N O I S E   I N   I N D U C T I O N   M O T O R S

A THESIS

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by

Isaac Attas

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C O N T E N T S

A K N O W L E D G E M E N T S

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INTRODUCTION

Psychologists have found that noise reduces human efficiency. This effect may be considerable even for the most phlegmatic individuals. Moreover, almost everyone objects to unnecessary noises. In recent years there has arisen a very insistent demand for quieter electrical apparatus. This applies particularly to household apparatus such as refrigerator motors, vacuum sweepers and the like. Practically noiseless motors are also required for various drives (refrigerating plants, circulation pumps of hot water systems, big ventilating systems for rooms, organ bellows etc.) in non industrial plants such as storerooms, business houses, hotels, hospitals, churches, theatres etc, where all noise should be suppressed. It must also be noted that the stage has been reached where it is taken for granted that the products of the better known manufacturers will give entirely satisfactory performance, and often, the deciding factor between various makes is only the quietness with which the equipment operates. However,noise and vibration are not merely annoyances which may affect the sale of a product or lower the efficiency of a worker. They are often evidences of defects in design and manufacture which seriously affect the life of the equipment and, in many cases, the safety of the user.

The problem of noise in electric machinery is becoming more important day by day, and in the recent years, more than 40 articles have been written on the subject. Most of these papers, however, are of a little help to the designer since each article covers only a few aspects of this most complicated problem. The subject of noise in electric machinery is still considered a new problem. Its solution is under development and no clear-cut rules have as yet been published which would lead to the design of a practically noiseless motor.

The general accepted theory of vibration is based on the premise that vibration and noise in electromagnetism arise from forces which act in a periodic manner upon the different component parts of the machine. Such forces are called " driving forces " and the actual mechanical displacement or vibration which results is often called the"vibrational response". The amplitude of the vibrational response depends upon the amplitude of the driving forces as well as upon the mass, stiffness and damping factors associated with the vibrating member. In general, this amplitude can be reduced either by decreasing the amplitude of the driving force which causes it, or else by modifying the mass, stiffness and damping factors associated with the vibrating member so that a smaller response occurs under the action of the same driving forces. In electric machinery there are many agencies which contribute to these factors and to the final vibrational response. As a rule it is not simple process to isolate the particular agencies responsible for excessive vibration, particularly when several agencies are cumulative in effect and are present simultaneously.

The driving forces which are present in electric machinery are of two general kinds, namely, mechanical and electromagnetic.

Mechanical driving forces are those which arise from mechanical or structural irregularities such as mechanical unbalance, nonsymmetrical mechanical parts, friction, loose bearings and so forth. Electromagnetic driving forces are those which arise by virtue of magnetostriction and magnetic attraction between different parts of the machine.

A third kind of noise, the noise produced by the ventilating fan and by the cooling air circulating through the machine is often considered as the major component of the total emitted noise. The problem of windage noise will not be studied in exten~~t~~o in this thesis because it is so important that it would make in itself the subject for a new study. Only a few general ideas will be expressed and suggestions for possible study and experimenting given.

In the first chapter of this thesis a general outline is given of the methods and units used for machinery noise measurements. The magnetic driving forces are studied in Chapter II and the mechanical driving forces in Chapter III. Chapter III also deals with the vibrational response of the various noise-emitting elements of an induction motor. In Chapter IV is studied the vibration and noise due to both mechanical and electrical causes. Chapter V comes as a conclusion giving rules for better design of quiet motors. The mathematical derivation of Chapter II leads to a general formula connecting the number of stator and rotor slots, the number of poles and the number of phases. With the help of this formula the possibility of vibration and noise in a certain motor can be predicted when the number of stator and rotor slots is known. The presence of dips in the speed-torque curve may be foretold by the use of the same formula. It is the writer's belief that it is the first time that such a general formula which covers all the aspects of interaction of harmonic fields in the air gap has been stated.

Most of the equipment used for the experiments is shown in Fig. 1. It consists of various stators and rotors that can be assembled

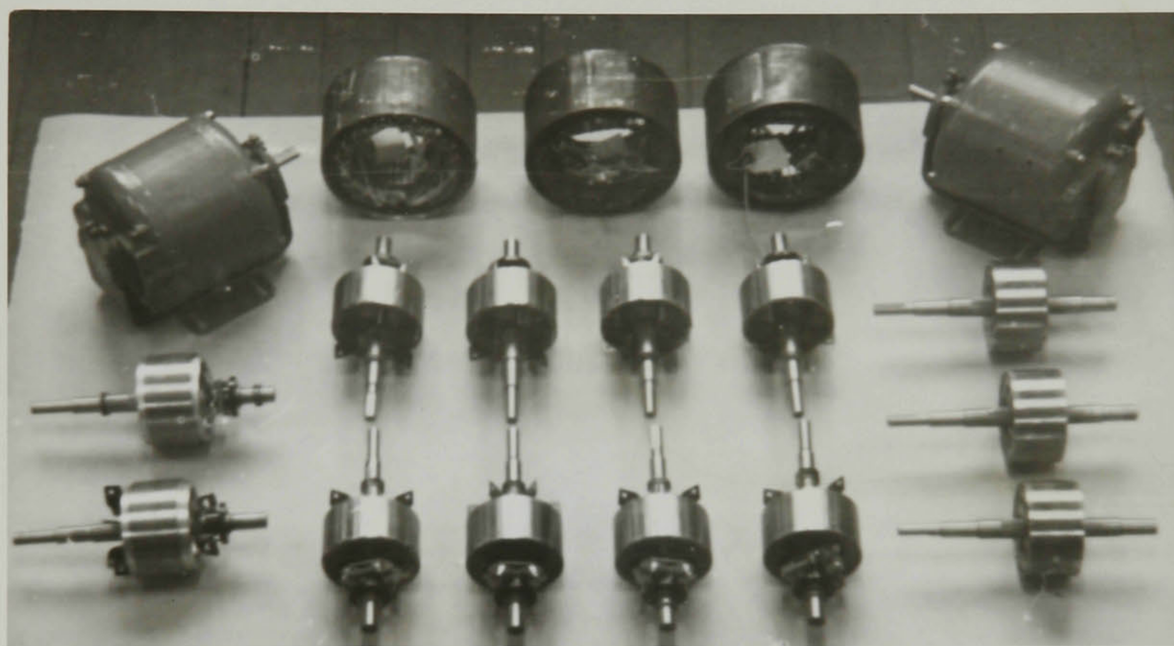

Fig. 1

into a single-phase, 110 V, 60 cycle, 0.25 HP, 4-pole, 1725 rpm motor. Five stators are supplied of which three have similar laminations with a different winding distribution, one is not baked and the fifth one has a different lamination design. 14 rotors are supplied: 4 rotors with 45 slots and 4 others with 48 slots, skewed from 1 to $2\frac{1}{2}$ slots. There are 4 other 48-slot rotors with different air gap lengths. Two aluminum rotors with a herringbone skew are also supplied. Experiments were also run on a three-phase, 220 V, 60 cycle, 0.5 HP, 12-pole, 570 rpm motor. This motor had a 45-slot rotor and was provided with three stators with 36 slots each, identical in every respect except for a different value of skew.

A 4-pole induction motor with 36 stator slots and having two rotors with 45 and 48 slots will be used as a numerical application throughout the paper to illustrate both the advanced theory and the mathematical derivations. This special case was chosen to agree with the equipment used.

The experiments were run in a very large room where the closest wall was so far from the motor that reflection was completely negligible. Different machines, scattered at random around the room, did not spoil the accuracy of the measurements because there were no symmetrical reflecting surfaces. The motors were fastened on a board mounted on four elastic supports. These supports rested on two wooden blocks covered on both sides (upper and lower) with rubber sheets $\frac{1}{4}$" thick. It is believed that most of the transmittable noise was thus avoided. Measurements on the .25 HP motor showed a noise level lower by 10 to 15 db with the use of the resilient mounting. In all the measurements, four readings of the sound level were recorded at a standard distance from the motor and on the four directions A, B, C, D, as shown in Fig. 2. The average of the four readings was then calculated. The sound level meter used was made by H. H. Scott, Inc. Cambridge, Mass. It is regretful that a noise analyzer was not available so that not only the sound level but also the sound pattern of the emitted noise would be found and the share of the various causes in the production of noise determined.

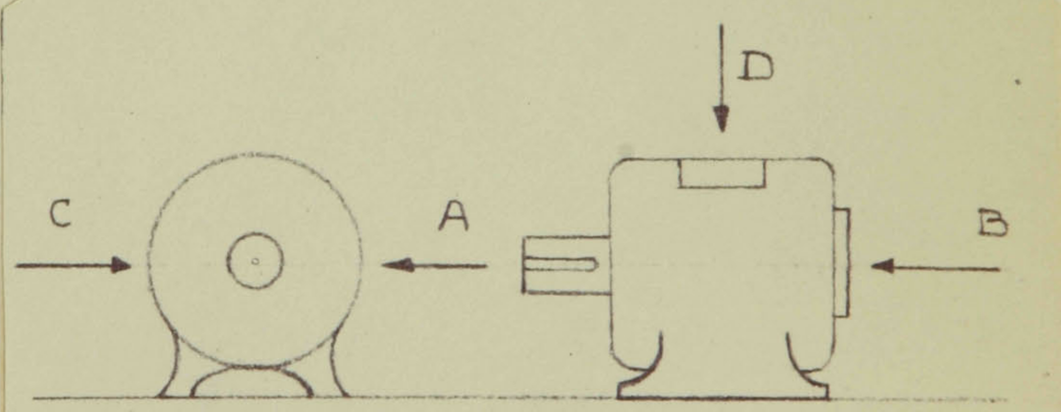

Fig. 2

CHAPTER I

THE MEASUREMENT OF MACHINERY NOISE

Often, after apparatus is installed, questions arise as to the true causes of the observed sound level, and the best way in which to reduce it. Also it is often desirable to design apparatus for a given installation to meet a specified sound level. These objectives require knowledge of the sound produced by the apparatus and of the effects on the sound level of reflections and absorption due to ambient conditions. In this connection it is necessary to make numerous measurements of sound levels, both in the factory and in the field.

The American Institute of Electrical Engineers, in its Test Code for Apparatus Noise Measurement, establishes reasonably uniform methods of conducting and reporting such sound level tests so that whatever or by whomever tests are made, the results will be of value for record and will be truly comparable with tests made at other times and places.

A general outline of the units and methods for machinery noise measurement will be given in this part of the paper in accordance to A.I.E.E. Standard 520[(27)*], to A.S.A. Standards Z24.1,[(32)] Z24.2,[(33)] Z24.3[(36)], and to personal observations made by the writer.

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* Superior numerals in parentheses refer to items in the Bibliography.

DEFINITIONS

1.- Sound Intensity

The sound intensity of a sound field in a specified direction at a point is the sound energy transmitted per unit of time in the specified direction through a unit area normal to this direction at the point. The unit is the erg per second per square centimeter but sound intensity may also be expressed in watts per square centimeter.

In the case of a plane or spherical wave having the effective sound pressure P (dynes per square centimeter), the velocity of propagation c (centimeters per second) in a medium of density ρ (grams per cubic centimeter), the intensity in the direction of propagation is given by:

$$I = \frac{P^2}{\rho c} \quad \text{(ergs per second per square centimeter)} \qquad (1)$$

This same relation can often be used in practice with sufficient accuracy to calculate the intensity at a point near the source with only a pressure measurement.

2.- Intensity Level

The intensity level in decibels of a sound is 10 times the logarithm to the base 10 of the ratio of the intensity I of this sound to the reference intensity I_0 .This reference intensity shall be 10^{-16} watt per square centimeter. In a plane or spherical wave in air, this intensity corresponds to an effective sound pressure P given by the formula:

$$P = 0.000207 \sqrt{\frac{H}{76}\sqrt{\frac{273}{T}}} \qquad (2)$$

where P is expressed in dynes per square centimeter, H is the height of the barometer in centimeters and T is the absolute temperature. At a temperature of 20C and a pressure of 76 centimeters of mercury, P = 0.000204 dyne per square centimeter.

3.- Pressure Level

The pressure level in decibels of a sound is 20 times the logarithm to the base 10 of the ratio of the pressure P of this sound to the reference pressure P_0 which is 0.0002 dyne per square centimeter. P_0 is approximately 16 db below the threshold of hearing.

4.- Velocity Level

The velocity level in decibels of a sound is 20 times the logarithm to the base 10 of the ratio of the particle velocity of the sound to the reference particle velocity which is 0.000005 centimeter per second.

5.- Loudness Level

The loudness level in phons of a sound is numerically equal to the intensity level in decibels of the 1000 cycle per second pure tone, which is judged by the listeners to be equivalent in loudness. In observing the loudness of the reference sound, the observer shall face the source which shall be small and listen with both ears at a position so that the distance from the source to a line joining the two ears is one meter. The manner of listening to the unknown sound may be considered as part of the charasteristics of that sound.

The reference loudness shall be that corresponding to a loudness level of zero phon and is taken as one loudness unit. The relation to be used in noise measurements between loudness level expressed in phons and loudness expressed in loudness units shall be that shown by the curve of Fig. 3. Loudness level in phons is equivalent for most practical purposes to sound level as measured by a sound level meter, assuming that the meter has exactly the correct frequency response for the particular level being measured.

GENERAL CONSIDERATIONS

6.- Radiation of Sound

The distribution of sound about sources other than a few simple types is usually very complex. The out-of-phase as well as the multi-frequency vibrations which exist on the surfaces of large sound producing bodies such as transformers, motors and generators and other apparatus

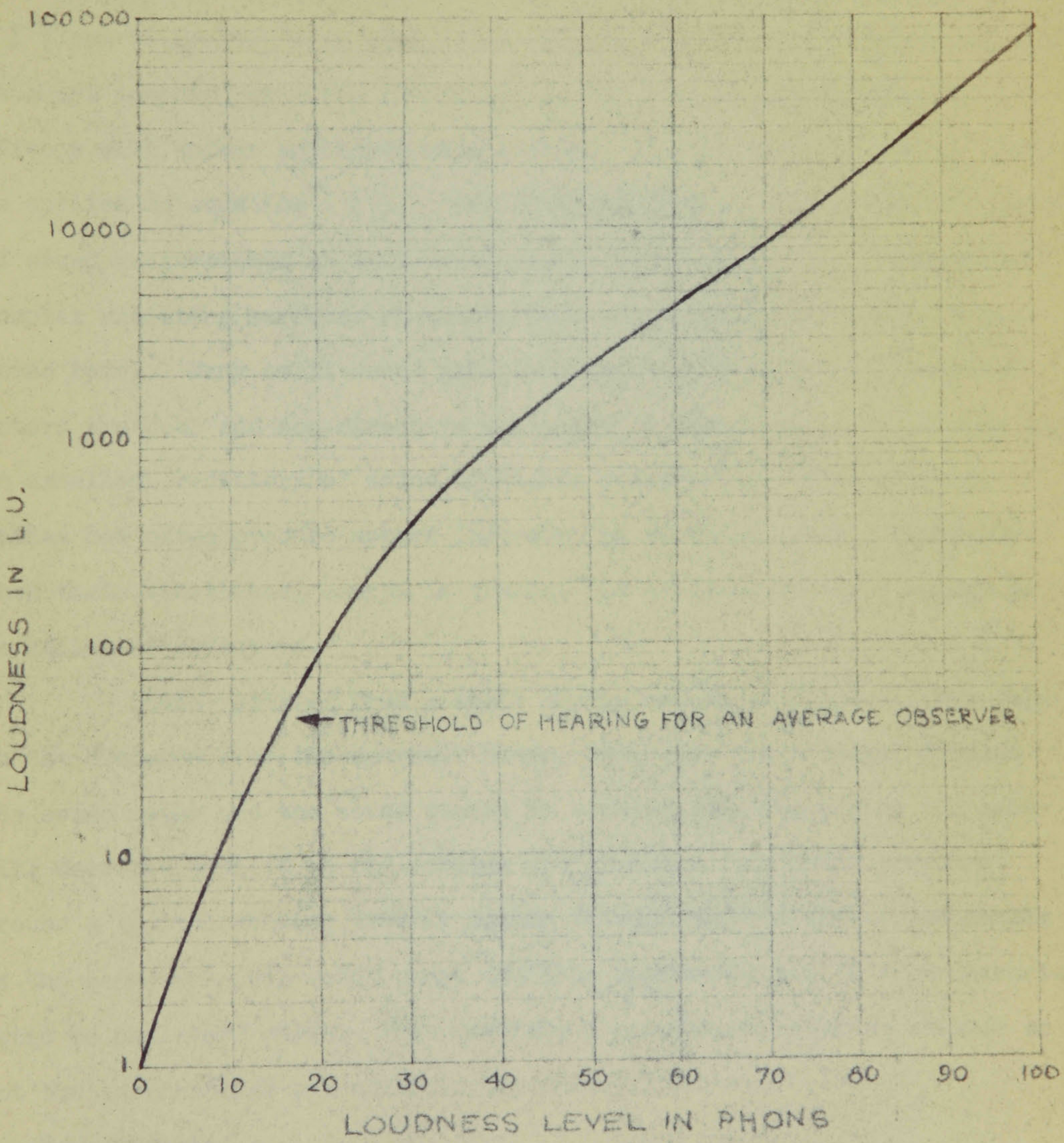

100000
10000
1000
100
10
1.
LOUDNESS IN L.U.
THRESHOLD OF HEARING FOR AN AVERAGE OBSERVER
0
10
20
30
40
50
60
70
80
90
100
LOUDNESS LEVEL IN PHONS

FIG. 3

give rise to sound patterns that defy mathematical analysis. In practice, however, several simplifying assumptions can often be applied to help us understand the nature of sound radiation from apparatus. If the wavelength of the emitted sound is large compared with the dimensions of the source, the sound will be radiated quite uniformly in all directions and equation (1) can be applied with sufficient approximation. But if the wavelength is short compared with the dimensions of the source, important directive effects will appear and correction factors, usually given by curves, must be applied to equation (1) . For reliable results a considerable number of sound measurements at different points about a large sound source with complex radiating surfaces should be made and averaged to determine the sound level. Many small sound generators such as fractional horsepower motors are poor and non-directive radiators of low frequency sound but can be excellent radiators of sound of higher frequencies. Such sounding bodies can often produce marked increases in sound of lower frequencies when their vibrational energy is transmitted to surfaces large enough to be efficient radiators.

Sound radiated from a small source varies in pressure inversely as the distance from the source. Hence, each time the distance between the sound meter and the sound source is doubled, the reading of the meter will decrease 6 db if no reflections are present. The sound pressure around a typical machine usually passes through several minima and maxima as the sound meter is moved away, and then approaches the inverse characteristic as explained above. This provides a convenient check on whether or not the surroundings are sufficiently non-reflective.

7.- Room Effects

When sound generators are placed within rooms the sound will usually be reflected back and forth, many times from the surfaces, and so give a sound intensity distribution within the space that is very complicated. At any point within the room the sound level is the resultant of two effects, namely, the direct sound which reaches the point without first being reflected, and the indirect sound which arrives after suffering one or more reflections from surrounding surfaces. In general, there will be little correlation between intensity and distance to the source and it may be found that at certain points remote from the source the sound is more intense than at positions much closer.

The effect of room reflections can usually be made small by placing the microphone close to the source, and by taking a number of measurements about the source. Reflections are entirely absent only when measurements are made outdoors at some distance from all buildings or other obstructions, or in a room, all interior surfaces of which are 100 % absorbent. In practice, the ideal location does not exist, but it is usually possible to minimize the effects of those reflections which cannot be avoided.

In any room the average sound level will build up to an intensity such that the sound energy absorbed equals the sound energy radiated by the source. Hence in a room with high reflecting interior surfaces the sound will build up to a very high level. This is obviously undesirable for ordinary measurements but in certain applications it is a real advantage. If such a room has irregular or non-parallel wall surfaces, errors due to standing waves are avoided and the level at any point is the result of multiple reflections of sound which originated from all sides of the machine under test. Hence the reading represents in effect an integration of thetotal sound radiated by the machine. This room therefore may be very useful for comparative measurements as when quieting a particular machine.

An analysis based on reverberation theory leads to the following equation :

$$(SL_1) - (SL_2) = 10 \log_{10}(\frac{A_1}{A_2}) \qquad (3)$$

where (SL_1) is the sound level in decibels in a room having a total absorption of A_1 and (SL_2) is the sound level when the absorption is A_2.

8.- Instrument Standards

Three response curves are provided for in the sound level meters, Curve A for a 40 db sound level, Curve B for a 70 db level and a flat response curve C for higher levels. Changing the weighing curve can produce variations in the results ranging from negligible differences in the medium and upper frequency range to variations of 20 or 30 decibels at low frequencies. The following table shows the sound level ranges and the weighing curve recommended:

Sound level range	Weighing curve
24 - 55 db	A (40 db)
55 - 85 db	B (70 db)
85 -140 db	C (equal response)

Strict use of this table will sometimes be impossible. Moreover, if the measurement is one of a series, most of which is well within the range of a particular curve, this setting should be used for all measurements. If no such clear-cut distinction exists, it is desirable to record measurements made with both curves. When actual loudness is important, rather than mere changes in loudness or physical values, it is sometimes desirable to make measurements on both curves and average the results.

The weighing curve should always be recorded as well as the decibell values.

Scale C is used for very high intensities where low-frequency noise is predominant and wherever measurements of actual sound pressure are derived.

To find the actual apparent loudness to the human ear, the sound level in db may be assumed approximately equal to loudness level in phons and the correspondind value of loudness read from the curve in Fig. 3.

FACTORY TESTS

9.- Microphone Placement

To promote uniformity and facilitate the comparison of measurements it is suggested in A.I.E.E. Test Code that one of three standard distances from the microphone to the apparatus be adopted in making factory noise measurements, these distances being six inches, one foot, and three feet. The distance should be measured to the nearest major surface, minor surface projections being disregarded. The microphone should be placed in a position free from disturbing air currents, vibration, electric or magnetic fields, or other external influences, which may affect the readings obtained.

To measure with great accuracy the total noise output from a machine, it would be necessary to take an infinite number of measurements all around the machine and integrate the results. In actual practice, measurements are made at equal intervals around a machine and at a fixed distance from it, the actual number of such measurements depending upon the complexity of the sound pattern and the importance of the results. A simpler procedure can generally be used when comparing similar machines as in production testing. A test position can be selected which gives a single reading that varies closely with the average noise as determined by the above method. This single test position is not necessarily one giving the same reading as the average. Usually it is the position providing the highest reading.

When obtaining an average sound level figure from a group of sound level redings taken around a machine, the average shall be obtained by the energy average method. In most cases, where the range of readings to be averaged is of the order of 10 db or below, the arithmetic average of the decibel readings is a sufficiently close approximation and may normally be used. The energy average is defined as 10 times the logarithm to the base 10 of the average of the antilogarithms of 1 / 10 of the separate decibel values. The following example will illustrate the method of obtaining the average:

Microphone position	Sound level (db)	Relative sound level (Antilog. 1/10 sound level)	
I	40	10,000	
II	50	100,000	
III	55	316,000	
IV	60	1,000,000	
V	70	10,000,000	
		5) 11,426,200	total
		2,285,240	average

$10 \log_{10} 2,285,240 = 61.7$ db

Where possible, to insure measurement of airborne noise only, resilient means (felt, celotex, rubber, springs etc.) should be interposed between the device being measured and its support, so that the transmission of vibration to the support and the reaction on the device are both negligible. The transmittable noise (or potential noise that may be transmitted through the mounting structure) depend on the place where the machine is going to be mounted and can be measured in the final position by taking the difference between the noise level measured and the level of airborne noise known from factory tests.

10.-Background Noise

Ordinarily, sound measurements should be made under conditions where the background noise level is negligible -- that is, at least 10 db below the level being measured. In general, in a factory test, this requires a special noise box or room with the efficient sound absorbing material placed on the floor, walls and ceiling, so that it would be accoustically dead that is should have as short a time of reverberation as possible. Also no two interior walls should be parallel in order to minimize errors from standing waves. The box should be large enough so that a reasonable spacing between microphone and machine can be used. The sound-proof room does not reduce low level noises but is very effective with high level noises. Experiments conducted by Mr. C. G. Veinott[19] have shown that while the sound level inside the room was reduced to 52 db for a factory noise level of 69 db, it was reduced by only 1 db for an outside noise level of 42 db. It has been proved , also by Mr. C. G. Veinott, that high frequency noises are reduced more effectively than low frequency ones.

Correction for background noises in factory tests are not generally permissible due to erratic variations which frequently occur in the ambient noise. When the ambient noise can be considered as constant the following equation,

$$\text{Error in db} = \tfrac{1}{2}\left\{ d - 20 \log_{10}\left[2 \sinh \left(0.1151\right) d\right]\right\} , \qquad (4)$$

where d is the difference in db between the readings for total noise and the readings for background noise alone, can be used to correct for ambient noise. The curve of Fig. 4 is more convenient for practical applications. The horizontal scale of the chart represents the difference in sound meter reading with and without the machine under test in operation. The vertical scale represents the number of db to be subtracted from the total reading (machine plus background noise) to obtain the noise level generated by the machine alone.

11.- Vibration Measurements

In order to eliminate difficulties produced by high ambient noise levels, or when measuring fairly quiet devices such as, for instance, electric clocks, a vibration pickup may be used in place of the microphone. By measuring surface vibration instead of sound produced, all needs for special surroundings is eliminated and direct comparison of machines is possible on the test floor during working hours. This principle also makes it possible to make representative measurements on the various elements of

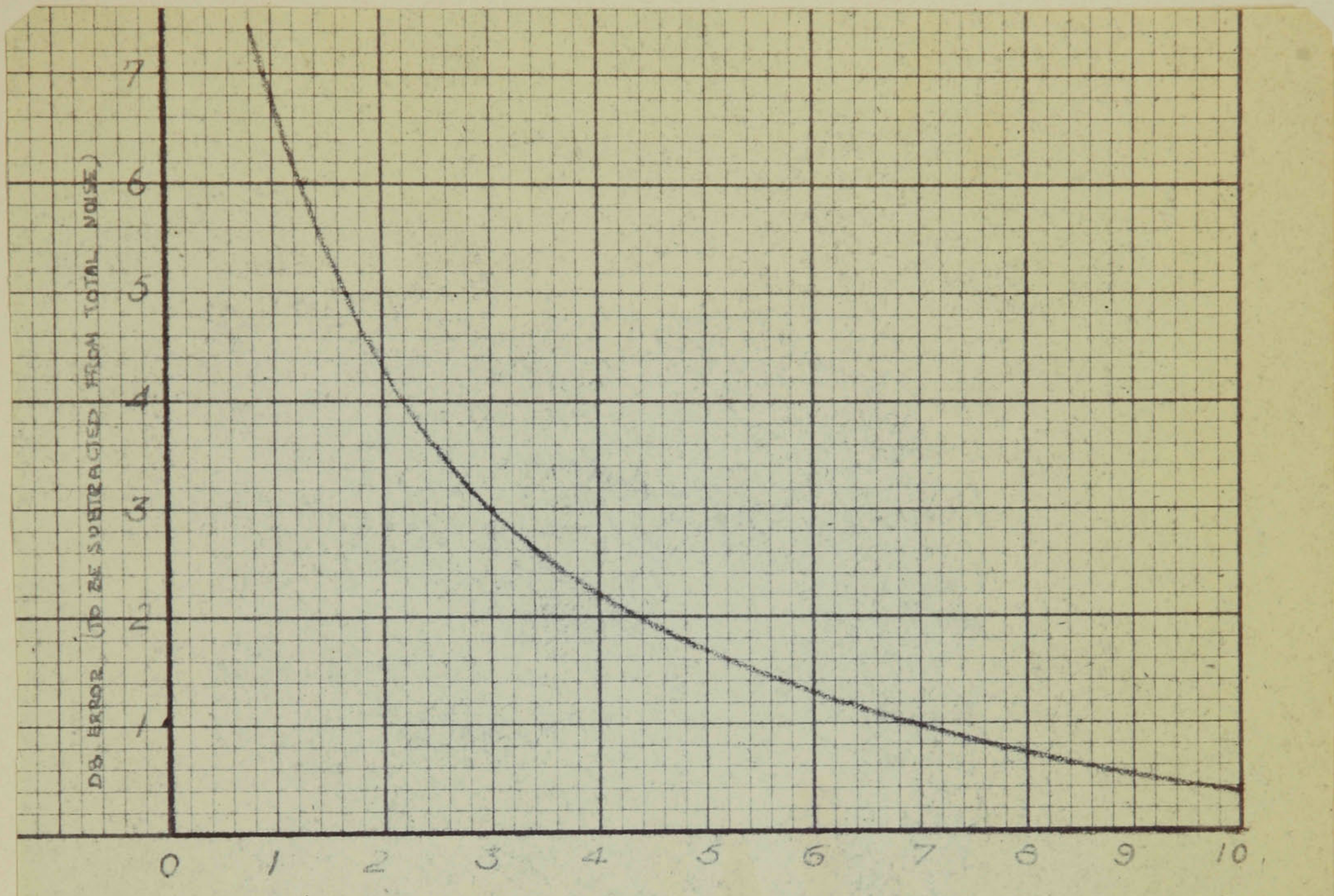


FIG. 4

the vibrating device tracing the vibration to its source and determining the effect of different conditions on the source.

12.- Reporting Measurements

When reporting sound measurements, the dimensions of the room and the nature of the floor, walls, and ceiling, the sound level of the room, the condition of operation of apparatus under test, the position of the microphone, and the frequency response of the sound level meter should be reported. The type of mounting and other information indicative of the degree of transmission of sound to the supports should also be reported.

CHAPTER II

MAGNETIC DRIVING FORCES

Magnetic noise is that in which the forces are those associated with the varying magnetic flux density. The study can be limited to the vibration of magnetic parts produced by the flux in the adjacent air parts of the magnetic circuit. The conductors themselves can usually be neglected as a noise source, as the forces on them, being situated in the relatively weak leakage field, are small. Likewise, whatever internal stress may exist in the magnetic parts can be neglected, as the rigid parts are not deformed appreciably.

In the theoretical study of induction motors it is assumed that the magnetic field is a sine curve and that the magnetic circuits are not saturated. Never is this ideal state of things attained in practice. The true situation in the air gap will be studied in this chapter and the parasitic torque and forces due to harmonic fields determined.

HARMONIC FIELDS IN AIR GAP

The resultant air gap flux wave in an induction motor is the differeace between the fundamental stator flux plus its harmonics and the fundamental rotor flux plus its harmonics. The difference between the two fundamental fluxes is usually less than the larger one and often less than either one, but the harmonics, with few exceptions, are neither of the same velocity or space order and, therefore do not add or subtract from each other. It follows that the resultant flux wave has more harmonics of much greater per unit amplitude than does either the stator or rotor flux wave alone. It is the interaction of two large harmonic flux waves of slightly different space order andtravelling at a high frequency with respect to each other that causes high frequency magnetic-force waves of long space pitch which are effective in producing noise. A list of these harmonic waves and their interaction will be given below.

13.- Nomenclature

A_r = amplitude of rotor fundamental wave

A_s = amplitude of stator fundamental wave

B_n = component of air gap density

B_δ = total air gap magnetic density

f_o = rated frequency in cycles per second

K = any positive integer

N = order of the harmonic

p = number of pair of poles

s = slip in per unit values

x = distance around the periphery

Z_r = number of rotor slots

Z_s = number of stator slots

Φ = number of phases

φ = angle

$\omega_0 = 2\pi f_0$

14.- Fundamental Field

This is the magnetomotive force wave which has a frequency f_0, a speed n_0 and a number of pair of poles p_0, the values of f_0 and of p_0 being those given on the nameplate of the motor. For this mmf wave $N_0 = 1$. n_0 is the synchronous speed of the motor.

Numerical application: $f_0 = 60$, $n_0 = 1800$, $p_0 = 2$

15.- Stator Space Harmonic Fields Due To Winding Distribution

Assuming smooth stator and rotor surfaces, the number of poles of the harmonic fluxes produced in the air gap by the distribution in the armature symmetrical windings is equal to the number of poles for which the motor is wound by the order of the harmonic. The space distribution of the magnetomotive force wave of a single armature coil can be expressed in the form: $A_s(\sin x + 1/3 \sin 3x + 1/5 \sin 5x + \ldots)$. In the case of a distributed winding the order of higher harmonics is given by:

$$N_1 = \pm(2\Phi K_1 \pm 1) \qquad (5)$$

and the number of pair of poles:

$$p_1 = p_0 N_1 = p_0 (2\Phi K_1 \pm 1) \qquad (5')$$

The time variation of all the harmonics is the same since they are all produced by the same current. Therefore, with sinusoidal impressed voltages, the revolving harmonic component fields in the air gap flux, due to windings, revolve more slowly than the fundamental field, the speed being:

$$n_1 = \frac{n_0}{\pm(2\Phi K_1 \pm 1)} \qquad (5'')$$

From (5) it can be seen that the third harmonics are absent from a three-phase machine. This is true because the third harmonics will be in space phase and will differ by 120° in time phase thus giving a zero resultant. The direction of rotation of harmonics is the same with the fundamental for $N_1 = (2\Phi K_1 + 1)$ and opposite that of the fundamental for $N_1 = (2\Phi K_1 - 1)$. The amplitude of these waves is A_s/N_1. This is reduced when the magnetomotive force wave approaches the shape of a pure sine wave.

Numerical application:

$\Phi = 3$ $N_1 = \pm(2x3xK_1 \pm 1) = -5, 7, -11, 13, -17, 19, -23, 25$ etc.

$p_1 = p_o N_1 = 2xN_1 = 10, 14, 22, 26, 34, 38, 46, 50$ etc.

$n_1 = n_o/N_1 = 1800/N_1 = -360, 256, -164, 138, -106, 96, -78, 72$ etc.

$\Phi = 2$ $N_1 = \pm(2x2xK_1 \pm 1) = -3, 5, -7, 9, -11, 13, -15, 17, -19$ etc.

$p_1 = \qquad = 6, 10, 14, 18, 22, 26, 30, 34, 38,$ etc.

$n_1 = \qquad = -600, 360, -256, 200, -164, 138, -120$ etc

16.- Stator Space Subharmonics Fields Due To Winding Distribution

A non-symmetrical winding produces fields having fewer than fundamental number of poles revolving at a higher than rated speed. However these fields will not betaken into account here because they are very easily eliminated by a " good " winding distribution.

17.- Stator Space Harmonic Fields Due To Stator Slots

Since the stator surface is slotted and not smooth as assumed in par. 15, the flux distribution will be different from the magnetomotive force distribution. Taking the general case of the N_1th stator harmonic with $p_o N_1$ poles, the distribution in a smooth air gap would be given by the expression: $\sin(p_o N_1 x - \omega t)$. As shown in Fig. 5, the presence of the teeth will introduce ripples in the sine wave of a frequency Z_s or of any multiple $K_2 Z_s$ of the number of teeth. The final distribution can be expressed by: $\sin(p_o N_1 x - \omega t) \cdot \sin K_2 Z_s x =$

$$= \frac{1}{2}\left[\cos(p_o N_1 x - \omega t - K_2 Z_s x) - \cos(p_o N_1 x - \omega t + K_2 Z_s x)\right] =$$

$$= \frac{1}{2}\left\{\cos\left[(p_o N_1 - K_2 Z_s)x - \omega t\right] - \cos\left[(p_o N_1 + K_2 Z_s)x - \omega t\right]\right\}$$

This expression represents two rotating waves produced by a pulsating wave. Their amplitude is half the amplitude of the pulsating wave and they rotate on both directions. The expression

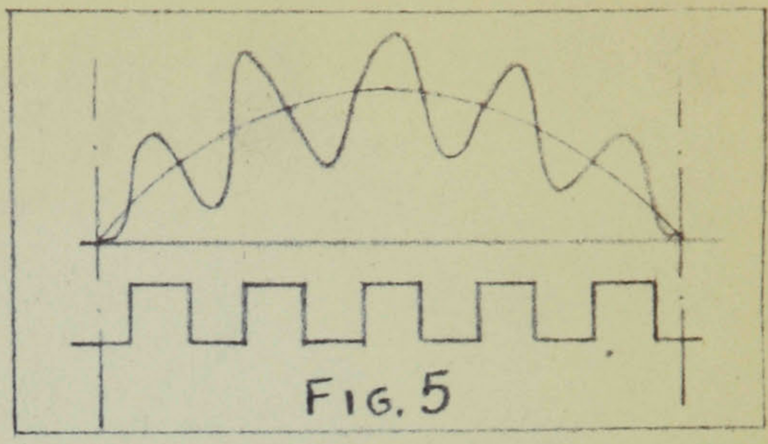


FIG. 5

can be written in the general form: $\frac{1}{2}\cos\left[(p_o N_1 \pm K_2 Z_s) \pm \omega t\right]$

which shows that the harmonic fields due to stator slots will have a number of pairs of poles equal to

$$p_2 = p_o N_1 \pm K_2 Z_s \qquad (6')$$

The order of the harmonic is given by

$$N_2 = p_2/p_o = N_1 \pm Z_s K_2/p_o \qquad (6)$$

Considering only the fundamental stator field, $N_1 = 1$, equation 6 becomes

$$N_2 = 1 \pm K_2 Z_s/p_o \qquad (6a)$$

and

$$p_2 = p_o \pm K_2 Z_s \qquad (6a')$$

The frequency is equal to the fundamental frequency and the speed of rotation is

$$\pm n_o/\left(1 \pm \frac{Z_s}{p_o}K_2\right) \qquad (6a'')$$

The amplitude of the harmonic waves will be equal to A_s/N_2. The magnitude of tooth harmonics cannot be eliminated or reduced by a " good " winding distribution because the distribution factor for the tooth harmonics is identical with that of the fundamental flux.

Numerical application: $Z_s = 36$

$N_2 = (1 \pm K_2 Z_s/p_o) = 1 \pm K_2 36/2 = 17, 19, 35, 37, 53, 55$ etc.

$p_2 = p_o N_2 = 2N_2 = \qquad = 34, 38, 70, 74, 106, 110$ etc.

$n_2 = n_o/N_2 = \pm 1800/N_2 = \pm 106, \pm 95, \pm 51.5, \pm 48, \pm 34, \pm 33$ etc.

18.- Rotor Space Harmonic Fields

The fundamental and harmonic fields of the stator act on the rotor winding to produce waves of rotor currents which in turn produce waves of rotor flux moving at slip frequency with respect to the rotor. The presence of the rotor teeth will introduce higher harmonics in these waves and an expression similar to (6) gives the order of these harmonics when the stator surface is assumed smooth. It is then:

$$N_3 = (N_1 \pm K_3 Z_r / p_o) \qquad (7)$$

Considering only the fundamental stator flux, $N_1 = 1$, and neglecting the influence of higher harmonics will give:

$$N_3 = (1 \pm K_3 Z_r / p_o) \qquad (7a)$$

The number of pairs of poles is:

$$p_3 = p_o N_3 = p_o \pm K_3 Z_r \qquad (7a')$$

and the frequency f_o. The speed of rotation relatively to the rotor is

$$n_{3r} = \pm \frac{n_o \mp n_r}{N_3} = \frac{n_o \mp n_r}{1 \pm K_3 Z_r / p_o} = \frac{s\, n_o}{K_3 \frac{Z_r}{p_o} \pm 1}$$

and relatively to the stator:

$$n_{3s} = n_{3r} + n_{rs} = \pm \frac{n_o \mp n_r}{N_3} + n_r = \pm \frac{n_o \mp (1 - s) n_o K_3 Z_r / p_o}{1 \pm K_3 Z_r / p_o} \qquad (7a'')$$

Only the upper or the lower signs are taken at a time. The amplitude of these rotor harmonics is: $\frac{A_r}{1 \pm K_3 Z_r / p_o}$. A negative sign such as always occurs with the lesser poled harmonics of each pair of tooth harmonics indicates that this harmonic is moving around the rotor in a direction opposite the motion of the fundamental flux.

Numerical application: $Z_r = 48$ or 45

$N_3 = 1 \pm K_3 Z_r / p_o = 1 \pm K_3 48/2 = 23, 26, 47, 49, 71, 73,$ etc. for 48 slots

$p_3 = p_o N_3 = 2 N_3 = 46, 50, 94, 98, 142, 146,$ etc.

$N_3 = \quad = 1 \pm K_3 45/2 = 21.5, 23.5, 44, 46, 66.5, 68.5$ etc. for 45 slots

$p_3 = \quad = 43, 47, 88, 92, 133, 137$ etc.

19.- Rotor Space Subharmonics Fields

When the number of squirrel-cage bars is small with respect to the pitch of the exciting wave of flux, it is possible for a squirrel-cage to produce a flux wave having fewer poles than the exciting flux wave. Such subharmonic waves become very important for the noise of a motor and must be taken into account by including them in the calculation along with the fundamental and the tooth harmonics. It will be, as in (7),

$$N_3' = N_1 \pm K_3 Z_r / p_o \qquad (8)$$

with $Z_r K_3$, at least for the lower values of K_3, smaller than N_1. For instance: $N_1 = 17$, $Z_r = 48$, $K_3 = 1$, $N_3' = 17 \pm 48 = 65 > 17$

$= 31 > 17$

but for $N_1 = 17$, $Z_r = 24$, $K_3 = 1$, $N_3' = 17 \pm 24 = 41 > 17$

$7 < 17$

The 7th harmonic of rotor flux is therefore a subharmonic wave. Equation (8) gives the order and direction (referred to the direction of N_1 which is always taken as positive) of the subharmonic fields.

If the stator winding is "good" and the number of rotor slots is greater than the number of stator slots by p_o, subharmonics may usually be neglected. This is usually the case in practice.

20.- Time Harmonics In Impressed Voltage

The mth time harmonic has a frequency mf_o and a number of pairs of poles $p = p_o$. Speed is therefore mn_o. There are no third harmonics (or multiples) because they do not exist in the line voltages of a balanced three phase supply. The harmonics in the impressed voltage cause harmonics in the currents taken by the motor but because of high impedance of the motor for the harmonic currents, these currents are very small. The magnitude of these time harmonics being very small their influence on the performance of the motor will be considered negligible.

21.- Harmonics Due To Permeance Variations

In the development of air gap fluxes described thus far it was assumed that at least one of the two, rotor and stator, surfaces was smooth. This is well justified in small electrical machinery where substancially closed slots are employed and where the problem of noise reaches its most acute form. In larger motors having open slots the permeance could still be considered constant because accompanying the use of open slots is the use of greatly increased air gaps which minimize such phenomena as may arise from effects due exclusively to permeance variations. It is believed, however that the theory advanced would not be complete if the influence of permeance variation was not included and, to this effect, the notion of " oscillating permeance " was borrowed from Mr. G. Kron [9]. Mr. Kron has assumed that the permeance could be represented at any instant as the superposition of a constant permeance P_c, plus two revolving permeances, one due to the stator teeth represented by $P_s = \sum \pm A_{sk} \cos K_s Z_s x$, and the other due to the rotor teeth and represented by $P_r = \sum \pm A_{rk} \cos \left[K_r Z_r x - K_r (1-s) \frac{Z_r}{p_o} \omega t \right]$. To give the true picture an oscillating permeance must be added which is equal to the product of the revolving permeances. Only the largest values need be considered, that is:

$$P_s = A_s \cos Z_s x$$

$$P_r = A_r \cos \left[Z_r x - (1-s) \frac{Z_r}{p_o} \omega t \right]$$

$$P_o = P_s P_r = \frac{A_s A_r}{2} \left[\cos \left\{ (Z_s + Z_r) x - (1-s) \frac{Z_r}{p_o} \omega t \right\} + \cos \left\{ (Z_s - Z_r) x - (1-s) \frac{Z_r}{p_o} \omega t \right\} \right]$$

and neglecting the high frequency term $(Z_s + Z_r)$:

$$P_o = \frac{A_s A_r}{2} \left[\cos \left\{ Dx - (1-s) \frac{Z_r}{p_o} \omega t \right\} \right] \qquad \text{where } D = Z_s - Z_r$$

The effect of P_s and P_r has been examined in paragraphs 17 and 18. To take into account the influence of P_o, it has been shown [9] that whenever a current density with p_1 pole pairs revolves along an air gap having a constant permeance plus a revolving permeance with p_2 pole pairs it produces a magnetomotive force space distribution with p_1 pairs of poles. This magnetomotive force produces with the permeance p_2 two revolving fluxes having $p_1 + p_2$ and $p_1 - p_2$ pairs of poles. Among the fluxes, only those produced by P_o can effectively interact with the fundamental. The fundamental magnetomotive force is $A_o \cos (p_o x - \omega t)$ and for $P_o = \frac{A_r A_s}{2} \cos \left[Dx + (1-s) \frac{Z_r}{p_2} \omega t \right]$ the produced fluxes are

$$F \cos \left\{ (p_o \pm D)x \pm \left[(1-s) \frac{Z_r}{p_o} \pm 1 \right] \omega t \right\}$$

with

$$p_4 = p_o \pm D \qquad (9')$$

pairs of poles revolving at a speed : $n_4 = \left[(1-s) \frac{Z_r}{p_o} \pm 1 \right] n_o \frac{p_o}{p_o \pm D}$

or

$$n_4 = \left[(1-s) Z_r \pm p_o \right] n_o / p_o \pm D \qquad (9'')$$

Numerical application: $D = 48 - 36 = 12$ or $D = 45 - 36 = 9$

$p_4 = p_o \pm D = 12 \pm 2 = 10$ or 14 or $p_o \pm D = 9 \pm 2 = 7$ or 11

INTERACTION OF AIR GAP FIELDS

22.- Magnetic Driving Forces

The driving force of a rotating electrical machine is a tangential force produced by the main flux wave and the main magnetomotive force wave. The flux harmonics and the harmonics of the magnetomotive force produce parasitic tangential forces (synchronous and asynchronous motor effects) which contribute to the noise. However the main source of noise is the group of radial forces produced by the flux harmonics.

The flux at every point in the air gap of the motor and at every instant of time is exactly representable as the sum of the fundamental fluxes and all the harmonic fluxes. Thus, we may write, using densities instead of fluxes,

$$B_\delta = \sum_{n=1}^{\infty} B_n \qquad (10)$$

The radial force density at any point in the gap due to B_δ is, according to Maxwell:

$$F / S = B_\delta^2 / 72{,}130{,}000 \text{ pounds per sq. in.} \quad (11)$$

For the purposes of this paper it is desirable to work with the components of the force density obtainable from the expansion of $(\sum_{n=1}^{\infty} B_n)^2$. This expansion can be written: $(\sum_{n=1}^{\infty} B_n)^2 = \sum_{n=1}^{\infty} B_n^2 + \sum_{n,n'=1}^{\infty} 2B_n B_{n'}$ for $n \neq n'$

Putting :

$$B_n = A \sin(p\varphi - \omega t - a_n)$$

$$B_{n'} = A' \sin(p'\varphi - \omega' t - a'_n) \qquad \text{where } a_n = \text{phase angle}$$

it will be:

$$B_n^2 = A^2 \sin^2(p\varphi - \omega t - a_n) =$$

$$= \frac{A^2}{2} - \frac{A^2}{2}\cos(2p\varphi - 2\omega t - 2a_n) \qquad (12)$$

and

$$B_n B_{n'} = AA' \sin(p\varphi - \omega t - a_n)\sin(p'\varphi - \omega' t - a'_n)$$

$$= \frac{AA'}{2}\left\{\cos\left[(p-p')\varphi - (\omega-\omega')t - (a_n - a'_n)\right] - \cos\left[(p+p')\varphi - (\omega+\omega')t - (a_n + a'_n)\right]\right\} \quad (13)$$

It will be seen from (13) that the product of the two flux waves is representable as the sum of two force waves, the first wave having a long wave length and the second one having a very short wave length. In general, very short force waves will not be important because the stator will be very stiff to short wave distortion. Similarly, terms like B_n^2 will not be studied here because, according to (12), they give rise to a constant force, which cannot produce any vibration, and to a force wave of double frequency with twice the number of poles which, as explained in par. 26 cannot be avoided by selecting a suitable number of slots or a " good " winding distribution. The only effective noise productive waves are therefore those with a number of poles equal to the difference of the number of poles of the component flux waves, revolving with an angular velocity equal to the difference of the component velocities. The frequency of the emitted sound will be equal to the frequency of the force wave.

23.- Magnitude Of Driving Forces

The amplitude of a driving force is, as shown in (13) equal to one half the product of the two interacting flux waves that give rise to this force. The magnitude of the magnetomotive force waves is ~~inversely~~ proportional to the current in the winding and is dependant upon the pitch and distribution constants, while the flux field they produce is proportional to the magnetomotive force wave and inversely proportional to the air gap length. It should be recognized that while the fundamental flux is proportional to the magnetizing current the harmonic flux is proportional to the total current. The noise will increase with the load because the amplitude of the field waves and therefore the magnitude of the driving forces increase with the load.

Experimental evidence:

The magnitude of the driving forces and therefore the sound level of the emitted noise should be decreased when a lower flux density (less current in the primary windings) or a larger air gap is used.

The 0.25 HP motor was run on no load with different values of the applied voltage thus giving different values of the current through the stator windings. Background noise level 36 db. Measurements at a distance of 6". Weighing curve 40 db.

Volts	Amps	Max. db	Aver. db
89	4.8	47.5	47.5
104	6.0	48.5	47.8
122	8.0	50.0	48.8

A change from 4.8 amps to 8.0 amps, or an increase in current of 170%, changes the noise level by 2.5 db (max. values) that is it increases the sound power output by 180%. This shows the influence of primary current.

The same motor was tried with 4 different rotors, the only difference between the rotors being in the air gap length. The readings are tabulated in the next page. Background noise level 36 db. Standard distance 6". Weighing curve 40 db.

Air gap length inches	Voltage volts	Current amps	Noise level Maximum	Average
0.010	110	3.7	55.5	54.2
	124	4.8	59.0	57.0
0.015	110	4.8	51.5	50.2
	122	6.0	53.0	52.0
0.020	100	4.8	50.5	49.0
	113	6.0	50.5	49.5
0.025	89	4.8	47.5	47.5
	104	6.0	48.5	47.8

It can be seen that with 0.015" air gap the noise level is at least 4 db lower, or the sound power output is 2.5 times less, than with the 0.010" air gap. A further increase in the air gap length again decreases the noise but by smaller amounts, 1 to 3 db. The air gap length exerts a double influence on the noise level. An increase of the air gap reduces the higher harmonic slot oscillations, thus reducing the driving forces. It also increases the reluctance of the magnetic path lowering, for the same ampereturns, the flux density which, as shown before, reduces the noise. A serious disadvantage in the use of large air gaps comes from the fact that the power factor drops to very low values. Thus, assuming that the imput power is the same for the four different rotors and expressing the power as $W = E\ I \cos\varphi$ it can be seen that $\cos\varphi$ is inversely proportional to the voltage required to circulate the same current through the stator. Assuming the power factor of the 0.010" gap rotor to be 100% the relative values of $\cos\varphi$ will be : 100% (0.010 ") , 89% (0.015") , 80% (0.020") , and 72% (0.025")

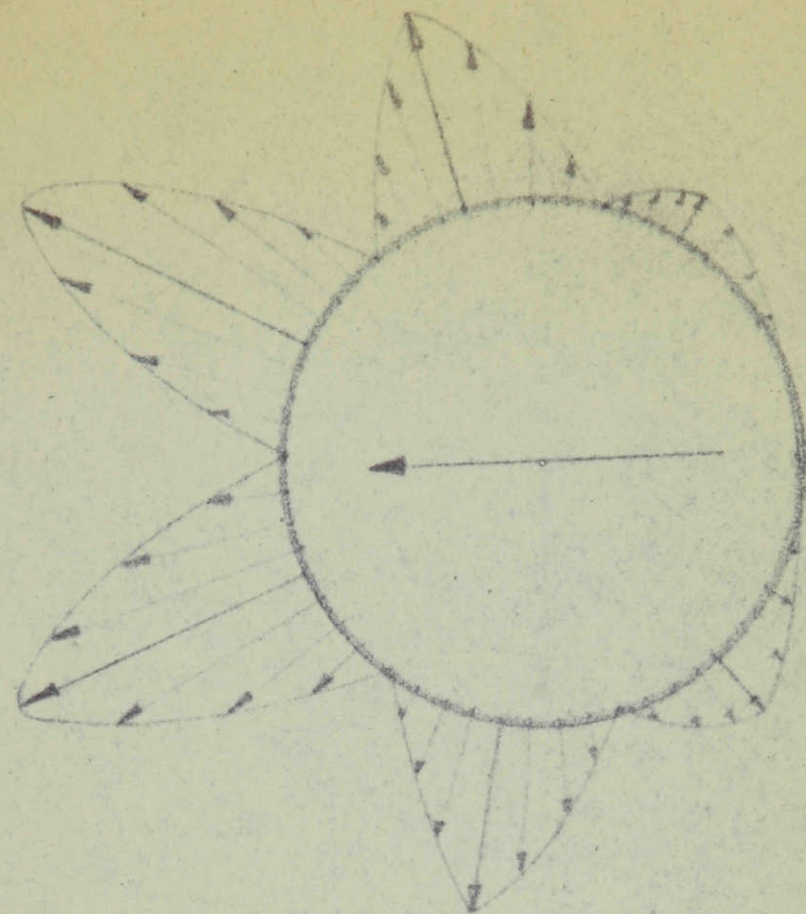
One fact that makes the analytical study of noise in induction motors particularly difficult is that the fundamental flux as well as certain of the harmonic fluxes are considerably affected by the damping action of the secondary while other harmonic fields, especially those caused by zig-zag leakage reactance are not subject to any damping. Damping is very effective in a squirrel cage motor with low resistance in the secondary. In this case all differencial leakage fluxes are damped out and all irregularities in the flux caused by irregularities in the air gap. With high resistance rotors the damping effect of the rotor on the flux distribution will be less effective. According to this a low starting torque motor should be quieter than one with a high stating torque. Furthermore the distribution of the total squirrel cage resistance between bars and rings has a certain influence on the various fields. It is evident that with partially damped main flux irregularities, partially damped differencial leakage fields and undamped zig-zag fields, the final outcome is rather difficult of correct analysis. For the purposes of this paper the damping influence of the rotor will not be taken into account.

24.- Fields Differing By Two Poles

If a 2p-pole field be connected with a ($2p \pm 2$) - pole field there is a strong zone the center of which is immediately opposite to that of a weak zone which leads to an unbalanced magnetic pull. This force can be represented by a radial vector rotating about the axis of the rotor with a frequency equal to the difference of the frequencies of the two fields and which will set up a rotor vibration. The phenomenon is similar to a mechanically unbalanced rotor revolving with a speed equal to the difference of the speeds of the two fields.

The unbalanced pull is shown by the graphical construction of Fig. 6 and Fig. 7. In both cases three sinusoidal curves with 4, 6, and 8 poles are plotted along the air gap periphery. The three curves of

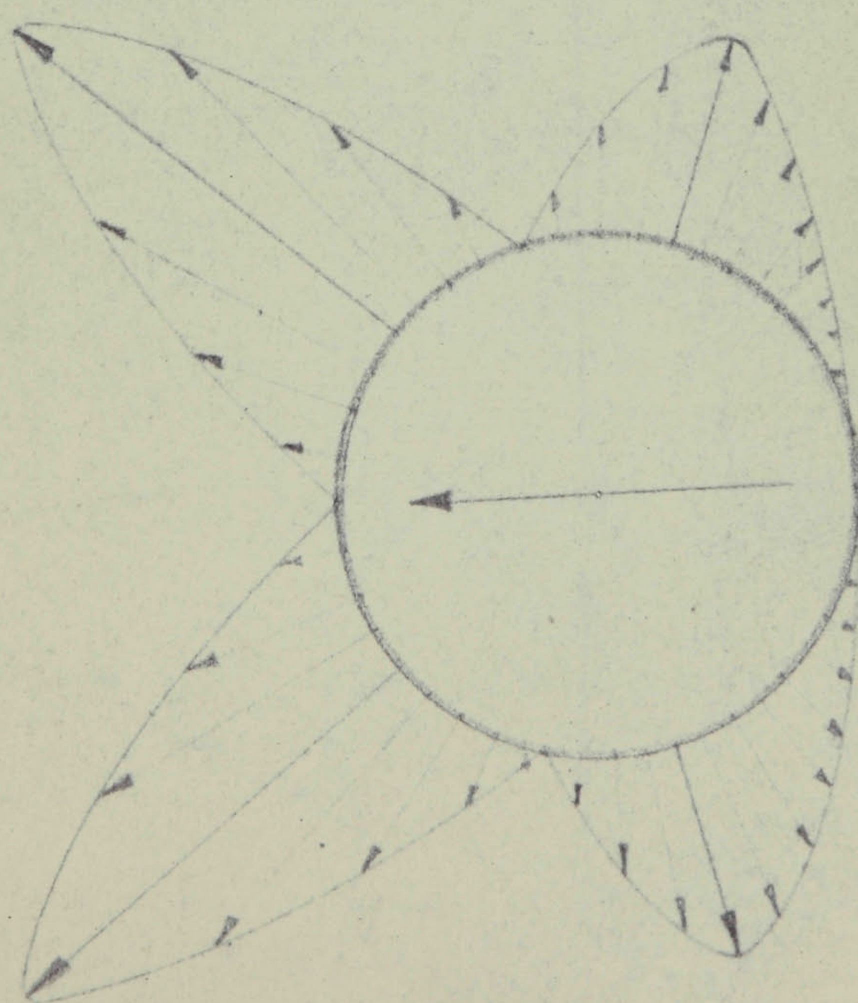
2-pole difference force distribution



Common zero value

6-8 poles

FIG. 7c

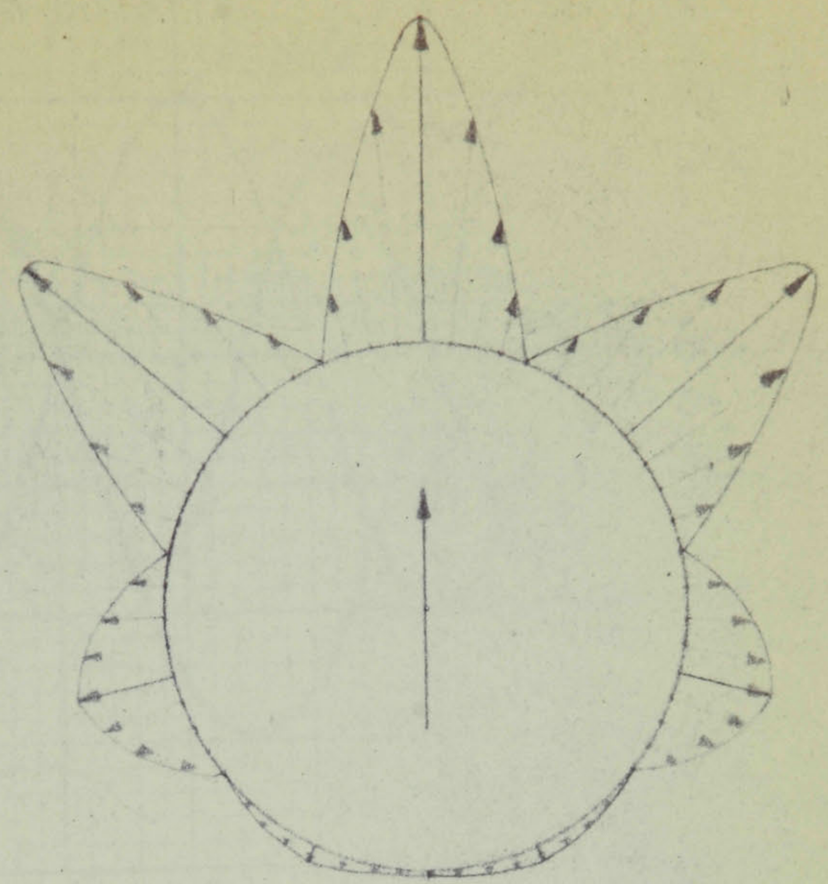


Common zero value

4-6 poles

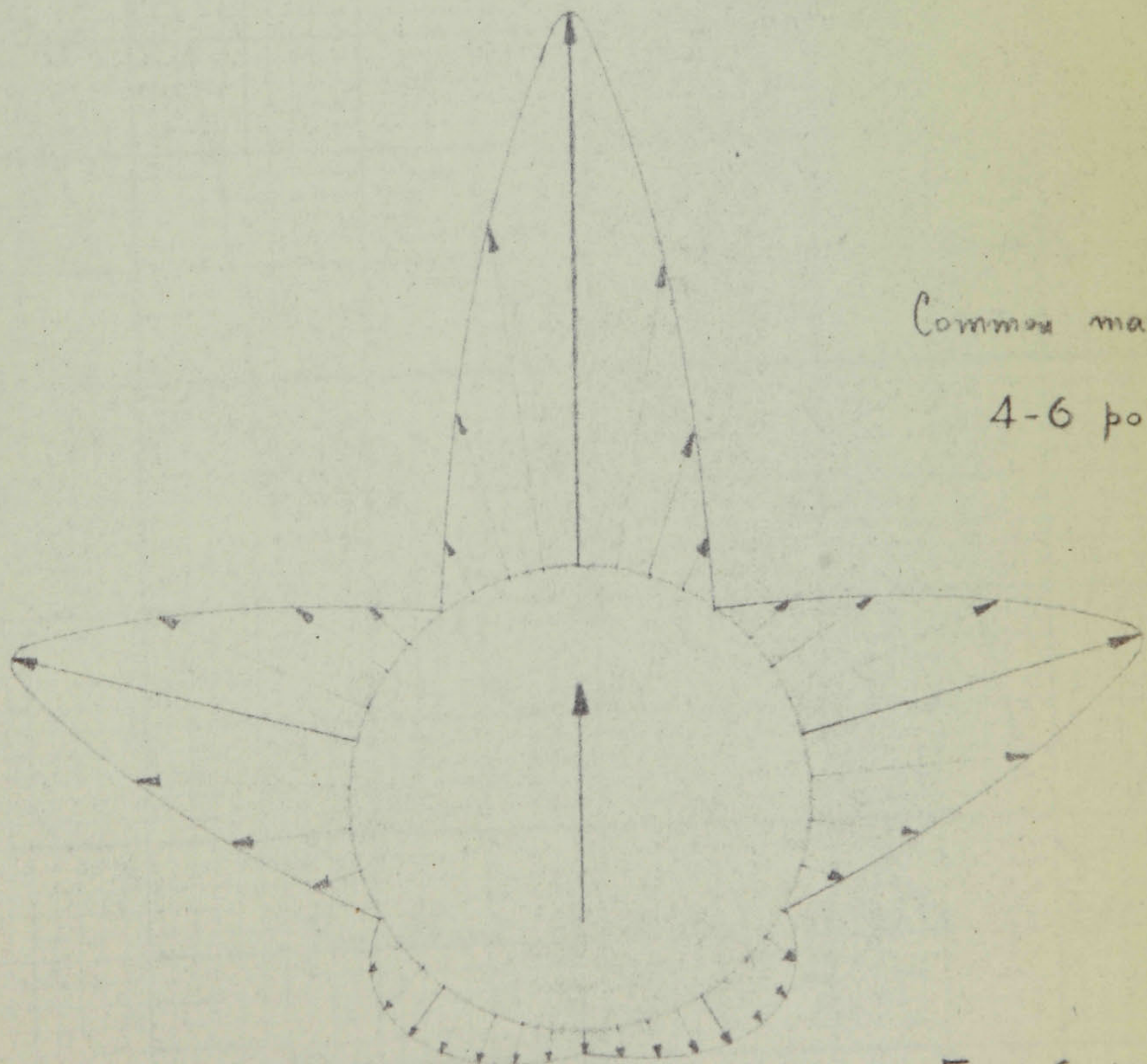
FIG. 7d

2-pole difference force distribution



Common max. value
6-8 polec

FIG. 6.c



Common max. value
4-6 polec

FIG. 6 d

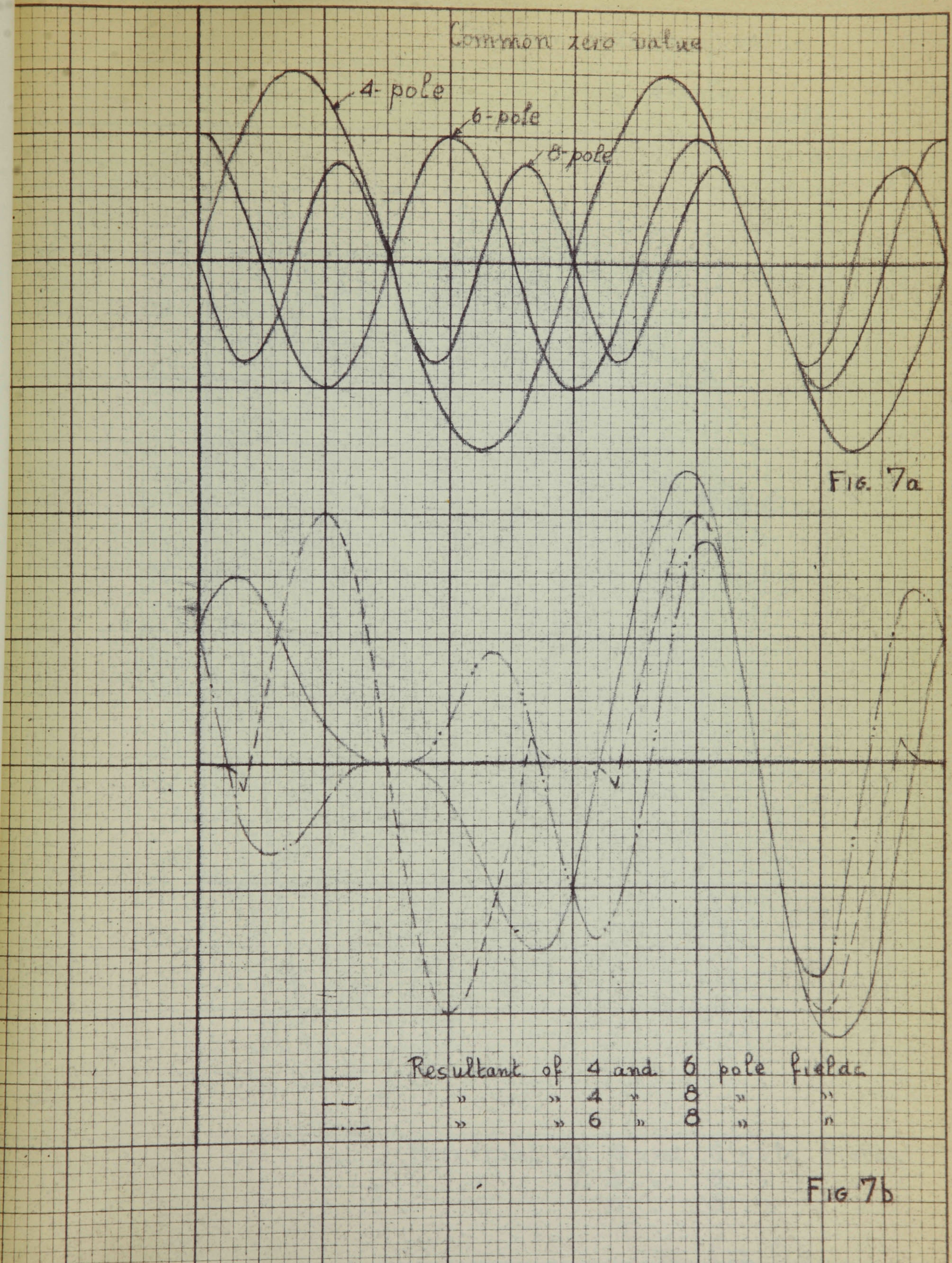
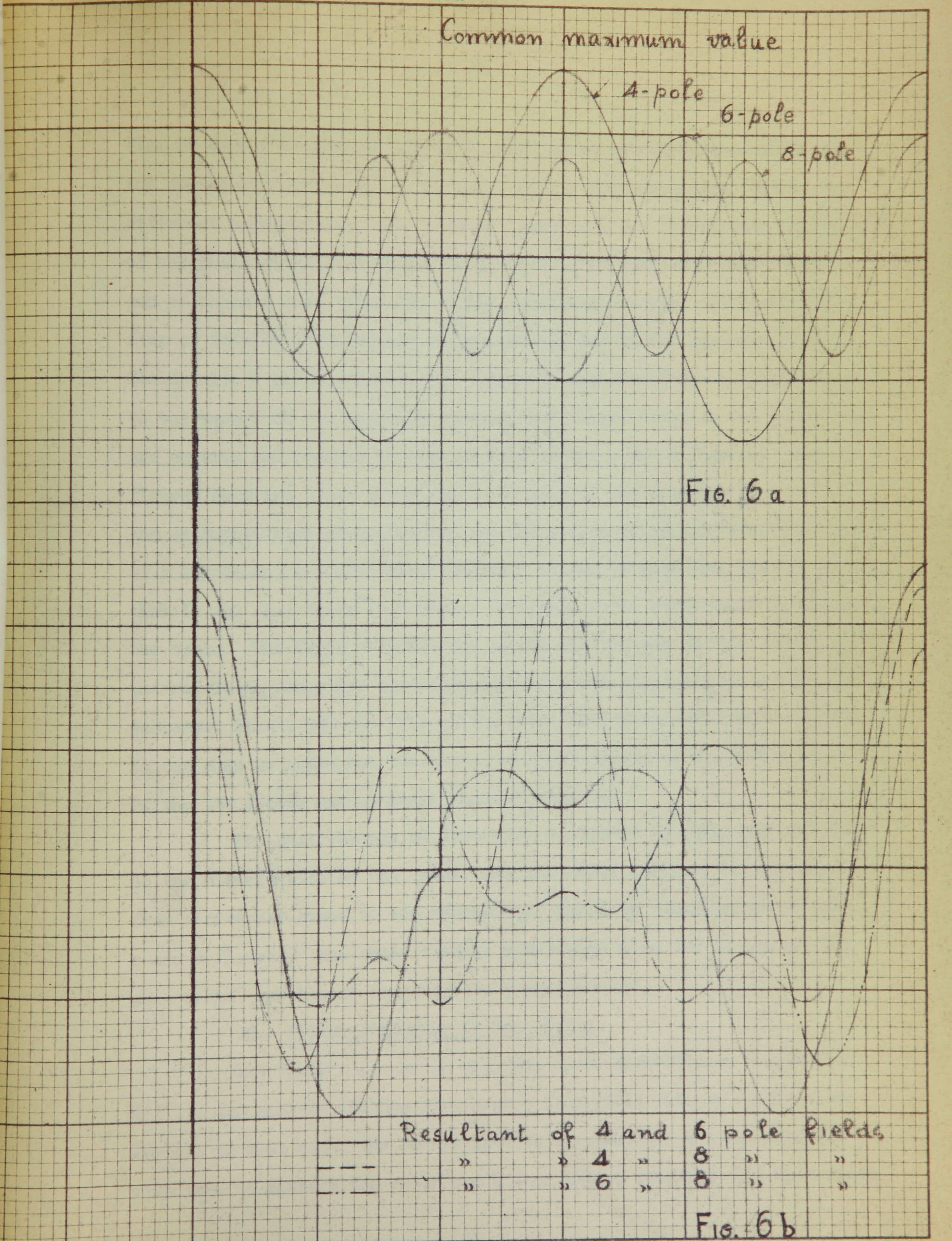

Common zero value
4-pole
6-pole
8-pole
Resultant of 4 and 6 pole fields
" " 4 " 8 " "
" " 6 " 8 " "

FIG. 7a

FIG 7b



Common maximum value
4-pole
6-pole
8-pole

Fig. 6a

Resultant of 4 and 6 pole fields
" " 4 " 8 " "
" " 6 " 8 " "

Fig. 6b

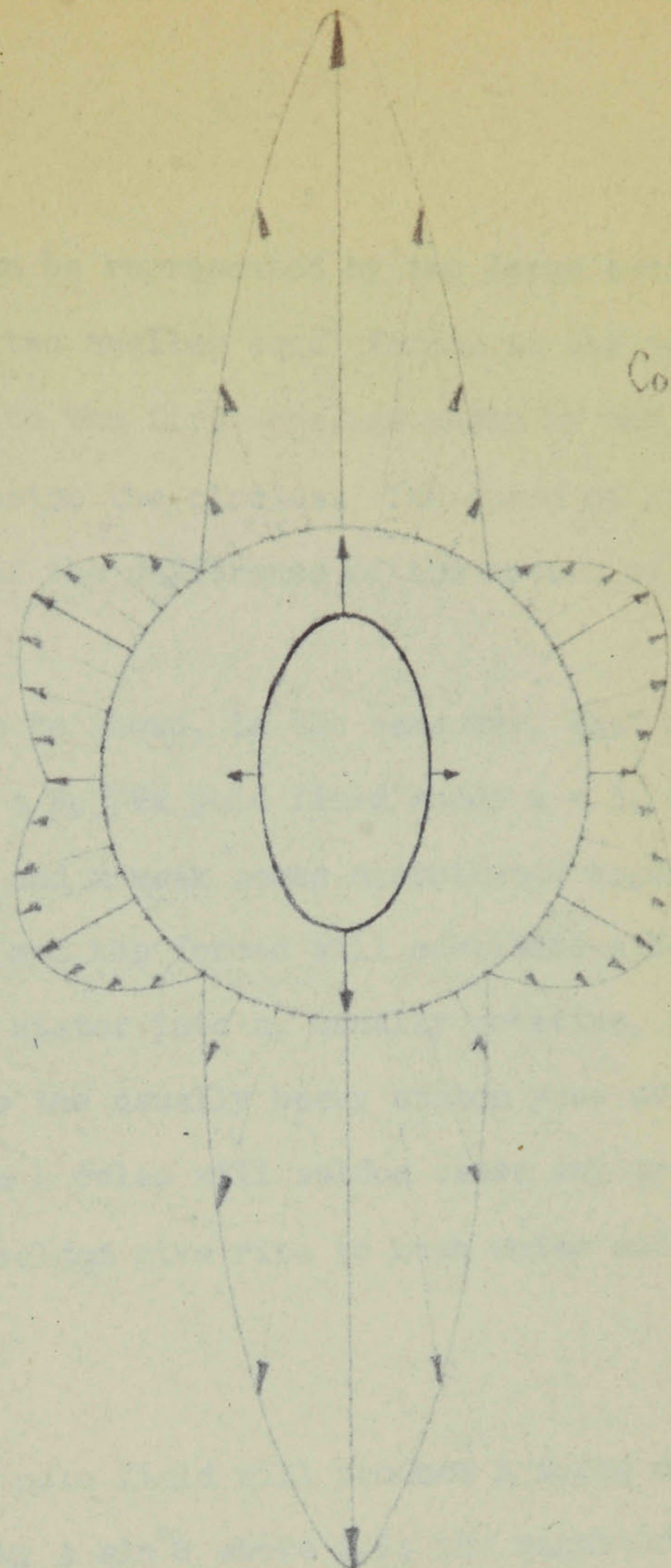
Fig. 6a have a maximum at the same point of the circumference while all the curves of Fig. 7a cross the zero line at the same point. These two different combinations were chosen to show that the results are independent of the phase angle betweenthe field curves. In figures 6b and 7b the sum of these fields taken two by two is plotted to give the resultant flux or density distribution in the air gap due to the presence of two fields with 2 poles difference. Since the magnetic force is proportional to the square of the magnetic density, the values found in the b figures are squared and plotted in a polar form in the c and d figures. The c figures show the force distribution because of the presence of a 6-pole and an 8-pole flux distribution, and the d figures because ofthe presence of a 4-pole and a 6-pole distribution. It can be seen from these figures that in any case there is always a resultant force which tends to distort the rotor. The force distribution around the air gap and the value of the resultant force are different for different phase angles between the component fluxes but the pull on the rotor will always be unidirectional. It can also be seen t hat the number of nodes depend on the number of poles of the two fields and on their phase difference.

The distribution of forces on the stator is equal and opposite to the distribution on the rotor, and the stator tends to be distorted from a circular shape to one ressembling the force distribution on the c and d figures. The shape changes with rotation of the fields but the same radial unbalanced component is always present.

25.- Fields Differing By Four Poles

The figures 6e and 7e show the force distribution around the air gap for the combination of a 4-pole flux field with an 8-pole flux field. In both cases the forces accross any diameter are balanced but they tend to distort the rotor and stator into a rotating ellipse because

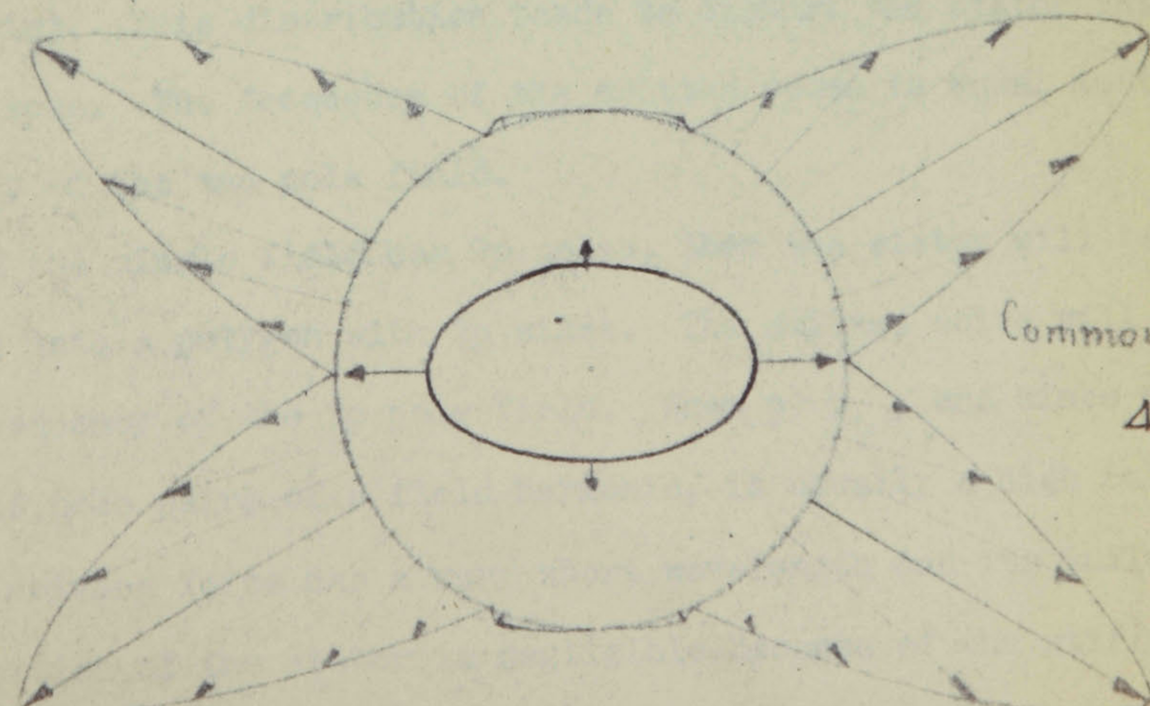
Four pole difference force distribution



Common max. value

4-8 poles

FIG. 6e.



Common zero value

4-8 poles

FIG. 7e

the resultant forces can be represented by two large equal forces at the ends of a diameter and two smaller equal forces at the two ends of a diameter perpendicular to the first one, as shown by the small diagrammatic representation inside the circles. The speed of rotation of this ellipse is equal to half the difference of the speeds of the two component fields.

It could also be shown, in the same way, that if a 2p-pole field be combined with a $2p \pm 2x$ pole field where $x = 3, 4, 5, \ldots$ etc, there will be x strong and x weak zones distributed regularly around the periphery of the rotor and the forces will costitute a balanced system tending to distort the stator into a, usually rotating, polygon with x sides. However, due to the usually heavy stator yoke section, a polarity difference of more than 4 poles will seldom cause any trouble. Even difference of 4 poles seldom give rise to much noise and can be neglected for most cases.

26.- Single Field

A single two pole field will produce a force distribution around the air gap expressed by $A \sin^2\theta$ where A is the magnitude of the force at its maximum point and θ is the angle around the periphery taken from a reference point. This distribution tends to distort the stator into a rotating ellipse. The frequency of the emitted sound is equal to twice the frequency of the two pole field.

If the single field has 2p poles, then the stator will tend to be distorted into a polygon with 2p sides. The emitted noise will have twice the frequency of the 2p pole field. When $p > p_o$, and since p being the number of pole pairs of a field harmonic, is usually a high multiple of p_o , the driving force has a very short wavelength and its influence on the distortion of the stator is negligible because of the stiffness of the yoke to high frequency driving forces. However, when $p = p_o$

then $\omega = \omega_o$ and the stator is distorted into a polygon with $2p$ sides emitting a twice fundamental frequency noise. This driving force arises principally from the radial magnetic attraction between stator and rotor. As the magnetic field revolves in the air gap it passes a given point twice during each cycle of voltage change, once as a flux entering the stator and once as a flux entering the rotor. A force of attraction results, regardless of flux polarity. This force therefore builds up and dies away twice during each cycle of voltage change and produces a driving force having a frequency exactly twice that of the voltage supply. This force cannot be eliminated since it is inherent in any electric machine but the vibration it causes can be controlled by constructing the stator core sturdy enough so that the resultant core response be small. This force does not depend on slot combinations or winding distribution.

27.- Synchronous And Asynchronous Motor Effects

When two of the harmonic air gap fluxes are travelling at the same speed and in the same direction, this distribution will produce either forward or backward torque depending on their relative position in the air gap but not upon their direction of motion. Only fluxes having the same number of poles can react to give a steady torque.

If the two fluxes are produced by magnetomotive forces of the same number of poles the torque is classed as induction. If, however, the two fluxes are produced by magnetomotive forces of a different number of poles, the torque is classed as synchronous. In the former case, the two component fields always travel at the same speed and at the latter they travel at the same speed and develop a steady torque only at some particular rotor speed.

The study of synchronous and asynchronous motor effects is beyond the scope of this paper. However, since they are always a deciding factor in the design of an induction motor, they will be considered here and rules to avoid fields with the same number of poles will be given.

28.- Single-phase Motor Torque Pulsations

The power imput to a single phase circuit is necessarily of a pulsating character, as each time the incoming current or voltage passes through zero, the power does likewise. Supposing:

$\sqrt{2}\, V \sin \omega t$ = voltage impressed on a single phase circuit

$\sqrt{2}\, I \sin(\omega t - \theta)$ = current flowing in a single phase

θ = power factor angle

The power at any instant is:

$$P = 2VI \sin \omega t \sin(\omega t - \theta) = VI\left[\cos\theta - \cos(2\omega t - \theta)\right]$$

where $VI\cos\theta$ = comstant uniform active power flowing in one direction

$VI\cos(2\omega t - \theta)$ = alternating power of double frequency

In a balanced polyphase circuit, at each instant, the sum of the alternating powers of the phases is zero so that only the active power flows in the circuit.

Since the power taken by the load is uniform, the incoming energy must be alternately stored during the peaks and supplemented by released energy during the depressions. This energy is supplied by the kinetic energy of the rotor that will continually fluctuate up and down in response to torque variations with an amplitude of variation inversely proportional to its kinetic energy. This torque pulsation reacts upon the stator and causes it to oscillate through a small angle about the motor shaft as an axis, thus transmitting vibrations to the base upon which the motor is mounted. The amplitude of motion of the stator under the inluence of torque variations is inversely proportional to its moment of inertia about the shaft as an axis.

The only way to avoid noise in the operation of a single phase motor is to so mount the stator as to allow free torsional vibration about the shaft as an axis, without imparting the vibration to the supporting structure. Ofcourse, all other magnetic causes of vibration should also be avoided. The less expensive way to secure noiseless operation is to mount the stator firmly on a support which is sufficiently flexible to take up the motion of the stator without itself producing noise. A good example of such a mounting is a long thin board of the same width as the motor base. If the motor is affixed to this board with the shaft lengthwise, quiet operation is secured, as the board twists readily with the motion of the stator and yet does not transmit any motion to its end supports. If the motor is placed with its shaft crosswise, a considerable amount of noise is produced as the motor alternately lifts up and pushes down on the ends of the board thus transmitting vibration through the whole structure. An increase in the moment of inertia of the rotor will give also smaller changes in speed and therefore less stator vibration.

All noises should be reduced practically to zero by polyphase operation or by the use of a proper capacitor, effective only under load. Open phase operation of a three phase machine shows this clearly. One phase of the 0.5 Hp motor was opened when the motor was operating on three phase and at full speed. The impression to the ear was of a louder noise of lower mean frequency. This was due to the fact that a 120 cycles component was introduced due to ~~2~~ single phase operation. The readings taken with the sound level meter are shown in the table below

Skew	Phases	Amps	A	B	C	D	Average
0	3	4.0	68	70	70	68	69.0
	2	4.0	77	81	75	73	74.5
1	3	2.3	59	66	60	58	60.8
	2	2.3	61.5	64.5	61	59	61.5

Background noise level 37 db. Standard distance 1 ft. Weighing curve 70 db.

It can be seen from the table that there is an increase in noise when one phase is opened which is appreciable with no skew but which is quite small with one slot skew. It is believed that if the load was increased and the current taken by the motor with the skewed stator brought up to 4 amps, the difference in the emitted sound would be larger. It was not possible to load the motor over 2.3 amps because, due to the skew, the torque was very low and the motor stalled.

SLOT COMBINATIONS

29.- Tabulation Of Flux Fields In Air Gap

The most important space harmonic flux fields, the number of their pairs of poles and their speed of revolution are given for ready reference in the table below.

	Characteristics	Pairs of poles	Speed
a	Stator field due to winding distribution	$p_a = p_o \pm 2\Phi K_1 p_o$	$\omega_a = \dfrac{\omega_o}{1 \pm 2\Phi K_1}$
b	Stator field due to stator slots	$p_b = \pm(p_o \pm K_2 Z_s)$	$\omega_b = \dfrac{\pm \omega_o}{1 \pm K_2 \frac{Z_s}{p_o}}$
c	Rotor field	$p_c = \pm(p_o \pm K_3 Z_r)$	$\omega_c = \pm \dfrac{\omega_o \mp \frac{Z_r}{p_o} K_3 \omega_o (1-s)}{1 \pm \frac{Z_r}{p_o} K_3}$
d	Permeance field $D = \lvert Z_s - Z_r \rvert$	$p_d = \pm(p_o \pm D)$	$\omega_d = \dfrac{(1-s) Z_r \pm p_o}{p_o \pm D} \omega_o$

30.- Partial Equations

It was shown in the preceding paragraphs that a magnetic driving force with p pairs of poles will tend to distort the stator and rotor into a polygon with p sides and that only for $p = 1$ and $p = 2$ the response of the stator and rotor will be effective in producing vibration and noise. The best procedure therefore for the determination of slot combinations that should be avoided because of the interaction of the as above tabulated harmonic fields will be to put $|p_n - p_{n'}| \neq m$ where $n = a, b, c, d$; $n' = a, b, c, d$; $n \neq n'$; $m = 1$ for a 2-pole difference, $m = 2$ for a 4-pole difference; higher values of m need not be considered in a noise investigation. $m = 0$ is also considered to include synchronous and asynchronous motor effects.

<u>a - b:</u> $p_a - p_b = (p_o \pm 2\Phi K_1 p_o) \pm (p_o \pm K_2 Z_s) = 2p_o \pm 2\Phi K_1 p_o \pm K_2 Z_s \neq m$

$$\text{or} = 2\Phi K_1 p_o \pm K_2 Z_s \neq m$$

from which

$$K_2 Z_s \neq |m \pm (2p_o \pm 2\Phi K_1 p_o)| \qquad (14)$$

$$\text{or} \neq |m \pm 2\Phi K_1 p_o| \qquad (14')$$

<u>a - c:</u> $p_a - p_c = (p_o \pm 2\Phi K_1 p_o) \pm (p_o \pm K_3 Z_2) \neq m$

This is a similar expression to the one in a - b with $K_3 Z_r$ in the place of $K_2 Z_s$. Substituting, therefore, these values in (14) and (14'),

$$K_3 Z_r \neq |m \pm (2p_o \pm 2\Phi K_1 p_o)| \qquad (15)$$

$$\text{or} \neq |m \pm 2\Phi K_1 p_o| \qquad (15')$$

<u>a - d:</u> Replacing $K_2 Z_s$ in a - b by D, a similar formula is derived for the interaction of the a and d fields

$$D \neq |m \pm (2p_o \pm 2\Phi K_1 p_o)| \qquad (16)$$

$$\text{or} \neq |m \pm 2\Phi K_1 p_o| \qquad (16')$$

<u>b - c:</u> $p_b - p_c = (p_o \pm K_2 Z_s) \pm (p_o \pm K_3 Z_r) = 2p_o \pm (K_2 Z_s \pm K_3 Z_r) \neq m$

$$\text{or} = (K_2 Z_s \pm K_3 Z_r) \neq m$$

Since Z_s is larger than $2p_o$ (there is always more than one tooth per pole), and $K_2 Z_s + K_3 Z_r$ is much larger than Z_s, there cannot be any 2m-pole difference between $2p_o$ and $Z_s + Z_r$. Therefore the + signs in the parentheses are dropped. It is then,

$$|K_2 Z_s - K_3 Z_r| \neq 2p_o \pm m \qquad (17)$$

$$\text{or} \quad \neq m \qquad (17')$$

b - d: $p_b - p_d = (p_o \pm K_2 Z_s) \pm (P_o \pm D) \neq m$

as in b - c, the terms containing $K_2 Z_s + D$ must not be taken into account and therefore,

$$|K_2 Z_s - D| = |K_2 Z_s - (Z_s - Z_r)| = |(K_2 - 1)Z_s - Z_r| =$$

$$= |K' Z_s - Z_r| \neq 2p_o \pm m \qquad (18)$$

$$\text{or} \quad \neq m \qquad (18')$$

K' is any integer minus one = 0, 1, 2, 3, ...

c - d: Replacing Z_r in b - d by Z_s a similar expression is derived for the interaction of the c and d fields:

$$|Z_s - K' Z_r| \neq 2p_o \pm m \qquad (19)$$

$$\text{or} \quad \neq m \qquad (19')$$

31.- General Formula

The writer believes that the mathematical derivation of paragraph 30 and the derived partial equations form a new approach to the problem of slot combinations. A general formula is derived therefrom which contains every case studied in par. 30. It is believed that it is the first time that such a general formula has been worked out. It can be written:

$$|K_\alpha Z_s - K_\beta Z_r| \neq |2\lambda p_o \pm 2\phi K_1 p_o \pm m| \qquad (20)$$

where: K_α, K_β, K_1, are parameters independent of each other and equal to 0, 1, 2, 3, 4, ... etc.

λ = 0 or 1

m = 0 for synchronous or asynchronous motor effects

= 1 for a 2-pole difference driving force

= 2 for a 4-pole difference driving force.

This relation not only applies to every combination of two harmonic fields in the air gap, but it also takes into consideration the harmonic fields in b, c, d, of the tabulation which are produced by the higher harmonics of the winding distribution. It had been assumed in the derivation that the harmonic fields in b, c, d, were produced only by the fundamental stator magnetomotive force distribution.

It will be shown, by giving special values to K_α, K_β, and K_1 that this formula applies to every case studied in parag. 30.

For: $K_\beta = 0$, $\lambda = 1$, $K_\alpha Z_s \neq |2p_o \pm 2\phi K_1 p_o \pm m|$ (14)

" $\lambda = 0$, $K_\alpha Z_s \neq |2\phi K_1 p_o \pm m|$ (14')

$K_\alpha = 0$, $\lambda = 1$, $K_\beta Z_r \neq |2p_o \pm 2\phi K_1 p_o \pm m|$ (15)

" $\lambda = 0$, $K_\beta Z_r \neq |2\phi K_1 p_o \pm m|$ (15')

$K_\alpha = 1$, $K_\beta = 1$, $\lambda = 1$, $D \neq |2p_o \pm 2\phi K_1 p_o \pm m|$ (16)

" " $\lambda = 0$, $D \neq |2\phi K_1 p_o \pm m|$ (16')

$K_1 = 0$, $\lambda = 1$, $|K_\alpha Z_s - K_\beta Z_r| \neq 2p_o \pm m$ (17)

" $\lambda = 0$, $|K_\alpha Z_s - K_\beta Z_r| \neq m$ (17')

$K_\beta = 1$, $K_1 = 0$, $\lambda = 1$, $|K_\alpha Z_s - Z_r| \neq 2p_o \pm m$ (18)

" " $\lambda = 0$, $|K_\alpha Z_s - Z_r| \neq m$ (18')

$K_\alpha = 1$, $K_1 = 0$, $\lambda = 1$, $|Z_s - K_\beta Z_r| \neq 2p_o \pm m$ (19)

" " $\lambda = 0$, $|Z_s - K_\beta Z_r| \neq m$ (19')

The magnitude of the driving force is inversely proportional to the order of the harmonic. Therefore, in checking if the slot combination used in an induction motor agrees with relation (20), a disagreement can be completely neglected for higher values of K but will have an important influence on the noise emitted if K = 0, 1, or even 2.

32.- Numerical Application

$p_o = 2$, $\phi = 3$, $Z_s = 36$

1.- $Z_r = 48$ 2.- $Z_r = 45$

(20) becomes : 1.- $|36\,K_\alpha - 48\,K_\beta| \neq 4\lambda \pm 12\,K_1 \pm m$

2.- $|36\,K_\alpha - 45\,K_\beta| \neq 4\lambda \pm 12\,K_1 \pm m$

Ia.- $\lambda = 0$ $\qquad \left| 36\,K_\alpha - 48\,K_\beta \right| \neq 12\,K_1 \pm m$

The inequality always holds for m = 1 or 2 because the three other terms are multiples of 12. An equation can be derived only for m = 0 and for $K_\alpha = K_\beta = K_1 = 1$, which means that there will be dips in the torque curve. The next in amplitude effect would happen when $K_\alpha = 0$, $K_\beta = 1$, $K_1 = 4$, but it has a negligible influence on the torque because it is produced by a high winding harmonic ($K_1 = 4$)

lb.- $\lambda = 1$ $\qquad \left| 36\,K_\alpha - 48\,K_\beta \right| \neq \left| 4 \pm 12\,K_1 \pm m \right|$

For m = 0, 1, or 2 the inequality will always hold because three of the terms are multiples of 12 while $4 \pm m$ is always smaller than 12.

The combination therefore of the 36 slots stator and the 48 slots rotor will give a very quiet motor since there are no solutions for m = 1 or 2. The torque curve however will have a dip since there are two fields with the same number of poles.

2a.- $\lambda = 0$ $\qquad \left| 36\,K_\alpha - 45\,K_\beta \right| \neq 12\,K_1 \pm m$

Since all the other terms are multiples of 3 there cannot be any equality for m = 0, 1 or 2. The solution for m = 3 is ineffective in producing noise.

2b.- $\lambda = 1$ $\qquad \left| 36\,K_\alpha - 45\,K_\beta \right| \neq \left| 4 \pm 12\,K_1 \pm m \right|$

For $K_\alpha = K_\beta = K_1 = m = 1$ the general formula gives 9 = 9 and therefore there is a 2-pole driving force which produce stator vibration and noise. Also for $K_\alpha = 2$, $K_\beta = 1$, $K_1 = 2$, m = 1 the formula gives 27 = 27. There is again a two-pole force but of smaller amplitude because of higher K's. For m = 0 the inequality always holds.

The combination therefore of the 36 slots stator and the 45 slots rotor will give a perfect torque curve since there are no solutions for m = 0. This combination however produces stator vibration which may give rise to very objectionable noise.

33.- Experimental Evidence

The four 45 slots rotors and the four 48 slots rotors were tested with two stators identical in every respect except for the tooth design. In the table below only the values of sound level in the A direction and the average values are recorded. The A direction is chosen because in this direction the noise is maximum, and the maximum value rather than the average is representative of the noise emitted. Background sound level was 38 db. Weighing curve 40 db. Standard distance 6".

Stator lamination	Rotor slots	Skew in rotor slot pitch							
		1.0		1.5		2.0		2.5	
		Max.	Aver.	Max.	Aver.	Max.	Aver.	Max.	Aver.
Obsolete	45	63.0	62.4	58.0	57.1	56.5	54.5	55.3	54.2
	48	59.0	57.1	56.0	54.2	53.5	52.8	53.5	52.8
New	45	63.0	60.2	58.0	56.2	56.0	55.0	55.0	54.0
	48	56.5	56.2	56.0	54.5	55.0	53.5	54.5	53.1

This table shows that the experimental results agree thoroughly with the theory advanced. The improvement in quietness when the 48 slots rotor is used is very important. This improvement is more noticeable in the smaller skews (4 to 6 db which represent 1 / 3 to 1 / 4 the sound power output with the 45 slots rotor), and would be even more important if the rotors were not skewed at all. It must be noted that not a single measurement disagreed with the expected results; this shows that the equipment has been manufactured carefully.

34.- Good Slot Combinations (From Bibliography)

Various writers have, at different times, expressed rules for the choice of the best slot combination that would cut the noise emitted from a motor to the lowest possible level. It will be shown in this paragraph how all these rules are included in the general formula (20).

Rule 1.- (G. Kron[9], Punga & Raydt[15]). " When the slots differ by one or by the number of poles ± 1 transverse vibration may occur ". This is given by (20) when $K_\alpha = K_\beta = 1$, $K_1 = 0$ and m = 1 (2-pole difference fields). It is: $|Z_s - Z_r| = 2\lambda p_o \pm 1$

Rule 2.- (Same). " When the slots differ by the number of poles torsional vibration and noise may occur ". For $K_\alpha = K_\beta = 1$, $K_1 = 0$, $\lambda = 1$, m = 0 (asynchronous motor effect), (20) becomes: $|Z_s - Z_r| = 2p_o$

Rule 3.- (G. Kron[9]). " When the slots differ by half the number of poles torsional vibrations and noise may occur ". $K_\alpha = K_\beta = 2$, $\lambda = 1$, $K_1 = 0$, m = 0, and (20) becomes: $2|Z_s - Z_r| = 2p_o$ Or $|Z_s - Z_r| = p_o$.

Rule 4.- (Same). " $Z_r/2p$ should not be an integer ". If $Z_r/2p$ is an integer and since $Z_s/2p$ is usually an integer, (20) will not hold, usually, for m = 0. The torque curve may have dips. When expressing this rule, Mr. Kron was thinking only of the torque characteristics of the motor because, for m = 1 or m = 2, if $Z_r/2p$ is an integer it does not mean that noise will occur. On the contrary, it will be shown in the study of skew that the number of rotor slots should be divisible by the number of poles.

Rule 5.- (Tesla & Lamme). " Use of prime number of rotor slots". Though with this design dead points and low torque would be avoided since the second part of (20) is an even number for m = 0, it will result in an unbalanced pull on the rotor and therefore will have a tendency towards noise and vibration

Rule 6.- (H. S. Specht[6]). " $Z_r \neq Z_s$ ". This is obvious because for m = 0, $K_1 = 0$ and $\lambda = 0$ it would always be $K_\alpha Z_s - K_\beta Z_r = 0$. For $K_\alpha - K_\beta$ there would be an infinite number of locking harmonics. This rule can be extended with the help of (20) : $K_\alpha Z_s \neq K_\beta Z_r$.

35.- Good Slot Combinations (From (20))

Neglecting the higher harmonics due to winding distribution, the following relations can be derived from (20):

a.- m = 0 $|K_\alpha Z_s - K_\beta Z_r| \neq 2p_o$

or $\neq 0$

for synchronous and asynchronous motor effects.

b.- m = 1 $|K_\alpha Z_s - K_\beta Z_r| \neq 2p_o \pm 1$

or $\neq 1$

for rotor vibration due to unbalance rotating force

c.- m = 2 $|K_\alpha Z_s - K_\beta Z_r| \neq 2p_o \pm 2$

or $\neq 2$

for rotating elliptical stator distortion

These results can be included in the following rule:

" The difference of the stator and rotor slots or the difference of the smaller multiples of the number of stator and rotor slots should not be equal to:

the number of poles or zero, to avoid harmonic torques

the number of poles ± 1 or 1, or the number of poles ± 2 or 2, to avoid noise and vibration ".

36.- Simpler Way To Find If A Motor Will Be Quiet

The most complete way to determine how good a slot combination would be, is to use the values of Z_s, Z_r, p_o, ϕ in the general formula and see if there are any driving forces of any importance (low values of K) for m = 0, 1, 2.

There is, however an easier way to check on the slot combination that saves time though the results are not so accurate. The number of poles for the four fields of the tabulation of par. 29, when neglecting the higher harmonics, become:

a.- $p_a = p_o \pm 2\phi K_1 p_o$ (21)

b.- $p_b = p_o \pm Z_s$

c.- $p_c = p_o \pm Z_r$

d.- $p_d = p_o \pm D$

b, c, d, may be included in one equation $p = p_o \pm r$ (22)

where $r = Z_s$ or Z_r or D.

Procedure: Tabulate qualitatively the fields given by (21) and (22) and examine for two fields differing by 0, 2, or 4 poles

Numerical application : $Z_s = 36$ $Z_r = 48$ or 45

(21) $\phi = 2$

N	2-pole	4-pole	6-pole
1	1	2	3
3	3	6	9
5	5	10	15
7	7	14	21
9	9	18	27
11	11	22	33
13	13	26	39

$\phi = 3$

1	1	2	3
5	5	10	15
7	7	14	21
11	11	22	33
13	13	26	39
17	17	34	51
19	19	38	57

(22) $Z_r = 48$

	2-pole	4-pole	6-pole
$r = Z_s$	35 , 37	34 , 38	33 , 39
$r = Z_r$	47 , 49	46 , 50	45 , 51
$r = D$	11 , 13	10 , 14	9 , 15

$Z_r = 45$

	2-pole	4-pole	6-pole
$r = Z_s$	35 , 37	34 , 38	33 , 39
$r = Z_r$	44 , 46	43 , 47	42 , 48
$r = D$	8 , 10	7 , 11	6 , 12

48 slots rotor.

For a 2-pole motor, the 11-pole and 13-pole harmonics of (21) lock with the 11-pole and 13-pole harmonics of (22) for two phase or three phase machines. There is no 2-pole or 4-pole difference fields.

For a 4-pole motor, the locking harmonics will be the 10th and 14th which have only a smaller effect because they are lower harmonics. There is also a locking of the 34-pole and the 38-pole harmonics but their amplitude is negligible. There are no 2-pole or 4-pole difference fields. These results apply to two and three phase motors.

For a 6-pole motor locking of the 9th and 15th (large amplitude) and 33rd and 39th, 45th and 51rst (negligible amplitude). No 2-pole or 4-pole field difference for two or three phase.

45 slots rotor

$p_o = 1$. There are no fields with the same number of poles but the 7th and 9th harmonics from (21) react on the 8th and 10th from (22) to give an unbalanced force that sets the rotor into vibration. This applies to two and three phase motors.

$p_o = 2$. In two phase motors the 6th and 10 th harmonics of (21) react on the 7th and 11th of (22) and produce noise. In three phase motors only the 10th harmonic reacts on the 11th. There are no harmonics with the same number of poles.

$p_o = 3$. There is only a locking point for the 33-pole and 39-pole harmonics which are quite small. There are no harmonic fields setting the rotor into vibration. Same for two or three phases.

It can be seen from this example that the results agree with what had been derived in par. 32, that is that the 36 - 48 combination will give a practically noiseless motor but there will be dips in the torque curve, while the 36 - 45 combination, though it has a smooth torque curve is liable to become very noisy. For a 6-pole motor however a 45 slots rotor is more advantageous than a 48 slots one.

The results obtained for a two phase motor apply as well to a single phase motor since the harmonic fields given by (21) are the same for both cases.

VIBRATION OF TEETH

When the noise problem in induction motors was first investigated there was a great deal of speculation concerning the particular part of the machine that produced the principal vibration. Suspicion was cast on stator teeth, on the tips of the rotor teeth and on the rotor teeth as a whole. It finally became evident that, as shown in the preceding paragraphs, the principal source of trouble was the vibration of the stator and of the rotor as a whole unit. The vibration, however, of the teeth, though of a minor importance, increases the noise emitted and complicates the sound pattern.

37.- Vibration Of Stator Teeth

Fig. 8 shows the rotor teeth and the different, relative to the rotor, positions occupied by a stator tooth during the rotation of the rotor. The flux in a stator tooth pulsates from a maximum when the tooth is in position "a" to a minimum for the position "e". The force exerted on

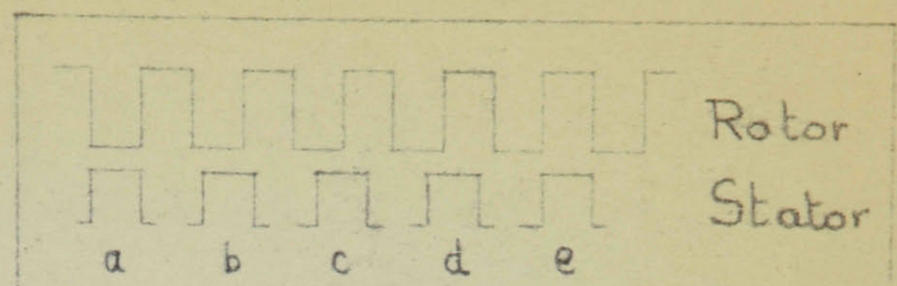


FIG. 8

the stator teeth can be analyzed into a radial component and a tangential component. The tangential component is zero at the position "a", passes through a maximum at "c" when the stator tooth has moved half the width of the rotor tooth, and becomes zero again at the position "e" where the forces of attraction from the two adjacent rotor teeth are equal. Passed the point "e", this force increases again but on the opposite direction to become zero when the stator tooth is in line with the next rotor tooth. Thus the tangential force fluctuates with a frequency equal to Z_r x rps (revolutions per second), and sets the stator tooth in a tangential vibration with the same frequency.

The natural period of vibration of the teeth in a tangential direction can be determined with a comparatively high degree of accuracy by calculation as if for a rod held at one end. For this case, the lowest natural frequency of vibration is given by the equation:

$$f = \frac{1}{l^2} \left(0.560 \sqrt{EIg/Aa} \right) \qquad (23)$$

where: l = length of bar, EI = flexural rigidity of the bar, A = area of cross section, d = weight per unit volume of the material of the bar. Calculations made for different tooth designs of standard motors and condensed in Fig. 9 show that the natural frequency is very high viz. above 3000 cycles per second. Short wide teeth have a higher natural frequency and therefore give less noise than long narrow ones

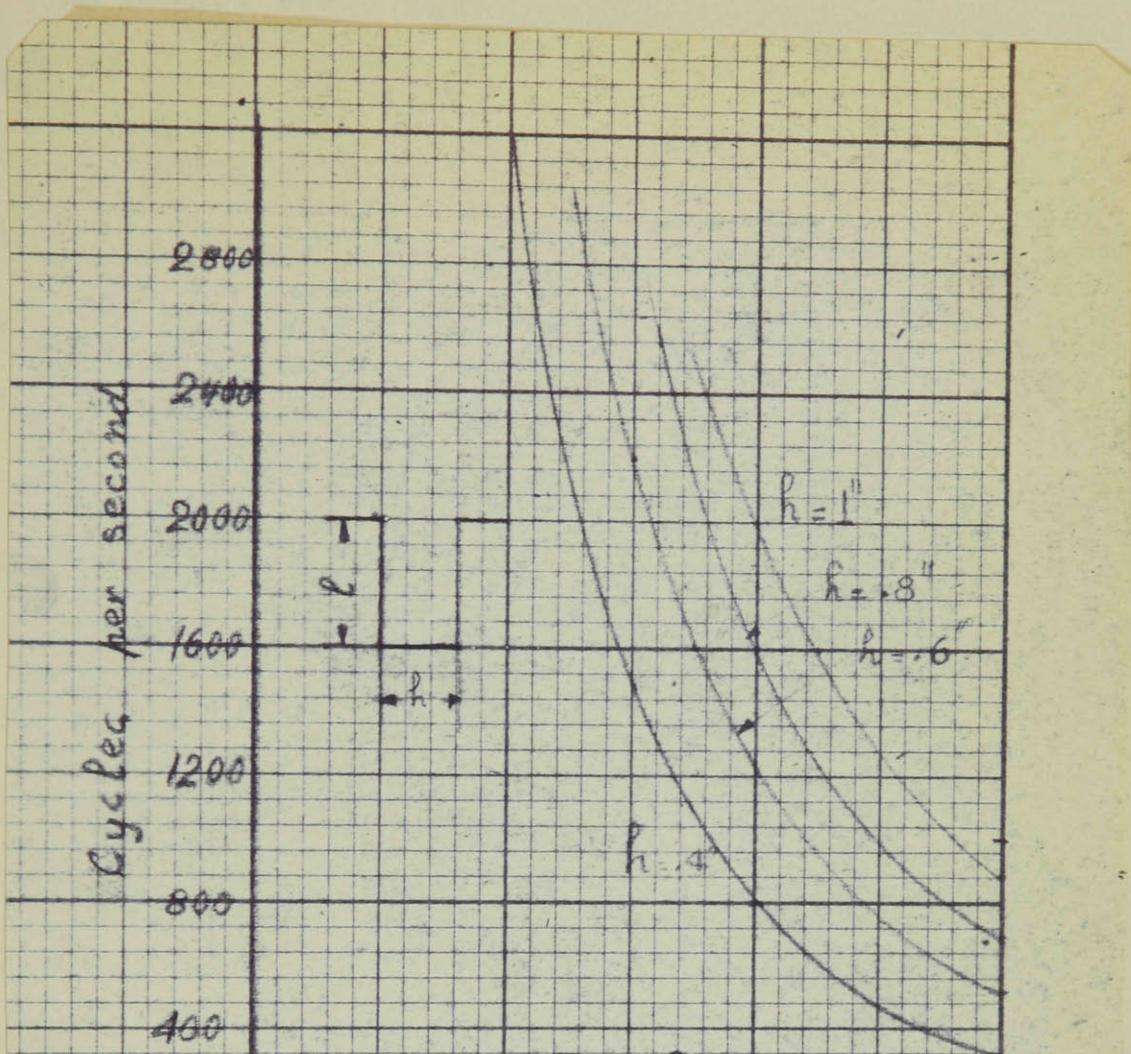


FIG. 9

Numerical application: The 0.25 HP motor studied in this paper has a tooth width of .17", a slot depth of .688" and a core stack of 1.75".Then :

$l = .688$ in.

$E = 29 \times 10$ lb/in.^{2}

$I = 1.75 \times .17 /12$ in.^{4}

$g = 32.2 \times 12$ in/sec.^{2}

$A = 1.75 \times .17$ in.^{2}

$d = 3.65$ lb/in.^{3}

Applying these values in (23) f is found to be equal to 3160 cycles per second.

In almost every case resonnance can be avoided since the impulse frequency of the tangential magnetic pull is usually lower than 2500 cycles per second. Thus , for the case of the 48 slots rotor, the impulse frequency is 48 x 60 = 2880 cycles per second for the two pole motor and 1440 cycles per second for twe four pole. In the case of large high speed machines a special study of the tangential stator tooth vibration must be made.

The tangential pull of the teeth is further modulated in amplitude by the fundamental frequency of the alternating or rotating field. It is thus possible for modulation frequencies to occur which may be either the sum or the difference of the two frequencies, i.e. N_r x rps $\pm$ f_o

The radial component of the force exerted on the tooth fluctuates from a maximum when a stator and a rotor tooth are in line to a minimum when the stator tooth has moved by half a rotor slot pitch. This force sets the stator tooth into radial vibration with a frequency N_r x rps which, though of a minor importance, adds to the noise due to tangential vibration.

38.- Vibration Of Rotor Teeth

What was said about the stator teeth in last paragraph applies equally well to the rotor teeth if the words stator and rotor are interchanged and N_s used instead of N_r. The number of stator teeth being usually smaller than the number of rotor teeth, there are fewer possibilities that the frequency of the magnetic force will reach the natural period of vibration of the rotor teeth.

39.- Vibration Of Rotor Tooth Tips

The force of attraction between the stator tooth and the rotor tooth tip is maximum in position A and zero in position B as shown in Fig. 10. The rotor tooth tip will be set in vibration with a frequency equal to the number of stator teeth x rps. This noise must be provided against by having the root y sufficiently thick to prevent bending, and by the use of a larger air gap which reduces the reactance variation. This factor is completely absent in rotors with totally closed slots but is very frequent in machines with a small air gap and thin rotor tooth tips.

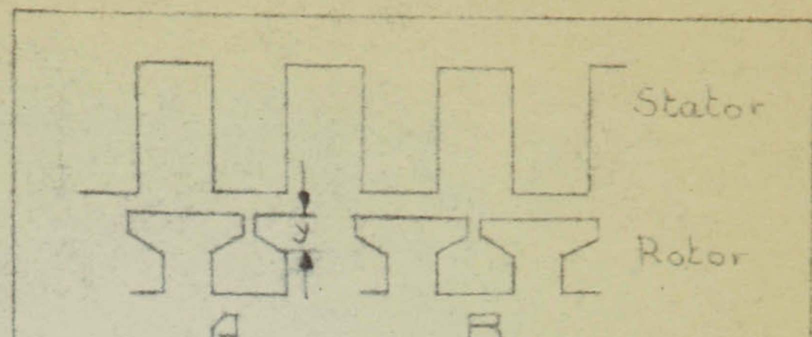


FIG. 10

40.- Axial Vibration Of Teeth

Certain authors, (Hildebrand[(6)]), have expressed the opinion that the teeth may produce noise by vibrating axially. However, nothing in the previous analysis shows that there should be an axial component of magnetic force that would produce axial vibration. The only driving force that would be responsible for axial vibration is the force of attraction between adjacent laminations which has twice fundamental frequency because it is proportional to the flux density squared. Tight clamping and impregnation should minimize the emitted note. However, Mr. H. C. Specht[(6)] has tried motors without dipping them in varnish or bakelite so that the laminations would move freely in the axial direction, and he noticed only a slight difference in noise level compared with motors that were dipped and baked. This is accepted by Mr. Specht as somewhat of a proof that the axial vibration of the teeth is bothering a little if at all.

The writer tried two stators of which one was baked while the other was not. The stators were identical in every other respect. The readings are tabulated below. Standard distance 6". Background noise level 37 db. Weighing curve 40 db.

	Volts	Amps	Max.	Aver.
Not baked	97	4.8	55.5	51.85
	108	6.0	54.5	51.65
Baked	97	4.8	53.5	53.50
	109	6.0	54.5	54.25

If the theory of axial vibration was true, the baked stator, having the laminations more tightly held together, should give less noise. This is not the case however, because the readings do not allow to come to any conclusions. Though there seem to be an improvement in the maximum values, the average values of the baked laminations, on the contrary, are much higher.

41.- Tooth Vibration Due To Zig Zag Leakage Flux

When the stator and rotor teeth are in the position B as shown in Fig. 10, the zig zag flux is a minimum, and when in A it is a maximum so that there is a pulsation of flux in the tooth tips and two notes are emitted which have frequencies equal to Z_r x rps and Z_s x rps. To reduce this noise the variation in the leakage flux must be a minimum. This is achieved by making the leakage flux as small as possible. A. Gray[4] states that the leakage flux is directly proportional to the ampereconductors per slot and inversely proportional to the air gap clearance. Gray suggests that to prevent excessive noise due to zig zag leakage, the ratio of the ampereconductors per slot at full load to the air gap clearance in inches must be less than 14000 for machines with open stator and practically closed rotor slots or 12000 for machines with partially closed slots for both stator and rotor. In the 0.25 HP motor it is:

$$\frac{\text{ampereconductors per slot}}{\text{air gap clearance in inches}} = \frac{109}{.015} = 7300 < 12000$$

The rotor slots being totally enclosed, the noise due to zig zag leakage flux pulsation is very small.

With a rotor differing one tooth from the stator the zig zag leakage fluxes are very much unbalanced and the unbalancing rotates 360° around the cicumference and fluctuates in amplitude with the motor moving one tooth pitch. This would cause pulsation and noise either by deflection of the shaft or looseness of the bearings. This phenomenon has been included in (20) and it can be seen from there that for $K_\alpha = K_\beta = m = 1$, $\lambda = K_1 = 0$ it is $|Z_s - Z_r| = 1$ and an unbalanced force sets the rotor into vibration. The case of one slot difference is an extreme one not likely to occur in practice but similar conditions although less extreme may exist with an odd number of slots in the rotor, a construction which has been frequently used in the past in order to avoid dead points under starting conditions. It is nearly always safe to use, especially in small motors, an even number of slots in the rotor in which case the zig zag leakage is always balanced.

42.- Experimental Evidence

From the table in paragraph 33 it may be seen that no conclusions can be drawn as to which of the two laminations is the best. Though the obsolete lamination has narrower and longer teeth than the new one, the noise emitted is in some cases higher and in some other cases lower than the noise emitted when the new lamination is used. It seems therefore that the tooth design has little influence on the noise level of this motor. This was expected because the rotor slots are closed and the stator slots have a small only opening. Moreover, the driving forces are out of the range of the resonant frequencies of the tooth vibration.

CHAPTER III

MECHANICAL CONSIDERATIONS

MECHANICAL DRIVING FORCES

An important agency which contributes to excessive vibration is static or dynamic unbalance of a rotating assembly. This produces an excessive mechanical driving force of fundamental frequency -- that is numerically equal to the speed of rotation in revolutions per second -- and this results in an excessive vibrational response of the same frequency. This phenomenon has long been realized and many kinds of balancing machines have been developped for testing and correcting the condition of balance of rotating parts.

In addition to the fundamental component, harmonic driving forces having frequencies which are multiples of the speed of rotation in revolutions per second frequently occur. These are caused by mechanical imperfections or dissymmetries in the construction of the machine. For this reason they are more difficult to control than the fundamental. The second harmonic component (double frequency than the fundamental) is most likely to give rise to troublesome vibration. This component arises from shafts having portions of non circular cross sectional area, caused by keyways, for instance, from slightly elliptical bearings and other mechanical imperfections such as rotor or stator out of round, bent shaft, too great clearance between shaft and bearings or uneven or eccentric air gap. There are number of elements that may cause the rotor or stator to be out of round. In the first place there is a slight variation due to punch-and-die work, which may amount to 0.005" between individual punchings. In the second place some allowance around the outside of the punching must be made in the fixture or frame in which they are built up so that they will assemble readily, and this allows the punchings to stagger more or less. In the third place when the punchings are actually assembled in the frame, the frame may spring out of shape slightly after machining owing to the release of casting strains when removing the material in the cut. Ofcourse none of these variations is in itself large but when they all accumulate in the same direction,perceptible eccentricity may result amounting to a good many thousands of an inch. Mechanical vibration may also be due to either the stator or the rotor windings. For example in a squirrel cage rotor there may be bad contacts between certain bars and the shortcircuiting rings, resulting in more resistance in some parts of the windings than in others. This in turn affects the distribution of current in the different bars and hence affects the magnetic field and varies the mechanical pull from point to point.

The only effective method by which harmonic driving forces arising from these causes may be controlled is by the use of rotating assemblies and shafts that are as nearly circular in cross section as practical considerations will permit, and by better workmanship and closer tolerances in the manufacture of the component parts of the machine.

VIBRATIONAL RESPONSE

43.- General Considerations

The airborne noise arising from an electric motor has as its principal source the vibration of stator frame and rotor structure. The natural and correct assumption is that the magnetic forces set up in the air gap of the motor, and the mechanical unbalance, cause the frame structure and the rotor to be distorted periodically with time and that these vibrating surfaces beat the air causing airborne noise. The distortion of the frame is very often transmitted to the supporting structure which, in turn, is set into vibration and may amplify the emitted noise if its natural period of vibration happens to be close to the period of the driving force. It is very difficult, if not impossible, to predict the way in which the supporting structure will react to the vibrations transmitted through the supports of the motors, as shown by the following case: In a substation, located in the basement of an office building, the vibration due to open phase operation of a converter was very pronounced only thirteen floors above, and yet the operator in charge was not aware of anything wrong.

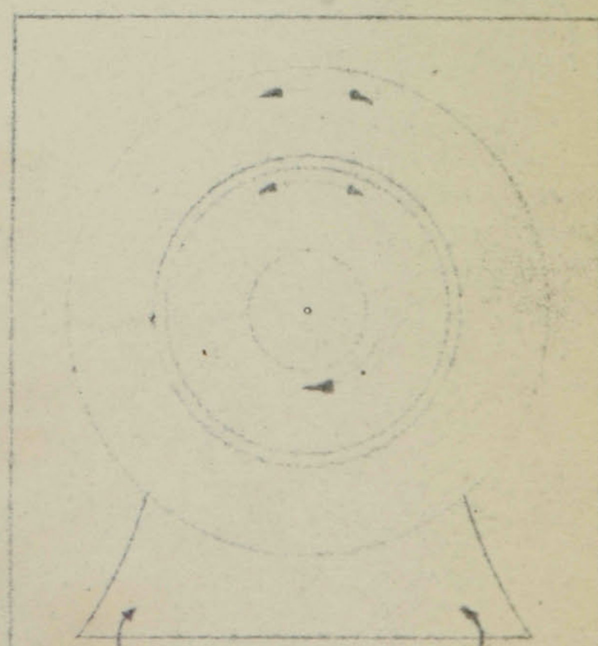


FIG. 11

The noise producing parts can vibrate in several ways. A torsional vibration of the stator and rotor as a whole results from periodic torque pulsation as in the case of single phase motors. Such a vibration may be particularly objectionable as it is transmitted directly through the motor feet to the supporting structure (Fig. 11). Due to the reaction of unsymmetrical supports, torque pulsation may also distort the stator from its circular shape. A circular vibration simulating a mechanically unbalanced rotor as shown in Fig. 12 results from an unbalanced magnetic pull. Such is the case when there are in the air gap two fields with a 2-pole difference. It has been shown that this unbalanced force may revolve in either direction and at other than synchronous or rotor speeds or it may even be stationary. If due to the interaction of two fields with a difference of four poles, the stator may be deflected as shown in Fig. 13, and as proved in par. 25, into a rotating elliptical shape. Similarly, the force may be balanced on more than two radii resulting in a tendency toward a rotating polygon.

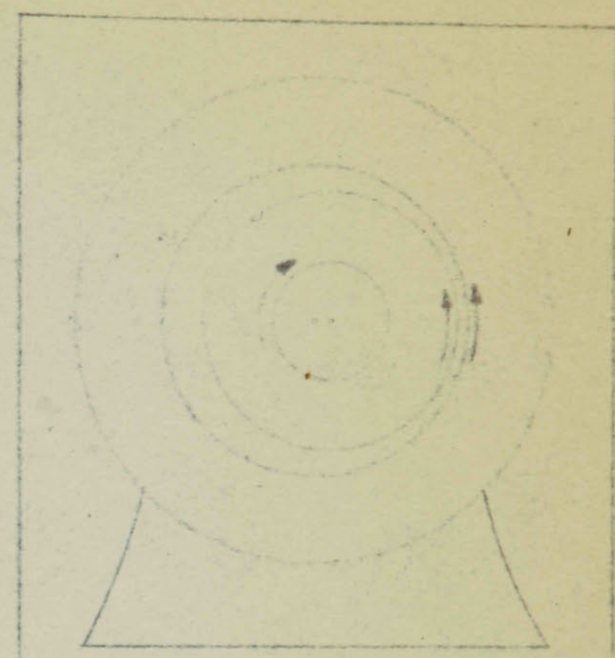


Fig. 12

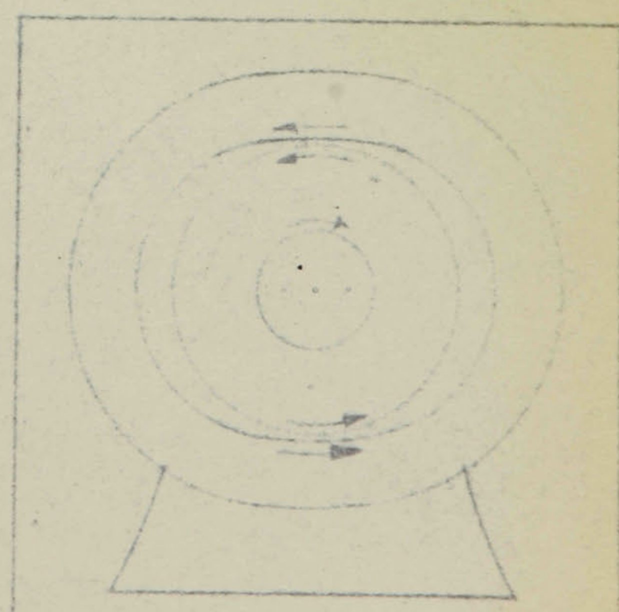


Fig. 13

Most vibrations are forced i.e. their frequency is not near a critical frequency. In this case the vibration is proportional to the force and inversely proportional to the rigidity of the parts. However, if the frequency of the driving force corresponds with one of the natural resonant frequencies of the rotating assembly, the latter is in indifferent equilibrium and a small component of driving force may cause excessive vibration and whipping or even mechanical failure of the rotating assembly. A thin cast iron housing, for instance, round in shape and with end brackets of a bell shape, will have periods of natural frequency which very often happen to be in the order of the magnetic noise frequencies thus giving rise to a very objectionable noise. For satisfactory operation, the ordinary critical speeds must be considerably outside the normal operating speed range of the machine. Since the pressure of economics preclude the possibility of obtaining quietness by the introduction of massive structures, ways must be found to obtain desired levels of quietness with the same or lighter structures than those previously used.

It was the purpose of Chapter II to determine the magnetic driving forces and to give rules for reducing these forces to the lowest possible values. Approximate formulae will be given in this Chapter so as to find the natural frequency of vibration of frames. One must be very careful in the use of these formulae because, as shown in some papers, experience with tests on many motors indicate that there is sufficient variation in the response characteristics of apparently identical stator structures, assembled in complete motors,to leave little hope that in the near future a reasonably exact expression for stator response can be presented. Thus it happens, very rarely however, that a number of slots recognized as being theoretically desirable and proved to be so on a given type of motor should not also be advantageous on another type of motor. This is not an absolute rule because every induction motor has its own complicated oscillatory system which may be excited by some upper harmonic, seamingly perfectly harmless in itself.

Calculations of stator response do have their place in a noise investigation but the results obtained thus far have been more qualitative than quantitative.

In the case where mechanical resonance is prominent, the only effective method of reducing the resultant vibration is a change in the machine design which will shift the natural resonant frequency outside the range of frequencies encountered during normal operation. It is not so easy to insure this result in a new design because of the inaccuracy of the frequency formulae. This, however, can be done after the final test of the machine is made. If high amplitudes of vibration do not arise from small driving forces the design is satisfactory. Otherwise stiffening of certain parts and modification of the shape of others are necessary.

Frequencies in the lower audio range are of major importance. For these frequencies the stiffness and damping factors of mechanical members are comparatively much smaller and a much greater resonant response can result. Calculations and measurements of the vibrating periods of stator structures have shown that it is quite possible to get natural frequencies in the range of 120 cycles per second which is the frequency of the major flux pulsation. Fortunately, very low resonant frequencies are not usually encountered in machines of standard design because the frame and other parts must be constructed with sufficient strength to withstand stresses which are likely to occur during normal and abnormal operation.

44.- Stator Vibration

It may not seem entirely obvious that a rotating deflection of the stator can be affected by resonance. It can be shown, however, that the rotating pull or deflection may be considered as the result of two stationary pulsating pulls which are 90 deg. apart in space and in time. Thus, resonance to a travelling elliptical deflection is quite possible.

Mr. J. P. Den Hartog[5] has worked out a formula which, taking into account the flexibility of the feet, gives the lowest natural period of vibration of the frame. The frame is assumed to have the simple shape shown in Fig. 14. Fig 15 represents the original and the distorted shape of the frame. The flexibility of the feet is expressed by the ratio $K = M/\psi$ i.e. the bending moment in in-lb at the feet necessary to cause a change in slope of one radian. Then the expression $\alpha RK/EI$ is to be computed, where:

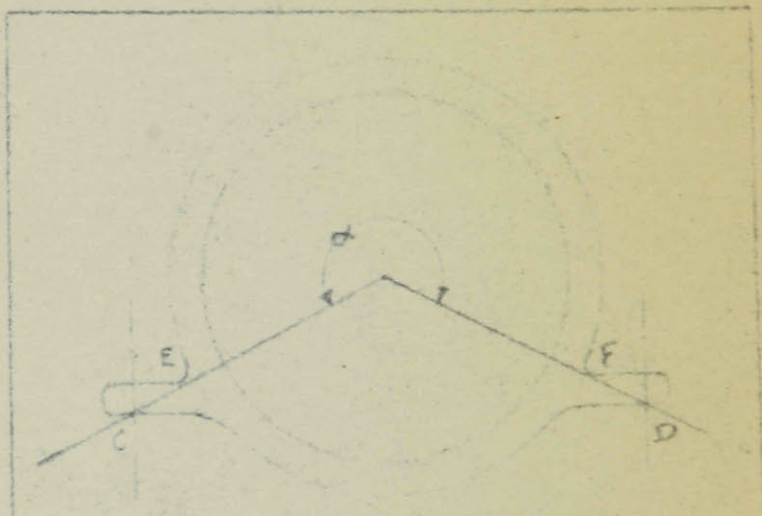


FIG. 14

R = mean radius of the ring in inches

E = modulus of elasticity in pounds per square inch.

I = moment of inertia of the cross section of the ring in inches^4

α = angle in radians as shown in Fig. 14.

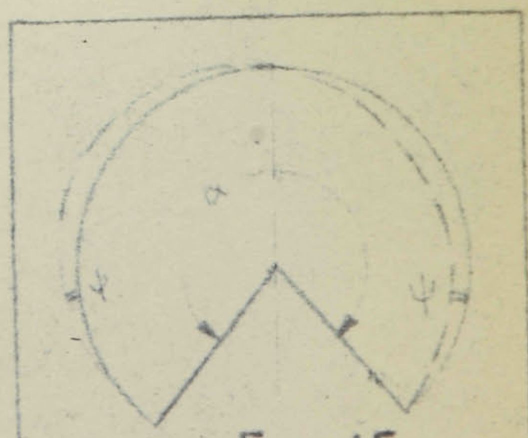


FIG. 15

With this expression the value of a parameter θ (expressed in degrees) can be taken from Fig. 16. This parameter θ determines the flexibility of the feet and varies between 0 deg. and 77.5 deg, these

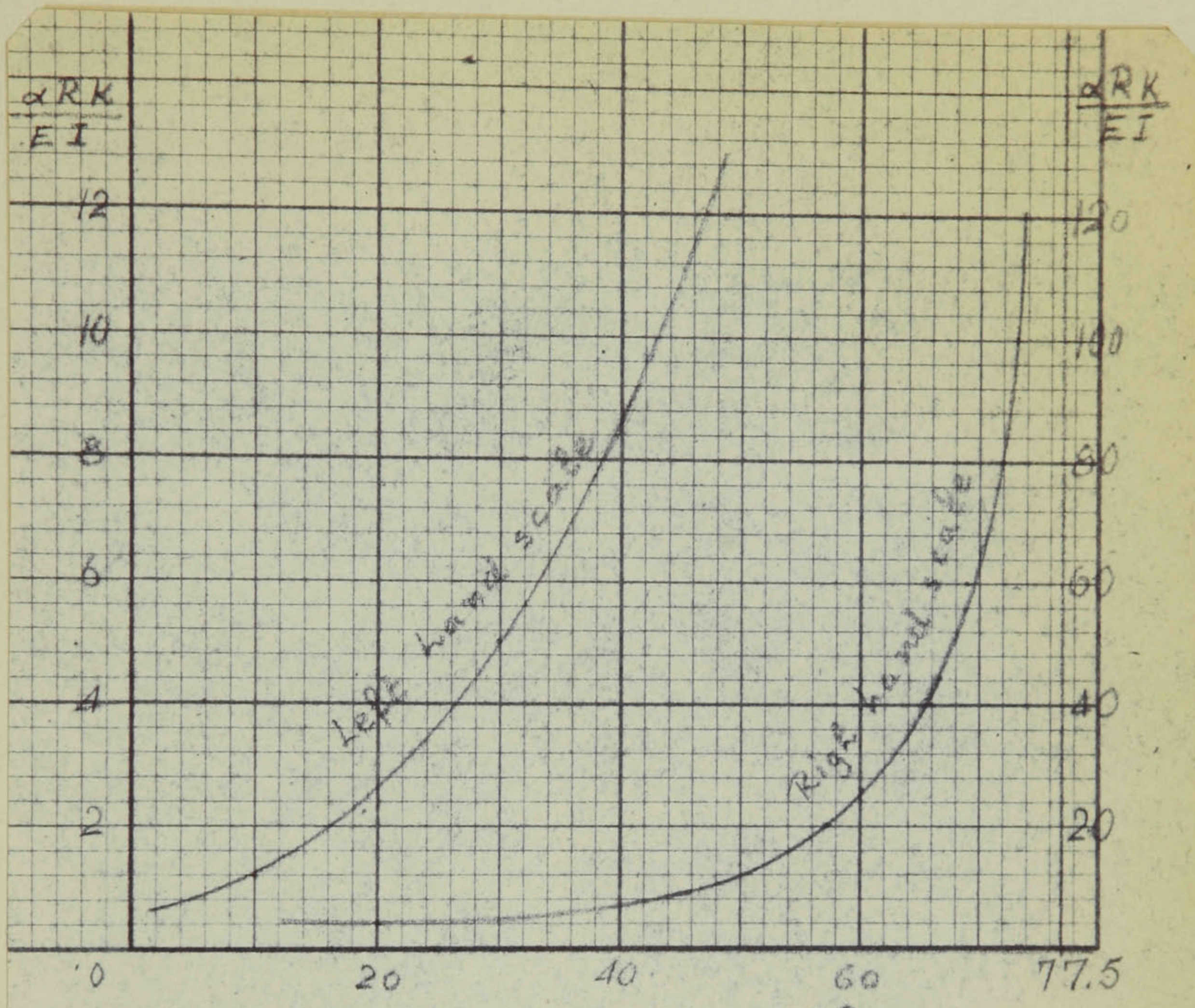


FIG. 16

values corresponding to hinged and clamped feet. The stiffer the feet, the larger the value of θ will be.

Fig. 17 fixes the magnitude of a constant C when θ and the central angle α are known. The natural frequency will be:

$$f = \frac{C}{2\pi R^2}\sqrt{\frac{EI}{\gamma}} \qquad (24)$$

where γ is the mass per inch of circumferencial length of the frame and is equal to 1/386 times the weight (lb) per inch. It must be noted that, as reported by some authors (Graham[13]), tests made in a number of machines have shown that resistance to a distortion in the plane of the laminations is usually dependant entirely upon the frame

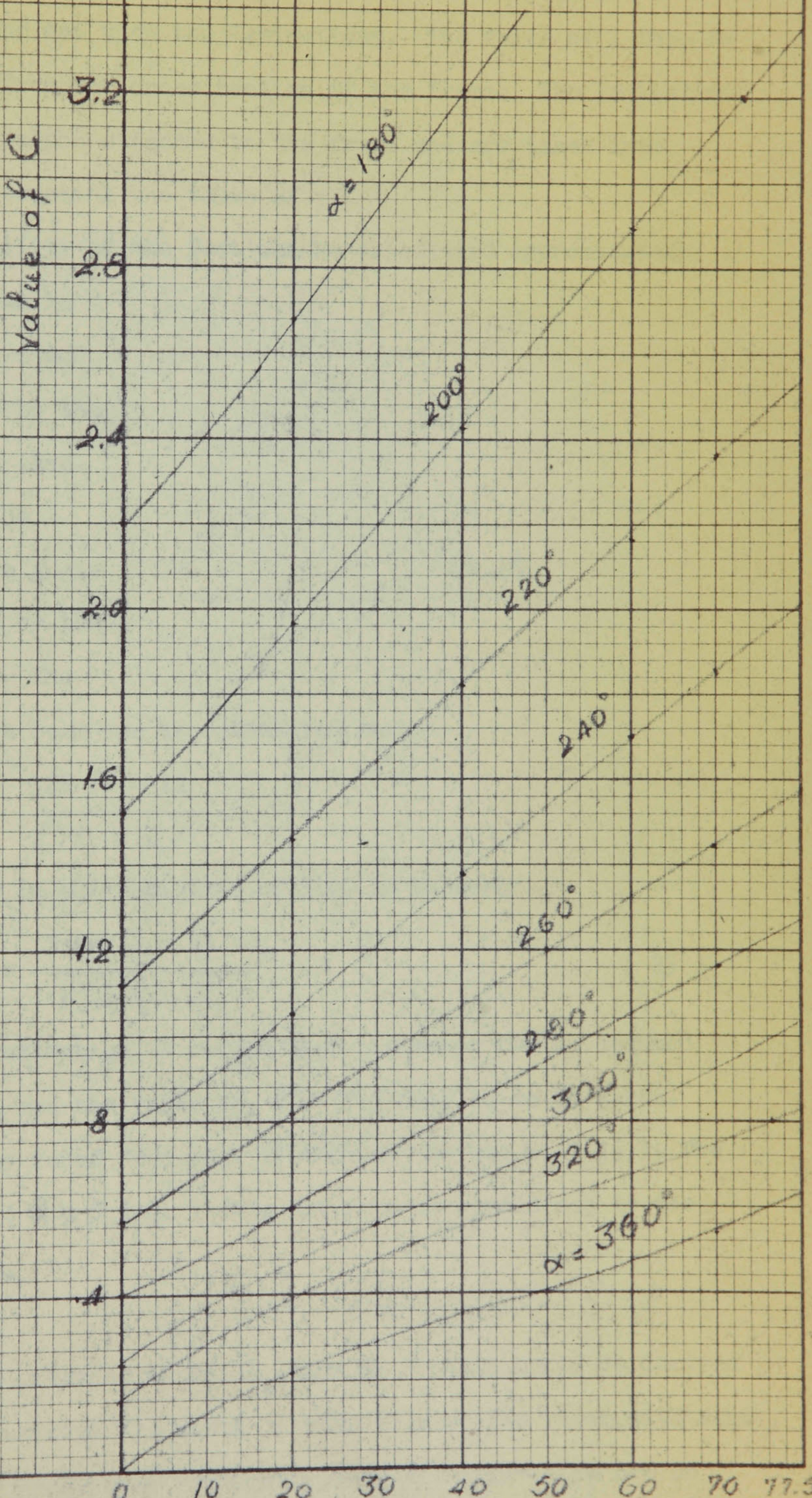
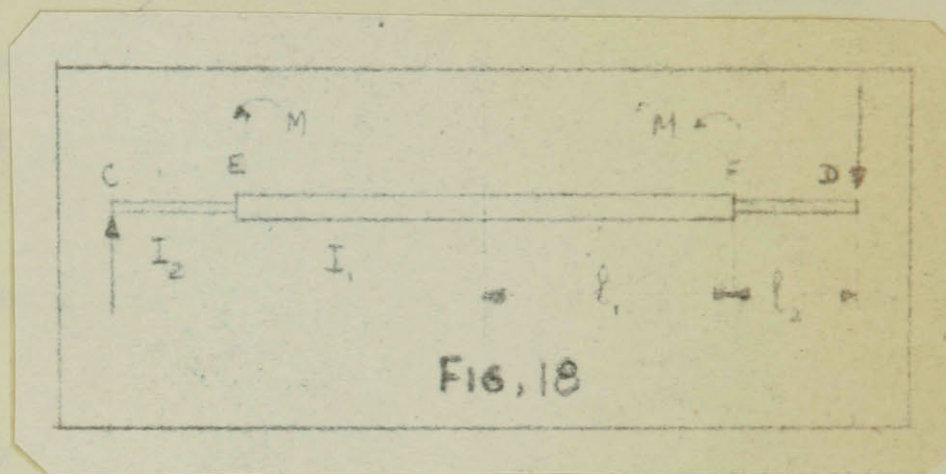


FIG. 17 ⟶ θ°

or clamping plates and not upon the laminations. The value of I then is to be computed only from the cross section of the frame. The laminations become so much dead weight attached to the frame and are taken into consideration when computing the value of γ .

The flexibility of the feet can be calculated approximately on the assumption shown in Fig. 18. The points C and D where the bolts go to the foundation have been assumed stationary and the structure outside the angle α has been replaced by the one shown in Fig 18. The flexibility of such a beam can be calculated to:



FIG, 18

$$K = \frac{3E\,(l_1 - l_2)^2}{l_1^3/I_1 + l_2^3/I_2}$$

so that the quantity in the ordinates of Fig. 16 becomes:

$$\frac{\alpha RK}{EI} = \frac{3\alpha R\,(l_1 - l_2)^3}{l_1^3 + l_2^3 I_1/I_2} \qquad (25)$$

If only a rough approximation is needed than the motor can be assumed to have clamped feet and the value of $\theta = 77.5$ deg. is to be taken. C is then immediately found from Fig. 17 without the use of (25). The case of hinged feet ($\theta = 0$ deg.) does not seem to come close to the real state of things.

45.- Rotor Vibration

Due to the presence of spider arms no formula could be derived that would give the natural frequency of vibration of the rotor with some accuracy. For small fractional horsepower motors, where there are no spider arms, the formulae given in any textbook of mechanics can be used. It is however unnecessary to calculate the natural period of vibration of a rotor because it is very easy to derive the same information by experimental means. For that, the rotor is taken out from the machine and mounted on rigid supports at the bearings. Then a blow is applied with a rubber hammer at the quarter point of the rotor assembly and the frequency of free vibration is recorded. This will be a little higher than the true one because bearings can never be like rigid supports.

46.- Vibration of Teeth

This has already been treated in par. 37.

47.- Bearing Noise

Even if the rotor is very rigidly designed it is always susceptible to oscillations because of the playing in the bearings which must always be reckoned with. Bells and rollers knock in the cage and in the inner raceways during the running. The ball bearing noise has its frequencies in the range of 1000 to 6000 cycles, that is, in the range of greatest audibility, without any dependence on speed. When the motors are required to be silent journal bearings are used. The noise level increases with the bore, nearly in a straight line and decreases with the bore diameter. It also increases with the speed and depend on the quality of the lubricant used. A tight fit in the bearings gives a silent operation. An increase in tolerance increases the noise level. A worn-out journal, for instance, causes a steep increase of noise.

48.- Phenomenon Of Beats.

In the case of a two-pole, 60 cycle, induction motor, the important electromagnetic force of lowest frequency is, as shown in par. 26, 120 cycles per second. At the same time a mechanical driving force, due to an unbalanced rotor for instance, the second harmonic component of which has a frequency double the speed of rotation in revolutions per second can exist. When these two forces exist simultaneously they alternately reinforce and interfere with each other and cause a response which builds up and dies away in amplitude giving rise to the phenomenon of beats. Since the difference in frequency is exactly twice the slip frequency of the induction motor, two cycles of the beat frequency correspond exactly with one revolution of motor slip. The emitted noise is therefore at twice slip frequency. The mechanical driving force of double frequency may also be due to a shaft bent, within the elastic limit, as the result of the centrifugal force of a slightly unbalanced weight accentuated by the increased magnetic pull on the side having the minimum air gap. In this case the center of the rotor travels in a small circle around the center of the stator and there is a point of minimum air gap which travels at rotor speed. As this coincides with the maximum magnetic field at twice slip frequency the characteristic beat wave is accounted for. If then the lowest frequency of the frame for which resonance occurs is in the neighborhood of 120 cycles per second the vibration amplitude may build up to an excessively high value.

To study this phenomenon a piece of wood was inserted into one of the ventilating holes of a rotor of the 0.25 HP motor. This weight being very light the beat noise was hardly audible. To increase the unbalance, heavy steel nails were driven into the wood thus increasing its weight. The beat noise then became very clear. It was noticed that an increase in beat noise did not result in an increase in the noise level. The very low frequency of the beats are, however, excessively annoying, and the listener is much more conscious to this slowly building up and dying away sound than to a constant sound even of higher intensity.

CHAPTER IV

OTHER FACTORS INFLUENCING NOISE

ECCENTRIC ROTOR

The imperfect centering of the rotor in the stator core gives rise to a second-harmonic electromagnetic driving force in a two-pole machine. As the magnetic flux in the air gap revolves, it passes the point where the air gap is shortest twice during each cycle of voltage change, once as a flux entering the rotor and once as a flux entering the stator. Simultaneously, a flux of opposite polarity exists in the air gap at the point diametrically opposite. Since the radial magnetic force exerted on the rotor increases as the air gap length decreases, the two attractive forces, acting at diametrically opposite points on the rotor, do not always cancel. When the flux is in the position where the air gap is largest or shortest (Fig. 19a), a net force results which acts towards the point where the gap is shortest. As the air gap fluxes move from the position, the unbalance between the two diametrically opposite forces diminishes and becomes zero at the point 90° from this position (Fig. 19b). As the flux rotates further, an unbalance again exists in the same direction and reaches a maximum when the flux has moved through another 90°. This unbalance builds up and dies away twice during each cycle of voltage change and produces a second harmonic driving force which acts to deflect the rotor towards the point where the air gap is shortest. This driving force is transmitted through the bearing supports and serves as a driving force to produce vibration in the frame of the machine.

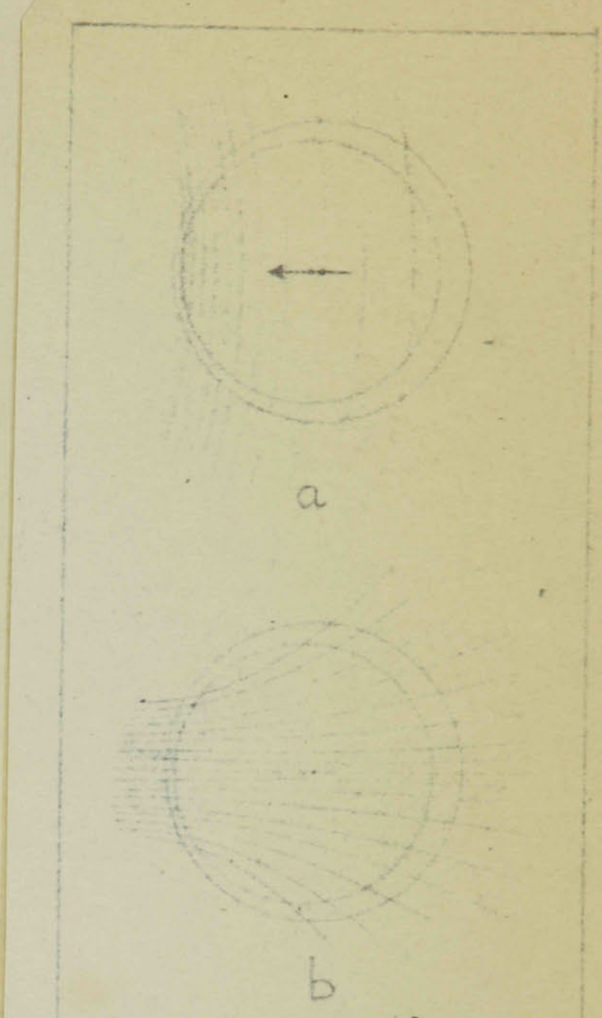


FIG. 19

In a four pole motor the eccentric pull on the rotor is always present and reasonably uniform in the same direction which therefore means that with a four pole motor less noise might be expected from this cause.(see Fig. 20). The driving force pulsates four times for each revolution but the speed being equal to half the speed of the two pole motor the emitted note is again twice fundamental frequency.

The same phenomenon would occur with a perfectly centered rotor and a stator that is not perfectly cylindrical. Vibration of the rotor ring is then possible since the rotor will be subjected to a varying magnetic pull at two points of the stator cicumference during each revolution owing to the difference in air gap. The frequency is again twice that of the supply if the air gap is larger at one point. If there are x, equally spaced such points around the stator circumference the rotor will be differmed x times per revolution after the fashion of a polygon with x corners. This can take place more especially in case where the x points on the stator coincide with x arms of the rotor or an equal number of cicumferencial fixings. If the rotor runs out of truth it can give rise to a similar polygon-like deformation of the stator with a frequency equal to twice the speed of revolution or a multiple of it.

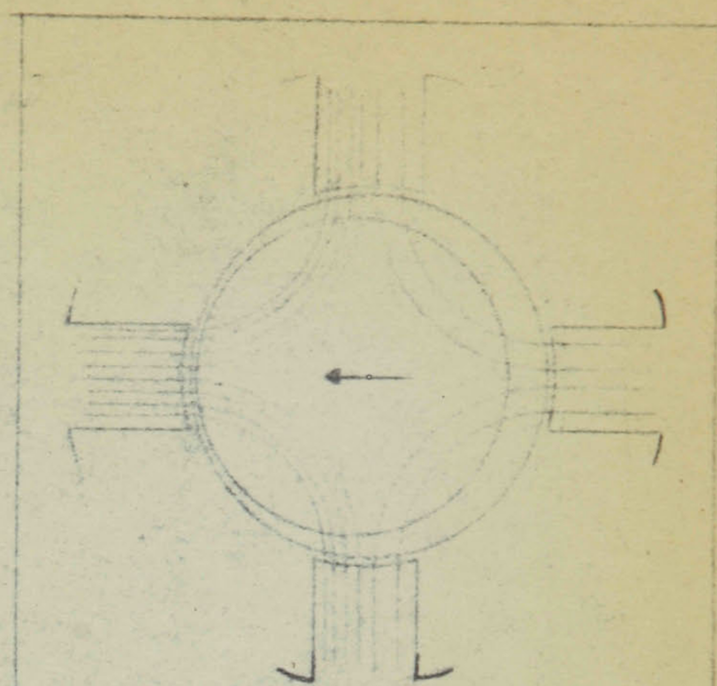


Fig. 20

The driving force produced by the eccentricity of the rotor may be eliminated by perfect centering of the rotor within the stator core. The driving force due to an eccentric rotor may be used in some instances to counteract or neutralize driving forces of the same frequency which arise from other causes since the phase and magnitude of the driving forces thus produced may be controlled by the use of an adjustable bearing support which will allow the rotor to be shifted off center in any direction. The magnitude of this driving force depends upon the amount by which the rotor is off center with respect to the stator and the phase (with respect to the impressed voltage) depends upon which direction it is off center, that is upon the angular position of the point in the air gap periphery where the gap is shortest.

THE EFFECT OF ROTOR SKEWING

The use of skewed rotor slots is an effective means for reducing the noise emitted by an induction motor. The presence of skewed slots exerts an influence on both the mechanical and electrical performance of the motor.

49.- Use Of Skew To Reduce Harmonics

To get a current flow in a wound armature one side of each coil must be placed under a north pole and the other a south pole. If both sides of the coil were placed under like poles no current would flow. Referring to the case of 9 slots per pole, it has been found in par. 17 that the 17th and the 19th harmonics are always present regardless of the method of winding. It is assumed that there are no other harmonics. Since the fundamental has two pair of poles these harmonics will have 34 and 38 pairs of poles. If one end of the rotor bar is placed under a north harmonic pole and the other end under the next north harmonic pole, the integral of the induced electromotive force along the bar will be zero. This prevents therefore the flow of current in the rotor caused by the harmonic poles.

The rotor should be skewed 1/34 of the entire rotor for the 17th harmonic and 1/38 for the 19th. For the 45-bar rotor the skew would be 1.32 bar skew for the 17th harmonic and 1.175 for the 19th. For the 48-bar rotor these values would accordingly be 1.41 and 1.27 bar skew. The same result would be achieved if the above values were doubled but that would lower the torque and the elctrical characteristics of the motor to very unsuitable values.

The worst skew exists when the rotor bar has one end under a north harmonic pole and the other under the next south pole. Thus the worst condition for the 17th would be 0.5 x 45 x 1/34 = 0.66 (45 slots) and 0.5 x 48 x 1/34 = 0.587 (48 slots), and for the 19th it would be 0.71 and 0.635. Another bad condition is a 3-harmonic bar skew. With this condition one end would be under a north pole and the other under a south pole three pole pitches away. However, since the point 1/3 from the beginning would be a pole of the same polarity with the other end the current would flow only because of 1/3 bar length.

The values of the skews in slot pitches for the three cases studied and for the two rotors are tabulated below.

	45 slots rotor		48 slots rotor	
	17th har.	19th har.	17th har.	19th har.
1 harmonic pole skew	0.66	0.587	0.71	0.635
2 harmonic pole skew	1.32	1.175	1.41	1.27
3 harmonic pole skew	1.98	1.76	2.12	1.91

50.- Use Of Skew To Reduce Vibration

The theory used in this paragraph is based on a paper by Mr. S. J. Mikina[17].

An extended elastic body is capable of vibrating in a large number of normal modes of vibration. Of these only a few are of any real importance in practical problems. In an electrical motor only four modes can be found. In the first mode the poles are situated at the antinodes and move in a purely radial direction. In the second mode the poles are situated at the nodes of the frame vibration and rotate from side to side as the slope of the elastic line of the frame at the nodes changes during the motion. In the third mode the poles are again at the antinodes of the frame motion. Each one rotates in a radial plane about a tangent to the frame cylinder midway between its nodes . The fourth mode involves simple extension and compression of the frame with practically no bending.

The energy imput per pole to the first, second and fourth modes of vibration, in percent of the maximum values, for different values of the skew, is given in Fig. 21. From this curve it may be seen that the energy imput to the first, second and fourth modes of vibration is zero with a skew of any integral number of slot pitches. The maximum energy imput with over one bar skew is never larger than 20%. The maximum imput energy occurs with zero skew.

The energy imput for the third mode of vibration for different values of skew is given in Fig. 22. Ordinates are in percent of the maximum imput energy to the third mode. This curve shows that the imput energy is zero for a skew of 0, 1.43, 2,48 slot pitch and that it is maximum for a skew of 2/3 = 0.66 slot pitch. This is the most critical of the three modes rising rapidly on either side of the zero energy point.

Mr. S. J. Mikina has proved that the third mode of vibration has a value very close to zero when the number of rotor bars is divisible by the number of poles or when the rotor is skewed in a symmetrical herringbone shape. A further advantage of the herringbone rotor construction is that the forces on the rotor due to skewed bars are symmetrical and that there is no axial component (as in the case of a straight line skew) that would tend to push the rotor out of the stator frame, especially under sudden application of load.

Some difficulty would be encountered in a practical design if the best value of skew had to be chosen because the requirements for zero energy imput to all the principal modes of vibration are mutually exclusive and cannot be simultaneously satisfied in a general case if they are based on a straight line skewing alone. Moreover, the nature of the energy curves shown in Fig. 21 and 22 is such that no satisfactory balance can be struck which would result in small imput to all the modes at once

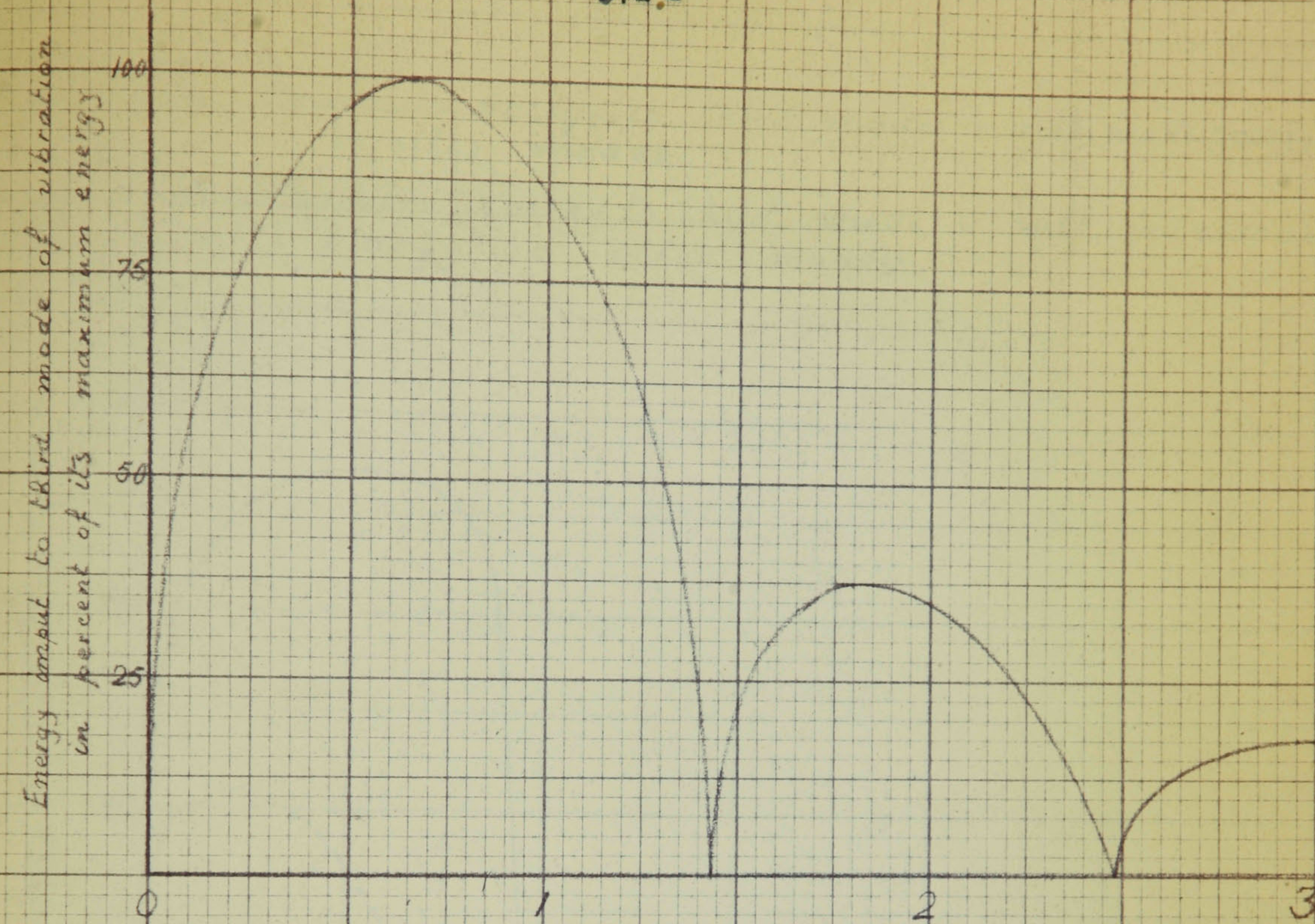

Energy input to third mode of vibration
in percent of its maximum energy
100
75
50
25
0
1
2
3

FIG. 21

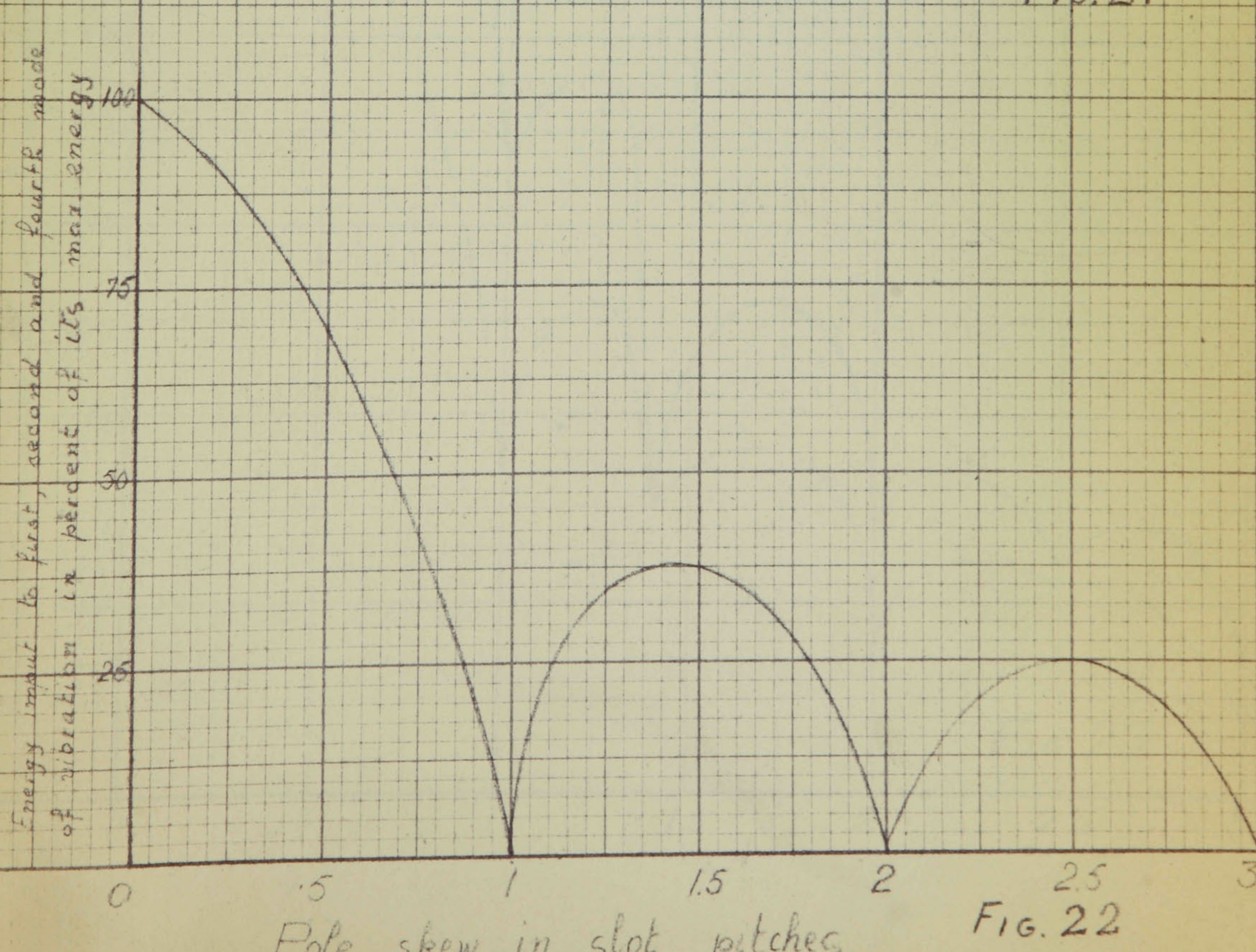

Energy input to first, second and fourth mode
of vibration in percent of its max energy
100
75
50
25
0
.5
1
1.5
2
2.5
3
Pole skew in slot pitches

FIG. 22

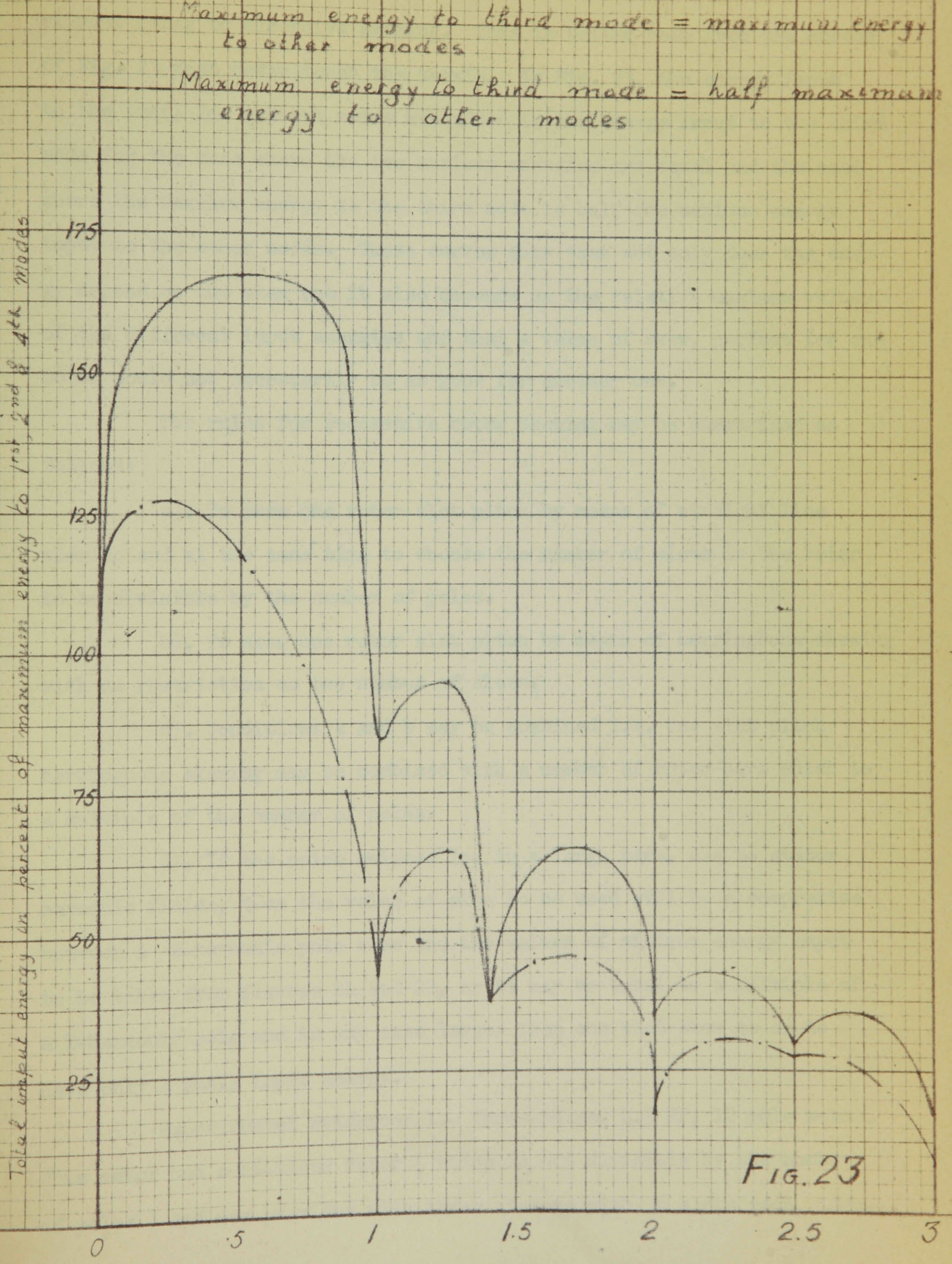

Maximum energy to third mode = maximum energy to other modes
Maximum energy to third mode = half maximum energy to other modes
Total input energy in percent of maximum energy to 1st, 2nd & 4th modes
175
150
125
100
75
50
25
0
·5
1
1.5
2
2.5
3

FIG. 23

without going to excessive skewing. This is better seen from the curves of Fig. 23 that never touch the x-axis. In Fig. 23, the energy imput to the 1rst, 2nd and 4th modes was added to the energy imput to the third mode for the case where the two maximum energies are equal and the case where the maximum energy to the third mode is half the maximum energy imput to the other modes. These curves show that not only does the energy depend on the skew but it also depends on the relative maximum values of the different imput energies and that a value of skew, perfect for one motor would not necessarily be as good for another one.

The rules for successive rotor skewing may be summarized as follows:

a). To skew one rotor slot pitch by means of straight line skewing and at the same time to choose the number of slots in the rotor to be divisible by the number of poles.

b). To skew one rotor slot pitch by means of herringbone skewing with no restriction on the number of slots.

c). Conditions a and b may be combined i.e. one slot pitch herringbone skewing may be combined with a number of rotor bars which is a multiple of the number of poles.

d). If the noise emitted is, in a large part, due to stator harmonic fields, then the value of skew to be used will be a compromise between a 2-harmonic pole skew and the values of minimum energy imput to be taken from figures 21 and 22, or it may be equal to a multiple of a 2-harmonic pole and in any case larger than one slot skew so as to have only a small imput energy.

Experimental evidence: 1.- Three-phase, 0.5 HP, 12 pOle motor. This motor had a 45-slots rotor (which is not divisible by the number of poles) and was provided with three stators with 36 slots each, identical in every respect except for the value of skew which is equal to 0, 0.25, and 1 slot pitch. The readings are given in a tabulated form below. Background sound level 37db. Weighing curve 70db. Standard distance 1 ft.

Skew	I amps	A	B	C	D	Averag.
0	0.0	61.5	62.5	61.0	60.0	61.3
	2.3	69.0	73.0	72.0	69.0	70.8
	3.0	69.0	73.0	70.5	68.0	70.1
	4.0	68.0	70.0	70.0	68.0	69.0
	5.0	68.0	70.0	70.0	68.0	69.0
0.25	0.0	59.0	60.5	60.5	59.0	59.8
	2.3	65.0	69.0	67.0	65.0	66.5
	3.0	..	..	..	..	..
	4.0	76.0	79.0	74.0	76.0	78.8
	5.0	79.0	82.0	78.0	76.0	78.8
1	0.0	57.0	59.5	58.5	57.0	58.0
	2.3	59.0	66.0	60.0	58.0	60.8
	3.0	61.5	67.0	63.0	60.0	63.9
	3.5	65.5	67.5	67.5	63.0	63.9

The noise level for I = 0 is very low and almost constant for the different values of skew. This noise is due only to windage, friction and mechanical unbalance, and is independent of the amount of skew. The slight decrease noticed with the increasing skew is due to the fact that to run the motor with I = 0 it was first brought up to full speed and then the switch was opened and readings taken of the noise level. The response of the noise meter being slow the value read was influenced by the noise level before the switch was opened.

With the stator 0.25 slot pitch the noise level increased disproportionately when the machine was loaded. As shown in Fig. 23 the total imput energy has a maximum at points around 0.25 slot skew and falls down steeply for smaller or larger values. The coincidence therefore of the skew with the value for maximum imput energy sets the stator into intense vibration. This fact alone does not account for the very high noise level produced; it was noticed that when the rotor was loaded with a current of 3 amps, corresponding to a certain speed of rotation, the whole set up started vibrating excessively and a howling noise was emitted. At a little higher or lower speed this vibration disappeared. It is believed that the double influence of an increased imput energy and of a speed of rotation coinciding with one of the resonant frequencies of the assembly is the cause for the howling noise emitted.

The noise level is much lower when the stator is skewed one slot and that because for this value the imput energy to the three modes of vibration is zero. The value of 1 slot skew is close to 1.175 and 1.32 which, as shown in the table of p.66 would eliminate the influence of the 19th and 17th stator harmonics.

An increase of noise when the machine is loaded means an increase of leakage flux which is only natural because of saturation of the teeth. The slight decrease noticed when the non skewed motor was loaded could not be explained

2.- 0.25 HP, 4-pole, 1725 rpm motor. The tabulation of the experimental results is given in par. 33. The table shows that as expected the maximum as well as the average values decrease when the skew increases The difference in noise level between 1 and $1\frac{1}{2}$ slot skew is approximately 5 db for the 45 slots rotor but it is only of the order of only 2 db or db or less for the 48 slots rotor. The number of slots in the 48 rotor is divisible by the number of poles and therefore, according to the theory exposed, there is no imput energy to the third mode of vibration. The imput energy for the other modes is higher for 1.5 slot skew and more noise would be expected at this value of the skew if the 17th and 19th harmonics were not almost completely wiped out by this skew, thus lowering the noise output to a level of one or two db below the output noise for 1 slot skew. In the case of the 45 slots rotor the number of slots is not divisible by the number of poles and therefore the third mode of vibration is present and, as seen from Fig. 23, the imput energy at $1\frac{1}{2}$ slots skew is smaller than the one at 1 slot skew thus accounting for the large difference.

It must also be noted that the difference in noise between 2 and $2\frac{1}{2}$ slot skew is of the order of 1 db or less. It is uneconomical to use a rotor skewed two and a half slots since very little is gained in quietness as compared to two slots skew but much is lost in performance. It is not advisable to skew less than 2 slots since the noise level at $1\frac{1}{2}$ slot skew is appreciably higher.

A rotor with a $1\frac{1}{2}$ herringbone skew was also available and was tried. The noise level was 56.5 for the maximum value and 54.4 for the average when tried with the obsolete lamination. These values are practically the same with values found with a straight line $1\frac{1}{2}$ skew and no improvement was achieved withtthe herringbone skew. These results were expected since the herringbone skew is used to eliminate the energy imput to the third mode of vibration which is absent in a 48 slots rotor 4-pole motor. Thus no advantage was gained by the herringbone skew while, on the contrary, the value of $1\frac{1}{2}$ skew coincides with a peak of the curve for the imput energy to the other modes. Had the skew been 1 slot and the rotor a 45 slots one, a quieter motor should be expected

WINDAGE NOISE

This is one of the most important sources of noise especially in high speed machines. When a motor is supplied with a ventilating fan it is usual to obtain appreciable noise due to such factors as : (1) Size of radial air gap between fan and casing; the noise increases with diminution of the air gap, (2) diameter of fan, (3) speed, (4) volume of air blow; noise level increases with an increase of (2), (3), (4). (5) number of blades; more blades give lower noise level. (6) pitch of blades, (7) section or profile of blades which must be as close to the aerofoil section as possible. The noise emitted is known as a siren frequency and often consists not only of the fundamental but also of the harmonics. This renders it the more important to extend the formulae derived from the studies of aerodynamics to the construction of the cooling fans. Unfortunately the difficulty is usually met in the design of the fan that the direction of rotation is not known. This must be specified for a good fan. A noisy fan is a sign of a low efficiency fan.

Experimental evidence: The same rotor of the 0.25 HP motor was tested with and without ventilating fans.

Stator	Fan	A	B	C	D	Aver.
Obsolete	Yes	56.5	53.0	55.0	53.0	54.4
	No	53.0	53.0	54.5	53.0	53.4
New	Yes	58.0	56.0	58.0	58.5	57.5
	No	57.0	52.0	56.5	54.5	55.0

The noise level decrease is in the range of 1 db to 2.5 db which represents a reduction in the sound energy output of 20% to 50%. The percentage of noise due to windage in this case is rather small but the amount of air being circulated in the machine is not very large. It must also be noted that this is a single phase motor and that magnetic vibrations are very pronounced. Credit must also be given to the good design of the fan. The sound level of a metadyne generator with intense cooling and running at 2800 rpm was measured and found to be 90 db which is entirely due to windage noise.

Even when the motor is not supplied with ventilating fans appreciable noise may be encountered due to the action of the rotor. As the rotor revolves air currents are set up in the radial direction and a puff of air will pass through the stator, between the stator coils every time a rotor tooth comes opposite a stator tooth (see Fig. 24) thus emitting a noise of a frequency equal to the number of puffs per second which is equal to the number of rotor slots times the revolutions per second. The intensity of the note depends on the peripheral velocity of the rotor. The disturbance becomes more objectionable with high pitched notes.

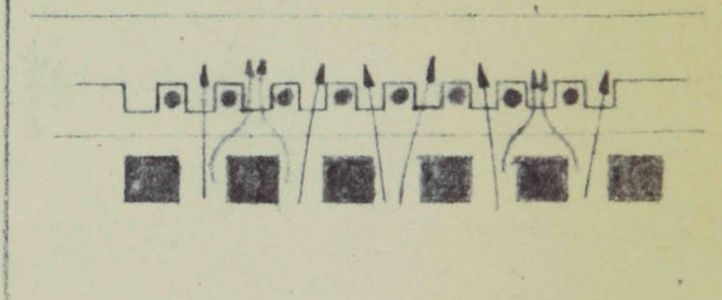


FIG. 24

Noise can be lowered in intensity by blocking up the motor vent ducts; where some doubts exist as to which ventilating duct is at fault the plan may be adopted of keeping away the air from the part suspected. This can sometimes be done by packing pieces of cotton cloth into the inside opening of the duct. By process of elimination the faulty part can be determined. The intensity of the note can be reduced by staggering the vent ducts as shown in Fig. 25 and since the air gap is large in high speed machines (those in which the windage noise reaches a critical value) the ventilation of such machines will not be seriously affected. It should be noted that for high speed machines a large number of narrow ducts will give quieter operation than a small number of wide ducts because the velocity of air through the ducts will be reduced.

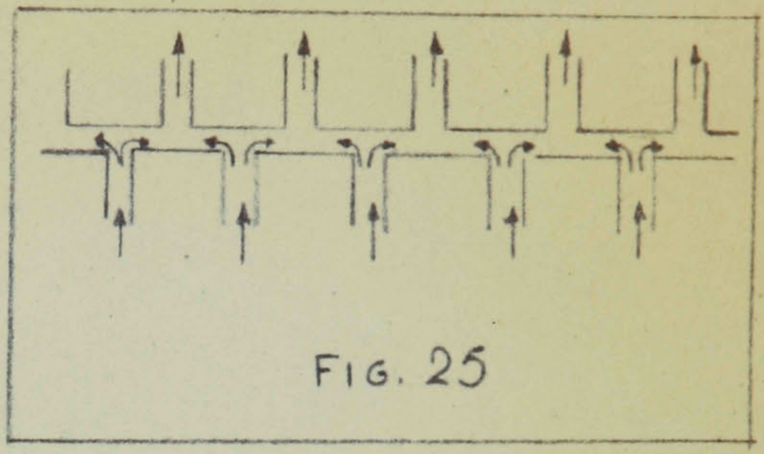


FIG. 25

Another phenomenon which complicates the noise level due to windage is resonance cavities which may prove exceedingly annoying. Every cavity, when filled with air, has one or more natural periods of vibration, a fact of which special use is made in the playing of wind instruments and organs. If a cavity of this kind is excited by air vibrations, the tone is amplified considerably. If the cavity is detected then small changes in the shape of the cavities by the introduction of pegs of bakelized paper well secured to the frame may be sufficient to stop the resonance and reduce the noise. For a permanent elimination of the cavity the design of the machine must be changed. It is sometimes enough to change the shape of the stator teeth because then the natural frequency of the cavities is changed.

THE INFLUENCE OF SATURATION

Three stators with a different winding distribution, producing a magnetomotive force of three different shapes,were tried with the same rotor and under the same conditions. The readings taken are tabulated in the next page.

	Iamps	Vvolts	Maximum	Average
Standard 14:28:26:20	4.8	110	52.5	51
	6.0	122	52.5	50.8
S-38 15:26:22:15	4.8	97	53.5	53.6
	6.0	109	54.5	54.2
S-39 14:22:22:10	4.8	92	49.0	48.5
	6.0	103	50.5	49.5

The magnetomotive curves for the three stators are plotted in Fig. 26. The ordinates of the mmf curve for the standard winding were multiplied by the ratio 78/88 because this stator has 88 turns while the other two have 78 turns.

The table shows that the S-39 winding distribution gives the lowest noise level while the S-38 is the most noisy. The mmf curve of X S-38 is the closets to a sine curve of the three while the standard has a higher peak and the S-39 a still more peaky shape. The effect of saturation is to cut or lower the top of the flux wave produced by the mmf distribution. When the peaky curve is reduced it comes closer to the shape of a sine curve while when the sine curve saturates a flat top flux curve results. Any change from a sine curve means harmonic fluxes and therefore noise and vibration. Saturation should be avoided or its influence counteracted by the choice of the proper winding distribution.

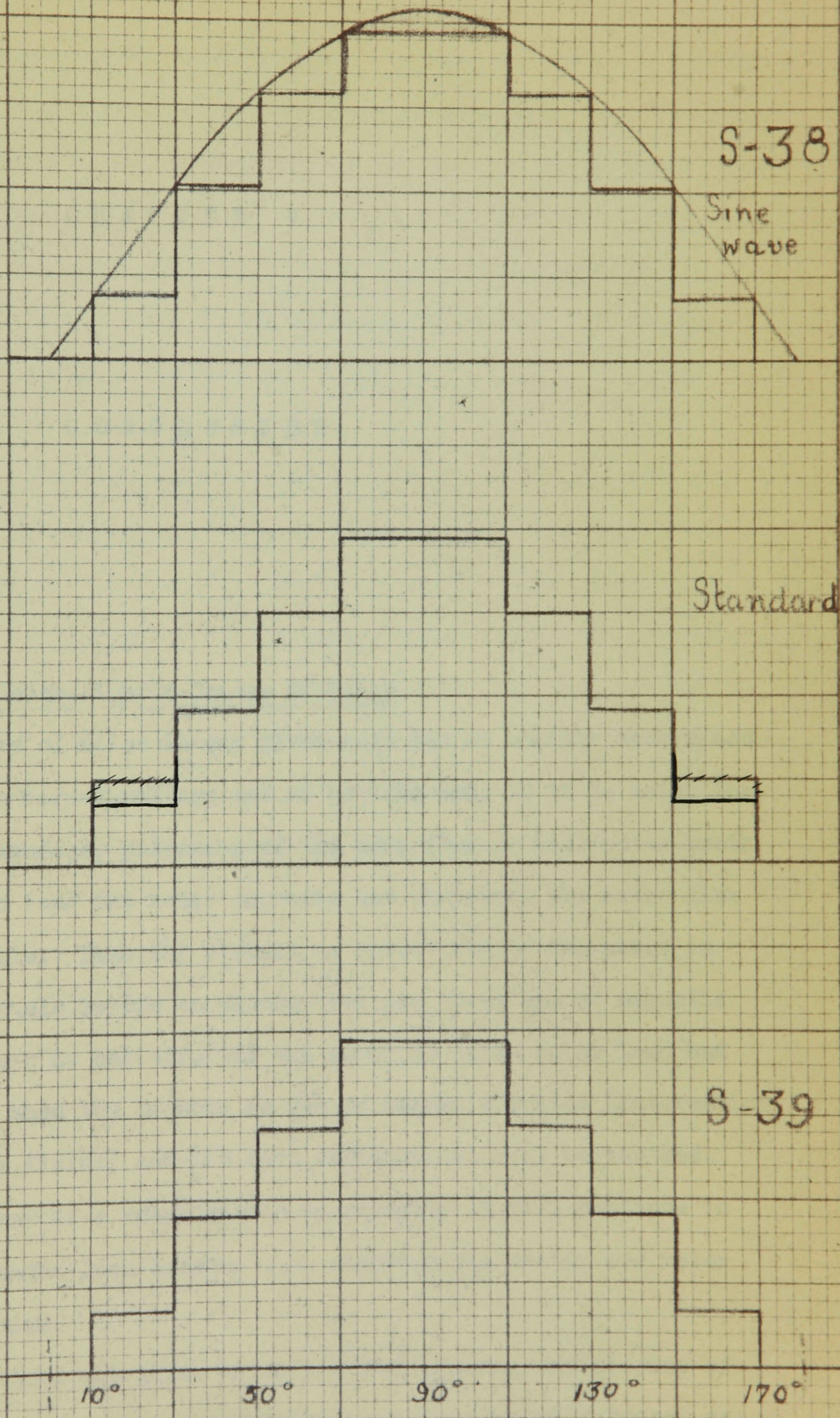


Fig. 26

CHAPTER V

CONCLUSION

51.- Experimental Separation Of Causes Of Noise

The causes of noise in any given case can be readily determined. The motor is run on normal voltage and at no load and, if it is noisy the trouble is due to windage, pulsation of the main flux or weak rotor teeth tips, because on no load the rotor currents and therefore the rotor flux fields are negligible. Then the circuit is opened and if the motor is still noisy the trouble is due to windage because there are no magnetic fields when the power is cut off. If the motor is quiet on no load but noisy when loaded, then the trouble is due to magnetic causes (interaction of stator and rotor fields) and the slot combinations must be checked. It must be noted that if a motor starts up silently it will continue to run silently even under load because when starting the stator and rotor currents are very high.

If mechanical vibration is noticeable, opening and closing of the switch will show whether the vibration is due to magnetic causes or to purely mechanical causes.

If the noise or vibration produced is considerably higher than what would be expected a small change in speed or in the applied frequency will show whether this is due to driving forces close to the natural frequency of vibration of the machine. If the noise is considerably decreased when the speed is changed, resonance is responsible for its high level. If the noise level is not altered it is to be deduced that the driving forces are excessively high.

52.- General Rules For Manufacture Of Quiet Motors

1.- The motor must be carefully built in production: careless or inaccurate manufacturing methods often show up in some manner as uneven air gaps, loose bearings or loose rotor bars. Exact centering and perfect balance of the rotor is a primary condition for quiet running. The stator and rotor cores must be carefully put together and pressed so that no loose laminations give rise to vibrations.

2.- Sleeve bearings instead of ball bearings must be used for practically noiseless motors. The tolerance in the bearings must be as small as possible.

3.-Use a larger shaft to reduce rotor vibration

4.-Increase of the stator stiffness so that the natural period of vibration be outside the range of frequencies of the driving forces.

5.- Use of closed slots to reduce higher slot harmonics

6.- There should be no magnetic part extremely light that would vibrate excessively (teeth and tooth tips).

7.- Preference of a squirrel-cage to a slip ring motor: with otherwise equivalent design a motor with phase wound rotor is usually more noisy than a similar squirrel cage machine. The slip-ring motor has generally fewer slots with wider slot openings and hence the permeance variations are greater and of a lower order

8.- The number of rotor bars should be carefully selected so that there will be no harmonics with a 2 or 4-pole difference. The general formula (20) is useful for the proper choice of the number of rotor slots.

9.- The number of rotor bars should be divisible by the number of poles or the rotor should be skewed in a herringbome shape.

10.- The winding should be distributed so that the primary electromotive force is as close to a sine wave as possible. Therefore as many stator as possible stator slots per pole should be used.

11.- Tooth harmonics should be eliminated by the proper choice of rotor skew. This applies to the lowest harmonics.

12.- The rotor should be skewed to any integral number of slots to avoid the 1rst, 2nd and fourth modes of vibration.

13.- A compromise between 11 and 12 must be made but the skew should never be less than 1 slot

14.- Increase of the air gap at the expense of power factor

15.- Lower flux density

16.- If possible,two coexisting noise producers should cancel each other: for instance, two-pole difference fields reacted upon by two four-pole difference fields may give noiseless motors.

17.- Liberal dimensioning of the machine to reduce the amount of cooling air to a minimum.

18.- Choice of best fan design

53.- Installation Of Motors For Quiet Operation

It has been found that even though special quiet operating motors are used, an installation is often objectionably noisy. These motors are subtancially quieter than standard motor in a noise room test but this is only a small part of the total noise produced by the combination of the motor and the driven machine and little or no overall improvement can be discerned between an installation with a standard and a special quiet motor. Moreover the possibilities of quieting a motor are limited; the total air sound output radiated by a source of sound can be taken as proportional to the surface and to the square of the velocity of the mechanical vibrations. Both the surface and the deflection and frequency and , consequently, the velocity of the mechanical vibrations increase with the size of the machine. The sound output therefore increases rapidly with the nominal output of the machines. That is why it is impossible, above a certain size of machines to reduce the noise to a permissible level under ordinary conditions. Therefore where large and high speed machines areconcerned an abatement of noise can be brought about by enclosing the machine directly with a sound damping cover. Special designs are made for ventilation with cooling air directly from the surrounding atmosphere or for ventilation through closed ducts. Special sound dampers can be enclosed in these ducts. A further step in the abatement of noise is achieved by sound proofing a machine room, baffles, sound-proof ceiling etc. Transmitted noise is however far more important than airborne noise because if a motor is to be mounted on a construction that has a natural frequency somewhere near the various frequencies of the vibrations in the motor the noise may be increased terrifically. This is especially true for buildings of reinforced concrete which carry sound extraordinarily well and where the best noiseless motor built is just good enough. Most manufacturers have, in order to avoid transmittable noise, adopted flexible mountings using springs or material of a flexible nature. Insulation of motors from their supports through air gaps filled with a resilient medium prevents the amplification and transmission of the produced noise.

Operation of a motor at 30% to 35% lower than its rating is recommended for very quiet operation. This reduces the magnetic noise.

54.- Conclusion

When the precautions recommended in the preceding paragraph are taken, little or no, overall improvement can be discerned between an installation with a standard and a special quiet operating motor. This small difference has further been reduced by the improvement in standard motors during recent years.

The purpose of this paper was to point out the factors responsible for the noise emitted by an induction motor and to give rules for the better design of these motors. When the conclusions to which this paper arrives be applied to the design of an induction motor, the sound level of the motor will be lower than if it was designed to meet only the electrical specifications. The proper choice of slot combinations, winding distribution, skew and mechanical design must be used with any standard motor. Where extreme quietness is desired this should be achieved not by a special design of the motor but by taking special precautions for sound proofing the motor and the room and by the use of a resilient mounting. Then a sufficiently quiet standard motor may be used.

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