# APPROXIMATE METHODS OF LONGITUDE DETERMINATION

M.Sc. THESIS.

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# AN INVESTIGATION OF PERSONAL EQUATION IN ASTRONOMICAL OBSERVATIONS WITH THE REPSOLD MICROMETER EYEPIECE.

—— J. B. Baird. ——

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## Approximate Methods of Longitude Determination.

## An Investigation of Personal Equation in Astronomical Observations with the Repsold Micrometer Eyepiece.

The subject matter of this thesis consists of results obtained from observations carried on at the McGill University Observatory, together with an account of the various calculations by which these results have been deduced.

The first section - Approximate Methods of Longitude Determination - contains an account of four of the more common methods by which Longitude may be determined in the field: and by way of illustration a value for the longitude of Montreal has in each case been calculated from actual observation.

In the second section an attempt has been made to establish a relation between the Personal Equation existing in Astronomical Transit observations as performed with (1) the Telegraphic Key, (2) the Repsold Micrometer Eyepiece, and (3) the ordinary "Eye and Ear" method.

The astronomical longitude of a station on the earth's surface is the angle between the meridian plane of the station and some arbitrarily chosen initial meridian plane. Usually the meridian of Greenwich is taken as this initial meridian, but sometimes that of Paris or of Berlin or of Washington, D.C., is employed. However, in all parts of this work it will be understood that the zero meridian is that which passes through the Greenwich Observatory. Astronomical latitude and longitude should be distinguished from Geodetic latitude and longitude, and care should be taken to prevent either one of these from becoming confused with Celestial latitude and longitude. Geodetic latitude and longitude differ from the astronomical in that instead of being referred to a gravity line at the station they are referred to a gravity line which has been corrected for local deflection and station error. Celestial latitude and longitude form a system of co-ordinates which, though frequently used by the astronomer, are seldom of use to the engineer. In this the ecliptic and vernal equinox play the same part as do the equator and right ascension. Of course, by "longitude" is meant the astronomical longitude, wherever the word has been used throughout this thesis.

Now it has been already stated that the difference in longitude of two points on the earth's surface is equal to the angle at the pole formed by the meridian planes passing through the two points. As the earth revolves uniformly on its axis, this becomes equal to the difference between the times of transit of the same star over the two meridians, and may be expressed either in degrees, minutes and seconds of arc, or in hours, minutes and seconds of time; for astronomical purposes the latter designation is generally preferred.

As an astronomical problem the determination of the difference of longitude between two places consists in an accurate determination of the local time at each place and the comparison of the times so determined; the difference between the times being the difference in longitude. It follows then that an accurate determination of time is absolutely essential for finding differences of longitude. That being the case, it may be well before going any further to briefly discuss the various methods by which time is reckoned.

Apparent solar time is a natural and direct measure of duration, inasmuch as it is indicated directly by the hour angle of the Sun, the most conspicuous of all the heavenly bodies. However, it is impossible to regulate a clock or chronometer so that it keeps this time accurately, since the different days are of unequal length; this inequality being due to the eccentricity of the earth's orbit and to the obliquity of the ecliptic. Though the solar day is not actually of constant length, it has a mean or average length than which it is never much shorter or much longer. This "mean day" may be conceived to be determined by the successive transits of an imaginary sun - called the Mean Sun over the meridian starting with the real sun at some assumed epoch and crossing the meridian at equal intervals of time. The interval between successive transits of the Meridian of this imaginary sun is called a Mean Solar Day; and a clock regulated so as to complete its period in a mean solar day is called a mean solar clock. For brevity, "mean solar time" is spoken of simply as "mean time". The difference between mean time and apparent time, i.e., between the mean sun and the real sun, is called the Equation of Time. It is the interval of time by which the mean sun is fast or slow of the real sun at a given instant. Its greatest value is about 16 minutes.

The Civil day commences and ends at midnight. The hours from midnight to noon are counted from 0 to 12 and are marked A.M. The remaining hours, from noon to midnight, are again numbered from 0 to 12 but are marked P.M.

The Astronomical day commences at noon on the civil day of the same date. Its hours are numbered from 0 to 24, from noon of one day to noon of the next. The astronomical time as well as the civil time may be either apparent solar or mean solar. The convenience of the astronomical day for the astronomer arises from the fact that he does not have to change the date on his record of observations in the midst of a night's work, as he would be obliged to do if he used civil dates. The day of the month in the Nautical Almanac - Page 1 excepted, being for apparent noon - is assumed to be the mean astronomical day. Seeing, therefore, the importance of being able to convert civil time into astronomical, the following simple rule may be useful.- In civil time, for P.M., make no change; for A.M., diminish the day of the month by 1 and add 12 to the hours. Thus, Jan. 2nd. 7h. 49m. P.M. civil time is 7h. 49m. astronomical time, but Jan. 2nd. 7h. 49m. A.M. civil time is Jan. 1st. 19h. 49m. astronomical time.

In consequence of the earth's diurnal motion the meridian of every place is constantly moving among the stars in such a way as to make a complete revolution in 23 hrs. 56 min. 4.09 secs. It may be easier to conceive of the celestial sphere as revolving from east to west, the terrestrial meridian remaining at rest. The effect will be the same geometrically whether the true or the apparent motion be considered. Hence, there are two sets of meridians on the celestial sphere. One set is fixed among the stars and is in constant apparent motion from east to west with the stars, while the other set is fixed by the earth and is apparently at rest. As differences of latitude are measured by angles in the heavens, so differences of terrestrial longitude are measured by the time it takes a celestial meridian to pass from one terrestrial meridian to another: while differences of right ascension are measured by the time it takes a terrestrial meridian to pass from one celestial meridian to another. Ordinary solar time would, however, be inconvenient for this measure, because a revolution does not take place in an exact number of hours. So it becomes necessary to adopt a different measure, and the measure adopted is known as Sidereal Time. The time required for one revolution of the celestial meridian is divided into 24 hours, and these hours are subdivided into minutes and seconds. Sidereal noon at any place is the instant at which the vernal equinox passes the meridian of that place. Since right ascensions are reckoned from the equinox, when it is sidereal noon, or 0 hour, all celestial objects on the medridian of the place are in $0^{\circ}$ of Right Ascension. At 1 hr. sidereal time the meridian has moved through $15^{\circ}$ and objects now on the meridian are in $15^{\circ}$ of right ascension. Throughout its entire diurnal course the right ascension of the meridian constantly increases at the rate of $15^{\circ}$ an hour, so that the right ascension is always found by multiplying the sidereal time by 15. But to avoid this constant multiplication it is customary to express both right ascensions and terrestrial longitudes in hours. Thus, the longitude of Quebec is $71^{\circ}$ 14.5' West, but in astronomical language the longitude is said to be 4h. 44m. 52.6s., meaning that it takes 4h.44m.52.6s. for any celestial meridian to pass from the meridian of Greenwich to that of Quebec. Consequently, when it is 0 hr. sidereal time at Quebec it is 4h. 44m. 52.6s. sidereal time at Greenwich.

It will follow then from what has been said that both the sidereal and the mean solar time differ for places in different longitudes at the same imstant of absolute time, the reason being that the commencement of a day is reckoned at each place in one case from the time of transit of the vernal equinox (first point of Aries), and in the other from that of the mean sun, over the meridian of the place.

Since the declination circle through the mean sun separates from any given meridian of the earth by equal angles in equal times, the difference of longitude of any two places is proportional to the interval elapsed between the times of transit of the mean sun over their meridians, i.e., to the time indicated by a mean solar clock at the first meridian at the instant when the sun is on the second meridian: thus the difference in the times of a phenomenon occurring at a given instant,as recorded by clocks at two places, is proportional to their difference of longitude. This is the principle of all the methods used for determining longitudes.

The principal methods by which differences of longitude may be determined are:-

1. By Signals.- These signals are either terrestrial (such as the sudden disappearance or reappearance of light or flashes of gunpowder), or celestial (such as the beginning or ending of an eclipse, a transit or an occultation).

2. By the Electric Telegraph,- method of star signals.
3. By obtaining the error of a mean time chronometer on local time, and knowing at the same time the error of the chronometer on some standard time.
4. By altitudes of the moon.
5. By transportation of chronometers.
6. By azimuths of the moon, or transits of the moon and a star over the same vertical circle.
7. By moon culminations.
8. By lunar distances.

All of the above cannot be rightly termed <u>field</u> methods, so only some of them will be discussed, and (1), (3), (7) and (8) have been selected, being probably the most commonly employed for the purpose in practice.

1. Of the methods which are included under "signals" the terrestrial ones are immediately excluded, since it is out of the question to attempt to obtain the longitude of Montreal with respect to Greenwich by any signal having its origin on the earth. Of those which may be termed "celestial signals" the most important depends upon the motions of Jupiter and his satellites. Of these there may be noted,-

(a) The eclipses of Jupiter's satellites by the shadow of the planet. The Greenwich times of the disappearance of each satellite and of its reappearance are accurately given in the Nautical Almanac; so that an observer,who has noted one of these phenomena, has only to take the difference between this observed local time of its occurrence and the Greenwich time given in the Almanac, to have his absolute longitude. With telescopes of different powers, however, the instant of a satellite's disappearance must evidently vary since the eclipse of the satellite takes place gradually, and the more powerful the telescope the longer will it continue to show the satellite. Since, however, the same causes which accelerate the disappearance will retard the reappearance, if both phenomena are observed on the same evening under nearly the same atmospheric conditions, the mean of the two resulting longitude values will be nearly free from error.

(b) The occultations of the satellites by the body of the planet.- The approximate Greenwich times of the disappearance behind the disc and the reappearance of each satellite are given in the Nautical Almanac.

(c) The transits of the satellites over Jupiter's disc.- The approximate Greenwich times of "ingress" and "egress", or the first and last instants when the satellite appears projected on the planet's disc, are given in the Almanac.

(d) The transits of the shadows of the satellites over Jupiter's disc.- The approximate Greenwich times of ingress and egress are given in the Almanac.

So in the Almanac (page 506) for (say) Jan. 1st. 1909 the following may be seen:-

| | | H. M. s. |
|---|---|---|
| I. | Tr. I. | 0-43 |
| I. | Sh. E. | 1-55 |
| I. | Tr. E. | 3-02 |
| I. | Ec.D. | 20-51-21 |
| II. | Ec. D. | 21-53-34. |

which means that at 0h. 43m. Greenwich time the first satellite (Io) begins a transit across the disc of Jupiter - 0h. 43m. is the "ingress of its transit."

The egress of the shadow of Io across Jupiter's disc takes place at 1h. 55m. Greenwich time.

The egress of the transit happens at 3h. 2m.

At 20h. 51m. 21s. Io disappears into the shadow of Jupiter.

At 21h. 53m. 34s. the second satellite (Europa) disappears into Jupiter's shadow.

It is apparent then, since the times of the eclipses only are given to the nearest second, the use of occultation, transits or transits of shadows cannot be availed of for determinations of longitude. The times of the other phenomena are given to the nearest minute only. They may be observed simultaneously by two observers using the Almanac merely to indicate when to be ready for an observation.

To find the time of the disappearance or reappearance for any other place than Greenwich, it is only necessary to add the difference of longitude (in time) if the place be east, or to subtract it, if it be west. Jupiter, however, may be below the horizon of the place at this time, or the intensity of sunlight or of twilight may render the satellites invisible. Hence it is necessary to ascertain the position of the sun and of Jupiter with respect to the earth.

Suppose, for example, it is required to find the Right Ascension and Declination of Jupiter at 6 p.m., mean civil time, on Jan. 15th. 1909, at Montreal, and also the mean time of Jupiter's transit of Montreal's meridian. It must not be assumed that the longitude of Montreal is known exactly, but it must be approximately known, and this will be sufficient to discover Jupiter's position as accurately as is required. If the longitude be assumed as 4h. 50m. West the Greenwich astronomical time corresponding to 6 p.m. at Montreal will be 10h. 50m.

From Nautical Almanac (Page 170),-

| | h. | m. | s. |
|---|---|---|---|
| Right Ascension on Jan. 15th.(noon) = | 11 | 3 | 25.96 |
| " " " " 16th. " = | 11 | 3 | 14.73 |
| showing a decrease in R.A. in 24 hours of | | | 11.23 s. |

∴ decrease corresponding to 10h.50m. $= \frac{10.833 \times 11.23}{24}$

$= 5.05$ s.

Hence, 11h. 3m. 20.82s. will be the Right Ascension at 6 p.m.

Similarly from page 170 of Nautical Almanac the correction to be added to the declination on Jan. 15th. (noon) is

$$\frac{10.833 \times 87.0}{24} = 39.1 \text{ s.}$$

giving N 7°-25'-16" as the declination at the required time.

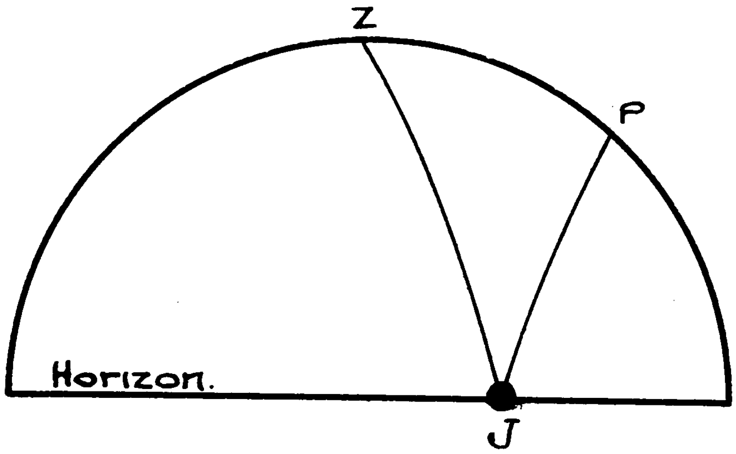
And from the same page of the Almanac,-

| | h. | m. |
|---|---|---|
| Jupiter passed meridian of Greenwich on Jan.15th. at | 15 | 23.6 |
| " " " " " " " 16th. " | 15 | 19.5 |
| Difference | | 4.1 |

so difference corresponding to 4h. 50m. (Montreal's assumed longitude) $= \frac{4.833 \times 4.1}{24} = 0.825$ s.

Hence, Jupiter passed Montreal's meridian at 15h. 22.77m. on Jan. 15th., or 3h. 22.77m. a.m. Jan. 16th. civil time.

The time of rising of Jupiter may also be required; if so, it becomes necessary to solve a spherical triangle to obtain the value of the hour angle.




Z is the zenith,
P is the pole,
J is Jupiter.

Jan. 19th.

In triangle ZPJ, ZP = 90°- latitude = 44°. 30'

ZJ = 90°+ refraction correction = 90°. 33'

PJ = 90°- declination = 82°. 29'

Knowing then the three sides it remains to solve for the angle at P. Calling the sides p, z, j; by spherical trigonometry

$$\sin P/2 = \sqrt{\frac{\sin(s-z)\ \sin(s-j)}{\sin z\ \ \sin j}}$$

(s being equal to half the sum of the sides.)
Substituting gives P = 98° 30' or 6h. 34m. as the hour angle.
But the time of meridian passage (as calculated above) was

15 h. 06.2 s.

Hence the time of rising on Jan. 19th. was 8.32 p.m.

Of course, all the above calculations are approximate, since the longitude assumed is not the correct value, but they are sufficiently accurate to serve the purpose of showing whether observations upon Jupiter's satellites are capable of being carried out on these dates.

After having completed the above short calculations, a would-be observer is in a position to know whether an observation on an eclipse of a satellite is feasible. If so, some means must be supplied whereby he is enabled to distinguish the various satellites. This means is available in the Nautical Almanac, where a diagram is shown representing at a given hour after mean noon of each day of the month, the relative positions of the images of Jupiter and of his satellites as they would appear in an inverting telescope. Jupiter is indicated by a white circle [○] in the centre of the page, the satellites by points. The numerals, 1,2,3,4, annexed to the points serve to distinguish the satellites, and their positions indicate the directions of the satellites' motions, which are towards the numerals. A clear circle beside a satellite denotes that the satellite is on the disc of Jupiter, and a black circle [●] that it is either behind the disc or within the shadow of Jupiter.

The satellites are in the superior parts of their orbits, or have Jupiter between them and the earth, when they are moving from west to east, and are in the inferior parts of their orbits, or between the earth and Jupiter, when moving from east to west. In the former case eclipses and occultations occur, and in the latter transits of the satellites and of their shadows. The configurations, as they are called, are given for mean astronomical time. In addition to these configurations further assistance is given by diagrams showing the phases of the eclipses of the satellites for an inverting telescope. These latter diagrams exhibit the position of each satellite with respect to the disc of the planet at the moment of disappearance and reappearance, as it would appear in an inverting telescope.

As an illustration of the actual work involved in an observation of an eclipse, the steps taken during a determination of the longitude made on Feb. 25th. 1909 are given below.

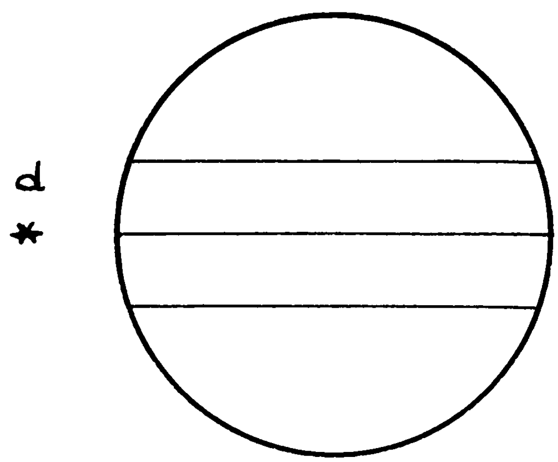
To begin with, a brief calculation of the time of Jupiter's rising and meridian passage indicated that it was possible to witness the phenomenon on that date. By reference to the Almanac (page 508) the following was found:-

| | | h. m. s. |
|---|---|---|
| Feb. 25. | I. Ec. D. | 17-29-19 |
| | I. Oc. R. | 19-50 |
| | II. Sh. I. | 23-26 |
| | II. Tr. I. | 23-44 |

As has been already pointed out, of these four only the first was of any practical value for the purpose of determining the longitude.

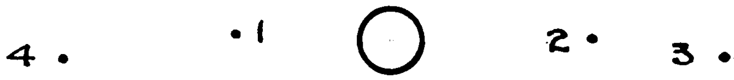
Also on page 509 of the Almanac under "Phases of the eclipses of the satellites for an inverting telescope" this figure was seen:-

I



The asterisk indicates the position of the satellite with respect to Jupiter at the instant it disappears into the shadow of the planet, as seen through an inverting telescope.

On the same page under "Configurations at 13 hrs. for an inverting telescope" opposite Feb. 25th., there was this diagram:-




from which is seen, since 2 and 1 were both moving towards the planet, and 2 was due to transit (at 23h. 44m.) and 1 was to be eclipsed, that 2 was in the inferior part of its orbit while 1 was in the superior part, as already explained.

Since 13 hrs. Greenwich time - the hour for which the configurations are shown - corresponds to some instant between 8 p.m. and 9 p.m. at Montreal, the satellite Io was easily distinguished at 8 p.m. by the aid of the configurations given. So, once his position was known, it was simply a matter of watching him until he disappeared, and noting the instant at which he was snuffed out. This instant as recorded was 10h. 55m. 30s. sidereal time (the time being taken with a sidereal chronometer). The phenomenon was noted only to the nearest half-minute, since the disappearance of the satellite was so gradual that the lapse of half a minute produced no apparent change in the satellite's position. The corresponding mean time was 12h. 33m. 48s. a.m. Feb. 26th.

So,

| | h. m. s. |
|---|---|
| mean local time of the eclipse at Greenwich was | 17-29-19 |
| and " " " " " " Montreal " | 12-33-48 |
| | 4-55-31 |

showing a difference in local times of 4h. 55m. 31s., which represents the longitude of Montreal.

The results by this method gave the following values for Montreal's longitude.-

| Date | Phenomenon | Greenwich time of occurrence. | Montreal time of occurrence. | Resulting Longitude. |
|---|---|---|---|---|
| | | h. m. s. | h. m. s. | h. m. s. |
| Jan. 30 | IV. Ec.R. | 16-21-27 | 11-26-41 | 4-54-46 |
| Feb. 8 | III. Ec.D. | 14-53-41 | 9-57-30 | 4-56-11 |
| " 11 | I. Ec.D. | 13-42-15 | 8-47-12 | 4-55-03 |
| " 18 | I. Ec.D. | 15-35-53 | 10-40-21 | 4-55-32 |
| " 25 | I. Ec.D. | 17-29-19 | 12-33-48 | 4-55-31 |
| " 27 | II. Ec.D. | 18-42-15 | 13-47-16 | 4-54-59 |

giving as a mean value 4h. 55m. 20.3s.

The results given, though differing widely, have been selected: other results gave values for the longitude showing an even wider variation than the above, the variation depending to a very marked degree (for a given telescope) upon the clearness of the atmosphere. On "semi-hazy" nights, when it was possible to distinguish the satellites through the telescope, the time of disappearance or reappearance as noted would precede the true time by as much as three minutes. This discrepancy was mainly due to the imperfect image of Jupiter's disc, and combined with that the presence of haze, so much so that the point on the planet's disc at which the satellite should disappear could not be sharply defined.

The mean value of the longitude as deduced from the results given - 4h. 55m. 20.3s. - shows a discrepancy of 1 m. 01·6s ~~or ,6 s.~~ from the value as obtained by the electric telegraph. In the course of explanation of this method it was pointed out that the accuracy of obtaining the instant at which the satellite was eclipsed depended almost entirely upon the telescope used - its magnifying power and (to an even greater extent) the amount of light admitted by its objective. This fact was clearly shown by the use of telescopes of varying power; a pair of field glasses completely failed to detect the presence of the satellites. The instrument with which the majority of these observations was made, was a transit manufactured by Watts, London. It had a magnifying power of about 30 diameters and an objective of $1\frac{3}{4}$" clear opening: in other words, it approached very closely the most powerful instruments found in field practice. On the other hand, chromatism was not entirely eliminated, so, as a result, instead of having a sharply-defined image of Jupiter's disc, there was a series of coloured edges exhibited. With these facts and the above results before us, it seems evident that longitudes as determined in the field by this means will not agree within half a minute (of time) with the values as determined by the more accurate methods, such as the electric telegraph. Moreover, this method has not the advantage possessed by lunar methods of being almost always available at times suited to the convenience of the observer.

The second method, which will be discussed - number (3) amongst the various methods used for determining longitude as given above - depends almost entirely upon the computation of a "Time Set". It is employed in the work of the Fourth Year Survey School to determine the longitude of the

Camp. The principle is somewhat similar to that of method (5), known as "Transportation of Chronometers". Essentially the process consists in finding the error of a mean time chronometer by observations of the transits of stars across the meridian, and also in finding its error on a standard clock, thus obtaining the relation between mean local time and standard time, and hence the longitude of the place of observation. The error of the chronometer on local mean time is obtained from the result of the Time Set, and the error on standard is found simply by comparison of the clock face with that of standard. The standard clock must, of course, be marking correct standard time.

If $\alpha$ denote the right ascension of a star,

$T$ " " observed time of its passage over the mean wire of the instrument,

$\Delta T$ denote the clock error,

then, if the instrument were adjusted so that the plane of the meridian passed through the mean wire, the relation $\alpha = T + \Delta T$ would be true. Practically, however, it is impossible for this condition to hold, and so the observed time $T$ is really subject to the corrections for azimuth, inclination of axis and collimation inherent in the instrument.

Now it can be easily proved that for an azimuth error, a, the error in time (hour angle) t, becomes a x sin z sec $\delta$, where z denotes the star's zenith distance, and $\delta$ its declination. This may be written as Aa, A being equal to sin z sec $\delta$ ; and by a similar process it can be shown that the effect of an inclination of the horizontal axis of b on the hour angle is given by cos z sec $\delta$ — written as Bb; and for a collimation error of c the resulting error in time becomes equal to sec $\delta$

Hence, though theoretically the equation of the Astronomical Transit instrument is $\alpha = T + \Delta T$, it becomes for practical purposes,-

$$\alpha = T + \Delta T + Aa + Bb + Cc$$

in which A, B and C are equal to sin z sec $\delta$, cos z sec $\delta$, and sec $\delta$, respectively.

A Time Set, as commonly employed, contains six stars, three being observed in the position "Lamp West" and three in "Lamp East", and so selected that for each position one north star is included amongst the three. The north stars, owing to the slowness of their apparent motion, do not give good results as "time stars"; they are observed merely to assist in the determination of the errors of azimuth and of collimation of the instrument. The error of horizontality of the axis is found by the use of a striding or hanging level.

When an observation of the transit of six such stars has been completed, the following quantities are known:- the observed times of transit over the mean wire, the declinations and the right ascensions. Then, the declinations being known, the quantities A, B and C can be found, if the latitude be assumed. The error <u>b</u> is obtained by reading the level in reversed positions and adopting the mean of the readings, the value of one division of the level having been previously determined on a level-trier. It must be noted that T, the observed time from the chronometer of the star's transit, is expressed in mean time, whereas $\alpha$ , the star's right ascension - the time when the star should have crossed the meridian - is given in sidereal units. Hence it is not correct to subtract $\alpha$ from T as recorded, but T must first be "corrected for rate". This rate correction is very simply applied, and depends on the fact that 1 hour of mean time is equivalent to 1 h. 0m. 9.86s. of sidereal time. Then it remains to choose some instant - this is done by an inspection of the observed times - at which the error of the chronometer is to be determined, and having chosen this instant, a proportional part of

9.86s. must be added to, or subtracted from, the observed T for every star to obtain the "T corrected for rate".

Having reached this stage, i.e., after T has been corrected for rate, an examination of the equation of the instrument will show that for any given star in the set, all the quantities composing the equation are known with the exception of $\Delta T$, <u>a</u> and <u>c</u>. Of these three, $\Delta T$ is the final result of the computation, and <u>a</u> and <u>c</u> are obtained by the solution of certain simultaneous equations. The formation of such equations and their solution will be best described by an example. When the values for <u>a</u> and <u>c</u> have been found and substituted in the equation, the clock error, $\Delta T$ is immediately obtained.

Owing to adverse weather conditions sufficient time-sets (to be of value) have not been computed, but rather than allow this method to pass without illustration, some sets taken during the Survey School at the observatory on Westmount Hill have been inserted. Of course, the longitude so found is that of the Westmount observatory, but the results given will serve to show the general character of and degree of accuracy peculiar to this second method.

<u>Time.</u> <u>Astronomical Transit.</u>

Station - Westmount Hill. Date.- Sept. 4. 1908.

Instrument.- No. 2. Clock.- No. 20. Observer . . . .

| | Lamp West | | | Lamp East | | |
|---|---|---|---|---|---|---|
| Declin. | 9°-26' | 48° 53' | -0°46' | 40°48' | 62°12' | -5°58' |
| A | 0.60 | - 0.09 | 0.70 | 0.12 | -0.62 | 0.79 |
| B | 0.82 | 1.52 | 0.71 | 1.32 | 2.05 | 0.63 |
| C | -1.01 | - 1.52 | -1.00 | 1.32 | 2.14 | 1.07 |
| T | 10-38-25.30 | 10-42-30.90 | 10-59-40.30 | 9-52-56.40 | 10-15-59.10 | 10-25-26.90 |
| Corr, for rate to 10h.30m. | 1.38 | 2.87 | 4.87 | -6.09 | -2.30 | -0.75 |
| Corr. T | 10-38-26.68 | 10-42-33.77 | 10-59-45.17 | 9-52-50.31 | 10-15-56.80 | 10-25-26.15 |
| b = -0.39 Bb = | -0.32 | -0.59 | -0.28 | -0.51 | -0.80 | -0.24 |
| T + Bb | 10-38-26.36 | 10-42-33.18 | 10-59-44.89 | 9-52-49.80 | 10-15-56.00 | 10-25-25.91 |
| α | 21-39-42.28 | 21-43-26.20 | 22-01-05.85 | 20-53-46.81 | 21-16-26.04 | 21-26-45.29 |
| α -(T + Bb) | 12-58-44.08 | 12-59 06.98 | 12-58-39.05 | 12-59-03.99 | 12-59-29.96 | 12-58-40.62 |
| a=35.11 Aa= | 21.04 | - 3.16 | 24.59 | 3.79 | -21.62 | 27.60 |
| α -(T+Bb+Aa) | 59-05.12 | 03.82 | 03.64 | 07.78 | 08.34 | 08.22 |
| c=1.34 : Cc= | 1.35 | 2.02 | 1.34 | -1.77 | -2.87 | -1.40 |
| α -(T+Aa+Bb+Cc)= ΔT= | 59-06.47 | 05.84 | 04.98 | 06.01 | 05.47 | 06.82 |

Lamp West: 59-05.76 Lamp East: 59-06.10

Hence ΔT = 12h. 59m. 05.93s.

This is the error of the chronometer at 10h. 30m. sidereal time, so the correct sidereal time of the observation was 21h. 30m. 54.07s.

| | | h. | m. | s. |
|---|---|---|---|---|
| Sidereal time of mean noon Sept,4.'08 | = | 10 | 53 | 37.09 |
| ∴ sidereal interval since mean noon up to the time of the observation | = | 10 | 37 | 26.98 |
| and the equivalent mean interval | = | 10 | 35 | 32.82 |
| but the mean time clock face was | | 10 | 30 | 00.00 |

Hence clock No. 20 was slow of local time (Westmount) by 5m. 32.82s.

A comparison of Clock 20 with the clock marking Eastern Standard time showed standard to be slow of Clock 20 by 0.50 sec. So mean local time at Westmount was fast ~~by~~ of Eastern Standard by 5m. 33.32s. But Eastern Standard time is the local time for a point in longitude 5 hrs. West. Therefore, the longitude of Westmount according to this calculation is 5 hrs. less 5m. 33.32 s., or 4h. 54m. 26.68s. Its probable error is $\pm$0.18S.

To find a and c :-

The equation of the transit instrument may be written:-

$$\alpha - (T + Bb) = \Delta T + Aa + Cc$$

By substituting values obtained from each star the following equations are obtained:-

| | m. s. | | |
|---|---|---|---|
| (1) | 58-44.08 | = | $\Delta T + 0.60a - 1.01c$ |
| (2) | 59-06.98 | = | $\Delta T - 0.09a - 1.52c$ |
| (3) | 58-39.05 | = | $\Delta T + 0.70a - 1.00c$ |
| (4) | 59-03.99 | = | $\Delta T + 0.12a + 1.32c$ |
| (5) | 59-29.96 | = | $\Delta T - 0.62a + 2.14c$ |
| (6) | 58-40.62 | = | $\Delta T + 0.79a + 1.01c$ |

Then assuming $c = 0$, from (1) and (2) $a = 35.0$

and from (2) and (3) $a = 35.4$

giving a mean value for L.W. of 35.2

Similarly the mean value for L.E. is 35.0; the average of the L.W. and L.E. values - 35.1 - is employed in the time set.

For the value of c the same six equations are used and stars similarly situated on each side are compared not much error in azimuth, and so assume $a \doteq 0$.

The results of four other sets are:-

**I.**

| | Lamp West. | | | Lamp East. | | |
|---|---|---|---|---|---|---|
| Decl. | 38°-42' | 59°-17' | 13°-44' | 67°-30' | 2°-56' | 27°-46' |
| T | 7-21-21.50 | 7-37-39.20 | 7-48-25.40 | 8-00-43.40 | 8-08-03.20 | 8-14-22.30 |
| $\alpha$ | 18-33-50.79 | 18-49-52.17 | 19-01-12.54 | 19-12-34.05 | 19-20-53.42 | 19-27-02.37 |
| $\Delta T$ | 12-47-31.23 | 47-30.62 | 47-30.65 | 47-29.50 | 47-31.31 | 47-31.03 |

Error of clock on Standard 9.10s. slow.

Resulting Longitude = 4h. 54m. 26.37s. ± 0.09s.

**II.**

| | | | | | | |
|---|---|---|---|---|---|---|
| Decl. | 10°-23' | 70°-02' | 19°-15' | -1°-05' | 39°-58' | 62°-41' |
| T | 8-29-00.5 | 8-36-33.5 | 8-41-48.00 | 8-53-35.90 | 9-06-17.50 | 9-15-49.50 |
| $\alpha$ | 19-41-55.01 | 19-48-31.80 | 19-54-41.81 | 20-06-35.53 | 20-18-57.59 | 20-28-04.91 |
| $\Delta T$ | 12-47-22.62 | 47-25.00 | 47-20.65 | 47-21.72 | 47-21.90 | 47-23.63 |

Error of clock on Standard 9.10s. slow.

Resulting Longitude 4h. 54m. 26.38s ± 0.42s.

III. Lamp West. Lamp East.

| | Lamp West. | | | | Lamp East. | |
|---|---|---|---|---|---|---|
| Decl. | - 8°-14' | - 1°-51' | 49°-49' | 29°-45' | 23°-43' | 65°-43' |
| T | 11-10-34.00 | 11-15-32.10 | 11-26-35.90 | 11-37-25.90 | 11-40-46.60 | 11-45-45.70 |
| α | 22-12-01.16 | 22-16-56.62 | 22-27-32.72 | 22-38-43.69 | 22-42-08.26 | 22-46-27.87 |
| ΔT | 12-58-56.02 | 58-56.46 | 58-55.18 | 58-56.21 | 58-56.34 | 58-55.00 |

Error on Standard 0.50s. fast.

Resulting Longitude 4h. 54m. 26.68s. ± 0.18s

IV.

| | | | | | | |
|---|---|---|---|---|---|---|
| Decl. | 77°-07' | 28°-35' | 14°-40' | 33°-13' | 56°-02' | 23°-46' |
| T | 12-35-37.50 | 1-02-07.50 | 1-06-51.90 | 1-30-29.00 | 1-34-08.50 | 1-40-51.90 |
| α | 23-35-40.11 | 24-03-40.17 | 24-08-32.10 | 24-32-00.26 | 24-35-19.90 | 24-42-29.85 |
| ΔT | 12-58-35.26 | 58-39.14 | 58-39.09 | 58-38.78 | 58-37.71 | 58-38.32 |

Error on Standard 0.50s. fast.

Resulting Longitude 4h. 54m. 26.33s. ± 0.40s

The "weighted mean" of these observations is 4h.54m.26.40s.

± 0.04s

Since the other two methods yet to be dealt with - "Lunar Culminations" and "Lunar Distances" - depend entirely upon the motion of the moon, a brief discussion of her motion, before these methods are described, will perhaps be useful.

It is a very widely known fact that the moon makes a revolution in the celestial sphere in about a month, and that during this revolution a number of different phases are presented, known as "new moon", "first quarter", "full moon", "last quarter", etc., depending on her position relative to the sun. A study of these phases during a single revolution will make it clear that the moon is a globular, dark body, illuminated by the light of the sun - a fact which has been evident for many generations. She is the nearest to us of all the heavenly bodies, and revolves in an orbit which is approximately an ellipse of small eccentricity with the centre of gravity of the earth and moon in the focus. Her mean distance is about 240,000 miles, being about 60 times the earth's radius. The actual motion of the moon in space results from the combination of the motion of the earth about the sun and of the moon about the earth. The actual curve described by her is concave to the sun in every part.

As the sun makes a revolution around the celestial sphere in a year, so the moon makes a similar revolution amongst the stars. Her apparent motion is, however, much more rapid, its sidereal period being about 27-1/3 days. Thus she completes 13 revolutions about the earth before the earth has completed one about the sun. This motion can be noticed on any clear night between "first quarter" and "full moon", if the moon happens to be near a bright star. If the position of the moon relatively to the star be noted from hour to hour, it will be found that she is constantly working towards the east by a distance about equal to her own diameter in one hour. The following night she will be found from 12° to 14° east of the star, and will rise, cross the meridian and set from half an hour to an hour later than she did the preceding night. At the end of 27 days, 8 hours, she will be back in the same position among the stars in which she was first seen. If, however, starting from one "new moon," we count forwards this period, we shall find that the moon, although she has returned to the same position among the stars, has not got back to "new moon" again. The reason is that the sun has moved forwards in virtue of his apparent annual motion so far that it will require more than two days for the moon to overtake him. So, although the moon really revolves around the earth in 27-1/3 days, the average interval between one "new moon" and the next is 29-1/2 days.

The plane of the moon's orbit is not coincident with the ecliptic, but is inclined to it at an angle of rather more than $5^{\circ}$. The points of intersection of the orbit with the ecliptic, or the nodes, have a rapid retrograde motion among the stars, the period of a revolution of the nodes being about 18½ years. Thus in one year the nodes retreat through about $19\frac{1}{2}^{\circ}$, and in one day through more than 3'. The general nature of the motion of the moon about the earth may, therefore, be conceived by supposing it to move in an ellipse, the plane of which moved so that its inclination to the ecliptic is pretty constant, while its line of intersection retreats on the ecliptic at the rate of 3' a day.

The line of apsides or major axis of the moon's ellipse is also in rapid motion: the motion being direct and at such a rate as to complete a revolution in about 9½ years. These irregularities are due to the disturbing attraction of the sun. There are many other "inequalities of the moon's motion," as they are called, besides the above, which should be stated in order to give a notion of the general nature of the moon's apparent path in the heavens.

Three conditions stand in the way of the attainment of great accuracy by any method involving the moon:

Firstly. Since the moon, as has already been pointed out, requires about 27-1/3 days to make one complete revolution in her orbit about the earth, the apparent motion of the moon amongst the stars is then about one-twenty-seventh as fast as the apparent motion of the stars relative to an observer's meridian, which furnishes his measure of time. Any error in determining the position of the moon is then multiplied by at least twenty-seven, when it is converted into time in the process of the computation. If then the time of transit of the moon, for example, could be observed as accurately as that of a star, one would expect the errors in a longitude computed from moon culminations to be twenty-seven times as great as the errors of the local time derived from the same number of star observations.

Secondly. The motion of the moon is so difficult to compute that her position at various times as given in the Ephemeris, and also of course the data there given in regard to lunar distances and occultations, are in error by amounts which become whole seconds when multiplied by twenty-seven.

Thirdly. The limb or edge of the visible disc of the moon is necessarily the object really observed and this is a "ragged edge" rather than a perfect arc for purposes of measurement.

Third Method.- By Moon Culminations.

The moon's motion in right ascension is so rapid that the change in this element, while the moon is passing from one meridian to another, may be used to determine differences of longitude. Owing to this motion she separates in right ascension from any fixed star by $360^{\circ}$ in 27.3 days. In one day, therefore, her motion in right ascension is rather more than $13^{\circ}$: at the beginning and end of an hour she transits two meridians nearly $15^{\circ}$ apart, and in this interval of time her motion in right ascension is more than half a degree. If then the right ascension of the moon be ascertained at her transit over the meridian of the place on any day, and if the rate of her motion in right ascension at the time be known, and the right ascension which she had at the instant of her preceding transit over the meridian of

Greenwich, the difference of the right ascensions at the place and at Greenwich determines the longitude of the place. Her right ascension at the instant of meridian transit is most accurately found by means of the interval of sidereal time between this transit and that of a neighbouring star. For this purpose, the Nautical Almanac contains a list of "moon-culminating" stars, two of which are selected for every day, and the mean of whose declinations is nearly the same as that of the moon. The Almanac also contains the right ascension of the moon's bright limb for each culmination, both upper and lower, and the variation of this right ascension in one hour of longitude, i.e., the variation during the interval between the moon's transit over two meridians whose difference in longitude is one hour. This variation is not uniform, and its value is given for the instant of the passage over the meridian of Greenwich. These quantities facilitate the reduction of corresponding observations, as will be seen below.

As regards the observation, let

T, T' denote the sidereal times of the culminations of the moon's limb and the star respectively, corrected for all the known errors of the transit instrument and for clock rate;

and $\alpha$ , $\alpha'$ denote the right ascensions of the moon's limb and the star at the instants of transit.

Then evidently,-

$$\alpha = \alpha' + T - T'$$

The star and the moon being nearly in the same parallel, the instrumental errors which affect T also affect T' by nearly the same quantity. It should not, however, for this reason be omitted to apply all the corrections for known instrumental errors, since by their omission there would be introduced an error into the longitude exactly equal to the uncorrected error of the instrument. For, suppose the instrumental error produces an error z in the time of the star's transit, the effect is the same as if the instrument were perfectly mounted in a meridian whose longitude west of the place is equal to z; but the sidereal time required by the moon to describe this interval z is equal to z plus the increase of the Moon's right ascension in this interval. Hence the longitude found would be in error by the quantity z.

It can readily be seen then that the chief aim in connection with the observation lies in obtaining the correct instant at which the moon's limb passes the meridian at the point of observation. If the right ascension of the limb at upper culmination at Greenwich, as given in the Almanac, be subtracted from the right ascension obtained as a result of the observation, the difference will represent the total variation in right ascension during the interval elapsing while the moon travels from the meridian of Greenwich to that of the point of observation. From the Almanac the variation in 1 hour of longitude corresponding to this total variation can be calculated, and hence the longitude of the place of observation is obtained by dividing the total variation by the variation in 1 hour of longitude as found.

To illustrate this, the observation made on March 3rd. 1909 will be given:-

From page 419 of the Nautical Almanac -

| Date | Name | Apparent Right Ascension. | Var. of Moon's Right Ascens. in 1 hr. of Longitude. | Apparent Declin. | Variation in moon's declin, in 1 hr. long. |
|---|---|---|---|---|---|
| | | h.m. s. | s. | | |
| Mar.3 | Moon IU | 8-43-15.00 | 131.08 | N 21°47' 30.1" | -376.5" |
| | Moon IL | 9- 9- 9.43 | 127.99 | N 20°24' 53.4" | -448.4" |
| | ξ Cancri | 9- 4- 8.55 | | 22°25' | |
| | B.A.C. 3138 | 9- 8- 26.33 | | 21°40' | |

The result of the observation of the transit of the moon's limb was:-

from "wires" *

| | h. | m. | s. | h. | m. | s. |
|---|---|---|---|---|---|---|
| | 8 | 55 | 18.2 | 8 | 57 | 09.4 |
| | | | 27.2 | | | 18.4 |
| | | | 35.9 | | | 27.5 |
| Means | 8 | 55 | 27.1 | 8 | 57 | 18.43 |

8 56 22.76

from "contacts"

| | h. | m. | s. |
|---|---|---|---|
| | 8 | 56 | 17.71 |
| | | | 19.37 |
| | | | 20.91 |
| | | | 22.63 |
| | | | 24.40 |
| | | | 25.74 |
| | | | 27.55 |
| Mean | 8 | 56 | 22.62 |

| | | | | | |
|---|---|---|---|---|---|
| So, | T from wires | = | 8 | 56 | 22.76 |
| | T from contacts | = | 8 | 56 | 22.62 |
| | Hence, mean T | = | 8 | 56 | 22.69 |

(giving "contacts" and "wires" equal values)

* An explanation of what is meant by "wires" and "contacts" will be found later on during the description of the Repsold Eyepiece.

For star $\xi$ Cancri,-

| | | h. | m. | s. |
|---|---|---|---|---|
| $\alpha$ | = | 9 | 4 | 8.55 |
| observed T | = | 9 | 6 | 37.08 |
| ∴ $\alpha$ - T | = | | -2 | 28.53 |

and for star B.A.C. 3138,-

| | | | | |
|---|---|---|---|---|
| $\alpha$ | = | 9 | 8 | 26.33 |
| observed T | = | 9 | 10 | 54.38 |
| so, $\alpha$ - T | = | | - 2 | 28.05 |

Mean value of $\alpha$ - T = - 2m. 28.29s.

But $\alpha - T = \Delta T + Aa + Bb + Cc$

and Aa + Bb + Cc for the instrument (as will be seen later) amounted to - 7.89s. of time.

∴ $-2^m\ 28^s.05 = \Delta T - 7.89$

so, $\Delta T = -$ 2m. 20.40s.

| | h.m. s. |
|---|---|
| Now observed time of transit of moon's limb was | 8 56 22.69 |
| and $\alpha$ - T was | 2 28.29 |
| ∴ corrected time of transit was | 8 53 54.40 |

Then,-

| Right Ascensions. | | | Var, in 1 hr. of Long. |
|---|---|---|---|
| h. | m. | s. | s. |
| 8 | 43 | 15.00 | 131.08 |
| 8 | 53 | 54.40 | ? |
| 9 | 9 | 9.43 | 127.99 |

With that data it can be shown that the "variation in 1 hr. of longitude" corresponding to 8h. 53m. 54.40s. is 129.74s. The mean of 129.74s. and 131.08s. is 130.41s., which may be assumed as the variation in 1 hr. of longitude during the interval the moon's right ascension was changing from 8h. 43m. 15s. to 8h. 53m. 54.40s. This is a total variation of 639.40secs.

But total vatiation in Right Ascension ÷ variation in 1 hr. of longitude

= longitude

So,

$$\text{longitude of Montreal} = \frac{639.40}{130.41} = 4h.\ 54m.\ 10.4s.$$

The results obtained from Moon culminations were:-

| Date | Observed T | α - T | Right Ascension | Total Change in Rt. Ascen. | Var.in 1 hr. of long. | Longi-tude. |
|---|---|---|---|---|---|---|
| | h.m. s. | m. s. | h.m. s. | s. | s. | h. |
| Jan.2 | 3-58-37.66 | -3-54.80 | 3-54-42.86 | 675.584 | 137.79 | 4.9030 |
| " 5 | 6-49-50.04 | -4-04.91 | 6-45-45.13 | 703.240 | 143.41 | 4.9037 |
| Feb.2 | 7-23-37.39 | -0-46.43 | 7-22-50.96 | 686.084 | 139.92 | 4.9034 |
| " 3 | 8-18-42.16 | -0-50.61 | 8-17-51.55 | 663.904 | 135.28 | 4.9040 |
| " 26 | 4-17-10.95 | -2-11.68 | 4-14-59.27 | 691.358 | 140.99 | 4.9036 |
| " 27 | 5-14-02.85 | -2-15.08 | 5-11-47.77 | 698.825 | 142.53 | 4.9030 |
| Mar.3 | 8-56-22.69 | -2-28.29 | 8-53-54.40 | 639.400 | 130.41 | 4.9029 |
| " 6 | 11-23-57.17 | -2-38.86 | 11-21-18.31 | 558.044 | 113.84 | 4.9021 |

Mean value ) = 4.9032 hrs.
for longitude ) = 4h 54m 11.6 ± 1.8 s.

It very often happens that the "moon-culminating" stars are very small and, in consequence of this, the brilliancy of the moon renders them invisible. When this happens, the clock error has to be determined in the ordinary way by observing the transits of other stars, but care should be taken to select stars whose mean declination is about equal to that of the moon.

"If the lunar tables were perfectly accurate, the "true longitude given by the observation would be found at "once by comparing the observed right ascension with that of "the Ephemeris. There are two methods of avoiding or elimin-"ating the errors of the Ephemeris. In the first, which has "heretofore been exclusively followed, the observation is com-"pared with a corresponding one on the same day at the first "meridian, or at some meridian the longitude of which is well "established. In this method the increase of the right "ascension in passing from one meridian to the other is "directly observed, and the error of the Ephemeris on the day "of observation is consequently avoided: but observations at "the unknown meridian are frequently rendered useless by a "failure to obtain the corresponding observation at the first "meridian.

"In the second method, proposed by Professor Peirce, "the Ephemeris is first corrected by means of all the observa- "tions taken at the fixed observatories during the semi- "lunation within which the observation for longitude falls. "The corrected Ephemeris then takes the place of the corres- "ponding observation, and is even better than the single "corresponding observation, since it has been corrected by "means of all the observations at the fixed observatories "during the semi-lunation."

(Chauvenet, Vol. I.)

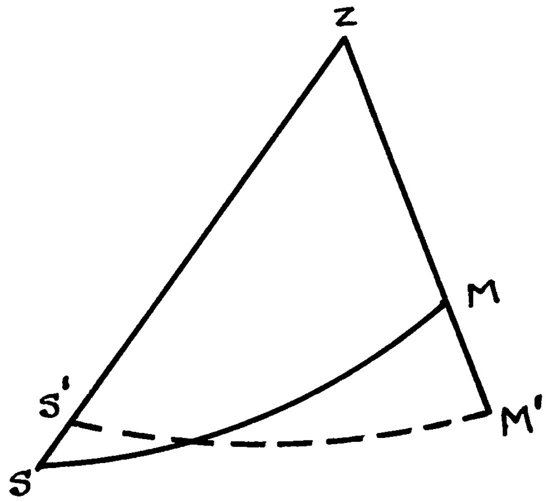
Fourth Method.- By Lunar Distances.

The moon making her monthly circuit of the heavens may be considered a sort of standard clock from which the observer may learn the Greenwich time in whatever part of the world he may find himself. This he does by observing her position amongst the stars. Knowing from the Nautical Almanac the right ascension and declination of the moon and of certain other bodies,- sun, planets and bright stars - it is possible to calculate for any hour the distance between the moon and those bodies. So, if an observer measures this distance with a sextant, he has the means of finding at what Greenwich time the distance was equal to that measured. Thus, suppose a lunar distance to be measured at 10 p.m.(local time) at a place whose longitude was known to lie between 5 hrs. W. and 6 hrs. W. of Greenwich. The corresponding Greenwich time will lie between 15 hrs. and 16 hrs. So it remains to compute the distance between the moon and the star at 15 hrs. and at 16 hrs. Greenwich time. It will then be found that the value of the angle as measured at the place will be between its value for 15 hrs. and its value for 16 hrs. So by interpolation the time can be found when the moon and the star were the measured distance apart, thus giving a means of comparing the Greenwich time with that of the place of observation.

The moon moves among the stars only about 13° in a day, a distance equal to her own diameter in an hour. If the observer wants his Greenwich time within half a minute, he must, therefore, determine the position of the moon within the hundred and twentieth of her diameter. Even this degree of accuracy can be obtained only by having the moon's position relatively to the star predicted with great accuracy, and this is a very complex problem.

In addition to this complexity, this method is open to the objection of being difficult, owing to the long calculation necessary to free the measured distance from the effects of the refraction of both bodies by the atmosphere and of the parallax of the moon. Until recent years the Nautical Almanac furnished tables showing the true distance of the moon's centre from the sun and from some nine bright fixed stars - known as "lunar distance" stars - selected in the path of the moon, for every third hour of mean Greenwich time. It was decided, however, to omit them as their use was not sufficient to justify their retention. Consequently, in all of the observations reduced below a great deal more time was consumed in the computations than would have been necessary had the work been done some years ago.

The actual observation, as given below, consisted in measuring the distance of the moon's bright limb from Arcturus ( $\alpha$ Boötes). The apparent distance of the moon's centre from the star was found by adding the moon's apparent semidiameter to the measured distance. The apparent distance found in this way differs from the true (geocentric) distance in consequence of the parallax and refraction, which affect the altitudes of the objects, and consequently their distance. The true distance was, therefore, obtained by computation, the general principle of which may be shown in a simple manner as follows:-




Let Z denote the zenith,

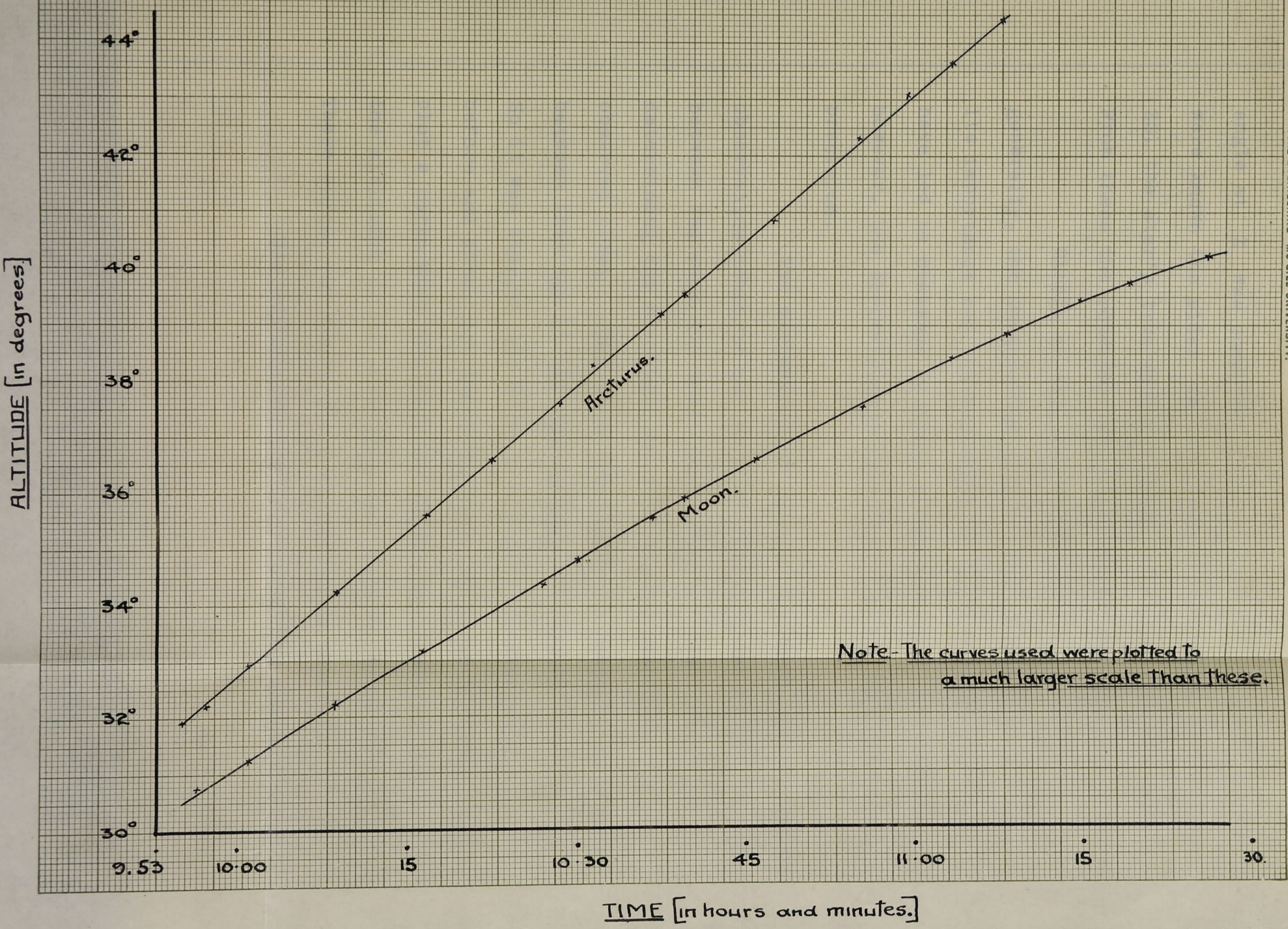
M' and S' the observed places of the moon and star,

MM' the parallax and refraction of the moon,

SS' the refraction of the star,

M and S are then the geocentric places.

The apparent altitudes of both were measured in addition to the angular distance between the moon and star. The apparent zenith distances and, consequently, the true zenith distances were, therefore, known. In the triangle Z M' S' there were known the three sides - M' S', the apparent distance of the objects, Z M', the apparent zenith distance of the moon, Z S', the apparent zenith distance of the star, - from which the



44°
42°
40°
38°
36°
34°
32°
30°
ALTITUDE [in degrees]
Arcturus.
Moon.
Note.- The curves used were plotted to a much larger scale than these.
9.53
10·00
15
10·30
45
11·00
15
30.
TIME [in hours and minutes.]

angle at Z was computed. Then in the triangle Z M S there were known the sides ZM, the moon's true zenith distance; ZS, the star's true zenith distance; and the angle at Z. From these the required true distance MS was computed.

Simple as the problem appears when stated generally, the computation of it is by no means brief; and its importance and the frequency of its application at sea have led to numerous attempts to abridge it. In most cases, however, the abbreviations have been made at the expense of precision.

The explanation of the method shows that a complete observation consists not only in measuring the angular distance between the moon and star, and noting the time, but also in determining the altitudes of the moon and star with their observed times. Now it is impossible to measure the angle between them and at the same instant to measure their altitudes So it was thought advisable to observe the altitudes both before and after the measurement of the lunar distance, and plot a curve from the readings taken, and from the curve find the altitudes at the time when the lunar distance was measured. These curves are shown on the accompanying sheet.

The sextant was used for measuring the altitudes was manufactured by T. Cooke and Sons, London, and read to 10 seconds. Its mark is S14 in the equipment of the Surveying Department. The lunar distances were measured with a German instrument, made by Tesdorf, which read to 20 secs.; marked S 13 in the equipment.

All the results given below were obtained on March 8th. 1909, and the following may be taken as a typical example of the calculation involved in a reduction to find the longitude.

| Altitude of Moon's Limb. | Altitude of Star. | Angle between Moon's Limb and Star. | Time. |
|---|---|---|---|
| | | | h.m. s. |
| 38°-45'-00" | 44°-24'-10" | 28°-33'-19" | 12-03-00 P.M. |

If the longitude of Montreal be assumed to lie between 4 hrs. 30 m. and 5 hrs. 30 m. West, it will be necessary to calculate the value for MS at 16 h. 20 m. and at 17 h. 20 m. Greenwich time in order that an interpolation may be used.

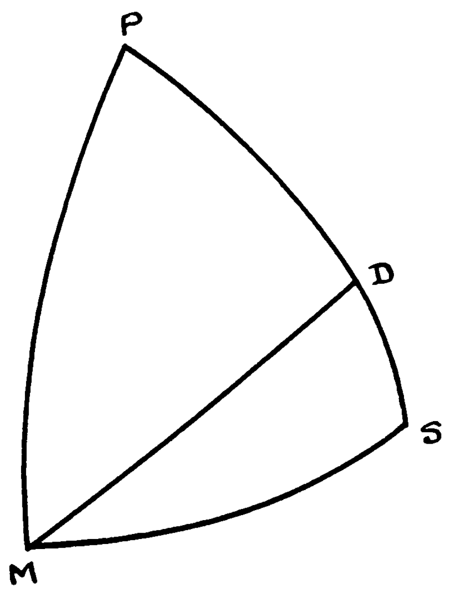
So the first problem is to find the true distance between the moon's centre and Arcturus at 16h. 20m. March 8th. Greenwich time.

From Nautical Almanac (page 31),-

| | | h. | m. | s. |
|---|---|---|---|---|
| Moon's right ascension at 16 hrs. | = | 12 | 44 | 58.69 |
| Variation in 20 mins. | = | | | 35.98 |
| Hence right ascension at 16 hrs. 20 m. | = | 12 | 45 | 34.67 |

Similarly for the declination,-

| | | | | | |
|---|---|---|---|---|---|
| At 16 hrs. moon's declination | = | N 0° | 17' | 54.1" |
| Variation in 20 m. | = | | 4 | 07.9 |
| giving as declination at 16 hrs. 20 m. | | N 0° | 13 | 46.2 |




In this spherical triangle, let M be the moon,

P be the pole,

S be Arcturus.

MD is perpendicular to PS.

The sides are the polar distances of the moon and Arcturus respectively, and the angle contained is the difference of the right ascensions of the moon and the star.

$$\cos MS = \cos PM \cos PS + \sin PM \sin PS \cos P$$

$$= \cos PM \cos PS (1 + \tan PM \tan PS \cos P)$$

but in the right angled triangle PMD

$$\tan PD = \tan PM \cos P,$$

therefore,

$$\cos MS = \cos PM \cos PS (1 + \tan PD \tan PS)$$

$$= \cos PM \cos (PS - PD) \sec PD.$$

---

March 8th. at 16h. 20m.

| | Right Ascension. | Declination. | |
|---|---|---|---|
| Moon | 12-45-34.67 | N $0^{\circ}$-13'-46.2" | tan PM = 12.3975459 |
| Arcturus | 14-11-31.17 | N $19^{\circ}$-39'-10.0" | cos P = 9.9687214 |
| Diff. | 1-25-56.50 | PS=$70^{\circ}$-20'-50.0" | tan PD = 12.3662673 |
| Diff(in arc) | $21^{\circ}$29'07.5" | PM=$89^{\circ}$-46'-13.8" | ∴ PD = $89^{\circ}$-45'-12" |
| | | P =$21^{\circ}$-29'-07.5" | PS = $70^{\circ}$-20'-50" |
| | | | diff. = 19 -24 -22 |

| | |
|---|---|
| cos diff. | = 9.9745979 |
| cos P M | = 7.6024506 |
| sec P D | = 12.3661766 |
| cos M S | = 9.9432251 |

∴ MS = $28^{\circ}$-39'-47."

Similarly it can be shown that at 17 hrs. 20 m.

|  |  |
|---|---|
|  | h. m. s. |
| Moon's right ascension was | 12-47-22.64 |
| and moon's declination was | N $0^{\circ}$-1'-22.5" |
| and hence MS at 17 hrs. 20 m. was | $28^{\circ}$-28'-26" |

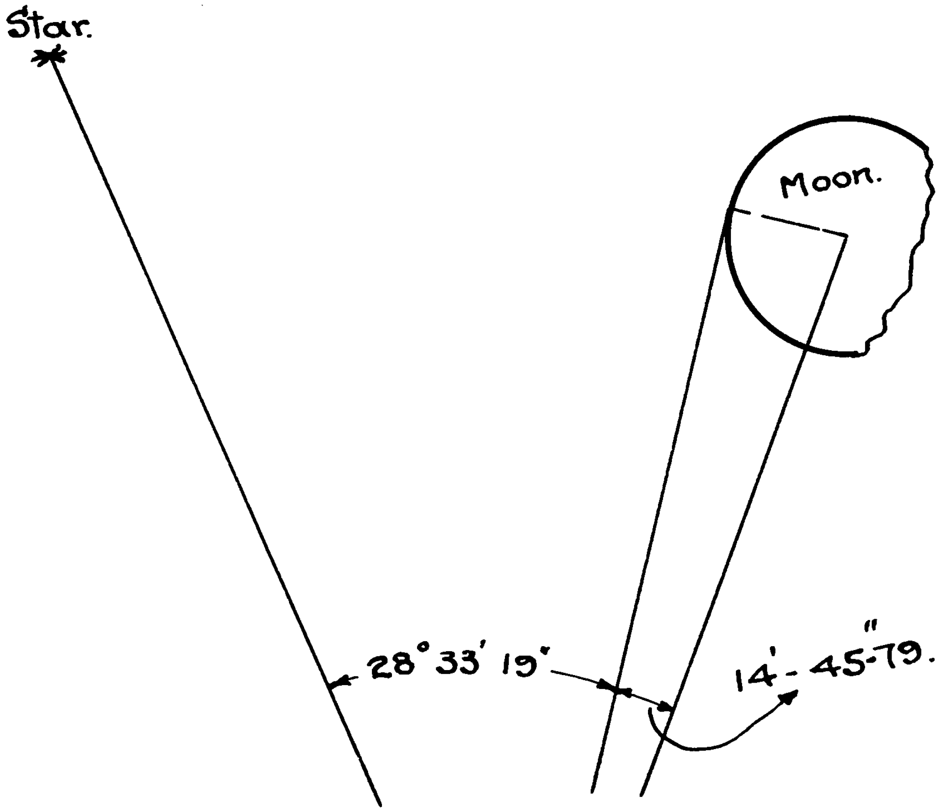
Reduction of the Observation.

Mean time of the observation was 12 hrs. 3m. 00 secs the corresponding Greenwich time will be ~~take~~ between 4 hrs. and 5 hrs. later, but as the Semidiameter variation is only 1.42" in 12 hrs. it will not make much difference, as far as the correction is concerned, what Greenwich instant is assumed as corresponding to 12.03 p.m. Montreal time. Suppose 16h. 53m. 00s. to be the corresponding Greenwich time (i.e. assume 4h. 50m. to be the longitude). Hence it remains to find the value of the semidiameter and the horizontal parallax at this time, which is 4 hrs. 53m. after midnight of March 8th.

From page 28 of the Almanac it follows that -

the semidiameter correction = 14'-45.79"

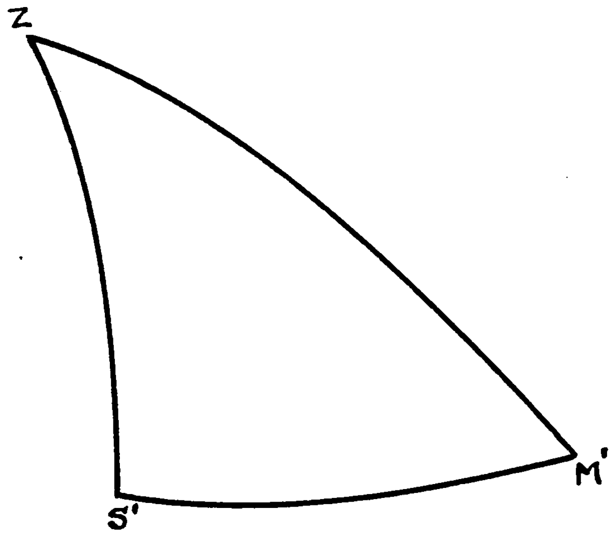
and equat. horizontal parallax correction = 54'-05.4




| | | | | |
|---|---|---|---|---|
| Measured angle between star and moon's limb | = | 28° | 33' | 19" |
| Semidiameter correction | = | | 14 | 46 |
| Hence distance between centres of moon and star | = | 28 | 48 | 05 |

| | | | | |
|---|---|---|---|---|
| Altitude of moon's limb | = | 38° | 45' | 00" |
| Semidiameter | = | | 14 | 46 |
| So altitude of moon's centre | = | 38 | 59 | 46 |

| | | | | |
|---|---|---|---|---|
| Altitude of star | = | 44° | 24' | 10" |




M' and S' are the observed positions of moon and star.

Z is the zenith.

| | | | | |
|---|---|---|---|---|
| M'S' | = | 28° | 48' | 05" |
| Z M' | = | 51 | 00 | 14 |
| Z S | = | 45 | 35 | 50 |
| 2a | = | 124 | 84 | 09 |

∴ a = 62° 42' 05" ( <u>a</u> being = half the sum of the sides of the spherical triangle.)

a - ZM' = 11° 41' 51" = a - s'

a - ZS' = 17° 06' 15" = a - m'

By spherical trigonometry,-

$$\sin Z/2 = \sqrt{\frac{\sin(a - m')\ \sin(a - s')}{\sin m'\ \sin s'}}$$

Substitution gives Z = 38° 15' 31"

If $\pi$ = the moon's equatorial horizontal parallax,

and $\pi_1$ = the moon's horizontal parallax corrected for latitude,

then $\pi_1 = \pi + \Delta\pi$

$\Delta\pi$ being a correction given in Table XIII. Chauvenet, Vol. II,, page 603.

And if h' denote the apparent altitude of the moon's centre,

r " " refraction for the altitude h',

and $h_1$ " " corrected altitude,

(from Chauvenet)

$$h_1 = h' - r + \pi_1 \cos(h' - r)$$

In the example,-

$$\pi_1 = 54' - 5.4'' + 5.0'' = 54' - 10''$$

$$h_1 = 38^\circ 59' 46'' - 1' 10'' + 54' 10'' \times \cos 38^\circ 58' 36''$$

$$= 39^\circ 40' 41''$$

So, moon's corrected zenith distance = 50° 19' 19"

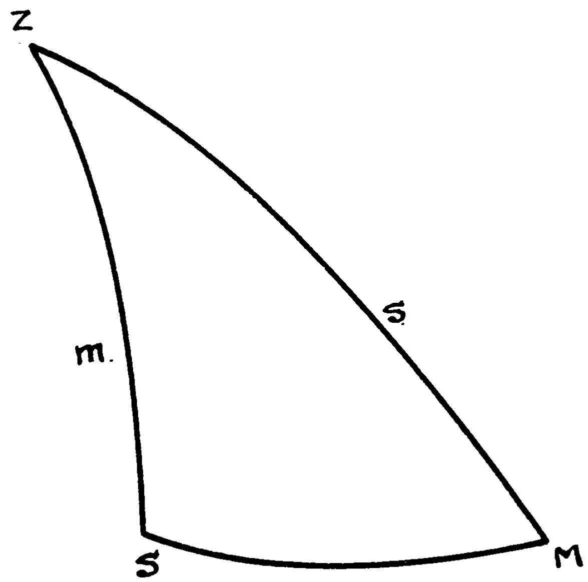
Altitude of Arcturus = 44° 24' 10"

Corr, for refraction = 0 58"

∴ Corrected altitude = 44 23 12

and corrected zenith distance = 45° 36' 48"

Now in the triangle ZSM




ZM = s = 50°-19'-19" (moon's corrected zenith distance)

ZS = m = 45°-36'-48 (star's corrected zenith distance)

and angle at Z = 38°-15'-31" (as already found).

From the relations,-

$$\tan \tfrac{1}{2}(S + M) = \frac{\cos \frac{1}{2}(s - m)}{\cos \frac{1}{2}(s + m)} \cot Z/2$$

$$\tan \tfrac{1}{2}(S - M) = \frac{\sin \frac{1}{2}(s - m)}{\sin \frac{1}{2}(s + m)} \cot Z/2$$

it follows that

$$S + M = 153^\circ\text{-}52'\text{-}02''$$

and $\underline{S - M = 17^\circ\text{-}30'\text{-}44''}$

so that $S = 85^\circ\text{-}41'\text{-}23''$

From the same triangle,-

$$\frac{\sin MS}{\sin Z} = \frac{\sin ZM}{\sin S}$$

from which sin MS = 9.6798243

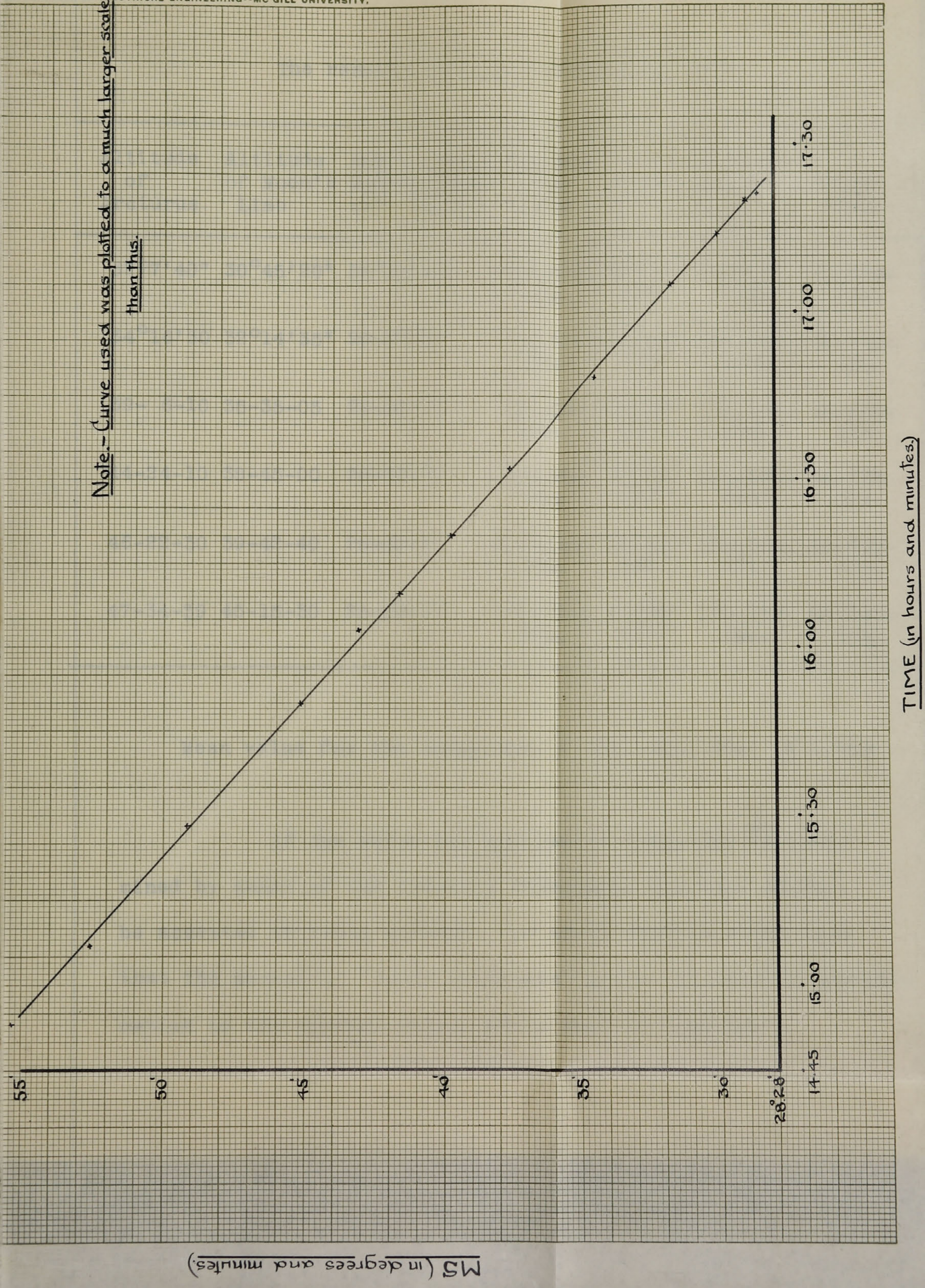
and MS = $28^{\circ}$-33'-00"

Having found MS it remains to interpolate amongst its known values at known times so as to find the Greenwich time when MS had a value of $28^{\circ}$-33'. Here again it was thought advisable to plot a curve showing MS (as calculated) on a "time" base. From the curve the Greenwich time when MS had this value was 16h. 57m. 30s.

So,

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | h. | m. | s. | |
| | MS = $28^{\circ}$-33' | at | | 16 | 57 | 30 | Greenwich time. |
| and also | MS = $28^{\circ}$-33' | at | | 12 | 03 | 00 | Montreal time. |

Hence longitude of Montreal is 4h. 54m. 30s. West.


Note.- Curve used was plotted to a much larger scale
than this.
TIME (in hours and minutes)
14·45
15·00
15·30
16·00
16·30
17·00
17·30
MS (in degrees and minutes)
55'
50'
45'
40'
35'
30'
28°28'

The results of "Lunar Distances" were,-

| Altitude of Arcturus | Altitude of Moon's Limb | Angle bet. Moon's Limb and Star | Montreal Mean Time | MS | Corres. Green-wich Time | Longi-tude |
|---|---|---|---|---|---|---|
| 32°07'40" | 30°45'20" | 28°38'17" | 10h.51m. 35s. | 28°45' 53" | 15h46m. 01s. | 4h.54m. 26s. |
| 34°16'30 | 32°14'35" | 28-37-22 | 11h.03m. 52s. | 28°43' 50" | 15h.58m. 11s. | 4h.54m. 19s. |
| 39- 6-10 | 35-35-05 | 28-35-19 | 11h.31m. 55s. | 28°38' 57" | 16h.26m. 06s. | 4h.54m. 11s. |
| 44-24-10 | 38-45-00 | 28-33-19 | 12h.03m. 00s. | 28°33' 00" | 16h.57m. 30s. | 4h.54m. 30s. |
| 46-22-30 | 39-42-40 | 28-32-34 | 12h.14m. 27s. | 28°30' 48" | 17h.08m. 52s. | 4h.54m. 25s. |
| 47-34-39 | 40-15-30 | 28-32-10 | 12h.21m. 36s. | 28°29' 21" | 17h.15m. 59s. | 4h.54m. 23s. |

Mean value for the longitude = 4h. 54m. 22s. ± 1.76s.

If the value for Montreal's longitude as determined by means of the electric telegraph, 4h. 54m. 18.7s., be accepted as being correct, the above results would show that the method of "lunar distances" gives the nearest approximation to the correct value, while that of "Jupiter's satellites" shows the greatest discrepancy. Perhaps the mean of only six or seven results is not sufficient to recommend or condemn a method, but it should at any rate convey some idea of the degree of accuracy one may expect from that method. Unfortunately, owing chiefly to adverse weather conditions, it was very difficult to obtain a great many more observations than those of which results have been given. Observations of the eclipses of Jupiter's satellites were not feasible until about the end of January, and after that time many of the eclipses took place at such instants that it was impossible to observe them.

The weighted mean value of the longitude obtained from observations of the moon is 4h. 54m. 17s., which represents an error of 1.7 secs. (of time) from the true longitude. In this latitude $1^\circ$ is equivalent to about 47 miles, so that 1.7 secs. of time means an error of approximately one-fifth of a mile in the distance between Greenwich and Montreal.

In addition to unfavourable weather conditions it must be pointed out that the season of the year was not by any means conducive to the obtaining of good results, even when the weather was fine. The work herein involved occupied a period commencing in November, when the T. & S. Astronomical Transit

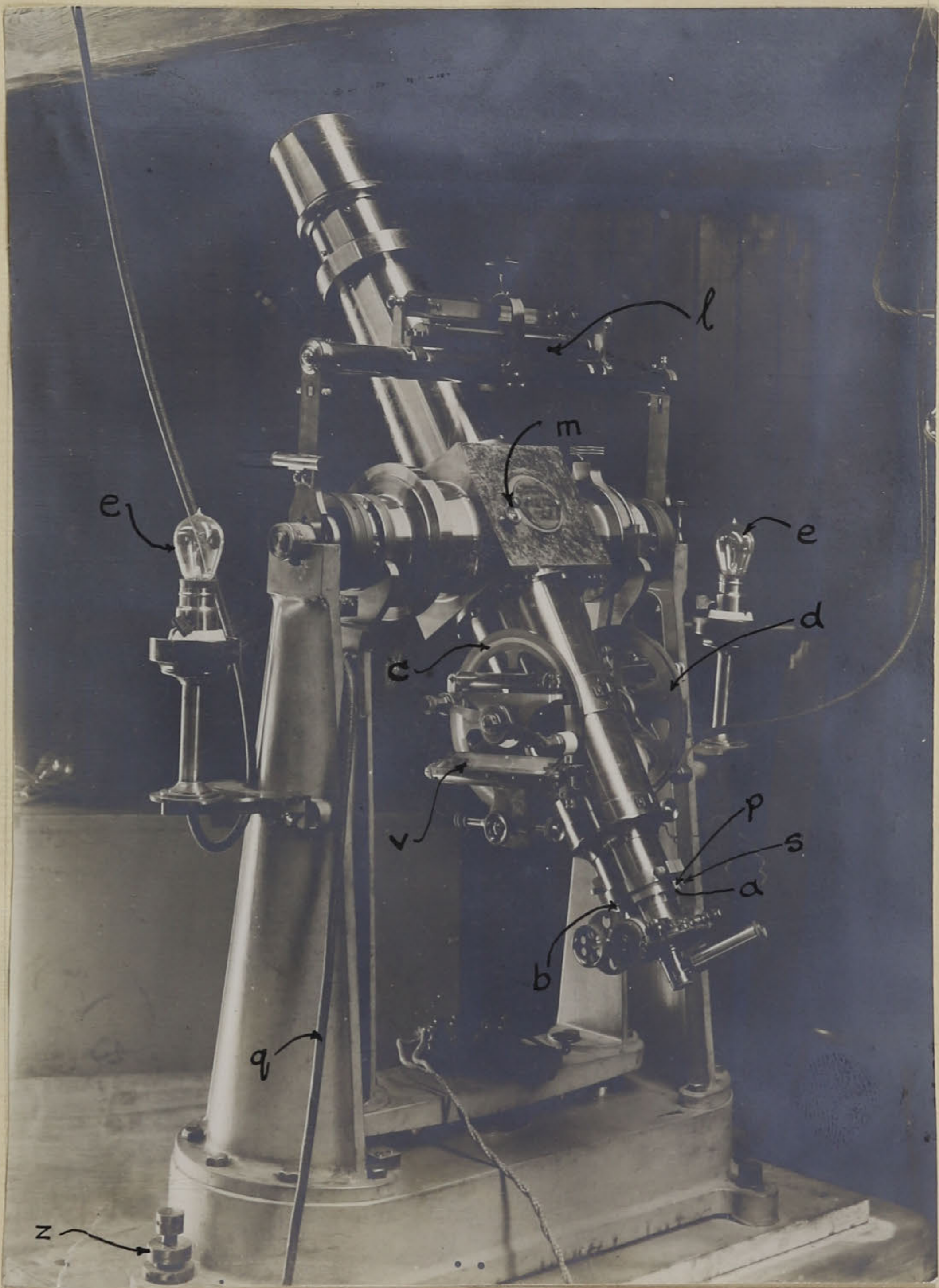



Fig. A.

arrived and was adjusted in the meridian, and ending in March, so that it included the entire winter months. And, moreover, it is to be noted that these methods of longitude determination are, even at their best, considered to be only approximate.

## Description of the Astronomical Transit.

The Astronomical Transit with which the results of Methods (2) and (3) have been found, as well as the comparisons in the personal equation of the second section of this thesis, was manufactured by Troughton and Simms, London. A photograph of the instrument is shown in Figure A. The telescope is 34 inches focal length and 3 inches aperture. It is provided with a diagonal eyepiece for observing transits of stars near the zenith, the magnifying power being about 42 diameters. In addition to this it is fitted with the Repsold recording micrometer, a description of which as well as a short discussion of the theory involved, will be given later. By unclamping a screw on the underside of the telescope opposite the one marked (s), Fig. A, the telescope can be revolved through a right angle, the stop (p) moving from its present position (a) to the position (b). In this second position the instrument can be used as a zenith telescope for the purpose of finding Latitude, and for this purpose an extra level vial (v) is provided, the value of one division of which is equal to 1.279 secs. The circle (c) is 6 inches diameter and reads to 10 secs. The corresponding circle (d), used simply for observing transits of stars, reads to 1 minute, and carries a 10 secs. bubble. The length of the horizontal axis is about 20 inches. The frame rests on three foot-screws by means of which it is levelled, the final adjustment in this direction being made with a fine screw (z) at the left end of the frame.

At the opposite end is a screw (which is hidden), by means of which the final adjustment in azimuth is made. The distance from left hand foot-screw to the azimuth screw is about 28 inches; this is the radius about which the end swings when an adjustment for azimuth is being made. The two electric lamps (e) at the opposite ends of the axis are for illuminating the field. The axis being perforated, the light enters it, falling on a small mirror at the intersection with the telescope, by which it is reflected down the tube to the eyepiece. This mirror may be adjusted by the screw (m). The error of horizontality of the axis is found by means of the striding level, (l).

The equatorial wire interval, i.e., the time taken by a star travelling on the equator to pass from one wire to the next, is 4.2 secs., and the time such a star takes to reach the centre of the comb after leaving the mean wire of a group is 52.6 secs.

## Determination of the Azimuth Error.

The instrument was moved in azimuth three times, but the result of just one of the determinations of the error will be given.

| $\delta$ | A | $\alpha$ | T | $\alpha - T = \Delta T + A\alpha$ | Combining | a |
|---|---|---|---|---|---|---|
| 10°23 | 0.584 | 19-41-31.54 | 20-13-30.8 | m. s. 31-59.2 = $\Delta T + 0.584a$ | 1.797a = 62.8 s. | 34.9 s. |
| 70°01 | -1.213 | 19-48-31.64 | 20-19-28.1 | m. s. 30-56.4 = $\Delta T + 1.213a$ | | |
| 62°11 | -0.615 | 21-16-25.92 | 21-47-33.7 | m. s. 31-07.7 = $\Delta T - 0.615a$ | 1.697a = 1.80 s. | 1.06 s. |
| -22°48 | 1.082 | 21-21-27.45 | 21-52-33.4 | m. s. 31-05.9 = $\Delta T + 1.082a$ | | |

1st. adjustment.

a = 34.9 secs. of time = 34.9 x 15 secs. of arc = 8'-44".

Distance between pivots(as stated above) = 28 inches.

Hence, lateral movement to put line of sight

in the plane of the meridian, $= 28 \times \tan 8'-44''$

$= .0728$ ins.

Since 1 revolution of azimuth adjusting screw = .033"

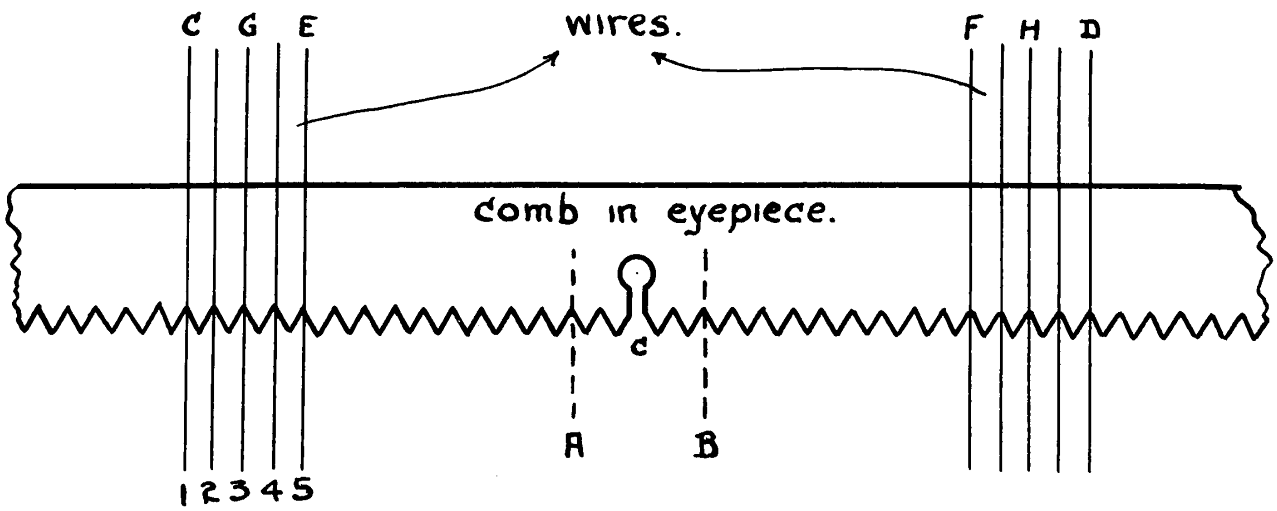
(30 threads to the inch)

it required, to give this lateral movement $\frac{.0728}{.033}$ or 2.2 revols.

<u>2nd. adjustment.</u> After giving what was supposed to be 2.2 revolutions and testing the adjustment, <u>a</u> was found to be 1.06 secs. of time. This error was so small that no further correction could be made.

## Determination of the Collimation Error.

The method adopted for finding <u>c</u> was by reversal on a slowly moving star.



As explained above, equatorial wire interval is 4.2 secs., so that an equatorial star would take 4.2 secs. to travel from wire 1 to wire 2, and 8.4 secs. to pass from 1 to 3. Also, since the equatorial "group interval" is 52.6 secs., the star would take 52.6 secs. to go from wire 3 to c, the centre of the comb.

Declination of star selected was $74^{\circ}$-28'.

| | h. m. s. |
|---|---|
| Time it was on mean wire in L.W. position was | 24-43-45.10 |
| " " " " " " " L.E. " " | 24-48-06.20 |

but since the instrument was reversed in the process it was the same wires that were traversed in each case.

sec. $\delta$ = 3.734, so that since 52.6 secs. is the equatorial group interval, the interval for a declination of $74^{\circ}$-28' is 52.6 x 3.734 secs., or 3m. 16s.

∴ time star should reach centre of comb for L.W. position was 24-43-45.10 (h. m. s.) plus 3-16 (m. s.) or 47-01.10 (m. s.)

also time it should reach the centre of comb for L.E. position was 24-48-06.20 (h. m. s.) minus 3-16 (m. s.) or 44-50.2 (m. s.)

The difference of these two times gives twice the collimation error (owing to reversal) for a declination of $74^{\circ}$-28',

∴ the collimation error, c, for an equatorial star =

$$\frac{\text{this difference}}{2 \times \sec.\delta} = \frac{2\text{m.}\,10.9\text{s.}}{2 \times 3.734} \text{ or } 17.6 \text{ secs.}$$

## Determination of the Level Error.

The value of 1 division of the level bubble tube was found to be 0.9033 secs. It was found by means of a level-trier.

Micrometer movement was 1 sec. for each setting.

Length of bubble, 26 divisions.

### Increased Readings.

| Bubble ends. | | Bubble Movement. | | Mean. |
|---|---|---|---|---|
| N | S | N | S | |
| 30 | 4 | | | |
| | | 2 | 2 | 2 |
| 32 | 6 | | | |
| | | 1 | 1 | 1 |
| 33 | 7 | | | |
| | | 2 | 2 | 2 |
| 35 | 9 | | | |
| | | 1 | 1 | 1 |
| 36 | 10 | | | |
| | | 1 | 1 | 1 |
| 37 | 11 | | | |
| | | 2 | 2 | 2 |
| 39 | 13 | | | |
| | | 1 | 1 | 1 |
| 40 | 14 | | | |
| | | 1 | 1 | 1 |
| 41 | 15 | | | |
| | | 1 | 1 | 1 |
| 42 | 16 | | | |
| | | 1 | 1 | 1 |
| 43 | 17 | | | |
| | | 1 | 1 | 1 |
| 44 | 19 | | | |
| | | 1 | 1 | 1 |

## Increased Readings (continued)

| Bubble ends. | | Bubble Movement. | | Mean. |
|---|---|---|---|---|
| N | S | N | S | |
| 45 | 20 | | | |
| | | 2 | 2 | 2 |
| 47 | 22 | | | |
| | | 1 | 1 | 1 |
| 48 | 23 | | | |
| | etc. | etc. | | |

The average movement of the bubble was 1.107 divisions.

Hence,

$$\text{Scale value} = \frac{\text{micrometer movement}}{\text{average bubble movement}} = \frac{1}{1.107} = 0.9033''$$

The mean value of the level readings of the horizontal axis were,-

| Lamp West. | E. | W. |
|---|---|---|
| | 12.75 | 50 |
| | 52.50 | 16.52 |
| Means | 32.63 | 33.26 |

$$b = \text{level error} = \frac{.62}{2} \times .903 \text{ secs. of arc} = .28''$$

<u>Lamp East.</u>

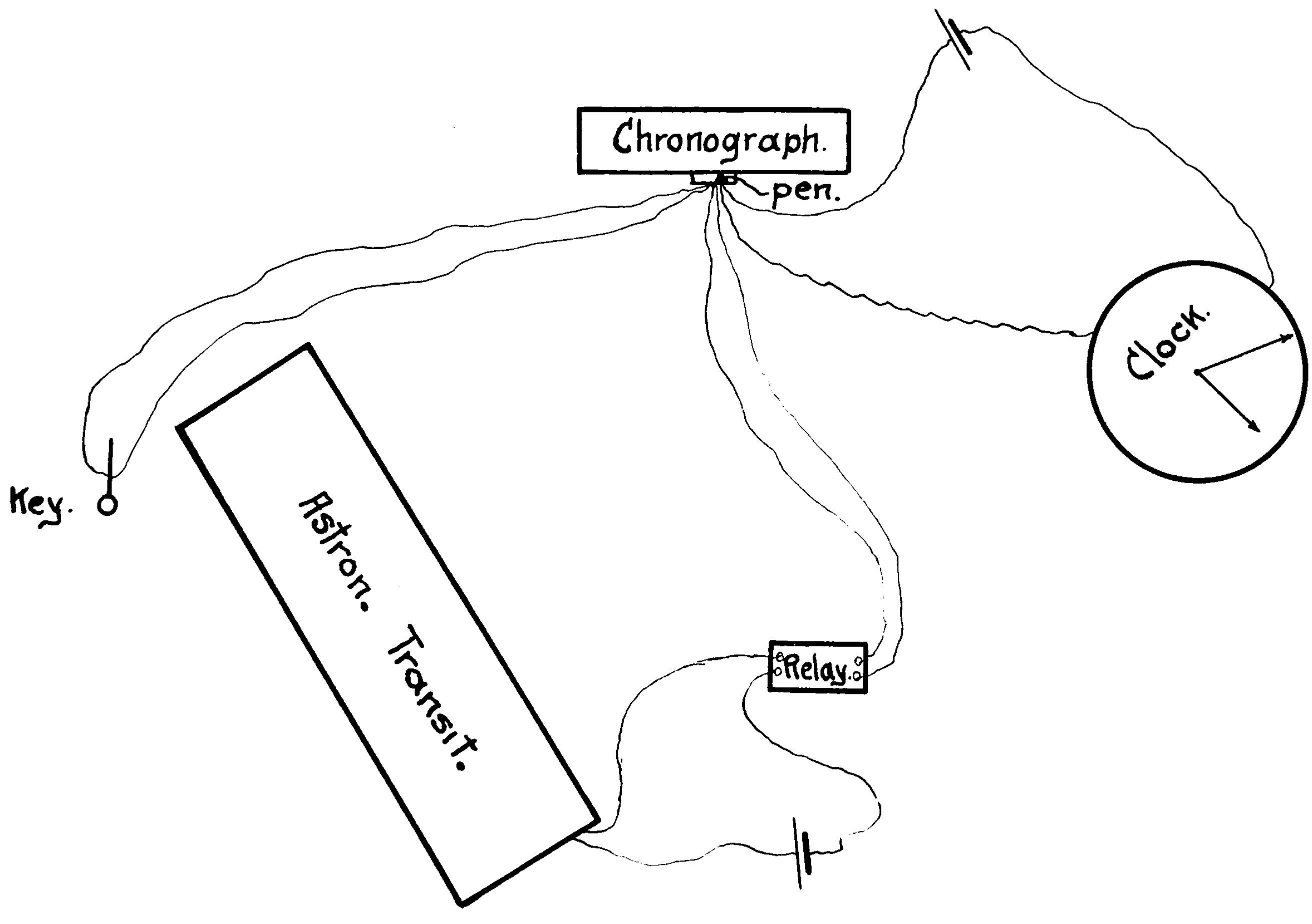
| | E. | W. |
|---|---|---|
| | 11.0 | 51.50 |
| | 49.0 | 9.75 |
| Means | 30.0 | 30.63 |

giving a value of .284" for <u>b</u>.

So, mean value of the level error = .282" of arc

or <u>.02 secs. of time.</u>

The lay-out of the circuit by means of which the taps of the key and the electric contacts were recorded was essentially thus:-



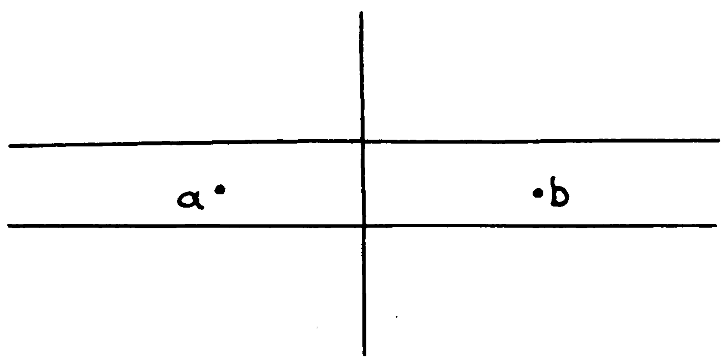
## SECTION II.

### Personal Equation.

When the results of transit observations made by different observers are compared, it is found that they differ generally by small but nearly constant quantities. One observer perhaps acquires a habit of noting the transit too early by a fraction of a second, while another will note it uniformly too late. This difference is called their "personal equation". It is customary to speak of the relative and the absolute personal equation, the former being the constant difference between the right ascensions, or clock corrections deduced from observations made by two different observers, and the latter the difference between the absolute value of the quantity and that obtained by an observer who notes the time uniformly too early or too late. When results obtained from observations of two different observers are compared, the personal equation must always be determined and the necessary correction applied.

Just as there is a personal equation in the results obtained by two observers, who have used the same method for their observation, so there is a personal equation in results obtained by the same observer, who has used different methods. The different methods employed in this case were the "eye and ear" method, the "chronographic" method and the "transit micrometer" method.

Eye and Ear Method.- A few seconds before the star reaches the wire the observer takes the time from the chronometer and watches the star as it approaches the wire, at the same time counting the beats of the chronometer. When the star crosses the wire the exact instant is noted: if the wire is crossed between two beats the fractional part of a second is estimated to the nearest tenth. This estimation is made more by the eye than the ear; thus, suppose when the observer counts 10 secs. the star is at a, and when 11 secs. at b; the distance from a to the wire will be compared with the distance from a to b, and the ratio will be expressed in tenths. In this case the time will be 10.4 secs.




This is done for every wire over which the star's transit is to be noted.

<u>Chronographic Method</u>.- By this method the observer registers the instant when the star is on the wire by simply pressing the key which breaks the electrical circuit. This instant is recorded by a mark on the cylinder of the chronograph, and may be read off at leisure. As the observer is not obliged to count the seconds, as in the"eye and ear" method, the wires may be placed much closer together and a larger number of readings taken.

The main advantages which the chronograph possesses over the methods employed before its introduction, are:-

<u>first</u>, a comparatively inexperienced observer can record astronomical phenomena by its use with a considerable degree of accuracy, and

<u>second</u>, the record is made by simply pressing a key with the finger: thus many more observations can be made in a given time than is possible when everything must be written down with a pencil.

<u>Transit Micrometer Method</u>.-

<u>Theory of the Transit Micrometer.</u>

The transit micrometer is a form of registering micrometer placed with its movable hair in the focal plane of an astronomical transit, and at right angles to the direction of the motion of the image of a star which is being observed as it crosses the meridian. Certain contact points on the micrometer head serve to break an electric contact as they pass a fixed contact spring, and they record upon a chronograph the instant at which the micrometer head, and, therefore, the moving micrometer hair, reached each of the positions corresponding to the contact point.

The movement necessary to make the micrometer hair follow a star image is given to the micrometer head by the hand of the observer.

The purpose of the transit micrometer is to furnish a means of determining time, that is, clock errors, which shall be sensibly free from any personal equation on the part of the observer.

The process of observing consists simply in bisecting the star image and in keeping it bisected continuously as it moves across the field. The record made upon the chronograph is similar to that which would appear there if a series of fixed lines were in the focal plane of the telescope, and the observer recorded the instant of transit of a star image across each line as seen by him by operating an electric key in the chronograph circuit. The computation is, with a possible exception in some minor details, made exactly as if the chronograph record had been produced by the use of fixed lines and a key.

The mental process of the observer is widely different in the two cases. In the first case he attempts to observe and record the particular instant of the transit of a star image across a certain fixed line. The result as recorded upon the chronograph sheet depends upon the rapidity of perception and action of the observer. In the other case, the time element does not enter directly into the mental process of the observer. He is not trying to note the particular instant at which any event occurs, but is intent simply upon keeping the star image bisected by a movable hair which is under his control. In general, he sees the star image at a given instant either slightly ahead of or slightly behind the moving hair, and determines to make the hair move more slowly or more rapidly, so as to improve the bisection. After an interval, which depends upon his rapidity of action, the bisection is perfected. He soon observes that the bisection is again imperfect and makes an attempt to improve it. This cycle of events, the noting that the bisection is imperfect, deciding to correct it, attempting to correct it, and again observing that it is imperfect, is repeated at a rate which is dependent upon the rapidity of perception and of action of the observer. His personal equation of the kind which affects the key method of observation is now effective in determining the amplitude and period of the oscillation of the moving hair forward and backward across the star image, but not in fixing the average error in the position of the moving hair. The latter is fixed mainly by a personal equation of the same form as if a series of bisections of a fixed star image were being made. The observer may be subject to a personal equation in estimating the position of the image which leads him to habitually place the line slightly to the right or to the left of the image. Such a personal equation in estimating the position of a stationary or slow-moving image has a much smaller effect upon the result of an observation of time with a transit micrometer than the personal equation in observing the instant of transit has upon the result of an observation of time with an electric key. This is the theory of the transit micrometer. The proof of the correctness of the theory lies in the results which have been secured with the transit micrometer by various observers.

Though it has been claimed that the accidental errors of observation have been reduced by the use of a transit micrometer in the place of a key, the principal claim, and in all cases the important one, is that it nearly, if not quite, eliminates from the results the effects of all personal equation on the part of the observer, and hence, also, the effects of variation of the personal equation, which would otherwise be present.

## Description of the Transit Micrometer.

Within the micrometer box and near to one side is mounted the micrometer screw. Upon the latter fits, by a thread and cylindrical bearing, a rectangular frame forming the slide. All play or lost motion, both of the slide upon the screw and the screw in its bearings, is taken up by means of a helical spring within the box, which, pressing from the inner end of the box against the slide and through it against the screw, holds the latter firmly against the point of an adjustable abutting screw, without impeding its free rotary motion. Upon the slide, at right angles to its line of motion, is mounted the single spider thread, which is used for bisecting the star during the passage across the field. A comb, the distances between whose teeth are equal to one turn

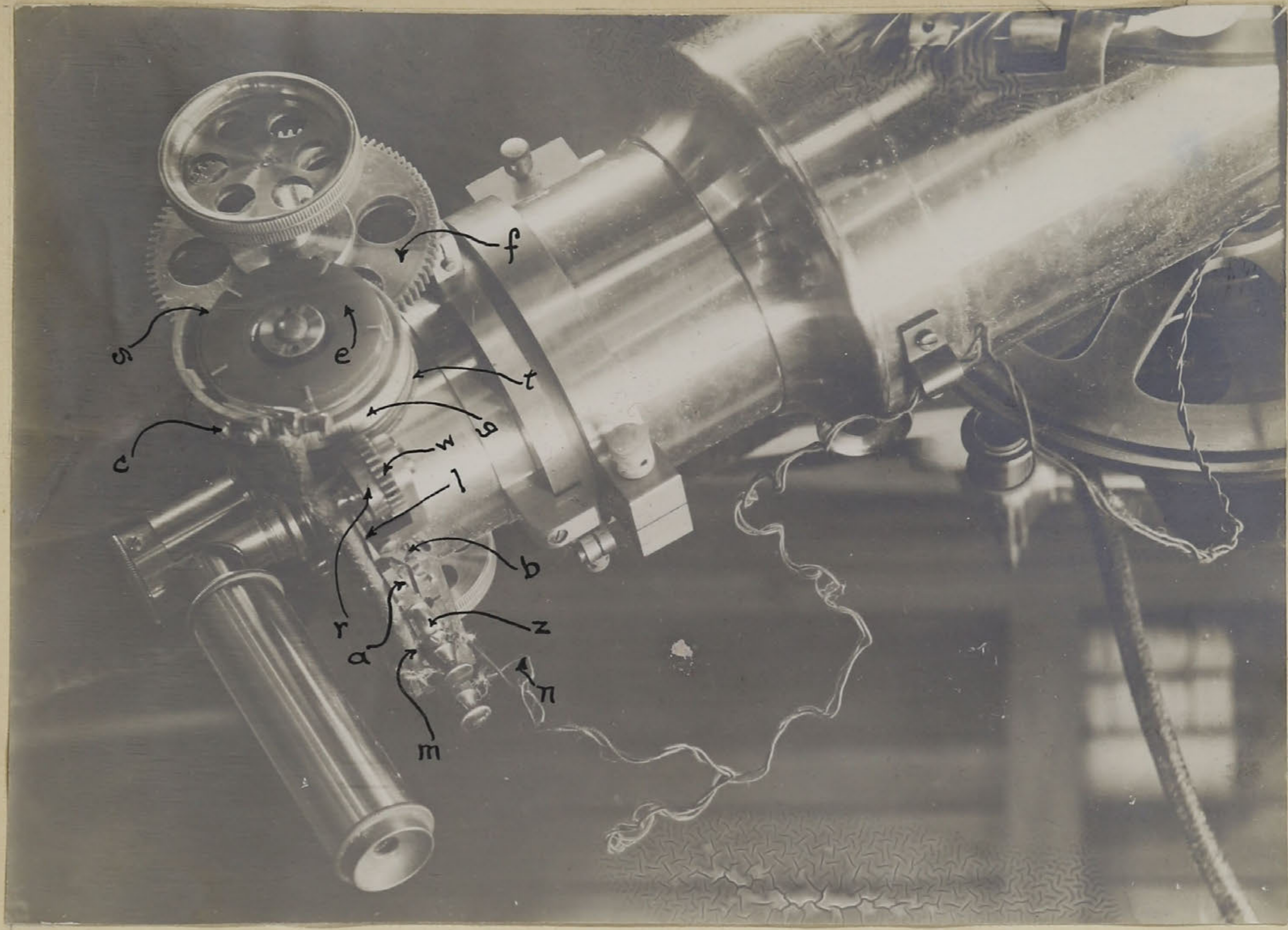



Fig. 1.

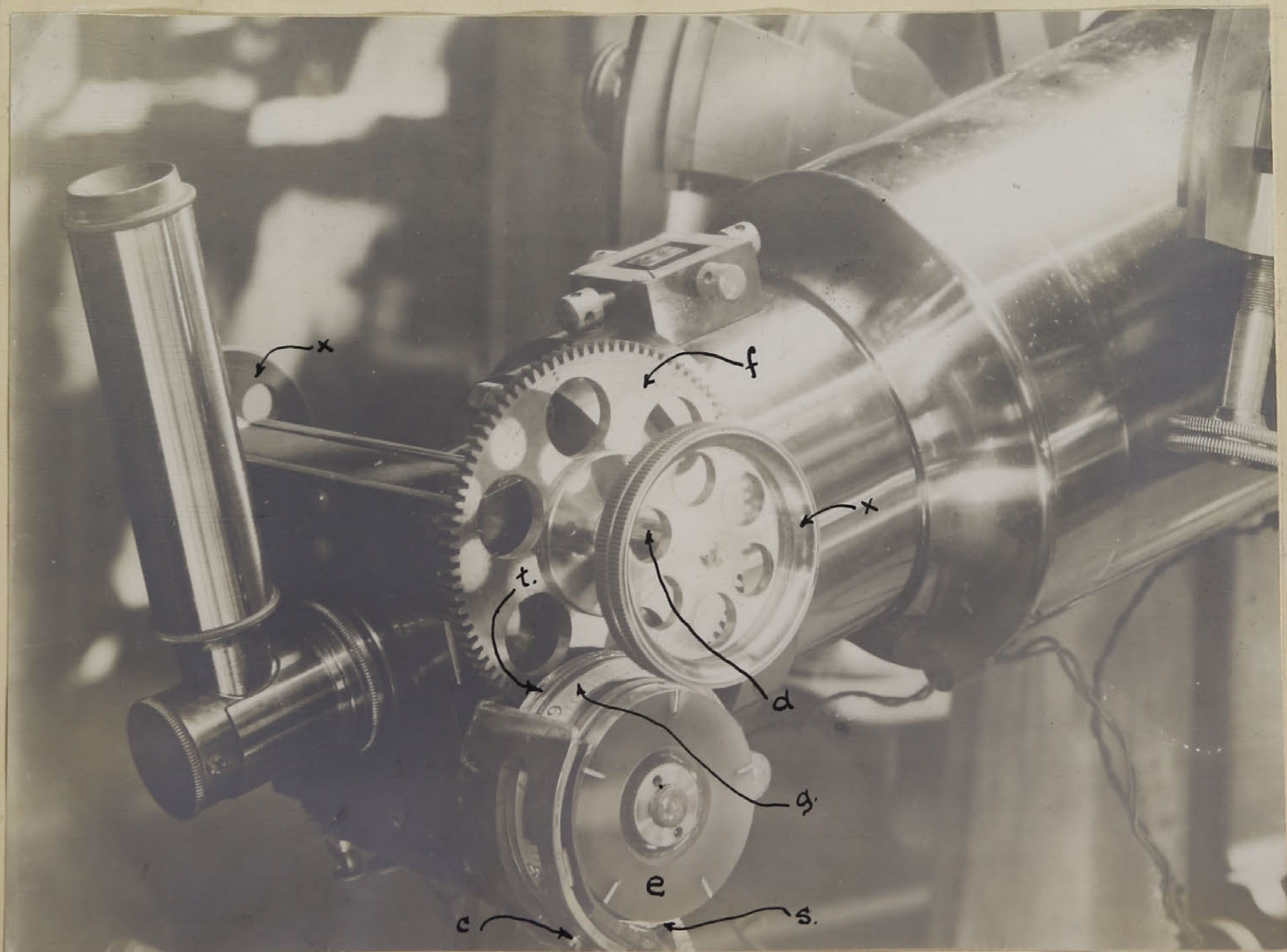



Fig. 2.

of the screw, is also provided and the five teeth at the centre of the comb indicate the whole turns of the screw within which the observations are to be made.

That portion of the micrometer screw, which projects through the box, has the micrometer head fitted upon it, and secured in position by a clamp nut. The edge of this head, which is graduated (g. Fig. 1), also carries at its opposite corner a screw thread (t. Fig. 1). Attached to the outer side of the head, and fitted concentrically with it, is an agate plate (e. Fig. 1). Five strips of platinum are slotted into the edge of the agate (see Figs. 1 and 2) and secured in such a manner as to make metallic contact with the micrometer head proper, and through it, with the screw, micrometer box, telescope and telescope pivots, and the iron uprights of the transit. By releasing the clamp nut within the agate plate, the graduated head (g) with its thread (t) can be adjusted, in a rotary sense, in relation to the thread of the screw, and, therefore, also of the spider thread upon the slide. At the same time the position of the platinum contact strips can be set to correspond to the zero of the graduation (g), which latter is read by the index, which is underneath the micrometer head in Fig. 1.

A small ebonite plate secured to the micrometer box carries upon its outer end the contact spring (s. Fig. 1). It ends in a piece of platinum which is turned over so as to rest radially upon the agate plate. A small screw (c), Fig. 1, serves to adjust the pressure of the spring upon the cylinder. Against one end of the micrometer box is fastened a small bracket, upon which is centred a small worm wheel (w), gearing into the screw thread (t) of the micrometer head. The rim of a cup-shaped cylinder (r), Fig. 1, which is secured to this worm wheel so that it can be turned and clamped in any position relative to the zero point of the micrometer head, has cut into it a notch with sloping ends and of a length corresponding to four teeth of the worm wheel (w) or four turns of the micrometer screw. From the end of a lever (l), Fig. 1, mounted against the side of the micrometer box, projects a small steel pin reaching over the rim of the cylinder (r). The other end of this lever carries a small tip which rests upon the end of a spring (b), Fig. 1, mounted on an ebonite plate and pressing at its middle point against a platinum-tipped screw (a). Whenever the small steel pin of the lever (l) rests in the notch of the cylinder (r), the spring (b) is in contact with (a) and allows the flow of an electric current through the wires (m and n) to the contact spring (s). But when the micrometer has been turned through two revolutions to either side of its zero position, and its motion is continued, the sloped ends of the notch in the cylinder (r) will engage the lever (l) and through it force the spring (b) away from the screw (a), thus breaking the circuit. It will be seen, therefore, that this arrangement permits of the motion of the spider thread across the entire field without transmitting records to the chronograph except during the four revolutions symmetrically placed about the line of collimation.

Against the inner side of the wheel (t) is fastened a spur wheel, into which the wheel (f), Figs. 1 and 2, gears, being mounted on the hand-wheel shaft (d), Fig. 2. The hand-wheels (x), Fig. 2, are equidistant from the centre of the telescope allowing lots of room for manipulating in any position of the eyepiece.

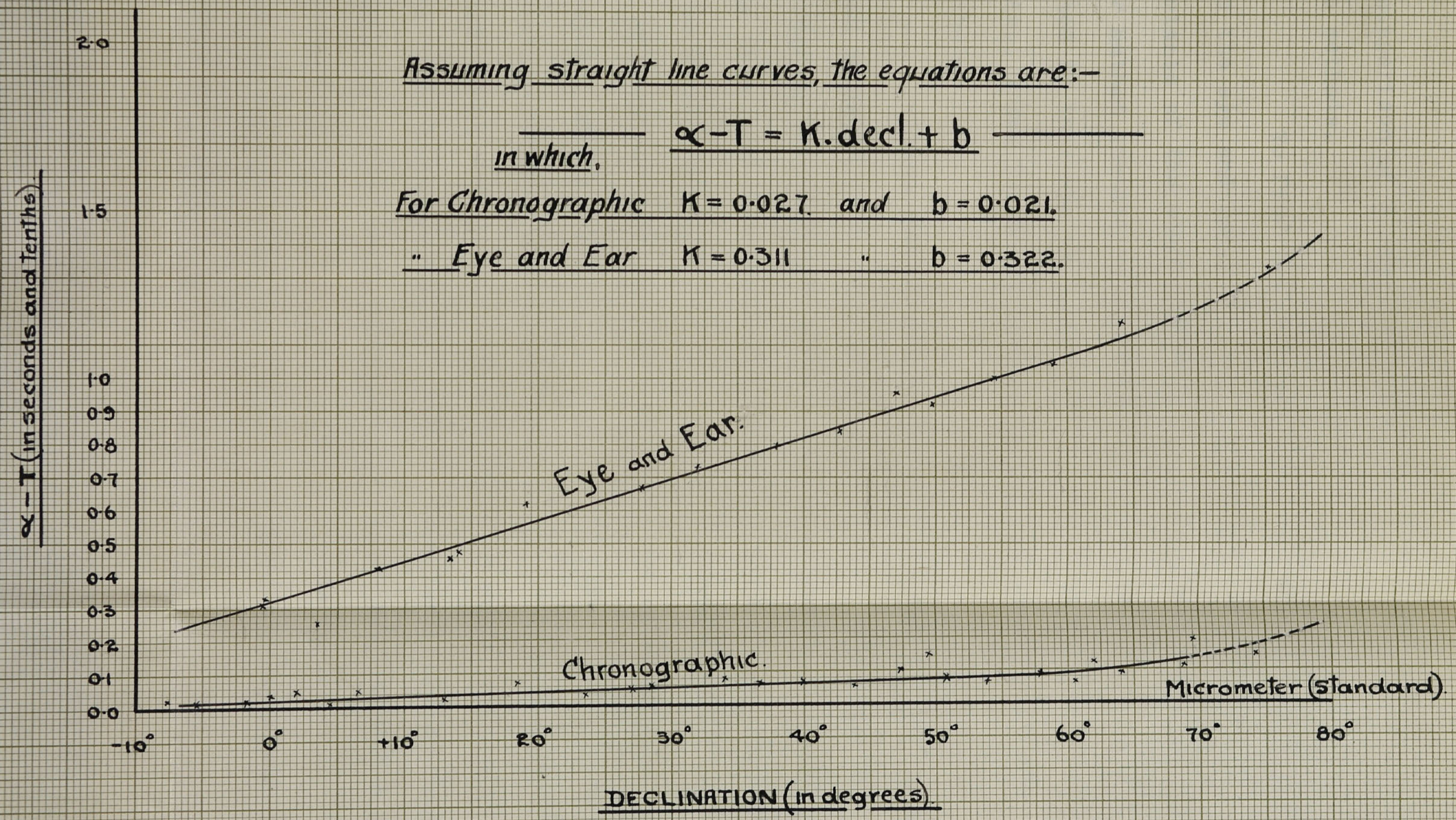
As indicated in the description of the agate head with its five platinum contact strips, the instrument itself is used as part of the electric conductor forming the relay circuit. Through the wire (q), Fig. A, the current is carried along the telescope to and into the telescope axis, within the latter to an insulated metal cylinder projecting from the transit pivot. Each of the wye bearings of the transit has fastened to it an insulated contact spring, which, being connected with an insulated binding post at the foot of the instrument, establishes the circuit whether the telescope lies in the position "Lamp west" or "Lamp east". In Fig. 1, (z) is a switch by means of which the circuit may be broken even though (b) and (a) were in contact. In its position as shown the circuit would be closed, so that if (a) and (b) were in contact records would be produced on the chronograph sheet.

In order to make a comparison of the personal equation in the results of the chronographic method with that of in the results of the micrometer method, it was necessary to observe the same star by the two methods and find the time of its transit of the meridian in each case. To effect this, groups of hairs were placed in the field on both sides of the comb's centre, each group at a distance of five teeth, i.e., an equatorial interval of 20 secs. of time, from the limits of the area over which the movable wire passed when contacts were being recorded on the chronograph sheet. So the field as viewed through the eyepiece appeared like this,-

This figure is shown on Page 62.

AB is the distance along the comb over which contacts when made were recorded. It can be seen that this immediately gave the time of transit of the star over the meridian, if the centre of the comb were in the meridian plane.

The groups of hairs were placed symmetrically about the comb's centre as shown. Hence, if the key of the circuit be pressed when the star was on the hair C and again when it was on D and the mean of these two instants be found, this mean instant will be the time when the star was on the centre of the comb. Similarly the means of the instants when the star was at E and F, and at G and H, will represent the instants when the star was on the comb's centre. So if all five hair passages be noted, the mean value will give the value of T for the star from the chronographic method. The value of T for the same star by the micrometer method was given by taking the mean of the contacts as recorded. Hence


Assuming straight line curves, the equations are:—
α−T = K.decl. + b
in which,
For Chronographic K = 0·027 and b = 0·021.
" Eye and Ear K = 0·311 " b = 0·322.
Eye and Ear.
Chronographic.
Micrometer (standard).
α − T (in seconds and tenths)
2·0
1·5
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0·0
−10°
0°
+10°
20°
30°
40°
50°
60°
70°
80°
DECLINATION (in degrees).

a comparison of T, and hence of $(\alpha - T)$, is obtained by the two methods.

To find the relation between the micrometer method and the eye and ear, it is only necessary to observe the instants of the star's passage of the hairs by the eye and ear method instead of by tapping the key as before. Having noted in this way the instants at which the star crossed the hairs, T was found and hence $(\alpha - T)$ for the eye and ear method.

Thus the chronographic and the eye and ear methods were compared with the micrometer method. Adopting the latter as the standard the accompanying curves were obtained. The points on the curves were found in this way. Suppose A to be the value of $(\alpha - T)$ from the micrometer for a star in declination $30^\circ$; B its value from the chronographic method; and C from the eye and ear method. Then at the $30^\circ$ point on the abscissa an ordinate of $(\alpha - T)$ was erected whose value was (B - A) and the point so found was a point on the "chronographic method" curve. Similarly an ordinate of value (C - A) erected from the same point represented a point on the "eye and ear" curve. Points found in this way for the various declinations and joined with a smooth line gave the curves as shown.

The principal conclusions from the work in this section are,-

(1) That the personal equation with a transit micrometer is less than with a key, which in turn is less than with the eye and ear method.

(2) Good observations can be secured at once with the transit micrometer without previous practice. There are so many contacts made that where in one case a contact is recorded too soon it is in another recorded too late; this is repeated for twenty contacts, so that the mean is very near the correct value.

(3) That the observations may be made much more rapidly with the micrometer than with a key.

(4) That on account of the perfect action of the cut-out switch, which insures that none but the required records shall appear on the chronograph sheet, the identification of the record on the sheets is no more difficult than for a key record.

## Errors.

It does not seem fitting that such a thesis as this should be brought to a close without some remarks on the various errors which affect the results it contains. Such errors may be grouped into three classes:-

(1) <u>external errors</u>, or errors arising from conditions outside the instrument and observer;

(2) <u>instrumental errors</u>, or errors due to the instrument, arising from lack of perfect adjustment, from imperfect construction, from instability of the relative positions of different parts, etc.;

(3) <u>observer's errors</u>, or errors due directly to the inaccuracies of the observer, arising from his unavoidable errors in judgment as to what he sees and hears, and from the fact that his nerves and brain do not act simultaneously.

Errors arising from all these sources may be termed "errors of observation".

<u>External errors</u>.- The two principal external errors in connection with astronomical transit work are the error in the assumed right ascension of the star and the lateral refraction of the light from the star.

If only such stars as are given in the various "star lists" are observed for time, the probable errors in the right ascensions will usually be on an average ± 0.04 sec.

or $\pm$ 0.05 sec. and no appreciable constant errors need be feared from this source.

It can be seen that the effect of lateral refraction upon transit time observations must be quite small in comparison with other errors, but the value of the error caused by it is very difficult to estimate.

Then with regard to sextant observations, the accuracy of a determination of a star's or the moon's altitude depends largely upon the time of the night at which the observations are made. Near culmination the altitude is changing quite slowly, whereas a few hours later or earlier the change of altitude is comparatively rapid. Evidently the effect of a given error in the measured altitude upon a computed time will be less the greater the rate of change of the altitude.

Instrumental errors.- Among these errors may be mentioned those arising from change in azimuth, collimation, and inclination, from non-verticality of the lines of the reticle, from poor focusing and poor centering of the eyepiece, from irregularity of pivots, and from variations in the clock rate.

The errors of azimuth and collimation being determined from the observations themselves are quite thoroughly cancelled out from the final result, provided they remain constant during the period over which the observations extend, and provided also that the stars observed are so distributed in declination as to furnish a good determination of these constants. Their changes, however, during that interval, arising from changes of temperature, shocks to the instrument or other causes produce errors in the final result. In general such errors are small but not inappreciable.

The changes in inclination during each half-set evidently produce errors directly. But the mean value of the inclination is determined from readings of the striding level, not from the time observations, and the level may give an erroneous determination of the mean inclination. "However "small this error may be under the best conditions and most "skilful manipulation, there can be no doubt that careless "handling and slow reading of the striding level or a little "heedlessness about bringing a warm reading lamp too near to "it, may easily make this error one of the largest affecting "the result." (Hayford).

An error of 0.0002 inch in the determination of the difference of elevation of the two pivots of the astronomical transit, which is in use in the U.S. Coast and Geodetic Survey for accurate time determinations, produces an error of 0.1 sec. or more in the deduced time of transit of a zenith star.

If the lines of the reticle are not carefully adjusted so as to define vertical planes, stars will be observed too early or too late if observed above or below the middle of the reticle. Errors arising from this source may be eliminated by always observing within the space given by the two horizontal hairs of the reticle.

Poor focusing of either the object glass or the eyepiece leads to increased accidental errors because of poor definition of the star's image. But poor focusing of the object glass is especially objectionable because it puts the reticle and the star's image in different planes, and so produces parallax.

If the inequality of the two pivots has been carefully determined the errors arising from defects in their shapes may ordinarily depend be depended upon to be negligible.

Changes in the rate of the clock during a set of observations evidently produce errors in the deduced clock correction at the mean epoch of the set. Under ordinary conditions such errors are exceedingly small. They will be less the more rapidly the observations are made.

All of the above refer chiefly to the transit, but like it, the sextant is subject to giving erroneous results. Any inclination of the index-glass or the horizon-glass to the sextant plane introduces errors, and the smaller the angle measured the greater is the error. Again, if the centre about which the graduated arc is described does not coincide with the centre about which the index-arm swings, an error due to this eccentricity will be introduced into every reading.

Observer's errors.- These are by far the most serious in transit time observations. Results are subject to both accidental and constant errors in the observer's estimate of the time of transit as well as in his estimate of the position of contact in the case of the sextant observations. "The accidental error depends mainly upon the personality and experience of the observer and the care with which he observes, but also to a certain extent upon the steadiness of the refraction, the power of the telescope, the brightness and definition of the images, and the physical conditions affecting the observer's comfort." (Hayford.)

"In addition still to these errors there is another "which is constant for all the observations of a set. Every "experienced observer, though doing his best to record the "time of transit accurately, in reality forms a fixed habit "of observing too late or too early, by a constant interval. "This interval between the time when the star image actually "transits across a line of the reticle and the recorded time "of transit is called the "absolute personal equation" of the "observer." . . . . . . (Hayford.)

To sum up, it may be stated that the accidental errors in the determination of a clock correction from observations with an astronomical transit may be reduced within the limits indicated by the probable error ± 0.02 sec. to ± 0.10 sec., but that the result is subject to a large constant error, the observer's absolute personal equation, which may be ten times as great as this probable error.