"A PHOTOELASTIC INVESTIGATION OF LIGHT-GAUGE ALUMINUM COMPRESSION MEMBERS IN THE POST-BUCKLING RANGE"

by

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"A PHOTOELASTIC INVESTIGATION OF LIGHT-GAUGE ALUMINUM COMPRESSION MEMBERS IN THE POST-BUCKLING RANGE"

UNE ETUDE SUR LE COMPORTEMENT DES COLONNES A AMES MINCES EN ALUMINIUM APRES VOILEMENT

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et de Mécanique Appliquée

M. Eng.
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RESUME

Le comportement des colonnes à âmes minces est étudié expérimentalement et théoriquement. La propagation des contraintes et des déplacements est obtenue partout dans la structure par les deux méthodes, et une bonne corrélation entre les deux méthodes est réalisée.

Les expériences ont été faites sur les profiles en "U" renforcés couverts avec du plastique "photoelastic". Une attention particulière est accordée à la manière d'avortement et la propagation des contraintes et des déplacements. Les contraintes et les déplacements ont été étudies aux points particuliers.

L'étude théorique est faite au moyen d'un programme d'ordinateur basé sur la méthode des éléments finis dans lequel les nonlinéarités géométriques matérieles sont introduites.

Il est démontré que le programme est valable pour l'analyse des structures en éléments minces, et peut être utilisé pour la prédiction des déplacements et des contraintes dont l'une des parois est en état d'instabilité, ou quand elle atteint sa charge critique.

A STUDY OF THE BEHAVIOUR OF LIGHT-GAUGE ALUMINUM COMPRESSION MEMBERS INTO THE POST-BUCKLING RANGE

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ABSTRACT

The behaviour of thin-walled structures loaded as columns was studied both experimentally and theoretically. Propagation of stresses and strains were obtained throughout the structure by both methods, and good correlation between the two methods of analysis was realized.

The experiments were conducted on stiffened channel sections coated with photoelastic plastic. Special attention was given to the mode of failure, and propagation of stresses and strains. Individual values of stress and strain were studied at particular points on the structure.

A Computer program based on the finite element method which incorporates both material and geometric non-linearities was used for the theoretical analysis.

It is shown that the computer program is valid for the analysis of thin-walled structures. It is also shown that the propagation of stresses and strains as well as individual values can be predicted accurately when individual elements become unstable or reach their buckling loads.

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Last, but definitely not least, to my mother who was kind enough to type this thesis.

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k	Sensitivity constant of Plastic
c	Velocity of light in a vacuum
N	Integer
λ	Average wavelength of light
ϵ_1, ϵ_2	Principal strains
t	Thickness of Plastic
u, v, w	Components of displacement in x, y, z directions
x, y, z	Rectangular cartesian coordinates of a point with respect to the local axes

CHAPTER 1

INTRODUCTION

A great deal of work has been done recently in the area of light gauge structural members. In members of this type, the question of both local and overall buckling is of primary importance. For the most part, existing building codes do not take into consideration the very significant reserve strength that these members may possess at the initial buckling load due to their behaviour in the post-buckling region. This is probably due to the lack of any widely accepted, relatively inexpensive analytical or empirical formulae. Nevertheless, some research has been carried out to investigate this phenomenon and a few empirical solutions have been developed. However, most are based upon a relatively small number of experimental tests and have therefore not enjoyed widespread acceptance. In addition, most existing solutions were developed and verified by means of tests that were conducted on specimens of structural steel. The materials of primary interest in this study are aluminum alloys, and the amount of previous work in this field is relatively meager. Thus, when initially considering this problem, it was necessary to decide exactly what course the study should take.

The study of any scientific phenomenon can be carried out in one of two ways: either the actual behaviour can be explained by means of a theory developed from a mathematical model or the behaviour can be postulated after having done some type of experimental study on the behaviour of similar specimens under comparable conditions. These two types of approach can be roughly catagorized as analytical and experimental studies. Either approach, if it incorporates the inherent characteristics of the problem, should give reasonable results. Each also has its own advantages and disadvantages. The analytical approach, while definitely more formal and often a great deal quicker and more practical, does not take each and every possible variable into consideration, usually because many of them are unknown and unexpected. Most analytical solutions only incorporate those variables which are expected to affect the results to some appreciable extent. Often, analytical solutions to certain types of problems are presented and accepted, only to be replaced by some different theory some years later. It is the very nature of many analytical developments that no exact solution is known and therefore some type of approximation must necessarily be employed. Likewise, experimental studies have their own inherent errors and can thus approach the correct result only if carefully planned and executed.

Thus it is that neither analytical nor experimental investigations alone can be trusted to provide the exact solution to any given problem. Both approaches must be utilized in order to verify that an acceptable solution to the problem has been achieved.

A.Q. Khan (31) at McGill University developed a geometrically non-linear finite element computer model which was intended to provide solutions for various loading conditions on thin-walled structural members. The development of the program and the method of solution made the analysis of folded plate structures a relatively straightforward process.

Y. Fabien (24) utilized the aforementionned program to obtain results for comparison with experimental tests which he had designed and executed. He tested thin-walled aluminum sections as beams and obtained results consisting mostly of data on strains and displacements. These results were obtained at particular points on the test specimens in question. H.P. Lee (32) has more recently completed the development of a geometrically non-linear finite element computer model which also incorporates the ability to analyze structures with non-linear material properties. It was decided, therefore, that in order to use Lee's program to the fullest, and to provide a basis for verifying future analytical models which may be developed, an extensive set of fairly accurate experimental data on a particular type of test specimen was needed. Since Fabien had provided relatively extensive strain and displacement data for thin-walled sections tested as beams, it was felt that data from a different type of test under other loading conditions would be desirable.

An important question that had to be resolved from the very start was exactly what type of test was to be employed. In almost all tests on thin-walled aluminum structures that have been reported in the literature, data has been accumulated using either electric strain gauges or mechanical extensometers. Although both of these devices can provide accurate results, they have one obvious shortcoming: they can only provide results at one particular point on the structure. This can be a drawback because normally using such a scheme, one would tend to be unaware of any unexpected areas of high stress which might develop. Another consideration in the choice of test method was that the experimental test data was to be compared with a finite element solution. Since one of the inherent advantages of the finite element method is that it provides results at a great number of points throughout the structure, it was desirable that the test method to be employed should also provide results at a great many points. After careful consideration, it was decided that the photoelastic method using a photoelastic coating would probably be the most appropriate since it would provide a great deal of information at any particular stage of loading. By means of the coloured fringe patterns which appear, the principle stress and strain differences at any point on the test specimen can easily be determined and, by recording the propagation of these fringes, the development of stress throughout the loading sequence can be followed. The distribution of stresses around any buckled regions can be recorded and studied. Also, because the entire pattern of stress distribution throughout the structure is available, the possibility of unexpected areas of stress concentration can be investigated and, perhaps, foreseen in subsequent investigations.

Another important consideration was the choice of coating. Normally, the type of test and the kind of material govern this decision.

Finally, the thickness of plastic coating is very important since the accuracy of the measurements depends to a very great extent on the thickness, especially if relatively high stresses and strains are expected. Here, the decision must be taken whether to use a thick plastic so that a large number of fringes can be observed or to use a thin plastic so that the reinforcing effect of the plastic on the test specimen is kept to a minimum. It was decided to use a relatively thick coating since fairly high stresses were expected. The manner in which the reinforcing effects were handled is dealt with in the following chapter.

There are naturally many areas that must be investigated and many questions concerning the type of material, thickness, general shape, loads to be expected and various other contributing factors that must be considered before a testing program can be successfully developed. The various factors which led to the choice of test specimens in this study will be discussed further in the following chapter. The aim of this study was to provide a fairly comprehensive set of data for a particular test specimen under a particular loading condition. The results, which consist of data obtained from both the finite element analysis and the experimental tests are compared and contrasted in an attempt to determine a range of acceptable values for this type of test.

This could form a sound basis for verifying any new developments in this field. Hopefully, new and more extensive methods of analysis will verify the results obtained herein. It should be emphasized, however, that the results obtained contain a certain margin of error. It has already been stated, in fact, that both of the methods utilized are subject to inherent shortcomings and disadvantages. However, it is the author's intention to establish that, where the results from the two methods correspond fairly well, the data is reliable and therefore could be used as a basis for the development of more refined methods of analysis.

CHAPTER 2

EXPERIMENTAL ANALYSIS

2.1 Test Specimens

The experimental investigation provided a rather extensive set of accurate data giving stresses and strains in a structural member loaded into the post-buckling range. The sections were formed from light gauge aluminum alloy sheets and the major considerations which governed the choice of section geometry were:

1) ease of fabrication, as all the specimens were to be fabricated in the laboratory by the author;

2) a geometry which would not only ensure that the desired buckling phenomenon would occur but that it would also be readily visible. It was intended that the specimens all be cut from a single sheet of aluminum and then bent into the desired shapes. It was therefore necessary that the section be made up of a series of continuous bends. Consequently sections such as wide flange, tee, or I sections were eliminated as possibilities. Cruciform sections and closed sections were likewise excluded. Sections which satisfied the fabrication requirements were channels, angles, and Z sections. It was felt that the increased torsional rigidity of the channel section would be an asset, and hence, it was decided that a stiffened channel section would most easily provide all the necessary requirements. A suitable thickness had to be chosen

that would facilitate the fabrication of the test specimens by means of a conventional handoperated bending machine. The dimensions of the section were chosen such that the plate section forming the webs of the channels would buckle locally at a load level well below the critical buckling load of any of the other plate elements and also well before the entire member became unstable. In choosing these dimensions, it was necessary to limit the upper bound to that which could be comfortably accomodated in the bending machine. Results and recommendations presented in a technical paper (19) describing research conducted at the University of Waterloo were used to establish an optimum flange to web width ratio. According to this research, a ratio of 0.375 or less should ensure both maximum buckling and ultimate loads. Another restraint which entered into the consideration was the total area of the specimen. Since the ultimate intention was to coat the specimens with photoelastic plastic, the area to be covered was of primary importance. The maximum size of plastic sheet that can be readily fabricated by an inexperienced worker with conventional equipment is 10 in. x 10 in. (See Photo. 2.1) Thus it was realized that it would be advantageous to keep the developed width of the cross-section below 10 in. This restriction was adhered to primarily for ease of fabrication. In actual fact, two pieces of plastic can be butted together to form a satisfactory joint. However, this operation presents some additional problems, not the least of which would be calibrating plastics of two different sensitivity factors. Actually, over the length of the column, adjacent sheets of plastic were butted together. It was decided to minimize the

Casting Plate
Photograph 2-1

number of joints in the longitudinal direction and eliminate joints in the transverse direction entirely. It was felt that since different pieces of plastic would have different thicknesses, sensitivities, and fringe values, the number of different sheets of plastic per specimens should be kept to a minimum. A thickness and cross-sectional profile were established to ensure that all of the aforementionned conditions were met. The length of the specimens was also given careful consideration. The specimens had to be long enough so that any stress concentrations due to end effects would have diminished in the areas where the buckling phenomenon was expected to take place. At the same time, however, they had to be short enough so that the question of overall instability would not enter into the problem. All of the above considerations led to the choice of cross-section as shown in Figure 4-1. The aluminum alloy from which the columns were fabricated was 5052-H34. Typical properties of this material are listed in Table 2-1. The dimensions of the test section were investigated using traditional methods to ensure that the web would buckle locally before any other instability or failure mode was exhibited. The ends of the specimens were cut very carefully and then ground flat so as to assure that they were exactly perpendicular to the axes of the columns. This ensured that proper alignment of the specimens could be obtained. Any significant skewness of the end faces would cause a tilt of the column which would naturally induce a flexural moment into the column and perhaps defeat the purpose of the test by masking the desired phenomena.

	Specified Minimum Value	Typical Values
Tensile Strength	34 ksi	38 ksi
Yield Strength	27 ksi	33 ksi
Shear Strength	21 ksi	21 ksi
Fatigue Endurance Limit	18 ksi	18 ksi
Elongation	6 %	9 %

Table 2-1

Typical Material Properties

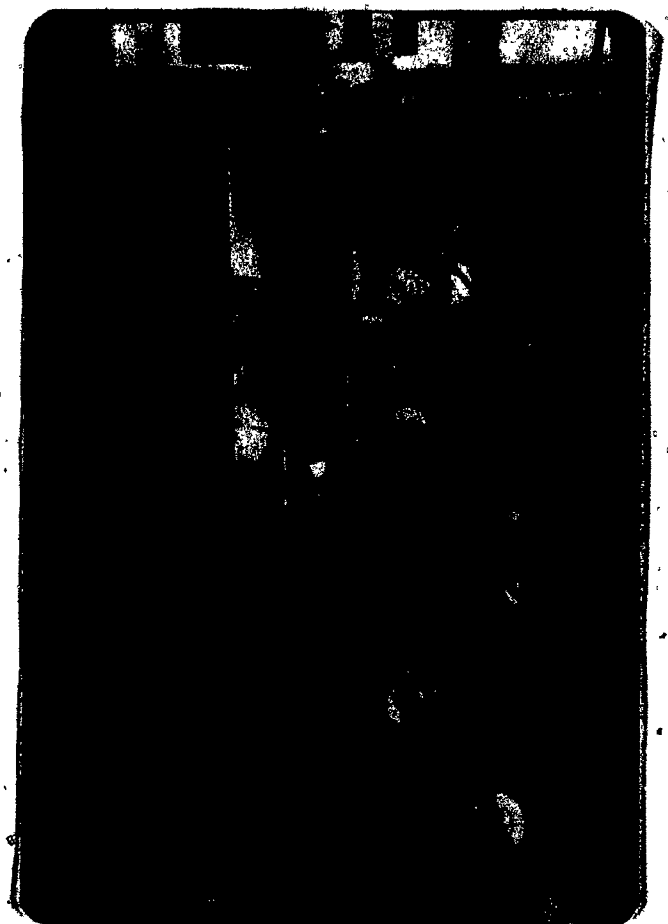
Aluminum Alloy

Once the geometry of the test specimens was decided and possible problems with the fabrication eliminated, two columns were fabricated. It was decided that these two should be tested without photoelastic coatings to establish the mode for the actual uncoated structure. In both tests dial gauges were used to measure out of plane deflections of the major plate elements of the specimens. (Photographs 2-2a, 2-2b).

In both of these tests, the general behaviour was similar and as expected. In each column, a series of buckles was visible and measurable. The usual pattern of square buckled panels was observed. (Photographs 2-3a, 2-3b). Both failed when the column became unstable after kinking at a distance of about one-third of the length from one end of the column. (Photograph 2-4).

The application of photoelastic coatings is actually an art which requires patience and, above all, practice. Initially, predetermined amounts of plastic resin and hardener are mixed at a specific temperature for a specific period of time. The amount naturally depends on the size and thickness of the sheet to be cast. The mixture is poured onto a cleaned and prepared teflon coated plate. The liquid finds its own level and, providing that the plate is level, it will naturally be of uniform thickness. The plastic compound is allowed to set for a time until it reaches a state where it is mechanically stable but highly flexible sheet. The plastic sheet is then molded onto a test specimen which has already been coated with mineral oil. The sheet is flexible enough to be molded around the edges with small radii so that the two are completely in contact over the whole area. The plastic is then left to harden. After it has completely cured, it is removed, trimmed, and thoroughly cleaned. The specimen itself is cleaned with a wire brush and a series of cleaning solvents. The contoured plastic sheet is glued to the specimen by means of a reflective cement and the specimen is ready for testing.

The process, although straightforward in theory, is often not so simple in actual practice. The time factor involved is critical. The sheet must be removed from the teflon slate at precisely the right degree of polymerization or otherwise the mechanical properties of the sheet will be altered during the contouring process or the sheet will not be flexible enough to bend easily and may shatter. Unfortunately, this is not always as easy to predict as might be expected. The rate of polymerization is sensitive to temperature, humidity, thickness of the sheet and possibly other unknown factors. Also, the entire plastic sheet cannot be used to coat the specimen. A strip about one half inch wide must be discarded since, due to the meniscus effect, it is not of uniform thickness. It is also necessary to remove a piece one inch by three inches from the sheet for calibrating the entire sheet. From this piece the sensitivity constant, k, is determined. This factor is slightly different for different thicknesses and types of plastic and is used when converting colour patterns into values of principal stress and strain differences. The calibration of the



Control Test Set-up

Photograph 2-2a

Control Test Set-up

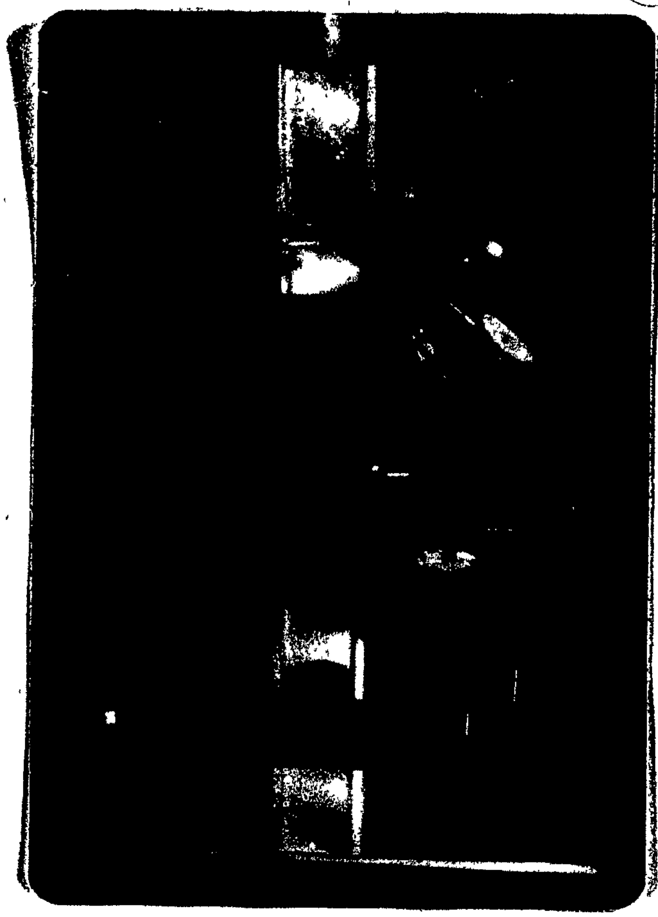
Photograph 2-2b

Buckle Patterns - Control Test

Photograph 2-3a

Buckle Patterns - Control Test

Photograph 2-3b



Failure Mode of Control Column

Photograph 2-4

plastic is accomplished by fixing the small piece of plastic to an aluminum cantilever beam. (See photographs in appendix). The number of fringes visible per inch of deflection is then estimated. This value and the plastic thickness are then used to determine the sensitivity. (Figure 2-1).

2.2 <u>Apparatus</u>

The entire series of experimental tests was conducted using an Instron tension-compression load cell, model No. D-212-260. This piece of equipment is a deflection-controlled testing machine of relatively high sensitivity. The instrument is equipped with an automatic time-load chart so that the applied load at any instant is easily visible and controlled. In this study, this feature was of prime importance since it was necessary that, at each load level, the load be held constant for a few minutes while photographs were being taken. The machine has a rated capacity of 50,000 lb., with the smallest load range from 0 to 100 lb.

The major instrument used in the experimental analysis was a highly refined refleotive polariscope, Photostress Universal Large Field Meter Model LF/MU. (Photograph 2-5). The instrument was mounted on a completely adjustable tripod and used in conjunction with the specified light source. *The large field meter consists of a polarizer, an analyzer, and two quarter wave plates. The quarter wave plates are used to generate circularly polarized light. With the instrument orientated in such a manner as to produce circularly polarized light directed onto a photostress plastic

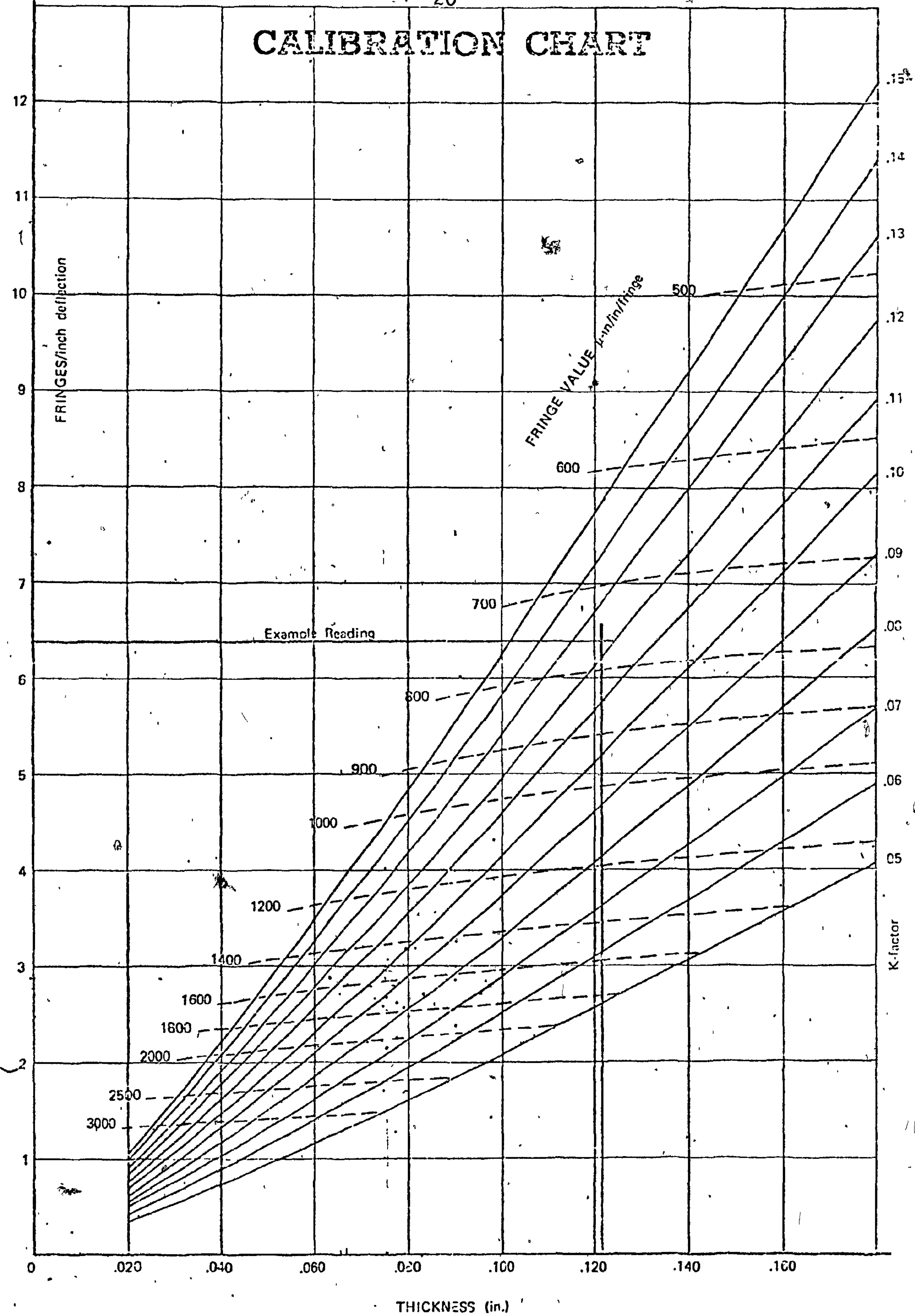
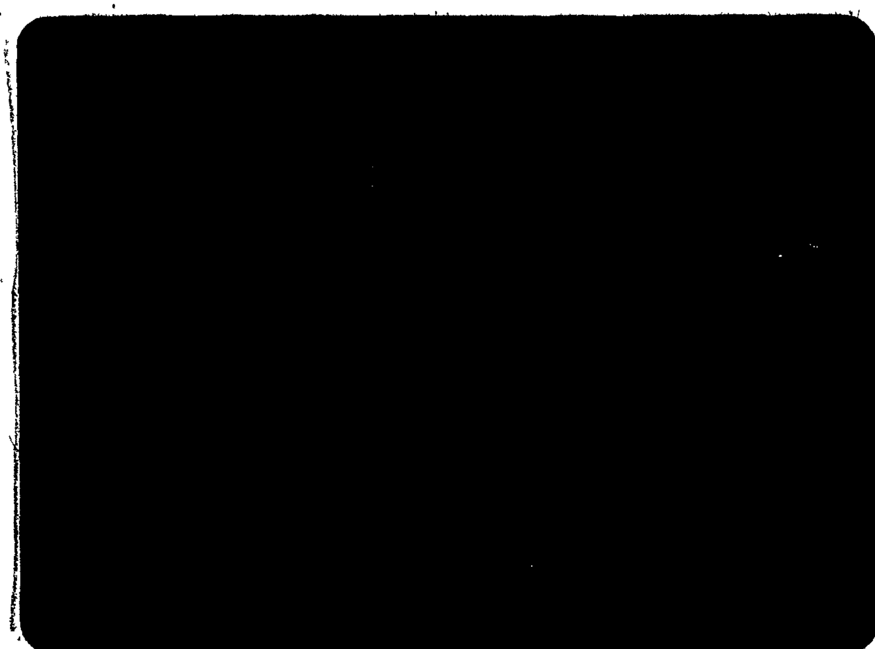

CALIBRATION CHART
FRINGES/inch deflection
FRINGE VALUE µ-in/in/fringe
Example Reading
K-factor
THICKNESS (in.)
500
600
700
800
900
1000
1200
1400
1600
1600
2000
2500
3000
12
11
10
9
8
7
6
5
4
3
2
1
0
.020
.040
.060
.080
.100
.120
.140
.160
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05

Figure 2-1



Large Field Meter - Model LF/MU

Photograph 2-5

coated specimen, bands of colour are readily visible with a circular polarizing filter. These bands of colour are called isochromatics and are lines of constant principal strain differences. By recording these isochromatics at regularly spaced load levels, it is possible to obtain a complete history of the propagation of principal strain differences throughout the entire structure.

The camera, also mounted on a tripod, which was used to record the colour patterns at the various load levels, was a Nikon, Model FN-2. The lens was a Nikkor Auto 50 mm f/1.4. This model is equipped with a light meter which allows the operator to measure the degree of light intensity directly through the lens, assuring an accurate reading and therefore greatly improving the reliability of the photographic results. This is a most important feature for this type of work since it is obviously necessary to obtain as accurate a representation as possible of the colour patterns in order to be able to properly interpret them. It is also equally important that the film used be one which gives extremely accurate colour reproduction. After much consultation with experts and some experimentation under various conditions, it was decided that Ektachrome EHB 135 would be the most suitable type. This is a high speed film with a colour sensitivity that is balanced to the tungsten light source in the Large Field Meter.

The best technique for recording the colour patterns was established only after a periodof trial and error. Many of the problems were simply effects which had not been foreseen and which had to be solved as they became apparent. The light source, for instance, is a tungsten lamp which has a tendency to create "hot spots" on the specimen unless it is placed far enough from the test specimen. Perhaps the most bothersome problem was that of reflection. Some light is reflected from the top surface of the plastic and thus makes the readings on the light meter practically useless. The problem was solved by tilting the light source in such a way that any light reflected from the top surface of the plastic was reflected away from the camera. Another problem, which caused a great deal of trouble, was that of light reflected from other light sources in the test area. This extra light has a tendency to upset the delicate balance required between the light source from the instrument and the film which is used. This problem was alleviated simply by removing all other light sources in the test area. The accuracy of the system was verified by checking the readings recorded on the film for a simple cantilever test with known and accepted theories. (See Photograph 2-7). The results were extremely accurate and thus verified the accuracy of this stage of the research. (See Table 2-2).

2.3 Material Control Tests

It was necessary to determine the properties and qualities of the individual materials used during the course of the study. It was felt that it would be not only beneficial but absolutely necessary that a test be performed to determine the stress-strain relationship of the material. This information was to be used in the computer idealization so that the non-linear properties of the material would be represented as accurately as possible. Two test coupons were cut from the same sheet of aluminum as the test specimens. A tension test was performed on each coupon and average values of stress for various levels of strain were established. These values were used to establish a theoretical stress-strain relationship, both linear and non-linear, for the material. The resulting relationship is shown in Figure 2-2.

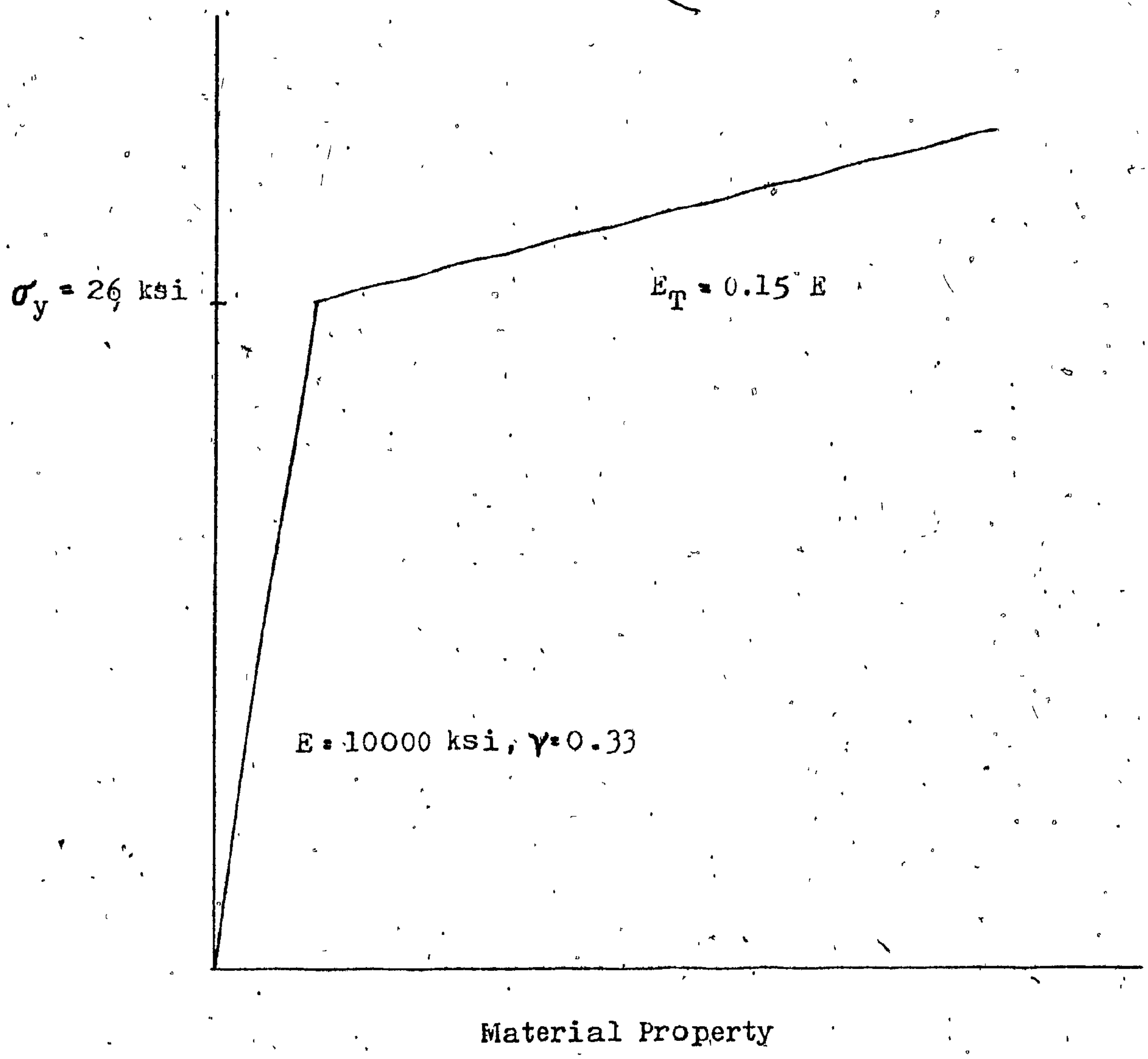
Some type of verification test was also needed to ensure that the quality and properties of the photoelastic plastic would be as expected. It was necessary, therefore, to devise some type of simple, sure test to accomplish this. A cantilever beam was chosen. A piece of aluminum cut from the same sheet as the test specimen, 12 in. long and 2 in. wide, was coated on both sides with a thickness of plastic approximately the same as that used on the actual test specimen. (Photograph 2-6). By performing the appropriate calculations, an equivalent section was established and the expected levels of stress for various load levels were determined. The plastic used in the verification test was then calibrated and a simple flexural test was performed. Readings for levels of stress at various load levels were taken. (Photograph 2-7). These were compared with the analytical solution and the results are shown in Table 2-2. It is apparent from these results that the accuracy was exceptionally good. Any errors which occurred were well within acceptable levels for experimental work. In short, the verification test proved beyond reasonable doubt that the methods of fabrication of the plastic and the system used to interpret the results were not only within acceptable bounds but actually of extremely high accuracy. This confirmed that the methods used to prepare the test specimens were entirely satisfactory.

2.4 Test Set-Up

The set-up for the experimental tests presented a number of problems. Initially, a relatively stiff plate was used to transmit the load to the column. By positioning the column so that the center of the cross-head of the machine coincided with the center of gravity of the cross-section of the column, it was felt that concentric loading would be guaranteed. Therefore relatively stiff steel plates were used at both the top and bottom ends of the column. However due to possible inaccuracies in the fabrication of the columns, unequal proportions of load might have been introduced into various parts of the cross-section. In an effort to eliminate the possibility of non-uniform load distribution the bottom of the column was mounted on a ball and socket support.

(See Page A-5)

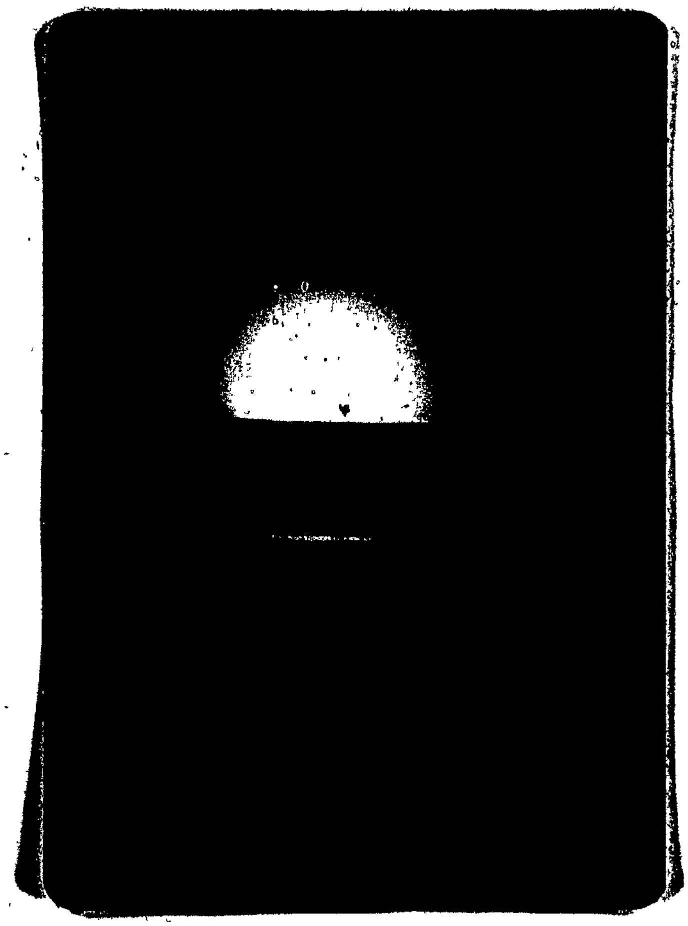
Various problems occurred during the preliminary stages of testing. For example, it was initially assumed that the best results could be achieved by butting the end of the plastic flush with the end of the aluminum. However, this proved not to be the case. The high local stresses caused cracking of the plastic and subsequent instability at the end of the column. It was then decided to apply the plastic to the main part of the column but to leave a length of one inch at each end free of plastic. It was felt that this would eliminate the cracking problem, but still



Material Stress-Strain Relationship
Figure 2-2

Cantilever Test Specimen

Photograph 2-6



Cantilever Test

Photograph 2-7

CANTILEVER TEST

Position From End (in)	Colour	Exp Strain µin/in	Corrected Exp Strain µin/in	Moment (in-lb)	Theoretical Strain µin/in	% Err
1	Light Yellow	500	480	5	475	1%
2	Tint of Passage	945	907	10	950	4%
3	Yellow	1520	1459	15	1425	2.4%
4	Red +	1850	1776	20	1900	6.5%
5	Green +	2360	2265	25	2375	[illegible].4%
6	Pink +	2800	2688	30	2850	5.6%

TABLE 2-2

allow the column to behave as expected. In retrospect this was a poor decision as the column became unstable at the ends due to the reduced composite cross-sectional area. It was finally decided that a satisfactory solution to the problem would be to apply the plastic to the very ends of the columns but bevel it. The benefits of this arrangement would be two-fold. First, the end of the plastic would not be flush with the support, so overstress and cracking would not be a problem. Second, since the plastic extended completely to the ends of the column, local instability of the ends should not occur. In fact, this set-up was successfully used for the remainder of the tests.

The primary objective of the research was to provide an extensive set of data for one particular type of local buckling in folded plate structures. Therefore, a few slightly different types of tests were tried and that which yielded the most extensive, consistent results was used to verify the accuracy of a previously developed computer program. In fact, the test set-up described herein was used to perform tests on concentrically and eccentrically loaded columns. It was soon discovered, however, that the effects of overall bending in the columns tended to mask the effects of local plate buckling. Therefore, the majority of the effort was directed towards concentrically loaded columns. Both loading conditions were, in fact, attempted, but the local buckling phenomenon soon proved to be much more readily visible when the loading was concentric. The tests were thereafter devoted to obtaining an extensive set of data for the concentrically loaded column.

CHAPTER 3

THEORETICAL SET-UP

3.1 Photoelasticity Theory

There are three areas which must be considered in order to describe the theory which has been used during the course of this research. The first is the basis of the photoelastic coating technique for which a detailed theoretical description is beyond the scope of this study. Anyone wishing to have more information concerning the technique may refer to the bibliography where some basic and traditional works are listed. There is also mention of some more recent developments as well as specialized case studies. It may be useful, however, to give a short, rather general, description in order to bring to mind the basics of the technique.

The theory of photoelasticity is based primarily on the theory of light and some of the special properties of certain types of plastic. Light is a series of waves containing vibrations in all directions perpendicular to the direction of propagation. The velocity of light in a vacuum is a constant, c. In transparent bodies, the velocity is slightly lower. The ratio of the velocity of light in a transparent body to the velocity in a vacuum is called the index of refraction. In most materials this index is constant, regardless of the direction of propagation. Crystals are an exception to this rule. The index depends upon the orientation of the vibration of the light with respect to the axis of the crystal. Most plastics, on the other hand, exhibit optical properties of both crystals and other transparent bodies. They behave isotropically when unstressed but anisotropically when stressed. Interestingly enough, the change in the index of refraction is a function of the strain induced. This behaviour is comparable to the resistance change in an electrical strain gauge.

The introduction of a polarizing filter into the system causes circularly polarized light to be generated. As the light beam strikes the plastic it splits and two plane polarized beams are formed which lie along the planes corresponding to the planes of principle strain at the point of entry. The time required for the two light beams to pass through the piece will be a function of the thickness of plastic at that point and the velocity of the beams. The relative retardation between the two beams is therefore simply a function of the thickness of the plastic and the difference in the indices of refraction. Since it was stated earlier that the index of refraction is proportional to the strain and a property of the plastic called the strain optical coefficient, the relative retardation is proportional to the principal strain difference at any point. In a study such as this, it is beneficial to produce circularly polarized light in order to study lines of equal colour. In this system the intensity of the emerging light is zero when the relative retardation between the waves is equal to an integral multiple of the wavelength of the light. For particular values of wavelength, a particular wave will disapppear and the complimentary wave will be seen. By refining this basic theory and evaluating specific cases, a "colour-stress conversion table" (See Table 3-1) has been developed. The expression for the difference in principal strain is:

$$(\epsilon_1 - \epsilon_2) = \frac{N\lambda}{2Kt}$$

Where N = Integer

λ = Average wavelength of white light = 22.7×10^{-6} in

$\epsilon_1 \& \epsilon_2$ = Principal strains

K = Strain Optical Coefficient or Sensitivity

t = Thickness of Plastic

3.2 Finite Element Program

The finite element program which was used in this study was developed and perfected by H.P. Lee (32). It would be beyond the scope of this study to explain it in any great detail but a general description will be included so as to familiarize the reader with its overall capabilities.

The general approach in the finite element method is to divide the structure into a series of elements which are given an assumed displacement function. The choice of element size and number is important because this is one factor which determines the degree of accuracy of the idealization. Usually, the more elements that are used, the more accurate is the solution. Unfortunately, the number of calculations which must be performed in order to solve the problem is directly proportional to the square of the number of

COLOUR	STRAIN μin/in	STRESS psi
Black	0	0
Grey	170	1300
White	430	3300
Very Pale Yellow	460	3500
Light Yellow	500	3800
Brown-Yellow	720	5500
Reddish-Orange	840	6500
Red	900	6900
Tint of Passage 1	945	7200
Indigo	980	7500
Blue	1100	8500
Green	1250	9600
Greenish-Yellow	1450	11200
Pure Yellow	1520	11700
Orange	1670	12800
Dark Red	1830	14100
Tint of Passage 2	1890	14400
Indigo	1910	14700
Green	2200	17100
Greenish-Yellow	2380	18300
Carmine Red	2550	19600
Tint of Passage	2835	21700

Colour-Stress Conversion Chart

Table 3-1

elements. Also the choice of displacement function must be carefully made so that the representation chosen accurately reflects the actual behaviour of the structure. Elements are defined by the coordinates of their nodes and their degrees of freedom.

The element which was used by Lee has three components of translation and three of rotation. The polynomials which correspond to each displacement function are:

u: $1, x, y, xy$

v: $1, x, y, x^2, xy, x^3, x^2y, x^3y$

w: $1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, xy^3$

This element was chosen to best describe the type of behaviour expected in folded plate structures. It is necessary to set up the elements and number the nodes in a certain manner in order to achieve the maximum efficiency of the program. The idealization which was used in this study is shown in Figure 3-1. The material properties which were used in this idealization were shown in Figure 2-2. It is necessary that for this type of study, where the material is assumed to have non-linear properties some type of material idealization be used which closely approximates the actual material behaviour. In this study, tests were performed on coupons cut from the actual test material and stress-strain curves were generated. This basic information was used to develop a mathematical curve which could be used by the computer.

The program gave results for stress and strain at various load levels and at various points on the structure. It also gave

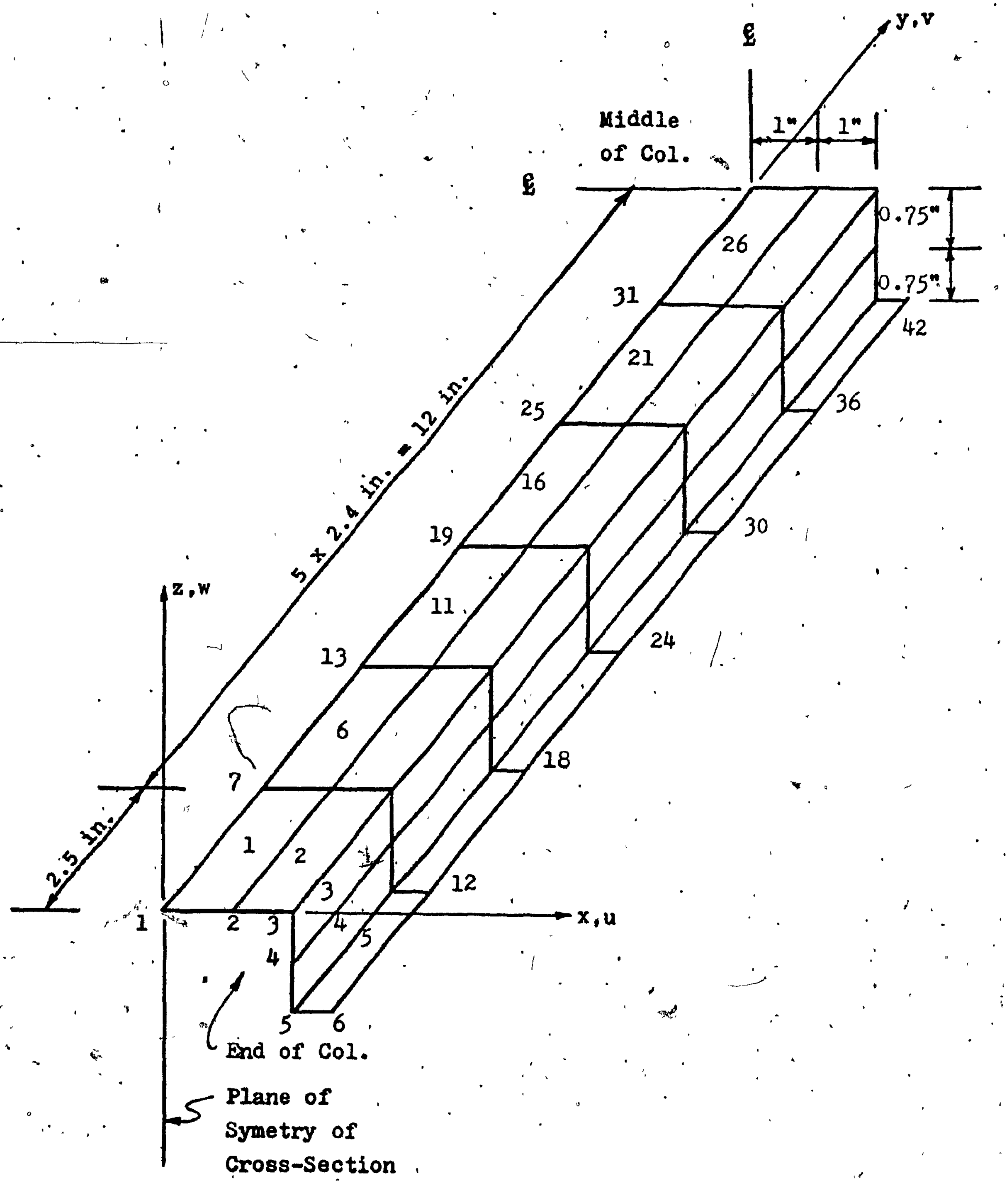

y,v
Middle of Col.
1"
1"
0.75"
0.75"
26
31
42
21
36
25
16
5 x 2.4 in. = 12 in.
19
30
z,w
11
24
13
6
18
7
2.5 in.
1
2
3
12
1
2
3
4
5
x,u
4
5
6
End of Col.
Plane of Symetry of Cross-Section

Figure 3-1

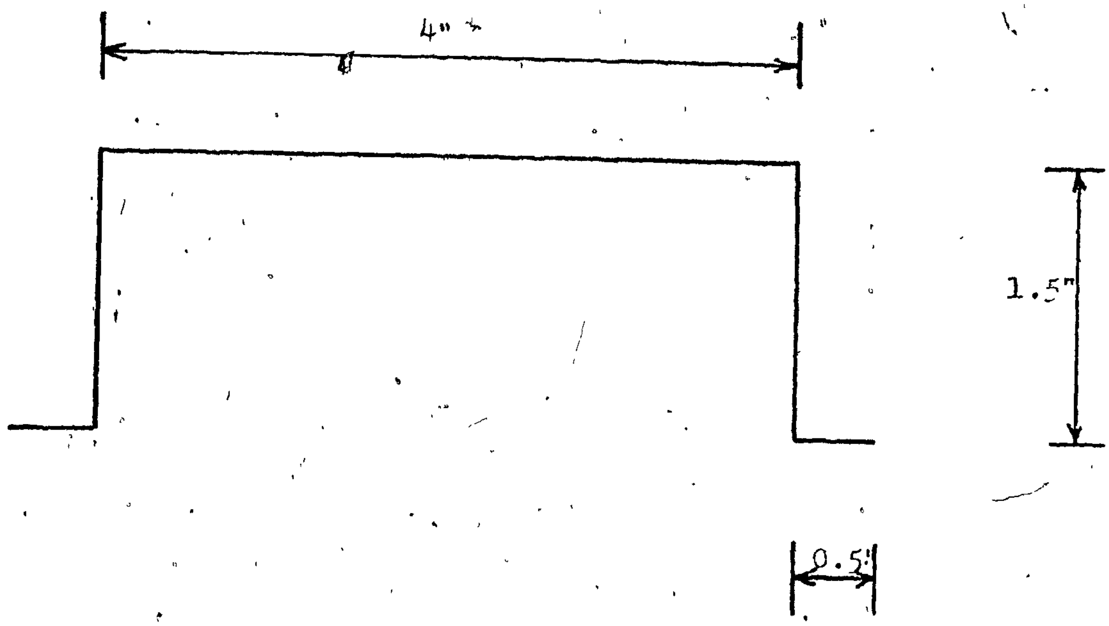
results at various levels throughout the thickness of the structure. It was therefore necessary to use the values for stress and strain at each outer surface corresponding to each point in order to separate the flexural and axial components of stress. Once this was done it was necessary to convert these individual values of stress and strain at a point into corresponding values for principal stress and strain differences. These results were then compared directly to the results obtained from the experimental program.

CHAPTER 4

DISCUSSION OF EXPERIMENTAL RESULTS

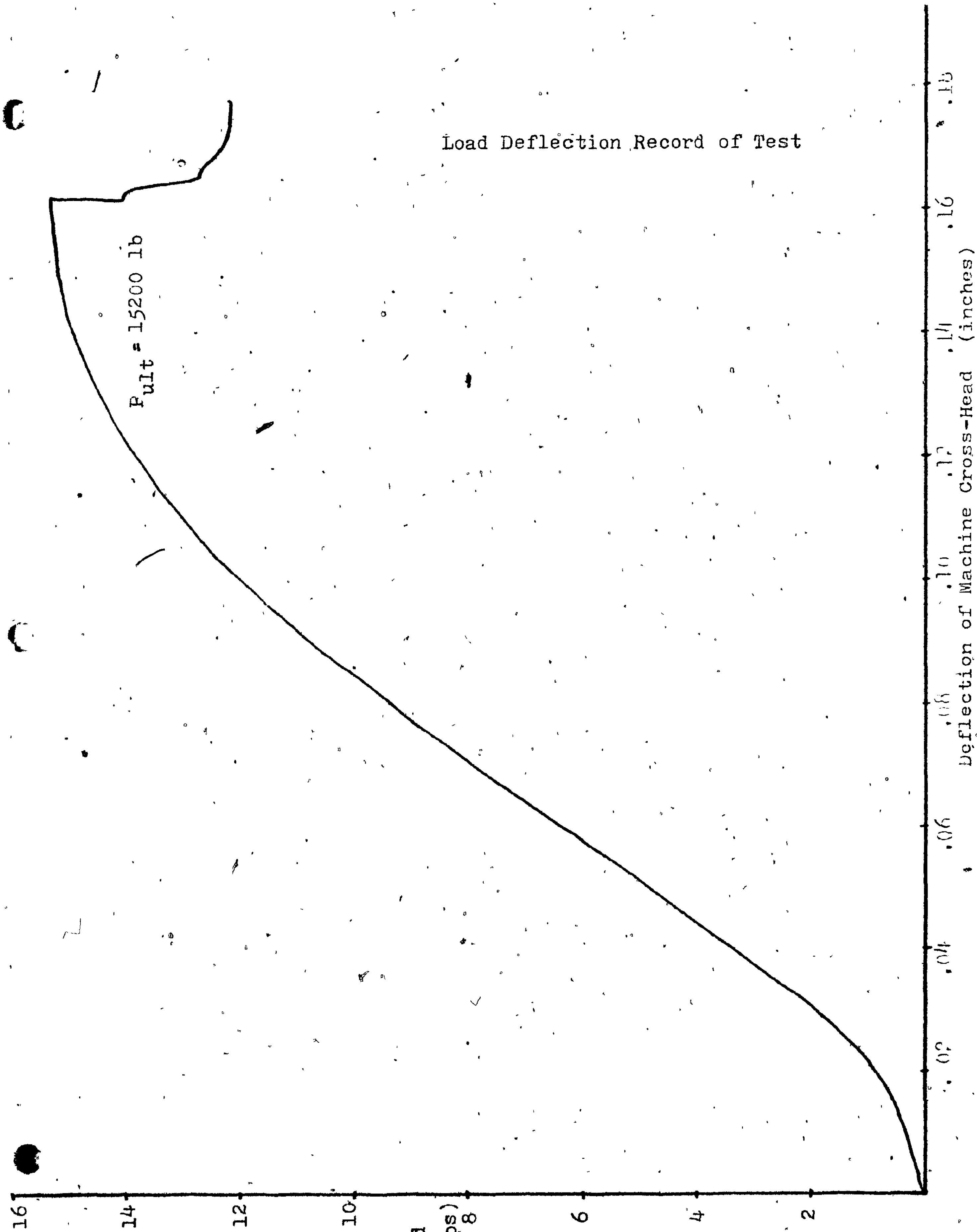
The column was tested under a concentric axial load. The cross-section was a shown in Figure (4-1). The ultimate load was 15,200 lb. The load-deflection record was as shown in Figure (4-2). The column behaviour throughout the experiment was similar to that of the uncoated specimens. There were definite indications that the previously observed buckle pattern in the channel web was generated. This was seen by the colour bands in the photoelastic coating and also by the external appearance of the top aurface of the plastic which exhibited definite high and low points although no actual deflection readings were taken. As previously stated, it was felt that a great deal of work had already been done in this area. The major aim in checking that this phenomenon occurred was to ensure that the general behaviour of the coated and uncoated specimens was similar.

The second major observation was the actual mechanism of failure. Both the coated and uncoated columns first showed signs of buckling and kinking at a section about one-third the length up from the bottom. Also, in both cases, the bottom of the column then rotated and the load-carrying capacity decreased.



Cross-Sectional Dimensions of Test Specimens

Figure 4-1



Load Deflection Record of Test

Figure 4-2

It was felt that, since the general behaviour throughout the loading sequence and the actual mechanism of failure were similar in both the uncoated and coated specimens, the test set-up and idealization were accurate.

Upon detailed examination of the photographic records of the experiment, one can easily observe the expected propagation of stresses. At the outset of the loading sequence, there is a fairly evenly distributed dark grey colour throughout the specimen. Some isolated areas of white and yellow are visible but these are at points of direct load application and can be attributed to the small initial loads used to hold the specimen in proper alignment. As the loading sequence progresses, definite bands of colour become increasingly discernible. As the load level is further increased, the colour patterns begin to repeat. There are definite areas, more noticeably at the ends directly under the point of application of the load, where the width of each colour band is very small. This signifies that in these areas the level of stress is changing very rapidly from one point on the cross-section to another a slight distance away. As the load level is increased, it becomes apparent that there are other areas where the condition of rapidly changing stresses is exhibited. From a relatively early stage in the loading sequence, narrow colour bands can be seen at a point about one third of the length up from the bottom of the specimen. The bands gradually widen both above and below this point. From a relatively early stage it can be deduced that this area is one where relatively high stresses and perhaps local instability may be exhibited. In retrospect, this is in fact the case. It is precisely at this point in the specimen that the failure sequence is initiated. The question that immediately comes to mind is why did these high stresses and the accompanying instability occur at this particular point in the structure? The question is definitely worthy of consideration and will be dealt with later in the discussion. However, one factor that should definitely be at least part of any explanation is that of initial imperfections. Either material or geometric imperfections, possibly some combination of the two, contributed to make this point in the structure particularly susceptible to high stresses under this type of loading. Finally, the possibility of human error in the test set-up, regardless of how small would definitely contribute to making some areas of the cross-section more critical than others.

The displacement of the cross-head of the testing machine during the test was plotted automatically against the load level. The resulting relationship is shown in Figure 4-2. As was to be expected, there is a certain section of the graph that is a straight line. This illustrates that the structure is deforming linearly during application of the load. At a load of about 800 lb there is a definite change in slope of the graph. At this point the structure, after having undergone some initial rate of deformation, begins to stiffen. The structure behaves non-linearly as it continues to stiffen, up to a load of about 2000 lb. From this point up to about 10,500 lb, the graph is once again a straight line indicating a basically linear behaviour. From 10,500 lb to the ultimate load of 15,200 lb, the behaviour is again non-linear as the structure starts to weaken and finally collapse. It is quite apparent that there are various phases that the structure passes through in the course of the test. The phases during which the structure exhibits a linear type of load-deflection history illustrates a condition wherein the structure is basically stable and most of the deformation is axial in nature. On the other hand, the sections which are non-linear indicate that the structure is unstable to some extent and the deformation which is recorded is due to combined axial and flexural effects. The first non-linear section (from 800 lb to roughly 2000 lb) is probably due to a "settling-in" effect. The structure has taken some load and has, as a result, undergone a certain amount of deformation. At this point, the stiffening effect under load becomes significant enough to become apparent in the load-deflection curve. This effect is transient, however, and as soon as the structure once again reaches a state of stable equilibrium, the linear relationship reappears. It is only when the effects of the buckling become significant that the behaviour again becomes non-linear. At this point and beyond, the bending stresses are starting to have a large effect on the behaviour of the structure. It is the natural stiffening effect of the plate elements which tend to allow the structure to stiffen after initial deflection. It is this phenomenon which gives folded plate structures the great reserves of post-buckling strength for which they are noted.

It is possible to select individual points in the specimen and try to gain some knowledge of their behaviour by the propagation of stresses and strains. In actuality, the curves in Figures 4-3, 4-4, and 4-5 are a plot of load verses principal strain differences. The curves have been plotted for three points. The principal strain differences were determined from the photographic records at the center of Elements 11, 16, and 21. These three points are all in the general area of the buckle which appeared in the test specimen. Therefore, it is probably correct to assume that the effects of buckling had a more long-lived and pronounced effect here than at any other point in the specimen. The bending effects should probably have been more noticeable and significant at low load levels at these points than at any others.

Taking first Element 16, and investigating the shape of the curve of plain strain difference versus load, it can be seen that the relationship is very nearly linear up to about 7500 lb. The curve starts to diminish in slope at about this point signifying that the deformation is starting to become non-linear and the structure more flexible. However, at a load slightly higher than this, actually somewhere between 8000 and 10,000 lb, there is a point of inflection. This could be an indication that the structure has buckled locally and, as a result, deflected out of plane. This deflection has caused the stiffening effect which is illustrated by the steeper slope of the curve. From this point on, the deformation is gradual but continuous, and constantly in the same direction. The stiffening effect is seen from the curve to continue to

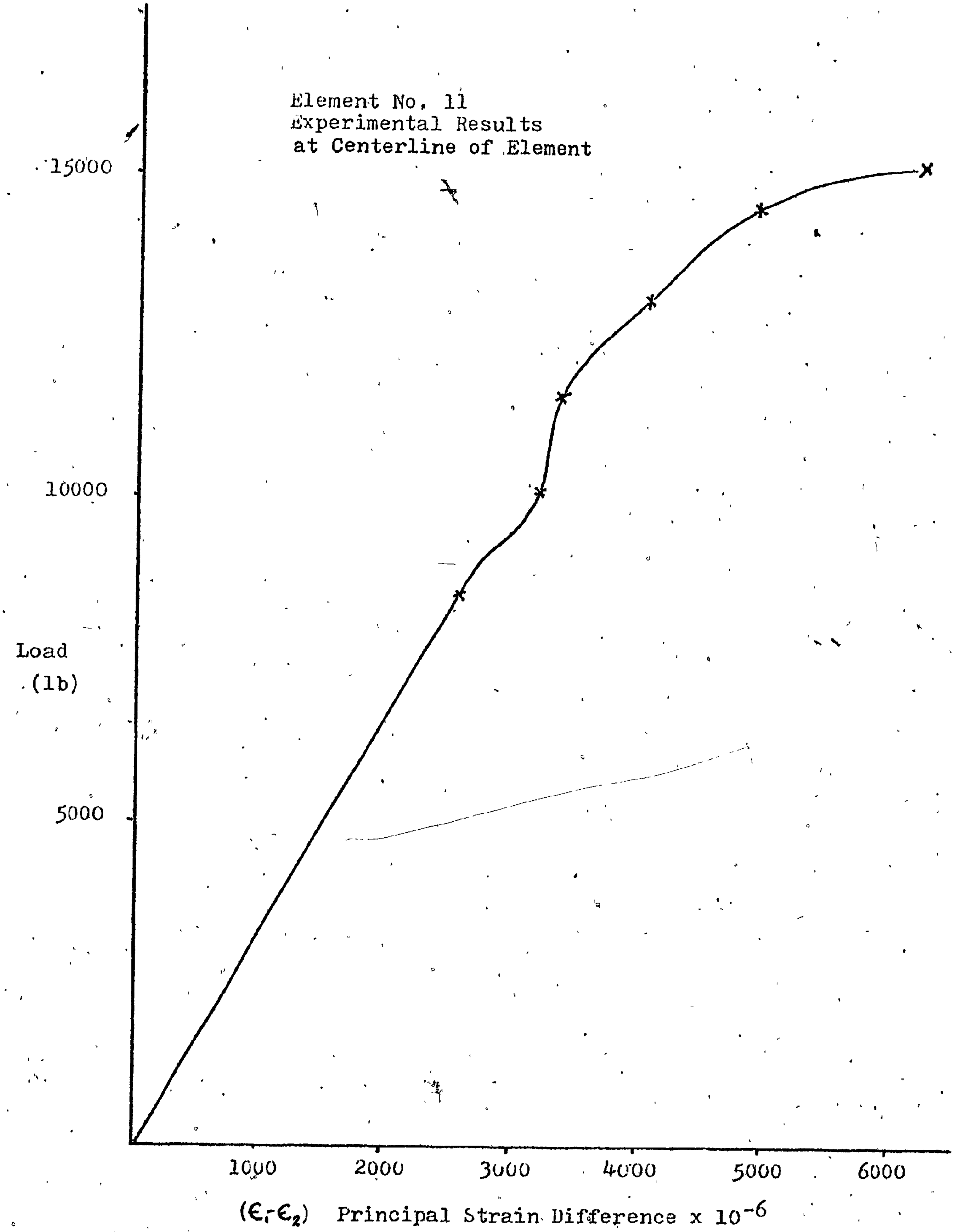

Element No. 11
Experimental Results
at Centerline of Element
15000
10000
5000
Load
(lb)
1000
2000
3000
4000
5000
6000
(ϵ_1-ϵ_2) Principal Strain Difference x 10^{-6}

Figure 4-3

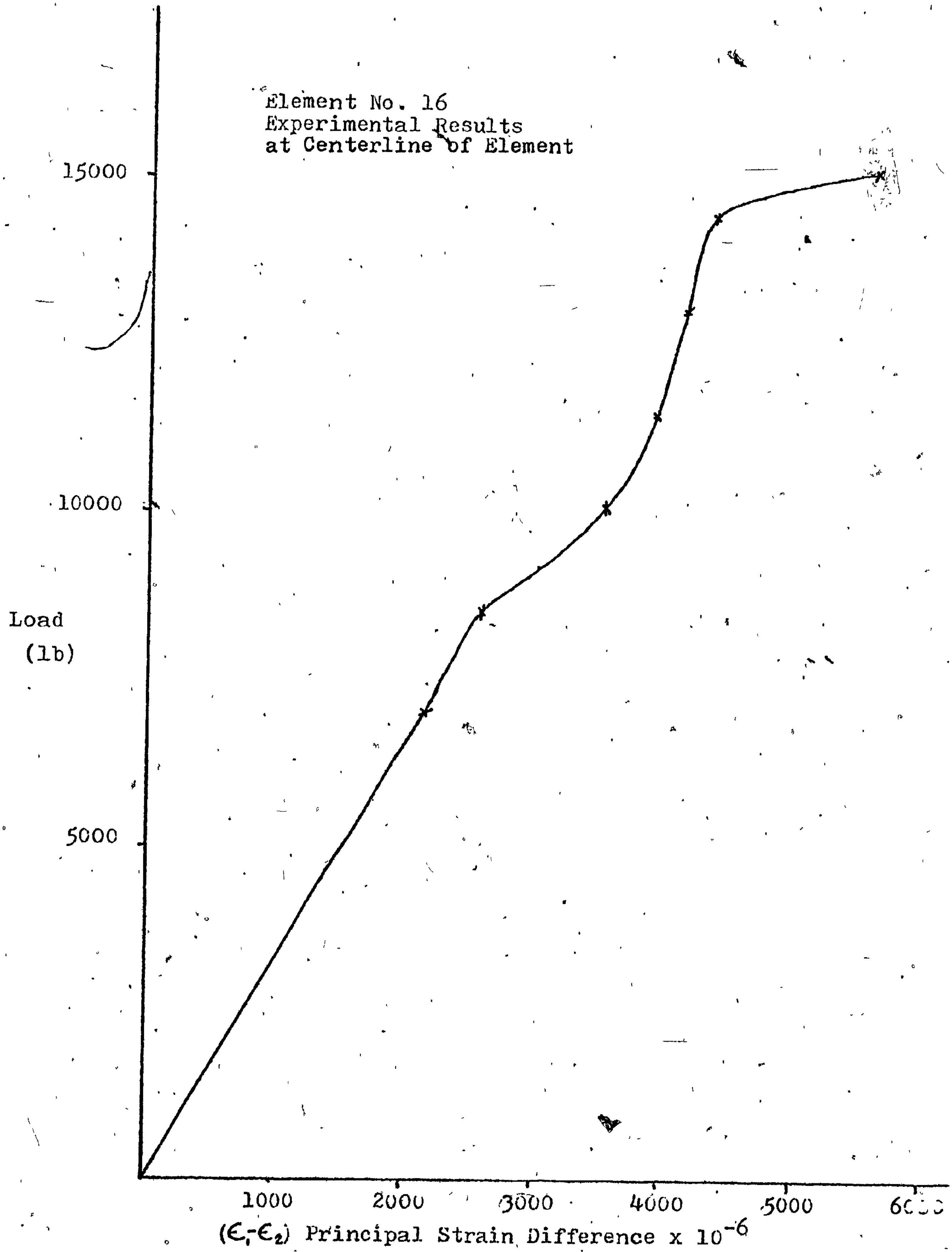

Element No. 16
Experimental Results
at Centerline of Element
15000
10000
5000
Load
(lb)
1000
2000
3000
4000
5000
$(\epsilon_1 - \epsilon_2)$ Principal Strain Difference x 10^{-6}

FIGure 4-4

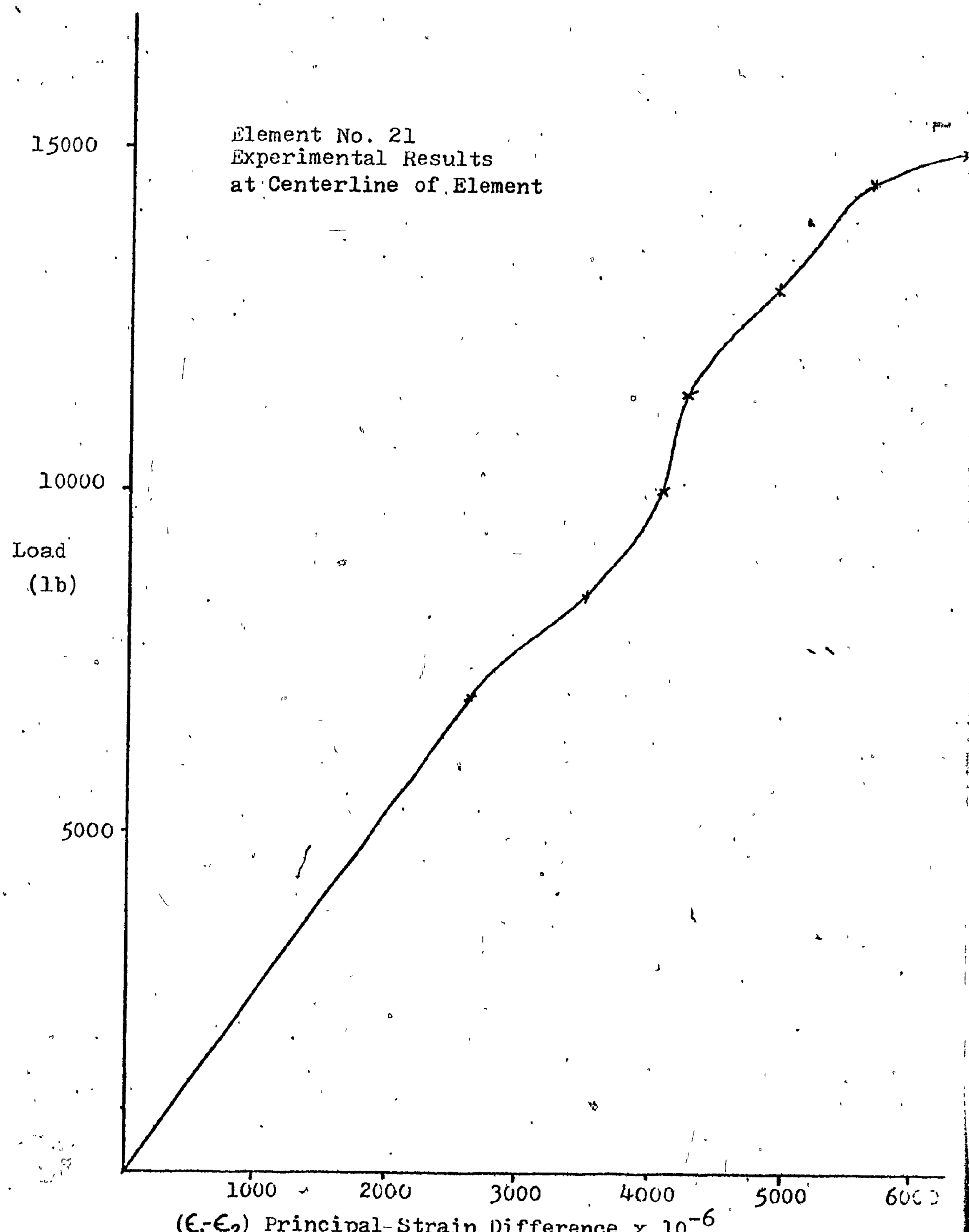

Element No. 21
Experimental Results
at Centerline of Element
15000
10000
5000
Load
(lb)
1000
2000
3000
4000
5000
(ϵ_1-ϵ_2) Principal-Strain Difference x 10^{-6}

Figure 4-5

a point at about 14,500 lb. At this point, the slope of the curve changes rapidly and the deformation and principal strain differences increase very rapidly with only a slight increase in load. Finally, at a load level of about 15,200 lb, the load-carrying capacity of the structure decreases sharply and large deformations take place.

The sequence of events which occurred during the test can be summarized as follows. From the outset of the test, the column behaves linearly and most of the resulting stresses and strains are the result of direct axial load. The second stage is short-lived. It is the stage where the column exhibits a non-linear behaviour. This is most likely due to a situation where bending effects have become significant with respect to axial effects. It is somewhere in this stage that the actual buckling of the plate elements takes place. The third stage occurs when the adjacent plate elements start to have a stiffening effect on the buckled and deformed element. This stiffening effect continues until the adjacent plate elements themselves become unstable due to the high combination of axial and bending stresses. It is not easy to determine the exact load at which the transition from one stage to the next occurs. This is understandable because the column is not ideal and therefore there are no sharp transition points but rather gradual transitions which take place over a significant load interval.

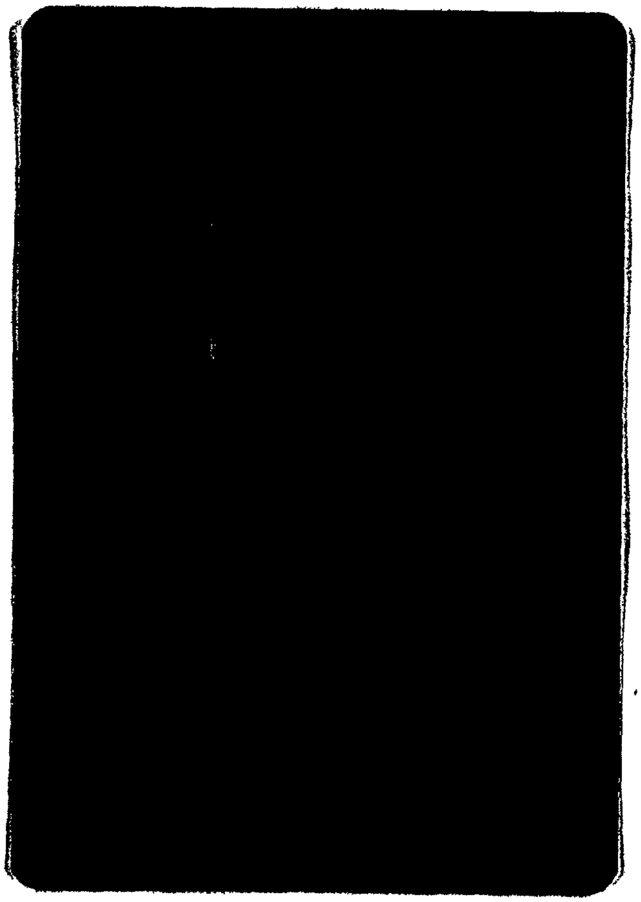
It is now appropriate to investigate some of the other points in the immediate vicinity of element number 16. This should give some indication as to the validity of the previous assumption that the buckling was initiated in this area. If buckling was initiated at this location, it would be natural to assume that other points on the cross-section would display the traditional signs of a buckled condition to a lesser degree. Investigating point number 11, which is situated along the length of the column at a point slightly lower than Point 16, it is evident that the expected results indeed occurred. The stages through which the point passed during the load sequence are once again evident. However, it is clear that the transitions from one stage to the next are more gradual and even less clearly defined. The curve of load versus principal strain difference is almost linear up to about 10,000 lb. At this point, there is a definite stiffening effect. However, from this point up to the eventual collapse load, it is evident that the bending strains become increasingly more dominant. From these observations, it therefore seems correct to conclude that although this point displays some of the traditional characteristics of a buckled situation, none of the signs appear to be so strong as to lead to the conclusion that buckling was initiated at this point.

Point 21 is the next to be examined. Once again, the familiar linear section of the load-displacement curve at a load somewhere between 8000 lb. and 10,000 lb. is evident. This initial stage is followed once again by a relatively short stage where stiffening is apparent. This stage is followed by a longer one where the bending effects become increasingly more significant until finally the load-carrying capacity decreases and collapse results. Once again, these stages are characteristic of the history of a plate element undergoing local buckling. However, these characteristics are not evident to the same extent as they were for Element 16. Point 21 is located higher on the length of the column than is Point 16. From the evidence that has been presented, there can be little doube that the local buckling of the major plate element of the column was initiated very close to Point 16. Points 11 and 21, situated above and below Point 16 on the column length, both show indications that they are in the general vicinity of a local buckling phenomenon. However, neither of them show these indications to as great a degree as Point 16.

The second effect that can be observed from these experiments is the propagation of the stress pattern. By studying the overall structure and observing the stress pattern from one load level to the next, it is possible to gain some knowledge of the actual behaviour of the specimen as a whole. Classical theory indicates that where an axially loaded structure made up of plate elements undergoes local buckling, the pattern that the buckles take up is that of a series of square panels. It can also be shown that adjacent panels in a series of square buckles deflect in opposite directions. Therefore, if one were to study the relative stresses and strains which are present in a loaded specimen of this type, one might expect to observe a series of circular or relatively circular contours. There should be concentric contours very close together in the case of rapidly changing stress or, conversely, farther apart in the case of stresses which are not changing as rapidly. In each case, the outer ring of each set of contours would be adjacent to the outer ring of the neighbouring set of contours. This pattern should be apparent whether the specimen is loaded concentrically or eccentrically. However, in the case of eccentric loading, this effect might be harder to discern depending on the degree oe eccentricity. In the case of high eccentricity, the overall bending effects could make the pattern of ring contours harder to recognize. However, upon close examination, it should always be possible to separate the two effects and isolate them.

Photograph 4-1 was taken at a load of 7000 lb during a test on a concentrically loaded column. It can be seen from the changing patterns of colour contours that, for the majority of this section of the structure, the changes in principle strain difference are rather gradual. Only near the very bottom of the colour is there any indication of rapidly changing levels of stress. This is undoubtedly due to stress concentrations created by the end effects due to bearing.

It is also interesting to note that, even though there is a small area that is obscured by glue which has squeezed between two adjoining pieces of plastic, the general colour patterns seem to be continuous and consistent on each side of the plastic joint. From this picture, it is quite evident that no apparent buckling has taken place at this load level. There is no indication of circular contours or even repeated colour bars which could signify colour contours in a less than ideal specimen.

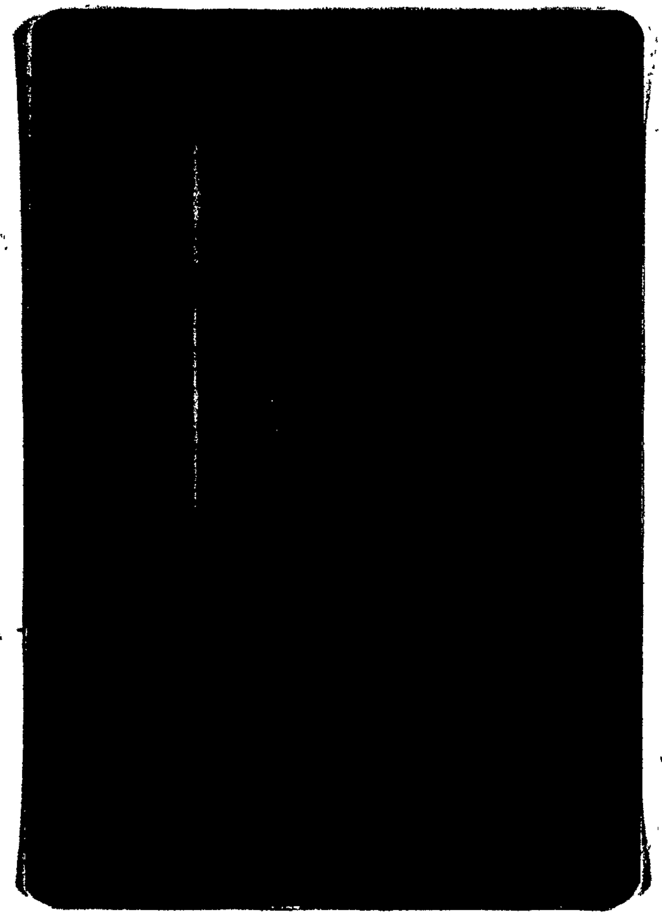


Colour Patterns at 7000 lb

Photograph 4-1

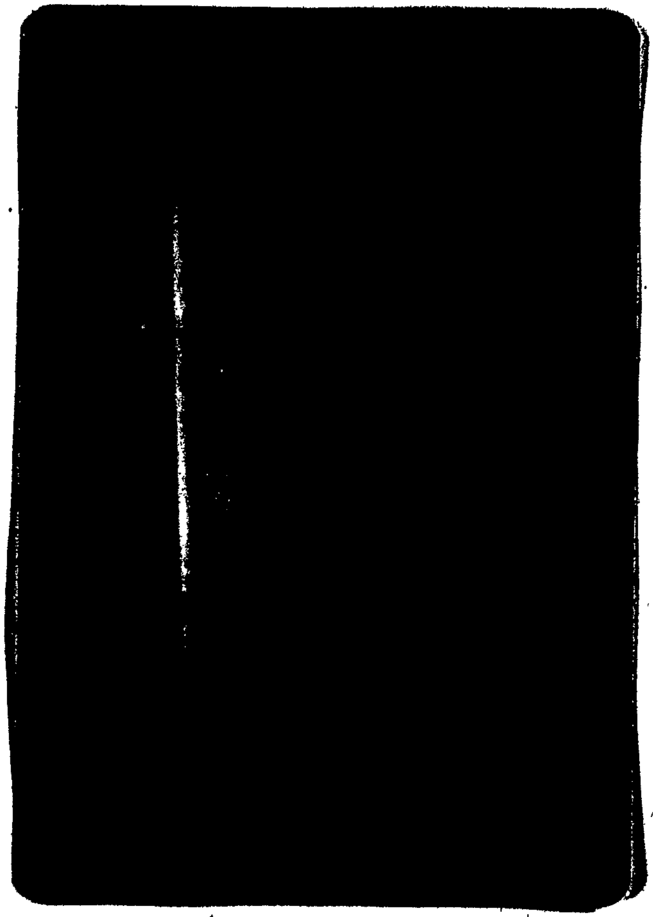
Photograph 4-2 shows the same area of the same specimen during the same testing sequence at a load level of 8500 lb. The same general effects are recognizable, although each seems to be present to a greater degree. The stress concentrations due to the end effects are very noticeable, even to the extent that it can be seen that these concentrations originate at the corners of the column and that the center of the largest plate element is relatively unstressed at the base of the column. Once again, there is relatively little indication that buckling has occurred. It would be safe to assume that no local buckling has, as yet, taken place. At this load level it is again noticeable that the continuity of colour patterns is quite good across the joint between adjacent pieces of plastic. This would lead one to believe that very little residual stress was introduced into the plastic during the bonding procedure.

Photograph 4-3 shows, once again, the same conditions as the previous two pictures but the load has been further increased to 10,000 lb. At first glance, the same pattern as the previous two photographs seems to be exhibited. However, upon closer examination, a subtle difference is evident. Whereas at loads of 7000 & 8500 lb. the colour pattern was rather scattered and actually did not seem to have any definite pattern, this is not quite the case at 10,000 lb. The colours seem to have separated into two major distinctive regions. The first extends from just slightly above the end effects and continues to just below the interface between the two pieces of plastic and it is basically all yellow.



Colour Patterns at 8500 lb

- Photograph 4-2

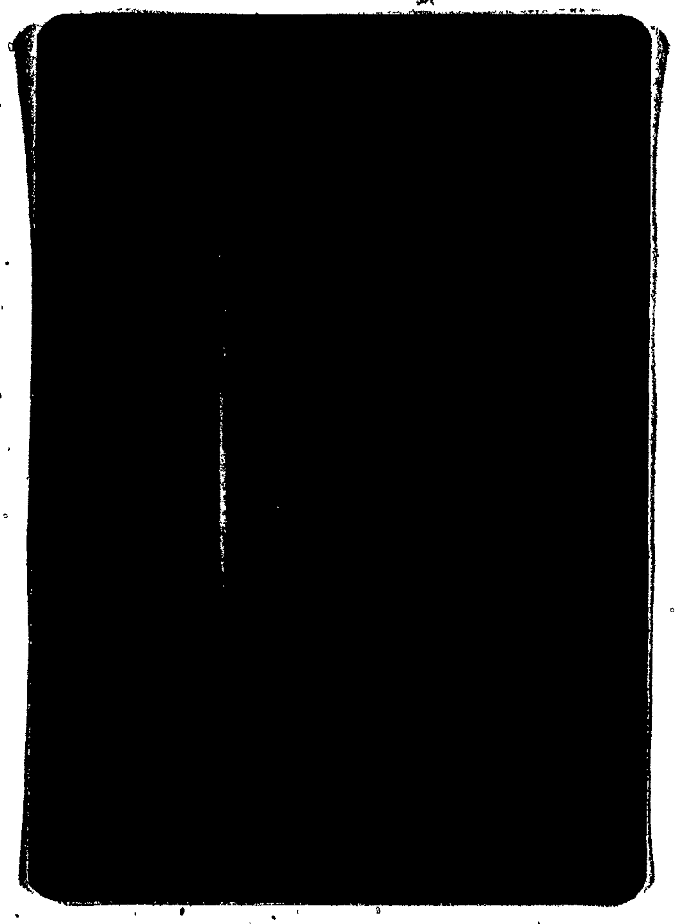


Colour Patterns at 10,000 lb

Photograph 4-3

This is interesting since it seems to indicate that this region has undergone some kind of change which has caused this whole area to be stressed to roughly the same level. The section of the column which is directly above this region is also practically all red, indicating that this region is also all stressed to relatively the same stress level, in this case a higher one. These observations could indicate that any one of several different conditions exist. First, they could point to the fact that some type of local buckling has taken place and that the out of plane deformations one would expect have actually taken place. The different colours give some indication of the location and extent of the buckle and could be used to estimate the extent of the deformation. The large areas of different colours could also indicate that the column loading is not actually concentric and that the load is slightly displaced in the out of plane direction of the major plate element. This would cause the entire column to bend in this direction and induce stresses which would increase toward the center of the column. It is quite possible that the effects which are illustrated in the photographs are actually a combination of these two possibilities. There is another condition which seems to have begun to develop in this photograph which is worthy of mention. If the end effects are studied closely it becomes obvious that the effects are more predominant on one side of the speciment. This could indicate that the loading on the column is displaced slightly in the transverse direction. This is not necessarily a detrimental situation in the light of the entire experiment. Actually, it would be quite unreasonable to expect that, in an experiment of this type, the loading would be perfectly concentric. It is, however, necessary to keep these points in mind when analyzing developments and conditions in subsequent photographs. It is also probable that some of these effects are due to the fact that the column was fabricated by the author on a hand operated press brake and naturally does not demonstrate all the qualities of an ideal column.

Photograph 4-4 shows the stress pattern at a load of 11,500 lb. Some of the same effects previously discussed are once again evident. They are, however, starting to become more readily visible. For example, in this photograph there are very strong indications that the column loading is definitely displaced in the transverse direction and that the column geometry is such that the column is possibly biased slightly toward one side. There is definitely more red on one side of the column than on the other and the end effects on that same side have extended much higher on the column. There is one interesting observation, however, that can be made concerning the buckling phenomenon. The expected series of circular colour contours seem to have started to develop. They are, however, developing not on the center line of the column as expected, but are displaced to one side. This would seem to be consistent with some of the observations. What this means is that because of the slightly eccentric load, the normally visible and centered stress contours due to local plate buckling are being somewhat obscured and mixed with the stresses generated by the unexpected overall biaxial bending of the column. Unfortunately, this makes detailed study of the buckling effects somewhat more difficult. It is still quite

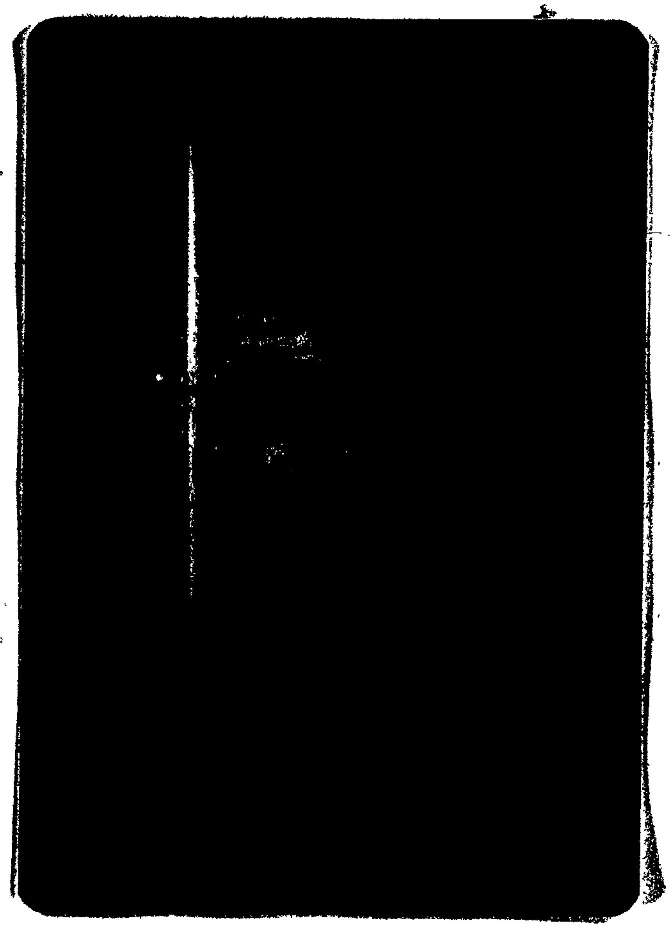


Colour Patterns at 11,500 lb

Photograph 4-4

possible, nevertheless, to make general observations and quite probably draw some reasonable conclusions concerning this phenomenon. It is interesting to note that the presence of the colour contours seems to be centered about 7 or 8 in. up from the bottom of the column. This is interesting for two reasons. First, when the graphs of load histories of individual points on the structure were discussed, it was determined that the local buckling was centered at just about the same location on the column. This would seem to verify the hypothesis that the colour patterns which are observed are actually being produced in part by the local buckling effect. It is also interesting to note that the load level at which these effects were first observed was somewhere between 8,500 lb and 10,000 lb in both cases. The second effect that is of interest is that, after disregarding end effects, the approximate length of the buckle pattern is roughly the same as the width of the plate element which has buckled. This is consistent with classical plate theory which predicts that plates should buckle in square panels.

Photograph 4-5 shows a further stage of the loading sequence at 13,000 lb. Once again the previously discussed effects are not only present but are in fact evident to an even greater degree. The colour contours reflecting the stress patterns are becoming even more refined. On close examination, green, yellow, orange, red, blue and finally a second green and a second yellow band are all visible. Once again the center of the set of bands is shifted to the side and is still located in approximately the same location. The end effects are very well defined and the stresses on one side



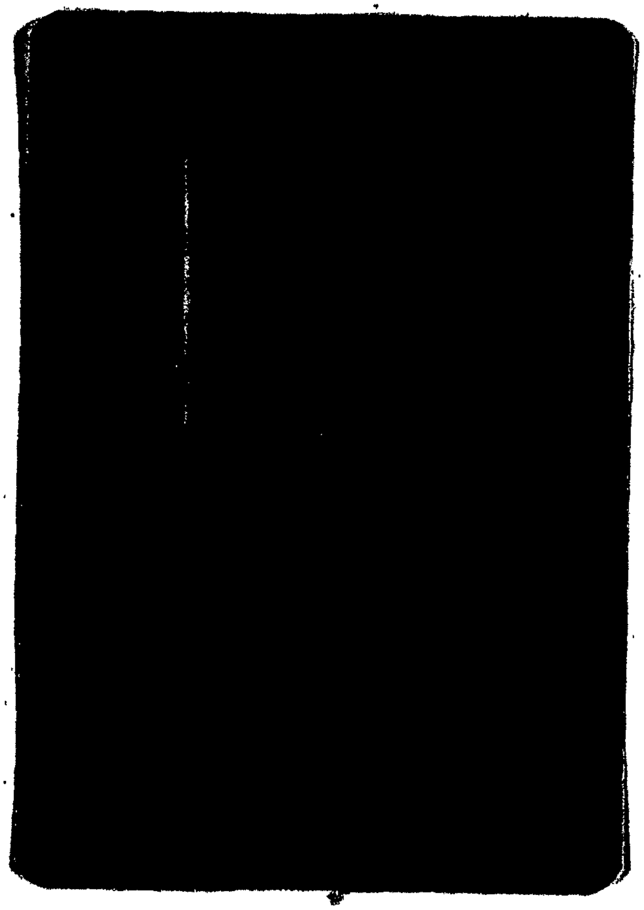
Colour Patterns at 13,000 lb

Photograph 4-5

seem to be quite significant. There are at least three green bands visible. However, the effects on the other side are substantially less. It is very probable that the stresses due to axial compression are being reduced by the tensile stresses induced by the overall bending of the column.

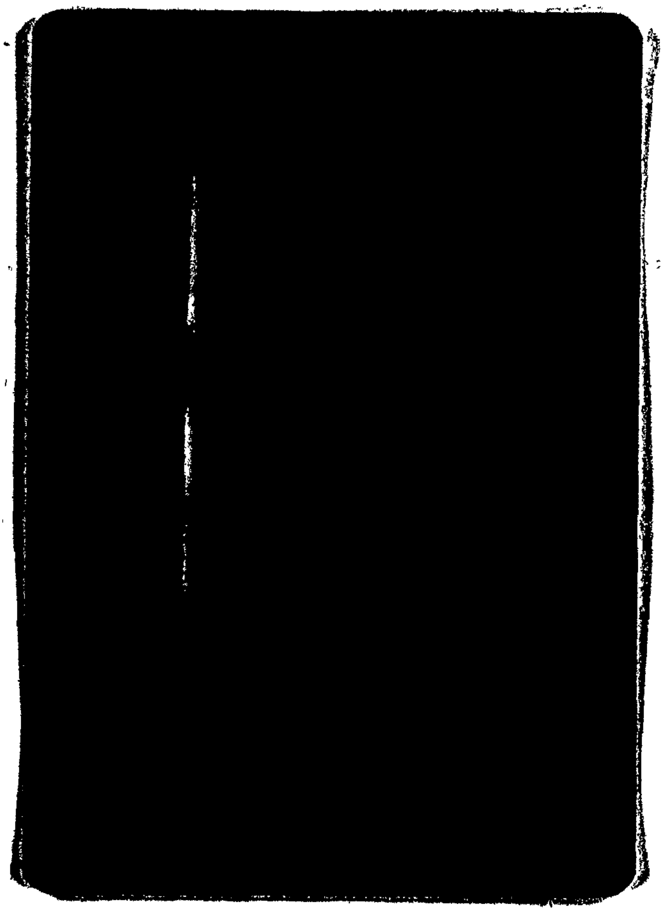
Photograph 4-6 shows the stress pattern at 14,500 lb. It is evident that some radical changes have taken place. The stress levels have definitely increased sustantially. It also seems that the effects of overall bending have become much more significant than those of local plate buckling. The colour bands have switched from being a series of relatively concentric circles to being wavy lines of colour which run mostly vertically over the length of the column. The bending of the column in the transverse direction would normally give rise to a series of straight parallel lines running the length of the column. The buckling effect causes a series of concentric circles. Combining these two conditions in the relative degrees that they exist gives rise to a series of wavy vertical lines. Therefore, even though at first glance the effects of localized plate buckling do not seem to be evident, upon closer examination it is evident that, not only are these effects present, but they are contributing substantially to the overall pattern.

Photograph 4-7 shows that the stress pattern at the time of failure at 15,200 lb. The wavy vertical lines are very much in evidence and in fact, they predominate. The column failed due to a



Colour Patterns at 14,500 lb

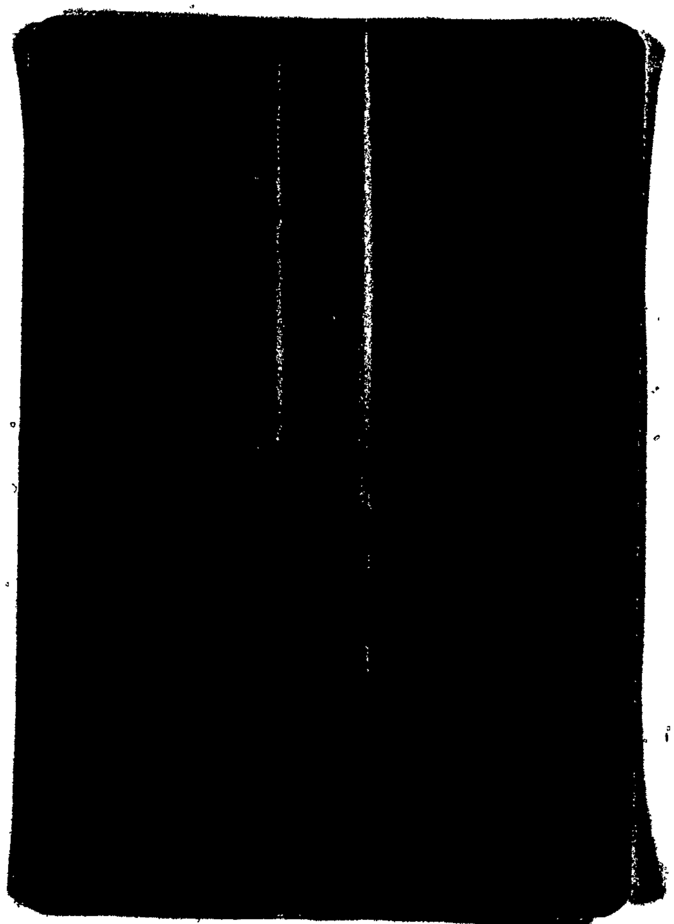
Photograph 4-6



Colour Patterns at 15,200 lb

Photograph 4-7

slight kinking effect which was located very close to the interface of the two pieces of plastic. In Photograph 4-8, it is evident that the stresses are very high in this region and that it was probably due to these high stresses that failure resulted. It is also quite possible that some initial imperfections existed in the original specimen at this point and that this imperfection led to the accumulation of high concentrated stresses in this region. Initial imperfections, possibly at the interface between the two pieces of plastic, could be responsible for giving rise to many of the different effects that have been discussed. However, the mode of failure of the column was the same as for the uncoated column tested previously in a similar fashion. Therefore, it is reasonable to assume that, although initial imperfections can slightly alter the effects at certain points and may even slightly affect the behaviour of the column, in no way did they cause the general overall behaviour to deviate from that of the control specimen. Further photographs showing different portions of the column at various loads are included in the appendices. Overall photographs of the column at failure are also included.



Colour Patterns at Failure Location

Photograph 4-8

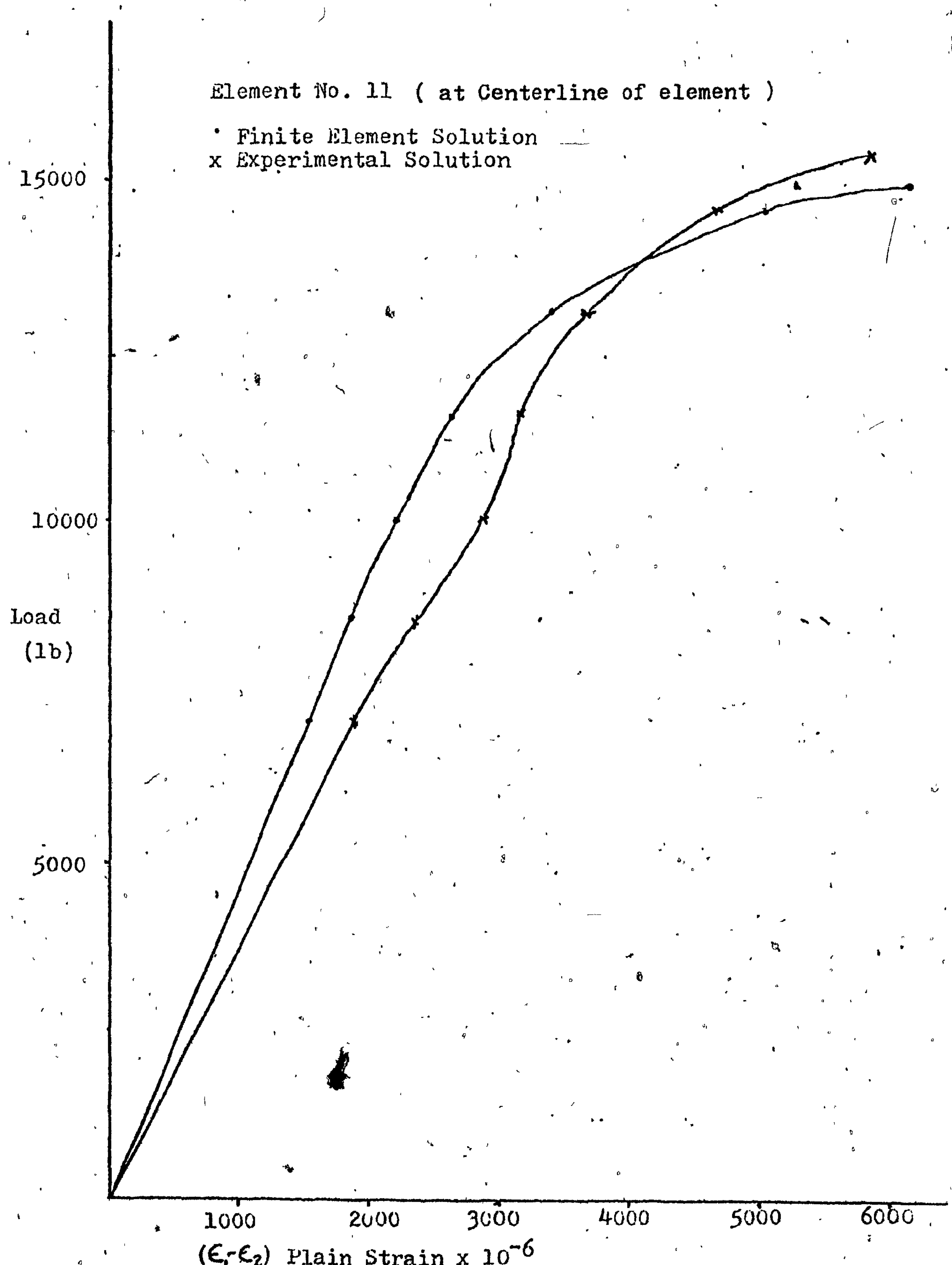
CHAPTER 5

COMPARISON OF EXPERIMENTAL & ANALYTICAL SOLUTIONS

During the course of the research, a finite element method of solution was developed by H.P. Lee (32) which was especially suited to this type of work. A detailed description of the method may be found in some of the references listed in the bibliography.

It is interesting to compare and contrast the results of this sophisticated computer idealization with those obtained from the actual experimental tests. In the early stages of this study, when the first computer and experimental test results were becoming available, it was realized that a simple comparison of the principal stress and strain differences at any particular point on the specimen did not seem very promising. Plots of principal strain differences versus load resulted in curves which seemed to be of an entirely different shape. It was decided, therefore, to try to separate the axial and flexural components of stress and then make individual comparisons. This was done because it seemed quite possible that flexural stresses were being introduced into the experiment that were not being introduced into the computer model. This was discussed in Chapter 4 and it has been shown that this is most probably what actually happened. It is possible to investigate both sides of the test specimen at any given point and, by performing the necessary arithmetic manipulations, separate the flexural and axial components of stress. Right from the beginning of this study, the experimental and analytical solution methods were both designed so that the stresses and strains could be determined on both exterior faces.

In order to proceed from the conclusions that have already been reached, it is the intention of the author to continue to discuss the same region of the column that was previously studied. The intention is to plot on the same graph the results of both the computer solution and the experimental results. The first point to be discussed, therefore, will be No. 11. Figure 5-1 shows the plot of principal plane strain difference versus load for both solutions. It is quite apparent the correlation is very good. In fact, the maximum difference is of the order of 20% and in places the curves actually coincide. It is interesting to note that even the general shapes of the two curves are almost identical. Both curves are relatively straight up to about 8,500 lb at which point the behaviour becomes slightly inelastic and consequently the lines start to curve. This curvature becomes more pronounced as the inelastic behaviour gradually dominates. There is some difference in the two curves at about 10,000 lb. Up to this point the experimental curve has bent over slightly and then started to stiffen and return more to the shape of analytic solution. There are two possible explanations for this. First, it could be that some unknown effect



Element No. 11 (at Centerline of element)
• Finite Element Solution
x Experimental Solution
15000
10000
5000
Load
(lb)
1000
2000
3000
4000
5000
6000
(ϵ_1-ϵ_2) Plain Strain x 10^{-6}

Figure 5-1

has caused the reading at this point to be slightly in error and that the curve should actually be a smooth one which approximates that of the computer solution. The second possibility is that the experimental specimen actually buckled at, or near, this point and then subsequently stiffened to produce the curve shown. Which of the two effects, if either, is the actual cause for the deviation is difficult to determine with any degree of certainty. However, it is interesting to note that, even though these indeterminate factors are present, the plots of the two solutions are actually quite similar. In fact, even at maximum load the difference is only about 7%.

The second point under consideration is point 16. The plot of principal strain difference against load is shown in Figure 5-2. At first glance, this plot does not seem to exhibit the same characteristics as in the previous case. There are, however, some marked similarities between the two solutions. For example, once again the two curves have the same general shape up to about 8,500 lb. At this point the curve showing the experimental solution has started to become inelastic to a slightly greater extent. At the same point, however, the computer model starts to show very strong tendencies to stiffen whereas the same effect, although present in the experimental results, is much more gradual. In fact the experimental test results seem to indicate that the specimen keeps stiffening until only shortly before failure. In the computer model, the structure undergoes a certain amount of stiffening and then continues to deform inelastically. This should

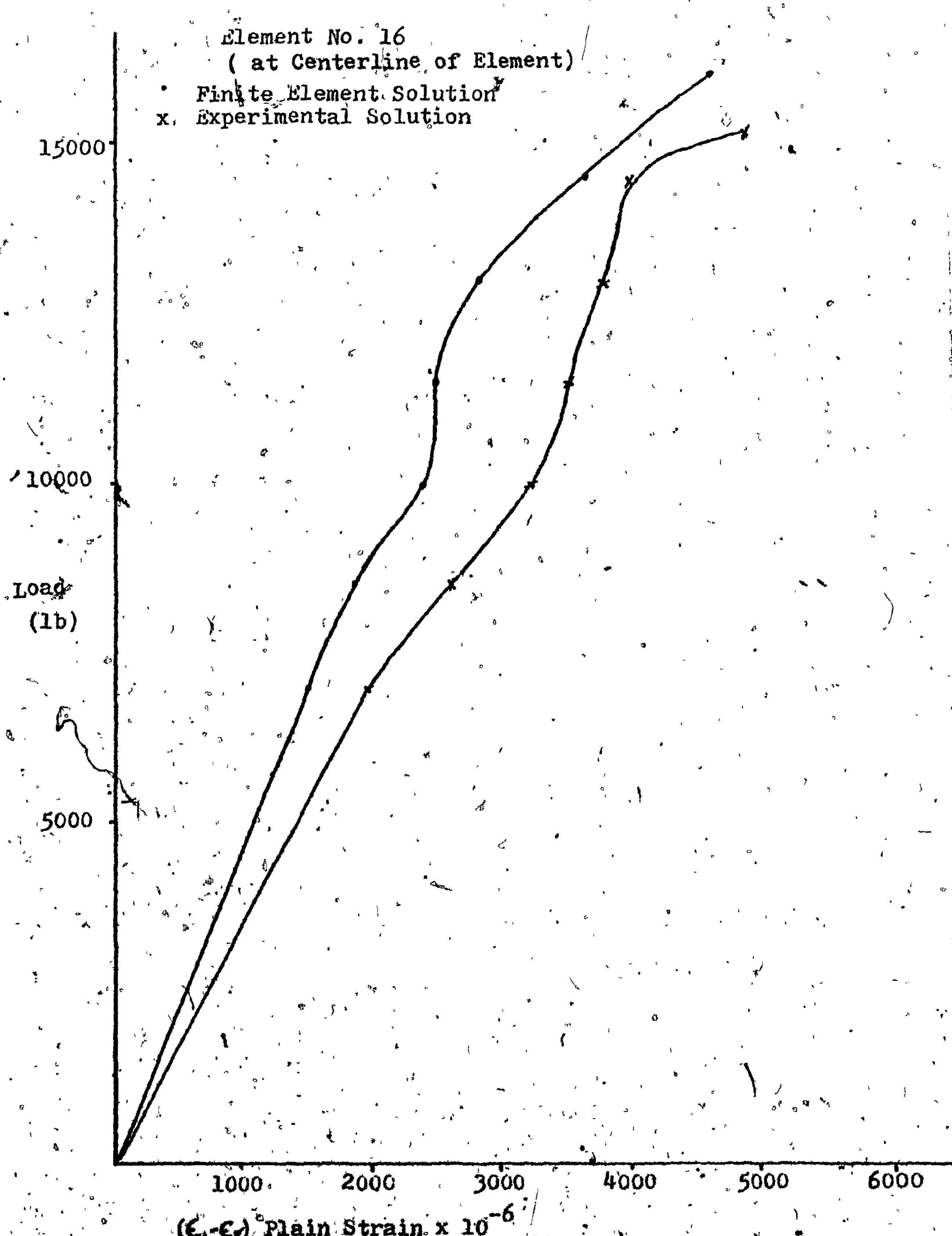
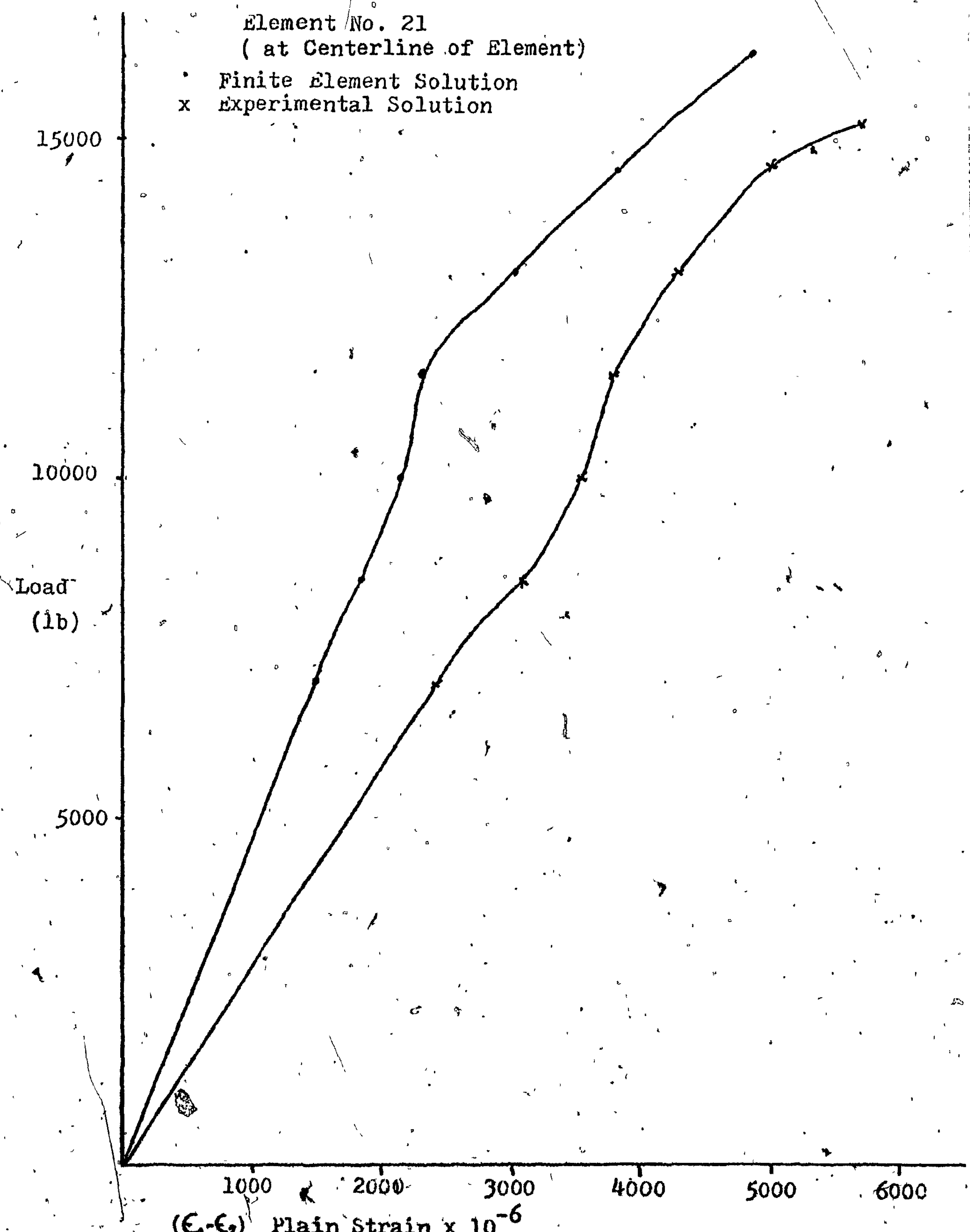

Element No. 16
(at Centerline of Element)
• Finite Element Solution
x. Experimental Solution
15000
10000
5000
Load
(lb)
1000
2000
3000
4000
5000
6000
$(\epsilon_1 - \epsilon_2)$ Plain Strain x 10^{-6}

Figure 5-2

not be surprising, however. It is quite natural that a more ideal model will behave in this manner. Due to the inherent imperfections in the physical test specimen, the behaviour will be gradual rather than sharp and pronounced as in an ideal case. It is of interest to note that although in one case the behaviour is gradual and in the other the effects are seen abruptly that at the very high load of 15,000 lb, the difference between the two levels of principal strain difference is less than 10%.

Point 21 the next under consideration. In this case the two curves, although they have basically the same shape, exhibit quite different levels of principle strain difference. As illustrated in Figure 5-3, the principle strain difference found in the experimental model are significantly larger than those found in the computer model. To a much smaller degree, this is also the case for the previous two points. There is a possible explanation for this, however. In the computer model, the specimen is modelled in such a way that the load is introduced into the cross-section in a perfectly even manner. The ends are assumed to be 100% perpendicular to the axis of the column. In practice, this can never actually be true. No matter how much care is taken, the ends of a test specimen can never be perfectly flat. This in itself would tend to introduce an uneven stress distribution into the cross-section. This effect alone would not, however, produce the degree of difference that is exhibited in Figure 5-3. In fact, the load would tend to distribute itself evenly over the cross-section. However, there is an effect which could be significant in producing this result.



Element No. 21
(at Centerline of Element)
• Finite Element Solution
x Experimental Solution
15000
10000
5000
Load
(lb)
1000
2000
3000
4000
5000
6000
$(\epsilon_1-\epsilon_2)$ Plain Strain x 10^{-6}

Figure 5-3

It is the fact that at the outside of the column, along the stiffening lips of the channel section, the plastic did not in all cases end flush with the aluminum. In most cases, in fact, the plastic ended slightly before the edge of the aluminum. This was due to the manner in which the plastic was molded onto the aluminum. It is very difficult to ensure a completely flush edge and, at the time, the author underestimated the importance of this effect. The effect of this situation would be to slightly reduce the load-carrying capacity of the outer edges of the cross-section, somewhat like reducing the cross-sectional area. This would naturally increase the stresses and strains in the central part of the cross-section slightly. With this in mind, it now becomes easier to see the similarities between the two curves. As in the case of Point 16, Point 21 illustrates perfectly the inherent differences between an ideal and an actual model. The effects present in the computer model are present in the experimental model but the transition from one stage to the next is much more gradual. The effects which consist of an initial elastic behaviour, some subsequent type of inelastic behaviour followed by a general stiffening, and finally inelastic behaviour and failure, are present in both models. Interestingly enough, upon close examination it becomes evident that these effects are each present at approximately the same corresponding load levels.

Point 26, shown in Figure 5-4, illustrates basically the same points. It is evident that this point is quite a distance from the point of the initial onset of buckling. All of the pre-

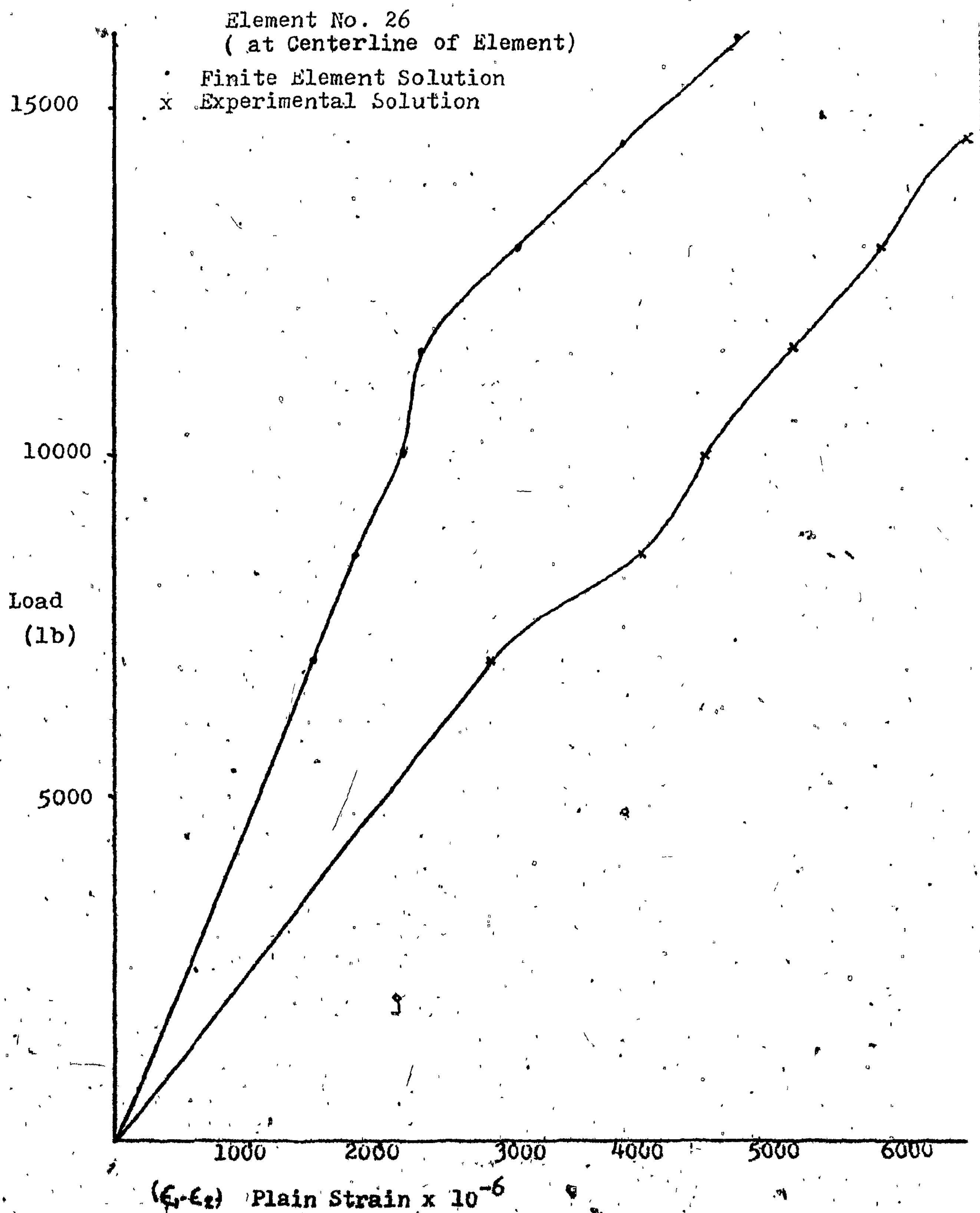

Element No. 26
(at Centerline of Element)
• Finite Element Solution
x Experimental Solution
15000
10000
5000
Load
(lb)
1000
2000
3000
4000
5000
6000
(ϵ_1-ϵ_2) Plain Strain x 10^{-6}

Figure 5-4

viously discussed effects, although present, are shown to a much lesser degree, almost to the point where the curve is a straight line. There are undoubtedly indications that buckling takes place and that a general stiffening of the specimen results.

It is evident, upon investigating the behaviour of all four points, from both the computer and experimental results, that the buckling was initiated at Point 16. In fact, there was an attempt made in the computer model to initiate buckling at this point. This was done after the initial tests were made on the uncoated specimens. These specimens buckled at this point and after the first test with a coated specimen it was noted that again buckling was initiated in this region.

It is of some interest to investigate the principal flexural strain difference to see if similar correlations exist. Figures 5-5, 5-6, and 5-7 are plots of principal flexural strain difference versus load at Points 11, 16, and 21 respectively. It is quite easy to see that the behaviour of the experimental test model is, at best, erratic. The strain differences seem to shift back and forth but there is some similarity between the two solutions. Both seem to be basically elastic up to a load of about 8,500 lb. At this point some inelastic behaviour takes place for a relatively short period whereupon a stiffening effect follows. The extent and duration of this stiffening seems to vary from one point to the next. Possibly the secret to the solution of the behaviour of the structure can be found by investigating the computer solution which

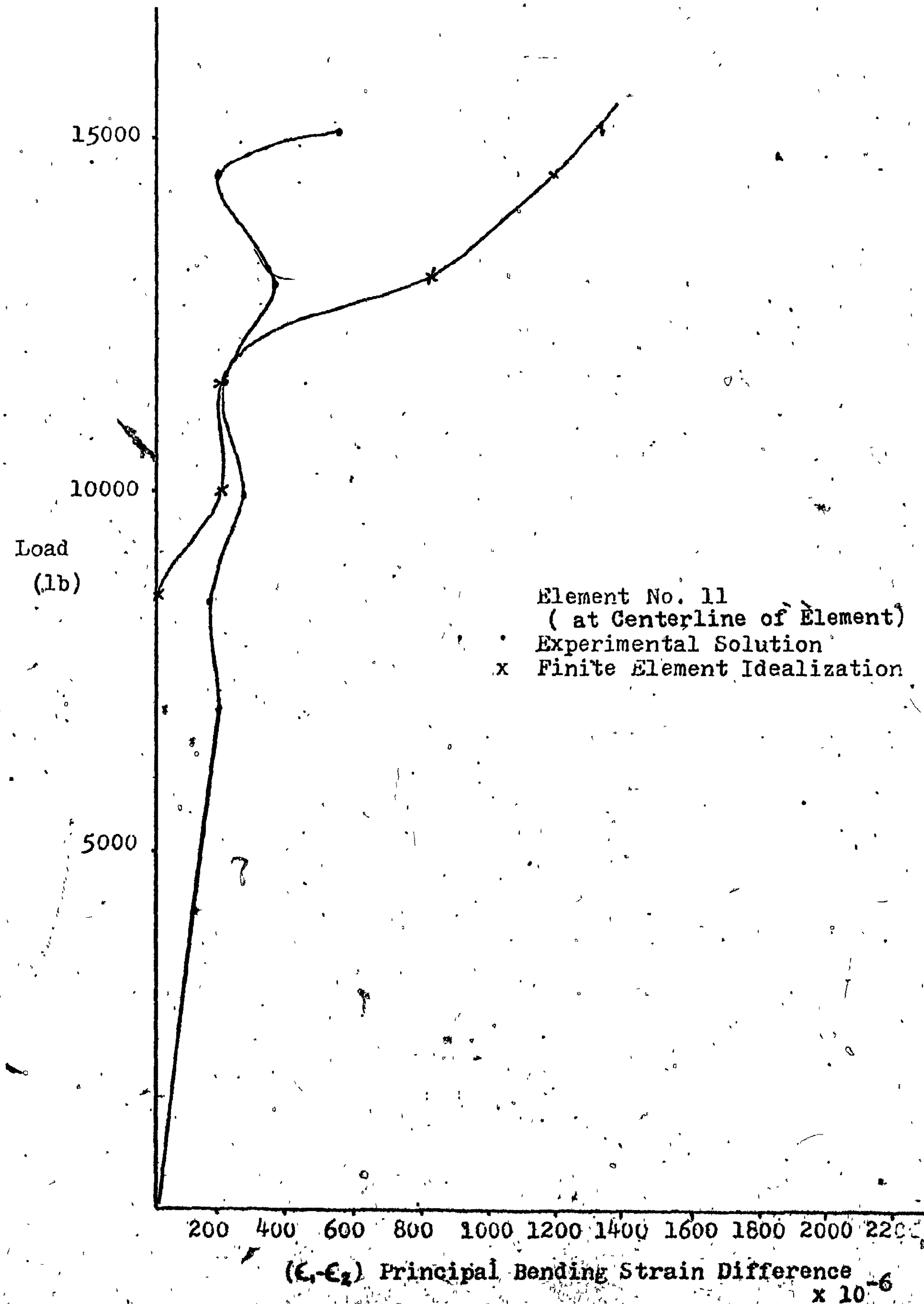

15000
10000
5000
Load
(lb)
Element No. 11
(at Centerline of Element)
• Experimental Solution
x Finite Element Idealization
200 400 600 800 1000 1200 1400 1600 1800 2000 2200
$(\epsilon_1-\epsilon_2)$ Principal Bending Strain Difference
$\times 10^{-6}$

Figure 5

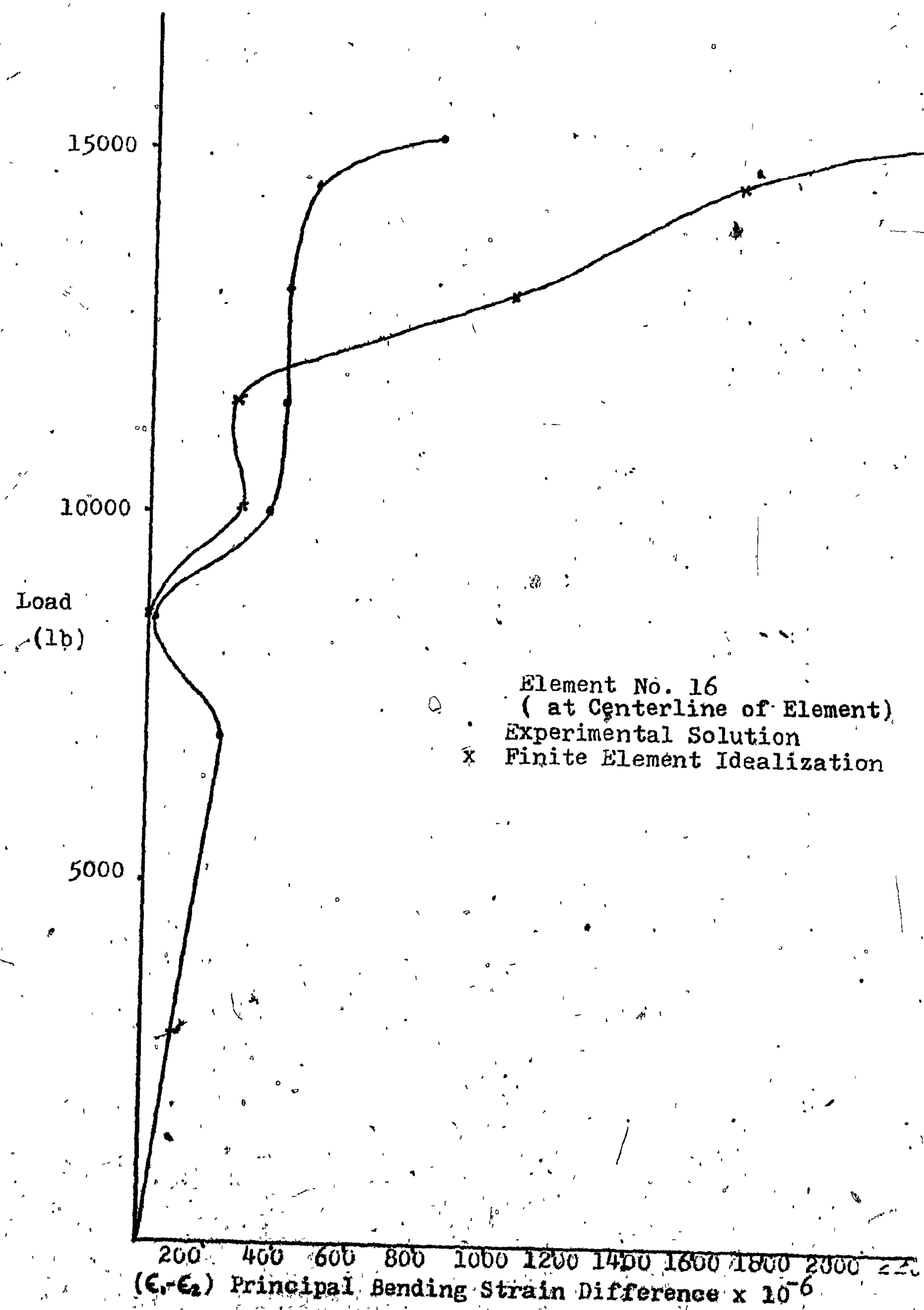

15000
10000
5000
Load
(lb)
Element No. 16
(at Centerline of Element)
• Experimental Solution
x Finite Element Idealization
200
400
600
800
1000
1200
1400
1600
1800
2000
$(\epsilon_1-\epsilon_2)$ Principal Bending Strain Difference x 10^{-6}

Figure 5-6

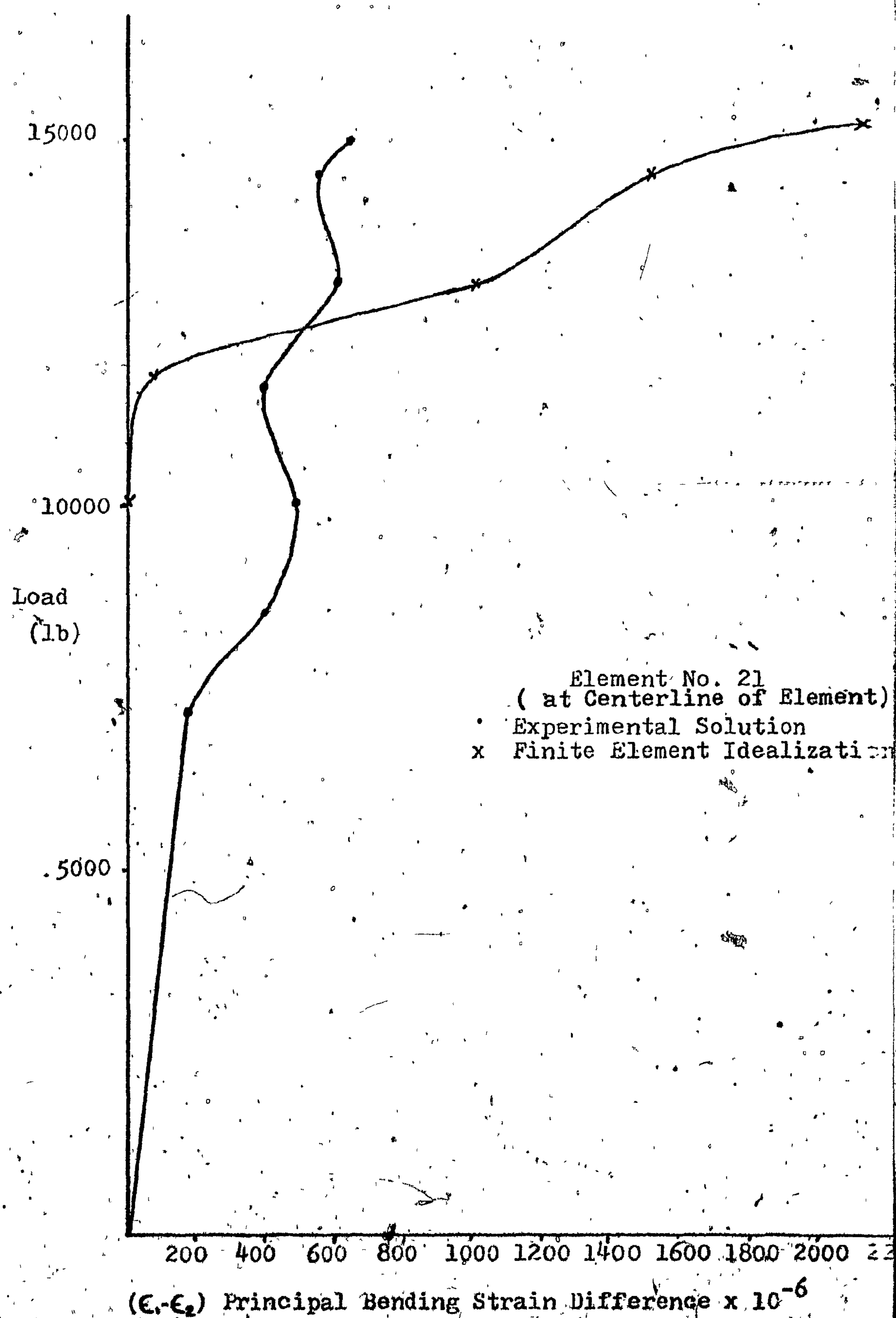

15000
10000
Load
(lb)
5000
Element No. 21
(at Centerline of Element)
• Experimental Solution
x Finite Element Idealizati
200 400 600 800 1000 1200 1400 1600 1800 2000
(ϵ_1-ϵ_2) Principal Bending Strain Difference x 10^{-6}

Figure 5-7

should be more basic and clearer than that illustrated by the experimental solution. At Points 11 and 16, no bending effect is evident until 8,500 lb, at which point inelastic behaviour commences. At about 10,000 lb the specimen starts to stiffen and continues to do so up to about 11,500 lb where inelastic behaviour again starts and continues until failure. The only solution that could possibly explain the seemingly erratic and certainly complicated behaviour of the experimental model is that there are many initial imperfections and secondary effects which are contributing to the behaviour illustrated in Figures 5-5, 5-6, and 5-7.

In short, it must be assumed that the behaviour of the experimental model is complicated by inherent fabrication inaccuracies. This makes comparison only possible in a general way. However, the basic behaviour is probably illustrated in its ideal condition by the plot of the computer solution.

It is of some interest to compare the overall stress pattern produced by the computer solution with that produced by the experimental test. First and foremost, the computer model assumes symmetry of the structure about its center line. In fact, only one quarter of the structure was analyzed. Naturally the buckle which is produced is likewise symmetrical with the center line of the column. This has the advantage of eliminating the masking effects which were found during the discussion in the previous section. There is no possibility of having slightly out of plane loading conditions which confuse the stress pattern. However, once again, the resulting solution is only applicable for the ideal column loading condition. It has been shown by investigating individual points on a buckle that the order of magnitude is similar and that the principal axial strain differences are within acceptable experimental errors. Points above and below the center of the buckle exhibit similar characteristics which seems to indicate that the symmetry is illustrated in the experimental solution. In general, the correlation between the experimental and computer results is quite good. The magnitudes of individual levels of principal strain differences are very close. The symmetry which is inherent in the computer model is also illustrated to a fairly high degree in the experimental model. Finally, both experimental and computer models behaved in a manner which was very similar to the test which was performed on an uncoated specimen.

Chapter 6

CONCLUSIONS

Good correlation between the experimental test results and the theoretical finite element computer results was realized in this study. The propagation of stresses and strains which were observed in the experimental tests were quite closely predicted and reproduced by the finite element computer model. Individual values of principal strain differences at specific load levels obtained by means of both methods were within acceptable limits of error. Principal flexural strain differences for the most part correlated quite closely. Small deviations which were obtained at higher load levels could be attributed to inaccuracies in either the test set-up, or the fabrication of the specimen.

It seems reasonable to conclude that the study was successful. The initial intent was to provide a set of accurate data concerning one particular type of structure subjected to one particular type of loading. In this respect, the main objectives were definitely realized. The data collected and reported is both extensive and accurate. A secondary benefit was also obtained from this study. The experimental data helped to verify the accuracy of the finite element program. It is therefore reasonable to assume that the computer program can be used to analyze similar types of structures.

A great deal of time and energy was expended during the course of this study in perfecting the plastic coating technique. It would be repetitive and somewhat wasteful to suggest that other researchers continue this line of study in the same manner. While it would definitely be of interest to study other shapes or different load conditions it would serve no benefit to have future researchers struggle through the pains of learning the fine points of the photoelastic coating method. It would, however, be beneficial, perhaps to employ a skilled technician who could perform this work. Thus interesting and perhaps beneficial studies of a similar nature could be carried out by qualified researchers without forcing them to become bogged down by the tedious and repetitive job of fabricating and applying coatings.

Future studies which could be initiated on this basis could include different types of sections, some simple to construct, such as angles and Zee sections and some more complicated, such as tees and cruciform sections. These could be tested experimentally and then compared to computer analyses. Further work could also include testing different sections under various types of loading conditions. Flexural loading, combined axial compression and flexure, and torsional loading, could all be investigated. An extensive set of data could then be compiled and used to verify new computer models and improve on existing ones.

It is necessary that the first in a line of researchers studying a problem struggle through all the elementary facets of that problem. For this reason, it was not only necessary but also beneficial that all phases of the work reported here be actually performed by the author. However, it will be the responsibility of future researchers of this topic to learn from the experiences reported herein and use this information as a stepping-stone to more complex and interesting studies.

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APPENDIX