CORRECTIONS TO PARTON MODEL FORMULAE

IN QUANTUM CHROMODYNAMICS

by

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ABSTRACT

In terms of the $O(\alpha_s^2)$ correction to the longitudinal structure function of deep inelastic scattering due to the subprocess $\gamma^* q \rightarrow qq\bar{q}$, a complete definition of the gluon density in a quark is introduced. This density is used to calculate the $O(\alpha_s^2)$ corrections due to $qq \rightarrow qq\gamma^*$ to the following quantities: (a) the inclusive cross section for dilepton production (Drell-Yan process) and, (b) the transverse momentum distribution of dileptons in proton-proton collisions. At presently available energies and dilepton masses, the correction (a) is found to be small, thus leaving unspoiled the successful Drell-Yan description of dilepton production, but the correction (b) is found to be significant. Both corrections (a) and (b) become more important as one approaches the kinematic boundaries. Other definitions of the gluon density are also discussed and compared.

RESUME

Une définition complète de la densité de gluon dans un quark est introduite en se basant sur la correction d'ordre α_s^2 apportée a la fonction de structure longitudinale par le sous-processus $\gamma^* q \to qq\bar{q}$, en diffusion fortement inélastique. Cette densité est utilisée pour évaluer dans les collisions proton-proton, l'effet du sous-processus $qq \to qq\gamma^*$ qui génère des corrections d'ordre α_s^2 aux quantités suivantes: (a) Section efficace pour la production inclusive de dileptons (processus de Drell-Yan) et, (b) distribution de l'impulsion transverse des dileptons. La correction (a) s'avère petite lorsque évaluée aux énergies et pour les masses de dileptons présentement disponibles; le succès de la description de la production de dileptons offerte par Drell-Yan reste donc intact. La correction (b), pour sa part, s'avère importante. Par contre, toutes deux augmentent lorsqu'on tend vers la limite cinématique. Différentes définitions de la densité de gluon dans un quark sont aussi considérées et comparées.

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CHAPTER I

INTRODUCTION

1-1 Status of Quantum Chromodynamics

In the last decade, there has been a growing conviction between physicists that the basis of an approach which will eventually become a complete theory of the strong interactions has been found. The beginning of this theory came in 1963, when research to find the underlying hadronic structure led physicists to postulate quarks as the fundamental constituents of the hadronic matter [1]. About ten years later, during which the concepts of gluon and color were introduced, the effort to understand strong interactions resulted in the development of quantum chromodynamics (QCD) as the suitable quantum field theory for the description of these interactions. Clearly, in advancing QCD as the theory of strong interactions, the hope was to repeat the striking success of another field theory, quantum electrodynamics (QED), that describes the interactions between electrons and photons.

The main idea of QCD has been to make the $SU_c(3)$ color symmetry a local, rather than just a global symmetry. To implement this local $SU_c(3)$ symmetry, one must introduce vector gauge fields, A_μ^a ($a = 1, 2, \ldots, 8$), called gluons together with the quark fermion fields, q_α^A (α = color index = 1, 2, 3 and A = flavour index = $1, 2, \ldots, N_f$). This leads to the

Lagrangian density

$$L_{QCD}(x) = L_1(x) + \nabla L(x) + L_{g.f.}(x) + L_g(x) \qquad (1.1)$$

where $L_1(x)$ is the minimal locally gauge invariant Lagrangian density implied by $SU_c(3)$ symmetry:

$$L_1(x) = i\, \bar{q}^A_\alpha(x)\, \not{D}_{\alpha\beta}\, q^A_\beta(x) - \frac{1}{4} F^a{}_{\mu\nu}(x)\, F^{a\mu\nu}(x) \qquad (1.2)$$

with

$$\not{D}_{\alpha\beta}\, q^A_\beta(x) \equiv (\delta_{\alpha\beta}\, \partial_\mu - i\, \frac{g}{2}\, \lambda^a{}_{\alpha\beta}\, A^a_\mu\, (x))\, \gamma^\mu\, q^A_\beta(x) \qquad (1.3)$$

$$F^a{}_{\mu\nu}(x) \equiv \partial_\mu\, A^a_\nu(x) - \partial_\nu\, A^a_\mu(x) + g\, f^{abc}\, A^b_\mu(x)\, A^c_\nu(x) \qquad (1.4)$$

g is the coupling constant ($g^2/4\pi = \alpha_s$) and f^{abc} are the structure constants of $SU_c(3)$. To this Lagrangian density one adds a quark mass term, ∇L, a gauge fixing term, $L_{g.f.}$, which is required to insure a proper quantization procedure, and a Faddeev-Popov ghost term, L_g, to preserve unitarity.

One of the main features of this QCD Lagrangian density is that, as the second term of L_1 shows, there is self-interaction between the non-abelian gauge fields, $A^a_\mu(x)$. Unlike QED, where photons can interact only via electron loops, the QCD Lagrangian density leads to gluons interacting directly with themselves in the form of three- and four-gluon couplings. Although this seems to bring additional complications, it turns out to be of great importance because it is directly responsible for the most unique property of asymptotic freedom.

It is well-known that in QED, as we approach the long distance regime, vacuum polarization introduces a shielding effect on the charge. In other words, the effective charge becomes larger at short distances (or equivalently, for large momentum transfers). The situation is totally different in QCD. The existence of self-interactions between the gluons produces an antishielding effect, and as we approach shorter and shorter distances the effective strong charge decreases. This phenomenon is called asymptotic freedom. The renormalization group equations [2] together with the operator product expansion [3] (OPE) techniques lead to a running coupling constant, α_s, behaving as:

$$\alpha_s(Q^2) \equiv \frac{\bar{g}^2(Q^2)}{4\pi} = \frac{12\pi}{(33 - 2N_f)\ln Q^2/\Lambda^2} \tag{1.5}$$

where N_f is the number of quark flavours, Q^2 is the large momentum transfer squared in the process, and Λ is a free parameter that has to be fixed by experiment (0.2 GeV $\lesssim \Lambda \lesssim$ 0.7 GeV). What makes QCD so attractive is that, it is the only known renormalizable gauge theory that possesses the property of asymptotic freedom.

So far, QCD has not been found in clear conflict with any existing phenomenology of the strong interactions. On the other hand, no one has proven within QCD the existence of a single bound state (hadron). There may be difficulties even in formulating the theory in precise terms, for the bound state physics. As a result, since the physical particles, the hadrons, are bound states, many predictions on physical processes must rely on models or approximations (such as the parton model or the impulse approximation).

Among the computational methods used in QCD, one of the most successful is the Wilson's OPE[3]. For instance, in the case of deep inelastic lepton-hadron scattering, one has been able to determine the scaling behaviour of the structure functions, the basic quantities involved in this process. This analysis, based on the OPE of currents for free field theory on the light cone, has led to the result that deep inelastic scattering in the Bjorken limit is dominated by leading light-cone singularities. This in return, leads to the scaling law and well-known sum rules. A more extensive analysis on the basis of the OPE has also successfully described scaling violations (Q^2-dependence) of leptoproduction structure functions.

Unfortunately, the applicability of the OPE is limited to rather few processes. For instance, the same kind of light-cone behaviour has also been studied for processes such as inclusive dilepton production in hadronic collisions ($h_1 + h_2 \to \ell^+\ell^- + X$); here, however, the results are inconclusive, because in the scaling limit cross sections are not dominated by light-cone singularities.

Perhaps the most important consequence of asymptotic freedom is that it renders possible the perturbative approach around α_s at short distances. Corrections to the free field behaviour can be computed perturbatively and the predictions of QCD can be tested for processes like dilepton production (Drell-Yan process) as well as for deep inelastic scattering (DIS), provided that we are dealing with short distance behaviour. Of course, when large distances ($\gtrsim$ hadronic radius) are involved, QCD becomes a strong coupling theory, and this perturbative approach fails to give a clear description or requires phenomenological parameters.

Perturbative QCD has already many applications to the lowest $O(\alpha_s)$ [4-16]. Such quantities, as the dilepton mass distribution, $d\sigma/dM^2$, and the transverse momentum distribution, $d\sigma/dM^2 d^2 q_T$ in dilepton production ($h_1 + h_2 \to \ell^+ \ell^- + X$) and quantities like the inclusive cross section $E d\sigma/d^3p$ for large transverse momentum pion or in real photon production ($h_1 + h_2 \to \pi + X$ or $h_1 + h_2 \to \gamma + X$), have been studied; and the results can be said to be in quantitative or semiquantitative agreement with data. In several cases, in which perturbative QCD is not in clear agreement, it does at least provide a qualitative description of the experimental data.

1-2 Purpose of this work

On the other hand, at present energies and momentum transfers, the QCD running coupling constant is not very small ($\alpha_s \simeq 0.3$ is a typical value). The study of higher order corrections becomes then of great importance. In cases of disagreement (or partial agreement) with experiment, calculation of higher order processes is, in general, of obvious necessity. But also in cases of complete agreement, theoritical consistency demands a proof that higher order effects are unimportant. Clearly, calculation of QCD corrections is an essential part of the theoretical effort towards an understanding of the physics of hadrons.

Inclusive production of dileptons in hadron collisions offers a particular example of corrections. Here, the lowest order contribution (the Born term) is determined by the Drell-Yan mechanism [17,18]

$$q + \bar{q} \rightarrow \gamma^* \tag{1.6}$$

(q=quark, $\bar{q}$=antiquark and γ^*=virtual photon $\rightarrow \ell^+\ell^-$) and the QCD corrections of first order in α_s come from the subprocesses:

$$g + q \rightarrow \gamma^* + q \tag{1.7}$$

$$q + \bar{q} \rightarrow \gamma^* + g \tag{1.8}$$

(g=gluon).

One of the problems arising from the perturbation calculation of the subprocesses (1.7) and (1.8) is the presence of mass singularities in the subprocess cross sections. These singularities are known to factorize, and can be absorbed by a process-independent redefinition of the initial parton (by parton, we mean quark or gluon) distribution functions. Then the remaining contribution to $O(\alpha_s)$ constitutes the correction term.

There is, however, some ambiguity in the process of redefinition. The absorption of the mass singularity proceeds through the introduction of a density, $G_{i/j}$, of a parton i in a parton j. This density is a sum of a leading logarithmic term arising from perturbation calculation plus non-leading terms. Although the leading logarithmic contribution is well-defined, the non-leading terms are to a great extent arbitrary, and this affects the magnitude of the correction term.

Perhaps the most attractive definition for the non-leading terms proceeds through comparison with DIS. Then the correction to any physical process due to a given QCD subprocess is determined by comparing to the contribution of the corresponding subprocess in DIS. This procedure has the important advantage that the resulting correction is regularization-prescription and gauge independent. Furthermore, this definition implies that DIS is free of corrections (i.e. all corrections are to be absorbed in the redefined parton distribution functions) and much of the information on parton distribution functions is known to come from direct measurements of DIS structure functions.

To be more specific, in terms of the example of the processes (1.6-1.8), the Born term contribution to DIS is:

$$\gamma^* + q \rightarrow q \tag{1.9}$$

and the $O(\alpha_s)$ contributions come from the subprocesses:

$$\gamma^* + g \rightarrow q + \bar{q} \tag{1.10}$$

$$\gamma^* + q \rightarrow g + q \tag{1.11}$$

Then by requiring the DIS structure function F_2 to be free of $O(\alpha_s)$ corrections, we completely fix the $O(\alpha_s)$ corrections to dilepton production arising from the subprocesses (1.7) and (1.8).

There is, now, a growing interest in the $O(\alpha_s^2)$ QCD subprocesses. In particular, we may expect the subprocess

$$q + q \rightarrow \gamma^* + q + q \tag{1.12}$$

to generate a large correction to dilepton production in proton-proton collisions, because of the presence of valence quarks in the initial state. This subprocess introduces two new parton densities, $G_{g/q}$ and $G_{\bar{q}/q}$. Unfortunately, the requirement that F_2 be free of $O(\alpha_s^2)$ corrections, defines only the parton density $G_{\bar{q}/q}$. The density $G_{g/q}$ of a gluon in a quark, then, still remains a problem. This is because, unlike quarks (and antiquarks), gluons do not couple directly with electromagnetic and weak current probes.

Nevertheless, there is a number of important physical quantities of which the QCD Born term involves gluons. In DIS, such a quantity is known to be the longitudinal structure function F_L; then a Born term contribution is provided by the subprocess (1.10). Furthermore, a correction to F_L would come from the $O(\alpha_s^2)$ $\gamma^* q \to q q \bar{q}$ subprocess. One may then obtain a complete definition of $G_{g/q}$ by requiring this correction to be zero. This is one of the original ideas proposed in this work.

The so-determined gluon density can be subsequently employed to calculate the correction to the Drell-Yan mechanism in $p + p \to \ell^+ \ell^- + X$, due to subprocess (1.12). This procedure also fixes the correction to the transverse momentum distribution of dileptons due to the same subprocess. Both corrections will be regularization-prescription and gauge independent, provided that all the calculations are carried in the same regularization scheme and in the same gauge.

The purpose of this work is to carry this program in all detail. In Chapter II, we describe the basic formalism that we shall use and we examine the lower order contributions to leptoproduction and dilepton production(6,7,8,10,11,17,18). We then proceed in Chapter III to the determination of the gluon density, from DIS. Chapter IV is devoted to the calculation of the $O(\alpha_s^2)$ correction, due to the subprocess (1.12), to the inclusive cross section $d\sigma/dM^2$ for dilepton production; and Chapter V, to the same correction to the transverse momentum distribution $d\sigma/dM^2d^2q_T$. Finally, in Appendix A, some of the matrix elements needed in this work are computed, and in Appendix B, we describe another possible convention for the gluon density $G_{g/q}$.

CHAPTER II

BASIC FORMALISM

This chapter is devoted to the main features of perturbative QCD used in the description of leptoproduction and dilepton production. We review some qualitative properties of the parton model and define the basic quantities that will be used later on to extend the calculations to a higher order.

Throughout this work, we adopt the notation of reference 19 and the natural unit system ($\hbar=c=1$); then, the momenta are written

$$p^{\mu}=(E;\vec{p})=(p^0;p^1,p^2,p^3). \tag{2.1}$$

For simplicity, we assume that the partons are massless. The regularization procedure will consist in taking the initial partons slightly off mass shell, with $p^2<0$.

2-1 Perturbative QCD

It is a well known fact that perturbative QCD is not appropriate to describe the confinement of quarks or, in general, any bound states. So, at first sight, it might appear hopeless to try to explain hadron collisions in terms of perturbation approximation series. However, in QCD, the strong coupling constant, α_s, varies with Q^2, the momentum transfer involved, and for large values of Q^2, α_s becomes small (provided that the number of quark flavors is smaller than 17). This property was called asymptotic freedom(Ch.I). It amounts to saying that quarks and gluons are virtually free, or, at least, weakly interacting when large momentum transfers, or equivalently small distances, are involved.

This property and the parton nature of quarks has led to the so-called impulse approximation. The impulse approximation assumes that if you probe hadrons with a sfficiently large momentum transfer, you can "freeze" them on a time scale much shorter than that characteristic of their strong interactions. According to that, collisions involving hadrons at high energy can be divided in two time scale: A short-time scale, of the order of the inverse of the large momentum in the process, that characterizes the hard collisions of the constituents, and a long-time scale, of the order of the hadron radius, that characterizes the binding and recombination of the constituents. The short-time scale physics depends on the involved parton subprocesses, but is calculable via perturbation methods. The long-time scale physics rules the bound states and is independent of the subprocess; it is there that perturbative QCD fails to give a clear description.

In order to describe inclusive reations such as dilepton production and leptoproduction, this picture requires that the hadronic cross section be composed of a parton cross section for the "observed" individual partons as well as a description of the parton structure of the hadrons. Such a cross section would take the form [20]

$$d\sigma^{hadron}(P_1,P_2,\ldots P_J)= \sum_{\substack{\text{parton types}\\ \text{and helicities}}} \int \prod_{j=1}^{J} dx_j\ G_j(x_j)$$

$$\times\ d\sigma^{parton}(p_1,p_2,\ldots p_J) \tag{2.2}$$

where P_j (p_j) are the observed hadron (parton) momenta, and, x_j is the fraction of the incoming hadron momentum carried by the corresponding incoming parton,

$$p_j = x_j\ P_j \tag{2.3}$$

with $0 \leq x_j \leq 1$, and $j=1,2,\ldots J$. The product runs over a number J of initial partons denoted by j. The functions $G_j(x_j)$ contain all the dependence on the incoming hadrons in the form of distribution functions, whereas, the parton cross section, $d\sigma^{parton}$, depends only on the subprocess considered. It can be computed as the perturbation series of Feynman diagrams.

However, a problem arises from these calculations. The parton cross section has infrared and mass singularities, and this renders the use of perturbation theory inadequate.

To solve this problem of singularities, a computational method has been introduced. It suggests that all infrared and mass singularities may appear in factors extracted from the complete naive parton cross section, $d\sigma^{parton}$, that will be absorbed in the functions G_j. Such a factorization is possible because of the convolution form of equation (2.2). The remaining part of the cross section $d\tilde{\sigma}^{parton}$, will then be well behaved, and still be calculable in terms of perturbation theory. The singularities disappear into renormalized (i.e. physically measurable and finite) quantities, $\tilde{G}_j$. When factorization of singularities is possible, the hadronic cross section can be written in a similar form as equation (2.2)

$$d\sigma^{hadron}(P_1,P_2,\ldots P_J) = \sum_{\substack{\text{parton types}\\ \text{and helicities}}} \int \prod_{j=1}^{J} dx_j\ \tilde{G}_j(x_j)\ d\tilde{\sigma}^{parton}(p_1,p_2,\ldots p_J) \tag{2.4}$$

To determine the correction terms we are interested in, we will have to factorize the mass singularity which arises from perturbation calculations. General proofs of factorization are now available [20-23]. Nevertheless, we will demonstrate how it is performed in our specific cases, showing also in detail how it can generate scale violations (Q^2 dependence) in the distribution functions.

Although factorization seems to be only a mathematical device, it has a precise meaning in the impulse approximation. The long-time scale only appears in terms of the inverse constituent masses. By letting the parton masses go to zero, the long- and short-time scale physics separate as the long-time scale goes to infinity. This can be seen clearly in configuration space. The divergences come from regions of integration corresponding to the possibility of propagation of internal particles over macroscopically long time and distances. Assuming that the interactions which take place over indefinitely long times are the ones that sum to give quark and gluon bound states in QCD, we find that these divergences really belong to the long-time scale. From that point of view, factorization is precisely associated with the separation of the short-time hard collisions from long-time scale interactions in parton cross section.

There exists actually no way of finding theoritically an exact analytical expression for the parton distribution functions, G_j, in a hadron. We must rely on the numerical analysis of physical cross sections, assuming we can approximate them reasonably well with the lower order subprocesses. However, one can certainly make qualitative remarks based on the fact that hadrons are bound states of "valence" quarks (or antiquarks) that are surrounded by a "sea" of quarks, antiquarks and gluons. The probability to find a valence quark in a given hadron, is clearly higher than that of finding gluons or sea quarks, so that we expect the valence distribution functions to dominate that of the gluon and the sea quarks. Furthermore, because it is most unlikely that a parton would come out of a hadron with all the hadron momentum, in high energy collisions, we also expect those distribution functions to get small when x_j goes to one. These two facts are reflected in the behaviour of the distribution functions in most of the parton distributions existing. For a fixed Q^2 ($Q^2=Q_0^2$), the physically measurable distribution functions for a parton i in a hadron h, $G_{i/h}$, have the general form

$$G_{i/h}(x,Q_0^2) \sim (1-x)^{\eta_i} \qquad (2.5)$$

where $\eta_{valence} \sim 3$, $\eta_{gluon} \sim 5$ and $\eta_{sea} \sim 7$. The difference in the powers η_i leads to "large" differences in the parton distribution functions that can generate contributions comparable in magnitude from subprocesses of different order in α_s. This would be expected , considering the fact that for the range of Q^2 available at present, α_s is not very small. Indeed, we do anticipate [(4)] that a subprocess of order α_s^2 involving only valence quarks, leads to a contribution of comparable magnitude with a subprocess of order α_s involving gluons , or, of order α_s^0 involving sea antiquarks. In this work, we evaluate the contributions of such $O(\alpha_s^2)$

subprocess, comparing with the $O(\alpha_s)$ and $O(1)$ known contributions.

2-2 Leptoproduction

A large part of this work is devoted to deep-inelastic lepton-hadron scattering (leptoproduction) represented schematically in figure [2.1]. This corresponds to the physical process

$$\ell + h \to \ell + X \tag{2.6}$$

Here ℓ stands for a lepton, h for a hadron and X for any set of final hadrons.

The basic quantities used to discuss deep inelastic process are the structure functions W_L (or W_1), W_2 and W_3. Our discussion of leptoproduction be restricted to spin-averaged processes. Then these functions are defined in terms of the well-known tensor of electromagnetic or weak currents (12):

$$W_{\mu\nu} = \int d^4z\, e^{iq\cdot z} \langle p|[J_\mu(z),J_\nu(0)]|p\rangle_{\text{spin averaged}}$$

$$= e_{\mu\nu}\,\frac{\nu W_L(\nu,Q^2)}{2x} + d_{\mu\nu}\,\frac{\nu W_2(\nu,Q^2)}{2x}$$

$$-i\varepsilon_{\mu\nu\alpha\beta}\,\frac{p^\alpha p^\beta}{\nu}\,\nu W_3(\nu,Q^2) \tag{2.7}$$

where $\nu = p \cdot q$, $Q^2 = -q^2 \geq 0$ and, $x = Q^2/2\nu$, q being the momentum of the virtual photon and p, the momentum of the incoming hadron.

In our case, as we can see from figure [2.1], we have chosen the mediating boson between the lepton and the hadron to be a virtual photon. Then, the current J_μ stands for the electromagnetic current. The W_3 term represents the vector-axial interference, and is therefore absent in electromagnetic processes. The tensors $e_{\mu\nu}$ and $d_{\mu\nu}$ are defined by the equations

$$e_{\mu\nu} \equiv g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \tag{2.8}$$

$$d_{\mu\nu} \equiv - \frac{p_\mu p_\nu}{\nu^2} q^2 + \frac{p_\mu q_\nu + p_\nu q_\mu}{\nu} - g_{\mu\nu} \tag{2.9}$$

Instead of W_L, we often use another structure function W_1, related to W_L and W_2 as follows

$$\nu W_L \equiv \nu W_2 - 2xW_1 \tag{2.10}$$

In general, the struture functions depend on both ν and Q^2. According to Bjorken scaling (24), however, when ν and Q^2 are taken sufficiently large so that mass scales can be neglected, the dimensionless functions $F_L = \nu W_L$ and $F_2 = \nu W_2$ depend only on one variable $x = Q^2/2\nu$. Although the simple parton model was first introduced [25] to give an intuitive picture of scaling, as it is now well-known, it can also account for scale violations. Careful calculations that have been done within QCD by means of the operator product expansion, have effectively predicted some small Q^2-dependence in the struture functions F_L and F_2. Experiments agree quite well with the predicted Q^2-evolution, however, we must say that the range of Q^2 available at present is not very large.

Let us now emphasize the importance of considering higher order effects in the DIS process. Many experimental results used in the determination of parton distribution functions come from lepton-hadron DIS. It is clear that these functions are easier to extract and that a better accuracy is expected when we probe hadrons with point-like particles such as the electron.

Of course, experiments give the struture functions of hadrons that include the contributions of subprocesses to all orders. However the distribution function calculations rely on the assumption that the partonic cross section is known up to a given order only, and that higher orders have little effects. If the higher order terms add important corrections, the distribution functions should be recomputed. Hence, in order to keep the same distribution functions in higher order calculations, it is crucial for self-consistency that the corrections to F_L and F_2 be small. As we shall see later, in some cases there is a way to minimize the corrections by adjusting an arbitrary function.

Let us now review the already well-known results for the contributions to DIS from subprocesses of order α_s^0 and α_s^1. The lowest order contribution to the DIS struture functions comes from the point-like quark-photon cross section shown in the diagram of figure [2.2]. The subprocess is

$$\gamma^* + q \to q \tag{2.11}$$

where γ^* represents the virtual photon.

The contribution to the physical struture function F_2^h takes the form of a convolution of the quark distribution function in a hadron with the subprocess cross section (a δ-function in this case) summed over the quark flavors. A similar contribution to order α_s comes from antiquarks, instead of quarks, to give the well-known result:

$$\frac{1}{x} F_2^h(x) = \sum_i e_{q_i}^2 \int_x^1 \frac{dy}{y} [G_{q_i/h}(y) + G_{\bar{q}_i/h}(y)] \, \delta(1-\frac{x}{y})$$

$$= \sum_i e_{q_i}^2 \, [G_{q_i/h}(x) + G_{\bar{q}_i/h}(x)] \tag{2.12}$$

From then on, when h appears as a superscript of F_2 or F_L, it denotes the hadronic structure function; otherwise, we deal with the partonic quantities.

Notice that so far we have indicated no Q^2-dependence. This is to emphasize the fact that up to this order, the parton cross section introduces no scale violations. F_L^h, the longitudinal struture function, gets no contribution from this graph.

The first order subprocesses introduce two major contributions. The subprocesses are

$$\gamma^* + q \to q + g \tag{2.13}$$

$$\gamma^* + g \to q + \bar{q}. \tag{2.14}$$

A first contribution arise from the diagrams in figure [2.3a-b] which contain one quark in the initial state. The diagrams of figure [2.3a] indicate the emission of a gluon and, a second set of diagrams (figure [2.3b]) introduces a wavefunction and a vertex function renormalization correction of order α_s. In the Feynman gauge, these diagrams give a contribution to $F_2^h(x,q^2)$ of the following form:

$$\frac{1}{x} F^h_{2,q}(x,q^2) = \sum_i e^2_{q_i} \int_x^1 \frac{dy}{y} \left[G_{q_i/h}(y) + G_{\bar{q}_i/h}(y)\right]$$

$$\times \frac{4}{3} \frac{\alpha_s}{2\pi} \Big\{ - \left[\left(\frac{1+z^2}{(1-z)}\right)_+ + \frac{3}{2}\,\delta(1-z)\right] \ln \frac{q^2}{p^2} + 1 + 3z - \frac{3}{2(1-z)_+}$$

$$- 2\left(\frac{1+z^2}{1-z}\right)_+ \ln z - \frac{2}{3}\,\pi^2\,\delta(1-z) \Big\}$$

$$\frac{1}{x} F^h_{2,q}(x,q^2) = \sum_i e^2_{q_i} \int_x^1 \frac{dy}{y} [G_{q_i/h}(y) + G_{\bar{q}_i/h}(y)]$$

$$\frac{\alpha_s}{2\pi} \{ P_{qq}(\tfrac{x}{y}) \ln \frac{q^2}{p^2} + f_{q,2}(\tfrac{x}{y}) \} \qquad (2.15)$$

where

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} \equiv \int_0^1 dz \left(\frac{f(z)-f(1)}{1-z} \right) \qquad (2.16)$$

and z=x/y. Here, P_{ij} is the well-known splitting function that determines the probability that a parton i comes out of a parton j with a fraction z of its momentum [26]. The function $f_{q,2}$ is obviously regularization-prescription dependent. The second contribution comes from the diagrams with a gluon in the initial state (figure [2.3c]). It takes the form of a gluon distribution function convoluting with a cross section.

$$\frac{1}{x} F^h_{2,g}(x,q^2) = \sum_i e^2_{q_i} \int_x^1 \frac{dy}{y} G_{g/h}(y) \frac{\alpha_s}{\pi} \left(\frac{1}{2} [z^2+(1-z)^2] \right.$$

$$\left. \times [\ln \frac{q^2}{p^2} - 2\ln z - 1] - \frac{1}{2}(1-2z)^2 \right)$$

$$= \sum_i e^2_{q_i} \int_x^1 \frac{dy}{y} G_{g/h}(y) \frac{\alpha_s}{\pi} \{P_{qg}(\tfrac{x}{y}) \ln \frac{q^2}{p^2} + f_{g,2}(\tfrac{x}{y})\}$$

$$(2.17)$$

Again, the non-leading logarithmic term, $f_{g,2}$, is regularization-prescription dependent. We observe a similarity between equations (2.17) and (2.15). Both parton cross sections have a splitting function as the coefficient of their leading logarithmic term. As a matter of fact, both contributions take the form of a parton i distribution function convoluting with the probability that a parton j comes out of a parton i. This suggests that at least a part of each contribution should be understood as a parton j-photon contribution. This idea has led to the introduction of parton densities $G_{q/q}$ and $G_{q/g}$.

$$G_{q/q}(z,q^2) = G_{\bar{q}/\bar{q}}(z,q^2) = \frac{\alpha_s}{2\pi} \{P_{qq}(z) \ln \frac{q^2}{p^2} + u_{qq}(z)\} \tag{2.18}$$

$$G_{q/g}(z,q^2) = \frac{\alpha_s}{2\pi} \{P_{qg}(z) \ln \frac{q^2}{p^2} + u_{qg}(z)\} \tag{2.19}$$

The functions u_{qq} and u_{qg} are to a great extent arbitrary. We use similar symbols for distribution functions and parton densities since these quantities are similar in many respects.

The parton densities, when convoluted with the proper distribution functions, determine a Q^2-evolution of the structure functions, F_2^h,

$$\frac{1}{x} F_2^h(x,q^2) \propto \int_x^1 \frac{dy}{y} [G_{q_i/h}(y) + G_{\bar{q}_i/h}(y)] \; (\delta(1-\frac{x}{y})$$

$$+ G_{q/q}(\frac{x}{y},q^2)) + 2 \int_x^1 \frac{dy}{y} G_{g/h}(y) \; G_{q_i/g}(\frac{x}{y},q^2)$$

$$(2.20)$$

Now, consider the term proportional to $G_{q/q}$; it can be cast in the form

$$G_{q/q}(z,q^2) = \int_z^1 \frac{d\alpha}{\alpha} G_{q/q}(\alpha,q^2) \; \delta(1-\frac{z}{\alpha}) \qquad (2.21)$$

and, this is interpreted as the convolution of the probability that a quark comes out of a quark (with a fraction α of its momentum) with a term proportional to the naive $\gamma^* q$ cross section. A similar interpretation also holds for $G_{q/g}$, so that these two terms really contribute to the subprocess $\gamma^* q \to q$. This justifies their absorption in redefined distribution functions, which is done by rewriting the expression (2.20) as

$$\int_x^1 \frac{dy}{y} \; (\; [G_{q_i/h}(y) + \int_y^1 \frac{d\alpha}{\alpha} \{ \; G_{q_i/h}(\frac{x}{\alpha}) \; G_{q/q}(\alpha,q^2) + G_{g/h}(\frac{x}{\alpha})$$

$$\times \; G_{q_i/h}(\alpha,q^2) \; \} \;] \; + \; [\bar{q}_i \leftrightarrow q_i] \;) \; \delta(1-\frac{x}{y})$$

$$(2.22)$$

Then, the non-scaling terms in the square brackets are chosen to be the new quark and antiquark distribution functions, respectively. Because such absorption may occur at all orders in α_s, expression (2.22) is not expected to provide the exact Q^2-dependence of the distribution functions. However, it clearly shows how Bjorken scaling can be violated within the parton model.

The remaining $O(\alpha_s)$ correction terms in equations (2.15) and (2.17) are then respectively

$$\frac{1}{x} F^h_{2,q}(x,q^2) = \sum_i e^2_{q_i} \int_x^1 \frac{dy}{y} [G_{q_i/h}(y,q^2) + G_{\bar{q}_i/h}(y,q^2)]$$

$$\times \frac{\alpha_s}{2\pi} \{f_{q,2}(\tfrac{x}{y}) - u_{qq}(\tfrac{x}{y})\} \tag{2.23}$$

$$\frac{1}{x} F^h_{2,g}(x,q^2) = \sum_i e^2_{q_i} \int_x^1 \frac{dy}{y} G_{g/h}(y,q^2) \frac{\alpha_s}{2\pi} \{f_{g,2}(\tfrac{x}{y}) - u_{qg}(\tfrac{x}{y})\} \tag{2.24}$$

where the Q^2-evolution of the distribution functions is determined by assuming that equation (2.12) holds now for all Q^2.

As we mentioned before, the corrections must be small in order that the distribution function calculations remain valid. The arbitrariness of the functions u_{qq} and u_{qg} can serve here to guarantee that such a condition is fulfilled. The $O(\alpha_s)$ corrections to F_2 are then set to zero with the choice:

$$u_{qq}(z) \equiv f_{q,2}(z) \tag{2.25}$$

$$= \frac{4}{3}\left[1+3z-\frac{3}{2(1-z)_+} - 2\left(\frac{1+z^2}{1-z}\right)_+ \ln z - \frac{2\pi^2}{3}\,\delta(1-z)\right]$$

$$u_{qg}(z) \equiv f_{g,2}(z) \tag{2.26}$$

$$= -\,2P_{qg}(z)\,\ln z + 3z(1-z) - 1$$

Obviously u_{qq} and u_{qg} determined this way, will show a regularization-prescription dependence, like $f_{q,2}$ and $f_{g,2}$.

The longitudinal struture function, F_L, receives its first contribution at this order in α_s; this comes from the subprocesses $\gamma^*g \to q\bar{q}$ and $\gamma^*q \to qg$. We are interested in the first that gives the well-known contribution [(27-29)]

$$\frac{1}{x} F^h_{L,g}(x) = \sum_i e^2_{q_i} \int_x^1 \frac{dy}{y}\, G_{g/h}(y)\, \frac{\alpha_s}{2\pi}\left[2\frac{x}{y}\left(1-\frac{x}{y}\right)\right]$$

$$= \int_x^1 \frac{dy}{y}\, G_{g/h}(y)\, \frac{\alpha_s}{2\pi}\, f\!\left(\frac{x}{y}\right) \tag{2.27}$$

The partonic term $f(\frac{x}{y})$ introduces no Q^2-dependence; therefore to this order, F_L scales. This result will be reproduced later on, in order to describe the computational method used in the next order calculations.

The $O(\alpha_s^2)$ contributions should be dominated by the subprocess $q\gamma^* \to qq\bar{q}$ (figure [2.4]) because it involves valence quark distribution functions. Following the same scheme as in the description of the $O(\alpha_s)$ corrections to F_2, the $O(\alpha_s^2)$ quark contribution to the partonic structure function F_2 can be cast in the form

$$\frac{1}{x} F_{2,q}(x,q^2) = \int_x^1 \frac{dy}{y}[G_{\bar{q}/q}(y,q^2)\; C_0(\tfrac{x}{y}) + G_{g/q}(y,q^2)\; C_1(\tfrac{x}{y})]$$

$$+ \quad C_2(x) \qquad (2.28)$$

Here C_0 is just the naive γ^*q cross section

$$C_0(x) = e_q^2\; \delta(1-x) \qquad (2.29)$$

and C_1 is related to the $O(\alpha_s)$ cross section for the subprocess $\gamma^*g \to q\bar{q}$; this was set to vanish i.e.

$$C_1 \equiv 0 \qquad (2.30)$$

As before the term from the integral really pertains to the lower order subprocess and can be absorbed in F_2^h as a Q^2-evolution. The correction C_2 is yet to be determined.

The density $G_{g/q}(x,q^2)$ can be written

$$G_{g/q}(x,q^2) = \frac{\alpha_s}{2\pi}\,\{P_{gq}(x)\;\ln\frac{q^2}{p^2} \;+\; u_{gq}(x)\} \qquad (2.31)$$

However, the density $G_{\bar{q}/q}$ does not have this simple form because an antiquark can come out of a quark only through the emission of a gluon. Thus $G_{\bar{q}/q}$ takes the form

$$G_{\bar{q}/q}(x,q^2) = \left(\frac{\alpha_s}{2\pi}\right)^2 \int_x^1 \frac{d\alpha}{\alpha} \left[P_{\bar{q}g}\left(\frac{x}{\alpha}\right) P_{gq}(\alpha) \ln^2 \frac{q^2}{p^2} + K(x,\alpha) \ln \frac{q^2}{p^2} + u_{\bar{q}q}(x,\alpha)\right] \tag{2.32}$$

Leading logarithmic and non-leading logarithmic coefficients can be specified but neither $u_{gq}(x)$ nor $u_{\bar{q}q}(x,\alpha)$ is unique and as we shall see in the next section they are both required for the determination of $O(\alpha_s^2)$ correction to dilepton production. From the equation (2.28), it is clear that by imposing the requirement of no $O(\alpha_s^2)$ correction to F_2, we fix only the function $u_{\bar{q}q}$.

The correction to the longitudinal structure function, however, has the form:

$$\frac{1}{x} F_{L,q}(x,q^2) = \int_x^1 \frac{dy}{y} G_{g/q}(y,q^2) B_1\left(\frac{x}{y}\right) + B_2(x) \tag{2.33}$$

where $B_1(z)$ is the Born term due to $\gamma^* g \to q\bar{q}$

$$B_1(z) \propto \frac{\alpha_s}{2\pi} z(1-z) \tag{2.34}$$

Then, u_{gq} can be determined by requiring F_L to be free of $O(\alpha_s^2)$ corrections. Again the integral term introduces scale violations to F_L.

The determination of the function $u_{\bar{q}q}$ and u_{gq} is a main subject of this work. We propose a different approach to the one above in appendix B. This approach of appendix B does not guarantee the smallness of the correction to the structure functions. However, it is found that this choice give rise to small corrections that can be neglected.

2-3 Dilepton production

The perturbative QCD approach will now be applied to the production of dilepton in hadron-hadron collisions. Each hadron produces a parton, the two partons interact and they physically produce a virtual photon (with $q^2=M^2>0$) plus a number of partons; the virtual photon transforms to a lepton pair and the partons produce a number of (unobserved) hadrons (see figure [2.5]).

Let s be the square of the c.m. energy of the two incoming hadrons. We introduce the usual scaling variable for dilepton production

$$\tau = \frac{q^2}{s} = \frac{M^2}{s} \tag{2.35}$$

Then the contribution to the inclusive cross section of a particular subprocess with initial partons i and j is, according to this picture:

$$\frac{d\sigma^{ij}_{h_1h_2}}{dq^2} = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} G_{i/h_1}(x_1,q^2)\, G_{j/h_2}(x_2,q^2)\, \frac{d\sigma_{ij}}{dq^2}$$

$$+ (1 \leftrightarrow 2) \qquad (2.36)$$

where $\tau_{12}=\tau/x_1x_2$.

The Born term for dilepton production is determined by the Drell-Yan mechanism [17] shown in figure [2.6]

$$q + \bar{q} \rightarrow \gamma^* \qquad (2.37)$$

The cross section for this subprocess is

$$\frac{d\sigma_{q\bar{q}}}{dq^2} = \frac{4\pi\alpha^2_{em}}{3Nsq^2}\; e^2_q \;\; \delta(1-\tau) \qquad (2.38)$$

where e_q is the fractional charge of the quark, and the 1/N factor comes from color-averaging (for $SU_c(N)$ with N=3 in QCD)

For hadron-hadron collisions, this leads to the well-known Drell-Yan formula

$$\frac{d\sigma^{DY}_{h_1h_2}}{dq^2} = \frac{4\pi\alpha^2_{em}}{3Nsq^2} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \{ \sum_i e^2_{q_i}\; G_{q_i/h_1}(x_1)\; G_{\bar{q}_i/h_2}(x_2)$$

$$+ \quad (1 \leftrightarrow 2) \} \, \delta(1-\tau_{12}) \tag{2.39}$$

where the sum is as before over the quark flavors.

The QCD corrections of first order in the coupling constant α_s arise from the subprocesses:

$$g + q(\bar{q}) \rightarrow \gamma^* + q(\bar{q}) \tag{2.40}$$

$$q + \bar{q} \rightarrow \gamma^* + g \tag{2.41}$$

shown in figure [2.7]. To both these subprocesses, one should also include vertex and quark self-energy corrections up to $O(\alpha_s)$ (figure [2.7c]). These terms together with $q\bar{q} \rightarrow \gamma^* g$ provide the large correction ($\sim \pi^2$) to $d\sigma/dq^2$ of Drell-Yan.

Perturbation calculation is known to introduce mass singularities. As we discussed at the beginning of this chapter, they are regularized by setting the momentum of the initial partons slightly off mass shell (i.e. $p_1^2, p_2^2 < 0$).

The structure of the correction term is as follows:

$$\frac{d\sigma^{(1)}_{h_1 h_2}}{dq^2} = \frac{4\pi\alpha^2_{em}}{3sq^2} \frac{1}{N} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \alpha_s \, \theta(1-\tau_{12}) \sum_i e^2_{q_i} \{ [G_{q_i/h_1}(x_1)$$

$$\times \, G_{\bar{q}_i/h_2}(x_2) + (1 \leftrightarrow 2)] \times \{P_{qq}(\tau_{12}) \ln \frac{(q^2)^2}{p_1^2 p_2^2} + f_q^{DY}(\tau_{12})\}$$

$$+ \left(\left[\left(G_{q_i/h_1}(x_1) + G_{\bar{q}_i/h_1}(x_1) \right) G_{g/h_2}(x_2) \right] \times \left[P_{qg}(\tau_{12}) \right. \right.$$

$$\left. \left. \times \ln \frac{q^2}{-p_2^2} + f_g^{DY}(\tau_{12}) \right] + (1 \leftrightarrow 2) \right) \Big) \tag{2.42}$$

The functions $P_{qq} = P_{\overline{qq}}$ and P_{qg} are the same splitting functions used in DIS. The presence of the splitting functions in front of the logarithmic term indicates that we can interpret this result in terms of the parton densities, $G_{q/q}$ and $G_{q/g}$. Then, equation (2.42) can be written as follows:

$$e_q^2 \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \Big(\left[G_{q/h_1}(x_1)\, G_{\bar{q}/h_2}(x_2) + (1 \leftrightarrow 2) \right] \times \left[2\pi \{ G_{q/q}(\tau_{12}, -q^2) \right.$$

$$\left. + G_{\bar{q}/\bar{q}}(\ _{12}, -q^2) \} + \alpha_s (f_q^{DY}(\tau_{12}) - u_{qq}(\tau_{12})) \right] + \left[G_{q/h_1}(x_1) \right.$$

$$\left. + G_{\bar{q}/h_1}(x_1) \right] G_{g/h_2}(x_2) \left[2\pi\, G_{q/g}(\tau_{12}, -q^2) + \alpha_s (f_g^{DY}(\tau_{12}) \right.$$

$$\left. - u_{qg}(\tau_{12})) \right] \Big) \times \theta(1 - \tau_{12}) \tag{2.43}$$

where we have omitted obvious factors and the sum over the quark flavors. In this form, we see how the mass singularities can be absorbed in the process independent distribution functions using the same arguments as for DIS. Then, the remaining correction terms to the partonic cross sections are [3,4]

$$\frac{d\sigma^{(1)}_{q\bar{q}}}{dq^2}(\tau) = \frac{4\pi\alpha^2_{em}}{3sq^2}\ \frac{e^2_q}{N}\ \alpha_s\ \{f^{DY}_q(\tau) - 2u_{qq}(\tau)\}\ \theta(1-\tau)$$

$$= \frac{4\pi\alpha^2_{em}}{3sq^2}\ \frac{e^2_q}{N}\ \frac{4}{3}\ \frac{\alpha_s}{2\pi}\ \{\ \frac{3}{(1-\tau)_+} - 6 - 4\tau + 2(1+\tau^2)$$

$$\times\ \left(\frac{\ln\ 1-\tau}{1-\tau}\right)_+ + \left(\frac{4\pi^2}{3} + 1\right)\ \delta(1-\tau)\}\ \theta(1-\tau) \tag{2.44}$$

$$\frac{d\sigma^{(1)}_{qg}}{dq^2}(\tau) = \frac{4\pi\alpha^2_{em}}{3sq^2}\ \frac{e^2_q}{N}\ \alpha_s\ \{\ f^{DY}_g(\tau) - u_{qg}(\tau)\ \}\ \theta(1-\tau)$$

$$= \frac{4\pi\alpha^2_{em}}{3sq^2}\ \frac{e^2_q}{N}\ \frac{\alpha_s}{2\pi}\ \{P_{qg}(\tau)\ \ln\ (1-\tau) + \frac{3}{4} - \frac{5}{2}\tau + \frac{9}{4}\tau^2\} \tag{2.45}$$

and α_s is now the running coupling constant. The functions f^{DY}_g, f^{DY}_q, u_{qg} and u_{qq} are all regularization-prescription dependent. However, because we are consistently using throughout this work the same method of regularization, such a dependence cancels out in the partonic cross sections $d\sigma^{(1)}_{q\bar{q}}/dq^2$ and $d\sigma^{(1)}_{qg}/dq^2$.

Corrections of order α_s arising form the subprocess $qg \rightarrow q\gamma^*$ are of rather minor importance. Even for the physical process $pp \rightarrow \ell^+\ell^-+X$ they are of the order of 10%, while they are expected to be negligible in proton-antiproton collisions. However the subprocess $q\bar{q} \to \gamma^* g$ yields surprisingly large contribution even in the proton-nucleon case, and should not be neglected.

We will now focus our attention on the order α_s^2 quark-quark subprocess (figure [2.8])

$$q + q \to q + q + \gamma^* \qquad (2.46)$$

This subprocess is particularly relevant in the study of proton-nucleon collisions where quarks are the valence partons. Because of the relative magnitude of the quark, antiquark and gluon distribution functions, the quark-quark term can prove to be significant.

The cross section for $qq \to qq\gamma^*$ can be written in the general form

$$\frac{d\sigma_{qq}}{dq^2} = \frac{4\pi\alpha_{em}^2}{3sq^2} \{ \int_\tau^1 \frac{dx}{x}([G_{\bar{q}/q_1}(x,-q^2)\ \kappa_0(\tfrac{\tau}{x}) + G_{g/q_1}(x,-q^2)$$

$$\times\ \kappa_1(\tfrac{\tau}{x})] + [q_1 \leftrightarrow q_2]) + \kappa_2(\tau) \} \qquad (2.47)$$

where $\kappa_0(\tau)$ is proportional to the $q\bar{q} \to \gamma^*$ Drell-Yan cross section

$$\kappa_0(\tau) = \frac{e_q^2}{2N}\ \delta(1-\tau) \qquad (2.48)$$

$\kappa_1(\tau)$ comes from the $O(\alpha_s)$ quark-gluon correction (equation (2.45))

$$\kappa_1(\tau) = \frac{e_q^2}{2N} \frac{\alpha_s}{\pi} \{P_{qg}(\tau) \ln (1-\tau) + \frac{3}{4} - \frac{5}{2}\tau + \frac{9}{4}\tau^2\} \tag{2.49}$$

The parton densities $G_{\bar{q}/q}$ and $G_{g/q}$ are defined by equations (2.31) and (2.32) respectively. Once again, the integral can be absorbed in redefined valence quark distribution functions $G_{q_i/h}(x,q^2)$.

As mentioned above, u_{gq} and $u_{\bar{q}q}$ are to a great extent arbitrary (in particular they are regularization-prescription dependent) and still, they play an important role in the determination of the quark-quark term $\kappa_2(\tau)$. Let us denote the non-logarithmic term of $d\sigma_{qq}/dq^2$ obtained from perturbation calculation, for a given gauge and regularization prescription, by

$$\int_\tau^1 \frac{dx}{x} L(x,\tau) \tag{2.50}$$

From figure [2.8] , we can see that the calculation of the cross section will generate terms proportional to $e_{q_1}^2$, $e_{q_2}^2$ and $e_{q_1}e_{q_2}$ where e_{q_1} and e_{q_2} are the respective charges of the quarks q_1 and q_2. Then, κ_2 will be given by

$$\kappa_2(\tau) = \int_\tau^1 \frac{dx}{x} \{ (\frac{\alpha_s}{2\pi})^2 [L(x,\tau) - \kappa_0(\frac{\tau}{x}) \int_x^1 \frac{d\alpha}{\alpha} u_{\bar{q}_2 q_1}(\tau,\alpha)]$$

$$- \left(\frac{\alpha_s}{2\pi}\right)\left[u_{gq_1}(x)\ \kappa_1\left(\frac{\tau}{x}\right)\right] \quad + \quad (q_1 \leftrightarrow q_2)\ \}$$

(2.51)

where for simplicity we have assumed that q_1 and q_2 are non-identical quarks. The result, κ_2, is gauge and regularization-prescription independent provided that $u_{\bar{q}q}$ and u_{gq} have been calculated in the same gauge and using the same regularization scheme as for $L(x,\tau)$.

CHAPTER III

LEPTOPRODUCTION

This chapter is devoted to the determination of the constant terms (independent of Q^2) of the gluon and antiquark densities in a quark, u_{gq} and $u_{\bar{q}q}$, respectively. This is done through the evaluation of the $O(\alpha_s^2)$ correction to the leptoproduction structure functions.

The first section contains the evaluation of the $O(\alpha_s)$ Born term of the longitudinal structure function. This will set the basis for the next order "quark" correction to F_L; this is calculated in the second section. The $O(\alpha_s^2)$ correction will then be incorporated into a redefined distribution function, setting the "net" $O(\alpha_s^2)$ correction to zero. This procedure completely fixes u_{gq}. A proof of the factorization of mass singularities is then provided in section 3-3. Finally we will briefly state the results of the perturbation calculation for the determination of $u_{\bar{q}q}$; this is done by requiring no $O(\alpha_s^2)$ correction to F_2^h. The details of this calculation have already been published (see references 15,30 and 31).

Throughout this chapter and even further on,many quantities are approximated for large x ($x \to 1$), x being the scaling variable in leptoproduction. This is because, in order to get the hadronic quantities, the partonic cross sections have to be convoluted with the valence quark distribution functions, and these distributions are known to be steep functions of x near 1. Therefore, one may drop higher powers of (1 - x) in the partonic cross sections without losing too much accuracy. This approximation will greatly simplify the discussion of the $O(\alpha_s^2)$ correction of dilepton production in the next chapters.

3-1 $O(\alpha_s)$ contribution to F_L

The first non-zero contribution to the longitudinal structure function F_L comes from the $O(\alpha_s)$ diagrams of figure [2.3]. Each diagram gives a contribution to the electromagnetic tensor $W_{\mu\nu}$. If we assign four-momentum p^μ to the initial massless parton (gluon or quark), the longitudinal structure function is projected out when $p^\mu p^\nu$ is contracted with $W_{\mu\nu}$. This procedure amounts to choosing the polarization vector of the virtual photon parallel to the four-momentum of the incoming parton

We are interested by $F_{L,g}$, the $O(\alpha_s)$ contribution from the subprocess $g\gamma^* \to q\bar{q}$, called from then on the "gluon" contribution. $F_{L,g}$ is given by

$$\frac{1}{x} F_{L,g}(x) = \sum_i e_{q_i}^2 \frac{\alpha_s}{\pi} x \int d^4k \; \delta_+[(q+k)^2] \; \delta_+[(p-k)^2] \; |M_g|^2 \tag{3.1}$$

where the momenta are defined in figure [2.3]. The δ-functions indicate that the final quarks must be on mass shell and of positive energy.

$$\delta_+(P^2) \equiv \delta(P^2)\ \theta(P_0) \tag{3.2}$$

$|M_g|$ is proportional to the $p^\mu p^\nu$-projected spin averaged matrix element of the subprocess $\gamma^* g \to q\bar{q}$. x is the usual scaling variable

$$x = \frac{-q^2}{2p\cdot q} \tag{3.3}$$

Let us now introduce a more convenient set of variables (Sudakov parameters [32]) instead of the four components of k.

$$k = \alpha p_1 - \beta p_2 - \ell \tag{3.4}$$

and,

$$d^4k = \frac{u}{4}\ d\alpha\ d\beta\ d\phi\ d\ell_T^2 \tag{3.5}$$

with $p_1^2=p_2^2=0$ and $u=(p_1+p_2)^2$. α and β are then scalars $(0\leq\alpha,\beta\leq 1)$ and ℓ is a four-momentum orthogonal to both p_1 and p_2, i.e. $\ell\cdot p_1 = \ell\cdot p_2 = 0$. To carry the numerical calculations we use the frame of reference where

$$p_1 = \frac{\sqrt{u}}{2}\,(1\ ;\ \vec{0}\ ,\ 1) \qquad p_2 = \frac{\sqrt{u}}{2}\,(1\ ;\ \vec{0}\ ,\ -1) \tag{3.6}$$

and,

$$\ell = (0\ ;\ \vec{\ell}_T\ ,\ 0) \tag{3.7}$$

As we shall discuss later, we are interested in the value of $F_{L,g}$ at large x. Then the kinematical relations are much simplified by using the relation[27]

$$p_2 = q + xp_1 \tag{3.8}$$

The requirement $p_2^2=0$ is in direct accord with the definition of x in equation (3.3), when applied to p_2 in (3.8).

The calculation of the graphes of figure [2.3c] when expressed in terms of Sudakov parameters, gives (see Appendix A)

$$|M_g|^2 = 1 - \alpha \tag{3.9}$$

The fact that the final partons are taken on mass shell implies

$$\begin{aligned}(p - k)^2 &= 2(1 - \alpha)\beta\ p\cdot p_2 + \ell^2 \\ &= (1 - \alpha)\beta u - \ell_T^2 = 0\end{aligned} \tag{3.10}$$

and for the other final quark line,

$$(q + k)^2 = [(\alpha-x)p + (1-\beta)p_2 - \ell]^2$$

$$= (\alpha-x)(1-\beta)u - \ell_T^2$$

$$= u(\alpha-x-(1-x)\beta) = 0 \qquad (3.11)$$

where in the last relation we have substituted ℓ_T^2 using relation (3.10). The condition that the energy of the final partons be positive leads to

$$(p - k)^0 = (1-\alpha)p^0 + \beta p_2^0 + \ell^0$$

$$= \frac{\sqrt{u}}{2}(1-\alpha+\beta) = \frac{\sqrt{u}}{2}(1-x+x\beta) \geqq 0 \qquad (3.12)$$

and,

$$(q + k)^0 = (\alpha-x)p^0 + (1-\beta)p_2^0 - \ell^0$$

$$= \frac{\sqrt{u}}{2}(\alpha-x+1-\beta) = \frac{\sqrt{u}}{2}(1-\beta x) \geqq 0 \qquad (3.13)$$

Therefore, β must obey the inequality

$$\frac{x-1}{x} \leqq \beta \leqq \frac{1}{x}. \qquad (3.14)$$

However, we already have $0 \leqq x \leqq 1$. Then, relation (3.14) is automatically satisfied because we had $0 \leqq \beta \leqq 1$.

Integrating over ℓ_T^2 and ϕ, the $O(\alpha_s)$ gluon contribution takes the form

$$\frac{1}{x} F_{L,g}(x) = 2 \sum_i e_{q_i}^2 \frac{\alpha_s}{\pi} x \int_0^1 d\beta \int d\alpha \; \delta[\alpha - x - (1-x)\beta] \; (1-\alpha) \tag{3.15}$$

Notice that the ϕ-integration is trivially carried, the matrix element as well as the argument of the δ-function being both ϕ-independent. Finally, integrating over α and then β, we obtain

$$\frac{1}{x} F_{L,g}(x) = \sum_i e_{q_i}^2 \frac{\alpha_s}{\pi} \; x(1-x) \equiv f(x) \tag{3.16}$$

We can reproduce now the relation (2.27) for the physical process $h+\ell \to X+\ell$, by convoluting this result with the gluon distribution function in the hadron h. The contribution from the subprocess $g\gamma^* \to q\bar{q}$ is then

$$\frac{1}{x} F_{L,g}^{h}(x) = \int_x^1 \frac{dx_1}{x_1} G_{g/h}(x_1) \; f(\frac{x}{x_1}) \tag{3.17}$$

3-2 Contribution of $q\gamma^* \to qq\bar{q}$ to F_L

In the previous chapter, we have emphasized the importance of considering the contribution to the next order in α_s. We anticipate in particular a large contribution from the subprocess $q\gamma^* \to qq\bar{q}$ (called subsequently the quark contribution), because of the presence of a valence quark in the initial state. Let us now proceed with the evaluation of this subprocess (figure [2.4]). In the notation of figure [2.4], the contribution $F_{L,q}$ of this subprocess to the longitudinal structure function is

$$\frac{1}{x} F_{L,q}(x,q^2) = e_q^2 \frac{\alpha_s}{\pi^4} \frac{x}{\nu} \int d^4k_1 \; d^4k_2 \; \delta_+[(p-k_1)^2]$$

$$\times \; \delta_+[(k_1-k_2)^2] \; \delta_+[(k_2+q)^2] \; |M_q|^2 \qquad (3.18)$$

where $\nu = p \cdot q$ and $|M_q|$ is proportional to the matrix element. Again it is more convenient to work with the Sudakov parameters. We then define these parameters with respect to the momenta of the final quarks:

$$p - k_1 = (1 - \alpha_1)p_1 + \beta_1 p_2 + \ell_1 \qquad (3.19)$$

$$k_1 - k_2 = (\alpha_1 - \alpha_2)p_1 + \beta_2 p_2 + \ell_2 \qquad (3.20)$$

with the requirements that $\ell_1 \cdot p_1 = \ell_2 \cdot p_2 = 0$ and $p_1^2 = p_2^2 = 0$. Again p_1, p_2 and ℓ_i $(i=1,2)$ are taken as follows:

$$p_1 = \frac{\sqrt{u}}{2} (1 \; ; \; \vec{0} \; , \; 1) \qquad p_2 = \frac{\sqrt{u}}{2} (1 \; ; \; \vec{0} \; , -1) \qquad (3.21)$$

$$\ell_i = (0\ ;\ \vec{\ell}_{Ti}\ ,\ 0) \tag{3.22}$$

and $0 \leqq \alpha_i, \beta_i \leqq 1$. In the $O(\alpha_s)$ calculation we chose $p_1 \equiv p$. A similar choice will not be made here, because we expect to get divergences that must be regularized by setting the initial parton slightly off shell (i.e. small $p^2 < 0$). However p_1 can still be expressed in terms of p as follows:

$$p_1 = p - (\frac{p^2}{u})p_2 \tag{3.23}$$

Again, for large x (near 1), we may keep the relation[27]

$$p_2 = q + xp_1 \qquad \text{or,} \qquad q \simeq p_2 - xp \tag{3.24}$$

for small p^2.

Let us now examine how the kinematical conditions imposed on the final quarks momenta come out in terms of these new variables. The first δ-function in equation (3.18) implies

$$(p - k_1)^2 = [(1-\alpha_1)p_1 + \beta_1 p_2 + \ell_1^2]^2$$

$$= (1-\alpha_1)\beta_1 u - \ell_{T1}^2 = 0 \tag{3.25}$$

where we have used the relations (3.21-22). A similar relation follows from the second δ-function.

$$(k_1 - k_2)^2 = (\alpha_1 - \alpha_2)\beta_2 u - \ell_{T2}^2 = 0 \qquad (3.26)$$

Finally, the last δ-function in equation (3.18) introduces the variable x in the integrand through the definition of q of equation (3.24). We have:

$$(q + k_2)^2 = q^2 + 2q\cdot k_2 + k_2^2 = 0 \qquad (3.27)$$

with,

$$2q\cdot k_2 = 2(p_2 - xp_1)(\alpha_2 p_1 - (\beta_1 + \beta_2 - \frac{p^2}{u})p_2 - \ell_1 - \ell_2)$$

$$\simeq [\alpha_2 + x(\beta_1 + \beta_2)]u \qquad (3.28)$$

and,

$$k_2^2 = [\alpha_2 p_1 - (\beta_1 + \beta_2 - \frac{p^2}{u})p_2 - \ell_1 - \ell_2]^2$$

$$\simeq -\alpha_2(\beta_1 + \beta_2)u - \ell_{T1}^2 - \ell_{T2}^2 - 2\ell_{T1}\ell_{T2}\cos\Phi \qquad (3.29)$$

In the last expressions (3.28-29), we have neglected $O(p^2)$ terms. Φ is the angle between $\vec{\ell}_{T1}$ and $\vec{\ell}_{T2}$. Then we can write equation (3.27) as

$$(\alpha_2 - x)(1 - \beta_1 - \beta_2)u - \ell_{T1}^2 - \ell_{T2}^2 - 2\ell_{T1}\ell_{T2}\cos\Phi = 0 \qquad (3.30)$$

Now, using the relation

$$d^4k_i = \frac{u}{4}\, d\alpha_i\; d\beta_i\; d\phi_i\; d\ell_{Ti}^2 \qquad i=1,2 \tag{3.31}$$

the contribution $F_{L,q}$ can be written

$$\frac{1}{x} F_{L,q}(x,q^2) = e_q^2 \frac{\alpha_s^2}{8\pi^3} \frac{u^2}{\nu} \int d\alpha_1\; d\beta_1\; d\alpha_2\; d\beta_2\; d\Phi\; |M_q|^2$$

$$\times\; \delta[(\alpha_2 - x)(1-\beta_1-\beta_2) - \ell_{T1}^2 - \ell_{T2}^2 - 2\ell_{T1}\ell_{T2}\cos\Phi\,] \tag{3.32}$$

where the first two δ-functions have been omitted, being understood that ℓ_{T1}^2 and ℓ_{T2}^2 obey relations (3.25) and (3.26) respectively.

It turns out that at least the leading terms of the matrix element $|M_q|^2$ are Φ-independent. For such terms, the integration over angle is trivially carried with the result (see Appendix A)

$$|M_q|^2 \simeq -\frac{2u}{\alpha_1^2} \left\{ \frac{1}{k_1^2} P_{gq}(\alpha_1) - C_F \frac{(2-\alpha_1)p^2}{(k_1^2)^2} \right\} \times \left|M_g\left(\frac{\alpha_2}{\alpha_1}\right)\right|^2 \tag{3.33}$$

for α_1 near 1. We will see later why this limit is of particular interest in our calculations. P_{gq} is the splitting function for a gluon in a quark

$$P_{gq}(z) = C_F \frac{1 + (1-z)^2}{z} \tag{3.34}$$

with $C_F=(N^2-1)/2N$ for $SU_c(N)$ $(N=3)$. The factor $1/k_1^2$ comes directly from the gluon propagator (figure [2.4]).

We expect that the most important contribution of the graphs will appear when $\beta_1 \simeq 0$, because we have

$$k_1^2 = -\beta_1 u + \alpha_1 p^2 \tag{3.35}$$

and thus, $k_1^2 \to O(p^2)$ as $\beta_1 \to 0$. In the limit of small β_1, we have also $\ell_{T1} \to 0$ as one can see from equation (3.25). This limit corresponds to the almost collinear emission of a gluon from a quark ($k_1 \simeq \alpha_1 p_1$). Then, for such a limit, the argument of the remaining δ-function can be reduced to

$$\alpha_2 - x - (\alpha_1 - x)\beta_2 = 0 \tag{3.36}$$

Carrying the integration over Φ yields

$$\frac{1}{x} F_{L,q}(x,q^2) \simeq e_q^2 \frac{\alpha_s}{4\pi^2} \, x \, \frac{u}{\nu} \int d\alpha_1 \; d\beta_1 \; d\alpha_2 \; d\beta_2 \; |M_q|^2$$

$$\times \; \delta[\alpha_2 - x - (\alpha_1 - x)\beta_2] \tag{3.37}$$

Before going further in the process of integrating expression (3.37), we must examine the conditions imposed on the energies of the final partons because they may affect the regions of integration. These conditions are:

$$(p - k_1)^0 \geqq 0 \qquad \Longrightarrow \qquad 1-\alpha_1+\beta_1 \geqq 0$$

$$(k_1 - k_2)^0 \geqq 0 \qquad \Longrightarrow \qquad \alpha_1-\alpha_2+\beta_2 \geqq 0$$

$$(q + k_2)^0 \geqq 0 \qquad \Longrightarrow \qquad \alpha_2-x+1-\beta_1-\beta_2+\frac{p^2}{u} \tag{3.38}$$

The first condition on β_1 is overcome by a stronger condition $\beta_1 \geqq 0$. The last inequalities together with equation (3.36) give the restrictions on the range of β_2,

$$\frac{x-\alpha_1}{1-\alpha_1+x} \leqq \beta_2 \lesssim \frac{1}{1-\alpha_1+x} \tag{3.39}$$

when $\beta_1 \to 0$ and p^2 is small. However, as we shall see below, $\alpha_1 \geqq x$. The last relation is therefore automatically satisfied ($0 \leqq \beta_2 \leqq 1$). Finally, the last condition we shall impose is related to the off-shell subprocess $\gamma^*g \to q\bar{q}$ considered as part of the subprocess of figure [2.4]. In order to compare with the contribution of $\gamma^*g \to q\bar{q}$ (calculated in section 3-1), we require the following subenergy to be positive:

$$(q + k_1)^2 = [(\alpha_1-x)p_1 + (1-\beta_1+\frac{p^2}{u})p_2 - \ell_1]^2$$

$$= (\alpha_1-x)(1-\beta_1+\frac{p^2}{u})u - \ell_{T1}^2$$

$$= -(1-\alpha_1)\beta_1 u + (\alpha_1-x)(1+\frac{p^2}{u})u \geqq 0 \tag{3.40}$$

Thus, neglecting terms of $O(p^2)$, we get a maximum value for β_1, i.e.,

$$\beta_1 \leqq \beta_1^m = \frac{\alpha_1 - x}{1-x} \tag{3.41}$$

But we must also have $\beta_1 \geqq 0$, so that

$$x \leqq \alpha_1 \leqq 1 \tag{3.42}$$

We see that the limit $x \to 1$, implies $\alpha_1 \to 1$ as well. This is the reason we approximated $|M_q|^2$ for large α_1 ($\alpha_1 \simeq 1$) in equation (3.33).

Integrating equation (3.37) over α_2 and β_2 we obtain:

$$\frac{1}{x} F_{L,q}(x,q^2) \simeq e_q^2 \frac{\alpha_s}{4\pi^2} x \frac{u}{\nu} \int_x^1 d\alpha_1 \int_0^{\beta_1^m} d\beta_1 \left(-\frac{2u}{\alpha_1^2}\right) \left(\frac{P_{gq}(\alpha_1)}{k_1^2} - \frac{C_F(2-\alpha_1)p^2}{(k_1^2)^2}\right) \times \left|M_g\left(\tfrac{1}{2}\left(1+\frac{x}{\alpha_1}\right)\right)\right|^2 \tag{3.43}$$

with k_1^2 given by equation (3.35). We now carry the integration with respect to β_1. Neglecting the p^2-terms with respect to β_1^m we will use the results

$$\int_0^{\beta_1^m} \frac{d\beta_1}{k_1^2} \simeq -\frac{1}{u} \ln \frac{u\beta_1^m}{-\alpha_1 p^2} \tag{3.44}$$

$$\int_0^{\beta_1^m} \frac{d\beta_1}{(k_1^2)^2} \simeq -\frac{1}{u\alpha_1 p^2} \tag{3.45}$$

Furthermore we have

$$|M_g(\tfrac{1}{2}(1+\tfrac{x}{\alpha_1}))|^2 = \tfrac{1}{2}(1-\tfrac{x}{\alpha_1}) \tag{3.46}$$

and,

$$\nu = p\cdot q = \frac{u}{2} + \frac{xp^2}{2} \simeq \frac{u}{2} \tag{3.47}$$

In terms of the function f(x) defined in section 3-1, we find the expression:

$$\frac{1}{x} F_{L,q}(x,q^2) \simeq \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\alpha_1}{\alpha_1} \left[P_{gq}(\alpha_1) \ln \frac{(\alpha_1-x)q^2}{\alpha_1 x(1-x)p^2} - C_F \frac{(2-\alpha_1)}{\alpha_1}\right] \times f(\tfrac{x}{\alpha_1}) \tag{3.48}$$

$$\equiv f_q(x,q^2)$$

Let us now recall relation (2.33), which gives the expected form of the $O(\alpha_s^2)$ quark correction for the longitudinal structure function in terms of the parton densities

$$\frac{1}{x} F_{L,q}(x,q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{gq}(y) \ln \frac{q^2}{p^2} + u_{gq}(y)\right] \times f(\tfrac{x}{y}) + B_2(x). \tag{3.49}$$

The integral term represents a correction to the $O(\alpha_s)$ subprocess that is absorbed into the gluon distribution function; the proof of the related factorization property is given in the next section. This correction provides a part of the Q^2-evolution of the gluon distribution in the hadron. Then, the requirement that F_L^h be free of $O(\alpha_s^2)$ quark corrections implies $B_2 \equiv 0$. Furthermore, equation (3.48) must be equivalent to equation (3.49). Thus the non-logarithmic term in the gluon density in a quark must obey

$$\int_x^1 \frac{dy}{y} \left[P_{gq}(y) \ln \frac{y-x}{yx(1-x)} - C_F \frac{2-y}{y}\right] f\left(\frac{x}{y}\right)$$

$$\equiv \int_x^1 \frac{dy}{y} u_{gq}(y) f\left(\frac{x}{y}\right) \qquad (3.50)$$

In this way, we find for $\alpha_1 \simeq 1$,

$$u_{gq}(\alpha_1) \simeq -P_{gq}(\alpha_1)\left[2\ln \alpha_1 + 2 - \frac{1}{\alpha_1}\right] - C_F \frac{2-\alpha_1}{\alpha_1} \qquad (3.51)$$

This function will be used in the next two chapters where we evaluate the $O(\alpha_s^2)$ $qq \to qq\gamma^*$ correction for dilepton production in proton-proton collisions.

3-3 Factorization of the mass singularity

In the previous section, we set the $O(\alpha_s^2)$ quark correction to the longitudinal structure function equal to zero ($B_2 \equiv 0$). In so doing, we assumed that the mass singularity in (3.48) could be absorbed in a redefined gluon distribution function in a hadron. Although general derivations of factorization justifying this assertion already exist [20-23], we consider it worthwile to provide a proof in our special case. Moreover , such a proof shows in detail how one can obtain a Q^2-dependent longitudinal structure function $F_L^h(x,q^2)$.

The contribution of the subprocess $q\gamma^* \to qq\bar{q}$ to the physical process $h + \ell \to X + \ell$, is

$$\frac{1}{x} F^h_{L,q}(x,q^2) = \int_x^1 \frac{dx_1}{x_1} \sum_i G_{q_i/h}(x_1)\, f_{q_i}(\frac{x}{x_1},q^2)$$

$$= \int_x^1 \frac{dx_1}{x_1} \sum_i G_{q_i/h}(x_1) \int_{x/x_1}^1 \frac{d\alpha_1}{\alpha_1} G_{g/q_i}(\alpha_1,q^2)\, f(\frac{x}{\alpha_1 x_1}) \tag{3.52}$$

Now, the relation

$$\sum_i \int_x^1 \frac{d\alpha}{\alpha} G_{g/q_i}(\alpha,q^2)\, G_{q_i/h}(\frac{x}{\alpha}) \equiv G^{\alpha_s}_{g/h}(x,q^2) \tag{3.53}$$

clearly represents an $O(\alpha_s)$ contribution to the gluon distribution function in the hadron h via the emission of a gluon from a quark inside the hadron.

Let us now introduce the Mellin transform $\tilde{w}(n)$ of a function $w(x)$

$$\tilde{w}(n) \equiv \int_0^1 dx\, x^{n-1}\, w(x) \tag{3.54}$$

We recall the convolution theorem for the Mellin transforms, namely if

$$w(x) = \int_x^1 \frac{dy}{y}\, u(y)\, v(\tfrac{x}{y})$$

then,

$$\tilde{w}(n) = \tilde{u}(n)\, \tilde{v}(n) \tag{3.55}$$

The Mellin transform of the function $\frac{1}{x} F^h_{L,q}(x,q^2)$ can then be written

$$\sum_i \tilde{G}_{q_i/h}(n,q^2)\, \tilde{G}_{g/q_i}(n,q^2)\, \tilde{f}(n) = \tilde{G}^{\alpha_s}_{g/h}(n,q^2)\, \tilde{f}(n) \tag{3.56}$$

This result is equivalent to

$$\frac{1}{x} F^h_{L,q}(x,q^2) = \int_x^1 \frac{dx_1}{x_1}\, G^{\alpha_s}_{g/h}(x_1,q^2)\, f(\frac{x}{x_1}) \tag{3.57}$$

Comparison with equation (3.17) shows that the mass singularity can be absorbed by redefining the gluon distribution function

$$G_{g/h}(x) + G^{\alpha_s}_{g/h}(x,q^2) \equiv G_{g/h}(x,q^2) \tag{3.58}$$

This completes the demonstration of factorization.

3-4 Determination of $u_{\overline{q}q}$

In order to apply our results to the problem of dilepton production, we need to specify the form of the density of an antiquark inside a quark, $G_{\overline{q}/q}$. The problem has already been treated with various approaches in references 15, 30 and 31. Here we proceed as follows: The general form of the $O(\alpha_s^2)$ quark contribution (figure [2.4]) to the partonic structure function, F_2, has been given in equation (2.28); in this, $C_2(x)$ represents a part containing no powers of $\ln q^2/p^2$. A perturbation calculation of the graphs of figure [2.4] gives in the limit of large x, which is of interest in our case (see also Appendix B):

$$C_2(x) \simeq \left(\frac{\alpha_s}{2\pi}\right)^2 e_q^2 \,\{C_F(\frac{7\pi^2}{24} + 2) - u_{\overline{q}q}(x,\alpha)\}\,(1-x) \tag{3.59}$$

Here, α in the argument of $u_{\overline{q}q}$ also goes to 1 as $x \to 1$.

We shall now require no $O(\alpha_s^2)$ correction to the hadronic structure function F_2^h from the subprocess $q\gamma^* \to qq\overline{q}$. This is possible if

$$C_2(x) \equiv 0 \tag{3.60}$$

which implies $(x \to 1)$

$$u_{\bar{q}q}(x,\alpha) \simeq C_F\left(\frac{7\pi^2}{24} + 2\right). \tag{3.61}$$

CHAPTER IV

INCLUSIVE DILEPTON PRODUCTION

We consider the inclusive production of dilepton in hadron collisions, $h_1 + h_2 \rightarrow \ell^+\ell^- + X$. The Born term (of order α_s^0) is given by the Drell-Yan subprocess [17,18] $q\bar{q} \rightarrow \gamma^*$. We have already mentioned that higher order subprocesses, via the absorption of the mass singularities, generate scale violations in the distribution functions. The remaining finite terms contribute to the hadronic cross section when convoluted with the distributions; but they are supressed by powers of the running coupling constant α_s compared to the leading Drell-Yan subprocess. However it has been argued[4] that in proton-proton collisions, a number of subprocesses may be equally important for dilepton production. The relative magnitude of the valence quark, gluon and sea antiquark distribution functions at large x suggests that in the region of high $\tau=\hat{q}^2/s$, large contributions may come from what we refer to as the "quark-gluon" subprocess, i.e. $qg \rightarrow q\gamma^*$ (to $O(\alpha_s)$) and from $qq \rightarrow qq\gamma^*$ (to $O(\alpha_s^2)$) which will be called the "quark-quark" contribution.

However, one should be careful with this kind of rough estimates. In the range of q^2 available, the running coupling constant is not very small. A priori, important contributions may come from other subprocesses as well. This is found to be the case for $q\bar{q} \to \gamma^* g$ subprocess(6,7,8,10,11) which yields an unexpectedly large contribution.

The $O(\alpha_s)$ correction to Drell-Yan has already been studied thouroughly(6,7,8,11,33), and we briefly recalled some results in section 3-3. In this chapter, we will examine the $O(\alpha_s^2)$ quark-quark correction. First, by using for parton densities the convention proposed in Chapter III, we calculate the magnitude of this contribution to $p + p \to \ell^+\ell^- + X$; then we compare with the results of other conventions.

4-1 Quark-quark corrections

The $O(\alpha_s^2)$ contributions of the subprocess $qq \to qq\gamma^*$ are indicated on figure [4.1a-b]. As we can see from these unitary diagrams these contributions can be divided in two parts: First, the "squared" terms which are proportional to the square of the charge of one of the incoming partons; these correspond to the diagrams of figure [4.1a]. Only these diagrams (and those with $q_1 \leftrightarrow q_2$) give rise to mass singularities. Second, the "interference" terms, which have the charge structure $e_{q_1} e_{q_2}$ and correspond to the diagrams of figure [4.1b]. These two particles irreducible diagrams are individually finite in a physical gauge (15). Furthermore, the sum of all the contributions of figure [4.1b] is gauge invariant and therefore finite in any gauge. This finiteness becomes even more clear if we consider the way the mass singularities are expected to be absorbed. Recalling equation (2.47)

$$\frac{d\sigma_{qq}}{dq^2} = \frac{4\pi\alpha_{em}^2}{3sq^2} \{ \int_\tau^1 \frac{dx}{x} [G_{\bar{q}/q_1}(x,-q^2)\kappa_0(\frac{\tau}{x}) + G_{g/q_1}(x,-q^2)$$

$$\times \kappa_1(\frac{\tau}{x})] + [q_1 \leftrightarrow q_2] + \kappa_2(\tau) \qquad (4.1)$$

with κ_0 and κ_1 proportional to $e_{q_2}^2$, we see that there is no way that mass singularities proportional to $e_{q_1}e_{q_2}$ can be absorbed. Furthermore, since the arbitrariness in the determination of the constant terms in κ_2 arises from the factorization of the singular terms, it follows that the interference contribution to κ_2 is unique as well as finite. This is not the case for the squared terms that have double logarithmic mass singularities. The non-leading logarithmic term coefficient and the finite term are then not unique, namely, they depend on the regularization prescription. However, when logarithmic terms (leading and non-leading) resulting from perturbation, are compared to expression (4.1), the parton densities being defined with respect to DIS, in the same gauge and regularization prescription, they are found to be identical. The term κ_2 is given by the subtraction of the finite term of the integral in equation (4.1) from the original finite term found in perturbation calculation. Then, κ_2 is regularization-prescription as well as gauge independent, the dependence being cancelled in the subtraction process.

In the case of identical quarks, additional singular contributions of the type of figure [4.1c] must also be taken into account. These extra terms are studied in a very recent work (34) and their contribution is found to be very small. In this work we restrict ourselves to non-identical quarks.

Consider now the finite $O(\alpha_s^2)$ quark-quark correction term κ_2. It can be written as:

$$\kappa_2(\tau)=\left(\frac{\alpha_s}{\pi}\right)^2 \left[(e_{q_1}^2 + e_{q_2}^2)\, C_A(\tau) + 2e_{q_1}e_{q_2}\, C_B(\tau)\right] \tag{4.2}$$

corresponding to the two different gauge-invariant contributions discussed before. The perturbation calculations leading to $C_A(\tau)$ and $C_B(\tau)$ are made in the Coulomb gauge which is more appropriate for the analysis of the singular contributions. Of course, this is a physical gauge so that none of the diagrams contributing to $C_B(\tau)$ will give rise to mass singularities. This gauge is defined in Appendix A, which also includes some details on the matrix elements considered in this section.

Consider now the squared terms of figure [4.1a]. We can see that these contributions will have in common the trace

$$\mathrm{Tr}[\not{p}_1\, \not{\epsilon}(k_1,\lambda')\, (\not{p}_1-\not{k}_1)\, \not{\epsilon}(k_1,\lambda'')] \tag{4.3}$$

This trace corresponds to the emission of a gluon off shell from the quark line q_1 of momentum p_1. This suggests that the quark-quark (qq) matrix element may be written in terms of the quark-gluon (qg) matrix element. Detailed calculations (see Appendix A) lead to

$$\int_0^{2\pi} d\phi \; |M_{qq}|^2 \simeq - \frac{4\pi}{\alpha_1^2} \left[\frac{1+(1-\alpha_1)^2}{k_1^2} - \frac{\alpha_1(2-\alpha_1)p_1^2}{(k_1^2)^2} \right] |M_{qg}|^2$$

$$+ \; 2^8\pi \; \frac{s}{(k_2^2)^2} \; \frac{(1-\alpha_1)\,\tau\,(\alpha_1-\tau)}{\alpha_1^4} \qquad (4.4)$$

and,

$$\frac{d\sigma_{qq}}{dq^2} \simeq \frac{4\pi\alpha_{em}^2}{3sq^2} \left(e_{q_1}^2 + e_{q_2}^2\right) \frac{N^2-1}{4N^2} \frac{s}{2^7\pi} \left(\frac{\alpha_s}{\pi}\right)^2 \int_0^1 \frac{d\alpha_1}{\alpha_1} \, d\beta_1 \, d\alpha_2$$

$$\times \; \theta(\alpha_1-\alpha_2) \; \theta\!\left(\alpha_2-\tau-\frac{\alpha_2}{\alpha_1}(\beta_1-\alpha_1 p_1^2/s)\right) \int_0^{2\pi} d\phi \; |M_{qq}|^2$$

$$(4.5)$$

$|M_{qg}|$ is related to the matrix element for $qg \to q\gamma^*$ and is defined in Appendix A (see also reference 15). The kinematics involved in this problem are fully described in Chapter V, where we are calculating the differential cross section $d\sigma/d^4q = d\sigma/MdMdyd^2q_T$ in order to get the qq correction to the q_T distribution of dileptons.

Upon integration, one finds that $d\sigma_{qq}/dq^2$ can be cast in the form

$$\frac{d\sigma_{qq}}{dq^2} = \frac{4\pi\alpha_{em}^2}{3sq^2} \frac{(e_{q_1}^2 + e_{q_2}^2)}{2N} (\frac{\alpha_s}{2\pi})^2 \int_\tau^1 \frac{d\alpha_1}{\alpha_1} F(\alpha_1,\tau;q^2) \tag{4.6}$$

$F(\alpha,\tau;q^2)$ contains the following terms:

$$F(\alpha,\tau;q^2) = \Lambda_2(\alpha,\tau/\alpha) \ln^2 \frac{q^2}{-\tau p^2} + \Lambda_1(\alpha,\tau/\alpha) \ln \frac{q^2}{-\tau p^2} + \Lambda_0(\alpha,\tau/\alpha) \tag{4.7}$$

where

$$\Lambda_2(\alpha,z) = P_{gq}(\alpha) P_{qg}(z)$$

$$\Lambda_1(\alpha,z) = P_{gq}(\alpha) [2P_{qg}(z) (\ln \frac{1-z}{z} - 1) + \frac{1}{2} + z - \frac{3}{2}z^2]$$
$$+ 8 C_F \frac{1-\alpha}{\alpha} z(1-z) - 2 C_F (\frac{2-\alpha}{\alpha}) P_{qg}(z)$$

$$\Lambda_0(\alpha,z) = P_{gq}(\alpha) \{P_{qg}(z) [\ln^2(1-z) - 2\ln(z)\ln(1-z)$$
$$+ 2\int_0^{1-z} \frac{d\beta}{\beta} \ln 1-\beta] + (\frac{1-z}{z})(1+3z) \ln(1-z) + \frac{3}{2} z^2 \ln z$$
$$- \frac{z}{2}(1-z)\} + 2[1-\ln(1-z)] [P_{gq}(\alpha) P_{qg}(z) - 4C_F \frac{1-\alpha}{\alpha}$$
$$z(1-z)] - 2C_F \frac{2-\alpha}{\alpha} [P_{qg}(z) (-2+\ln(\frac{1-z}{z})) + \frac{1-z}{4} (1+3z)] \tag{4.8}$$

As we mentioned before, we do not include the interference terms for identical quarks. These give only non-leading contributions for $\tau \to 1$ and may be discarded. Then, the expressions for Λ_2 and Λ_1 are exact. This is not the case for Λ_0 which, however, contains all the terms contributing for $\tau \to 1$.

The coefficient of the leading logarithmic term $\Lambda_2(\alpha,z)$ corresponds exactly to the leading term in the antiquark density in a quark $G_{\bar{q}/q}(x,-q^2)$, as we expect from equation (4.1). Furthermore, the non-leading $O(\ln q^2/-p^2)$ term coefficient

$$- 2\, \Lambda_2(\alpha,\tau/\alpha)\, \ln \tau + \Lambda_1(\alpha,\tau/\alpha) \tag{4.9}$$

is just the same as the term

$$K(\tau,\alpha) + 2\, P_{gq}(\alpha)\, [P_{qg}(\tau)\, \ln(1-\tau) + \frac{3}{4} - \frac{5}{2}\tau + \frac{9}{4}\tau^2] \tag{4.10}$$

of equation (4.1), where $K(\tau,\alpha)$ is defined in Appendix B. The equality of these two terms guarantees that all the mass singularities are absorbed in parton distribution functions.

The constant (non-logarithmic) term is simply

$$\Lambda(\alpha,\tau) = \Lambda_2(\alpha,\tau/\alpha)\, \ln^2\tau \;- \Lambda_1(\alpha,\tau/\alpha)\, \ln^2\tau \;+ \Lambda_0(\alpha,\tau/\alpha) \tag{4.11}$$

Now, according to equation (4.2), this term contains the contributions arising from the constant terms of the parton densities as well as the $O(\alpha_s^2)$ qq correction term proportional to $(e_{q_1}^2 + e_{q_q}^2)$ in $\kappa_2(\tau)$ (i.e. $C_A(\tau)$). To obtain the correct expression for C_A, we eliminate these contributions from the perturbation calculation result as it is shown in equation (2.51). Then, in terms of (2.48) and (2.49), we get

$$C_A(\tau) \simeq \frac{1}{8N} \int_\tau^1 \frac{d\alpha}{\alpha} \{\Lambda_2(\alpha,\tau/\alpha) \ln^2\tau - \Lambda_1(\alpha,\tau/\alpha) \ln^2\tau + \Lambda_0(\alpha,\tau/\alpha)$$

$$-[2P_{qg}(\tau/\alpha) \ln(1-\tau/\alpha) + \frac{3}{2} - \frac{5\tau}{\alpha} + \frac{9\tau^2}{2\alpha^2}] u_{gq}(\alpha) - u_{\overline{q}q}(\tau,\alpha)\} \tag{4.12}$$

This is where the convention adopted for u_{gq} and $u_{\overline{q}q}$ plays an important role. In Chapter III, we specified $u_{\overline{q}q}$ by subtracting the corresponding $O(\alpha_s^2)$ correction to the DIS structure function $F_2^h(x,q^2)$ due to the subprocess $q\gamma^* \to qq\overline{q}$. The same procedure has been used to determine u_{gq} but this time , with respect to the longitudinal structure function $F_L^h(x,q^2)$. These conventions lead to a complete determination of the $O(\alpha_s^2)$ correction term C_A. In the limit of large τ ($\tau \to 1$), which determines here the dominant contribution as we shall see later in this section, we obtain

$$C_A(\tau) \simeq \frac{C_F}{8N} (1-\tau) [\frac{1}{2} \ln^2(1-\tau) - \ln(1-\tau) + 4 - \frac{7\pi^2}{24}]$$

$$+ \quad O((1-\tau)^2 \ln(1-\tau)) \qquad (4.13)$$

Consider now the interference terms of the type of figure [4.1b]. As we discussed above they give a contribution proportional to $e_{q_1} e_{q_2}$ that is finite and, therefore, the problem of regularization-prescription dependence do not arise. This has already been calculated [15,30]. The expression for C_B is long and tedious; we shall only state the result for the region of interest, i.e. near $\tau = 1$,

$$C_B(\tau) \simeq (1-\tau)^3 \qquad (4.14)$$

C_B turns out to be suppressed by two powers of $(1-\tau)$ relative to C_A. Since it is the region $\tau \simeq 1$ that controls the correction under consideration, the interference term contribution ($\sim C_B(\tau)$) is not important.

The complete functions C_A and C_B as calculated in reference 15, are shown on figure [4.2].

The contribution of the subprocess $qq \to qq\gamma^*$ to the physical process $h_1 + h_2 \to \ell^+\ell^- + X$, is

$$\frac{d\sigma^{qq}_{h_1h_2}}{dq^2} = \frac{4\pi\alpha^2_{em}}{3sq^2} \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} G_{q_i/h_1}(x_1,q^2)\, G_{q_j/h_2}(x_2,q^2)$$

$$\times \kappa_2(\tau/x_1x_2) \qquad (4.15)$$

To obtain an idea of the order of magnitude of this correction, we have carried a calculation for proton-proton collisions, where the contribution is expected to be dominated by valence quarks ($p + p \rightarrow \ell^+ \ell^- + X$).

We use the distribution functions $G_{q_i/p}$ ($\sim (1-x)^3$) of reference 35, which are based on a well-known counting rules[36,37] and on the parametrizations for q^2-dependence suggested in reference 38. As a result, this solution uses as inputs

$$x\ G_{\bar{q}/p}(x,\ q_0^2) \sim (1-x)^7 \tag{4.16}$$

$$x\ G_{g/p}(x,\ q_0^2) \sim (1-x)^5 \tag{4.17}$$

with $q_0^2 = 1.8\ \mathrm{GeV}^2$. Clearly, we can see that all the distribution functions converge more or less rapidly to zero as $x \rightarrow 1$. Therefore we expect that the correction to the physical process will get most of its contributions from small values of x_1 and x_2. But for such values, the argument of the correction κ_2 (i.e. τ/x_1x_2) is large and near the kinematic limit $\tau/x_1x_2 \leqq 1$. Carrying the integrations in equation (4.15) the dominant contribution, by far, comes from this region and this justifies the use of the approximate form of $C_A(\tau)$ given in equation (4.13) and of $C_B(\tau)$ discussed before. Such an approximation has been shown [15] to be good raising an error $\lesssim$ 40% down to values of $\tau \simeq 0.2$, when compared to the full expression for κ_2.

The results of our numerical calculations are presented in the form of the ratio

$$R(\tau,s) = \frac{(d\sigma_{pp}^{qq}/dq^2)}{(d\sigma_{pp}/dq^2)_{DY}} \qquad (4.18)$$

Here $(d\sigma_{pp}/dq^2)_{DY}$ denotes the Drell-Yan cross section corresponding to equation (2.39) calculated with the non-scaling q and $\bar{q}$ distribution functions of reference 35. The ratio $R(\tau,s)$ is plotted in figure [4.3] (solid lines) at energies $\sqrt{s}$ = 6.5 GeV and $\sqrt{s}$ = 27 GeV (Brookhaven and Fermilab energies respectively).

The conclusion that we can draw from this figure is that, at presently available dilepton masses ($\tau \lesssim 0.3$), the correction arising from the $O(\alpha_s^2)$ $qq \rightarrow qq\gamma^*$ subprocess is very small and can be neglected. This is certainly true at Fermilab energies ($\sqrt{s}$ = 27 GeV). This conclusion is consistent with the fact that the Drell-Yan mechanism explains all the basic features of the experimental data. It is also true at Brookhaven energies ($\sqrt{s}$ = 6.5 GeV) although the contribution is more significant. This may somewhat affect tests of scaling within the Drell-Yan model. Also the presence of a large $O(\alpha_s^2)$ qq correction should be taken into account when extracting antiquark distribution functions near x = 1.

The general behaviour of $R(\tau,s)$ is easily understood qualitatively. For fixed τ, the decrease of $R(\tau,s)$ with increasing s, is mainly due to the decrease of α_s, the running coupling constant. For fixed s, $R(\tau,s)$ undergoes a rapid increase with τ. The reason for such an increase lies on the form of the distribution functions

$$x\, G_i\,(x,-q_0^2) \sim (1-x)^{n_i} \tag{4.19}$$

The valence distribution functions involve a power $n_v \sim 3$-4, while the sea distribution function used in Drell-Yan is significantly smaller with $n_s \geqq 7$.

In fact, we should stress that $R(\tau,s)$, in particular for large τ, is sensitive to the assumption made about the exact form of the sea distribution function as well as the gluon distribution function, which through mixing effects is known to influence the evolution of the $\bar{q}$ distribution. Therefore, the results for $R(\tau,s)$ should really be taken as an indication of the order of magnitude and of the qualitative features of the $O(\alpha_s^2)$ $qq \to qq\gamma^*$ contribution. On the other hand, the correction itself, $d\sigma_{pp}^{qq}/dq^2$, since it involves quark valence distributions, does not show much sensitivity to changes in $G_{\bar{q}/p}(x,\, q_0^2)$ and $G_{g/p}(x,q_0^2)$.

4-2 Comparison with other conventions

As discussed before, the correction term $C_A(\tau)$ as well as the ratio $R(\tau,s)$ are sensitive the convention regarding the choice of the non-logarithmic terms u_{gq} and $u_{\bar{q}q}$. We recall here briefly some of the conventions that have already been proposed.

The question of magnitude of the quark-quark correction to dilepton production was first studied in reference 15. In this work, the densities $G_{g/q}(x,q^2)$ and $G_{\bar{q}/q}(x,q^2)$ were defined in a way different from that of section 2-2. The convention adopted in reference 15 amounts to choosing:

$$u_{gq}(x) = - P_{gq}(x) \ln x - C_F \frac{2-x}{x} \qquad (4.20)$$

which leads to the following limiting form of $C_A(\tau)$ (for $\tau \simeq 1$)

$$C_A(\tau) \simeq \frac{C_F}{8N} (1-\tau) [\frac{1}{2} \ln^2(1-\tau) - 2 \ln(1-\tau) + 3] \qquad (4.21)$$

where the function $u_{\bar{q}q}$ has been fixed by requiring no $O(\alpha_s^2)$ correction from the $\gamma^* q \to qq\bar{q}$ subprocess as in section 3-4. The results in terms of the ratio $R(\tau,s)$ defined in equation (4.18) are presented in figure [4.3] (dash-dotted lines). The corection is somewhat bigger, but with the same qualitative features and of the same order of magnitude as before.

A different convention is proposed in reference 16. It is related with the definition of a quark density in a quark.

$$G_{q/q}(x,q^2) = \frac{\alpha_s}{2\pi} \{P_{qq}(x) \ln \frac{q^2}{p^2} + u_{qq}\} \qquad (4.22)$$

where

$$P_{qq}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\, \delta(x-1) \right] \qquad (4.23)$$

The function u_{qq} is specified by requiring [6,7,16] no $O(\alpha_s)$ correction generated by the subprocess $\gamma^* q \to gq$ to leptoproduction; and this implies

$$u_{qq}(x) = C_F \left[-2 \frac{1+x^2}{(1-x)_+} \ln x + 1 + 3x - \frac{3}{2} \frac{1}{(1-x)_+} - \frac{2\pi^2}{3} (1-x) \right]$$

$$(4.24)$$

as we can see from equation (2.15). The authors then invoke the conservation-of-momentum sum rule by the gluon field, i.e.

$$\int_0^1 dx\, x \left[\sum_i G_{q_i/h}(x,q^2) + G_{g/h}(x,q^2) \right] = 1 \qquad (4.25)$$

This imposes the following condition on the second moments ($n = 2$) of $u_{gq}(x)$ and $u_{qq}(x)$:

$$\tilde{u}_{gq}(n=2) = \tilde{u}_{qq}(n=2) \qquad (4.26)$$

where

$$\tilde{w}(n) = \int_0^1 dx\, x^{n-1}\, w(x) \qquad (4.27)$$

To obtain a complete definition for the gluon, reference 16 proposes that this relation be extended to all moments n. This implies:

$$u_{gq}(x) = -u_{qq}(x) \tag{4.28}$$

One also needs to specify $u_{\bar{q}q}(x,\alpha)$, and for this, one may require no $O(\alpha_s^2)$ correction to leptoproduction, as in section 3-4 ($C_2(x) \equiv 0$). Then, for not too small τ, this convention leads to a correction term $C_A(\tau)$ that has form:

$$C_A(\tau) \simeq \frac{C_F}{8N} \Big[\frac{3}{2} \ln^2(1-\tau) - \frac{2\pi^2}{3} \ln(1-\tau) - \frac{5}{12} \pi^2$$
$$+ (1-\tau) \Big[-\frac{5}{2} \ln^2(1-\tau) + \Big(\frac{1}{2} + \frac{4\pi^2}{3}\Big) \ln(1-\tau)\Big]\Big] \tag{4.29}$$

The resulting $R(\tau,s)$ appears in figure [4.3] for $\sqrt{s}$=27 GeV (dash-dot-dotted line ; notice that the result is multiplied by 10^{-2}). Clearly, the convention (4.28) introduces a too large correction to dilepton production. This comes from the fact that $C_A(\tau) \to \infty$ when approaching $\tau = 1$, whereas the previous convention gave $C_A(\tau) \to O(1-\tau)$. Such divergent behaviour near $\tau = 1$ is due to the singular terms $\sim 1/(1-x)_+$ and $\sim \delta(1-x)$ in $u_{gq}(x)$. Since the correction to the physical cross section $d\sigma_{pp}^{qq}/dq^2$ is dominated by the region of integration where $\tau_{12} \simeq 1$, the correction is found to be very large.

Such a large correction renders useless the successful phenomenology of dilepton production based on Drell-Yan mechanism. We may conclude that the extension of (4.26) to all moments is a condition too strong and unnecessary.

We consider now the convention of reference 31 which is described in Appendix B. This is based on the relation used in the procedure of absorbing the mass singularities

$$G^{\alpha_s}_{\bar{q}/h}(x,q^2) = \int_0^1 \frac{d\alpha}{\alpha}\, G_{\bar{q}/g}(\tfrac{x}{\alpha},q^2)\; G_{g/h}(\alpha,q^2) \qquad (4.30)$$

and corresponds to extending this relation to h=quark:

$$G_{\bar{q}/q}(x,q^2) \equiv \int_x^1 \frac{d\alpha}{\alpha}\, G_{\bar{q}/g}(\tfrac{x}{\alpha},q^2)\; G_{g/q}(\alpha,q^2) \qquad (4.31)$$

Last condition specifies both $u_{gq}(x)$ and $u_{\bar{q}q}(x,\alpha)$ and leads to a quark-quark correction term near $\tau = 1$ with:

$$C_A(\tau) \simeq \frac{C_F}{8N}\,(1-\tau)\,[\tfrac{1}{2}\ln^2(1-\tau) + \ln(1-\tau) + 2] \qquad (4.32)$$

The corresponding ratio $R(\tau,s)$ is also presented in figure [4.3] (dashed lines). We see that now the result is very similar to that of $R(\tau,s)$ calculated with the conventions of Chapter III (solid lines) differing by less than a factor of 2. The reason is that near $\alpha = 1$, the convention of Chapter III implies $u_{gq}(\alpha) \sim -2C_F$, whereas the convention of reference 31 gives $u_{gq}(\alpha) \sim -4C_F$. Notice that this is also the case for the conventions of reference 15 (dash-dotted lines) i.e. $R(\tau,s)$ has a similar shape but differs from the solid lines by a factor of ~ 2; in reference 15, for large α, $u_{gq}(\alpha) \sim -C_F$.

We must mention that with the conventions of references

15 and 31, the $O(\alpha_s^2)$ qq corrections to leptoproduction do not vanish. However, in both cases, they are found to be very small.

The last convention that we shall examine is that of reference 30. In order to subtract the mass singularities, they use essentially a slight extension of the method of reference 26 for the calculation of leading logarithms of transition functions. They apply this method to the subprocess $q + B \rightarrow q + X$, where both B and X (anything) are chosen to be scalar. The initial quark emits a gluon and this is followed by a process of the type $g + B \rightarrow X$. This fixes the non-logarithmic term u_{gq} of the gluon density in a quark. The function $u_{\bar{q}q}$ is again determined with respect to leptoproduction as in section 3-4. Near $\tau = 1$, they find

$$C_A(\tau) \simeq \frac{C_F}{8N} (1-\tau) \left[\frac{1}{2} \ln^2(1-\tau) - 3\ln(1-\tau)\right] \tag{4.33}$$

Notice that the coefficient of the leading logarithmic term is the same as that of the conventions of section 3-2, Appendix B and reference 15. Because the qq correction is dominated by contributions near $\tau = 1$, we expect that this convention will lead to results for $R(\tau,s)$ similar to those generated by the aforementioned conventions. Furthermore, the coefficient to the next leading logarithmic term in equation (4.33) is larger; this should slightly raide $R(\tau,s)$ above those already calculated except the one from the convention suggested by reference 16 which have the limit $C_A(\tau) \to \infty$ as $\tau \to 1$.

The results of reference 30 are presented in a different fashion. Furthermore, they use different distribution functions which have the effect of reducing the Drell-Yan contribution near $\tau = 1$. Taking this into consideration, however, we find that their convention leads to the same order of magnitude for the quark-quark correction to the Drell-Yan process. The same general features of $R(\tau,s)$ are also observed.

Finally, let us just mention that reference 30 finds for the contribution of the interference term near $\tau = 1$:

$$C_B(\tau) \simeq -\frac{7}{8}(1-\tau)^2 \tag{4.34}$$

This is in some disagreement with our result (we find $C_B(\tau) \simeq (1-\tau)^3$). As we already discussed this interference term do not give any mass singularity and is convention independent. Anyway, $C_B(\tau)$ of reference 30 is still suppressed by one power of $(1-\tau)$ (neglecting logarithms) with respect to $C_A(\tau)$ and therefore does not significantly affect the contribution to the physical cross section.

CHAPTER V

TRANSVERSE MOMENTUM DISTRIBUTION OF DILEPTONS

As a second application, we consider the transverse momentum (q_T) distribution of dileptons in proton-proton collisions, $p + p \to \ell^+\ell^- + X$. Experiment shows that dileptons are sometimes produced with high transverse momenta ($q_T \sim M$ = dilepton mass). In the Drell-Yan picture the q_T distribution can be accounted for only by the intrinsic transverse motion of the quark and antiquark (due to gluon Bremsstrahlung) in the hadron. This intrinsic motion together with higher order effects due to soft (multiple) gluon Bremsstrahlung [5,39,40] are known to be very important in determining the low q_T distribution of dileptons. However, because the intrinsic transverse momentum of the partons is beleived to be rather small ($\sim$ 300 MeV) compared to the high $\langle q_T \rangle$ observed ($\sim$ 1 GeV) the Drell-Yan mechanism alone is inadequate to describe the q_T behaviour of dileptons in this region. One must then proceed with the calculation of higher order subprocesses.

Neglecting the intrinsic transverse motion effects, the QCD subprocesses

$$q + g \to q + \gamma^* \tag{5.1}$$

$$q + \bar{q} \to g + \gamma^* \tag{5.2}$$

are known to provide the Born terms to the q_T distribution of the dileptons and they have been shown to account partly for the experimental q_T distribution [41-45].

However, even at the highest available q_T these predictions fall somewhat below the data. It is therefore essential to examine the next order contributions. This is important in particular for the $O(\alpha_s^2)$ contribution from the $qq \to qq\gamma^*$ subprocess for proton-proton collisions This contribution may well be comparable to those from the subprocesses (5.1) and (5.2) because it involves valence instead of gluon and sea distribution functions.

In the first section, we briefly recall the calculation of the contribution from the subprocess (5.1). This will set the basis of our calculations for the $O(\alpha_s^2)$ quark-quark subprocess to be performed in section 5-2. In section 5-3, the mass singularities arising from the perturbation calculation are absorbed through the redefinition of the gluon distribution function. With the use of the complete definition of the gluon density in a quark, this procedure will set unambiguously the correction terms due to the $qq \to qq\gamma^*$ subprocess. Finally the last section examinesthe contribution to the physical process $p+p \to \ell^+\ell^- + X$ brought by these correction terms. Different conventions for the parton densities are then compared and discussed.

The quantity we are interested in is the differential cross section

$$\frac{d\sigma}{d^4q} = \frac{d\sigma}{MdMd^2q_T dy} \tag{5.3}$$

This definition is related to the four-momentum of the virtual photon, q, with components

$$q = ((M^2 + q_T^2)^{1/2} \cosh y \; ; \; \vec{q}_T \; , \; (M^2 + q_T^2)^{1/2} \sinh y) \tag{5.4}$$

where M is the dilepton mass, $\vec{q}_T$ is the transverse momentum and y is the rapidity in the c.m. frame of the colliding partons.

5-1 Quark-gluon contribution

As we just discussed, subprocess (5.1) contributes to the Born term of the q_T distribution of dileptons. Furthermore this subprocess is included (with the gluon off mass shell) in the $O(\alpha_s^2)$ qq subprocess that we are interested in. We will see in the next section that the leading contribution of this subprocess is closely related to that of $qg \to q\gamma^*$.

We now briefly consider the subprocess $qg \to q\gamma^*$. The differential cross section $d\sigma/d^4q$ in the notation of figure [2.7a] is given by

$$\frac{d\sigma_{qg}}{d^4q} = \frac{4\pi\alpha_{em}^2}{9sq^2} \; e_q^2 \; \frac{\alpha_s}{16\pi^2} \int d^4k \; \delta_+[(p_1-k)^2] \; \delta^{(4)}(p_2+k-q)|M_{qg}|^2 \tag{5.5}$$

where $|M_{qg}|$ is proportional to the matrix element. Both initial partons can be taken slightly off shell. However, this is not really necessary because no mass singularity arises from the integral. The first δ-function indicate that the final quark must be taken on mass shell with positive energy and the second δ-function simply states the energy-momentum conservation for the subprocess.

Perturbation calculation gives for $|M_{qg}|$

$$|M_{qg}|^2 = 8\,[-\frac{s}{k^2} + \frac{k\cdot(p_1+p_2)\ (p_1-k)\cdot p_2}{k^2 s} - \frac{2p_1\cdot k}{s} + \frac{2p_1^2\ k\cdot p_2}{(k^2)^2}] \tag{5.6}$$

where $s=(p_1 + p_2)^2$ (see Appendix A for details of calculations). The $O(p_1^2)$ term is unimportant in calculating the q_T distribution of dilepton and it will be neglected.

Introducing the dimensionless invariants τ, ξ, and η for the quark-gluon subprocess, we write:

$$\begin{aligned} \tau &\equiv \frac{q^2}{s} \equiv \frac{M^2}{s} \\ \xi &\equiv \frac{2p_1\cdot q}{s} = (\frac{M^2+q_T^2}{s})^{1/2}\ e^{-y} \\ \eta &\equiv \frac{2p_2\cdot q}{s} = (\frac{M^2+q_T^2}{s})^{1/2}\ e^{y} \end{aligned} \tag{5.7}$$

Furthermore we work in the c.m. of the colliding gluon and quark, and we may write:

$$p_1 = \frac{\sqrt{s}}{2}\,(1\ ;\ \vec{0}\ ,\ 1)$$

$$p_2 = \frac{\sqrt{s}}{2}\,(1\ ;\ \vec{0}\ ,\ -1) \qquad (5.8)$$

Then using the first δ-function, we find that the requirement that the final quark be on mass shell leads to

$$\begin{aligned}(p_1 - k)^2 &= (p_1 + p_2 - q)^2 \\ &= (p_1 + p_2)^2 - 2(p_1 + p_2)\cdot q + q^2 \\ &= s\,(1 + \tau - \eta - \xi) = 0 \qquad (5.9)\end{aligned}$$

Its energy is required to be positive, i.e.

$$\begin{aligned}(p_1 - k)^0 &= (p_1^0 + p_2^0 - q^0) \\ &= \frac{\sqrt{s}}{2}\,(2 - (\xi + \eta)) \geq 0\end{aligned}$$

or, $\qquad \xi + \eta \geq 2 \qquad (5.10)$

where we have used equations (5.8) and the fact that the energy component of q may be written:

$$q^0 = (\xi + \eta)\,\frac{\sqrt{s}}{2} \tag{5.11}$$

The integration in equation (5.5) can be performed with the help of $\delta^{(4)}(p_2+k-q)$. Then, writing $|M_{qg}|^2$ in terms of the variables τ, ξ and η, we obtain for the differential cross section of the $O(\alpha_s)$ quark-gluon subprocess

$$\frac{d\sigma_{qg}}{d^4q}(s,\tau,\xi,\eta;\alpha_s) = \frac{4\pi\alpha_{em}^2}{9sq^2}\, e_q^2\, \frac{\alpha_s}{2\pi^2}\, \delta(1+\tau-\xi-\eta)\; F(\tau,\xi,\eta) \tag{5.12}$$

where

$$F(\tau,\xi,\eta) \equiv \frac{1-2\tau(1-\eta)}{\eta-\tau} + 1 - \xi \tag{5.13}$$

Consider then the physical process $h_1 + h_2 \to \ell^+\ell^- + X$ where h_1 and h_2 are hadrons. The contribution of the subprocess $qg \to q\gamma^*$ to the hadronic differential cross section is:

$$\frac{d\sigma^{qg}_{h_1h_2}}{d^4q} = \int dx_1\, dx_2\, G_{q/h_1}(x_1)\, G_{g/h_2}(x_2)\, \frac{d\sigma_{qg}}{d^4q}(\tilde{s},\tau_{12},\frac{\xi}{x_2},\frac{\eta}{x_1};\alpha_s)$$

$$+ (1\leftrightarrow 2) \tag{5.14}$$

where

$$\tilde{s} = x_1x_2s \qquad\qquad \tau_{12} = \tau/x_1x_2 \tag{5.15}$$

The change in the argument of the cross section is simply due to the change of variables $p_1 \to x_1p_1$ and $p_2 \to x_2p_2$, where p_1

and p_2 are now the momenta of the hadrons involved. The relation (5.10) is then written

$$\frac{\xi}{x_1} + \frac{\eta}{x_2} \geqq 2 \qquad (5.16)$$

It determines the region of integration of the variables x_1 and x_2, together with the condition $0 \leqq x_1, x_2 \leqq 1$.

5-2 $O(\alpha_s^2)$ quark-quark contribution

We are interested in the contribution to the q_T distribution coming from the subprocess $qq \to qq\gamma^*$. We shall consider only the "squared" terms of figure [4.1a] that have the charge structure e_q^2. The contribution of the "interference" terms, as well as of identical quarks, will not be included. They are expected to be small compared to the squared terms, as the results of Chapter IV indicate .

In the notation of figure [2.8], the differential cross section $d\sigma/d^4q$ is given by:

$$\frac{d\sigma_{qq}}{d^4q} = \frac{4\pi\alpha_{em}^2}{9sq^2} \left(e_{q_1}^2 + e_{q_2}^2\right) \frac{\alpha_s^2}{32\pi^4} \int d^4k_1 \; \delta_+((p_1-k_1)^2)$$

$$\times \; \delta_+((k_1-k_2)^2) \; |M_{qq}|^2 \qquad (5.17)$$

where $|M_{qq}|$ is proportional to the matrix elements of the squared terms. The δ-functions indicate that the final quarks must be taken on mass shell with a positive energy.

Again, the regularization procedure consists of taking the initial partons slightly off mass shell, i.e., p_1^2 and p_2^2 are small and negative.

Let us now introduce the appropriate set of Sudakov variables to facilitate the integrations. We write,

$$p_1 - k_1 = (1-\alpha)p_1' + \quad p_2' + \ell \tag{5.18}$$

with the element of four-momentum

$$d^4k_1 = \frac{s}{4}\, d\alpha\ d\beta\ d\ell_T^2\ d\phi \tag{5.19}$$

The momenta p_1' and p_2' have the usual property

$$p_1'\cdot\ell = p_2'\cdot\ell = 0 \tag{5.20}$$

Then, α and β are scalars running from zero to one, $\vec{\ell}_T$ is the transverse component of ℓ and, ϕ defines the direction of $\vec{\ell}_T$ in the transverse plane. s is the c.m. energy squared

$$s = (p_1^2 + p_2^2)^2 \tag{5.21}$$

We can write p_1' and p_2' in terms of p_1 and p_2 as follows:

$$p_1' \equiv p_1 - \frac{p_1^2}{s} p_2 \quad \text{or} \quad p_1 = p_1' + \frac{p_1^2}{s} p_2' + O(\frac{p^4}{s^2} p_2')$$

$$p_2' \equiv p_2 - \frac{p_2^2}{s} p_1 \quad \text{or} \quad p_2 = p_2' + \frac{p_2^2}{s} p_1' + O(\frac{p^4}{s^2} p_1') \tag{5.22}$$

where p_1^2, $p_2^2 \sim p^2$, so that in the c.m. of the colliding partons

$$p_1' \simeq \frac{\sqrt{s}}{2} (1 \; ; \vec{0} \; , 1) \qquad p_2' \simeq \frac{\sqrt{s}}{2} (1 \; ; \vec{0} \; , -1)$$

$$\ell = (0 \; ; \vec{\ell}_T \; , 0) \tag{5.23}$$

Notice that here

$$p_1'^2 = O((\frac{p^2}{s})^2) \neq 0 \qquad p_2'^2 = O((\frac{p^2}{s})^2) \neq 0 \tag{5.24}$$

However, these values can certainly be neglected for small p^2 and, from then on, we will simplify the calculations using $p_1'^2 = p_2'^2 = 0$.

We shall work with the same dimensionless invariants as in section 5-1 (i.e. τ, ξ and η) except that p_1 is now assigned to the initial quark instead of the gluon. The virtual photon momentum, q, still have the form of equation (5.4), with $\vec{q}_T$ and y representing the transverse momentum and the rapidity in the c.m. of the colliding quarks.

Consider now the first δ-function. It implies:

$$(p_1 - k_1)^2 = (1-\alpha)\beta s - \ell_T^2 = 0 \tag{5.25}$$

and

$$(p_1 - k_1)_0 = \frac{\sqrt{s}}{2}\left(1-\alpha+\beta-(1-\alpha)\frac{p_1^2}{s} - \beta\frac{p_2^2}{s}\right)$$

$$\simeq \frac{\sqrt{s}}{2}(1-\alpha+\beta) \geqq 0 \tag{5.26}$$

for small p_1^2 and p_2^2.. Notice that equation (5.25) implies $\ell_T^2=0$ and

$$k_1 \simeq \alpha p_1 \tag{5.27}$$

when $\beta=0$, i.e. the gluon is emitted collinearly with the initial quark. The second δ-function impose conditions on (k_1-k_2); here,

$$k_1 - k_2 = \left(\left(\alpha+\frac{p_2^2}{s}\right)p_1' + \left(1-\beta+\frac{p_1^2}{s}\right)p_2' - \ell - q\right)$$

$$\simeq \alpha p_1' + (1-\beta)p_2' - \ell - q \tag{5.28}$$

if we neglect the $O(\frac{p^2}{s})$ terms. Then, the on-mass-shell condition gives for small p^2

$$(k_1 - k_2)^2 \simeq \alpha(1-\beta)s - \ell_T^2 + q^2 - 2\alpha p_1'\cdot q - 2(1-\beta)p_2'\cdot q + 2\ell\cdot q$$

$$\simeq \tau s - \alpha\xi s - (1-\beta)\eta s - 2\ell_T q_T \cos\phi = 0 \tag{5.29}$$

where we have used equation (5.25). The angle $\phi=0$ is chosen to coincide with the direction of $\vec{q}_T$ in the transverse plane.

It is necessary, so that the energy of the final quark be positive, that

$$(k_1 - k_2)_0 \simeq \alpha p'_{10} + (1-\beta)p'_{20} - \ell_0 - q_0$$

$$\simeq \frac{\sqrt{s}}{2} (1+\alpha-\beta-\xi-\eta) \geqq 0 \qquad (5.30)$$

We can now carry easily the integration over ℓ_T^2 using relation (5.25). Then,

$$J \equiv \int d^4k_1 \; \delta_+((p_1-k_1)^2) \; \delta_+((k_1-k_2)^2) \; |M_{qq}|^2$$

$$\simeq \frac{1}{4} \int d\alpha \; d\beta \; d\phi \; \delta(\alpha-\beta+\tau-\alpha\xi-(1-\beta)\eta-\frac{2}{\sqrt{s}} ((1-\alpha)\beta)^{1/2} q_T \cos\phi)$$

$$\times |M_{qq}|^2 \qquad (5.31)$$

The limits of integration of α and β are specified by relations (5.26) and (5.30) as well as $0 \leqq \alpha,\beta \leqq 1$. Relation (5.26) is automatically satisfied because of the more restraining condition that both α and β runs from zero to one. Relation (5.30) however, imposes an upper limit on β,

$$\beta_m = 1 + \alpha - \xi - \eta \qquad (5.32)$$

In DIS, when we considered the $O(\alpha_s^2)$ quark contribution, we refered to diagrams that are symmetric with respect to the exchange of the external lines with momenta p_2 and q to that of figure [2.8]. In that case we saw that, because of the gluon propagator, the most important contribution came from the region near $\beta=0$, i.e. where the gluon is emitted collinearly with the quark. For reason that will become apparent later, this is also the case for the $O(\alpha_s^2)$ quark-quark contribution to the q_T distribution of dileptons. Thus the argument of the δ-function in J can be rewritten as follows:

$$J \simeq \frac{1}{4} \int d\alpha \int_0^{\beta_m} d\beta \; \delta(\alpha+\tau-\alpha\xi-\eta) \int_0^{2\pi} d\phi \; |M_{qq}|^2 \qquad (5.33)$$

In this way, we have eliminated the angular dependence in the argument of the δ-function. The last integral has been calculated in reference (15). We reproduce the calculations in Appendix A. The result is

$$\int_0^{2\pi} d\phi \; |M_{qq}|^2 \simeq -\frac{4\pi}{\alpha} \left(\frac{1}{k_1^2} P_{gq}(\alpha) - C_F \frac{(2-\alpha)}{(k_1^2)^2} p_1^2 \right) |M_{qg}|^2$$

$$+ 2^8 \pi \, C_F \frac{s}{(k_2^2)^2} \; \frac{(1-\alpha) \; \tau^2 \; (\alpha-\tau)}{\alpha^4} \qquad (5.34)$$

where $|M_{qg}|$ is proportional to the quark-gluon matrix element defined in section 5-1. This proportionality of the matrix elements is easily understood; a part of the qq contribution is attributed to an off-shell qg contribution, where the gluon has been emitted from the initial quark.

The last term of equation (5.34) contains only a part of the non-dominant terms in the region of $\beta=0$; comparable contributions are left out. It has very little effect on the q_T distribution of $p+p \to \ell^+\ell^-+X$, and has been included merely as an indication of the non-dominant effects.

In terms of the Sudakov variables k_1^2 and k_2^2 are:

$$k_1^2 = (p_1 - (1-\alpha)p_1' - \beta p_2' - \ell)^2$$

$$\simeq \alpha p_1^2 - \beta s \qquad (5.35)$$

$$k_2^2 = (q - p_2)^2$$

$$\simeq (\tau - \eta)s \qquad (5.36)$$

where we have neglected $O(\frac{p^4}{s^2})$ and $O(\frac{p^2}{s})$ terms respectively.

We found in section 5-1, that $|M_{qg}|^2$ can be written in terms of the invariants τ, ξ and η as follows;

$$|M_{qg}(k,p_1,p_2)|^2 = 8\left(\frac{1-2\tau(1-\eta)}{(\eta-\tau)} + 1 - \xi \right) = 8F(\tau,\xi,\eta) \qquad (5.37)$$

However, now, we must substitute $k \to k_2$ and $p_1 \to k_1 \simeq \alpha p_1$. With this replacement, by going from the c.m. of the initial quarks with p_1 and p_2 to the c.m. of the quark with p_2 and the gluon with k_1, we find easily

$$\tau \to \tau/\alpha \quad , \quad \xi \to \xi \quad , \quad \eta \to \eta/\alpha \tag{5.38}$$

so that in equation (5.34)

$$|M_{qg}|^2 = 8\ F(\frac{\tau}{\alpha},\xi,\frac{\eta}{\alpha}) \tag{5.39}$$

The only β-dependence of the matrix element $|M_{qq}|$ originates from the k_1^2 and $(k_1^2)^2$ denominators. This justifies the statement made above that the major contribution comes from integrating near $\beta=0$.

We can now carry the integration over β. The integral J is then,

$$J \simeq \frac{\pi}{s} \int \frac{d\alpha}{\alpha}\, \delta(\alpha+\tau-\alpha\xi-\eta)\ \{\ (\ P_{gq}(\alpha)\ \ln \frac{s\beta_m}{-\alpha p_1^2} - C_F\ \frac{2-\alpha}{\alpha} + O(\frac{p_1^2}{s})\ )$$

$$\times\ 8\ F(\frac{\tau}{\alpha},\xi,\frac{\tau}{\alpha}) + 2^6 C_F\ \beta_m\ \frac{(1-\alpha)\ \tau^2\ (\alpha-\tau)}{(\tau-\eta)^2\alpha^3}\ \} \tag{5.40}$$

where we have used the approximations (3.44) and (3.45).

The integration over α is straightforward. However, the mass singularity arising near $p_1^2 = 0$ must be factorized in order to determine the correction terms to the q_T distribution of dileptons due to the subprocess $qq \to qq\gamma^*$. We proceed with the factorization in the next section.

5-3 Correction terms and factorization of the mass singularity

The splitting function P_{gq} in front of the logarithmic term in equation (5.40), indicates that the singularity can be removed with the help of the gluon density in a quark,

$$G_{g/q}(x,q^2) = \frac{\alpha_s}{2\pi}\{P_{gq}(x)\ \ln \frac{q^2}{-p_1^2} + u_{gq}(x)\} \qquad (5.41)$$

where the constant (non-logarithmic) piece, u_{gq}, has been specified in Chapter III. This function carries the same dependence as before (i.e. gauge and regularization-prescription dependence). But as we discussed in the previous chapters the dependences cancel out when compared to the dilepton production cross sections if we have been consistent in the gauge and regularization procedure used.

Let us now rewrite the differential cross section due to $qq \to qq\gamma^*$ in terms of this gluon in a quark density. We obtain:

$$\frac{d\sigma_{qq}}{d^4q} = \sum_{i=1}^{3} \frac{d\sigma_{qq}^{(i)}}{d^4q}(s,\tau,\xi,\eta;\alpha_s) \qquad (5.42)$$

where

$$\frac{d\sigma_{qq}^{(i)}}{d^4q}(s,\tau,\xi,\eta;\alpha_s) = \frac{4\pi\alpha_{em}^2}{9sq^2}\, e_q^2\, \frac{\alpha_s}{2\pi^2}\, \frac{1}{s} \int \frac{d\alpha}{\alpha^2}\, \delta(1+\frac{\tau}{\alpha}-\xi-\frac{\eta}{\alpha})$$

$$\times\ K^{(i)}(\alpha;\tau,\xi,\eta) \qquad (5.43)$$

and

$$K^{(1)}(\alpha;\tau,\xi,\eta) = G_{g/q}(\alpha,-q^2)\; F(\frac{\tau}{\alpha},\xi,\frac{\eta}{\alpha})$$

$$K^{(2)}(\alpha;\tau,\xi,\eta) = \frac{\alpha_s}{2\pi}\{P_{gq}(\alpha)\ \ln\frac{\beta_m}{\tau\alpha} - u_{gq}(\alpha) - C_F\ \frac{2-\alpha}{\alpha}\}\ F(\frac{\tau}{\alpha},\xi,\frac{\eta}{\alpha})$$

$$K^{(3)}(\alpha;\tau,\xi,\eta) = \frac{\alpha_s}{2\pi}\ 8C_F\ \beta_m\ \frac{(1-\alpha)\ \tau^2\ (\alpha-\tau)}{(\tau-\eta)^2\alpha^2} \qquad (5.44)$$

The term with i=1 contains the mass singularity and will be absorbed in a redefined gluon distribution function. The terms i=2,3 provide the correction due to the quark-quark subprocess that we consider.

Consider now the physical process of two colliding hadrons $h_1 + h_2 \to \ell^+\ell^- + X$; each term gives a contribution

$$\frac{d\sigma^{(i)}_{h_1h_2}}{d^4q} = \int dx_1\ dx_2\ G_{q_1/h_1}(x_1)\ G_{q_2/h_2}(x_2)\ \frac{d\sigma^{(i)}_{q_1q_2}}{d^4q}(\tilde{s},\tau_{12},\frac{\xi}{x_2},\frac{\eta}{x_1};\alpha_s)$$

$$+ \quad (1\leftrightarrow 2) \qquad (5.45)$$

Then the factorization of the singularity is performed as follows: Consider the expression (5.45) with i=1. The integral over x_1 can be written

$$S \equiv \int \frac{dx_1}{x_1}\ G_{q_1/h_1}(x_1) \int \frac{d\alpha}{\alpha}\ \frac{1}{x_1\alpha}\ G_{g/q_1}(\alpha,q^2)\ \delta(1-\frac{\xi}{x_2}+\frac{\tau}{\alpha x_1 x_2} - \frac{\eta}{\alpha x_1})$$

$$\times\ F\left(\frac{\tau}{\alpha x_1 x_2},\frac{\xi}{x_2},\frac{\eta}{\alpha x_1}\right). \tag{5.46}$$

We notice that the function F appears as a function of x_2 and of the product αx_1. Let us write

$$F\left(\frac{\tau}{\alpha x_1 x_2},\frac{\xi}{x_2},\frac{\eta}{\alpha x_1}\right) \equiv H(\alpha x_1, x_2) \tag{5.47}$$

Furthermore, we set

$$\frac{\eta x_2 - \tau}{x_2 - \xi} \equiv f(x_2) \tag{5.48}$$

Then we obtain the expression for S

$$S = \frac{1}{(1 - \xi/x_2)} \int \frac{d\alpha}{\alpha}\, G_{g/q_1}(\alpha,-q^2) \int \frac{dx_1}{x_1}\, G_{q_1/h_1}(x_1)$$

$$\times\ \delta(\alpha x_1 - f(x_2))\ H(\alpha x_1, x_2)$$

$$= \frac{H(f(x_2),x_2)}{(1-\xi/x_2)\ f(x_2)} \int \frac{d\alpha}{\alpha}\, G_{g/q_1}(\alpha,-q^2)\ G_{q_1/h_1}\left(\frac{f(x_2)}{\alpha}\right) \tag{5.49}$$

The last integral represents an $O(\alpha_s)$ contribution to the gluon distribution function in a hadron h_1 (via the emission of a quark). This suggests that we write,

$$\int \frac{d\alpha}{\alpha} G_{g/q}(\alpha,-q^2)\ G_{q/h}(\tfrac{x}{\alpha}) = G^{(\alpha_s)}_{g/h}(x,-q^2) \tag{5.50}$$

Using this last statement, we find that S can take the forms:

$$S = \frac{H(f(x_2),x_2)}{(1-\xi/x_2)\ f(x_2)}\ G^{(\alpha_s)}_{g/h_1}(f(x_2),-q^2)$$

$$= \frac{1}{(1-\xi/x_2)} \int \frac{dx_1}{x_1}\ G^{(\alpha_s)}_{g/h_1}(x_1,-q^2)\ \delta(x_1-f(x_2))\ H(x_1,x_2)$$

$$= \int dx_1\ G^{(\alpha_s)}_{g/h_1}(x_1,-q^2)\ \delta(1-\frac{\xi}{x_2}-\frac{\eta}{x_1}+\frac{\tau}{x_1x_2})\ F(\frac{\tau}{x_1x_2},\frac{\xi}{x_2},\frac{\eta}{x_1})$$

(5.51)

The physical contribution to the differential cross section can be written in terms of the quark-gluon differential cross section defined by equation (5.12). We finally obtain:

$$\frac{d\sigma^{(1)}_{h_1h_2}}{d^4q} = \int dx_1\ dx_2\ G^{(\alpha_s)}_{g/h_1}(x_1,-q^2)\ G_{q_2/h_2}(x_2)\ \frac{d\sigma_{qg}}{d^4q}(\tilde{s},\tau_{12},\frac{\xi}{x_2},\frac{\eta}{x_1};\alpha_s)$$

$$+ \quad (1\leftrightarrow 2) \tag{5.52}$$

It is now clear that the contribution has the form of the quark-gluon contribution of equation (5.14). The mass singularity contained in $G^{(\alpha_s)}_{g/h_1}$ can therefore be absorbed by redefining the gluon distribution function as follows:

$$G_{g/h}(x) + G_{g/h}^{(\alpha_s)}(x,-q^2) \equiv G_{g/h}(x,-q^2) \tag{5.53}$$

Notice that this result is identical to the one obtained in section 3-3.

5-4 Correction to the q_T distribution of dileptons

We consider the q_T distribution of the dileptons for the process $p + p \to \ell^+\ell^- + X$. The results will be presented for the case of zero-rapidity (y=0) where we have

$$\xi = \eta = \omega = \left(\frac{M^2 + q_T^2}{s}\right)^{1/2} \tag{5.54}$$

The $O(\alpha_s)$ dominant Born term is given by equation (5.14). As discussed in section 5-1 the region of integration for the variables x_1 and x_2 is delimited by the relations

$$\omega\left(\frac{1}{x_1} + \frac{1}{x_2}\right) \geqq 2 \qquad \text{and, } 0 \leqq x_1, x_2 \leqq 1 \tag{5.55}$$

Furthermore, the δ-function implies

$$1 + \frac{\tau}{x_1 x_2} - \frac{\omega}{x_1} - \frac{\omega}{x_2} = 0 \tag{5.56}$$

According to relations (5.55) and (5.56), we must take the lower limit on x_2:

$$x_2 \geqq \frac{\tau}{x_1} \tag{5.57}$$

From the kinematical point of view, this condition is obvious. One can not produce a dilepton with a mass M larger than the energy available from the collision of the two partons.

Relation (5.56) can be rewritten as:

$$x_1 = \frac{x_2\omega - \tau}{x_2 - \omega} \qquad (5.58)$$

But, because $0 \leqq x_2 \leqq 1$, this relation gives a lower limit for x_1 that is:

$$x_1 \geqq \frac{\omega - \tau}{1 - \omega} = x_m \qquad (5.59)$$

The limit (5.57) is not really needed because the integration over the variable x_2 can be performed with the δ-function. The differential cross section for the physical process of colliding protons due to the qg → qγ* subprocess is therefore:

$$\frac{d\sigma^{qg}_{pp}}{d^4q} = \sum_i \int_{x_m}^{1} dx_1\, G_{q_i/p}(x_1)\, G_{g/p}\left(\frac{x_1\omega-\tau}{x_1-\omega}\right) \frac{4\pi\alpha^2_{em}}{9sq^2}\, e^2_{q_i}\, \frac{\alpha_s}{2\pi^2}$$

$$\times\, F\left(\tau_{12}, \frac{\omega}{x_2}, \frac{\omega}{x_1}\right) \quad + \quad (1 \leftrightarrow 2) \qquad (5.60)$$

The correction terms due to the subprocess $qq \rightarrow qq\gamma^*$ are given by equations (5.43-45). The relations that determine the region of integration are the following (for y=0):

$$1 + x_1\alpha - \frac{\omega}{x_2} - \frac{\omega}{x_1} \geqq 0 \tag{5.61}$$

$$1 + \frac{\tau}{\alpha x_1 x_2} - \frac{\omega}{x_2} - \frac{\omega}{\alpha x_1} = 0 \tag{5.62}$$

and $0 \leqq x_1, x_2, \alpha \leqq 1$. The first relation imposes a lower limit on α which is overcome by the more restaining condition of the δ-function. This condition (equation (5.62)) leads to:

$$x_1 = \frac{\omega x_2 - \tau}{x_2 - \omega} \tag{5.63}$$

But, both x_1 and α are smaller than one; it follows that,

$$x_1 \geqq \frac{\omega x_2 - \tau}{x_2 - \omega} = x_{1m} \tag{5.64}$$

and that

$$\frac{\omega x_2 - \tau}{x_2 - \omega} \leqq 1 \qquad \text{or,} \qquad x_2 \geqq \frac{\omega - \tau}{1 - \omega} = x_{2m} \tag{5.65}$$

The correction terms for proton-proton collisions are then for i=2,3:

$$\frac{d\sigma_{pp}^{(i)}}{d^4q} = \frac{4\pi\alpha_{em}^2}{9s^2q^2} \sum_{j,k} \left\{ e_{q_j}^2 \frac{\alpha_s}{2\pi} \int_{x_{2m}}^1 dx_2 \int_{x_{1m}}^1 dx_1\, G_{q_j/p}(x_1) \right.$$

$$\left. \times\, G_{q_k/p}(x_2) \frac{K^{(i)}(\alpha_0;\tau_{12},\omega/x_2,\omega/x_1)}{\alpha_0(1 - \omega/x_2)} \right\} + \binom{1\leftrightarrow 2}{j\leftrightarrow k} \tag{5.66}$$

where

$$\alpha_0 = \frac{\omega x_2 - \tau}{x_1(x_2 - \omega)} \tag{5.67}$$

We have mentioned before that the parton distribution function decreases fast as $x \to 1$, so that most of the contribution to the integral comes from x_1 and x_2 as small as possible. But, the lower limit of integration for x_1 was found by setting $\alpha = 1$. Therefore, the dominant contribution will come from $\alpha \sim 1$. This justifies the approximate formulae (valid for $\alpha \sim 1$) for the partonic cross section, in particular for the term involving the function u_{gq}.

As the expression for $K^{(2)}$ in equations (5.44) shows, the correction depends on the function u_{gq} which is to some extent a matter of convention. In Chapter IV, where we also considered dilepton production, the correction to $O(\alpha_s^2)$ due to the $qq \to qq\gamma^*$ subprocess, was somewhat sensitive to another convention-dependent function $u_{\bar{q}q}$. This function need not be specified here because the subprocess $gq \to q\gamma^*$ provides the $O(\alpha_s)$ dominant Born term.

We first consider the correction that results from the convention proposed in section 3-2. The calculations of the differential cross section are performed with the non-scaling parton distribution functions of reference 35, in which $q_0^2 = 1.8\ \mathrm{GeV}^2$. The results are presented in the form of the ratio:

$$R_{+-} = R_{\ell^+\ell^-}(x,M,q_T,y) \equiv \frac{d\sigma^{(2)}_{pp}/d^4q \; + \; d\sigma^{(3)}_{pp}/d^4q}{d\sigma^{qg}_{pp}/d^4q} \tag{5.68}$$

for the energy $\sqrt{s}$ = 27.4 GeV, rapidity y = 0 and dilepton masses M = 5.5, 7.5 and 9.5 GeV (figure [5.1], solid lines).

We first notice that the corrections are positive, and this is a step in the right direction, since, as we mentioned, the predictions of $O(\alpha_s)$ fall somewhat below the experimental data. These corrections are already of the order of 50% near q_T = 5 GeV and increase with increasing dilepton mass M and transverse momentum q_T.

The general behaviour of the ratio R_{+-} is easily understood . The production of dileptons with large q_T and/or large M, is possible only for large x_1 and x_2. Therefore, only a small region near x_1 and x_2=1 controls the behaviour of R_{+-} for large values of q_T and M. However, in the numerator of R_{+-}, a valence distribution function appears, whereas the denominator involves a gluon distribution, and those are known to behave for $x \to 1$ as $\sim (1-x)^3$ and $\sim (1-x)^5$ respectively. From this point of view, R_{+-} is expected to increase.

As mentioned before, the two corrections $d\sigma^{(2)}_{pp}/d^4q$ and $d\sigma^{(3)}_{pp}/d^4q$ do not constitute the total correction due to the subprocess $qq \to qq\gamma^*$. We have left out non-dominant contributions to the matrix element because they were believed to be unimportant with respect to the correction $d\sigma^{(2)}_{pp}/d^4q$ in the region of $\beta \simeq 0$. However, we did perform the calculations for one of the non-dominant terms, $d\sigma^{(3)}_{pp}/d^4q$, as an indication of the order of magnitude of these contributions. For $q_T \simeq 5$ GeV the contribution was found to be less than 1%, decreasing rapidly with increasing q_T. At low q_T ($q_T \lesssim 1$ GeV), the contributions are comparable; however, in this region the overall contribution due to the $O(\alpha_s^2)$ quark-quark subprocess is quite small compared to that of the $O(\alpha_s)$ quark-gluon subprocess. Furthermore, at low q_T, one must also consider the intrinsic transverse momentum effects. Hence, this approximation is totally justified in the region of q_T we are interested in.

Let us consider a second convention for u_{gq}. This convention is described in Appendix B (see also reference 31). We recall that this function generated only negligible corrections to the leptoproduction structure function $F_2(x,q^2)$. Moreover, we found in section 4-2, that its effect on the Drell-Yan formula was comparable to that of the convention of section 3-2. The results for the q_T distribution of dileptons appears on figure [5.1] (dashed lines) for the same energy, masses and rapidity as before(49). Clearly R_{+-} is of similar shape and magnitude. This is not surprising since the correction is dominated by the region near $\alpha = 1$ and that, as $\alpha \to 1$, $u_{gq}(\alpha) \to -4C_F$ whereas in the first convention we had $u_{gq}(\alpha) \to -2C_F$.

This q_T distribution of dileptons due to $qq \to qq\gamma^*$ subprocess has also been studied in a very recent paper(39).

The authors consider various conventions, both for on-mass-shell and off-mass-shell cases, including the convention of reference 31. The conclusions are the same as ours, i.e. that the $O(\alpha_s^2)$ qq subprocess gives large corrections at high transverse momentum q_T and should not be neglected. Furthermore the authors also finds by considering many choices of u_{gq} that the results did not present larges differences throughout almost all the kinematic region; clearly, this as well is in support of our conclusions.

CHAPTER VI

SUMMARY AND CONCLUSIONS

In this work, we have studied two important corrections arising from higher order terms in QCD perturbation theory:

The first is the $O(\alpha_s^2)$ correction to the Drell-Yan cross section for dilepton production in proton-proton collisions, due to the subprocess $q + q \rightarrow q + q + \gamma^*$. The possible importance of this correction has been emphasized long ago [4]. Our conclusion is that this correction is small at presently available dilepton masses ($\tau \lesssim 0.3$); thus it leaves practically unspoiled the successes of the Drell-Yan mechanism. However, the relative magnitude of this correction increases with τ, and becomes quite significant as one approaches the kinematic boundary $\tau = 1$.

The second is the $O(\alpha_s^2)$ correction to the transverse momentum distribution of dileptons, due again to the subprocess $q + q \rightarrow q + q + \gamma^*$. We find that for proton-proton collisions this is rather large with respect to the contribution of the $O(\alpha_s)$ subprocess $q + g \rightarrow q + \gamma^*$; and that it increases with increasing transverse momentum and dilepton mass, i.e. again as one approaches the kinematic boundary. This result explains, at least partially, the gap that appears to exist between experiment and theoretical predictions based on $O(\alpha_s)$ subprocesses only.

The study of these corrections required a complete definition of the gluon density in a quark, $G_{g/q}(x,q^2)$. Such a definition was also one of the main objectives of this thesis. The basic idea was to realize that the density $G_{g/q}(x,q^2)$ is also involved in the $O(\alpha_s^2)$ contribution to the DIS longitudinal structure function $F_L(x,q^2)$, due to the subprocess $\gamma^* + q \to q + q + \bar{q}$. The requirement that $F_L(x,q^2)$ be free of $O(\alpha_s^2)$ corrections led to a complete specification of $G_{g/q}(x,q^2)$.

There are of course, other processes which also require a complete definition of $G_{g/q}(x,q^2)$. Clearly, any process involving the subprocess $q + q \to q + q + \gamma^*$ is affected, since this gluon density fixes the $q + q \to q + q + \gamma^*$ parton cross section. One example of such a process is large-p_T direct photon production up to $O(\alpha_s^2)$, which involves a bremsstrahlung correction (due to $q + q \to q + q + \gamma$); for $p + p \to \gamma + X$, this correction was also found to be important [47,48]. Another example is large-p_T hadron production calculated up to $O(\alpha_s^3)$.

As an overall conclusion, the corrections studied in this thesis do not constitute, at present, any crucial test of perturbative QCD. On the other hand, they do not contradict any of its successes; and they offer a better understanding of the role of higher order terms of the QCD perturbation expansion.

APPENDIX A

We outline here the calculation of the matrix elements for certain subprocesses involved in this work. The results are most easily obtained and interpreted in the Coulomb gauge

$$A_0^a = 0, \qquad \nabla\vec{A}^a = 0. \tag{A.1}$$

The corresponding gluon propagator is:

$$D_{\mu\nu}^{ab}(k) = \frac{\delta^{ab}}{k^2 + i\varepsilon}\left[-g_{\mu\nu} - \frac{k_\mu k_\nu - (\eta\cdot k)(\eta_\mu k_\nu + \eta_\nu k_\mu)}{(\eta\cdot k)^2 - k^2}\right] \tag{A.2}$$

where k is the gluon four-momentum and η is a timelike unit vector. This propagator can also be written in the form:

$$D_{\mu\nu}^{ab}(k) = \frac{\delta^{ab}}{k^2 + i\varepsilon}\left[\sum_{\lambda=1}^{2} \varepsilon_\nu(k,\lambda)\,\varepsilon_\mu(k,\lambda) + \frac{k^2\eta_\mu\eta_\nu}{(\eta\cdot k)^2 - k^2}\right] \tag{A.3}$$

where λ is the helicity of the gluon. The transverse polarization vectors $\varepsilon = (0;\vec{\varepsilon})$ obeys

$$\varepsilon(k,\lambda)\cdot\varepsilon(k,\lambda') = -\delta_{\lambda\lambda'}$$

$$\varepsilon(k,\lambda)\cdot k = 0 \tag{A.4}$$

The second term in equation (A.3) is of the form of an instantaneous Coulomb interaction and cancels a similar contribution hidden in the first term.

A-1 Leptoproduction

The $O(\alpha_s)$ $g\gamma^* \to q\bar{q}$ contribution to the longitudinal structure function F_L involves the unitarity graphs of figure [A.1]. As we shall see, the contribution to F_L generated by these graphs is finite and therefore the gluon can be taken on mass shell. Then for $p_1^2 = 0$, the projection of the contribution onto F_L coming from the "squared" graphs (figure [A.1a-b]) vanishes. The only non-zero contribution comes from the "interference" graphs (figure [A.1c-d]), and is:

$$\frac{4(p_1 \cdot p_2)}{k^2(p_1+p_2)^2} \{ 2(k \cdot p_1)(k \cdot p_2) - k^2(p_1 \cdot p_2) \} \tag{A.5}$$

Using the Sudakov parameters defined in section 3-1, this expression can be rewritten as:

$$|M_g|^2 u = (1 - \alpha) u \tag{A.6}$$

This result is obviously finite, and it does agree with that of reference 46.

The $O(\alpha_s^2)$ $q\gamma^* \to qq\bar{q}$ subprocess gets its contributions from the unitarity graphs of the type of those on figure [A.2].

Consider first, the part of the contribution coming from the term $\sim \sum \varepsilon_\mu \varepsilon_\nu$ in the gluon propagator. Then the contribution to F_L involves the quantity

$$\sum_\lambda \sum_{\lambda'} T_1(\lambda,\lambda')\, T_2(\lambda,\lambda')\, (k_1^2)^{-2}\, (k_2^2)^{-2} \tag{A.7}$$

where

$$T_1(\lambda,\lambda') = \mathrm{Tr}\,[\not{p}\ \not{\varepsilon}(k_1,\lambda)\ (\not{p}-\not{k}_1)\ \not{\varepsilon}(k_1,\lambda')]$$

$$T_2(\lambda,\lambda') = \mathrm{Tr}\,[\not{p}_2\ \not{p}\ \not{k}_2\ \not{\varepsilon}(k_1,\lambda)\ (\not{k}_1-\not{k}_2)\ \not{\varepsilon}(k_1,\lambda')\ \not{k}_2\ \not{p}] \tag{A.8}$$

Using the relations (A.4), we find in terms of the Sudakov parameters defined in section 3-2

$$T_1(\lambda,\lambda') = \frac{2}{\alpha_1^2}\,[-k_1^2\,[1+(1-\alpha_1)^2] + \alpha_1(2-\alpha_1)p^2]\ \delta_{\lambda\lambda'} \tag{A.9}$$

We are interested in the leading part of the $O(\alpha_s^2)$ contribution of the $q\gamma^* \to qq\bar{q}$ subprocess, which comes from $\beta_1 \simeq 0$ (so that $k_1^2 \simeq 0$). This corresponds to the configuration where the gluon of momentum k_1 is almost collinear with the quark of momentum p. After summing over λ', using (A.9), the remaining sum is most easily calculated and interpreted in the configuration in which also the quark of momentum k_2 is collinear with the gluon k_1. Again, using the relations

[A.4], we find,

$$\sum_{\lambda} T_2(\lambda,\lambda) \simeq \frac{4u}{\alpha_1} \; k_2^2 \; (1 - \frac{\alpha_2}{\alpha_1}) \tag{A.10}$$

We now indicate the effects of the remaining part of the propagator. In the same configuration, the trace $T_1(\lambda,\lambda)$ receives a contribution

$$\frac{k_1^2}{(\eta\cdot k_1)^2 - k_1^2} \; Tr[\not p \not \eta \; (\not p - \not k_1) \not \eta] \simeq 8 \; \frac{1-\alpha_1}{\alpha_1^2} \; k_1^2 \tag{A.11}$$

However, we are interested in $F_L(x)$ as $x \to 1$ and this implies $\alpha_1 \to 1$; this last contribution is therefore negligible. Finally, the sum $\sum_{\lambda} T_2(\lambda,\lambda)$ is modified by terms of $O(\frac{p^2}{u})$ which are unimportant and/or by terms of $O(k_1^2/u)$ which give finite but non-leading contributions [15] near $\alpha_1 = 1$. Combining these results, we obtain

$$|M_g|^2 \simeq - \frac{2C_F}{\alpha_1^2} \left[\frac{1+(1-\alpha_1)^2}{\alpha_1 k_1^2} - \frac{2-\alpha_1}{(k_1^2)^2} \, p^2 \right] u \; (1 - \frac{\alpha_2}{\alpha_1}) \tag{A.12}$$

and equation (A.6) leads to equation (3.33).

A-2 Dilepton production

The calculation of the matrix elements and of the partonic cross sections for the $qg \to q\gamma^*$ and $qq \to qq\gamma^*$ subprocesses have been done in reference 15. We merely state, here, some of the details of the calculations.

Refering to the kinematics of section 5-2, the singular contribution to the squared matrix element is calculated using

$$\sum_{\lambda=1}^{2} \not{k}_2 \not{\epsilon}(k_1,\lambda) (\not{k}_1-\not{k}_2) \not{\epsilon}(k_1,\lambda) \not{k}_2 \simeq - \frac{2}{\alpha_1} (k_2^2 - \frac{\alpha_2}{\alpha_1} k_1^2)$$

$$\times [\alpha_2^2 + (\alpha_1-\alpha_2)^2] \not{p}_1 \quad \text{(A.13)}$$

and relation (A.9). Both expressions are valid within the first order in the small momentum transfers. Some terms that would vanish after integration over the angle ϕ between $\vec{\ell}_{T1}$ and $\vec{\ell}_{T2}$, have been dropped.

The total $O(\alpha_s)$ qg contribution is given by:

$$|M_{qg}|^2 = 8 [- \frac{s}{k^2} + \frac{8k\cdot(p_1+p_2)(p_1-k_1)\cdot p_2}{k^2 s} - \frac{2p_1\cdot k}{s}$$

$$+ \frac{2p_1^2 \; k\cdot p_2}{(k^2)^2}] \quad \text{(A.14)}$$

where the kinematical variables are assigned as in figure [2.7a]. In terms of the invariants τ, ξ and η defined in section 5-1, the denominator takes the form

$$k^2 = s(\tau - \eta + \frac{p_2^2}{s}) \quad \text{(A.15)}$$

A mass singularity would arise if we were to integrate over all possible k^2, or equivalently over all transverse momentum, q_T, of the virtual photon. However, if we are interested in the q_T distribution of the dileptons, k^2 is fixed for a given q_T and, as far as $q_T > 0$, $|M_{qg}|^2$ is finite and the parton can be taken on mass shell. Then, neglecting all terms of $O(\frac{p^2}{s})$, we obtain the expression

$$|M_{qg}|^2 = 8 \left[\frac{1-2\tau(1-\eta)}{(\eta-\tau)} + 1 - \xi \right] \tag{A.16}$$

However, the determination of the cross section $d\sigma/dq^2$ involves an integration over all possible q_T and partons have to be taken off mass shell. Then, the cross section for the $gq \to q\gamma^*$ subprocess is

$$\frac{d\sigma_{qg}}{dq^2} = \frac{4\pi\alpha_{em}^2}{3sq^2} \frac{e_q^2}{2N} \frac{\alpha_s}{16} \int_{-\tau p_1^2/s}^{1-\tau} d\left(\frac{-k^2}{s}\right) |M_{qg}|^2 \tag{A.17}$$

where

$$|M_{qg}|^2 \equiv |M_{qg}(\tau,p_1,k,s)|^2 = [\tau^2+(1-\tau)^2] \left(\frac{\tau p_1^2 s}{(k^2)^2} - \frac{s}{k^2}\right) + 2\tau - \frac{k^2}{s} \tag{A.18}$$

We have neglected terms leading to contributions of $O(p_1^2/s)$ in the cross section $d\sigma_{qg}/dq^2$.

The main objective, here, is to find the matrix element for the $O(\alpha_s^2)$ $qq \to qq\gamma^*$ subprocess. This is done more easily by establishing a relation between $|M_{qq}|^2$ and the $qg \to q\gamma^*$ matrix element. This relation is suggested by the presence of the off-shell qg subprocess in the $qq \to qq\gamma^*$ subprocess.

Looking at the unitarity graphs of figure [4.1a], we notice that the trace $T_1(\lambda, \lambda')$ is common to all graphs. The contribution to $|M_{qq}|^2$ will then be proportional to the product of $T_1(\lambda, \lambda')$ with an off-shell $qg \to q\gamma^*$ matrix element. An additional contribution will come from the instantaneous Coulomb term in the gluon propagator. Strictly speaking, we should also include the contribution that comes from graphs of the type of figure [4.1b]; however, those were found to be non-dominant in the limit we are interested in ($\alpha_1 \to 1$).

We finally obtain an expression for $|M_{qq}|^2$ in terms of the variables defined in section 5-2:

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi\ |M_{qq}|^2 \simeq -\frac{2}{\alpha_1^2}\left[\frac{1+(1-\alpha_1)^2}{k_1^2} - \frac{\alpha_1(2-\alpha_1)}{(k_1^2)^2}\right]$$

$$\times\ |M_{qg}(\frac{\tau}{\alpha_1}, k_1, k_2, \alpha_1 s)|^2 \ + \ \frac{2^7 s}{(k_2^2)^2}\ \frac{(1-\alpha_1)\ \tau^2\ (\alpha_1-\tau)}{\alpha_1^4} \tag{A.19}$$

and the total cross section for $qq \to qq\gamma^*$ is given by:

$$\frac{d\sigma_{qq}}{dq^2} \simeq \frac{4\pi\alpha_{em}^2}{3sq^2}\ \frac{(e_{q_1}^2+e_{q_2}^2)}{2N}\ \frac{C_F s}{2^7\pi}\ (\frac{\alpha_s}{\pi})^2 \int_0^1 \frac{d\alpha_1}{\alpha_1}\, d\beta_1\, d\alpha_2\ \theta(\alpha_1-\alpha_2)$$

$$\times\ \theta(\alpha_2 - \tau - \frac{\alpha_2}{\alpha_1}(\beta_1 - \alpha_1 p_1^2/s)) \times \int_0^{2\pi} d\phi\ |M_{qq}|^2 \tag{A.20}$$

APPENDIX B

It is clear from the foregoing analysis that, in general, there is some ambiguity in the definition of a parton density inside another parton, and in particular, of the gluon density inside a quark. Since, at present, our predictions concerning corrections to physical quantities depend on these definitions, it is important to consider approaches leading to different definitions, and compare the results.

An approach for the determination of u_{gq} (along with $u_{\bar{q}q}$) different from that of sections 3-2 and 3-4 is suggested by the form of the $O(\alpha_s)$ correction to the antiquark distribution function in a hadron,

$$G_{\bar{q}/h}^{(\alpha_s)}(x,q^2) = \int_x^1 \frac{d\alpha}{\alpha} G_{\bar{q}/g}(\frac{x}{\alpha},q^2)\ G_{g/h}(\alpha) \qquad \text{(B.1)}$$

This is the basic relation used in the absorption of mass singularities by a redefinition of the parton distribution. The approach consists in extending this relation to the case of a quark instead of a hadron, namely,

$$G_{\bar{q}/q}(x,q^2) = \int_x^1 \frac{d\alpha}{\alpha} G_{\bar{q}/g}(\frac{x}{\alpha},q^2)\ G_{g/q}(\alpha,q^2) \qquad \text{(B.2)}$$

with all parton densities defined as in Chapter II. On the basis of those definitions, the condition (B.2) is automatically satisfied with respect to $O(\ln^2 q^2/-p^2)$ terms. For terms of $O(\ln q^2/-p^2)$ and of $O(1)$ the equation (B.2) implies:

$$\int_x^1 \frac{d\alpha}{\alpha} \left[u_{qg}\left(\frac{x}{\alpha}\right) P_{gq}(\alpha) + P_{qg}\left(\frac{x}{\alpha}\right) u_{gq}(\alpha) \right] = \int_x^1 \frac{d\alpha}{\alpha} K(x,\alpha) \tag{B.3}$$

and $u_{q\bar{q}}(x,\alpha) = u_{qg}\left(\frac{x}{\alpha}\right) u_{gq}(\alpha)$ (B.4)

where P_{gq} and P_{qg} are the well-known splitting functions. The function $K(x,\alpha)$ is determined as follows: The cross section for $qq \to qq\gamma^*$ can be written in the general form:

$$\frac{d\sigma_{qq}}{dq^2} = \frac{4\pi\alpha_{em}^2}{3sq^2} \int_\tau^1 \frac{dx}{x} \left[G_{\bar{q}/q_1}(x,-q^2)\ \kappa_0\left(\frac{\tau}{x}\right) + G_{g/q_1}(x,-q^2) \times \kappa_1\left(\frac{\tau}{x}\right) + (q_1 \leftrightarrow q_2)\right] + \kappa_2(\tau) \tag{B.5}$$

Now κ_0 and κ_1 are known (see section 2-3). Furthermore, the leading logarithmic term of G_{g/q_1} is known exactly. Therefore, the only unknown in the coefficient of the non-leading logarithmic term of the cross section $d\sigma_{qq}/dq^2$ is $K(x,\alpha)$. Comparing expression (B.5) with perturbation calculations of the parton cross section (section 4-1), we obtain:

$$K(x,\alpha) = P_{gq}(\alpha) \left[2P_{qg}\left(\frac{x}{\alpha}\right) \left(\ln\left(\frac{\alpha}{x^2}\right) - 1\right) + 6\, \frac{x}{\alpha}\left(1-\frac{x}{\alpha}\right)\right] + 8C_F\, \frac{1-\alpha}{\alpha}\, \frac{x}{\alpha} \left(1-\frac{x}{\alpha}\right) - 2C_F\, P_{qg}\left(\frac{x}{\alpha}\right) \frac{2-\alpha}{\alpha} \tag{B.6}$$

Now, the non-leading logarithmic term in $d\sigma_{qq}/dq^2$ is regularization-prescription dependent, and a fortiori, so will it be for u_{gq} and $u_{\bar{q}q}$. However, in the process of subtracting the mass singularities to determine the correction terms, the regularization-prescription dependence falls.

$K(x,\alpha)$ also appears in leptoproduction for the $O(\alpha_s^2)$ $q\gamma^* \to qq\bar{q}$ subprocess. It has been calculated in the limit of large x ($x \to 1$) and has been found to be consistent with the expression (B.6), taking into consideration that the calculation of $K(x,\alpha)$ was performed with the same regularization procedure; this last result constitutes a check of equation (B.6).

In order to find the solution of equation (B.3), for the function u_{gq}, we invoke the convolution theorem for Mellin transform as stated in section 3-3. Then, taking the Mellin transform (or the n^{th} moment) of the expression (B.3), we get:

$$\tilde{u}_{qg}(n)\,\tilde{P}_{gq}(n) + \tilde{P}_{qg}(n)\,\tilde{u}_{gq}(n) = \tilde{k}(n) \tag{B.7}$$

where

$$\tilde{k}(n) \equiv \int_0^1 dx\, x^{n-1} \int_x^1 \frac{d\alpha}{\alpha} K(x,\alpha) \tag{B.8}$$

The function $u_{gq}(\alpha)$ is then obtained by taking the inverse Mellin transform of:

$$\tilde{u}_{gq}(n) = \frac{1}{\tilde{P}_{qg}(n)} [\tilde{k}(n) - \tilde{u}_{qg}(n)\,\tilde{P}_{gq}(n)] \tag{B.9}$$

Straightforward, but lengthy, calculations lead to a uniquely specified function:

$$u_{gq}(\alpha) = 2C_F \left[-\frac{2}{3}\alpha^2 - \alpha + \frac{1}{6\alpha} - \frac{2(1-\alpha)^2}{\alpha} \ln \alpha - \frac{\sqrt{\alpha}}{2} \{\cos(\theta_0 \ln \alpha) - \frac{3}{2\theta_0} \sin(\theta_0 \ln \alpha) \} \right] \qquad (B.10)$$

where $\theta_0 = \sqrt{7}/2$. The other function to be determined, $u_{\bar{q}q}$, then comes directly from substituting u_{gq} in expression (B.4).

As discussed in Chapter III, if we want the relation between distribution and structure functions to remain valid, it is necessary that this convention for u_{gq} and $u_{\bar{q}q}$, generates little or no $O(\alpha_s^2)$ correction to leptoproduction.

The function $u_{\bar{q}q}$ will affect the leptoproduction structure function F_2 as follows: Direct perturbation calculations of the graphs of figure [2.4] lead to the expression:

$$\frac{1}{x} F_{2,q}(x,q^2) = \int_x^1 \frac{dy}{y} G_{\bar{q}/q}(y,q^2)\, C_0\left(\frac{x}{y}\right) + C_2(x) \qquad (B.11)$$

where $G_{\bar{q}/q}$ is given in section 2-2.

The non-logarithmic piece, C_2, is of the form:

$$C_2(x) = e_q^2 \left(\frac{\alpha_s}{2\pi}\right)^2 \left[\Phi(x) - \int_x^1 \frac{d\alpha}{\alpha} u_{\bar{q}q}(x,\alpha) \right] \qquad (B.12)$$

where the function $\Phi(x)$ is the non-logarithmic piece of the aforementioned perturbation calculations,and for large x ($x \simeq 1$):

$$\phi(x) \simeq C_F \left(\frac{7\pi^2}{24} + 2\right)(1-x) \tag{B.13}$$

As explained in Chapter III, this limiting form obtained for $x \to 1$, is sufficient for our purpose. Then we get for C_2, for not too small x:

$$C_2(x) \simeq e_q^2 \left(\frac{\alpha_s}{2\pi}\right)^2 C_F \left(\frac{7\pi^2}{24} - 2\right)(1-x). \tag{B.14}$$

The correction to the hadronic structure function, $F^h_{2,q}$, is then:

$$\frac{1}{x} F^h_{2,q}(x,q^2) = \sum_i e^2_{q_i} \int_x^1 \frac{dy}{y} G_{q_i/h}(x,q^2)\, C_2\left(\frac{x}{y}\right) \tag{B.15}$$

To evaluate the magnitude of this correction, we use the parton distributions parametrized by Owens and Reya[35]. The correction is found to be very small. For example, near x=0.6 and for $q^2 = -10\ \text{GeV}^2$, it represents a 2% change in the structure function. The degree of accuracy to which the structure function is actually known is of the order of 2%. We may then conclude that the subprocess $q\gamma^* \to qq\bar{q}$ leaves leptoproduction practically unaffected with this convention, which is required in order that the relations between distribution and structure functions remain unaltered.

FIGURES

FIGURE [2.1]

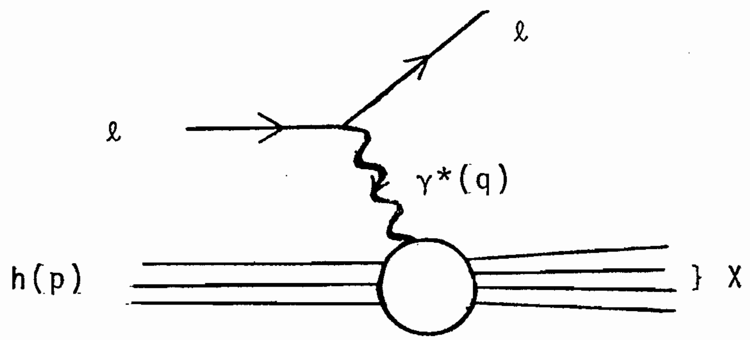


Figure [2.1]. Lepton-hadron deep inelastic scattering: ℓ stands for a charged lepton, γ^* for a virtual photon of momentum q, h for a hadron of momentum p, and X for any set of final hadrons.

FIGURE [2.2]

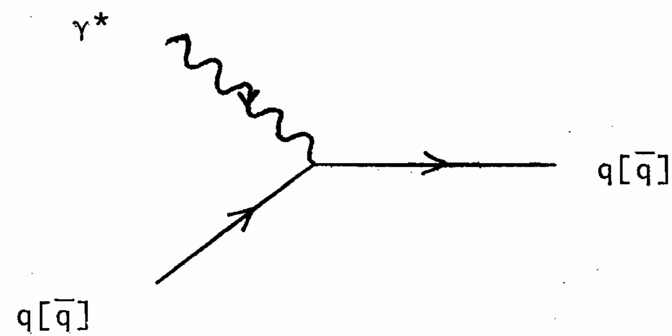


Figure [2.2]. $O(\alpha_s^0)$ subprocess contributing to leptoproduction ($q[\bar{q}]$ denotes a quark[antiquark]).

FIGURE [2.3]

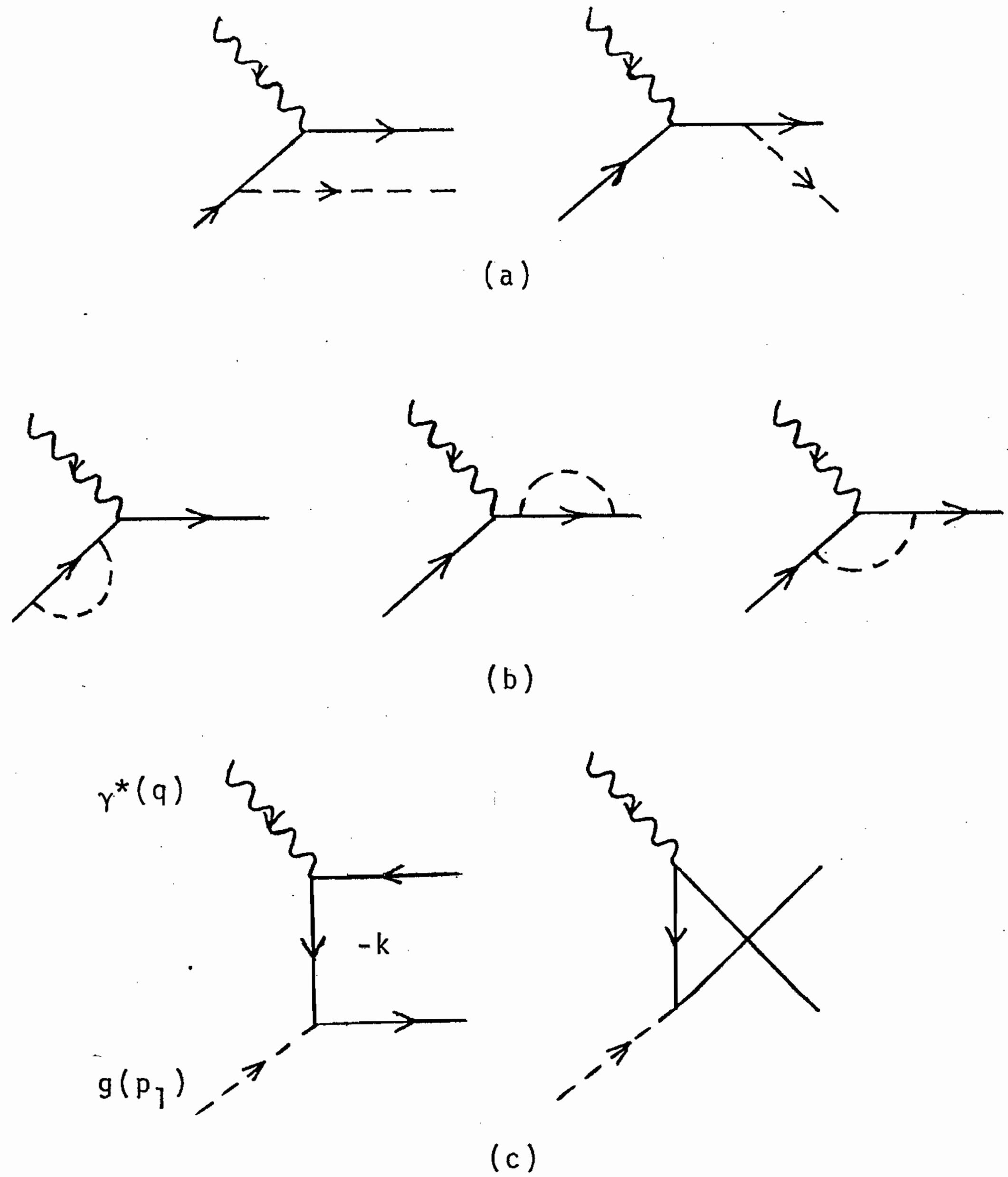


Figure [2.3]. Diagrams contributing to leptoproduction to order α_s. Figure [2.3a] shows the emission of a gluon in the subprocess. In figure [2.3b], we find the diagrams considered when we determined the correction to $O(\alpha_s)$ due to renormalization. Finally a contribution comes from diagrams with a gluon in the initial state, figure [2.3c].

FIGURE [2.4]

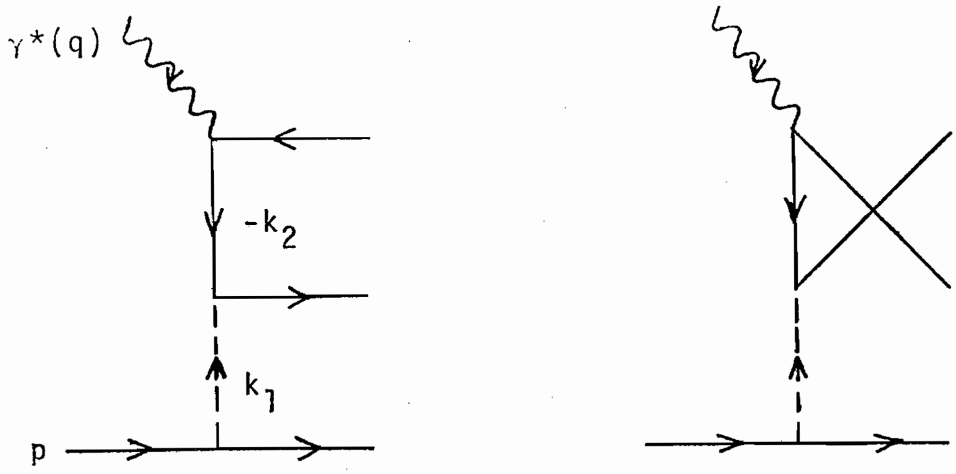


Figure [2.4]. Quark $O(\alpha_s^2)$ diagrams contributing to leptoproduction.

FIGURE [2.5]

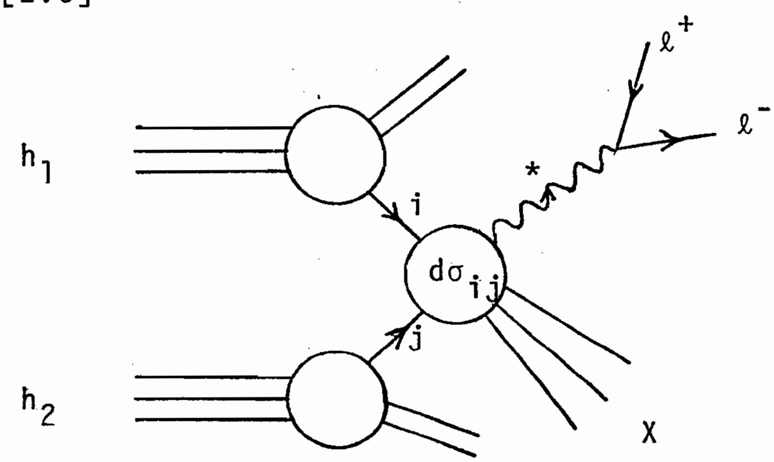


Figure [2.5]. Parton picture of dilepton production. i and j represent partons coming out of hadron h_1 and h_2, and $d\sigma_{ij}$ is the parton cross section of the subprocess.

FIGURE [2.6]

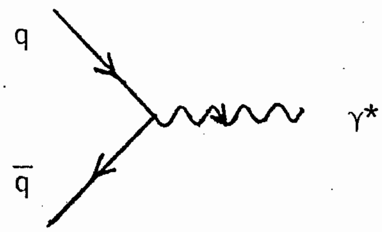
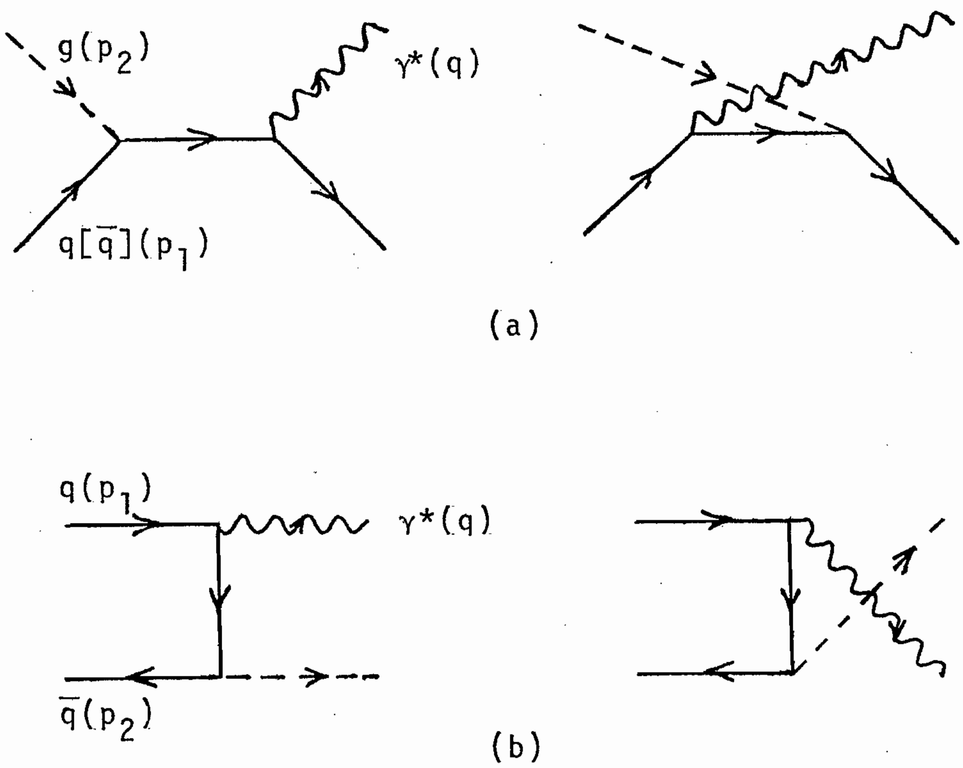
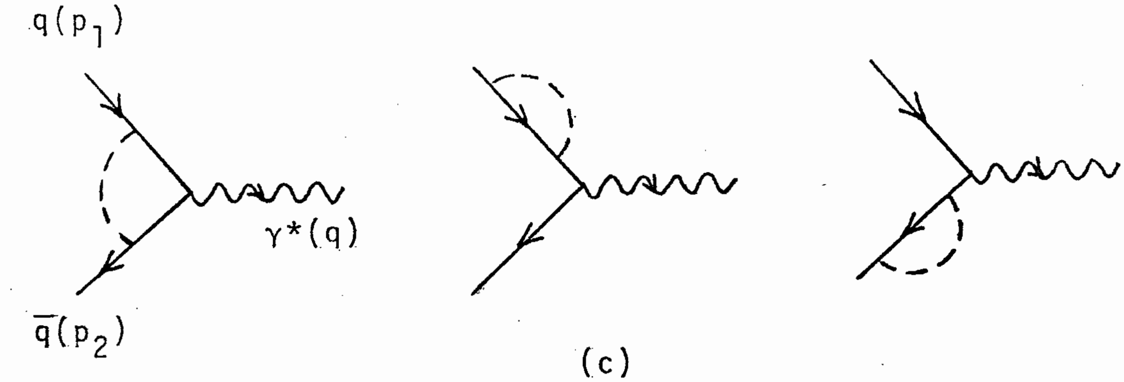


Figure [2.6]. Drell-Yan mechanism: $O(\alpha_s^0)$ contribution to dilepton production.

FIGURE [2.7]





(c)

Figure [2.7]. $O(\alpha_s)$ diagrams that contribute to dilepton production: Figure [2.7a,b and c] show respectively, the diagrams with a gluon in the initial state, the diagrams with a gluon in the final state and finally, the diagrams from which arise renormalization corrections, all contributing to order α_s.

FIGURE [2.8]

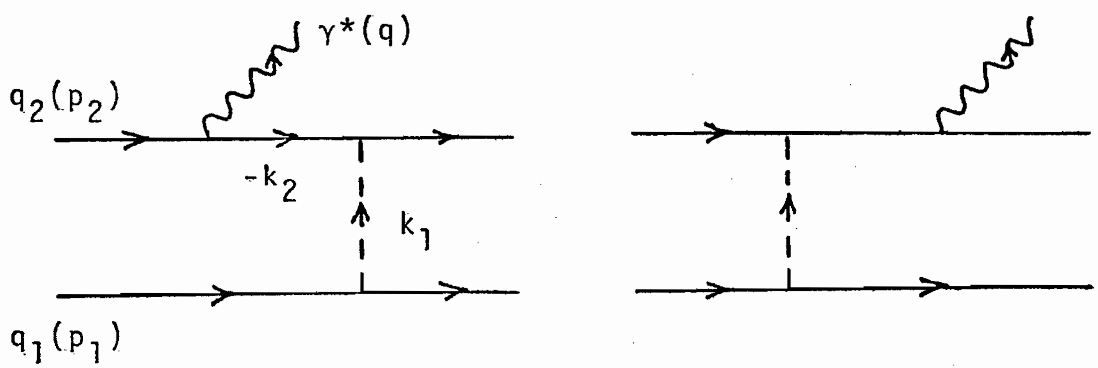
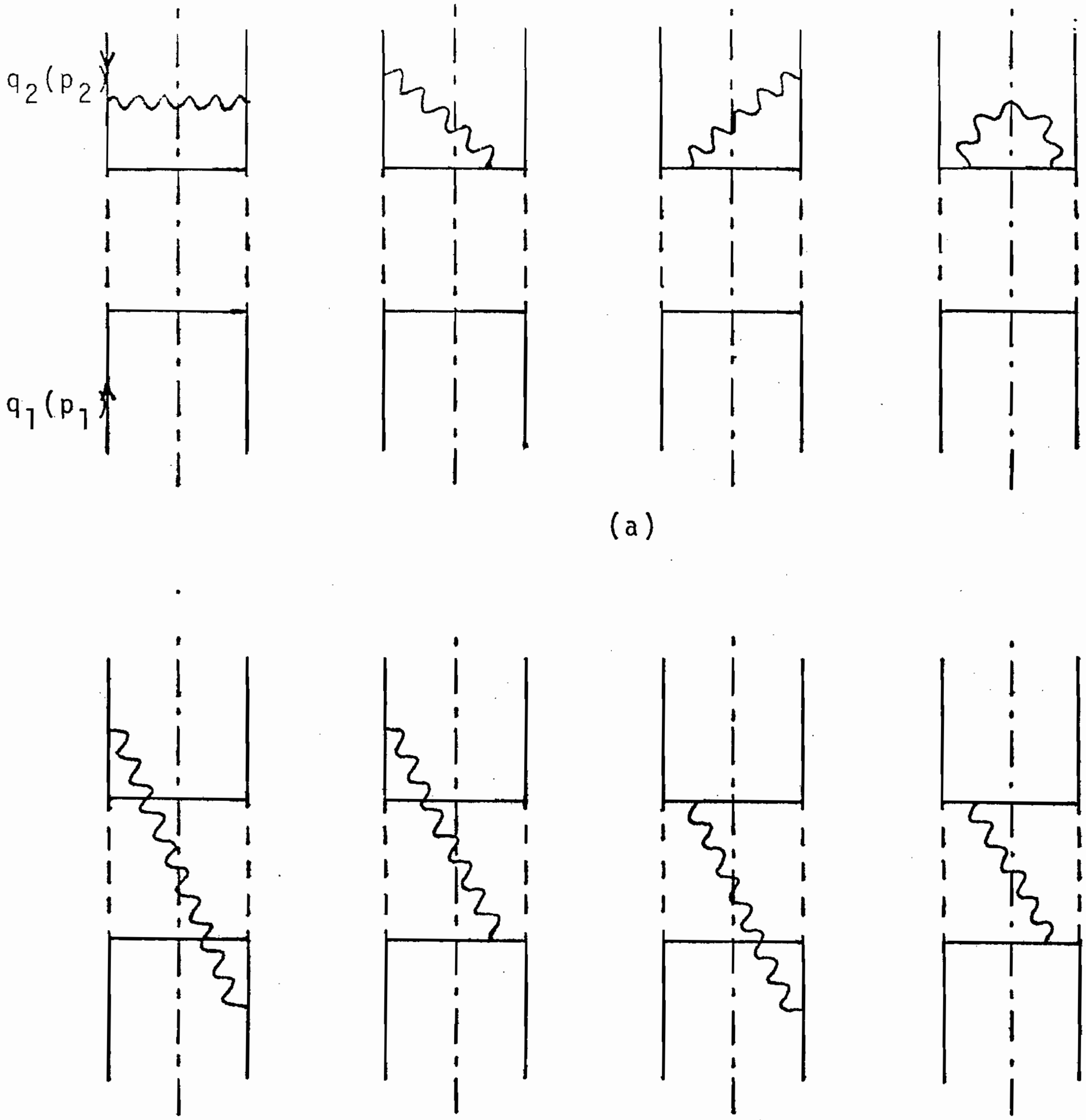


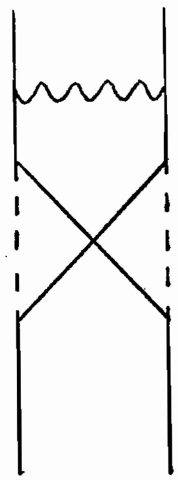
Figure [2.8]. $O(\alpha_s^2)$ quark-quark diagrams contributing to dilepton production.

FIGURE [4.1]



(a)

(b)



(c)

Figure [4.1]. Unitarity diagrams contributing to dilepton production due to the $qq \rightarrow qq\gamma^*$ subprocess. The total contribution can be divided in two categories: The "squared" diagrams (figure [4.1a] and those with $q_1 \leftrightarrow q_2$) and the "interference" diagrams (figure [4.1b] and those with $q_1 \leftrightarrow q_2$). In the case of identical initial quarks, diagrams of the type of figure [4.1c] must also be considered.

FIGURE [4.2]

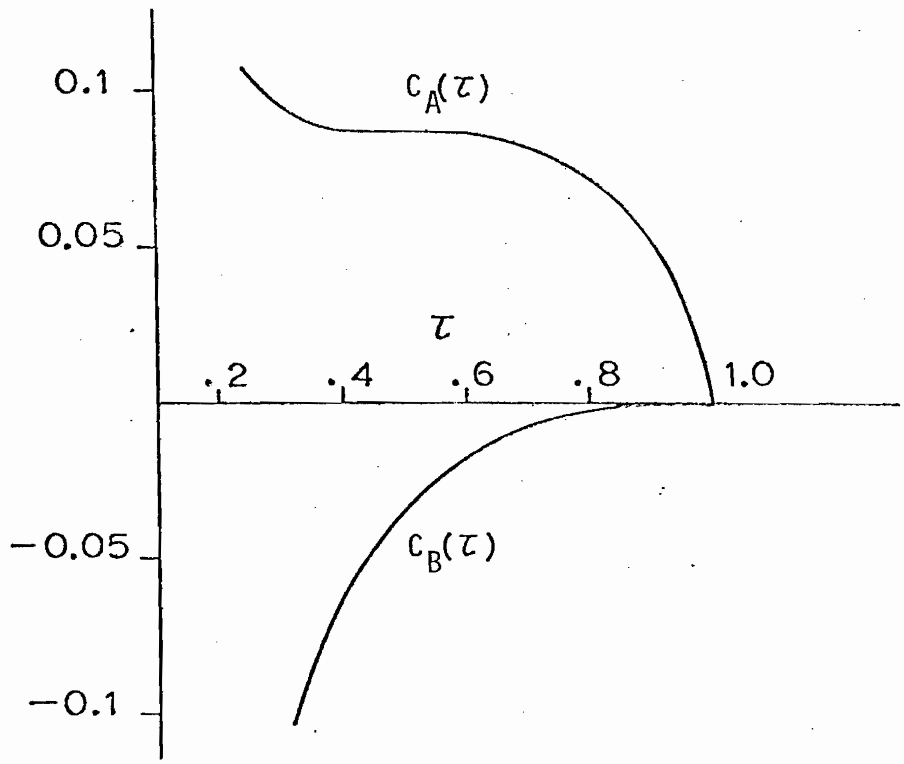


Figure [4.2]. The functions $C_A(\tau)$ and $C_B(\tau)$. The calculation includes all powers of $(1-\tau)$ as shown in reference 15.

FIGURE [4.3]

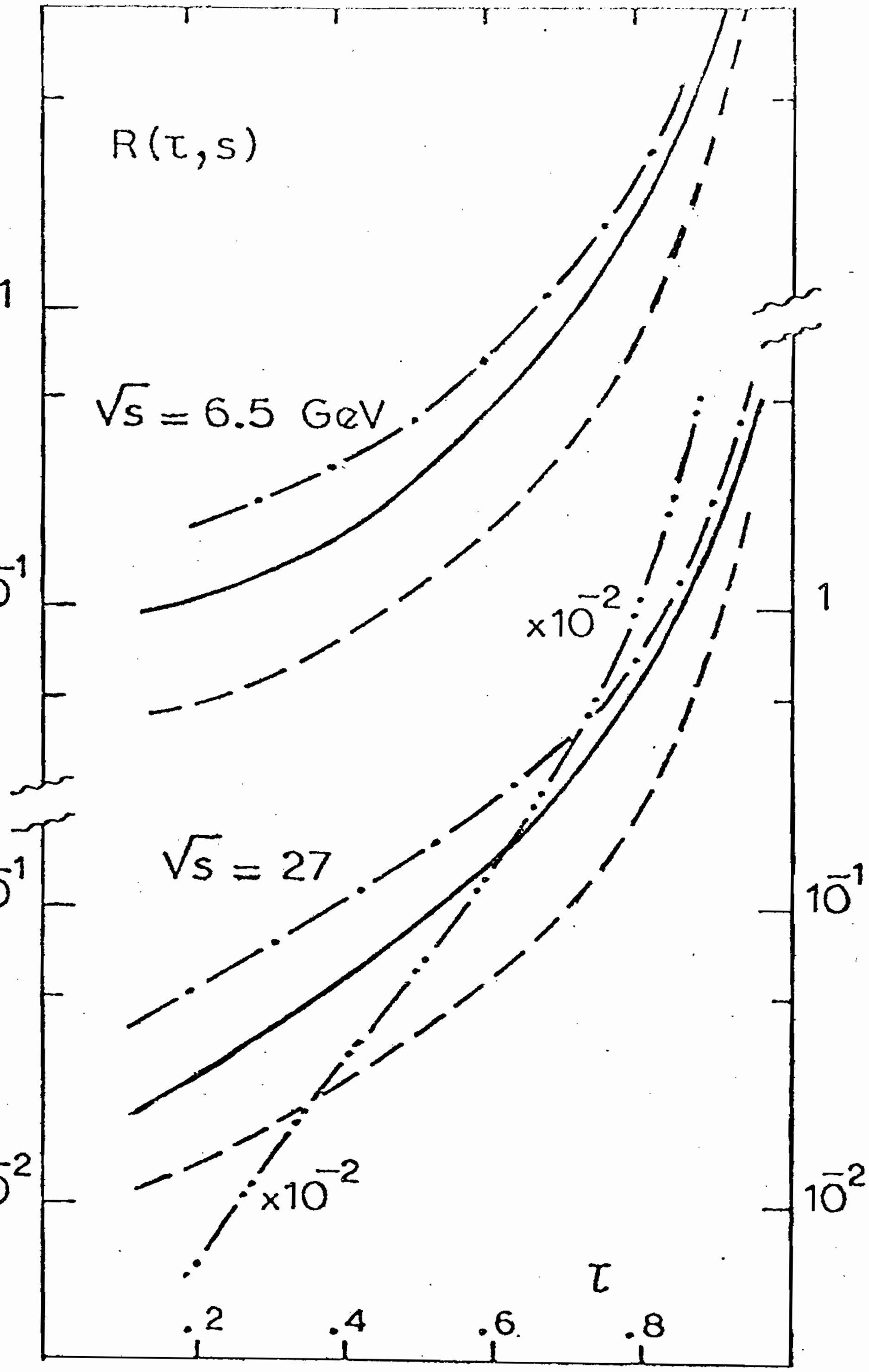


Figure [4.3]. The ratio $R(\tau,s)\equiv(d\sigma_{pp}^{qq}/dq^2)/(d\sigma_{pp}/dq^2)_{DY}$. Solid lines correspond to equation (4.12), dash-dotted lines to equation (4.21), dash-dot-dotted lines to equation (4.29) and dashed lines to equation (4.32).

FIGURE [5.1]

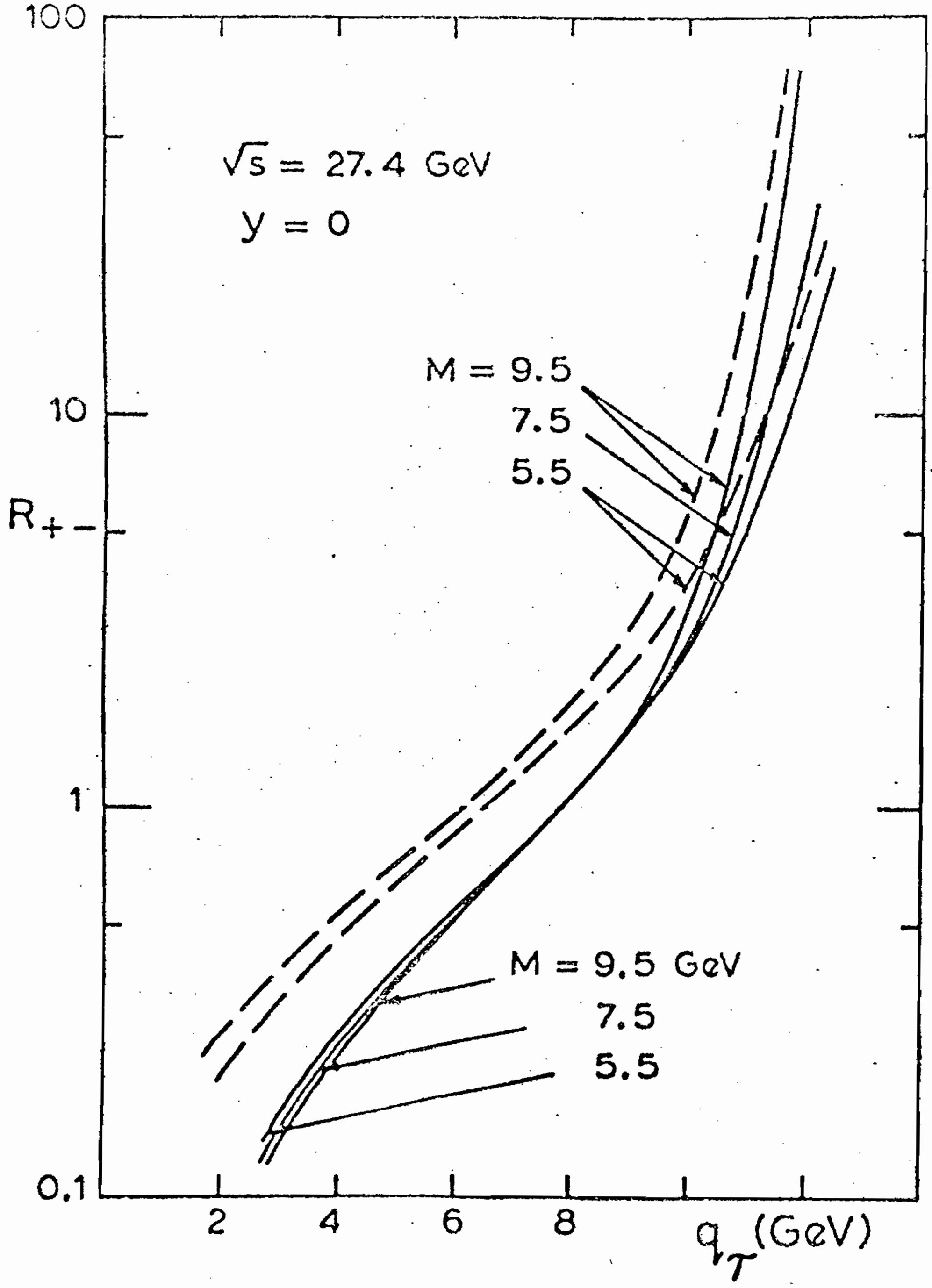


Figure [5.1]. The ratio R_{+-} is plotted for $\sqrt{s}$ = 27.4, y = 0 and M = 5.5, 7.5 and 9.5 GeV. Solid lines correspond to the convention for u_{gq} of Chapter III, dashed lines to that of reference 31 (Appendix B).

FIGURE [A.1]

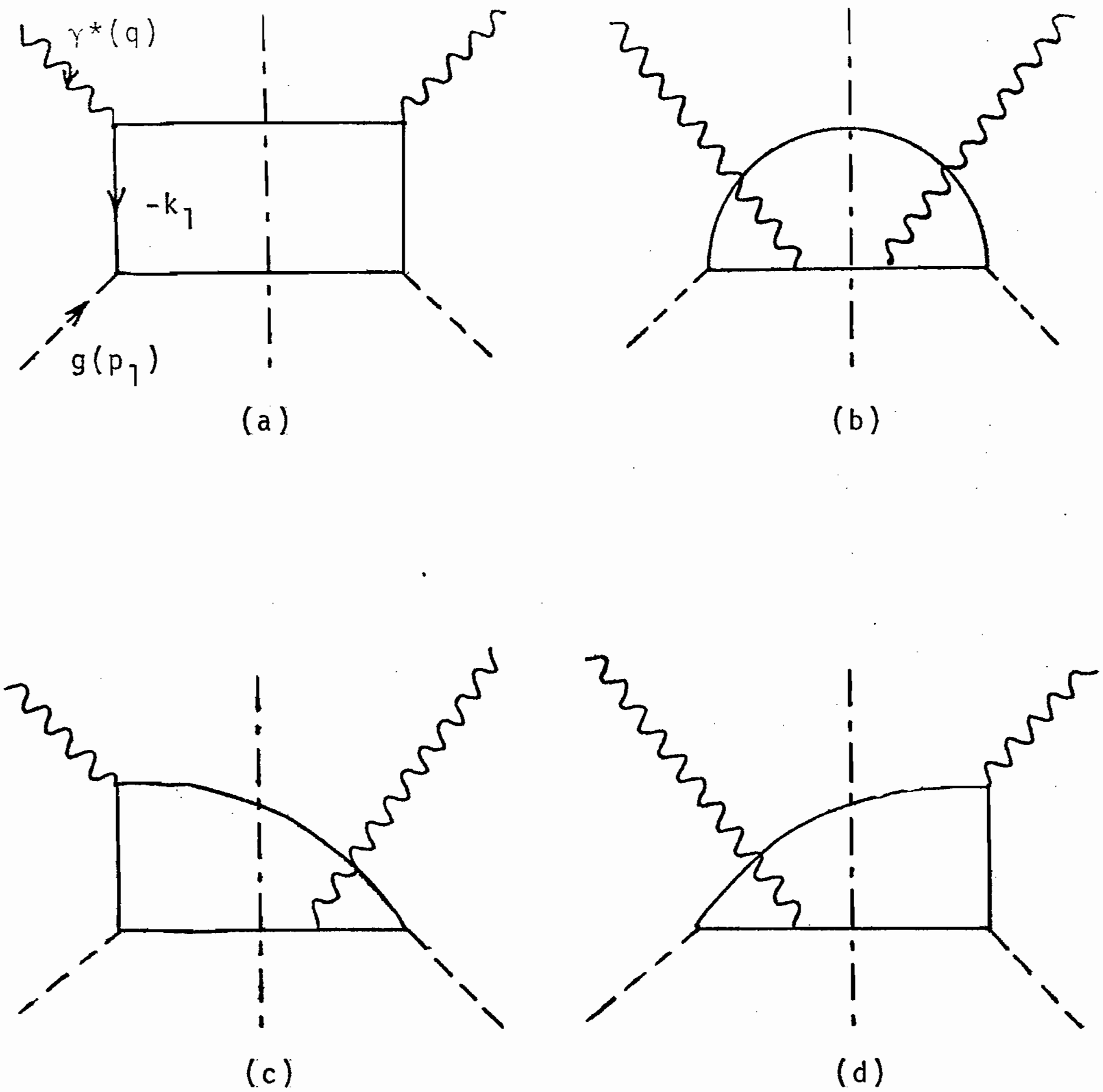


Figure [A.1]. $O(\alpha_s)$ $g\gamma^* \to q\bar{q}$ unitarity graphs contributing to F_L. Figure [A.1a,b] correspond to the "squared" terms and figure [A.1c,d], the "interference" terms.

FIGURE [A.2]

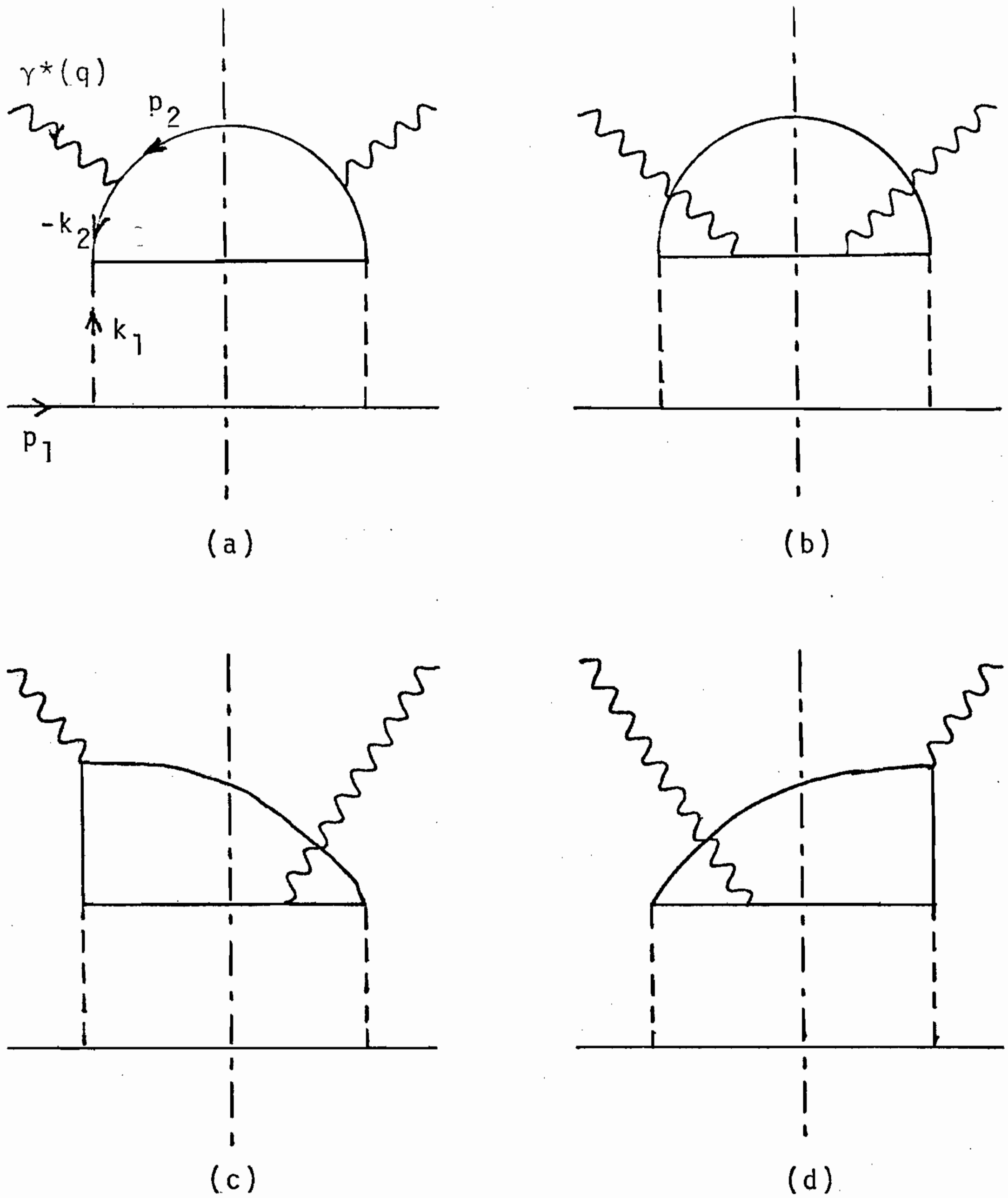


Figure [A.2]. $O(\alpha_s^2)$ $q\gamma^* \to qq\bar{q}$ unitarity graphs contributing to F_L.

REFERENCES

1-M. Gell-Mann, Y. Ne'emann, The eightfold way,(W.A. Benjamin Inc., Publisher, New York, 1964)

2-M. Gell-Mann, F. Low, Phys. Rev., 95, 1300 (1954)

E. Stueckelberg, A. Peterman, Helv. Phys. Acta., 26, 499 (1953)

3-K.G. Wilson, Phys. Rev., 179, 1499 (1969)

4-H. Georgi, Phys. Rev., D17, 3010 (1978)

5-V. Dokshister, D. D'yankonov, S. Troyan, Materials of the 13th Winter School of the Leningrad Institute of NUclear Physics (1978), SLAC Translation No 183 (unpublished); Phys. Lett. 79B, 269 (1978)

6-G. Altarelli, R.K. Ellis, G. Martinelli, Nucl. Phys.,B143, 521 (1978)

7-G. Altarelli, R.K. Ellis, G. Martinelli, Nucl. Phys., B146, 544 (1978)

8-J. Kubar-André, F.E. Paige, Phys. Rev., D19, 221 (1979)

9-J. Abad, B. Humpert, Phys. Lett., 78B, 627 (1978)

10-J. Abad, B. Humpert, Phys. Lett., 80B, 433 (1979)

11-A.P. Contogouris, J. Kripfganz, Phys. Rev., D19, 2207 (1979)

12-A.J. Buras, Rev. Mod. Phys., 52, 199 (1980)

13-W. Bardeen, A.J. Buras, D. Duke, T. Muta, Phys. Rev., D18, 3998 (1978)

E. Floratos, D. Ross, C. Sachrajda, Nucl. Phys., B129, 66 (1977); B152, 493 (1978)

J. Kodaira, T. Uematsu, Nucl. Phys., B141, 497 (1978)

14-K. Harada, T. Kaneko, N. Sakai, Nucl. Phys., B155, 169 (1979)

15-A.P. Contogouris, J. Kripfganz, Phys. Rev., D20, 2295 (1979)

16-G. Altarelli, R.K. Ellis, G. Martinelli, Nucl. Phys., B157, 461 (1979)

17-S.D. Drell, T.M. Yan, Phys. Rev. Lett., 25, 316 (1970)

18-S.D. Drell, T.M. Yan, Am. Phys. (N.Y.), 66, 578 (1971)

19-J.D. Bjorken, S.D. Drell, Relativistic Quantum Mechanics (McGraw-Hill, New York, 1965)

20-R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer, G.G. Ross, Phys. Lett., 78B (1978); Nucl. Phys., B152, 285 (1979)

21-A.H. Mueller, Phys. Rev., D9, 963 (1974); Phys. Rev., D18, 3705 (1978)

22-D. Amati, R. Petronzio, G. Veneziano, Nucl. Phys., B140,(1978) B146, 29 (1978)

23-I. Antoniadis. L. Beaulieu, C. Kounas, A proof of factorization of the mass singularities in the Bjorken limit, (preprint) LPTENS 79/28

24-J.D. Bjorken, Phys. Rev., 179, 1547 (1969)

25-R.P. Feynman, Phys. Rev. Lett., 23, 1415 (1969)

26-G. Altarelli, G. Parisi, Nucl. Phys., B126, 298 (1977)

27-C. Llewellyn-Smith, Acta Phys. Austriaca Suppl., 19, 331 (1978); Phys. Lett., 79B, 83 (1978)

28-G. Altarelli, G. Martinelli, Phys. Lett., 76B, 1, 89 (1978)

29-S. Treiman, F. Wilczek, A. Zee, Phys. Rev., D10, 2881 (1974)

30-A.N. Schellekens, W.L. Van Neerven, Phys. Rev., D21, 2619 (1980)

31-A.P. Contogouris, L. Marleau, Phys. Rev., D22, 1102 (1980)

32-V.V. Sudakov, Zh. Eksp. Teor. Fiz., 30, 87 (1956) (Sov. Phys. JETP, 3, 65 (1956))

33-J. Abad, B. Humpert, Phys. Lett., 80B, 286 (1979)

34-A.N. Schellekens, W.L. Van Neerven, Phys. Rev., D22, 1623 (1980)

35-J.F. Owens,E. Reya, Phys. Rev., D17, 3003 (1978)

36-G.R. Farrar, Nucl. Phys., B77, 429 (1974)

37-J.F. Gunion, Phys. Rev., D10, 242 (1974)

38-A.J. Buras, K.J.F. Gaemers, Nucl. Phys., B132, 249 (1978)

39-G. Parisi, R. Petronzio, Nucl. Phys., B154, 427 (1979)

40-C. Lo, J. Sullivan, Phys. Lett., 86B, 327 (1979)

41-K. Kajantie, R. Raitio, Nucl. Phys., B139, 72 (1978)

42-H. Fritzsh, D. Minkowski, Phys. Lett., 69B, 316 (1977)

43-F. Halzen, D. Scott, Phys. Lett., 40, 1117 (1978); Phys. Rev., D18, 3378 (1978)

44-G. Altarelli, G. Parisi, R. Petronzio, Phys. Lett., 76B, 351 and 356 (1978)

45-C. Michael, T. Weiler, Phenomenology of Quantum Chromodynamics, Proceedings of the XIII Rencontre de Moriond (1978) edited by Trân Thanh Vân, Editions Frontières, Gif-sur-Yvette (1978) pp.179

46-I. Hinchliffe, C. Llewellyn-Smith, Nucl. Phys., B128, 93 (1977)

47-A.P. Contogouris, S. Papadopoulos, C. Papavassiliou, Nucl. Phys., (in press)

48-A.P. Contogouris, J. Kripfganz, L. Marleau, S. Papadopoulos, McGill preprint, Presented at the XV Rencontre de Moriond, Les Arcs-Savoie, 8-15 March 1980

49-A.P. Contogouris, R. Gaskell, L. Marleau, Phys. Rev., D22, 1109 (1980)