FLOW-INDUCED VIBRATION OF CYLINDRICAL STRUCTURES:
A REVIEW OF THE STATE OF THE ART

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FOREWORD

This paper has been prepared for presentation at the Second International Topical Meeting on Nuclear Reactor Thermohydraulics, sponsored by ANS, ASME and AIChemE, to be held in Santa Barbara, California, on 11-14 January 1983, under the title of "A review of flow-induced vibrations in reactors and reactor components".

As discussed in the Introduction, however, the paper confines itself to a review of flow-induced vibration of singular cylinders or arrays thereof subjected to either cross or axial flow, which represent structures and flow conditions commonly found in nuclear reactors and reactor components. Nevertheless, such structures are equally commonly found in non-nuclear power generation, in the chemical processing industry, and elsewhere.

It was therefore decided that, although the conference title is topical and valid for that meeting, the title of this Report should be changed to "Flow-Induced Vibration of Cylindrical Structures: A Review of the State of the Art", to better reflect its wider applicability.

AVANT-PROPOS

La présente étude a été préparée pour le colloque "Second International Topical Meeting on Nuclear Reactor Thermo-hydraulics", qui aura lieu à Santa Barbara (Californie) en janvier 1983, sous le titre: "Etude des vibrations induites par des écoulements dans les réacteurs et leurs équipements auxiliaires".

Cependant, cette étude se limite aux vibrations des cylindres, soit isolés, soit en faisceaux, induites par des écoulements transversaux ou axiaux (voir l'introduction). Quoique l'on retrouve très souvent ce type de structure dans les systèmes nucléaires, il n'est pas moins omniprésent dans les systèmes de production d'électricité conventionnels, les systèmes de traitement chimiques et manufacturiers, etc.

On a donc décidé de donner à ce rapport un autre titre qui serait à la fois plus spécifique et plus général, à savoir: "Vibrations des structures cylindriques induites par des écoulements: étude critique de l'état actuel de la question."

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A REVIEW OF FLOW-INDUCED VIBRATIONS IN REACTORS AND REACTOR COMPONENTS

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ABSTRACT

This paper presents a review of the state of the art of two classes of vibration problems encountered in reactors and reactor peripherals: namely, vibration of cylindrical structures induced by cross flow and by axial flow. A historical perspective is given first, in which the milestone contributions that have advanced the state of the art are highlighted. Then recent developments in the last decade, with emphasis on those in the last three years, are discussed, concluding with an assessment of the state of understanding of the fluidelastic mechanisms involved, on the one hand, and of predictive tools available to the designer, on the other.

NOMENCLATURE

- A_y transverse displacement amplitude due to Strouhal resonance
- D cylinder diameter
- D_h hydraulic diameter
- EI flexural rigidity of cylinder
- f frequency (Hz)
- f_a acoustic resonance frequency
- f_n natural frequency of cylinder
- f_s Strouhal frequency
- f_{vs} vortex-shedding frequency
- K fluidelastic instability constant in equation (7)
- K_{min} minimum value of K for design purposes
- m mass of cylinder per unit length (usually includes the added mass)
- p pitch of equilateral triangular arrays (Fig.1)
- Re Reynolds number
- s_t transverse spacing of cylinder array, T/D (Fig.1)
- s_ℓ longitudinal spacing of cylinder array, L/D (Fig.1)
- S Strouhal number
- U flow velocity; for arrays in cross flow U is usually the "reference gap velocity", related to the array-less (empty) upstream velocity U_∞ by $U_\infty s_t/(s_t-1)$
- U_c critical flow velocity for fluidelastic instability and acoustic resonance
- u dimensionless flow velocity for axial flow, $= (\rho A/EI)^{\frac{1}{2}}UL$
- y_{rms} rms vibration amplitude due to buffeting
- L length of cylinder
- γ mode shape factor, = 1 for rigid-body motion, 1.305 for cantilevered cylinder, and 1.155 for simply-supported cylinder
- δ logarithmic decrement
- δ_r $= 2m\delta/\rho D^2$, in Table 1 (δ_r^*: based on measurements in still fluid)
- ζ damping factor (ratio of actual to critical damping)
- ρ fluid density
- $\phi_n(x)$ nth mode eigenfunction of cylinder

1. INTRODUCTION

Consideration of flow-induced vibration has traditionally not been an integral part of design of nuclear plant in the same sense as, for instance, reactor physics and thermohydraulics. Design deficiency, in terms of the latter, may mean that the plant will not work at all, while inadequacy in terms of flow-induced vibration considerations may simply mean that smooth operation will be short-lived. This distinction, coupled with inexcusable disregard of one of Murphy's laws, caused flow-induced vibration to be relegated to the status of a secondary design parameter; indeed, prior to the mid-1960's, it was often totally ignored: if a structure could stand up to the steady fluid loads, then a healthy safety factor ought to take care of the unsteady ones underlying vibration, it was optimistically reasoned.

Some spectacular failures, some of which are described and analyzed in another review [1], have since put flow-induced vibration in nuclear reactors and peripherals on the map, so to speak; this was aided by the fact that all incidents and malfunctions in nuclear plant have to be reported and thoroughly studied. The realization that flow-induced vibration problems can cause complete shutdowns of nuclear plants for prolonged periods of time, ranging from days to years, and the enormous power-replacement costs entailed, have convinced designers that they may only ignore flow-induced vibration at their peril.

There are a host of possible vibration problems in nuclear plant, in the reactor itself, as well as in the associated piping, heat exchangers and steam generators; ancillary equipment, such as in-core instrument tubes, "secondary" piping, etc., are often involved [1]. Most of these diverse vibration problems may be classified under four broad categories [1], namely those caused by (i) cross flow, (ii) internal, or (iii) external axial flow, and (iv) annular or leakage flow — although other classifications and approaches for reviewing the state of the art are clearly also possible; *vide* Ref. [2,3].

This paper will only examine two of these classes of vibration. Specifically, it will deal with vibration of structures, and especially cylindrical structures, induced by either external axial or cross flow.[1] The paper will thus provide an up-dated and expanded version of another recent review of the state of the art [4], in which emphasis will be placed on more recent developments in the field, and where flow-induced vibration of solitary cylinders, in addition to cylinder clusters, will also be reviewed. Because of rapid development and topicality, the lion's share of this review goes to cross-flow-induced vibration of cylinder arrays.

The obvious fact should be stressed that any structural component placed in flow will vibrate, to a greater or lesser extent. Vibration *per se* is not bad; indeed recent studies, e.g. [5,6], attempt to take advantage of vibration to enhance heat transfer! Only sometimes are the effects of vibration bad: either catastrophic, involving almost immediate failure of components, or more subtle, involving longer-term damage by fretting wear and fatigue which may limit the life span of components. In the former case, it is sufficient for the designer to know how to avoid conditions leading to such catastrophic failures. In the latter case, however, prediction of vibration behaviour is not enough: the link between impact level and fretting wear for different materials and environments, or between vibration-induced stress and fatigue, as the case may be, are essential blocks of knowledge that a designer must equally have at his disposal.

[1] It is obvious, of course, especially to the participants of this Topical Meeting, that this classification involves drastic idealization of the actual flow field in reactors and heat exchangers. It is, nevertheless, retained because, as will become obvious later, the state of knowledge is less than adequate, even for such highly idealized flow conditions, let alone for more realistic flow fields.

2. SOLITARY CYLINDER IN CROSS FLOW

The forces exerted on a solitary bluff body, and especially on a circular cylinder, subjected to cross flow have been studied quite extensively for a very long time, starting with Strouhal's work [7] in 1878. Extensive reviews are available on this topic, e.g. [8-16], mostly treating the classical problem of vortex shedding and the vibrations induced thereby. Interested readers are also referred to Ref. [17,18].

Two phenomena are of interest insofar as flow-induced vibration is concerned: vortex shedding and turbulent buffeting.

2.1 Vibration induced by vortex shedding

The vortex-shedding mechanism is well known and has been experienced by any fast swimmer on his arms. In the subcritical and supercritical flow régimes,[2] characterized by $40 \leq Re \leq 2 \times 10^5$ and $Re \geq 3.5 \times 10^6$, respectively, periodic shedding of vortices — with Strouhal numbers $S \simeq 0.2$ and $S \simeq 0.3$, respectively, for circular cylinders — may cause large amplitude oscillations if system damping is small. In the intermediate, critical range of Reynolds numbers, periodicity is not sharply defined; the values of S, if at all definable, vary widely and can be as high as 0.50. All the foregoing, and especially in the critical régime, are highly dependent on flow confinement (blockage), surface roughness, incident free-stream turbulence intensity and scale, aspect ratio, two dimensionality of flow, and vibration amplitude. The same is true for the fluctuating components of lift and drag induced by periodic vortex shedding; a tabulation of most anterior studies on fluctuating loads on circular cylinders has recently been compiled [19].

If vibration amplitude is not negligibly small, i.e. if it is greater than 0.01D to 0.02D, and if the parameter $m\delta/\rho D^2$ is not too large, then lock-in may arise, where the vortex-shedding frequency, f_{vs}, is entrained to follow the cylinder natural frequency, f_n, over a range of flow velocities; the width of this lock-in range depends on the vibration amplitude. Several studies have been conducted, where the cylinder is either forced to move in a prescribed manner (usually transversely to the flow) or left free to vibrate in response to excitation by the flow. Studies with amplitudes ranging from 0.01D to several diameters have been conducted, e.g. [20-24]. In the lock-in region, there is an increase in the strength of the vortices shed and in the span-wise correlation — hence also in the fluctuating loads exerted on the cylinder.

In addition to normal resonance, harmonic and subharmonic resonances have also been observed, e.g. [20, 24]. Moreover, oscillation predominantly in the streamwise direction at $2f_{vs}$ is also possible [25-27], in contrast to the usual case where the oscillation is predominantly in the cross-stream direction.

The literature on vortex shedding about solitary cylinders and on the forces and motions induced thereon is truly overwhelmingly extensive and cannot possibly be properly reviewed in the limited space available here — considering that there literally exist hundreds of important papers on the various aspects of the topic. Extensive work has been done on various parameters affecting vortex shedding and on the steady and fluctuat-

[2] There is some confusion in nomenclature here: some authors define the three régimes as subcritical, supercritical and transcritical, in ascending Reynolds number sequence, rather than subcritical, critical and supercritical.

ing loads induced on the cylinder, e.g. on the effect of confinement (blockage) [28], end conditions [29,30], aspect ratio [30,31], mass ratio [32], upstream turbulence [33-35], surface roughness [36-38], proximity to a wall [39,40], angle of attack of the flowing fluid [41], and phase composition of the fluid [42], to mention some of the principal ones, where the references quoted are illustrative, rather than exhaustive. These parameters constitute a complex multi-dimensional space, affecting all the foregoing discussion on vortex shedding and on vortex-excited vibration, both quantitatively and qualitatively.

Having said all this, the designer must nevertheless be provided with some tools for dealing with this vibration problem. The usual design strategy is to avoid resonance and lock-in, if possible, e.g. by altering f_n and keeping it ±40% away from f_{vs}. Another possibility, if space permits, is to suppress vortex shedding or reduce its strength, e.g. by splitter plates, helical strakes or a perforated shroud[3] — although some or all of these may be impractical in industrial environments. If neither of the above is possible, then the only option is to increase the damping, so as to reduce the vibration amplitude.

In Blevins' review [2], the expressions for vibration amplitude, A_y, according to three analytical models are given in tabular form, reproduced here as Table 1; however, since most of these models draw upon Hartlen and Currie's remarkable work [43,44], that model has also been added to the Table. The various quantities are defined in the Nomenclature.

Author	Predicted resonant amplitude
Hartlen & Currie [43,44]	$\frac{A_y}{D} = \frac{0.0505}{[3.36 + \delta_r^2]^{\frac{1}{2}} S^2}$
Griffin, Skop & Ramberg [45]	$\frac{A_y}{D} = \frac{1.29\gamma}{[1 + 0.43(2\pi S^2 \delta_r^*)]^{3.35}}$
Iwan & Blevins [46]	$\frac{A_y}{D} = \frac{0.07\gamma}{(\delta_r + 1.9)S^2} \times \left\{0.3 + \frac{0.72}{(\delta_r + 1.9)S}\right\}^{\frac{1}{2}}$
Sarpkaya [47]	$\frac{A_y}{D} = \frac{0.32}{[0.06 + (2\pi S^2 \delta_r)^2]^{\frac{1}{2}}}$

Table 1. Predictions of resonant vortex-induced vibration amplitude of circular cylindrical structures as a function of damping.

All four models give similar answers and correctly predict the self-limiting vortex-induced vibration at large amplitudes. For $\delta_r = \pi^2$, $S = 0.2$, for example, A_y/D is found to be 0.126,[4] 0.113γ, 0.116γ, 0.128, respectively, according to these four models, where it is noted that $1.00 < \gamma < 1.30$ typically. These expressions, however, overpredict the amplitude of vortex-induced vibration if it is small, because, as the vibration becomes of order < 0.1D, the span-wise correlation of vortex shedding becomes smaller, while in these expressions it is assumed to extend over the total length of the cylinder. (For a stationary cylinder, the correlation length varies from 2D to 12D, according to various studies [2], but for vibrating cylinders, it is strongly dependent on the amplitude of motion [48].) Finally, it should be stressed that Table 1 gives the *resonant* amplitude of vibration. For a more general discussion the reader is also referred to Ref.[16,49].

All the above relate to circular cylinders. There also exists a whole body of literature on vortex-shedding and its effects on non-circular bluff bodies of triangular, rectangular, semi-profiled, elliptical and other cross sections [16,18]. These will not be discussed here, nor will the interesting effects of sound on vortex shedding, e.g. [50,51]. Consequently, this review will also not deal with galloping, which is an important consideration in the design of structural members of non-circular cross section subjected to cross flow [2].

2.2 Vibration induced by turbulent buffeting

In addition to vortex-induced forces, a cylinder in cross flow is also subjected to turbulent buffeting due to the random pressure perturbations of the fluid stream acting on the cylinder surface. Buffeting is operative at all flow velocities and flow régimes. The cylinder acts somewhat like a band-absorption filter, extracting energy from the turbulent field in narrow windows about the natural frequencies of the cylinder (mostly about the lowest natural frequency). In this respect, this vibration, which is generally of small amplitude, is similar to that excited by turbulence in axial flow situations and in cross flow through cylinder arrays, to be discussed later.

Recent work by So and Savkar [35] shows that the distinction between vortex shedding and buffeting becomes rather arbitrary when the intensity of incident turbulence exceeds a certain level. Thus, for turbulent intensities of the order of 10% the spectrum of fluctuating lift on the cylinder becomes very much broader than in laboratory situations; indeed, there is no longer an identifiable vortex-shedding "peak". Also, the fluctuating lift coefficients diminish drastically, as compared to those for turbulence intensities of the order of 1% or less.[5] Most importantly, there is some experimental evidence that, under conditions of high turbulence, lock-in is entirely eliminated [52]; indeed, a very recently proposed design procedure [53] for the response of a solitary cylinder to turbulent cross flow takes advantage of this factor — recognizing nevertheless that it has not yet incontrovertibly been established.

2.3 Shell-type vibrations

If the cylinder is sufficiently thin, then in addition to beam-like motions, it may also be subjected to shell-type vibrations [54-57]. Although it was originally thought that these vibrations were sub-harmonically excited by vortex shedding [54] it now seems more likely that they are an aeroelastic flutter phenomenon [56].

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[3] The writer has belatedly become aware of an excellent review of means for suppressing vortex shedding by Zdravkovich [240].

[4] In the Hartlen & Currie model, the numerical value of 3.36 is approximate [44], but the possible error is small, so long as δ_r is not too small.

[5] These findings are important additional reasons why the models of Table 1 are likely to overestimate the amplitude of vibration in industrial environments.

2.4 State of the art for vibration of a solitary cylinder in cross flow

The degree of understanding of the unsteady fluid mechanics of flow about a solitary cylinder in cross flow is far more advanced than any of the other phenomena discussed in this paper. Yet, new insights are still possible, e.g. even in conjunction with the very process of rolling up of vortices behind the cylinder [58].

From the point of view of design data, a great deal remains to be done and, in the context of this paper, especially for cylinders in high turbulence level, "industrial" cross flows. Savkar and So's work [35,52] shows how different the forces acting on a cylinder in such environments can be, as compared to those under normal laboratory conditions, and that even the lock-in phenomenon may be suppressed. Nevertheless, some design guides, as well as compilations of data that can be used as design guides, are available [1-3, 16-19, 53] for judicious use by the seasoned designer.

3. CYLINDER ARRAYS IN CROSS FLOW. I. VIBRATION INDUCED BY BUFFETING AND "VORTEX SHEDDING"

The dynamical behaviour of two cylinders in cross flow, positioned less than n diameters apart, may be radically different from that of a solitary one, where n depends on the geometrical disposition of the pair *vis-à-vis* the flow [59,60]; this is because the overall flow field is affected by the presence of the additional cylinder, and also because of wake interference effects. Several studies have been made — especially in connection with applications to structural vibration induced by wind and marine currents — in which the two cylinders are in tandem, side-by-side or in staggered arrangement. An excellent review has been given by Zdravkovich [60].

The present review, however, will mainly deal with the case of *cylinder arrays*, involving many cylinders in repeated geometric pattern, such as shown in Fig.1.

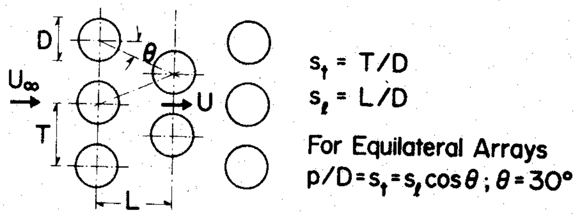

Fig.1. Definition diagram for cylinder arrays in cross flow.

Traditionally, four distinct mechanisms have been thought to be operative, giving rise to flow-induced vibration: turbulent buffeting, vortex shedding, jet switching and fluidelastic instability — *vide* [1-4, 61-72]. This Section will deal with the first two, as well as with the related problem of acoustic resonances.

3.1 Buffeting and "vortex shedding" in cylinder arrays in cross flow: developments to *circa* 1978

Early studies on vibration of tube banks in heat exchangers proceeded on the assumption that vortex shedding was the main, if not the only, mechanism exciting vibration. Hence, the main concern was to determine the pertinent Strouhal numbers for various geometrical layouts of the tubes [73-79], so as to enable designers to avoid resonance under operating conditions. Two kinds of resonance were recognized as having to be avoided: resonance with the tube natural frequencies, and acoustical resonance with the fluid medium contained by the outer walls of the system — the former potentially damaging the tubes, and the latter the walls of the system as well as the hearing of those working in the vicinity [1].

Buffeting excitation of cylinders in arrays was also recognized as giving rise to vibration, similarly to the case of solitary cylinders. The difference here is that the incident turbulent field in a tube bank is generated mostly by the upstream rows of tubes, rather than by free-stream turbulence. It was generally believed that the vibration generated by buffeting excitation is of very small amplitude, and hence considerably less worrisome than that produced by the other phenomena.

Thus, prior to the mid-1960's, buffeting and vortex shedding were considered to be two distinct and unrelated mechanisms giving rise to vibration in cylinder arrays. Moreover, the vibration in both cases was considered to be forced by the flow field, and the possibility of lock-in was not discussed.

In 1964, in a remarkable paper, Owen [80] disputed the very existence of "vortex shedding" within closely spaced cylinder arrays, as well as the separate identity of the two mechanisms of vortex shedding and buffeting. Indeed, it is rather difficult to visualize shear layers separating from the cylinders and neatly rolling up into vortices in the classical manner, in view of the narrowness of the flow passages — let alone the formation of conventional Kármán-Bénard vortex streets. According to Owen, "*Deep enough within a bank of tubes, the cumulative growth of random irregularities in the labyrinthine-like, high-Reynolds-number flow must lead to a state of complete incoherence on which it is difficult to imagine any superposed regular pattern....to be discernible.*" He then went on to propose that the observed periodicity in cylinder arrays may simply be associated with the dominant frequency, f_b, in the fairly broad-band turbulent energy spectrum. Thus, the observed resonances, according to Owen, would be generated by coincidence of f_b with f_n or f_a, for the two resonances mentioned in the first paragraph of this Section. By sound physical reasoning, he then went on to predict that in a cylinder array, characterized by the geometric parameters s_t and s_ℓ, this predominant frequency f_b may be found from the corresponding Strouhal number S_b, as follows:

$$S_b = [3.05(1-1/s_t)^2 + 0.28](s_\ell s_t)^{-1}. \qquad (1)$$

Owen's work did not gain wide acceptance, and Y.N. Chen and his followers [79] maintained that there exists vortex shedding as such, even deep in a cylinder array — although later conceding [81,82] that it may not be of the classical form, and proposing instead the so-called "jet switch", "wake swing" and "jet instability" models, which, although conceptually appealing, were not experimentally substantiated.

In due course, by collecting data from various sources, maps were constructed of Strouhal numbers associated with "vortex shedding", S_{vs}, for various values of s_t and s_ℓ, notably by Fitz-Hugh [83,18] and by Chen [84]. These maps are quite intricate, displaying pockets of high S_{vs} in otherwise low-lying areas — resembling somewhat the map of Europe at the time of the Thirty Years' War. One of the reasons for this was suspected to be [1] that the data in which there was acoustic reinforcement are not separated from those in which there was not. (In a very recent paper, an attempt has been made to expurgate the data [85], and the maps begin to look more orderly.)

Fig.2 shows the Strouhal numbers for different layout angles (*vide* Fig.1) according to Y.N. Chen's and Fitz-Hugh's maps and Owen's equation (1). It is noted that, although Chen and Fitz-Hugh draw their data mostly from the same sources, large discrepancies do nevertheless exist between them. Moreover, these differences are of the same order as those between the vortex-shedding S_{vs} of Chen's and Fitz-Hugh's and the buffeting S_b of Owen's. It was concluded [1] that there is but *one* resonance mechanism at work, but one that has been explained in terms of different hypotheses, by the vortex-shedding advocates, on the one hand, and the buffeting disciples, on the other. For this reason, in this paper the frequency of a dominant periodic component in the fluid medium which varies proportionately to the flow velocity will simply be called Strouhal frequency, f_s, whatever its cause; the corresponding Strouhal number will be denoted by S.

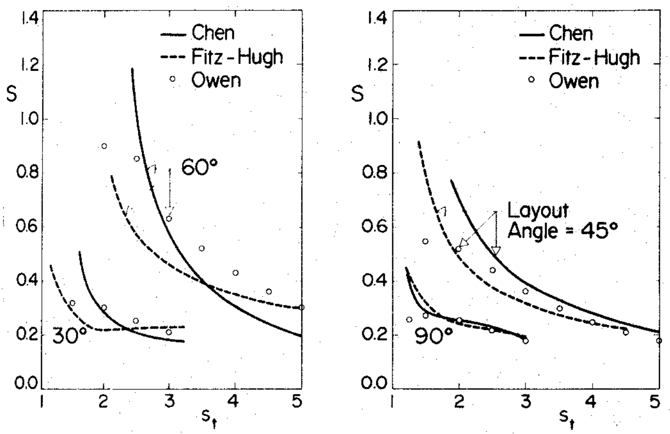

Fig.2. Strouhal numbers of dominant flow periodicity in an array in cross flow, according to Y.N. Chen, Fitz-Hugh and Owen. For definition of layout angle θ, see Fig.1.

However, whatever is the true mechanism underlying periodicity in the flow within cylinder arrays, matters are even more complicated than indicated so far: it was found that flow periodicity does not always arise and that, when it does, it does not necessarily occur throughout the array. The following observations have been made.

(i) For gaseous flows, Gibert *et al.* [86] found that resonances may occur only if $m\delta^{\frac{1}{2}}/\rho D^2 < 30$, and they suggest that lock-in actually takes place. However, the phenomenon was observed at maximum intensity only in the first few rows [87].

(ii) For gaseous flows at low Reynolds numbers (Re < 700), Grover and Weaver [88] found that discrete periodicity in the flow could be observed in the first 15 rows; but at higher Re, increased turbulence levels overwhelm or suppress any discrete f_s.

(iii) For gaseous flows, Zdravkovich and Namork [89] found that, for a closely packed double-row array, the flow structure around the cylinders in the first row was intrinsically different from that in the second — with one Strouhal frequency (which they equate to $2f_s$) appearing in both rows, and another, equal to f_s, only in the second row.

(iv) For liquid flows, Pettigrew *et al.* [90] found that resonances between f_n and f_s occur only in the first few rows, with $0.3 < S < 0.7$, and suggest that such resonances need be considered only if $(U/f_nD)^{-1} < 1$.

(v) For two-phase flows, Pettigrew *et al.* [90] report that they have never observed flow-induced resonances of this type.

These observations were not claimed by the researchers involved to be general, having been made over limited ranges of geometries and Reynolds numbers, specific types of construction (and hence mass ratios and damping parameters), and so on.

All the foregoing make it obvious that *circa* 1978, there was not only a rather confused state of understanding of the unsteady fluid mechanics of interstitial flow in cylinder arrays, but also an incomplete and often contradictory data base available for practical design purposes.

Nevertheless, it was necessary to develop *some* design guidelines, and a good beginning was made by Pettigrew *et al.* [90-92]. They proposed sound analytical models (a) for buffeting vibration, assuming the excitation to be a random process, and (b) for "vortex-shedding" response at resonance, assuming the excitation to be harmonic. Using these models and their extensive measurements of vibration in cylinder arrays, they worked backwards and extracted (i) the pertinent components of the power-spectral-densities of turbulent buffeting excitation and (ii) the fluctuating lift coefficients at resonance between f_s and f_n. This data, together with the analytical models, could then be used as conservative design tools. The same analytical model for buffeting response, in a more elaborate form, was also presented by Blevins [93]. These will be presented, in their up-dated forms, in Sections 3.3 and 3.4.

Before presenting more recent developments, the question of acoustic resonances should be discussed. Periodicity in the flow through cylinder arrays is greatly enhanced if there is coincidence with an acoustic mode of the flow-containing vessel or duct [1], resulting in intense sound amplification and large fluctuating pressure levels on the containing walls. The deafening sound levels that may result from this type of resonance are a very real problem, both for the personnel and for the integrity of the system, although not necessarily causing large vibrations of the cylinders. This phenomenon is associated mainly with gas flows. In water, the speed of sound is higher and equipment dimensions generally smaller, both implying high f_a; at the same time flow velocities are smaller, so that f_s tends to be lower; hence, resonance conditions are less likely to materialize in water. The interested reader is referred to Ref.[79,94-97,1]. The pertinent available design guidelines will be presented in Section 3.5.

3.2 Buffeting and "vortex shedding" in cylinder arrays in cross flow: recent developments

Zdravkovich and Namork pursued their studies of the interstitial flow structure in closely packed normal equilateral arrays (p/D = 1.375) in air, taking surface pressure readings on several cylinders and making hot-wire anemometer traverses between rows [98,99]. Large turbulence intensities, as high as 50% were observed. It was found that the flow structure in the first row is radically different from that in the second, and that the third-row cylinders are affected not only by the narrow wakes behind the second-row cylinders but also by the turbulence generated by the first-row wakes. From the third row onwards, the interstitial flow pattern remains almost unchanged. Moreover, it was found that there exists a strong correlation between velocity and turbulence profiles in between the rows. In these experiments f_n and f_s were far removed from each other, allowing the study of buffeting response of the cylinders away from resonance. Second-row cylinders were found to be excited most.

Chen [100] conducted an extensive set of experiments with both closely-packed and widely-spaced staggered arrays (s_t/s_ℓ = 4.00/3.76, 2.00/3.76; 4.66/1.40; 1.66/2.80; 2.33/1.40) in air, measuring cylinder response, as well as velocity fluctuations in the fluid with hot-wire anemometers. In most cases several f_s were found in the array, one of which was generally dominant; acoustic resonances were also clearly observed, as well as lock-in of f_s to the dominant cavity f_a. For large longitudinal spacings (s_ℓ = 3.76), where it is supposed that there is space enough for something akin to "classical" vortex shedding to occur, the cylinder response displays (i) an important peak corresponding to resonance between f_n and f_s, and (ii) a sizable vibration of the cylinder at both f_n and f_a, in the region of acoustical lock-in of f_s. For arrays with smaller s_ℓ, the observed f_s are supposed to arise from Chen's "jet instability" (s_t/s_ℓ = 4.66/1.40), "wake swing" (s_t/s_ℓ = 1.66/2.88) and "jet switch" (s_t/s_ℓ = 2.33/1.40) mechanisms. What is most important, however, is that in these arrays no peaks were observed in the vibration amplitude *versus* flow-velocity curves at the point of coincidence of f_s with f_n; although the increase in amplitude is not smooth, Chen concedes that the response may practically be computed as due to buffeting. Significantly, the computed fluctuating lift coefficients are in all cases small, generally less than 0.2 and often less than 0.1. The maximum lift coefficient varies from row to row; for s_ℓ = 3.76, for example, the maximum occurs in the fifth row. Finally, it should be said that the significance of this work transcends the important results obtained, in that it represents, in a certain sense, a reconciliation of two divergent schools of thought.

Fluctuating lift coefficients were determined by Gibert *et al.* [101] on single- and double-row arrays in air (p/D = 1.24), by taking pressure measurements on cylinders which were either rigidly held or forced to vibrate with amplitude 10^{-3}D to 10^{-2}D. At resonance ($f_s \simeq f_n$), the fluctuating lift coefficient peak was found to be much broader and six times lower than for a solitary cylinder. For cylinders free to vibrate, lock-in was found to occur, with a lock-in range (in terms of U/fD) several times wider than for a solitary cylinder.

Similar measurements were made by Blevins *et al.* [102] for quasi-in-line (s_t = 1.86, $s_\ell \simeq$ 1.4) twelve-row arrays in air; in addition to fluctuating lift spectra, the span-wise correlation coefficients were also measured, on both immobile and force-vibrated cylinders. The amplitude of the lift force was found to increase from the first, to the second and third rows, but then to remain virtually unchanged (in agreement with Zdravkovich's observations [98]). It was also found that when the spectra, $S_L(f)$, are plotted in dimensionless diagrams of $S_L(f)[\frac{1}{2}\rho DU^2]^{-2}(U/D)$ *versus* fD/U, a collapse of the data for different U is achieved, on sensibly the same curve. This data may be used to complement Blevins' analytical modeling of buffeting response [93]. The spectra were found to display a very weak hump at 0.12 < fD/U < 0.20, most notable in the second row, which was identified "with the remnants of organized vortex shedding". As expected, similarly to a solitary cylinder, the span-wise correlation drastically increases with cylinder motion.

Gorman [103] studied the effect of upstream turbulators on deep cylinder arrays, taken from actual heat exchangers, in water flow (normal equilateral, p/D = 1.36; square, $s_t = s_\ell$ = 1.37, 1.47). These arrays "were known to have large vortex resonances in their inlet row tubes", most pronounced in the first and second rows. Various types of turbulators were tried: screens, flat-strip grids and rigid-tube grids. The first were found to have little effect, while the latter two, with axes perpendicular to the cylinder axes, were found to be remarkably effective in suppressing vibration; this was attributed to their ability to break up the correlation of the fluid forces on the cylinders.

Gorman later extended his experiments to multi-span, multi-pass arrangements [104,105] to model more accurately the situation in real-life heat exchangers.

A similar study to Gorman's [103] on the effect of upstream turbulators on arrays of rigidly held cylinders in air (p/D = 1.2, 1.5, 1.7) was conducted by Savkar and Litzinger [106]. In this case the fluctuating lift and drag coefficients in different rows and the turbulence and other flow characteristics were carefully measured — both without and with turbulators. The following observations, among others, were made: (i) there appears to be a transition from a subcritical to a supercritical régime with increasing Re, akin to that for a solitary cylinder, with a sudden drop in the fluctuating lift coefficient at a critical Re; (ii) the dominant Strouhal peak for both lift and drag spectra occurs at the same frequency, implying that the mechanism underlying interstitial flow periodicity in the array is not that of "classical" vortex shedding; (iii) square-mesh grid turbulators entirely eliminate the Strouhal peak, but the buffeting spectra remain otherwise similar.

Nakamura *et al.* [107] conducted systematic experiments on in-line and staggered arrays in air-water mixtures, measuring the buffeting forces in drag and lift directions. Some very interesting observations were made; e.g., the drag coefficients, typically 2.5 times the corresponding lift ones, display power spectral densities of distinctly different patterns in the slug- and bubbly-flow régimes.

Donaldson and McKnight [108] studied the development of turbulence and acoustic signals in a model cross-flow heat exchanger. No correlation was found between turbulence and acoustic signals, until the cavity fundamental, f_a, is excited, whereupon they become correlated and the sound-pressure level rises rapidly to 165 db. It is suggested that the signals are such that discrete vortex shedding is an unlikely forcing function for the excitation of the acoustic resonance; rather, it is thought that, at a threshold flow velocity, the turbulent energy in the neighbourhood of f_a becomes sufficient to initiate resonance and *thence* organize periodicity ("vortex shedding") in the interstitial flow.

Eisinger [109], among other things, discusses practical measures for avoiding acoustic resonances.

Finally, a series of studies has been published on the speed of sound in passages containing arrays of rigid or flexible cylinders, and on the pertinent acoustic natural frequencies [110-113].

3.3 Buffeting response of cylinder arrays in cross flow: state of knowledge and design guidelines

Turbulence-induced vibration, or buffeting, occurs at all flow velocities and in every conceivable situation. There is basically one analytical model for predicting vibration amplitude, based on random vibration theory; however, in developing this model and rendering it practicable for simple use, two different forms have emerged: one due to Pettigrew and Gorman [91,92,3] and another due to Blevins [93,102].

The Pettigrew and Gorman model, in its initial form, recognizes (i) a finite correlation length for the excitation field, (ii) general boundary conditions, and (iii) general response in all modes of the system. In its simplest form, however, it takes the field to be fully correlated span-wise, the cylinders to be simply supported and the response to be principally in the first mode. Since the data with which it must be used correspond to these same simplifying assumptions, this provides a neat and effective way of obtaining a conservative estimate of vibration amplitude; the mid-span r.m.s. amplitude is given by the remarkably simple form

$$y_{rms}(\tfrac{1}{2}L) = S^{\frac{1}{2}}(f_1)/[4\pi^5 f_1{}^3 m^2 \zeta_1]^{\frac{1}{2}}, \qquad (2)$$

where $S(f_1)$ is the power-spectral-density of the excitation force per unit length, and f_1 and ζ_1 are the cylinder first-mode frequency and damping ratio, respectively; $S(f)$ may be written in the form

$$S^{\frac{1}{2}}(f) = \tfrac{1}{2}\rho DU^2 C_R \qquad (3)$$

where $C_R(f)$ is the effective random excitation coefficient. Based on the data collected by Pettigrew and Gorman, C_R should be read from either of the two lines of Fig.3, corresponding to a frequency on the abscissa equal to f_1.

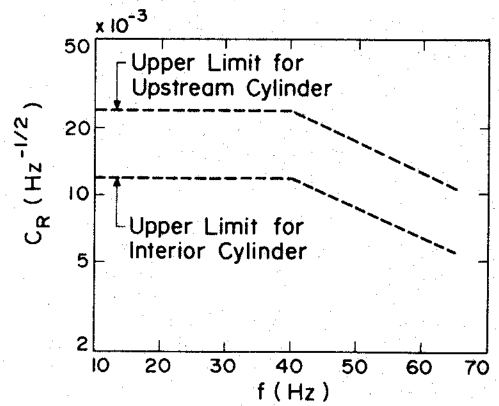

Fig.3. The random excitation coefficient $C_R(f)$ for the calculation of buffeting response of a cylinder in a cylinder array in cross flow, according to Pettigrew and Gorman [equations (2) and (3)].

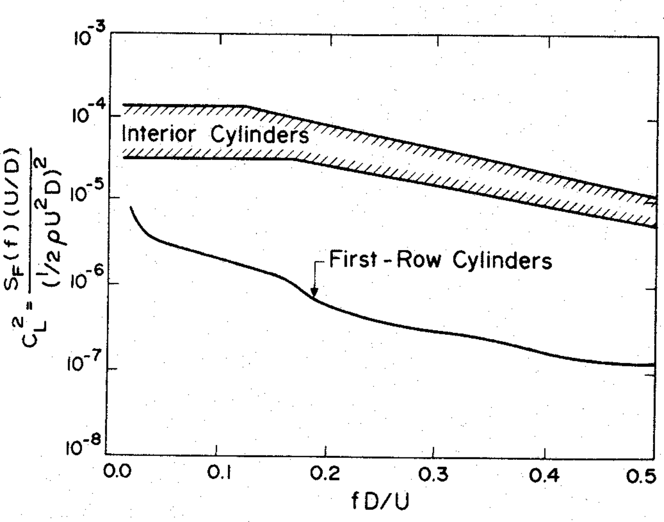

Fig.4. The lift coefficient C_L for buffeting response of a cylinder in a cylinder array in cross flow, according to Blevins, Gibert and Villard [equations (4) and (5)].

Blevins' expression for the mid-span vibration amplitude is equally simple:

$$y_{rms}(\tfrac{1}{2}L) = S_F^{\frac{1}{2}}(f_n)\ J\ \phi_n(\tfrac{1}{2}L)/[64\pi^3 f_n{}^3 m^2 \zeta_n]^{\frac{1}{2}}, \qquad (4)$$

where $S_F(f_n)$ is the power-spectral-density of the excitation field per unit length, J is the joint acceptance which is equal to 1.0 for a perfectly correlated field, $\phi_n(\tfrac{1}{2}L)$ is the eigenfunction at mid-span; the other quantities are the same as usual. Typical values of $S_F(f)$ were measured by Blevins *et al.* [102], and are given in dimensionless form in Fig.4. To be able to easily utilize this figure, it is convenient to write

$$[(U/D)S_F(f)]^{\frac{1}{2}} = \tfrac{1}{2}\rho DU^2 C_L. \qquad (5)$$

Hence, it is easy to see that the two expressions are very similar, except for the nondimensionlizing factor U/D on both ordinate and abscissa of Fig.4.

As this has not been done before, it was considered useful to compare the two expressions for two arbitrarily chosen cases, as shown in Table 2.

Fluid	D (mm)	f_n (Hz)	U (m/s)	$(y_{rms})_{PG} / (y_{rms})_{BGV}$
Water	19	40	2.0	2.6
Air	25	60	50	0.5

Table 2. Comparison of mid-span buffeting amplitudes of inner-row cylinders, assuming simply-supported ends and fully correlated excitation field, according to the Pettigrew & Gorman and Blevins, Gibert & Villard variants of the analytical model.

The agreement is remarkably close, considering that Pettigrew and Gorman determined C_R from experiments in water, while Blevins *et al.* from experiments in air. This is not an extensive comparison, however, and no doubt cases may be found where the two are further apart.

One important disagreement that should be noted is that in Fig.3 the upstream cylinders are shown to be subjected to higher excitation, while the opposite is true in Fig.4. This likely reflects differences in upstream turbulence intensities in the two sets of experimental data.

Pettigrew and Gorman also give equivalent design guides for two-phase flow [91,92].

It should finally be noted that in using equations (2)-(4) f_n(or f_1) and ζ_n(or ζ_1) should be the values measured in stationary fluid, rather than *in vacuo*.

3.4 Strouhal resonance in cylinder arrays in cross flow: state of knowledge and design guidelines

Although the work of the past three years' cannot be said to have answered all outstanding questions by any means, some solid progress has nevertheless been achieved.

It appears clear that Strouhal periodicity in the interstitial flow within the first few rows of cylinder arrays does commonly occur, unless the upstream turbulence level is sufficiently high to suppress it. Deep within the array, on the other hand, Strouhal periodicity may or may not survive, depending on Reynolds number, cylinder layout geometry, fluid-to-cylinder mass ratio and cylinder vibration amplitude.

In all cases, however, if resonance with an acoustic mode should be established, then periodicity is strong and omnipresent.

From the point of view of cylinder vibration, Strouhal resonance — defined as amplified vibration, over and above the buffeting background, when f_s coincides with f_n — may occur

(i) for tightly spaced arrays in gaseous flows, in the first couple of rows — provided that upstream turbulence is kept quite low;

(ii) for tightly spaced arrays in liquid flows, in the first few rows — unless turbulence levels are very high;

(iii) for widely spaced arrays, throughout the array;

(iv) for any type of array if there should exist a *triple* coincidence of f_s, f_a and f_n.[6]

Clearly, these are tentative generalizations, since what is "very high" or "quite low","tight" or "loose" has not quantitatively been specified. Hence, until this is done, the designer is faced with having to design to avoid Strouhal resonances, "just in case".

If the Strouhal number is known fairly accurately, then one can design such that U/f_nD be ±40 or ±50% away from S^{-1}. If S is not known with sufficient accuracy, one may use Pettigrew's design guide [3,92]: noting that for typical cases $1/3 < S < 2/3$, it is suggested that the design be such that $U/f_nD < 1$, which is sufficiently less than $(2/3)^{-1}$ "to take care of possible 'locking-in' of the wake."

As noted previously, however, the design criterion $U/f_nD < 1$ may lead to impossibly over-conservative designs [3,4]. Hence, a second step in the design procedure is to calculate the vibration amplitude at the point of the presumed resonance: it might be sufficiently small that the cylinder array could live with it. Pettigrew [3] gives the very simple expression for the amplitude, A_y, in the lift direction

$$A_y(x) = \frac{\phi_n(x)}{8\pi^2 f_n^{\,2}\zeta_n} \int_0^L F(x')\phi_n(x')dx', \qquad (6)$$

where F(x) is the correlated periodic wake shedding force along the cylinder, which may be expressed as

$$F(x) = \tfrac{1}{2}\rho D U^2(x)\, C_L,$$

C_L being the lift coefficient. It may be seen that, especially for gases, where ρ is small, F(x) may also be quite small. Unfortunately values of C_L are not sufficiently well documented, although Pettigrew *et al.* [90-92,3] and Y.N. Chen [82,100] do give some values. It is noted that C_L is generally smaller than 0.10 and can be as low as 0.01 for $p/D \le 1.5$ [3,4,92], while it can take much larger values for more widely spaced arrays, where s_ℓ, $s_t \sim O(3)$ or $O(4)$ [82,100]. These lift coefficients (and the span-wise correlation integrated into them) are likely to be strongly dependent on vibration amplitude; however, there is insufficient information at present to be able to take this factor into account. Pettigrew [3] recommends that the maximum vibration amplitude be kept below 0.02D.

Finally, the use of upstream turbulators is recommended, wherever feasible, at least for tightly spaced arrays, so that Strouhal resonances may be suppressed altogether [103,105,106]. However, for more widely spaced arrays, it is unlikely that the same effect may be achieved by such turbulators.

[6] Under such circumstances the array becomes defunct within hours [1].

3.5 Acoustic resonances in cylinder arrays in cross flow: state of knowledge and design guidelines

As stated previously, if f_s should coincide with f_a, it is possible to excite acoustic resonance in the system. However, there exists evidence suggesting that resonance may also occur if $f_a \approx 2f_s$, where the Strouhal frequency is captured by an acoustic one — *vide*, e.g., [114]. Moreover, it has also been found [97,85] that resonance may be initiated with $0.4 < f_s/f_a < 0.8$, depending on the number of rows in the array. Hence, the problem is by no means simple.

Furthermore, even in the simplest case of $f_s \approx f_a$, frequency coincidence is a necessary but not sufficient condition for acoustic resonance to occur. It is additionally necessary for the strength of the excitation to be sufficiently high to overcome acoustic damping, which is a function of system construction as well as of geometry [85,115,116].

Chen [79,82,94] has proposed empirical guidelines for the occurence of acoustic resonance for in-line arrays, which may be stated as follows:

(i) resonance *may* occur if $U \ge U_c$, where U_c is calculated such that $f_s \ge f_a$ through

$$U_c = f_a D/S\ ;$$

(ii) resonance *will* occur if $U \ge U_c$ and additionally

$$\Psi \equiv (Re/S)[1 - 1/x_\ell]^2(1/s_t) > 600 \text{ or } 2{,}000,$$

where 600 applies to laboratory cases and 2,000 to real heat exchangers (where acoustic damping is likely to be higher).

A simpler criterion has been proposed by Bryce *et al.* [97], stating that resonance will only occur if

$$U/f_a D > 2(s_\ell - 0.5).$$

Ulrich, more recently, has proposed another criterion [116].

Unfortunately no cross-comparisons of the relative effectiveness of these criteria have been published, and the designer is left unaided in this respect [117]. In cases where acoustic resonance appears to be unavoidable, then the use of baffles to acoustically detune the system seems the most effective design strategy [96,109]; Zdravkovich and Nuttall's suggestion [114] of (i) using unequal longitudinal pitches in successive rows and (ii) removing some of the tubes in the array, to detune the excitation, should also be mentioned.

Finally, it should be reiterated that f_a will vary with the geometric density of cylinders in the array through variations in the sonic speed [97,110-113], and this should be taken into account.

4. CYLINDER ARRAYS IN CROSS FLOW. II. FLUIDELASTIC INSTABILITIES

This Section deals with self-excited vibrations of cylinder arrays in cross flow, otherwise known as fluidelastic instabilities.

4.1 Fluidelastic instabilities: developments to *circa* 1980

In 1962 Roberts uncovered the possibility of self-excited oscillations of a staggered row of cylinders in cross flow, where only alternate cylinders were free to move in the in-flow direction [118,119]. The mechanism proposed to explain the phenomenon was a Coanda-like "jet switching", where the jet pairing between adjacent cylinders switches direction in synchronism with cylinder motions, in such a manner that energy is extracted by the cylinders from the flow, in the course of a cycle of oscillation; if this energy is sufficiently high to offset dissipation, then motion is amplified to a self-sustaining limit cycle. Roberts developed a complex semi-empirical analytical model for predicting critical flow velocity for the onset of instability. Moreover, he suggested that the proposed mechanism may only occur if $U/f_nD > 12$ approximately, which would allow sufficient time for jet switching to actually take place.

In 1970 Connors studied the problem of a single row of cylinders, all of which were free to vibrate [120]. Drawing on Roberts' work, he proposed a quasi-static semi-empirical analytical model to explain the observed self-excited oscillations. In this model the displacement of cylinders with respect to their neighbours results in modified forces coming into being *vis-à-vis* those of the original configuration; for a given pattern of motions, sufficient energy may be extracted from the flow, independently of jet switching, to overcome damping and sustain the vibration. For his particular single-row array and an assumed intercylinder modal pattern, Connors obtained the following relationship for the critical flow velocity, U_c:

$$U_c/f_nD = 9.9(m\delta/\rho D^2)^{\frac{1}{2}}. \qquad (6)$$

While Roberts' work was largely ignored, Connors' equation became widely accepted. Moreover, unfortunately, equation (6) was subsequently misused to calculate U_c for systems for which it was never intended : notably, for multi-row cylinder arrays, for which it has subsequently been shown to be non-conservative.

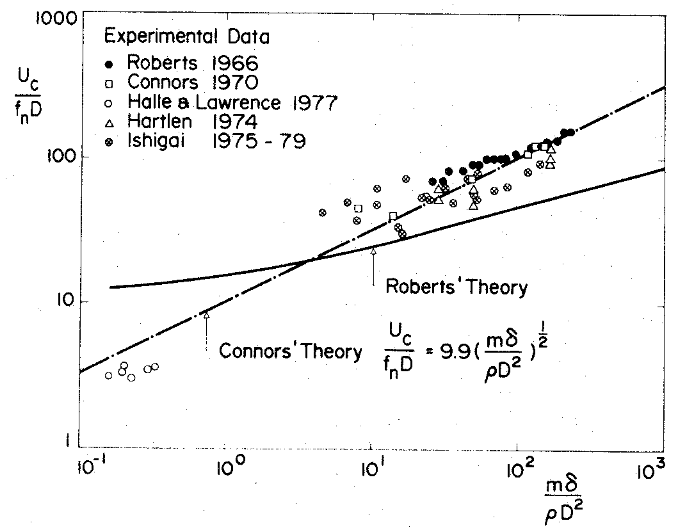

Fig.5. The threshold of fluidelastic instability of a single row of cylinders in cross flow.

An interesting comparison of Connors' and Roberts' theoretical U_c/f_nD and their own and others' experimental data [119-121] was later undertaken [1] and is reproduced here in Fig.5. It appears that, irrespective of whether U_c/f_nD is greater or less than 12, the two theories are relatively close to each other; more importantly, all the experimental results are self-consistent and agree fairly well with both theories, although those for small U_c/f_nD agree much better with equation (6). This suggests that, basically, the same type of phenomenon occurs in all the experiments, but that it was explained in terms of different mechanisms by Roberts and Connors.

Connors' model was mathematically formalized by Blevins [124] and extended, conceptually at least, to deal with cylinder arrays [125], as well as to include the effect of changing damping with flow [126][7]. However, the general form of equation (6) survived, except that it was recognized that the constant 9.9 had to be different for cylinder arrays; i.e., the criterion for instability should have the form

$$U_c/f_nD = K\ (m\delta/\rho D^2)^{\frac{1}{2}}. \qquad (7)$$

The general response of a cylinder in an array subjected to cross flow is typically that shown in Fig. 6, where response to flow periodicity ("vortex shedding") may or may not develop, as pointed out in Section 3. Fluidelastic instability, on the other hand, apparently always develops and it is the most serious and damaging phenomenon to contend with [1]; yet its prediction remained rather uncertain.

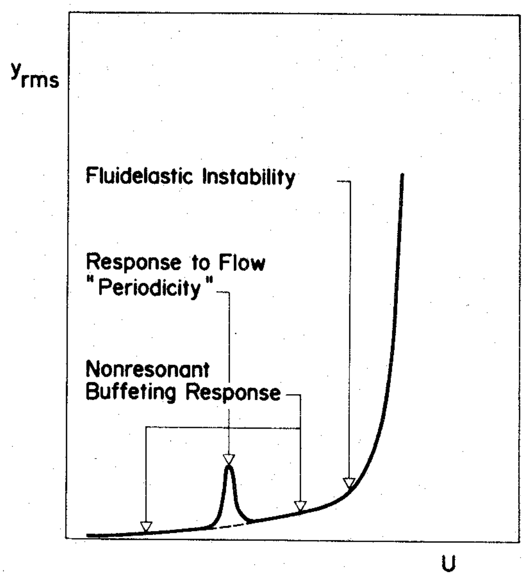

Fig.6. Idealized response of a cylinder in an array subjected to cross flow.

A great deal of experimental data was gathered to obtain suitable values of K, applicable to cylinder arrays of different geometries, different fluids, etc. [121,127-137,86-92] — without wishing to imply that this was the only, or even the principal, concern of most of these papers. However, it was found impossible to define a unique K for all systems. Soper attempted to define $K = K(s_t, s_\ell)$; although his own results were self-consistent, nevertheless large discrepancies remained when comparing with other researchers' data [135].

Hence, the only sensible thing to do seemed to be to define a *minimum* value of K, K_{min}, such that all occurrences of fluidelastic instability would occur at higher U_c/f_nD than those computed with equation (7) and $K = K_{min}$. Connors, from work on square-geometry arrays, proposed [131] $K = 0.37 + 1.76\ s_t$ for $1.41 \le s_t \le 2.12$, yielding $K_{min} = 2.9$. Soper's experiments [135] suggest $K_{min} \simeq 2.0$ for rotated (parallel) triangular arrays, and higher values for other geometries. How-

[7] This inclusion is controversial, and prominent authorities in the field dispute its validity, on fundamental grounds.

ever, it was the value of $K_{min} = 3.3$ proposed by Pettigrew *et al.* [90-92,3] which gained most widespread acceptance; indeed, when tested against actual heat exchangers which developed, or not, fluidelastic instabilities, it was shown to stand up very well [90-92]. However, it should be noted that important disagreements have arisen as to exactly how the various terms in equation (7) should be interpreted (Section 4.2).

Some more fundamental work was undertaken by Zdravkovich and his colleagues [138-140,98,99] on single- and multi-row arrays in airflow, measuring (i) pressure distributions about the cylinders, thereby deriving force coefficients in the lift and drag directions (in both original and displaced geometries), (ii) flow velocity distributions in the interstitial passages, (iii) vibration amplitudes, and (iv) patterns of inter-cylinder motions through high-speed cinematography. The observed patterns of motion were found to be well correlated with those suggested by the forces measured in statically "deformed" array configurations, corresponding to different parts of a cycle of oscillation. Moreover, it was found that (a) systems with only one flexible cylinder display relatively small vibration amplitude, the amplitude increasing with the number of flexible cylinders in the array, being a maximum when all cylinders are flexible; (b) turbulators, with axes normal to the cylinder axes greatly affect the fluidelastic behaviour of the system and delay the onset of instability; (c) cylinders in the second row are most susceptible to oscillation.

However, such fundamental studies were quite rare. But with very few exceptions, equation (7) was considered to be valid for any system, if only the magic K could be determined — a view tenaciously held, even in the face of accumulating contrary evidence. *It is important to recognize here that all fluid-mechanical aspects of the instability are hidden within K.* As shown by Heinecke [141], for a one-degree-of-freedom system excited aerodynamically by a force of the type $\frac{1}{2}C\rho U^2 \sin \Omega t$, energy considerations will inevitably lead to resonant conditions when an equation of the form of (7) is satisfied: for the energy input per cycle of oscillation is proportional to $\rho U^2/m$ and energy dissipation per cycle is proportional to $f_n^2\delta$; hence, balance between these two energies is obtained when $(U/f_n) \propto (m\delta/\rho)^{\frac{1}{2}}$, which leads directly to equation (7).

In this light, it is not surprising that some purely empirical design guides for avoiding excessive vibration in heat exchangers [142-144], developed independently of the aforementioned work, can be re-cast in the form of equation (7), as pointed out in Ref.[1,4]. However, the system of cylinders in an array is generally a multi-degree-of-freedom system; also, the exciting force is known to be displacement- and velocity-dependent. Hence, it is by no means obvious that the simple form of equation (7) is necessarily valid.

Indeed, towards the end of the 1970's solid experimental evidence began to be gathered against the Connors-Blevins semi-analytical model and the simple form of equation (7) for predicting U_c, as follows.

(i) Southworth and Zdravkovich [138] and Weaver and Grover [145] have shown that even if all cylinders but one are held rigidly immobile, then that one becomes unstable at the prescribed flow, contrary to Connors' theory which requires a certain pattern of inter-cylinder displacements.

(ii) Gorman [130] showed that in arrays of mixed brass and stainless steel tubes, the instability develops for the more flexible brass tubes, while the others remain immobile, once again contrary to the Connors-Blevins theoretical model.

(iii) Whereas theory predicts that detuning the cylinders should result in a monotonic stabilizing effect, at least for a single row [124], Weaver and Lever [146] found that this effect disappears if detuning is greater than a small percentage of the basic frequency.

(iv) Experiments by Weaver and Grover [145], in which δ was varied whilst the other parameters were kept constant, suggest that $U_c/f_nD \propto \delta^{0.21}$; it was proposed that, for an expression of the form of equation (7), the exponents on the two dimensionless numbers, $m/\rho D^2$ and δ, need not be the same.

(v) Evidence by Nicolet *et al.* [87] points to $U_c/f_nD \propto \delta^{0.1}$ to $\delta^{0.3}$.

(vi) Other experimental evidence by Nicolet and Gibert *et al.* [86,87], in which ρ was varied whilst δ remained constant, suggests that $U_c/f_nD \propto (m/\rho D^2)^{0.3}$.

Hence, by 1977/78 it became obvious that all was not well with fluidelastic instability modelling. Yet, in the absence of a better predictive tool, designers had little choice but to use some form of equation (7) for predicting U_c.

Two attempts were made to devise new analytical models, in this period, by Balsa [147] and S.S. Chen [148]. Faced with the problem of dealing with the separated flow in the arrays, they both considered the flow to be irrotational and modelled it using potential flow theory; thus, the presence of wakes was totally ignored.

Balsa's theory is quite involved, utilizing the method of matched asymptotic expansions; in the end it results in an expression for *divergence* instability, which, in the notation used here, may be written as $U_c/f_nD = C(m/\rho D^2)^{\frac{1}{2}}$, where C is a function of the characteristic gap between cylinders and an "interaction parameter". The absence of δ is noteworthy: as the flow field is totally conservative, the system must lose stability by divergence, which is independent of velocity-dependent forces.

Chen [148] formulates the system of cylinder-to-cylinder interactions and obtains expressions for coupled inertial, damping and stiffness (displacement-dependent) terms. When these are evaluated by potential flow theory, the results are shown to compare poorly with experiments. This work, however, is important in view of later developments, as will be seen in Section 4.4.

4.2 On the utilization of equation (7)

In addition to questions about its validity, questions also arose on the interpretation of most of the terms of equation (7).

It was generally accepted that f_n should be the frequency of oscillation in the quiescent fluid medium. However, for multi-span cylinders there exist groups of natural frequencies with the same mode shape per span (*vide* Section 5, [90,104]). If the flow velocity is non-uniform over the total multi-span length of the cylinder, it is not always the mode with the lowest natural frequency that will be excited with minimum U [90]. Hence, equation (7) should be applied to all modes of the multiply supported structure, taking account of non-uniform flow velocity distribution U(x) by calculating an effective U, as follows [129,18]:

$$U_{eff} = \left[\int_0^L U^2(x)\phi^2(x)dx \Big/ \int_0^L \phi^2(x)dx \right]^{\frac{1}{2}},$$

where L is the total length.

For closely packed arrays in dense fluids there exist *bands* of frequencies, containing many first-mode-shape natural frequencies. Normally, the lowest one within this band is utilized, and methods for calculating it are reviewed in Section 5. It has to be recognized that in such situations the hydrodynamic or "added" mass may be considerably higher than that of a solitary cylinder in unconfined fluid — which is equal to ρA. Here, there is divergence of practice: some do utilize this higher added mass, whilst others use the one pertaining to a solitary cylinder ($=\rho A$), which can be several times smaller.

U is usually defined as the reference gap velocity. Some researchers, however, prefer to use the so-called minimum-gap velocity, which is the velocity at the minimum diagonal space between staggered arrays, and which for certain layout geometries may be considerably higher than the reference gap velocity.

The question of which δ to use is perhaps the most serious. Some researchers use δ *in vacuo* (approximately measured in still air), some in the pertinent quiescent fluid medium, while others [126] the predicted δ that might exist under flow and vibration conditions — despite the fact that $\delta = 0$ at $U = U_c$. The values of δ, according to these three interpretations, may differ from one another by as much as one order of magnitude. Here, the difficulty of measuring δ in liquid should be noted: if a cylinder is excited in quiescent liquid, then, through the effects of hydrodynamic coupling, energy "leaks away" to excite the surrounding cylinders, giving false estimates of δ. The matter is usually resolved by rigidly clamping all surrounding cylinders before measuring δ, although it is uncertain what exactly this value of δ represents.

The value of m is usually taken to be the effective mass, including the "added" or hydrodynamic mass — *vide* Section 5. Here, however, there is the same divergence of practice referred to in the paragraph discussing f_n, as to which added mass is actually used.

The interpretation of D has remained unchallenged, so far: it is the outside diameter of the cylinder!

Finally, from the point of view of interpreting experimental results, the onset of instability, and hence U_c, is defined in different ways. Some define it in terms of a threshold amplitude, when it exceeds a certain fraction of the diameter; others from the intersection of the tangent to the fast-rising portion of Fig.6 with the buffeting response tangent; others [145] in terms of the onset of coherent sinusoidal oscillation; others still, from the clanging sound of cylinders impacting.

4.3 Recent, mainly experimental studies on fluidelastic instability

Blevins *et al.* [102] conducted an extensive experimental programme. They confirmed that a single flexible cylinder among rigid ones in an array does become unstable. Detuning effects were studied and it was found that progressive detuning of whole rows of cylinders is most effective; however, detuning was found to be less effective in stabilizing the system than predicted by existing theory. Among other things, it is reported that in these tests (i) $K_{min} \simeq 2.5$, although for slightly spiralling arrays $K \simeq 1.7$; (ii) there is no discernible pattern of inter-cylinder motions.

Gibert *et al.* [101] report on a massive experimental programme with arrays of different geometries and pitch-to-diameter ratios, in air and water flows. Among other things, it was found that (i) for $m\delta/\rho D^2 > 20$ and $20 > m\delta/\rho D^2 > 7$, the experimental data suggest that $U_c/f_nD \propto (m\delta/\rho D^2)^{\frac{1}{2}}$ and $U_c/f_nD \propto (m\delta/\rho D^2)^{0\cdot 3}$, respectively; (ii) for $m\delta/\rho D^2 < 7$, there is interference with Strouhal lock-in for very lightly damped cylinders; (iii) the total set of their data suggest a relationship of the form $U_c/f_nD = 2.7(m\delta/\rho D^2)^{0\cdot 34}$.

Another extensive experimental study was conducted by Heilker and Vincent [149] for various types of arrays in water and air-water mixtures. It is found that $U_c/f_nD = K(m\delta/\rho D^2)^{\frac{1}{2}}$ approximately, with $K_{min} = 5.0$. In the presentation of the results, however, the values of δ measured in the *flowing fluid* are used; at least for U close to U_c, these values of δ would be lower than δ in quiescent fluid. Hence, had the latter been used, then a lower K_{min} would have been obtained.

Chen and Jendrzejczyk [150] studied the evolution of measured δ with increasing U and report that (i) initially, damping increases with U; (ii) it then displays a dip associated with Strouhal resonance, and subsequently increases again; (iii) as U_c is approached, δ decreases rapidly, suggesting that it will eventually become zero, at which point structural modal damping would be equal to the *negative* flow-induced damping.

The same authors have also presented an extensive set of experimental data [151] for twelve different arrays of slightly varying spacings and different geometries, mass ratios, damping and progressive row-by-row detuning, in water flow. It was found that (i) detuning stabilizes the system, which agrees with previous work [138,146,124], albeit to a lesser degree than previously suggested [146,124]; (ii) cross-grid-type upstream turbulators stabilize the system, whilst grids parallel to the cylinder axes destabilize it, where both stabilizing [139] and destabilizing [129] effects had previously been reported; (iii) Strouhal resonance may interact with and reinforce fluidelastic instability; (iv) the amplitude-velocity graphs (Fig.6) display multiple U_c's, which are attributed to different coupled modes of the system; (v) the most critical cylinders in this study lie in the first row. Finally, it is suggested that the results obtained would indicate that $U_c/f_nD = 2.49(m\delta/\rho D^2)^{0\cdot 52}$.

Weaver and co-workers have studied a series of specialized questions relating to fluidelastic instabilities [152-156]. It was found [152] that the minimum number of cylinder rows necessary to study fluidelastic instabilities is six — at least for parallel triangular arrays and $p/D = 1.375$ in air. It was also found that the instability first develops in the third and fourth rows, and that the presence of downstream rows does not significantly affect the instability threshold.

Weaver and El-Kashlan [153] extended previous work in which δ was varied independently [145] and also varied $m/\rho D^2$ while keeping δ constant, finding that

$$U_c/f_nD = C(m/\rho D^2)^{0\cdot 29}\,\delta^{0\cdot 21}$$

is more appropriate than equation (7). The 0.21 exponent agrees with that of previous studies [145], while $(m/\rho D^2)^{0\cdot 29}$ agrees with the work of Ref.[86,87], and to some extent with that of Ref.[101].

Yeung and Weaver [154] studied the effect of approach-velocity direction on triangular arrays with $p/D = 1.5$, in water. The flow direction was varied through an angle $0° \le \alpha \le 30°$, where $\alpha = 0°$ corresponds to a parallel triangular array and $\alpha = 30°$ to a normal triangular array. It was found that as α is increased from 0°, U_c/f_nD rises slowly, up to $\alpha \simeq 20°$, beyond which there is an upwards jump to the U_c/f_nD value associated with $\alpha = 30°$, pointing to the importance of the interstitial flow pattern in the array.

Weaver and Koroyannakis [155] studied the effect of different ("asymmetric") stiffnesses in the in-flow and cross-flow directions — partly in order to simulate the situation in the U-bend region of heat exchangers. These experiments, with parallel triangular arrays ($p/D = 1.375$) in water flow, show that U_c/f_nD may be up to 20% higher for asymmetric stiffnesses as compared to the symmetric case, this effect being independent of the orientation in which the system is stiffer. Whirling, diagonal and figure-8 motions of the cylinders were observed.

The same authors also studied the geometrically identical parallel triangular array (p/D = 1.375) both in air and water flow [156]. It was found that the behaviour of arrays in water is much less "regular" than in air, due to hydrodynamic effects — similarly to Ref. [151]; the cylinders first become unstable in one of the lowest frequencies in the band of coupled frequencies (Section 5), and then progressively higher-frequency modes become involved.

Heinecke and Mohr [157] studied the response of in-line arrays ($s_t = s_\ell$ = 2.88,2.30,1.44,1.15) in air. Some of the cylinders were instrumented around their circumference, to yield unsteady pressure distributions about the cylinders and static and dynamic force coefficients; the corresponding vibration amplitudes were also measured. From these measurements it is concluded that (a) for the s_t = 2.88 and 2.30 arrays, the dominant vibration-inducing mechanisms, at least in the flow-velocity range studied, are Strouhal resonance and buffeting, and (b) only for the smaller two spacings do fluidelastic instabilities take place. Moreover, two distinct types of instability are suggested: (i) a "galloping" instability, for the s_t = 1.44 array, which is akin to wake-galloping of transmission lines, and which may occur even if only one cylinder is flexible among rigid ones; (ii) an aerodynamic-stiffness-controlled mechanism, termed "fluidelastically coupled instability", which occurs for the s_t = 1.15 array, and which develops only if all cylinders (or at least a sufficiently large group) are free to vibrate.

Rémy [158] studied an in-line array ($s_t = s_\ell$ = 1.44) in air and water, as well as in air-water mixtures. Contrary to others' findings [90,91,137,149], no fluidelastic instabilities in two-phase flow were found, because "the existence of two phases limits ... coupling between the tubes"; instead, vibration amplitudes were found to increase with flow to high levels, but with no distinct instability threshold. Hence, this is perhaps a question of definition of fluidelastic instability (*vide* Section 4.2): if its occurrence had been defined in terms of exceeding a certain vibration level, then Rémy's conclusion might have been different.

Fujita *et al.* [159] and Connors [160] study the effect on fluidelastic instability of two unusual flow conditions: a thin two-dimensional jet impinging on multiply-supported cylindrical arrays [159] and a "corridor" flow on the outer edge of the array [160], in both cases the cross flow eventually diffusing into the array.

Gorman [104,105] and Halle *et al.* [161,162] examine the more realistic (for heat exchangers) configuration of multi-span arrays under multi-pass flow conditions. It was found that fluidelastic instabilities occur at higher U_c/f_nD than in the equivalent single-span system; it is concluded that equation (7) with K_{min} = 3.3 is a good conservative criterion.

Finally, a cross section of the very real problems faced by designers and experimenters working on large facilities, that researchers should be aware of, are highlighted in Ref. [158,163-165]. Bai and Rémy [158, 163,164] discuss a large number of experimental data obtained in large scale facilities. Reflecting the formidable difficulties in determining δ therein, they propose a design criterion of the form

$$U_c/f_nD = K'(m/\rho D^2)^{\frac{1}{2}},$$

where $K' = K\delta^{\frac{1}{2}}$. Thus, in 1981, we come full circle back to Thorngren's criterion for avoiding excessive vibration in heat exchangers [143], except that now K'_{min}= 0.70, as compared to Thorngren's $K' = 2.76[p/D-1]^{\frac{1}{2}}$; taking p/D = 1.31 to 1.44, which covers the range of Bai-Rémy experiments, values of 1.54 and 1.83, respectively, are obtained for K', and it is easily seen that the Thorngren criterion is non-conservative. Indeed, as previously shown [1,4] most of these older design criteria [142-144] are non-conservative.

4.4 Analytical developments on fluidelastic instabilities: 1980-82

The theoretical development of the subject has been both rapid and exciting.

Whiston and Thomas [166] re-examine Blevins' analysis of a single row of cylinders in cross flow and, using his data and geometry, extend it to arrays, considering only position-dependent forces. The single-row analysis is extended to deal with patterns of motion of wavelength λ, other than the Connors-Blevins λ=4T (i.e. such that the pattern periodically repeats itself every four cylinder pitches); it is found that λ = 4T is the least stable pattern. The effects of frequency detuning are discussed, and it is found that Blevins' large detuning effect applies only to small δ; for large δ it is very small. The model is successfully extended to staggered cylinder arrays, by considering a union-jack kernel of nine cylinders and recognizing that the interstitial flow velocity in the minimum gaps (taking account of the wakes) may be larger than the maximum in the gap between two cylinders in the same row; the extension to in-line arrays is more tenuous. In all cases a relationship of the form of equation (7) is obtained, with K_{min}= 2.8 for arrays. The discussion on mechanical coupling and detuning effects is extensive, interesting and illuminating.

In 1980, Tanaka and Takahara published a remarkable paper [167]. Considering a kernel of five cylinders of an in-line array, a full unsteady aerodynamic force formulation was utilized; e.g., the force on cylinder i in the x-direction is given by

$$F_{xi} = \tfrac{1}{2}\rho U^2 \sum_{j=1}^{5} \{C_{xxj}x_j + C_{xyj}y_j\},$$

where C_{xxj} and C_{xyj} are the aerodynamic force coefficients on cylinder i in the x-direction due to vibration of cylinder j in the x- and y-directions, respectively; x_j and y_j are the amplitudes of motion of the two directions. Similar expressions obtain for the other forces acting on the five cylinders. The C's are complex numbers and functions of the reduced velocity of motion, U/fD. Hence, they contain acceleration-, velocity- and displacement-dependent components: i.e., added-mass, aerodynamic damping and aerodynamic stiffness components. Solving the resultant matrix equation yields the complex eigenfrequencies of the system, from which the stability may be determined.

The force coefficients were measured experimentally by forcing each cylinder in the five-cylinder kernel to vibrate in water, over a range of U/fD, for the specific in-line arrangement of $s_t = s_\ell$ = 1.33. Using these in the analysis yields values of U_c/f_nD in excellent agreement with experimental values, both in air and water [167,168]. Further measurements of the C's for other s_t and s_ℓ yields, once more, excellent agreement with experiment [169]. For the $s_t = s_\ell$ = 1.33 in-line array, it was found [168] that

$U_c/f_nD \propto (m/\rho D^2)^{\frac{1}{2}}\delta^{\frac{1}{2}}$ for low-density flow, and

$U_c/f_nD \propto (m/\rho D^2)^{1/3}\delta^{1/5}$ for high-density flow

which agrees with others' experimental observations [145, 153,86,87] — whilst for $s_t = s_\ell = 2.0$ [169] $U_c/f_nD \propto (m\delta/\rho D^2)^{3/4}$. Moreover, as s_t, s_ℓ are increased, U_c/f_nD becomes larger, but beyond $s_t = s_\ell = 2.0$ it remains almost constant. Different types of detuning were found to variously stabilize and even to destabilize the system.

Chen [170,171] developed further his previous model [148] in the light of Tanaka and Takahara's work [167, 168]. Instead of considering very large arrays, now five-cylinder sub-systems were examined and, when it came to numerical calculations, Tanaka and Takahara's measured force coefficients were utilized, rather than those previously obtained by potential flow theory. Some remarkable insights were obtained thereby. Two distinct mechanisms are distinguished:

(i) "Fluid-damping-controlled" instability, induced by negative aerodynamic damping forces, which may occur even if one cylinder is flexible among rigid ones — cf. [157]; this mechanism is dominant for low $m\delta/\rho D^2$, i.e. for high density flows. For the pure form of this instability, $U_c/f_nD = K^*(m\delta/\rho D^2)$, where K^* is a function of U/fD, and f_n, m and δ are all appropriately evaluated either in vacuum or in the quiescent fluid medium.

(ii) "Fluidelastic-stiffness-controlled" instability, which is dependent on coupling and phase differences in the motions of neighbouring cylinders; this mechanism is dominant for high $m\delta/\rho D^2$, i.e. for gaseous flows. For the pure form of this instability, $U_c/f_nD \propto (m\delta/\rho D^2)^{\frac{1}{2}}$. In general, both mechanisms will be implicated and the instability criterion will have the functional form

$$U_c/f_nD = F(m\delta/\rho D^2, m/\rho D^2, s_t, s_\ell, \text{turbulence characteristics}).$$

In addition to oscillatory instabilities, divergence is also predicted to occur.

Calculated U_c/f_nD *versus* $m\delta/\rho D^2$ diagrams exhibit the jumps found by Tanaka and Takahara, and are in good agreement with their and some others' experimental results.

The effects of some parameters are also clarified [171]. Thus, since the fluid-damping-controlled mechanism depends basically on the motion of one cylinder, it is insensitive to detuning, while in the stiffness-controlled mechanism (large $m\delta/\rho D^2$) it may be important. On the effect of $m/\rho D^2$ and δ, Chen finds that, for the cases investigated, they are complementary, such that effectively they may be dealt with as effects of $m\delta/\rho D^2$ *over narrow ranges* of that parameter. Thus, for the specific in-line array analyzed ($s_t = s_\ell = 1.33$), it is found that

for $0.01 < m\delta/\rho D^2 < 0.5$: $U_c/f_nD = 3.4(m\delta/\rho D^2)^{0.18}$;

for $0.5 < m\delta/\rho D^2 < 3.5$: $U_c/f_nD = 4.0(m\delta/\rho D^2)^{0.41}$;

for $3.5 < m\delta/\rho D^2$: $U_c/f_nD = 7.6(m\delta/\rho D^2)^{0.42}$;

where m, δ and f_n here are the values in vacuum.

Price and Paidoussis [173,174] developed a quasi-static model for a staggered double row of cylinders, which nevertheless incorporates velocity-dependent terms in the manner successfully utilized by Den Hartog for galloping oscillations [175]. The force coefficients associated with a kernel of three cylinders are measured statically — a much simpler proposition to Tanaka and Takahara's measurements. The critical flow velocity for fluidelastic instability is found to have the form

$$U_c/f_nD = A\{1 + [1 + B(m\delta/\rho D^2)]^{\frac{1}{2}}\},$$

where A and B are functions of $m/\rho D^2$, s_t and s_ℓ. Thus, for small $m\delta/\rho D^2$, the effective exponent in an equation of the form (7) is quite small, i.e. system stability is not very sensitive to $m\delta/\rho D^2$; on the other hand, for large $m\delta/\rho D^2$ the effective exponent tends to $\frac{1}{2}$. Clearly these agree with Chen's findings [170,171]. The theoretical results were found to be in reasonably good agreement with the experimental data of Weaver and his co-workers [145,153]. The effect of various types of detuning and mechanical coupling were also studied, and it was found that both effects depend strongly on $m/\rho D^2$, δ and the specific form of coupling and detuning utilized: although usually producing a stabilizing effect, they may also lead to destabilization of the system.

Lever and Weaver [176] developed a very different and promising, truly analytical model, based on a number of reasonable assumptions and requiring virtually no input from measurements. The theory is based on the observation that, both in air and water cross flows, a single flexible cylinder among rigid ones does become unstable at essentially the same U_c as an all-flexible array [138,145,102,156]. A model is then developed considering the motions of just one cylinder, surrounded by immobile neighbours. The forces leading to destabilization are considered to arise from flow redistribution in the "channel" formed by the dividing streamlines of sub-channel flows, where the flow redistribution is caused by cylinder motion, with an appropriate phase lag; thus, both velocity- and displacement-dependent forces are part of the model.

This model was applied to a parallel triangular array (p/D = 1.375). The U_c/f_nD *versus* $m\delta/\rho D^2$ diagram exhibits several instability bounds, depending on the various values the aforementioned phase lag may take. Below $m\delta/\rho D^2 \simeq 3$ there exist several instability lines, giving rise to jumps, similar to those found by others [167-171]. Agreement with experiment is remarkably good, especially for high $m\delta/\rho D^2$.

4.5 Fluidelastic instability: state of knowledge and design guidelines

Although it cannot yet be said that the fluid mechanics underlying fluidelastic instability are well understood, the new set of analytical and semi-analytical studies [166-176] and extensive experimental investigations [101,102,134,149-158] have begun to shed light and impose order in what appeared to be, *circa* 1980, a rather chaotic state of affairs.

There is a great deal yet to be clarified, however. For example: is it true that there are two distinct mechanisms at work? As contended by Chen [170,171] and by Heinecke and Mohr [157], on the one hand, based on their work with in-line arrays, the "fluid-damping-controlled" or "galloping" instability is operative at low $m\delta/\rho D^2$ and/or small s_t, s_ℓ, whilst the "fluidelastic-stiffness-controlled" instability at high $m\delta/\rho D^2$ and/or larger s_t, s_ℓ. Lever and Weaver [176], on the other hand, suggest that there is but one mechanism at work, modified by a phase lag between cylinder motion and flow redistribution (and hence modified forces on the cylinder), which effectively gives different stability bounds for low and high $m\delta/\rho D^2$. The allied question that must be clarified is whether a single flexible cylinder among rigid ones can become fluidelastically unstable for high $m\delta/\rho D^2$ in all cases; is the divergence of views between Chen and Heinecke and Mohr, on the one hand, and Lever and Weaver, on the other, related to the different geometries investigated, both theoretically and experimentally?

Tanaka *et al.* [167-169] and Chen [170,171] have shown that, if one is prepared to measure all the complex force coefficients for a given array, then excellent prediction of U_c in that array is possible. However, it should be recognized that, going this route, it would be necessary to undertake these extensive measurements for each and every array geometry one is interested in, over the pertinent range of U/fD. Clearly, the question must be asked if, under the circumstances, it would not be simpler to measure U_c/f_nD directly and by-pass the use of the model altogether. Price and Paidoussis' [173,174] approach involves a less onerous measurement task, but has basically the same drawback; moreover, their analysis may be inappropriate for very low U_c/f_nD, where the quasi-static assumption breaks down. In this respect, Lever and Weaver's proposition [176] looks most promising, for it requires minimal empirical input and requires no force coefficient measurements.

above discussion. The true relationship between U_c/f_nD and the parameters involved is likely to be rather complex and to have the form [170-176]

$$U_c/f_nD = F(m\delta/\rho D^2,\ m/\rho D^2,\ s_t,\ s_\ell,\ Re,\ u.t.c.),$$

where u.t.c. stands for upstream turbulence characteristics, which may affect the onset of instability in the most critical first few rows of the array. Clearly, the above relationship implies that the simple form $U_c/f_nD = F'(m\delta/\rho D^2)$ can only be a rough guide; the fact that it is at all useful simply suggests that $m\delta/\rho D^2$ is a very important parameter in determining fluidelastic instability.

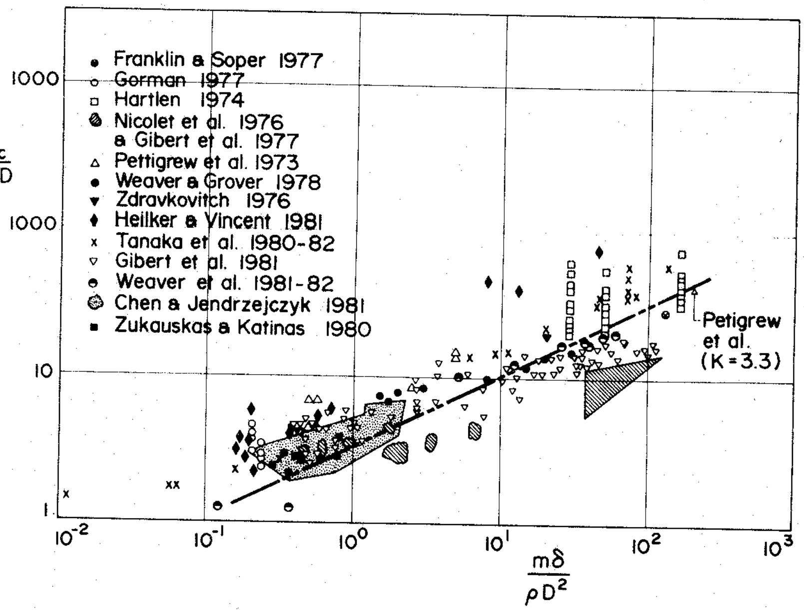

Fig.7. Experimental values, from various sources, for the threshold of fluidelastic instability of cylinder arrays in cross flow, plotted in the conventional manner; also shown is Pettigrew's *et al.* empirical guide.

Matters should be clarified in the next few years, but, until they do, designers may have to continue utilizing whatever semi-empirical guidelines they have been using, provided that the parameters in their new designs do not stray too far away from those of tested designs. In an attempt to help matters in this way, most of the available experimental data have been plotted as shown in Fig.7, despite the fact that this data suffers from the various interpretational difficulties discussed in Section 4.2, which are doubtlessly partly responsible for the large scatter. Other factors contributing to the scatter are (i) differing layout geometries and spacings, (ii) different degrees of tuning of the cylinders, (iii) mechanical coupling effects, (iv) upstream turbulence and (v) Reynolds numbers involved, all of which, it is now well established, affect the stability of the system, in addition to the parameters involved in Fig.7.

Despite all the above, it is clear from this figure that, as suggested by the work of Ref.[170-176], equation (7) cannot possibly apply over the whole range of $m/\rho D^2$ and $m\delta/\rho D^2$: the line shown is simply too steep, for one thing. Thus, for low $m\delta/\rho D^2$, the slope is almost horizontal; for $2 \times 10^{-1} < m\delta/\rho D^2 < 2 \times 10^1$, roughly, the slope is nearer 1/3 than 1/2, gradually rising with increasing $m\delta/\rho D^2$. One could draw a curved lower-bound envelope through the lowest data points in the figure and utilize it as a new design guideline. However, this temptation was resisted in view of the

5. NATURAL FREQUENCIES AND HYDRODYNAMIC COUPLING IN TIGHTLY-SPACED ARRAYS AND MULTIPLY SUPPORTED CYLINDERS

In most of the foregoing, the cylinders have been characterized by a single natural frequency, f_n, generally presumed to be the lowest natural frequency of a cylinder in the system: i.e., that of its first beam mode. This is so for two reasons: first, this mode is likely to be the least damped and hence the most easily excited; second, for Strouhal resonance and fluidelastic instability, the lowest f_n would be the one excited with the minimum possible U.

However, this need not always be the case. For example, in cases of high impingement velocity over one half the cylinder span, the mechanical receptance (admittance) of the second mode is higher for this loading, and hence the system may develop vibration or instability mostly, or exclusively, in that mode. Such considerations are especially pertinent to multiply supported cylinders, a common arrangement in heat exchangers and in nuclear reactors. In such cases it is generally improper to consider each span in isolation; rather, the natural frequencies of the whole system should be considered.

For a cylinder supported at its extremities and at an intermediate point, for example, there will be *two* natural frequencies, both with a first-beam-mode shape within each of the two spans. The overall mode shapes will be different, however; in one mode the two "humps" will be on the same side, while in the other on opposite sides. For a larger number of spans this number of distinct natural frequencies multiplies factorially. There exists, of course, a similar number of modes with higher mode shape within each span. The reader is referred to Ref.[90,104,177,178].

In cases where the fluid is dense, e.g. for liquid flows, and especially in the case of tightly spaced arrays, there exists non-negligible hydrodynamic coupling in cylinder motions; i.e., it may no longer be assumed that motions of one cylinder do not affect those of adjacent ones, even in still fluid. Motion of one cylinder accelerates the fluid, which creates a pressure field around adjacent cylinders, causing them to also vibrate. Hydrodynamic coupling has extensively been studied by S.S. Chen and his associates and by others [179-187].

One important consequence of hydrodynamic coupling is the following. A *solitary* cylinder immersed in dense fluid would have one first-beam-mode natural frequency, one second-mode natural frequency, and so on. The same would be true for an array of cylinders *in vacuo*. However, for a N-cylinder array *immersed in dense fluid,* there would be *a group* of 2N first-beam-mode natural frequencies, with values clustered about that of the solitary cylinder in the same fluid: thus, there is a band of first-mode frequencies, in place of a single discrete value; the tighter the array, the wider this band would generally be. Similarly, there would be a band of second-mode natural frequencies, and so on.

For cylinder arrays of relatively few cylinders, the individual frequencies (and the bands referred to above) may be found by the "classical" means of Ref. [179-184,186] or by finite-element techniques [185,186, 188,189]. For large arrays, however, a more attractive proposition is to determine the frequency bands by the so-called homogenization technique [190,191].

It should finally be mentioned that in two-phase flows hydrodynamic coupling effects are not well understood and cannot yet be properly modelled [192,193].

In this paper the difficulties of dealing with real cylinder supports, with partial contact, flexible constraints and other nonlinear effects will not be considered (*vide*, e.g., [178]),nor will the important effects of mechanical coupling [166].

We next turn our attention to vibration of solitary cylinders and arrays induced by axial flow.

6. CYLINDERS IN AXIAL FLOW

Problems associated with axial-flow-induced vibration are rather rare [1]. Nevertheless, because of the absence of separated flow regions, it has been possible to develop analytical methods to a much higher level than for cross-flow-induced vibration. Furthermore, there exists broad agreement as to the dynamical behaviour of cylinders in axial flow. Hence, it is felt to be unnecessary to give here a historical review for this topic, which in any case may be found elsewhere [194-196].

The behaviour of the systems with increasing flow is much like that of Fig.6, except that (i) the resonance due to flow periodicity has a different meaning in this case, (Section 6.2),and (ii) the instability is usually of the divergence type, rather than an oscillatory instability. The nonresonant buffeting response is essentially similar in axial and cross flow; it will simply be referred to here as buffeting, although it is often called subcritical vibration.

6.1 Buffeting of cylinders in axial flow

It is known that at any given flow velocity, U, there exist in the flow stochastic pressure fluctuations due to turbulence and far-field disturbances. It is now widely accepted that these, acting on the surface of the cylinders, constitute the excitation force field for buffeting vibration — although other possible mechanisms have also been advanced in the past [195]. Similarly to nonresonant buffeting in cross flow, the cylinders act as shaped band-pass filters, vibrating predominantly in their first-mode.

The vibration amplitudes y_{rms}/D, typically less than 10^{-2}, rarely exceed 10^{-1}. Such small vibrations would ordinarily be neglected, were it not for the often extremely close spacing of the cylinders in the array, with intercylinder gap-to-radius ratios of the order of 10^{-1}; hence, although very small, these vibrations may cause intercylinder impact, which may result in wear and fretting damage.

There exist well developed analytical tools for predicting the vibration amplitude of cylinders in axial flow [197-205]; strictly, they really apply to *solitary cylinders,* rather than clusters. The various theories developed are essentially similar, but nevertheless differ in some important respects which should be pointed out.

Some theories [199,200,202,203] consider the cylinder to be a damped beam immersed in still fluid, completely ignoring the effect of mean flow on the vibrational characteristics; the mean flow, according to this approach, is viewed simply as a vehicle for the generation and transmission of the pressure fluctuations which excite the vibration.

The main effects of the mean flow are the following:

(i) flow-induced damping is generated, which in the range of most practical situations is proportional to U [204,206-208,201];

(ii) the natural frequencies of the cylinders are diminished [204,206,207];

(iii) the eigenmodes are no longer normal.

The theory of Ref.[201] takes some of these effects into account, and the theory of Ref.[204] takes them fully into account. It should be added, nevertheless, that if the dimensionless flow velocity $u \ll 0.5$, these effects are not very large. As in most engineering applications $u \leq 0.5$, neglecting mean flow effects is not at unreasonable approximation.

Other assumptions made are the following: (a) there is no correlation between the pressure fields on adjacent cylinders in the array [199-205]; (b) the motion of a cylinder has no effect on the pressure field, nor on the motion of adjacent cylinders [199-205]; (c) the process is ergodic and the pressure field homogeneous [199-202,204].

In most cases, the characteristics of the pressure field are left as input parameters to the analytical model. As shown by Reavis [199] the excitation field contains much more energy than that generated by boundary-layer pressure fluctuations. Indeed, the pressure field exciting the vibration has near-field and far-field components. The near field comprises the local pressure fluctuations associated with the boundary layer *and* the non-propagating part of disturbances associated with singularities (e.g. valves, bends, protuberances, supports, etc.). Propagating disturbances in the form of acoustic waves constitute the far field. The theory of Ref.[203] takes far field characteristics into account.

It has been shown that, generally, both the non-propagating part of singularity-related disturbances and the far field (i.e., the so-called "system characteristics") are important [209]. That is the reason why, in cases where the pressure field characteristics have been measured *in situ,* i.e., in the very same facility in which cylinder vibration is measured, agreement between measured and predicted vibration amplitudes is so good [200-203]; otherwise, prediction and measurement may differ by as much as one order of magnitude.

Clearly, the proposition of having to measure the pressure field characteristics *in situ* in order to predict the vibration is not practical, for then the vibration amplitude itself might as well be measured *in situ*, with at least equal ease!

Of course, if the pressure field could be characterized and predicted, say as a result of a comprehensive series of measurements of surface pressure fluctuations on cylinders in clusters, then the existing analytical models would become much more useful. Progress is being made in this regard. Measurements of turbulence and wall-pressure fluctuations for a single cylinder in narrow annular flow and in seven-cylinder arrays have now been made [210,211], with and without upstream turbulators. For the single cylinder, it was found that turbulence intensity eventually dies out to a constant level, no matter what the upstream turbulence level is [210]; the vibration amplitude is higher for higher upstream turbulence intensities, provided that D_h/D is high; for $D_h/D = 0.85$, however, the vibration response was found to be insensitive to upstream turbulence. For cylinder arrays ($D_h/D = 0.63$) it was found that the effect of upstream turbulence almost disappears for $x/L > 0.125$, where x is the axial coordinate measured from the upstream support, so far as the wall-pressure p.s.d., $\Phi_p(f)$, is concerned [211]. Although the pressure field was found to be a function of geometry, as well as D_h, it is nevertheless possible to characterize it in terms of $\Phi_p(f)(U/D_h)(\frac{1}{2}\rho U^2)^{-2}$ *versus* f/U at a given axial location — similarly to equation (5). The theory of Ref.[205] takes into account a varying $\Phi_p(f)$ with x, assuming it to decay exponentially.

At present, there is no theory taking into account hydrodynamic coupling on buffeting response — other than the added mass effect. It has recently been shown that the dynamics of the system in such circumstances may be quite different from that of a single cylinder [212], especially at high flow velocities: the response spectrum is much broader, corresponding to the band of first-mode frequencies referred to in Section 5, the breadth increasing with U. Mechanical coupling effects may also be important, although rarely taken into account [213,214].

It should also be mentioned that buffeting by two-phase flow may be quite different than predicted by assuming it to be an equivalent homogenized mixture [192, 193,215-218].

Because of the limitations of the analytical models discussed above, it is often necessary for the designer to predict buffeting vibration amplitudes by means of empirical and semi-empirical relationships [199,197,219-222], especially if there is no means at hand for predicting the pressure field characteristics in the system being designed. As discussed in Ref.[195, 4], these relationships, applied indiscriminately to data for both solitary cylinders and to clusters, are capable of prediction of vibration amplitude to no better than one order of magnitude. This degree of success is similar to that achieved by analytical methods, unless the characteristics of the system are very well known. The interested reader is referred to Ref.[195, 1-4,205] for details of the various models and comparison between them [4].

6.2 Vibrations due to flow periodicity

If the mean flow velocity in an array of cylinders is harmonically perturbed, then parametric resonances may be excited provided that the perturbation frequency, f, lies in the neighbourhood of $2f_n/k$, $k = 1,2,3\ldots$ The physical nature of these resonances is similar to that induced in columns subjected to a time-varying compressive load.

These parametric resonances are likely to materialize, especially the principal one, where $f = 2f_n$, if δ is not too large, the amplitude of flow perturbations not too small and the upstream turbulence level not too high. These resonances have been studied both experimentally and theoretically by Paidoussis, Hara and their associates [223-226]. Prediction of the instability zones, at least for a single cylinder in axial flow, is very good [226].

6.3 Fluidelastic instabilities in axial flow

A solitary cylinder in axial flow of sufficiently high flow velocity will be subject to fluidelastic instabilities: divergence, followed by flutter at higher flow velocities [206,207,204,217,227-230]. The mechanism involves the reduction of the effective flexural rigidity of the system through a centrifugal (otherwise viewed as a compressive) load on the instantaneously bent cylinder, where this force is proportional to u^2.

Cylinder arrays similarly develop fluidelastic instabilities, but at lower u for dense fluid flows, through the effect of hydrodynamic coupling [179,231-233,187]; the tighter the array, the lower is the critical flow velocity.

The transition from the buffeting régime to instability occurs through progressive broadening of the hydrodynamically coupled band of vibration frequencies (Section 5) as the flow velocity is increased; loss of stability by divergence then occurs when the lower bound of this band reaches zero frequency [233,212]. Agreement between theory and experiment is excellent [231-233,212].

However, even in cylinder arrays, these instabilities materialize usually only for $u > 1.5$, which lies beyond the operating range of typical engineering systems. This is the reason for not dwelling in this review on this interesting phenomenon.

6.4 Axial-flow-induced vibration: state of knowledge and design guidelines

Understanding of the various mechanisms underlying vibration of cylinders in axial flow may be said to be good, at least in single-phase fluid flows.

The design procedure is straight forward. *First,* it is ascertained that $u < 1.5$, so that fluidelastic instabilities are not likely to occur. *Second,* if the system is known to contain sizable harmonic flow-velocity perturbations, the design should be such that their frequency f be ±25% away from $2f_n$; alternatively upstream turbulators may be utilized to suppress parametric resonances. *Third,* the buffeting vibration amplitude is computed to ensure that no intercylinder impact occurs, or that this impact does not exceed fretting criteria — or, for that matter, that the vibration amplitude does not exceed fatigue criteria.

The buffeting vibration amplitude may be estimated either semi-empirically [197,199,219-222], or, if there exists reliable information on the excitation field for the hydraulic system at hand, analytically [197-205] — remembering that the reliability of prediction may be as low as within one order of magnitude, as discussed in greater detail in Section 6.1.

6.5 Related studies in axial-flow-induced vibration

The dynamics of so-called fuel strings has also been studied, *vide* e.g. [234-237]. Fuel strings consist of bundles of cylinders stacked vertically and held together by a central rod, supported only at the top or bottom of the assembly and lying within a close-fitting tubular channel containing the flow. In this case, it is the vibration of the string as a whole, rather than of individual cylinders, that is of interest.

Similar are the vibrations of AGR-type fuel "stringers" [238,62,63], where the string is sheathed with a tubular component, although the mechanism of excitation is much more complex, because of the narrowness of the flow passage between this sheath and the surrounding flow tube.

7. CONCLUSION

It may safely be said that some very solid progress has been achieved in the past few years. Although the period cannot honestly be described as one of orgiastic creativity, it has nevertheless been characterized by what may be referred to as inspired persistence in attacking the very questions where the state of the art, as perceived in 1979, had been weakest [239].

For vibration induced by axial flow, the greatest weakness had been, and still is, in the characterization of the pressure field exciting buffeting, so as to permit the well-developed analytical models to be successfully applied; here some significant progress has been made, and it may confidently be expected that in a few years buffeting response in axially disposed arrays will be predictable with the same degree of success as in piping systems [1].

There has also been significant progress in the development of analytical models for axial-flow-induced fluidelastic instabilities. This phenomenon *per se* is not practically important. However, the development of successful analytical methods for its prediction decidedly *is*, for it underpins the analytical models for buffeting, which are of considerable practical concern.

For cross-flow-induced vibration of a single cylinder the state of affairs is fairly good. From the perspective of this review, the most exciting developments are related to the dynamical behaviour of a cylinder in high-turbulence-intensity "industrial" environments, which has been shown to be considerably different from that under well controlled, low-turbulence laboratory conditions.

The most spectacular progress has been made in the case of cross-flow-induced vibration of cylinder arrays, where, *circa* 1980, there existed a state of restrained chaos, arising from the realization that the theretofore accepted understanding of the phenomena (e.g., "vortex shedding" in arrays) and semi-empirical design guidelines (e.g., the Connors-Blevins expression for predicting fluidelastic instability) were inadequate — yet, with nothing in sight to take their place. Buffeting response may now be predicted well, but further characterization of the excitation field for a greater variety of geometries and flow conditions is still needed. Resonant buffeting, or "vortex shedding", response may also be predicted with some confidence; however, the conditions under which this phenomenon will materialize remain to be clarified, as well as more extensive and reliable data banks to be assembled on the Strouhal number and lift coefficients, for a variety of system parameters. Similarly, a great deal remains to be done on acoustic resonances, both at the fundamental level and in cross-testing the available empirical design guidelines.

Exciting developments have occurred with respect to fluidelastic-instability analytical modelling for cylinder arrays in cross flow; however, the state of the art is still in a state of flux.

The weakest link in all the above remains [239] its application to two-phase flows, despite the fact that such flows are quite common.

Having completed this review, the writer is filled with a sense of awe and admiration for the designer: despite serious gaps in codified knowledge, contradictory sets of design guides and researchers' claims (exacerbated by the latter's apparent reluctance to read one another's work), the designer has been able, through his legendary intestinal sixth sense, common sense and experience, to put together so many systems that have given satisfactory service for so long.

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