Determining optimal open pit to underground mine transition depth using stochastic mine planning techniques

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Contribution of Authors

This section states the contribution of the co-author of the papers that comprise the present work. The author of this thesis is the primary author of all the work presented herein. The work was completed with the supervision and advice of his advisor Prof. Roussos Dimitrakopoulos, who is also the co-author on the two papers that comprise this thesis.

Chapter 2: MacNeil, J., and Dimitrakopoulos, R. (2015). A stochastic optimization formulation for the transition from open pit to underground mining.

Chapter 3: MacNeil, J., and Dimitrakopoulos, R. (2015). Determining the optimal open pit to underground mining transition depth at Geita gold mine using stochastic mine planning.

Abstract

Years of open pit mine production results in a pit of increased width and depth. This causes the cost of producing deeper ore to increase. The ore produced can also be more heavily diluted by surrounding waste. In order to increase the amount of economic reserves and mine life, a transition to underground mining can be made. There is a threshold where mining through underground methods becomes more profitable than open pit, and it is important to effectively identify this threshold as it can have a great impact on a mine's profits. This thesis uses stochastic mine planning methods to identify the optimal open pit to underground mining (OP-UG) transition depth. The method proposed herein decomposes the problem by identifying a series of candidate scenarios where it is feasible to make an OP-UG transition. The economic viability of each member of the set of candidate transition depths is then evaluated by producing uncertainty-based life-of-mine production plans which are used to outline expected yearly cash flows. An initial application of this proposed method is presented in Chapter 3 where the benefits of using stochastic mine planning to provide well-informed long-term strategic decision-making criteria are observed. Specifically, an application of the stochastic approach produces operational schedules which lead to a 9% or \$43 M increase in net present value (NPV) over the corresponding deterministic framework. A second work presented in Chapter 4 describes the application of the proposed method at Geita gold mine, a large gold mine in Eastern Africa. At this operation, future ore production is forecasted to fall well below the mill's capacity, and to supplement this deficiency a transition from open pit to underground mining is being considered. The resulting analysis from the proposed stochastic framework shows that the most profitable decision involves forgoing underground mine development and continuing to produce through solely open pit mining for the foreseeable future. Valuable insights towards the risk associated with the proposed mine design are gained through stochastic risk analysis. Results show a 23% NPV increase for the stochastic mine plans when compared to the conventional deterministic equivalent.

Résumé

Années de ouvertes résultats de la production de la mine à ciel dans une fosse de l'augmentation de largeur et de profondeur. Cela provoque le coût de production de minerai plus profond à augmenter. Le minerai produit peut aussi être plus fortement dilué par les déchets environnante. Afin d'augmenter le montant des réserves économiques et vie de la mine, une transition à l'exploitation souterraine peut être faite. Il existe un seuil où l'exploitation minière par des méthodes souterraines devient plus rentable que la mine à ciel ouvert, et il est important d'identifier efficacement ce seuil car elle peut avoir un grand impact sur les profits d'une mine. Cette thèse utilise des méthodes de planification de la mine stochastiques pour identifier le ciel ouvert optimale à l'exploitation souterraine (OP-UG) la profondeur de transition. La méthode proposée ici se décompose le problème en identifiant une série de scénarios de candidats où il est possible de faire une transition OP-UG. La viabilité économique de chaque membre de l'ensemble des profondeurs de transition candidats est ensuite évaluée en produisant des plans fondés sur l'incertitude de la vie de la mine de production qui sont utilisés pour décrire les flux de trésorerie annuels attendus. Une première application de cette méthode proposée est présentée dans le chapitre 3, où les avantages de l'utilisation planification de la mine stochastique de fournir des critères de prise de décisions stratégiques à long terme bien informés sont observées. Plus précisément, une application de l'approche stochastique produit des horaires de fonctionnement qui conduisent à un ou 43 M \$ en hausse de 9% en valeur actuelle nette (VAN) sur le cadre déterministe correspondant. Un deuxième travail présenté dans le chapitre 4 décrit l'application de la méthode proposée à Geita mine d'or, une grande mine d'or en Afrique de l'Est. A cette opération, la future production de minerai est prévu de bien tomber en dessous de la capacité de l'usine, et de compléter cette lacune une transition à ciel ouvert à l'exploitation souterraine est envisagée. L'analyse résultant du cadre stochastique proposé montre que la décision la plus rentable implique de renoncer au développement de la mine souterraine et de continuer à produire par l'exploitation minière à ciel ouvert uniquement pour l'avenir prévisible. Des idées précieuses vers le risque associé à la conception de la mine proposée sont obtenus grâce à l'analyse de risque stochastique. Les résultats montrent une

augmentation de la VAN de 23% pour les plans de la mine stochastiques par rapport à l'équivalent déterministe classique.

Goals and Objectives

The goal of this thesis is to develop and apply a method to determine the optimal transition depth from open pit to underground mining which utilizes stochastic mine planning techniques and effectively describes the transition depth in an operationally implementable manner in three dimensions. In order to achieve this goal, the following objective must be fulfilled:

- 1. Perform a critical review of the recent literature that is relevant to the open pit to underground transition decision. This involves developments on the topics of: traditional open pit mine planning, underground mine planning, incorporating uncertainty into the mine planning process and the open pit to underground transition problem itself.
- 2. Develop a framework to determine optimal transition depth from open pit to underground mining which utilizes stochastic mine planning techniques and perform a field test of proposed method through a case study.
- 3. Apply developed approach to a currently operating, full-scale mining operation which is currently considering the decision to transition for open pit to underground mining.

Thesis Outline

This thesis is organized into the following chapters:

Chapter 1 provides a literature review of traditional open pit and underground mining practices, advanced stochastic mine planning methods, and previous attempts to solve the open pit to underground mining transition problem.

Chapter 2 presents the developed methodology to identify the optimal open pit to underground mining transition depth which utilizes stochastic mine planning techniques. An application of the method at a gold deposit is also discussed.

Chapter 3 discusses an application of the developed methodology at a full-scale, operating gold mine in Tanzania. Details of the benefits of stochastic mine planning in long-term strategic decision-making are provided.

Chapter 4 summarizes the contributions made by this work and outlines suggestions for future research on this topic.

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1. Literature Review

1.1 Overview

Any approach to determine a mine's optimal OP to UG transition depth can be viewed by its ability to address a set of specific issues. The first task involves discretizing the material above and below ground. For surface mining, material is typically discretized into blocks, where each block is characterized by its metal content, mining cost, processing cost and recovery (Hustrulid & Kutcha, 2013). Below ground, material is grouped into stopes that vary in shape and size based on the mining method chosen. After discretizing material into selective mining units, a long-term mine plan is produced which outlines the production schedule for both the OP and UG components of the mine. This mining sequence is created through use of optimization frameworks that aim to maximize discounted cash flows profits. Frameworks exist to incorporate sources of uncertainty into the planning process that create value while mitigating risk. It has been shown that geological uncertainty can have a large impact on the profits generated by mine plans. In this thesis geological uncertainty refers to grade or metal content uncertainty and it is important to incorporate this type of uncertainty into mine planning by considering a set of equally-probably orebody simulations (Godoy, 2003; Ramazan & Dimitrakopoulos, 2007; Jewbali, 2006; Albor & Dimitrakopoulos, 2010; Goodfellow, 2014; Montiel, 2014). These mine plans can then be used to accurately calculate the value of a mine by considering the amount of valuable material extracted on a yearly basis while incorporating time value of money. From there, the interaction between the OP and UG components can be modeled to accurately value the act of making a transition between mining methods. It is worthwhile to review the important past work done in these issues pertinent to the OP-UG problem, beginning with long-term OP planning and UG planning methods, then notable works that attempt to solve the OP-UG transition problem, and finally techniques that incorporate geological uncertainty into advanced mine planning frameworks.

1.2 Traditional Open Pit Planning

Traditional mine planning practices utilize estimation methods that produced a single (deterministic) orebody model which makes assumptions about metal content and contains no information related to variability and the magnitude of geological risk. The conventional planning process begins by using this estimated model to determine the ultimate pit limits which designate the extent to which is it economically feasible to mine. To determine the optimum pit limits for an open-pit mine an implementation of the Lerchs and Grossman algorithm is typically used (Lerchs & Grossman, 1965). This algorithm is run with the objective of maximizing the economic value of the deposit, and discounted cash flows can be approximated by using a bench-phase heuristic which aims to fill the processing or mining capacity, whichever comes first, on a yearly basis (Whittle, 1998). A parameterized implementation of the Lerchs and Grossman algorithm (1965) can be run to produce several ultimate pit contours or varying size, known as nested pits, by applying a dynamic factor to the economic value of each block (Hustrulid & Kuchta, 1996). Nested pits of material can be grouped into pushbacks which provide an operational guideline as to how to progress from a pit design standpoint as production progresses. The pushbacks also provide a basis for production scheduling, a process which determines the period that a given mining unit should be extracted within in order to maximize the mine's discounted annual cash flows (Johnson, 1969; Dagdelen, 1985). Annual discounted cash flows can be projected based on the yearly schedule and the estimated value of each mining block. These disjoint processes are consistently executed as today's current mine planning practices. More advanced work aims to improve upon current practices by unifying the process and determining the schedule first. The extent of the portion of the orebody that has been schedule then outlines the ultimate pit contour (Stone et al., 2007). This methodology has the ability to integrate a mining complex consisting of several mines within complex processing streams (Menabde, 2011; Menabde & Stone, 2010).

1.3 Stochastic Mine Planning

In addition to the limitations associated with the sequential nature of traditional mine planning techniques, it has been demonstrated that using a single estimated orebody model for planning can result in a sub-optimal mine plan along with an in-accurate valuation of a mining asset (Vallee, 2000; Ravenscroft, 1992). To overcome this, geological uncertainty is integrated throughout the planning process using a set of equally probably orebody simulations. The benefits of stochastic mine planning where geological uncertainty is considered has been extensively demonstrated (Greico & Dimitrakopoulos, 2007; Leite & Dimitrakopoulos, 2007; Ramazan & Dimitrakopoulos, 2013). These practices differ from the previously discussed traditional methods by utilizing a resource model comprised of several equally probable orebody simulations which encapsulate the variability within a given deposit.

In its simplest form, stochastic mine planning techniques can be used to conduct a risk analysis that quantifies the impact of geological uncertainty on an existing production schedule by observing the schedule's ability to meet key production targets (Ravenscroft, 1992). Dimitrakopoulos, Farrelly and Godoy (2002) perform a risk analysis on a conventional mine plan for a gold deposit and observe several shortcomings. The authors show an application where the originally projected NPV using a single estimated orebody appears to be misleading, as the risk analysis shows that under geological uncertainty the produced mine plan has only a 5% chance of reaching this original NPV (Dimitrakopoulos et al., 2002). Additionally, the authors conclude that the expected, NPV derived through risk analysis is 25% below what is originally projected by the deterministic optimizer (Dimitrakopoulos et al., 2002). The results of risk analysis highlight the uncertainty surrounding key project parameters in the mining business. The root of these shortcoming stems from a deficiency in ore production on a yearly basis, which can be attributed to the inability of a single estimated orebody model to accurately represent the spatial variability and connectivity of high grades. Moreover, the economic value of each block based on its mineral contents is determined through a non-linear transfer function. Therefore, the resulting profits of an optimized schedule produced by using a single input orebody model that is an average of a set of simulations will not be identical to the average profits of a stochastic schedule built considering the same set of simulations. This non-linear transfer function makes it imperative that an entire set of orebody simulations – not just the average - are considered for mine planning and decision-making purposes.

Given the impact geological uncertainty has on the cash flows a given production schedule is able to generate, a focus towards considering such geological simulations during the scheduling process is made by Godoy (2003), Ramazan and Dimitrakopoulos (2007), Goodfellow (2014), and Montiel (2014) among others. Godoy (2003) presents an approach towards production scheduling which aims to maximize value and mitigate risk. In this work, the authors initially produce a set of production schedules using conventional techniques on a set of orebody simulations for a deposit. From there the results of the initial scheduling runs provide the optimizer with information relating to the probability that a given block should be scheduled in a certain period. A simulated annealing optimizer then perturbs such blocks with the objective of minimizing expected deviations from ore and waste targets across all of the simulations to produce a single production schedule which is risk resilient. The result of applying this method at the Superpit in Western Australia shows a benefit of 28% for the perturbed schedule over the starting deterministic solution. In addition to this increase in value, the authors report a reduction in risk whereby the perturbed schedule has a 3% chance of failing to meet yearly ore production targets, compared to the starting schedules chances of 13%. The author notes the increase in NPV is largely due to the optimizer's ability to extract more metal earlier on in the project life, and defer mining waste until later periods. On a yearly basis, the optimizer schedules a combination of blocks with a high grade values along with those which have a high probability of having a high value. This results efficient risk blending over each period produces a schedule that realistically projects cash flows under geological uncertainty. The approach has a number of benefits, but the method has a few downfalls which include not explicitly aiming to maximize NPV in the simulated annealing algorithms objective function, and a time-consuming beginning step of producing a production schedule for each simulation.

A mine production scheduling method utilizes mathematical programming techniques to incorporate geological uncertainty is presented by Ramazan and Dimitrakopoulos (2004). A mixed-integer-programming (MIP) formulation produces unique schedules for a set of simulated orebodies to obtain a probability distribution outlining the likelihood of a given block being mined in a certain period. The authors then attempt to maximize the expected net present value which incorporates the probability for each block. Also included in the optimization's objective is a term which aims to limit deviations from creating a spatially smooth schedule. Dimitrakopoulos and Ramazan (2004) propose a similar MIP methodology with the objective of limiting deviations of mined material from having 100% probability of being a certain specification. The objective function also tries to produce a smooth and operationally implementable schedule and limit deviations from key production targets. The authors introduce the concept of geological risk discounting, where a factor is applied to the unit cost of deviating from a specified target. This factor penalizes deviations early on in the project more heavily than those that occur later. In the capital-intensive business of mining, it is important to ensure that early year targets will be meet in order to produce cash flow to pay back creditors. When the proposed MIP is applied to a nickel laterite deposit, the formulation performs well at reducing the impact of geological risk on a production schedule. When compared to the corresponding deterministic schedule, the stochastic schedule has a much higher probability of hitting yearly ore production targets. The downfall of the two mentioned MIP approaches both put forth by Ramazan and Dimitrakopoulos revolves around its reliance on a probabilistic representation of the uncertainty within each block independently of those surrounding it. This measure ignores the variability and uncertainty associated with neighbouring material. Such joint local uncertainty can be better represented by considering a set of equallyprobable geological simulations.

To overcome the limits of probabilistic methods, Menabde et al. (2007) develop an MIP approach that simultaneously optimizes cut-off grade and production sequence while considering geological uncertainty through use of a set of orebody simulations. The approach implements a formulation that allows the optimizer to select a cut-off grade for each production year. The complexity of the MIP is reduced by aggregating blocks into panels and considering a single binary decision variable for scheduling each panel. When compared to the results gathered using a deterministic optimization with a marginal cut-off grade, the schedule produced through a stochastic optimization with a variable cut-off grade increases the NPV of a mining project by 26%.

Ramazan and Dimitrakopoulos (2007, 2013) expand on this uncertainty-based scheduling work and develop a two-stage stochastic programming (SIP) formulation, which receives a set of orebody simulations as an input for production scheduling. The SIP formulation has the objective of maximizing discounted cash flow produced by a mine and minimizing deviations from key targets on a yearly basis while considering geological uncertainty. The authors utilize a two-stage optimization framework where anticipative (first-stage) and adaptive (second-stage) programming models are combined to form a unique formulation with recourse variables (Birge & Louveaux, 2011). In the work by Ramazan and Dimitrakopoulos (2013), scheduling decisions are made through first-stage decision variables based on the expected economic value of a given block when considering a set of orebody simulations. These mining-related decisions are scenario-independent to provide an operationally implementable schedule. The second-stage, or recourse linear variables are scenario-dependent and govern the amount of material that should be reclaimed from a stockpile in a given year, for a specific simulation. Such decisions are made once the uncertainty is revealed in a given simulation, and allow for the mitigation of risk across the scenarios considered. In a case study at a small gold deposit, the SIP formulation produces a result with an NPV that is 10% higher than the deterministic-equivalent that uses a single estimated orebody model to make scheduling decisions (Ramazan & Dimitrakopoulos, 2013). Although, practicality issues arise as large SIP problems that rely on mathematical programming techniques take a large amount of time to solve, the case study mentioned took 40 hours to solve.

Leite and Dimitrakopoulos (2007) apply the developed SIP to a copper deposit and report a 29% increase in the expected NPV of a stochastic schedule over the deterministic equivalent after performing risk analysis. The authors see a familiar trend as the stochastic schedule outperform the deterministic schedule when observing how the impact of geological uncertainty affects the schedule's ability to meet production targets on a yearly basis. Benndorf and Dimitrakopoulos (2013) then further test the SIP methodology on a multiple-element deposit that has complex metal grade targets and blending requirements. Along with an increased chance of meeting metal grade targets for the stochastic schedule, the authors investigate the impact of the magnitude of a penalty unit cost in the objective

function. The presented work concludes that increasing this unit cost will help the optimizer reduce yearly deviations up to a certain threshold, after which the optimizer will not have the ability to further mitigate risk.

Montiel (2014) and Goodfellow (2014) expand upon the initiative of incorporating geological uncertainty by developing methods that take a global optimization approach to planning mining complexes. A mining complex is an interactive system comprised of several producing mines and multiples processing streams located in a close proximity. Taking a holistic approach to optimizing a large number of decision variables results in a globally optimal solution which has been shown to be more optimal than a series of disjoint optimization runs (Whittle, 2010). Both authors deviate from traditional block-based valuation techniques, as revenue is instead declared when a stream of valuable material reaches a processing destination. Shifting the revenue to the processor overcomes limitations of defining the economic value of a discrete unit of mined material since the impact of blending multiple units and optimal processing streams can be considered. This also allows for the inclusion of non-linear recovery curves, and more accurate valuation of the metal content. In addition to this non-linear transfer function, the authors are both able to model complex constraints such as blending restrictions and the impact of deleterious elements. The authors also implement the ability to effectively route mined material through a mining complex in order to maximize value. First-stage processing-related decisions are modeled as binary decision variables which govern the material flow of each unit of material as it leaves the mine, and then second-stage continuous recourse variables are utilized to mitigate risk at tertiary destinations, such as stockpiles, once uncertainty has been revealed.

To solve the large problem related to globally optimizing a mining complex Montiel (2014) uses a simulated annealing solution method that relies on three perturbation methods that occur on different operational levels. The author also explores the impact of controlling different operating modes. Here, it is discovered that having the ability to oscillate between operational modes at the processing plant which alter the grinding size and consequently throughput can add value. Within the developed framework the author demonstrates the ability to simultaneously schedule both an open pit and underground cut-

and-fill mine. Further improvements can be made to this work by including a diversification strategy within the simulated annealing algorithm.

Goodfellow (2014) allows the optimizer to make decisions related to capital expenditures and produces a destination policy which is robust to uncertainty. To overcome the complex task of assigning a binary processing destination decision variable to each unit of mined material, Goodfellow clusters blocks of similar attribute values and assigns and optimal destination to this cluster. In real-time, once mining commences and a block's cluster membership is known, so is its optimal destination based on the processing policy derived for each cluster during the optimization process. This provides mine operators with a detailed and implementable plan as they gather more data related to the material mined in the short-term. To overcome complexity issues the author implements a hybrid approach that alternates between particle swarm optimization and simulated annealing. The complex heuristic takes a long time to solve and demands a high degree of knowledge when tuning optimization related parameters.

In addition to the simulated annealing, other metaheuristics can be used to overcome the computational complexity issues of large realistic production scheduling of deposits that require unpractical amounts of time to solve. Metaheuristics are able to provide a high quality solution to the mine production scheduling problem in an efficient amount of time. Lamghari and Dimitrakopoulos (2012) propose a metaheuristic method based on tabu search which is further improved upon by Senecal (2015). This method starts with an initial feasible solution that is iteratively modified by performing perturbations within a specified neighborhood. Results show are encouraging with small gaps of less than 4% between the solution produced by the metaheuristic and the optimal solution reached using mathematical programming (Lamghari & Dimitrakopoulos, 2012).

1.4 Underground Planning

Optimization techniques for underground mining are not as advanced as what has been developed for open-pit. This is largely because of the case-specific nature of each underground mine. As well, the scheduling optimization formulation for an underground mine tends to be substantially more complex than what is typically seen for open pit (O'Sullivan, Brickey & Newman, 2015). A number of factors contribute to this increased complexity, most notably constraints that contain several variables, which leads to a dense constraint matrix. As well, there are several activities associated with extracting a single stope, all of which need to be independently scheduled and many underground mines do not stockpile ore, therefore scheduling decisions must immediately satisfy all blending requirements (O'Sullivan et al., 2015).

In general underground long-term planning is divided into two phases: stope design and stope sequencing. Determining an optimal stope design is the first phase of this procedure, and from there the stope extraction sequence can be produced. For the purposes of the large-scale strategic decision of determining the optimal point to transition from open pit to underground mining, efforts mainly focus on scheduling stope extraction to produce guidelines which allow for the accurate valuation of underground mines.

The floating stope algorithm was the first stope design algorithm to be developed and is implemented in a commercially available software product (Alford, 1995). In this algorithm, a minimum cut-off grade is set and a resulting minimum stope size is floated around the deposit to outline the minimum dimension of stopes, where the average grade within the stope is above the cutoff. Two envelopes are then formed: the inner envelope and outer envelope. The inner envelope contains the union of all highest grade stope positions for each block above the cutoff grade. The outer envelope contains the union of all possible stope positions for a block above the cutoff grade. It is then up to a mining engineer to decide where between the inner and outer envelope the optimal stope design lies. Issues with this method arise when stopes overlap, meaning that two or more stopes share one or more high grade blocks. When using this method, the result's optimality is highly related to the interpretation of the engineer as the envelopes merely provide a guideline for stope design.

Geological uncertainty was incorporated into stope design by Grieco and Dimitrakopoulos (2007). This probabilistic method uses mixed integer programming techniques to determine stope and pillar designs. A predetermined grid of rings is grouped together by the optimizer as it aims to maximize metal content. The potential drawback of this method is that the probabilistic nature of the optimizer uses strict binary variables and is not able to accurately capture the joint local uncertainty of the orebody.

Newman and Kuchta (2007) attempt to solve the underground production scheduling problem at a large underground sublevel caving operation. The model aims to decide when the extraction of large panels should occur through the use of Load-Haul-Dump (LHD) machine placements. The problem is large and heavily constrained through vertical and horizontal sequencing requirements along with blasting capacity issues. The objective function of this formulation aims to minimize deviations from monthly ore demand for the two different types of ore. The scheduling process is important because no stockpiles exist at the mine under investigation. Due to the size and complexity of the formulation, the authors attempt to solve an aggregation version of the original formulation where time periods are combined to form phases. Further efforts to overcome the computational complexity of scheduling at the large sublevel caving operation are presented by Martinez and Newman (2011). In this work the authors utilize a decomposition heuristic to simplify the original formulation by creating a series of sub problems that each includes a component from the original objective function. The results of this run produce a solution with a higher objective value and in less time than the original than what is seen for the earlier work (Martinez & Newman, 2010). Shortcomings of both works include failing to incorporate maximizing profits as an objective in the optimization, and solely focusing on minimizing deviations from ore demand. It has been demonstrated that both objective can be incorporated into an optimization formulation as seen in Ramazan and Dimitrakopoulos (2013).

Another underground stope production scheduling method that includes modifications to mathematical programming formulation in order to address solving speed issues is developed by Epstein et al. (2012), who effectively schedule a copper mining complex that is comprised of multiple open pit and underground mines, processors and products. The formulation takes a holistic approach to simultaneous scheduling separate open pit and underground mines at once. The underground method considered implements sub-level caving, where mining units are combined to build vertical columns where the highest grades are located at the bottom of the column and grades decrease upwards vertically. The authors perform an LP relaxation on the originally NP-hard underground scheduling formulation, which works favorably because the layout of the high grade ore at the bottom of each column. This relaxation provides a near-integer solution, and a rounding heuristic is utilized in an iterative fashion to improve the integer quality of the relaxation. A similar rounding heuristic is used to solve the open pit. When benchmarked against traditional practices where a disjoint approach is taken to sequentially schedule neighbouring mines, the advanced integrated model shows an 8% NPV benefit. The method is impressive in its ability to model an entire mining complex simultaneously and reduce solving time of the underground scheduling to a few hours using the rounding heuristic compared to a process that took several weeks using the traditional approach. Although, further testing through case studies where the grade distribution does not play favorably into the architecture of the formulation are required to demonstrate versatility of the solving speed measures implemented.

Bley and Terblanche (2012) present two formulations to schedule the extraction of underground stopes. Beginning with a resource production and consumption framework which models the different mineral products produced while considering consumption activities such as the labour required to extract to stope. The authors, then present a lowresolution resource model with micro-selectivity, which makes utilizes a piece-wise linear approximation of a mining method specific grade tonnage curve to approximate the metal tonnage generated by mining a given stope. The authors aim to model and schedule stopes in the short term on a monthly basis and conclude that the low-resolution resource model is able to handle the relevant short-term constraints and problem size more effectively than the production/consumption framework. Although, relying on a grade tonnage curve to approximate the metal tonnage mined, the method neglects the local heterogeneity and uncertainty of mineral grades associated with mineral deposits.

Nehring, Topal and Little (2009) present a way to substantially reduce the solution time of underground MIP optimizers by limiting the number of binary decision variables included in the formulation. Their approach involves decreasing the number of binary variables for each mining stope from the number of extraction related activities to one per block. Instead of independently scheduling extraction activities for each block, a single binary decision variable is used to define the commencement of mining activities at that given block. Subsequent extraction activities are then carried out on a predetermined timeline. This technique is used in the method presented in this paper to increase the solving speed of the scheduler. Despite the benefits in solving speed, combining these activity decisions may have an impact of the overall optimality of the solution. Little, Knight and Topal (2013) present a profound approach where an operation's stopes are design and scheduled simultaneously. Little et al. (2013) determine the optimal stope layout and stope extraction sequence within the same optimizer. It has been shown in the past that this type of simultaneous optimization leads to better solutions (Whittle, 2010).

Due to a limited number of underground scheduling software products, Roberts and Bloss (2014) alter the scheduling optimization formulation of an underground stoping operation so that it can be accommodated by BLASOR, an existing software product that produces long term schedules for open pit mines (Stone et al., 2007). The relevant UG constraints are formatted much like slope constraints commonly seen in OP scheduling. As well, each stope is divided into grade bins to allow for cut-off grade optimization. In order to keep the mill operating at full capacity, the highest projected value corresponds to the lowest cut-off grade. The results show that this type of scheduling can be successfully executed and in the case study examined.

1.5 Combined Open Pit to Underground Planning for Transition

The increasing frequency at which the decision of when to transition from open pit to underground mining has motivated several research studies in the past. Since some of the world's largest mines will reach their ultimate pit in the coming years, it has become increasingly important to develop methods that optimize and explore the possibility of transitioning from open-pit to underground (Fuentes, 2004). For example, the world's deepest copper mine Codelco's Chuquicamata in Chile will complete a transition in 2019 from OP to UG mining (block caving), gaining access to approximately 1.7 billion tonnes of copper ore reserves which lie beneath the existing pit (Kjetland, 2012). Fuentes and Caceres (2004) also conclude that there is no algorithm that can simultaneously manage and generate an optimal mine plan that outlines the transition from open-pit mining to underground.

The relevant literature can be grouped into two main focuses: indicator-based and scheduling-based. Indicator-based methods make use of comparison between key indicators and parameters for the open pit and underground portions of the deposit. Others approaches focus on looking further into the scheduling aspect which then allows for comparison between financial values of open pit and underground portion. As the capability of computers has advanced, efforts have shifted away from indicator-based and towards a more computationally cumbersome task of scheduling-focused analysis.

Methods to determine an optimal transition depth began with a heavy reliance on the use and comparison of mining indicators. This trend begun with developing the term allowable stripping ratio (ASR) which compares the additional cost incurred for underground mining over open pit versus the unit cost per unit of waste (Popov, 1971). The author states that if the value of the conventionally user Overall Stripping (OSR) exceeds, the ASR then the transition to underground, because at this point it is more expensive to extract through open pit than underground despite the cost differences, because of the amount of external dilution. Musendu (1995) expands this analysis to include several parameters in his analysis in determining an optimal transition level. Notably, the author believes that if the following relationships are met, then it is permissible to make a transition to underground. The required relationships include: lower recovery for OP, low grades, higher OP cost, high stripping costs, low production rate, and low UG dilution. These guidelines provide a conceptual description of an optimal open pit to underground transition point, but for such a capital-intensive decision more analysis is required.

Opoku and Musingwini (2013) builds upon this idea by incorporating geological uncertainty into the benchmarking of key parameters to determine when a mine should make the transition. As a change from other works which take a similar approach, the authors describe the transition point in as a production year instead of a depth, stating that this allows a mine operator to capture the dynamic nature of the decision. The author observes the impact of geological uncertainty on several key project indicators, such as cost, processed gold, average grade, NPV and gold price to cost ratio. Such indicators are benchmarked against the value of these other projects that have previously successfully made the transition. Focusing on the impact of indicators forces the author to lose resolution on the problem and is not accurately able to determine how varying the transition year impacts the decision at each mine investigated, which can have a great impact on profits.

There are several shortcomings associated with indicator-based approaches to solving the OP-UG transition problem. Methods which focus on stripping ratio neglect the revenue component generated by extracting valuable material. Stripping ratio fails to incorporate all components which contribute to the economic value of a block. The revenue generated may change between open pit and underground mining methods and thus this revenue difference should be incorporated in any study that is aiming to maximize a mine's profits through making an OP-UG transition. Marginal fluctuations in strip ratio will be seen in NPV calculations as all of the parameters included in strip ratio are also tracked in NPV. It is apparent that NPV is a superior metric since an NPV calculation incorporates all of the information that is conveyed by indicators such as strip ratio. As well, apart from Opoku and Musingwini's work (2013), indicator-based approaches fail to account for the impact of uncertainty can have. For instance, there is inherent geological uncertainty associated with the classification of a block and as its classification changes between ore and waste this can impact the stripping ratio, head grade, cost of mining ore and recovered metal.

Scheduling efforts began with Nilsson (1992), who investigated scheduling within pushbacks of growing size to determine which transition depth is optimal. Four different pushback designs are evaluated, along with their corresponding underground orebodies. The author notes that the pushback deem optimal for the combined OP-UG method is different from the ultimate pit if the operation went solely through OP methods. In the hypothetical case study presented, the option of combined OP-UG mining increases the value of the operation by 19%. A hybrid approach that includes both scheduling and indicator analysis is developed by Visser and Ding (2006), who aim to solve the transition problem by jointly observing the impact of transition depth and open pit slope angle on profits and other

key project indicators. The objective of perturbing the open pit slope angle in order to decrease strip ratio and operating cost is performed in conjunction with iterative bench-wise scheduling. The OP optimization process iterates downwards through a deposit, observing NPV, IRR and unit costs for progressively deeper benches until a pre-set criteria is met. Scheduling UG is done after OP, as the author looks at a block caving setup which progresses upwards from the deepest strata available for underground mining. The author applies this method to Pallabora Copper Mine in South Africa and observes a \$28M or 4% increase in NPV for the case of transitioning to underground mining as opposed to solely focusing on open pit mining for the duration of the life-of-mine. The author lays out a framework that provides analysis on financial impact and encourage observing NPV and IRR, but no clear decision criteria is stated and the final decision is left up to the user. The approaches by Nilsson (1992) and Visser and Ding (2006) fail to capture the local complexity of a resource model, as a constant homogenous grade is assumed throughout a deposit.

Carli and Peroni (2013) present an approach similar to Nilsson (1992), but in 3dimensions. The authors begin by classifying each block based on marginal and breakeven cut-off grades as developed by Rendu (2013). Each block is assigned an economic value based on where its grade lies in relation to the cut-off thresholds. Once these economic values are established, Datamine Studio 3 NPV Scheduler (Datamine Software, 2013) is used to produce five open pit shells with price factors ranging from 60%-100% on 10% increments. These shells are used as ultimate pit contours for the candidate transition depths evaluated. Below each pit, a similar procedure of identify portions of the orebody that are ore using a breakeven cut-off grade (Rendu, 2013) using the underground parameters is carried out for the portion of material not included in the extent of the open pit contour. The author then evaluates the financial viability of solely open pit mining, underground mining and combined open pit and underground mining. The option to transition to UG mining increases the NPV over the option of solely mining through OP by 9%. Within the options of where to transition, each transition depth has quite a similar NPV, as the optimal depth chosen has an NPV 4% higher than the worst-case decision. The method fails to consider the location and impact of the crown pillar. As well, by neglecting geological uncertainty and

imposing a strict cut-off grade to classify ore, the optimizer has strict guidelines which limit optimality based on predetermined economic parameters.

Roberts, Elkington, van Olden, and Maulen (2013) build a strategic plan for a large copper mine which currently mines through a stoping operation and is investigating the benefit of expanding underground production. The mine is also considering the potential of an open pit mine which has already been planned for future production. The authors evaluate how the opportunity cost of mining a block through underground methods impacts the overall profitability of the mine. After investigating scheduling both OP and UG mines in an overlapping section, the authors conclude that UG stopes that lie within the optimal OP ultimate pit should not be scheduled because they will reduce the profitability of the project. The authors mention that expanding the UG mine to extract high-grade material lying below the open pit is advantageous because UG methods can mine this material earlier in the project life and make up for potential shortfalls in OP ore production in order to ensure that the processor is operating at its full capacity. Although, the authors do not evaluate the decision at a high resolution, in terms of how varying the size of the underground expansion will impact the increase in profits.

A movement towards optimization is made by Bakhtavar and Shahriar (2009), who present a heuristic method that compares the economic value of mine blocks when extracted through OP versus their value if extracted by UG techniques. The method iterates progressively downwards through a deposit, comparing the profit of the two techniques until the value of a certain mining progression underground exceeds the value of mining it through open-pit. Once this threshold is reached, the method concludes that this level is the optimal transition depth. The author moves to an integer programming approach in a later work (Bahktavar, 2012), where binary decision variables are used to determine whether or not a unit of material is extracted through OP or UG methods. Constraints governing open pit slope requirements and a minimum size threshold for underground stopes are included in the formulation to model the problem. A soft reserve constraint is included to prevent the optimizer from assigning a unit more than one method of extraction. This approach overcomes the previous shortfalls of a heuristic method since using mathematical programming guarantees an optimal solution. Although, the optimization does not schedule blocks in a specific year, only a decision on what method to mine is determined which affects how accurately the operation can be valued. Newman, Yano and Rubio (2013) format the transition depth problem as a longest-path network flow. This approach determines the optimal transition depth by creating a network that outlines possible mining sequences, their corresponding transition depths along with the associated NPV (Newman et al., 2013). The orebody is discretized into horizontal strata for the above a below-ground mining components, and it is assumed that a worst-case bench mining schedule is adopted for openpit production, and a bottom-up schedule for the underground block caving component of the mine (Newman et al., 2013). Both Bahktavar and Shahriar (2009) and Newman et al. (2013) choose to discretize the space into strata and effectively reduce all solutions to twodimensions, since the optimal transition depth is described as a strata or plane in the x-y dimension. This result is not implementable, since a two-dimensional plane considered to be the transition depth cannot be used as an ultimate pit guideline. In addition to this, studies have shown that bench-wise mining is not optimal, and mining schedules outlined by nested pits generate more value (Godoy, 2003). Fixing the production schedule to progress from the top downwards for OP and from the bottom upwards for UG may not adequately value the asset, since the schedule will not be optimal. More realistic selective mining units and an optimized schedule can also provide a more accurate representation of a mine's value. In addition, the attempts to solve the OP-UG problem discussed above also fail to consider geological uncertainty.

Realistic selective mining units and an optimized schedule can provide a more accurate representation of a mine's value, and this is the approach taken by Dagdelen and Traore (2014) who further extend this OP to UG transition idea to the context of a mining complex. In this work, the authors investigate the transition decision at a currently operating open pit mine that exists within the context of a mining complex that is comprised of five producing open pits, four stockpiles and one processing plant. The authors take an iterative approach by evaluating a set of selected transition depths through optimizing the life-ofmine production schedules of both the open pit and underground mines using mixed linear integer programming techniques. The authors begin by using Geovia's Whittle software product (Geovia, 2012) to generate a series of pits which provide an ultimate pit contour. The crown pillar, a large portion of undisturbed host material serves as protection between the lowest OP working and the highest UG levels, is located below the ultimate pit. The location of the ultimate pit and crown pillar provide a basis for the underground mine design. Optimized life-of-mine production schedules are then created to determine yearly cash flow and resulting NPV. This procedure is repeated for progressively deeper transition depths until the NPV observed in the current iteration is less than what was seen for a previously considered transition depth, at which point the authors conclude that the previously considered depth, with a higher NPV, is optimal.

Whittle et al. (2015) make an initial attempt to accurately model the optimal placement of a crown pillar which is a large portion of undisturbed host material which lies between the ultimate pit and the high underground opening for geotechnical stability reasons. The work to expand on a network-flow formulation used to determine the optimal ultimate pit for an open pit mine (Lerchs and Grossman, 1965), to solve the OP UG transition decision. The authors introduce an alternate opportunity cost approach in a formatted digraph. The model alters the flow of arcs in the open pit portion of the digraph in order to effectively control the shape and size of the crown pillar which is a paramount feature that needs to be addressed when considering the OP-UG transition. This work makes an initial attempt at solving one of the largely neglected topics involved with the OP-UG transition problem, but fails to consider sources of uncertainty.

All previous attempts to solve the transition problem fail to address the impact of geological uncertainty. Such deterministic frameworks produce misleading NPV projects and have a low chance of meeting production targets once mining commences (Dimitrakopoulos et al., 2002). Stochastic mine planning techniques have been shown to produce favorable results in the presence of uncertainty (Godoy, 2003; Goodfellow, 2014; Montiel, 2014). Before outlining a method that incorporates geological uncertainty into the OP UG decision making framework, it is useful to review past efforts to developed risk-based resource modeling frameworks that are relied upon to produce the geological simulations required for stochastic planning.

1.6 Modeling Geological Uncertainty

Previously mentioned stochastic mine planning methods highlighted the benefits of considering geological uncertainty throughout the planning process. In order to represent the inherent variability of a given deposit selected for scheduling, discrete equally-probable simulations are produced. Orebody simulations are created in a sequential fashion based on the hard conditioning data available, which in mining typically is in the form of drillhole assays or geological interpretations. In order to confirm that a given simulation is an accurate realization of the deposit, validations must be performed to ensure that the statistics of the original hard data are reproduced.

The distribution of mineral contents throughout a deposit is considered to be a random process. Models to characterize such processes in space rely on a distribution of certain attributes over a random spatial field. Equally probable simulation can then be made sequentially by drawing realizations from the conditional probability distribution functions associated with such a spatial random field at different locations. The most common method that implements this sequential approach is Sequential Gaussian Simulation (SGS), which assumes a random field is governed by a Gaussian distribution (Isaaks, 1991). SGS among other methods, use the conditioning data point to build a Gaussian distribution over each node in the random spatial field. In a randomly generated order, these nodes are visited and a randomly generated realization is drawn from the distribution. This local realization then becomes a member of the conditional data set. While creating multiple simulation realizations, the path through simulated nodes changes, as well does the specific values drawn from local distributions, which results in a set of simulations that have local differences from each other. The inclusion of recently simulated nodes within the conditioning data set was shown to demand a considerable amount of memory and lead to a performance decline (Godoy, 2003). In order to overcome these issues, Godoy (2003) proposes direct block simulation (DBSIM) which simulates on a point-support scale in the same manner seen for GSGS (Dimitrakopoulos & Luo, 2004). DBSIM begins by simulating the internal nodes within a selective mining unit. A block value is then calculated as the average of the internal simulated nodes and the simulated points are discarded as the average block value is entered into the conditioning data set. This algorithm effectively integrates

conditioning data on both the block and point support scale. The DBSIM algorithm is efficient because utilizes the advantages capitalized upon by GSGS, where the adjacent nodes are considered to share a common neighbourhood. As well DBSIM performs fewer searches than strictly point-wise simulation algorithms. Assuming a Gaussian distribution throughout the random field and relying on two-point statistical measures such as variance are the main shortcomings of SGS, GSGS and DBSIM. Particularly when modeling complex geological phenomena that deviate from a Gaussian distribution, other simulation methods are required to overcome the limitations of two-point simulation frameworks.

To expand upon the limitations of conventional simulation techniques that only consider first and second-order statistics, multiple point (MP) models have been developed that utilize high-order spatial statistics. High-order statistics are important measures that require consideration in order to model and reproduce the connectivity of complex geometries often seen in mineral deposits. It has been demonstrated that MP methods are able to accurately model such complex spatial features that deviate from Gaussianity (Arpat, 2005; Mariethoz, Renard, & Straubhaar, 2010). MP sequential simulation methods store the frequency of repeatable spatial patterns and determine the probability associated with such patterns occurring on a simulation grid. A commonly used MP simulation technique is a mining setting is single normal equation simulation or SNESIM (Strebelle, 2002). MP statistics require closely spaced conditioning data (Guardiano & Srivstava, 1993) and therefore rely on the use of a training image, or geological interpretation of the schematic of the structures within a deposit, to serve as an input to the MP simulation algorithm. Measures have been taken to reduce solving speed issues such as storing data event probability distributions in a tree and optimizing size of the template (Strebelle, 2013). In a mining setting, MP simulation algorithms such as SNESIM are often called upon to simulate categorical variables such as lithologies based on such a training image.

2. A Stochastic Optimization Formulation for the Transition from Open Pit to Underground Mining

2.1 Introduction

The transition from open pit (OP) to underground (UG) methods requires a large capital cost for development and potential delays in production but can provide access to a large supply of reserves and subsequently extend a mine's life. Additionally, an operating mine may benefit from such a transition because of the opportunity to utilize existing infrastructure and equipment, particularly when in a remote location. Optimization approaches towards the open pit to underground transition decision (or OP-UG) may commence with discretizing the space above and below ground into selective units. For surface mining, material is typically discretized into mining blocks, while underground material is frequently grouped into stopes of varying size depending on the mining method chosen. From there and through production scheduling optimization, the interaction between the OP and UG components can be modeled to realistically value the asset under study.

Historically, operations research efforts in mine planning have been focused on open pits as opposed to underground operations. Most commonly, the open pit planning process begins by determining the ultimate pit limits and industry standard is the nested implementation of the Lerchs - Grossman's (LG) algorithm (Lerchs and Grossman, 1965; Whittle, 1988, 1999). This algorithm utilizes a maximum closure concept to determine optimal pit limits, and a nested implementation facilitates economic discounting. For underground mine planning, optimization techniques are less advanced as when compared to those employed for open pit mines and heavily depend on the mining method used. In practice, underground long-term planning is divided into two phases: stope design and production sequencing. For stope design methods, the Floating Stope algorithm (Alford, 1995) is the oldest computerized design tool available, although not an optimization algorithm. Mine optimization research has developed methods that schedule the extraction of discretized units in underground mines (e.g. Trout, 1995; Nehring and Topal, 2007) based on mixed integer programming (MIP) approaches. Nehring, Topal and Little (2009) and Little and Topal (2011) extend MIP approaches to reduce the solution times by combining decision variables. Adaptation of open pit MIP approaches for optimization of underground strategic mine planning have also been proposed (Roberts and Bloss, 2014).

Some of the world's largest mines are expected to reach their ultimate pit in the next 15 years (Kjetland, 2012). Despite the importance of the topic, there is no well-established algorithm to simultaneously generate an optimal mine plan that outlines the transition from open pit mining to underground (Fuentes and Caceres, 2004). The first attempt to solve the OP to UG transition problem was made by Popov (1971). More recently a movement towards applying optimization techniques to the problem has been made starting with Bakhtavar, Shahriar, and Oraee (2008) who present a heuristic method that compares the economic value of mine blocks when extracted through OP versus their value if extracted by UG techniques. The method iterates progressively downwards through a deposit, concluding that the optimal transition is the depth reached when the value of a block mined by UG methods exceeds the corresponding OP mining value. Drawback of this method is that it provides a transition depth only described in two-dimensions which is unrealistic from a practical standpoint. A 2D plane is not an implementable guideline for open pit production, which relies upon pushbacks and ultimate pit contours to govern production progress. A main effort is presented in Newman, Yano and Rubio (2013) where the transition depth problem is formulated as a longest-path network flow problem. Each path within the network has a unique extraction sequence, a transition depth and a corresponding net present value (NPV). Major limitation of this development is that it amounts to a 2D solution of what is a 3D problem, as the orebody is discretized into horizontal strata for the above and below ground mining components. At the same time worst-case bench-wise mining schedule is adopted for open pit production and a bottom-up schedule for the underground block caving component of the mine. These highly constrained mining bench-wise progressions have been demonstrated to be far from optimal (Whittle, 1988) and are rarely implemented in practice. More realistic selective mining units and an optimized schedule can provide a more accurate representation of a mine's value, and this is the approach taken by Dagdelen and Traore (2014) who further extend this OP to UG transition idea to the context of a mining complex. In this work, the authors investigate the transition decision at a currently operating open pit mine that exists within the context of a mining complex that

is comprised of five producing open pits, four stockpiles and one processing plant. Dagdelen and Traore (2014) take an iterative approach by evaluating a set of selected transition depths through optimizing the life-of-mine production schedules of both the open pit and underground mines using mixed linear integer programming techniques. The authors begin by using Geovia's Whittle (Geovia, 2012) software to generate a series of pits which provide an ultimate pit contour. The crown pillar, a large portion of undisturbed host material that serves as protection between the lowest OP working and the highest UG levels, is located below the ultimate pit. The location of the ultimate pit and crown pillar provide a basis for the underground mine design. Optimized life-of-mine production schedules are then created to determine yearly cash flow and resulting NPV. This procedure is repeated for progressively deeper transition depths until the NPV observed in the current iteration is less than what was seen for a previously considered transition depth, at which point the authors conclude that the previously considered depth, with a higher NPV, is optimal.

All above mentioned attempts to optimize the OP-UG transition depth fail to consider geological uncertainty, a major cause for failure in mining projects (Vallee, 2000). Stochastic optimizers integrate and manage geological uncertainty (eg grades, material types, metal, and rock properties) throughout the scheduling process. Such scheduling optimizers have been long shown to increase the net present value (NPV) of an operation, while providing a schedule that has a high probability of meeting metal production and cash flow targets (e.g. Godoy, 2003; Ramazan and Dimitrakopoulos, 2007; Jewbali, 2006; Albor and Dimitrakopoulos, 2010; Goodfellow, 2014; Montiel, 2014; and others). Implementing such frameworks is extremely valuable when making long-term strategic decisions because of their ability to accurately value assets.

In this paper, the financial viability of a set of candidate transition depths is evaluated in order to identify the most profitable transition depth. In order to get an accurate projection of the yearly cash flows each candidate transition depth is capable of generating, a yearly life-of-mine extraction schedule is produced for both to OP and UG components of the mine. Here, a two-stage stochastic integer programming (SIP) formulation of the production scheduling problem is used, which is similar to the work developed by Ramazan and Dimitrakopoulos (2013). The proposed method improves upon previous works related to the OP-UG transition problem by simultaneously incorporating geological uncertainty into long-term decision-making while providing a transition depth described in threedimensions that can be implemented and understood by those who operate the mine.

In the following sections, the method of evaluating a set of pre-selected candidate transition depths to determine which is optimal is discussed. Then a stochastic integer programming formulation used to produce a long-term production schedules for each of the pre-selected candidate transition depths is presented. Finally, a field test of the proposed method is analyzed as the method is applied to a gold mine.

2.2 Method

2.2.1 The general set up: Candidate transition depths

The method proposed herein to determine the transition depth from OP to UG mining is based on the discretization of the orebody space into different selective units and then accurately assessing the value of OP and UG portions of the mine based on optimized yearly extraction sequences of these discretized units. More specifically, this leads to a set of several candidate transition depths being assessed in terms of value and then the candidate depth that corresponds to the highest total discounted profit is deemed optimal for the mine being considered. Stochastic integer programming (SIP) provides the required optimization framework to make an informed decision, as this optimizer considers stochastic representations of geological uncertainty while generating the OP and UG long-term production schedules that accurately predict discounted cash flows.

For each transition depth being considered, the OP optimization process begins with discretizing the OP orebody space into blocks, sized based on operational selectivity. Candidate transition depths can be primarily identified based on feasible crown pillar locations. A crown pillar envelope outlined by a geotechnical study delineates an area that the crown pillar can be safely located within. In the proposed method, it is assumed that the crown pillar is not mined. As the crown pillar location changes within this envelope, the extent of the OP and UG orebody also changes and the impact this has on yearly discounted cash flow can be investigated (Figure 2.1). The year in which the transition is planned to

occur varies across the candidate transition depths. Since the orebodies vary in size across the candidate transition depths, it is logical to allocate more years of open pit production for transition depths with a larger OP orebody and vice versa. In addition to a unique transition year, each candidate transition depth corresponds to a unique ultimate open pit limit, crown pillar location and underground orebody domain, all of which are described in the threedimensional space.



Figure 2.1 - Generating candidate transition depths



Figure 2.2 - Schematic representation of the proposed optimization approach
An optimization solution outlining a long-term schedule that maximizes NPV is produced separately for the OP and UG operations at each of the candidate transition depths considered. Once optimal extraction sequences for the open pit and underground portions have been derived for each depth, the value of transitioning at a certain depth can be determined by summing the economic value of the OP and UG components. From here, the combined NPVs at each depth can be compared to easily identify the most favorable transition decision. This process is outlined in Figure 2.2.

2.2.2 Stochastic integer programming: Mine scheduling optimization

The proposed stochastic integer program (SIP) aims to maximize discounted cash flow and minimize deviations from key production targets while producing an extraction schedule that abides by the relevant constraints. The OP optimization produces a long-term schedule that outlines a yearly extraction sequence of mining blocks, while UG optimization adopts the same two-stage stochastic programming approach for scheduling stope extraction. The formulation for both OP and UG scheduling are extremely similar, as such only OP formulation is shown. The only difference for the UG formulation is that stopes are being scheduled instead of blocks, and yearly metal is being constrained instead of yearly waste as seen in the OP formulation.

2.2.3 Developing Risk-Based Life-of-Mine Plans: Open Pit Optimization Formulation

The objective function for the OP SIP model shown in equation (1) maximizes discounted cash flows and minimizes deviations from targets, and is similar to what is presented by Ramazan and Dimitrakopoulos (2013). Part 1 of the objective function contains first-stage decision variables, b_i^t , which govern what year a given block *i* is extracted within. These are scenario independent decision variables and the metal content of each block is uncertain at the time this decision is made. The terms in Part 1 of equation (1) represent the profits generated as a result of extracting certain blocks in a year and these profits are appropriately discounted based on which period they are realized in.

Part 2 of equation (1) contains second-stage decision variables that are used to manage the uncertainty in the ore supply during the optimization. These recourse variables (*d*) are decision variables determined once the geological uncertainty associated with each scenario has been unveiled. At this time, the gap above or below the mine's annual ore and waste targets is known on a scenario-dependent basis and these deviations are discouraged throughout the life-of-mine. This component of the objective function is important because it is reasonable to suggest that if a schedule markedly deviates from yearly ore and waste targets, then it is unlikely that the projected NPV of the schedule will be realized throughout a mine's life. Therefore, including these variables in the objective function and reducing deviations allows the SIP to produce a practical and feasible schedule along with cash flow projections that have a high probability of being achieved once production commences.

The following notation is used to formulate the first-stage of the OP SIP objective function:

i is the block identifier;

t is a scheduling time period;

 $b_i^t = \begin{cases} 1 & \text{Block } i \text{ is mined through OP in period } t; \\ 0 & \text{Otherwise} \end{cases}$

Rec is the mining and processing recovery of the operation;

T_i is the weight of block *i*;

 $NR_i = T_i \times g_i^s \times Rec \times (Price - Selling Cost)$ is the net revenue generated by selling all the metal contained in block *i* in simulated orebody *s*;

MC_i is the cost of mining block *i*;

PC_i is the processing cost of block *i*;

$$E\{V_i\} = \begin{cases} NR_i - MC_i - PC_i & \text{if } NR_i > PC_i \\ -MC_i & \text{if } NR_i \le PC_i \end{cases}$$
 is the economic value of a block *i*;

r is the discount rate;

 $E\{(NPV_i^t)\} = \frac{E\{V_i^0\}}{(1+r)^t}$ is the expected NPV if the block *i* is mined in period *t*;

N is the number of selective mining units available for scheduling;

z is an identifier for the transition depth being considered;

 P_z is the number of production periods scheduled for candidate transition depth z.

The following notation is used to formulate the second-stage of the OP SIP objective function:

s is a simulated orebody model;

S is the number of simulated orebody models;

w and *o* are target parameters, or type of production targets; *w* is for the waste target; *o* if for the ore production target;

u is the maximum target (upper bound);

l is the minimum target (lower bound);

 d_{su}^{to} , d_{su}^{tw} are the excessive amounts for the target parameters produced;

 d_{sl}^{to} , d_{sl}^{tw} are the deficient amounts for the target parameters produced;

 $c_u^{to}, c_l^{to}, c_u^{tw}, c_l^{tw}$ are unit costs for $d_{su}^{to}, d_{sl}^{to}, d_{su}^{tw}, d_{sl}^{tw}$ respectively in the optimization's objective function.

OP Objective function

$$Max \underbrace{\sum_{t=1}^{P_z} \sum_{i=1}^{N} E\{(NPV_i^t)\}b_i^t}_{Part \ 1} - \underbrace{\sum_{s=1}^{S} \sum_{t=1}^{P_z} \frac{1}{S}(c_u^{to} d_{su}^{to} + c_l^{to} d_{sl}^{to} + c_u^{tw} d_{su}^{tw} + c_l^{tw} d_{sl}^{tw})}_{Part \ 2} \quad (1)$$

OP Constraints

The following notation is required for the constraints:

 W_{tar} is the targeted amount of waste material to be mined in a given period;

 ${\cal O}_{tar}$ is the targeted amount of ore material to be mined in a given period;

*O*_{si} is the ore tonnage of block *i* in the orebody model *s*;

 $Q_{UG,tar}$ is the yearly metal production target during underground mining;

MCap_{min} is the minimum amount of material required to be mined in a given period;

 $MCap_{max}$ is the maximum amount of material that can possibly be mined in a given period;

 l_i is the set of predecessor for block *i*.

Scenario-Dependent:

Waste constraints for each time period t

$$\sum_{i=1}^{N} W_{si} b_{i}^{t} - d_{su}^{tg} + d_{sl}^{tg} = W_{tar} \qquad s = 1, 2, \dots, S; t = 1, 2, \dots, P_{z} \quad (2)$$

Processing constraints

$$\sum_{i=1}^{N} O_{si} b_i^t - d_{su}^{to} + d_{sl}^{to} = O_{tar} \qquad s = 1, 2, \dots, S; t = 1, 2, \dots, P_z$$
(3)

Scenario-Independent:

Precedence constraints

$$b_i^t - \sum_{k=1}^t b_h^k \le 0$$
 $i = 1, 2, ..., N; t = 1, 2, ..., P_z; h \in l_i$ (4)

Mining capacity constraints

$$MCap_{min} \leq \sum_{i=1}^{N} T_i b_i^t \leq MCap_{max} \qquad t = 1, 2, \dots, P_z \quad (5)$$

Reserve constraints

$$\sum_{t=1}^{P_z} b_i^t \le 1 \qquad i = 1, 2, \dots, N \quad (6)$$

Constraints (2) and (3) are scenario-dependent constraints that quantify the magnitude of deviation within each scenario from waste and ore targets based on first-stage decision variables (b_i^t). Constraints (4) – (6) contain only first-stage decision variables (b_i^t) and thus are scenario-independent. The precedence constraint (4) ensures that the optimizer mines the blocks overlying a specific block *i* before it can be considered for extraction. The reserve constraint (6) prevents the optimizer from mining a single block *i* more than once.

The size of OP mine scheduling problems cause computational issues when using commercial solvers since it can take long periods of time to arrive at or near an optimal solution (Lamghari et al 2013). In order to overcome these issues, metaheuristics can be used. These are algorithms which efficiently search the solution space and have the proven ability to find high quality solutions in relatively small amounts of time (Ferland, Amaya and Djuimo, 2007; Lamghari and Dimitrakopoulos, 2012; Lamghari, Dimitrakopoulos and Ferland, 2014). To be effective these algorithms must be specifically tailored to match the nature of the problem being solved. In the context of mine production scheduling, the tabu search algorithm is well suited and a parallel implementation is utilized here to schedule the open pit portion of the deposit for each transition depth that is considered (Lamghari and Dimitrakopoulos, 2012; Senecal, 2015). For more details on tabu seach, the reader is referred to the appendix.

2.2.4 Developing Risk-Based Life-of-Mine Plans: Underground Optimization Formulation

The UG scheduling formulation is very similar to the OP formulation. Both have objective functions which aim to maximize discounted profits, while minimizing deviations from key production targets. The UG objective function is similar to what is proposed for the OP scheduling function in equation (1), except the binary decision variables can be represented using a_i^t which designates the period in which extraction-related activities occur for each stope *i*. As well, recourse variables in the second portion of the objective function aim to limit deviations from ore and metal targets, as opposed to ore and waste targets in the OP objective function. Since UG mining methods have a higher level of selectivity than OP mining, waste is often not mined, but rather left in situ and only valuable material is produced. Therefore it is more useful to constrain the amount of yearly metal produced in a UG optimization. Underground cost structure is viewed from a standpoint of cost per ton of material extracted. This standard figure contains expenses related to development, ventilation, drilling, blasting, extracting, backfilling and overhead. In terms of size and complexity, the UG scheduling model presented here is simpler than what is seen for the OP model. The reduced size is due to only considering long-term extraction constraints and a small number of mining units that require scheduling. This allows for the schedule to be conveniently solved using IBM ILOG CPLEX 12.6 (IBM, 2011), a commercially available software which relies on mathematical programming techniques to provide an exact solution.

UG Constraints

Scenario-Dependent:

N /

Metal constraints for each time period t

$$\sum_{j=1}^{M} g_{sj} O_{sj} a_j^t - d_{su}^{tm} + d_{sl}^{tm} = Q_{UG,tar} \qquad s = 1, 2, \dots, S; t = 1, \dots, P_z \quad (7)$$

Processing constraints

$$\sum_{j=1}^{M} O_{sj} a_i^t - d_{su}^{to} + d_{sl}^{to} = O_{tar} \qquad s = 1, 2, \dots, S; t = 1, \dots, P_z$$
(8)

Scenario-Independent

Precedence constraints

$$a_j^t - \sum_{k=1}^t a_h^k \le 0$$
 $j = 1, 2, ..., M; t = 1, ..., P_z; h \in l_j$ (9)

Mining capacity constraints

$$MCap_{min}^{UG} \le \sum_{j=1}^{M} T_j \ a_j^t \le MCap_{max}^{UG} \qquad t = P_{OP} + 1, \dots, P$$
 (10)

Equations (7) – (10) show the constraints included in the UG SIP formulation. In equation (9), the set of predecessors for each stope (l_j) are defined by considering the relevant geotechnical issues which constrain the sequencing optimization. These precedence relationships are created using Enhanced Production Scheduler (EPS) software from Datamine (Datamine Software, 2013). In the case of the application presented in this paper, the precedence relationships implemented were passed along by industry-based collaborators who operate the mine.

Once the optimization for both the OP and UG components is completed for each candidate transition depth, the optimal transition depth can then be identified as the depth *z* that leads to a maximum value of the expression below.

$$NPV_z^{OP} + NPV_z^{UG} \quad z = 1, \dots, D \quad (11)$$

2.3 Application at a Gold Deposit

In order to evaluate the benefits of the proposed method, it is applied to a gold deposit that has been altered to suit an OP-UG transition scenario. In this case study, the optimal transition depth from open pit to underground mining of a gold operation is investigated. 35 The mine's life begins with open pit mining and will transition to production through underground mining by implementing the underhand cut and fill method. Underground production is planned to commence immediately after open pit production ceases. On the mine site there is one mill processing stream with a fixed recovery curve. No stockpile is considered. A crown pillar envelope for the deposit is identified a-priori along with four crown pillar locations within this envelope leading to four distinct candidate transition depths which are evaluated. Each transition depth possesses a unique above and below ground orebody, dictated by a varying crown pillar location in the vertical plane. The year in which the transition between mining methods occurs varies throughout the candidate transition depths to accommodate for increased reserves in the OP or UG orebody as the location of the crown pillar shifts. The combined OP and UG mine life is 14 years for all candidate transition depths tested. The discrepancy in orebody size and reserves that can be accessed by OP and UG methods for each candidate transition depth along with the transition year is shown in Figure 2.3. As the size of the OP deepens and the number of OP blocks increases, the amount of UG stopes within the accessible underground resource decreases. A schematic of how the crown pillar location varies can be seen in Figure 2.4.

	Transition Depth 1	Transition Depth 2	Transition Depth 3	Transition Depth 4
Number of OP Blocks	64,255	72,585	80,915	89,245
Number of UG Stopes	418	356	340	311
Production Years through OP	7	8	9	10
Production Years through UG	7	6	5	4

Figure 2.3 - Size of potential orebody at each transition depth



Figure 2.4 - Schematic of transition depths for case study based on crown pillar location

The relevant economic and technical parameters used to generate the optimization models are shown in Figure 2.5.

Metal Price	\$900/oz
Crown Pillar Height	60ft
Economic Discount Rate	10%
Processing cost/ton	\$31.5
OP Mining cost/ton	\$1.5
UG Mining cost/ton	\$135
OP Mining Rate	18,500,000t/year
UG Mining Rate	350,000t/year
OP Mining Recovery	0.95
UG Mining Recovery	0.92

Figure 2.5 - Economic and technical parameters

2.4 Stochastic Optimization Results and Risk Analysis

The transition depth determined to be optimal for the proposed stochastic optimization framework is Transition Depth 2 (TD 2) as seen in Figure 2.6. This transition depth can be described by having a crown pillar located at an elevation of 760ft, and access to 72,585 open pit blocks and 356 stopes. The optimal transition depth in this case study provides a 5% higher NPV than the next best candidate transition depth and a 13% NPV improvement over the least optimal depth. Such a large impact on the financial outcome of a mine confirms that in-depth analysis before making this type of long-term strategic decision is beneficial.



Figure 2.6 – Risk profile on NPV of stochastic schedules

In order to evaluate to risk associated with stochastic decision making, a risk analysis is performed on the life-of-mine plans corresponding to the optimal transition depth stated above in. Similar analysis has been done extensively on open pit case studies (Dimitrakopoulos, Farrelly and Godoy, 2002; Godoy, 2003; Leite and Dimitrakopoulos, 2007; Ramazan and Dimitrakopoulos, 2013). To do so, a set of 20 geological simulations are used and passed through the long-term production schedule determined for the optimal transition depth, which is this case is Transition Depth 2. This process provides the yearly figures for mill production tonnages, metal production and cash flow projections for each

simulation if the schedule was implemented and the grades within a given simulation were realized. Figure 2.7 and Figure 2.8 show that the stochastic schedule produced for Transition Depth 2 has a high probability of meeting mill input tonnage targets on a yearly basis. The ability to meet targets translates into a high level of certainty with regards to realizing yearly cash flow projections once production commences; this is expanded upon later. Stochastic schedules perform well during risk analysis because the inherent geological variability within the deposit is captured within the simulations and then considered while making scheduling decisions in a stochastic framework.



Figure 2.7 - Risk profile of yearly cumulative cash flow of stochastic result



Figure 2.8 - Performance of stochastic schedule in meeting yearly ore targets

In Figure 2.8 there are large deviations from the target yearly ore production targets in period 7 and 8, before the Transition Depth 2 schedule shifts to underground production in period 9. This is because geological risk discounting (Ramazan and Dimitrakopoulos, 2005) is utilized as a risk management technique during OP scheduling, which penalizes deviation from production targets more heavily in the early years of production. This is valuable in the capital-intensive mining sector to increase certainty within early year project revenue and potentially decrease the length of a project's payback period. In addition to this, common long term scheduling practices within the mining industry involves updating the schedule on a yearly basis as new information about the orebody is gathered so the large deviations later in the open pit mine life are not a large cause for concern. After the transition is made to underground mining in year 9, a high penalty incurred on deviations from ore targets to ensure that ore targets are met in the early years of the underground mine. This leads to a tight risk profile throughout the underground life of mine (periods 9 to 14). Figure 2.9 shows the stochastic schedule's ability to produce metal at a steady rate throughout the entire life-of-mine.



Figure 2.9 - Risk profile on cumulative metal produced by stochastic schedule

2.4.1 Comparison to Deterministic Optimization Result

To showcase the benefit of incorporating geological uncertainty into long-term strategic decision making, the SIP result is benchmarked against a deterministic 40

optimization that uses the same formulation. The deterministic optimization process however receives an input of only a single orebody model containing estimated values for the grade of each block and stope. Yearly production scheduling decisions are made based on these definitive grade estimates, and from there yearly cash flows streams are projected. This procedure is followed for each of the four transition depths considered, as was done for the stochastic case. Geovia's Whittle mine planning software is used to schedule the openpit portion of the mine, while an MIP is used for the underground scheduling (Geovia, 2012). This underground scheduling utilizes the deterministic equivalent of the stochastic underground schedule formulation seen earlier. The projected yearly discounted cash flows can be seen in Figure 2.10 and suggest that Transition Depth 2 (TD 2) is also optimal from a deterministic perspective.



Figure 2.10 - Analysis of NPV's projected by deterministic schedules

To assess the deterministic framework's ability to manage geological uncertainty, risk analysis is performed on the deterministic schedule for the optimal transition depth 2. The 20 geological simulations mentioned earlier are passed through the deterministic schedule produced for Transition Depth 2 and yearly cash projections based on each simulation are summarized in Figure 2.11. The results are compared to identical analysis on the stochastic schedule, also for Transition Depth 2. The P50 (median) NPV of the simulations when passed through the stochastic schedule is 9% or \$42M higher than the P50 observed for the deterministic case. Further to that point, this analysis suggests that there is a 90% that the deterministic schedule's NPV falls below the NPV of the stochastic schedule.



Figure 2.11 - Risk analysis of projected deterministic NPV

In Figure 2.11, the NPV projected by risk analysis is 5% below what the optimizer originally predicted. Along with this, there is a large variation in yearly cash generated. Figure 2.11 also concludes that there is a 70% chance that once production commences, the realized NPV will be less than the original projection. Figure 2.11 shows that the P50 of the stochastic risk profiles for transition depth 2 are higher than both the deterministic projected NPV and the P50 of the deterministic risk profiles by 4% and 9% respectively. This trend of increased value for the stochastic framework extends to other transition depths as well.



Figure 2.12 - Comparison of NPV at different transition depths

Figure 2.12 shows that in addition to the stochastic schedule at the optimal transition depth (TD 2) generating a higher NPV than the optimal deterministic result, also TD 2, the next best transition depth in the stochastic case (TD 3) is \$17M or 3.4% than the optimal deterministic result. The transition depth is shown as Location 2 in Figure 2.6. The increased NPVs seen for the stochastic approach are due to the method's ability to consider sources of geological uncertainty while making scheduling decisions. Trends within the gold grades are captured within the simulations, and making scheduling decisions while have more information on the spatial continuity and variability of the grades allows the optimizer to capitalize on such trends. Overall, the stochastic scheduler is motivated to mine high grade areas with low variability early in the mine life and defer extraction of low grade and risky material to later periods.

Figure 2.13 shows the median (P50) of deviations from yearly mill tonnage targets for the stochastic and deterministic schedules with respect to the 20 simulated orebody models. Throughout the entire life of mine, the stochastic schedule limits these deviations from targets while the deterministic schedule has no control over such risk. The deterministic schedule's inability to meet yearly mill input tonnage is a cause for concern and suggests that the mine is unlikely to meet important targets once production commences if such a schedule is implemented.



Figure 2.13 – Magnitude of deviation from yearly mill input tonnage target



Figure 2.14 - Two cross sectional views of deterministic schedule produced by Whittle (right) and the schedule obtained by the proposed SIP (left) for Transition Depth 2

Figure 2.14 shows a visual comparison between the stochastic and deterministic schedules produced for Transition Depth 2. The shading in the Figure 2.14 describes which period a mining block is scheduled to be extracted in. Overall, the stochastic schedule appears to be smoother and more mineable than the deterministic schedule, meaning that large groups of near-by blocks are scheduled to be extracted within the same period. As well,

both cross sections reveal that the stochastic schedule mines more material than the deterministic schedule produced by Whittle, resulting in a larger ultimate pit for the stochastic case. These differences stem from Whittle determining the ultimate pit before scheduling by utilizing a single estimated orebody model containing smoothed grade values. In the stochastic case, the task of determining the ultimate pit contour is done while having knowledge of 20 geological simulations which provide detailed information on the high and low grade areas within the deposit. In this case the stochastic scheduler identifies profitable deep-lying high grade material that cannot be captured using traditional deterministic methods.

2.5 Discussion

A new method for determining the optimal OP-UG transition depth is presented. The proposed method improves upon previously developed techniques by jointly taking a truly three-dimensional approach to determining the optimal OP-UG transition depth, through the optimization of extraction sequences for both OP and UG components while considering geological uncertainty. The optimal transition decision is effectively described by a transition year, three-dimensional optimal open pit contour, a crown pillar location and a clearly defined underground orebody. In the examined case study, it was determined that the second of four transition depths evaluated is optimal which involves transitioning to underground mining in period 9. Making the decision to transition at the second candidate transition depth evaluated results in a 13% increase in NPV over the worst-case decision, as predicted by the stochastic framework. Upon closer inspection through risk analysis procedures, the stochastic framework is shown to provide a more realistic valuation of both the OP and UG assets. In addition to this, the stochastic framework produces operationally implementable production schedules that lead to a 9% NPV increase and reduction in risk when compared to the deterministic. It is shown that the yearly cash flow projections outlined by the deterministic optimizer for the underground mine life are unlikely to be met resulting to misleading decision criteria. Overall, the proposed stochastic framework has proven to provide a robust approach to determining an optimal open pit to underground mining transition depth. Future studies should aim to improve on this method by more effectively searching the solution space of the problem.

3. Determining the optimal open pit to underground mining transition depth at Geita gold mine using stochastic mine planning

3.1 Introduction

The Geita gold mine is a large gold mining complex located in northern Tanzania. The mine is completely owned and operated by AngloGold Ashanti. The mining complex is comprised of six operating open pits which feed a carbon-in-leach processing plant and produces roughly 500,000 ounces of gold per year. Geita is widely considered to be AngloGold Ashanti's flagship African gold mine. Current mine plans show a concerning deficit in ore production below the mill's capacity in a few years. In order to supplement this deficit and to keep the mill operating at full capacity, a transition to underground mining is being considered to provide additional ore. The developed approach which relies on stochastic mine planning techniques will be applied to evaluate the decision to go underground at the Nyangkanga pit within the Geita gold mining complex.

Motivation for considering the transition to underground mining includes gaining access to a large supply of reserves and subsequently prolonging a mine's life. As well, existing production facilities can be utilized during underground production. The task of determining an optimal transition depth from open pit to underground mining incorporates several aspects of mine planning which are combined to address the decision. Extensive work has been done on long-term mine planning for open pit and underground mines separately (Dimitrakopoulos, 2011; Newman, 2010) but to date work on the transition problem has been limited. Recent efforts have been aimed to incorporate the decision to transition from open pit to underground into an optimization framework. A work by Dagdelen and Traore (2014) attempts to determine the optimal transition depth for an open pit mine currently operating in the context of a mining complex which contains several other mines and processors. The authors test the financial viability of a number of transition scenarios by iteratively increasing the size of the open pit resource until they have identified a maximum value. Carli and Peroni (2015) present a similar approach to Dagdelen and Traore (2014), by considering five separate candidate transition depths based on pit shells produced using price factors that range from 60%-100%. The authors report a 9% increase in NPV for making the optimal transition when compared to mining through solely open pit.

Newman, Caro and Rubio (2013) decompose the problem using a network flow formulation, where each possible extraction sequence is represented as an arc in the flow. The method assumes top-down bench-wise mining above ground and bottom-up below for a block caving operation which is limits the optimality of the schedule (Whittle, 1988). The work by Newman et al. (2013) is limited as the proposed method provides a solution which is restricted to 2-Dimensions and is not operationally implementable.

Each of the mentioned works fail to consider geological uncertainty throughout the planning process which has been extensively demonstrated to have a large impact on a mine's profits (Godoy, 2003; Leite, 2007; Ramazan and Dimitrakopoulos, 2013). Geological uncertainty can be effectively integrated into the planning process through the use of several conditionally simulated orebody models. A set of orebody simulations encapsulates the inherent variability within the grades of deposit. Developed geostatistical algorithms SNESIM (Strebelle, 2002) and DBSIM (Dimitrakopoulos and Luo, 2004) can be used to efficiently simulate lithologies (categorical attributes) and mineral content (continuous attributes).

The method developed begins with identifying number of viable candidate transition depths where the switch can be made from open pit to underground mining. Stochastic production schedules dictating yearly extraction for both the open pit and underground portions of a deposit are then produced for each candidate transition depth. These stochastic schedules provide an accurate projection of yearly cash flows under uncertainty, by utilizing a resource model which is comprised of several equally-probable geological simulations. The summation of the underground and open pit mine's NPV results in the NPV of the act of transitioning at a given candidate transition depth. The most profitable of all depths considered is deemed optimal. This approach improves upon previous works by jointly describing the optimal transition depth in 3-dimensions, predicting cash flows with optimized schedules, while incorporate geological uncertainty into the decision-making framework.

3.2 Method

3.2.1 Quantifying Geological Uncertainty

Before the scheduling process commences, orebody simulations are produced to quantify geological uncertainty. These simulations encapsulate the inherent variability within the orebody which allows for an accurate valuation along with a quantitative description of the level of technical risk associated with a certain deposit. Simulations can be created for continuous or categorical attributes. Three separate types of simulations are produced for this case study: lithology simulations, open pit gold grade simulations, and underground stope gold grade simulations. The lithologies are represented as a categorical attributes during the simulation process, while grades are continuous.

Lithologies are simulated first using an implementation of SNESIM (Strebelle, 2002) within SGeMS (Remy, Boucher and Wu, 2005). Three different lithologies were simulated: Banded Iron Formation, Diorites and Feldspars. Figure 3.1 shows a strong visual reproduction of the drill hole data in a simulations. Further validation of the simulations through comparing indicator variograms from these simulations to that of the original drill hole data was also completed.



Figure 3.1 - Visual reproduction of lithology simulations

Once these lithological boundaries have been simulated, gold grade can be separately simulated within each lithological domain. In Figure 3.2, the different simulated portions of the orebody for each lithology can be seen. These gold grade simulations were produced

using direct-block simulation (Boucher and Dimitrakopoulos, 2009). Variograms produced from the simulations showed a strong reproduction of the original drill hole data.



Figure 3.2 - Simulating gold grade within separate lithologies

Stope simulations are created by filtering gold grade simulations on a point support into the volume of each stope using Datamine Studio 3 (Datamine Software, 2013). Once points have been grouped together for each stope and across each simulation, the average is taken. This provides a series of simulated values for each stope.

3.2.2 Candidate Transition Depths

To decompose this large problem, the solution space is broken down into a finite number of scenarios where the mine will be able to make the transition. It is important to describe these transition situations in 3-dimension, so they can be easily understood and implemented at the mine site. Therefore, transition depths are described through three components: the ultimate pit contour, extent of underground orebody, and crown pillar location.



Figure 3.3 - Layout of a candidate transition depth

3.2.3 Stochastic Mine Planning

In order to accurately assess the economic value associated with transitioning at each of the identified transition depths, optimized schedules are produced under geological uncertainty which accurately project yearly cash flows. In the proposed method production scheduling uses an optimization framework that relies on Stochastic Integer Programming (SIP) to maximize value while minimizing the risk of failing to meet productions targets on a yearly basis. The SIP optimizer is unique in its ability to accommodate several orebody simulations, as opposed to a single estimated orebody model required for deterministic optimization. In order to gauge the benefits of including uncertainty, a similar deterministic optimization can be executed in parallel so the results can be benchmarked against the stochastic framework's outputs.

The proposed formulation utilized for scheduling the OP and UG are quite similar. Both require an input of 20 geological simulations, and share common objectives of maximizing discounted value while minimizing deviations from targets as seen in equations (1) and (2). The objective function for the OP scheduling formulation can be seen in equation (1). Part (1) of equation (1) represents the discounted profit ($E(NPV_i^t)$) generated by mining an OP block *i* in period *t*, an action governed by the binary decision variable b_i^t . The second part of equation (1) is included to minimize deviations from ore and waste tonnage

production targets on a yearly basis. Here, the d variables relate to the magnitude of deviation within each scenario from yearly ore and waste targets. The magnitude of such deviations is based upon the scheduling variables in part (1) - b_i^t . Within each simulation extracting a block will correspond to a different amount of ore, metal and waste since the grade values vary between simulations. Minimizing these unique deviations above (d_{su}) and below (d_{sl}) targets across all simulations aims to mitigate geological risk throughout the scheduling process. The parameter *c* in part (2) of equation (1) represents the unit cost of deviation. Altering this *c* parameter value shifts the focus of the optimizer between maximizing NPV (part 1) and minimizing deviations from yearly production targets (part 2). The objective function for the UG scheduling optimization seen in equation (2) is similar to what was seen in the OP objective function, except stopes are scheduled for extraction through the binary decision variable a_i^t . The OP and UG scheduling formulations contain constraints that govern the logistics of the mining process. Such expressions constrain the optimizer to mine every unit, meet yearly targets, and acknowledge precedence relationships between mining units. Further details of the two-stage SIP formulation implemented for open pit scheduling can be found in Ramazan and Dimitrakopoulos (2013).



The cardinal difference between the OP and UG optimization processes lies in the solution method, since their size varies greatly. Despite recent work that has suggested UG optimization is more cumbersome due to complex constraints (O'Sullivan, Brickley and Newman, 2015), in the case study evaluated within this paper the OP problem is much larger as only long-term scheduling constraints are considered for the UG portion. To overcome the complexity of the OP scheduling problem, metaheuristics can be used (Lamghari and Dimitrakopoulos 2012). Here, a parallel implementation of tabu search (Lamghari and Dimitrakopoulos, 2012; Senecal, 2015) is utilized which has been demonstrated to provide a high quality solution in a reasonable amount of time. Since the UG scheduling optimization is smaller, a commercially available tool, IBM ILOG CPLEX (IBM, 2011), is conveniently used which relies on mathematical programming techniques to find the optimal solution.

3.3 Case Study at Geita Gold Mine

3.3.1 Introduction

The proposed methodology for determining an open pit to underground mining transition depth is tested at Geita gold mine, a currently operating mining complex in northern Tanzania. It has been discovered that in the next few years the ore production from the currently operating pit will not be able to meet the yearly mill tonnage target. In order to make up for this deficit, those operating the mine within AngloGold Ashanti are considering a transition to underground mining to provide supplemental ore production.

3.3.2 Site Specifics

Within the area of the deposit identified for underground potential, there are four zones. Zones 1 and 2 have preliminary stopes designs completed and are being targeted for immediate production, while zones 3 and 4 are areas for future production. The current thinking at the mine site is to use the cash flow generated through mining zones 1 and 2 in order to fund further delineation drilling in zones 3 and 4. Since stopes have not yet been designed for zones 3 and 4, they are not directly incorporated into the following financial analysis, but the upside potential of this area is kept in mind when making a final recommendation.



Figure 3.4 - Mine layout

To construct a set of candidate transition depths, three provided pushback designs are utilized that serve as potential ultimate pit contours. These pushback designs, labelled Cut 7, Cut 8 and Cut 9, along with each of their own unique corresponding underground orebodies will be considered for transitioning. These candidate transition depths will be referred to as Cut 7, Cut 8 and Cut 9. Open pit production is planned to continue until the extent of a given pushback design is reached, while underground production is scheduled to commence in production year 4 across all candidate depths. Figure 3.5 below shows the three candidate transition depths tested.



Figure 3.5 - Candidate transition depths to be evaluated

The number of stopes that are able to be mined underground varies as the size of the ultimate pit changes. The details of the dimensions of the open pit and underground orebody for each candidate transition depth can be seen in Figure 3.6. Since only a preliminary scoping study has been completed on the underground mine, these design stopes do not contain a significant tonnage and therefore lead to a short underground mine life.

	Ope	en-Pit	Underground	
	Mine Life (years)	Number of Blocks	Mine Life (years)	Number of Stopes
Cut 7	5	13,000	3	92
Cut 8	7	20,000	1	37
Cut 9	9	25,000	1	42

Figure 3	.6 - De	scription	of candida	ate transition	depths

3.3.3 Results and Analysis

Figure 3.7 shows the risk profiles of the cash flow resulting from the decision to transition at Cut 7, 8 or 9. It is apparent that transitioning at Cut 9 is optimal, the transition depth which corresponds to the largest open pit and longest combined mine life.



Figure 3.7 –Risk profiles on cumulative cash flows of stochastic framework

In order to evaluate to risk associated with stochastic decision making, a risk analysis is performed on the life-of-mine plans corresponding to the optimal transition depth stated above. Similar analysis has been done extensively on open pit case studies (Godoy, 2003; Leite and Dimitrakopoulos, 2007; Ramazan and Dimitrakopoulos, 2013). To do so, a set of 20 geological simulations are used and passed through the long-term production schedule determined for the optimal transition depth, which is this case is Cut 9. This process provides the yearly figures for mill production tonnages and cash flow projections for each simulation if the schedule was implemented and the grades within a given simulation were realized throughout mining. Figure 3.8 shows that the stochastic schedule produced for transitioning at Cut 9 has a high probability of meeting mill input tonnage targets on a yearly basis. This ability to meet ore targets solves the issue of a deficiency in future ore production which was the original motivation for considering to transition to underground mining. As well, this result translates into a high level of certainty with regards to realizing yearly cash flow projections once production commences.



Figure 3.8 - Yearly mill tonnage input for stochastic schedule at Cut 9

To benchmark the benefits of stochastic decision-making, a deterministic framework using a similar optimization formulation is applied to the mentioned case study. The difference between stochastic and deterministic frameworks lies within the orebody model input that the optimizer recieves. In stochastic optimization, the optimizer recieves a set of twenty equally probably simulations which quantifies the uncertainty associated with a deposit. For the deterministic framework, a single estimated orebody is used which contains smoothed grades and minimal information about uncertainty. Currently, the vast majority of the mining industry relies on deterministic frameworks to make their long-term strategic decisions.



Figure 3.9 - Cumulative cash flows of deterministic framework for each candidate transition depth

Figure 3.9 shows that transitioning at Cut 9 is also optimal for the deterministic framework. As was done for the stochastic case, risk analysis can be performed on the production schedule created by the deterministic optimizer for Cut 9. In doing this analysis, there are two different cumulative cash figures, the projected cash which is the amount of cash the deterministic scheduler expects to produce based on the grades within the single orebody used for scheduling. The second and more accurate figure is the cash risk profile, where the value that would be seen if a given simulation was realized and the proposed schedule was implemented is shown. It is useful to summarize the resulting values for each simulation using P10, P50, and P90 curves. Figure 3.10 shows the results of cash flow risk analysis on the deterministic schedule produced for transitioning at Cut 9. Here a projected NPV of \$831M is seen, while P50 of the risk profile, or the expected value based on risk analysis, of \$765M. This implies that the expected value of the deposit when considering geological uncertainty is 8% and \$66M less than what was originally projected by the optimizer based on a single estimated orebody used for scheduling. Further to that point, 17 of the 20 simulations provide an NPV through risk profiles that is below what the originally

projected NPV, which leads us to conclude that there is a 85% chance that this projected cash value will not be realized once production commences.



Figure 3.10 - Risk profiles of cumulative cash flow for deterministic schedules at Cut 9

In addition to this uncertain financial valuation, the deterministic schedule produced for transitioning at Cut 9 fails to meet a key project indicator, the annual mill tonnage. Figure 3.11 shows that on a yearly basis, the deterministic schedule struggles to meet the yearly mill production target. Upon closer inspection, a drastic decrease in input tonnage in production year 6 and 7 can be seen, and as was mentioned earlier this deficit is the original motivation for considering the transition from open pit to underground mining. Therefore, within the deterministic framework, making a transition to UG mining at Cut 9 does not solve the important ore deficiency issue. Conversely, Figure 3.8 shows the stochastic schedule has the ability to meet the ore production target throughout the entire life of mine.



Figure 3.11 - Mill tonnage forecast for Cut 9 deterministic schedule

Along with the ability to meet annual ore targets, the stochastic schedule also significantly increases the value of the mine. When considering the schedules produced for making the transition at Cut 9, there is a 23% or \$145M increase in the stochastic NPV as predicted by risk profiles when compared to the NPV risk profiles of the deterministic schedule. This comparison is seen in Figure 3.12.



Figure 3.12 - NPV comparison for stochastic and deterministic results at Cut 9

The above risk-based evaluation of the candidate transition depths has led us to conclude that transitioning at Cut 9 is optimal. Although, the underground mine below Cut 9 has an NPV of \$1.1M over its one year mine life, while the open pit has an NPV of \$908M. As well, the magnitude of variation between P90 and P10 for the UG mine is \$11.9M. This low NPV and high variation of cash are a cause for concern in the capital intensive process of underground mining and suggest that there is a very high level of technical risk associated with the current underground mine design.

3.4 Discussion

Based on the previous analysis, the decision of making the transition from open pit to underground mining at Cut 7 and Cut 8 is ruled out. Therefore the recommendation is to continue mining through open pit until Cut 9 since cash is being consistently generated late in the mine life. As well, there is a stable mill feed during risk analysis on the stochastic schedule which resolves a key issue currently facing the mine's operators. Since the underground mine beneath Cut 9 has low profits and high technical risk, a further investigation towards improving the financial benefits are required before a decision to begin underground mining is recommended. One area for future investigation is zones 3 and 4 which were originally deemed to be of interest for future production.

In this paper, a risk based approach is applied to a currently operating mine facing a strategic decision with a large financial impact. The decision to forgo underground mining and continue producing through open pit was reached after in-depth analysis. As well, the benefit of stochastic mine planning over conventional deterministic methods has once again been shown with an increase in NPV of 23%. In addition to this, the schedule produced by the stochastic optimizer is able to meet the mill requirement throughout the life-of-mine thus providing a low-risk alternative to transitioning to underground mining.

4. Conclusions

The presented and tested method developed within this thesis improves upon previous works by jointly: considering geological uncertainty, describing the optimal transition depth in three dimensions, thoroughly searching the solution space through a simultaneous optimization formulation, and producing an optimized schedule which incorporates time value of money to give an accurate valuation of the mining complex.

Improvements can be made on the existing OP UG transition problem methods by simultaneously optimizing production scheduling decisions for both the open pit and underground portions of the deposit. It has been extensively demonstrated that such a simultaneous approach leads to a globally optimal solution, as opposed to running several disjoint optimizations which can be trapped in a series of local optima (Whittle, 2010; Montiel, 2014; Goodfellow, 2014). Once an orebody has been simulated and discretized into mining blocks for open pit mining and stopes for underground production, the authors propose an approach to maximize value achieved throughout the act of transition between mining methods. The proposed approach relies upon stochastic mine planning methods that have been effectively demonstrated on the mine production scheduling problem to increase value and reduce risk. A two-stage stochastic integer programming formulation that aims to maximize discounted cash flow while limiting the risk of failing to meet key project targets on a yearly basis, as seen by Ramazan and Dimitrakopoulos (2013) can be expanded to schedule mining blocks and stopes on a yearly basis, with a few added constraints to govern the logistics of transitioning between mining methods. The resulting extent of mining blocks and stopes scheduled will outline the ultimate pit contour, crown pillar location, and the extent of the underground orebody, which are three metrics that have been demonstrated throughout this thesis to effectively describe an open pit to underground transition depth on an operational level. In addition to the constraints included in the Ramazan and Dimitrakopoulos (2013), a crown pillar constraint is necessary to prevent closely neighbouring stopes and blocks from being mined out by separate methods. This will result in a large portion of undisturbed host material between the final open pit and the underground openings, known as the crown pillar, which is important for stability reasons.

Appendix

In the presented work, a parallel implementation of Tabu Search is used to solve the large open pit mine scheduling problem in a reasonable amount of time. This metaheuristic method takes advantage of the multi-core processing architecture in modern computers to effectively distribute tasks and find high quality solutions. Essentially, the algorithm perturbs an initial feasible production schedule by changing the yearly scheduling decision for a given block, then impact of these perturbations is evaluated and they are accepted based on their ability to increase the value of the solution. As the algorithm accepts perturbation and progresses through the solution space, it prohibits itself from repeatedly visiting the same solution by labeling these previously visited solutions as tabu (forbidden) for a certain amount of time. The Tabu Search procedure stops after a specified number of proposed perturbations have been evaluated which fail to improve the solution. In order to prevent the algorithm for getting trapped in a locally (as opposed to globally) optimal solution, a diversification strategy is included in the metaheuristic to generate new, unique starting solutions to that can then be improved.

The specific implementation used in the work presented here is known as Parallel Independent Tabu Search (Senecal, 2015) where the Master-Slave (Hansen, 1993) parallel algorithm design is used. In this scheme, a master thread delegates the task of performing Tabu Search to each available thread and provides them with a unique starting solution. These threads then operate independently to identify the best solution possible using Tabu Search. These solutions for each are then compared to identify the optimal solution. With this efficient implementation of Tabu Search, more instances of the algorithm can be run simultaneously to thoroughly cover the solution space in less time than a purely sequential and single threaded approach. More algorithmic details can be found in the work by Lamghari and Dimitrakopoulos (2012).

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