} + q_{-i}^j) - w^j]q + [\alpha - \beta_{-i}(q_{-i}^{-j} + q_{-i}^j) - \gamma(q_i^{-j} + q) - w^j]q_{-i}^j$;
where q_i^{-j}, q_{-i}^{-j}, and q_{-i}^j represent the quantity of product i carried by the rival retailer, the quantity of substitute product (or product $-i$) carried by the rival retailer, and the quantity of substitute product carried by the retailer j, respectively; w^j denote the unit cost incurred by retailer j ($j = 1, 2$). The profit for retailer j by carrying product i for a quantity of q_i^j can be expressed as below:

$\Pi_i^j = [p_i(q_i^j, q_{-i}^j, q_i^{-j}, q_{-i}^{-j}) - w^j]q_i^j$;

where $p_i(.)$ is the price function for product i (described in Equation 1) that depends on the total quantities of both products (the product itself and the substitute) carried by both retailers.

Given that $0 \leq \Delta \leq w < \alpha$ and $0 < \gamma < \beta_1 < \beta_2$, we are able to derive retailers' pay-offs in the simultaneous game for the quantity setting (SQ) corresponding to each of the six possible assortment decisions listed in Table 2–3.

• **Price competition (SP)**

For the price competition model, we assume that the retailers first make their assortment decisions and then simultaneously choose the retail prices of the products they decide to carry, and their relative market shares will determine the quantities. In addition, we assume if a retailer chooses to carry full assortment, both products are offered at a unique price, i.e. $p_1^j = p_2^j = p^j$ for $j \in \{1, 2\}$, where p^j is the retail price by retailer j for the category of products[7] .

To describe the demand function in the price competition settings, we take a similar approach to Kurtuluş and Toktay [2011]; that is, the demand for product i ($i = 1, 2$) if offered by both retailers, is given as follows.

$$\begin{aligned} q_i^1 &= \lambda(\mathcal{A} - \mathcal{B}_i p^1 + \Gamma(p^2 - p^1)) \\ q_i^2 &= (1 - \lambda)(\mathcal{A} - \mathcal{B}_i p^2 + \Gamma(p^1 - p^2)) \end{aligned} \tag{2}$$

where $0 < \Gamma < \mathcal{B}_1 < \mathcal{B}_2 \leq 1$; p^1 and p^2 are the retail prices set by the two retailers; and, λ and $(1 - \lambda)$ are the market shares of $R1$ and $R2$, respectively.

In Equation 2, the intercept $\mathcal{A}$ can be interpreted as the potential market demand, because if both retailers set their price to zero, the total market demand for each product would be $\mathcal{A}$. The parameters $\mathcal{B}_1$ and $\mathcal{B}_2$ are the price-sensitivities of the demand for products 1 and 2. We assume $\mathcal{B}_1 < \mathcal{B}_2$ to ensure that there will be more sales of product 1 (popular) compared to the product 2 (less popular) if both products are priced equally. Also, to ensure that both products are potentially profitable, we assume $w \leq \mathcal{A}/\mathcal{B}_2$ in both price competition settings (i.e. SP and DFP). The parameter Γ is the cross-price sensitivity parameter that measures the relative increase in the product i's demand for retailer j as the unit price of his rival increases. For the sake of tractability, we assume a fixed Γ for all $i = 1, 2$ and $j = 1, 2$. Evidently, as Γ increases, the demand at retailer j becomes more sensitive to the

[7] This apparently restrictive assumption is quite realistic as we observe that many retailers offer some categories of products, such as T-shirts, tooth pastes, and most daily commodities with different attributes (categorized based on color, size, and taste), at the same price.

price changes at the other retailer. This means Γ measures the sensitivity of consumers to price changes at a particular retailer. We assume Γ is positive to ensure that there is competition between the two retailers in their retail price, and also $\Gamma \leq min\{\mathcal{B}_1, \mathcal{B}_2\}$ to guarantee downward demand functions for both products.

Clearly, if product i is only offered by retailer j, Equation 2 would not hold and the demand for the two retailers would be:

$q_i^j = (\mathcal{A} - \mathcal{B}_i p^j)$ and $q_i^{-j} = 0$, $i = 1, 2$.

The profit for retailer j by carrying product i would be $\Pi_i^j = (p^j - w^j)q_i^j$. Computing the total payoff of retailers would then be straightforward. These payoff functions of the two retailers are provided in Appendix 5.2.

In the next subsection, we provide the frameworks for dominant-fringe games.

2.3.2 Dominant- Fringe Games

In the previous section, we assumed $R1$ has unit cost advantage over the weak retailer, but neither of the retailers was the price leader in the market. Now, we assume that $R1$ is able to set a universal market price for his own product(s) and the weak retailer has to follow the price if he decides to carry the same product(s). This assumption is particularly valid in practical situations where a retailer loses his sales if he sets a considerably higher price that the market leader's.

Similarly, we develop three-stage dominant-fringe competition models to investigate the impact of price leadership on the assortment decision made by the market dominant retailer. The decisions in the first two stages in the dominant-fringe games are exactly the same as those in the simultaneous-move games; i.e. both retailers make their assortment decisions simultaneously in stage 1, and unit costs become known in stage 2 with $R1$'s cost lower than $R2$'s by Δ, which corresponds to the cost advantage of $R1$. The only difference between simultaneous-move and dominant-fringe games is that in stage 3, the dominant retailer first determines the market price of his chosen products anticipating the reactions from $R2$. Given the price chosen by $R1$, the weak retailer ($R2$) chooses the price/quantity of the product that he exclusively decides to carry but not $R1$ (if any).

In general, there are two modeling approaches for determining retailers' market shares at dominant-fringe games. First, many papers in the marketing literature (e.g. Chen 2003, Raju and Zhang 2005) assume that each retailer maintains a fixed, exogenous share of the market demand Q^m. For instance, the shares of dominant and fringe retailers can be measured by λQ^m and $(1-\lambda)Q^m$ respectively, where λ is fixed and not a function of market price. In the second approach, which is mainly used in the economics literature (for instance Samuelson and Marks 2008, Riordan 1998), retailers' relative market shares are functions of the market price. In this study and especially in the quantity competition setting, we focus on the approach taken by Raju and Zhang [2005] who specify that the dominant retailer sets the retail price and his sales revenue is proportionate to his relative market share. Similarly for a commonly carried product, we assume the product quantity offered by $R1$ and $R2$ will be determined based on their market shares of λ and $1-\lambda$, respectively. However, if a product is carried by only one retailer, then the total market demand would be satisfied by that retailer regardless of the potential value of λ. Now, we provide the details of dominant-fringe competition in quantity and price settings, respectively.

- **Quantity Competition (DFQ)**

To model dominant-fringe competition under quantity setting, we divide the six possible assortment decisions, illustrated in Table 2–3, into two groups: (i) the products carried by $R2$ are also offered by $R1$, that is, $< F, F >$, $< F, P >$, $< F, LP >$, and $< P, P >$; (ii) $R2$ carries the less popular product but not $R1$, that is, $< P, LP >$ and $< P, F >$. For the assortments in the first group, the total market demand of product i (the sum of quantities offered by both retailers), based on previously defined linear demand function in SQ model, is as below.

$$Q_i^m = \frac{\gamma p_{-i} - \gamma\alpha - \beta_{-i}\, p_i + \beta_{-i}\alpha}{-\gamma^2 + \beta_1\beta_2} \qquad \forall i \in \{1, 2\} \tag{3}$$

where p_{-i} and β_{-i} are the retail price and the price-sensitivity of the other product, respectively. For the product(s) carried by both retailers, the demand for the dominant retailer and the fringe are $q_i^1 = \lambda Q_i^m$ and $q_i^2 = (1-\lambda)Q_i^m$, respectively; where $0 < \lambda < 1$ is the fraction of the market share reserved by the power retailer. If the product is only carried by retailer j, then $q_i^j = Q_i^m$ and $q_i^{-j} = 0$. Note that similar to SQ model, we assume that both retailers offer a product $i \in \{1, 2\}$ at a unique price, i.e. $p_i^1 = p_i^2 = p_i$.

Table 2–3: Assortment Choices by Two Retailers.

Variety outcome	$R1$'s choice	$R2$'s choice
$< F, F >$	Both products 1 and 2	Both products 1 and 2
$< F, P >$	Both products 1 and 2	Only product 1
$< F, LP >$	Both products 1 and 2	Only product 2
$< P, F >$	Only product 1	Both products 1 and 2
$< P, P >$	Only product 1	Only product 1
$< P, LP >$	Only product 1	Only product 2

For the assortments in the second group provided in the above paragraph, when the dominant retailer does not offer the less popular product and $R2$ decides to carry it, the dominant retailer determines the retail price of the popular product, p_1, and the fringe selects the total quantity for less popular product ($Q_2^m \equiv q_2^2$). Then, following the demand function in SQ model, the total market demand for the popular product (Q_1^m) and the market price for the less popular product (p_2) are derived as follows, respectively:

$$Q_1^m = \frac{\alpha - p_1 - \gamma Q_2^m}{\beta_1};$$

$$p_2 = \frac{-\gamma\alpha + \beta_1\alpha - \beta_1\beta_2 Q_2^m + p_1\gamma + \gamma^2 Q_2^m}{\beta_1}.$$

Similar to SQ model, the profit for retailer j from the sale of product i is: $\Pi_i^j = [p_i(q_i^1, q_{-i}^1, q_i^2, q_{-i}^2) - w^j]q_i^j$. As mentioned earlier, the main difference between retailers' payoffs in SQ and DFQ models are the market shares of retailers that are determined exogenously in DFQ model but not in SQ model. Retailers' payoff in DFQ game for each variety outcome are stated in Appendix 5.2.

- **Price Competition (DFP)**

In this section, we consider another competition framework where the power retailer selects the market price of his carried products and the weak retailer may only determine the retail price of the product not carried by the dominant retailer. Note again that similar to SP game, retailers set a unique price for the whole category, i.e. $p_1^j = p_2^j = p^j$, if they decide to carry both products. Moreover, since there is only one price for the common product carried by both retailers, the cross-price sensitivity effect in the demand function presented in SP model, Γ, will disappear in DFP model. Therefore, the total quantity of product i offered to the market by the two retailers can be easily computed as below:

$$Q_i^m = (\mathcal{A} - \mathcal{B}_i p^1)$$

where the intercept $\mathcal{A}$ is the potential market demand for product i; p^1 is the price selected by the dominant retailer, $R1$; and $\mathcal{B}_i$ is the slope of downward demand function for product i. If both retailers carry product i, the sale of each retailer will be determined proportionate to their market share; hence, for the common product ($i \in \{1, 2\}$), retailers' demand would be: $q_i^1 = \lambda Q_i^m$; $q_i^2 = (1 - \lambda) Q_i^m$ where λ is the market share of the dominant retailer.

We would like to highlight the assortment outcomes of $< P, F >$ and $< P, LP >$ when the less popular product is offered only by the weak retailer. For $< P, LP >$, it is plausible to assume that the weak retailer may choose a different price for the less popular product (p^2). Hence, $R2$ will meet the market demand by the following quantity:

$$Q_2^m = q_2^2 = (\mathcal{A} - \mathcal{B}_2.p^2).$$

For $< P, F >$ when $R2$ carries both products and $R1$ carries only popular product, $R2$ may either choose to set a different price for the less popular product or keep the same price for both products. In the equilibrium analyses, we assume that $R2$ takes a different price for the second product, even though either of these two assumptions does not change the results significantly. The payoff functions of both retailers, calculated based on this assumption, are presented in Appendix 5.2.

2.4. Equilibrium analysis: Impacts on the Assortment Choice of the Power Retailer

In this section, we derive the equilibrium outcomes of the games and identify potential impacts of some underlying factors on the assortment decisions by the power retailer.

These factors include: (1) retail power (cost advantage, price leadership, and market share) possessed by the power retailer; and (2) non-power factors including product characteristics (popularity distribution, substitutability degree, and product expensiveness for competitors) and the degree of price competition (only in SP model). We capture all of these factors, except price leadership, by special parameters/ratios that are summarized in Table 2–4. On the other hand, we measure the effect of price leadership by comparing the variety choices under simultaneous-move to dominant-fringe games.

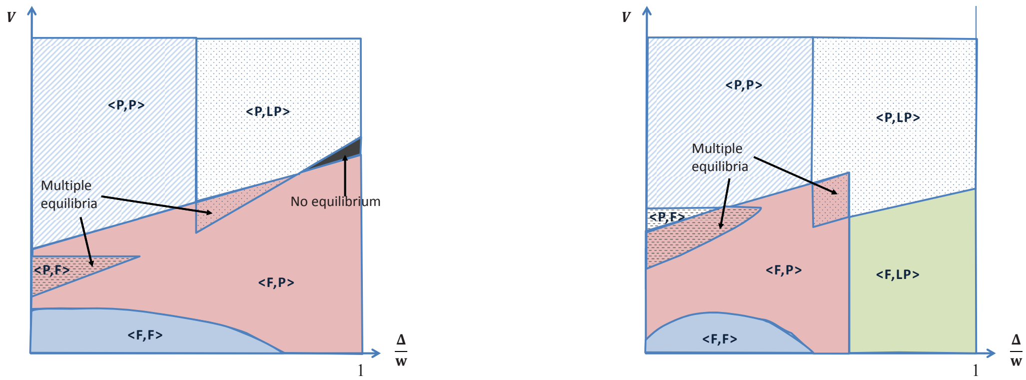
The variety choices of the retailers in any specific game might vary by changing any of those underlying factors. Figure 1 demonstrates an example of variety choices of retailers in equilibrium when all the parameters are fixed except variety cost (V, vertical axis) and $R1$'s cost advantage ($\frac{\Delta}{w}$, horizontal axis). Note that these graphs are just a static demonstration of variety outcomes for fixed values of other parameters outlined in Table 2–4, and, using any other combination of these factors may change the final variety outcomes.

A quick investigation in Figure 2–1 reveals the following important facts:

1. For relatively low variety cost (V), both retailers $R1$ and $R2$ choose to take full assortment [see the area indicated by $< F, F >$]; but as the variety cost increases to higher levels, both retailers switch to limited assortments indicated by the area of $< P, P >$ and $< P, LP >$. However, it is usually the weak retailer who first limits his product variety [i.e. $< F, P >$ and $< F, LP >$] especially when the degree of cost advantage by $R1$ ($\frac{\Delta}{w}$) is high enough.
2. In the dominant-fringe games (DFP, DFQ), when the cost advantage is very high but variety cost is relatively low, $R1$ carries full assortment and $R2$ chooses to carry less popular product [see the area indicated by (F, LP)]. One might wonder what happens that $R2$ prefers less popular product more than the popular product when $R1$ takes the full assortment. The answer is in the pricing decision of the power retailer $R1$. The variety outcome $< F, LP >$ happens primarily when $R1$'s market share and cost advantage are relatively low and high, respectively. In those cases, even though $R2$ can sell a significantly larger quantity by offering popular product, he takes the other

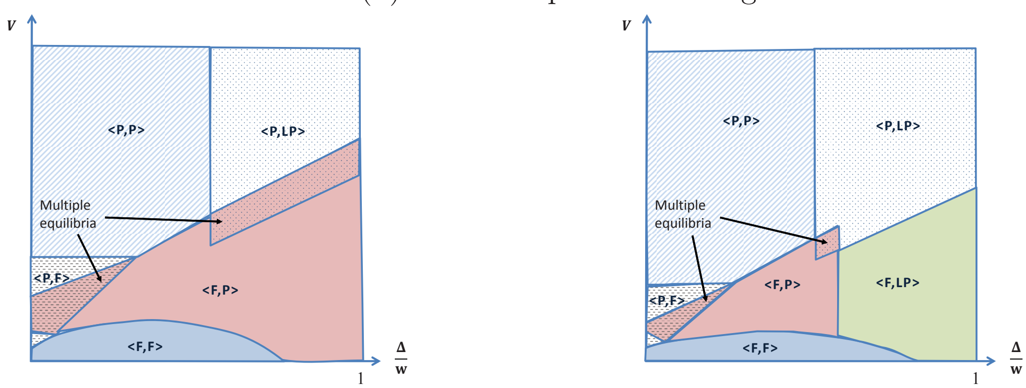
Figure 2–1: Equilibrium Variety Choices of Retailers for a Particular Instance of Parameters/Ratios.

(a) Quantity competition setting:



SQ Model DFQ model

(b) Price competition setting:



SP Model DFP model

Note. These graphs show variety outcomes of different games in equilibrium for a setting where $\lambda = 0.5$, $\frac{\beta_1}{\beta_2} = \frac{\mathcal{B}_1}{\mathcal{B}_2} = 0.5$, $\frac{w}{\mathcal{A}/\mathcal{B}_2} = \frac{w}{\alpha} = 0.8$, and $\frac{\gamma}{\beta_1} = \frac{\Gamma}{\mathcal{B}_1} = 0.1$. The horizontal and vertical axes are with respect to $R1$'s cost advantage (Δ/w) and variety cost (V), respectively, in both price and quantity settings.

product since otherwise $R1$ triggers an aggressive price competition by lowering the market price. In fact, by taking less popular product, $R2$ seeks to end up with less competition, a higher price, and consequently a higher profit. $< F, LP >$ can be equilibrium variety in SP game as well eespecially when the cross-sensitivity degree $(\frac{\Gamma}{\mathcal{B}_1})$ is very high. High degree of price competition enables $R1$ to have a significant influence on $R2$'s quantity, price, profit, and consequently his assortment choice by severely reducing his price. Delegating the details of the analysis to the Appendix 5.3, in the following lemma, we show that this outcome never happens in SQ game (proofs for all lemmas and propositions are provided in Appendix 5.3).

Lemma 1 *The outcome $< F, LP >$ never prevails as variety equilibrium in SQ game.*

This result is because under SQ game, the pricing power of $R1$ is strongly limited and he cannot start a fierce competition to make the popular product unfavorable for $R2$ in any circumstance. Therefore, in SQ game, $R2$ always prefers to carry popular product rather than the other product if he notices that $R1$ carries full assortment.

3. As can be seen from Figure 1, there might be more than one (at most two) Nash equilibria at some points. The following lemma shows a general case of this situation in SQ game.

Lemma 2 *The outcome $< P, F >$ never prevails as a unique equilibrium in SQ game.*

As a general rule, at these multi-equilibrium points, any outcome may happen because one retailer may play one equilibrium strategy while the other retailer chooses another equilibrium strategy (see Cachon and Netessine 2004, for more details). Consequently, we decide to direct our attention towards the unique equilibrium in this chapter to capture all parameter spaces for which the final variety outcome of the game is predictable. In our proofs, rather, because we are mainly concerned about the possibility of assortment reduction by the power retailer, we focus on the cases when the power retailer carries full assortment even as one of the possible equilibria in a multi-equilibrium point. In fact, as the variety cost increases, the powerful retailer would switch from full assortment (either unique or multi-equilibrium) to limited assortment (unique equilibrium) in certain levels of variety cost. These switching points form a boundary to distinguish $R1$'s decision changes. We later use these boundaries to analytically investigate the possibility of assortment increase or decrease.

The main objective of this chapter is to understand the effects of different power and non-power attributes (five continuous factors in each game plus market price leadership, stated in Table 2–4) on the assortment choice by the power retailer. To this aim, we study the changes in $R1$'s equilibrium assortment as those underlying factors change. All of these ratios in Table 2–4 are defined in the interval of (0,1). Note that due to the high degree of freedom in this problem on one hand, and the complexity of payoff functions on the other hand, conducting a series of insightful numerical analyses is inevitable. To do so in a relatively efficient and expositionally simple way, we operationalize these impacts by fixing all the ratios except one to see the pure effect of that particular factor on $R1$'s assortment in a defined setting. By doing so for any instance of the parameters, one of the following six general patterns are likely to happen:

1. *Assortment Increase (PF):* his assortment increases from popular product to full assortment.
2. *Assortment Reduction (FP):* his assortment decreases from full assortment to popular product.
3. *First Increase, then Reduction (PFP):* his assortment increases from popular to full assortment up to a a certain level; then it decreases again to popular product.
4. *First Reduction, then Increase (FPF):* his assortment decreases from full assortment to popular product; then it increases again to the full assortment.
5. *No Change - Full Assortment (FF):* he takes full assortment and does not decrease it to popular product.
6. *No Change - Limited Assortment (PP):* he takes popular product and does not increase it to full assortment.

Repeating this approach for numerous instances of parameter combinations enables us to characterize the overall behavior of $R1$'s assortment with respect to each substantial factor. Note that this behavior in equilibrium might be different depending on the setting (values of other parameter ratios). For instance, depending on the size of his market share, $R1$ might either decrease or increase his assortment when the characteristics of the product category change. Therefore, the following steps are necessary in order to correctly measure the assortment impacts of each underlying attribute (i.e. any of those ratios mentioned in Table 2–4 except price leadership, e.g. $\frac{\Delta}{w}$ when studying cost advantage's effects).

- First, find the pattern of $R1$'s assortment in the underlying attribute for all possible combinations of other factors.
- Second, count the frequency of each pattern. Relative frequencies of different patterns can approximately reveal the impact of that attribute on $R1$'s assortment.
- Third, for each pattern, characterize the conditions under which that pattern (assortment increase or decrease) happens. This information let us find the necessary conditions for any assortment change by the power retailer.

Theoretically, there are infinite numbers of possibilities for each factor, but in order to practically explore the possibility of assortment reduction or increase, we consider about twenty different instances of the values for λ, $\frac{\gamma}{\beta_1}$, $\frac{\beta_1}{\beta_2}$,and $\frac{w}{\alpha}$ in quantity settings (SQ and DFQ) and λ, $\frac{\Gamma}{\mathcal{B}_1}$, $\frac{\mathcal{B}_1}{\mathcal{B}_2}$,and $\frac{w}{\mathcal{A}/\mathcal{B}_2}$ in price settings (SP and DFP) in the range of (0-0.95) [8] . The exhaustive experiment gives a comprehensive view of the assortment choices by the retailers. We have summarized the results of the above procedure for different attributes in Appendix 5.1 (tables 5–1, 5–3, 5–4, 5–5, 5–6, 5–7). Note that although for the sake of expositional efficiency, we mainly provide the patterns of $R1$'s assortment choice (see tables 5–1 to 5–7), in most of the cases in order to explore the real cause for assortment change of $R1$, we should also consider the reactions from the weak retailer. Therefore, whenever needed, we present the transitions in overall variety outcomes of the games, which is often informative of the real incentives behind the assortment decisions by the power retailer.

Note that for the pure impact of price leadership, we would compare $R1$'s assortment in simultaneous and dominant-fringe models point-by-point and find the conditions under which his assortment increases or decreases. The summary of this impact is stated in Table

[8] We also consider different values of variety cost (V) in all the models. Even though a small increase in the value of variety cost might have a significant impact on the assortment choice, the most important issue is to ensure that the patterns are discovered for the same variety cost levels at both models in price or quantity settings (i.e. in the pairs of SQ-DFQ and SP-DFP). Overall, we consider about twenty different values for variety costs and it turns out that these values are relatively high since the majority of cases leads to PP pattern (popular product with no change in assortment). However, this does not create any serious problem in interpreting the results as we only focus on the relative frequency of those patterns that correspond to any assortment change (i.e. FP, PF, FPF, and PFP).

5–2. In addition, we are able to validate the numerical results by exploring more tractable models (SQ and DFP) and deriving analytical results.

Now, we focus on the roles of power and non-power factors that may affect the assortment choice of the power retailer and outline our findings regarding each essential attribute.

2.4.1 Cost Advantage

We measure the assortment effects of $R1$'s cost advantage by observing the behavior of his variety choice as the ratio $\frac{\Delta}{w}$ changes. According to Table 5–1, more cost advantage generally gives the power retailer the incentive to increase his product line to benefit from different products. Obviously, this is true only when the variety cost is lower than the profit that he can gain from the less popular product. However, in the table, there are some cases that correspond to assortment decrease when the degree of cost advantage is very high (pattern PFP) especially in dominant-fringe games. This slight difference between simultaneous-move and dominant-fringe games suggests that pricing power may increase the possibility of assortment reduction in response to increasing cost advantage. In what follows, we summarize the most important points.

1. In *simultaneous-move* games where the power retailer cannot directly force the weak retailer to take his price and is under price competition, as his cost advantage increases, $R1$ tends to take the full assortment to gain higher profits. Delegating the details of the analysis to Appendix 5.4, the following proposition addresses this effect in SQ game where there is no exception.

   Proposition 1 *In SQ game for any given $\frac{w}{\alpha}$, $\frac{\gamma}{\beta_1}$, $\frac{\beta_1}{\beta_2} \in (0,1)$ and V, as the cost advantage of the powerful retailer ($\frac{\Delta}{w}$) increases, the power retailer never switches from full assortment (F) to limited product variety (P).*

   This proposition states that higher cost advantage of a powerful retailer in SQ game would not provide any incentive for him to reduce his assortment as he can gain more profit by offering more products. Note that unlike $R1$'s, the weak retailer $R2$'s assortment decision does not follow a fixed pattern. Indeed, he adjusts his assortment level primarily based on the popularity distribution: he decreases (resp. increases) his assortment if products' demand levels are highly different (resp. similar).

2. Similarly in SP model, we find that a powerful retailer with high enough market share never decreases his assortment as his cost advantage increases. Assortment reduction is possible only under very rare situations where $R1$ has enough cost advantage and all the following conditions hold: $\lambda \leq 0.4$; $w \geq 0.9\mathcal{A}/\mathcal{B}_2$; $\mathcal{B}_1 \geq 0.9\mathcal{B}_2$; and $\Gamma \geq 0.5\mathcal{B}_1$. In other words, assortment reduction is possible only when $R1$'s market share in common products (λ) is very low for a category with low popularity difference that is already expensive for the weak retailer in a market that is highly sensitive to the retail prices. In these cases, the variety outcome of the game moves from $< F, P >$ to $< P, LP >$; and it is mainly because as $R1$'s cost advantage increases he has more manipulating power to decrease his retail price to a very low level. With a low price in a sensitive market to the price difference in the two retailers, $R1$ can push $R2$ to avoid carrying popular product and engaging in a severe price competition. As a result, in these situations, $R1$ reduces his assortment but is under less pressure to decrease the price and obtains the total market demand for the popular product. Retailer $R2$ adopts less popular product only when he finds it popular enough not to lose much quantity and profit.

3. In the *dominant-fringe* games (DFQ and DFP) where $R1$ has considerable pricing dominance, he increases his assortment in most of the cases in order to keep his competitive advantage and profitability. However, there is some possibility for assortment reduction too when the degree of cost advantage is very high. This result reveals that even when $R1$ has the full pricing supremacy, a significant cost advantage is required for assortment reduction. Similar to SP game, almost all the assortment reductions for $R1$ happens when the variety outcome of the game switches from $< F, P >$ to $< P, LP >$. When $R1$ is the price leader and his cost power is very high, he can charge a very low price that makes it infeasible for the weak retailer to carry the same products as $R1$. Under such condition, the weak retailer prefers to carry less popular

product instead and consequently, $R1$ takes popular product. According to Table 5–1, this case happens primarily when $R1$'s market share in common products λ is relatively low, and he can threaten the weak retailer with a severe loss-making price competition. This finally leads to $R1$'s sole supply of the popular product as $R2$ focuses on the less popular product with more freedom from the pricing perspective. Figure 1 provides an illustrative example of retailers' assortment decision as $R1$'s cost advantage increases.

2.4.2 Price Leadership

To characterize the impact of market dominance on the power retailer's assortment choice, we perform a one-to-one comparison of the equilibrium variety outcomes between dominant-fringe and simultaneous-move games in both quantity and price settings. As can be observed from the payoff functions provided in Appendix 5.2, finding the exact analytical results about when assortment reduction or increase may happen in SP and DFQ games is difficult. Nevertheless, our observations (a summary is presented in Table 5–2) suggest that as we move from simultaneous-move to dominant-fringe model, $R1$ would either increase or decrease his assortment and his decision mainly depends on the amount of the cost advantage and characteristics of product category.

We now provide the details of the comparisons in both quantity and price settings.

- **Quantity competition settings: DFQ and SQ**

As described before, the main difference between SQ and DFQ lies in defining of an exogenous parameter for the market shares in DFQ model, while the market shares in SQ are endogenous and are determined based on the quantity choice by the retailers[9] . The main objective of this comparison is to find conditions under which assortment reduction for $R1$ as the market dominant retailer is possible (if any).

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[9] Note that in order to increase the validity of this comparison between SQ and DFQ, one best strategy is to compare variety choices under SQ with those under DFQ with a large enough market share to be comparable to SQ model (for instance $\lambda \geq 0.5$). However, for the price setting analysis, we do not restrict our attention to this range and include all the range of $0 < \lambda < 1$.

Remark 1 *Comparison of SQ and DFQ Games: The power retailer $R1$ reduces his assortment under DFQ compared to SQ only if (1) his cost advantage ($\frac{\Delta}{w}$) is high enough, (2) the product category is considerably expensive for the weak retailer (equivalently when $R2$ has a potentially low profit margin, i.e. large $\frac{w}{\alpha}$), and (3) the products' demands are nearly independent (low substitutability degree).*

As the above remark indicates (see the details in Table 5–2), the market power in DFQ reduces retailer $R1$'s assortment compared to SQ model only if he has enough power to influence the market by reducing his price, i.e. when his market share is big enough, his cost advantage is significant, and also his opponent has low profit margin. These conditions put $R2$ in a situation to seek a way to possibly escape the competition, mainly by carrying only the products not carried by $R1$ once he decides to carry popular product. In this situation, a low substitutability degree is in the favor of $R1$ as he can affect R2 significantly by setting the price for the popular product and not being affected by the less popular product. This indeed leads to a market division: the popular product is offered by the dominant retailer and the less popular product by the weak retailer. This strategy guarantees highest profit for the dominant retailer in the competition.

Note that since we limit our attention only on $\lambda \geq 0.5$, the effect of low market share in Table 5–2 is not clearly observable for quantity settings. However, a further analysis reveals that assortment reduction is only possible for considerably large market share of the dominant retailer (λ). The exact intervals for λ and the ratio of $\frac{w}{\alpha}$ under which assortment reduction becomes possible comes from:

$$\{\lambda, \frac{w}{\alpha} : \quad \frac{4\lambda^2\beta_1\beta_2 - 3\lambda^2\gamma^2 - 2\lambda\gamma^2 + \gamma^2}{4\lambda^2\beta_1\beta_2 - 2\lambda^3\gamma^2 - \lambda^2\gamma^2 - \gamma^2} < \frac{w}{\alpha - w}\} \tag{4}$$

For any λ and $\frac{w}{\alpha}$ in above inequality, the assortment reduction can happen only for high enough cost advantage ($\frac{\Delta}{w}$) as below:

$$0 \leq A^{\tau} \leq \frac{\Delta}{w} \leq 1$$

where $A^\tau = \frac{(\alpha-w)(4\lambda^2\beta_1\beta_2-3\lambda^2\gamma^2-2\lambda\gamma^2+\gamma^2)}{w(4\lambda^2\beta_1\beta_2-2\lambda^3\gamma^2-\lambda^2\gamma^2-\gamma^2)}$. This assortment reduction by $R1$ is also observable in Figure 1 when $\frac{\Delta}{w}$ is relatively high. In contrast, the dominant retailer may also decide to increase his product variety depending on the payoff functions and the anticipated reaction of the weak retailer. In fact, for a wide range of parameters where $R1$ cannot force $R2$ to carry less popular product either because he is not large enough or $R2$'s profit margin is high, he does not reduce his assortment compared to the simultaneous game (SQ).

For $\{\lambda, \frac{w}{\alpha} : \quad A^\tau > 0\}$ (roughly equivalent to $\frac{w}{\alpha} < 0.5$ for high enough market shares), the assortment choice by $R1$ in SQ is always less or equal to that at DFQ for the whole possible range of $0 < \frac{\Delta}{w} < 1$. In other cases when (4) holds, for low enough cost advantages [lower than A^τ], $R1$ would increase his assortment when he possesses the price leadership.

- **Price competition settings: DFP and SP**

In order to identify the assortment effects of market price leadership in price setting, we compare the equilibrium variety choices of the power retailer in SP and DFP games for the same values of the parameters λ, $\frac{\Gamma}{\mathcal{B}_1}$, $\frac{\mathcal{B}_1}{\mathcal{B}_2}$, $\frac{\Delta}{w}$, V, and $\frac{w}{\mathcal{A}/\mathcal{B}_2}$. A summary of the numerical results is presented in Table 5–2. The following remark highlights the main finding.

Remark 2 *In price competition setting, the price leadership (market power) generally leads to assortment reduction for $R1$. The only exceptions are when his market share is very small, the product category has a potentially large profit margin, and popularity difference of products is low.*

This result is highly stronger than its counterpart in quantity setting as we find that a big enough power retailer *always decreases* his assortment when he becomes privileged with market dominance. However, our observations (see Table 5–2) reveals that even in price competition case under rare conditions, the assortment of $R1$ may increase from SP to DFP. As the above remark states, this basically happens when the products are equally popular (to be exact, $\frac{\mathcal{B}_1}{\mathcal{B}_2} \geq 0.9$) and the price-leader retailer lacks a very strong position in the market in terms of his share of the target market and the unit cost advantage; in such circumstances limited assortment indeed restricts $R1$ to benefit from the other profitable product (LP).

In summary, market dominance, in general, brings more power and leads to more flexibility in assortment planning (increase or reduction) for a power retailer; however, the final assortment choice of the price leader depends on the extent to which he can benefit from his pricing power.

2.4.3 Market Share

When analyzing the assortment impacts of cost and pricing power of a retailer, we find that the relative market share (represented by λ in all models except SQ in which market shares are set endogenously) is indeed another source of manipulating power in both supply channel and marketplace for a power retailer. Thus, in this section, we investigate whether or not increasing the market size of a power retailer can lead to his assortment reduction. To summarize, based on our analysis of simultaneous-move (only SP model) and dominant-fringe models, we find that increasing market share of the power retailer in all the scenarios can lead to assortment reduction under certain conditions, which we express in detail in what follows.

- **Simultaneous-move: SP model**

According to Table 5–3, increasing market share (λ) may result in assortment reduction or increase for the powerful retailer in SP game regardless of the degree of his cost advantage. The consequent assortment choice of $R1$ mainly depends on $\frac{\mathcal{B}_1}{\mathcal{B}_2}$, which is illustrated in the following remark.

Remark 3 *In a simultaneous-move game (SP) as the market share (λ) increases, the dominant retailer sets his assortment depending on the similarity in popularity of products, that is, $\frac{\mathcal{B}_1}{\mathcal{B}_2}$, as below:*

1. *When products are highly different in popularity (low $\frac{\mathcal{B}_1}{\mathcal{B}_2}$), $R1$ reduces his assortment (the variety outcome of the game switches from $< F, P >$ to $< P, P >$),*
2. *When products become similar in popularity (high $\frac{\mathcal{B}_1}{\mathcal{B}_2}$), $R1$ increases his assortment (the variety outcome of the game switches from $< P, F >$ to $< F, P >$).*

As the above remark suggests, the most important factor influencing the assortment decision of $R1$ when his market share increases is the degree of popularity difference of the products in the category ($\frac{\mathcal{B}_1}{\mathcal{B}_2}$). When products are demanded almost equally in the market, the powerful retailer may find it more profitable to stick with full assortment forcing $R2$ to carry limited assortment (only popular product). On the contrary, when one product is much more popular than the other, retailer $R1$ switches to a limited assortment to increase his profitability primarily by saving in variety costs. These results suggest that in a simultaneous-move game, an increase in the market share of a retailer (with or without significant cost advantage) gives him more flexibility in manipulating the market and gaining benefits by means of his variety selections.

• **Dominant-Fringe: DFQ and DFP models**

We now examine the impact of larger market share on the equilibrium assortment decisions when the power retailer possesses the market dominance (price leadership). Based on the results presented in Table 5–3, $R1$ gains more flexibility in choosing profitable assortments as his market share λ increases, and depending on the product characteristics, he may decrease or increase his assortment choice.

A further analysis shows that the tendency of $R1$ to offer full assortment when $R2$ carries popular product decreases in DFQ model because when λ increases, his profit from popular product increases and he does not have to offer less popular product to keep his profitability. This implies that assortment reduction can certainly happen when λ is large enough in the range of $\max\{0, \frac{2\beta_1\beta_2-\gamma^2+2\sqrt{\beta_1^2\beta_2^2-\beta_1\beta_2\gamma^2}}{\gamma^2}\} \leq \lambda < 1$ eespecially when products are very different in the potential popularity (low $\frac{\mathcal{B}_1}{\mathcal{B}_2}$); nevertheless, the effect of market share is not fully detectable in DFQ model and no regular pattern can be observed. This is to say, the variety can either increase or decrease by a dramatic increase in the market share.

Similarly in DFP game, an increase in the market share of the dominant retailer can affect his assortment decision either to increase or decrease the product varieties. Even though increasing λ makes $R1$ less tempted to offer full assortment when $R2$ carries popular product, he becomes willing to offer full assortment when $R2$ chooses less popular product.

This phenomenon in fact provides the possibility of occurrence for both assortment reduction and increase.

2.4.4 Non-power Attributes

We now explore the impact of the parameters that characterize the category of products and the nature of retail competition on the assortment decisions by the power retailer. We measure the sensitivity of retailer $R1$'s variety choices to different parameter ratios that correspond to product characteristics including the demand popularity distribution in category, substitutability of products (in quantity setting), and potential unit cost at the weak retailer (being a potentially low- or high-margin product category). We also investigate the assortment impacts of price competition effect (only in SP game). The full list of notations for these variables are presented in Table 2–4.

Table 2–4: List of all underlying factors.

Underlying Factor	Description (Measurement Ratio in Quantity/Price setting)
Power factors:	
1. Cost Advantage	Relative advantage of $R1$ in terms of unit wholesale price, $\left[\frac{\Delta}{w}; \frac{\Delta}{w}\right]^*$.
2. Market Share	Share of $R1$ from the total market demand of the commonly carried product(s), $[\lambda; \lambda]$.
3. Price Leadership	Ability to impose the market price to the weak retailer, [we measure this factor by comparing the equilibrium outcomes in simultaneous-move games, where no retailer has special price leading power, with dominant-fringe games, where the power retailer has the full power to dictate his price to the weak retailer].
Non-power factors:	
3. Popularity Similarity	Relative similarity of products in terms of market demand, $\left[\frac{\beta_1}{\beta_2}; \frac{\mathcal{B}_1}{\mathcal{B}_2}\right]$.
4. Category Expensiveness	Degree of expensiveness of the product for the weak retailer (there is an inverse relationship between this ratio and potential profit margin of the category), $\left[\frac{w}{\alpha}; \frac{w}{\mathcal{A}/\mathcal{B}_2}\right]$.
5. Substitutability Degree	Relative cross-sensitivity of the demand for one product to the quantity of other product in quantity settings, $\left[\frac{\gamma}{\beta_1}; \text{N.A.}\right]$.
6. Price Competition Degree	Relative cross-sensitivity of the demand for one retailer to the competitor's price in price settings, $\left[\text{N.A.}; \frac{\Gamma}{\mathcal{B}_1}\right]$.

* Note: $[X, Y]$ indicates the way we measure the factor in quantity (X) and price (Y) settings.

In what follows, we address their potential effects on the final variety choice by the power retailer $R1$ and provide numerical and analytical results. In summary, our findings suggest that these non-power factors can play significant roles on the retailers' assortment decisions.

• **Popularity distribution of products**

As explained in model framework, the popularity of a product in our model is assessed by the sensitivity of its demand to retail price; for instance, the demand for the more popular product is expected to be less sensitive to the price. Therefore, popularity distribution (the extent to which products are similar in popularity) can be measured using $\frac{\beta_1}{\beta_2}$ and $\frac{\mathcal{B}_1}{\mathcal{B}_2}$ ratios in quantity and price settings, respectively. Since $0 < \beta_1 < \beta_2$ and $0 < \mathcal{B}_1 < \mathcal{B}_2$, we know that $0 < \frac{\beta_1}{\beta_2} < 1$ and $0 < \frac{\mathcal{B}_1}{\mathcal{B}_2} < 1$. As $\frac{\beta_1}{\beta_2}$ (resp. $\frac{\mathcal{B}_1}{\mathcal{B}_2}$) increases and approaches to one, products become more equally popular and on the contrary, when it approaches zero, products are different in terms of popularity and potential demand.

Intuitively, when a few products are so popular that the major portion of the total demand comes only from them, retailers are believed to limit their assortment to those more demanded products. Our observations summarized in Table 5–4 and analysis of different games support the idea that the power retailer (with high enough market share) limits his assortment when the popularity difference of the products increases, i.e. when trendy or fashionable products are present. Now we present some major results of our analyses in different games as follows.

1. In SQ game where the relative market shares are set endogenously, there is a strong association between popularity distribution and assortment choice by the power retailer $R1$ as presented in the following proposition.

   Proposition 2 *In SQ model, the powerful retailer reduces his assortment when the popularity difference of products increases (equivalently when the ratio of $\frac{\beta_1}{\beta_2}$ decreases).*

   This proposition states that as the popularity distribution becomes more and more asymmetric, retailers have more incentive to focus only on popular products.

2. A similar result holds in DFQ model. If the market share of the dominant retailer is big enough, he definitely increases his assortment as $\frac{\beta_1}{\beta_2}$ increase. However, if he does not possess a big share of market demand, he might instead decrease his assortment. This assortment reduction policy works well especially when he has a significant cost

advantage. According to Table 5–4, when his market share is low, he changes the rule of the game by switching the variety outcome from $< F, P >$ to $< P, LP >$. He indeed pushes $R2$ to take only less popular product when he himself carries only popular product. He does that by an aggressive price reduction in $< F, P >$ that makes the popular product unprofitable for $R2$ even though his market share is considerably bigger.

3. Likewise, in the price competition settings, an increase in similarity between the two products (increase in $\frac{\mathcal{B}_1}{\mathcal{B}_2}$) leads to assortment increase for the power retailer. However, there are a few exceptions where reductions are also possible. Similar to DFQ game, assortment reductions happen mainly when the overall variety outcome shifts from $< F, P >$ to $< P, LP >$. This case generally happens when $R1$'s market share in common product λ is relatively low and as a result, he tries to provide a situation not to share any common product with $R2$, an outcome that only happens under $< P, LP >$. Interestingly, in SP where $R1$ cannot impose his price directly on $R2$, he can does so if he has a significant cost advantage and the market is relatively sensitive to the price difference (high $\frac{\Gamma}{\mathcal{B}_1}$).

- **Substitutability of products (quantity setting)**

The parameter γ in quantity competition settings refers to the cross-sensitivity of products' demand to their substitutes; therefore, we let the ratio $\frac{\gamma}{\beta_1} \in [0, 1]$ measure the substitutability degree between the two products. It is intuitively expected that retailers decrease their assortment if the products become more substitutable to each other. This effect is clearly extractable in the simultaneous-move model, but it becomes difficult to find a clear relationship in DFQ game because of its complex non-linearity. In what follows, we present the potential impact of products' substitutability on the assortment choice of $R1$.

Proposition 3 *In SQ model, the powerful retailer would switch to the limited assortment (popular product) if he faces a significant increase in substitutability between the two products (measured by the ratio of $\frac{\gamma}{\beta_1}$).*

This is also true in the presence of market power for $R1$. As it can be observed from Table 5–5, when $R1$'s market share is relatively high, with an increase in $\frac{\gamma}{\beta_1}$, $R1$ becomes less willing to carry full assortment in response to $R2$'s limited assortment, which is generally expected to result in assortment reduction for the dominant retailer. However, our observations in Table 5–5 reveals that assortment increase can also happen when $R1$'s market share is low and his cost advantage is relatively high. In these cases, the assortment outcome of the game moves from $< P, LP >$ to $< F, P >$. In fact, $R1$ carries full assortment and decreases the price, which makes $R2$ to carry the popular product. This assortment outcome change brings $R1$ a significantly larger quantity at a lower price that leads to a higher profit in the end.

- **Price competition effect (price setting)**

In simultaneous-move model in price competition setting (SP), we assume that the demand for a retailer can be sensitive to the price adopted by his competitor. The parameter $\Gamma(\Gamma < \mathcal{B}_1)$ denotes this cross-sensitivity; and therefore, we let the ratio $\frac{\Gamma}{\mathcal{B}_1} \in (0, 1)$ measure the price competition degree. We only have the chance to investigate its effect in SP model because as mentioned in §2.3.2, this parameter is taken off in DFP model since the final prices for the common products are the same for both retailers. Based on our observations in Table 5–6, its impact on the assortment decision by the power retailer $R1$ is outlined as follows.

1. In general, an increase in the sensitivity of demand to the competitor's price leads to assortment increase for the power retailer in SP model. This is perhaps because as the retail price competition degree increases, it amplifies the cost superiority of $R1$ in the market in a way such that he can cannibalize the demand from the weak retailer by decreasing his price. In these situations, increasing variety can also lead to an increase in the total quantity sold in the market and the cost of this assortment increase can be easily offset by the increased sale of the products. That is why the effect of higher degree of price competition on the assortment choice for the power retailer is very similar to the effect of increasing cost advantage in simultaneous-move competitions.

2. Although higher price competition often leads to assortment increase, there are also situations in which assortment reduction happens. This situation takes place eespecially when $R1$'s market share is relatively low. Under these circumstances, he might instead decrease his assortment to benefit from lower variety cost and higher retail price (as a result of slight reduction in price competition) even though he loses some quantity. The overall variety outcome of the game shifts from $< F, F >$ to $< P, F >$ in these cases.

- **Actual expensiveness of the category for the weak retailer**

In this section, we investigate the assortment impact of the actual expensiveness of the category for the weak retailer $R2$. To do so, we let the ratios of $\frac{w}{\alpha} \in (0, 1)$ and $\frac{w}{\mathcal{A}/\mathcal{B}_2} \in (0, 1)$ measure this factor in quantity and price settings, respectively. Note that these ratios are also inversely related to the potential profit margin of the category because normally, if the potential unit cost of the products relative to the maximum possible price is high (resp. low), the potential profit margin of the category is low (resp. high). Therefore, when these ratios increase (resp. decrease) and approach to one (resp. zero), the category becomes potentially less (resp. more) profitable. The results of our analysis (see Table 5–7) show that this feature of product categories can have a substantial role in determining the assortment decisions by a power retailer. In what follows a summary of our observations is expressed.

1. When the retailer is competing through variety and quantity choices in a simultaneous-move game (SQ), evaluating this effect is analytically possible. The following proposition provides the key results in SQ model.

   Proposition 4 *In SQ model, as the potential profitability of product category decreases ($\frac{w}{\alpha}$ increases), the powerful retailer would reduce his assortment choice.*

   This finding is quite intuitive because when offering a product becomes expectedly less profitable, the retailers may decrease their investment on that particular category and reduce the space (number or volume of shelves) assigned to that category to save in variety costs.

2. This is to some extent true in all other games (SP, DFP, DFQ) too. When the power retailer's market share λ is relatively high, an increase in the actual unit cost at the weak retailer ($\frac{w}{\alpha}$ or $\frac{w}{\mathcal{A}/\mathcal{B}_2}$) makes $R1$ more sensitive towards the variety cost and leads to his assortment reduction. However, for lower values of λ, there are some possibilities for assortment increase. When $R1$'s market share is very low, he might find it more profitable to increase his assortment (and consequently his quantity) by pushing $R2$ to decrease his assortment. This is usually true when he has significant cost advantage to be able to manipulate the market price as much as possible. In these cases, the equilibrium variety choice transfers from $< P, F >$ to $< F, P >$. Hence, as the weak retailer becomes more vulnerable to the low profit margin products and takes a limited assortment, the power retailer with high cost advantage can offset the losses of lower margin by increasing variety (and total quantity) of products.

In the next section, we provide a summary of results along with the managerial implications that can be helpful for understanding the effect of retail power and other non-power attributes on the variety choices taken in the practice.

2.5. Conclusion and Future Direction

In this chapter, we examine whether strong retail power can be a cause for assortment reduction, a practice widely taken by many big-box retailers. To capture some important elements of retail power, we consider two specific powers: (i) unit cost advantage and (ii) market dominance represented by price leadership. We develop game theoretical models to consider competitions between two independent retailers in making variety and quantity/price decisions either simultaneously or sequentially. The two retailers are asymmetric in power: the power retailer can have either market power to set the market price and/or product cost advantage over the weak retailer. In addition to these two retail powers, we are able to examine the pure impact of power retailer's market share on his assortment as the third element of retail power. Moreover, we measure the impact of some other non-power factors including 1- popularity distribution, 2- product substitability, 3- cross-sensitivity of demand to the competitor's price, and 4- potential profit margin of the product category.

Our analysis sheds some light on the effects of these power and non-power attributes on the assortment choice by a power retailer and helps retailers in identifying situations where they may actually choose to reduce their assortment as a result of their special power. We first provide a brief summary of the results in Table 2–5 and then provide further remarks.

As can be seen from Table 2–5, both analytical and numerical studies demonstrate that assortment reduction is almost impossible when the power retailer gains only cost advantage with no market dominance (price leadership). On the other hand, price leadership when supported with large market share and cost advantage, would lead to the retailer's assortment reduction. This finding shows the strategic importance of pricing power when making retail assortment decisions in competitive environments. Therefore, we can interpret retail pricing and assortment as two complementary tools in competitive marketplaces, and only when a retailer has a strong standing in pricing aspect, he may choose to decrease his assortment in the competition.

Table 2–5: Possibility of Assortment Reduction for Power Retailer as Each Attribute Increases.

	Quantity setting		Price setting	
	SQ	DFQ	SP	DFP
Power factors:				
1- Price leadership	Highly Possible		Highly Possible	
2- Cost advantage	Impossible	Possible	Rare	Possible
3- Market share	-	Highly Possible	Highly Possible	Possible
Non-power factors:				
1- Popularity similarity	Impossible	Possible*	Possible*	Possible*
2- Product substitability	Always	Highly Possible	-	-
3- Sensitivity to competitor's price	-	-	Possible*	-
4- Potential profit margin of product category	Always	Highly Possible	Highly Possible	Highly Possible

The possibility terms in this table correspond to the percentage of assortment reduction out of all the cases with assortment change in tables 5–1 to 5–7, i.e. (> 20%: Highly possible); (1%-20%: Possible); (< 1%: Rare); (No observation: Impossible).
* The majority of these cases happen only when the market share of the power retailer is significantly low.

Our results reveal that assortment reduction is a special decision and only takes place if the power retailer has enough power to manipulate the market and earn the high levels of profits. This condition holds only when first, the power retailer has the cost advantage that enables him to lower his prices; second, he is the price leader in the market that makes him to dictate his low prices to the weak retailer, sometimes forcing him not to carry the popular product(s); and third, his market share is large enough that guarantees a high enough profit from limited number of products. For example, Home Depot and Walmart are certainly market-price leaders for many categories of products. Both of them have significant market shares and benefit from recent technological advances and are famous for their cost management efficiency. Therefore, they are roughly speaking, good examples for big market dominant retailers with cost advantage who reduce their assortment level.

In addition to price leadership and cost advantage, we find that market share of a power retailer may have significant impact on his assortment choice. In fact, a larger market share for a power retailer can result in his assortment reduction at both simultaneous-move and dominant-fringe settings at any degree of cost advantage. Assortment reduction happens mainly when the products have highly asymmetric popularity degrees; i.e. under the presence of a highly popular product if the market share of a retailer (with or without cost advantage and price leadership) increases, he is likely to decrease his assortment. To mention a rough example of this situation, we may consider the recent assortment reductions by some big-box retailers while their market shares have expanded. For instance, in 2009, Walmart removed thousands of SKUs from its stores, even though it decided later to bring a portion of them back to its assortment (Kahn et al. 2013). A future empirical study on the relationship between the degree of assortment reduction in Walmart stores with the characteristics of the product categories can verify this impact of popularity distribution.

The relative market share of the power retailer may also have a great indirect impact on the effects of non-power factors. Large enough market shares let the power retailer to focus on the pure impact of his pricing and assortment decisions on his profitability, whereas when his market share is comparatively low, he might put more emphasis on the competition effect and set his pricing and assortment decisions so that he can push the other retailer to limit or expand his assortment. Note that, indeed, market share as a source of power should be taken into consideration along with the degrees of price leadership and cost advantage. For instance, a market dominant retailer with low market share but high cost advantage, can force the weak retailer to some extent to carry a limited or full assortment. This might also be true under the lack of price leadership, as a low-market-share retailer can still manipulate the decisions of the other retailer when he has a considerably high cost advantage in a market that is highly sensitive to the retail price differences.

Our results show that the characteristics of the product category can have significant influence on the assortment choice by the power retailer. As expected in majority of the cases considered in this chapter, if one product becomes significantly popular and more profitable, the power retailer would quickly switches to limited assortment for lower variety cost levels. Likewise, when the substitutability between the two products becomes very high, the dominant retailer becomes more sensitive towards variety cost and shifts to limited assortment. Also, if the potential unit cost of a product category increases with no increase in the popularity (lower potential profit margin), the power retailer is more likely to focus only on the popular products. These effects mainly hold when the market share of the power retailer is relatively large. However, when the market share of the power retailer in common product(s) is low, he might act inversely by increasing his assortment. In such situations, he might trigger a severe price competition while he increases his assortment (and consequently his quantity), which usually pushes the weak retailer to limit his assortment (either to popular or less popular products depending on the situation).

Lastly, we investigate the assortment impact of price competition degree in simultaneous-move game (SP model) and find that if the sensitivity of demand to the competitor's price increases, the power retailer with cost advantage would increase his assortment level. This is because he can reduce the price to an unprofitable level for the weak retailer and attract the majority of the market demand. This increase in the demand (and consequently in profit if price competition degree is high enough) in many cases can be very large such that he finds it profitable to carry even less popular products. Nevertheless, this result might significantly change for a power retailer with a low market share. In fact, when his share of the common product(s) is low, he might instead decrease his assortment to benefit from both lower price competition (and a higher price as a result) and lower variety costs while he loses some of his total quantity.

To outline some potential future research in this stream, the first possibility is to consider different sources of retail power other than cost advantage and market dominance, such as retail image and special access to consumers or suppliers. Further research needs to be done to establish the impact of these sorts of retail power on the assortment decision by a power retailer. Moreover, relaxing the restrictive assumption that a retailer can get both products at identical cost and considering more complicated situations where retailers and suppliers make their decisions in a bargaining contractual framework can lead to more realistic situations and perhaps more general implications for retailers and suppliers. Another interesting extension would be to consider more complicated demand functions such as non-linear utility-based ones (e.g. MNL model) in order to check the robustness of the results in other settings.

In our analysis, we attempt to capture the most important factors affecting the assortment decisions of a power retailer that were never addressed in a unified framework in the literature. Nevertheless, other retail constraints and considerations, such as budget and shelf space constraint, can be effective on assortment planning provisions. In addition, retailers may have some strategic decisions other than quantity, pricing, and variety selection that we have not modeled in this analysis. Advertising, service, and promotional actions are likely to shift by a change in the power structure of retail network and this may alter the dominant retailer's assortment decisions. It would be interesting to investigate the impact of the combination of these factors on variety choice in future studies. Another additional research is needed to investigate the dominant retailer's assortment behaviors in the case of complement products rather than substitute products.

Our model considers a one-shot static competitive game between two asymmetric retailers. It would be interesting to focus on more dynamic settings and repeated games that may also diminish the difficulty of analyzing the outcomes (points) with multiple equilibria. For the future studies, it might also be possible to consider more categories of products to inspect the effect of retail power on cross-category assortment choices and management regimes adopted by a dominant retailer. Another interesting extension would be to consider Oligopoly and Monopoly situations as the benchmarks and investigate how the assortment choices are different from Dominant-fringe and Duopoly settings.

Lastly, we believe that the joint analysis of the retail power and assortment reduction presents fruitful research opportunities and hope that this study will fuel future research in this direction.

CHAPTER 3
Quality Scores in Reverse Auctions: Motivations, Information Sharing and Credibility

3.1. Introduction

An increasing number of buyers nowadays use (electronic) B2B platforms to manage their sourcing processes.There are two main benefits of e-sourcing. First, it considerably helps to streamline the end-to-end procurement process. According to a survey conducted among more than 200 companies, the deployment of e-sourcing cuts in half the sourcing cycle, which averages from 3.3 to 4.2 months (Minahan 2005). Popular B2B online services such as AliSourcePro literally enable anyone to receive several quotes within 48 hours after posting a simple buying request to a vast network of suppliers and help to finalize the procurement contract in a few days (BusinessWire 2014). Second benefit of e-sourcing comes in the form of reduction in purchase price due to the increased nature of supplier competition, which presents itself as direct savings to the bottom-line (BuyIT 2004). Furey [2009] reported that in the early 2010s, e-sourcing through Ariba, a company specialized in online procurement services, saved companies 5% to 7% of their procurement costs. Similarly, General Electric (GE) alone claimed a saving of about $680 million and a net saving of more than 8% in 2001 by using SourceBid, a reverse auction tool and a part of GE's Global Exchange Network (GEN). In addition, the U.S. General Services Administration attributed savings of 12%-48% to the use of procurement auctions (Sawhney 2003).

A popular format employed in e-sourcing corresponds to the buyer-initiated one, known as e-Reverse Auctions, i.e., eRA (Jap 2002). This format has been used for a wide range of products and services varying from machined metal components and printed circuit boards to marketing and legal services (Furey 2009). There are two versions of eRAs: price-only and price-plus. In the price-only eRAs, the contract is awarded to the bidder with the lowest price, whereas in the price-plus eRAs, other non-price factors, such as quality, reliability, timely delivery, etc., are also factored into the buyer's decision calculus to determine who wins the business resulting from a competitive bidding process (Jap and Haruvy 2008). One of the commonly used procedures for incorporating these non-price factors is to assign a score, so-called *Quality Score* (QS), that represents the buyer's evaluation of suppliers on these non-price factors. The buyer then adjusts the bidding prices of the suppliers using these scores and awards the contract to the bidder whose QS-adjusted price, so-called *generalized price*, is the lowest (Anderson and Frohlich 2001, Jap 2002)[1] .

Even though price-plus eRAs provide the buyers with the increased level of flexibility in the sense that the contract is not awarded only on the basis of purchase price, it leads to two inter-related problems. One is informational, and the other one is related to its impact on the degree of competition. Informational problem arises among the partners because the suppliers do not know exactly how the buyer calculates QS. This comes from two main sources: (1) uncertainty on which non-price factors are used by the buyer (attribute uncertainty), and (2) uncertainty on how these non-price factors are combined to evaluate suppliers' final quality scores (procedural uncertainty). As noted in the literature, these two uncertainties result in informational asymmetries between buyers and suppliers regarding the rules of auctions (Beall et al. 2003). On the other hand, the buyer may attempt to alleviate this information asymmetry by communicating both attributes and procedure (together known as the scoring formula) to the suppliers. However, this can adversely impact the degree of competition among the suppliers. This is the source of the second problem. Namely, when the suppliers know the exact scoring formula and their relative quality scores,

[1] The notion of quality scores is also widely used in keyword auctions (a generalized type of forward auctions) as a way to incorporate non-price attributes in the bid of advertisers (Geddes 2014).

they can reverse-engineer the winning price and intentionally adjust their bid prices so as to win the auction (Haruvy and Katok 2013)[2] .

Even though the reverse-auctions have been extensively explored in the literature, to the best of our knowledge, the informational and strategic issues regarding the quality scores in eRAs have not received due attention. In this context, the objectives of this chapter are to address the following research questions:

(i) *When* does the buyer share the QS information with the suppliers in eRAs?

(ii) If the buyer decides to share QS with the suppliers, *how* can it be *credibly* shared?

(ii) *What* is the impact of sharing QS information on the decisions, and profits/cost of channel parties?

In order to address these issues, we develop a bi-level supply chain model in which the downstream party, the buyer, uses a first-price and sealed-format reverse auction to procure from two competing sellers (hereinafter refereed to as supplier H and supplier L) at the upstream level. Knowing that suppliers' cost information plays an important role in the result of the auction, in this essay, we consider two extreme cases: 1) public cost information, where the cost information of each supplier is known to the buyer and the competing supplier, and 2) fully private cost, where the marginal cost of each supplier is known only to herself and other players are in full uncertainty about it. In our main model, we start with the public cost information setting in order to better focus on the implications of QS sharing in a relatively more straightforward setting. Suppliers are assumed to be heterogeneous in terms of both marginal costs and quality scores. Namely, both marginal cost and QS of supplier H are higher than those of supplier L. The parties interact with each other in the following order: First, the buyer evaluates the non-price attributes of the suppliers H and

[2] Indeed, in 2002 when Google first introduced AdWords, it published the exact formula with which each advertiser's bid was scored. However, Google later introduced both attribute and procedural uncertainties to the auction rules by defining, in addition to click-through rates (CTR), some ambiguous non-price factors such as the quality of the advertisement text. Over the years, many search engines follow the footsteps of Google which in turn increased the uncertainty of quality scores from the perspective of advertisers. See Geddes [2014] for the historical account of Google's AdWord auctions.

L and assigns to each[3] a QS, which is privately known only to the buyer. Second, the buyer decides whether or not to share QS information with the suppliers. Then, suppliers *competitively* submit their bids (quotations). Finally, the buyer calculates the generalized prices of suppliers based on the QS-adjusted bids, and awards the order to the one with the lowest generalized price.

We characterize decisions (prices and quantities), and pay-offs (profits and costs) of all the supply chain parties under both pooling and separating equilibria. Their comparative analyses allow us to evaluate when and how the QS information can be credibly shared by the buyer with the suppliers and how it affects the degree of price competition among the suppliers and the payoffs of supply chain partners. First of all, the information asymmetry on the relative quality scores between the suppliers leads to an uncertainty on them regarding what price to charge. We show that this leads to higher prices than those under symmetric information particularly when the degree of information asymmetry is high. This in turn generates an incentive for the buyer to share relative QS information with the suppliers. That said, we also show that the buyer has always incentive to distort the relative QS information, which puts the credibility of QS information at stake. In this context, we show that the buyer can share relative QS credibly with the help of a commitment device such as an advance revenue guarantee. This increases the degree of competition among the suppliers (which in turn lowers the equilibrium bid prices) when they are heterogeneous. However, it comes at a [signalling] cost for the buyer. Hence, the buyer opts for sharing QS information only when the degree of information asymmetry between the buyer and the suppliers is sufficiently high. Moreover, the analysis of a general case with more than two suppliers allows us to show that an increase in the number of participants in the eRAs naturally intensifies the price competition and hence reduces the need for sharing QS information credibly.

Finally, by relaxing the suppliers' public cost assumption and considering the private cost setting, we are able to comment about the effects of their cost information privacy

[3] Throughout this chapter, we use masculine and feminine pronouns for the buyer and suppliers, respectively.

on buyer's and suppliers' incentives and actions in an auction where participating suppliers are heterogeneous in QS. Regardless of the technical differences in analysis, we find similar results by this extension: the buyer prefers not to share the relative QS with the suppliers if they believe that they are quite homogeneous in QS as this condition increases the degree of price competition; however, as the degree of QS uncertainty increases, the buyer rather incur a cost and signal this information with the aim of lowering the bid prices.

3.2. Related Literature

Our work is primarily related to two streams of research: (i) papers analyzing various sourcing mechanisms in the context of auctions, and (ii) papers dealing with informational issues in the context of supply chains.

Most of the papers in the first stream can be categorized into three groups. First group of the papers study the impact of auction formats (first- vs. second-price, open vs. closed, etc) on the buyer's surplus and suppliers' decisions in the bidding process (see Kostamis et al. 2009, Elmaghraby et al. 2012, and Budde and Minner 2014). The papers in the second group compare price-plus with price-only both experimentally and analytically and show that price-plus auctions are more effective in increasing buyer's utility compared to price-only auctions (see Chen-Ritzo et al. 2005, Bichler 2000, and Asker and Cantillon 2008). Finally, the papers in the third group compare supplier- vs. buyer-determined price-plus auctions and show that the outcome depends on the number of bidders (Engelbrecht-Wiggans et al. 2007), and the relative cost differences among the suppliers and the number of non-price attributes (Santamaría 2015). Note that in a buyer-determined auction, similar to our setting, it is the buyer who decides on non-price attributes based on some pre-determined scoring rules, whereas in supplier-determined auction, suppliers bid on not only prices but also non-price attributes, such as delivery time, quality etc. In the former, buyers might evaluate non-price attributes before or after the bidding process. Wan and Beil [2009] analytically compare the performance of pre- and post-evaluations and show how the buyer's choice depends on the evaluation (qualification screening) cost. The latter is also called a menu auction, in which the bidder is allowed to offer a menu of price and non-price attributes. We refer the readers to Bernheim and Whinston [1986] for a detailed analysis of menu auctions.

Recent empirical studies provide strong evidence that information asymmetry can have significant impacts on the auction's results and certainly affects the buyer's surplus (Mithas and Jones 2007). Haruvy and Katok [2013] using experiments show that in on-line procurement auctions with open-bid format where exogenous bidder quality affects determination of the winner, the buyer surplus significantly decreases when the information about bidders' quality is publicly known. This work is also related to the stream of research on (forward) ad auctions in which the auction-holder assigns a QS to each bidder in order to capture critical non-price features. The key research question in this stream is to analyze the impact of auction formats on the truth-telling property (see Liu et al. 2010 and references therein). To the best of our knowledge, no paper in this stream analytically explores the impact of information asymmetry about the non-price attributes on bidding behaviour of the suppliers as well as the buyer's surplus.

Second stream that is related to this chapter is on information asymmetry and the resulting credibility issues in reverse auctions and supply chain. The papers are divided into two groups depending on whether the asymmetric information is on the supply side such as costs and reliabilities of the suppliers (Kim and Netessine 2013, Beil and Wein 2003, Corbett and De Groote 2000, Ha 2001, Yang et al. 2009 , Yang et al. 2012, Gümüs et al. 2012) or on the demand side (Cachon and Lariviere 2001, Li and Scheller-Wolf 2011, Wang et al. 2014, Gümüş 2014, Özer and Wei 2006). This chapter differs from this literature in two ways: (i) by considering buyer-determined reverse auctions and (ii) by dealing with the credibility of information on the suppliers' quality scores. The papers in this stream are also methodologically divided into mechanism design and signalling depending on whether it is informed or uninformed party who offers the contract. In our analysis, we use the latter framework to model an informed principal who utilizes the advance revenue guarantee with the aim of sharing QS information in a credible fashion (see Riley 2001 for an extensive literature review on signalling games).

Lastly, a stream of research related to our study is about advance commitments in the supply chain. Similar to our use of advance guarantee for signalling quality score information, Klotz and Chatterjee [1995] use an advance quantity guarantee to the incumbent supplier in exchange for participating in the procurement auction. Signalling characteristics of commitment contracts have been analyzed in both operations and marketing literature. Cachon [2004] studies the use of advance purchase discount contracts as a mechanism for sharing the inventory risk in a supply chain. Özer and Wei [2006] and Gümüs et al. [2012] analyze the role of advance guarantee contracts in enabling credible forecast and supply information sharing, respectively, between supply chain parties. Tang et al. [2004] and Yu et al. [2014] study the role of advance selling commitments between retailers and consumers in updating demand forecast and signalling product quality, respectively.

3.3. Model Framework

In order to investigate the impact of information sharing in buyer-determined price-plus reverse auctions, we model a stylized two-level supply chain consisting of one buyer and two suppliers, among which the buyer holds a simple (first-price, sealed-format) *electronic* reverse auction.

The suppliers differ in terms of both marginal costs and quality scores. Let c_i, and QS_i denote the marginal cost and quality score of supplier $i \in \{L, H\}$. Throughout this chapter, except in §3.8, we assume $c_L < c_H$. Also, we assume that quality score of supplier H is known to be higher than that of supplier L. However, the suppliers H and L do not know how much their quality scores assessed by the buyer, i.e., QS_H and QS_L, respectively, differ from each other. In order to capture the relative difference between quality scores of the suppliers, we define α and let it equal to QS_L/QS_H. Note that α essentially captures the degree of *relative similarity* between suppliers H and L in terms of their quality scores. That means, the closer α is to 1 (resp, 0), the more similar (resp., dissimilar) supplier L becomes to supplier H in terms of their quality scores.

In order to model the asymmetric information regarding the quality scores, we assume that the true value of relative QS, i.e., α is known only to the buyer. For the sake of tractability, we assume that suppliers hold a-priori belief on α in the sense that it is uniformly distributed between $\underline{\alpha}$ and $\overline{\alpha}$, where without loss of generality, $0 < \underline{\alpha} \leq \overline{\alpha} \leq 1$. Note that the difference between upper and lower bounds denoted by $\Delta = \overline{\alpha} - \underline{\alpha}$ represents the degree of information asymmetry on the quality scores between the buyer and the suppliers L and H.

Upon receiving the bid prices, the buyer then decides on the winner. There are various ways to merge price and quality score data of bidders. In this essay, we focus on one of the simplest rule employed in practice, which is called multiplicative generalized price rule[4] . Briefly, in the context of multiplicative form, the buyer first computes generalized price for each supplier i, denoted by p'_i, by computing price-to-QS ratio, i.e., $p'_i = p_i/QS_i$. The generalized prices of all the suppliers are then sorted in ascending order. The supplier whose generalized price is lowest wins the order. Note that in the case of two suppliers, the above multiplicative form boils down to the following simplified comparison between relative price and QS of suppliers H and L. Namely, supplier L wins if $p_L/p_H < \alpha$, otherwise, supplier H wins.

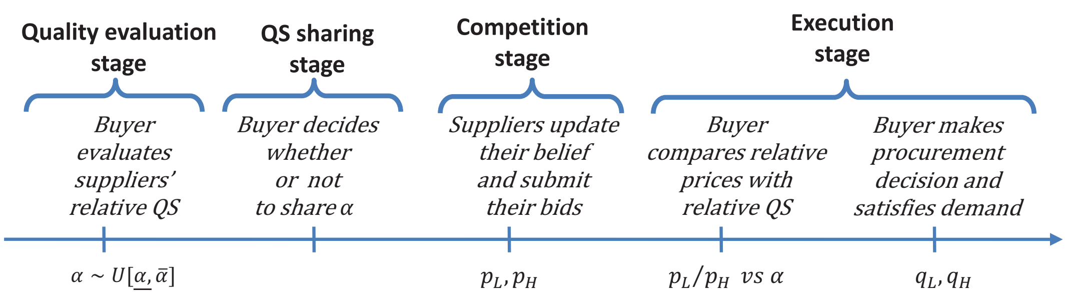
We also define a cap on the bid prices offered by the suppliers denoted by p_r to create a more realistic situation by preventing the price to approach to infinity in special cases. We assume that p_r is greater than c_H. Essentially, one can interpret this cap as the maximum (reserve) price the buyer is willing to pay to the suppliers or price in the spot market to which he has always access. In other words, supplier i loses automatically if $p_i \geq p_r$ irrespective of the above comparison rule. Finally, in order to focus on the main research questions related to the quality score, we assume that buyer's demand is deterministic and equal to Q units.

[4] In practice, buyers may use different linear or non-linear formulas for incorporating price and QS. For instance, GE's Commercial Finance devision uses a non-linear rule in auctioning their legal services (refer to Tunca et al. 2014, for a description of the important factors and the scoring rule). However, linear models and specially simple additive models are amongst the most popular ones and are widely used in both practice and academia [e.g. Kulp and Randall 2005, Kostamis et al. 2009]. In addition, there is another simple rule employed in practice, which is called multiplicative generalized price rule that theoretically leads to the same result as simple additive models (refer to Anderson and Frohlich 2001 for a detailed illustration of this rule). In this essay, we focus on multiplicative rule mainly due to the expositional simplicity.

Given the above order allocation rule, the cost of the buyer and the profits of the suppliers H and L (denoted by κ_B, π_H, and π_L, respectively) can be expressed as follows[5] :

$$(\kappa_B, \pi_H, \pi_L) = \begin{cases} (Q \times p_H, Q \times [p_H - c_H], 0) & \text{if } \frac{p_L}{p_H} \geq \alpha, p_H \leq p_r \\ (Q \times p_L, 0, Q \times [p_L - c_L]) & \text{if } \frac{p_L}{p_H} < \alpha, p_L \leq p_r \\ (Q \times p_r, 0, 0) & \text{otherwise} \end{cases} \tag{1}$$

Figure 3–1: Timeline of Decisions and Events.



The timing of decisions and events is shown in Figure 3–1 and provided as follows.

1. Buyer evaluates α, i.e., the ratio between quality scores of the suppliers L and H.
2. Buyer decides whether or not to share α with the suppliers.
3. In response to the buyer's decision, suppliers update their prior beliefs on α via Bayesian updating and submit their bid prices p_i.
4. Buyer compares the relative price and QS of the suppliers H and L. Based on the comparison, the buyer decides on the winner and orders Q units from her.

[5] An important point in this profit function is that the buyer does not enter the QS into his actual cost function. In practice, there might be two main reasons why a buyer considers QS only in the allocation stage but not in his utility function: 1) buyer knows himself that QS can be extremely subjective as it is mainly based on his evaluation and opinions whereas the winning price is the actual objective amount that he is committed to pay, 2) Many buyers may use QS as a device for discriminating local suppliers from cheap foreign suppliers which might be mandatory by the local or regional regulations and policies. For the analysis of the separating games when the buyer adjusts his profit function with QS, refer to Proposition 11.

Before we start the analysis, we summarize the list of notations used for parameters and decision variables in this chapter in Table 3–1.

Table 3–1: Notation used for model parameters and decision variables.

Model parameters	
$c_L; c_H$	Marginal cost of suppliers L and H, respectively.
α	The true value of relative ratio between quality scores of suppliers L and H, i.e., QS_L/QS_H.
$\underline{\alpha}; \overline{\alpha}$	Lower and upper bounds on suppliers' uniform a priori belief distribution for the true value of α.
$\pi_L; \pi_H$	Expected profit of the suppliers L and H, respectively.
κ_B	Expected cost of the buyer
Q	Total demand
p_r	Reserve price (e.g., spot market price)
Decision variables	
η_L, η_H	Advance revenue guarantee offered to the suppliers L and H, respectively.
$p_L; p_H$	Unit prices quoted by the suppliers L and H, respectively.
$q_L; q_H$	Buyer's order allocation decisions for the suppliers L and H, respectively.

3.4. Symmetric Information: Benchmark

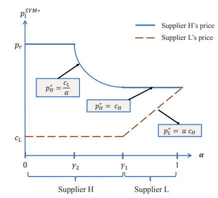
To establish a benchmark, we first consider the case where the true value of α is known to all the parties in the supply chain. The problem, therefore, transforms to a symmetric information Bertrand price competition between suppliers H and L, in which the suppliers would undercut each other until one of them hits her marginal cost. Delegating the details of the analysis to Appendix 5.4, we provide full characterization of equilibrium points under symmetric information in the following proposition.

Proposition 5 *Let $\gamma_1 = c_L/c_H$ and $\gamma_2 = c_L/p_r$ (where $\gamma_2 < \gamma_1$ as $p_r > c_H$). Under symmetric information equilibrium, the buyer orders from supplier L if $\alpha > \gamma_1$, and from supplier H if $\alpha \leq \gamma_1$. For when $\alpha < \gamma_2$, the buyer orders from H at the reserve price (p_r). The complete characterization of equilibrium decisions, profits and costs for the supply chain parties under symmetric information is provided in table 3–2.*

Note that when α is relatively high ($\gamma_1 < \alpha$), i.e., suppliers L and H are relatively similar in terms of their quality scores, supplier L becomes the sole supplier for the buyer due to her cost advantage. This enables her to always set a price p_L such that $\frac{p_L}{\alpha}$ is infinitesimally less than supplier H's marginal cost c_H. Letting $\frac{p_L}{\alpha} = c_H$ and solving for p_L would yield the equilibrium price p_L^* for supplier L. Likewise, when α is between γ_1 and γ_2, supplier H wins

Table 3–2: Equilibrium decisions, profits, and cost under symmetric information.

Regions		$0 < \alpha < \gamma_2$	$\gamma_2 < \alpha < \gamma_1$	$\gamma_1 < \alpha < 1$
Prices (bids)	p_H^*	p_r	$\frac{c_L}{\alpha}$	c_H
	p_L^*	c_L		$c_H\alpha$
Order Alloc.	q_H^*, q_L^*	$Q, 0$		$0, Q$
Suppliers'	π_H^*	$Q(p_r - c_H)$	$Q(\frac{c_L}{\alpha} - c_H)$	0
Profits	π_L^*	0		$Q(\alpha c_H - c_L)$
Buyer's Cost	κ_B^*	Qp_r	$Q\frac{c_L}{\alpha}$	$Qc_H\alpha$



Throughout this chapter, equilibrium profits/costs and decision variables are annotated with asterisks.

and her equilibrium price can be characterized in a similar fashion. Finally, when α is less than γ_2, then supplier H would always win by charging infinitesimally less than the reserve price. In the following corollary, we characterize the sensitivity of equilibrium prices and profits of the suppliers with respect to α and their marginal costs:

Corollary 1 *Under symmetric information setting,*

1. *The equilibrium price and profit of supplier H (p_H^*, π_H^*) (weakly) decrease in α, while those of supplier L (i.e., p_L^* and π_L^*) (weakly) increase in α.*
2. *By an increase in c_L for a fixed c_H, both suppliers L and H weakly increase their bid prices.*

The first part of this corollary in fact implies that α influences the degree of price competition among suppliers, which in turn affects their prices in equilibrium. The second part shows the impact of suppliers' relative cost efficiency on their pricing decisions in the competition. In the next section, we show that this impact of α will be important in determining the equilibrium points under asymmetric information.

3.5. Asymmetric information

In this section, we analyze the equilibria under asymmetric information setting where the true value of α is known only to the buyer. We also assume that any information shared by the buyer regarding the true value of α is non-verifiable by the suppliers. The following Lemma implies that in the absence of a credible signal, the buyer would always have incentive to distort the true value of α:

Lemma 3 *There is no separating equilibrium under which the buyer discloses the true value of α to the suppliers.*

The above result is an artefact of Corollary 1. Recall that the buyer can indirectly influence the equilibrium price of the auction by manipulating the value of α. Hence, in order to credibly share the true value of α with the suppliers, the buyer would need a costly signal. In this chapter, we consider a commitment contract ("revenue guarantee"), which is commonly used in the context of reverse-auctions. Basically, this contract provides supplier i with a guarantee of minimum level of revenue that is expressed as a pre-determined percentage η_i of total cost of procuring demand Q at some fixed reserve price p_r. More specifically, under such contract, the supplier i's revenue would be equal to $\eta_i Q p_r$ if she loses the auction. If she wins the auction, then her revenue would be either $p_i Q(1-\eta_{-i})$ or $\eta_i Q p_r$, whichever is the higher[6] . Taking into account the marginal cost, one can write down the profit expressions for suppliers H and L under revenue guarantee contract as follows:

$$(\pi_H, \pi_L) = \begin{cases} \eta_H Q(p_r - c_H), \max\left[Q(1-\eta_H)p_L, Q\eta_L p_r\right] - Q(1-\eta_H)c_L & \text{if supplier } L \text{ wins, i.e., } \frac{p_L}{p_H} < \alpha \ \& \ p_L \leq p_r, \\ \max\left[Q(1-\eta_L)p_H, Q\eta_H p_r\right] - Q(1-\eta_L)c_H, \eta_L Q(p_r - c_L) & \text{if supplier } H \text{ wins, i.e. } \frac{p_L}{p_H} \geq \alpha \ \& \ p_L \leq p_r. \end{cases}$$

Note that suppliers H and L's profits depend on not only their bid prices p_H and p_L but also the guaranteed revenue levels η_H and η_L. Because of this dependency, the reverse auction with revenue guarantee contracts leads to a costly signalling game (Fudenberg and Tirole 1991), where the buyer can potentially share α with the suppliers in a credible fashion. As in any signalling game, an equilibrium can be of three types: (i) pooling, where $\eta_i(\alpha) = \eta_i$ for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, (ii) separating, where $\eta_i(\alpha) \neq \eta_i(\alpha')$ for all $\alpha \neq \alpha'$, or (iii) semi-separating, where $\eta_i(\alpha)$ satisfies pooling and separating conditions for different subsets of α. In what follows, we analyze each type separately.

3.5.1 Pooling Equilibrium: No information sharing

Note that in a pooling equilibrium, $\eta_i(\alpha) = \eta_i$ for all α, where $0 \leq \eta_i \leq 1$. In other words, the buyer provides the same level of revenue guarantee regardless of the true value of α. Therefore, suppliers' a-posterior beliefs are same as their a-priori beliefs. In this

[6] Throughout this chapter, we use the standard game theory notation to represent the index of supplier i's opponent with $-i$.

situation, suppliers' bidding prices will be solely based on their prior beliefs. As it happens in a typical signalling game, we obtain multiple pooling equilibria by varying the value of revenue guarantee η_i from 0 to 1. However, we can eliminate all but the one with $\eta_i = 0$ by employing the least cost refinement argument that is commonly used in signalling literature for equilibrium refinement purposes. More specifically, under a constant but non-zero revenue guarantee $\eta_i > 0$, we can always show that supplier i will never offer a bid, which is smaller than p_i^{low}, where

$$p_i^{low} = c_i + \tfrac{\eta_i}{1-\eta_{-i}}(p_r - c_i) \text{ for } i = L \text{ and } H$$

Note that there is no incentive for the supplier i to offer a bid price less than p_i^{low}, otherwise, she would earn more by simply charging more than her opponent and delivering only the guaranteed portion η_i. This suggests that among all the pooling equilibria, where the buyer sets a constant revenue guarantee for all α, the one with $\eta_i = 0$ for both $i = L$ and H is the *least costly pooling equilibrium* for the buyer. Therefore, throughout this chapter, we refine all the pooling equilibria and focus only on the least costly one. Given $\eta_i = 0$ for both supplier $i = L, H$, we can then characterize the price competition between supplier L and H under asymmetric quality score information as follows.

Lemma 4 *Under the pooling equilibrium (PE), the price competition between suppliers leads to one of the following four different equilibrium prices:*

- *PE-1: Interior solution of* $p_H^{int} = \frac{\overline{\alpha}c_H + \sqrt{\overline{\alpha}^2 c_H^2 + 8\underline{\alpha}c_H c_L}}{4\underline{\alpha}}$ *and* $p_L^{int} = \frac{(\overline{\alpha}^2 c_H + \overline{\alpha}\sqrt{\overline{\alpha}^2 c_H^2 + 8\underline{\alpha}c_H c_L} + 4\underline{\alpha}c_L)}{8\underline{\alpha}}$*;*
- *PE-2: Boundary solution of* $p_H = p_r$ *and* $p_L = max(\frac{\overline{\alpha}p_r + c_L}{2}, c_L)$*;*
- *PE-3: Boundary solution of* $p_H = c_H$ *and* $p_L = c_H\underline{\alpha}$*;*
- *PE-4: Boundary solution of* $p_H = \min(\frac{c_L}{\bar{\alpha}}, p_r)$ *and* $p_L = c_L$*.*

The above lemma shows that except for the case of PE-1, the price competition under pooling equilibrium leads to a boundary solution. Under these boundary cases, similar to Bertrand competition under symmetric information, suppliers undercut each other until one of them hits her marginal cost. On the other hand, in the case of PE-1, the price competition under asymmetric information leads to an interior equilibrium pair (p_H^{int}, and p_L^{int}), which is sustained by the inherent uncertainty regarding the quality score among the suppliers (as opposed to price-undercutting forces). Therefore, the equilibrium prices in PE-1 depend on both lower and upper ranges of relative QS information uncertainty (i.e., $\underline{\alpha}$, and $\overline{\alpha}$).

Given the possible equilibrium bid prices, we can now characterize the regions for $\underline{\alpha}$, and $\overline{\alpha}$ under which each one of the above equilibria is sustained as a unique pooling equilibrium.

Proposition 6 *Complete characterization of equilibrium decisions (prices and quantities), and pay-offs (profits and costs) for the supply chain parties under pooling equilibrium for each case is provided in Table 3–3.*

There are two take-aways from the above Proposition.

- When the degree of QS uncertainty is relatively small, then either PE_3 or PE_4 appears in the equilibrium. This case leads to an intensive price competition, in which the suppliers sustain the equilibrium only by undercutting each other's offers. Similar to Bertrand competition under symmetric information, this in turn results in an equilibrium where one of the suppliers undercuts her opponent's offer at the marginal cost. There is however a crucial difference between equilibria under symmetric and asymmetric information scenarios due to the fact that under the latter scenario, the suppliers do not know the exact value of QS. Hence, a supplier cannot decide on the exact price that would undercut her opponent's offer at the marginal cost. The equilibrium analysis shows that when the range of uncertainty on QS (i.e., $\overline{\alpha}-\underline{\alpha}$) is sufficiently small, the winner sets a bidding price that would undercut her opponent even under the worst-case scenario. For example, in region PE_3, in equilibrium, the winner is supplier L, and she undercuts supplier H assuming that her QS is equal to $\underline{\alpha}$. Note that under this *worst-case* bidding strategy, supplier L still wins the auction even if her true QS turns out to be higher than $\underline{\alpha}$.
- As the range of uncertainty on QS uncertainty increases, then the worst-case bidding strategy becomes very costly for the suppliers. Therefore, they instead use *expected-case* bidding strategy, under which each maximizes the likelihood of winning the auction

multiplied by the bid price assuming that true value of QS is distributed uniformly between $\underline{\alpha}$ and $\overline{\alpha}$. Consequently, either PE_1 or PE_2 appears in the equilibrium. Also, because of the expected-case bidding strategy, the winner of the auction is determined based on not only bids but also the true value of QS. Namely, under both PE_1 and PE_2, *ceteris paribus*, supplier L wins the auction if her QS is larger than a threshold (γ_1 for PE_1 and γ_2 for PE_2), and supplier H wins otherwise.

Given these observations, we conduct sensitivity analyses first in Proposition 7 about how the equilibrium bid prices change with respect to system parameters such as costs, i.e., c_L, c_H, and lower and upper bounds of QS, i.e., $\overline{\alpha}$ and $\underline{\alpha}$, and then in Proposition 8 about how they compare with respect to their symmetric information counterparts.

Proposition 7 *Sensitivity analyses with respect to the effect of parameters on the suppliers' optimal prices in pooling equilibria lead to the following.*

1. ***Effect of the Marginal Costs:*** *As the marginal cost of supplier L converges to that of supplier H, equilibrium bid prices of both suppliers increase and the regions for PE_4 increase while those for PE_3 decrease.*
2. ***Effect of QS Uncertainty:*** *In regions for PE_1 and PE_2, the equilibrium bid prices p^*_H and p^*_L increase in $\overline{\alpha}$ (for a fixed $\underline{\alpha}$) and decrease in $\underline{\alpha}$ (for a fixed $\overline{\alpha}$). However, in regions PE_3 and PE_4, the bid prices decrease in $\overline{\alpha}$ (for a fixed $\underline{\alpha}$) and weakly increase in $\underline{\alpha}$ (for a fixed $\overline{\alpha}$).*

The first part of Proposition 7 shows that the equilibrium bid prices increase as the suppliers become similar in terms of their marginal costs. Furthermore, we also compare the extent of increase in supplier L's bid price to that of supplier H, and find that supplier L increases her bid price more than supplier H does (e.g. if supplier L has a %10 increase, supplier H has %8 increase). This implies that an increase in the degree of cost homogeneity among the suppliers does not lead to higher profit for supplier L, which partly reflects itself in the form of region PE_3 becoming smaller. This result is consistent with the symmetric information setting.

Figure 3–2: Equilibrium characterization under asymmetric information: Pooling Equilibrium.

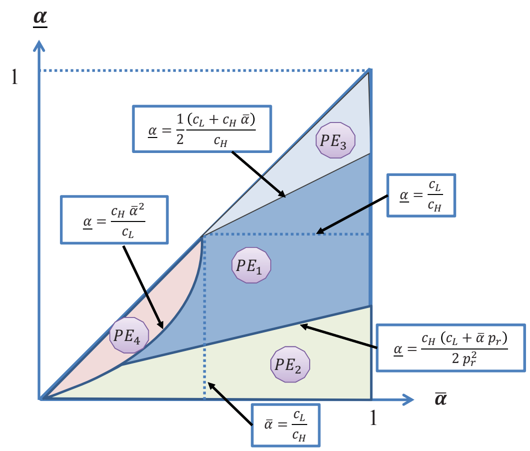


Table 3–3: Equilibrium characterization under asymmetric information: Pooling Equilibrium.

Regions		$(\underline{\alpha},\overline{\alpha}) \in PE_1$		$(\underline{\alpha},\overline{\alpha}) \in PE_2$		$(\underline{\alpha},\overline{\alpha}) \in PE_3$	$(\underline{\alpha},\overline{\alpha}) \in PE_4$
Range of α		$\alpha \leq \gamma_1$	$\alpha > \gamma_1$	$\alpha \leq \gamma_2$	$\alpha > \gamma_2$	$\underline{\alpha} \leq \alpha \leq \overline{\alpha}$	$\underline{\alpha} \leq \alpha \leq \overline{\alpha}$
Prices	p_H^*	$\frac{\overline{\alpha}c_H+\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}}{4\underline{\alpha}}$		p_r		c_H	$\min(\frac{c_L}{\overline{\alpha}}, p_r)$
	p_L^*	$\frac{(\overline{\alpha}^2c_H+\overline{\alpha}\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}+4\underline{\alpha}c_L)}{8\underline{\alpha}}$		$max(\frac{\overline{\alpha}p_r+c_L}{2}, c_L)$		$c_H\underline{\alpha}$	c_L
Alloc.	$q_H^*,\ q_L^*$	$Q, 0$	$0, Q$	$Q, 0$	$0, Q$	$0, Q$	$Q, 0$
Suppliers'	π_H^*	$Q(\frac{\overline{\alpha}c_H+\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}}{4\underline{\alpha}} - c_H)$	0	$Q(p_r - c_H)$	0	0	$Q(\min(\frac{c_L}{\overline{\alpha}}, p_r) - c_H)$
Profits	π_L^*	0	$\frac{(\overline{\alpha}^2c_H+\overline{\alpha}\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}+4\underline{\alpha}c_L)}{8\underline{\alpha}} - c_L)$	0	$Q(max(\frac{\overline{\alpha}p_r+c_L}{2}, c_L) - c_L)$	$Q(c_H\underline{\alpha} - c_L)$	0
Cost	κ_B^*	$Q\frac{\overline{\alpha}c_H+\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}}{4\underline{\alpha}}$	$Q\frac{(\overline{\alpha}^2c_H+\overline{\alpha}\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}+4\underline{\alpha}c_L)}{8\underline{\alpha}}$	Qp_r	$Q(max(\frac{\overline{\alpha}p_r+c_L}{2}, c_L))$	$Qc_H\underline{\alpha}$	$Q\min(\frac{c_L}{\overline{\alpha}}, p_r)$

Let $\gamma_1 = \frac{(\overline{\alpha}^2c_H+\overline{\alpha}\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}+4\underline{\alpha}c_L)}{2(\overline{\alpha}c_H+\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L})}$ and $\gamma_2 = \frac{max(\frac{\overline{\alpha}p_r+c_L}{2}, c_L)}{p_r}$ for PE-1 and PE-2, respectively.

In equilibrium, $\eta_i^* = 0$ for $i \in \{H, L\}$. For this equilibrium to be sustainable, we assume suppliers will not update their belief if they observe out of equilibrium signals, i.e. they continue to believe $\alpha \in \mathcal{U}(\underline{\alpha}, \overline{\alpha})$ if $\eta_i > 0$.

The second part of Proposition 7 addresses the effect of information asymmetry in relative QS on the suppliers' bid prices. Reducing uncertainty generally leads to lower bid prices from both suppliers as long as the range of uncertainty in QS is sufficiently large. On the other hand, when the range of uncertainty is low, reducing uncertainty further would lead to higher bid prices. In fact, for each α, there are thresholds for $\underline{\alpha}$ and $\overline{\alpha}$ at which the equilibrium bid prices are at their lowest possible levels under asymmetric information scenario. The following proposition explores these thresholds and identify the circumstances where asymmetric information results in lower bid prices than symmetric information.

Proposition 8 *The winning price under the pooling equilibrium is lower than that under the symmetric information for all values of* $\alpha \in [\underline{\alpha}, \overline{\alpha}]$*, where the difference between* $\overline{\alpha}$ *and* $\underline{\alpha}$ *is sufficiently small, i.e.,* $\overline{\alpha} - \underline{\alpha}$ *is smaller than* $\frac{\overline{\alpha}(c_L - c_H\overline{\alpha})}{c_L}$ *if* $\overline{\alpha} \leq \frac{c_L}{c_H}$*, and* $\frac{c_H\overline{\alpha} - c_L}{2c_H}$ *otherwise.*

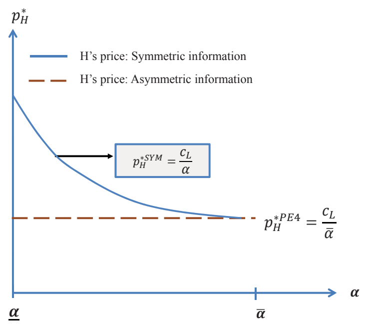
Proposition 8 is illustrated in Figure 3–3. Left panel corresponds to $\overline{\alpha} \leq c_L/c_H$, i.e., the relative QS of supplier L is low, whereas the right panel corresponds to the opposite case, i.e., $\overline{\alpha} > c_L/c_H$. Recall that in both cases, suppliers use worst-case bidding strategy under pooling equilibria, which results in bid prices that are lower than those under symmetric information. Hence, without further investigation, we can conclude that if the difference between $\overline{\alpha}$ and $\underline{\alpha}$ is sufficiently small, the buyer is always better off by keeping QS information private information from the suppliers in order to induce them to apply the worst-case bidding strategy.

3.5.2 Separating Equilibrium

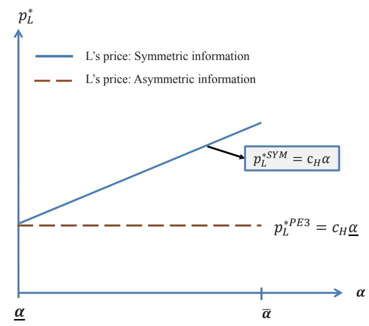
Under a separating equilibrium, by its definition, the buyer must have no incentive to deviate from telling the true value of α, and consequently, suppliers L and H would be able to correctly infer the true value of α, and act accordingly. In order to achieve the consistency between buyer's and the suppliers' decisions, the resulting *Perfect Bayesian Equilibrium* must meet the following two requirements.

First, the buyer's advance guarantee $\eta_i(\alpha)$ must be a one-to-one mapping between the true value of α and the level of guarantee. Technically speaking, this means that if $\alpha \neq \alpha'$,

Figure 3–3: The impact of low uncertainty on the equilibrium prices under symmetric information vs. pooling equilibrium.



(a) **Supplier H's price;** $\frac{c_H \overline{\alpha}^2}{c_L} \leq \underline{\alpha} < \overline{\alpha} < \frac{c_L}{c_H}$



(b) **Supplier L's price;** $\overline{\alpha} > \frac{c_L}{c_H}$ **and** $\underline{\alpha} \geq \frac{c_L + c_H \overline{\alpha}}{2 c_H}$

then $(\eta_H(\alpha), \eta_L(\alpha)) \neq (\eta_H(\alpha'), \eta_L(\alpha'))$. This ensures that the buyer sends different signals for different α information. Suppliers then correctly infer the true value of α when they observe a pair of $(\eta_H(\alpha), \eta_L(\alpha))$. Second, the choice of guarantees $\eta_H(\alpha)$ and $\eta_L(\alpha)$ should be incentive compatible for the buyer so that the buyer has no incentive to deviate from the equilibrium. To validate if these two requirements are satisfied in the equilibrium, we first characterize the best response of the suppliers given $\eta_H^*(\alpha)$ and $\eta_L^*(\alpha)$. We then verify that the buyer has no incentive to deviate from equilibrium $\eta_i^*(\alpha)$, $i \in \{H, L\}$.

The next lemma characterizes the first part, i.e., the suppliers' optimal bids and the equilibrium cost and profits given that the buyer offers $\eta_H^*(\alpha)$ and $\eta_L^*(\alpha)$.

Lemma 5 *Assume* $\gamma_1 = \frac{p_L^{low}}{p_H^{low}} < 1$ *,*$\gamma_2 = \frac{p_L^{low}}{p_r}$ $(\gamma_2 < \gamma_1)$*, where* $p_i^{low} = c_i + \frac{\eta_i}{1-\eta_{-i}}(p_r - c_i)$ *for* $i = L$ *and* H*. In a separating equilibrium, the suppliers would charge the equilibrium bids characterized in Table 3–4.*

This lemma follows directly from the fact that after receiving the signal and correctly inferring α, the suppliers bidding strategy would be very similar to the symmetric information case except that the minimum price would be the price implied by revenue guarantees p_i^{low} instead of their marginal costs c_i for $i = H, L$. Consider for example the case where the buyer offers the advance revenue guarantee only to supplier H, i.e. $\eta_L = 0$. If the supplier L's QS is very low, i.e., $\alpha \leq \gamma_1$, then there is no need for supplier H to lower her price in order to outbid

Table 3–4: Best responses of parties after receiving the signal: Separating Equilibrium.

Equilibrium decisions		$(\eta_H, \eta_L) \in [0,1]^2, \quad 0 \leq \eta_H + \eta_L \leq 1$		
		$0 < \alpha \leq \gamma_2$	$\gamma_2 \leq \alpha \leq \gamma_1$	$\gamma_1 \leq \alpha \leq 1$
Prices (bids)	p_H^*	p_r	$\frac{p_L^{low}}{\alpha}$	p_H^{low}
	p_L^*	p_L^{low}		αp_H^{low}
Order Alloc	q_H^*, q_L^*	$(1-\eta_L)Q, \eta_L Q$		$\eta_H Q, (1-\eta_H)Q$
Suppliers' Profits	π_H^*	$Q(1-\eta_L)(p_r - c_H)$	$Q(1-\eta_L)(\frac{p_L^{low}}{\alpha} - c_H)$	$\eta_H Q(p_r - c_H)$
	π_L^*	$\eta_L Q(p_r - c_L)$		$(1-\eta_H)Q(\alpha p_H^{low} - c_L)$
Buyer's Cost	κ_B^*	Qp_r	$Q\left(\eta_L p_r + (1-\eta_L)\frac{p_L^{low}}{\alpha}\right)$	$Q\left(\eta_H p_r + (1-\eta_H)\alpha p_H^{low}\right)$

† Note that $p_i^{low} = c_i + \frac{\eta_i}{1-\eta_{-i}}(p_r - c_i)$ for $i = L$ and H if suppliers L and H are offered a minimum revenue of $\eta_L Q p_r$ and $\eta_H Q p_r$, respectively.

supplier L and win the entire order from the buyer. Hence, in this case, the equilibrium will be exactly same as the one under symmetric information ($\frac{c_L}{\alpha}$). When the QS of supplier L becomes closer to that of supplier H, i.e., $\gamma_1 \leq \alpha \leq 1$, supplier H needs to engage in an intensive price competition with supplier L in order to win the entire order allocation from the buyer. There is however another option for supplier H if she is content with winning only the partial order. Namely, in this case, she would simply offer $p_H^{low} = c_H + \frac{\eta_H}{1-\eta_L}(p_r - c_H)$, receive $\eta_H(\alpha)Q$ from the buyer and share the remaining $(1-\eta_H(\alpha))Q$ with supplier L. Similar case holds when the buyer offers the advance revenue guarantee only to supplier L (instead of supplier H). The main difference in this case is that the buyer's order is shared among the suppliers when the QS of supplier L is sufficiently lower than that of supplier H because this is the case when supplier L has to lower her price in order to outbid supplier H if she wants to win the total order. Since this becomes more costly for supplier L as α decreases, she would rather prefer to charge $p_L^{low} = c_L + \frac{\eta_L}{1-\eta_H}(p_r - c_L)$, receive $\eta_L(\alpha)Q$ from the buyer and share the remaining $(1-\eta_L(\alpha))Q$ with supplier H when $\alpha < \gamma_1$.

Given the suppliers' best response strategies, there still remains to show that the buyer must have no incentive to misreport the true value of α. In other words, given α, the buyer must prefer or at least be indifferent to offer the advance guarantees of $[\eta_H(\alpha), \eta_L(\alpha)]$ to any other guarantee of $[\eta_H(\alpha'), \eta_L(\alpha')]$, where $\alpha' \neq \alpha$. The necessary condition for this requirement is expressed in terms of the buyer's total cost function, denoted by κ_B, as follows:

$$\kappa_B(\eta_H(\alpha), \eta_L(\alpha), p_H(\alpha), p_L(\alpha), \alpha) \leq \kappa_B(\eta_H(\alpha'), \eta_L(\alpha'), p_H(\alpha'), p_L(\alpha'), \alpha) \quad \forall \alpha, \alpha' : \alpha \neq \alpha',$$

For the regions of α in which the total cost function is smooth (i.e., differentiable in the first-order sense) the above condition can be rewritten *locally* as follows:

$$\frac{\partial \kappa_B(\eta_H(\alpha'), \eta_L(\alpha'), p_H(\alpha'), p_L(\alpha'), \alpha)}{\partial \alpha'} \Big|_{(\alpha'=\alpha)} = 0$$

We can use the above condition to derive non-linear ordinary differential equations (ODE) to characterize incentive-compatible advance revenue guarantee $\eta_i(\alpha)$ for supplier $i \in \{L, H\}$ – refer to the Appendix 5.4 for the ODEs. Also, in order for the above equations to possess the feasible solutions, we need to impose appropriate boundary conditions at $\underline{\alpha}$ and $\overline{\alpha}$. Delegating the details of the analysis to the Appendix 5.4, in the following proposition, we provide the conditions under which the above ODEs yield equilibrium solutions as well as characterize the equilibria in the closed-form.

Proposition 9 1. *Existence of Separating Equilibrium: There exists incentive-compatible solutions only when $\frac{c_L}{c_H} \leq \overline{\alpha} \leq 1$ for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$.*

2. *Characterization of Separating Equilibrium: Among all separating equilibria, the least-costly one to the buyer is characterized as follows:*

$$\eta_L(\alpha) = 0 \textit{ for all } \alpha \in [\underline{\alpha}, \overline{\alpha}] \textit{ and } \eta_H(\alpha) = \begin{cases} f(\tilde{\alpha}) & \underline{\alpha} \leq \alpha < \tilde{\alpha} \\ f(\alpha) & \max(\underline{\alpha}, \tilde{\alpha}) \leq \alpha \leq \overline{\alpha} \end{cases}$$

where $f(\alpha) = \frac{\alpha p_r + p_r - 2\alpha c_H - \sqrt{\alpha^2 p_r^2 + 2\alpha p_r^2 + p_r^2 - 4\alpha c_H p_r - 4\alpha c_H \overline{\alpha}(p_r - c_H)}}{2\alpha(p_r - c_H)}$ *and* $\tilde{\alpha} = \frac{(p_r - c_L)c_L}{c_H \overline{\alpha} p_r - \overline{\alpha} c_H^2 + c_H p_r - p_r c_L}$.
Observing upon $\eta_H(\alpha)$, the suppliers update their belief on α according to the following function:

$$\alpha(\eta_H) = \begin{cases} \alpha, & \eta_H \in [\eta_H(\overline{\alpha}), \min(f(\underline{\alpha}), f(\tilde{\alpha}))] \\ \sim \textit{Uniform}(\underline{\alpha}, \tilde{\alpha}), & \eta_H > \min(f(\underline{\alpha}), f(\tilde{\alpha})) \end{cases}$$

and set their bidding prices as follows:

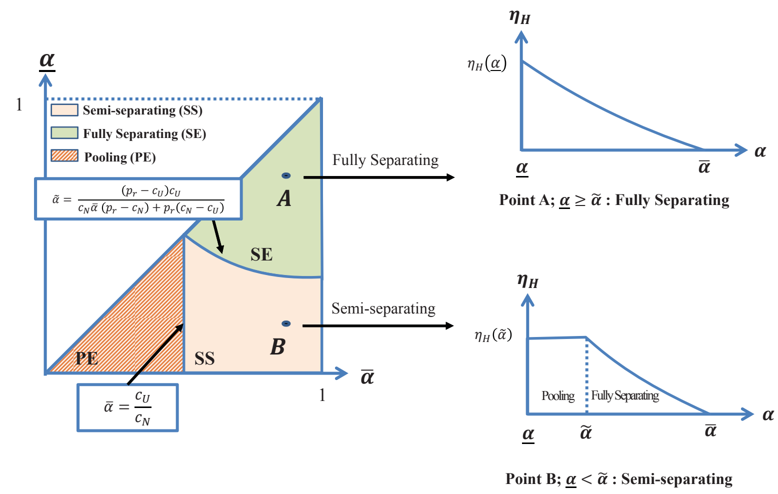
$$(p_H^*, p_L^*) = \begin{cases} (c_H + \eta_H(p_r - c_H), \alpha(c_H + \eta_H(p_r - c_H))) & \eta_H \in [\eta_H(\overline{\alpha}), \min(f(\underline{\alpha}), f(\tilde{\alpha}))] \\ (c_H + \eta_H(\tilde{\alpha})(p_r - c_H), c_L) & \eta_H > \min(f(\underline{\alpha}), f(\tilde{\alpha})) \end{cases}$$

Note that a separating equilibrium is sustainable only when upper bound of true value of α is sufficiently large, i.e., $\frac{c_L}{c_H} \leq \overline{\alpha}$. Depending on whether the lower bound $\underline{\alpha}$ is greater than $\tilde{\alpha}$ or not, the equilibrium is either fully separating or semi-separating. In the first case, the buyer can share the true value of α with the suppliers in credible fashion for all values of $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, whereas in the second case, he signals true value only when α is on the upper region. Both fully and semi-separating equilibria are illustrated in Figure 3–4. The above result also implies that the signalling takes place only when the degree of homogeneity between suppliers (in terms of their quality scores) is not too low.

The rationale behind this comes from the analysis of signalling cost incurred by the buyer when he shares the true value of α via advance revenue guarantee. In the next section, we will see that the signalling cost is driven by two factors: (i) degree of homogeneity among the suppliers in terms of their quality scores and (ii) the degree of information asymmetry between the buyer and the suppliers. Here, we focus on the former and discuss the impact of the latter in the next section.

We know from the symmetric information case, when the quality scores of the suppliers are close to each other, i.e., α is sufficiently high, they engage in an intensive competition. This suggests that the buyer would prefer to share the true value of $\alpha \in [\underline{\alpha}, \overline{\alpha}]$ as long as the upper bound $\overline{\alpha}$ is sufficiently high, i.e., $\overline{\alpha} \geq c_L/c_H$. Proposition 9 also shows that $\eta_L(\alpha) = 0$ for all values of α, which suggests that the buyer would prefer to use η_H (as opposed to η_L) to signal the true value of α. The rationale behind this does indeed relate to the signalling characteristics of η_L and η_H contracts. As shown in Lemma 5, η_L can be used to signal low α values, whereas η_H works for the high α values. But since the intensity of price competition between suppliers decreases and consequently bid prices increase for low α values, signalling

Figure 3–4: Characterization of separating equilibria.



via η_L (as opposed to via η_H) creates highly unfavourable conditions for the buyer, which in turn makes signalling too costly to be sustained in equilibrium.

In the following corollary, we explore the sensitivity of equilibrium contracts and decisions with respect to α. Bid prices' sensitivity with respect to α under separating equilibria follows quite closely to its counterpart under symmetric information. Also, as shown in Figure 3–4, $\eta_H(\alpha)$ decreases in α. This reinforces our finding that as the suppliers become quite similar to each other in terms of their quality scores, suppliers' price competition becomes more intensified. This reduces the need for (hence cost of) signalling from the perspective of the buyer.

Corollary 2 *The equilibrium advance guarantee to supplier H (η_H^*), and Supplier H's price (p_H^{*SE}) decrease in α, whereas Supplier L's price (p_L^{*SE}) increases in α.*

Before we analyze the impact of credible information sharing in our main model, in what follows, we extend our exploration of signalling possibility by relaxing the following two important assumptions one by one:

1. Cost superiority assumption for supplier L, i.e. when $c_H < c_L$,

2. Independence of buyer's cost from suppliers' QS, i.e. when

$$\kappa_B = Q(\frac{p_w}{QS_w})$$

where subindex w corresponds to the winning supplier.

Proposition 10 *When supplier H's marginal cost is less than or equal to the entrant supplier's cost, i.e. $c_H \leq c_L$, the buyer will never be able to truthfully share the true value of α by revenue guarantees to suppliers.*

According to the above proposition, when supplier H is more *cost efficient* than supplier L, separating equilibria become unsustainable. This can be explained mainly by the increased distance in the qualifications of the suppliers. Indeed, supplier H becomes the ideal candidate for the buyer in any situation and this makes supplier H skeptical about the signal that he receives, unless the buyer offers the whole contract at the highest possible price, which makes it very costly to the buyer and unstable from an equilibrium perspective.

Throughout this chapter, we assumed that even though the buyer uses suppliers' QS in his order assignment rule, his cost function is *independent* from the scores that he gives to the suppliers. This assumption can be justified based on the fact that the buyer knows his scores are subjectively assigned and might be untrue in many cases because of his actual limitations in evaluating non-price attributes. In addition, some non-price attributes that the buyer consider in evaluating suppliers' QS are indeed unconnected (or hardly connected) with his final costs. A good example of the latter case may correspond to suppliers' years of experience. Even though the buyer may consider a score for this factor by giving more chance to the more experienced suppliers, in the end, he actually cares about the bidding prices of suppliers as long as they have a minimum acceptable experience. Now, if we relax this assumption, the result completely changes according to Proposition 11.

Proposition 11 *If the buyer considers suppliers' QS as true measures for the extra costs in addition to the bidding prices (i.e. if he incorporates those scores in his cost function), he will not be able to truthfully share suppliers' relative QS with them.*

The buyer will not be able to share suppliers' QS anymore because of the extreme level of interest conflicts between the suppliers and the buyer given the shared value of QS. In a hypothetical scenario where the buyer shares the true value of relative QS with the suppliers, the winning supplier increases her price to a level where her QS-adjusted price is marginally lower than the other supplier. But, this opportunistic action of suppliers indeed makes the buyer almost indifferent between the suppliers as he in this setting considers QS-adjusted price as his final unit cost instead of only price. This overall price increase by the suppliers, therefore, makes the buyer worse off if he decides to share suppliers' relative QS.

3.6. The Impact of Credible Information Sharing

As we raised in the research questions, our ultimate goal is to analyze the impact of sharing QS information with the suppliers on the buyer's cost and suppliers' profits. In order to understand this impact, we first evaluate how it affects the equilibrium decisions of the supply chain parties (i.e., suppliers' prices and buyer's order allocation), and then compare the cost and profits of the buyer and the suppliers, respectively, under pooling or separating equilibria.

3.6.1 On Equilibrium Decisions: Prices and Quantities

We characterize the pooling and separating equilibria in Propositions 6 and 9, respectively. Therefore, in order to evaluate the impact of sharing QS information on equilibrium decisions, we compare the equilibria characterized in these Propositions.

We identify two dimensions of quality score information that affect the comparison between the pooling and separating equilibria: (i) the degree of QS information asymmetry between the buyer and the suppliers (measured by the difference between $\overline{\alpha}$ and $\underline{\alpha}$), and (ii) the degree of homogeneity of the suppliers' QS. First of all, as shown in Proposition 9, the separating equilibrium is not sustainable when $\overline{\alpha}$ is less than c_L/c_H. Excluding this case (denoted as region C_0 in Figure 3–5), in what follows, we compare the equilibrium decisions (prices and quantities) between pooling and separating equilibria:

Proposition 12 *Assuming that $\overline{\alpha} \geq c_L/c_H$, when the degree of information asymmetry is low, i.e., $(\underline{\alpha}, \overline{\alpha}) \in C_1$:*

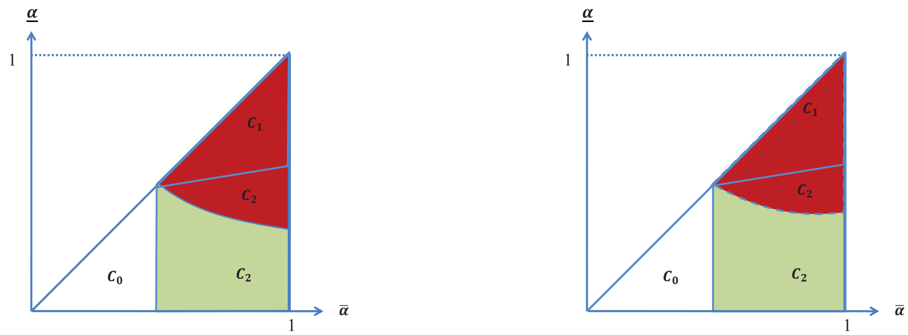
- *The expected equilibrium bid prices are lower under the pooling equilibrium than under the separating equilibrium: $\bar{p}_L^{*PE} < \bar{p}_L^{*SE}$ and $\bar{p}_H^{*PE} < \bar{p}_H^{*SE}$.*
- *The expected equilibrium order quantity to supplier H (resp. L) is lower (resp. higher) under the pooling equilibrium than under the separating equilibrium: $\bar{q}_L^{*PE} > \bar{q}_L^{*SE}$ and $\bar{q}_H^{*PE} < \bar{q}_H^{*SE}$.*

The above Proposition implies that when the degree of QS information asymmetry is low, and $\underline{\alpha}$ is bounded below (region C_1), i.e., the QS of supplier L is similar to that of supplier H, the bid prices under separating equilibrium indeed become larger than those under pooling equilibrium. Furthermore, supplier L becomes the sole winner under pooling equilibrium, whereas she has to share the order with supplier H under separating equilibrium.

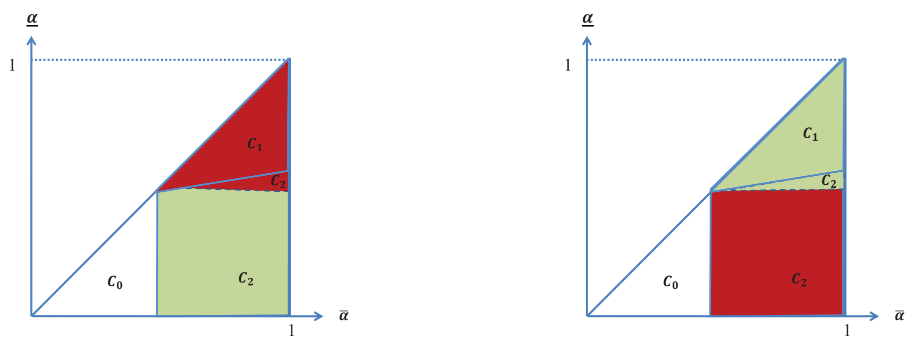
The rationale behind these results relate to the differences between bidding strategies under pooling and separating equilibria. Namely, under a pooling equilibrium, supplier L employs worst-case bidding strategy, i.e., undercuts her opponent assuming that her QS is equal to its lower bound $\underline{\alpha}$. This definitely benefits to the buyer and ensures that supplier L would be the sole winner irrespective of the true α value. However, when the true value of α is signalled by the buyer (via advance revenue guarantee), then supplier L does not have to lower her bid price too much. Hence, the buyer is always better off with a pooling equilibrium in region C_1. There is a caveat for supplier L. Due to the presence of advance revenue guarantee η_H, supplier L has to share the order with supplier H. Therefore, it is not clear whether supplier L is better off with pooling or separating equilibria. In the next subsection, we take into account both equilibrium quantities and prices and evaluate the impact of signalling QS information on supplier L.

Next, we illustrate the impact of information sharing on equilibrium decisions in region C_2, where the difference between $\overline{\alpha}$ and $\underline{\alpha}$ is sufficiently high (see Figure 3–5). Note that as opposed to region C_1, in region C_2, the equilibrium prices under pooling are not guaranteed to be lower than those under separating equilibria. Even though we have closed-form

Figure 3–5: Effects of Pooling vs. Separating on price/quantities in asymmetric information setting.



Regions	Supplier H's price	Supplier L's price
$(\underline{\alpha}, \overline{\alpha}) \in C_1$	$\bar{p}_H^{PE} < \bar{p}_H^{SE}$	$\bar{p}_L^{PE} < \bar{p}_L^{SE}$
$(\underline{\alpha}, \overline{\alpha}) \in C_2$	$\bar{p}_H^{PE} < \bar{p}_H^{SE}$ if $\Delta_{p_H}^{SE1} > 0$; $\bar{p}_H^{PE} \geq \bar{p}_H^{SE}$ o.w.	$\bar{p}_L^{PE} < \bar{p}_L^{SE}$ if $\Delta_{p_L}^{SE1} > 0$; $\bar{p}_L^{PE} \geq \bar{p}_L^{SE}$ o.w.



Regions	Supplier H's quantity	Supplier L's quantity
$(\underline{\alpha}, \overline{\alpha}) \in C_1$	$\bar{q}_H^{PE} < \bar{q}_H^{SE}$	$\bar{q}_L^{PE} > \bar{q}_L^{SE}$
$(\underline{\alpha}, \overline{\alpha}) \in C_2$	$\bar{q}_H^{PE} < \bar{q}_H^{SE}$ if $\Delta_{q_H}^{SE1} > 0$; $\bar{q}_H^{PE} \geq \bar{q}_H^{SE}$ o.w.	$\bar{q}_L^{PE} > \bar{q}_L^{SE}$ if $\Delta_{q_L}^{SE1} < 0$; $\bar{q}_L^{PE} \leq \bar{q}_L^{SE}$ o.w.

Note. The different colored regions in the above figure denote the following impacts of pooling vs. separating equilibria on decision variables: green (light shaded) regions - decision variable lower under pooling; red (dark shaded) regions - decision variable lower under separating; and, white regions - separating equilibria not sustainable. $\Delta_{p_H}^{SE1}$, $\Delta_{p_L}^{SE1}$, $\Delta_{q_H}^{SE1}$, and $\Delta_{q_L}^{SE1}$ are characterized in Appendix 5.4.

expressions for equilibrium prices under both pooling and separating equilibria, it is too complicated to compare these expressions. Through extensive numerical studies, we can derive some general insights. Keeping $\overline{\alpha}$ constant, if we increase the information asymmetry by reducing $\underline{\alpha}$, we observe two things: First, the equilibrium pooling prices become greater than equilibrium separating prices for both suppliers L and H. Secondly, supplier L's expected order allocation under separating equilibria becomes more than that under pooling equilibria.

The rationale behind these observations again comes from the change in bidding strategy. Namely, when the information asymmetry is high, both suppliers L and H use a bidding strategy under pooling equilibrium, where each supplier charge a price that maximizes her own expected profit (as opposed to worst-case profit). This makes bid prices under pooling equilibrium higher than those under separating equilibrium. That said, it turns out that supplier L receives smaller order quantity under pooling equilibrium than under separating equilibrium *in expected sense* even though she has to share the order with supplier H under separating equilibrium.

3.6.2 On Equilibrium Profits/Costs

In previous subsection, we analyzed the impact of information sharing on equilibrium decisions. In order to evaluate the full impact on supply chain parties' payoffs, we need to consider equilibrium prices and quantities together. In what follows, we establish this by taking into account regions one at a time. As before, region C_0 is excluded from the analysis because separating equilibria are not sustainable in this region. Therefore, we first consider region C_1:

Proposition 13 *When the degree of information asymmetry is low, and the degree of homogeneity is high, i.e., $(\underline{\alpha}, \overline{\alpha}) \in C_1$, then:*

- *The equilibrium profit of supplier H is lower under pooling than under the separating equilibria, i.e., $\overline{\pi}_H^{*PE} \leq \overline{\pi}_H^{*SE}$.*
- *The equilibrium profit of supplier L is lower under pooling than under separating equilibria if and only if $c_L \geq \overline{c}_L = \frac{(2c_H - p_r)(\sigma_{\alpha,\eta_H} + \mu_\alpha \mu_{\eta_H}) - c_H \frac{\overline{\alpha} - \underline{\alpha}}{2} - (p_r - c_H)E(\eta_H^2 \alpha)}{\mu_{\eta_H}}$.*
- *The unit cost of the buyer and the total supply chain' cost are lower under pooling than under separating equilibria, i.e., $\overline{\kappa}_B^{*PE} \leq \overline{\kappa}_B^{*SE}$ and $\overline{\kappa}_{SC}^{*PE} \leq \overline{\kappa}_{SC}^{*SE}$.*

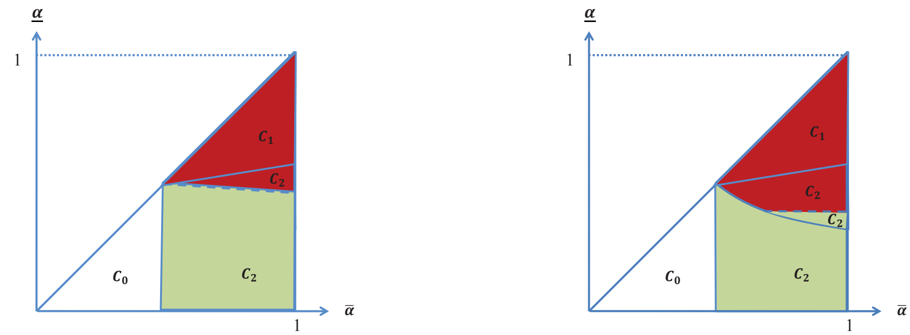
We discuss the above results for each party below:

- First of all, supplier H is always better off with a separating equilibrium. This is because irrespective of the true value of α, she always loses out the order allocation against supplier L under pooling equilibria, whereas under separating equilibria, there exists at least some cases under which she shares the order allocation with supplier L. Hence, she would always prefer to receive QS information in this case.

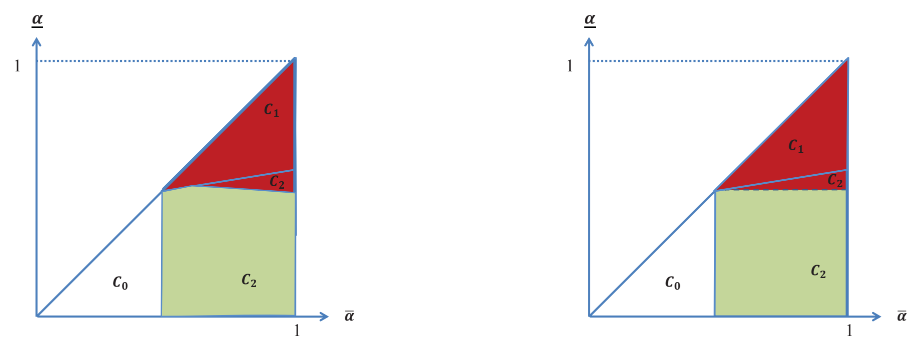
- Second, the buyer and the total supply chain are always better off with a pooling equilibrium. The buyer is better off because of the lower price levels under pooling equilibria due to a more intensive price competition among the suppliers. Interestingly, no information sharing would also benefit to the total supply chain because sharing QS information leads to inefficiency as it shifts some of the order to less efficient supplier (i.e., supplier H).
- Finally, supplier L is generally better off with a pooling equilibrium (unless her marginal cost is sufficiently high). This is because even though she can increase her bid price under a separating equilibrium, she would need to share the buyer's order with supplier H. The decrease in quantity under the separating equilibrium leads to a lower total cost for supplier L as well. When she is highly cost efficient this cost saving under the separating equilibrium is not very significant and since the reduction in order quantity offsets the increase in the bid prices, she would be eventually worse off with the separating equilibrium.

  On the other hand, when supplier L's marginal cost is sufficiently high (low cost efficiency), sharing the order with supplier H does not hurt too much. Therefore, the benefit gained due to the increase in the bid price exceeds the cost due to the reduction in total order received by supplier L. To summarize, if $c_L \geq \bar{c}_L$, supplier L is better off with the separating equilibrium. Note that the threshold for supplier L's marginal cost $\bar{c}_L$ involves not only first order statistics of the signal (i.e., the mean of advance revenue guarantee μ_{η_H}) but also the second order statistics (i.e., its covariance σ_{α,η_H} with a-priori distribution on α). Delegating the details to Appendix 5.4, the rationale behind this relates to the fact that η_H decreases non-linearly in α.

Figure 3–6: Effects of Pooling vs. Separating on supply chain partners' cost/profits in asymmetric information setting.



Regions	Supplier H's profit	Supplier L's profit
$(\underline{\alpha}, \overline{\alpha}) \in C_1$	$\bar{\pi}_H^{PE} < \bar{\pi}_H^{SE}$	$\bar{\pi}_L^{PE} < \bar{\pi}_L^{SE}$ if $c_L > \bar{c}_L$; $\bar{\pi}_L^{PE} \geq \bar{\pi}_L^{PE}$ o.w.
$(\underline{\alpha}, \overline{\alpha}) \in C_2$	$\bar{\pi}_H^{PE} < \bar{\pi}_H^{SE}$ if $\Delta_{\pi_H}^{SE1} > 0$; $\bar{\pi}_H^{PE} \geq \bar{\pi}_H^{SE}$ o.w.	$\bar{\pi}_L^{PE} < \bar{\pi}_L^{SE}$ if $\Delta_{\pi_L}^{SE1} > 0$; $\bar{\pi}_L^{PE} \geq \bar{\pi}_L^{SE}$ o.w.



Regions	Buyer's cost	Total supply chain cost
$(\underline{\alpha}, \overline{\alpha}) \in C_1$	$\bar{\kappa}_B^{PE} < \bar{\kappa}_B^{SE}$	$\bar{\kappa}_{SC}^{PE} < \bar{\kappa}_{SC}^{SE}$
$(\underline{\alpha}, \overline{\alpha}) \in C_2$	$\bar{\kappa}_B^{PE} < \bar{\kappa}_B^{SE}$ if $\Delta_{\kappa_B}^{SE1} > 0$; $\bar{\kappa}_B^{PE} \geq \bar{\kappa}_B^{SE}$ o.w.	$\bar{\kappa}_{SC}^{PE} < \bar{\kappa}_{SC}^{SE}$ if $\Delta_{\kappa_{SC}}^{SE1} > 0$; $\bar{\kappa}_{SC}^{PE} \geq \bar{\kappa}_{SC}^{SE}$ o.w.

Note. The different colored regions in the above figure denote the following impacts of pooling vs. separating equilibria on decision variables: green (light shaded) regions - costs/profits lower under pooling; red (dark shaded) regions - costs/profits lower under separating; and, white regions - separating equilibria not sustainable. $\Delta_{\pi_H}^{SE1}$, $\Delta_{\pi_L}^{SE1}$, $\Delta_{\kappa_B}^{SE1}$, $\Delta_{\kappa_{SC}}^{SE1}$, and $\bar{c}_L$ are characterized in Appendix 5.4.

Next, we consider the region C_2 where the degree of information asymmetry is high. Using closed-form characterizations of supply chain parties' payoffs under pooling and separating equilibria, we illustrate in Figure 3–6 when each stake-holder is better off with separating equilibria, i.e., sharing QS information. In consistent with the previous subsection, the increase in the degree of information asymmetry flips the impact of sharing QS on the payoffs for all the supply chain parties. For example, when $(\underline{\alpha}, \overline{\alpha}) \in C_1$, suppliers are generally better off with a separating equilibrium (except for supplier L under certain conditions as illustrated in Proposition 7). However, as $\underline{\alpha}$ decreases (while $\overline{\alpha}$ is kept constant), i.e., as the degree of information asymmetry increases, the suppliers prefer pooling over separating equilibria, whereas both buyer and total supply chain prefer separating over pooling equilibria.

The rationale behind these observations is mainly due to the nature of the suppliers' bidding strategy under pooling equilibria. When the QS information is not shared, each supplier chooses a bid that maximizes her expected profit. As we have shown in the previous section, this reduces the degree of price competition among the suppliers in region C2, which results in high bid prices. Therefore, it is preferable for the buyer to signal the QS information (via advance revenue guarantee) in order to prevent the suppliers from adopting the expected-profit maximizing bidding strategy and engage them instead in Bertrand-type undercutting price competition. However, advance revenue guarantee contract is a costly signalling device for the buyer. Nevertheless, as shown in Figure 3–6, the buyer still benefits from sharing QS information with the suppliers even if it entails a costly signal.

3.7. Extension 1: Multiple Suppliers in the Auction

In order to further explore the effect of *competition* on the equilibrium decisions and buyer's choice of information sharing, in this section, we extend the main model in §3.3 to the case where there are more than two suppliers. In order to simplify the analysis, we assume that there are $m + 1$ suppliers, m of them are of L-type and one of them is of H-type. We define the relative quality scores of all the L-type suppliers with respect to H-type supplier. Let α_i denote the relative QS of i^{th} L-type supplier, where $i = 1, \ldots, m$. To be consistent with the main model, we keep all the remaining assumptions stated in §3.3 unchanged. More specifically, we assume that all H- and L-type suppliers share the same marginal costs, i.e., c_H, and c_L, respectively, α_i's are unknown to all the suppliers, and it is a common knowledge that α_i's are uniformly distributed between some publicly known $\underline{\alpha}$ and $\overline{\alpha}$, where $0 \leq \underline{\alpha} \leq \overline{\alpha} \leq 1$. That means, the suppliers face potentially m-dimensional asymmetric information problem. We can follow same line of equilibrium analysis conducted for the main model. However, in order to avoid the repetition, we focus only on the key differences between the main and extended models.

Symmetric Information Analysis: First, we consider the equilibrium points under *symmetric information* where all the suppliers know the true values of α_i. Without loss of generality, we can assume that L-type suppliers are indexed such that the first L-type supplier has the highest QS, the second L-type supplier has the second highest QS, etc, i.e., $\underline{\alpha} \leq \alpha_m \leq \ldots \alpha_2 \leq \alpha_1 \leq \overline{\alpha}$. Under such setting, we can characterize equilibria under symmetric information via Bertrand price undercutting argument. For instance, when the winner is supplier L with the highest QS, then she would set her bid to undercut all the other suppliers by charging $\min(c_H\alpha_1, \frac{c_L\alpha_1}{\alpha_2})$. The key observation from the above characterization is that even though there are m L-type suppliers with different quality scores, the equilibrium bid price depends only on the first two highest (relative) quality scores, i.e., α_1 and α_2.

Asymmetric Information Analysis: First of all, based on the same argument used under symmetric information case, it suffices for the buyer to signal only the two highest relative quality scores under asymmetric information. In order to explore how the buyer can signal α_1 and α_2 in a credible fashion, we consider the same revenue guarantees η_i, where $i = \{L, H\}$. Recall that the buyer never uses η_L (i.e., $\eta_L = 0$) in a separating equilibrium under two supplier case. However, this result does not extend to the case where there are more than one supplier L. Let $\eta_L(\alpha_1, \alpha_2)$ and $\eta_H(\alpha_1, \alpha_2)$ denote the signals employed by the buyer in a separating equilibrium. In order for these signals to be incentive compatible for the buyer and hence credible for the suppliers, they need to satisfy the following system of *nonlinear partial differential equations* (PDEs):

$$\frac{\partial \kappa_B(\eta_L(\alpha_1', \alpha_2'), \eta_H(\alpha_1', \alpha_2'), \alpha_1, \alpha_2)}{\partial \alpha_i'} \Big|_{(\alpha_i' = \alpha_i)} = 0 \qquad \forall i \in \{1, 2\} \text{ and } \forall (\alpha_1, \alpha_2) \tag{2}$$

where $\kappa_B(\eta_L, \eta_H, \alpha_1, \alpha_2)$ denotes the total cost of the buyer. In order for these PDEs to possess solutions, we also need to impose appropriate boundary conditions. Below proposition characterizes the separating equilibria:

Proposition 14 *Suppose that $m > 1$, and $\underline{\alpha} \leq \alpha_2 < \alpha_1 \leq \overline{\alpha}$. Then, the following is true for a separating equilibrium:*

- *If the relative QS of second-ranked supplier L is low (i.e., $\alpha_2 < \frac{c_L}{c_H}$), it is sufficient for the buyer to signal only the true value of the highest unknown relative quality score, i.e., α_1. This can be done through a revenue guarantee η_H as characterized in Proposition 9.*
- *Otherwise, i.e., $\alpha_2 \geq \frac{c_L}{c_H}$, the buyer can credibly signal the true value of the relative quality score $\frac{\alpha_1}{\alpha_2}$ by the following guarantee η_L to supplier L that satisfies the system of PDEs provided in (2):*

$$\eta_L^*(\alpha_1, \alpha_2) = \frac{c_L(m-1)\left(\frac{\overline{\alpha}}{\underline{\alpha}} - \frac{\alpha_1}{\alpha_2}\right)}{p_r(\frac{\alpha_1}{\alpha_2} - 1) + m(p_r - c_L\frac{\alpha_1}{\alpha_2})}$$

  Observing the value of $\eta_L(\alpha_1, \alpha_2)$, suppliers update their belief on $\frac{\alpha_1}{\alpha_2}$ and set their bidding prices as characterized in Appendix 5.4.

Note that signalling equilibria depend on the true value of the second highest relative quality score α_2. If α_2 is less than c_L/c_H, the second-ranked supplier L can not compete with highest-ranked supplier L and supplier H. Therefore, the buyer would simply need to signal the relative ratio between quality scores of highest-ranked supplier L and supplier H via η_H as characterized in Proposition 5. However, if α_2 is greater than c_L/c_H, then the second-ranked supplier L always outbids supplier H. In this case, instead of signalling α_1 and α_2 separately, the buyer would need to signal only the ratio between α_1 and α_2. This is consistent with the previous case in the sense that it always suffices for the buyer to signal the ratio between the quality scores of the two competing suppliers. Since in this case, the competition effectively takes place between the first- and the second-ranked suppliers L, the above result shows that it suffices to signal their relative quality score α_1/α_2.

Analysis of the impact of information sharing: Here, we only focus on the number of suppliers L and evaluate its impact on the equilibrium prices and the buyer's payoff when he signals the QS information. Note that η_L increases in m (refer to Appendix 5.4 for the details of sensitivity analysis). The rationale behind this is as follows. As m increases, the supplier L with the highest QS becomes more differentiated from the supplier L with an average QS. Therefore, the cost of signal that needs to be sent to the highest-ranked supplier L (as measured by η_L) increases in m. There is also another impact. Namely, the increase in m intensifies the degree of competition among the suppliers, which lowers the equilibrium bid prices. Hence, as the number of suppliers increases, the buyer prefers not to signal the true values of quality scores with the suppliers in expectation that the competitive forces among the suppliers naturally keep the prices down.

3.8. Extension 2: Suppliers' Private Cost Information

In order to analyze the impact of *cost information privacy* on the equilibrium decisions and buyer's choice of information sharing, in this section, we now extend the main model in §3.3 by relaxing suppliers' public cost assumption. In what follows, we summarize the key characteristics of this setting compared to the basic model presented in §3.3.

- To investigate QS information sharing when suppliers have private cost information, we consider a model with two suppliers, among which the buyer holds a simple (first-price, sealed-format) reverse auction. Similar to the basic model, upon receiving the bid prices, the buyer uses a multiplicative generalized price rule to adjust suppliers' bids with their quality scores in order to select the winner. Suppliers differ in terms of quality scores (QS_i). They know that $QS_L \leq QS_H$, but they do not know the exact value of the relative QS (i.e. $\alpha = QS_L/QS_H$). To make the model more tractable, we assume that suppliers' a-priori belief on α follows a discrete distribution in the sense that it can only take either $\underline{\alpha}$ or $\overline{\alpha}$, each with equal probability of 1/2.
- Suppliers' cost information is privately known to themselves. In other words, they know the exact value of their own cost, but they do not know the marginal cost of their competitor. Without loss of generality, we assume that their prior belief regarding their competitor's cost is $c_{-i} \in \mathcal{U}(0,1)$, where $\mathcal{U}$ is the continuous uniform distribution. The buyer has no better information, i.e. he also a-priori believes that $c_i \in \mathcal{U}(0,1), \quad \forall i \in \{L, H\}$. Since, the focus of this study is on QS information sharing,

we do not consider any attempt from suppliers to share their cost information with the buyer or their competitors.

- The buyer considers a reserve price of 1 on the bid prices offered by the suppliers. This is the maximum (reserve) price that the buyer is willing to pay to the suppliers. In other words, supplier i loses automatically if $p_i \geq 1$ irrespective of other supplier's price.
- The timing of decisions and events is the same as that in the basic model (provided in Figure 3–1).

In the rest of this section, we compute suppliers' bid functions and buyer's expected cost under symmetric and asymmetric QS information settings. This allows us to analyze the buyer's strategic decision regarding sharing or not sharing quality scores with the competing suppliers.

Symmetric QS Information Analysis: First, we consider the equilibrium under symmetric information where both suppliers know the true value of α. Assuming that $0 < \alpha \leq 1$, the game switches to a private-cost reverse auction with two QS-asymmetric players. Delegating the detail of the analysis to Appendix 5.4, the equilibrium bid prices of suppliers are as follows:

$$p_L^{SYM} = \begin{cases} \frac{\alpha}{6} + \frac{1}{3} & \text{if } 0 \leq c_L \leq \frac{1}{3} - \frac{\alpha}{3} \\ \frac{c_L}{2} + \frac{\alpha}{3} + \frac{1}{6} & \text{if } \frac{1}{3} - \frac{\alpha}{3} \leq c_L \leq \frac{4}{3}\alpha - \frac{1}{3} \\ \alpha & \text{if } \frac{4}{3}\alpha - \frac{1}{3} \leq c_L \leq \alpha \\ c_L & \text{if } \alpha \leq c_L \leq 1 \end{cases}$$

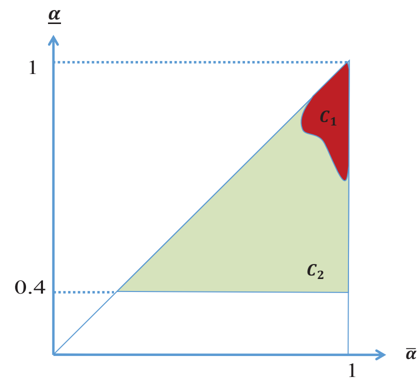
$$p_H^{SYM} = \begin{cases} \frac{c_H}{2} + \frac{1}{3\alpha} + \frac{1}{6} & \text{if } 0 \leq c_H \leq \frac{5}{3} - \frac{2}{3\alpha} \\ 1 & \text{if } \frac{5}{3} - \frac{2}{3\alpha} \leq c_H \leq 1 \end{cases}$$

In the symmetric equilibrium when α is known, each supplier considers their actual marginal cost plus their information regarding other supplier's marginal cost. Similar to the public cost information setting, supplier L increases her price as α increases while supplier H decreases her price. This supports the idea that price competition generally grows as suppliers become more homogeneous in QS.

Asymmetric Information (Pooling Equilibrium): Under information asymmetry assumption, we first characterize pooling equilibria under which the buyer does not attempt to share QS information with the suppliers. In this setting, suppliers are aware of their own cost, but they do not know their competitor's cost and the true value of α. Therefore, each supplier makes their pricing decisions solely based on their prior belief on α and their competitor's marginal cost. Maximizing suppliers' payoffs (details are provided in Appendix 5.4), we first find their bid functions under pooling equilibria. The bid functions enable us to compute parties' expected cost/profits under pooling equilibria. Comparative analysis of buyer's expected cost under pooling and symmetric QS information settings allows us to comment on the buyer's incentive in sharing suppliers' QS with them when the QS information is verifiable (i.e. costless to be shared).

According to Figure 3–7, the buyer is better off to disclose suppliers' quality scores unless they are quite homogeneous in QS and in addition, they are aware of this fact. In most of the cases, this result is in accordance with our previous results in the basic model with public cost information. The only apparent contradiction happens in the case of heterogeneous suppliers with low information asymmetry. A closer look at the assumptions of the models, however, can explain this phenomenon and show that there is actually no significant contradiction. The key point is that the buyer's expected cost highly depends on supplier H's price in the case of QS-heterogeneous suppliers. Therefore, he does anything to assures that supplier H will not increase her price. Even though the buyer generally prefers a very low level of QS information asymmetry than information symmetry, he chooses to disclose α in the case of heterogeneous suppliers, because supplier H's price would be much higher under QS information asymmetry as in that case, she bids based on her expected payoff functions considering expected α and expected cost of supplier L.

Figure 3–7: Effect of Pooling vs. Symmetric QS Information on Buyer's Cost.



Regions	Buyer's cost
$(\underline{\alpha}, \overline{\alpha}) \in C_1$	$\overline{\kappa}_B^{PE} < \overline{\kappa}_B^{SYM}$
$(\underline{\alpha}, \overline{\alpha}) \in C_2$	$\overline{\kappa}_B^{PE} > \overline{\kappa}_B^{SYM}$

Asymmetric Information (Separating Equilibrium): In order to explore how the buyer can signal α in a credible fashion, we consider the same signalling devices: advance revenue guarantees to suppliers. Let $\eta_L(\alpha)$ and $\eta_H(\alpha)$ denote the signals employed by the buyer in a separating equilibrium. In order for these signals to be incentive compatible for the buyer and hence credible for the suppliers, they need to satisfy the following system of equations:

$$\kappa_B(\eta_L(\overline{\alpha}), \eta_H(\overline{\alpha}), p_L(\overline{\alpha}), p_H(\overline{\alpha}), \overline{\alpha}) \leq \kappa_B(\eta_L(\underline{\alpha}), \eta_H(\underline{\alpha}), p_L(\underline{\alpha}), p_H(\underline{\alpha}), \overline{\alpha})$$

$$\kappa_B(\eta_L(\underline{\alpha}), \eta_H(\underline{\alpha}), p_L(\underline{\alpha}), p_H(\underline{\alpha}), \underline{\alpha}) \leq \kappa_B(\eta_L(\overline{\alpha}), \eta_H(\overline{\alpha}), p_L(\overline{\alpha}), p_H(\overline{\alpha}), \underline{\alpha})$$

where $\kappa_B(\eta_L(\alpha'), \eta_H(\alpha'), p_H(\alpha'), p_L(\alpha'), \alpha)$ denotes the total cost of the buyer if he signals α' and his true type is α.

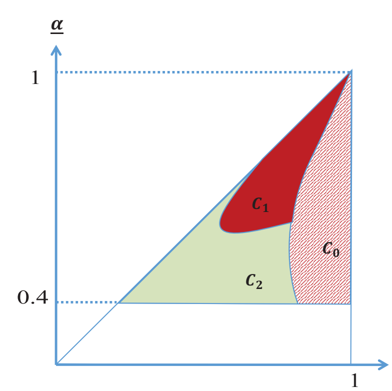
Skipping the details of the analysis (which are provided in the Appendix 5.4), in this section, we focus mainly on those cases where the buyer uses revenue guarantees to supplier H to signal the true value of α, i.e. $\eta_L(\alpha) = 0 \quad \forall \alpha \in \{\underline{\alpha}, \overline{\alpha}\}$. In that case, suppliers' bid response to the buyer's signal $(\eta_H(\alpha))$ can be characterized as below:

$$p_L = \begin{cases} \frac{\alpha}{6} + \frac{1}{3} + \frac{\alpha\eta_H}{2} & \text{if } 0 \le c_L \le c_L^w \\ \frac{c_L}{2} + \frac{\alpha}{3} + \frac{1}{6} & \text{if } c_L^w \le c_L \le c_L^1 \\ \alpha & \text{if } c_L^1 \le c_L \le \alpha \\ c_L & \text{if } \alpha \le c_L \le 1 \end{cases}$$

$$p_H = \begin{cases} \left(\frac{1-\eta_H}{2}\right) c_H + \frac{1}{3\alpha} + \frac{1}{6} + \frac{\eta_H}{2} & \text{if } 0 \le c_H \le c_H^1 \\ 1 & \text{if } c_H^1 \le c_H \le 1 \end{cases}$$

where $c_H^1 = \frac{3\alpha\eta_H - 5\alpha + 2}{3\alpha(-1+\eta_H)}$; $c_L^w = \frac{1}{3} - \frac{\alpha}{3} + \eta_H\alpha$; $c_L^1 = \frac{4}{3}\alpha - \frac{1}{3}$; $c_L cor = \frac{1}{3} - \frac{\alpha}{3} + c_H\alpha - \alpha c_H\eta_H + \alpha\eta_H$, and $c_H cor = \frac{3\alpha\eta_H - \alpha - 3c_L + 1}{3\alpha(-1+\eta_H)}$. Given this response from the suppliers, buyer uses $\eta_H(\underline{\alpha}) = \underline{\eta_H} > 0$ and $\eta_H(\overline{\alpha}) = \overline{\eta_H} = 0$ to signal his true type. This signal along with the bid functions allow us to compute the players' expected cost/profits under separating equilibria. Comparing buyer's expected cost under pooling with that under separating equilibria helps us to answer one of the most important research questions of this chapter: what should be the buyer's strategic decision for sharing relative QS with the suppliers when they have private access to their cost information?

Figure 3–8: Effect of Pooling vs. Separating Equilibria on Buyer's Cost.



Regions	Buyer's cost (Pooling, Separating)
$(\underline{\alpha}, \overline{\alpha}) \in C_0$	$(\overline{\kappa}_B^{PE}, N.A.)$
$(\underline{\alpha}, \overline{\alpha}) \in C_1$	$(\overline{\kappa}_B^{PE}, \overline{\kappa}_B^{SE})$; $\overline{\kappa}_B^{PE} < \overline{\kappa}_B^{SE}$
$(\underline{\alpha}, \overline{\alpha}) \in C_2$	$(\overline{\kappa}_B^{PE}, \overline{\kappa}_B^{SE})$; $\overline{\kappa}_B^{PE} > \overline{\kappa}_B^{SE}$

Figure 3–8 shows buyer's preference between pooling and separating for different combinations of $(\underline{\alpha}, \overline{\alpha})$ for the case of non-verifiable QS information. Concentrating on the points where signalling is possible using advance guarantees to supplier H, we can summarize the results in three different points:

- *When the degree of information asymmetry is high,* the buyer's best preference is to share the true value of α since otherwise the QS uncertainty paired with cost uncertainty can lead to increased bid prices of suppliers, which is very harmful to the buyer.
- In the case of *homogeneous suppliers with low QS uncertainty,* the buyer is better off to keep the suppliers in uncertainty with respect to the exact value of the relative QS. In this case, the more severe price competition happens under pooling equilibria as the suppliers discretionarily reduce their bid prices with the aim of increasing their winning chance.
- Lastly, in the case of *heterogeneous suppliers with low QS uncertainty*, since the buyer relies mainly on supplier H as his main qualified supply source, it is very important to ensure that she will not increase her price. As a result, the buyer prefers to incur the signalling cost and share α because supplier H's price would be much higher under QS information asymmetry as in that case, she bids on the basis of her consideration for the expected α and the expected cost of supplier L.

3.9. Conclusion

In this chapter, we analyze (i) how a buyer can credibly share the private QS information with the upstream suppliers, (ii) how it impacts the equilibrium decisions and the profits/cost of channel parities, and (iii) what factors induces the buyer to share (and not to share). To address these questions, we develop a decentralized supply chain model with a buyer and two heterogeneous suppliers that are competing for the buyer's order quantity. Buyer decides on the order allocation based on not only the bid prices offered competitively by the suppliers but also their quality scores, which are private information for the buyer.

In our basic model where the cost information of suppliers is publicly known to all parties, we first characterize the equilibrium under symmetric information (i.e., when the true value of the relative quality score is known to the suppliers). In this case, suppliers L and H strategically adjust their bid prices in upward and downward directions, respectively, as the true score of supplier L gets closer to that of supplier H. Under asymmetric information, this very impact of the quality scores on the bid prices also makes it impossible for the buyer to credibly share the quality score information with the suppliers without the use of a costly signal. Along these lines, advance revenue guarantees provide the buyer with such a signalling device.

The comparative analyses of the equilibrium bid prices under pooling and separating equilibria show that the bidding strategy is affected by whether or not the true value of quality score is shared with the suppliers. We identify two cases. First, when the range of true value of quality score α is sufficiently small, both suppliers would aggressively undercut each other's offer irrespective of α. This increases the intensity of price competition among the suppliers, which lowers the equilibrium pooling prices. Hence, the buyer has no incentive to share the true value of quality score. Second, when the degree of information asymmetry is high, i.e. the range of uncertainty on α is high, this bidding strategy becomes very costly for the suppliers. In this case, rather than attempting to win the order for all values of quality score, each supplier charges a price that would maximize her expected profit based on a-priori beliefs. This however reduces the degree of competition among the suppliers, which in turn increases the equilibrium prices.

In the second case, the buyer can signal the true value of relative quality score at a (signalling) cost. That is, the buyer has to offer advance revenue contract to one of the suppliers, which indirectly sets a lower bound on the equilibrium prices. Our comparative analysis between pooling and separating equilibria from the buyer's perspective leads to two results. First, if the buyer decides to offer an advance revenue contract, he would always offer it to supplier H. Second, the comparison of the cost (due to increase in prices) under pooling equilibria with that (due to signalling) under separating equilibria implies that the buyer would prefer to share the true value of the relative quality score only when the degree of information asymmetry becomes sufficiently high.

Next, we extend our analysis to the case when there are more than two suppliers with unknown quality scores, and show that the qualitative nature of our results mostly holds true. Our main findings in this extension are twofold. First, as in the main model, the buyer needs to signal only the true value of relative quality scores to the competing suppliers. Second, since the price competition among multiple suppliers naturally reduces the equilibrium bid prices, there is not much incentive left to the buyer in signalling quality scores to the suppliers. Qualitatively speaking, most of our results holds true under this setting.

Finally, we relax the assumption regarding suppliers' public cost information and extend our basic model to the case where suppliers privately know their own costs. Our similar results from this section support the idea that the degree of information asymmetry and the actual homogeneity of suppliers in QS are the two most important factors for the buyer when strategically deciding about sharing QS information with the suppliers. More precisely, in this case, the buyer attempts to keep suppliers' ambiguity in QS only if they are quite homogeneous in QS and the degree of QS uncertainty is low enough.

The above-mentioned results give some managerial insights on the use and sharing of quality scores in reverse supply auctions. First, when the range of uncertainty is small, the buyer would intentionally keep the assessment of bid prices ambiguous for the suppliers. This can be done in many different ways. For example, some procurement auctions start with the buyer's issuing to the set of potential suppliers a formal request for interest (RFI), followed by request for bid (RFB). If the buyer wants to make the entire process more ambiguous, he can skip RFI and RFP stages and directly issue RFB. RFP stage can be also customized specifically to adjust the amount of information sharing regarding the supplier selection process. For example, in certain RFPs, the buyer provides not only how the scoring is done under each category but also what weights are assigned to each category and how they are combined (see BC-Procurement 2010). Yet, in other RFPs, the buyer provides only limited information regarding either scoring or weighting and leaves the other one undetermined (see Trent 2010). The latter makes the ultimate assessment formula more uncertain from the suppliers' perspective. As suggested by our results, this would prevent the suppliers from adjusting strategically their bid prices at the expense of the buyer.

Along these lines, many B2B e-commerce platforms such as AliSourcePro, AribaWeb, etc. streamline the procurement process for the companies by enabling them to create a simple buying request without going through the hurdles of RFx processes. These are particularly effective in lowering the procurement cost of commodity-type products for small and medium size companies. From the lenses of our results, this streamlining has also an increasing effect on the degree of uncertainty regarding the procurer's deliberation process of the winner among the bidders.

Our results also imply that the procurement process needs to be customized according to the evolution of the supply base. When the supply base consists of a few qualified suppliers, and a new entrant joins, the buyer would open only a limited portion of his total demand to the competitive bids from the untested supplier and keep procuring the remaining part for the qualified suppliers. However, as the supply base enlarges with the inclusion of new entrants, the buyer starts opening the entire order to the competition among all the suppliers.

The model presented in this chapter can be extended in different directions. First possible extension is to add supply- and demand-side risks to the model. For example, in a situation where the suppliers face uncertain capacities, the buyer may want to diversify his supply base. This leads to multi-unit reverse auction models, where the buyer procures from multiple suppliers all at once. Similar to our base model in this chapter, quality scores affect both bidding strategies of the suppliers as well as the equilibrium bids in this case. Particularly, the fact that the bidder with not only the lowest quality-score adjusted bid but also the second lowest, third lowest, etc. receives some portion from the total order would create more complicated bidding strategies for the bidders. Similarly, the demand risks may change the buyer's incentive for sharing quality score information with the suppliers. Another important aspect is that both buyers and suppliers may have budget limitations, which affects the design of procurement auctions. Another interesting extension would be to consider more realistic situations where a supplier is not aware of the number and the cost/QS characteristics of her competitors. These scenarios will add to the level of the uncertainty that a supplier bears before participating in the auction. Finally, there is also a large literature that considers the impact of quality scores on forward auctions. Even though throughout this chapter we focus on reverse auctions, our work may shed some light on the future research in this direction as well. Lastly, we believe that the analysis of the quality scores presents fruitful research opportunities and hope that this model will fuel potential research in the future.

CHAPTER 4
Concluding Remarks

In this dissertation, we focused on two issues related to the asymmetries in operations and supply chain management in order to shed some light on their impacts on operational practice and performance of channel parties. First, we addressed the issue of power asymmetry in the retail sector and investigated the effects of superior market and channel power of a dominant retailer on his assortment choice. Second, we investigated the issue of QS information asymmetry in buyer-determined reverse auctions and examined buyers' best strategies in dealing with this asymmetry.

In Chapter 2, we addressed the phenomenon of assortment reduction by dominant retailers and explored potential power-related causes for adoption of this strategy. To capture important elements of retail power and their underlying effects on the retail variety choice of a dominant retailer, we developed game-theoretical competitive models between two asymmetric retailers: one dominant and one weak. The dominant retailer can have either price leadership to set the market price, product cost advantage over the weak retailer, or both. The analysis suggests that possessing only unit cost advantage cannot lead to assortment reduction, and in fact, in addition to the cost advantage by the dominant retailer, price-leadership is a necessary condition for assortment reduction. This indeed proves a strategic relation between pricing and assortment decisions as two operational and tactical tools for retailers in today's competitive markets. In addition to these sorts of retail power, the analysis shows that large market share (as a third source of retail power) intensifies the effect of price leadership by enabling the dominant retailer to influence the entire market more strongly with a lower number of products.

In Chapter 3, we addressed the role that non-price attributes play in buyer-determined auctions and intended to provide normative recommendations to the buyers regarding sharing QS with the suppliers before holding a procurement auction. We considered a two-level supply chain with one buyer and two suppliers, where the buyer uses a first-price, sealed-format reverse auction to select among the suppliers. We characterized equilibrium decisions and cost/profits of all parties in the channel under both symmetric and asymmetric information settings (pooling, separating, and semi-separating).

By comparing the buyer's equilibrium cost under different settings, we found two important factors that affect his strategy choice regarding QS sharing: suppliers' prior information asymmetry with regard to QS, and the actual QS homogeneity of suppliers. From the buyer's perspective, in general, a low degree of information asymmetry in QS is more effective than both extreme cases of high uncertainty and/or information symmetry. For this reason, buyers usually tend to make the scoring rule as clear and understandable as possible before holding the auction. We also found that in the case where suppliers know that they are quite similar in terms of the non-price attributes, the buyer is better off under pooling equilibria, as this information asymmetry makes the suppliers to decrease their bid prices more aggressively. These results demonstrate a strong robustness as they not only held true under a more general setting with multiple incumbent and entrant suppliers, but also were supported when the suppliers' public cost assumption was relaxed.

There are several interesting ways in which this work can be extended. First, we only considered pricing power and cost advantage for the dominant retailers in Chapter 2. It would be useful to explore the impacts of other sources of retail power such as bargaining power, technology dominance, and retail image on the assortment choice of a dominant retailer. It would be also interesting to see how the results in Chapter 2 are robust when other realistic constraints such as budget and shelf space constraints are imposed. We studied the problem using a set of static competitive games between two asymmetric retailers. It would be also interesting to explore the causal relationship between power and assortment in more dynamic settings and repeated games. Finally, we modeled the game using linear demand functions. Using utility-based demand functions such as locational or MNL models would be helpful in commenting on the effects of these asymmetries on final consumers' utility and welfare.

In Chapter 3, we analytically studied the problem of QS information sharing by a buyer to some upstream suppliers using game theory models. It would be interesting to study QS information asymmetry under more realistic situations with actual behavioral and/or communicational realities. Our research suggests that the buyer is better off when suppliers have little uncertainty (neither high uncertainty nor full symmetry) regarding their non-price scores. Validity of this finding in behavioral context can be tested through human-subject experiments on a series of multi-player games. Also, we assumed that all the actions and messages of the buyer are publicly and equally shared to the suppliers. Changing the communication channels to a more general case where the buyer can send private messages to each supplier regarding their own and others' QS may alter the results. In addition, it would be interesting to conduct separate sets of experiments in countries with different cultural background (for instance, North American vs. Asian countries) and compare the participants' behaviors in these locations in order to find if culture can play a significant role in the willingness of buyers to share the information.

CHAPTER 5
Appendices: Numerical Analyses and Proofs

5.1. Numerical analyses for Chapter 2

In order to numerically test the assortment impact of power and non-power factors, as explained in §2.4, we assign different values to each parameter and examine whether $R1$'s assortment increases or decreases following the six different patterns (FF, FP, FPF, PP, PF, PFP).

The values assigned to each parameters are as follows:

- $\frac{\Delta}{w}$, from 0 to 0.95 in equal increments of 0.05 (twenty distinct values);
- λ, from 0.05 to 0.95 in equal increments of 0.05 (ninety distinct values);
- $\frac{\beta_1}{\beta_2}$ and $\frac{\mathcal{B}_1}{\mathcal{B}_2}$, from 0.05 to 0.95 in equal increments of 0.05 (ninety distinct values);
- $\frac{\gamma}{\beta_1}$ and $\frac{\Gamma}{\mathcal{B}_1}$, from 0.05 to 0.95 in equal increments of 0.05 (ninety distinct values);
- $\frac{w}{\alpha}$ and $\frac{w}{\mathcal{A}/\mathcal{B}_2}$, from 0.05 to 0.95 in equal increments of 0.05 (ninety distinct values).
- We also considered twenty distinct values for variety cost (V) in the form of $x \times \frac{\mathcal{A}^2}{\mathcal{B}_2}$ and $x \times \frac{\alpha^2}{\beta_2}$ in quantity and price settings, with x from 0 to 0.95 in equal increments of 0.05.

In tables 5–1 to 5–2, we characterize the special patterns that correspond to any assortment change. However, to reduce the size of tables, we group the values in two levels: 1- *Low*: when the corresponding ratio is lower than 0.5, and 2- *High*: when the measurement ratio is equal to or higher than 0.5.

Table 5–1: Frequency of $R1$'s Assortment Behavior as his Cost Advantage (Δ/w) Increases.

Increase in Δ/w	PP	FF	FP	FPF	PF	PFP
SQ	2,407,290	46,394	0	0	138,662 (100%)†	0
SP	2,289,167	72,302	0	0	226,315 (99.8%)	390[1] (0.2%)
DFQ	2,403,093	55,595	0	0	97,871 (94%)	5,708[2] (6%)
DFP	2,421,104	50,043	0	0	122,501 (98%)	2,867[3] (2%)

Characterization of Special Cases:

	Potential-Cost*		Demand-Similarity**		Cross-Sensitivity***		Market-Share****	
	Low	High	Low	High	Low	High	Low	High
[1] PFP in SP	0	100%	0	100%	20%	80%	100%	0
[2] PFP in DFQ	0	100%	80%	20%	85%	15%	86%	14%
[3] PFP in DFP	45%	55%	0	100%	-	-	100%	0%

† Percentages show the relative frequency of a particular pattern w.r.t. the total number of all cases that correspond to any assortment change (i.e. all except PP and FF).

* Measured by $\frac{w}{\alpha}$ and $\frac{w}{\mathcal{A}/\mathcal{B}_2}$ in quantity and price settings, respectively.

** Measured by $\frac{\beta_1}{\beta_2}$ and $\frac{\mathcal{B}_1}{\mathcal{B}_2}$ in quantity and price settings, respectively.

*** Measured by $\frac{\gamma}{\beta_1}$ (resp. $\frac{\Gamma}{\mathcal{B}_1}$) in quantity (resp. price) settings as substitutability (resp. price competition) degree.

**** Measured by λ in both quantity (DFQ) and price settings.

Table 5–2: Frequency of $R1$'s Assortment Change in Simultaneous-move vs Domiannt-fringe.

	P → P	F → F	P → F	F → P
SQ → DFQ	12,802,012	391,686	82,757[1] (61.3%)†	52,303[2] (38.7%)
SP → DFP	48,041,202	1,722,891	3,270[3] (0.4%)	797,659[4] (99.6%)

Characterization of Special Cases:

	Potential-Cost*		Demand-Similarity**		Cross-Sensitivity***		Cost-Advantage****		Market-Share*****	
	Low	High	Low	High	Low	High	Low	High	Low	High
[1] $(P \to F)_Q$	95%	5%	39%	61%	17%	83%	39%	61%	-	100%
[2] $(F \to P)_Q$	0	100%	47%	53%	90%	10%	1%	99%	-	100 %
[3] $(P \to F)_P$	100%	0	0	100%	7%	93%	72%	28%	100%	0
[4] $(F \to P)_P$	58%	42%	62%	38%	49%	51%	25%	75%	80%	20%

† Percentages show the relative frequency of all the patterns that correspond to any assortment change (i.e. $P \to F$ and $F \to P$).

* Measured by $\frac{w}{\alpha}$ and $\frac{w}{\mathcal{A}/\mathcal{B}_2}$ in quantity and price settings, respectively.

** Measured by $\frac{\beta_1}{\beta_2}$ and $\frac{\mathcal{B}_1}{\mathcal{B}_2}$ in quantity and price settings, respectively.

*** Measured by $\frac{\gamma}{\beta_1}$ (resp. $\frac{\Gamma}{\mathcal{B}_1}$) in quantity (resp. price) settings as substitutability (resp. price competition) degree.

**** Measured by $\frac{\Delta}{w}$ in both quantity and price settings.

***** Measured by λ in both quantity (DFQ) and price settings.

Table 5–3: Frequency of $R1$'s Assortment Behavior as his Market Share (λ) Increases.

Increase in λ	PP	FF	FP	FPF	PF	PFP
SP	2,429,508	100,964	114,533[1] (54%)	25[2] (0%)	96,625[3] (46%)	224[4] (0%)
DFQ	2,533,814	37,238	37,413[5] (22%)	3,442[6] (2%)	112,014[7] (65%)	19,155[8] (11%)
DFP	2,551,299	31,676	12,720[9] (8%)	0	146,907[10] (92%)	0

Characterization of Special Cases:

	Potential-Cost*		Demand-Similarity**		Cross-Sensitivity***		Cost-Advantage****	
	Low	High	Low	High	Low	High	Low	High
[1] FP in SP	62%	38%	91%	9%	54%	46%	31%	69%
[2] FPF in SP	0%	100%	0%	100%	0%	100%	0%	100%
[3] PF in SP	96%	4%	13%	87%	50%	50%	43%	57%
[4] PFP in SP	100%	0%	100%	0%	22%	78%	100%	0%
[5] FP in DFQ	64%	36%	77%	23%	53%	47%	23%	77%
[6] FPF in DFQ	1%	99%	96%	4%	79%	21%	0%	100%
[7] PF in DFQ	74%	26%	40%	60%	79%	21%	35%	65%
[8] PFP in DFQ	48%	52%	38%	62%	1%	99%	10%	90%
[9] FP in DFP	63%	37%	49%	51%	-	-	18%	82%
[10] PF in DFP	71%	29%	7%	93%	-	-	31%	69%

† Percentages show the relative frequency of a particular pattern w.r.t. the total number of all cases that correspond to any assortment change (i.e. all except PP and FF).

* Measured by $\frac{w}{\alpha}$ and $\frac{w}{\mathcal{A}/\mathcal{B}_2}$ in quantity and price settings, respectively.

** Measured by $\frac{\beta_1}{\beta_2}$ and $\frac{\mathcal{B}_1}{\mathcal{B}_2}$ in quantity and price settings, respectively.

*** Measured by $\frac{\gamma}{\beta_1}$ (resp. $\frac{\Gamma}{\mathcal{B}_1}$) in quantity (resp. price) settings as substitutability (resp. price competition) degree.

**** Measured by $\frac{\Delta}{w}$ in both quantity and price settings.

Table 5–4: Frequency of $R1$'s Assortment Behavior as $\frac{\beta_1}{\beta_2}$ and $\frac{\mathcal{B}_1}{\mathcal{B}_2}$ in Quantity and Price Settings Increase.

Increase in $\frac{\beta_1}{\beta_2}/\frac{\mathcal{B}_1}{\mathcal{B}_2}$	PP	FF	FP	FPF	PF	PFP
SQ	2,613,154	95,777	0	0	11473[1] (100%)	0
SP	2,395,986	0	125[2] (0%)	0	286,332[3] (85%)	51,929[4] (15%)
DFQ	2,589,759	99,194	12,396[5] (30%)	100[6] (0%)	25,540[7] (62%)	3,442[8] (8%)
DFP	2,517,128	0	0	0	211,257[9] (93.5%)	14,716[10] (6.5%)

Characterization of Special Cases:

	Potential-Cost*		Cross-Sensitivity**		Cost-Advantage***		Market-Share****	
	Low	High	Low	High	Low	High	Low	High
[1] PF in SQ	59%	41%	33%	67%	15%	85%	50%	50%
[2] FP in SP	100%	0%	0%	100%	0%	100%	100%	0%
[3] PF in SP	57%	43%	50%	50%	25%	75%	42%	58%
[4] PFP in SP	87%	13%	49%	51%	46%	54%	99%	1%
[5] FP in DFQ	43%	57%	22%	78%	7%	93%	100%	0%
[6] FPF in DFQ	100%	0%	0%	100%	100%	0%	100%	0%
[7] PF in DFQ	80%	20%	23%	77%	31%	69%	47%	53%
[8] PFP in DFQ	36%	64%	9%	91%	2%	98%	100%	0%
[9] PF in DFP	70%	30%	-	-	28%	72%	29%	71%
[10] PFP in DFP	44%	56%	-	-	15%	85%	86%	14%

† Percentages show the relative frequency of a particular pattern w.r.t. the total number of all cases that correspond to any assortment change (i.e. all except PP and FF).

* Measured by $\frac{w}{\alpha}$ and $\frac{w}{\mathcal{A}/\mathcal{B}_2}$ in quantity and price settings, respectively.

** Measured by $\frac{\gamma}{\beta_1}$ (resp. $\frac{\Gamma}{\mathcal{B}_1}$) in quantity (resp. price) settings as substitutability (resp. price competition) degree.

*** Measured by $\frac{\Delta}{w}$ in both quantity and price settings.

**** Measured by λ in both quantity (DFQ) and price settings.

Table 5–5: Frequency of $R1$'s Assortment Behavior as Products become More Substitutable ($\frac{\gamma}{\beta_1}$ Increases.

Increase in $\frac{\gamma}{\beta_1}$	PP	FF	FP	FPF	PF	PFP
SQ	2,472,981	0	270,619[1] (100%)	0	0	0
DFQ	2,499,245	37,562	188,835[2] (92%)	1,571[3] (1%)	8,106 (4%)[4]	6,385[5] (3%)

Characterization of Special Cases:

	Potential-Cost*		Demand-Similarity**		Cost-Advantage***		Market-Share****	
	Low	High	Low	High	Low	High	Low	High
[1] FP in SQ	57%	43%	53%	47%	24%	76%	50%	50%
[2] FP in DFQ	67%	33%	61%	39%	28%	72%	37%	63%
[3] FPF in DFQ	22%	78%	27%	73%	10%	90%	89%	11%
[4] PF in DFQ	17%	83%	81%	19%	3%	97%	100%	0%
[5] PFP in DFQ	68%	32%	84%	16%	42%	58%	100%	0%

† Percentages show the relative frequency of a particular pattern w.r.t. the total number of all cases that correspond to any assortment change (i.e. all except PP and FF).

* Measured by $\frac{w}{\alpha}$ and $\frac{w}{\mathcal{A}/\mathcal{B}_2}$ in quantity and price settings, respectively.

** Measured by $\frac{\beta_1}{\beta_2}$ and $\frac{\mathcal{B}_1}{\mathcal{B}_2}$ in quantity and price settings, respectively.

*** Measured by $\frac{\Delta}{w}$ in both quantity and price settings.

**** Measured by λ in both quantity (DFQ) and price settings.

Table 5–6: Frequency of $R1$'s Assortment Behavior as Demand Sensitivity to Competitor's Price ($\frac{\Gamma}{\mathcal{B}_1}$) Increases.

Increase in $\frac{\Gamma}{\mathcal{B}_1}$	PP	FF	FP	FPF	PF	PFP
SP	2,525,733	160,800	7,059[1] (26.9%)	474[2] (1.8%)	18,507[3] (70.6%)	175[4] (0.7%)
DFP	2,593,974	94904	0	0	0	0

Characterization of Special Cases:

	Potential-Cost*		Demand-Similarity**		Cost-Advantage***		Market-Share****	
	Low	High	Low	High	Low	High	Low	High
[1] FP in SP	69%	31%	49%	51%	37%	63%	99%	1%
[2] FPF in SP	26%	74%	26%	74%	5%	95%	79%	21%
[3] PF in SP	62%	38%	74%	26%	32%	68%	58%	42%
[4] PFP in SP	100%	0%	0%	100%	57%	43%	100%	0%

† Percentages show the relative frequency of a particular pattern w.r.t. the total number of all cases that correspond to any assortment change (i.e. all except PP and FF).

* Measured by $\frac{w}{\alpha}$ and $\frac{w}{\mathcal{A}/\mathcal{B}_2}$ in quantity and price settings, respectively.

** Measured by $\frac{\beta_1}{\beta_2}$ and $\frac{\mathcal{B}_1}{\mathcal{B}_2}$ in quantity and price settings, respectively.

*** Measured by $\frac{\Delta}{w}$ in both quantity and price settings.

**** Measured by λ in both quantity (DFQ) and price settings.

Table 5–7: Frequency of $R1$'s Assortment Behavior as Potential Profit Margin of Products ($\frac{w}{\alpha}$ and $\frac{w}{\mathcal{A}/\mathcal{B}_2}$ in Quantity and Price Settings) Increase.

Increase in $\frac{w}{\alpha}\Big/\frac{w}{\mathcal{A}/\mathcal{B}_2}$	PP	FF	FP	FPF	PF	PFP
SQ	2,591,954	61,856	89,791[1] (100%)	0	0	0
SP	2,435,169	82,158	150,225[2] (69%)	2,644[3] (1%)	23,944[4] (11%)	40,431[5] (19%)
DFQ	2,494,930	24,618	158,854[6] (82%)	100[7] (0%)	2,619[8] (1%)	32,699[9] (17%)
DFP	2,539,327	43,898	121,467[10] (79%)	499[11] (0%)	2,494[12] (2%)	30,180[13] (20%)

Characterization of Special Cases:

	Demand-Similarity*		Cross-Sensitivity**		Cost-Advantage***		Market-Share****	
	Low	High	Low	High	Low	High	Low	High
[1] FP in SQ	60%	40%	89%	11%	54%	46%	50%	50%
[2] FP in SP	45%	55%	52%	48%	60%	40%	46%	54%
[3] FPF in SP	5%	95%	56%	44%	0%	100%	100%	0%
[4] PF in SP	28%	72%	52%	48%	0%	100%	98%	2%
[5] PFP in SP	31%	69%	51%	49%	36%	64%	100%	0%
[6] FP in DFQ	49%	51%	72%	28%	44%	56%	35%	65%
[7] FPF in DFQ	100%	0%	0%	100%	0%	100%	100%	0%
[8] PF in DFQ	77%	23%	79%	21%	0%	100%	100%	0%
[9] PFP in DFQ	67%	33%	83%	17%	39%	61%	100%	0%
[10] FP in DFP	10%	90%	-	-	56%	44%	27%	73%
[11] FPF in DFP	50%	50%	-	-	0%	100%	75%	25%
[12] PF in DFP	60%	40%	-	-	0%	100%	100%	0%
[13] PFP in DFP	11%	89%	-	-	27%	73%	98%	2%

† Percentages show the relative frequency of a particular pattern w.r.t. the total number of all cases that correspond to any assortment change (i.e. all except PP and FF).

* Measured by $\frac{\beta_1}{\beta_2}$ and $\frac{\mathcal{B}_1}{\mathcal{B}_2}$ in quantity and price settings, respectively.

** Measured by $\frac{\gamma}{\beta_1}$ (resp. $\frac{\Gamma}{\mathcal{B}_1}$) in quantity (resp. price) settings as substitutability (resp. price competition) degree.

*** Measured by $\frac{\Delta}{w}$ in both quantity and price settings.

**** Measured by λ in both quantity (DFQ) and price settings.

5.2. Profit functions and variety graphs at Chapter 2

In any of the studied games, each variety choice by retailers leads to a different set of payoff functions in equilibrium. Here in Table 5–8 to 5–11, we only provide these equilibrium payoff functions.

Note that with a backward operation, once the payoff functions are determined (in the end of stage 3 of the games), we can easily characterize the conditions under which each variety outcome (out of those listed in Table 2–3) happens in the equilibrium (in stage 1 of the games). Below, we capture the relationship between payoff functions under each equilibrium variety outcome:

- $< F, F >$ where $\Pi^1(F, F) \geq \Pi^1(P, F)$; $\Pi^2(F, F) \geq \Pi^2(F, P)$; and $\Pi^2(F, F) \geq \Pi^2(F, LP)$.
- $< F, P >$ where $\Pi^1(F, P) \geq \Pi^1(P, P)$; $\Pi^2(F, F) \leq \Pi^2(F, P)$; and $\Pi^2(F, P) \geq \Pi^2(F, LP)$.
- $< F, LP >$ where $\Pi^1(F, LP) \geq \Pi^1(P, LP)$; $\Pi^2(F, F) \leq \Pi^2(F, LP)$; and $\Pi^2(F, P) \leq \Pi^2(F, LP)$.
- $< P, F >$ where $\Pi^1(F, F) \leq \Pi^1(P, F)$; $\Pi^2(P, F) \geq \Pi^2(P, P)$; and $\Pi^2(P, F) \geq \Pi^2(P, LP)$.
- $< P, P >$ where $\Pi^1(F, P) \leq \Pi^1(P, P)$; $\Pi^2(P, F) \leq \Pi^2(P, P)$; and $\Pi^2(P, P) \geq \Pi^2(P, LP)$.
- $< P, LP >$ where $\Pi^1(F, LP) \leq \Pi^1(P, LP)$; $\Pi^2(P, F) \leq \Pi^2(P, LP)$; and $\Pi^2(P, P) \leq \Pi^2(P, LP)$.

Given any pair of the above equilibrium variety outcomes, we can define boundary conditions for which retailers are indifferent between those distinct variety outcomes. Therefore, for the sake of analytical proofs and following the critical values in Figure 1, if we fix values of $\frac{\Delta}{w}, \lambda, \frac{\gamma}{\beta_1}, \frac{\beta_1}{\beta_2}, \frac{w}{\alpha}$ for quantity settings and $\frac{\Delta}{w}, \lambda, \frac{\Gamma}{\mathcal{B}_1}, \frac{\mathcal{B}_1}{\mathcal{B}_2}, \frac{w}{\mathcal{A}/\mathcal{B}_2}$ for price settings, we can find boundary variety cost levels as a function of those ratios/parameters for each combination of variety outcomes. Here, since all the propositions are w.r.t. SQ model, we only focus on quantity competition setting and define the bounds correspondingly, as follows:

$$\mathcal{Y}^j_{(v^1,v^2),(u^1,u^2)} = \left\{ V\left(\frac{\Delta}{w}, \lambda, \frac{\gamma}{\beta_1}, \frac{\beta_1}{\beta_2}, \frac{w}{\alpha}\right) \quad : \quad \Pi^j(v^1, v^2) = \Pi^j(u^1, u^2) \right\}$$

where $\mathcal{Y}^j_{(v^1,v^2),(u^1,u^2)}$ is the locus of points in which retailer j is indifferent between variety outcomes of $< v^1, v^2 >$ and $< u^1, u^2 >$ in a quantity competition setting; v^j and u^j are variety choice of retailer $j \in \{1, 2\}$. As a result, nine sets of variety bounds can be defined that are used to determine equilibrium variety outcomes of the games.

$$\begin{aligned}
&\mathcal{Y}^1_{(F,F),(P,F)}: && \text{where } \Pi^1(F,F) = \Pi^1(P,F) \\
&\mathcal{Y}^1_{(F,P),(P,P)}: && \text{where } \Pi^1(F,P) = \Pi^1(P,P) \\
&\mathcal{Y}^1_{(F,LP),(P,LP)}: && \text{where } \Pi^1(F,LP) = \Pi^1(P,LP) \\
&\mathcal{Y}^2_{(F,F),(F,P)}: && \text{where } \Pi^2(F,F) = \Pi^2(F,P) \\
&\mathcal{Y}^2_{(F,F),(F,LP)}: && \text{where } \Pi^2(F,F) = \Pi^2(F,LP) \\
&\mathcal{Y}^2_{(P,F),(P,P)}: && \text{where } \Pi^2(P,F) = \Pi^2(P,P) \\
&\mathcal{Y}^2_{(P,F),(P,LP)}: && \text{where } \Pi^2(P,F) = \Pi^2(P,LP) \\
&\mathcal{Y}^2_{(F,P),(F,LP)}: && \text{where } \Pi^2(F,P) = \Pi^2(F,LP) \\
&\mathcal{Y}^2_{(P,P),(P,LP)}: && \text{where } \Pi^2(P,P) = \Pi^2(P,LP)
\end{aligned}$$

We use these bounds when proving the propositions in Appendix 5.3.

Table 5–8: Payoff Functions for Retailers in Different Variety Outcomes in SQ Game.

Variety Outcome	Retailer $R1$' Payoff (Π^1)	Retailer $R2$'s Payoff (Π^2)
$< F, F >$	$\frac{(\alpha-w+2\Delta)(-2\gamma\alpha+\beta_2\alpha+2\gamma w-4\gamma\Delta-\beta_2 w+2\beta_2\Delta+\beta_1\alpha-\beta_1 w+2\beta_1\Delta)}{9(\beta_2\beta_1-\gamma^2)} - V$	$\frac{(\alpha-w-\Delta)(-2\gamma\alpha+\beta_2\alpha+2\gamma w+2\gamma\Delta-\beta_2 w-\beta_2\Delta+\beta_1\alpha-\beta_1 w-\beta_1\Delta)}{9(\beta_2\beta_1-\gamma^2)} - V$
$< F, P >$	$\Pi^1(F,P) = (-18\beta_1^2 w\Delta + 18\beta_1^2\alpha\Delta + 9\beta_1^2 w^2 + 9\beta_1^2\Delta^2 + 9\beta_1^2\alpha^2 - 18\beta_1^2\alpha w + 16\beta_1\alpha\beta_2\Delta - 16\beta_2\beta_1 w\Delta + 4\beta_1\alpha^2\beta_2 + 4\beta_2\beta_1 w^2 + 16\beta_2\beta_1\Delta^2 - 8\beta_1\alpha\beta_2 w - 18\beta_1\alpha^2\gamma - 18\beta_1\Delta^2\gamma + 36\beta_1\alpha\gamma w + 36\beta_1 w\gamma\Delta - 18\beta_1 w^2\gamma - 36\beta_1\alpha\gamma\Delta + 5\gamma^2 w^2 + 2\alpha\gamma^2\Delta - 2\gamma^2 w\Delta + 5\alpha^2\gamma^2 - 7\gamma^2\Delta^2 - 10\alpha\gamma^2 w)/(36\beta_1(\beta_2\beta_1 - \gamma^2)) - V$	$\frac{(\alpha-w-\Delta)^2}{9\beta_1}$
$< F, LP >$	$(16\beta_1\alpha\beta_2\Delta - 16\beta_2\beta_1 w\Delta + 4\beta_1\alpha^2\beta_2 + 4\beta_2\beta_1 w^2 + 16\beta_2\beta_1\Delta^2 - 8\beta_1\alpha\beta_2 w - 18\beta_2^2 w\Delta + 18\alpha\beta_2^2\Delta + 9\beta_2^2 w^2 + 9\beta_2^2\Delta^2 + 9\alpha^2\beta_2^2 - 18\alpha\beta_2^2 w - 18\alpha^2\beta_2\gamma - 18\beta_2\Delta^2\gamma + 36\beta_2\alpha\gamma w + 36\beta_2 w\gamma\Delta - 18\beta_2 w^2\gamma - 36\beta_2\alpha\gamma\Delta + 5\gamma^2 w^2 + 2\alpha\gamma^2\Delta - 2\gamma^2 w\Delta + 5\alpha^2\gamma^2 - 7\gamma^2\Delta^2 - 10\alpha\gamma^2 w)/(36\beta_2(\beta_2\beta_1 - \gamma^2)) - V$	$\frac{(\alpha-w-\Delta)^2}{9\beta_2}$
$< P, F >$	$\frac{(\alpha-w+2\Delta)^2}{9\beta_1}$	$(9\beta_1^2\alpha^2 + 9\beta_1^2 w^2 - 18\beta_1^2\alpha w + 8\beta_2\beta_1 w\Delta + 4\beta_1\alpha^2\beta_2 - 8\beta_1\alpha\beta_2\Delta + 4\beta_2\beta_1 w^2 - 8\beta_1\alpha\beta_2 w + 4\beta_2\beta_1\Delta^2 - 18\beta_1\alpha^2\gamma - 18\beta_1 w^2\gamma + 36\beta_1\alpha\gamma w - 8\gamma^2 w\Delta + 5\alpha^2\gamma^2 + 5\gamma^2 w^2 - 4\gamma^2\Delta^2 - 10\alpha\gamma^2 w + 8\alpha\gamma^2\Delta)/(36\beta_1(\beta_2\beta_1 - \gamma^2)) - V$
$< P, P >$	$\frac{(\alpha-w+2\Delta)^2}{9\beta_1}$	$\frac{(\alpha-w-\Delta)^2}{9\beta_1}$
$< P, LP >$	$\frac{\beta_1(2\beta_2\alpha-2\beta_2 w+2\beta_2\Delta-\gamma\alpha+\gamma w)^2}{(-\gamma^2+4\beta_2\beta_1)^2}$	$\frac{\beta_1(2\beta_2\alpha-2\beta_2 w+2\beta_2\Delta-\gamma\alpha+\gamma w)^2}{(-\gamma^2+4\beta_2\beta_1)^2}$

Table 5–9: Payoff Functions for Retailers in Different Variety Outcomes in DFQ Game.

Variety Outcome	Retailer $R1$' Payoff (Π^1)	Retailer $R2$'s Payoff (Π^2)
$<F,F>$	$(\frac{\alpha-w+\Delta)\lambda(-2\gamma\alpha+2\gamma w-2\gamma\Delta+\beta_2\alpha-\beta_2 w+\beta_2\Delta+\beta_1\alpha-\beta_1 w+\beta_1\Delta)}{4(-\gamma^2+\beta_2\beta_1)}-V$	$\frac{(1-\lambda)(\alpha-w-\Delta)(-2\gamma\alpha+2\gamma w-2\gamma\Delta+\beta_2\alpha-\beta_2 w+\beta_2\Delta+\beta_1\alpha-\beta_1 w+\beta_1\Delta)}{4(-\gamma^2+\beta_2\beta_1)}-V$
$<F,P>$	$\lambda(-2\beta_1\Delta w-2\beta_1\alpha w+\beta_1\alpha^2+\beta_1 w^2+\beta_1\Delta^2+2\beta_1\alpha\Delta+\beta_2\alpha^2\lambda+\beta_2\lambda w^2-2\beta_2\Delta\lambda w+\beta_2\Delta^2\lambda-2\beta_2\alpha\lambda w+2\beta_2\alpha\Delta\lambda-\Delta^2\gamma\lambda-\gamma w^2-\alpha^2\gamma\lambda-2\alpha\Delta\gamma\lambda+2\gamma\Delta\lambda w-\gamma\lambda w^2-2\alpha\Delta\gamma+2\gamma\alpha w+2\gamma\Delta w-\Delta^2\gamma+2\gamma\alpha\lambda w-\alpha^2\gamma)/(4\beta_1\lambda\beta_2-\lambda^2\gamma^2-2\lambda\gamma^2-\gamma^2)-V$	$-(2\beta_2\alpha\lambda-2\beta_2\lambda w+2\beta_2\Delta\lambda+\gamma w-\gamma\alpha+\gamma\lambda w-\gamma\Delta\lambda-\gamma\Delta-\gamma\alpha\lambda)(-\beta_1\lambda\gamma\alpha-\beta_1\gamma w+2\beta_1\lambda\beta_2\alpha-2\beta_1\lambda\beta_2 w-2\beta_1\lambda\beta_2\Delta-\lambda\gamma^2\alpha+\beta_1\gamma\Delta+\lambda^2\gamma^2\Delta+\beta_1\lambda\gamma w-\beta_1\lambda\gamma\Delta-\alpha\gamma^2+\lambda\gamma^2 w+\lambda\gamma^2\Delta+\beta_1\gamma\alpha+\gamma^2 w)(-1+\lambda)/(4\beta_1\lambda\beta_2-\lambda^2\gamma^2-2\lambda\gamma^2-\gamma^2)^2$
$<F,LP>$	$\lambda(\beta_1\alpha^2\lambda+\beta_1\lambda w^2+2\beta_1\alpha\Delta\lambda-2\beta_1\alpha\lambda w+\beta_1\Delta^2\lambda-2\beta_1\Delta\lambda w+\beta_2\alpha^2+\beta_2 w^2+2\beta_2\alpha\Delta-2\beta_2\alpha w+\beta_2\Delta^2-2\beta_2\Delta w+2\gamma\Delta\lambda w-\alpha^2\gamma-\alpha^2\gamma\lambda-\Delta^2\gamma\lambda-\gamma\lambda w^2-\gamma w^2-2\alpha\Delta\gamma\lambda-\Delta^2\gamma+2\gamma\Delta w-2\alpha\Delta\gamma+2\gamma\alpha\lambda w+2\gamma\alpha w)/(4\beta_1\lambda\beta_2-\lambda^2\gamma^2-2\lambda\gamma^2-\gamma^2)-V$	$-(2\beta_1\alpha\lambda-2\beta_1\lambda w+2\beta_1\Delta\lambda-\gamma\Delta\lambda+\gamma w-\gamma\Delta+\gamma\lambda w-\gamma\alpha\lambda-\gamma\alpha)(-\gamma\beta_2 w+\gamma\beta_2\Delta-\gamma\lambda\beta_2\alpha-\gamma\lambda\beta_2\Delta+2\beta_1\lambda\beta_2\alpha-2\beta_1\lambda\beta_2 w-2\beta_1\lambda\beta_2\Delta+\lambda\gamma^2 w-\alpha\gamma^2-\lambda\gamma^2\alpha+\gamma\beta_2\alpha+\gamma\lambda\beta_2 w+\lambda\gamma^2\Delta+\lambda^2\gamma^2\Delta+\gamma^2 w)(-1+\lambda)/(4\beta_1\lambda\beta_2-\lambda^2\gamma^2-2\lambda\gamma^2-\gamma^2)^2$
$<P,F>$	$\frac{(-\beta_1\gamma\alpha+\beta_1\gamma w-2\Delta\gamma^2+\lambda\gamma^2\Delta-\alpha\gamma^2+\gamma^2 w+2\beta_2\alpha\beta_1+2\beta_2\beta_1\Delta-2\beta_2\beta_1 w)^2\lambda}{((\lambda\gamma^2+4\beta_2\beta_1-4\gamma^2)^2\beta_1)}$	$-V+(-8\gamma\beta_1^2\alpha^2\beta_2+8\beta_1\beta_2\Delta^2\gamma^2-4\alpha\gamma^4\Delta-4\gamma^2 w^2\beta_1\beta_2-2\gamma^4 w\Delta\lambda+4\alpha\beta_2\beta_1\Delta\gamma^2-12\gamma^2\lambda w\alpha\beta_2\beta_1+10\beta_1 w^2\gamma^3+10\alpha^2\beta_1\gamma^3+2\beta_1\beta_2 w\Delta\gamma^2\lambda+10\gamma^2\beta_1^2 w\alpha-8\alpha\beta_2^2\beta_1^2 w-8\alpha\beta_2\beta_1^3 w+\beta_1 w^2\gamma^3\lambda^2-4\beta_1\beta_2 w\Delta\gamma^2-8\beta_1^2\gamma\lambda w\alpha\beta_2+2\beta_1\beta_2\gamma^2\lambda^2 w\alpha+\alpha^2\beta_1\gamma^3\lambda^2-20\gamma^3 w\beta_1\alpha-5\beta_1^2\alpha^2\gamma^2-4\beta_1^2\beta_2^2\Delta^2+\beta_1^2\gamma^2\lambda w^2+4\gamma^4 w\Delta-4\gamma^3\beta_1 w\Delta-8\gamma\beta_1^2 w^2\beta_2+\beta_1^2\alpha^2\gamma^2\lambda+4\lambda\beta_1^2\beta_2^2\Delta^2+2\alpha\gamma^4\Delta\lambda-4\alpha^2\gamma^2\beta_2\beta_1+4\gamma^3\beta_1\alpha\Delta-2\alpha\beta_2\beta_1\Delta\gamma^2\lambda-\alpha^2\gamma^4+2\gamma^4 w\alpha-5\gamma^2\beta_1^2 w^2-4\lambda\beta_1^2\beta_2^2 w^2+16\gamma\beta_1^2 w\alpha\beta_2-7\gamma^3\lambda w^2\beta_1-7\alpha^2\gamma^3\lambda\beta_1-4\lambda\alpha^2\beta_2^2\beta_1^2+4\alpha^2\beta_2^2\beta_1^2+4\beta_1^2\beta_2^2 w^2-\Delta^2\gamma^4\lambda^2+4\Delta^2\gamma^4\lambda-\beta_1\beta_2\gamma^2\lambda^2 w^2-\beta_1\alpha^2\beta_2\gamma^2\lambda^2+8\gamma^2 w\alpha\beta_2\beta_1+14\gamma^3\lambda w\beta_1\alpha+6\gamma^2\lambda w^2\beta_1\beta_2+4\gamma\beta_1^2 w\beta_2\Delta-4\gamma\beta_1^2\alpha\beta_2\Delta-8\beta_1\beta_2\Delta^2\gamma^2\lambda+4\alpha^2\beta_2\beta_1^3+4\beta_1^3\beta_2 w^2-4\Delta^2\gamma^4-\gamma^4 w^2-2\alpha\beta_1\gamma^3\lambda^2 w+4\beta_1^2\alpha^2\gamma\lambda\beta_2+\lambda^2\beta_1\beta_2\Delta^2\gamma^2+8\lambda\alpha\beta_2^2\beta_1^2 w-2\beta_1^2\gamma^2\lambda w\alpha+4\beta_1^2\gamma\lambda w^2\beta_2+6\alpha^2\gamma^2\lambda\beta_2\beta_1)/((\lambda\gamma^2+4\beta_2\beta_1-4\gamma^2)^2\beta_1)$
$<P,P>$	$\frac{(\alpha-w+\Delta)^2\lambda}{4\beta_1}$	$\frac{(1-\lambda)(\alpha-w-\Delta)(\alpha-w+\Delta)}{4\beta_1}$
$<P,LP>$	$\frac{(-\Delta\gamma^2-\alpha\gamma^2+\gamma^2 w+\beta_1\gamma w-\beta_1\gamma\alpha+2\beta_2\alpha\beta_1+2\beta_2\beta_1\Delta-2\beta_2\beta_1 w)^2}{((4\beta_2\beta_1-3\gamma^2)^2\beta_1)}$	$(-\gamma\alpha\beta_2\beta_1+\alpha\gamma^3+2\alpha\beta_2\beta_1^2-2\alpha\gamma^2\beta_1-2\beta_1^2\beta_2 w+\gamma\beta_1\beta_2 w-\gamma\beta_1\beta_2\Delta+2\gamma^2 w\beta_1-\gamma^3 w+\Delta\gamma^3)(-2\beta_1 w+\gamma w-\gamma\alpha+2\alpha\beta_1-\gamma\Delta)/((4\beta_2\beta_1-3\gamma^2)^2\beta_1)$

Table 5–10: Payoff Functions for Retailers in Different Variety Outcomes in SP Game.

Variety Outcome	Retailer $R1$' Payoff (Π^1)	Retailer $R2$'s Payoff (Π^2)
$<F,F>$	$(1/4)(2\mathcal{B}_1\Delta\mathcal{B}_2 - 3\Gamma w\mathcal{B}_1 + 4\Gamma\Delta\mathcal{B}_2 - 2\mathcal{B}_1 w\mathcal{B}_2 - 3\Gamma w\mathcal{B}_2 + 4\Gamma\Delta\mathcal{B}_1 - w\mathcal{B}_1^2 + \Delta\mathcal{B}_1^2 - w\mathcal{B}_2^2 + \Delta\mathcal{B}_2^2 + 2\mathcal{B}_1\mathcal{A} + 2\mathcal{B}_2\mathcal{A} + 6\Gamma\mathcal{A} + 2\Delta\Gamma^2)\lambda(-10\Gamma w\mathcal{B}_1\mathcal{B}_2 + 12\Gamma\Delta\mathcal{B}_1\mathcal{B}_2 - w\mathcal{B}_1^3 + \Delta\mathcal{B}_1^3 - w\mathcal{B}_2^3 + \Delta\mathcal{B}_2^3 + 12\mathcal{A}\Gamma^2 + 2\mathcal{A}\mathcal{B}_1^2 + 2\mathcal{A}\mathcal{B}_2^2 + 4\Gamma^3\Delta + 6\Gamma\Delta\mathcal{B}_1^2 - 6\mathcal{B}_1 w\Gamma^2 + 4\mathcal{B}_1\mathcal{A}\mathcal{B}_2 + 10\Delta\Gamma^2\mathcal{B}_2 + 3\Delta\mathcal{B}_1^2\mathcal{B}_2 + 10\mathcal{B}_1\Delta\Gamma^2 + 3\mathcal{B}_1\Delta\mathcal{B}_2^2 + 6\Gamma\Delta\mathcal{B}_2^2 - 5\Gamma w\mathcal{B}_1^2 + 10\mathcal{B}_1\mathcal{A}\Gamma - 3w\mathcal{B}_1^2\mathcal{B}_2 - 3\mathcal{B}_1 w\mathcal{B}_2^2 + 10\Gamma\mathcal{A}\mathcal{B}_2 - 6\mathcal{B}_2 w\Gamma^2 - 5\Gamma w\mathcal{B}_2^2)/(3\Gamma^2 + 4\mathcal{B}_1\Gamma + 2\mathcal{B}_1\mathcal{B}_2 + \mathcal{B}_1^2 + 4\Gamma\mathcal{B}_2 + \mathcal{B}_2^2)^2 - V$	$-(1/4)(6\Gamma\mathcal{A} - 2\Delta\Gamma^2 - 3\Gamma w\mathcal{B}_1 - \Gamma\Delta\mathcal{B}_1 - w\mathcal{B}_1^2 - 3\Gamma w\mathcal{B}_2 - \Gamma\Delta\mathcal{B}_2 + 2\mathcal{B}_1\mathcal{A} - w\mathcal{B}_2^2 + 2\mathcal{B}_2\mathcal{A} - 2\mathcal{B}_1 w\mathcal{B}_2)(-1+\lambda)(10\mathcal{B}_1\mathcal{A}\Gamma + 4\mathcal{B}_1\mathcal{A}\mathcal{B}_2 + 10\Gamma\mathcal{A}\mathcal{B}_2 - 3w\mathcal{B}_1^2\mathcal{B}_2 - \Gamma\Delta\mathcal{B}_1^2 - 5\Gamma w\mathcal{B}_1^2 - 3\mathcal{B}_1 w\mathcal{B}_2^2 - 4\mathcal{B}_1\Delta\Gamma^2 - 6\mathcal{B}_1 w\Gamma^2 - 4\Delta\Gamma^2\mathcal{B}_2 - \Gamma\Delta\mathcal{B}_2^2 - 5\Gamma w\mathcal{B}_2^2 - 6\mathcal{B}_2 w\Gamma^2 + 12\mathcal{A}\Gamma^2 + 2\mathcal{A}\mathcal{B}_1^2 + 2\mathcal{A}\mathcal{B}_2^2 - w\mathcal{B}_1^3 - 4\Gamma^3\Delta - w\mathcal{B}_2^3 - 10\Gamma w\mathcal{B}_1\mathcal{B}_2 - 2\Gamma\Delta\mathcal{B}_1\mathcal{B}_2)/(3\Gamma^2 + 4\mathcal{B}_1\Gamma + 2\mathcal{B}_1\mathcal{B}_2 + \mathcal{B}_1^2 + 4\Gamma\mathcal{B}_2 + \mathcal{B}_2^2)^2 - V$
$<F,P>$	$(2\Gamma\mathcal{A} + 3\Gamma\lambda\mathcal{A} + \lambda\Delta\Gamma^2 - 2\Gamma w\mathcal{B}_2 + 2\Gamma\Delta\mathcal{B}_2 + 2\mathcal{B}_1\mathcal{A} - 3\Gamma w\lambda\mathcal{B}_1 + 4\Gamma\lambda\Delta\mathcal{B}_1 - 2\lambda\mathcal{B}_1^2 w - 2\mathcal{B}_1 w\mathcal{B}_2 + 2\lambda\mathcal{B}_1\mathcal{A} + 2\lambda\Delta\mathcal{B}_1^2 + 2\mathcal{B}_1\Delta\mathcal{B}_2)(2\mathcal{B}_1\mathcal{A}\mathcal{B}_2 + 2\mathcal{B}_1\Delta\mathcal{B}_2^2 + 2\Gamma\Delta\mathcal{B}_2^2 - 2\mathcal{B}_1 w\mathcal{B}_2^2 + 2\Gamma\mathcal{A}\mathcal{B}_2 - 2\Gamma w\mathcal{B}_2^2 + 2\mathcal{A}\lambda^2\mathcal{B}_1^2 + 2\lambda^2\Delta\mathcal{B}_1^3 + 3\mathcal{A}\lambda^2\Gamma^2 + \lambda^2\Delta\Gamma^3 - 2\lambda^2\mathcal{B}_1^3 w + 2\mathcal{A}\lambda\Gamma^2 + 2\mathcal{A}\lambda\mathcal{B}_1^2 - 7\Gamma w\mathcal{B}_2\lambda\mathcal{B}_1 + 8\Gamma\mathcal{B}_2\lambda\Delta\mathcal{B}_1 + 4\lambda\mathcal{B}_1\mathcal{A}\Gamma + 3\Gamma\mathcal{B}_2\lambda\mathcal{A} + 3\mathcal{B}_2\lambda\Delta\Gamma^2 - 2\mathcal{B}_2 w\lambda\Gamma^2 - 4\mathcal{B}_2\lambda\mathcal{B}_1^2 w + 2\mathcal{B}_2\lambda\mathcal{B}_1\mathcal{A} + 4\mathcal{B}_2\lambda\Delta\mathcal{B}_1^2 + 5\mathcal{A}\lambda^2\mathcal{B}_1\Gamma + 5\mathcal{B}_1\lambda^2\Delta\Gamma^2 - 3\mathcal{B}_1 w\lambda^2\Gamma^2 - 5\Gamma w\lambda^2\mathcal{B}_1^2 + 6\Gamma\lambda^2\Delta\mathcal{B}_1^2)/(3\lambda\Gamma^2 + 8\lambda\mathcal{B}_1\Gamma + 4\lambda\mathcal{B}_1^2 + 4\Gamma\mathcal{B}_2 + 4\mathcal{B}_1\mathcal{B}_2)^2 - V$	$-(3\Gamma\lambda\mathcal{A} - \lambda\Delta\Gamma^2 - 3\Gamma w\lambda\mathcal{B}_1 - \Gamma\lambda\Delta\mathcal{B}_1 - 2\lambda\mathcal{B}_1^2 w + \Gamma\mathcal{A} - \Gamma w\mathcal{B}_2 - \Gamma\Delta\mathcal{B}_2 - 2\mathcal{B}_1 w\mathcal{B}_2 + 2\lambda\mathcal{B}_1\mathcal{A} + 2\mathcal{B}_2\mathcal{A})(-1+\lambda)(-2\lambda\mathcal{B}_1^3 w - \lambda\Delta\Gamma^3 - 2\lambda\mathcal{B}_1\Delta\Gamma^2 - 5\Gamma w\lambda\mathcal{B}_1^2 - 3\mathcal{B}_1 w\lambda\Gamma^2 - \Gamma\lambda\Delta\mathcal{B}_1^2 + \mathcal{B}_1\mathcal{A}\Gamma + 2\mathcal{B}_1\mathcal{A}\mathcal{B}_2 + 2\Gamma\mathcal{A}\mathcal{B}_2 - 2w\mathcal{B}_1^2\mathcal{B}_2 - \Delta\Gamma^2\mathcal{B}_2 - \mathcal{B}_2 w\Gamma^2 + \mathcal{A}\Gamma^2 - 3\Gamma w\mathcal{B}_1\mathcal{B}_2 - \Gamma\Delta\mathcal{B}_1\mathcal{B}_2 + 3\mathcal{A}\lambda\Gamma^2 + 2\mathcal{A}\lambda\mathcal{B}_1^2 + 5\lambda\mathcal{B}_1\mathcal{A}\Gamma)/(3\lambda\Gamma^2 + 8\lambda\mathcal{B}_1\Gamma + 4\lambda\mathcal{B}_1^2 + 4\Gamma\mathcal{B}_2 + 4\mathcal{B}_1\mathcal{B}_2)^2$
$<F,LP>$	$(2\Gamma\mathcal{A} - 2\Gamma w\mathcal{B}_1 + 2\Gamma\Delta\mathcal{B}_1 + 3\Gamma\lambda\mathcal{A} + \lambda\Delta\Gamma^2 + 2\mathcal{B}_2\mathcal{A} - 3\Gamma w\mathcal{B}_2\lambda + 4\Gamma\mathcal{B}_2\lambda\Delta - 2\mathcal{B}_1 w\mathcal{B}_2 - 2\lambda\mathcal{B}_2^2 w + 2\mathcal{B}_2\lambda\mathcal{A} + 2\mathcal{B}_1\Delta\mathcal{B}_2 + 2\lambda\Delta\mathcal{B}_2^2)(2\mathcal{A}\lambda^2\mathcal{B}_2^2 + 2\Gamma\Delta\mathcal{B}_1^2 + 2\mathcal{B}_1\mathcal{A}\mathcal{B}_2 + 2\Delta\mathcal{B}_1^2\mathcal{B}_2 - 2\Gamma w\mathcal{B}_1^2 + 2\mathcal{B}_1\mathcal{A}\Gamma - 2w\mathcal{B}_1^2\mathcal{B}_2 + 3\mathcal{A}\lambda^2\Gamma^2 + \lambda^2\Delta\Gamma^3 + 2\mathcal{A}\lambda\Gamma^2 - 7\Gamma w\mathcal{B}_2\lambda\mathcal{B}_1 + 8\Gamma\mathcal{B}_2\lambda\Delta\mathcal{B}_1 + 3\lambda\mathcal{B}_1\mathcal{A}\Gamma + 4\Gamma\mathcal{B}_2\lambda\mathcal{A} + 2\mathcal{B}_2\lambda\mathcal{B}_1\mathcal{A} + 3\lambda\mathcal{B}_1\Delta\Gamma^2 - 2\mathcal{B}_1 w\lambda\Gamma^2 - 4\lambda\mathcal{B}_1\mathcal{B}_2^2 w + 4\lambda\mathcal{B}_1\Delta\mathcal{B}_2^2 + 5\Gamma\mathcal{B}_2\lambda^2\mathcal{A} + 5\mathcal{B}_2\lambda^2\Delta\Gamma^2 - 3\mathcal{B}_2 w\lambda^2\Gamma^2 - 5\Gamma w\mathcal{B}_2^2\lambda^2 + 6\Gamma\mathcal{B}_2^2\lambda^2\Delta - 2\lambda^2\mathcal{B}_2^3 w + 2\lambda^2\Delta\mathcal{B}_2^3 + 2\mathcal{A}\lambda\mathcal{B}_2^2)/(3\lambda\Gamma^2 + 4\mathcal{B}_1\Gamma + 4\mathcal{B}_1\mathcal{B}_2 + 8\Gamma\mathcal{B}_2\lambda + 4\lambda\mathcal{B}_2^2)^2 - V$	$-(\Gamma\mathcal{A} - \Gamma w\mathcal{B}_1 - \Gamma\Delta\mathcal{B}_1 + 3\Gamma\lambda\mathcal{A} - \lambda\Delta\Gamma^2 - 3\Gamma w\mathcal{B}_2\lambda - \Gamma\mathcal{B}_2\lambda\Delta - 2\mathcal{B}_1 w\mathcal{B}_2 + 2\mathcal{B}_1\mathcal{A} - 2\lambda\mathcal{B}_2^2 w + 2\mathcal{B}_2\lambda\mathcal{A})(-1+\lambda)(-\lambda\Delta\Gamma^3 - 2\lambda\mathcal{B}_2^3 w - 5\Gamma w\mathcal{B}_2^2\lambda - \Gamma\mathcal{B}_2^2\lambda\Delta + 2\mathcal{B}_1\mathcal{A}\Gamma + 2\mathcal{B}_1\mathcal{A}\mathcal{B}_2 + \Gamma\mathcal{A}\mathcal{B}_2 - 2\mathcal{B}_1 w\mathcal{B}_2^2 - \mathcal{B}_1\Delta\Gamma^2 - \mathcal{B}_1 w\Gamma^2 + \mathcal{A}\Gamma^2 - 3\Gamma w\mathcal{B}_1\mathcal{B}_2 - \Gamma\Delta\mathcal{B}_1\mathcal{B}_2 + 3\mathcal{A}\lambda\Gamma^2 + 5\Gamma\mathcal{B}_2\lambda\mathcal{A} - 2\mathcal{B}_2\lambda\Delta\Gamma^2 - 3\mathcal{B}_2 w\lambda\Gamma^2 + 2\mathcal{A}\lambda\mathcal{B}_2^2)/(3\lambda\Gamma^2 + 4\mathcal{B}_1\Gamma + 4\mathcal{B}_1\mathcal{B}_2 + 8\Gamma\mathcal{B}_2\lambda + 4\lambda\mathcal{B}_2^2)^2$
$<P,F>$	$(-3\Gamma w\lambda\mathcal{B}_1 + 4\Gamma\lambda\Delta\mathcal{B}_1 - 2\mathcal{B}_1\Delta\mathcal{B}_2 + 3\Gamma w\mathcal{B}_1 - 2\Gamma\Delta\mathcal{B}_2 + 2\mathcal{B}_1 w\mathcal{B}_2 + \Gamma w\mathcal{B}_2 - 4\Gamma\Delta\mathcal{B}_1 + \lambda\Delta\Gamma^2 + 3\Gamma\lambda\mathcal{A} - 2\lambda\mathcal{B}_1^2 w + 2\lambda\mathcal{B}_1\mathcal{A} + 2\lambda\Delta\mathcal{B}_1^2 + 2w\mathcal{B}_1^2 - 2\Delta\mathcal{B}_1^2 - 2\mathcal{B}_1\mathcal{A} - 2\mathcal{B}_2\mathcal{A} - 4\Gamma\mathcal{A} - \Delta\Gamma^2)\lambda(-6\Gamma\Delta\mathcal{B}_1^2 - 2\mathcal{B}_1\mathcal{A}\mathcal{B}_2 - 2\Delta\mathcal{B}_1^2\mathcal{B}_2 + 5\Gamma w\mathcal{B}_1^2 - 6\mathcal{B}_1\mathcal{A}\Gamma + 2w\mathcal{B}_1^2\mathcal{B}_2 - 2\Gamma\mathcal{A}\mathcal{B}_2 - 5\mathcal{B}_1\Delta\Gamma^2 + \lambda\Delta\Gamma^3 + 3\mathcal{B}_1 w\Gamma^2 + 3\Gamma w\mathcal{B}_1\mathcal{B}_2 - 4\Gamma\Delta\mathcal{B}_1\mathcal{B}_2 - \Gamma^3\Delta - 2\mathcal{A}\mathcal{B}_1^2 + 2w\mathcal{B}_1^3 - 2\Delta\mathcal{B}_1^3 + 3\mathcal{A}\lambda\Gamma^2 - 5\Gamma w\lambda\mathcal{B}_1^2 + 6\Gamma\lambda\Delta\mathcal{B}_1^2 + 5\lambda\mathcal{B}_1\mathcal{A}\Gamma + 5\lambda\mathcal{B}_1\Delta\Gamma^2 - 3\mathcal{B}_1 w\lambda\Gamma^2 - 4\mathcal{A}\Gamma^2 - 2\lambda\mathcal{B}_1^3 w + \mathcal{B}_2 w\Gamma^2 - 2\Delta\Gamma^2\mathcal{B}_2 + 2\lambda\Delta\mathcal{B}_1^3 + 2\mathcal{A}\lambda\mathcal{B}_1^2)/(3\lambda\Gamma^2 - 8\mathcal{B}_1\Gamma + 4\lambda\mathcal{B}_1^2 - 4\mathcal{B}_1\mathcal{B}_2 - 4\Gamma\mathcal{B}_2 + 8\lambda\mathcal{B}_1\Gamma - 3\Gamma^2 - 4\mathcal{B}_1^2)^2$	$-(-5\Gamma\mathcal{A} + \Delta\Gamma^2 + 2w\mathcal{B}_1^2 - 4\mathcal{B}_1\mathcal{A} + \Gamma\Delta\mathcal{B}_1 + 3\Gamma w\mathcal{B}_1 + 3\Gamma\lambda\mathcal{A} + 2\Gamma w\mathcal{B}_2 - \lambda\Delta\Gamma^2 + 2\lambda\mathcal{B}_1\mathcal{A} - 3\Gamma w\lambda\mathcal{B}_1 + 2\mathcal{B}_1 w\mathcal{B}_2 - \Gamma\lambda\Delta\mathcal{B}_1 - 2\lambda\mathcal{B}_1^2 w)(-\Gamma\Delta\mathcal{B}_1^2 + 4\mathcal{B}_1\mathcal{A}\mathcal{B}_2 - 5\Gamma w\mathcal{B}_1^2 + 9\mathcal{B}_1\mathcal{A}\Gamma - 4w\mathcal{B}_1^2\mathcal{B}_2 + 5\Gamma\mathcal{A}\mathcal{B}_2 - 2\mathcal{B}_1\Delta\Gamma^2 - 2\Gamma w\mathcal{B}_2^2 - 2\mathcal{B}_1 w\mathcal{B}_2^2 + 2\lambda\Delta\Gamma^3 - 3\mathcal{B}_1 w\Gamma^2 - 7\Gamma w\mathcal{B}_1\mathcal{B}_2 - \Gamma\Delta\mathcal{B}_1\mathcal{B}_2 - \Gamma^3\Delta + 4\mathcal{A}\mathcal{B}_1^2 - 2w\mathcal{B}_1^3 + 2\mathcal{A}\lambda^2\mathcal{B}_1^2 - 2\lambda^2\mathcal{B}_1^3 w + 3\mathcal{A}\lambda^2\Gamma^2 - \lambda^2\Delta\Gamma^3 - 8\mathcal{A}\lambda\Gamma^2 + 5\mathcal{A}\lambda^2\mathcal{B}_1\Gamma - 2\mathcal{B}_1\lambda^2\Delta\Gamma^2 - 3\mathcal{B}_1 w\lambda^2\Gamma^2 - 5\Gamma w\lambda^2\mathcal{B}_1^2 - \Gamma\lambda^2\Delta\mathcal{B}_1^2 + 7\Gamma w\mathcal{B}_2\lambda\mathcal{B}_1 + \Gamma\mathcal{B}_2\lambda\Delta\mathcal{B}_1 + 10\Gamma w\lambda\mathcal{B}_1^2 + 2\Gamma\lambda\Delta\mathcal{B}_1^2 - 14\lambda\mathcal{B}_1\mathcal{A}\Gamma - 3\Gamma\mathcal{B}_2\lambda\mathcal{A} - 2\mathcal{B}_2\lambda\mathcal{B}_1\mathcal{A} + 4\lambda\mathcal{B}_1\Delta\Gamma^2 + 6\mathcal{B}_1 w\lambda\Gamma^2 + 4\mathcal{B}_2\lambda\mathcal{B}_1^2 w + \mathcal{B}_2\lambda\Delta\Gamma^2 + 2\mathcal{B}_2 w\lambda\Gamma^2 + 5\mathcal{A}\Gamma^2 + 4\lambda\mathcal{B}_1^3 w - 2\mathcal{B}_2 w\Gamma^2 - \Delta\Gamma^2\mathcal{B}_2 - 6\mathcal{A}\lambda\mathcal{B}_1^2)/(3\lambda\Gamma^2 - 8\mathcal{B}_1\Gamma + 4\lambda\mathcal{B}_1^2 - 4\mathcal{B}_1\mathcal{B}_2 - 4\Gamma\mathcal{B}_2 + 8\lambda\mathcal{B}_1\Gamma - 3\Gamma^2 - 4\mathcal{B}_1^2)^2 - V$
$<P,P>$	$((2\mathcal{B}_1\mathcal{A} + 4\Gamma\Delta\mathcal{B}_1 - 2w\mathcal{B}_1^2 - 3\Gamma w\mathcal{B}_1 + 2\Delta\mathcal{B}_1^2 + 3\Gamma\mathcal{A} + \Delta\Gamma^2)\lambda(3\mathcal{A}\Gamma^2 + 5\mathcal{B}_1\mathcal{A}\Gamma + 2\mathcal{A}\mathcal{B}_1^2 + 6\Gamma\Delta\mathcal{B}_1^2 - 2w\mathcal{B}_1^3 - 5\Gamma w\mathcal{B}_1^2 + 2\Delta\mathcal{B}_1^3 + 5\mathcal{B}_1\Delta\Gamma^2 - 3\mathcal{B}_1 w\Gamma^2 + \Gamma^3\Delta))/(3\Gamma^2 + 8\mathcal{B}_1\Gamma + 4\mathcal{B}_1^2)^2$	$(-(3\Gamma\mathcal{A} - \Delta\Gamma^2 - 3\Gamma w\mathcal{B}_1 - \Gamma\Delta\mathcal{B}_1 - 2w\mathcal{B}_1^2 + 2\mathcal{B}_1\mathcal{A})(-1+\lambda)(3\mathcal{A}\Gamma^2 + 5\mathcal{B}_1\mathcal{A}\Gamma + 2\mathcal{A}\mathcal{B}_1^2 - 2\mathcal{B}_1\Delta\Gamma^2 - 5\Gamma w\mathcal{B}_1^2 - 3\mathcal{B}_1 w\Gamma^2 - \Gamma\Delta\mathcal{B}_1^2 - 2w\mathcal{B}_1^3 - \Gamma^3\Delta))/(3\Gamma^2 + 8\mathcal{B}_1\Gamma + 4\mathcal{B}_1^2)^2$
$<P,LP>$	$\frac{(\mathcal{A}-\mathcal{B}_1 w+\mathcal{B}_1\Delta)^2}{4\mathcal{B}_1}$	$\frac{(\mathcal{A}-\mathcal{B}_2 w)^2}{4\mathcal{B}_2}$

Table 5–11: Payoff Functions for Retailers in Different Variety Outcomes in DFP Game.

Variety Outcome	Retailer $R1$' Payoff (Π^1)	Retailer $R2$'s Payoff (Π^2)
$< F, F >$	$\frac{(2\mathcal{A}-\mathcal{B}_1 w+\mathcal{B}_1\Delta-\mathcal{B}_2 w+\mathcal{B}_2\Delta)^2\lambda}{4(\mathcal{B}_1+\mathcal{B}_2)} - V$	$\frac{(2\mathcal{A}-\mathcal{B}_1 w-\mathcal{B}_1\Delta-\mathcal{B}_2 w-\mathcal{B}_2\Delta)(1-\lambda)(2\mathcal{A}-\mathcal{B}_1 w+\mathcal{B}_1\Delta-\mathcal{B}_2 w+\mathcal{B}_2\Delta)}{4(\mathcal{B}_1+\mathcal{B}_2)} - V$
$< F, P >$	$\frac{(\mathcal{A}\lambda-\mathcal{B}_1 w\lambda+\lambda\mathcal{B}_1\Delta+\mathcal{A}-\mathcal{B}_2 w+\mathcal{B}_2\Delta)^2}{4(\lambda\mathcal{B}_1+\mathcal{B}_2)} - V$	$\frac{(\mathcal{A}\lambda-\mathcal{B}_1 w\lambda-\lambda\mathcal{B}_1\Delta+\mathcal{A}-\mathcal{B}_2 w-\mathcal{B}_2\Delta)(1-\lambda)(\lambda\mathcal{B}_1\mathcal{A}+2\mathcal{B}_2\mathcal{A}-\lambda\mathcal{B}_1^2 w+\lambda\Delta\mathcal{B}_1^2-\mathcal{B}_1\mathcal{A}-\mathcal{B}_1 w\mathcal{B}_2+\mathcal{B}_1\Delta\mathcal{B}_2)}{4(\lambda\mathcal{B}_1+\mathcal{B}_2)^2}$
$< F, LP >$	$\frac{(\mathcal{A}-\mathcal{B}_1 w+\mathcal{B}_1\Delta+\mathcal{A}\lambda-\lambda\mathcal{B}_2 w+\lambda\mathcal{B}_2\Delta)^2}{4(\mathcal{B}_1+\lambda\mathcal{B}_2)} - V$	$\frac{(\mathcal{A}-\mathcal{B}_1 w-\mathcal{B}_1\Delta+\mathcal{A}\lambda-\lambda\mathcal{B}_2 w-\lambda\mathcal{B}_2\Delta)(1-\lambda)(2\mathcal{B}_1\mathcal{A}+\lambda\mathcal{B}_2\mathcal{A}-\mathcal{B}_2\mathcal{A}-\mathcal{B}_1 w\mathcal{B}_2+\mathcal{B}_1\Delta\mathcal{B}_2-\lambda\mathcal{B}_2^2 w+\lambda\Delta\mathcal{B}_2^2)}{4(\mathcal{B}_1+\lambda\mathcal{B}_2)^2}$
$< P, F >$	$\frac{(\mathcal{A}-\mathcal{B}_1 w+\mathcal{B}_1\Delta)^2\lambda}{4\mathcal{B}_1}$	$\frac{(\mathcal{B}_2\mathcal{A}^2-4\mathcal{B}_2\mathcal{A}\mathcal{B}_1 w-\mathcal{B}_2\mathcal{A}^2\lambda+2\lambda\mathcal{B}_2\mathcal{A}\mathcal{B}_1 w+\mathcal{B}_2\mathcal{B}_1^2 w^2-\mathcal{B}_2\mathcal{B}_1^2 w^2\lambda-\mathcal{B}_2\mathcal{B}_1^2\Delta^2+\lambda\mathcal{B}_2\mathcal{B}_1^2\Delta^2+\mathcal{B}_1\mathcal{A}^2+\mathcal{B}_1\mathcal{B}_2^2 w^2)}{4(\mathcal{B}_1\mathcal{B}_2)} - V$
$< P, P >$	$\frac{(\mathcal{A}-\mathcal{B}_1 w+\mathcal{B}_1\Delta)^2\lambda}{4\mathcal{B}_1}$	$\frac{(\mathcal{A}-\mathcal{B}_1 w-\mathcal{B}_1\Delta)(1-\lambda)(\mathcal{A}-\mathcal{B}_1 w+\mathcal{B}_1\Delta)}{4\mathcal{B}_1}$
$< P, LP >$	$\frac{(\mathcal{A}-\mathcal{B}_1 w+\mathcal{B}_1\Delta)^2}{4\mathcal{B}_1}$	$\frac{(\mathcal{A}-\mathcal{B}_2 w)^2}{4\mathcal{B}_2}$

5.3. Proofs of propositions and lemmas for Chapter 2

Proof. Lemma 1:

As presented in Appendix 5.2, the outcome $< F, LP >$ in any game is the locus of points where $\Pi^1(F, LP) \geq \Pi^1(P, LP)$, $\Pi^2(F, F) \leq \Pi^2(F, LP)$, and $\Pi^2(F, P) \leq \Pi^2(F, LP)$. We can find the corresponding bounds for the first two inequalities as follows:

$\mathcal{Y}^1_{(F,LP),(P,LP)} = (144\alpha\beta_2^3\Delta w\gamma^2\beta_1 + 144\alpha\beta_2^2\gamma^3\Delta w\beta_1 - 432\alpha\beta_2^3\gamma\Delta w\beta_1^2 + 9\beta_2^2w^2\gamma^4 - 18\beta_2w^2\gamma^5 - 18\alpha^2\beta_2\gamma^5 - 2\gamma^6w^2\Delta - 10\alpha\gamma^6w + 9\alpha^2\beta_2^2\gamma^4 - 256\beta_2^3\beta_1^3w^2\Delta + 12\beta_1^2\alpha^2\beta_2^2\gamma^2 + 12\beta_2^2\beta_1^2w^2\gamma^2 + 256\beta_2^3\beta_1^3\Delta^2w^2 - 128\beta_1^3\alpha\beta_2^3w - 18\beta_2^2w^2\Delta\gamma^4 + 72\beta_2^3w^2\gamma^2\beta_1 + 9\beta_2^2\Delta^2w^2\gamma^4 + 72\alpha^2\beta_2^3\gamma^2\beta_1 - 18\alpha\beta_2^2w\gamma^4 - 144\alpha^2\beta_2^3\gamma\beta_1^2 - 18\beta_2\Delta^2w^2\gamma^5 + 36\beta_2\alpha\gamma^5w + 36\beta_2w^2\gamma^5\Delta - 144\beta_2^3w^2\gamma\beta_1^2 + 2\alpha\gamma^6\Delta w + 5\gamma^6w^2 + 5\alpha^2\gamma^6 - 96\beta_1^2\alpha\beta_2^2\Delta w\gamma^2 + 256\beta_1^3\alpha\beta_2^3\Delta w + 96\beta_2^2\beta_1^2w^2\Delta\gamma^2 + 72\beta_2\beta_1\Delta^2w^2\gamma^4 - 240\beta_2^2\beta_1^2\Delta^2w^2\gamma^2 - 24\beta_1^2\alpha\beta_2^2w\gamma^2 - 144\beta_2^3w^2\Delta\gamma^2\beta_1 + 18\alpha\beta_2^2\Delta w\gamma^4 + 72\beta_2^3\Delta^2w^2\gamma^2\beta_1 - 144\alpha\beta_2^3w\gamma^2\beta_1 + 144\beta_2^2\Delta^2w^2\gamma^3\beta_1 - 288\beta_2^3\Delta^2w^2\gamma\beta_1^2 + 288\alpha\beta_2^3\gamma w\beta_1^2 - 144\beta_2^2w^2\gamma^3\Delta\beta_1 + 432\beta_2^3w^2\gamma\Delta\beta_1^2 - 36\beta_2\alpha\gamma^5\Delta w + 64\beta_1^3\alpha^2\beta_2^3 + 64\beta_2^3\beta_1^3w^2 - 7\gamma^6\Delta^2w^2)/(36\beta_2(\beta_2\beta_1 - \gamma^2)(-\gamma^2 + 4\beta_2\beta_1)^2)$

$$\mathcal{Y}^2_{(F,F),(F,LP)} = \frac{(\alpha - w - \Delta w)(-2\beta_2\gamma\alpha + \alpha\beta_2^2 + 2\beta_2\gamma w + 2\beta_2\gamma\Delta w - \beta_2^2 w - \beta_2^2\Delta w + \alpha\gamma^2 - w\gamma^2 - \Delta w\gamma^2)}{9\beta_2(\beta_2\beta_1 - \gamma^2)}$$

And it is easy to see that for any values of $\frac{\Delta}{w}, \lambda, \frac{\gamma}{\beta_1}, \frac{\beta_1}{\beta_2}, \frac{w}{\alpha}$:

- $\forall\ V \geq \mathcal{Y}^1_{(F,LP),(P,LP)} : \Pi^1(F, LP) \leq \Pi^1(P, LP)$
- $\forall\ V \leq \mathcal{Y}^1_{(F,LP),(P,LP)} : \Pi^1(F, LP) \geq \Pi^1(P, LP)$ *
- $\forall\ V \geq \mathcal{Y}^2_{(F,F),(F,LP)} : \Pi^2(F, F) \leq \Pi^2(F, LP)$ *
- $\forall\ V \leq \mathcal{Y}^2_{(F,F),(F,LP)} : \Pi^2(F, F) \geq \Pi^2(F, LP)$

But, for the third condition ($\Pi^2(F, P) \leq \Pi^2(F, LP)$), it is impossible to happen if $\beta_1 < \beta_2$ since:

$$\beta_1 < \beta_2 \to \frac{1}{\beta_1} > \frac{1}{\beta_2} \to \frac{(\alpha - w - \Delta)^2}{9\beta_1} > \frac{(\alpha - w - \Delta)^2}{9\beta_2} \to \Pi^2(F, P) > \Pi^2(F, LP)$$

which means that $R2$ is always better off to offer popular product when facing the full assortment by $R1$. Therefore, there is no point that satisfies all the three inequality conditions, and hence, it is impossible for $< F, LP >$ to be an equilibrium at SQ game.

Proof. **Lemma 2:** In order to show that $< P, F >$ never prevails as a unique equilibrium in SQ game and it always comes with $< F, P >$, it is sufficient to do the following steps:
(1) To capture all the bounds for $< P, F >$ and $< F, P >$ regions (note that because each variety outcome is to be defined by three inequalities, there are at most three binding constraints for each case plus a non-negativity constraint: $V \geq 0$);
(2) To show that at least one of the lower (resp. upper) bounds for $< P, F >$ is always higher (resp. lower) than the lower (resp. upper) bound for $< F, P >$ for the whole possible values of $\frac{\Delta}{w}, \lambda, \frac{\gamma}{\beta_1}, \frac{\beta_1}{\beta_2}$, and $\frac{w}{\alpha}$.

First, to capture the bounds for each region:

The outcome $< F, P >$ is defined by the three inequalities: $\Pi^1(F, P) \geq \Pi^1(P, P)$; $\Pi^2(F, F) \leq \Pi^2(F, P)$; and $\Pi^2(F, P) \geq \Pi^2(F, LP)$. We showed that because $\beta_1 \leq \beta_2$, we always have $\Pi^2(F, P) \geq \Pi^2(F, LP)$ for the entire range of parameters. Therefore, $\mathcal{Y}^2_{(F,P),(F,LP)}$ is not binding and $< F, P >$ outcome is only defined by $\mathcal{Y}^1_{(F,P),(P,P)}$ and $\mathcal{Y}^2_{(F,F),(F,P)}$.

It is easy to see that

1. $\forall\ V \geq \mathcal{Y}^1_{(F,P),(P,P)} : \Pi^1(F, P) \leq \Pi^1(P, P)$
2. $\forall\ V \leq \mathcal{Y}^1_{(F,P),(P,P)} : \Pi^1(F, P) \geq \Pi^1(P, P)$ *
3. $\forall\ V \geq \mathcal{Y}^2_{(F,F),(F,P)} : \Pi^2(F, F) \leq \Pi^2(F, P)$ *
4. $\forall\ V \leq \mathcal{Y}^2_{(F,F),(F,P)} : \Pi^2(F, F) \geq \Pi^2(F, P)$

Therefore, at a fixed point (fixed values of $\frac{\Delta}{w}, \lambda, \frac{\gamma}{\beta_1}, \frac{\beta_1}{\beta_2}, \frac{w}{\alpha}$), for any variety cost level such that

$$\mathcal{Y}^2_{(F,F),(F,P)} \leq V \leq \mathcal{Y}^1_{(F,P),(P,P)}$$

$< F, P >$ would be the equilibrium outcome. Therefore, $\mathcal{Y}^2_{(F,F),(F,P)}$ and $\mathcal{Y}^1_{(F,P),(P,P)}$ are the lower and upper bounds for the outcome $< F, P >$ respectively.

The same procedure applies for finding the lower and upper bounds for $< P, F >$. This would be the variety outcome of the game only if $\Pi^1(F, F) \leq \Pi^1(P, F)$; $\Pi^2(P, F) \geq \Pi^2(P, P)$;

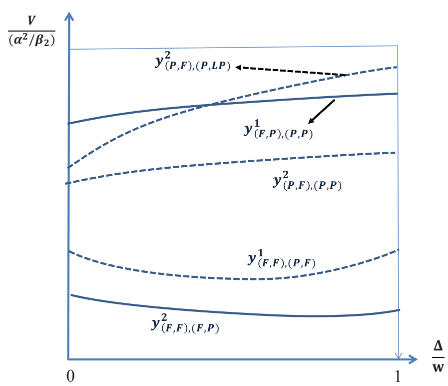
and $\Pi^2(P,F) \geq \Pi^2(P,LP)$. These three inequalities lead to the definition of $\mathcal{Y}^1_{(F,F),(P,F)}$, $\mathcal{Y}^2_{(P,F),(P,P)}$ and $\mathcal{Y}^2_{(P,F),(P,LP)}$ as follows:

$$\mathcal{Y}^1_{(F,F),(P,F)} = \frac{(\alpha - w + 2\Delta w)(-2\beta_1\gamma\alpha + 2\beta_1\gamma w - 4\beta_1\gamma\Delta w + \beta_1^2\alpha - \beta_1^2 w + 2\beta_1^2\Delta w + \alpha\gamma^2 - w\gamma^2 + 2\Delta w\gamma^2)}{(9(\beta_2\beta_1 - \gamma^2)\beta_1)}$$

$$\mathcal{Y}^2_{(P,F),(P,P)} = \frac{(\beta_1^2 w^2 + \beta_1^2\alpha^2 + \gamma^2 w^2 + \alpha^2\gamma^2 - 2\beta_1 w^2\gamma - 2\alpha\gamma^2 w - 2\beta_1^2\alpha w - 2\beta_1\alpha^2\gamma + 4\beta_1\gamma\alpha w)}{(4(\beta_2\beta_1 - \gamma^2)\beta_1)}$$

$\mathcal{Y}^2_{(P,F),(P,LP)} = (128\beta_2^3\beta_1^3 w^2\Delta + 12\beta_1^2\alpha^2\beta_2^2\gamma^2 + 12\beta_2^2\beta_1^2 w^2\gamma^2 + 64\beta_2^3\beta_1^3\Delta^2 w^2 - 128\beta_1^3\alpha\beta_2^3 w + 8\alpha\gamma^6\Delta w + 72\beta_1^3\alpha^2\gamma^2\beta_2 + 72\beta_1^3 w^2\gamma^2\beta_2 - 18\beta_1^2\alpha w\gamma^4 - 144\beta_1^3\alpha^2\gamma\beta_2^2 - 144\beta_1^3 w^2\gamma\beta_2^2 + 36\beta_1\alpha\gamma^5 w + 144\beta_2^2\beta_1^3\alpha\gamma\Delta w - 144\beta_2\beta_1^2\alpha\gamma^3\Delta w + 120\beta_1^2\alpha\beta_2^2\Delta w\gamma^2 + 5\gamma^6 w^2 + 5\alpha^2\gamma^6 + 64\beta_2^3\beta_1^3 w^2 + 64\beta_1^3\alpha^2\beta_2^3 - 8\gamma^6 w^2\Delta - 18\beta_1 w^2\gamma^5 - 18\beta_1\alpha^2\gamma^5 + 9\beta_1^2 w^2\gamma^4 + 9\beta_1^2\alpha^2\gamma^4 - 4\gamma^6\Delta^2 w^2 - 10\alpha\gamma^6 w - 128\beta_1^3\alpha\beta_2^3\Delta w - 120\beta_2^2\beta_1^2 w^2\Delta\gamma^2 + 72\beta_2\beta_1\Delta^2 w^2\gamma^4 - 132\beta_2^2\beta_1^2\Delta^2 w^2\gamma^2 - 24\beta_1^2\alpha\beta_2^2 w\gamma^2 - 144\beta_1^3\alpha w\gamma^2\beta_2 + 288\beta_1^3\alpha\gamma w\beta_2^2 + 144\beta_2\beta_1^2 w^2\gamma^3\Delta - 144\beta_1^3 w^2\gamma\beta_2^2\Delta)/(36(\beta_2\beta_1 - \gamma^2)\beta_1(-\gamma^2 + 4\beta_2\beta_1)^2)$

It is also easy to see that at fixed values of $\frac{\Delta}{w}, \lambda, \frac{\gamma}{\beta_1}, \frac{\beta_1}{\beta_2}, \frac{w}{\alpha}$, a variety cost level such that $\mathcal{Y}^1_{(F,F),(P,F)} \leq V$, $\mathcal{Y}^2_{(P,F),(P,P)} \geq V$, and $\mathcal{Y}^2_{(P,F),(P,LP)} \geq V$ will lead to $< P,F >$ equilibrium outcome. Therefore, $\mathcal{Y}^1_{(F,F),(P,F)}$ is the lower bound and the other two are upper bounds for the region $< P,F >$ as presented hypothetically in the following graph.



Second, we show that for any point such that $0 < \Delta < w < \alpha$ and $0 \leq \gamma < \beta_1 < \beta_2$: $\mathcal{Y}^1_{(F,F),(P,F)} \geq \mathcal{Y}^2_{(F,F),(F,P)}$; and $\mathcal{Y}^2_{(P,F),(P,P)} \leq \mathcal{Y}^1_{(F,P),(P,P)}$.

(1) It is easy to see that $\mathcal{Y}^1_{(F,F),(P,F)} \geq \mathcal{Y}^2_{(F,F),(F,P)}$ because

$$\mathcal{Y}^1_{(F,F),(P,F)} - \mathcal{Y}^2_{(F,F),(F,P)} = \frac{3w\Delta(-2w\gamma^2 + \Delta w\gamma^2 + \beta_1^2\Delta w - 2\beta_1^2 w + 2\beta_1^2\alpha - 2\beta_1\gamma\Delta w + 2\alpha\gamma^2 + 4\beta_1\gamma w - 4\beta_1\alpha\gamma)}{9(\beta_1\beta_2 - \gamma^2)\beta_1} \geq$$

given $0 \leq \gamma < \beta_1 < \beta_2$ and $0 < w < \alpha$.

(2) Note that $\mathcal{Y}^2_{(P,F),(P,P)}$ is fixed at $\frac{\Delta}{w}$. Besides, $\mathcal{Y}^1_{(F,P),(P,P)}(\frac{\Delta}{w} = 0) = \mathcal{Y}^2_{(P,F),(P,P)}$. Hence, $\mathcal{Y}^2_{(P,F),(P,P)} \leq \mathcal{Y}^1_{(F,P),(P,P)}$ because $\mathcal{Y}^1_{(F,P),(P,P)}$ is always increasing at $\frac{\Delta}{w}$ as the first order condition implies it:

$$\frac{\partial \mathcal{Y}^1_{(F,P),(P,P)}}{\partial\left(\frac{\Delta}{w}\right)} = -\frac{\left((\frac{\gamma}{\beta_1}) - 1\right)^2 \left((\frac{w}{\alpha})(\frac{\Delta}{w}) - (\frac{w}{\alpha}) + 1\right)\left(\frac{w}{\alpha}\right)}{2\left((\frac{\gamma}{\beta_1})^2(\frac{\beta_1}{\beta_2}) - 1\right)} \geq 0$$

given that $0 \leq \gamma < \beta_1 < \beta_2$ and $0 < \Delta < w < a$.

Therefore, considering these facts, there is no unique equilibrium of $< P, F >$ in SQ game as it always comes with the variety choice of $< F, P >$.

□

To complete the proofs for propositions 1-4, in the following lemma, we first show that the full assortment equilibrium for $R1$ can be characterized only by $\mathcal{Y}^1_{(F,P),(P,P)}$.

Lemma 6 ***In SQ game, at any point such that*** $0 \leq \gamma < \beta_1 < \beta_2$ ***and*** $0 < \Delta < w < a$***, for variety cost values of***

$$0 \leq V \leq \mathcal{Y}^1_{(F,P),(P,P)},$$

the powerful retailer $R1$ ***would carry the full assortment in the equilibrium.***

Lemma 6: The entire points where $R1$ carries full assortment are captured by the equilibrium outcomes of $< F, F >$, $< F, P >$, and $< F, LP >$. We already showed that $< F, LP >$ never happens and $< F, P >$ outcome is defined by $\mathcal{Y}^1_{(F,P),(P,P)}$ and $\mathcal{Y}^2_{(F,F),(F,P)}$(see the proofs of Lemma 1 and 2). Here, we find the effective bounds for $< F, F >$ outcome as follows:
We know that the outcome $< F, F >$ is defined where $\Pi^1(F, F) \geq \Pi^1(P, F)$; $\Pi^2(F, F) \geq \Pi^2(F, P)$; and $\Pi^2(F, F) \geq \Pi^2(F, LP)$. It is easy to see that at a fixed $\frac{\Delta}{w}, \lambda, \frac{\gamma}{\beta_1}, \frac{\beta_1}{\beta_2}$, and $\frac{w}{\alpha}$:

- $\forall\ V \leq \mathcal{Y}^1_{(F,F),(P,F)} : \Pi^1(F, F) \geq \Pi^1(P, F)$ *
- $\forall\ V \leq \mathcal{Y}^2_{(F,F),(F,P)} : \Pi^1(F, F) \geq \Pi^1(F, P)$ *

- $\forall\ V \leq \mathcal{Y}^2_{(F,F),(F,LP)}$: $\Pi^2(F,F) \geq \Pi^2(F,LP)$ *

Therefore, all $\mathcal{Y}^1_{(F,F),(P,F)}$, $\mathcal{Y}^2_{(F,F),(F,P)}$, $\mathcal{Y}^2_{(F,F),(F,LP)}$ are the upper bounds and obviously$V \geq 0$ is the only lower bound for $< F, F >$. We already showed that for any point such that $0 \leq \gamma < \beta_1 < \beta_2$ and $0 < \Delta < w < a$, we have $\mathcal{Y}^2_{(F,F),(F,P)} \leq \mathcal{Y}^1_{(F,F),(P,F)}$ (see the proof of Lemma 2). Here we show that for all the possible range of parameters, $\mathcal{Y}^2_{(F,F),(F,P)} \leq \mathcal{Y}^2_{(F,F),(F,LP)}$ (this simplifies $< F, F >$ as a region below the boundary $\mathcal{Y}^2_{(F,F),(F,P)}$):

It would be much easier to check, if we transfer the equations to the format of $A(x-B)^2+C$, as below:

$\mathcal{Y}^2_{(F,F),(F,P)} = \frac{w^2(\beta_1^2-2\beta_1\gamma+\gamma^2)(\Delta-(\alpha-w)/w)^2}{9\beta_1(\beta_1\beta_2-\gamma^2)}$
$\mathcal{Y}^2_{(F,F),(F,LP)} = \frac{w^2(\beta_2^2-2\beta_2\gamma+\gamma^2)(\Delta-(\alpha-w)/w)^2}{9\beta_2(\beta_1\beta_2-\gamma^2)}$

Since $0 < \gamma < \beta_1 < \beta_2$ the followings hold: $1 < \frac{\beta_2}{\beta_1} < \frac{\beta_2-\gamma}{\beta_1-\gamma} \rightarrow 1 < \frac{\beta_2}{\beta_1} < \frac{(\beta_2-\gamma)^2}{(\beta_1-\gamma)^2}$. Therefore, it is easy to see that $\mathcal{Y}^2_{(F,F),(F,P)} \leq \mathcal{Y}^2_{(F,F),(F,LP)}$. As a result, among the three upper bounds for $< F, F >$ outcome only $\mathcal{Y}^2_{(F,F),(F,P)}$ is effective. Hence, we can define the outcome $< F, F >$ as follows:

For any $0 \leq \gamma < \beta_1 < \beta_2$ and $0 < \Delta < w < a$, any variety cost level less than

$$0 \leq\ V \leq \mathcal{Y}^2_{(F,F),(F,P)}$$

would lead to $< F, F >$ outcome.

Hence, the whole region where $R1$ carries full assortment can be characterized by $\mathcal{Y}^2_{(F,F),(F,P)}$ and $\mathcal{Y}^1_{(F,P),(P,P)}$. Now we make it even simpler by showing that for any point such that $0 \leq \gamma < \beta_1 < \beta_2$ and $0 < \Delta < w < a$, we have $\mathcal{Y}^2_{(F,F),(F,P)} \leq \mathcal{Y}^1_{(F,P),(P,P)}$. To do so, we simplify the equations for $\mathcal{Y}^1_{(F,P),(P,P)}$ and $\mathcal{Y}^2_{(F,F),(F,P)}$ to $A(x-B)^2+C$ format, as follows:

$$\mathcal{Y}^2_{(F,F),(F,P)} = \frac{w^2\beta_2(\beta_1^2-2\beta_1\gamma+\gamma^2)(\Delta-\frac{\alpha-w}{w})^2}{9\alpha^2(\beta_1\beta_2-\gamma^2)\beta_1}$$

$$\mathcal{Y}^1_{(F,P),(P,P)} = \frac{w^2\beta_2(\beta_1^2-2\beta_1\gamma+\gamma^2)(\Delta+\frac{\alpha-w}{w})^2}{4\alpha^2(\beta_1\beta_2-\gamma^2)\beta_1}$$

Since for any $0 \leq \frac{\Delta}{w}$, $(\Delta+\frac{\alpha-w}{w})^2 \geq (\Delta-\frac{\alpha-w}{w})^2$ then it is easy to find that $\mathcal{Y}^2_{(F,F),(F,P)} \leq \mathcal{Y}^1_{(F,P),(P,P)}$.

This simplifies the full assortment region by $R1$ as for all the parameter values such that $0 \leq \gamma < \beta_1 < \beta_2$ and $0 < \Delta < w < a$, any variety cost lower than

$$0 \leq \ V \leq \mathcal{Y}^1_{(F,P),(P,P)}$$

would lead to full assortment for the powerful retailer $R1$. □

Proof. **Proposition 1:**

As stated before, determining the final outcome of the game in regions with multiple equilibria is impossible. To prove, therefore, we just focus on the situations where the powerful retailer carries full assortment and show that the area where $R1$ carries full assortment always expands in his cost advantage ($\frac{\Delta}{w}$).

Lemma 6 extremely simplifies the proof, as we only need to show that $\mathcal{Y}^1_{(F,P),(P,P)}$ is increasing in $\frac{\Delta}{w} \in [0,1]$, and it is very straightforward using first-order condition as follows:

$$\mathcal{Y}^1_{(F,P),(P,P)} = \frac{w^2(-2\beta_1\gamma + \gamma^2 + \beta_1^2)(\Delta + \frac{\alpha - w}{w})^2}{4(\beta_2\beta_1 - \gamma^2)\beta_1}$$

$$\frac{\partial \mathcal{Y}^1_{(F,P),(P,P)}}{\partial\left(\frac{\Delta}{w}\right)} = -\frac{\left((\frac{\gamma}{\beta_1}) - 1\right)^2 \left((\frac{w}{\alpha})(\frac{\Delta}{w}) - (\frac{w}{\alpha}) + 1\right)\left(\frac{w}{\alpha}\right)}{2\left((\frac{\gamma}{\beta_1})^2(\frac{\beta_1}{\beta_2}) - 1\right)} \geq 0$$

□

Proof. **Proposition 2:**

According to Lemma 6, the proof reduces to showing that $\mathcal{Y}^1_{(F,P),(P,P)}$ increases at β_2/β_1. We can show it simply by first-order derivative as follows:

$$\frac{\partial \mathcal{Y}^1_{(F,P),(P,P)}}{\partial(\frac{\beta_1}{\beta_2})} = \frac{\left((\frac{\gamma}{\beta_1}) - 1\right)^2 \left((\frac{w}{\alpha})(\frac{\Delta}{w}) - (\frac{w}{\alpha}) + 1\right)^2 (\frac{\gamma}{\beta_1})^2}{4\left((\frac{\gamma}{\beta_1})^2(\frac{\beta_1}{\beta_2}) - 1\right)^2} \geq 0$$

Hence, $\mathcal{Y}^1_{(F,P),(P,P)}$ and the full assortment region increase at β_2/β_1. □

Proof. **Proposition 3:** In order to show that the powerful retailer would switch to the limited assortment if he faces an increase in substitutability between the two products, we should show that the full assortment region of $R1$ shrinks when the ratio of $\frac{\gamma}{\beta_1}$ increases. Given the result of Lemma 6, this is equivalent to show that $\mathcal{Y}^1_{(F,P),(P,P)}$ is decreasing at $\frac{\gamma}{\beta_1}$. And, it is simply shown by the first order derivative as follows:

$$\frac{\partial \mathcal{Y}^1_{(F,P),(P,P)}}{\partial(\frac{\gamma}{\beta_1})} = -\frac{\left((\frac{w}{\alpha})(\frac{\Delta}{w}) - (\frac{w}{\alpha}) + 1\right)^2 \left((\frac{\gamma}{\beta_1})^2(\frac{\beta_1}{\beta_2}) - (\frac{\gamma}{\beta_1})(\frac{\beta_1}{\beta_2}) - (\frac{\gamma}{\beta_1}) + 1\right)}{2\left((\frac{\gamma}{\beta_1})^2(\frac{\beta_1}{\beta_2}) - 1\right)^2} \leq 0$$

Therefore, $\mathcal{Y}^1_{(F,P),(P,P)}$ as the only upper bound for full assortment region for $R1$ decreases at $\frac{\gamma}{\beta_1}$.

□

Proof. **Proposition 4:**

Similar to Propositions 2 and 3, since the whole region of full assortment by R1 is characterized by $\mathcal{Y}^1_{(F,P),(P,P)}$ (according to Lemma 6), we only should show that $\mathcal{Y}^1_{(F,P),(P,P)}$ is decreasing in w/α. And it can be done by first-order derivative for the continuous function of $\mathcal{Y}^1_{(F,P),(P,P)}$ as follows:

$$\frac{\partial \mathcal{Y}^1_{(F,P),(P,P)}}{\partial(\frac{w}{\alpha})} = -\frac{\left((\frac{\gamma}{\beta_1}) - 1\right)^2 \left((\frac{w}{\alpha})(\frac{\Delta}{w}) - (\frac{w}{\alpha}) + 1\right)\left(\frac{\Delta}{w} - 1\right)}{2\left((\frac{\gamma}{\beta_1})^2(\frac{\beta_1}{\beta_2}) - 1\right)} \leq 0$$

Therefore, when the ratio of $\frac{w}{\alpha}$ increases (decrease in potential profit margin), the powerful retailer in SQ model would reduce his assortment choice.

□

5.4. Proofs of propositions and lemmas for Chapter 3

Proof. **Proposition 5:**

Under symmetric information, suppliers know the exact value of their relative quality score (QS), i.e. $\frac{QS_L}{QS_H}$, evaluated and assigned by the buyer. In this case, the problem can be analyzed backward by starting from the buyer's order allocation in the last stage. The buyer identifies the supplier with the lowest generalized price (i.e. $\frac{p_i}{QS_i}$ in multiplicative generalized price rule) and if her bid price is less than or equal to the reserve price (spot market price), he orders the whole demand from that supplier, otherwise he procures from external sources. Note that comparison of generalized prices, i.e. $\frac{p_i}{QS_i}$ vs $\frac{p_{-i}}{QS_{-i}}$, can also be done through comparing suppliers' relative price with their relative QS, i.e. $\frac{p_i}{p_{-i}}$ vs $\frac{QS_i}{QS_{-i}}$. That is indeed why the suppliers only need to know their relative QS rather than their actual scores assigned by the buyer.

Let supplier H and L propose p_H and p_L per unit and their relative quality score is α (i.e. $\frac{QS_L}{QS_H} = \alpha$). Then, the buyer's expected cost can be expressed as below:

$$\kappa_B(p_H, p_L, \alpha, p_r) = \begin{cases} Q \times p_H & \text{if supplier H wins, i.e. } \alpha \leq \frac{p_L}{p_H}, p_H \leq p_r \\ Q \times p_L & \text{if supplier L wins, i.e. } \alpha > \frac{p_L}{p_H}, p_L \leq p_r \\ Q \times p_r & \text{otherwise} \end{cases} \tag{1}$$

Therefore, the buyer's optimal allocation policy can be specified as:

$$(q_H^*(p_H, p_L, \alpha, p_r), q_L^*(p_H, p_L, \alpha, p_r)) = \begin{cases} (Q, 0) & \text{if } \alpha \leq \frac{p_L}{p_H}, p_H \leq p_r \\ (0, Q) & \text{if } \alpha > \frac{p_L}{p_H}, p_L \leq p_r \\ (0, 0) & \text{otherwise} \end{cases} \tag{2}$$

Given this order allocation policy, the two suppliers engage in a Bertrand price competition in which each supplier undercuts the other until one of them reaches to her minimum price (marginal cost). Each supplier wants to propose a price that maximizes her expected profit given the decision of the other supplier and the allocation policy of the buyer, as follows:

$$\pi_H(\alpha, p_L) = Q \times max_{p_H \leq p_r} \left((p_H - c_H) Prob[\alpha \leq \tfrac{p_L}{p_H}]; 0 \right)$$
$$\pi_L(\alpha, p_H) = Q \times max_{p_L \leq p_r} \left((p_L - c_L) Prob[\alpha > \tfrac{p_L}{p_H}]; 0 \right)$$

The second term in the profit functions of suppliers L and H, zero, ensures that $p_i \geq c_i$ for $i = H$ and L. This Bertrand competition results in an equilibrium in which the supplier with lower generalized price undercuts the other by just epsilon (i.e. infinitesimal) amount and wins the auction. In addition, they both know that the maximum bid price they can offer is p_r.

Given the suppliers' profit functions, their best response functions would be:

$$p_H^*(p_L) = min(max(c_H; \tfrac{p_L}{\alpha}); p_r)$$
$$p_L^*(p_H) = min(max(c_L; p_H\alpha); p_r)$$

Consequently, based on the above pricing schemes in equilibrium, one of the followings would be true:

1. If $\frac{c_L}{c_H} \leq \alpha \leq 1$, the buyer orders from supplier L. This is because supplier H will not offer a lower price than c_H in this range of relative QS, and supplier L can win the auction at a price infinitesimally lower than $c_H\alpha$.

2. If $0 < \alpha \leq \frac{c_L}{c_H}$, the buyer procures from supplier H. This is because supplier H can undercut supplier L until she reaches her marginal cost c_L and win the auction at a price infinitesimally

lower than $min(\frac{c_L}{\alpha}; p_r)$. Hence, when $\frac{c_L}{p_r} \leq \alpha \leq \frac{c_L}{c_H}$, supplier H's price would be $\frac{c_L}{\alpha}$ and when $0 < \alpha < \frac{c_L}{p_r}$, she would offer p_r.

Based on the supplier's optimal bidding policies, we can now fully characterize the buyer's order allocation, suppliers' profits, and the buyer's cost under symmetric information in the three regions of $0 < \alpha < \frac{c_L}{p_r}$, $\frac{c_L}{p_r} \leq \alpha \leq \frac{c_L}{c_H}$, and $\frac{c_L}{c_H} \leq \alpha \leq 1$ as presented in Table 3–2 in Chapter 3.

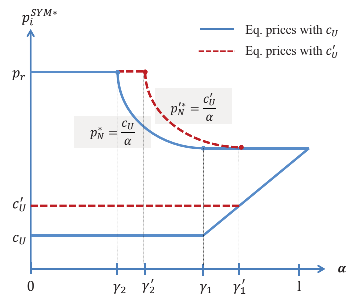
□

Proof. **Corollary 1:**

First, according to Table 3–2, for $\alpha > \frac{c_L}{c_H}$ the expected bid and profit of supplier L ($c_H\alpha$ and $Q[c_H\alpha - c_L]$ respectively) are strictly increasing in α, while $p^*_H = c_H$ and $\pi^*_H = 0$ stay unchanged. In contrast, when $\frac{c_L}{p_r} \leq \alpha \leq \frac{c_L}{c_H}$, the expected bid and profit of supplier H ($\frac{c_L}{\alpha}$ and $Q[\frac{c_L}{\alpha} - c_H]$ respectively) strictly decrease in α, but $p^*_L = c_L$ and $\pi^*_L = 0$ remain unaffected. Also, in the last region of $0 < \alpha < \frac{c_L}{p_r}$, the expected bids and profits of both suppliers stay unchanged with respect to α ($p^*_L = \pi^*_L = 0$, $p^*_H = p_r$, and $\pi^*_H = Q(p_r - c_H)$). To summarize, the equilibrium price and profit of supplier H are (weakly) decreasing in α, while those of supplier L are (weakly) increasing in α.

Second, a similar procedure applies for verifying the effect of cost homogeneity. By fixing c_H and increasing c_L to c'_L (where $c_H > c'_L > c_L$), the threshold $\gamma_1 = \frac{c_L}{c_H}$ increases to $\gamma'_1 = \frac{c'_L}{c_H}$, which in turn increases and decreases the range of α for which supplier H and L win the auction, respectively. In that case, it is easy to see that supplier L's price *increases* from c_L and $c_H\alpha$ to c'_L for when $\alpha \in (0, \gamma_1)$ and $\alpha \in (\gamma_1, \gamma'_1)$, respectively. And, supplier H's price *increases* from c_H and $\frac{c_L}{\alpha}$ to $\frac{c'_L}{\alpha}$ in the intervals of $\alpha \in (\gamma_1, \gamma'_1)$ and $\alpha \in (0, \gamma_1)$, respectively. In addition, $\gamma_2 = \frac{c_L}{p_r}$ increases to $\gamma'_2 = \frac{c'_L}{p_r}$, which implies that supplier H offers p_r for a larger interval of $\alpha \in (0, \frac{c'_L}{p_r}]$. Overall, by an increase in c_L for a fixed c_H, both suppliers L and H weakly increase their bid prices. This is also illustrated in the following graph.

□



Proof. Lemma 3:

Assume suppliers a-priori believe that $\alpha \in [\underline{\alpha}, \overline{\alpha}]$ (let $\overline{\alpha} > \gamma_2$ in order to focus only on cases where the shared information is likely to influence suppliers' pricing decisions). We now show that the buyer always has enough incentive to distort the QS information shared with the suppliers in a way to enforce the winner to reduce her winning price. To do so, we consider two cases as follows:

- If the true value of relative QS is $\alpha > \gamma_1 = \frac{c_L}{c_H}$, supplier L is supposed to win the auction under symmetric information setting. Now if the buyer distorts and shares an $\alpha' < \alpha$, his unit cost would be $c_H\alpha'$ which is lower than that under truthful sharing, $c_H\alpha$. In this case, by manipulating the relative QS, the buyer can order from his preferred supplier at a lower cost.

- If the true value of relative QS is $\alpha < \gamma_1 = \frac{c_L}{c_H}$, supplier H would instead be the winner if the buyer discloses α. However, the buyer's unit cost if he shares an $\alpha' > \alpha$ would be $\frac{c_L}{\alpha'}$ which is lower than his cost if he discloses the true α, i.e. $\frac{c_L}{\alpha}$. Therefore, the buyer would have enough incentive to distort the QS information.

To summarize the above points, the buyer always has an incentive to distort the relative QS and share a value that enforces the winning supplier to decrease her price. Note that the buyer's benefit from this distortion depends on the suppliers' prior belief (degree of uncertainty) and the true value of α.

□

Proof. Lemma 4

First, we find the internal solution for the optimal pricing policy of the profit-maximizing suppliers under the least costly pooling equilibrium ($\eta_i = 0$) by assuming no limit on their prices. Their expected profit functions under the internal solution would be:

$$\pi_H(p_H, p_L, \underline{\alpha}, \overline{\alpha}) = Q(p_H - c_H)\int_{\underline{\alpha}}^{p_L/p_H} f(\alpha)d\alpha \tag{3}$$

$$\pi_L(p_H, p_L, \underline{\alpha}, \overline{\alpha}) = Q(p_L - c_L)\int_{p_L/p_H}^{\overline{\alpha}} f(\alpha)d\alpha \tag{4}$$

where $f(\alpha)$ is the suppliers' prior belief regarding the true value of α. Assuming a uniform distribution, $f(\alpha)$ for $\underline{\alpha} \le \alpha \le \overline{\alpha}$ would be $f(\alpha) = \mathcal{L}[\underline{\alpha}, \overline{\alpha}] = \frac{1}{\overline{\alpha}-\underline{\alpha}}$. After plugging $f(\alpha)$ into the suppliers' profit functions and taking the first derivative, we can derive the best response functions of the suppliers as: $p_H^*(p_L) = \sqrt{\frac{c_H p_L}{\underline{\alpha}}}$ and $p_L^*(p_H) = \frac{\overline{\alpha}p_H + c_L}{2}$.

By solving this system of equations, we can find the internal equilibrium (PE-1) of the game, as below:

$$p_H^{int} = \frac{\overline{\alpha}c_H + \sqrt{\overline{\alpha}^2 c_H^2 + 8\underline{\alpha}c_H c_L}}{4\underline{\alpha}} \quad , \qquad p_L^{int} = \frac{(\overline{\alpha}^2 c_H + \overline{\alpha}\sqrt{\overline{\alpha}^2 c_H^2 + 8\underline{\alpha}c_H c_L} + 4\underline{\alpha}c_L)}{8\underline{\alpha}}.$$

This internal point will be the equilibrium only if it satisfies the upper and lower pricing bounds, i.e. $c_H \le p_H^{int} \le p_r$ and $c_L \le p_L^{int} \le p_r$. Now, we focus on the cases when the internal solution hits the boundary conditions. Technically, the following cases might happen:

- $p_H^{int} > p_r$: this case happens when $\underline{\alpha} < \frac{c_H(c_L + \overline{\alpha}p_r)}{2p_r^2}$. In this case, supplier H cannot bid above the reserve price, p_r. Supplier L's best response to this price (the one that maximizes 4) is to offer $p_L^*(p_r) = \frac{\overline{\alpha}p_r + c_L}{2}$ as long as it is not lower than her marginal cost, c_L. Note that in this boundary solution, $\pi_L(\frac{\overline{\alpha}p_r + c_L}{2}) > \pi_L(p_r\underline{\alpha})$; therefore, supplier L earns a higher expected profit by offering $\frac{\overline{\alpha}p_r + c_L}{2}$ than the worst-case bidding price that guarantees her winning in the auction, $p_r\underline{\alpha}$.

- $p_H^{int} < c_H$: this case happens when $\underline{\alpha} > \frac{(c_L + c_H\overline{\alpha})}{2c_H}$. Obviously, supplier H does not bid a price lower than her marginal cost c_H. Supplier L's best response function based on (4) would be $p_L(c_H) = \frac{\overline{\alpha}c_H + c_L}{2} > c_L$. However, this price is lower than the price under the worst-case bidding strategy that guarantees her winning for any value of α, i.e. $c_H\underline{\alpha} > \frac{\overline{\alpha}c_H + c_L}{2}$. As a result, whenever $\underline{\alpha} > \frac{(c_L + c_H\overline{\alpha})}{2c_H}$, supplier L follows the worst-case bidding policy and offers $c_H\underline{\alpha}$.

- $p_L^{int} < c_L$: this case happens when $\underline{\alpha} > \frac{c_H \overline{\alpha}^2}{c_L}$. Similarly, supplier L does not offer a price lower than her marginal cost c_L. Maximizing (3) when $p_L^* = c_L$ would suggest a bidding price of $p_H = \sqrt{\frac{c_H c_L}{\underline{\alpha}}} > c_H$ for the known supplier. But, this price is lower than the price under the worst-case bidding policy for supplier H, which is $\frac{c_L}{\overline{\alpha}}$ (of course, provided that it is lower than p_r). Therefore, whenever $\underline{\alpha} > \frac{c_H \overline{\alpha}^2}{c_L}$, supplier H can guarantee her winning and the maximum profit by offering $min\{p_r, \frac{c_L}{\overline{\alpha}}\}$.

- $p_L^{int} > p_r$: this case actually never happens because for any point such that $0 < c_L < c_H$ and $0 < \underline{\alpha} < \overline{\alpha} \leq 1$, we have always $p_H^{int} > p_L^{int}$; therefore, before p_L^{int} hits the upper bound p_r, p_H^{int} does that and this case corresponds to the first boundary solution above.

To summarize, the price competition between the two suppliers under the asymmetric information setting (least-costly pooling equilibrium) leads to one of the following four different equilibrium prices:

- PE-1: Internal solution of p_H^{int} and p_L^{int} if both prices are in the acceptable ranges.

- PE-2: Boundary solution of $p_H = p_r$ and $p_L = max\{c_L, \frac{\overline{\alpha} p_r + c_L}{2}\}$.

- PE-3: Boundary solution of $p_H = c_H$ and $p_L = c_H \underline{\alpha}$.

- PE-4: Boundary solution of $p_H = min\{p_r, \frac{c_L}{\overline{\alpha}}\}$ and $p_L = c_L$.

□

Proof. Proposition 6:

Given the possible equilibrium bid prices analyzed in Lemma 4, we first characterize the regions for $\underline{\alpha}$ and $\overline{\alpha}$ under which each one of those equilibrium outcomes is sustainable as a unique pooling equilibrium.

1- The internal solution (PE-1) would be the equilibrium only if $c_H \leq p_H^{int} \leq p_r$ and $c_L \leq p_L^{int} \leq p_r$. By solving these equations, we find the effective ranges of $\underline{\alpha} \leq \frac{c_L + c_H \overline{\alpha}}{2c_H}$, $\underline{\alpha} \leq \frac{c_H \overline{\alpha}^2}{c_L}$, and $\underline{\alpha} \geq \frac{c_H(c_L + \overline{\alpha} p_r)}{2p_r^2}$. Hence, when $(\underline{\alpha}, \overline{\alpha})$ belongs to the resulting region (as illustrated in Figure 3–2 in Chapter 3), the suppliers would bid according to the internal equilibrium (PE-1). Based on these bid prices, supplier H wins the auction if $0 < \alpha < \frac{p_L^*}{p_H^*} = \frac{(\overline{\alpha}^2 c_H + \overline{\alpha}\sqrt{\overline{\alpha}^2 c_H^2 + 8\underline{\alpha} c_H c_L} + 4\underline{\alpha} c_L)}{2(\overline{\alpha} c_H + \sqrt{\overline{\alpha}^2 c_H^2 + 8\underline{\alpha} c_H c_L})}$; otherwise for large enough α, supplier L takes the entire order.

2- The boundary solution PE-2 prevails if $p_r < p_H^{int}$. This would be the case for the values of $\underline{\alpha} < \frac{c_H(c_L+\overline{\alpha}p_r)}{2p_r^2}$. In this region, supplier H wins the auction if the true value of relative QS is $0 < \alpha < \frac{p_L^*}{p_H^*}$; and otherwise if α is high enough, supplier L wins the contract.

3- The boundary solution PE-3 would be the equilibrium outcome if $p_H^{int} < c_H$. This happens when $\underline{\alpha} > \frac{(c_L+c_H\overline{\alpha})}{2c_H}$. It is easy to see that when the suppliers' prior belief $(\underline{\alpha}, \overline{\alpha})$ takes place in this region, the buyer always orders from supplier L since $\alpha \geq \frac{p_L^*}{p_H^*} = \underline{\alpha}$ for the entire range of $\underline{\alpha} \leq \alpha \leq \overline{\alpha}$.

4- The boundary solution PE-4 prevails as the equilibrium outcome if $p_L^{int} < c_L$. This leads to a region where $\underline{\alpha} > \frac{c_H\overline{\alpha}^2}{c_L}$. In this region, the buyer always orders from supplier H because $\alpha \leq \overline{\alpha} \leq \frac{p_L^*}{p_H^*}$ for the whole range of $\underline{\alpha} \leq \alpha \leq \overline{\alpha}$.

Now that the bid prices and the buyer's allocation are characterized for any value of $\underline{\alpha}$ and $\overline{\alpha}$ $(0 < \underline{\alpha} < \overline{\alpha} \leq 1)$, we can easily find the cost/profits functions of the buyer and the suppliers. The winner in the auction (supplier i) supplies the whole demand Q at her bidding price (say p_i^*) and her profit would be $\pi_i = Q(p_i^* - c_i)$. The other supplier $(-i)$ fails in the competition and as a result, $\pi_{-i} = 0$. The buyer's total cost would then be $\kappa_B = Qp_i^*$.

A summary of equilibrium bid prices, order allocation, and cost/profits functions for all the regions of $(\underline{\alpha}, \overline{\alpha})$ is presented in Table 3–3.

□

Proof. **Proposition 7:** In order to prove these statements, we focus on the equilibrium prices of suppliers under pooling equilibria (Table 3–3 in Chapter 3). First, we capture the effect of cost homogeneity of suppliers by increasing c_L at a fixed c_H. As can be seen from Table 3–3, an increase in c_L at a fixed c_H would increase the equilibrium prices in all regions except PE-3 that stays unchanged.

Also, this change in the cost homogeneity of suppliers can slightly move the boundary regions. In Figure 3–2, as c_L increases (at a fixed c_H), the region $\underline{\alpha} > \frac{(c_L+c_H\overline{\alpha})}{2c_H}$ covers a smaller triangle for PE-3, while the region $\underline{\alpha} > \frac{c_H\overline{\alpha}^2}{c_L}$ expands that leads to an increase in PE-4. The effects of these changes on other two regions (PE-1 and PE-2) depend on the relative increase and decrease in regions PE-4 and PE-3, respectively. As a result of these boundary changes, there are some points for which the equilibrium outcome changes from a region to another region. For instance, as the region PE-4 expands, it includes new points that were originally in regions PE-1, PE-2, and PE-3.

In total, five different cases might happen as the boundaries change when the marginal cost of supplier L increases from c_L to c'_L:

- PE-1 to PE-2: (p^*_H, p^*_L) increases from $(p^{int}_H(c_L), p^{int}_L(c_L))$ to $(p_r, max(c'_L, \frac{\overline{\alpha}p_r+c'_L}{2}))$;
- PE-3 to PE-1: (p^*_H, p^*_L) increases from $(c_H, c_H\underline{\alpha})$ to $(p^{int}_H(c'_L), p^{int}_L(c'_L))$;
- PE-1 to PE-4: (p^*_H, p^*_L) increases from $(p^{int}_H(c_L), p^{int}_L(c_L))$ to $(min(p_r, \frac{c'_L}{\overline{\alpha}}), c'_L)$;
- PE-2 to PE-4: (p^*_H, p^*_L) increases from $(p_r, max(c_L, \frac{\overline{\alpha}p_r+c_L}{2}))$ to $(min(p_r, \frac{c'_L}{\overline{\alpha}}), c'_L)$;
- PE-3 to PE-4: (p^*_H, p^*_L) increases from $(c_H, c_H\underline{\alpha})$ to $(min(p_r, \frac{c'_L}{\overline{\alpha}}), c'_L)$.

In order to avoid unnecessary operations, here we only focus on the fourth case (PE-2 to PE-4), for example, and show how the bidding prices by the two suppliers *increase* at a point $(\underline{\alpha}, \overline{\alpha})$ that was originally in region PE-2 and now is located at region PE-4 as the marginal cost of supplier L increases from c_L to c'_L. The proof is a follows:

The point $(\underline{\alpha}, \overline{\alpha})$ is located in region PE-2 (resp. PE-4) when marginal cost of supplier L is c_L (resp. c'_L). According to Proposition 6 and Table 3–3, this happens only when

$$\forall\ (\underline{\alpha}, \overline{\alpha})\ :\ \frac{c_H\overline{\alpha}^2}{c'_L} < \underline{\alpha} < \frac{c_H\overline{\alpha}^2}{c_L}\quad \&\quad \underline{\alpha} < \frac{c_H(c_L+\overline{\alpha}p_r)}{2p_r^2}.$$

From $\frac{c_H\overline{\alpha}^2}{c'_L} < \frac{c_H(c_L+\overline{\alpha}p_r)}{2p_r^2}$, if we isolate $\overline{\alpha}$, we find

$$\overline{\alpha} < \frac{c'_L + \sqrt{{c'_L}^2 + 8c'_Lc_L}}{4p_r}. \tag{5}$$

By multiplying c'_L in the inverse of both sides of inequality 5, we find: $\frac{c'_L}{\overline{\alpha}} > \frac{4c'_Lp_r}{c'_L+\sqrt{{c'_L}^2+8c'_Lc_L}}$. Then, since $\sqrt{{c'_L}^2 + 8c'_Lc_L} < 3c'_L$, we find $\frac{c'_L}{\overline{\alpha}} > p_r$, which implies that for all $(\underline{\alpha}, \overline{\alpha})$ supplier H's price p_r does not change.

Also, by multiplying inequality (5) at $\frac{p_r}{2}$ and adding $\frac{c_L}{2}$, we find: $\frac{\overline{\alpha}p_r+c_L}{2} < \frac{c'_L+\sqrt{{c'_L}^2+8c'_Lc_L}}{8} + \frac{c_L}{2}$. It is easy to see that the right hand side is less than c'_L, which results in $\frac{\overline{\alpha}p_r+c_L}{2} < c'_L$ that implies that supplier L increases her bid price. Overall, both suppliers' prices (weakly) increase in this region as L's marginal cost increases to c'_L(where $c_L < c'_L < c_H$).

Second, the equilibrium bid prices p^*_L and p^*_H in regions PE-1 and PE-2 (resp. in regions PE-3 and PE-4) weakly increase (resp. decrease) in $\overline{\alpha}$ (for a fixed $\underline{\alpha}$) and weakly decrease (resp. increase) in $\underline{\alpha}$(for a fixed $\overline{\alpha}$). The effect of uncertainty (change in $\underline{\alpha}$ and $\overline{\alpha}$) on the equilibrium bidding prices is obvious in regions PE-2, PE-3, and PE-4 (see Table 3–3); but for the region PE-1, we can show this effect by first-order derivative as follows:

$$\frac{\partial p_H}{\partial \underline{\alpha}} = -\frac{c_H\left(4\underline{\alpha}c_L + \sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L}\,\overline{\alpha} + c_H\overline{\alpha}^2\right)}{4\underline{\alpha}^2\sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L}} < 0$$

$$\frac{\partial p_H}{\partial \overline{\alpha}} = \frac{c_H\left(c_H\overline{\alpha} + \sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L}\right)}{4\underline{\alpha}\sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L}} > 0$$

$$\frac{\partial p_L}{\partial \underline{\alpha}} = -\frac{c_H\overline{\alpha}\left(4\underline{\alpha}c_L + \overline{\alpha}\sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L} + c_H\overline{\alpha}^2\right)}{8\underline{\alpha}^2\sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L}} < 0$$

$$\frac{\partial p_L}{\partial \overline{\alpha}} = \frac{\left(c_H\overline{\alpha} + \sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L}\right)^2}{8\underline{\alpha}\sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L}} > 0$$

Therefore, both suppliers' prices in region PE-1 are strictly decreasing in $\underline{\alpha}$ and increasing in $\overline{\alpha}$. Taking into account the overall impact of QS information uncertainty on suppliers' prices in all the four regions, we conclude the optimal level of uncertainty is:

- $\underline{\alpha} = \frac{c_L + c_H\overline{\alpha}}{2c_H}$ (boundary between PE-3 and PE-1/PE-2), when $\alpha > \frac{c_L}{c_H}$;
- $\underline{\alpha} = \frac{c_H\overline{\alpha}^2}{c_L}$ (boundary between PE-4 and PE-1/PE-2), when $\alpha < \frac{c_L}{c_H}$.

□

Proof. **Proposition 8:** In order to show that the boundary solutions of PE-3 and PE-4 in pooling equilibria lead to lower bid prices than those under symmetric information, we first refer to Propositions 5 and 6 to find the equilibrium bid prices under each setting. Then, it is easy to check that when the degree of information asymmetry is sufficiently low, the bid prices under pooling equilibria are lower than those under symmetric information as follows.

- The boundary solution PE-3 happens when $\overline{\alpha} > \frac{c_L}{c_H}$ and $\underline{\alpha} \geq \frac{c_L + c_H\overline{\alpha}}{2c_H}$, and under PE-3 the bid prices are $p_H^{*PE} = c_H$ and $p_L^{*PE} = c_H\underline{\alpha}$. Since $\underline{\alpha} > \frac{c_L}{c_H}$, the bid prices under symmetric information are $p_H^{*SYM} = c_H$ and $p_L^{*SYM} = c_H\alpha$ for any α. It is now easy to verify that $p_i^{*PE} \leq p_i^{*SYM}$ for $\forall i \in \{H, L\}$ under PE-3.

- Same procedure applies to the region PE-4. It happens when $\frac{c_H \overline{\alpha}^2}{c_L} \leq \underline{\alpha} < \overline{\alpha} < \frac{c_L}{c_H}$ and the bid prices are $p_H^{*PE} = min\left(\frac{c_L}{\overline{\alpha}}, p_r\right)$ and $p_L^{*PE} = c_L$. Also, when the true value of relative QS is less than $\frac{c_L}{c_H}$, the bid prices under symmetric information are $p_H^{*SYM} = \min\left(\frac{c_L}{\alpha}, p_r\right)$ and $p_L^{*SYM} = c_L$. As a result, $p_i^{*PE} \leq p_i^{*SYM}$ for $\forall i \in \{H, L\}$ under PE-4.

□

Proof. **Lemma 5:**

In a separating equilibrium, after observing the guarantees $\eta_L Q p_r$ and $\eta_H Q p_r$ to suppliers L and H, respectively, and correctly inferring the relative value of quality scores $\frac{QS_L}{QS_H}$ (i.e. α), suppliers engage in a Bertrand price competition similar to the case of symmetric information setting with some changes as follows.

Under the presence of advance revenue guarantees, the buyer's total cost function would be:

$$\kappa_B(p_H, p_L, \alpha, p_r, \eta_H, \eta_L) = \begin{cases} max\{\eta_H Q p_r, (1-\eta_L) Q p_H\} + \eta_L Q p_r & \text{if } \alpha \leq \frac{p_L}{p_H}, p_H \leq p_r \\ max\{\eta_L Q p_r, (1-\eta_H) Q p_L\} + \eta_H Q p_r & \text{if } \alpha > \frac{p_L}{p_H}, p_L \leq p_r \\ Q \times p_r & otherwise \end{cases} \tag{6}$$

Note that in the above cost function, for example in the first case when supplier H wins, she will be offered an order of $Q(1-\eta_L)$ (total demand minus the guaranteed portion for the rival supplier), and her total revenue will not be lower than the guaranteed revenue ($\eta_H Q p_r$). Therefore, she either takes the order at her winning price or at a price that makes her revenue equal to the guaranteed revenue, whichever is higher. The above cost function corresponds to the following allocation policy.

$$(q_H^*(p_H, p_L, \alpha, p_r, \eta_H, \eta_L), q_L^*(p_H, p_L, \alpha, p_r, \eta_H, \eta_L)) = \begin{cases} (Q(1-\eta_L), \eta_L Q) & \text{if } \alpha \leq \frac{p_L}{p_H}, p_H \leq p_r \\ (\eta_H Q, (1-\eta_H) Q) & \text{if } \alpha > \frac{p_L}{p_H}, p_L \leq p_r \\ (0, 0) & otherwise \end{cases} \tag{7}$$

After observing the guarantees (η_H and η_L) and the reserve price (p_r), suppliers' objective is to maximize their expected profits, as below:

$$\pi_H(\alpha, p_L, \eta_H, \eta_L) = \max_{p_H}\{Prob[\alpha < \frac{p_L}{p_H}] \max\left[Q(1-\eta_L)(p_H - c_H), Q(\eta_H p_r - (1-\eta_L) c_H)\right]; Q\eta_H(p_r - c_H)\} \tag{8}$$

$$\pi_L(\alpha, p_H, \eta_H, \eta_L) = \max_{p_L}\{Prob[\alpha > \frac{p_L}{p_H}] \max[Q(1-\eta_H)(p_L - c_L), Q(\eta_L p_r - (1-\eta_H)c_L)]\, ; Q\eta_L(p_r - c_L)\} \tag{9}$$

The terms $Q(1-\eta_{-i})(p_i - c_i)$ $(i = H, L)$ in suppliers' profit functions denote the profit of supplier i if she wins the auction at a price that makes her revenue greater than the guaranteed revenue. However, if her winning price is so low that her revenue becomes lower than the guaranteed value, she ultimately receives a profit of $Q(\eta_i p_r - (1-\eta_{-i})c_i)$. The last terms in the profit functions of suppliers L and H correspond to the case when they fail in the competition. This last term was zero in the symmetric information case. As a result, under the presence of guarantees, suppliers feel less pressure to engage in an intense price competition to win the auction. Indeed, a supplier is better off to lose the auction than to win at a very low price simply because if she loses, she is only responsible to procure $\eta_i Q$, whereas if she wins at a very low price, she obtains the same revenue for procuring $(1-\eta_{-i})Q$. And, since $\eta_L + \eta_H < 1$, then $\eta_i Q < (1-\eta_{-i})Q$ for $i = H$ and L; and hence, $Q(\eta_i p_r - (1-\eta_{-i})c_i) \leq Q\eta_i(p_r - c_i))$. Therefore, their profit functions would actually simplify to:

$$\pi_H(\alpha, p_L, \eta_H, \eta_L) = \max_{p_H}\{Prob[\alpha < \frac{p_L}{p_H}]\, [Q(1-\eta_L)(p_H - c_H)]\, ; Q\eta_H(p_r - c_H)\}$$
$$\pi_L(\alpha, p_H, \eta_H, \eta_L) = \max_{p_L}\{Prob[\alpha > \frac{p_L}{p_H}]\, [Q(1-\eta_H)(p_L - c_L)]\, ; Q\eta_L(p_r - c_L)\}$$

And, the minimum price that makes a supplier indifferent between winning or losing the auction would be (we denote it as p_i^{low}):

$p_i^{low} = \{p_i \mid Q\eta_i(p_r - c_i) = Q(1-\eta_{-i})(p_i - c_i)\} = c_i + \frac{\eta_i}{1-\eta_{-i}}(p_r - c_i)$

Now, the equilibrium turns out to be very similar to that under symmetric information (proposition 5) except that suppliers' minimum price would be p_i^{low} instead of c_i. Similarly, we can find the best response functions of the suppliers as below.

$$p_H^*(p_L) = min(max(p_H^{low}; \tfrac{p_L}{\alpha}); p_r)$$
$$p_L^*(p_H) = min(max(p_L^{low}; \alpha p_H); p_r)$$

where $p_i^{low} = c_i + \frac{\eta_i}{1-\eta_{-i}}(p_r - c_i)$ for $i = L$ and H.

It is easy now to see that the winner of the auction in the equilibrium is

- supplier H for $0 < \alpha < \frac{p_L^{low}}{p_H^{low}}$
- supplier L for $\frac{p_L^{low}}{p_H^{low}} < \alpha \leq 1$

Knowing the suppliers' optimal pricing and the buyer's optimal order allocation policy, we can now characterize the equilibrium similar to the symmetric information setting as presented in Table 3–4.

Note that we have made an implicit assumption that the buyer offers the guarantees such that $\frac{p_L^{low}}{p_H^{low}} < 1$. This assumption should logically hold because otherwise it enforces supplier L out of the competition even for $\alpha = 1$, and obviously that would be a very costly strategy for the buyer as we will see in Proposition 9.

□

Proof. Proposition 9:

Assume the true value of relative QS is α $(0 < \alpha < 1)$ and the buyer uses η_H and η_L to credibly share this value. The corresponding actions of the suppliers and the total profits/cost functions can be derived according to Lemma 5. As can be observed, all suppliers' prices, buyer's allocation, and parties' profits/cost depend on the true value of α. Based on Table 3–4, there are three different regions in total defined by γ_1 and γ_2 (note that these thresholds also depend on the guarantees) with different sets of actions. Since in one region, supplier L wins the auction, whereas in the other two, supplier H wins the auction, we can capture two sets of policies for the buyer: I) offering guarantees so as to make supplier H win the auction (H-winning policy), II) providing guarantees in order to make L win the auction (L-winning policy). Below, we consider an arbitrary α belongs to each region (where the buyer uses an L-winning or H-winning strategy) in separate scenarios and check to see how the buyer should change the guarantees in order to credibly share a small neighborhood around α (say $[\alpha - \epsilon, \alpha + \epsilon]$). The morale here is to see if the buyer can credibly share α by offering different informative guarantees while keeping the same winner in the whole region of $[\underline{\alpha}, \overline{\alpha}]$. Then, if the QS sharing was possible by both L-winning and H-winning strategies, we check to see if sharing is also possible in bigger regions potentially from 0 to 1 by a combination of both strategies. Note that the buyer can also change the regions by changing the guarantees.

Therefore, here we would like to see when and which policy (making L the winner or H) leads to sustainable equilibria.

1. Let $0 < \alpha < \gamma_2$: Obviously, the buyer has no interest in sharing the QS information in this region because p_r is the maximum possible price for the suppliers to offer and even under a pooling equilibrium, suppliers never bid above this threshold.

2. Let $\gamma_2 < \alpha < \gamma_1$: In this case the buyer offers the guarantees in such a way that makes supplier H the winner of the auction. In the equilibrium, when the buyer chooses $\eta_H(\hat{\alpha})$ and $\eta_L(\hat{\alpha})$, suppliers infer $\hat{\alpha}$ and respond accordingly by their price choices. We should find $\eta_H(\hat{\alpha})$ and $\eta_L(\hat{\alpha})$ such that the buyer with true α never finds any incentive to deviate and signal an $\alpha' \neq \alpha$, i.e.

$$\kappa_B(\eta_H(\alpha), \eta_L(\alpha), p_H(\alpha), p_L(\alpha), \alpha) \leq \kappa_B(\eta_H(\hat{\alpha}), \eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha)$$

   The buyer's total cost if his real type is α and he signals $\hat{\alpha}$ would be:

$$\kappa_B(\eta_H(\hat{\alpha}), \eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha) = \begin{cases} \eta_L(\hat{\alpha})Qp_r + (1-\eta_L(\hat{\alpha}))Q\frac{p_L^{low}}{\hat{\alpha}} & \text{if } \hat{\alpha} = \alpha \\ \eta_L(\hat{\alpha})Qp_r + (1-\eta_L(\hat{\alpha}))Q\frac{p_L^{low}}{\hat{\alpha}} & \text{if } \hat{\alpha} > \alpha \\ Qp_L^{low} & \text{if } \hat{\alpha} < \alpha \end{cases} \quad (10)$$

   According to the proof of Lemma 3, we know that when H is potentially the winner of the auction, the buyer is usually tempted to deviate by disclosing a larger α to decrease the confidence of supplier H in increasing her price. In order to prevent the buyer with a true type of α to signal an $\hat{\alpha} > \alpha$, we should characterize the guarantees η_H and η_L such that

   $\kappa_B(\eta_H(\alpha), \eta_L(\alpha), p_H(\alpha), p_L(\alpha), \alpha) \leq \kappa_B(\eta_H(\hat{\alpha}), \eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha) \quad \forall \hat{\alpha} : \alpha < \hat{\alpha} \leq \alpha + \epsilon$

   Because of the continuity of cost function in this region, we can find the values of guarantees by the first-order condition, as follows:

   $\frac{\partial \kappa_B}{\partial \hat{\alpha}} |_{\hat{\alpha}=\alpha} \quad = 0 \quad \Rightarrow$

$$\frac{\partial \eta_L(\hat{\alpha})}{\partial \hat{\alpha}} p_r - \frac{\partial \eta_L(\hat{\alpha})}{\partial \hat{\alpha}} . \frac{c_L + \frac{\eta_L(\hat{\alpha})(p_r - c_L)}{1-\eta_H(\hat{\alpha})}}{\hat{\alpha}} + \frac{(1-\eta_L(\hat{\alpha}))}{\hat{\alpha}} \left(\frac{\partial \eta_L(\hat{\alpha})}{\partial \hat{\alpha}} . \frac{p_r - c_L}{1-\eta_H(\hat{\alpha})} + \frac{\eta_L(\hat{\alpha})(p_r - c_L)\left(\frac{\partial \eta_H(\hat{\alpha})}{\partial \hat{\alpha}}\right)}{(1-\eta_H(\hat{\alpha}))^2} \right)$$
$$- \frac{(1-\eta_L(\hat{\alpha}))\left(c_L + \frac{\eta_L(\hat{\alpha})(p_r - c_L)}{(1-\eta_H(\hat{\alpha}))}\right)}{\hat{\alpha}^2} = 0$$

Now, if we solve the above non-linear ODE for $\eta_H(\alpha)$, we find the following:

$$\eta_H(\alpha) = \frac{\alpha p_r \eta_L(\alpha) + \eta_L(\alpha) p_r - 2\eta_L(\alpha) c_L + \eta_L(\alpha)^2 c_L - \eta_L(\alpha)^2 p_r + c_L + K\alpha}{-\eta_L(\alpha) c_L + \alpha p_r \eta_L(\alpha) + c_L + K\alpha}$$

where $K \in \mathbb{R}$ is a constant real value. A further investigation reveals that $\eta_L(\alpha)$ is increasing in α, while η_H can be set to zero, and the buyer's minimum unit cost by implementing these guarantees would be $\frac{c_L}{\underline{\alpha}}$ (details skipped). These signals eliminate the buyer's incentive to magnify the true value of α. However, according to (10), if the buyer signals a lower value of relative QS, his final unit cost turns out to be p_L^{low}. In that case, the buyer indeed misleads supplier H and makes supplier L win the auction. The lowest value of α that the buyer can signal is $\underline{\alpha}$ with a signal of $\eta_H = \eta_L = 0$, which leads to $p_L^{low} = c_L$. This price (c_L) is significantly lower than the buyer's cost under truthful information sharing ($\frac{c_L}{\underline{\alpha}}$) that provides enough incentive for the buyer to deviate. Therefore, *the buyer cannot share the QS information credibly only by advance guarantees to supplier L* (i.e. with an H-winning policy).

3. Let $\gamma_1 < \alpha < 1 :$ Contrary to the previous case, here we focus on the revenue guarantees that make supplier L win the auction. In the equilibrium, when the buyer chooses $\eta_H(\hat{\alpha})$ and $\eta_L(\hat{\alpha})$, suppliers infer $\hat{\alpha}$ and respond accordingly by their price choices (according to Lemma 5). We should now find these guarantees in such a way that the buyer never finds any incentive to cheat, i.e.

$$\kappa_B(\eta_H(\alpha), \eta_L(\alpha), p_H(\alpha), p_L(\alpha), \alpha) \leq \kappa_B(\eta_H(\hat{\alpha}), \eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha).$$

The buyer's total cost if his real type is α and he signals $\hat{\alpha}$, would be:

$$\kappa_B(\eta_H(\hat{\alpha}), \eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha) = \begin{cases} \eta_H(\hat{\alpha})Qp_r + (1-\eta_H(\hat{\alpha}))Q\hat{\alpha}p_H^{low} & \text{if } \hat{\alpha} = \alpha \\ Qp_H^{low} & \text{if } \hat{\alpha} > \alpha \\ \eta_H(\hat{\alpha})Qp_r + (1-\eta_H(\hat{\alpha}))Q\hat{\alpha}p_H^{low} & \text{if } \hat{\alpha} < \alpha \end{cases}$$

We know that according to the proof of Lemma 3, when L is potentially the winner of the auction based on the relative QS and the degree of cost heterogeneity, the buyer is usually inclined to share a lower α to influence supplier L's price while keeping her the actual winner of the contract. In order to prevent the buyer with a true type of α to signal an $\hat{\alpha} < \alpha$, we should characterize the guarantees η_H and η_L such that:

$$\kappa_B\left(\eta_H(\alpha), \eta_L(\alpha), p_H(\alpha), p_L(\alpha), \alpha\right) \leq \kappa_B(\eta_H(\hat{\alpha}), \eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha) \quad \forall \hat{\alpha} : \alpha - \epsilon < \hat{\alpha} \leq \alpha$$

Or, equivalently, the minimum value of $\kappa_B(\eta_H(\hat{\alpha}), \eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha)$ should be always at $\hat{\alpha} = \alpha$ for the whole range of $\underline{\alpha} \leq \alpha \leq \overline{\alpha}$. Because of the local continuity of the cost function, the guarantees can also be derived by the first-order condition of $\kappa_B = \eta_H(\hat{\alpha})Qp_r + (1-\eta_H(\hat{\alpha}))Q\hat{\alpha}p_H^{low}$.

$$\frac{\partial(\eta_H(\hat{\alpha})Qp_r + (1-\eta_H(\hat{\alpha}))Q\hat{\alpha}p_H^{low})}{\partial\hat{\alpha}} \Big|_{\hat{\alpha}=\alpha} = 0$$

This leads to the following differential equation (ODE):

$$\left(\frac{\partial\eta_H(\hat{\alpha})}{\partial\hat{\alpha}}\right)\left(p_r - \hat{\alpha}\left(c_H + \eta_H(\hat{\alpha})\frac{p_r - c_H}{1-\eta_L(\hat{\alpha})}\right)\right) + (1-\eta_H(\hat{\alpha}))\left(c_H + \eta_H(\hat{\alpha})\frac{p_r - c_H}{1-\eta_L(\hat{\alpha})}\right)$$
$$+ (1-\eta_H(\hat{\alpha}))\hat{\alpha}\left(\left(\frac{\partial\eta_H(\hat{\alpha})}{\partial\hat{\alpha}}\right)\frac{p_r - c_H}{1-\eta_L(\hat{\alpha})} + \eta_H(\hat{\alpha})\frac{\partial\eta_L(\hat{\alpha})}{\partial\hat{\alpha}}\frac{(p_r - c_H)}{(1-\eta_L(\hat{\alpha}))^2}\right) = 0 \quad (11)$$

Now, if we solve the above non-linear ODE and isolate $\eta_L(\alpha)$, we find the following:

$$\eta_L(\alpha) = \frac{-\alpha\eta_H(\alpha)^2 p_r - 2\alpha c_H\eta_H(\alpha) + \alpha\eta_H(\alpha)^2 c_H + \alpha p_r\eta_H(\alpha) + \eta_H(\alpha)p_r + \alpha c_H + K}{\eta_H(\alpha)p_r - \alpha c_H\eta_H(\alpha) + \alpha c_H + K} \quad (12)$$

where $K \in \mathbb{R}$ is a constant real value. Two important points are noticeable here:

(a) If the buyer wants to share the QS information by making supplier L win the auction, he has to offer some positive guarantee to supplier H, i.e. $\eta_H \neq 0$; this is because otherwise η_L should be one, which does not make any sense.

(b) Interestingly, if we plug these guarantees (12) into the buyer's cost function, his total cost turns out to be: $\kappa_B(\eta_H(\alpha), \eta_L(\alpha), p_H(\alpha), p_L(\alpha), \alpha) = -K$.

Based on these two points, we refine the equilibria by reducing the signalling dimensions only to η_H (a special case where $\eta_L = 0$) as this leads to the same cost for the buyer. Equation (12) has to hold for all $\underline{\alpha} \leq \alpha \leq \overline{\alpha}$; therefore, by solving ODE (11) when $\eta_L(\alpha) \to 0$, $\eta_H(\alpha)$ would be in the following form:

$$\eta_H(\alpha) = \frac{\alpha p_r + p_r - 2\alpha c_H - \sqrt{\alpha^2 p_r^2 + 2\alpha p_r^2 + p_r^2 - 4\alpha c_H p_r + 4\alpha K(p_r - c_H)}}{2\alpha(p_r - c_H)}.$$

This $\eta_H(\alpha)$ transforms the buyer's cost function to a fixed constant value equal to $\kappa_B = -K$. Therefore, the problem switches to finding the maximum K such that $\eta_H(\alpha)$ is - first, between 0 and 1 for all values of α ($\underline{\alpha} < \alpha < \overline{\alpha}$); -second, one-to-one (because the cost function is continuous in this range, it is sufficient to have $\eta'_H(\alpha)$ non-negative or non-positive for all the values of α). We later show in Corollary 2 that $\eta_H(\alpha)$ is strictly decreasing in α and takes the highest value at $\alpha = \underline{\alpha}$. Therefore, the least costly signalling equilibrium would ideally make $\eta_H(\overline{\alpha}) = 0$. It is easy to verify that $K = -c_H\overline{\alpha}$ satisfies all these conditions and provides the most efficient signalling tool for the buyer, i.e. $\eta_H(\overline{\alpha}) = 0$ and $0 < \eta_H(\underline{\alpha}) < 1$ (even for $\alpha = 0$, $\lim_{\alpha \to 0^+} \eta_H(\alpha) = \frac{\overline{\alpha} c_H}{p_r} < 1$). The buyer's total cost would be then $c_H\overline{\alpha}$, and the revenue guarantee to supplier H, η_H, would be:

$$\eta_H(\alpha) = \frac{\alpha p_r + p_r - 2\alpha c_H - \sqrt{\alpha^2 p_r^2 + 2\alpha p_r^2 + p_r^2 - 4\alpha c_H p_r - 4\alpha c_H \overline{\alpha}(p_r - c_H)}}{2\alpha(p_r - c_H)}.$$

This minimum revenue guarantee to supplier H prevents the buyer to signal a lower value of α than his true type when he chooses a L-winning strategy. Interestingly, the buyer will never choose to signal a higher value of $\hat{\alpha} > \alpha$ because in that case:

$$\kappa_B(\eta_H(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha) = Qp_H^{low} \geq Qc_H \geq \kappa_B(\eta_H(\alpha), p_H(\alpha), p_L(\alpha), \alpha) = Qc_H\overline{\alpha} \quad \forall \hat{\alpha} > \alpha.$$

By utilizing this costly signalling tool, supplier L's price would be increasing in α, while supplier H's price decreases until it reaches c_H (the details are provided in Corollary 2).

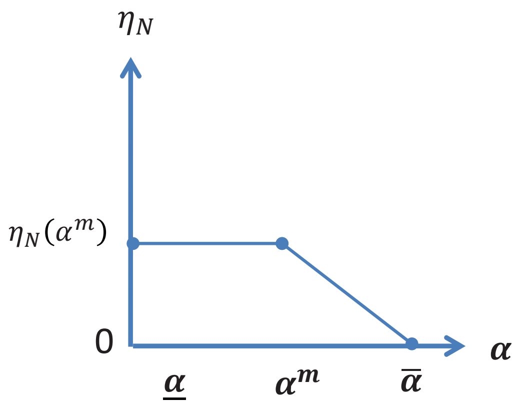
There is only one note here: supplier L will not offer a price lower than c_L. Therefore, if for any instance of α, the value of $\alpha[c_H + \eta_H(\alpha)(p_r - c_H)]$ becomes lower than c_L (this happens when $\alpha < \tilde{\alpha} = \frac{(p_r - c_L)c_L}{c_H\overline{\alpha}p_r - \overline{\alpha}c_H^2 + c_H p_r - p_r c_L}$), the buyer does not need to signal the exact value of α, and instead it suffices to show that α is very low, i.e. $\alpha \leq \tilde{\alpha}$. This is because the buyer becomes worse off by continuing to increase the guarantee for α lower than $\tilde{\alpha}$ as the supplier L does not decrease her price anymore, while supplier H continues to increase her price. Therefore, the buyer would not offer a guarantee more than a threshold $\eta_H(\tilde{\alpha})$, where $\tilde{\alpha}$ is the point in which $\alpha[c_H + \eta_H(\alpha)(p_r - c_H)] = c_L$. To summarize, the optimal guarantees would be:

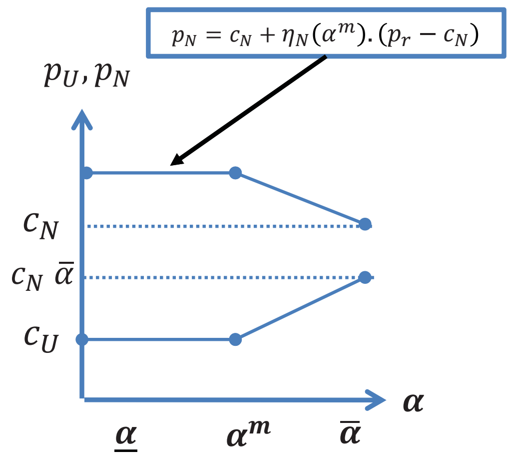
$$\eta_L^*(\alpha) = 0 \text{ for all } \alpha \in [\underline{\alpha}, \overline{\alpha}] \text{ and } \eta_H^*(\alpha) = \begin{cases} f(\tilde{\alpha}) & \underline{\alpha} \leq \alpha < \tilde{\alpha} \\ f(\alpha) & \max(\underline{\alpha}, \tilde{\alpha}) \leq \alpha \leq \overline{\alpha} \end{cases}$$

where $f(\alpha) = \frac{\alpha p_r + p_r - 2\alpha c_H - \sqrt{\alpha^2 p_r^2 + 2\alpha p_r^2 + p_r^2 - 4\alpha c_H p_r - 4\alpha c_H \overline{\alpha}(p_r - c_H)}}{2\alpha(p_r - c_H)}$ and $\tilde{\alpha} = \frac{(p_r - c_L)c_L}{c_H\overline{\alpha}p_r - \overline{\alpha}c_H^2 + c_H p_r - p_r c_L}$.

The following graph illustrates this possibility for semi-separating equilibria. Note that signalling through a L-winning strategy is possible only when $\tilde{\alpha} < \overline{\alpha}$, or equivalently, when $\overline{\alpha}(c_H + \eta_H(\overline{\alpha})(p_r - c_H) \geq c_L$, which translates to a situation where $\overline{\alpha} > \frac{c_L}{c_H}$.

Since the buyer has no incentive to manipulate this signal, the suppliers will update their belief in the following fashion. If they observe $0 \leq \eta_H \leq \min(\eta_H(\underline{\alpha}), \eta_H(\tilde{\alpha}))$, they update their belief using $\eta_H^{-1}(\eta_H(\alpha))$; but if they observe any value $\eta_H > \min(\eta_H(\underline{\alpha}), \eta_H(\tilde{\alpha}))$, they believe that $\underline{\alpha} \leq \alpha \leq \tilde{\alpha}$. In the former case, the equilibrium prices by the suppliers would be $p_H = c_H + \eta_H(p_r - c_H)$ and $p_L = \alpha(c_H + \eta_H(p_r - c_H))$, whereas in the latter, $p_H = c_H + \eta_H(p_r - c_H)$ and $p_L = c_L$.





□

Proof. Corollary 2:

We know that under information sharing, whenever the buyer uses different values of advance guarantees to supplier H, the equilibrium prices of suppliers H and L are, respectively, $p_H^{*SE}(\alpha) = c_H + \eta_H(\alpha)[p_r - c_H]$ and $p_L^{*SE}(\alpha) = \alpha[c_H + \eta_H(\alpha)[p_r - c_H]]$ at the range of $\alpha \in [\tilde{\alpha}, \overline{\alpha}]$. Below, we show that the bid prices by the suppliers H and L are strictly decreasing and increasing in α, respectively, in this range, while they remain unchanged in the range of $\alpha \in [\underline{\alpha}, \tilde{\alpha}]$.

<u>1- $p_H^{*SE}(\alpha)$ is decreasing in α:</u> In order to show that the supplier H's equilibrium price is decreasing (which is equivalent to showing that the signal η_H is decreasing in α), we take the first derivative of p_H^{*SE} with respect to α (for $\alpha \in [\tilde{\alpha}, \overline{\alpha}]$) and show that it is always negative given our assumptions on parameters α, $\underline{\alpha}$, $\overline{\alpha}$, $\frac{c_L}{c_H}$, and p_r. But before doing so, in order to simplify the form of derivatives, we denote $K_1 = \frac{1}{c_H(\overline{\alpha}p_r - \overline{\alpha}c_H + p_r)} > 0$ that makes the guarantee equal to: $f(\alpha) = \frac{\alpha p_r + p_r - 2\alpha c_H - \sqrt{\alpha^2 p_r^2 + 2\alpha p_r^2 + p_r^2 - 4\alpha/K_1}}{2\alpha(p_r - c_H)}$; therefore,

$$\frac{\partial p_H^{*SE}}{\partial \alpha} = -\frac{\sqrt{K_1^2 p_r^4 (1+\alpha)^2 - 4K_1\alpha p_r^2} - K_1 p_r^2(1+\alpha) + 2\alpha}{2\alpha^2\sqrt{K_1^2 p_r^2(1+\alpha)^2 - 4K_1\alpha}}, \qquad \forall\ \alpha \in [\tilde{\alpha}, \overline{\alpha}] \tag{13}$$

First, we find the acceptable domain where the above derivative is meaningful. Domain is the range of parameters where $K_1 p_r^2(1+\alpha)^2 \geq 4\alpha$. If we replace K_1 with its real value, it would transforms to $\frac{p_r^2(1+\alpha)^2}{c_H[p_r + \overline{\alpha}(p_r - c_H)]} \geq 4\alpha$. The left-hand side is always greater than $\frac{p_r^2(1+\alpha)^2}{c_H[2p_r - c_H]}$.

And, it is easy to show that $p_r^2(1+\alpha)^2 \geq 4\alpha c_H(2p_r - c_H)$ since $p_r^2 \geq c_H(2p_r - c_H)$ (for any $c_H \leq p_r$) and $(1+\alpha)^2 \geq 4\alpha$ (for any $0 < \alpha \leq 1$). Therefore, (13) is meaningful at any point of α and any upper and lower bounds such that $0 < \underline{\alpha} \leq \alpha \leq \overline{\alpha} \leq 1$.

Since the denumerator in (13) is always positive; it is sufficient to show that the numerator is always negative, or

$$\sqrt{K_1^2 p_r^4 (1+\alpha)^2 - 4K_1\alpha p_r^2} - K_1 p_r^2(1+\alpha) + 2\alpha > 0. \tag{14}$$

Let denote $A = K_1 p_r^2(1+\alpha) > 0$ and $B = 4K_1\alpha p_r^2 > 0$; hence, (14) follows, if:

$$\sqrt{A^2 - B} - A + 2\alpha > 0. \tag{15}$$

And, this holds only if:

$$B < 4A\alpha - 4\alpha^2 \tag{16}$$

If we replug the original values at (16), we find that it holds only if $K_1 p_r^2 > 1$, or equivalently when:

$$\frac{p_r^2}{c_H[p_r + \overline{\alpha}(p_r - c_H)]} > 1. \tag{17}$$

We know that $\frac{p_r^2}{c_H[p_r+\overline{\alpha}(p_r-c_H)]} \geq \frac{p_r^2}{c_H[2p_r-c_H]}$ for any $\overline{\alpha}$ ($0 < \overline{\alpha} \leq 1$). Also, $\frac{p_r^2}{c_H(2p_r-c_H)} > 1$, because the maximum value of $c_H(2p_r - c_H)$ would be when $c_H = p_r$; hence, for any c_H lower than p_r, we have $p_r^2 > c_H(2p_r - c_H)$. To summarize, p_H^{*SE} is strictly decreasing in α for $\alpha \in [\tilde{\alpha}, \overline{\alpha}]$ ($\tilde{\alpha}$ is given in Proposition 9). Also, supplier H's price would be constant in the range of $[\underline{\alpha}, \tilde{\alpha}]$ as it is not a function of α.

2- $p_L^{*SE}(\alpha)$ is increasing in α: A similar approach applies to prove that p_L^{*SE} is increasing in α.

$$\frac{\partial p_L^{*SE}}{\partial \alpha} = \frac{\sqrt{K_1^2 p_r^4(1+\alpha)^2 - 4K_1\alpha p_r^2} - K_1 p_r^2(1+\alpha) + 2}{2\sqrt{K_1^2 p_r^2(1+\alpha)^2 - 4K_1\alpha}}, \qquad \forall\ \alpha \in [\tilde{\alpha}, \overline{\alpha}]$$

It is sufficient to show that the numerator is always positive, or

$$\sqrt{K_1^2 p_r^4(1+\alpha)^2 - 4K_1\alpha p_r^2} - K_1 p_r^2(1+\alpha) + 2 > 0.$$

Therefore, by the same transformation of the variables to A and B, it follows if:

$$\sqrt{A^2 - B} - A + 2 > 0.$$

This holds only if $B < 4A - 4$. If we replug the original values, it transforms to $K_1 p_r^2 > 1$, or equivalently $\frac{p_r^2}{c_H[p_r + \bar{\alpha}(p_r - c_H)]} > 1$. And, this is always true as we showed it in the previous part. Therefore, supplier L's price would be increasing in α; in fact it remains fixed for $\alpha < \tilde{\alpha}$ and strictly increases for $\alpha \geq \tilde{\alpha}$. $\square$

Proof. Proposition 10:

Before analyzing the separating equilibrium, it is worthwhile to study the equilibrium under the symmetric information setting (as benchmark) when $c_L \geq c_H$. Under symmetric information, the equilibrium bids would be $p_L^* = c_L$ and $p_H^* = \min(c_L/\alpha, p_r)$ for $0 < \alpha \leq 1$. In that case, in the equilibrium, supplier H always wins at a price of $\min(c_L/\alpha, p_r)$ and makes some profit ($p_H^* \geq c_H$).

As we discussed in our previous analysis (proposition 9), under asymmetric information, if a supplier is guaranteed to have at least an order of $\eta_i Q$, he may feel less pressure on him to win the auction at very low prices. As a result, supplier i never bids below $p_i^{low} = c_i + \eta_i(p_r - c_i)$ where $p_i^{low} > c_i$ for $i = U$ or H. The following lemma characterizes the suppliers' optimal bids and the equilibrium profits if $c_L \geq c_H$ and they receive truthful information regarding α.

Lemma 7 *Let assume $\gamma^U = p_L^{low}/p_r$,$\gamma_1^N = c_L/p_H^{low}$,and $\gamma_2^N = c_L/p_r$. In the separating equilibrium, after observing the revenue guarantee $\eta_i Q p_r$ to supplier i (i=U or N) and correctly inferring α, the suppliers' optimal bid would be as presented in the Table 5–12.*

The proof of this lemma directly follows from the definition of p_i^{low} and the similarity to the symmetric information setting when the suppliers become informed of the true type.

A separating equilibrium must satisfy all the requirements mentioned before: a one-to-one

Table 5–12: Best responses of the suppliers and the buyer after receiving the signal: Separating Equilibrium ($c_H \leq c_L$).

Advance Guarantee (buyer's signal)		$0 \leq \eta_H \leq 1$			$0 \leq \eta_L \leq 1$	
Range of α		$0 < \alpha \leq \gamma_2^N$	$\gamma_2^N \leq \alpha \leq \min(1, \gamma_1^N)$	$\min(1, \gamma_1^N) \leq \alpha \leq 1$	$0 < \alpha \leq \gamma^U$	$\gamma^U \leq \alpha \leq 1$
Prices (bids)	p_H^*	p_r	$\frac{c_L}{\alpha}$	p_H^{low}	p_r	$\frac{p_L^{low}}{\alpha}$
	p_L^*	c_L	c_L	αp_H^{low}	p_L^{low}	p_L^{low}
Order Alloc	q_H^*, q_L^*	$Q, 0$	$Q, 0$	$\eta_H Q, (1-\eta_H)Q$	$(1-\eta_L)Q, \eta_L Q$	$(1-\eta_L)Q, \eta_L Q$
Suppliers'	π_H^*	$Q(p_r - c_H)$	$Q(\frac{c_L}{\alpha} - c_H)$	$\eta_H Q(p_r - c_H)$	$(1-\eta_L)Q(p_r - c_H)$	$(1-\eta_L)Q(\frac{p_L^{low}}{\alpha} - c_H)$
Profits	π_L^*	0	0	$(1-\eta_H)Q(\alpha p_H^{low} - c_L)$	$\eta_L Q(p_r - c_L)$	$\eta_L Q(p_r - c_L)$
Buyer's Cost	TC_B^*	Qp_r	$Q\frac{c_L}{\alpha}$	$Q(\eta_H p_r + (1-\eta_H)\alpha p_H^{low})$	Qp_r	$Q(\eta_L p_r + (1-\eta_L)\frac{p_L^{low}}{\alpha})$

† Note that $p_i^{low} = c_i + \eta_i(p_r - c_H)$ for $i = U$ and N only if the supplier i is offered an advance revenue guarantee of $\eta_i Q p_r$.

signal by the buyer who should have no incentive to deviate. We first, assume an interior separating equilibrium exists. Then, according to the Lemma 7, the suppliers' best response p_L and p_H and order allocation can be characterized depending on the buyer's choice of signal. In the equilibrium, when the buyer chooses $\eta_i(\hat{\alpha})$, the suppliers infer $\hat{\alpha}$ and bid accordingly. In the following, we analyze each signalling tool separately.

1) **Let $i = U$, i.e. the buyer guarantees a minimum revenue of $\eta_L Q p_r$ to supplier L:**

In the equilibrium, when the buyer chooses $\eta_L(\hat{\alpha})$, suppliers infer $\hat{\alpha}$ and respond accordingly by their price choice. We should find $\eta_L(\hat{\alpha})$ in such a way that the buyer never finds any incentive to cheat, i.e. $TC_B(\eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha) \leq TC_B(\eta_L(\alpha), p_H(\alpha), p_L(\alpha), \alpha)$.
The buyer's profit would be

$$TC_B(\eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha = \hat{\alpha}) = \eta_L(\hat{\alpha})Qp_r + (1 - \eta_L(\hat{\alpha}))Qp_H(\hat{\alpha})$$

$$TC_B(\eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha < \hat{\alpha}) = \eta_L(\hat{\alpha})Qp_r + (1 - \eta_L(\hat{\alpha}))Qp_H(\hat{\alpha})$$

$$TC_B(\eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha > \hat{\alpha}) = Qp_L$$

For this to be an equilibrium, the buyer's total cost should be minimized at $\hat{\alpha} = \alpha$ if the true type of the buyer is α. First, let find the optimal advance guarantee in which the buyer never signals a higher value of α. In that case, the following first order condition should be satisfied at $\hat{\alpha} = \alpha$:

$$\frac{\partial TC_B(\eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha)}{\partial \hat{\alpha}} |_{\hat{\alpha}=\alpha=0}$$

And, it leads to the following guarantee function:

$$\eta_L(\alpha) = \frac{-\alpha p_r + 2c_L - p_r + \sqrt{\alpha^2 p_r^2 - 4c_L \alpha p_r + 2\alpha p_r^2 + p_r^2 + 4p_r C_1 \alpha - 4c_L C_1 \alpha}}{2(-p_r + c_L)}$$

where C_1 is a constant and should be chosen such that $\eta_H(\alpha)$ satisfies all the required conditions and gives the most efficient signalling tool. It is easy to see that $\eta_H(\alpha)$ should be increasing in α and takes the highest value at $\alpha = \overline{\alpha}$. By Using this guarantee, if the buyer sticks to the truthful signalling of $\alpha = \hat{\alpha}$, his total cost would be fixed equal to $\min(p_r, \frac{c_L}{\underline{\alpha}}) > c_L$. But, using this signalling tool, the buyer always prefer to guarantee $\eta_L(\hat{\alpha})$ to signal a QS of $\hat{\alpha}$ far lower than α. In fact by $\eta_L \simeq 0$, the buyer signals that α is very low close to $\underline{\alpha}$ to receive bids of $p_L = c_L$ and $p_H = min(p_r, c_L/\underline{\alpha})$. In that case, supplier H loses the auction while supplier L wins at its lowest possible price c_L. Therefore, $TC_B(\eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha > \hat{\alpha}) < TC_B(\eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha = \hat{\alpha})$ which makes the separating equilibrium fail to exist and work.

2) Let $i = N$, i.e. the buyer guarantees a minimum revenue of $\eta_H Q p_r$ to the supplier N:

In the equilibrium, the guarantee to supplier N, $\eta_H Q p_r$, cannot be lower than what makes $p_H^{low} = c_L/\overline{\alpha}$, otherwise it transforms no information as supplier H can always takes $p_H = c_L/\overline{\alpha}$ and wins the auction with no need for more information. By a similar reasoning as previous part, we can show that $\eta_H(\alpha)$ should be decreasing at α to make sure that the buyer never signals a $\hat{\alpha}$ lower than α.

$$TC_B(\eta_H(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha = \hat{\alpha}) = \eta_H(\hat{\alpha}) Q p_r + (1 - \eta_H(\hat{\alpha})) Q p_L(\hat{\alpha})$$

$$TC_B(\eta_H(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha > \hat{\alpha}) = \eta_H(\hat{\alpha}) Q p_r + (1 - \eta_H(\hat{\alpha})) Q p_L(\hat{\alpha})$$

$$TC_B(\eta_H(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha < \hat{\alpha}) = Qp_H$$

But, in the same fashion as before, with a decreasing signal, the buyer would be always motivated to signal a higher value of α (by a lower guarantee) to mislead both suppliers so that supplier L offers a higher price than expected in the equilibrium while supplier H offers his lowest possible price and wins the auction at $p_H = \frac{c_L}{\alpha}$. Therefore, $TC_B(\eta_H(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha < \hat{\alpha}) < TC_B(\eta_H(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha = \hat{\alpha})$, which makes the separating equilibrium fail to exist and work.

□

Proof. Proposition 11:

Following the same procedure taken in the proof of Proposition 9, we focus on the three regions that are defined based on values in Table 3–4. But, in contrast, we show that signalling fails in all those three regions given the new definition for the buyer's cost function.

1. Let $0 < \alpha < \gamma_2$: Again, the buyer is reluctant to share the relative QS in this region because p_r is the worst cost for him, which cannot be reduced by sharing.
2. Let $\gamma_2 < \alpha < \gamma_1$: Under this region, the buyer's total cost if his real type is α and he signals $\hat{\alpha}$ would be:

$$\kappa_B(\eta_H(\hat{\alpha}), \eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha) = \begin{cases} \eta_L(\hat{\alpha})Q\frac{p_r}{\hat{\alpha}} + (1-\eta_L(\hat{\alpha}))Q\frac{p_L^{low}}{\hat{\alpha}} & \text{if } \hat{\alpha} = \alpha \\ \eta_L(\hat{\alpha})Q\frac{p_r}{\hat{\alpha}} + (1-\eta_L(\hat{\alpha}))Q\frac{p_L^{low}}{\hat{\alpha}} & \text{if } \hat{\alpha} > \alpha \\ Qp_L^{low} & \text{if } \hat{\alpha} < \alpha \end{cases} \quad (18)$$

   Similarly, we can find an increasing $\eta_L(\alpha)$, which makes the buyer not to signal a higher value. But obviously, the buyer's actual cost will be minimal if he signals the lowest possible value of α, which in the end distorts buyer's incentive in sharing suppliers' QS.

3. Let $\gamma_1 < \alpha < 1$: Here we focus on the revenue guarantees that make supplier L win the auction. The buyer's total cost if his real type is α and he signals $\hat{\alpha}$, would be:

$$\kappa_B(\eta_H(\hat{\alpha}), \eta_L(\hat{\alpha}), p_H(\hat{\alpha}), p_L(\hat{\alpha}), \alpha) = \begin{cases} \eta_H(\hat{\alpha})Qp_r + (1-\eta_H(\hat{\alpha}))Q\frac{\hat{\alpha}p_H^{low}}{\alpha} & \text{if } \hat{\alpha} = \alpha \\ Qp_H^{low} & \text{if } \hat{\alpha} > \alpha \\ \eta_H(\hat{\alpha})Qp_r + (1-\eta_H(\hat{\alpha}))Q\frac{\hat{\alpha}p_H^{low}}{\alpha} & \text{if } \hat{\alpha} < \alpha \end{cases}$$

We can easily find a decreasing $\eta_H(\alpha)$ that removes buyer's incentive in signalling a lower value than his true type α. However, the buyer can actually deviate and signal $\overline{\alpha}$ and guarantee a winning price of $\frac{c_L}{\overline{\alpha}}$ by the supplier H, which always leads to the lowest total QS-adjusted cost for him, i.e.

$$\kappa_B(\eta_H(\overline{\alpha}), \eta_L(\overline{\alpha}), p_H(\overline{\alpha}), p_L(\overline{\alpha}), \alpha) \leq \kappa_B(\eta_H(\alpha), \eta_L(\alpha), p_H(\alpha), p_L(\alpha), \alpha).$$

This leads to a general failure in signalling α with other parties.

Proof. Proposition 12:

First, we characterize the expected value of equilibrium decisions in pooling and separating equilibria. The expectation in both cases is only with respect to α. The equilibrium decisions in the region C_1 do not depend on α under the pooling equilibria. Therefore, we can easily capture them from Proposition 6 as presented again in Table 5–13 below.

Note that in region C_1 (i.e. where $\underline{\alpha} \geq \frac{c_L + c_H\overline{\alpha}}{2c_H}$), all the values of α ($\underline{\alpha} \leq \alpha \leq \overline{\alpha}$) can be separately signaled by the buyer (fully separating in Figure 3–4). To show this, it is sufficient to verify that the region $\underline{\alpha} \geq \frac{(p_r - c_L)c_L}{c_H\overline{\alpha}p_r - \overline{\alpha}c_H^2 + c_Hp_r - p_rc_L}$ (the locus of points where fully separating is possible) always includes the whole region of $\underline{\alpha} \geq \frac{c_L + c_H\overline{\alpha}}{2c_H}$; and it is true if the line $\underline{\alpha} = \frac{(p_r - c_L)c_L}{c_H\overline{\alpha}p_r - \overline{\alpha}c_H^2 + c_Hp_r - p_rc_L}$ is always lower than $\underline{\alpha} = \frac{c_L + c_H\overline{\alpha}}{2c_H}$ for $\frac{c_L}{c_H} < \overline{\alpha} < 1$ (see Figure 3–4 in Proposition 6). Both lines are continous and start from the point ($\underline{\alpha} = \frac{c_L}{c_H}, \overline{\alpha} = \frac{c_L}{c_H}$); but the former and the latter are strictly decreasing and increasing in $\overline{\alpha}$, respectively; i.e.,

$$\underline{\alpha} = \frac{(p_r - c_L)c_L}{c_H\overline{\alpha}p_r - \overline{\alpha}c_H^2 + c_Hp_r - p_rc_L} \quad \rightarrow \quad \frac{\partial\underline{\alpha}}{\partial\overline{\alpha}} = -\frac{(p_r - c_L)c_L(c_Hp_r - c_H^2)}{(c_H\overline{\alpha}p_r - \overline{\alpha}c_H^2 + c_Hp_r - p_rc_L)^2} < 0$$

$$\underline{\alpha} = \frac{c_L + c_H\overline{\alpha}}{2c_H} \quad \rightarrow \quad \frac{\partial\underline{\alpha}}{\partial\overline{\alpha}} = \frac{1}{2} > 0$$

Hence, the region $\underline{\alpha} \geq \frac{(p_r - c_L)c_L}{c_H\overline{\alpha}p_r - \overline{\alpha}c_H^2 + c_H p_r - p_r c_L}$ always completely includes the whole region of $\underline{\alpha} \geq \frac{c_L + c_H\overline{\alpha}}{2c_H}$, which implies that the buyer fully shares all the values of α ($\underline{\alpha} \leq \alpha \leq \overline{\alpha}$) by distinct signals (no semi-separating in C_1). Therefore, under separating equilibria (Proposition 9), the expected values of equilibrium decisions with respect to α can be easily derived as expressed in Table 5–13.

Table 5–13: Expected values of equilibrium decisions under pooling and separating equilibria for the points in region C_1.

Equilibrium		Pooling	Separating
Prices (bids)	$\bar{p}_H^*$	c_H	$c_H + \mu_{\eta_H}(p_r - c_H)$
	$\bar{p}_L^*$	$c_H\underline{\alpha}$	$c_H\mu_\alpha + (p_r - c_H)[\sigma_{\alpha,\eta_H} + \mu_\alpha\mu_{\eta_H}]$
Order Alloc	$\bar{q}_H^*$	0	$Q\mu_{\eta_H}$
	$\bar{q}_L^*$	Q	$Q(1 - \mu_{\eta_H})$

Let σ_{α,η_H} denote the covariance of α and η_H and $\mu_{\eta_H} = \int_{\underline{\alpha}}^{\overline{\alpha}} \eta_H(\alpha)\frac{1}{\overline{\alpha}-\underline{\alpha}}d\alpha$.

Observing the values in this table and considering that $\mu_\alpha = \frac{\underline{\alpha}+\overline{\alpha}}{2} > \underline{\alpha} \geq 0$, it is easily verifiable that $\bar{p}_H^{*PE} < \bar{p}_H^{*SE}$, $\bar{q}_H^{*PE} < \bar{q}_H^{*SE}$, $\bar{q}_L^{*PE} > \bar{q}_L^{*SE}$, and $\bar{p}_L^{*PE} < \bar{p}_L^{*SE}$.

□

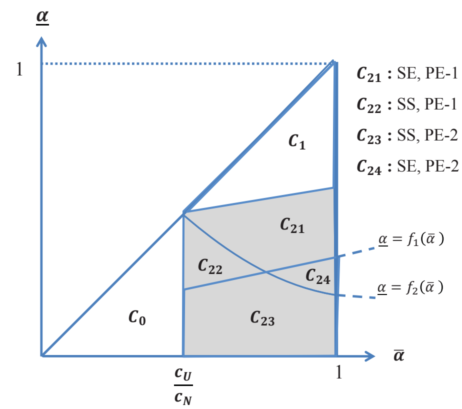
Proof. Impact of Information Sharing On Prices and Quantities (Region C_2 in Figure 3–5):

Here, we focus on the region C_2 in Figure 3–5 and compare the parties' decisions in pooling and separating equilibria. In total, this region can be divided at most to four sub-regions depending on the final outcome for separating (either fully- or semi-separating, referred to as SE and SS, respectively) and pooling (either PE-1 or PE-2) equilibria, as illustrated in Figure 5–1. Note that among these four regions, C_{24}, where PE-2 and fully separating are the final outcomes of pooling and separating equilibria respectively, would emerge only if:

$$f_2(\overline{\alpha} = 1) \equiv \frac{(p_r - c_L)c_L}{c_H p_r - c_H^2 + c_H p_r - p_r c_L} < \frac{c_H(c_L + p_r)}{2p_r^2} \equiv f_1(\overline{\alpha} = 1).$$

All the other cases (i.e. C_{21}, C_{22}, C_{23}) would prevail regardless of the values of parameters p_r, c_L, and c_H, as long as $0 \leq c_L < c_H < p_r$.

Figure 5–1: Sub-regions of region C_2 depending on different possible pooling and separating outcomes.



Below, we provide the expected values of equilibrium prices and quantities with respect to α under each region.

- Let $(\underline{\alpha}, \overline{\alpha}) \in C_{21}$: The expected equilibrium decisions in pooling and separating equilibria in region C_{21} are characterized based on Propositions 6 and 9, respectively, and are provided in Table 5–14. Note that the expected quantities under pooling equilibria for all the sub-regions in C_2 are calculated as below:
$\bar{q}_H^* = Q \times Prob(Hwins) = Q\frac{\gamma-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}$ and $\bar{q}_L^* = Q \times Prob(Lwins) = Q\frac{\overline{\alpha}-\gamma}{\overline{\alpha}-\underline{\alpha}}$ where $\gamma = \frac{p_L^{*PE}}{p_H^{*PE}}$.

Table 5–14: Expected values of equilibrium decisions under pooling and separating equilibria for the points in region C_{21}.

Equilibrium		Pooling	Separating
Prices (bids)	$\bar{p}_H^*$	$\frac{\overline{\alpha}c_H+\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}}{4\underline{\alpha}}$	$c_H + \mu_{\eta_H}(p_r - c_H)$
	$\bar{p}_L^*$	$\frac{(\overline{\alpha}^2c_H+\overline{\alpha}\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}+4\underline{\alpha}c_L)}{8\underline{\alpha}}$	$c_H\mu_\alpha + (p_r - c_H)[\sigma_{\alpha,\eta_H} + \mu_\alpha\mu_{\eta_H}]$
Order Alloc	$\bar{q}_H^*$	$\frac{Q}{2}.\frac{c_H\overline{\alpha}^2+(\overline{\alpha}-2\underline{\alpha})\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L}+4c_L\underline{\alpha}-2\underline{\alpha}c_H\overline{\alpha}}{(c_H\overline{\alpha}+\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L})(\overline{\alpha}-\underline{\alpha})}$	$Q\mu_{\eta_H}$
	$\bar{q}_L^*$	$\frac{Q}{2}.\frac{c_H\overline{\alpha}^2+\overline{\alpha}\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L}-4c_L\underline{\alpha}}{(c_H\overline{\alpha}+\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L})(\overline{\alpha}-\underline{\alpha})}$	$Q(1-\mu_{\eta_H})$

Let σ_{α,η_H} denote the covariance of α and η_H and $\mu_{\eta_H} = \int_{\underline{\alpha}}^{\overline{\alpha}} \eta_H(\alpha)\frac{1}{\overline{\alpha}-\underline{\alpha}}d\alpha$.

According to this table, in the sub-region C_{21}, the followings hold true:

1. $\bar{p}_H^{PE} < \bar{p}_H^{SE}$ if $\Delta_{p_H}^{SE1} = c_H + \mu_{\eta_H}(p_r - c_H) - \frac{\overline{\alpha}c_H+\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}}{4\underline{\alpha}} > 0$
2. $\bar{p}_L^{PE} < \bar{p}_L^{SE}$ if $\Delta_{p_L}^{SE1} = c_H\mu_\alpha+(p_r-c_H)[\sigma_{\alpha,\eta_H}+\mu_\alpha\mu_{\eta_H}]-\frac{(\overline{\alpha}^2c_H+\overline{\alpha}\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}+4\underline{\alpha}c_L)}{8\underline{\alpha}} > 0$
3. $\bar{q}_H^{PE} < \bar{q}_H^{SE}$ if $\Delta_{q_H}^{SE1} = Q\mu_{\eta_H} - \frac{Q}{2}.\frac{c_H\overline{\alpha}^2+(\overline{\alpha}-2\underline{\alpha})\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L}+4c_L\underline{\alpha}-2\underline{\alpha}c_H\overline{\alpha}}{(c_H\overline{\alpha}+\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L})(\overline{\alpha}-\underline{\alpha})} > 0$

4. $\bar{q}_L^{PE} > \bar{q}_L^{SE}$ if $\Delta_{q_L}^{SE1} = Q(1-\mu_{\eta_H}) - \frac{Q}{2}.\frac{c_H\overline{\alpha}^2+\overline{\alpha}\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L}-4c_L\underline{\alpha}}{(c_H\overline{\alpha}+\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L})(\overline{\alpha}-\underline{\alpha})} < 0$

- Let $(\underline{\alpha}, \overline{\alpha}) \in C_{22}$: The expected values of equilibrium outcomes in pooling and separating equilibria are provided in Table 5–15 below.

  In a semi-separating equilibrium (a combination of pooling and separating equilibria), suppliers' decisions follow a mixed-type distribution (a mix of discrete and continuous distributions). More specifically, with a probability of $\left(P_D = \frac{\tilde{\alpha}-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}\right)$, the outcome is a fixed, discrete value, whereas it can take distinct continuous values with the probability of $\left(P_C = \frac{\overline{\alpha}-\tilde{\alpha}}{\overline{\alpha}-\underline{\alpha}} = 1 - P_D\right)$. For such a mixed-type variable, the expected value would be computed as follows:

$$E(x) = P_D E(x_D) + P_C E(x_C) \tag{19}$$

  where $E(x)$ is the expected value of the mixed-type random variable x; and P_D and $E(x_D)$ [P_C and $E(x_c)$] denote the probability of occurrence and the expected value of the discrete [continuous] part, respectively. We use (19) to calculate the expected values of prices and quantities at regions C_{22} and C_{23} under which the buyer uses a mix of pooling and separating (semi-separating) strategies.

Table 5–15: Expected values of equilibrium decisions under pooling and separating equilibria for the points in region C_{22}.

Equilibrium		Pooling	Semi-Separating
Prices (bids)	$\bar{p}_H^*$	$\frac{\overline{\alpha}c_H+\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}}{4\underline{\alpha}}$	$c_H + (p_r - c_H)(P_D\tilde{\eta}_H + P_C\tilde{\mu}_{\eta_H})$
	$\bar{p}_L^*$	$\frac{(\overline{\alpha}^2c_H+\overline{\alpha}\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}+4\underline{\alpha}c_L)}{8\underline{\alpha}}$	$P_Dc_L + P_C\left[\tilde{\mu}_\alpha c_H + \tilde{E}(\alpha\eta_H)(p_r-c_H)\right]$
Order Alloc	$\bar{q}_H^*$	$\frac{Q}{2}.\frac{c_H\overline{\alpha}^2+(\overline{\alpha}-2\underline{\alpha})\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L}+4c_L\underline{\alpha}-2\underline{\alpha}c_H\overline{\alpha}}{(c_H\overline{\alpha}+\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L})(\overline{\alpha}-\underline{\alpha})}$	$Q\left[P_D\tilde{\eta}_H + P_C\tilde{\mu}_{\eta_H}\right]$
	$\bar{q}_L^*$	$\frac{Q}{2}.\frac{c_H\overline{\alpha}^2+\overline{\alpha}\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L}-4c_L\underline{\alpha}}{(c_H\overline{\alpha}+\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L})(\overline{\alpha}-\underline{\alpha})}$	$Q\left[P_D(1-\tilde{\eta}_H) + P_C(1-\tilde{\mu}_{\eta_H})\right]$

Let $\tilde{\eta}_H = \eta_H(\tilde{\alpha}) = \eta_H(\frac{(p_r-c_L)c_L}{c_H\overline{\alpha}p_r-\overline{\alpha}c_H^2+c_Hp_r-p_rc_L})$; $\tilde{\mu}_{\eta_H} = \int_{\tilde{\alpha}}^{\overline{\alpha}}\eta_H(\alpha)\frac{1}{\overline{\alpha}-\tilde{\alpha}}d\alpha$; $\tilde{E}(x) = \int_{\tilde{\alpha}}^{\overline{\alpha}}x\frac{1}{\overline{\alpha}-\tilde{\alpha}}d\alpha$; and $\tilde{\mu}_\alpha = \frac{\tilde{\alpha}+\overline{\alpha}}{2}$.

According to this table, in this region:

1. $\bar{p}_H^{PE} < \bar{p}_H^{SE}$ if $\Delta_{p_H}^{SE1} = c_H + (p_r - c_H)(P_D\tilde{\eta}_H + P_C\tilde{\mu}_{\eta_H}) - \frac{\overline{\alpha}c_H+\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}}{4\underline{\alpha}} > 0$
2. $\bar{p}_L^{PE} < \bar{p}_L^{SE}$ if $\Delta_{p_L}^{SE1} = P_Dc_L + P_C\left[\tilde{\mu}_\alpha c_H + \tilde{E}(\alpha\eta_H)(p_r-c_H)\right] - \frac{(\overline{\alpha}^2c_H+\overline{\alpha}\sqrt{\overline{\alpha}^2c_H^2+8\underline{\alpha}c_Hc_L}+4\underline{\alpha}c_L)}{8\underline{\alpha}} > 0$
3. $\bar{q}_H^{PE} < \bar{q}_H^{SE}$ if $\Delta_{q_H}^{SE1} = Q\left[P_D\tilde{\eta}_H + P_C\tilde{\mu}_{\eta_H}\right] - \frac{Q}{2}.\frac{c_H\overline{\alpha}^2+(\overline{\alpha}-2\underline{\alpha})\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L}+4c_L\underline{\alpha}-2\underline{\alpha}c_H\overline{\alpha}}{(c_H\overline{\alpha}+\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L})(\overline{\alpha}-\underline{\alpha})} > 0$

4. $\bar{q}_L^{PE} > \bar{q}_L^{SE}$ if $\Delta_{q_L}^{SE1} = Q\left[P_D(1-\tilde{\eta}_H) + P_C(1-\tilde{\mu}_{\eta_H})\right] - \frac{Q}{2}.\frac{c_H\overline{\alpha}^2+\overline{\alpha}\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L}-4c_L\underline{\alpha}}{(c_H\overline{\alpha}+\sqrt{c_H^2\overline{\alpha}^2+8\underline{\alpha}c_Hc_L})(\overline{\alpha}-\underline{\alpha})} < 0$

- Let $(\underline{\alpha}, \overline{\alpha}) \in C_{23}$: Table 5–16 presents the expected equilibrium decision values under pooling and separating equilibria derived from Propositions 6 and 9, respectively.

Table 5–16: Expected values of equilibrium decisions under pooling and separating equilibria for the points in region C_{23}.

Equilibrium		Pooling	Semi-Separating
Prices (bids)	$\bar{p}_H^*$	p_r	$c_H + (p_r - c_H)(P_D\tilde{\eta}_H + P_C\tilde{\mu}_{\eta_H})$
	$\bar{p}_L^*$	$\frac{\overline{\alpha}p_r+c_L}{2}$	$P_Dc_L + P_C\left[\tilde{\mu}_\alpha c_H + \tilde{E}(\alpha\eta_H)(p_r - c_H)\right]$
Order Alloc	$\bar{q}_H^*$	$\frac{Q}{2}.\frac{\overline{\alpha}p_r+c_L-2\underline{\alpha}p_r}{p_r(\overline{\alpha}-\underline{\alpha})}$	$Q\left[P_D\tilde{\eta}_H + P_C\tilde{\mu}_{\eta_H}\right]$
	$\bar{q}_L^*$	$\frac{Q}{2}.\frac{\overline{\alpha}p_r-c_L}{p_r(\overline{\alpha}-\underline{\alpha})}$	$Q\left[P_D(1-\tilde{\eta}_H) + P_C(1-\tilde{\mu}_{\eta_H})\right]$

Let $\tilde{\eta}_H = \eta_H(\tilde{\alpha}) = \eta_H(\frac{(p_r-c_L)c_L}{c_H\overline{\alpha}p_r-\overline{\alpha}c_H^2+c_Hp_r-p_rc_L})$; $\tilde{\mu}_{\eta_H} = \int_{\tilde{\alpha}}^{\overline{\alpha}} \eta_H(\alpha)\frac{1}{\overline{\alpha}-\tilde{\alpha}}d\alpha$; $\tilde{E}(x) = \int_{\tilde{\alpha}}^{\overline{\alpha}} x\frac{1}{\overline{\alpha}-\tilde{\alpha}}d\alpha$; and $\tilde{\mu}_\alpha = \frac{\tilde{\alpha}+\overline{\alpha}}{2}$.

According to this table, the followings hold true:

1. $\bar{p}_H^{PE} \geq \bar{p}_H^{SE}$
2. $\bar{p}_L^{PE} < \bar{p}_L^{SE}$ if $\Delta_{p_L}^{SE1} = P_Dc_L + P_C\left[\tilde{\mu}_\alpha c_H + \tilde{E}(\alpha\eta_H)(p_r - c_H)\right] - \frac{\overline{\alpha}p_r+c_L}{2} > 0$
3. $\bar{q}_H^{PE} < \bar{q}_H^{SE}$ if $\Delta_{q_H}^{SE1} = Q\left[P_D\tilde{\eta}_H + P_C\tilde{\mu}_{\eta_H}\right] - \frac{Q}{2}.\frac{\overline{\alpha}p_r+c_L-2\underline{\alpha}p_r}{p_r(\overline{\alpha}-\underline{\alpha})} > 0$
4. $\bar{q}_L^{PE} > \bar{q}_L^{SE}$ if $\Delta_{q_L}^{SE1} = Q\left[P_D(1-\tilde{\eta}_H) + P_C(1-\tilde{\mu}_{\eta_H})\right] - \frac{Q}{2}.\frac{\overline{\alpha}p_r-c_L}{p_r(\overline{\alpha}-\underline{\alpha})} < 0$

- Let $(\underline{\alpha}, \overline{\alpha}) \in C_{24}$: The expected equilibrium prices and quantities in pooling and separating equilibria are stated in Table 5–17.

Table 5–17: Expected values of equilibrium decisions under pooling and separating equilibria for the points in region C_{24}.

Equilibrium		Pooling	Separating
Prices (bids)	$\bar{p}_H^*$	p_r	$c_H + \mu_{\eta_H}(p_r - c_H)$
	$\bar{p}_L^*$	$\frac{\overline{\alpha}p_r+c_L}{2}$	$c_H\mu_\alpha + (p_r - c_H)[\sigma_{\alpha,\eta_H} + \mu_\alpha\mu_{\eta_H}]$
Order Alloc	$\bar{q}_H^*$	$\frac{Q}{2}.\frac{\overline{\alpha}p_r+c_L-2\underline{\alpha}p_r}{p_r(\overline{\alpha}-\underline{\alpha})}$	$Q\mu_{\eta_H}$
	$\bar{q}_L^*$	$\frac{Q}{2}.\frac{\overline{\alpha}p_r-c_L}{p_r(\overline{\alpha}-\underline{\alpha})}$	$Q(1 - \mu_{\eta_H})$

Let σ_{α,η_H} denote the covariance of α and η_H and $\mu_{\eta_H} = \int\limits_{\underline{\alpha}}^{\overline{\alpha}} \eta_H(\alpha)\frac{1}{\overline{\alpha}-\underline{\alpha}}d\alpha$.

According to this table, we can easily compare the equilibrium decisions under pooling and separating equilibria, as follows:

1. $\bar{p}_H^{PE} \geq \bar{p}_H^{SE}$

2. $\bar{p}_L^{PE} < \bar{p}_L^{SE}$ if $\Delta_{p_L}^{SE1} = c_H\mu_\alpha + (p_r - c_H)[\sigma_{\alpha,\eta_H} + \mu_\alpha\mu_{\eta_H}] - \frac{\overline{\alpha}p_r + c_L}{2} > 0$
3. $\bar{q}_H^{PE} < \bar{q}_H^{SE}$ if $\Delta_{q_H}^{SE1} = Q\mu_{\eta_H} - \frac{Q}{2}.\frac{\overline{\alpha}p_r + c_L - 2\underline{\alpha}p_r}{p_r(\overline{\alpha}-\underline{\alpha})} > 0$
4. $\bar{q}_L^{PE} > \bar{q}_L^{SE}$ if $\Delta_{q_L}^{SE1} = Q(1-\mu_{\eta_H}) - \frac{Q}{2}.\frac{\overline{\alpha}p_r - c_L}{p_r(\overline{\alpha}-\underline{\alpha})} < 0$

□

Proof. Proposition 13: We first characterize the expected values of equilibrium payoffs with respect to α, according to Propositions 6 and 9, as expressed in the following table.

Table 5–18: Expected values of equilibrium profits/costs under pooling and separating equilibria for the points in region C_1.

Equilibrium		Pooling	Separating
Suppliers'	$\overline{\pi}_H^*$	0	$Q\mu_{\eta_H}(p_r - c_H)$
Profits	$\overline{\pi}_L^*$	$Q(c_H\underline{\alpha} - c_L)$	$Q[c_H\mu_\alpha + (p_r - 2c_H)(\sigma_{\alpha,\eta_H} + \mu_\alpha\mu_{\eta_H}) + (p_r - c_H)E(\eta_H^2\alpha) + c_L\mu_{\eta_H} - c_L]$
Buyer's Cost	$\overline{\kappa}_B^*$	$Qc_H\underline{\alpha}$	$Qc_H\overline{\alpha}$
Total Supply chain Cost	$\overline{\kappa}_{SC}^*$	Qc_L	$Q(\mu_{\eta_H}c_H + (1-\mu_{\eta_H})c_L)$

Based on the table and given that $\mu_{\eta_H} > 0$, it is easy to verify that $\overline{\pi}_H^{*PE} < \overline{\pi}_H^{*SE}$, $\overline{\kappa}_B^{*PE} < \overline{\kappa}_B^{*SE}$, and $\overline{\kappa}_{SC}^{*PE} < \overline{\kappa}_{SC}^{*SE}$. But for supplier L, since $q_L^{*SE} < q_L^{*PE}$ and $p_L^{*SE} > p_L^{*PE}$, her profit can be lower or higher under the pooling compared to the separating equilibrium. However, it is easy to see that when the unit cost of supplier L is high enough, her profit would be lower under pooling equilibria, i.e.

$$\forall c_L : c_L \geq \overline{c}_L = \frac{(2c_H - p_r)(\sigma_{\alpha,\eta_H} + \mu_\alpha\mu_{\eta_H}) - c_H\left(\frac{\overline{\alpha}-\underline{\alpha}}{2}\right) - (p_r - c_H)E(\eta_H^2\alpha)}{\mu_{\eta_H}} \Rightarrow \overline{\pi}_L^{*PE} < \overline{\pi}_L^{*SE}$$

□

Proof. Impact of Information Sharing On Supply Chain Parties' Profits/Cost (Region C_2 in Figure 3–6):

Here, we provide $\Delta_{\pi_H}^{SE1}$, $\Delta_{\pi_L}^{SE1}$, $\Delta_{\kappa_B}^{SE1}$, and $\Delta_{\kappa_{SC}}^{SE1}$ for the points in the region C_2. Note that depending on the sub-region (C_{xx}) for $(\underline{\alpha}, \overline{\alpha})$, these thresholds may differ.

- Let $(\underline{\alpha}, \overline{\alpha}) \in C_{21}$: The expected equilibrium profits/costs in pooling and separating equilibria, derived based on Propositions 6 and 9, are presented in Table 5–19.

Let $S = c_H\overline{\alpha} + \sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L}$. Now, according to this table, we can compare supply chains' payoffs in this sub-region, as follows:

Table 5–19: Expected values of equilibrium profits/costs under pooling and separating equilibria for the points in region C_{21}.

Equilibrium		Pooling	Separating
Suppliers'	$\overline{\pi}_H^*$	$\frac{Q}{8}\frac{(S-4\underline{\alpha}c_H)(\overline{\alpha}S+4c_L\underline{\alpha}-2\underline{\alpha}S)}{\underline{\alpha}S(\overline{\alpha}-\underline{\alpha})}$	$Q\mu_{\eta_H}(p_r - c_H)$
Profits	$\overline{\pi}_L^*$	$\frac{Q}{16}\frac{(\overline{\alpha}S-4c_L\underline{\alpha})^2}{\underline{\alpha}(\overline{\alpha}-\underline{\alpha})S}$	$Q[c_H\mu_\alpha + (p_r - 2c_H)(\sigma_{\alpha,\eta_H} + \mu_\alpha\mu_{\eta_H}) + (p_r - c_H)E(\eta_H^2\alpha) + c_L\mu_{\eta_H} - c_L]$
Buyer's Cost	$\overline{\kappa}_B^*$	$\frac{Q}{8}\frac{\psi(S)}{S\underline{\alpha}(\overline{\alpha}-\underline{\alpha})}$	$Qc_H\overline{\alpha}$
Total SC Cost	$\overline{\kappa}_{SC}^*$	$\frac{Q}{2}\frac{c_H\overline{\alpha}S-2c_H\underline{\alpha}S+4\underline{\alpha}c_Lc_H+\overline{\alpha}c_LS-4c_L^2\underline{\alpha}}{S(\overline{\alpha}-\underline{\alpha})}$	$Q(\mu_{\eta_H}c_H + (1-\mu_{\eta_H})c_L)$

where $S = c_H\overline{\alpha} + \sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L}$ and $\psi(S) = 2c_H\overline{\alpha}^2S + 8c_L\underline{\alpha}c_H\overline{\alpha} - 4\underline{\alpha}c_L\overline{\alpha}S - 16\underline{\alpha}^2c_Hc_L + 4Sc_L\underline{\alpha} + c_L\overline{\alpha}^3S + 4c_H\overline{\alpha}^2c_L\underline{\alpha} - 8c_L^2\underline{\alpha}^2$.

1. $\bar{\pi}_H^{PE} < \bar{\pi}_H^{SE}$ if $\Delta_{\pi_H}^{SE1} = Q\mu_{\eta_H}(p_r - c_H) - \frac{Q}{8}\frac{(S-4\underline{\alpha}c_H)(\overline{\alpha}S+4c_L\underline{\alpha}-2\underline{\alpha}S)}{\underline{\alpha}S(\overline{\alpha}-\underline{\alpha})} > 0$
2. $\bar{\pi}_L^{PE} < \bar{\pi}_L^{SE}$ if $\Delta_{\pi_L}^{SE1} =$
$$= Q[c_H\mu_\alpha + (p_r - 2c_H)(\sigma_{\alpha,\eta_H} + \mu_\alpha\mu_{\eta_H}) + (p_r - c_H)E(\eta_H^2\alpha) + c_L\mu_{\eta_H} -$$
$c_L] - \frac{Q}{16}\frac{(\overline{\alpha}S-4c_L\underline{\alpha})^2}{\underline{\alpha}(\overline{\alpha}-\underline{\alpha})S} > 0$
3. $\overline{\kappa}_B^{PE} < \overline{\kappa}_B^{SE}$ if $\Delta_{\kappa_B}^{SE1} =$
$$= Qc_H\overline{\alpha} - \frac{Q}{8}.\frac{2c_H\overline{\alpha}^2S+8c_L\underline{\alpha}c_H\overline{\alpha}-4\underline{\alpha}c_L\overline{\alpha}S-16\underline{\alpha}^2c_Hc_L+4Sc_L\underline{\alpha}+c_L\overline{\alpha}^3S+4c_H\overline{\alpha}^2c_L\underline{\alpha}-8c_L^2\underline{\alpha}^2}{S\underline{\alpha}(\overline{\alpha}-\underline{\alpha})} > 0$$
4. $\overline{\kappa}_{SC}^{PE} < \overline{\kappa}_{SC}^{SE}$ if $\Delta_{\kappa_{SC}}^{SE1} = Q(\mu_{\eta_H}c_H + (1-\mu_{\eta_H})c_L) - \frac{1}{2}.\frac{c_H\overline{\alpha}S-2c_H\underline{\alpha}S+4\underline{\alpha}c_Lc_H+\overline{\alpha}c_LS-4c_L^2\underline{\alpha}}{S(\overline{\alpha}-\underline{\alpha})} > 0$

- Let $(\underline{\alpha}, \overline{\alpha}) \in C_{22}$: The expected values of equilibrium profits/costs under the pooling and separating equilibria in the sub-region C_{22} are stated in Table 5–20. According to this table, the comparison of the profits/costs functions would be as follows:

Table 5–20: Expected values of equilibrium profits/costs under pooling and separating equilibria for the points in region C_{22}.

Equilibrium		Pooling	Separating
Suppliers'	$\overline{\pi}_H^*$	$\frac{Q}{8}\frac{(S-4\underline{\alpha}c_H)(\overline{\alpha}S+4c_L\underline{\alpha}-2\underline{\alpha}S)}{\underline{\alpha}S(\overline{\alpha}-\underline{\alpha})}$	$(p_r - c_H)Q\left[P_D\tilde{\eta}_H + P_C\tilde{E}(\eta_H)\right]$
Profits	$\overline{\pi}_L^*$	$\frac{Q}{16}\frac{(\overline{\alpha}S-4c_L\underline{\alpha})^2}{\underline{\alpha}(\overline{\alpha}-\underline{\alpha})S}$	$Q\left[c_H\tilde{E}(\alpha(1-\eta_H)) + (p_r - c_H)\tilde{E}(\alpha\eta_H(1-\eta_H)) - c_L\tilde{E}(1-\eta_H)\right]$
Buyer's Cost	$\overline{\kappa}_B^*$	$\frac{Q}{8}\frac{\psi(S)}{S\underline{\alpha}(\overline{\alpha}-\underline{\alpha})}$	$QP_D\left[p_r\tilde{\eta}_H + c_L(1-\tilde{\eta}_H)\right] + QP_C\left[p_r\tilde{E}(\eta_H) + c_H\tilde{E}(\alpha(1-\eta_H)) + (p_r - c_H)\tilde{E}(\alpha\eta_H(1-\eta_H))\right]$
Total Supply Chain Cost	$\overline{\kappa}_{SC}^*$	$\frac{Q}{2}\frac{c_H\overline{\alpha}S-2c_H\underline{\alpha}S+4\underline{\alpha}c_Lc_H+\overline{\alpha}c_LS-4c_L^2\underline{\alpha}}{S(\overline{\alpha}-\underline{\alpha})}$	$QP_D\left[c_H\tilde{\eta}_H + (1-\tilde{\eta}_H)c_L\right] + QP_C\left[(c_H - c_L)\tilde{E}(\eta_H) + c_L\right]$

where $S = c_H\overline{\alpha} + \sqrt{c_H^2\overline{\alpha}^2 + 8\underline{\alpha}c_Hc_L}$; $\psi(S) = 2c_H\overline{\alpha}^2S + 8c_L\underline{\alpha}c_H\overline{\alpha} - 4\underline{\alpha}c_L\overline{\alpha}S - 16\underline{\alpha}^2c_Hc_L + 4Sc_L\underline{\alpha} + c_L\overline{\alpha}^3S + 4c_H\overline{\alpha}^2c_L\underline{\alpha} - 8c_L^2\underline{\alpha}^2$; $\tilde{\eta}_H = \eta_H(\tilde{\alpha}) = \eta_H(\frac{(p_r-c_L)c_L}{c_H\overline{\alpha}p_r-\overline{\alpha}c_H^2+c_Hp_r-p_rc_L})$; $\tilde{\mu}_{\eta_H} = \int_{\tilde{\alpha}}^{\overline{\alpha}}\eta_H(\alpha)\frac{1}{\overline{\alpha}-\tilde{\alpha}}d\alpha$; $\tilde{E}(x) = \int_{\tilde{\alpha}}^{\overline{\alpha}}xf(x)dx$; $\tilde{\mu}_\alpha = \frac{\tilde{\alpha}+\overline{\alpha}}{2}$; $P_D = \frac{\tilde{\alpha}-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}$ and $P_C = 1 - P_D$.

1. $\bar{\pi}_H^{PE} < \bar{\pi}_H^{SE}$ if $\Delta_{\pi_H}^{SE1} = (p_r - c_H)Q\left[P_D\tilde{\eta}_H + P_C\tilde{E}(\eta_H)\right] - \frac{Q}{8}\frac{(S-4\underline{\alpha}c_H)(\overline{\alpha}S+4c_L\underline{\alpha}-2\underline{\alpha}S)}{\underline{\alpha}S(\overline{\alpha}-\underline{\alpha})} > 0$

2. $\bar{\pi}_L^{PE} < \bar{\pi}_L^{SE}$ if $\Delta_{\pi_L}^{SE1} =$

$$= Q\left[c_H\tilde{E}(\alpha(1-\eta_H)) + (p_r - c_H)\tilde{E}(\alpha\eta_H(1-\eta_H)) - c_L\tilde{E}(1-\eta_H)\right] -$$

$\frac{Q}{16}\frac{(\overline{\alpha}S-4c_L\underline{\alpha})^2}{\underline{\alpha}(\overline{\alpha}-\underline{\alpha})S} > 0$

3. $\overline{\kappa}_B^{PE} < \overline{\kappa}_B^{SE}$ if $\Delta_{\kappa_B}^{SE1} = QP_D\left[p_r\tilde{\eta}_H + c_L(1-\tilde{\eta}_H)\right] + QP_C\left[p_r\tilde{E}(\eta_H) + c_H\tilde{E}(\alpha(1-\eta_H)) + (p_r - c_H)\tilde{E}(\alpha\eta_H\right.$

$$-\frac{Q}{8}.\frac{2c_H\overline{\alpha}^2S+8c_L\underline{\alpha}c_H\overline{\alpha}-4\underline{\alpha}c_L\overline{\alpha}S-16\underline{\alpha}^2c_Hc_L+4Sc_L\underline{\alpha}+c_L\overline{\alpha}^3S+4c_H\overline{\alpha}^2c_L\underline{\alpha}-8c_L^2\underline{\alpha}^2}{S\underline{\alpha}(\overline{\alpha}-\underline{\alpha})} > 0$$

4. $\overline{\kappa}_{SC}^{PE} < \overline{\kappa}_{SC}^{SE}$ if $\Delta_{\kappa_{SC}}^{SE1} = QP_D\left[c_H\tilde{\eta}_H + (1-\tilde{\eta}_H)c_L\right] + QP_C\left[(c_H - c_L)\tilde{E}(\eta_H) + c_L\right]$

$$-\frac{1}{2}.\frac{c_H\overline{\alpha}S-2c_H\underline{\alpha}S+4\underline{\alpha}c_Lc_H+\overline{\alpha}c_LS-4c_L^2\underline{\alpha}}{S(\overline{\alpha}-\underline{\alpha})} > 0$$

- Let $(\underline{\alpha}, \overline{\alpha}) \in C_{23}$: The expected equilibrium profits/costs in this sub-region are presented in Table 5–21. Given this table, the following statements hold true:

Table 5–21: Expected values of equilibrium profits/costs under pooling and separating equilibria for the points in region C_{23}.

Equilibrium		Pooling	Separating
Suppliers'	$\overline{\pi}_H^*$	$\frac{Q}{2}\frac{(p_r-c_H)(\overline{\alpha}p_r+c_L-2\underline{\alpha}p_r)}{p_r(\overline{\alpha}-\underline{\alpha})}$	$(p_r - c_H)Q\left[P_D\tilde{\eta}_H + P_C\tilde{E}(\eta_H)\right]$
Profits	$\overline{\pi}_L^*$	$\frac{Q}{4}\frac{(\overline{\alpha}p_r-c_L)^2}{p_r(\overline{\alpha}-\underline{\alpha})}$	$Q\left[c_H\tilde{E}(\alpha(1-\eta_H)) + (p_r - c_H)\tilde{E}(\alpha\eta_H(1-\eta_H)) - c_L\tilde{E}(1-\eta_H)\right]$
Buyer's Cost	$\overline{\kappa}_B^*$	$\frac{Q}{4}\frac{2\overline{\alpha}p_r^2+2p_rc_L-4\underline{\alpha}p_r^2+\overline{\alpha}^2p_r^2-c_L^2}{p_r(\overline{\alpha}-\underline{\alpha})}$	$QP_D\left[p_r\tilde{\eta}_H + c_L(1-\tilde{\eta}_H)\right] + QP_C\left[p_r\tilde{E}(\eta_H) + c_H\tilde{E}(\alpha(1-\eta_H)) + (p_r - c_H)\tilde{E}(\alpha\eta_H(1-\eta_H))\right]$
Total SC Cost	$\overline{\kappa}_{SC}^*$	$\frac{Q}{2}\frac{c_H\overline{\alpha}p_r+c_Hc_L-2c_H\underline{\alpha}p_r+\overline{\alpha}p_rc_L-c_L^2}{p_r(\overline{\alpha}-\underline{\alpha})}$	$QP_D\left[c_H\tilde{\eta}_H + (1-\tilde{\eta}_H)c_L\right] + QP_C\left[(c_H - c_L)\tilde{E}(\eta_H) + c_L\right]$

Let $\tilde{\eta}_H = \eta_H(\tilde{\alpha}) = \eta_H(\frac{(p_r-c_L)c_L}{c_H\overline{\alpha}p_r-\overline{\alpha}c_H^2+c_Hp_r-p_rc_L})$; $\tilde{\mu}_{\eta_H} = \int_{\tilde{\alpha}}^{\overline{\alpha}}\eta_H(\alpha)\frac{1}{\overline{\alpha}-\tilde{\alpha}}d\alpha$; $\tilde{E}(x) = \int_{\tilde{\alpha}}^{\overline{\alpha}}xf(x)dx$; $\tilde{\mu}_\alpha = \frac{\tilde{\alpha}+\overline{\alpha}}{2}$; $P_D = \frac{\tilde{\alpha}-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}$ and $P_C = 1 - P_D$.

1. $\bar{\pi}_H^{PE} < \bar{\pi}_H^{SE}$ if $\Delta_{\pi_H}^{SE1} = (p_r - c_H)Q\left[P_D\tilde{\eta}_H + P_C\tilde{E}(\eta_H)\right] - \frac{Q}{2}.\frac{(p_r-c_H)(\overline{\alpha}p_r+c_L-2\underline{\alpha}p_r)}{p_r(\overline{\alpha}-\underline{\alpha})} > 0$
2. $\bar{\pi}_L^{PE} < \bar{\pi}_L^{SE}$ if $\Delta_{\pi_L}^{SE1} = Q\left[c_H\tilde{E}(\alpha(1-\eta_H)) + (p_r - c_H)\tilde{E}(\alpha\eta_H(1-\eta_H)) - c_L\tilde{E}(1-\eta_H)\right] -$
   $\frac{Q}{4}.\frac{(\overline{\alpha}p_r-c_L)^2}{p_r(\overline{\alpha}-\underline{\alpha})} > 0$
3. $\overline{\kappa}_B^{PE} < \overline{\kappa}_B^{SE}$ if $\Delta_{\kappa_B}^{SE1} = QP_D\left[p_r\tilde{\eta}_H + c_L(1-\tilde{\eta}_H)\right] + QP_C\left[p_r\tilde{E}(\eta_H) + c_H\tilde{E}(\alpha(1-\eta_H)) + (p_r - c_H)\tilde{E}(\alpha\eta_H\right.$
   $$-\frac{Q}{4}.\frac{2\overline{\alpha}p_r^2+2p_rc_L-4\underline{\alpha}p_r^2+\overline{\alpha}^2p_r^2-c_L^2}{p_r(\overline{\alpha}-\underline{\alpha})} > 0$$
4. $\overline{\kappa}_{SC}^{PE} < \overline{\kappa}_{SC}^{SE}$ if $\Delta_{\kappa_{SC}}^{SE1} = QP_D\left[c_H\tilde{\eta}_H + (1-\tilde{\eta}_H)c_L\right] + QP_C\left[(c_H - c_L)\tilde{E}(\eta_H) + c_L\right]$
   $$-\frac{Q}{2}.\frac{c_H\overline{\alpha}p_r+c_Hc_L-2c_H\underline{\alpha}p_r+\overline{\alpha}p_rc_L-c_L^2}{p_r(\overline{\alpha}-\underline{\alpha})} > 0$$

- Let $(\underline{\alpha}, \overline{\alpha}) \in C_{24}$: Finally, Table 5–22 summarizes the equilibrium profits/costs under pooling and separating equilibria for this sub-region. And, based on the values of profits/costs, we can characterize the thresholds for which separating and pooling equilibria lead to equal payoff for supply chain parties.

1. $\bar{\pi}_H^{PE} < \bar{\pi}_H^{SE}$ if $\Delta_{\pi_H}^{SE1} = Q\mu_{\eta_H}(p_r - c_H) - \frac{Q}{2}.\frac{(p_r-c_H)(\overline{\alpha}p_r+c_L-2\underline{\alpha}p_r)}{p_r(\overline{\alpha}-\underline{\alpha})} > 0$

Table 5–22: Expected values of equilibrium profits/costs under pooling and separating equilibria for the points in region C_{24}.

Equilibrium		Pooling	Separating
Suppliers'	$\overline{\pi}^*_H$	$\frac{Q}{2}\frac{(p_r-c_H)(\overline{\alpha}p_r+c_L-2\underline{\alpha}p_r)}{p_r(\overline{\alpha}-\underline{\alpha})}$	$Q\mu_{\eta_H}(p_r-c_H)$
Profits	$\overline{\pi}^*_L$	$\frac{Q}{4}\frac{(\overline{\alpha}p_r-c_L)^2}{p_r(\overline{\alpha}-\underline{\alpha})}$	$Q[c_H\mu_\alpha+(p_r-2c_H)(\sigma_{\alpha,\eta_H}+\mu_\alpha\mu_{\eta_H})+(p_r-c_H)E(\eta_H^2\alpha)+c_L\mu_{\eta_H}-c_L]$
Buyer's Cost	$\overline{\kappa}^*_B$	$\frac{Q}{4}\frac{2\overline{\alpha}p_r^2+2p_rc_L-4\underline{\alpha}p_r^2+\overline{\alpha}^2p_r^2-c_L^2}{p_r(\overline{\alpha}-\underline{\alpha})}$	$Qc_H\overline{\alpha}$
Total SC Cost	$\overline{\kappa}^*_{SC}$	$\frac{Q}{2}\frac{c_H\overline{\alpha}p_r+c_Hc_L-2c_H\underline{\alpha}p_r+\overline{\alpha}p_rc_L-c_L^2}{p_r(\overline{\alpha}-\underline{\alpha})}$	$Q(\mu_{\eta_H}c_H+(1-\mu_{\eta_H})c_L)$

Let σ_{α,η_H} denote the covariance of α and η_H; $\mu_{\eta_H}=\int_{\underline{\alpha}}^{\overline{\alpha}}\eta_H(\alpha)\frac{1}{\overline{\alpha}-\underline{\alpha}}d\alpha$; and $E(x)=\int_{\underline{\alpha}}^{\overline{\alpha}}x\frac{1}{\overline{\alpha}-\underline{\alpha}}d\alpha$.

2. $\bar{\pi}_L^{PE} < \bar{\pi}_L^{SE}$ if $\Delta^{SE1}_{\pi_L} = Q[c_H\mu_\alpha + (p_r-2c_H)(\sigma_{\alpha,\eta_H}+\mu_\alpha\mu_{\eta_H}) + (p_r-c_H)E(\eta_H^2\alpha) + c_L\mu_{\eta_H} - c_L]$
$$-\frac{Q}{4}.\frac{(\overline{\alpha}p_r-c_L)^2}{p_r(\overline{\alpha}-\underline{\alpha})} > 0$$
3. $\overline{\kappa}_B^{PE} < \overline{\kappa}_B^{SE}$ if $\Delta^{SE1}_{\kappa_B} = Qc_H\overline{\alpha} - \frac{Q}{4}.\frac{2\overline{\alpha}p_r^2+2p_rc_L-4\underline{\alpha}p_r^2+\overline{\alpha}^2p_r^2-c_L^2}{p_r(\overline{\alpha}-\underline{\alpha})} > 0$
4. $\overline{\kappa}_{SC}^{PE} < \overline{\kappa}_{SC}^{SE}$ if $\Delta^{SE1}_{\kappa_{SC}} = Q(\mu_{\eta_H}c_H + (1-\mu_{\eta_H})c_L) - \frac{Q}{2}.\frac{c_H\overline{\alpha}p_r+c_Hc_L-2c_H\underline{\alpha}p_r+\overline{\alpha}p_rc_L-c_L^2}{p_r(\overline{\alpha}-\underline{\alpha})} > 0$

□

Proof. Pooling equilibrium in the presence of multiple suppliers

It is easy to observe that the best response of the suppliers in a pooling equilibrium where they cannot infer any new information regarding the quality scores, would be as follows:

$$p_L^{i*} = \min(p_r, \max[c_L, f_L^i(p_H^*, p_L^{1*}, \ldots, p_L^{i-1*}, p_L^{i+1*}, \ldots, p_L^{m*})]) \qquad \forall i = 1, \ldots, m$$

$$p_H^* = \begin{cases} c_H & n > 1 \\ \min(p_r, \max[c_H, f_H(p_L^{1*}, \ldots, p_L^{m*})]) & n = 1 \end{cases}$$

To establish the internal solution of the bid prices in pooling equilibrium, first let assume that $n \geq 2$. In this case, all the incumbent known suppliers will offer $p_H^* = c_H$ because in a non-cooperative game they just undercut each other to a level where they cannot decrease their price anymore. Then, the profit function for an entrant supplier with the rank $i \in \{1, \ldots, m\}$ is as follows:

$$\pi_L^i = (p_i - c_L)\int_{p_i/c_H}^{\overline{\alpha}} \frac{1}{\overline{\alpha}-\underline{\alpha}}d\alpha_i \prod_{j\in\{1,\ldots,m\},j>i}\int_{\underline{\alpha}}^{\overline{\alpha}}\int_{\frac{\alpha_j p_i}{p_j}}^{\overline{\alpha}} \frac{2}{(\overline{\alpha}-\underline{\alpha})^2}d\alpha_i d\alpha_j \prod_{j\in\{1,\ldots,m\},j<i}\int_{\underline{\alpha}}^{\overline{\alpha}}\int_{\underline{\alpha}}^{\frac{\alpha_i p_j}{p_i}} \frac{2}{(\overline{\alpha}-\underline{\alpha})^2}d\alpha_j d\alpha_i \quad \forall i \in 1,$$

As above, there are three integration terms in the suppliers' profit function. They, respectively, represent the probability of having a better QS-adjusted price than known suppliers, suppliers with lower QS's, and suppliers with higher QS ranks. Suppliers' difference in their QS ranking brings a source of asymmetry in their pricing decisions. By taking the first derivative of the profit functions of suppliers with respect to their prices, we come up to a non-linear system of equation with m variables $\{p_L^1, \ldots, p_L^m\}$:

$$\frac{\partial \pi_L^i}{\partial p_i} = 0 \quad \forall i \in \{1, \ldots, m\}.$$

There are some established mathematical and evolutionary algorithms to solve such non-linear system of equations. For instance, we used *fsolve* function in MATLAB that can use three different algorithms of 'trust-region-dogleg' (default), 'trust-region-reflective', and 'levenberg-marquardt'. Solving this system of equation will provide the internal solution for the pooling equilibrium, which should be adjusted with the boundary conditions.

Now, assume $n = 1$, in this case the profit function of the known supplier and entrant suppliers is as follows.

$$\pi_H = (p_H - c_H) \prod_{i \in \{1,\ldots,m\}} \int_{\underline{\alpha}}^{p_i/p_H} \frac{1}{\overline{\alpha} - \underline{\alpha}} d\alpha_i$$

$$\pi_L^i = (p_i - c_L) \int_{p_i/p_H}^{\overline{\alpha}} \frac{1}{\overline{\alpha} - \underline{\alpha}} d\alpha_i \prod_{j \in \{1,\ldots,m\}, j>i} \int_{\underline{\alpha}}^{\overline{\alpha}} \int_{\frac{\alpha_j p_i}{p_j}}^{\overline{\alpha}} \frac{2}{(\overline{\alpha} - \underline{\alpha})^2} d\alpha_i d\alpha_j \prod_{j \in \{1,\ldots,m\}, j<i} \int_{\underline{\alpha}}^{\overline{\alpha}} \int_{\underline{\alpha}}^{\frac{\alpha_i p_j}{p_i}} \frac{2}{(\overline{\alpha} - \underline{\alpha})^2} d\alpha_j d\alpha_i \quad \forall i \in 1,$$

The only difference between π_L^i when $n > 1$ with it when $n = 1$ is that in the latter untested suppliers should consider an internal price $p_H^* \geq c_H$ for the incumbent supplier. Again, we should take the first derivative of profit function of suppliers w.r.t their prices and then solve the non-linear system of equations with $m + 1$ variables $\{p_H, p_L^1, \ldots, p_L^m\}$.

□

Proof. Proposition 14:

We assume all H- and L-type suppliers share the same marginal costs, i.e., c_H, and c_L, respectively. We also assume that the buyer assigns identical QS to all suppliers N. Let α_i be the true value of relative QS of i^{th} supplier L with respect to supplier(s) H, where $i = 1, \ldots, m$. α_i's are unknown to all the suppliers, and it is a common knowledge that α_i's are uniformly distributed between some publicly known $\underline{\alpha}$ and $\overline{\alpha}$, where $0 \leq \underline{\alpha} \leq \overline{\alpha} \leq 1$. That means, the suppliers face potentially m-dimensional asymmetric information problem. Before analyzing separating equilibria, we would characterize suppliers' decisions under the symmetric information.

Symmetric Information Setting: In this setting, the buyer and all the suppliers know the exact QS of each participating supplier. Under the symmetric QS information, suppliers will engage in a Bertrand competition setting in which they reduce their prices to a level that guarantees their winning provided that this price is not lower than their marginal cost. When there are multiple known suppliers $n \geq 2$, because only one supplier can win the contract, all the known suppliers offer $p_H^* = c_H$ in a non-cooperative competition setting. However, if $n = 1$, the incumbent supplier may find situations to increase her price to $p_H^* = max(c_H, \frac{c_L}{\alpha_1})$ where α_1 is the ratio between the scores of the first-ranked supplier L and supplier H. Note that $\frac{c_L}{\alpha_1}$ is the price that makes known suppliers' generalized price equal to that of first-ranked supplier L if she bids at her minimum price (marginal cost c_L). The first-ranked unknown supplier competes with supplier(s) H on one hand, and with other unknown suppliers (more importantly with the second-rank supplier with a relative score of α_2) on the other hand. So, she would definitely win the auction if she offers an epsilon lower than $min\{\frac{c_L \alpha_1}{\alpha_2}, c_H \alpha\}$ given this is not lower than c_L. If supplier(s) H and the first-ranked supplier L follow the Bertrand competition and bid as stated above, the other unknown suppliers cannot beat them in the competition even if they bid at their marginal cost c_L.

Therefore, one can characterize the equilibrium bidding prices under symmetric information only by knowing the exact values of the two random variables α_1 and α_2. Indeed, a closer look reveals that those prices can be characterized even by one useful piece of knowledge, that is, the ratio of the quality scores assigned to the two suppliers with the highest chance of winning based on the combination of both quality scores and unit costs; i.e. $\frac{\alpha_1}{\alpha_2}$ given that $\alpha_2 \geq \frac{c_L}{c_H}$, or α_1 if $\alpha_2 < \frac{c_L}{c_H}$. Obviously, if suppliers know that $\alpha_1 < \frac{c_L}{c_H}$, there is no separating equilibrium as previously verified in Proposition 9. Now, we investigate the accuracy of this reasoning and seek to find possible, efficient ways of signalling QS information from the buyer to the suppliers when $m \geq 2$.

Asymmetric Information Setting: Under the presence of multiple known suppliers, when the suppliers know that true values of relative quality scores are such that the final winner of the auction is/are known supplier(s) (i.e. $0 < \alpha_m < \cdots < \alpha_1 < \frac{c_L}{c_H}$), the buyer has no need to share this information as the competition under asymmetric information leads to the lowest possible price c_H. Also, we showed in the main model with two suppliers that offering guarantees with an H-winning policy is very costly to be sustainable, which makes it incredible for the suppliers. Here instead, we focus again on the cases where the buyer seeks a L-winning strategy by offering guarantees to the known and unknown suppliers.

As we showed in the symmetric information setting, among all m different values of relative QS for suppliers L, only α_1 and α_2 matter for determining the equilibrium prices and the order allocation. This is also true about the separating equilibria as the suppliers are supposed to gain the required knowledge for making their decisions. Therefore, we assume suppliers a priori believe that α_1 and α_2 are uniformly distributed on the lower triangle $\underline{\alpha} < \alpha_2 < \alpha_1 < \overline{\alpha}$. In order to be consistent with the analysis of the main model, we assume the buyer would use revenue guarantees to known and unknown suppliers for the signalling purpose. Let η_H and $\frac{m}{m-1}\eta_L$ denote the total revenues to known and unknown suppliers[1] , respectively. For the sake of analytical tractability and without loss of generality, we assume the buyer equally divides the total guarantees among the suppliers; i.e. if he guarantees a total revenue of

[1] Needless to mention, the total guarantees to suppliers H and L are functions of n and m, respectively. When the buyer employs an L-winning strategy, he would ultimately orders: 1- an $\frac{m-1}{m}$ portion of the total guarantees to the suppliers L at the price of p_r; 2- the total guarantees to the known suppliers at the spot market price p_r; and 3- the remaining needed quantity to the first-ranked supplier L at her winning price. Therefore, this function for the total guarantees to suppliers L does not affect the generality of the results and is only to simplify the form of buyer's total cost at equation (20).

$Q\eta_H p_r$ (resp. $\frac{m}{m-1}Q\eta_L p_r$) to known (resp. unknown) suppliers, each supplier H (resp. L) is guaranteed a revenue of $\frac{Q\eta_H p_r}{n}$ (resp. $\frac{Q\eta_L p_r}{m-1}$). The buyer's cost then would be:

$$\kappa_B = p_r Q[\eta_H(\alpha_1,\alpha_2)+\eta_L(\alpha_1,\alpha_2)] + [1-\eta_H(\alpha_1,\alpha_2)-\eta_L(\alpha_1,\alpha_2)]Qp_L^{1*} \tag{20}$$

where p_L^{1*} is the equilibrium price of the first-ranked untested supplier, which would be:

$$p_L^{1*} = \min\left[\frac{\alpha_1}{\alpha_2}p_L^{low}; \alpha_1 p_H^{low}\right] \tag{21}$$

This is very similar to her equilibrium price under symmetric information except that instead of marginal costs of c_L and c_H, she considers the fact that the other suppliers logically increase their minimum prices in the auction because of the revenue guarantees. We can find the new minimum prices (p_i^{low} for $i = L, H$ in equation 21) in the same way as we calculated in Lemma 4, which leads to the followings:

$$p_L^{low} = c_L + \frac{\eta_L(\alpha_1,\alpha_2)(p_r - c_L)}{(m-1)(1-\eta_H(\alpha_1,\alpha_2)-\eta_L(\alpha_1,\alpha_2))}$$

$$p_H^{low} = c_H + \frac{\eta_H(\alpha_1,\alpha_2)(p_r - c_H)}{n\left[1-\frac{m.\eta_L}{m-1}-\frac{(n-1)\eta_H}{n}\right]}$$

Note that in (21), if:

1. $\alpha_2 \geq \frac{p_L^{low}}{p_H^{low}}$, then $p_L^{1*} = \frac{\alpha_1}{\alpha_2}p_L^{low}$ and $\kappa_B = p_r Q(\eta_H+\eta_L) + [1-\eta_H-\eta_L]Q\frac{\alpha_1}{\alpha_2}p_L^{low}$,
2. $\alpha_2 < \frac{p_L^{low}}{p_H^{low}}$, then $p_L^{1*} = \alpha_1 p_H^{low}$ and $\kappa_B = p_r Q(\eta_H+\eta_L) + [1-\eta_H-\eta_L]Q\alpha_1 p_H^{low}$.

The second point implies that if the two unknown suppliers become too separated such that the second-ranked supplier L is out of the competition, the buyer's cost becomes only a function of the ratio of the first-ranked supplier L's and supplier H's quality scores (i.e. α_1), and there is no need for signalling α_2. In fact, the actual competition would be between supplier H and the first-ranked supplier L in this case. Therefore, the result of Proposition 9 would apply to the separating equilibria with multiple unknown suppliers if $\frac{\alpha_2}{\alpha_1}$ is relatively low. To continue, we first assume $\underline{\alpha} > \frac{p_L^{low}}{p_H^{low}}$ to ensure that $\frac{\alpha_1 p_L^{low}}{\alpha_2} < p_H^{low}\alpha_1$. This makes the buyer's cost function as below:

$$\kappa_B = p_r Q[\eta_H(\alpha_1,\alpha_2)+\eta_L(\alpha_1,\alpha_2)]+[1-\eta_H(\alpha_1,\alpha_2)-\eta_L(\alpha_1,\alpha_2)]Q\frac{\alpha_1}{\alpha_2}\left(c_L + \frac{\eta_L(\alpha_1,\alpha_2)(p_r - c_L)}{(m-1)(1-\eta_H(\alpha_1,\alpha_2)-\eta_L(\alpha_1,\alpha_2))}\right) \tag{22}$$

Now, in order to find $\eta_H^*(\alpha_1, \alpha_2)$ and $\eta_L^*(\alpha_1, \alpha_2)$, we should solve the following system of Partial Differential Equations (PDE):

$$\left.\frac{\partial \kappa_B(\eta_L(\alpha_1', \alpha_2'), \eta_H(\alpha_1', \alpha_2'), \alpha_1, \alpha_2)}{\partial \alpha_1'}\right|_{(\alpha_1', \alpha_2')=(\alpha_1, \alpha_2)} = 0 \qquad \forall(\alpha_1, \alpha_2)$$

$$\left.\frac{\partial \kappa_B(\eta_L(\alpha_1', \alpha_2'), \eta_H(\alpha_1', \alpha_2'), \alpha_1, \alpha_2)}{\partial \alpha_2'}\right|_{(\alpha_1', \alpha_2')=(\alpha_1, \alpha_2)} = 0 \qquad \forall(\alpha_1, \alpha_2) \tag{23}$$

In addition to the complicated nature of the above system of PDEs, another difficulty is defining the boundary conditions for η_H and η_L. Instead, based on the results of Proposition 9, we conjecture that the buyer's total cost under the separating equilibrium would be constant for all the instances of (α_1, α_2), that is:

$$\kappa_B^{*SE}(\alpha_1, \alpha_2) = K, \qquad \forall \alpha_1, \alpha_2 : \underline{\alpha} \leq \alpha_2 \leq \alpha_1 \leq \overline{\alpha} \tag{24}$$

where K is a fixed value that does not depend on α_1 and α_2. By making (22) equal to the fixed value of K ($\kappa_B = K$), we find the relationship between $\eta_L^*(\alpha_1, \alpha_2)$ and $\eta_H^*(\alpha_1, \alpha_2)$, as follows:

$$\eta_L^*(\alpha_1, \alpha_2) = \frac{p_r\alpha_2\eta_H^*(\alpha_1, \alpha_2) - p_r\alpha_2 m\eta_H^*(\alpha_1, \alpha_2) + \alpha_1 c_L - \alpha_1 c_L\eta_H^*(\alpha_1, \alpha_2) - \alpha_1 c_L m + \alpha_1 c_L m\eta_H^*(\alpha_1, \alpha_2) - K\alpha_2 + K\alpha_2 m}{p_r\alpha_2 m - \alpha_1 c_L m + \alpha_1 p_r - p_r\alpha_2} \tag{25}$$

Based on (25), we conjecture that $\eta_L^*(\alpha_1, \alpha_2)$ and $\eta_H^*(\alpha_1, \alpha_2)$ should be of the following forms:

$\eta_L^*(\alpha_1, \alpha_2) = \frac{A_1\alpha_1 + A_2\alpha_2}{A_3\alpha_1 + A_4\alpha_2}$

$\eta_H^*(\alpha_1, \alpha_2) = \frac{B_1\alpha_1 + B_2\alpha_2}{B_3\alpha_1 + B_4\alpha_2}$

If we plug these functions in the system of PDEs (23), we find an infinite number of functions that technically work as signalling tools for the buyer. But among all these functions, there are two interesting forms of functions: one with $\eta_L^* = 0$ and the other with $\eta_H^* = 0$ for all α_1 and α_2. These two are among the simplest ones to be implemented from the buyer's perspective. This in fact shows that when both α_1 and α_2 are high enough, the buyer can signal the required information (the ratio of $\frac{\alpha_1}{\alpha_2}$) by a one-dimensional signal η_i, $\forall i \in \{L, H\}$.

Proposition 9 shows that when the actual competition is between supplier H and the first-ranked supplier L (when α_2 is very low), the buyer is able to signal α_1 (the relative QS of supplier L compared to that for supplier H) only with revenue guarantees to known supplier $\eta_H \geq 0$. Therefore, in this section, we focus on the case where $\eta_H = 0$ to make sure that the buyer uses different signals for different types, as this one-to-one projection between types and signals is required by the Bayesian equilibrium concept.

Now, if we make $\eta_H = 0$ for all α_1 and α_2 in (25), we find the form of $\eta_L^*(\alpha_1, \alpha_2)$ as below:

$$\eta_L^*(\alpha_1, \alpha_2) = \frac{K\alpha_2 m - K\alpha_2 - \alpha_1 c_L m + \alpha_1 c_L}{p_r\alpha_2 m - p_r\alpha_2 - \alpha_1 c_L m + \alpha_1 p_r} = \frac{K(m-1) - c_L(m-1)\frac{\alpha_1}{\alpha_2}}{p_r(m-1) - (c_L m - p_r)\frac{\alpha_1}{\alpha_2}} = \eta_L^*(\frac{\alpha_1}{\alpha_2}) \quad (26)$$

Equation (26) shows that what indeed matters when the actual competition is between the first-ranked and the second-ranked suppliers L (i.e. both α_1 and α_2 are high enough), is the relative QS between them, i.e. $\frac{\alpha_1}{\alpha_2}$. Now, in order to find K, we need to check the followings:

(a) $0 < \eta_L^*(\frac{\alpha_1}{\alpha_2}) < 1 \quad \forall\ \alpha_2 \leq \alpha_1$;

(b) $\eta_L^*(\frac{\alpha_1}{\alpha_2}) \neq \eta_L^*(\frac{\alpha_1'}{\alpha_2'})$ for any $\frac{\alpha_1}{\alpha_2} \neq \frac{\alpha_1'}{\alpha_2'}$.

It is easy to verify that with any K such that $\frac{c_L\overline{\alpha}}{\underline{\alpha}} \leq K \leq p_r$, the second condition is satisfied as the signal becomes strictly *decreasing* at $\frac{\alpha_1}{\alpha_2}$. However, the least costly signal corresponds to $K = \frac{c_L\overline{\alpha}}{\underline{\alpha}}$ which makes $\eta_L^*(\overline{\alpha}, \underline{\alpha}) = 0$, therefore,

$$\eta_L^*(\alpha_1, \alpha_2) = \frac{c_L(m-1)\left(\frac{\overline{\alpha}}{\underline{\alpha}} - \frac{\alpha_1}{\alpha_2}\right)}{p_r(m-1) - (c_L m - p_r)\frac{\alpha_1}{\alpha_2}} = \frac{c_L(m-1)\left(\frac{\overline{\alpha}}{\underline{\alpha}} - \frac{\alpha_1}{\alpha_2}\right)}{p_r(\frac{\alpha_1}{\alpha_2} - 1) + m(p_r - c_L\frac{\alpha_1}{\alpha_2})} \quad (27)$$

Now, we find the conditions for α_2 under which separating equilibria are always possible. According to (21), in order to make one-to-one signalling possible by sharing only the ratio of $\frac{\alpha_1}{\alpha_2}$, the only important condition is $\alpha_2 > \frac{p_L^{low}}{p_H^{low}}$. By utilizing $\eta_L^*(\frac{\alpha_1}{\alpha_2})$ and $\eta_H^* = 0$, we would have $p_H^{low} = c_H$ and $p_L^{low} \geq c_L$; therefore, $\frac{p_L^{low}}{p_H^{low}} \leq \frac{c_L}{c_H}$ [the equality takes place at $(\alpha_1, \alpha_2) = (\overline{\alpha}, \underline{\alpha})$]. As a result, *if* $\alpha_2 \geq \frac{c_L}{c_H}$, *for all the values of* $1 \leq \frac{\alpha_1}{\alpha_2} \leq \frac{\overline{\alpha}}{\underline{\alpha}}$, *the buyer is able to offer a different informative revenue level to unknown suppliers (fully separating).* In contrast, if $\alpha_2 < \frac{c_L}{c_H}$, the buyer needs to signal only the relative QS of the first-ranked supplier L compared to supplier H's QS (α_1) as illustrated in Proposition 9. In that case, the equilibrium bid price of the first-ranked supplier L would be $p_L^{1*} = \alpha_1 p_H^{low}$ instead of $\frac{\alpha_1}{\alpha_2} p_L^{low}$.

Since the buyer has no incentive to manipulate the signal (27), the suppliers will update their belief in the following fashion. If they observe any η_L such that $0 \leq \eta_L \leq \eta_L(1)$, they update their belief using $\eta_L^{-1}(\eta_L(\frac{\alpha_1}{\alpha_2}))$; however, if they observe any off-equilibrium value $\eta_L > \eta_L(1)$, they logically believe that $\frac{\alpha_1}{\alpha_2} = 1$. In the former case, the equilibrium prices would be: $p_H^* = c_H$, $p_L^{1*} = \frac{\alpha_1}{\alpha_2}\left(c_L + (p_r - c_L)\frac{\eta_L}{(m-1)(1-\eta_L)}\right)$, and $p_L^{i>1*} = c_L + (p_r - c_L)\frac{\eta_L}{(m-1)(1-\eta_L)}$, respectively by supplier H, the first-ranked entrant supplier, and other unknown suppliers. In the latter case, $p_H^* = c_H$ and all entrant suppliers (including the first-ranked) offer $c_L + (p_r - c_L)\frac{\eta_L(1)}{(m-1)(1-\eta_L(1))}$.

- Sensitivity Analysis of the Number of Entrant Suppliers: To verify that the signal (27) is increasing in the number of entrants (m), we can take the first-order derivative with respect to m as below:

$$\frac{\partial \eta_L^*(\frac{\alpha_1}{\alpha_2})}{\partial m} = \frac{c_L \alpha_1(\overline{\alpha}\alpha_2 - \alpha_1\underline{\alpha})(p_r - c_L)}{\underline{\alpha}(-p_r\alpha_1 + pr\alpha_2 - mp_r\alpha_2 + mc_L\alpha_1)^2} \geq 0 \tag{28}$$

  And, (28) is non-negative because $\frac{\alpha_1}{\alpha_2} \leq \frac{\overline{\alpha}}{\underline{\alpha}}$ (i.e. $\overline{\alpha}\alpha_2 - \alpha_1\underline{\alpha} \geq 0$). Given the continuity of $\eta_L^*(\frac{\alpha_1}{\alpha_2})$, this proves that $\eta_L^*(\frac{\alpha_1}{\alpha_2})$ is increasing in m.

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Proof. Private Cost Information: Symmetric QS Information Analysis (Benchmark)

In this setting, all the components of the model other than the realized values of marginal costs are assumed to be commonly known to all bidders. Especially, they do know the exact value of $0 < \alpha < 1$. The buyer uses a first-price sealed-bid auction, where the supplier with the lowest QS-adjusted price gets the object and receives the amount she bids. A summery of the assumptions in the setting is as follows:

$$\begin{cases} QS_L = \alpha; \quad c_L \in \mathcal{U}(0,1); \quad c_L \text{ known only to supp. L;} \\ QS_H = 1; \quad c_H \in \mathcal{U}(0,1); \quad c_H \text{ known only to supp. H.} \end{cases} \tag{29}$$

Suppliers are risk neutral; they seek to maximize their expected profits. Given the bids of p_L and p_H, the payoffs are:

$$\pi_i = \begin{cases} p_i - c_i & \text{if} \frac{p_i}{QS_i} \leq \frac{p_{-i}}{QS_{-i}}, \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

Clearly, no supplier would bid an amount lower than her cost, since this would lead to a loss. Fixing the bidding behavior of others, at any bid that will neither win for sure nor lose for sure, a supplier faces a simple trade-off. A decrease in the bid will increase the probability of winning while, at the same time reducing the gains from winning.

Considering this trade-off, we can write the expected profit of suppliers as follows:

$$\pi_L = (p_L - c_L) Prob\left[c_H > p_H^{-1}(p_L/\alpha)\right] = (p_L - c_L)(1 - F_H(p_H^{-1}(p_L/\alpha)))$$
$$\pi_H = (p_H - c_H) Prob\left[c_L > p_L^{-1}(p_H\alpha)\right] = (p_H - c_H)(1 - F_L(p_L^{-1}(p_H\alpha)))$$

where F_H and F_L are the distributions of c_H and c_L respectively. Since we only consider uniform distribution for c_i in the interval of $(0,1)$, $F(c_i) = c_i \quad \forall i \in \{L, H\}$. In addition, conjecturing a linear form for bid functions under non-boundary conditions, i.e. $p_L(c_L) = A + Bc_L$ and $p_H(c_H) = D + Ec_H$, payoff functions can be rewritten as follows:

$$\pi_L = (p_L - c_L)\left(1 - \frac{p_L - D\alpha}{E\alpha}\right)$$
$$\pi_H = (p_H - c_H)\left(1 - \frac{p_H - A/\alpha}{B/\alpha}\right)$$

By taking the first order derivative of payoff functions, we can find the bid functions as follows:

$$\frac{\partial \pi_L}{p_L} = 0 \Rightarrow p_L = \frac{D\alpha + E\alpha + c_L}{2}$$
$$\frac{\partial \pi_H}{p_H} = 0 \Rightarrow p_H = \frac{\alpha c_H + A + B}{2\alpha} \tag{31}$$

By considering the original linear forms for bid functions (i.e. $p_L(c_L) = A + Bc_L$ and $p_H(c_H) = D + Ec_H$) and above relationship, we can simply characterize the bid functions (internal equilibrium) as follows:

$$p_L = \frac{c_L}{2} + \frac{\alpha}{3} + \frac{1}{6}$$

$$p_H = \frac{c_H}{2} + \frac{1}{3\alpha} + \frac{1}{6}$$

Now, by enforcing the boundary conditions (i.e. $c_i \leq p_i \leq 1$), the bidding prices of suppliers will be:

$$p_L = \begin{cases} \frac{\alpha}{6} + \frac{1}{3} & \text{if } 0 \leq c_L \leq \frac{1}{3} - \frac{\alpha}{3} \\ \frac{c_L}{2} + \frac{\alpha}{3} + \frac{1}{6} & \text{if } \frac{1}{3} - \frac{\alpha}{3} \leq c_L \leq \frac{4}{3}\alpha - \frac{1}{3} \\ \alpha & \text{if } \frac{4}{3}\alpha - \frac{1}{3} \leq c_L \leq \alpha \\ c_L & \text{if } \alpha \leq c_L \leq 1 \end{cases}$$

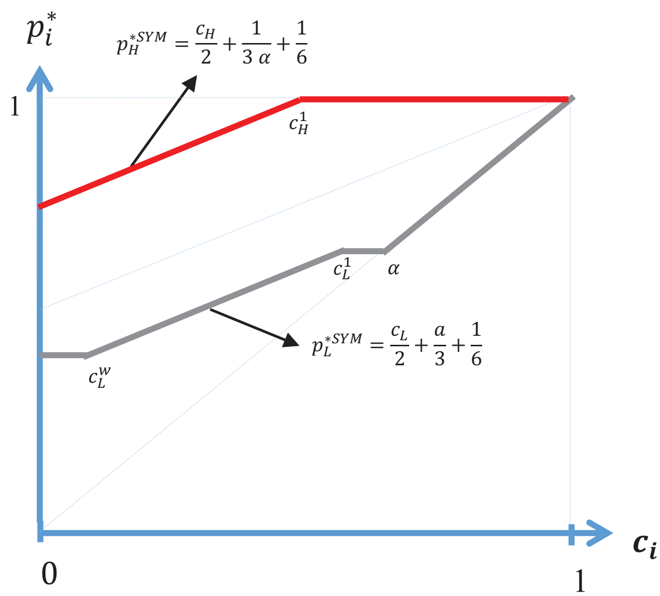
$$p_H = \begin{cases} \frac{c_H}{2} + \frac{1}{3\alpha} + \frac{1}{6} & \text{if } 0 \leq c_H \leq \frac{5}{3} - \frac{2}{3\alpha} \\ 1 & \text{if } \frac{5}{3} - \frac{2}{3\alpha} \leq c_H \leq 1 \end{cases}$$

Given the bid function for supplier H, she will bid $p_H = 1$ for any value of $0 \leq c_H \leq 1$ if α is very low (i.e. $\alpha < 0.4$). Therefore, we assume $\alpha \geq 0.4$ in order to focus on more reasonable and interesting results. In a special case where $\alpha = 1$, the game switches to a symmetric private-cost auction game for which $p_i = \frac{1}{2} + \frac{c_i}{2}$.

Now, in order to find $\kappa_B(\alpha)$, we need to take into consideration different ranges of c_L and c_H for which each supplier may win the auction:

$\kappa_B = (1 - c_H^1)(1 - \alpha) + \frac{c_L^w(\alpha+2)}{6} + (1 - c_L^1)\left(\int_0^{c_H^1} p_H dc_H\right) + (\alpha - c_L^1)(1 - c_H^1)\alpha + (1 - c_H^1)\int_{c_L^w}^{c_L^1} p_L dc_L + \int_{c_L^w}^{c_L^1}\int_0^{c_H cor} p_H dc_H dc_L + \int_0^{c_H^1}\int_{c_L^w}^{c_L cor} p_L dc_L dc_H = \frac{11\alpha^4+2\alpha^3+9\alpha^2+40\alpha-8}{81\alpha^2}$

where $c_H^1 = \frac{5}{3} - \frac{2}{3\alpha}$; $c_L^w = \frac{1}{3} - \frac{\alpha}{3}$; $c_L^1 = \frac{4}{3}\alpha - \frac{1}{3}$; $c_L cor = \frac{1}{3} - \frac{\alpha}{3} + c_H \alpha$, and $c_H cor = \frac{1}{3} - \frac{1}{3\alpha} + \frac{c_L}{\alpha}$.



□

Proof. Private Cost Information: Asymmetric QS Information Analysis (Pooling Equilibrium)

Except the realized values of marginal costs and the true value of α, all components of the model are assumed to be commonly known to all the parties. Each supplier privately knows their own cost and the buyer does not take any effort to share the true value of α. Similar to the symmetric model, if we consider a linear form for bid function of the suppliers (i.e. $p_L(c_L) = A + Bc_L$ and $p_H(c_H) = D + Ec_H$), the payoffs will be of the following forms:

$$\pi_L = (p_L - c_L)\left[\frac{1}{2}\left(1 - \frac{p_L - D\overline{\alpha}}{E\overline{\alpha}}\right) + \frac{1}{2}\left(1 - \frac{p_L - D\underline{\alpha}}{E\underline{\alpha}}\right)\right]$$
$$\pi_H = (p_H - c_H)\left[\frac{1}{2}\left(1 - \frac{p_H - A/\overline{\alpha}}{B/\overline{\alpha}}\right) + \frac{1}{2}\left(1 - \frac{p_H - A/\underline{\alpha}}{B/\underline{\alpha}}\right)\right]$$

By taking the first order derivative of payoff functions and considering the linear form of the bid functions, we can characterize non-boundary bid price functions as below:

$$p_L = \frac{c_L}{2} + \frac{\underline{\alpha}\overline{\alpha}(1+\overline{\alpha}+\underline{\alpha})}{2(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)}$$

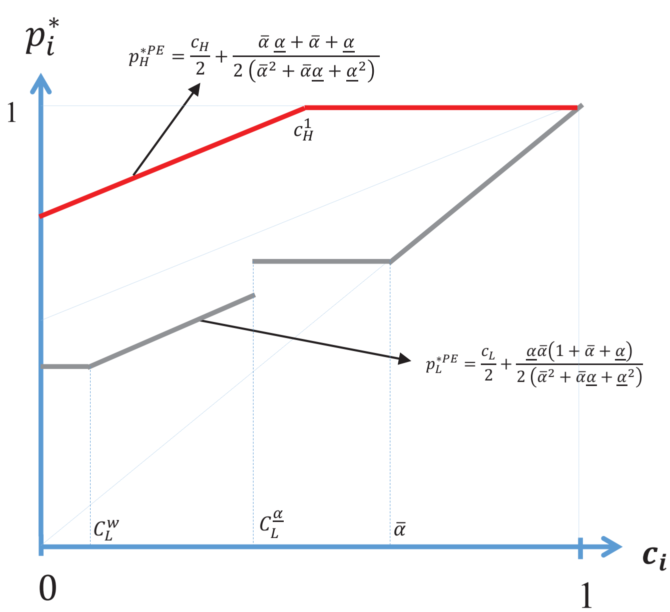
$$p_H = \frac{c_H}{2} + \frac{\overline{\alpha}\underline{\alpha}+\overline{\alpha}+\underline{\alpha}}{2(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)}$$

Now, if we impose the boundary conditions, the final suppliers' bid functions will be as follows:

$$p_H = \begin{cases} \frac{c_H}{2} + \frac{\overline{\alpha}\underline{\alpha}+\overline{\alpha}+\underline{\alpha}}{2(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)} & \text{if } 0 \le c_H \le c_H^1 \\ 1 & \text{if } c_H^1 \le c_H \le 1 \end{cases}$$

$$p_L = \begin{cases} \frac{\underline{\alpha}(\overline{\alpha}\underline{\alpha}+\overline{\alpha}+\underline{\alpha})}{2(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)} & \text{if } 0 \le c_L \le \max\{0, c_L^w\} \\ \frac{c_L}{2} + \frac{\underline{\alpha}\overline{\alpha}(1+\overline{\alpha}+\underline{\alpha})}{2(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)} & \text{if } \max\{0, c_L^w\} \le c_L \le c_L^{\underline{\alpha}} \\ \overline{\alpha} & \text{if } c_L^{\underline{\alpha}} \le c_L \le \overline{\alpha} \\ c_L & \text{if } \overline{\alpha} \le c_L \le 1 \end{cases}$$

where $c_H^1 = \frac{2\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+2\underline{\alpha}^2-\overline{\alpha}-\underline{\alpha}}{\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2}$; $c_L^w = \frac{\underline{\alpha}(\underline{\alpha}-\overline{\alpha}^2)}{\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2}$; $c_L^{\underline{\alpha}} = \frac{\underline{\alpha}(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+2\underline{\alpha}^2-\overline{\alpha})}{\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2}$. Note that $c_L^{\underline{\alpha}}$ is the point where $p_L(c_L) = \underline{\alpha}$, and $c_L^w > 0$ only if $\underline{\alpha} > \overline{\alpha}^2$, which qualitatively applies to those situations where the range of the distribution is small.



Lastly, to find buyer's expected cost under a pooling equilibrium ($\kappa_B(\alpha)$), we need to break down different ranges of c_L and c_H for which each supplier may win the auction:

<u>If $\overline{\alpha}^2 < \underline{\alpha}$:</u>

$$\kappa_B = \tfrac{1}{2}(\kappa_B^{\underline{\alpha}} + \kappa_B^{\overline{\alpha}}) = \tfrac{-1}{24(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)^3\overline{\alpha}^2}(12\overline{\alpha}^{10} + 12\overline{\alpha}^9\underline{\alpha} + 16\overline{\alpha}^8\underline{\alpha}^2 - 31\overline{\alpha}^7\underline{\alpha}^3 - 42\overline{\alpha}^6\underline{\alpha}^4 - 61\overline{\alpha}^5\underline{\alpha}^5 - 2\overline{\alpha}^4\underline{\alpha}^6 + 24\overline{\alpha}^3\underline{\alpha}^7 + 40\overline{\alpha}^2\underline{\alpha}^8 + 16\overline{\alpha}\underline{\alpha}^9 - 24\overline{\alpha}^9 - 32\overline{\alpha}^8\underline{\alpha} - 45\overline{\alpha}^7\underline{\alpha}^2 + 11\overline{\alpha}^6\underline{\alpha}^3 + 11\overline{\alpha}^5\underline{\alpha}^4 - 9\overline{\alpha}^4\underline{\alpha}^5 - 56\overline{\alpha}^3\underline{\alpha}^6 - 56\overline{\alpha}^2\underline{\alpha}^7 - 24\overline{\alpha}\underline{\alpha}^8 - 8\underline{\alpha}^9 + 12\overline{\alpha}^8 + 3\overline{\alpha}^7\underline{\alpha} + 12\overline{\alpha}^6\underline{\alpha}^2 + 24\overline{\alpha}^5\underline{\alpha}^3 + 42\overline{\alpha}^4\underline{\alpha}^4 + 33\overline{\alpha}^3\underline{\alpha}^5 + 18\overline{\alpha}^2\underline{\alpha}^6 - 23\overline{\alpha}^7 - 69\overline{\alpha}^6\underline{\alpha} - 104\overline{\alpha}^5\underline{\alpha}^2 - 104\overline{\alpha}^4\underline{\alpha}^3 - 69\overline{\alpha}^3\underline{\alpha}^4 - 23\overline{\alpha}^2\underline{\alpha}^5 + 4\overline{\alpha}^6 + 16\overline{\alpha}^5\underline{\alpha} + 24\overline{\alpha}^4\underline{\alpha}^2 + 16\overline{\alpha}^3\underline{\alpha}^3 + 4\overline{\alpha}^2\underline{\alpha}^4)$$

where,

$$\kappa_B^{\overline{\alpha}} = (1-\overline{\alpha})\left(1 - c_H^1 + \int_0^{c_H^1} p_H dc_H\right) + (\overline{\alpha} - c_L^{\underline{\alpha}})\left((1-c_H^1)\overline{\alpha} + \int_0^{c_H^1} p_H dc_H\right) + (1-c_H^1)\left(\int_{c_L^w}^{c_L^{\underline{\alpha}}} p_L dc_L\right) + \tfrac{c_L^w(\overline{\alpha}\underline{\alpha}+\overline{\alpha}+\underline{\alpha})\underline{\alpha}}{2(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)} + (c_H^1 - c_H^2)\left(\int_{c_L^w}^{c_L^{\underline{\alpha}}} p_L dc_L\right) + c_H^2\left(\int_{c_L^w}^{c_L^{w,\overline{\alpha}}} p_L dc_L\right) + \int_0^{c_H^2}\int_{c_L^{w,\overline{\alpha}}}^{c_L^{cor,\overline{\alpha}}} p_L dc_L dc_H + \int_{c_L^{w,\overline{\alpha}}}^{c_L^{\underline{\alpha}}}\int_0^{c_H^{cor,\overline{\alpha}}} p_H dc_H dc_L$$

and

$$\kappa_B^{\underline{\alpha}} = (1-\overline{\alpha})\left(1 - c_H^1 + \int_0^{c_H^1} p_H dc_H\right) + (\overline{\alpha} - c_L^{\underline{\alpha}})\left(1 - c_H^1 + \int_0^{c_H^1} p_H dc_H\right) + (1-c_H^1)\left(\int_{c_L^w}^{c_L^{\underline{\alpha}}} p_L dc_L\right) + \tfrac{c_L^w(\overline{\alpha}\underline{\alpha}+\overline{\alpha}+\underline{\alpha})\underline{\alpha}}{2(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)} + \int_0^{c_H^1}\int_{c_L^w}^{c_L^{cor,\underline{\alpha}}} p_L dc_L dc_H + \int_{c_L^w}^{c_L^{\underline{\alpha}}}\int_0^{c_H^{cor,\underline{\alpha}}} p_H dc_H dc_L$$

<u>Otherwise, if $\overline{\alpha}^2 > \underline{\alpha}$:</u>

$$\kappa_B = \tfrac{1}{2}(\kappa_B^{\underline{\alpha}} + \kappa_B^{\overline{\alpha}}) = \tfrac{-1}{24(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)^3\overline{\alpha}^2}(12\overline{\alpha}^{10} + 12\overline{\alpha}^9\underline{\alpha} + 20\overline{\alpha}^8\underline{\alpha}^2 - 28\overline{\alpha}^7\underline{\alpha}^3 - 36\overline{\alpha}^6\underline{\alpha}^4 - 61\overline{\alpha}^5\underline{\alpha}^5 - 2\overline{\alpha}^4\underline{\alpha}^6 + 24\overline{\alpha}^3\underline{\alpha}^7 + 40\overline{\alpha}^2\underline{\alpha}^8 + 16\overline{\alpha}\underline{\alpha}^9 - 24\overline{\alpha}^9 - 31\overline{\alpha}^8\underline{\alpha} - 45\overline{\alpha}^7\underline{\alpha}^2 + 2\overline{\alpha}^6\underline{\alpha}^3 + 5\overline{\alpha}^5\underline{\alpha}^4 - 21\overline{\alpha}^4\underline{\alpha}^5 - 56\overline{\alpha}^3\underline{\alpha}^6 - 56\overline{\alpha}^2\underline{\alpha}^7 - 24\overline{\alpha}\underline{\alpha}^8 - 8\underline{\alpha}^9 + 12\overline{\alpha}^8 + 6\overline{\alpha}^7\underline{\alpha} + 12\overline{\alpha}^6\underline{\alpha}^2 + 24\overline{\alpha}^5\underline{\alpha}^3 + 48\overline{\alpha}^4\underline{\alpha}^4 + 36\overline{\alpha}^3\underline{\alpha}^5 + 24\overline{\alpha}^2\underline{\alpha}^6 - 23\overline{\alpha}^7 - 69\overline{\alpha}^6\underline{\alpha} - 110\overline{\alpha}^5\underline{\alpha}^2 - 107\overline{\alpha}^4\underline{\alpha}^3 - 69\overline{\alpha}^3\underline{\alpha}^4 - 24\overline{\alpha}^2\underline{\alpha}^5 + 4\overline{\alpha}^6 + 16\overline{\alpha}^5\underline{\alpha} + 24\overline{\alpha}^4\underline{\alpha}^2 + 19\overline{\alpha}^3\underline{\alpha}^3 + 6\overline{\alpha}^2\underline{\alpha}^4)$$

where,

$$\kappa_B^{\overline{\alpha}} = (1-\overline{\alpha})\left(1 - c_H^1 + \int_0^{c_H^1} p_H dc_H\right) + (\overline{\alpha} - c_L^{\underline{\alpha}})\left((1-c_H^1)\overline{\alpha} + \int_0^{c_H^1} p_H dc_H\right) + (1-c_H^1)\left(\int_{c_L^{w,\overline{\alpha}}}^{c_L^{\underline{\alpha}}} p_L dc_L\right) + \int_0^{c_L^{w,\overline{\alpha}}} p_L dc_L + (c_H^1 - c_H^2)\left(\int_{c_L^{w,\overline{\alpha}}}^{c_L^{\underline{\alpha}}} p_L dc_L\right) + \int_0^{c_H^2}\int_{c_L^{w,\overline{\alpha}}}^{c_L^{cor,\overline{\alpha}}} p_L dc_L dc_H + \int_{c_L^{w,\overline{\alpha}}}^{c_L^{\underline{\alpha}}}\int_0^{c_H^{cor,\overline{\alpha}}} p_H dc_H dc_L;$$

$$\kappa_B^{\underline{\alpha}} = (1-\overline{\alpha})\left(1 - c_H^1 + \int_0^{c_H^1} p_H dc_H\right) + (\overline{\alpha} - c_L^{\underline{\alpha}})\left(1 - c_H^1 + \int_0^{c_H^1} p_H dc_H\right) + (1-c_H^1)\left(\int_0^{c_L^{\underline{\alpha}}} p_L dc_L\right) + c_L^{\underline{\alpha}}\left(\int_0^{c_H^w} p_H dc_H\right) + \int_{c_H^w}^{c_H^1}\int_0^{c_L^{cor,\underline{\alpha}}} p_L dc_L dc_H + \int_0^{c_L^{\underline{\alpha}}}\int_{c_H^w}^{c_H^{cor,\underline{\alpha}}} p_H dc_H dc_L;$$

$c_L^{cor,\overline{\alpha}} = \frac{\overline{\alpha}(\overline{\alpha}^2 c_H + \overline{\alpha}\underline{\alpha} c_H + \underline{\alpha}^2 c_H - \underline{\alpha}^2 + \overline{\alpha})}{\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2}$; $c_H^{cor,\overline{\alpha}} = \frac{\overline{\alpha}^2 c_L + \overline{\alpha}\underline{\alpha}^2 + \overline{\alpha}\underline{\alpha} c_L + \underline{\alpha}^2 c_L - \overline{\alpha}^2}{(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)\overline{\alpha}}$; $c_L^{cor,\underline{\alpha}} = \frac{\underline{\alpha}(\overline{\alpha}^2 c_H + \overline{\alpha}\underline{\alpha} c_H + \underline{\alpha}^2 c_H - \overline{\alpha}^2 + \underline{\alpha})}{\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2}$; $c_H^{cor,\underline{\alpha}} = \frac{\overline{\alpha}^2\underline{\alpha}+\overline{\alpha}^2 c_L+\overline{\alpha}\underline{\alpha}c_L+\underline{\alpha}^2 c_L-\underline{\alpha}^2}{(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)\underline{\alpha}}$; $c_H^1 = \frac{2\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+2\underline{\alpha}^2-\overline{\alpha}-\underline{\alpha}}{\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2}$; $c_L^w = \frac{\underline{\alpha}(\underline{\alpha}-\overline{\alpha}^2)}{\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2}$; $c_H^w = \frac{\overline{\alpha}^2-\underline{\alpha}}{\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2}$; $c_L^{al} = \frac{\underline{\alpha}(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+2\underline{\alpha}^2-\overline{\alpha})}{\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2}$; $c_H^2 = \frac{\overline{\alpha}^2\underline{\alpha}+2\overline{\alpha}\underline{\alpha}^2+2\underline{\alpha}^3-\overline{\alpha}^2-\overline{\alpha}\underline{\alpha}}{(\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2)\overline{\alpha}}$; $c_L^{w,\overline{\alpha}} = \frac{\overline{\alpha}(-\underline{\alpha}^2+\overline{\alpha})}{\overline{\alpha}^2+\overline{\alpha}\underline{\alpha}+\underline{\alpha}^2}$.

□

Proof. Private Cost Information: Asymmetric QS Information Analysis (Separating Equilibrium)

In this section, we focus on the possibility of credible signalling for the buyer when suppliers' cost information is privately known only to themselves. We continue to use advance revenue guarantees to the suppliers as the signalling tool for the buyer. Let $\eta_L(\alpha)$ and $\eta_H(\alpha)$ denote the signals employed by the buyer in a separating equilibrium. In order for these signals to be incentive compatible for the buyer and hence credible for the suppliers, they need to satisfy the following system of equations:

$$\kappa_B(\eta_L(\overline{\alpha}), \eta_H(\overline{\alpha}), p_L(\overline{\alpha}), p_H(\overline{\alpha}), \overline{\alpha}) \leq \kappa_B(\eta_L(\underline{\alpha}), \eta_H(\underline{\alpha}), p_L(\underline{\alpha}), p_H(\underline{\alpha}), \overline{\alpha})$$

$$\kappa_B(\eta_L(\underline{\alpha}), \eta_H(\underline{\alpha}), p_L(\underline{\alpha}), p_H(\underline{\alpha}), \underline{\alpha}) \leq \kappa_B(\eta_L(\overline{\alpha}), \eta_H(\overline{\alpha}), p_L(\overline{\alpha}), p_H(\overline{\alpha}), \underline{\alpha})$$

where $\kappa_B(\eta_L, \eta_H, p_L, p_H, \alpha)$ denotes the total cost of the buyer.

Similar to §3.5.2, the first step in analyzing the separating equilibria is to understand suppliers' behavior once they receive a signal of $[\eta_L(\alpha), \eta_H(\alpha)]$ and infer the value of α. Again, by focusing on the linear bid functions ($p_L = A + Bc_L$ and $p_H = D + Ec_H$), we can write suppliers' payoff functions as follows:

$$\pi_L = (p_L - c_L)(1-\eta_H)\left(1 - \frac{p_L - D\alpha}{E\alpha}\right) + (1-c_L)\eta_L\left(\frac{p_L - D\alpha}{E\alpha}\right)$$
$$\pi_H = (p_H - c_H)(1-\eta_L)\left(1 - \frac{p_H - A/\alpha}{B/\alpha}\right) + (1-c_H)\eta_H\left(\frac{p_H - A/\alpha}{B/\alpha}\right)$$

Considering that α in this setting can take either of the two distinct values of $\underline{\alpha}$ or $\overline{\alpha}$, there is no need for complicated signalling devices; and indeed, guarantees to only one supplier is enough for signalling α. Therefore, in this section, to simplify the analysis, we focus only on the cases where $\eta_L = 0$ and $\eta_H \geq 0$. More specifically, the buyer uses $\underline{\eta_H}$ and $\overline{\eta_H}$ for signalling

$\underline{\alpha}$ and $\overline{\alpha}$ respectively. After an analysis of internal and boundary conditions, similar to the symmetric information setting, we can characterize the bid prices of suppliers as below:

$$p_L = \begin{cases} \frac{\alpha}{6} + \frac{1}{3} + \frac{\alpha\eta_H}{2} & \text{if } 0 \leq c_L \leq c_L^w \\ \frac{c_L}{2} + \frac{\alpha}{3} + \frac{1}{6} & \text{if } c_L^w \leq c_L \leq c_L^1 \\ \alpha & \text{if } c_L^1 \leq c_L \leq \alpha \\ c_L & \text{if } \alpha \leq c_L \leq 1 \end{cases}$$

$$p_H = \begin{cases} \left(\frac{1-\eta_H}{2}\right) c_H + \frac{1}{3\alpha} + \frac{1}{6} + \frac{\eta_H}{2} & \text{if } 0 \leq c_H \leq c_H^1 \\ 1 & \text{if } c_H^1 \leq c_H \leq 1 \end{cases}$$

where $c_H^1 = \frac{3\alpha\eta_H - 5\alpha + 2}{3\alpha(-1+\eta_H)}$; $c_L^w = \frac{1}{3} - \frac{\alpha}{3} + \eta_H\alpha$; and $c_L^1 = \frac{4}{3}\alpha - \frac{1}{3}$.

These bid functions allow us to find buyer's expected cost under different conditions.

$$\kappa_B(\underline{\eta_H}, \underline{\alpha}) = \frac{(27\underline{\alpha}^4\underline{\eta_H}^4 - 54\underline{\alpha}^4\underline{\eta_H}^3 + 36\underline{\alpha}^4\underline{\eta_H}^2 - 20\underline{\alpha}^4\underline{\eta_H} - 18\underline{\alpha}^3\underline{\eta_H}^2 + 11\underline{\alpha}^4 + 24\underline{\alpha}^3\underline{\eta_H} - 18\underline{\alpha}^2\underline{\eta_H}^2 + 2\underline{\alpha}^3 - 15\underline{\alpha}^2\underline{\eta_H} + 9\underline{\alpha}^2 - 16\underline{\alpha}\underline{\eta_H} + 40\underline{\alpha} - 8)}{81\underline{\alpha}^2(1-\underline{\eta_H})}$$

$$\kappa_B(\overline{\eta_H}, \overline{\alpha}) = \frac{(27\overline{\alpha}^4\overline{\eta_H}^4 - 54\overline{\alpha}^4\overline{\eta_H}^3 + 36\overline{\alpha}^4\overline{\eta_H}^2 - 20\overline{\alpha}^4\overline{\eta_H} - 18\overline{\alpha}^3\overline{\eta_H}^2 + 11\overline{\alpha}^4 + 24\overline{\alpha}^3\overline{\eta_H} - 18\overline{\alpha}^2\overline{\eta_H}^2 + 2\overline{\alpha}^3 - 15\overline{\alpha}^2\overline{\eta_H} + 9\overline{\alpha}^2 - 16\overline{\alpha}\overline{\eta_H} + 40\overline{\alpha} - 8)}{81\overline{\alpha}^2(1-\overline{\eta_H})}$$

$\kappa_B(\underline{\eta_H}, \overline{\alpha}) = (27\overline{\alpha}^4\underline{\alpha}^3\underline{\eta_H}^4 + 81\overline{\alpha}^2\underline{\alpha}^5\underline{\eta_H}^4 - 216\overline{\alpha}^2\underline{\alpha}^5\underline{\eta_H}^3 - 18\overline{\alpha}^4\underline{\alpha}^3\underline{\eta_H}^2 + 54\overline{\alpha}^4\underline{\alpha}^2\underline{\eta_H}^3 - 108\overline{\alpha}^3\underline{\alpha}^3\underline{\eta_H}^3 + 162\overline{\alpha}^2\underline{\alpha}^5\underline{\eta_H}^2 + 54\overline{\alpha}^2\underline{\alpha}^4\underline{\eta_H}^3 - 8\overline{\alpha}^4\underline{\alpha}^3\underline{\eta_H} - 18\overline{\alpha}^4\underline{\alpha}^2\underline{\eta_H}^2 - 54\overline{\alpha}^3\underline{\alpha}^3\underline{\eta_H}^2 - 72\overline{\alpha}^2\underline{\alpha}^5\underline{\eta_H} - \overline{\alpha}^4\underline{\alpha}^3 - 138\overline{\alpha}^4\underline{\alpha}^2\underline{\eta_H} + 36\overline{\alpha}^4\underline{\alpha}\underline{\eta_H}^2 + 108\overline{\alpha}^3\underline{\alpha}^3\underline{\eta_H} - 108\overline{\alpha}^3\underline{\alpha}^2\underline{\eta_H}^2 + 45\overline{\alpha}^2\underline{\alpha}^5 + 18\overline{\alpha}^2\underline{\alpha}^4\underline{\eta_H} + 108\overline{\alpha}\underline{\alpha}^5\underline{\eta_H} + 102\overline{\alpha}^4\underline{\alpha}^2 - 24\overline{\alpha}^4\underline{\alpha}\underline{\eta_H} + 2\overline{\alpha}^3\underline{\alpha}^3 + 216\overline{\alpha}^3\underline{\alpha}^2\underline{\eta_H} + 120\overline{\alpha}^2\underline{\alpha}^4 - 36\overline{\alpha}^2\underline{\alpha}^3\underline{\eta_H} - 216\overline{\alpha}\underline{\alpha}^5 - 216\overline{\alpha}\underline{\alpha}^4\underline{\eta_H} - 12\overline{\alpha}^4\underline{\alpha} + 8\overline{\alpha}^4\underline{\eta_H} - 204\overline{\alpha}^3\underline{\alpha}^2 + 144\overline{\alpha}^2\underline{\alpha}^3 - 72\overline{\alpha}^2\underline{\alpha}^2\underline{\eta_H} + 108\underline{\alpha}^5 - 8\overline{\alpha}^4 + 24\overline{\alpha}^3\underline{\alpha} + 144\overline{\alpha}^2\underline{\alpha}^2 + 16\overline{\alpha}^3 - 48\overline{\alpha}^2\underline{\alpha}) \Big/ \left(324\overline{\alpha}^2\underline{\alpha}^3(1 - \underline{\eta_H})\right)$

$\kappa_B(\overline{\eta_H}, \underline{\alpha}) = (108\overline{\alpha}^5\underline{\alpha}\overline{\eta_H}^4 - 108\overline{\alpha}^5\underline{\alpha}\overline{\eta_H}^3 - 108\overline{\alpha}^4\underline{\alpha}^2\overline{\eta_H}^3 - 36\overline{\alpha}^5\underline{\alpha}\overline{\eta_H}^2 + 54\overline{\alpha}^5\overline{\eta_H}^3 + 180\overline{\alpha}^4\underline{\alpha}^2\overline{\eta_H}^2 - 54\overline{\alpha}^3\underline{\alpha}^2\overline{\eta_H}^3 + 28\overline{\alpha}^5\underline{\alpha}\overline{\eta_H} - 36\overline{\alpha}^4\underline{\alpha}^2\overline{\eta_H} - 90\overline{\alpha}^4\underline{\alpha}\overline{\eta_H}^2 + 18\overline{\alpha}^3\underline{\alpha}^2\overline{\eta_H}^2 + 8\overline{\alpha}^5\underline{\alpha} - 18\overline{\alpha}^5\overline{\eta_H} - 36\overline{\alpha}^4\underline{\alpha}^2 + 60\overline{\alpha}^4\underline{\alpha}\overline{\eta_H} + 81\overline{\alpha}^4\overline{\eta_H}^2 - 162\overline{\alpha}^3\underline{\alpha}^2\overline{\eta_H} - 36\overline{\alpha}^3\underline{\alpha}\overline{\eta_H}^2 + 216\overline{\alpha}^2\underline{\alpha}^3\overline{\eta_H} - 117\overline{\alpha}^2\underline{\alpha}^2\overline{\eta_H}^2 - 216\overline{\alpha}\underline{\alpha}^4\overline{\eta_H} - 4\overline{\alpha}^5 + 30\overline{\alpha}^4\underline{\alpha} - 18\overline{\alpha}^4\overline{\eta_H} - 18\overline{\alpha}^3\underline{\alpha}^2 + 12\overline{\alpha}^3\underline{\alpha}\overline{\eta_H} - 270\overline{\alpha}^2\underline{\alpha}^2\overline{\eta_H} + 216\overline{\alpha}\underline{\alpha}^4 + 432\overline{\alpha}\underline{\alpha}^3\overline{\eta_H} - 15\overline{\alpha}^4 + 24\overline{\alpha}^3\underline{\alpha} + 36\overline{\alpha}^3\overline{\eta_H} + 243\overline{\alpha}^2\underline{\alpha}^2 + 8\overline{\alpha}^2\underline{\alpha}\overline{\eta_H} - 432\overline{\alpha}\underline{\alpha}^3 - 180\overline{\alpha}\underline{\alpha}^2\overline{\eta_H} - 12\overline{\alpha}^3 - 8\overline{\alpha}^2\underline{\alpha} + 252\overline{\alpha}\underline{\alpha}^2 + 4\overline{\alpha}^2 - 36\underline{\alpha}^2) \Big/ (324\overline{\alpha}^2\underline{\alpha}^2(1 - \overline{\eta_H}))$

Given the complexity of bid functions, finding a general, yet simple closed-form solution for all the combinations of $\underline{\alpha}$ and $\overline{\alpha}$ is difficult. However, similar to the basic model with suppliers' public cost information, it is evident that the buyer has more incentive to deviate when $\alpha = \underline{\alpha}$ because of his potential influence on supplier H as the most qualified player in the game. Therefore, to build a separating equilibrium under which the buyer signals his true type, we impose $\overline{\eta_H} = 0$ and try to find solvable equations for $\underline{\eta_H}$.

$$\kappa_B(\overline{\eta_H} = 0, \overline{\alpha}) \leq \kappa_B(\underline{\eta_H}, \overline{\alpha})$$

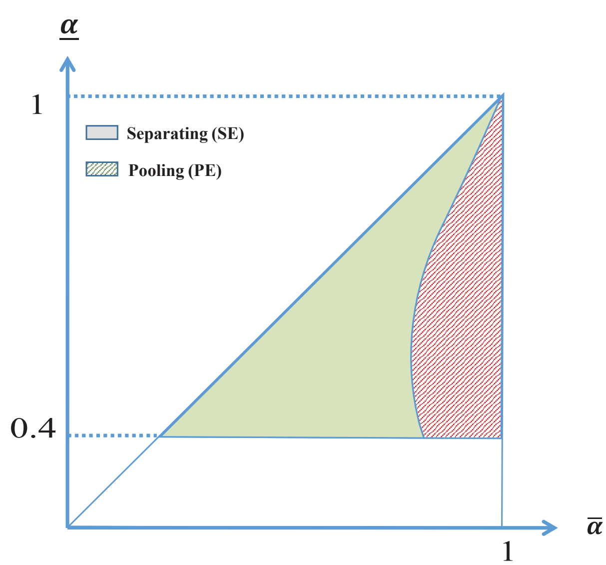
$$\kappa_B(\underline{\eta_H}, \underline{\alpha}) \leq \kappa_B(\overline{\eta_H} = 0, \underline{\alpha})$$

Conjecturing from typical signalling games with two types for the informed principal, the least costly $\underline{\eta_H}$ should make one of the above inequalities binding (i.e. equality should hold) while the other one is also satisfied (often in non-equality form). Since $\kappa_B(\overline{\eta_H}, \overline{\alpha})$ is fixed at $\underline{\eta_H}$, we should logically find $\underline{\eta_H}$ for which $\kappa_B(\overline{\eta_H} = 0, \overline{\alpha}) = \kappa_B(\underline{\eta_H}, \overline{\alpha})$ and then check to see if $\kappa_B(\underline{\eta_H}, \underline{\alpha}) \leq \kappa_B(\overline{\eta_H} = 0, \underline{\alpha})$.

$\kappa_B(\overline{\eta_H} = 0, \overline{\alpha}) = \kappa_B(\underline{\eta_H}, \overline{\alpha}) \Rightarrow$ Quartic Polynomial:

$(27\overline{\alpha}^4\underline{\alpha}^3+81\overline{\alpha}^2\underline{\alpha}^5)\underline{\eta_H}^4+(-216\overline{\alpha}^2\underline{\alpha}^5+54\overline{\alpha}^4\underline{\alpha}^2-108\overline{\alpha}^3\underline{\alpha}^3+54\overline{\alpha}^2\underline{\alpha}^4)\underline{\eta_H}^3+(-18\overline{\alpha}^4\underline{\alpha}^3+162\overline{\alpha}^2\underline{\alpha}^5-18\overline{\alpha}^4\underline{\alpha}^2-54\overline{\alpha}^3\underline{\alpha}^3+36\overline{\alpha}^4\underline{\alpha}-108\overline{\alpha}^3\underline{\alpha}^2)\underline{\eta_H}^2+(36\overline{\alpha}^4\underline{\alpha}^3-72\overline{\alpha}^2\underline{\alpha}^5-138\overline{\alpha}^4\underline{\alpha}^2+116\overline{\alpha}^3\underline{\alpha}^3+18\overline{\alpha}^2\underline{\alpha}^4+108\overline{\alpha}\underline{\alpha}^5-24\overline{\alpha}^4\underline{\alpha}+216\overline{\alpha}^3\underline{\alpha}^2-216\overline{\alpha}\underline{\alpha}^4+8\overline{\alpha}^4-72\overline{\alpha}^2\underline{\alpha}^2+160\overline{\alpha}\underline{\alpha}^3-32\underline{\alpha}^3)\underline{\eta_H}-45\overline{\alpha}^4\underline{\alpha}^3+45\overline{\alpha}^2\underline{\alpha}^5+102\overline{\alpha}^4\underline{\alpha}^2-6\overline{\alpha}^3\underline{\alpha}^3+120\overline{\alpha}^2\underline{\alpha}^4-216\overline{\alpha}\underline{\alpha}^5-12\overline{\alpha}^4\underline{\alpha}-204\overline{\alpha}^3\underline{\alpha}^2+108\overline{\alpha}^2\underline{\alpha}^3+108\underline{\alpha}^5-8\overline{\alpha}^4+24\overline{\alpha}^3\underline{\alpha}+144\overline{\alpha}^2\underline{\alpha}^2-160\overline{\alpha}\underline{\alpha}^3+16\overline{\alpha}^3-48\overline{\alpha}^2\underline{\alpha}+32\underline{\alpha}^3 = 0$

Figure 5–2: Characterization of Separating Equilibrium under Suppliers' Private Cost Information.



In fact, according to our numerical analysis, in most of the cases, the first root of the above quartic function ($\underline{\eta_H^*}$) satisfies the conditions and provides possibility for credibly signalling α. To summarize, in those cases where signalling is possible (refer to Fig 5–2), the buyer uses $(\eta_H(\underline{\alpha}), \eta_H(\overline{\alpha})) = \left(\underline{\eta_H^*}, 0\right)$ to signal the true value of $\alpha \in \{\underline{\alpha}, \overline{\alpha}\}$.

Observing upon $\eta_H(\alpha)$, suppliers update their belief on α according to the following function:

$$\alpha(\eta_H) = \begin{cases} \overline{\alpha}, & \eta_H \in \left[0, \underline{\eta_H^*}\right) \\ \underline{\alpha}, & \eta_H \geq \underline{\eta_H^*} \end{cases}$$

and update their bidding prices accordingly.

□

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